Barclays Risk Analytics and Index Solutions
17 December 2015

Interpretation of the equity risk factor returns

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A recurring theme in the research of the cross section of equity returns refers to the interaction and relative importance of the different kind of factors that explain these returns. We compare two alternative model designs in relation to this issue. Specifically, we analyze the properties of linear risk factor models resulting from a one-step versus a multi-step cross sectional regression in equity markets. Both sets of models have "similar" factors that span industry, country, and technical/fundamental risk factors.

We focus on analysing the results in the European equity market as this market incorporates a country dimension that makes it more interesting. We do, however, also present results for the US market, which are similar.

The POINT European equity risk model utilizes a multi-step estimation methodology where the industry factors are estimated in the first step, fundamental and technical factors in the second, followed by residual country factors in the third step. This allows for a cleaner interpretation of risk factors, especially the industry factors. We also construct a one-step model where all such factors (industry, fundamental/technical, and country) are estimated jointly in a one-step regression. Moreover, to highlight the importance of having accurate estimates of industry factor sensitivities (betas), we also construct an alternative formulation of the one-step model where securities have a unit loading to the industry factors. We then compare the interpretation and characteristics of factors coming out of these three different models and assess their performance in a back-testing framework.

We find that these different model formulations provide generally comparable results. Industry and country factors have quite similar properties across these models, suggesting a relatively low level of interaction among those two types of factors. Fundamental and technical factors also have a fairly similar interpretation across the models but differences can be larger, especially for certain technical factors, such as realized volatility.

The models also show a similar back-testing performance for long-only industry and country portfolios. However, the one-step model that is missing industry sensitivities clearly underperforms other models in the case of long/short portfolios, highlighting the importance of accurate beta estimates.

We present our findings across the full data period under analysis, 2001-15. However as Europe has experienced major regime shifts in the context of financial markets since 2011, we also show that our results still hold if we focus our analysis onto the European sovereign crisis period.

1. Introduction

In finance, there is a large body of literature that addresses the relative importance and economic interpretation of different types of equity factors in explaining the cross section of equity returns. The standard set of factors studied is aligned along three main dimensions: industry, country, and investment style (fundamental and technical factors).

We analyze this issue by comparing the characteristics of these commonly used factors estimated using two different econometric approaches: a one-step and a multi-step cross-sectional regression. These approaches are designed to highlight the interactions in the joint distribution of the equity factor returns, changing their economic interpretation.

In the one-step approach, all factors are estimated in a single step cross-sectional regression. This method is the standard approach and allows the data to speak freely and drive the interactions between the three sets of factors under study. It is also agnostic to the relative importance of the factors (except for cases where the regression includes certain constraints that changes the interpretation of factors). In the multi-step approach, we take a strong stance in what regards the relative importance of the different types of factors. Specifically, we perform a multi-step regression using the same set of factors, but using the residuals of each step as the dependent variable on the next step. In particular, we use the following order: industry, style, and country. Factors estimated under the two approaches should deliver different results and interpretations – except when the three set of effects are perfectly orthogonal.

We focus on the European equity market, where the three dimensions referred to above can be well explored. However, we also find similar results in the case of the US equity market.

The following section describes the setup and calibration of each approach. We then discuss how the industry, fundamental, and country factors resulting from these different approaches can be interpreted. Finally, the last section illustrates the back-testing performance of these models for portfolios based on industry, country, and various fundamental/technical investment themes.

2. Setup of the different models

The POINT continental European equity risk model (please see Ural (2010)) discussed in this paper consists of a collection of country, industry, fundamental, and technical factors. The industry factors correspond to the Global Industry Classification Standard (GICS) level-2 classification (24 industries). The set of fundamental/technical factors include market value (size), corporate default probability (distress), momentum, earnings to price, earnings forecast, realized volatility, and discretionary accruals (earnings management). Finally, there are 15 residual country factors corresponding to the largest continental European equity markets (there is a separate model for the UK equity market in POINT).

The estimation set consists of the largest 1,000 stocks in continental Europe and we use a history of 15 years in our analysis (2001-15). The regression methodology used in both models (one-step versus multi-step) is a cross-sectional weighted least squares regression.

The multi-step model (MS)

In the multi-step model, industry factors are estimated in the first step, fundamental and technical factors are estimated in the second, and residual country factors are estimated in the third step. This is the estimation technique POINT European equity risk model utilizes. The following equations demonstrate this step-wise regression technique:

STEP 1: $$Ret_i^t = \sum_j^{24\ indus} \beta_i^{t-1} * I^t{}_j + \boldsymbol{\vartheta}_i^t$$

STEP 2: $\vartheta_i^t = \sum_k^{7styles} l_{i,k}^{t-1} * S_k^t + \boldsymbol{\omega_i^t}$

STEP 3: $\omega_i^t = \sum_l^{15\,cc} C_l^t + \boldsymbol{\varepsilon_i^t}$

Where Ret_i^t is the return of the stock i at the time t.

$\mathrm{I}^t{}_j$ is the set of (dummy) industry factors spanning the 24 GICS industries.

β_i^{t-1} is the stock specific industry beta.

$\mathrm{S}^t{}_k$ is the set of style factors.

$l_{i,k}^{t-1}$ is the relevant exposure of the stock i to the style factor k.

$\mathrm{C}^t{}_l$ is the set of (dummy) country residual factors.

$\boldsymbol{\varepsilon_i^t}$ is the residual return for the stock i at time t.

Although another sequence of steps could be chosen, we believe that the order described above is the most intuitive for the majority of market participants. Industry factors are widely considered the major drivers of risk for most equity portfolios in Europe, especially for large/diversified portfolios. On the contrary, country factors were assumed much less important up to the spark of the European crisis in 2011. These factors picked up in importance due to country-specific nature of the crisis but industry factors continued to dominate in terms of explaining the risk of equity portfolios.

In the first step of the estimation process, industry factors are estimated in isolation from all other factors on a univariate basis. Under this approach, these factors have very straightforward interpretations and are statistically very close to the market value-weighted industry indices in Europe[1]. Exposures to the industry factors (betas) are estimated using higher frequency data (daily data) in POINT equity models to enhance the responsiveness and accuracy of these estimates. Please refer to Silva, Staal, and Ural (2009) for more details on the beta estimation methodology used in POINT risk models.

In the second step, we regress residual returns from the first step to the loadings of fundamental and technical factors on a multivariate basis to estimate the monthly realizations for these factors. These factors represent major investment themes for portfolio managers and are estimated after fully taking into account the industry effect. Therefore, by design, the industry and fundamental/technical effects are orthogonal. The loading to any of these factors is a function of the corresponding firm characteristic (eg, the loading of a stock to the E/P factor is a function of its E/P ratio). These fundamental/technical factor loadings are normalized to have a mean of 0 and a standard deviation of 1, which makes them comparable across different factors that have very different units. Figure 1 provides a description of the fundamental/technical factors in the POINT European equity risk model.

[1] Before running the first step of the regression, we need the stock specific industry betas as inputs to this regression. We estimate these betas through a proprietary methodology that utilizes time-series regressions with daily data. Each stock has exposure to only one industry, as specified by its GICS-2 classification; hence, ALL other industry exposures are zero. Once we estimate betas for each stock, we start the first step of the regression. We regress total returns of stocks to the industry betas on a cross-sectional basis to estimate the industry factor returns. As we estimate industry factors in isolation in the first step and each stock belongs to only one industry, this is essentially a univariate regression for each industry factor.

