Research Technical Note

Credit Risk Models

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1 Introduction

Valuation of a complex security typically requires a **pricing model** (a pricing function or procedure). This uses market data and parameters defining the specific model used for the valuation of the security. Estimating the security's risk requires a realistic **risk model** of possible market movements and the revaluation of the security under these hypothetical scenarios. The goal of this document is to describe the different *risk* models, supported by Risk Server, available to securities which are exposed to credit risk, and to clarify the important relations between specific pricing models and the credit risk models used in RiskServer.

The list of securities relevant to our discussion includes bonds (regular or callable), credit default swaps (single names, indices, and options on CDS), convertible bonds (and options on CB), bond futures and bond futures options.

Credit risk has been traditionally linked to the concepts of credit spreads, transition probabilities (the probability that an entity with a given credit rating will transition to a different credit rating over a certain period of time), and default probabilities. Historically, default and transition probabilities have been provided by rating agencies or other research firms including banks' internal credit departments. They are estimated by using a range of statistical methods, employing data over long time periods, 30 to 50 years depending on the data available, and by studying the frequency of credit transitions (of which default is an extreme case) for similar obligors or obligor types. They represent actual ('real world' or 'physical' as sometimes they are referred to, as opposed to 'risk neutral') probabilities and, historically, they have been criticized because they suffered from a lack of responsiveness that prevented their effective use for short term risk forecasting.

As the need increased to extract default probabilities and credit spreads from observable prices on the market to generate more responsive measures of credit risk that could be used in pricing and risk calculations, two classes of risk models emerged: structural models and reduced form intensity models.

Structural (Merton type) models use equity and balance sheet information to derive hazard rates, i.e. the intensity associated with the default event,[1] for a given issuer. Hence they use equity market data as a proxy for credit risk: declining equity prices prompt worsening of credit. Moody's Expected Default Frequency model also belongs to the same class, as the **Credit Grades** model available in Risk Server.

Reduced form intensity models assume an exogenous default process and imply default probabilities from market prices of credit sensitive instruments, typically bond prices and CDS instruments. The hazard rate term structure can then be extracted from market quotes using a recovery rate assumption, and its evolution and risk characteristics are linked to the behavior of curves built from market quotes. The **CDS Spread Model**, the **CDS Upfront Model**, and the **Hull-White Credit Model** all belong to this class. Together with the Credit Grade Model, they are collectively referred to, in RM4 and RiskServer, as **Obligor Default Models**.

A simpler model, the **Bond Spread Model**, is also available in RiskServer. It derives credit spreads from bond quotes, and does not use hazard rates or recovery rates parameters.

The following table summarizes the salient features of each model. Details about the additional inputs are provided in the definition section, while methodologies are described in more details in the corresponding sections specified in the table. Recovery rates are included as model data to indicate that they are an important part of the model definition, and that they play a role in risk forecasting even if they are not used as risk factors. The 'Unspecified' model refers to cases where no credit risk model is selected, in which case risk is estimated without separating risk free rates and credit spreads, and hence there is no possibility of distinguishing between interest rate and credit contributions to risk. The 'Available to' column refers to the position types where the model can be used; 'all' refers to CDS, CDSIndex (CDX), and CDS options (CDSO), GenericBond (GB), convertible (CB) and convertible options (CBO), bond future (BF) and bond future option (BFO).

[1] see Sec. [3.3.1] for a simple exposition of the relation between hazard rates and default probabilities.

Model	Obligor Model	Model Data	Credit Factors	Available to	Features
Bond Spread Curve		discount curve (issuer/sector)	bond spreads	GB, CB, CBO BF, BFO	Inputs [2.1] Methodology [3.2]
Bond Spread Hull-White	✓	discount curve (issuer/sector), recovery rates	bond spreads	GB, CDS, CDSO BF, BFO	Inputs [2.2] Methodology [3.3]
CDS Spread Curve	✓	CDS ISDA spreads, recovery rates	CDS fair spreads	GB, CDS, CDX, CB, CDSO BF, BFO	Inputs [2.3] Methodology [3.4]
CDS Upfront Curve	✓	CDS ISDA spreads, recovery rates	CDS upfront prices	GB, CDS, CB BF, BFO	Inputs [2.4] Methodology [3.5]
Credit Grades	✓	equity price, recovery rates	equity	All	Inputs [2.5] Methodology [3.6]
Unspecified		discount curve (issuer/sector)	discount yields	GB, CB BF, BFO	Methodology [3.2]

Note that the terminology used in RM4/RML4 and RM3/RML3 for Bond Spread Curve and the Bond Spread Hull-White models can be confusing. In particular the RM4/RML4 interface does not use the Hull-White Credit Model name, since that model is selected there as the Bond Spread Model choice, among different Obligor Default Models.

1.1 Pricing Models and Risk Models

A *pricing model* calculates the price and sensitivities of a security given market data, terms and conditions and, depending on the security, additional parameters defining the valuation model. Market data can be directly observable (e.g. equity and commodity prices) or derived from market prices based on well established quotation conventions (volatilities and Libor curves). Other parameters might depend on the specific choice of the selected numerical procedure and, more importantly, on the properties of the underlying drivers of the security price, which are reflected in the choice of model adopted. The former are "implementation" parameters; the latter are the true "model" parameters defining distributional assumptions and, more generally, the stochastic processes determining the evolution of the underlying drivers of price change.

To be concrete, examples of implementation parameters include the number of tree steps defining a tree construction, the choice between binomial or trinomial tree methodology, the number of simulated scenarios in a Monte Carlo simulation, and the choice between different type of random number sequences used in Monte Carlo simulation; examples of model parameters for credit sensitive instruments include the parameters associated to a) the yield curve model describing interest rate changes[2]; b) the selection of the term structure of credit spreads appropriate for the security and the choice of the drivers of change for the spreads; c) the link between the spreads and the equity associated to the issuer.

Irrespective of the model chosen to value a security the distinction between model parameters and market data is blurred by the fact that model parameters are often calibrated to market data, and that any pricing model can be adjusted to match a given price through the process of price calibration. By *model* calibration we refer to the determination of relatively few model parameters that are shared between securities on the same underlying. By *price* calibration we refer to a final adjustment of the model which is specific to a given security and aligns the model price of the security to the market price, when the latter is available. Some of these credit related model parameters will play an important role in the linkage between pricing and risk models.

1.2 Credit Spreads and Default Probabilities

Consider pricing and risk forecasting of a fixed coupon bond with no embedded optionality. Its price is the sum of the expected discounted values of coupons and principal paid at bond maturity. If the bond were risk free, coupons and principal would be discounted at the risk free rate. The fact that the value of the bond is typically lower than the sum of risk free discounted cash flows reflects a non-zero probability of default over the lifetime of the issue. Related quantitative measures of the credit risk of the bond are its (credit) spread above the risk free rate term structure (i.e. the spread that, if used for discounting the bond cashflows in addition to the risk free rate, reproduces the price of the bond) and the probability that the issue would survive until each of the payment dates. The two measures can be quantitatively linked in a way described in Section [3.3] by specifying a recovery-rate parameter.

By providing a way to a) estimating the spread or default term structure for the current market scenario (i.e. the level of credit "risk factors") and, b) modeling the changes of credit risk factors and their co-movement with other market factors (interest rates in this case) we are setting up a framework to price and forecast the risk of a simple bond. This can be done in several ways but the core decision is the identification of the main drivers of risk for the specific bond: we recognize that interest rates are a common source of risk for all bonds; that the same term structure of spreads or default probabilities links many issues of the same

[2]The Hull-White one factor interest rate model assumes the term structure of rates is driven by a single factor, the "short" rate, which follows a mean reverting Brownian motion, and can be fully determined by specifying two parameters, the volatility of the short rate and the mean reversion speed; the Black-Derman-Toy model also assumes that a short rate drives the yield curve, but its process follows a geometric Brownian motion instead, etc. Note that the **Hull-White interest rate model** should not be confused with **Hull-White Credit pricing** (see Section [3.3]) to which we will extensively refer to in the remaining of the document. The former is a model of the yield curve used for pricing of FI securities, the latter is a framework relating credit spreads, hazard rates, and recovery rates.

issuer (in the case of a corporate bond, with a relatively liquid set of traded bonds); and that the individual bond has a residual risk determined by its idiosyncratic characteristics.

The potential granularity of the credit risk description depends on the choice of the risk factors: if we select credit spreads, and because the spreads combine together information about default probabilities and recovery rate assumptions (see Section [3.3]), the risk of the two components cannot be disentangled; if, on the other hand, we select default probability as the principal source of risk, we can, at least in principle, decide whether to promote recovery rates to risk factors, or, by keeping them unchanged, to neglect that source of risk. In RiskManager, credit risk models built around default probability risk factors allow the specification of recovery rate parameters, but do not treat them as risk factors.

The choice is not unique and depends on many factors including data availability and quality, and our choice of a hedging universe in relation to our overall trading strategy. For instance we might decide that liquid CDS spreads on a similar bond provide a better proxy for credit risk than a term structure of spreads derived from illiquid bond data, or that good quality credit data about the corporate issuer should be obtained through the properties of the associated equity (especially if we are interested in linking equity and spread risk). When making this choice we are implicitly selecting the risk factors that are going to drive credit spread changes, i.e. we are selecting a **credit risk model**. Any choice of the credit model could ultimately result in a spread term structure that can be used to price the bond in the same way, i.e. as the sum of risky discounted cash flows. [3] The pricing model, in this case, is essentially the same, but the scenario generation of the spread term structure is different because the drivers of spread changes (the credit risk factors) are different, i.e. the underlying credit risk model is different.

1.3 Credit Pricing and Credit Risk Models

The example of the simple bond is instructive because it can be conceptually extended to more complex securities, and it highlights the connection between the available choices of pricing models and credit models. For all credit sensitive securities credit risk can be represented in terms of credit spread or default probabilities (whose modeling in Risk Server is obtained by using hazard rates.)

When the pricing model for a security is based on hazard rates, the naturally associated credit risk models are those that create scenarios for hazard rates; when the pricing model is based on spreads, the credit models are those driving directly spread changes. As mentioned earlier, spreads can be turned into hazard rates via a recovery rate assumption. As a result, at least in principle, a bridge can be provided to cross the two types of pricing and credit models, and for some securities and methodologies, the functionality is available in Risk Server that allows a pricing models based on spreads to use a credit model based on hazard rates and vice

[3] In reality, when a credit risk model in Risk Manager is defined in terms of default probabilities (or an equivalent hazard rate term structure), pricing of expected cashflows is done directly without going through the calculation of an equivalent spread.

versa[4] In this document we'll spell out the details of the available credit risk models, and clarify the impact of the various input parameters defining the risk models. The different pricing models appropriate for each security type are discussed in detail in separate documents, and will simply be referred to here as specific examples.

