Estimating Extreme Negative Portfolio Returns: A Portfolio Manager's Guide to Barclays Capital Tail Risk Model

In this paper, we present our methodology for portfolio tail risk measurement. The Barclays Capital Tail Risk Model delivers to portfolio managers and traders the complete probability distribution of their monthly portfolio's return (or P&L) that is then summarized by two risk measures: value at risk and expected shortfall.

The Tail Risk Model comes with detailed tail risk reports that permit the portfolio manager to examine the particular sources of portfolio tail risk. Our Tail Risk Model is implemented within, and is fully consistent with, the Barclays Capital Global Risk Model available in POINT®.

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PART I – INTRODUCTION[1]

Portfolio managers have long known that the forces driving their world are not always smooth or symmetrical. While the familiar symmetrical, bell-shaped normal distribution may describe some natural phenomena, managers have long known or suspected that this does a poor job describing market returns. Why does this matter? Because investors tend to care more about losses than gains, and they have a particularly strong aversion to large negative losses. In addition, large losses are particularly damaging to a manager's reputation and business. Consequently, managers care very much about the extreme (or, "tail") behavior of their portfolio's returns. In turn, clients are increasingly demanding tail risk measures from their managers.

Unfortunately, the traditional portfolio risk measure, standard deviation, gives equivalent weight to a loss as to a gain. In addition, it does not adequately measure the likelihood of tail events, so using it to measure a portfolio's risk is unlikely to be fully satisfactory for managers or their clients.

Many investors manage portfolios against an index or benchmark. For them, less attention is given to the portfolio's absolute return and than to the portfolio's return *difference* (i.e., tracking error) versus the benchmark. Again, clients are more sensitive to negative tracking errors than positive ones and very sensitive to large deviations from the benchmark. Traditionally, portfolio risk in this context has been measured by the portfolio's standard deviation of tracking errors (or tracking error volatility, TEV). Once again, the standard deviation is likely to fall short of being an adequate measure of portfolio tail risk.

So what is a portfolio manager to do? There are other statistics that can capture the asymmetry or extreme behavior of a portfolio's returns or tracking error. However, to measure these statistics correctly, a way must be found to fully represent a portfolio's (and its benchmark's) expected return performance. In other words, what is the expected distribution of a portfolio's returns and tracking errors over the next month? With such a distribution at hand, a portfolio manager can then measure the likelihood of extreme outcomes with greater accuracy.

The goal of the Barclays Capital Tail Risk Model is to give managers a description of their portfolio's return distribution. From this distribution, we calculate two measures of tail risk: Value at Risk (VaR) and Expected Shortfall (ES). The Tail Risk Model is fully integrated in the Barclays Capital Multi-factor Global Risk Model, which provides measures of a portfolio's tracking error volatility. Now, not only can investors obtain a portfolio's TEV, VaR, and ES, but they can also use the portfolio's (and benchmark's) entire return distribution to obtain any other desired risk measure, all within the same consistent modeling framework. In addition, managers can use the Tail Risk Model to respond to client and regulatory demands for more rigorous modeling of portfolio tail risk.

In Part I, we describe Barclays Capital's methodology for modeling a portfolio's distribution of monthly returns and provide some examples. Part II illustrates the Tail Risk Model's reports for several portfolios, first for a portfolio benchmarked against the Barclays Capital Global Aggregate Index. We then discuss the tail risk report for the U.S. High Yield Index against a cash benchmark. Our final example is a highly skewed, highly non-normal negatively convex portfolio, allowing us to highlight the flexibility of our Tail Risk Model.

[1] This paper has been previously published in October 2007 as "Estimating Extreme Negative Portfolio Returns: A Portfolio Manager's Guide to Lehman Brothers' Tail Risk Model" by Attilio Meucci, Yingjin Gan, Anthony Lazanas, and Bruce Phelps. Michael Bos, Bruce Moskowitz, Nancy Roth, Andy Sparks, and Gary Wang provided help for that paper. The authors would like to thank Arne Staal for his help with the current revision.

Part III discusses our tail risk modeling methodology. To obtain the distribution of the portfolio's return (or P&L), four steps are necessary: identification of risk factors; pricing the securities at the investment horizon; producing a portfolio return distribution by aggregating the securities' distribution; and summarizing the wealth of information contained in the return (or P&L) distribution by means of a few significant statistics. We discuss each step in detail.

1.1 Portfolio Tail Risk – Examples

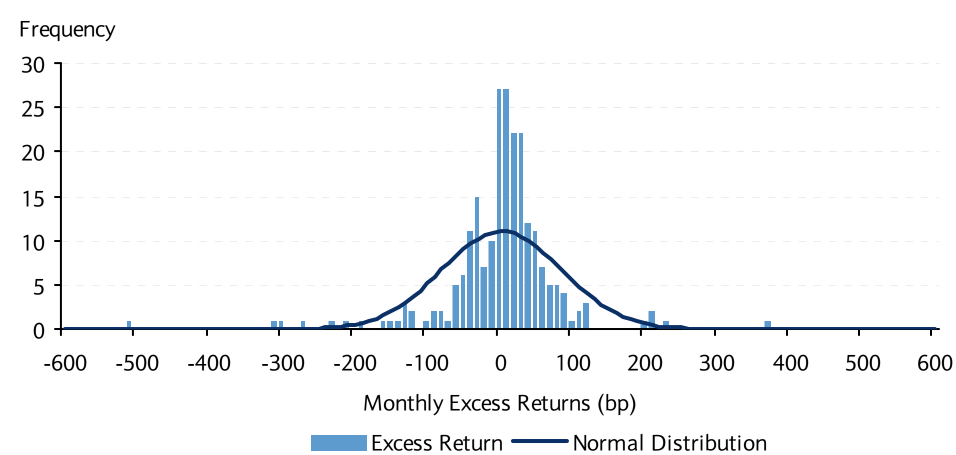
Using standard deviation or tracking error to sufficiently express portfolio risk implicitly assumes that the portfolio's returns follow a normal distribution. This is approximately correct if the individual positions in the portfolio are normally distributed, or the portfolio is sufficiently diversified: indeed, due to the law of large numbers, the distribution of a large portfolio of independent positions approaches the normal distribution regardless of the distribution of the individual positions.

However, portfolio returns can significantly deviate from normality; credit asset returns provide a good example. Most of the time, a credit portfolio will produce a return slightly above a corresponding duration-matched Treasury portfolio. However, occasionally there is an event that can produce substantial portfolio losses. Figure 1 presents the distribution of monthly excess returns for the U.S. Corporate Baa Index from August 1988 through July 2007. In this example, we chose to exclude the period after July 2007 to investigate tail risk behaviour over periods not overwhelmed by the 2008-09 crisis.

This index is broad and well diversified: 425 issuers with 1,223 issues as of August 2007. Yet, as seen in the chart, its distribution of excess returns is quite different from a normal distribution with the same mean and standard deviation.

During this period, six of the 228 (2.63%) monthly excess return observations are lower than 2.5 standard deviations below the mean monthly excess return. If these returns were normally distributed with the same mean (-3bp) and standard deviation (82bp) as the empirical sample, only 1.4 of the 228 (0.62%) of the observations would be this low. The distribution of Baa excess returns displays fat negative tails, despite the fact that the portfolio is very well diversified and there is a large number of monthly return observations.

Figure 1: Distribution of Monthly Excess Returns, U.S. Corporate Baa Index vs. Normal Distribution, August 1988-July 2007

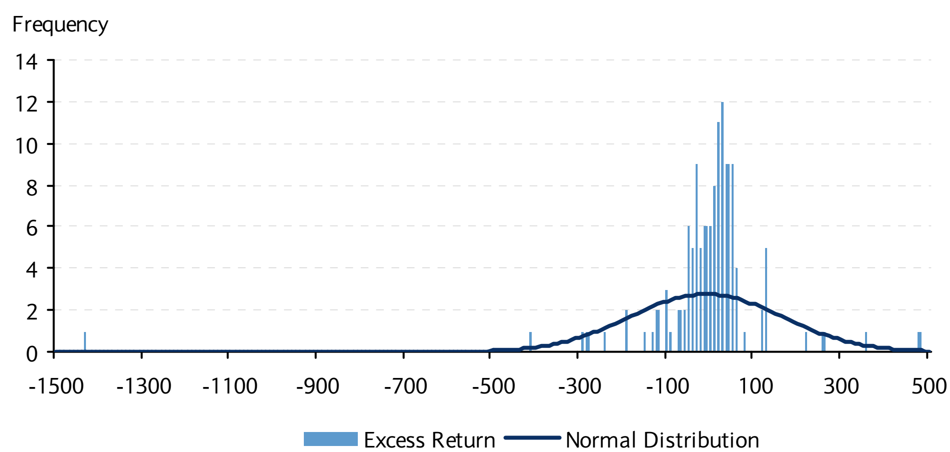


Source: Barclays Capital

For credit portfolios, the deviation from normality usually becomes accentuated as the portfolio becomes less diversified or of lower quality. Figure 2 shows the distribution of monthly excess returns for the Utility Baa Index since August 1997 until July 2007. For this somewhat less-diversified index (101 issuers with 268 issues as of August 2007), its distribution of excess returns is very different than a corresponding normal distribution with the same mean and standard deviation.

Figure 2 shows that during August 1997-July 2007, only one of the 120 (0.83%) observations of the monthly excess return of the Baa Utility Index is lower than 2.5 standard deviations below the mean monthly excess return (-446bp). If the distribution were normal, we would expect about 0.7 such months (0.60%) – close to actual experience. However, this lowest observation is such an extreme tail event – a monthly excess return for July 2002 of -14.25%, or 8.5 standard deviations below the mean, arising from the turmoil of the California energy crisis – that it would be highly unlikely ever to occur if returns were normally distributed. Indeed, a normal model would predict that such an extreme observation would occur only once every 87 trillion centuries, equal to about half a million times the lifetime of the universe.[2]

Figure 2: Distribution of Monthly Excess Returns U.S. Corporate Baa Utility Index vs. Normal Distribution, August 1997-July 2007



Source: Barclays Capital

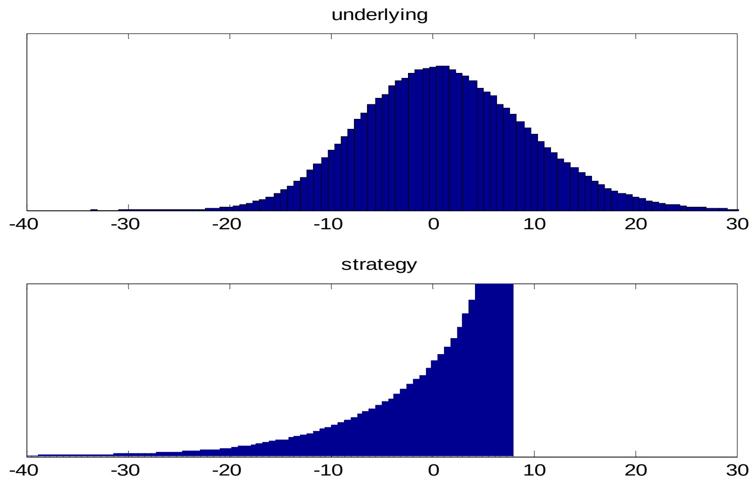
Besides credit and other market factors, another significant source of tail risk in a portfolio is derivative instruments such as options. Consider the probability distribution of returns of two alternative strategies.

The first strategy ("asset only") is a long position in a stock. Assume the stock's initial value is $100 and its annualized compounded return has an expected value of 6%, with a volatility of 20%. The second strategy ("covered call", or "derivative") is long two shares of the same stock but also short six call options with a strike price of $110 expiring in five months.

[2] What we actually show here is that the *unconditional* distribution of excess returns cannot possibly follow the normal distribution. Beyond the obvious fact that excess returns depend on spread duration, there may be other variables that affect the distribution of excess returns. One such variable we use in our risk models is the level of spreads. However, even if one generates the distribution of excess returns normalized by duration and spreads, the July 2002 return is still 7.2 standard deviations below the mean, implying a frequency of once in every 14 lifetimes of the universe under the normality assumption.

Consider an investment horizon of two months. The P&Ls of both strategies have the same expected value ($1.35) and standard deviation ($8.30). If these were the only quantities used to describe the P&L distribution, we would consider these two strategies as equally risky.

Figure 3: P&L Distributions for "Asset Only" and "Derivative" Strategies



Source: Barclays Capital

Figure 3 shows the distribution of the P&L at a two-month horizon for each strategy. Of course, as the figure indicates, these are fundamentally different strategies. While investing in the asset strategy provides a standard return profile with similar exposures to upside or downside risk, the derivative strategy has been structured to provide a small positive return in most scenarios while eliminating the probability of large positive returns and increasing the probability of large negative ones.

Each strategy's complete risk profile is provided by its entire probability distribution of returns (and any risk model should seek to represent it). Naturally, the left tail of the distribution – the one representing extreme losses – is of particular interest to investors. To summarize the information provided by the return distribution with regard to extreme negative returns, portfolio managers typically rely on two measures:

Value at Risk (VaR): This is a portfolio's return (or tracking error) *threshold value* such that the portfolio is expected to outperform this value a specified percentage of the time. VaR is typically reported as a loss, i.e., the negative of the above definition. For example, if a portfolio's tracking error VaR (at a 99% confidence level) is reported as 20bp, then 99% of the time the portfolio is expected not to underperform its benchmark by more than 20bp. Alternatively, one could say that the portfolio's tracking error is expected to be worse than -20bp 1% of the time. If we represent the probability distribution with a set of, say, 100,000 equally likely scenarios, the 99% confidence VaR, $VaR_{99\%}$, represents the *best* among the worst 1,000 scenarios.

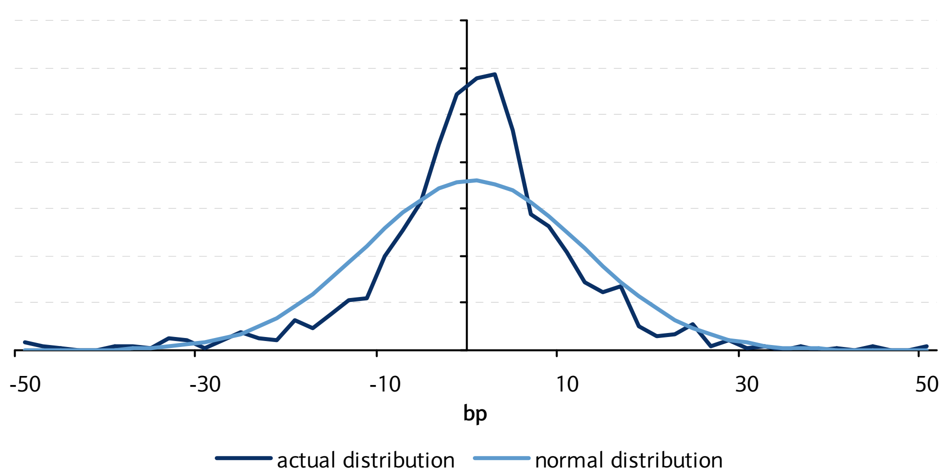
Some portfolio managers feel that VaR is an inadequate measure of tail risk because it is only a threshold value and does not provide information about the extent of the losses beyond that. To highlight this shortcoming, imagine a bond shortly before its maturity that has a small chance to default, say 0.9%, in which case it would be almost worthless. If it does not default, it would be worth 100. If the current value of the bond is 99 and we represent its P&L distribution with 100,000 scenarios, then in 900 of these scenarios the P&L would be -99, and in the remaining 99,100 scenarios it would be +1. The $VaR_{99\%}$ of a portfolio holding this bond – the (negative of) the P&L of the 1,000th worst scenario – is -1, a number that completely ignores the possibility of default. In other words, VaR lacks certain fundamental properties that a portfolio manager would expect from a good risk measure.[3]

Expected Shortfall (ES): To overcome the shortcomings of VaR, many portfolio managers have turned to ES, the average *loss* of all the worst-case scenarios beyond the threshold. Using the same example above, in a set of 100,000 equally likely scenarios, the 99%-confidence ES, $ES_{99\%}$, is the average loss among the worst 1,000 scenarios. For the above example, the $ES_{99\%}$ is 89, which gives the portfolio manager a better representation of the potential losses faced in the worst 1% realizations of portfolio performance.

