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## INDEX RULES FOR CURRENCY HEDGING AND CURRENCY RETURNS: A PRIMER

The popularity of the Global Aggregate Index and other multi-currency indices provided by Lehman Brothers has led to substantial interest in the index rules for calculating currency returns for both hedged and unhedged indices. In particular, investors frequently ask how to hedge portfolio positions so as to best match index currency exposures. We discuss the currency aspects of index return calculations, with special emphasis on the procedure that the index uses to hedge foreign exchange exposure in currency-hedged indices.

### Unhedged Returns

Consider an investor who buys foreign currency at the beginning of the month and sells the position back into base currency at the end of the month. The realized capital gain return from this investment in foreign currency is:

$$\text{FXapprec} = \frac{\text{FX}_{\text{end}} - \text{FX}_{\text{beg}}}{\text{FX}_{\text{beg}}} \qquad (1)$$

$\text{FX}_{\text{beg}}$ and $\text{FX}_{\text{end}}$ are the base-currency values of one unit of the local at the beginning and the end of the return period, respectively.

Consider an investor who buys a bond denominated in a foreign currency at the beginning of the month and, at the end of the month, sells the bond and converts the foreign currency proceeds back into the base currency. The investor's (base-currency) realized return on this investment is:

$$\begin{aligned}\text{base-currency total return} &= (1 + \text{local return})(1 + \text{FXapprec}) - 1. \\ &= \text{local return} + \text{FXapprec} \\ &\quad + (\text{local return})(\text{FXapprec})\end{aligned} \qquad (2)$$

To keep analytics as intuitive and tractable as possible, Lehman decomposes index returns into additive components. Accordingly, currency return is defined as follows:

$$\text{currency return} = \text{base currency total return} - \text{local return} \qquad (3)$$

where

$$\text{local return} = \text{price return} + \text{coupon return} + \text{paydown return}.$$

Currency return is the difference between base-currency return and local return. Substituting Equation (2) into Equation (3) provides:

$$\begin{aligned}\text{currency return} &= \text{FXapprec} + (\text{local return})(\text{FXapprec}). \\ &= (1 + \text{local return})\,(\text{FXapprec})\end{aligned} \qquad (4)$$

Note that the currency return on the bond is not equal to the capital gain on a pure currency investment. From Equation (1), the base-currency capital gain on a pure currency investment is FXapprec, while the bond's currency return is FXapprec + (local return)(FXapprec). The bond's currency return contains an interaction component (local return)(FXapprec) that drives a wedge between the capital gain on a currency investment in the cash market and a bond's currency return.

### Example: Unhedged Returns

Consider an index in which the base currency is the euro that contains the USD-denominated Ford bond described in Figure 1. The local return on this bond in April 2003 was 14.87%. The total euro return on an unhedged position in this bond was 12.32%. From Equation (3), the index currency return for an unhedged position in this bond was 12.32% - 14.87% which comes to -2.55%.

The currency return on this bond can also be calculated from Equation (4). In this case, we note that the EUR/USD exchange rate was marked at 1.0912 at the beginning of April and 1.116 at the end of April. Thus, the value of one U.S. dollar fell from 0.9164 euros at the start of April to 0.8961 euros at the end of April. (These values are arrived at by taking the reciprocal of the quoted exchange rate.)

Here FXapprec is (0.8961 - 0.9164)/0.9164, or -2.22%. During April, the U.S. dollar lost 2.22% of its value when measured in euros. From Equation (4), the currency return is (-0.0222)(1.1487), which comes to -2.55% and matches the previous currency return calculation.

The difference between the -2.22% euro return for investments in U.S. dollar cash and the -2.55% currency return on the Ford bond is 33 bp, which exactly equals the value of the interaction term: (0.1487)(-2.22%).

