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We explore the use of an option selling overlay to improve portfolio rebalancing. Within a multi-asset class portfolio, portfolio weights deviate from targets as asset values fluctuate. Investors typically use a rebalancing process to bring portfolio weights back to their desired strategic allocations. However, between rebalances, investors are exposed to unintentional timing bets as weights deviate from targets. These timing bets introduce basis risk to their policy portfolio. A short option overlay can assist with hedging this unintentional exposure. We solve for the overlay construction that provides the desired rebalancing trade upon option expiration and back test an illustrative overlay. Our analysis shows significant reduction in the portfolio's uncompensated timing exposure. Furthermore, by selling options, the overlay earns the volatility risk premium and thereby adds alpha to the portfolio. Lastly, we show that an option overlay for rebalancing is implementable even when considering transaction costs and real-world constraints.

Strategic asset allocation is at the heart of many institutional investors' portfolio construction. Investors often start by defining their policy portfolio, with strategic fixed percentage target weights¹ to each asset class within their portfolio. Deviations from these long-term strategic allocations may thus be considered inadvertent, tactical exposures that create basis risk or tracking error to the policy portfolio.

Until a portfolio rebalance brings allocations back in line with strategic targets, portfolio weights deviate from these targets as asset prices change. The value of winning investments in the portfolio increases and the value of losing investments in the portfolio decreases. This effect leads to short-term cross-asset momentum exposure. Relative to the strategic targets, the portfolio is overweight winners and underweight losers until the portfolio is rebalanced.

With all of the empirical evidence that shows trend-following has performed well historically (see Hurst, et al. [2017]), could the momentum exposure arising from a non-rebalanced portfolio be a net positive for investors? We think the answer is no. Portfolio managers should prefer exposures that are explicitly constructed and sized over those that arise unintentionally as a byproduct of a process. The likelihood that the byproduct exposure is similar in size and direction to the explicit exposure is small.² In fact, we find that the momentum exposure embedded in a monthly-rebalanced equity and bond portfolio (between rebalances) has, in fact, modestly detracted from performance.

Institutional investors put significant thought into choosing the strategic weights of their policy portfolios. Therefore, it makes sense that they would often put a rebalancing process in place in order to trade the portfolio

back toward its strategic allocation. Options can be used systematically to improve portfolio rebalancing. An option has time-varying exposure to its underlying instrument. The change in a long option position's equity exposure is positively related to equity returns, exhibiting momentum. Thus, a short option position's time-varying equity exposure provides short-term reversal.

Option traders typically choose to hedge options with the underlying instrument to remove this timing exposure because it is usually an undesired source of risk. However, within a rebalancing process, an option's equity exposure becomes desirable because it may be used to hedge the overall portfolio's exposure as portfolio weights deviate from long-term targets.

In addition to stabilizing the portfolio's equity exposure, a short option overlay can also add alpha to the portfolio. Those who purchase options typically do so as a hedge against market drawdowns, transferring their tail risk to option sellers. Sellers rightfully seek compensation for accepting this undesirable tail risk. This compensation is generally referred to as the volatility risk premium. By selling options, the overlay thereby harvests the well-documented volatility risk premium embedded in equity index options that are expensively priced on average. But what is particularly interesting in the case of selling options as an overlay for rebalancing is that the short option positions are a hedge for the seller too, reducing the basis risk their imperfectly rebalanced portfolio has to their policy portfolio.³

Ilmanen and Maloney [2015] examine the risk, return, and cost implications of illustrative rebalancing strategies that use both calendar-based and exposure-based rebalance triggers. Gort and Burgener [2014] look at similar rebalancing strategies, as well as portfolio properties after adding option-writing overlays to the rebalancing process. Co and Hatzopoulos [2009] also highlight that options can be used as a tool to improve rebalancing. Our article extends this literature by focusing on the return and risk properties of an option overlay within a rebalancing process, as well as analyzing the portfolio's undesired tactical exposures between rebalances. Furthermore, we examine the portfolio construction of an option overlay, compare it to existing derivative instruments such as variance swaps, and study the impact of the option strike range and strike increment on hedging efficacy when implementing the overlay.

MOTIVATION

Some investors may consider whether adding an option overlay to their portfolio justifies the additional operational complexity. To address this question, we compare transaction costs of using an option overlay relative to monthly rebalancing and daily rebalancing. Unlike traditional approaches, we show that an option overlay can not only cover its transaction costs, but also add alpha to the portfolio by earning the volatility risk premium (VRP).

An investor incurs transaction costs to rebalance a portfolio. On the other hand, a VRP harvesting strategy typically pays transaction costs for delta hedging. In this use case, neither the explicit rebalancing trades nor the delta hedging trades is required because the short options' market exposure hedges the portfolio's timing exposure. This exposure netting saves transaction costs that each approach would otherwise independently pay. However, in order to realize these netting benefits, short option positions must be integrated into the rebalance process, which may be operationally complex.

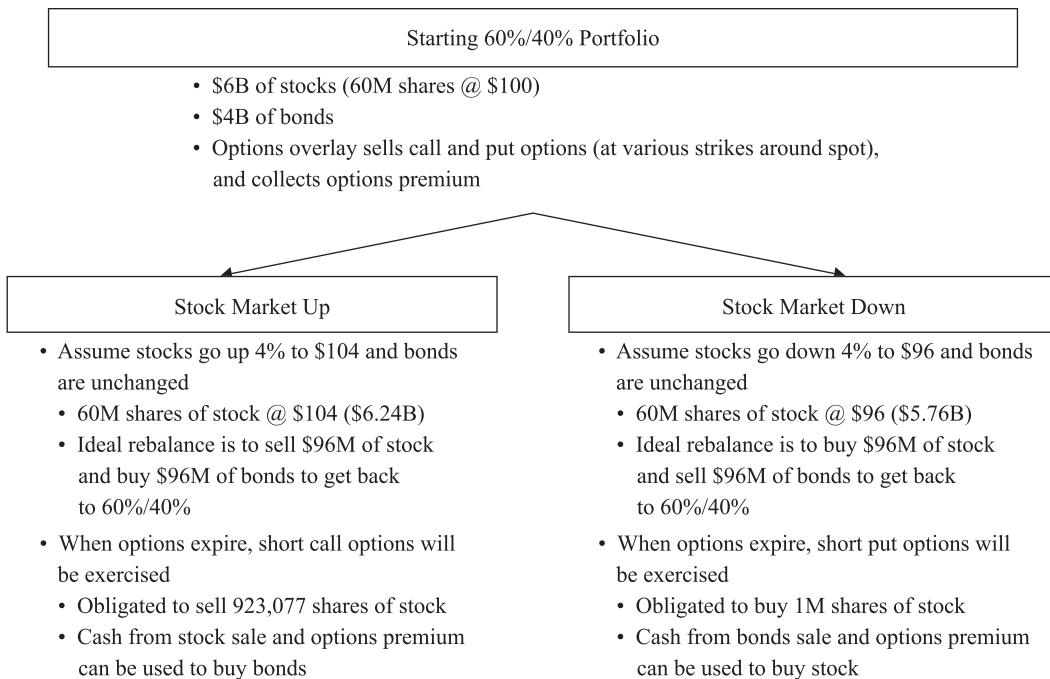
If an investor sells physically settled options, another benefit of this approach is to enforce discipline within a portfolio rebalancing process. Assuming the buyers of the physically-settled options exercise rationally, the short option overlay mechanically obligates the investor to purchase the underlying asset after market declines and sell the underlying asset after market rallies. Many institutions dedicate investment personnel to focus on implementing rebalances. Rebalancing may be evaluated discretionarily. An option overlay provides a systematic approach that avoids potentially arbitrary, on-the-fly decision making—particularly relevant after large market moves.

ILLUSTRATIVE EXAMPLE

We start with an illustrative example to demonstrate how options can assist with portfolio rebalancing. An investor has a \$10B portfolio with a 60%/40% strategic allocation to stocks and bonds. Therefore, the portfolio holds \$6B in stocks (specifically, 60 million shares of a \$100 stock) and \$4B in bonds. The investor initiates an option overlay by selling a portfolio of call and put options with strike prices near the current stock price (specifics of options selection will be discussed in the *Option Overlay Construction* section). The required

EXHIBIT 1

Illustrative Portfolio Rebalancing Example Using Options



Notes: The exhibit shows illustrative portfolio rebalance trades for a \$10B portfolio with a 60%/40% allocation to stocks and bonds. The portfolio also includes an option overlay that sells a portfolio of call and put options with strike prices near the current stock price. The "Stock Market Up" scenario assumes that stocks go up 4% and bonds are unchanged. The "Stock Market Down" scenario assumes that stocks go down 4% and bonds are unchanged.

Source: AQR.

margin for the short option positions can be collateralized by the stock and/or bond positions, and therefore no additional cash is required. The options are European and physically settled. Put and call option assignment results in either the purchase or sale of the underlying stock, respectively. Exhibit 1 illustrates two different scenarios that can occur at option expiration.

In the *Stock Market Up* scenario, the stock rises 4% to a price of \$104 and bond prices are unchanged. In this case, the investor holds \$6.24B in stocks (60 million shares of a \$104 stock) and continues to hold \$4B in bonds. The portfolio has a total value of \$10.24B, with a 61%/39% weight in stocks and bonds. The portfolio is overweight stocks and a traditional rebalancing approach would sell \$96M of stock (specifically 923,077 shares) and buy \$96M of bonds to get back to the 60%/40% strategic allocation.

However, the investor has sold call options. As a result, he is obligated to sell 923,077 shares of the stock

to the option-holders who exercise their call options. The stock is sold at a price equal to the strike price of the exercised call options. The cash raised from the stock sale and from the option premium initially collected when the options were sold can be used to buy \$96M of bonds.

In the *Stock Market Down* scenario, the stock declines 4% to a price of \$96 and bond prices are unchanged. In this case, the investor holds \$5.76B in stocks (60 million shares of a \$96 stock) and \$4B in bonds. The portfolio is valued at \$9.76B, with a 59%/41% weight between stocks and bonds. Because the portfolio is underweight stocks, the investor needs to buy \$96M of stock (specifically, 1,000,000 shares) and sell \$96M of bonds to get back to the strategic allocation.

However, because the investor has sold put options, he is obligated to buy 1,000,000 shares of stock from the option-holders who exercised their put options.⁴ The cash required for this stock purchase is funded by

selling bonds, as well as the option premium initially collected when the options were sold.

In each scenario, the option overlay systematically rebalances the portfolio to its strategic allocation based on the price changes of the equity market. The investor did not explicitly choose to trade shares of the stock, which removes concerns such as discretion of rebalance trade timing and potential market impact. The short options committed the investor to buy or sell shares of stock depending on market moves. In addition to trading the appropriate quantity of stock at option expiration, we will show that an option overlay also reduces the portfolio's unintended timing exposures prior to expiration.

ILLUSTRATIVE MOMENTUM AND REVERSAL TACTICAL EXPOSURES

The top-left panel of Exhibit 2 plots the return of the equity market over four sample option expiration cycles (September to December 2014). The top-right panel illustrates the tactical equity exposure of a 60%/40% portfolio over the same time period. The portfolio is rebalanced back to its strategic allocation at option expiration. Between rebalances, the portfolio's equity exposure deviates from its strategic allocation. Due to its embedded momentum exposure, the 60%/40% portfolio was overweight equities in months one and three as equity markets rallied and underweight equities in months two and four as equity markets declined.

On the other hand, the bottom-left panel illustrates the tactical equity exposure of a short option overlay. Due to its embedded reversal exposure, the overlay was underweight equities in months one and three and overweight equities in months two and four. The bottom-right panel shows that the tactical positioning of the un-rebalanced portfolio and the short option positions offset. The option overlay naturally hedges the portfolio's tactical equity exposure between rebalances. We now turn to the details of how the option overlay is constructed.

DATA DESCRIPTION

We use the S&P 500 Total Return Index and the Barclays U.S. Aggregate Bond Index from Bloomberg to construct equity and bond returns, respectively.

All returns, and associated performance decompositions, shown in this article are excess of the risk-free rate. We use 3-month USD LIBOR from Bloomberg as the risk-free rate. To calculate option returns and exposures for the option overlay, we use the OptionMetrics IVY database, which provides daily closing bid and ask quotes and option deltas for the S&P 500 options analyzed in this article.

OPTION OVERLAY CONSTRUCTION

To construct an option overlay, we first need to determine the quantity of shares needed to rebalance the stock component⁵ of the portfolio to its target allocation as a function of the stock price. If we assume that bond returns are small relative to stock returns and that the portfolio begins at its strategic allocation, the number of required shares is provided by Equation 1 (see the Appendix for derivation):

$$q_t^s - q_{t-1}^s \cong q_{t-1}^s(1 - w^s) \left(\frac{p_{t-1}^s}{p_t^s} - 1 \right) \quad (1)$$

where p^s = stock price, q^s = quantity of stocks, w^s = target equity allocation.

