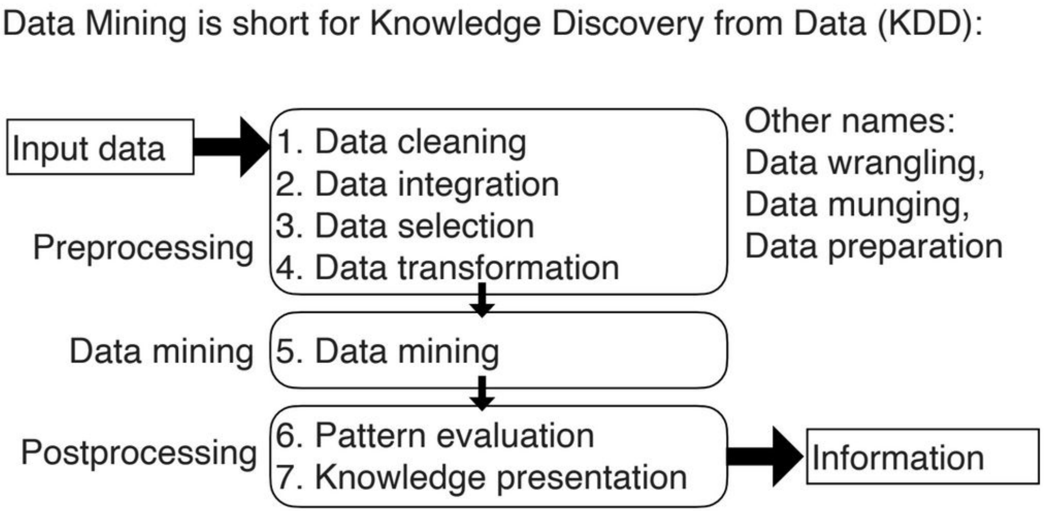
Introduction to network science

Data science course LPI, 2023/2024
Marc Santolini
Liubov Tupikina

Outline of the day

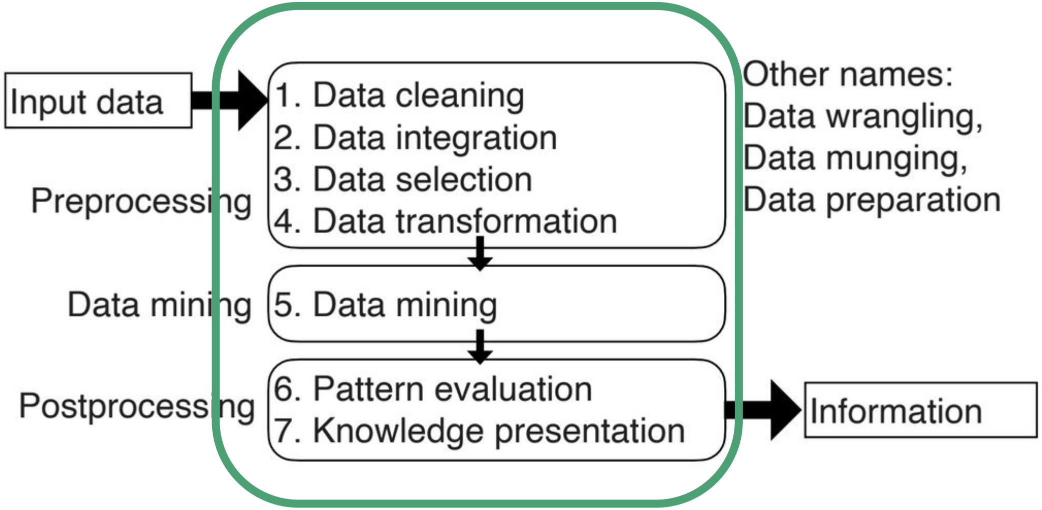
1. Introduction to networks
2. Practical part: notebooks

Personal projects



Personal projects

Data Mining is short for Knowledge Discovery from Data (KDD):



Personal projects

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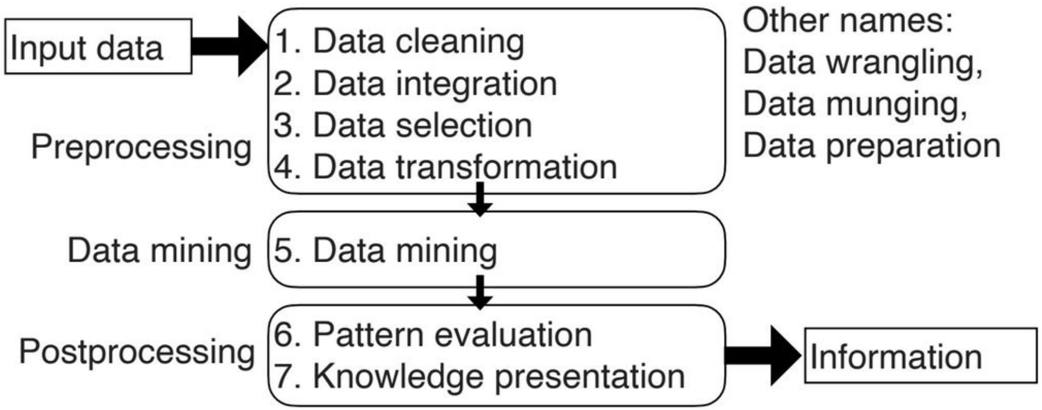


Structure of the folder:

- figures
- notebooks
- raw_data
- report
- results
- README.md

Github of the course

Personal projects example

Directions for projects:
- Networks-oriented projects
- Statistics of data oriented projects

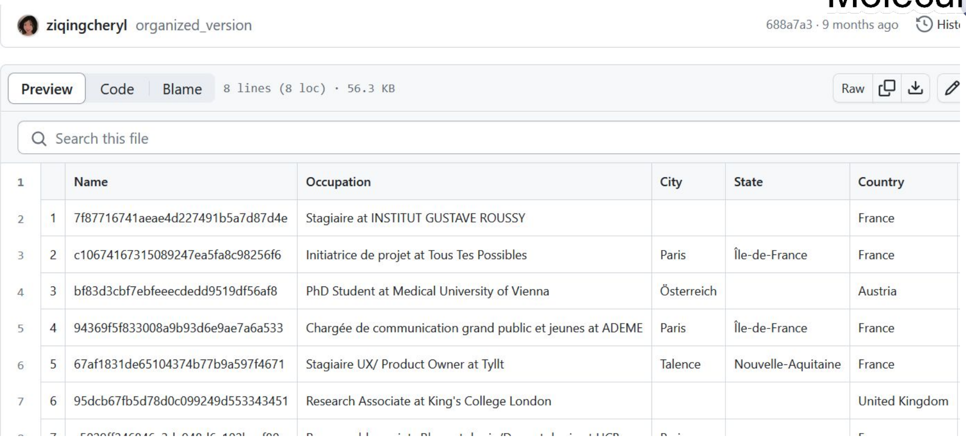
Process data: what is important to do with data?
Depersonalisation, data ethics issues

Development of specific research questions:
- Descriptive questions: what is the distribution, organisation of subcategories
- Hypothesis building: properties of subclasses etc.

Personal projects example

Process data: what is important to do with data?

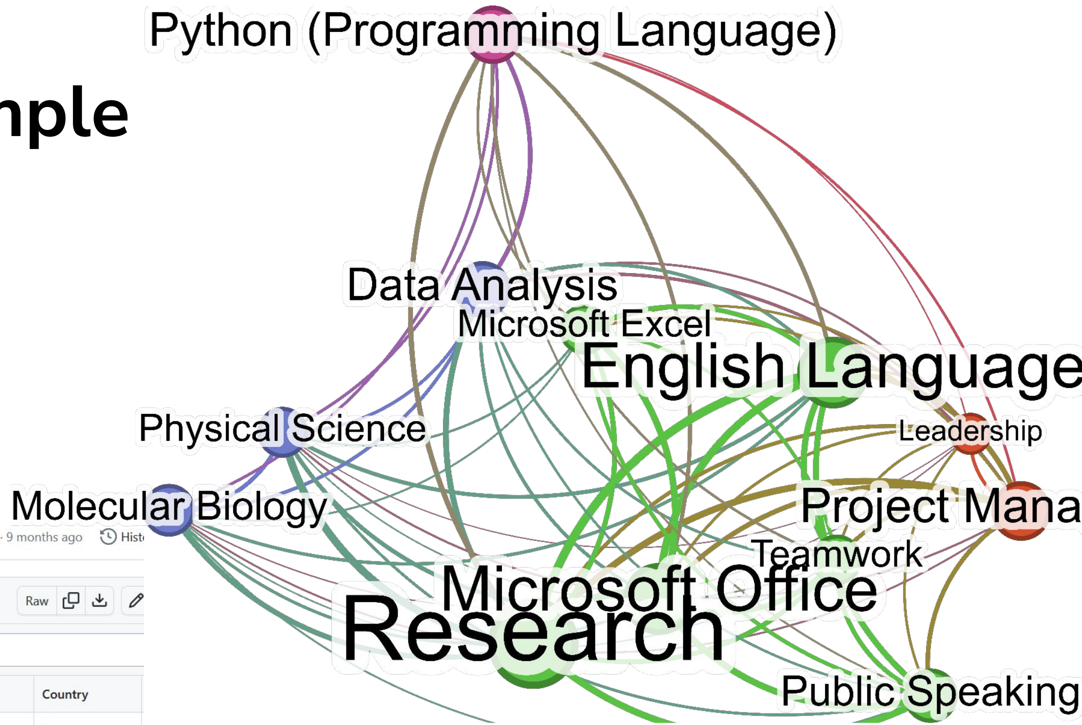
[Github of the course]


ziqingcheryl organized_version 688a7a3 · 9 months ago

Preview Code Blame 8 lines (8 loc) · 56.3 KB Raw

Search this file

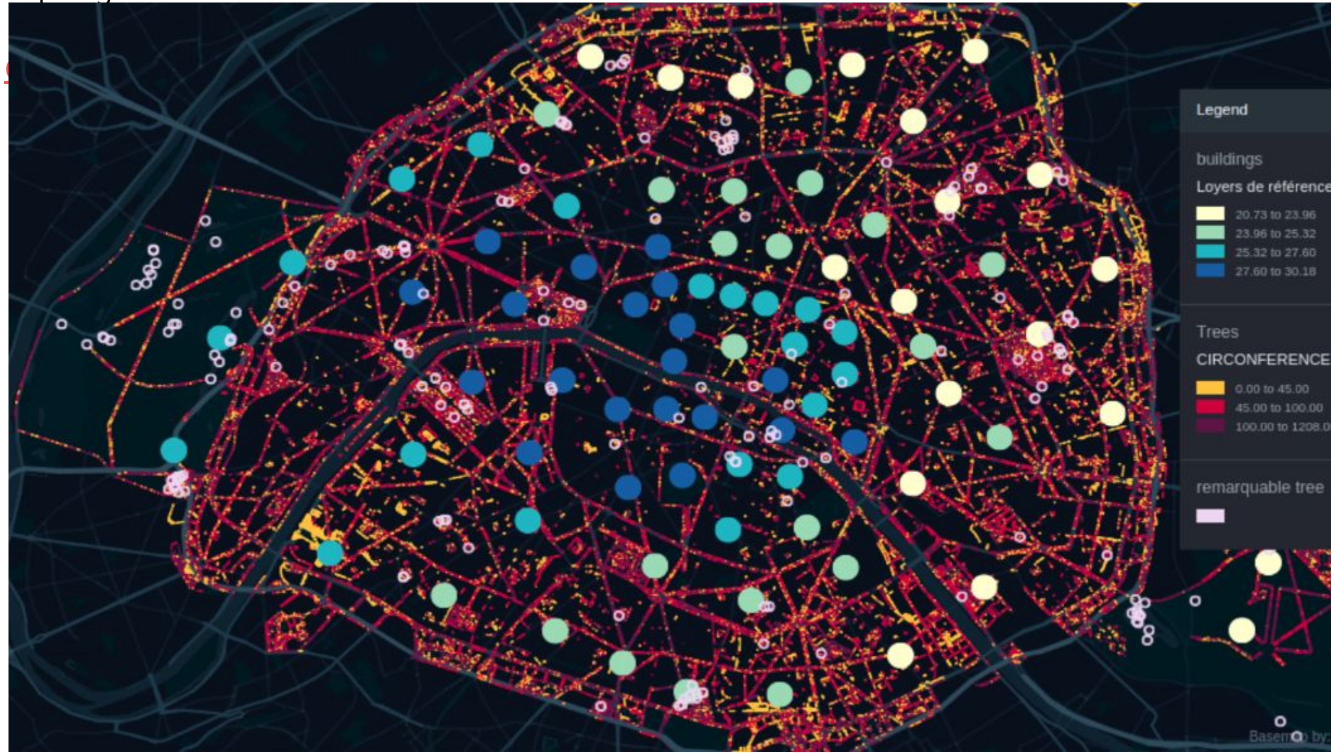
	Name	Occupation	City	State	Country
1	7f87716741aeae4d227491b5a7d87d4e	Stagiaire at INSTITUT GUSTAVE ROUSSY			France
2	c10674167315089247ea5fa8c98256f6	Initiatrice de projet at Tous Tes Possibles	Paris	Île-de-France	France
3	bf83d3cbf7ebfeeecdedd9519df56af8	PhD Student at Medical University of Vienna	Österreich		Austria
4	94369f5f833008a9b93d6e9ae7a6a533	Chargée de communication grand public et jeunes at ADEME	Paris	Île-de-France	France
5	67af1831de65104374b77b9a597f4671	Stagiaire UX/ Product Owner at Tyllt	Talence	Nouvelle-Aquitaine	France
6	95dcb67fb5d78d0c099249d553343451	Research Associate at King's College London			United Kingdom



Personal projects example

Some ideas for future projects: Olympics, statistics, the Guardian data stories, Humanitarian data HDX

Kepler.gl visualisation



Assignment for personal projects (on google classroom)

1. Choose the dataset (already existing or mining resource if to get the data)
2. Choose the research questions, motivation (directions)
3. Feel free to consult us (office hours or within the emails) for recommendations or suggestions :)
4. Submit in classroom the link to dataset or description of 1., 2.

Resources and libraries

Standard libraries (Python): numpy, matplotlib, scikit learn, seaborn
Network libraries: networkx, osmnx (open street data)

Support materials

- Big data course Marc and Liubov https://github.com/Big-data-course-CRI/
- Correlaid, Complex system conference CSS 2023 and TidyTuesday https://github.com/rfordatascience/tidytuesday
- Network science book http://networksciencebook.com/
- Network repository https://networks.skewed.de/
- Visualisation tools https://gephi.org/users/download/

Why networks?

