PORTFOLIO MODELING

# The US Equity Options Risk Model

**This paper describes the US Equity Options Risk Model, part of the Barclays Capital Global Risk Model (GRM). The model is available through POINT, the Barclays Capital portfolio analytics and modeling platform. It is a multi-factor model that incorporates equity cash, implied volatility, and convexity risk factors.**

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## 1. Introduction[1]

The US Equity Options Risk Model in POINT is a monthly model that covers US equity options trading in USD. It allows portfolio managers to quantify different sources of risk in their option positions through the use of economically intuitive factors. It is also fully integrated with all the other asset classes through the Global Risk Model (Joneja, Dynkin *et al*. (2005)).

The model has three main components: 1) the cash equity factors, used to capture the first order risk of the option associated with its underlying stock; 2) the implied volatility risk factors; and 3) convexity risk factors, to capture other dependencies. Please refer to Silva, Staal, and Ural (2009) for details regarding the US cash equity risk model.

We start by briefly describing the structure of the model: The Taylor expansion for the monthly change in price of equity options can be formulated as:

$$\partial c = \Delta\,\partial S + \Lambda\,\partial\sigma + \frac{1}{2}\Gamma\left(\partial S\right)^2 + \ldots \qquad (1)$$

where c is the option price, S is the underlying stock price, $\sigma$ is the implied volatility, $\Delta$ (delta) is the sensitivity of the option price to the changes in the stock price, $\Gamma$ (gamma) is the sensitivity of delta to the changes in the stock price, and $\Lambda$ (vega) is the sensitivity of the option price to the changes in the volatility of the underlying. Ignoring the higher order terms, we can get from (1) the rate of return on the option prices as:

$$\frac{\partial c}{c} \approx \frac{\Delta\,S}{c} * \frac{\partial S}{S} + \frac{\Lambda\,\sigma}{c} * \frac{\partial\sigma}{\sigma} + \frac{\Gamma S^2}{2c} * \left(\frac{\partial S}{S}\right)^2 \qquad (2)$$

which can be rewritten as

$$r_c \approx \frac{\Delta\,S}{c} * \mathrm{r_S} + \frac{\Lambda\,\sigma}{c} * \mathrm{r}_\sigma + \frac{\Gamma S^2}{2c} * r_S^2 \qquad (3)$$

where $r_S$ is the rate of return on the underlying price, $r_\sigma$ is the percentage change in the implied volatility, and $r_S^2$ is the squared return on the underlying. Hence, an equity option loads onto three sets of factors in POINT GRM:

[1] The authors would like to thank Anthony Lazanas for his valuable guidance in developing the model. Fatih Karakurt, Chandra Koppella, and Gary Wang contributed to the implementation of the model in POINT.

- Underlying Equity
- Equity Implied Volatility
- Equity Convexity

where the first set of factors is defined through the US equity risk model[2] and the last two are risk factors specific to equity options. For unhedged option positions, the underlying component is generally by far the most important part of the model in explaining the variation in option returns. For delta hedged options, volatility (vega) and convexity (gamma) components become the major drivers of risk.

In addition to the option-specific characteristics (e.g., time to maturity and moneyness), we found industry to be the most important variable in explaining systematic cross sectional differences in volatility across equity options. Therefore, the US equity options risk model employs 24 industry implied volatility risk factors, as defined by the standard GICS level 2 classification. This is in line with the industry factors used in the risk model for the underlying stocks. We therefore can capture the well-documented negative correlation between the returns of the underlying and the change in implied volatility of the option. Figure 1 documents this relationship for the systematic risk factors in our model: correlations between underlying and volatility industry factors are usually negative at about 30% for each industry.

