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The New Lehman Brothers High Yield Risk Model

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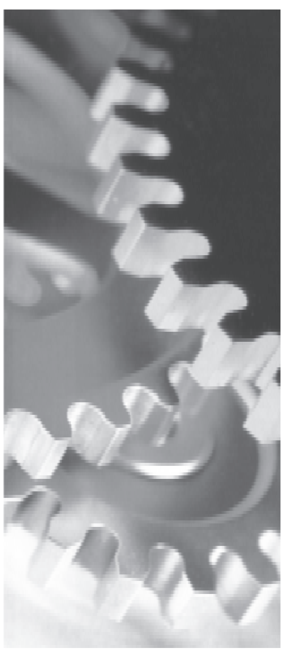

LEHMAN BROTHERS

The New Lehman Brothers High Yield Risk Model[1]

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We describe the new Lehman Brothers high yield risk model, which simultaneously models market spread risk and default risk. The market spread risk arises from changes in the level of credit spreads (or yields) in the mark-to-market of a credit product; default risk is the risk that an obligor fails to meet its contractual obligation. Our risk model provides an integrated framework for quantifying both risks.

1. INTRODUCTION

For years, the Lehman Brothers multifactor risk model has served as an effective risk management tool for fixed income money managers. It provides investors with the means to quantify the risk that portfolio returns will deviate from those of their benchmark. Such a risk measure is known as Tracking Error Volatility (TEV) – the "predicted" standard deviation of the relative return of the portfolio with respect to a benchmark. The current version of the risk model covers all the sectors of the U.S. Aggregate Index including Treasury, Agency, Credit, MBS, CMBS and ABS. In this article we introduce a new component of the risk model which covers a non-index asset class: High Yield.

Although high yield is not included in the U.S. Aggregate Index, its importance has grown rapidly in recent years due to the economic downturn and corporate debt overhung from the late 1990s. In the last two years, we have witnessed a steady stream of corporate credit rating downgrades, and the default rate has hit a historic high. Many issuers that had investment grade ratings have been downgraded to high yield or even distressed. Portfolio managers who used to hold investment grade bonds were left with a large amount of high yield bonds in their portfolios. All of this called for a credit risk model which not only covers investment grade, but also the high yield sector.

We begin our analysis with a brief review of our current investment grade risk model, and discuss its limitations when we apply the same methodology to high yield securities. We then lay out our new high yield risk model and illustrate some of the output available on our portfolio analytics platform – POINT.

1.1. Review of Lehman Brothers Investment Grade Credit Risk Model

The current investment grade risk model only covers investment grade corporate bonds – those rated BBB or higher. We will review this model briefly and relate it to the high yield model. A detailed description of the investment grade risk model can be found in Naldi, Chu and Wang (2002).

The investment grade risk model first decomposes the total return of a bond into a deterministic part and different stochastic parts:

$$Tot\ R_t = Carry\ R_t + YC\ R_t + Vol\ R_t + Sprd\ R_t \qquad (1)$$

[1] We would like to thank Michael Anderson, Arthur Berd, Lev Dynkin, Michael Guarnieri, Mark Howard, Jay Hyman, Dev Joneja, Alex Kirk, Roy Mashal, Marco Naldi, Claus Pedersen and Stuart Turnbull for their valuable comments and suggestions. Gary Wang and Larry Chen contributed to the development and implementation of the risk model in Lehman Brothers portfolio analytic platform - POINT.

The deterministic part is the carry of the bond, and includes its coupon return and the return due to the passage of time. The stochastic part of the return, which depends on changes in market conditions, is further decomposed into yield curve return, volatility return and spread return. Each component will be modeled as a linear combination of a set of risk factors.

The yield curve return is modeled as depending on changes in six benchmark yields, corresponding to bonds with a maturity of 6 months and 2, 5, 10, 20 and 30 years. The sensitivity of a bond's returns to each of these key rates is measured by its key-rate duration. Therefore, the model of the yield curve return is:

$$YC\ R_t \approx -\sum_{i=1}^{6} KRD_{i,t-1} * \Delta KR_{i,t} + OAC_{t-1} * \left(\overline{\Delta KR}_t\right)^2 \tag{2}$$

where the six key-rate changes, $\Delta KR_{i,t}$, and the squared average key-rate change, $\left(\overline{\Delta KR}_t\right)^2$, represent observable systematic factors, loaded by the bond's key-rate durations (*KRD*) and option-adjusted convexity (*OAC*) respectively.

The volatility return is non-zero only for bonds with embedded options. It is modeled as:

$$Vol\ R_t \approx \frac{100}{Price_{t-1}} * Vega_{t-1} * F_t^{Vol} = -VolDur_{t-1} * F_t^{Vol} \tag{3}$$

where F_t^{Vol} is a latent, non-observable volatility factor with loading as volatility duration.

The spread return for investment grade bonds is modeled primarily on the basis of its industry sector and credit rating. We use nine industry sectors and three credit qualities. Each of the 27 buckets has its own spread factor which can be interpreted as the average OAS change of the bucket. Beyond those 27 bucket factors, we also consider maturity effect, OAS effect and country effect. We have the following setup for spread return:

$$\mathrm{SprdR_t} \approx -OASD_{t-1}\left[F_t^{cell} + \left(TTM_{t-1} - \overline{TTM}_{t-1}^{cell}\right)F_t^{Twist} + \left(OAS_{t-1} - \overline{OAS}_{t-1}^{cell}\right)F_t^{OAS} + F_t^{nonUS}\right] \tag{4}$$

where TTM is the time to maturity and $\overline{x}^{cell}$ represents the median value of variable x in the corresponding issuer group. The realizations of the unobserved latent factors $F_t^{Vol}, F_t^{Cell}, F_t^{Twist}$, F_t^{OAS} and F_t^{nonUS} are all estimated by cross-sectional regressions.

Finally we combine the three individual linear factor models for yield curve, volatility and spread return to provide a grand multi-factor linear risk model for total return as following:

$$R_t^i = Carry_{t-1}^i + L_{t-1}^i F_t' + \varepsilon_t^i \tag{5}$$

where F_t is the (1 x K) vector of the systematic factors while L_{t-1}^i is the corresponding vector of risk exposures; and ε_t^i is the idiosyncratic return. The idiosyncratic component is the part of risk which cannot be explained by the systematic factors: idiosyncratic risks are issuer-specific and independent of each other.