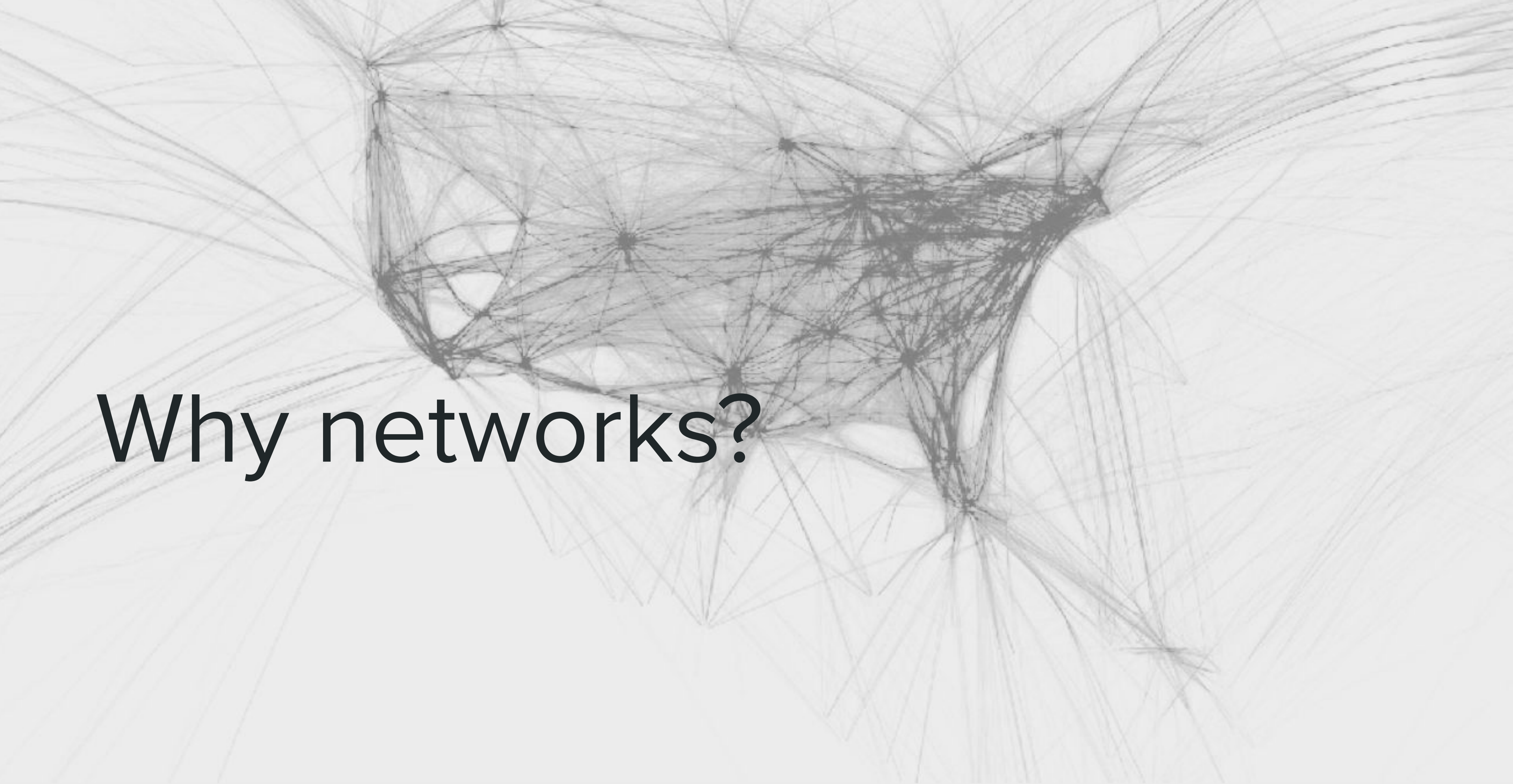
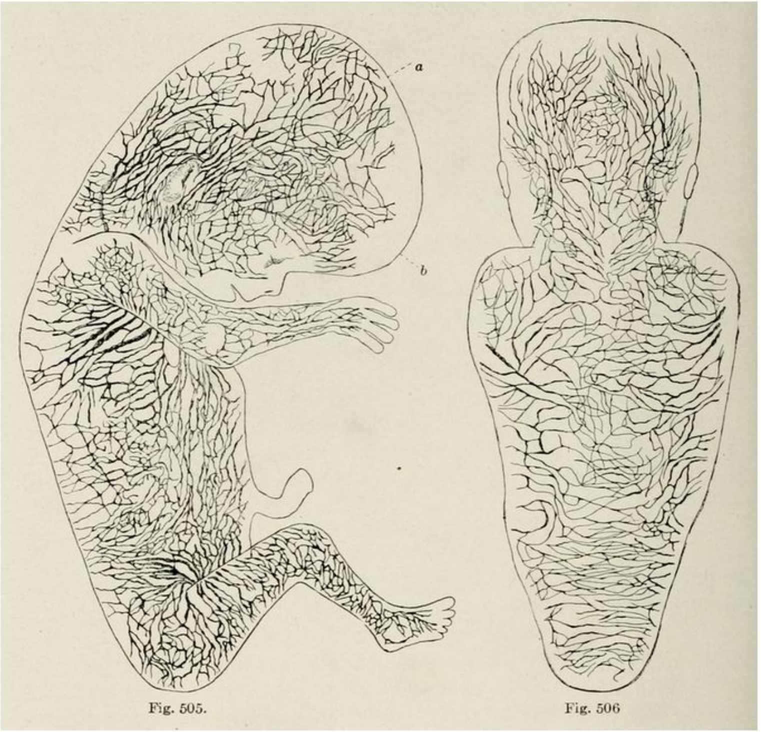


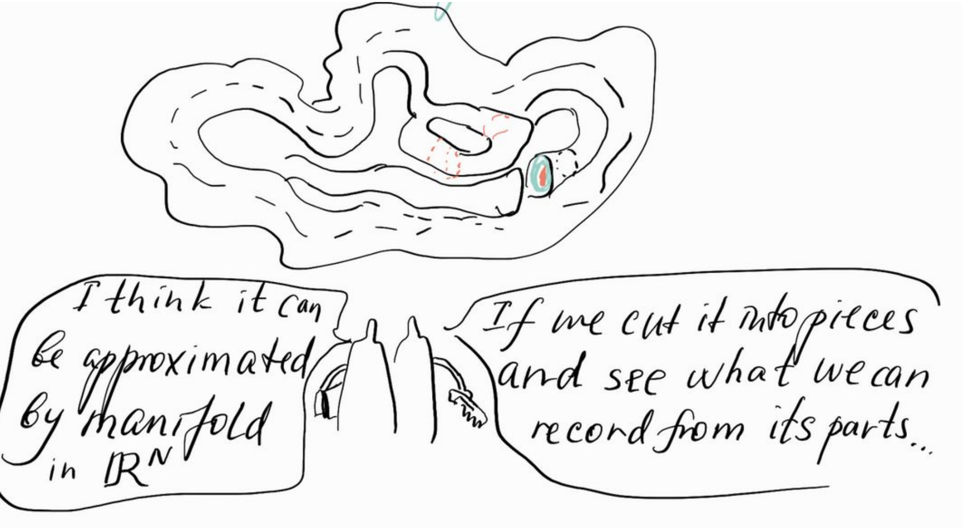
Figure 7.11

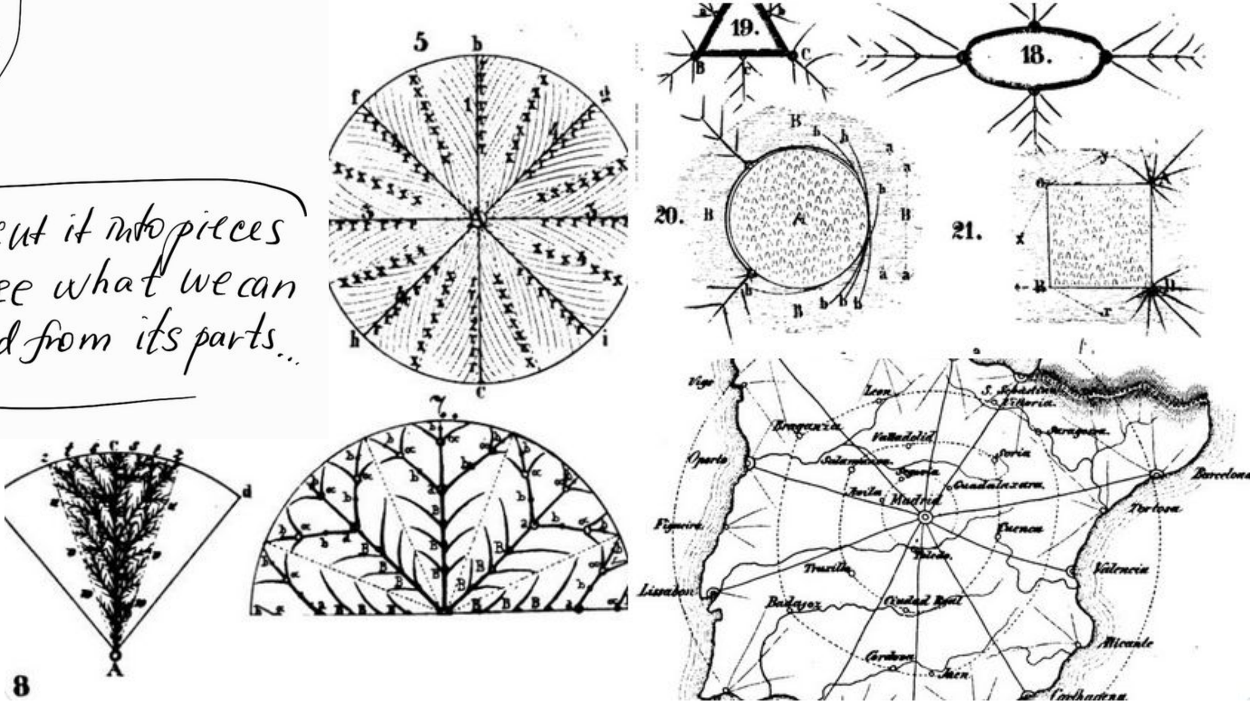
NETWORKS DESCRIBE HOW THINGS CONNECT AND INTERACT



Distension of the lymphatic vessels in the human foetus, from Franz Kreibel, *Manual of human embryology*, 1910

NETWORKS DESCRIBE HOW THINGS CONNECT AND INTERACT





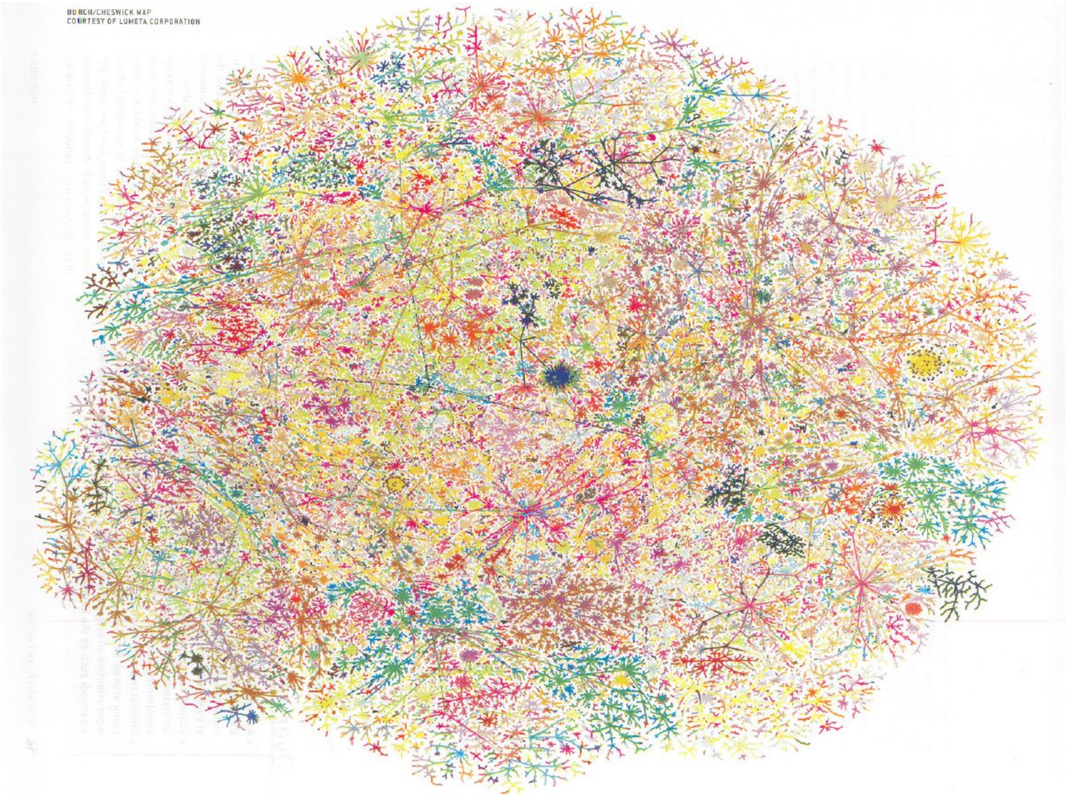
Network theory, graph theory

1. Implementation of algorithms using networks
2. Representing your data using networks
3. Ways of solving problems using networks representation

What is network science?

One idea in network science is that any node can influence other nodes, not only their direct connections.

Such indirect influence happens through some external phenomenon—travel in a transportation network, information transfer in the Internet, vibrations in a spiderweb, etc.—and depends on how the network is connected.
(P. Holme)



Network science

1. Network **measures and network types**
2. Networks in **time and space**
3. Networks from **data**

Figure 7.11

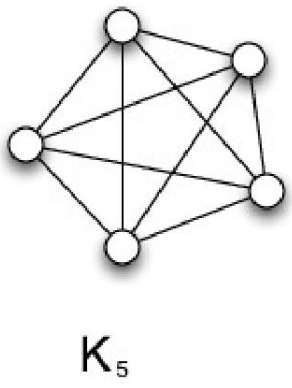
How did the network science start?

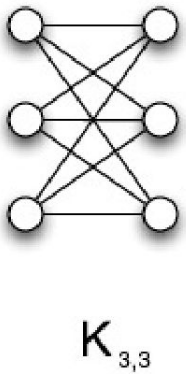
http://networksciencebook.com/

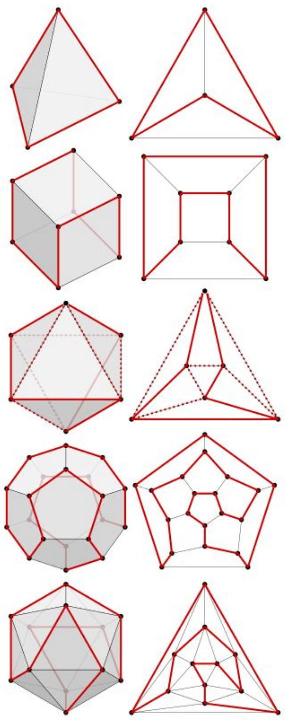
Network science

- Graph theory:
  Koenigsberg problem 1736
  Eurlerian paths algorithms 1873

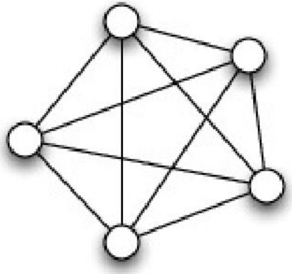
- Soft matter physics

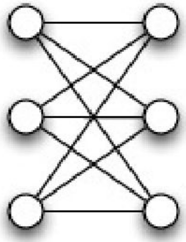






Network science and graph theory



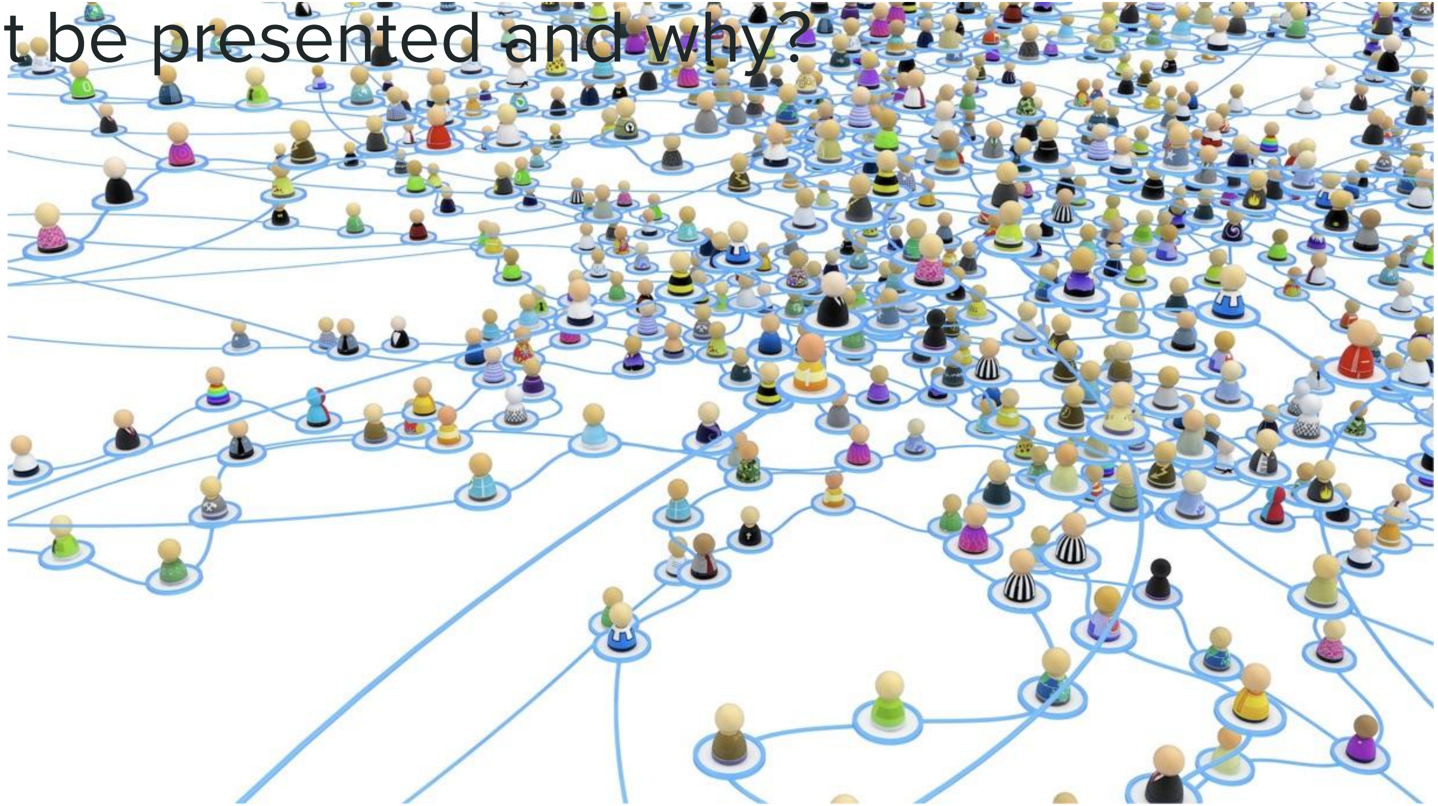


Graph (discrete mathematics), a structure made of vertices V and edges E.

Networks (from real world) can be represented as graphs.

