

# Quantitative Portfolio Strategy

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Lev Dynkin 212-526-6302

ldynkin@lehman.com

Jay Hyman 972-3-623-8745

jhyman@lehman.com

Vadim Konstantinovsky, CFA 212-526-8290

vkonstan@lehman.com

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## A CASE FOR CREDIT INVESTING IN BUY-AND-HOLD PORTFOLIOS

Credit spreads compensate investors for various types of risk associated with credit instruments. In addition to the fundamental risk of default, these include market risks such as sector-wide spread volatility, credit deterioration, and ratings downgrades of specific issuers. Spreads also include a liquidity premium, reflecting the risk that in a crisis, the issue may be difficult to sell. Every investment decision is based on the investor's perception of this risk and reward tradeoff. Yet not all investors are necessarily subject to all risks. To the extent that one is protected from a particular risk that is built into the "compensation" part, i.e., spread, there is a potential for superior risk-adjusted returns. Volatility and liquidity risks, for example, are relevant only to total-return investors who need to mark to market or trade the security. This is certainly the case for active managers and enhanced indexers. Buy-and-hold investors, on the other hand, could view credit spreads from a very different perspective. For such investors, short-term spread volatility and liquidity squeezes are of little importance. The only risk that affects them directly is default risk.<sup>1</sup> Yet they still stand to benefit from spreads that presumably compensate active managers for their exposures to short-term market risk.

We conducted a simple analysis of the tradeoff between spread and default risk for buy-and-hold investors. We show that current spread levels provide a significant cushion against default risk and that in order for a diversified corporate portfolio to underperform Treasuries over a 10-year holding period, the default rate would need to be significantly higher than historical averages. We do not propose a method for allocating portions of spreads to particular risks. Rather, we suggest a simple framework that can be used by buy-and-hold investors to decide whether the market presents them with good buying opportunities.

In our analysis, we compare holding period returns on portfolios of non-callable 10-year credits with those of 10-year Treasuries. We consider two rating categories: single-A and Baa. We use a simple, stylized analysis that ignores the effects of reinvestment and inflation. Our interpretation of the buy-and-hold assumption is that, in the absence of defaults, we expect the annualized total return to equal yield to maturity. Default is modeled as a one-period problem—each bond either defaults during the next ten years or survives to maturity. Bonds that default within that period do not contribute anything to the cumulative performance beyond their recovery value. Essentially, we assume that all bonds that default do so at the very beginning. This makes our analysis more conservative because the coupons received prior to default are ignored.

If we let  $y$  denote the average portfolio yield (in decimal),  $D$  the realized portfolio default rate, and  $R$  the average recovery rate on defaulted bonds, then our simple model for the terminal value of the portfolio after 10 years is

$$term\_value = (1 - D)(1 + y)^{10} + D \cdot R = (1 + ann\_ret)^{10}$$

We differentiate between the market-wide default rates, such as those published by rating agencies, and the realized *portfolio* default rates. The default and recovery rates based on historical averages may be used in this equation to project expected returns, but the

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<sup>1</sup> We realize that this is a simplification. Certain book value investors, for example, may be subject to downgrade risk (e.g., insurance companies with risk-based capital requirements).

analysis of portfolio default risk must include the possibility that a given portfolio may experience worse defaults than the market as a whole. We assume that the number of defaults  $n_{\text{default}}$  in a portfolio follows the binomial distribution with parameters  $n$  and  $p$ , where  $n$  is the number of bonds in the portfolio,  $p$  is the market-wide default rate, and  $D$  in the return equation is given by  $n_{\text{default}}/n$ . Then we determine what market-wide default rates can be tolerated while still maintaining a high level of confidence that our portfolio will outperform Treasuries. The Treasury total return is obtained by setting  $D$  to 0 and  $y$  to an appropriate Treasury yield. The number of bonds in the portfolio is thus an important parameter of the study.