**Figure 1. Correlations between Underlying and Volatility Industry Factors**

| Factor | Correlation | Factor | Correlation |
|---|---|---|---|
| energy | -0.25 | NCY_Household | -0.32 |
| materials | -0.37 | HLT_Health_Care | -0.31 |
| capital_goods | -0.34 | HLT_Pharmaceuticals | -0.22 |
| commercial | -0.53 | FIN_Banks | -0.37 |
| transportation | -0.32 | FIN_Div_Financials | -0.21 |
| automobiles | -0.27 | FIN_Insurance | -0.50 |
| cons_durables | -0.15 | FIN_Real_Estate | -0.01 |
| cons_services | -0.05 | TEC_Software | -0.45 |
| media | -0.27 | TEC_Hardware | -0.44 |
| retailing | -0.29 | TEC_Semiconductors | -0.47 |
| ncy_retailing | -0.16 | Telecommunication | -0.27 |
| NCY_Food | -0.44 | Utilities | -0.31 |

Source: Barclays Capital Portfolio Modeling

Finally, every equity option loads onto the equity convexity factor. The loading to this factor is a function of gamma, the sensitivity of delta to the changes in the underlying price, which gives portfolio managers a gauge for determining how frequently they need to update their delta hedges on option positions.

[2] This model includes industry (GICS level 2), fundamental/technical, and residual market volatility risk factors.

In Section 2, we describe the three sets of systematic factors in detail. We also briefly discuss the data used in the calibration. Section 3 then describes the idiosyncratic risk model for US equity options. Finally, Section 4 illustrates the implementation of the model in POINT.

## 2. The Systematic Risk Model

The return of an equity option, as described in (3), depends on three components: underlying equity, equity volatility, and equity convexity. The volatility of the return therefore depends on the volatility of these three components (and their correlations). In this section, we discuss each of these components in detail.

### Underlying Equity Factors

The underlying component of the model captures the risk in the price of the option due to its sensitivity to the changes in the underlying price. As mentioned in Section 1, this part of the model is generally the most significant one in explaining the systematic risk for unhedged option positions. Its importance is more pronounced when the option is more in-the-money. In (3), we can see that the volatility of this component is driven by two sources: the loading of an equity option to the underlying equity return – function of delta, underlying price, and the option price – and the volatility of the return of the underlying:

$$\frac{\Delta S}{c} * \sigma_{r_S}$$

We use the US equity risk model to gauge the volatility of the return of the underlying - $\sigma_{r_S}$.

See Silva, Staal, and Ural (2009) for a detailed description of the US equity model in POINT. Figure 2 summarizes the set of systematic risk factors used in that model:

Figure 2. The US Equity Risk Model Factors

| Factor | Definition |
|---|---|
| Industry | GICS level 2 industries |
| Market Value | Log of market capitalization |
| Book to Price | Book value over current market capitalization |
| Earnings to Price | Last 4Q earnings over current market capitalization |
| POINT Corporate Default Probability | Next one-year corporate default probability from the proprietary Barclays Capital model |
| Change in Discretionary Accruals | Measures the degree of earnings management |
| Total Yield | Last 4Q dividends plus stock repurchases minus resales divided by market capitalization |
| Residualized Forward E/P | Next 4Q earnings forecast over current market capitalization – residualized |
| Share Turnover | Daily volume traded over common shares outstanding averaged the over the past month |
| Momentum | Cumulative stock return from month t=-10 to t=-1 |
| Residualized Realized Volatility | Realized volatility of daily returns over the past three months – residualized |
| Residual Market Volatility | Excess return factor that captures residual systematic risk |

Source: Barclays Capital Portfolio Modeling

## Equity Volatility Factors

The volatility component of the model captures the risk in the option price due to its sensitivity to the changes in the implied volatility of the underlying. As before, the volatility associated with this component depends on the sensitivity of the option return to changes in implied volatility – a function of vega, implied volatility, option price – and the "volatility" of implied volatility. Using (3), the corresponding component is:

