QUANTITATIVE PORTFOLIO STRATEGY

Assembling Portfolios of Alpha Strategies: Adjusting for Volatility and Correlation Clustering

The risk profile of a portfolio of systematic alpha strategies can be very unstable when volatilities and correlations of individual strategies vary over time. This can be detrimental to portfolio performance. We consider a dynamic rebalancing mechanism based on estimated short-term volatilities and correlations and show that this can help stabilize portfolio risk and improve efficiency.

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Introduction

How should portfolio managers allocate a target risk budget across a set of alpha strategies? Due to the dynamic nature of most systematic strategies, the risk of any portfolio of such strategies is unlikely to remain constant over time. In fact, the short-term risk of individual strategies changes and often displays clustering, with high volatility regimes followed by subdued periods. In addition, correlations among active strategies can vary significantly and even change sign due to asynchronous changes in the underlying positions.

All these factors can cause instability in a portfolio's short-term volatility. Such instability is undesirable, as it makes the portfolio exceed its risk budget in volatile periods and miss its alpha target in steady times. Fortunately, short-term volatilities and correlations are often predictable and can be estimated using statistical techniques.

In this article, we propose a mechanism that helps stabilize portfolio risk through dynamic re-balancing based on estimated short-term volatilities and correlations of strategy returns. In order to make our approach robust, we assume that each strategy generates identical ex-ante Sharpe ratios. Therefore, the task of optimal allocation to individual strategies resembles optimal risk diversification.

In Section I, we consider the case of a single systematic alpha strategy. We use an example to discuss the phenomenon of volatility clustering and propose a dynamic re-balancing mechanism that smoothes the risk profile of isolated strategy returns over time. Section II discusses the effect of dynamic correlations on volatility clustering at the portfolio level with a simple example of two systematic alpha strategies. We illustrate the unstable nature of correlations when strategy signals change asynchronously and discuss adjustments to the rebalancing mechanism that take dynamic correlations into account. In section III, we discuss practical considerations. Section IV discusses why these techniques are particularly important for portfolios of alpha strategies.

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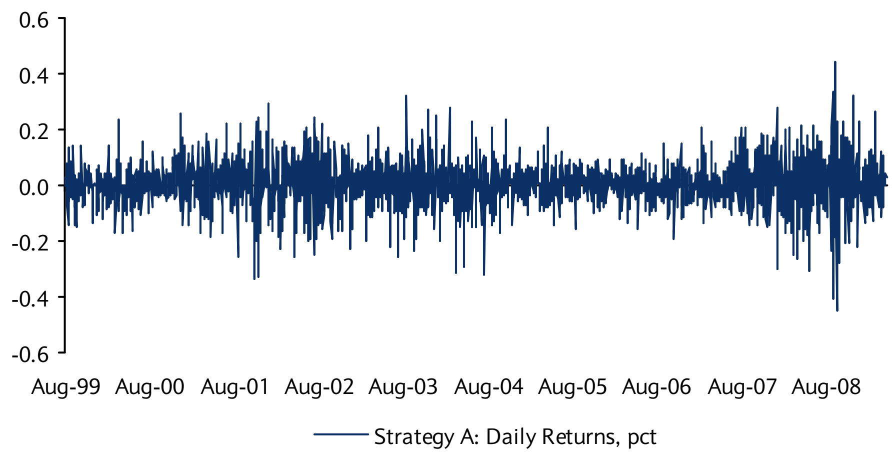
I. Volatility Clustering

Systematic alpha strategies are usually designed without explicit controls for short-term risk. For example, duration constraints, frequently implemented in rate strategies, fail to prevent significant changes in risk associated with volatility clustering in interest rate markets. As an illustration, we refer throughout this article to a hypothetical trading strategy that we call strategy A. It is assumed to take positions of fixed magnitude, but varying sign, in underlying asset A. The time series of daily returns of strategy A are shown in Figure 1.