FIGURE 1

Fundamental/Technical factors available in the POINT European equity risk model

Fundamental/Technical factor	Definition
Market value	Log of market capitalization
Earnings to price	Last one year earnings over market capitalization
Corporate default probability	Next one year CDP from the proprietary POINT model
Change in discretionary accruals	Measures the degree of earnings management
residualized earnings forecast	Next one year earnings forecast over market capitalizations (residualized)
Momentum	Cumulative stock return for the 9M period from month t=-10 to t=-1
Residualized Realized volatility	Residualized volatility of daily returns net of industry over the past three months

Source: BRAIS

In the third step of the estimation process, we regress the residuals from the second step to a unit loading to estimate the residual country factors. Given the lack of an intercept in the previous steps, the residuals from the second step are not mean zero. This last step assures this to be the case. We will refer to this model with a multi-step estimation as the **MS model**.

One-Step (OS) Models

The first model (OS1)

This one-step model incorporates the same set of factors mentioned in the MS model. However, here all equity factors (industry, fundamental/technical, and country) are estimated jointly, in a one-step cross-sectional regression. The following equation demonstrates the systematic and idiosyncratic components of the model:

$$Ret_i^t = \sum_{24\ indus} \beta_i^{t-1} * I^t{}_j + \sum_{7\ style} l_{i,k}^{t-1} * S_k^t + \sum_{15\ cc} C_l^t + \boldsymbol{\varepsilon_i^t}$$

The exposures to both industry (betas) and to the fundamental/technical factors remain similar to the ones used for the MS model previously described. To prevent potential multi-co linearity issues, we impose a standard regression constraint, namely that the market value weighted sum of the country factors to be zero. This also provides the interpretation of the country factors to be residual factors. Such a similar set-up between MS and this model allows us to evaluate the change of interpretation and interaction between the industry, fundamental, and country factors that is purely due to differences between a one-step versus multi-step estimation technique. We refer to this model as OS1. The comparison between MS and OS1 models is the central focus of this paper but we also construct alternative one-step formulations to illustrate other aspects of equity risk models.

The second model (OS2)

To quantify the importance of accurately estimating industry exposures (betas), we introduce a second version of the one-step model. The loadings to the industry factors are unit loadings.

The following equation demonstrates the systematic and idiosyncratic components of such a model:

$$Ret_i^t = \sum_{24\ indus} I^t{}_j + \sum_{7\ style} l_{i,k}^{t-1} * S_k^t + \sum_{15\ cc} C_l^t + \varepsilon_i^t$$

The exposures to the fundamental/technical and country factors remain the same. Similar to the OS1 model, to prevent the dummy variable trap and to keep the interpretation of country factors in line with MS and OS1 models, we impose the market value weighted sum of the country factors to be zero. In what follows we refer to this model as OS2.

This OS2 model can actually be reformulated into a model with an intercept (model OS2*):

$$Ret_i^t = Intercept + \sum_{24\ indus} I^t{}_j + \sum_{7\ style} l_{i,k}^{t-1} * S_k^t + \sum_{15\ cc} C_l^t + \varepsilon_i^t$$

To prevent the dummy variable trap in this formulation, we have to impose that the market value weighted sum of the industry and country factors separately to be zero. Under this set-up, the intercept turns out to be a market factor, statistically identical to a market value-weighted index in continental Europe and the interpretation of industry factors are now residual industry factors net of the market (and other) factors. It is interesting to note that the country and fundamental/technical factors between OS2 and OS2* turn out to be identical. Furthermore, each industry factor from the OS2 model is identical to the sum of the intercept and the relevant residual industry factor from OS2*. We will only focus on the first variation of this formulation, OS2.

3. Comparison of the resulting factors

In this section, we compare the characteristics of the resulting estimated factors in the three models specified (MS, OS1, and OS2), by analyzing their volatility and correlation profiles over the 2002-15 sample. We compute two sets of statistics in terms of time weighting: an equal-weighted (UW) and an exponentially weighted moving average (EWMA) model with one year half-life using monthly data. The former provides long-run average properties of the factors whereas the latter focuses on the more recent period of our sample, providing a more up to date picture in terms of the characteristics of the factors.

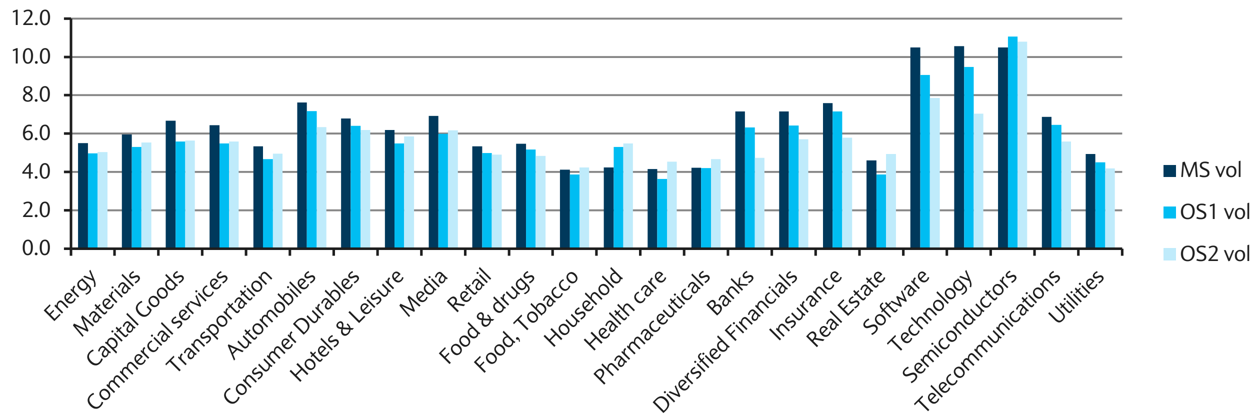
As European financial markets have experienced some drastic regime shifts over the past three to five years, we also focus our attention on the European Sovereign crisis period (January 2011 to August 2013) where the importance of certain peripheral European country factors increased significantly.

Industry Factors

Figures 2 and 3 show that the UW and EWMA volatilities of the monthly industry factor realizations under the one-step model OS1 are quite similar to that of the MS model. The volatilities of the industry factors from the OS2 model are also typically similar to the other two, expect for certain industries that have a high beta to the market such as technology (Figure 2) and Banks (Figure 3) and low-beta industries such as real estate and utilities (Figure 3). This seems to be a direct consequence of the OS2 model having only binary industry loadings as opposed to properly estimated betas.

