우림 9-6

2019년 8월 14일 수요일 오후 7:21

$$\lim_{n\to\infty} a_n = \left(1+\frac{1}{2}+\frac{1}{2^2}+\frac{1}{2^3}+\cdots\right)\cdot\left(1+\frac{1}{3}+\frac{1}{3^2}+\frac{1}{3^3}+\cdots\right)\cdots$$

Since a_n includes $\left(1+\frac{1}{p_m}+\frac{1}{p_m^2}+\cdots\right)$ for any m th prime,

all natural reciprocal appears on $\lim_{n\to\infty} a_n$

$\therefore \displaystyle\sum_{n=1}^{\infty}\frac{1}{n} = \lim_{n\to\infty} a_n$ diverges.

Now, $\displaystyle\lim_{n\to\infty}\log a_n = \sum_{n=1}^{\infty}\log\frac{1}{1-\frac{1}{p_n}} = \sum_{n=1}^{\infty}\left(\log(p_n)-\log(p_n-1)\right) = \infty$

Since $\log n - \log(n-1) \le \frac{2}{n}$ (This is easy integral inequality),

$$\lim_{n\to\infty}\log a_n \le \sum_{n=1}^{\infty}\frac{2}{p_n} = \infty.$$