Figure 1: Daily returns of Strategy A



Source: Barclays Capital

The risk of strategy A does not seem to be constant over time. Stable periods are followed by more volatile ones. Such phenomena are documented for many markets and have been extensively studied in the academic literature.[1]

Short-term volatility of strategy returns can be reliably estimated from historical data by statistical techniques that vary from the rolling window approach to the more sophisticated GARCH and EGARCH models proposed in the literature.[2] For the purposes of our study, we use weighted standard deviation based on a rolling window of 60 daily observations.[3,4] Daily returns are weighted to ensure that recent observations contribute more to the volatility. We use a decay parameter that corresponds to a half-life of 34 days, meaning that the contribution of a 34-day-old return is half that of the most recent observation. This method provides estimates of short-term volatilities that are generally very close to those produced by GARCH or EGARCH models. At the same time, the method is more robust if there are large outliers in the sample. The decay parameter in the exponentially weighted scheme is fixed, while parameters in GARCH and EGARCH are estimated from historical returns under the normality assumption and therefore potentially affected by sporadic behavior of the data.

[1] See, for example, Mandelbrot, B., 1963, "The variation in certain speculative prices," *Journal of Business,* 36, 394-419.
[2] For ARCH models see Engle, R., 1982, "Autoregressive Conditional Heteroskedasticity with Estimates of Variance of United Kingdom Inflation," *Econometrica,* 50, 987-1008. GARCH models are introduced in Bollerslev, T., 1986, "Generalized Autoregressive Conditional Heteroskedasticity," *Journal of Econometrics,* 31, 307-327.
[3] See the appendix for a definition of weighted volatility.
[4] This 60-day trailing weighted standard deviation is an ad hoc choice. Ideally, we would like to use the best available estimator of near-term volatility for the strategy in question. If information is available regarding the current positions held by the strategy, this can help refine the volatility estimate. For the particular example of strategy A, information about the strategy's holdings is very useful in projecting its correlations with other strategies, but not in estimating its volatility. This is because the strategy maintains a position of constant magnitude in a single asset while changing the allocation sign according to a signal. The estimated volatility of such a position will be the same whether it is long or short.

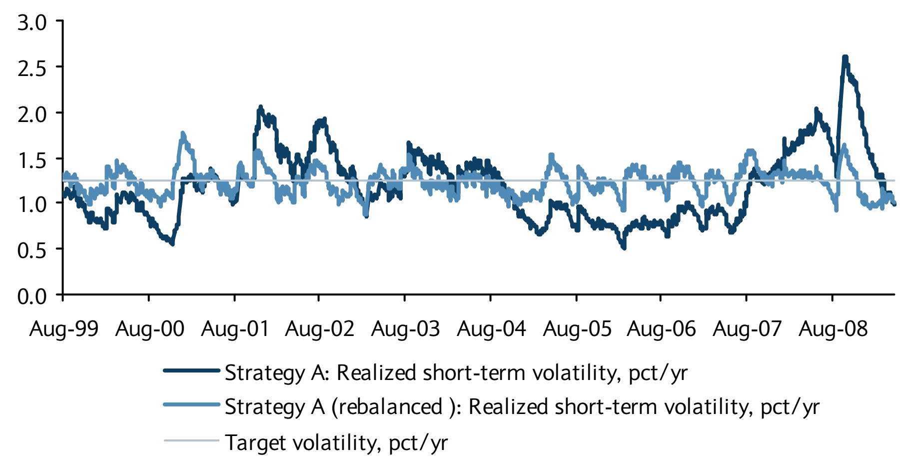
The volatility clustering observed in the daily returns of strategy A can be significantly reduced by dynamically adjusting exposure to the strategy according to its estimated short-term volatility. Exposure is reduced when volatility is high and increased when volatility is low; this can be expressed by making the notional of the strategy inversely proportional to the estimated short-term volatility:

$$N_{A,t} = \frac{\sigma}{\sigma_{A,t}}, \tag{1}$$

where σ is the target volatility level, $\sigma_{A,t}$ is the estimated short-term volatility, and $N_{A,t}$ is the dynamically adjusted notional of the strategy.

The volatility estimated from weighted standard deviation is used to dynamically rebalance the strategy notional according to equation (1). In the example shown in Figure 2, the strategy is re-balanced daily with a target volatility set at 1.25%/year: the short-term realized volatility of the dynamically re-balanced strategy is more stable than that of the original strategy.

