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"Tactical portfolio choices, alternative asset classes and the global financial crisis"

RELATORE:

CH.MO PROF. MASSIMILIANO CAPORIN

LAUREANDA: SOLDÀ MICHELA

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Firma dello studente

Michela Soldà

Ai miei genitori

ABSTRACT/RIASSUNTO

The recent financial crisis has put into question the returns obtained from a large part of the institutional investors. In general, the managers construct their portfolios by considering only stocks and bonds and by maintaining unchanged over time their asset allocation. The idea behind this work is to understand whether investing in a portfolio composed with alternative asset classes (other than the usual common asset classes) and providing frequent rebalancing may achieve better performances than what could be obtained with the classical stocks and bonds portfolio. This work will focus its attention on several alternative asset classes and constructs a global portfolio (the regional areas considered are United States, Europe and Japan). The models taken into account to find the portfolio construction inputs are two: the traditional model of Markowitz and the Black and Litterman model. First of all, some strategic portfolios composed with stocks, bonds and only marginally hedge funds, are presented. At a later stage, these portfolios are managed following the tactical asset allocation strategy. The tactical asset allocation was created considering two variations range (a wider range and a smaller one). Finally, the two types of portfolio construction are compared by taking into consideration numerous performance indicators.

La recente crisi finanziaria ha messo in discussione i rendimenti ottenuti da un gran numero di investitori istituzionali. Solitamente, gli investitori costruiscono i loro portafogli considerando solo azioni e obbligazioni e mantenendo inalterata nel tempo la loro allocazione nelle varie classi di investimento. L'idea alla base di questo lavoro è quella di capire se investendo in un portafoglio composto da classi di investimenti alternativi (diverse dalle solite azioni e obbligazioni) e aggiustando frequentemente i pesi assegnati a ciascuna *asset* si possono ottenere prestazioni migliori nei rendimenti rispetto a quanto si è verificato in passato. Questo lavoro pone l’attenzione su diverse *alternative asset classes* e costruisce un portafoglio globale (le aree geografiche considerate sono: Stati Uniti, Europa e Giappone). I modelli presi in considerazione per trovare gli input per costruire i portafogli sono due: il modello di Markowitz e quello di Black e Litterman. In primo luogo, sono stati costruiti alcuni portafogli strategici composti solo da azioni, obbligazioni e solo marginalmente, da *hedge funds*. In un secondo momento, questi portafogli sono stati gestiti seguendo la strategia di allocazione tattica. L’allocazione tattica è stata effettuata considerando due bande di variazione (una banda più ampia e una più contenuta) rispetto al portafoglio strategico. Infine, i due tipi di *asset allocation* sono stati confrontati tra loro considerando numerosi indicatori di performance.

CONTENTS

INTRODUCTION

The recent financial crisis has put into question the way in which the institutional investors managed their clients' wealth. The institutional investors are professional financial institutions that work on behalf of small, or private investors in order to achieve their clients' specific objectives in terms of risk and returns. They have more information and they are more skilled than their clients and it is for these reasons that the private investors trust them to manage their wealth. The number of the institutional investors' clients has increased in the last decade also because of the continuous increment of financial products placed on the market and an increasing number of regulations on which the investors must comply. Hence, these investors have the knowledge necessary to find an efficient allocation for their clients savings and can supply a method to diversify the risk that may arise[1]. Their main characteristic, in fact, is to give to their clients a method of risk pooling and so doing obtain a better risk-return performance. The process in which the investors decide how to allocate their wealth is called asset allocation. The asset allocation is one of the most important steps, or even the most important because it enables investors to reach their clients' goals.

The financial crisis situation in financial markets has dropped, in a dramatic way, the interest rate levels. In this period, in fact, the interest rates have experienced very low levels and in some cases even negative. This is the reason why institutional investors have focused their attention on finding different approaches for constructing investments portfolios. They have begun to change their traditional asset allocation which was generally equal to 70% of their total wealth invested in bonds (40% to government bonds and 30% to corporate bonds) and the remaining 30% invested in equities[2] by increasing the use of alternative asset classes in their portfolios construction. In addition, investors usually rebalance their asset allocation very rarely, but with the global financial crisis, the losses that they have suffered has forced them to make more frequent rebalances. The aim of this work is to understand if there could be a better solution than a portfolio constructed with only stocks and bonds and whose asset allocation is rebalanced only when there are some losses. In particular, this work is going to explain how a portfolio constructed with the stocks, bonds and alternative asset classes tactically managed, with monthly rebalancing, can better exploit the several market possibilities. The market gives some clues to the investors and by efficiently exploiting these it is possible for them to

[1] Report submitted by a Working Group established by the Committee on the Global Financial System, (2007).
[2] Pozen, Palmer, & Saphiro, Strategic Asset Allocation in Times of Financial Repression, (2010).

understand the undervalued or overvalued asset classes and take benefit from these mispriced securities.

During the last decade the financial markets experienced a period in which investing in stocks and bonds generated low interest rates. Hence, this situation has induced the creation of considerable literature concerning possible solutions to the problems that the financial markets were undergoing. Some of the papers described here concern the inclusion of the alternative asset classes into the portfolio asset allocation and some others are related to the use of the tactical asset allocation. This thesis can be considered as an innovative point of view because it takes into account a high number of alternative asset classes and combines them in a tactical way. The papers analysed, instead, have considered only one, or more, alternative asset class at a time in order to construct their portfolios.

By looking at the existing literature concerning the construction of portfolios in which the alternative asset classes have been included in the asset allocation, some papers analyse this argument. In particular, they study, for different purposes, the effect and the motivation behind the inclusion of asset classes different from stocks and bonds. The papers that introduce in their asset allocation only one alternative asset class are several. The alternative asset classes that are mostly used in a portfolio construction with stocks and bonds are hedge funds and real estate. The papers concerning the inclusion of hedge funds are *Hedge Fund Performance 1990–2000: Do the "Money Machines" Really Add Value?* and *Who Should Buy Hedge Funds? The Effects of Including Hedge Funds in Portfolios of Stocks and Bonds*, both written by Amin G. S. and Kat H. M. and they all result in the same conclusions. The portfolios obtained with stocks, bonds and hedge funds provide a better risk-return trade-off than investing in an only stocks and bonds portfolio, but also than investing in hedge funds as stand-alone investments. The inclusion of hedge funds supplies a higher level of diversification. The benefit of including this asset class in the portfolio construction is higher for institutional investors thanks to their longer time horizon than for private investors. The paper that takes into account the portfolios obtained with stocks, bonds and real estate is that written by Lee S. and Stevenson S., *The Case for REITs in the Mixed-Asset Portfolio in the Short and Long Run*. The result that comes from the analysis made in the paper is that the alternative asset class, real estate, positively affects the performance of the portfolio in which it has been included. However, the longer this alternative asset class is held in the portfolios, the higher the diversification benefits the investors receive.

There are other papers that consider useful the addition of some alternative asset classes in the portfolio construction because their inclusion results in an increase of portfolios performance values. One of the papers that analyses this point of view is the paper written in

2010 by Pozen R. C., Palmer B. and Shapiro N., *Asset Allocation by Institutional Investors after the Recent Financial Crisis*. It analyses the behaviour of institutional investors after the recent financial crisis. In particular, the authors saw that, after the years of the crisis, the institutional investors all over the world have increased the use of the alternative asset classes in their portfolio construction. In addition, investment managers, by starting from the classical stocks and bonds portfolio, have reduced the presence of equities and increased the presence of high-quality bonds in their portfolios. This type of investors has invested a higher amount of wealth not only in domestic securities, but they have also increased their exposure to non-domestic securities in order to hedge their returns from the home country risk. The authors consider as alternative investments, securities such as hedge funds, private equities and real estates. Their findings have confirmed that the tendency of increasing alternative investments is verified for all of the types of institutional investors and it is true in each geographical area that they have taken into account. In addition, Pozen, Palmer and Shapiro state that institutional investors include alternative asset classes in their portfolios construction because the alternative and common asset classes are low correlated. The authors see in this low correlation the possibility to hedge returns from market fluctuations and to increase the diversification effect. Another paper that moves in the same direction is *Strategic Asset Allocation and Role of Alternative Investments* written by Cumming D., Hass L. H. and Schweizer D.. This paper introduces in the classical stocks and bonds portfolio some type of alternative asset classes, such as real estate, private equity, commodities and hedge funds. The authors verify that a portfolio constructed by adding to the stocks and bonds these alternative asset classes provides a positive diversification benefits to who invest in this portfolio.

The tactical asset allocation is the other element in which this work is focused. There is a consistent existing literature about this argument. The papers concerning this theme that have been analysed are briefly presented below. All these papers agree on the fact that the tactical asset allocation is a good opportunity in order to increase the portfolios returns. In particular, Gratcheva E. M. and Falk J. E. discuss in their paper *Optimal deviations from an asset allocation* the importance of the asset allocation in the process of achieving the objectives of the investments. The authors highlight the higher benefits that a tactical asset allocation may achieve with respect to the strategic asset allocation. The investors can benefit from an increase in long-term returns by using the tactical asset allocation process. This increase in returns generates a higher average return than what the strategic asset allocation can provide. Also the paper *Fundamental-driven and Tactical Asset Allocation: what really matters?* written by Boulier J.-F. and Hartpence M. proves that the tatcical asset allocation results in a better performance. These authors focus their attention on simulating the behaviour of several types of asset allocation management of a portfolio constructed with bonds and equities. The results they obtained are that the tactical asset allocation provides a stronger positive impact on the returns of the portfolio than those that are achieved with the other types of asset allocation management. The last paper analysed concerning the advantages of using the tactical asset allocation strategy was written in 1998 by Goodsall W. A.R., *Tactical Asset Allocation.* The author states that in the market there are a lot of opportunities for an investor to beat the market and the investors need to discover these opportunities. This is not easy, but it is sufficient to capture some of these opportunities to achieve a better performance than those that may be obtained by following the passive management style. In this paper, the possible effects on constructing a portfolio with the tactical asset allocation are examined and the results found are that the tactical asset allocation has a particular advantage given by the fact that the investors can change their portfolios asset allocation frequently, even day-by-day. In this way they have the opportunity to constantly monitor and evaluate the behaviour of the assets on which they are investing. Although the tactical asset allocation has several drawbacks it is a good way to capture opportunities that may arise in the markets and to take advantage of the diversification effect.

This thesis hopefully contributes to the existing literature by analysing the different types of portfolio construction and the results and by looking at the level of performance obtained by each type of portfolio in the periods analysed. It tries to understand if adopting one type of asset allocation in a period of financial instability could generate better results than using another asset allocation strategy. In particular, this work combines together the use of the alternative asset classes into the asset allocation and the tactical management of the standard strategic portfolios. By trying to develop these arguments the work has been divided in five chapters.

Chapter 1 and **Chapter 2** are review chapters, which aim to clarify and uniquely define the concepts that will be used. **Chapter 1**, in fact, contains the notion concerning the portfolio management, in particular, it explains what steps are used to compose this process and which management type can be applied in these steps. In addition, the concepts of several types of management (active, passive and a combination of the two) are explained. The definition and the process generally used to construct the strategic and the tactical asset allocation are also specified.

