QUANTITATIVE PORTFOLIO MANAGEMENT CONFERENCE

Scenario Analysis in Large Covariance Matrices

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Agenda

- Types of scenarios and why scenarios on covariance matrices
- Current practice on covariance scenarios
- Review a simple methodology
  - Application: Covariance consistent with a short stress period
- Conclusion and extensions

Why Are Scenarios Useful?

- Analyze the behavior of portfolios across different paths / conditions
  - Historical scenarios
  - Scenarios our portfolio may be specifically sensitive to
  - Scenarios we are generally concerned with
- Scenarios analysis on returns
  - Pricing models: Perturb inputs
  - Linear approximations (e.g., linear factor models): Perturb factors
  - Correlated scenarios: Using covariances to complete scenarios
- Scenarios on covariances: Our focus today
  - Complements the scenarios on the first moment
  - Better characterization of the return distribution under scenarios
    - “Correlations in the tails,” “Correlation breakdowns”
  - Better measure of the risk / return trade-off under a scenario
    - Efficiency of particular hedges

Practical Dilemma

- Scenarios are usually defined at a high / aggregate level
  - E.g., relationship between broad macro variables or risk factors
- Risk systems usually involve a high number of factors and covariances
  - Make sure we capture all imbalances our portfolio may have
    - E.g., we may be neutral US equities, but long on US financials

Example: Portfolio of US credit and equities

- Scenarios on the join behavior of yield curve, credit spreads and equities
  - Express our scenario in a 3 x 3 matrix
- Risk models for these asset classes have tens of factors (~60 in POINT)
  - Hard to express scenarios on this larger dimension

Typical Solutions

How is the dilemma solved?

1. Work in the high dimensional space
   - Changing specific covariates, one by one or block by block
     - Consistent with the granularity of imbalances our portfolios may have
     - Change a large number of correlations, without specific views
   - Hard to control actual changes once you impose positive definiteness
   - Large body of literature: How to change “the least” possible (Defeng Sun)
2. Significantly reduce the dimension with which we work
   - Have views on a small set of correlations at the aggregate level
   - Define exposures to these aggregate scenarios
   - Typical use of these scenarios stop here

➢ Can we recover richer dimensionalities?

Example (cont'd)

Start with the previous example

- Go back to December 2006
- Looking at broad correlations between Treasury, credit and equities
- Credit strategist forecasts a flight to quality in the credit markets

Correlations 1990–2006		
	TSY	CRD
CRD	-0.43	
EQT	0.12	0.44

Scenario 2007–2009		
	TSY	CRD
CRD	-0.65	
EQT	?	?

- How can we complete this scenario?

- Let's start with some notation

Note: The source for all figures in this presentation is Barclays Capital.

Some Notation

- Suppose we have a set of *M* factors *F.* Divide them in two sets
  - F_1: *K* factors for which we have views on their covariance
  - F_2: All other *M-K* factors

- Question: How to propagate the views to the entire set?

- Start by regressing: $F_2 = \beta F_1 + \varepsilon$

- We can then re-construct the covariance matrix as

$$\Sigma = V(F) = \begin{bmatrix} \Sigma_1 & \Sigma_1\beta \\ \beta\Sigma_1 & \beta\Sigma_1\beta' + \Omega \end{bmatrix} \quad \text{where} \quad \Sigma_1 = V(F_1), \quad \Omega = \text{var}(\varepsilon)$$

Implementing Views

- Suppose we have views on

$$\Sigma_1 \rightarrow \Sigma_1^*$$

- Can we represent the new covariance matrix as?

$$\Sigma^* = \beta \Sigma_1^* \beta' + \Omega$$

  - Many situations where this approach delivers good results
  - In general, it delivers intuition about scenario construction
- The approach can be improved by taking into consideration how our views affect
  - The regressed (unconditional) betas
  - Total volatility of the factor
  - Different methodologies have been proposed to address these issues
    - Global macro funds concerned with changes in betas
    - Merger arbitrage funds concerned with changes in idiosyncratic volatility

Example (cont'd)

- In our example

$$F = \begin{bmatrix} F_1 \\ F_2 \end{bmatrix} = \begin{bmatrix} F_{TSY} \\ F_{CRD} \\ \hdashline F_{EQT} \end{bmatrix}$$

