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A Sector Based Approach to Fixed Income Performance Attribution

Fixed income requires a unique approach to attribution since much of its return is driven by structural risk factors that are specific to the bond market. The challenge in terms of performance attribution is to create a robust model that evaluates the investment process fairly without introducing too much complexity. Otherwise, the results may be meaningless to clients who may get lost in the details while missing the key performance messages. To develop the right performance model we need to answer a few key questions: Which structural factors should be included? How can we leverage the existing performance attribution methodologies where they are appropriate? And how can we balance simplicity, clarity and understandability with rigor and accuracy? This article addresses these challenges by employing a sector-based approach where market weightings are adjusted by the systematic risk of bonds, where the income component of bond return is recognized and where the true drivers of price volatility are included. The result is an attribution model that reflects the active bond management process with rigor, accuracy and clarity and which accomplishes this with straightforward calculations that are easy to grasp and to communicate to clients.

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OVERVIEW

Fixed income investments are an important part of many investment portfolios. In fact, many believe that fixed income investments are increasing in importance due to increasing worldwide demand for the liquidity, stability and diversification that bonds provide. Insurance companies have always been bond investors, since their projected claims payments are easily matched against their bond portfolios' principal and interest payments. Similarly, pension funds seek to match projected liabilities with the predictable cash flows that bond portfolios provide. And, as the world's population ages, individual investors tend to invest a greater proportion of their capital in bonds, since these provide the income and safety that they require, especially at retirement.

Just as investors' needs for bonds are increasing, we see that fixed income performance attribution methodologies also continue to evolve. Beginning with the use of simple equity models, bond attribution models have moved through several stages. Overly general models based on the Security Market Line dominated the early stages. Subsequent innovations included "modified equity" models that simply subtracted an impact from changes in Treasury rates and then employed the familiar equity model, treating the remaining return as a price residual. These simplistic models were inadequate because they failed to identify the structural factors that represented the bond portfolio manager's active investment process. Most recently we have seen a swing to the other extreme of complexity, with several versions of highly mathematical, multi-factor models that incorporate both structural and complex issue-specific factors. By mixing risk factors with issue selection parameters, these models produced results that were often incomprehensible due to their complexity and lack of intuitiveness – sometimes even for bond portfolio managers! The cost and complexity of these recent models have put them out of the reach of smaller companies and prevented them from gaining broad acceptance in the marketplace. In addition, there is some doubt as to whether these models are simply mathematical abstractions or are truly representative of the bond management process.

This paper outlines a reasonable balance between the simple intuition of the familiar sector-based approach and the inclusion of those unique, structural factors that are the key drivers of fixed income investing. The result is a straightforward approach to attribution that is truly representative of the fixed income portfolio manager's decision process. It explains the results of the active management of the portfolio's structural factors and identifies the active residual or "selection effect." Of critical importance is the benefit of a highly practical and intuitive model that is easy to explain to non-technical users and especially to clients. In addition, the model requires a minimum of data to represent the portfolio and to customize the benchmark, bringing this robust approach within the grasp of smaller firms that may have limited resources to devote to performance attribution.

GUIDELINES FOR PERFORMANCE ATTRIBUTION METHODOLOGIES

The purpose of performance attribution is to explain the relative benefits of the investment manager's active decision process. Therefore, any valid performance attribution methodology must reflect the investment process of the manager. Simply stated, the manager's decisions must be the basis for the attribution model. Second, the methodology must allow for the creation of a benchmark that represents the systematic risk and style characteristics of the manager's long-term investment strategy. Only a representative benchmark can evaluate the portfolio's risk-adjusted return and provide the basis for evaluating the impact of the manager's tactical allocation shifts. After explaining the return due to these structural factors, the residual return legitimately represents the remaining active return, which is generally due to idiosyncratic risk. This residual term is usually referred to as a "selection effect." In a bond portfolio, this selection effect is relatively small, since bonds are homogenous investments after adjusting for differences in risk and style. Unlike equities, where individual companies reflect a high degree of differentiation (even when they belong to the same industrial sector) bonds with common characteristics tend to sell at comparable prices and have equivalent returns. As a result, we can see that differences in performance between bond portfolios tend to be the result of different structural characteristics. A good bond performance methodology will control for these structural factors and will produce a relatively small selection effect.

A REVIEW OF BOND BASICS

The bond market is significantly larger than the stock market, both in terms of number of securities as well as money value. Stocks are only issued by public companies, whereas bonds are issued by both public and private companies, and by all levels of government (as well as by agencies of the government.) And where a company usually issues only a single series of common stock, that same company may issue several (and sometimes dozens of) series of bonds, each with its own unique characteristics. It has been said that one counts stocks by the thousands and counts bonds by the millions. Notwithstanding their importance as investments, bonds rarely make the news, whereas equities dominate the nightly news. This is understandable, since equities are leading indicators of an economy. Given their relative unfamiliarity compared with stocks, and given the emerging awareness of the drivers of bond performance, it is prudent to begin with a review of a few fundamental bond concepts and to compare bonds with stocks. This will help to explain why bonds require their own attribution model, and will also help to define the foundations of the bond performance model.

Bonds are distinguished from stocks by five factors: their investment horizon, their return potential, their markets, their risk factors and their opportunity for a true selection return. We will provide a brief review of each of these characteristics. A more in-depth discussion of bonds can be found in many investment texts, some of which are devoted exclusively to bonds.

1. Bonds are usually issued with a fixed term to maturity, at which time the amount borrowed has been fully repaid. By comparison, stocks are permanent investments.

2. Bonds provide a fixed return from required interest payments to the bondholders who are simply lenders. When companies prosper, this increased value goes to stockholders, who bear the risks of ownership. Where stocks have an unlimited upside, bonds have a limited return potential that is generally in line with their interest rates.

3. Bonds are often "buy and hold" investments, meaning that once purchased, they are held until maturity. As a result, the secondary market for bonds is quite limited. Aside from Treasury bonds, only a small proportion of bonds are bought and sold on a given day. As a result, bonds are less liquid than stocks, and they reflect a lower degree of reliability regarding their true market prices, especially for any individual bond. By comparison, stocks enjoy a highly active secondary market and reliable pricing.

4. Bond returns are driven by their expected interest income and also by changes in market yields. As market yields rise above a bond's stated income yield, the bond becomes relatively unattractive and its price falls. The opposite effect happens when market yields fall. Therefore, the main risk factor for a bond is its sensitivity to yield changes. By comparison, stock returns are driven by the performance of the market's economic sectors, and the main risk for a stock is its sensitivity to the overall market, measured by its beta statistic.

It is clear that the performance of bonds and stocks are driven by different factors, and that both bonds and stocks require their own performance attribution models. Stock performance is explained by general market sensitivity, sector exposure and issue selection, whereas bond performance is explained by interest income and the price sensitivity with respect to changes in market yields. Where the idiosyncratic selection effect in stock portfolios may be relatively large, the selection effect in bond portfolios is generally relatively small.

DECOMPOSING BOND RETURN

The return for any investment may be stated as its income return plus its price return. For a stock, this is represented by its dividend yield plus the return from its change in market price over the performance period. For a bond, this is represented by its coupon return and its return from price change (where price change is driven by yield change.) The coupon return is simply stated as the periodic coupon rate divided by the beginning period bond price. This periodic coupon rate is calculated as the annual coupon rate divided by the horizon of the performance period. For example, the monthly income return for a 6% coupon bond selling at $105.25 would be:

$$\textit{Income return} = \textit{Coupon/Frequency} \div \textit{Price} = [6 \div 12] \div 105.25 = 0.50 \div 105.25 = 0.475\%$$

The price return for any investment is driven by the ratio of ending price to beginning price. If this bond's price increased from 105.25 to 107.25 during the month, its price return would be:

$$\textit{Price return} = [\textit{Ending Price} \div \textit{Beginning Price}] - 1 = 107.25 \div 105.25 - 1 = 1.90\%$$

The total monthly return for our bond would be:

$$\textit{Total return} = \textit{Income return} + \textit{Price return} = 0.475\% + 1.900\% = 2.375\%$$

However, to understand the sources of bond price change, an understanding of bond yield and the sensitivity of a bond to this yield change is required.

