```
SetDirectory[NotebookDirectory[]];

(* Import data *)
NamData0 =
  Import["RAW_data/MaxLik_OUTPUT_TNLCd_anal_test_BestPerm_wait_TT4.txt", "Table"];

Rho[i_, NamData0_] := {{NamData0[[4 + 16 i, 1]], NamData0[[5 + 16 i, 1]]},
    {NamData0[[6 + 16 i, 1]], NamData0[[7 + 16 i, 1]]}} +
  {{NamData0[[9 + 16 i, 1]], NamData0[[10 + 16 i, 1]]},
    {NamData0[[11 + 16 i, 1]], NamData0[[12 + 16 i, 1]]}} * I

RhoListALL = ParallelTable[Rho[i, NamData0], {i, 0, NamData0[[1, 1]] - 1}];

RhoListH = ParallelTable[Rho[i, NamData0], {i, 0, NamData0[[1, 1]] - 1, 6}];
RhoListV = ParallelTable[Rho[i, NamData0], {i, 1, NamData0[[1, 1]] - 1, 6}];
RhoListD = ParallelTable[Rho[i, NamData0], {i, 2, NamData0[[1, 1]] - 1, 6}];
RhoListA = ParallelTable[Rho[i, NamData0], {i, 3, NamData0[[1, 1]] - 1, 6}];
RhoListR = ParallelTable[Rho[i, NamData0], {i, 4, NamData0[[1, 1]] - 1, 6}];
RhoListL = ParallelTable[Rho[i, NamData0], {i, 5, NamData0[[1, 1]] - 1, 6}];

RhoListL // Dimensions
{100, 2, 2}

(* Target HVDARL states *)
StateH = {{1}, {0}};
StateV = {{0}, {1}};
StateD = (1 / (2 ^ (1 / 2))) {{1}, {1}};
StateA = (1 / (2 ^ (1 / 2))) {{1}, {-1}};
StateR = (1 / (2 ^ (1 / 2))) {{1}, {I}};
StateL = (1 / (2 ^ (1 / 2))) {{1}, {-I}};
States = {StateH, StateV, StateD, StateA, StateR, StateL};

StateToRho[i_] := States[[i]].ConjugateTranspose[States[[i]]]
Rhos = ParallelTable[StateToRho[i], {i, 1, Dimensions[States][[1]]}];
Dimensions[Rhos]
{6, 2, 2}

sigma2 = {{0, 1}, {1, 0}};
sigma3 = {{0, -I}, {I, 0}};
sigma1 = {{1, 0}, {0, -1}};
sigma = {sigma1, sigma2, sigma3};
blochStates[i_, x_] := ConjugateTranspose[States[[x]]].sigma[[i]].States[[x]]
blochlistStates =
  ParallelTable[Re[blochStates[i, x][[1, 1]]], {x, 1, Dimensions[States][[1]]}, {i, 1, 3}];

RhoToBloch[Rho_, i_, j_] := Re[Tr[Rho[[i]].sigma[[j]]]]
```

```
(* Fidelity *)

Fidelity[Rho1_, Rho2_, i_, j_] :=
 Re[Tr[MatrixPower[MatrixPower[Rho1[[i]], 1/2].Rho2[[j]].MatrixPower[Rho1[[i]], 1/2], 1/2]]^
  2

FidelityListH = ParallelTable[
   Fidelity[Rhos, RhoListH, i, j], {j, 1, Dimensions[RhoListH][[1]]}, {i, 1, 1}];
FidelityListV = ParallelTable[Fidelity[Rhos, RhoListV, i, j],
   {j, 1, Dimensions[RhoListV][[1]]}, {i, 2, 2}];
FidelityListD = ParallelTable[Fidelity[Rhos, RhoListD, i, j],
   {j, 1, Dimensions[RhoListD][[1]]}, {i, 3, 3}];
FidelityListA = ParallelTable[Fidelity[Rhos, RhoListA, i, j],
   {j, 1, Dimensions[RhoListA][[1]]}, {i, 4, 4}];
FidelityListR = ParallelTable[Fidelity[Rhos, RhoListR, i, j],
   {j, 1, Dimensions[RhoListR][[1]]}, {i, 5, 5}];
FidelityListL = ParallelTable[Fidelity[Rhos, RhoListL, i, j],
