RAPIDRATINGS®

CECL & Term Probability of Default Whitepaper

WHITEPAPER

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CECL & Term Probability of Default

This year many US GAAP filers, especially in the banking industry, are focused on the formidable new accounting challenges coming their way from the Federal Accounting Standards Board's (FASB's) revolutionary prescriptions on **Current Expected Credit Loss (CECL)**. Beginning after December, 2019, for SEC registrants (and later on for others) CECL will impose an unprecedented requirement on all respondents to maintain life-of-instrument estimates of credit losses (ECL) on financial assets, both performing and non-performing. Under the new rules, every quarter respondents will have to defend such estimates as being **"reasonable and supportable"** as far forward as they care to make them for specific assets and pools of assets, before reverting to plausible statistical histories for the remaining tenors of these assets. Although banks and others may be well-accustomed to conducting internally-directed predictive analyses of some sorts on their financial assets any such analysis falls far short of the formal, explicit, comprehensive and recurring representations to both auditors and regulators that CECL will compel.

Solving for all of CECL's prescriptions will be a profound exercise in enterprise risk management for a great many financial institutions. They will likely craft their responses from both internal and a variety of external resources. RapidRatings brings superlative credentials to one of CECL's most critical challenges – making tenable, consistent and continual financial health judgments about corporate borrowers at scale.

The CECL Revolution in Financial Assets

For many respondents with longer-term assets, especially banks, FASB's new ECL exercises will have to begin with "reasonable and supportable" calculations of Term PDs for internal workflows and for delivery, in turn, to auditors and others.. But, as the BIS recently cautioned[1], such PDs must be "point-in-time" (PIT) and not "through-the-cycle," so that many banks will need to adjust their longstanding Basel-based metrics accordingly.

Outside the US, the International Accounting Standard is about to impose its own credit-impairment standard on January 1, 2018 – IFRS 9. Here the ECL-calculation interval will extend only to 12 months, except for the deteriorated asset on which the respondent will have to make a CECL-style expected-loss calculation over the instrument's lifetime.

Global banks will become dual reporters, filing under both IFRS 9 and CECL. Many of them intend to adapt their IFRS 9 tools and processes to their subsequent CECL needs.

[1] In a recent BIS Quarterly Review, Dr. Benjamin Cohen, Head of Financial Markets, and co-author Gerald A. Edwards, Jr. address "[The new era of expected credit loss provisioning](#)," pointing out that banks will need to adjust their Basel-related calculations of ECL (p. 46).

RapidRatings Introduces the Term PD Model

RapidRatings uses a combination of advanced analytics and proprietary algorithms to provide accurate financial health assessments of public and private companies for clients, which include several G-SIBs, along with other financial institutions and a number of the largest non-financial companies in the US. In order to help our clients meet their pressing CECL and IFRS-9 obligations, RapidRatings has developed an algebraic Term Probability of Default (PD) Model with a forecast horizon of up to ten years. The new Term PD Model, a PIT system with a range of portfolio-support services surrounding it, is designed to serve as an authoritative primary model to meet both CECL's analytical needs and its reporting obligations to auditors and others. For those banks that have other systems in place, the RapidRatings Term PD Model can serve as a rigorously independent challenger model to better achieve these new goals. What is most important is that the Model and its underlying analytics, described below, are deliberately free of size bias, so that the Model will serve equally well in grading Small Business, Middle Market and Large Corporate C&I borrowers.

Shorter-Term Measurements of Corporate Strength

RapidRatings has built the Term PD Model on its two existing, shorter-term measurements of corporate strength:

> **The Core Health Score** (CHS) combines 62 financial ratios in an econometric model that differentiates exceptionally well between weak and strong firms in assessing efficiency and competitiveness over the next 2-3 years. It runs on a scale of 0/worst to 100/best.
>
> **The Financial Health Rating** (FHR®) is composed of the CHS and its interactions with 11 additional Resilience Indicators in a probabilistic statistical classification model that produces an estimate of the Probability of Default over 12 months. The Core Health Score explains 88% of the default-prediction accuracy level achieved by the FHR Model, while the 11 Resilience Indicators are the source of the remaining 12%. The FHR likewise runs on a scale of 0/worst to 100/best.

(Appendix A5 contains an overview of the approach: Methodological Overview of the Financial Health Rating, Measuring Default Risk with the FHR, Core Health Model Overview (including ratios employed) and FHR Transition Matrix and Gini Analysis)

RapidRatings refreshes these Core Health Scores and Financial Health Ratings on 11,000 public companies of all descriptions around the globe, as often as they report. We apply the same metrics and industry-specific weightings to more than 25,000 private companies that we, in turn, rate confidentially for individual subscribers. Since our rating exercises are entirely automated, they can be applied readily on any scale a subscriber may require. In order to assure complete objectivity in its work, RapidRatings takes no rating fees or management guidance from any company in coverage.

As the appendices illustrate, the analytics are hard, quantitative and non-discretionary. They involve fundamentals-based analysis of financial statements only. They are also without geographic bias, so that they can apply to C&I borrowers anywhere around the world.

Distinguishing Between Defaulters and Survivors

Establishing the Term PD Model's classification accuracy in sorting between defaulters and survivors involved the careful examination of the 3,458 firms in the Markit 3000 for which RapidRatings had issued an FHR in calendar 2006 (mostly based on financial information for the year ended 12/31/2005). The exercise can be illustrated by grouping companies according to four FHR cut-offs[2] -- 25, 40, 60 and 90. The percentage figures in Table 1 outline the performance through calendar 2007, 2008, etc. The row for FHR under 40, for example, displays the cumulative default experience of companies rated below 40 in 2006.

Table 1: Cumulative Default Observations 2007-2013

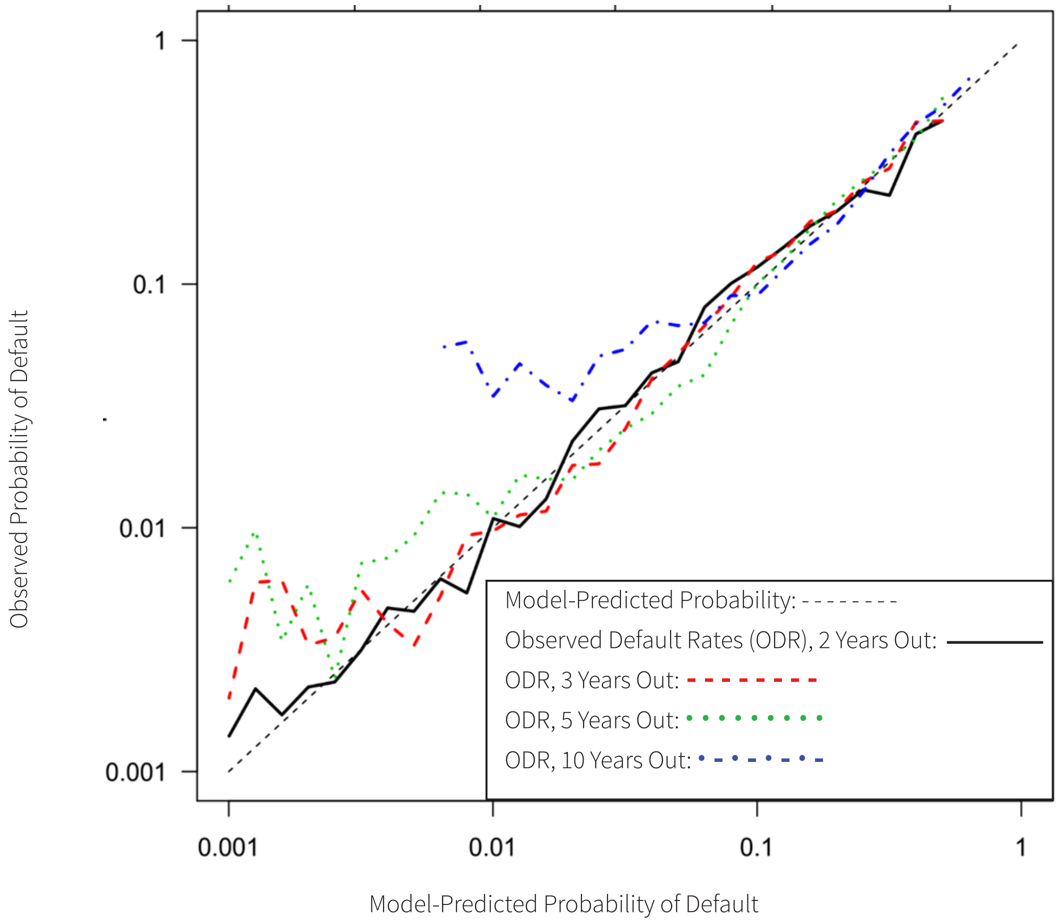
FHR Under	2007	2008	2009	2010	2011	2012	2013
25	14%	25%	32%	32%	34%	38%	50%
40	6%	10%	17%	21%	22%	26%	32%
60	3%	5%	9%	11%	12%	15%	19%
90	2%	3%	5%	6%	7%	9%	11%

In turn, we assessed the Model against multiple criteria of robustness and predictive value over the course of these same years. One of the main conclusions in the Model was that poor corporate financial health persists for a long period of time: groups with lower FHRs in 2006 were always more likely to default, even seven years later.

Figure 1 reveals the strength of the selected Term PD Model. It compares the Model-predicted (shown by the 45-degree dotted line) and the observed default rates over horizons of 2 years, 3 years, 5 years, and 10 years. The estimated probability is close to the observed probability over the whole range for which the observed probabilities are accurate enough for meaningful comparison (there are very few firms with less than a 1% probability of default over 5 years and virtually none over 10 years).

[2] This band of FHR values was chosen to give a good spread of high, medium and low values. Because of the small number of firms with scores below 25 or above 90, observational results are very noisy beyond the extremes presented.

Figure 1: Model-Predicted Probabilites vs. Observed Default Rates



Key Benefits

1. The probability of default increases smoothly as the time horizon lengthens, for every value of FHR.
2. The probability of default declines smoothly as FHR increases, at every time horizon.
3. The classification accuracy declines gradually at longer horizons, but remains high out to 10 years.
4. The expected probability of default matches the observed default rates for all FHRs out to 10 years (with the possible exception that the expected probability may be too low for the highest-rated firms for horizons over 5 year).

Benefits Beyond CECL

In addition to serving banks struggling to meet CECL's demands, RapidRatings also expects its present subscribers in third-party risk management (TPRM), corporate-credit extension and capital markets to put the new Term PD Model to immediate use. It will tell them if and when to begin their mitigation-planning on counterparties and if and when the financial marketplace properly reflects a focus company's longer-term prospects versus those of its competitors. Since our data-ingestion, ratings-generation and archiving workflows are entirely automated we can address TPRM and bond-portfolio surveillance needs on any scale and frequency necessary, as with bank loans.