FIGURE 2

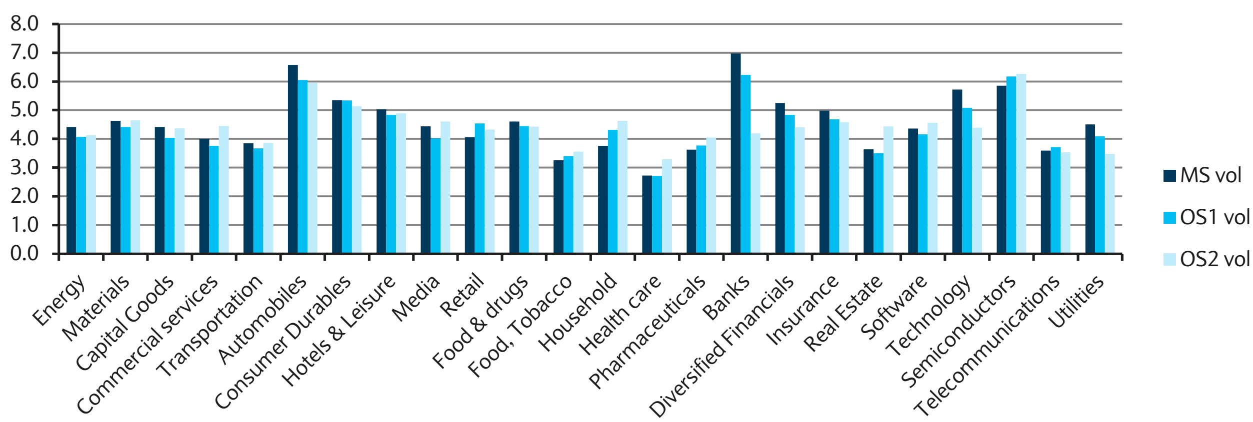
EW GICS-2 industry volatilities across the three models for 2002-15



Source: BRAIS

FIGURE 3

EWMA weighted GICS-2 industry volatilities across the three models for 2002-15



Source: BRAIS

If we look at correlations of each industry factor across the three different formulations MS, OS1, and OS2, Figure 4 shows that the correlations are quite high across the board, especially when comparing the MS and OS1 models. The factors from the OS2 model still show very high correlations with the real estate and food & tobacco having the lowest ones at 89% (EW).

FIGURE 4

Equal weight and EWMA time weighted correlations of the industry factors across models

	Equal Weight Correlations		EWMA weighted Correlations	
GICS-2 Sector	MSvOS1	OS1vOS2	MSvOS1	OS1vOS2
Energy	98%	96%	98%	97%
Materials	99%	97%	99%	99%
Capital Goods	99%	97%	99%	98%
Commercial services	97%	94%	98%	97%
Transportation	98%	96%	99%	98%
Automobiles	98%	96%	99%	99%
Consumer Durables	99%	97%	99%	97%
Hotels & Leisure	98%	97%	97%	96%
Media	99%	97%	98%	97%
Retail	97%	95%	96%	94%
Food & drugs	99%	94%	99%	98%
Food, Tobacco	96%	89%	98%	94%
Household	95%	94%	95%	94%
Health care	96%	93%	96%	92%
Pharmaceuticals	96%	94%	97%	97%
Banks	98%	96%	98%	96%
Diversified Financials	98%	97%	99%	96%
Insurance	99%	95%	98%	97%
Real Estate	95%	89%	98%	93%
Software	98%	96%	99%	98%
Technology	99%	96%	97%	94%
Semiconductors	94%	92%	96%	96%
Telecommunications	99%	96%	99%	96%
Utilities	98%	95%	95%	92%

Source: BRAIS

Investment Style Factors

We now continue the analysis with the second set of factors, namely fundamental and technical. Figures 5 and 6 show the volatility of these factors across the different calibrations and how they correlate to each other where there are larger differences compared with the case of industry factors. First of all, factor volatilities in the MS model are somewhat smaller than those of other models for UW and EWMA, which suggests some degree of interaction between the industry and the style factors in the one-step models. This means that, as expected, some of the interactions between industry and style factors are captured by the univariate industry factors under the MS model. Certain factors such as realized volatility exhibit significantly higher volatility in the OS2 model (no beta), compared with OS1 (with beta), mainly due to the effect of a beta being captured partially by style factors when beta is missing (beta is essentially correlation times the ratio of volatilities and literature suggests that the cross-sectional variation across stocks in terms of their industry/market betas is mainly driven by the variation in the volatility of stocks rather than their correlation to industry/market). This result reveals certain biases embedded in the style factors extracted from a no-beta model (OS2).

FIGURE 5

Equal weight volatilities and correlations of the style factors across models

	Equal Weight Volatilities			Equal Weight Correlations	
Factor name	MS	OS1	OS2	MSvOS1	OS1vOS2
CDP	0.4	0.5	0.7	84%	67%
Momentum	0.9	1.1	1.5	91%	88%
Discretionary accruals	0.4	0.5	0.5	87%	87%
Market Value	0.9	1.0	0.8	96%	55%
Realized Volatility	0.6	0.9	1.4	64%	48%
Earnings to Price	0.5	0.6	0.6	93%	89%
Earnings Forecast	0.6	0.7	0.8	87%	85%

Source: BRAIS

FIGURE 6

EWMA time weighted volatilities and correlations of the style factors across models

	EWMA Volatilities			EWMA Correlations	
Factor name	MS	OS1	OS2	MSvOS1	OS1vOS2
CDP	0.3	0.4	0.5	79%	60%
Momentum	0.7	0.9	1.1	92%	88%
Discretionary accruals	0.6	0.7	0.7	86%	91%
Market Value	0.7	0.8	0.7	91%	58%
Realized Volatility	0.6	0.9	1.4	58%	33%
Earnings to Price	0.4	0.5	0.5	90%	87%
Earnings Forecast	0.9	1.0	1.1	88%	87%

Source: BRAIS

In term of correlations, the investment style factor correlations between the MS and OS1 are generally quite high with the exception of realized volatility (64% EW and 58% EWMA). This can be attributed to the interaction between the loadings of this factor (defined as the standard deviation of the security's daily residual returns) and the loadings of the industry factors (betas).

The correlations between the OS1 and OS2 style factors are typically lower with momentum, realized volatility, and market value factors showing the largest differences. This suggests that the interpretation of these factors changes substantially with the switch from industry beta loadings to unit loadings.

In economic factor models, we would expect factors to show a certain degree of correlation but, ideally, these correlations should be low. Another way to compare style factors across different model formulations is to analyze the correlations between style factors and industry factors. Since there are too many combinations in such an exercise, we instead explore correlations between style factors and a hypothetical market factor (defined as the value weighted market index). Figure 7 presents correlations that are more different for OS2 (and generally higher) compared with the MS and OS1 models. CDP and realized volatility are two factors with the largest positive correlations to the market factor, in line with these factors exhibiting relatively low correlations in the previous analysis (Figures 5 and 6). Again, this suggests that certain style factors try to capture the beta effect when beta is missing in the model specifications. This is not very surprising as the volatility of stock returns is the major component of the beta and equity/asset volatility is an important component to the default probability computation (please see Asvanunt and Staal (2009) for the POINT corporate default probability model).