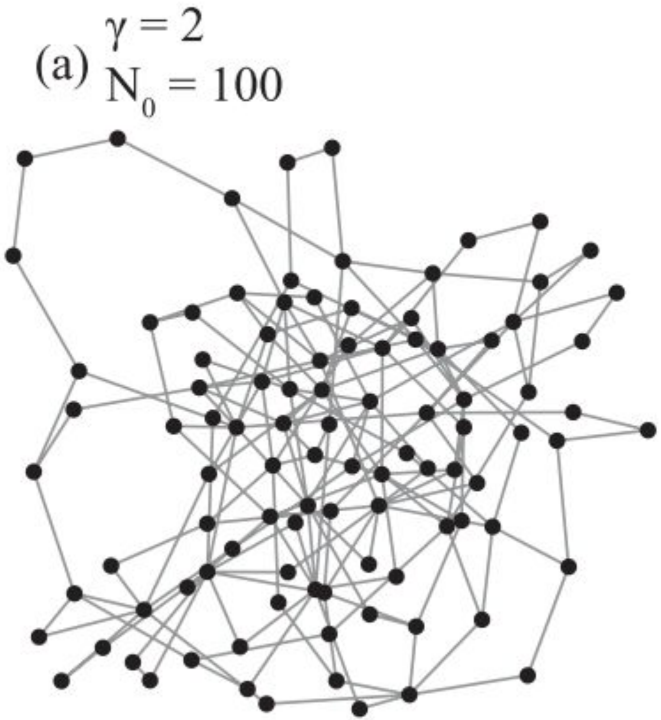
Can anything be presented as a network?

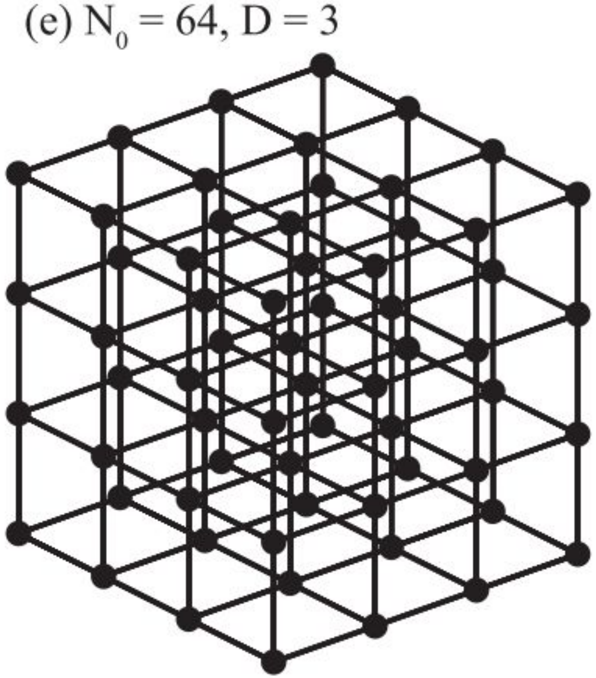
What cannot be presented and why?



Examples of network representations

M.Stella et al.





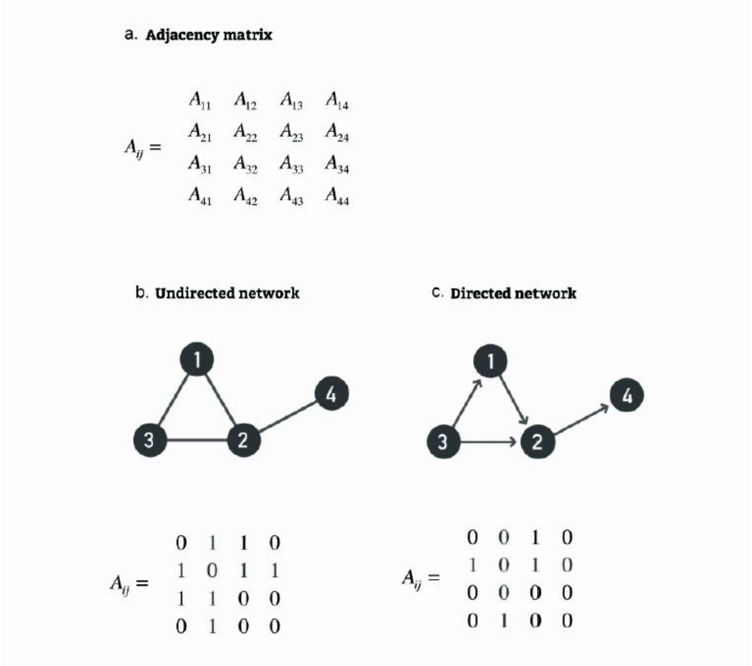
What we will look at in network science?

1. **Network definition and measures**
2. Networks in time and space
3. Networks from data

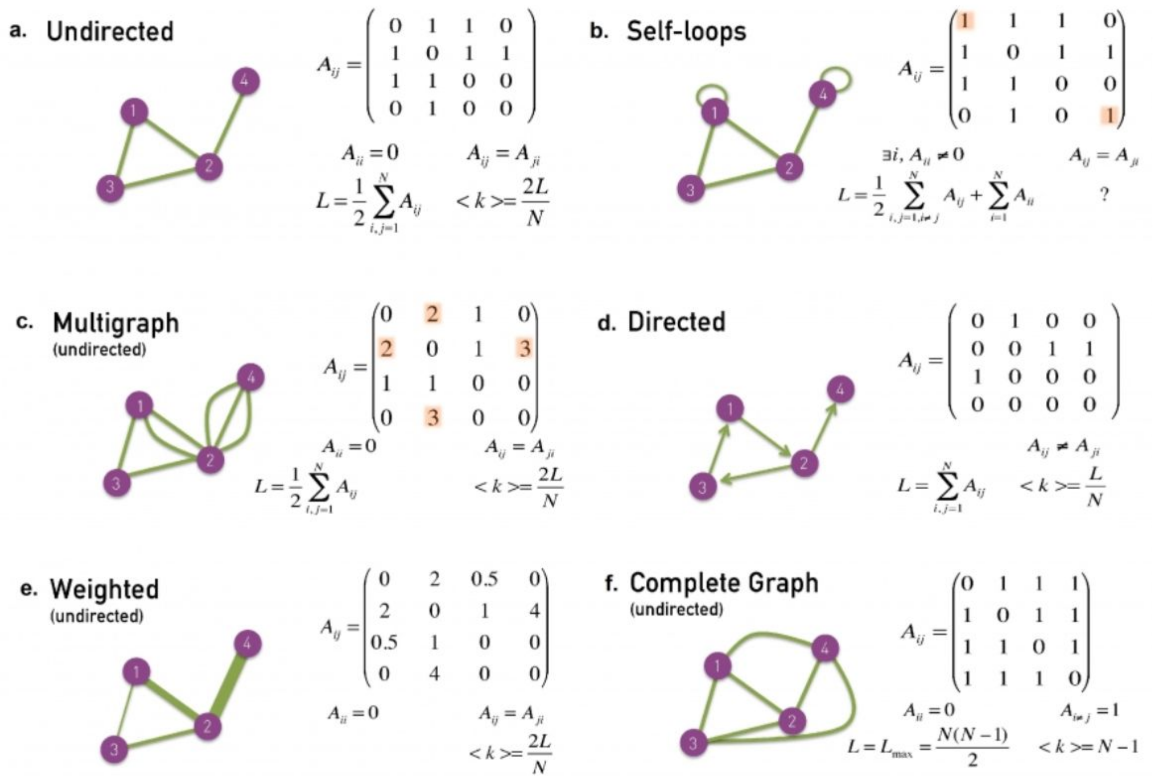
1. Network definitions

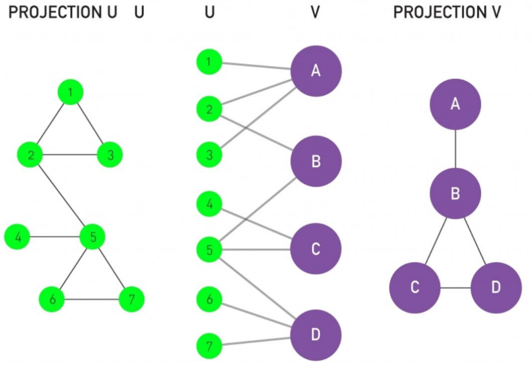
How to represent a network: **edgelist** and **adjacency matrix.**

Adjacency matrix encodes the same information about the network as edgelists.



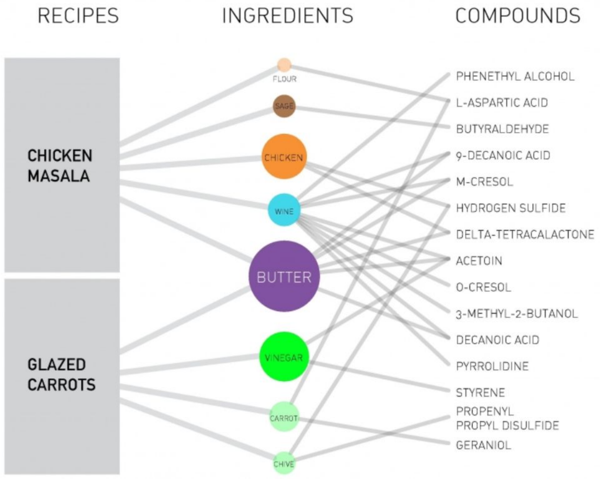
Network types based on links, nodes properties

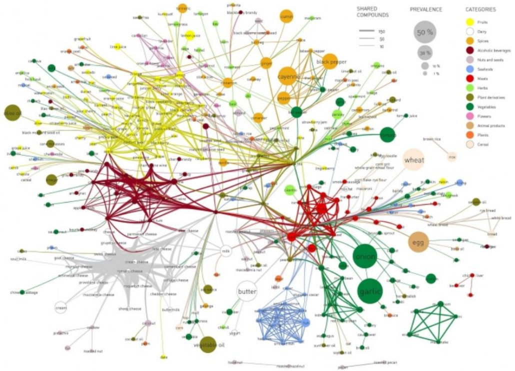


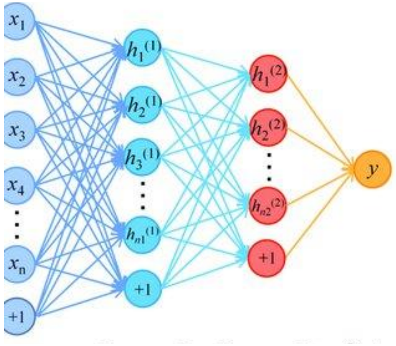


Network types based on links, nodes properties

bipartite networks





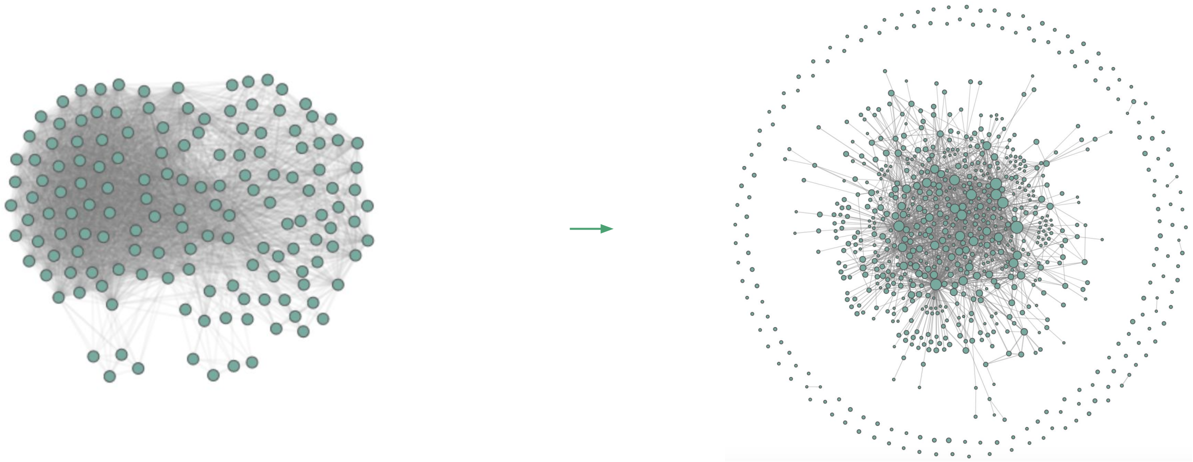


Network measures

Main idea is to characterise their properties.
Local measures for each node (degree)
Global measures for the whole network (**density** - number of links normalised by number of links in a **complete graph)**

Network measures and layout

Local measures for each node. **Global measures** for the whole network.
Layouts of the same network (left, right).



Network statistics

Network statistics

What are nodes with highest centrality?

Size n = 34

Volume m = 78

Loop count l = 0

Triangle count t = 45

Square count q = 154 4-

Maximum degree dmax = 17

Average degree d = 4.588 24

Size of Large Connected Component N = 34

Diameter δ = 5 50

Median distance δM = 2

Mean distance δm = 2.443 26

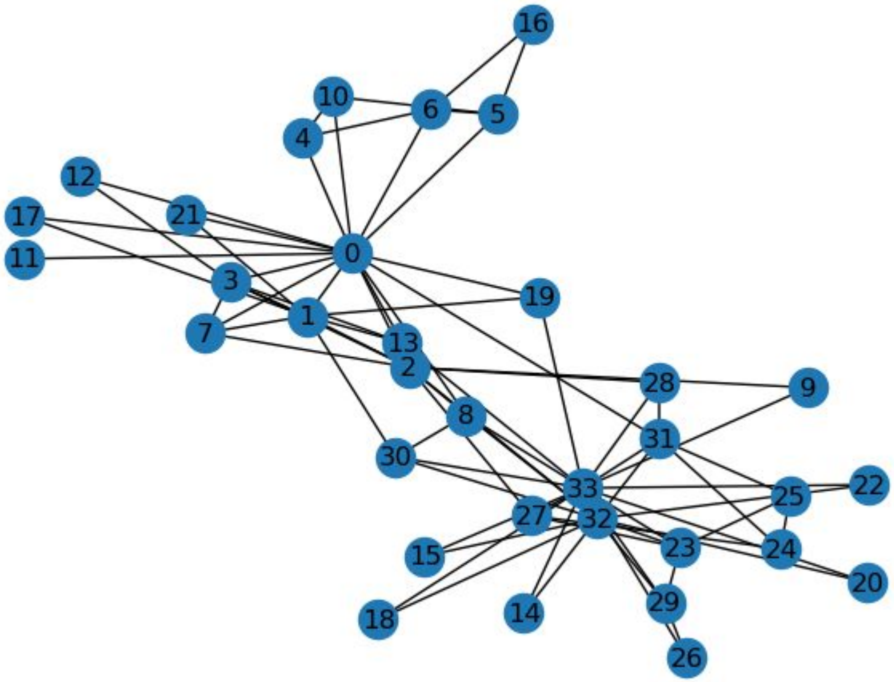
Gini coefficient G = 0.385 370

Power law exponent γ = 1.780 96

Quick check-in

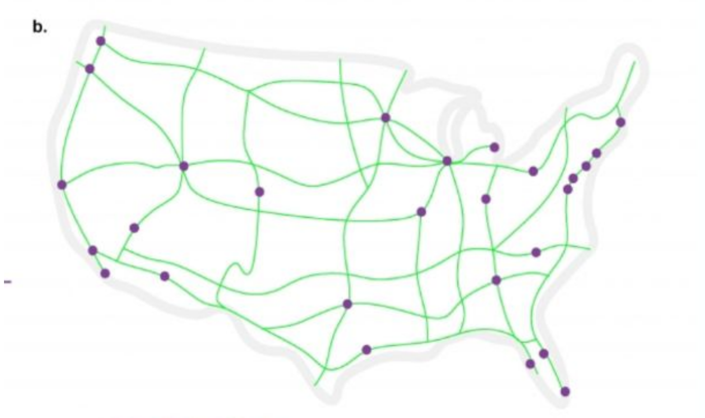
What are network measures for this network?

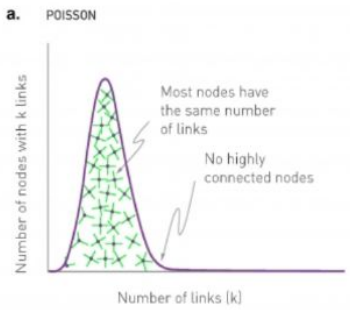
What node would have the highest betweenness centrality? What would be the best spreader?



Network statistics

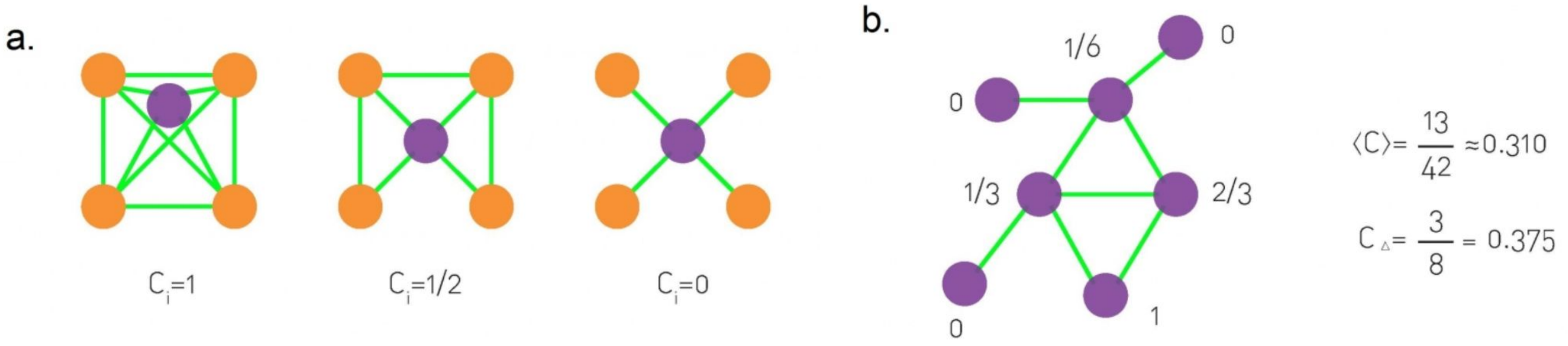
Degree measure - is a local measure to characterise how many nodes each node is connected to.
How to look into degree for N nodes?
Looking into the degree distribution: plotting how many nodes have degree=k.





