"Measuring Factor Exposures: Uses and Abuses"[1]

Ronen Israel, AQR Capital Management
ronen.israel@aqr.com
2 Greenwich Plaza, Greenwich CT. 06830

Adrienne Ross, AQR Capital Management
adrienne.ross@aqr.com
2 Greenwich Plaza, Greenwich CT. 06830

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Abstract

A growing number of investors have come to view their portfolios (especially equity portfolios) as a collection of exposures to risk factors. The most prevalent and widely harvested of these risk factors is the market (equity risk premium); but there are also others, such as value and momentum (style premia).

Measuring exposures to these factors can be a challenge. Investors need to understand how factors are constructed and implemented in their portfolios. They also need to know how statistical analysis may be best applied. Without the proper model, rewards for factor exposures may be misconstrued as alpha, and investors may be misinformed about the risks their portfolios truly face.

This paper should serve as a practical guide for investors looking to measure portfolio factor exposures. We discuss some of the pitfalls associated with regression analysis, and how factor design can matter a lot more than expected. Ultimately, investors with a clear understanding of the risk sources in an existing portfolio, as well as the risk exposures of other portfolios under consideration, may have an edge in building better diversified portfolios.

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Introduction: Why Should Investors Care About Factor Exposures?

Investors have become increasingly focused on how to harvest returns in an efficient way. A big part of that process involves understanding the systematic sources of risk and reward in their portfolios. "Risk-based investing" generally views a portfolio as a collection of return-generating processes or factors. The most straightforward of these processes is to invest in asset classes, such as stocks and bonds (asset class premia). Such risk taking has been rewarded globally over the long term, and has historically represented the biggest driver of returns for investors. However, asset class premia represent just one dimension of returns. A largely independent, separate source comes from style premia. Style premia are a set of systematic sources of returns that are well researched, geographically pervasive and have been shown to be persistent. There is a logical, economic rationale for why they provide a long-term source of return (and are likely to continue to do so).[2] Finally, they can be applied across multiple asset classes.[3]

The common feature of risk-based investing is the emphasis on improved risk diversification, which can be achieved by identifying the sources of returns that are underrepresented in a portfolio. Investors who understand what risk sources their portfolios are exposed to (and the magnitude of these exposures) may be better suited to evaluate existing and potential managers. Without an understanding of portfolio risk factor exposures, how else would investors be able to tell if their value manager, for example, is actually providing significant value exposure? Or whether a manager is truly delivering alpha, and not some other factor exposure? Or even, whether a new manager would be additive to their existing portfolio?

These are important questions for investors to answer, but quantifying them may be difficult. There are many ways to measure and interpret the results of factor analysis. Some investors may use a holdings-based approach, while others use returns-based regression analysis.[4] There are also many variations in

[2] See "How Can a Strategy Still Work if Everyone Knows About It?" accessed September 23, 2015, www.aqr.com/cliffs-perspective/how-can-a-strategy-still-work-if-everyone-knows-about-it.

[3] Applying styles across multiple asset classes provides greater diversification. In addition, the effectiveness of styles across asset classes helps dissuade criticisms of data mining. Asness, Moskowitz and Pedersen (2013); Asness, Ilmanen, Israel and Moskowitz (2015). Past performance is not indicative of future results.

[4] While a holdings-based approach may provide more precise estimates at each point in time (and may also indicate how managers vary their exposures over time), it may not be a feasible approach for many investors. It requires significant resources and infrastructure and often leads to challenges with

portfolio construction and factor portfolio design. Even a single factor such as value has variations that an investor should consider — it can be applied as a tilt to a long-only equity portfolio,[5] or it can be applied in a "purer" form through long/short strategies; it can be based on multiple measures of value, or a single measure such as book-to-price; or it can span multiple asset classes or geographies. Simply put, even two factors that aim to capture the same economic phenomenon can differ significantly in their construction — and these differences can matter.

In this paper, we discuss some of the difficulties associated with measuring and interpreting factor exposures. We look at a regression-based approach and explore some common pitfalls of regression analysis; we also describe the differences associated with academic versus practitioner factors, and outline various choices that can affect the results. We hope that after reading this paper investors will be better able to measure portfolio factor exposures, understand the results of factor models and, ultimately, determine whether their portfolios are accessing the sources of return they want in a diversified manner.

A Brief History of Factors

Asset pricing models generally dictate that risk factors command a risk premium. Modern Portfolio Theory quantifies the relationship between risk and expected return, distinguishing between two types of risks: idiosyncratic risk (that which can be diversified away) and systematic risk (such as market risk that cannot be diversified away). The Capital Asset Pricing Model (CAPM) provides a framework to evaluate the risk premium of systematic market risk.[6] In the CAPM single-factor world, we can use linear regression analysis to decompose returns into two components: alpha and beta. Alpha is the portion of returns that cannot be explained by exposure to the market, while beta is the portion of returns that can be attributed to the market.[7] But studies have shown that single-factor models may not

data – an investor may not always have access to the holdings, or holdings may be available at a lag or may only be partially-available (i.e., 13F filings do not reflect short positions). In contrast, a regression-based approach is more accessible and still provides useful information in analyzing exposures. Ultimately, where possible, investors should look at both approaches for a more robust sense of portfolio factor exposures.

[5] The long-only style tilt portfolio will still have significant market exposure. This type of style portfolio is often referred to as a "smart beta" portfolio.

[6] CAPM says the expected return on any security is proportional to the risk of that security as measured by its market beta.

[7] More generally, the economic definition of alpha relates to returns that cannot be explained by exposure to common risk factors (Berger, Crowell, Israel and Kabiller, 2012).

adequately explain the relationship between risk and expected return, and that there are other risk factors at play. For example, under the framework of Fama and French (1992, 1993) the returns to a portfolio could be better explained by not only looking at how the overall equity market performed but also at the performance of size and value factors (i.e., the relative performance between small- and large-cap stocks, and between cheap and expensive stocks). Adding these two factors (value and size) to the market created a multi-factor model for asset pricing. Academics have continued to explore other risk factors, such as momentum[8] and low-beta or low risk,[9] and have shown that these factors have been effective in explaining long-run average returns.