Finally, the base-currency total return for the Ford bond can be directly calculated from Equation (2). In the current example, the product of (1 + local return) and (1 + FXapprec) is (1.1487)(0.9778), which comes to 1.1232. Subtracting one provides the bond's 12.32% total return in euros.

### Sample Implementation: Unhedged Returns

Figure 2 provides the set of actual transactions that an investor could use to generate the unhedged index return for the Ford bond in April 2003. The only potential discrepancy

Figure 1. **Unhedged Index Return for April 2003: Ford 7.5% Coupon Maturing August 1, 2026**

| | | |
|---|---|---|
| Price Return | 14.08% | |
| Coupon Return | 0.79% | |
| Paydown Return | 0% | |
| Local Return | 14.87% | |
| Currency Retun | -2.55% | |
| Total Unhedged Return in Euros | 12.32% | |
| | | |
| EUR/USD Spot March 31, 2003 | 1.09120 | |
| EUR/USD Spot April 30, 2003 | 1.11600 | |
| | | |
| $FX_{beg}$ | 0.91642 | |
| $FX_{end}$ | 0.89606 | |
| FXapprec | -2.22% | |
| Currency Return | -2.55% | = (-2.22%)(1.1487) |
| Total Return (Unhedged) in Euros | 12.32% | = (1-.0222)(1.1487) - 1 |

between the investor's realized euro return from these transactions and the index euro return is the investor's inability to ensure that trades are executed exactly at index prices.[1]

To replicate the index return, the investor uses 72.29 EUR to buy 78.89 dollars at the spot FX rate at the close of business on March 31. The U.S. dollar proceeds are used simultaneously to purchase the Ford bond. At the close of business on April 30, the investor sells the Ford bond and in return receives 90.62 dollars, which is simultaneously sold in the spot FX market for 81.20 EUR. These transactions result in a euro return of 12.32%.

### Hedged Returns

Now consider the currency-hedged version of the index containing the Ford bond. For currency-hedged indices, as well as unhedged indices, currency return satisfies the relation:

currency return = base-currency total return - local return.

One can verify this using the numbers in Figure 3. The Lehman indices report a currency-hedged total return of 14.67% (EUR) for this bond. Subtracting the 14.87% local return comes to -0.21%, which equals the reported currency return for the bond in the currency-hedged version of the index.

While the definition of currency return remains unchanged, the calculation of base-currency total return becomes somewhat more complicated for currency-hedged indices. The index position in a bond denominated in a foreign currency for a hedged

[1] If the bond paid a coupon during the month, then another discrepancy would arise because of the index convention of not reinvesting cash payments until the end of the month.

Figure 2. **Unhedged Returns, Implementation for April 2003: Ford 7.5% Coupon Maturing August 1, 2026**

| | March 31 | April 30 | | |
|---|---|---|---|---|
| Price | 77.635 | 88.742 | | |
| Accrued Interest | 1.250 | 1.875 | | |
| EUR/USD Spot | 1.0912 | 1.1160 | | |
| Value of One USD in Euros | 0.9164 | 0.8961 | | |
| | | | | |
| **Transaction** | **EUR** | **USD** | **FX Rate** | **Comment** |
| **March 31** | | | | |
| Initial EUR Balance | 72.292 | | | |
| Purchase Dollars at Spot | -72.292 | 78.885 | 0.9164 | USD Cash Price = 77.635 + 1.250 |
| | | | | Cost in EUR Is (0.9164)(USD Cash Price) |
| Purchase Bond | | -78.885 | | |
| | | | | |
| **April 30** | | | | |
| Sell Bond | | 90.617 | 0.8961 | USD Cash Price = 88.742 + 1.875 |
| Sell USD into EUR | 81.198 | -90.617 | | |
| Final EUR Balance | 81.198 | | | |
| | | | | |
| Total Return (Unhedged EUR) | 12.32% | | | (EUR Proceeds/Initial EUR Investment) - 1 |

