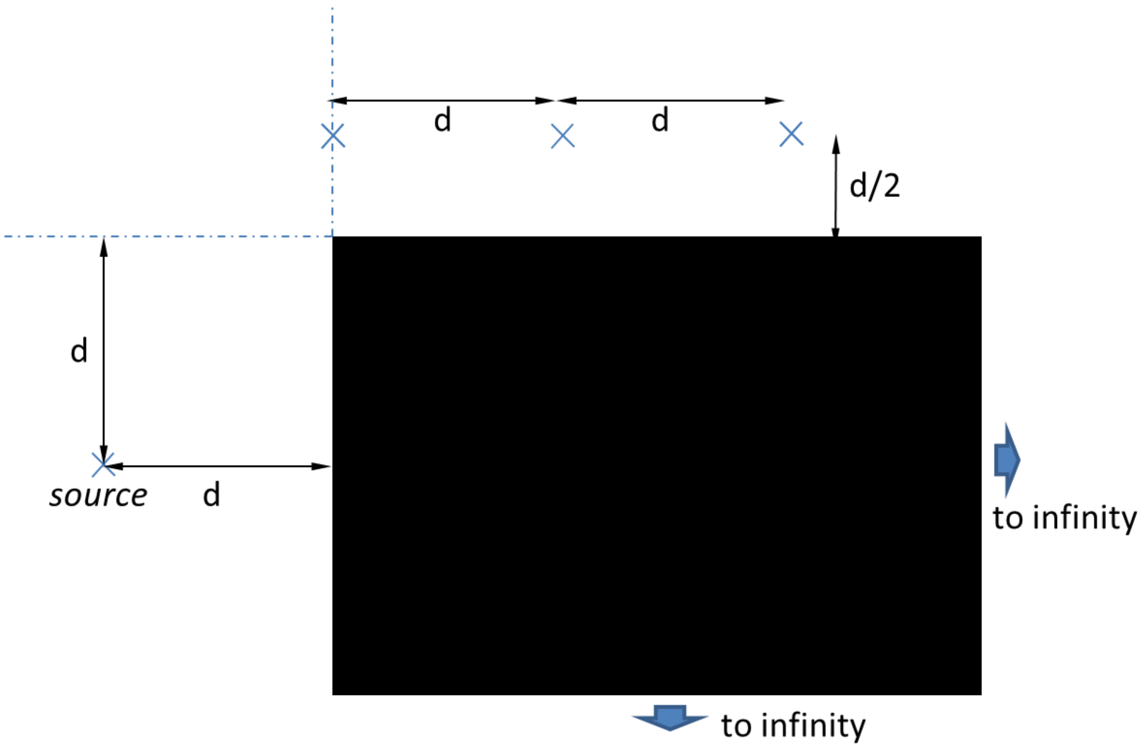
Computational solution of wave problems
Project: FDTD simulation of diffraction at a 90-degree wedge

Goal:

This project aims at providing insight in the finite difference time domain (FDTD) technique with particular focus on non-reflecting boundary conditions and handling non-Cartesian surfaces. The balance between numerical efficiency and accuracy, in particular with regard to the approximation of the non-Cartesian surface will be explored.

Problem:

An omnidirectional point source (line in 3D) is located at a distance *d* from one of the surfaces of an infinitely extended 90-degree wedge. The distance below the second surface delimiting the wedge is also *d*. Three receivers are located at a distance *d*/2 from the other face of the wedge with distances *d* between them. The area around this structure is filled with a medium where the wave speed is c. This area is unbounded upward and towards the left-hand side. The wedge material continues to infinity to the right and the bottom.



The frequencies of interest range up to *k.d*=4 (lower limit *k.d*=0.1) where k is the wave number $=2\pi/\lambda$.

1. Implement a Cartesian staggered grid FDTD to obtain the ratio of the p-field at the receivers to the p-field at the same distance in open space as a function of frequency. Choose the grid step such that the phase error is acceptable an the distances are implemented as accurately as possible. Initially the surfaces are perfectly reflecting ($\sigma_n=0$). Look for an analytical solution for this problem and compare numerical with analytical results. This part of the project will highlight the accuracy of the PML that you implement.

2. Now cover both surfaces of the wedge with a locally reacting material with surface impedance $Z=2c+j\omega$ (conventions of the general problem in the course, frequency conventions that assure stability). Compare the results with part 1.

3. To explore the effect of boundaries not matching the grid cell boundaries, we now tilt the wedge over 30 degrees compared to the Cartesian grid also moving the source and the receivers in such a way that their relative position to the wedge stays the same. This should not affect the field nor the results. But numerical discretisation error may occur. Explore the effect for at least two grid cell sizes and for the two boundary conditions.

4. Finally select and implement one possible way to improve the description of the boundary that is tilted relative to the cartesian cells and compare (accuracy, cpu-time, and memory usage) to the result obtained for the Cartesian grid for the same average grid cell size. In this “toy”-problem, the horizontal and vertical Cartesian discretisation should be kept intact.

Practical

Students can work on this project in groups of 3. The result is a software code and a discussion of the results obtained and the choices made in the form of a brief report. During the open book exam the report will be a starting point to query the degree of knowledge and insight obtained by the student regarding the FDTD technique. (Note that the exam will obviously involve other topics as well)