In general, style premia have been most widely studied in equity markets, with some classic examples being the work of Fama and French referenced above. For each style, they use single, simple and fairly standard definitions — they are described in **Exhibit 1**.[10]

Exhibit 1: Common Academic Factor Definitions

HML	**"High Minus Low": a long/short measure of value that goes long stocks with high book-to-market values and short stocks with low book-to-market values**
UMD	**"Up Minus Down": a long/short measure of momentum that goes long stocks with high returns over the past 12 months (skipping the most recent month) and short stocks with low returns over the same period**
SMB	**"Small Minus Big": a long/short measure of size that goes long**

[8] Jegadeesh and Titman (1993); Asness (1994).
[9] Black (1972); Frazzini and Pedersen (2014).
[10] Specifically, these factors are constructed as follows: SMB and HML are formed by first splitting the universe of stocks into two size categories (S and B) using NYSE market-cap medians and then splitting stocks into three groups based on book-to-market equity [highest 30%(H), middle 40%(M), and lowest 30%(L), using NYSE breakpoints].The intersection of stocks across the six categories are value-weighed and used to form the portfolios SH(small, high book-to-market equity (BE/ME)), SM(small, middle BE/ME), SL (small, low BE/ME), BH(big, high BE/ME), BM(big, middle BE/ME), and BL (big, low BE/ME), where SMB is the average of the three small stock portfolios (1/3 SH + 1/3 SM + 1/3 SL) minus the average of the three big stock portfolios (1/3 BH + 1/3 BM + 1/3 BL) and HML is the average of the two high book-to-market portfolios (1/2 SH+ 1/2 BH) minus the average of the two low book-to-market portfolios (1/2 SL + 1/2 BL). UMD is constructed similarly to HML, in which two size groups and three momentum groups [highest 30% (U), middle 40% (M), lowest 30% (D)] are used to form six portfolios and UMD is the average of the small and big winners minus the average of the small and big losers.

small-market-cap stocks and short large-market-cap stocks

Assessing Factor Exposures in a Portfolio

Using these well-known academic factors, we can analyze an illustrative portfolio's factor exposures. But before we do, we should emphasize that the factors studied here are not a definitive or exhaustive list of factors. We should also emphasize that different design choices in factor construction can result in very different measured portfolio exposures. Indeed, the fact that you can still get large differences based on specific design choices is much of our point; we will revisit these design choices later in the paper.

A common approach to measuring factor exposures is linear regression analysis; it describes the relationship between a dependent variable (portfolio returns) and explanatory variables (risk factors). Static (full sample) regression analysis provides information on average exposures over a given period, but will not provide any insight into whether a manager varies factor exposures over time. To understand how factor exposures vary over time you can look at dynamic (rolling window) regression betas, ideally using at least 36 months of data.[11]

Regression analysis can be done with one risk factor or as many as the portfolio aims to capture. If the portfolio captures multiple styles, then multiple factors should be used. If the portfolio is a global multi–asset style portfolio, then the relevant factors should cover multiple assets in a global context. Ideally, the factors used should be similar to those implemented in the portfolio, or at least one should account for those differences in assessing the results. For example, long-only portfolios are more constrained in harvesting style premia as underweights are capped at their respective benchmark weights. In contrast,

[11] The tradeoff is that some, perhaps a lot, of this variation may in fact be random noise. Past performance is not indicative of future results.

long/short factors (and portfolios) are purer in that they are unconstrained. These differences should be accounted for when performing and interpreting factor analysis.[12]

For this analysis, we examine a hypothetical long-only equity portfolio that aims to capture returns from value, momentum and size. Specifically, the portfolio is constructed with 50/50 weight on simple measures of value (book-to-price, using current prices[13]) and momentum (price return over the last 12 months) within the small-cap universe.[14] In practice an investor may not know the portfolio exposures in advance, but since our goal is to illustrate how to best apply the analysis, we will proceed as if we do.

We start with a simple one-factor model and then add the additional factors that the portfolio aims to capture. We analyze style exposures using academic factors (over practitioner factors) for simplicity and illustrative purposes. The performance characteristics of the portfolio and factors used are shown in **Exhibit 2**, which shows that the portfolio returned an annual 13.5% in excess of cash on average from 1980–2014.

It's important to note that we are analyzing a very long history, which may not be available in practice. In general it's important to include as many observations as possible, with a general rule of thumb being a minimum of 36 months.

Exhibit 2: Hypothetical Performance Statistics

January 1980–December 2014

	Portfolio	Market	Value (HML)	Momentum (UMD)	Size (SMB)
Annual Excess Returns	13.5%	7.8%	3.6%	7.3%	1.6%
Volatility	17.8%	15.6%	10.5%	15.8%	10.6%
Correlation with Portfolio		0.84	-0.06	-0.08	0.53

[12] Because unconstrained long/short factors can capture the underlying styles more efficiently, long-only portfolios are essentially penalized when long/short factors are used in the regression; this is because the regression expects the long-only portfolio to harvest returns to the same extent that long/short factors can. In these cases, investors should be cautious with their interpretation of alpha.
[13] Fama-French HML uses lagged prices. See section on "other factor design choices."
[14] See Frazzini, Israel, Moskowitz and Novy-Marx (2013), for more detail on how to construct a multi-style portfolio. Note that we have followed a similar multi-style portfolio construction approach here. To build our portfolio, we rank stocks based on simple measures for value (book-to-price using current prices) and momentum (price return over the last 12 months) within the U.S. small-cap universe (Russell 2000). We compute a composite rank by applying a 50% weight to value and 50% to momentum. We then select the top 25% of stocks with the highest combined ranking and weight the stocks in the resulting portfolio via a 50/50 combination of each stock's market capitalization and standardized combined rank. Portfolio returns are gross of transaction costs, un-optimized and undiscounted. The portfolios are rebalanced monthly.

Note: All returns are arithmetic averages. Returns are in excess of cash.
Source: AQR, Ken French Data Library. The portfolio is a hypothetical simple 50/50 value and momentum long-only small-cap equity portfolio, gross of fees and transaction costs, and excess of cash. The portfolio is rebalanced monthly. The academic explanatory variables are the contemporaneous monthly Fama-French factors for the market (MKT-RF), value (HML), momentum (UMD) and size (SMB). The market is the value-weight return of all CRSP firms. Hypothetical data has inherent limitations some of which are discussed herein.

We can use these returns and betas from regression analysis to decompose portfolio excess of cash returns $(R_i - R_f)$.[15] The first regression model we look at is the CAPM with the market as the only factor.[16]

(1) $(R_i - R_f) = \alpha + \beta_{MKT}(R_{MKT} - R_f) + \varepsilon$

Or roughly,

Portfolio Returns in Excess of Cash =

Alpha + Beta x Market Risk Premium[17]

The results in **Exhibit 3** show that the portfolio had a market beta of 0.96 (based on Model 1 in Part A). This means — not surprisingly, as the portfolio is long-only — that the portfolio had meaningful exposure to the market. We also know (from Exhibit 2) that over this period the equity market has done well, delivering 7.8% excess of cash returns. As a result, we can see (in Part B of Exhibit 3) that the portfolio's positive exposure to the market contributed 7.4% to overall returns,[18] and that 6.1% was "alpha" in excess of market exposure.

[15] One of the most common mistakes in running factor analysis is to forget to take out cash from the returns of the left- and right-hand side variables. For a long-only factor or portfolio, such as the market, one must explicitly do that. A long/short factor is a self-financed portfolio whose returns are already in excess of cash.

[16] We have also included an error term (ε), which is the difference between actual realized returns and expected returns. More specifically, the error term captures the unexplained component of the relationship between the dependent variable (e.g., the portfolio excess returns) and explanatory variables (e.g., the market risk premium).

[17] All risk premia in this paper are returns in excess of cash.

[18] Market beta × market risk premium = 0.96 × 7.8%.

