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Visibility in differential phase-contrast imaging with partial coherence source*

Liu Xin(刘 鑫)[a)b)], Guo Jin-Chuan(郭金川)[b)],
Peng Xiang(彭 翔)[b)], and Niu Han-Ben(牛憨笨)[b)†]

[a)] *Institute of Optoelectronics Science and Engineering, Huazhong University of Science and Technology, Wuhan* 430074, *China*

[b)] *Institute of Optoelectronics, Shenzhen University, Key Laboratory of Optoelectronic Devices and Systems of Ministry of Education, Shenzhen* 518060, *China*

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This paper gives theoretical analysis of visibility of fringes, which is influenced by distances, temporal and spatial coherence of source, in hard x-ray differential phase-contrast imaging with microfocus x-ray source. According to the character of longitudinal periodicity of the interferogram, the setup is insensitive to mechanical drift and vibrations. The effect of temporal coherence of x-ray source is investigated and its related bandwidth is derived. Based on the theory of partially coherent light, it shows that the requirement for the spatial coherence of x-ray source is not strict and can be met by the general microfocus x-ray tube for x-ray differential phase-contrast imaging.

Keywords: visibility, differential phase-contrast, interference fringes, phase grating

PACC: 0785, 4280F, 6180C

1. Introduction

The cross section for elastic scattering of hard x-rays in matter, which causes phase shift of incident x-rays, is usually much greater than that for absorption, especially for biological soft tissues. So x-ray phase imaging methods are several thousand times more sensitive than conventional absorption imaging in principle. Various phase-sensitive imaging methods have been developed during the past ten years,[1−10] such as the propagation-based method,[3−5] interference method[6] and diffraction enhanced imaging.[7] Recently, the x-ray differential phase-contrast imaging method[8−10] mainly consisting of two transmission gratings, which can efficiently yield quantitative differential phase-contrast images, has attracted increasing attention. However relative research of this technique is mainly implemented with synchrotron radiation sources, which are unavailable to most laboratories. If a conventional compact x-ray generator, for instance x-ray tubes, can be used as the light sources of differential phase-contrast imaging, as reported quite recently,[11] there will be widespread applications in many fields.

In differential phase-contrast imaging, the distribution of intensity obtained by detector is an interferogram, so the visibility of fringes is an important factor. Weitkamp *et al*[8] and Momose *et al*[9] have considered the effect of temporal and spatial coherency, which almost can be neglected on synchrotron radiation source, on visibility. But the quantitative result of visibility influenced by various distance, bandwidth and focal spot of x-ray source is still unknown.

In this paper, three aspects consisting of distance, temporal and spatial coherency, which greatly influence the visibility of fringes, are discussed. Analysing the characteristics of the longitudinal periodicity of the interferogram and the variations of visibility with distance, we give quantitative result of the requirement with respect to precision of distance. From the periodic variation of visibility of the interferogram versus wavelength, the requirement of bandwidth can be obtained. In the case of a point source of finite size, the displacement of interferogram caused by size of the focal spot is considered instead of the spatial coherence length. Finally phase retrieval of the partially coherent light source is discussed.

2. Intensity distribution of the interferogram

The complex transmission function $T(x)$ of the grating can be expressed with a Fourier series:

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†Corresponding author. E-mail: hbniu@szu.edu.cn

$$T(x) = \sum_{n=-\infty}^{\infty} a_n \exp\left(\mathrm{j}n\frac{2\pi}{d}x\right), \tag{1}$$

where a_n is the amplitude of the nth harmonic and d is the period of the grating. The complex amplitude of the propagating field at a distance z behind the grating illuminated by a spherical beam is given by

$$U(x,z) = \exp\left[\mathrm{j}\pi\frac{x^2}{\lambda}\left(\frac{1}{z}-\frac{1}{Mz}\right)\right]\sum_{n=-\infty}^{\infty} a_n \times \exp\left(-\mathrm{i}\pi n^2\frac{z\lambda}{Md^2}\right)\exp\left(2\pi\mathrm{i}\frac{n}{Md}x\right) \tag{2}$$