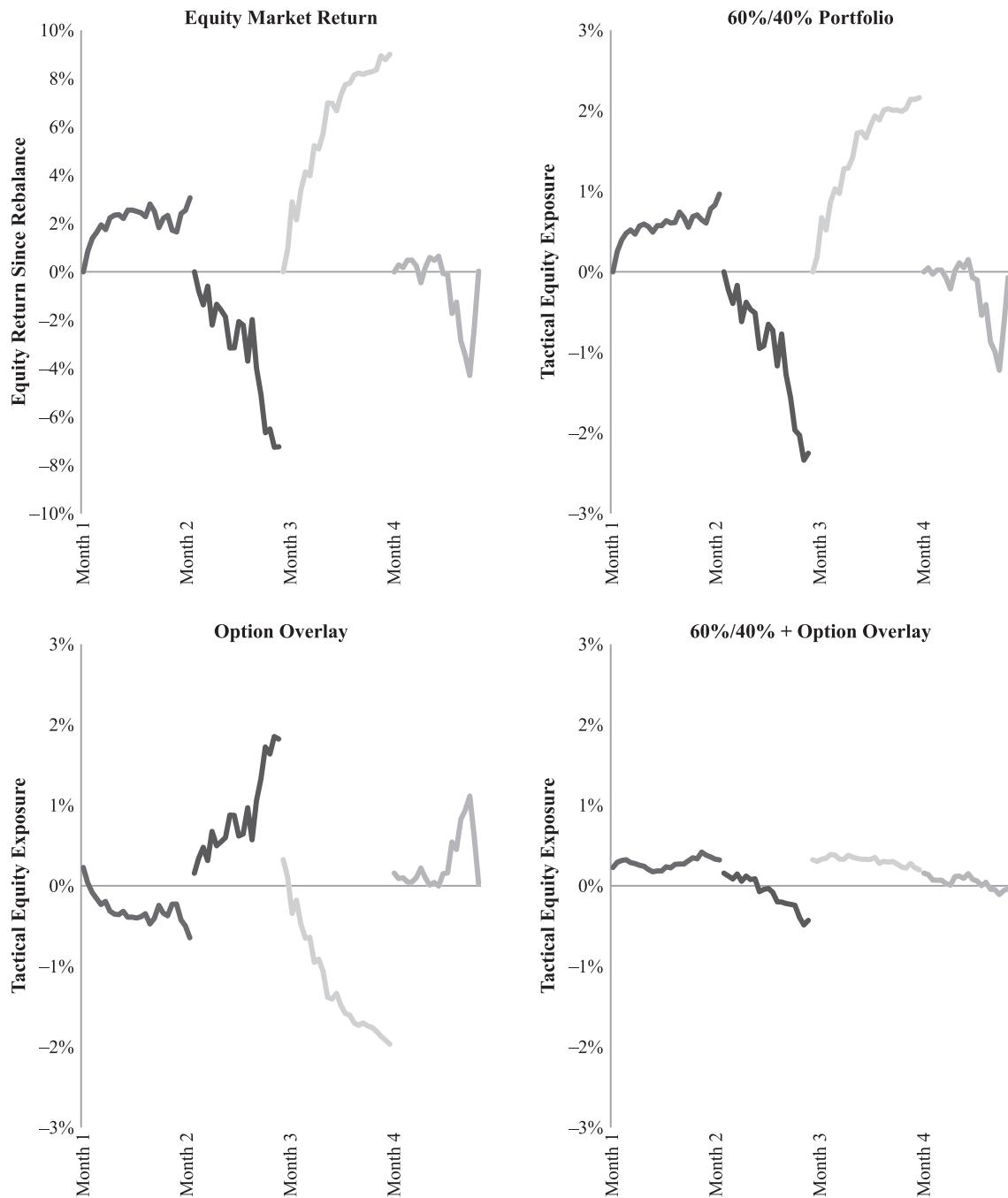
Consistent with the illustrative example, if the stock price increases ($p_t^s > p_{t-1}^s$), shares must be sold. Conversely, if the stock price decreases ($p_t^s < p_{t-1}^s$) shares must be bought. We also see that the larger the target equity allocation, the smaller the required rebalance trade is because stocks would represent the majority of the portfolio. In the limit with a 100% stock portfolio, w^s is 1, and clearly, no rebalance trade is required regardless of stock price changes.

The trade required to return the portfolio to its target equity allocation is similar to the rebalancing trade required by a leveraged exchange-traded fund (ETF) that maintains a leverage of w^s . The above formula is consistent with the leveraged ETF rebalance processes derived in Ivanov and Lenkey [2014] and Cheng and Madhavan [2009].

We choose to size the option overlay such that the correct number of shares is traded at option expiration. In this case, there is no need to rely on a model to estimate an option's equity exposure. The number of shares transferred at expiration is known mechanically if the option buyer rationally exercises.

EXHIBIT 2

Illustrative Tactical Equity Exposures of 60%/40% Portfolio and Option Overlay



Notes: The top-left panel plots equity market returns over four sample option expiration cycles (September to December 2014). The top-right panel shows the tactical equity exposure of a 60%/40% portfolio over the same time period. The portfolio is rebalanced to its strategic allocation at option expiration. The bottom-left panel shows the tactical equity exposure of an illustrative, 1-month short option overlay initiated on each expiration date. The bottom-right panel shows that the tactical exposure of the un-rebalanced portfolio and the short option overlay offset.

Sources: AQR, Bloomberg, OptionMetrics.

The option overlay only sells out-of-the-money options. Therefore, if the stock rallies (declines), call (put) options are exercised and the investor is obligated to sell (buy) shares. The option overlay does not perfectly hedge the tactical equity exposure at all points in time. This is because an option's delta changes with the passage of time.⁶

Because the magnitude of market moves is unknown ahead of time, the range of option strikes to choose is a portfolio construction decision. Therefore, we provide a generalized solution that allows for selling multiple option strikes within the overlay. To do so, our derivation (see the Appendix) accounts for rebalance contributions provided by all strikes sold in the overlay. Equation 2 defines how many out-of-the-money options should be sold at each specific strike, and is intended to be evaluated independently for out-of-the-money calls and puts:

$$\text{Options Sold}_i = \text{Notional}_{t-1}^s (1 - w^s) \frac{\text{Abs}(K_{i-1} - K_i)}{K_i K_{i-1}} \quad (2)$$

where $1 \leq i \leq N$, $K_0 = p_t^s$, K = Option Strike and $i = \text{Nth furthest option strike from current stock price}$.

Alternatively, this equation can be rewritten as Equation 3:

$$\frac{\text{Options Sold}_i}{q_{t-1}^s} = (1 - w^s) \frac{p_{t-1}^s \text{Abs}(K_{i-1} - K_i)}{K_i K_{i-1}} \quad (3)$$

There is an interesting relationship between the derived option overlay and existing instruments in derivatives markets. Because the number of options sold is approximately inversely proportional to the strike squared, the relative weighting among strikes is similar to that of a variance swap replication portfolio (Demeterfi et al. [1999]). However, variance swaps do not help with portfolio rebalancing because they are inherently delta hedged (typically on a daily basis) and are cash-settled. The overlay works because of the options' delta and their physical settlement.

Exhibit 3 plots the number of options sold within an option overlay assuming a stock notional of \$6B, an initial stock price of \$100, and a stock weight of 60%. These parameters are intentionally selected to match the illustrative example presented earlier. We also assume

that options are sold at strikes spaced every \$1 and show a range of strike prices between \$80 and \$120.

At a strike of 101, the overlay sells 237,624 call options. The sum of all call options sold between strikes of 101 and 104 is 923,077. If the stock is at \$104 at expiration, the overlay will lead to the sale of 923,077 shares—matching the Stock Market Up 4% scenario in the illustrative example.

More out-of-the-money put options are required than out-of-the-money calls. Specifically, at a strike of \$99, the overlay sells 242,424 put options. The sum of all put options sold between strikes of \$96 and \$99 is 1,000,000. Therefore, if the stock is at \$96 at expiration, the overlay will require us to buy 1,000,000 shares—once again exactly matching the number of shares required in the Stock Market Down 4% scenario in the illustrative example. No at-the-money option contracts are sold at the current stock price because if the stock finishes at \$100 at expiration, there is no need to rebalance the portfolio.

We arbitrarily selected the distance between option strikes to be \$1 and ranges of strikes sold to be $\pm \$20$ for Exhibit 3, but these are important portfolio construction decisions that impact the efficacy of the option overlay. We analyze these decisions in a later section.

60%/40% PORTFOLIO PROPERTIES

Before analyzing the implications of adding an option overlay, we start by examining the risk, return, and exposure characteristics of the 60%/40% portfolio itself. We back test a strategy that rebalances the portfolio to the strategic weights monthly on standard S&P 500 option expiration dates (typically the 3rd Friday of the month).^{7,8} Because the portfolio's weights deviate from long-term strategic weights between monthly rebalances, we define the 60%/40% portfolio's *tactical equity exposure* in Equation 4 and the portfolio's *tactical bond exposure* in Equation 5:

Tactical Equity Exposure,

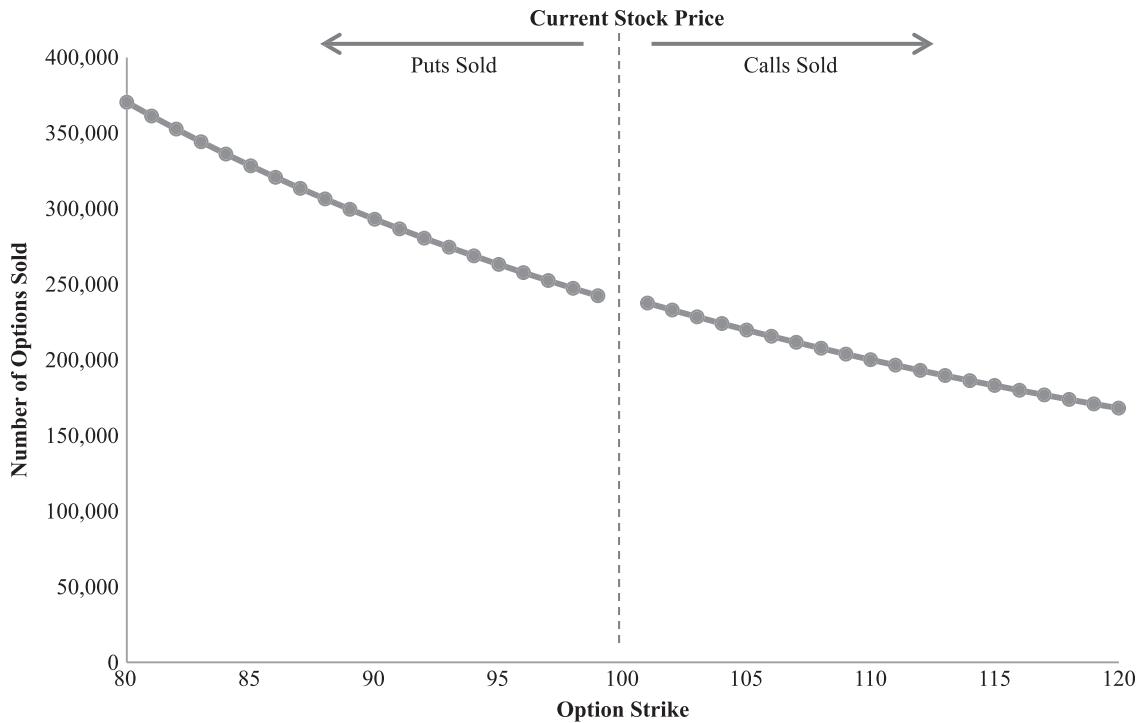
$$= w_t^s - w^s = \frac{q_t^s p_t^s}{q_t^s p_t^s + q_t^b p_t^b} - w^s, \text{ where } w^s = 0.6 \quad (4)$$

Tactical Bond Exposure,

$$= w_t^b - w^b = \frac{q_t^b p_t^b}{q_t^s p_t^s + q_t^b p_t^b} - w^b, \text{ where } w^b = 0.4 \quad (5)$$

EXHIBIT 3

Options Sold in Illustrative Overlay



Notes: The exhibit plots the number of options sold within an option overlay assuming a stock notional of \$6B, an initial stock price of \$100, and a stock weight of 60%. Option strikes are spaced by \$1 increments, and are shown over a range from \$80 and \$120. The number of options sold at each strike is determined by the following equation (Equation 2):

$$\$6B * (1 - 0.6) \frac{\text{Abs}(K_{i-1} - K_i)}{K_i K_{i-1}}.$$

where K = option strike, i = N th furthest option strike from current stock price, i ranges from 1 to 20, and $K_0 = 100$.

Source: AQR.

Exhibit 4 shows the distribution of tactical equity exposure from a 20-year backtest run between 1996 and 2015. While the stock tactical exposure is close to zero (0.02%) on average, it varies significantly over time. Its minimum was -7.6%, its maximum was +2.9%, and the 5th and 95th percentiles were -1.3% and +1.2%, respectively. We also show that the stock timing exposure had a 0.24 beta to the stock return since the last portfolio rebalance. As expected, the tactical exposure exhibits momentum. Particularly at its extremes, this exposure can be economically meaningful.