About RapidRatings

RapidRatings is transforming the way the world's leading companies manage enterprise and financial risk. RapidRatings provides the most sophisticated analysis of the financial health of public and private companies in the world. The company's analytics system provides predictive insights into borrowers, third-party partners, suppliers, vendors, credit customers and securities issuers. Every business conversation becomes more productive, transparent and efficient with the RapidRatings Financial Health System. To learn more, visit us at www.RapidRatings.com.

Appendix to the White Paper on the Term Probability of Default

Appendix A1 to A4 contains detailed information on the algebraic modelling used to derive Term PDs and the performance of the model. Appendix A5 to A8 contains an overview of the Methodological Overview of the Financial Health Rating, Measuring Default Risk with the FHR, Core Health Model Overview (including ratios employed) and FHR Transition Matrix and Gini Analysis.

A1. Problem Statement

The Financial Health Rating (FHR) indicates the probability of default of a firm over the coming 12 months. It is built upon the Core Health Score (CHS) and a number of resilience variables (x1, x2, ...).

The probability of default within the next year is estimated as

$$p_D \cong \frac{1}{1+e^{-Z}} \qquad (1)$$

where Z is defined by

$$Z = a + a_0 CHS + b_0 CHS^2 + a_1 x_1 + b_1 x_1^{\,2} + a_2 x_2 + b_2 x_2^{\,2} + \ldots \qquad (2)$$

and Z is expressed for convenience as an FHR by

$$FHR = \frac{100}{1+8.52e^{0.369Z}} \qquad (3)$$

The central variable in this formulation is Z; equation (1) links Z to the probability of default over the next year, while equation (3) expresses Z in the form of a rating from 0 to 100. Z can take any positive, negative or zero value, while pD can take values between 0 and 1 only.

Previous work by Rapid Ratings has shown how to choose both the resilience variables and the coefficients in equation (2) in order that equation (1) gives the most accurate estimate of the probability of default over the next 12 months. Figure A1 shows that the FHR estimated for a 1-year horizon has predictive power for some years into the future; the idea of the Term PD is that it will estimate the actual probability of default, given the FHR, over these longer time horizons.

Extending the idea used to develop the FHR, it is convenient to express the default probability over a time horizon t, pD(t), in terms of a Z(t) defined by equivalents to equations (1)-(3):

$$p_D(t) \cong \frac{1}{1+e^{-Z(t)}} \qquad (1a)$$

$$Z\,(t) = a\,(t) + a_0(t)CHS) + b_0(t)CHS^2 + a_1(t)x_1 + b_1(t)x_1^{\,2} + \ldots \qquad (2a)$$

$$FHR(t) = \frac{100}{1+8.52e^{0.369Z(t)}} \qquad (3a)$$

FHR(t) will be referred to as the Term FHR, to distinguish it from the standard FHR. The Term Probability of Default model uses the term FHR to compute the probability of default using equation (1a). It is obvious from the definitions that $p_D(1) = p_D$, $Z(1) = Z$, and $FHR(1) = FHR$. It is also obvious that $p_D(0) = 0$ and so $Z(0) = -\infty$.

The CHS and the resilience variables in equation (2a) are not time-dependent; they are measured at the date that the financial data is released. Forecasting what they might be at some future date is beyond our ability. The coefficients may change with the time horizon.

For some purposes, such as computing the Currently Expected Credit Loss (CECL), what is required is the current estimate of the probability that default will occur in some future interval, such as in the year from t-1 to t:

$$\triangle p_D(t) = p_D(t) - p_D(t-1) \tag{4}$$

The problem is therefore to approximate Z(t) as a suitable function of FHR and t. It is apparent from Figure A1 that the FHR contains a substantial amount of information for this purpose.

A2. Algebraic Modelling: The Mathematics

The first step is to consider what approximations might be "suitable".

The hazard function is the probability per unit time that a firm will default at time t given that it has not defaulted previously (in contrast to $p_D(t)$, which is the probability that a firm defaults at time t or earlier). The hazard function is:

$$h(t) = \frac{1}{1-p_D(t)} \frac{\mathrm{d}p_D}{\mathrm{d}t} = p_D(t) \frac{\mathrm{d}Z(t)}{\mathrm{d}t} \tag{5}$$

by making explicit use of equation (1a).

Eventually, every surviving firm is expected to revert to the mean and its hazard function will become that of firms in general, regardless of whether the firm was strong or weak in its distant past. That is, for large enough t, h(t) will become constant and independent of CHS and the resilience variables. Likewise, pD(t) will become constant (it will become 1 if no firms drop out of the database); eventually, every firm will either fail or drop out of the database.

That is, equation (5) tells us that $\frac{\mathrm{d}Z(t)}{\mathrm{d}t}$ eventually becomes constant and independent of CHS and the resilience variables; equation (2a) tells us that

$$\frac{\mathrm{d}Z(t)}{\mathrm{d}t} = \frac{\mathrm{d}a}{\mathrm{d}t} + \frac{\mathrm{d}b_0}{\mathrm{d}t} CHS^2 + \frac{\mathrm{d}a_1}{\mathrm{d}t} x_1 + \frac{\mathrm{d}b_1}{\mathrm{d}t} x_1^{\,2} + \dots \tag{6}$$

and so the time derivatives of every ai(t) and bi(t) must become zero while the derivative of a(t) becomes constant. We can meet these requirements, and also the requirement that $Z(0) = -\infty$, by assuming that

$$\left.\begin{aligned} a(t) &= ag(t) + \beta \ln t + f(t) \\ a_i(t) &= a_i g(t) \\ b_i(t) &= b_i g(t) \end{aligned}\right\} \tag{7}$$

where f(t) and g(t) are unspecified but finite functions whose time derivatives become constant and zero respectively as t becomes infinitely large. Equations (7) assert that all coefficients a, a_i and b_i are multiplied by the same time-dependent factor g(t). Inserting these assumptions into equation (2a) and comparing with equation (2) immediately gives

$$Z(t) = \beta \ln t + f(t) + Zg(t) \tag{8}$$

The condition Z(1) = Z requires that g(1) = 1 and f(1) = 0. Equation (8) may be combined with equation (1a) to give

$$p_D(t) = \frac{1}{1+e^{-\beta \ln t - f(t) - Zg(t)}} = \frac{1}{1+t^{-\beta} e^{-f(t) - Zg(t)}} \tag{9}$$

For t close to zero, this is approximately $t^{\beta} e^{f(0) + Zg(0)}$, and since the probability of default for short periods must be proportional to t we can infer that β must be 1.

Thus, the probability of default over a time horizon t is

$$p_D(t) = \frac{1}{1+e^{-\ln t - f(t) - Zg(t)}} \tag{10}$$

For any given horizon t, equation (10) is a standard logit model where (ln t + f(t)) and g(t) are constants to be estimated. The functions can be traced out by estimating equation (10) for different values of t.

The estimated values of the functions f(t) and g(t) are independently determined for each t, and all of the estimates are subject to sampling variability. As a consequence, it can often happen that the estimated value of Z(t) and hence of $p_D(t)$ declines when going from one value of t to the next higher one; that is, $\Delta p_D(t)$ can be negative. There are, of course, no actual cases where (the unobservable) $\Delta p_D(t)$ is negative, so if this appears to happen it must be caused by sampling variability, not by the fundamental properties of the data.

The solution adopted for the Term Probability of Default is to approximate the functions by suitable mathematical forms. Note at once that this cannot be achieved for all cases. In differential terms, from equation (8),

$$\frac{dZ(t)}{dt} = \frac{\beta}{t} + \frac{df}{dt} + Z\frac{dg}{dt} \tag{11}$$

and since the possible values of Z run from negative to positive infinity, sufficiently extreme values of Z must be able to drive this derivative to negative values for at least some values of t. This applies for any possible way of defining the function g(t), unless g(t) is to be made constant with a derivative of zero. We may, however, be able to ensure that $\Delta p_D(t)$ will be non-negative for all values of Z encountered in practice.

An algebraic approximation of the functions that meets the requirements at t=1 and as t becomes large is

$$\left.\begin{aligned} f(t) &= a_f + b_f e^{-c_f t} + d_f t \\ g(t) &= a_g + b_g e^{-c_g t} \end{aligned}\right\} \tag{12}$$

Here a_f etc. are constants to be fitted to the functions f(t) and g(t) estimated from the default data over various horizons using logit analysis and equation (10). Note that the term $d_f t$ allows the derivative of f(t) to be non-zero for large t, while there is no corresponding term for g(t) because its derivative must be zero for large t.

Equation (8) has the consequence that g(1) = 1 and f(1) = 0 to ensure that the FHR is reproduced over the 1-year horizon. From equation (12) these imply respectively that $a_f + b_f e^{-cf} + d_f = 0$ and $a_g + b_g e^{-cg} = 1$. There is also the condition $\beta = 1$.

The estimated values (with standard errors in parentheses) are $a_f + b_f e^{-cf} + d_f =$ 0.003 (0.015), $a_g + b_g e^{-cg} =$ 0.982 (0.005) and $\beta =$ 1.37 (0.07). These are close to the required values (although the difference for the last two is statistically significant). To improve the efficiency of estimation, the constraints were then imposed for the final parameter estimation. The resulting parameters are

Parameter	Estimate	Parameter	Estimate
a_f	-2.7092	a_g	0.3901
b_f	3.4422	b_g	0.8520
c_f	0.2841	c_g	0.3342
d_f	0.1182		

The quality of the resulting function may be seen in Figure A2. The probability of default will be a strictly decreasing function of FHR provided that g(t) is positive. Equation (12) with these parameters ensures that this condition is met.

