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A Hybrid Credit–Rates Model With Applications

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Abstract

We describe a model to incorporate credit risk into the valuation of defaultable fixed income instruments, in which a stochastic interest rate process governed by one-factor Hull-White dynamics has its state space augmented by an absorbing default state. At any point in time in the life of the instrument, the deterministic term structure of credit spreads for the issuer dictates the risk-neutral probability of entering the default state, in which the the ex-default value of the instrument is a predetermined, deterministic recovery level. Although lacking credit volatility and correlation, this model provides a valuation adequate for risk management and relative value identification.

Keywords. Credit risk, default probability, credit default swap, LGM, Hull-White, credit-rates hybrid, bonds, structured notes.

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1 Model

The classical one-factor Hull-White model — one of the most commonly used models to price interest rate derivatives — has a short rate $r(t)$ with risk-neutral dynamics given by

$$r(t) = \theta(t) + x(t);$$
$$dx(t) = \kappa(t)x(t)dt + \sigma(t)dW(t); x(0) = 0,$$

where the short rate reverts at speed $\kappa(t)$ to a long term mean level $\theta(t)$ and diffuses with instantaneous volatility $\sigma(t)$, and W(t) denotes standard Brownian motion. This analytically solvable stochastic differential equation results in a normally distributed short rate process $r(t)$ and a log-normally distributed zero-coupon bond price process $P(t;T)$, $t \leq T$. Given a numeraire $N(t; x(t))$, the value $V(t)$ of a default-free contingent claim $C(T; x(T))$ is obtained via standard arbitrage pricing theory as the risk-neutral expectation

$$V(t) = N(t; x(t))\mathbb{E}\left[\frac{C(T; x(T))}{N(T; x(T))} \mid F(t)\right], \tag{1}$$

where $F(t)$ denotes the filtration, or the information accumulated up to time t. Depending on the complexity of the instrument, the method of choice to implement this valuation formula may be an analytic formula, numerical quadrature, a binomial tree, a partial differential equation or a Monte Carlo method.

One approach to incorporating credit risk into this model hinges on augmenting the interest rate dynamics with an independent Poisson default process and the interest rate state space with an absorbing default state. Any contingent claim is then priced as a risk-neutral expectation of partial prices that condition upon the default event. We illustrate this approach using a simple example of a bond. Consider the valuation of a defaultable bond of maturity T, let $\tau > 0$ denote its random default time, and assume that upon default, the instrument recovers a fixed fraction R of par. By assumption, the default time is independent of the state variable $x(t)$. Then the defaultable bond value $V^D(t)$ (the superscript D denoting the presence of default risk, in contrast with the default-free value in (1) per unit notional of the defaultable bond is simply

$$\begin{aligned} V^D(t) &= Q_t(\tau > t)V^D(t \mid \tau > t) + (1 - Q_t(\tau \leq t))V^D(t \mid \tau \leq t) \\ &= Q_t(\tau > t)V(t) + (1 - Q_t(\tau \leq t))R, \end{aligned} \tag{2}$$

where Q_t denotes the risk-neutral default distribution of the issuer at time t and $V(t)$, as obtained in (1), is the default-free valuation of the bond. To obtain the defaultable present value $V^D(0)$, we create a discrete timeline $0 = t_0 < t_1 < \ldots < t_n = T$ including the dates corresponding to events of interest such as coupon cashflows, exercises of embedded option, etc. A rollback from the terminal bond maturity date T, applying (2) to determine $V^D(t_i)$ at each time step t_i, eventually yields the defaultable present value $V^D(0)$ of the bond. In the current implementation of this model, credit volatility is ignored; Q_t is the same distribution for all t and is obtained by stripping the bond issuers credit curve; see [FNS09] for details. The default-free value $V(t)$ is obtained using Bloombergs one-factor LGM (Linear Gauss Markov) model [Hag05], which essentially employs Hull-White dynamics, but with a nonstandard (non-money market) numeraire $N(t; x(t))$ and an analytically obtained transition density for the state variable $x(t)$, to compute the risk-neutral expectation in (1).

2 Credit Relative Value

This model can be used for rich/cheap analysis across different bonds from the same issuer, or from different issuers of similar credit quality, in the following manner. In the same spirit as the classical option-adjusted spread (OAS), define the default-adjusted spread (DAS) as the spread above the yield curve needed to make the model price $V^D(0)$ match the quoted price in the market. For two bonds differing in issuer credit ratings (and possibly differing also in maturity, coupon or embedded optionality), the DAS serves as an indication of relative value adjusted for credit risk; the bond with the higher DAS is relatively cheaper. This kind of analysis as implemented in the Bloomberg function **VCDS<GO>**is shown in Figure 1. Note that the DAS as denoted in that function is defined as the spread below the yield curve at which the model price and market price coincide, and as such, will have the opposite sign compared to the typical convention used in OAS analysis.

