QUANTITATIVE PORTFOLIO STRATEGY

Approximating Index Returns

Lev Dynkin
212-526-6302
ldynkin@barcap.com

Jay Hyman
+972 3 623 8745
jay.hyman@barcap.com

Michael Ng
212-526-6685
mikeng@barcap.com

www.barcap.com

To help streamline the management process, portfolio managers often find it convenient to boil down the characteristics of portfolios and indices to a very small number of key risk factor sensitivities, such as Treasury and spread duration. When reviewing portfolio performance over a given month, these can be combined with market statistics such as changes in yields and spreads to provide a back-of-the-envelope analysis of returns.

Sometimes, however, simple models of this type can be subject to large errors. As a litmus test for the usefulness of such a model, we examine the extent to which simple approximations can explain the returns on Barclays Capital indices. We focus on several months in which unusual events could lead to large approximation errors and show that subtle differences in the way such models are specified can have a great effect on their robustness.

We do not mean to suggest, by any means, that the simple models discussed here are the best available. Barclays Capital indices are fully transparent. The monthly returns of an index can be traced down to the return contributions of each security; these, in turn, are supported by complete information on their beginning and ending pricings. The Barclays Capital POINT system offers investors a powerful suite of portfolio analytics that include very detailed models for *ex-ante* risk analysis and *ex-post* performance attribution. The simple approximations discussed here are no substitute for these sophisticated models. Nevertheless, while fully cognizant of the fact that this approach cannot match the precision of more sophisticated models, many managers use simple approximations of the type discussed here to form an intuitive overview of performance.

A Simple Model … but Not as Simple as It Looks

We begin by presenting, in rather bare-bones form, a simple approximation of returns based on duration, convexity, and yield.

$$\frac{\Delta P}{P} \approx Y\Delta t - D\Delta Y + \tfrac{1}{2}C(\Delta Y)^2$$

The approximate return of a bond is, thus, expressed simply as a carry term proportional to yield, a first-order approximation of the return due to yield change, given by duration, and a second-order correction, given by convexity, reflecting the non-linearity of the price-yield relationship, which can become significant in months with large yield changes.

For portfolios with significant credit exposures, it is often desirable to expand this somewhat to separate the effects of changes in Treasury yields from those of spread changes. This can potentially give us two duration terms and three convexity terms:

$$\frac{\Delta P}{P} \approx Y\Delta t - D_T\Delta Y_T - D_S\Delta S + \tfrac{1}{2}C_T(\Delta Y_T)^2 + C_{TS}(\Delta Y_T\Delta S) + \tfrac{1}{2}C_S(\Delta S)^2$$

Certainly, if we ignore the convexity terms and focus on the first-order terms, these approximations both appear very simple at first glance. Yield gives us a carry return as time elapses, and changes in yield give us additional returns in proportion to duration. Yet when we try to carry out the calculation in practice, we quickly realize that these simple equations are written in such abstract terms that there are many ways to implement them. Let us begin with the simplest case: analyzing a single bond over a single month using equation (1). As soon as we try to plug in numbers, we realize that the simple notation used in equation (1) obscures the fact that we have mixed several point-in-time measures (yield, duration, and convexity) with several measures of change over time ($\Delta t, \Delta Y, \Delta S$). For each of the point-in-time measures, we will have two possible values at our disposal – one from the beginning of the month, and one from the end of the month. Which ones belong in our approximation?

Mathematically, the duration and convexity of a bond can be used to build a second-order approximation to the price-yield behavior of a bond to instantaneous changes in yield. That is, if we reduce equation (1) to a single point in time by removing the carry term, it correctly uses the duration of the bond to measure its sensitivity to yield changes. However, with the passage of time, the maturity of the bond shortens, and typically its duration will shorten as well, reducing its sensitivity to yield change at the horizon. Using the beginning-of-month duration in this case will leave us with a bias towards overstating the return effect of any yield change. One could therefore suggest that the most precise implementation of approximation (1) would be to use the beginning-of-month yield to calculate the carry return, but then to use the end-of-month duration and convexity to measure the effects of yield change.

$$\frac{\Delta P}{P_{beg}} \approx Y_{beg}\Delta t - D_{end}\Delta Y + \tfrac{1}{2} C_{end}\left(\Delta Y\right)^2$$

While the use of end-of-month sensitivities may provide more precise results, some applications preclude their use. If the goal of this exercise is to give a simple analysis of how the *ex-post* returns relate to the *ex-ante* risk exposures, then the measures of risk exposure (here, duration and convexity) must be numbers available at the beginning of the period, and only the risk factor changes (here, yield change) can be numbers that remain unknown until the end of the month. In this case, managers would use beginning-of-month values for duration and convexity and accept that this can cause some additional noise in the estimation.[1]

When the return to be explained is not a single bond, but a portfolio or an index, more questions arise. How exactly should one compute the index yield, duration, and convexity for this purpose? How exactly should one define “change in yield” or “change in spread” for the purposes of equation (2)? We take a look at some numerical examples to illustrate how different implementation decisions can give very different results in challenging months.

