Enhancing Risk Parity by Including Views

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Abstract

Within the finance literature there is an apparent gap between the inherent ignorance of expected returns of a risk parity approach on the one hand and the assumed certainty of expected returns in a mean variance approach on the other. We propose a portfolio selection framework that allows an investor to position herself between these two extremes. Depending on the confidence in one's expected return estimates, the optimal portfolio will be tilted more towards the risk parity portfolio or to the mean variance portfolio. We illustrate the framework for an investor in an asset allocation context.

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Recently there has been increased interest in applying risk control techniques in an asset allocation context. Risk control strategies, such as 1/N or equal-weighting[1], volatility weighting[2], maximum diversification[3], and equal risk contribution or risk parity[4], serve to control the risk profile of an investment portfolio. Their primary goal is to avoid pockets of risk concentration and to achieve portfolio diversification against losses. Apart from this risk budgeting context, risk control – and especially risk parity – has gained popularity as a full-fledged investment criterion.[5] Under this "new paradigm" of investing, the significance of risk control is extended to offering opportunities to reap risk-adjusted outperformance. Back test results seem to suggest that controlling the risk dimension is sufficient to build a portfolio and as a result, risk control techniques started to compete with the standard mean-variance approach.

As investment criteria, mean-variance optimization (aiming to maximize the portfolio's Sharpe Ratio) on the one hand and risk control on the other, can be considered as two extremes. The latter criterion is expected returns agnostic whereas the former presupposes knowledge of expected returns. At the one extreme, the use of expected returns in the mean-variance portfolio optimization process is problematic in practice. Firstly, expected returns are notoriously hard to estimate *ex-ante*. Secondly, the mis-estimation of expected returns has a great impact on the composition of a mean-variance optimized portfolio[6]. So from this perspective, reducing the reliance on expected returns is certainly attractive. At the other extreme, it is questionable whether

[1] See DeMiguel, Garlappi and Uppal [2009].
[2] See Fleming, Kirby and Ostdiek [2001] and Hallerbach [2012].
[3] See Choueifaty and Coignard [2008].
[4] See Qian [2005] and Maillard, Roncalli and Teïletche [2010].
[5] See for example Amenc and Martellini [2014].
[6] Best and Grauer [1991] and Chopra [1993] show that slightly different expected returns may imply very different mean-variance optimized portfolios. Michaud [1989] shows that mean-variance optimization overweighs assets with a large ratio of expected returns to estimated variance; precisely these assets are likely to have large estimation errors. Michaud coined this effect the "error maximizing property of mean-variance optimization".

any historical outperformance of risk control strategies can be extrapolated into the future. So going forward, the use of risk control as the sole investment objective may have little appeal and falls short of including available information on expected returns.

In an almost natural way, these considerations suggest striking a balance between the extremes of the unsatisfactory and the utopian. In particular, we propose to use the Black and Litterman [1991, 1992] optimization process. Black and Litterman [1991, p.14] invoke an equilibrium argument as they start from a global market-cap weighted portfolio and use the implied equilibrium views "*except to the extent explicitly stated otherwise*". In our set-up however, , we use the risk parity portfolio as the reference portfolio, attuned to the specific investment universe of the investor. We consider it as the neutral starting point when only risk information is available. In line with Carhart, Cheah, De Santis, Farrell and Litterman [2014], who use a smart beta reference portfolio, we do not require macro-consistency[7] and consider the risk parity portfolio as the anchor in subsequent risk-return optimizations.

We therefore suggest the following step-wise portfolio selection procedure. Start by calculating the risk parity portfolio, which equalizes each asset's contribution to portfolio risk. This portfolio serves as a reference for our ex-ante views on expected returns. In step 2, apply reverse portfolio optimization and derive the implied expected returns (cf. Sharpe [1974]). These expected returns maximize the portfolio's Sharpe ratio. If the implied expected returns are equal to the perceived views, the portfolio at hand maximizes the Sharpe ratio and we are done. If not, then move to step 3: adjust the implied expected returns according to the confidence placed in one's ex-ante views and apply mean-variance optimization. After this process, the resulting

[7] A macro-consistent portfolio is a portfolio that everyone in the world can hold while markets clear. The global market cap weighted portfolio is the only macro-consistent portfolio and holding this self-rebalancing portfolio is the only macro-consistent investment strategy.

portfolio is somewhere in between the initial risk parity portfolio (with no confidence at all in one's views) and the maximum Sharpe ratio portfolio (with full confidence in one's views).

Our paper is closely related to the later work of Jurczenko and Teiletche [2015] who also apply the Black-Litterman approach. In our approach we focus on using the risk parity portfolio as the neutral starting point and on illustrating the changes in the portfolio weights when the confidence in the views changes. Jurczenko and Teiletche [2015] on the other hand build an analytical framework starting from the inverse volatility portfolio (where the portfolio weights are inversely proportional to volatilities) with a volatility target.

Roncalli [2015] presents an alternative way to incorporate expected returns into risk parity portfolios. He considers a generalized risk measure that comprises not only the risk dimension (volatility) but also the performance dimension (expected return). Examples are Value-at-Risk and Expected Shortfall under normality assumptions, which depend on the expected return. By using this generalized risk measure instead of volatility alone, risk parity portfolios can be interpreted as mean-variance portfolios that are optimized subject to a weight diversification constraint. A scaling parameter governs the trade-off between performance contributions and volatility contributions and determines whether the return component or the volatility component dominates in the risk contribution. Although this approach is analytically elegant, how the scaling parameter should be set is open for debate (comparable to choosing the risk aversion parameter in mean-variance optimization).

This paper is organized as follows. We start with a summary of the theoretical background of risk parity and the Bayesian portfolio revision procedure. We next apply these theoretical insights to an asset allocation set-up with three asset classes: equities, bonds and commodities.

Finally, we discuss the implications for the investment decision process and provide some suggestions for further research.

Risk parity portfolios

It is surprising that, while ignoring information on expected returns, risk control portfolios appear to have historically outperformed market cap weighted or mean-variance optimized portfolios, see for example Asness, Frazzini and Pedersen [2012]. However, some studies tune down the apparent outperformance of risk-based strategies by criticizing back-tests on technical grounds, see for example Anderson, Bianchi and Goldberg [2012] (who comment on Asness et al. [2012]) and Goldberg and Mahmoud [2013]. Even when back-test assumptions are realistic and fair, the outperformance of risk parity strategies can be linked to overweighting (and leveraging) asset classes that in the rear view mirror have paired high historical returns with low risk levels. This is most obvious for bonds over the past three decades. Given the current low interest rate environment, we deem extrapolating historical bond returns into the future not representative. The large losses incurred by risk parity funds in June 2013 following the US interest rate increases[8] should support a healthy skepticism about the past being a mirror of the future.

Nevertheless, risk control strategies can be an objective starting point in the absence of (confidence in the) views on expected returns. When an investor is ignorant about expected returns, the only advice one can give is to diversify. Under naïve diversification, the investor applies equal money weights to the assets in his portfolio. However, in a multi asset portfolio, there are marked differences between the riskiness of different asset classes. The volatility of equities, for example, is a multiple of the volatility of government bonds. So although an equally-weighted portfolio is perfectly balanced in terms of money weights, it can be very unbalanced in terms of risk weights.

