Strategic Rebalancing

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ABSTRACT

A mechanical rebalancing strategy, such as a monthly or quarterly reallocation towards fixed portfolio weights, is an active strategy. Winning asset classes are sold and losers are bought. During crises, when markets are often trending, this can lead to substantially larger drawdowns than a buy-and-hold strategy. Our paper shows that the concavity (or "negative convexity") induced by rebalancing can be substantially mitigated, taking the popular 60-40 stock-bond portfolio as our use case. One alternative is an allocation to a trend-following strategy. The positive convexity of this overlay tends to counter the impact on drawdowns of the mechanical rebalancing strategy. The second alternative we call *strategic rebalancing*, which uses smart rebalancing timing based on trend-following signals –without a direct allocation to a trend-following strategy. For example, if the trend-following model suggests that stock markets are in a negative trend, rebalancing is delayed.

Keywords: *Asset allocation, Smart rebalancing, Market timing, Active management, Buy and hold, Overlays, 60-40 portfolio, Balanced portfolio, Stock-bond portfolio, Rebalancing, Trend, Momentum*

JEL codes: *E32, E44, G11, G12, G13*.

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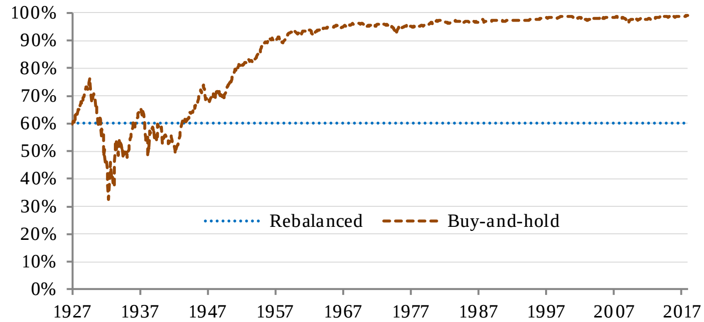
Introduction

A pure buy-and-hold portfolio has the drawback that the asset mix tends to drift over time and, as such, is untenable for investors who seek diversification. As illustrated in Figure 1 for a US stock-bond portfolio, an initial 60% of capital allocated to stocks in 1927 drifts to a 76% allocation by 1929, a 32% allocation by 1932, and a level close to 100% over time, as stocks tend to outperform bonds over the long run.

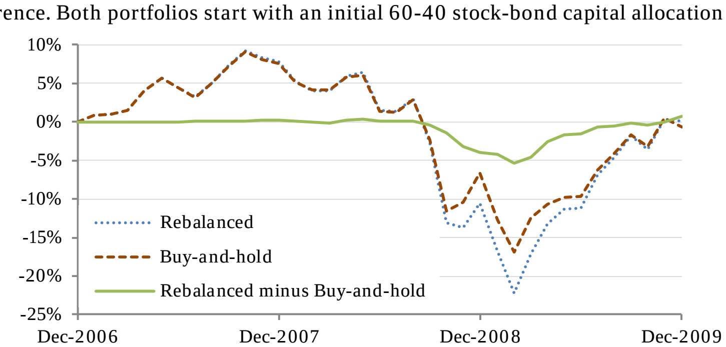
Figure 1: Allocation to Stocks for a Monthly-Rebalanced and Buy-and-Hold Portfolio
The figure shows the percent allocated to stocks for a monthly-rebalanced and a buy-and-hold portfolio. In both cases, at the start 60% of capital is allocated to stocks and 40% to bonds. We use monthly US data from January 1927 to December 2017. The stock data are from Kenneth French's website. The bond data are from the Federal Reserve, prepended with Global Financial Data (GFD).



However, a stock-bond portfolio that regularly rebalances tends to underperform a buy-and-hold portfolio at times of continued outperformance of one of the assets. Using a simple two-period model, we explain the main intuition behind this effect: rebalancing means selling (relative) winners, and if winners continue to outperform, that detracts from performance.

As stocks typically have more volatile returns than bonds, relative returns tend to be driven by stocks. Hence, of particular interest are episodes with continued negative (absolute and relative) stock performance, such as the 2007-2009 global financial crisis. In Figure 2, we contrast the monthly-rebalanced and buy-and-hold cumulative performance over the financial crisis period, where both start with an initial 60-40 stock-bond capital allocation. The maximum drawdown of the monthly-rebalanced portfolio is 1.2 times (or 5 percentage points) worse than that of the buy-and-hold portfolio, right at the time when financial markets turmoil is greatest.

Figure 2: Performance Monthly-Rebalanced and Buy-and-Hold Portfolio (2007-2009)
The figure shows the cumulative return for a monthly-rebalanced and buy-and-hold performance for the 2007-2009 financial crisis period, as well as the difference. Both portfolios start with an initial 60-40 stock-bond capital allocation in January 2007.



In earlier work, Granger et al. (2014) formally show that rebalancing is similar to starting with a buy-and-hold portfolio and adding a short straddle (selling both a call and a put option) on the relative value of the portfolio assets. The option-like payoff to rebalancing induces concavity (or *negative convexity*) by magnifying drawdowns when there are pronounced divergences in asset returns.[2]

We show that time-series momentum (or trend) strategies, applied to futures on the same stock and bond markets, are natural complements to a rebalanced portfolio. This is because the trend payoff tends to mimic that of a long straddle option position, or exhibits *positive convexity*, see, e.g., Martin and Zou (2012) and Hamill, Rattray, and Van Hemert (2016).[3]

Our main analysis is for the 1960-2017 period, which includes the bond bear market of the 1960 and 1970s, but omits the different bond regime before 1960.[4] We evaluate how 1-, 3-, and 12-month trend strategies perform during the five worst drawdowns for the 60-40 stock-bond portfolio. Allocating 10% to a trend strategy and 90% to a 60-40 monthly-rebalanced portfolio improves the average drawdown by about 5 percentage points, compared to a 100% allocation to a 60-40 monthly rebalanced portfolio. The trend allocation has no adverse impact on the average return over our sample period. That is, while one would normally expect a drag on the overall (long-term) performance when allocating to a defensive strategy, in our sample, the trend-following premium earned offsets the cost (or insurance premium) paid.[5]

An alternative to a trend allocation is strategically timing and sizing rebalancing trades, which we label *strategic rebalancing*. We first consider a range of popular heuristic rules, varying the rebalancing frequency, using thresholds, and trading only partially back to the 60-40 asset mix. Such heuristic rules reduce the average maximum drawdown level for the five crises considered by up to 1 percentage point. However, using strategic rebalancing rules based on either the past stock or past stock-bond relative returns gives improvements of 2 to 3 percentage points.

The literature on rebalancing dates back to at least Perold and Sharpe (1988).[6] Our main contribution is that we show that the concavity induced by rebalancing is effectively countered with a trend exposure, which exhibits convexity and can be either implemented directly via an allocation to a trend product, or alternatively with a strategic trend-based rebalancing rule. The five worst drawdowns for a 60-40 portfolio over the 1960-2017 period considered are materially reduced.

Our paper is organized as follows. In Section 1, we show that the return difference between a rebalanced and a buy-and-hold portfolio is concave in the relative stock-bond performance, both analytically in a stylized two-period model and empirically for the 1960-2017 period. In Section

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[2] Among practitioners, the term "negative convexity" rather than concavity is often used. This stems from reading position exposures on risk sheets and so preferring a measure that can be either positive or negative, like beta or delta, rather than the more cumbersome switching between convex and concave when the direction changes.

[3] While time-series momentum applied to macro markets (like a broad equity index or Government bond) considered in this paper tends to display positively convex returns, Daniel and Moskowitz (2016) argue that cross-sectional momentum applied to individual stocks is subject crash risk.

[4] See also Harvey et al. (2018) for a discussion on the different US bond regimes.

[5] We find that the performance of trend strategies is consistent over time, not driven by any particular sub-period.

[6] See also Fernholz and Shay (1982), Booth and Fama (1992), Erb and Harvey (2006), and Brown (2015).