The same framework can be applied for multiple risk factors. Our first multivariate regression adds the value factor.

$$(2)\ (R_i - R_f) = \alpha + \beta_{MKT}\left(R_{MKT} - R_f\right) + \beta_{HML}(R_{HML}) + \varepsilon$$

The results under Model 2 show that the portfolio had positive exposure to value (with a beta of 0.43), which means that the portfolio on average bought cheap stocks.[19] Because value is a historically rewarded long-run source of returns, having positive exposure benefited the portfolio, with value contributing 1.6% to portfolio returns ($HML\ beta \times HML\ risk\ premium = 0.43 \times 3.6\%$).

Next we add the momentum factor in Model 3.

$$(3)\ (R_i - R_f) = \alpha + \beta_{MKT}\left(R_{MKT} - R_f\right) + \beta_{HML}(R_{HML}) + + \beta_{UMD}(R_{UMD}) + \varepsilon$$

As one would expect, we see that the momentum loading is positive (with a beta of 0.09), which means that the portfolio on average bought recent winners. But the magnitude of this exposure is smaller than expected for a portfolio that aims to capture returns from momentum investing. It seems that momentum only contributed 0.6% to portfolio returns ($UMD\ beta \times UMD\ risk\ premium = 0.09 \times 7.3\%$), while value contributed 1.7%. This may seem odd for a portfolio that is built with a 50/50 combination of value and momentum. But we should keep in mind that we’re still looking at an incomplete model — one without all the risk factors in the portfolio. Let’s see what happens when we add the size variable in our next model (Model 4 in Exhibit 3).

$$(4)\ (R_i - R_f) = \alpha + \beta_{MKT}\left(R_{MKT} - R_f\right) + \beta_{HML}(R_{HML}) + + \beta_{UMD}(R_{UMD}) + \beta_{SMB}(R_{SMB}) + \varepsilon$$

[19] Even though value has a negative univariate correlation with the portfolio (as seen in Exhibit 2), we can see that after controlling for market exposure (in Exhibit 3), the portfolio loads positively on value. We will discuss the importance of multivariate over univariate regressions for a multi-factor portfolio later in the paper.

In our final model (which includes all the sources of return that the portfolio aims to capture), we still see a small beta on momentum, with the factor contributing 0.5% to portfolio returns over the period ($UMD\ beta \times UMD\ risk\ premium = 0.07 \times 7.3\%$). However, this unintuitive result can be largely explained by factor design differences. Stay tuned and we will come back to this issue later in the paper.[20]

The good news is that when it comes to the other factors in Model 4, the results are consistent with intuition. For size, we see a large positive exposure (beta of 0.74), which means the portfolio had meaningful exposure to small-cap stocks. This exposure contributed 1.2% to portfolio returns over the period. We also see that after controlling for size, value had an even larger beta, which means that it contributed 2.4% to portfolio returns.

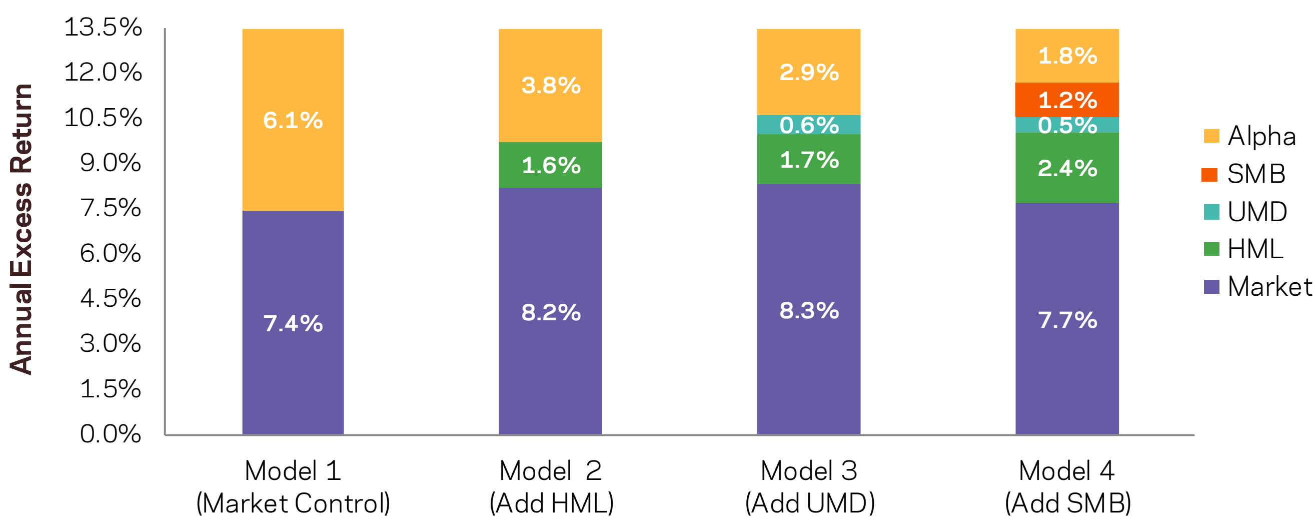
[20] See the section on "other factor design choices" where we discuss how HML can be viewed as an incidental bet on UMD; this affects regression results by lowering the loading on UMD (as HML is eating up some of the UMD loading that would otherwise exist). We correct for this in Appendix B, and show a higher loading on UMD. Also see Frazzini, Israel, Moskowitz and Novy-Marx (2013) and Asness, Frazzini, Israel and Moskowitz (2014) for more information on the most efficient way to gain exposure to momentum.

Exhibit 3: Decomposing Hypothetical Portfolio Returns by Factors
January 1980–December 2014

Part A: Regression Results

	Model 1 (Market Control)	Model 2 (Add HML)	Model 3 (Add UMD)	Model 4 (Add SMB)
Alpha (ann.)	**6.1%**	**3.8%**	**2.9%**	**1.8%**
t-statistic	*3.6*	*2.5*	*1.9*	*2.2*
Market Beta	**0.96**	**1.05**	**1.07**	**0.99**
t-statistic	*31.1*	*35.7*	*36.0*	*61.5*
HML Beta		**0.43**	**0.46**	**0.65**
t-statistic		*9.8*	*10.3*	*26.4*
UMD Beta			**0.09**	**0.07**
t-statistic			*3.0*	*4.6*
SMB Beta				**0.74**
t-statistic				*32.2*
R^2	0.70	0.75	0.76	0.93

Part B: Portfolio Return Decomposition



Note: All returns are arithmetic averages. The bar chart in Part B uses the factor returns (from Exhibit 2) and factor betas (from Part A) to decompose portfolio returns. Numbers may not exactly tie out due to rounding.

Source: AQR, Ken French Data Library. AQR analysis based on a hypothetical simple 50/50 value and momentum long-only small-cap equity portfolio, gross of fees and transaction costs, and excess of cash. The portfolio is rebalanced monthly. The academic explanatory variables are the contemporaneous monthly Fama-French factors for the market (MKT-RF), value (HML), momentum (UMD) and size (SMB). The market is the value-weight return of all CRSP firms. Hypothetical data has inherent limitations some of which are discussed herein.