Example 1: January 2009 – Non-parallel Yield Curve Movement

In January 2009, the Barclays Capital US Credit Index earned a total return of -0.01%. This unremarkable final result masks several unusual events that happened simultaneously: Treasury yields increased dramatically, but in a highly non-parallel manner, and corporate spreads tightened significantly.

[1] An additional possibility would be to form beginning-of-month estimates of the end-of-month sensitivities, simply to reflect the additional passage of time. It is not clear, however, that this non-standard procedure would justify the effort involved.

Figure 1 shows a first attempt at implementing just the first three terms of equation (2), ignoring the convexity issues for now. The first row shows the key index statistics as of the end of December 2008: yield-to-worst, which will determine the carry; OAD, which will give the sensitivity to changes in Treasury yield; and option-adjusted spread duration (OASD), which will give the sensitivity to changes in spread.

Figure 1: Approximating the Credit Index Return for Jan 2009 (Naïve Approach)

	Yield	OAD	OASD	
Sensitivity	6.83	6.03	5.99	
	Time Elapsed	Trs Yld Chg	Spread Chg	
Change	0.08	0.40	-0.57	
	Carry	Rates	Spread	Total
Return	0.57	-2.41	3.41	1.57

Source: Barclays Capital

The change in Treasury yield is calculated in Figure 1 based on the yield-to-worst of the Treasury Index. This rose 40bp during January 2009, from 1.55 to 1.95. The spread change is taken as the difference between the beginning and ending index OAS levels for the Credit Index, which is equivalent to the market-value-weighted change in OAS across all bonds in the index. By this measure, spreads tightened 57bp. Multiplying the sensitivities by the appropriate risk factor changes (and by -1 as well for the duration terms), we arrive at the approximate return contributions from each of the three factors considered. Astonishingly, when we add them up, we find that we have estimated a return of 1.57% for the month – quite a far cry from the -0.01% that was actually earned.

While we are fully aware that this is only an approximation, and we expect some noise in our estimates, we would not expect this back-of-the-envelope calculation to be so completely misleading. What went wrong?

First of all, as we are all well aware, portfolio duration is a measure of sensitivity to a parallel shift in yields across the whole portfolio. When rates change in a non-parallel manner, it is critical to measure the change in rates that is most relevant to the portfolio or index of interest. In particular, the yield change of the Treasury Index may not necessarily be the most appropriate representation of the Treasury yield change that will affect the Credit Index. Figure 2 details six key rate duration exposures of the Credit Index as December 31, 2008, as well as the fitted Treasury yields for those six maturities as of that date and the subsequent yield change over January 2009 for each of these rates.

Figure 2: Treasury Yield Changes for Six Key Rates, January 2009

	6M	2Y	5Y	10Y	20Y	30Y
Credit Index KRD	0.07	0.69	1.62	1.67	1.08	0.90
Key Rate Bond OAD	0.41	1.90	4.77	8.98	14.75	19.27
Yield (beginning)	0.52	0.73	1.36	2.77	3.12	2.63
Yield (ending)	0.54	0.92	1.73	3.43	3.91	3.66
Yield Change	0.02	0.19	0.37	0.66	0.79	1.03

Source: Barclays Capital

A quick glance at Figure 2 reveals that the change in the Treasury yield curve during this month was about as far from parallel as it could be. Rates were virtually unchanged at the short end, but went up more than 100bp at the long end. Under these conditions, it is easy to see the shortcomings of an approach in which just a single duration number is used to represent the sensitivity to changes in rates. To get the fairest results, the number we use for the change in Treasury yields should be calculated based on the KRD profile of the Credit Index. If we weight the yield changes by these KRD weights, we find that the relevant Treasury yield change was +60bp, not +40bp as measured by the Treasury Index.

Similarly, we can ask what is the most appropriate way to measure the change in spread. It is clear that spread changes in bonds to which we have a greater spread duration exposure will have a greater effect on portfolio return. This is not reflected when we use market-value-weighting to measure the spread change. We are better off using dollar duration weighting to aggregate the spread change across all the bonds in the index.[2] If we do this for January 2009, we find that the average spread change of the Credit Index was -48bp, not -57bp as reported in Figure 1.