To put our findings in historical perspective, we obtained 10-year cumulative default rates from Moody's. Figure 1 shows such rates for two investment-grade rating categories, A and Baa, issued from 1970 through 1992 (so that the last 10-year frame is 1992-2002). While for some groups the realized default rates could be as high as 9.7% (1982 Baa cohort), the average annual rates are relatively modest at 1.57% for single-A and 5.09% for Baa.<sup>2</sup> We will proceed to show that these rates would not be nearly enough to justify the current spread levels if defaults were the only source of risk in the market. Figure 1 shows 3-year default rates as well; we will return to these later. Another critical assumption, besides the default rate, is the recovery rate. According to Moody's, while historical recovery rates for defaulted bonds span from 0% to 100%, they report an average historical recovery rate of 41%, with a standard deviation of 28%; the median recovery rate was 35%. In our analysis, we will use a fixed recovery rate assumption; to be conservative, we will use values of 0%, 20%, and 40%.

To illustrate our method, let us examine the distribution of holding period returns in two single-A portfolios. As of November 30, 2002, the yield on the non-callable 10-year single-A credits was 5.69%. The yield on the 10-year on-the-run Treasury, which is our benchmark holding period return, was 4.22%. Figure 2 plots the return distributions for a 20-bond and a 50-bond portfolio under our binomial model assumption, using the conservative 5% market-wide default rate and 20% recovery rate. The solid line indicates the probability of a given number of defaults out of the 20 (or 50) bonds in the portfolio over ten years; the shaded bars indicate the annualized total returns corresponding to this event given current yield levels. As Figure 2a shows, as long as two or fewer bonds out of twenty default, the portfolio will outperform Treasuries. The cumulative probability of this is 92.4%. (With three defaults, the portfolio returns 4.20%, just below the Treasury return. Including this case, the cumulative probability is 98.4%.) In the 50-bond portfolio, the return exceeds that of Treasuries as long as no more than seven bonds default. The cumulative probability of this outcome is considerably higher at 99.7%. Another surprising consequence of today's high spread levels is that even if as many as 9 out of 20 bonds default, the portfolio earns a positive return.

As we increase the market default rate, the distribution shown in Figure 2 will change, with higher probabilities assigned to the "tail" events in which the portfolio suffers from a very high realized default rate. The "worst-case" default rate is the highest percentage

<sup>2</sup> Note that Moody's weights the cohort default rates by cohort size (number of issuers) when calculating the long-term average default rates, so that a simple average will give slightly different numbers.

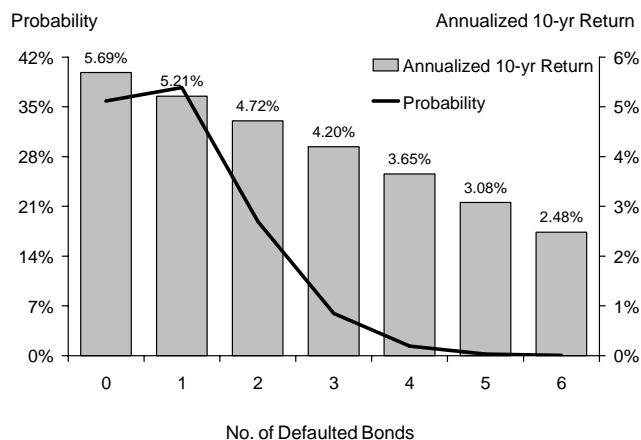
Figure 1. **Average 10-Year and 3-Year Cumulative Default Rates for Yearly Cohorts, %**