[4]Note that there is no guarantee that a sensible spread term structure could always be mapped into a sensible hazard rate structure, for a given recovery rate, and vice versa. This is why we speak of a 'natural' relation between pricing and credit models of the same type.

2 Inputs and Definitions

2.1 Bond Spread Model

RM Name	*Req'd*	*Definition*	*Example*
useBondSpread-CurveModel	No	Unary tag specifying that the Bond Spread Curve credit model is used.	
risklessCurve	No	Risk free curve used for stress test. Must be a yield curve name that matches one currently defined in the Market Data database.	USD Swap
discountCurve	**Yes**	Yield curve used for discounting coupon and principal payments. Must be a yield curve name that matches one currently defined in the Market Data database.	USD Swap

Note that the names above correspond to the RML3 schema supported by RiskServer (corresponding to the RiskManager3 interface). In the RML4 schema (corresponding to the RiskManager4 interface) the useBondSpreadCurveModel is still present, and it activates the Bond Spread Model if an ObligorDefaultModel is not specified. If the ObligorDefaultModel → BondSpreadCurveModel choice is selected, the credit model used is the Hull-White Credit Model or the Bond Spread Model depending on whether the useHullWhite flag in ObligorDefaultModel → BondSpreadCurveModel is specified or not.

2.2 Hull-White Credit Model

RM Name	*Req'd*	*Definition*	*Example*
useBondSpread-HullWhite-CreditModel	No	Unary tag specifying that the Hull-White Credit model is used.	
hullWhitePricing-RecoveryRate	No	Recovery rate, in percent, used in the Hull-White pricing calculation only. It refers to the specific security. If omitted it defaults to the value specified by hullWhiteCalibrationRecoveryRate. If both vaues are omitted it is set to 40%. Available for genericBond, convertibleBond, and genericConvertibleBond.	
hullWhiteCalibration-RecoveryRate	No	Recovery rate in percent used when calibrating the hazard rate term structure to the risky discount curve. It refers to the issuer or bond class, depending on the specified risky curve. If omitted it defaults to 40%.	
risklessCurve	No	Risk free curve used for stress test. Must be a yield curve name that matches one currently defined in the Market Data database. Note that in the RML4 schema the riskless curve used for hazard rate calibration is the riskFreeDefinition input, part of the ObligorDefaultModel→ BondSpreadCurveModel.	USD Swap
discountCurve	**Yes**	Yield curve used in the calibration of the hazard rate term structure, see Eq. (21). Must be a yield curve name that matches one currently defined in the Market Data database. Note that in the RML4 schema the risky yield curve used for hazard rate calibration is the riskyCurveName input, part of the ObligorDefaultModel→ BondSpreadCurveModel.	USD Swap

The names in the table above correspond to the RML3 schema supported by RiskServer (corresponding to the RiskManager3 interface). In the RML4 schema, the Hull-White Credit Model is selected by specifying the ObligorDefaultModel → BondSpreadCurveModel choice (i.e. the useBondSpreadHullWhiteCreditModel input is not available in RLM4), **and** by specifying the useHullWhite flag as part of the BondSpreadCurveModel input.

Note that in the RM4 UI, this obligor credit model is used only if the useCreditModel checkbox is ticked, and the useBondSpreadCurveModel checkbox is not.

The hullWhiteCalibrationRecoveryRate parameter can be specified in the RML4 schema through the ObligorDefaultModel → BondRecoveryRate input.

2.3 CDS Spread Curve Model

RM Name	*Req'd*	*Definition*	*Example*
useCDSSpread-CurveModel	No	If specified, the CDS Spread Curve Model, defined by the following inputs, is used.	
hullWhitePricing-RecoveryRate	No	Recovery rate, in percent, used in the Hull-White pricing calculation only. It refers to the specific security. If omitted it defaults to the value specified by bondRecoveryRate. If both vaues are omitted it is set to 40%. Available for genericBond, convertibleBond, and genericConvertibleBond.	
cdsSpread-CurveModel	No	*Collection of inputs defining the CDS Spread Curve credit model*	
CDSIndexData	No	CDS index spread time series used for hazard rate calculation. Must be a CDS Index name that matches one currently defined in the Market Data database. If specified, it takes precedence on *cdsSpreadCurve*. The model uses the Observed CDS Index Spread time series associated with the specified CDS Index name, and calibrates a flat hazard rate curve. (For more details on Observed CDS Index Spread and CDS Index data in general see [11].)	
cdsSpreadCurve	Yes/No	CDS Spread curve used for calculation of an issuer hazard rate structure. Must be a CDS Spread curve name that matches one currently defined in the Market Data database. Required if *CDSIndexData* input is not specified. In general, a piecewise constant hazard rate curve is calibrated to match the full term structure of CDS spreads. If the CDS Spread curve name refers to a CDS Index (including series, version and maturity), the model will use Markit theoretical spreads of the given index series. In this case, the spread curve will only contain a single spread corresponding to the maturity given in the curve name, so the hazard rate curve will be flat. (For more details on theoretical Markit spreads and CDS Index data in general see [11].)	MX.USD.-CDS.UNITED MEXICAN STATES.SEN

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RM Name	*Req'd*	*Definition*	*Example*
discountYield-Curve	No	Yield curve to use for the discounting of cds expected losses and spread payments. Must be a yield curve name that matches one currently defined in the Market Data database. If unspecified and a risklessCurve input is specified at the position level, discountYieldCurve defaults to that value. If unspecified and no risklessCurve input is specified, it defaults to the government curve associated to the currency of the position.	USD Swap
bondRecovery-Rate	No	Recovery rate in percent used when calibrating the hazard rate term structure to the cdsSpreadCurve input. It refers to the bond's debt class. If left blank, it defaults to 40%.	30
horizonFor-HazardRates	No	The first CDS Spread curve term after the supplied horizon (in years), will be the last term made available for use in calibration of the hazard curve. The hazard rate curve for terms beyond the requested horizon is set to a constant level corresponding to the specified horizon. This can be used to reduce credit model exposure to excess volatility in default probability at long horizons, where spread information can be sporadic. If left blank, all terms of the CDS Spread curve are available for use in the computation of hazard rates.	5.5
cdsSpread-CalibrationLimit	No	Maximum absolute value of the spread, in basis points, allowed in price calibration for any given market price or fair spread. When specified, it overrides the default value defined in RiskServer INI file. This field is currently used in credit default swap and synthetic CDO models. It must be positive when specified.	3000
beta	No	Multiplier to be applied to Monte Carlo simulated values of the CDS spread returns. It defaults to one if unspecified. If CDS spreads are of type *log-return*, r_i is the log-return associated to the spread at node i obtained via the simulation methodology, s_i is the CDS spread in the base scenario, and s'_i is the CDS spread in the simulated scenario, then $s'_i = s_i e^{\beta r_i}$. If CDS spreads are of type *difference return*, $s'_i = s_i + \beta r_i$. For additional details on how r_i is defined see Appendix A in this document.	1.2
CDSSpread-Curve-Proxy.single-NodeProxy	No	*Collection of inputs defining the proxy historical returns used when CDS Spread Curve data on two observation dates is not sufficient to define a proper historical return. See [8] for a description of when proxy returns are used.* The *singleNodeProxy* is specified through an additional *cdsSpreadCurve* and a *curveNode* input defining the node of the proxy curve used to compute the return. An optional *proxyBeta* input can be specified to modify the return as described under *beta*; for instance if CDS spreads are of type *difference return*, if β' denotes *proxyBeta*, and r is the proxy return, $s'_i = s_i + \beta' r$.	

In the RML4 schema, the CDS Spread Model is selected through the ObligorDefaultModel → CDSSpread-CurveModel choice, which corresponds to the RML3 useCDSSpreadCurveModel input. Note that in the RM4 UI, this obligor credit model is used only if the useCreditModel checkbox is ticked, and the useBond-SpreadCurveModel checkbox is not.

If this credit model is selected, the discountCurve input specified under the position might not be used, even when that input is required. This is the case for instance for convertibleBonds, or for the genericBond position when the discountCurveSpread input is not specified.

2.4 CDS Upfront Curve Model

RM Name	*Req'd*	*Definition*	*Example*
useCDSUpfront-SpreadCurveModel	No	If specified, the CDS Upfront Curve Model, defined by the following inputs, is used.	
hullWhitePricing-RecoveryRate	No	Recovery rate, in percent, used in the Hull-White pricing calculation only. It refers to the specific security. If omitted it defaults to the value specified by bondRecoveryRate. If both vaues are omitted it is set to 40%. Available for genericBond, convertibleBond, and genericConvert-ibleBond.	
cdsUpfront-CurveModel	No	*Collection of inputs defining the CDS Upfront Curve credit model.*	
cdsSpreadCurve	**Yes**	CDS Spread curve used for calculation of an issuer hazard rate structure. Must be a CDS Spread curve name that matches one currently defined in the Market Data database	MX.USD.-CDS.UNITED MEXICAN STATES.SEN
discountYield-Curve	No	Yield curve to use for the discounting of cds expected losses and spread payments. Must be a yield curve name that matches one currently defined in the Market Data database.If unspecified and a risklessCurve input is specified at the position level, discountYieldCurve defaults to that value. If unspecified and no risklessCurve input is specified, it defaults to the government curve associated to the currency of the position.	USD Swap
bondRecovery-Rate	No	Recovery rate in percent used when calibrating the hazard rate term structure to the cdsSpreadCurve input. It refers to the bond's debt class. If left blank, it defaults to 40%.	30
horizonFor-HazardRates	No	The first CDS Spread curve term after the supplied horizon (in years), will be the last term made available for use in calibration of the hazard curve. The hazard rate curve for terms beyond the requested horizon is set to a constant level corresponding to the specified horizon. The flat filling can be used to reduce credit model exposure to excess volatility in default probability at long horizons, where spread information can be sporadic. If left blank, then all terms of the CDS Spread curve will be available for use in computation of hazard rates.	5.5
spread	No	The spread that will be used to generate the upfront time series (e.g., 100 or 500).	31
bondCoupon	No	Coupon of the underlying bond, as an annual percentage. Must be greater than or equal to zero. If left blank, a par coupon will be calculated on the bond assuming the same frequency and maturity of the swap.	6.5
parCouponYield-Curve	No	Yield curve to use for the calculation of the par coupon if necessary. Must be a yield curve name that matches one currently defined in the market data database.	USD Swap
cdsSpread-CalibrationLimit	No	Maximum absolute value of the spread, in basis points, allowed in price calibration for any given market price or fair spread. When specified, it overrides the default value defined in RiskServer INI file. This field is currently only used in the Credit Default Swap. Must be positive.	3000

In the RML4 schema, the CDS Upfront Model is selected through the ObligorDefaultModel → CDSUpfrontCurve-Model choice, which corresponds to the RML3 useCDSUpfrontSpreadCurveModel input. Note that in the RM4 UI, this obligor credit model is used only if the useCreditModel checkbox is ticked, and the useBond-SpreadCurveModel checkbox is not.

