```
(** Data import **)
SetDirectory[NotebookDirectory[]];

NamData0 = Import["0_11h38min__LCPA .txt", "Table"];
NamData15 = Import["15_12h12min__LCPA .txt", "Table"];
NamData30 = Import["30_12h47min__LCPA .txt", "Table"];
NamData45 = Import["45_11h41min__LCPA .txt", "Table"];
S1HN0 = Table[NamData0[[n, 2]], {n, 2, Dimensions[NamData0][[1]] - 1, 2}];
S2HN0 = Table[NamData0[[n, 3]], {n, 2, Dimensions[NamData0][[1]] - 1, 2}];
S3HN0 = Table[NamData0[[n, 4]], {n, 2, Dimensions[NamData0][[1]] - 1, 2}];
S1HN15 = Table[NamData15[[n, 2]], {n, 2, Dimensions[NamData15][[1]] - 1, 2}];
S2HN15 = Table[NamData15[[n, 3]], {n, 2, Dimensions[NamData15][[1]] - 1, 2}];
S3HN15 = Table[NamData15[[n, 4]], {n, 2, Dimensions[NamData15][[1]] - 1, 2}];
S1HN30 = Table[NamData30[[n, 2]], {n, 2, Dimensions[NamData30][[1]] - 1, 2}];
S2HN30 = Table[NamData30[[n, 3]], {n, 2, Dimensions[NamData30][[1]] - 1, 2}];
S3HN30 = Table[NamData30[[n, 4]], {n, 2, Dimensions[NamData30][[1]] - 1, 2}];
S1HN45 = Table[NamData45[[n, 2]], {n, 2, Dimensions[NamData45][[1]] - 1, 2}];
S2HN45 = Table[NamData45[[n, 3]], {n, 2, Dimensions[NamData45][[1]] - 1, 2}];
S3HN45 = Table[NamData45[[n, 4]], {n, 2, Dimensions[NamData45][[1]] - 1, 2}];
Naps0 = Table[NamData0[[n, 14]], {n, 1, Dimensions[NamData0][[1]] - 1, 2}];
Naps15 = Table[NamData15[[n, 15]], {n, 1, Dimensions[NamData15][[1]] - 1, 2}];
Naps30 = Table[NamData30[[n, 15]], {n, 1, Dimensions[NamData30][[1]] - 1, 2}];
Naps45 = Table[NamData45[[n, 13]], {n, 1, Dimensions[NamData45][[1]] - 1, 2}];
step = 1;
od = 1;
BlochMer0 =
  Table[{S1HN0[[n]], S2HN0[[n]], S3HN0[[n]]}, {n, od, Dimensions[S1HN0][[1]], step}];
BlochMer15 = Table[{S1HN15[[n]], S2HN15[[n]], S3HN15[[n]]},
   {n, od, Dimensions[S1HN15][[1]], step}];
BlochMer30 = Table[{S1HN30[[n]], S2HN30[[n]], S3HN30[[n]]},
   {n, od, Dimensions[S1HN30][[1]], step}];
BlochMer45 = Table[{S1HN45[[n]], S2HN45[[n]], S3HN45[[n]]},
   {n, od, Dimensions[S1HN45][[1]], step}];
Naps0a = Table[Naps0[[n]], {n, od, Dimensions[Naps0][[1]], step}];
Naps15a = Table[Naps15[[n]], {n, od, Dimensions[Naps15][[1]], step}];
Naps30a = Table[Naps30[[n]], {n, od, Dimensions[Naps30][[1]], step}];
Naps45a = Table[Naps45[[n]], {n, od, Dimensions[Naps45][[1]], step}];

(** Theoretical  model **)
```

```
states = {{{1}, {0}}, {{0}, {1}}, 1 / Sqrt[2] * {{1}, {1}}, 1 / Sqrt[2] * {{1}, {-1}},
    1 / Sqrt[2] * {{1}, {I}}, 1 / Sqrt[2] * {{1}, {-I}}}; (* All base states *)
stateIn = states[[1]]; (* Input state - pol. output *)
rot[x_] := {{Cos[x], Sin[x]}, {-Sin[x], Cos[x]}} (* rot. matrix *)
gamma[u_] := retfun[u]
X[u_] := Sqrt[(phi ^ 2) + gamma[u] ^ 2]
```

$$\text{retfun}[u\_] := a + \frac{1}{b + \text{Exp}[d - c * (u \wedge e)]} \text{(* Logistic function *)}$$

$$\text{lcmat}[u\_] := \left\{\left\{\text{Cos}[X[u]] + \frac{i\ \text{gamma}[u]\ \text{Sin}[X[u]]}{X[u]}, -\frac{\text{phi Sin}[X[u]]}{X[u]}\right\},\right.$$
$$\left.\left\{\frac{\text{phi Sin}[X[u]]}{X[u]}, \text{Cos}[X[u]] - \frac{i\ \text{gamma}[u]\ \text{Sin}[X[u]]}{X[u]}\right\}\right\} \text{(* Final TNLCc matrix *)}$$

```
total[beta_, u_] := rot[-beta].rot[phi].lcmat[u].