As defined in Equation 6, a monthly rebalanced 60%/40% portfolio can be decomposed into three components: a *daily* rebalanced 60%/40% portfolio that closely tracks the desired strategic allocation and the tactical equity and bond positioning:

$$\begin{aligned} \text{MonthlyRebalancedPortfolio}_t &= \text{DailyRebalancedPortfolio}_t + \text{TacticalEquity}_t \\ &\quad + \text{TacticalBond}_t, \end{aligned} \quad (6)$$

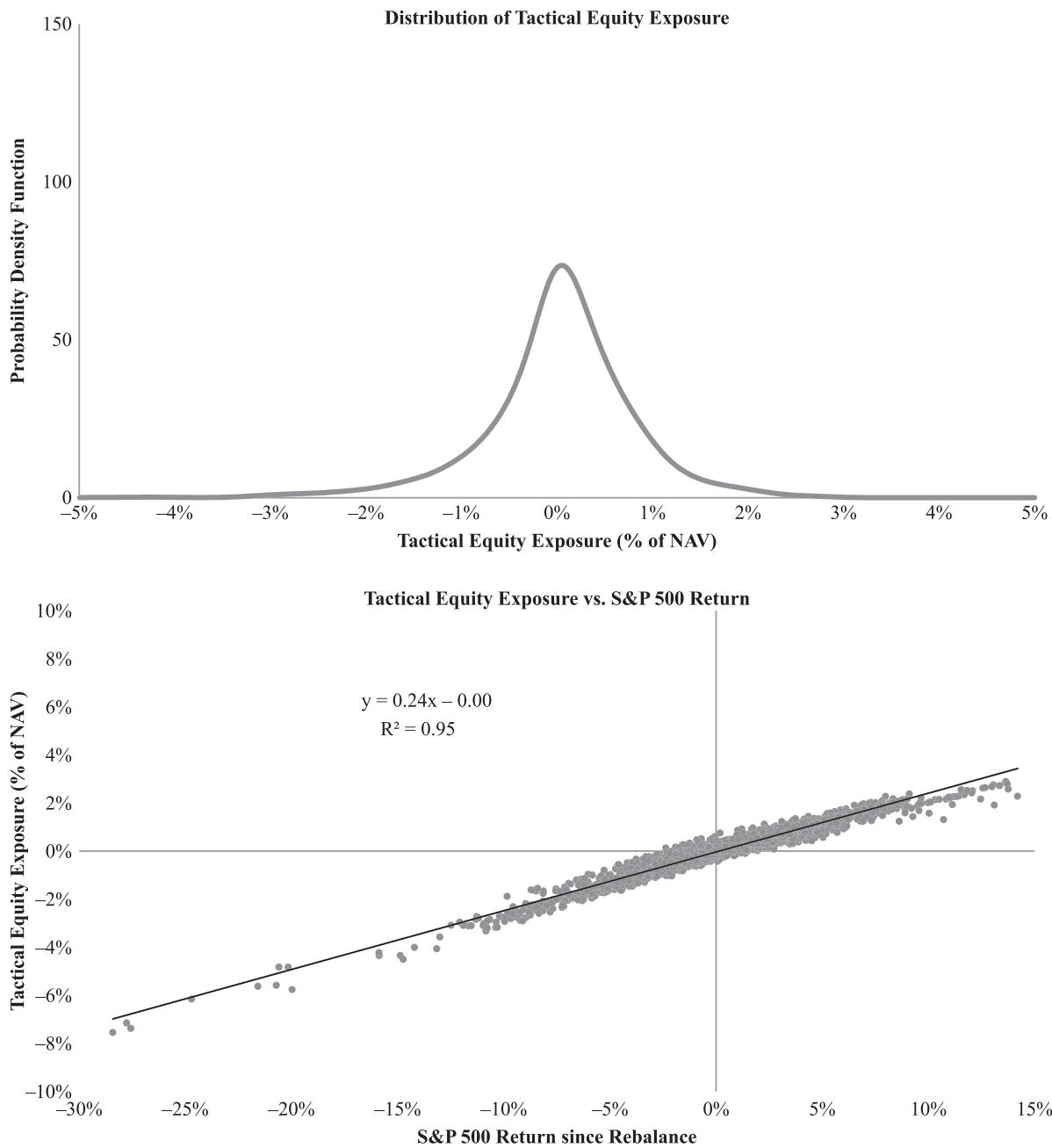
We use the variables r_t^s and r_t^b to represent equity and bond returns between $t - 1$ and t , respectively, and calculate the returns of each of these components in Equations 7, 8, and 9:

$$\text{DailyRebalancedPortfolio}_t = w^s * r_t^s + w^b * r_t^b \quad (7)$$

$$\begin{aligned} \text{Tactical Equity}_t &= (w_{t-1}^s - w^s) * r_t^s = \left(\frac{q_{t-1}^s p_{t-1}^s}{q_{t-1}^s p_{t-1}^s + q_{t-1}^b p_{t-1}^b} - w^s \right) * r_t^s \end{aligned} \quad (8)$$

EXHIBIT 4

60%/40% Portfolio Tactical Equity Exposure (1996–2015)



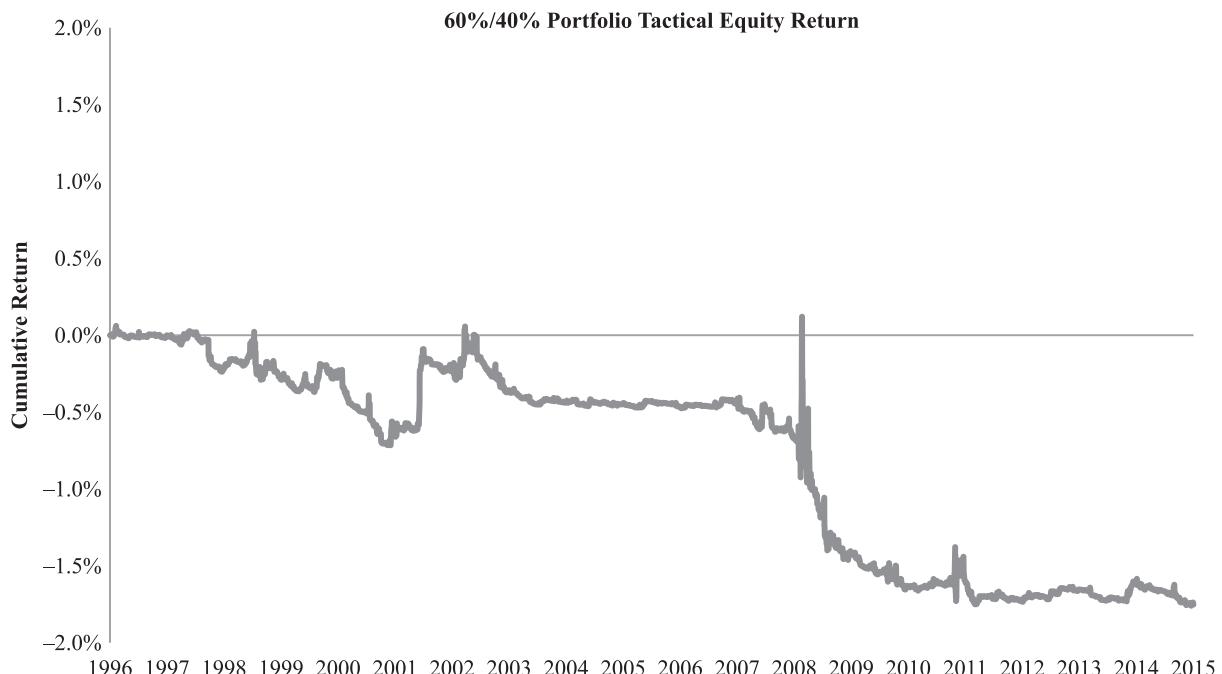
Notes: The top panel plots the distribution of a 60%/40% portfolio's tactical equity exposure between rebalances from a 20-year backtest between 1996 and 2015. Tactical equity exposure is defined as the deviation between the portfolio's percentage equity allocation and the 60% strategic allocation. The bottom panel shows the relationship between tactical equity exposure and the S&P 500 return since the last rebalance.

Sources: AQR, Bloomberg.

EXHIBIT 5

Performance Decomposition of 60%/40% Portfolio

1996–2015	60%/40% Monthly Rebalanced	60%/40% Daily Rebalanced	Tactical Equity	Tactical Bond
Annualized Excess Return	5.12%	5.23%	-0.08%	-0.03%
Annualized Volatility	9.81%	9.85%	0.21%	0.06%
Sharpe Ratio	0.52	0.53	-0.41	-0.55



Notes: The top panel reports a performance decomposition of a monthly rebalanced 60%/40% portfolio into three components: a daily rebalanced 60%/40% portfolio that closely tracks the desired strategic allocation, tactical equity, and tactical bond. Tactical equity is defined as the return attributable to the deviation between the portfolio's percentage equity allocation and the 60% strategic allocation. Tactical bond is defined as the return attributable to the deviation between the portfolio's percentage bond allocation and the 40% strategic allocation. The bottom panel shows the cumulative tactical equity return over the backtest period.

Sources: AQR, Bloomberg.

*TacticalBond*_t

$$= (w_{t-1}^b - w^b) * r_t^b = \left(\frac{q_{t-1}^b p_{t-1}^b}{q_{t-1}^s p_{t-1}^s + q_{t-1}^b p_{t-1}^b} - w^b \right) * r_t^b \quad (9)$$

On monthly rebalance dates, tactical equity and bond exposures are zero because the equity allocation is equal to w^s and the bond allocation is equal to w^b . Therefore, corresponding tactical returns are zero on dates immediately following monthly rebalances. On subsequent dates, tactical equity and bond returns are typically non-zero due to deviation from strategic weights.

Exhibit 5 reports the results of this performance decomposition. The monthly rebalanced 60%/40% portfolio realized 5.12% annualized excess return with 9.81% annualized volatility, resulting in a 0.52 Sharpe ratio. The “pure” daily rebalanced 60%/40% portfolio had a higher return of 5.23% and 9.85%⁹ annualized volatility, resulting in a 0.53 Sharpe ratio.

The tactical stock component, which is the primary focus of our analysis, realized –8 basis points of annualized return and 21 basis points of annualized volatility over the sample period. Although the Sharpe ratio of the stock timing component was –0.4, we do

not believe there is any economic rationale for negative (or positive) expected returns from this component and indeed the returns are statistically insignificant (*t*-stat of -1.8). The 21 basis points of tactical stock return volatility is uncompensated tracking error, and the option overlay seeks to reduce this exposure. The bond timing component realized -3 basis points of annualized return and a relatively low annualized volatility of 6 basis points due to bonds being less volatile than stocks.

OPTION OVERLAY PROPERTIES

Within the *Option Overlay Construction* section, we solved for the number of option contracts to sell at a given strike. However, we still have to decide which maturity, strike range, and rebalancing rules to apply to fully specify an overlay. Although there are a number of permutations that could be explored, we seek to define simple rules as an illustration.

We start by selecting front-month S&P 500 options expiring on the standard monthly expiration date (3rd Friday of the month).¹⁰ We initially select all available out-of-the-money call and put option strikes up to a cutoff of 10 delta. Because Black–Scholes delta can also be loosely interpreted as the probability of an option finishing in-the-money at expiration, we expect the S&P 500 to be within the selected strike range 80% of the time when the options expire.¹¹

After the initial option sale, the overlay may no longer hold options that extend out to 10 delta due to market moves. Therefore, on each day we evaluate whether additional options with greater than 10 delta are available beyond the strike range currently held. If so, we sell these incremental options to ensure coverage out to 10 delta for strikes both above and below the current S&P 500 level. Once options are sold, they are held until expiration, at which point the process repeats with the sale of the next month's options.

We report the properties of an overlay constructed using this methodology in Exhibit 6. Over the 20-year backtest period,¹² on average, the overlay held 32 options on a given day and traded options on 16.4% of days. The number of option strikes held is high because *all* out-of-the-money options greater than 10 delta were selected, and the strategy traded frequently because the overlay was evaluated daily to ensure the strike range extended to 10 delta.

EXHIBIT 6

Properties of Option Overlay (1996–2015)

Property	Full-Period Average
Number of Options Held	32.1
% of Days Traded	16.4%
Calendar Days To Expiration	15.4
Moneyness (% of Spot)	98.5%
Option Notional/NAV	4.0%
Option Notional/Equity NAV	6.6%
Vega/NAV	-0.2 bps
\$Gamma/NAV	-22.1 bps

Notes: The exhibit reports the properties of an option overlay constructed using S&P 500 options. The overlay selects front-month options expiring on the standard monthly expiration date (3rd Friday of the month). The overlay initially selects all available out-of-the-money call and put option strikes up to a cutoff of 10 delta. After the initial option sale, the overlay may no longer hold options that extend out to 10 delta due to market moves. Therefore, on each day the overlay seeks to sell additional options with greater than 10 delta if any are available beyond the strike range currently held. Once options are sold they are held until expiration, at which point the process repeats with the sale of the next month's options. The quantity of options sold at each strike is determined by the following equation (Equation 2):

$$\text{Options Sold}_i = \text{Notional}_{t-1}^s (1 - w^s) \frac{\text{Abs}(K_{i-1} - K_i)}{K_i K_{i-1}},$$

where K = Option Strike, $K_0 = P_t^s$, and $i = N$ th furthest option strike from current stock price.

Source: AQR, Bloomberg, OptionMetrics.

For the purpose of this article, we choose an option selection and rebalancing methodology that is easily defined and interpretable. We also construct the overlay to provide a wide range of coverage in order to illustrate the overlay's potential efficacy. Both the number of options held and the frequency of trading could be reduced through an optimization process. We examine strike range and increment choices in a later section, but refining an optimization process for the option overlay is beyond the scope of this article.

The average maturity of the option portfolio is approximately 15 days, which is intuitive because the overlay sells front-month options and holds them until expiration. The average moneyness, weighted by the number of contracts held, is 98.5%, which is also in line with our expectation because the quantity of out-of-the-money puts sold is greater than that of out-of-the-money calls.

The size of the option overlay is fairly small. The notional of options sold is 4% of NAV (or 6.67% of the 60% equity allocation's notional), on average. As a point of reference, a typical covered call strategy sells a call option that has a notional equivalent to the NAV. The option overlay's vega is also quite small. If implied volatility rises by 1 point, we expect the option overlay's PL to be -0.2 basis points on average.