For the example offered in Figure 3, the asset strategy (long stock) has a $VaR_{99\%}$ of \$16 and an $ES_{99\%}$ of \$19, while the derivatives strategy (long stock, short calls) has a $VaR_{99\%}$ of \$29 and an $ES_{99\%}$ of \$40, representing significantly higher tail risk. Even though both strategies have the same expected returns and standard deviations, they have very different tail risks.

While credit assets and derivatives are well-known sources of tail risk, there are others that may be less familiar. Indeed, many asset returns exhibit non-normal, "fat-tailed" behavior.[4] For example, Figure 4 presents the empirical distribution of realized one-week changes in the 6m USD Libor rate (in bp) between 1987 and 2007. It then overlays the normal probability distribution with the same mean and variance as the historical distribution of weekly rate changes.

Figure 4: Weekly Changes in 6m USD Libor: Actual vs. Normal Distribution, 1987-2007



Source: Barclays Capital

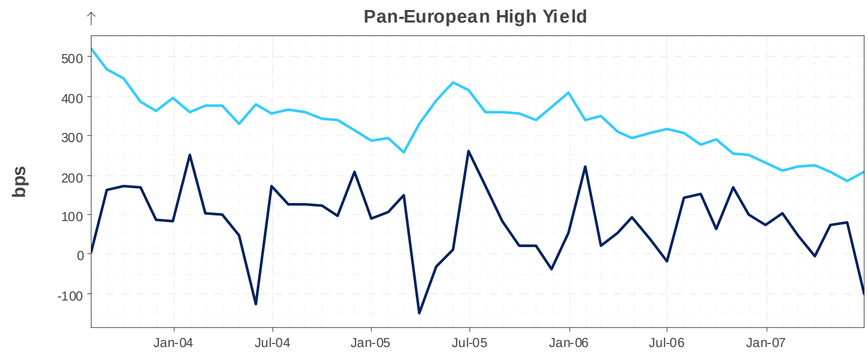
The empirical distribution is significantly different from the normal. While both have means of about 0 and standard deviations of about 12 bp, the normal distribution would expect weekly moves larger than 40 bp in magnitude to occur roughly twice every 20 years. In contrast, over our 20-year sample period, such moves have occurred 12 times, or six times as often.

The above observations have significant implications for portfolio tail risk measurement. The recent turmoil in the credit markets, after a long period of healthy returns and relatively low volatility, provides an illustrative, and cautionary, case study.

[3] For a detailed discussion on the subject, please see the seminal Artzner, Delbean, Eber, and Heath (1999) paper.
[4] See Purzitsky (2006).

As of June 2007, at the very start of the current volatile period, the Pan-European High Yield Index had a spread of 210 bp (just above its historical low of 187 bp in May 2007) and a four-year monthly standard deviation of returns of 88 bp. Models that measure risk by analyzing returns over a rolling historical window would tell a manager that the worst month in the previous four years had produced a loss of 151bp (Figure 5).

Figure 5: Pan-Euro High Yield Index OAS and Monthly Total Return July 2003-June 2007



Key	Axis	Name	Last	Minimum		Maximum		Mean	SD
—	Left	Pan-European High Yield - Monthly Total Return	-102	-151	03/31/2005	262	06/30/2005	83	88
—	Left	Pan-European High Yield - OAS	210	187	05/31/2007	522	07/31/2003	331	71

Source: Barclays Capital

The Barclays Capital Global Risk Model ran as of June 29, 2007, estimated the monthly total return volatility of the index to be 90bp, in line with the four-year historical estimate. A naïve calculation of tail risk based on this estimate and assuming that the index monthly total return follows the normal distribution indicates that the average return in the worst 1% of scenarios ($ES_{99\%}$) would be about 240bp below the expected carry of the index (57 bp/ month) – a loss of 183bp, close to the worst monthly loss over the previous four years.

Figure 6: Risk of the Pan-Euro HY Index vs. Cash on June 29, 2007 according to the Barclays Capital Tail Risk Model

POINT® — Global Risk Model

VaR Summary — 6/29/2007

Portfolio: Pan Euro HY (Statistics) - EUR
Benchmark: EUR Cash
Reporting Units: Returns in bps/month

	Portfolio vs. Benchmark		Portfolio		Benchmark	
	VaR 99.0%	ES 99.0%	VaR 99.0%	ES 99.0%	VaR 99.0%	ES 99.0%
Total (EUR)	2,298,965.5	3,419,571.1	2,298,965.5	3,419,571.1	0.0	0.0
Total (bp)	265.8	395.4	265.8	395.4	0.0	0.0
Carry		23.0		57.0		34.0

Source: POINT®

The estimate of the Tail Risk Model for the average return in the worst 1% of scenarios is 395bp below the expected carry (Figure 6, ES 99%)[5], i.e. a loss of 338 bp – about double the size of the naïve estimates. In July 2007, the index experienced a loss of 340bp, very close to the 99% prediction of the Tail Risk Model, but more than double the amount of the other estimate.

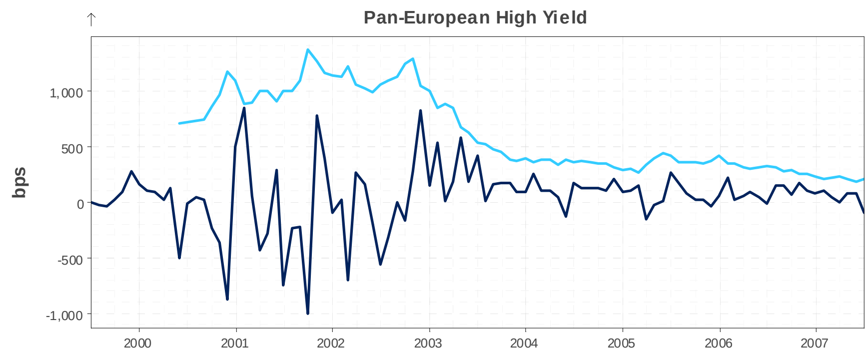
[5] Notice that the Tail Risk Model reports both VaR and ES as *losses net of carry*. To estimate the predicted magnitude of losses, the expected carry must be subtracted from VaR or ES.

How is our risk model able to produce a reasonable estimate of extreme losses while producing a volatility estimate that is consistent with recent history?

1. The first step is to decouple regular volatility – typically consistent with recent history – from tail risk, which is driven by extreme infrequent events such as market crashes and defaults.

2. Since such events are infrequent by nature, it is likely that they may not be present in recent history. For this reason, the second step is to scan as much history as possible for the occurrence of such events. Indeed, if we take a look at the full history of the index (Figure 7), we discover a wildly volatile period in the early 2000s. The full sample estimate for the total return volatility is 293bp, and the worst observed loss is 1008bp.

Figure 7: Pan-Euro High Yield Index OAS and Monthly Total Return May 1999-June 2007



Key	Axis	Name	Last	Minimum		Maximum		Mean	SD
	Left	Pan-European High Yield - Monthly Total Return	-102	-1008	09/28/2001	850	01/31/2001	44	293
	Left	Pan-European High Yield - OAS	210	187	05/31/2007	1373	09/28/2001	616	355

Source: Barclays Capital

3. Estimating tail risk with simple risk measures based on the relatively short full history of this index may overestimate the true exposure to extreme losses, since almost half of the historical observations come from the extraordinarily volatile period of 2000-03. For this reason, the third step entails the use of a model that has the richness to adapt to low regular volatility and at the same time allow for the possibility of large extreme losses. Our model allows separate calibration of the tails of the returns distribution to the entire history of returns, while the body of the distribution can be calibrated to recent history. In addition, our default risk model accounts for the possibility of default events that further drive extreme losses without a commensurate effect on regular, i.e., normal market, volatility.

4. The final step is to ensure that historical results are interpreted in their proper context. For example, we know that the risk of credit securities increases with the level of spreads. Although spread levels of the Pan-European High Yield Index skyrocketed to 1000+ bp during the volatile period after the turn of the century, those in July 2007 were significantly lower, so we reduced our estimates of risk commensurately. Further, since the history of this index is short, we may have to adjust the estimated frequency of extreme events (and, as a result, the probability of extreme losses) to make it more consistent with that of similar indices with a longer history.

1.2. Barclays Capital Tail Risk Modeling Framework

The Barclays Capital Tail Risk Model builds on the risk modeling framework of the Barclays Capital Global Risk Model.[6] Intuitively, the Global Risk Model first decomposes a bond's return into changes in risk factors and the bond's "loading" or "exposure" to each of them. For example, an intermediate USD bond is likely exposed to changes in the 5y USD key rate (a risk factor) and its exposure to that risk factor is the bond's 5y key rate duration. The Global Risk Model identifies the factors (both systematic and security-specific) and exposures (e.g., KRD and OASD) that drive individual securities and, hence, portfolio monthly return volatility.

Care is taken to ensure that we identify factors that exhibit stability in their behavior. In other words, we want to be as certain as possible that the historical behavior of a risk factor is likely to be its behavior in the future. Using the Barclays Capital Index database, we employ statistical techniques to generate long time series of historical observations for each of these factors, as well as their volatilities and correlations with other factors.

To generate a complete distribution for each factor, the Tail Risk Model treats each risk factor as a random variable. However, we do not necessarily assume it is a normal random variable. Instead, we find a distribution (typically a "fat-tailed" *t*-distribution) that best describes each factor's historical behavior, including its tail behavior.

Of course, it is possible that in our sample history for any given factor there is an absence of extreme events. However, this does not necessarily imply that an extreme event will never occur. Similarly, an occurrence of a recent extreme event does not imply that it will continue occurring with the same frequency. To deal with these circumstances, we use an estimation technique that imposes structure on each risk factor to make use of all available data as much as possible. Given the rarity of extreme events, our estimation of tail risk makes use of the entire available history. However, since the bulk of a risk factor's distribution (i.e., non-extreme events) represents "regular" risk in a normal market environment, our estimate of a factor's regular market behavior relies more heavily on recent history, as opposed to an equal weighting of all historical observations.

Having specified the behavior of risk factors, the Tail Risk Model generates a security's return distribution by conducting many simulations, or "draws," from the factors' distributions. For example, for one simulation or draw, we sample from each of the security's risk factors (including security-specific risk). In other words, we make a random draw from each risk factor's fitted distribution. After incorporating the correlation among the risk factors, we then multiply these "simulated" factor observations by the bond's current corresponding factor exposures. Summing across the products of all the security's risk factors multiplied by their factor exposures, we arrive at the security's total return for this one particular scenario. We then repeat this simulation many thousands of times to generate an entire distribution of possible returns for the bond.

For a portfolio, the Tail Risk Model conducts this exercise for all the securities in the portfolio and then aggregates the results scenario by scenario to get the entire return distribution for the portfolio. To produce the portfolio's entire tracking error distribution, we also perform the same simulation for the portfolio's benchmark and take the difference in returns (portfolio minus benchmark) for each simulation run.

Once we have generated the entire distributions for total return and tracking error, we can easily calculate VaR, ES and any other risk measure at any chosen confidence interval. Further, the Tail Risk Model breaks down portfolio risk into additive contributions of various

[6] See Portfolio Modeling Group research.

risk factors, allowing portfolio managers to quickly identify sources of tail risk in the portfolio and see if it matches their desired risk profile. As such, it is an invaluable tool for portfolio and risk managers, helping them to understand portfolio risk better and be more prepared to respond to ever-increasing regulatory scrutiny.

Several regulatory jurisdictions have issued guidelines for the measurement of portfolio tail risk.[7] Our model is generally consistent with such guidelines and has the flexibility to adapt easily to specific interpretations. We are working with clients who wish to use it for regulatory purposes and to have it explicitly approved by regulators for this purpose.

1.3. The Barclays Capital Advantage

Portfolio managers have been using the Barclays Capital Global Risk Model for over a decade. The model is continually refined and enhanced to reflect the latest developments in risk modeling techniques and new security coverage.[8] The new Barclays Capital Tail Risk Model is built on the same underlying modeling foundation, which offers the manager several advantages.

First, for over 30 years, Barclays Capital Indices have held a dominant position in the fixed-income marketplace among U. S. domestic and global investors. As a result, we have bond-level trader-supplied price and indicative data that have been very closely scrutinized by many investors benchmarked to our indices. We use this high-quality historical information at the individual security level to calibrate and test the Global Risk Model. All the parameters of the Global Risk Model, and the new Tail Risk Model, are estimated using the best and most comprehensive market data available.

Second, for many years Barclays Capital has worked closely with portfolio managers who have Barclays Capital indices as benchmarks. As a result of these relationships, we are working toward developing tools that a portfolio manager can easily use and that generate specific market actions to achieve a desired goal. Our modeling efforts ultimately have the portfolio manager in mind: we aim to make the interpretation of risk factors and our risk reports as intuitive as possible. As shown in Part I, the output from the Tail Risk Model is simple to interpret, which ought to help a portfolio manager quickly identify sources of tail risk and take action, if desired.

Third, as briefly described above, our Tail Risk Model is built on the foundation of our Global Risk Model. Therefore, tail risk – similar to portfolio or tracking error volatility – is explained in terms of securities' analytical sensitivities to market risk factors, computed using Barclays Capital security valuation models. These risk factors, in turn, can be intuitively interpreted by portfolio managers as changes of rates, spreads, implied volatility, equity returns, etc. We see this as a clear advantage of our methodology over purely statistical methods such as principal component analysis of the return history.

Finally, diversification risk is very high on the investor agenda, especially in credit portfolios. Barclays Capital portfolio risk modeling also quantifies security-specific risk by taking advantage of our index data to analyze historical returns of individual securities. We then estimate a security's idiosyncratic risk using the residual returns of individual securities, i.e., returns unexplained by a combination of all the systematic risks.

[7] See, e.g., European Union (2004) and (2005) and Elvinger et al. (2003)
[8] Among others, the Global Risk Model now handles municipal securities, equities, equity index futures, equity options, convertibles, corporate loans, and credit derivatives including standard CDO tranches. Plans exist for coverage to include other instruments in due course.

PART II: PORTFOLIO APPLICATIONS

The best way to become familiar with the Tail Risk Model is to review the model's risk reports. We present and discuss sample risk reports for three different types of portfolios.

The first is a traditional long-only cash portfolio benchmarked against the Barclays Capital U.S. Aggregate Index. We will walk through each section of the report (and, for the interested reader, relate specific output to the corresponding equations presented in Part II of this publication). The second is the U.S. High Yield Index benchmarked against cash. In the third, we construct a very negatively convex portfolio to highlight how tracking error volatility alone is not sufficient to properly describe risk. Instead, the measures provided by the Barclays Capital Tail Risk Model, namely VaR and ES, provide a much more accurate description of the portfolio's return (P&L) distribution and possible extreme behavior.

2. MEASURING THE TAIL RISK OF A LARGE, DIVERSIFIED PORTFOLIO

Here, we examine a sample tail risk report for a traditional long-only cash portfolio benchmarked against the U.S. Aggregate Index.