In the second chapter, the methodology used in the empirical analyses is presented by highlighting the limits and the advantages of the models adopted. The Markowitz's mean-variance theory and the Black-Litterman model are explained. These models are the methodology in which the following chapter bases all its inputs calculations. **Chapter 3**, in fact, has the objective to make clear what asset classes have been used based on the distinction of common and alternative asset classes and, in particular, how all the inputs for the empirical analysis have been calculated. In this chapter, the different types of asset classes and their characteristics are described and it has been highlighted the way in which all the data have been retrieved and calculated.

The data obtained in **Chapter 3** are the inputs for the asset allocations, which are developed in **Chapter 4** where the asset allocation process has been constructed. In particular, several types of strategic portfolios have been built. These portfolios are those in which the asset allocation has been decided and maintained unchanged for the entire period considered and, in a later stage, the portfolios obtained have been tactically managed. The tactical management of portfolios implies that some constraints are given and these portfolios are rebalanced every month by taking into account those constraints, the models used to combine the data inputs (Markowitz or Black and Litterman) and the methods in which the data are achieved (rolling sample moments or exponentially weighted moving average methods).

In **Chapter 5**, several performance estimations are presented by considering four time periods. The first time period is the whole period taken into account in the entire analysis and the other three periods are: the years before the financial crisis (from 2003 to 2006), the years of the global financial crisis (from 2007 to 2009) and the period of the sovereign debt crisis (from 2010 until the end of the period, 2014).

In the last part of this work, the conclusions and the results obtained in the previous analysis are summarized and examined. In particular, in the conclusive part it has been considered if the portfolios managed in a tactical way and by including the alternative asset classes in their asset allocation can perform better than the strategic stocks and bonds portfolios.

1. INVESTMENT MANAGEMENT

The concept of investment management refers to all techniques and strategies considered for the purpose of allowing to take decisions about which assets to include in the portfolio in analysis, how to evaluate its performance, and how to modify the decisions in response to changes in market trends. These choices are made in order to meet investment goals for the profit of investors. There are two main investors' categories: institutional and private investors. In this work, the institutional investors are the type taken into account. The asset management is made with the so-called *asset allocation*. The asset allocation is one of the components of the Investment Policy Statement, which is a guide to planning and implementing an investment process.

Moreover, talking about some of the active portfolio management strategies bring in mind their analysis with respect to a benchmark. A benchmark is a reference portfolio with respect to which the institutional investor has to compare their portfolio performances. It might be composed by considering a market index or by mixing several asset classes indexes. The aim of the benchmark is to highlight both the composition features and the existing links between the portfolio management and index. In order to be efficient, a benchmark has to be endowed with transparency, representativeness and it must be investable. All these characteristics mean that an investor has to be able to replicate the benchmark performance.

By looking at the relationship between the benchmark and the constructed portfolio, there are two types of portfolio management that can be found: *active* and *passive management*. The passive portfolio managers want to replicate the benchmark returns, whereas the active portfolio managers try to beat the benchmark. The classification of a portfolio into an actively or passively managed portfolio is not always easy. For this purpose, a helpful element is the *tracking error* (also called the *active return*), that allows to recognize one management style to the other. The tracking error is the difference between the active or passive portfolio return and the benchmark return. Therefore, whether the active return is close to zero, the analysed portfolio is managed in a passive way, on the other hand, if it is different from zero, the portfolio is an active managed portfolio. The tracking error can be positive and it states that the analysed portfolio over-performs the benchmark, otherwise it can be negative by meaning that the portfolio under-performs the benchmark. The reasons of the tracking error might be several: transaction costs, differences in securities prices and a different asset mix with respect to the benchmark.

In the following paragraphs, the management types and the process of an asset allocation construction seen above will be further analysed. In the very first part of this chapter, there is the explanation and distinction between the passive and active management. At a later stage, the focus will be on the description of how an asset allocation procedure has to be driven.

1.1. PASSIVE MANAGEMENT

Passive management aims to replicate the benchmark return, because the passive managers believe in the market efficiency[3] and they do not try to beat the market. They hold securities for long periods and make unfrequently changes. They modify the managed portfolio only when the consensus opinion of the market portfolio changes[4] or by adapting to a variation in the benchmark composition.

To replicate the benchmark, the passive portfolio managers can choose between the *full replication* and the *sampling replication*.

Portfolio managers implement the *full replication* by choosing a benchmark, then they construct a portfolio by including all the securities contained in the benchmark with the same weights. They hold this composition until anything in the benchmark changes. This strategy ensures the complete similarity between the benchmark and the passive portfolio performances, at least in theory. In practice, the results cannot be taken for granted so easily because there are numerous challenges that have to be taken into account. Some of these problems are linked to the number of assets included in the benchmark. In particular, the larger is the number of asset classes in the benchmark, the larger could be the trading costs, the operating costs, the executive costs, the management fees and so on. Therefore, the passive portfolio is more likely to not properly replicate the benchmark selected.

On the other hand, the *sampling replication* consists in a selection of the most representative securities that compose the benchmark and try to reproduce the benchmark performances by investing in them. The portfolio returns as obtained with the sampling replication can never precisely track the benchmark returns as the replication can. The sampling replication of the benchmark can be done by using different techniques, as *stratified sampling* (or *linear optimization*) and *sampled optimization* (or *quadratic optimization*).

A mix of the previous techniques is also possible. There can be many possible combinations and they can be called blended approach. A blend of the best characteristics from different

[3] In an efficient market, the market prices reflect immediately the information included in their time series and those publicly or privately available, then there is no way to obtain a better return than the benchmark.
[4] *Investments*, Sharpe, (1985), pp. 655, 660.

sampling replications strategies can create the best index management process. The *stratified sampling* method consists of choosing a homogeneous criterion (geographical area, sector, industry, and so on) to select the sample securities of the benchmark. Once the criterion has been decided, the passive managers need to select the most representative securities for each sample group and then construct the portfolio. Another way to obtain the sampling-replication of the benchmark is the *sampled* or *quadratic optimization* that adopts the mean-variance model with the help of computer programs in order to find the portfolio that minimizes the return deviations from the benchmark[5].

Furthermore, by analysing the passive management, another type of classification that can be done concerns three different approaches helpful in this management style: *buy and hold*, *constant mix* and *constant proportion*.

The *buy-and-hold* approach is a passive and static investment strategy. Once the portfolio has been created, its asset mix is never modified, no portfolio rebalancing is allowed. The portfolio variations are entirely attributed to the movements in the market.

The second approach mentioned above, the *constant mix*, is a passive and dynamic investment strategy. The portfolio needs to be periodically rebalanced, following countertrend market variations. The weights of the asset classes do not change over time.

Finally, the last method is the *constant proportion strategy*, another dynamic investment strategy. The portfolio asset mix is continuously adjusted by modifying the weights of the asset classes in response of market fluctuations.

By summing up the peculiar characteristics of a passively managed portfolio is possible to say that these type of portfolio is characterized by infrequent changes in its composition with consequently low costs (transaction costs, or analysis costs, etc.). This happens because once the passive managers have chosen a benchmark they believe in market behaviour and they try to replicate it as much as possible. On the contrary, the active managers move in the opposite direction. They do not believe in market efficiency and for this reason they try to take profits from market mispricing.

[5] Investment Analysis & Portfolio Management, Reilly & Brown, (2012), pp. 552.

1.2. ACTIVE MANAGEMENT

Another type of investment management is the active management. Active managers rely on market inefficiency: they believe that in the market there are single securities or groups of securities mispriced. They adjust quite frequently their portfolio securities[6]. There are two definition of active management. An active portfolio in a broad sense is composed by using same asset classes with different weights with respect to those used in the benchmark portfolio composition. An active portfolio in a restricted sense refers to a portfolio constructed with short or long positions on asset classes that may be or may not be included in the benchmark portfolio.

Active managers make repeated adjustments in the active portfolio. These adjustments create higher analysis and transaction costs with respect to the passive portfolio. These higher costs are justified only if the managers construct a portfolio that performs better than the benchmark and the passive portfolios. Active managers believe in a careful search of mispriced securities and an appropriated strategy allows them to beat the market since market prices do not fully reflect all the information that they have. For these reasons, important concepts like *market timing* and *security selection* should be considered.

Market timing represents all the techniques that allow the portfolio managers to choose the best moment to invest and divest. The shorter the time horizon, the more important the use of market timing. More precisely, this strategy tries to anticipate market trends by amplifying the positive trends and containing the negative ones. Once portfolio managers have forecasted the market trends, they have to adequate the portfolio composition. If they have estimated a positive trend, they increase the weights of the risky assets. On the other hand, if the forecasted trend is negative the weights that have to be increased are those of the riskless asset.

To better analyse the market timing, the expected return of a general portfolio *p* can be found with the formula of the Capital Asset Pricing Model (CAPM):

$$\mathrm{R}'_{\mathrm{p}} = \alpha_{\mathrm{p}} + \beta_{\mathrm{p}}\mathrm{R}'_{\mathrm{B}} + \varepsilon_{\mathrm{p}}.$$

R'_p represents the portfolio differential return (the difference between the portfolio return and the return of the risk free asset, $R_p - R_f$), R^i_B the benchmark differential return (the difference between the benchmark return and the return of the risk free asset, $R_B - R_f$), β_p is the level of risk bore by the portfolio (it is the level of sensibility of the portfolio return with respect to the benchmark return), α_p is the additional value that the investment may achieve as a result of the skills of the managers (it is also called the Jensen's alpha) and ε_p the estimation error. The component of this formula that is linked with the market timing is β_p. When portfolio

[6] *Investments*, Sharpe, (1985), pp. 655.

managers raise β_p, this increase is expected to have two implications on the portfolio: a higher risk and a higher return (the concept of return means both a possible loss and a possible profit). Moreover, if the evaluations show that the benchmark return is expected to decrease, portfolio managers need to reduce the β_p before the benchmark value decline takes place. On the opposite, if the benchmark is expected to rise beforehand, the β_p has to be increased.

The aim of the *security selection* (or *stock picking*) is to beat the benchmark not by changing the portfolio level of risk, but by identifying mispriced securities. In particular, when managers discover an undervalued security, they put it in the portfolio composition and hold it until the market values it correctly. At the opposite, whether they find an overpriced security in their portfolio, they try to sell it. Therefore, in the security selection strategy, portfolio managers try to reduce the weights of the overpriced securities and increase the weights of the under-priced ones without changing the portfolio risk.

By referring to the CAPM formula seen above, in the security selection strategy, the managers are not interested in changing the β_p, but to achieve a higher portfolio return by raising α_p. In fact, α_p highlights the security selection effect, in particular, it reveals the quality and the accuracy of the managers' information and forecasts. Consequently, the parameter α_p is linked to the managers' skill and for this reason is difficult to modify.

The market timing and the security selection may be combined together, but it is very hard to obtain specific and enough skills for both. Therefore, generally, managers focus their attention on only one of them.

In an active portfolio construction, portfolio managers can choose to use two types of approaches: the *top-down* and the *bottom-up*.

With the *top-down approach* managers considers the market in which they are willing to invest and then it follows a more detailed and precise style. More precisely, portfolio constructors begin their analysis by taking into account the asset classes in which they could carry out the investment (stock, bond, etc.), then they focus their attention by considering the countries in which the portfolio has to be split and, at the end, they select individual securities that will be used in the portfolio construction.

On the contrary, the *bottom-up approach* selects the best securities without making a preliminary evaluation about the sectors or the markets from which they come. There is no distinction between domestic or foreign securities as well. This approach creates a more aggressive portfolio, in terms of level of risk.