The option overlay's delta reflects its exposure to the underlying market, and therefore the overlay's tactical equity exposure is simply its delta. Exhibit 7 shows the distribution of this exposure, which was slightly positive, on average ($+0.08\%$). Its minimum was -3.0% , its maximum was $+8.6\%$, and its 5th and 95th percentiles were -1.0% and $+1.2\%$, respectively.

The option overlay's tactical stock exposure exhibits reversal and had a -0.22 beta to the stock return since the last portfolio rebalance. The magnitude of this beta is close to the $+0.24$ beta arising from the 60%/40% portfolio's momentum exposure. This is perhaps unsurprising because the option overlay is specifically tailored to hedge the 60%/40% portfolio's tactical equity exposure.

As defined in Equation 10, we decompose the option overlay's performance into two components: a delta-neutral option portfolio and tactical equity exposure.

$$\text{OptionOverlay}_t = \text{Delta Neutral Options}_t + \text{TacticalEquity}_t \quad (10)$$

We use the variable Δ to represent the option portfolio's delta and calculate the returns of each of these components in Equation 11 and Equation 12:

$$\text{DeltaNeutralOptions}_t = \text{OptionOverlay}_t - \Delta_t r_t^s \quad (11)$$

$$\text{TacticalEquity}_t = \Delta_t r_t^s \quad (12)$$

Exhibit 8 shows this performance decomposition over the 20-year backtest period. The option overlay had 11 basis points of annualized excess return and 26 basis points of annualized volatility, resulting in a 0.4 Sharpe ratio. The volatility of the tactical equity component was 20 basis points, again similar to the 21 basis points of volatility realized by the 60%/40% portfolio's tactical equity component. The annualized return was 5 basis points, but this return was statistically insignificant (t -stat of 1.1).

The delta-neutral short options component realized 6 basis points of annualized return (excess of the risk-free rate, and measured relative to the overall NAV) and had a 0.5 Sharpe ratio. Not only were the returns from this component statistically significant (t -stat of 2.3), but we also believe they are backed by economic intuition because delta-neutral short options harvest the well-documented volatility risk premium (see Fallon et al. [2015]). Although this component only provided a very light allocation to the volatility risk premium with 12 basis points of realized volatility, the overlay provides a source of alpha as an added bonus to the primary objective of reducing equity timing exposure.¹³

Israelov and Tummala [2017] show that the risk-adjusted returns of a volatility risk premium harvesting portfolio can potentially be improved through selecting more compensated options. In particular, they find that shorter-dated options are more compensated than longer-dated options, which supports the use of front-month options to facilitate portfolio rebalances. They also find that options with strikes at and moderately below the current index level are more compensated. The option overlay does have exposure to these option strikes, but it also has exposure to other strikes that are less well compensated. Although it may be possible to slightly overweight options with higher expected risk-adjusted returns, we believe it makes sense to at least begin with weights that optimally hedge the portfolio as a benchmark case.

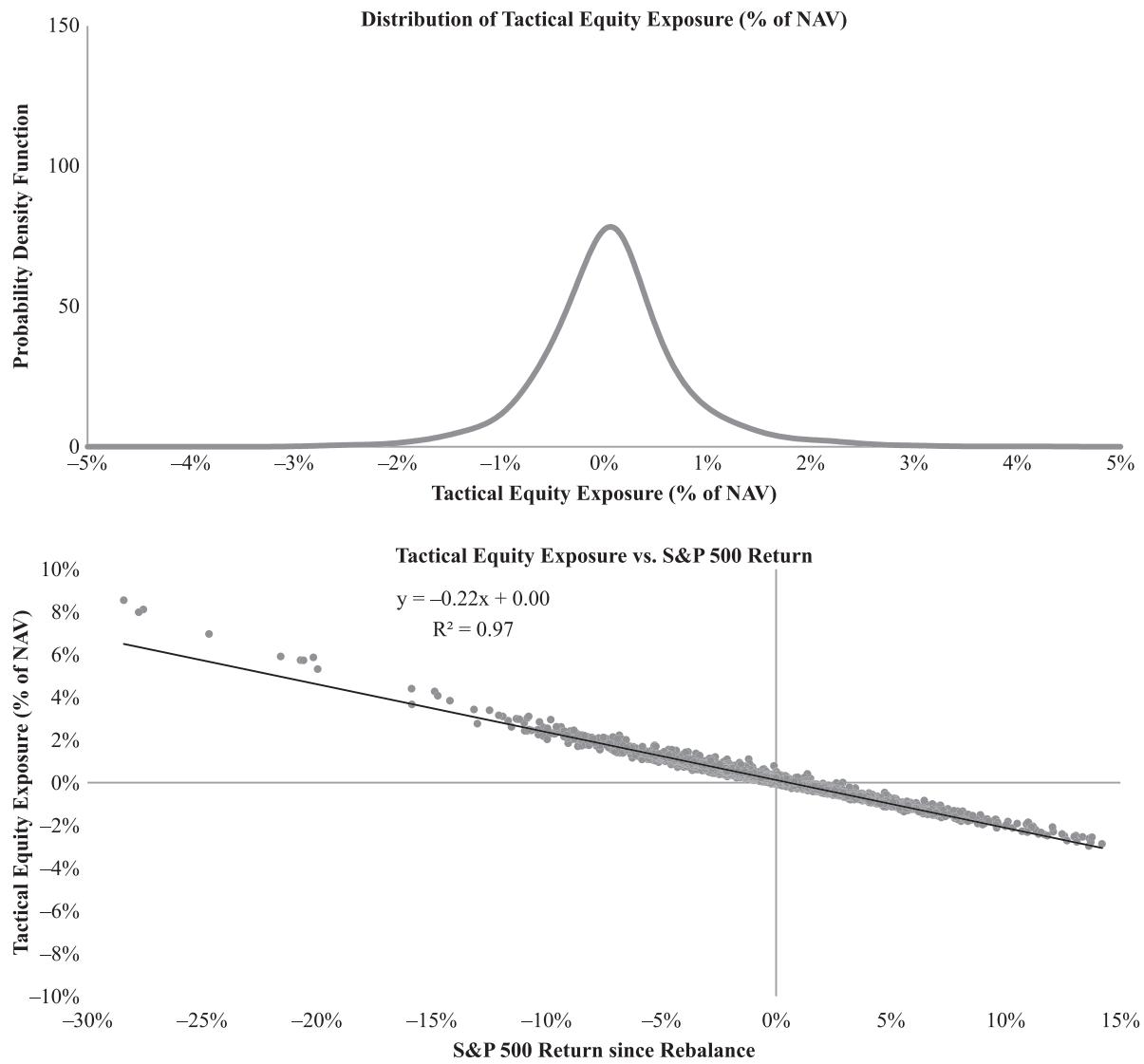
60%/40% PORTFOLIO + OVERLAY PROPERTIES

We now turn to the exposure, risk, and return properties of the 60%/40% portfolio with the option overlay. The goal of the option overlay is to reduce the portfolio's tactical equity exposure. Exhibit 9 shows the impact of the overlay on the portfolio's tactical equity exposure. The top panel plots the distribution of the portfolio's tactical equity exposure with and without the overlay. The bottom panel plots the tactical equity exposure in relation to the equity return since the last portfolio rebalance.

The option-overlaid portfolio's tactical equity exposure was 0.09% , on average, with a minimum of -1.0% and a maximum of 0.9% . The range of tactical equity exposure was significantly narrower than the -7.6% to 2.9% range seen for the standalone 60%/40% portfolio. Its 5th and 95th percentiles were -0.3% and 0.4% , again

EXHIBIT 7

Option Overlay Tactical Equity Exposure (1996–2015)



Notes: The top panel plots the distribution of an option overlay's tactical equity exposure from a 20-year backtest between 1996 and 2015. Tactical equity exposure is defined as the option overlay's Black–Scholes delta exposure. The bottom panel shows the relationship between tactical equity exposure and the S&P 500 return since the last rebalance. The option overlay is constructed by selecting front-month, S&P 500 options that expire on the standard monthly expiration date (3rd Friday of the month). The overlay initially selects all available out-of-the-money call and put option strikes up to a cutoff of 10 delta. After the initial option sale, the overlay may no longer hold options that extend out to 10 delta due to market moves. Therefore, on each day, the overlay seeks to sell additional options with greater than 10 delta if any are available beyond the strike range currently held. Once options are sold, they are held until expiration, at which point the process repeats with the sale of the next month's options. The quantity of options sold at each strike is determined by the following equation (Equation 2):

$$\text{Options Sold}_i = \text{Notional}_{i-1} (1 - w^s) \frac{\text{Abs}(K_{i-1} - K_i)}{K_i K_{i-1}},$$

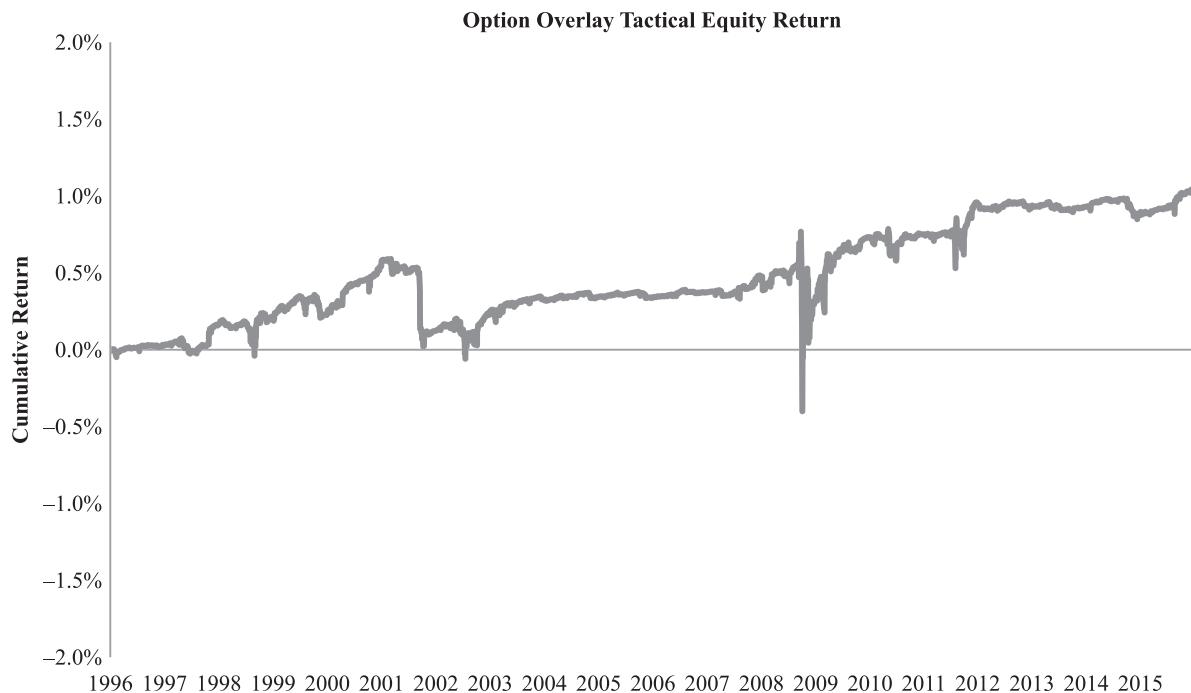
where K = Option Strike, $K_0 = p_t^s$, and $i = N$ th furthest option strike from current stock price.

Sources: AQR, Bloomberg, OptionMetrics.

EXHIBIT 8

Performance Decomposition of Option Overlay Portfolio

1996–2015	Option Overlay	Delta Hedged Short Options	Tactical Equity
Annualized Excess Return	0.11%	0.06%	0.05%
Annualized Volatility	0.26%	0.12%	0.20%
Sharpe Ratio	0.42	0.52	0.25



Notes: The top panel reports a performance decomposition of an option overlay into two components: a delta-neutral option portfolio and tactical equity. Tactical equity is defined as the return attributable to the option overlay's Black–Scholes delta exposure. The bottom panel shows the cumulative tactical equity return over the backtest period. The option overlay is constructed by selecting front-month, S&P 500 options that expire on the standard monthly expiration date (3rd Friday of the month). The overlay initially selects all available out-of-the-money call and put option strikes up to a cutoff of 10 delta. After the initial option sale, the overlay may no longer hold options that extend out to 10 delta due to market moves. Therefore, on each day, the overlay seeks to sell additional options with greater than 10 delta if any are available beyond the strike range currently held. Once options are sold, they are held until expiration, at which point the process repeats with the sale of the next month's options. The quantity of options sold at each strike is determined by the following equation (Equation 2):

$$\text{Options Sold}_i = \text{Notional}_{i-1}^s (1 - w^s) \frac{\text{Abs}(K_{i-1} - K_i)}{K_i K_{i-1}},$$

where K = Option Strike, $K_0 = p_t^s$, and $i = N$ th furthest option strike from current stock price.

Sources: AQR, Bloomberg, OptionMetrics.

