TD n°2

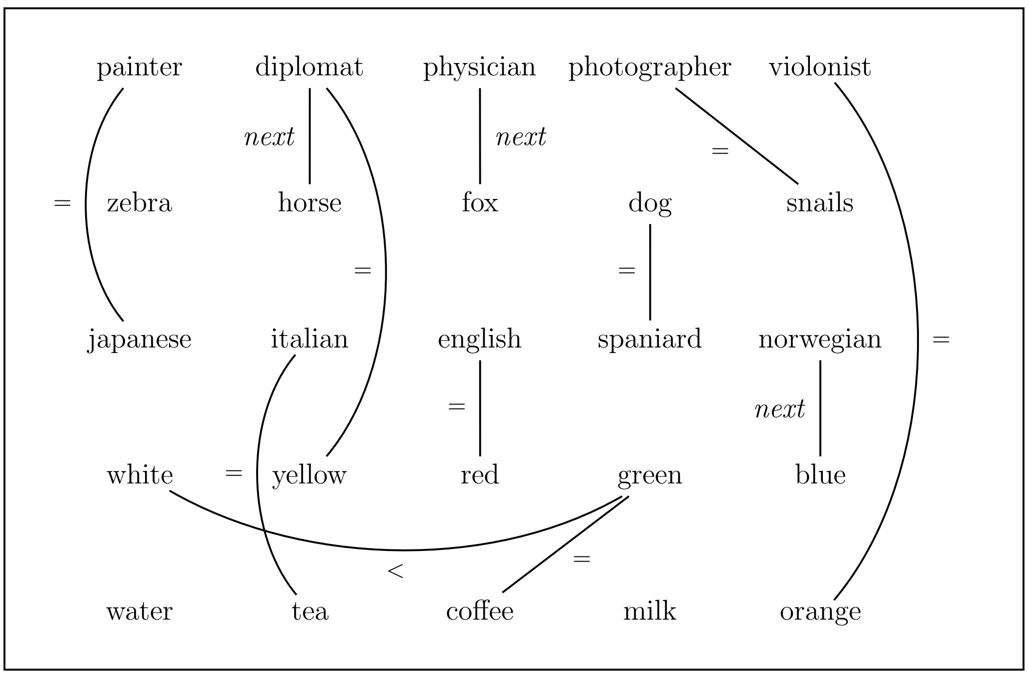
Exercise 1 (arc and bounds consistency)

Let the problem $(\{x, y\}, \{D_x = 0..2, D_y = 0..4\}, \{x \neq 1, y = x + 1\})$.

1. It is arc consistent? If not, calculate an equivalent arc consistent problem.
2. It is bounds consistent? If not, calculate an equivalent bounds consistent problem.

Exercise 2 (Zebra puzzle)

The constraint graph of the Zebra puzzle is given below. Constraint *next* between two variables means that the corresponding houses are adjacent. Furthermore, we know that the Norwegian lives in the first house on the left (house 1) and that milk is drunk by the owner of the middle house (house 3). Calculate an equivalent arc consistent problem by constraint propagation.



Exercise 3 (3-arc consistency)

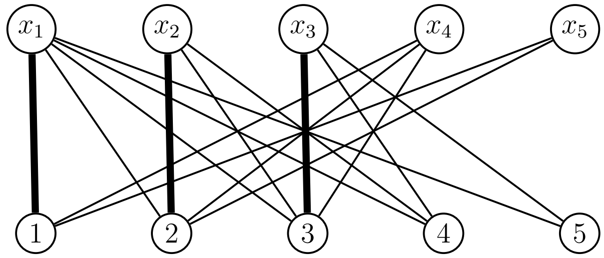
Definition: A CSP (C, X, D) is 3-arc consistent if for every variable x_i and every $a_i \in D_i$, the problem $(C, X, \{D_1, \ldots, D_{i-1}, \{a_i\}, D_{i+1}, \ldots, D_n\})$ is arc consistent.

Let the problem $\{x, y, z\}, \{1..2, 1..2, 1..2\}, \{x \neq y, x \neq z, y \neq z\}$

1. It is consistent?
2. It is arc consistent? If not, calculate an equivalent arc consistent problem.
3. It is 3-arc consistent? If not, calculate an equivalent 3-arc consistent problem.