Ultimately, in interpreting the results of regression analysis investors should focus on the model that includes the systematic sources of returns that the portfolio aims to capture; in this case, it would be Model 4. For portfolios that capture styles in an integrated way, it's important to include multiple factors to control for the correlation between factors. In other words, to take into account how factors are related to each other. It is well known that value and momentum are negatively correlated, and portfolios formed in an integrated way can take advantage of this. Focusing on how value performs stand-alone (i.e., Model 2) has little implication on how value adds to a portfolio that combines value with other factors synergistically (i.e., Model 4). One of the benefits of multi-factor investing is the relatively low correlations factors have with each other, making the "whole" more efficient than the sum of its parts.

Alpha vs. Beta

While betas are important in understanding factor exposures in a portfolio, alpha can be useful in analyzing manager "skill." It's important that investors are able to tell whether a manager is actually providing alpha, above and beyond their intended factor exposures. But this means that they need to be sure that they're using the correct model when analyzing factor exposures. Without the proper model, rewards for factor exposures may be misconstrued as alpha. This can lead to suboptimal investment choices, such as hiring a manager that seems to deliver "alpha," but really just provides simple factor tilts.

To illustrate this point we can look at the alpha estimates in Exhibit 3.[21] By looking at each model on a step-wise basis, we can see how the inclusion of additional risk factors reduces alpha significantly; in other words, alpha has been replaced by factor exposures. When the market is the only factor (Model 1) it seems as though the portfolio has significant alpha at 6.1%, but when we add the other risk factors we see that alpha is reduced to 2.9% with value and momentum, and finally to 1.8% with all four factors.[22] These results have important implications — if you don't control for multiple exposures in a multi-factor portfolio, then excess returns will look as if they are mostly alpha.

[21] It's important to caveat that even with a large number of observations (i.e., more than five years), alpha can be difficult to assess with conviction.
[22] Note that alpha goes away when you include a "purer" measure of value based on current price; this is shown in Appendix B and described in the section "other factor design choices."

But it's also important to note that "alpha" depends on what is already in your portfolio. For *any* portfolio, adding positive expected return strategies that are uncorrelated to existing risk exposures can provide significant portfolio alpha. For the market portfolio, adding value and momentum exposures can have the same effect as adding alpha (as both represent new, more efficient sources of portfolio returns).[23] Along the same lines, adding momentum to a value portfolio can provide significant alpha.

The main point is this: in order to determine whether such a factor can be "alpha to you," an investor must first determine which factors are already present in their existing portfolio — those that are not can potentially be alpha.

Factor Differences: Academics vs. Practitioners

So far we have focused on factor analysis and how to interpret the results. But the results of factor analysis are highly influenced by how factors are formed. There are many differences between the ways factors are measured from an academic standpoint versus how they get implemented in portfolios. Investors should be aware that not all factors are the same, even those attempting to measure essentially the same economic phenomenon — and these differences can matter. We focus here on design decisions that can have a meaningful impact on the results of factor analysis.

Implementability

At a basic level, academic factors do not account for implementation costs. They are gross of fees, transaction costs and taxes. They do not face any of the real-world frictions that implementable portfolios do. Essentially, they are not a perfect representation of how factors get implemented in practice.

Differences in implementation approaches may be reflected in factor model results. Even if a portfolio does a perfect job of capturing the factors, it could still have negative alpha in the regression, which would represent implementation differences associated with capturing the factors. For example, if you

[23] See Berger, Crowell, Israel and Kabiller (2012) in which they discuss the concept of "alpha to you."

compare a portfolio that faces trading costs versus one that doesn't, clearly the former will underperform the latter, possibly implying negative alpha. In fact, this is exactly what we see when we look at a composite of mutual funds — these results are shown in Appendix D. When looking at a live portfolio against academic factors, investors should not be surprised by negative alpha. In these cases, investors should either use practitioner factors on the RHS, or just focus on beta comparisons because trading costs and other implementation issues do not affect these estimates.[24]

Investment Universe

Academic factors (such as those used here) span a wide market capitalization range and are, in fact, overly reliant on small-cap stocks or even micro-cap stocks (we will explain this in greater detail in the next section). The factors include the entire CRSP universe of approximately 5,000 stocks. Many practitioners would agree that a trading strategy that dips far below the Russell 3000 is not a very implementable one, and this is likely where most of the bottom two quintiles in the academic factors fall.

Practitioners mainly focus on large- to mid-cap universes for investability reasons. For portfolios that provide exposure to the large-cap universe, academic factors may not be an accurate representation of desired exposures. Given that academic factors span a wide range of market capitalization, factor analysis results will be highly impacted by the influence of some other part of the capitalization range — a range that is not being captured in the portfolio by design.

Factor Weighting

Generally, academic factors are formed using an intersection of size and their particular factor (value, in the case of HML).[25] For the factors described in Exhibit 1, a stock's size is determined by the median market capitalization, which means a roughly equal number of stocks are considered "big" and

[24] Specifically, these implementation issues drop out of the covariance. Implementation issues, such as fees and transaction costs, are relatively stable components (constants), which mathematically don't matter much for higher moments such as covariance.

[25] See footnote 8 for more information on how the academic factors are constructed.

"small."[26] The factors are formed by giving equal capital weight to each universe, which given the higher risk of small stocks likely means that an even greater risk weight and contribution comes from small stocks. This means that betas tend to be underestimated because of stale prices for microcap stocks.

Practitioners generally take views on the entire universe, assigning larger positive weights to the stocks that rank more favorably on some measure, and larger negative weights to the less favorable stocks. For example, practitioners may weight stocks by accounting for the relative cheapness, or how "strong" in value each stock is. This approach assigns larger positive weights to the stocks that rank more favorably on B/P, for example. Practitioners would weight stocks in the resulting portfolio via each stock's standardized rank (i.e., signal-weighting) or they might use a blend – say 50/50 - of the standardized rank and market capitalization weight. Both weighting schemes result in increased exposure to stocks with high value ranks as compared to a simple value portfolio that weights the top 50% B/P stocks based on market-cap.

Industries

Academic factors do a simple ranking across stocks, and in doing so implicitly take style bets within and across industries (also across countries in international portfolios), without any explicit risk controls on the relative contributions of each. In contrast, the factors implemented by practitioners may differentiate stocks within and across industries (i.e., industry views). They are designed to capture and target risk to both *independently*. This distinction can result in a more diversified portfolio, one without unintended industry concentrations.

Risk Targeting

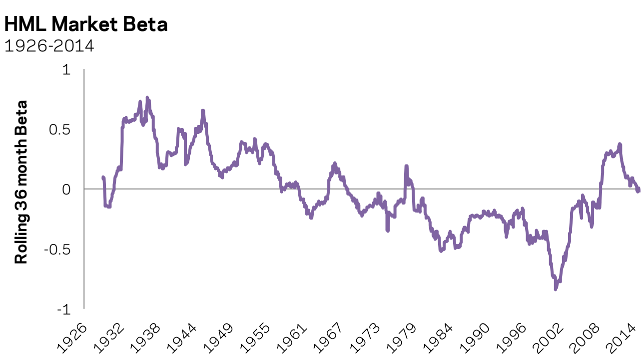
Risk targeting is a technique that many practitioners use when constructing factors; this approach dynamically targets risk to provide more consistent realized volatility in changing market conditions.

[26] Despite its large number of stocks, the small-cap group contains roughly 10% of the market-cap of all stocks (Fama and French, 1993).