2.5 Credit Grade Model

RM Name	*Req'd*	*Definition*	*Example*
useCreditGrade-Model	No	If specified, the Credit Grade Model, defined by the following inputs, is used.	
creditGradeModel	No	*Collection of inputs defining the CreditGrades credit model*	
issuerBond-RecoveryRate	No	Recovery rate specific to the bond's debt class, in percentage term. If left blank, default will set to that in DPS timeseries if available, or 50% otherwise.	
issuerEquityName	**Yes**	Name of the underlying equity (or best proxy, if actual equity is not available). Must be an equity name that matches one currently defined in the Market Data database. If left empty, the whole creditGrade model input will be invalid.	
issuerBeta	No	Beta of this underlying equity with respect to Equity Name. If left blank, default value is 1.0.	
equityReturnBooster		Multiplier which applies to percentage returns with respect to the Equity Name. Default value is 1.0.	
issuerEquityPrice	No	Latest closing market price for one share of the underlying equity. If specified, RiskServer calibrates its risk model to match this price. If left blank, current market price is computed from appropriate historical time series.	
issuerEquityCurrency	No	Currency in which the underlying equity is denominated. If specified, this field overrides the currency declared for the equity time series.	
proxyName	No	Name of proxy to be used to fill in data before the first data available for the real equity time series. Must be an equity name that matches one currently defined in the Market Data database.	
proxyBeta	No	Beta to be applied to proxy time series returns in order to form proxy levels. Defaults to 1.0 if not specified.	
issuerVolatility	No	Implied equity volatility of the underlying equity, as an annualized percentage. Must be a positive number. If left blank, volatility is computed from appropriate historical time series.	
issuerDebtPerShare	No	Debt per share of the issuer. Must be a non-negative number. If DPS timeseries data is available for the equity name, this field may be left blank. The currency is the equity currency is specified else the currency of the equity time series.	
issuerGlobal-RecoveryRate	No	Average recovery rate of all of firm's debt, used when calibrating the model to the security market price. If left blank, default will set to that in DPS timeseries if available, or 50% otherwise.	
issuerDefault-BarrierStdev	No	Percent standard deviation of recovery value of all of firm's debt. If left blank, default is 30%.	

In the RML4 schema, the Credit Grade Modelis selected through the ObligorDefaultModel → CreditGradeModel choice, which corresponds to the RML3 useCreditGradeModel input. Note that in the RM4 UI, this obligor credit model is used only if the useCreditModel checkbox is ticked, and the useBondSpreadCurveModel checkbox is not.

3 Methodology

As we have already mentioned in the Introduction, credit models use market data to define risk factors which drive the market scenarios that ultimately determine the risk of a security. Since different securities use pricing models whose model parameters do not always coincide with the market data underlying the selected credit model, introducing a credit risk model requires the following:

1. Establishing a map between market data (risky curves, risk-free curves, CDS spreads, CDS Upfronts, equity data, etc) in the base scenario and the model parameters used when pricing the given security.

2. Identifying a model parameter that is best suited for price calibration of the security, given its market price.

3. Defining how the risk factors of the credit model generate new market scenarios.

To be specific and to fully illustrate the approach we'll consider several examples of credit sensitive securities, depending on the credit model. Examples will include the case of coupon bonds with and without embedded optionality, convertible bonds, CDS contracts, and option on a CDS. In many of these cases we'll refer to a common set of recurring pricing formulas and notations.

We'll denote by P and $\tilde{P}$ the market price and model price of a given security in the base scenario, where by base scenario we refer to the market scenario on analysis date. The market price might be given, while the model price can be computed via a pricing model, given proper inputs. We'll indicate by primed variables model prices under a scenario different from the base scenario (which might be coming from historical or Monte Carlo simulation, or from stress test); hence if the model price $\tilde{P}$ of a security in the base scenario is a function of market data (M) and model data (m), $\tilde{P} = \tilde{P}(M,m)$, $\tilde{P}' = \tilde{P}(M',m')$ indicates the model price of the security under the new scenario identified by (M',m').

When discussing bond with optionality and convertible bonds, we'll sometimes refer to the model value of the same security without optionality, the stripped security, and we'll denote it by $\tilde{P}_S$ or $\tilde{P}'_S$. For instance, in the case of a convertible bond stripped of its optionality and reduced to a simple coupon bond, we have:

$$\tilde{P}_S = \sum_i CF_i \times D_i. \tag{1}$$

where the sum is over all futures cashflows CF_i at time t_i, $D_i = \exp[-r_i t_i]$ is the model discount factor, and r_i the risky discount rate for time t_i. We'll also need to extract a constant credit spread from the stripped model value; if s is such credit spread and by rewriting $\tilde{P}_S$ in terms of *risk free* rates we have:

$$\tilde{P}_S = \sum_i CF_i \times \exp\left[-(r_i^{rf} + s)\, t_i\right], \tag{2}$$

or its corresponding primed version:

$$\tilde{P}'_S = \sum_i CF_i \times D'_i = \sum_i CF_i \times \exp\left[-(r_i'^{rf} + s')\, t_i\right], \tag{3}$$

3.1 A note on recoveries for default-based methods

The following logic can disabled for the affected instrument types by switching on the tag *recoveryCap/disable* under the respective Pricing Model Setting of the Valuation Spec.

In case of default-probability-based pricing methods, when the model accounts for a recovery in the default, we cap the pricing recoveries based on the market prices; i.e. when default probabilities are calibrated to external sources, the internal price information does not override the recovery of those external pricing methodologies, e.g. a CDS contract on a different seniority tranche may be used to obtain default probabilities.

While recoveries, in theory, might be received in the far future, in practice recoveries are low enough that prices around the value is very likely indicative that actually the (almost) imminent recovery is being traded. To also account for the fact that the bond is not yet in default, we actually impose the following cap to price the low-probability regular cash flows as well:

$$\text{Recovery} = \max\{p \times \text{Market Price}, \text{Recovery Input}\}, \tag{4}$$

where

- p is the percentage defined by the tag *recoveryCap/marketPricePercentage* of the Pricing Model Settings, which defaults to 95%,
- the Market Price is the dirty price equalling the calibration target PV for the model (e.g. implying from discount curve spread, and taking settlement period into account), and
- the Recovery Input might be user supplied, defaulted from other inputs, obtained from a time series, or simply defaulted: as the recovery would be determined without the capping logic.

The recovery is determined for each Valuation Spec independently, and then it is kept constant for all the scenarios. This means that the same recovery is used for all simulated VaR settings, all stress tests, time stresses, etc.

If the result of the capping is different to the Recovery Input, we issue a warning for the position.

Currently the CreditGrade model does not impose the above capping logic.

3.2 Bond Spread Model

The Bond Spread Model represents credit risk in terms of associated changes in credit spreads defined above a risk-free curve. Credit spreads are defined as the difference between issuer-specific or credit-rating curves ('Risky Curves'), and a risk-free curve, where both curves can be user defined.

An advantage of this model is its relative simplicity—risk estimates with the Bond Spread Model require the same analytics as the pricing of a conventional bond, but a clear separation of credit spread and risk-free term structures allow for meaningful risk attribution along these two risk categories.

The risky curves used as market data are computed from bonds with no embedded optionality, and are calibrated using the available universe of bonds corresponding to an issuer or a credit sector, see [12]. Risky curves do not describe, therefore, all the credit features of a particular security but they might provide a good proxy to them. The process of price calibration allow us to extract the specific risk characteristics of a given security which, in the case of the Bond Spread Model, is captured by the idiosyncratic credit spread s_ε.

3.2.1 Price Calibration

Bond *without* embedded optionality

We denote by P and $\tilde{P}$ the market price and model price of the bond. We have:

$$\tilde{P} = \sum_i CF_i \times D_i, \tag{5}$$

where the sum is over all futures cashflows CF_i at time t_i, and $D_i = \exp[-r_i t_i]$ is the appropriate model discount factor, with r_i the risky discount rate for time t_i.

We now define s_ε as the additional idiosyncratic constant credit spread to be added to the risky curve term structure to reproduce the market price[5] :

$$P = \sum_i CF_i \times \exp[-(r_i + s_\varepsilon)t_i]. \tag{6}$$

[5] Note that if a discountCurveSpread s_d input is specified for the bond, the market price is *defined* by the equation, and $s_\varepsilon = s_d$.

Considering the case of scenario prices we have:

$$\begin{aligned} \tilde{P}' &= \sum_i CF_i \times \exp\left[-r_i' t_i\right], \\ P' &= \sum_i CF_i \times \exp\left[-(r_i' + s_\varepsilon)\, t_i\right] \end{aligned} \tag{7}$$

where the assumption made in the derivation of P' is that idiosyncratic risk is a characteristic of the bond and does not change rapidly with changing market conditions, i.e. it can be estimated in the base scenario and kept constant during risk estimation.

Notice that the OAS related statistics (creditSpread, issuerOAS, and idiosyncraticOAS) are defined in a different way, i.e. by using the *risk free* rates as the base curve. If we refer to them as OAS, OAS_{issuer}, and OAS_{idio} their definition is:

$$P = \sum_i CF_i \times \exp\left[-(r_i^{rf} + OAS)\, t_i\right],$$

$$\tilde{P} = \sum_i CF_i \times \exp\left[-(r_i^{rf} + OAS_{issuer})\, t_i\right],$$

and $OAS_{idio} = OAS - OAS_{issuer}$.

Details about the maximum allowed values for s_ε and OAS are spelled out in [14].

Bond *with* embedded optionality

In the case of a more complex security, like a callable bond, the analysis is conceptually similar but, instead of having an analytic formula that can be used to price the bond, like Eq. (5), the model value $\tilde{P}$ is a complex function (implemented via a BDT tree on the short rate for bonds, see [14]) of a term structure of rates

$$\tilde{P} = \tilde{P}(\boldsymbol{r}, \Theta), \tag{8}$$

where $\boldsymbol{r}$ is the risky rate term structure, and Θ denotes the collection of other parameters related to T&C, implementation, and market conditions.

