INVESTING WITH RISK PREMIA FACTORS

Return sources, portfolio construction, and tail risk management

Concerns about the past and future performance of portfolios constructed based on traditional views of equity versus fixed income allocations have led to closer scrutiny of the available structural sources of return and risk. In addition, diversification is increasingly hard to achieve in an economic environment that is driven mostly by sharp swings in investor risk appetite. While proposed solutions to address aspects of these issues abound, a comprehensive discussion of this important evolution in the investment process is lacking. We analyze factor-based investing, from choosing the building blocks to portfolio construction to tail hedging.

The main points:

- **Increasingly, investors view the investment universe as being organized by systematic sources of returns and risk, often thought of as risk premia, that transcend asset class classifications. We welcome this development because we believe that the investment universe should be organized in units with intuitive performance and risk properties such as risk premia.**
- **In the absence of explicit performance views, risk diversification across risk premia factors is a sensible and robust allocation strategy. This simple observation leaves many complicated questions; both "risk" and "diversification" are concepts that can be addressed in numerous ways. We discuss the most popular approaches, introduce new suggestions, and highlight underlying assumptions.**
- **We propose tail-risk hedging overlays as the last step in the allocation process. Since true tail-risk hedging comes at a cost, directly integrating tail hedging in the primary allocation process complicates and obfuscates the ultimate investment purpose of optimally accessing sources of positive excess return.**

We illustrate many of these considerations in a detailed allocation example. We address practical concerns related to liquidity, capacity, and estimation error.

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The evolution of asset allocation

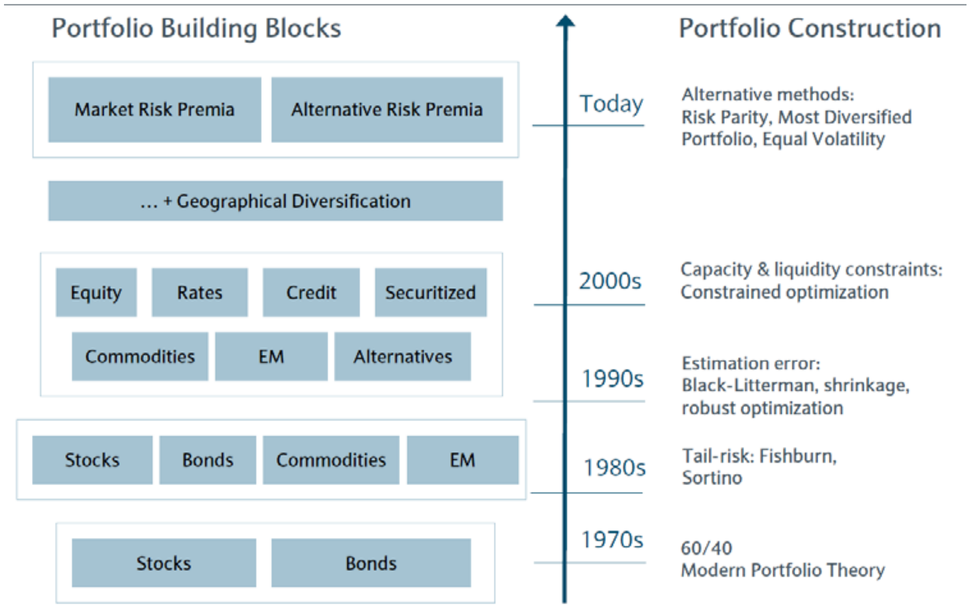
Underlying every portfolio are two major decisions: the choice of "building blocks" and the method for allocating investments. In recent years, both of these underlying concepts have come under increased scrutiny by the investment community. There has been a strong movement toward defining building blocks along risk types rather than asset types and an increased focus in portfolio construction on explicitly diversifying risk types. The latter is often presented as a move away from what is thought of as "Modern Portfolio Theory." We discuss this evolution in the investment process and the powerful drivers behind it. We explain how this evolution draws on recent insights into risk and return behavior to more effectively use existing investment frameworks (it is a reinterpretation of old methods, not the revolution it is sometimes portrayed to be). Furthermore, we apply these insights to a real-life global portfolio example. We suggest solutions that are general in nature, with applicability beyond specific problems.

From asset classes and instruments to risk factors

To understand the recent evolution of mainstream investment processes, we first look at how these two steps evolved historically, as illustrated in Figure 1. For much of recent history, macro-level allocation decisions were driven by equity versus fixed income choices, resulting in a "60/40" view of the world. Commodities were a more recent addition to the allocation classification of most investors. Other asset types that historically play smaller roles, such as real estate, hedge funds, and emerging markets, were included in the mainstream allocation process at various points in time. In addition, investors increasingly take a global approach to their allocation decisions, with geographical dispersion of investment performance providing another dimension of potential diversification.

It has long been understood that the return and risk behavior of most asset classes can be captured through a limited number of common factors. These factors capture the systematic sources of return and risk within the asset class. In both fixed income and equities, practitioners have long made use of factor models that capture major sources of risk (e.g., interest rate risk, credit risk, inflation risk, country risk) to describe common risk exposures in portfolios. The greater focus on "risk premia factors" is a natural evolution of the use of risk factors in the investment process, one that primarily (if often implicitly) aims for more consistency between allocation, risk, and attribution decisions. While we are great believers in this move toward consistency, it should be noted that semantics matter in this context: large differences exist between investors in the interpretation and definition of risk premia factors. Some will see their investment universe as a collection of tradable factors consisting of traditional beta factors and additional alternative risk premia, others will take a deeper macro-based approach and relate these tradable factors to growth, inflation, and policy developments in underlying economies and treat these as the ultimate risk factors on which to base decisions. Investment results can be hugely different depending on the investor's view of how to define investment factors. Should returns be defined, for example, on nominal Treasuries in excess of returns on inflation-linked bonds? How are excess returns defined? Choices such as these can dramatically affect the portfolio allocation and risk-budgeting process.

Figure 1: Historical evolution of the initial steps of asset allocation



Source: Barclays Research

Despite the many possible interpretations of this approach, the benefits of a risk premia-based, or factor-based, investment process are substantial:

- A factor-based investment process facilitates transparency and parsimony, a great benefit, especially in the light of increasing complexity of investment opportunities.
- A focus on the systematic sources of return and risk allows for better economic intuition about the risk and return properties of major investment decisions.
- Views are more easily expressed directly on the drivers of portfolio returns.
- Factor-based investing allows for robust methodologies, from risk estimation to portfolio optimization.

From mean-variance-optimization to "post" modern portfolio theory

While of central importance in mainstream investment approaches, some major failings of the standard risk/return maximization methodology for portfolio construction have long been identified (we refer to mean-variance optimization and related methodologies as Modern Portfolio Theory (MPT)). First, investors arguably dislike only downside risk, especially the risk of extreme negative returns. MPT typically takes a symmetric treatment of risk as the starting point, and although many solutions to this issue have been proposed, they often lead to complex and fragile practical outcomes. A second issue with MPT stems more from practice than theory: MPT requires forecasts of the risk and return of the investment building blocks. Often those forecasts are obtained from historical estimates. These estimates do not always make for good forecasts, and different approaches to, e.g., risk estimation can affect the final portfolio dramatically.[1] The third major issue with MPT comes from capacity constraints and liquidity limitations. While of major practical concern, MPT is silent on how best to address the practical limitations of marketplaces and financial instruments. Efforts to incorporate, for example, capacity constraints and transaction costs in the MPT construction exercise have led to complex constrained optimization approaches. The resulting allocations are often significantly at odds with an optimal risk/return objective. Fourth and perhaps most important, unconstrained risk/return optimization typically does not deliver well-diversified solutions. For example, as many have pointed out, more than 95% of the risk in a typical 60/40 portfolio is driven by the equities allocation, a fact that has become painfully obvious to many investors over the past few years. Lack of diversification in allocations is often driven by some form of the error-maximization problem, in which mistaken assumptions about future expected returns and correlations lead to extreme allocation outcomes.

A variety of allocation methods have been advocated to address the various shortcomings of MPT. These are typically presented as revolutionary and often specifically focused on risk diversification arguments. Prominent examples are Minimum Variance Portfolio, Maximum Sharpe Ratio Portfolio, and the ubiquitous Risk Parity Portfolio (sometimes implemented as an Equal Volatility-Weighted Portfolio). Within asset classes, yet more alternatives to traditional allocation schemes have been created at the instrument level, ranging from equal-weighted to fundamentally weighted portfolios and indices. Justification for any of these methods is often based on ex-post empirical results rather than an understanding of the principal assumptions and conditions that underlie the potential success of each method. We aim to highlight the investment thesis underlying each method, rather than the *ex-post* performance realized in historical examples.

For example, while we believe that risk diversification is a useful practice whose importance and applicability have attracted a lot of attention and generated a lot of enthusiasm recently, we must judge cautiously its impact by comparing the historical performance of risk-diversified portfolios relative to portfolios constructed using standard allocation methods. Indeed, proponents like to highlight the superior performance of risk-diversified portfolios (e.g., risk-parity portfolios) relative to standard portfolios over the past 40 years. However, this outperformance is mostly due to the strong performance of rates over the period rather than evidence for the risk parity concept, as it is typically advertised. Therefore, empirical results do not provide much evidence for equal risk solutions as the panacea to the asset allocation problem they are purported to be. Understanding under what assumptions and conditions Risk Parity and other methods would succeed or fail provides a much better footing in a forward-looking investment approach. Most of the standard implementations of these alternative methods fall squarely into the MPT paradigm: they are optimal MPT solutions under certain assumptions on risk and return properties. Lastly, none of these methods offers a complete solution to common concerns in portfolio construction. For example, most do not explicitly address tail risk.

In the next two sections, we discuss considerations for defining building blocks for a risk premia-based investment process and outline portfolio construction frameworks that are well suited to a factor-based investment approach. In the third section, we provide an example of a liquid asset allocation portfolio that builds on the conclusions from our framework discussion.

[1] Black and Litterman (1992) made a notable effort to address this.

Building blocks: From asset classes and instruments to risk premia

Although often taken for granted, any investment decision starts with the definition and selection of the investable universe. Increasingly, investors aggregate the "atoms", or building blocks, of their investment universe along types of risk rather than asset classes and individual instruments. Not everyone will agree on the definition of or which factors are the most relevant for their investment purposes. Regardless of the exact implementation, a systematic factor-based investment approach has many advantages, not in the least a more direct and transparent approach to diversification.

Risk premia factors as "building blocks"

To understand why it is important to construct appropriate investment blocks, we look at their role in the construction process. Investment objectives are often (implicitly or explicitly) centered on achieving the best tradeoff between portfolio performance (return) and risk. Maximizing this tradeoff requires views and forecasts of each block's performance, of its risk, and of the joint behavior of all blocks. However, these forecasts and views are often based on extrapolation from historical data. Unfortunately, history provides limited guidance to future return behavior; some risk measures, such as volatility, are often forecast with a useful degree of accuracy. Correlations are more difficult to forecast, in part because of the curse of dimensionality. Future performance (expected returns) is notoriously hard to forecast, and more so solely from historical data. Given the sensitivities to and potential for forecast errors, the definition of investment blocks matters because we require building blocks that have intuitive properties, whose risk and performance we can measure well and have economically informed views on, independently from historical data. Risk premia factors are appealing building blocks, not in the least because they tend to bring transparency through simplification. Since the number of risk types is fairly limited and historically does not grow as fast as the types of assets available, risk and return can be better forecast from existing data. For example, time-varying correlations (which drive diversification potential) are more easily modeled and understood using a limited number of factors that capture available investment opportunities.

Analyzing the investment universe in terms of risk premia allows us to cover the systematic investment opportunities within and across asset classes. For example, within developed equity markets, most investors will recognize investment factors related to value, momentum, size, etc. that describe additional investment dimensions beyond the pure asset class exposure. This approach also highlights that asset classes may contain risks that do not have an associated premium; hence, investors may not find it desirable to have them in their portfolios. For example, a European stock has both equity and FX risk from the perspective of a US investor. The EUR/USD FX risk has a questionable premium, simply because this investor is not intrinsically different from a European investor in a US stock that faces the opposite risk. The risk premia approach can be applied more broadly than asset class allocation because it focuses on the identification of systematic sources of return and risk beyond asset class exposure.

How to identify risk premia

One approach to identifying intuitive investment factors relies on the insight that in a world without pure arbitrage opportunities (or in which they are very few and hard to get), any consistent source of return should be a compensation for a particular risk, hence the notion of "risk premium" defined as a source of returns. It is (compensated) risk factors, not asset classes per se, that determine investment performance.

How to identify risk premia factors? Since a risk premium represents compensation for an assumed risk, a good place to start are the risk factors (i.e., types of risk) embedded in the investment universe. Risk factors can be defined in various ways, but for our purpose, we prefer *tradable* risk factors that follow economic intuition. The tradability requirement excludes other useful factors, such as macroeconomic ones. A mapping from tradable factors to macroeconomic ones is a worthwhile exercise beyond the scope of this report (an approach that could be used to form views on tradable factors).

Which risks carry a premium?[2] Based on empirical and economic arguments, we can say that the broad asset classes, each a component of a broad "market portfolio," carry a premium over holding cash, the riskless asset. The market portfolio theoretically contains all (liquid) assets that produce a return. In our model portfolio, we will focus on liquid broad asset classes: equities, rates, credit, and commodities.