  rot[beta](* Final TNLCc matrix of a rotated TNLCc by betha *)
```

```
ident = {{1, 0}, {0, 1}};
sigma2 = {{0, 1}, {1, 0}};
sigma3 = {{0, -I}, {I, 0}};
sigma2cc = {{0, I}, {-I, 0}};
sigma1 = {{1, 0}, {0, -1}};
sigma = {sigma1, sigma2, sigma3}; (* Definition of the Sigma matrices *)

bloch[rho_] := Table[ Re[Tr[sigma[[i]].rho]], {i, 1, 3}] (* density matrix → bloch vec. *)
ketOut[beta_, u_] := total[beta, u].stateIn (* Final output pol.state - ket *)
braOut[beta_, u_] := ConjugateTranspose [total[beta, u].stateIn]
(* Final output pol.state - bra *)
rhoOut[beta_, u_] := KroneckerProduct [braOut[beta, u], ketOut[beta, u]]
(* Final output pol.state - density matrix *)

(** Def. states for fit - rotated TNLCc by angles 4 **)
VysTupTHJ0 = ketOut[theta, V];
rhoVysTup0 = rhoOut[theta, V];
VysTupTHS0 = bloch[rhoVysTup0];

VysTupTHJ15 = ketOut[theta + N[11 °], V];
rhoVysTup15 = rhoOut[theta + N[11 °], V];
VysTupTHS15 = bloch[rhoVysTup15];

VysTupTHJ30 = ketOut[theta + N[26 °], V];
rhoVysTup30 = rhoOut[theta + N[26 °], V];
VysTupTHS30 = bloch[rhoVysTup30];

VysTupTHJ45 = ketOut[theta + N[41 °], V];
rhoVysTup45 = rhoOut[theta + N[41 °], V];
VysTupTHS45 = bloch[rhoVysTup45];

VysTupTHSNap0 = Table[VysTupTHS0 /. {V → Naps0a[[n]]}, {n, 1, Dimensions [Naps0a][[1]]}];
VysTupTHSNap15 = Table[VysTupTHS15 /. {V → Naps15a[[n]]}, {n, 1, Dimensions [Naps0a][[1]]}];
VysTupTHSNap30 = Table[VysTupTHS30 /. {V → Naps30a[[n]]}, {n, 1, Dimensions [Naps0a][[1]]}];
