Banc of America Securities

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# CMS Excerpt: Quantitative Focus

## Comparing Cash Bond to CDS Spreads — The Par Equivalent CDS Spread Methodology

- Another pressing question for credit investors is: "What's the best way to compare bond to CDS spreads?" When bond prices move significantly from par, interpolated spread to the LIBOR curve or Asset Swap spread introduces bias. **Par Equivalent CDS spreads** provide a more robust, bias-free methodology for comparing CDS to cash bond spreads. As a result, we will be shifting our spread methodology in Credit OAS from Asset Swap to Par Equivalent CDS effective Wednesday, July 21.

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# Quantitative Focus: Comparing Cash Bond to CDS Spreads — The Par Equivalent CDS Spread Methodology

## Credit Spread as a Measure of Compensation for Credit Risk

Credit spreads are an integral part of the lexicon of credit investors. Beyond the simple spread to Treasury, however, the various other spread measures are both infrequently used and even less frequently understood. In most cases, the simple approach works best: calculate the YTM and subtract a benchmark interest rate—either Treasury or Swap rate—to determine the spread. But sometimes that approach introduces bias into the comparison of cash bond spreads to credit default swap spreads. To solve that problem, we turn to the Par Equivalent CDS Spread methodology, which we'll introduce here along with a comprehensive review of the other spread alternatives.

### Yield Spread (and I-Spread)

Arguably the simplest way to define and understand credit spread is to compare the yield to maturity of a straight corporate bond to a matched maturity Treasury yield (or swap yield). For example, if the yield to maturity of a five-year corporate bond is 7% while the yield to maturity of a five-year Treasury bond is 5%, the yield spread is simply 200 bps (or 2%). In fact, most high-grade corporate bonds are quoted on benchmark Treasury yield. If there is no default, a credit investor earns 2% more per annum on the corporate bond than he could earn on a matched-maturity Treasury bond. Of course, the reason for the 2% extra yield is the risk that the corporate bond issuer may fail to pay.

In reality, the maturity date of a corporate bond that is traded is rarely the same as the benchmark Treasury (usually on-the-run Treasury) that is used to quote the spread. When the maturity dates of the corporate and the benchmark Treasury are far apart, the concept of yield spread is less than well defined, especially when the Treasury yield curve is steep (either upward sloping or inverted). The practical solution to this problem is to calculate yield spread against interpolated matched-maturity Treasury yield (derived from on-the-run Treasury instruments). This is the so-called I-Spread. It could either be against interpolated Treasury yield or interpolated Swap yield.

### Z-Spread

Another often-used and easily misunderstood spread terminology is the Z-Spread. This is just a variation of the I-Spread and belongs to the category of yield spread. While I-Spread (to LIBOR) is the incremental yield over the yield-to-maturity of a matched maturity swap, Z-Spread (to LIBOR) is the incremental yield over the entire spot curve (or zero curve). The origin of the Z-Spread concept is the mortgage market. Z-Spread can be interpreted as spread income on a corporate bond when the investor finances her purchase with a bunch of zero-coupon borrowings. That is, each of the bond cash flow payments is financed separately at matched-maturity zero rate. This is obviously a highly theoretical construct.

Mathematically, Z-Spread is determined by equating the price of a corporate bond to the sum of discounted cash flows (coupon plus principal) at rates that are calculated as the sum of zero coupon LIBOR rate and a fixed spread on each cash flow payment date. In contrast, I-Spread is calculated as the simple difference between two numbers: the yield-to-maturity of the underlying bond, and the matched maturity swap yield. Both Z-Spread and I-Spread are yield spreads and they are normally very close to each other.

When Z-Spread and I-Spread are different, it is usually due to the shape of the yield curve. If the LIBOR yield curve is flat, Z-Spread is exactly the same as I-Spread since all the zero-coupon LIBOR rates coincide with the LIBOR Swap rates. However, when the LIBOR yield curve is upward slopping, Z-Spread tends to be slightly higher than I-Spread. The opposite is true when the yield curve is downward slopping. For those who are curious, this is because on balance, zero rates are lower than the level of yield-to-maturity when the yield curve is upward sloping. The slightly higher Z-Spread is needed to compensate for the lower zero rates.

Another factor that could affect the relationship between Z-Spread and I-Spread is the level of coupon rate. In the extreme case, when the coupon rate is zero, Z-Spread is lower (higher) than I-Spread when the yield curve is upward (downward) sloping.

### Asset Swap Spread

Most corporate bonds are structured as fixed rate bullets with flat coupon payment plus par at maturity. For leveraged investors who fund the purchase of corporate bonds with floating rate liability there is an interest rate mismatch. Asset Swap transactions are designed to solve this problem. That is, the investor buys the fixed rate corporate bond, swaps the fixed rate payment into LIBOR-based floating payments to meet cash flow requirements on the funding side.

A par interest rate swap is normally utilized in this type of transaction. Under a standard interest rate swap, the two counter-parties exchange floating LIBOR payments with fixed payments based on the swap rate for the maturity of the swap contract. However, in an asset swap transaction there is no guarantee that the fixed rate payment from the underlying cash bond is the same as the swap rate. As a result, the swap counter-party needs to pay the investor LIBOR floating payment plus a spread to compensate for the difference between the actual fixed payment (plus the impact of discount or premium if there is any) and the fixed payment required under the par interest rate swap. This spread is the so-called Asset Swap Spread.

