How the Wealth Was Won: Factor Shares as Market Fundamentals*

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Abstract

We provide novel evidence on the driving forces behind the sharp increase in equity values over the post-war era. From the beginning of 1989 to the end of 2017, 23 trillion dollars of real equity wealth was created by the nonfinancial corporate sector. We estimate that 54% of this increase was attributable to a reallocation of rents to shareholders in a decelerating economy. Economic growth accounts for just 24%, followed by lower interest rates (11%) and a lower risk premium (11%). From 1952 to 1988 less than half as much wealth was created, but economic growth accounted for 92% of it.

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1 Introduction

The U.S. stock market has done exceptionally well in the post-war era, driven mostly by rapid growth in the last three decades. For the 29 years from the beginning of 1989 to the end of 2017, the real value of market equity for the nonfinancial corporate sector (NFCS) grew at an annual rate of 6.9% per annum. This may be compared to an annual growth of just 3.2% over the previous 29 years, from the beginning of 1959 to the end of 1988. By contrast, the real value of what was actually produced by the sector exhibits the opposite temporal pattern: real net value added of the NFCS grew 4.4% per annum in the earlier subsample, compared to just 2.5% per annum in the most recent subsample.[1]

The upshot of these trends is a widening chasm between the stock market and the broader economy, a phenomenon displayed in Figure 1, which plots the ratio of market equity for the NFCS to three different measures of aggregate economic activity: gross domestic product, personal consumption expenditures, and net value added of the NFCS. (To make the units comparable, each series has been normalized to unity in 1989:Q1.) Despite substantial volatility in these ratios, each is either at or near its post-war high by the end of 2017. Notably, the ratio of market equity to after-tax profits (earnings) for the NFCS is not near its post-war high. In terms of the two subsamples considered above, earnings grew 4.8% per annum from 1989-2017, compared to 3.4% per annum from 1959-1988.

What should financial economists make of these trends? After all, textbook economics teaches us that the stock market and the broader economy should share a common trend, so that the very factors that boost economic growth are also the key to rising equity values over long periods of time. Yet a cursory examination of Figure 1 suggests that this basic tenet of macroeconomic theory has not been borne out by data.[2] What then is responsible

[1] Growth rates are annualized changes over the subsample, e.g., $(P_{2017:4}/P_{1988:1})^{29} - 1$, in real terms, deflated by the implicit price deflator for NFCS net value added.

[2] This tenet goes back to at least Kaldor (1957), followed by a vast literature in macroeconomic theory that presumes balanced growth among economic aggregates over long periods of time. For a more recent variant, see Farhi and Gourio (2018). Previous studies have noted the apparent disconnect between economic growth and the rate of return on stocks over long periods of time, both domestically and internationally (see e.g., Estrada (2012); Ritter (2012); Siegel (2014)). Our study provides evidence on the foundations of this

for the boom in equity values over the post-war period?

Addressing this question empirically requires not only data on what has occurred, but an economic model of how investors value equity. That's because theoretical factors other than economic growth could predominate as causal forces in an equity boom even over sustained periods of time, if they are persistent enough and large enough to overturn the drag from a slower economy. These include changes in how economic growth is expected to be linked to the future growth in cash payments to shareholders, as well as changes in how those payments are discounted back to the present. The latter in turn depend not only on the expected path of future short rates, but also on how risk-tolerant or optimistic investors feel when making equity investments, i.e., on risk premia. Finally, it's possible that economic growth has been the key driving force behind the market's rise over the whole post-war period after all, even if the last thirty years have been a striking exception.

In this paper we investigate the origins of sharply rising equity values in the post-war era by estimating a model of the U.S. equity market. Although the specification of a model necessarily imposes some structure, our approach is intended to let the data speak as much as possible. We do this by estimating a flexible parametric model of how equities are priced that allows for influence from a number of mutually uncorrelated latent factors, while at the same time inferring what values those factors must have taken over our sample to explain the data. The identification of mutually uncorrelated components and the specification of a log linear model allow us to precisely decompose the market's observed growth into distinct component sources.

Equity in our model is priced, not by a representative household, but by a representative shareholder, akin in the data to a wealthy household or large institutional investor. The remaining agents supply labor, but play no role in asset pricing. Shareholder preferences are subject to a shock that alters their appetite for risk, but investors understand that the state variables that drive equity values are subject to transitional dynamics and take these into account when forming expectations. We estimate the full dynamic model, which allows not only for time variation in the expected growth of rents generated from productive activity,

disconnect in post-war U.S. data.

but also in how those rents are apportioned between shareholders and other claimants, principally labor. In addition, the model incorporates time variation in the share of rents paid to taxes and interest, in the equity risk premium, and in the expected future path of interest rates over both the near- and long-term. We apply this model to the NFCS over the period 1952:Q1-2017:Q4.

Our main results may be summarized as follows. First, we find that neither economic growth, risk premia, nor short term interest rates has been the foremost driving force behind the market's sharp gains over the last several decades. Instead, the single most important factor has been a *factors share* shock that reallocates the rewards of production without affecting the size of those rewards. Our estimates imply that the realizations of this shock persistently reallocated rents to shareholders and away from labor compensation, to such an extent that they account for 54% of the market increase since 1989 and 36% over the full sample. Second, equity values were also boosted since 1989 by persistently declining interest rates and a decline in risk premia, but these factors had much smaller quantitative effects, with each contributing 11% to the increase in stock values. Third, growth in the real value of what was produced by the sector contributed just 23% to the increase in equity values since 1989 and 50% over the full sample. By contrast, from 1952 to 1988, economic growth accounted for 92% of the rise in equity values, but that 37 year period created less than half the equity wealth generated over the 29 years since 1989. Fourth, taxes and interest are estimated to have played a negligible role in equity market fluctuations throughout the sample.

An implication of these findings is that the high returns to holding equity over the post-war period have been in large part attributable to good luck, driven primarily by a string of favorable factors share shocks that reallocated rents to shareholders. We estimate that roughly 2.1 percentage points of the post-war average annual log return on equity in excess of a short-term interest rate is attributable to this string of favorable shocks, rather than to genuine compensation for bearing risk. These results imply that the common practice of averaging return, dividend, or payout data over the post-war sample to estimate an equity risk premium is likely to overstate the true risk premium by about 50%.

The rest of this paper is organized as follows. The next section discusses related literature.

Section 3 describes the theoretical model. Section 4 describes the econometric procedure and data. Section 5 presents our findings. Section 6 concludes.

2 Related Literature

The asset pricing literature has traditionally focused on explaining stock market expected returns, typically measured over monthly, quarterly or annual horizons. Surprisingly little attention has been given to understanding what drives the real (adjusted for inflation) level of the stock market, i.e., stock price variation. Two papers concerned with these questions and those closest to this one are Lettau and Ludvigson (2013) and our previous paper entitled "Origins of Stock Market Fluctuations," (Greenwald, Lettau, and Ludvigson (2014), GLL), which this paper supplants. These papers emphasized the relevance of factors share shocks in the data for explaining stock market values, but they differ in a number of substantive ways from the present study. Lettau and Ludvigson (2013) was a purely empirical exercise that investigated three shocks from a VAR, while GLL presented a model of the stochastic discount factor (SDF) to interpret these VAR shocks. But neither paper undertook a formal estimation of an equity pricing model, and the theoretical framework in GLL was less general and less flexible than that of this paper. In this paper, and in contrast to GLL, we estimate the full dynamic equity pricing model over time, as well as the latent states. The model of the SDF in this paper also adds additional state variables not present in the model of GLL that allow for low- and high-frequency components to time-varying interest rates, low- and high-frequency fluctuations in the share of rents accruing to shareholders, as well as a time-varying tax and interest share. Finally, the model in this paper also does away with a commonplace but implausible assumption that cash payments to shareholders are equal to earnings, by allowing for reinvestment.

Like GLL and Lettau, Ludvigson, and Ma (2018), the model of this paper adopts a heterogeneous agent perspective characterized by two types of agents and imperfect risk sharing between them: wealth is concentrated in the hands of a few investors, or "shareholders," while most households are "workers" who finance consumption primarily out of wages and salaries. This aspect adds an important element of realism to the model, since only about half of households report owning stocks either directly or indirectly in 2016. More importantly, even among those households that own equity, most own very little: the top 5% of the stock wealth distribution owns 76% of the stock market value and earns a relatively small fraction of income as labor compensation.[3] In this sense our model relates to a classic older literature emphasizing the importance for stock pricing of limited stock market participation and heterogeneity (Mankiw (1986), Mankiw and Zeldes (1991), Constantinides and Duffie (1996), Vissing-Jorgensen (2002), Ait-Sahalia, Parker, and Yogo (2004), Guvenen (2009), and Malloy, Moskowitz, and Vissing-Jorgensen (2009)). In contrast to this literature, our results suggest the relevance of frameworks in which investors are concerned about shocks that have opposite effects on labor and capital. Such redistributive shocks play no role in the traditional limited participation literature.

Besides Lettau and Ludvigson (2013), GLL, and Lettau, Ludvigson, and Ma (2018), a growing body of literature considers the role of redistributive shocks in asset pricing or macro models, most in representative agent settings (Danthine and Donaldson (2002); Favilukis and Lin (2013a, 2013b, 2015), Gomez (2016), Marfe (2016), Farhi and Gourio (2018)). In this literature, labor compensation is a charge to claimants on the firm and therefore a source of cash-flow variation in stock and bond markets. In contrast to the limited participation/heterogeneous agent paradigm pursued here, representative agent models imply that a variant of the consumption CAPM using aggregate consumption still prices equity returns, so those frameworks cannot not account for the evidence in Lettau, Ludvigson, and Ma (2018) that the capital (i.e., nonlabor) share of aggregate income exhibits significant explanatory power for expected returns across a range of equity characteristic portfolios and non-equity asset classes.

[3]Source: 2016 Survey of Consumer Finances (SCF). In the 2016 SCF, 52% of households report owning stock either directly or indirectly. Stockowners in the top 5% of the net worth distribution had a median wage-to-capital income ratio of 27%, where capital income is defined as the sum of income from dividends, capital gains, pensions, net rents, trusts, royalties, and/or sole proprietorship or farm. Even this low number likely overstates traditional worker income for this group, since the SCF and the IRS count income paid in the form of restricted stock and stock options as "wages and salaries." Executives who receive substantial sums of this form would be better categorized as "shareholders" in the model below, rather than as "workers" who own no (or very few) assets.

The factors share element of our paper is related to a separate macroeconomic literature that examines the long-run variation in the labor share (e.g., Karabarbounis and Neiman (2013), and the theoretical study of Lansing (2014)). The factors share findings in this paper also echo those from previous studies that use very different methodologies but find that returns to human capital are negatively correlated with those to stock market wealth (Lustig and Van Nieuwerburgh (2008); Lettau and Ludvigson (2009); Chen, Favilukis, and Ludvigson (2014))).

Farhi and Gourio (2018) extend a representative agent neoclassical growth model to allow for time varying risk premia and study the sources of macro-finance trends in recent data. They find a large role for rising market power in the high returns to equity, echoing our findings regarding the importance of the factors share shock for driving equity values over the post-war period. An appealing feature of their approach is that it specifies a structural model of production that takes a firm stand on the sources of variation in the earnings share, attributable in this case to time-varying markups in a standard oligopolistic competition, constant returns to scale framework. But its fair to say that the literature has not yet reached a consensus on the key structural features of the data that drive variation over time in the aggregate labor/profit share (e.g., see the differing explanations in Autor, Dorn, Katz, Patterson, and Van Reenen (2017), Hartman-Glaser, Lustig, and Xiaolan (2016), and Kehrig and Vincent (2018)). Our modeling and estimation approach obviates the need to take a stand on why the corporate earnings share has fluctuated the way it has over time, by furnishing estimates that match the observed earnings share exactly (i.e., without error) over the sample and at each point in time. We also model and estimate full transition dynamics for each factor that drives asset returns, rather than making inferences by estimating parameters assumed to be fixed over time on different subsamples of the data and then making comparisons of parameters across subsamples. It is this aspect of the approach that allows us to precisely decompose the role of each component source for explaining all of the observed variation in equity values over our sample and at each point in time.

3 The Model

The economy is populated by a representative firm that produces aggregate output, and two types of households. The first type are investors who typify those that own the majority of equity wealth in the U.S. These could be wealthy households or large institutional investors. They may borrow and lend amongst themselves in the risk-free bond market. We refer to these investors simply as "shareholders." The second type are hand-to-mouth "workers" who finance consumption out of wages and salaries. The model is stylized, as we suppose that workers own no assets and consume their labor earnings. Although taken literally these assumptions inaccurately describe the real world, we argue that they are a good first approximation of the data given the high concentration of wealth at the top, evidence that the wealthiest earn the overwhelming majority of their income from ownership of assets or firms and that households outside of the top 5% of the stock wealth distribution own far less financial wealth of any kind.[4]

Aggregate output is governed by a constant returns to scale process:

$$Y_t = A_t N_t^{\alpha} K_t^{1-\alpha}, \tag{1}$$

where A_t is a mean zero factor neutral total factor productivity (TFP) shock, N_t is the aggregate labor endowment (hours times a productivity factor) and K_t is input of capital, respectively. Workers inelastically supply labor to produce output. Capital grows deterministically at a gross rate $G = 1 + g$, which permits deterministic growth of labor productivity at rate G. Hours of labor supplied are fixed and normalized to unity. Taken together, these assumptions imply that $Y_t = A_t G^{t\alpha} K_0 G^{t(1-\alpha)} = A_t K_0 G^t$, where K_0 is the fixed initial value of the capital stock.

In the accounting framework of the data, a fraction τ_t of Y_t is devoted to taxes and interest (and a catchall of "other" charges against earnings). We refer to τ_t simply as the "tax and

[4]See discussion above. In the 2016 SCF, the median household in the top 5% of the stock wealth distribution had $2.97 million in nonstock financial wealth. By comparison, households with no equity holdings had median nonstock financial wealth of $1,800, while all households (including equity owners) in the bottom 95% of the stock wealth distribution had median nonstock financial wealth was $17,480. Additional evidence is presented in Lettau, Ludvigson, and Ma (2019).

interest" share for brevity. The remaining $1-\tau_t$ is divided between labor compensation and after-tax profits (earnings). Labor compensation in the model is equal to W_tN_t, where W_t is an aggregate wage per unit of productivity. Define $Z_t \equiv 1-\tau_t$ and let S_t denote the after-tax profit share of combined after-tax profit and labor compensation. Since the share of combined after-tax profit and labor compensation in total output is S_t, total earnings E_t are identically equal to[5]

$$E_t \equiv S_t Z_t Y_t. \tag{2}$$

Thus labor compensation is identically equal to

$$W_t N_t \equiv (1 - S_t)\, Z_t Y_t,$$

where $(1-S_t)$ is the labor share of combined after-tax profits and labor compensation. We refer to E_t/Y_t as the earnings share and $(W_tN_t)/Y_t$ as the labor share. Although in principle these shares can be influenced by fluctuations in the tax/interest share, we shall see that, in the data, the vast majority of variation in these shares is attributable to other sources, captured here by S_t.

The variable S_t is modeled as an exogenous stochastic process with an innovation that we shall refer to as a *factor share shock*. This shock captures changes that may occur for any reason in the allocation of rewards between firms and workers in an imperfectly competitive environment, while holding fixed the size of those rewards. Possible sources of variation in S_t could include changes in industry concentration structure that alter the labor intensivity of aggregate production, changes in the bargaining power of U.S. workers due to international competition the prevalence of worker unions, an increase in offshoring, or technological factors that alter how substitutable are labor and capital inputs.[6]

[5] This identity follows the NIPA accounting for the nonfinancial corporate sector.