$$\frac{\Lambda\,\sigma}{c} * \sigma_{r_\sigma}$$

To understand the intuition behind this expression, let's go back to (1) and to the fact that the change in the price of the option depends on $\Lambda_i \times \partial\sigma_i$. Therefore its return depends on $\Lambda_i / c_i \times \partial\sigma_i$. We can have such an expression for each option trading on a particular underlying. However, our goal is to identify systematic risk factors, that is, systematic patterns that affect the return of a large cross section of options. We do this by looking more closely into the pattern of the implied volatilities. For an equity option from a given underlying stock, there are two major parameters that affect the implied volatility of the option (hence the option price and its volatility risk): strike price and time to maturity. We can therefore construct a surface of implied volatility along these two dimensions and look for systematic patterns for the change in this surface across all options. Instead, we first reduce the dimensionality of the problem. Specifically, with respect to the first parameter, strike price, our equity volatility model is calibrated only to at-the-money options (5% around ATM) and therefore our risk factors do not account for the variation in implied volatilities for different levels of moneyness of the option[3]. Regarding the second parameter, maturity, our research shows that volatility returns of options across different maturities (for the same strike and underlying) are highly correlated, although exhibiting different volatilities (short maturity options tend to exhibit higher volatility of volatility). This suggests using a single risk factor to represent the risk associated with the changes in implied volatility, but adjusting its loading as a function of time to maturity (using the adjustment factor *f*(*T*)). This rationale allows us to represent the partial return (with respect to the volatility component) of all (ATM) options for a particular underlying as

$$\frac{\Lambda_i f(T_i)}{c_i} \times \partial\sigma$$

where now $\sigma$ represents the implied volatility of a specific underlying and not of the specific option. Moreover, not surprisingly, our research shows that the change in the implied volatility is highly correlated with the level of implied volatility. That is, a time series factor representing the percentage change in implied volatility, instead of its change would tend to be more invariant, therefore better for forecasting purposes. This suggests the following last transformation as seen in (3):

$$\frac{\Lambda_i \sigma_i f(T_i)}{c_i} \times \frac{\partial\sigma}{\sigma} = \frac{\Lambda_i \sigma_i f(T_i)}{c_i} \times r_\sigma$$

We have now separated the return of the option due to changes in implied volatility into an option-specific loading and a more generic implied volatility return for each underlying.

[3] This means the model is especially suited to deal with ATM options. We plan to generalize this approach in the future.

To implement this set-up, we start by constructing at the beginning of each month the short term – 2 month (this is the reference tenor used in our model) – ATM implied volatility for each underlying equity. For a particular tenor *T*, using options from different strikes (5% around ATM), the ATM implied volatility for a particular underlying is estimated as $a_T$ in the following regression:

$$\sigma_i = a_T + b_T \frac{\log\left(\frac{K_i}{S}\right)}{\sqrt{T}} \tag{4}$$

where σ is the implied volatility, *K* is the strike price, *S* is the underlying price, and *T* is time to maturity. Then using these ATM implied volatilities for different tenors, we compute a 2-month ATM implied volatility via linear interpolation in the maturity space. Using a "sticky strike" approach, we compute the implied volatility for this underlying at the end-of-month[4] using a similar technique. This allows us to compute monthly percentage changes in implied volatilities for each underlying - $r_\sigma$ .

To factorize this percentage change in volatilities into a systematic and an idiosyncratic component, we search for risk factors that explain broad movements in the cross section of options on a monthly basis. As is the case with the return of the underlying, we found industry to be the major systematic driver of such changes. Therefore, using the percentage change in implied volatility for all underlying equities, we then perform a cross-sectional regression each month to estimate the volatility industry factor realizations:

$$r_{\sigma,S} = F + \varepsilon_{\sigma,S} \tag{5}$$

where $r_{\sigma,S}$ is the percentage change in the implied volatility (return on volatility) for underlying equity *S* and *F* is the set of industry volatility factors. Each option loads onto a single industry factor (out of 24) with a unit loading that corresponds to the GICS level 2 classification of the issuing company. The volatility factors are estimated via weighted least squares regression where the weights are proportional to the square root of the market capitalization of the underlying equity.

Finally, note that we estimate $f(T)$, that is the loading dependency on the tenure of the option by fitting the ATM implied volatilities for all tenures, using the approach described earlier. Because this function is remarkably robust across the cross section, we use the same function for all options. This function is decreasing in time to maturity.