If the underlying bond were priced at par, its coupon rate would be the same as the yield to maturity. The difference between the bond's coupon rate and matched-maturity swap rate is just the I-Spread. So, when the investor swaps the bond's fixed coupon for LIBOR floating payments, the swap counter-party would have to pay floating LIBOR plus the difference between the bond's coupon and the swap rate (which happens to be the I-Spread of the bond). In other words, when a bond is trading at par, Asset Swap Spread is the same as the I-Spread (ignoring day count conventions).

Things are not nearly as simple when the underlying bond is trading away from par. Market convention dictates that Asset Swap Spread be calculated in two steps. First, we calculate the difference between the bond's coupon rate and the fixed swap rate (for par interest rate swap). This is the mathematically trivial step and it forms the base level of the Asset Swap Spread. Second, we calculate the spread adjustment that needs to be added on top of the base level. The adjustment is positive (negative) when the bond is trading at a discount (premium). The amount of adjustment is equal to the level annuity payment that is equivalent to the amount of price discount (premium) in present value terms, calculated at the matched-maturity swap yield.

Because of the fact that the amount of discount (premium) is amortized at the swap yield instead of the yield to maturity of the bond, Asset Swap Spread could be different from the I-Spread, which is an imperfect but much better proxy for the equivalent CDS spread. For bonds trading at deep discount, Asset Swap Spread could be much lower than I-Spread.

### CDS Spread

When a credit investor buys a corporate bond, the investment is really a combination of a risk-free Treasury instrument and the credit risk of the underlying issuer. Investors who wish to invest in credit risk only can either buy a corporate bond and short the Treasury or enter a credit default swap contract. CDS Spread is the spread level stated on a credit default swap contract as compensation to investors for the risk of default or related credit event as defined in the contract. The CDS contract serves to separate credit risk from a cash bond.

### Connecting Cash Bond Spreads to CDS Spreads

Market efficiency (no arbitrage) dictates that compensation for credit risk an investor could earn on a CDS contract should not be materially different from the credit spread that is available in the cash market. If an investor's cost of funding is LIBOR flat, the spread compensation from a cash bond is the I-Spread against the matched maturity swap rate. For example, if the yield to maturity on a five-year corporate bond is 7% while the five-year swap rate is 5.5%, the I-Spread is 150 bps. It would seem obvious that the investor would be indifferent regarding cash versus CDS if the five-year CDS spread is also 150 bps. Unfortunately, the above statement is not always true, especially when the underlying bond is trading away from par.

For example, if the underlying cash bond is trading at a premium (say $120), the investor will earn 150 bps per annum on the entire $120 investment if there is no credit event during the life of the bond. Similarly, the investor could sell protection in the CDS market on $120 of notional exposure and earn the same 150 bps of spread on the same amount of investment. However if there is a credit event and if we assume that recovery rate is 40%**,** the investor would end up losing $80 ($120 investment minus $40 recovery on par) in the cash bond. This is simply because the par value of the bond is still $100 even though it's traded at $120. In contrast, the same 40% recovery rate means that the loss under the CDS contract is only $72 ($120 minus $120 times 40%).

### Par CDS Equivalent Spread

In the example we just illustrated, the 150 bps of I-Spread on the cash bond provides the same spread compensation as the CDS contract (with a CDS spread of 150 bps). However, in case of a credit event, the cash bond stands to lose more than the CDS contract. So the 150 bps of I-Spread is only as good as, say, 143 bps of CDS spread. That is, the investor is indifferent between the 150 bps of I-Spread for the cash bond and the 143 bps of CDS spread. The 143 bps of spread for the cash bond is calculated based on the 150 bps of I-Spread and incorporates the potential incremental loss upon default. This is the Par CDS Equivalent Spread.

So, for bonds trading at premium, Par CDS Equivalent Spread is the I-Spread minus recovery adjustment. In other words, I-Spread for a premium bond overstates the "true level" of credit spread. Similarly, for cash bonds trading at discount (say $80 price), Par CDS Equivalent Spread is higher than the I-Spread. That is, I-Spread understates the "true level" of credit spread.

Note that if the underlying cash bond is trading at par ($100 price), the difference in recovery becomes a non-issue. Consequently, for cash bonds trading at par, Par CDS Equivalent Spread is the same as the I-Spread, provided that we ignore any day count and other minor operational conventions.

### Summary

To summarize, below are the definitions of various forms of credit spreads.

- *I-Spread,* or interpolated spread to swap, is the yield difference between a cash corporate bond and a matched-maturity swap yield. The economics of I-Spread for a par bond are identical to those of CDS Spread except that there is no funding requirement under the credit default swap contract.
- *Z-Spread* is a variation of the I-Spread. It incorporates the shape of the yield curve in its calculation. When the yield curve is flat, Z-Spread is the same as I-Spread. Usually, the Z-Spread is slightly higher (lower) than the I-Spread when the yield curve is upward (downward) sloping.
- *Asset Swap Spread* is the per annum incremental income (measured in bps) on top of LIBOR that an investor gets when she swaps a fixed rate corporate bond to LIBOR-based floating payments. It is calculated as an adjusted difference between the coupon rate of the bond and

par interest rate swap rate, where the adjustment is the amortized price discount (or premium) at the swap rate.