[6] A literature has evolved to explain the decline in the aggregate labor share in the past 30 years. Some have argued that the aggregate labor share has fallen due to a reallocation of value added toward a few "superstar" firms with low labor shares (e.g., Autor, Dorn, Katz, Patterson, and Van Reenen (2017) Hartman-Glaser, Lustig, and Xiaolan (2016); Kehrig and Vincent (2018)). In our model this would show up as a decline in $1-S_t$.

In summary, earnings can vary over time for three reasons. First, the level of output and economic growth may change, which affects the size of the economic pie Y_t. Second, shifts in S_t alter the allocation of rewards between shareholders and workers independently from the magnitude of Y_t. Third, the tax and interest wedge Z_t may vary over time.

The firm makes cash payments to shareholders, which differs from earnings by net new investment. Net new investment is required to attain long term (steady state) growth in output. To achieve this, the firm reinvests a fixed fraction ωY_t of current output each period, so that the remaining funds left over for shareholders are given by

$$C_t = E_t - \omega Y_t = (S_t Z_t - \omega) Y_t. \tag{3}$$

The variable C_t is net payout, defined as the sum of net dividend payments and net equity issuance. It encompasses any cash distribution to shareholders including share repurchases, which have become the dominant means of returning cash to shareholders in the U.S. For brevity, we shall refer to these payments more simply as shareholder "cashflows."

This model of reinvestment implicitly presumes that the firm has access to a technology or market that allows it to swap the stochastic stream ωY_t for a fixed increment g to the net growth rate of Y_t forever. More generally, it is a simple way to capture the empirical fact that firms in aggregate retain part of their revenue for reinvestment, so that C_t is invariably less than E_t on average.

Let C_{it}^s denote the consumption of an individual stockholder indexed by i at time t. Identical shareholders maximize the function

$$U = E \sum_{t=0}^{\infty} \prod_{k=0}^{t} \beta_k u\left(C_{it}^s\right) \tag{4}$$

with

$$u\left(C_{it}^s\right) = \frac{\left(C_{it}^s\right)^{1-x_{t-1}}}{1-x_{t-1}}, \tag{5}$$

where β_t is a time-varying subjective time discount factor with $\beta_0 = 1$. An important aspect of these preferences is that the parameter x_t is not constant but instead varies stochastically over time. This variable, which can be thought of as a time-varying sentiment or preference shifter that shareholders take as given, drives the price of risk in the stochastic discount factor (SDF). Shareholder preferences are also subject to an externality in the subjective discount factor β_t. A time-varying specification for the subjective time discount factor is essential for obtaining a stable risk-free rate along with a volatile equity premium. If instead the subjective time discount factor were itself a constant, shocks to x_t and cashflow growth would generate counterfactual volatility in the risk-free rate.

Worker preferences play no role in asset pricing since they hold no assets. We assume that equities are priced by a representative *shareholder* who owns the entire corporate sector. In equilibrium, identical shareholders will have identical consumption, equal to per capita aggregate cashflows.[7] This should be distinguished from the more common approach of modeling a representative *household* in which aggregate consumption is the source of systematic risk. In the model of this paper, aggregate per capita shareholder cashflows are the appropriate source of systematic risk. We therefore drop the i subscript and simply denote the consumption of a representative shareholder $C_t^s = C_t$ from now on.

The intertemporal marginal rate of substitution of shareholder consumption is the SDF and takes the form

$$\begin{aligned} M_{t+1} &= \beta_t \left(\frac{C_{t+1}}{C_t} \right)^{-x_t} \\ \ln M_{t+1} &= -\mathbf{1}'\delta_t - d_t - x_t \Delta \ln C_{t+1} \end{aligned} \tag{6}$$

where the subjective time discount factor $\beta_t \equiv \frac{\exp(\delta_t)}{\exp(d_t)}$ is specified below. This SDF is a more general version of the lognormal models considered in previous work (e.g., Campbell and Cochrane (1999)) and (Lettau and Wachter (2007)). As in these models, the preference shifter x_t is taken as an externality (akin to an external habit) that is the same for each stockowner and represents the market's willingness to bear risk.

The preference shifter x_t is a latent state variable that will be estimated. We allow it to vary independently of the macroeconomic state and shareholder fundamentals. Since an SDF

[7] This does not mean that individual shareholders are hand-to-mouth households. They may borrow and save in the risk-free bond and could have idiosyncratic investment income drawn from an identical distribution. But they are assumed to be able to perfectly share any identical idiosyncratic risk with other shareholders so that, in equilibrium, they each consume per capita aggregate shareholder cashflowsC_t. See the Appendix for a simplified model.

always reflects both preferences and beliefs, an increase in x_t may be thought of as either an increase in effective risk aversion or an increase in pessimism about market fundamentals. The variable may occasionally go negative, reflecting the possibility that investors sometimes behave in a confident or risk tolerant manner. This does not imply a negative equity risk premium, however, since investors in the model can occasionally behave in a risk tolerant manner while still being averse to risk on average. Indeed, our estimates imply a substantial positive equity risk premium over samples of the size currently available.

3.1 A Loglinear Model

We work with a loglinear approximation of the model that can be solved analytically. Throughout the rest of the paper we use lower case letters denote log variables, e.g., $\ln(Y_t) = y_t$.

A loglinear model facilitates estimation by permitting the resulting system of equations to be written in state space form so that the Kalman filter may be used to infer unobserved states. We have also taken the alternative approach of solving the non-linear model numerically and using the Hamilton filter in conjunction with a large number of discrete states to estimate it. (This alternative approach is presented in the Appendix.) Solving the nonlinear model obviates the need for two linear approximations, one to obtain a linear expression for the log return (Campbell and Shiller (1989)), and another to obtain a linear expression for log payout. It can also be used to restrict the earnings share from going above unity. The results using this alternative methodology are very similar to those using the log linear model, because the approximation to the dynamics of returns and payout are quite accurate and because, although theoretically possible, at our estimated parameter values the earnings share in practice never exceeds unity even in very long simulations of the model.[8]

The model we explore has six latent state variables and seven latent shocks whose evo-

[8]These statements are based on simulations of the model at the posterior mode parameter values and simulations of length 10,000 periods. The lower bound of zero is never violated because the specified stochastic process for the earnings share is in log units. The upper bound is effectively never violated because the mode estimate of the mean of the log earnings share is more than 3.6 standard deviations in absolute terms away from zero.

lution are described below. These state variables include a factors share process S_t that we specify as subject to two shocks due to low- and high-frequency components, a subjective time-discount factor process δ_t that we specify as subject to two shocks due to low- and high-frequency components, a latent price of risk process x_t, and the tax and interest share process Z_t. Although data on taxes and interest are directly observable, we filter these data to estimate a latent stochastic process for Z_t, since the equilibrium asset returns in the model depend not just on today's tax and interest share, but on the entire expected future path of τ_t. In addition to shocks to each of these latent state components, a seventh latent shock is the i.i.d. innovation to TFP growth.

All shocks are modeled as Gaussian and independent over time. In order to precisely decompose the market's growth into distinct component sources, we directly specify the shocks to the latent state variables as mutually uncorrelated. This does not mean that there can be no correlated components among the latent state variables, it merely means that the shocks we identify represent only the mutually uncorrelated components. If the mutually uncorrelated components are negligible, our econometric procedure is free to estimate a small or even zero variance parameter for the mutually uncorrelated shocks. The estimates below therefore give a sense of the quantitative importance of the orthogonal pieces of the latent states.

3.1.1 Earnings and Cashflow Growth

We model TFP as a mean-zero unit root process in log levels, implying that the log difference is independently and identically distributed (i.i.d.):

$$\Delta a_{t+1} = \varepsilon_{a,t+1}, \quad \varepsilon_{a,t+1} \sim N\, i.i.d. \left(0, \sigma_a^2\right). \tag{7}$$

Since $Y_t = A_t K_0 G$, we have $y_t = a_t + k_0 + g$ and log output growth is

$$\Delta y_{t+1} = g + \varepsilon_{a,t+1}. \tag{8}$$

Since earnings $E_t = S_t Z_t Y_t$, log earnings growth is

$$\Delta e_t = \Delta s_t + \Delta z_t + \Delta y_t.$$

Define a variable $Q_t \equiv S_t Z_t$. Cash payments to shareholders are $C_t = (Q_t - \omega) Y_t$, or in log terms $c_t = \ln(Q_t - \omega) + y_t$. Loglinearizing the first term around $q_t = \ln(Q_t)$, we obtain an approximate equation for log payout $c_t = \bar{c} + \xi(s_t + z_t) + y_t$, where $\xi = \frac{\overline{SZ}}{\overline{SZ} - \omega}$ and $\overline{SZ}$ is the average value of $S_t Z_t$. Thus log payout growth is given by

$$\Delta c_t = \xi(\Delta s_t + \Delta z_t) + \Delta y_t.$$

An inspection of the time series of the observed corporate earnings share plainly suggests that it contains both lower and higher frequency sources of variation (as shown below in several figures). We accommodate this in the model by allowing growth in the earnings share to contain two components that are multiplicative in levels or linear in logs. One log component, $s_{LF,t}$ will be subject to lower frequency variation compared to the other, less persistent, component, denoted $s_{HF,t}$. Thus the total log earnings share is the sum of two components, i.e., $s_t = \mathbf{1}'(s_{LFt}, s_{HFt})'$, where $\mathbf{1}' \equiv (1, 1)$.

Denote a vector $\mathbf{s}_t = (s_{LFt}, s_{HFt})'$. We specify the dynamics of cashflow and earnings share growth with the following equations:

$$\Delta c_{t+1} = \xi \mathbf{1}' \Delta \mathbf{s}_{t+1} + \xi \Delta z_{t+1} + \Delta y_{t+1} \tag{9}$$

$$\mathbf{s}_{t+1} = (\mathbf{I} - \mathbf{\Phi}_s)\bar{\mathbf{s}} + \mathbf{\Phi}_s \mathbf{s}_t + \boldsymbol{\varepsilon}_{s,t+1}, \qquad \boldsymbol{\varepsilon}_{s,t+1} \sim N(\mathbf{0}, \mathbf{\Sigma}_s) \tag{10}$$

$$\Delta \mathbf{s}_{t+1} = -(\mathbf{I} - \mathbf{\Phi}_s)\widetilde{\mathbf{s}}_t + \boldsymbol{\varepsilon}_{s,t+1}, \qquad \widetilde{\mathbf{s}}_t \equiv \mathbf{s}_t - \bar{\mathbf{s}} \tag{11}$$

where $\mathbf{I}$ is the 2×2 identity matrix, $\mathbf{\Phi}_s$ is a 2×2 diagonal matrix with the first-order autocorrelation coefficient of each $\mathbf{s}_t$ element in the diagonal entries, $\bar{\mathbf{s}}$ is a vector containing the means of the two components, and $\mathbf{\Sigma}_s$ is a diagonal covariance matrix. We model an AR(1) process for z_{t+1} in an analogous way with a persistence parameter ϕ_z:

$$z_{t+1} = (1 - \phi_z)\bar{z} + \phi_z z_t + \varepsilon_{z,t+1}, \quad \varepsilon_{z,t+1} \sim N\, i.i.d.\left(0, \sigma_z^2\right).$$

The specification of Gaussian shocks implies that the level of the earnings share can occasionally go above unity. As mentioned, however, the estimated parameters of the $\mathbf{s}_{t+1}$ process imply that these shares go above unity less than one percent of the time in a long simulation of the model. The Appendix of the paper presents results for a nonlinear version of the model that prevents the share process from ever going above unity. Since the shares rarely go above unity in the linear model, we find the results to be very close to those presented below.

3.1.2 Stochastic Discount Factor and Risk Free Rate

The log of the SDF is an affine function of log cashflow growth times the price of risk, but also depends on the subjective time discount factor $\beta_t \equiv \frac{\exp(\delta_t)}{\exp(d_t)}$. Variation in the subjective time discount factor affects the return on the risk-free asset whose value is known with certainty at time t and given in gross units by

$$R_{f,t+1} \equiv \left(\mathbb{E}_t\left[M_{t+1}\right]\right)^{-1}.$$

An inspection of the data on short rates of interest, e.g., the three-month Treasury bill (T-bill) rate, suggest that short rates also contain both lower and higher frequency sources of variation. As above, we accommodate this in the empirical model by allowing δ_t to contain two components that are multiplicative in levels or linear in logs, i.e., $\delta_t = \mathbf{1}'\boldsymbol{\delta}_t$, where $\boldsymbol{\delta}_t = (\delta_{LFt}, \delta_{HFt})'$. One component, δ_{LFt}, will be subject to lower frequency variation compared to the other less persistent component, denoted δ_{HFt}. We assume $\boldsymbol{\delta}_{t+1}$ follow a multivariate Gaussian process so that the log SDF becomes

$$\begin{aligned} m_{t+1} &\equiv \ln M_{t+1} = -\mathbf{1}'\boldsymbol{\delta}_t - d_t - x_t \Delta c_{t+1} \qquad (12) \\ \boldsymbol{\delta}_{t+1} &= (\mathbf{I} - \boldsymbol{\Phi}_\delta)\bar{\boldsymbol{\delta}} + \boldsymbol{\Phi}_\delta \boldsymbol{\delta}_t + \boldsymbol{\varepsilon}_{\delta,t+1},\ \boldsymbol{\varepsilon}_{\delta,t+1} \sim N(0, \boldsymbol{\Sigma}_\delta), \end{aligned}$$

where $\boldsymbol{\Phi}_r$ is a 2×2 diagonal matrix with the first-order autocorrelation coefficient of each component in the diagonal entries, $\bar{\boldsymbol{\delta}}$ is the vector of means of the two components of $\boldsymbol{\delta}_t$ and $\boldsymbol{\Sigma}_\delta$ is a diagonal covariance matrix. The stochastic process $\boldsymbol{\delta}_t$ is a shock to the subjective time-discount factor that moves the risk-free rate independently of cashflow growth variation. The parameter d_t is a compensating factor chosen to ensure a log risk-free rate $r_{f,t} = -\ln \mathbb{E}_t \exp(m_{t+1})$ of

$$r_{f,t} = \mathbf{1}'\boldsymbol{\delta}_t.$$

With Gaussian shocks, the SDF is conditionally lognormal, which implies that the log risk-free rate takes the form

$$r_{f,t+1} = \mathbf{1}'\boldsymbol{\delta}_t + d_t + x_t \left[g - \xi \left(\phi_z \tilde{z}_t + \mathbf{1}'(\mathbf{I} - \boldsymbol{\Phi}_s)\widetilde{\mathbf{s}}_t\right)\right] - \frac{1}{2} x_t^2 \left(\sigma_a^2 + \xi \left(\mathbf{1}'\boldsymbol{\Sigma}_s \mathbf{1}\right)\right).$$

It follows that

$$d_t = -x_t \left[g - \xi \left(\phi_z \tilde{z}_t + \mathbf{1}'(\mathbf{I} - \boldsymbol{\Phi}_s)\widetilde{\mathbf{s}}_t\right)\right] + \frac{1}{2} x_t^2 \left(\sigma_a^2 + \xi \left(\mathbf{1}'\boldsymbol{\Sigma}_s \mathbf{1}\right)\right).$$

The SDF also depends on the price of risk state variable x_t, which we assume follows a first-order autoregressive (AR(1)) process:

$$x_{t+1} = (1 - \phi_x)\,\overline{x} + \phi_x x_t + \varepsilon_{x,t+1}, \quad \varepsilon_{x,t+1} \sim N\, i.i.d. \left(0, \sigma_x^2\right).$$

The log SDF (12) has the implication that growth in the consumption of *shareholders*, rather than all households, is the fundamental source of systematic risk for shareholders. Unlike most models, redistributive shocks that shift the share of income between labor and capital shift shareholder consumption and are a source of systematic risk for asset owners. This implication has been explored by (Lettau, Ludvigson, and Ma (2019)) who study risk pricing in a large number of cross-sections of return premia.