REVIEW OF BOND YIELD

A bond's yield has two components:

1. A *risk free interest rate*, usually represented by a Treasury bond. Interest rates generally increase with the maturity of the bond, implying that longer-term

bonds carry greater price volatility then shorter-term bonds. When we plot the graph of yields relative to their maturities, this curve is usually "upward sloping" to compensate investors for the higher risk inherent in longer maturity bonds.

2. A *risk premium* that compensates investors for any uncertainties related to the receipt of interest and principal payments, as well as changes in this risk premium that could cause a decline in bond price. This premium is usually called a "Spread over Treasuries" or simply a "Spread." An increase in spread is called a "widening" while a decrease in spread is called a "tightening."

Bonds may carry any of four types of risk and each risk provides its own risk premium.

- *Interest rate risk*: This is the risk that bond prices will fall as Treasury rates rise. As current Treasury rates rise above the yields of existing bonds, these bonds become relatively undesirable. This lower demand results in a lower bond price. All bonds have interest rate risk, since all yields are based on the Treasury rate.

- *Credit risk*: This is the risk that either the borrower will default on payments or that the market will demand a higher risk premium for the existing risks of the bond. Bond risk is evaluated by rating agencies that assign bond ratings using a letter system. Investment grade bonds have a high likelihood of paying their interest and principal payments and are rated AAA, AA, A and BBB (in declining order of creditworthiness.) Credit risk is found in Corporate bonds, Asset Backed bonds (bonds backed by pools of card loans, credit card loans and home equity loans) and Commercial Mortgage Backed bonds (bonds backed by pools of mortgages on apartment houses, hotels, retail stores and other commercial buildings.)

- *Prepayment risk*: This is essentially the risk of a loss of expected income. As market yields fall, investors tend to refinance their existing bonds at lower rates, thereby lowering their interest costs. This leaves lenders to reinvest the proceeds at lower rates than their original bonds. Prepayment risk is primarily found in Mortgage Backed bonds where the price of the bond can only rise to a ceiling level of the price at issue ($100 or "Par.") Without a corresponding increase in price to compensate for lower future income levels, prepayment risk results in losses for bondholders.

- *Equity or Idiosyncratic risk*: The risk inherent in below investment grade bonds ("junk" bonds) where the probability of default is significantly higher than for investment grade bonds. Examples include low-rated corporate bonds as well as sovereign bonds from emerging economies. Ratings for speculative grade bonds are BB, B, CCC and D. These bonds share the idiosyncratic nature of equity because company-specific or country-specific issues dominate their pricing. For example, market yields may decline, causing general bond prices to rise, and yet an individual company may slip into bankruptcy and see a severe decline in the price of its bonds. Therefore, these lower-rated bonds reflect a higher likelihood of default, higher potential price volatility and a greater selection effect than investment grade bonds.

Bond yield is therefore the combination of three components:

- A short-term *risk free rate* that compensates the investor for inflation and provides a small real return

- A *maturity premium* to compensate for potential losses from unanticipated inflation.The sum of the risk free rate and the maturity premium is the *Treasury rate*.

- A *risk premium* to compensate for potential losses from defaults and/or prepayments

BOND DURATION: THE LINK BETWEEN YIELD AND PRICE

As stated, a bond's price changes inversely with changes in yields. As yields rise, bond prices fall; as yields fall, bond prices rise. The degree of price change depends on the bond's sensitivity to yield change. This sensitivity is referred to as the bond's "Duration" or more precisely it's "Modified Effective Duration." In practical terms, a bond's price will change approximately by its Duration for every 1% change in yield. For example, a 5-year duration bond will see its price rise approximately 5% if yields decline by 1% (100 basis points or 100 bps.) Conversely, this same bond will see its price decline by approximately 5% if yields increase by 100 bps. Bond Duration is the systematic risk measure for bonds. It is comparable to Beta for stocks. Just as market risk cannot be diversified away for equities, the risk of price declines from rising yields cannot be diversified away in a bond portfolio.

The calculation of bond duration is beyond the scope of this article, but this can be found in many investment textbooks. However, for definitional purposes a bond's duration can be calculated as the weighted average timing of its cash flows, using the present value of each cash flow as the weighting factor. This result is then divided by (1 + Yield) which creates a measure of the bond's price sensitivity. There is a small and generally immaterial error in this price change estimate, since the graph of bond price at each yield level is a curve rather than a straight line. Essentially, duration is the first derivative of the bond's price/yield curve and therefore a linear estimate of price change. The use of this linear estimate of price change implies a tendency to slightly overestimate price declines and slightly underestimate price increases. However, this estimation error is quite small for most bonds. Fortunately, any estimation error with regard to the Treasury component of price change will be reflected in the Spread component of price change.

Therefore, the general form for bond price change is:

Change in Bond Price = -Duration x Change in Yield

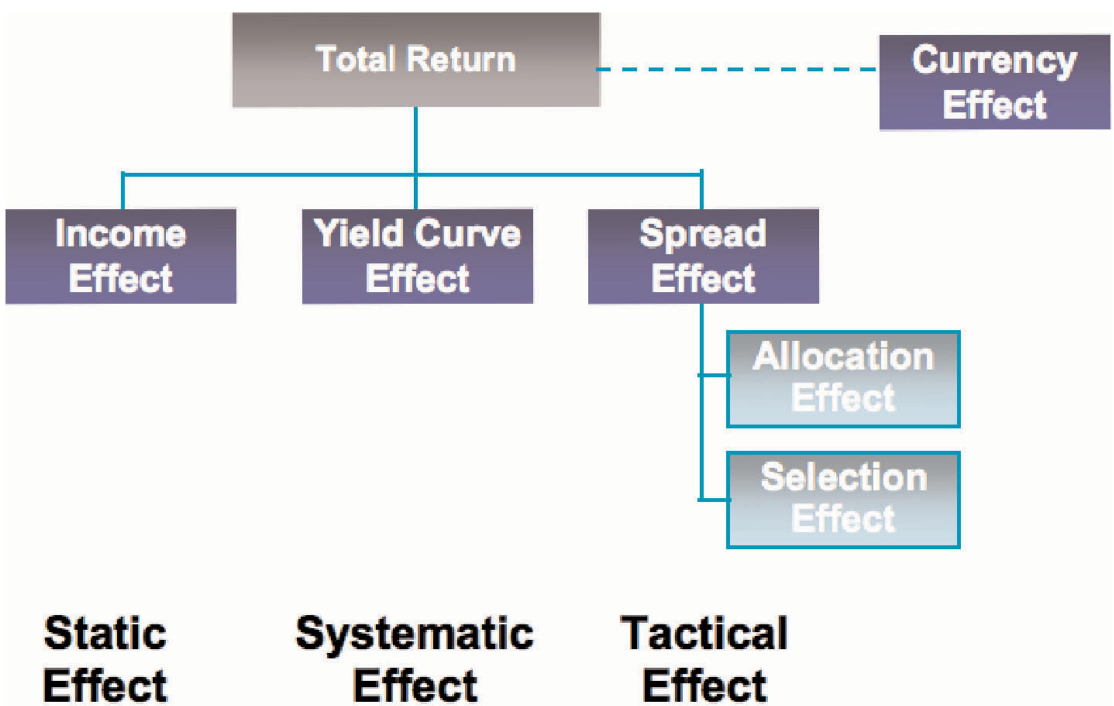
Two things are worth noting in this calculation. First, we employ the negative of Duration so that the inverse relationship between Price and Yield is preserved. Second, we see that a bond's yield may change due to changes in the Treasury rate, changes in the Risk Premium or both. We can decompose bond price return into its two components, Treasury return and Spread return, where:

- *Treasury return = -Duration x Treasury change*
- *Spread return = -Duration x Spread change*

DECOMPOSING BOND RETURN

Bond total return can be decomposed into its main structural components: Income return, Treasury return and Spread return. Any residual return not explained by these structural components is considered a Selection return, where:

Figure 1: Bond Attribution Model



- Selection return = Total return – Treasury return – Spread return

Therefore, the attribution model for bonds is:

- *Total return = Income return + Treasury return + Spread return + Selection return.*

The components of the attribution methodology are represented in Figure 1.

