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A QUICK LOOK AT INDEX TAILS

When looking at an asset class at the macro level, investors and analysts often characterize its return distribution by just two numbers: mean and standard deviation. Various forms of analysis (e.g., information ratios, Sharpe ratios, and mean-variance optimization) have been built on this assumption that the mean and standard deviation fully describe an asset's return distribution. Often, the distribution is assumed to be Normal, as this symmetric distribution is fully defined by its mean and standard deviation and because it is both relatively tractable for researchers and widely familiar to readers. However, how many times have we read footnotes and disclaimers pointing out that if the returns are not normally distributed, then all bets are off?

Yet it is widely accepted that for many asset classes, notably those with embedded options or substantial credit risk, their excess returns are not normally distributed. These asset classes have return distributions that are both asymmetric and with "fat tails"—higher probabilities of extreme events. As we have pointed out in a recent paper,[1] credit by its very nature is an extremely asymmetric investment. An investor who buys a corporate bond and holds it to maturity earns a relatively modest spread over Treasuries, with a very high probability, in return for taking the risk of a catastrophic loss (i.e., default) with a very low probability. When viewed in this framework, all of the variability in the return lies in the extreme negative tail; the upside is very limited. Since the excess return distribution in not Normal, the mean and standard deviation are insufficient to fully describe the distribution. Consequently, the standard deviation of return is not an adequate risk measure for a buy-and-hold credit investor.[2]

The situation is different for a total return investor who is concerned with monthly excess returns. For such relatively short holding-period horizons, the excess return distribution for credit assets is much more symmetric as the bulk of the excess return is driven primarily by the widening and tightening of spreads. Viewed on a monthly basis, these returns tend to look much more balanced, with occasional large positive returns, as well as large negative returns.

In this short piece, we take a quick peek at the empirically observed monthly excess return distributions for various credit classes over the last ten years. We attempt to address the following questions:

- How fat are the tails?
- How asymmetric is the distribution? Are the negative tails much fatter than the positive ones?
- Is there a substantial difference between investment grade and high-yield credit in this regard?
- Can we measure how this phenomenon changes with the holding-period return horizon?

In Figure 1, we present a summary of the last ten years of monthly excess returns for a small set of Lehman Brothers indices. For each index shown, in addition to the mean and

[1] *Optimal Credit Allocation for Buy-and-Hold Investors*, Lehman Brothers, January 2004
[2] If the investor's utility function were "quadratic," then the distribution's mean and standard deviation would be sufficient for the investor's financial decision making.

standard deviation of monthly excess returns, we present two measures of the degree to which the distributions are non-Normal: skewness and kurtosis. Skewness measures the extent to which a distribution is asymmetric. A perfectly symmetric distribution, like the Normal, will have a skewness of zero. Negative skewness indicates that the negative tail of the distribution is larger than the positive tail, that the extreme events are more likely to be losses than gains. Kurtosis measures the fatness of the tails, both positive and negative. Positive kurtosis indicates that the distribution has fatter tails than the Normal distribution (higher probability of extreme results, either positive or negative); negative kurtosis indicates a distribution with thinner tails.[3] The combination of positive kurtosis and negative skewness thus means that the tails overall are larger than Normal and that the negative tail is larger than the positive one.

To help put some intuition behind these measurements, Figure 1 also provides some intuitive measurements of the extremes of the excess return distributions. We show the minimum and maximum observed monthly excess returns for each series, as well as the averages of the worst 5% and best 5% of the observed months.[4] Last, we show an average number of issuers over the last few years for each of these indices (for the Emerging Markets Index, this is the number of countries represented).