2, we show that the return to a trend strategy applied to stocks and bonds is convex in the relative stock-bond performance. We illustrate that a modest allocation to a trend strategy can effectively counter the concavity induced by rebalancing and as such reduce drawdowns. In Section 3, we explore different heuristics as well as trend-based strategic rebalancing rules and show that in particular the strategic rebalancing rules are helpful for reducing drawdowns for a 60-40 stock-bond portfolio. In Section 4, we compare a direct allocation to trend and an indirect trend exposure obtained with a trend-based strategic rebalancing rule. We provide concluding remarks in Section 5.

1. Comparing Rebalanced and Buy-and-Hold Portfolio Returns

The notion that 60/40 equity/bond is a good asset mix has been around for decades, see e.g. Ambachtsheer (1987) for an early reference.[7] From a general equilibrium point of view this makes sense, as the ratio of equity and bond value in e.g. the US has been around 60:40 over the past decades, even though this ratio is subject to considerable variability.[8] Large pension plans and sovereign wealth funds often explicitly target a fixed 60-40 asset mix. For example, the Norwegian Government Pension Fund Global in 2007 adopted a 60% target allocation to equities, with the remainder mostly invested in fixed income, see e.g., Chambers, Dimson, and Ilmanen (2012). In this section, we start by considering a two-period model to illustrate the difference between monthly rebalancing to a constant asset mix ('Rebal') and buy-and-hold ('Hold').

Writing the return for stocks and bonds in period $t = 1, 2$ as R_t^S and R_t^B respectively, the return of a portfolio that rebalances between period 1 and 2 to allocations w^S and w^B for stocks and bonds is:

$$1 + R^{REBAL} = \left(1 + w^S R_1^S + w^B R_1^B\right)\left(1 + w^S R_2^S + w^B R_2^B\right).$$

For a portfolio that starts with the same weights as above, but does not rebalance after period 1, we get:

$$1 + R^{HOLD} = w^S\left(1 + R_1^S\right)\left(1 + R_2^S\right) + w^B(1 + R_1^B)(1 + R_2^B).$$

We can rewrite the return for stocks and bonds in terms of the average returns $R_t^{Avg} = 0.5R_t^S + 0.5R_t^B$, and the stock-bond return difference, $\kappa_t = R_t^S - R_t^B$:

$$\begin{aligned} R_t^S &= R_t^{Avg} + 0.5\kappa_t, \\ R_t^B &= R_t^{Avg} - 0.5\kappa_t. \end{aligned}$$

[7] The focus of this paper is on 60/40 stocks/bonds in terms of capital allocation. More recently, volatility targeting has been gaining traction and we defer to Harvey et al. (2018) for a discussion on a 60/40 stocks/bond portfolio in terms of risk allocation.

[8] US government debt has averaged around 60% of GDP since 1966: https://fred.stlouisfed.org/series/GFDEGDQ188S. The stock market capitalization-to-GDP ratio, also known as the Buffett Indicator, has averaged around 90% since 1975: https://fred.stlouisfed.org/series/DDDM01USA156NWDB. Note that both government debt and the stock market cap have trended upwards similarly as fraction of GDP since the 1970s.

Substituting in these terms in above Rebal and Hold return expressions, taking the difference, using the fact that the allocations sum to one, and rearranging gives:

$$R^{REBAL} - R^{HOLD} = -w^S w^B \kappa_1 \kappa_2.$$

So if the relative performance is trending ($\kappa_{1,} \kappa_2$ are either both positive or both negative), then the rebalanced portfolio underperforms. Intuitively, rebalancing means selling winners, and if winners continue to outperform, that detracts from performance. And vice versa, if there is reversal of relative performance, then the rebalanced portfolio outperforms.[9,10]

Notice some special cases. For an equally weighted portfolio (50% of capital allocated to stocks and 50% to bonds), the Rebal-Hold return difference is $-0.25\kappa_1\kappa_2$, while for the 60-40 stock-bond portfolio it is slightly less: $-0.24\kappa_1\kappa_2$. In case of a 100% allocation to either stocks or bonds, we have a zero return difference, which is intuitive, as there can be no drift in the relative allocation for a one-asset portfolio.

Also notice that the formula allows us to measure the order of magnitudes. If stocks underperform bonds by 40% in both periods 1 and 2 (i.e., $\kappa_1 = \kappa_2 = -40\%$) then a 60-40 rebalanced portfolio has a 3.84 percentage point more negative return than the buy-and-hold portfolio. These numbers roughly correspond to what happened between October 2007 and February 2009, when stocks were down 50.3% (or two periods of -29.5% returns, taking into account compounding) and bonds were up 17.9% (or two periods of +8.6% returns). In Figure 1, the Rebal-Hold return difference is 5.3 percentage points per February 2009, which is slightly more, as compounding in a setting with more than two periods will exacerbate the rebalancing effect.

In a multi-period setting, the return difference between a monthly-rebalanced and buy-and-hold portfolio is similar to that of a short straddle written on the relative performance of stocks and bonds.[11] Granger et al. (2014) make this point and provide both analytical expressions and simulation results.

For our empirical analysis, we use we use monthly value-weighted returns of firms listed on the NYSE, AMEX, and NASDAQ from Kenneth French's website.[12] For bonds, we use US Treasury bond data from the Federal Reserve.[13]

In Figure 3, we plot the Rebal-Hold return difference when both have a 60-40 stock-bond mix at the start of the period (vertical axis) versus the stock-bond relative return (horizontal axis). Each dot in the figure corresponds to a 1-year (rolling 12-month) window, where data run from January

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[9] Similarly, Perold and Sharpe (1988) note that constant-mix portfolios have less downside protection and less upside than a buy-and-hold strategy, while performing better in relatively trendless but volatile environments.

[10] Goetzmann et al. (2007) discuss show that rebalancing strategies can make traditional performance metrics less reliable and suggest using manipulation-proof performance measures.

[11] Fung and Hsieh (2001) argue that trend following strategies are theoretically more related to lookback straddles, but find that empirically standard straddles explain trend-following returns as well as lookback straddles.

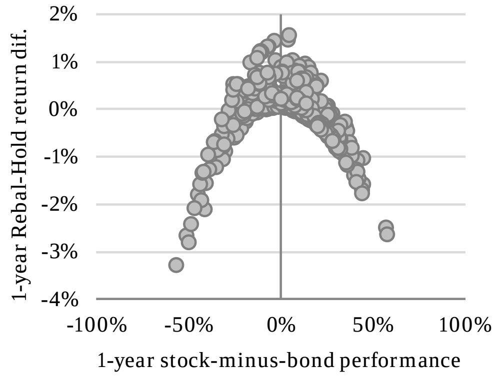
[12] See: http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data_library.html.

[13] Federal Reserve Economic Data (FRED), see https://fred.stlouisfed.org.

1960 to December 2017.[14] Indeed, the Rebal-Hold return difference looks a lot like the payoff of a short straddle on the relative performance.

Figure 3: Rebal-Minus-Hold versus Stock-Minus-Bond 1-Year Returns

The figure shows the Rebal-Hold return difference when both have a 60-40 stock-bond mix at the start of the period (vertical axis) versus the stock-bond relative return (horizontal axis). Each dot in the figure corresponds to a 1-year (rolling 12-month) window, where data run from January 1960 to December 2017.



2. Impact of a Simple Trend Strategy Allocation

We define a simple time-series momentum (trend) signal for asset k, as the return over the past N months, divided by the standard deviation of the past 12 monthly returns and the square-root of N to make it approximately unit standard deviation:[15]

$$mom_{t-1}^{k}(N) = \frac{\sum_{i=1}^{N} \tilde{R}_{t-i}^{k}}{\sigma_{t-1}^{k}\sqrt{N}}.$$

For the asset returns, we will use the stock and bond data used before, but in excess of 1-month Treasury bill returns (denoted by a tilde), which is a proxy of the return on an unfunded futures contract on the stock or bond index. We cannot use stock and bond futures data directly, as they are not available as far back as 1960.

