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Practical Portfolio Optimization

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POINT Portfolio Modeling
Index, Portfolio, and Risk Solutions

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Contents

A Historical Perspective

- Mean-variance framework (Markowitz [1952,1959])
  - Widely used in a variety of settings (especially in equities)
  - A number of practical extensions
- Combating the estimation errors – Robust portfolio optimization
  - Multiple risk / analytical models, scenario analytics, stressed risk models
  - Portfolio constraints
  - Uncertainty sets around model estimates (Iyengar and Goldfarb [2005])
  - Re-sampled efficient frontiers (Michaud [1998])
  - Stochastic programming-based approach (impractical)
- Tail risk optimization
  - CVaR, VaR, Omega, drawdown, etc. optimization
  - Not common in practice, primarily due to estimation problems in the tails

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Mean-Variance Optimization Framework

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Mean-Variance Optimization Foundations

- Markowitz (1952,1959)
- Point estimates of covariance matrix and expected returns
- Efficient frontiers

Quadratic Programming

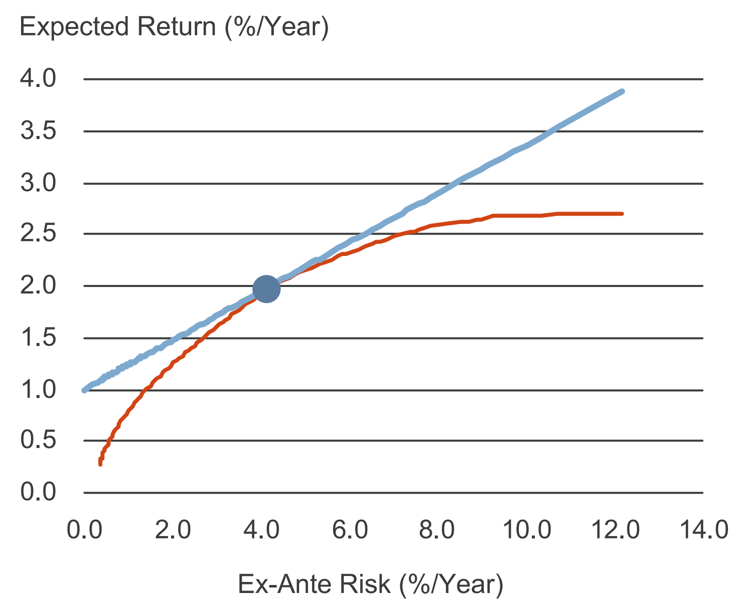
$$\underset{x \in \Re^N}{Max} \quad U(x) = \mu'(x - x^B) - \lambda (x - x^B)'\Sigma(x - x^B) - g(x)$$

$$s.t. \quad \left.\begin{array}{r} e'x = 1, \\ x \geq 0, \\ A'x \leq b \end{array}\right\} \Leftrightarrow x \in IS$$

Budget

Long-only

Generic Linear Constraints



N investable securities
μ: the vector of expected returns of asset
Σ: the variance-covariance matrix of the returns of asset
x: the vector of portfolio weights (decision variables)
$g(x)$: the portfolio penalty (e.g. liquidity, age, size etc penalties)
Examples of generic linear constraints: security/sector/loading (over)-weights
$\lambda > 0 \Rightarrow$ risk averse investors

Computational Challenges

- Predictable and practical solution time
- Scale of the underlying optimization model
  - A large-scale portfolio optimization problem can have hundreds of thousands of variables and thousands of constraints
  - Running time grows non-linearly with the problem size
  - Convex optimization is “easy” both in theory and in practice
- Combinatorial constraints
  - Constraints on number of positions, number of trades, round lots, etc.
  - Integer programming formulations
- Non-convexity (local vs. global solutions)
  - Separate exposure constraints on the long and short side of the portfolio
  - Lower bound on leverage, risk, etc.

Computational Challenges – Combinatorial Constraints

- Integer variables are needed to model combinatorial constraints
- Such problems are provably NP-hard (exponential solution time)
- In theory, can be solved to optimality using Branch and Bound schemes
- Lagrangian / duality-based heuristics (iteratively solve the convex relaxation)
- Examples – Modeling using the "big M" method
  - # of securities no more than SE