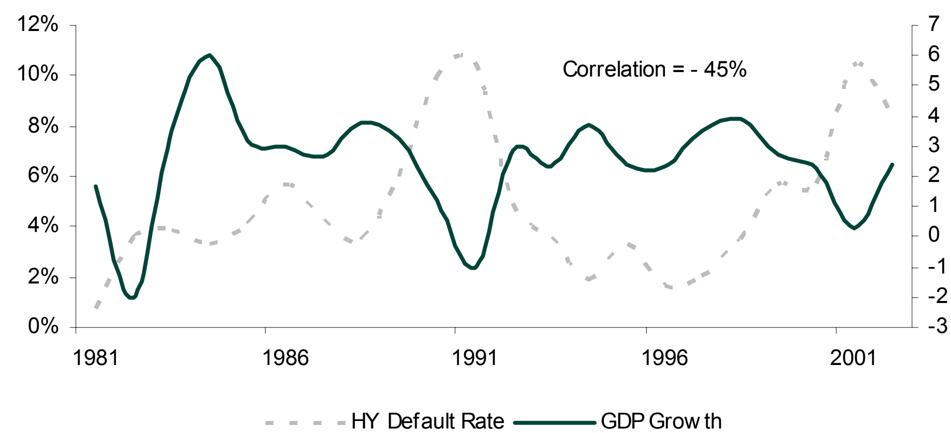
1.2. Motivation behind the New High Yield Risk Model

Having reviewed the current investment grade risk model, we now briefly discuss its limitations when applied to high yield securities. In general, the broad concept of *credit risk* includes *market spread risk* and *default risk*. *Default risk* is the risk that an issuer fails to meet its contractual obligation in a given period of time. *Market spread risk* is the risk due to fluctuation in the level of credit spread, i.e. the price of the bond, supposing that the issuer does not default in the given period. The *market spread risk* depends on investors' assessment of *future* default risk and the liquidity of the underlying bonds.

In addition, when portfolio effects are calculated, we have to take correlation effects into account. Like *market spread risk*, it is widely acknowledged that *default risk* is also a systematic risk, and empirical evidence shows the existence of credit contagion[2]. A number of studies, for example Duffee (1998) and Keenan (2000), have found that aggregate default rates are related to general macro-economic factors and business cycle indicators. The financial distress of one firm can directly trigger the distress of other firms. Intuitively, it is easy to imagine that Global Crossing is more likely to default in a period when WorldCom is in Chapter 11.

Figure 1 shows the aggregate annual default rate for US high yield issues from 1981 to 2002. On the same figure we also plot the annual GDP growth for the US. The negative correlation between default rate and GDP growth is obvious: during boom periods when the economy is doing well, the aggregate default rate is low; during recession periods, the aggregate default rate is high and corporates tend to fall into financial difficulties. For instance, during the recessions of 1990–91 and 2001–03, the aggregate default rate was at historic highs, illustrating that default risk is systematic and relates to macro-economic conditions.

Figure 1. HY Default Rate and GDP Growth (1981 – 2002)



Source: Moody's Investor Service and Lehman Brothers calculations.

Systematic default risk has important implications for high yield credit portfolios. It indicates that default risk is not idiosyncratic and cannot totally be diversified simply by adding new issues to the portfolio. It should be noted that the issue of default is not important when we are calculating the Tracking Error Volatility of investment grade portfolios. The likelihood of

[2] *Credit contagion refers to the propagation of economic distress from one firm to another.*

default is very small for an issuer rated BBB or higher. However, the problems are more serious for high yield bonds, for which the annual probability of default might be as high as 35% for a CCC issuer. Understanding and quantifying such a systematic risk becomes an important issue for credit money managers.

The current investment grade risk model focuses on market spread risk and does not explicitly model default risk. As Naldi, Chu and Wang (2002) mentioned, the investment grade model uses a robust technique to estimate factor realizations and eliminate sample outliers. As a consequence, the investment grade risk model implicitly treats default events as uncorrelated idiosyncratic events and ignores the systematic correlation between different issuers. In this paper, we extend the current investment grade risk model to create a framework which not only accounts for movement of credit spreads, but also takes default risk into account. We develop a very simple conceptual model of default in which issuer default is unpredictable and random, and can occur over the next time period with a probability that depends on current conditions. We keep the traditional multi-factor framework for the component of market spread risk. In the next section, we give a detailed description of the new high yield risk model.

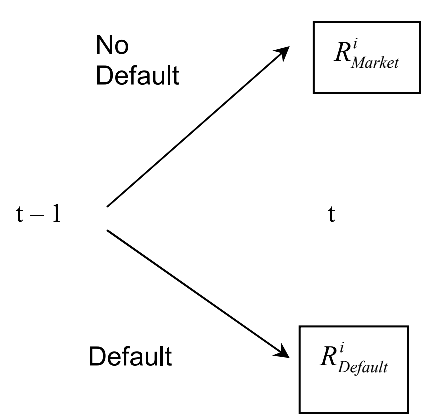
2. AN INTEGRATED HIGH YIELD RISK MODEL

The starting point for the high yield risk model is return decomposition conditional on the event of default, with the return of the bond written as:

$$R_t^i = (1 - I^i) R_{Market}^i + I^i R_{Default}^i \tag{6}$$

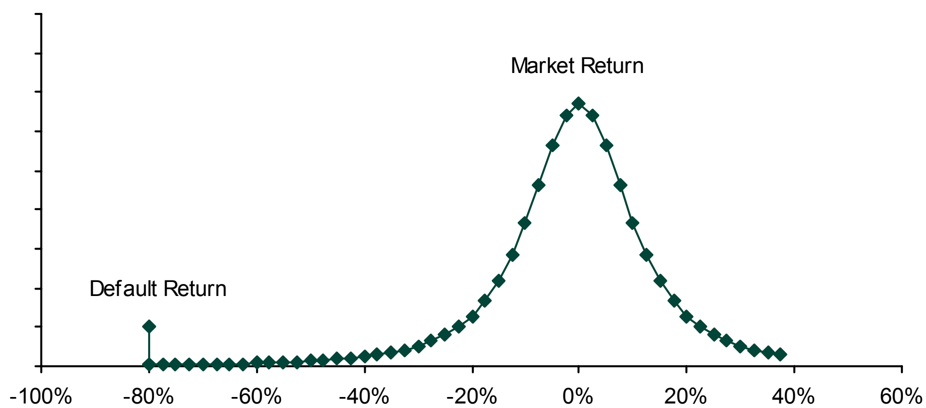
Here I^i is an indicative random variable for default event – it has a value of 1 if the issuer defaults in period t, and 0 otherwise. At the end of the period from t–1 to t, conditional on no default, the return for the bond will be R_{Market}^i. However, if the firm does default during the period, the return will be $R_{Default}^i$. From a continuous time perspective, equation (6) states that the price of a credit can be viewed as the combination of a diffusion process R_{Market}^i and a jump process I^i. Figure 2 illustrates this concept.

