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EQUITY MARKET NEUTRAL PORTFOLIOS
Hedging out market factors

Summary

Large shocks in an equity portfolio are typically driven by correlated (and hence collective) moves of its constituents. This accords correlation matrices a historically central place in numerous studies on portfolio construction and risk management [1].

In this note, we illustrate how certain statistical methods enable us to identify the main market factors (or 'modes') that an equity market neutral portfolio should hedge, in order to extract value from signals, while avoiding exposure to large, collective market moves. These methods rely on the processing of stock returns correlation matrices.

However, because time series are finite, measured correlations are subject to the effects of noise: a fact that one must take into account when employing empirical correlation matrices in portfolio construction. Comparing the properties of empirical correlation matrices to those obtained in random cases, and using results from the theory of random matrices, enables us to distinguish genuine characteristics of the dependence structure of a set of stocks from noisy and unreliable features.

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Market neutral portfolios: hedging out the market modes

Equity market neutral portfolios seek to profit by capturing pricing anomalies in a relative manner. Regardless of the dataset (e.g., price-based or fundamental) from which the predictive signals derive, the portfolio is short stocks that are considered (temporarily) overpriced, and long those considered (temporarily) underpriced, with a targeted zero net exposure.

There is, however, more to true market neutrality than cash neutrality. Market neutral portfolios should aim to be as insensitive as possible to market moves, profiting from relative pricing anomalies without being exposed to shocks resulting from correlated moves of underlying stocks. It is necessary, then, to identify the main factors (or 'modes') that are expected to generate such moves.

Being 'orthogonal' to the market...

Intuitively, the portfolio must be orthogonal to (or statistically independent from) 'the market', that is, the tendency for all stocks to move in the same direction, since most have positive beta to the market index. To illustrate, let $P = (p_1, p_2, \ldots, p_N)$ be a portfolio of dimension N, with each pi the position (in dollars, long or short) of each stock, i. P is the portfolio that results from a combination of various signals (e.g. mean reversion, trending, fundamentals, etc.), before neutralization. A simple, intuitive 'market' mode is a vector equally weighted to each stock: $M = (1, 1, \ldots, 1)$. Neutralizing P with respect to M is then a simple transformation[1], so that $P_{neutral} * M = p_{1neutral} + p_{2neutral} + \ldots + p_{Nneutral} = 0$. In this first, rudimentary example, market neutrality is equivalent to cash neutrality.

...but also to other factors...

Beyond this intuitive market mode, it is easy enough to think of other factors with different co-movement properties that should also be hedged out as sources of undesirable variance. The Fama-French [2] factors are candidates, since they are known to explain a substantial portion of the variance of stock portfolios. From

observation or intuition, others also arise, such as cyclical vs. non-cyclical, or commodities vs. financials, which are also good variance explainers, which we might also seek to neutralize.

...that we extract in a systematic manner

As with all of the building blocks of our strategies, CFM favours a statistical, data-based approach, enabling a direct, systematic and self-adapting definition of various market modes that evolve with market conditions.[2]

Principal Component Analysis: identifying the market modes

Principal Component Analysis

Principal Component Analysis (PCA) is a statistical method that extracts statistically independent factors (or 'modes') that best explain the variance of a set of stock returns data. In the market neutral portfolio context, exposure to these modes should be reduced to zero, in order to minimize exposure to the variance that they generate.

A 2-stock illustration

The figure below geometrically illustrates this in the simple case of two stocks (in this case, normalized daily returns of Apple and IBM from 2000 to 2014), where we identify maximal inertia orthogonal axes in the scatterplot of returns.

Performing a PCA is equivalent to 'diagonalizing' the correlation matrix of stock returns. In short, PCA provides us with an ordered set of independent modes (the 'eigenvectors') and the part of the variance that each explain (the 'eigenvalues').

This is illustrated in the **Figure 1** below for the simple 2-stock market case. The main market mode $M = (1, 1)$, in red, corresponds to simultaneous co-movements of the 2 stocks. It accounts for 70.4% of the variance (with an eigenvalue of 1.41). The second mode (in blue) is

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[1] Which is projecting out the component of P that is parallel to M.