$$\sum_{i=1}^{n} \delta_i \leq SE$$

  - All trades greater than MT

$$\left| x_i - x_i^0 \right| \geq \gamma_i \cdot MT$$

$i, j = 1, \ldots, N$: denotes the set of investable risky securities

$\delta_i \in \{0,1\}$: Holding indicator variable for security i

$\gamma_i \in \{0,1\}$: Trading indicator variable for security i

x_i : the size of security i in the portfolio

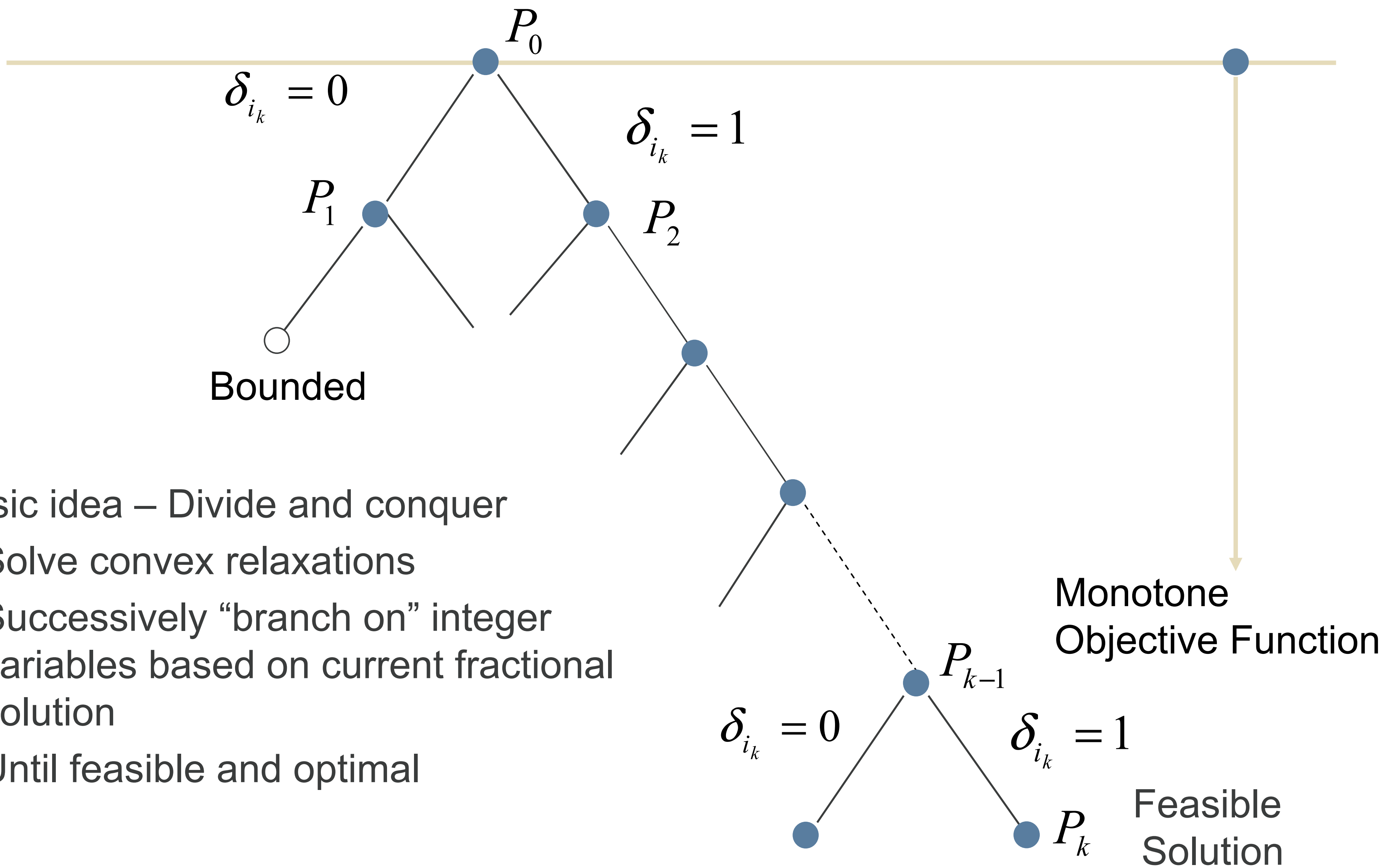
$(x_i - x_i^0)$: the trading size of security i

$\gamma_i M \leq (x_i - x_i^0) \leq \gamma_i M,$

$\delta_i M \leq x_i \leq \delta_i M$

Note that $|x_i| > 0$ if and only if $\delta_i = 1$ and $|x_i - x_i^0| > 0$ if and only if $\gamma_i = 1$

Branch and Bound Algorithms



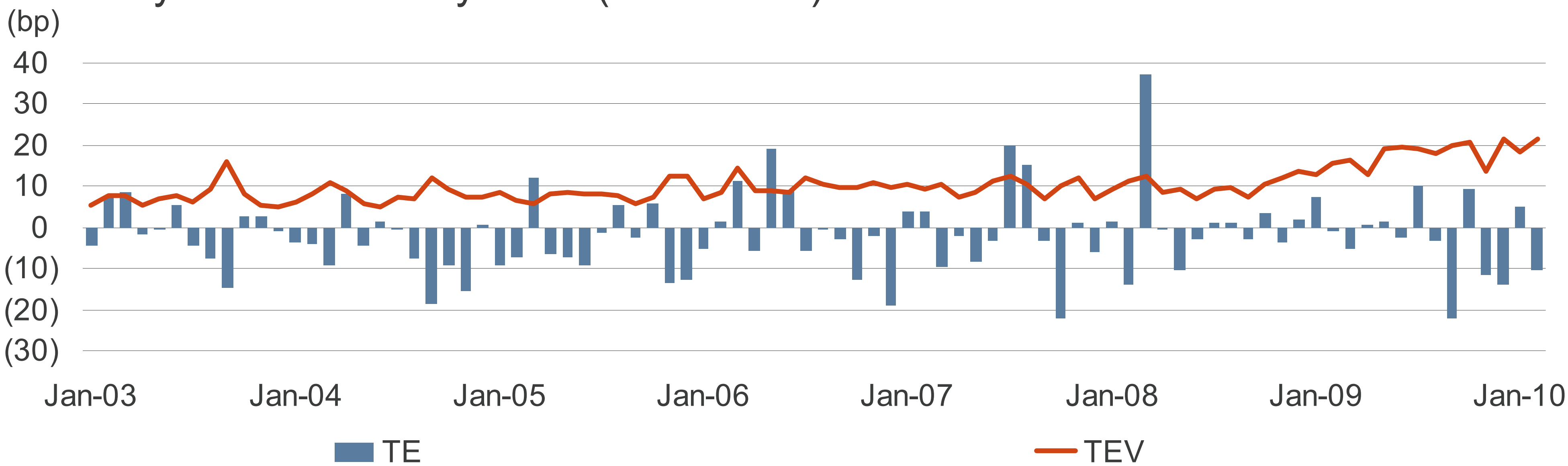
- Basic idea – Divide and conquer
  - Solve convex relaxations
  - Successively "branch on" integer variables based on current fractional solution
  - Until feasible and optimal