3.1.3 Equilibrium Stock Market Values

Let P_t denote total market equity, i.e., price per share times shares outstanding. Then with C_t equal to total equity payout, we write the return on equity from the end of t to the end of $t+1$ as

$$R_{t+1} = \frac{P_{t+1} + C_{t+1}}{P_t}.$$

Define $pc_t \equiv \ln\left(\frac{P_t}{C_t}\right)$. The log return obeys the following approximate identity (Campbell and Shiller (1989)):

$$r_{t+1} = \kappa_0 + \kappa_1 pc_{t+1} - pc_t + \Delta c_{t+1}, \tag{13}$$

where $\kappa_1 = \exp\left(\overline{pc}\right) / \left(1 + \exp\left(\overline{pc}\right)\right)$, and $\kappa_0 = \exp\left(\overline{pc}\right) + 1 - \kappa_1 \overline{pc}$.

The first-order-condition for optimal shareholder consumption implies the following Euler equation:

$$\frac{P_t}{C_t} = \mathbb{E}_t \exp\left[m_{t+1} + \Delta c_{t+1} + \ln\left(\frac{P_{t+1}}{C_{t+1}} + 1\right)\right]. \tag{14}$$

The relevant state variables for the equilibrium pricing of equity are the two components of s_t (low and high frequency), the two components of δ_t (low and high frequency), the price of risk state variable x_t, and the taxes/interest share state variable Z_t. We conjecture a solution to (14) taking the form

$$pc_t = A_0 + \mathbf{A}_s'\widetilde{\mathbf{s}}_t + \mathbf{A}_r'\widetilde{\boldsymbol{\delta}}_t + A_x\tilde{x}_t + A_Z\tilde{z}_t, \tag{15}$$

where $\mathbf{A}_s'$ and $\mathbf{A}_r'$ are 1×2 vectors and "tildes" indicate deviations from the mean. The solution verified in the Appendix implies that the coefficients on these state variables take the form

$$\begin{aligned}
\mathbf{A}_s' &= -\xi\mathbf{1}'(\mathbf{I} - \mathbf{\Phi}_s)(\mathbf{I} - \kappa_1\mathbf{\Phi}_s)^{-1} \\
A_x' &= -\left[\left(\xi^2\left(\mathbf{1}'\mathbf{\Sigma}_s\mathbf{1} + \sigma_Z^2\right) + \sigma_g^2\right) + \xi\kappa_1\left(A_s'\mathbf{\Sigma}_s\mathbf{1} + A_z\sigma_Z^2\right)\right](1 - \kappa_1\phi_x)^{-1} \\
\mathbf{A}_\delta' &= -\mathbf{1}'\left(\mathbf{I} - \kappa_1\mathbf{\Phi}_\delta\right)^{-1} \\
A_z &= -\xi(1 - \phi_z)(1 - \kappa_1\phi_z)^{-1}
\end{aligned}$$

All terms on the left-hand-side are negative. The coefficients $\mathbf{A}_\delta'$ and A_x' imply that an increase in the risk-free rate or an increase in the price of risk x_t reduces the price-cashflow ratio because either event increases the rate at which future payouts are discounted. The size of these effects depend on the persistence of the movements in the risk-free rate and the price of risk, as captured by $\mathbf{\Phi}_\delta$ and ϕ_x. The more persistent the shocks, the larger the effects. The elements of $\mathbf{A}_s'$ are also both negative, since the elements of $\mathbf{\Phi}_s$ are both less than unity. Thus any increase in the earnings share, while potentially persistent, is ultimately transitory and translates into a transitory increase in cashflows to shareholders. In equilibrium, equity values rise proportionally less than current cashflows in anticipation of eventual mean reversion in payout, so pc_t is negatively related to changes in $\widetilde{\mathbf{s}}_t$. The size of these effects depend on the persistence of the $\widetilde{\mathbf{s}}_t$ process, captured by the elements of $\mathbf{\Phi}_s$, with more persistent affects translating into smaller movements in pc_t, where the effects approach zero as the elements of $\mathbf{\Phi}_s$ approach unity.

As shown in the Appendix, the model solution implies that the log equity premium is given by

$$\mathbb{E}_t[r_{t+1}] - r_{f,t} = \left[\left(\xi^2\left(\mathbf{1}'\mathbf{\Sigma}_s\mathbf{1}+\sigma_z^2\right)+\sigma_a^2\right)+\xi\kappa_1\left(\mathbf{A}_s'\mathbf{\Sigma}_s\mathbf{1}+A_z\sigma_z^2\right)\right]x_t - \frac{1}{2}\mathbb{V}_t(r_{t+1}),$$

$$\mathbb{V}_t(r_{t+1}) = \kappa_1^2\left(\mathbf{A}_s'\mathbf{\Sigma}_s\mathbf{A}_s + A_z^2\sigma_z^2 + A_x^2\sigma_x^2 + \mathbf{A}_\delta'\mathbf{\Sigma}_\delta\mathbf{A}_\delta\right) + \left[\xi^2\left(\mathbf{1}'\mathbf{\Sigma}_s\mathbf{1}+\sigma_z^2\right)+\sigma_a^2\right] + 2\xi\kappa_1\left(\mathbf{A}_s'\mathbf{\Sigma}_s\mathbf{1}+A_z\sigma_z^2\right),$$

where $\mathbb{V}_t(\cdot)$ denotes variance conditional on time t information. The conditional variance is constant due to homoskedasticity of the shocks, but the equity premium varies over time with the price of risk x_t.

4 Estimation and Data

The model just described consists of a vector of primitive parameters

$$\boldsymbol{\theta} = \left(\xi, g, \sigma_a^2, \operatorname{vec}\left(\mathbf{\Phi}_s\right), \operatorname{vec}\left(\mathbf{\Phi}_\delta\right), \phi_x, \phi_Z, \operatorname{vec}\left(\mathbf{\Sigma}_s\right), \operatorname{vec}\left(\mathbf{\Sigma}_\delta\right), \sigma_x^2, \sigma_z^2, \overline{s}, \bar{\delta}, \bar{x}, \overline{z}\right)',$$

where vec $(\cdot)$ denotes the vectorization of a matrix and variables with "bars" indicate means.

The parameters are divided into two groups. One group consists of a small number that are calibrated rather than estimated. These include $\bar{s}$, ξ and ϕ_x. The calibration of these parameters is discussed below. The second group consists of the remaining parameters that are freely estimated.[9]

We estimate the model's primitive parameters using Bayesian methods with flat priors. The latent states are recovered by a filtering algorithm and inferred jointly with the primitive parameters of the model. The model has six latent state variables: the two components each of $\mathbf{s}_t$ and $\boldsymbol{\delta}_t$, the risk price process x_t, and the tax and interest process $\ln Z_t$. Since the model is linear in logs, the Kalman filter may be used to estimate the latent states.

We use observations on five series: the log share of rents going to earnings (the earnings share) $e_t - y_t \equiv ey_t$, a measure of short term real interest rates as a proxy for the log risk-free rate $r_{f,t+1}$, observations on taxes and interest share of the NFCS embedded in $\ln Z_t$, growth

[9] Although $\mathbf{s}$ and $\boldsymbol{\delta}$ each have two components, we set the mean of one to zero and therefore only estimate or calibrate a single parameter for the mean of each.

in output for the NFCS as measured by net value added, and our target variable, the log market equity market to output ratio $p_t - y_t \equiv py_t$. The model implies that these observed variables are related to the primitive parameters and latent state variables according to the following system of equations:

$$\begin{aligned}
ey_t &= \mathbf{1}'(\tilde{\mathbf{s}}_t + \bar{\mathbf{s}}) \\
r_{ft} &= \mathbf{1}'(\widetilde{\boldsymbol{\delta}}_t + \bar{\boldsymbol{\delta}}) \\
py_t &= pc_t + cy_t \\
&= \overline{py} + (\mathbf{A}_s' + \xi\mathbf{1}')\,\widetilde{\mathbf{s}}_t + \mathbf{A}_r'\widetilde{\boldsymbol{\delta}}_t + A_x\tilde{x}_t + (A_z + \xi)\,\tilde{z}_t \qquad (16) \\
\tilde{z}_{t+1} &= \phi_T\tilde{z}_t + \varepsilon_{z,t+1} \\
z_t &= \tilde{z}_t + \overline{z} \\
\Delta y_t &= g + \Delta\tilde{y}_t
\end{aligned}$$

where $cy_t \equiv c_t - y_t$, and $\overline{py} \equiv A_0 + \bar{c} + \xi\mathbf{1}'\overline{\mathbf{s}} + \xi\overline{z}$. Note that the last two equations are identities that exactly pin down the values of $\varepsilon_{z,t}$ and $\varepsilon_{a,t}$.

The above equations may be written in state space form as follows:

$$\mathcal{Y}_t = \mathbf{H}'\boldsymbol{\beta}_t + \mathbf{G}'\mathbf{1} \qquad (17)$$

$$\boldsymbol{\beta}_t = \mathbf{F}\boldsymbol{\beta}_{t-1} + \mathbf{v}_t, \qquad (18)$$

where the observation vector $\mathcal{Y}_t \equiv (ey_t, r_{ft}, py_t, \Delta z_t, \Delta y_t)'$, the latent state vector $\boldsymbol{\beta}_t \equiv \left(\tilde{s}_{LF,t}, \tilde{s}_{HF,t}, \tilde{\delta}_{LF,t}, \tilde{\delta}_{HF,t}, \tilde{x}_t, \tilde{z}_t, \Delta\tilde{y}_t\right)'$, $\mathbf{v}_t = (\varepsilon_{s,LF,t}, \varepsilon_{s,HF,t}, \varepsilon_{\delta,LF,t}, \varepsilon_{\delta,HF,t}, \varepsilon_{x,t}, \varepsilon_{z,t}, \varepsilon_{a,t})'$, where $\mathbf{F}$, $\mathbf{H}'$, and $\mathbf{G}'$ are matrices of primitive parameters of dimensions (7×7), (5×7), and (5×1), respectively, and "LF" and "HF" denote the low- and high-frequency elements of the vectors $\widetilde{\mathbf{s}}_t$ and $\widetilde{\boldsymbol{\delta}}_t$.[10] Note that the i.i.d. shock $\varepsilon_{a,t}$ shows up in the "state equation"

[10] The matrices take the form

$$\mathbf{F} = \begin{bmatrix}
\phi_{s,LF} & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & \phi_{s,HF} & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & \phi_{\delta,LF} & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & \phi_{\delta,HF} & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & \phi_x & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & \phi_z & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0
\end{bmatrix}$$

(18) even though it is not a theoretical "state variable" that must be conditioned upon to solve (14), because it is latent and must still be inferred in estimation. Given values for the primitive parameters in the combinations $\mathbf{F}$, $\mathbf{H}'$, and $\mathbf{G}'$, we use the Kalman filter to obtain smoothed estimates of the state vector $\boldsymbol{\beta}_t$, which we denote $\boldsymbol{\beta}_{t|T}$.

The observation equation of the state space representation (17) could in principle be subject to measurement error. In our application, however, we are able to drive measurement error effectively to zero, because we use a sufficiently flexible loglinear model and because we fit observations on five variables with seven latent states. As a consequence, we can filter the five series to perfectly match the observations on the earnings share, the tax/interest share, the log real risk-free rate, log output growth Δy_t, and the market equity-to-output ratio $p_t - y_t$, comparable to an $\overline{R}^2$ of one in a linear regression setting. Moreover, because we perfectly match $p_t - y_t$ and Δy_t, we also perfectly match the growth in market equity Δp_t over time and at each point in time, a property we exploit when calculating the growth decompositions discussed below.

The posterior distribution of $\boldsymbol{\theta}$ is obtained by computing the likelihood using the Kalman filter and combining it with priors. Since we use flat priors, the posterior coincides with the likelihood and the posterior mode estimate of $\boldsymbol{\theta}$ coincides with the maximum likelihood estimate (MLE). Uncertainty about the parameters $\boldsymbol{\theta}$ is characterized using a random walk Metropolis-Hastings (RWMH) algorithm, while uncertainty about the latent state $\boldsymbol{\beta}_t$ is char-

$$\mathbf{H}' = \begin{bmatrix} 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 & 0 \\ A_{s,LF} + \xi & A_{s,HF} + \xi & A_{r,LF} & A_{r,HF} & A_x & (A_Z + \xi) & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\mathbf{G}' = \begin{bmatrix} \mathbf{1}'\overline{s} \\ \mathbf{1}'\overline{\delta} \\ \overline{py} \\ \overline{z} \\ g \end{bmatrix},$$

where $\phi_{\cdot,HF}$, and $\phi_{\cdot,LF}$ refer to the elements of the matrix $\boldsymbol{\Phi}_{\cdot}$ corresponding to the high- and low-frequency components, respectively.

acterized using the simulation smoother of Durbin and Koopman (2002). We use the RWMH algorithm to generate 110,000 draws of $\boldsymbol{\theta}$. We discard the first 10,000 draws and retain every 10th draw, leaving a total of 10,000 draws over which the posterior is computed. For every one of these 10,000 draws of parameters, we simulate one draw of the latent states using the simulation smoother. The plots below therefore reflect both parameter and latent state uncertainty.[11]

Our data consist of quarterly observations spanning the period 1952:Q1 to 2017:Q4. Researchers often examine joint trends in financial markets and aggregate economic quantities by using data on stock market indexes comprised of publicly traded stocks (such as the CRSP value-weighted index or the S&P 500 index), along with data on aggregate measures of output and the labor share (e.g., GLL, Farhi and Gourio (2018)). A weakness of this approach is that it pairs data on stock prices with data on output and labor compensation that are not directly relevant for one another because they do not pertain to the same sector of the economy. This creates the potential for confounding compositional effects over time. For example if publicly traded firms have experienced larger shifts in their labor shares and/or larger shifts in their output compared to non-public firms, movements in the *aggregate* quantities for output and labor compensation would not correctly describe the firms for which market equity is measured. (Aggregate quantities such as GDP and the labor share data from the Bureau of Economic Analysis (BEA) includes estimates for privately held firms and firms that have not issued equity all.) To address the empirical questions posed by our investigation it is important that earnings, output, labor compensation, taxes and interest, and the market value of equity all pertain to the same sector of the economy. For this reason, we focus on the non-financial corporate sector (NFCS), where all of these variables can be measured directly for that sector. The use of the corporate sector also has the advantage that the labor share, $1 - S_t$, is not affected by the statistical imputation of labor income from total income reported by sole proprietors and unincorporated businesses.

[11] The latent state space includes components that differ according to their degree of persistence. With flat priors, a penalty to the likelihood is required to ensure that the low frequency component actually has greater persistence than the higher frequency component. This is accomplished setting a penalty that forces the likelihood to negative infinity if the parameter search wanders into a space where $\phi_{j,LF} \leq \phi_{j,HF}$.

We use total market equity for NFCS to measure p_t. Total output is observed as net value added for the sector and measures y_t. Output is measured in real, per capita terms. Labor compensation, earnings, taxes and interest are also directly observed for this sector. The real risk-free rate is measured as the three-month T-bill rate less the fitted value from a regression of inflation on lags of inflation. The Appendix provides a detailed description of these data and our sources. Unless otherwise noted, the sample spans the period 1952:Q1-2017:Q4.