As in the case of a simple bond, we account for the specific characteristics of the security by calibrating a single credit spread parameter to the actual market price of the security, if the latter is provided, i.e. we solve for s_ε that satisfies:

$$P = \tilde{P}(\boldsymbol{r} + s_\varepsilon, \Theta), \tag{9}$$

where $\boldsymbol{r} + s_\varepsilon$ denote the risky curve shifted by the constant idiosyncratic spread level s_ε.

Given an arbitrary market scenario, scenario prices are computed as:

$$P' = \tilde{P}(\boldsymbol{r}' + s_\varepsilon, \Theta'), \tag{10}$$

from which all risk measures and risk estimations can be derived.

Convertible Bond

For details about various pricing models available for convertibles see [7], but here it is sufficient to point out that, among the models available, two (the Simple Spread Pricing Model and the Local Spread Pricing Model) are using an equity based binomial tree with discounting of cashflows using both a risk free curve and a risky curve, while one (the Default Based Model) uses an equity based binomial tree constructed using risk free rates and an hazard rate structure to allow explicitly for the possibility of default for the underlying bond. Only Simple Spread and Local Spread Pricing model support the Bond Spread Credit model. If the client wants to use the pricing model using risk free rates and an hazard rate structure , the appropriate credit model in that case is an obligor credit model like CDS, Upfront CDS or CreditGrades models.

Below, and in the remaining of the document, we denote by $\tilde{P}_{ss}(\boldsymbol{r}_{rf},\boldsymbol{r},\Theta)$ the tree algorithm corresponding to the simple spread model, which returns a model price $\tilde{P}_{ss}$ given a risk free curve, a risky curve, and other parameters used for tree construction (Θ), including the equity price S and its volatility σ. Selection of the Bond Spread Model instructs the model to use the risky curve input, which would otherwise be ignored for pricing if any other hazard rate based credit model were selected.

In the absence of a market price, scenario prices are simply obtained by using:

$$P' = \tilde{P}_{ss}(\boldsymbol{r}'_{rf},\boldsymbol{r}',\Theta') \tag{11}$$

When a security price (P) is specified, we calibrate the idiosyncratic spread by solving for s_{ε}: [6]

$$P = \tilde{P}_{ss}(\boldsymbol{r}_{rf},\boldsymbol{r}+s_{\varepsilon},\Theta) \tag{12}$$

we then generate scenario pricing by using

$$P' = \tilde{P}_{ss}(\boldsymbol{r}'_{rf},\boldsymbol{r}'+s_{\varepsilon},\Theta') \tag{13}$$

where the primed variables indicate scenario rates, and Θ' incorporates scenario volatility (σ'), and equity price (S').

3.2.2 Risk Decomposition

Use of the Bond Spread Model allows for separation of credit and interest rate risk. This can be done by writing $\Delta P = P' - P \approx \Delta P_c + \Delta P_{rf}$ where the two terms are obtained by keeping risk-free rates and credit spreads constant in each scenario, respectively.

[6] As described in details in [7], s_{ε} is not always the model parameter used for calibration to the convertible price. If s_{ε} is specified by the user via the input discountCurveSpread, the equity volatility parameter σ is calibrated instead. Here we focus on s_{ε} because of its relevance to credit risk models.

In the case of bonds where risky and risk-free curves are associated to the same currency, we have:

$$\begin{aligned}\Delta P_c &= \tilde{P}(\boldsymbol{r}_{rf} + (\boldsymbol{r}' - \boldsymbol{r}'_{rf}) + s_\varepsilon, \Theta) - \tilde{P}(\boldsymbol{r} + s_\varepsilon, \Theta), \\ \Delta P_{rf} &= \tilde{P}(\boldsymbol{r} + (\boldsymbol{r}'_{rf} - \boldsymbol{r}_{rf}) + s_\varepsilon, \Theta) - \tilde{P}(\boldsymbol{r} + s_\varepsilon, \Theta)\end{aligned} \tag{14}$$

In the case of convertibles we have:

$$\begin{aligned}\Delta P_c &= \tilde{P}_{ss}(\boldsymbol{r}_{rf}, \boldsymbol{r}_{rf} + (\boldsymbol{r}' - \boldsymbol{r}'_{rf}) + s_\varepsilon, \Theta) - \tilde{P}_{ss}(\boldsymbol{r}_{rf}, \boldsymbol{r} + s_\varepsilon, \Theta), \\ \Delta P_{rf} &= \tilde{P}_{ss}(\boldsymbol{r}'_{rf}, \boldsymbol{r} + (\boldsymbol{r}'_{rf} - \boldsymbol{r}_{rf}) + s_\varepsilon, \Theta) - \tilde{P}_{ss}(\boldsymbol{r}_{rf}, \boldsymbol{r} + s_\varepsilon, \Theta)\end{aligned} \tag{15}$$

3.3 Hull-White Credit Model

The Hull-White Credit Model for credit-sensitive instruments is similar to the Bond Spread Model in that it uses as market data the same risky curves used by the Bond Spread Model. However, this model allows the user to specify recovery rate information that is used in the process of model calibration and pricing. Recovery rates are needed to convert spreads to hazard rate structure and vice versa as described in the following section. In the following we denote by RR_c the recovery rate used for calibration, and by RR_p the recovery rate used for pricing, see Sec. [2.2].

We first describe the hazard rate calibration procedure corresponding to model calibration. Then we'll cover price calibration by considering two examples: a coupon bond with no optionality and a bond with embedded optionality.

3.3.1 Hazard Rate Calibration

Hazard rates are calibrated by using the RR_c parameter to back out the equivalent (piecewise constant) hazard rate term structure that would generate the same risky discounted values of bond equivalent cashflows with maturities corresponding to the nodes of the risk free and risky curve inputs.

Let's first recall the definition of hazard rates and their relation to probability of default. The hazard rate $h(t)$ is defined (see [3]) as the instantaneous probability density of default at time t given that no default has occurred prior to time t. If the unconditional probability density of default at time t is denoted by $f(t)$, then, by definition, the hazard rate is given by:

$$h(t) = \frac{f(t)}{1 - F(t)} \tag{16}$$

where $F(t) = \int_0^t f(s)ds$ is the cumulative probability of default from time 0 to t. By using $f(t) = F'(t)$ in

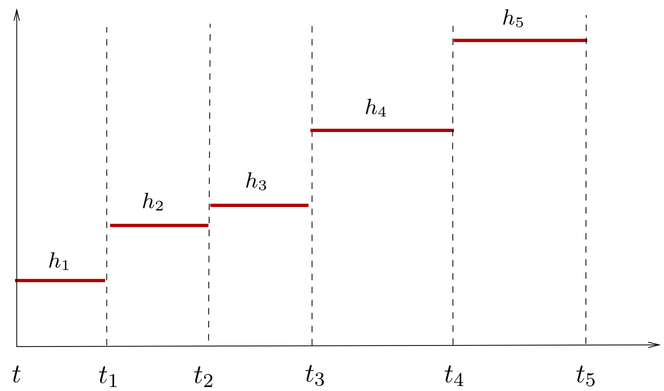

Figure 1: *Piecewise Constant Hazard Rates.*

Eq. ((16)):

$$h(t) = \frac{F'(t)}{1 - F(t)}.$$

and integrating both sides, we obtain:

$$-\int_0^t h(s)ds = \ln(1 - F(t)).$$

This gives

$$F(t) = 1 - \exp(-\int_0^t h(\tau)d\tau). \tag{17}$$

In RiskServer we make the assumption that $h(t)$ is piece-wise constant between the nodes on the hazard rate term structure[7], see Fig. (1).

A useful interpretation of hazard rates is obtained by rewriting Eq. (17) as:

$$S(t) = 1 - F(t) = \exp(-\int_0^t h(\tau)d\tau). \tag{18}$$

where $S(t)$ is the cumulative probability of survival to time t. From this we see that $h(t)$ can be interpreted as the instantaneous forward interest rate adjustment that is equivalent to a given default probability structure. To understand how, consider the present value of a unit risk-free cashflow at time t_i, $D_i^{rf} = \exp\left[-r_i^{rf} t_i\right]$, and 'turn on' the possibility of default for the issuer; if we assume that the default probability is described by

[7]The t-values for the nodes on the hazard rate term structure are a union of those from the risky and riskless curves.

$h(t)$, we obtain an estimate of the present value of the risky cashflow as[8]:

$$D_i = S(t_i)D_i^{rf} = \exp\left[-(r_i^{rf} + s_i)\,t_i\right], \tag{19}$$

where the credit spread s_i is defined in terms of the 'instantaneous spread' $h(t)$ as $s_i = 1/t_i \int_0^{t_i} h(\tau)d\tau$.

How might[9] we use this to calibrate a hazard rate structure from a given risky curve? Given a bond (the optionality stripped bond in the case of a bond with embedded optionality) and a risky curve, we consider the value of similar bonds with maturities $T_1, T_2, \ldots, T_N$ corresponding to the union of the nodes of the risk free and risky curve and we bootstrap the values $h_1, h_2, \ldots, h_N$ defining the calibrated piece-wise constant hazard rate structure.

Let's assume the bond with maturity T_i has notional N, and that there are n_i payment dates with coupon payment of C_k on the k^{th} payment date. We denote by RR_c the recovery rate associated with the hypothetical bond. Given any of the n time periods $[t_{k-1}, t_k]$ (t_0 could be in the past) we have to consider three possibilities: default occurring before the period, after the period, and during the period. If the bond defaults before the period, there is no cash flow. If the bond defaults after the period, there is a payment of C_k at t_k. If the bond defaults during the period, there is a payment of accrued interest plus notional, adjusted by the recovery rate assumption. the model value of the bond corresponding to maturity T_i can be written as:

$$\begin{aligned}\tilde{V}_i = \sum_{k=1}^{n_i} D_k^{rf} \cdot \Big\{ [1 - F(t_k)] \cdot C_k + [F(t_k) - F(t_{k-1})]\,(C_k + N)\,RR_c \Big\} \\ + D_{n_i}^{rf} \cdot N \cdot (1 - F(t_{n_i}))\end{aligned} \tag{20}$$

In the Bond Spread Hull-White model, the hazard rate structure is calibrated to a discount curve using the associated zero-coupon bond prices. For a zero-coupon bond maturing at time t, Eq. 20 simplifies to

$$Z(t) = D^{rf}(t) \cdot [(1 - F(t)) + RR_c \cdot F(t)] \tag{21}$$

Let $z(t)$ denote the risky zero coupon rate so that $Z(t) = e^{-z(t)t}$, and divide both sides of (21) by $D^{rf}(t) = e^{-r^{rf}(t)t}$:

$$e^{-s(t)t} = e^{-(z(t) - r^{rf}(t))t} = 1 - (1 - RR_c) \cdot F(t), \tag{22}$$

where $s(t) = z(t) - r^{rf}(t)$ is the term structure of bond spreads.