Figure 3: Approximating the Credit Index Return for January 2009 (Revised Approach)

	Yield	OAD	OASD	
Sensitivity	6.83	6.03	5.99	
	Time Elapsed	Trs Yld Chg	Spread Chg	
Change	0.08	0.60	-0.48	
	Carry	Rates	Spread	Total
Return	0.57	-3.62	2.88	-0.17

Source: Barclays Capital

In Figure 3, we revisit the approximation of Figure 1 using our revised estimates of Treasury yield change and spread change. We find that our three first-order return components now sum to -17bp and that the losses due to the rise in Treasury yields are greater in magnitude than the gains from spread tightening – and even large enough to wipe out the spread carry return as well.

Finally, we should briefly address the convexity issue. In most months, the convexity terms will all be very close to zero. The last two terms in equation (2) are difficult to measure precisely, as our index data provide only the Treasury convexity; for bullet bonds, it is a fair approximation to assume that all three convexities are equal. It should be noted that the convexity numbers that we report are scaled down by a factor of 100 to facilitate their use with yield changes expressed as percentages. For our example, using a yield change of 0.60% with the reported index convexity (OAC) of 0.70, the estimated return due to yield curve convexity is 0.5 * 0.70 * (0.60)^2 = 0.13. That is, the yield curve convexity effect would add about 13bp to the return. Spread convexity would generate a small positive return as well; but as yield and spread move in opposite directions, the cross-convexity term would provide a negative contribution that would probably mostly cancel out the two positives. Our conclusion: for a back-of-the-envelope type calculation such as this, the magnitudes of the convexity return terms are sufficiently small that they do not warrant the additional complexity that would be added to the calculations. If additional insight is warranted, it would probably be better to keep track of portfolio yield curve exposures at a

[2] We have previously demonstrated that duration-weighted spread changes correspond most directly to excess returns. See "Computing Excess Return of Spread Securities," in *Quantitative Management of Bond Portfolios*, L. Dynkin *et al*, Princeton Press, 2007.

more detailed level – if not a full key rate duration profile, then perhaps a coarse partitioning of duration contributions to long and short rates.

As a mid-point between keeping track of a single portfolio duration and six distinct KRD exposures, we tried a simple partition of the Credit Index and the Treasury Index into three maturity buckets: 1-5y, 5-10y, and over 10y to maturity. The monthly yield change in each of these buckets would be represented, as in our original naïve approach, by the market-weighted yield change of the appropriate component of the Treasury Index. The yield change applied to the portfolio, however, would combine these three yield changes after weighting them by the duration contributions of the three components of the Credit Index. This should be sufficient to capture, in a rough sense, any drastic reshaping of the curve. We then applied this approximation method, as well as the two methods illustrated above, to approximate the Credit Index returns on a monthly basis from July 2001 to October 2009. That return, according to each of these methods, was separated into a Treasury part and an excess return part by dividing the carry return into yield and spread components and adding these to the return components based on yield change and spread change, respectively. This allowed us to compare the estimated excess returns with those reported for the index and the estimated Treasury returns to the difference between the reported total and excess returns. In Figure 4, we report the tracking errors of each of the estimation techniques for the Treasury return, the excess return and the total return.

Figure 4: Tracking error volatilities, by return component, for three different estimation techniques, July 2001-October 2009

		Tracking Error Volatilities (bp/mo)		
Key Measure of Interest Rate Sensitivity	**Weighting of Spread Change**	**Treasury Return**	**Excess Return**	**Total Return**
Single duration	Market-weighted	40.3	28.5	43.2
Six key rate durations (KRDs)	OASD-weighted	7.0	15.7	14.2
Three bucketed duration contributions	OASD-weighted	11.2	15.7	17.2

Source: Barclays Capital

As we distinguish more clearly among our exposures to yield changes at different points along the curve, we see a clear improvement in our ability to approximate the actual performance of the target index or portfolio based on its macro characteristics. Similarly, the use of OASD-weighted spread changes gives a closer approximation than market-weighted spread changes. Furthermore, even the use of a simple approximation based on three maturity buckets can improve the accuracy substantially, even if it cannot quite achieve the performance obtainable using 6 KRDs.