[8] See for example “Fashionable ‘Risk Parity’ Funds Hit Hard” in The Wall Street Journal (June 27, 2013).

When we instead equate the risk contributions of the portfolio components, we obtain the risk parity portfolio.[9] In such a portfolio, no single asset dominates the portfolio risk profile. As a result, a risk parity portfolio is perfectly diversified in terms of risk (or equivalently: loss) contributions.[10]

In a risk parity portfolio, the weights of all N assets are proportional to the inverse of their betas with respect to the portfolio return:

$$w_i^{RP} \sim \frac{1}{\beta_{ip}} \tag{1}$$

where β_{ip} is the slope coefficient from a regression of the excess returns of asset i on the portfolio excess returns (for details, see the Appendix). Since by definition the contribution of each asset to portfolio risk must equal 1/N, the composition of the risk parity portfolio can easily be calculated by requiring that for each asset $w_i \beta_{ip} = 1/N$.[11] Note that we can rewrite β_{ip} as the product of (1) the correlation with the portfolio and (2) the quotient of the asset and portfolio volatility, so:

$$\beta_{ip} = \rho_{ip}\, \sigma_i / \sigma_p \tag{2}$$

Hence, when correlations are uniform or when ignored altogether, eq.(1) implies setting each weight proportional to the stand-alone volatility of the corresponding asset. Often normalization is applied to ensure the weights sum up to unity. This yields the inverse volatility portfolio (IVP).[12] Neglecting correlation information makes IVP a "naïve" risk parity strategy. Volatility weighting has been applied for long by practitioners to improve cross-asset comparability and to reduce portfolio or strategy risk. This practice may be inspired by statistics, where inverse variance

[9] The term "risk parity" is confusingly also used in a generic sense for other risk controlled portfolios, but here we reserve this term for equal risk contribution portfolios. The seminal paper on risk parity is Qian [2005]. For an in-depth overview of risk controlled portfolios, we refer to Roncalli [2013]. Hallerbach [2015] provides a theoretical overview, an empirical illustration and a critical evaluation of risk control strategies.
[10] See Qian [2006].
[11] For an overview of algorithms to calculate the risk parity portfolio composition, we refer to Roncalli [2013].
[12] See Maillard et al [2010]. The IVP is sometimes confusingly denoted as the Equal Risk Budget portfolio, see Leote de Carvalho et al. [2012].

weighting is used to minimize the variance of the sum of two or more random variables.[13] The IVP is equivalent to the risk parity portfolio when there are only two assets (in the two-asset case, the correlation is irrelevant), or when correlations are uniform. When both correlations and volatilities are uniform, the IVP is the 1/N portfolio. Except for the impact of (markedly different) correlations, IVPs will be quite similar to risk parity portfolios.

Some characteristics of risk parity portfolios are worth mentioning. Firstly, noting the definition of beta in eq.(2), eq.(1) implies that risk parity portfolios favor assets with low levels of volatility and low correlations with other assets (these assets are "portfolio diversifiers"). Consequently, leverage is needed to boost the low risk and return of risk parity portfolios in order to match any risk budgets or return targets. Secondly, the composition of the risk parity portfolio, like any non-market cap weighted portfolio, depends on choosing the number of assets and hence on any pre-grouping of assets (see Lee [2011]). For example, when a hypothetical portfolio consists of 5 asset classes, each asset class will receive a 20% risk weight. However, when we next aggregate investment grade and high yield corporate bonds within this portfolio into a single credits portfolio, the risk allocations shift to 25%.

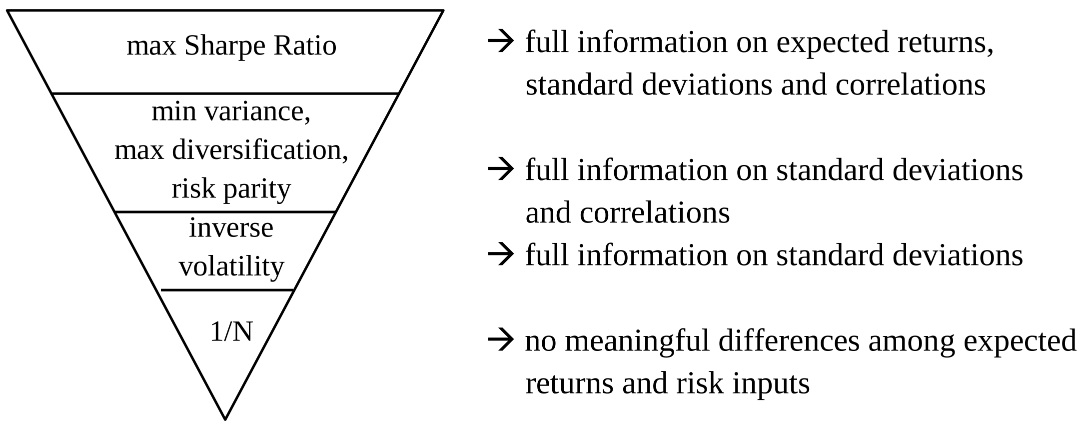
Portfolio construction and information demands

As a neutral starting point for our proposed stepwise portfolio selection procedure, we favor the risk parity portfolio over other risk control portfolios. To corroborate our choice, we first discuss the link between several portfolio rules and the information burden placed on the investor when adopting that rule.

[13] Inverse variance weighting is used in Weighted Least Squares regression, and in meta-analyses (see for example Harrison [2010]), among others.

The portfolio decision pyramid in Figure 1 illustrates the increasing requirements that apply to portfolio optimization inputs when moving from a naively diversified portfolio to the maximum Sharpe ratio portfolio. At the bottom of this inverted pyramid, we have the equally-weighted portfolio (1/N). Equal weighting can be an *ex cathedra* choice, or motivated by the historical performance of equally-weighted portfolios. However, in the current context of mean-variance portfolio optimization, equal weighting corresponds to the case where one cannot indicate any meaningful differences among expected returns, standard deviations and correlations. The best one can do is to naively diversify and equate money weights within the portfolio. On the next level, one can only put reliable trust in differences among standard deviations. This extra information allows one to shift from naïve money weight diversification to naïve risk weight diversification applying volatility weighting. This yields the IVP. On the third level, one has full risk information. The full covariance matrix is available and one has confidence in differences between standard deviations and correlations. This information allows one to build the full minimum variance portfolio (MVP), the maximum diversification portfolio (MDP), or risk parity portfolios. Finally, at the top level, one is able to indicate meaningful differences between all relevant inputs, i.e. the ex-ante covariance matrix and the expected returns. In that case, one can perform a full-fledged mean-variance optimization and obtain the maximum Sharpe ratio portfolio.

Figure 1: The portfolio decision pyramid. From bottom to top, it shows the increasing informational burden when using various portfolio construction rules. Source: Hallerbach [2015].



Having full risk information but no information on (differences among) expected returns places an investor on the third level. We favor the risk parity portfolio over other risk control portfolios as starting point for our stepwise portfolio selection procedure for a number of reasons. First of all, the risk parity portfolio is perfectly diversified in terms of risk or loss contributions. In addition, it is less concentrated than the MVP and the MDP[14] and it contains all *N* assets. Finally, the risk parity portfolio is more robust, i.e. less error maximizing, than the MVP. The intuitive reason is that the MVP is found by means of optimization, i.e. by equating marginal risk contributions, whereas the risk parity portfolio is found by a restriction on the product of weights and marginal risk contributions.[15]

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[14] See Choueifaty and Coignard [2008]. Leote de Carvalho et al. [2012] find that there is a surprisingly high commonality between the MVP and the MDP.