There are three parameters that are calibrated rather than estimated. The first, ξ, relates payout growth to earnings growth according to (9). The National Income and Product Account (NIPA) estimate of net payout for the U.S. corporate sector is noisy and subject to large swings due to temporary factors such as changes in tax law.[12] For this reason, we avoid targeting the post-war sequence of observations on payout and instead calibrate ξ to match the relative sample (unconditional) standard deviation of payout growth to earnings growth over the full sample, implying $\xi \approx 2$. The second calibrated parameter is $\bar{s}$, which contains the means of the factor share process. To make sure we exactly match the observed mean earnings share, we set the mean of the high-frequency component to zero and the mean of the low-frequency component to match the mean of after-tax profit share observed in the full sample of data, equal to 0.092. The third calibrated parameter is ϕ_x, which measures the persistence of the price of risk state variable x_t. Unlike the earnings/labor share and risk-free rate, there is no corresponding observable series for x_t that could be used to discipline its persistence in estimation. This is important because, if ϕ_x were freely estimated under a flat prior, the procedure just described would choose parameters of the $\mathbf{s}_t$ and $\boldsymbol{\delta}_t$ processes to exactly match the observed series on s_t and $r_{f,t}$, while setting the undisciplined free parameter ϕ_x to be whatever value is required to explain all of the variation in our target variable py_t. Since the requisite estimate of ϕ_x for that purpose is a value that is close to unity, this would imply an equity risk premium that is a unit root process, with the implausible

[12] For a recent example, see NIPA Table 4.1, which shows an unusually large increase in 2018:Q1 in net dividends received from the rest of the world by domestic businesses, which generated a very large decline in net payout. BEA has indicated that these unusual transactions reflect the effect of changes in the U.S. tax law attributable to the Tax Cut and Jobs Act of 2017 that eliminated taxes for U.S. multinationals on repatriated profits from their affiliates abroad.

implication that all shocks to risk premia are permanent. Any presumed permanent shock to the discount rate (emanating from either risk premia or the risk-free rate) will have extremely large effects on equity valuation ratios. But other evidence on the persistence of risk premia in equity markets suggests that permanent shocks are counterfactual. For example, the risk-premium consumption-wealth proxy cay_t (Lettau and Ludvigson (2001)) has a first-order autocorrelation coefficient of about 0.90 in recent data. Martin (2017) uses options data to estimate the stock market risk premium and finds that the risk-premium proxy (called the SVIX) has a first-order autocorrelation coefficient of 0.8.[13] For our baseline estimation, we therefore choose $\phi_x = 0.85$ as a happy medium between these estimates. We assess robustness to using $\phi_x = 0.80$ or 0.90 below. This calibration can be interpreted as the imposition of a dogmatic prior that the equity risk premium is not so persistent that it is nearly a unit root random variable.

5 Results

We begin our discussion of the results by returning to Figure (1), which shows that the ratio of market equity (ME) for the NFCS relative to several measures of macroeconomic activity have trended upward since about 1989, and risen sharply since 2009. The most apt comparison between equity values and aggregate economic activity is given by the ratio of ME for the NFCS to output for the NFCS, since this relates market equity to economic activity for the same economic sector. Of the three series, this ratio has risen the most over time.

[13]The dividend yield as its usually measured from CRSP is slightly more persistent than cay_t (with autocorrelation around 0.93-0.95 depending on the sample) but there are at least two problems with using this variable to proxy for risk-premia fluctuations. First, for the last several decades firms have distributed cash to shareholders via repurchases rather than dividends. In this case, net payout is the appropriate measure of cashflows to shareholders. Larrain and Yogo (2008) show that the payout yield is substantially less persistent than the dividend yield, with an autocorrelation coefficient around 0.81. For our data on the NFCS, the payout yield has a first order autocorrelation coefficient of 0.82. Second, Lettau and Ludvigson (2005) find that the CRSP dividend yield omits important transitory components in risk premium variation that, when accounted for, imply that the persistence of the dividend yield is an inaccurate measure of the persistence of risk premia.

There were large decreases in these ratios during the tech bust in the stock market around 2000, and again during the Great Financial Crisis in 2008. But even these large declines were subsequently more than completely reversed, and hence the upward trend since 1989 has continued. The figure also shows the ratio of ME to after-tax profits (E) in the NFCS. This ratio was high just before the tech bust in 2000, but unlike that time, the most recent boom in equities has not been accompanied by a large increase in the price-earnings ratio for the sector. That is because earnings have grown much more quickly in the post-2000 period than it did in the tech boom years. These fundamental trends offer important clues to our empirical results discussed next.

5.1 Parameter Estimates

Table 1 presents the estimates of our primitive parameters based on the posterior distribution obtained with flat priors.

The mode estimate of the risk price parameter x_t is about 4.5, a modest value that reflects the volatility in cash payments to shareholders that are the source of systematic risk to shareholders in the model. Hence outsized aversion to risk or ambiguity is not needed to explain the data.

The persistence parameters of various components are of interest, since they determine the magnitude that changes in each latent component have on market equity values. The mode estimate of the high-frequency persistence process that drives the risk-free rate, $\phi_{\delta,HF} = 0.16$, whereas the persistence of the low-frequency component is estimated to be $\phi_{\delta,LF} = 0.93$. While the latter is clearly persistent, it is quite far from permanent. This explains why the large declines in real interest rates observed over the last 30 years are not, by themselves, likely be an important factor in the boom in equity values. Even though rates have declined, it is not today's rate but the expected future path of rates that matters for discount rates and equity values. This evidence implies that the low rates of recent years are highly unlikely to persistent indefinitely.

The mode estimates for the factors share persistence parameters are $\phi_{s,HF} = 0.9250$ and $\phi_{s,LF} = 0.9997$. These estimates indicate that factors share changes are substantially more persistent than interest rate changes, though still not permanent. The tax/interest share is also persistent, $\phi_Z = 0.95$, but relatively stable with a standard deviation of 0.018. Finally, the mode estimate of the volatility of the productivity shock (output growth) is 0.16, which is roughly equal to that of quarterly real personal consumption growth.

5.2 Asset Pricing Moments

Table 2 presents the model's implications for asset pricing moments and compares them to data for the NFCS. The columns labeled "Model" are computed using the mode parameter estimates and then simulating the model 1,000 times using a sample length equal to that of our historical sample. The asset pricing moments in the "Model" columns are averages across the simulations. The columns labeled "Fitted," compute moments using the mode parameter estimates but combine these with the estimated latent states obtained by fitting the model to the observed historical sample. These fitted moments are the model's implications for the asset pricing moments conditional on the observed sequence of shocks that actually generated the historical data, so they are directly comparable to the sample "Data" moments that are also reported in the table.

Table 2 shows that the model fitted moments do a reasonably good job of matching the sample moments for the equity premium and the log price-payout ratio (Log earnings growth, the log earnings share growth, and the risk-free rate are matched exactly by construction since we use these series as observables.) The moment that is modestly off is the fitted mean log excess return, which is 6.4% per annum and lower than the 7.3% per annum for the average excess return in historical data. The reason is that the model's fitted means for log payout growth and log payout share growth (which were not targets of our estimation), understate the observed sample means for the payout series, though the model gets the fitted volatilities about right. Despite this understatement, the fitted means are in the right ballpark of the actual payout data, which are highly volatile. Since no data on payout were used, this provides a check on the estimation and indicates that it is not producing a wildly inaccurate account of the data.

An important aspect of the results reported in Table 2 is that the model mean log equity premium is 4.3% per annum, whereas the fitted equity premium generated by the model is 6.4%. This large divergence is attributable to the good luck equities have enjoyed over the post-war period, driven primarily, according to our estimates, by a string of favorable factors share shocks that redistributed rents to shareholders. This is also reflected in the finding that the fitted means for earnings growth and payout growth are both substantially larger than the model means for these series. Moreover, since the earnings and payout shares have a constant mean, the mean of the growth in the shares in the model is zero, while the fitted model means for these series are estimated to be large and positive. Taken together, these estimates imply that roughly 2.1% of the post-war mean log return on stocks in excess of a T-bill rate is attributable to a string of favorable factors share shocks, rather than to genuine compensation for bearing risk. The findings present a cautionary tale for the common practice of using the sample mean excess return, (or components of the sample mean return such as the dividend-price ratio or dividend-earnings ratio, as suggested by identities exploiting the Gordon growth formula), to infer an equity risk premium, even over samples as long as that of the post-war period. The results here suggest that such sample averages overstate the true equity risk premium by about 50%.

5.3 Dynamics of Equity Values

The results in Table 2 tell us about the sample moments but are silent on the dynamic forces that have given rise to sharply increasing equity values over time in the sample. To consider these dynamic forces, it is instructive to study a graphical representation of our observed variables over time, juxtaposed by the estimated portion of variation attributable to each latent component in the model. Figures 2 through 7 present these results. These figures display the time variation in a series (e.g., the earnings share of output, the price-output ratio. etc.) along with the component attributable to different sources of variation. This is accomplished by either fixing one component at its mean during a particular period, or by allowing only one component to vary. The shaded areas around each estimated component source are 90% credible sets that take into account both parameter and latent state uncertainty.

Figure 2 shows the time-variation in the log earnings share ey_t over our sample, along with the portion of this variation attributable to each estimated factors share component s_t. Because we use the earnings share data as an observable and because there is no measurement error in the observation equations, these two components explain all of the variation in the earnings share over the sample and at each point in time. The plot shows that the log earnings share was high in the 1950s and 1960s, low in the 1970s and 1980s, and then began an upward trajectory starting around 1990 that continues to the end of the sample, interrupted only temporarily by the tech bust in 2000-2001. Panel (a) juxtaposes observations on ey_t with the model's implications for this series if only the high-frequency factors share component $s_{HF,t}$ were varying over our sample. ("Factor share (HF) Only" means the low frequency component is fixed at its estimated mean value $\overline{s}$). This high frequency component captures well the transitory variation in the earnings share, but cannot account for the full rise in the earnings share over the sample. Panels (b) shows that the low-frequency $s_{LF,t}$ component captures the longer term swings in the earnings share, which increases over the sample, and sharply since the year 2000.

Figure 3 shows the evolution of the risk-free rate over time, along with the portion of this variation attributable to the estimated risk-free rate components. Again, because we use the risk-free rate data as an observable and because there is no measurement error in the observation equations, these two components explain all of the variation in our measure of the risk-free interest rate over the sample and at each point in time. The figure shows that real rates were low at times in the 1950s and late 1970s, then rose under the Volcker disinflation around 1979 and remained elevated for over a decade before declining to their current low values. The high frequency component $\delta_{HF,t}$ picks up the transitory variation in the series while the low frequency component $\delta_{LF,t}$ captures the trend in this series. Panel (b) shows that fixing the high frequency component matters little for capturing the low-high-low pattern of interest rates, which is well fit by the low frequency component alone. Panel (c) confirms that the low frequency component has trended downward over time and especially since 1989. At the same time, it is important to observe that although rates are low today, they are not unusually so by historical standards. They were as low at times in the early part of the sample, a fact well captured by the $\delta_{LF,t}$ component, as exhibited in panel (c).

Our model and estimates may be used to quantify the importance of low recent real rates for driving equity values.

The next several figures decompose our target variable, the log market equity-to-output ratio, py_t, into sources of variation attributable to different latent components. Figure 4 shows the observed py_t series, which has a low frequency upward trend in it over our sample that is captured by the low frequency factors share component $s_{LF,t}$, as exhibited in panel (a). The high frequency component $s_{HF,t}$, produces some "wiggles" in the series but contributes nothing to the trend (panel (b)). Panel (c) shows that if we fix the low frequency factors share component at its value since 1952:Q1, the model is unable to capture any of the upward trajectory since about 1980. Panel (d) exhibits a similar pattern by showing the actual series since 1989 against what the model would imply if $s_{LF,t}$ were fixed at its value in 1989. Taken together the results show a large role for factors share shifts in driving upward the market value of equity relative to output.

Figure 5 shows the part of the observed equity value-to-output ratio variation that can be attributed to movements in the risk-free rate. Panels (a)-(d) show that the estimates attribute only a modest role to risk-free rate variation in explaining the rise in equity values relative to output over our sample. The last two panels show that shutting down either component does little to the model's ability to match the trend movements in py_t, which are driven primarily by factors share variation. Panel (d) does indicate that some role for lower risk-free rates in the last 30 years, especially since around 2000: by the end of the sample, the price-output ratio would be about two-tenths of a log point lower had there been no change in risk-free rates since 1989.

These findings may be related to those in Bianchi, Lettau, and Ludvigson (2016), who find evidence for a monetary policy role in low-frequency variation of short term interest rates, which is linked to low-frequency time-variation in the consumption-wealth variable cay_t. But they also find that this monetary policy component is correlated with risk-premia variation, such that lower policy rates are associated with a reach-for-yield that is associated with a lower present discouted value of future equity risk premia. These findings are therefore not directly comparable to those here, since we identify only the mutually uncorrelated components of risk-free rate and equity premium variation.

Figure 6 shows the part of the observed py_t variation that can be attributed to fluctuations in the one-step-ahead equity risk premium $\mathbb{E}_t[r_{t+1}] - r_{f,t}$, driven in the model by x_t. Panel (a) shows that risk premium variation explains almost all of the transitory booms and busts in equity values relative to output over our sample, including the technology boom/bust, the boom in equity values leading up to the financial crisis of 2008-2009, and the sharp decline in those values during the financial crisis. In particular, Panel (a) shows that an estimated decline in the risk premium explained almost all the boom in equity values during the run-up to the tech bust in 2000. (Recall from (15) and (16) that a decline in x_t translates into a *rise* in py_t.) But risk premium variation does not explain the trend component of py_t, as seen from Panel (b), which fixes x_t at its mean value. The longer term swings in py_t are captured by the factors share component. Panels (c) and (d) show what happens if x_t is fixed at the value it took either at the beginning of the sample, or at its value in 1989:Q1. From these figures we see that a small portion of the increase in ey_t is explained by a decline in x_t, since if the variable is fixed at its either its 1952 or 1989 value the model does not match the full increase in ey_t over those time periods.

These findings on the equity risk premium may be compared with those of Farhi and Gourio (2018), who conclude that it has increased since about the year 2000. Panel (a) of Figure 6 indicates that we also find higher risk premia compared to the year 2000. This panel shows that, if the risk premium were the only factor that varied over our sample, the price-output ratio would be lower today than it was in 2000, indicating that risk premia are higher today than they were at the peak of the tech boom. Panel (b) reinforces this conclusion by showing that, py_t would have risen more since 2000 (starting from a lower level) had risk premia been fixed over the sample. Nevertheless, an inspection of the time series of estimated one-step-ahead equity risk premium values (not shown) indicates that, while the equity premium is still higher than the record low values it took at the height of the tech boom, is quite low by historical standards at the end of the sample in 2017:Q4.

Finally, Figure 7 shows the contribution of the tax/interest share component to the dynamics of ey_t. All panels indicate that variation in this component has had a negligible effect on equity values relative to output over time.