Mean-Variance Optimization – Applications

- MVO can be used for asset allocation and/or security selection decisions
- Portfolio construction examples
  - Cash and derivatives index replication
  - Maximizing risk-adjusted returns (overlay alpha strategies, equity long-short optimization [130/30 strategies])
  - Implementing views – Factor mimicking portfolio with clean exposure (or tilt) to certain factors / sectors (e.g., momentum, value, size, etc. portfolios)
- Portfolio rebalancing examples
  - Recommend few trades with low turnover to re-align the risk profile of a portfolio to benchmark
- Portfolio hedging
  - Hedging specific components (e.g., curve, FX, etc.) of portfolio risk using specific (derivatives) securities
- Other applications include benchmark construction, cash flow matching (immunization), testing for mean-variance efficiency

Index Replication I – *Global Treasury Index* Using a 20-bond Minimum TEV Portfolio

- January 2003–February 2010 (86 months)



Realized Tracking Error Statistics

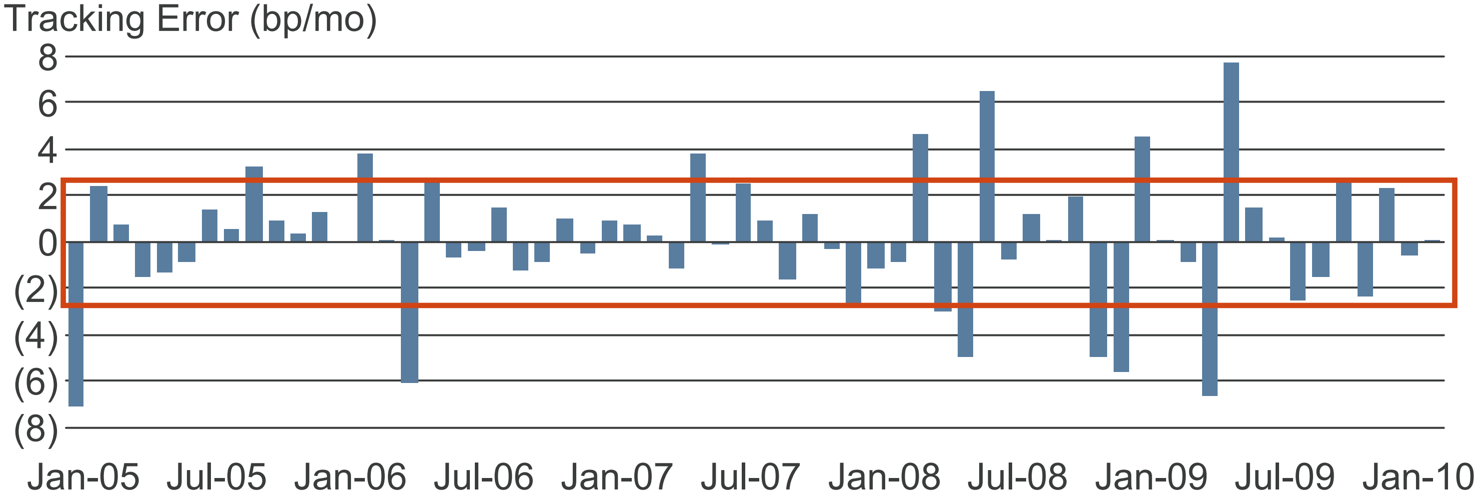
	Volatility Forecast	Active Return	Standardized TE
Median	9.3		
Stdev		9.4	0.9
Max	21.7	37.3	2.97
Min	4.9	(22.0)	(2.14)

Source: Barclays Capital POINT.

Index Replication II – *Global Treasury G7 Ex USD Index* Using Stratified Sampling

- Only trade bonds with OAS between -10 and 10bps in Japan, Germany, UK, and Canada (to avoid specialness due to deep discount or premium)
- Match market weights and OAD exposure in the following cells
  - Three maturity buckets (0 to 5yrs, 5yrs to 10yrs, and 10yrs and above) in Germany, France, Italy, UK, and Japan (15 cells)
  - Canada (1 cell)
- Minimize turnover and reinvest the cash generated by the portfolio

Tracking Error Analysis – Jan 05–Feb 2010 (62 months)



TE Statistic (bp/mo)	
Average	0.0
Median	0.1
Std Dev	2.8
Max	7.8
Min	(7.1)
Avg. Turnover	9%

Source: Barclays Capital POINT.