As has been explained so far, an active management strategy implies a portfolio having a high risk and high transaction, analysis and forecasting costs. On the other hand, passive management type only slightly differs from the results obtained by the index chosen. This type of managers usually does not exploit the possibility of higher performances. Therefore, active or passive management taken alone cannot be considered the best portfolio management. These two portfolio management styles have some limits so the right direction might be to adopt a mix of both.

Another strategy is the so-called *core-satellite strategy*, it tries to combine the advantages of passive and active management style. In order to make this combination, managers create the general portfolio which is split in two additional portfolios: the *core portfolio* and the *satellite portfolio*. The *core-portfolio* represents the passive management style and it replicates the benchmark. It is the component that is included most in the general portfolio. Its aim is to monitor the levels of risk. The *satellite-portfolio* represents the active management style. It aims to obtain a better performance with respect to the benchmark and to the portfolio performance that would have been obtained by adopting only the core component. It represents only a little portion of the entire general portfolio component in order to reduce transaction, analysis and management costs. The advantages of the core-satellite strategy is that the portfolio obtained with this strategy is a general portfolio that can reach high levels of active returns and consequently high levels of risks with low costs. In this way, the decrease in transaction costs, the risks reduction and the higher returns objectives are reached at the same time.

Regardless the type of management, active, passive or a mix of the two, the investment management process is the same for all types of portfolios, irrespective of its size and/or aim. In order to explain how it works, the CFA (Chartered Financial Analyst) Institute divides the portfolio management process into three main steps: planning, execution, and feedback[7].

Planning refers to the choice of the inputs required (i.e. assets classes, market conditions, etc.) and the investment objectives. It lays the foundations for an early asset allocation process. In this very first step of the portfolio management the Investment Policy Statement (IPS) needs to be constructed. It is employed as a strategic guide to the planning and the implementation of an investment program. The IPS is important because it describes the scope, the objectives (in terms of returns and risks) and the performance measurement of the investment process to which it refers.

The execution is the second step of the process and is aimed at finding the optimal asset allocation and it states the used asset classes. The asset allocation, in fact, is the selection of investments' mix which means that each component is an asset class rather than an individual security. In addition, in this part of the process, the weights (the division of funds among the

[7] *Investments*, Bodie, Kane, & Marcus, (2014), pp. 978.

asset classes chosen) and the rules needed to manage several types of investments within the portfolio are defined.

The portfolio management final step is the feedback, it is a constant and periodical check and eventual update of introductory decisions. This is necessary because market conditions and investor needs are in constant transformation.

The most interesting part, for the purpose of this work, is the asset allocation, which will be described in the following paragraphs.

1.3. Asset Allocation

The asset allocation is not a single step decision. In fact, in order to build an investment strategy, it determines the different types of asset classes, how to divide the wealth between them and what are the limits established by the policy weights. These decisions aim to provide the best investment return available for the level of risk the investor is willing to accept. It is one of the most critical points in the portfolio construction. In addition, the asset allocation is responsible for the major part of return and risk levels; it explains the highest part of the variations in risk and return[8].

The asset allocation process can be carried out by using different methods. To determine the asset mix of a portfolio, there are four general approaches which are related one to the other (*integrated*, *strategic*, *tactical* and *insured*[9]). In what follows all these strategies will be briefly introduced, but the analysis will be focus only on the strategic and tactical asset allocation approaches that are more pertinent for this work.

The *integrated asset allocation*, from a general perspective point of view, contains the other three asset allocation procedures. It takes into account market conditions and the investors' goals and constraints in a separate way. Then, the information concerning these elements is combined together to obtain the optimal asset mix for the analysed investor. Afterwards, the optimal portfolio constructed and the initial expectations of the investor are combined and whenever there are some differences or a fundamental change in market conditions or in the investors' behaviours, the asset mix is modified by including the new information.[10]

The *insured asset allocation* is the most dynamic strategy. It implies continuous adjustments in the asset mix in order to manage investors' objectives and constraints, assuming unchanged

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[8] Several studies have examined the importance of the asset allocation in the investment management in determining the investment returns and/or risks. *Determinants of Portfolio Performance,* Brinson, Hood, & Beebower, (1986), *Determinants of Portfolio Performance II: An Update*, Brinson, Singer, & Beebower, (1991) and *Does Asset Allocation Policy Explain 40, 90, or 100 Percent of Performance?*, Ibbotson & Kaplan, (2000) are only some of these studies.
[9] Investment Analysis & Portfolio Management, Reilly & Brown, (2012), pp. 577.
[10] *Integrated Asset Allocation*, Sharpe, (1987), pp. 25-27.

market conditions. In addition, an insured asset allocation approach allocates usually the wealth in only two asset classes, the most common two are stocks and bonds[11].

The two approaches that will be analysed below are strictly linked one to the other. In fact, as Statman M. wrote in 2000 in his article *The 93.6% Question of Financial Advisor*, both strategic and tactical asset allocation are movements regarding the efficient frontier: the strategic asset allocation represents movements along the frontier, on the other hand, the tactical asset allocation denotes changes of the frontier. In particular, the choice of an appropriate technique of tactical allocation strategy arises from fluctuations in prices of the assets included in the portfolio, which gives origine to a redistribution of the weights. Therefore, the tactical asset allocation changes the risk-return defined when the strategic portfolio was built and selected. Considering the purpose of this work, the most interesting aspect to examine is the strategic portfolio construction and its tactical management. The strategic asset allocation chooses a target asset allocation and the tactical strategy varies the weights assigned to each asset arounf the pre-determined target.

1.3.1. STRATEGIC ASSET ALLOCATION

Strategic asset allocation is an approach used to determine the amount of wealth that should be invested in an asset mix based on risk and return asset classes characteristics. To make decisions in the strategic asset allocation, the investors' risk aversion is taken into account, but the market conditions are not. In general, each amount of wealth is expressed as a percentage of total value invested in each asset class. The strategic asset allocation describes the creation of a long-term portfolio realised with this asset mix. The portfolio managers predict returns, risks and correlations by using the historical data for the selected asset classes and then they construct a portfolio. Hence, with this approach, a needed assumption is that the past performance will be replicated for future performance, thus the estimates of these parameters remain constant over time or, in other words, these estimates are not sensitive to recent information.

Generally, the strategic asset allocation exploits common asset classes (such as cash equivalents, stocks and bonds); sometimes, the institutional investors (in particular, pension funds) add also hedge funds to create their strategic portfolio. This choice is engendered because the common asset classes have more copious documentation about historical data than alternative asset classes. In addition, the asset mix constructed with stocks, bonds, cash equivalents and eventually with hedge funds, is the one that most fits to the features that asset

[11] *Integrated Asset Allocation*, Sharpe, (1987), pp. 29-31 and *Investment Analysis & Portfolio Management*, Reilly & Brown, (2012), pp. 582-583.

classes must have to be included in strategic asset allocation. In fact, these are the asset classes that have the lowest level of volatility.

A strategic asset allocation horizon is usually of 10 to 25 years[12], but the portfolio can be adjusted by rebalancing the asset weights. This portfolio rebalancing may happen when some changes in the expected returns of the asset arise in the medium-term (ca. 5 years).

A good strategic asset allocation assures a well-constructed optimal portfolio, which is built by considering investors' constraints, investors' risk preferences and the long-run expectations about risk and return that are usually taken as constant values. Hence, the strategic asset allocation is the most important step in building the portfolio.

1.3.2. TACTICAL ASSET ALLOCATION

Tactical asset allocation involves short-term decisions that imply a dynamic application with frequent adjustments in the strategic portion of the portfolio. These adjustments are driven by changes in predictions concerning asset returns. In fact, the tactical asset mix aims at exploiting variations between and within the different asset classes. In other words, tactical asset allocation regularly adjusts the strategic asset mix when changes in short-term market forecasts take place. The changes in the asset return predictions can arise in a relatively short period (for example, about one or three to six months) and generally they are the consequences of large changes in market prices. The investors receive some information from the market, if they overact to this news, they could change their asked risk premium and this could cause variations in the market prices.

The investor's risk aversion and his constrains are assumed to remain constant over time. In this step of the investment process, the only thing that is considered is the asset return forecast changes. Tactical asset allocation tries to obtain extra returns by exploiting inefficiencies or temporary imbalances in equilibrium values among different asset classes. Thus, tactical asset allocation requires more frequent trading than strategic asset allocation.

Whereas the goal of the tactical asset allocation is to maximize the portfolio returns, taking into account the assumption that, in several asset classes, the returns diverge from their equilibrium levels. This strategy may pursue excess returns by exploiting changes in current market conditions.

To construct tactical asset allocation, it is necessary to find asset classes that are volatile and change their value quite often. Alternative asset classes have these characteristics. Hence, they are the asset classes that could add value in the investment process.

[12] *Fundamental-driven and Tactical Asset Allocation: what really matters?*, Boulier & Hartpence, (2004), pp. 2.

2. METHODOLOGY

The description of the Markowitz and Black and Litterman models are briefly illustrated in this chapter. These models will be used in **Chapter 4** for the construction of the portfolios.

The first is the Markowitz model, also known as the mean-variance theory. It is one of the most used and famous model for the portfolio selection. Its aim is to obtain a methodology, which is able to select a better portfolio, among several portfolios, with respect others, until the investors find their optimal portfolio. The model lays the basis for many further theories because it has simple assumptions and is easy to understand. One of the models that comes from the development of the mean-variance theory is the Capital Asset Pricing Model. It adds a risk-free asset to the Markowitz method background and it introduces the concept of beta. The addition of the riskless asset implies constructing a new efficient frontier, which is an upward sloping line. The beta measures the sensitivity of the analysed portfolio with respect to the market movements. Another model that can be considered as a development of the mean-variance theory is the one that turns out to be the equally weighted portfolio. All the securities have the same weight. It will be described only briefly in this part of the work, but it is only a way to assign weights to each asset class and not a real theory.

The Black and Litterman model is the last model analysed in this chapter. It tries to solve some of the Markowitz's model issues. The Black-Litterman model is noteworthy because it pulls together different types of information such as the market equilibrium and the investors' opinions concerning the future market trend. This process is made by using the theorem of Bayes and it creates an optimal variance-covariance matrix for the analysed assets. Then, the results obtained with this procedure are used in the classical Markowitz optimization approach, in order to obtain the investors' optimal portfolio. The Black and Litterman model can be considered as a development of the Markowitz mean-variance theory and will be explained in a separated subchapter with respect to the Markowitz's model because the Black and Litterman model is an innovative theory. It involves in the portfolio construction process some subjective inputs. In particular, by considering the investors' views as an input, the authors of the model are willing to include in the optimal portfolio the investors' opinions about the future performance of the asset classes used. If the investors have no information about each or some assets, they will hold a segment of the market portfolio; otherwise, they will own a portfolio which moves away from the equilibrium portfolio. This moving away phenomenon is stronger the more extreme are investors' views and their trust in their own opinions.