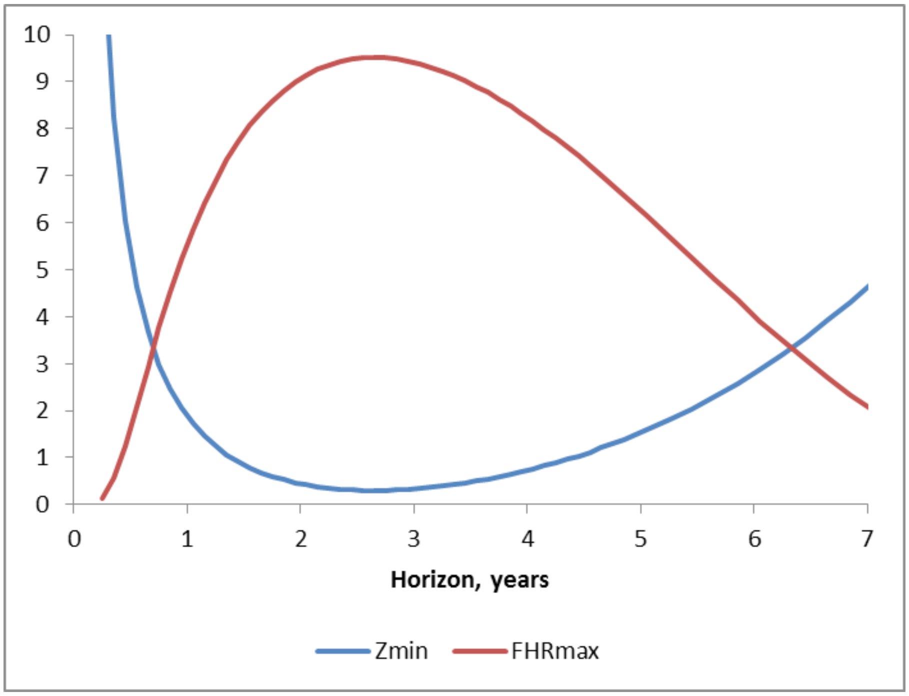
Equations (11) and (12) (with β=1) lead to

$$\frac{\mathrm{d}Z(t)}{\mathrm{d}t} = \frac{1}{t} - b_f c_f e^{-c_f t} + d_f - b_g c_g e^{-c_g t} Z \tag{13}$$

and this is negative, making $\Delta p_D(t)$ negative, if

$$Z > \left(\frac{1}{t} + d_f\right)\frac{e^{c_g t}}{b_g c_g} - \frac{b_f c_f}{b_g c_g} e^{(c_g - c_f)t} \tag{14}$$

So Z(t) (and thus p_D(t)) are decreasing functions of t only for large enough values of Z, that is, for small enough values of FHR. The following plot shows how the limiting values of Z given by inequality (14) and the corresponding values of FHR depend on the horizon t, using the actual values of the parameters:



It is evident that the values of Z for which $p_D(t)$ decreases over some range of t correspond to values of FHR below about 9.5; there is a problem, but it applies only to the worst few firms (less than 0.001% of the whole data set). The ad hoc adjustment, to be discussed next, eliminates the problem down to FHR<1, meaning that it would never be observed in practice.

To fix the problem when it occurs, the simplest approach is to adjust equation (13) directly in the problematic region of t. The right-hand side of equation (13) approaches d_f as $t \to \infty$ and approaches positive infinity as $t \to 0$; since d_f is positive, any region in which $\frac{dZ(t)}{dt}$ is negative must be bounded between t_1 and t_2, say. Near its minimum, it can be approximated by a parabola, and so the negative derivative can be eliminated by adding an offsetting parabola

$$\frac{d\triangle Z(t)}{dt} = \alpha\,(t - t_1)(t_2 - t) \qquad \text{for } t_1 < t < t_2 \tag{15}$$

for a suitable α. The values of α, t_1 and t_2 depend on Z and can be approximated[3] by a quadratic function of Z as shown in this table:

Variable	α	t_1	t_2
Constant	0.0308	1.9065	3.6761
Coefficient of Z	0.0614	-0.5955	2.2064
Coefficient of Z^2	0.0068	0.0548	-.01585

[3] These coefficients are conservative, guaranteed to prevent pD(t) decreasing at the expense of making the adjustment slightly larger than it needs to be. Because the adjustment hardly ever applies in any case, this is not a significant issue.

Z(t) is then increased by the integral of this adjustment:

$$\triangle Z(t) = \begin{cases} 0 & t < t_1 \\ \frac{\alpha}{3}(t - t_1)^2 \left(\frac{3t_2 - t_1}{2} - t\right) & \text{for} \quad t_1 \leq t \leq t_2 \\ \frac{\alpha}{6}(t_2 - t_1)^3 & t > t_2 \end{cases} \tag{16}$$

and the final model (with $\beta = 1$) is

$$Z(t) = \ln t + f(t) + Z\,g(t) + \triangle Z(t) \tag{17}$$

A3. Performance of the Model

This section examines the performance of the Term Probability of Default model using three standard tests that measure different aspects of performance: classification accuracy (that is, how successful the model is at ordering firms from best to worst); the accuracy of estimation of the probability of default; and the accuracy of the estimation of expected loss.

A note about holdout samples: In the absence of a strong theory that makes unambiguous predictions, the only protection against self-deception by over-fitting data is to replicate a study, that is, to repeat it later using different data[4]. When we cannot afford to replicate the study because we need to apply the results immediately, a common substitute is to perform an "in-sample replication" by dividing the sample in two, developing a model using half the sample and testing it with the other half. The only purpose of such a "holdout" approach is to assess whether the results seem likely to disappear when subject to replication. The model must not be chosen using the results of the holdout sample, because then it is not being held out at all. The best estimates of the model parameters, and the best comparison of alternative models, come from using the entire available data set rather than from the estimation/holdout subsamples. Some performance estimates are favorably biased when calculated using the sample on which the model parameters are calculated, and the holdout design may reduce the bias; however, resampling approaches such as the bootstrap are better designed to estimate both bias and variability. In this note, the models are evaluated based on the results of the full sample, but performance statistics from a holdout sample are also supplied in order to give a sense of the credibility of the results. The full sample is split in two, with one half being used for estimating the coefficients and the other half being used to compute the performance statistics. Whether the model is the best can only be assessed with new data several years from now, that is, with a proper replication.

[4] Consider an imaginary study in which a coin was tossed randomly each day of a month. Given a 5% significance level, it is very likely that on one day (say the 11th of the month) the coin will show significantly more heads than normal. One might develop a betting strategy from this evidence, since the result is statistically significant, but of course such a strategy would fail. If the study were to be replicated in another month, it would be found that there was nothing special about tossing a coin on the 11th of the month.

a. Classification Accuracy

The ability of an indicator to separate firms into groups that will or will not default over a given time horizon is conventionally summarized[5] by AUC, the area under the ROC curve. This is a number between 0 and 1; a value of 1 means that the indicator perfectly separates defaulters and non-defaulters, with all defaulters having a lower Term FHR (higher probability of default) than all non-defaulters. A value of 0.5 is equivalent to random guessing, where the Term FHR has no ability to indicate likely defaulters. (Values of less than 0.5 correspond to worse-than-random-guess performance, with defaulters tending to have higher FHR than non-defaulters.)

The following table shows the AUC for various time horizons. The classification accuracy declines as the horizon gets longer. Even after 10 years, however, the Term FHR gives a respectably good ordering of firms from defaulters to non-defaulters. The accuracy is a little less for the holdout sample, as it should be, but the difference is small, suggesting that the result would be robust under replication.

	1	2	3	4	5	6	7	8	9	10	Years
Full sample	0.921	0.885	0.853	0.829	0.811	0.799	0.786	0.774	0.765	0.756	
Holdout sample	0.912	0.874	0.842	0.819	0.803	0.790	0.780	0.770	0.762	0.754	

b. Estimated Default Probabilites

For short horizons, clients are interested in the ability to classify firms as potential defaulters or non-defaulters, because this gives them the opportunity to take managerial actions in the short term. Over longer horizons, however, the interest is rather in making estimates of the probability of default because it is this probability that gets fed into valuation models. Thus, the AUC is of less interest than the accuracy with which $p_D(t)$ can be estimated.

Figure A4 and Figure A5 present summaries in graphical form. For each figure, cases are placed in non-overlapping groups according to their estimated probability of default (within a range of + 10%), and the actual probability of default of each group is observed. Figure A4 is based on the entire data set, while Figure A5 is based on the holdout sample. For a model which exactly reported the probability of default, the result would lie on the diagonal line for which the observed probability is equal to the estimated probability.

Where there are few firms with a given expected probability of default (small size of the sub-sample), the observed probability of default has large sampling fluctuations. This is evident from the variability of the results at the lower-left and upper-right ends of the plots. There are very few or no firms predicted to have probabilities of default of 1% or less over 5 or 10 years, and so the results are particularly subject to fluctuation there, or no results at all can be reported.

[5] The AUC summarises all of the Type 1 and Type 2 error rate information in a single number. It contains the same information as the Gini coefficient (although the numerical values are different), and either may be used.

For the 2-year and 3-year horizon in Figure A4, the results are close to the perfect-estimation diagonal line. For the 5-year and 10-year horizon, the estimated probability of default seems consistently too low for the safest firms (bottom left corner).

Figure A5 broadly confirms this result using a holdout sample, suggesting again that the results would be robust under replication.

c. Estimated Default Probabilites

The final test simulates the results of a common application of these estimated probabilities: forecasting the default loss from a portfolio of investments, possibly net of profits from non-defaulted investments. This is a direct test of the economic accuracy of the model, whether applied to portfolio investment decisions, to policies governing which suppliers or customers to work with, or to estimation of expected credit losses for financial reporting (including reserve adequacy calculations for banks). Each of these applications requires an estimate of the expected credit losses from a portfolio, and a good estimator of default probabilities will produce an estimate which is close to reality in dollar terms.

In this paragraph only, the time horizon will be measured in quarters rather than years, to avoid awkward notation in the summations. Consider a portfolio of firms, indexed by i, where there is an annual profit of X_i (quarterly profit of $X_i/4$) as long as the firm does not default and a one-time loss of L_i if the firm defaults. The cumulative probability of default of firm i up to the end of quarter t is $p_{Di}(t)$. The probability that firm i will fail after t quarters (that is, during quarter t+1) is $p_{Di}(t+1)$ $p_{Di}(t)$, and so we can write down the expected survival time for this firm in quarters. Note these outcomes are not exhaustive, since nothing is said about probabilities beyond 40 quarters; to correct for this, we must divide the usual formula by $p_{Di}(40)$. For clarity, $p_{Di}(t)$ is written as pt in equation (18) below. Assuming that each default occurs in the middle of the corresponding quarter, the expected survival time is

$$
\begin{aligned}
S_i &= \{0.5p_1 + 1.5(p_2 - p_1) + 2.5(p_3 - p_2) + \ldots + 39.5(p_{40} - p_{39})\}/p_{40} \\
&= \{-p_1 - p_2 - p_3 - \ldots - p_{39} + 39.5p_{40}\}/p_{40} \\
&= 39.5 - \sum_{t=1}^{39} \frac{p_t}{p_{40}}
\end{aligned}
\tag{18}
$$

The expected profit or loss for the whole portfolio over 10 years is then

$$
\sum_i \{X_i \frac{S_i}{4} - L_i p_{Di}(10 \text{ years})\} \tag{19}
$$

This is not a present value; the default loss is assumed to have the same value whenever it occurs, and profits until the default are given equal value. Computing a present value would add to the complexity and introduce another parameter for little benefit.