- Sensitivity analysis

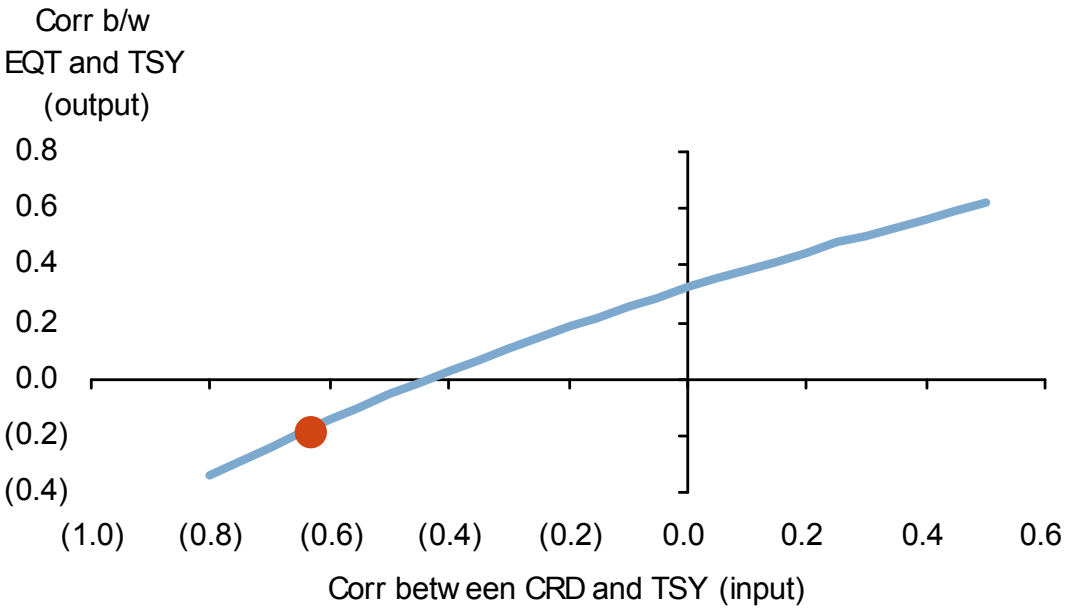
  Good diagnosis tool

- Fixed betas $\neq$

  Fixed correlations

Results (Correlations)

Pairs	1990–2006 Actual	2007–2009 Actual	2007–2009 Scenario
TSY, CRD	-0.43	-0.65	-0.65 ← Input
EQT, TSY	0.12	-0.24	-0.19
EQT,CRD	0.44	0.66	0.64 → Output



Application: Stressed Correlation

- Suppose you have three equity regional risk models
  - US, EU and JP, each with 24 industry factors (72 factors)
  - Monthly data from the past 16 years (1994–2009)
- Question: Can we construct the large covariance matrix (72 x 72) consistent with the correlation behavior from August 2008 to February 2009 (seven months)?
  - We can't use the same dimension (72) and frequency (monthly)
    - Need to decrease dimensionality
  - Use the framework and look at the scenario using instead
    - Country's average factor (three factors)
    - Weekly returns (30 weeks)

Note: straightforward to include assumptions on betas and residual volatility

Application: Scenario

- Actual data for the stress-scenario and the whole sample

Volatilities	All Sample	Stress
US	4.97	8.17
EU	5.62	6.30
JP	5.08	8.06

Correlations	US	EU	JP
US		0.78	0.41
EU	0.81		0.46
JP	0.67	0.82	

All

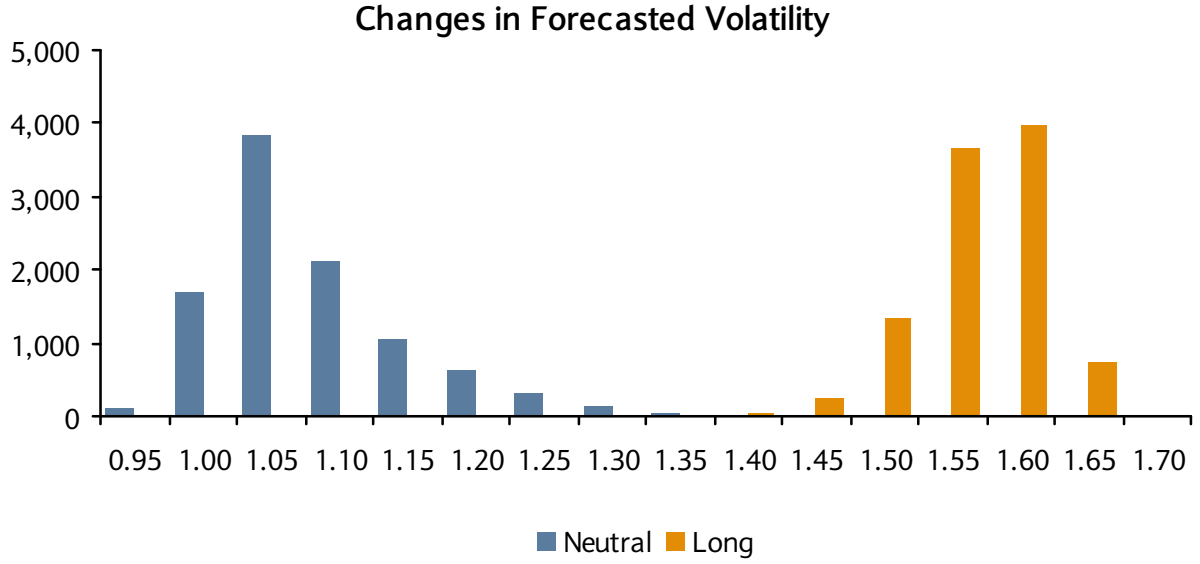
Stress

$$\underset{(72 \times 72)}{\Sigma^*} = \beta \underset{(3 \times 3)}{\Sigma_1^*} \beta' + \Omega$$

- We have now a stress-consistent large covariance matrix
- Do we gain particular insights from it?