- *CDS Spread* is a pure and simple credit risk premium that an investor is paid to take on credit risk in the credit default swap market. It's a percentage of the notional amount (expressed in terms of bps) that the investor is paid per annum. If there is a credit event during the life of the transaction, this payment stops and the investor pays the buyer of protection a one-time credit loss that is equal to par minus recovery.
- *Par CDS Equivalent Spread* is the spread level calculated for a cash bond that makes it indifferent for an investor (whose cost of funding is LIBOR flat) to choose between the cash instrument and CDS.

Theoretically, in the absence of arbitrage, the level of CDS Spread should be the same as the I-Spread and the Asset Swap Spread if the underlying cash bond is trading at par and both the cash and CDS instruments mature at the same time. The relationship breaks down when the cash bond is trading away from par. Both I-Spread and Asset Swap Spread end up overstating the actual level of spread when the cash bond is trading above par (for different reasons). In the case of I-Spread, the overstatement is due to the higher potential loss for a premium bond in case of default. For the Asset Swap Spread, it is because of the understatement of spread adjustment related to the amortization of price premium at the "risk-free" swap yield.

**Figure 1. The ABC's of Credit Spreads**

| Credit Spreads | Definition | Interpretation | Things to Know | Defect(s) |
|---|---|---|---|---|
| Benchmark Spread (Spread to Treasury) | Difference between YTM of the cash bond and that of a particular benchmark Treasury issue (on-the-run or curve depending on trader) | Incremental yield (as a compensation for credit risk) relative to a comparable Treasury instrument | Traditional quoting mechanism of the corporate bond market; problematic as a measure of relative value | Potential maturity mismatch; ignores funding cost on short Treasury |
| I-Spread (to LIBOR) | Difference between YTM of the cash bond and interpolated swap yield with matched-maturity | Spread income when an investor purchases a corporate bond with fixed rate funding at LIBOR flat | For par bond, I-Spread is a good proxy for matched-maturity CDS Spread; yet for premium (discount) bond it overstates (understates) CDS spread | Ignores recovery impact when a bond is priced away from par |
| Z-Spread (to LIBOR) | Constant level of spread added on top of the zero swap curve to price the bond to its current market value | Spread income when an investor purchases a bond with a bunch of fixed rate funding at LIBOR spot rates | With a flat yield curve, Z-Spread equals I-Spread; else it could be higher/lower than I-Spread depending on the shape of the yield curve | Ignores recovery impact when a bond is priced away from par |
| Asset Swap Spread | Constant level of spread the investor receives when swapping fixed rate coupons to floating LIBOR payments | Spread income on a fixed rate bond funded at LIBOR flat floating source combined with an interest rate swap to hedge interest rate risk | For a par bond, it's no different from I-Spread; else, it's higher(lower) than I-Spread if the bond is trading above(below) par | Ignores recovery impact when a bond is priced away from par; and amortizes amount of price disounts/premium at risk-free rate |
| Par CDS Equivalent Spread | Calculated spread level of a cash bond that economically equates the cash investment to a matched-maturity CDS transaction | When Par CDS Equivalent Spread of a cash bond equals the CDS Spread of matched-maturity, it's virtually impossible to arbitrage the two markets | The best proxy to CDS Spread of a cash bond; it equals I-Spread for par bonds; for bonds trading away from par, it removes the recovery bias imbedded in I-Spread; ideal measure for relative value and cash-CDS basis | None |

Source: Banc of America Securities LLC estimates.

Z-Spread is a more sophisticated form of I-Spread. It is calculated based on discounted cash flows at a constant spread against the spot curve (or zero curve). It thus incorporates both the convexity of the cash bond and the shape of the yield curve. Yet it does not add much to its simplistic counterpart (I-Spread). At an intuitive level, it captures the spread income for an investor who purchases a fixed rate cash bond with a collection of fixed rate funding sources that match one-to-one with each cash flow from the bond.

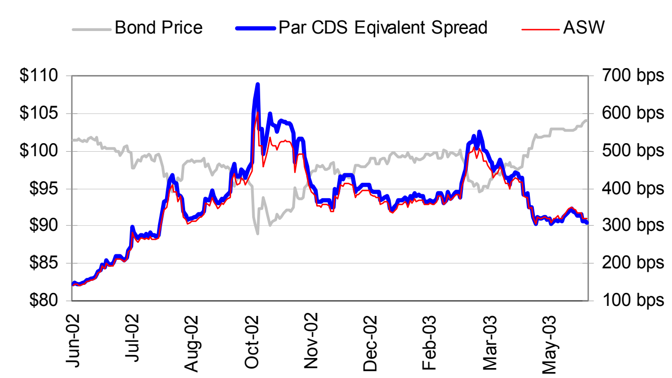
The compromising solution to realign the spread level of a cash bond with that of an otherwise identical CDS contract is to calculate the Par CDS Equivalent Spread of a cash bond. The idea of the calculation is to first calibrate the "risk-neutral" probability of survival of the underlying issuer (with an assumed level of recovery) from the price level of the cash bond. We then derive the CDS spread level that corresponds to the pre-specified recovery rate and survival probability.