Figure 2. Market Spread Return and Default Return



The market return, R^i_{Market}, depends on the movement of interest rates, credit spreads and implied volatility, and the default return, $R^i_{Default}$, depends on the recovery value upon default. Figure 3 illustrates hypothetical distributions for the market spread return and the default return for a single issue.

Figure 3. Distributions of Market Return and Default Return (Single Issue)



We now show the detailed setup for the market spread return and default return separately.

2.1. Market Risk

The setup for market spread return is similar to the investment grade risk model. As we have discussed in section 1, we first decompose the total market return into time return, treasury return, volatility return and spread (residual) return, as in equation (1). The setup for volatility return and treasury return are exactly the same as all other risk models as in equations (2) and (3). However, we should point out that for high yield bonds, only spread (residual) return is significant. The dependence on the Treasury curve or even the volatility surface is relatively small. We retain the yield curve return and volatility return in the split only for the sake of consistency with the other risk models.

The spread return is modeled similarly to that in the investment grade risk model, based on sector and rating. The high yield sector and rating partitions are shown in Figure 4. The partitions are chosen so that there exist enough issues in each bucket along the history (to ensure that the estimation of systematic factors is not dominated by a couple of large issuers) and the issues in each bucket are highly correlated (to ensure that the systematic factors represent the common co-movements for a certain industrial group).[3] In general, the high yield sector partition is different from the investment grade partition.

[3] *We thank Michael Guarnieri for his input on HY partition.*

Figure 4. High-Yield Partition

BB/B	Below B
Basic Industry Cyclical Capital Goods Communication Energy Financial Media Non - Cyclical Technology Transportation Utility	Distressed Group

Bonds rated single-B and above are grouped into 11 sector buckets. The factor loading for each sector factor is simply the option-adjusted spread duration (OASD) of the bond. The corresponding bucket factor can be interpreted as the average OAS change for the sector. Beside the 11 sector factors, we have a twist factor to capture the slope of the spread curve and an OAS factor to differentiate high OAS bonds from low OAS bonds. The loadings for these two factors are the bond's relative time to maturity and OAS with respect to its sector median. The following equation summarizes the details of the spread model for bonds rated B and above:

$$\mathrm{SprdR_t} = -OASD_{t-1}\left[F_t^{Sector} + \left(TTM_{t-1} - \overline{TTM}_{t-1}^{Sector}\right)F_t^{Twist} + \left(OAS_{t-1} - \overline{OAS}_{t-1}^{Sector}\right)F_t^{OAS}\right] + \varepsilon_t \qquad (7)$$

All bonds with a rating of CCC and below are grouped into one single distressed bucket. In general, the spread return for a bond can be roughly approximated by $OASD \times \Delta OAS$. However, such a first order approximation no longer works for the distressed sector because ΔOAS is no longer a small number. In our risk model, we use unit instead of OASD as the loading for the distressed bucket factor, and the factor itself should be interpreted as the average *excess return* of the distressed group.

In addition to the bucket factor, we have four more factors for the distressed sector: a twist factor, a price factor, a leverage factor and a collateral-type factor. The twist factor measures the maturity effect, and its loading is simply the time to maturity of the bond relative to the median of the group. The price factor is similar to the OAS factor for non-distressed bonds and its loading is the relative price of the bond. The leverage factor differentiates between the returns of highly leveraged firms and those of low leveraged firms. Its loading is the relative leverage ratio[4] with respect to its industry peers. Finally, we have a collateral factor which is designed to capture the common movement for subordinated bonds. Only subordinated bonds are exposed to this factor. The following equation summarizes the spread model for distressed bonds:

$$\begin{aligned}\mathrm{SprdR_t} &= F_t^{Distress} + \left(TTM_{t-1} - \overline{TTM}_{t-1}^{Distress}\right)F_t^{Twist} + \left(P_{t-1} - \overline{P}_{t-1}^{Distress}\right)F_t^{\mathrm{Pr}ice} \\ &\quad + (LEV_{t-1} - \overline{LEV}_{t-1}^{Sector})F_t^{LEV} + I^{Sub}F_t^{Sub} + \varepsilon_t\end{aligned} \qquad (8)$$

4 *Leverage Ratio is defined as:*
(Long-Term Debt + Short-Term Debt) / (Long-Term Debt + Short-Term Debt + Market Value).

As with all other risk models, all spread factors are latent factors and will be estimated by a monthly cross sectional regression. Figure 5 summarizes all high yield market spread factors.

Figure 5. Systematic Market Spread Factors

BB & B		Below B	
Basic Industry	OASD	Distressed	Unit
Cyclical	OASD	Twist Distressed	MAT
Capital Goods	OASD	Price Distressed	Price
Communication	OASD	Leverage	Leverage
Energy	OASD	Subordinated	Unit
Financial	OASD		
Media	OASD		
Non-Cyclical	OASD		
Technology	OASD		
Transportation	OASD		
Utility	OASD		
Twist (BB&B)	OASD*MAT		
OAS (BB&B)	OASD*OAS		

The systematic risk factors (including all yield curve, volatility and spread factors) explain the systematic portion of the market return. The part of the market return which cannot be explained by those systematic factors are idiosyncratic risks. The idiosyncratic risks are issuer-specific and independent of each other. We use the factor regression residuals to estimate the idiosyncratic variances based on issuers' industrial sector and OASD (for non-distressed sectors only). Compared with investment grade bonds, high yield bonds in general have a much higher idiosyncratic variance. Figure 6 shows the idiosyncratic spread volatility for different investment grade and high yield buckets over the period of 1997–2003.