with paraxial approximation, where λ is the x-ray wavelength, $M=(R+z)/R$ is the magnification and R represents the radius of the illuminating beam. For $R \gg z$, $M \approx 1$, the phase factor of Eq.(2) can be negligible, then we have

$$U(x,z) \approx \sum_{n=-\infty}^{\infty} a_n \exp\left(-\mathrm{i}\pi n^2\frac{z\lambda}{Md^2}\right) \exp\left(2\pi\mathrm{i}\frac{n}{Md}x\right). \tag{3}$$

When $\frac{z\lambda}{Md^2} = 2m (m=0,1,2,\ldots)$, $U(Mx,z)=T(x)$, this implies that self-images are generated at the distance. X-ray amplitude grating will absorb much of incoming x-ray, thus the attenuation causes a requirement of high brilliant x-ray sources in order to ensure a sufficient signal-to-noise ratio. But present micro-focus x-ray tube cannot provide high enough brilliant illumination. Compared with the amplitude grating, the phase grating shows the almost negligible absorption and exhibits high transmission to the income x-ray. The phase shift of phase grating is π selected by maximizing diffraction efficiency.[12] From Eq.(2) we obtain the intensity distribution at the detector plane

$$I(x,z) = \frac{16}{\pi^2}\sum_{m,n=1} C(m,n)B(m,n;x) \times \exp\left[-\mathrm{i}\pi 8(m+n-1)(m-n)\frac{2z}{Mz_{\mathrm{T}}}\right], \tag{4}$$

where

$$C(m,n) = \frac{(-1)^{m+n}}{(2m-1)(2n-1)},$$
$$B(m,n;x) = \cos\left[(2m-1)\frac{2\pi}{d}\frac{x}{M}\right] \times \cos\left[(2n-1)\frac{2\pi}{d}\frac{x}{M}\right]. \tag{5}$$

In Eq.(4) we denote the Talbot distance for self-imaging plane as $z_{\mathrm{T}} = 2d^2/\lambda$.

3. Drift distance

From Eqs.(4) and (5) the lateral period and longitudinal period can be obtained. When

$$z' = \frac{Rz_k}{R-z_k}, \tag{6}$$

where $z_k = kz_{\mathrm{T}}/16 (k=0,1,2,\ldots)$, k is defined as the coefficient of longitudinal period, we have

$$I(Mx, z+z') = I(x,z), \tag{7a}$$
$$I\left(x+kM\frac{d}{2}, z\right) = I(x,z). \tag{7b}$$

From Eq.(7b) we obtain the result that the lateral period is wavelength independent, which means that polychromatic radiation may equally be used.

Although the periods of interference fringes can be obtained from Eq.(7), the visibility of fringes varying with the distance between grating and detector is still unknown. However the irradiance distribution varies periodically with the longitudinal distance, that is to say the visibility has the same periodicity. According to Eq.(4), irradiance distribution given by numerical simulations is shown in Fig.1. Obviously, the maximum and minimum visibility of fringes pattern occur when the distances z accord with Eqs.(8a) and (8b) respectively

$$\frac{zR}{z+R} = (2k+1)\frac{z_{\mathrm{T}}}{16}, \tag{8a}$$
$$\frac{zR}{z+R} = k\frac{z_{\mathrm{T}}}{8}. \tag{8b}$$

At the same time, from left image of Fig.1 we can calculate the envelope of the fringes visibility, shown in right hand side of Fig.1. From the plot the result can be obtained that the visibility will not be less than 0.5 so long as the drift distance from the centre distance selected by the Eq.(8a) does not exceed one-sixth of the longitudinal periodicity. We denote the drift distance as Δz, which is given by

$$\Delta z = \frac{1}{3}\left[\frac{R^2 z_{\mathrm{T}}}{16R^2-(4k+1)Rz_{\mathrm{T}}+k(2k+1)z_{\mathrm{T}}^2/8}\right] \tag{9}$$

from the Eqs.(8a) and (8b).