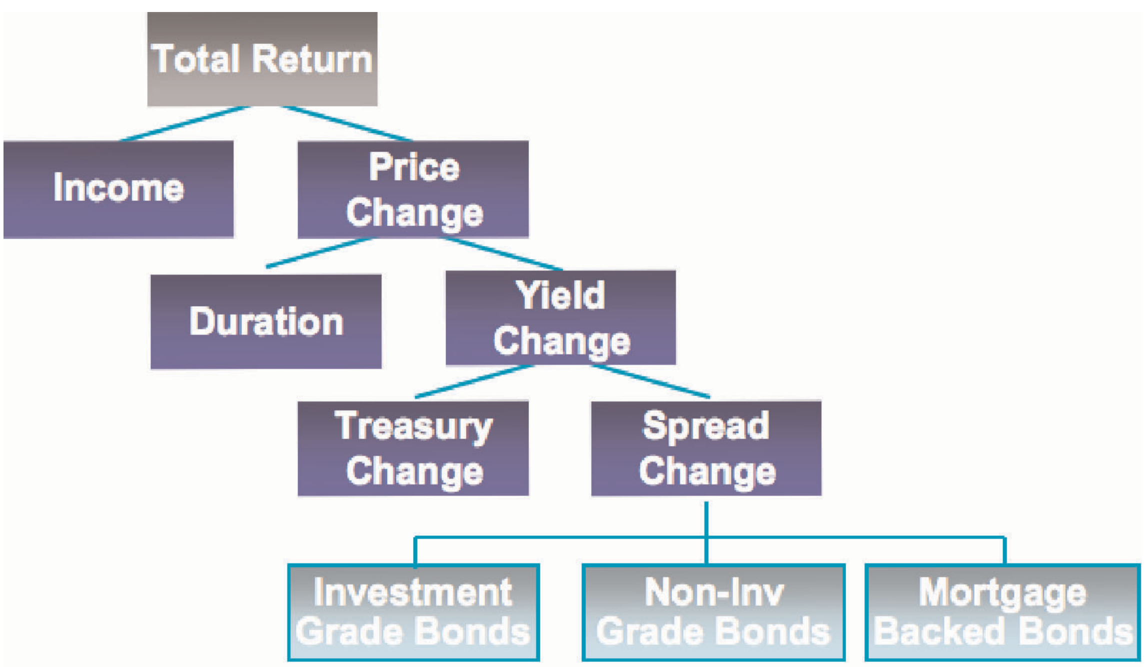
The data and general calculation process of the model are represented in Figure 2.

A SECTOR-BASED VIEW OF FIXED INCOME PERFORMANCE

Describing the sector groupings in a bond portfolio is more involved than for an equity portfolio. Since bonds are described using a set of structural factors and risk factors that occur simultaneously, it is possible to describe these bond sectors by several characteristics and to arrange these characteristics in any order of priority. For example, each bond belongs to a given market or issuer sector, which may be described as Governments, Corporates, Mortgage Backed, Asset Backed, Commercial Mortgage Backed, High Yield, Foreign and Emerging Markets. Each sector has a given maturity, making it more or less responsive to changes in yields. Additionally, each sector also has a rating. Collectively, these parameters fully describe or classify each bond. The potential for numerous groupings of bonds is potentially problematic. Therefore, it is essential to group the bonds in the way that best represents the manager's investment process.

Some analysts choose to view yield curve exposure (the weightings to the different points along the Treasury curve) as the primary investment decision factor, with the sector weightings taking a secondary position in the decision process. Others view the sector weightings as the primary investment decision, with yield curve exposure seen as a secondary decision that may be the result of active bets, issue selection or simply the availability of bonds in an illiquid market. A reconciliation of these positions is sometimes reflected in the "cell based" approach to sectors, where each intersection of sector and risk is treated as a unique "cell." Here, the portfolio's sectors are represented by the matrix of sector and yield curve exposures. Unfortunately, such schemes usually result in an incomprehensible array of dozens of individual cells without any clear and cohesive message.

Figure 2: Return Decomposition



In the straightforward model presented here, the durations of the various sectors are the basis for the portfolio's yield curve exposures. Rather than select the portfolio's yield curve exposures through arbitrary points (1-year, 2-year, 3-year, 5-year, 7-year, 10-year, 30-year) we will consider the sector durations to represent these "key rate durations" for the portfolio. There are two important reasons for this. First, a portfolio's yield curve distribution is a critical aspect of its structure and its return attribution, and so the values used to represent this curve distribution should be the result of observable decisions, rather than arbitrary points chosen for the convenience of data gathering. The portfolio manager selects bonds with specific durations to implement the sector strategy. Therefore, each sector's duration is the result of specific manager decisions and specific bond purchases. It is a delightful curiosity that it is easy to gather the durations of the portfolio's sectors and the corresponding sectors in the benchmark using readily available public data. However, it is difficult to gather all of the benchmark data sorted by the set of arbitrarily chosen yield curve points. In fact, such an approach may require having access to all of the benchmark's constituent bond holdings, which often number in the thousands or tens of thousands of bonds. This places such a model out of the reach of smaller firms. And so, the relevant, decision-based set of sector durations is more representative, less costly and less difficult to implement, while the arbitrary set of duration exposures is less relevant while involving higher cost and difficulty in implementation.

The second reason for joining the portfolio's sector durations and its yield curve exposures is because this approach reflects how bond managers determine sector weightings. A bond's weighting includes aspects of both money and time, so that the true weighting of a bond is not simply its market weighting, but rather the combined effect of weighting and duration, which bond managers refer to as *"Contribution to Duration."* Therefore, sector exposures can only be understood in terms of the simultaneous consideration of both weighting and duration. Since sector weightings cannot be separated from their durations, then sector durations must be a critical component of the portfolio's yield curve exposure. At a more practical level, a portfolio manager can only gain exposure to the yield curve through the purchase of bonds and so the yield curve and sector/issue decisions are made *simultaneously*, rather than in the bifurcated manner implied by the arbitrary selection of key rate durations. We conclude that using sector durations to represent Treasury curve exposure provides a methodology that is representative in its design and both practical and efficient in its implementation.

CALCULATIONS AND CASE STUDY

We will explain and illustrate the bond attribution model in the context of an analysis of an actual portfolio relative to its benchmark. Typical of most active bond strategies, this portfolio reflects significant tactical structural differences from its benchmark. These include different sector weightings, different levels of systematic risk in each sector (measured by duration) and a different set of exposures along the yield curve and a different level of overall systematic risk when compared with the benchmark. Not surprisingly, this reflects a different level of interest income and differences in the price risk resulting from different exposures to the drivers of yield change, namely Treasury rate and spread changes. We will evaluate the differences in performance caused by these structural differences and explain the remaining selection return.