[15] It can be shown that $\sigma_{MVP} \leq \sigma_{RiskParity} \leq \sigma_{1/N}$, where the MVP is error maximizing and the 1/N portfolio focuses on money allocation, not risk allocation. Hence, the ex-ante volatility of the risk parity portfolio is between the lowest level (from the MVP) and the volatility of the naively diversified 1/N portfolio. See Maillard et al. [2010].

Approaches to reduce the impact of uncertainty surrounding views

Although Markowitz's [1952] mean-variance portfolio optimization is widely used, one of the major issues with this approach is its sensitivity to the inputs and especially the return forecasts. Chopra and Ziemba [1993] note that "*errors in means are about ten times as important as errors in variances and covariances*". Small changes in the inputs can result in large and often non-intuitive changes in the optimized portfolio. Several authors have proposed solutions ranging from a Bayesian approach to including additional constraints (see e.g. Jorion [1986], Michaud [1989], Black and Litterman [1991, 1992], Chopra [1993] and DeMiguel, Garlappi, Nogales and Uppal [2009]).

The Black-Litterman approach is an often used example of a Bayesian method to reduce the sensitivity of the optimal portfolio weights to the return expectations. In general terms, the approach starts with a reference portfolio in order to derive implied views.[16] These views will ensure that the reference portfolio is the mean-variance optimal portfolio if there are no other views. Subsequently the investor has a set of (linear combinations of) views for the assets as well as the confidence in each of the views. The Black-Litterman method then combines these two sets of views into a new, posterior view. The weights on the two views depend on the confidence in each of them. Mean-variance optimization based on this new view will generally result in a more intuitive and diversified portfolio. For technical details, we refer to the Appendix. Idzorek [2005] provides a good overview of the issues regarding the implementation of the Black-Litterman approach.

We emphasize that we deviate from Black and Litterman [1991, 1992] by choosing the reference portfolio different from market portfolio. In this respect, we follow Sefton, Bulsing and

[16] If the market portfolio is chosen as the reference portfolio, as do Black and Litterman [1991], then the implied views are equal to the equilibrium returns that clear the markets.

Scowcroft [2004], Pessaris [2012] and Carhart et al. [2014] who also choose a different reference portfolio as an anchor for the optimization procedure. In the previous section, we motivated why we take the risk parity portfolio as the reference portfolio. The risk parity portfolio is merely a natural starting point to determine the implied views. If a particular investor has no other views or no confidence in her ex ante views, then the mean-variance optimization based on the implied views will by default result in the risk parity portfolio being his optimal portfolio.

Data

To resemble an institutional investment portfolio, we consider three common asset classes[17]: US equities, Treasuries and commodities. For US equities we use the market proxy based on the CSRP data base from Kenneth French' website.[18] For bonds we compute the equally weighted average of the Barclays Capital US Treasury Intermediate and Long index. In this way we better match the liability structure of a typical pension fund. For commodities we use the S&P GSCI Ultra Light Energy index. This index is better diversified than the generic S&P GSCI index, as the weight of energy related commodities is substantially smaller. All returns are in excess of the risk-free rate, which is the 1-month T-Bill from the Kenneth French website. The average annual risk-free return over the sample amounted to 4.9%. All data series run from February 1978 to December 2013, on a monthly frequency. Using a higher frequency would result in too much noise for the purpose of strategic asset allocation, and lower frequencies have the disadvantage of too few observations to efficiently estimate the models.

[17] Credits and alternative assets are also commonly part of institutional investment portfolios. However, to strike a good balance between tractability of our results and practical relevance we selected these three asset classes.
[18] http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data_library.html

Table 1: Summary statistics (Panel A, annualized) and correlations (Panel B), February 1978 – December 2013

Panel A: excess return statistics	Average	Volatility	Sharpe ratio
Equities	7.9%	15.8%	0.50
Bonds	3.1%	7.4%	0.43
Commodities	1.1%	13.3%	0.08

Panel B: correlations	Equities	Bonds	Commodities
Equities	100%		
Bonds	7%	100%	
Commodities	31%	-7%	100%

In Table 1 we present excess return summary statistics and correlations. These form the starting point for our analyses. Panel A shows the univariate return characteristics. Over the full sample period equities showed the largest annualized excess return (7.9%). Bonds and commodities generated an excess return of just over 3% and 1% respectively. Equities had the highest volatility (15.8%), while the volatility of bonds was lowest (7.4%). Panel B shows full sample correlation statistics. Bonds had relatively low correlations with both equities (7%) and commodities (-7%), indicating diversification opportunities. The correlation between equities and commodities was moderately positive (31%).

Main results

In this section we describe the results from our empirical analysis. We start with the portfolio weights and implied views from the risk parity portfolio. Next we show how posterior views and portfolio weights change, depending on the confidence in one's views.

Table 2: Risk parity portfolio weights, implied views and implied Sharpe ratios

	Portfolio weight	Implied view	Implied Sharpe ratio
Equities	20.4%	3.7%	0.23
Bonds	53.4%	1.4%	0.19
Commodities	26.3%	2.9%	0.21

The risk parity portfolio, the derived implied views, and the implied Sharpe ratios are summarized in Table 2. We assume a risk aversion parameter δ equal to 5.[19] The portfolio allocates to a large extent to bonds (53.4%), followed by commodities (26.3%) and equities (20.4%). The large allocation to bonds is the result of the low volatility, as well as the low correlation of bonds with the other two assets. The higher allocation to commodities compared to equities can largely be attributed to the lower volatility of commodities and its negative correlation with bonds. The corresponding implied views for equities, bonds and commodities are 3.7%, 1.4% and 2.9%, respectively, following from eq.(8) in the Appendix. Implied Sharpe ratios are calculated by dividing the implied excess views by the historical standard deviations. The implied Sharpe ratios turn out to be relatively close to each other. This is due to the mean-variance optimality underlying the implied views. Note that the risk parity portfolio is the maximum Sharpe ratio portfolio when individual Sharpe ratios are the same and correlations are uniform. Since correlations are not uniform we observe some differences across Sharpe ratios.

[19] See Campbell and Viceira [2002], e.g.