FIGURE 7

EWMA time weighted correlations of the style factors with the market factor

	EWMA Correlations with the market factor		
Factor name	**MS**	**OS1**	**OS2**
CDP	30%	34%	69%
Momentum	-36%	-41%	-40%
Discretionary accruals	4%	-2%	0%
Market Value	-59%	-62%	29%
Realized Volatility	30%	29%	73%
Earnings to Price	-3%	3%	-21%
Earnings Forecast	14%	8%	27%

Source: BRAIS

Country Factors

Finally, we analyze the characteristics of the country factors across models. Figure 8 shows the EWMA volatilities of the country factors for the different models and correlations of country factors across these models. The characterization of volatilities across countries for the different models is remarkably similar and correlations are quite high (typically above 85%). This result strongly suggests that country and industry/style factors are relatively orthogonal across all models studied. These country factors are also quite uncorrelated among themselves. The median correlation between country factors is about -8% (with 90% of correlations between -21% and 18%) for the MS model, and the picture is very similar for the other models as well.

FIGURE 8

Equal weight volatilities and correlations of the country factors across models

	Equal Weight Volatilities			Equal Weight Correlations	
Country factor	**MS**	**OS1**	**OS2**	**MSvOS1**	**OS1vOS2**
Austria	1.5	1.4	1.5	94%	92%
Belgium	1.3	1.3	1.2	93%	93%
Denmark	2.1	2.1	2.1	98%	97%
Finland	1.6	1.7	1.8	98%	93%
France	0.8	0.9	0.9	97%	96%
Germany	1.1	1.1	1.1	96%	96%
Greece	6.4	6.2	5.8	92%	91%
Ireland	2.5	2.5	2.6	98%	97%
Italy	1.6	1.8	2.0	98%	95%
Netherlands	1.2	1.3	1.3	94%	90%
Norway	1.4	1.6	1.8	89%	89%
Portugal	3.3	2.8	2.9	87%	82%
Spain	1.9	2.0	2.1	97%	93%
Sweden	1.4	1.7	1.7	98%	96%
Switzerland	1.0	1.5	1.7	90%	82%

Source: BRAIS

As Europe has experienced some major regime shifts in the context of financial markets since 2011, we analyze whether these full sample results would still hold if we were to focus on the period around the European sovereign crisis (we define such period as January 2011 to August 2013). This crisis had a very country-specific nature; hence, it is especially important to understand the characteristics of country factors during this period. Figure 9 shows the same volatility and correlation statistics presented in Figure 8 where we see that the results are in line with the full data period, similar volatility profiles and very high correlations of country factors across models.

FIGURE 9

Equal weight volatilities/correlations of the country factors (2011-13) across models

	Equal Weight Volatilities			Equal Weight Correlations	
Country factor	MS	OS1	OS2	MSvOS1	OS1vOS2
Austria	1.7	1.6	1.7	96%	93%
Belgium	1.4	1.4	1.4	96%	94%
Denmark	1.8	1.9	2.0	98%	98%
Finland	1.6	1.7	1.9	98%	92%
France	0.9	1.0	1.0	97%	96%
Germany	1.2	1.2	1.2	96%	95%
Greece	7.2	7.6	7.1	96%	95%
Ireland	2.5	2.5	2.7	99%	97%
Italy	1.5	1.8	2.0	98%	95%
Netherlands	1.4	1.4	1.4	95%	91%
Norway	1.2	1.4	1.5	97%	95%
Portugal	3.8	2.8	2.9	88%	79%
Spain	2.2	2.4	2.5	97%	94%
Sweden	1.5	1.8	1.8	97%	95%
Switzerland	0.8	0.9	1.1	96%	86%

Source: BRAIS

All in all, the evidence presented here suggests that the country factor characterization across the different models is quite similar. Moreover, as the country factors are relatively uncorrelated among themselves, they can be interpreted as "residual" country factors – clustered idiosyncratic returns within individual countries.

4. Back-testing performance of the different models

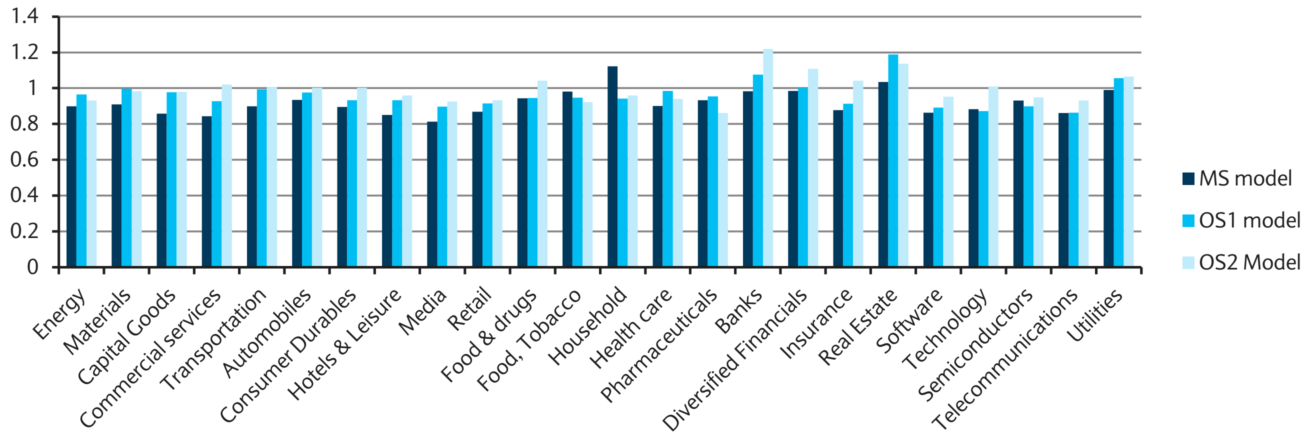
We now turn our focus to the back-testing performance of the different model formulations in terms of their ability to accurately forecast the volatility of various types of equity portfolios. This exercise allows us to better understand the characteristics – and potential biases – of the different formulations. We use the same data period (2002-15) for this analysis. To perform this exercise, for every month in the history, we estimate the covariance matrix of the systematic risk factors based on the monthly factor realizations up until that month. We use a standard EWMA (exponentially time weighted moving average with one-year half life) method to estimate factor volatilities and correlations. We then use this covariance matrix (along with the corresponding factor exposures as described in previous sections) to estimate, every month, the volatility of each of the portfolios under consideration and gauge how accurate each model formulation estimates these volatilities.

To construct our test statistic, for a given portfolio, we compute its standardized return each month. The standardized return is defined as the ratio of the realized return in a given month divided by its estimated volatility at the beginning of that month. If the volatility estimates for the portfolio return are accurate, on average, the standard deviation of this standardized return should be close to one (within the confidence interval). If the test statistic is significantly larger than one, this signals an underestimation of volatility and vice versa for the test statistic smaller than one. This test on standardized returns is ubiquitous in this kind of analysis and we perform such tests across all POINT multi-factor risk models.

We perform this test across a large set of value- and equal-weighted portfolios along the same dimensions risk factors are constructed, industry, country, and investment style. Specifically, we start with the analysis of long-only industry (GICS level-2) portfolios. Figure 10 illustrates the performance of all three models for such industry portfolios.

