우림 10-4

2019년 8월 14일 수요일 오후 6:22

(i) Put $x=1$ on $(1+x)^n \to 2^n = {}_nC_0 + {}_nC_1 + \cdots + {}_nC_n$

$x=-1$ on $(1+x)^n \to 0 = {}_nC_0 - {}_nC_1 + \cdots + (-1)^n {}_nC_n$

Adding to results and dividing by 2, we get

$${}_nC_0 + {}_nC_2 + \cdots = 2^{n-1} \left(= {}_nC_1 + {}_nC_3 + \cdots\right)$$

(ii) For a polynomial $p(x) = a_n x^n + \cdots + a_1 x + a_0$, and ξ (imaginary root of $x^3=1$),

$$\begin{aligned}
P(1) &= a_n 1 + a_{n-1} 1 + \cdots + a_1 1 + a_0 \\
P(\xi) &= a_n \xi^n + a_{n-1}\xi^{n-1} + \cdots + a_1 \xi + a_0 \\
P(\xi^2) &= a_n \xi^{2n} + a_{n-1}\xi^{2n-2} + \cdots + a_1 \xi^2 + a_0
\end{aligned}$$

Since 3 is prime, $m(0,1,2) \bmod 3 \equiv (0,1,2)$ for $m \not\equiv 0 \pmod 3$

and since $1+\xi+\xi^2=0$,

$$\frac{P(1)+P(\xi)+P(\xi^2)}{3} = a_0 + a_3 + \cdots$$

we can do the same thing for the other two, on $xP(x)$ and $x^2P(x)$.

Let $P(x) = (1+x)^n$. $P(1) = 2^n$, $P(\xi) = (1+\xi)^n = (-1)^n \xi^{2n}$,

$P(\xi^2) = (-1)^n \xi^n$. Therefore we get three numbers:

$$\frac{2^n + (-1)^n(\xi^{2n} + \xi^n)}{3}, \quad \frac{2^n + (-1)^n(\xi^{2n+2} + \xi^{n+4})}{3}, \quad \frac{2^n + (-1)^n(\xi^{2n+1} + \xi^{n+2})}{3}$$

Since $\xi^3 = 1$, we only have to test for $n = 0, 1, 2$,

which produces the desired result.