Figure 2: Short-term realized volatilities of strategy A and its rebalanced version



Note: Short-term realized volatilities are estimated as weighted standard deviations using a rolling window of 60 daily return observations. The decay parameter is 0.98 (34-day half-life). The target annual volatility for the rebalanced strategy is 1.25%. Source: Barclays Capital

Although the short-term risk of the rebalanced strategy does not become constant over time, its stability is significantly improved. To quantify this improvement, we use the standard deviation of estimated short-term volatilities – a measure of the stability of strategy risk.[5] We compare standard deviations of short-term volatilities, as well as other performance statistics of the original and rebalanced strategies, in Figure 3. The standard deviation of short-term volatilities of the rebalanced strategy drops significantly – by more than 50% – confirming the visual evidence provided by Figure 2. In addition, the Sharpe ratio of the rebalanced strategy is higher than that of the original strategy. This is evidence that the considered strategy tends to deliver higher returns in a stable market environment. Since dynamic rebalancing increases strategy leverage in quiet times and decreases it in volatile periods, it has the effect of improving the Sharpe ratio of the strategy overall. This result should not be expected in general because the relationship between performance and volatility of an active strategy can be either positive or negative.

[5] See the appendix for a definition of this measure of stability of strategy risk.

Figure 3: Return statistics and reduction in volatility clustering for rebalanced strategy A

Statistics	Strategy A	Strategy A (rebalanced)
Standard deviation of short-term volatility (%/y)	**0.39**	**0.14**
Average return (%/y)	1.43	1.52
Volatility (%/y)	1.25	1.25
Sharpe ratio	1.14	1.22

Note: All statistics are calculated from daily returns over the period from 18-Aug-99 to 11-May-09. The target annual volatility for the rebalanced strategy is 1.25%. Rebalancing transaction cost is assumed to be 1bp. Short-term volatilities are estimated as weighted standard deviations in rolling sample of 60 daily return observations. The decay parameter is 0.98. Source: Barclays Capital

II. Combining Strategies into a Portfolio – Correlation Clustering

How do we combine systematic alpha strategies into a portfolio? Several issues need to be addressed to answer this question. First, volatilities of individual strategies can be different in magnitude. For example, the volatility of a strategy linked to the equity market could be 15%/y or more, while that of a strategy based on interest rates could be as low as 1%/y. Combining these two strategies into a portfolio based on equal notional weights will create a bias in portfolio risk and performance towards the characteristics of the more volatile equity strategy. To maximize efficiency, it is more desirable to take equal risk allocations to all independent sources of risk and return. Second, short-term volatilities of individual strategies can change significantly over time, presenting the problem of volatility clustering discussed in the previous section. Third, when strategy returns are correlated, the equal allocation of isolated risk does not lead to optimal diversification; more risk should be applied to those strategies that are least correlated with the others. Finally, the dynamic nature of the correlations among strategy returns can make portfolio volatility unstable even when all of the previous issues are addressed.

In this section, we discuss how to build a balanced portfolio of systematic alpha strategies given the dynamic nature of their short-term volatilities and correlations. We illustrate the method with a simple example of two quantitative strategies that trade two related assets based on different signals. In addition to strategy A introduced in the previous section, we consider strategy B, which takes long or short positions in asset B according to its own trading signal. Assets A and B are closely related, so that their returns are positively correlated. These two strategies are used to build a dynamic portfolio with stable risk characteristics and improved performance. We first present a mechanism that addresses the volatility-related issues alone and then discuss the treatment of dynamic correlations.

Both volatility-related issues – volatility dispersion across strategies and volatility clustering – can be addressed by an extension of the volatility-targeting approach presented in the previous section. A portfolio manager can first rebalance the strategies individually, targeting the same volatility level for each strategy, and then combine the resulting strategies into an equally weighted portfolio. In other words, we assign equal *isolated* risk contributions to each individual strategy. Allocations to strategies A and B are, thus, driven by a minor variation of equation (1):

$$N_{A,t} = \frac{\sigma_{PF}}{\sqrt{2}\sigma_{A,t}}, \qquad N_{B,t} = \frac{\sigma_{PF}}{\sqrt{2}\sigma_{B,t}}, \tag{2}$$

where σ_{PF} is the target volatility of the portfolio, $\sigma_{A,t}$ is the estimated short-term volatility of strategy A, $\sigma_{B,t}$ is the estimated short-term volatility of strategy B, $N_{A,t}$ is the dynamic notional of strategy A, and $N_{B,t}$ is the dynamic notional of strategy B. As the estimated volatilities of the two strategies fluctuate, we continually rebalance the allocations such that each strategy brings a risk of $\sigma_{PF}/\sqrt{2}$. Assuming that the strategies are uncorrelated, this approach will maintain both the total portfolio risk of σ_{PF} and the equal allocation of risk among the two strategies.