3 Credit Risk Management

This model and the DAS computation above also provide a straightforward handle on credit risk in a bond portfolio. For each bond in the portfolio, we first compute the DAS, then shift the yield curve (up or down, depending on the DAS convention used); this will adjust for the model-implied baseline level of default risk by aligning the model price with the market price of the bond. Then we may calculate, for instance, various metrics of credit exposure such a Credit DV01 (CR DV01), time-bucketed Credit Key-Rate Deltas (CR KRD), Credit Scenario Anaysis or even a Credit Value-at-Risk (CR VaR) by simply perturbing the credit curve appropriately (small bumps for sensitivities, large shocks for scenario analysis, or in accordance to a historical or model-implied distribution for VaR), and revaluing the bond.

The Bloomberg function **HGBD<GO>**shown in Figure 2 uses the CR DV01 and the IR DV01 provided by this model to calculate hedging positions in credit default swaps and interest rate swaps that make the combined portfolio delta neutral with respect to credit and interest rate risk.

4 Other Methods

An intuitive and even simpler approach to incorporating credit risk is to simply employ risky discounting from an issuer-specific default-adjusted par curve as described in [NS07]. In this approach, the model price V D(0) of a bond is simply computed as a sum of risky cashflows

$$V^D(0) = R\int_0^T P(0,t)\,dQ(\tau \le t) + C\sum_{i=1}^{n} \alpha_i P(0,t_i)(1-Q_{\tau\le t_i}) + P(0,T)(1-Q(\tau\le T)$$

where $P(0;t)$ is the risk-free discount factor to time t. Setting $V^D(0) = 1$ and solving for C yields a risky par coupon, and when repeated for increasing maturities, a risky par curve. As explained in [Fle07], this approach would produce pricing errors that are accentuated as credit spreads get higher, or as the bond drifts away from par, or with significant term structure in credit spreads. The hybrid model described earlier has the following advantages:

- credit spread term structure is properly handled;
- by coupling with a full-fledged dynamic interest rate model, embedded options are properly handled; and
- credit risk impact on the exercise of embedded options are better handled than simply with an "all-in" risky discount factor.

However, this model as presented lacks credit volatility and correlation of default risk with rates, and will likely not produce a valuation sufficiently accurate for trading applications. One of the main difficulties in addressing these shortcomings is the lack of liquid market in single-name credit default swaptions to which the credit volatility and correlation to rates may be calibrated.

References

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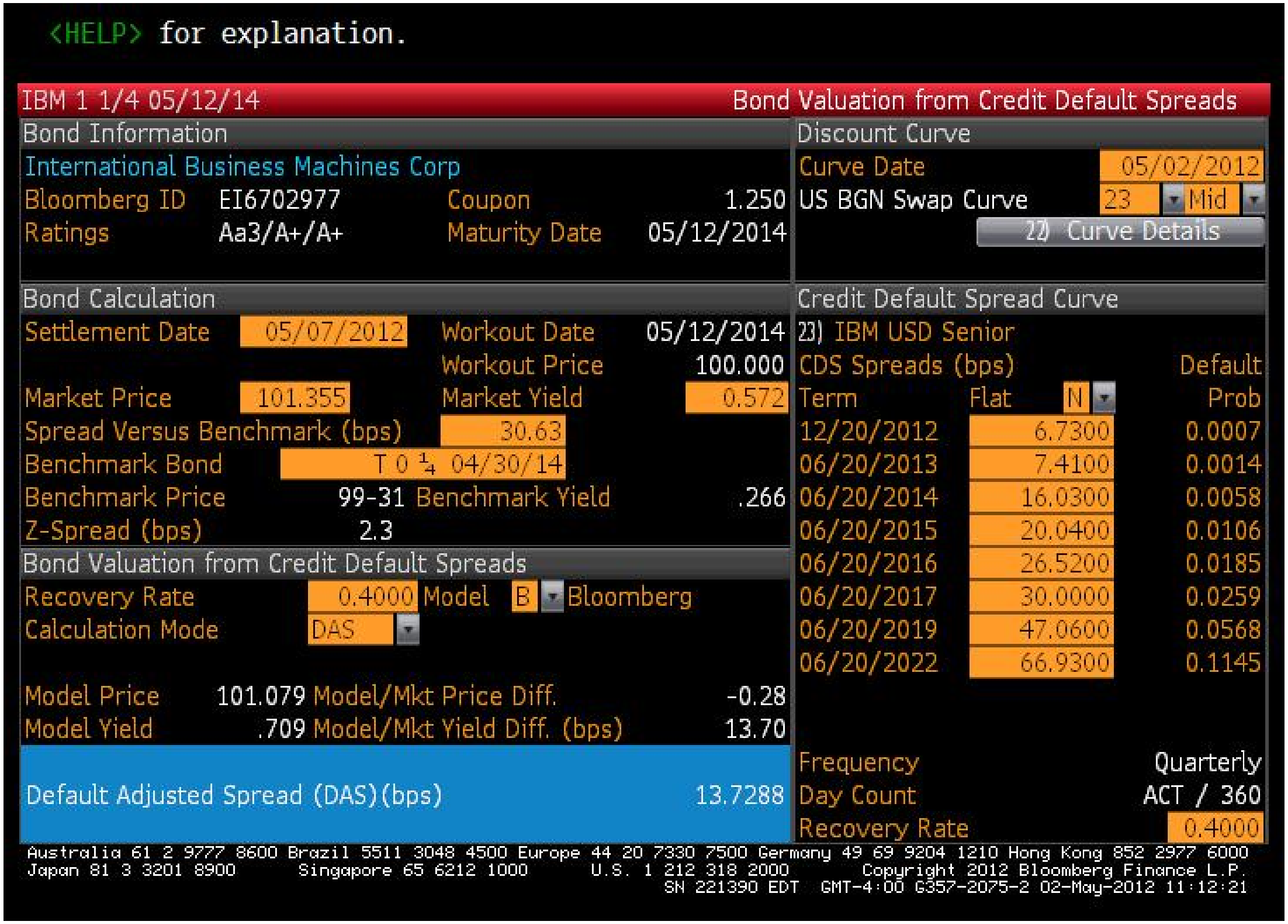