Figure 2 presents histograms for several of these excess return distributions. For the Corporate Index, we see quite clearly the effect of the positive kurtosis. The return distribution is peaked in the center but has extreme events well beyond what would be expected in a Normal distribution with the same mean and standard deviation (which is shown as the smooth curve in the histogram). Consistent with the relatively small negative skewness, these tail events are somewhat more extreme on the negative side, but a clear positive tail can be seen, as well. The High Yield Index is similarly characterized by fat tails that are fairly symmetric. The Emerging Markets Index displays a substantially larger (negative) skewness and (positive) kurtosis, largely due to a single huge event: the excess return of -28% in the Russian crisis of August 1998. Just for comparison, Figure 2(d) shows the distribution of excess returns over cash for the 3- to 7-year portion of the Treasury index, with skewness and kurtosis both close to zero. This distribution can be seen to line up quite nicely with the Normal.

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[3] Technically, the Normal distribution has a kurtosis of 3, and one should evaluate the relative fatness of tails by comparing the kurtosis of a given distribution to 3. In practice, it is customary to use "excess kurtosis," obtained by subtracting 3 from the kurtosis of the distribution. Throughout this piece, we ignore this distinction, and use the term "kurtosis" to refer to "excess kurtosis."

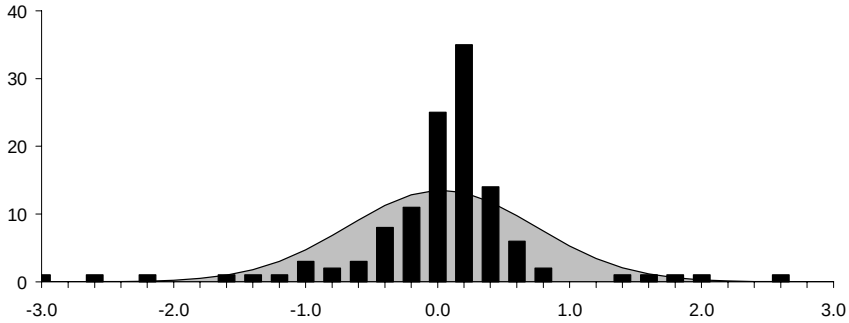
[4] For each monthly excess return series, we sorted the 121 realized return observations, then took the average of the best 6 monthly returns and the worst 6. This corresponds roughly to an empirical version of the expected shortfall measurement at a 95% confidence level.

Figure 1. **Statistical Properties of the Distributions of Monthly Excess Returns,** January 1994 - January 2004

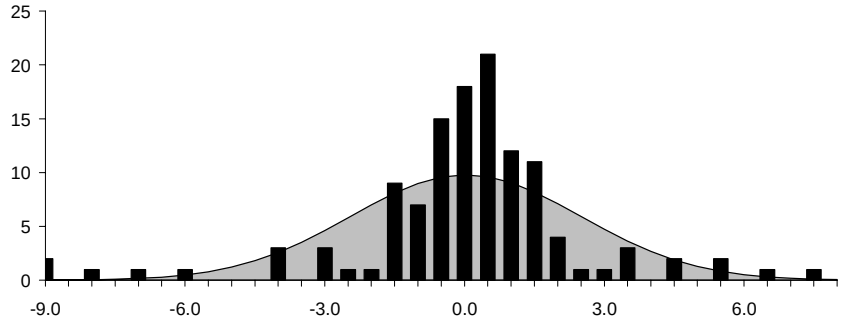
Index	Mean (%/mo)	Stdev (%/mo)	Skewness	Kurtosis	min	max	Average of worst 5%	Average of best 5%	Issuers
Corporates	0.03	0.70	-0.70	6.25	-2.93	2.70	-1.98	1.72	715
Consumer Cyclical	0.05	1.01	-0.17	5.31	-3.89	3.71	-2.67	2.73	58
Industrials	0.02	0.80	-0.59	4.52	-2.78	2.77	-2.34	1.94	412
Energy	0.08	0.68	-1.24	7.57	-3.41	2.03	-1.89	1.67	52
Pipelines	-0.15	1.99	-5.03	41.61	-16.75	5.79	-6.26	3.01	16
High Yield	0.03	2.46	-0.82	4.00	-8.85	7.38	-7.14	5.66	764
Emerging Markets	0.41	4.42	-2.40	13.11	-27.88	9.01	-11.42	7.40	29