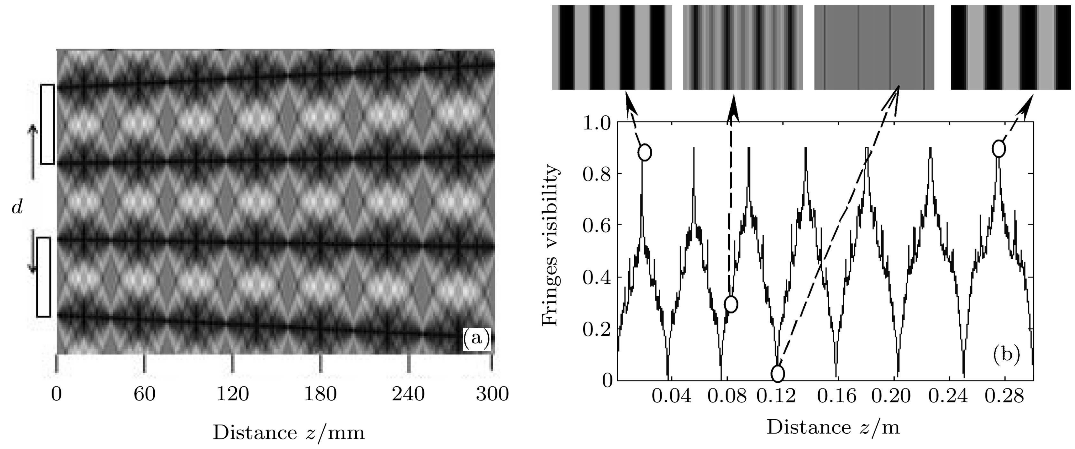

Fig.1. (a) Intensity distribution versus z distance, where the radius is $R = 1.5\,\text{m}$, the energy of x-ray is 50 keV, the periodicity of phase grating is $4\,\mu$m. (b) Fringes visibility as a function of distance z. Four images above the plot are patterns of fringes at different distances. The periodicity of phase grating is $4\,\mu$m, and R is 1.5 m.

The plot of Δz versus x-ray photon energy is shown in Fig.2, which shows that the higher the x-ray photon energy, the lower the requirement on the distance precision. We can find from the plot of Fig.2 that Δz can be as large as several centimetres when x-ray energy is 40 keV. So the requirements on the mechanical stability and the precision of movement are not critical.

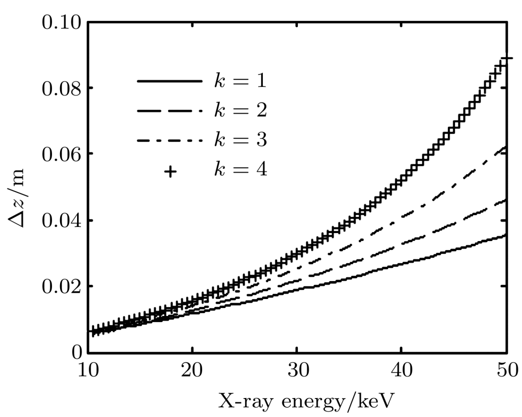

Fig.2. Drift distance as a function of x-ray energy and coefficient of longitudinal period k.

4. Coherence

So far a perfectly monochromatic x-ray source has been assumed. In practice the x-ray source always has a finite bandwidth, which can cause a decrease in the visibility of fringe patterns. Although the lateral periodicity is independent of the wavelength, two aspects are influenced by the bandwidth, which are 1) Talbot condition and 2) zero-order diffraction of the phase grating. First, the position of detector cannot meet the Talbot condition for all wavelengths at the same time. Second, the value of phase shift designed for a certain wavelength will deviate from π because of different wavelength. So the amplitude of zero-order diffraction cannot be zero with a chromatic x-ray source. However, to some extent, differential phase-contrast can tolerate finite bandwidth.