Figure 6. Idiosyncratic Spread Volatility (bp/month) (1997–2003)

	AAA/AA	A	BBB		BB/B
Banking And Brokerage	11	19	25	Basic Industry	93
Financial Companies, Insurance And Reits	10	21	30	Cyclical	87
Basic Industry	8	13	28	Capital Goods	83
Communication And Technology	17	26	33	Communication	144
Consumer Cyclical	8	16	31	Energy	65
Consumer Non-Cyclical	10	13	25	Financial	91
Energy And Transportation	12	13	18	Media	90
Utility	14	19	41	Non – Cyclical	74
Non-Corporate	10	22	32	Technology	126
				Transportation	109
				Utility	86
Average	11	18	29		95

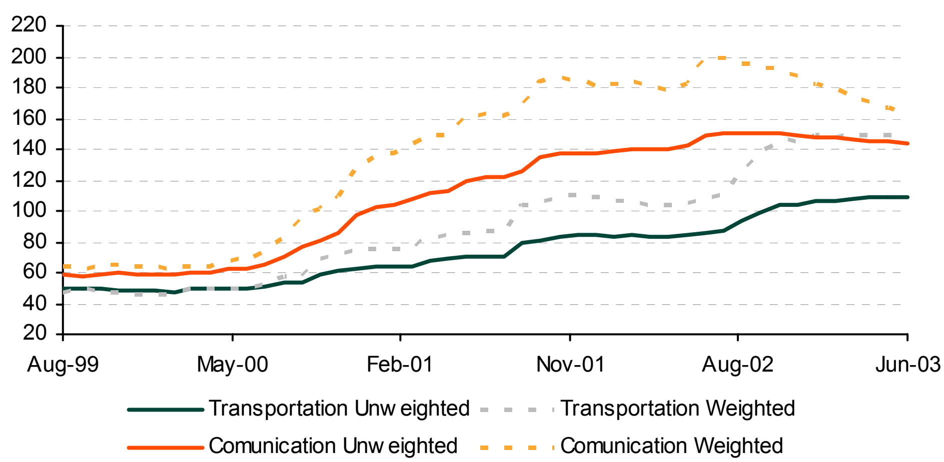
On average, the idiosyncratic spread volatility of high yield (BB/B) bonds is eight times that of AAA/AA bonds (correspondingly, five and three times that of A and BBB bonds). Such a difference has a major impact on portfolio diversification strategies. In general, the idiosyncratic volatility of a portfolio is proportional to $1/\sqrt{N}$, where N is the number of issues in the portfolio[5]. Hence, to diversify away idiosyncratic risk, a BB/B portfolio needs as many as 25 (= 5^2) times the issues as a single-A portfolio.

Another important feature of our high yield risk model is that we offer investors two options to calculate the market spread volatility[6]: a time-weighted scheme and an equal-weighted scheme. The equal-weighted scheme puts equal weight on all historical observations whereas the time-weighted scheme puts more weight on recent observations. Time-weighting is extremely important for idiosyncratic risk because of recent credit blow-ups. In 2001−02, we witnessed a lot of company-specific financial turmoil, for example WorldCom and Enron. As a consequence, the issuer-specific idiosyncratic variance increased dramatically for some industrial sectors. By offering investors two alternative ways to calculate volatility, our risk model is flexible enough to address the issue of time-varying volatility.

In Figures 7a and 7b, we show the time-varying idiosyncratic spread volatility for different industrials. The time variability of volatility is clearly sector and industry dependent. For instance, the time-weighted volatility is much higher than the unweighted volatility for such sectors as Transportation and Communication due to the idiosyncratic events that took place in 2001; whereas for such sectors as Consumer Non-cyclical and Energy, the difference between time-weighted volatility and unweighted volatility is small, which implies relatively stable spread volatility in the last decade.

The time variability for the systematic covariance matrix is not as profound as for the idiosyncratic variance. Nevertheless, we still offer investors two alternative time weighting schemes to calculate the systematic covariance matrix.

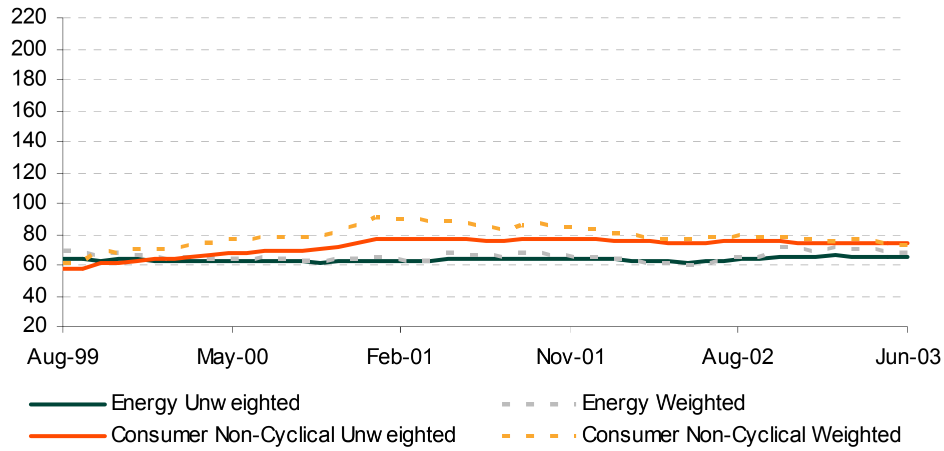
Figure 7a. Idiosyncratic Spread Volatility (Transportation and Communication)



5 A detailed description of the relationship of idiosyncratic risk variance and number of issues in the portfolio can be found in "Testing the Lehman Brothers Agency Risk Model", Chang and Naldi (2003).

6 The time-varying idiosyncratic volatility was first introduced by A. Berd and M. Naldi (2002) in the Lehman Brothers Investment Grade Credit Risk Model.

Figure 7b. Idiosyncratic Spread Volatility (Energy and Consumer Non-Cyclical)



2.2. Default Risk

Having discussed the component of market risk, we will focus on default risk in this section. The default risk for a single corporate issue depends on the default event and the recovery rate upon default. For instance, ignoring the term of market return, R^i_{Market}, in equation (6) yields:

$$R^i_t \approx I^i R^i_{Default} \tag{6a}$$

For default event, we use a simple binominal distribution where the default probability will be calibrated *ex ante* at the beginning of each month. Figure 8 shows the estimated annual default probability for different rating categories.

Figure 8. Annual Default Probability

BB	B	CCC
1.3%	8.7%	32.8%

In general, the recovery rate will also be a random variable at the beginning of the period and its realization depends on the seniority, sector or other firm-specific characteristics[7]. In our model, we will use only a deterministic value based on the issue's seniority and sector for the sake of tractability. The expected recovery rate is calculated as the weighted average of the industry with the same seniority[8]. This recovery model was developed by Arthur Berd and Gaurav Tejwani from the Lehman Brothers Credit Strategy Group. Figure 9 shows the estimated recovery rate for senior unsecured bonds.