[2] CFM has published several papers on the dynamics of market modes, which are available at www.cfm.fr

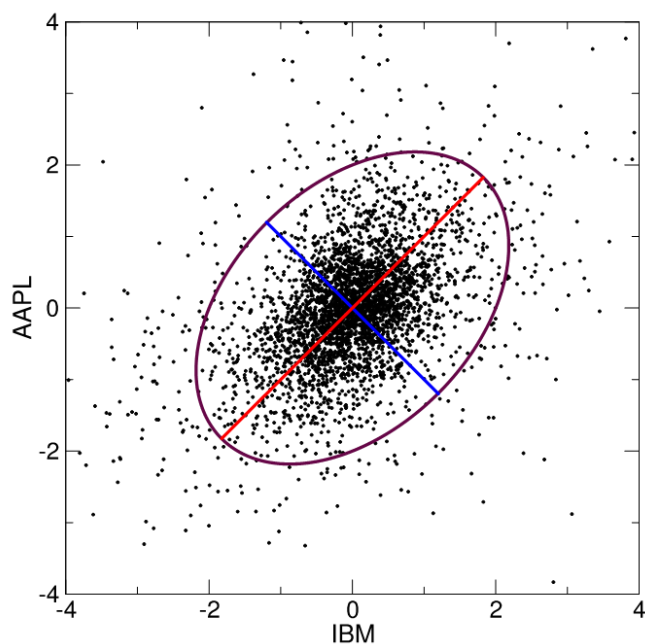orthogonal to the first, and accounts for 29.6% of the variance (with an eigenvalue of 0.59).

Figure 1

The case of an N-stock portfolio

This example is simple to extend to the more relevant multidimensional case of a global equity market neutral portfolio. A set of N stocks will result in N modes and their N corresponding eigenvalues. **Figure 2** is a histogram of the eigenvalues of the correlation matrix estimated with the normalized returns, from August 2008 to June 2014, of a pool comprised of the $N = 500$ most liquid US stocks on the period.

The bulky part indicates a myriad of small eigenvalues, individually of minor importance in terms of generated variance. A few larger eigenvalues stand out of the bulk on the right, representing the modes that dominate the variance.

The market and secondary modes

To the far right on the inset is the largest eigenvalue, associated with the market mode[3], which accounts for 41% of the total variance. The other large eigenvalues are associated with sector-related modes. Of note, secondary market modes vary a lot in nature, depending on periods and market zones. The second most important mode

tends to be 'Commodities vs. Others' in the Australian market, whereas it can be 'Financials and Utilities vs. Others' in the European markets. Changes in the amplitude and nature of these modes make it inefficient to try and guess them, and advocate in favour of a systematic and data-based extraction.
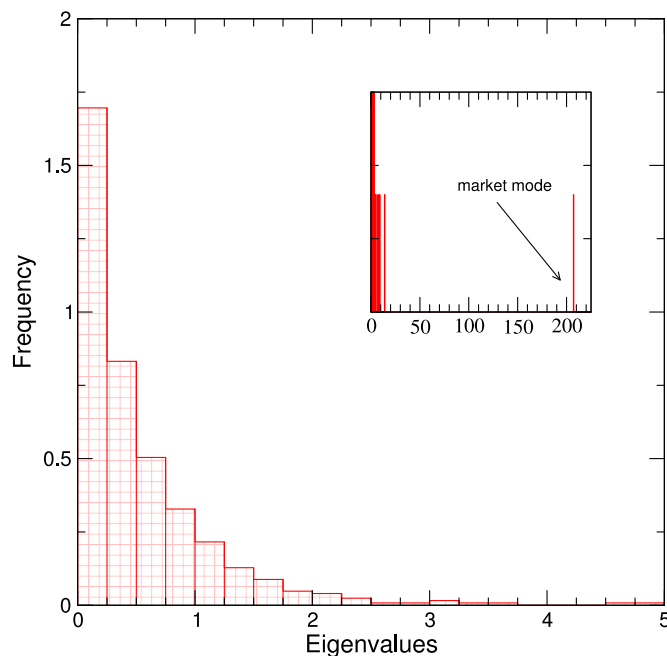

Figure 2

Noisy measurements

One could naively neutralize the portfolio with respect to several modes, but there is some subtlety in determining the appropriate number of modes to hedge. The finite nature of the time series and the large dimensionality of the correlation matrices introduce significant noise in their estimation[4]. Those are, to a large extent, random, and one should be careful in applications. The smallest eigenvalues are particularly sensitive, it is thus important to be able to distinguish signal from noise; that is, to determine which eigenvalues contain real information and which result primarily from randomness, and hence are expected to be unstable.