FIGURE10

Performance of the different models GICS-2 industry portfolios for 2002-15



Source: BRAIS

Model	MS	OS1	OS2
Average Standard deviation	94%	96%	100%

Source: BRAIS

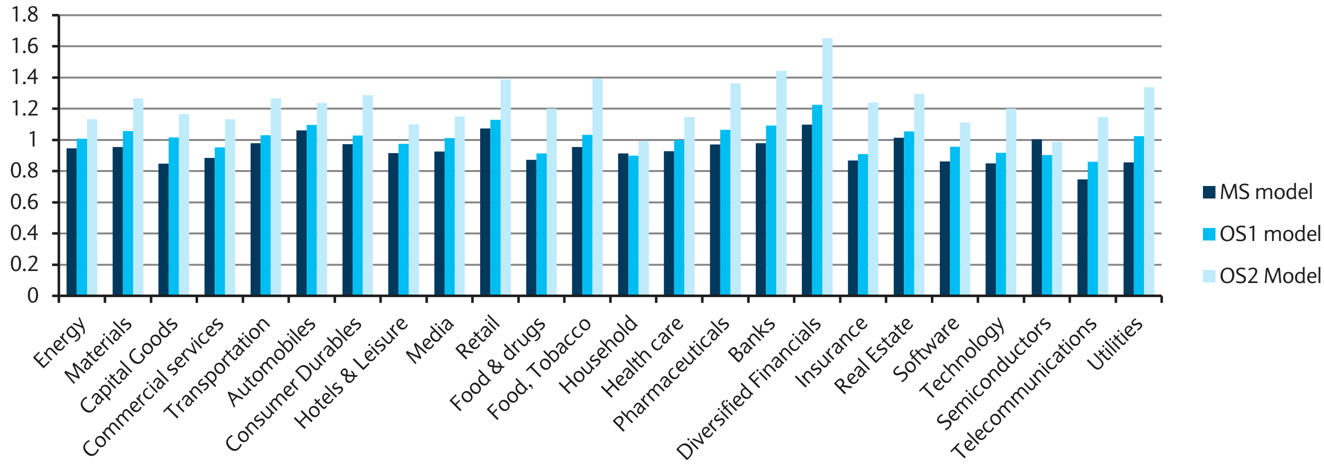
The results of the long-only industry portfolios seem inconclusive as all models perform very well over the sample period. In particular, the volatility of standardized returns is very close to one for all calibrations. Similar results (not shown here) were obtained for long-only country-based portfolios.

To better differentiate the performance of the different models, we then turn our attention to long/short portfolios along the same dimensions. The long/short industry portfolios are defined as the equal notional long the specific industry and short the market. The long/short country portfolios are defined similarly (equal notional long the country and short the market).

Figure 11 illustrates the performance of all three models on long/short industry portfolios. The OS2 model clearly underestimates the volatility of high/low beta industries such as diversified financials (165%), banks (146%), and utilities (136%). This underestimation of volatility for such industries is statistically significant in these long-short portfolios; the importance of having accurate beta estimates (which the OS2 model lacks) is magnified as the first order effect of the market is cancelled out. On the other hand, the results under the MS and OS1 models are quite robust, with no relevant bias in the treatment of these portfolios. Notably, the results across all industries are quite stable for those models, with the test statistic at 0.8-1.2 almost all of the time.

FIGURE 11

Performance of the different models long/short GICS-2 industry portfolios for 2002-15



Source: BRAIS

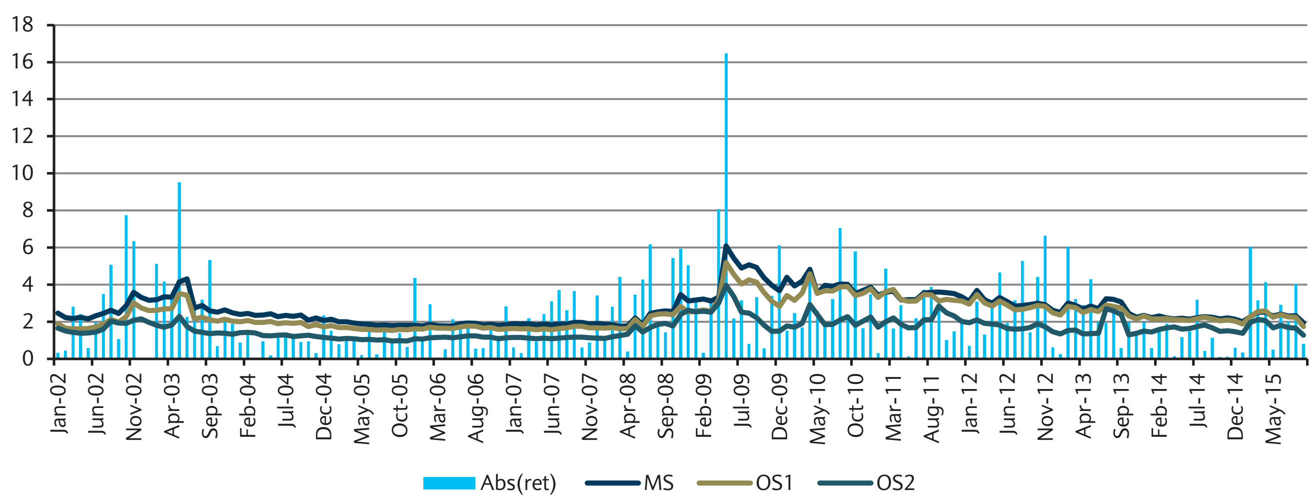
Model	MS	OS1	OS2
Average Standard deviation	94%	101%	124%

Source: BRAIS

Figure 12 shows the absolute returns of the long-short portfolio and different risk forecasts coming from the three models for the diversified financials industry. The underestimation of volatility from the OS2 model (no beta loadings) is clear, especially in early 2000s and around the financial crisis. The volatility forecasts coming from the MS and OS1 models are very similar (in terms of level and moving in tandem over time) throughout the whole period for this portfolio.

FIGURE 12

The absolute return of the diversified financials sector net market long short portfolio vs. the different risk forecasts

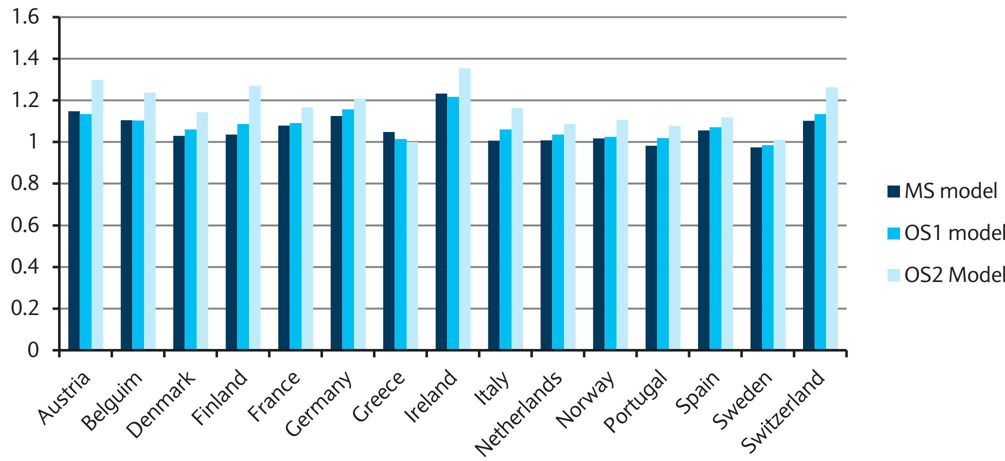


Source: BRAIS

For the long-short country portfolios, the results are not too different, Figure 13. OS2 underperforms the other two models, where this underperformance is not as severe as in the case of industries. This is understandable because what OS2 lacks compared with other models is appropriate industry exposures and given the similar nature of the country factors across calibrations. The performance of MS and OS1 models is even more noteworthy here, given that the country factor under all models is a residual factor.