[7] *Researchers and practitioners typically use a β-distribution to model this risk associated with recovery rate.*

[8] *The basic assumption of our recovery model is that the recovery rate is proportional to the par value instead of the market value of the bond. As Duffie and Singleton (1999) and Jarrow, Lando and Turnbull (1997) shows, the assumption of recovery value plays a vital role in credit pricing.*

Figure 9. Recovery Rate (Senior Unsecured)

Banking And Brokerage	64
Financial Companies, Insurance And Reits	64
Basic Industry	27
Communication And Technology	21
Consumer Cyclical	30
Consumer Non-Cyclical	33
Energy And Transportation	32
Utility	51
Non-Corporate	37

Source: Moody's Investor Service and Lehman Brothers calculations.

Given the default probability and expected recovery rate upon default for a single issue, we are then able to approximate the standard deviation of the risk due to default as following:

$$Std \approx \sqrt{\text{Default Prob}} \times \text{Loss Upon Default} \tag{9}$$

To measure the default risk for a portfolio, we still need default correlation among different issues besides default probability and recovery rate. Our setup for default correlation follows a structural framework based on the value of the firm, along the lines of Merton (1974). Under this approach, a default event is triggered whenever the firm's asset value falls below a threshold defined by the firm's liabilities. Dependence of the default of one firm on the default of other firms can be modeled through the correlation among firms' asset value processes.

In reality, we cannot directly observe a firm's asset value. However, in this framework, equity is a call option on the asset value of the firm. Hence we can use equity correlation as a proxy for asset correlation – so the dependence between defaults will be driven by equity correlations. We use historical time-series of equity returns to estimate the equity (asset) correlation for a given pair of credits by building a separate sector-based equity risk model. The equity risk model covers the G-7 equity market and has 70 systematic factors which are based on the seven G-7 countries and 10 MSCI industrial sectors. Using such an equity risk model, we are able to estimate the pairwise equity (asset) correlation for different credits. The correlation structure and the information on default probability enable us to calculate the default correlation using a simulation methodology. Technical details about the simulation can be found in the Appendix.

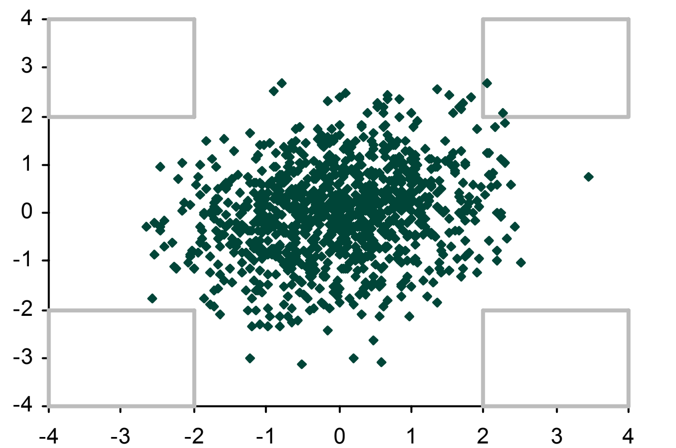
The underlying asset-based structural methodology for default is very popular in standard credit models such as CreditMetrics and KMV, which use a multivariate Gaussian distribution to model the joint equity (asset) return. The unique feature of our setup is that we use a joint Student-t distribution for the underlying equity return.

First of all, the Student-t distribution is a more general form of the Gaussian distribution. Empirical research shows that joint distribution of equity returns more closely follows a Student-t distribution with 10-12 degrees of freedom than a multivariate Gaussian distribution. For instance, R. Mashal and M. Naldi, A. Zeevi (2003) rejected the hypothesis of infinite degrees of freedom (corresponding to Gaussian) using likelihood ratio tests.

Secondly, the joint Student-t distribution has an important feature: *tail-dependence*. Under the joint t-distribution, extreme co-movements are more likely, a property that is crucial for generating default correlation since default is a rare event[9]. In contrast, under a joint Gaussian distribution, the default correlation will be too small for any reasonable correlation between the underlying Gaussian asset value returns. Figure 10 shows the scatter plots of a Gaussian distribution and a Student-t distribution with five degrees of freedom, both with 20% correlation. Clearly, under joint t-distribution, we see more incidents of joint extreme events.

Figure 10. Joint Events under Gaussian and Student-t Distributions

Gaussian



Source: Lehman Brothers.

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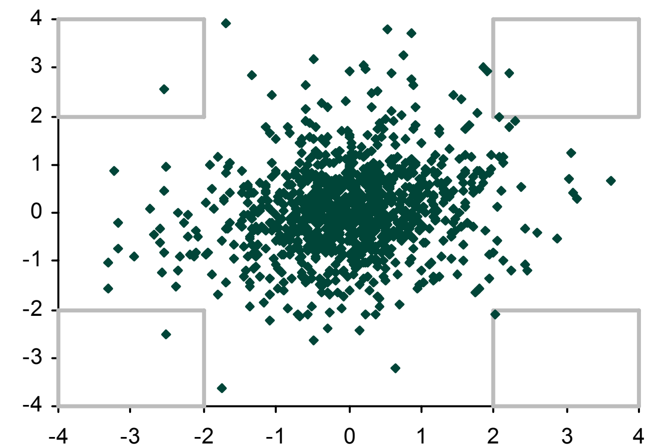

Source: Lehman Brothers.

In summary, we have discussed how to use the default probability and recovery rate to model the default risk for a single issue, and how to use a t-dependence structure to model default correlation among different issuers. However, to get the full picture of the distribution for total return for a credit portfolio, we still need to know the correlation between the market spread return, R^i_{Market}, and the default event, I^i. In reality, it is very natural to believe there is a positive correlation between these two risks: when the economy is doing well, market spread returns tend to be higher and there are few defaults. However, in our risk model, for the sake of tractability we will make a seemingly bold assumption: that these two risks are independent of each other. We would argue that for our purpose, any correlation assumptions will not have material impact on the output of our risk model.

The risk model is typically used over a short to medium time horizon – from a few months to a few years. First, note that the current level of defaults and spreads will be explicit inputs for the risk model. Default risk is primarily driven by these levels. The only remaining concern then is how *changes* in default rates conditional on spread changes are accounted for. The output of the risk model will focus on two quantities: the *Tracking Error Volatility* which is the standard deviation of the return distribution and other tail properties of the distribution. For Tracking Error Volatility, the effect of this correlation is minimal. Intuitively, we can

[9] *For details about extreme events and joint-t distribution, interested readers can refer to R. Mashal and M. Naldi, A. Zeevi (2003), and D. O'Kane and L. Schloegl (2002).*

imagine a Poisson process which governs the default event and a Brownian Motion process which governs market returns. For relatively small time intervals and low default probabilities, it is well understood that the correlation between a Poisson process and Brownian Motion is small even if their underlying factors are highly correlated[10]. For tail properties, results such as Expected Shortfall and VAR are primarily driven by the distribution of default event. Hence, for model tractability, we will assume that default events are independent of the market spread return, R^i_{Market}.