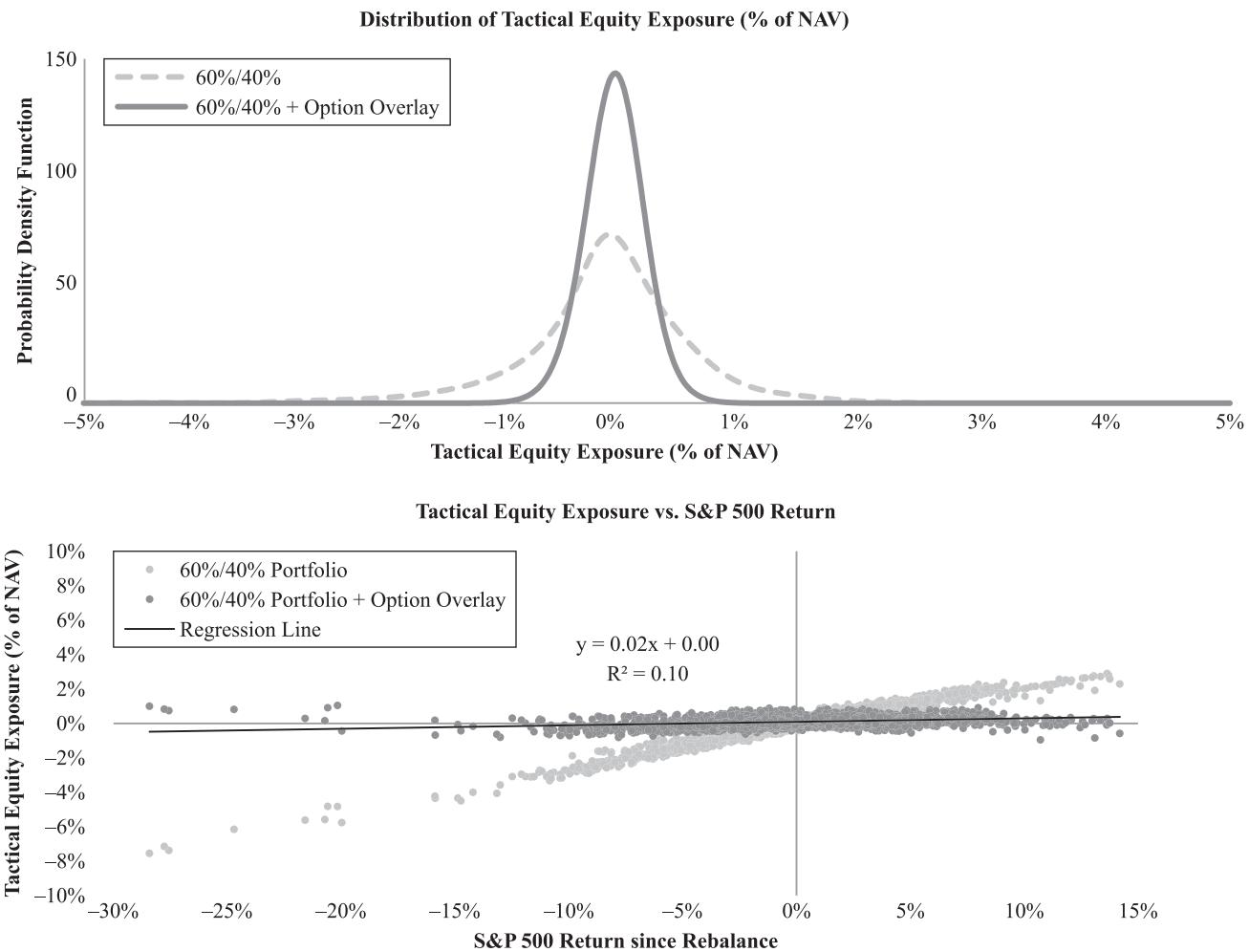
considerably narrower than the analogous -1.3% and 1.2% values seen for the standalone 60%/40% portfolio. As shown in the bottom panel, unlike the monthly-rebalanced 60%/40% portfolio, the option-overlaid portfolio's tactical equity exposure had no discernable relationship to equity returns since the last portfolio rebalance. The addition of the overlay almost entirely

removed the 60%/40% portfolio's uncompensated momentum exposure.

Exhibit 10 decomposes the overlaid portfolio's performance into its passive and tactical exposures. The first table in the Exhibit repeats the decomposition of the non-overlaid, monthly-rebalanced 60%/40% portfolio for reference. The addition of the option overlay

EXHIBIT 9

60%/40% Portfolio + Option Overlay Tactical Equity Exposure (1996–2015)



Notes: The top panel plots the distribution of tactical equity exposure for a 60%/40% portfolio, with and without an option overlay, using a 20-year backtest between 1996 and 2015. The 60%/40% portfolio's tactical equity exposure is defined as the deviation between the portfolio's percentage equity allocation and the 60% strategic allocation. The option overlay's tactical equity exposure is the overlay's Black–Scholes delta exposure. The bottom panel shows the relationship between tactical equity exposure and the S&P 500 return since the last rebalance. The option overlay is constructed by selecting front-month, S&P 500 options that expire on the standard monthly expiration date (3rd Friday of the month). The overlay initially selects all available out-of-the-money call and put option strikes up to a cutoff of 10 delta. After the initial option sale, the overlay may no longer hold options that extend out to 10 delta due to market moves. Therefore, on each day, the overlay seeks to sell additional options with greater than 10 delta if any are available beyond the strike range currently held. Once options are sold, they are held until expiration, at which point the process repeats with the sale of the next month's options. The quantity of options sold at each strike is determined by the following equation (Equation 2):

$$\text{Options Sold}_i = \text{Notional}_{i-1}^s (1 - w^s) \frac{\text{Abs}(K_{i-1} - K_i)}{K_i K_{i-1}},$$

where K = Option Strike, $K_0 = p_t^s$, and $i = N$ th furthest option strike from current stock price.

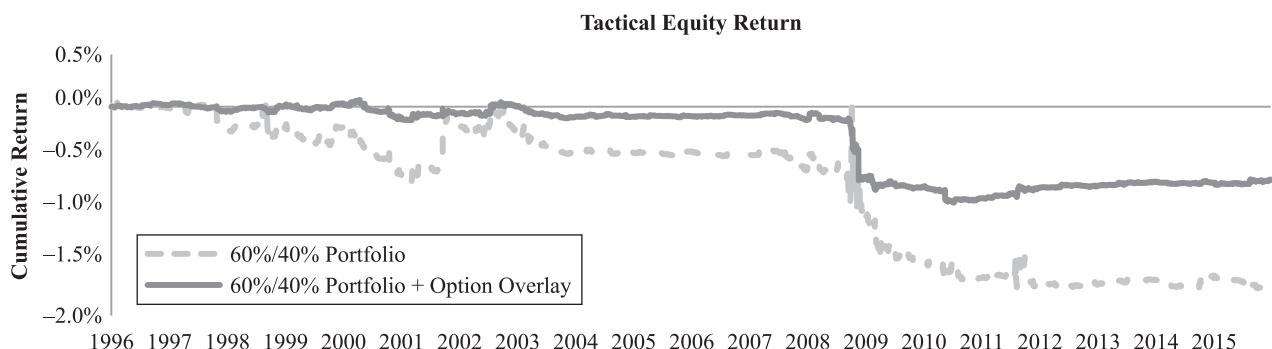
Sources: AQR, Bloomberg, OptionMetrics.

EXHIBIT 10

Performance Decomposition of 60%/40% Portfolio + Overlay

		60%/40% Portfolio				
1996–2015		60%/40% Monthly Rebalanced	60%/40% Daily Rebalanced	Tactical Equity	Tactical Bond	Short Volatility
Annualized Excess Return		5.12%	5.23%	-0.08%	-0.03%	—
Annualized Volatility		9.81%	9.85%	0.21%	0.06%	—
Sharpe Ratio		0.52	0.53	-0.41	-0.55	—

		60%/40% Portfolio + Option Overlay				
1996–2015		With Options Overlay	60%/40% Daily Rebalanced	Tactical Equity	Tactical Bond	Short Volatility
Annualized Excess Return		5.23%	5.23%	-0.03%	-0.03%	0.06%
Annualized Volatility		9.92%	9.85%	0.10%	0.06%	0.12%
Sharpe Ratio		0.53	0.53	-0.35	-0.55	0.52



Notes: The top panel reports a performance decomposition of a monthly rebalanced 60%/40% portfolio into three components: a daily rebalanced 60%/40% portfolio that closely tracks the desired strategic allocation, tactical equity, and tactical bond. The 60%/40% portfolio's tactical equity is defined as the return attributable to the deviation between the portfolio's percentage equity allocation and the 60% strategic allocation. The 60%/40% portfolio's tactical bond is defined as the return attributable to the deviation between the portfolio's percentage bond allocation and the 40% strategic allocation. The middle panel reports the same performance decomposition with the addition of an option overlay. The option overlay's tactical equity is defined as the return attributable to the unhedged Black–Scholes delta exposure. The option overlay's short volatility attribution is the return of the delta-hedged short option portfolio. The bottom panel shows the cumulative tactical equity return over the backtest period. The option overlay is constructed by selecting front-month, S&P 500 options that expire on the standard monthly expiration date (3rd Friday of the month). The overlay initially selects all available out-of-the-money call and put option strikes up to a cutoff of 10 delta. After the initial option sale, the overlay may no longer hold options that extend out to 10 delta due to market moves. Therefore, on each day, the overlay seeks to sell additional options with greater than 10 delta if any are available beyond the strike range currently held. Once options are sold, they are held until expiration, at which point the process repeats with the sale of the next month's options. The quantity of options sold at each strike is determined by the following equation (Equation 2):

$$\text{Options Sold}_i = \text{Notional}_{i-1}^s (1 - w^s) \frac{\text{Abs}(K_{i-1} - K_i)}{K_i K_{i-1}},$$

where K = Option Strike, $K_0 = p_t^s$, and $i = N$ th furthest option strike from current stock price.

Sources: AQR, Bloomberg, OptionMetrics.

reduced the tactical equity component's annualized volatility by more than half, from 21 basis points to 10 basis points. The remaining tactical equity exposure is due to the options' delta not perfectly offsetting the 60%/40% portfolio's tactical equity exposure on the days prior to option expiration. Although it is beyond the scope of this article, it may be possible to further reduce the tactical equity exposure using futures or by dynamically rebalancing the overlay after accounting for current option deltas. As expected, the bond timing component was unrelated to the addition of the equity option overlay.

The option-overlaid portfolio had an annualized return of 5.23%, which was 11 basis points higher than the monthly-rebalanced 60%/40% portfolio's 5.12% annualized return. The outperformance can be attributed to two sources: (1) +5 basis points from reduced tactical equity exposure, which happened to contribute negatively during this period and (2) +6 basis points from short volatility exposure, which is a source of alpha due to the volatility risk premium. The improved returns led to a marginally higher Sharpe ratio (0.53 versus 0.52).

The option-overlaid portfolio's annualized volatility was 9.92%, slightly higher than the monthly-rebalanced 60%/40% portfolio's 9.81% annualized volatility.¹⁴ Although the combined portfolio had lower tactical equity exposure, the overlay also added short volatility exposure. The overlay effectively substituted compensated exposure to the volatility risk premium for uncompensated tactical equity exposure.

IMPLEMENTATION CONSIDERATIONS

In our backtests, we defined an illustrative option overlay that selected every available out-of-the-money option above a minimum delta threshold. This illustrative example was defined as such for its simplicity, but an investor implementing an option overlay likely wishes to make deliberate decisions regarding option selection. For instance, should they sell physically- or cash-settled options? Should they sell index or single-stock options? How wide of a strike range should be implemented? What strike increment should be used? We explore the efficacy of an option overlay program along these dimensions while adhering to practical, real-world constraints.

Physical vs. Cash Settlement

Physically settled SPY ETF options are preferable for investors who have passive equity exposure and who obtain that exposure using SPY ETFs because option assignment physically aligns with the intended underlying instrument. Other investors who use S&P 500 futures to obtain their equity exposure could similarly sell physically-settled S&P 500 futures options.

However, some investors, such as those seeking to add alpha from stock selection, hold individual stocks. These investors may prefer cash settlement to physical settlement so that they can buy/sell individual stocks without having to liquidate the assigned position. Correspondingly, these investors can construct an overlay using cash-settled S&P 500 index options, instead of physically-settled SPY ETF or S&P 500 futures options.

Index vs. Single-Stock Options

We do not believe implementing our approach using single-stock options is advisable for three reasons. First, once an equity portfolio is established, the investor may want the weights of individual names to change in proportion to changes in market capitalization and in proportion to changes in active views. Our rebalancing methodology does not allow for this, as it is intended to control the absolute weight of the underlying asset. Second, as shown by Israelov and Santoli [2017], selling single-stock options has historically been less rewarding than selling equity index options. And third, selling a portfolio of single-name options adds considerable operational complexity.

Strike Range

We focus our attention on physically settled SPY ETF options. Exhibit 11 shows a snapshot of the first 40 out-of-the-money SPY call and put options expiring on December 16, 2016. The snapshot was captured as of November 21, 2016, when the options had approximately a month left to expiration.