While this technique can ensure that a portfolio stays diversified over time (so portfolio risk does not fluctuate with market volatility), it can also help when building a risk-balanced multi-style portfolio. That is, this technique can help practitioners ensure that their desired risk weights are maintained (in a similar vein to rebalancing portfolios to preserve strategic asset allocations). Practitioners can also build market (or beta) neutral long/short portfolios.

Academic factors typically do not utilize risk targeting as their factors are returns to a \$1 long/\$1 short portfolio, whose risk and market exposures can vary. The effect of this can be seen in **Exhibit 4,** which shows how HML has significant variation in market exposure over time.[27]

Exhibit 4: Varying Market Exposure of HML Over Time



Source: AQR, Ken French Data Library. Analysis based on the market (MKT-RF) and HML portfolios. The market is the value-weight return of all CRSP firms.

[27] Note that this graph is meant to be descriptive of the types of issues that may arise when analyzing non risk-targeted portfolios. We cannot say for certain how much of the relation shown here is noise, or if it is predictable.

Multiple Measures of Styles

While stocks selected using the traditional academic value measure perform well in empirical studies, there is no theory that says book-to-price is the best measure for value. Other measures can be used and applied simultaneously to form a more robust and reliable view of a stock's value. For example, investors can look at a variety of other reasonable fundamentals, including earnings, cash flows, and sales, all normalized by some form of price. Factors that draw on multiple measures of value can significantly improve performance, as shown in **Exhibit 5**.[28]

The same intuition applies for other styles. For example, momentum factors that include both earnings momentum and price momentum may be more robust portfolios.

Other Factor Design Choices

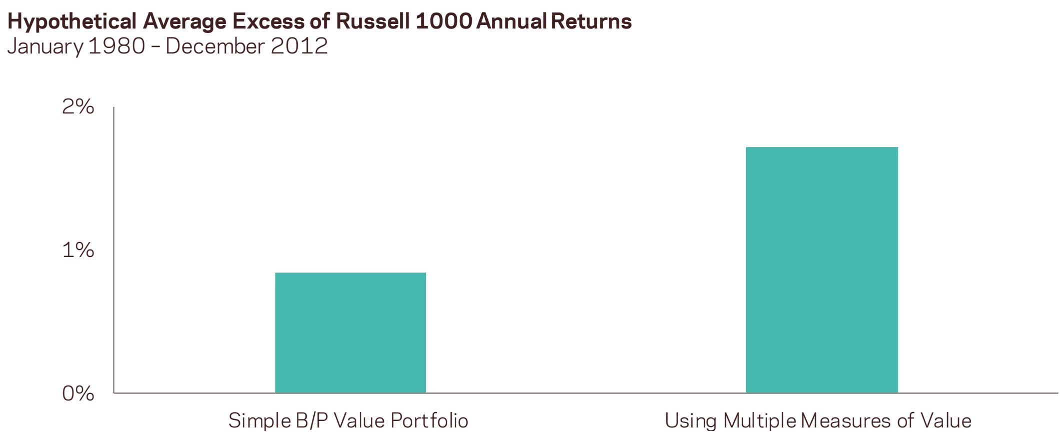
Other design decisions can have a meaningful impact on returns. Looking at the case of value, Fama–French construct HML using a lagged value for price that creates a noisy combination of value and momentum. When forming their value portfolio on book-to-price, they use the price that existed contemporaneously with the book value, which due to financial reporting can be lagged by 6 to 18 months. So a company that looked expensive based on its book value and price from six months ago and whose stock has fallen over the past six months should look better from a valuation perspective (since the price is lower, and holding book value constant[29]). Yet, in a traditional definition (using *lagged prices*) the stock is viewed the same way irrespective of the price move.

An alternative way of looking at it is to define value with the *current price*, which means the stock is now cheaper. On the other hand if you incorporate momentum into the process the stock doesn't look any better since its price has fallen over the past six months. Putting the two together, the stock looks more attractive from a value perspective, but less attractive from a momentum perspective, with the net

[28] Asness, Frazzini, Israel and Moskowitz (2014); Asness, Frazzini, Israel and Moskowitz (2015); Israel and Moskowitz (2013).
[29] This is a reasonable assumption. See Asness and Frazzini (2013).

effect ending up potentially in the same place as the traditional definition of value. As a result, the traditional definition can be viewed as an incidental bet on both value and momentum; in fact, empirically the traditional definition of value ends up being approximately 80% pure value (current price) and 20% momentum.[30]

Exhibit 5: Design Decisions Are Important in Portfolio Construction



Source: Frazzini, Israel, Moskowitz and Novy-Marx (2013). Book-to-price is defined using current price. The multiple measures of value include book-to-price, earnings-to-price, forecasted earnings-to-price, cash flow-to-enterprise value, and sales-to-enterprise value.

In order to correct for this noisy combination of value and momentum, Asness and Frazzini (2013) suggest replacing the 6- to 18-month lagged price with the current price to compute valuation ratios that use more updated information. Measuring HML using current price (what they call "HML Devil") eliminates any incidental exposure to momentum, resulting in a better proxy for true value, while still using information available at the time of investing.

This factor design choice is especially important when interpreting regression results. When regressing a portfolio of value and momentum on UMD and HML (using the traditional academic definition), it will appear that UMD has a lower loading, as HML is eating up some of the UMD loading that would otherwise exist. This is exactly what we saw in Exhibit 3, where UMD had a very low loading.

[30] Asness and Frazzini (2013).

However, if HML is defined using current price (as is the case with HML Devil), the value loading will no longer have exposure to momentum and any momentum exposure in the portfolio will go directly into UMD, thus raising its loading. This is consistent with what we see when we make the HML Devil correction to the analysis from Exhibit 3: the UMD loading increases from 0.07 to 0.32; these results are shown in the Appendix in Exhibit B1.

In this section we have discussed a few factor differences that can meaningfully affect the results of factor analysis. As a result, we encourage investors to be aware of these differences when interpreting regression results.

Concluding Remarks

Market exposure has historically rewarded long-term investors, but market risk is only one exposure among several that can deliver robust long-term returns. Measuring exposure to risk factors can be a challenge: factors can be formed multiple ways and statistical analysis is ridden with nuances. Ultimately investors who understand how to measure factor exposures may be better able to build truly diversified portfolios.

The following summary points are useful for investors to think about when decomposing portfolios into risk factors:

- Even a single factor such as value has variations that an investor should consider: there are many differences between how factors are constructed from an academic standpoint versus how they are implemented in portfolios. In conducting factor analysis, investors should ask themselves: What exactly are these factors I'm using? Are they the same as those I'm getting in my portfolio? The answers to these questions affect beta and alpha estimates. Factor loadings are highly influenced by the design and universe of factors; and alpha estimates reflect implementation differences associated with capturing the factors. For example, if you compare a portfolio that faces trading costs versus one that doesn't, it is not surprising the former will underperform the latter, and possibly show negative alpha. When investors want to compare alphas and betas across managers they should be sure they are using the factors being captured in the portfolios. Ultimately, not accounting for factor exposures properly can lead to suboptimal investment choices, such as hiring an expensive manager that seems to deliver "alpha," but really just provides simple factor tilts.