2.3. Tracking Error Volatility

One of the key outputs of the risk model is the Tracking Error Volatility: the *predicted* standard deviation of the relative (or total) return of a portfolio. Under our framework, the total return for a portfolio which includes high yield securities can be written as:

$$\begin{aligned} R &= \sum_{i(nonHY)} \theta_i R(nonHY)_i + \sum_{i(HY)} \theta_i R(HY)_i \\ &= \sum_{i(nonHY)} \theta_i (L_i F + \varepsilon_i) + \sum_{i(HY)} \theta_i [(1 - I^i)(L_i F + \varepsilon_i) + I^i \alpha^i] \\ &= \underbrace{\sum_{i(all)} \theta_i (L_i F + \varepsilon_i)}_{R_1(withoutDefault)} + \underbrace{\sum_{i(HY)} \theta_i I^i [\alpha^i - (L_i F + \varepsilon_i)]}_{R_2(Default)} \end{aligned} \tag{10}$$

The first component R_1 is identical to our standard multi-factor risk model without default.

Hence, the variance for the return of the portfolio can be written as the following:

$$\mathrm{var}(R) = \mathrm{var}(R_1) + \mathrm{var}(R_2) + 2\,\mathrm{cov}(R_1, R_2) \tag{11}$$

The first part of the variance is exactly the same as the standard multi-factor risk model excluding default. It will be fully determined by the variance and covariance matrix of the systematic factors, Σ, and idiosyncratic variance, Ω_i. The second and third parts are default related and depend on joint default probability $E(I^i I^j)$ [or default correlation $\rho(I^i, I^j)$][11]. The details of the calculation are shown in the Appendix.

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10 *We have tried a simulation to gauge the effect on Tracking Error Volatility of using different correlation assumptions. We are able to show that using reasonable parameters, the Tracking Error Volatility is affected by only 1~3% when we relax the independent assumption.*

11 *The relationship between the joint default probability,* $E(I^i I^j)$*, and default correlation,* $\rho(I^i, I^j)$*, is given by:*

$$\rho(I^i, I^j) = \frac{E(I^i I^j) - p^i p^j}{\sqrt{(1-p^i)p^i(1-p^j)p^j}}$$

where p^i *is the default probability for firm i and* p^j *is the default probability for firm j.*

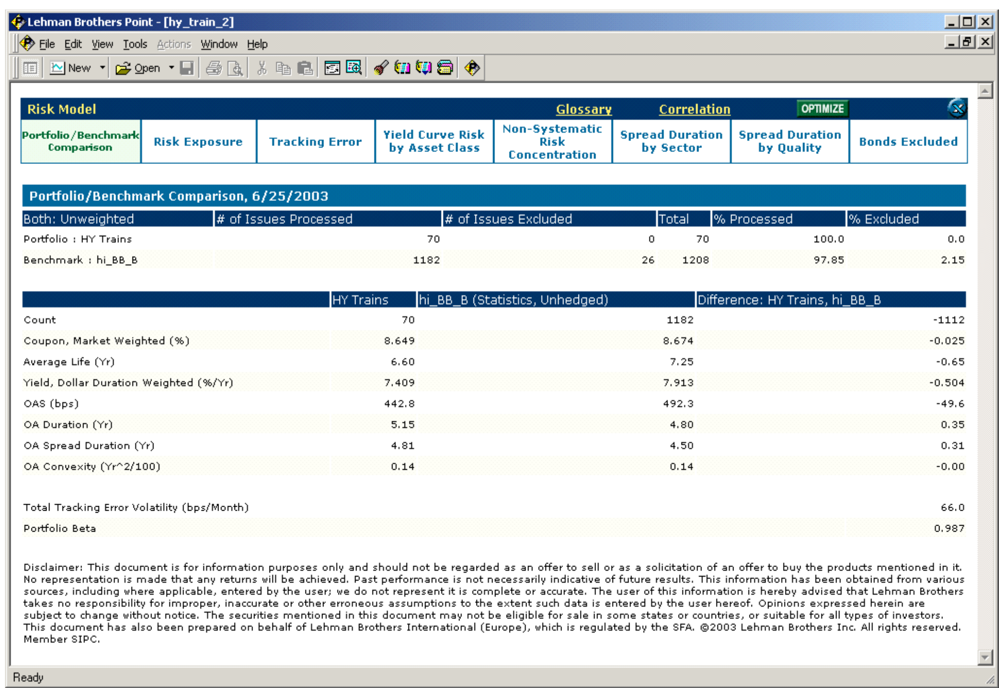
3. POINT IMPLEMENTATION

In this section, we use one example to show the implementation of the high yield risk model on our portfolio analytical platform: POINT. Currently POINT can calculate the total Tracking Error Volatility for a portfolio with high yield issues, and then decompose the total TEV into market component and default component.

The sample portfolio we use in this paper is the Lehman Brothers High Yield TRAINS. TRAINS is a product developed by Lehman Brothers that allows credit investors to invest in a diversified portfolio of credit risk in one single instrument. The high yield TRAINS includes 70 issues across different sectors and different maturity buckets. The following report compares the TRAINS portfolio to the selected benchmark: Lehman Brothers High Yield Index (BB & B). The portfolio has a slightly lower average maturity, lower duration and lower average OAS, but higher spread duration than the index. The bottom of the report displays the total Tracking Error Volatility of 66bp/month for this example.