## Equity Convexity Factor

The change in the option price cannot be explained by just a linear function of the change in the underlying price as delta also changes when the underlying price moves. Gamma, the second derivative of the option price with respect to the underlying price, captures this non-linearity in the relationship between the price of the option and its underlying. This component

[4] Because we use a sticky strike, the end-of-month implied volatility may be not at the money, depending on the evolution of the underlying price during the month.

of the model becomes more significant as the option is closer to being at-the-money. As we can see in (3), we model $r_S^2$ in the convexity component of the model.

$$\frac{\Gamma S^2}{2c} * r_S^2$$

In a portfolio context, modeling the volatility of this term directly makes the problem intractable, as the covariance matrix would increase linearly with the number of options in the portfolio. To simplify the problem, we start with a single factor model for the underlying stock return as

$$r_S = \beta_S F_M + \varepsilon_S \tag{6}$$

where $\beta_S$ is the sensitivity of the return of stock $S$ to the market factor, $F_M$. When we square both sides in (6), we get

$$r_S^2 = \beta_S^2 F_M^2 + \gamma_S \tag{7}$$

where $\gamma_S$ captures the residual term. This approach allows us to reduce the number of systematic factors under this section to one, the square of the market return. Note that from (3) and (7), we see that the loading to the equity convexity factor is a function of gamma, market beta, underlying price, and the option price.

### Data

To develop our model, we used monthly options data from OptionMetrics starting from 1994. To calibrate the model each month, we use the cross-section of ATM options from OptionMetrics.

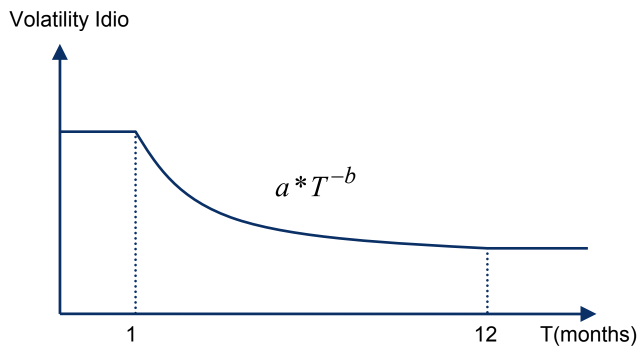
## 3. The Idiosyncratic Risk Model

The idiosyncratic risk model for US equity options is built on the same structure as in the systematic part of the model, using delta and vega approximation. Thus the idiosyncratic risk of an option is a function of the idiosyncratic risk of its underlying equity and its volatility idiosyncratic risk as given in the following equation:

$$\varepsilon_c = \frac{\Delta\, S}{c} * \varepsilon_S + \frac{\Lambda\, \sigma\, f(T)}{c} * \varepsilon_\sigma \tag{8}$$

In the US Equity Risk Model, we estimate the underlying idiosyncratic risk using daily residual returns in the past three months overlaid by certain stock characteristics. On the other hand, we model the volatility idiosyncratic risk as a function of time-to-maturity of the option. Figure 3 illustrates this functional form where shorter maturity options are assigned higher volatility idiosyncratic risk.

**Figure 3. Volatility Idiosyncratic Risk as a Function of Time to Maturity**




Source: Barclays Capital Portfolio Modeling

In the calculation of the idiosyncratic volatility and the idiosyncratic correlations for an equity option, we found empirical evidence to assume that

$$\rho(\varepsilon_S, \varepsilon_\sigma) = 0$$

Thus the idiosyncratic risk of an equity option is a weighted sum of its underlying idiosyncratic risk and volatility idiosyncratic risk. As in the case of cash equities, we assume that equity options from different issuers have zero idiosyncratic correlation but options from the same issuer have some degree of idiosyncratic correlation as a function of delta, vega and other parameters in (8).

## 4. Illustrations

In this section, we illustrate the implementation of the US equity options risk model in POINT. We run the risk model as of August 31, 2009, with a call option on Microsoft (MSFT UW - 2010 JAN 16 - 25 C) with no benchmark. Figure 4 illustrates the factor exposure report in POINT where we see that this equity option loads onto all three sets of factors described in the previous sections.