Neglecting the deviation of phase shift designed for a certain wavelength, we can obtain that fringes vary periodically with the variation of wavelength for the same distance z. Like the analysis of longitudinal period, we can obtain $I(x,\lambda) = I(x,\lambda+\lambda_{\mathrm{T}})$, when

$$\lambda_{\mathrm{T}} = k\frac{Md^2}{8z}. \qquad (10)$$

The symbol λ_{T} denotes wavelength periodicity. The intensity distribution versus wavelength is show in Fig.2. Suppose that the source has a normalized intensity distribution, $w(\lambda)$. Intensity distribution is given by[13]

$$I(x) = \int_{\lambda}^{\lambda+\Delta\lambda} w(\lambda)I(x,\lambda)\mathrm{d}\lambda, \qquad (11)$$

where $\Delta\lambda$ is the bandwidth. From the variations of fringes in Fig.3, the bandwidth should be limited in one wavelength periodicity for distinct fringes. For example, substituting the wavelength λ and distance z selected by Eq.(8a) into Eq.(14) ($k = 1$), we can obtain $\lambda_{\mathrm{T}} = \lambda/3$. That is to say that the maximum visibility occurs at λ and $\lambda + \lambda_{\mathrm{T}}$. When the deviation of wavelength for the centre wavelength is $\Delta\lambda = \lambda_{\mathrm{T}}/3$, the visibility of fringes pattern decreases to 0.5 ,which is an acceptable value. Finally we can obtain $\Delta\lambda/\lambda = 15\%$. For general condition that the phase shift ε of the phase grating is not π, Eq.(4) can be rewritten as

$$U(x,z)=\frac{1}{2}\left[1+\exp(\mathrm{i}\varepsilon)\right]+4[1-\exp(\mathrm{i}\varepsilon)]\sum_{n=1}^{\infty}\frac{\sin(n\pi/2)}{n}\exp\left(-\mathrm{j}2\pi n^{2}\frac{2z}{z_{\mathrm{T}}}\right)\cos\left(n\frac{2\pi}{d}x\right), \tag{12}$$

where the phase grating is illuminated by plane wave. According to the deduction above, diffraction image of numerical simulation is shown in Fig.4, where the material of phase grating is silicon. The best visibility occurs at 25.7 keV. However the visibilities is acceptable from 24.45 keV to 27.58 keV and thus the bandwidth is 12%. The compact x-ray generator such as a microfocus beam x-ray tube can meet the requirement of bandwidth with a suitable filter.

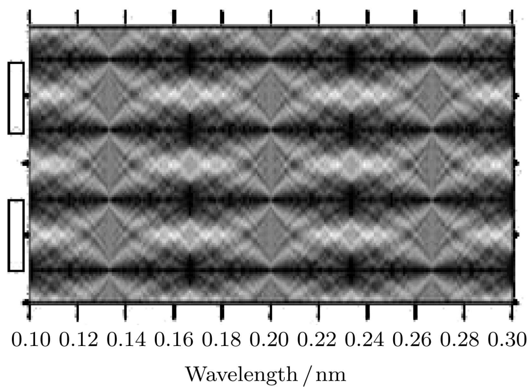

Fig.3. Interferogram versus the wavelength, where the radius R is 1.5 m, and $z=0.015$ m.

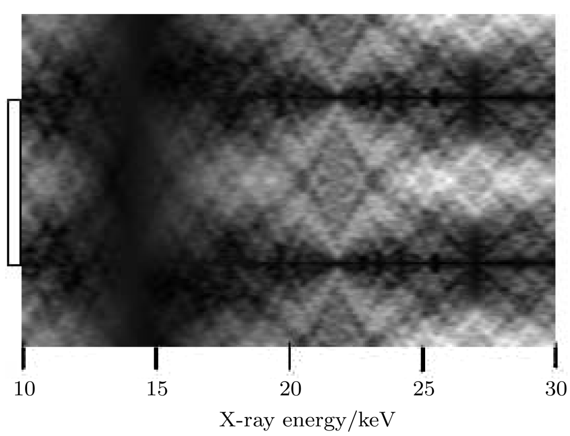

Fig.4. Interferogram of silicon grating with different x-ray energy. The thickness of grating is 32 μm, and distance of z is 31 mm.