[8] We are assuming here that no recovery occurs in the event of default and also that default events for the issuer are negligibly correlated to the risk-free interest rate structure.

[9] In the implementation, we follow the simplified approach described following Eq. (20).

Using (17) and rearranging the equation we get:

$$\int_0^t h(s)ds = -\ln \frac{e^{-s(t)t} - R}{1-R}. \tag{23}$$

This result allows us to determine the hazard rates, if the bond spread curve $s(t)$ is known.

Assume that the risky curve and risk free curve are known at certain reference maturities. Let $T_1 < T_2 < ... < T_N$ denote the union of these maturities (expressed as Act/365 year fractions), so that for any $k = 1,2,...,N$ either or both curves have a node at T_k. Then the nodes of the bond spread curve are defined for each k as $s_k = s(T_k) = z(T_k) - r^{rf}(T_k)$, where $z(T_k)$ and $r^{rf}(T_k)$ are computed by linear interpolation, when the respective curves have no node at T_k.

Using the piecewise constant hazard rate curve assumption, and choosing the curve nodes to match T_1, ..., T_N, we can re-write (23) for each $k = 1,2,...,N$ as

$$\sum_{j=1}^{k} h_j(T_j - T_{j-1}) = -\ln \frac{e^{-s_k T_k} - R}{1-R}, \tag{24}$$

where $T_0 = 0$. This allows us to bootstrap the hazard rate structure by first computing h_1, then h_2, and so forth based on the following recursion:

$$\begin{aligned} h_1 &= -\frac{1}{T_1 - T_0} \ln \frac{e^{-s_1 T_1} - R}{1-R}, \\ h_k &= -\frac{1}{T_k - T_{k-1}} \left[\ln \frac{e^{-s_k T_k} - R}{1-R} + \sum_{j=1}^{k-1} h_j(T_j - T_{j-1}) \right]. \end{aligned} \tag{25}$$

From (23) it is clear that $R \leq e^{-s_k T_k} \leq 1$ must hold for all $k = 1,2,...N$. On one hand, this means that it is possible to observe discount rates that are higher than those allowed by the hazard-rate model. In this case we set the hazard rate to a large positive number (5000% per year) on the interval under consideration. It is also possible for the discount curve to imply negative hazard rates, when the spread of the discount curve above the riskless curve decreases too quickly. In this case the hazard rate is set to zero on the interval under consideration.

It is possible for the hazard rate calibration to succeed, meaning that the hazard rates exactly reproduce the observed risky discount rates, but the resulting credit model is unusable for pricing other instruments. This occurs when the model generates unrealistic sensitivity statistics, such as CS01 and VaR. This problem is associated with large hazard rates, for reasons explained in [13]. It can be avoided by replacing the flat 5000% hazard rate cap with a dynamic cap based on the width of the interval under consideration: $h_k^{\text{cap}} = 1/dT_k$, where $dT_k = T_k - T_{k-1}$ is the width of the interval between nodes at T_{k-1} to T_k.[10] Users

[10]This particular value for the cap is chosen by requiring that the Delta Equivalents of an instrument vary monotonically between

may enable the dynamic hazard rate cap in the valuation spec by including the tag *creditModelSettings* → *bondSpreadHullWhiteCreditModelSettings* → *hazardRateCalibrationSettings* → *useDynamicCap*. Note that this setting may also insert additional nodes in the hazard rate term structure to guarantee that the spacing between nodes is never more than one year — i.e., the hazard rate cap is never less than 1.0.

3.3.2 Hull-White Credit Bond Pricing

Given a hazard rate structure bond pricing under the Hull-White credit model is obtained by using Eq. (20), but using the pricing recovery rate RR_p:

$$\tilde{P} = \sum_{k=1}^{n} D_k^{rf} \cdot [(1 - F(t_k)) \cdot C_k + (F(t_k) - F(t_{k-1}))(C_k + 1) RR_p] + D_n^{rf}(1 - F(t_n)). \tag{26}$$

The idea behind the specification of two possibly different recovery rates is simple. The hazard rate structure obtained by the risky curve input via calibration is meant to represent the default term structure of the bond debt class, and RR_c is the recovery rate parameter reflecting this. Bond of similar seniority of the same issuer will be calibrated to the same hazard rate structure. In pricing a security, however, the user should specify the recovery rate RR_p that is specific to that security. RR_c plays therefore a role similar to the one played by the bondRecoveryRate input in the CDS Spread Model and CDS Upfront Model, and the issuerGlobalRecoveryRate in the Credit Grade Model. The RR_p parameter is specific to the security in all the obligor based credit models, and it is specified through the hullWhitePricingRecoveryRate input in all models except the CGs, where the issuerBondRecoveryRate is used.

3.3.3 Price Calibration

The model price of a bond in the Hull-White Credit Model is not based on risky discounting, but on explicitly using hazard rates to compute the price of a bond as the value of its cash flows, discounted at the *risk-free* rate, minus the value of expected credit losses due to default, also discounted at the *risk-free* rate.

By following the same principles as in the Bond Spread Model we consider two examples: a coupon bond with no optionality and a bond with embedded optionality.

Bond *without* embedded optionality

Once the hazard rate structure has been computed, the model value of the bond $\tilde{P}$ is obtained following the methodology presented in Sec. [3.3.2]. The basic idea, however, is simple:

$$\tilde{P} = \sum_i D_i^{rf} (CF_i - L_i(RR_p)), \tag{27}$$

nodes on the spread curve. See [13] for details.

where, unlike in Eq. (5) where D_is are risky discount factors, in Eq. (27) we use risk-free discount factors, i.e. $D_i^{rf} = \exp\left[-r_i^{rf} t_i\right]$, and L_i are the value of credit losses also discounted with risk-free rates. We have made the dependence on RR_p explicit to emphasize that, while RR_c is used when calibrating hazard rates, RR_p is used when pricing the bond.

We can write Eq. (27) equivalently as

$$\begin{aligned} \tilde{P} &= \sum_i (CF_i - L_i) \times D_i^{rf} \\ &= \sum_i CF_i \times D_i^{rf} \times \exp\left[-s\, t_i\right], \end{aligned} \tag{28}$$

where s is the constant *issuer spread* that is calibrated to match the price in Eq. (27). The parameter used for market calibration is, as in the Bond Spread Model , the *idiosyncratic spread* (s_ε), which is defined by:

$$P = \sum_i CF_i \times D_i^{rf} \times \exp\left[-(s + s_\varepsilon)\, t_i\right]. \tag{29}$$

Pricing in other scenarios is then obtained as:

$$P' = \sum_i CF_i \times D_i'^{rf} \times \exp\left[-(s' + s_\varepsilon)\, t_i\right], \tag{30}$$

where D'^{rf}s are the scenario risk-free discount factors, and s' is the issuer spread that is calibrated to match the expected value of credit losses in the new scenario. (These are obtained by recalibrating the hazard rate structure using scenario values for risky and risk-free rates, see Sec. [3.3.1].)

Bond *with* embedded optionality

The approach here is conceptually similar to what described in the Bond Spread Model case, but the pricing formula uses a risky curve (compare with Eq. (8)) defined in terms of risk-free rates $\boldsymbol{r}_{rf}$ and a constant issuer credit spread s calibrated as described below:

$$\tilde{P} = \tilde{P}(\boldsymbol{r}_{rf} + s, \Theta). \tag{31}$$

Model and price calibration are then described by the following steps:

1. We calibrate a hazard rate structure for the base scenario as we did in the simple bond case (using RR_c)

2. We compute the model value of the stripped bond $\tilde{P}_S$ using Eq. (27) and RR_p

3. We calibrate s using $\tilde{P}_S$ and Eq. (28)

4. We compute the model value $\tilde{P}$ using Eq.(31)

5. We calibrate s_ε, if a market price P is given, using $P = \tilde{P}(\boldsymbol{r}_{rf} + s + s_\varepsilon, \Theta)$

Scenario prices P' are obtained by repeating steps 1 to 4 with scenario data, i.e. by deriving the scenario hazard rate structure from $\boldsymbol{r}'_{rf}$ and $\boldsymbol{r}'$, by recalibrating the issuer spread s' for the given scenario, and by using the pricing formula

$$P' = \tilde{P}(\boldsymbol{r}'_{rf} + s' + s_\varepsilon, \Theta'). \tag{32}$$

In both cases of bonds with and without embedded optionalities, with RiskServer 5.4 Phase 2, we are supporting the possibility of using a term structure of risky spreads, instead of the flat spread s defined in Eq. (28). Within the new approach the idiosyncratic spread (s_ε) is still flat, but it applies to a terms structure of risky spreads. For more details on the new approach see [15].

3.4 CDS Spread Model

The CDS Spread Model calibrates the hazard rate structure for a security class based on CDS spread data and generates scenarios based on changes in such spreads. The use of hazard rates and the calibration of the model to the security market price will depend on the characteristics of the security and the pricing model selected. In this section we'll describe the hazard rate structure calibration procedure adopted in this model, and we'll consider the example of model calibration to the security market price for simple bonds, bonds with optionality, convertibles, CDS, and CDS options.

Since the Spring of 2008 the CDS market has evolved to standardizing the spread payments[11], and has started effectively using an 'upfront price' as the market quote. Standardization of the payments makes it easier to enter into contracts with counter-parties where fixed spread liabilities exactly cancel.

To avoid confusion, here is some of the relevant terminology used in this document:

- Fair spread, par spread: the fixed (annualized) spread that makes it fair to enter into the CDS contract with zero upfront payment. Calculation of the fair spread is done by using a piecewise constant hazard rate term structure that is consistent with values of CDS contracts with shorter terms on the same entity, and with a recovery rate which can be specific to the entity.
- ISDA spread: the fixed (annualized) spread that makes it fair to enter into the CDS contract with zero upfront payment. Calculation of the ISDA spread is obtained by assuming a constant hazard rate for the entire term of the CDS, and a standard recovery rate assumption. For standard spreads, recovery rates, and ISDA standard model conventions see `http://www.cdsmodel.com/cdsmodel/fee-computations.page`

[11] The fixed-leg payment days are March 20, June 20, September 20, and December 20, or close to these dates, depending on weekends and market holidays.