To identify other premia – let's call them alternative premia – we start from a mainstream explanation of why a risk has an associated premium: a risk must be disliked economically by a broad set of market participants who are exposed to it and are willing to pay a premium to get rid of it. Rationally, market participants dislike a risk because it cannot be diversified away and it has the potential to negatively affect their wealth, especially if risk increases when they value their wealth more than usual. These periods of heightened investor risk aversion can be driven by increased expectations of future losses, which often follow periods of acute realized losses. This line of reasoning can, for example, explain the long-run profitability of FX carry strategies as a risk premium: providing short-run funding to currency areas with relatively high interest rates (which an FX carry trade implicitly achieves) exposes investors to local and global macro growth risk factors. Another example is merger arbitrage strategies, which are exposed primarily to individual deal failure risk. Systematically, however, risk of deal failure is most acute in times of general market unrest. A systematic discussion of our approach to identifying and understanding alternative risk premia across asset classes was published by Rennison et al. (2011), whose paper analyzes the set of risk premia along the style and asset-class dimensions shown in Figure 2, to which we add the traditional beta premium of each asset class.

Figure 2: Stylized matrix representation of risk premia

	Traditional Beta	Carry	Curve	Value	Momentum	Liquidity	Event Driven
Equities							
Rates							
Credit							
Commodities							
Currencies							
Volatility							

Source: Barclays Research

Other rational explanations for the existence of alternative risk premia rely on institutional and market inefficiencies, which typically arise from the separation between asset managers and asset owners. One such inefficiency requires a large segment of bond investors to sell corporate securities downgraded below investment grade status. The resulting selloff often pushes prices below fundamental values, potentially contributing to a higher premium being priced into high yield bonds, especially bonds that have recently been downgraded to high yield status (a "fallen angel" risk premium, see Ng and Phelps (2011)). In addition to rational explanations for risk premia existence, there are a host of explanations based on

[2] For a comprehensive treatment of sources of returns, including their historical dynamic, we recommend the monograph by Ilmanen (2011).

behavioral patterns, such as investors' refusing to realize "paper" losses, herding, and extrapolation (for a recent review, see Subrahmanyam (2008)). The behavioral-based premia earn their compensation for the risk Keynes referred to when he said "The market can stay irrational longer than you can stay solvent."

Once the potential premia are identified, investors have to select which ones are appropriate for their portfolios. This is one of the most important decisions investors can make, as investment performance tends to be driven by macro-level allocation decisions rather than individual security selection, especially in the current uncertain economic climate. Not all investors will want to get exposure to all risk premia, as their sensitivities to various risk types differ, mainly because they have different horizons and institutional arrangements. Through-the-cycle investors may opt for a maximally diversified risk premia portfolio, while more tactical investors will find that certain risks do not carry enough compensation at particular points in time to compensate for the exposure. Certain investment factors may be so valuable to investors that they may even pay a premium to have them as part of their portfolios, particularly those risks that have positive returns during periods when most other ones have negative returns and investors value their wealth more (volatility premia come to mind).

How to trade risk premia factors

Traditional risk premia factors (i.e., equity, rates, commodities, credit) are easily accessible through index swaps, ETFs/ETNs, and futures. Extracting alternative premia requires non-conventional techniques such as short positioning, leverage, and the use of derivatives. However, the skills and knowledge required have become more widely available, and a convenient way to access these premia is through the increasing scope and breadth of alternative risk premia strategies available in the form of tradable indices (such as Barclays' extensive family of strategy indices). For example, while a large segment of the hedge fund industry focuses on merger arbitrage (MA) strategies, it has become possible to invest in systematic MA indices with lower fees, higher liquidity, and full transparency (the Barclays Q-MA family of indices successfully captures this event-driven risk premium). It is becoming increasingly common to manage strategic and tactical allocation investments through a wide array of investment vehicles capturing directly the systematic sources of risk and return across asset classes, without direct participation in instrument-level decisions.

Risk-centered portfolio construction

Most investors aim for their portfolio to maximize the tradeoff between future performance and risk when they combine the portfolio building blocks, with the possible exception of investors with liability-driven mandates. We center our portfolio construction framework on this tenet.

There are various ways to express this tradeoff mathematically; we use the ratio of expected excess return to risk[3]; namely, the portfolio weights *w* solve:

$$\max_{w} \text{Portfolio Excess Return} / \text{Portfolio Risk},$$

where excess return is measured over cash.

Typically, risk scales linearly, e.g., if we double all allocations, then portfolio risk doubles. In that case, the ratio of portfolio return to risk is scale free, e.g., if we double all allocations, the ratio does not change. Therefore, to pinpoint the absolute level of allocations, we need an extra constraint on the magnitude of the weights, which can be achieved by setting a target for portfolio performance, portfolio risk, or portfolio leverage.[4] We treat the leverage issue at the end of this section; until then, we set the weights to sum to one.

If we use the typical definition of risk as portfolio volatility, then absent any other considerations, the portfolio chooses weights *w* of risky assets such that

$$\max_{w} \frac{\mu_p}{Vol(r_p)} = \max_{w} \frac{w'\mu}{\sqrt{w'\Sigma w}} = \max_{w} \text{Portfolio Sharpe Ratio} \text{ and } w' i_N = 1,$$

where Σ is the assets' covariance matrix forecast, μ is the vector of assets' expected returns forecast, $\mu_p = w'\mu$ is the portfolio return, $Vol(r_p) = \sqrt{w'\Sigma w}$ is its volatility, N is the number of assets or building blocks, and i_N is a vector of ones. Portfolio construction based on this method is known as Mean Variance Optimization (MVO), in part because it employs only the mean and variance of the portfolio. Therefore, for MVO, we must forecast the blocks' expected returns and their covariance matrix, which can be broken down into volatilities and correlations. In the case of more general risk measures (e.g., expected shortfall), constructing the portfolio requires forecasts of its excess return and risk, which are built from the forecasts of each block's return, its risk, and the joint behavior among blocks. More general treatments of risk and "joint behavior" typically lead to significant mathematical complexity and potentially unstable empirical results because of estimation issues. We focus on risk expressed as variance, but address tail risk and asymmetric returns through an additional tail risk hedging step.

The MVO portfolio has appealing theoretical and practical properties. The ratio of a block's marginal portfolio return contribution to its marginal portfolio risk contribution is the same for all blocks and equals the portfolio Sharpe ratio. Interpreting this result, in the final portfolio, each position has the same marginal tradeoff between risk and return, a sensible outcome, and it equals the portfolio's tradeoff. If that is not the case, then we may prefer to increase (decrease) the allocation to assets that have a better (worse) tradeoff than the

[3] This definition is more general than it seems. Should we define performance versus a benchmark, then we can treat the difference between our portfolio and the benchmark as the portfolio we analyze. Moreover, should we define the tradeoff as the difference between performance and risk, we can transform that problem into the one we analyze.
[4] We assume that leverage can be employed and the analysis is conducted on excess return over cash. This is a reasonable assumption for the synthetic assets case discussed in our examples. For more general cases, if leverage is not available, then the details of various formulae change, but the main conclusions remain the same.

portfolio one. On the practical side, the final portfolio weights are easily computed as $w = k\Sigma^{-1}\mu$, where *k* is a scalar that makes weights sum to one.

However, over time, several critical problems were identified with MVO, both theoretical and practical. The three most important:

1. Volatility is not an appropriate definition of risk: Investors dislike only negative performance – in particular, extreme negative performance.

2. Mean and covariance are forecast with error. These errors tend to affect allocations in a particular pattern, and their effects do not diminish in large portfolios.

3. Assets are not in unlimited supply, and adding them to the portfolio affects their performance.

Addressing these problems complicates the MVO framework dramatically and makes it unintuitive, as mentioned in the first section's historical discussion. Below, we discuss a framework to address such considerations and in the process incorporate some of the alternative methods to MVO that have become popular recently, such as risk-parity.

A two-step approach to address extreme events

Investors dislike large negative performance surprises much more than small or positive ones. Risk, as measured by volatility or downside deviation, assigns importance to an extreme outcome directly proportional to its ratio to the risk measure. For example, for investors that define risk as volatility, a negative return four times as large as volatility affects risk (and the compensation they demand to carry it) four times more than a negative return the same size as volatility. However, the large negative return may have irreparable implications for portfolio managers and, therefore, be much more costly than its 4:1 ratio to volatility implies. Because investors care more about large negative outcomes than their relation to volatility entails, we have to consider separately the effect of extreme events located in the negative tail of returns distribution, often referred to as tail events.

Two popular metrics for extreme returns are Value at Risk (VaR) and Conditional Value at Risk (CVaR, otherwise known as Expected Shortfall). VaR measures the best of the worst outcomes below a certain threshold, and CVaR measures the average of these worst outcomes. Several techniques are available to forecast them, and they often involve forecasting the entire joint distribution of asset returns, a very complex exercise.

These measures can be used directly as replacements for volatility in the definition of portfolio risk. This approach, while appealing, makes the optimization problem difficult to solve and unintuitive, as it generally lacks closed-form solutions. In particular, when we substitute the VaR metric, portfolio construction loses some key consistency properties (e.g., sub-additivity) and optimization becomes a mathematically challenging exercise (under all but trivial assumptions on return distributions). An optimization problem based on CVaR has better properties, but still lacks closed-form solutions.

As an alternative to defining risk directly as CVaR, we may proceed with the portfolio construction by defining risk as volatility and address tail-risk at a subsequent second step with an additional allocation to an asset that is expected to perform well during these tail events, a "tail-hedge overlay." We can size the overlay by treating it as a portfolio construction problem with two assets, the unhedged portfolio and the tail hedge. The goal of the portfolio becomes to maximize performance versus tail risk, in line with our objective and in contrast to the first step. This problem is much easier to solve than directly defining portfolio risk as CVaR because we have only one variable: the allocation to the overlay.

Step 1: $$\max_w UnhedgedPortfolioSharpeRatio = \max_w \frac{w'\mu}{\sqrt{w'\Sigma w}} = \max_w \frac{\mu_p}{Vol(r_p)}$$

Step 2: $$\max_\lambda \frac{\mu_p + \lambda\mu_o}{CVaR(r_p + \lambda r_o)}$$

Where λ is the allocation to the overlay, r_o the overlay return, and μ_o its average return (which in most cases will be negative, reflecting the price of insurance).

In step 2, the overlay may be alternatively sized by imposing a tail-risk target, e.g., the allocation to the overlay is such that the combination with the portfolio has a (C)VaR smaller than portfolio volatility times an appropriately chosen multiplier.

The allocation to the hedge overlay should not depend on dynamically forecasted volatility. In periods of stressed markets the overlay has large-magnitude returns, which lead to position downsizing under a risk-centered approach. However, the overlay should be fully allocated and not downsized during stressed periods. If we use solely tail risk to allocate to the overlay, the allocation remains almost constant over long periods because it is difficult to assess tail risk on a short-term basis.

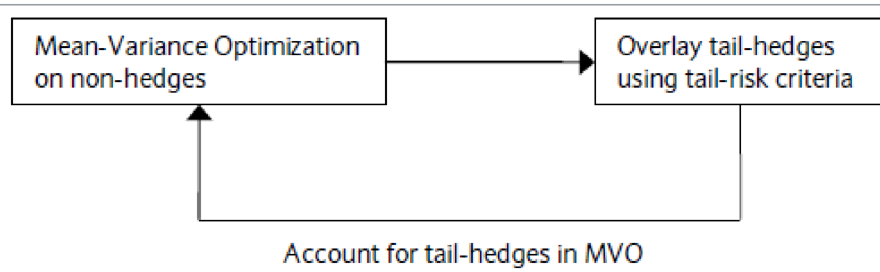
The overlay approach described above presents a subtle yet important issue: when we created the unhedged portfolio, we did not consider that the portfolio has a tail hedge. Because of the tail hedge, certain risky assets may contribute to portfolio risk less than we considered when we constructed the unhedged portfolio. Under our principle of maximizing returns versus risk, we may increase allocations to assets that are positively related to the hedged extreme events (e.g., equity) and decrease them to negatively related assets (e.g., rates). For that, we can re-compute the first step with the tail-hedge overlay included:

Step 1': $$\max_w HedgedPortfolioSharpeRatio = \max_w \frac{\mu_p + \lambda\mu_o}{Vol(r_p + \lambda r_o)}$$

Because we obtain the overlay size λ only at step 2 as a function of the unhedged portfolio weights *w*, we may iterate over the two steps to find the final allocations to the hedged portfolio, as shown in Figure 3.

The overlay approach represents only one of several intuitive methods to deal with tail hedging. Another promising avenue retains the MVO framework but constructs the covariance forecast from distressed scenarios, delivering an optimal portfolio in a stressed

Figure 3: Tail-hedge overlay procedure



Source: Barclays Research

environment. The approach essentially assumes that returns are still normally distributed, but with time-variable covariances. Ignoring time variation may only give the impression that the distribution has fat tails.[5] We can modify this approach further by combining the stressed forecasts with regular forecasts to get a portfolio that is typically close to optimal but with a tilt toward performing during stressed environments.

