Quantitative Portfolio Strategies

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THE GLOBAL AGGREGATE: RETURN REPLICATION WITH DERIVATIVES

The risk diversification effect offered by the Lehman Brothers Global Aggregate Index makes it an attractive benchmark for investors. Its comprehensive coverage of global investment grade fixed income markets, the broad acceptance of its underlying component indices, and the impending addition of Asian corporate bonds make it a benchmark of choice for asset-allocation funds and investors interested in global spread products.[1]

The management of a bond fund benchmarked to the Global Aggregate requires particular skills and resources. Decision-making in the case of the Global Aggregate benchmark has many more dimensions than in the single-currency case: there are 10 exchange rates, 11 yield curves, and often several credit quality and sector components for each currency's index. In addition, the potential for currency hedging opens up another decision dimension that allows the manager to separate the currency view from the fixed income investment decision. Resources are needed at the macro, yield curve, and credit levels, along with institutional expertise at the individual currency level, in order to manage all these aspects of the decision process. Only very large asset-management organizations will be able to do this. Outside of such organizations, the possibility of sub-contracting the management of some of the individual currency portfolios exists. This, though, can lead to a blurring of responsibilities and introduce informational inefficiencies between plan sponsors, the main manager, and the specialist managers acting in the individual markets.

Other important issues raised by a Global Aggregate benchmark are the efficiency by which asset-allocation moves can be best accomplished, and the investment of cash balances in such funds. Moving large balances from one currency's bond market to another bond market can be a time-consuming and expensive task. Investment into some of the smaller or less liquid markets and sectors can be taxing. If large cash balances are allowed to accumulate (e.g., from cash inflows or coupon payments), there is a potential for large tracking errors.

Building on our research on index replication with derivatives,[2] we will show how replication techniques using futures and swaps can provide an efficient way to support the management of such a portfolio. Replication with derivatives of some of the indices that are part of the Global Aggregate can provide an effective way for managers to track very closely the markets in which they have little expertise. This allows them to concentrate on actively managing the markets where they have expertise and in which active management matters. Similarly, liquid exchange-traded derivatives can help facilitate rapid asset allocation shifts between currencies and yield curves. If a fund has to keep a significant amount of cash, it may experience substantial deviations from index returns. Derivatives, which require essentially zero cash outlays up front, can be used to duplicate the index returns for the amount of the cash holdings to achieve closer overall index tracking.

In this report, we provide insight into the replication of the Global Aggregate Index. First, we explain our replication methodology, utilizing baskets consisting of Treasury futures, Eurodollar futures, and swaps (and their equivalents in the euro, yen, and pound sterling). Building on historical simulation, we show which set of derivatives provides the lowest tracking error portfolio, and we show the composition of such a replication portfolio for June 2000.

Methodology and Instruments

Figure 1 shows the composition of the Global Aggregate Index by currency as of August 31, 2000. The four most important issuer currencies account for more than 96% of index capitalization. Just replicating the four most important indices should, therefore, result in a low tracking error.

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[2] See the soon-to-be published Lehman Brothers publication, *The Global Aggregate Index.*

[2] See the Lehman Brothers publications, *Replicating Index Returns with Treasury Futures*, by Lev Dynkin, Jay Hyman, and Wei Wu, November 1997, and *Replication of Index Returns with Treasury Futures, Eurodollar (Euribor) Futures, and Swaps*, by Lev Dynkin and Peter Lindner, March 2000.

Figure 1. **Composition of the Global Aggregate Index by Currency,** as of August 31, 2000

	Market Value ($ million)	% of Index
U.S. Dollar	5741200	48.4
Euro	3440925	29.0
Yen	1661369	14.0
Sterling	550471	4.6
Danish, Norwegian Krone, Swedish Krona, Greek Drachma	228730	1.9
Australian, Canadian, New Zealand Dollar	229943	1.9
Global Aggregate	11852638	100.0