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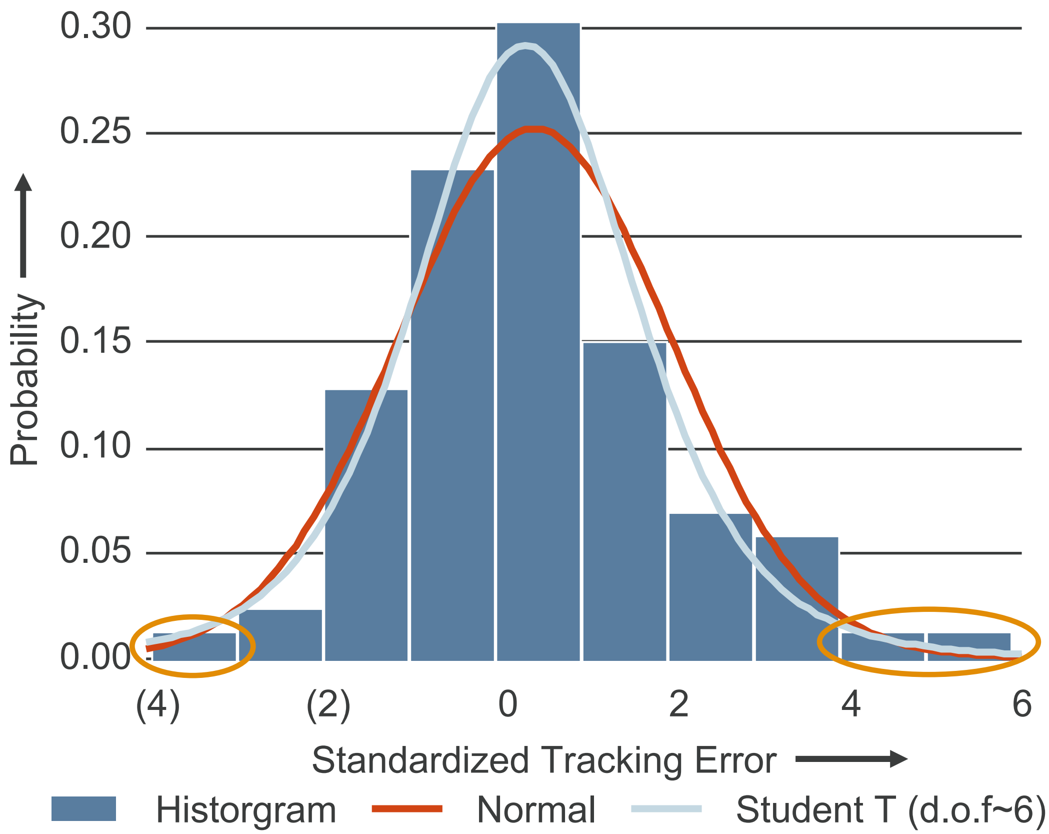
Index Replication III – Idiosyncratic and Tail Risk

- Minimum TEV portfolios to replicate US Credit Index
- Limit number of positions to 75
- Jan 2003–Feb 2009 (86 months)



	Tracking Error	Standardized Tracking Error
Std Dev	24bps	1.58
Max	1.19%	5.36
Min	0.41%	(3.02)

Source: Barclays Capital POINT.

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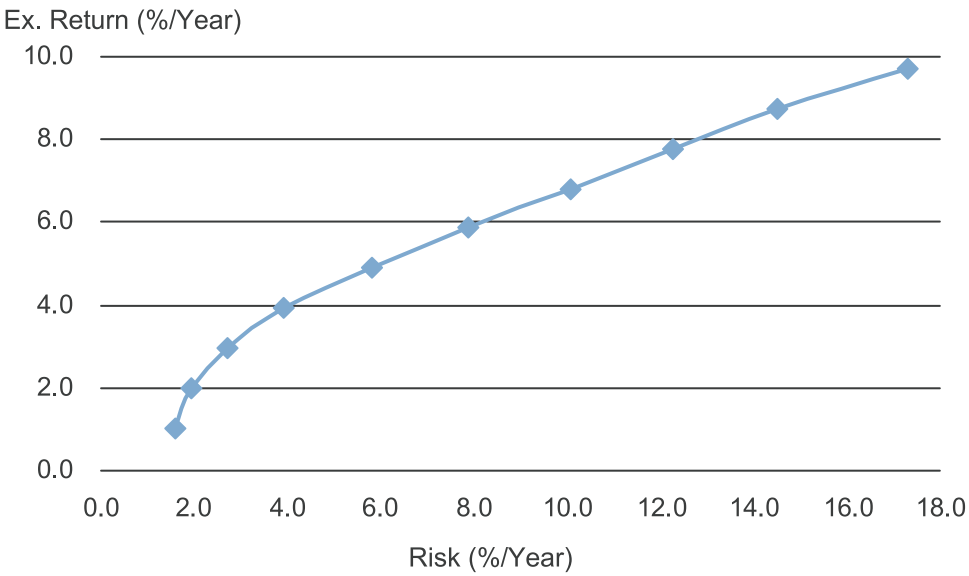
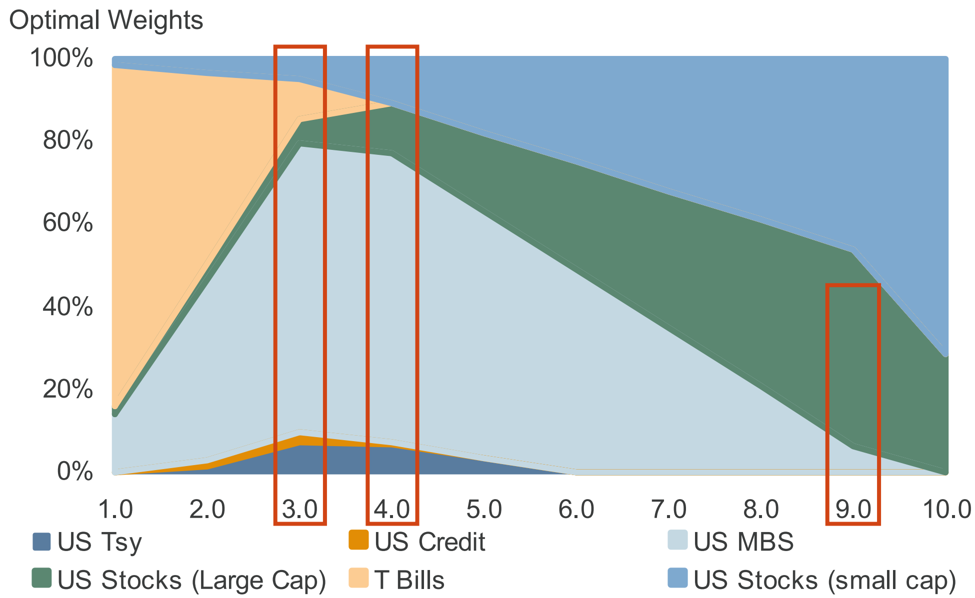
Application Pitfall – Estimation Errors