VysTupTHSNap45 = Table[VysTupTHS45 /. {V → Naps45a[[n]]}, {n, 1, Dimensions [Naps0a][[1]]}];

(** FIT **)

SeedRandom[1];

initials = {{-8.314031137459752` ,
    0.07364228363744871` , 174.92047651524567` , -2.9721437986637023` ,
    -2.8536892547196415` , -5.145593598118274` , -2.56022951974189` }};
```

```
FitPar =
 NMinimize[{ParallelSum[Sum[(BlochMer0[[m, n]] - VysTupTHSNap0[[m, n]])^2, {n, {1, 2, 3}}],
     {m, 1, Dimensions[BlochMer0][[1]]}] +
    ParallelSum[Sum[(BlochMer30[[m, n]] - VysTupTHSNap30[[m, n]])^2, {n, {1, 2, 3}}],
     {m, 1, Dimensions[BlochMer30][[1]]}] +
    ParallelSum[Sum[(BlochMer15[[m, n]] - VysTupTHSNap15[[m, n]])^2, {n, {1, 2, 3}}],
     {m, 1, Dimensions[BlochMer15][[1]]}] +
    ParallelSum[Sum[(BlochMer45[[m, n]] - VysTupTHSNap45[[m, n]])^2, {n, {1, 2, 3}}],
     {m, 1, Dimensions[BlochMer45][[1]]}], Min[ParallelSum[retfun[v], {v, 0.1, 10.1}]] ≥ 0},
  {a, b, c, d, e, phi, theta}, Method → {"Automatic", "InitialPoints" → initials},
  MaxIterations → 200(*,
  StepMonitor:→Print["Step to theta = ",theta]*)]
stop = AbsoluteTime[];
stop - start
```

```
{10.275, {a → -7.82057, b → 0.0761821, c → 176.156,
  d → -2.89523, e → -2.86832, phi → -5.15607, theta → -2.58144}}
```

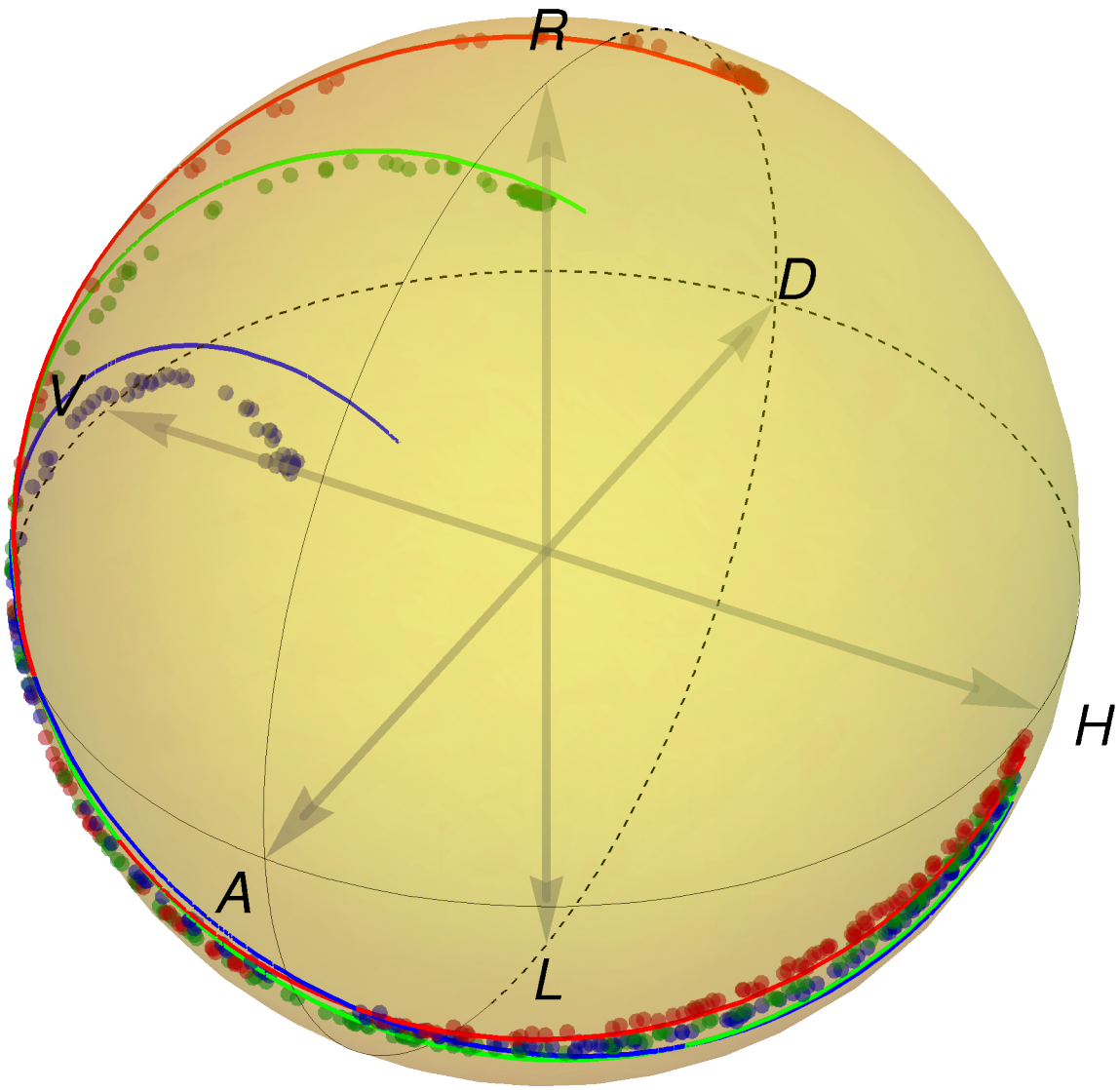
```
4716.075161
```

```
(** Export **)
```

```
FitDATA = {a → -7.820573268569158` , b → 0.07618212773600479` ,
   c → 176.1560023971697` , d → -2.89522737626486` , e → -2.8683220566787044` ,
   phi → -5.156073407255145` , theta → -2.581437502776475` };
VysTupTHS0a = VysTupTHS0 /. FitDATA;
