우림 9-3

2019년 8월 14일 수요일 오후 6:11

$$\sum_{n=0}^{\infty} \log\left(1+\frac{1}{2^{3^n}}+\frac{1}{2^{2\cdot 3^n}}\right) = \log \prod_{n=0}^{\infty}\left(1+\frac{1}{2^{3^n}}+\frac{1}{2^{2\cdot 3^n}}\right)$$

The Infinite Product is

$$\left(1+\frac{1}{2^{3^0}}+\frac{1}{2^{2\cdot 3^0}}\right)\left(1+\frac{1}{2^{3^1}}+\frac{1}{2^{2\cdot 3^1}}\right)\left(1+\frac{1}{2^{3^2}}+\frac{1}{2^{2\cdot 3^2}}\right)\cdots$$

Choosing one from each parenthesis, (and multiply) we get one term of the infinite product.

Now, since we can construct all natural number by 3-base, it is equal to

$$1+\frac{1}{2}+\frac{1}{2^2}+\frac{1}{2^3}+\cdots = 2.$$

Thus the answer is $\log 2$.