Figure 8: Portfolio and Benchmark Comparison: Traditional Portfolio with U.S. Aggregate Benchmark, July 2009

Net Market Weight % (Portfolio-Benchmark)			
Sector	**PORT**	**BMK**	**Net**
Treasury/Agency/Muni	35.0	34.9	0.1
Credit - Inv. Grade	26.2	22.0	4.2
Credit - High Yield	1.0	0.0	1.0
Securitized	36.1	41.9	-5.8
Inflation-Linked	0.0	0.0	0.0
Emerging Markets	1.7	1.1	0.6
Other (Cash, Deriv. etc)	0.0	0.0	0.0
Total	**100.0**	**100.0**	**0.0**

Summary Analytics			
	PORT	**BMK**	**Net**
OAD	4.7	4.3	0.4
OAS	93.9	85.5	8.4
OASD	4.8	4.4	0.3
OAC	-0.1	-0.3	0.3
Vega	-0.0	-0.0	0.0

USD Net Key Rate Duration Exposures			
Key Rate	**PORT**	**BMK**	**Net**
6 month	0.2	0.2	0.0
2 year	0.5	0.6	-0.1
5 year	1.0	1.2	-0.1
10 year	1.3	1.2	0.1
20 year	1.5	0.7	0.8
30 year	0.2	0.5	-0.2

Portfolio Risk Summary Report

Portfolio: **TAIL : Agg Portfolio**
Index: **US Aggregate (Statistics)**
Reporting Units: **Returns in bps/month**

Factor Partition Summary			
	Contrib to TEV	**Isolated TEV**	**Systematic Beta**
Total	**27.2**	**27.2**	**1.1**
Systematic	24.0	25.5	1.1
Curve	14.5	18.6	1.0
Swap Spreads	0.2	3.6	0.0
Volatility	1.9	5.3	-0.0
Spread Gov-Related	0.0	1.8	0.0
Spread Credit and EMG	0.1	6.8	0.1
Spread Securitized	7.3	12.2	-0.0
Idiosyncratic	2.8	8.8	-
Credit default	0.3	3.0	-

Source: Barclays Capital

In Figure 8, we show a comparison of the portfolio and the benchmark, with respect to sector exposures, summary analytics and key-rate durations, allowing us to get an overview of the major risk exposures of the net portfolio return. We can see that the portfolio is underweighted in the securitized sector, in particular to CMBS (not shown), and overweighted in credit, including a small out-of-index exposure to the risky high yield sector. In addition, it is long duration, convexity and spread exposure with respect to spread level and spread duration. The key-rate profile reveals that the portfolio is not uniformly long duration along the curve and that it also has significant exposure to curve reshaping, especially long exposure to the 20y point and short exposures to the short and long ends of the curve. We also show the tracking error volatility (TEV) of the portfolio vs. the benchmark (27.2bp/month), along with its breakdown to major risk factors. The major contributors to TEV are curve factors and securitized spread factors. Credit spreads risk has minimal contribution to TEV because of its strong negative correlation to curve risk.

2.1. VaR Summary Report

Figure 9 is the VaR Summary Report, which gives the overall risk, as well as the breakdown of risk into systematic, idiosyncratic and default risk components. The systematic risk component is further broken down into contributions from broad sub-categories of risk factors such as currency and yield curve. There are three separate panels with the same structure to present information on three different universes: “Portfolio vs. Benchmark,” “Portfolio,” and “Benchmark.” At the bottom of the page are histograms of the three total return simulations.

The first row of the report is expressed in P&L space, which is obtained by scaling the basis point returns by the beginning market value of the portfolio. All other numbers in the table are presented in return space (in bp/month).

Numbers reported in the large first block of rows are contributions to risk as defined in Section 9. These (in bp/month) provide an easy way to identify the main drivers of tail risk. By definition, contributions from all the individual categories of risk sum to the total risk. So all the contributions to $VaR_{99\%}$ of tracking errors (i.e., the “Portfolio vs. Benchmark” universe) sum to 67.8bp/month. The second block of rows reports isolated systematic, idiosyncratic, and default risks. We assume independence between these three categories. The last row of the report shows the monthly carry return for each universe.[9]

Within each of the panels, the first column shows the VaR and its breakdowns (see Equation *(54)*). In this example, the $VaR_{99\%}$ is 67.8bp/month, which means that among the worst 1% of the scenarios (i.e., simulation runs), the best net return (i.e., the “best-worst return”) is -67.8bp/month.[10] To understand what is happening in those worst 1% of scenarios, we must go beyond VaR and look at the ES value, which summarizes the average net return of all those 1% worst-case scenarios. The total ES and contributions to it by various blocks of factors (see Equation *(57)*) are reported in the third column. In this example, $ES_{99\%}$ is 89.6bp/month. Both numbers are slightly greater than what a normal distribution with volatility equal to the estimated TEV would have predicted (about 63bp/month and 82bp/month, respectively), suggesting a mildly thick-tailed distribution.

Using the breakdown of contributions, we can drill down and find the main sources of tail risk. For example, for this particular portfolio, the systematic risk contributions from “Curve,” “Spread Credit and EMG,” and “Spread Securitized” are the most significant components of VaR and ES.

[9] The monthly carry is calculated as the annual yield-to-worst divided by 12.
[10] Note that reported VaR and ES are all returns excluding carry. Therefore, to get the total return, carry should be added back. For simplicity, we refer to ex-carry return as return.

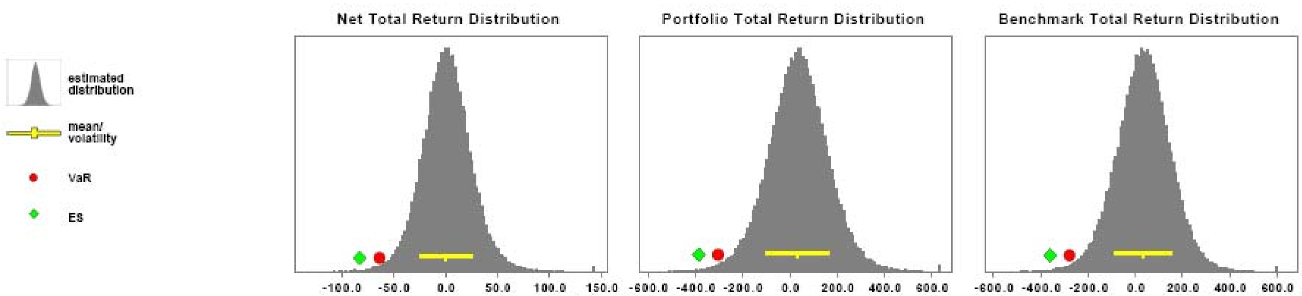
Figure 9: VaR Summary for the Traditional U.S. Aggregate Portfolio, July 2009

POINT® Global Risk Model

VaR Summary **7/31/2009**

Portfolio: TAIL : Agg Portfolio - USD
Benchmark: US Aggregate (Statistics)
Reporting Units: Returns in bps/month

	Portfolio vs. Benchmark		Portfolio		Benchmark	
	VaR 99.0%	ES 99.0%	VaR 99.0%	ES 99.0%	VaR 99.0%	ES 99.0%
Total (USD)	340,023.8	449,153.9	1,683,142.7	2,094,473.3	1,562,542.1	1,989,961.4
Total (bp)	67.8	89.6	335.6	417.6	311.5	396.8
Systematic Contributions	58.7	80.9	334.1	416.1	311.0	396.3
Curve	22.9	22.9	276.0	298.4	228.3	216.8
Swap Spreads	0.8	0.1	5.5	2.5	6.4	4.2
Volatility	6.7	11.0	-0.5	-0.5	0.9	0.8
Spread Gov-Related	0.1	0.3	3.0	4.1	4.4	6.0
Spread Credit and EMG	11.3	15.2	54.9	97.9	68.5	128.9
Spread Securitized	17.0	31.3	-4.7	13.6	2.5	39.7
Idiosyncratic Contributions	5.4	4.4	0.9	0.6	0.3	0.2
Default Contributions	3.8	4.2	0.6	0.8	0.3	0.3
Systematic Isolated	63.7	86.0	334.8	416.8	311.2	396.5
Idiosyncratic Isolated	21.3	24.7	23.0	26.8	11.7	13.7
Default Isolated	13.6	30.3	16.6	35.3	5.7	9.9
Carry		-0.5		31.8		32.4



Source: Barclays Capital

It is interesting to note in Figure 9 that securitized spreads contribute roughly 25% to overall VaR (17.0bp/67.8bp) but approximately 35% of the net return $ES_{99\%}$ (31.3bp/89.6bp). This is not surprising, as the portfolio has a significant underweight to the securitized sector (Figure 8), which is well known for its tail risk, especially in the non-agency sub-sector.

It is also interesting to note that while credit spreads ("Spread Credit and EMG") contributed very little to TEV (Figure 8), due to their negative correlation to rates, they contribute significantly to tail risk. The reason is that the distribution of credit spreads is significantly thicker tailed than the distribution of rates, causing significant tail risk from credit spreads even when their contribution to TEV is minimal.

Estimated distributions of net returns (not 'net losses') are plotted at the bottom of Figure 9 separately for each universe. For this well-diversified aggregate portfolio, the distributions of all the universes are bell-shaped but are not close to normal distribution. Volatility, VaR and ES are highlighted. Since we plot the returns instead of losses, both 99% confidence $VaR_{99\%}$ and $ES_{99\%}$ fall on the left tails.

2.2. VaR Systematic Details Report

To help managers drill down into the contributions of particular systematic risk factors, we provide the VaR Systematic Details Report (Figure 10), which lists all of the portfolio's and benchmark's active risk factors. The length of this report varies depending on the portfolio and benchmark holdings, which determine the set of active risk factors to display. Risk factors are grouped into broad categories for ease of navigation.

In this example, the VaR Systematic Details Report expands to four pages. To save space, we present in Figure 10 a truncated version containing factors with relatively large contributions to VaR. This report is structured very similarly to the VaR Summary Report. The only difference is that besides absolute contributions to VaR and ES, we also display the contributions as percentages.

Since curve and MBS/securitized factors are the main contributors to the systematic risk, we will drill down into these categories (credit risk of the benchmark seemed well matched in this portfolio, with the contribution to TEV of the “Spread credit” factors of 0.2bp/month). Under the "Key Rates and Convexity" category, the contributions of the various key rates to tail risk are commensurate to the exposures we observed in Figure 8. The 20y key rate is the largest contributor to VaR and ES for the portfolio. Under the "MBS Spread & Vol" category, "USD MBS New Current" and "USD MBS New Premium" are the top contributors to tail risk; “USD CMBS AAA” is also a significant contributor. The "USD MBS Seasoned Premium” factor serves as a hedge and reduces tail risk.

Figure 10: VaR Systematic Details for the Traditional U.S. Aggregate Portfolio, July 2009

POINT® Global Risk Model

VaR Systematic Details 7/31/2009

Portfolio: TAIL : Agg Portfolio - USD
Benchmark: US Aggregate (Statistics)
Reporting Units: Returns in bps/month

	Portfolio vs. Benchmark				Portfolio				Benchmark			
	VaR 99.0%	%	ES 99.0%	%	VaR 99.0%	%	ES 99.0%	%	VaR 99.0%	%	ES 99.0%	%
Regular Systematic	58.7	100.0	80.9	100.0	334.1	100.0	416.1	100.0	311.0	100.0	396.3	100.0
Key Rates and Convexity												
USD 6M key rate	0.0	0.1	0.1	0.1	0.3	0.1	-0.7	-0.2	0.5	0.2	-0.9	-0.2
USD 2Y key rate	-2.9	-4.9	-3.5	-4.4	12.1	3.6	10.1	2.4	15.1	4.8	9.8	2.5
USD 5Y key rate	-5.4	-9.3	-6.1	-7.5	49.2	14.7	46.6	11.2	51.4	16.5	41.0	10.3
USD 10Y key rate	6.2	10.5	6.5	8.0	99.7	29.8	111.0	26.7	84.4	27.1	85.3	21.5
USD 20Y key rate	39.0	66.4	41.6	51.4	100.0	29.9	114.2	27.4	44.6	14.4	46.5	11.7
USD 30Y key rate	-11.0	-18.7	-11.8	-14.6	13.8	4.1	16.0	3.8	27.0	8.7	28.4	7.2
USD Convexity	-3.0	-5.2	-3.9	-4.8	0.9	0.3	1.3	0.3	5.2	1.7	6.8	1.7
MBS Spread & Vol.												
USD MBS Short/Derivative Volatility	1.9	3.2	2.9	3.6	-0.1	-0.0	-0.1	-0.0	0.3	0.1	0.3	0.1
USD MBS Long/Derivative Volatility	4.5	7.7	7.9	9.7	-0.4	-0.1	-0.4	-0.1	0.6	0.2	0.5	0.1
US 30-Year Mortgage Spread	-1.0	-1.7	-1.9	-2.3	-2.1	-0.6	-3.8	-0.9	-3.1	-1.0	-6.4	-1.6
US 15-Year Mortgage Spread	1.0	1.7	1.5	1.8	-0.1	-0.0	-0.7	-0.2	-0.1	-0.0	-0.5	-0.1
US Hybrid Mortgage Spread	-0.1	-0.1	-0.1	-0.2	-0.0	-0.0	-0.1	-0.0	-0.1	-0.0	-0.3	-0.1
USD MBS New Discount	-0.0	-0.0	0.1	0.1	0.0	0.0	0.0	0.0	0.6	0.2	0.8	0.2
USD MBS New Current	7.1	12.2	14.4	17.8	0.0	0.0	0.0	0.0	1.0	0.3	5.3	1.3
USD MBS New Premium	14.1	24.0	23.4	28.9	-0.2	-0.0	0.6	0.1	0.2	0.1	10.8	2.7
USD MBS Seasoned Discount	-0.0	-0.0	-0.0	-0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
USD MBS Seasoned Current	-1.3	-2.2	-2.4	-2.9	1.0	0.3	2.1	0.5	0.6	0.2	1.1	0.3
USD MBS Seasoned Premium	-12.0	-20.5	-19.5	-24.1	-8.2	-2.5	9.0	2.2	-2.6	-0.8	14.2	3.6
USD MBS GNMA 30Y	0.5	0.8	0.5	0.6	0.9	0.3	0.7	0.2	0.4	0.1	0.2	0.1
USD MBS Conv 15Y	1.3	2.3	1.6	1.9	2.2	0.7	2.3	0.5	0.9	0.3	0.8	0.2
USD MBS GNMA 15Y	-0.0	-0.0	-0.0	-0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
US Agency Hybrid Premium	0.1	0.2	0.2	0.3	0.2	0.1	0.5	0.1	0.4	0.1	0.9	0.2
CMBS Spread												
USD CMBS AAA	5.6	9.5	10.8	13.4	0.1	0.0	1.7	0.4	3.1	1.0	10.4	2.6

Source: Barclays Capital

Note that the MBS factors contribute more to the $ES_{99\%}$ than to $VaR_{99\%}$, which shows that the extreme observations experienced by these factors are larger in magnitude than the ones experienced by other factors. The relative contribution to tail risk may differ dramatically from the contribution to volatility: for example, the "USD MBS New Current" and "USD MBS New Premium" have a contribution of 2.4% and 5.8% to tracking error volatility (from “Factor Exposure – Full Details” report, not shown), or about one-fourth of their contribution to VaR.

2.3. VaR Default Details Report

The VaR Default Details Report (Figure 11) presents contributions of default risk to $VaR_{99\%}$ and $ES_{99\%}$ for the top 15 issuer tickers.[11] Tickers are sorted by contribution of default risk to VaR. For our portfolio, TXU Energy (TXU), Mexican Government (MEX), CBS (CBS), and the Mexican Government Oil Company (PEMEX) are the main contributors to tracking error arising from default risk. For this portfolio, overall default risk is relatively low. It is interesting to note that the contribution of CBS to $VaR_{99\%}$ is bigger than its contribution to $ES_{99\%}$. This occurs because even if the CBS default probability may be high, its loss-given-default is probably lower than that of the other securities because of either higher recovery rate assumptions or depressed current valuations.