Next we consider the effect of spatial coherence by using the theory of partially coherent light.[14] In the case of an extended incoherent source, the intensity distribution is given by[15]

$$I(x)=S\left(\frac{x}{z/R}\right)\otimes I_p(x), \tag{13}$$

where $S(x)$ represents the intensity distribution of source and $I_p(x)$ denotes the imaging of object with point source whose coordinate is (0,0). The symbol $\otimes$ denotes convolution operation. Supposing that the x-ray source consists of two point sources, $S(x)=\delta(x-a/2)+\delta(x+a/2)$, where a represents the distance between two point sources, we can express Eq.(13) as

$$I(x)=I_p\left(x-\frac{z}{2R}a\right)+I_p\left(x+\frac{z}{2R}a\right). \tag{14}$$

In general, the lateral periodicity is approximately $d/2$ by using self-image effect. So the distance of two images given by Eq.(14) can not exceed the quarter periodicity of the fringes. Then the distance a should meet the condition

$$a<\frac{Rd}{8z}. \tag{15}$$

For instance, with the periodicity of phase grating $d=2\,\mu$m, the radius $R=1.5$ m and the distance $z=0.015$ m, the distance between two point source must meet the condition: $a<25\,\mu$m. In the case of microfocus source, its diameter can not exceed 25 μm, which is not a critical requirement for general microfocus x-ray tube.

5. Discussion

Since the periodicity of the interference fringes does not exceed a few microns, the image detector existing at present has not sufficiently spatial resolution to resolve the fringes. Therefore, an analyser grating placed in the detection plane can generate a Moiré pattern, which has much larger periodicity than the interference fringes. A phase object placed before the phase grating can distort the incoming x-ray wavefront which causes the distortion of the moiré pattern. The inclination angle of incoming x-ray $\varphi(x,y)$ is related to the wave-front phase profile $\exp[\mathrm{i}\Phi(x,y)]$

$$\varphi(x,y) = \frac{\lambda}{2\pi}\frac{\partial \Phi(x,y)}{\partial x}. \tag{16}$$

The intensity distribution has the form of $I[x - z\varphi(x,y), z]$ after a tedious calculation. From the moiré pattern distorted by object, the phase distribution of object can be retrieved. More detail discussion was described in Refs.[8] and [16].

There is nothing in the literature regarding the effect of spatial coherence on the retrieval of phase. The approach to eliminate the effect of spatial coherence can be obtained from Eq.(13). First we can remove the phase object from the setup; the Moiré pattern only concludes the information of light source, from which the intensity distribution of source can be resolved. Then, when the object exists, the image is deconvolved by the intensity distribution of source. The phase distribution of object is not unique because of different wavelengths, so how to define the phase shift is a problem, let alone the retrieval of phase. In future work we will study this issue.

6. Conclusion

Three aspects including various distances, bandwidth and diameter of source, which are most crucial factors on contrast of differential phase-contrast image, are discussed. The analysis of longitudinal periodicity shows that the precise requirement of distance and movement of detector is not strict, especially for higher x-ray energy, because the decrease of contrast is tolerable in the infinite variation of distance as mention above. The method of differential interference requires finite bandwidth, but micro-focus source can meet the requirement with a suitable filter. We assume that the x-ray source accords with the van Cittert–Zernike theorem[14] that the source consists of independent elements which are statistically independent. Then the image can be considered as the superposition of the images caused by point source with different displacements. The maximum displacement of image, which is determined by various distance and dimension of source, should not exceed the quarter of the periodicity of the interference fringes. For the general microfocus source the requirement of spatial coherence is easily met.[17,18]

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