FIGURE 13

Performance of the different models for long/short country portfolios (2002-15)



Source: BRAIS

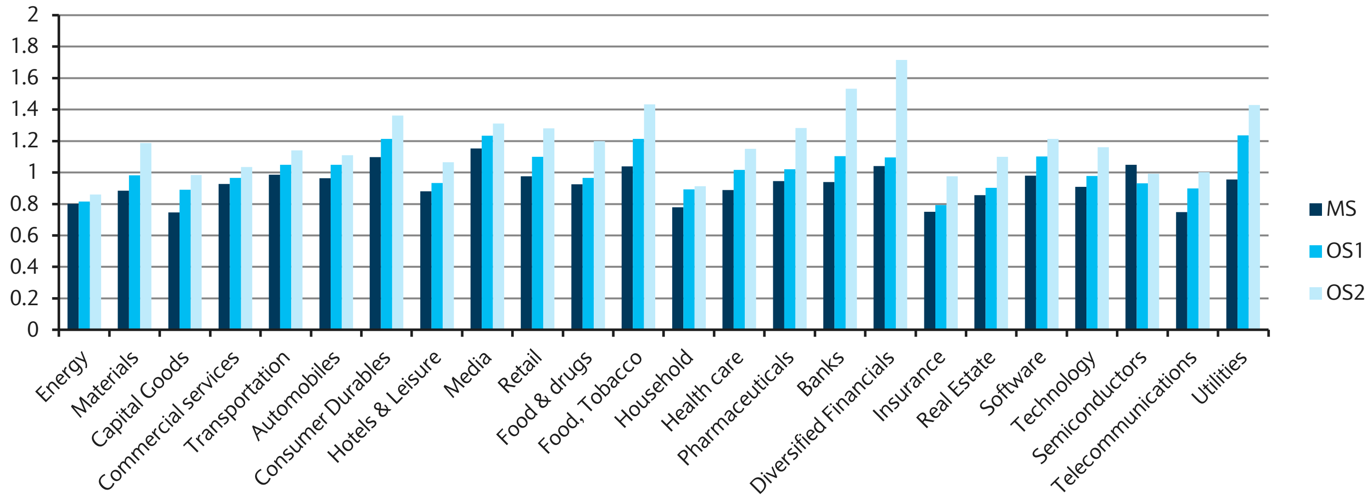
Model	MS	OS1	OS2
Average Standard deviation	106%	108%	119%

Source: BRAIS

Figures 14 and 15, which focus on the performance of the different models during the EU sovereign crisis (January 2011 - August 2013), shows that the behaviour we have observed for the full time period for industry and country portfolios also holds if we were to focus on the crisis period. The OS2 model still underperforms the other two models in general and tends to underpredict volatility, especially for long-short industry portfolios for this period. Again, results are more similar across models in the case of country portfolios. It is not surprising to see that there is more variation across countries in terms of the performance of models for this crisis period due to the country-specific nature of the crisis, as compared to the whole time period in Figure 13.

FIGURE 14

Performance of the different models long/short GICS-2 industry portfolios for the 2011-13 EU crisis



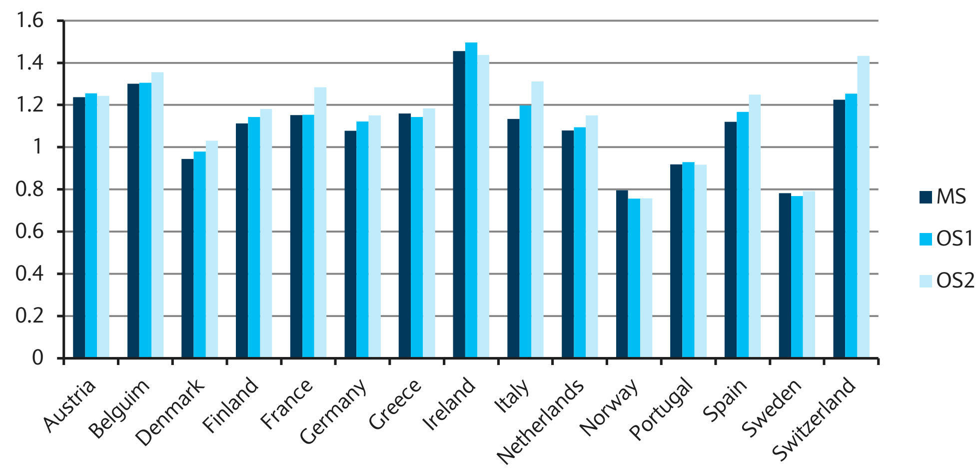
Source: BRAIS

Model	MS	OS1	OS2
Average Standard deviation	92%	101%	118%

Source: BRAIS

FIGURE 15

Performance of the different models for long/short country portfolios (2011-13)



Source: BRAIS

Model	MS	OS1	OS2
Average Standard deviation	110%	111%	116%

Source: BRAIS

To enrich the back-testing analysis we have presented so far using industry and country-based portfolios, we also construct portfolios based on standard investment themes such as momentum, market value (size), book to market (value), as well as other style factors that are in the POINT European Equity Risk Model. These portfolios are constructed each month, by going long the stocks that are in the top 30% of the value of the relevant firm characteristic and going short stocks that are in the bottom 30%. Stocks in these portfolios are equal weighted, a more standard approach in constructing such style portfolios.

Figure 16 shows the test statistic for all style factors. All models seem to perform well and in a comparable fashion across such style portfolios where the test statistics are in the confidence interval.

FIGURE 16

Performance of the different models for long/short style portfolios (2002-15)

Tilted Portfolio	MS model	OS1 model	OS2 Model
CDP	98%	98%	109%
Momentum	104%	99%	91%
Discretionary Accruals	115%	117%	115%
Market Value	104%	91%	94%
Realized Volatility	96%	100%	85%
Earnings to Price	108%	112%	115%
Earnings Forecast	114%	116%	122%

Source: BRAIS

Finally, we construct a market-beta tilted portfolio, similar to the above style factors, by going long stocks that have the highest 30% market beta and going short the lowest 30%, with equal weights. Such a portfolio exacerbates the effect of an accurate beta in the model and, not surprisingly, the OS2 model shows a severe under-prediction of volatility (by 78%). Nevertheless it is worth noting that the MS and OS1 models both perform well for such portfolios, as can be seen in Figure 17. Such a beta tilt is actually a widely popular theme in the equity market, in the construction of the so called low-beta or low-volatility strategies (see Karyda, Staal, and Ural (2014)).

FIGURE 17

Performance of the different models for long/short beta portfolios (2002-15)

Model	MS	OS1	OS2
Long/Short Beta portfolio performance	99%	111%	178%

Source: BRAIS

References

A. Asvanunt and A. Staal (2009), *The Corporate Default Probability Model in the Barclays Capital POINT Platform*, Barclays, April 2009.