Network statistics

Clustering coefficient is a measure of the degree to which nodes in a graph tend to cluster together.



Network statistics

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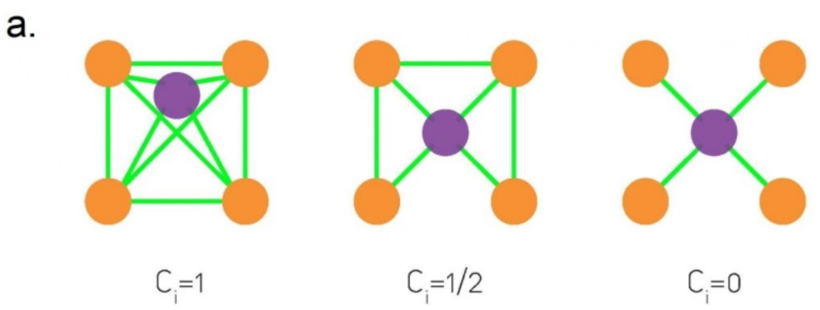


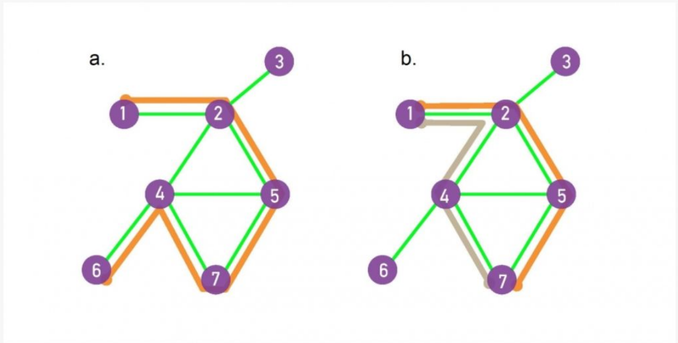


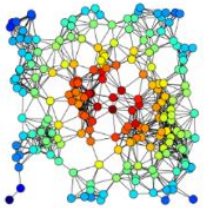
Image 2.12

Paths

- A path between nodes i_0 and i_n is an ordered list of n links P = {(i_0, i_1), (i_1, i_2), (i_2, i_3), ... ,(i_{n-1}, i_n)}. The length of this path is n. The path shown in orange in (a) follows the route 1→2→5→7→4→6, hence its length is n = 5.

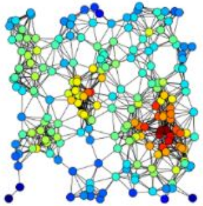
Shortest path finding between nodes are used in many algorithms for networks.
Example path between node 1 and node 6 in a graph is then encoded as a sequence of nodes, e.g. (1,2,5,7,4,6). One of the most known shortest path algorithm is Dijkstra’s algorithm (1956).

Network measures



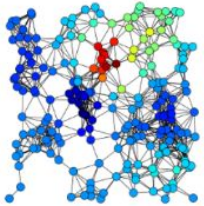
Closeness centrality

B



Degree centrality

D



Katz centrality

F

TABLE 2: Definitions of network science terms and variables.

Term/variable	Definition
N	number of nodes, N, in graph
E	number of edges, E, in graph
network density	ratio of the number of edges to the maximum number of possible edges $\frac{2E}{N(N-1)}$
$d(n_i, n_j)$	shortest path between node i and node j $d(n_i, n_j)$ where $n_i, n_j \in N$
hortest path length, L	average length of shortest path between pairs of nodes $L = \frac{1}{N(N-1)} \cdot \sum_{i \neq j} d(n_i, n_j)$
, D	largest shortest path between nodes $D = \max_{n_i \in N, n_j \in N} d(n_i, n_j)$
centrality	inverse of the sum of the length of the shortest paths between node i and all other nodes in the graph $C_i = \frac{1}{\sum_j d(n_i, n_j)}$
i	number of edges attached to node i
legree, $\langle k \rangle$	average number of edges per node in network $\langle k \rangle = \frac{1}{N} \sum_{n=i}^{N} k_i$
tering coefficient, c_i	number of edges between the neighbors of node i divided by the maximum number of edges between those neighbors $c_i = \frac{2\lvert e_{jk} \rvert}{k_i(k_i - 1)}$ where $n_j, n_k \in N_i$, $e_{jk} \in E$
lustering coefficient, $\langle C \rangle$	average clustering coefficient of nodes in the network $\langle C \rangle = \frac{1}{N} \sum_{n=i}^{N} c_i$
ity, Q	proportion of edges that fall within subgroups of nodes minus the expected proportion if edges were randomly distributed, range $[-1, 1]$
fficiency, E_G	measure of how efficiently information is exchanged in the network $E_G = \frac{1}{n(n-1)} \sum_{i \neq j \in N} \frac{1}{d(n_i, n_j)}$
nnected component	largest group of nodes in the network that are connected to each other in a single component
stribution, $P(k)$	probability distribution of node degrees in the network
	power-law exponent for the degree distribution
rld structure	network with short average path lengths and relatively high clustering coefficient (relative to a random graph with similar density)
network	network with a degree distribution that is power-law distributed

Network measures

Most of the measures can be estimated directly using networkx python library.

Table 2: Definitions of network science terms and variables.

Term/variable	Definition
N	number of nodes, N, in graph
E	number of edges, E, in graph
network density	ratio of the number of edges to the maximum number of possible edges $\frac{2E}{N(N-1)}$
distance, $d(n_i, n_j)$	shortest path between node i and node j $d(n_i, n_j)$ where $n_i, n_j \in N$
average shortest path length, L	average length of shortest path between pairs of nodes $L = \frac{1}{N(N-1)} \cdot \sum_{i \neq j} d(n_i, n_j)$
diameter, D	largest shortest path between nodes $D = \max_{n_i \in N, n_j \in N} d(n_i, n_j)$
closeness centrality	inverse of the sum of the length of the shortest paths between node i and all other nodes in the graph $C_i = \frac{1}{\sum_j d(n_i, n_j)}$
degree, k_i	number of edges attached to node i
average degree, $\langle k \rangle$	average number of edges per node in network $\langle k \rangle = \frac{1}{N} \sum_{n=1}^{N} k_i$
local clustering coefficient, c_i	number of edges between the neighbors of node i divided by the maximum number of edges between those neighbors $c_i = \frac{2\|e_{jk}\|}{k_i(k_i - 1)}$ where $n_j, n_k \in N_i$, $e_{jk} \in E$
average clustering coefficient, $\langle C \rangle$	average clustering coefficient of nodes in the network $\langle C \rangle = \frac{1}{N} \sum_{n=i}^{N} c_i$
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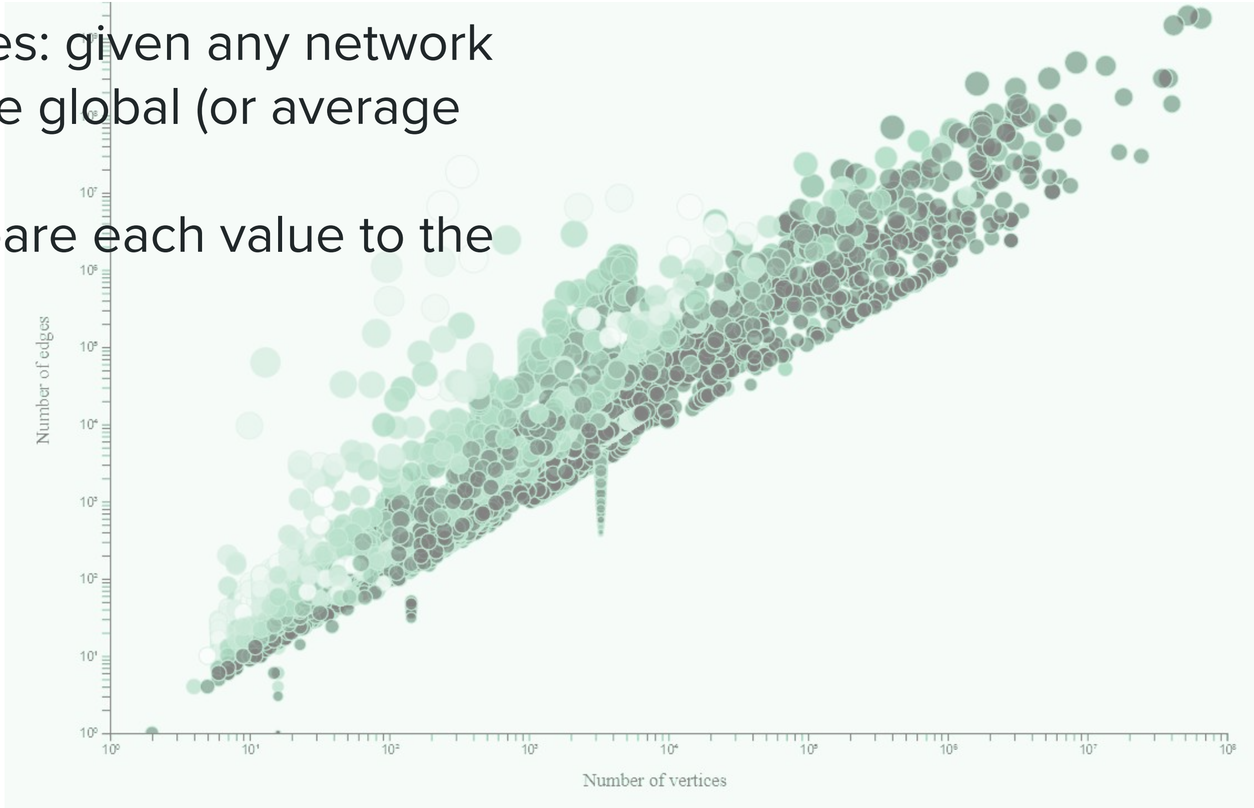
average efficiency, E_G	measure of how efficiently information is exchanged in the network $E_G = \frac{1}{n(n-1)} \sum_{i \neq j \in N} \frac{1}{d(n_i, n_j)}$
largest connected component	largest group of nodes in the network that are connected to each other in a single component
degree distribution, $P(k)$	probability distribution of node degrees in the network
γ	power-law exponent for the degree distribution
Small world structure	network with short average path lengths and relatively high clustering coefficient (relative to a random graph with similar density)
scale-free network	network with a degree distribution that is power-law distributed

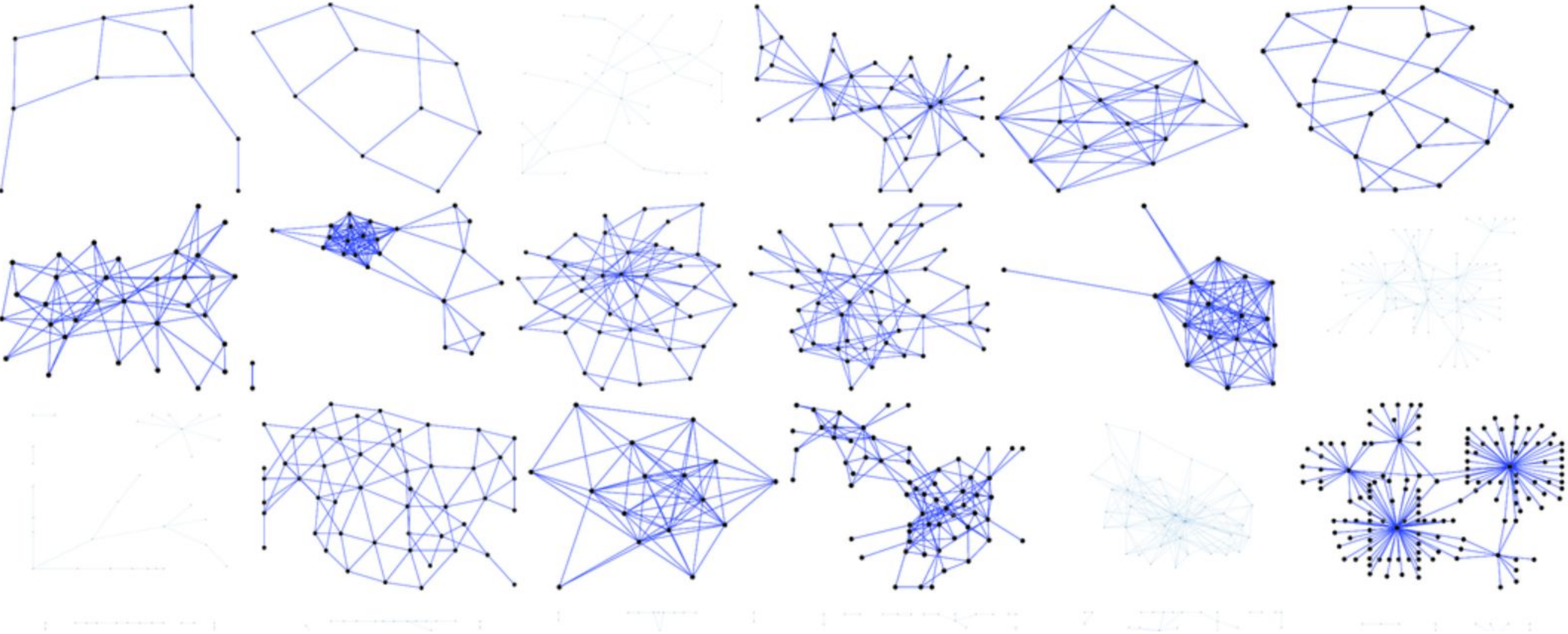
Choosing a network to consider

Global networks measures: given any network centrality we can estimate global (or average network measure).
The main idea is to compare each value to the average.



[Datasets]()

Random networks



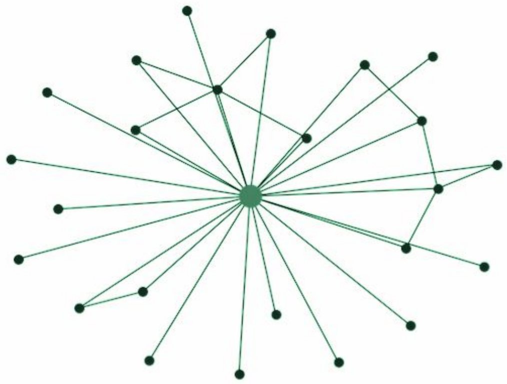
Random networks: model by Erdős (1913-1996) and Rényi (1921-1970)

1. Create N nodes
2. Connect each pair of N labeled nodes with probability p. You can do it yourself by tossing a coin each time.