VysTupTHS15a = VysTupTHS15 /. FitDATA;
VysTupTHS30a = VysTupTHS30 /. FitDATA;
VysTupTHS45a = VysTupTHS45 /. FitDATA;
Vod = 0.01;
Vto = 10;
```

```
Angle1 = Pi / 8 - Pi / 20;
Angle2 = -Pi / 8 - Pi / 20;
```

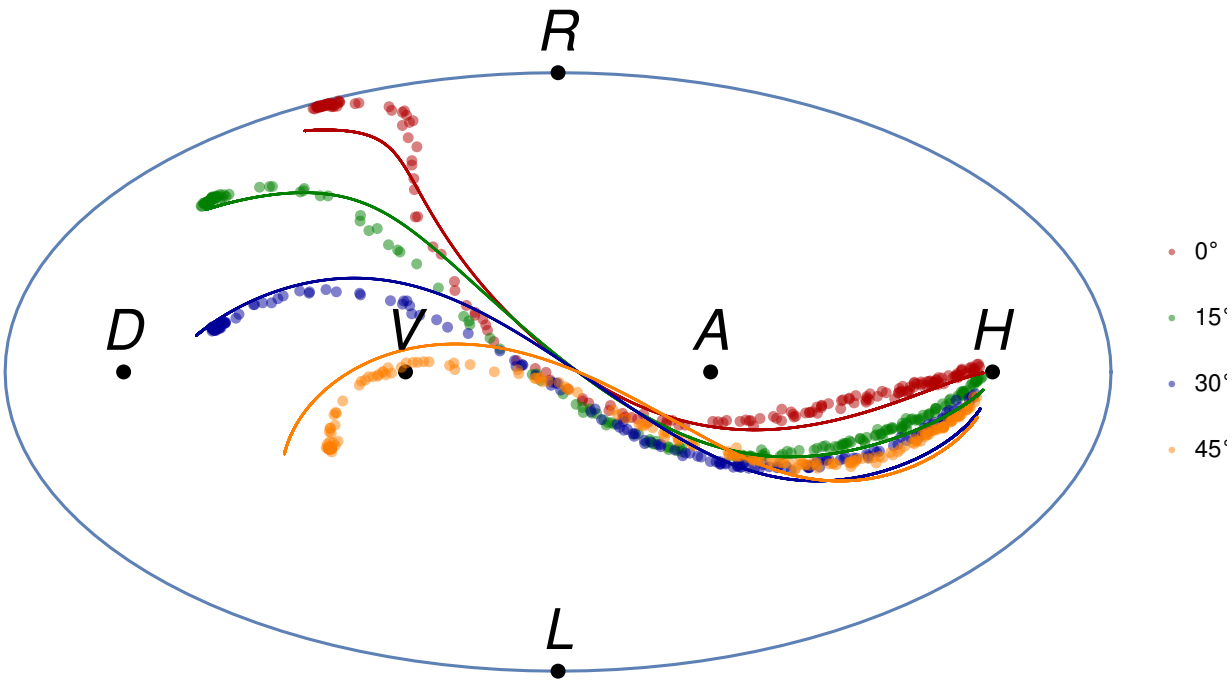
```
ForExport = Show[ListPointPlot3D [BlochMer15 ,
    BoxRatios → {1, 1, 1}, PlotRange → {{-1, 1}, {-1, 1}, {-1, 1}},
    PlotStyle → Directive [PointSize [0.015], Opacity [.5], RGBColor [.7, 0, 0]],
    Axes → False , Boxed → False], ListPointPlot3D [BlochMer30 ,
    BoxRatios → {1, 1, 1}, PlotRange → {{-1, 1}, {-1, 1}, {-1, 1}},
    PlotStyle → Directive [PointSize [0.015], Opacity [.5], RGBColor [0, .5, 0]],
    Axes → False , Boxed → False], ListPointPlot3D [BlochMer45 ,
    BoxRatios → {1, 1, 1}, PlotRange → {{-1, 1}, {-1, 1}, {-1, 1}},
    PlotStyle → Directive [PointSize [0.015], Opacity [.5], RGBColor [0, 0, .6]],
    Axes → False , Boxed → False],
  ParametricPlot3D [VysTupTHS15a , {V, Vod , Vto}, PlotStyle → {RGBColor [1, 0, 0], Thick}],
  ParametricPlot3D [VysTupTHS30a , {V, Vod , Vto}, PlotStyle → {RGBColor [0, 1, 0], Thick}],
  ParametricPlot3D [VysTupTHS45a , {V, Vod , Vto}, PlotStyle → {RGBColor [0, 0, 1], Thick}],
  ParametricPlot3D [{0, Sin[a], Cos[a]}, {a, 0 + Angle1 , Pi + Angle1},
   PlotStyle → {Black , Dashed , Thin}], ParametricPlot3D [{0, Sin[a], Cos[a]},
   {a, Pi + Angle1 , 2 Pi + Angle1}, PlotStyle → {Black , Thin}],
  ParametricPlot3D [{Sin[a], Cos[a], 0}, {a, -Pi / 2 + Angle2 , Pi / 2 + Angle2},