   {j, 1, Dimensions[RhoListL][[1]]}, {i, 6, 6}];

(* Ommit the best and the worse *)
RhoListH = Delete[RhoListH, Position[FidelityListH, Max[FidelityListH]][[1]][[1]]];
FidelityListH = ParallelTable[
   Fidelity[Rhos, RhoListH, i, j], {j, 1, Dimensions[RhoListH][[1]]}, {i, 1, 1}];
RhoListH = Delete[RhoListH, Position[FidelityListH, Min[FidelityListH]][[1]][[1]]];

RhoListV = Delete[RhoListV, Position[FidelityListV, Max[FidelityListV]][[1]][[1]]];
FidelityListV = ParallelTable[
   Fidelity[Rhos, RhoListV, i, j], {j, 1, Dimensions[RhoListV][[1]]}, {i, 2, 2}];
RhoListV = Delete[RhoListV, Position[FidelityListV, Min[FidelityListV]][[1]][[1]]];

RhoListD = Delete[RhoListD, Position[FidelityListD, Max[FidelityListD]][[1]][[1]]];
FidelityListD = ParallelTable[
   Fidelity[Rhos, RhoListD, i, j], {j, 1, Dimensions[RhoListD][[1]]}, {i, 3, 3}];
RhoListD = Delete[RhoListD, Position[FidelityListD, Min[FidelityListD]][[1]][[1]]];

RhoListA = Delete[RhoListA, Position[FidelityListA, Max[FidelityListA]][[1]][[1]]];
FidelityListA = ParallelTable[
   Fidelity[Rhos, RhoListA, i, j], {j, 1, Dimensions[RhoListA][[1]]}, {i, 4, 4}];
RhoListA = Delete[RhoListA, Position[FidelityListA, Min[FidelityListA]][[1]][[1]]];

RhoListR = Delete[RhoListR, Position[FidelityListR, Max[FidelityListR]][[1]][[1]]];
FidelityListR = ParallelTable[
   Fidelity[Rhos, RhoListR, i, j], {j, 1, Dimensions[RhoListR][[1]]}, {i, 5, 5}];
RhoListR = Delete[RhoListR, Position[FidelityListR, Min[FidelityListR]][[1]][[1]]];
```

```
RhoListL = Delete[RhoListL , Position[FidelityListL , Max[FidelityListL ]][[1]][[1]]];
FidelityListL = ParallelTable [
    Fidelity[Rhos , RhoListL , i, j], {j, 1, Dimensions[RhoListL ][[1]]}, {i, 6, 6}];
RhoListL = Delete[RhoListL , Position[FidelityListL , Min[FidelityListL ]][[1]][[1]]];

FidelityListH = ParallelTable [
    Fidelity[Rhos , RhoListH , i, j], {j, 1, Dimensions[RhoListH ][[1]]}, {i, 1, 1}];
FidelityListV = ParallelTable [Fidelity[Rhos , RhoListV , i, j],
    {j, 1, Dimensions[RhoListV ][[1]]}, {i, 2, 2}];
FidelityListD = ParallelTable [Fidelity[Rhos , RhoListD , i, j],
    {j, 1, Dimensions[RhoListD ][[1]]}, {i, 3, 3}];
FidelityListA = ParallelTable [Fidelity[Rhos , RhoListA , i, j],
    {j, 1, Dimensions[RhoListA ][[1]]}, {i, 4, 4}];
FidelityListR = ParallelTable [Fidelity[Rhos , RhoListR , i, j],
    {j, 1, Dimensions[RhoListR ][[1]]}, {i, 5, 5}];
FidelityListL = ParallelTable [Fidelity[Rhos , RhoListL , i, j],
    {j, 1, Dimensions[RhoListL ][[1]]}, {i, 6, 6}];
```

```
(* H *)
Mean[FidelityListH ]
StandardDeviation [FidelityListH ]
```