The instruments we use in the replication are liquid exchange-traded futures contracts or swaps. Treasury futures allow the tracking of the part of an index return that is induced by yield curve movements, whereas Eurodollar futures and swaps are better able to replicate the part of total return due to spreads and changes in spreads. For the replication of the U.S. Aggregate Index, we use the 2-, 5-, and 10-year Treasury futures and the bond futures contract. We also use Eurodollar futures out to 5 years in expiration and 2-, 5-, 10-, and 30-year swaps. For the euro currency zone, we use the 2-, 5-, and 10-year German treasury futures contracts and LIFFE-traded Euribor contracts with expirations out to 4 years. Eurodollar and Euribor contracts are used out to 5 and 4 years to expiration, respectively, due to their superior liquidity characteristics. For Japan and the U.K., only their highly liquid 10-year treasury futures contracts are used in the replication. We do not replicate the indices of any one of the remaining seven bond markets. Besides their small size, derivatives markets in these countries are mostly either not very liquid or non-existent. Our results suggest that a replication of these remaining indices would not have offered much improvement on the tracking errors observed. One strategy that we evaluate is *proxy replication* of the bond markets that are not replicated with local currency instruments. The returns of these markets are replicated with the replication portfolios of the larger markets that are closest to them in nature.

For the replication with futures contracts and swaps, we utilize a cell-based duration matching approach. We divide the bonds in an index into a number of duration cells, whose boundaries bracket the duration of each of the contracts. The return of each cell is then replicated on a dollar duration-matched basis with the return of the corresponding number of Treasury futures contracts. The cash in all derivatives replication portfolios is invested at each currency's 1-month LIBOR rate. The tracking errors from the individual cells are then combined to give the tracking error of the full index. Swaps are treated similarly.

For the replication with Eurodollar and Euribor futures contracts, we utilize only the "stripped hedge." In a stripped index hedge, the part of an index consisting of all the bonds with cash flows up to five years (or four years in case of the euro) is divided into four buckets per year, corresponding to the expiration of each of the Eurodollar (or Euribor) contracts used. Each cash flow bucket is then PVBP-hedged with the appropriate number of Eurodollar (or Euribor) futures contracts, upon whose rate movements the value of the cash flows depends. Using only one or two Eurodollar (Euribor) contracts (the stacked hedge) proved insufficient.

One method that worked particularly well for both the dollar and the euro is *hybrid replication*. Hybrid replication in general comprises the replication of different parts of a portfolio with distinct sets of instruments. Specifically, in the context of our work, it is the combination of Eurodollar (Euribor) futures at the short end of the curve and Treasury futures at the long end. The correlation of the errors from replicating the shorter bonds in an index with Eurodollar/Euribor futures or swaps and the replication of the longer bonds in that index with Treasury futures is usually very small or negative. One, therefore, often reaps substantial diversification benefits from the replication of an index with distinct sets of instruments. For both the U.S. and the Euro-Aggregate, this results in lower tracking errors than those found using Treasury futures or swaps alone in the replication.

Similarly, we find that the correlations between the replication errors from the individual markets and their volatilities play an important role. Particularly, the instruments used in the replication of the U.S. Aggregate, which is responsible for almost 50% of the Global Aggregate's market capitalization, play an important role in determining the overall tracking error.

What does a replication portfolio look like? How does it perform? Figure 2 provides detailed information for a derivatives portfolio that was designed to track the Lehman Brothers Global Aggregate Index during June 2000. The results are for a portfolio with a market value of $1 billion as of May 31, 2000.

We investigated a number of different replication strategies, combining pure Treasury futures replication and swaps replication for the U.S. with pure treasury futures replication and, alternatively, hybrid replication for the euro zone. As can be seen in Figure 4, the lowest tracking error on a historical basis was achieved with hybrid Eurodollar futures/swaps and Euribor/treasury futures portfolios for the U.S. and the Euro-Aggregate, respectively. Since with hybrid replication all bonds with cash flows out to five (or four) years are being replicated with Eurodollar (or Euribor) futures, respectively, 2- and 5-year swaps (or futures) are not used for either the U.S. Aggregate or the Euro-Aggregate replication. When considering the number of contracts in the replication, one should keep in mind that the notionals of the 10-year and the bond contract in the U.S. are $100,000, the notional of the German 10-year futures is EUR100,000, and the notional of the U.K. 10-year contract is GBP100,000. The Japanese 10-year contract has a notional of JPY100,000,000, which, at current exchange rates, is about 10 times larger than the notional of the U.S. 10-year contract. For the two largest markets, the replication portfolios underperform their indices; for three of the remaining four markets, they outperform the indices. This exemplifies the diversification effect that often occurs in the replication of the Global Aggregate Index.