C. Karyda, A.D. Staal, and C. Ural (2014), *Low Volatility Equity Strategies: Anomaly or Capital Structure Effect*, Barclays, August 2014.

A.B. Silva, A.D. Staal, and C. Ural (2009), *The US Equity Risk Model*, Barclays, July 2009.

C. Ural (2010), *The European Equity Risk Model*, Barclays, July 2010.

Appendix

We provide the results of a similar analysis run on the US equity market. In this case, all models are composed of only industry and fundamental/technical factors (there are no country residual factors as the model only covers the US market). The industry factors similarly span the 24 GICS level-2 industries and the list of fundamental/technical factors include: market value, corporate default probability, momentum, book to price, earnings to price, earnings forecast, realized volatility, share turnover, dividend yield, and discretionary accruals. Please refer to Silva, Staal, and Ural (2009) for a detailed description of the POINT US equity risk model.

The techniques used in the POINT US equity risk model are in line with those of the European equity model described previously, including the estimation of industry betas, exposures to fundamental/technical factors, a multi-step estimation methodology, and the weighted least squares regression.

Figures 18-25 show that the results are in line with those of the European equity model. Industry factors show very high correlations across the models, with the MS model factors having slightly higher volatilities in general as they are estimated in isolation in the first step. The picture for style factors is similar to the European model, with realized volatility, momentum, and share turnover (this factor does not exist in the European model) exhibiting the largest differences in the volatility or correlation dimensions; these are all technical factors. In terms of back-testing results, the OS2 model tends to underperform the MS or OS1, especially in the case of industry or beta tilted long-short portfolios.

Multi-step Model (MS)

For the US, in the multi-step model, industry factors are estimated in the first step while fundamental and technical factors are estimated in the second (there is also a third step but that factor has negligible importance compared with the others and we do not use that to simplify our analysis). The following equations demonstrate this step-wise regression technique:

STEP 1: $$Ret_i^t = \sum_j^{24\,indus} \beta_i^{t-1} * I^t{}_j + \boldsymbol{\vartheta_i^t}$$

STEP 2: $$\vartheta_i^t = \sum_k^{10 styles} l_{i,k}^{t-1} * S_k^t + \boldsymbol{\varepsilon_i^t}$$

Where Ret_i^t is the return of the stock i at the time t.

$\mathrm{I^t}_j$ is the set of (dummy) industry factors spanning the 24 GICS industries.

β_i^{t-1} is the stock specific industry beta.

$\mathrm{S^t}_k$ is the set of style factors.

$l_{i,k}^{t-1}$ is the relevant exposure of the stock i to the style factor k.

ε_i^t is the residual return for the stock i at time t.

First one-step Model (OS1)

The OS1 model in the case of the US market defined similarly to the OS1 model in the European market. The difference here again is that there are no country residual factors.

$$Ret_i^t = \sum_{24\,indus} \beta_i^{t-1} * I^t{}_j + \sum_{10 style} l_{i,j}^{t-1} * S_j^t + \varepsilon_i^t$$

Where Ret_i^t is the return of the stock i at the time t.

$\mathrm{I^t}_j$ is the set of residual industry factors spanning the 24 gics2 industries.

S^t_j is the set of fundamental/technical factors and $I^{t-1}_{i,j}$ is the relevant exposure of the stock i to the technical factor j.

Second one-step Model (OS2)

The OS2 model in the case of the US market is fairly similar to the one described in the European market. The difference here again is that there are no country residual factors.

$$Ret_i^t = \sum_{24\ indus} I_j^t + \sum_{10\ style} I_{i,j}^{t-1} * S_j^t + \varepsilon_i^t$$

Where Ret_i^t is the return of the stock i at the time t.

I^t_j is the set of residual industry factors spanning the 24 gics2 industries.

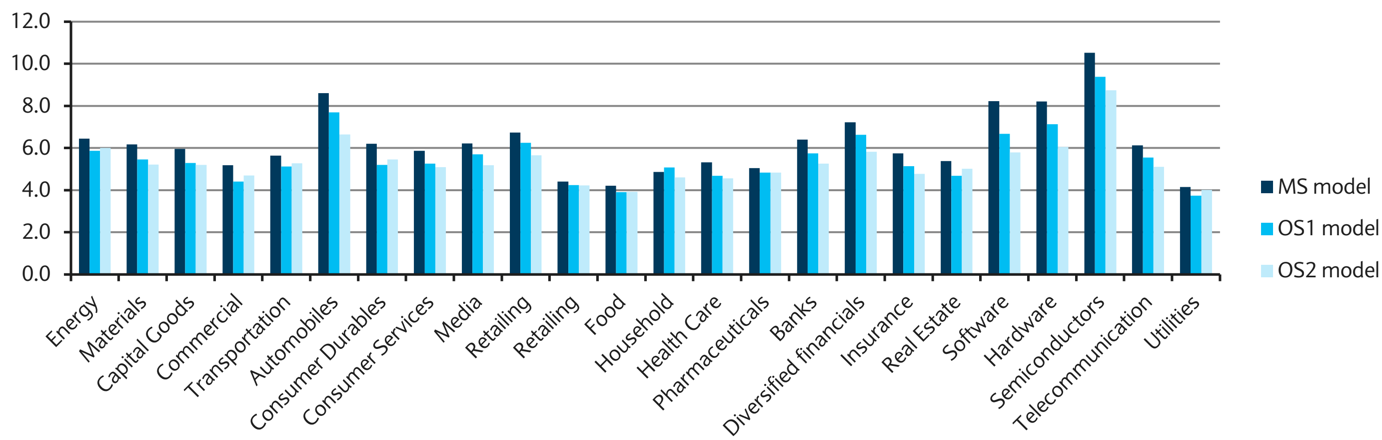
S^t_j is the set of style factors and $I^{t-1}_{i,j}$ is the relevant exposure of the stock i to the style factor j.

ε_i^t is the residual return for the stock i at time t.

Comparison of the resulting factors

FIGURE 18

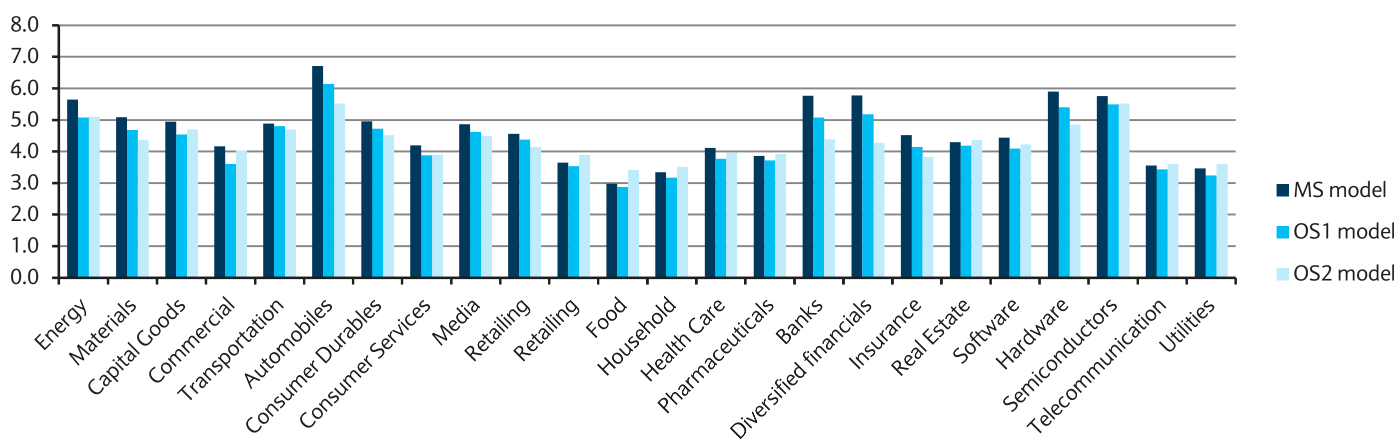
Equal weighted **GICS-2 industry volatilities across the four models for 2002-15**