{0.993872}

{0.000302101 }

```
(* V *)
Mean[FidelityListV ]
StandardDeviation [FidelityListV ]
```

{0.998131}

{0.000277094 }

```
(* D *)
Mean[FidelityListD ]
StandardDeviation [FidelityListD ]
```

{0.998753}

{0.0000165457 }

```
(* A *)
Mean[FidelityListA ]
StandardDeviation [FidelityListA ]
```

{0.999365}

$\{6.25959 \times 10^{-6}\}$

```
(* R *)
Mean[FidelityListR]
StandardDeviation[FidelityListR]
```

```
{0.999792}
```

$$\{9.73719 \times 10^{-6}\}$$

```
(* L *)
Mean[FidelityListL]
StandardDeviation[FidelityListL]
```

```
{0.999856}
```

$$\{3.76065 \times 10^{-6}\}$$

```
(* Purity *)

Purity[Rho_] := Re[Tr[MatrixPower[Rho, 2]]]

PurityListH = ParallelTable[Purity[RhoListH[[i]]], {i, 1, Dimensions[RhoListH][[1]]}];
PurityListV = ParallelTable[Purity[RhoListV[[i]]], {i, 1, Dimensions[RhoListV][[1]]}];
PurityListD = ParallelTable[Purity[RhoListD[[i]]], {i, 1, Dimensions[RhoListD][[1]]}];
PurityListA = ParallelTable[Purity[RhoListA[[i]]], {i, 1, Dimensions[RhoListA][[1]]}];
PurityListR = ParallelTable[Purity[RhoListR[[i]]], {i, 1, Dimensions[RhoListR][[1]]}];
PurityListL = ParallelTable[Purity[RhoListL[[i]]], {i, 1, Dimensions[RhoListL][[1]]}];

(* H *)
Mean[PurityListH]
StandardDeviation[PurityListH]
```

```
0.989041
```

```
0.000591314
```

```
(* V *)
Mean[PurityListV]
StandardDeviation[PurityListV]
```

```
0.997846
```

```
0.000380086
```

```
(* D *)
Mean[PurityListD]
StandardDeviation[PurityListD]
```

```
1.
```

$$1.27307 \times 10^{-14}$$

```
(* A *)
Mean[PurityListA]
StandardDeviation[PurityListA]
```

1.

2.67788×10^{-14}

```
(* R *)
Mean[PurityListR]
StandardDeviation[PurityListR]
```

1.

1.06734×10^{-12}

```
(* L *)
Mean[PurityListL]
StandardDeviation[PurityListL]
```

1.

7.26198×10^{-15}

```
(* Angles *)
BlochListH = ParallelTable[RhoToBloch[RhoListH, i, j],
   {i, 1, Dimensions[RhoListH][[1]]}, {j, 1, Dimensions[sigma][[1]]}];
BlochListV = ParallelTable[RhoToBloch[RhoListV, i, j],
   {i, 1, Dimensions[RhoListV][[1]]}, {j, 1, Dimensions[sigma][[1]]}];
BlochListD = ParallelTable[RhoToBloch[RhoListD, i, j],
   {i, 1, Dimensions[RhoListD][[1]]}, {j, 1, Dimensions[sigma][[1]]}];
BlochListA = ParallelTable[RhoToBloch[RhoListA, i, j],
   {i, 1, Dimensions[RhoListA][[1]]}, {j, 1, Dimensions[sigma][[1]]}];
BlochListR = ParallelTable[RhoToBloch[RhoListR, i, j],
   {i, 1, Dimensions[RhoListR][[1]]}, {j, 1, Dimensions[sigma][[1]]}];
BlochListL = ParallelTable[RhoToBloch[RhoListL, i, j],
   {i, 1, Dimensions[RhoListL][[1]]}, {j, 1, Dimensions[sigma][[1]]}];
```

```
AnglesH = Table[VectorAngle[BlochListH[[i]], blochlistStates[[1]]],
   {i, 1, Dimensions[BlochListH][[1]]}];
AnglesV = Table[VectorAngle[BlochListV[[i]], blochlistStates[[2]]],
   {i, 1, Dimensions[BlochListV][[1]]}];
AnglesD = Table[VectorAngle[BlochListD[[i]], blochlistStates[[3]]],
   {i, 1, Dimensions[BlochListD][[1]]}];
AnglesA = Table[VectorAngle[BlochListA[[i]], blochlistStates[[4]]],
   {i, 1, Dimensions[BlochListA][[1]]}];
AnglesR = Table[VectorAngle[BlochListR[[i]], blochlistStates[[5]]],
   {i, 1, Dimensions[BlochListR][[1]]}];
AnglesL = Table[VectorAngle[BlochListL[[i]], blochlistStates[[6]]],
   {i, 1, Dimensions[BlochListL][[1]]}];

(* H *)
Mean[AnglesH];
UnitConvert[% rad, "AngularDegrees"] / 2
StandardDeviation[AnglesH];
UnitConvert[% rad, "AngularDegrees"] / 2
```

```
1.43252°
```

```
0.0286624°
```

```
(* V *)
Mean[AnglesV];
UnitConvert[% rad, "AngularDegrees"] / 2
StandardDeviation[AnglesV];
UnitConvert[% rad, "AngularDegrees"] / 2
```

```
1.6115°
```

```
0.0768598°
```

```
(* D *)
Mean[AnglesD];
UnitConvert[% rad, "AngularDegrees"] / 2
StandardDeviation[AnglesD];
UnitConvert[% rad, "AngularDegrees"] / 2
```

```
2.02381°
```

```
0.0134183°
```

```
(* A *)
Mean[AnglesA];
UnitConvert[% rad, "AngularDegrees"] / 2
StandardDeviation[AnglesA];
UnitConvert[% rad, "AngularDegrees"] / 2
```

