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Axioma Research Paper
No. 008

Portfolio Construction Strategies Using More Than One Risk Model

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Using more than one risk model in a portfolio construction strategy allows a portfolio manager to exploit the fact that different risk models measure and capture risk differently. Having both a fundamental and statistical risk model simultaneously in the strategy ensures that the optimized portfolio reflects both points of view. Two risk model strategies can produce just as conservative portfolios and better overall performance than one risk model alone provided that the strategies are calibrated so that both risk models affect the optimal portfolio solution.

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Portfolio Construction Research by

Portfolio Construction Strategies Using More Than One Risk Model

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August 2008

Summary

A systematic calibration procedure is described for incorporating more than one risk model in a portfolio construction strategy. The addition of a second risk model can lead to just as conservative portfolios and better overall performance than one risk model alone provided that the strategy is calibrated so that both risk models affect the optimal portfolio solution.

We report results using Axioma's Japanese risk models and two risk constraints in the portfolio construction strategy. In all cases, Axioma's daily fundamental factor Japanese risk model is the primary risk model which is used to define the tracking error of the portfolio, the primary risk constraint in the strategy. Three different second risk model constraints are considered:

    a) Constrain active risk using Axioma's statistical factor risk model.

    b) Constrain total risk using Axioma's statistical factor risk model.

    c) Constrain specific risk using Axioma's fundamental factor risk model.

Calibration of the portfolio construction parameters is essential since the region over which both risk models affect the solution is difficult to predict a priori and can be relatively narrow. In many cases, calibration results give a clear, optimum set of portfolio construction parameters in which the second risk model synergistically improves portfolio performance.

Introduction

The question addressed by this paper is how best to incorporate a second risk model into an existing portfolio construction strategy that already utilizes a primary risk model. The primary and secondary risk models could be fundamental factor risk models, statistical factor risk models, dense, asset-asset covariance matrices computed from the historical time series of asset returns, or any combination of these. The second risk model could also be simply a diagonal specific variance matrix, the "alpha estimation error" matrix associated with robust optimization (Ceria and Stubbs 2006; Renshaw 2008), or even the identity matrix, in which case the second risk prediction may be difficult to compare with the primary risk prediction. How do we determine if the second risk model is beneficial, superfluous, or deleterious? Should the second risk model be incorporated into the portfolio construction strategy at all, and if so, how should the strategy parameters be calibrated (or re-calibrated, in the case of the existing strategy parameters) to best take advantage of the second risk model?

Several authors have argued that one of the contributors to the poor performance of quantitative hedge funds over the past 12 months has been the fact that many quantitative managers use the same commercially available risk models (Ceria, 2007; Foley, 2008). The use of such a small number of similar risk models may have led to these managers taking similar portfolio positions. Using more than one risk model may be helpful in ameliorating this problem since the second risk model diversifies portfolio positions.

When the two risk models are comparable, simple "averaging" approaches are possible. The portfolio manager could create two distinct trade lists by performing separate rebalancings using each risk model independently. The portfolio manager could then trade the average of the two trade lists, assuming the averaged trades and portfolios were acceptable[1]. Alternatively, the portfolio manager could explicitly average the risk predictions of both models, and construct a portfolio whose risk target was limited by the average model prediction. Both of these approaches only make sense if the risk predictions of the two risk models are comparable, and

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[1] The average holdings and trades will only satisfy the linear constraints imposed in the portfolio construction process. Nonlinear constraints such as risk and number of names held or traded are usually not satisfied by the averaged portfolio. In any event, the averaged portfolio and trade list will generally not be optimal.

both risk models are of equal quality. There would seem to be little motivation for averaging a "good" risk model and a "poor" risk model.

When the two risk models are not comparable, of unequal quality, or if the portfolio's tracking error limit is defined explicitly by the primary risk model, then averaging methods are not attractive. In these cases, the second risk model must be incorporated into the portfolio construction strategy by either adding it to the objective term (either as risk or variance) or by adding a new risk constraint using the second model. The obvious question that arises is calibration. When the second risk model is added to the objective, a risk aversion constant must be selected; when it is added as a risk constraint, a risk limit must be chosen.

The most common use of multiple risk models for portfolio construction is robust portfolio construction (Ceria and Stubbs, 2006). In robust portfolio construction, a second risk model is constructed that measures the estimation error in expected asset returns. This second risk model is not comparable to the primary risk model, and, often, the second risk model is diagonal. The second risk model is added to the objective function (as risk, not variance). Although some encouraging results have been reported, a number of studies have found that the robust portfolios obtained in this approach are too conservative (Zhu et al., 2008).

In this paper, we incorporate the second risk model into the portfolio construction strategy as a second, independent risk limit constraint and propose a calibration strategy that ensures that both risk model constraints (primary and secondary) are binding for the optimized portfolio. If the risk limits are not calibrated in this fashion, then it is possible (and often likely) that only one risk model constraint, the more conservative one, will be binding for the optimal portfolio solution. The other risk model will be superfluous. Although one can imagine scenarios where this might be desirable – say, over the course of a backtest where the most conservative solution is desired – the calibration procedure described here is superior in at least three respects. First, the procedure enables a portfolio manager to ensure his or her intended outcome regardless of whether that intention is to have one, both, or neither risk models binding. Second, as illustrated in this paper, in many cases there is substantial, synergistic benefit when both risk models are simultaneously binding. And third, we explicitly avoid overly conservative solutions. In fact, throughout this paper, we only consider portfolios that are just as conservative (i.e., have identical risk) as the portfolios obtained using one risk model alone. More conservative solutions are not considered.

The second risk constraint interacts with both the primary risk constraint and with the other constraints in the portfolio construction strategy. Most common portfolio construction strategies use linear constraints such as bounds on asset holdings or trades, exposures to risk factors, and limits on portfolio turnover. Linear constraints are easily understood, numerically efficient to solve, and provide direct, rigid control of the portfolio's asset bets and trades sizes as well as the portfolio's exposures. Quadratic risk constraint are more challenging to solve[2], but they permit more flexible control of the portfolio characteristics in that larger asset bets are allowed if their contribution to risk is sufficiently small. As shown by the results reported here, properly calibrating a portfolio construction strategy with multiple risk models often leads to superior portfolio performance over using any one single risk model alone.