DATA REQUIREMENTS

We begin by gathering and calculating the basic data required for the model. This data will form the basis for the attribution analysis. While this model can be analyzed at any level of granularity (down to the issue level) we will proceed at the sector level. It is worth noting that since this is a linear model, the attribution results will be the same whether the model is implemented at the issue level or any other level of aggregation. The basic data for the model includes:

- Market value weighting (%)
- Return
- Coupon
- Beginning price
- Beginning duration
- Treasury curve at beginning and end of performance period

Using the basic data in Tables 1 and 2 we will calculate the remaining data needed for the attribution analysis:

- Par value weighting (%)
- Treasury change at the point matching each sector duration ("Duration Matched Treasury")
- Spread change for each benchmark sector

IDENTIFYING THE APPROPRIATE WEIGHTING FACTORS

Risk and return values are weighted by market value weightings. However, by definition coupon and price values are weighted by par value. So, for each sector we calculate the par value in money terms and then find the percentage weighting of each sector's par value where:

Par value = Market value ÷ Price

We also note that Yield changes (Treasury, spread and total yield) are weighted by the compound factor of Market Percentage x Duration (or "Duration Contribution") and this weighted product is then divided by the average Duration. We will also express the Duration Contribution of each sector as a percentage of total Duration. *Therefore, there are three relevant weighting factors for fixed income attribution: Market, Par and Duration Contribution.*

Table 3 contains the Treasury curves and the rate changes over the performance period. Beginning with the published Treasury returns (in bold font) we used linear interpolation to calculate the intermediary points along the curve in 1/8 year increments. The changes in rates are simply the differences between the two Treasury curves. The final step in identifying the basic data is to find the Treasury change that most closely matches each sector's duration for each sector. (One may also interpolate to find the yield curve point to match each sector's duration exactly; this is a matter of personal judgment and will not have a material effect on the attribution results.)

CALCULATING THE COMPONENT RETURNS

Now that we have the required data for the benchmark, we will calculate each of the component returns ("Effects") that sum to the total return for each sector. We will illustrate each calculation using the data from the benchmark's Corporate sector.

- **Income Effect** or Interest earned on Market Value Invested = **Periodic Coupon ÷ Price**

  0.49 ÷ 105.57 = 0.4641% or 0.46% (result expressed to one bps of precision)

- **Treasury Effect** or Price change from Treasury change = **- Duration x Treasury change**

Table 1: Basic Benchmark Data

Sector	Market (%)	Return	Coupon	Price	Duration
Governments	36.50%	1.47	0.38	104.55	4.76
MBS	34.40%	0.83	0.45	100.42	3.05
ABS	1.30%	1.10	0.36	99.91	2.96
CMBS	3.50%	1.61	0.46	102.49	4.58
Corporates	24.30%	1.52	0.49	105.57	5.82
Total	100.00%	1.26	0.43	103.20	4.40

Table 2: Basic Portfolio Data

Sector	Market (%)	Return	Coupon	Price	Duration
Governments	20.50%	1.85	0.45	112.58	6.31
MBS	23.00%	0.83	0.49	102.75	2.87
ABS	6.50%	1.10	0.36	101.75	2.53
CMBS	8.00%	1.71	0.49	103.12	4.67
Corporates	42.00%	1.26	0.47	103.26	4.15
Total	100.00%	1.31	0.47	104.81	4.23

Table 3: Yield Curves at Beginning and End of Performance Period

Maturity	Begin	End	Change
0.250	**1.125**	**0.975**	**-0.1500**
0.500	**1.640**	**1.420**	**-0.2200**
1.000	**1.850**	**1.600**	**-0.2500**
1.500	2.067	1.817	-0.2500
2.000	**2.290**	**2.070**	**-0.2200**
2.500	2.390	2.160	-0.2300
2.625	2.415	2.183	-0.2325
2.750	2.440	2.205	-0.2350
2.875	2.465	2.228	-0.2375
3.000	**2.49**	**2.25**	**-0.2400**
3.125	2.524	2.283	-0.2413
3.250	2.558	2.315	-0.2425
3.500	2.625	2.380	-0.2450
4.000	2.760	2.510	-0.2500
4.125	2.794	2.543	-0.2513
4.250	2.828	2.575	-0.2525
4.375	2.861	2.608	-0.2538
4.438	2.88	2.62	-0.2544
4.500	2.895	2.640	-0.2550
4.625	2.929	2.673	-0.2563
4.750	2.963	2.705	-0.2575
4.875	2.996	2.738	-0.2588
5.000	**3.03**	**2.77**	**-0.2600**
5.125	3.049	2.786	-0.2623
5.250	3.067	2.803	-0.2645
5.375	3.086	2.819	-0.2668
5.500	3.104	2.835	-0.2690
5.625	3.123	2.851	-0.2713
5.750	3.141	2.868	-0.2735
5.875	3.160	2.887	-0.2735
6.000	3.178	2.900	-0.2780
6.125	3.198	2.923	-0.2757
6.250	3.219	2.945	-0.2735
6.500	3.259	2.990	-0.2690
7.000	**3.340**	**3.080**	**-0.2600**
7.500	3.414	3.145	-0.2690
8.000	3.488	3.210	-0.2780
8.500	3.562	3.275	-0.2870
9.000	3.636	3.340	-0.2960
9.500	3.703	3.380	-0.3230
10.000	**3.770**	**3.420**	-0.3500
20.000	**4.490**	**4.100**	-0.3900
30.000	**4.490**	**4.130**	-0.3600

-5.82 x -0.2735% = 1.5918% or 1.59%

- **Spread Effect** or Price change from change in Spread

  Since benchmarks contain a fixed set of securities there is no selection effect. Therefore, all remaining return is the result of spread change. So, for the benchmark:

  Spread Effect = Total return minus Income effect minus Treasury effect

  1.52% – 0.4641% – 1.5918% = -0.5359% or -0.54%

- **Spread Change = Spread Effect ÷ - Duration**

  -0.5359 ÷ -5.82 = +0.0921% or 9 bps
  This *Benchmark Sector Spread Change* is the relevant market data that we will use in calculating the Spread Effect for the corresponding sector in the portfolio, where:

  Portfolio Spread Effect = - Duration x Benchmark Spread Change

Tables 4 and 5 include the data for the benchmark and the portfolio including the totals for each of the factors (return, coupon, price, duration, Treasury change, spread change.)

Before preparing the analysis of return and excess return (or "contribution to return" and "attribution analysis") we will first examine the relative exposures in terms of both money and risk (see Figures 3 and 4). This will provide significant guidance to the subsequent analysis.

Sector exposure is also determined by degree of systematic risk, measured by duration. For bond managers, the more meaningful exposure is in terms of both market value and systematic risk, or "contribution to duration."

The sector durations also illustrate the yield curve exposures. That is, the sector durations represent the key rate duration exposures of the benchmark and the portfolio. This graph provides additional insight into the

Table 4: Benchmark Data

Sector	Market (%)	Par (%)	Duration Contribution (%)	Return	Coupon	Price	Duration	Treasury Change	Spread Change
Governments	36.50%	36.0%	39.5%	1.47	0.38	104.55	4.76	-0.2575	0.0250
MBS	34.40%	35.4%	23.8%	0.83	0.45	100.42	3.05	-0.2400	0.1148
ABS	1.30%	1.3%	0.9%	1.10	0.36	99.91	2.96	-0.2400	-0.0099
CMBS	3.50%	3.5%	3.6%	1.61	0.46	102.49	4.58	-0.2563	0.0028
Corporates	24.30%	23.8%	32.1%	1.52	0.49	105.57	5.82	-0.2735	0.0921
Total	100.00%	100.0%	100.0%	1.26	0.43	103.20	4.40	-0.2583	0.0669

Table 5: Portfolio Data

Sector	Market (%)	Par (%)	Duration Contribution (%)	Return	Coupon	Price	Duration	Treasury Change	Spread Change
Governments	36.50%	36.0%	39.5%	1.47	0.38	104.55	4.76	-0.2575	0.0250
MBS	34.40%	35.4%	23.8%	0.83	0.45	100.42	3.05	-0.2400	0.1148
ABS	1.30%	1.3%	0.9%	1.10	0.36	99.91	2.96	-0.2400	-0.0099
CMBS	3.50%	3.5%	3.6%	1.61	0.46	102.49	4.58	-0.2563	0.0028
Corporates	24.30%	23.8%	32.1%	1.52	0.49	105.57	5.82	-0.2735	0.0921
Total	100.00%	100.0%	100.0%	1.26	0.43	103.20	4.40	-0.2583	0.0669

manager's decisions as to where to take interest rate risk and spread risk. Clearly the portfolio and the benchmark have different average durations, and different exposures along the yield curve. In terms of spread exposure, the manager has a greater weighting to the most of the "spread sectors" but takes less price risk by using shorter durations relative to the benchmark. In contrast, the manager takes greater price risk in the Treasury sector, which is also used to align the portfolio's duration more closely with the duration of the benchmark. This is a common bond management technique. This is illustrated in Figure 5.