This approach can be shown to be optimal for a set of independent strategies. However, it does not account for correlations among strategies and for dynamic fluctuations in those correlations, which can significantly influence risk at the portfolio level. Will the risk and performance profile of such a portfolio be stable? The answer to this question is generally not. Even after adjusting weights of individual strategies to counteract changes in their volatilities, correlation changes can still contribute significantly to volatility clustering at the portfolio level.

Our ability to improve performance by adjusting for strategy correlations will depend on the accuracy with which we can estimate these correlations. This, in turn, will depend on the amount of information that we have about the strategies. In this section, we assume that we have complete knowledge of strategy positions. Our estimates of strategy correlations can therefore change due to changes in the estimates of asset correlations or to changes in the positions of one or more of the strategies. In the next section, we will discuss the situation in which the strategies offer less transparency and the manager needs to treat each strategy as a "black box" that produces a time series of returns.

It is important to keep in mind that when we build a portfolio of active strategies, risk diversification occurs through two distinct mechanisms. First, we invest in different underlying assets, which are less than perfectly correlated ("asset diversification"). Second, each strategy uses a different signal to control the timing of long and short positions ("signal diversification").[6] Therefore, the correlations between strategy returns are typically lower than those between the returns of the underlying assets. This is illustrated in Figure 4, which compares the pair of active strategies A and B with the pair of underlying assets. The correlation between the former is significantly lower than the correlation between the latter.

Figure 4: Performance of active strategies vs. long positions in underlying assets

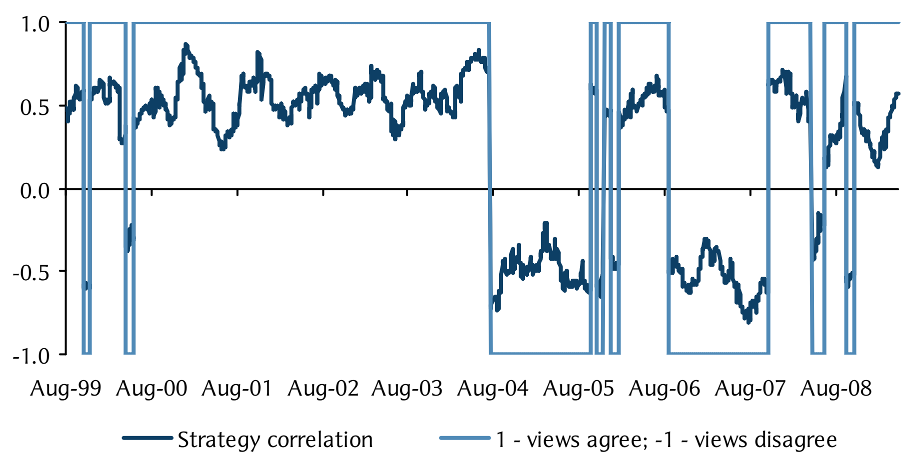
Statistics	Strategy A (signal-based timing of asset A)	Strategy B (signal-based timing of asset B)	Long Asset A	Long Asset B
Average return (%/y)	1.43	1.30	1.24	0.65
Volatility (%/y)	1.25	0.93	1.25	0.93
Sharpe ratio	1.14	1.40	0.99	0.70
Correlation	--	**0.30**	--	**0.51**

Note: All statistics are calculated from daily returns over the period 18 August 1999 to 11 May 2009.
Source: Barclays Capital

[6] For a detailed discussion of diversification mechanisms in portfolios of active strategies we refer to A. Desclée., J. Hyman, and S. Polbennikov, 2009, *Horizon Diversification: Reducing Risk in a Portfolio of Active Strategies*, Barclays Capital.

This reduction in correlation is due to the signal diversification mechanism. The trading signals used in the two strategies are related, but not perfectly correlated. There are periods when strategy positions in the underlying assets agree (eg, we are long assets A and B) and periods when they disagree (eg, long asset A and short asset B). The short-term correlation between the returns of Strategy A and Strategy B is positive in the former and negative in the latter case. Over the whole sample, however, positive and negative correlations are averaged, reducing the long-term strategy correlation. Figure 5 illustrates the estimation of short-term correlations between the daily returns of the two strategies. First, the dynamic asset correlation is estimated from daily returns of the two underlying assets over a trailing window. If the positions in the two assets take the same sign, the estimated strategy correlation is equal to the asset correlation. If the two strategies take opposing positions, we multiply the asset correlation by -1 to obtain the strategy correlation. This can result in large swings in strategy correlations. Figure 5 implies that the strategy signals are known. This allows us to immediately adjust portfolio weights as needed whenever signals are updated. The situation becomes more difficult when the details of strategy mechanics are not available to investors, so that correlation swings due to switches in strategy positions have to be inferred from the data. This situation is discussed in Section III.