The role of historical data and forecast errors

The forecasts of assets' expected returns and covariance are typically at least partially based on historical data. Extrapolating from historical data often leads to poor forecasts. To understand the effect of forecast error in the case of MVO, we decompose the covariance matrix into volatilities and correlations[6] and we normalize expected returns by volatilities (i.e., we analyze Sharpe ratios rather than expected returns). These three quantities – asset volatilities, correlations, and Sharpe ratios – have more intuitive and stable properties over time and across assets than covariances and means.

The effect of forecast errors can be understood from the expression for MVO portfolio weights as a product of four quantities:

$$w_{optim} = k\Lambda^{-1}\Omega^{-1}S \,,$$

where Λ is a diagonal matrix of asset volatilities, Ω is asset correlation matrix, S is a vector of asset Sharpe ratio, and k is a previously considered leverage-dependent scalar. The scalar depends on all other quantities; hence, any estimation error associated with them should have an effect on it as well. For example, errors in asset volatilities may lead to over-/under-leveraging the portfolio to achieve a risk target, making the portfolio volatility too high or too low.

These three quantities are forecast from historical data with different degrees of error, and these errors affect allocations differently. Historical data forecast volatilities well (for a framework and discussion, see Gabudean and Schuehle (2011)). Correlations are more difficult to forecast and they may require fairly involved modeling techniques to capture the details of the matrix (for a framework and discussion, see Gabudean (2012)). Lastly, Sharpe ratios are notoriously difficult to forecast from historical data, mainly because of their expected return component.

Unfortunately, errors in difficult-to-forecast quantities, such as correlations and expected returns, have a significant effect on allocations, which increases the importance of addressing forecasting errors in these quantities. As we show in the appendix, we can summarize the effects of forecast errors in the three quantities as follows:

- **Errors in volatility forecasts** create inversely proportional errors in allocations. For example, an underestimation by 10% of a given asset volatility increases its allocation relative to other assets by 10%. The typically small magnitude of errors in volatility forecasting makes their effects limited and well understood in the context of MVO portfolios.

- The effects of **errors in correlation forecasts** depend on the structure of the correlation matrix. Even in the simple case of a portfolio of three assets, a 1% error in one correlation measure – estimated to be 1% instead of 0 – may change some allocations

[5] For a discussion and application of this point in the context of time-varying volatility, see Gabudean and el-Khanjar (2010).
[6] This decomposition makes sense from both a practical and a theoretical point: the two quantities are better modeled with different models, and one refers to the individual distribution of assets while the other only to the joint behavior of the assets.

by 0.3% or by as much as 3%, depending on the value of the other two correlations. The potentially high magnitude and the uncertainty of the effect of correlation errors on the allocations compel a reduced reliance on historical estimates.

- The effect of **errors in Sharpe ratio forecasts** depends in large part on the correlation matrix. The maximum effect of errors on the final allocations matches the condition of the correlation matrix. Thus, the more extreme the correlation matrix eigenvalues (i.e., poorer condition), the more affected the allocations may be. In practice, using less correlated building blocks reduces the effect of correlation forecasting errors.

These results about the effects of forecast errors generalize beyond the MVO framework to any portfolio construction method that maximizes the ratio of performance versus risk, as long as risk scales linearly. Portfolio weights depend on at least three sets of quantities that have correspondents in the MVO case: the scale parameter of each asset return distribution (volatility in the MVO case), the ratio of average return to the scale parameter for each asset (Sharpe ratio), and the dependency (e.g., copula) parameters that describe the multivariate dependence among all assets returns (the correlation matrix). The effects of errors in these quantities match the effects of errors in corresponding MVO quantities.

Risk Parity and other methods that implicitly address forecast errors

Most methods to address forecasting errors essentially combine or replace historical estimates with predefined values for forecasting purposes. Methods vary in their choices of predefined values and how they combine them with the estimates. Below, we discuss some popular choices, with technical details left for the appendix. Many of these methods have been proposed in the literature with arguments outside of the MVO framework, but as has been pointed out by others, the methods are mean-variance optimal under certain assumptions about the risk and return parameters of the underlying assets.

Because mean-variance stems from the maximization of performance versus risk, we contend that these methods are appropriate for investors only when the mean-variance optimality assumptions are satisfied. Any construction method employed by investors must be justified by appealing to the maximization of performance versus risk tenet, regardless of how performance and risk are defined.

Some prominent examples:

Equal-weights portfolio: This portfolio is obtained as a MVO allocation by replacing historical forecasts for all three quantities – volatilities, correlations, and Sharpe ratios – with the assumption that they are the same for all assets.

The Global Minimum Variance portfolio: This portfolio is obtained by replacing historical forecasts for Sharpe ratios with the assumption that they are inversely proportional to volatilities, or that the expected returns are the same. Volatility and correlation forecasts are the same as historical values. We can write its allocations as $w_{GMV} = \Lambda^{-1}\Omega\Lambda^{-1}i_N$

This portfolio does not make any assumptions about volatilities and correlations; thus, it may be exposed to forecasting errors, particularly in correlations. Moreover, the identical expected return assumption may be questionable in many environments, because expected returns tend to be related to the magnitude of risk, which is often captured by volatilities. Therefore, this portfolio construction may be appropriate if we have a strong incentive to believe that assets' expected returns will be the same in the future and that historical estimates provide a reliable and robust forecast of the correlation matrix.

Equal-volatility portfolio: This portfolio is obtained by replacing historical forecasts for correlations and Sharpe ratios with the assumption that they are the same for all assets. Volatility forecasts are the same as historical values. Allocations are inversely proportional to assets' volatilities. The name "equal-volatility" comes from the fact that all allocations have the same volatility estimate (allocation's volatility = weight * asset volatility).

Given the good record at forecasting volatilities from historical data, this portfolio promises to be a significant improvement over the equal-weights portfolio. Furthermore, it underscores the individual nature of volatilities and the direct way it affects allocations. If we replace each asset with a version that is dynamically invested to keep forecast volatility constant over time and across assets, our portfolio can be specified as an equal-weighted combination of these constant-volatility assets. In this interpretation, we can use the equal-volatility as a benchmark to analyze the effect of historical correlations and Sharpe ratios.

We rewrite the general problem in terms of allocations to constant-volatility assets $\tilde{w}_{optim}$

$$\tilde{w}_{optim} \equiv \Lambda w_{optim} = \Omega^{-1} S$$

We can write the equal-volatility portfolio allocations as $\tilde{w}_{EV} = i_N$.

The "Maximum Sharpe Ratio" or "Most-Diversified" portfolio: This portfolio is obtained by replacing historical forecasts for Sharpe ratios with the assumption that they are the same for all assets. Volatilities and correlation forecasts are the same as historical values. If we return to the specification where assets have constant volatility, then the equal Sharpe ratio forecast translates into equal expected returns. Noting that we use the historical volatilities and correlations, this portfolio can be interpreted as the Global Minimum Variance portfolio of the constant-volatility assets. We can write its allocations as $\tilde{w}_{MSR} = \Omega^{-1} i_N$.

The moniker Maximum Sharpe Ratio (MSR), given by Martellini (2008), comes from the fact that this portfolio is primarily justified in the literature under the MVO framework, which maximizes the portfolio Sharpe ratio. The second moniker, Most Diversified, given by Choueifaty and Coignard (2008), comes from the fact that it minimizes risk, but after it makes all building blocks have the same expected return.

The appeal of this portfolio comes from its use of historical data for volatilities and its assumption of equal Sharpe ratios, which seems reasonable in the absence of a reliable expected return forecast. However, the MSR portfolio's weakness comes from its full reliance on historical correlations. As discussed, these are difficult to forecast, and forecasting errors have a potentially large effect on allocations.

The shrunk-matrix portfolio: This portfolio linearly combines the historical correlation matrix with a predefined one, which is typically a zero-correlation, or more generally an equi-correlation matrix, thus addressing the particular weakness of the MSR portfolio. The weight assigned to historical estimates match the level of confidence in these values. While the original implementation by Ledoit and Wolf (2003) uses historical values as forecasts for volatilities and Sharpe ratios, this portfolio can be easily adapted to use forecasts from the methods described above, e.g., constant values, something particularly appealing for Sharpe ratios.

The shrinkage method we discussed works with correlation matrix Ω; however, optimal weights are a direct function of the inverse of the correlation matrix Ω^{-1}. Shrinking the correlation matrix may still leave a poorly behaved matrix. A potentially better approach is to shrink the inverse of the correlation matrix. The pre-defined value can be the same equi-correlation or identity matrix we originally employed to shrink Ω, because their inverse has a similar structure. Therefore, if we interpret the original shrinkage matrix as an arithmetic average of two correlation matrices, then shrinking the inverse can be interpreted as the harmonic average of the same two matrices.

Furthermore, we can extend the shrinkage principle, ubiquitous in statistics, to Sharpe ratios as well. For example, we can shrink the historical values to some predefined, possibly identical ones. Another popular method, similarly designed to reduce the effects of extreme realizations, replaces the actual values of Sharpe ratios with their rank.

The Equal Risk Contribution (ERC, or Risk-Parity) portfolio: This portfolio is obtained by replacing the correlation matrix with a combination of historical values and the identity matrix. Moreover, it replaces historical forecasts for Sharpe ratios with the assumption that they are the same, but keeps historical estimates for volatility. For the correlation matrix, the combination of historical estimates and zero is a complex non-linear one, in contrast to the shrunk matrix described above.

The other two characteristics, constant Sharpe ratios and historical volatilities, are common to two of the portfolios above: the equal-volatility and the Maximum Sharpe Ratio. Noting that one portfolio uses the zero-correlation matrix and the other the historical matrix, we can understand risk-parity as a method that blends the two portfolios. Indeed, for all allocations, the values given by risk-parity are between the ones given by the two portfolios.

While we relegate most technical details to the appendix, we find it illuminating to write the equal-risk contribution conditions as

$$\left(\tilde{w}_{ERC}\right) \bullet \left(\Omega \tilde{w}_{ERC}\right) = \left(i_N\right) \bullet \left(i_N\right),$$

where the $\bullet$ represents element-by-element multiplication. This expression shows that risk parity conditions are simply the product of the equations satisfied by the equal-volatility portfolio $\tilde{w}_{EV} = i_N$ and the MSR portfolio $\Omega \tilde{w}_{MSR} = i_N$.

In ERC formulae, there is no parameter controlling the importance of historical forecasts versus the prior of zero correlations. In contrast to the shrinkage methods that have an explicit weight parameter controlling the combination of the historical estimates and the pre-defined ones, ERC applies a particular penalty to historical correlations. To understand it, we note that the MSR component of ERC equations aims to make all allocations have the same correlation with the final portfolio. However, if this goal results in allocations too different from equal weights – abstracting for volatility – then the ERC method moves the MSR allocations toward equal weights. Given that ERC combines the two methods using product as opposed to sum, larger weights compared with equal ones are more drastically shrunk back toward equal weights. Therefore the larger the effect of correlations on final weights, the more ERC tones down that effect. The strength of this approach is that it potentially reduces more the correlation errors that have a larger effect on final allocations.

Extensions to Risk-Parity that allow for varying Sharpe ratios: The above interpretation of ERC points toward ways to generalize the ERC method. For example, if we depart from the constant Sharpe ratio assumption, then a Sharpe ratio tilted ERC, more aptly named Sharpe ratio-proportional Risk Contributions (SRRC), may be specified as the solution to

$$\left(\tilde{w}_{SRRC}\right) \bullet \left(\Omega \tilde{w}_{SRRC}\right) = S \bullet S\,.$$

ERC can be viewed also as a risk-budgeting exercise, where in its standard form all risks are given the same budget. Sensibly, if we expect an asset to perform better (i.e., have a higher

Sharpe ratio), then we want to increase its risk budget, which is what SRRC achieves as well: total contribution to risk of each asset is proportional to its Sharpe ratio squared.

If, in addition, we have a different prior for the correlation matrix than the identity one, then an intuitively modified version of ERC (let's call it fully-adjusted risk contributions, or FRC) solves

$$\left(\Omega_{prior}\tilde{w}_{FRC}\right)\bullet\left(\Omega_{historic}\tilde{w}_{FRC}\right)=S\bullet S$$

Figure 4 summarizes the various approaches discussed above and how they translate into MVO by listing what forecasts need to be employed in the MVO to obtain the respective portfolios.

Black and Litterman (1992), based on work by Theil and Goldberger (1961), proposed a more sophisticated method to deal with forecast uncertainty, as well as incorporate views about risk and returns. The method is comprehensive, but it leads to a level of complexity seen as a potential drawback. Moreover, the method relies heavily on forecasting errors measured statistically, which may be inappropriate. For example, as we observe more returns over time, Black-Litterman relies more and more on the historical Sharpe ratios, as it sees them more precisely forecasted. However, Sharpe ratios may experience secular changes that make historical observations less relevant. In that case, a better choice is to rely more on a value obtained in a less statistical way by gauging the current economic and policy fundamentals. While similar in spirit to the other methods discussed, we do not detail it further because its complexity puts it beyond the framework we discuss here.