CALCULATING THE RETURN DECOMPOSITION ("CONTRIBUTION TO RETURN")

Using the data and calculations above, we can now prepare an analysis of the return of each sector in the benchmark and in the portfolio. We also provide the

Figure 3: Sector Allocations by Market Weight

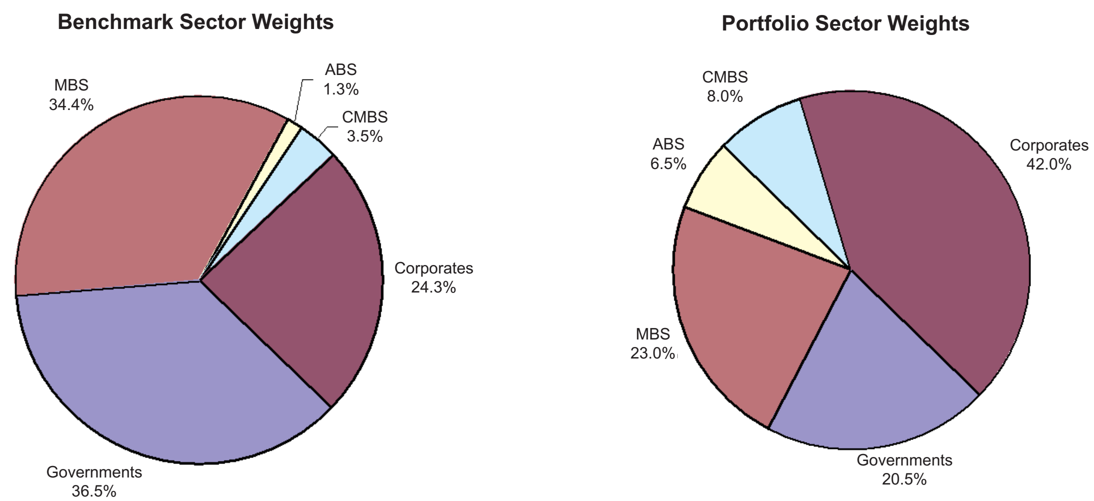
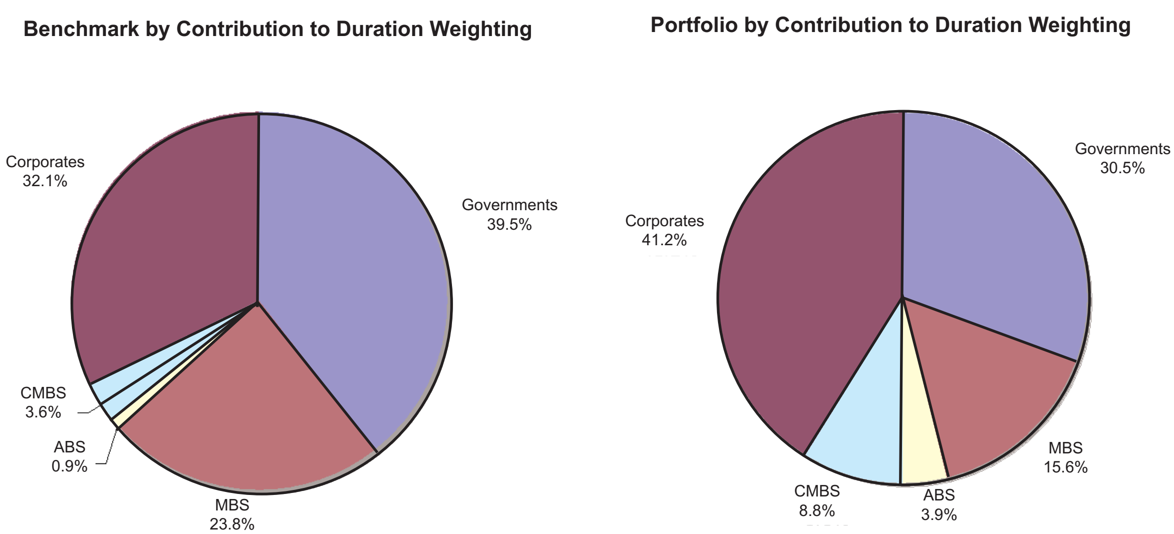


Figure 4: Sector Allocations by Contribution to Duration Weight



summary return for each model factor (*Income, Treasury, Spread, and Selection*.) These are illustrated in Tables 6 and 7.

The linear nature of this model allows for unlimited drill down capability. That is, each sector can be further broken out into its meaningful subcomponents or sub-sectors and then rolled up to the aggregate sector level. For example, Governments can be broken out into Treasuries vs. Agencies, and Corporates can be broken out by industrial sectors. Customizing the benchmark is a simple matter of rearranging the data and calculating weighted average factor values for the revised groupings. The flexibility of the model is unlimited with regard to groupings by sector or any other combination of relevant factors.

The totals for each return component can be calculated in either of two ways. First, these may be derived by simply taking the weighted average of the sector effects using market value as the weightings. Or they may be calculated directly using the total values at the portfolio level for the basic data. Bear in mind that each component of return has its own weighting factor. Coupon and price are weighted by percentage of par value. Duration is weighted by percentage of market value. Treasury change and spread change are weighted by percentage contribution to duration, or stated differently, they are

Figure 5: Yield Curve Exposures

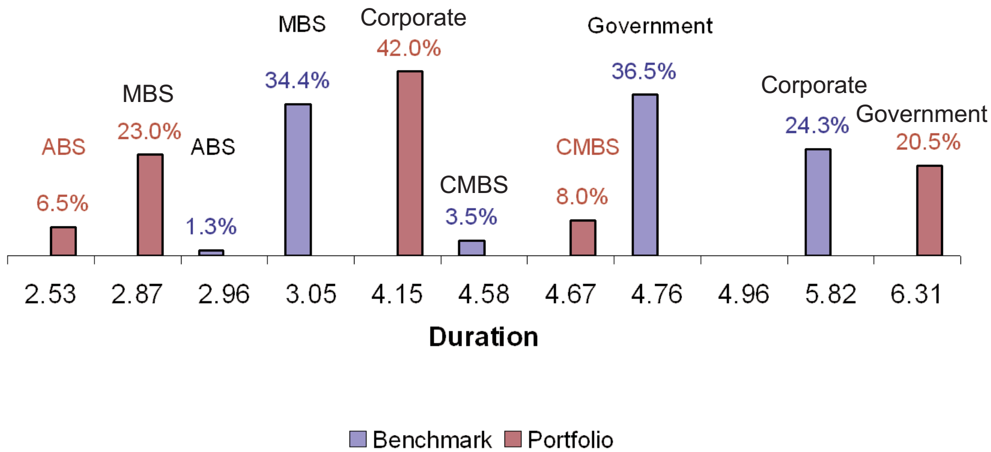


Table 6: Benchmark Return Analysis

	Income Effect	Treasury Effect	Spread Effect	Selection Effect	Total
Governments	0.36	1.23	-0.12	0.00	1.47
MBS	0.45	0.73	-0.35	0.00	0.83
ABS	0.36	0.71	0.03	0.00	1.10
CMBS	0.45	1.17	-0.01	0.00	1.61
Corporates	0.46	1.59	-0.54	0.00	1.52
Total	***0.42***	***1.14***	***-0.29***	***0.00***	***1.26***

Table 7: Portfolio Return Analysis

	Income Effect	Treasury Effect	Spread Effect	Selection Effect	Total
Governments	0.40	1.65	-0.16	-0.04	1.85
MBS	0.48	0.69	-0.33	-0.01	0.83
ABS	0.35	0.57	0.03	0.15	1.10
CMBS	0.48	1.21	-0.01	0.04	1.71
Corporates	0.46	1.06	-0.38	0.13	1.26
Total	***0.44***	***1.07***	***-0.27***	***0.06***	***1.31***

Table 8: Total Values for Benchmark and Portfolio

	Benchmark	Portfolio
Coupon	0.43	0.47
Price	103.20	104.81
Duration	4.40	4.23
Treasury Change	-0.2583	-0.2538
Spread Change	0.0669	0.0633

weighted by the product of percentage market times duration, and the sum is then divided by the average duration. These total values are shown in Table 8.