5.4 Growth Decompositions

In this section we quantify the importance of differing drivers of equity values over the post-war period by calculating a set of growth decompositions that decompose the total growth in equity values into distinct sources attributable to each latent state variable in the model. The contributions are computed by taking the total growth in the target variable and dividing it into parts attributable to only a single component (fixing all other components at their values at the beginning of the sample). By construction, these components sum to 100% of the observed variation in equity values, since the model along with the fitted latent components is able to perfectly match at each point in time the observed log market equity-to-output ratio, py_t, as well as output growth Δy_t. Panel A of Table 3 presents the decompositions for the total change in the real log of market equity p_t either over the whole sample or over the period since 1989, while Panel B presents the same decompositions for the total change in py_t.[14]

The estimates reported in Panel A of Table 3 indicate that about 54% of the market increase since 1989 and 36% over the full sample is attributable to the sum of the two factors share components $s_{LF,t}$ and $s_{HF,t}$, with the vast majority of this coming from the low frequency component. Over the recent period since 1989, the roles of the other components are much smaller. For example, persistently declining interest rates and a decline in the equity risk premium each contributed about 11% to the rise in equity values since 1989. Over the full sample, real interest rates contributed a much smaller 2.6%, while declining risk premia again contributed about 11%.

What about the low recent real rates? Considering the period since 2000, as exhibited in Figure 3, it is evident that short term interest rates are low relative to the 1984-2000 period but are not unusually low by the standards of the whole sample. This is echoed in the estimated value for the persistence parameter $\phi_{\delta,LF}$ obtained under flat priors, which shows that short rate changes are not nearly as persistent as would be required to warrant

[14] The growth decompositions for the real log level of market equity p_t are computed by adding back in the growth Δy_t in real output (net value added) to the growth Δpy_t. Since Δy_t is deflated by the implicit price deflator for net value added, the decomposition for p_t pertains to the value of market equity deflated by the implicit NVA price deflator.

an expectation of permanent shifts in rates. Once these aspects of the data are taken into account, it is not hard to understand why the low interest rates of recent years are not, by themselves, a force for unusually high equity valuation ratios since 2000.

In contrast to factors shares movements, growth in the real value of what was actually produced by the sector is a far less important driver of equity values since 1989: economic growth explains just 23% to the increase in equity values since 1989 and, as a result of this weak contribution, explains only 50% over the full post-war sample. Economic growth explains a large fraction of the rise in equity values over the previous subsample, from 1952 to 1988, where it accounted for 92% of the increase, while factors share movements explained relatively little (just 11.34% of the rise is explained by the sum of the two factors share components $s_{LF,t}$ and $s_{HF,t}$). But that 37 year period created less than half the wealth generated in the 29 years 1989:Q1-2017:Q4. These findings underscore a striking aspect of post-war equity markets: in the 37 year subsample for which equity values grew comparatively slowly, economic growth was the engine of market growth while factors share played virtually no role. By contrast, in the shorter recent subsample for which equity values grew much more sharply, factors share shocks were the engine of market growth while economic growth played a small role. Evidently, factors shares rather than economic growth are a more salient measure of fundamental value in the stock market over our sample.

The numbers reported in Panel B of Table 3 decompose the growth in the log market equity-to-output ratio py_t over time. It is notable that py_t was almost constant over the early subsample from 1952-1988. Yet the estimated contribution of $s_{LF,t}$ is quite large in this subperiod, since even the small estimated movements in factors shares over this subsample are still large by comparison to the negligible change in py_t. These estimates show that the anemic growth in py_t came about, in part, due to substantial offsetting affects from a declining risk premium and higher interest rates, which each contributed 61% and -115%, respectively. The high frequency component of the factors share process, $s_{HF,t}$, is also estimated to have played a sizable role in holding down growth in py_t with a contribution of -68%, indicating that the earnings share temporarily fell over this period, while the labor share temporarily increased.

The above results are based on our baseline calibrated value for the persistence of the risk premium component $\phi_x = 0.85$. Table 4 shows the growth decompositions for the real value of market equity p_t (comparable to those in Panel A of Table 3), for $\phi_x = 0.9$ and $\phi_x = 0.8$. Not surprisingly, the $\phi_x = 0.9$ case (Panel A) shows a modestly higher contribution from declining risk premia than do the baseline estimates. Declining risk premia are now estimated to explain 17% of the rise in equity values in both the full sample and the sample since 1989, compared to 11% under the baseline parameter value of $\phi_x = 0.85$. Conversely, the $\phi_x = 0.8$ case (Panel B) shows a modestly lower contribution from declining risk premia than do the baseline estimates, which explain about 8% in both the full sample and the sample since 1989. These changes have small effects on the role of the factors share components. For the $\phi_x = 0.9$ case, about 48% of the increase in market equity since 1989 and 30% over the full sample has been attributable to the sum of the two factors share components $s_{LF,t}$ and $s_{HF,t}$, compared to 54% and 36% for the baseline estimation. For the $\phi_x = 0.8$ case, about 57% of the market increase since 1989 and 39% over the full sample has been attributable to the sum of the two factors share components $s_{LF,t}$ and $s_{HF,t}$. For both alternative parameterizations, the role of economic growth is very close to what is estimated for the baseline case.

6 Conclusion

We investigate the reasons for rising equity values over the post-war period. We do this by estimating a flexible parametric model of how equities are priced that allows for influence from a number of mutually uncorrelated latent components, while at the same time inferring what values those components must have taken over our sample to explain the data. The identification of mutually uncorrelated components and the specification of a log linear model allow us to precisely decompose the observed market growth into distinct component sources. The model is flexible enough to explain 100% of the rise in equity values over our sample and at each point in time.

We confront our model with data on equity values, output, the earnings share of output (or conversely, one minus the labor share), interest rates, and the share of taxes and interest paid by corporations. We find that the high returns to holding equity over the post-war era have been attributable in large part to good luck, driven by a string of favorable factors share shocks that reallocated rents away from workers and toward shareholders. Indeed, our estimates suggest that at least 2.1 percentage points of the post-war average annual log equity return in excess of a short-term interest rate is attributable to this string of reallocative shocks, rather than to genuine compensation for bearing risk. This estimate implies that the sample mean log excess equity return overstates the true risk premium by at least 50%.

Factors share shocks account for about 54% of the market's rise since 1989 and 36% of the increase over the entire post-war period. Equity values were modestly boosted since 1989 by persistently declining interest rates and a decline in risk premia, both of which contributed 11% to rising equity values. Yet growth in the real value of aggregate output contributed just 23% since 1989 and just 50% over the full sample. By contrast, economic growth was overwhelmingly important for rising equity values from 1952 to 1988, where it explained 92% of the market's rise. But that 37 year period generated less than half the wealth created in the 29 years since 1989. In this sense, factors shares have been more relevant than economic growth as a measure fundamental value in the stock market.

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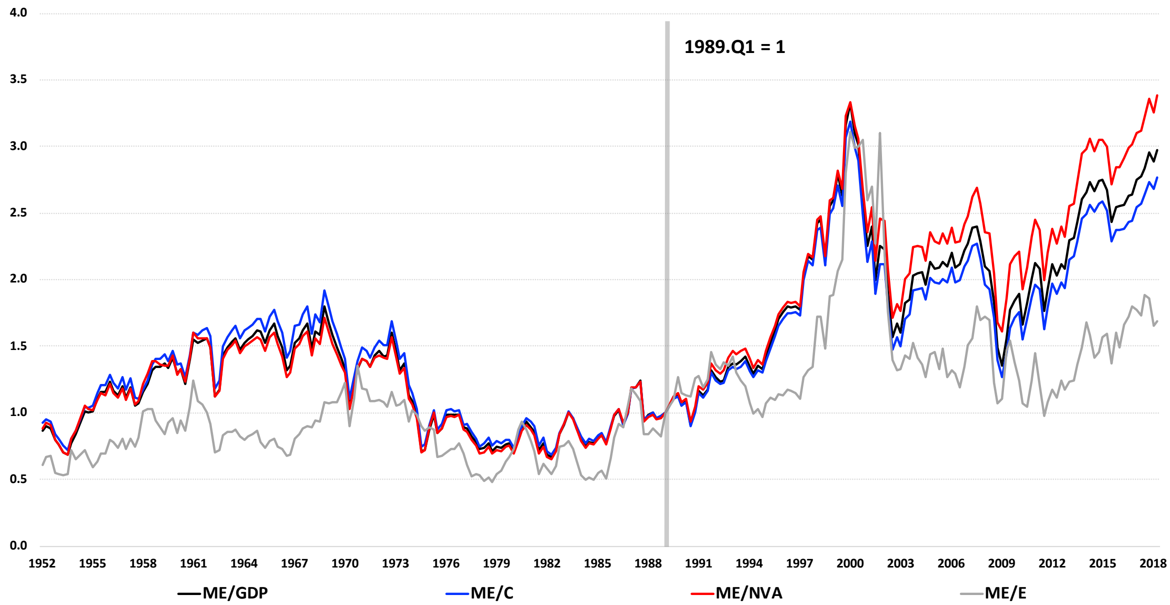
Figures and Tables

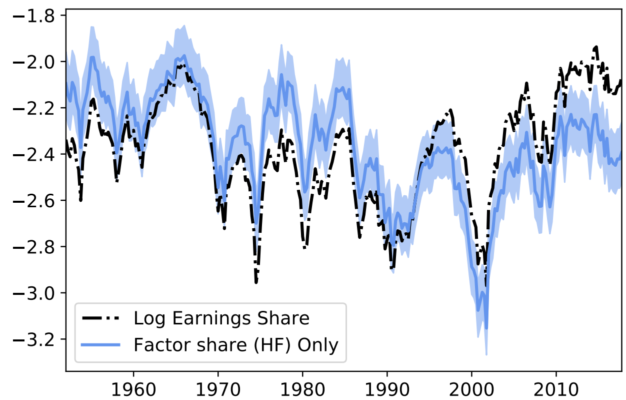
Figure 1: Stock Market Ratios



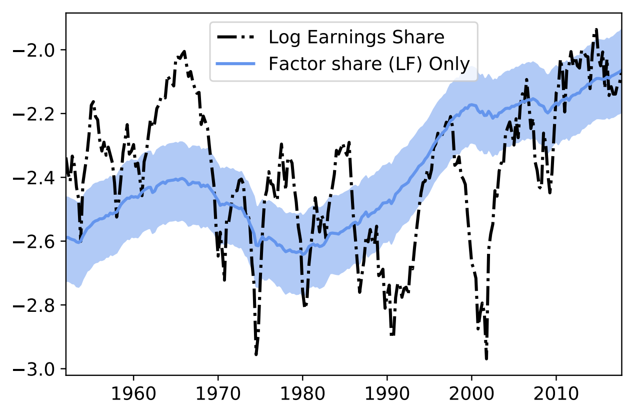
Notes: The sample spans the period 1952:Q1-2017:Q4. **ME:** Nonfinancial Corporate Sector Stock Value. **E:** Nonfinancial Corporate Business After-Tax Profits. **GDP & C:** Current Dollars GDP and personal consumption expenditures. **NVA:** Net Value Added of Nonfinancial Corporate Sector.

Figure 2: Earnings Share Components

(a)



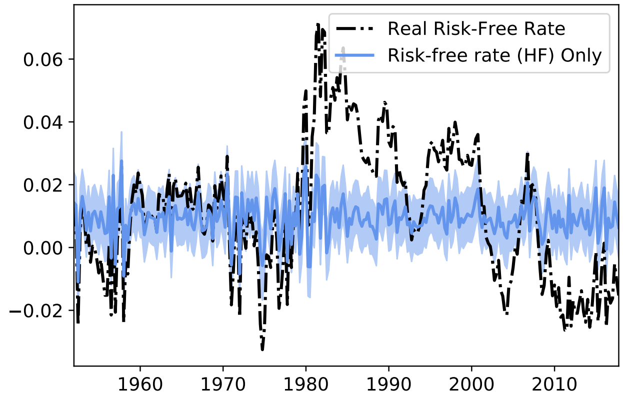
(b)



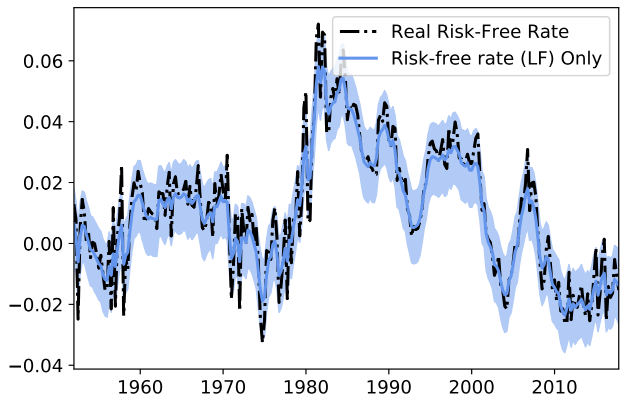
Notes: The figure exhbits the observed earnings share series along with the model-implied variation in the series attributable to certain latent components. The shaded areas surrounding each estimated component are 90% credible sets that take into account both parameter and latent state uncertainty. The sample spans the period 1952:Q1-2017:Q4.

Figure 3: Risk-Free Rate Components

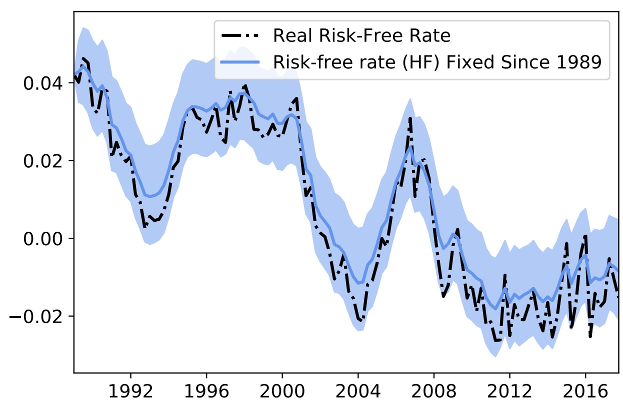
(a)



(b)

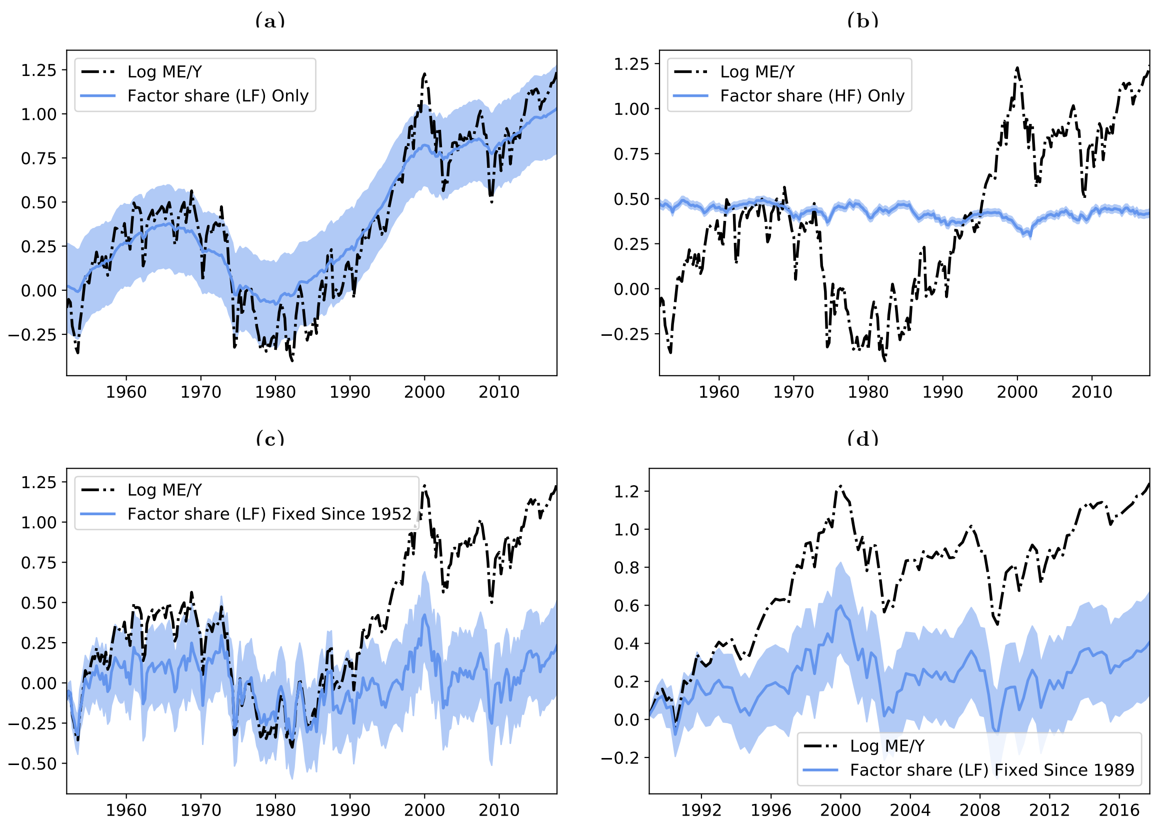


(c)