- There are many things to consider when performing statistical analysis on portfolios. For portfolios with more than one risk factor, multivariate models are most appropriate. Investors should consider *t*-statistics, not just betas, to assess factor exposures, especially when comparing portfolios with different volatilities.
- In order to determine whether a certain factor exposure can be "alpha to you," an investor must first determine which factors are already present in their existing portfolio — those that are not can potentially be alpha.

Appendix A | The Statistics of Regression Analysis

We hope these details will help investors better understand and interpret the results of regression models.

The Mechanics of Beta

Investors looking to analyze portfolio exposures often look at betas of regression results. Beta measures the sensitivity of the portfolio to a certain factor. In the case of market beta, it tells us how much a security or portfolio's price tends to change when the market moves. From a mathematical standpoint, the beta for portfolio *i* is equal to its correlation with the market times the ratio of the portfolio's volatility to the market's volatility.[31]

$$\beta_{i,m} = \rho_{i,m}(\sigma_i/\sigma_m)$$

or,f

$$factor\ beta = factor\ correlation\ with\ portfolio \times \left(\frac{portfolio\ volatility}{factor\ volatility}\right)$$

Since volatility varies considerably across portfolios, comparisons of betas can be misleading. Using the equation above, we can see that for the same level of correlation, the higher a portfolio's volatility, the higher its beta. Let's see why this matters. Suppose an investor is comparing value exposure for two different portfolios: portfolio A is a defensive equity portfolio (with lower volatility) and portfolio B is a levered equity portfolio (with higher volatility). It could be the case that portfolio B has a higher value beta, which would seem to indicate that it has higher value exposure. However, the higher beta could be a result of portfolio B's higher volatility, rather than more meaningful value exposure (assuming the same level of correlation between both portfolios and the value factor). When investors fail to account for different levels of volatilities between portfolios, they may conclude that one portfolio is providing

[31] This equation applies for betas using a univariate regression, i.e., with a single right-hand side variable. Multivariate regression betas may differ from univariate betas because they control for the other right-hand side variables, which means that they take correlations into account.

more value exposure than another, which it does in notional terms — but in terms of exposure per unit of risk, that may not be the case.

This approach can also be extended to comparisons of different factors for the same portfolio. Looking back at Exhibit 3 under Model 4, we can compare the loadings on value and momentum. One would expect similar betas on these factors as the portfolio is built to target each equally (with 50/50 weight).[32] But even with similar correlation with the portfolio, value has a meaningfully higher loading (looking at Model 4). Does this mean that value contributes more than momentum? Not necessarily as we need to account for their differing levels of volatility. For the same level of correlation, the higher a factor's volatility, the lower its beta. Put differently, the lower beta on UMD versus HML is partly driven by differing volatility levels[33] — from Exhibit 2 we see that UMD had volatility of 15.8%, while HML had volatility of 10.5%.

But investors can make adjustments to allow for more direct beta comparisons. When comparing factors for the same portfolio, the impact of differing volatilities should be eliminated; this can be done by volatility scaling the right-hand side (RHS) factors such that they all realize the same volatility. And for those looking to compare betas across portfolios (on a risk-adjusted basis), they can look at correlations, which are invariant to volatility and can be compared more directly across portfolios with different volatilities.[34]

Portfolio Risk Decomposition

[32] Some investors may be familiar with the work that Sharpe did on style analysis (1988, 1992). This approach constrains the regression so that the coefficients are positive and sum to one. In this case, the coefficients (or betas) can be used as weights in building the 'replicating' portfolio. In other words, a portfolio with factor weights equal to the weighted average of the coefficients should behave similar in terms of its returns.

[33] The lower relative loading is partly driven by differing volatilities, but it is also a result of the fact that HML can be viewed as an incidental bet on both value and momentum. We correct for this by using a "purer" measure of value; this is shown in Appendix B and described in the section "other factor design choices."

[34] Though for a multi-factor portfolio, investors should focus on partial correlations, which provide insight into the relationship between two variables while controlling for a third. Alternatively, for a long-only portfolio investors can look at correlations using active returns; that is, net out the market or benchmark exposure.

Betas from regression analysis can also be used in portfolio risk attribution. This approach is best thought of as variance decomposition, and is done by using factor beta, factor volatility, portfolio volatility and factor correlations.[35] For example, from Exhibit 2 and 3 we see that the market factor had average volatility of 15.6% and a market beta of 0.96 (based on Model 1). This tells us that the market factor dominates the risk profile of the portfolio, contributing an estimated 14.9% risk to the portfolio ($\sqrt{\text{market beta}^2 \times \text{market volatility}^2} = \sqrt{0.96^2 \times 15.6\%^2}$).[36] Given that overall portfolio risk is 17.8%, we can estimate the proportion of variance that is being driven by market exposure $\left(\frac{\text{market variance contribution}}{\text{portfolio variance}}\right) = \left(\frac{14.9\%^2}{17.8\%^2}\right) = 0.70$. This means that roughly 70% of portfolio variance can be attributed to the market risk factor.[37] But there is an interesting application of this result: 0.70 is the same as the R^2 measure for Model 1 (shown in the final row of the regression table in Exhibit 3). We will now discuss R^2 in more detail.

The R^2 Measure: Model Explanatory Power

The R^2 measure provides information on the overall explanatory power of the regression model. It tells us how much of returns are explained by factors included on the right-hand side of the equation. Generally, the higher the R^2 the better the model explains portfolio returns. We can see from the R^2 measure at the bottom of the table in Exhibit 3 that multivariate analysis is more effective (than univariate) at explaining returns for a multi-factor portfolio. In particular, we see in the final column of the table that the inclusion of additional risk factors has improved the explanatory power of the model (that is, how much of portfolio variance is being captured by these factors), with the R^2 improving from 0.70 to 0.93.[38]

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[35] This approach is similar to decomposing portfolio risk by using portfolio weights, correlation and volatility estimates. We have included an example of how to do this for a simple two factor portfolio in Appendix C.

[36] Note that volatility is the square root of variance.

[37] In this case the idiosyncratic, asset-specific risk would account for 30% of the overall variance of the portfolio. This example focuses on a single-factor model where we can ignore factor correlations. If we were to apply the same approach for a multi-factor model, factor correlations would matter and we would need to incorporate the covariance matrix. This approach requires matrix algebra and is computationally intensive, so we have omitted the calculation.

[38] Note that it's more accurate to look at the adjusted R^2 when comparing models with a different number of explanatory variables. By construction, the R^2 will never be lower and could possibly be higher when additional explanatory variables are included in the regression; and the adjusted R^2 corrects for that. When there are a large number of observations the two measures will be similar; this is the case with our regression as we use monthly data over 35

The *t*-statistic: A Measure of Statistical Significance

While beta tells us whether a factor exposure is economically meaningful (and how much a factor may contribute to risk and returns), it doesn't tell us whether the factor exposure is statistically significant. Just because a portfolio has a high beta coefficient to a factor doesn't mean it's statistically different than a portfolio with a zero beta, or no factor exposure. As such, it's important to look at the *t*-statistic. This measure tells us whether a particular factor exposure is statistically significant. It is a measure of how confident we are about our beta estimates.[39] When the *t*-statistic is greater than two, we can say with 95% confidence (or a 5% chance we are wrong) that the beta estimate is statistically different than zero.[40] In other words, we can say that a portfolio has significant exposure to a factor.