1.44429°

0.00712572°

```
(* R *)
Mean[AnglesR];
UnitConvert[% rad, "AngularDegrees"] / 2
StandardDeviation[AnglesR];
UnitConvert[% rad, "AngularDegrees"] / 2
```

0.825799°

0.0192889°

```
(* L *)
Mean[AnglesL];
UnitConvert[% rad, "AngularDegrees"] / 2
StandardDeviation[AnglesL];
UnitConvert[% rad, "AngularDegrees"] / 2
```

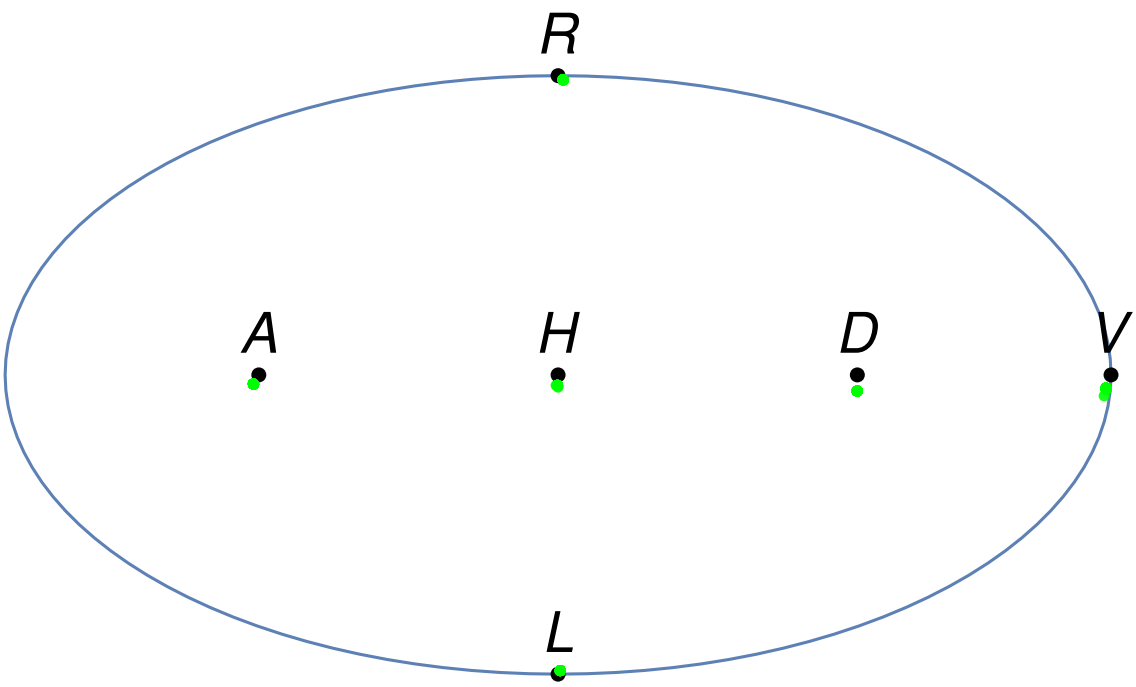
0.686937°

0.00898179°

```
(* Hammer projection *)
GetSpherical[Bloch_] := {ArcTan[Sqrt[Bloch[[2]]^2 + Bloch[[1]]^2], Bloch[[3]]],
  Limit[ArcTan[x, Bloch[[2]]], {x -> Bloch[[1]]}]}

HammerCoordinates[bloch_] :=
 ArrayReshape[{(2 * Sqrt[2] * Cos[elev] * Sin[az / 2]) / Sqrt[1 + Cos[elev] Cos[az / 2]],
    (Sqrt[2] * Sin[elev]) / Sqrt[1 + Cos[elev] Cos[az / 2]]} /.
   {elev -> GetSpherical[bloch][[1]], az -> GetSpherical[bloch][[2]]}, {2}]
```

```
Show[ListPlot[
  Table[ArrayReshape [{(2 * Sqrt[2] * Cos[elev] * Sin[az / 2]) / Sqrt[1 + Cos[elev] Cos[az / 2]],
      (Sqrt[2] * Sin[elev]) / Sqrt[1 + Cos[elev] Cos[az / 2]]} /.
     {elev -> Range[0., 2 * Pi, 2 * Pi / 120][[i]], az → Pi}, {2}], {i, 1, 121}],
  Joined → {True, False}], ListPlot[Table[HammerCoordinates [blochlistStates [[i]]],
   {i, 1, 6}], PlotStyle → {PointSize[0.013], RGBColor[0, 0, 0]}],
 ListPlot[Table[HammerCoordinates [BlochListH [[i]]], {i, 1, Dimensions [BlochListH ][[1]]}],
  PlotStyle → {PointSize[0.0095], RGBColor[0, 1, 0], Opacity[0.9]}],
 ListPlot[Table[HammerCoordinates [BlochListV [[i]]], {i, 1, Dimensions [BlochListV ][[1]]}],
  PlotStyle → {PointSize[0.0095], RGBColor[0, 1, 0], Opacity[0.9]}],
 ListPlot[Table[HammerCoordinates [BlochListD [[i]]], {i, 1, Dimensions [BlochListD ][[1]]}],
  PlotStyle → {PointSize[0.0095], RGBColor[0, 1, 0], Opacity[0.9]}],
 ListPlot[Table[HammerCoordinates [BlochListA [[i]]], {i, 1, Dimensions [BlochListA ][[1]]}],
  PlotStyle → {PointSize[0.0095], RGBColor[0, 1, 0], Opacity[0.9]}],