In this paper, we performed calibrations for three different, secondary risk model constraints from October 31, 2005 to October 31, 2006. We then test the performance of the calibrated portfolio construction strategies out-of-sample from October 31, 2006 to October 31, 2007. The primary risk model in each case is Axioma's Japanese daily, fundamental factor model. For the first example, the second risk model is Axioma's Japanese daily, statistical factor model which is used to constrain active risk. Hence, in this example, there are two different, but comparable risk models in the portfolio construction strategy. In the second example, the Axioma Japanese statistical factor model is used as the second risk model but to constrain total portfolio risk instead of active risk. In the third example, the second risk model is the active specific risk prediction from the Japanese fundamental model. All three examples lead to superior portfolio performance both in- and out-of-sample tests. The results of the calibration procedure illustrate the parameter regions where both risk models are binding, and give guidance on how to adjust the other portfolio parameters in the portfolio construction strategy.

Example 1. Constraining Active Risk with Fundamental and Statistical Risk Models

In our first example, we calibrate a simple portfolio construction strategy using Axioma's two Japanese risk models. The primary risk model is the fundamental factor model. The second risk

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[2] In fact, quadratic optimization algorithms are unable to solve quadratic constraints. With these algorithms, risk terms must be embedded in the objective function as variance with a risk aversion coefficient. Modern second-order cone programming (SOCP) optimizers such as Axioma's handle quadratic constraints directly and efficiently.

model is the statistical factor model. We take the largest 1000 assets in the TOPIX exchange as our universe and benchmark. This is a broad-market benchmark which is similar to (but not identical to) the TOPIX 1000 index. We rebalance the portfolio monthly from October 31, 2005 to October 31, 2006. This consists of only 12 rebalancings, which is small but nevertheless illustrates the second risk model calibration procedure.

The portfolio construction strategy maximizes expected, active return. Our naïve expected return estimates are the product of the assets' factor exposures to Axioma's Japanese fundamental factor risk model and the factor returns over the 20 days prior to rebalancing. These expected return estimates are used for illustration only and are not expected to be particularly accurate.

At each rebalancing, we impose the following constraints, parameterized using the three variables X, TO, and Y:

- Long-only holdings.
- Maximum tracking error[3] of 4% (The primary risk model constraint.)
- Active asset holding bounds of ± X%
- Maximum, monthly, round trip portfolio turnover of TO%.
- Maximum active statistical risk model risk of Y% (The second risk model constraint)

The portfolio starts from an all cash position, so the turnover constraint is not applied in the first rebalancing.

With X and TO fixed, at each rebalancing we determine the range of Y over which both risk constraints are binding. We then compute various statistics such as the number of asset held and the realized portfolio return for the optimal portfolios within this range.

Figure 1 shows results for TO = 30%. The horizontal axis gives the values for the maximum asset bound constraint (X), and the vertical axis gives the level of active risk (Y) from the statistical factor model. The average number of assets held is shown by the colored region. The average number of assets held varies from 112 to 175. The white regions in the figure indicate regions in which at least one risk model is not binding. The white region above the colored region represents solutions in which the statistical factor model's risk constraint is sufficiently large that it is not binding and therefore superfluous. The optimal solutions in this region are

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[3] Tracking error is the predicted active risk with respect to the market-cap weighted universe.

identical to those on the upper edge of the colored region. The white region below the colored region is the region in which the statistical factor model's risk constraint is sufficiently tight that the tracking error constraint from the primary, fundamental factor model is no longer binding. In this region, the primary tracking error constraint is irrelevant, and the portfolios are more conservative than indicated.

Even though both risk models are comparable, the colored region lies entirely below 4% of active risk. If we had naively created a portfolio construction strategy in which both risk models were limited to 4%, then the second risk model would have been superfluous (non-binding) in all optimized solutions.

Although the results may look like an efficient frontier, they are not. The entire colored region corresponds to optimal portfolios, not just those on its boundaries. The results are the averages over the 12 rebalancings, so the boundaries for any particular rebalancing may be somewhat different than those shown in the Figure. We have used a 12 month calibration period to limit these differences. The maximum tracking error of 4% is shown by the dashed line.
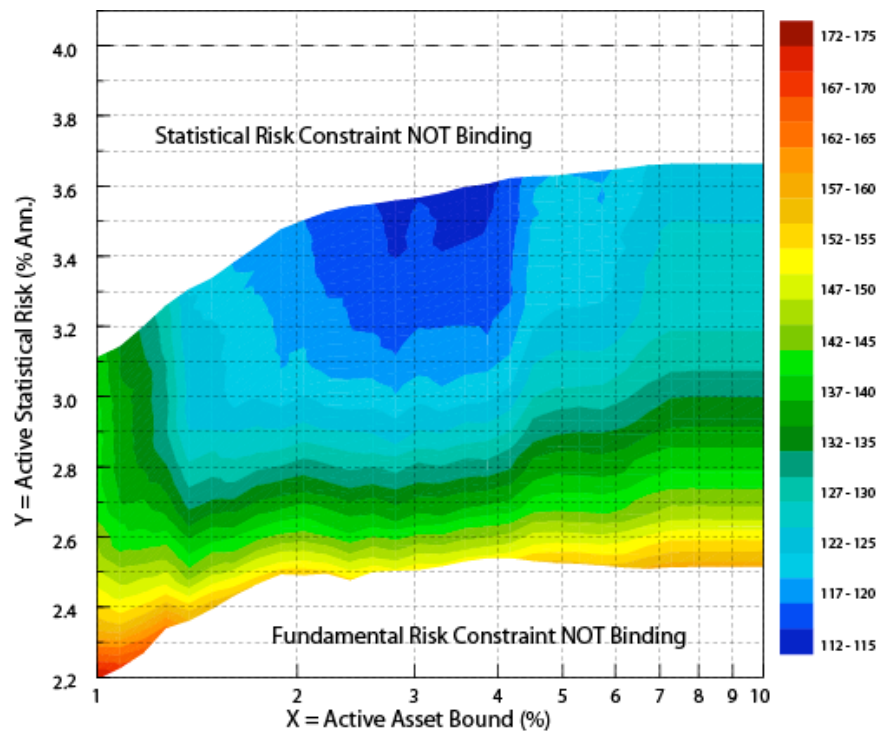

Figure 1. The narrow, colored contour gives the average number of asset held when both risk models are binding. TO = 30%.