The minimum tick size for SPY options is \$0.01. The best bid/offer quote eventually becomes \$0.00/\$0.01 for deep out-of-the-money call options (greater than 112% of the current spot level). Because investors cannot sell options with a zero bid, the minimum tick size

EXHIBIT 11

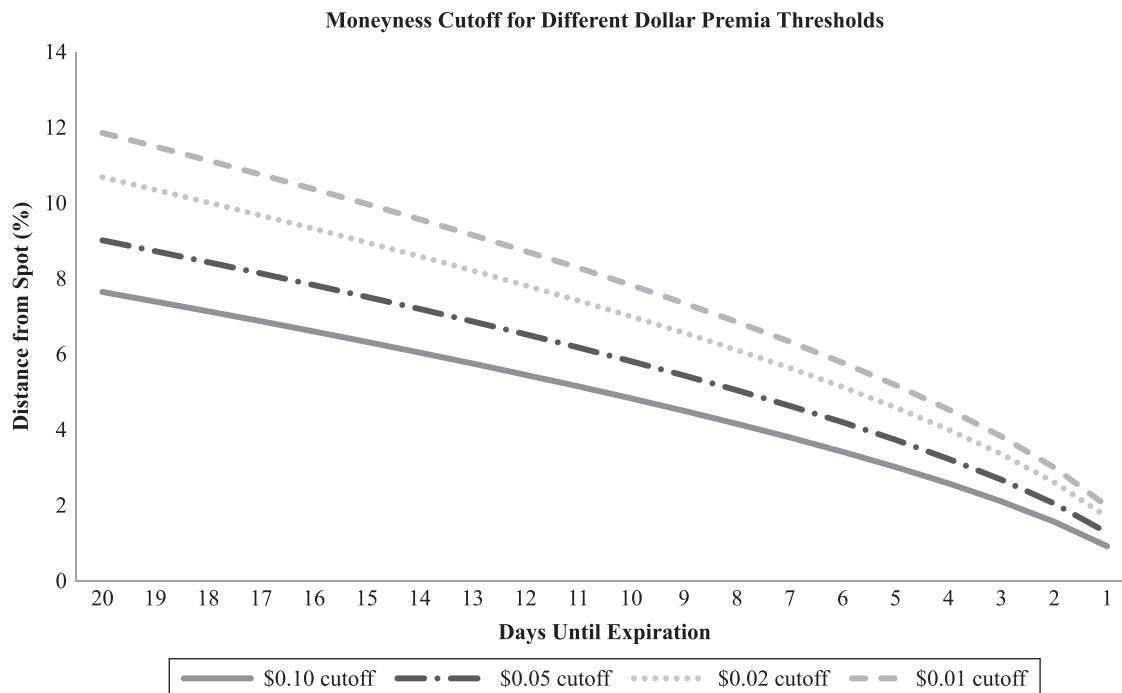
SPY Options Expiring December 16, 2016, as of November 21, 2016

Calls					Puts				
Strike	% Strike	Bid	Ask	Volume	Strike	% Strike	Bid	Ask	Volume
220 1/2	100.2	2.08	2.10	5,520	220	99.9	2.85	2.87	7,297
221	100.4	1.80	1.82	14,426	219 1/2	99.7	2.61	2.64	1,851
221 1/2	100.6	1.55	1.56	5,730	219	99.5	2.40	2.42	14,780
222	100.8	1.31	1.32	9,496	218 1/2	99.3	2.21	2.23	503
222 1/2	101.1	1.10	1.11	5,372	218	99.0	2.03	2.04	11,609
223	101.3	0.91	0.92	15,200	217 1/2	98.8	1.86	1.87	1,866
223 1/2	101.5	0.75	0.76	5,174	217	98.6	1.71	1.72	16,255
224	101.7	0.61	0.62	5,489	216 1/2	98.3	1.56	1.58	2,887
224 1/2	102.0	0.49	0.50	874	216	98.1	1.45	1.46	6,177
225	102.2	0.40	0.41	6,602	215 1/2	97.9	1.32	1.34	1,322
225 1/2	102.4	0.32	0.33	2,950	215	97.7	1.23	1.24	17,574
226	102.7	0.25	0.26	4,969	214 1/2	97.4	1.12	1.13	2,429
226 1/2	102.9	0.21	0.22	1,615	214	97.2	1.03	1.04	11,860
227	103.1	0.17	0.18	6,219	213 1/2	97.0	0.95	0.96	689
227 1/2	103.3	0.14	0.15	258	213	96.8	0.88	0.89	7,543
228	103.6	0.12	0.13	4,581	212 1/2	96.5	0.80	0.81	1,828
228 1/2	103.8	0.10	0.11	1,980	212	96.3	0.74	0.75	16,061
229	104.0	0.09	0.10	5,474	211 1/2	96.1	0.68	0.70	311
229 1/2	104.2	0.08	0.09	2,542	211	95.8	0.63	0.64	11,239
230	104.5	0.07	0.08	2,400	210 1/2	95.6	0.58	0.59	3,541
231	104.9	0.06	0.07	283	210	95.4	0.54	0.55	15,438
232	105.4	0.05	0.06	1,499	209 1/2	95.2	0.50	0.51	1,700
232 1/2	105.6	0.04	0.05	3,658	209	94.9	0.46	0.47	3,174
233	105.8	0.04	0.05	522	208 1/2	94.7	0.43	0.44	495
234	106.3	0.03	0.04	6	208	94.5	0.40	0.41	6,078
235	106.7	0.03	0.04	281	207 1/2	94.3	0.37	0.38	1,007
236	107.2	0.03	0.04	231	207	94.0	0.34	0.35	2,272
237	107.7	0.02	0.03	107	206 1/2	93.8	0.32	0.33	277
237 1/2	107.9	0.02	0.03	0	206	93.6	0.30	0.31	3,255
238	108.1	0.02	0.03	20	205 1/2	93.3	0.28	0.29	1,553
239	108.6	0.02	0.03	0	205	93.1	0.26	0.27	11,404
240	109.0	0.02	0.03	200	204 1/2	92.9	0.24	0.25	466
241	109.5	0.01	0.02	0	204	92.7	0.23	0.24	7,797
242	109.9	0.01	0.02	0	203 1/2	92.4	0.21	0.22	306
242 1/2	110.2	0.01	0.02	2	203	92.2	0.20	0.21	2,632
243	110.4	0.01	0.02	0	202 1/2	92.0	0.19	0.20	1,584
245	111.3	0.01	0.02	2	202	91.8	0.18	0.19	1,791
247 1/2	112.4	0.00	0.01	142	201 1/2	91.5	0.17	0.18	850
250	113.6	0.00	0.01	10	201	91.3	0.16	0.17	1,690
252 1/2	114.7	0.00	0.01	0	200 1/2	91.1	0.15	0.16	747

Notes: The exhibit reports the properties of SPY options with an expiration date of December 16, 2016. The data are as of November 21, 2016. The first 40 out-of-the-money call and put options were selected. Options are sorted by distance from the SPY close price (220.15).

EXHIBIT 12

Strike Range Cutoff for Different Option Premia Thresholds



Notes: The exhibit plots the out-of-the-money option strike corresponding to different option prices for a range of days until expiration. This analysis uses Black–Scholes pricing, assuming a flat 16% implied volatility surface.

Source: AQR.

essentially bounds the implementable strike range for the option overlay.

Even if the bid price of an option is \$0.01, an investor may not wish to sell because the minimum transaction cost, which can be approximated as crossing the minimum half spread of \$0.005, is a large percentage (50%) of the option execution price. This trading cost consideration also limits the practical range of strikes that may be implemented in an overlay program.

Our analysis restricts the options that an investor sells to those with a price of at least \$0.05. In this case, the bid-to-mid trading cost is 10% of the bid price. Exhibit 12 plots the out-of-the-money option strike corresponding to different option prices according to Black–Scholes. For parsimony, we ignore the volatility skew and assume a flat 16% implied volatility surface. The \$0.05 option with 20 days until expiration is approximately 9% out-of-the-money. With only one day until expiration, a \$0.05 option is only 1.3%

out-of-the-money due to the decreased time value. Therefore, for an investor who wishes to only execute options above a premium threshold, the range of available strikes is inherently restricted because deep out-of-the-money options have little value.

We now consider the impact of strike range on hedging efficacy via simulations. Equity and bond returns are simulated under a lognormal distribution, with zero mean and 16% and 4% annualized volatility, respectively. Options are priced with 16% implied volatility, and the option overlay's delta is computed using Black–Scholes. In each of our 10,000 simulations, we construct option overlays with strikes that are 0.25% apart.

We begin by defining our "100% coverage" range, which selects out-of-the-money options up to the defined \$0.05 minimum premium threshold. We then consider the effect of restricted strike ranges by limiting the coverage to 75%, 50%, and 25% of the full 100% coverage range. We vary the number of days to expiration between one and twenty, and plot

the associated simulation results in Exhibit 13. For reference, we also show the simulation results without any option overlay positions. For the No Overlay case, tactical equity exposure is unaffected by the number of days to expiration because there are no option positions.

The first panel in the exhibit shows the 90% confidence interval of the tactical equity exposure. In all cases the median is close to zero because we assume zero expected return for both stocks and bonds. The magnitude of the tactical equity exposure is negatively related to the strike range, decreasing as the strike range approaches the “100% Coverage” case for the widest strike range.

With 20 days left to expiration, the 90% confidence interval in the No Overlay case was -69 to $+66$ basis points. The 25% Coverage case had a comparable range of -46 to $+43$ basis points, reducing exposure by around 33%. The 50% Coverage case’s range was -28 to $+28$ bps, reducing exposure by approximately 40%. Lastly, the 100% Coverage case’s range was -17 to $+18$ basis points, resulting in approximately 75% exposure reduction.¹⁵

In general, we observe higher tactical equity exposure as the stock nears the edge of the overlay’s strike range. With fewer days until expiration, the tactical equity exposure increases for all strike range coverages because the \$0.05 premium threshold prevents the overlay from selling further out-of-the-money options.

The second panel in the exhibit plots the expected tracking error (ETE) from tactical equity exposure. With 20 days until expiration, the ETE for No Overlay was 6.6 basis points. The 25% Coverage case reduced ETE by about 33% to 4.4 basis points. The 100% Coverage case reduced ETE by around 75% to 1.7 basis points.¹⁶ Overall, both simulated stock timing exposure and associated tracking error were significantly reduced by implementing an option overlay—consistent with our previously shown backtested results.

Strike Increment

The market snapshot in Exhibit 12 also indicates that near-the-money SPY options have strikes that are spaced \$0.50 apart (about 0.25% relative to SPY’s price of \$220.15). This 0.25% strike increment places a minimum bound on how far apart listed strikes can

be selected. As seen for deeper out-of-the-money calls, the strike increment eventually widens to \$1.00 (or 0.45% of SPY’s price).

Although we may prefer to have continuous strike resolution (in an ideal setting), the actual market is considerably more discrete. It is worth investigating how this discreteness affects hedging efficacy. We again turn to simulations to do so, using the same parameters for modeling equity and bond returns and for implied volatility. In this case, for each of the 10,000 simulations the range of strikes sold is restricted to options with prices of at least \$0.05 (consistent with the 100% Coverage strike range we previously defined).

We test three implementable strike increments in our simulations: 0.25%, 0.5%, and 1% of the index value at the time of option sale. We also consider a non-implementable 0.01% strike increment as a “best-case” scenario. Finally, we include the monthly rebalanced portfolio with no option overlay as a benchmark. Results are plotted in Exhibit 14.

The top panel plots the 90% confidence interval for tactical equity exposure as it relates to days until expiration. The magnitude of the tactical equity exposure is only moderately reduced with finer strike increments. In the No Overlay case, the 90% confidence interval ranged from -69 to $+68$ basis points. For the overlay implemented with 100 basis point strike increments, the comparable range was -19 to $+18$ basis points, resulting in around 73% exposure reduction. There was little subsequent reduction in equity exposure when using 0.01% strike increments (comparable range was -16 to $+18$ basis points).

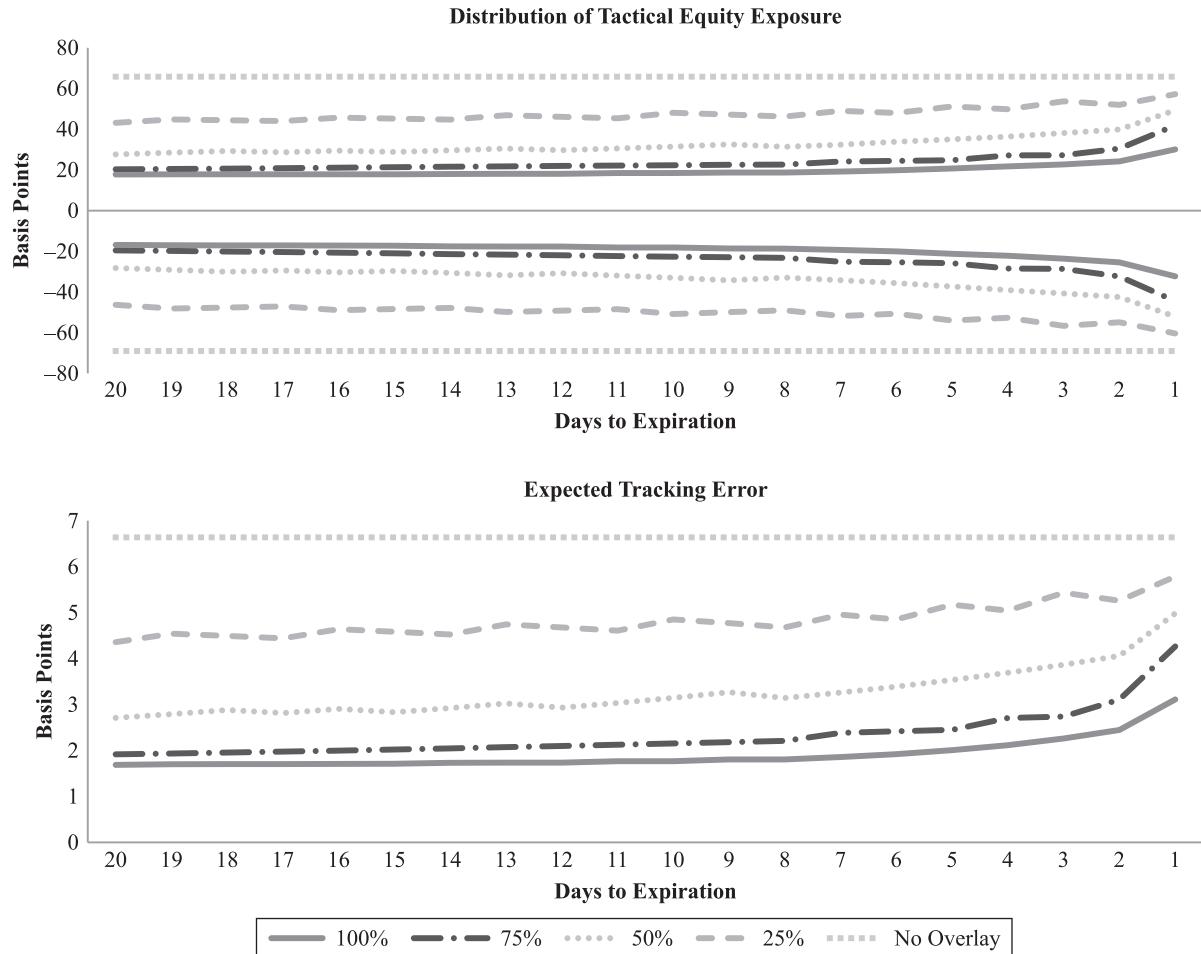
The second panel in Exhibit 14 plots the expected tracking error (ETE) from tactical equity exposure. With 20 days until expiration, the ETE for No Overlay was 6.7 basis points. The 1% strike increment overlay reduced ETE by around 73% to 1.8 basis points. Once again, there was only marginal subsequent reduction when using 0.01% strike increments (ETE was 1.7 basis points).