The dollar returns of the non-dollar indices that are part of the Global Aggregate are strongly influenced by the currency returns. The outperformance of the replicating portfolios is only slightly affected by the currency return, though. Since the outperformance is the difference of two portfolio returns, the currency return is essentially subtracted. It is for this reason, also, that the outperformance is not affected by currency hedging, as long as the currency exposure of the replication portfolio is hedged in a manner identical to that of the currency exposure of the index being replicated. The one number that stands out in the last column of Figure 2 is the combined outperformance of the three non-U.S. dollar markets: Canada, Australia, and New Zealand. The smaller markets in terms of market capitalization—non-euro Scandinavia and Greece and Canada, Australia, and New Zealand—are grouped into two blocks that are replicated with the Euro-Aggregate and the U.S. Aggregate replication portfolio, respectively. The severe underperformance of the U.S. Aggregate replication portfolio versus the combined market returns of these three markets is due largely to the unusually large appreciation that the Australian and the New Zealand dollar experienced during June 2000. Over the full sample, hedging the bond markets or just the currencies of the smaller markets does not have much of an effect.

Performance

Using different instruments to replicate different sectors of an index can result in replication portfolios with superior tracking error characteristics. This is the essence of hybrid replication. We can make use of different hybrid replication techniques for different markets. A variety of strategies can, therefore, be used in the replication of the Global Aggregate. We tested a number of

Figure 2. **Results of Global Aggregate Index Replication for June 2000**

	Market Value ($ million) (as of 5/31/00)	**%** **of Index** (as of 5/31/00)	**No. of Futures Contracts/Swaps Used** **Eurodollar/ Euribor Futures**	**10-yr. Treasury Futures**	**Swaps** (per mn. notional) **10 yr.**	**30 yr.**	**Local Curr. Returns** (bp) **Index**	**Replication Portfolio**	**Outperform.**
U.S. Dollar	5441550	47.0	1414		162	64	208.0	181.7	-26.3
Euro	3532490	30.5	1160	1125			39.0	30.6	-8.4
Yen	1604971	13.9		97			-29.2	-9.2	20.1
Sterling	559610	4.8		253			21.4	47.1	25.7
Danish, Norwegian Krone, Swedish Krona, Greek Drachma	227647	2.0					10.3*	30.6	20.3
Australian, Canadian, New Zealand Dollar	221868	1.9					336.9**	181.7	-155.2
Global Aggregate Return (in $)							253.5	240.2	-13.3

* Return in euros.
** Return in dollars.

replication strategies on a historical basis with data from February 1999 to June 2000. This period was chosen because the euro came into existence in January 1999, which is also when trading in Euribor futures started on the LIFFE. We present the results for the best-performing strategy, involving Eurodollar futures and swaps for the U.S. and Euribor and treasury futures for the euro zone. We also present results for the worst performing strategy, involving solely Treasury futures for the U.S. and the Euro Aggregate. Figure 3 contains the means and volatilities of the returns of the Global Aggregate Index for four currencies: dollar, euro, yen, and pound sterling.

Figure 4 contains the means and the volatilities of the outperformance in dollars and euros. The mean index returns of the dollar and the euro differ by 100 bp a month. This is due mainly to the degree by which each currency appreciated over the period. On the other hand, the index volatility expressed in the two currencies is not that different. Index volatility in the local currency is higher for the markets that represent a smaller share of the Global Aggregate.