Figure 1: Relative value analysis of corporate bonds using **VCDS<GO>**.

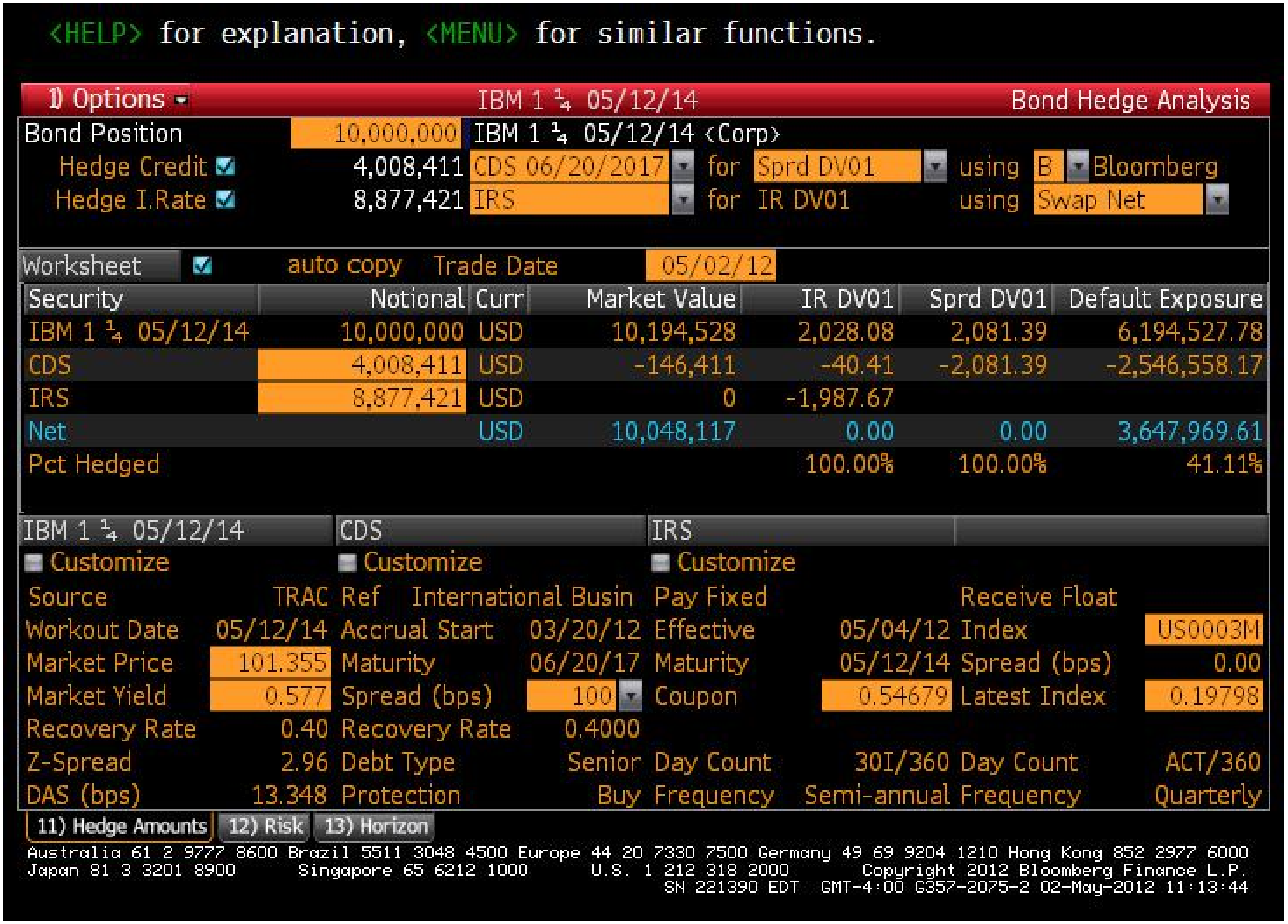

Figure 2: Delta hedging credit and interest rate risk in a corporate bond using **HGBD<GO>**.