Annual Cohort	10-Year		3-Year	
	A	Baa	A	Baa
1970	0.45	2.86	0.00	0.28
1971	0.84	2.74	0.00	0.82
1972	0.00	3.10	0.00	0.75
1973	0.40	3.55	0.00	0.96
1974	0.80	3.73	0.00	0.74
1975	0.39	3.77	0.00	0.26
1976	0.67	4.90	0.00	0.57
1977	1.30	5.04	0.00	0.58
1978	1.33	4.84	0.00	0.00
1979	2.78	4.79	0.00	0.31
1980	3.26	5.82	0.29	0.99
1981	3.08	8.56	0.29	1.93
1982	3.82	9.70	0.27	1.35
1983	3.55	7.99	0.00	1.61
1984	4.21	6.61	0.47	0.79
1985	4.85	6.70	1.37	1.24
1986	2.40	8.62	0.82	3.09
1987	1.75	7.20	0.42	1.47
1988	1.45	5.25	1.02	1.07
1989	0.58	3.50	0.58	1.93
1990	0.00	1.17	0.00	0.65
1991	0.28	1.18	0.00	0.29
1992	0.49	3.37	0.00	0.00
1993			0.00	0.27
1994			0.00	0.22
1995			0.00	0.00
1996			0.00	0.18
1997			0.00	0.77
1998			0.10	1.02
1999			0.20	1.46
Average	1.57	5.09	0.21	0.87
Worst case	4.85	9.70	1.37	3.09

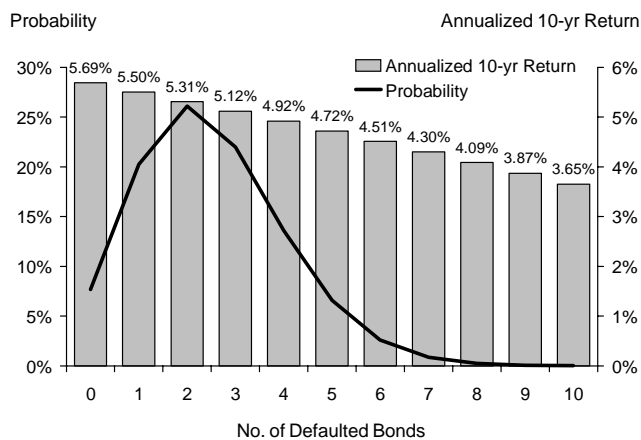
Source: Moody's Investors Service.

Figure 2. **Distribution of Expected Returns in a Single-A Portfolio , 5% Market Default Rate, 20% Recovery**

a. **20-Bond Portfolio**



b. **50-Bond Portfolio**

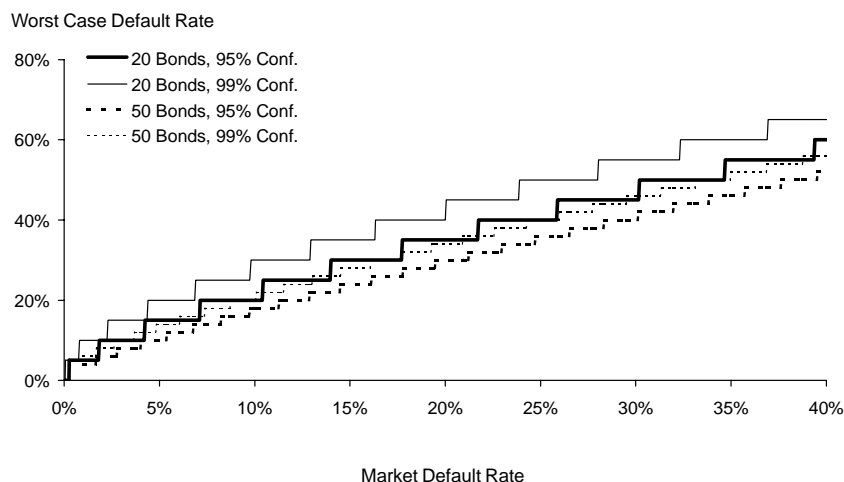


of defaulted bonds in the portfolio for which the cumulative probability of such an event is still within the confidence interval of 95% or 99%. The more bonds there are in the portfolio, the more closely the distribution concentrates around its mean and the smaller the worst-case default rate. Figure 3 shows the worst-case default rates with both 95% and 99% confidence for portfolios of 20 and 50 single-A bonds. At a market-wide default rate of 10%, we can be 95% sure that realized defaults on a 20-bond portfolio will be no worse than 20% (4 defaults) and 99% sure that they will be no worse than 30%. For a 50-bond portfolio, the worst-case default rates are 18% and 20%, respectively. (Note that the difference between the worst-case default rates of a 50-bond portfolio and a 20-bond portfolio is much more pronounced at the 99% confidence level. With small portfolios, it is much harder to rule out the chance of what seem to be very unlikely events.) The staircase shape evident in Figure 3 is due to the discrete nature of the problem—an integer number of bonds that default.