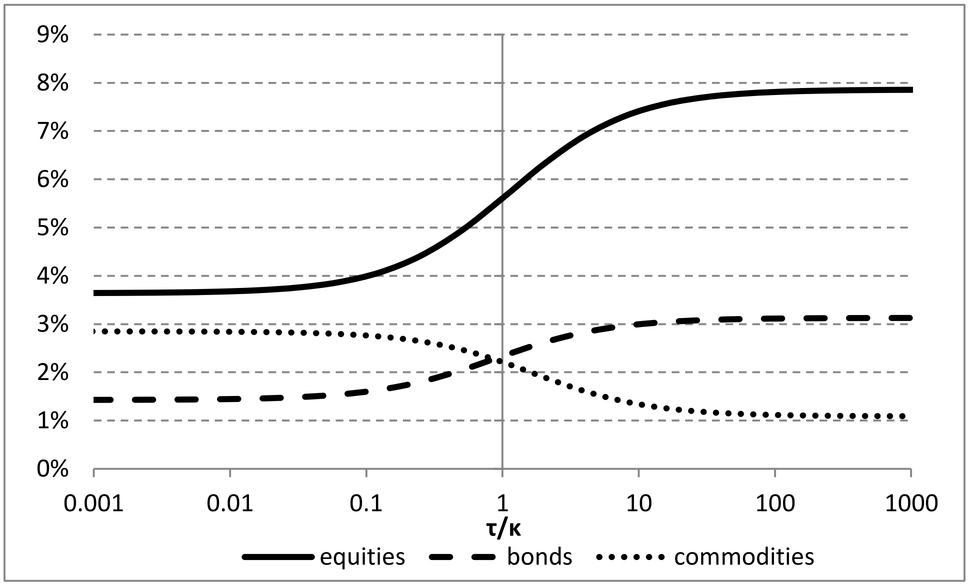
Next, the investor has a set of views which she will combine with the implied views. In this study we use the historical excess returns on equities, bonds and commodities from Table 1 as the investor's views. Furthermore, we assume that the investor has no (explicit) views on the risk regime.[20] The use of historical returns as investor's views serves for illustrative purposes only. In practice, an investor may want to employ economic or statistical analysis to derive her views. The Black-Litterman approach then combines the two sets of views into a posterior view. Depending on the confidence in each of the views, the new view will be tilted more towards the implied view or towards the investor's view. The relative confidence in each of the two sets of views will be reflected by the ratio τ/κ. In most applications the default value for τ is given by the inverse of the length of the historical period used for the estimation of the covariance matrix ($\tau = 1/T$). The value of κ, however, is provided by the investor and is an indication for the confidence in the investor's view. If the view is certain (i.e. there is no uncertainty surrounding it) the value for κ will be low ($\kappa \downarrow 0$) whereas if there is no confidence in the view, the value for κ will be high ($\kappa \rightarrow \infty$).[21] Mean-variance optimization based on the posterior views results in a new portfolio allocation.

Figures 2 and 3 show the distribution of the posterior views and the corresponding portfolio weights respectively, for different values of the relative confidence in the implied and the investor's views τ/κ. Table 3 reports the exact numbers from the figures for the polar cases of large and low confidence in the investor's views, as well as for the case with equal confidence in the implied and investor's views. We start with the posterior views in Figure 2. On the left hand side $\tau/\kappa \downarrow 0$ ($\kappa \rightarrow \infty$), meaning that the uncertainty around the investor's views is large. The posterior views correspond to the implied views of the reference (risk parity) portfolio as described in Table

[20] See e.g. Qian and Gorman [2001] for an example in case the investor has views on the covariance matrix.
[21] We refer to the Appendix for a more thorough explanation.

2. On the right hand side $\tau/\kappa \to \infty$ ($\kappa \downarrow 0$) we observe the investor's views that correspond to the historical views, similar to the average returns in Table 1. In the middle, at $\tau/\kappa = 1$, the posterior view is approximately equal to the average of the implied and investor's views.[22] Hence, an investor who has equal confidence in the implied and her own views will use expected returns equal to 5.6%, 2.3% and 2.2% for equities, bonds and commodities, respectively.

Figure 2: Full sample posterior views



[22] If we had used the full variance-covariance matrix Σ, as outlined in the section 'Black-Litterman approach' in the Appendix, the posterior views at $\tau/\kappa = 1$ would be *exactly* similar to the average of the risk parity and the investor's views. However, to generate the results in this section we have assumed the investor uses a diagonal matrix for Ω, i.e. only variances. This assumption matches practice more closely, as investors usually have views on the means and consider uncertainty around these means on a stand-alone basis.

Figure 3: Full sample portfolio weights

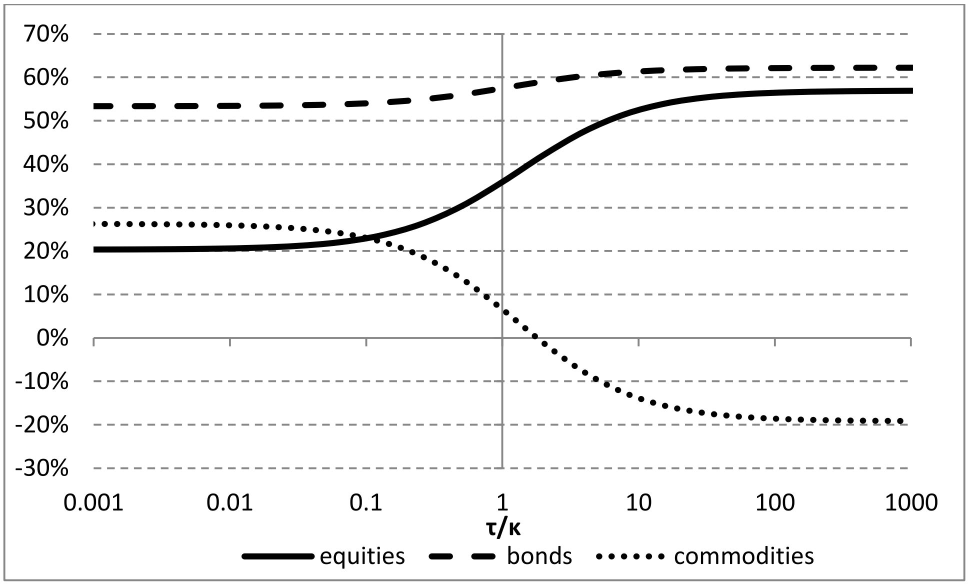


Table 3: Posterior views and portfolio weights over the full sample

	Posterior views			Portfolio weights		
Full sample	$\tau/\kappa\downarrow 0$	$\tau/\kappa=1$	$\tau/\kappa\to\infty$	$\tau/\kappa\downarrow 0$	$\tau/\kappa=1$	$\tau/\kappa\to\infty$
Equities	3.7%	5.6%	7.9%	20.4%	35.9%	56.9%
Bonds	1.4%	2.3%	3.1%	53.4%	57.5%	62.2%
Commodities	2.9%	2.2%	1.1%	26.3%	6.6%	-19.1%

From these views the portfolio allocations naturally follow. The portfolio weights are depicted in Figure 3. Because of lack of confidence in the investor's views when $\tau/\kappa \downarrow 0$, on the left hand side in Figure 3 the investor sticks with the risk parity portfolio, which spreads the risk contributions equally among the three asset classes. This results in the portfolio allocation as described in Table 3. On the right hand side, when $\tau/\kappa \to \infty$, we have (close to) full confidence in the investor's views. This portfolio corresponds to the classical mean-variance portfolio, which would be obtained when only historical data are used. At this end, we see that the largest portfolio allocation (62%) goes to bonds, as was the case for the risk parity portfolio, but now closely followed by equities (57%), which according to the view offer a relatively large Sharpe ratio. The allocation to commodities is -19%, due to the low Sharpe ratio of just above zero and the positive correlation with equities.