2.1. MARKOWITZ MODEL AND CAPM

Harry Markowitz was the first who tried to solve the problem of the optimal portfolio selection and his model is the pillar of the current asset allocation theory. Markowitz has been the author of the modern portfolio theory and in 1952 he developed his theory based on two elements: the mean and the variance of an assets portfolio. This model takes into account not only the input parameters, but also the investors' behaviour. Those elements, combined together, engender the choice of the optimal portfolio for the analysed investors. The concept of optimality has a specific meaning in this context. Markowitz, in fact, with the word optimal intended to identify the portfolio that has the highest expected return value with the same level of risk or the portfolio with the lowest risk level with the same expected return value. This idea leads to the formulation of a concave curve (the so called *efficient frontier*), from which the investors can choose their favourite portfolio, according to their preferences. The efficient frontier is constructed by choosing among all feasible combinations of asset classes which match the Markowitz optimal criteria. Then, the efficient frontier can be built by maximizing the investors' expected return with a consequent modification on the risk level portfolio or by minimizing the portfolio's level of risk. Therefore, this efficient frontier is constructed only with risky assets and it is not affected by the investors' risk aversion. To choose the optimal portfolio, it is necessary to find the point of tangency between the indifference curve obtained by the investors' utility function and the efficient frontier. This tangent point is, according to Markowitz, the optimal portfolio.

Nevertheless, the mean-variance model is based not only on the choice relating to which kind of securities are included in the portfolio, but also on the fraction of wealth that has to be given to each security. One of the most important features of the securities is to see whether they move following others securities or not. If an asset moves in the same direction as another one, it is said that they are correlated. The correlation is a significant element regarding the portfolio construction because it gives an advantage in terms of risk. Lower is the covariance between two securities, smaller the variance portfolio and higher its return. This phenomenon is called *diversification* and it is a powerful way to obtain a decrease in total portfolio risk. The efficient frontier is composed, to the greater extent, from well-diversified portfolios.

What follows is a more detailed explanation of the Markowitz model by highlighting which are the hypothesis on which the model is based, how it has been constructed and what are its limitations and advantages.

MODEL HYPOTHESIS

The model is based on a set of several hypotheses, some of them directly come up in the equations' explanation on the Markowitz theory paper, but some others have to be implicitly disclosed in the model.

One hypothesis that appears clear in the mean-variance model is that investors are concerned with two parameters: the expected return value (mean) and the risk level (variance). They want to maximize the portfolio expected return for a given level of risk or maximize the portfolio return by minimizing its risk. Applying both hypothesis implies that the efficient frontier obtained is the same only if the investors' utility function is quadratic or when the utility function is exponential and returns are normally distributed.

Since the mean and variance are unknown, they need to be estimated by looking at their historical data. The investors have homogenous expectations about expected returns, variances and covariances of securities.

The market on which the model is based is assumed to be an efficient market. This means that the market operators have the same information at the same time and they cannot affect price movements (i.e. they are price takers). In addition, the investors are rational, risk averse and want to maximize their utility function. In other words, they want to earn as much money as possible with a lower possible risk. Thus, the investors' utility function is concave and increasing (this shape is due to their risk aversion and to their preference to increase consumption).

Moreover, the investors are characterized by a single time horizon; at the beginning of this investment period, they allocate their wealth among several securities. They have to decide how much wealth (the weight) to assign to each asset class; whether it is assumed that short selling does not exist, all weights must be positive and in any case, since the weights are a percentage of the whole wealth, they must sum to 1.

Another hypothesis in the Markowitz model is that the fees and transaction costs are not considered. The investors are hypothesized to have no credit limits and the securities exchanged in the market are divisible (it can be bought or sold all fractional parts of securities and there are no minimum boundaries for the orders).

The hypothesis referred in this part are the basis of the Markowitz theory, which will be explained.

MODEL CONSTRUCTION

The Markowitz model is also known as mean-variance model because it states a way to choose between a couple of portfolios only by looking at their risk and return values. The assumptions concerning the portfolio construction, focalizing only on mean and variance and on the fact that the investors are risk averse and want to maximise their utility, produce the mean-variance principle. This principle declares that, between two investment strategies, it is preferable the one that has a higher expected return and a lower standard deviation.

As long as the investors are risk averse and prefer a portfolio with higher expected return and as low as possible level of risk, the main inputs they need are returns. Then, if the investors want to construct their portfolio including a number N of asset classes[13], they have to compute the expected return μ of each asset class. The amount of wealth, ω, to allocate in each asset class must be chosen for the construction of a portfolio. Thus, the expected return of a portfolio, is given by: $\overline{R_p} = \omega'\mu$.

Once the expected return of each asset class has been found, the next step is to determine the distance between a single asset class return and its expected return, the variance. The variance, σ_i^2, of each asset class i $(i = 1,2, \dots, N)$ measures the density with which the distribution is grouped around the expected returns of the asset class considered. In the Markowitz model, the variance represents the uncertainty, or better, the risk taken on investing in that asset class. This parameter is not dimensionally homogeneous to the values of the sample, then, it is better to consider its square root, the standard deviation, σ_i. In particular, since the investors have to consider several asset classes to construct their portfolio, it is more usual to talk about the symmetric and positive definite variance-covariance matrix, $\hat{\Sigma}$:

$$\hat{\Sigma} = \begin{bmatrix} \sigma_1^2 & \cdots & \sigma_{1,N} \\ \vdots & \ddots & \vdots \\ \sigma_{N,1} & \cdots & \sigma_N^2 \end{bmatrix} = \begin{bmatrix} \sigma_1^2 & \cdots & \sigma_1\sigma_N\rho_{1,N} \\ \vdots & \ddots & \vdots \\ \sigma_N\sigma_1\rho_{N,1} & \cdots & \sigma_N^2 \end{bmatrix}$$

where $\sigma_{j,k}$ is the covariance between the asset class j and k. The covariance is a measure that represents how much the returns on two assets change together and in its formula is also used the concept of correlation $\rho_{j,k}$. The correlation indicates whether two variables are related one to the other. Since the standard deviation, being a square root, has to be always positive, the only element inside it that can be negative is the correlation and then the covariance. Covariance and correlation concepts, in fact, are linked one to the other; the covariance is calculated as

[13] Markowitz's theory does not suffer from any change if the optimal portfolio calculated is constructed by two or by N securities. For this reason, in this work, it is explained how to calculate the optimal portfolio including N asset classes.

$\sigma_{j,k} = \sigma_j \sigma_k \rho_{j,k}$, then the correlation is $\rho_{j,k} = \frac{\sigma_{j,k}}{\sigma_j \sigma_k}$. The inclusion of the correlation, through the covariance in the standard deviation of the portfolio, highlights the presence of two different components of the risk. One component is the market risk, better known as systematic risk, the part of the risk that cannot be eliminated. The other component is the risk specific of each individual security and it can be reduced by increasing the number of assets included in the portfolio. It is the diversifiable risk and represents the asset return component that is uncorrelated with the market fluctuations. For this reason, adding a large number of assets in the portfolio generates a partial reduction of the portfolio risk, if and only if, the asset classes included in the portfolio are not correlated each other.

Having the values of asset classes expected returns and risks it is possible to construct several portfolios just by changing the weights of asset classes included in each portfolio. The *two-funds separation theorem*, in fact, states that a portfolio can be obtained as a combination of only two risky assets. All the possible combinations compose the set of feasible portfolios.

Markowitz, with his model, aims to find for every level of return, a portfolio with the lowest level of risk and for every level of risk, a portfolio with the highest level of return. This means that he wants to maximize the portfolio return level with a given level of risk or to minimize the portfolio risk given a level of return. So, the Markowitz model can be seen as a minimization or maximization problem, both subject to a constraint. This constraint is that the wealth the investors want to invest is finite and they wish to fully invest their wealth. Hence, by considering the amount of wealth invested in each asset class like a percentage, the whole fraction of wealth has to amount to one: $\omega_1 + \omega_2 + \ldots + \omega_3 = \sum_{i=1}^{m} \omega_i = 1$.

The maximization and minimization problems of Markowitz can be seen analytically as:

- minimization problem: $\min_{\omega \in \mathbb{R}} \omega' \hat{\Sigma} \omega$

$$\text{subject to } 1'\omega = 1$$

- maximization problem: $\max_{\omega} \frac{\omega' \mu}{\sqrt{\omega' \Sigma \omega}}$

$$\text{subject to } 1'\omega = 1.$$

Since the investors considered by Markowitz prefer higher expected returns with an as low as possible level of risk or lowest risk with an as high as possible expected return, the set of feasible portfolios has to take into account only the portfolios that have these characteristics. Graphing all the possible portfolios on a plane that has on the horizontal axe the standard deviation and the expected return on the vertical axe, the upper edge delimits the set of feasible portfolios. It takes the form of a branch of hyperbola or a broken line which represents the set of portfolios preferred as dominant and it is the *efficient frontier*. Hence, the efficient frontier represents all the portfolios that can be considered optimal portfolios because, by looking to all the feasible set of portfolios achievable with the given asset classes, there are no portfolios with better conditions than those on the efficient frontier. The entire efficient frontier can be obtained as a combination of only two risky portfolios and, as stated by the two-funds separation theorem each portfolio that composes the efficient frontier is nothing more than a mix between two risky portfolios. In particular, by trying to solve the Markowitz minimization and maximization problems, it is possible to find two portfolios from which the entire efficient frontier takes place: the *global minimum variance portfolio* and the *maximum trade-off portfolio*.

The *global minimum variance portfolio*, or the *global minimum portfolio*, is the portfolio with the lowest level of risk and it represents the vertex of the hyperbola arm. The efficient portfolios are only the points that lie on the upper part of the frontier, with respect to the global minimum variance portfolio. This specific portfolio can be constructed by solving the minimization problem which gives as result: $\omega_V = \frac{\hat{\Sigma}^{-1}1}{1'\hat{\Sigma}^{-1}1}$, where $\hat{\Sigma}^{-1}$ is the inverse of the variance-covariance matrix. The vector ω_V represents the weights vector of the global minimum variance portfolio.

The *maximum trade-off portfolio* plays another important role in the efficient frontier construction. This portfolio can be seen as the result of the maximization and the vector of weights that results from the maximum trade-off portfolio is $\omega_T = \frac{\hat{\Sigma}^{-1}\mu}{1'\hat{\Sigma}^{-1}\mu}$.

So far, only efficient portfolios have been found, but the optimal portfolio has not been constructed yet. Selecting a portfolio considered optimal for specified investors implies the introduction of their risk preferences, the investors' utility function. The utility function gives a value for all choices that the investors may have to make. The higher the utility function that the choice gives, the greater the numeric value obtained by this choice; in the case of investments, each combination of risk-return gives different levels of utility. Representing the utility functions in a graph, they take the form of curves (the *indifference curves*) and each point in a particular curve shows a portfolio with different risk-return combinations that provide the same satisfaction level. Higher the indifference curve, greater the satisfaction given to the investors; for this reason, since they want to maximize their satisfaction they have to move on the highest indifference curve without forgetting that they have to submit to their constraints.

Since the two-funds theorem holds, the investors' optimal portfolio is a combination between the global minimum variance portfolio and the maximum trade-off portfolio. In particular, it is located on the tangent point between the efficient frontier and the highest as possible indifference curve. It represents the highest level of satisfaction obtainable by the investors. Any other portfolio, different from the point of tangency, cannot be the optimal portfolio. The reason behind this statement is that even though they lay on the same indifference curve, they could be outside the feasible set of portfolios available for those investors, or they could lay on the same efficient frontier but outside of the indifference curve and as a consequence they will give back a lower level of satisfaction.

The Markowitz theory comes to the conclusion that investors with the same level of risk aversion, caring only for mean and standard deviation, will invest in the same portfolio (the tangent point between their indifference curve and the efficient frontier). In addition, his theory demonstrates that to invest does not mean just pick securities, but choose the correct mix among asset classes. Moreover, if the correlation between securities included in the asset allocation of the portfolio is negative, the total portfolio risk will be reduced by the effect of the diversification.