Total monthly volatility estimate for the option is 5,374bp. The last column in the factor exposure report is the contribution of each factor to total volatility. We see that the "US Equity TEC Software" factor – underlying industry factor – has the largest contribution to volatility by a very significant margin. This is not surprising as delta for this option is 0.4982 and the industry factors are the major component of the underlying US equity risk model. We see that the exposure to this factor is 7.790, which is much higher than the industry beta of Microsoft (1.02 - not shown below) owing to the leveraged nature of option positions. We also see that the volatility industry factor (US Equity Implied Vol TEC Software) has a significant negative contribution to volatility due to its negative correlation with the underlying industry factor (US Equity TEC Software).

### Figure 4. Factor Exposure Report in POINT

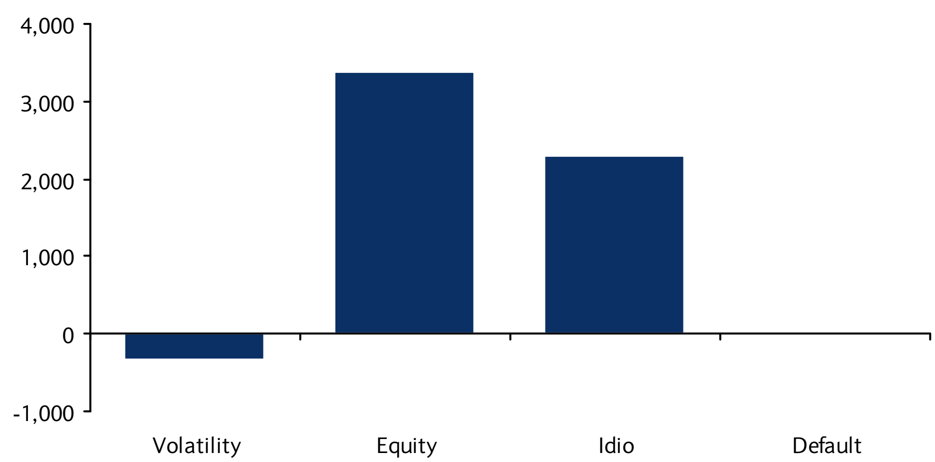
**Portfolio:** MSQ+AE
**Benchmark:** USD Cash
**Reporting Units:** Returns in bps/month

| Factor name | Sensitivity/ Exposure | Portfolio exposure | Benchmark exposure | Net exposure | Factor volatility | TE impact of an isolated 1 std. dev. up change | TE impact of a correlated 1 std. dev. up change | Marginal contribution to TEV | Percentage of tracking error variance (%) | Contribution to TEV |
|---|---|---|---|---|---|---|---|---|---|---|
| CURRENCY | | | | | | | | | | |
| USD (US Dollar) | MW% | 100.00 | 100.00 | 0.00 | 0.00 | 0.00 | | 0.000 | 0.00 | 0.00 |
| EQUITIES DEVELOPED MARKETS | | | | | | | | | | |
| US Equity TEC Software | Empirical Beta | 7.790 | 0.000 | 7.790 | 640.14 | 4,986.62 | 3,891.79 | 463.522 | 67.18 | 3,610.78 |
| US Equity Total Yield | Total Yield | 2.685 | 0.000 | 2.685 | 65.05 | 174.64 | 603.36 | 7.302 | 0.36 | 19.61 |
| US Equity Corporate Default Probability | HDP | -1.453 | 0.000 | -1.453 | 112.41 | -163.36 | 1,310.31 | 27.405 | -0.74 | -39.83 |
| US Equity Share Turnover Rate | Share Turnover | -5.503 | 0.000 | -5.503 | 75.07 | -413.11 | -898.93 | -12.556 | 1.29 | 69.09 |
| US Equity Momentum (9m) | Momentum | 0.168 | 0.000 | 0.168 | 167.71 | 28.21 | -1,272.24 | -39.698 | -0.12 | -6.68 |
| US Equity Discretionary Accruals | Accruals | 10.017 | 0.000 | 10.017 | 50.11 | 501.92 | -1,181.30 | -11.013 | -2.05 | -110.32 |
| US Equity Market Value | Size | 2.889 | 0.000 | 2.889 | 170.24 | 491.76 | -10.34 | -0.327 | -0.02 | -0.95 |
| US Equity Realized Volatility | Realized Volatility | -0.902 | 0.000 | -0.902 | 138.35 | -124.78 | -635.92 | -16.369 | 0.27 | 14.76 |
| US Equity Earnings to Price | Earnings/Price | 1.132 | 0.000 | 1.132 | 86.57 | 97.98 | -552.07 | -8.892 | -0.19 | -10.06 |
| US Equity Book to Price | Book/Price | -3.363 | 0.000 | -3.363 | 75.15 | -252.75 | 1,379.95 | 19.295 | -1.21 | -64.89 |
| US Equity Earnings Forecast | Earnings Forecast | 0.315 | 0.000 | 0.315 | 82.56 | 26.04 | 853.25 | 13.107 | 0.08 | 4.13 |
| US Equity Other Market Volatility | MW | 7.652 | 0.000 | 7.652 | 41.30 | 315.99 | -999.21 | -7.677 | -1.09 | -58.75 |
| EQUITY VOLATILITY | | | | | | | | | | |
| US Equity Implied Vol TEC Software | Equity Volatility | 0.832 | 0.000 | 0.832 | 1,205.64 | 1,003.01 | -1,754.12 | -393.480 | -6.09 | -327.35 |
| US Equity Option Convexity | Equity Convexity | 2.06 | 0.00 | 2.06 | 78.90 | 162.29 | -1,198.54 | -17.593 | -0.67 | -36.19 |