Figure 11: VaR Default Details for the Traditional U.S. Aggregate Portfolio, July 2009

POINT® Global Risk Model

VaR Default Details 7/31/2009

Portfolio: Agg Portfolio - USD
Benchmark: US Aggregate (Statistics)
Reporting Units: Returns in bps/month

	Portfolio vs. Benchmark				Portfolio				Benchmark			
	VaR 99.0%	%	ES 99.0%	%	VaR 99.0%	%	ES 99.0%	%	VaR 99.0%	%	ES 99.0%	%
Total Credit	1.3	100.0	1.1	100.0	0.4	100.0	0.6	100.0	0.3	100.0	0.3	100.0
TXU	0.3	20.0	0.2	22.0	0.2	42.3	0.2	27.1	0.0	3.0	0.0	2.2
MEX	0.4	34.2	0.5	43.4	0.0	0.5	0.1	22.5	0.0	0.5	0.0	11.6
CBS	0.2	13.9	0.1	8.5	0.0	1.2	0.0	0.0	0.0	0.1	0.0	0.8
PEMEX	0.1	10.5	0.1	11.3	0.1	21.9	0.0	5.9	0.0	2.0	0.0	1.9
DAIGR	0.0	3.7	0.1	6.8	0.0	0.1	0.0	4.9	0.0	0.1	0.0	1.9
BRAZIL	-0.0	-0.0	-0.0	-0.0	-0.0	-0.0	-0.0	-0.0	0.0	0.0	0.0	0.0
DVN	0.1	8.9	0.0	0.0	0.0	0.1	0.0	4.4	0.0	0.0	0.0	1.2
KPN	0.1	6.1	0.1	5.0	0.0	1.1	0.0	4.7	0.0	0.6	0.0	0.5
AMD	0.1	6.8	0.1	8.0	0.0	9.8	0.0	5.6	-0.0	-0.0	-0.0	-0.0
BRK	0.0	0.0	0.0	0.0	0.0	0.7	0.0	3.4	0.0	0.3	0.0	0.9
UBS	0.0	2.8	0.0	3.3	0.0	1.0	0.0	3.1	0.0	0.7	0.0	0.4
TWC	0.0	2.4	0.0	3.4	0.0	9.1	0.0	7.3	0.0	16.6	0.0	4.5
CMCSA	-0.0	-0.0	-0.0	-0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
F	0.0	1.8	0.0	1.9	0.0	1.5	0.0	3.4	-0.0	-0.0	-0.0	-0.0
NGGLN	0.0	0.0	0.0	0.0	0.0	3.1	0.0	1.9	0.0	0.0	0.0	0.2

Source: Barclays Capital

2.4. VaR Partitions Reports

The VaR Security Partition Report (top of Figure 12) presents the breakdown of net VaR (i.e., portfolio net of benchmark) by types of securities (rows) and by major types of risk (columns). As the note at the bottom of the report mentions, the partition used can be customized. The ES equivalent of the VaR Security Partition Report is ES Security Partition Report (not shown here).

Moreover, we can use a factor partition instead of a security partition; the results of such an exercise are reported in the VaR Risk Factor Partition Report (bottom of Figure 12). This matches in structure the top of the VaR Summary Report (Figure 9), adding more details and allowing the user to customize it. For example, the 17bp/month of the securitized sector contribution to $VaR_{99\%}$ can be split into contributions of 9.7bp/month from the US MBS sector, 6.5bp/month from the US CMBS sector and 0.8bp/month from the US ABS sector.

[11] N=15 by default, but a user can change this value.

Figure 12: VaR Security Partition Report and VaR Risk Factor Partition Report for the Traditional U.S. Aggregate Portfolio, July 2009

POINT® Global Risk Model

Value At Risk - Security Partition 7/31/2009

Portfolio: TAIL : Agg Portfolio
Benchmark: US Aggregate (Statistics)
Reporting Units: Returns in bps/month
Partition : GRM Default

		Net Contribution to VaR									
Security Partition Bucket	Net Market Weight (%)	Systematic							Idiosyncratic	Default	Total
		Curve	Swp Sprd	Vol	Gov Sprd	Crd & EMG	Sec'ed Sprd	Systematic Net VaR 99.0%	Net VaR 99.0%	Net VaR 99.0%	Net VaR 99.0%
Total	**.00**	**22.85**	**.82**	**6.66**	**.08**	**11.27**	**16.97**	**58.66**	**5.38**	**3.76**	**67.80**
Debt	.00	22.85	.82	6.66	.08	11.27	16.97	58.66	5.38	3.76	**67.80**
USD	.00	22.85	.82	6.66	.08	11.27	16.97	58.66	5.38	3.76	**67.80**
Treasuries	3.65	36.01	.00	-.00	.01	.00	.00	36.02	1.75	.00	**37.77**
Gov Agencies	-1.67	3.10	-.25	.20	.07	.76	.00	3.88	1.02	.00	**4.90**
Gov Non-Agencies	3.85	10.91	-.49	.02	.00	1.19	.00	11.62	6.32	1.03	**18.98**
Corporates	.00	-10.02	.63	.01	.00	9.32	.00	-.06	-1.09	2.72	**1.58**
Inv Grade	-.97	-11.18	.62	.02	.00	12.20	.00	1.66	-13.35	1.90	**-9.80**
High Yield	.97	1.16	.01	-.01	.00	-2.88	.00	-1.72	12.26	.83	**11.37**
MBS	-3.49	-15.32	1.09	6.44	.00	.00	9.70	1.91	1.08	.00	**2.98**
ABS	-.48	-.52	-.01	.00	.00	.00	.81	.29	-2.03	.00	**-1.74**
CMBS	-1.86	-1.31	-.14	.00	.00	.00	6.46	5.01	-1.68	.00	**3.33**

Note. A different partition can be defined to generate this report on the options screen.

POINT® Global Risk Model

VaR - Risk Factor Partition 7/31/2009

Portfolio: TAIL : Agg Portfolio
Benchmark: US Aggregate (Statistics)
Reporting Units: Returns in bps/month
Partition : GRM Global Factors

	Tail risk contributions					
	Portfolio vs. Benchmark		Portfolio		Benchmark	
Factor Partition Bucket	VaR 99.0%	ES 99.0%	VaR 99.0%	ES 99.0%	VaR 99.0%	ES 99.0%
Total	67.8	89.6	335.6	417.6	311.5	396.8
Systematic Contributions	58.7	80.9	334.1	416.1	311.0	396.3
Curve	22.9	22.9	276.0	298.4	228.3	216.8
YC USD-Yield/Swap Curve	22.9	22.9	276.0	298.4	228.3	216.8
Swap Spreads	0.8	0.1	5.5	2.5	6.4	4.2
Volatility	6.7	11.0	-0.5	-0.5	0.9	0.8
Yield Curve	6.7	11.0	-0.5	-0.5	0.9	0.8
Spread Gov-Related	0.1	0.3	3.0	4.1	4.4	6.0
Treasury Spreads	0.0	0.0	0.0	0.1	0.1	0.1
Agency Spreads	0.1	0.3	2.9	4.0	4.3	6.0
Spread Credit and EMG	11.3	15.2	54.9	97.9	68.5	128.9
Credit Investment Grade	13.2	18.0	52.1	92.0	66.2	124.9
Credit High Yield	-2.0	-2.5	-0.2	1.4	0.0	0.0
Emerging Markets	0.0	-0.2	3.0	4.5	2.4	4.0
Spread Securitized	17.0	31.3	-4.7	13.6	2.5	39.7
US-MBS	9.7	17.8	-6.2	10.6	-1.8	26.9
US-CMBS	6.5	12.3	1.5	3.0	4.2	12.0
US-ABS	0.8	1.2	0.0	0.0	0.1	0.7
Idiosyncratic Contributions	5.4	4.4	0.9	0.6	0.3	0.2
Default Contributions	3.8	4.2	0.6	0.8	0.3	0.3

Note. A different partition can be defined to generate this report on the options screen.

Source: Barclays Capital

3. TAIL RISK OF THE U.S. HIGH YIELD INDEX

In this section, we discuss the tail risk report of the U.S. High Yield Index against USD cash. As with the Global Risk Model, a manager can use the Tail Risk Model to compare a portfolio versus an index, a portfolio versus another portfolio, or an index versus another index.

In Figure 13, the VaR Summary Report as of July 31, 2009, we observe that the return distribution for this index vs. cash has $VaR_{99\%}$ of 1,741bp/month and $ES_{99\%}$ of 2,747bp/month. The standard deviation of the distribution as estimated by the TEV model is 507bp/month (see Figure 14) This is not a normal distribution. If it were, given a standard deviation of 507bp/ month, the $VaR_{99\%}$ and the $ES_{99\%}$ had been 1,181 and 1,354bp/month, respectively, which are significantly smaller than the ones produced by the model.

Figure 13: VaR Summary Report for the U.S. High Yield Index, July 2009

POINT®

VaR Summary

Portfolio: US HY (Statistics) - USD
Benchmark: USD Cash
Reporting Units: Returns in bps/month

	Portfolio vs. Benchmark	
	VaR 99.0%	**ES 99.0%**
Total (USD)	106,713,452.9	168,353,072.2
Total (bp)	1,741.2	2,746.9
Systematic Contributions	1,690.3	2,703.9
Curve	-130.7	-161.9
Swap Spreads	-1.2	-4.0
Volatility	1.6	2.0
Spread Credit and EMG	1,820.5	2,867.8
Idiosyncratic Contributions	7.1	6.2
Default Contributions	43.7	36.8
Systematic Isolated	1,699.7	2,710.2
Idiosyncratic Isolated	161.9	195.3
Default Isolated	263.4	342.0
Carry		94.1

Source: Barclays Capital

Figure 14: TEV Risk Summary Report for the U.S. High Yield Index, July 2009

Factor Partition Summary			
	Contrib to TEV	Isolated TEV	Systematic Beta
Total	506.8	506.8	0.0
Systematic	470.4	488.3	0.0
Curve	-39.5	144.2	0.0
Swap Spreads	1.4	37.1	0.0
Volatility	0.6	4.6	0.0
Spread Credit and EMG	507.9	547.1	0.0
Idiosyncratic	8.4	65.3	-
Credit default	28.1	119.3	-

Source: Barclays Capital

Somewhat surprisingly, the majority of risk is contributed by systematic factors – 97.1% (1,690.3bp/month) of $VaR_{99\%}$ and 98.4% (2,703.9 bp/month) of $ES_{99\%}$, while default risk contributes 2.5% (43.7bp/month) of $VaR_{99\%}$ and 1.3% (36.8 bp/month) of $ES_{99\%}$, and idiosyncratic risk contributions are very small. In the aftermath of the 2008 economic crisis, one might expect default risk to constitute a significant component of the risk of high yield securities. While it did increase significantly after the crisis, systematic market risk capturing credit, liquidity, and risk premia increased even more, thus reducing the contribution percentage of default (and idiosyncratic) risk.

To illustrate these points, we estimate the tail risk of the US HY index pre-crisis, as of June 29, 2007 (using the composition, loadings, and factors at the time) and as of July 31, 2009 (with the composition, loadings, and factors available as of that date). The results are summarized in Figure 15. Even with the spread duration of the index falling from 4.7 to 4.2 years, the rising spreads (from 290bp/month to 830bp/month) and increased spread volatility have caused the isolated systematic risk component of the $VaR_{99\%}$to increase more than 7.5 times (1700 vs. 220) and 10 times for $ES_{99\%}$ (2710 vs. 258). On the other hand, isolated default risk has increased by less than a factor of 2 (263 vs. 154 for $VaR_{99\%}$ and 342 vs. 240 for $ES_{99\%}$).

Figure 15: The Tail Risk of US High Yield Index before the Crisis (June 29, 2007) and during It (July 31, 2009)

	June 29, 2007				July 31, 2009			
	$VaR_{99\%}$		$ES_{99\%}$		$VaR_{99\%}$		$ES_{99\%}$	
	Contrib.	Isolated	Contrib.	Isolated	Contrib.	Isolated	Contrib.	Isolated
Total	266.8	266.8	330.9	330.9	1741.2	1741.2	2746.9	2746.9
Systematic	190.1	219.8	181.7	258.1	1690.3	1699.7	2703.9	2710.2
Idiosyncratic	6.0	39.6	4.6	46.8	7.1	161.9	6.2	195.3
Default	70.8	154.0	144.6	239.6	43.7	263.4	36.8	342.0

Source: Barclays Capital

To further understand why high-yield default risk has a relatively small increase, given the magnitude of the crisis, one must recall the inputs used to calculate the default risk. It is a combination of the risk of the default event as measured by the issuers' default probabilities and the loss given default that is the difference between the current valuation of bonds and their recovery value. While default probabilities have significantly increased and recovery rates decreased, loss given default has decreased because of the depressed valuation of the high yield bonds in the aftermath of the crisis. In other words, much of the potential default losses have already been realized. This also explains why isolated default $ES_{99\%}$ has increased less in percentage terms than isolated default $VaR_{99\%}$. Even though there are more defaults, losses after each default are smaller.

A particular feature of the default risk distribution is its very asymmetric nature, producing significant tail risk even when its contribution to TEV is quite small: most of the time, there are no defaults, but occasionally a big loss occurs (see equation *(19))*. Therefore, the effect of default events is more pronounced deep in the tails than in the body of the P&L distribution.

Returning to the VaR Summary Report in Figure 13 we notice that the majority of systematic risk is contributed by "Spread credit and EMG" factors. The curve component reduces tail risk due to the negative correlation between curve and credit spreads.

Because of the large number and diversity of issuers in the index (780, with a total of 1,594 issues), the idiosyncratic risk contribution to overall tracking error risk is small, but not negligible (contributing 7.1bp/month to $VaR_{99\%}$ and 6.2bp/month to $ES_{99\%}$). Notice that the contribution of idiosyncratic risk to $ES_{99\%}$ is less than its contribution to $VaR_{99\%}$. This does not mean that the *isolated* $ES_{99\%}$ of idiosyncratic risk is less than the *isolated* $VaR_{99\%}$ (161bp/month vs. 195bp/month, Figure 13). The result is due to the fact that the size of risk contribution of each risk source depends on the risk contributions of the other sources. As we move from the VaR risk measure to ES, systematic risk becomes more prominent relative to the other contributors to risk (default and idiosyncratic risk), reducing the magnitude of their respective contributions. The same applies to default risk.

To identify the specific names that contribute to default risk, we look into the Default Details Report (Figure 16). The top fifteen contributors to the default risk of the U.S. HY index are listed in descending order of contribution to tracking error volatility arising from default risk. For this index, Ford (F) is the largest contributor to default risk in both risk measures. This is the result of a combination of Ford's large market value weight in the index (3.64%) and its high default probability.

It merits discussing here large differences that may occur between contributions to default volatility and contributions to default tail risk. All positions that may produce credit losses – large or small – contribute to default volatility. On the other hand, $VaR_{99\%}$ and $ES_{99\%}$ are measured over the worst 1% of outcomes, where naturally the positions with the biggest exposure to default losses (because of the combination of position size, default probability and recovery rate) are disproportionately represented. Indeed, scenarios with default losses coming from small positions or those with low default probability or high recovery rates may not even appear in the worst 1% of scenarios. Such positions will have zero contribution to tail risk, allowing the riskier positions to contribute a larger percentage to tail risk than they do for volatility.

Figure 16: VaR Default Details for the U.S. High Yield Index, July 2009

POINT®

VaR Default Details

Portfolio: US HY (Statistics) - USD
Benchmark: USD Cash
Reporting Units: Returns in bps/month

	Portfolio vs. Benchmark			
	VaR 99.0%	%	ES 99.0%	%
Total Credit	43.7	100.0	36.8	100.0
F	3.0	6.8	1.5	4.0
S	2.5	5.7	1.4	3.9
GMAC	1.1	2.4	1.2	3.2
MGM	1.2	2.8	0.8	2.2
HCA	0.7	1.6	0.6	1.5
HET	2.1	4.7	1.4	3.9
FDC	1.3	3.0	1.2	3.2
CIT	1.1	2.6	0.6	1.7
CTX	0.4	0.9	0.3	0.9
INTEL	0.5	1.2	0.4	1.0
RAD	0.6	1.3	0.4	1.0
CHK	0.1	0.2	0.3	0.9
NXPBV	0.5	1.1	0.5	1.3
FSL	0.4	0.9	0.4	1.1
THC	0.3	0.8	0.2	0.6

Source: Barclays Capital

4. TAIL RISK OF A NEGATIVELY CONVEX PORTFOLIO

In this section, we analyze the risk of a portfolio with very large negative convexity. Such a portfolio would typically display a "non-normal" asymmetric return profile. A bond that is negatively convex has the property that its duration increases as interest rates rise and its duration decreases as rates fall. Consequently, for similar up and down moves in rates, the bond's positive returns will be smaller in magnitude than its negative returns. As we will see, the tracking error volatility alone is not sufficient to capture the risk of this portfolio: VaR and ES are indispensable for understanding portfolio risk.