Looking back at the momentum factor, even though the portfolio may not have an economically meaningful beta (at 0.07 in Model 4), we can see that it is statistically significant (with a *t*-statistic greater than two). The *t*-statistic is an especially important measure for comparing portfolios with different volatilities.

At the end of the day, both beta and *t*-statistics provide valuable information when assessing factor exposures. A factor exposure that is both economically meaningful and statistically significant (reliable) means you can count on it impacting your portfolio in a big way. An exposure that is only economically meaningful but not reliable *could* impact you in a big way, but with a high degree of uncertainty. Finally, an exposure that is small but reliable means you can expect (with greater certainty) that it will impact your portfolio, but only in a small way. While an investor may not care a lot about this last application, it's still worth understanding when analyzing the regression output.

years (meaning a large sample size with 420 observations).

[39] It's important to note that the *t*-statistic increases with more observations; that is, as the sample size grows very large we are more certain about our beta estimates.

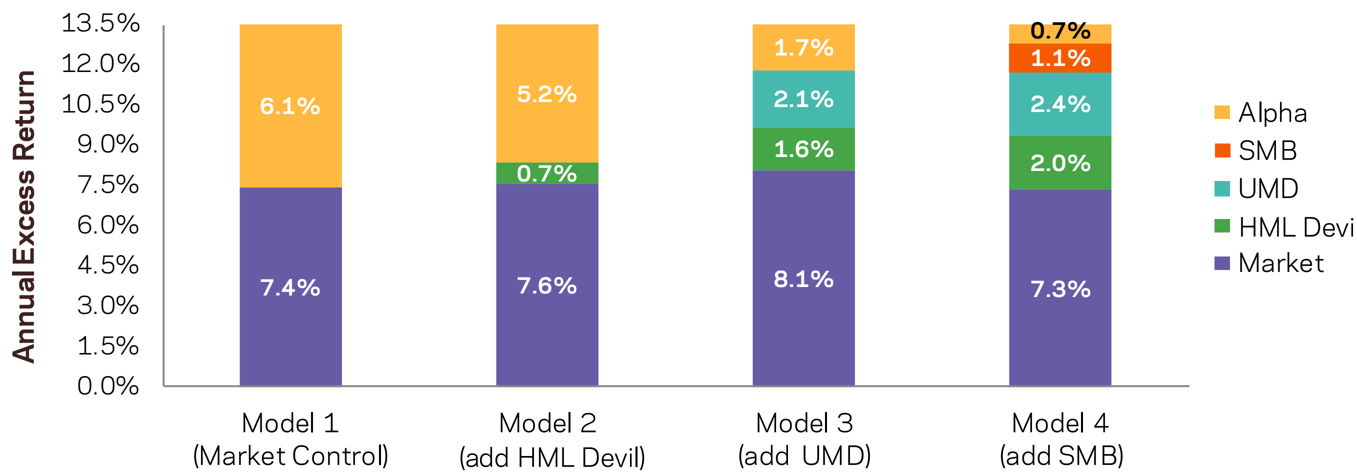
[40] A *t*-statistic of two generally represents a reasonable standard of significance (implies statistical significance at a 95% confidence interval under the assumption of a normal distribution) if no look-ahead bias. Generally, the higher the *t*-statistic the more confident we can be about our beta estimates.

Appendix B | Correcting for HML Devil

Exhibit B1: Decomposing Hypothetical Portfolio Returns by Factors
January 1980–December 2014
Part A: Regression Results

	Model 1 (Market Control)	Model 2 (Add HML Devil)	Model 3 (Add UMD)	Model 4 (Add SMB)
Alpha (ann.)	**6.1%**	**5.2%**	**1.7%**	**0.7%**
t-statistic	*3.6*	*3.2*	*1.1*	*0.7*
Market Beta	**0.96**	**0.98**	**1.04**	**0.94**
t-statistic	*31.1*	*32.8*	*35.5*	*50.0*
HML Devil Beta		**0.22**	**0.48**	**0.61**
t-statistic		*5.9*	*9.6*	*19.0*
UMD Beta			**0.29**	**0.32**
t-statistic			*7.3*	*12.9*
SMB Beta				**0.68**
t-statistic				*25.1*
R^2	0.70	0.72	0.75	0.90

Part B: Return Decomposition



Source: AQR analysis based on a hypothetical simple 50/50 value and momentum long-only small-cap equity portfolio, gross of fees and transaction costs, and excess of cash. The portfolio is rebalanced monthly. The academic explanatory variables are the contemporaneous monthly academic factors for the market (MKT-RF), value (HML Devil), momentum (UMD), and size (SMB). The portfolio returned 13.5% in excess of cash on average over the period, the market returned 7.8% excess of cash, HML Devil returned 3.3%, UMD returned 7.3% and SMB returned 1.6%. The market is the value-weight return of all CRSP firms. Hypothetical data has inherent limitations some of which are discussed herein.

Appendix C | Alternate Method of Hypothetical Portfolio Risk Decomposition

For this example, we use a simple 50/50 value/momentum long/short portfolio.

Step 1: Determine the covariance matrix

Using assumptions on volatility and correlation[41] (inputs in blue), we create the covariance matrix.

Portfolio Inputs

	Volatility
Value (HML)	11%
Momentum (UMD)	16%

Correlation Matrix

	Value (HML)	Momentum (UMD)
Value (HML)	1.0	-0.2
Momentum (UMD)	-0.2	1.0

Covariance Matrix

	Value (HML)	Momentum (UMD)
Value (HML)	0.012	-0.003
Momentum (UMD)	-0.003	0.012

$$\begin{aligned}\textbf{Covariance(HML, UMD)} &= \textbf{Correlation(HML, UMD)} \times \textbf{Volatility(HML)} \times \textbf{Volatility (UMD)}\\ &= -0.2 \times 11\% \times 16\% \\ &= -0.003\end{aligned}$$

Step 2: Determine the variance contribution of each factor

Using capital weights and the covariance matrix from step 1 (shown by the inputs in blue below), we can determine the variance contribution (VAR Contrib.) of each factor.

Portfolio Inputs

	Volatility	Capital Weights
Value (HML)	11%	50%
Momentum (UMD)	16%	50%

Covariance Matrix

	Value (HML)	Momentum (UMD)
Value (HML)	0.012	-0.003
Momentum (UMD)	-0.003	0.012

	Variance
Value (HML)	0.23%
Momentum (UMD)	0.57%
Portfolio	0.80%

$$\begin{aligned}&\textbf{VAR Contrib. (HML)}\\ &= \textbf{Weight(HML)}^2 \times \textbf{Volatility(HML)}^2 + \textbf{Weight(HML)} \times \textbf{Weight (UMD)}\\ &\times \textbf{Covariance(HML, UMD)}\\ &\quad = 50\%^2 \times 11\%^2 + 50\% \times 50\% \times -0.003\\ &\quad = 0.23\%\end{aligned}$$

Note: unlike volatility, portfolio variance is additive:

$$\begin{aligned}\textbf{VAR(Portfolio)} &= \textbf{VAR Contrib. (HML)} + \textbf{VAR Contrib. (UMD)}\\ &= 0.23\% + 0.57\%\\ &= 0.80\%\end{aligned}$$

[41] Note that we have used assumptions that are broadly representative of the historical volatilities and correlations for HML and UMD. But the example applies for any set of assumptions. It is for illustrative purposes only.