Figure 11. Portfolio/Benchmark Comparison

Portfolio = Lehman Brothers High Yield TRAIN
Benchmark = US High Yield Index



Portfolio/Benchmark Comparison, 6/25/2003

Both: Unweighted	# of Issues Processed	# of Issues Excluded	Total	% Processed	% Excluded
Portfolio : HY Trains	70	0	70	100.0	0.0
Benchmark : hi_BB_B	1182	26	1208	97.85	2.15

	HY Trains	hi_BB_B (Statistics, Unhedged)	Difference: HY Trains, hi_BB_B
Count	70	1182	-1112
Coupon, Market Weighted (%)	8.649	8.674	-0.025
Average Life (Yr)	6.60	7.25	-0.65
Yield, Dollar Duration Weighted (%/Yr)	7.409	7.913	-0.504
OAS (bps)	442.8	492.3	-49.6
OA Duration (Yr)	5.15	4.80	0.35
OA Spread Duration (Yr)	4.81	4.50	0.31
OA Convexity (Yr^2/100)	0.14	0.14	-0.00
Total Tracking Error Volatility (bps/Month)			66.0
Portfolio Beta			0.987

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The next report decomposes the total TEV into different components. In this example, we see that the biggest contributors for the systematic market TEV are interest rate risk and high yield spread risk, with isolated contributions of 16.6bp/month and 27.7bp/month respectively. The total systematic TEV is 28bp/month, which is smaller than the sum of the yield curve TEV and the high yield spread TEV due to the negative correlation between spread and interest rates. Figure 12 also shows that the idiosyncratic TEV is about 41bp/month in this example. As we have discussed earlier, high yield issues have larger idiosyncratic variance, and investors need more issues to achieve the same diversification goal for a high yield portfolio. This example clearly illustrates this point: although we have 70 issues in TRAINS, the idiosyncratic TEV is still very high and indeed is much larger than the systematic component. Finally, this report also displays the TEV due to default risk which is 43.8bp/month in this example.

Figure 12. Tracking Error Volatility (bp/month)

Portfolio = Lehman Brothers High Yield TRAIN
Benchmark = US High Yield Index

Lehman Brothers Point - [hy_train_2]

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Source	Isolated	Cumulative	Change in Cumulative
Key Rates	16.6	16.6	16.6
Convexity Factor	0.0	16.6	-0.0
Treasury Volatility Factor	0.0	16.6	0.0
Agency Volatility Factor	0.0	16.6	0.0
Credit Investment Grade Volatility Facto	0.0	16.6	0.0
Credit High Yield Volatility Factor	0.8	16.2	-0.4
MBS Volatility Factors	0.0	16.2	0.0
Treasury Spread Factors	0.0	16.2	0.0
Agency Spread Factors	0.0	16.2	0.0
Credit Investment Grade Spread Factors	2.9	14.8	-1.4
Credit High Yield Spread Factors	27.7	28.0	13.2
MBS Spread Factors	0.0	28.0	0.0
CMBS Spread Factors	0.0	28.0	0.0
ABS Spread Factors	0.0	28.0	0.0
Systematic Tracking Error Volatility (bps/Month)			28.0
Idiosyncratic Tracking Error Volatility (bps/Month)			41.0
Default Tracking Error Volatility (High Yield Only) (bps/Month)			43.8
Total Tracking Error Volatility (bps/Month)			66.0
Portfolio Expected Return (bps/Month)			62.3
Benchmark Expected Return (bps/Month)			64.7
Difference of Expected Return (bps/Month)			-2.4

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Figure 13 lists the 20 largest sources of idiosyncratic risk concentration relative to the index. It is interesting to note that the largest contribution comes not from any bond in the TRAINS portfolio, but from the Barrett Resources bonds which form 2.26% of the Lehman Brothers High Yield Index.

Figure 13. Non-Systematic Concentration

Portfolio = Lehman Brothers High Yield TRAIN
Benchmark = US High Yield Index

Lehman Brothers Point - [hy_train_2]

Concentration Source	Issuer Name	Sector	Rating	Issues in Portfolio	% of Portfolio	% of Benchmark	Net Contrb. to OASD	Specific TE Vol (bps/month)
Credit(IG/HY):								

WMB	BARRETT RESOURCES CORP	Other Utilities	B3 B1	0	0.0	2.26	-0.156	13.6
FLEX	FLEXTRONICS INTL LTD	Industrial	BA2	1	1.41	0.24	0.086	11.0
EP	ANR PIPELINE CO.	Other Utilities	B1 B2	0	0.0	1.36	-0.107	9.3
BTU	PEABODY ENERGY CORP	Industrial	BA3	1	1.43	0.17	0.076	7.1
TPC	TRITON PCS INC	Industrial	B3 B2	1	1.48	0.53	0.047	6.8
ABC	AMERISOURCEBERGEN CORP	Industrial	BA3	1	1.43	0.22	0.088	6.5
BYD	BOYD GAMING CORP	Industrial	B1	1	1.42	0.15	0.074	6.4
TCN	TELUS CORPORATION	Industrial	BA1	1	1.46	0.88	0.043	6.3
BLL	BALL CORP	Industrial	BA3 B1	1	1.39	0.23	0.075	6.2
SANM	SANMINA-SCI CORP	Industrial	BA2	1	1.43	0.22	0.048	6.1
DRRA	DURA OPERATING	Industrial	B1 B2	1	1.46	0.2	0.069	6.0
AYE	ALLEGHENY ENERGY INC	Other Utilities	B1	1	1.44	0.39	0.067	5.8
BOW	BOWATER	Industrial	BA1	1	1.41	0.41	0.062	5.8
SPW	SPX CORPORATION	Industrial	BA3	1	1.46	0.22	0.067	5.6
SBGI	SINCLAIR BROADCASTING	Industrial	B2	1	1.45	0.24	0.060	5.5
TRWAUT	TRW AUTOMOTIVE INC	Industrial	B2 B1	1	1.48	0.34	0.061	5.3
RCL	ROYAL CARIBBEAN	Industrial	BA2	1	1.51	0.48	0.058	5.0
DTV	DIRECTV HOLDINGS/FINANCE	Industrial	B1	1	1.44	0.4	0.055	5.0
SQA	SEQUA CORP	Industrial	B1	1	1.45	0.19	0.060	5.0
SPOT	PANAMSAT CORP	Industrial	BA2 BA3	1	1.45	0.34	0.055	5.0

Ready

To assess the effect of our t-dependence structure on the estimation of default TEV, we report the TEV under different structural assumptions for the dependence of defaults: first, assuming that all default events are independent, and next under a Gaussian dependence structure.

Figure 14. Tracking Error Volatility (bp/month) under Different Default Structures

Portfolio = Lehman Brothers High Yield TRAIN
Benchmark = US High Yield Index

	Student – t	Gaussian	Independent
Default TEV	43.8	38.4	25.2
Total TEV	66.0	62.5	55.4

Figure 14 shows that the t-dependence structure produces a higher risk estimate than either the Gaussian or the independence assumption. The default TEV estimate is reduced from 43.8bp/month to 38.4bp/month and to 25.2 bps/month when we use a Gaussian dependence structure and an independent structure respectively. This reduction was solely due to default correlation. Under an independent assumption, the default risk is essentially idiosyncratic because there is no correlation between default events. With a Gaussian assumption, we underestimate the default correlation due to the lack of tail dependence.