Example 2: June 2009 – Credit Events

In our second example, we highlight certain types of events for which spread changes do not provide a good explanation of return and could generate misleading results. We examine a customized index of the US High Yield market with a 2% issuer cap. If we try to explain the returns of this index in June 2009, we obtain some very large errors, due to a number of irregular OAS changes. We suggest a practical approach that can provide more robust results in the case of similar events.

June 2009 was good for credit in general and High Yield in particular, characterized by a small rise in Treasury yields and a significant tightening of credit spreads. Our issuer-capped

High Yield Index turned in a return of 2.94%. If we carry out our analysis of return based on Treasury yield changes and spread changes, as in the previous section, we find that an index yield of 13.75% gives us a carry return of about 1.15% for the month and that a rate increase of about 0.11%, coupled with a beginning OAD of 4.36, gives a rates return contribution of -0.48%. The lion's share of index return for that month comes from the spread change component. Depending on how we measure the spread change, though, we can obtain wildly different results (Figure 5).

Figure 5: Approximating the Returns of a High Yield Issuer-Capped Index for June 2009

Weighting Method for Index Spread Change	OASD	Spread Change	Estimated Spread Return	Estimated Total Return
MV (beg)	4.28	-1.20	5.14	5.81
MV (beg) * OASD (beg)	4.28	-0.75	3.21	3.88
MV (beg) * OASD (end)	4.28	-0.53	2.27	2.94

Source: Barclays Capital

The first two rows depict the two ways to calculate spread changes that were compared in the previous discussion of the Credit Index. Once again, using simple market weighting to calculate the index spread change gives a large distortion; in this case, the estimated total return is nearly double the actual return of the index. Switching to weighting by beginning-of-month spread duration contributions gives an improvement, but we still find ourselves overstating returns by nearly 100bp. The third method, which relies on ending spread durations, gives a much better result. To understand why, we will need to take a closer look at several index events.

There is one bond in the index, from E*Trade Financial Corp, that underwent a major price recovery over the course of several months, from a dollar price of about 30 in March to about 60 at the start of June to over 100 at the end of the month. As a result, its OAS changed from 2783bp at the start of the month to -30130bp at the end. This nonsensical, large negative OAS is a result that occasionally occurs when a currently callable bond is priced significantly above the call price. While the market is clearly ignoring the call feature when pricing the bond, the option model still sees the call as active and near, and a large negative OAS is the only way to justify a price that is inconsistent with the call feature (YTW is negative as well for the same reason). Due to this effect, this one bond exhibits an OAS change of -32913 bp. Unfortunately, this is large enough to have a significant effect on the average OAS change at the index level as well.

In addition, there are quite a number of bonds, from GM and other issuers, that officially defaulted during the month. As the market largely anticipated this outcome, there is no big negative return associated with this event. We find that the returns for these bonds are mixed, and the average actually comes out positive for the group. However, this leads to a technical problem. Since the bonds have defaulted, analytics are no longer calculated at the end of the month, so the durations and spreads are all populated with zeros at month-end. If we calculate spread change mechanically for such a bond, we may find that the beginning spread was 6000bp and the ending spread is zero, such that the monthly OAS change for this bond shows up as -6000bp.

In Figures 6 and 7, we break down the index into three categories (the two mentioned above and all the rest) and show how they contribute to the calculation of OAS change at the index level when we use market-weighted averaging (Figure 6) and dollar-duration-weighted averaging (Figure 7). In particular, we chose to apply dollar-duration weights obtained using the product of beginning percentage of MV and ending OASD.

Figure 6: Calculating Market-Weighted Spread Change in the High-Yield Issuer-Capped index for June 2009

	Num Bonds	Percentage of Beginning MV	Change in OAS Weighted by MV Beg	Contrib to Index OAS Change
E-Trade Bond	1	0.06%	-32913.3	-18.7
Defaulted Bonds	38	0.79%	-5639.3	-44.4
Rest of Index	1527	99.15%	-57.3	-56.9
Total	1566	100.00%		-120.0

Source: Barclays Capital

Figure 7: Calculating Reweighted Spread Change in the High-Yield Issuer-Capped index for June 2009

	Num Bonds	Index Weight by MV_{beg} * OAD_{end}	Change in OAS Weighted by MV_{beg} * OAD_{end}	Contrib to Index OAS Change
E-Trade Bond	1	0.0003%	-32913.3	-0.1
Defaulted Bonds	38	0.0000%	N/A	0.0
Rest of Index	1527	99.9997%	-52.7	-52.7
Total	1566	100.0000%		-52.8

Source: Barclays Capital

On the bulk of the index (1527 bonds) for which spreads are positive both beginning and end, the differences between the MV-weighted spread change (-57.3 bp) and the DDur-weighted spread change (-52.7 bp) are not particularly great. The MV-weighted spread change, however, is very sensitive to these large reported OAS changes in less than 1% of the index; these bring the overall average spread change from -57.3 to -120. The DDur-weighted calculation happens to be robust to these errors, since in the cases in which the ending OAS goes to zero, the ending duration goes to zero as well. Similarly, for the immediately callable bond for which the OAS becomes very negative, the ending duration is also very short; OASD for this bond is 0.02, giving it a near zero contribution to the duration-weighted calculation.