How high can the market-wide default rate be if we still want to maintain a given confidence level that the portfolio default rate will be below a certain bound? This relationship can be shown by drawing a horizontal line across Figure 3 at a given level of realized defaults and finding the corresponding market rate for a given confidence level and portfolio size. For instance, if we can tolerate a portfolio default rate of up to 20%, we can ensure with 99% confidence that this level will not be reached in a 20-bond portfolio as long as the market default rate does not exceed 4.35%; for 95% confidence, we can have a market default rate up to 7.10%. For a 50-bond portfolio, we can achieve these confidence levels at higher market default rates: up to 8.70% for 99% confidence and up to 11.25% for 95%.

In Figure 4, we combine the binomial model for portfolio defaults with our simple model of portfolio return to show what market default assumptions would be needed to break even with Treasuries at a given spread level. The figure is based on a Treasury yield of 4%,

Figure 3. **Worst-Case Portfolio Default Rate as a Function of Market Default Rate, Binomial Model**



but will change only slightly with small changes in Treasury yield. At this level, using simple annual compounding, a 10-year Treasury investment of \$1 will have a terminal value of 1.48, while a credit investment with a spread of 200 bp will have a terminal value of 1.79 (if it does not default). Assuming a 20% recovery rate, our simple return equation tells us that a realized portfolio default rate of 19.5% would make the return on the credit portfolio equal to the Treasury return.<sup>3</sup> This break-even default rate demonstrates just how much cushion can be generated by credit spreads—with a spread of 200 bp, we can experience 9 defaults in a 50-bond portfolio and still outperform Treasuries! We then use the binomial model, as shown in Figure 3, to look up the upper limits on market default rates that would allow us to maintain 95% (or 99%) confidence that the actual default rate will be below this value for a portfolio of 20 (or 50) bonds.

Figure 5 presents these break-even market default rates as a function of spread in graphical form and indicates the current average spread levels for A and Baa credits, along with their historical default rates. Spreads for single-A credits in the 10-year range currently average 122 bp. If we assume that the single-A market default rate over the coming 10 years will follow the historical average of 1.57%, we can have greater than 99% confidence of outperforming Treasuries over the 10-year holding period. If the market default rate matches that of the worst observed A cohort, 4.85%, we will be just under the 95% confidence level for a 20-bond portfolio, but will still be 99% sure of outperforming with a 50-bond portfolio. For Baa credits, with an average spread of 200 bp, the average

<sup>3</sup> Note that the breakeven realized default rate shown in Figure 4 depends only on our approximate formula for portfolio return. The binomial model and the assumption of uncorrelated defaults do not come into play until we translate this to break-even market default rates for a given portfolio size and confidence level.

