우림 8-2

2019년 8월 14일 수요일 오후 7:09

a_n is equal to n+1th Fibbonachi number

(By recurrence relation, $n+1 = \begin{cases} 1+(\quad\quad)-a_n \\ 2+(\quad\quad)-a_{n+1} \end{cases}$)

$\rightarrow a_{n+1} = a_n + a_{n+1}$

Now,

$$\frac{1}{a_n a_{n+2}} = \frac{1}{a_n(a_{n+1}+a_n)} = \left(\frac{1}{a_n} - \frac{1}{a_n + a_{n+1}}\right)\frac{1}{a_{n+1}}$$

$$= \frac{1}{a_n a_{n+1}} - \frac{1}{a_{n+2}a_{n+1}}.$$

$$\therefore \sum_{n=1}^{\infty} \frac{1}{a_n a_{n+2}} = \frac{1}{1\cdot 2} - \cancel{\frac{1}{2\cdot 3}} + \cancel{\frac{1}{2\cdot 3}} - \cancel{\frac{1}{3\cdot 5}} + \cancel{\frac{1}{3\cdot 5}} - \cdots$$

$$= \frac{1}{2}.$$