Using these summary statistics, we can now calculate the contribution to return for each component, and then simply subtract the benchmark value from the portfolio value to calculate the contribution to excess return for each component. We illustrate these summary-level calculations for the benchmark:

Benchmark Income Effect = 0.43 ÷ 103.2 = 0.42%
Benchmark Treasury Effect = - 4.40 x - 0.2583 = 1.14%
Benchmark Spread Effect = - 4.40 x 0.0669 = -0.29%
(this is also the residual value)
Total Return 1.26%
(difference due to rounding of display)

PREPARING THE ATTRIBUTION ANALYSIS AT THE PORTFOLIO LEVEL

Once the return decomposition has been prepared for the benchmark and the portfolio, we simply subtract the values to produce the attribution results. This is illustrated in Table 9.

What if we had used the traditional equity method to prepare the attribution analysis? Simply stated, we would have ignored the significant income component of return, and we would have ignored the differences in risk that were the key drivers of the changes in price. Whenever risk is incorrectly specified the attribution results will be incorrect and we will reward or punish managers for differences in risk and style rather than for their skill. We see this clearly in our example, where the use of the equity method produces a negative selection effect instead of a significant positive effect. This is illustrated in Table 10.

ATTRIBUTION RESULTS AT THE SECTOR LEVEL

We can extend our analysis to show each sector's contribution to excess return. In fact, we can analyze each sector's contribution to the excess return from every return effect: Income, Treasury, Spread and Selection. This process is a simple matter of treating each return effect as its own entity. (We can visualize each return effect as though it were a total portfolio return.) We then apply the standard equity attribution model to derive each sector's contribution to that return effect. Since the return effects have already been created using the proper income and risk factors, we are free to apply the equity model to these results as a means of assigning the excess return already calculated. To do this, we will make the familiar references to an "allocation" compo-

Table 9: Summary Level Attribution Analysis

	Benchmark	Portfolio	Alpha
Income	0.42	0.44	0.02
Treasury	1.14	1.07	-0.06
Spread	-0.29	-0.27	0.03
Selection	0.00	0.06	0.06
Total	***1.26***	***1.31***	***0.05***

Table 10: Attribution Results Using the Equity Method

	Allocation	Selection	Total
Governments	-0.03	0.08	0.04
MBS	0.05	0.00	0.05
ABS	-0.01	0.00	-0.01
CMBS	0.02	0.01	0.02
Corporates	0.05	-0.11	-0.06
Total	**0.07**	**-0.02**	**0.05**

nent" and a "selection" component. The "allocation" component will be driven by the portfolio's differences in sector weighting (relative to the benchmark) and the relative advantage that each sector brings within the benchmark return for the respective fixed income factor. The "selection" component is calculated as the portfolio weighting times the difference in return between the portfolio sector and its corresponding benchmark sector. We will illustrate by calculating each sector's contribution to excess return from exposure to the Treasury curve.

We note that the standard attribution "ingredients" are used to assign the contributions to excess return to each of the fixed income return factors: Income, Treasury and Spread. These ingredients include:

- Weighting Difference: Portfolio weighting minus Benchmark weighting
- Sector Opportunity: Benchmark sector return minus Benchmark total return (this is stated for each fixed income return component)
- Portfolio Opportunity: Portfolio sector return minus benchmark sector return

Using these terms, we can create the "allocation" and "selection" components for each sector's contribution to the excess return for each respective fixed income factor (Income, Treasury, and Spread.) We note that the "allocation" portion measures the impact from differences in money weighting, while the "selection" component measures the impact from risk or otherwise differently structured investments. When these two components are added together they sum to each sector's contribution to excess return for each fixed income factor. Of course, the sum of all of the sectors' excess return contributions equals the total contribution to ex-

Table 11: Assigning Contributions to Excess Return using "Brinson" Equity Approach

	(a) Portfolio Weighting	(b) Benchmark Weighting	(c) Weighting Difference	(d) Portfolio TSY Return	(e) Benchmark TSY Return	(f) Sector Opportunity: Benchmark Sector Return minus Benchmark Total Return	(g) Portfolio Opportunity: Portfolio Sector Return minus Benchmark Sector Return	(h) = (c) x (f) "Allocation Effect:" Weighting Difference x Sector Opportunity	(i) = (a) x (g) "Selection Effect:" Portfolio Weighting x Portfolio Opportunity	(j) = (h) + (i) "Alloction plus Selection:" Total Contribution to Excess Return
Governments	20.50%	36.50%	-16.00%	1.6532	1.2257	0.0894	0.4275	-0.0143	0.0876	0.0733
MBS	23.00%	34.40%	-11.40%	0.6885	0.7320	-0.4043	-0.0435	0.0461	-0.0100	0.0361
ABS	6.50%	1.30%	5.20%	0.5680	0.7104	-0.4259	-0.1424	-0.0221	-0.0093	-0.0314
CMBS	8.00%	3.50%	4.50%	1.2086	1.1739	0.0375	0.0347	0.0017	0.0028	0.0045
Corporates	42.00%	24.30%	17.70%	1.0566	1.5918	0.4555	-0.5352	0.0806	-0.2248	-0.1442
Totals				**1.0746**	**1.1363**			**0.0919**	**-0.1536**	**-0.0617**

cess return. In our example illustrating the Treasury return, we can see that this sum (- 6 bps) is equal to the difference in the Treasury return for the portfolio and for the benchmark (as illustrated in Table 9.) With all this in mind, we can now complete the analysis for each of the fixed income return factors within all of the sectors. This provides a robust and granular explanation of the sources of excess return. For example, we may examine the portfolio's contribution to excess return either by fixed income effect (Income, Treasury, Spread and Selection) or by sector (Governments, MBS, ABS, CMBS and Corporates) or by both. These results are illustrated in Table 12.

SOME INSIGHTS INTO THE ATTRIBUTION ANALYSIS

Some of these attribution results may seem obvious, while others may seem somewhat counter-intuitive. For example, it seems obvious that the portfolio should have underperformed the benchmark with regard to its Treasury return. Given a decline in rates accompanied by a lower duration relative to the benchmark, we would expect a smaller price increase in the portfolio than in the benchmark. This intuition is confirmed by the 6 bps of underperformance caused by the mismatch of duration in the portfolio, most of which was caused by the shorter duration of the Corporate sector.

However, given the increase in spreads and given the portfolio's higher allocation to corporate bonds, we might have expected to see a corresponding degree of underperformance in the bond sector's contribution to excess return from Spread Effect. Instead, we see that corporate bonds made a positive contribution to relative spread effect. How could this be? The simple explanation is that, as previously stated, the corporate bonds in the portfolio had a lower duration than the corresponding bonds in the benchmark. The manager made a deliberate decision to have less "spread duration" and as a result, the portfolio was somewhat shielded from the price decline experienced in the benchmark. An additional insight can be gained by examining the "allocation" and "selection" components of this detailed sector analysis. In the case of corporates, we can see that the "allocation" effect (driven simply by market exposure) was negative, since we overweighted a sector that experienced a negative return from spread widening. However, the lower duration produced a smaller price decline that is reflected in the "selection" effect. The positive selection component of about 6 bps (reflecting lower risk) dominated the negative allocation component of about 4 bps (reflecting higher market weighting) to produce an overall positive impact of 2 bps on the relative performance of the portfolio. Once again, this shows that exposure in a bond portfolio is the result of both weighting and risk, and that if risk is ignored then attribution results will be incorrect (especially with regard to sector allocation effects.)