Figure 3. **Hedged Index Return for April 2003: Ford 7.5% Coupon Maturing August 1, 2026**

| | | |
|---|---|---|
| Price Return | 14.08% | |
| Coupon Return | 0.79% | |
| Paydown Return | 0% | |
| Local Return | 14.87% | |
| Currency Retun | -0.21% | |
| Total Hedged Return in Euros | 14.67% | |
| | | |
| Yield (March 31, 2003) | 9.99% | |
| Hedge Magnitude per Dollar Invested (H) | 1.008157 | $= (1 + (9.99\%/2))^{(1/6)}$ |
| | | |
| EUR/USD Spot March 31, 2003 | 1.09120 | |
| EUR/USD Spot April 30, 2003 | 1.11600 | |
| 1-Month Forward Value in Euros of One U.S. Dollar | 0.91740 | |
| $FX_{beg}$ | 0.91642 | = 1 / (1.09120) |
| $FX_{end}$ | 0.89606 | = 1 / (1.11600) |
| FXapprec | -2.22% | = (0.89606 - 0.91642) / (0.91642) |
| Forward Return | 2.33% | = (0.91740 - 0.89606) / (0.91642) |
| | | |
| Total Hedged Return in Euros | 14.67% | = 12.32% + (1.008157)(2.33%) |
| | | |
| Expected Currency Return | 0.11% | = (1.008157)(0.91740 - 0.91642)/(0.91642) |
| Residual Currency Return | -0.31% | = (1 + 14.67% - 1.008157)(-2.22%) |
| Currency Return (Total) | -0.21% | = -0.31% + 0.11% |

index is actually a position in two instruments: the bond plus a position in one-month currency forwards:

$$\text{base currency total return (hedged)} = \text{base currency total return (unhedged)} + H\,(\text{Forward Return}) \quad (5a)$$

where

base currency total return (unhedged) is provided in Equation (2)
H is the size of the hedge measured in local currency

$$\text{Forward Return} = \frac{\text{ForwardValue} - FX_{end}}{FX_{beg}} \quad (5b)$$

Forward Value is the number of base-currency units to be received for each unit of the local currency delivered in the forward contract. This value is set in the marketplace at the beginning of the month and is received at delivery at the end of the month.

Equation (3) can be used to re-express Equation (5a) as follows:

$$\text{base currency total return (hedged)} = \text{local return} + \text{currency return (unhedged)} + H\,(\text{Forward Return}) \quad (6)$$

Note that the second term on the right hand side of Equation (6) is increasing in the end-of-month FX rate and the third term on the right hand side of Equation (6) is

decreasing in the end-of-month FX rate. If the hedge were perfect, these term terms would reduce to a constant and remove all sensitivity to the exchange rate at the end of the month.[2]

Equation (5) is general. It provides the base-currency total return under any hedging rule. By setting H according to Lehman index rules, Equation (5) becomes a full specification of base-currency total return for Lehman indices.

For securities other than mortgages, Lehman indices set H as follows:[3]

$$H = (1 + \text{yield}/2)^{(1/6)} \qquad (7)$$

H is a projected end-of-month market value per unit of local currency invested at the beginning of the month. In Equation (7), local-currency security value is projected to grow at the rate implied by its yield. For non-bullet bonds, Equation (7) uses yield to worst.

The perfect currency hedge for the April return would set H equal to the bond's end-of-April local-currency value. However, the perfect hedge could not be obtained by an actual investor at the beginning of April: The local currency value of the index at the end of April is not knowable on March 31, when the hedge must be implemented. Index construction at Lehman Brothers always stresses the importance of investability. In this spirit, the index does not use end-of-month values in determining the currency hedge. The yield in Equation (7) is the yield at the beginning of the month.

Returning to the Ford 7.5% coupon bond, Figure 3 reports that the yield on this bond at the beginning of April was 9.99% and that 0.9174 euros will be received for every U.S. dollar delivered in current 1-month currency forwards. Therefore, H is 1.00816 and Forward Return is 2.329%. Using these values in Equation (5), together with the bond's 12.32% unhedged base-currency return, results in 14.67%, the bond's currency-hedged total euro return.