We consider a number of trend strategies, combining trend signals for stocks, bonds, and a long stocks, short bonds spread position. The formula below illustrates how we determine the strategy return for the case of putting equal risk on stocks and bonds trend. We set a reference volatility level, σ^{Ref}, that leads to about 15-20% annualized volatility for the strategy returns. We conservatively assume that 20% of capital needs to be posted for margin and earns no interest, while the remaining 80% earns the T-bill return, R_t^F. So we get the following expression for the strategy returns of an N-month trend strategy:

$$R_t^{mom(N)} = 0.5mom_{t-1}^{S}(N)\frac{\sigma^{Ref}}{\sigma_{t-1}^{S}}\tilde{R}_t^S + 0.5mom_{t-1}^{B}(N)\frac{\sigma^{Ref}}{\sigma_{t-1}^{B}}\tilde{R}_t^B + 0.8R_t^F.$$

[14] Like Hamill, Rattray, and Van Hemert (2016), we argue it is important to start as far back as 1960, so that the sample period includes a bond bear market environment (pre-1982).

[15] Moskowitz, Ooi, and Pedersen (2012) and Levine and Pedersen (2016) use similar formulations.

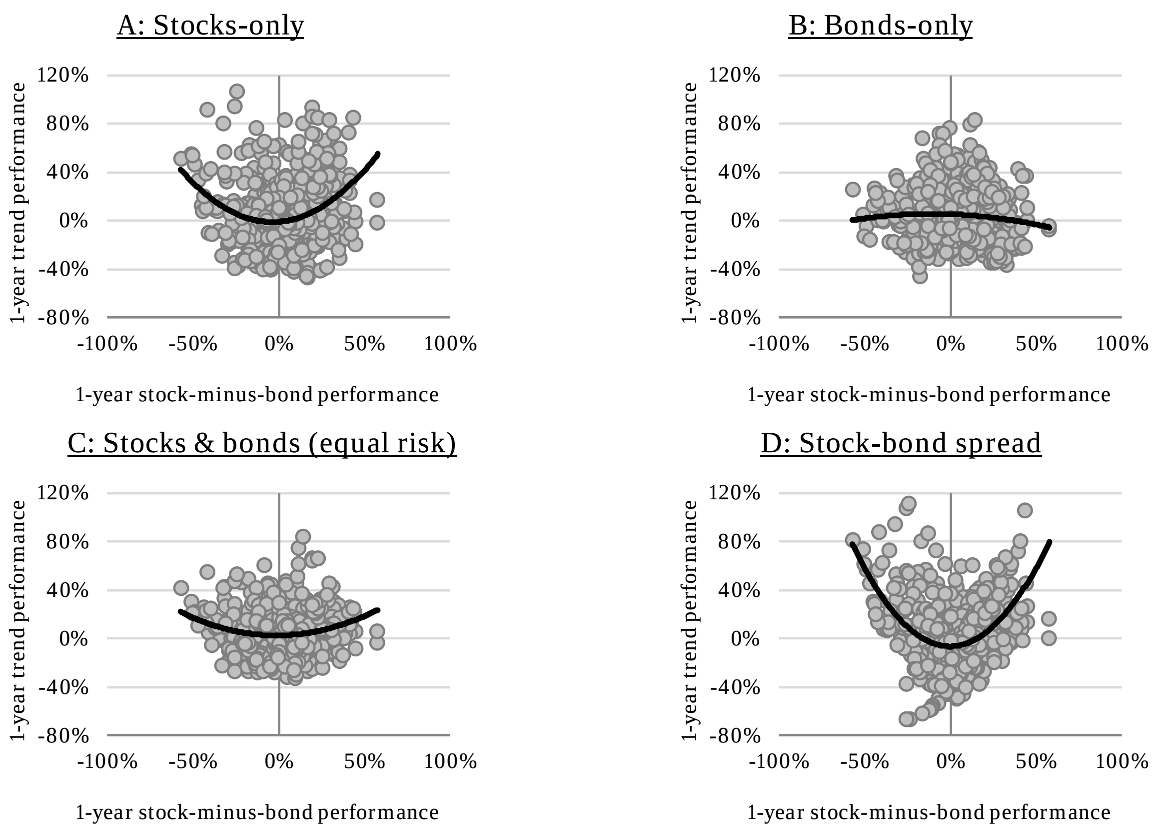
We introduce two more features so that the simple trend strategy has more bounded long and short positions in stock and bond futures. These features are consistent with how practitioners implement trend strategies. First, we put a floor and cap on the signal value of -1 and +1 respectively. Second, for the annualized security volatility used in the second equation, we use a floor of 10% in case of stocks and the stock-bond spread asset, and 5% in case of bonds, which corresponds to about two-thirds of the full-sample realized value. This will limit the leverage at times of low realized asset volatilities. Neither of these additional features changes the return dynamics much, but they do achieve more bounded positions.[16]

For the trend strategy, implemented with futures, we follow Harvey et al. (2018) and assume a transaction cost of 1bp (0.01%) of the traded notional for equities and 0.5bp for bonds (an estimate for the current trading conditions). For rebalancing of the 60-40 target portfolio, implemented with cash equity and bond holdings, we follow a recent Norges Bank (2018) report and use 30bp for equities and 13bp for bonds (reflective of the 2015-2016 period).

In Figure 4, we plot the 1-year return of various trend strategies (vertical axis) versus the 1-year relative stock-bond return (horizontal axis), with the best quadratic fit added as a solid line. We use 3-month trend signals, but we confirmed results are similar for 1- and 12-month formulations.

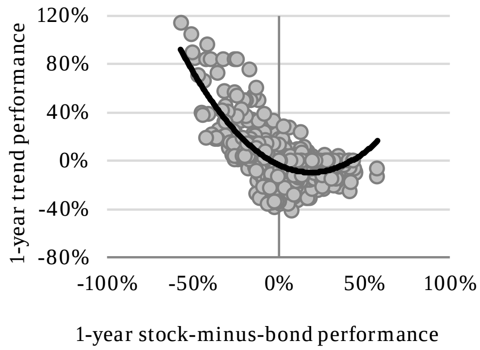
Figure 4: 3-Month Trend versus Stock-Minus-Bond 1-Year Return

This figure plots 1-year (rolling 12-month) performance of various 3-month trend strategies (vertical axis) versus the 1-year relative stock-bond performance (horizontal axis) for the period 1960 to 2017. The solid line represents the best quadratic fit.

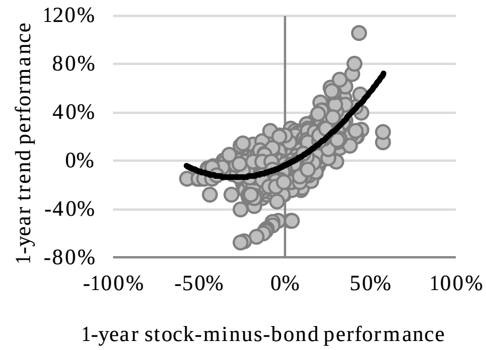


[16] This is consistent with how practitioners implement such trend strategies.

E: Stock-bond spread (if trend negative)



F: Stock-bond spread (if trend positive)



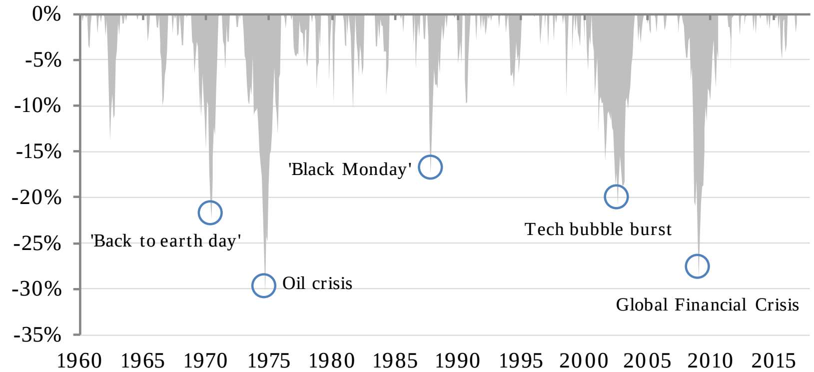
In case of a stock-bond spread trend strategy (Panel D), a very pronounced smile pattern is visible, which is expected given the known convexity property of trend strategy returns when evaluated against the returns of the traded asset (i.e., the horizontal axis in all panels is the stock-bond spread return). Also, for stock-only (Panel A) and stocks and bond, equal risk (Panel C), the trend strategy returns are convex, which is unsurprising, given that the stock and stock-bond excess returns are 0.9 correlated over the 1960-2017 period (stock return variance dominates the stock-bond return spread).[17] As such, a trend strategy with a stock weighting looks like a natural complement to a rebalancing strategy, as we have shown before that the Rebal-Hold return difference displays concavity. A bond-only trend strategy (Panel B) does not show a clear relation against the stock-bond relative return. Finally, we consider a stock-bond spread trend strategy that trades only in case of negative (Panel E) and positive (Panel F) trends. The payoff of these strategies mimics not so much a long straddle (put plus call) anymore, but rather that of just a put (in case of negative trend) and call (in case of positive trend).