   PlotStyle → {Black , Dashed , Thin}], ParametricPlot3D [{Sin[a], Cos[a], 0},
   {a, Pi / 2 + Angle2 , 3 Pi / 2 + Angle2}, PlotStyle → {Black , Thin}],
  Graphics3D [{{RGBColor [0.5 , 0.5 , 0.5 , .5], Arrowheads [0.04], Thickness [0.008],
     Arrow[{{{0, 0, 0}, {1, 0, 0}}, {{0, 0, 0}, {-1, 0, 0}}, {{0, 0, 0}, {0, 1, 0}},
       {{0, 0, 0}, {0, -1, 0}}, {{0, 0, 0}, {0, 0, 1}}, {{0, 0, 0}, {0, 0, -1}}}]},
    {Opacity [.3], Yellow , Sphere [{0, 0, 0}]}}], Graphics3D [
   Style[Text["R", {0, 0, 1.1}], Black , Italic , 30]],
  Graphics3D [Style[Text["L", {0, 0, -1.1}], Black , Italic , 30]],
  Graphics3D [Style[Text["D", {0, 1.1, 0}], Black , Italic , 30]],
  Graphics3D [Style[Text["A", {0, -1.1, 0}], Black , Italic , 30]],
  Graphics3D [Style[Text["H", {1.1, 0, 0}], Black , Italic , 30]],
  Graphics3D [Style[Text["V", {-1.1, 0, 0}], Black , Italic , 30]],
  Background → Transparent , ImageSize → 600]
```




```
(** Hammer projection **)
GetSpherical[Bloch_] := {ArcTan[Sqrt[Bloch[[2]]^2 + Bloch[[1]]^2], Bloch[[3]]],
   Limit[ArcTan[x, Bloch[[2]]], {x -> Bloch[[1]]}]}

blochlistStates = {{1, 0, 0}, {-1, 0, 0}, {0, 1, 0}, {0, -1, 0}, {0, 0, 1}, {0, 0, -1}};

(* Rotate Hammer proj @90deg {0,0,1} *)

rotVec = {0, 0, 1};(* Rot. axis *)
angleRot = 3 Pi / 4(*-Pi/4*);(* Rot. angle *)
```

```
HammerCoordinates [bloch_] :=
 ArrayReshape [{(2 * Sqrt[2] * Cos[elev] * Sin[az / 2]) / Sqrt[1 + Cos[elev] Cos[az / 2]],
    (Sqrt[2] * Sin[elev]) / Sqrt[1 + Cos[elev] Cos[az / 2]]} /.