- Contractual spread, standard spread: the fixed (annualized) spread associated with a given entity. The market upfront is the upfront payment that makes it fair to enter into the CDS given the specified standard spread.

The methodology described in this section uses market fair spreads as risk factors. The methodology in the following section (Sec. [3.5]) uses upfront quotes (derived from the same market fair spreads) as risk factors.

3.4.1 Hazard Rate Calibration

For a brief introduction to the concept of hazard rates see Sec. [3.3.1]

To bootstrap the hazard rate structure for a given reference name we use the quoted fair spreads (Markit composite spreads) of CDS contracts on that name, maturing on different dates. The hazard rates are determined so that the spread of each reference CDS contract match the market quote, under the following assumptions and conventions:

- Reference CDS contracts settle on a T+1 basis, i.e. one business day after the analysis date.
- The CDS contracts pay coupon and mature on standard ISDA dates (the 20$^{\text{th}}$ of March, June, September and December). The value of the actual coupon is not required for the hazard rate calibration.
- The maturity date of the contract is determined by adding the CDS tenor to the settlement date, and moving forward to the next ISDA date. For instance, if the settlement date is 5 April 2015, and the the tenor is 5Y, then the maturity date will be 20 June 2020.
- The loss given default per unit notional is $1-R$ where R is the quoted (Markit composite) recovery rate of the reference name (fetched from the database).
- The hazard rate curve is assumed to be piecewise constant and right-continuous, where the curve nodes coincide with the maturities of the reference CDS contracts.
- CDS cashflows are discounted using the swap curve associated with the currency of the contract. (E.g. USD Swap for USD.)
- We employ the CDS pricing formulas described in the Credit Default Swap technical note [9].

It is important to point out that none of these assumptions can be changed or overridden by the user for hazard rate calibration. In particular, it is not possible to change the recovery rate assumption.

Assume that the CDS spread curve available in the database contains k fair spread quotes $s_1, s_2, \ldots, s_k$ for maturity dates $T_1, T_2, \ldots, T_k$. The hazard rate curve will then take the form

$$h(t) = \begin{cases} h_1 & \text{if } t \le T_1, \\ h_2 & \text{if } T_1 < t \le T_2, \\ \ldots \\ h_{k-1} & \text{if } T_{k-2} < t \le T_{k-1}, \\ h_k & \text{if } t > T_{k-1}. \end{cases} \quad (33)$$

We measure time on an Act/365 basis, so that T_i represents the number of days, divided by 365, that elapsed between some common reference point and the time to maturity of the i^{th} CDS contract (falling on a standard ISDA date).

The objective is to determine the hazard rate levels h_1, ..., h_k so that the fair spread computed by the CDS pricing model is equal the quoted spread for each of the k reference contracts. This can be done in an iterative way, by solving k univariate equations.

First, we solve for h_1 by searching for the hazard rate that makes the computed fair spread of the contract maturing on T_1 equal to s_1. We have one unknown: h_1, and one equation: the CDS is priced to its market quote; and we employ the secant method to solve it. Once h_1 is known, we can determine h_2 by matching the computed fair spread of the contract maturing on T_2 to the quoted spread s_2. Again, we have a single unknown and a single equation, because the second curve node is lined up with the maturity of the second reference contract.

We repeat the same procedure for the third, fourth, etc. curve node, until the k^{th} one. Note that in some rare cases, one or more of the equations in the bootstrapping procedure may not have solution or it may be negative.[12] In order to make sure that the algorithm always produces some finite, non-negative hazard rates, we constrain their values to the $[0, 50]$ closed interval. In case there is no solution with this range, the hazard rate set to the lower or upper boundaries of 0 or 50, respectively.

Let us express explicitly the equations to be solved during the bootstrapping process[13] . It is important to remember, that the settlement date of the reference contracts is one business day after the valuation date, i.e. $t_V < t_S$, so we formally need to perform forward pricing to calibrate the hazard rates. Specifically, assuming that the first $m-1$ hazard rates have already been determined, hazard rate h_m is derived by solving the equation $0 = DEL_m - DES_m(s_m) + AP(s_m)D(t_S)(1 - F(t_S))$, where DEL_m and $DES_m(s_m)$ represent the present value of the protection leg and premium leg, respectively; while $AP(s_m)$ represents the accrued

[12] A typical case when there may be no solution is when the fair spread is extremely high, and the recovery rate assumption is not consistent with the spread level. Extremely high spreads means that the market prices a high default probability **and** high loss severity, i.e. low recovery rate. So if our recovery rate assumption is not sufficiently low (e.g. 40% instead of, say, 20%) it would require default probabilities above 1 to recover the market spread from the model.

[13] Additional details and full description of notation can be found in [10].

interest amount receivable by the protection buyer on the settlement date, given a spread of s_m. More explicitly:

$$0 = (1-R)\left[D(t_1)(F_1 - F(t_S)) + \sum_{i=2}^{n} D(t_i)(F_i - F_{i-1})\right] -$$
$$- s_m \times \zeta \left[(t_S - t_0)D(t_1)(1-F(t_S)) + \zeta \sum_{i=1}^{n_m} D(t_i) \sum_{j=1}^{k_i} \frac{F_{i,j} - F_{i,j-1}}{h_{i,j}}\right] + s_m \times \zeta (t_S - t_0) D(t_S)(1 - F(t_S)), \quad (34)$$

where n_m is the number of coupon dates to the maturity date T_m. After some rearrangement, this is equivalent to

$$s_m = \frac{1-R}{\zeta} \frac{D_1(F_1 - F(t_S)) + \sum_{i=2}^{n} D(t_i)(F_i - F_{i-1})}{(t_S - t_0)(D(t_1) - D(t_S))(1 - F(t_S)) + \zeta \sum_{i=1}^{n_m} D(t_i) \sum_{j=1}^{k_i} \frac{F_{i,j} - F_{i,j-1}}{h_{i,j}}}. \quad (35)$$

The implementation solves this last equation for each reference contract.

3.4.2 Price Calibration

In all the examples below we assume that the model calibration in the base scenario has yielded a hazard rate structure $\boldsymbol{h}$. RR_p indicates the *hullWhitePricingRecoveryRate* input.

Bond *without* embedded optionality The parameter used for price calibration is the idiosyncratic credit spread s_ε.

1. We compute the model value of the bond $\tilde{P}$ using Eq. (27) and RR_p
2. We calibrate s using $\tilde{P}(\cdot)$ and Eq. (28)
3. We define s_ε, if a market price P is given, using Eq. (29)

Scenario prices P' are generated by recalibrating the hazard rate structure $\boldsymbol{h}'$ for each scenario and repeating steps 1-2 with scenario data while keeping s_ε fixed.

Bond *with* embedded optionality The parameter used for price calibration is the idiosyncratic credit spread s_ε.

1. We compute the model value of the stripped bond $\tilde{P}_S$ using Eq. (27) and RR_p
2. We calibrate s using $\tilde{P}_S(\cdot)$ and Eq. (28)
3. We compute the model value $\tilde{P}$ using Eq. (31)
4. We calibrate s_ε, if a market price P is given, using $P = \tilde{P}(\boldsymbol{r}_{rf} + s + s_\varepsilon, \Theta)$

Scenario prices P' are obtained by recalibrating the hazard rate structure for each scenario, repeating steps 1 to 3 with scenario data, and by using Eq. (32).

In both cases of bonds with and without embedded optionalities, with RiskServer 5.4 Phase 2, we are supporting the possibility of using a term structure of risky spreads, instead of the flat spread s defined in Eq. (28). Within the new approach the idiosyncratic spread (s_ε) is still flat, but it applies to a terms structure of risky spreads. For more details on the new approach see [15].

Convertible Bond

Here we use the terminology defined in Sec. [3.2.1]. Note that, depending on the selected pricing model and the inputs provided, an extensive list of parameter can be calibrated. Credit risk models are not always calibrated. For a full description of the available functionality for convertibles see [7].

Spread based models

The only time when credit model gets calibrated is when (i) market price (MP) is given and spread calibration is selected *or* (ii) user inputs non zero *discountCurveSpread*. In these cases we calibrate credit model to the total credit spread, s_{tot} which is constructed as follows:

1. we compute the model value of the stripped bond $\tilde{P}_S$ using Eq. (27) and RR_p
2. we define s using $\tilde{P}_S$ and Eq. (2)
3. $s_{tot} = s + s_\varepsilon$ where s_ε is the calibrated idiosyncratic spread or the input *discountCurveSpread*.

The following steps are necessary to calibrate the CDS spread model.

1. Calculate the stripped price of the bond $\tilde{P}_S$ using the total spread, s_{tot} using Eq. (2)
2. In the CDS spread model calibrate a constant shift on top of CDS spreads which will result in such a hazard rate term structure which will make the stripped bond price match $\tilde{P}_S$ using Eq. (27)

Scenario prices P' are obtained by deriving a new scenario hazard rate structure from $\boldsymbol{r}'_{rf}$ and scenario CDS spread data (plus the calibrated CDS shift), calculating the total spread s'_{tot} for the given scenario, and using $\tilde{P}_{ss}(\boldsymbol{r}'_{rf}, s'_{tot}, \Theta')$, $\tilde{P}_{ls}(\boldsymbol{r}'_{rf}, s'_{tot}, \Theta')$.

Default based model The model requires specification of a base scenario hazard rate structure which is provided by the credit model. Parameters that can be calibrated according to the schema described above are, listed according to priority: volatility (σ), recoveryRate (R), *credit risk model*. If the calibrateHazardRateOnly input is specified instead, only the idiosyncraticHazardRate (h_ε) parameter is calibrated to match MP. The credit model is only calibrated if calibrateHazardRateOnly is not set, and volatility and recovery rate calibration is not enought to match market price.

In the CDS credit model the constant CS shift (added on top of market CDS quotes) are calibrated so that the resulting hazard rate term structure $\boldsymbol{h}$ satisfies $P = \tilde{P}_{db}(\boldsymbol{r}_{rf}, \boldsymbol{h}, R, \sigma, \Theta)$ where P is the market price of the convertible bond.

We create scenario prices P' by simulating CDS spreads, calculating a simulated hazard rate term structure of $\boldsymbol{h}'$ and evaluating $\tilde{P}_{db}(\boldsymbol{r}_{rf}, \boldsymbol{h}', R, \sigma, \Theta)$.

For additional details and special cases see [7].