The large differences in allocations resulting from either $\tau/\kappa \downarrow 0$ (risk parity) or $\tau/\kappa \to \infty$ (mean-variance) show the strong impact of the views in the allocation process. In between the risk parity and mean-variance portfolio we have the special case $\tau/\kappa = 1$, when the investor has equal confidence in the implied view and the investor's view. The resulting portfolio is invested 36% in equities, 58% in bonds and 7% in commodities.

Asset allocation in different states of the world

When the investor's views become more distinct, differences between portfolio allocation methods may also become more extreme. Therefore, in this section we will show results for views that are dependent on the state of the world. There are several ways of defining states of the world. Different econometric techniques could be used to identify these states, for instance a Markov switching model (Hamilton [1989]). The views can also be based on fundamental views. However, our goal is not to ex-ante predict which state of the world prevails at any point in time. Rather we want to show how different views – and especially uncertainties around these views – affect the optimal asset allocation. The business cycle classification by NBER makes a recognizable classification that clearly distinguishes between two different states: contractions and expansions (see e.g. Lustig and Verdelhan [2012]).[23]

[23] See www.nber.org

Figure 4: NBER classification and cumulative asset excess returns (logarithmic scale)

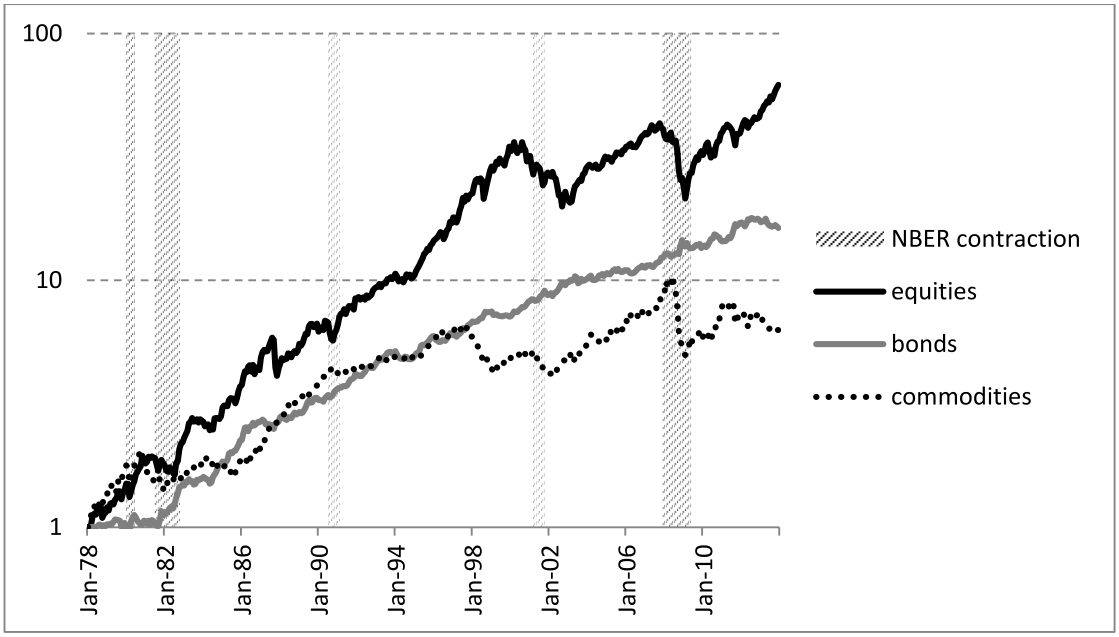


Figure 4 depicts NBER contraction periods (the periods that are not highlighted are classified as expansions) together with cumulative asset returns. Particularly the contractions in 2001 (IT bubble burst) and 2008-2009 (credit crisis) show severe negative excess returns for equities and commodities. This could greatly affect the investor's views and corresponding portfolio allocation, as we will see next.

Table 4: Annualized excess returns during expansions and contractions

	Expansions	Contractions	Full Sample
Equities	10.8%	-12.0%	7.9%
Bonds	2.4%	7.9%	3.1%
Commodities	3.2%	-13.1%	1.1%
# Observations	375	56	431

Table 4 shows the annualized excess returns of the asset classes during expansions and contractions. In NBER expansions equities were clearly superior, with an average return of 10.8%. Commodities performed better in expansions compared to the full sample. During NBER contractions, returns are very different from those during expansions. Equities and commodities returned about -12% and -13% respectively. Bonds on the other hand performed well with almost 8% on average, which is commonly associated with a flight-to-quality effect in times of crises. In line with the full sample analysis, we use the historical excess returns on equities, bonds and commodities as the views. For our illustrative purposes it is not an impediment that the NBER classifications are determined with hindsight, neither that views based on contractions are based on only 56 observations.

We first consider the investor who assumes she is in an expansion state on the world. She has constructed her views accordingly (Table 4). In Figure 5 we observe the posterior views. On the left hand side, we again see for small values of τ/κ that the posterior views resemble the implied views from the reference portfolio. On the right hand side, when the investor has full confidence in her own views, these views prevail. Interestingly, the figure shows that the range of views for commodities and bonds is rather limited, while the views on equities range from 3.7% (risk parity) to 10.8% (investor's view). The numbers are reported in Table 5. The right hand side of Figure 6 shows the portfolio allocations when the investor fully believes in the views that correspond to an expansion state of the world. We see larger allocations to equities and commodities, compared to the full sample. Nevertheless, the portfolio weight for commodities is still negative in expansions, when higher confidence is attached to the investor's views, due to the large allocation to equities. On the other hand, the weight of bonds has become much smaller at about 29% (compared to 62% in the full sample), which can be attributed to bonds being relatively unattractive during expansions.