Corresponding class in networkx:

```
G_er = nx.erdos_renyi_graph(n, p2)
```

Random networks: model by Erdős (1913-1996) and Rényi (1921-1970)



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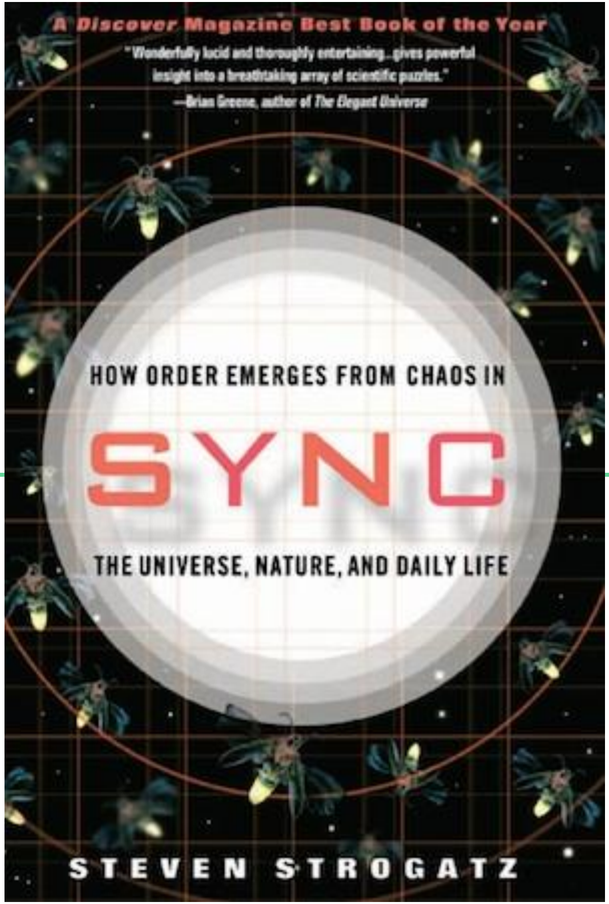
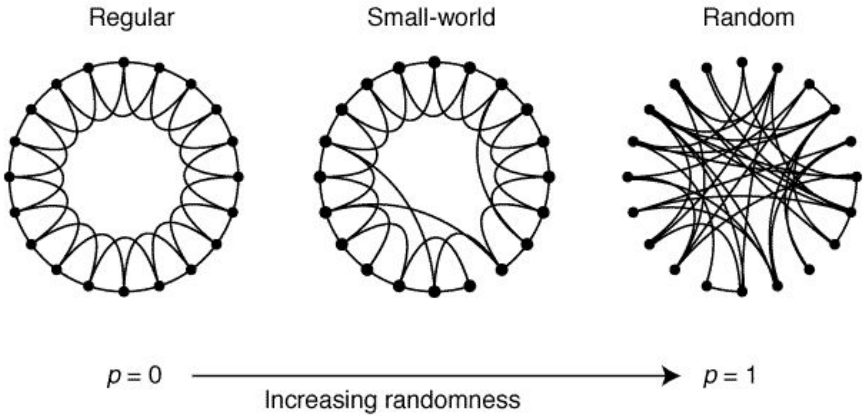
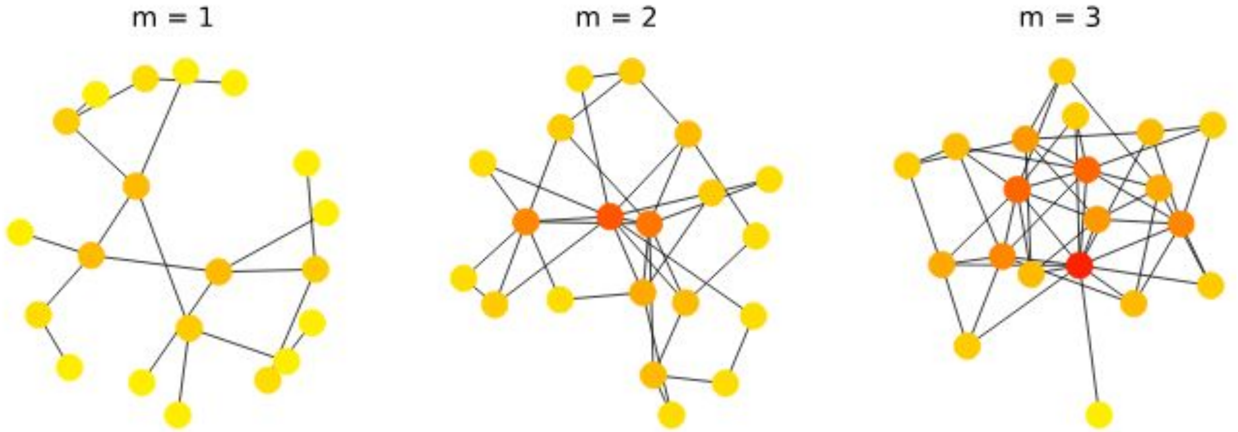
G_er = nx.erdos_renyi_graph(n, p2)
```

Random networks: Watts-Strogatz network





Random networks: model by Barabasi Albert



A graph of
nodes is grown by attaching new nodes each with m edges that are preferentially attached to existing nodes with high degree.

A. L. Barabási and R. Albert "Emergence of scaling in random networks", Science, 1999.

Random networks: model Barabasi Albert

Network G on N nodes and m edges preferential attachment.
Model by Barabasi and Albert creates a random network with algorithm:

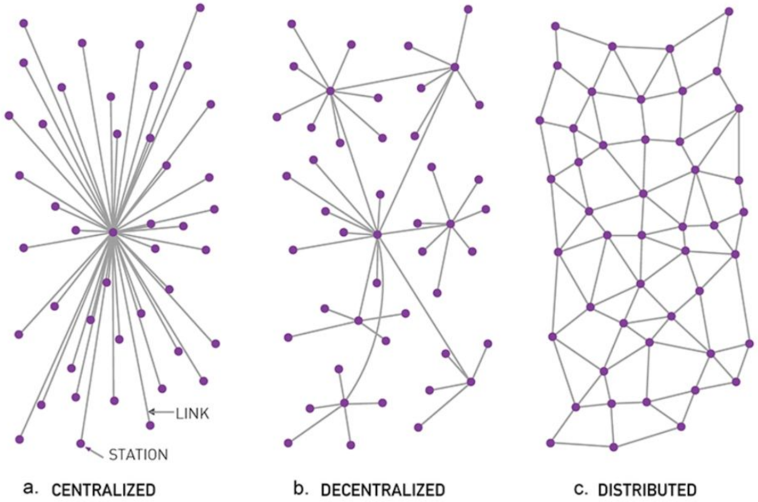
```
1. Create starting nodes
2. Connecta new node with m edges to existing
nodes
3. Repeat (2.) x times for all non existing
nodes
```

Network and robustness

Network science requires intuition, e.g. how to construct a network, such that it would have a specific property, e.g robustness, or particular distribution?

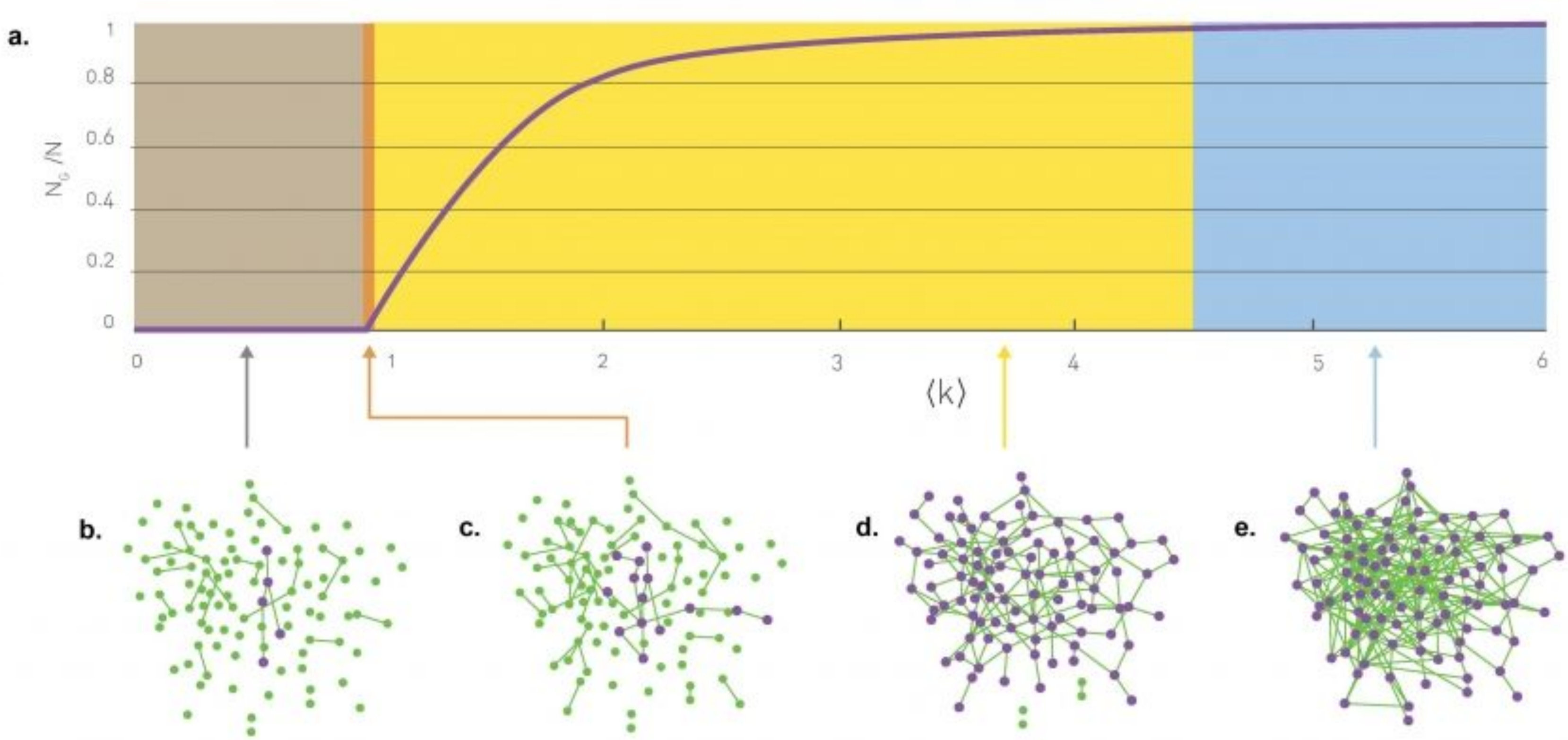
Are random networks robust?

Fig. credits P. Barran



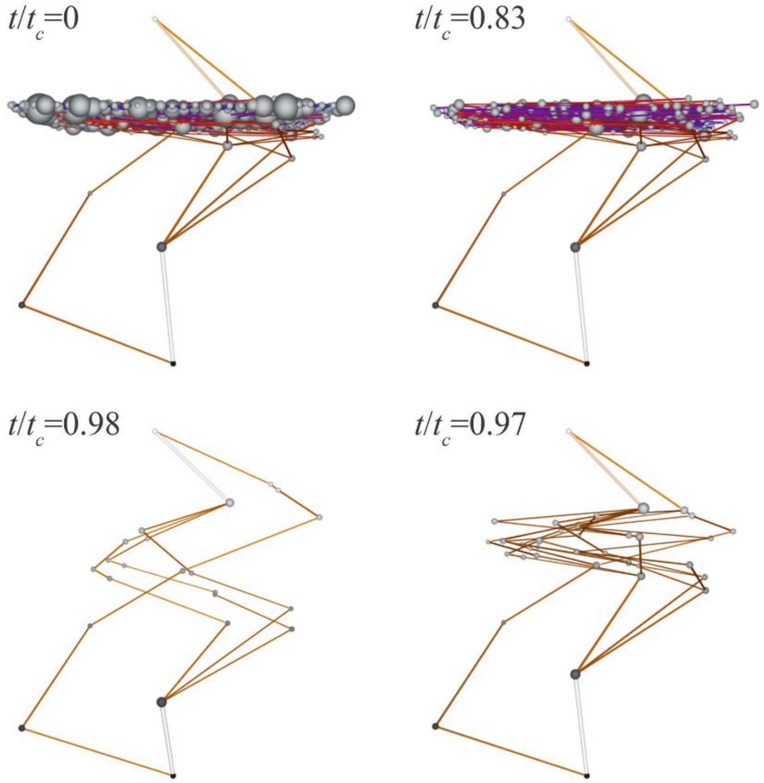
Example of a universal law at the collective scale

Emergence of a giant component in a network above a threshold of number of links

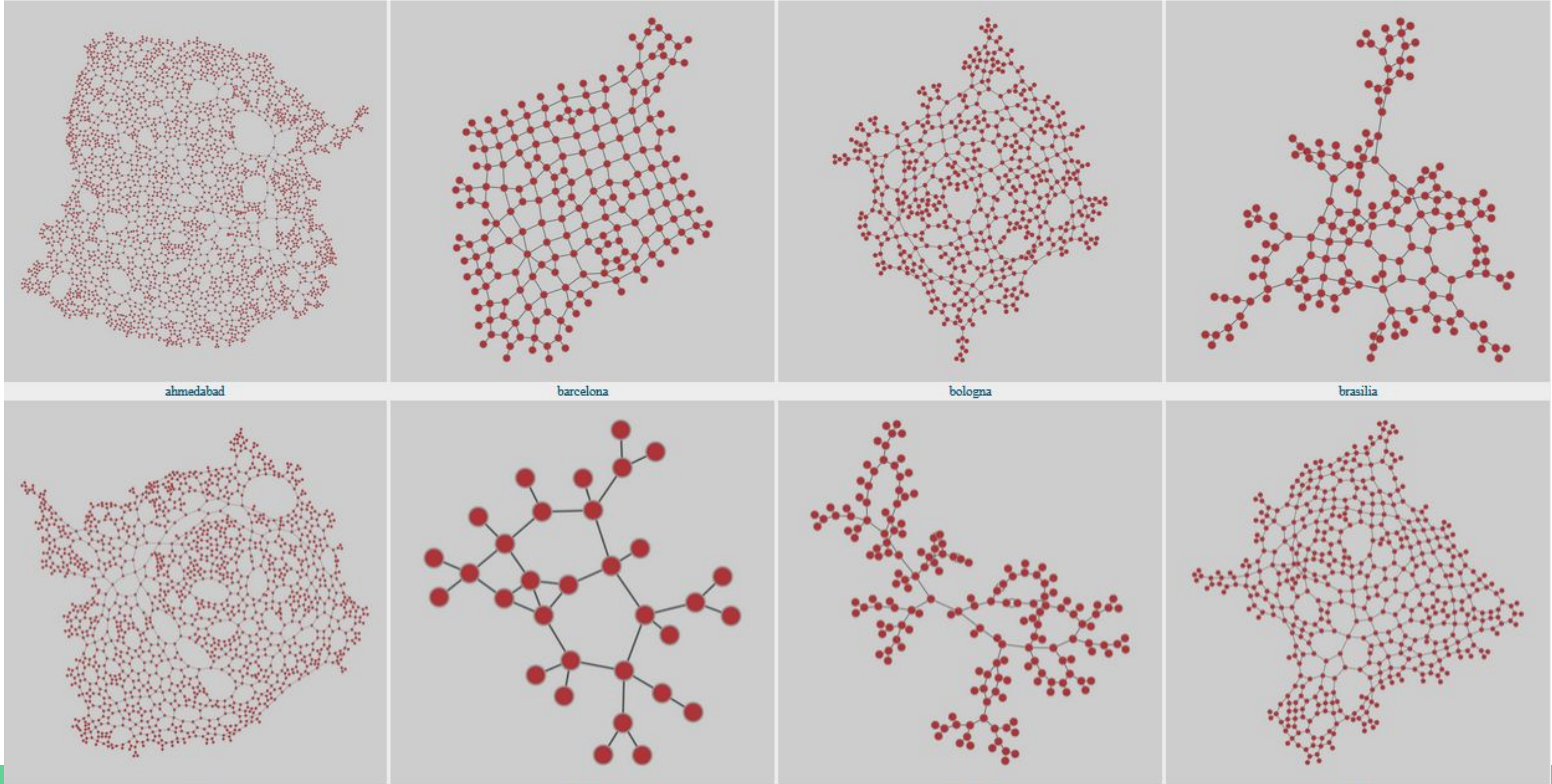


Networks in time and space

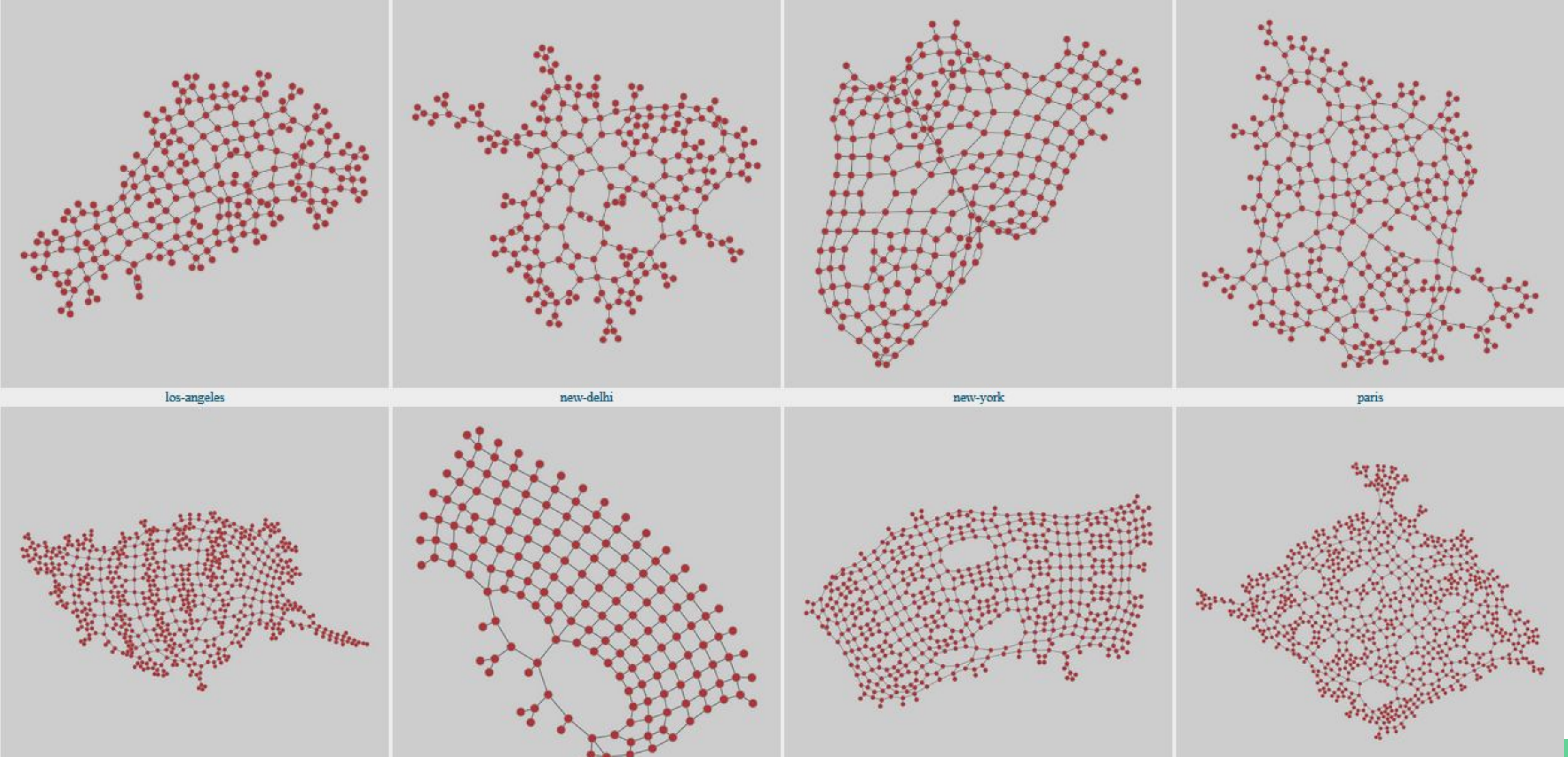
Percolation of networks in time
[Nat.Comm. 2020]



Networks in time and space



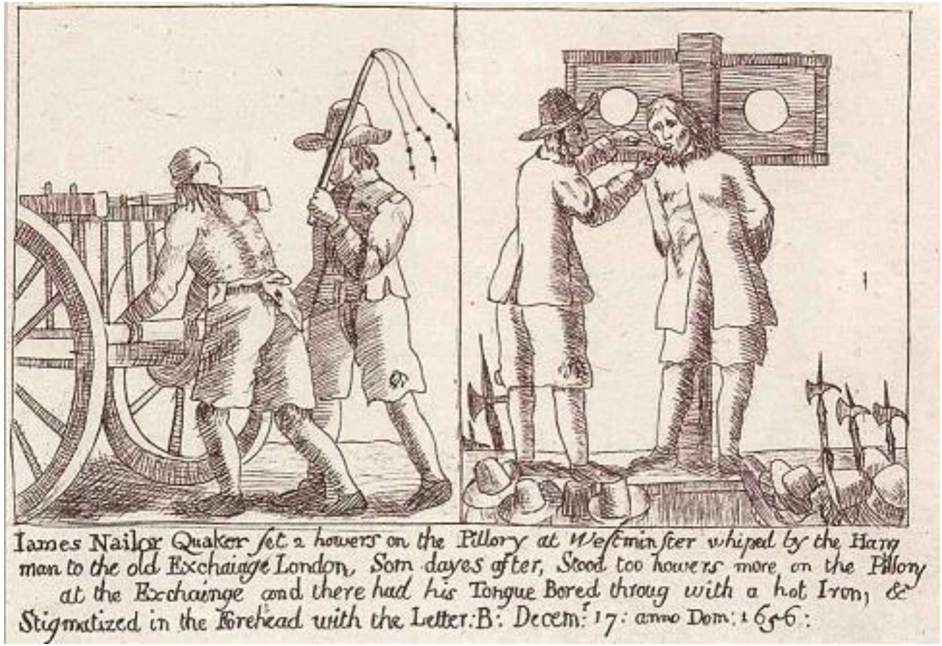
Networks in time and space



Quick check-in

Can networks tell a new story about your data?

Typical examples:
Quakers, people who belong to a historically Protestant Christian set of denominations known formally as the Religious Society of Friends.



Hypergraphs

Non-binary interactions in nature

Higher-order motif analysis in hypergraphs

Quintino Francesco Lotito, Federico Musciotto, Alberto Montresor & Federico Battiston

Communications Physics **5**, Article number: 79 (2022) | Cite this article

Abstract

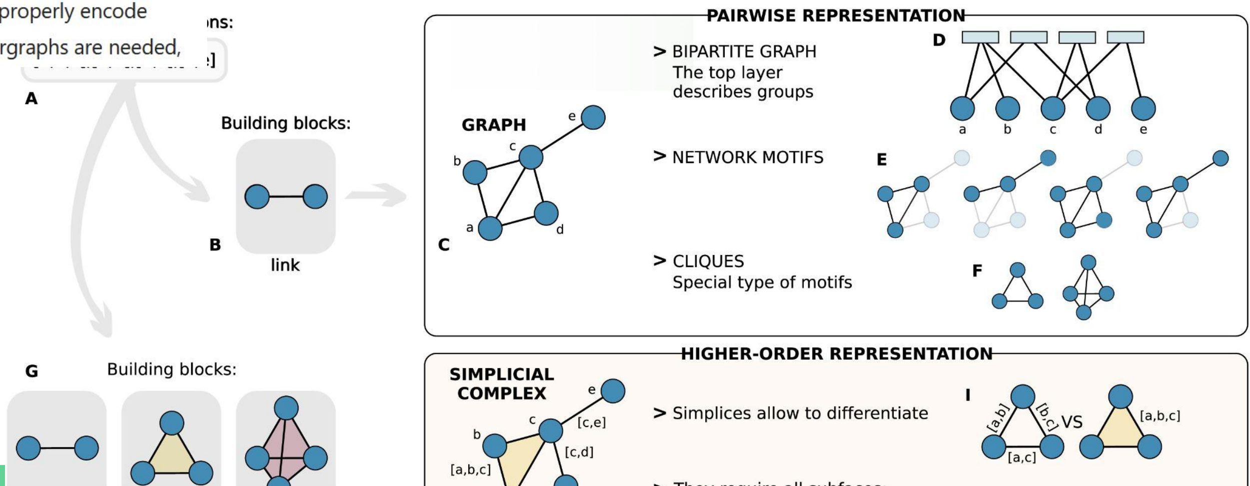
A deluge of new data on real-world networks suggests that interactions among system units are not limited to pairs, but often involve a higher number of nodes. To properly encode higher-order interactions, richer mathematical frameworks such as hypergraphs are needed,

https://doi.org/10.1038/s42005-021-00710-4 OPEN

Detecting informative higher-order interactions in statistically validated hypergraphs

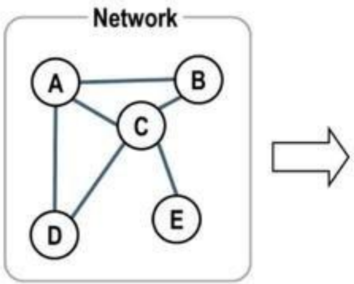
Federico Musciotto [1✉], Federico Battiston [2] & Rosario N. Mantegna [1,3]

Recent empirical evidence has shown that in many real-world systems, successfully represented as networks, interactions are not limited to dyads, but often involve three or more agents at a time. These data are better described by hypergraphs, where hyperlinks encode higher-order interactions among a group of nodes. In spite of the extensive literature on



Main idea and main problematics

Graphs operations, **binary** relations vs.   Hyper-graphs operations **higher-order** relations



Connectivity Matrix

	A	B	C	D	E
A	0	1	1	1	0
B	1	0	1	0	0
C	1	1	0	1	1
D	1	0	1	0	0
E	0	0	1	0	0

Simple Connectivity Matrix

C2

	A	B	C	D	E
A	3	1	2	1	1
B	1	2	1	2	1
C	2	1	4	1	0
D	1	2	1	2	1
E	1	1	0	1	1

=

C1

	A	B	C	D	E
A	0	1	1	1	0
B	1	0	1	0	0
C	1	1	0	1	1
D	1	0	1	0	0
E	0	0	1	0	0

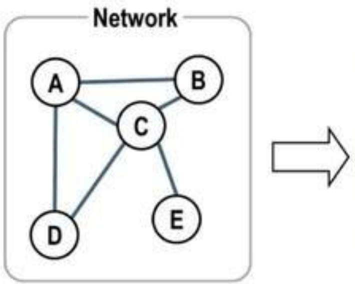
X