4. UPCOMING ENHANCEMENTS

Tracking Error Volatility is a very important measure for portfolio risk, but by no means the only one. From a statistical perspective, TEV only specifies the second moment of the return distribution without considering higher moments. In reality, it is quite possible that the same standard deviation for two different distributions does not necessarily define the same risk profile. For example, consider two distributions with the same standard deviation but different tail properties: one is Gaussian and the other is binominal with a very large probability of a small win and a small probability of a big loss. Even with the same standard deviation, we would argue that the latter has a more risky profile of returns. For credit, and even more so for high yield securities, the return distribution certainly has the flavor of the latter distribution because of default. Hence, risk managers have used other risk metrics beyond Tracking Error Volatility, for example value-at-risk (VAR) and Expected Shortfall (ES)[12]. Those risk measures are designed to capture the tail properties of the return distribution.

To obtain risk metrics such as VAR and Expected Shortfall, we need to specify a certain distribution for the return because different distributions will have different tail properties and different estimates for VAR and Expected Shortfall. The traditional multi-factor risk model cannot handle this issue because of its structural limitation with regard to default event. However, the new high yield risk model gives us a natural framework to estimate risk measures such as VAR and Expected Shortfall. As we have shown in Figure 3, the default component in the return decomposition essentially provides an explicit structural assumption on the left tail of the return distribution.

In this paper, we apply a simulation-based methodology to obtain those risk metrics. In particular, we simulate the default events for issuers in a portfolio based on the correlation structure as discussed in section 2. We then obtain risk measures like VAR and Expected Shortfall for this portfolio. Figure 15 shows the Expected Shortfall for the high yield TRAINS. For comparison, we also report the Expected Shortfall under different dependence assumptions: Gaussian and independent.

Figure 15. Expected Shortfall in bp/month

Portfolio = Lehman Brothers High Yield TRAIN

Confidence Level	5%	1%	0.10%
Student – t	-6.1	-10.88	-21.99
Gaussian	-4.76	-6.31	-8.51
Independent	-4.61	-6.01	-7.4
Relative Difference (Gaussian)	128%	172%	258%
Relative Difference (Independent)	132%	181%	297%

As with the TEV analysis, we see in Figure 15 that our t-dependence structure produces a higher Expected Shortfall across different confidence levels. Furthermore, the difference is greater for lower confidence levels. This qualitative result is expected because the Expected Shortfall mainly depends on the tail of the return distribution of the portfolio, and that is where the t-dependence structural assumption makes a difference.

12 *VAR at a confidence level of β% is the worst loss in the best β% of all scenarios. For example a VAR of 100 MM at a 99% level of confidence means that the probability of the realized loss wider than 100 MM is only 1%. However, VAR does not distinguish how severe the loss might be in the 1% tail. Unlike VAR, the Expected Shortfall risk measure takes this into account and gives the expected loss in the 1 - β tail.*

5. CONCLUSIONS

In this paper, we described the new Lehman Brothers high yield risk model. The unique feature of the model is that it combines a multi-factor model of market movements with a model of default. In this model, default risk is systematic and default correlation is modeled via a t-dependence structure. Such an integrated framework allows investors to obtain not only the Tracking Error Volatility of the portfolio, but also risk measures such as VAR and Expected Shortfall, which depends on the tail properties of the return distribution.

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APPENDIX

Simulation for Default Correlation

In this section, we detail the simulation methodology for calculating the default correlation. Let us look at firm i and j with default probability p^i and p^j. As discussed in section 3, we assume that their underlying asset returns follow a joint student t distribution with degree of freedom ν and correlation ρ_{ij}. The correlation ρ_{ij} and degree of freedom ν will be given by our Equity Risk Model. The simulation procedure works as follows:

1. Derive the thresholds of default $\hat{t}_i$ and $\hat{t}_j$ from default probability p^i and p^j based on the cumulative student t distribution.
2. In simulation path #n, draw $t_i(n)$ and $t_j(n)$ from a joint t distribution with degree of freedom ν and correlation ρ_{ij}. Compare $t_i(n)$ and $t_j(n)$ to the threshold $\hat{t}_i$ and $\hat{t}_j$ to get the default event $I_i(n)$ and $I_j(n)$.
3. Repeat step 2 for N times.
4. Compute the sample correlation of I_i and I_j to get default correlation $\rho(I^i, I^j)$.

Tracking Error Volatility

In this section, we detail the calculation of the Tracking Error Volatility with the presence of default risk. As we have shown in equation (10), the total Tracking Error Volatility can be written as three components:

The first component will be exactly the same as the result of a traditional linear multi-factor risk model:

$$\text{var}(R_1) = L\Sigma L' + \sum_i \theta_i^2 \Omega_i$$

where $L = \sum_i L_i \theta_i$ is the (relative) risk exposure of the portfolio, Σ is the covariance matrix of the systematic factors and Ω_i is the idiosyncratic variance for issuer i.

The second and third components depend on the default risk. The following equations detail the expression for $\text{cov}(R_1, R_2)$ and $\text{var}(R_2)$:

$$\begin{aligned}\text{cov}(R_1, R_2) &= \text{cov}(LF + \sum_{i(HY)} \theta_i \varepsilon_i, \sum_{i(HY)} \theta_i I^i [\alpha^i - (L_i F + \varepsilon_i)]) \\ &= \ldots \\ &= \ldots \\ &= -\sum_{i(HY)} L\Sigma(\theta_i p^i L_i)' - \sum_{i(HY)} \theta^2{}_i p^i \Omega_i\end{aligned}$$

where p^i is the default probability for firm i.

$$
\begin{aligned}
\operatorname{var}(R_2) &= \operatorname{var}\Big(\sum_{i(HY)} \theta_i I^i [\alpha^i - (L_i F + \varepsilon_i)]\Big) \\
&= \sum_{ij(HY)} \theta_i \theta_j [\alpha^i \alpha^j - (\alpha^i L_i + \alpha^j L_j)\overline{F} + L^i E(FF')L^{j'} + \Omega^i \delta_{ij}] E(I^i I^j) \\
&\quad - [\sum_{i(HY)} \theta_i (\alpha^i - L_i \overline{F}) p^i]^2
\end{aligned}
$$

where the joint default probability $E(I^i I^j)$ will be derived through simulation as we have discussed earlier. It is evident that the second and third parts will converge to 0 if the default probability p^i go to 0 for all issues in the portfolio (and benchmark). Hence for a portfolio (and the corresponding Benchmark) without High Yield securities, these two components do not contribute to total Tracking Error Volatility.

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