Consistent with the strike range simulations, we see a similar improvement in hedging efficacy relative to not using an option overlay. However, if an investor prefers to limit the number of options sold (perhaps due to operational considerations), these results suggest that it is better to do so by decreasing the strike increment rather than the strike range.

EXHIBIT 13

Simulated Tactical Equity Exposure for Varying Strike Ranges

Strike increment fixed at 0.25%; \$0.05 premium cutoff for 100% strike range



Notes: The exhibit shows the impact of an option overlay's strike range on its hedging efficacy via simulations. The option overlay hedges a 60%/40% equity/bond portfolio. Equity and bond returns are simulated under a lognormal distribution, with zero mean and 16% and 4% annualized volatility, respectively. Options are priced with 16% implied volatility and the option overlay's delta is computed using Black-Scholes. In each of our 10,000 simulations, we construct option overlays with strikes that are 0.25% apart. We begin by defining the "100% coverage" strike range, which selects out-of-the-money options up to a \$0.05 minimum premium threshold. We then consider the effect of restricted strike ranges by limiting the strike range coverage to 75%, 50%, and 25% of the full 100% coverage range. The number of days to option expiration varies between one and twenty. The quantity of options sold at each strike is determined by the following equation (Equation 2):

$$\text{Options Sold}_i = \text{Notional}_{i-1}(1-w^s) \frac{\text{Abs}(K_{i-1} - K_i)}{K_i K_{i-1}},$$

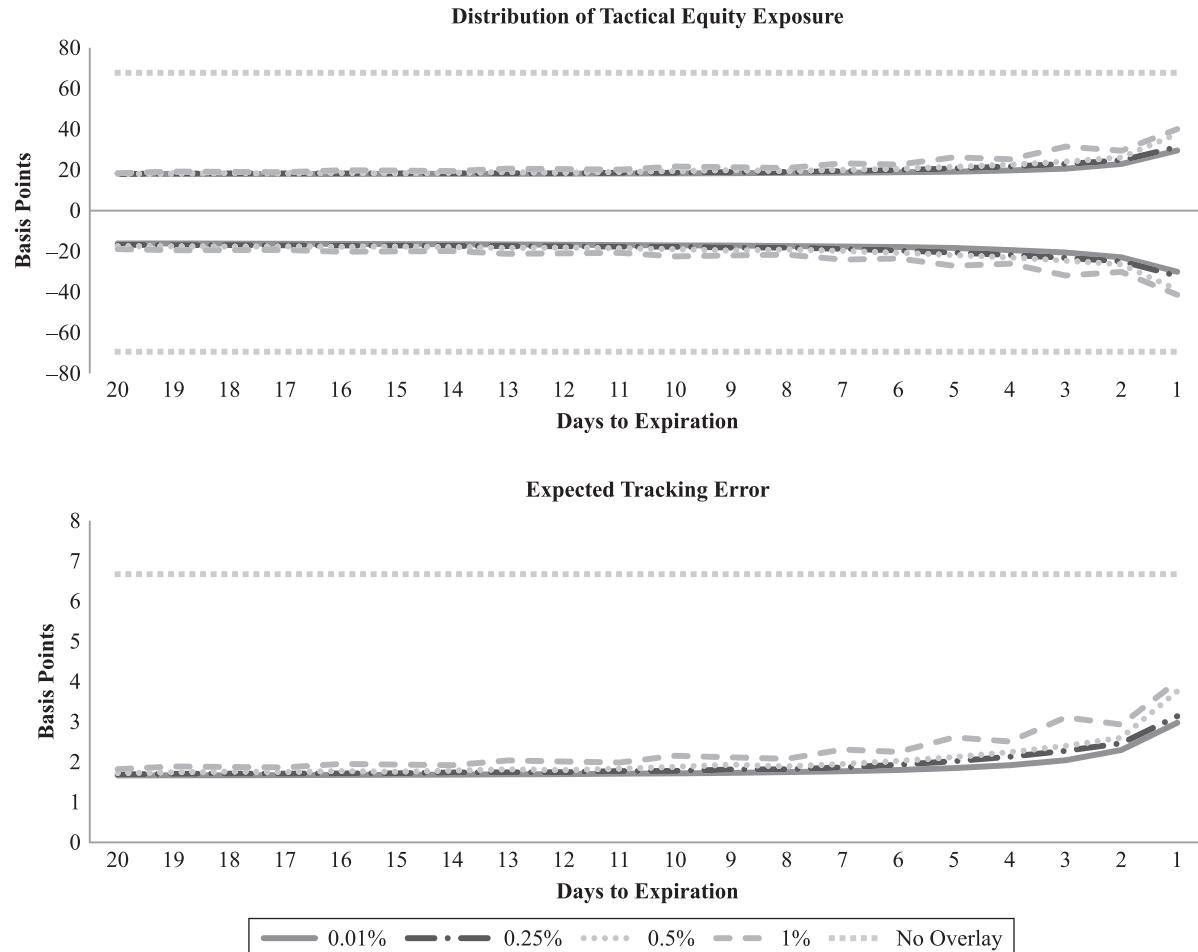
where K = Option Strike, $K_0 = p_t^s$, and $i = N$ th furthest option strike from current stock price.

The option-overlaid portfolio's tactical equity exposure is the sum of the 60%/40% portfolio's tactical equity exposure and the option overlay's tactical equity exposure. The 60%/40% portfolio's tactical equity exposure is defined as the deviation between the portfolio's percentage equity allocation and the 60% strategic allocation. The option overlay's tactical equity exposure is its Black-Scholes delta exposure. The top panel shows the 90% confidence interval of the option-overlaid portfolio's tactical equity exposure. The bottom panel plots the expected tracking error of the option-overlaid portfolio's tactical equity exposure, assuming 16% annualized equity volatility.

Source: AQR.

EXHIBIT 14

Simulated Tactical Equity Exposure for Varying Strike Increments Strike range fixed at \$0.05 premium cutoff



Notes: The exhibit shows the impact of an option overlay's strike increment on its hedging efficacy via simulations. The option overlay hedges a 60%/40% equity/bond portfolio. Equity and bond returns are simulated under a lognormal distribution, with zero mean and 16% and 4% annualized volatility, respectively. Options are priced with 16% implied volatility. The option overlay's delta is computed using Black-Scholes. In each of our 10,000 simulations, the range of strikes sold is restricted to options with prices of at least \$0.05. We test three implementable strike increments in our simulations: 0.25%, 0.5%, and 1%. We also consider a non-implementable 0.01% strike increment as a "best-case" scenario. Finally, we include the monthly-rebalanced portfolio with no option overlay as a benchmark. The number of days to option expiration varies between one and twenty. The quantity of options sold at each strike is determined by the following equation (Equation 2):

$$\text{Options Sold}_i = \text{Notional}_{i-1} (1 - w^s) \frac{\text{Abs}(K_{i-1} - K_i)}{K_i K_{i-1}},$$

where K = Option Strike, $K_0 = p_t^s$, and $i = N$ th furthest option strike from current stock price.

The option-overlaid portfolio's tactical equity exposure is the sum of the 60%/40% portfolio's tactical equity exposure and the option overlay's tactical equity exposure. The 60%/40% portfolio's tactical equity exposure is defined as the deviation between the portfolio's percentage equity allocation and the 60% strategic allocation. The option overlay's tactical equity exposure is its Black-Scholes delta exposure. The top panel shows the 90% confidence interval of the option-overlaid portfolio's tactical equity exposure. The bottom panel plots the expected tracking error of the option-overlaid portfolio's tactical equity exposure, assuming 16% annualized equity volatility.

Source: AQR.

EXHIBIT 15

Turnover and Estimated Transaction Costs for Different Rebalancing Approaches

1996–2015	60%/40% Monthly Rebalanced	60%/40% Daily Rebalanced	60%/40% + Option Overlay
Annual Turnover (% of NAV)	11%	54%	51%
Annual Stock Trading Costs (bps)	0.3	1.3	—
Annual Option Trading Costs (bps)	—	—	1.3

Notes: The exhibit reports turnover and trading costs for the following three rebalancing approaches: monthly-rebalanced to 60%/40%, daily rebalanced to 60%/40%, and overlaying options on the 60%/40% portfolio. Transaction cost calculations assume 2.5 basis points for stock transaction costs and 2.5 basis points of notional for option transaction costs. The option overlay is constructed by selecting front-month, S&P 500 options that expire on the standard monthly expiration date (3rd Friday of the month). The overlay initially selects all available out-of-the-money call and put option strikes up to a cutoff of 10 delta. After the initial option sale, the overlay may no longer hold options that extend out to 10 delta due to market moves. Therefore, on each day, the overlay seeks to sell additional options with greater than 10 delta if any are available beyond the strike range currently held. Once options are sold, they are held until expiration, at which point the process repeats with the sale of the next month's options. The quantity of options sold at each strike is determined by the following equation (Equation 2):

$$OptionsSold_i = Notional_{i-1}^s (1 - w^s) \frac{Abs(K_{i-1} - K_i)}{K_i K_{i-1}},$$

where K = Option Strike, $K_0 = p_t^s$, and $i = N$ th furthest option strike from current stock price.

Because the option overlay's strike range was extended daily to ensure strike range coverage to 10 delta, all turnover and trading costs for option-overlaid rebalancing approach are solely for options trading, and we assume no trading in the underlying stock.

Source: AQR, Bloomberg, OptionMetrics.

Turnover and Trading Costs

Trading costs are an important consideration in any rebalancing program. We now analyze the turnover and trading costs of the option overlay. We compare the following three approaches¹⁷ and report results in Exhibit 15: (1) monthly rebalanced to 60%/40%, (2) daily rebalanced to 60%/40%, and (3) overlaying options on the 60%/40% portfolio. For the last approach we use the same backtest setup as specified earlier in the *Option Overlay Properties* section.

The annualized stock turnover was 11% for the monthly rebalanced approach. As expected, we see

a considerably higher turnover of 54% for the daily rebalanced approach. For the approach using the option overlay, for consistency we report turnover using the option notional traded and calculate an annualized turnover of 51%. However, the delta-adjusted option notional¹⁸ traded is much lower at 2% annualized due to offsetting delta from put and call option trades. Although beyond the scope of this article, it is possible to use optimization to reduce the annualized turnover for each approach.

To estimate transaction costs, we assume that trading stocks costs 2.5 basis points and trading options costs 2.5 basis points of the option notional.¹⁹ Under these assumptions, the monthly rebalanced approach incurs 0.3 basis points of annualized trading cost. Both the daily rebalanced and option overlay²⁰ approaches incur approximately 1.3 basis points of annualized cost.

The option overlay approach to rebalancing is not advantageous in terms of trading costs. It has four times higher trading costs than the monthly rebalanced portfolio and similar trading costs as the daily rebalanced portfolio. However, as reported in Exhibit 10, the option overlay has earned 6 basis points annualized from the volatility risk premium. Therefore, estimated option trading costs are approximately 20% of the historical gross returns earned from the volatility risk premium. This source of return more than covers the cost of execution, netting the portfolio almost 5 basis points per year of alpha.

Adding 5 basis points per year of alpha may not appear to be a significant contribution. Is an option overlay really worth the effort and operational complexity? Appearances may be deceiving. Consider for a moment the traditional approach to adding alpha to a portfolio: investing with a long-only active equity manager. If a portfolio invested 5% of its NAV (or 8.3% of its 60% equity allocation) with a long-only active equity manager who provides 4% annualized tracking error, that investment would contribute 20 basis points of tracking error to the portfolio. The manager would then have to deliver a 0.25 information ratio to match the 5 basis points of alpha provided by the option overlay.

CONCLUSION

Investors who seek to hedge against market crashes often turn to equity index options, buying put options for protection. Unfortunately, this hedge negatively

impacts the portfolio's expected returns because the investor pays the volatility risk premium. In this article, we show that investors can *sell* equity index options to hedge a different risk in their portfolio. Because the investor earns the volatility risk premium, this hedge adds alpha to the portfolio—a statement that can rarely be made about a portfolio hedge.

