Pieris virginiensis progress report 06

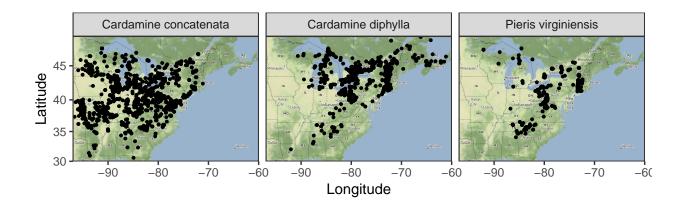
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12 September, 2019

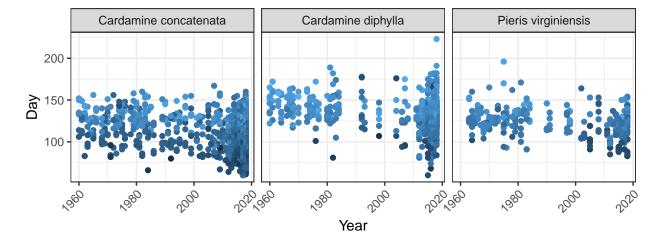
Retrieval from iNaturalist and GBIF returned 3369 total observations.

Following date and sample size filtering, observation counts for species:

- Pieris virginiensis: 544 observations
- Cardamine concatenata: 1963 observations
- Cardamine diphylla: 862 observations

Observations for 1960 - 2018 (total of 3369 observations following date and sample size filtering):





Note in lower plot, darker points are from lower latitudes, while lighter points are from higher latitudes.

Models allowing species x latitude interactions

Previous models only included species x year interactions, but since these things all have different altitudinal ranges, would be useful to include that interaction as well.

$$Julian \ day = \beta_0 + \beta_1 Year + \beta_2 Latitude + \beta_3 Species + \beta_4 Year \times Species + \beta_5 Latitude \times Species$$

Expanding Species because there are three levels, this becomes:

However, because there are three levels to Species, a more accurate representation of this model would be

 $Julian\; day = \beta_0 + \beta_1 Year + \beta_2 Latitude + \beta_3 concatenata + \beta_4 diphylla + \beta_5 Year \times concatenata + \beta_6 Year \times diphylla + \beta_7 Latitude \times diphy$

(so long as P. virginensis is reference).

The model for P. virginensis remains the same:

Julian day =
$$\beta_0 + \beta_1 Year + \beta_2 Latitude$$

And rearranging:

$$Julian \ day = \beta_0 + \beta_2 Latitude + \beta_1 Year$$

For a single value of *Latitude*, the first two terms constitute the intercept $(\beta_0 + \beta_2 Latitude)$ and the last coefficient is the slope (β_1) .

For the host lines, there are two models, one for each species of host. For C. concatenata:

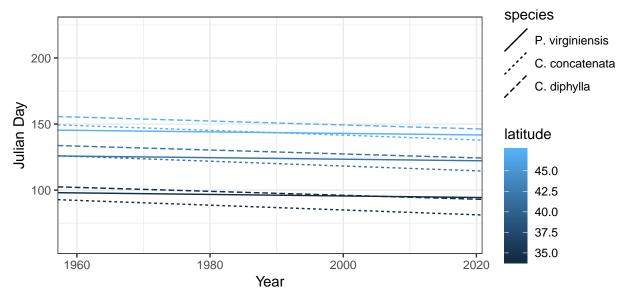
$$\label{eq:Julian day = beta beta day for the formula} Julian \ day = \beta_0 + \beta_1 Year + \beta_2 Latitude + \beta_3 + \beta_5 Year + \beta_7 Latitude$$

Rearranging:

Julian day =
$$\beta_0 + \beta_3 + (\beta_1 + \beta_5)Year + (\beta_2 + \beta_7)Latitude$$

So for a single value of *Latitude*, the model for *C. concatenata* has an intercept of $\beta_0 + \beta_3 + (\beta_2 + \beta_7)Latitude$ and a slope of β_1 and β_5 .

Similarly, the model for *C. diphylla* can be simplified to an intercept of $\beta_0 + \beta_4 + (\beta_2 + \beta_8)Latitude$ and a slope of $\beta_1 + \beta_6$.



Considering the slopes, we see that controlling for latitude, observations are getting earlier, but the change is happening faster in the host plants than in *P. virginiensis*:

| Species | Change |
|--------------------------------------------|-------------------------------------------------------|
| P. virginiensis C. concatenata C. diphylla | 0.058 days/year 0.182 days/year 0.149 days/year |