We consider a portfolio consisting of securities from the U.S. Agency Index with large negative convexity and with key-rate durations close to those of the index. We assume the benchmark for this portfolio is the Agency Index itself, with a convexity close to zero (0.1). We use the POINT Optimizer (see Kumar and Lazanas 2009) to create such a portfolio as of July 31, 2009, by minimizing its OAC and setting a +/-0.1y bound for the difference between portfolio and index KRDs and a +/-0.01y bound for the difference in vegas.

The portfolio bonds are callable securities, and their returns will be influenced by changes in the implied volatility risk factors, which have an effect on the value of the embedded call options in the bonds. The portfolio's returns will also be driven by the convexity risk factor (one of the yield curve risk factors), which captures the effect of realized average changes in yield. While most risk factors have a symmetric effect on a portfolio's returns and tracking error, this is not true if there is a large net convexity exposure. For example, if a portfolio is duration neutral but more positively convex than its benchmark, it will outperform if interest rates move up by a small amount and also outperform if rates move similarly in the opposite direction. Consequently, a portfolio with net positive convexity will contribute a positive return owing to convexity, regardless of the direction of the yield curve movement, which skews the tracking errors to be greater than zero.

However, the opposite is true for a portfolio that is duration neutral but more negatively convex than its benchmark. If rates decline, the portfolio's net duration exposure decreases, leading to smaller returns than the benchmark (i.e., negative tracking errors). If rates increase, the portfolio's net duration exposure increases, leading to larger negative returns than the benchmark (i.e., also negative tracking errors). Consequently, although changes in rates are roughly symmetrical, the portfolio's net negative convexity skews the tracking errors to be less than zero. When a portfolio's net convexity exposure is an important driver of its return, the distribution of its total returns and tracking errors will show strong skewness. As we will see below, in such cases, TEV alone cannot give a sufficient description of the portfolio's risk exposure.

Figure 17: Market Structure Report for the Negatively Convex Portfolio, July 2009

POINT® — **Market Structure**

Port : ConvexAgy — Run Date: 08/20/2009
Index : US Agencies (Statistics, Unhedged) — As Of : 7/31/2009
Diff : Difference: ConvexAgy, US Agencies (Statistics, Unhedged) — Base Currency: USD

	OAD	Count	KRD 6mo	KRD 2yr	KRD 5yr	KRD 10yr	KRD 20yr	KRD 30yr	OAC
Port	2.43	7	0.09	0.78	0.98	0.44	0.13	0.01	-1.57
Index	3.44	935	0.16	0.98	1.18	0.64	0.33	0.16	0.10
Diff	-1.01	-928	-0.07	-0.20	-0.20	-0.20	-0.20	-0.15	-1.67

Source: Barclays Capital

The Market Structure Report (Figure 17) shows the curve exposure of the portfolio relative to its benchmark. Overall, the portfolio has a slightly shorter duration than the benchmark and roughly matches the key rate exposure of the benchmark. However, there is a large convexity mismatch. The portfolio has an OAC of -1.57, compared with 0.1 for the benchmark.

According to the results displayed in Figure 18, the total TEV is 47.4bp/month. If we infer the tail risk behaviour of the portfolio from this number, assuming a normal distribution, we conclude that the average return in the 1% worst case scenarios ($-ES_{99\%}$) is -126bp/month (-47.4*2.67). Moreover, the average return in the 1% best case scenarios ($-ES_{1\%}$) is 126bp/month, the symmetric value of $ES_{99\%}$.

Figure 18: TEV Risk Summary Report for the Negatively Convex Portfolio, July 2009

Portfolio Risk Summary Report

Portfolio: ConvexAgy
Index: US Agencies (Statistics)
Reporting Units: Returns in bps/month

Factor Partition Summary			
	Contrib to TEV	Isolated TEV	Systematic Beta
Total	47.4	47.4	0.7
Systematic	43.3	45.3	0.7
Curve	34.0	40.0	0.5
Swap Spreads	1.9	10.3	0.1
Volatility	0.5	2.1	-0.0
Spread Gov-Related	5.2	14.0	0.1
Spread Credit and EMG	1.6	11.3	0.0
Idiosyncratic	4.1	14.0	-
Credit default	0.0	1.2	-

Source: Barclays Capital

However, there are two issues with these inferences: 1) as described above, our intuition says that the return distribution is negatively skewed, hence expecting in absolute terms a larger $ES_{99\%}$ than $ES_{1\%,}$ and 2), the distribution is unlikely to be normal given that most risk factors have thick tails.

The tail risk model is well suited to address these issues. We can see that by analyzing the VaR Summary Report (Figure 19). First, the graphs at the bottom of Figure 19 show that the net return distribution is negatively skewed, as we expected. Second, the $ES_{99\%}$ reported in Figure 19 is 159bp/month, significantly higher than the naïve estimate of 126. On the other side of the return distribution, the $ES_{1\%}$ (the average best 1% scenarios) is 116bp/month, lower than the naïve estimate. To further underscore the difference from normality, note that the range between $ES_{99\%}$ and $ES_{1\%}$ is 275, higher than the 252 (126*2) a normal distribution implies from TEV. An analysis using VaR instead of ES reaches similar conclusions. Thus, the forecasted distribution from our tail risk model is both skewed, and wider than what is implied by the TEV.

To better understand the difference between the TEV and tail risk results, we look at the role of individual risk factors. Figure 19 shows that yield curve risk factors are dominant in explaining tail risk. For the "Portfolio vs. Benchmark" universe (i.e., tracking error) the yield curve factors contribute 87.5/123.1 = 71% to $VaR_{99\%}$ and 115.6/159.1 = 73% to $ES_{99\%}$. However, as we saw in the Market Structure Report (Figure 17), the portfolio and the benchmark have somewhat similar OAD and key rates profile. So what is the source of this yield curve risk? Figure 20 shows the VaR Systematic Details Report, which provides more information about each yield curve factor. In addition to the key rate factors, there is "convexity", which measures the effect on the portfolio of the average change in interest rates. For tracking error, the convexity factor contributes 18.3% to systematic volatility (not shown), 25.9% to $VaR_{99\%}$ and 30.7% to $ES_{99\%}$, whereas the individual net key rate exposures are, as expected, a smaller source of risk. For the portfolio returns alone (i.e., not tracking error), the contribution of the convexity factor drops sharply (13.7% for $VaR_{99\%}$ and 15.3% for $ES_{99\%}$) because the key rate exposures are larger, as they are no longer netted against the benchmark. Therefore, as expected, the convexity yield curve factor plays a much larger role for the "Portfolio vs. Benchmark" universe than for the "Portfolio" universe.

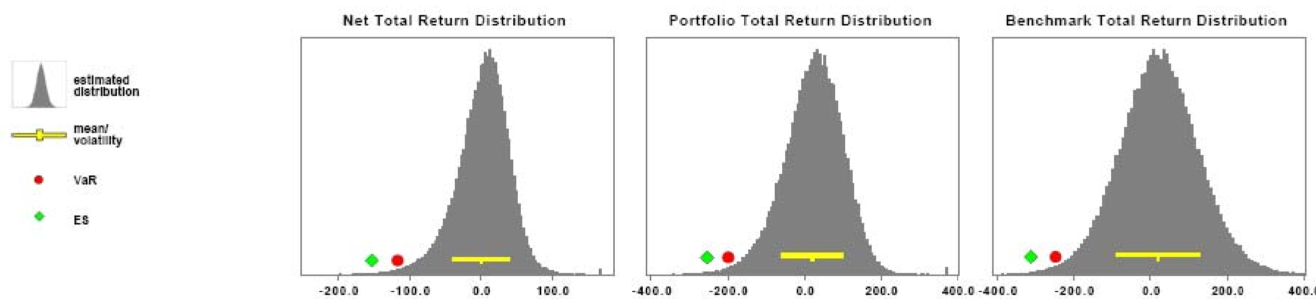
Figure 19: VaR Summary Report for the Negatively Convex Portfolio, July 2009

POINT® Global Risk Model

VaR Summary 7/31/2009

Portfolio: ConvexAgy - USD
Benchmark: US Agencies (Statistics)
Reporting Units: Returns in bps/month

	Portfolio vs. Benchmark		Portfolio		Benchmark	
	VaR 99.0%	ES 99.0%	VaR 99.0%	ES 99.0%	VaR 99.0%	ES 99.0%
Total (USD)	1,231,424.8	1,591,128.4	2,246,113.2	2,802,489.1	2,678,501.6	3,319,419.8
Total (bp)	123.1	159.1	224.6	280.2	267.9	331.9
Systematic Contributions	114.9	151.1	220.5	276.2	267.5	331.6
Curve	87.5	115.6	173.7	197.3	198.7	199.7
Swap Spreads	4.9	3.2	16.5	28.4	20.9	39.1
Volatility	1.3	1.5	-1.4	-1.6	-0.8	-0.7
Spread Gov-Related	16.8	25.5	31.7	52.1	47.7	90.5
Spread Credit and EMG	4.4	5.4	0.0	0.0	0.9	3.1
Idiosyncratic Contributions	8.3	8.0	4.1	4.1	0.2	0.1
Default Contributions	-0.1	0.0	0.0	0.0	0.2	0.2
Systematic Isolated	119.5	155.4	222.6	278.4	267.6	331.8
Idiosyncratic Isolated	37.3	48.4	36.6	47.5	6.9	8.1
Default Isolated	0.0	0.0	0.0	0.0	0.1	6.0
Carry		6.2		25.6		19.4



Source: Barclays Capital

Figure 20. VaR and ES Systematic Details for the Large Convexity Portfolio, July 2009

POINT® Global Risk Model

VaR Systematic Details 7/31/2009

Portfolio: ConvexAgy - USD
Benchmark: US Agencies (Statistics)
Reporting Units: Returns in bps/month

	Portfolio vs. Benchmark				Portfolio				Benchmark			
	VaR 99.0%	%	ES 99.0%	%	VaR 99.0%	%	ES 99.0%	%	VaR 99.0%	%	ES 99.0%	%
Regular Systematic	114.9	100.0	151.1	100.0	220.5	100.0	276.2	100.0	267.5	100.0	331.6	100.0
Key Rates and Convexity												
USD 6M key rate	1.0	0.9	1.2	0.8	3.1	1.4	3.3	1.2	3.5	1.3	2.8	0.8
USD 2Y key rate	7.7	6.7	8.3	5.5	36.8	16.7	37.0	13.4	42.0	15.7	37.5	11.3
USD 5Y key rate	11.3	9.9	12.6	8.4	61.6	28.0	64.9	23.5	71.5	26.7	68.5	20.6
USD 10Y key rate	14.8	12.8	18.2	12.0	33.5	15.2	39.6	14.4	51.0	19.1	56.4	17.0
USD 20Y key rate	13.2	11.5	16.6	11.0	8.1	3.7	9.8	3.6	22.3	8.3	25.1	7.6
USD 30Y key rate	9.7	8.4	12.4	8.2	0.4	0.2	0.4	0.2	10.3	3.8	11.8	3.6
USD Convexity	29.8	25.9	46.3	30.7	30.1	13.7	42.1	15.3	-1.9	-0.7	-2.4	-0.7

Source: Barclays Capital

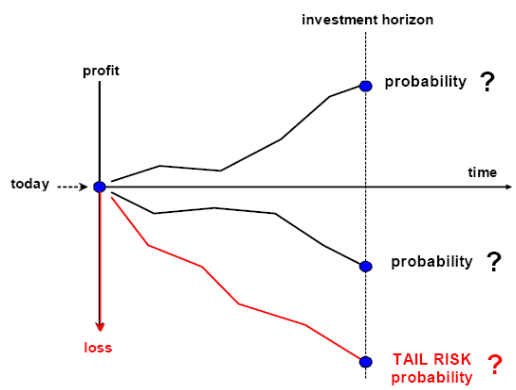
From this example, an important point is that tail risk measures increase our understanding of the portfolio risk (over TEV), especially when certain risk exposures that can produce asymmetrical returns play a large role in driving a portfolio’s risk and return. Several of the derivative instruments that are widely used in today’s portfolio management have markedly asymmetric return distributions. Their addition can often alter the risk and return profile of a traditional portfolio. It may be relevant for a manager to have the additional information provided by VaR and ES to obtain a better understanding of the portfolio’s risks.

PART III: BARCLAYS CAPITAL TAIL RISK MODELING FRAMEWORK

5. THE FOUR STEPS OF RISK MODELING

Portfolio managers and traders need to monitor the probability distribution of their returns or P&L associated with their investment strategies, (Figure 21). To deliver and interpret the P&L distribution, four steps are necessary.[12] Differences in tail risk models arise from different approaches to implementing these four steps.

Figure 21: Purpose of a Risk Model: the P&L Distribution



Source: Barclays Capital

Step 1. "Identifying Risk Factors": Generating their Joint Distribution

In order to model the random behavior of security returns, we must identify the risk factors that drive this erratic behavior. These sources should display a similar behavior across different time periods, in such a way that we can learn about the future from the past: In other words, these risk factors should constitute "invariants" of the market.[12]

For example, in the Treasury market an appropriate set of risk factors are key-rate changes over non-overlapping time periods.

Once the risk factors (or, invariants) have been identified, their joint distribution needs to be *estimated* from the available history of data and *represented* in a tractable way.

Regarding *estimation*, one can use either parametric or non-parametric approaches. The fully non-parametric approach models the joint distribution of the factors in terms of their realized empirical distribution: Under this distribution, only past joint occurrences of the factors can take place in the future and the probabilities of all these outcomes are equal. Alternatively, the parametric approach models the joint distribution of the factors in terms of a parsimonious, analytical multivariate distribution. Typically, this distribution belongs to the elliptical family of which the normal distribution is a member.

For instance, as a first approximation, and neglecting such features as "fat tails," the key-rate changes over non-overlapping time periods can be modelled as a jointly normal distribution.

[12] See Meucci (2005) for more details.

Regarding the *representation* of the market distribution, one can either opt for an analytical representation or for a scenario-based representation. In the analytical approach, the distribution is described by a mathematical formula. In the scenario-based approach, the market distribution is represented by a discrete set of scenarios. These scenarios are not necessarily historical realizations but are often Monte Carlo simulations generated under analytical, semi-parametric, or fully non-parametric assumptions.

For instance, a normal distribution can be represented analytically in terms of its probability density function which involves the exponential function; alternatively, it can be represented by a large set of simulations. Similarly, the realized empirical distribution can be represented by a formula; alternatively, it can be represented by the historical realizations themselves, or by a larger set of simulations bootstrapped from the historical realizations.

The most suitable choice among the above approaches is dictated by the nature of the market being estimated and modelled. The Tail Risk Model maintains the extensive security coverage of the Barclays Capital Global Risk Model. Security types supported in the model include government and credit securities in 25 currencies, MBS, ABS, and CMBS, inflation-linked securities, interest rate and credit derivatives, and equities. The risk factors for such a diverse and extended set of asset classes display very different behaviors; therefore, we chose the simulation approach to model the market. This approach is general and in principle can accommodate any distribution. However, it is important to impose structure on this distribution during the estimation process. Indeed, the amount of information contained in the final joint distribution of risk factors far exceeds the available information contained in the time series of the market risk factors; therefore, only by imposing structure can we derive meaningful estimates of the joint distribution of the market.

Step 2. "Pricing": From Risk Factors to the Distribution of a Security's Returns

Ultimately, in order to generate portfolio returns, we need security prices at a given investment horizon. The randomness contained in the joint distribution of the risk factors must ultimately be translated into a joint distribution of security prices.