The test portfolio comprised every Markit 3000 firm for which data was available, weighted by total assets. The loss parameter was the ratio X_i/L_i, the ratio between the annual profit and the loss caused by default. If this ratio is zero, then profits are disregarded, and the result is a pure expected loss measure suitable for calculating CECL. The largest ratio considered was 1, which might be appropriate for evaluating a customer credit policy where losses are not large relative to profit opportunities.

The results of this analysis are shown in Table A5. The Term Probability of Default performs well for the CECL case where the ratio X_i/L_i is zero. At the other extreme, where X_i/L_i is 0.1 or larger, the model underestimates the profit by some 30%. (Of course, it might be thought miraculously good to be able to estimate profit to within 30% over a 10-year horizon!) Again, the holdout sample results suggest that the results would be robust under replication.

A4. Summary

The Term Probability of Default has a number of desirable properties:

1. The Term FHR exactly matches the FHR at the 1-year horizon.
2. Its hazard rate behaves appropriately over long horizons.
3. The probability of default increases smoothly as the time horizon lengthens, for every value of FHR.
4. The probability of default declines smoothly as FHR increases, at every time horizon.
5. The classification accuracy declines gradually at longer horizons, but remains good out to 10 years.
6. The expected probability of default matches the observed default rates for all FHR out to 10 years, with the possible exception that the expected probability may be too low for the safest firms (with the highest FHR) for horizons over 5 years.
7. The estimate of economic profit or loss over a 10-year horizon is accurate to within about 5% when considering expected credit losses, but less accurate when considering profits from dealing with non-failed clients as well as losses from defaults.
8. The results of holdout tests are consistent with these findings, suggesting that they are not sample-specific and that the Term Probability of Default should be useful in other populations than the Markit 3000 set of firms used to develop it.

Table A1.

Economic accuracy of the Term PD, over a 10-year horizon. Based on all firms for which 10 years of data was available. The column headings represent the annual profit from dealing with a firm divided by the loss if the firm defaults; if this is zero, the column presents a pure expected loss concept suitable for calculating CECL. The first row shows the actual 10-year profit for the Markit 3000 portfolio as a fraction of the loss from default; the second row shows the same value estimated by the model. Top panel uses all data; bottom panel uses a holdout sample.

All data	X/L=0	X/L=0.01	X/L=0.02	X/L=0.05	X/L=0.1	X/L=0.2	X/L=0.5	X/L=1
Actual	-0.19	-0.10	-0.01	0.25	0.69	1.57	4.22	8.63
Term PD	-0.18	-0.12	-0.05	0.14	0.45	1.09	2.99	6.16

Holdout sample	X/L=0	X/L=0.01	X/L=0.02	X/L=0.05	X/L=0.1	X/L=0.2	X/L=0.5	X/L=1
Actual	-0.20	-0.11	-0.02	0.25	0.69	1.57	4.21	8.61
Term PD	-0.18	-0.11	-0.05	0.15	0.48	1.13	3.09	6.35

Figure A1.

Proportion of firms with FHR below 25, 40, 60 or 90 in 2006 (before the Global Financial Crisis) that actually defaulted during each subsequent calendar year. Even during and after the GFC, the FHR measured in 2006 had predictive power for identifying the probability of default of different firms.

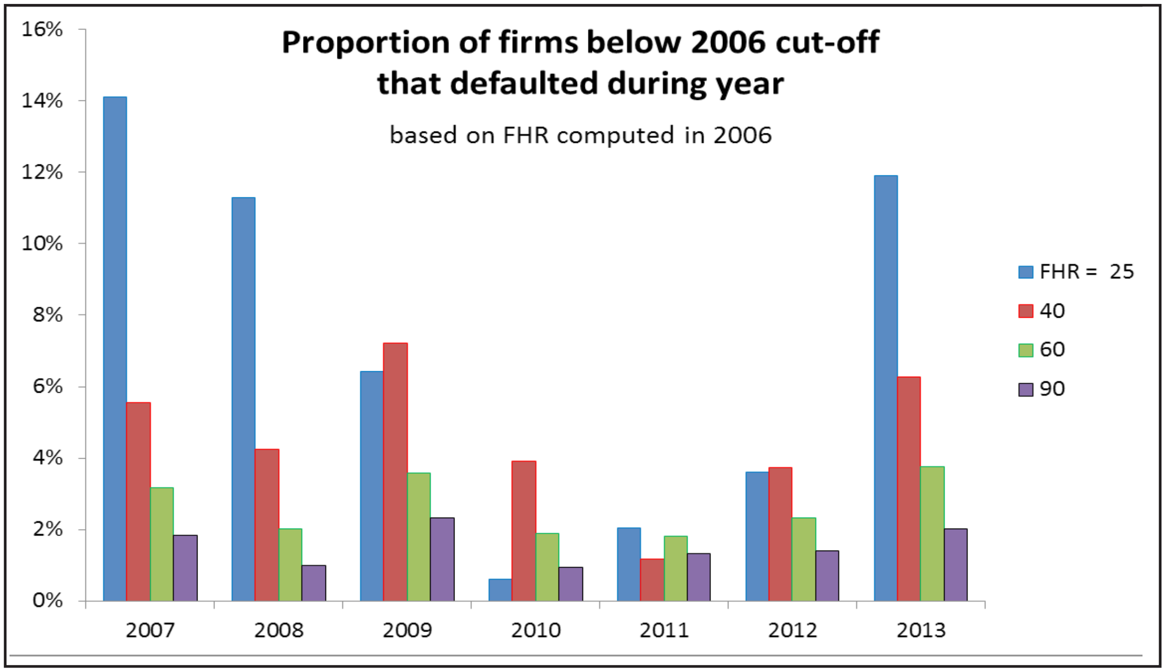


Figure A2.

Fits to the functions ln(t) + f(t) (black) and g(t) (red). Points are estimated by logit, using equation (10), for each quarterly horizon from 0.25 years to 10 years, with error bars corresponding to one standard error. The lines are the finally fitted functions from equation (12).

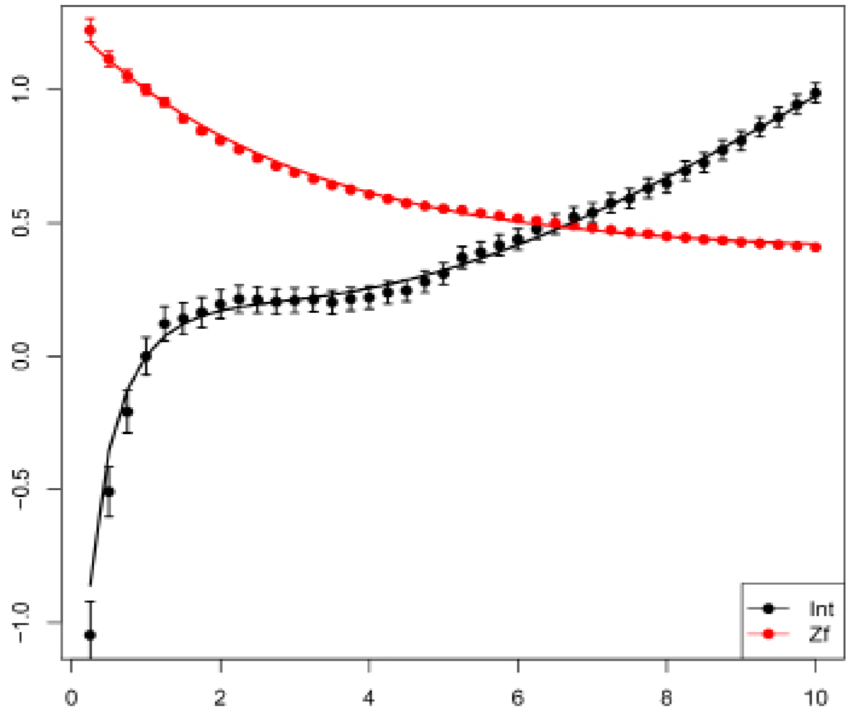


Figure A3.

Illustration of the adjustment process to prevent p_D from decreasing, for the case FHR = 9. The dashed black line shows the unadjusted values of Z(t) given by equations (8) and (12); the scale is on the left vertical axis. The red dashed line is the derivative of Z(t); the scale is on the right vertical axis. Between about 2 and 4 years, the derivative is negative and Z(t) is decreasing (and therefore $p_D(t)$ is decreasing also). The red solid line is the adjustment to the derivative, which makes it positive everywhere. The solid black line is the adjusted Z(t), which is always increasing. After 4 years, the adjustment increases Z(t) by a constant amount of about 0.13, which for example increases the estimated p_D over a 4-year horizon from 62.8% to 65.8%.