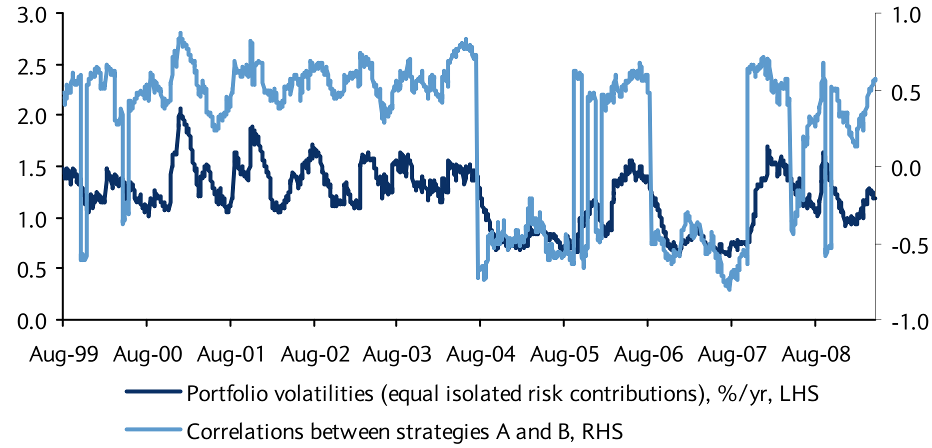
Figure 5: Estimated short-term correlations between daily returns of strategies A and B



Note: Short-term correlations between assets A and B are calculated from weighted covariance matrices in a trailing sample of 60 daily return observations. The decay parameter is 0.98 (34-day half-life). The correlation becomes negative when both strategies have opposite views. Source: Barclays Capital

Given the large swings in the correlation between strategies A and B, we need to revisit our approach to portfolio construction. As a first step, let us examine how this correlation relates to the volatility of the simple portfolio that we constructed according to Equation (2) with equal isolated risk contributions to each strategy. Figure 6 shows the realized short-term volatility of the portfolio alongside the estimated short-term correlations between the daily returns of strategies A and B. We find that when the strategy correlations are negative, the observed strategy volatility is much lower than the targeted risk level of 1.25%/year; conversely, when the correlation is significantly positive, the portfolio volatility tends to overshoot the risk target.

Figure 6: Estimated correlations between daily returns of strategies A and B and realized volatility of the portfolio based on equal isolated volatility contributions



Note: Short-term correlations between daily returns of Assets A and B are estimated from weighted covariance matrices in a trailing sample of 60 daily return observations. The decay parameter is 0.98 (34-day half-life). The strategy correlation is set equal to the asset correlation when the signals driving strategies A and B take the same sign; this is multiplied by -1 when the two signals disagree. Allocations to strategies A and B in the portfolio are inversely proportional to their short-term volatilities. Source: Barclays Capital

This example shows that portfolio allocation based on equal isolated risk contributions does not fully solve the problem of volatility clustering at the portfolio level. A more complete way to combine alpha strategies in a portfolio is to weight the individual strategies based on their correlated contributions to overall portfolio risk. As short-term volatilities and correlations change, so do risk contributions. Therefore, the method allows targeting short-term volatility contributions at the portfolio level.

In our simple example based on two strategies, allocations to strategies A and B are still inversely proportional to their short-term volatilities, but they are also adjusted for short-term correlation. Exposures are increased when correlation is low and decreased when correlation is high, as shown below:[7]

$$N_{A,t} = \frac{\sigma_{PF}}{\sqrt{2\left(1+\rho_{AB,t}\right)}\sigma_{A,t}}, \qquad N_{B,t} = \frac{\sigma_{PF}}{\sqrt{2\left(1+\rho_{AB,t}\right)}\sigma_{B,t}}, \qquad (3)$$

where σ_{PF} is the target volatility of the portfolio, $\sigma_{A,t}$ is the estimated short-term volatility of strategy A, $\sigma_{B,t}$ is the estimated short-term volatility of strategy B, and $\rho_{AB,t}$ is the estimated correlation between daily returns of strategy A and B.