Given their seemingly complementary return profile in most cases considered, we next combine a monthly-rebalanced portfolio (90%) and the various trend strategies (10%). We are particularly interested in whether the addition of a trend reduces drawdowns. In Figure 5, we depict the drawdown level for the monthly-rebalanced 60-40 portfolio, computed as the return from the peak (highest cumulative return up to that point in time). On five occasions, the drawdown is worse than -15%, where the trough months are June 1970 ('Back to earth day' for tech stocks), September 1974 (Oil crisis/Yom Kippur War, Collapse Bretton Woods, Watergate), November 1987 ('Black Monday', program trading), June 2002 (Tech bubble burst), and February 2009 (Global Financial Crisis).[18] In subsequent analyses, we will report the change in the drawdown level at these five worst moments for the 60-40 portfolio.

[17] Asvanunt, Nielsen, and Villalon (2015) also study the impact of adding a two-asset (equity and bond) trend strategy, like we show in Figure 4 (Panel C), to a 60/40 stock/bond portfolio.
[18] Note that Black Monday refers to October 19, 1987, but that the market trough (using monthly data) is Nov 1988.

Figure 5: Drawdown Level Monthly-Rebalanced 60-40 Portfolio

The figure shows the drawdown level for a monthly-rebalanced 60-40 stock-bond allocation. Data are monthly from 1960 to 2017.



In Table 1, we report how a 90% monthly-rebalanced 60-40 portfolio (Rebal) plus 10% allocation to a trend strategy performs when compared with the benchmark of 100% Rebal. We cover the same six trend variations as before in Figure 4, and look at 1-, 3-, and 12-month trend windows. For trend systems applied to stocks-only (Panel A) and bonds-only (Panel B), the average return is similar to the 9.1% of the 100% Rebal portfolio – but with lower volatility – even though the trend system is applied to just one asset.[19] The drawdowns tend to be less severe, with an average improvement (ΔDD average) ranging from 3.6% to 5.7%. In case of stocks (Panel A), the notable exception is the drawdown around Black Monday when a slow, 12-month trend strategy actually exacerbates the drawdown.

Table 1: Rebal Plus Trend Performance Statistics

We contrast a 100% allocation to a monthly-rebalanced portfolio with a 60-40 stock-bond capital allocation to one where 10% of the portfolio is replaced with an allocation to various 1-, 3-, and 12-month trend specifications. Performance statistics reported on are the average stock, bond, and total allocation (block 1), the annualized notional trading (as percentage of the total portfolio value) in the 60-40 rebalanced portfolio, stock and bond futures, as well as annualized trading costs (block 2), average return, standard deviation, the ratio of these two, and the Sharpe ratio (block 3), and the change in the drawdown level for the five worst drawdowns (ΔDD) at the trough, compared to a 100% allocation to the monthly-rebalanced 60-40 portfolio. The data are from 1960 to 2017.

A: Trend applied to stocks-only

	Rebal (100%)	Rebal (90%) 1m trend (10%)	Rebal (90%) 3m trend (10%)	Rebal (90%) 12m trend (10%)
Stock allocation (avg)	60.0%	57.7%	60.1%	64.5%
Bond allocation (avg)	40.0%	36.0%	36.0%	36.0%
Total allocation (avg)	100.0%	93.7%	96.1%	100.5%
Rebal trade (ann)	10.2%	9.1%	9.1%	9.1%
Stock fut. trade (ann)	0.0%	207.3%	106.7%	45.1%
Bond fut. trade (ann)	0.0%	0.0%	0.0%	0.0%
Cost estimate (ann)	4.4 bps	6.0 bps	5.0 bps	4.4 bps
Return, (ann)	9.1%	9.0%	9.1%	9.6%
Volatility (ann)	9.8%	8.7%	9.1%	9.7%
Ret./Vol (ann)	0.92	1.03	1.00	0.99
Sharpe ratio (ann)	0.47	0.52	0.51	0.53
ΔDD Jun 1970	0.0%	3.6%	1.9%	5.4%
ΔDD Sep 1974	0.0%	9.7%	8.9%	9.8%
ΔDD Nov1987	0.0%	4.9%	-0.8%	-3.0%
ΔDD Sep 2002	0.0%	4.2%	4.2%	6.0%
ΔDD Feb 2009	0.0%	6.1%	7.9%	9.0%
ΔDD average	0.0%	5.7%	4.4%	5.4%

B: Trend applied to bonds-only

	Rebal (100%)	Rebal (90%) 1m trend (10%)	Rebal (90%) 3m trend (10%)	Rebal (90%) 12m trend (10%)
Stock allocation (avg)	60.0%	54.0%	54.0%	54.0%
Bond allocation (avg)	40.0%	39.9%	41.7%	45.3%
Total allocation (avg)	100.0%	93.9%	95.7%	99.3%
Rebal trade (ann)	10.2%	9.1%	9.1%	9.1%
Stock fut. trade (ann)	0.0%	0.0%	0.0%	0.0%
Bond fut. trade (ann)	0.0%	438.0%	245.4%	111.6%
Cost estimate (ann)	4.4 bps	6.1 bps	5.2 bps	4.5 bps
Return, (ann)	9.1%	9.4%	9.1%	9.3%
Volatility (ann)	9.8%	9.2%	9.2%	9.2%
Ret./Vol (ann)	0.92	1.02	0.99	1.01
Sharpe ratio (ann)	0.47	0.54	0.50	0.53
ΔDD Jun 1970	0.0%	5.1%	3.5%	5.6%
ΔDD Sep 1974	0.0%	6.0%	5.0%	6.2%
ΔDD Nov1987	0.0%	0.9%	0.8%	1.1%
ΔDD Sep 2002	0.0%	6.4%	5.2%	7.7%
ΔDD Feb 2009	0.0%	2.1%	3.8%	6.0%
ΔDD average	0.0%	4.1%	3.6%	5.3%

[19] Using data from 1960 (like in this paper), Hamill, Rattray, and Van Hemert (2016) find that a simple trend strategy applied to 55 securities realizes a Sharpe ratio well above one.

C: Trend applied to stocks & bonds (equal risk)

	Rebal (100%)	Rebal (90%) 1m trend (10%)	Rebal (90%) 3m trend (10%)	Rebal (90%) 12m trend (10%)
Stock allocation (avg)	60.0%	55.8%	57.0%	59.3%
Bond allocation (avg)	40.0%	37.9%	38.8%	40.7%
Total allocation (avg)	100.0%	93.8%	95.9%	99.9%
Rebal trade (ann)	10.2%	9.1%	9.1%	9.1%
Stock fut. trade (ann)	0.0%	103.7%	53.4%	22.6%
Bond fut. trade (ann)	0.0%	219.0%	122.7%	55.8%
Cost estimate (ann)	4.4 bps	6.1 bps	5.1 bps	4.4 bps
Return, (ann)	9.1%	9.2%	9.1%	9.4%
Volatility (ann)	9.8%	8.8%	9.0%	9.3%
Ret./Vol (ann)	0.92	1.04	1.01	1.02
Sharpe ratio (ann)	0.47	0.54	0.51	0.54
ΔDD Jun 1970	0.0%	4.4%	2.7%	5.6%
ΔDD Sep 1974	0.0%	7.9%	7.0%	8.0%
ΔDD Nov1987	0.0%	2.9%	0.0%	-1.0%
ΔDD Sep 2002	0.0%	5.4%	4.7%	6.8%
ΔDD Feb 2009	0.0%	4.4%	6.5%	7.6%
ΔDD average	0.0%	5.0%	4.2%	5.4%