- Extreme sensitivity to estimates of expected returns
- Inconsistency between risk and expected return forecasts
- Concentrated optimal portfolios / not necessarily well diversified
- No distinction between downside and upside volatility
- Tail risk

Mean-Variance Asset Allocation to US Markets





Source: Barclays Capital.

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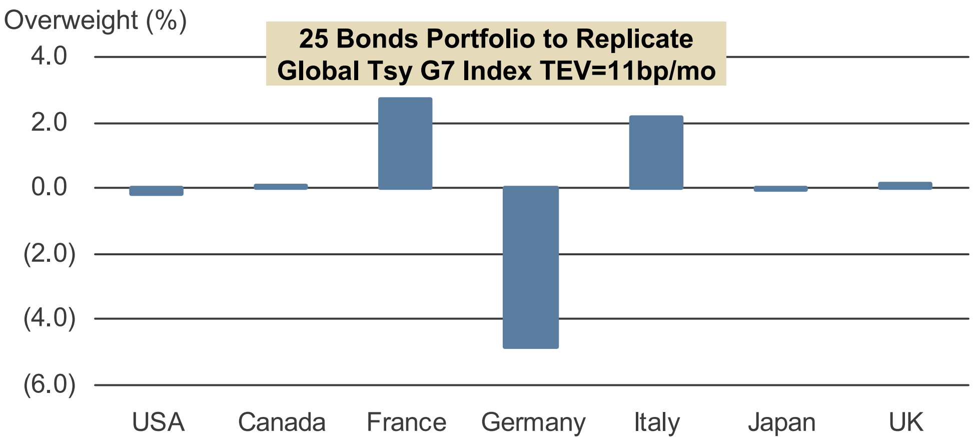
Application Pitfall – Exploit Risk Model Weakness

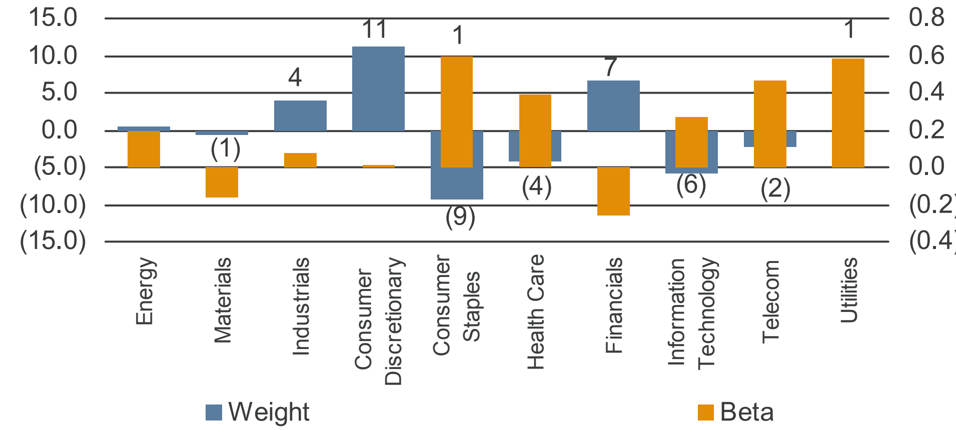
- MVO maximizes exposures to risk factors not modeled in the risk model
  - Exploitation of detailed factor correlation estimates in a large-scale risk model (e.g., FX vs. local factors, curve vs. credit factors)
  - Substitution among "similar" risky exposures, which may diverge, creating a large basis risk (e.g., overweight (lever up) short duration exposure to replicate longer duration, etc.)

Minimum TEV Portfolio 1 (FI)



- Constrained optimization (no country overweights) achieve a TEV of 12/mo

Minimum TEV Portfolio 2 (Equities)



- Optimizer underweights positive active beta and vice versa

Source: Barclays Capital.