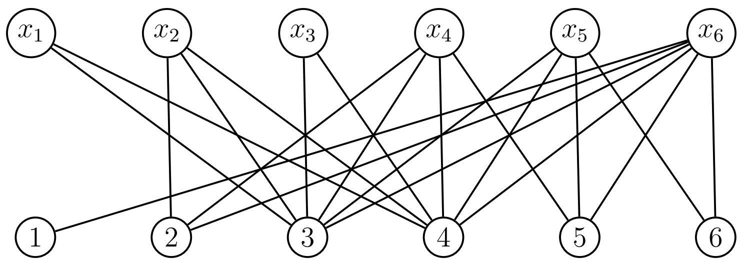
Exercise 4 (matching)

Consider the following bipartite graph and let $M = \{(x_1, 1), (x_2, 2), (x_3, 3)\}$ be a matching. Calculate a maximum matching.



Exercise 5 (alldifferent)

Consider the CSP with six variables $x_1, \ldots, x_6$ with domains $D_1 = 3..4$, $D_2 = 2..4$, $D_3 = 3..4$, $D_4 = 2..5$, $D_5 = 3..6$, $D_6 = 1..6$ and a constraint alldifferent$(x_1, \ldots, x_6)$.



1. Find a trivial solution.
2. Enforce arc consistency on the binary decomposition of the constraint, namely the set of constraints $\{x_i \neq x_j : 1 \leq i < j \leq n\}$.
3. Enforce bounds consistency on the constraint.
4. Find all the solutions.

Exercise 6 (global constraint element)

Consider the problem of assigning workers to products such that the profit is greater than 19, given the following profit matrix.

	p_1	p_2	p_3	p_4
w_1	7	1	3	4
w_2	8	2	5	1
w_3	4	3	7	2
w_4	3	1	6	3

1. Propose a model with binary variables.
2. The global constraint element(y, T, x) applies to x, y and a table $T = (a_1, a_2, \ldots, a_m)$. It states that $x = a_y$, i.e., x is the y-th value in T. Propose a new model with integer variables w_i $(1 \leq i \leq 4)$ representing the product assigned to the i-th worker.

3. Propose a new model with integer variables p_i ($1 \leq i \leq 4$) representing the worker assigned to the i-th product.

4. Propose a filtering algorithm enforcing arc consistency on the element constraint.

Exercise 7 (magic square)

A magic square of order N is an arrangement of distinct numbers $1..N^2$ in a square grid $N \times N$ such that the sums of numbers in each row, each column, the main diagonal and the secondary diagonal are all equal.

$$\begin{array}{cccccc}
 & & & & & 34 \\
 & & & & \nearrow & \\
\hline
7 & 12 & 1 & 14 & \rightarrow & 34 \\
2 & 13 & 8 & 11 & \rightarrow & 34 \\
16 & 3 & 10 & 5 & \rightarrow & 34 \\
9 & 6 & 34 & 4 & \rightarrow & 34 \\
\hline
\downarrow & \downarrow & \downarrow & \downarrow & \searrow & \\
34 & 34 & 34 & 34 & & 34
\end{array}$$

1. Remark that the sum of all cells must be equal to the sum of the sums of all rows. Then define the sum in each row.

2. Propose a model with integer variables.

3. Let the assignment $\{x_{ij} \rightarrow a_{ij}\}$ ($1 \leq i, j \leq N$) be a magic square. Prove that $\{x_{ij} \rightarrow a_{ji}\}$ is also a magic square.

4. How to eliminate the symmetry with respect to the main diagonal?

Exercise 8 (inversion)

Define the operators enforcing bounds consistency on the constraints $x_3 = x_1/x_2$, $x_2 = x_1^2$, $x_2 = \exp(x_1)$ respectively considering intervals of integers and floating point intervals.

Exercise 9 (parallel robot)

Propose a model for the planar robot 3-RPR such that the points A_i are fixed and the moving platform is known (distances l_j and angle β). For this robot, the commands resize the legs. Define the forward kinematics problem and the inverse kinematics problem.