3.5 CDS Upfront Model

CDS Spread Model and CDS Upfront Model use the same market data but they use different risk factors to generate new market scenarios. In the former case we use spreads, in the latter we use upfronts.

Since upfront prices can be negative or zero, however, we cannot use them in a straightforward way as ordinary price factors, i.e. we cannot always define their log return. In order to address this issue, we introduce a transformation L mapping upfronts into positive numbers. L is an approximate (linearized) upfront-to-spread map:

$$L(t) = U(t) + s_c(T + 0.25).$$

Here $U(t)$ is the regularized upfront price at time t[14], s_c is the (annualized) contract spread payment (specified by the *spread* input in the cdsUpfrontCurveModel) and T is the length of the term of the CDS in years. We need to add 0.25 because of market conventions on payments, for instance the CDS value for the 1-year node has 5 corresponding spread payments.

$L(t)$ can be interpreted as the value of the original CDS assuming all of the future payments are paid immediately with no discounting and no effort to determine if there is a default. For this reason the value is always positive.

To compute an actual (i.e. *not* transformed) simulated upfront corresponding to a simulated scenario we first draw the log return (or the relative return $L(t)/L(t-1)$ for historical simulations) and we apply it to today's transformed upfront $L(t = 0)$. We then apply the inverse of the L transform, i.e. we subtract the constant contribution $s_c(T + 0.25)$, to obtain the clean upfront corresponding to the new scenario.

3.5.1 Hazard Rate Calibration

The hazard rate calibration algorithm is conceptually identical to the one described in Sec. [3.4.1]. However, while the calibration of each h_i is done by matching the value of the corresponding fair spread (to maturity T_i) expressed as a function of the hazard rate structure in the CDS Spread Model, when credit scenarios are

[14] See[6] for details about the concept of regularized upfronts and for specifics about how the time series of upfronts are defined.

specified in terms of upfront nodes, calibration of h_i is done by matching the value of the upfront computed as a function of the hazard rate structure. For details about the explicit formulas used in the two cases see [9].

3.5.2 Price Calibration

The price calibration procedures are identical as in the cases described in Sec. [3.4.2].

3.6 CreditGrades Model

CreditGrades[15] model is a structural model [2]. These models derive from work of Black and Scholes [1] and Merton [4] who observed that both equity and debt can be viewed as options on the value of a firm's assets, implying that equity option pricing techniques can be adapted for assessing the credit quality of an issuer.

The CreditGrades model assumes a geometric Brownian motion model for the firm value, and a log-normal distribution for the firm recovery value [5].

The model is designed to track credit spreads well (as opposed to produce accurate probabilities of default) and to provide a timely indication of when a firm's credit becomes impaired. Parameter estimates and other model decisions were made based on the model's ability to reproduce historical default swap spreads.

One departure we make from the standard structural model is to address the artificially low short-term credit spreads that are follow from the standard model. These low spreads occur because assets that begin above the barrier cannot reach immediately the barrier by diffusion only. We model the uncertainty in the default barrier (*issuerDefaultBarrierStdev* in the input section), motivated by the fact that we cannot expect to know the exact level of leverage of a firm except at the time the firm actually defaults. The uncertainty in the barrier admits the possibility that the firm's asset value may be closer to the default point than we might otherwise believe, leading to higher short-term spreads than are produced without the barrier uncertainty.

Another difference is that the CreditGrades approach is more practical, bypassing strict definitions in favor of simple formulas tied to market observables. As a result, the model can be stated as a simple formula depending on a small number of input parameters, and sensitivities to these parameters can be easily ascertained.

The survival probability of the issuer up to time t is given by

$$S(t) = \Phi\left(\frac{-A_t}{2} + \frac{\log(d)}{A_t}\right) - d\Phi\left(\frac{-A_t}{2} - \frac{\log(d)}{A_t}\right), \tag{36}$$

[15] In the current section we quote from [2] to highlight the most important properties and objectives of the model.

where

$$d = \frac{E+\bar{L}D}{\bar{L}D}\exp(\lambda^2), \quad (37)$$

$$A_t^2 = \sigma_A^2\, t+\lambda^2. \quad (38)$$

where $\Phi(x)$ the standard normal cumulative distribution function. The following Table relates the parameters in the formula above to the inputs of the Credit Grade Model:

Symbol	Parameter	Description
$\bar{L}$	*issuerGlobalRecoveryRate*	Average recovery rate of all of firm's debt.
E	*issuerEquityPrice*	Equity or proxy price (adjusted by beta and return).
σ_A	Issuer Asset Volatility	Related to *issuerVolatility* by $E\sigma_E/(E+\bar{L}D)$.
σ_E	*issuerVolatility*	Issuer volatility, used to calculate asset volatility.
D	*issuerDebtPerShare*	Debt per share of the issuer.
λ	*issuerDefaultBarrierStdev*	Percent standard deviation of recovery value of all of firm's debt.

Table 6: Relation between input parameters and Credit Grades model parameters

The cumulative default probability function and the cumulative probability of survival to time t are related by

$$F(t) = 1-S(t) \quad (39)$$

3.6.1 Hazard Rate Calibration

Unlike the other credit risk models using hazard rates, which allow for calibration of a full hazard rate structure from bonds or CDS market data, and therefore generates a 'risk neutral' hazard rate structure which is appropriate when pricing credit sensitive securities, the Credit Grade Model starts with a 'structural' hazard rate structure that must be calibrated to the security price. Two parameters in the credit model are used for this purpose, the asset volatility σ_A and the *issuerBondRecoveryRate* RR_p. We'll consider the specifics of the hazard rate structure calibration for each case in the Price Calibration section below.

3.6.2 Price Calibration

In the following RR_p indicates the *issuerBondRecoveryRate* input. Note that calculation of the hazard rate structure is done using Eq. (36) which uses a fixed value for $\bar{L}$ = *issuerGlobalRecoveryRate*.

Bond *without* embedded optionality

1. We compute the model value of the bond $\tilde{P}$ using Eq. (27) and RR_p, where the credit loss is obtained from the hazard rate structure of the Credit Grade Model

2. If a market price P is given, we calibrate the asset volatility σ_A until the resulting hazard rate structure gives $P = \tilde{P}(\sigma_A, RR_p)$. The procedure is detailed below in Sec. [3.6.3], and gives values $\bar{\sigma}_A$ and $\overline{RR}_p$

3. We create scenario prices P' by repeating step 1 with scenario data, i.e. by using the hazard rate structure $\boldsymbol{h}'$ derived from Eq. (36) with E', $\bar{\sigma}_A$, and $\bar{L}$, and using Eq. (27) with $\overline{RR}_p$

Bond *with* embedded optionality As in cases previously considered for other credit models, we need to map the hazard rate structure to a constant spread s in order to calibrate the model to the security price.

1. We calibrate the spread s solving $P = \tilde{P}(s)$ using Eq. (31)

2. We define the model value of the stripped bond $\tilde{P}_S(s)$ using Eq. (2) and s

3. We calibrate the asset volatility σ_A until the resulting hazard rate structure reproduces the stripped bond model value using Eq. (27) and RR_p, where the credit loss is obtained from the hazard rate structure . The procedure is detailed below and gives values $\bar{\sigma}_A$ and $\overline{RR}_p$.

Scenario prices P' are generated by repeating step 1 with scenario data ($\boldsymbol{r}'_{rf}$ and s'), i.e. by using the hazard rate structure $\boldsymbol{h}'$ derived from Eq. (36) with E', $\bar{\sigma}_A$,and $\bar{L}$; by using Eq. (27) with $\overline{RR}_p$ to generate a stripped bond scenario price, $\tilde{P}'_S$; and by calibrating the appropriate scenario spread s' by matching $\tilde{P}'_S$ using Eq. (3).

Convertible Bond

Following the notation introduced in Sec. [3.2.1], we denote by $\tilde{P}_{ss}(\boldsymbol{r}_{rf}, s, \Theta)$, $\tilde{P}_{ls}(\boldsymbol{r}_{rf}, s, \Theta)$ and $\tilde{P}_{db}(\boldsymbol{r}_{rf}, \boldsymbol{h}, \Theta)$ the tree algorithms corresponding to simple spread, local spread, and default based pricing models, respectively. For these models the Θ inputs represent other parameters used for tree constructions including the equity price S and its volatility σ. Note that for the Credit Grade Model, like for the CDS Spread Model and CDS Upfront Model, the risky curve input is not used for pricing, but the pricing tree currently requires specification of a *constant* credit spread s for spread based pricing models.

Spread based models A detailed example is given here for the case when the market price is supplied by the user and spread calibration is selected. Other cases are similar, and the full specification of the calibration possibilities is given in [7].

1. We estimate the total credit spread s. This is done by solving for s using the equation $P = \tilde{P}_{ls}(\boldsymbol{r}_{rf}, s, \Theta)$ for local spread model; $P = \tilde{P}_{ss}(\boldsymbol{r}_{rf}, s, \Theta)$, for simple spread model. Note that no Credit Grade Model information is used in this step.

2. We define a stripped bond model value $\tilde{P}_S(s)$ using Eq. (2) and s

3. We calibrate the Credit Grade Model using Eq. (27) and RR_p, where the credit loss is obtained from the hazard rate structure of the Credit Grade Model. Calibration is done by solving for σ_A. The procedure is detailed below and gives values $\bar{\sigma}_A$ and $\overline{RR}_p$

Scenario prices P' are generated by using $\tilde{P}_{ss}(\boldsymbol{r}'_{rf}, s', [\Theta]')$, $\tilde{P}_{ls}(\boldsymbol{r}'_{rf}, s', [\Theta]')$, where s' is obtained by: a) computing the hazard rate structure $\boldsymbol{h}'$ from Eq. (36) with E', $\bar{\sigma}_A$, and $\bar{L}$; b) using Eq. (27) and $\overline{RR}_p$ to get $\tilde{P}'_S$; c) computing s' given $\tilde{P}'_S$ using Eq. (3).

Default based model

The model requires specification of a base scenario hazard rate structure which is provided by the credit model. Parameters that can be calibrated according to the schema described above are, listed according to priority: volatility (σ), recoveryRate (R), *credit risk model*. If the calibrateHazardRateOnly input is specified instead, only the idiosyncraticHazardRate (h_ε) parameter is calibrated to match MP. The credit model is only calibrated if calibrateHazardRateOnly is not set, and volatility and recovery rate calibration is not enought to match market price.