For instance, if a specific steepening scenario (short maturities down, long maturities up) in the above joint normal distribution for the Treasury key rate changes has a probability p of materializing, with the same probability p the value of a short-maturity bond will increase and at the same time the value of a long-maturity bond will decrease by amounts that can be calculated precisely from the size of the steepening and the bond characteristics.

Pricing can be exact, namely "full-repricing", or approximate, using the "Greeks." Typically the pricing function for a fixed-income security or for an exotic derivative is complex, making it difficult to carry forward the analytical method to produce the distribution of prices. Hence, full-repricing can only be performed when the simulation approach is chosen to model the risk factors. Since the computational cost of repricing a security in each scenario is high, full-repricing is only feasible when the number of scenarios is low, as in the historical simulation case, or when the number of securities is limited, i.e., in a very specific market.

On the other hand, a first-order "Greeks" (or "theta-delta-vega," or "carry-duration") pricing approximation can be used with both the analytical and simulation approaches. Unfortunately, in general, using the first-order approximation to produce the distribution of prices is not satisfactory.

A second-order "Greeks" (or "gamma" or "convexity") approximation is better for most securities, see below for more details. If you make the extreme assumption that the risk factors are normally distributed, then the second-order approximation can be performed analytically. However, under very general assumptions for the risk factors, the second-order "Greeks" approximation can also be handled numerically with a large number of simulations. This is the approach we take in the Tail Risk Model.

Step 3. "Aggregation": from Single-Securities to the Portfolio's Distribution

The joint distribution of all the individual security returns in a portfolio must be aggregated to produce the portfolio's distribution of returns or P&L (Figure 22).

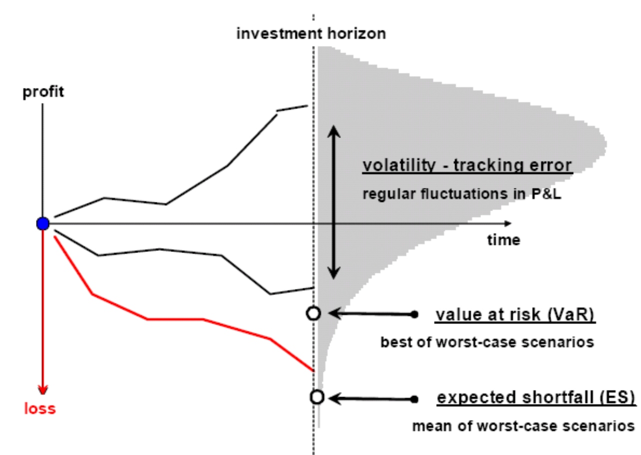
For instance, a long-short position in the above long-maturity and short-maturity bonds gives rise to a specific P&L with probability p.

Again, the first- or second-order analytical approximation can be aggregated analytically, but the results only apply to extremely restrictive (e.g., normal) markets. On the other hand, the simulation-based approach can be easily aggregated scenario-by-scenario, which is the approach we take.

Step 4. Summarizing Information: Portfolio Volatility and Tail Risk

The wealth of information contained in the final distribution of a portfolio's returns or P&L is not easy to interpret. A few significant statistics can help summarize this information. The Tail Risk Model summarizes the returns or P&L distribution in terms of the standard deviation, the value at risk (VaR), and the expected shortfall (ES) (Figure 22).

Figure 22: Summarizing the Return (P&L) Distribution-Standard Deviation, VaR and ES



Source: Barclays Capital

The standard deviation is formally defined as the normalized expectation of the square deviations from the mean.[13] Denoting a portfolio's P&L by Π, the portfolio's P&L standard deviation is defined as:

$$Sd \equiv \sqrt{\mathbb{E}\left\{(\Pi - \mathbb{E}\{\Pi\})^2\right\}} \tag{1}$$

[13] When measuring a portfolio's returns net of its benchmark's return the standard deviation is known as tracking error volatility.

Intuitively, the standard deviation is a measure of the potential variability of the P&L under normal market conditions.

In order to analyze the tails of a distribution, the Tail Risk Model also calculates the VaR and the ES. The VaR is defined as a percentile of the loss:

$$VaR_c \equiv Q_{-\Pi}(c), \tag{2}$$

Where $Q_X(c)$ denotes the c×100-percentile of the distribution of X, and the confidence c is typically set very high (e.g., c ≈ 99%). Intuitively, in a set of, say, 100,000 simulations, the 99%-confidence VaR, $VaR_{99\%}$, is the best among the worst 1,000 scenarios.

Since the VaR is insensitive to the distribution of the remaining 999 worst-case scenarios, we also provide the expected loss, or Expected Shortfall (ES), conditioned on the loss exceeding the VaR:

$$ES_c \equiv \mathbb{E}\{-\Pi | -\Pi \geq VaR_c\}. \tag{3}$$

Intuitively, in a set of 100,000 simulations, the 99%-confidence ES, $ES_{99\%}$, is the average P&L among the worst 1,000 P&L scenarios.

The standard deviation, the value at risk and the expected shortfall of the P&L constitute the output of the Tail Risk Model which is described in detail in Part I of this report.

6. MODELING THE DISTRIBUTION OF RISK FACTORS

Total market risk is modelled as the combinations of three broad classes of risk factors: a set **X** of systematic factors which affect all the securities; a set **ε** of idiosyncratic factors which affect each security individually; and a set **B** of default factors which affect credit-risky bonds.

6.1 Systematic Risk Factors

The exhaustive and yet parsimonious set of systematic factors, such as key rate changes, that span the large market covered by the Barclays Capital Global Risk Model is detailed in Portfolio Modeling Group research. Recently, several new factors have been added to better model credit risk and to cover new asset classes such as municipal securities, equities, equity options, etc. The number of systematic factors is now approximately ***K*** ≈ 800.

We model and estimate the joint distribution of the factors according to a marginal-copula factorization. We represent the marginal distribution of each factor by means of its cumulative distribution function (cdf):

$$F_k(x) \equiv \text{Prob}\{X_k \leq x\}, \quad k = 1, \ldots, K. \tag{4}$$

Periodically[14], we use the information available in the time series of each factor to estimate all the cumulative distribution functions.

Currently, the estimation process fits each factor to a Student-*t* distribution with zero expected value, a factor-specific degrees of freedom and a factor-specific scatter parameter. Our current method is as follows. First, we use the whole time series of a given factor to fit the degrees of freedom, which represent the tail behavior, or extreme events, of that factor. Next, we offer two choices: (a) *weighted estimation,* where we use an exponentially

[14] Monthly as of the time of this publication

smoothed quasi-maximum-likelihood approach with a half-life of one year to fit the scatter parameter to the most recent observations[15], and (b) *unweighted estimation* where we use a standard maximum-likelihood approach to fit the scatter parameter to the entire sample.

For instance, weighted estimates of the degrees of freedom ν_k and the scatter parameter σ_k of the distribution of the monthly changes in the six-month and ten-year key rates of the Treasury curve as of July 2009 are:

$$X_{6m}: \quad \upsilon_{6m} \approx 3.3 \quad \sigma_{6m} \approx 21bp \tag{5}$$

$$X_{10y}: \quad \upsilon_{10y} \approx 7.3 \quad \sigma_{10y} \approx 31bp \tag{6}$$

In a *t*-distribution, the lower the degrees of freedom the farther away the distribution is from normality. In particular, a distribution with 5 degrees of freedom is significantly "non-normal."

Similarly, the weighted estimates of the parameters for the banking percentage credit spread change or the utilities percentage credit spread change risk factors[16] are:

$$X_{ba}: \quad \upsilon_{ba} \approx 2.5 \quad \sigma_{ba} \approx 12.6\% \tag{7}$$

$$X_{ut}: \quad \upsilon_{ut} \approx 2.6 \quad \sigma_{ut} \approx 8.6\% \tag{8}$$

For these factors, the degrees of freedom are even lower and the occurrence of extreme events, or "fat tails" is much more likely than would be the case if they were normally distributed.

As discussed below, the Student-*t* assumption does not play any role in the subsequent steps of the model. Therefore, further refinements that account for skewness or other features can easily be included in the Tail Risk Model.

Since the marginal distributions are determined in *(4)*, the full joint distribution of the systematic factors **X** is completely determined by the choice of a dependence structure, also known as a *copula*. We model the dependence among the factors by means of a normal copula. More precisely, consider a normal vector with correlation matrix $\mathbf{\Gamma}$, as estimated by the Global Risk Model:

$$\mathbf{Y} \sim \mathrm{N}\left(\mathbf{0}, \mathbf{\Gamma}\right). \tag{9}$$

We model the joint distribution of the systematic risk factors as follows:

$$\begin{pmatrix} X_1 \\ \vdots \\ X_K \end{pmatrix} \stackrel{d}{=} \begin{pmatrix} F_1^{-1}\left(\Phi\left(Y_1\right)\right) \\ \vdots \\ F_K^{-1}\left(\Phi\left(Y_K\right)\right) \end{pmatrix}, \tag{10}$$

where Φ denotes the cdf of the standard normal distribution. This joint structure is consistent with the marginal specification described above. Indeed, it turns out that the cdf of the generic ***k***-th factor implied by *(10)* is precisely *(4)*.

[15] See Purzitsky (2006) for more details.
[16] Percentage credit spread change factors are usually called DTS (duration-times-spread) factors

Consider the monthly change of the six-month rate X_{6m}. Although as in *(5)* this variable is not normal, we transform it into a standard normal random variable:

$$Y_{6m} \equiv \Phi^{-1}\left(F_{6m}\left(X_{6m}\right)\right) \sim \mathrm{N}\left(0,1\right). \quad (11)$$

This non-linear transformation is similar in nature to the computation of the z-score, whereby a random variable is de-meaned and is divided by its standard deviation[17]. If we apply a similar transformation to the change of the ten-year rate X_{10y} we obtain from *(6)* another standard normal distribution:

$$Y_{10y} \equiv \Phi^{-1}\left(F_{10y}\left(X_{10y}\right)\right) \sim \mathrm{N}\left(0,1\right). \quad (12)$$

We assume that the transformed variables Y_{6m} and Y_{10y} are jointly normal:

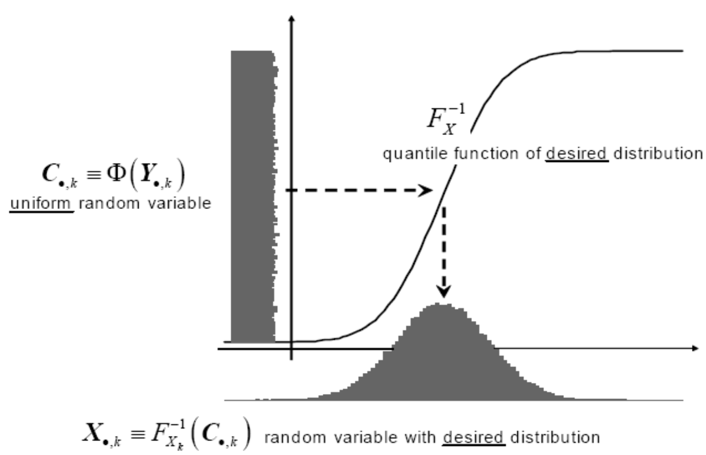
$$\begin{pmatrix} Y_{6m} \\ Y_{10y} \end{pmatrix} \sim \mathrm{N}\left(\begin{pmatrix} 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 1 & \rho \\ \rho & 1 \end{pmatrix}\right), \quad (13)$$

where the only free parameter ρ represents the correlation of the transformed variables.

We represent the joint distribution of the factors **X** in terms of a *J*×*K* panel $\mathcal{X}$ of *J* joint Monte Carlo simulations: the generic *j*-th row represents a joint scenario for the factors **X** and the generic *k*-th column represents the marginal distribution of the *k*-th factor X_k. Numerical tests show that the quality of the simulations is roughly independent of the number *K* of factors. On the other hand, the quality improves with the number of simulations, but so does the computational cost. We have chosen the number of simulations appropriately to achieve a balance between quality and computational cost.

To produce $\mathcal{X}$ in practice we proceed as in Figure 23. First we generate a *J*×*K* panel $\mathcal{Y}$ of *J* joint Monte Carlo simulations from the normal distribution *(9)*. Then we apply the standard normal cdf Φ to each entry of the panel $\mathcal{Y}$, thereby obtaining a *J*×*K* panel $\mathcal{C} \equiv \Phi\left(\mathcal{Y}\right)$. The columns of this panel have a uniform distribution and represent the copula. Finally, we apply the suitable quantile function F_k^{-1} to each column of the copula panel $\mathcal{C}$. The joint distribution of the systematic factors **X** is fully represented by the panel of Monte Carlo scenarios $\mathcal{X}$.

Figure 23: Uniform Distribution is Transformed into Desired Marginal Distribution with Desired Joint Structure



Source: A. Meucci, Risk and Asset Allocation, Springer 2005

[17] See Meucci (2005).

We conclude this section with a remark on our choice of a normal copula *(9)* to model co-dependence. This structure is fully described by a correlation matrix. However, there is evidence that certain pairs of factors exhibit greater tail co-dependence than that implied by their correlation. Therefore, one could think of introducing a richer model. For instance, in the same way as we currently use a Student-*t* distribution to de-couple the "regular" risk from the "tail" risk of a factor, we could use a *t*-copula to de-couple "regular" co-dependence from "tail" co-dependence. Indeed, the *t*-copula features higher co-dependence among extreme events than the normal copula. However, a *t*-copula offers only a single parameter to express the excess tail co-dependence of all the variables; therefore, it is appropriate only for small sets of variables that display similar tail behavior such as the credit factors used to price CDOs. In the context of the Barclays Capital Tail Risk model, where the number of factors is large and the relationship between most factors weak, using a *t*-copula would not be appropriate.

6.2 Idiosyncratic Risk Factors

Idiosyncratic shocks are security-specific sources of risk that are in general independent of one another. However, there might be non-zero correlations among some securities, such as those securities belonging to the same issuer.[18] Therefore, there exist small clusters of non-zero idiosyncratic correlation in the market.

Consider the generic *m*-th cluster, e.g., the generic *m*-th issuer. We model the joint distribution of the idiosyncratic shock $\boldsymbol{\epsilon}_m$ for the *m*-th cluster by means of a multivariate *t* distribution:

$$\boldsymbol{\epsilon}_m \sim \mathrm{St}\left(\nu_m, \mathbf{0}, \boldsymbol{\Psi}_m\right), \quad m = 1, \ldots, M. \tag{14}$$

In this expression, $\boldsymbol{\epsilon}_m$ is a vector with a cluster-specific number of entries N_m, one for each security in the cluster; ***M*** is the total number of clusters; ν_m are the cluster-specific degrees of freedom (d.o.f.); **0** is the expected value; and $\boldsymbol{\Psi}_m$ is the cluster-specific scatter matrix.

The d.o.f. ν_m in *(14)* are estimated as follows: First we partition the market into mutually exclusive macro subsectors that include several clusters. Then, for each bucket we estimate the d.o.f. of the idiosyncratic shock of each security by maximum likelihood. Next, for each bucket we consider the cross-sectional distribution of these estimates, and finally we estimate the d.o.f. for all the clusters in the bucket as the median of this distribution.

In Table *(15)* we report the degrees of freedom for a few macro-buckets.

Bucket	d.o.f.
Treasuries	10
Investment grade	8
High-yield distressed	4

(15)

As for the estimation of the scatter matrix $\boldsymbol{\Psi}_m$ in *(14),* we first refer to the covariance, which is estimated as in the Global Risk Model. Then, the covariance and the degrees of freedom unequivocally determine the $N_m \times N_m$ scatter matrices $\boldsymbol{\Psi}_m$ through the relationship $\mathrm{Cov}\left\{\boldsymbol{\epsilon}\right\} = \boldsymbol{\Psi}_m \nu_m / \left(\nu_m - 2\right)$.

[18] In the credit market these correlations are modelled as a function of the spread of the securities involved, see Silva (2009) for the intuition and the details on this approach.