   {elev -> GetSpherical [RotationMatrix [angleRot , rotVec].bloch][[1]],
    az -> GetSpherical [RotationMatrix [angleRot , rotVec].bloch][[2]]}, {2}]

zero = -1 * 10 ^(-16); (* correction only for state plotting *)

HammerPlot = Show[ListPlot[
   Table[ArrayReshape [{(2 * Sqrt[2] * Cos[elev] * Sin[az / 2]) / Sqrt[1 + Cos[elev] Cos[az / 2]],
       (Sqrt[2] * Sin[elev]) / Sqrt[1 + Cos[elev] Cos[az / 2]]} /.
      {elev -> Range[0. , 2 * Pi , 2 * Pi / 120][[i]], az → Pi}, {2}],
    {i , 1 , 121}], Joined → {True , False}], ListPlot[
   Table[HammerCoordinates [blochlistStates [[i]] + {zero , zero , zero}], {i , 1 , 6}],
   PlotStyle → {PointSize [0.013], RGBColor [0 , 0 , 0]}],
  ListPlot[Table[HammerCoordinates [BlochMer0 [[i]]], {i , 1 , Dimensions [BlochMer0 ][[1]]}],
   PlotStyle → {PointSize [0.0095], RGBColor [.7 , 0 , 0], Opacity [0.5]},
   PlotLegends → {"0°"}],
  ListPlot[Table[HammerCoordinates [BlochMer15 [[i]]], {i , 1 , Dimensions [BlochMer15 ][[1]]}],
   PlotStyle → {PointSize [0.0095], RGBColor [0 , .5 , 0], Opacity [0.5]},
   PlotLegends → {"15°"}],
  ListPlot[Table[HammerCoordinates [BlochMer30 [[i]]], {i , 1 , Dimensions [BlochMer30 ][[1]]}],
   PlotStyle → {PointSize [0.0095], RGBColor [0 , 0 , .6], Opacity [0.5]},
   PlotLegends → {"30°"}],
  ListPlot[Table[HammerCoordinates [BlochMer45 [[i]]], {i , 1 , Dimensions [BlochMer45 ][[1]]}],
   PlotStyle → {PointSize [0.0095], RGBColor [1 , .5 , 0], Opacity [0.5]},
   PlotLegends → {"45°"}],
  ListPlot[ParallelTable [HammerCoordinates [VysTupTHS0a ], {V , Vod , Vto , 0.001}],
   PlotStyle → {PointSize [0.00095], RGBColor [.7 , 0 , 0], Opacity [0.9]}],
  ListPlot[ParallelTable [HammerCoordinates [VysTupTHS15a ], {V , Vod , Vto , 0.001}],
   PlotStyle → {PointSize [0.00095], RGBColor [0 , .5 , 0], Opacity [0.9]}],
  ListPlot[ParallelTable [HammerCoordinates [VysTupTHS30a ], {V , Vod , Vto , 0.001}],
   PlotStyle → {PointSize [0.00095], RGBColor [0 , 0 , 0.6], Opacity [0.9]}],
  ListPlot[ParallelTable [HammerCoordinates [VysTupTHS45a ], {V , Vod , Vto , 0.001}],
   PlotStyle → {PointSize [0.00095], RGBColor [1 , .5 , 0], Opacity [0.9]}],
  (*ListPlot[Table[HammerCoordinates [BlochListR [[i]]],{i,1,6}],
   PlotStyle→{PointSize [0.0095],RGBColor [0,1,0],Opacity [0.9]}],
  ListPlot[Table[HammerCoordinates [BlochListL [[i]]],{i,1,6}],
   PlotStyle→{PointSize [0.0095],RGBColor [0,1,0],Opacity [0.9]}],*)
  Graphics[Style[Text["H", HammerCoordinates [{1, zero , zero}] + {0, 0.2}],
    Black , Italic , 30]],