It is our belief that Par CDS Equivalent Spread for cash bond is the best spread measure for relative value analysis for integrated cash-CDS portfolios and for the purpose of exploring cash-CDS basis.

## ASW vs. Par CDS Equivalent Spread — A Real World Example

The following figure illustrates the difference between ASW and Par CDS Equivalent Spread on a particular bond—F 6.5% '07. The time frame we picked was from June 2002 to June 2003. This was the time when this bond issue traded away from par.

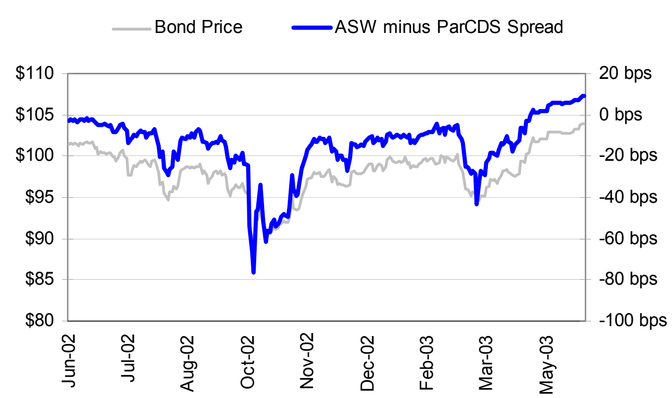
**Figure 2. Bond Price, ASW, and Par CDS Equivalent Spread**




Source: Banc of America Securities LLC estimates.

The following chart highlights the difference between the two spread measures.

**Figure 3.. Bond Price, and the ASW Par CDS Equivalent Spread difference**




Source: Banc of America Securities LLC estimates.

Clearly, the error introduced by ASW is closely related to the dollar price of the bond. When the bond was trading at around a $90 price, ASW understated Par CDS Equivalent Spread by almost 80 bps.

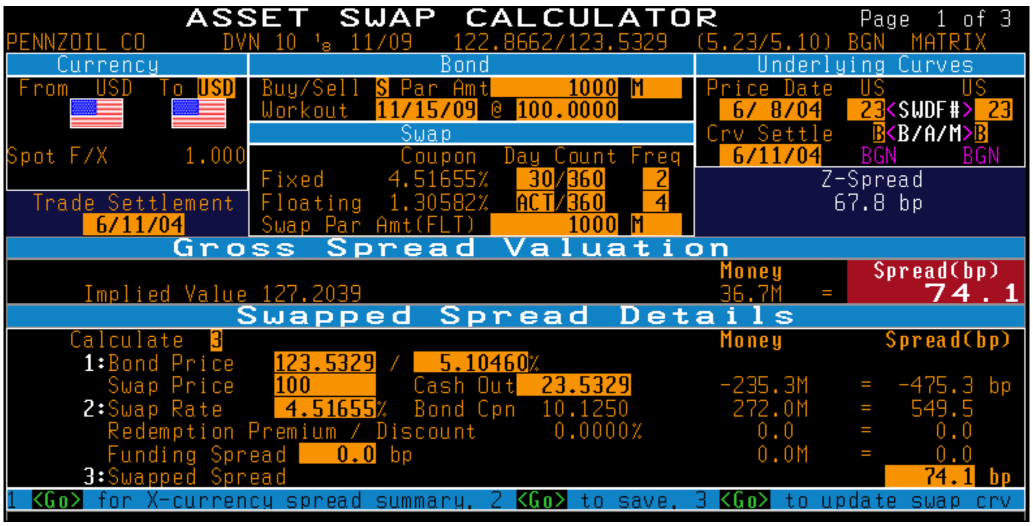
# Appendix A: Illustrating Credit Spreads with a Real World Example

In this section, we illustrate the various credit spreads with a real world example.

## Asset Swap Spread (Bloomberg ASW Screen)

To better understand how asset swap spread is calculated, we illustrate the process on an actual bond using the Bloomberg ASW screen. Shown below is the calculation of Asset Swap Spread for a bond issued by Devon Energy Corp back in 1989. The bond carries a 10.125% coupon and a maturity date of November 15, 2009.

**Figure 4. ASW Calculation on Bloomberg**




Source: Bloomberg.

On June 8, 2004, this bond is trading at $123.53 with a yield to maturity of 5.1046%. The 74 bps of asset swap spread on this bond is calculated in two steps. First, the bond is paying a coupon of 10.125% while the par swap rate is only 4.517%. The difference is 5.608% (or 561 bps). The reason that we are seeing 549.5 bps in the above screen-shot is because we have used a different (quick and dirty) day-count convention than what Bloomberg used (a more accurate day-count).