The MVO interpretation of these methods allows for consistent comparison across them. When investors choose a construction method, they can identify which method's implicit forecasts match their situation more closely, underscoring yet again the need for intuitive building blocks. Thus, any method may be valid under the right circumstances. For example, if building blocks can assumed to have similar Sharpe ratios, similar correlations, and varying volatilities that can be reasonably forecast from historical data, then the equal-volatility portfolio is a reasonable choice for investors that prefer a simple method. In a similar setup but with correlations varying across assets, the Maximum Sharpe Ratio Portfolio may be more appropriate. If historical correlations have some forecasting power, then a risk-parity approach may be more appropriate.

Figure 4: Summary of select portfolio construction methods

		Metrics definition			
Name	**Tradeoff definition**	**Sharpe**	**Volatility**	**Correlations**	**VaR/Tail risk**
Equal weights portfolio	Mean / Volatility	Identical	Identical	Identical	Normal distribution
Equal volatility portfolio	Mean / Volatility	Identical	Historical	Identical	Normal distribution
Maximum Sharpe Ratio portfolio	Mean / Volatility	Identical	Historical	Historical	Normal distribution
Equal Risk Contributions portfolio	Mean / Volatility	Identical	Historical	Avg of historical and zero	Normal distribution
Sharpe Ratio proportional RC portfolio	Mean / Volatility	Historical	Historical	Avg of historical and zero	Normal distribution
Fully-adjusted RC portfolio	Mean / Volatility	Historical	Historical	Avg of historical and predefined	Normal distribution
Tail-hedge overlay portfolio	Mean / Volatility and Mean / CVaR	Historical	Historical	Historical	Empirical distribution
Tail-optimization portfolio	Mean / CVaR	Historical	Historical	Historical	Empirical distribution

Source: Barclays Research

These methods can be integrated into a more general framework than MVO by, e.g., adding tail-hedge overlays as described in the previous section and shown in Figure 3. Thus, other solutions to other limitations to MVO may be combined with setups described in this section for a better constructed portfolio.

Capacity constraints in balanced portfolios

Many dimensions of a portfolio construction exercise are not captured by the performance versus risk trade-off framework described thus far. For example, market capacity affects the portfolio through the effect of position size on future returns, which contradicts the implicit assumption we have held thus far that asset performance does not depend on our own allocation. In practice, acquiring a large position may adversely affect the price at which it is acquired and, hence, its future returns. Therefore, portfolio weights depend on the portfolio size, making size an additional parameter in the portfolio construction exercise.

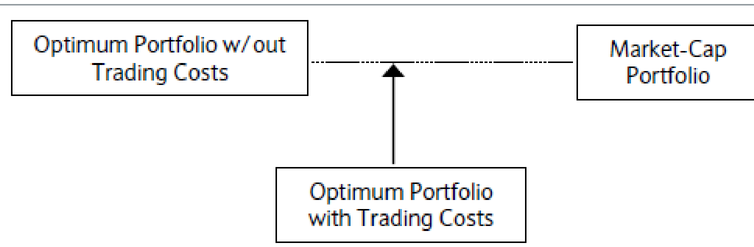
One simple way to address problems with capacity is by increasing it through additional assets that are similar to existing ones (high correlation, similar performance). These related assets must get a special treatment; otherwise, the MVO may try to arbitrage them versus related assets by picking extreme long-short allocations among them. To prevent that, similar assets may be combined into one building block before the portfolio is constructed. The combination may follow equal-volatility weighting, i.e., assume that correlations and performance are identical to prevent any arbitraging effects in the MVO framework.

In absence of additional capacity, to address the problem, we observe that the portfolio least affected by capacity is the market cap-weighted one, or the market portfolio. If the size effect on each asset's performance is proportional to how big the allocation is compared with the asset's market size, then an investor minimizing the size effect will hold the market portfolio. Moreover, the market portfolio is buy and hold; hence, once invested, it requires no trading, absent new issuance. The buy-and-hold feature minimizes rebalancing costs as well.

An investor balances the effect of size versus maximizing the (size-unadjusted) risk-return tradeoff; thus, the final portfolio is a combination of the market portfolio and the optimal portfolio, where by "optimal" we mean the portfolio that maximizes performance versus risk disregarding size, as discussed in the previous sections. We illustrate this setup in Figure 5.

The tilt toward the market portfolio depends on the portfolio size, so the bigger the investor, the closer the portfolio is to the market one. In traditional Modern Portfolio Theory, every investor holds a leveraged version of the market portfolio – the CAPM result. In our framework, the market portfolio remains special, but investors' portfolios are not simple leveraged versions of it. Investors come up with different solutions to the portfolio construction problem because they have different preferences and different forecasts about risk and performance. Forecasts may differ because of the different information that investors possess or because of their investment horizons. This discussion about the market

Figure 5: Optimum portfolio after accounting for size



Source: Barclays Research

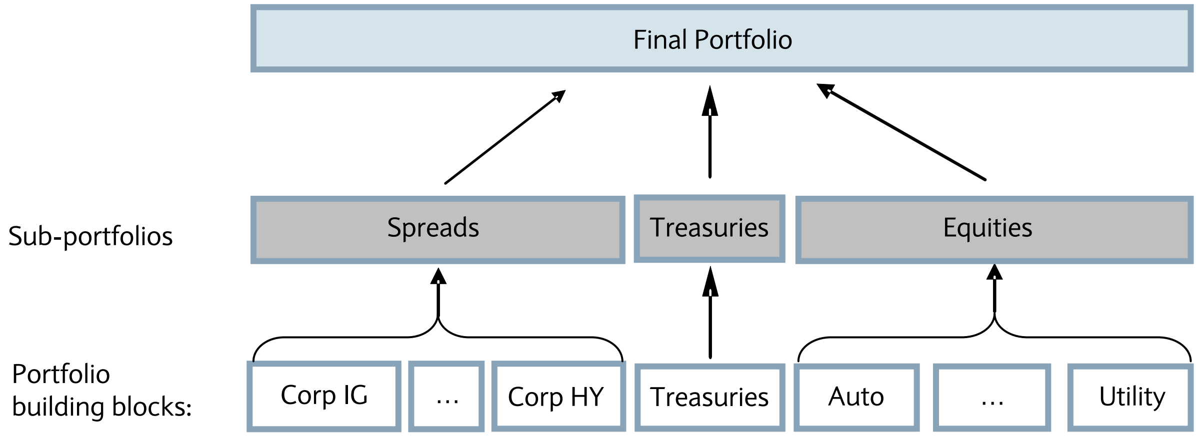
portfolio also informs the choice between a market weight and equal weight portfolios for agnostic investors. An investor without any information to forecast risk and return may appropriately invest in the market portfolio, as it may combine all the views from various investors across all horizons that have been expressed through their investment choices. In contrast, an equal-weighted portfolio may be appropriately chosen by an investor that has an informed belief that the building blocks should be similar and is small enough not to be affected by capacity issues.

Typically, capacity constraints are incorporated into portfolio construction through notional caps on allocations. The caps may be imposed as constraints in the portfolio optimization, but that makes the problem complex and unintuitive to solve. Alternatively, they can be imposed ex-post: unconstrained optimal allocations above their caps are re-allocated to other positions. This simplification may provide an outcome close to the constrained optimal only for well-behaved problems, so it should be used cautiously.

Imposing notional caps, we implicitly assume that capacity will not affect future performance as long as the allocation is below the cap, but for every dollar above the cap, the future performance will suddenly drop to zero. This seems drastic. One method to lessen this effect is to group smaller building blocks before the portfolio is constructed such that the resulting blocks have a similar capacity constraint. Intuitively, assets with smaller capacity get a decreased risk allocation, which should reduce their weight toward the market portfolio weight. The lower risk allocation acts as if the smaller capacity blocks have lower expected performance than their counterparts; thus, we implicitly assume that capacity has a similar effect on every dollar invested regardless of the amount invested. Judging how much we should penalize an asset by decreasing its risk allocation can be an iterative process – we start by building a portfolio while ignoring capacity and then group blocks with notional weights close to or above capacity, effectively decreasing their risk allocation in a second iteration.

The portfolio construction with buckets proceeds hierarchically, first building sub-portfolios and then creating a portfolio of them. When choosing the components of a sub-portfolio, apart from capacity considerations, we may prefer risk premia of similar types to create a sub-portfolio with a few well-defined risk-premia and intuitive properties, a nice feature since they become building blocks of the final portfolio. For example, consider a portfolio of ten US equity industry sectors, one US Treasuries index, and some fixed income spreads. These building blocks have different capacities, with individual US equity industries being much smaller than the US Treasury market. Therefore, we may group US equity industry and spreads sectors into sub-portfolios before combining them with the US Treasuries. Naturally, the equity sectors may be grouped into one bucket, since they all include the equity risk premium, and the spreads sectors into another bucket. These two sub-portfolios each represent well-defined risk-premia types, in contrast to other possible, but unnatural sub-portfolios that mix equity and spreads. Thus, in this example, we create the buckets based on two considerations: capacity and similarity.

Figure 6: Hierarchical portfolio construction to address capacity constraints



Source: Barclays Research

We can treat each sub-portfolio construction as a separate problem – a portfolio construction in its own right – and apply the MVO with previously discussed enhancements. We may take advantage of the similar nature of sub-portfolio building blocks and assume that all correlations and Sharpe ratios are identical, thus using the equal-volatility method to construct the sub-portfolio.

An additional issue potentially affecting performance is turnover, which may be reduced by lowering rebalancing frequency. Apart from rebalancing costs, optimum rebalancing frequency depends on the stability of various risk and return metrics and on the drift of the portfolio weights from the optimum allocation. On the first point, if we have economic and statistical reasons to believe that the metrics are stable, implying stable allocations, then an infrequent rebalancing schedule may prevent excessive turnover from historical estimates' moving around true values. However, even for stable allocations, we have to consider the drift from optimum values due to changes in the market value between rebalancing periods. During volatile periods, the market value of various positions may diverge significantly from the optimum values, thus arguing for a more frequent rebalancing. Furthermore, during such periods, it is likely that forecasts will change as well – particularly for volatilities and Sharpe ratios - changing the optimal allocations.

Leverage

The last issue to treat is leverage, or the cash allocation in the portfolio, which is expressed in the optimum weights equation through the parameter k. The amount of cash borrowed or lent in the final portfolio is simply one minus the sum of risky asset weights w. Leverage can be an explicit target in the portfolio, or often a consequence of explicit return or risk targets. Even when leverage is fixed, the level may be chosen based on some long-term target of risk or performance. In utility-based frameworks, leverage is a consequence of investor risk aversion.

Some investors want to include both a leverage constraint and a risk or performance target. In such cases, leverage typically has a ceiling, often with a constraint of "no borrowing." When borrowing is not allowed, issues arise when the risk or performance target requires borrowing. The constraint decreases the portfolio Sharpe ratio, in contrast to our results at the beginning of this section, because the portfolio is forced to over-allocate to high risk (returns) assets to increase its risk (returns) to the target level.

Leverage is often employed dynamically in Risk Parity approaches to target a fixed level of risk over time. This approach may trap investors, because keeping portfolio risk fixed requires leveraging up during calm periods and deleveraging during volatile ones. Typically, at market tops, volatility is low, meaning investors may enter a downturn leveraged; and at market bottoms, volatility is high, meaning they may enter the recovery unleveraged. To avoid this, investors may target long-run risk or, alternatively, be very conservative with the risk forecast by limiting how much the forecast drops during calm times.

The biggest issue related to leverage comes from large sudden drops in liquidity. These drops are tail events that are difficult to capture in the risk forecast. Traditional securities such as stocks and bonds have limited liability; hence, their value almost never drops to zero, as they have some residual optional value in almost any scenario. Leverage may be applied to these assets because, e.g., investor risk tolerance is higher than the current risk of the asset and adding more risk increases future returns. Once leverage is applied, a temporary drop in asset price, regardless if due to fundamentals or liquidity, may push the value of the leveraged position below zero, forcing the investor to liquidate and lose its entire capital. Moreover, if the market contains many leveraged investors, they may all be forced to liquidate at once, further increasing the probability of a wipe-out. Alternatively, unleveraged investors can weather the storm until the liquidity scare disappears; however, they may still have to take the loss due to the worse fundamentals that triggered the liquidity scare.[7]

[7] Such a scenario played out in the CMBS market during the financial crisis of 2007-09. Before the crisis, the extremely low realized risk prompted many investors to leverage up in order to increase their potential return. Prices dropped during the crisis, initially because of fundamentals. Many leveraged investors were forced to liquidate, dropping prices much lower than fundamentals (e.g., they dropped by 50% in the fall of 2008, while the worst predictions of default losses did not exceed 10%), wiping out many investors in the process. The CMBS market recovered two-thirds of the losses within a year and all of the losses within two.

Asset allocation with risk premia – A practical example

In this section, we apply the principles developed so far to construct a tradable liquid cross-asset portfolio. Instead of trying to create an ideal portfolio, we approach the portfolio construction exercise from the practitioner's point of view, emphasizing practicality over intellectual purity while aiming to justify our decisions.

We use a hierarchical method of portfolio construction, in which we first combine individual investments into investable major asset buckets and then combine these major assets into the portfolio. Tail risk protection is added separately following the method discussed in the previous section.