Source: BRAIS

FIGURE 19

EWMA weighted GICS-2 industry volatilities across the four models for 2002-15



Source: BRAIS

FIGURE 20

Equal weight and EWMA time weighted correlations of the industry factors across models

	Equal Weight Correlations		EWMA weighted Correlations	
GICS-2 Sector	**MSvOS1**	**OS1vOS2**	**MSvOS1**	**OS1vOS2**
Energy	98%	97%	99%	98%
Materials	97%	97%	99%	99%
Capital Goods	98%	97%	99%	99%
Commercial services	97%	96%	98%	97%
Transportation	97%	96%	99%	99%
Automobiles	96%	96%	99%	99%
Consumer Durables	97%	96%	99%	98%
Hotels & Leisure	97%	95%	98%	98%
Media	98%	97%	100%	99%
Retail	99%	98%	99%	98%
Food & drugs	96%	92%	99%	96%
Food, Tobacco	95%	90%	98%	95%
Household	96%	92%	98%	95%
Health care	97%	96%	99%	99%
Pharmaceuticals	98%	97%	98%	97%
Banks	98%	97%	99%	98%
Diversified Financials	99%	98%	99%	98%
Insurance	98%	95%	99%	98%
Real Estate	94%	91%	99%	98%
Software	96%	93%	98%	98%
Technology	97%	96%	99%	98%
Semiconductors	98%	97%	99%	99%
Telecommunications	97%	95%	99%	97%
Utilities	94%	92%	98%	95%

Source: BRAIS

FIGURE 21

Equal weight volatilities and correlations of the style factors across models

	Equal Weight Volatilities			Equal Weight Correlations	
Factor name	**MS**	**OS1**	**OS2**	**MSvOS1**	**OS1vOS2**
Total Yield	0.4	0.4	0.5	85%	75%
CDP	0.7	0.8	0.8	96%	93%
Share Turnover	0.6	0.8	1.1	92%	68%
Momentum (9m)	1.1	1.3	1.5	96%	93%
Discretionary Accruals	0.4	0.5	0.5	92%	89%
Market Value	0.8	1.0	0.9	98%	62%
Realized Volatility	0.8	1.0	1.6	85%	62%
Earnings to Price	0.5	0.5	0.5	94%	92%
Book to Price	0.4	0.5	0.6	87%	83%
Earnings Forecast	0.7	0.9	0.9	93%	90%

Source: BRAIS

FIGURE 22

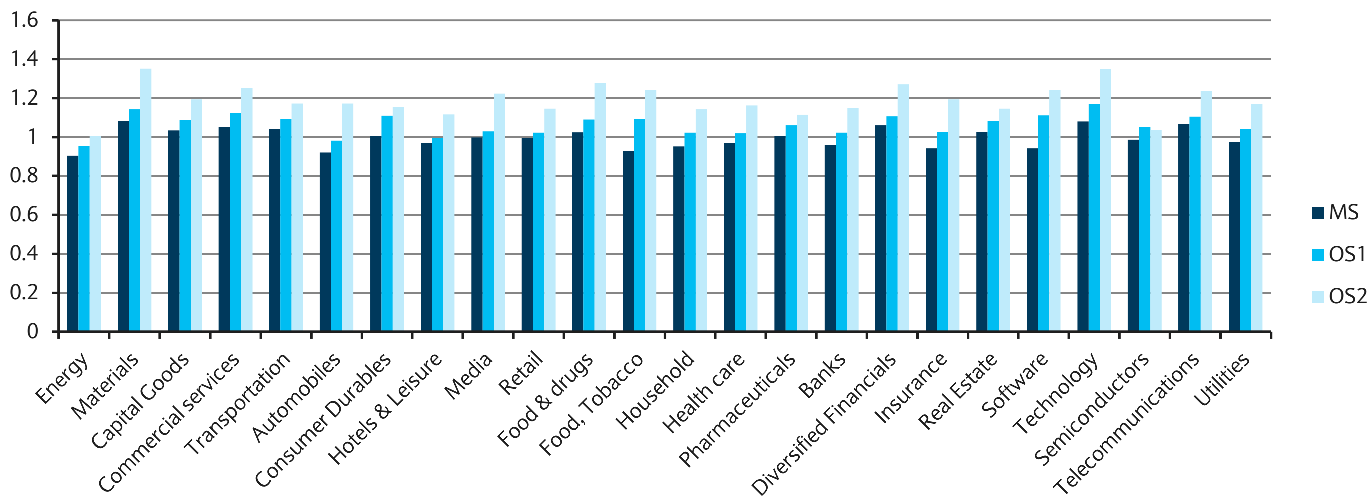
EWMA volatilities and correlations of the style factors across models

	EWMA Volatilities			EWMA Correlations	
Factor name	**MS**	**OS1**	**OS2**	**MSvOS1**	**OS1vOS2**
Total Yield	0.3	0.3	0.3	91%	84%
CDP	0.3	0.3	0.4	94%	80%
Share Turnover	0.4	0.4	0.5	95%	89%
Momentum (9m)	0.8	0.9	0.9	99%	95%
Discretionary Accruals	0.4	0.4	0.4	94%	92%
Market Value	0.4	0.6	0.6	98%	88%
Realized Volatility	0.7	0.9	1.6	90%	54%
Earnings to Price	0.4	0.4	0.4	96%	93%
Book to Price	0.3	0.4	0.4	90%	80%
Earnings Forecast	0.4	0.5	0.5	93%	89%

Source: BRAIS

FIGURE 23

Performance of the different models long/short GICS-2 industry portfolios for 2002-15



Source: BRAIS

FIGURE 24

Performance of the different models for long/short style portfolios (2002-15)

	MS	OS1	OS2
Total Yield	1.2	1.18	1.26
CDP	1.17	1.16	1.14
Share Turnover	1.06	1.08	1.05
Momentum (9m)	1.07	1.11	1.15
Discretionary Accruals	1.15	1.18	1.20
Market Value	1.10	1.08	1.12
Realized Volatility	1.09	1.09	1.08
Earnings to Price	1.24	1.22	1.16
Book to Price	1.08	1.04	1.06
Earnings Forecast	1.13	1.17	1.16

Source: BRAIS

FIGURE 25

Performance of the different models for long/short beta portfolios (2002-15)

Model	MS	OS1	OS2
Long/short beta portfolio performance	1.06	1.15	1.46

Source: BRAIS

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