Source: Barclays Capital POINT

Figure 5 depicts the contribution of different risk factor classes to the total volatility of the Microsoft call option. We see that the "Equity" (includes equity underlying and convexity factors) and the "Idio" are the two major contributors to total volatility where a very significant part of the idiosyncratic risk is due to the underlying idiosyncratic risk.

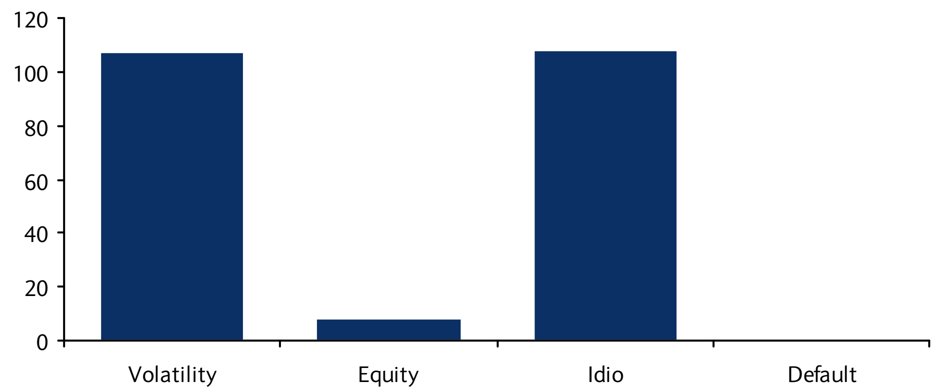
### Figure 5. Contribution of Different Factor Classes to TEV (in bp) for MSFT Call Option




Source: Barclays Capital POINT

Figure 6 illustrates the same analysis for a delta hedged position on the same call option. The total volatility of the position comes down to 224bp and now the major contributors are "Volatility" (equity volatility factors) and "Idio" components of the model. "Equity" category now incorporates only the equity convexity factor and "Idio" consists of only volatility idiosyncratic risk. Please note that the idiosyncratic risk is a significant part of the total volatility in both cases as they incorporate a single option position. As the number of option positions increases in an options portfolio, the idiosyncratic risk would diversify away, becoming a smaller part of the total portfolio risk.

**Figure 6. Contribution of Different Factor Classes to TEV (in bp) for Delta Hedged MSFT Call Option**




Source: Barclays Capital POINT

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**Analyst Certification(s)**

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