For each currency, the currency-hedge for the entire index is the weighted average of the security hedges weighted by beginning-of-the-month index weight. Thus, the hedge for the April index return is the weighted average of the individual April security hedges using security market weights from the beginning of April.

For individual securities, the following equation can also be used to calculate currency returns according to hedged index rules:[4]

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[2] While the sum of the second and third terms in Equation (6) would be a constant in a perfect hedge, this constant would generally be non-zero. The constant would be the carry return for the perfect hedge, which is related to the divergence between the local and base-currency deposit rates.

[3] If the security is scheduled to mature or be called during the month, then the exponent in Equation (7) is set to the time remaining until the maturity or call date, where time remaining is expressed as a fraction of a six-month period. This also applies to the analogous exponent in Equation (9).

[4] Forward Value – $FX_{beg}$ is referred to as the one-month forward point. PC Product does not directly report forward exchange rates. Instead it contains current spot exchange rates and forward points which can then be used to calculate forward exchange rates.

$$\text{currency return} = H\,\frac{\text{ForwardValue} - FX_{beg}}{FX_{beg}} + (1 + \text{local return} - H)\,(\text{FXapprec}) \qquad (8)$$

All of the components of the first term in Equation (8) are known at the time the hedge is implemented. This first term is often referred to as the carry from the hedge. In the current Ford example, its value is (1.00816)(0.9174 - 0.9164)/(0.9164), which comes to 10.8 bp. In April 2003, USD deposit rates were lower than EUR deposit rates, which resulted in the hedge's positive carry return.

Since index hedges are designed to be implementable by investors at the beginning of the month, they are not perfect hedges: the bond will have residual currency exposure in the index after hedging. This residual currency exposure is equal to the difference between the size of the hedge and the market value of the security at the end of the month. The FX appreciation realized by this exposure is measured by the second term in Equation (8). The exact value of this term will not be known until the end of the month. In our example, this term is (1.1487 – 1.00815)(-2.22%), which equals -31.2 bp. These two terms sum to the reported -21 bp currency return for the bond in the hedged index. Note that while 21 bp is a fairly large magnitude for a hedged currency return, it is less than one-tenth the size of the unhedged currency return.

### Sample Implementation: Hedged Returns

Figure 4 lists the transactions an investor could use to generate the hedged index return for the Ford bond in April 2003. At the close of business on March 31, the investor executes the following transactions:

1) Pay 72.29 EUR to buy 78.89 U.S. dollars at the spot FX rate.
2) Pay 78.89 U.S. dollars to purchase the Ford bond.
3) Go short 79.53 one-month dollars in the one-month EUR/USD forward market.

At the close of business on April 30, the investor closes out these positions as follows:

1) Sell the Ford bond for USD proceeds of 90.62.
2) Deliver 79.53 USD to the one-month forward market and receive 72.60 EUR.
3) Sell the remaining 11.09 USD in the spot currency market for 9.94 EUR.

These transactions result in a euro return of 14.67%.

### Mortgages

For U.S. mortgages, H is set to

$$\left(1 - \frac{\text{projected principal payments}}{100}\right)\left(1 + \frac{\text{mortgage yield}}{2}\right)^{1/6} + (\text{projected paydown}) \qquad (9)$$

