우림 5-2

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In $(0,\infty)$, differentiate both side with respect to y

$$\left(\int_y^{y^2} f(x)dx\right)' = 2yf(y^2) - f(y) = \left(\int_1^y f(x)dx\right)' = f(y)$$

So we get $yf(y^2) = f(y)$. And we let $f(1) = k$.

For $y>0$,

$$f(y) = \frac{f(\sqrt{y})}{\sqrt{y}} = \frac{f(\sqrt[4]{y})}{\sqrt{y}\,\sqrt[4]{y}} = \cdots = \frac{f(\sqrt[2^n]{y})}{\sqrt{y}\,\sqrt[4]{y}\cdots\sqrt[2^n]{y}}$$

Since the assumption says f is continuous,

$$f(y) = \lim_{n\to\infty} \frac{f(\sqrt[2^n]{y})}{\sqrt{y}\cdots\sqrt[2^n]{y}} = \frac{f(1)}{y} = \frac{k}{y}.$$