Figure 5: Posterior views in expansion states of the world

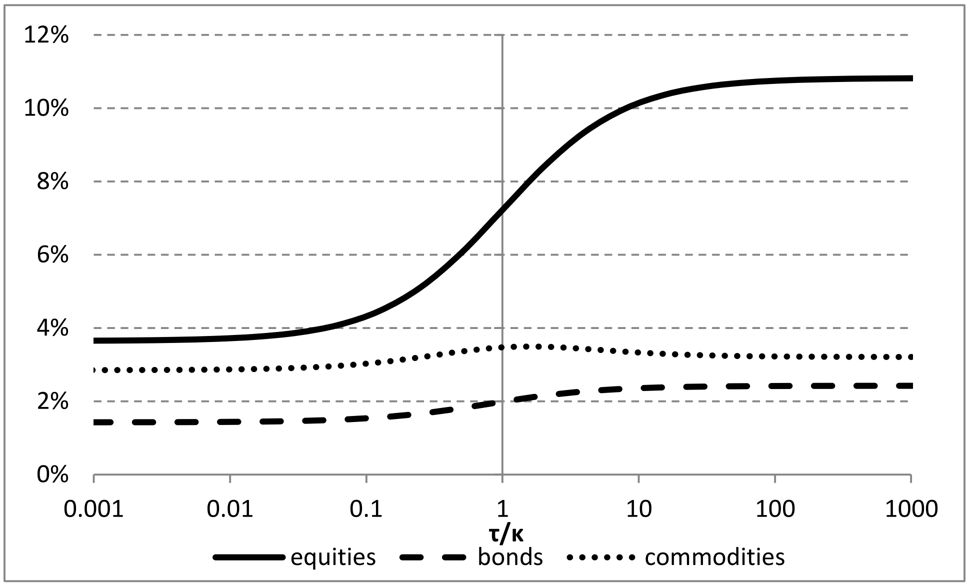


Figure 6: Portfolio weights in expansion states of the world

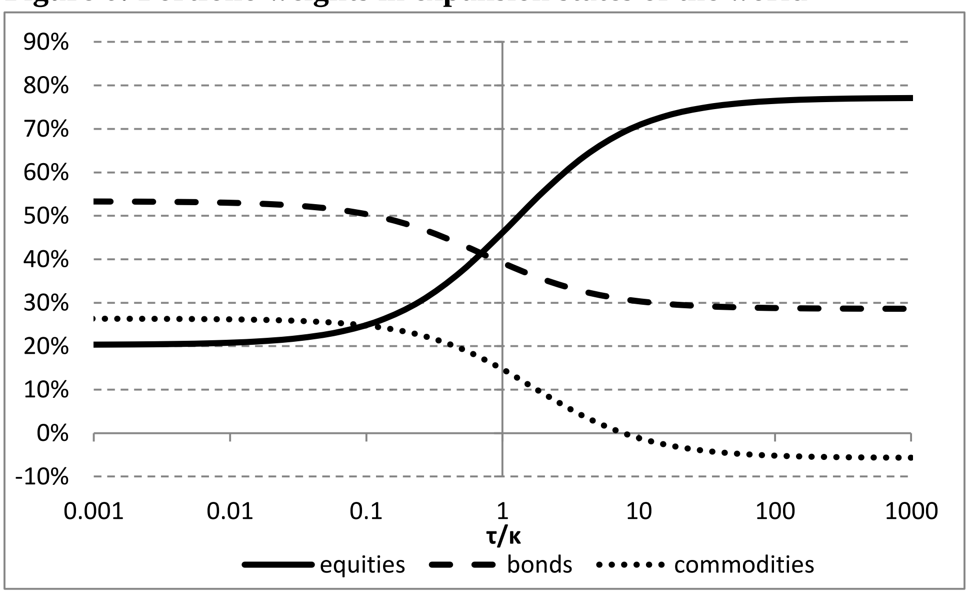


Table 5: Posterior views and portfolio weights in NBER expansions and contractions

	Posterior views			Portfolio weights		
Expansions	**$\tau/\kappa\downarrow 0$**	**$\tau/\kappa=1$**	**$\tau/\kappa\to\infty$**	**$\tau/\kappa\downarrow 0$**	**$\tau/\kappa=1$**	**$\tau/\kappa\to\infty$**
Equities	3.7%	7.2%	10.8%	20.4%	46.1%	77.1%
Bonds	1.4%	2.0%	2.4%	53.4%	39.2%	28.6%
Commodities	2.9%	3.5%	3.2%	26.3%	14.7%	-5.7%
	Posterior views			**Portfolio weights**		
Contractions	**$\tau/\kappa\downarrow 0$**	**$\tau/\kappa=1$**	**$\tau/\kappa\to\infty$**	**$\tau/\kappa\downarrow 0$**	**$\tau/\kappa=1$**	**$\tau/\kappa\to\infty$**
Equities	3.7%	-5.2%	-12.0%	20.4%	-32.5%	-78.0%
Bonds	1.4%	4.7%	7.9%	53.4%	179.9%	287.1%
Commodities	2.9%	-6.2%	-13.1%	26.3%	-47.4%	-109.1%

Next we consider the case for contraction states of the world. During contractions equities and commodities heavily underperformed, while bonds boomed. This is also visible in the views in Figure 7. Since equities and commodities are both unattractive in this scenario, the allocation to these assets is relatively small, or negative, depending on the confidence in the views, as shown in Figure 8 and Table 5. The allocation to bonds on the other hand is large, varying between 53% (risk parity) and 287% (investor's views). In general, the range of optimal portfolio weights is large. Thus, depending on the confidence of the investor in her views, substantially different optimal portfolios can emerge.

Figure 7: Posterior views in contraction states of the world

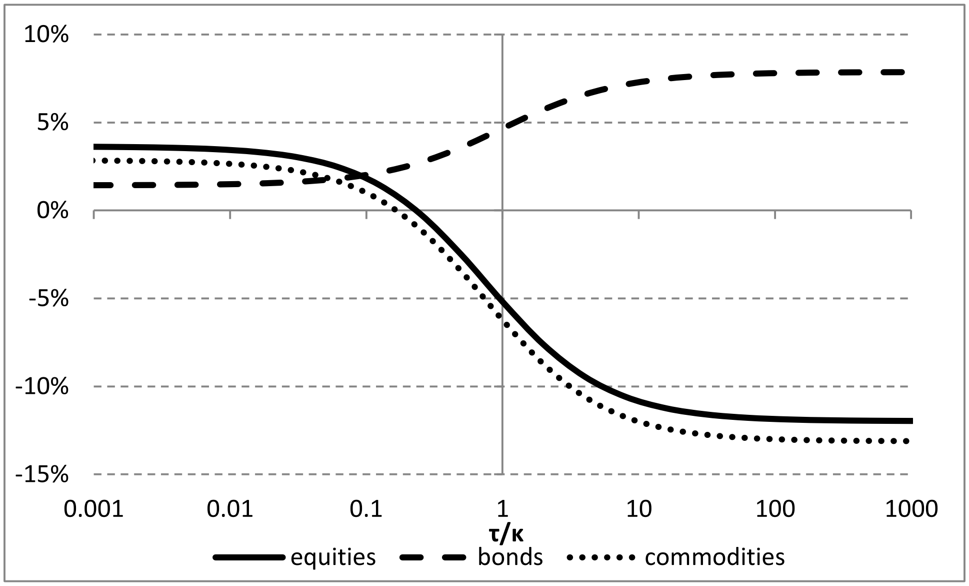
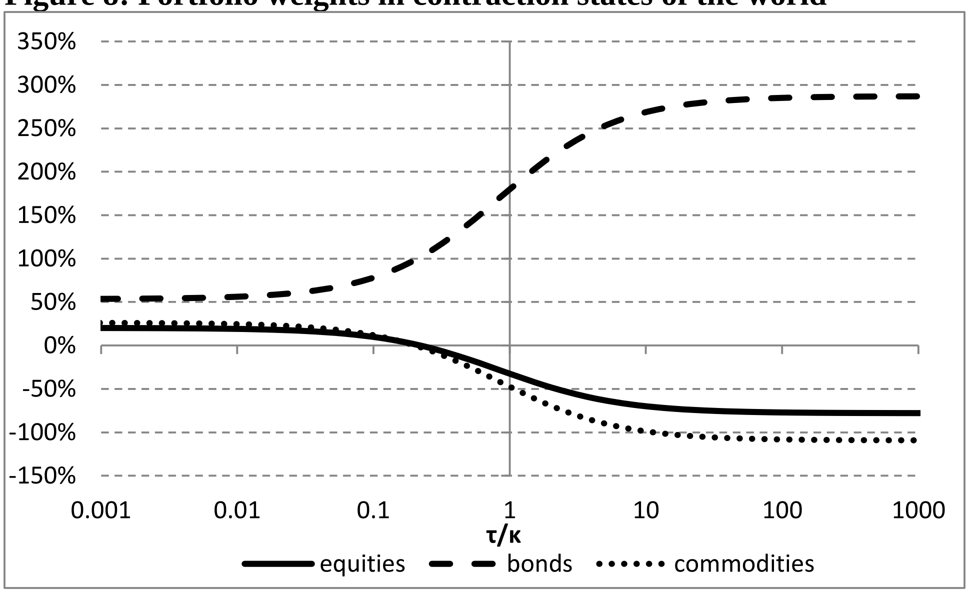


Figure 8: Portfolio weights in contraction states of the world



Impact of risk aversion and short-sale constraints

In this section we perform some complementary analyses. First we analyze the effect of the risk aversion on the optimal portfolios. Secondly, we add restrictions on short selling and leverage to the framework, as many investors may have investment restrictions in practice.