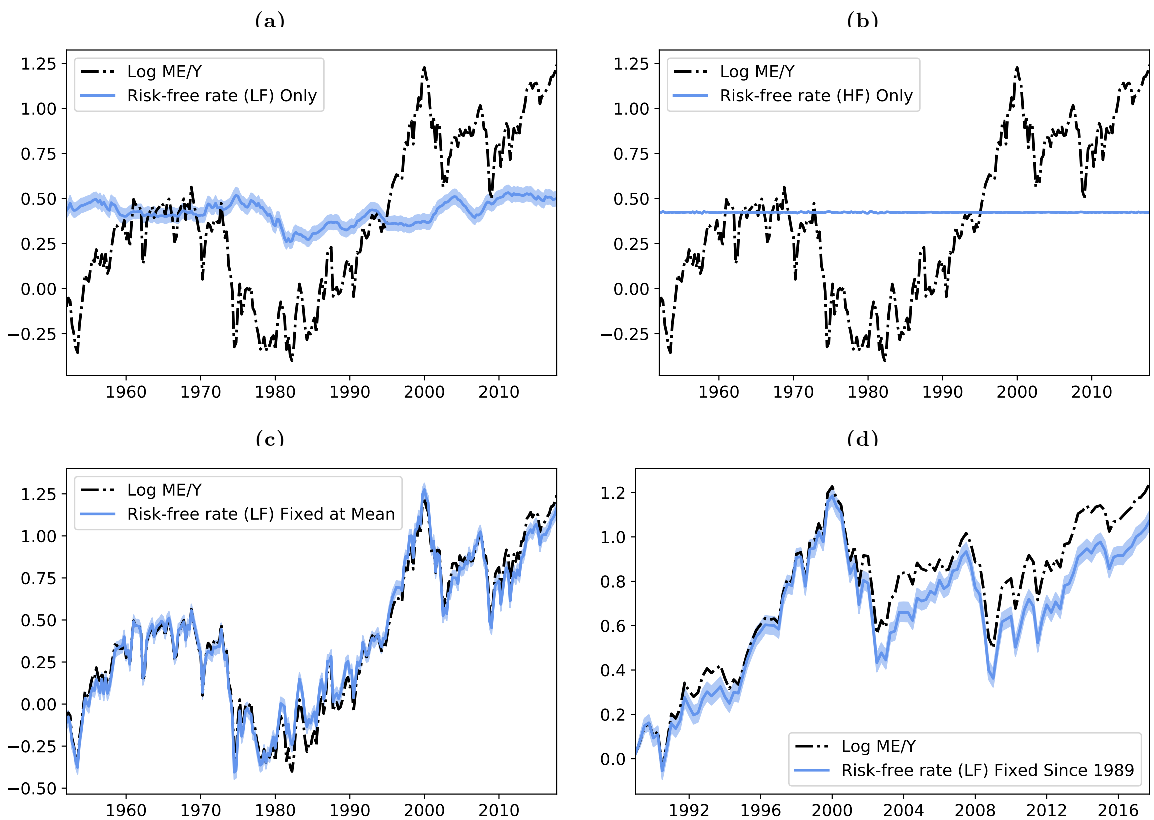
Notes: The real risk-free rate is computed as the three-month T-bill rate minus the fitted value from a regression of GDP deflator inflation on lags of inflation and interest rates. The figure exhbits the observed risk-free rate series along with the model-implied variation in the series attributable to certain latent components. The shaded areas surrounding each estimated component are 90% credible sets that take into account both parameter and latent state uncertainty. The sample spans the period 1952:Q1-2017:Q4.

Figure 4: Market Equity-Output Ratio and Factors Share Component



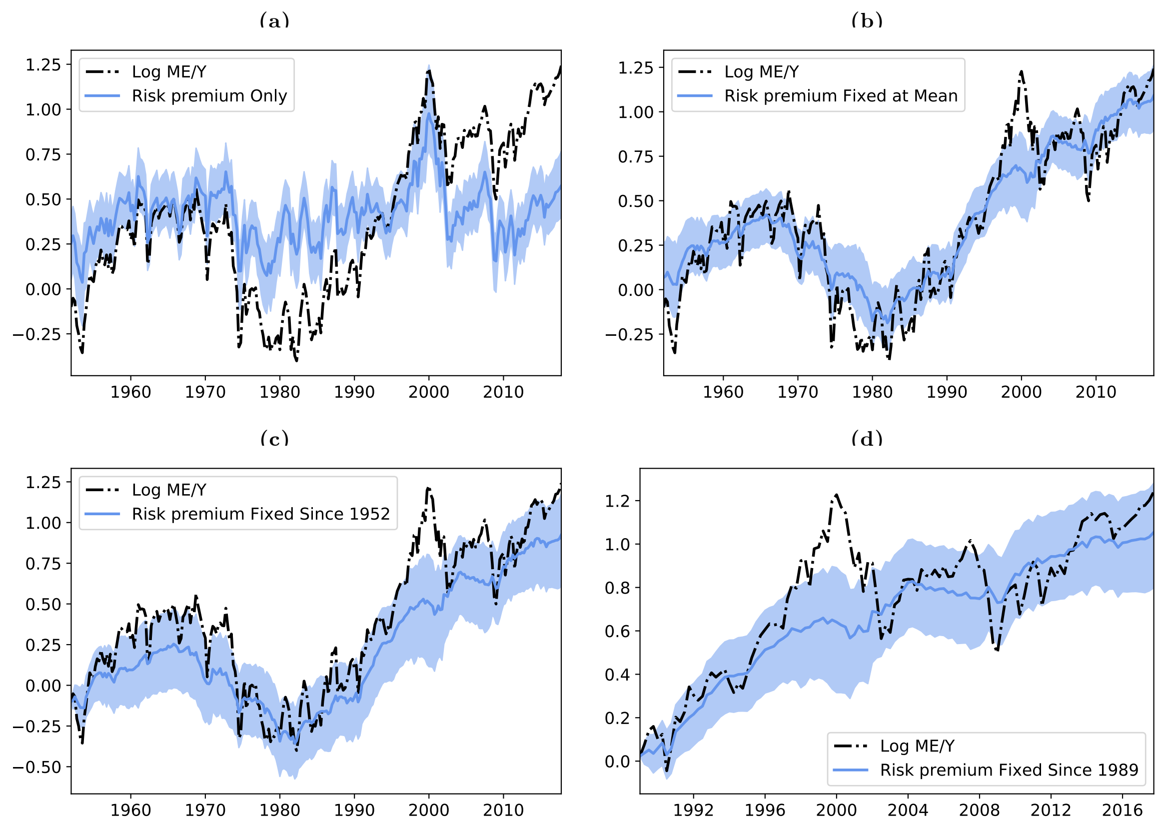
Notes: The figure exhbits the observed market equity-to-output series along with the model-implied variation in the series attributable to certain latent components. The shaded areas surrounding each estimated component are 90% credible sets that take into account both parameter and latent state uncertainty. The sample spans the period 1952:Q1-2017:Q4.

Figure 5: Market Equity-Output Ratio and Risk-Free Rate Component



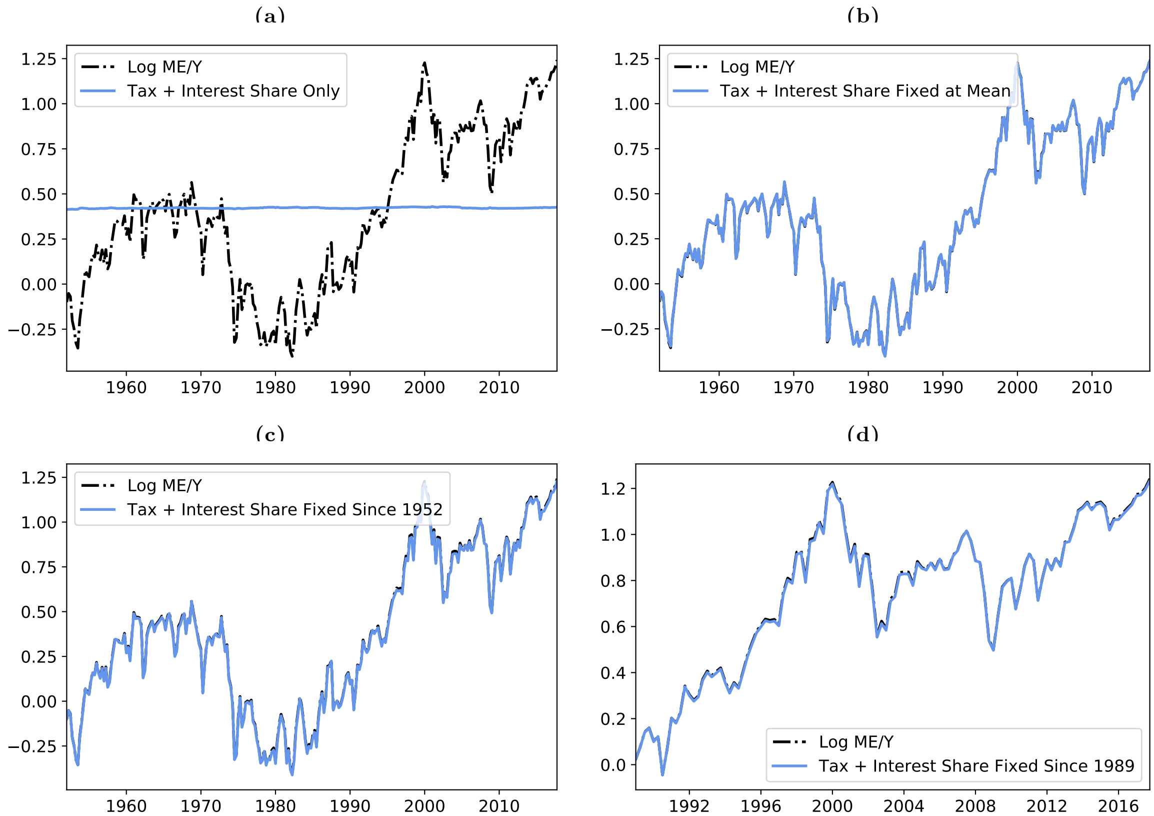
Notes: The figure exhbits the observed market equity-to-output series along with the model-implied variation in the series attributable to the risk-free rate component. The shaded areas surrounding each estimated component are 90% credible sets that take into account both parameter and latent state uncertainty. The sample spans the period 1952:Q1-2017:Q4.

Figure 6: Market Equity-Output Ratio and Risk Premium Component



Notes: The figure exhbits the observed market equity-to-output series along with the model-implied variation in the series attributable to the risk premium component. The shaded areas surrounding each estimated component are 90% credible sets that take into account both parameter and latent state uncertainty. The sample spans the period 1952:Q1-2017:Q4.

Figure 7: Market Equity-Output Ratio and Tax/Interest Component



Notes: The figure exhbits the observed market equity-to-output series along with the model-implied variation in the series attributable to the tax/interest component. The shaded areas surrounding each estimated component are 90% credible sets that take into account both parameter and latent state uncertainty. The sample spans the period 1952:Q1-2017:Q4.

Table 1: Parameter Estimates

Variable	Parameter	Mode	5%	Median	95%
Risk Price Mean	$\bar{x}$	4.4832	3.3174	4.3791	5.8452
Risk Price Vol.	σ_x	3.8086	2.8981	3.8307	5.1905
Risk-Free Rate Mean	$\bar{r}_f$	0.0023	0.0008	0.0027	0.0048
Risk-Free (HF) Pers.	$\phi_{\delta,HF}$	0.1587	0.0290	0.1928	0.4109
Risk-Free (HF) Vol.	$\sigma_{\delta,HF}$	0.0019	0.0016	0.0019	0.0022
Risk-Free (LF) Pers.	$\phi_{\delta,LF}$	0.9321	0.8949	0.9314	0.9558
Risk-Free (LF) Vol.	$\sigma_{\delta,LF}$	0.0015	0.0012	0.0015	0.0019
Factor Share (HF) Pers.	$\phi_{s,HF}$	0.9250	0.8981	0.9245	0.9455
Factor Share (HF) Vol.	$\sigma_{s,HF}$	0.0680	0.0633	0.0683	0.0734
Factor Share (LF) Pers.	$\phi_{s,LF}$	0.9997	0.9984	0.9996	0.9999
Factor Share (LF) Vol.	$\sigma_{s,LF}$	0.0179	0.0132	0.0179	0.0230
Tax + Interest Share Pers.	ϕ_Z	0.9545	0.9244	0.9583	0.9875
Tax + Interest Vol.	σ_Z	0.0041	0.0038	0.0041	0.0044
Productivity Vol.	σ_a	0.0160	0.0148	0.0159	0.0171

Notes: The table reports parameter estimates from the posterior distribution. The sample spans the period 1952:Q1-2017:Q4.

Table 2: Asset Pricing Moments

Variable	Model Mean	Model SD	Fitted Mean	Fitted SD	Data Mean	Data SD
Log Equity Return	5.264	16.868	7.516	17.203	8.671	16.872
Log Risk-Free Rate	0.942	1.515	1.110	1.998	1.110	1.998
Log Excess Return	4.322	16.957	6.410	17.191	7.576	16.710
Log Price-Payout Ratio	3.507	0.334	3.486	0.456	3.392	0.493
Log Earnings Growth	2.065	11.198	2.450	15.041	2.450	15.041
Log Payout Growth	2.064	21.952	3.095	28.167	4.243	30.558
Log Earnings Share Growth	0.000	10.897	0.405	13.337	0.405	13.337
Log Payout Share Growth	0.000	21.804	1.106	26.607	2.254	28.678

Notes: All statistics are computed for annual (continuously compounded) data. "Model"numbers are averages across 1000 simulations of the model of the same size as our data sample. "Fitted"numbers use the estimated latent states fitted to observed data in our historical sample. The sample spans the period 1952:Q1-2017:Q4.

Table 3: Growth Decomposition

Contribution	1952-2017	1952-1988	1989-2017
Panel A: Market Equity			
Total	1381.05%	190.38%	394.03%
Factor share (LF)	37.60%	16.57%	52.17%
Factor share (HF)	-1.89%	-5.23%	1.92%
Tax + Interest Share	0.49%	0.55%	0.54%
Risk premium	11.02%	4.75%	10.96%
Risk-free rate (LF)	2.47%	-8.91%	10.60%
Risk-free rate (HF)	0.09%	0.02%	0.12%
Real Output Growth	50.22%	92.25%	23.69%
Panel B: Market Equity/NVA			
Contribution	1952-2017	1952-1988	1989-2017
Total	282.58%	8.61%	238.35%
Factor share (LF)	75.53%	213.96%	68.37%
Factor share (HF)	-3.79%	-67.51%	2.51%
Tax + Interest Share	0.98%	7.05%	0.71%
Risk premium	22.14%	61.26%	14.36%
Risk-free rate (LF)	4.97%	-115.01%	13.89%
Risk-free rate (HF)	0.18%	0.26%	0.16%

Notes: The table presents the growth decompositions for the real value of market equity (top panel) or the market equity-output ratio (bottom panel). The persistence parameter of the risk price is set to its baseline value of 0.85. The sample spans the period 1952:Q1-2017:Q4.