Figure 2. **Comparing Observed Index Excess Returns to the Normal Distribution**

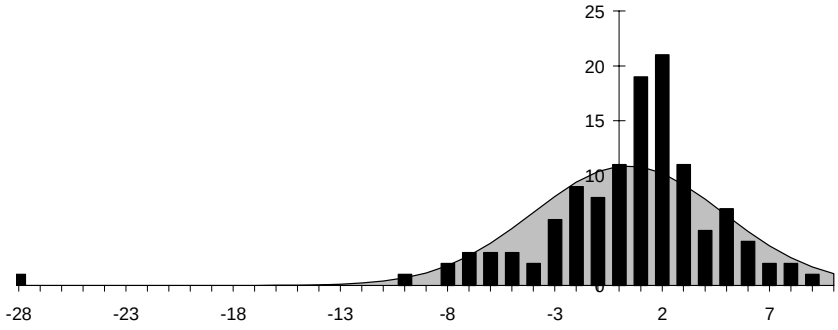
a) Corporates (skewness -0.70, kurtosis 6.25)



b) High Yield (skewness -0.82, kurtosis 4.0)



c) Emerging Markets (skewness -2.4, kurtosis 13.1)



d) U.S. Treasuries 3-7 Years, monthly excess returns over cash, Aug 1988 - Dec 2003 (skewness -0.13, kurtosis -0.3)

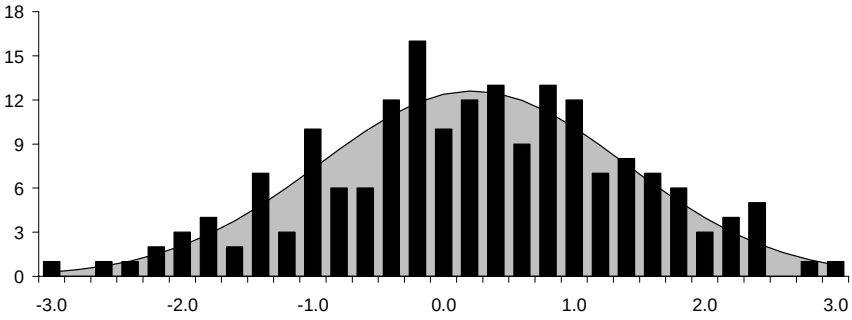
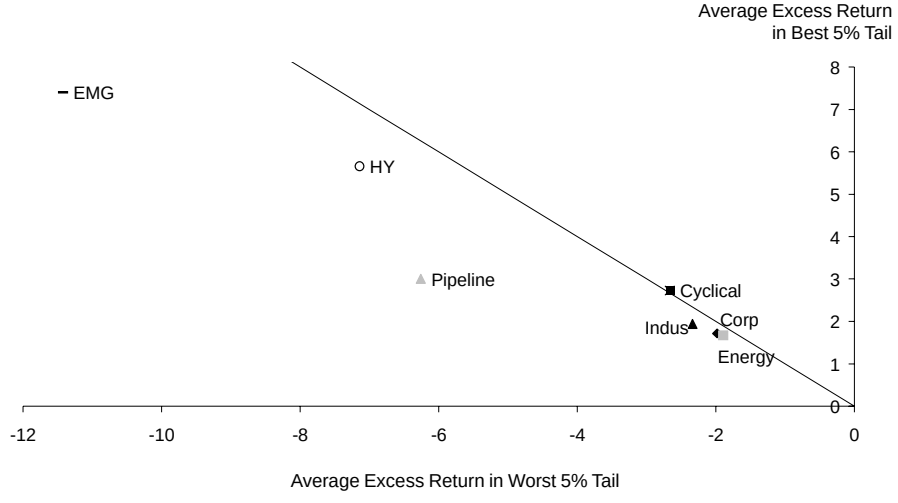