In Table 6 we present posterior views and portfolio weights for a lower ($\delta = 2$) and higher ($\delta = 10$) level of risk aversion. If the investor's risk aversion changes, all views change by the same factor, as Σw will remain the same (see eq.(8) in the Appendix). When $\tau/\kappa \downarrow 0$,the posterior view will be equal to the implied view, which in turn implies that the optimal portfolio equals the reference portfolio, irrespective of the risk aversion parameter. For instance, when $\delta = 2$ the investor is more offensive, and would already invest 20.4% in equities at an implied view of 1.5%. The more defensive investor would demand a much higher equity premium of 7.3% to govern a similar allocation of 20.4% to equities. For the other polar case $\tau/\kappa \to \infty$, or the traditional mean-variance framework, we see a different picture. The allocations for risk aversions $\delta = 2$ and $\delta = 10$ differ substantially, although the posterior views are equal. This equality is implied by the full confidence in the investor's views. For a given posterior view, the more offensive investor takes larger (in absolute terms) positions in the more risky assets.

Table 6: Posterior views and portfolio weights for different risk aversion parameters

	Posterior views			**Portfolio weights**		
δ=2	**$\tau/\kappa\downarrow 0$**	**τ/κ=1**	**$\tau/\kappa\to\infty$**	**$\tau/\kappa\downarrow 0$**	**τ/κ=1**	**$\tau/\kappa\to\infty$**
Equities	1.5%	4.7%	7.9%	20.4%	71.5%	131.3%
Bonds	0.6%	1.9%	3.1%	53.4%	47.1%	48.3%
Commodities	1.1%	1.5%	1.1%	26.3%	-18.5%	-79.6%
	Posterior views			**Portfolio weights**		
δ=10	**$\tau/\kappa\downarrow 0$**	**τ/κ=1**	**$\tau/\kappa\to\infty$**	**$\tau/\kappa\downarrow 0$**	**τ/κ=1**	**$\tau/\kappa\to\infty$**
Equities	7.3%	7.1%	7.9%	20.4%	24.0%	32.1%
Bonds	2.9%	3.1%	3.1%	53.4%	61.0%	66.8%
Commodities	5.7%	3.5%	1.1%	26.3%	15.0%	1.1%

Next we investigate restrictions on short selling and leverage. Up to this stage, the only restriction on the portfolio weights was that the weights should at up to 100%. However, many institutional investors may be restricted in the use of derivatives to use leverage and implement short positions.

Figure 9: Portfolio weights under long-only constraint

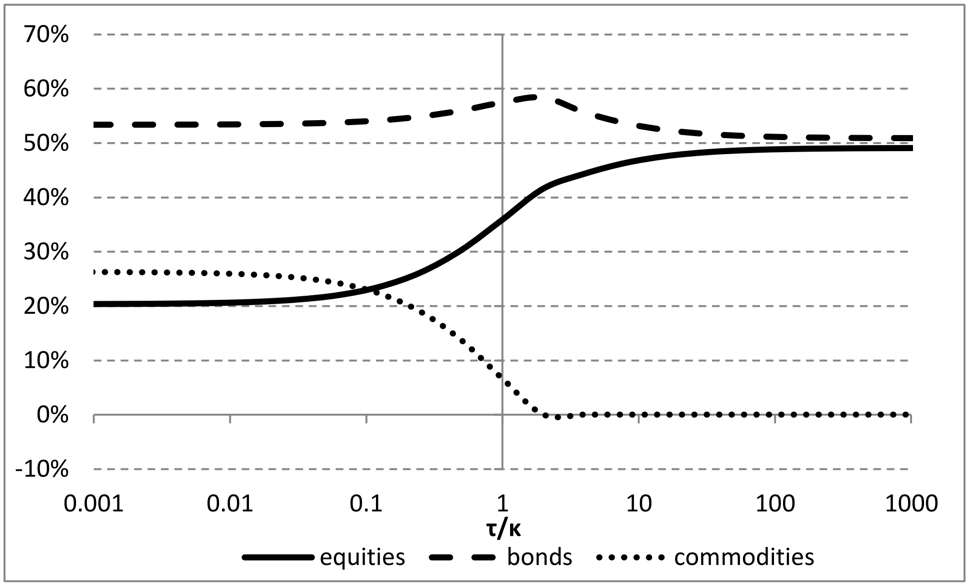


Table 7: Posterior views and portfolio weights under long-only constraint

	Posterior views			Portfolio weights		
Long-only	**$\tau/\kappa\downarrow 0$**	**$\tau/\kappa=1$**	**$\tau/\kappa\to\infty$**	**$\tau/\kappa\downarrow 0$**	**$\tau/\kappa=1$**	**$\tau/\kappa\to\infty$**
Equities	3.7%	5.6%	7.9%	20.4%	35.9%	49.1%
Bonds	1.4%	2.3%	3.1%	53.4%	57.5%	50.9%
Commodities	2.9%	2.2%	1.1%	26.3%	6.6%	0.00%

Therefore we show in Figure 9 and Table 7 results when short selling and leverage are not allowed. The analysis is carried out for a risk aversion parameter $\delta = 5$. As the restrictions only apply to the optimization, the posterior views are given by the ones in Table 3. The portfolio weights, however, are different as the long-only restriction becomes binding. For example, when $\tau/\kappa \to \infty$, the investor has full confidence in her views, a short position of 19.1% in commodities emerges in the unrestricted base case. With long-only restriction, the weight for commodities is limited to a lower bound of zero percent. The weights of equities and bonds are reduced from 56.9% and 62.2% to 49.1% and 50.9% respectively. Figure 9 provides a graphical representation of the evolvement of the portfolio weights as function of the confidence in the investor's views, when leverage and short selling are not allowed. If the constraints are not binding, the portfolio weights are the same as in the unrestricted case in Figure 3. Around $\tau/\kappa = 2$ the unrestricted weight for commodities becomes negative. As the confidence in the investor's view further increases, the weights for both equities and bonds get close to 50%.

Conclusions

The key problem in portfolio optimization is not per se the optimization itself, but the specification of the inputs, notably the views on expected returns. Noting that full confidence in expected returns in mean-variance optimization on the one hand and full ignorance of expected returns in risk-controlled strategies on the other hand are two extremes, we propose an approach to bridge the gap between the two environments. The use of the Black-Litterman method allows an investor to position herself in the continuum between the extremes. In our approach we use the risk parity portfolio as the reference portfolio in order to determine the implied views. These views can then be adjusted by means of the investor's view regarding the expected returns. The higher the confidence in her views, the more the investor's portfolio will be shifted towards the maximum Sharpe ratio portfolio that is consistent with her views.