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Implementing Views – Bayesian Approach

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Implementing Views – The BL Approach

- The MVO requires a complete set of expected returns
- The BL approach allows us to incorporate manager's views on
  - The expected returns of a partial set of securities
  - The sectors, or the spreads between securities or sectors
- Often times, the views on expected return are "inconsistent" with the risk profile of the assets
  - A security with low risk (e.g., beta) and low correlation has much smaller expected return than a higher risk and highly correlated security
  - Some of this inconsistency may be explained by tail risk considerations (e.g., FX carry strategy)
- The model proposes to take a middle ground between an equilibrium market portfolio and the optimal portfolio based on the views in a Bayesian framework

The Theory

- Starting point

  $\Sigma_{n \times n}$: The covariance matrix forecast (e.g. based on a factor risk model)

  $P_{k \times n}$: The LHS matrix used to represent views (e.g., identity matrix for views on point estimates)

  $q_{k \times 1}$: The RHS vector to represent the views

  w_{eq} : Weights of the assets in the equilibrium market portfolios (e.g., S&P 500 index)

  Ω : (Diagnal) matrix representing the variance in the given views

- Posteriors

$$\mu \equiv N(\pi, \tau\Sigma) \;\; where \;\; \pi = \delta \cdot \Sigma \cdot w_{eq}$$

$$\mu \equiv N(\pi, \tau\Sigma) \,|\, P\mu \equiv N(q, \Omega)$$

$$\Downarrow$$

$$\mu \equiv N(\hat{\pi}, \hat{\Sigma}) \;\; where \;\; \begin{cases} \hat{\mu} = \pi + \tau\Sigma P'(\tau P\Sigma P' + \Omega)^{-1} \cdot (q - P \cdot \pi') \\ \hat{\Sigma} = (1+\tau)\Sigma - \tau^2 \Sigma P'(\tau P\Sigma P' + \Omega)^{-1} P\Sigma \end{cases}$$

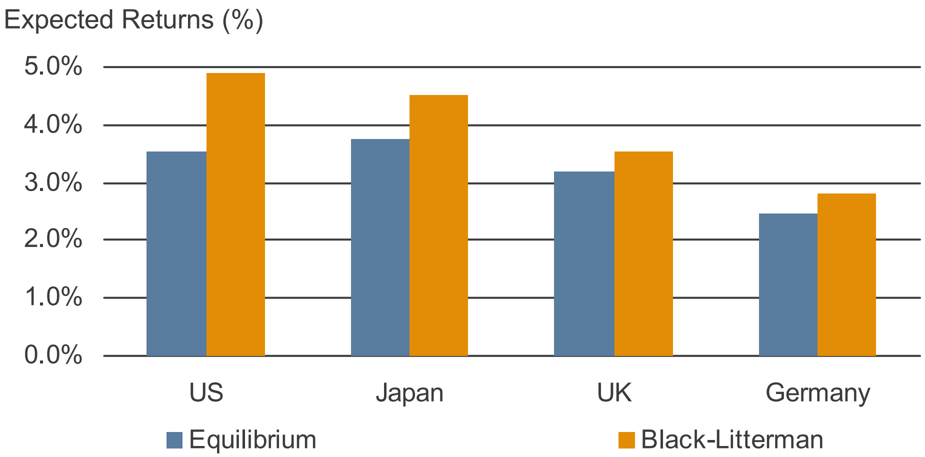
- What does τ and δ represent?

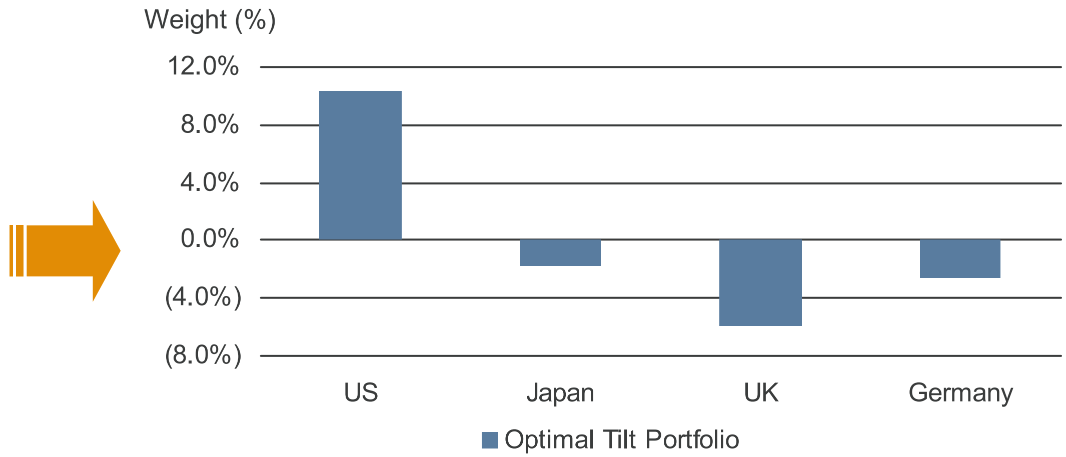
An Illustrative Example – Asset Allocation to G4 Equity Markets

- Estimates of correlation, volatilities, and expected returns

	Implied Exp. Ret.	Vol (%/YR)	Correlation US	Correlation Japan	Correlation UK
US	4%	15%			
Japan	3%	17%	0.45		
UK	3%	16%	0.30	0.40	
Germany	3%	15%	0.25	0.45	0.30

- Views – US equities to outperform equilibrium expectation by 1% and Japan to outperform UK by 1%





Source: Barclays Capital, Litterman et al. (2004).