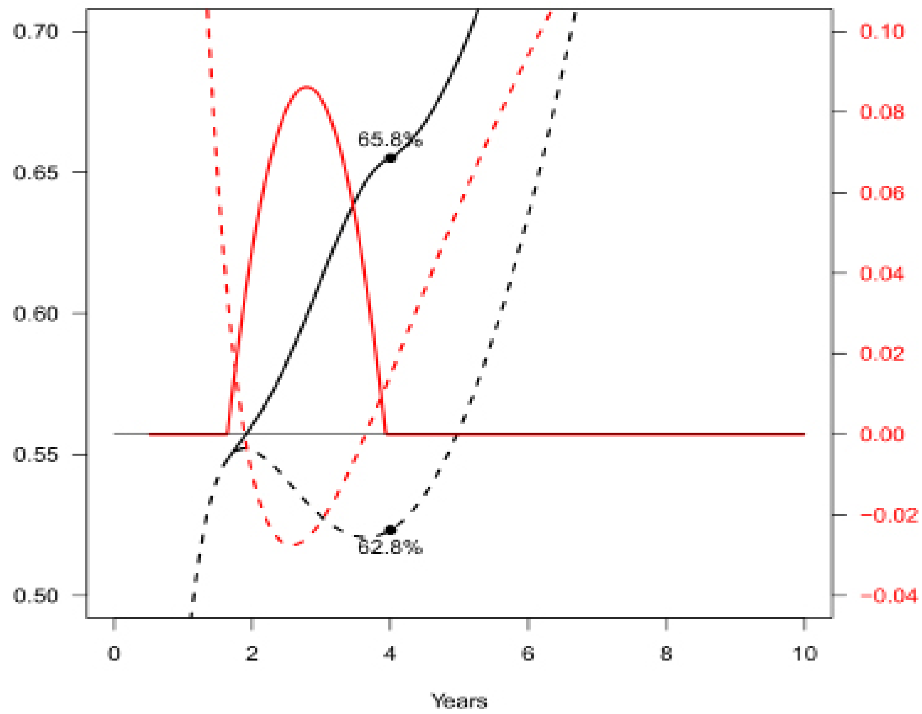
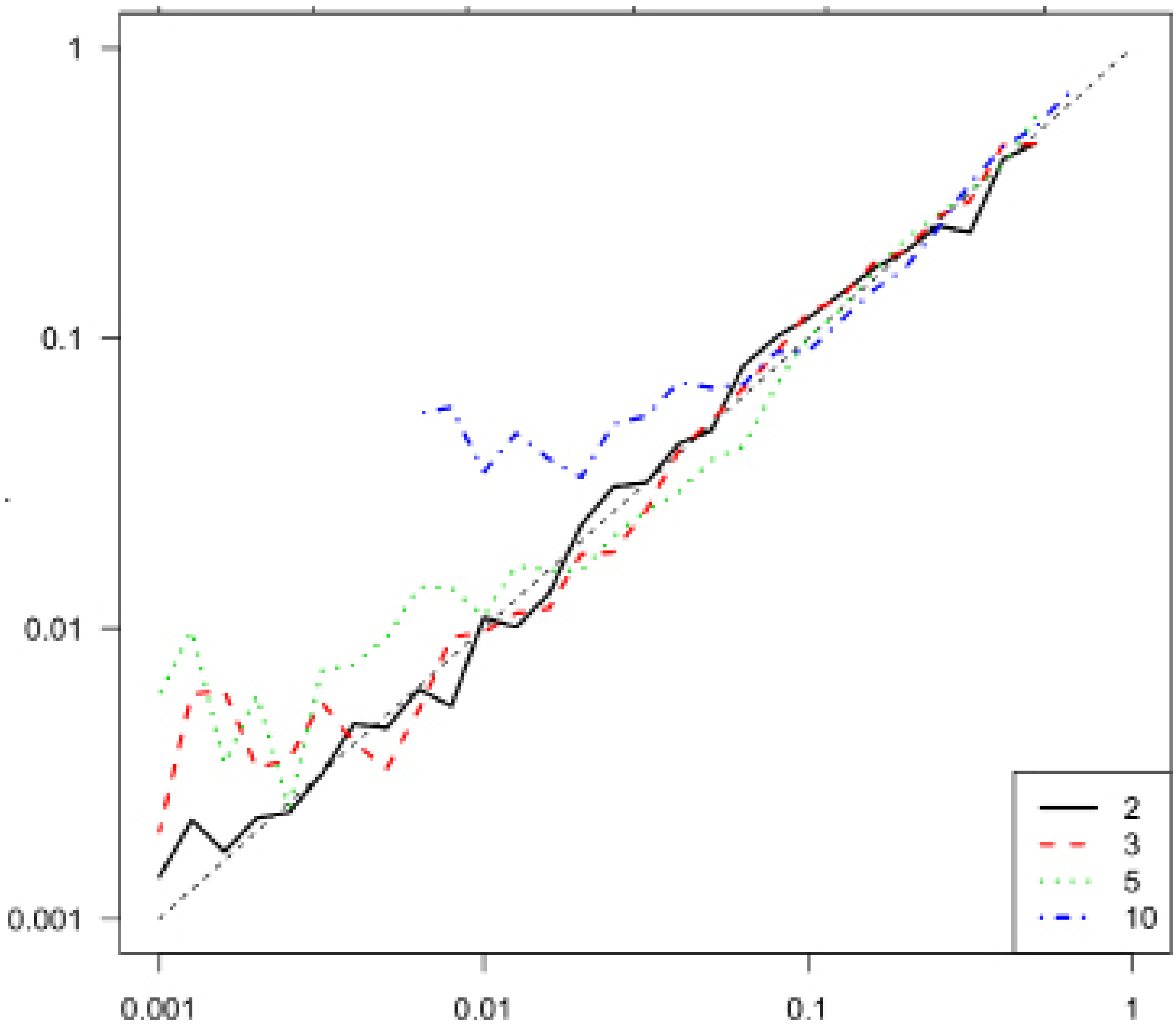


Figure A4.

Observed default rates based on the full sample. The chart shows results for horizons of 2, 3, 5 and 10 years. The diagonal dashed line shows where the predicted and observed rates are the same. The vertical axis shows the observed probability of default.



A5. Methodological Overview of the Financial Health Rating

Introduction

The Financial Health Rating (FHR®) is Rapid Ratings' proprietary and innovative measure of risk that estimates the financial strength and weakness of firms. This is achieved by combining the Core Health Score models that focus on financial health over the medium term (next 2-3 years) with 11 additional ratios that are specifically focused on short-term (within the next year) default estimation. The FHR system is sensitive to balance sheet performance by providing additional emphasis on total debt and liquidity, and has sensitivity to managing obligations through earnings performance, which together help sharpen the default prediction accuracy and improve the discrimination between high risk survivors and high risk defaulters in the next 12 months.

All internal and external back testing has confirmed that there is a very strong inverse correlation between the Core Health level and the probability of default; i.e., a highly rated firm has a consistently low probability of default, while a poorly rated firm has a consistently high probability of default. The FHR is well aligned with decision-making processes aimed at managing default risk, particularly with a 12-month horizon. Risk managers with a wider scope or longer horizon are well served by considering the intersection between Core Health and default risk outlined in our Quadrant Analysis.

Model Overview

The Core Health Model is a multivariate econometric pseudo panel data model which measures the financial health of a firm along a continuum from 0 to 100. The FHR Model is a probabilistic statistical classification model that estimates a firm's probability of default along a continuum from 0 to 100. The FHR model is designed to fine-tune the Core Health model and sharpen our estimates of the probability of default by combining the Core Health model with 11 Resilience Indicators. On its own the Core Health score (CHS) explains 88% of the default prediction accuracy level achieved by the FHR Model. The 11 resilience indicators are the source of most of the remaining 12%. Of those 11 ratios, additional predictive value was extracted from 5 ratios already in the FHR model. The remaining 6 significant ratios are new to the FHR system. Thus, the FHR model employs a total of 68 financial ratios (62 original ratios plus 6 new ratios). All 11 resilience ratios are dynamically interactive and a change in one changes the others. Further refinement in the FHR model is, to a small extent, based on an industry standardization adjustment which corrects for differences in ratios across industries so that, for example, a firm with an FHR of 60 will have the same probability of default across all industries (this new adjustment is completely different from the industry specific risk adjustment in the original models).

The Core Health model, our traditional rating approach, measures a firm's efficiency and competitiveness with a horizon of the next 2 to 3 years. As presented in table A2, the Core Health model:

1. Employs 62 financial ratios that are divided into (a) Operating Profitability (11 ratios); (b) Net Profitability (12 ratios); (c) Cost Structure Efficiency (18 ratios); (d) Capital Structure Efficiency (11 ratios); and Other Ratios (10 ratios, largely measuring liquidity)
2. Benchmarks each of the 62 ratios on an industry-specific basis for each firm against a global distribution of peers in 24 industries covering up to 40 years of performance data
3. Generates a score for each ratio, which is then multiplied by the specific ratio weight based on the ratio's predictive significance (determined by econometric modeling)
4. Produces a single weighted risk score, which we now call our Core Health Score (CHS)

The Resilience Indicators measure a company's leverage, liquidity and earnings performance. They interact dynamically with Core Health to indicate lower or higher short-term risk. Strong Core Health makes a company less sensitive to the impact of its Resilience Indicators, which will have less impact on the final rating. However, as a firm's Core Health deteriorates, its Resilience Indicators become increasingly important, and will have a more significant impact. This dynamic interaction between Core Health and Resilience is a key pillar of the FHR's predictive ability.

Leverage is a solvency metric that depicts the extent to which a firm's assets are dependent on debt as compared to equity (Total Debt to Total Assets).

Liquidity measures the ability of the firm to survive any short-term crises that drain its asset reserves (Working Capital to Total Assets; Cash to Current Liabilities; and CFO to Current Liabilities).

Earnings Performance assesses the firm's efficiency in managing internal constraints (using COGS to Sales) and internal opportunities to generate upstream (Operating Profit to Total Assets), and down-stream (NPAT to Total Assets Retained Earnings to Total Assets) profitability to permit the firm to meet internal obligations (salaries, wages, etc.) and external obligations (CFO to Current Debt Service; Operating Profit to Interest Expense; and Operating Profit to Current Debt Service).

Probability of Default

The output of the model, from which the probability of default is also derived, is monotonically transformed into the FHR. This transformation ensures that FHR is a score between 0 and 100, and that lower values correspond to a higher probability of default. Based on testing of over 1,300 defaults between 1991 and 2013, a particular transformation was chosen so that the probability of default is about 1% for an FHR of 40 and 0.1% for an FHR of 60. The threshold at 40 between Medium Risk and High Risk is an important workflow lever for many users, and the group of companies whose probability of default is greater than 1% encompasses over 90% of defaults rated since 1991.

Once a company passes the High Risk threshold with a rating below 40, it demonstrates not only a material level of default risk, but that default risk increases geometrically with each subsequent reduction in FHR, approximately doubling for each 5-point FHR deterioration. The relationship between probability of default and FHR is presented with a logarithmic scale in figure A5 and with a linear scale in figure A6.