The colored contours in Figure 1 form two distinct regions. For a max asset holdings bound (X) of less than 3%, the tighter asset bounds increase the number of assets held. In addition, tighter statistical factor model risk constraints also increase the number of assets held. For a max asset holdings bound (X) greater than 3%, more assets are held by either tightening the statistical factor model risk constraint or by raising the max asset bound (X) constraint. The most diversified portfolios are obtained by doing both simultaneously.

Figures 2 and 3 show contours of the cumulative (in this case, annual), active return and the single, worst, monthly active return over the course of the 12 rebalancings. Neither of these results includes transaction costs or market impact.
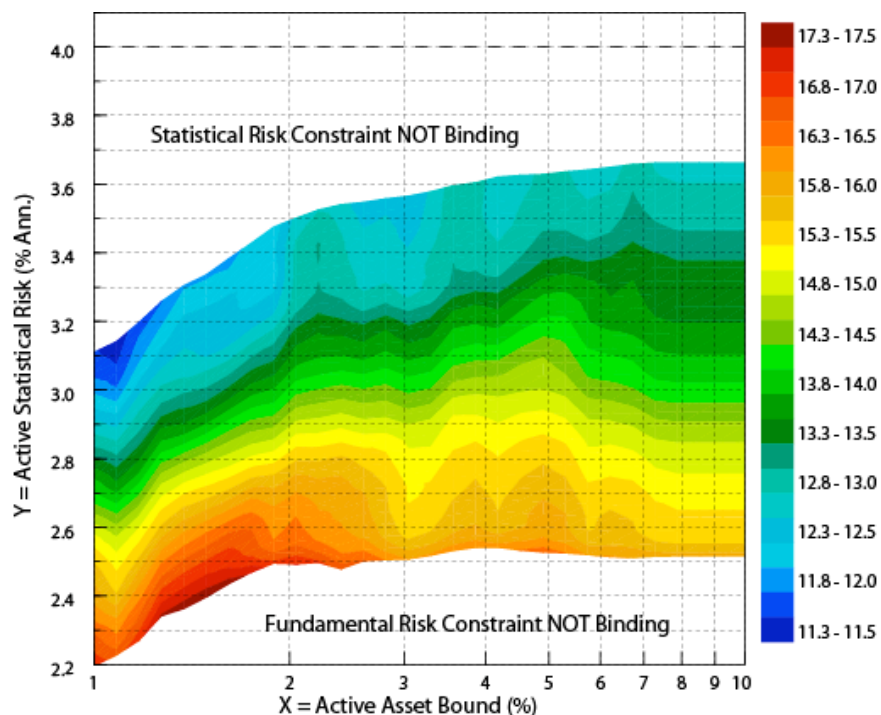

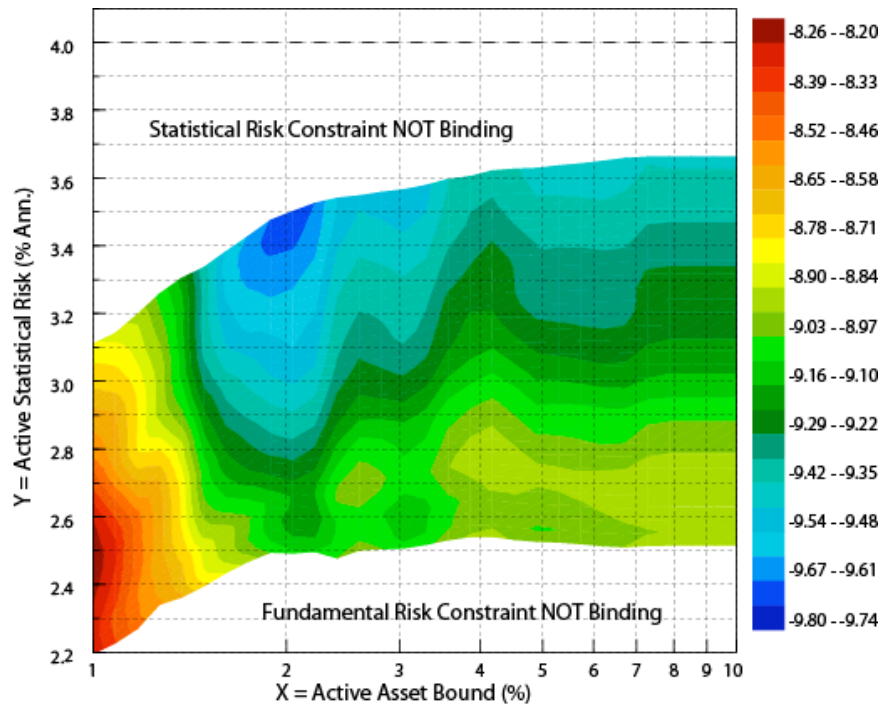**Figure 2.** Cumulative, active return (%) when both risk models are binding. TO = 30%.

Figure 3. The worst, monthly, active return (%) when both risk models are binding. TO = 30%.

Adding the Japanese statistical factor model risk constraint generally increases the portfolio return and improves the worst monthly return. The strategy parameters of X = 1.0% and Y = 2.3% produce a large cumulative return and the smallest worst monthly return. This solution performs better than any of the solutions without the secondary statistical factor model risk constraint (the white region above the colored regions).   These parameter values are the suggested strategy calibration for TO = 30%.