Figure 4. **Break-Even 10-Year Market Default Rate, 20% Recovery**

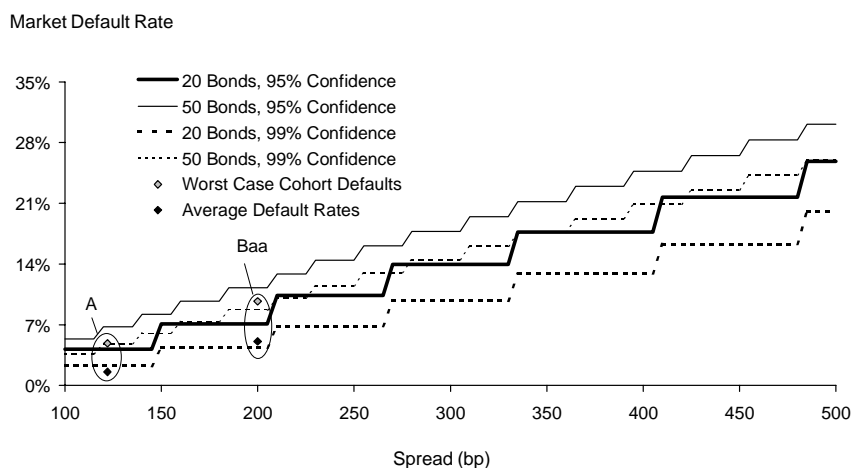
Corporate Spread (bp)	Corporate Yield (%)	Corporate Terminal Val.	Breakeven Realized Def.	Market Default Rate			
				20 Bonds, 95% Confid.	50 Bonds, 95% Confid.	20 Bonds, 99% Confid.	50 Bonds, 99% Confid.
100	5.00	1.63	10.4%	4.2%	5.4%	2.3%	3.7%
125	5.25	1.67	12.8%	4.2%	6.8%	2.3%	4.8%
150	5.50	1.71	15.1%	7.1%	8.2%	4.4%	6.1%
175	5.75	1.75	17.4%	7.1%	9.7%	4.4%	7.4%
200	6.00	1.79	19.5%	7.1%	11.3%	4.4%	8.7%
225	6.25	1.83	21.6%	10.4%	12.9%	6.9%	10.1%
250	6.50	1.88	23.7%	10.4%	14.5%	6.9%	11.5%
275	6.75	1.92	25.6%	14.0%	16.1%	9.8%	13.0%
300	7.00	1.97	27.6%	14.0%	17.8%	9.8%	14.5%
325	7.25	2.01	29.4%	14.0%	19.5%	9.8%	16.1%
350	7.50	2.06	31.2%	17.7%	21.2%	12.9%	17.7%
375	7.75	2.11	33.0%	17.7%	23.0%	12.9%	19.3%
400	8.00	2.16	34.6%	17.7%	24.7%	12.9%	20.9%
Treasury Yield			4%				
Treasury Terminal Value			1.48				
Recovery Rate			20%				

historical default rate of 5.09% would place us between the 95% and 99% levels for a 20-bond portfolio and well above 99% confidence for a 50-bond portfolio. If we assume the worst-case market default rate of 9.7%, a 50-bond portfolio would still give us greater than 95% confidence of outperforming Treasuries, even under an assumed recovery rate of only 20%.

The points marked in Figure 5 for A and Baa can be viewed only as a gross approximation. The credit markets have lately been characterized by spread bifurcation. It is not the case that all Baa spreads are tightly clustered around the average spread of 200 bp—in fact, the truth is quite the opposite. Many Baa bonds have spreads that are multiples of this average, and many are much tighter. It is also not clear exactly how to project default probabilities for these very different populations of bonds. Certainly, it would seem rather absurd to use a single dataset of historical Baa average default rates to project the same expected default rates for bonds with spreads of 100 and 800 bp. Various tools are available for projecting expected default frequencies of specific issuers based on issuer fundamentals, including debt and equity market valuations and volatilities. Figure 5 presents a general framework for evaluating the spread of a given group of bonds against a particular assumption for its worst-case default rate. Our colleagues in the Credit Strategy group have discussed the merits of a diversified “outlier portfolio” consisting of the highest-spread bonds in a given rating category. Our analysis would show just how extreme a default scenario would be required to cause underperformance when using this strategy on a buy-and-hold basis.