LIMITS AND ADVANTAGES OF THE MODEL

The mean-variance model has been criticized a lot for the assumptions on which the model lays the foundation because they appear to be too simplistic. As long as the model remains in the theoretical sphere, it works correctly enough, but when it is applied to the real world there are many assumptions that are too strong and simplistic. One of the hypothesis that appear completely unrealistic is that the investors can buy or sell any fraction of securities without any minimum order size. In the reality this is not true, there are some securities that cannot be traded in fractions and have to be subject to a minimum order boundary. This problem is not a real problem for institutional investors because, by having a huge amount of money, they have not many reasons to buy less than the minimum order size or a fraction of a security. In addition to that, investors (all types of investors) are not allowed to lend or borrow an infinite amount of shares because they have a credit limit, their wealth is not inexhaustible.

The assumption about the inexistence of fees and transaction costs is another unrealistic hypothesis because, for example, investors often take advantages of brokers' experience and have to pay their commission fees or by trading financial products, they have to pay taxes to which those products are subject.

The hypothesis concerning that investors operate in an efficient market cannot be accepted in a real context. Even though the investors have correct ideas of potential returns, they do not act rationally because there are some emotional decisions that need to be considered in their behaviour. Since the type of market operators analysed are institutional investors, they buy and sell great amount of securities. Their trades move the price value of the securities they buy or sell in large part. Consequently, the correlation between the asset classes cannot be constant. In addition, real markets are characterized by information asymmetry meaning that the investors cannot have the same information at the same time. In particular, institutional investors, usually, are better informed than private investors. Hence, the market efficiency hypothesis do not hold in real world.

One of the most crucial problems of the Markowitz theory is the calculation of the theory inputs. To calculate mean and variance, the only elements on which the investors can rely is the time series of the securities. By looking only on assets' past performance does not lead to a correct view of future performances. Furthermore, small changes in the calculation of the expected returns can lead to big swings in portfolio positions, causing incorrect weights in some asset classes. Hence, the Markowitz model's weights are too sensitive of the errors that may occur in the mean-variance model inputs.

By listing the hypothesis of the mean-variance model, it has been highlighted that the theory holds only if the utility function of investors is presumed to be quadratic or if it is supposed to be exponential with normal distributed returns. The assumption of quadratic utility function is enough because the solution obtained using it is irrespective of the distribution of returns. Since the investors want to maximize their utility function with respect to their wealth, they can just modify the share of wealth invested in each asset. Then, the result of the maximization problem[14] is $\omega^* = \frac{\overline{R_E}}{A(W)\sigma^2}$ (where $\overline{R_E}$ is the risk premium, equal to the difference between the expected return of the asset and the risk-free asset return, $A(W)$ is the absolute risk aversion, as measured by Arrow-Pratt and σ^2 is the variance of the asset return). ω^* is increasing in $\overline{R_E}$, decreasing in σ^2 and since $A(W)$ is increasing in W, the previous formula is decreasing in $A(W)$. Therefore, for high levels of W the utility function becomes concave, it starts to decrease gradually and the wealth increases, violating the assumption that investors consider that more is better than less.

On the other hand, the exponential utility function has to take into account the returns' distribution. The assumption about the utility function holds only if the returns are normally distributed. In reality, several studies reject the hypothesis of normal distributed returns. Nevertheless, the Central Limit Theorem states that with a large number of observations it makes sense to consider the data regarding returns normally distributed.

Despite the fact that the Markowitz model has all these limits, however, it appears to be the most famous and easiest portfolio selection model because it explains in a very simple way such a sophisticated problem by using only readily available elements. It introduces a very important

[14] Eeckhoudt, Gollier, & Schlesinger, (2005), pp. 32-35

concept as the efficient frontier, which exhibits what is the link between the return that the portfolio will probably give and its riskiness. This idea is useful in explaining the diversification benefits: the diversification can decrease the volatility of a portfolio without decreasing its expected return, or it can increase the return without increasing the risk. Beyond the highlighted benefit of this theory regarding the diversification investment process, the Markowitz theory is highly appreciated by the financial risk managers. They exploit the fact that, diversifying a portfolio through several asset classes amply reduce the risk of a financial ruin even if some asset classes included in the portfolio do not go in the hoped direction.

Harry Markowitz has laid the foundation for the so-called modern portfolio theory and, despite the limitations his model has, it is taken as inspiration even today and it is examined for subsequent models. Although many researchers have tried to solve the problems relating to the estimates for the mean-variance inputs, the procedure for choosing the optimal portfolio (the efficient frontier construction and the selection of the portfolio) have been left unchanged even in later models[15].

EQUALLY WEIGHTED PORTFOLIOS

A portfolio constructed with equal weights does not follow a real theory, but it just abides by a particular way to weigh the securities included in it. In fact, it is a type of weighing that provides same weights to each security in the portfolio. The equally weighted portfolio is the simplest and unconstrained procedure that can be adopted in creating a portfolio. Therefore, the technique for calculating the inputs for the asset allocation does not change from that of the model chosen; the only thing that is modified is the weights assigned to each asset class. Whether there are *N* asset classes, the weight given to each asset is:

$$\omega_1 = \omega_2 = \ldots = \omega_N = \frac{1}{N}.$$

Although it is not a real model that is developed by Markowitz like the CAPM, it has been added in the methodology and in the procedures adopted for the construction of various portfolios. Because the principle of mean-variance, due to its limits, is not as effective in portfolio construction as an equally weighted portfolio[16].

[15] This statement is not intended to refer to the totality of the models existing in literature, but it refers to those analysed in the present work.
[16] As Richard O. Michaud explained in the introduction of its work *The Markowitz Optimization Enigma: Is 'Optimized' Optimal?* (1989).

2.1.1. CAPITAL ASSET PRICING MODEL

The Capital Asset Pricing Model (CAPM) is an extension, suggested by William Sharpe in 1964, to the Markowitz model. It introduces a new concept of risk (the beta) and a riskless asset. The beta represents the sensitivity of the risky asset on the movements of the market. The larger the beta, the higher the expected return of the portfolio constructed.

The CAPM is based on the same assumptions explained in the Markowitz model and an additional one. The mean-variance theory as explained by Markowitz, in fact, is amplified by considering that the investors can lend or borrow at the same risk-free interest rate. So the portfolio constructed, by following this model, may contain an activity that has no risk, as well as all risky activities. The risk-free asset, or riskless asset, R_f, is an asset for which its future return is known with certainty in the day in which the portfolio is constructed[17]. This kind of asset is usually identified in the asset class called cash equivalents[18].

Being the CAPM a development of the mean-variance theory, it is based on the same principle with the additional inclusion of the risk-free asset in the portfolio construction. Including the riskless asset means to calculate a different efficient frontier with respect to Markowitz's. The new efficient frontier is no more a part of the hyperbola, but a straight line. This is because, until the risk-free asset has a risk equal to zero, in the return-standard deviation graph, it is located in the vertical axe. Hence, to link all the feasible risky portfolios (the Markowitz's efficient frontier) with the riskless portfolio[19] comes out a new efficient frontier, the *capital market line* (CML).

The CML is a straight line expressed with the formula $\overline{R_p} = R_f + \left(\overline{R_M} - R_f\right)\frac{\sigma_{p,M}}{\sigma_M^2}$, where $\overline{R_M}$ and σ_M^2 are, respectively, the expected return and variance of the market portfolio and $\sigma_{p,M}$ is the covariance between the market portfolio and the portfolio calculated. The market portfolio is the one that is situated on both the Markowitz's efficient frontier and the CML. It is the tangent point between these two lines and therefore represents the most efficient portfolio.

The CML includes any efficient combination between the risk-free assets and the market portfolio. It is an upward sloping line, because as long as the Markowitz's assumption of not satiety principle for risk-averse investors holds, the investors will be more willing to choose a portfolio with a higher risk if the portfolio return will be greater. Hence, the slope of this line is

[17] This is a theoretical assumption. In the real world, it is almost impossible to find an asset with variance, or standard deviation as well, equal to zero.

[18] The definition of the different types of asset classes used in this work and the asset class chosen as risk-free asset is argument of **Chapter 3**.

[19] It is possible to consider the risk-free portfolio as a portfolio obtained by combining all the available riskless securities.

positive and it is called the Sharpe ratio, $\frac{(\overline{R_M}-R_f)}{\sigma_M}$, which represents the reward for holding a risky portfolio instead of a risk-free one ($\overline{R_M} - R_f$, which is the risk premium) calculated per unit of market risk. The Sharpe index is used to evaluate the efficiency of any portfolio. By comparing two Sharpe ratios it is possible to say that the higher the Sharpe ratio, the higher the efficiency of the portfolio chosen, because it means that its CML is more sloped.

The previously seen formula of CML can also be written as $\overline{R_p} = R_f + \left(\overline{R_M} - R_f\right)\beta_p$, where $\beta_p = \frac{\sigma_{p,M}}{\sigma_M^2}$ measures the dependence of the portfolio that has been considered with respect to the market portfolio. By looking at the beta in terms of an investment risk, it highlights the two risk components: the specific and systematic risk. In fact, writing the portfolio risk as $\sigma_p^2 = \beta_p^2\sigma_M^2 + \varepsilon_p^2$ means that $\beta_p^2\sigma_M^2$ is the part of the risk that cannot be eliminated, because it depends on both the market fluctuations and the beta of the portfolio analysed, and ε_p^2 is the idiosyncratic risk that can be eliminated with the diversification procedure. The only way to eliminate completely the ε_p^2 is by including the market portfolio in the portfolio.

To find the optimal portfolio that investors may obtain by adopting the CAPM it is necessary to take into account also the investors' preference. As long as the Markowitz's model assumptions hold, the risk aversion and the preferences of investors are characterized by quadratic functions. Hence, the optimal portfolio is the tangent point between the CML and the highest investors' indifference curve.

The CAPM which is based on the same assumptions of the Markowitz's model, has also the same limitations and advantages that these hypotheses imply[20].

One of the assumptions of the CAPM concerns the introduction of a risk-free asset which is an unrealistic hypothesis. Whatever type of security is chosen in the analysis, every asset suffers from daily changes, which imply volatility, hence the concept of riskless asset is violated.

The concept of beta is a useful concept, but it cannot be directly observable in the realty. Thus, the beta needs to be found through an estimation which implies some problems that may affect the reliability of the model.

The CAPM is very helpful because it links the value of a security to a single risk factor, the risk of the market. In addition, it is one of the few models that offers a method for calculating the idiosyncratic risk and the possibility to eliminate the specific risk by investing on the market portfolio even though it is little realistic.

[20] To avoid repetition, in this part there will be analyzed only the drawbacks and benefits that have not already been considered in the chapter dedicated to Markowitz.

2.2. BLACK & LITTERMAN MODEL

Fischer Black and Robert Litterman developed this model in 1990. Their aim was to try to find some solutions for the problems that arise from the Markowitz's model. The theory they constructed combines the mean-variance theory with some ideas from CAPM in order to create a better way for the investors to catch their optimal portfolio weights.