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[3] Of note, when N is larger than 2, the market mode can significantly deviate from the equally weighted vector (1, 1,..., 1), depending on market conditions.

[4] In our example, N*(N-1)/2=124,750 correlation coefficients are estimated using time series of around T=1,000 days. This dimensionality causes that among many measured coefficients, some will appear

much greater/smaller than their 'true' value, hence introducing randomness in the correlation mix. The ratio N/T is a crucial feature in estimating correlation matrices.

Random Matrix Theory: selecting the modes that matter

As already pointed out, the finiteness of time series introduces noise into the empirical correlations. PCA enables us to identify the modes that best explain the variance of the portfolio. Those corresponding to the largest eigenvalues seem to clearly indicate modes to be hedged out, whereas at the smaller end, noise becomes dominant. It would be good to have an objective criterion to distinguish the modes which correspond to the true correlation structure from those that are a result of randomness.

Comparison to a random case

It is then informative to compare the properties of an empirical correlation matrix (computed on realized stock return data) to a random correlation matrix that one would obtain from strictly independent assets. Deviations from the random matrix would suggest the presence of real information on the dependence structure of a set of stocks. The theory of random matrices, which flourished in the 1960s, has provided many interesting results, some of genuine interest in financial applications [3].

A cut-off between noise and information

In particular, it provides us [4] with a formula for the density of eigenvalues of the correlation matrices obtained from N independent stocks on a sample of size T, where both N and T are large. This is illustrated by **Figure 3**, showing both the empirical distribution of eigenvalues (in red), and the prediction of random matrix theory (in black). Several of the empirical eigenvalues exceed the limit predicted for a purely random matrix (at the right of the black curve), making the case for considering only a subset of the empirical modes in the portfolio construction process. Further refinements can be (and are) made in order to better filter out eigenvalues [4].
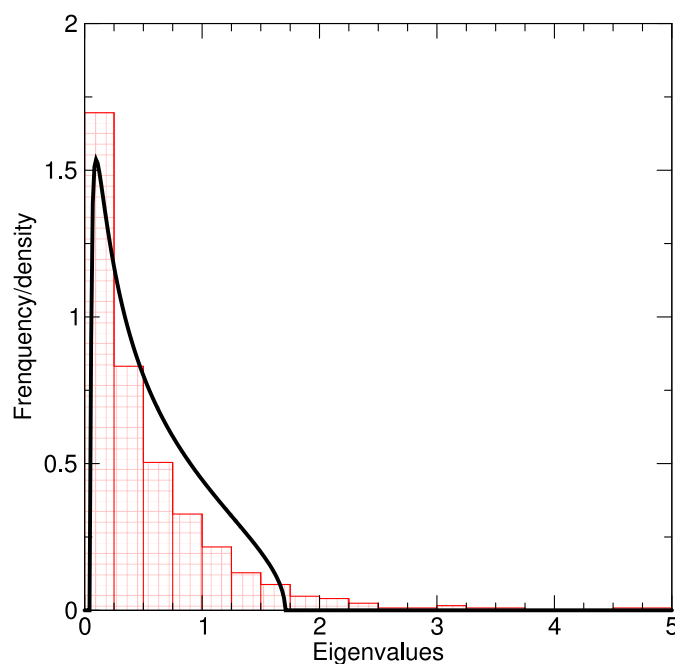

Figure 3

Conclusion

This note outlines some of the motivations for using advanced quantitative methods in portfolio construction, particularly in an equity market neutral context.

PCA is a powerful statistical tool used with empirical correlation matrices to identify primary market modes to be hedged out, in order to protect against undesirable factor volatility.

Many practitioners 'neutralize' their portfolio by maintaining a zero net exposure and possibly controlling a couple of the usual risk factors. However, secondary market modes vary a lot (in amplitude and in nature) across periods and zones. This advocates for a systematic extraction of market factors, which in general cannot be intuited, and which evolve with market conditions.

Finally, the naïve use of empirical correlation matrices and PCA ignores a significant random component, distorting the process. Random matrix theory provides a method to determine a cut-off between real information and noise, making possible a more robust portfolio construction.

The methodology described applies equally well to portfolio optimization and risk assessment. Pure empirical correlation matrices are inadequate, and may lead to an underestimate of risk [3].

Works cited

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Important disclosures

CFM has pioneered and applied an academic and scientific approach to financial markets, creating award winning strategies and a market leading investment management firm.

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