Table 4: Growth Decomposition

Contribution	1952-2017	1952-1988	1989-2017
Panel A: Market Equity, $\phi_x = 0.80$			
Total	1381.05%	190.38%	394.03%
Factor share (LF)	41.48%	21.16%	55.61%
Factor share (HF)	-2.18%	-5.58%	1.65%
Tax + Interest Share	0.48%	0.54%	0.53%
Risk premium	7.54%	0.16%	8.20%
Risk-free rate (LF)	2.38%	-8.55%	10.19%
Risk-free rate (HF)	0.09%	0.02%	0.12%
Real PC Output Growth	50.22%	92.25%	23.69%
Panel A: Market Equity, $\phi_x = 0.90$			
Contribution	1952-2017	1952-1988	1989-2017
Total	1381.05%	190.38%	394.03%
Factor share (LF)	30.78%	10.14%	45.07%
Factor share (HF)	-1.35%	-4.68%	2.45%
Tax + Interest Share	0.49%	0.55%	0.54%
Risk premium	17.18%	11.12%	16.99%
Risk-free rate (LF)	2.58%	-9.40%	11.12%
Risk-free rate (HF)	0.09%	0.02%	0.13%
Real PC Output Growth	50.22%	92.25%	23.69%

Notes: The table presents the growth decompositions for the real value of market equity with persistence parameter of the risk price set to 0.80 (top panel) and set to 0.90 (bottom panel). The sample spans the period 1952:Q1-2017:Q4.

Appendix: For Online Publication

Data Description

CORPORATE EQUITY

Corporate equity is obtained from the Flow of Funds Table B103, series code LM103164103, nonfinancial corporate business; corporate equities; liability. Unadjusted transactions estimated by Federal Reserve Board (Capital Markets and Flow of Funds Sections), using data from the following commercial sources: cash mergers and acquisitions data from Thompson Financial Services SDC database; public issuance and share repurchase data from Standard and Poor's Compustat database; and private equity issuance data from Dow Jones Private Equity Analyst and PriceWaterhouseCoopers Money tree report. Level at market value is obtained separately as the sum of the market value of publicly-held equity of U.S. domestic nonfinancial corporations excluding intercompany holdings (FOF series FL103164115) and the market value of closely-held equity (FOF series FL103164123).

CORP. NET VALUE ADDED, CORP. LABOR COMPENSATION, CORP. AFTER-TAX PROFITS, TAXES AND INTEREST

These variables are obtained from NIPA. Data for the net value added (NVA) comes from NIPA Table 1.14 (nonfinancial corporate sector series code A457RC1). We use per capita real net value added, deflated by the implicit price deflator for net value added. After tax profits (ATP) come from NIPA Table 1.14 (nonfinancial corporate sector series code: W328RC1). Nonfinancial corporate Sector Labor compensation (LC) comes from Table 1.14 (series code A460RC). The after-tax profit share ($ATPS$) of NVA is identically equal to

$$\begin{aligned} ATPS &= \frac{ATP}{ATP+LC}\frac{ATP+LC}{NVA} = \underbrace{\frac{ATP}{ATP+LC}}_{\equiv S_t}\frac{NVA-(\text{taxes and interest})}{NVA} \\ &= S_t\left[1-\underbrace{\left(\frac{\text{taxes and interest}}{NVA}\right)}_{\equiv \tau_t}\right]. \end{aligned}$$

where S_t is the after-tax profit share of combined profit plus labor compensation, "taxes and interest" is the sum of taxes on production and imports less subsidies (W325RC1), net interest and miscellaneous payments (B471RC1), business current transfer payments (Net) (W327RC1), and taxes on corporate income (B465RC1). We define corporate earnings E_t as

$$E_t = S_t \left(1 - \tau_t\right) NVA_t,$$

which is equivalent to

$$E_t = \left[1 - \underbrace{\frac{LC_t}{ATP_t + LC_t}}_{\text{Labor share of labor+profit}}\right] \left(1 - \tau_t\right) NVA_t.$$

NET DIVIDENDS PLUS NET REPURCHASES (EQUITY PAYOUT)

"Net dividends plus net repurchases" is computed using the Flow of Funds Table F.103 (nonfinancial corporate business sector) by subtracting Net Equity Issuance (FA103164103) from Net Dividends (FA106121075). Net repurchases are repurchases net of share issuance, so net repurchases is the negative of net equity issuance. Net dividends consists of payments in cash or other assets, excluding the corporation's own stock, made by corporations located in the United States and abroad to stockholders who are U.S. residents. The payments are netted against dividends received by U.S. corporations, thereby providing a measure of the dividends paid by U.S. corporations to other sectors.

PRICE DEFLATORS

NFCS implicit price deflator and GDP deflator. A chain-type price deflator for the nonfinancial corporate sector (NFCS) is obtained implicitly by dividing the net value added of nonfinancial corporate business by the chained real dollar net value added of nonfinancial corporate business from NIPA Table 1.14. This index is used to deflate net value added of the NFCS. The GDP deflator is used to construct a real returns and a real interest (see below). GDPDEF is retrieved from FRED. Our source is the Bureau of Economic Analysis.

INTEREST RATE

The nominal risk-free rate is measured by the 3-Month Treasury Bill rate, secondary market rate. We take the (average) quarterly 3-Month Treasury bill from FRED [TB3MS]. A real rate is constructed by subtracting the fitted value from a regression of GDP deflator inflation onto lags of inflation from the nominal rate. Our source is the board of governors of the Federal Reserve System and the Bureau of Economic Analysis.

A Simple Model of Workers and Shareholders

We consider a stylized limited participation endowment economy in which wealth is concentrated in the hands of a few asset owners, or "shareholders," while most households are "workers" who finance consumption out of wages and salaries. The economy is closed. Workers own no risky asset shares and consume their labor earnings. There is no risk-sharing between workers and shareholders. A representative firm issues no new shares and buys back no shares. Cashflows are equal to output minus a wage bill,

$$C_t = Y_t - w_t N_t,$$

where w_t equals the wage and N_t is aggregate labor supply. The wage bill is equal to Y_t times a time-varying labor share α_t,

$$w_t N_t = \alpha_t Y_t => C_t = (1 - \alpha_t) Y_t. \tag{19}$$

We rule out short sales in the risky asset:

$$\theta_t^i \geq 0.$$

Asset owners not only purchase shares in the risky security, but also trade with one another in a one-period bond with price at time t denoted by q_t. The real quantity of bonds is denoted B_{t+1}, where $B_{t+1} < 0$ represents a borrowing position. The bond is in zero net supply among asset owners. Asset owners could have idiosyncratic investment income ζ_t^i, which is idependently and identically distributed across investors and time. The gross financial assets of investor i at time t are given by

$$A_t^i \equiv \theta_t^i (V_t + C_t) + B_t^i.$$

The budget constraint for the ith investor is

$$\begin{aligned} C_t^i + B_{t+1}^i q_t + \theta_{t+1}^i V_t &= A_t^i + \zeta_t^i \\ &= \theta_t^i (V_t + C_t) + B_t^i + \zeta_t^i, \end{aligned} \tag{20}$$

where C_t^i denotes the consumption of investor i.

A large number of identical nonrich workers, denoted by w, receive labor income and do not participate in asset markets. The budget constraint for the representative worker is therefore

$$C^w = \alpha_t Y_t. \tag{21}$$

Equity market clearing requires

$$\sum_i \theta_t^i = 1.$$

Bond market clearing requires

$$\sum_i B_t^i = 0.$$

Aggregating (20) and (21) and imposing both market clearing and (19) implies that aggregate (worker plus shareholder) consumption C_t^{Agg} is equal to total output Y_t. Aggregating over the budget constraint of shareholders shows that their consumption is equal to the capital share times aggregate consumption C_t^{Agg}:

$$C_t^S = C_t = \underbrace{(1-\alpha_t)}_{KS_t} C_t^{AGG}.$$

A representative shareholder who owns the entire corporate sector will therefore have consumption equal to $C_t^{Agg} \cdot KS_t$. This reasoning goes through as an approximation if workers own a small fraction of the corporate sector even if there is some risk-sharing in the form of risk-free borrowing and lending between workers and shareholders, as long as any risk-sharing across these groups is imperfect. While individual shareholders can smooth out transitory fluctuations in income by buying and selling assets, shareholders as a whole are less able to do so since purchases and sales of any asset must net to zero across all asset owners.

Model Solution

This section derives the coefficients of the main asset pricing equation (15). Since the variable z_t behaves exactly like a component of $\mathbf{s}_t$ in the model, for notational convenience, we assume without loss of generality that z_t is simply an extra element of the vector $\mathbf{s}_t$ (whose length is arbitrary in this derivation), and do not explicitly include it as a separate state variable, until substituting it back in for the final formulas. We also maintain some expressions (e.g., A_x) using matrix notation for generality even though in practice we use a single (scalar) element for x_t.

To begin, define for convenience the variables

$$u_{t+1} = \log(PD_{t+1} + 1) - pd_t$$
$$q_{t+1} = m_{t+1} + \Delta d_{t+1}$$

so that $m_{t+1} + r_{t+1} = u_{t+1} + q_{t+1}$. Applying the log linear approximation to $\log(PD_{t+1} + 1)$ and substituting in our guessed functional form (15) yields

$$\begin{aligned}
u_{t+1} &\equiv \log(PD_{t+1} + 1) - pd_t \\
&= \kappa_0 + \kappa_1 \Big(A_0 + \mathbf{A}_s' \tilde{\mathbf{s}}_{t+1} + A_x' \tilde{x}_{t+1} + \mathbf{A}_r' \tilde{\boldsymbol{\delta}}_t \Big) - \Big(A_0 + \mathbf{A}_s' \tilde{\mathbf{s}}_t + A_x' \tilde{x}_t + \mathbf{A}_r' \tilde{\boldsymbol{\delta}}_t \Big) \\
&= \kappa_0 + (\kappa_1 - 1) A_0 + \mathbf{A}_s' \left(\kappa_1 \boldsymbol{\Phi}_s - I \right) \tilde{\mathbf{s}}_t + A_x' (\kappa_1 \Phi_x - I) \tilde{x}_t + \mathbf{A}_r' (\kappa_1 \Phi_r - I) \tilde{\boldsymbol{\delta}}_t \\
&\qquad + \kappa_1 \mathbf{A}_s' \varepsilon_{s,t+1} + \kappa_1 A_x' \varepsilon_{x,t+1} + \kappa_1 \mathbf{A}_r' \varepsilon_{r,t+1}.
\end{aligned}$$

Now turning to q_{t+1}, we can expand the expression to yield

$$q_{t+1} = -\delta_t + x_t g - \xi x_t \mathbf{1}'(\mathbf{I} - \boldsymbol{\Phi}_s) \tilde{\mathbf{s}}_t - \frac{1}{2} x_t^2 \Big(\xi (\mathbf{1}' \boldsymbol{\Sigma}_s \mathbf{1}) + \sigma_g^2 \Big) + (1 - x_t) \Delta d_{t+1}.$$

Next, we apply our fundamental asset pricing equation $0 = \log \mathrm{E}_t \left[q_{t+1} + u_{t+1} \right]$, which under lognormality implies

$$0 = \mathrm{E}_t[q_{t+1}] + \mathrm{E}_t[u_{t+1}] + \frac{1}{2} \mathrm{Var}_t(q_{t+1}) + \frac{1}{2} \mathrm{Var}_t(u_{t+1}) + \mathrm{Cov}(q_{t+1}, u_{t+1}).$$

These moments can be calculated as

$$\begin{aligned}
\mathrm{E}_t[q_{t+1}] &= -\delta_t + g - \xi \mathbf{1}'(\mathbf{I} - \boldsymbol{\Phi}_s) \tilde{\mathbf{s}}_t - \frac{1}{2} x_t^2 \Big(\xi (\mathbf{1}' \boldsymbol{\Sigma}_s \mathbf{1}) + \sigma_g^2 \Big) \\
\mathrm{E}_t[u_{t+1}] &= \kappa_0 + (\kappa_1 - 1) A_0 + \mathbf{A}_s' \left(\kappa_1 \boldsymbol{\Phi}_s - \mathbf{I} \right) \tilde{\mathbf{s}}_t + A_x' (\kappa_1 \Phi_x - \mathbf{I}) \tilde{x}_t + \mathbf{A}_r' (\kappa_1 \Phi_r - \mathbf{I}) \tilde{\boldsymbol{\delta}}_t \\
\mathrm{Var}_t(q_{t+1}) &= (1 - x_t)^2 \left(\xi^2 \left(\mathbf{1}' \boldsymbol{\Sigma}_s \mathbf{1} \right) + \sigma_g^2 \right) \\
\mathrm{Var}_t(u_{t+1}) &= \kappa_1^2 \Big(\mathbf{A}_s' \boldsymbol{\Sigma}_s \mathbf{A}_s' + A_x' \boldsymbol{\Sigma}_x A_x + \mathbf{A}_r' \boldsymbol{\Sigma}_r \mathbf{A}_r \Big) \\
\mathrm{Cov}_t(q_{t+1}, u_{t+1}) &= \xi \kappa_1 (1 - x_t) (\mathbf{A}_s' \boldsymbol{\Sigma}_s \mathbf{1}).
\end{aligned}$$

Substituting, we obtain

$$
\begin{aligned}
0 = & -\mathbf{1}'\bar{\delta} + g + \kappa_0 + (\kappa_1 - 1)A_0 \\
& + \frac{1}{2}(1-2\bar{x})\left(\xi^2\left(\mathbf{1}'\mathbf{\Sigma}_s\mathbf{1}\right) + \sigma_g^2\right) + \frac{1}{2}\kappa_1^2\Big(\mathbf{A}_s'\mathbf{\Sigma}_s\mathbf{A}_s' + A_x'\mathbf{\Sigma}_x A_x + \mathbf{A}_r'\mathbf{\Sigma}_r\mathbf{A}_r\Big) + \kappa_1(1-\bar{x})\mathbf{A}_s'\mathbf{\Sigma}_s\mathbf{1} \\
& + \Big[-\xi\mathbf{1}'(\mathbf{I} - \mathbf{\Phi}_s) + \mathbf{A}_s'\left(\kappa_1\mathbf{\Phi}_s - I\right)\Big]\tilde{\mathbf{s}}_t \\
& + \Big[A_x'(\kappa_1\Phi_x - \mathbf{I}) - \Big(\left(\xi^2\left(\mathbf{1}'\mathbf{\Sigma}_s\mathbf{1}\right) + \sigma_g^2\right) + \xi\kappa_1\left(\mathbf{A}_s'\mathbf{\Sigma}_s\mathbf{1}\right)\Big)\mathbf{1}'\Big]\tilde{x}_t \\
& + \Big[-\mathbf{1}' + \mathbf{A}_r'(\kappa_1\Phi_r - I)\Big]\tilde{\boldsymbol{\delta}}_t
\end{aligned}
$$

Applying the method of undetermined coefficients (now reintroducing z_t analogously to s_t) now yields

$$
\begin{aligned}
\mathbf{A}_s' &= -\xi\mathbf{1}'(\mathbf{I} - \mathbf{\Phi}_s)(\mathbf{I} - \kappa_1\mathbf{\Phi}_s)^{-1} \\
A_x' &= -\Big[\Big(\left(\xi^2\left(\mathbf{1}'\mathbf{\Sigma}_s\mathbf{1}\right) + \sigma_g^2\right) + \xi\kappa_1\left(\mathbf{A}_s'\mathbf{\Sigma}_s\mathbf{1}\right)\Big)\Big]\left(\mathbf{I} - \kappa_1\Phi_x\right)^{-1} \\
\mathbf{A}_r' &= -\mathbf{1}'\left(\mathbf{I} - \kappa_1\Phi_r\right)^{-1}
\end{aligned}
$$

while the constant term must solve

$$
\begin{aligned}
0 = & -\mathbf{1}'\bar{\delta} + g + \kappa_0 + (\kappa_1 - 1)A_0 \\
& + \frac{1}{2}(1-2\bar{x})\left(\xi^2\left(\mathbf{1}'\mathbf{\Sigma}_s\mathbf{1}\right) + \sigma_g^2\right) + \frac{1}{2}\kappa_1^2\Big(\mathbf{A}_s'\mathbf{\Sigma}_s\mathbf{A}_s' + A_x'\mathbf{\Sigma}_x A_x + \mathbf{A}_r'\mathbf{\Sigma}_r\mathbf{A}_r\Big) + \xi\kappa_1(1-\bar{x})(\mathbf{A}_s'\mathbf{\Sigma}_s\mathbf{1}).
\end{aligned}
$$

Separately splitting off the z_t component of $\mathbf{s}_t$ yields the formulas in the text.