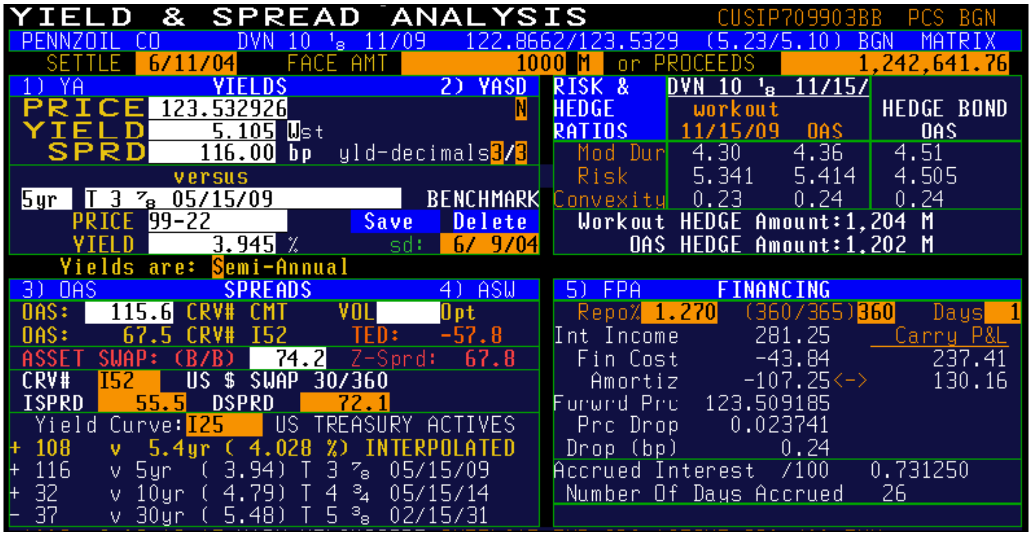
Swap market convention dictates that the swap be constructed at par. This means that the swap counter-party needs to amortize the $23.53 premium into the swap spread. To do so, we calculate the equivalent spread payment per coupon period that will produce a present value of $23.53 at the interest rate of 4.517%. To put it differently, if we calculate the sum of present values of an annuity cash flow in the amount of $4.75 per annum (paid semiannually) at 4.5175%, we would end up with $23.53.

All the highlighted cells can be altered to provide "what if" calculations.

## ASW, I-Spread, and Z-Spread (YAS Screen)

The YAS screen on Bloomberg (shown below) for this bond provides I-Spread and Z-Spread in addition to benchmark spread and Asset Swap Spread.

**Figure 5. Bloomberg YAS Screenshot**




Source: Bloomberg.

The various spread levels are shown in the SPREADS section in the lower left-hand corner. At the current price level of $123.5329, the bond has an Asset Swap Spread of 74 bps, an I-Spread of 56 bps, and a Z-Spread of 68 bps. A corporate bond trader would have quoted this bond at 116 bps against T 3 7/8 2009 (on the run 5-year Treasury).

The market price of this particular bond issue is is far above par (due to its high coupon rate of 10.125%). As we would expect, Asset Swap Spread (74 bps) is higher than I-Spread (56 bps) due to the “risk-free” amortization on the $23.53 of price premium. Meantime, Z-Spread (68 bps) is higher than I-Spread (56 bps) because of the upward sloping yield curve. The size of the Z-Spread I-Spread difference (12 bps) is driven by both the slope of the yield curve and the level of coupon rate. On the YAS page for DVN 6.875% 2011, we see that Z-Spread and I-Spread are 116 bps and 108 bps, respectively. The difference between Z-Spread and I-Spread is smaller (8 bps) on a similar bond with lower coupon.

Again, all the highlighted cells can be changed to recalculate. As an example, we produced the following table to illustrate the potential problem with Asset Swap Spread when the underlying bond is trading away from par.

**Figure 6. Potential Problem With the Asset Swap Spread**

| DVN 10 1/8 '09 | | Benchmark | Benchmark | Interpolated | Interpolated | Asset Swap | Spread |
|---|---|---|---|---|---|---|---|
| Px | YTM | Yield | Spread | Swap Yield | Spread | Spread | Difference |
| **$124** | 5.105% | 3.945% | 116 bps | 4.550% | **56 bps** | **75 bps** | **20 bps** |
| **$120** | 5.778% | 3.952% | 183 bps | 4.550% | **123 bps** | **148 bps** | **25 bps** |
| **$110** | 7.826% | 3.952% | 387 bps | 4.550% | **328 bps** | **353 bps** | **25 bps** |
| **$100** | 10.121% | 3.952% | 617 bps | 4.550% | **557 bps** | **557 bps** | **0 bps** |
| **$90** | 12.727% | 3.952% | 878 bps | 4.550% | **818 bps** | **764 bps** | **-54 bps** |
| **$80** | 15.733% | 3.952% | 1178 bps | 4.550% | **1118 bps** | **969 bps** | **-149 bps** |
| **$70** | 19.269% | 3.952% | 1532 bps | 4.550% | **1472 bps** | **1174 bps** | **-298 bps** |

Source: Banc of America Securities LLC estimates.

In constructing the above table, we assumed that the day count convention and payment frequency on the interest rate swap follows those of the cash bond (that is 30/360, semiannual). We did this to prevent the day-count and payment frequency from becoming an issue when we compare different spread levels.

As the price of the bond moves away from par, Asset Swap Spread starts to deviate from I-Spread. The problem is more severe when the bond is trading at a discount. When the bond is trading at a high premium, the difference begins to diminish as the levels of YTM and swap yield are getting closer.

In the following table, we examine the differences between Asset Swap Spread, I-Spread, Z-Spread and Par CDS Equivalent Spread.