Figures 4, 5, and 6 below show the same calibration results for TO = 15%, 30%, and 60%.   The Figures show that TO has a profound effect on the portfolios obtained: tighter TO constraints narrow the region over which both risk models are simultaneously binding. As a practical matter, for strategies similar to TO = 15%, it may be difficult to keep a strategy in the dual-binding region unless it is calibrated explicitly. (The colored regions shown here are averages over all 12 rebalancings.)
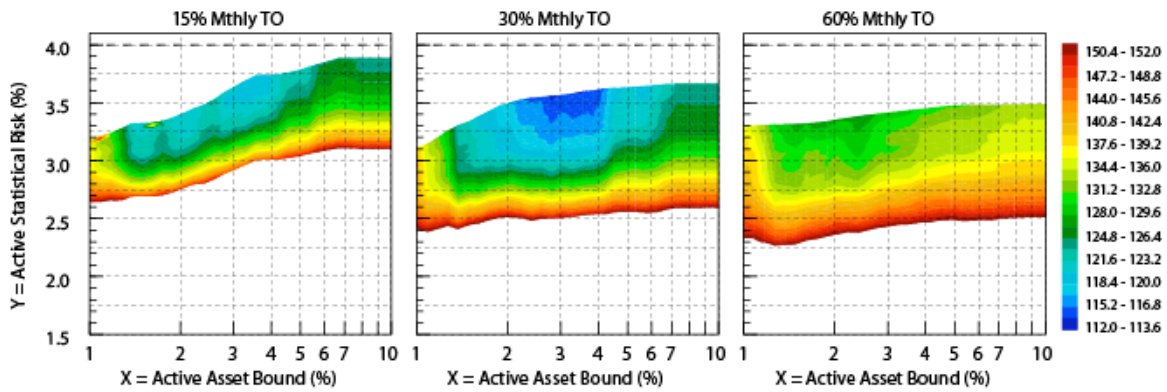
Figure 4. The number of asset held versus asset bound
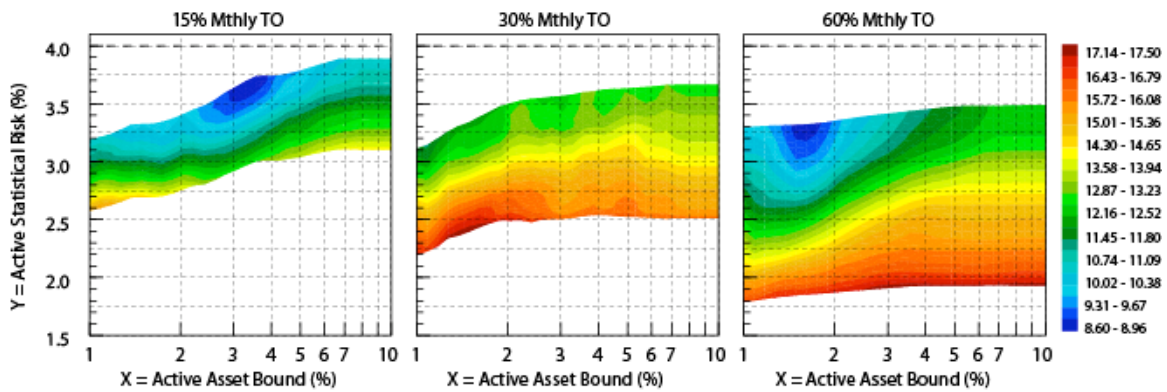and the statistical risk model risk constraint for TO = 15, 30, and 60%.

Figure 5. The cumulative, active return (%) versus asset bound
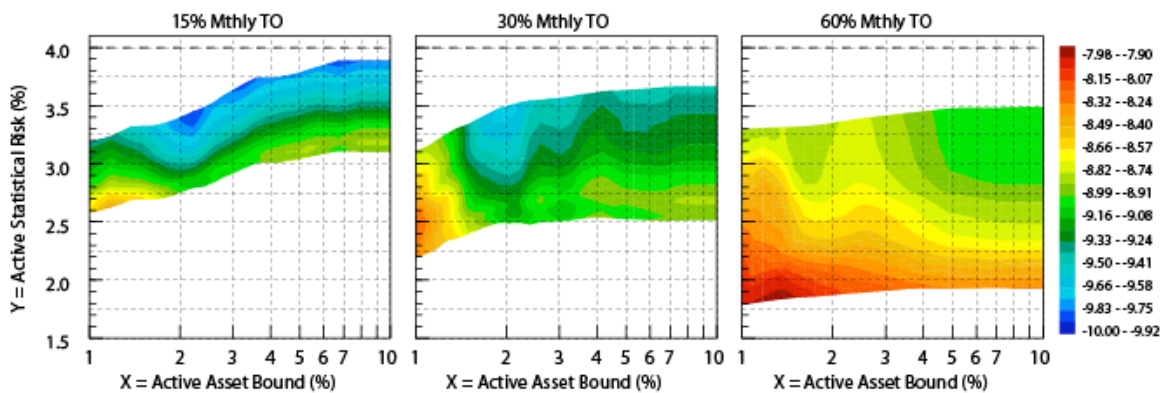and the statistical risk model risk constraint for TO = 15, 30, and 60%.

Figure 6. The worst case monthly active return (%) versus asset bound
and the statistical risk model risk constraint for TO = 15, 30, and 60%.

The case with TO = 30% is shown here in the same way as previously shown in Figs. 1 to 3, and is shown in order to facilitate comparison with the other TO cases.

In all three TO cases, portfolio performance is improved by adding the second risk model constraint and making it as tight as possible without making the primary risk constraint non-binding.

For TO = 15%, the best cumulative returns are given by either a max asset holdings bound constraint (X) = 1.0% or X = 10%. The best, worst case return is achieved for similar values of X, with X = 1.0% being slightly better.   We therefore conclude that the best calibration is X = 1.0% and Y = 2.6%. This solution has the largest number of assets held (about 150).