D: Trend applied to stock-bond spread

	Rebal (100%)	Rebal (90%) 1m trend (10%)	Rebal (90%) 3m trend (10%)	Rebal (90%) 12m trend (10%)
Stock allocation (avg)	60.0%	57.2%	59.1%	62.6%
Bond allocation (avg)	40.0%	32.8%	30.9%	27.4%
Total allocation (avg)	100.0%	90.0%	90.0%	90.0%
Rebal trade (ann)	10.2%	9.1%	9.1%	9.1%
Stock fut. trade (ann)	0.0%	200.1%	110.8%	49.7%
Bond fut. trade (ann)	0.0%	200.1%	110.8%	49.7%
Cost estimate (ann)	4.4 bps	6.9 bps	5.6 bps	4.7 bps
Return, (ann)	9.1%	9.0%	8.9%	9.1%
Volatility (ann)	9.8%	8.8%	9.0%	9.4%
Ret./Vol (ann)	0.92	1.02	0.99	0.97
Sharpe ratio (ann)	0.47	0.51	0.49	0.49
ΔDD Jun 1970	0.0%	3.9%	1.9%	4.9%
ΔDD Sep 1974	0.0%	7.9%	7.4%	6.9%
ΔDD Nov1987	0.0%	0.7%	-3.9%	-4.5%
ΔDD Sep 2002	0.0%	5.1%	6.2%	7.5%
ΔDD Feb 2009	0.0%	7.1%	8.2%	8.9%
ΔDD average	0.0%	4.9%	4.0%	4.7%

E: Trend applied to stock-bond, negative only

	Rebal (100%)	Rebal (90%) 1m trend (10%)	Rebal (90%) 3m trend (10%)	Rebal (90%) 12m trend (10%)
Stock allocation (avg)	60.0%	48.6%	49.2%	50.4%
Bond allocation (avg)	40.0%	41.4%	40.8%	39.6%
Total allocation (avg)	100.0%	90.0%	90.0%	90.0%
Rebal trade (ann)	10.2%	9.1%	9.1%	9.1%
Stock fut. trade (ann)	0.0%	89.6%	45.0%	18.0%
Bond fut. trade (ann)	0.0%	89.6%	45.0%	18.0%
Cost estimate (ann)	4.4 bps	5.3 bps	4.6 bps	4.2 bps
Return, (ann)	9.1%	8.4%	8.5%	8.6%
Volatility (ann)	9.8%	8.1%	8.0%	8.2%
Ret./Vol (ann)	0.92	1.04	1.06	1.04
Sharpe ratio (ann)	0.47	0.49	0.51	0.50
ΔDD Jun 1970	0.0%	5.2%	4.6%	6.2%
ΔDD Sep 1974	0.0%	9.3%	9.6%	8.5%
ΔDD Nov1987	0.0%	2.3%	2.3%	1.8%
ΔDD Sep 2002	0.0%	8.0%	7.9%	8.8%
ΔDD Feb 2009	0.0%	8.2%	10.7%	10.4%
ΔDD average	0.0%	6.6%	7.0%	7.1%

F: Trend applied to stock-bond, positive only

	Rebal (100%)	Rebal (90%) 1m trend (10%)	Rebal (90%) 3m trend (10%)	Rebal (90%) 12m trend (10%)
Stock allocation (avg)	60.0%	62.6%	63.9%	66.2%
Bond allocation (avg)	40.0%	27.4%	26.1%	23.8%
Total allocation (avg)	100.0%	90.0%	90.0%	90.0%
Rebal trade (ann)	10.2%	9.1%	9.1%	9.1%
Stock fut. trade (ann)	0.0%	110.5%	65.8%	31.7%
Bond fut. trade (ann)	0.0%	110.5%	65.8%	31.7%
Cost estimate (ann)	4.4 bps	5.6 bps	4.9 bps	4.4 bps
Return, (ann)	9.1%	9.1%	8.9%	9.1%
Volatility (ann)	9.8%	9.5%	9.8%	10.0%
Ret./Vol (ann)	0.92	0.95	0.91	0.91
Sharpe ratio (ann)	0.47	0.49	0.45	0.46
ΔDD Jun 1970	0.0%	1.5%	0.2%	1.5%
ΔDD Sep 1974	0.0%	2.0%	1.4%	1.8%
ΔDD Nov1987	0.0%	0.1%	-4.4%	-4.5%
ΔDD Sep 2002	0.0%	-0.3%	0.8%	1.3%
ΔDD Feb 2009	0.0%	1.7%	0.4%	1.3%
ΔDD average	0.0%	1.0%	-0.3%	0.3%

In case of bonds (Panel B), the average stock allocation is much reduced at 54% (versus 60% stocks for the 100% Rebal portfolio) and so the better performance during stock market drawdowns is intuitive, while it is noteworthy that the average return is not reduced with the lower stock allocation (the bond trend strategy payoff fully compensates for the reduced equity-premium capture).

In case of stocks and bonds trend, equal risk (Panel C), a 10% trend allocation again leads to average returns at least as good as the benchmark and an improved average drawdown level. Also, in particular for 12-month trend, the average allocation to stocks and bonds is close to that of the 60% stocks and 40% bonds of the benchmark.

In Panel D, we consider trend on the stock-bond spread asset. Note that this will lead to a 90% total stock+bond allocation by construction, as the total (stock+bond) allocation from the 10% trend strategy is zero. Here, the average return is slightly lower than that of the benchmark. In addition, the drawdown around Black Monday is more impacted in case of 3- and 12-month trend compared to the specifications considered in Panel A, B, and C. In panels E and F we consider a version where we only trade on the stock-bond signal if it is negative and positive, respectively. Just trading on negative signals helps for reducing drawdowns, but leads to a much lower stock allocation and also a lower average return compared to the benchmark.

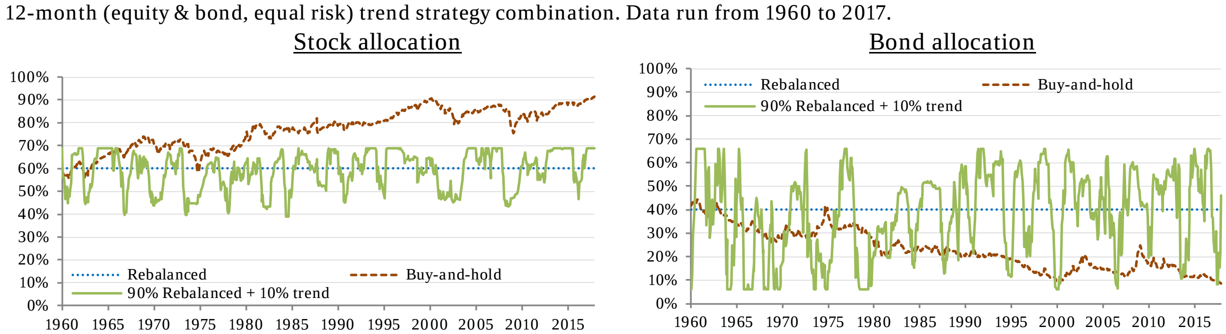
The Sharpe ratios reported in Table 1 for strategies with a 10% trend allocation range from 0.45 to 0.54, which is similar to the 0.47 Sharpe ratio for the 100% Rebal baseline case. However, the benefit of the trend allocation is not so much a higher Sharpe ratio, but rather a more benign risk profile with more shallow drawdowns. To illustrate this further, we employ the Goetzman et al. (2007) performance measure that controls for higher moments. This measure is the annualized certainty-equivalent return (CEQ) in a power utility framework and so penalizes negative skewness and excess kurtosis relative to the Sharpe ratio metric. The Appendix considers four different risk aversion parameters applied to the results in Panel C of Table 1 (see Appendix Table A1, Panels A and B). The results consistently show higher CEQs for the 10% to trend allocation. A similar insight can be gained by applying leverage to match the maximum drawdown size of the 100% rebalanced portfolio. Our results show that leveraged portfolios with 10% allocation to trend have significantly higher returns than the 100% rebalanced portfolio (Appendix Table A1, Panel C).

We checked that allocating 5% (rather than the 10% considered in Table 1) to a trend strategy gives, not surprisingly, about half this reduction in the average drawdown. That is, the impact scales about proportionally with the trend allocation (for modest values).