**Figure 7. ASW, I-Spread, Z-Spread, and Par CDS Equivalent Spread**

| DVN 10 1/8 '09 | | Benchmark | Asset Swap | I-Spread | Z-Spread | Par CDS | I-Spread minus |
|---|---|---|---|---|---|---|---|
| **Px** | **YTM** | Spread | Spread | (to LIBOR) | (to LIBOR) | Eqv. Spread | Par CDS |
| **$124** | 5.105% | 116 bps | 75 bps | 56 bps | 68 bps | 55 bps | **1 bps** |
| **$120** | 5.778% | 183 bps | 148 bps | 123 bps | 134 bps | 120 bps | **3 bps** |
| **$110** | 7.826% | 387 bps | 353 bps | 328 bps | 340 bps | 317 bps | **11 bps** |
| **$100** | **10.121%** | 617 bps | **557 bps** | **557 bps** | 571 bps | **557 bps** | **0 bps** |
| **$90** | 12.727% | 878 bps | 764 bps | 818 bps | 833 bps | 836 bps | **-18 bps** |
| **$80** | 15.733% | 1178 bps | 969 bps | 1118 bps | 1135 bps | 1203 bps | **-85 bps** |
| **$70** | 19.269% | 1532 bps | 1174 bps | 1472 bps | 1491 bps | 1711 bps | **-239 bps** |

Source: Banc of America Securities LLC estimates.

The first thing to notice is that when the bond is trading at par, all three forms of spread (ASW, I-Spread, and Par CDS Equivalent Spread) are the same. As expected, Z-Spread is different because of the upward sloping yield curve in the current interest rate environment (as of June 8, 2004).

When the bond is trading in discount (say, $80 price), ASW, I-Spread and Par CDS Spread are at 969 bps, 1,118 bps and 1,203 bps, respectively. The 149 bps of difference between ASW and I-Spread is due to the "risk-free" discounting of the $20 discount under ASW calculation. The 85-bp additional difference between I-Spread and Par CDS Spread is due to the recovery bias embedded in I-Spread calculation.

When the bond is trading at premium (for example, $110 price), ASW, I-Spread and Par CDS Spread are at 353 bps, 328 bps and 317 bps, respectively. In this case, ASW is higher than I-Spread, while I-Spread is higher than Par CDS Spread. However, as the price of the bond goes up from $110 to $120 the difference between I-Spread and Par CDS Spread drops from 11 bps to a mere 3 bps. This somewhat counterintuitive result is actually what we should expect. The key driver of the Par CDS Spread and I-Spread difference is recovery bias. As the price premium gets larger, the amount of recovery bias grows larger. But the probability of running into default is much smaller as the spread level goes down while bond price goes up. So, on balance, recovery bias begins to drop after the bond price reaches a certain level.

# Appendix B: The Mathematics of PAR CDS Equivalent Spread Calculation

In this section, we explain in detail how to calculate the Par CDS Equivalent Spread from the pricing of a cash bond. There are two steps to the process: first, the calibration of risk neutral survival probability of the underlying issuer implied in the pricing of the cash bond; and second, the calculation of equivalent CDS spread level that is consistent with the implied survival probability along with the assumed recovery rate. The reason that we need an assumed recovery rate is because we want to capture the impact of differential recovery in case the cash bond is not pried at par. Here, we follow the industry standard assumption of 40% recovery across all issuers.

## Survival Probability

Suppose the random default time for an issuer is $\tau$, and $\lambda(t)dt$ is the probability of default during the time frame of $[t, t+dt]$, provided default had not happened up to time $t$. λ(t) is often called default intensity. We define the risk neutral survival probability as

$Z(T)$ = Probability of no-default before $T$, that is, P( $\tau > T$ ).

It can be verified that $Z$ satisfies the following ordinary differential equation

$$Z'(t) = -\lambda(t)Zdt \text{ with } Z(0) = 1.$$

The solution of the above equation is the survival probability, that is,

$$Z(t) = \exp(-\int_0^t \lambda(u)du).$$

The probability of default is 1- Z.

## Calibrating Survival Probability from Cash Bond Pricing

Assume that the discount index is the LIBOR spot curve and $P(t)$ is the discount factor to time $t$. The price of a cash bond with survival probability Z(t) and recovery rate RR can be written as:

$$P_{cash} = c\sum_{i=1}^{N} \delta_i Z(t_i)P(t_i) + V_R Z(t_N)P(t_N) + RR\int_0^{t_N} P(u)Z(u)\lambda(u)du. \qquad \text{(A1)}$$

where $\delta_i$ is a day-count function between $t_{i-1}$ and $t_i$, $c$ is the coupon rate, $RR$ is the recovery rate, and $V_R$ is the redemption value. In the above equation we can use a standard numerical routine to back out the risk neutral default intensity $\lambda$ for any cash price.

