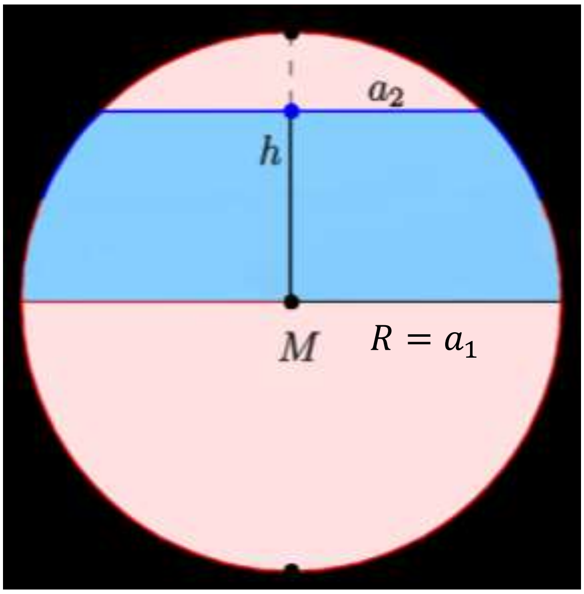
Alternative Lösung zu 4.2 (siehe Wikipedia Kugelschicht)

Lösungsschritte:

1. Obere blaue Mantelfläche berechnen
2. Obere Kreisfläche berechnen
3. Alles x2 nehmen



$$A_{Gesamt} = 2 \cdot A_{Mantelfläche} + 2 \cdot A_{Kreis}$$

$$r = \sqrt{h \cdot (2 \cdot R - h)}$$

$$A_{Mantelfläche} = 2 \cdot \pi \cdot R \cdot h = 2 \cdot \pi \cdot h \cdot \sqrt{a_1^{\,2} + \left(\frac{a_1^{\,2} - a_2^{\,2} - h^2}{2 \cdot h}\right)^2}$$

$$r = \sqrt{15\,mm \cdot (2 \cdot 45\,mm - 15\,mm)} = 33{,}54\,mm$$

$$A_{Mantelfläche} = 2 \cdot \pi \cdot 30\,mm \cdot \sqrt{(45\,mm)^2 + \left(\frac{(45\,mm)^2 - (33{,}54\,mm)^2 - (30\,mm)^2}{2 \cdot 30\,mm}\right)^2} = 8482\,mm^2$$

$$A_{Kreis} = \pi \cdot r^2 = \pi \cdot (h \cdot (2 \cdot R - h))^2 = \pi \cdot \left(\sqrt{15\,mm \cdot (2 \cdot 45\,mm - 15\,mm)}\right)^2 = 3534\,mm^2$$

$$A_{Gesamt} = 2 \cdot 8482\,mm^2 + 2 \cdot 3534\,mm^2 = 24032\,mm^2$$

5.1

$$c^2 = a^2 + b^2 - 2 \cdot a \cdot b \cdot \cos(\gamma)$$

$$c = \sqrt{a^2 + b^2 - 2 \cdot a \cdot b \cdot \cos(\gamma)} = \sqrt{(120\,m)^2 + (180\,m)^2 - 2 \cdot 120\,m \cdot 180\,m \cdot \cos(\gamma)} = 137{,}95\,m \approx 138\,m$$

$$\frac{a}{c} = \frac{\sin(\alpha)}{\sin(\gamma)}$$

$$\sin(\alpha) = \frac{a \cdot \sin(\gamma)}{c} = \frac{120\,m \cdot \sin(50)}{138\,m} = 0{,}6661 \rightarrow \alpha = 41{,}77\,°$$

$$\beta = 180\,° - \alpha - \gamma = 180\,° - 41{,}77\,° - 50\,° = 88{,}23\,°$$

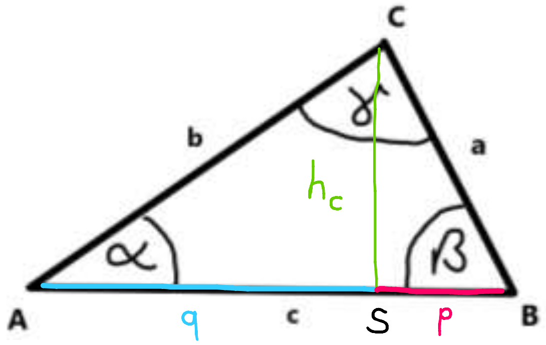
5.2

$$A_{\Delta ABC} = 0{,}5 \cdot a \cdot b \cdot \sin(\gamma)$$

$$A_{\Delta ABC} = 0{,}5 \cdot 120\,m \cdot 180\,m \cdot \sin(50) = 8273\,m^2$$

Alternative Lösung zu 5.2

$$A_{\Delta ABC} = A_{\Delta ASC} + A_{\Delta SBC}$$



$A_{\Delta ASC}$:

$$\sin(\alpha) = \frac{h_c}{b}$$

$$h_c = \sin(\alpha) \cdot b = \sin(41{,}77) \cdot 180\,m = 119{,}9\,m \approx 120\,m$$

$$b^2 = q^2 + h_c^2$$

$$q = \sqrt{b^2 - h_c^2} = \sqrt{(180\,m)^2 - (120\,m)^2} = 134{,}16\,m \approx 134\,m$$

$$A_{\Delta ASC} = 0{,}5 \cdot q \cdot h_c = 0{,}5 \cdot 134\,m \cdot 120\,m = 8040\,m^2$$

$A_{\Delta SBC}$:

$$p = c - q = 138\,m - 134\,m = 4\,m$$

$$A_{\Delta SBC} = 0{,}5 \cdot p \cdot h_c = 0{,}5 \cdot 4\,m \cdot 120\,m = 240\,m^2$$

$$A_{\Delta ABC} = 8040\,m^2 + 240\,m^2 = 8280\,m^2$$

6.
$$P(x)=P_0\cdot \mathrm{e}^{-k\cdot x}$$

$$\rightarrow P(x)=755\cdot e^{-0{,}05\cdot x}$$

6.1
$$P(x)=755\cdot e^{-0{,}05\cdot 120}=1{,}87$$

6.2
$$5=755\cdot e^{-0{,}05\cdot x}$$

$$\frac{1}{151}=e^{-0{,}05\cdot x}$$

$$\ln\left(\frac{1}{151}\right)=-0{,}05\cdot x$$

$$x=\frac{\ln\left(\frac{1}{151}\right)}{-0{,}05}=100{,}35$$

6.3
$$5^x+5^{x+2}=8$$

$$5^x+5^x\cdot 5^2=8$$

$$5^x+5^x\cdot 25=8$$

$$26\cdot 5^x=8$$

$$5^x=\frac{4}{13}$$

$$x=\log_5\left(\frac{4}{13}\right)=-0{,}7323$$