This model is focused on finding possible solutions for, in particular, the high sensitivity of mean-variance inputs, the estimation error maximization and highly concentrated portfolios. Another Black and Litterman objective in creating their model is to build an intuitive theory in order to construct investors' optimal portfolio. To achieve their purpose, they used a Bayesian approach to combine investors' views with the market equilibrium portfolio. This combination creates model inputs which are a set of the expected returns and the variance of each asset. These inputs are used in the classical mean-variance optimization process in order to find the optimal portfolio and its composition.

The Black and Litterman theory looks like, in some senses, to be a Markowitz's model development because their theory does not deviate from the mean-variance optimization process, the changes appear only in the inputs considered for the model construction.

MODEL HYPOTHESIS

One of the strictest assumption made by Black and Litterman involves the distribution of the inputs of the model. The two authors assume, as in Markowitz theory, that the return distribution is normal. In particular, the mean of the return follows the normal distribution ($\mu \sim N(\Pi; \tau\Sigma)$), where the mean and the variance of this distribution is derived by the CAPM and the inverse optimization. Resulting from this assumption is the hypothesis pertaining to the expected returns that follow the normal distribution, $\tilde{R} \sim N(\mu; \Sigma)$.

Another important hypothesis on which the model is based is that the prior covariance is proportional but independent to that of the returns.

The Black and Litterman model concerns the investors' views, in particular, they have to be unique, not correlated one to the other and they must be fully invested (the sum of the weights in a view has to be zero or one). However, it is not required that investors have a view of all the assets.

Another hypothesis of this model is that, whether the market is an efficient market (all the investors have the same information about it), the market portfolio will be the equilibrium portfolio.

In the Black and Litterman model, the two authors assume that the utility function used in the model construction is a quadratic utility function.

MODEL CONSTRUCTION

The model analysed in this part of the chapter explains the Black and Litterman's expected return formula by combining two sources of information. The first is a quantitative source. It is the implied expected returns that results from the CAPM, when the market is in equilibrium. This expected return vector is the portfolio that the investors would have if they had had no views (in other words, no ideas about the future asset returns performance). The second source is a qualitative source that concerns the information at which the investment managers have access. They use this kind of information to get ideas about expected return of the assets considered in the construction of the portfolio.

These two sources combined by the Bayes theorem[21] result in a new expected returns vector (the posterior return), which softens the equilibrium views by also using those of the investors. The portfolios that result with the introduction of the posterior distribution are intuitive portfolios with sensible weights.

The implied return are the equilibrium returns derived using the reverse optimization method. The creator of the optimization model is Sharpe[22], who studied a method to provide an additional tool to investors to improve the efficiency in the optimal portfolio composition. This method allows to calculate implicit expected returns by the adoption of the CAPM, or to use the formulas that will be explained in the follows.

Deducing the reverse optimization equation implies to specify the form of the utility function used in the model. The utility function used is the quadratic utility function (U), which is expressed by the equation:

$$U = \omega'\Pi - \frac{\delta}{2}\omega'\Sigma\omega,$$

where Π is a vector of the equilibrium excess return calculated for each asset, δ is the risk aversion parameter, ω is, as in the Markowitz model, the vector that represents the weights invested in each asset and Σ is the variance-covariance matrix of the excess returns for the assets. After having specified the form of the utility function, to calculate the excess returns

[21] Bayes' theorem is a mathematical formula that allows calculating the conditional probabilities. In statistics, the conditional probability refers to the ability to calculate the possibility that a particular event will occur given the likelihood that another event connected to the first takes place. A more detailed focus on the Bayes' theorem is reachable in Bayes, (1763).

[22] Sharpe, (1974).

vector, Π, it is necessary to maximize this utility function and solve the optimization problem for the parameter required. The way to find Π is shown below:

$$\frac{dU}{d\omega} = \Pi - \delta\Sigma\omega = 0$$

$$\Pi = \delta\Sigma\omega.$$

This formula contains some unspecified parameters, the risk aversion parameter (δ) and the weights vector (ω). The parameter Σ is the variance-covariance matrix that is computed from the historical data, as in the Markowitz model. The risk aversion parameter can be give arbitrarily or it can be calculated in the following way:

$$\Pi\omega' = \delta\omega'\Sigma\omega$$

$$\Pi\omega' + R_f = \delta\omega'\Sigma\omega + R_f$$

as long as $\Pi\omega' + R_f = \overline{R_M}$ and $\omega'\Sigma\omega = \sigma_M^2$, the equation can be rewritten as $\overline{R_M} = \delta\sigma_M^2 + R_f$ and then the risk aversion parameter is given by:

$$\delta = \frac{\overline{R_M} - R_f}{\sigma_M^2}.$$

Another way to see the δ value is by using the Sharpe ratio, $\delta = \frac{SharpeRatio}{\sigma_M}$.

The other unknown parameter is ω, the vector of the weights exploited to construct the market portfolio, which is calculated by considering all the assets included in the market in their capitalization proportion relative to market capitalization.

Then, by substituting the values found in the formula $\Pi = \delta\Sigma\omega$ it is possible to obtain the equilibrium asset returns.

The next step in Black-Litterman model is to specify the investors' view. The views can be expressed as asset classes return linear combinations:

$$P\mu = Q + \varepsilon_v,$$

where P is a matrix with dimension $(k \times n)$[23] in which the investors summarize the weight of each view, μ is the vector $(n \times 1)$ of the expected returns average, Q is the vector $(k \times 1)$ containing the expected returns of each view and ε_v is a $(k \times 1)$ vector which contains the error concerning the views.

In each row of the P matrix there are the views, the rows assume value 0 if the investors have not opinions about the corresponding asset, otherwise it is different from 0. There are two kinds of views: relative and absolute views. The relative views represent the investors' opinion concerning a different return (higher or lower) between one asset and another one; the absolute views, instead, represent the investors' opinion regarding the return of the same asset but in

[23] The parameters k represents the number of views and n is the number of asset classes included in the portfolio construction.

different periods of time. Whether the view considered is a relative one, the sum of its weights is equal to 0; if the view is absolute, its weights sum to 1.

One of the crucial points of the Black and Litterman model is the vector of the errors concerning the views. ε_v represents and quantifies the investors trust on their opinions about returns. The parameter follows a normal distribution and, in particular, $\varepsilon_v \sim N(0; \Omega)$, where Ω is the variance-covariance matrix of the views. Thus, also the views distribution is the normal distribution, $P\mu \sim N(Q; \Omega)$. In addition, since the views must be uncorrelated and independent to each other, the Ω matrix is a diagonal matrix.

It is not possible to observe the variance-covariance matrix of the views; however, there are two solutions that can be followed to find its value. The first way is to consider the variance among the views ($\hat{\Sigma}$) proportional to the variance among the equilibrium returns (Σ). The proportionality is given by the parameter τ, which is a scalar number. The value of τ is not uniquely derived because it is a subjective parameter, which represents the uncertainty of the CAPM distribution. Usually, it is set to belong to the interval that goes from 0 to 1 and, often, it takes a number within the range of 0.025 – 0.050[24]. Although it is a subjective number, there are some statistical ways to calculate it. In fact, τ can also be seen as the uncertainty that arises when the distribution mean is calculated. In statistics there are two estimators that can represent this parameter: the maximum likelihood estimator and the best quadratic unbiased estimator.

The formula $\Omega = diag(P(\tau\Sigma)P')$ is the result of this type of approach. Since the parameter τ is difficult to estimate, and it is contained in the variance-covariance matrix, this matrix is in general substituted by its inverse, the confidence matrix (Ω^{-1}). An alternative way to calculate the matrix Ω is to set a confidence interval among the estimated mean return, by exploiting the normality of the views.

So far it has been explained the way to find the two sources needed in order to obtain the posterior distribution of the model. Combining the sources means to apply the Bayes theory to the equilibrium-implied returns and to the investors' views. In the Black and Litterman model, the prior distribution is founded on the equilibrium excess returns, which follow a normal distribution ($P(A) \sim N(\Pi; \tau\Sigma)$); then the conditional distribution, which is necessary in the Bayes theory, is a normal distribution. As a consequence of these assumptions, the posterior distribution follows a normal distribution, too. The Bayesian model is a tool used to join the investors' view with the empirical analysis of the real world and the result at which it leads is the expected posterior distribution of the asset returns ($\overline{R_{post}}$). This variable has mean and variance equal to $\widehat{\Pi}$ and M that are described in the following equations:

[24] Walters, (2014), pp. 63.

$$\widehat{\Pi} = [(\tau\Sigma)^{-1}\Pi + P'\Omega^{-1}Q]^{-1}[(\tau\Sigma)^{-1} + P'\Omega^{-1}P]^{-1} =$$
$$= \Pi + \tau\Sigma P'[(P\tau\Sigma P') + \Omega]^{-1}[Q - P\Pi]$$
$$M = ((\tau\Sigma)^{-1} + P'\Omega^{-1}P)^{-1};$$

then, $\overline{R_{post}} \sim N(\widehat{\Pi}; M)$.

In this way, the Black and Litterman inputs have been obtained and the next step is to use them in the standard Markowitz optimisation process resulting in the optimal investors' portfolio.

LIMITS AND ADVANTAGES OF THE MODEL

The Black and Litterman model well succeeded in its attempt to solve some of the Markowitz's model existing problems, but it failed to find a solution for all the optimization problems. In particular, the model of Black and Litterman is based on the assumption that the returns are normally distributed. This hypothesis is a simplistic assumption. In the real world the returns distribution follows rarely, or even hardly ever, a normal distribution. In addition, the optimization process constructed by Markowitz is still the driving force of the Black-Litterman model.

Moreover, in this model some parameters (τ, Ω and δ) are not easily specified. In the literature there are several ideas about a way to specify them, but they are in contradiction one to the other. In fact, the parameter δ, which represents the risk aversion coefficient of the investors considered, is assumed to be constant and it can also be calculated considering two ways. In one of the two ways, to calculate the parameter δ it is required to know an implied market portfolio return, a not so easily computable element. The parameter Ω represents the uncertainty that investors have of their views. Ω and τ are the elements of the model that create the most problems because they are subjective parameters. τ is a scalar that takes values between 0 and 1; it should be close to zero if the point of view of Black and Litterman is accepted or it should be close to one if others studies are preferred.

The model does not have only drawbacks, it has also some advantages. More precisely, the Black-Litterman model is a way to specify the investors' view on returns and to combine the equilibrium returns with the investors' views. These can be partial or complete and the investors are not obliged to specify views for each asset. The certainty of the investors' view regarding the expected returns has to be explained in the confidence matrix; this addition makes the model more flexible. Then, the expected returns base their estimations on an intuitive and informative prior.

In this model, the equilibrium market portfolio is considered being the neutral starting point for the asset return estimates and it is estimated by the CAPM. The CAPM takes into account the market capitalizations for each asset; it allows to create a more stable portfolio with more reasonable weights than the traditional mean-variance portfolio.

3. DATA

The asset allocation is the first thing to do to construct a portfolio, as it has been explained in **Chapter 1**. Then, in order to choose the amount of wealth to allocate in each asset class, it is important to understand what an asset class is and what are their main characteristics. Thus, the first two parts of this chapter aim to explain the meaning and the characteristics of each asset class and analyse their time series. In the final part of the chapter, the principal descriptive statistic elements are calculated and two methods to forecast data (with smoothed trends and short-term fluctuations) are described.

The term *asset classes* refers to a group of elements that are linked by similar financial characteristics, attributes, risk/returns relationships and they are regulated by the same laws and rules. In other words, the components of each asset class must be homogenous one to the others. Each asset class has various risk and return profiles (generally, the higher is the return, the riskier is the asset class).