Figure A5: Probability of Default and the FHR (Logarithmic Scale)

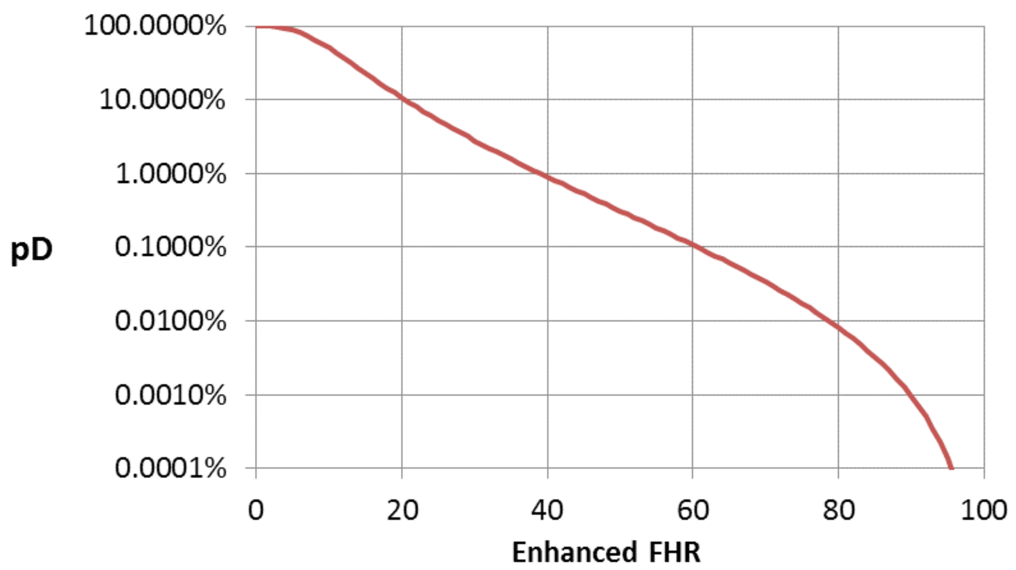
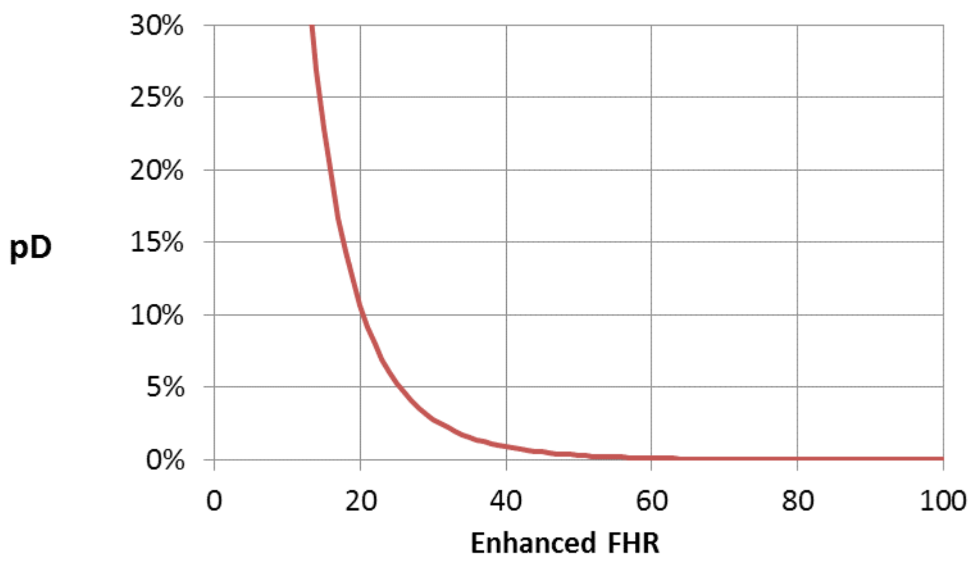


Figure A6: Probability of Default and the FHR (Linear Scale)



A6. Measuring Default Risk with the FHR

Introduction

This section describes the Financial Health Rating (FHR) model, a probabilistic statistical classification model for default prediction offered by Rapid Ratings. The Financial Health Rating combined with the analytical power of the Core Health Score provides a medium term (next 2-3 years) assessment of corporate efficiency and competitiveness with a set of 11 short term resilience indicators that together provide a sharp focus on estimating the probability of default within the next 12 months.

In line with Rapid Ratings' philosophy, the FHR uses only information from a firm's financial statements, so that it can be computed rapidly and objectively and can be applied to both listed and unlisted firms.

Description of the Data

Default is defined as a bankruptcy occurring at any time within the next 12 months following an annual or quarterly rating. This is the most useful definition in practice, since it means that whenever a new rating is issued the FHR indicates the risk of continuing to deal with the rated firm over the coming year. Non-default was defined as the situation where a firm has not defaulted within that time (even if it defaulted later). When it could not be determined whether a firm had defaulted within 12 months, that case was excluded from the analysis, except that firms that had not defaulted by the end of 2013 were assumed to be non-defaulters (this slightly understates the default rate, but since almost all such firms are in fact non-defaulters it improves the efficiency of the estimation).

The data set to which the model was fitted comprised all 4,613 firms that were or had been in the Markit 3000 index from 2000 to 2013. These firms had 163,171 ratings and 380 observed defaults. This choice of time period avoids contaminating the data with possibly irrelevant patterns from earlier times.

Methodology

The modelling approach was a standard logit model to estimate the probability of default as a function of the predictor variables (i.e. the Core Health Score (CHS) and a set of resilience variables), expressed as:

$$P_D = \frac{1}{1 + \exp(-Z)}$$

where

$$Z = a + a_0 * CHS + b_0 * CHS^2 + a_1 x_1 + b_1 x_1^{\,2} + a_2 x_2 + b_2 x_2^{\,2} + \ldots$$

and the x_i are "resilience" variables which affect the extent to which a firm with a particular CHS is more or less at risk of default compared to its peers. These resilience variables are equivalent to accounting ratios in the sense described below.

The logit model has well-known properties, including consistency with Bayes' Theorem in the way that the effects of additional variables are incorporated. (Probit modelling gave very similar results.)

Compared to standard logit models for default prediction, the FHR applies several methodological improvements:

1. To develop the resilience variables, an inverse tangent transformation was first applied to the equivalent ratio. That is, instead of the variable being y/z it was taken to be y.z = tan-1(y/z), ensuring that the result was in the correct quadrant. (The result was further scaled to the range [0, 1], but this was only for convenience of presentation and has no fundamental effects on the modelling.) This has several benefits:
    a. The result is always finite, which ensures that Z for a firm can never be dominated by an extreme value of a single variable; that is, the model is protected against outliers.
    b. It ensures that small changes in a denominator variable cannot cause large changes in the result (which may happen for ratios whose denominator can be negative).
    c. It preserves the ordering of firms by each variable.
    d. It avoids questions about which way up the ratio should go, since y.z is linearly related to z.y and so the two are perfectly equivalent. In contrast, y/z and z/y and not linearly related and might both be needed in a model; and even if only one is needed, then the model performance will be impaired if the wrong one is used.
    e. The effects of errors in data are minimized. Even such extreme cases as a negative reported value of Sales, which would cause serious problems with a ratio such as Cost of Sales/Sales, means only that the value of the resilience variable would be slightly outside the range [0,1]. Rather than deleting these cases, the values of x_i were simply Winsorized at 0 and 1. This does not matter for estimating the model, but is crucially important when applying the model to estimate the probability of default of an individual firm with suspect data.
2. The scarcity of default data across different industries globally was an important factor in the choice of an industry adjustment factor to produce one coefficient across all industries for each financial ratio among the resilience indicators. This is complementary to the industry specific estimation of risk in the Core Health Model.[6] The values from step 1 were standardized by industry. Many ratios differ greatly by industry, for good economic reasons which may not cause differing levels of default risk. The industry standardization compensates for this by monotonically rescaling the resilience variables within each industry to make the means and variances the same for every industry (or as near as possible so as to ensure compatibility with monotonicity and with retaining the range [0,1] for the values). Note that the Core Health Score is itself industry-standardized, with each component ratio being compared to the distribution for the firm's industry and with the weights given to the 62 ratios reflecting the combined difference in risk by industry.

[6] This approach differs from approach applied in the Core Health models in which the difference in individual ratio performance across industries directly reflects industry-specific risk.

3. Each xi appears quadratically rather than linearly in Z. This generalization was motivated by a preliminary Bayesian analysis for one resilience variable at a time, in which the posterior probability of default was derived from a logit model using the Core Health Score (known to be a good predictor of default) and indicator variables for the vintiles of that resilience variable. The coefficients of the indicator variables show the Bayesian update due to values in that vintile. Because this approach uses indicator variables, the plots of these coefficients show the natural functional form imposed by the data, not some prior expectation of the analyst. These plots typically show strong non-linear behavior for which a parabola is a reasonable (though not perfect) fit.

Missing values of any xi were substituted using a predicted value from a regression of the ratio on the Core Health Score, since values of many xi vary strongly with the CHS but not with industry, because of the industry standardization. (Using the mean conditional on the CHS would have introduced more noise, particularly for extremely large and small values of the CHS where there are few cases so that the mean is unstable. The regression-based predictor is more stable.) The ability to impute sensible values for missing data is again important when rating individual firms, and is a further benefit of building the model on top of the Core Health Score.

The initial set of 26 ratios included many that had been found important in prior research. In particular, they included all of the ratios that had been found important in classic studies by Altman (1993), Ohlson (1980) and Zmijewski (1984). Discontinuous/indicator variables used by Ohlson were not included, as they imply that an infinitesimal change in some variable (such as profit in a previous year) can trigger a step change in the probability of default. (The use of quadratic functions of the resilience variables achieves much of the purpose for which these discontinuous variables were introduced into Ohlson's model.)

The initial model involving the Core Health Score and the 26 resilience variables was then refined by removing variables that appear to duplicate others, leaving the CHS and a final set of 11 important variables. This refinement occurred in two steps:

1. In the initial model, several variables entered with what was unambiguously the wrong sign. For example, the univariate results show that large values of Current Liabilities/Current Assets are associated with default, and this is of course to be expected. However, in a multivariate model the corresponding resilience variable is highly significant, but the coefficients imply that firms with small values of this ratio are more likely to default than firms with very large values.

   Sign errors probably occur because the variables are not all independent of one another, so that various variables partly offset one another. For example, short-term debt is included in Current Liabilities and is a component of Current Debt Service. An extra dollar of short-term debt causes both Current Liabilities and Current Debt Service to increase by a dollar, and it is not clear to which of these the corresponding change in default risk should be attributed. If the contribution through Current Debt Service is over-estimated, then the estimated contribution from Current Liabilities will tend to be in the wrong direction to compensate. The related resilience variables need not be and in fact are not collinear: this is a problem of identification, not of multicollinearity.

The occurrence of a wrong sign was therefore taken as evidence that the variable does not belong in the model because it duplicates evidence from others in the set; such variables were dropped. Consistent with this explanation, their removal appeared to have a negligible effect on the model's predictive ability, and the remaining variables did not acquire significant incorrect signs in the refined model.