Step 3: Determine the percent risk/variance contribution of each factor

Finally, using the variance from step 2 we can determine the percent of portfolio variance coming from each factor.

	Volatility	Capital Weights	Variance
Value (HML)	11.0%	50%	0.23%
Momentum (UMD)	16.0%	50%	0.57%
Portfolio	8.9%	100%	0.80%

	% Contribution to Variance
Value (HML)	30%
Momentum (UMD)	70%
Portfolio	100%

$$\%\ \textbf{Contribution to Variance (HML)} = \frac{\textbf{VAR Contrib. (HML)}}{\boldsymbol{VAR\ (Portfolio)}}$$

$$= \frac{0.23\%}{0.80\%}$$

$$\approx 30\%$$

Appendix D | Applications for a Live Portfolio

In this paper we have focused on a hypothetical portfolio that aims to capture returns from value and momentum. We have done this for simplicity and illustrative purposes, but the same framework can be applied for any portfolio. So, what about a live portfolio? Should we expect the same results? In this section we use the Morningstar style boxes to identify and analyze the universe of small-cap value managers. That is, we look at a composite of all small-cap value managers as identified by Morningstar.[42]

The factor exposures shown here are directionally similar to those shown for the hypothetical portfolio we analyzed in the paper. As expected, we see positive and significant exposure to the market, value and size.[43] But an interesting result comes from a comparison of alpha, where we see that alpha goes from zero to negative in the final model. While this result is different than the stylized example we examined in the paper, it is consistent with our section on implementable factors. Ultimately, live portfolios face fees, transaction costs and taxes — all of which fall out of alpha.

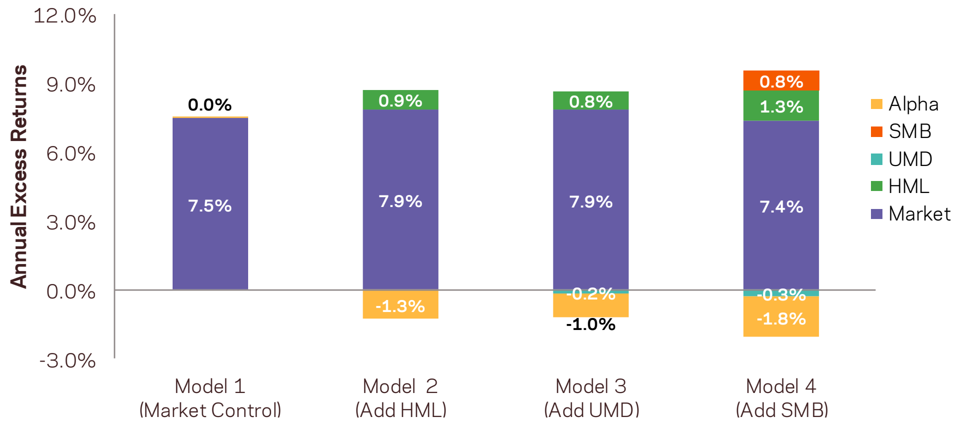
Exhibit C1: Analyzing a Composite of Small-Cap Value Managers
January 1980–December 2014

Part A: Regression Results

	Model 1 (Market Control)	Model 2 (Add HML)	Model 3 (Add UMD)	Model 4 (Add SMB)
Alpha (ann.)	**0.0%**	**-1.3%**	**-1.0%**	**-1.8%**
t-statistic	*0.0*	*-1.0*	*-0.8*	*-2.1*
Market Beta	**0.96**	**1.01**	**1.01**	**0.95**
t-statistic	*40.5*	*42.2*	*41.3*	*57.1*
HML Beta		**0.23**	**0.23**	**0.36**
t-statistic		*6.6*	*6.2*	*14.3*
UMD Beta			**-0.02**	**-0.04**
t-statistic			*-1.0*	*-2.3*
SMB Beta				**0.54**
t-statistic				*22.4*
R^2	0.80	0.82	0.82	0.92

Part B: Hypothetical Portfolio Return Decomposition

[42] This composite was obtained from Morningstar as of June 2015.
[43] Note that it is not surprising to see a low negative momentum loading as we are only looking at a value portfolio, rather than a 50/50 value/momentum portfolio (as we did earlier in the paper).



Source: AQR analysis based on the Morningstar universe of small-cap value mutual funds. The composite returns are net of management and performance fees. The academic explanatory variables are the contemporaneous monthly Fama-French factors for the market (MKT-RF), value (HML), momentum (UMD), and size (SMB). The portfolio returned 7.5% in excess of cash on average over the period, the market returned 7.8% excess of cash, HML returned 3.6%, UMD returned 7.3% and SMB returned 1.6%.

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Biographies

Ronen Israel, *Principal*

Ronen's primary focus is on portfolio management and research. He was instrumental in helping to build AQR's Global Stock Selection group and its initial algorithmic trading capabilities, and he now also runs the Global Alternative Premia group, which employs various investing styles across asset classes. He has published in *The Journal of Portfolio Management*, *The Journal of Financial Economics* and elsewhere, and sits on the executive board of the University of Pennsylvania's Jerome Fisher Program in Management and Technology. He is an adjunct professor of finance at New York University, has been a guest speaker at Harvard University, the University of Pennsylvania, Columbia University and the University of Chicago, and is a frequent conference speaker. Prior to AQR, Ronen was a senior analyst at Quantitative Financial Strategies Inc. He earned a B.S. in economics from the Wharton School at the University of Pennsylvania, a B.A.S. in biomedical science from the University of Pennsylvania's School of Engineering and Applied Science, and an M.A. in mathematics, specializing in mathematical finance, from Columbia.

Adrienne Ross, *Vice President*

Adrienne is a member of AQR's Portfolio Solutions Group, where she writes white papers and conducts investment research. She is also involved in the design of multi-asset portfolios and engages clients on portfolio construction, risk allocation and capturing alternative sources of returns. She has published research on how different investments respond to economic environments in *The Journal of Portfolio Management*, regional economic factors in *The Journal of Economic Geography* and on the Web site of the Federal Reserve Bank of New York. Prior to AQR, she was a senior associate at PIMCO. She began her career as a researcher at a macroeconomic think tank in Canada. Adrienne earned a B.A. in economics and mathematics from the University of Toronto and an M.A. in quantitative finance from Columbia University.

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The information presented is supplemental to a GIPS-compliant presentation; however, because AQR does not currently manage accounts in the strategy presented, a GIPS-compliant presentation is not available. A complete list and description of all firm composites is available upon request.

The white papers discussed herein can be provided upon request.