For instance, Cisco as of July 2007 had three different bonds outstanding, namely 17275RAA, 17275RAB, 17275RAC. Therefore Cisco corresponds to a three-dimensional cluster $\epsilon'_{CIS} \equiv \left(\epsilon^{(\star A)}_{CIS}, \epsilon^{(\star B)}_{CIS}, \epsilon^{(\star C)}_{CIS}\right)'$, where

$$\epsilon_{CIS} \sim \mathrm{St}\left(\nu_{CIS}, \mathbf{0}, \boldsymbol{\Psi}_{CIS}\right), \tag{16}$$

The degrees of freedom are estimated $\nu_{CIS} \approx 8$. The covariance matrix is estimated as

$$\mathrm{Cov}\left\{\boldsymbol{\epsilon}_{CIS}\right\} \approx \begin{pmatrix} 95 & 88 & 226 \\ \cdot & 402 & 465 \\ \cdot & \cdot & 2640 \end{pmatrix}, \tag{17}$$

where the units are squared basis points per month. Therefore the scatter matrix reads:

$$\boldsymbol{\Psi}_{CIS} = \frac{\nu_{CIS} - 2}{\nu_{CIS}} \mathrm{Cov}\left\{\boldsymbol{\epsilon}_{CIS}\right\} \approx \begin{pmatrix} 72 & 66 & 170 \\ \cdot & 301 & 349 \\ \cdot & \cdot & 1980 \end{pmatrix}. \tag{18}$$

The different idiosyncratic variance of these three bonds is due to their different maturities of two, four, and nine years, respectively.

The joint distribution of the idiosyncratic factors is fully represented by the set of parameters *(14)*.

6.3. Default risk factors

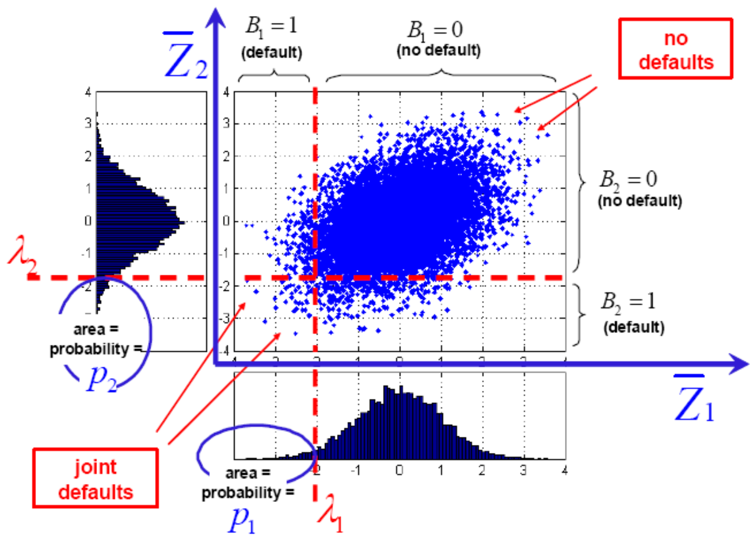
Securities such as high yield bonds are exposed to default risk. For the generic *n*-th security, the event of default is a Bernoulli variable, i.e., a variable B_n which can only assume the values 1 or 0 with probabilities p_n and $1 - p_n$, respectively:[19]

$$B_n \begin{array}{l} \overset{p_n}{\nearrow} \quad 1, \text{ if } n\text{-th security defaults} \\ \\ \underset{1-p_n}{\searrow} \quad 0, \text{ if } n\text{-th security does not defaults} \end{array} \tag{19}$$

These Bernoulli variables are not independent across securities. For instance, issuers in the same industry are more likely to default together. Also, bonds issued by the same issuer (or, obligor) are assumed to default together. The dependence structure among the defaults (Figure 24) is modelled according to a multivariate generalization of the structural approach by Merton (1974).

[19] See Chang (2003)

Figure 24: Credit Default Scenarios Triggered by Issuer's Low Asset Value



Source: Barclays Capital

First, the default of a company occurs when its value falls below a given threshold. Equivalently, letting a firm's equity proxy its value, the company defaults when its equity return falls below a given threshold. The threshold must be set in a way consistent with *(19)*: the probability of the *n*-th return being lower than the threshold must equal exactly p_n. Letting $\overline{Z}_n$ denote the de-meaned and normalized equity return of the *n*-th company; $\overline{F}_n(z) \equiv \mathbb{P}\{\overline{Z}_n < z\}$ its cdf; and λ_n the threshold which satisfies $\overline{F}_n(\lambda_n) \equiv p_n$, we obtain:

$$B_n \begin{array}{l} \overset{p_n}{\nearrow} \quad 1,\ \overline{Z}_n < \lambda_n \\ \underset{1-p_n}{\searrow} \quad 0,\ \overline{Z}_n \geq \lambda_n \end{array} \tag{20}$$

At this stage, the dependence among the default events is driven by the dependence among the standardized equity returns $\overline{\mathbf{Z}}$. Therefore the true market default risk factors are $\overline{\mathbf{Z}}$. We model these factors as a normal distribution which is fully specified by a correlation matrix:

$$\overline{\mathbf{Z}} \sim \mathrm{N}(\mathbf{0}, \mathbf{C}). \tag{21}$$

In Figure 24 we consider the case of two issuers whose normalized stock returns $\overline{Z}_1$ and $\overline{Z}_2$ display a correlation of 70%. The joint normal scenarios of $\overline{Z}_1$ and $\overline{Z}_2$ are shown in the upper-right portion of the scatter-plot figure. The projection of the scatter-plot on the axes gives rise to the marginal distributions $\overline{Z}_1$ and $\overline{Z}_2$, as represented by the respective standard-normal histograms. The area in the lower tail of the first marginal pdf below the threshold is the normalized number of scenarios such that $\overline{Z}_1 < \lambda_1$. Therefore

$$\mathbb{P}\{\overline{Z}_1 < \lambda_1\} = \overline{F}_1(\lambda_1) = p_1. \tag{22}$$

A similar argument holds for $\overline{Z}_2$

The joint distribution of the default triggers $\overline{\mathbf{Z}}$ is fully represented by the parametric form *(21)*. In order to make this distribution compatible with the simulation-based representation $\mathcal{X}$ of the systematic component, we represent the joint distribution of the default triggers

$\overline{\mathbf{Z}}$ in terms of a *J*×*N* panel of Monte Carlo scenarios $\overline{\mathcal{Z}}$. As for the systematic panel $\mathcal{X}$, the generic *j*-th row represents a joint scenario for the factors $\overline{\mathbf{Z}}$ and the generic *n*-th column represents the marginal distribution for the default trigger of the *n*-th issuer $\overline{Z}_n$, which is standard normal.

7. PRICING: THE SECURITIES' DISTRIBUTION

The P&L (return) of the generic *n*-th security Π_n is approximated as the sum of a systematic term S_n, which is completely determined by the scenarios of the systematic factors; a security-specific term ϵ_n, namely the idiosyncratic shock; and a negative term LGD_n, the loss given default, in case the *n*-th security defaults. Using the Bernoulli variables (19) this means:

$$\Pi_n \approx S_n + B_n LGD_n + \epsilon_n. \tag{23}$$

Assuming that the loss given default is deterministic,[20] pricing becomes a matter of expressing S_n and B_n in terms of the systematic factors $\mathbf{X}$ in *(10)* and the default factors $\overline{\mathbf{Z}}$ in *(21)*.

7.1. Systematic Factors

For small realizations of the systematic factors $\mathbf{X}$ in the Global Risk Model, the systematic P&L is approximated by a second-order (gamma or convexity) expansion:

$$S_n \approx \theta_n + \mathbf{L}_n' \mathbf{X} + \mathbf{X}' \mathbf{Q}_n \mathbf{X}. \tag{24}$$

In this expression θ_n is the deterministic component, known as the "theta" in the derivatives world or as the "carry" in fixed income; $\mathbf{L}_n$ is a K-dimensional vector of linear exposures, the "deltas-vegas" or the "durations," which account for the linear effects of the systematic factors on the market; and $\mathbf{Q}_n$ is a $K \times K$ matrix that accounts for the quadratic, non-linear effects of the systematic factors on the market, some of which are known as the "gammas" or "convexities."

For instance, let us assume that the generic n-th security is a Treasury bond. Then the linear exposures $\mathbf{L}_n$ have non-zero entries corresponding to the key-rates, such as *(5)* or *(6)*. We call these exposures the key-rate durations. Regarding the matrix $\mathbf{Q}_n$ of the quadratic exposures, we assume that the only non-zero entries lie on the diagonal corresponding to the key-rates; furthermore, we assume that these entries are all equal. This is equivalent to replacing the quadratic term in *(24)* with one single additional factor:

$$\begin{aligned} S_n \quad \approx \quad & L_n^{6m} X_{6m} + L_n^{2y} X_{2y} + L_n^{5y} X_{5y} \\ & + L_n^{10y} X_{10y} + L_n^{20y} X_{20y} + L_n^{30y} X_{30y} + Q_n X_{Cv}^2. \end{aligned} \tag{25}$$

In this expression, the single exposure $\mathbf{Q}_n$ is called convexity and the additional factor is fully determined by the key-rates:

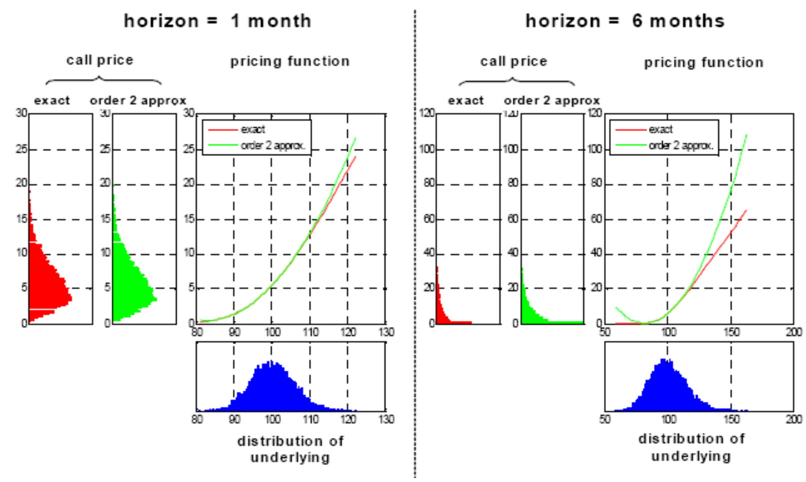
$$X_{Cv}^2 \equiv \frac{1}{6}\left(X_{6m}^2 + X_{2y}^2 + X_{5y}^2 + X_{10y}^2 + X_{20y}^2 + X_{30y}^2\right). \tag{26}$$

For each security, simple matrix manipulations of the panel $\mathcal{X}$ of systematic simulations that reflect *(24)* yield the distribution of the systematic P&L in terms of a large number *J* of Monte Carlo scenarios.

[20] This assumption can be relaxed

The quality of the quadratic approximation *(24)* depends on the relationship between the scale of the factors and the curvature of the pricing function. For some securities, the combination scale-curvature makes the approximation *(24)* invalid. For these securities it is best to implement a grid interpolation of the full repricing function.

Figure 25: Quadratic Approximation with Different Horizons



Source: Barclays Capital

For instance, consider the plain-vanilla Black-Scholes call option pricing function:

$$C = C_{BS}\left(S, K, \tau, \sigma, r_f\right), \tag{27}$$

where S is the value of the underlying at the investment horizon, K is the strike price, τ is the time to maturity at the investment horizon, σ is the implied volatility and r_f is the risk-free rate. Assume as in Black and Scholes (1973) that the compounded returns are normal:

$$\ln\left(\frac{S_T}{s_0}\right) \sim \mathrm{N}\left(T\mu, T\sigma^2\right), \tag{28}$$

where $\mu \approx 0$ and $\sigma \approx 20\%$ and time is measured in years. Then the second-order quadratic approximation is appropriate for horizons of the order of one month, see the left portion of Figure 25. However, as we see in the right portion, the same pricing function *(27)* is not appropriate for horizons of six months, due to the "square-root" propagation of risk *(28)* as the horizon T increases.

7.2. Idiosyncratic Factors

The idiosyncratic term *(14)* already operates directly at the security level *(23);* therefore, the pricing step is unnecessary in this case. In principle, it is immediate to generate a large number *J* of Monte Carlo simulations from this distribution for each security. However, as for the systematic component, from a computational point of view this step would be tremendously costly. Fortunately, we can bypass this step and simulate directly the portfolio-aggregate idiosyncratic component of the P&L, see Section 8.2 below.

7.3. Default Factors

Finally, regarding the default contribution, $B_n LGD_n$, in *(23)*, we again refer to Figure 24. The joint normal assumption *(21)* on the default triggers $\overline{\mathbf{Z}}$ implies that the cdf $\overline{F}_n$ of the generic trigger $\overline{Z}_n$ is actually Φ, the cdf of the standard normal distribution. Recalling that the threshold λ_n satisfies $\overline{F}_n(\lambda_n) \equiv p_n$, we can write *(20)* as:

$$B_n \begin{array}{l} \overset{p_n}{\nearrow} \quad 1,\ \Phi\left(\overline{Z}_n\right) < p_n \\ \underset{1-p_n}{\searrow} \quad 0,\ \Phi\left(\overline{Z}_n\right) \geq p_n \end{array} \tag{29}$$

Therefore, we define the joint distribution of the default events **B** as follows:

$$\begin{pmatrix} B_1 \\ \vdots \\ B_N \end{pmatrix} \stackrel{d}{=} \begin{pmatrix} Q_{B_1}\left(\Phi\left(\overline{Z}_1\right)\right) \\ \vdots \\ Q_{B_N}\left(\Phi\left(\overline{Z}_N\right)\right) \end{pmatrix}, \tag{30}$$

where Q_{B_n} is the inverse cdf of the n-th Bernoulli variable: $Q_{B_n}(u) \equiv 1$ if $u \leq p_n$ and otherwise $Q_{B_n}(u) \equiv 0$.

For instance, the distribution of the first default trigger in Figure 24 is:

$B_1 \equiv 1$	$B_1 \equiv 0$
$p_1 \equiv 2.25\%$	$1 - p_1 \equiv 97.75\%$

(31)

The distribution of the second default trigger is:

$B_2 \equiv 1$	$B_2 \equiv 0$
$p_2 \equiv 3.55\%$	$1 - p_2 \equiv 96.45\%$

(32)

Their joint distribution reads:

	$B_1 \equiv 1$	$B_1 \equiv 0$
$B_2 \equiv 1$	0.42%	3.13%
$B_2 \equiv 0$	1.83%	94.62%

(33)

The two default triggers, B_1 and B_2, are not independent. Indeed, the joint probability of default, i.e. the probability of both B_1 being one and B_2 being one, is 0.42% which is not equal to the product of the probability of B_1 being one and the probability of B_2 being one, which is $p_1 p_2 < 0.01\%$.

Given the stochastic representation *(30)* of the default events **B** we can generate joint Monte Carlo default simulations by applying the standard normal cdf and the appropriate Bernoulli quantile functions to the panel $\overline{\mathcal{Z}}$ of default trigger simulations obtained in Section 6.3. The result is a *J*×*N* panel of Monte Carlo scenarios **B**: The generic *j*-th row represents a joint scenario for the default events in the market and the generic *n*-th column represents the marginal distribution of default for the *n*-th issuer.

