



It suffices to prove that

$$f(\lambda a + (1-\lambda)b) \leq \frac{\lambda f(a) + (1-\lambda)f(b)}{1}$$

for any  $0 \leq \lambda \leq 1$ .

$\swarrow$

$$= \lambda(\lambda a + (1-\lambda)b)$$

$f'(x) \geq 0$  means that  $f(x)$  increases.

and by Mean Value Theorem,

$$\frac{f(\lambda a + (1-\lambda)b) - f(a)}{(1-\lambda)(b-a)} = f'(c) \quad (c \in (a, \lambda a + (1-\lambda)b))$$

$$\frac{f(b) - f(\lambda a + (1-\lambda)b)}{\lambda(b-a)} = f'(d) \quad (d \in (\lambda a + (1-\lambda)b, b))$$

Now from  $f'(d) \geq f'(c)$ , we get

$$\lambda f(\lambda a + (1-\lambda)b) - \lambda f(a) \leq (1-\lambda)f(b) - (1-\lambda)f(\lambda a + (1-\lambda)b)$$

$$\rightarrow \lambda f(a) + (1-\lambda)f(b) \geq f(\lambda a + (1-\lambda)b)$$