Quantum Programming (4ETI) **Spring 2020**

Assignement #1

Instructor: Dr. Fedrici — Due date: 13^{th} March, 2020

Problem 1
Read the section 2.2.5 of the Nielsen and Chuang and then answer the following questions.

a. What is the probability that when measuring the first bit of a system in the state $|\psi\rangle_{ab} = (|00\rangle + |10\rangle + |11\rangle)/\sqrt{3}$ the outcome is one ? What is the state of the system after such a measurement ?

b. What is the average value of the measurement along the X-axis for a qubit in state $|0\rangle$? What is the standard deviation ?

c. Show that the average value of the operator $M \coloneqq X_1 Z_2$ (i.e. applying X to the first subsystem and Z to the second subsystem) for a two-qubit system measured in the maximally entangled-state $|\psi\rangle_{ab} = (|00\rangle + |11\rangle)/\sqrt{2}$ is zero.

Problem 2
Read the section 2.1.8 of the Nielsen and Chuang and then answer the following question. The rotation gates are defined by the following operators:

$$R_x(\theta) \coloneqq e^{-i\theta X/2} = \cos(\theta/2)I - i\sin(\theta/2)X = \begin{bmatrix} \cos(\theta/2) & -i\sin(\theta/2) \\ -i\sin(\theta/2) & \cos(\theta/2) \end{bmatrix}; \tag{1}$$

$$R_y(\theta) \coloneqq e^{-i\theta Y/2} = \cos(\theta/2)I - i\sin(\theta/2)Y = \begin{bmatrix} \cos(\theta/2) & -\sin(\theta/2) \\ \sin(\theta/2) & \cos(\theta/2) \end{bmatrix}; \tag{2}$$

$$R_z(\theta) \coloneqq e^{-i\theta Z/2} = \cos(\theta/2)I - i\sin(\theta/2)Z = \begin{bmatrix} e^{-i\theta/2} & 0 \\ 0 & e^{i\theta/2} \end{bmatrix}. \tag{3}$$

Show that for any real number γ and matrix A such that $A^2 = I$:

$$e^{iA\gamma} = \cos(\gamma)I + i\sin(\gamma)A.$$

Use this to verify the definitions of the rotation gates.

Problem 3
Write a Qiskit program that allows for an input quantum circuit `qc` with a single-qubit register to draw the corresponding Bloch vector. To facilitate your work, Qiskit already comes with a function `plot_bloch_vector`, see Qiskit documentation. Use `'qasm_simulator'` for backend.

Hint: `bloch_vector[0]` is $\langle X\rangle$, `bloch_vector[1]` is $\langle Y\rangle$ and `bloch_vector[2]` is $\langle Z\rangle$, where $\langle i\rangle \equiv \langle\psi|i|\psi\rangle$ is the average value of the measurement along the i-axis for a qubit in sate $|\psi\rangle$, see **Problem 1** for average value computation examples.

Problem 4

Similarly to classical computation, it is common to represent numbers in quantum memory in registers of qubits. For example the binary number 00110 is represented by the quantum state $|00110\rangle$. In this problem you will construct Qiskit programs to implement binary addition on quantum bits and registers.

a. (Quantum Half-Adder) A half-adder takes two bits, x and y, as inputs, and outputs the modulo two sum of the two bits, together with a carry bit set to one if x and y are both one, or zero otherwise:

$$|x, y, 0\rangle \mapsto |x, s, c_{out}\rangle$$

where $s = x \oplus y$ and $c_{out} = xy$. Draw a quantum circuit implementing this operation and write the corresponding Qiskit program.

b. (Quantum Full-Adder) Two cascaded half-adders may be used to build a full-adder. A full-adder takes as input three bits, x, y and c_{in}. The bits x and y should be thought of as data to be added, while c_{in} is a carry bit from an earlier computation. The circuit outputs two bits. The first output bit is the modulo two sum of all three input bits, while the second output bit is a carry bit, which is set to one if two or more of the inputs is one, and is zero otherwise:

$$|x, y, c_{in}, 0\rangle \mapsto |x, y, s, c_{out}\rangle$$

where $s = x \oplus y \oplus c_{in}$ and $c_{out} = xy \oplus (x \oplus y)c_{in}$. Draw a quantum circuit implementing this operation and write the corresponding Qiskit program.