8. AGGREGATION: THE PORTFOLIO DISTRIBUTION

The joint distribution of the securities' P&L, as provided by the pricing step, is aggregated at the portfolio level, scenario by scenario, thus producing a full Monte Carlo simulation of the portfolio's P&L. Indeed, the portfolio P&L is a linear combination of the securities' P&L:

$$\Pi \equiv \sum_{n=1}^{N} w_n \Pi_n, \tag{34}$$

where w_n represents the amount of the n-th security.[21] From *(23)* we can write the portfolio P&L as the sum of three terms: systematic, idiosyncratic, and default P&Ls:

$$\Pi = \Pi^S + \Pi^I + \Pi^D. \tag{35}$$

8.1 Systematic P&L

From *(23)*, *(24)* and *(34)*, the systematic P&L is defined as:

$$\Pi^S \equiv \theta_\Pi + \mathbf{L}_\Pi' \mathbf{X} + \mathbf{X}' \mathbf{Q}_\Pi \mathbf{X}, \tag{36}$$

where the portfolio-specific scalar θ_Π, vector $\mathbf{L}_\Pi$, and matrix $\mathbf{Q}_\Pi$ read respectively:

$$\theta_\Pi \equiv \sum_{n=1}^{N} w_n \theta_n, \quad \mathbf{L}_\Pi \equiv \sum_{n=1}^{N} w_n \mathbf{L}_n, \quad \mathbf{Q}_\Pi \equiv \sum_{n=1}^{N} w_n \mathbf{Q}_n. \tag{37}$$

Simple matrix manipulations according to *(36)* of the panel $\mathcal{X}$ of simulations for the systematic factors obtained in Section 6.1 yield the distribution of Π^S in terms of the *J* Monte Carlo scenarios in the panel.

8.2 Idiosyncratic

From *(23)* and *(34)*, the idiosyncratic P&L in *(35)* is defined as:

$$\Pi^I \equiv \sum_{m=1}^{M} \eta_m, \tag{38}$$

where $\eta_m \equiv w_n \epsilon_n$ is the idiosyncratic P&L of the sub-portfolio relative to the generic m-th correlation cluster. From *(14)* the cluster-level shock is *t*-distributed:

$$\eta_m \sim \mathrm{St}\left(\nu_m, 0, \mathbf{w}_m' \mathbf{\Psi}_m \mathbf{w}_m\right), \quad m = 1, \ldots, M, \tag{39}$$

where the vector $\mathbf{w}_m$ represents the weights of the sub-portfolio relative to the *m*-th correlation cluster. Therefore the idiosyncratic P&L *(38)* is the sum of independent *t*-distributed random variables. When the portfolio only contains one cluster, Π^I is *t*-distributed and when the number of clusters in the portfolio is very large, diversification makes Π^I normal. In intermediate cases, the distribution of Π^I is computed semi-analytically and a large number *J* of Monte Carlo scenarios are generated.[22]

[21] The present discussion applies to both total return portfolios and benchmark-relative allocations. In the latter case, w is to be interpreted as the difference between the portfolio weights and the benchmark weights.
[22] For more details on the entropy-based methodology applied in this context the interested reader is referred to Meucci (2007).

For instance, consider the Cisco cluster described in *(16)*. Assume that our portfolio consists of an equally weighed combination of these three C bonds. Then from *(18)* the idiosyncratic term η of the Cisco cluster has a *t* distribution with 8 degrees of freedom and scatter parameter 390 bp^2 per month. If bonds from a different issuer were present in the portfolio, they would give rise to an independent cluster and thus an independent *t* distribution. Suppose that the d.o.f. of this second distribution were also 8. Then the total portfolio would have a *t* distribution with a higher d.o.f., say 12. In other words, the total portfolio would be more "normal" than the two independent clusters.

8.3 Default P&L

As for the default P&L Π^D in *(35)*, from *(23)* this term is defined as:

$$\Pi^D \equiv \sum_{n=1}^{N} w_n LGD_n B_n. \tag{40}$$

Assuming the loss given default LGD_n is deterministic, simple matrix manipulations of the panel B of ***J*** joint default Monte Carlo simulations obtained in Section 6.3 yield the distribution of Π^D in terms of a large number ***J*** of Monte Carlo scenarios.

9. ANALYZING INFORMATION: CONTRIBUTIONS TO RISK

With *J* Monte Carlo simulations for the systematic component of the portfolio P&L *(36)*, *J* Monte Carlo simulations for the idiosyncratic component of the portfolio P&L *(38)*, and *J* Monte Carlo simulations for the default component of the portfolio P&L *(40)*, it is immediate to obtain *J* Monte Carlo simulations of the whole portfolio P&L distribution as in *(35)*.

As discussed in the Introduction, the Tail Risk Model summarizes the P&L distribution by volatility, value at risk, and expected shortfall. In order to help portfolio managers actively manage the distribution of their P&L, the Model also decomposes volatility, VaR and ES into the contributions from the various risk factors. To perform this decomposition, note that the P&L can be written in general as the product of a vector $\widetilde{\mathbf{F}}$ of ***S*** risk sources times the corresponding manager's active risk exposures $\widetilde{\mathbf{b}}$:

$$\Pi = \sum_{s=1}^{S} \widetilde{b}_s \widetilde{F}_s. \tag{41}$$

This formulation includes *(34)*, where $S \equiv N$, the number of securities; $\widetilde{F}_n \equiv \Pi_n$ represents the P&L of the securities; and $\widetilde{\mathbf{b}} \equiv \mathbf{w}$ represents the respective portfolio weights. However, *(41)* also covers a factor-level decomposition. Indeed from *(35)* - *(36)* we obtain that $\widetilde{\mathbf{b}}$ contains the entries of $\mathbf{L}_\Pi$ and $\mathbf{Q}_\Pi$; $\widetilde{\mathbf{F}}$ represents the *K* systematic factors **X**, the few *C* linear combinations of their cross products that give rise to the convexity terms, the deterministic component, and the idiosyncratic and default components Π^I and Π^D. The total number of factors is *S*≡*K*+*C*+3. In particular, it is easy to build a *J*×*S* panel of joint Monte Carlo simulations $\widetilde{\mathbf{F}}$ of the factors $\widetilde{\mathcal{F}}$ out of the systematic panel $\mathcal{X}$ and the idiosyncratic and default simulations, respectively.

Ideally, we would like to express portfolio risk, as measured by volatility, VaR or ES, as the product of the risk factor exposures times the factor-specific "isolated" volatility, VaR or ES of the individual sources of risk, in a way fully symmetrical to *(41)*. Unfortunately, such an identity does not hold. Consider the standard deviation *(1)*:

$$Sd_{\Pi} \equiv \sqrt{\mathbb{E}\left\{(\Pi - \mathbb{E}\{\Pi\})^2\right\}}. \tag{42}$$

It is well known that:

$$Sd_{\Pi} \neq \sum_{s=1}^{S} \widetilde{b}_s Sd_s. \tag{43}$$

This fact is true in every market, even the simplest ones. Indeed, consider a normal market with only two factors:

$$\widetilde{\mathbf{F}} \sim \mathrm{N}(\boldsymbol{\mu}, \boldsymbol{\Sigma}), \tag{44}$$

where

$$\boldsymbol{\mu} \equiv (0,0)', \quad \boldsymbol{\Sigma} \equiv \begin{pmatrix} \sigma_1^2 & \rho\sigma_1\sigma_2 \\ \rho\sigma_1\sigma_2 & \sigma_2^2 \end{pmatrix}. \tag{45}$$

Then

$$\Pi \sim \mathrm{N}\left(\mu_{\Pi}, \sigma_{\Pi}^2\right), \tag{46}$$

where $\mu_{\Pi} \equiv 0$ nd

$$\sigma_{\Pi} \equiv \sqrt{\widetilde{b}_1^2\sigma_1^2 + \widetilde{b}_2^2\sigma_2^2 + 2\widetilde{b}_1\widetilde{b}_2\rho\sigma_1\sigma_2}. \tag{47}$$

We immediately verify that, as in *(43)*, unless $\rho \equiv 1$ obtain:

$$Sd_{\Pi} = \sigma_{\Pi} \neq \widetilde{b}_1\sigma_1 + \widetilde{b}_2\sigma_2 = \widetilde{b}_1 Sd_1 + \widetilde{b}_2 Sd_2. \tag{48}$$

The theory behind risk contributions rests on the observation that volatility, VaR and ES are all homogenous of degree one: By doubling the exposures $\widetilde{\mathbf{b}}$ in *(41)*, we double the risk in the portfolio.[23]

Although the decomposition *(43)* is not feasible, since the volatility is homogeneous the following identity holds true:

$$Sd_{\Pi} \equiv \sum_{s=1}^{S} \widetilde{b}_s \frac{\partial Sd}{\partial \widetilde{b}_s}. \tag{49}$$

where

$$\frac{\partial Sd_{\Pi}}{\partial \widetilde{\mathbf{b}}} = \frac{\mathrm{Cov}\left\{\widetilde{\mathbf{F}}\right\}\widetilde{\mathbf{b}}}{\sqrt{\widetilde{\mathbf{b}}'\,\mathrm{Cov}\left\{\widetilde{\mathbf{F}}\right\}\widetilde{\mathbf{b}}}}, \tag{50}$$

[23] See Meucci (2005).

Notice that *(49)* is an exact identity, not a first-order approximation. Total risk can still be expressed as the sum of the contributions from each factor, where the generic *s*-th contribution is the product of the "per-unit" marginal contribution $\partial Sd/\partial \widetilde{b}_s$ times the "amount" of the *s*-th factor in the portfolio, as represented by the exposure $\widetilde{b}_s$. Unfortunately, the per-unit marginal contribution $\partial Sd/\partial \widetilde{b}_s$ is not a truly "isolated" factor-specific quantity, as it depends on the factor correlations within the entire portfolio. However, *(49)* does indeed provide an additive decomposition of risk.

For instance, applying *(50)* to our example *(44)* - *(48)* we obtain:

$$\frac{\partial Sd_{\Pi}}{\partial \widetilde{b}_1} = \frac{\widetilde{b}_1\sigma_1^2 + \widetilde{b}_2\rho\sigma_1\sigma_2}{\sqrt{\widetilde{b}_1^2\sigma_1^2 + \widetilde{b}_2^2\sigma_2^2 + 2\widetilde{b}_1\widetilde{b}_2\rho\sigma_1\sigma_2}} \quad (51)$$

$$\frac{\partial Sd_{\Pi}}{\partial \widetilde{b}_2} = \frac{\widetilde{b}_2\sigma_2^2 + \widetilde{b}_1\rho\sigma_1\sigma_2}{\sqrt{\widetilde{b}_1^2\sigma_1^2 + \widetilde{b}_2^2\sigma_2^2 + 2\widetilde{b}_1\widetilde{b}_2\rho\sigma_1\sigma_2}}. \quad (52)$$

Quite clearly, neither of these "per unit" marginal contributions is factor-specific. For instance, the contribution from the first factor depends on the volatility of the second factor σ_2 , the correlation ρ between the factors, and the portfolio weights $\widetilde{b}_1$ and $\widetilde{b}_2$. However, the weighted sum of *(51)* and *(52)* adds up to the total volatility *(47)*.

From a computational point of view, the covariance of $\widetilde{\mathbf{F}}$ that appears in the partial derivatives *(50)* is provided by the sample covariance of the panel $\widetilde{\mathcal{F}}$.

As for the standard deviation *(43),* the portfolio's VaR is not the weighted average of the isolated VaRs:

$$VaR_c \neq \sum_{s=1}^{S} \widetilde{b}_s VaR_s. \quad (53)$$

However, since VaR is homogeneous we can therefore write it as the sum of the contributions from each factor:

$$VaR_c \equiv \sum_{s=1}^{S} \widetilde{b}_s \frac{\partial VaR_c}{\partial \widetilde{b}_s}. \quad (54)$$

Again, total risk can still be expressed as the sum of the contributions from each factor, where the generic *s*-th contribution is the product of the "per-unit" marginal contribution $\partial VaR_c/\partial \widetilde{b}_s$ times the "amount" of the *s*-th factor in the portfolio, as represented by the exposure $\widetilde{b}_s$.

In our normal example *(44)* - *(48)* the VaR is simply a multiple of the standard deviation. Therefore from *(51)* and *(52)* we obtain:

$$\frac{\partial VaR_c}{\partial \widetilde{b}_1} = \kappa_c \frac{\widetilde{b}_1\sigma_2^2 + \widetilde{b}_2\rho\sigma_1\sigma_2}{\sqrt{\widetilde{b}_1^2\sigma_1^2 + \widetilde{b}_2^2\sigma_2^2 + 2\widetilde{b}_1\widetilde{b}_2\rho\sigma_1\sigma_2}} \tag{55}$$

$$\frac{\partial VaR_c}{\partial \widetilde{b}_2} = \kappa_c \frac{\widetilde{b}_2\sigma_2^2 + \widetilde{b}_1\rho\sigma_1\sigma_2}{\sqrt{\widetilde{b}_1^2\sigma_1^2 + \widetilde{b}_2^2\sigma_2^2 + 2\widetilde{b}_1\widetilde{b}_2\rho\sigma_1\sigma_2}}. \tag{56}$$

where κ_c is the VaR with confidence c of a standard normal distribution.

In non-normal markets the volatility does not fully determine the VaR. However, the partial derivatives that appear in *(54)* can be expressed conveniently as in Hallerbach (2003), Gourieroux, Laurent, and Scaillet (2000), Tasche (2002):

$$\frac{\partial VaR_c}{\partial \widetilde{\mathbf{b}}} \equiv -\mathbb{E}\left\{\widetilde{\mathbf{F}}|\Pi \equiv -VaR_c\right\}. \tag{57}$$

In turn, these expectations can be approximated numerically as in Mausser (2003), Epperlein and Smillie (2006):

$$\frac{\partial VaR_c}{\partial \widetilde{\mathbf{b}}} \approx -\mathbf{k}_c' \mathcal{S}_{\mathbf{b}}. \tag{58}$$

In this expression $\mathcal{S}_{\mathbf{b}}$ is a *J*×*S* panel, whose generic *j*-th column is the *j*-th column of the panel $\widetilde{\mathcal{F}}$, sorted as the order statistics of the *J*-dimensional vector $-\widetilde{\mathcal{F}}\widetilde{\mathbf{b}}$; and $\mathbf{k}_c$ is a Gaussian smoothing kernel peaked around the rescaled confidence level cJ. This is how we obtained the numbers in Tail Risk Model reports.

Finally, the ES is also homogeneous and thus we can write also the ES as the sum of the contributions from each factor:

$$ES_c \equiv \sum_{s=1}^{S} \widetilde{b}_s \frac{\partial ES_c}{\partial \widetilde{b}_s}. \tag{59}$$

In our normal example *(44)* – *(48)* the expected shortfall, like the VaR, is a multiple of the standard deviation. Therefore from *(51)* and *(52)* we obtain:

$$\frac{\partial ES_c}{\partial \widetilde{b}_1} = \xi_c \frac{\widetilde{b}_1\sigma_2^2 + \widetilde{b}_2\rho\sigma_1\sigma_2}{\sqrt{\widetilde{b}_1^2\sigma_1^2 + \widetilde{b}_2^2\sigma_2^2 + 2\widetilde{b}_1\widetilde{b}_2\rho\sigma_1\sigma_2}} \tag{60}$$

$$\frac{\partial ES_c}{\partial \widetilde{b}_2} = \xi_c \frac{\widetilde{b}_2\sigma_2^2 + \widetilde{b}_1\rho\sigma_1\sigma_2}{\sqrt{\widetilde{b}_1^2\sigma_1^2 + \widetilde{b}_2^2\sigma_2^2 + 2\widetilde{b}_1\widetilde{b}_2\rho\sigma_1\sigma_2}}. \tag{61}$$

where ξ_c is the expected shortfall with confidence *c* of a standard normal distribution.

In non-normal markets the volatility does not fully determine the expected shortfall. However, the partial derivatives that appear in *(59)* can be expressed as:

$$\frac{\partial ES_c}{\partial \mathbf{b}} \equiv -\mathbb{E}\left\{\mathbf{F}|\Pi \leq -Q_{-\mathbf{b}'\mathbf{F}}\left(c\right)\right\}. \tag{62}$$

In turn, we can approximate numerically these expectations as:

$$\frac{\partial \mathcal{R}\left(\mathbf{b}\right)}{\partial \mathbf{b}} \approx -\mathbf{q}_c' \mathcal{S}_{\mathbf{b}}, \tag{63}$$

where $\mathbf{q}_c$ is a step function that jumps from 0 to $1/cJ$ at the rescaled confidence level cJ of the ES. This is how we obtain the numbers in the Tail Risk Model reports.

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