## Calculate CDS Equivalent Spread

Next, we are ready to price the floating leg and the fixed leg of a CDS with given default intensity. For the protection seller, the value of potential payment in case of a credit event is,

$$V_{floating}(T) = \int_0^T (1-RR)\lambda(t)P(t)Z(t)dt.$$

Meanwhile the protection buyer pays a periodic premium to the protection seller, the present value of the total premium is

$$V_{fixed}(T) = Spread \sum_{i=1}^{N} P(t_i) Z(t_i) \delta_i \ .$$

Again $\delta_i$ is a day-count function between $t_{i-1}$ and $t_i$, and Spread is the premium for default protection. Intuitively the *par CDS spread* is the fair value of risk premium, i.e. if the protection buyer pays the par CDS to the protection seller, then $V_{floating}$ equals $V_{fixed}$. More precisely it is,

$$Spread = \frac{V_{floating}(T)}{\sum_{i=1}^{N} P(t_i) Z(t_i) \delta_i} \ . \tag{A2}$$

## Calculation Example

To illustrate the calculation process, let's look at a hypothetical example. First, to simplify the calculation we assume that the swap curve is flat at 5% (a proxy of the risk-free interest rate), and the recovery rate in case of default is 40%. Assume also that the hypothetical corporate bond issue has a coupon of 12%. The bond matures in five years, pays its coupon on a semiannual basis, and is priced at 120. It's easy to calculate the yield to maturity of this bond (7.17%). Since the LIBOR curve is assumed to be flat at 5%, I-Spread for this bond is 217 bps.

To calculate the Par CDS Equivalent Spread for this bond, we need to first calibrate default intensity (and the survival probabilities for each coupon period) that is consistent with the 120-price level (that is, equation (A1) above).

**Figure 8. Step One - Use Bond Price to Calibrate Default Intensity**

| Coupon Period | Survival Probability | Probability of Default | Bond Cash Flow | Expected Cash Flow | Discount Factor | PV of Cash Flow |
|---|---|---|---|---|---|---|
| 1 | 98.5% | 1.54% | $6 | $6.52 | 97.5% | $6.36 |
| 2 | 96.9% | 1.52% | $6 | $6.42 | 95.1% | $6.11 |
| 3 | 95.4% | 1.50% | $6 | $6.33 | 92.8% | $5.86 |
| 4 | 94.0% | 1.47% | $6 | $6.23 | 90.5% | $5.63 |
| 5 | 92.5% | 1.45% | $6 | $6.13 | 88.2% | $5.41 |
| 6 | 91.1% | 1.43% | $6 | $6.04 | 86.1% | $5.19 |
| 7 | 89.7% | 1.41% | $6 | $5.94 | 83.9% | $4.99 |
| 8 | 88.3% | 1.38% | $6 | $5.85 | 81.9% | $4.79 |
| 9 | 86.9% | 1.36% | $6 | $5.76 | 79.9% | $4.60 |
| 10 | 85.6% | 1.34% | $106 | $91.26 | 77.9% | $71.07 |
| | | | | | | **$120** |

Source: Banc of America Securities LLC estimates.

Survival Probability is the chance that the issuer does not default by the end of a particular coupon period. It's entirely determined by the default intensity (that is, the chance of default per unit of time). Probability of Default refers to the chance of default during a particular coupon period. In other words, it's the chance that the issuer survived to the prior coupon period and defaults during the current coupon period.

The column under Expected Cash Flow is the probability weighted cash flows. For example, $6.52 for period one is the sum of 98.5%x$6 and 1.54%x$40. Discount Factor is the LIBOR discount rate for a particular coupon period, and it is used to generate the PV of Cash Flow column. Finally, the price of the bond is the sum of the present values of all the cash flows.

Depending on the level of default intensity, the sum of PV of Cash Flows may not necessarily be equal to 120. In the above example, the table is showing the survival probabilities that happen to make the sum of PVs equal to 120. And it turns out that 3.11% annual default intensity is exactly the number that makes everything work out perfectly.

Now that we have derived the default intensity for this bond, we proceed to use it to price both the floating and fixed legs of the credit default swap. This step will help us determine the Par CDS Equivalent Spread.

**Figure 9. Step Two – Use Default Intensity to Calculate Par CDS Equivalent Spread**

| Coupon Period | Survival Probability | Probability of Default | Loss on Default | Par CDS Spread | Discount Factor | PV of Floating Leg | PV of Fixed Leg |
|---|---|---|---|---|---|---|---|
| 1 | 98.5% | 1.54% | $60 | 188 bps | 97.5% | $0.90 | $0.90 |
| 2 | 96.9% | 1.52% | $60 | 188 bps | 95.1% | $0.87 | $0.87 |
| 3 | 95.4% | 1.50% | $60 | 188 bps | 92.8% | $0.83 | $0.83 |
| 4 | 94.0% | 1.47% | $60 | 188 bps | 90.5% | $0.80 | $0.80 |
| 5 | 92.5% | 1.45% | $60 | 188 bps | 88.2% | $0.77 | $0.77 |
| 6 | 91.1% | 1.43% | $60 | 188 bps | 86.1% | $0.74 | $0.74 |
| 7 | 89.7% | 1.41% | $60 | 188 bps | 83.9% | $0.71 | $0.71 |
| 8 | 88.3% | 1.38% | $60 | 188 bps | 81.9% | $0.68 | $0.68 |
| 9 | 86.9% | 1.36% | $60 | 188 bps | 79.9% | $0.65 | $0.65 |
| 10 | 85.6% | 1.34% | $60 | 188 bps | 77.9% | $0.63 | $0.63 |
| | | | | | | **$7.6** | **$7.6** |

Source: Banc of America Securities LLC estimates.