Figure 4. **Hedged Returns, Implementation for April 2003: Ford 7.5% Coupon Maturing August 1, 2026**

| | March 31 | April 30 |
|---|---|---|
| Price | 77.635 | 88.742 |
| Accrued Interest | 1.250 | 1.875 |
| EUR/USD Spot | 1.0912 | 1.1160 |
| Value of one USD in Euros | 0.9164 | 0.8961 |
| Yield | 9.99% | 8.63% |
| 1-Month Forward Value in Euros of One U.S. Dollar | 0.9174 | |
| Projected Growth In Local Bond Value over Coming Month | 0.8157% | |

| Transaction | EUR | USD | FX Rate | Comment |
|---|---|---|---|---|
| **March 31** | | | | |
| Initial EUR Balance | 72.292 | | | |
| Purchase Dollars at Spot | -72.292 | 78.885 | 0.9164 | USD Cash Price = 77.635 + 1.250<br>Cost in EUR is (0.9164)(USD Cash Price) |
| Purchase Bond | | -78.885 | | |
| Sell 79.528 One-Month Dollars | | | | (78.885)(1.008157) = 79.528 |
| **April 30** | | | | |
| Sell Bond | | 90.617 | | USD Cash Price = 88.742 + 1.875 |
| Close Forward Contract | 72.959 | -79.528 | 0.9174 | |
| Sell Residual USD into EUR | 9.936 | -11.089 | 0.8961 | |
| Final EUR Balance | 82.895 | | | |
| Total Return (Unhedged EUR) | 14.67% | | | (EUR Proceeds/Initial EUR Investment) - 1 |

where projected paydown =

$$(\text{projected principal payments})\ \frac{100+(\text{coupon}/12)}{\text{price}+(\text{coupon}/12)} \qquad (10a)$$

and projected principal payments =

$$\text{scheduled principal payment} + (\text{smm})(100-\text{scheduled principal payment}). \qquad (10b)$$

The mortgage yield in Equation (9), price in Equation (10a), and single-month mortality (smm) in Equation (10b) are beginning-of-the-month values. Thus, for the April 2003 return, the values used would be as of the close of trading on March 31. The scheduled principal payment value is the value scheduled for the end of the month (April 30 for April index returns).

From the first term in Equation (9), the component of the mortgage that is projected to survive is assumed to grow over the coming month at the rate given by the mortgage yield. In Equation (10a), the part of the mortgage that is projected to be repaid generates a return of

$$\frac{100+(\text{coupon}/12)}{\text{beginning of the month cash price}}.$$

The beginning-of-the-month cash price is the beginning-of-the-month price plus one-twelfth of an annual coupon.

In Equation (10b), the most recent smm number is used as an estimate of the upcoming smm value, and the unscheduled prepayments are calculated in the standard manner by multiplying the principal outstanding after scheduled repayment by single-month mortality.

### End-of-Month Roll of Currency Hedging Positions

Lehman is in the process of adding a new statistic, Hedge Market Value,[5] to portfolio analytics to help investors who wish to match index hedging procedures. Hedge Market Value applies index currency hedging rules to determine the one-month hedge for the portfolio position. These hedges are calculated based on market values from the current close of business. Hedge Market Value provides the hypothetical hedge that the index would implement today for a position of this magnitude if the current day were a month-end.

At the actual month-end, Hedge Market Value reports the exact hedge that the index would use to hedge the portfolio position over the upcoming month. Prior to month-end, Hedge Market Value can be viewed as an approximation of the hedge to be implemented at the beginning of the next month. This approximate becomes more precise as the month progresses and becomes exact at the actual month-end; it is designed to help investors roll forward their currency hedges at the end of the month.

### Using Hedges of Longer Tenors

For many investors in hedged global indices, the amounts that need to be hedged in each currency are fairly stable from month to month. Therefore, although the index hedges with one-month currency forwards, many of these investors choose to hedge with three-month contracts. While the use of three-month currency forwards introduces tracking error relative to a currency-hedged benchmark, these investors are willing to bear this tracking error for a variety of reasons. One justification is the perception that rolling a hedge four times a year in the three-month market entails lower execution costs than rolling the position twelve times a year in the one-month market. Another possible motivation, depending on the shapes of the two forward curves, is that the hedging cost (which is based on the deposit rate differential) can sometimes be significantly lower at longer tenors. At the beginning of last year,[6] we explored the risk/return tradeoff of using hedges of longer tenors, ranging from three months to two years. We concluded that while such strategies can add value, investors need to be aware that by extending the tenor of the hedge, they are essentially adding a view on short-term interest rates to their portfolio.

