우림 7-1

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First, $f(n\cdot 1) = f(n)\cdot 2 - f(n+1) + 1 \rightarrow f(n+1) = f(n) + 1$

Thus, for $x \in \mathbb{Z}$, $f(x) = x+1$. Also,

$f(x+k) = f(x) + k \quad (k \in \mathbb{Z})$.

Now, for $n \in \mathbb{Z}/\{0\}$,

$f(1) = 2 = f(n\cdot\frac{1}{n}) = f(n)f(\frac{1}{n}) - f(n+\frac{1}{n}) + 1$

$= (n+1)f(\frac{1}{n}) - f(\frac{1}{n}) - n + 1 \rightarrow \frac{n+1}{n} = 1 + \frac{1}{n} = f(\frac{1}{n})$.

Now, $f(\frac{m}{n}) = \frac{m}{n} + 1$ turns out to be true easily

$(m \in \mathbb{Z}, n \in \mathbb{Z}/\{0\})$ by the formulas previously discussed