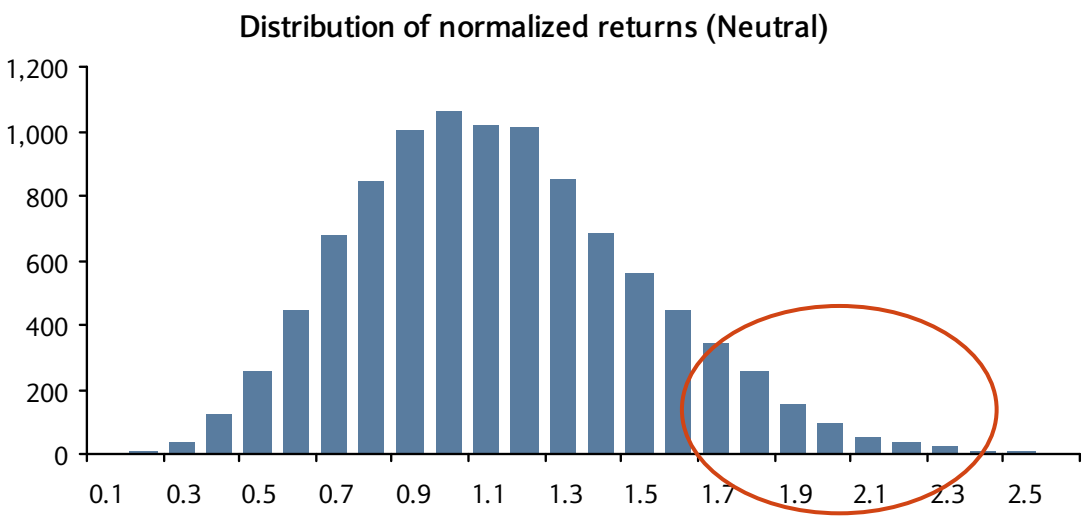
Application: Testing

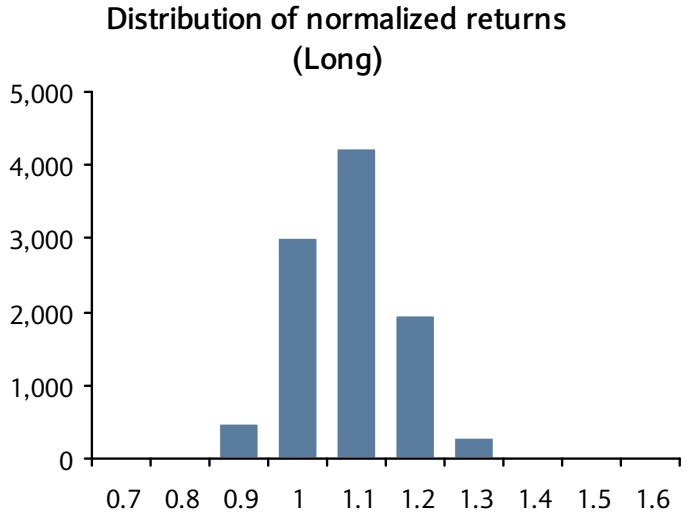
- Apply the new covariance matrix to two kinds of portfolios
  - Neutral: Portfolio that is neutral in each region
    - Short equally all 24 industries in the region
    - Long on four of the 24 industries in that region
    - Zero region aggregate exposures – need detailed exposure information
  - Long: long on four industries in each region
  - We simulated 10,000 random portfolios of each type



Application: Results

- Did the new covariance matrix perform well over this period?
  - We use monthly returns over the period to calculate actual volatility
  - We use only three series to correct the forecasted volatility for the whole covariance matrix





- The scenario seems to be describing this stress environment well
- Investigate the outliers: In this case, US Financials

Conclusions and Extensions

- Review of current methodology to incorporate scenarios on covariance matrices
  - Limitations
- How the scenario-consistent framework can help understand
  - The behavior of our portfolios under different scenarios
  - Shortcomings of focusing on historical analysis
  - Improve stress-testing in general
- There are many extensions not covered in this presentation, such as
  - Perturb covariance matrix for use in robust optimization
  - Globalizing regional risk models
  - Term structure of volatilities
- The analysis can be equally done at the asset / asset class / portfolio level

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