Finally, in Figure 6, we contrast the allocation to stocks and bonds (index plus futures) for a monthly-rebalanced portfolio with a 60-40 stock-bond capital allocation, a buy-and-hold portfolio that starts with the same allocation mix, and an allocation of 90% to the rebalanced portfolio and 10% allocation to a 12-month stocks and bonds (equal risk) trend strategy. Because of the signal caps and volatility floors used, the stock futures position is guaranteed to be between -15% and +15% and the bond futures position is between -30% and +30%. The Rebal plus trend combination has a fluctuating stock and bond allocation, but no long-term drift. It has a slight long bias with, on average, a 5.3% long stock and a 4.7% long bonds futures position, coincidentally almost exactly replacing the 10% reduction in the 60-40 rebalanced portfolio (see Table 1, Panel C).

Figure 6: Allocation to Stocks and Bonds for the Various Portfolios Considered

In this figure, we show the (index plus futures) allocation to stocks (left panel) and bonds (right panel) for a monthly-rebalanced 60-40 stock-bond portfolio, a buy-and-hold portfolio that starts with the same allocation mix, and a 90% monthly-rebalanced and 10% 12-month (equity & bond, equal risk) trend strategy combination. Data run from 1960 to 2017.



3. Strategic Rebalancing

Rather than allocating to a trend strategy to counteract the tendency of rebalanced portfolios to underperform during equity market drawdowns, we now study if one can get similar benefits by smartly timing and sizing rebalancing trades, which we call ‘strategic rebalancing’. We consider both commonly-used heuristics and trend-based rules.

There is a large literature on evaluating heuristic rebalancing rules, where typically one varies the rebalancing frequency, takes a threshold-based method, or combines these two approaches. Also, rather than rebalancing fully toward the target asset mix, one can rebalance partially and so reduce turnover (and save on trading costs), providing yet another knob to turn. See, e.g., Arnott and Lovell (1993) for early work on this topic.[20]

In Table 2, we produce statistics for combinations of these oft-used rules, with again our emphasis on whether drawdowns are less severe compared to rebalancing fully back to 60-40 every month (left-most column). Rebalancing monthly, but only half or a quarter toward 60-40 mostly reduces drawdowns and moreover leads to lower turnover, with rebalancing trades per year of 6.2% and 4.2% respectively, versus 10.2% with a full rebalance each month. The actual transaction cost savings will vary greatly across investors, but for the aforementioned transaction assumptions, the impact is arguably small at 0.4 to 4.4 basis points per annum. Quarterly and annual rebalancing further improves drawdowns, except with the 1987 drawdown (Black Monday), which is a drawdown that quickly reversed.[21,22]

Table 2: Rebalancing With Frequency- and Threshold Rules

We show results for frequency- (monthly, quarterly, annual) and threshold-based (60±2% and 60±4%) rebalancing rules. We consider a full, half, or quarter rebalancing toward the 60-40 capital allocation mix. We report in the different blocks: the average and standard deviation of the stock allocation, noting that the bond allocation is 100% minus the bond allocation (block 1); the fraction of months and annualized amount of rebalancing, and annualized trading costs (block 2); the average and standard deviation of the return, as well as the ratio (block 3); and the change in the drawdown level for the five worst drawdowns (ΔDD) at the trough, compared to a 100% allocation to the monthly-rebalanced 60-40 portfolio (block 4). Data are from 1960 to 2017.

	Monthly			Quarterly			Annual			2% threshold			4% threshold		
	Full	Half	Quarter	Full	Half	Quarter	Full	Half	Quarter	Full	Half	Quarter	Full	Half	Quarter
Stock allocation (avg)	60.0%	60.1%	60.2%	60.1%	60.3%	60.6%	60.4%	61.1%	62.2%	60.2%	60.4%	60.5%	60.5%	61.1%	61.4%
Stock allocation (std)	0.0%	0.7%	1.4%	1.3%	1.7%	2.5%	2.9%	3.3%	4.1%	0.9%	1.2%	1.7%	1.7%	1.9%	2.3%
%months rebal	100.0%	100.0%	100.0%	33.3%	33.3%	33.3%	8.3%	8.3%	8.3%	15.8%	23.0%	34.5%	6.6%	9.3%	14.2%
Rebal trade (ann)	10.2%	6.2%	4.2%	6.6%	3.9%	2.5%	3.6%	1.8%	1.1%	5.2%	3.9%	3.1%	3.7%	2.6%	2.1%
Cost estimate (ann)	4.4 bps	2.7 bps	1.8 bps	2.9 bps	1.7 bps	1.1 bps	1.6 bps	0.8 bps	0.5 bps	2.3 bps	1.7 bps	1.4 bps	1.6 bps	1.1 bps	0.9 bps
Return (ann)	9.1%	9.2%	9.2%	9.2%	9.2%	9.2%	9.3%	9.2%	9.1%	9.1%	9.2%	9.2%	9.2%	9.2%	9.2%
Volatility (ann)	9.8%	9.8%	9.8%	9.8%	9.8%	9.8%	9.8%	9.8%	10.0%	9.9%	9.8%	9.8%	9.8%	9.9%	9.9%
Ret/Vol (ann)	0.93	0.94	0.94	0.94	0.95	0.94	0.95	0.94	0.92	0.93	0.94	0.94	0.93	0.94	0.94
ΔDD Jun 1970	0.0%	0.1%	0.1%	0.1%	0.3%	0.2%	0.5%	0.2%	-0.7%	-0.2%	0.0%	0.1%	-0.2%	0.2%	-0.2%
ΔDD Sep 1974	0.0%	0.2%	0.6%	0.5%	0.9%	1.3%	1.3%	1.7%	1.4%	0.0%	0.4%	0.7%	0.4%	0.4%	0.6%
ΔDD Nov1987	0.0%	0.1%	-0.3%	0.5%	-0.2%	-1.0%	-2.3%	-2.2%	-2.4%	-0.4%	-0.2%	-0.3%	-0.7%	-0.7%	-0.7%
ΔDD Sep 2002	0.0%	0.4%	0.9%	1.2%	1.5%	1.9%	1.7%	1.9%	-0.1%	-0.2%	0.5%	1.2%	0.0%	1.1%	1.3%
ΔDD Feb 2009	0.0%	0.8%	1.8%	0.9%	2.1%	3.0%	2.8%	3.2%	2.9%	0.2%	1.1%	2.0%	0.0%	1.1%	1.9%
ΔDD average	0.0%	0.3%	0.6%	0.6%	0.9%	1.1%	0.8%	0.9%	0.2%	-0.1%	0.4%	0.8%	-0.1%	0.4%	0.6%

Threshold-based rules seem slightly less potent, where we consider rebalancing if the fraction of stocks is outside of the 60±2% and 60±4% range (but we considered other ranges also, which did

[20] Donohue and Yip (2003) and Masters (2003) link rebalancing rules to considerations like sensitivity to the tracking error with the constant-mix portfolio and transaction costs.

[21] Driessen and Kuiper (2017), Ilmanen and Maloney (2015), Huss and Maloney (2017) argue that rebalancing less frequently is a way to exploit predictability in asset returns.

[22] Here we use quarterly rebalancing per quarter ends (Dec, Mar, Jun, Sep) and annual rebalancing per year end (Dec). Using different months of the year leads to similar results.

not materially improve performance). In all cases, the correlation to the monthly, fully-rebalanced strategy is near 1 (not reported in the table) and the average return is barely impacted.

Next, we turn our attention to trend-based strategic rebalancing rules. In Table 3, we show results when rebalancing is delayed if the stock-bond spread trend is negative, positive, or continues to be in the same direction (i.e., to rebalance only when the trend direction now is in the opposite direction of a month ago, which likely corresponds to a not so strong or inconsistent trending environment).[23] In months with no delay, there is a rebalancing halfway toward the 60-40 asset mix (mirroring the middle case considered Table 2). The trend direction is determined by comparing the return over the past 1-, 3-, and 12-months to the typical (average) return over 1-, 3-, and 12-month windows.[24]

Table 3: Strategic Rebalancing With Stock-Bond Trend Rules

We show results when rebalancing is delayed if the stock-bond trend is negative, positive, or continues to be of the same sign (in which case rebalancing only occurs if the trend just changed sign). In months with no delay, there is a rebalancing half-way toward the 60-40 asset mix. The trend direction is determined by comparing the return over the past 1-, 3-, and 12-months to the typical (average) return over 1-, 3-, and 12-month windows. We report in the different blocks: the average and standard deviation of the stock allocation, noting that the bond allocation is 100% minus the bond allocation (block 1); the fraction of months and annualized amount of rebalancing, and annualized trading costs (block 2); the average and standard deviation of the return, as well as the ratio (block 3); and the change in the drawdown level for the five worst drawdowns (ΔDD) at the trough, compared to a 100% allocation to the monthly-rebalanced 60-40 portfolio (block 4). Data are from 1960 to 2017.