BL Model Extensions

- Choosing the model parameters
  - Explicit estimation of the variance of the prior estimator
  - Setting confidence in views based on the market estimates
    - (e.g., $\Omega = \rho P \Sigma P$ for some constant ρ)
- Using priors other than the market equilibrium
- Allowing views on risk factors driving the returns in a linear factor model
  - Incorporate views on risk scenarios (e.g., correlation between credit and treasury switches sign)
- Incorporating generic chance statements (“80% chance that X returns 50bps more than Y”) as views
- Extensions to other elliptical return distribution

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Tail Risk Optimization

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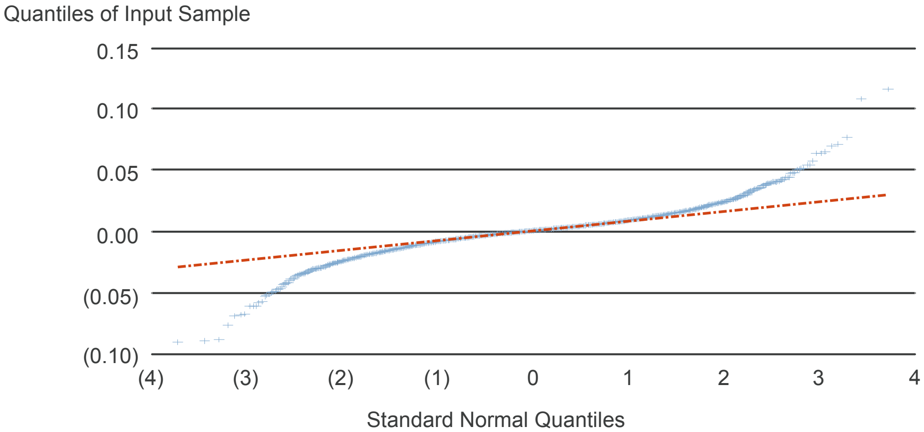
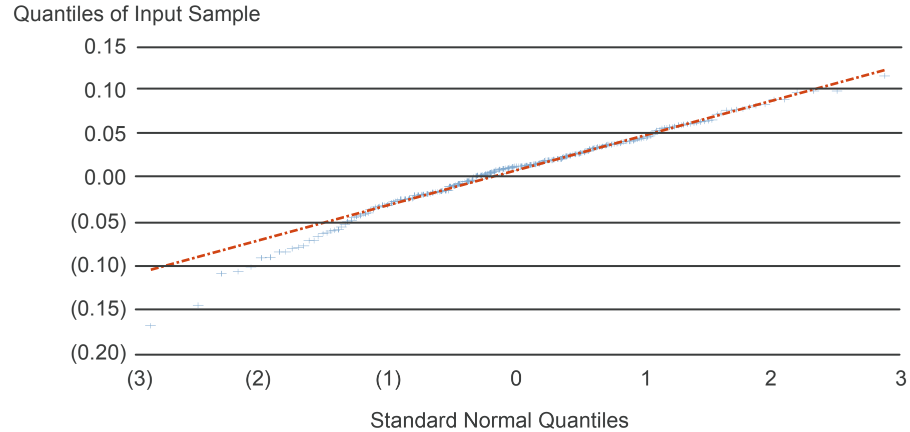
Distributions of Portfolio Returns

Consider the time series of S&P 500 (TR) index (Oct 1989–Jan 2010)

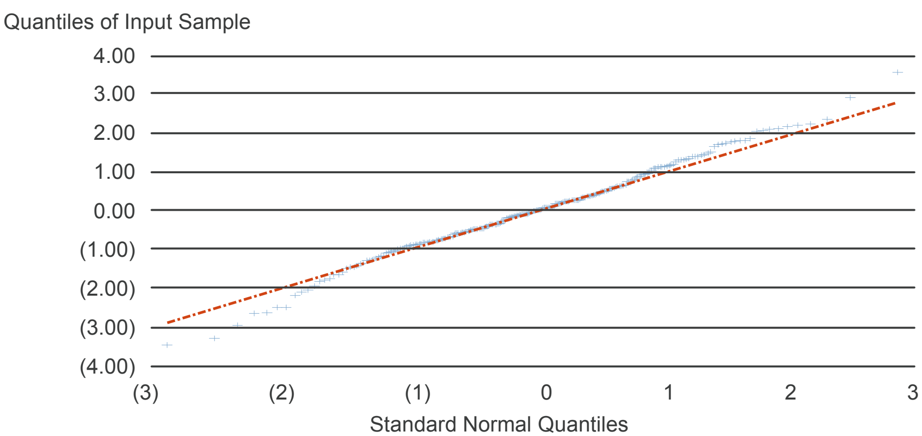
QQ Plot of Sample Daily Returns vs. Standard Normal



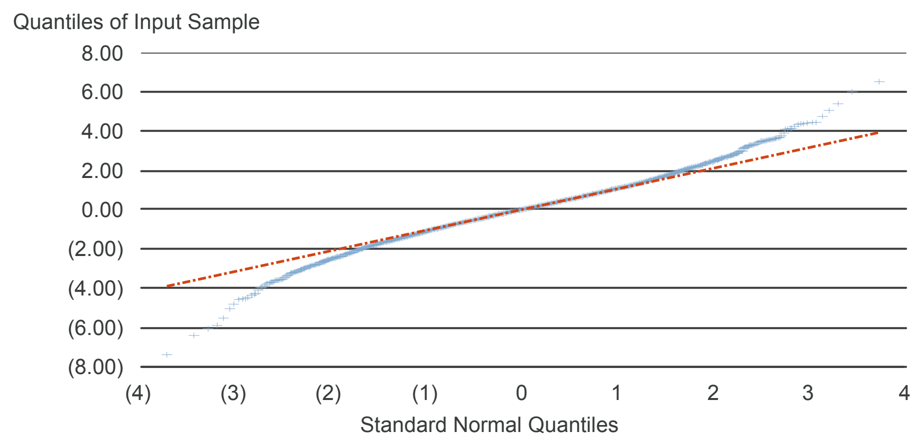
QQ Plot of Monthly Returns vs. Standard Normal



QQ Plot of Garch(1,1) Filtered Monthly vs. Standard Normal



QQ Plot of Garch(1,1) Filtered Daily Returns vs. Standard Normal



Source: Barclays Capital.