The performance advantage from adjusting for strategy volatilities and correlations is demonstrated in Figure 7, which compares three portfolios that combine strategies A and B. The first naively combines the two strategies in equal notional amounts. The second adjusts for strategy volatility and combines strategies according to Equation (2), ie, on the basis of isolated risk contributions. The last combines strategies A and B according to Equation (3), using correlated risk contributions.

[7] The derivation of this equation is provided in the appendix.

Figure 7: Reduction in volatility clustering for strategy combinations

Statistics	Equal weights	Equal isolated risk	Equal correlated risk
Std dev of short-term volatility (%/y)	**0.48**	**0.30**	**0.14**
Average return (%/y)	1.92	2.19	2.26
Volatility (%/y)	1.25	1.25	1.25
Sharpe ratio	1.54	1.75	1.82

*Note: All statistics are calculated from daily returns over the period from 18-Aug-99 to 11-May-09. The target annual volatility for all portfolios is 1.25%. Rebalancing transaction costs is assumed to be 1bp. Short-term volatilities are estimated as weighted standard deviations in rolling sample of 60 daily observations. The decay parameter is 0.98.
Source: Barclays Capital

All three portfolios were scaled to achieve an annual volatility of 1.25%/year. The one based on correlated risk contributions has the most stable risk profile and the highest Sharpe ratio.

The portfolio allocations specified by Equations (2) and (3) can be seen as special-case solutions to a more general portfolio optimization problem. We seek to maximize the expected portfolio return subject to a constraint on portfolio return volatility. The solution would involve the expected returns of each strategy in addition to their volatilities and correlations. In fact, it is the sensitivity of this optimization problem to the assumed expected returns, and the difficulty in estimating these, that has led us to avoid this formulation. Instead, we assume that each strategy will achieve the same Sharpe ratio and that the expected returns from each strategy will therefore be proportional to the amount of risk budgeted to them. This assumption transforms the problem from one of return maximization to one of optimal diversification.

The full effect of the correlation adjustment that results from this optimization is not adequately reflected in the two-asset example that we have explored so far. If we compare the correlation-adjusted solution in Equation (3) with the purely volatility-adjusted solution of Equation (2), we see that the correlation adjustment affects the weights assigned to Strategies A and B symmetrically. When correlation is positive, we decrease the strategy weights; when it is negative, we increase them. In every case, though, the isolated risk allocations to both strategies remain equal. When we apply this approach to three or more strategies, this will not necessarily be the case. The extension of this approach to *N* strategies is addressed in the Appendix. In this case, correlations among strategies will help dynamically target overall portfolio risk, as well as adjust the relative weights of the various strategies. For example, if strategies A and B are positively correlated to each other, but both uncorrelated with Strategy C, the weights will be adjusted to give Strategy C the largest risk allocation. This will ensure the optimal diversification of risk among the available strategies.

III. Practical Considerations

The main premise underlying our motivation for dynamic rebalancing is the persistence of volatility regimes. For our approach to be useful, historical data must hold information on the volatility of future returns. In most cases, this requirement holds, as asset returns tend to exhibit volatility clustering.[8]

However, scaling up active risk when volatility is expected to remain low can make the portfolio vulnerable to sudden large negative return realizations. This possibility should be carefully considered when evaluating the potential benefits of dynamic rebalancing. One

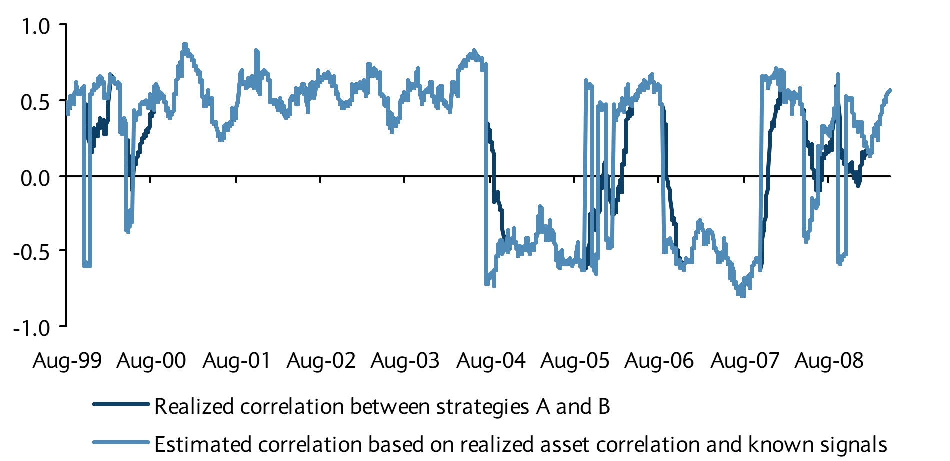
[8] See R. Cont, 2001, “Empirical Properties of Asset Returns: Stylized Facts and Statistical Issues”, *Quantitative Finance*, 1, pp. 1-14.

way to control for such extreme events is to include exposure constraints. In practice, the incremental benefit from taking additional leverage can be evaluated against the incremental loss associated with an extreme event. The appropriate balance between these two quantities will determine where to set exposure limits.