	Delay if 1m trend			Delay if 3m trend			Delay if 12m trend		
	Negative	Positive	Continues	Negative	Positive	Continues	Negative	Positive	Continues
Stock allocation (avg)	59.3%	61.0%	60.0%	58.9%	61.5%	60.2%	58.1%	61.6%	59.9%
Stock allocation (std)	2.2%	1.6%	2.3%	2.6%	2.3%	2.8%	3.9%	2.5%	4.9%
%months rebal	49.3%	50.7%	48.3%	48.0%	52.0%	25.6%	42.2%	57.8%	12.6%
Rebal trade (ann)	4.0%	4.5%	3.8%	3.5%	4.4%	2.7%	4.5%	4.3%	1.7%
Cost estimate (ann)	1.7 bps	1.9 bps	1.7 bps	1.5 bps	1.9 bps	1.2 bps	1.9 bps	1.9 bps	0.7 bps
Return (ann)	9.1%	9.2%	9.1%	9.1%	9.2%	9.1%	9.0%	9.2%	9.1%
Volatility (ann)	9.6%	9.9%	9.7%	9.5%	10.0%	9.7%	9.4%	10.0%	9.7%
Ret/Vol (ann)	0.95	0.93	0.94	0.96	0.93	0.94	0.96	0.92	0.94
ΔDD Jun 1970	0.5%	-0.1%	0.3%	0.7%	-0.1%	0.4%	0.7%	0.0%	0.3%
ΔDD Sep 1974	1.6%	0.2%	1.2%	1.6%	0.1%	1.1%	3.0%	0.2%	2.5%
ΔDD Nov1987	0.3%	-2.2%	-1.8%	0.3%	-2.2%	-1.7%	0.5%	-1.9%	-1.7%
ΔDD Sep 2002	1.8%	0.2%	1.3%	2.0%	0.2%	1.4%	5.2%	0.2%	4.8%
ΔDD Feb 2009	4.2%	0.7%	3.5%	4.8%	0.7%	3.6%	5.6%	0.8%	5.6%
ΔDD average	1.7%	-0.2%	0.9%	1.9%	-0.2%	0.9%	3.0%	-0.1%	2.3%

For all three trend windows, delaying of the rebalancing when there is a negative trend in the stock-bond spread is most beneficial for reducing drawdowns. This is intuitive, as drawdowns typically occur when stock returns are negative and so a delay of rebalancing means not buying back stocks to bring it back inline with the 60-40 mix. This result is also consistent with Figure 4 (Panel E), which shows that the payoff of an explicit allocation to a stock-bond spread trend strategy that is constrained to be negative mimics that of a put option (on the stock-bond spread return), or with Table 1 (Panel E), which shows that the same explicit trend allocation much reduces drawdowns. Delaying of the rebalancing when the 12-month trend is negative leads to a reduction of more than 5 percentage points in case of the Tech bubble burst (Sep 2002 trough) and financial crisis (Feb 2009 trough), which is comparable to that of the 10% trend allocations considered before in Table 1.

[23] We find that using a rule based on just the stock trend, rather than the stock-bond trend, leads to very similar results.

[24] We set this at 0.8%, 2.3%, and 9.1% for 1-, 3-, and 12-month trend horizons, corresponding to the empirical evidence over the 1960-2017 period.

The opposite case -- delaying of the rebalancing when there is a positive trend in the stock-bond spread return -- does not tend to reduce drawdowns. The case of delaying if the trend continues to be of the same sign sits in between the negative and postive trend-based rule.

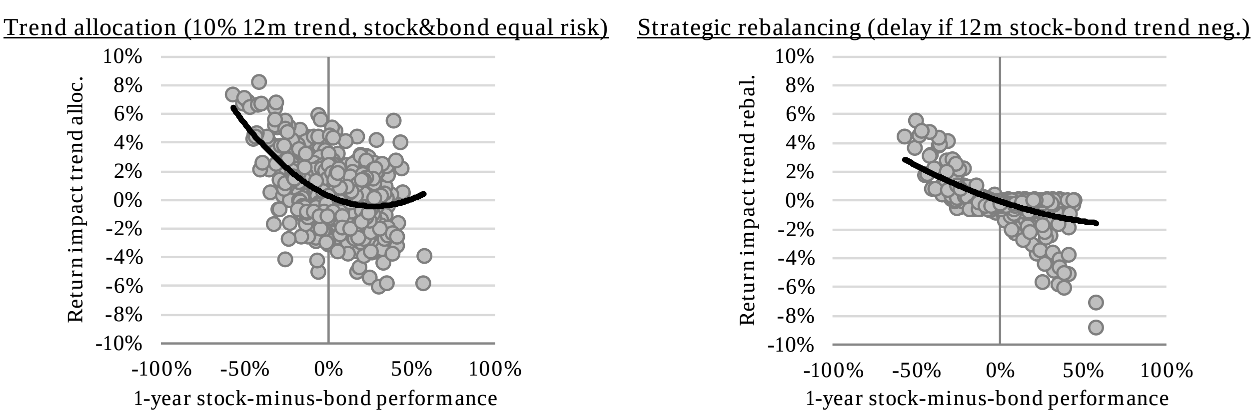
4. Strategic Rebalancing Versus a Direct Allocation to Trend

In Figure 7, we show the impact of adding the two different types of trend exposures considered in this paper to a monthly-rebalanced 60-40 stock bond portfolio, as a function of the 1-year stock-minus-bond return. In the left panel, we show the change in the 1-year return with a 10% allocation to a 12-month stocks and bonds (equal risk) trend strategy (reported on in Table 1, Panel C). In the right panel, we show the same output for a strategic rebalancing rule to delay rebalancing if the 12-month stock-bond spread trend is negative (reported on in Table 3).

Note that in both cases the trend exposure tends to be particularly helpful when stocks underperform bonds. Given that stocks are much more volatile than bonds, this usually means the exposure helps when stocks returns are negative. In case of a trend allocation (left panel), the asymmetry of the impact compares to a much more symmetric effect for the standalone trend performance as shown in Figure 4, Panel C. The reason is that in order to allocate to the trend strategy, we reduced the allocation to the 60-40 stock-bond portfolio from 100% to 90%, and this tends to help when the stock-bond performance is negative. In case of the strategic rebalancing rule (right panel), the asymmetric impact of the exposure comes directly from the rule itself being asymmetric, i.e., riding negative trends in the relative stock-bond return and not riding positive trends. The correlation between the two trend exposures in term of their return difference with the 100% monthly-rebalanced 60-40 stock-bond portfolio is 0.62, again suggesting they behave similarly.

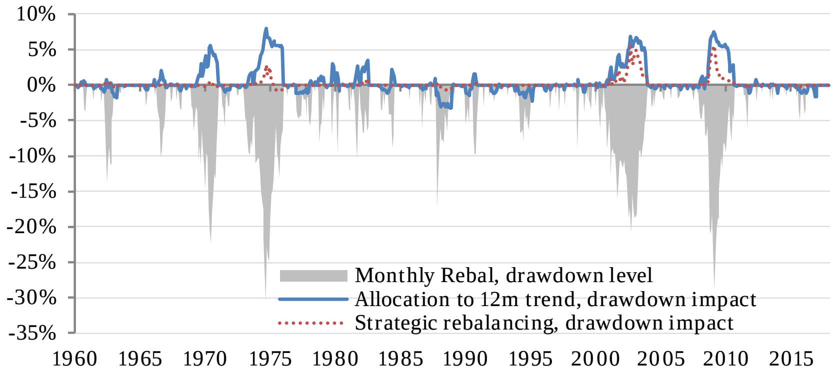
Figure 7: Impact of Adding a Trend Exposure versus the Stock-Minus-Bond 1-Year Return

This figure plots the 1-year (rolling 12-month) performance impact of adding a trend exposure to a monthly rebalanced 60-40 stock-bond strategy (vertical axis) versus the one-year relative stock-bond performance (horizontal axis) for the period 1960 to 2017. We consider a 10% allocation to a 12-month stocks and bonds (equal risk) trend strategy (left panel) and a strategic rebalancing rule to delay rebalancing when the 12-month stock-bond trend is negative (i.e., the return is below the average stock-bond 12-month return). Other statistics for these trend exposures can be found in Table 1, Panel C for the trend allocation and Table 3 for the stock-bond trend rebalancing rule.