Individual assets are grouped together based on the primary market risk premium they capture. Within each major asset class bucket, we assume that the constituent Sharpe ratios, as well as all correlations between the constituents, are equal, something that allows us to combine them using the equal volatility method. For the combination of the major asset buckets, we employ the equal risk contribution method while still assuming that Sharpe ratios are equal. This renders the selection of the major asset blocks an important decision affecting the resulting portfolio.

We choose the major asset classes to be global equities, global rates, commodities (without the precious metals sector), global spreads, and alternatives. The separate tail risk bucket holds long volatility products and gold. The resulting average allocation to global rates is about 30%. In contrast, the current common behavior of all risky assets – i.e., equities, commodities, and spreads – may prompt investors to group them together and treat rates as a separate, "safe" group. This approach would allocate a larger notional weight to rates, even in the presence of a tail risk hedge. Even though this method may seem more appropriate in the current financial landscape, we must not forget that the ongoing "lack-of-growth/low inflation" state of the economy is just one of its possible states. Historically, under different circumstances, the returns of the assets in the "risky" bucket are decoupled. For example, under a supply shock, equities and commodities acquire negative correlation. Even more important, there are scenarios under which rates and equities can move in the same direction, such as the lack of growth/high inflation landscape during the 1970s.

For all of these reasons, we decided to generate an example that treats the asset classes corresponding to major components of the economy as separate blocks and constructs a portfolio that works well on average over many possible states of the economy. Users that have specific views about the current and future state of the economy can, of course, adjust the portfolio construction process accordingly.

The building blocks

The portfolio building blocks are listed in Figure 7. Each major asset class is accessed through a combination of market cap-weighted indices from various geographies. Together, all indices from a given asset class provide the exposure to the global risk premium of that asset.

As discussed, the alternative premia occupy a separate bucket. To the extent possible, we try to avoid including alternative premia in the other major asset class buckets. For example, we use market value-weighted instruments in equities to avoid including the small minus big risk premium. However, sometimes trying to separate the pure market risk premium from alternative risk premia can be costly and not entirely practical. In those cases, we allow certain alternative risk premia to be included within the major asset blocks.

Figure 7: Diversified beta portfolio

Name	Construction	Bucket	Backfill Start Date	Bloomberg Ticker	Strategy Start Date
US Equity	Russell 1000 - 1moLIBOR	Equity	Jan-90	RU10INTR Index	Jan-90
Developed Equity ex-US	A basket of index futures	Equity	Jan-90	BXIIEIUE, BXIIGIUE, BXIIJTER, BXIIAUEU	Jan-00
EM Equity	MSCI EM USD - 1moLIBOR	Equity	Jan-99	NDUEEGF Index	Jan-99
Japan Rates	Barclays JP 10yr Treasury Futures	Rates	Jul-96	BXIIJTEA Index	Jul-96
US Rates	Barclays US 10yr Treasury Futures	Rates	Jan-90	BXIIUS10 Index	Feb-97
Euro Rates	Barclays Euro-Bund Futures	Rates	Jan-90	BXIIEU10 Index	Jan-99
UK Rates	Barclays UK Long Gilt Futures Index	Rates	Jan-96	BXIIUK10 Index	Jan-96
US Real Rates	Barclays US Inflation Linked - 1moLIBOR	Rates	Jan-90	BCIT1T Index	Mar-97
UK Real Rates	Barclays Sterling Gilt Inflation Linked - 1moGBPL	Rates	Jul-92	BSIG0T Index	Jul-92
Euro Real Rates	Barclays Germany Inflation Linked Bonds - 1moEuroL	Rates	Jan-99	BCIE1T Index	Jan-00
US MBS	Barclays REMIX Portfolio TBA Proxy	Rates	Jan-90	BRTPTRUU Index	Oct-05
Agriculture	Barclays Pure Beta Agriculture Futures	Commodities	Jan-90	BCC1AGPP Index	Jan-00
Energy	Barclays Pure Beta Energy Futures	Commodities	Jan-90	BCC1ENPP Index	Jan-00
Livestock	Barclays Pure Beta Livestock Futures	Commodities	Jan-90	BCC1LSPP Index	Jan-00
Industrial Metals	Barclays Pure Beta Industrial Metals Futures	Commodities	Jan-90	BCC1IMPP Index	Jan-00
US Investment Grade	Barclays CDX IG 5 yr Basket OTR	Spreads	Jan-90	LX01TUUU Index	Apr-04
Euro IG	Barclays iTraxx EUR 5yr Basket OTR	Spreads	Jan-99	LX02TUEU Index	Oct-04
US High Yield	Barclays CDX HY Basket OTR	Spreads	Jan-90	LX13TUUU Index	Oct-05
EM Sovereigns USD	Barclays EM Tradable USD Sovereign – DurH * US Tsy	Spreads	Jul-92	EMXUTRUU Index	Jul-04
Alternative Strategies	Barclays Cross-Asset Risk Premia 5% Index (XARP)	Alternative beta	Jul-92	BXIIX1P5 Index	Jun-06
Gold	Barclays Gold Nearby S2 ER	Tail-hedge	Jan-90	BCC2GC0P Index	Jan-00
Equity Volatility	S&P 500 VIX Mid-Term Futures	Tail-hedge	Jun-92	SPVXMP Index	Jan-06

Source: Barclays Research.

The separation of alternative risk premia into a separate bucket is based on the assumption that such premia have low correlations among themselves and with traditional market risk premia, and leverages the availability of the Barclays Cross-Asset Risk Premia index which spans a wide array of alternative risk premia. A more involved approach could attempt to separate risk premia according to their style (e.g. pro-cyclical carry, counter-cyclical value, market neutral strategies, and trend premia) or by asset class and attempt to group them in a different way. However, such approach would require an array of additional decisions the analysis of which is beyond the scope of this paper. For this reason we decided to follow the simpler method where alternative risk premia are grouped together in a separate bucket.

Equities bucket

We capture the equities market risk premium using broad indices from the US market, non-US developed markets, and emerging markets. The US market is accessed through the Russell 1000 index. The non-US developed markets index is a fixed-weight portfolio 1.3/1/1.3/1 of futures on four broad indices: Eurostoxx 50, FTSE 100, Nikkei 225, and Australia's ASX 100. Weights are chosen roughly in line with the long-term volatility of the indices. This portfolio closely and parsimoniously replicates the excess-of-funding returns of the MSCI EAFE Local Index (monthly correlation 99%). Thus, for risk purposes, we can proxy non-US developed equities with the MSCI EAFE Local index, which we do before

1999. The use of futures that are unfunded eliminates the primary exposure to FX risk.[8] This is desirable because we generally do not believe that the FX risk among developed markets is compensated.

On the other hand, EM FX has shown persistent positive excess returns and is widely accepted to contain a risk premium, as EM countries need external capital to fuel their economic growth. Accessing EM equities through a funded index, the MSCI EM index denominated in USD, allows the EM FX risk premium to be incorporated into the returns.[9] In addition, from a practical standpoint, it would be difficult and costly to form FX hedges for a multi-currency EM index.

Rates bucket

For rates, we consider the G4 developed countries: US, Europe (Germany), UK, and Japan. We access the rates premium through nominal and inflation-protected treasuries. From a risk perspective, nominal rates contain two types of risk: real rates risk and inflation risk. The inflation-linked bonds contain only real-rates risk. In the US, over the past 10 years our analysis of TIPS and inflation swaps shows that nominal treasuries contain similar amounts of the two types of risk. Admittedly, the past decade has been characterized by extremely low inflation; hence, in the more distant past, inflation risk may have dominated real rates risk.

To keep the construction simple, we do not separate the nominal rates into the two risk types and we give equal risk weights to the nominal and TIPS rates. Implicitly, we assign twice the risk weight to real rates than to inflation. Even though we make this choice for practical reasons, we may justify it ex-post by the importance that real rates hold in the economy. Inflation – while important as well – should theoretically affect the economy mainly over a short term, until expectations adjust. Furthermore, we do not include the Japanese inflation protected bonds because a persistently low inflation environment makes nominal rates almost identical to real ones.

In addition to nominal rates and TIPS, we can access another large fixed-income asset class that is assumed to be backed by the full faith and credit of the US government, agency mortgage backed securities (MBS).[10] MBS embed both the nominal rates premia (real rates and inflation) and an MBS-specific premium coming in part from selling homeowners an implicit prepayment option. However, to isolate the MBS-specific premium, investors have to short (hedge with) a dynamic basket of rates, which tend to be expensive to trade, particularly at the short end. Moreover, the composition of the basket depends on a very complex model of how rates affect the prepayment option, making such hedging a less than robust exercise. For these reasons, we add the MBS exposure in the portfolio without separating the two premia and place it in the rates bucket. The fact that the MBS investment contains a premium we have already included, US rates, means that we implicitly choose a higher risk exposure to US rates compared with other G4 currencies. This can be justified by the larger global importance of the US market; however, the alternative method of isolating the MBS risk premium and including it in the spread bucket is also perfectly reasonable.

Unlike equities, we do not include local EM rates premia, which would entail investing in local currency bonds, because of the short data history and higher difficulty of accessing them for foreign investors.

[8] A second-order component capturing the interaction between FX returns and the equity return remains, but its magnitude is much smaller that the first-order return of equity.
[9] The EM FX risk premium is not fully captured in the alternatives bucket. The XARP index contains the World FX Carry index, which is related but not identical to the EM FX risk premium.
[10] Ginnie Mae is backed explicitly by the US government. Fannie Mae and Freddie Mac are government-sponsored enterprises.

Commodities bucket

For commodities, we consider the broad production-weighted futures indices for the main commodity types: energy, agriculture, industrials, and livestock. Given the lack of correlation among these major types, we presume that there is almost no common premium across commodities and each type carries a separate risk that is compensated, except for precious metals. We choose not to include precious metals because their risk premium potential is dubious: gold had a slow unrelenting decline of 80% from 1983 to 2001. Furthermore, their contribution to the production process is marginal at best.

Spreads bucket

For the credit spreads bucket, we consider exposures to the US, European, and emerging markets. For investment grade credit, we use indices capturing the return of 5y CDX and 5y iTraxx, respectively, for the US and Europe. High yield credit is accessed through the US 5y HY CDX only because its European counterpart does not have sufficient liquidity. Emerging market spreads are accessed through the Barclays EM Tradable USD Sovereign index. This index contains only USD-denominated securities and, therefore, has no FX risk. Unlike our treatment of the MBS premium, here we explicitly hedge out interest rate risk using a short position in US Treasury futures that is netted out with the long futures position in the rates bucket. The rationale is that the magnitude of rates risk contained in EM bonds is significant, and we do not want it to affect the spreads bucket. At the same time, EM bonds contain a significant amount of credit spread risk, which is the reason we do not want to include them in the rates bucket like we did MBS.

Alternatives bucket

The Barclays XARP index (see Khambatta 2012) contains a broad selection of alternative premia accessible in a liquid format. The premia captured span various types across many asset classes, such as carry in FX, rates and equity volatility, value in commodities, curve in rates, congestion in rates, inflation and commodities, trend in FX and commodities, as well as event risk premia in equities. While we could have used a hierarchical structure for the construction of this bucket (e.g., organizing premia by style or asset class), we prefer the simplicity of accessing all alternative risk premia through a single index.

Tail risk hedge bucket

For tail hedging, we retain a long-volatility index and gold. Historically, these assets did not perform over long periods. In addition to the noted underperformance of gold, the long-volatility index loses approximately 30% annually when there are no jumps in equity volatility. Therefore, we do not include them in the portfolio for their expected returns, but for their potential to reduce tail risks. The building blocks we described thus far are exposed to three major macroeconomic risks: growth, inflation, and liquidity drops. While there is potential for diversification across individual premia, the common macroeconomic risks may bring about large negative portfolio returns. To limit the effects of low growth and lack of liquidity, we add the instrument with exposure to options-implied equity volatility that performs particularly well during sharp equity selloffs, which can be taken as indicators of any of the two macro risks. The second hedging instrument, gold, performs well during increased inflationary expectations, as gold is viewed the ultimate universal hard currency.

Additional risk premia exist across various asset classes, but most of them lack either liquidity or reliable historical data. For example, CMBS may carry a risk premium; however, the asset class as it exists today has arguably existed only after the 2008 financial crisis, rendering prior data irrelevant.[11] Since in this exercise we seek to understand the performance of the portfolio over a long period of time, we exclude such asset classes with short history.

Portfolio construction

Portfolio construction follows a hierarchical approach. We first combine individual constituents into the major asset class buckets and then combine the major asset class buckets into the portfolio. The tail risk hedge bucket is added later using the special method described above. The issue of capacity is addressed by carefully selecting the underlying building blocks, as well as by combining “smaller” building blocks into the major asset class buckets.

The major asset class buckets are constructed assuming that 1) each constituent will have the same Sharpe ratio and 2) correlations are similar across all constituents. We employ the equal Sharpe ratio assumption to sidestep the complex discussion about forecasting expected returns, as it is beyond the scope of this paper. Under these assumptions, the optimal portfolio construction choice is equal-volatility. For the tail-hedge bucket, we do not employ a dynamic volatility estimation methodology because the bucket must provide portfolio protection in market events during which volatility may spike unexpectedly. As a result, both the bucket and its constituents have fixed notional allocations. In particular, the two bucket constituents are combined in a 50/50 fixed proportion, roughly matching their long-term volatility.