Despite the imprecision of using average spreads to denote a rating bracket, it is interesting to look at the evolution over time of the break-even default rates implied by these average spreads. Figures 6 and 7 show these rates at the 95% confidence level as a function of time for two 50-bond portfolios, single-A and Baa. In both rating categories, the break-even levels have increased dramatically since 1998. While, prior to 1998, they

Figure 5. Break-Even 10-Year Market Default Rate, 20% Recovery



were at levels roughly comparable with the historical realized defaults, post-1998, they clearly exceed them by a wide margin. As we pointed out, the reason for this phenomenon is that the current market, to a much larger extent than before, is pricing in other sources of risk such as spread volatility and liquidity. This high-spread, high-volatility environment presents an attractive investment opportunity for buy-and-hold investors concerned primarily with default risk.

Is it possible that the market is pricing in an expectation that market default rates over the next ten years will exceed even the highest observed cumulative 10-year default rate for any cohort since 1970? The time series of realized 10-year default rates does not include enough recent data to provide any insight into this question. To help compare the most recent period with the wave of defaults in the mid-80s, we turn to the series of cumulative 3-year default rates. These are shown in Figure 1 alongside the 10-year rates. We see that while the 3-year default rate for Baa credits from 1999 through the end of 2001

Figure 6. **Break-Even 10-Year Market Default Rate as a Function of Time**  
95% Confidence, Single-A, 50 Bonds

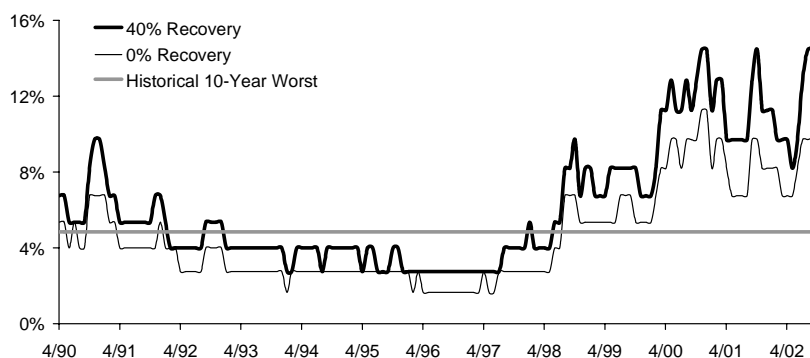
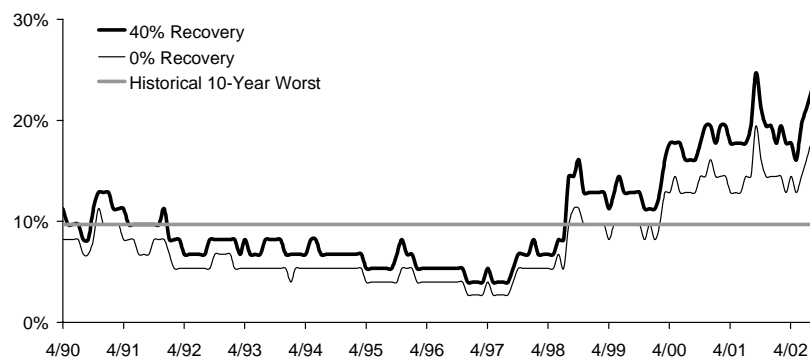


Figure 7. **Break-Even 10-Year Market Default Rate as a Function of Time**  
95% Confidence, Baa, 50 Bonds



increased to 1.46%—well over the long-term average of 0.87%—it still falls far short of the peak level of 3.09% observed for the 1986 cohort. This leads us to believe that even the high-volatility environment of the past few years does not warrant an expectation that 10-year cumulative defaults will exceed the peak rates of the 1980s.

In general, we have tried to keep our assumptions as conservative as possible, and found that even so, the probability of a diversified credit portfolio outperforming Treasuries on a buy-and-hold basis is remarkably high. In particular, we have made some pessimistic assumptions about what happens when a bond defaults. For defaulted bonds, we have assumed a recovery rate of only 20%, and we have allowed for no additional income on these bonds—no coupon income before a default and no reinvestment of the recovered proceeds. As far as the default rate, we have used the highest cumulative 10-year default rate observed in all annual cohorts originating from 1970 through 1992.