For TO = 60%, the story is similar. The region over which both risk models are binding is substantially broader.   Cumulative return is maximized by taking a relatively low asset holdings bound (X = 1.1%) and a tight statistical risk model risk constraint (Y = 1.7%).

Example 2. Constraining the Total Risk Predicted by the Statistical Risk Model

For our next example, the second risk model is, once again, Axioma's Japanese statistical factor model. In this case, however, this second model is used to constrain the total risk of the portfolio instead of the active risk as was done in the first example.   The first risk model, Axioma's fundamental factor model, is still used as the primary risk model and constrains the active risk (tracking error) of the portfolio to 4%, as in the above case.

The motivation for this strategy is the fact that statistical factor risk models normally have fewer factors than fundamental factor risk models. Since the minimum half-life of the historical time series is affected by the number of factors, statistical models can use a shorter half-life and can therefore respond more quickly to market movements, which, in turn, may be captured more by total risk predictions than by active risk predictions.

We test the same universe and rebalancing frequency as in the previous example. For sake of brevity, we will only report results for TO = 30%. Figure 7 shows contour plots of the

cumulative and worst monthly active returns as functions of the asset bound X and the specific risk model total risk constraint level Y.
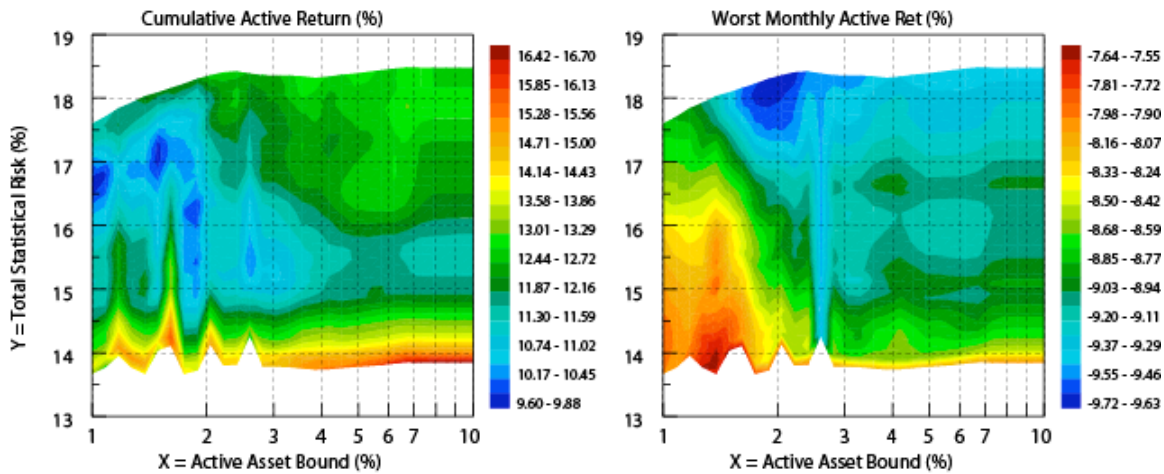

Figure 7. The cumulative and worst case monthly active returns versus asset bound and total, statistical risk limit for TO = 30%.

The results are similar to those reported in the previous example. The best performance generally occurs as the second risk model constraint is tightened until the primary risk model constraint is almost non-binding. The best solution for this case is approximately X = 1.2% and Y = 14.5%.

Example 3. Constraining Specific Risk as the Second Risk Model Constraint

For our third example, the second risk model is the specific risk predicted by Axioma's Japanese, fundamental factor model. This is a diagonal (uncorrelated) risk model which is meant to give results that are similar to those obtained using classical robust portfolio construction with a diagonal, estimation error matrix. Specific risk is a convenient "second" risk model to use since it is already specified by any factor model (both fundamental and statistical models). Of course, stock-picking investment strategies that maximize specific risk may not benefit by constraining specific risk, as done here. Nevertheless, such strategies may benefit from a well-chosen, diagonal, second risk model.

Figure 8 shows contour plots of the cumulative and worst monthly active returns as functions of the asset bound X and the specific risk model risk constraint level Y for TO = 30%.
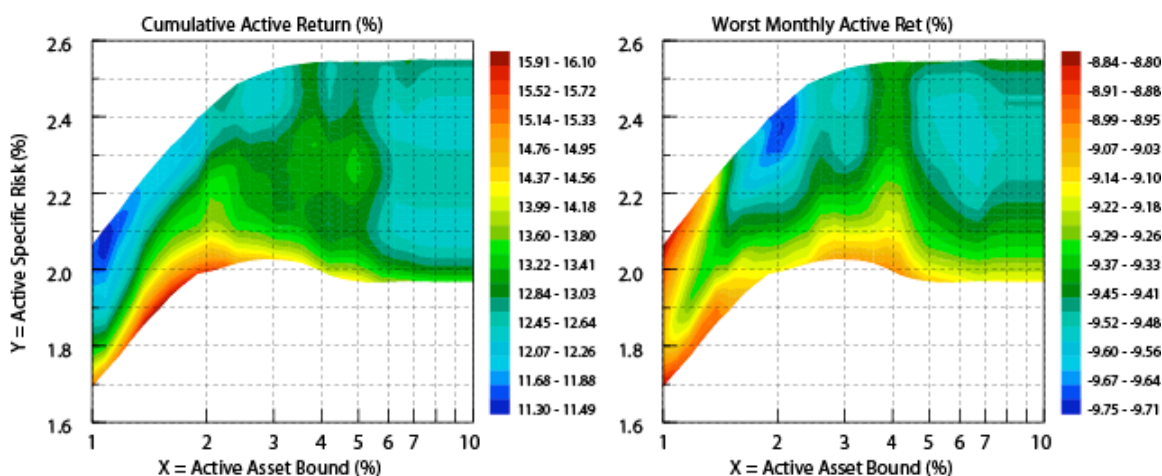
Figure 8. The cumulative and worst case monthly active returns versus asset bound and active specific risk limit for TO = 30%.