In the Credit Grade credit model σ_A and $\overline{RR}_p$ are calibrated so that the resulting hazard rate term structure $\boldsymbol{h}$ satisfies $P = \tilde{P}_{db}(\boldsymbol{r}_{rf}, \boldsymbol{h}, R, \sigma, \Theta)$ where P is the market price of the convertible bond.

We create scenario prices P' by simulating underlyings of Credit Grade model, calculating a simulated hazard rate term structure of $\boldsymbol{h}'$ and evaluating $\tilde{P}_{db}(\boldsymbol{r}_{rf}, \boldsymbol{h}', R, \sigma, \Theta)$.

3.6.3 Calibration Details and Issues

If a solution of the asset volatility cannot be found within a pre-set boundary (maximum allowable asset volatility, as defined by RiskServer INI file), and if RR_p is not user-specified, the default value for RR_p is adjusted downward at 5% intervals until an asset vol is found or until the recovery rate cannot be adjusted down further (cannot be negative) in which case the calibration will be declared failed.

Generally there are three categories of difficulties when calibratiing the Credit Grade Model. The first two are

1. The spread in the instrument over the **risk-free** curve is *negative*, which is not permitted under CG model.

2. The market price is too low to calibrate a reasonable (in the sense of not exceeding the pre-set maximum) value for the asset volatility.

For instruments with user-entered recovery rate, the second category may happen in high yield (junk) bonds and distressed bonds (convertibles).

The third category of calibration difficulties is related to the possible existence of a gap between the zero asset volatility price and the risk-free price. This gap is a technical artifact of CG modeling assumptions, specifically the log-normality of the default barrier (for details please refer to CreditGrades Technical Document [2]). The gap is generally small and a non-issue for medium and long dated bonds. For short dated bonds, however, this can sometimes pose a problem.

In cases where we cannot calibrate to a given market price, the following procedure is followed:

1. The asset volatility is estimated from the equity volatility, either user specified or historically estimated, as specified in Table [6].

2. We calculate the hazard rate structure $\boldsymbol{h}$ from Eq. (36), and use it to value the bond (or the stripped bond in case of a complex security) in the base market scenario using Eq. (27) with RR_p. We then derive the equivalent discount spread for that bond value, denoted by s_{CG}. We also calculate a discount spread corresponding to the given market price in the base scenario, without using the Credit Grade Model, and we denote it by s_0. We calculate the difference $s_\delta = s_{CG} - s_0$.

3. For each new scenario, we calculate the new discount spread s'_{CG} as in 2. For spread based pricing models the price in the new scenario is then calculated by setting $s' = s'_{CG} - s_\delta$.

The purpose of a fixed s_δ here is to model the non-credit spread component (assumed to be fixed) of a market discount spread, say a liquidity premium.

4 Stress Statistics with Credit Models

In this section we describe how the selection of credit models interact with the calculation of the following statistics:

- stressPVUserDefined
- deltaStressPVUserDefined
- generalizedPVBP

The first two statistics compute the Present Value (or the change in PV, i.e. deltaPV) under a stress scenario. They are available in RM3, but they are not directly available in RM4 as separate statistics; rather their functionality is obtained through the Present Value statistics run through a custom report where a user can specify a stress scenario, and the result can be run in Stress Value or Delta Value modes. The generalizedPVBP statistics reports the same type of information (PV or change in PV) under specified changes (typically expressed in basis points) of the curves underlying the position.

As described in Sec. [3], risk scenarios for positions using obligor default models are generated using a term structure of hazard rates which is recalibrated following the methodology described in the Methodology section (see, for instance, Eqs. (30) and (32)). This is what happens as well when stressed scenarios are computed for the three statistics above, according to their default configuration. However, the same statistics can be configured in a way to (partially) turn off the effect of the credit model in the following way:

- *stressPVUserDefined, deltaStressPVUserDefined*
  When a riskFactorTypeStressTest scenario is specified with the *fixedHazardRates* tag, risk-free rates are stressed, but the same hazard rate term structure calibrated in the base scenario is used in the stressed scenario, and hence expected future values of risky cash flows are not affected by the stress;
- *generalizedPVBP, stressPVUserDefined, deltaStressPVUserDefined*
  If the *doNotUseCreditGradeModels* is specified in the statistic definition (or if the 'Do Not Use Credit Model' box is checked in the RM3 or RM4 interface to the statistics) we have two cases: a) if the market price of the bond is specified, the position is stripped of the obligor default model and the bond spread model is used (with the specified discount curve) to calibrate the idiosyncratic spread associated to the position. The stressed scenario is then computed by changing the discount curve and using the calibrated idiosyncratic spread. b) if the market price of the bond is *not* specified, the model price of the bond is first computed using the selected credit model, and this model price is then used as the market price as described in case a).

5 Appendix

5.1 Simulation of interest rates and spreads

In this appendix we discuss how the simulated values of interest rate curves and spread curves used in several formulas quoted in this document are generated by RiskServer.

We begin by considering the equation defining scenario prices when the Bond Spread Model is used. For concreteness we consider the case of bond without optionality but nothing changes for bonds with optionality or other positions using this credit model, except for the pricing functions that are described in Sec[3.2]. The price of a bond without optionality is (see Eq. [7])

$$P' = \sum_i CF_i \times \exp\left[-(r'_i + s_\varepsilon)\, t_i\right]. \tag{40}$$

The simulated risky curve $\boldsymbol{r}'$ is obtained from the risky curve in the base scenario[16] $\boldsymbol{r}$ in way that depends

[16]Specification of the fully modeled position identifies the curve used for $\boldsymbol{r}$ (for instance US Govt, US Swap, Corporate BB rating, issuer xxx, etc). The *pricing date* specified in riskSettings via the pricingDateType input determines the date defining the values of the rate curve used in the analysis.

on the methodology used (Historical or Monte Carlo) and on the risk settings specified by the user. A full account of the how scenario rates are obtained in Monte Carlo simulation is described in [16], but here are the most important points:

- The return type associated to interest rates can be specified as allTypesToReturnType in the valuationSpec portion of the query (webservice clients), or *Time Series Return Definitions* in *riskSettings* (RM4 interface). Its default value is *differenceReturn*, which implies that, by default, the series of factor returns associated to each maturity node is constructed by using the difference of rates at two points in time. More precisely if $f_{T,n}$ denotes the factor return for maturity node T at a time point indexed by n, $f_{T,n} = -T(r_{T,n} - r_{T,n-1})$ where $r_{T,n}$ is the historical level of the rate a time n.[17]

- The set of time points used to define the time series of returns $f_{T,n}$ is defined by the user via the specification of the riskSetting inputs corresponding to

  - A lookback period, defining the time period used to define historical returns.
  - If the lookback period is defined as a trailing period, the *pricing date* specifies the most recent date in that period, i.e. the time indexed by N. If unspecified *pricing date* defaults to analysis date.
  - A return sampling period, with the possibility of overlap if the return period is longer than 1 day.

  This results in the definition of N factor returns, $f_{T,n}\, n = 1,...,N$, which are used in the generation of simulated scenarios.

- In Historical simulation N historical scenarios are generated for each maturity node: $r'_{T,n} = r_T - f_{T,n}/T = r_T + (r_{T,n} - r_{T,n-1})$; this defines $\boldsymbol{r}'$.[18] The required value of r'_i at the i^{th} cash flow date, needed in Eq. (40), is obtained via linear interpolation.

- In Monte Carlo simulation the N factor returns $f_{T,n}$ are used to generate N_{sim} simulated returns, $f^{\text{MC}}_{T,\alpha}\, \alpha = 1,...,N_{\text{sim}}$. Full details are given in[16]. Generation of scenario values for $\boldsymbol{r}'$ is then identical to historical simulation but using $f^{\text{MC}}_{T,\alpha}$ instead of the historical factor returns $f_{T,n}$.

- Interest rates in any simulated scenario are floored to zero to guarantee that rates have non-negative values.

[17] Interest rate levels that are used as risk factors are always continuously compounded. This means that the default choice of difference returns can be thought of equivalently as log-returns to discount factors; the two differ only by a factor of $-T$. On the other hand, the *display* rates that are input by the user and output in the application follow the convention of the particular curve under question. For example, US Government curve is semi-annually compounded. This display convention is also followed when changes in rates are specified e.g. in a stress test. In RiskManager 3 the display frequency is visible in the Market Data screen (add the PAYFREQ column under *Customize Columns*). In RiskManager 4 the display frequency is visible in the Market Data screen, under Market Data List in the Payment Frequency column (visible by default). In Web Services the display frequency is viewable through a *getMarketDataTypeInfo* query.

[18] With RS 5.4 Phase 1, it is possible to define historical simulation in a way that adjusts past historical scenarios to the current volatility environment. This can be done by using the risksettings input volatilityScaling→localVolatilityModel.

- If the specific interest rate curve is associated to factor returns defined as *logReturn* instead of *differenceReturn*, the definition of historical and simulated returns is correspondingly changed to $f_{T,n} = \log(r_{T,n}/r_{T,n-1})$, and $r'_{T,n} = r_T \exp(f_{T,n})$ for historical simulation, $r'_{T,\alpha} = r_T \exp(f^{\mathrm{MC}}_{T,\alpha})$ for Monte Carlo simulation.

Conceptually things are very similar for obligor default credit models. Consider for instance scenario pricing when the Hull-White Credit Model is used, where pricing depends on risk-free rate scenarios ($\boldsymbol{r}'_{rf}$) and a new value for the flat spread (s'). To be specific the formula for a bond with no optionality is given by (see Eq. (30)):

$$P' = \sum_i CF_i \times D_i'^{rf} \times \exp\left[-(s' + s_\varepsilon)\, t_i\right]. \tag{41}$$

Scenario values for $\boldsymbol{r}'_{rf}$ necessary to compute the risk-free discount factors in each scenarios, are obtained as described above, while scenario values for s' are obtained as described in detail in Sec. [3.3]. The calculation of s' for any given obligor default credit model relies on scenario values for $\boldsymbol{r}'$, for CDS spreads, for CDS upfronts, and for equity values, if the Hull-White Credit Model, the CDS Spread Model, the CDS Upfront Model, or if the Credit Grade Model is selected, respectively.

Above we have described scenario generation for interest rates. Scenario generation for CDS spreads, from which a hazard rate structure is derived which is used to compute s' used in Eq. (41), follows the same procedure described above. The default return type for CDS spreads is *logReturn*.

Scenario generation for CDS Upfronts is described in Sec[3.5].

Finally scenario generation for the Credit Grade Model model is obtained by defining scenarios for the equity price in the way described above for factors whose returnType is *logReturn*, and by deriving the new hazard rate structure and s' as described in Sec. [3.6].

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