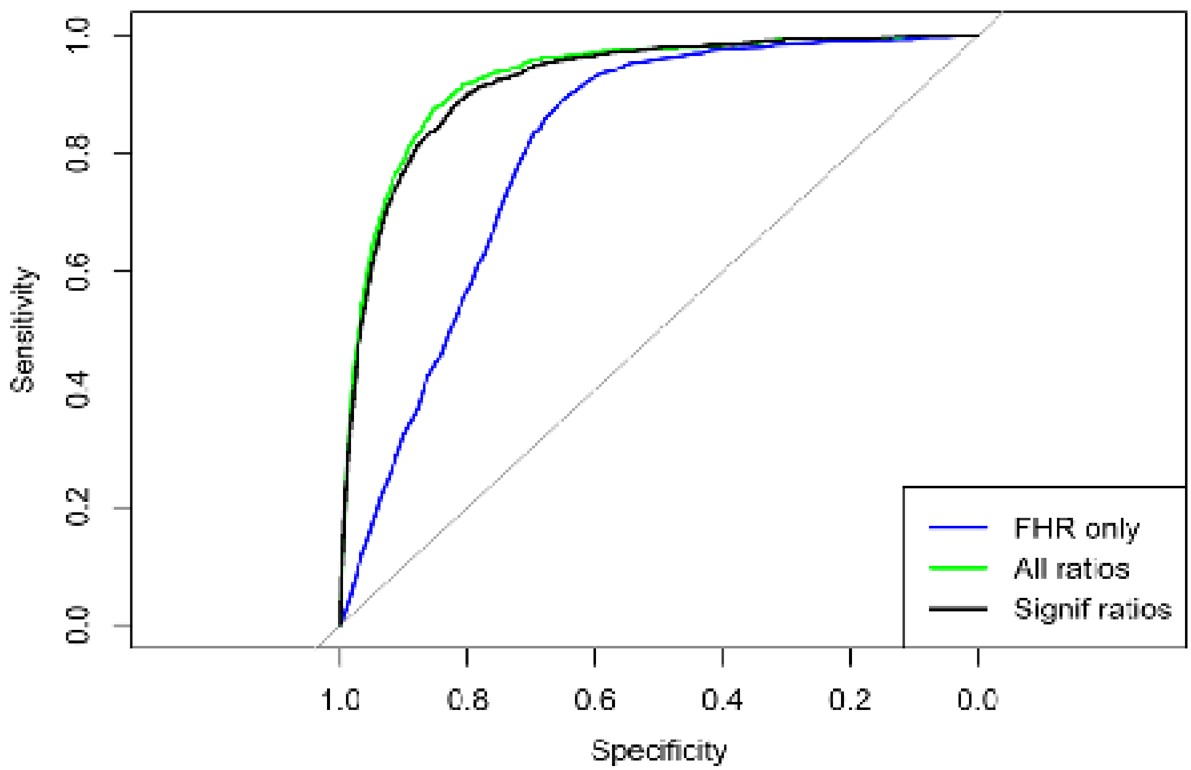
2. Resilience variables that had little effect on Z were removed, since whatever contribution they made was already explained by others in the model.

Findings

Removing variables from a statistical classification model always reduces classification accuracy, even if the variables do not belong in the model and their contribution is spurious; this is purely a mechanical result. However, the apparent loss of accuracy is very slight: the AUC was 0.809 using the Core Health Score only, but increased to 0.928 using the Core Health Score with all resilience variables, and fell only to 0.920 for the final model using the 11 resilience variables. This was considered an acceptable trade-off against the reduced risk of over-fitting the model to a specific sample.

Some of the resilience variables corresponding to ratios from each of the classic models survived in the final 11, but some from each classic model were dropped as well. The FHR thus includes better substitutes for some of the ratios in each classic model; of course, part of the substitution is provided by the Core Health Score itself. Performance of the final model was tested by standard methods: the Receiver Operating Characteristics (ROC) curve, summarized by the area under that curve (AUC). The AUC value has been mentioned above; the ROC is shown below.

Figure A7: Receiver Operating Characteristics (ROC) Curve



The ROC curve plots the sensitivity – the fraction of defaults that are correctly classified – against the specificity – the fraction of non-defaulting cases that are correctly classified; the specificity scale runs from right to left. A perfect classifier would have an ROC that went straight up to the top-left corner then straight across to the top-right corner. The improvement in changing from the Core Health Score to the FHR (the "significant vars" line) is apparent, as is the negligible loss from discarding the other 15 variables.

To test for over-fitting, the model was tested in two out-of-sample settings. First, the sample was randomly divided in two, with one half being used to fit the model parameters and the other half being used to compute the AUC. The values of AUC were 0.815 using CHS as the predictor and 0.909 using the final model. Second, the model was fitted with data through 2011 and the AUC was computed using data from 2012-2013. In this test, the values of AUC were 0.818 using the CHS as the predictor and 0.945 using the final model. These results straddle the in-sample result AUC=0.920, suggesting that there is no serious bias from over-fitting and that the model retains good power to estimate the probability of default in future years. For convenience of presentation, the Z from the final model is monotonically transformed to the score reported to clients:

$$FHR = \frac{1}{1 + 8.52\exp\,(0.369\,Z)}$$

This transformation ensures that FHR is a score between 0 and 100, and that lower values correspond to a higher probability of default. The particular numbers were chosen so that the probability of default is about 1% for a score of 40 and 0.1% for a score of 60. Because this transformation is monotonic, it leads to identical classification performance to that of Z.

The model has an interesting and desirable property: if the Core Health Score is high, suggesting little risk of default, the FHR is not very sensitive to changes in the resilience variables. If the Core Health Score is poor, then the FHR is much more sensitive to the values of these variables. That is, a healthy firm can afford modest deterioration in any resilience variable, whereas a weak firm sees its probability of default rise sharply if a resilience variable deteriorates. For weak firms, new or aggravated weaknesses compound existing problems or deficiencies.

To illustrate, consider two firms W and S, with CHS of 20 and 80 respectively. Firm S is at little risk of default, while firm W has a much higher risk. The most important of the resilience variables is NPAT.TotAss (corresponding to the ratio Net Profit to Total Assets). The mean values of NPAT.TotAss are 0.441 and 0.538 for firms with CHS=20 and 80 respectively; roughly speaking, these correspond to ratios NPAT/Total Assets of -19% and +12% respectively, although these must be treated with caution because the NPAT.TotAss values will correspond to different ratios in different industries. Keeping all other variables at their means for firms with the given Core Health scores, if we set NPAT.TotAss for firm W to be either 0.441 or 0.538 then the FHR will be 40.1 or 46.5 respectively (a difference of 6.4). But the same values for firm S will give an FHR of 84.1 or 87.3 respectively (a difference of 3.2). So the weak firm W has twice the sensitivity to changes in this ratio than the strong firm S has.

In summary, the FHR has important benefits as an estimator of short-term default risk. It shows very high classification accuracy over the next 12 months. This high accuracy appears to extend to future periods beyond that used for estimation. The FHR is built on the established Core Health Score plus 11 resilience variables that are known to be predictive of short-term default.

Moreover,

- Each variable complementing the CHS appears with the correct sign and has substantial predictive value;
- The FHR can be applied to firms with missing or dubious data, and is protected against being dominated by outlying values of any ratio; and
- The resilience variables have little effect on the default risk of firms that are fundamentally healthy but greater effects on weak firms.

The FHR adds substantial capabilities to Rapid Ratings' collection of tools to measure current and emerging financial and operational efficiency and competitiveness performance of, and related default risks facing, public and private companies.

A7. Core Health Model Overview

Purpose of the Core Health Models

The purpose of Rapid Ratings 25 industry models is to measure the core health of firms by grading a company on a global industry basis. The underlying theory is that a company's core health can be well measured by carefully selected financial ratios.

Model Structure

An econometric, stochastic series of models, the Rapid Ratings system produces a core health rating based on how a company performs relative to its global industry peers utilizing financial ratio analysis (outlined below). Each financial ratio is described by a metric that ranks firms in order of performance. The weight of each financial ratio in the overall core health rating is a function of the slope of its distribution, and may differ across industries. Those ratios are presented below:

Table A2: The 62 Financial Ratios in the Industrial FHR Model

LIQUIDITY RATIOS

1.	current assets / current liabilities
2.	quick assets / current assets
3.	quick assets / current liabilities
4.	quick assets / total assets
5.	quick assets / sales
6.	quick assets / total capital employed
7.	working capital / total assets
8.	working cap / total capital employed
9.	working capital / sales
10.	current assets / total sales
11.	current assets / total liabilities
12.	current liabilities / total assets
13.	current liabilities / total liabilities

ACTIVITY RATIOS

14.	sales / total assets
15.	sales / total shareholder equity
16.	sales / total capital employed
17.	sales / working capital
18.	sales / inventory
19.	sales / labour costs

DEBT MANAGEMENT

20.	net investment expense / EBIT
21.	net investment expense / NOPAT
22.	net investment expense / labour costs
23.	net invest expense / net debt levels

COST STRUCTURE

24.	cost of goods sold / total expenditure
25.	staff costs / total expenditure
26.	depreciation / total expenditure
27.	other op expenses / total expend
28.	net investment expense / total expend
29.	income tax / total expenditure
30.	cost of goods sold / sales
31.	staff costs / sales
32.	depreciation / sales
33.	other operating expenses / sales
34.	net investment expense / sales
35.	income tax / sales

SOLVENCY RATIOS

36.	total liabilities / sales
37.	term liabilities / total capital employed
38.	total liabilities / total assets
39.	equity / total assets

PROFITABILITY RATIOS

40.	gross profit / sales
41.	gross profit / total assets
42.	gross profit / total capital employed
43.	EBITD / sales
44.	EBITD / total assets
45.	EBITD / equity
46.	EBITD / total capital employed
47.	EBIT / sales
48.	EBIT / total assets
49.	EBIT / equity
50.	EBIT / total capital employed
51.	NOPAT / sales
52.	NOPAT / total assets
53.	NOPAT / equity
54.	NOPAT / total capital employed
55.	NPBT / sales
56.	NPBT / total assets
57.	NPBT / equity
58.	NPBT / total capital employed
59.	NPAT / sales
60.	NPAT / total assets

61.	NPAT / equity
62.	NPAT / total capital employed

A8. FHR Transition Matrix and Gini Analysis

Introduction

This study presents transition matrix and default rate analysis for listed US non-financial firms rated by Rapid Ratings between 1/1/2001 and 12/31/2016. We present 1 and 3 year perspectives and the default rates associated with each risk category within each time window.

The universe for the analysis includes all listed non-financial firms that have qualified for the Russell 3000 index at any time during the 16-year period. This includes 5234 entities, 448 of which defaulted during the period, and for which we issued a total of 194,136 ratings. 'Default' is defined as a missed principal or interest payment on a debt obligation or Chapter 11 filing in a US Bankruptcy Court.

Default Rates and Rating Transition Matrix

Within this section Rapid Ratings employs a static pool methodology commonly used in credit rating analysis. This methodology establishes pools of rated entities as of the start of each calendar year where the start and end rating for that calendar year (or subsequent calendar years for a 3-year view) is included in the analysis.

Tables A3 to A7 present transition rates and default rates by rating category (20 point bands). Table 4.5 presents default rates each year by rating deciles (10 point bands).