The results are similar to those reported in the previous examples. The best solution occurs when X = 1.1% and Y = 1.8%.

Out of Sample Results

We performed an out-of-sample backtest from October 31, 2006 to October 31, 2007. We considered six different strategies, three without the second risk model, and three with the second risk model. The cases correspond to the optimal parameters determined by the results above. These different cases are summarized in the top of Table 1 below. TO = 30% and the primary tracking error from the fundamental factor model is 4%. When running the backtest, we start from October 31, 2005 and run 24 monthly rebalancings so that the turnover for the first, out-of-sample month is meaningful. The results reported are only for the 12 out-of-sample rebalancings.

Table 1 shows the performance statistics for these six cases.

	Second Risk Model					
	Statistical Act. Risk		**Statistical Tot. Risk**		**Specific Risk**	
Asset Bound	1.0%	1.0%	1.2%	1.2%	1.1%	1.1%
Second Risk Bound	None	2.3%	None	14.5%	None	1.8%
Ave Assets Held	128.4	149.33	128.4	126.17	127.3	169.33
Annual Active Ret	3.71%	4.23%	3.58%	8.34%	3.528%	5.218%
Worst Mthly Ret	-1.68%	-2.00%	-1.79%	-1.98%	-1.866%	-1.888%
Mthly Vol	1.43%	1.29%	1.55%	1.59%	1.464%	1.500%
Information Ratio	0.748	0.949	0.668	1.518	0.696	1.004
Transfer Coefficient	41.6%	42.9%	42.0%	41.5%	42.1%	42.6%

Table 1. Out of Sample Results for October 31, 2006 to October 31, 2007.

In all cases, the addition of a properly calibrated second risk model constraint lead to superior portfolio performance as measured by either the strategy's annual active return or its information ratio.   In general, the risk metrics – volatility or worst monthly return – changed only slightly (in either direction) while the return increases substantially. Of course, there are only 12 out-of-sample returns, so the performance statistics have relatively large standard errors. Nevertheless, the performance statistics have moved in a favorable direction when the second risk model was added to the strategy and all the parameters were calibrated adequately.

For the three different, second risk model constraints examples considered, the largest out-of-sample improvement over the use of a single risk model occurred in the second case, when the second risk constraint, using the statistical factor model, was used to limit the total risk of the portfolio.

Discussion

Although we have only shown results incorporating a second risk model, three or more risk models could be used simultaneously in a portfolio construction strategy. Since this would add additional strategy parameters, the calibration procedure would become more complicated. Nevertheless, the same procedure of ensuring that all risk model constraints are binding can be used.

In our examples, we have added the second risk model as a risk constraint. It could also be incorporated into the objective function of the portfolio construction strategy, as could be done with the primary risk model constraint as well.   When a risk model is added to the objective function, it *always* affects the solution. There are no solutions corresponding to the non-binding

risk constraint case. This can be advantageous in that it is impossible to choose calibration parameters that render a risk model superfluous. In fact, many portfolio managers mistakenly believe that the only way to obtain solutions on the efficient frontier is to include the risk term(s) in the objective function[4].

There are, however, disadvantages to including risk in the objective function. First and foremost, the actual risk predicted is unknown. It may or may not be close to the targeted tracking error. In addition, although the risk terms always affect the solution, it can be difficult to determine by how much they affect the solution or when the other portfolio construction constraints are dominating the optimization portfolio solution. In the examples above, the predicted risk could be directly compared to their constraint limits. When risk is in the objective function, however, the only way to determine its significance is to solve the problem twice – once with the risk in the objective function, and a second time with the risk aversion parameter altered (or omitted entirely) – and then compare the resulting portfolios. The portfolio manager would then have to estimate whether or not the differences in the portfolios were meaningful.

Second, when risk is included in the objective function, the proper calibration values can vary significantly, depending on the expected returns used, and often makes little intuitive sense. If a portfolio manager is given a 4% tracking error target, he or she must first translate this 4% target into a corresponding risk aversion value. This risk aversion value may vary considerably from one portfolio construction strategy to another even if all the strategies target 4% tracking error. Next, if a second risk model is added to the objective function, the portfolio manager will have to calibrate *both* risk aversion constants once again. The risk aversion found using one risk model is unlikely to be the same when two risk models are present as the objective maximizes the weighted sum of the variances.

Third, if the objective also includes corrections due to transaction costs, market impact, the cost of shorting, and the like, the risk aversion constants will change again and recalibration will be needed.

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[4] This is likely a legacy of the successful marketing of quadratic optimization programs that could *only* handle risk in the objective function. SOCP solvers do not suffer this handicap.

On balance, even though the same solutions can be found incorporating risk into the objective function, the lack of intuition and the inability to use previously obtained risk aversion values appear to be practical drawbacks for incorporating risk into the objective function.

Using more than one risk model in a portfolio construction strategy allows a portfolio manager to exploit the fact that different risk models measure and capture risk differently. Having both a fundamental and statistical risk model simultaneously in the strategy ensures that the optimized portfolio reflects both points of view. The benefit is derived by the differences captured by both risk models, not which risk model is "better." If a portfolio manager believes one risk model is "better" than another, then he or she can simply use the "better" risk model as the primary risk model. When properly calibrated, the final two-risk model portfolio is not any more conservative than the one-risk model portfolio.

Of course, as illustrated in the third example, a strategy can incorporate two different risk constraint based on only one risk model. The second example could also have been performed with one risk model. This can be appealing if a second risk model is not available. However, this approach relies entirely on the risk factors captured by the single risk model. This may not fully capture all the diversification that would be captured by having two distinct risk models.

Finally, as illustrated by the results, the best strategy parameters involve the interaction of all the constraints in the portfolio construction strategy. The results presented here illustrate that interaction of four constraints – tracking error, asset holdings bounds (X), second risk model's risk constraint (Y), and turnover (TO). When calibrating a second risk model, it may be necessary to alter (even loosen) previously calibrated constraint values in order to obtain the best results.