C1

	A	B	C	D	E
A	0	1	1	1	0
B	1	0	1	0	0
C	1	1	0	1	1
D	1	0	1	0	0
E	0	0	1	0	0

Main idea and main problematics

Graphs operations, **binary** relations vs. Hyper-graphs operations **higher-order** relations



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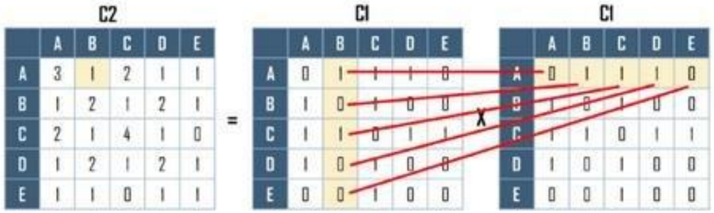
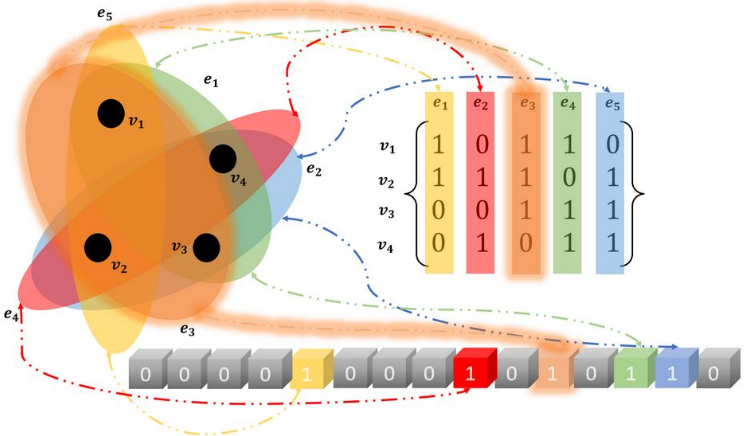
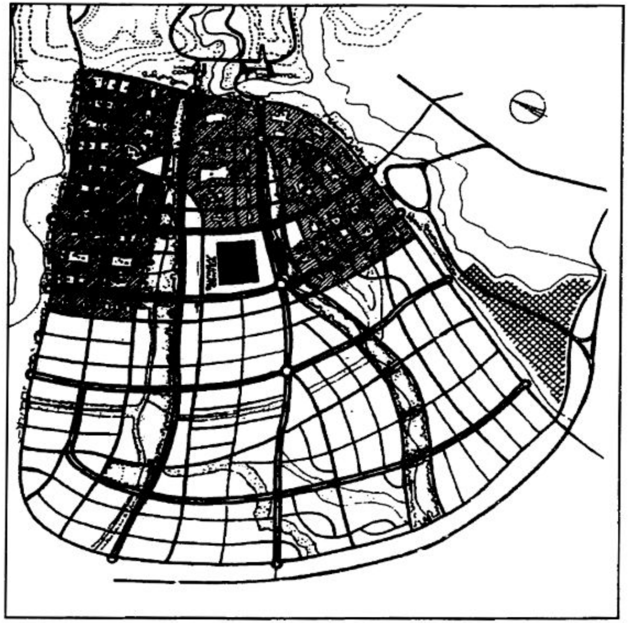
Simple Connectivity Matrix



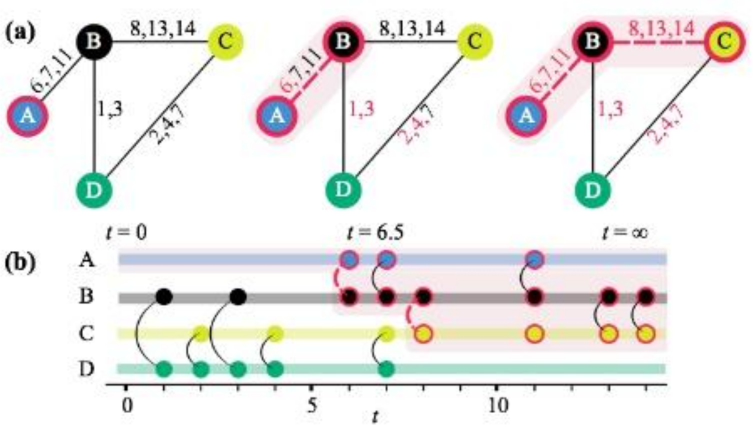


Networks in time and space

Master Plan for Chandigarh by Albert Mayer RAIC Journal, 1955 (Evenson Norma, Chandigarh, 1966)

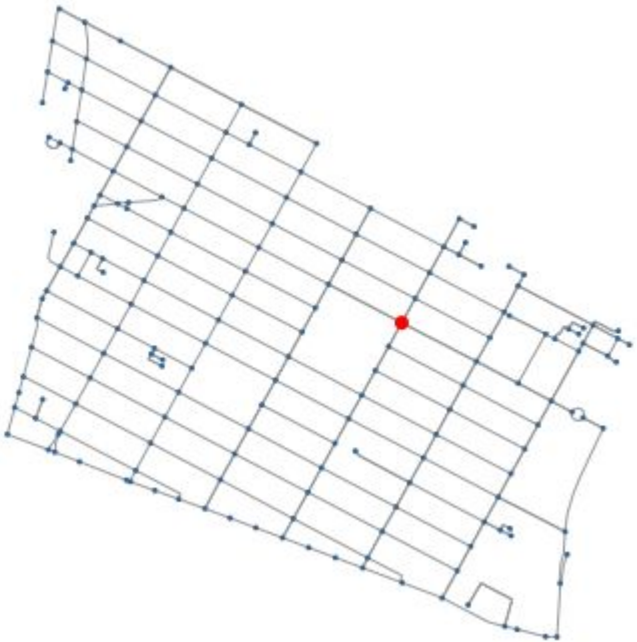


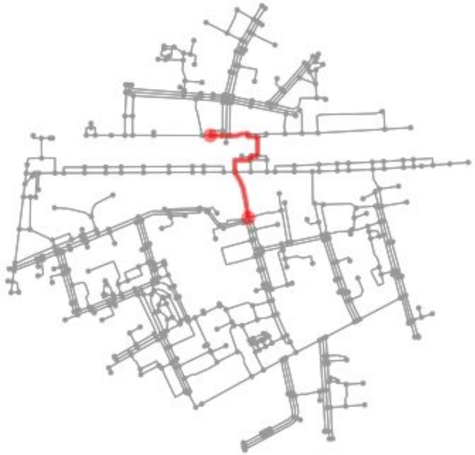
Temporality matters: reachability issue



Review Holme-Saramaki, Phys. Rep. (2012), arXiv:1108.1780

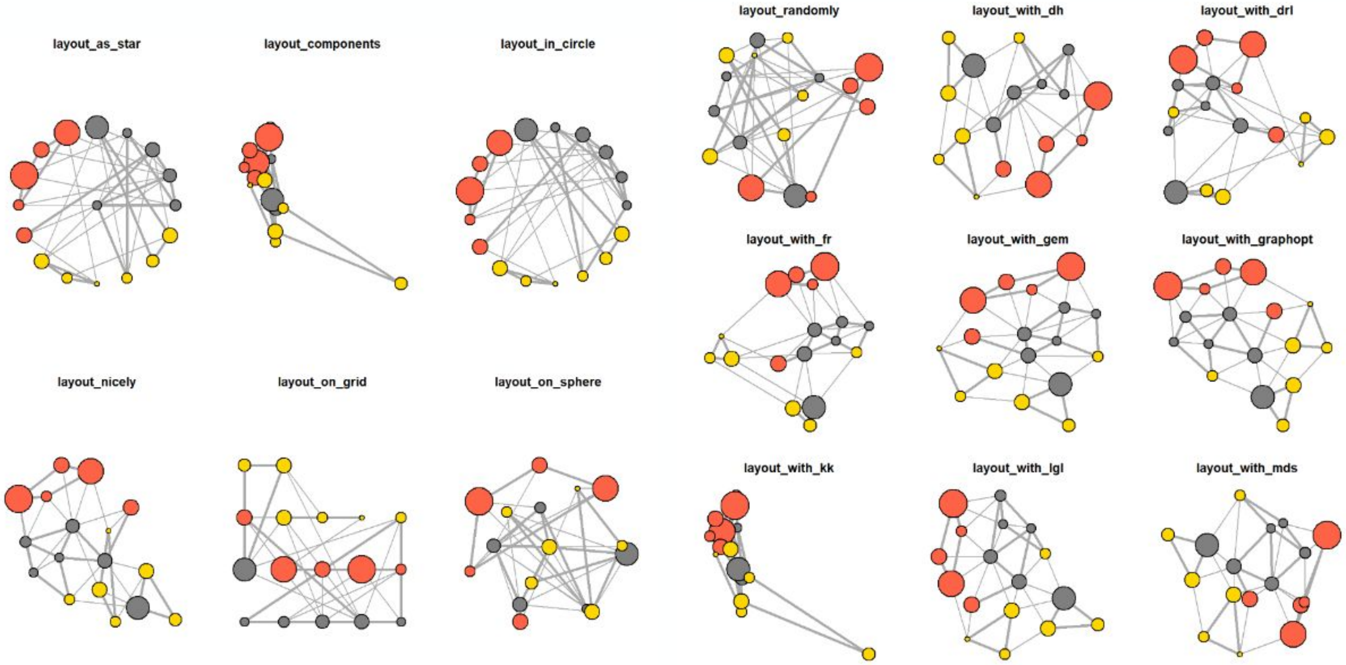
Networks in time and space





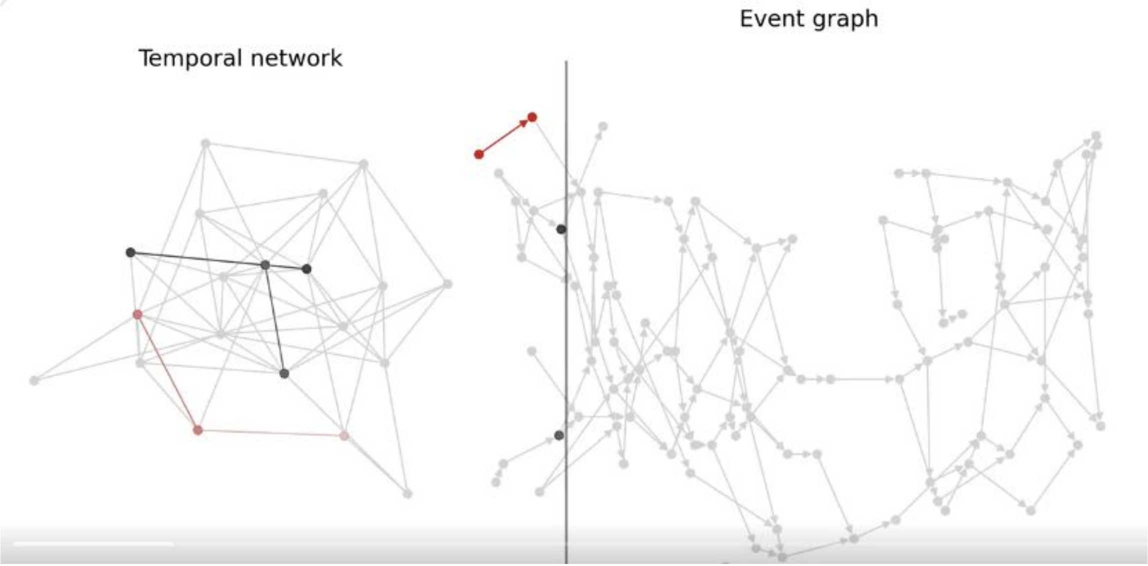
Osmnx for spatial networks analysis
https://arxiv.org/abs/1010.0302

Networks layout



Networks sonification

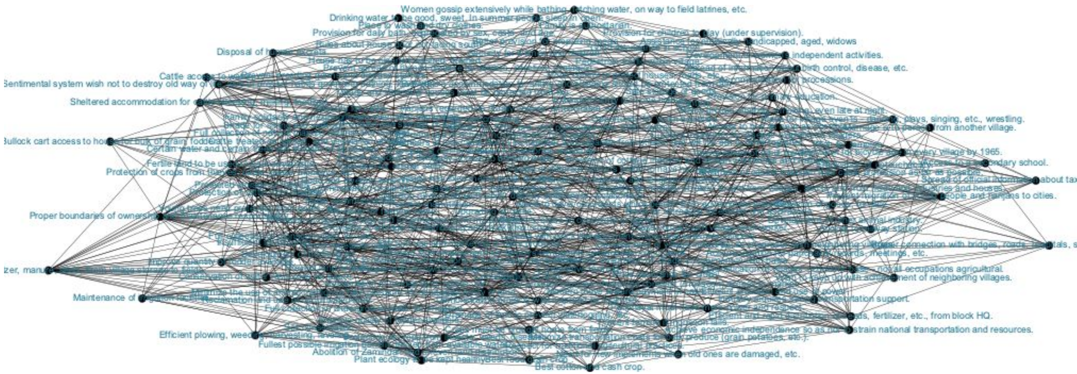
Directed percolation in temporal networks PRR 2022



Networks in time and space

Good resource on spatial networks

M.Barthelemy “Spatial networks”



Good resource on temporal networks

P.Holme, J.Saramaki “Temporal networks”

Holme blog https://petterhol.me/

What we will look at in network science?

1. Network measures and network types
2. Networks in time and space
3. Networks from data

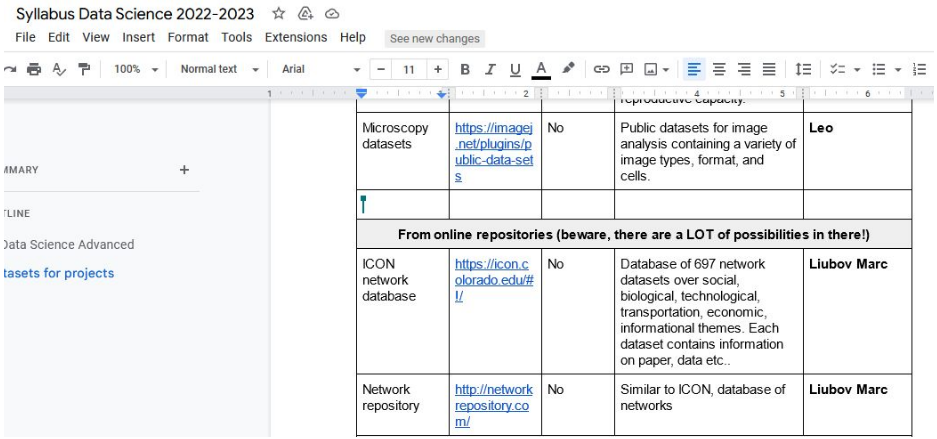
Figure 7.11

Aaron Koblin's *Flight Patterns* (2005): visualization of the flight paths of aircraft crossing North America

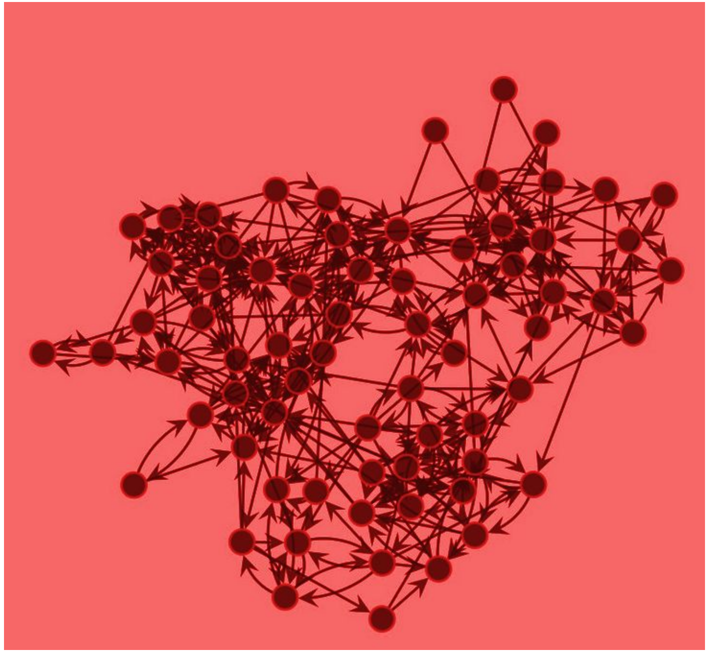
Where can I get network data?

Example:
Highschool: Illinois high school students (1958). A network of friendships among male students in a small high school in Illinois from 1958. 70 nodes, 366 edges.
https://networks.skewed.de/net/highschool

Example:
Facebook or wikipedia data
https://snap.stanford.edu/data/wiki-meta.html



Microscopy datasets	https://imagej.net/plugins/public-data-sets	No	Public datasets for image analysis containing a variety of image types, format, and cells.	**Leo**
From online repositories (beware, there are a LOT of possibilities in there!)				
ICON network database	https://icon.colorado.edu/#!/	No	Database of 697 network datasets over social, biological, technological, transportation, economic, informational themes. Each dataset contains information on paper, data etc..	**Liubov Marc**
Network repository	http://networkrepository.com/	No	Similar to ICON, database of networks	**Liubov Marc**



Social networks analysis

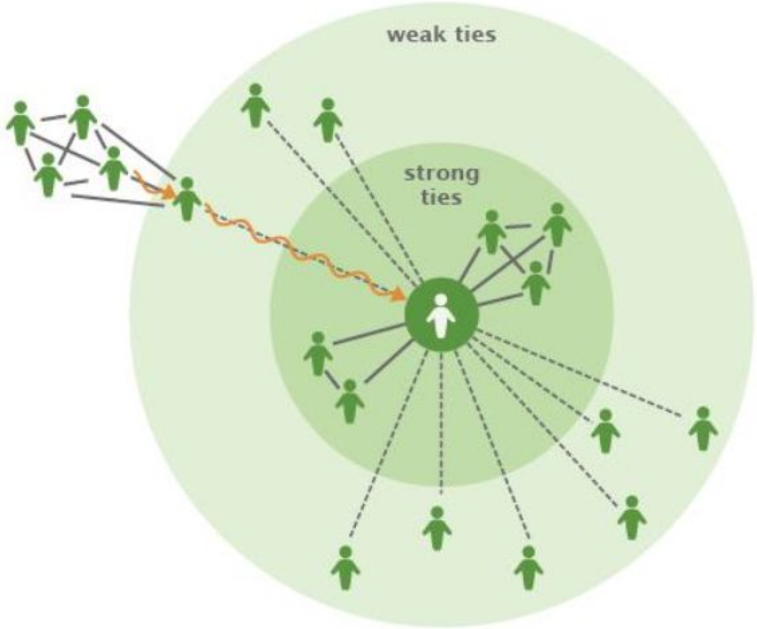
The Strength of Weak Ties[1]

Mark S. Granovetter
Johns Hopkins University

Analysis of social networks is suggested as a tool for linking micro and macro levels of sociological theory. The procedure is illustrated by elaboration of the macro implications of one aspect of small-scale interaction: the strength of dyadic ties. It is argued that the degree of overlap of two individuals' friendship networks varies directly with the strength of their tie to one another. The impact of this principle on diffusion of influence and information, mobility opportunity, and community organization is explored. Stress is laid on the cohesive power of weak ties. Most network models deal, implicitly, with strong ties, thus confining their applicability to small, well-defined groups. Emphasis on weak ties lends itself to discussion of relations *between* groups and to analysis of segments of social structure not easily defined in terms of primary groups.