 ListPlot[Table[HammerCoordinates [BlochListR [[i]]], {i, 1, Dimensions [BlochListR ][[1]]}],
  PlotStyle → {PointSize[0.0095], RGBColor[0, 1, 0], Opacity[0.9]}],
 ListPlot[Table[HammerCoordinates [BlochListL [[i]]], {i, 1, Dimensions [BlochListL ][[1]]}],
  PlotStyle → {PointSize[0.0095], RGBColor[0, 1, 0], Opacity[0.9]}],
 Graphics[Style[Text["H", {0, 0.2}], Black, Italic, 30]],
 Graphics[Style[Text["V", {2 √2, 0.2}], Black, Italic, 30]],
 Graphics[Style[Text["D", {2 / √(1 + 1/√2), 0.2}], Black, Italic, 30]],
 Graphics[Style[Text["A", {-2 / √(1 + 1/√2), 0.2}], Black, Italic, 30]],
 Graphics[Style[Text["R", {0, √2 + 0.2}], Black, Italic, 30]],
 Graphics[Style[Text["L", {0, -√2 + 0.2}], Black, Italic, 30]],
 PlotRange → All, Axes → False, ImageSize → 600]
```




```
(* Average *)

(* Fidelity *)
Mean[Join[FidelityListH , FidelityListV ,
  FidelityListD , FidelityListA , FidelityListR , FidelityListL ]]
StandardDeviation [Join[FidelityListH , FidelityListV ,
  FidelityListD , FidelityListA , FidelityListR , FidelityListL ]]
```

```
{0.998295}
```

```
{0.00207507}
```

```
(* Purity *)
Mean[Join[PurityListH , PurityListV ,
  PurityListD , PurityListA , PurityListR , PurityListL ]]
StandardDeviation [Join[PurityListH , PurityListV ,
  PurityListD , PurityListA , PurityListR , PurityListL ]]
```

```
0.997815
```

```
0.00401525
```

```
(* Angle *)
Mean[Join[AnglesH , AnglesV , AnglesD , AnglesA , AnglesR , AnglesL ]];
UnitConvert[% rad , "AngularDegrees "]/2
StandardDeviation [Join[AnglesH , AnglesV , AnglesD , AnglesA , AnglesR , AnglesL ]];
UnitConvert[% rad , "AngularDegrees "]/2
```

```
1.33748°
```

```
0.458453°
```

```
(* States Bloch vectors *)
(* H *)
Mean[Transpose [BlochListH ][[1]]]
StandardDeviation [Transpose [BlochListH ][[1]]]
Mean[Transpose [BlochListH ][[2]]]
StandardDeviation [Transpose [BlochListH ][[2]]]
Mean[Transpose [BlochListH ][[3]]]
StandardDeviation [Transpose [BlochListH ][[3]]]
```

```
0.987743
```

```