Preferences Over the Distributions

- Empirical stylized facts about financial asset returns
  - Time varying volatilities
  - Non-zero skew and excess kurtosis
  - Tail contagion – Correlations in the tail are higher than in the belly
- Risk measures – A risk measure is a mapping from a return (loss) distribution to a real number

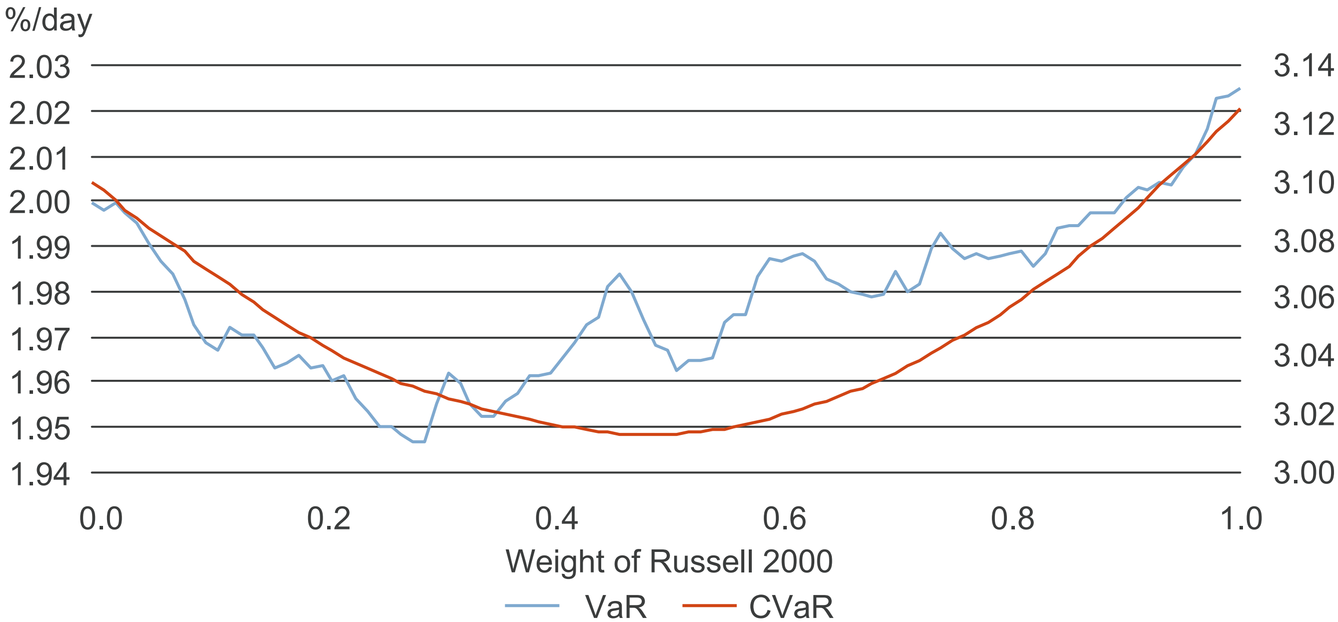
$$\rho(X) \mapsto \Re_{+}$$
$$\rho(X) = std(X), VaR(X), ES(X), \Omega(X) \ etc.$$

$$\Omega(w) = \frac{\int_H^{\infty} 1 - F(t; w)\,dt}{\int_{-\infty}^{H} F(t; w)\,dt}$$

- A number of popular approaches to model tails of return distributions
  - T-marginals with copulas in factor model framework
  - Extreme value theory
- The tradeoff between expected returns and tail risk
  - Hard to measure, instead used as a risk control tool

Tail Risk Optimization – Formulations

- Given the joint distribution of the investable assets as a (large) panel of simulated returns
  - CVaR, omega, and drawdown optimization can be modeled as a linear programming (LP) problem (Uryasev et al. [2002, 2003])
  - VaR is not a convex risk measure (not sub-additive)
    - Smoothing heuristics are used
- Tail risk optimal portfolios are very sensitive to the model (e.g., portfolio of Russell 2000 and Russell 1000 indices [Jan 1990–Mar 2010])



- Returns are vol-scaled before constructing the portfolio

Source: Barclays Capital.

Conclusion

- "Mean variance framework" with appropriate practical tool for handling well-known pitfalls remains state of the art in practice
- Non-convex and combinatorial problems are difficult to solve to optimality, and heuristics are commonly used
- Robust portfolio optimization remains impractical because of the difficulty in estimating the uncertainty set around unobservable estimates
- Practical tools (multiple risk models, flexible constraints, scenario constraints, etc.) are effective in combating the uncertainties in the inputs
- Black-Litterman model and its extensions provide a useful and practical asset allocation methodology for implementing views in a risk-consistent fashion
- Tail-risk optimization is getting traction and can be useful in explicit control of downside tail risk

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