How does this work? A diversified portfolio that is partially invested in equities, such as a 60%/40% equity/bond portfolio, has momentum exposure between rebalance periods. An option selling overlay provides offsetting reversal exposure that can significantly reduce this unintended momentum bet. Furthermore, by trading options the investor can systematically rebalance and avoid executing equity trades frequently.²¹

We solve for the option overlay's portfolio weights and investigate its efficacy as a cross-asset momentum hedge. Our findings show that options offer a compelling opportunity to improve portfolio rebalancing by reducing uncompensated tactical exposures and increasing expected returns through earning the volatility risk premium.

APPENDIX

OPTION OVERLAY DERIVATION

We start by defining the NAV of the portfolio in terms of current asset prices and quantities held in Equation A1:

$$NAV_t = q_{t-1}^s p_t^s + q_{t-1}^b p_t^b \quad (\text{A1})$$

where p^s = stock price, p^b = bond price, q^s = quantity of stocks, q^b = quantity of bonds.

As defined in Equation A2, the objective of rebalancing the portfolio is to solve for q^s such that the dollar weight of stocks in the portfolio meets our target, w^s .

$$w^s = \frac{q_t^s p_t^s}{NAV_t} \quad (\text{A2})$$

This rebalance has the constraint that we can only reallocate dollars invested in existing investments (there are no inflows or outflows). This constraint is defined in Equation A3:

$$NAV_t = q_t^s p_t^s + q_t^b p_t^b = q_{t-1}^s p_t^s + q_{t-1}^b p_t^b \quad (\text{A3})$$

We ultimately seek to solve for the needed stock trade $q_t^s - q_{t-1}^s$ that satisfies the objective while adhering to this

constraint. We start by plugging in the two definitions of NAV_t (Equation A3) into the definition of the target weight (Equation A2). This yields Equations A4 and Equation A5:

$$w^s = \frac{q_t^s p_t^s}{q_t^s p_t^s + q_t^b p_t^b} \quad (\text{A4})$$

$$w^s = \frac{q_t^s p_t^s}{q_{t-1}^s p_t^s + q_{t-1}^b p_t^b} \quad (\text{A5})$$

Solving Equation A4 for q_t^s , we arrive at Equation A6:

$$q_t^s = \frac{w^s}{1 - w^s} q_t^b \frac{p_t^b}{p_t^s} \quad (\text{A6})$$

If we make the assumption that the portfolio started at its target equity weight yesterday, then Equation A6 also provides Equation A7 for the number of shares held yesterday:

$$q_{t-1}^s = \frac{w^s}{1 - w^s} q_{t-1}^b \frac{p_{t-1}^b}{p_{t-1}^s} \quad (\text{A7})$$

Solving Equation A5 for q_t^s , we arrive at Equation A8:

$$q_t^s = w^s \left\{ q_{t-1}^s + q_{t-1}^b \frac{p_t^b}{p_t^s} \right\} \quad (\text{A8})$$

We then plug Equation A7 into Equation A8 and get Equation A9:

$$q_t^s = w^s q_{t-1}^b \frac{p_{t-1}^b}{p_{t-1}^s} \left(\frac{w^s}{1 - w^s} + \frac{p_{t-1}^s}{p_t^s} \frac{p_t^b}{p_{t-1}^b} \right) \quad (\text{A9})$$

Equation A10 solves for the needed equity rebalance trade by subtracting Equation A7 from Equation A9:

$$q_t^s - q_{t-1}^s = w^s q_{t-1}^b \frac{p_{t-1}^b}{p_{t-1}^s} \left(\frac{p_{t-1}^s}{p_t^s} \frac{p_t^b}{p_{t-1}^b} - 1 \right) \quad (\text{A10})$$

This equity rebalance trade can be rewritten using Equation A7. We also introduce variables r_t^s and r_t^b that represents stock and bond returns between $t - 1$ and t , respectively, to arrive at Equation A11:

$$q_t^s - q_{t-1}^s = q_{t-1}^s (1 - w^s) \left(\frac{p_{t-1}^s}{p_t^s} - 1 + \frac{p_{t-1}^s}{p_t^s} r_t^b \right) \quad (\text{A11})$$

Note that $\frac{p_{t-1}^s}{p_t^s} r_t^b = \frac{r_t^b}{1+r_t^s}$. We assume that $r_t^b \ll 1 + r_t^s$,

and therefore arrive at the following approximation in Equation A12:

$$q_t^s - q_{t-1}^s \approx q_{t-1}^s(1 - w^s) \left(\frac{p_{t-1}^s}{p_t^s} - 1 \right) \quad (\text{A12})$$

We now solve for the option overlay that provides the stock shares needed to rebalance the portfolio to its target equity weight at option expiration. We assume that at option expiration if the stock price is exactly at an option's strike (K), the option buyer will exercise the option. Therefore, the number of options that should be sold at a given strike is specified by Equation A13:

$$\text{Options Sold} = q_{t-1}^s(1 - w^s) \left(\frac{p_{t-1}^s}{K} - 1 \right) \quad (\text{A13})$$

However, we must also account for the number of options that were already sold at strikes closer to the current stock price. We assume that only out-of-the-money calls and out-of-the-money calls are sold. Index i in Equation A14 represents the n th furthest option strike from the current stock price (referenced independently for calls and puts):

$$\text{Options Sold}_i = \text{Abs} \left(p_{t-1}^s q_{t-1}^s (1 - w^s) \left(\frac{1}{K_i} - \frac{1}{K_{i-1}} \right) \right) \quad (\text{A14})$$

where $1 \leq i \leq N$ and $K_0 = p_t^s$.

Equation A14 can be re-written as Equation A15, which matches Equation 2 in the main article:

$$\text{Options Sold}_i = \text{Notional}_{t-1}^s (1 - w^s) \frac{\text{Abs}(K_{i-1} - K_i)}{K_i K_{i-1}} \quad (\text{A15})$$

ENDNOTES

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¹Strategic allocation based on fixed notional weights is clearly not the only way to construct a portfolio. Other

approaches, such as risk-based allocation (risk parity), may be used. However, many investors define their policy portfolio using notional weights and employ a rebalancing approach based on these weights.

²For example, the portfolio is exposed to momentum since the last portfolio rebalance. Therefore, the historical period over which momentum is measured is constantly changing. One day after the last rebalance, the momentum horizon is one day; twenty days since the last rebalance, the momentum horizon is twenty days. A monthly rebalanced portfolio is exposed to shorter-term trends of less than a month. But the empirical evidence shows that trend-following tends to be more effective over longer periods.

³We must emphasize that option selling is not a free lunch. Israelov and Nielsen [2014] identify the free lunch myth (Myth 8) that option sellers get paid to do what they would have done anyway. Selling the option commits the seller to trade the underlying equity at an undesirable price conditional on option exercise. For instance, a short \$99 strike put option commits the option seller to buy the stock for \$99 when the stock price is below \$99. This reason this must be the case is the buyer of the put option will only rationally exercise if the option is in-the-money. While selling options earns the volatility risk premium and has been profitable on average historically, the strategy is exposed to tail risk because it can have poor performance during large, rapid market moves.

⁴Note that in the *Stock Market Down* scenario the investor needed to buy more shares (1,000,000) than he needed to sell (923,077) in the *Stock Market Up* scenario. The quantity of options needed at various strikes varies, and will be addressed in the *Option Overlay Construction* section.

⁵The same methodology could be applied to the bond component of the portfolio, but for simplicity, we focus solely on the stock component of the portfolio. Furthermore, in a 60%/40% portfolio the majority of the portfolio's risk comes from stocks because they have a larger strategic weight and are the more volatile asset. Therefore, we believe that focusing on the stock component is a reasonable approximation.

⁶The delta $\left(\frac{\partial P}{\partial S} \right)$ of an option is its exposure to changes

in the underlying instrument's price. However, an option's delta changes over time and can be calculated through "charm", a higher-order Greek defined as $-\left(\frac{\partial \Delta}{\partial \tau} \right)$. As an example, consider an out-of-the-money option that is initially traded at a delta of 0.1. Assuming nothing else changes and we measure the option's delta right before expiration it would be close to 0.0.

⁷Rebalances are specifically chosen to coincide with option expiration dates to align exposures with an option overlay.

⁸An investor may also have to address capital inflows or outflows. If those flows are aligned with the portfolio's monthly rebalance, then the quantity of options sold changes (because $Notional_{t-1}^*$ changes), but the overall construction of the option overlay remains the same.

⁹The monthly rebalanced 60%/40% portfolio's realized volatility is slightly lower than the "pure", daily rebalanced 60%/40% portfolio's realized volatility due to negative correlation between tactical equity and the daily rebalanced portfolio. The ex ante correlation of these two returns is zero. Thus, we expect the daily rebalanced portfolio to have lower volatility than the monthly rebalanced portfolio.

¹⁰S&P 500 index options are cash-settled. Although physically settled options (such as SPY options) are preferable when used for rebalancing, S&P 500 index options have longer data availability. We expect analysis and findings to be very similar in either case because these options have nearly equivalent exposure.

¹¹Implied volatility is higher than subsequent realized volatility, on average, due to the volatility risk premium. Because of this effect, we actually expect the S&P 500 to be within the selected strike range greater than 80% of the time.

¹²The 20-year backtest period provides 240 monthly rebalances. While this is a limited number of data points, we further examine the hedging efficacy of the overlay using simulations in the *Implementation Considerations* section.

¹³An investor seeking to allocate to the volatility risk premium (VRP) in a standalone implementation would likely seek more exposure. However, the primary objective of the designed option overlay is to hedge the multi-asset class portfolio's equity timing exposure. While the resulting VRP allocation is modest, this does not preclude the investor from adding additional exposure through a VRP-dedicated strategy. As mentioned in the subsequent paragraph, that strategy may have a different construction (for example, it may select different options).

¹⁴The monthly rebalanced 60%/40% portfolio's tactical equity exposure is negatively correlated to the daily rebalanced 60%/40% portfolio. However, the option-overlaid portfolio's tactical equity exposure is positively correlated to the daily-rebalanced 60%/40% portfolio, leading to a slightly higher volatility than the non-overlaid portfolio. On an ex ante basis we expect zero correlation between these components and therefore do not expect the option-overlaid portfolio to realize higher volatility going forward.

¹⁵The 75% tactical equity exposure reduction in the simulation of the 100% Coverage case is comparable to the reduction seen in the option-overlaid portfolio backtest. However, the level of the exposure in the simulations (both with No Overlay and 100% Coverage) is roughly half that seen in the backtested results. Simulation results are reported for 20 days to expiration. Backtest exposure is higher because

options are held to expiration and tactical equity exposure increases closer to expiration. Furthermore, backtest exposure is highest during Q4 2008, which may not be modeled well by the log-normal return assumption in the simulation.

¹⁶The 75% reduction in ETE in the simulation of the 100% Coverage case is higher than the roughly 50% reduction in tracking error seen in the option-overlaid portfolio backtest. In the real world, implied volatilities are not constant. This effect could explain why the backtested option overlay does not reduce tracking error as much as seen in the simulation. The level of tracking error is also significantly higher in the backtest relative to the simulation. This is partially explained by the fact that backtest exposure was higher than the simulated exposure. It is also due to real-world effects that are not captured by the simulations, such as the fact that volatility can be significantly higher after large equity moves (when tactical equity exposure is large).

¹⁷For the non-option overlay approaches, we look at calendar-based rebalancing. Although beyond the scope of this article, it may be possible to use more sophisticated threshold-based rebalancing approaches based on notional deviations or tracking-error deviations.

¹⁸Delta-adjusted option notional is one way of measuring option exposure. It is defined as the notional of an option position, multiplied by the option's delta. For example, a short at-the-money put option has a delta of around 0.5. If an investor is short one at-the-money put option and the spot price of the underlyer is \$100, the option notional is \$100. The delta-adjusted option notional is \$50. In the context of option transaction costs, an investor is typically charged for two exposures: (1) delta exposure to the underlying market and (2) exposure to the underlyer's volatility. Delta-neutral trades are less costly because the first exposure is eliminated. For example, for the same number of option contracts, trading a 50-delta, at-the-money put option is more expensive than trading a delta-neutral, at-the-money straddle. Using delta-adjusted option notional to measure turnover isolates the net delta exposure of a strategy's trades.

¹⁹We estimate transaction costs relative to the options' vega exposure. As a rough approximation, we assume that it costs 0.2 volatility points to execute SPY options. We estimate the vega of an option using the following approximation: Vega = Notional * 40 bps * $\text{SQRT}(T)$. We then calculate dollar transaction costs as follows: Option Transaction Cost = Vega * 0.2 volatility points. Given that 1-month options are traded in the illustrative option overlay, we plug in $T = 1/12$ and calculate Option Transaction Cost $\sim = 2.31$ basis points, which we round higher to 2.5 basis points. SPY and S&P 500 index options are very liquid. For options on other equity underlyings, an investor would likely incur higher transaction costs. Alternatively, an investor could still use SPY options and accept basis risk.

²⁰The transaction cost estimate for the option overlay includes the selling of incremental options to ensure strike coverage out to 10 delta. Because we assume the investor trades physically settled options, we do not assume any transaction costs for trading the stock within this approach.

²¹As seen from the simulation results, ensuring that the option overlay's strike range coverage is sufficiently wide improves the overlay's hedging properties. Therefore, the investor may need to trade to extend the range of options sold if equities move significantly. We believe a process can be defined to occasionally execute incremental option trades without resulting in significant operational burden.

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