Figure 3 depicts the tail symmetry of these indices. Using the average of the best 5% and the average of the worst 5% numbers from Figure 1, we represent each index in terms of the tradeoff between upside and downside risk. The diagonal line marks the situation in which the upside gains are equal and opposite to the downside losses. We see that for the investment-grade corporate index and several of its subcomponents, the positive and negative tails are almost identical. The one glaring exception to this is the Pipelines index, which was home to Enron at the time of its downfall. The inclusion of the Pipelines index in this grouping is meant to illustrate the effect of non-systematic risk. We believe that the symmetric properties we are observing in the corporate index tails is due to the effect of diversification, which enables the index to avoid a huge return shock in a single month due to default-related losses. For less diversified portfolios and indices, the risk of default threatens to bring very asymmetric losses. This relates to the observed asymmetry in the Emerging Markets returns as well. The concentration of the risk among a relatively small number of countries in this index, coupled with the similar exposures of these countries to global economic events, creates the potential for very large negative returns. The high yield index, while it has a very high standard deviation of excess return, can be seen to be fairly close to the line, providing investors with the potential for large gains in return for taking the risk of large losses.

We have shown that well-diversified credit indices tend to have relatively symmetric (if fat-tailed) excess return distributions over a monthly horizon. Yes, there are fat negative tails, but they are compensated for by fat positive tails. How does this picture change as we increase the holding-period return horizon? This is a relatively tricky thing to measure, as the longer the return horizon, the fewer data points one has available. Thus,

Figure 3. **Symmetry of Positive and Negative Tails Using Monthly Excess Returns**



the 10-year period that we have chosen to study can provide 120 independent monthly data points, 10 independent annual data points, or a single 10-year return observation. With a small number of data points, though, no robust statistical conclusions can be reached. We decided to content ourselves with a second look at the data assuming a 3-month holding-period horizon. We grouped the data into 40 quarterly observations of cumulative excess returns, and repeated the analysis of Figure 1. The results are shown in Figure 4. The most striking result here is that the kurtosis (the fatness of the tails) falls drastically for every index shown: for the Corporate Index, from 6.25 to 0.85; and for the Emerging Markets Index, from 13.11 down to 1.37. The averaging of returns over a 3-month period smoothes out the extremes and pulls in the fat tails of the distribution. However, the skewness of the return distributions remains negative, in some cases becoming even more negative than in the monthly case.

As we go from a monthly to a quarterly horizon, what happens to the positive tail and the negative tail of the distribution? It is hard to understand this based on the skewness and kurtosis. It is also difficult to directly compare the worst 5% and best 5% results in Figures 1 and 4—these numbers are not normalized, and it is hard to compare monthly returns with quarterly ones. To address this, we suggest the following measure of the fatness of the tails: we first normalize the worst 5% and best 5% numbers by subtracting the mean and dividing by the standard deviation; we then divide by 2.063, the average number of standard deviations in the worst 5% tail of the standard Normal distribution. The result is a one-sided measure of the fatness of the positive (or negative) tail at the 95% confidence level. Numbers greater than 1 indicate fatter-than-normal tails, while numbers less than one indicate thinner-than-normal tails. These measures are shown in Figure 5a using monthly and in Figure 5b using quarterly data. We can see that while the monthly data tend to indicate fat tails on both the positive and negative sides of the distribution, the quarterly data indicate fat negative tails and thin positive tails for all of the asset classes shown.

We draw the following conclusions from this brief study:

1. The distributions of monthly excess returns for well-diversified credit asset classes tend to be somewhat fatter than normal, but exhibit a rough symmetry between the positive and negative tails.