A fundamental weakness of current sociological theory is that it does not relate micro-level interactions to macro-level patterns in any convincing way. Large-scale statistical, as well as qualitative, studies offer a good deal of insight into such macro phenomena as social mobility, community organization, and political structure. At the micro level, a large and increasing body of data and theory offers useful and illuminating ideas about what transpires within the confines of the small group. But how interaction in small groups aggregates to form large-scale patterns eludes us in most cases.

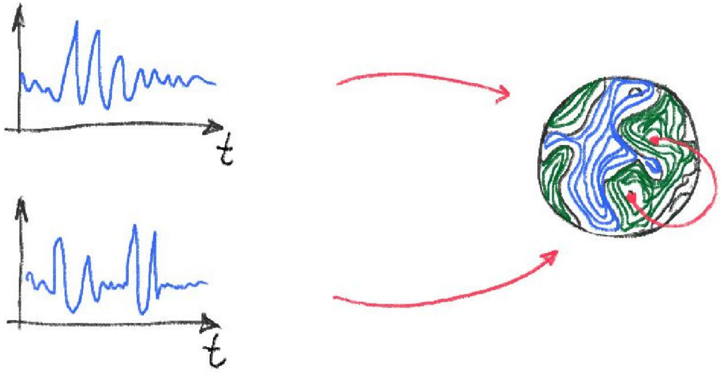
I will argue, in this paper, that the analysis of processes in interpersonal networks provides the most fruitful micro-macro bridge. In one way or another, it is through these networks that small-scale interaction becomes



How to construct networks from data?

1. Directly build correspondence between links and edges with data (social networks, flights data)
2. Preprocess data (first build correlation from data)

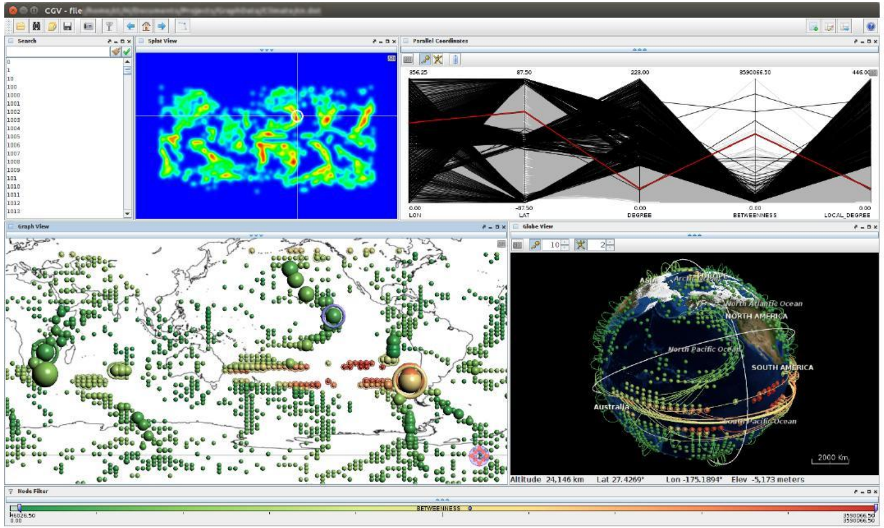
(example of project from www.pik-potsdam.de Climate networks)

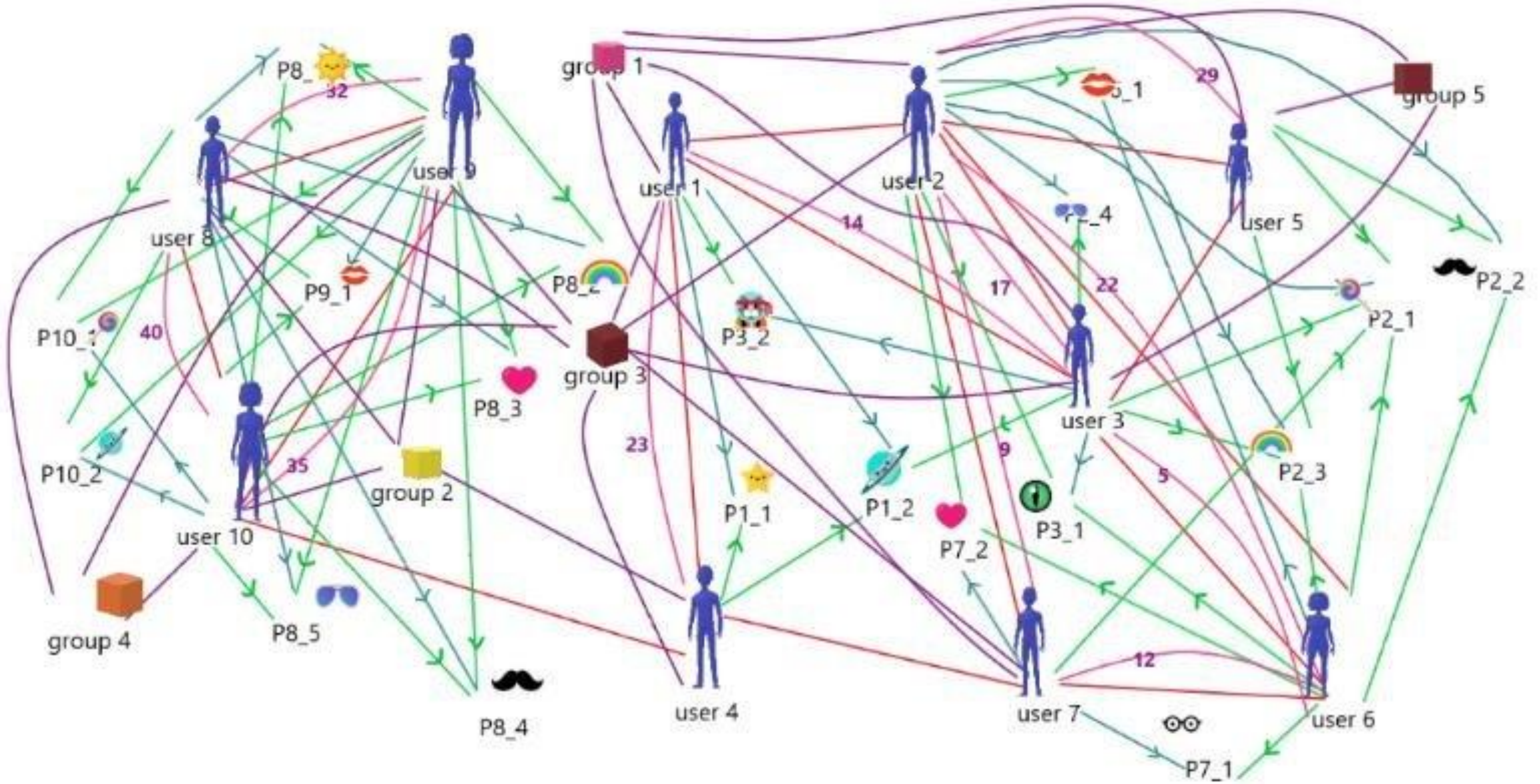


How to construct networks from data?

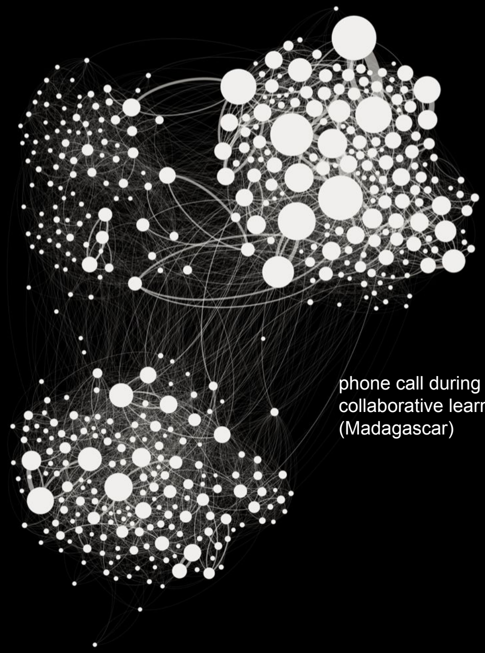
1. Directly build correspondence between links and edges with data (social networks, flights data)
2. **Preprocess data (first build correlation from data)**

   Working with data from big systems

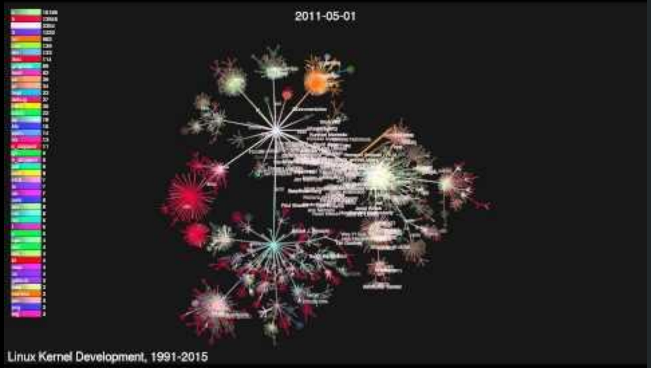




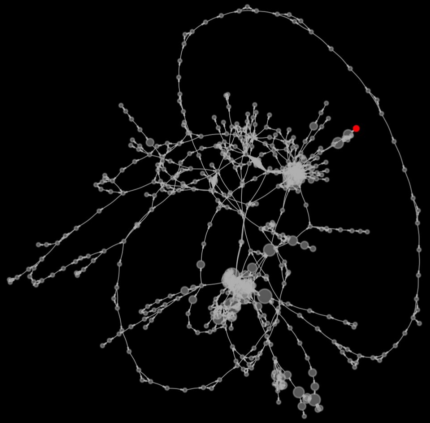
group 1
user 1
user 2
group 5
user 5
user 9
user 8
P8_2
P9_1
P10_1
P10_2
P3_2
group 3
P8_3
group 2
user 10
group 4
P8_5
P8_4
user 4
P1_1
P1_2
P7_2
P3_1
user 3
P2_1
P2_2
P2_3
user 7
user 6
P7_1
32
40
35
23
14
17
22
29
9
5
12



phone call during
collaborative learning
(Madagascar)



Linux collaborations



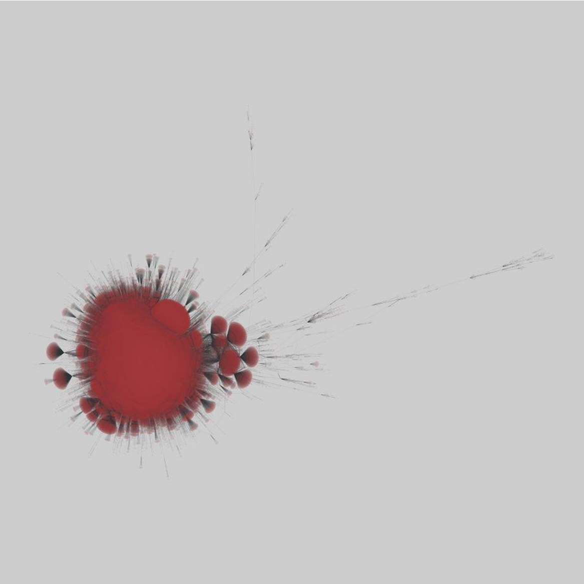
Activity network of a
construction project

Organisational networks
growth process
structure vs performance
contagion models
resilience
community structure

Network visualisaion

Can hairy ball be a good visualisation?
Discussions for class on visualisations

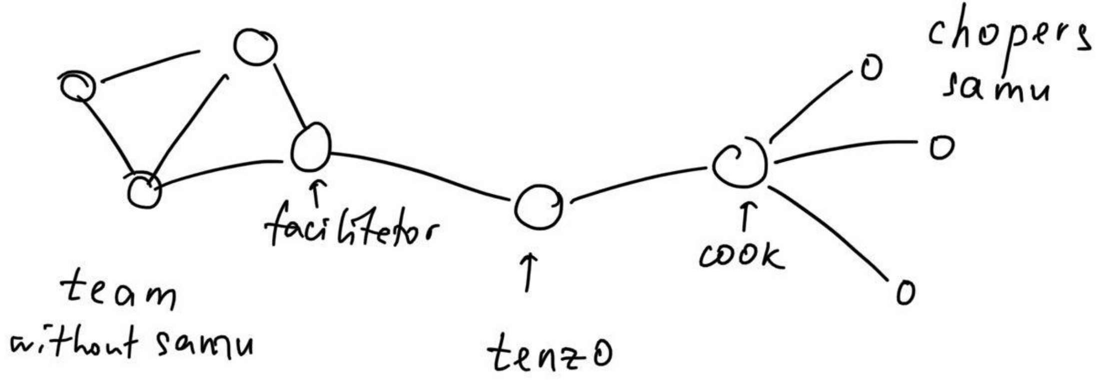
Linux kernel mailing list. A bipartite network of contributions by users to threads on the Linux kernel mailing list. 379554 nodes, 1565683 edges. https://networks.skewed.de/net/lkml_thread



Hands-on session

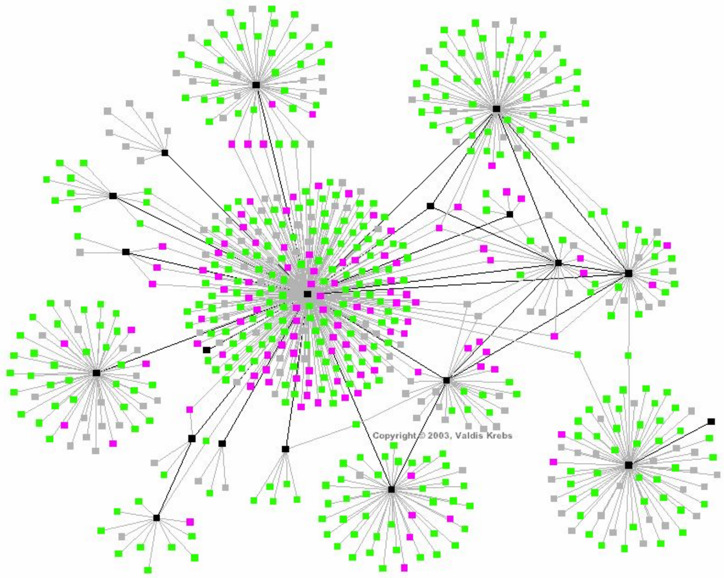
1. How to generate your own network?
2. How to create networks using models?
3. How to load your network using network-dataset?
4. How to study your data using the network representation of data?
5. What if the data is not in the network format? Preparing network format.
6. What are ways to create statistics and geometry on your newly represented data as a network?

Analysis of communities



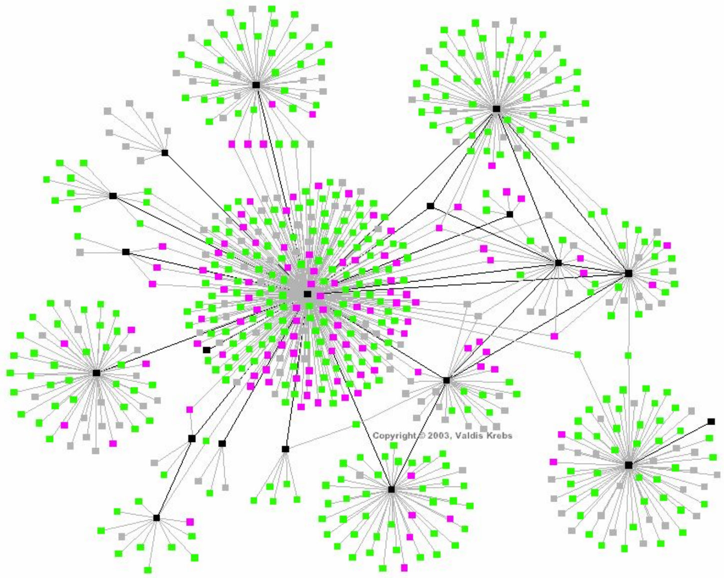
What we will look at in network science?

1. Network measures
2. Networks in time and space
3. Networks from data



What are spreading processes on networks?

Any examples of spreading processes?



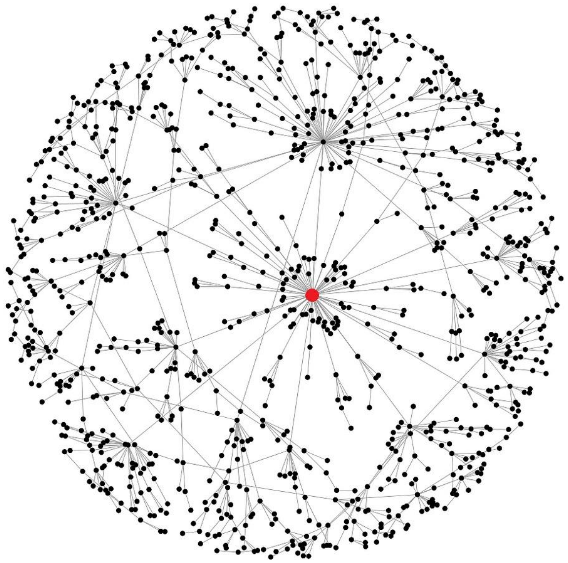
How to construct networks from data?

1. Decide what are links and nodes in your data?
2. Preprocess data (first build correlation from data)

Examples: social networks, flights data in the practical part

How to construct networks from data?

1. Directly build correspondence between links and edges with data (social networks, flights data)
2. Preprocess data (first build correlation from data)



How to construct networks from data?

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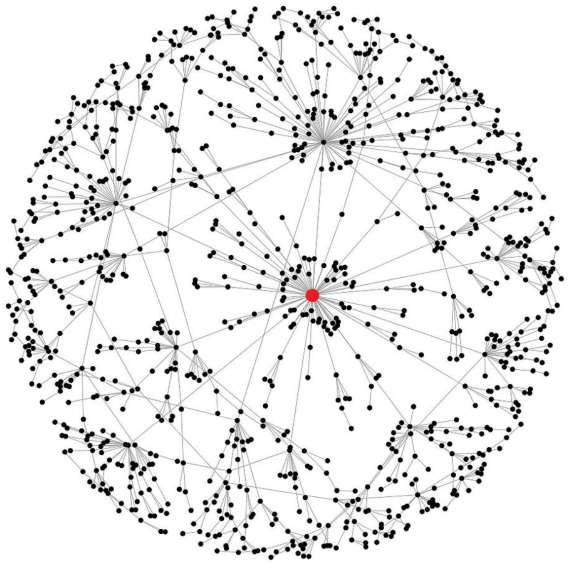