SUMMARY AND CONCLUSIONS

Bonds are a significant part of almost every investment portfolio and clients require an understanding of the absolute and relative performance of their bond portfolios. While equities have enjoyed a general consensus on how to analyze relative performance, bonds have begun their evolution toward an attribution methodology that is representative of the manager's investment process, robust in its results, reasonable in its cost and responsive to clients' information needs. Above all, the model must deliver answers to clients' questions in an intuitive man-

Table 12: Detailed Attribution Analysis by Sector

	Income	Treasury	Spread	Selection	Total
Governments	0.02	0.07	-0.04	-0.01	0.04
MBS	0.00	0.04	0.01	0.00	0.05
ABS	0.00	-0.03	0.02	0.01	-0.01
CMBS	0.00	0.00	0.01	0.00	0.02
Corporates	0.00	-0.14	0.02	0.05	-0.06
Totals	**0.02**	**-0.06**	**0.03**	**0.06**	**0.05**

ner that is easy to understand.

This straightforward bond attribution model meets all of these requirements. It identifies the key structural drivers of bond performance and of the manager's active investment process. These drivers include income, yield curve positioning and relative duration, sector exposure and spread management, and bond selection. Given the linear characteristic of its design, the model is unlimited in its flexibility in adapting to various sector breakouts and any sector/sub-sector hierarchies that drive the investment process.

Equally important, this method allows complete customization of the relevant benchmark, so that differences of risk and style are incorporated into the attribution analysis. Additionally, the attribution analysis may be calculated and presented in a variety of ways, using either granular factors at the sector/sub-sector level or at the total portfolio level. This ability to "drill down" or "roll up" the analysis is key to satisfying the needs of the various constituencies that will use this attribution analysis. As a result, this model can be used to provide meaningful results that are a critical part of the investment management process.

APPENDIX: ADDITIONAL INSIGHTS

A) Calculating parallel vs. non-parallel Treasury effects

We had already noted that each sector's economic exposure is described in terms of both money and risk. That is, a sector's true weighting is essentially the product of its market value percentage and its duration. This distribution of sector durations also forms the effective Treasury curve distribution, and this curve distribution is then consolidated into a single duration statistic. Stated differently, each portfolio or benchmark has an average duration around which its sector durations are distributed. This approach unifies the sector weighting and the yield curve distribution decisions and ensures that the "partial durations" along the yield curve are the result of deliberate decisions rather than arbitrarily chosen points. (These decisions include both the money weighting of each sector as well as the amount of systematic risk in the sector resulting from the selection of bonds with specific durations as chosen by the manager.)

We have calculated the impact of this distribution of durations or "curve exposure" in terms of the price return due to changes in Treasury rates along the curve, where each sector's (negative) duration is multiplied by the corresponding yield curve change to compute the respective price change. This price change can be further broken out into the change attributed to parallel and non-parallel yield curve movements. That is, we can evaluate the impact in a general change in the level of rates (the "parallel" or "shift" effect) as well as the impact of changes of different magnitude at the short end vs the long end of the curve (the "non-parallel" or "twist" effect.)

To calculate the parallel yield curve effect, we must first identify the point on the yield curve that will be used to represent the parallel change in rates. This is sometimes treated as an arbitrary decision, often selecting the 10-year point as the most representative measure of yield curve change. Others simply select the 10-year point because it is widely quoted, producing a readily available number. However, our viewpoint is that it is wise to select a widely-quoted point on the yield curve that corresponds closely to the long term average duration of the bond portfolio. For example, many diversified bond portfolios have durations closer to the 5-year point than to the 10-year point, and so we feel that, for these portfolios, the 5-year point on the curve is the more representative and therefore a better estimate for a parallel change in rates. Notwithstanding, any point selected along the curve will do, so long as one realizes that the split between "parallel" and "non-parallel" return is largely driven by the selection of the point on the yield curve chosen to represent the parallel change in rates. (*To visualize this, imagine a "flattening" non-parallel yield curve shift where short rates rise and long rates decline, and where the 5-year point is the "pivot point" for this yield curve movement. If the 5-year point is selected to represent the amount of parallel movement, then all of the Treasury curve impact will be considered non-parallel in nature. However, if the 10-year point is selected to represent the parallel shift, then the majority of the Treasury impact will be parallel in nature.*)

We recall that the Treasury return is calculated as:

Treasury return = - Duration x Treasury Change (at the point of the yield curve matching that duration)

We then define the following additional terms which decompose the Treasury change:

- "Key Rate Duration Change" or "Δ KRD" is the Treasury change corresponding to the selected parallel shift point (the 5-year point)
- "Duration Matched Treasury Change" or "Δ DMT" is the Treasury change corresponding to the sector duration

With these terms in mind, we can calculate the parallel return ("Shift") and the non-parallel return ("Twist") as follows:

- Shift return = - Duration x Change in Key Rate Duration or ***[-D x Δ KRD]***
- Twist return = - Duration x (Change in Duration Matched Treasury minus Change in Key Rate Duration) or ***[-D x (Δ DMT – Δ KRD)]***

We can easily see that these calculations consolidate into the total Treasury Return calculation as previously stated.

Let's demonstrate the breakout of the Treasury return for the Governments sector.

	Duration	Δ DMT	Δ 5yr KRD	Δ[DMT- ΔKRD]
Portfolio	6.31	-0.2620	-0.2600	-0.0020
Benchmark	4.76	-0.2576	-0.2600	+0.0025

- Portfolio Sector Shift Return =
  -6.31 x -0.26 = +1.6406
- Benchmark Sector Shift Return =
  -4.76 x +0.26 = +1.2376
- Portfolio Sector Twist Return =
  -6.31 x -0.0020 = +0.0126
- Benchmark Sector Twist Return =
  -4.76 x +0.0025 = - 0.0119

We can now sum these effects to the total Treasury return as previously stated:

- Portfolio Treasury Return =
  +1.6406 + 0.0126 = +1.6532
- Benchmark Treasury Return =
  +1.2376 + (-0.0119) = +1.2257

We can calculate the parallel and non-parallel effects for each sector, and then calculate each sector's contribution to excess return from the "shift" and "twist" components of the Treasury return, using the same attribution methodology we applied to the other performance factors. These return and attribution results are outlined in Tables 13 and 14.

B) Additional commentary on the Spread Effect

The three return components identified in this model are complete in describing the active decisions of the manager in terms of how bond portfolios are structured and managed. These components include income, price sensitivity to changes in the risk free rate through a portfolio's exposure to the yield curve ("Treasury" effect or "Duration" effect) and the price change from exposure to risk premia through the Spread effect. The sum of these return effects produces the "explained" return. The

Table 13: Treasury Return Analysis

	Portfolio Treasury Return	Portfolio Shift Return	Portfolio Twist Return	Benchmark Treasury Return	Benchmark Shift Return	Benchmark Twist Return
Governments	1.6532	1.6406	0.0126	1.2257	1.2376	-0.0119
MBS	0.6885	0.7462	-0.0577	0.7320	0.7930	-0.0610
ABS	0.5680	0.6578	-0.0898	0.7104	0.7696	-0.0592
CMBS	1.2086	1.2142	-0.0056	1.1739	1.1908	-0.0169
Corporates	1.0566	1.0790	-0.0224	1.5918	1.5132	0.0786
Total	**1.0746**	**1.1010**	**-0.0264**	**1.1363**	**1.1439**	**-0.0076**

Table 14: Treasury Return Attribution Analysis

Attribution Analysis	Treasury	Shift	Twist
Governments	0.0733	0.0676	0.0057
MBS	0.0361	0.0292	0.0068
ABS	-0.0314	-0.0267	-0.0047
CMBS	0.0045	0.0040	0.0005
Corporates	-0.1442	-0.1170	-0.0272
Total	**-0.0617**	**-0.0429**	**-0.0188**

residual of the observed return compared with the explained return is the Selection effect. In a valid fixed income model where a representative benchmark is used so that risks are clearly identified, this selection effect should be small, given the homogenous nature of individual bonds within each sector category.