  Graphics[Style[Text["V", HammerCoordinates [{-1, zero , zero}] + {0, 0.2}],
    Black , Italic , 30]],
  Graphics[Style[Text["D", HammerCoordinates [{zero , 1, zero}] + {0, 0.2}],
```

```
    Black, Italic, 30]],
  Graphics[Style[Text["A", HammerCoordinates [{zero, -1, zero}] + {0, 0.2}],
    Black, Italic, 30]],
  Graphics[Style[Text["R", HammerCoordinates [{zero, zero, 1}] + {0, 0.2}],
    Black, Italic, 30]],
  Graphics[Style[Text["L", HammerCoordinates [{zero, zero, -1}] + {0, 0.2}],
    Black, Italic, 30]],
  PlotRange → All, Axes → False, ImageSize → 600]
```




```
Export["Fit_Hammer_ROT.pdf", HammerPlot, ImageResolution → 600]
```

```
Fit_Hammer_ROT.pdf
```

```
(** Fidelity - model vs. data **)
Fidelity[u_, v_] := Re[1 / 2 (1 + u.v + Sqrt[(1 - Norm[u]) (1 - Norm[v])]) ]
```

```
BlochModel0 =
  ParallelTable [VysTupTHS0a /. V -> Naps0a[[i]], {i, 1, Dimensions [Naps0a][[1]]}];
BlochModel15 = ParallelTable [VysTupTHS15a /. V -> Naps15a[[i]],
   {i, 1, Dimensions [Naps15a][[1]]}];
BlochModel30 = ParallelTable [VysTupTHS30a /. V -> Naps30a[[i]],
   {i, 1, Dimensions [Naps30a][[1]]}];
BlochModel45 = ParallelTable [VysTupTHS45a /. V -> Naps45a[[i]],
   {i, 1, Dimensions [Naps45a][[1]]}];
```

```
Fidel0 = ParallelTable[
   Fidelity[BlochMer0[[i]], BlochModel0[[i]]], {i, 1, Dimensions[BlochMer0][[1]]}];
Fidel15 = ParallelTable[Fidelity[BlochMer15[[i]], BlochModel15[[i]]],
   {i, 1, Dimensions[BlochMer15][[1]]}];
Fidel30 = ParallelTable[Fidelity[BlochMer30[[i]], BlochModel30[[i]]],
   {i, 1, Dimensions[BlochMer30][[1]]}];
Fidel45 = ParallelTable[Fidelity[BlochMer45[[i]], BlochModel45[[i]]],
   {i, 1, Dimensions[BlochMer45][[1]]}];

Mean[Join[Fidel0, Fidel15, Fidel30, Fidel45]]
```

```
0.996788
```

```
StandardDeviation[Join[Fidel0, Fidel15, Fidel30, Fidel45]]
```

```
0.00423908
```

```
(** Angle - model vs. data **)
Angle[u_, v_] := 1/2 * (ArcCos[u.v/(Norm[u] * Norm[v])])

Angle0 = ParallelTable[
   Angle[BlochMer0[[i]], BlochModel0[[i]]], {i, 1, Dimensions[BlochMer0][[1]]}];
Angle15 = ParallelTable[Angle[BlochMer15[[i]], BlochModel15[[i] s]],
   {i, 1, Dimensions[BlochMer15][[1]]}];
Angle30 = ParallelTable[Angle[BlochMer30[[i]], BlochModel30[[i]]],
   {i, 1, Dimensions[BlochMer30][[1]]}];
Angle45 = ParallelTable[Angle[BlochMer45[[i]], BlochModel45[[i]]],
   {i, 1, Dimensions[BlochMer45][[1]]}];

Mean[Join[Angle0, Angle15, Angle30, Angle45]/Degree
```

```
2.7249
```

```
StandardDeviation[Join[Angle0, Angle15, Angle30, Angle45]/Degree
```

```
1.77509
```