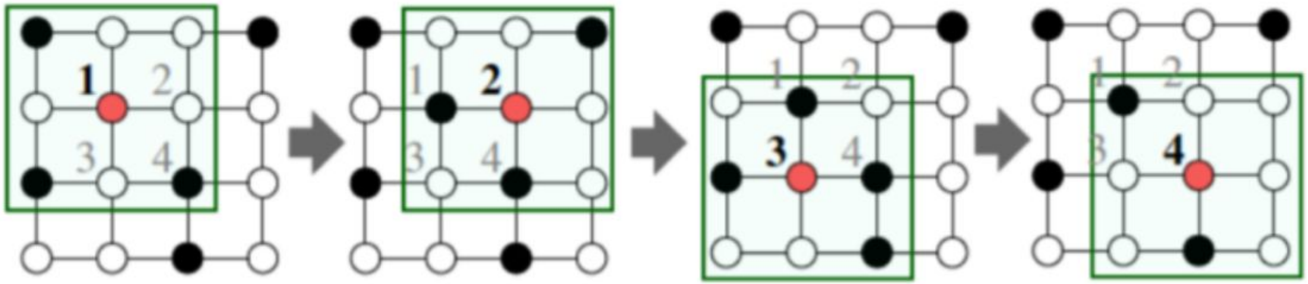


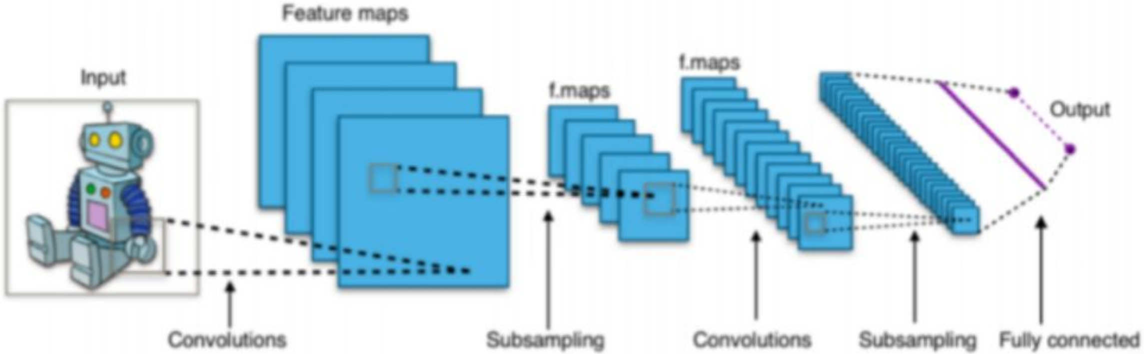
Remember that networks do not give one-to-one Correspondence of your data.
Hence do not generalize

Algorithms on graphs (examples like GNNs)

The most fundamental part of GNN is a Graph. In computer science, a **graph is a data structure consisting of two components: [nodes](https://neptune.ai/blog/graph-neural-network-and-some-of-gnn-applications) (vertices) and edges**. A graph ***G*** can be defined as ***G = (V, E)***, where ***V*** is the set of nodes, and ***E*** are the edges between them

https://neptune.ai/blog/graph-neural-network-and-some-of-gnn-applications





Network resources

http://networkrepository.com/networks.php
http://networksciencebook.com/chapter/3#advanced-b

Network resources

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http://networksciencebook.com/chapter/3#advanced-b

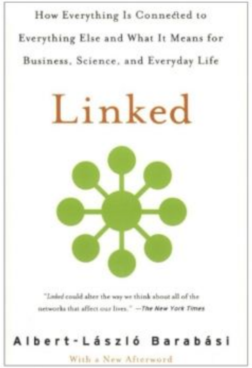
Spatial Networks

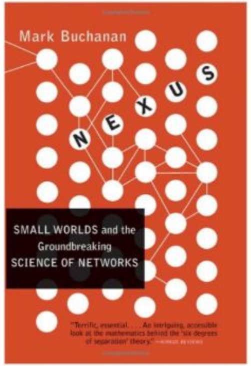
Marc Barthélemy*

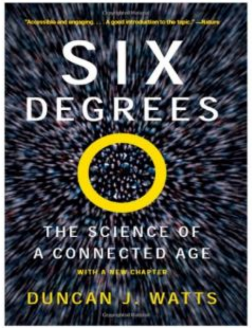
Institut de Physique Théorique, CEA, IPhT CNRS, URA 2306 F-91191 Gif-sur-Yvette France and Centre d'Analyse et de Mathématique Sociales (CAMS, UMR 8557 CNRS-EHESS) Ecole des Hautes Etudes en Sciences Sociales, 54 bd. Raspail, F-75270 Paris Cedex 06, France.

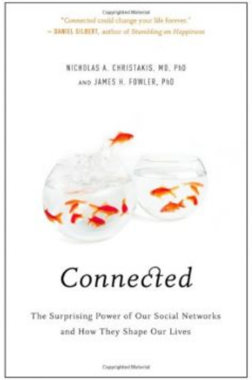
Complex systems are very often organized under the form of networks where nodes and edges are embedded in space. Transportation and mobility networks, Internet, mobile phone networks, power grids, social and contact networks, neural networks, are all examples where space is relevant and where topology alone does not contain all the information. Characterizing and understanding the structure and the evolution of spatial networks is thus crucial for many different fields ranging from urbanism to epidemiology. An important consequence of space on networks is that there is a cost associated to the length of edges which in turn has dramatic effects on the topological structure of these networks. We will expose thoroughly the current state of our understanding of how the spatial constraints affect the structure and properties of these networks. We will review the most recent empirical observations and the most important models of spatial networks. We will also discuss various processes which take place on these spatial networks, such as phase transitions, random walks, synchronization, navigation, resilience, and disease spread.

Contents









Jupyter notebooks and resources

https://classroom.google.com/u/0/w/NTQ0NTIwOTczMTQ2/t/all

GNNs

Why GNNs?

For more resources:

https://towardsdatascience.com/an-introduction-to-graph-neural-network-gnn-for-analysing-structured-data-afce79f4cfdc

GNNs

Traditional Graph Analysis Methods

Traditional methods are mostly algorithm-based, such as :

1. searching algorithms, e.g. BFS, DFS
2. shortest path algorithms, e.g. Dijkstra’s algorithm, Nearest Neighbour
3. spanning-tree algorithms, e.g. Prim’s algorithm
4. clustering methods, e.g. Highly Connected Components, k-mean

GNNs

Graph Neural Network, as how it is called, is a neural network that can directly be applied to

graphs. It provides a convenient way for node level, edge level, and graph level prediction task.

There are mainly three types of graph neural networks in the literature:

Recurrent Graph Neural Network

Spatial Convolutional Network

Spectral Convolutional Network

1	1	1	0
1	1	1	0
1	1	1	1
0	0	1	1

Adjacency matrix (A)

1	0	0
0	1	0
0	0	1
0	1	1

Feature matrix (X)