It is important to note that the risk for each unique sector ("credit," "prepayment" and "equity") is the primary driver of the Spread effect. However, it is also important to note that there are certain other factors that contribute to the model's Spread effect. These factors tend to have a minor, inconsequential and sometimes even barely perceptible impact on return, especially in the short run. They also share a common characteristic of being different from the structural decisions that drive the portfolio management process. Rather, they tend to be related more to issue selection, rather than to deliberate decisions around the structure of the portfolio relative to the benchmark. And finally, these factors increase in complexity relative to the key active decision factors included in this model. For these reasons, these incidental factors are summarized and included in the Spread effect and are part of the change in spread for each sector. These factors include roll, prepayment impact, amortization impact and changes in option volatility.

These additional factors reflect the inherent error in complex fixed income models: they confuse issue selection factors with structural factors. As a result, these models are unrepresentative of the portfolio management process and bring a complexity, cost and confusion that severely limit the intuitiveness and understandability of the output, therefore rendering the output unusable to any but the most knowledgeable fixed income investors. Additionally, these models are generally out of reach of all but the largest investor shops, since their cost in terms of required data and analytical rigor is so high.

As a result, we may be confident that the true decision process of the investment manager is adequately and accurately represented in this fixed income attribution model. Further, we may be assured that the impact of these minor additional factors is represented identically in both the benchmark and the portfolio. We know this because the impact that these factors have on each of the benchmark sector's spread change is translated exactly to the portfolio: after all, the spread change for the benchmark sector is used for the corresponding portfolio sector. Any difference in the portfolio's minor factors will appropriately be reflected in the selection effect where it belongs.

C) The Issue of Convexity in Fixed Income Attribution

We understand that the duration measure assumes that price changes resulting from changes in yield may be estimated via a linear relationship. This estimation is usually very close to the actual price change, so long as the change in yield is relatively small. However, the estimated price change using only duration will tend to overstate losses and understate gains, because the true relationship between price and yield is not linear, but rather is represented by a yield curve that is convex in nature (with both duration and gains increasing at an increasing rate as yields fall and both duration and losses decreasing at a decreasing rate as rates rise.) To address this estimation error, some include a measure of convexity in the calculation of estimated price change, noting that this convexity adjustment improves the estimated price change, but still leaves a very small residual error.

In this model, convexity is addressed by its contribution to spread change. This is especially evident in Mortgage Backed Securities (MBS) where negative convexity drives price down for any change in yield. This negative convexity is expressed through an additional widening of spread in the MBS sector, and this wider spread produces the same negative price impact that one would achieve using a "duration + convexity" construct. In positively-convex sectors (such as corporates) the convexity impact is reflected in a slight tightening of spread change, reconciling to the slightly higher price change observed for that sector. Therefore, we can see that the model's spread change already conveys information about each benchmark sector's convexity impact to the corresponding sector in the portfolio. Differences in convexity between the portfolio and the benchmark are therefore reflected in the Portfolio Selection effect. However, this impact will generally be quite small, since a) the portfolio and its appropriate benchmark will have convexities that are not materially different from each other and b) the impact of convexity is a very small component of price, as indicated in the following calculation:

Estimated Price Change = - Duration x Yield Change + .5 x Convexity x Yield Change Squared

Given the squaring of yield change for the convexity impact, the result is always quite small. In the sample portfolio used here, the impact of the greater portfolio convexity resulted in about ¼ of one basis point, clearly an immaterial impact that is already reflected in the Selection effect.

While the impact of convexity is small, the effect of its inclusion in the model is significant: *it turns a flexible and highly intuitive model into a severely limited one that actually produces less useful information about the investment process.* It is important to note that one cannot apply the breakout of yield change into its price components using a "duration + convexity" approach, since there is no mathematical equivalence when these components of yield change are separated. This is clearly demonstrated in the following calculation:

$$(-D \times \Delta Y + C/2 \times \Delta Y^2) \neq [-D \times \Delta T + C/2 \times \Delta T^2] + [-D \times \Delta S + C/2 \times \Delta S^2]$$

Where: ΔY = Yield Change, ΔT = Treasury Change, ΔS = Spread Change

Including convexity in the attribution model means that the critical breakout of a manager's decisions around both Treasury yield curve positioning and spread exposure would be lost in the hopes of pursuing a purely academic argument for "precision" that is clearly immaterial in its impact. Since including convexity into the core attribution model results in a significant loss of information, convexity should be evaluated only at the total yield level as a means of understanding its contribution to the selection effect. Although this can be accomplished at the sector level, we recommend analyzing the impact of convexity at the total portfolio level. Table 15 illustrates the impact of convexity at the total portfolio level.

We see clearly that the impact of differences in convex-

Table 15: Impact of Convexity as Part of Selection Effect

	Benchmark	Portfolio	Difference
Total Return	1.262	1.308	0.046
Income Return	0.420	0.444	0.024
Price Return (Includes Selection Effect)	0.842	0.864	0.022
** Duration	4.400	4.235	
** Convexity	-36.71	-23.27	
Implied Yield Change using Duration + Convexity model	-0.193	-0.205	
Duration Return	0.849	0.869	0.020
Convexity Return (Difference Reflected in Selection Effect)	-0.007	-0.005	**0.002**

Table 16: Impact of Convexity on Spread Change

Spread Change Differences: Duration Only vs Duration + Convexity Approaches (percentage basis)	Dur Only	Dur + Conv	Difference
Governments	0.0250	0.0273	-0.0022
MBS	0.1148	0.1092	0.0056
ABS	-0.0099	-0.0086	-0.0013
CMBS	0.0028	0.0048	-0.0020
Corporates	0.0921	0.0939	-0.0018
Total	0.0669	0.0671	-0.0002

ity is a mere 2/10ths of a bps and this is already reflected in the portfolio selection effect. As a result, we would note that the portfolio's previously stated selection return of 5.7 bps (.057%) contains a positive convexity impact of 0.2 bps, or that the "true" Selection effect is 5.5 bps (.055%) which nonetheless rounds to a 6 bps impact.

For those wishing to calculate the exact impact from convexity for each sector, and to roll this information up to the portfolio level, this can be accomplished by a two-step process:

a) Using the known total return and income return, calculate the implied total price change.

b) Using the implied total price change, solve for the implied total yield change using the "Duration + Convexity" model. This implicitly calculates the implied spread change, given the known change in the Treasury rate for each sector.

We can show that the impact on spread change resulting from including convexity in the model is quite small, even for the MBS sector, where negative convexity is most evident. The spread change for the "duration + convexity" approach is quite close to the spread change derived using only duration, as stated earlier in the article. For the total portfolio, the difference in spread change is a mere 2/100ths of a basis point (the difference between a 6.71 bps widening vs a 6.69 bps widening.) Even in the highly negatively convex MBS sector the difference is a mere ½ basis point. Table 16 illustrates the impact of including convexity in the calculation of spread change at the sector level, as well as at the total portfolio level.

This demonstrates that convexity is already accounted for adequately within this model by making a widening adjustment for negatively convex sectors, and making a corresponding tightening adjustment in positively convex sectors. Since both the benchmark and the portfolio are treated identically in the calculation of spread change, the impact of spread change on the difference in resulting price returns is as accurate for the "duration-only" approach as it is for the "duration + convexity" approach. However, with the duration-only approach of this model, we have the significant added benefit of gaining the ability to separate differences in price change to decisions around both yield curve positioning and spread exposure through sector exposure. Clearly, convexity impact is sometimes of interest and this can be calculated explicitly and most appropriately as an additional analysis to supplement the critical results of this model.