Another requirement of our approach is that strategy correlations be predictable. In our examples, we have assumed transparency of alpha strategies, meaning that we are precisely informed of any changes in active positions on a timely basis. This, however, may not be a very realistic assumption, and without sufficient information on strategy positions, short-term correlations need to be estimated from realized strategy returns. In this case, correlation estimates will reveal strategy positions with a lag. Such directly estimated correlations can differ significantly from correlations based on fully transparent strategy positions, in particular when different active strategies are related to correlated assets and rebalanced on a frequent basis.

Figure 8 compares strategy correlations estimated directly from realised strategy returns with those presented in the previous section, which were estimated from strategy decompositions into asset returns and active positions. It relates to our example Strategies A and B, which trade two highly correlated underlying assets.

Figure 8: Estimated correlations between strategies A and B calculated solely from strategy returns or based on knowledge of trading signals



Note: Short-term correlations between strategies A and B are calculated over a trailing sample of 60 daily return observations. The decay parameter is 0.98 (34 day half-life). These are compared with the correlations estimated earlier based on the trailing asset correlation between A and B and knowledge of the trading signals for the two strategies. Source: Barclays Capital

Correlations that are directly estimated from strategy returns react with a lag to swings in strategy positions and only gradually catch up with those estimated from the analysis of underlying constituents and associated signals. In our example, the difference between both correlations is generally small, so even directly estimated correlations help improve portfolio efficiency. The small magnitude of the difference is due to the fact that Strategies A and B do not change views very frequently. When signals change frequently, extracting correlations from historical returns is less informative, as illustrated in the small middle section of Figure 8 related to late 2005 returns. In such cases, using an assumption of zero strategy correlations and following the allocation outlined in Equation (2) can be more practical.

If the strategies under consideration use liquid instruments, dynamic rebalancing is not prohibitively expensive in terms of transaction costs.[9] However, our technique is also applicable to a broad universe of actively managed portfolios. Higher transaction costs do not make the concept of dynamic rebalancing irrelevant, but introduce an extra dimension in this analysis, which is the evaluation of efficiency gains against such costs. A simple practical approach is to make the rebalancing of less liquid instruments less frequent to ensure that transaction costs do not place a significant drag on portfolio return. One obvious possibility is to synchronize rebalancing of the portfolio with "signal re-positioning" of individual strategies. This requires an appropriate alignment of risk and investment horizons. Consider, for example, a credit strategy with a monthly investment horizon. The horizon of volatility estimation for dynamic rebalancing should be monthly as well in this case, so that the notional adjustment captures the strategy's volatility until the next rebalancing point.

IV. Conclusion

When allocating to a systematic trading strategy, the main unit of exposure can be expressed as return volatility. Indeed, an active strategy can be long or short various assets, change exposure over time and exploit leverage to various extents. While traditional asset allocation can be based on more objective measures of exposures such as percentage of market weights or, in the case of fixed income markets, measures of duration, risk units allocated to active strategies are not immediately and objectively observable. However, active risk budgeting is what matters when allocating to a single strategy or a portfolio of strategies. It is therefore important that the realized allocation of active risk correspond to the intended one.

As strategy returns may exhibit unstable volatilities and correlations, a dynamic allocation mechanism is required to stabilise the risk profile at the isolated strategy level, as well as at the portfolio level. Otherwise, the active risk of a portfolio can significantly exceed the allocated risk budget in a volatile environment, while in benign periods it can be too small to achieve the target alpha level. In addition, portfolios assembled without consideration for the dynamic risk of individual strategies can deliver inferior Sharpe ratios. The mechanism we have discussed in this article relies on the fact that over a short horizon, both volatility and correlation exhibit clustering and can be predicted using simple statistical techniques.