Table A3: 2001 - 2016 Average, 1-Year Transition Matrix, Financial Health Rating, US Non-Financials

	Very Low Risk (80-100)	Low Risk (60-79)	Medium Risk (40-59)	High Risk (20-39)	Very High Risk (0-19)	Default	Total
Very Low Risk (80-100)	71.7%	21.4%	5.6%	1.4%	0.0%	0.0%	21.4%
Low Risk (60-79)	14.6%	60.9%	20.7%	3.6%	0.1%	0.0%	30.8%
Medium Risk (40-59)	3.4%	21.1%	59.5%	15.4%	0.3%	0.3%	31.1%
High Risk (20-39)	2.2%	7.4%	29.4%	55.2%	3.0%	2.8%	15.8%
Very High Risk (0-19)	2.0%	4.0%	9.6%	40.0%	30.4%	14.0%	0.9%
Total	21.3%	31.1%	30.8%	15.3%	0.9%	0.7%	100.0%

Table A4: 2001 - 2016 Average, 3-Year Transition Matrix, Financial Health Rating, US Non-Financials

	Very Low Risk (80-100)	Low Risk (60-79)	Medium Risk (40-59)	High Risk (20-39)	Very High Risk (0-19)	Default	Total
Very Low Risk (80-100)	56.8%	28.4%	11.1%	3.3%	0.1%	0.2%	21.9%
Low Risk (60-79)	19.3%	48.3%	24.8%	6.9%	0.2%	0.4%	30.2%
Medium Risk (40-59)	7.9%	25.8%	46.5%	17.4%	0.6%	1.8%	30.8%
High Risk (20-39)	7.2%	13.8%	32.8%	35.5%	2.4%	8.2%	16.2%
Very High Risk (0-19)	7.5%	8.0%	16.9%	33.1%	10.4%	24.1%	1.0%
Total	22.0%	31.0%	29.7%	14.2%	0.8%	2.3%	100.0%

Table A5: 2016 Pool, 1-Year Transition Matrix, (1/1/2016 - 12/31/2016), Financial Health Rating, US Non-Financials

	Very Low Risk (80-100)	Low Risk (60-79)	Medium Risk (40-59)	High Risk (20-39)	Very High Risk (0-19)	Default	Total
Very Low Risk (80-100)	72.6%	20.9%	5.3%	1.2%	0.0%	0.0%	18.3%
Low Risk (60-79)	10.5%	60.8%	24.4%	3.9%	0.1%	0.2%	30.2%
Medium Risk (40-59)	1.0%	16.7%	63.0%	18.7%	0.0%	0.5%	35.3%
High Risk (20-39)	0.2%	3.8%	22.4%	62.8%	4.0%	6.8%	15.2%
Very High Risk (0-19)	0.0%	0.0%	3.8%	23.1%	50.0%	23.1%	0.9%
Total	16.9%	28.7%	34.0%	17.8%	1.1%	1.5%	100.0%

Table A6: 2014 Pool, 3-Year Transition Matrix, (1/1/2014 - 12/31/2016), Financial Health Rating, US Non-Financials

	Very Low Risk (80-100)	Low Risk (60-79)	Medium Risk (40-59)	High Risk (20-39)	Very High Risk (0-19)	Default	Total
Very Low Risk (80-100)	55.8%	29.3%	11.2%	3.4%	0.1%	0.2%	21.8%
Low Risk (60-79)	19.3%	48.6%	24.8%	6.8%	0.2%	0.4%	30.6%
Medium Risk (40-59)	8.2%	26.3%	46.3%	17.0%	0.6%	1.7%	30.8%
High Risk (20-39)	7.7%	14.5%	33.1%	35.0%	2.3%	7.6%	15.9%
Very High Risk (0-19)	8.2%	7.7%	14.2%	35.6%	9.9%	24.4%	0.9%
Total	21.9%	31.7%	29.7%	13.9%	0.7%	2.1%	100.0%

Table A7: Annual Default Rates by Rating Decile

	Very Low Risk		Low Risk		Medium Risk		High Risk		Very High Risk	Lower Risk	Higher Risk	Overall
Pool Start Date	90-100	80-89	70-79	60-69	50-59	40-49	30-39	20-29	0-19	60-100	0-39	0-100
1/1/2001	0.0%	0.0%	0.2%	0.0%	0.4%	1.6%	2.2%	5.5%	16.3%	0.1%	4.3%	1.2%
1/1/2002	0.0%	0.0%	0.0%	0.2%	0.0%	0.5%	2.9%	5.7%	26.3%	0.1%	5.5%	1.4%
1/1/2003	0.0%	0.0%	0.0%	0.2%	0.4%	0.0%	1.5%	6.7%	16.7%	0.1%	4.4%	1.0%
1/1/2004	0.0%	0.0%	0.0%	0.0%	0.2%	0.6%	0.7%	4.3%	11.8%	0.0%	2.3%	0.6%
1/1/2005	0.0%	0.0%	0.0%	0.0%	0.0%	0.6%	1.1%	3.3%	14.3%	0.0%	2.4%	0.5%
1/1/2006	0.0%	0.0%	0.0%	0.0%	0.0%	0.4%	0.6%	1.2%	3.8%	0.0%	1.0%	0.2%
1/1/2007	0.0%	0.0%	0.0%	0.0%	0.4%	0.2%	0.8%	6.0%	3.6%	0.0%	2.4%	0.5%
1/1/2008	0.0%	0.0%	0.0%	0.0%	0.2%	0.6%	2.1%	6.9%	12.0%	0.0%	4.1%	0.7%
1/1/2009	0.0%	0.0%	0.4%	0.0%	0.8%	1.1%	3.6%	10.9%	32.3%	0.1%	7.8%	1.7%
1/1/2010	0.0%	0.0%	0.0%	0.0%	0.0%	0.4%	1.8%	0.9%	6.8%	0.0%	1.9%	0.4%
1/1/2011	0.0%	0.0%	0.2%	0.0%	0.0%	1.0%	1.3%	5.9%	15.0%	0.1%	3.2%	0.6%
1/1/2012	0.0%	0.0%	0.0%	0.0%	0.0%	0.2%	2.7%	6.6%	6.3%	0.0%	3.9%	0.6%
1/1/2013	0.0%	0.0%	0.0%	0.0%	0.0%	0.2%	1.7%	3.7%	19.0%	0.0%	3.1%	0.5%
1/1/2014	0.0%	0.0%	0.2%	0.0%	0.0%	0.5%	1.8%	5.8%	4.5%	0.1%	3.1%	0.6%
1/1/2015	0.0%	0.0%	0.0%	0.0%	0.2%	1.1%	2.2%	5.6%	14.3%	0.0%	3.8%	0.7%
1/1/2016	0.0%	0.0%	0.0%	0.5%	0.2%	0.9%	3.0%	16.1%	23.1%	0.1%	7.8%	1.5%
Weighted Average	0.00%	0.00%	0.06%	0.05%	0.17%	0.61%	1.86%	5.78%	15.14%	0.03%	3.81%	0.79%

Note: Ratings for the bottom two deciles are combined into one Very High Risk (0-19) category due to the scarcity of ratings issued with a FHR below 20.

Lorenz Curve and Gini Coefficient

The Lorenz Curve and associated Gini coefficient are commonly used methods of measuring a model's efficacy at determining separation between two groups, in this case defaulters and survivors. The blue trend represents the performance of Rapid Ratings' FHR at issuing ratings in the higher risk level rating categories for companies which default within a time horizon versus issuing ratings in lower risk rating categories for companies which do not default within the time horizon. If a model randomly allocated ratings to defaulters and survivors it would plot along the grey trend line, and a perfect model would plot along the green trend line. The Rapid Ratings model aspires to match the 'perfect model' trend line and the Gini coefficient is a measure of the extent to which this is achieved.

In constructing the Lorenz Curve, Rapid Ratings employs a methodology that includes all ratings issued during the time period for the 'Universe' sample, and all ratings which were issued during the time period and within 12 months of default are included in the Defaulter sample for the 1-yr Gini (or within 36 months for the 3-yr Gini).

Figures A8 and A9 present the one year and three year Lorenz curves and Gini coefficients.

Figure A8: 2001 - 2016 Average 1-Year Lorenz and Gini Coefficient, US Non-Financials

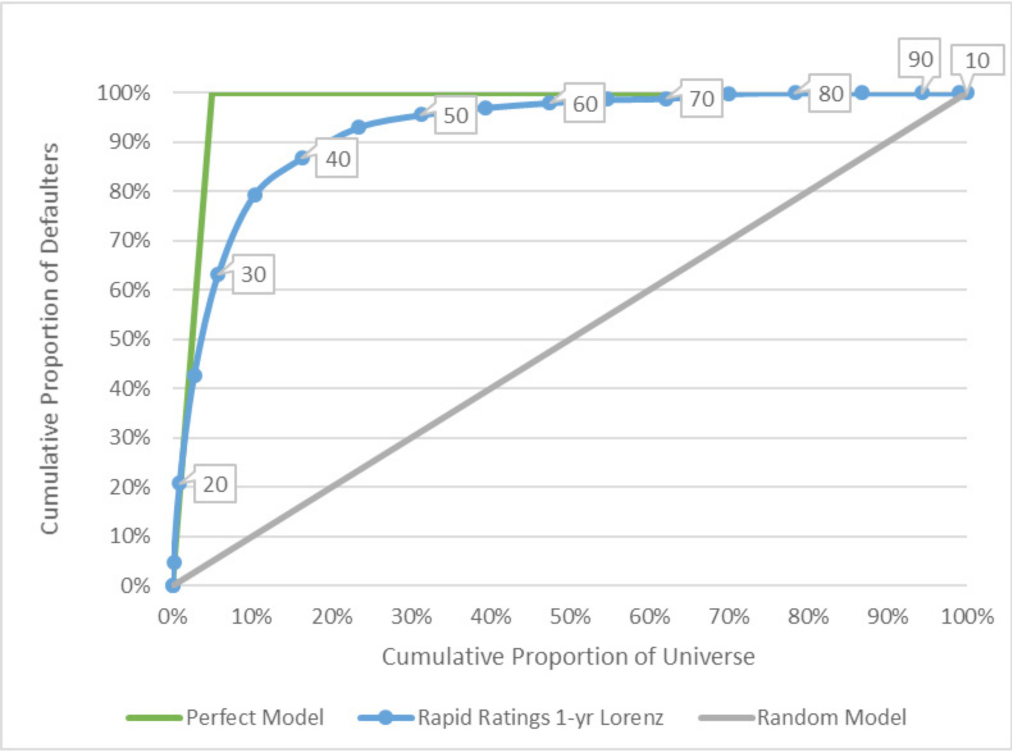


Figure A9: 2001 - 2014 Average 3-Year Lorenz and Gini Coefficient, US Non-Financials