Giving a definition of the different types of asset classes is hard. The literature about this argument is not uniform and there are several classification types regarding asset classes. In what follows, the most appropriate and useful classification for this work has been presented. The first type of division concerns common and alternative asset classes. Even if in literature there are several types of asset classes' classification, in this chapter this type of separation takes into account which asset classes were usually exploited in the portfolio construction before the financial crisis from an institutional investors' point of view.

Almost all the data used in this work have been retrieved from DataStream, one of the largest and most important financial database. It contains daily updated time series, which cover the entire world information and they may have different time span.

Only the data of the real estate index have been directly given by the provider, the LPX Group.

In order to standardize the data analysed, the data from the 1st of January 1998 to the 31st of December 2014 has been used; in this way, a dataset composed by 204 months has been obtained. The data used for the whole analysis takes into account the total return index rather than the price index. The latter reflects only the capital gain; on the other hand, the first considers in addition to the capital gain also interests, dividends and lending fees. These cash distributions are assumed to be reinvested into the index. The total return index is a more accurate way to represent the index performance with respect to the price index.

The process generated from the total return prices is usually considered a *random walk*. It belongs to the category of non-stationary processes because their changes cannot be predicted and this is due to its long memory. More precisely, past trends cannot predict future movements. For this reason, the historical time series returns will be used. The decision to choose returns instead of prices is also due to the fact that, in this work, financial markets are considered to be close to perfect competition and then prices are not affected by the size of investment. Moreover, returns contain all the information available and have no problems concerning the unit of measure. Returns also have more interesting statistical properties than prices have, they are stationary and ergodic[25].

The formula that will be used to calculate historical returns is the formula of simple returns:

$$\mathrm{R_t} = \frac{\mathrm{P_t}}{\mathrm{P_{t-1}}} - 1$$

where R_t is the simple return at time t and P_t and P_{t-1} are, respectively, the total return index at time t and at time $t-1$. Thus, the initial dataset is reduced by one observation because the first value, the one related to January 1998, has been lost with a consequent new dataset composed by 203 months.

In this chapter, the asset classes that will be used in the empirical part of this work will be analysed and all the inputs that will be utilized at a later stage will be calculated by employing the models explained in **Chapter 2**.

3.1. COMMON ASSET CLASSES

Stocks, bonds, and cash equivalents are the most common asset classes. They are the category of asset classes that contains assets, which are quite highly correlated one to the other and provide low risks and returns. These types of asset classes have generally been used in the institutional investors' portfolio construction. Only in the most recent period, an increasing use of alternative asset classes has been started, it is mainly due to the financial crisis that hit the global financial market around 2008; earlier, portfolios were built mostly with stocks and bonds.

The subchapters concerning stocks and bonds will be divided in several regional segments: United States, Europe and Japan[26]. The reason behind this choice refers to the institutional

[25] This means that a return is a stochastic process with some statistical properties (such as mean and variance), which can be inferred from a single, sufficiently long sample of the process. In other words, any sample from the process must represent the average statistical properties of the entire process.

[26] The corporate bond analysis will be divided only in the US and European area because no reliable Japanese corporate bond index with a long enough historicity has been found. For this reason, the Japan will not be considered in the corporate bonds analysis.

investors' behaviour; in fact, they usually divide markets in these regional areas. Since the indexes analysed in this part of the work are divided in geographical areas, they will be considered in their local currency and then each of them will be converted in a common currency. In order to make this conversion the exchange rate is necessary. In particular, the exchange rates used in this work are those provided by WM Company and Thomson Reuters; more precisely, the adopted exchange rates are those that convert from euro to US dollar and from yen to US dollar. The choice of converting the currencies in a common one permits taking into account also the exchange risk. This risk affects the investors who do not invest in their domestic securities, but in securities issued in a different currency. The value of these assets has to consider also the changes in the exchange rate between the investors' home currency and the issued securities currency. In this work, the US dollar is chosen as the common currency and all the assets expressed in other currency will be transformed in dollars. In particular, considering that investors are US investors, whenever they invest in non-US markets their investment returns are affected by the changes in their investment performance and by the exchange rate of the local currency against the dollars. In other words, currency risk gives important evidence about the fact that the fluctuations in the exchange rate may change the foreign investment total return, irrespective of how well an investment is performing.

In what follows, the common asset classes will be defined and analysed in respect of their risks and returns levels, from the riskless to the riskiest one.

3.1.1. CASH EQUIVALENTS

Cash equivalents or *cash* represents a group of assets consisting of the money market instruments such as treasury bills, certificates of deposit, commercial paper and so on[27].

The first instrument considered is the *treasury bill*. Treasury bills are liquid assets and they are issued with very short maturities of 4, 13, 26 or 52 weeks. Governments issue them in order to borrow money from the public.

Certificates of deposit are contracts pledged between a bank and an investor. The investors deposit their savings in a bank, but they cannot withdraw their money when they want. In fact, there is a time period in which the deposit cannot be collected and the bank pays back the interests and the principal at the pre-fixed time. Usually, certificates of deposits have short maturity (3 months or more) and they are quite safe because they are insured against the risk of bankruptcy of the issuing bank.

[27] The information about cash equivalents has been retrieved from the analysis of pp. 29-34 of Bodie, Kane, & Marcus, *Investments*, (2014).

The last money market instrument cited is *commercial paper*. Commercial papers are short-term debt notes and their length is up to 270 days. Generally, they are considered a kind of safe asset because the issuers' conditions can be checked on a time horizon lower than one month.

In general, cash equivalents are investments whose maturity is short (less than 12 months) and they have a high level of liquidity. To be considered as a part of this category the financial instrument has to be immediately available. Cash is the least risky asset class, this does not mean that investment losses do not occur, but they are very rare. Consequently, the expected return level is very low, it is close to zero. For these reasons, one of the cash equivalents' component is considered the risk-free asset and, in this work, it is the US Treasury Index. This index is provided by the Barclays company and it measures US dollar-denominated fixed-rate and nominal debt issued by the US Treasury. Therefore, the cash-equivalent asset class has been adopted only in the Capital Asset Pricing Model and in the construction of some performance indicators. It has not been used in the construction of any portfolios because, by including it in the portfolio construction, it will have absorbed all the wealth. In fact, since the cash is the least risky asset class the portfolio composition process will put a large amount of wealth in it by giving to it a high value of weight.

3.1.2. BONDS

Bonds are debt instruments emitted between the investors and the issuer that pay coupons in a pre-defined interval of time. Bonds change their features according to who their issuer is and the periodicity on which their coupons are paid.

One of the main characteristics of bonds is that they are riskier and have higher returns than cash equivalents, but have a lower risk and return level than stocks. In addition, there are several types of bonds; the classification taken into account varies considering the kind of institution that issued the bond: government or corporate.

There could be also other categories of bonds that may have higher returns and higher levels of risk. This group of bonds is the high-yield and, since they are riskier than corporate and government bonds, they have been included in the alternative asset classes.

SOVEREIGN BONDS

Sovereign bonds are securities issued by governments. This type of securities allows governments to borrow money from the investors who buy these bonds. Sovereign bonds provide a coupon with a specific expiration date and the repayment of the principal at their maturity. The coupon reflects the level of creditworthiness of the bond issuer, in fact, government bond coupons are higher than corporate bond coupons. Hence, this class of bonds is considered the one with the highest quality in the market because who invests in sovereign bonds are quite sure to be paid back, since these bonds are issued by central government banks. The uncertainty behind the government bond repayment arises from the fact that, even though they are high quality securities, there are some risks that the issuers may have to manage. Some of the risks affecting the government bonds are interest-rate risk, credit rating risk, inflation risk and exchange risk. *Interest rate risk* (or *market risk*) represents the possibility that the value underlined in bonds might change following the interest rate. In other words, if governments issue a bond and then its interest rate increases (decreases), the value of the bond issued follows an opposite behaviour, its price decreases (increases).

Each existing bond in the market is rated; every time a sovereign bond changes its rating, investors are faced with a *credit rating risk.*

The *inflation risk* links the returns of the investment with the level of current inflation. The effect of this risk is the most evident especially nowadays.

The last is the *exchange rate risk*; this type of risk represents the decrease or increase in value that the investors may suffer by investing in a market different from their own domestic market. The returns obtained can be affected by the changes in exchange rates from the domestic currency to the one of the market in which they were invested.

The time series used to examine the sovereign bonds have been provided by J.P. Morgan company. These indexes consider the government bonds in three regional areas: US, Europe and Japan. The figure below shows the behaviour of government bonds in the period in analysis.

The evidence behind **Figure 3.1** is that there are infrequent peaks. In fact, only one decreasing peak appears in 1999 in the historical data considered for the returns of Japan.

The figure exhibits high variations in returns in the periods before 2006 and after 2009. In the first and last part of this graph all of these three time series considered for sovereign bonds suffer from frequent and high changes in their returns.

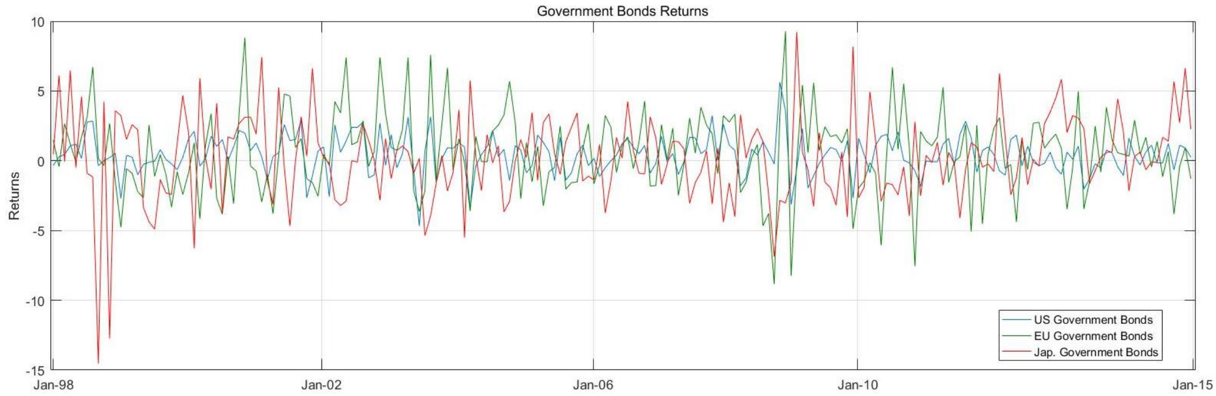


Figure 3.1: Sovereign bonds returns based on the common currency total return index of the three geographical area considered (US, Europe and Japan).

In what follows, the time series of returns are analysed separately in the three geographical areas considered.

UNITED STATES

The J.P. Morgan United States Government Bond Index is analysed and graphed here. This index is a comprehensive indicator for the developed markets local sovereign bonds.