A simple way to reduce volatility clustering in portfolio returns is to re-balance the portfolio dynamically according to the estimated short-term volatilities and correlations of individual strategies. In this method, the weight allocated to an alpha strategy is inversely proportional to its correlated risk contribution adjusted for expected performance. Dynamically re-balanced portfolios of alpha strategies exhibit improved stability of short-term risk over time. Being better balanced, they are also likely to outperform their static counterparts.

Appendix

Calculation of Realized Short-Term Volatility

We provide a formula for calculation of weighted realized short-term volatility of returns:

$$\sigma_t^2 = \frac{1}{60 \times \overline{w}} \times \sum_{i=1}^{60} w_{t-i} \left(r_{t-i} - \overline{r} \right)^2 ,$$

[9] The transaction cost on a change in notional of the underlying assets in our examples is assumed to be 1bp.

where σ_t is the short-term volatility, r_t is the return at time t, $\bar{r}$ is the weighted average return over the preceding 60 days, w_{t-1} is the weight of the observation occurred i periods ago in the past, and $\overline{w}$ is average of all weights over the preceding 60 days (normalization coefficient).

Calculation of Standard Deviation of Short-Term Volatilities

Standard deviation of short-term volatilities is a measure of stability of portfolio risk over time. It is defined through the following expression:

$$V_{stab} = \sqrt{\frac{1}{T} \times \sum_{i=1}^{T} \left(\sigma_t - \overline{\sigma}\right)^2} \ ,$$

where σ_t is the realized short-term volatility at time t and $\overline{\sigma}$ is the average volatility.

Portfolio Allocation for the Case of 2 Active Strategies

The portfolio allocation in the case of 2 correlated strategies provided by equation (3) can be derived as follows: first, there is a volatility risk budget σ_{PF} at the portfolio level that should not be exceeded at any point in time; second, the risk allocated to both strategies should identical. This is represented with two equations with two unknowns:

$$N_{A,t}^2 \sigma_{A,t}^2 + N_{B,t}^2 \sigma_{B,t}^2 + 2\rho_{AB,t} \sigma_{A,t} \sigma_{B,t} N_{A,t} N_{B,t} = \sigma_{PF}^2 \qquad \text{(3a)}$$

$$N_{A,t} \sigma_{A,t} = N_{B,t} \sigma_{B,t} \qquad \text{(3b)}$$

Substituting (3.b) into (3.a), we obtain:

$$2N_{B,t}^2 \sigma_{B,t}^2 \left(1 + \rho_{AB,t}\right) = \sigma_{PF}^2 \ .$$

This directly leads to equation (3).

Portfolio Allocation for the General Case of *N* Active Strategies

The proposed approach can be easily generalized to the case of *N* systematic strategies. If we assume that all strategies have identical Sharpe ratios, the allocation problem becomes a constrained optimization that maximizes the expected return of the portfolio of strategies per unit of portfolio risk:

$$\max_{\{N_t\}} \sigma_t N_t$$

$$s.t. \ \ N_t^{\,T} \Omega_t N_t \le \sigma_{PF}^2$$

where N_t is the notional allocation vector in period *t*, σ_{PF} is the target volatility of the portfolio, σ_t is the vector of short-term strategy volatilities in period *t*, and Ω_t is the short-term covariance matrix of strategy returns.

The optimal allocation is then given by the solution to this problem, which can be shown to be as follows:

$$N_t = \frac{\sigma_{PF}}{\sqrt{\sigma_t^T \Omega_t^{-1} \sigma_t}} \Omega_t^{-1} \sigma_t \,. \qquad \text{(4a)}$$

This solution can be re-written in terms of the correlation matrix R_t of strategy returns:

$$N_{i,t} = \frac{\sigma_{PF}}{\left(e^T R_t^{-1} e\right)^{1/2}} \frac{\left\{R_t^{-1} e\right\}_i}{\sigma_{i,t}} \,, \qquad \text{(4b)}$$

where e is a unit vector of ones. We see that the optimal allocation for strategy *i* is determined by the correlation matrix, its own volatility, and the targeted portfolio volatility. Note that it does not depend on the volatility of any other strategy.

Analyst Certification(s)

We, Albert Desclee, Jay Hyman and Simon Polbennikov, hereby certify (1) that the views expressed in this research report accurately reflect our personal views about any or all of the subject securities or issuers referred to in this research report and (2) no part of our compensation was, is or will be directly or indirectly related to the specific recommendations or views expressed in this research report.

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