If instead of weighting the individual bond spread changes themselves by beginning MV and ending OASD, we just take the dollar-duration-weighted average OAS at both the beginning and end of the month and subtract the two, we get a spread change of -55.1 bp, not much different from the calculation shown here.

Conclusion

Simple approximations of index returns using aggregate index characteristics will always be imperfect. However, to the extent investors insist on having them, a few simple steps can be taken to make these approximations better.

First, if the entire complex change in the shape of the Treasury curve has to be reduced to a single yield shift, this should be done based on the relevant part of the yield curve that best corresponds to the rates exposures of the index in question, not necessarily the Treasury

Index. For example, if we are trying to approximate the return of the Intermediate Credit Index, the changes in the long part of the Treasury curve should be irrelevant. The best way to do this would be to use multiple exposures to different parts of the curve; we have illustrated the use of six KRDs (key-rate durations), as well as a simpler approximation based on three maturity buckets. Either of these methods is clearly preferable to using a single duration number, as they allow the measurement of sensitivity to non-parallel yield curve movements.

Second, when multiplying the OASD of the index by an average spread change, one should be very careful about the definition of that average. The simple market-weighted average change in OAS is not very robust and may be easily distorted by extreme outliers. As we illustrated, these can occur either due to credit events such as default or because of currently callable bonds trading above the call price with high negative OAS when the market knows that the issuer will not exercise the call. Weighting the change in OAS by dollar duration (OASD*MV) reduces the effect of these outliers on the average number because the OASD for a bond in distress or a currently callable bond is low. Also, dollar-duration-weighted yield (and spread) are much better approximations of the portfolio IRR.

Following these recommendations may drastically improve the precision of a simple approximation of the index return and, in many instances, give answers close to the actual index return.

Analyst Certification(s)

We, Lev Dynkin, Jay Hyman and Michael Ng, hereby certify (1) that the views expressed in this research report accurately reflect our personal views about any or all of the subject securities or issuers referred to in this research report and (2) no part of our compensation was, is or will be directly or indirectly related to the specific recommendations or views expressed in this research report.

Important Disclosures

For current important disclosures regarding companies that are the subject of this research report, please send a written request to: Barclays Capital Research Compliance, 745 Seventh Avenue, 17th Floor, New York, NY 10019 or refer to https://ecommerce.barcap.com/research/cgi-bin/all/disclosuresSearch.pl or call 212-526-1072.

Barclays Capital does and seeks to do business with companies covered in its research reports. As a result, investors should be aware that Barclays Capital may have a conflict of interest that could affect the objectivity of this report. Any reference to Barclays Capital includes its affiliates. Barclays Capital and/or an affiliate thereof (the "firm") regularly trades, generally deals as principal and generally provides liquidity (as market maker or otherwise) in the debt securities that are the subject of this research report (and related derivatives thereof). The firm's proprietary trading accounts may have either a long and / or short position in such securities and / or derivative instruments, which may pose a conflict with the interests of investing customers. Where permitted and subject to appropriate information barrier restrictions, the firm's fixed income research analysts regularly interact with its trading desk personnel to determine current prices of fixed income securities. The firm's fixed income research analyst(s) receive compensation based on various factors including, but not limited to, the quality of their work, the overall performance of the firm (including the profitability of the investment banking department), the profitability and revenues of the Fixed Income Division and the outstanding principal amount and trading value of, the profitability of, and the potential interest of the firms investing clients in research with respect to, the asset class covered by the analyst. To the extent that any historical pricing information was obtained from Barclays Capital trading desks, the firm makes no representation that it is accurate or complete. All levels, prices and spreads are historical and do not represent current market levels, prices or spreads, some or all of which may have changed since the publication of this document. Barclays Capital produces a variety of research products including, but not limited to, fundamental analysis, equity-linked analysis, quantitative analysis, and trade ideas. Recommendations contained in one type of research product may differ from recommendations contained in other types of research products, whether as a result of differing time horizons, methodologies, or otherwise.

US13838