Conclusions

We have presented a systematic calibration procedure for assessing the potential benefit of adding a second risk model to a portfolio construction strategy. Bearing in mind that a limited number of tests were performed, the results suggest the following:

    (1) The addition of a second risk model can lead to better performance than using one risk model alone when the second risk model constraint is calibrated so that both risk model

constraints are simultaneously binding. In many cases, a good calibration tightens the second risk model constraint until the primary risk model constraint just remains binding.

(2) Calibration of the other portfolio construction parameters is essential. In particular, since the region over which both risk models are binding can be narrow, guessing what value to use (for X and Y in our examples) can lead to only one risk model being binding for the optimal solution, even when both risk models are comparable. The other constraints in the portfolio construction strategy can greatly influence the region over which both risk models are binding.

(3) In many cases, calibration results give a clear, optimum set of portfolio construction parameters in terms of return and risk.

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Appendix – Calibration Using Axioma Portfolio

Here we illustrate how to calibrate the second risk model using Axioma Portfolio. We use active specific risk as the second risk model constraint as it is readily available. Suppose we have a portfolio construction strategy that maximizes expected return with the following constraints:

- Fully invested budget constraint.
- Long Only
- Maximum tracking error of 4%

- Sector Neutral

- Active asset holding bounds ± 4%.

- Maximum active specific risk – TBD.

The figure below shows the original strategy with the specific risk limit of 99%, a value that is unlikely to be binding.


Name	Active	Type	Selection	Scope	Unit	Min	Max	Benchmark
budget	✓	Budget			CURRENCY			
Asset Bound	✓	Limit Holding	MASTER	ASSET	PERCENT	-4	4	REBALANCING.BENCHMARK
TrackingError	✓	Limit Risk	MASTER	AGGREGATE	PERCENT		4	REBALANCING.BENCHMARK
Specific Risk	✓	Limit Risk	MASTER	AGGREGATE	PERCENT		99	REBALANCING.BENCHMARK
Sector Neutral	✓	Limit Holding	Multiselection	SELECTION	PERCENT	0	0	REBALANCING.BENCHMARK

Initially, we only have a general idea what the specific risk constraint should be (say, 30 – 70% of the total active risk).   We determine the range over which both risk models are binding by using Axioma Portfolio's Frontier functionality.
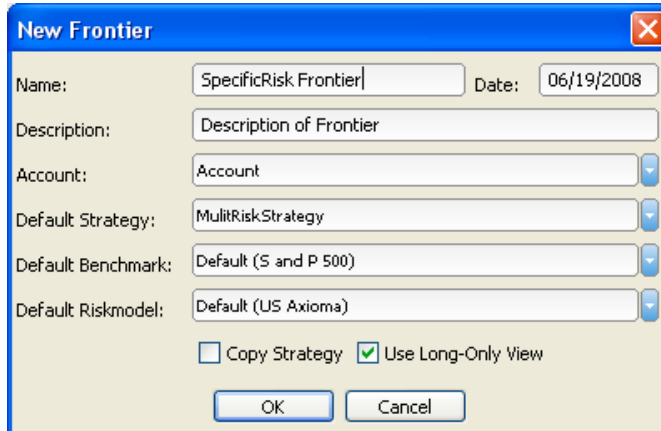
- Change to the Frontier Perspective

- Create a New Frontier
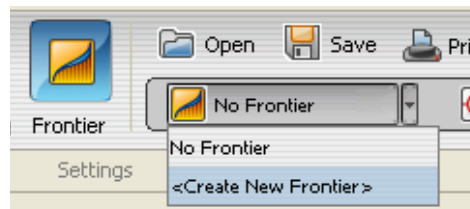
- Fill out the New Frontier information form.
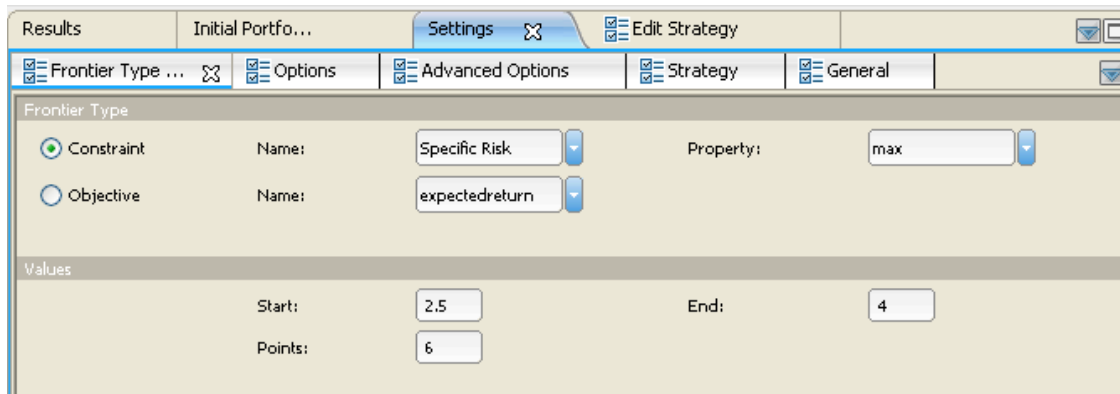
- Under the Setting/Frontier Type Tabs: select:

  - Constraint

  - Name: Specific Risk

  - Start: 2.5

  - End: 4.0

  - Points: 6

We are simply guessing that the range of specific risk limits over which both risk constraints will be binding is between 2.5% and 4.0%. Once we have the Frontier results, we can verify whether or not this assumption is true.