We illustrate our framework for a US investor whose opportunity set consists of equities, bonds and commodities. Of course, an investor is free to stipulate her own distinct views on expected returns. In our analysis we assume that the investor's views are equal to the historically observed average returns, while the uncertainty surrounding the views is related to the historical volatility. These views and the corresponding uncertainty around them can then be used to derive the posterior views. These views are subsequently used in the optimization. As with the general Black-Litterman methodology, the elicitation of the confidence in (or uncertainty regarding) the views remains a challenge for further research. Our results show how the optimal portfolio gradually changes from the risk parity portfolio towards the mean-variance portfolio associated with the investor's views. These insights offer investors practical support to improve their asset allocation decisions in challenging market environments (viz. rising interest rates).

Appendix

The risk parity portfolio

The contribution of an asset to portfolio risk equals its portfolio weight multiplied with its marginal contribution to portfolio risk. To derive this result, note that the portfolio volatility is linearly homogeneous in the portfolio weights: multiplying portfolio weights $\{w_i\}$ with a constant k multiplies the portfolio volatility with the same constant k. Euler's theorem then implies that:

$$\sigma_p = \sum_i w_i \frac{\partial \sigma_p}{\partial w_i} \tag{3}$$

where $\frac{\partial \sigma_p}{\partial w_i}$ is the marginal contribution of asset i to portfolio excess return volatility σ_p. It readily follows that:[24]

$$\frac{\partial \sigma_p}{\partial w_i} = \frac{\sigma_{ip}}{\sigma_p} \tag{4}$$

where σ_{ip} is the covariance between the excess returns on asset i and the portfolio. Note that $\frac{\sigma_{ip}}{\sigma_p} = \beta_{ip} \cdot \sigma_p$ where β_{ip} is the slope coefficient from a regression of the excess returns of asset i on the portfolio excess returns. The term $w_i \frac{\sigma_{ip}}{\sigma_p}$ is the component contribution of asset i to portfolio volatility. The sum of all component contributions to volatility equals total portfolio volatility. Hence, the weights of the risk parity portfolio satisfy:

[24] See Qian [2006] or Roncalli [2013]. Alternatively, in matrix notation we obtain $\frac{\partial \sigma_p}{\partial w} = \frac{\Sigma w}{\sqrt{w'\Sigma w}}$, where w is the weight vector and Σ is the covariance matrix.

$$w_i \frac{\partial \sigma_p}{\partial w_i} = \frac{\partial \sigma_p}{\partial w_j} w_j \quad \Leftrightarrow \quad w_i \beta_{ip} = w_j \beta_{jp} \tag{5}$$

This implies that the weights in the risk parity portfolio are proportional to the inverse of the corresponding betas:

$$w_i^{RP} \sim \frac{1}{\beta_{ip}} \tag{6}$$

Black-Litterman approach

Assume that the excess returns r on the N asset classes are normally distributed with μ the expected returns and Σ the covariance matrix:[25]

$$r \sim N(\mu, \Sigma) \tag{7}$$

The implied views are the views which if the investors hold these, the optimal portfolio in a mean-variance context will be equal to a given reference portfolio w. If the risk aversion parameter equals δ[26], the implied views Π are given by:

$$\Pi = \delta \Sigma w \tag{8}$$

The prior is given by $\mu \sim N(\Pi, \tau\Sigma)$ where τ reflects the uncertainty around the mean. In practice τ is given by *1/T* with T the number of observations used to determine the covariance matrix Σ.[27] The covariance matrix Σ is in general estimated using a set of historical data. This implicitly assumes that the returns are i.i.d. distributed and that there is no mean reversion or aversion in multiple period returns.

Suppose we have K ($\leq N$) linear views for (part of) the N assets with the confidence represented by Ω:

[25] The derivation is based on He and Litterman [1999].

[26] The utility function of the investor is given by $\max_x \; x'\Pi - \frac{1}{2}\delta x'\Sigma x$, with x the portfolio weights.

[27] Satchell and Scowcroft [2000] discuss an extension of the method where τ is unknown and stochastic.

$$P\mu = Q + \epsilon, \epsilon \sim N(0, \Omega) \tag{9}$$

where P is the set of K linear combinations of assets for which we have views, Q is the set of K views for these combinations and ϵ contains the errors around the views. These views will be combined with the prior view in a Bayesian framework.

The resulting posterior distribution of the expected returns is given by $N(\bar{\mu}, \bar{M}^{-1})$ where:

$$\bar{\mu} = [(\tau\Sigma)^{-1} + P'\Omega^{-1}P]^{-1}[(\tau\Sigma)^{-1}\Pi + P'\Omega^{-1}Q] \tag{10}$$

and:

$$\bar{M}^{-1} = [(\tau\Sigma)^{-1} + P'\Omega^{-1}P]^{-1} \tag{11}$$

As the expected returns are random variables, the distribution of the returns has to be adjusted to:

$$r \sim N(\bar{\mu}, \bar{\Sigma}) \tag{12}$$

with $\bar{\Sigma} = \Sigma + \bar{M}^{-1}$.

Assume further $P = I_N$, i.e. for each asset we have a view and let $\Omega = \kappa\Sigma$ represent the confidence in the views. The expected returns will then be distributed as

$$\bar{\mu} = [\tau^{-1} + \kappa^{-1}]^{-1}[\tau^{-1}\Pi + \kappa^{-1}Q] \tag{13}$$

and

$$\bar{M}^{-1} = [\tau^{-1} + \kappa^{-1}]^{-1}\Sigma \tag{14}$$

We will briefly look at the results for three different values of τ/κ. The ratio τ/κ reflects the confidence in the investor's view relative to the confidence in the implied view. If $\tau/\kappa \to \infty$ ($\kappa \downarrow 0$) the views become more and more certain, whereas for $\tau/\kappa \downarrow 0$ ($\kappa \to \infty$) the uncertainty is very large. If $\tau/\kappa = 1$ ($\kappa = \tau$) both sources of information (implied and investor's view) are given equal weight. For $\tau/\kappa \to \infty$ the distribution of the expected return in eq. (13)-(14) is given by $\bar{\mu} \to Q$ and $\overline{M}^{-1} \to 0_{N\times N}$. The returns are distributed as $r \sim N(Q, \Sigma)$. This is in fact the commonly used approach within the asset management industry to incorporate views. If there is however no confidence in the investor's view ($\tau/\kappa \downarrow 0$) the distribution of the expected returns in eq.(13)-(14) is given by $\bar{\mu} = \Pi$ and $\bar{M}^{-1} = \tau\Sigma$. The expected returns will not change. The returns are distributed as $r \sim N(\Pi, (1+\tau)\Sigma)$. The variance of the returns is thus higher than in the original setting in eq.(7). This is due to the explicit recognition that there is already uncertainty around the mean in the implied view. This uncertainty is given by $\tau\Sigma$. Finally, if $\tau/\kappa = 1$ then $\bar{\mu} = \frac{1}{2}[\Pi + Q]$ and $\bar{M}^{-1} = \frac{1}{2}\tau\Sigma$. The expected returns are in this setting equal to the average of the implied and the investor's view.

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