The final portfolio has six top-level components: five risk-premia buckets and a tail-hedge bucket. We create the final portfolio with the two-step tail-hedge overlay: combine the five premia buckets using a robust mean-variance approach and add a fixed allocation to the hedge bucket as an overlay to the mean-variance portfolio. The mean-variance portfolio is constructed using the risk-parity approach, implicitly assuming that each bucket will maintain the same Sharpe ratio and that the historical correlations are not robust forecasts of future correlations. Consistent with the overlay setup, we account for the tail-hedge allocation in the risk-parity portfolio; hence, the contribution to risk of each bucket is considered versus the tail-hedged portfolio and not of the portfolio that contains only the five premia buckets. Furthermore, in order to avoid excessive notional imbalances between investments, we introduce notional caps post-construction, both within bucket and across buckets at two times equal-weights across premia buckets. This capping is common among practitioners seeking to avoid idiosyncratic risk, i.e., excessive notional exposure to individual constituents that can allow those constituents to dominate the portfolio realized risk and return should their volatility become higher than estimated historically. Such realized/estimated volatility discrepancies occur regularly; AAA CMBS and other securitized products and European sovereign debt serve as vivid reminders.

Before returns data for tradable indices became available (the “strategy start date” in Figure 7), we used suitable proxies. The “backfill start date” shows when such proxies become available. The historical volatility of each block is estimated monthly with an exponentially weighted moving average (EWMA) model using monthly data with a fixed 12-month half-life parameter. The historical correlation across blocks is estimated using an EWMA model of monthly data with a fixed 48-month half-life parameter. We use a longer window to estimate correlations in order to improve the robustness of the estimation, as the number of parameters is much larger than in the case of volatilities. The volatility of a bucket is computed from the current allocation across the blocks within the bucket, their historical volatility, and their average correlation. The forecasting models in this example are chosen for their simplicity and can surely be improved by using more sophisticated covariance estimation methods.

[11] For a discussion of risk drivers of US CMBS securities and how they evolved over time, see Gabudean (2010).

We construct the portfolio monthly from January 1999 to March 2012 with data available at that time, assuming end-of-month trading prices and a reasonable estimate for trading cost. Funding for cash instruments is assumed to be the one-month Libor rate in the respective currency; hence, any FX effect is second order, as only the P&L is affected by it. Lastly, the "EM Sovereign USD" spreads are duration-hedged with the US Treasury 10y futures. The short futures position is netted in the final portfolio with the long US Treasury 10y futures position in the "Rates" bucket.

Results

This section details the results from our portfolio construction exercise and investigate the benefits of various model features, such as geographical diversification, tail hedging, and risk weighting. All returns are shown in excess over cash and after estimated trading costs.

Figure 8 shows the performance of the top-level buckets, contrasting them with the performance of broad US market indices capturing market risk premia in the four major asset classes. Over the period of our analysis, equities performed poorly and had a large drawdown. Our geographically diversified equities bucket achieved a higher return than the Russell 1000 index, but its volatility was similar because any diversification benefits were countered by the higher volatility of EM equities. Rates performed remarkably well in this period. Our diversified rates bucket achieved a Sharpe ratio of 0.70 on the back of dramatically lower volatility and drawdown relative to 0.59 for the US nominal rates index. Commodities diversified across sectors also performed well, with a Sharpe ratio of 0.59, a significant improvement over the 0.22 of the DJ UBS commodity index, but without avoiding a significant drawdown. Spreads performed poorly, but the diversified bucket again did slightly better than the comparable US market (US CDX IG).

The alternative premia index performed extremely well, as various components diversified each other to lower volatility; one must always bear in mind, though, that the performance of this bucket may be overstated because of survivorship bias with respect to the bucket constituents. The tail hedge had a positive return due to the strong performance of gold of about 10% annually, while the VIX position lost approximately 5% annually.

Figure 8: Monthly return statistics of various buckets and the associated broad markets

1999-2012Q1	Equity	Rates	Commodities	Spreads	Alternative premia	Tail hedge	Russell 1K (equity)	US 10y fut (rates)	DJ UBS (commods)	US CDX IG (spreads)
Annual excess ret	2.38%	2.49%	7.48%	0.19%	15.31%	3.74%	0.17%	3.85%	3.88%	-0.11%
Annual volatility	16.7%	3.58%	12.8%	3.62%	5.21%	17.7%	16.4%	6.54%	17.4%	2.67%
Worst month	-19.2%	-2.5%	-17.0%	-3.9%	-1.9%	-10.0%	-17.8%	-5.9%	-21.4%	-3.0%
Best month	11.4%	3.5%	8.7%	3.0%	5.3%	20.5%	11.2%	9.0%	12.9%	2.6%
Max drawdown	-54.5%	-6.9%	-45.6%	-13.1%	-3.3%	-30.9%	-57.6%	-10.7%	-54.7%	-10.2%
Correl w/ bucket							95%	88%	87%	94%
Sharpe ratio	**0.14**	**0.70**	**0.59**	**0.05**	**2.94**	**0.21**	**0.01**	**0.59**	**0.22**	**-0.04**

Source: Barclays Research, Bloomberg

Buckets have a complex correlation pattern, yet were stable over this period, as shown in Figure 9, which compares correlations estimated at two different points in time ten years (and one major crisis) apart. Spreads, equities, and commodities tend to have high correlations among themselves. Rates have low to negative correlation with the three buckets, while alternative betas have almost zero correlation with every other bucket. Tail hedge has a negative correlation with equity and spreads, even more so than rates do, showing that an explicit tail-hedge bucket may be better at hedging these two risks than rates are.

Figure 9: Correlation among buckets as of various dates, using a EWMA (48-month half-life)

F'cast date	Equity	Rates	Commodities	Spreads	Alternatives	Tail hedge
Mar-02		-1%	27%	64%	-4%	-47%
	-1%		-2%	-24%	46%	2%
	27%	-2%		17%	5%	-6%
	64%	-24%	17%		-7%	-23%
	-4%	46%	5%	-7%		29%
	-47%	2%	-6%	-23%	29%	
Mar-12		-18%	44%	75%	-7%	-53%
	-18%		-7%	-26%	34%	28%
	44%	-7%		23%	7%	-14%
	75%	-26%	23%		-21%	-58%
	-7%	34%	7%	-21%		31%
	-53%	28%	-14%	-58%	31%	

Source: Barclays Research, Bloomberg

The fact that correlations are not all the same and seem to have remained stable over the past ten years suggests that incorporating correlation estimates in portfolio construction would be useful. Of course, in the more distant past, the economy went through phases that were very different from the past ten years. During those phases, correlations among major asset classes looked very different. For example, during supply shocks, commodity and equity returns show negative correlation: efficient production of commodities may help the economy grow and contribute to good equity returns while commodity prices drop; conversely, geopolitical events may cause spikes in commodity prices while causing equity prices to drop. Furthermore, in high inflation / low growth states of the economy, both equities and rates have negative returns. A transition into a new state of the economy, one very different from the past ten years, is very possible, even anticipated by many practitioners. For this reason, we want a portfolio construction method that uses correlation estimates but is not overly sensitive to estimation errors, such as the Equal Risk Contribution method (ERC). Thus we choose ERC to combine the major asset class buckets. Furthermore, the correlations within buckets (not shown) are either all high (equity, rates, spreads) or close to zero (commodities, alternatives) and not expected to change dramatically in different economic regimes, justifying the equal-volatility approach.

The ERC method chosen to construct the example portfolio is meant to be more agnostic with respect to the future behavior of financial markets than methods that assume that the recent behavior will continue unchanged. By using equal expected Sharpe ratios, ERC professes complete inability to rank asset classes with respect to risk-adjusted returns, regardless of their recent performance. The method relies on correlation estimates because economic regimes that drive major asset class correlations are persistent over long periods of time. However, it shifts historical correlations toward zero, a level consistent with complete lack of information. Lastly, asset volatilities are taken to be the historical estimates (except for tail-risk hedging) because even a time-varying volatility can be forecast with reasonable accuracy in a dynamic setting.

Of course, portfolio managers with views on both expected returns and correlations can always incorporate such views in the above methodology by replacing the Sharpe ratios and correlations with their own forecasts. A method to incorporate views considers several possible macro-economic scenarios and estimates the statistical properties of assets (returns, volatility, and correlations) conditional on the particular scenario. Using these conditional forecasts, various “scenario-optimal” portfolios can be constructed and then merged into a final portfolio with certain scenario weights.

In order to evaluate the diversification benefits of the ERC method separately from the effect of the inclusion of the alternatives bucket and the tail-risk hedge, we will construct and compare three different portfolios:

(a) The **base portfolio** is constructed by combining the four major asset class buckets (equity, rates, commodities, spreads) using the ERC methodology described above. The portfolio is rebalanced monthly.

(b) The **base + hedge portfolio** adds tail risk protection to the base portfolio. Using estimates of the effect of the tail hedge allocation on the base + hedge portfolio's Sharpe ratio, Sortino ratio, and skewness, we size the tail hedge bucket at 6% of notional.

(c) The **complete** portfolio is constructed by applying the ERC methodology to five buckets, the four major asset classes and alternatives, and maintains the 6% allocation to the tail hedge bucket.

Figure 10 shows statistics of bucket weights. In notional terms, the complete portfolio is dominated by rates, spreads, and alternative premia, each in the range of 15-33%. Equity and commodities are at 6-9%. Weights vary significantly over time; the highest monthly allocation is more than double the lowest one for several buckets. Moreover, spreads hit their 37.6% cap (2x equal weight except for tail hedges) on some months. However, the turnover of the portfolio is low, at less than 3%/month on average. The turnover of the individual positions is shown on relative basis, e.g., a 3% turnover on an average notional exposure of 30% means that the exposure may increase to 31% = 30%*(1+3%).

Figure 10: Weight and relative turnover averages for various portfolios

1999-2012Q1	Portfolio	Equity	Rates	Commodities	Spreads	Alternatives	Tail
			Complete portfolio				
Weight	100.0%	6.1%	29.8%	8.5%	33.0%	16.6%	6.0%
Relative turnover	2.6%	4.5%	2.1%	3.6%	2.2%	2.8%	3.8%
Lowest weight	100.0%	4.5%	24.8%	5.0%	22.0%	12.8%	6.0%
Largest weight	100.0%	9.6%	37.6%	13.0%	37.6%	24.0%	6.0%
			Base portfolio				
Weight	100.0%	6.3%	45.7%	10.9%	37.1%		
Relative turnover	2.1%	4.6%	1.6%	3.7%	2.0%		
			Base + hedge portfolio				
Weight	100.0%	6.8%	40.8%	9.8%	36.6%		6.0%
Relative turnover	2.3%	4.5%	1.7%	3.7%	1.9%		3.8%

Source: Barclays Research, Bloomberg

Contrasting the three portfolios, the base portfolio contains a large allocation to rates at 45.7%, which would have been even higher if not for the 50% cap (2x equal weight). When we add a 6% allocation to tail hedge, almost all of it (4.9% on average) comes from the rates bucket, leaving the other allocations essentially unchanged. Moreover, rates hit the cap less often.

This result illustrates that the rates bucket in the portfolio has tail-risk hedging properties. In the presence of an explicit tail risk hedge, a risk-parity allocation method drops the exposure to rates significantly as their diversification role diminishes. This reduced exposure matters more in situations in which yields are particularly low (as they are currently) and investors are uncomfortable with large exposures to rates because of the potential of significant negative returns should yields mean-revert to higher levels. Moreover, tail hedging allows for an increased allocation to "growth" assets, such as equities and credit. The secret to an ERC (i.e., risk-parity) portfolio is adding enough unrelated risks so the portfolio does not rely solely on rates as diversifiers.

Next, we investigate the realized contributions to portfolio risk and return over the 1999-2012 sample. In Figure 11 we show the results for the three main portfolios: the complete one, the base, and the base + hedge. We note that half of the complete portfolio return comes from alternatives, while equities and spreads have a very small contribution, in line with the general performance of these two asset classes over the period. Risk contributions are very similar across buckets, as we intended, showing that over this decade, the correlations and volatility forecasts were accurate. The below-average contribution from rates in the base portfolio comes from the 50% cap mentioned above.