Figure 4. **Statistical Properties of the Distributions of Quarterly Excess Returns,** 1994-2003

Index	Mean (%/mo)	Stdev (%/mo)	Skewness	Kurtosis	min	max	Average of worst 5%	Average of best 5%	Issuers
Corporates	0.08	1.34	-0.82	0.85	-3.38	2.70	-3.22	2.49	715
Consumer Cyclical	0.14	1.95	-0.34	0.53	-4.44	4.41	-4.23	3.91	58
Industrials	0.06	1.59	-0.50	0.79	-3.68	3.83	-3.37	3.21	412
Energy	0.23	1.33	-1.56	3.23	-4.14	2.11	-3.66	2.10	52
Pipelines	-0.49	3.62	-3.41	14.57	-18.14	3.30	-12.98	3.30	16
High Yield	0.05	4.79	-1.05	0.85	-11.07	7.73	-11.06	7.15	764
Emerging Markets	1.27	8.43	-1.10	1.37	-25.51	14.73	-20.37	13.71	29

2. Contrary to what might have been expected, we have found this to be true for high-yield as well as investment-grade credit.
3. For the emerging markets index, and potentially for any highly concentrated index or portfolio, the returns are more asymmetrically distributed, dominated by a large negative tail.
4. As we lengthen the return horizon from monthly to quarterly, we find a noticeable increase in the asymmetry of the tails. The beneficial positive tails tend to get smaller, while the negative tails remain. Presumably (although it is hard to measure), this effect increases as the return horizon increases.

Figure 5a. **Symmetry of Standardized Positive and Negative Tails using Monthly Excess Returns**

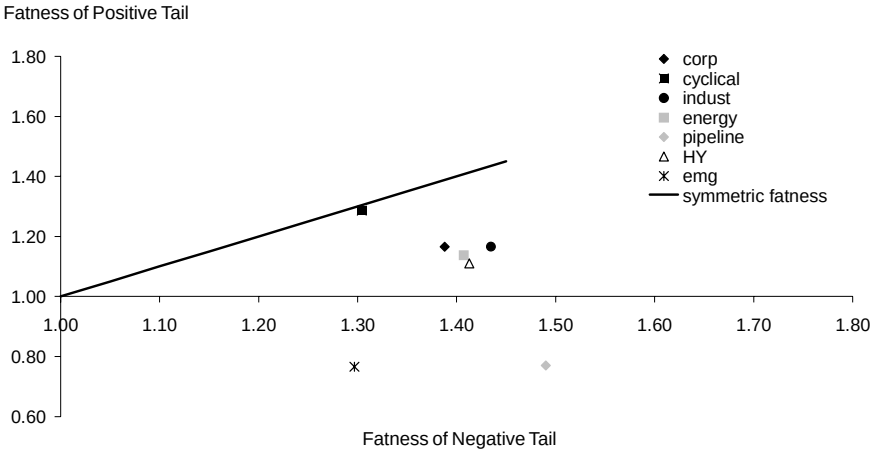
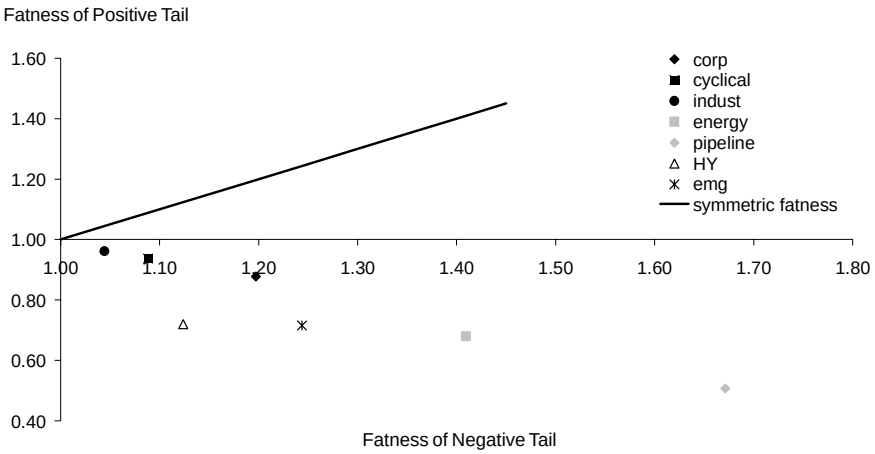


Figure 5b. **Symmetry of Standardized Positive and Negative Tails using Quarterly Excess Returns**



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