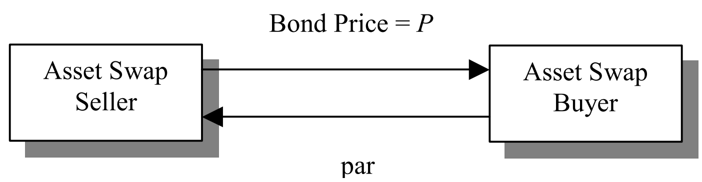
In the context of credit default swap, the floating leg represents potential payoff when there is a credit event, while the fixed leg is associated with the regular premium payments. In the table above, PV of Floating Leg is calculated by multiplying Probability of Default, Loss on Default, and Discount Factor.

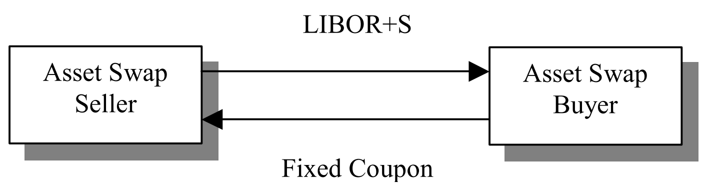
The column PV of Fixed Leg is the present value of expected premium payments (that is, Survival Probability times Spread times Discount Factor). It turns out that when the spread level is 188 bps (per annum), both the fixed leg and the floating leg have the same value. As a result, the 188 bps is the Par CDS Equivalent Spread that is consistent with the default intensity implied by the bond price.

Notice that the I-Spread for this bond is 217 bps. As we highlighted before, when the bond is trading at premium (120 price) I-Spread overstates the spread level because of the higher potential loss in case of default relative to CDS contracts.

# Appendix C: The Mathematics of Asset Swap Spread Calculation

At the settlement of an asset swap transaction, a seller agrees to swap a fixed coupon corporate bond with a floating cash flow on pre-defined notional amount. This transaction consists of two steps. First, the buyer pays the seller par value of the notional of the swap. The seller delivers the fixed coupon bond with face value equal to the swap notional to the buyer. Next, both parties enter an interest rate swap—that is, the buyer pays the periodic fixed coupon to the seller, and the seller pays the buyer forward LIBOR plus a premium to cover the financing cost and the credit risk that the buyer takes. Usually the premium is called the asset swap spread. The following diagrams illustrate the transaction of the asset swap described above.




Based on the assumption that the present values of the two (buyer and seller) cash flows are equal, the asset swap spread satisfies the following equation,

$$1 + C_{fixed} \sum_{i=1}^{N_{FIXED}} \delta_i df_i = P + \sum_{i=1}^{N_{FLOATING}} \Delta_i (L_i + S) df_i$$

where $df_i$ is the LIBOR discount factor, $L_i$ is the LIBOR forward rate (usually 3-month or 6-month), $\delta_i$ and $\Delta_i$ are day-count functions for the fixed coupon bond and the floating forward LIBOR. Notice that the interest rate swap yield with same tenor of the asset swap is,

$$C_{swap} \sum_{i=1}^{N_{FIXED}} \delta_i df_i = \sum_{i=1}^{N_{FLOATING}} \Delta_i L_i df_i .$$

If we ignore the difference in the day-count functions (fixed and floating), we can re-write the asset swap spread level as,

$$S = C_{fixed} - C_{swap} + (1 - P) / \sum_{i=1}^{N_{FLOATING}} \Delta_i df .$$

It is easy to see that the asset swap spread is just the difference of the fixed coupon and interest rate swap yield when the price of the fixed coupon bond is par.

# Appendix D: Implied Correlation Methodology[1]

In order to estimate implied correlation we use the following identity:

$$\sigma_{I,t}^2 = \sum_{i=1}^{N} w_{i,t}^2 \sigma_{i,t}^2 + 2\sum_{i=1}^{N-1}\sum_{j>i} w_{i,t} w_{j,t} \rho_{ij,t} \sigma_{i,t} \sigma_{j,t}$$

$\sigma_{I,t}^2$ = volatility of the index

$w_{i,t}$ = weight of index component i

$\rho_{ij,t}$ = pairwise correlation between two index components

$\sigma_{i,t}$ = volatility of index component i

NB: For simplicity we keep the index weights of each stock constant through the analysis (the error caused by this is small).

Here we see that standard deviation (volatility) of the index (or portfolio) can be expressed as a function of the volatilities of the underlying and correlation. We use call options on the index to extract the 3M implied volatility for the index and single name options on each of the underlying to extract name level volatilities. The measure is similar in spirit to implied correlations for structured credit products, although when pricing volatility on an index option, a full pairwise correlation matrix would be used. Here we find the single correlation number that solves the above equation—i.e., the correlation that produces the volatility number observed in the index option market.

As such:

$$\rho = \frac{\sigma_{I,t}^2 - \sum_{i=1}^{N} w_{i,t}^2 \sigma_{i,t}^2}{2\sum_{i=1}^{N-1}\sum_{j>i} w_{i,t} w_{j,t} \sigma_{i,t} \sigma_{j,t}}$$

In more intuitive terms this can be thought of as the following:

Implied Correlation equals -

Variance of the index – Variance at zero correlation

divided by

Variance at perfect correlation – Variance at zero correlation

$$\rho = \frac{\sigma_{I,t}^2 - \sigma_{Corr=0,t}^2}{\sigma_{Corr-1,t}^2 - \sigma_{Corr-0,t}^2}$$

[1]Note this description first appeared in *Credit Prism: "No Turn Unstoned"*, May 18, 2004.

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