### PIE: Perfect Index Execution

The beginning and ending values for spot and forward FX rates are provided by W.M. Reuters, based on valuation as of 4 PM London time on the last business day of the month.

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[5] This field is denoted MVHedgeE in PC Product.

[6] "Long-Horizon Currency Hedging in Global Index Portfolios," *Global Relative Value*, January 7, 2002, Lehman Brothers.

However, real-world investors are not able to transact all their hedges exactly at month-end, nor can they be sure that their forward contracts will be struck at exactly the closing rates used for index calculations. This hedge pricing noise can give rise to noticeable differences between the currency returns of the portfolio and the benchmark relative to the small size of currency returns on hedged indices. To help investors resolve this problem, Lehman Brothers recently introduced the Perfect Index Execution (PIE) FX execution service. PIE guarantees execution of one-month hedges at the exact index forward rate in the one-month forward market. As the end of the month approaches, the investor could calculate the total portfolio hedges to be implemented for the upcoming month based on the Hedge Market Value field described above and have these amounts executed through PIE. The resulting hedge should exactly match index hedging without concerns related to trade synchronization, market impact, and possible illiquidity at the time index prices are determined (4 pm London).

## Common Questions about Currency Returns on Lehman Brothers Indices

Q: Why aren't unhedged currency returns exactly the same for all bonds denominated in a given currency?
A: All bonds in a given currency will be subject to the same change in FX rates, which is denoted FXapprec in this article. However, as Lehman expresses currency returns using an additive convention, the reported currency return also depends on the local return of a bond, as shown in Equation (4). Thus, different bonds (or indices) denominated in the same currency can have different currency returns.

Q: Why do hedged indices have currency returns? Isn't the currency risk hedged away?
A: Hedging can eliminate the majority of currency risk, but there is no way for foreign investors to earn exactly the same return as local investors, for two reasons. First, there is no way to know at the beginning of a month exactly what the market value of a given investment will be at the end of the month. The amount by which the size of the hedge is different from the ultimate end-of-month market value (either too big or too small) results in a (usually small) exposure to changes in FX rates. Second, even if we assume that the hedge is set up in exactly the right size, there is a carry component (which can be either positive or negative) that corresponds to the difference between spot and forward rates.

The carry is proportional to the difference between the short-term deposit rates in the two currencies. One will pay up to shift cash to a higher-yielding currency on a hedged basis; this negative carry is sometimes referred to as the cost of the hedge. A shift to a lower-yielding currency on a hedged basis will yield a positive carry, essentially compensating the investor for the extra yield that he has foregone.

Both of these components of hedged currency return are seen explicitly in the formulation of Equation (8), in which the first term corresponds to the carry and the second to the FX return on the market value that remains unhedged.

Q: How is the size of the hedge determined in Lehman indices?
A: The size of the hedge is not assumed to be the ending market value, which cannot be known at the start of the month. It is also not assumed to be beginning market value, since this would result in a systematic bias toward under-hedging. Lehman uses a simple projection of end-of-month market value based on the combination of beginning-of-month market value

and yield. For mortgage-backed securities, this projection is adjusted for prepayment as well. The exact calculations are set forth in equations (7), (9), and (10).

Q: I cannot always implement my FX transactions at the levels assumed by the index. How can I reduce the tracking error that results from this effect?
A: Lehman Brothers has recently introduced a new service, Perfect Index Execution (PIE), which provides guaranteed execution of one-month hedges at the same rates used for index calculations (as quoted by W.M. Reuters). Use of this service should eliminate FX pricing noise as a source of portfolio tracking error.

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