0.000604202
```

```
-0.00326669
```

```
0.000962047
```

```
-0.049315
```

```
0.00100588
```

```
(* V *)
Mean[Transpose [BlochListV ][[1]]]
StandardDeviation [Transpose [BlochListV ][[1]]]
Mean[Transpose [BlochListV ][[2]]]
StandardDeviation [Transpose [BlochListV ][[2]]]
Mean[Transpose [BlochListV ][[3]]]
StandardDeviation [Transpose [BlochListV ][[3]]]
```

```
-0.996262
```

```
0.000554189
```

```
0.0316107
```

```
0.00109886
```

```
-0.0463308
```

```
0.0026837
```

```
(* D *)

Mean[Transpose[BlochListD][[1]]]
StandardDeviation[Transpose[BlochListD][[1]]]
Mean[Transpose[BlochListD][[2]]]
StandardDeviation[Transpose[BlochListD][[2]]]
Mean[Transpose[BlochListD][[3]]]
StandardDeviation[Transpose[BlochListD][[3]]]
```

```
-0.0030677

0.000617479

0.997506

0.0000330913

-0.0705161

0.000461088
```

```
(* A *)

Mean[Transpose[BlochListA][[1]]]
StandardDeviation[Transpose[BlochListA][[1]]]
Mean[Transpose[BlochListA][[2]]]
StandardDeviation[Transpose[BlochListA][[2]]]
Mean[Transpose[BlochListA][[3]]]
StandardDeviation[Transpose[BlochListA][[3]]]
```

```
-0.0306371

0.000664396

-0.998729

0.0000125192

-0.0400013

0.000665846
```

```
(* R *)
```

```
Mean[Transpose[BlochListR][[1]]]
StandardDeviation[Transpose[BlochListR][[1]]]
Mean[Transpose[BlochListR][[2]]]
StandardDeviation[Transpose[BlochListR][[2]]]
Mean[Transpose[BlochListR][[3]]]
StandardDeviation[Transpose[BlochListR][[3]]]
```

```
0.0226118
```

```
0.000394869
```

```
0.0178666
```

```
0.000698559
```

```
0.999584
```

```
0.0000194744
```

```
(* L *)
```

```
Mean[Transpose[BlochListL][[1]]]
StandardDeviation[Transpose[BlochListL][[1]]]
Mean[Transpose[BlochListL][[2]]]
StandardDeviation[Transpose[BlochListL][[2]]]
Mean[Transpose[BlochListL][[3]]]
StandardDeviation[Transpose[BlochListL][[3]]]
```

```
0.0227633
```

```
0.000325128
```

```
0.00752461
```

```
0.000266272
```

```
-0.999712
```

```
7.5213 × 10^-6
```