Expected Returns

Log equity returns are given by

$$r_{t+1} = u_{t+1} + \Delta d_{t+1}.$$

Since

$$
\begin{aligned}
\Delta d_{t+1} &= \xi\mathbf{1}'\Delta s_{t+1} + \Delta y_{t+1} \\
&= \xi\mathbf{1}'\Big(-(I - \mathbf{\Phi}_s)\tilde{\mathbf{s}}_t + \varepsilon_{s,t+1}\Big) + g + \varepsilon_{y,t+1} \\
u_{t+1} &= \kappa_0 + (\kappa_1 - 1)A_0 + \mathbf{A}_s'\left(\kappa_1\mathbf{\Phi}_s - I\right)\tilde{\mathbf{s}}_t + A_x'(\kappa_1\Phi_x - I)\tilde{x}_t + \mathbf{A}_r'(\kappa_1\Phi_r - I)\tilde{\boldsymbol{\delta}}_t \\
&\quad + \kappa_1\mathbf{A}_s'\varepsilon_{s,t+1} + \kappa_1 A_x'\varepsilon_{x,t+1} + \kappa_1\mathbf{A}_r'\varepsilon_{r,t+1}
\end{aligned}
$$

we have

$$\begin{aligned}
\mathrm{E}_t[u_{t+1}] &= \kappa_0 + (\kappa_1 - 1)A_0 + \mathbf{A}_s' \left(\kappa_1 \mathbf{\Phi}_s - \mathbf{I}\right) \tilde{\mathbf{s}}_t + A_x'(\kappa_1 \Phi_x - \mathbf{I})\tilde{x}_t + \mathbf{A}_r'(\kappa_1 \Phi_r - \mathbf{I})\tilde{\boldsymbol{\delta}}_t \\
\mathrm{E}_t\left[\Delta d_{t+1}\right] &= -\xi \mathbf{1}'(I - \mathbf{\Phi}_s)\tilde{\mathbf{s}}_t + g \\
\mathrm{Var}_t(u_{t+1}) &= \kappa_1^2 \Big(\mathbf{A}_s' \mathbf{\Sigma}_s \mathbf{A}_s + A_x' \mathbf{\Sigma}_x A_x + \mathbf{A}_r' \mathbf{\Sigma}_r \mathbf{A}_r\Big) \\
\mathrm{Var}_t(\Delta d_{t+1}) &= \xi^2 \mathbf{1}' \mathbf{\Sigma}_s \mathbf{1} + \sigma_y^2 \\
\mathrm{Cov}_t(u_{t+1}, \Delta d_{t+1}) &= \xi \kappa_1 \mathbf{A}_s' \mathbf{\Sigma}_s \mathbf{1}
\end{aligned}$$

and so

$$\begin{aligned}
\log \mathrm{E}_t\left[R_{t+1}\right] = \kappa_0 &+ (\kappa_1 - 1)A_0 + \mathbf{A}_s' \left(\kappa_1 \mathbf{\Phi}_s - \mathbf{I}\right) \tilde{\mathbf{s}}_t + A_x'(\kappa_1 \Phi_x - \mathbf{I})\tilde{x}_t + \mathbf{A}_r'(\kappa_1 \Phi_r - \mathbf{I})\tilde{\boldsymbol{\delta}}_t \\
&- \xi \mathbf{1}'(\mathbf{I} - \mathbf{\Phi}_s)\tilde{\mathbf{s}}_t + g \\
&+ \frac{1}{2}\kappa_1^2 \Big(\mathbf{A}_s' \mathbf{\Sigma}_s \mathbf{A}_s + A_x' \mathbf{\Sigma}_x A_x + \mathbf{A}_r' \mathbf{\Sigma}_r \mathbf{A}_r\Big) \\
&+ \frac{1}{2}\Big(\xi^2 \mathbf{1}' \mathbf{\Sigma}_s \mathbf{1} + \sigma_y^2\Big) \\
&+ \xi \kappa_1 \mathbf{A}_s' \mathbf{\Sigma}_s \mathbf{1}.
\end{aligned}$$

Since

$$\begin{aligned}
&\kappa_0 + (\kappa_1 - 1)A_0 = \mathbf{1}'\bar{\delta} - g \\
&\quad - \frac{1}{2}(1 - 2\bar{x})\left(\xi^2\left(\mathbf{1}' \mathbf{\Sigma}_s \mathbf{1}\right) + \sigma_g^2\right) - \frac{1}{2}\kappa_1^2 \Big(\mathbf{A}_s' \mathbf{\Sigma}_s \mathbf{A}_s' + A_x' \mathbf{\Sigma}_x A_x + \mathbf{A}_r' \mathbf{\Sigma}_r \mathbf{A}_r\Big) - \xi \kappa_1 (1 - \bar{x})(\mathbf{A}_s' \mathbf{\Sigma}_s \mathbf{1})
\end{aligned}$$

substituting yields

$$\begin{aligned}
\log \mathrm{E}_t\left[R_{t+1}\right] &= \mathbf{1}'\bar{\delta} + \mathbf{A}_s' \left(\kappa_1 \mathbf{\Phi}_s - I\right) \tilde{\mathbf{s}}_t + A_x'(\kappa_1 \Phi_x - I)\tilde{x}_t + \mathbf{A}_r'(\kappa_1 \Phi_r - I)\tilde{\boldsymbol{\delta}}_t \\
&\quad - \xi \mathbf{1}'(I - \mathbf{\Phi}_s)\tilde{\mathbf{s}}_t \\
&\quad + \bar{x}\Big(\xi^2 \mathbf{1}' \mathbf{\Sigma}_s \mathbf{1} + \sigma_y^2 + \xi \kappa_1 \mathbf{A}_s' \mathbf{\Sigma}_s \mathbf{1}\Big) \\
&= \mathbf{1}'\bar{\delta} + \mathbf{1}'\delta_t - \rho \xi \mathbf{1}'(I - \mathbf{\Phi}_s)\tilde{\mathbf{s}}_t + \Big[\left(\xi^2\left(\mathbf{1}' \mathbf{\Sigma}_s \mathbf{1}\right) + \sigma_g^2\right)\mathbf{1}' + \xi \kappa_1 \left(\mathbf{A}_s' \mathbf{\Sigma}_s \mathbf{1}\right)\mathbf{1}'\Big] x_t \\
&= r_{f,t} + \Big[\left(\xi^2\left(\mathbf{1}' \mathbf{\Sigma}_s \mathbf{1}\right) + \sigma_g^2\right)\mathbf{1}' + \xi \kappa_1 \left(\mathbf{A}_s' \mathbf{\Sigma}_s \mathbf{1}\right)\mathbf{1}'\Big] x_t \\
\log \mathrm{E}_t\left[R_{t+1}/R_{f,t}\right] &= \Big[\left(\xi^2\left(\mathbf{1}' \mathbf{\Sigma}_s \mathbf{1}\right) + \sigma_g^2\right)\mathbf{1}' + \xi \kappa_1 \left(\mathbf{A}_s' \mathbf{\Sigma}_s \mathbf{1}\right)\mathbf{1}'\Big] x_t.
\end{aligned}$$

Alternatively, taking the expectation and variance in logs yields

$$
\begin{aligned}
\mathrm{E}_t[r_{t+1}] &= \kappa_0 + (\kappa_1 - 1)A_0 + \mathbf{A}_s' \left(\kappa_1 \mathbf{\Phi}_s - \mathbf{I}\right) \tilde{\mathbf{s}}_t + A_x'(\kappa_1 \Phi_x - \mathbf{I})\tilde{x}_t + \mathbf{A}_r'(\kappa_1 \Phi_r - \mathbf{I})\tilde{\boldsymbol{\delta}}_t \\
&\qquad - \xi \mathbf{1}'(\mathbf{I} - \mathbf{\Phi}_s)\tilde{\mathbf{s}}_t + g \\
&= r_{f,t} - \frac{1}{2}\mathrm{Var}_t(r_{t+1}) + \left[\left(\xi^2 \left(\mathbf{1}'\mathbf{\Sigma}_s \mathbf{1}\right) + \sigma_g^2\right) \mathbf{1}' + \xi\kappa_1 \left(\mathbf{A}_s' \mathbf{\Sigma}_s \mathbf{1}\right) \mathbf{1}'\right] x_t \\
\mathrm{Var}_t(r_{t+1}) &= \kappa_1^2 \Big(\mathbf{A}_s' \mathbf{\Sigma}_s \mathbf{A}_s + A_x' \mathbf{\Sigma}_x A_x + \mathbf{A}_r' \mathbf{\Sigma}_r \mathbf{A}_r\Big) + \Big(\xi^2 \mathbf{1}'\mathbf{\Sigma}_s \mathbf{1} + \sigma_y^2\Big) + 2\xi\kappa_1 \mathbf{A}_s' \mathbf{\Sigma}_s \mathbf{1}.
\end{aligned}
$$

Again, splitting off z_t from $\mathbf{s}_t$ yields the formulas in the main text.

Estimation Details

This section describes the procedure used to obtain the parameter draws. First, because some of our variables are bounded by definition (e.g., volatilities cannot be negative), we define a set of parameter vectors satisfying these bounds denoted Θ. We exclude parameters outside of this set, which formally means that we apply a Bayesian prior

$$
p(\boldsymbol{\theta}) = \begin{cases} \text{const} & \text{for } \boldsymbol{\theta} \in \Theta \\ 0 & \text{for } \boldsymbol{\theta} \notin \Theta \end{cases}
$$

Our restrictions on Θ are as follows: all volatilities (σ), the average risk price $\bar{x}$, the average growth rate g, and the average real risk-free rate $\bar{\delta}$ are bounded below at zero. All persistence parameters (ϕ) and the average profit share $\exp(\bar{s})$ are bounded between zero and unity.

With these bounds set, we can evaluate the posterior by

$$
\pi(\boldsymbol{\theta}) = L(y|\boldsymbol{\theta})p(\boldsymbol{\theta}).
$$

so that the posterior is simply proportional to the likelihood over Θ and is equal to zero outside of Θ.

To draw from this posterior, we use a Random Walk Metropolis Hastings algorithm. We initialize the first draw $\boldsymbol{\theta}_0$ at the mode, and then iterate on the following algorithm:

1. Given $\boldsymbol{\theta}_j$, draw a proposal $\boldsymbol{\theta}^*$ from the distribution $\mathcal{N}(\boldsymbol{\theta}_j, c\mathbf{\Sigma}_\theta)$ for some scalar c and matrix $\mathbf{\Sigma}_\theta$ defined below.

2. Compute the ratio

$$\alpha = \frac{\pi(\boldsymbol{\theta}^*)}{\pi(\boldsymbol{\theta}_j)}.$$

3. Draw u from a Uniform [0, 1] distribution.

4. If $u < \alpha$, we accept the proposed draw and set $\boldsymbol{\theta}_{j+1} = \boldsymbol{\theta}^*$. Otherwise, we reject the draw and set $\boldsymbol{\theta}_{j+1} = \boldsymbol{\theta}_j$.

Because consecutive draws are highly serially correlated, for efficiency we save every fifth draw, meaning that our 110,000 saved draws correspond to 550,000 iterations of this procedure. For the covariance term, we initialize $\boldsymbol{\Sigma}_\theta$ to be the inverse Hessian of the log likelihood function at the mode. Once we have saved 10,000 draws, we begin updating $\boldsymbol{\Sigma}_\theta$ to be the sample covariance of the draws to date, following Haario, Saksman, Tamminen, et al. (2001), with the matrix re-computed after every 1,000 saved draws. For the scaling parameter c, we initialize it at 2.4/length($\boldsymbol{\theta}$) as recommended in Gelman, Stern, Carlin, Dunson, Vehtari, and Rubin (2013). To target an acceptance rate for our algorithm of 25%, we adapt the approach of Herbst and Schorfheide (2014) in updating

$$c_{new} = c_{old} \cdot \left(0.95 + 0.1 \frac{\exp(16(x - 0.25))}{1 + \exp(16(x - 0.25))}\right)$$

after every 1,000 saved draws, where c_{old} is the pre-update value of c.

Numerical Solution and Hamilton Filter Estimation

This appendix describes an alternative solution and estimation approach for the nonlinear model.

The price-dividend ratio satisfies

$$\begin{aligned} \frac{P_t}{D_t}(s_t) &= E_t\left[M_{t+1}\left(\frac{P_{t+1}}{D_{t+1}}(\mathfrak{s}_{t+1}) + 1\right)\frac{D_{t+1}}{D_t}\right] \\ &= E_t \exp\left(m_{t+1} + \Delta d_{t+1} + \ln\left(\frac{P_{t+1}}{D_{t+1}}(s_{t+1}) + 1\right)\right), \end{aligned}$$

where $\mathfrak{s}_t$ is a vector of state variables, $\mathfrak{s}_t \equiv (\Delta \ln a_t, S_t, \delta_t, x_t)'$. We therefore solve the function numerically on an $n \times n \times n \times n$ dimensional grid of values for the state variables, replacing the continuous time processes with a discrete Markov approximation following the approach in Rouwenhorst (1995). The continuous function $\frac{P_t}{D_t}(\mathfrak{s}_t)$ is then replaced by the $n \times n \times n$ functions $\frac{P_t}{D_t}(i,j,k)$, $i,j,k = 1,...,n$, each representing the price-dividend ratio in state $\Delta \ln a_i$, Z_j, and x_k, where the functions are defined recursively by

$$\frac{P}{D}(i,j,k) = \sum_{l=1}^{n}\sum_{m=1}^{n}\sum_{n=1}^{n} \pi_{i,l}\pi_{k,n}\pi_{j,m} \exp\left(m(l,m,n) + \Delta d(l,m,n) + \ln\left(\frac{P}{D}(l,m,n)+1\right)\right),$$

where $m(l,m,n)$ refers to the values m_{t+1}can take on in each of the states, and analogously for the other terms. We set $n = 35$.