Figure 11: Contributions to realized portfolio return and risk, by bucket

1999-2012Q1	Equity	Rates	Commodities	Spreads	Alternatives	Tail hedge
			Complete portfolio			
Return	5%	17%	17%	1%	53%	7%
Risk	19%	18%	21%	19%	17%	6%
			Base portfolio			
Return	9%	50%	40%	1%		
Risk	26%	19%	28%	27%		
			Base + hedge portfolio			
Return	10%	42%	36%	1%		12%
Risk	25%	23%	26%	24%		1%

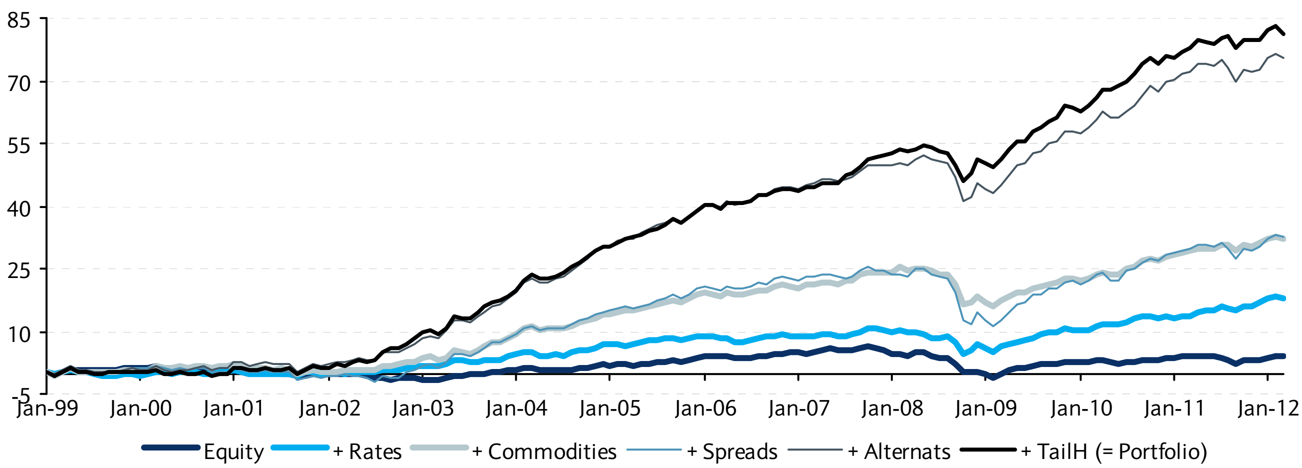
Source: Barclays Research, Bloomberg

Interestingly, once we add the hedge to the base portfolio, the contribution to return of equities increases slightly and its risk contribution goes down, while the total portfolio return increases and portfolio risk decreases. The rates contribution to risk increases to the targeted level as we hit the cap less often, while the lower allocation reduces the contribution to return. The 46% average allocation to rates in the base portfolio is not enough to make the rates contribution to risk equal to the other buckets, while the lower 40% allocation in the base + hedge hits the target. These results again show the strong benefits of tail hedging by reducing the reliance on rates as diversifiers.

Figure 12 details how various buckets contribute to the complete portfolio P&L over time, assuming an initial investment of 100 units. Initially, from 1999 to 2001, each asset class contributed little to P&L and their performances canceled each other. From 2003 on, equities had a meaningful contribution until the 2008 crisis, when all the gains were wiped out, followed by a rebound after 2009. Rates added value in 2002, but then the gain over equities remained flat until the crisis, meaning they did not have any additional contribution to the portfolio P&L over the period. However, after late 2008, that behavior changed, and rates added significant value. Commodities enjoyed strong years until 2008, when they lost a significant part of their gains, only to recover them post-crisis. Spreads had a miniscule contribution until the crisis, in spite of their large notional allocation, at which point they became a small drag on portfolio performance until the crisis passed. The alternatives were strong throughout; during the crisis they reduced the loss generated by other allocations (in the graph, the alternatives line drops less than the spreads line). Lastly, the tail hedge starts as a drag on performance until mid-2001, when it contributed two units for the portfolio over the next year and a half to compensate for the underperformance of equities and rates.

The contribution then stays flat until mid-2007, when it starts to increase significantly until 2012, particularly during the equity market routs of 2010 and 2011.

Figure 12: Cumulative contribution to complete portfolio P&L by bucket (stacked, portfolio notional on December 1998 = 100)



Source: Barclays Research, Bloomberg

Figure 13 introduces two more portfolios and compares various historical performance metrics of all five portfolios. The new portfolios are:

(d) The **major assets portfolio** is constructed by applying the ERC method to the four indices representing the major US asset classes: equities (Russell 1000), rates (10y US government bond futures index), commodities (DJ-UBS Commodities index), and credit (US CDX investment grade index). By comparing this portfolio with the base portfolio, we can assess the value of geographical and asset diversification in the bucket construction.

(e) The **base + alternatives portfolio** adds the alternatives bucket to the base portfolio. The comparison between these two portfolios provides better insight on the role of the alternatives bucket.

Figure 13: Statistics of various portfolios

1999-2012Q1	Major assets portfolio	Base portfolio	Base + hedge portfolio	Base + alternatives portfolio	Complete portfolio
Annual return	2.10%	2.46%	2.58%	4.84%	4.60%
Annual volatility	3.34%	3.14%	2.87%	2.92%	2.80%
Worst month	-4.4%	-5.4%	-4.3%	-3.5%	-2.8%
Best month	2.3%	2.4%	2.3%	2.3%	2.3%
Max drawdown	-8.9%	-10.2%	-7.9%	-6.6%	-5.7%
Skewness	-0.9	-1.7	-1.1	-0.8	-0.4
Sortino ratio	**0.9**	**1.1**	**1.4**	**3.0**	**3.2**
Correlation w/ Complete	81%	86%	95%	95%	100%
Sharpe ratio	**0.63**	**0.78**	**0.90**	**1.65**	**1.64**

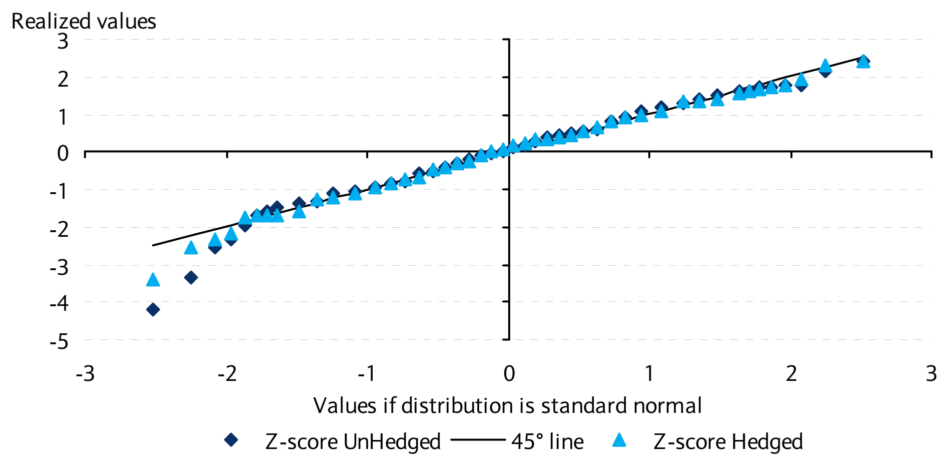
Note: Sortino ratio is defined over a 0% threshold. Source: Barclays Research, Bloomberg.

The complete portfolio has significantly better performance than all of its constituents (see Figure 8) except the alternatives bucket. Its historical net-of-cost Sharpe ratio is 1.64. On the other end of the spectrum, the major assets portfolio has a Sharpe ratio of 0.63. This is higher than the Sharpe ratios of all of its constituents (Figure 8), highlighting the benefits of risk diversification. Its historical volatility is 3.34%, significantly lower than all constituent asset classes except credit. Diversifying the exposure within an asset class by adding geographic and other premia as in the base portfolio (e.g., MBS, High Yield) increases the performance to a Sharpe ratio of 0.78, lowers volatility further to 3.14%, and even improves the performance versus tail risk tradeoff, as evidenced by a Sortino ratio of 1.1 versus 0.9 for the major assets portfolio.

The tail-hedge addition to the base portfolio significantly improves its properties: the Sortino ratio increases from 1.1 to 1.4, and the skew drops from -1.7 to -1.1. Given the stellar performance of the alternatives bucket in our sample, adding it to the base portfolio greatly increases performance, with all metrics improving dramatically. However, in evaluating these results, we have to take into account the survivorship bias that may affect the XARP index representing the alternatives bucket. Tail hedging remains useful even after the inclusion of the alternatives bucket: while the hedge cost causes the Sharpe ratio of the complete portfolio to be slightly lower than the Sharpe ratio of the base + alternatives portfolio, tail risk metrics (worst month, maximum drawdown, skewness, Sortino ratio) improve markedly.

Figure 14 shows the distribution percentiles of monthly Z-scored returns (i.e., subtracting the realized mean and dividing by realized volatility) for the base and base + tail-hedge portfolios versus the standard normal distribution. If the returns follow a standard normal distribution, then the percentiles should align on the 45° line. Notice the larger-than-expected extreme negative base portfolio returns (well below the line) and the significant reduction once we add tail hedging (percentiles closer to the line).[12]

Figure 14: Distribution of base and base + hedge portfolios monthly Z-scored returns versus standard normal distribution; select percentiles shown



Source: Barclays Research, Bloomberg

All above results are based on data from the past 13 years. To showcase the benefits of a risk-balanced portfolio construction approach over multiple economic regimes, we seek to expand the analysis over a longer historical period. However, many of the risk premia included in the

[12] As a fine point, we compute the realized volatility for the purpose of Z-scoring without the extreme events because they increase the realized volatility and reduce the Z-scores. The approach is validated by the fact that all other percentiles follow the 45° line, which means they follow a normal distribution with our chosen level of volatility.

complete portfolio have short histories, particularly the non-US ones. To facilitate the long-term analysis, we retain only risk premia with reliable long-term histories; in equities, just the US component; in rates, US nominals, US TIPS, MBS, and European nominals; in commodities, all four sectors are retained; in credit, we retain only US IG. We then combine these limited major asset class buckets into a portfolio using the ERC methodology similar to the base portfolio described above. We call this portfolio, for which we can obtain historical performance data since 1979, the **long-history base portfolio** and compare it with its constituents and composite portfolios using alternative methodologies.

Figure 15: Statistics of long-term portfolios, buckets, and markets

1979-2012Q1	Long-history base portfolio	Major assets portfolio	40/30/20/10	Equity (= Russell 1000)	Rates	Commodities	Spreads (= US CDX IG)	US 10y	DJ UBS
Annual return	1.67%	1.68%	3.52%	5.09%	1.77%	3.19%	0.14%	2.49%	1.34%
Annual volatility	3.2%	3.4%	8.1%	15.7%	4.9%	12.0%	3.6%	7.5%	14.1%
Worst month	-5.1%	-4.4%	-12.0%	-22.3%	-5.4%	-17.0%	-4.2%	-7.8%	-21.4%
Best month	2.7%	2.8%	6.4%	12.4%	7.7%	11.9%	3.9%	9.0%	12.9%
Max drawdown	-12.2%	-14.1%	-30.3%	-57.6%	-32.3%	-53.3%	-19.3%	-40.2%	-54.7%
Correl w/ Long-history base	100%	90%	82%	69%	44%	62%	48%	31%	54%
Sharpe ratio	**0.53**	**0.50**	**0.44**	**0.32**	**0.36**	**0.27**	**0.04**	**0.33**	**0.09**
1979-1988									
Annual return	0.92%	0.88%	2.48%	4.90%	0.02%	2.66%	0.61%	-0.53%	0.69%
Annual volatility	3.7%	3.9%	9.1%	16.9%	6.9%	13.8%	4.0%	9.7%	13.0%
Sharpe Ratio	**0.25**	**0.22**	**0.27**	**0.29**	**0.00**	**0.19**	**0.15**	**-0.05**	**0.05**
1989-1998									
Annual return	1.17%	1.93%	5.88%	12.17%	1.71%	-1.74%	-0.21%	3.77%	-1.32%
Annual volatility	2.2%	2.7%	6.4%	13.4%	3.4%	8.5%	2.4%	6.1%	9.7%
Sharpe ratio	**0.52**	**0.71**	**0.92**	**0.91**	**0.51**	**-0.20**	**-0.09**	**0.62**	**-0.14**
1999-2012Q1									
Annual return	2.63%	2.10%	2.55%	0.17%	3.15%	7.48%	0.06%	3.85%	3.88%
Annual volatility	3.3%	3.3%	8.4%	16.4%	4.0%	12.8%	4.1%	6.5%	17.4%
Sharpe ratio	**0.80**	**0.63**	**0.30**	**0.01**	**0.78**	**0.59**	**0.01**	**0.59**	**0.22**

Source: Barclays Research, Bloomberg

The long-term performance of the various buckets and the US market indices are shown in Figure 15. Note that Russell 1000 and US CDX IG are identical to the equities and spreads buckets, respectively. We notice the better performance of the diversified rates and commodities buckets versus the broad market indices during the 2000s, while the comparison for 1980s and 1990s is inconclusive. However, volatility is significantly lower for all sub-periods, showing the effects of diversification. Furthermore, the risk varies dramatically over time for all assets: elevated levels in 1980s, low levels in 1990s, and back to high levels in 2000s. One exception is US rates, for which risk in the 2000s was still much lower than in the 1980s. This variation in risk argues against the static weights portfolios often favored by practitioners.

We show one such portfolio, the 40 (equities)/30 (rates)/20 (commodities)/10 (spreads). For comparison, we add the portfolio of the major assets constructed using ERC, which bests the 40/30/20/10, showcasing the advantage of risk diversification across asset types. Furthermore, the additional diversification in the rates and commodity buckets of the long-history base portfolio enables this portfolio to improve upon the simpler major assets portfolio over the entire sample history.

The ranking among these three portfolios does not hold for each sub-period, because different asset classes performed within various sub-periods, and whichever portfolio was tilted more toward the winning asset benefited accordingly. For example, the 40/30/20/10 is tilted more toward equities than the other two, benefiting from the 1990s bull market, only to underperform in 2000s. However, across the entire sample and the various environments it contains, the portfolio best diversified across environments came out on top, underscoring the fundamental diversification premise: it balances performance across various risk premia without making a call on which premium will deliver best over the next period.