- Run the Frontier:

Once the frontier has finished, examine the Results/Summary window:

	Account	Solution 1	Solution 2	Solution 3	Solution 4	Solution 5	Solution 6
Frontier Value		2.5000	2.8000	3.1000	3.4000	3.7000	4.0000
Exp Return	0.00%	288.11%	307.72%	322.83%	329.89%	330.01%	330.01%
Transfer Coefficient (Over Local Universe)	-0.0025	0.4765	0.4542	0.4769	0.4876	0.4877	0.4877
Predicted Beta	0.0000	1.0239	1.0390	1.0265	1.0159	1.0148	1.0148
Total Risk	0.00%	14.30%	14.61%	14.45%	14.31%	14.30%	14.30%
Total Risk with Alpha Factor (40.0)	0.00%	14.95%	15.32%	15.34%	15.42%	15.44%	15.44%
Total Factor Risk	0.00%	14.07%	14.33%	14.10%	13.89%	13.87%	13.87%
Total Specific Risk	0.00%	2.59%	2.85%	3.14%	3.44%	3.48%	3.48%
Active Risk	13.53%	3.57%	4.00%	4.00%	4.00%	4.00%	4.00%
Active Risk with Alpha Factor (40.0)	13.65%	5.62%	6.14%	6.54%	6.98%	7.04%	7.04%
Active Factor Risk	13.47%	2.54%	2.86%	2.53%	2.11%	2.04%	2.04%
Active Specific Risk	1.30%	2.50%	2.80%	3.10%	3.40%	3.44%	3.44%
Long Count	0	114	98	80	63	62	62
Short Count	0	0	0	0	0	0	0
Cash Value	$0.00	$0.00	$0.00	$0.00	$0.00	$0.00	$0.00
Cash Imbalance	$100,000,000.00	-$0.00	$0.00	$0.00	$0.00	$0.00	$0.00
Long Value	$0.00	$100,000,000....	$100,000,000.00	$100,000,00...	$100,000,000.00	$100,00...	$100,00...
Short Value	$0.00	$0.00	$0.00	$0.00	$0.00	$0.00	$0.00
Net Value	$0.00	$100,000,000....	$100,000,000.00	$100,000,00...	$100,000,000.00	$100,00...	$100,00...
Reference Value	$100,000,000.00	$100,000,000....	$100,000,000.00	$100,000,00...	$100,000,000.00	$100,00...	$100,00...
L/S Ratio		∞	∞	∞	∞	∞	∞

Three rows have been highlighted:

• Frontier Value: the Active Specific Risk Constraint Limit applied.

• Active Risk: the primary risk constraint value (limited to 4%).

• Active Specific Risk, the second risk constraint value (limited to the Frontier Value)

By examining "Active Specific Risk" row, we see that the unconstrained active specific risk is 3.44%. This is determined by the fact that the Active Specific Risk is unchanged in Solutions 5 and 6, and in both solutions the Active Specific risk is less than the constraint limit imposed listed on the first line as Frontier Value.   Therefore, 3.44% corresponds to the top of the colored region in the calibration charts shown in this paper.

The lowest Active Specific Risk constraint we can apply and still maintain a 4% Tracking Error is somewhere between Solutions 1 and 2, with active specific risk constraints (Frontier Values) of 2.50% and 2.80% respectively.   This is determined by the fact that the active risk is 3.57% for Solution 1, which is below the targeted limit of 4%.   We can refine the result by resetting the Frontier and re-running it using specific risk limits between 2.5 and 2.8%.

Once we have determined the range of active specific risk limits over which both risk models are binding, we can further examine the characteristics of the solutions in this range. For example, the figure above shows the number of names held (Long Count) ranging from 62 to 114.

Now let's consider the same strategy as above except that we change the asset bound constraint to be ± 1%, a very tight limit. When we re-run the frontier, the Results/Summary is:

	Account	Solution 1	Solution 2	Solution 3	Solution 4	Solution 5	Solution 6
Frontier Value		2.5000	2.8000	3.1000	3.4000	3.7000	4.0000
Exp Return	0.00%	243.55%	243.55%	243.55%	243.55%	243.55%	243.55%
Transfer Coefficient (Over Local Universe)	-0.0025	0.3879	0.3879	0.3879	0.3879	0.3879	0.3879
Predicted Beta	0.0000	1.0643	1.0643	1.0643	1.0643	1.0643	1.0643
Total Risk	0.00%	14.84%	14.84%	14.84%	14.84%	14.84%	14.84%
Total Risk with Alpha Factor (40.0)	0.00%	15.27%	15.27%	15.27%	15.27%	15.27%	15.27%
Total Factor Risk	0.00%	14.63%	14.63%	14.63%	14.63%	14.63%	14.63%
Total Specific Risk	0.00%	2.47%	2.47%	2.47%	2.47%	2.47%	2.47%
Active Risk	13.53%	3.69%	3.69%	3.69%	3.69%	3.69%	3.69%
Active Risk with Alpha Factor (40.0)	13.65%	5.09%	5.09%	5.09%	5.09%	5.09%	5.09%
Active Factor Risk	13.47%	2.80%	2.80%	2.80%	2.80%	2.80%	2.80%
Active Specific Risk	1.30%	2.40%	2.40%	2.40%	2.40%	2.40%	2.40%
Long Count	0	100	100	100	100	100	100
Short Count	0	0	0	0	0	0	0
Cash Value	$0.00	$0.00	$0.00	$0.00	$0.00	$0.00	$0.00
Cash Imbalance	$100,000,000.00	$0.00	$0.00	$0.00	$0.00	$0.00	$0.00
Long Value	$0.00	$100,000,000....	$100,000,000.00	$100,000,00...	$100,000,000.00	$100,00...	$100,00...
Short Value	$0.00	$0.00	$0.00	$0.00	$0.00	$0.00	$0.00
Net Value	$0.00	$100,000,000....	$100,000,000.00	$100,000,00...	$100,000,000.00	$100,00...	$100,00...
Reference Value	$100,000,000.00	$100,000,000....	$100,000,000.00	$100,000,00...	$100,000,000.00	$100,00...	$100,00...
L/S Ratio		∞	∞	∞	∞	∞	∞

In this case, we get the same solution for all levels of active risk (3.69%) and active specific risk (2.40%). This means that neither risk constraint is binding when we set X = 1%. With this information in hand, a portfolio manager can perform another Frontier on this strategy's constraints (i.e., Asset Holding Bounds) to determine the optimal value.

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