Nevertheless, there are several simplifications in this analysis that may result in an overly optimistic assumption. The formula we use for return assumes that today's yields on corporates and Treasuries will be compounded throughout the 10-year period. This neglects the role of reinvestment risk. While both the credit portfolio and its equivalent Treasury counterpart are subject to reinvestment risk, the possibility that spreads may tighten over the horizon may reduce the overall return advantage over the period.

The use of the binomial model involves several simplifying assumptions. First, it assumes that the portfolio is equally weighted among a homogenous set of bonds with identical expected default rates. This makes it a theoretical construct that cannot be applied directly to real portfolios, but does not necessarily bias the results in either direction. Second, and more critically, we assume independence of individual defaults. An assumption of independence in a case in which defaults are highly correlated can cause a serious underestimation of default risk.

We do not believe that such underestimation is involved here, because all analysis is conditioned on the realized market default rate. In fact, this quantity is not a constant, but is itself a random variable. Given a particular outcome of the market default rate over the next ten years, we model the distribution of the portfolio default rate. Conditioned on the market default rate, all bonds are assumed to default independently with the same probability. However, if we look at the unconditional default probabilities, we will find that the defaults of any two bonds are highly correlated events. Both bonds will be likely to default if the market default rate is high and unlikely to default if it is low. As a result, bond A will be more likely to default when bond B does than when it does not.

The correlations among the defaults of different issuers reflect two types of factors: general macroeconomic trends that affect all issuers and industry-specific circumstances that can affect a particular sector of the market. The first of these is addressed by the dependence on the market default rate. While our model cannot account for industry-specific correlations, these can be in large part avoided by diversification of industry exposures in the portfolio. If lack of liquidity in the market makes such diversification impossible, our break-even default rates would have to be adjusted upward for industry correlations. Nevertheless, we believe it is feasible under most market conditions to construct a corporate portfolio of twenty or fifty names well diversified across industries.



One application of this work could be in portfolio dedication. The cash flows of a dedicated portfolio are designed to match a set of known or projected future liabilities. As such, the bonds in a dedicated portfolio are typically intended to be held to maturity. Our results indicate that credit could be used in a very risk-efficient manner in this application. The problem, of course, is that the notion of dedication is directly opposed to the use of bonds with uncertain cash flows. True, one could purchase a portfolio of corporate bonds to match a given cash flow stream much more cheaply than an all-Treasury portfolio. Yet we do not want to face the possibility that bond defaults in our portfolio will leave us unable to meet our liabilities. A solution could be to purchase a credit portfolio that over-funds the liability stream. For less than the cost of a Treasury portfolio, one should be able to purchase a credit portfolio with enough of a cushion against default to make the probability of a shortfall extremely low.

### Conclusion

The most basic of our assumptions is the existence of pure buy-and-hold investors. Is any investor totally immune from market risk? Are there no circumstances under which parts of the portfolio would need to be sold before maturity? Certainly, if a portfolio's mandate requires the liquidation of any position that falls below investment-grade, then this analysis should be adjusted to reflect downgrade risk. An insurance company may be required by regulators to mark to market bonds in distress and realize a charge against their earnings. A downgrade of an asset may result in an increase in risk-based capital requirements. Each investor must analyze to what extent our strict buy-and-hold assumptions relate to the actual operating environment. Nevertheless, we believe our results should be of interest to a wide range of buy-and-hold credit investors.

The precise levels of break-even corporate default rates and the implied outperformance potential may be rather difficult to determine. Yet our simplistic analysis indicates quite strongly that in a high-spread, high-volatility environment, corporate spreads reward buy-and-hold investors with a return premium that more than adequately compensates them for default risk. The comparison of the break-even market default rate to historical worst-case default rates offers a method for generating a buy signal for buy-and-hold investors. The signal is generated when volatility has driven spreads beyond levels that could be justified by historical defaults. Even under very conservative recovery assumptions, we find that this signal has indicated the attractiveness of credit for buy-and-hold investors since the major spread widening of 1998.