In this example, we see the importance of having a diverse set of premia in a portfolio, particularly the alternative ones, to capture the benefits of diversification. Tail hedging, even if unrewarded by itself, may dramatically reduce risk – thus allowing for a higher, or leveraged allocation to typical premia – and reduces the need for a high allocation to rates to diversify the equity exposure beyond what is justified by rates premium. Furthermore, tail-hedging overlays work to make the return distribution closer to normal and allow investors to use mean-variance techniques to construct the rest of the portfolio. Lastly, this risk-balanced approach to portfolio construction delivers results even over long periods, and one possible explanation comes from the fact that the identical Sharpe ratio assumption has been appropriate over extended periods.

Conclusion

Asset management has become a complex exercise that requires good understanding of an increasing number of accessible investments, as well as a variety of portfolio construction methods, along with their underlying assumptions and interactions. We have discussed several important principles that investors should keep in mind during the portfolio construction process.

While the topics we raised are complex at both conceptual and practical levels, our main points are intuitive and practical. We have argued for a risk premium factor-based investment approach based on the considerable merits of 1) economic intuition of the portfolio components and 2) robustness of portfolio construction. For portfolio construction, we remain within the spirit of the Modern Portfolio Theory of maximizing performance versus risk while addressing the shortcomings of the traditional implementations. To deal with tail risk intuitively, we presented a two-step approach that computes the allocation to tail hedges separately from the allocations to other blocks with positive expected returns and accounts for the cross-effects between the two types of blocks. A second shortcoming, the forecast errors from historical estimates of performance and risk, is implicitly addressed by several methods seemingly separate from MPT, yet their underlying assumptions put them squarely within it. Lastly, we examine the effect of capacity constraints on allocations. In traditional implementations, these constraints lead to intractable problems, but we propose addressing them using a hierarchical approach whereby blocks with smaller capacity are combined into larger ones.

The framework presented is meant to be intuitive to use and flexible enough to accommodate investors' particular preferences and conditions. The resulting portfolios should be practical and the effect on them of each investment decision fully transparent.

Using the above principles, we constructed a tradable liquid cross-asset portfolio and studied its performance over the past 13 years in relationship to portfolios from alternative construction methods: within this period, our example portfolio exhibits superior risk-adjusted performance. Furthermore, we analyzed the effect of our suggested improvements in the investment process and found that each adds significant value to our example portfolio along the lines expected. Because we chose a construction method that is robust over various macro-economic states and agnostic about future asset returns, the resulting portfolio can be improved upon if we incorporate tactical information about the current economic environment. However over longer time horizons, we expect it to have competitive risk-adjusted performance, as observed.

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Appendix: Detailed results

General portfolio construction setup

We consider a portfolio composed of N risky asset and cash. Cash can be freely borrowed and lent at the risk-free rate. Let *w* be the vector of allocations to risky assets. It follows that the allocation to cash is $1-w'i_N$ where i_N is a vector of ones. If we do not impose the constraint that risky weights *w* sum to one and we define expected returns to be in excess of the risk-free rate, we can ignore the cash allocation from now on.

We formulate the portfolio problem as:

(A1) $$\max_w \frac{\mu_p}{V_p} = \max_w \frac{w'\mu}{V_p}$$

Where $\mu_p = w'\mu$ is the portfolio expected return, V_p is a measure of portfolio risk, and μ is the vector of assets expected returns forecast.

This formulation is identical to a formulation where *w* solves

(A2) $$\max_w \mu_p - 0.5*b*V_p^2$$

The two equations have the same first-order conditions, regardless of the parameter *b*.

We impose the condition that risk scales linearly with the allocations, i.e., if a portfolio q has the weights $w_q = k*w$, where *k* is a scalar, then its risk $V_q = k*V_p$. Most risk measures considered satisfy this condition, e.g., volatility, VaR, CVaR, Downside Deviation.

Because portfolio expected returns also scale linearly with allocations, we find that $\mu_p/V_p = \mu_q/V_q$. If the portfolio p solves the portfolio problem (A1), then portfolio q solves it as well, regardless of level of *k*. Hence, we need an additional constraint on *k*. We can set the level of k by choosing either μ_p, V_p, or the allocation to cash $1-w'i_N$. Once we set one of these three quantities, the other two are determined by the maximum ratio μ_p/V_p; thus, it does not matter which one we set. In the alternative formulation (A2), the level of *k* is determined by the parameter *b*, which can be interpreted as the level of an investor's risk aversion.

From the first-order conditions and the degree-one homogeneity of the risk and returns functions, we get for the optimum portfolio:

$$0 = \frac{\partial \mu_p}{\partial w_i} V_p - \frac{\partial V_p}{\partial w_i}\mu_p \Leftrightarrow \frac{\mu_i}{MRC_i} = \frac{\mu_p}{V_p}$$ where $MRC_i \equiv \frac{\partial V_p}{\partial w_i}$, the marginal risk contribution of asset *i*. Note that the marginal contribution to portfolio return from asset *i* is its expected return μ_i. The ratio of marginal contribution to return to MRC is the same for all assets and equals the portfolio ratio of expected return to risk (maximum possible).

The special case of risk defined as volatility

If we set portfolio risk to be its volatility, then we can write

$$\max_w \frac{\mu_p}{Vol(r_p)} = \max_w \frac{w'\mu}{\sqrt{w'\Sigma w}} = \max_w S_p$$ and $w'i_N = 1$,

where Σ is the assets covariance matrix forecast ,$Vol(r_p) = \sqrt{w'\Sigma w}$ is portfolio volatility, and S_p its Sharpe ratio. The maximization implies that $\mu - k\Sigma w = 0$, where k is such that $w' i_N = 1$.

We can re-write the equations in terms of volatilities, correlations, and Sharpe ratios:

$$\mu - k\Sigma w = \mu - k\Lambda\Omega\Lambda w = \Lambda\left(\Lambda^{-1}\mu - k\Omega(\Lambda w)\right) = \Lambda(S - k\Omega\Lambda w) = 0 \Rightarrow S - k\Omega\Lambda w = 0,$$

where Λ is a diagonal matrix of assets volatilities, Ω is assets correlation matrix, *and S* is a vector of assets Sharpe ratio. Then the optimum weights are:

$$w_{MVO} = k^{-1}\Lambda^{-1}\Omega^{-1}S$$

From the no-leverage constraint, we get $k = i'_N\Lambda^{-1}\Omega^{-1}S$. From their definitions, the portfolio mean is $\mu_p = k^{-1}S'\Omega^{-1}S$ and volatility $V_p = k^{-1}\sqrt{S'\Omega^{-1}S}$. If instead of imposing no leverage, we choose a target for μ_p or V_p, then we modify *k* accordingly, e.g., $k = \mu_p^{-1}S'\Omega^{-1}S$. Furthermore, the portfolio Sharpe is $S_p = \sqrt{S'\Omega^{-1}S}$, which does not depend on *k*. If we use the utility specification (A2) for the tradeoff, then *b*=*k*.

Specializing the results from the previous section, we get $MRC_i = \dfrac{\mu_i}{S_p}$ or $\dfrac{\mu_i}{MRC_i} = S_p$.

The ratio of marginal contribution to return to MRC is the same for all assets and equals the optimum portfolio Sharpe ratio (maximum possible).

Effect of forecasting errors on optimal allocations

To see the effect of errors, we look at the expression $w_{MVO} = \Lambda^{-1}\Omega^{-1}S$, where we dropped *k*. For more clarity, we can define the volatility-adjusted weights $\tilde{w}_i = w_i\sigma_i$ or $\tilde{w} = \Lambda w$. Then $\tilde{w}_{MVO} = \Omega^{-1}S$.

The effect on *w* of errors in Λ (assuming no influence on Ω and *S*) is straightforward: it does not affect $\tilde{w}$; hence, any error in σ_i is canceled by the opposite effect on w_i.

The effect on *w* of errors in Ω (assuming no influence on Λ and *S*) can be implied from their effect on $\tilde{w}$. There are no analytical results we are aware of, but we can study the effect numerically, and in the text, we mention the results of a simple numerical study.

The effect on *w* of errors in S (assuming no influence on Ω and Λ) can be implied from their effect on $\tilde{w}$. Given the algebra results about the condition number of a matrix, we can state that the maximum effect on $\tilde{w}$ is the condition number of Ω. Note that errors in Ω tend to increase its condition number; hence, they may compound the effect of errors in S.

Note that the scaling parameter *k* is a function of Ω, Λ, and *S;* hence, any error in these metrics will affect *k* as well. This means that the optimum leverage is affected by errors in correlations, volatility, and return forecasts.

For cases in which portfolio risk is not defined as volatility, then we can replace volatility in the definition of Sharpe ratios and $\tilde{w}$ with a measure of scale, making the final portfolio depend only on scale-free measures of returns, the joint dependence of assets, and some potential shape parameters of the individual distribution of each asset.

Portfolio allocations for various methods

Below, we show the results of various portfolio allocations using the definitions above for various metrics. The metrics Ω, Λ, and *S* are assumed to be estimated historically if not specified otherwise.

Mean Variance (MVO): $\tilde{w}_{MVO} = \Omega^{-1}S$. Portfolio forecast Sharpe ratio is $S_p = \sqrt{S'\Omega^{-1}S}$, which does not depend on volatility. No-borrowing scaling is $k_{MVO} = i'_N \Lambda^{-1}\Omega^{-1}S$. Note that the *k* scaling depends on volatility even for $\tilde{w}$ formulation.

Global Minimum Variance (GMV): $w_{GMV} = \Sigma^{-1}i_N \Rightarrow \tilde{w}_{GMV} = \Omega\Lambda^{-1}i_N$ matching the special MVO case when we replace S by $m*\Lambda^{-1}i_N$, or μ by $m*i_N$. The parameter *m* is the asset return. Portfolio forecast Sharpe ratio is $S_p = m\sqrt{i'_N \Lambda^{-1}\Omega^{-1}\Lambda^{-1}i_N}$. No-borrowing scaling is $k_{GMV} = i'_N \Lambda^{-1}\Omega^{-1}\Lambda^{-1}i_N$

Equal volatility (EV): $\tilde{w}_{EV} = i_N = E^{-1}i_N$, matching the special MVO case when we replace Ω by $E \equiv (1-c)*I_N + c*i_N i'_N$ and S by $s*i_N$. The properties of *E* are further discussed in the "Shrinkage" case. Portfolio Sharpe is $S_p = s\sqrt{\frac{N}{1+(N-1)c}}$, which decreases with *c*. No-borrowing scaling is $k_{EV} = i'_N \Lambda^{-1}i_N$, which does not depend on *s* or *c*.

Maximum Sharpe Ratio (MSR): $\tilde{w}_{MSR} = \Omega^{-1}i_N$, matching the special MVO case when we replace S by $s*i_N$. Portfolio Sharpe is $S_p = s\sqrt{i'_N \Omega^{-1}i_N}$. The term $\sqrt{i'_N \Omega^{-1}i_N}$ shows the effect of diversification. No-borrowing scaling is $k_{MSR} = i'_N \Lambda^{-1}\Omega^{-1}i_N$.

Risk Parity: $TRC_i = c \Rightarrow w_i\sigma_i\left(\sigma_i w_i + \sum_{j\neq i}\rho_{i,j}\sigma_j w_j\right) = c \Rightarrow \tilde{w}_i\left(\tilde{w}_i + \sum_{j\neq i}\rho_{i,j}\tilde{w}_j\right) = c$ for all *i*. As shown in Maillard et al. (2010), the portfolio forecast Sharpe ratio is between the MSR and equal volatility.

We define the element by element operator $\bullet$ so we can write the equations above as

$$(\tilde{w}_{ERC}) \bullet (\Omega\tilde{w}_{ERC}) = (i_N) \bullet (i_N)$$

Note that the EV and MSR portfolios satisfy $\tilde{w}_{EV} = i_N$ and $\Omega\tilde{w}_{MSR} = i_N$. If we multiply separately the right and left sides of the two sets of equations and impose the two sides to be the same, we get the ERC equations.

Alternatively, we may re-write the ERC conditions with the equal weight and GMV portfolios in the relations above, which implies that we shrink the covariance matrix Σ towards the identity one. However, its implied treatment of volatilities makes it an intuitive approach.

Shrinkage: We want to investigate what is the inverse of an equi-correlation matrix. Let $E = (1-c)*I_N + c*i_N i'_N$, then $EE^{-1} = I_N$ and E^{-1} has a similar structure with *E*:

$E^{-1} = \frac{1}{1-c}*I_N - \frac{c}{(1-c)(1+(N-1)c)}*i_N i'_N$. Further, $Ei_N = const*i_N$, a result used for the equal-volatility portfolio. Hence, if we shrink Ω^{-1} to a matrix with constant diagonal and non-diagonal elements E^{-1}, it can be interpreted as taking the harmonic average of Ω and an equi-correlation matrix, for appropriately-chosen E^{-1}. In practice, we can choose an average correlation level *c* and use the expression derived above for E^{-1}.

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