Figure 3. **The Global Aggregate Index: Mean and Volatility of Returns,** February 1999-June 2000

	Returns in			
	Dollar	**Euro**	**Yen**	**Sterling**
Mean (bp/m)	-24.7	75.2	-78.3	23.7
Std. Dev. (bp/m)	151.6	145.1	241.5	178.1

Figure 4. **Historical Replication: Tracking Error and Mean Outperformance,** February 1999-June 2000

	Replication Errors of Returns			
	$ Returns		**Euro Returns**	
	Method 1*	**Method 2****	**Method 1***	**Method 2****
Mean (bp/m)	-6.5	4.6	-6.6	4.6
Volatility (bp/m)	13.0	23.4	13.0	23.6

* Eurodollar futures and swaps for U.S. Aggregate; Euribor and Treasury futures for the Euro Aggregate; 10-year Treasury futures for Japan and the Sterling Aggregate.
** 2-, 5-, 10-year, and bond futures for U.S. Aggregate; 2-, 5-, and 10-year German Treasury futures for the Euro Aggregate; 10-year Treasury futures for Japan and the Sterling Aggregate.

The means of the outperformance in the euro and the dollar, and the tracking errors, are virtually the same for identical replication methods across different base currencies. Using Eurodollar futures and swaps for the U.S. and Euribor and treasury futures for the euro zone leads to significantly reduced tracking errors compared with using Treasury futures only.

The smaller markets were not replicated individually for these results. Underlying those numbers is "proxy replication." For this, the non-euro and non-sterling European countries represented by the Euro Aggregate and the "dollar-block" markets of Australia, Canada, and New Zealand were represented by the U.S. Aggregate replication portfolio. How much of an error can be expected from this procedure? Given the small share of the Index that these seven countries represent, probably not much. Figure 5 provides answers to this question.

The first two columns in Figure 5 display mean outperformance and tracking errors when proxy replication is used versus the results from the replication when ignoring the market capitalization of the indices that are not replicated (columns 3 and 4). Tracking errors can actually increase when proxy replication is used.

The rows of Figure 5, from top to bottom, reflect the improvement in tracking error when we go from replicating only the U.S. Aggregate and the Euro Aggregate to replicating these two indices plus the Yen Aggregate Index and then to replicating the U.S. Aggregate, Euro Aggregate, Yen Aggregate, and Sterling Aggregate indices. When we consider proxy replication in these cases, the yen is proxied with the U.S. Aggregate and the Sterling Aggregate with the Euro Aggregate. As would be expected, the tracking error decreases less with the addition of indices with smaller market cap weightings. Explicitly adding the Yen

Figure 5. **Effects of Increasing Replication Coverage**
February 1999-June 2000

	With Proxy Replication		**No Proxy Replication**	
Indices Replicated	**Mean** ($ bp/m)	**Tracking Error** ($ bp/m)	**Mean** ($ bp/m)	**Tracking Error** ($ bp/m)
$, Euro	-17.7	40.5	-19.2	39.5
$, Euro, Yen	-9.0	16.5	-4.4	15.7
$, Euro, Yen, Sterling	-6.5	13.0	-5.7	13.4

Aggregate to the replication reduces the tracking error from 40 to about 16 bp. Adding the Sterling Aggregate to the replication reduces it by only another 3 bp. Trying to replicate the remaining seven indices explicitly, if necessary in the cash markets, would probably not lead to decreases in tracking errors that are worthy of the effort. Using the proper methodology for the replication of the important markets offers more from a cost/benefit perspective. These results, therefore, also lend support to an asset management approach that neglects smaller markets that are part of a multi-currency index and instead concentrates resources on the more important markets in the benchmark.

More details on the replication strategies and results for the U.S. and the Euro Aggregate can be found in our publications on these topics.[2] We are also in the process of writing a more detailed study of the replication of the Global Aggregate Index. Please contact the authors or other members of our Quantitative Portfolio Strategies Group for more information.

Publications—L. Pindyck, A. DiTizio, B. Davenport, W. Lee, D. Kramer, J. Threadgill, R. Madison, A. Acevedo, K. Kim, C. Rial, J. Batstone