In Figure 8, we show the impact on the drawdown level of the same two trend exposures that we considered before in Figure 7. Results at the trough of the five worst crises correspond to those reported before in Tables 1, Panel C and Table 3. This figure compares the impact of the different trend exposures on the drawdown level alongside and over time. The main takeaway is that either a direct allocation to a trend strategy or using trend signals as a basis of a rebalancing rule tends to reduce the drawdown materially. The performance around Black Monday is the only exception here in case of an allocation to trend (less so for the strategic rebalancing rule).

Figure 8: Impact of Adding a Trend Exposure on the Portfolio Drawdown Level
In this figure, we show the drawdown level of a monthly rebalanced 60-40 stock-bond portfolio (blue line) and the impact (change in drawdown level) when a adding a trend exposure for the period 1960 to 2017. We consider a 10% allocation to a 12-month stocks and bonds (equal risk) trend strategy and a strategic rebalancing rule to delay rebalancing when the 12-month stock-bond trend is negative (i.e., the return is below the average stock-bond 12-month return). Other statistics for these trend exposures can be found in Table 1, Panel C for the trend allocation and Table 3 for the stock-bond trend rebalancing rule.



5. Concluding Remarks

A pure buy-and-hold portfolio is untenable for most investors as it leads to a highly concentrated, undiversified portfolio. However, a 60-40 stock-bond portfolio (our use case) that rebalances every month to the 60:40 target ratio loses several percentage points more than a buy-and-hold portfolio during periods of continued stock market drawdowns. In essence, rebalancing to a constant asset mix means selling winners and buying losers, which hurts at times when the stock performance (relative to that of bonds) is trending. We show that the concavity induced by rebalancing is effectively countered with a trend exposure, which exhibits convexity and can be either implemented as a direct allocation to a trend strategy or with a strategic trend-based rebalancing rule.

While our focus is on countering the concavity induced by rebalancing, other considerations matter in practice as well. For example, investors can also use monthly in- and out-flows to move back toward the target asset mix. For example, Chambers, Dimson, and Ilmanen (2012) mention that the Norwegian Government Pension Fund Global directs monthly inflows into the asset class that is most underweight relative to the benchmark. For taxable investors, rebalancing using income has the added benefit that no assets need to be sold, which can be tax efficient; see Colleen, Kinniry, and Zilbering (2010).

Finally, we note that a stock-bond trend exposure is just one way to mitigate drawdowns at times of continued stock market losses. An investor has more arrows in her quiver. A good starting point is a more diversified portfolio that includes more asset classes and has an international exposure.

An allocation to a broader a broader trend strategy that benefits from trends in other macro assets at times of equity market distress may further dampen equity market losses, see Hamill, Rattray, and Van Hemert (2016). And Harvey et al. (2018) study volatility targeting and show that it can help manage the risk of a 60-40 stock-bond portfolio.

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Appendix

We employ the Goetzman et al. (2007) manipulation-proof performance measure. This measure is the annualized certainty-equivalent return (CEQ) in a power utility framework and so penalizes negative skewness and excess kurtosis relative to the Sharpe ratio metric. For a given risk aversion parameter γ, it is defined as:

$$CEQ(\gamma) = \frac{1}{(1-\gamma)\Delta t} \ln\left(\frac{1}{T}\sum_{t=1}^{T}\left(\frac{1+R_t}{1+R_t^F}\right)^{1-\gamma}\right)$$

In Table A1, we report the CEQ using monthly (Panel A) and annual (12-month overlapping) data (Panel B), for the case of trend applied to stocks and bonds (equal risk) which corresponds to Panel C in Table 1. The $\gamma = 0$ case corresponds to a risk-neutral setting, and the CEQ simply equals the annualized excess return. Using risk aversion parameter values of 2, 5, and 10, we see the benefit from a 10% trend allocation. For $\gamma = 10$, the CEQ is around 1% higher using monthly returns and 2% in case of annual returns. The higher value in case of annual returns shows that the benefits of a trend allocation are more pronounced at lower frequencies, where the intuition is that trend strategies require some time to start picking up on sustained market moves.

Table A1: Alternative performance metrics (stocks and bonds trend, equal risk)

The table reports the certainty-equity return (CEQ) using monthly (Panel A) and annual (12-month overlapping) returns (Panel B) for different levels of power-utility risk aversion, γ. in Panel C we also report returns unleveraged versus returns leveraged (taking into account the cost of borrowing) so that the average drawdown across the five episodes considered is equalized to that of the 100% Rebal benchmark. For all panels, the trend strategy is applied to stocks and bonds (equal risk), which is what the case covered in Table 1, Panel C. The data are from 1960 to 2017.

Panel A: certainty-equivalent return (CEQ), using monthly returns

	100% Rebal	90% Rebal 10% TsMom1m	90% Rebal 10% TsMom3m	90% Rebal 10% TsMom12m
		Certainty equivalent return (CEQ)		
γ=0 (risk neutral)	4.6%	4.7%	4.6%	4.9%
γ=2	3.7%	3.9%	3.8%	4.1%
γ=5	2.2%	2.8%	2.6%	2.8%
γ=10	-0.4%	0.8%	0.4%	0.5%
		CEQ vs. 100% Rebal		
γ=0 (risk neutral)	0.0%	0.1%	0.0%	0.3%
γ=2	0.0%	0.3%	0.1%	0.4%
γ=5	0.0%	0.6%	0.4%	0.6%
γ=10	0.0%	1.2%	0.8%	0.9%

Panel B: certainty equivalent (CEQ), using annual returns

	100% Rebal	90% Rebal 10% TsMom1m	90% Rebal 10% TsMom3m	90% Rebal 10% TsMom12m
		Certainty equivalent return (CEQ)		
γ=0 (risk neutral)	4.7%	4.8%	4.7%	5.0%
γ=2	3.6%	3.9%	3.7%	4.1%
γ=5	1.7%	2.4%	2.2%	2.6%
γ=10	-2.0%	-0.2%	-0.6%	0.0%
		CEQ vs. 100% Rebal		
γ=0 (risk neutral)	0.0%	0.1%	0.0%	0.3%
γ=2	0.0%	0.3%	0.1%	0.5%
γ=5	0.0%	0.8%	0.5%	0.9%
γ=10	0.0%	1.8%	1.4%	2.0%

Panel C: unleveraged versus leverage to match 100% Rebal average drawdown

	100% Rebal	90% Rebal 10% TsMom1m	90% Rebal 10% TsMom3m	90% Rebal 10% TsMom12m
		Return unleveraged and leveraged (matched DD)		
Unleveraged	4.6%	4.7%	4.6%	5.0%
Lev., matched DD	4.6%	5.8%	5.4%	6.2%
		Return vs 100% Rebal		
Unleveraged	0.0%	0.1%	0.0%	0.3%
Lev., matched DD	0.0%	1.1%	0.8%	1.5%
		T-stat return vs 100% Rebal		
Unleveraged	n.a.	0.3	-0.1	1.2
Lev., matched DD	n.a.	4.0	2.7	4.2

Note that the CEQ is still a measure that doesn't explicitly account for longer-term behavior, like the drawdown characteristic we have focused on. So in Panel C we also report returns unleveraged versus returns leveraged (taking into account the cost of borrowing) so that the average drawdown across the five episodes considered is equalized to that of the 100% Rebal benchmark. The leverage applied is around 1.2, as drawdowns are around a factor 1.2 lower without the leverage. A 10% trend allocation results in a 0.8% to 1.5% higher annualized return when leverage is applied to match the average drawdown depth. The t-stat on the return differential between the with-trend strategy and the 100% Rebal baseline case comes out as 2.7 to 4.2.