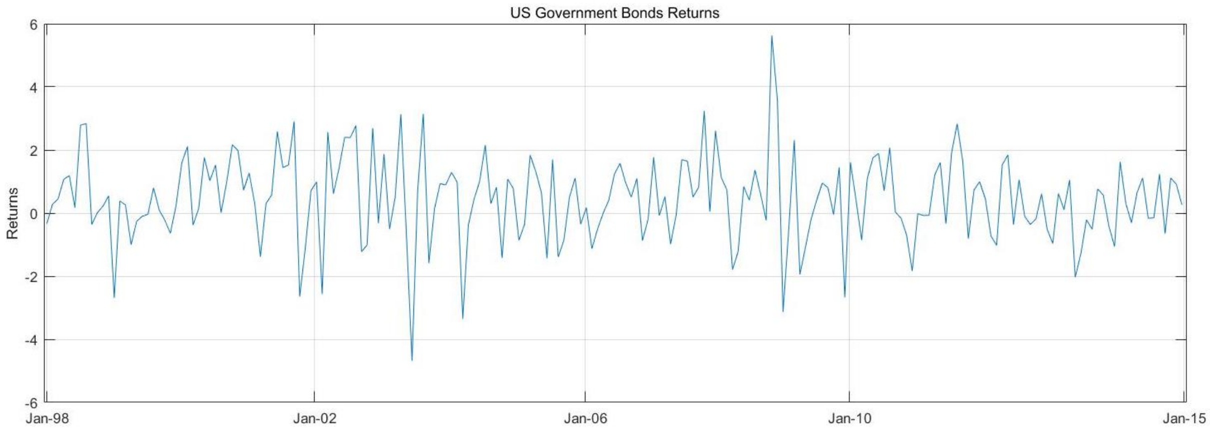


Figure 3.2: JPM United States Government Bond returns.

In the US government bonds returns nothing seems to have happened that may have destabilised the behaviour of this time series. What is interesting to see is that the return level of the government bonds is restrained. The maximum and minimum return value is, respectively, 5.63% and -4.68%. There are two anomalous values with respect to the entire time series. One extreme negative return appeared in 2004 and an anomalous positive return occurred in 2009. However, aside from these two peaks, the US government bonds returns do not show any peculiarity.

EUROPE

The JPM Europe Government Bond is the index used to analyse the government bonds behaviour in the European area in the period between 1998 and 2014. J.P. Morgan provided the index which is expressed in euro, the local currency. In order to standardise all the data, the index has been converted in US dollars. **Figure 3.3** represents the comparison between the returns calculated from the total return index in local currency and the returns obtained from the total return index in the common currency. As long as nothing changes in the performance measurement, apart from the currency in which the index is expressed, calculating the returns in local and in common currency allows to graphically understand how much of the government bond performance is due to the currency risk instead of the amount due to the real improvement or worsening of its performance.

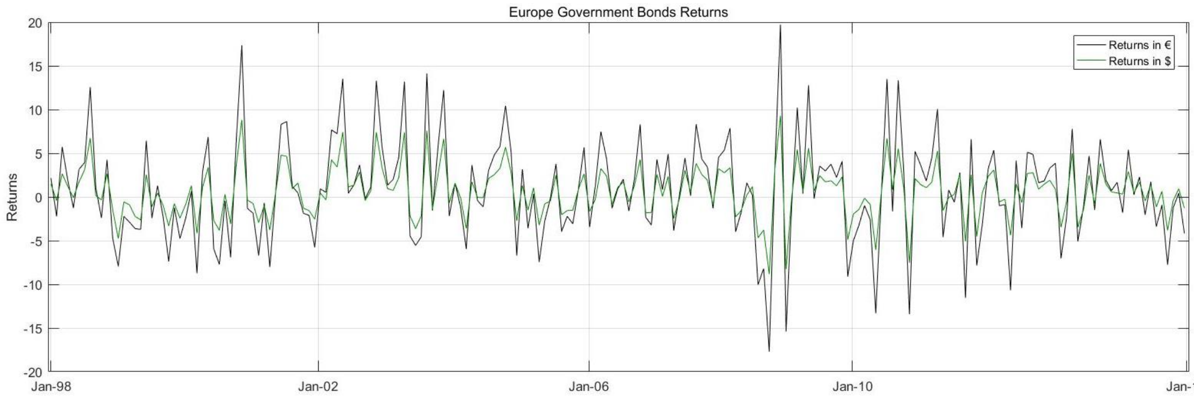


Figure 3.3: JPM Europe Government Bond returns (obtained from total return index in local, €, and common, $, currency).

Figure 3.3 shows the returns evaluated for the European government bonds total return index in € and in \$. The green line represents the time series obtained calculating the simple return of the total return index expressed in \$. The two lines show some negative and positive peaks. Looking at the negative peaks, the first one happens on the 31st of October 2008 and it is followed by other highly negative peaks (higher than -8%). All these negative points have happened during the financial crisis. On the other hand, the positive peaks (higher than 8%) are more frequent and spread almost in the entire period analysed. However, what appears evident and interesting to be examined is that the two time series do not coincide, they move in the same direction but with a different magnitude. This phenomenon happens because the two returns are calculated on the total return index obtained in a different currency. In the case of the European sovereign bonds, **Figure 3.3** shows several disparities between the returns expressed in dollar with respect to those expressed in euro. To understand how the currency risk affects the European government bonds it is sufficient to look at the difference between the two curves in **Figure 3.3**. In fact, whenever the green curve (returns obtained by calculating them on the total return index in $) is above the black one (returns of the total return index in local currency) it means that the investors' return is negatively affected by the currency risk. In particular, to better clarify this concept it has been taken into consideration the year 2010 as an example. At the beginning of the year 2008, the green line is above the black one, meaning that the exchange rate between the euro and the dollar, by the US investors' point of view, has depreciated. The consequence of this phenomenon is that US investors seem to have obtained a higher return, but in reality, their investment value is reduced. In the last semester of 2010, instead, the returns curve calculated on the total return index in dollar is below the one calculated in euro, meaning the exchange rate has appreciated. The effect that the currency risk has on US investors when they invest in foreign markets is that to reduce the real returns if the exchange rate depreciates and to increase them in the opposite case.

The currency risk for the index used in this work for the European government bond takes place in the entire time series path. This means that there is an impact on the changes in the exchange rate values for the US investors willing to invest in the European sovereign bonds. In particular, the correlation between the European sovereign bonds expressed in local currency and the exchange rate euro-dollar is about 0,0812. The positive but low correlation means that the two variables move in the same direction and the impact of the exchange rate on the sovereign bonds returns expressed in local currency is not very strong. Since to calculate the data in local currency from the data in common currency (and vice versa) the exchange rate is needed, both the correlation between the exchange rate and the observations in local currency and the one between the data in common and local currencies have to be taken into account. These parameters are exhibited in **Figure 3.4**. The low impact is confirmed also by the difference between the variance of the asset returns in local currency and the variance of the exchange rate. In this way, if this difference is positive (the variance of the asset in local currency is higher than the variance of the exchange rate) the currency risk has low impact; if the difference between the two variances is negative or equal to zero, the currency risk strongly affect the asset returns.

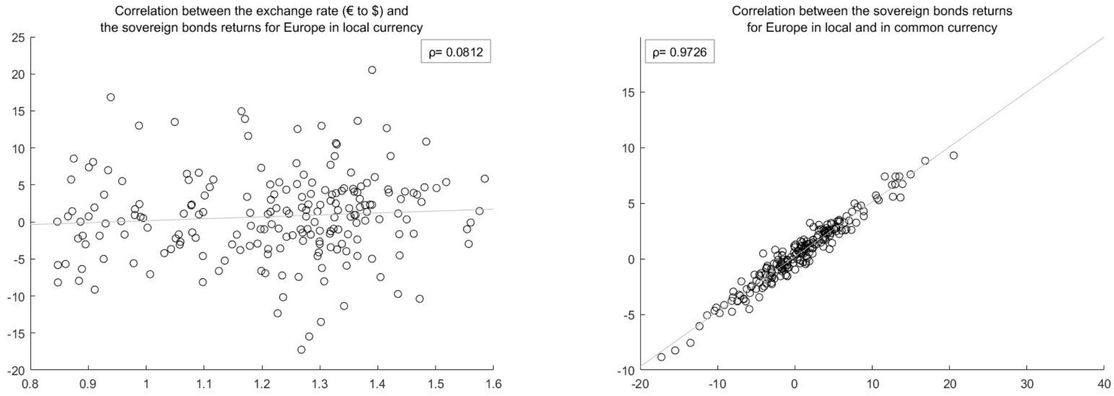


Figure 3.4: On the left, the correlation between the exchange rate and the data in local currency. On the right, the correlation between data in common and local currencies.

JAPAN

The J.P. Morgan provides the JPM Japan Government Bond Yen which is the index used for government bonds in the Japan area analysis.

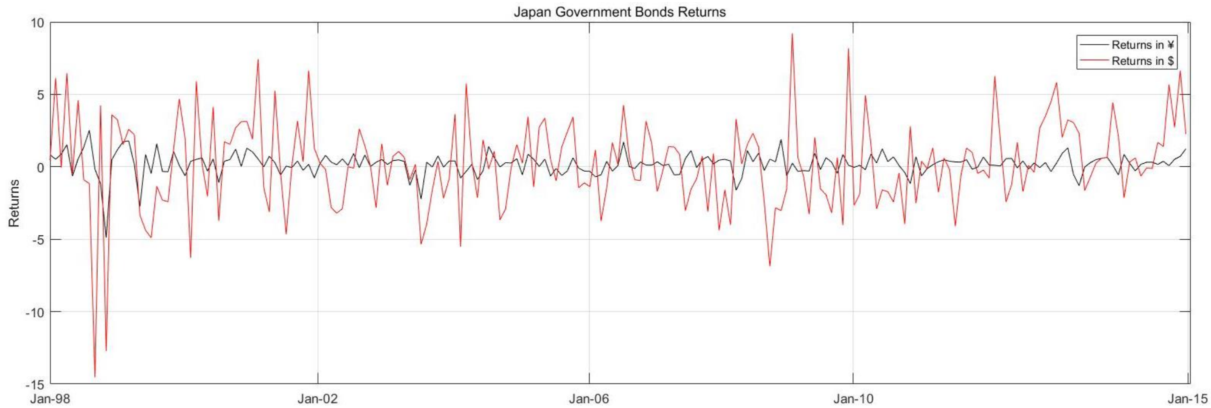


Figure 3.5: JPM Japan Government Bond returns (obtained from total return index in local, ¥, and common, $, currency).

Figure 3.5 shows a very high difference between the two lines. The returns calculated from the total return index in yen is quite stable, they do not differ much from the values between -2.74% and 2.52% with only an exception on the 31st of December 1998 of a negative value of -4.90%.

In the period after the 2009 the Japanese government and Central Bank of Japan used all the tools in their hands to stop the deflation. They tried to make government bonds as less attractive as possible by increasing the government bonds prices in order to push investors to riskier assets. The high changes in value are a confirmation of what was done and this phenomenon is more evident in the returns calculated by the data in common currency. The political decision concerning the freeze of deflation modifies the government bond returns from nominal returns to real returns, adjusted from the inflation rate.

The returns expressed in $ are less stable; they have some frequent, but not really wide, positive and/or negative peaks with respect to the returns calculated from the total return index in local currency. Hence, the currency risk is present and high in the whole period. The correlation between the exchange rate and the data expressed in local currency does not have a strong impact, it is close to zero. On the other hand, the correlation between the two time series data is stronger. The returns in common and in local currencies are positively correlated to each other. The values for the two correlations are not high, this means that the currency risk strongly affects the returns of sovereign bonds of Japan. The impact of the currency risk on the returns of the sovereign bonds is confirmed by the difference between the two variances. The variance of the assets returns in local currency is lower than the exchange rate variance.

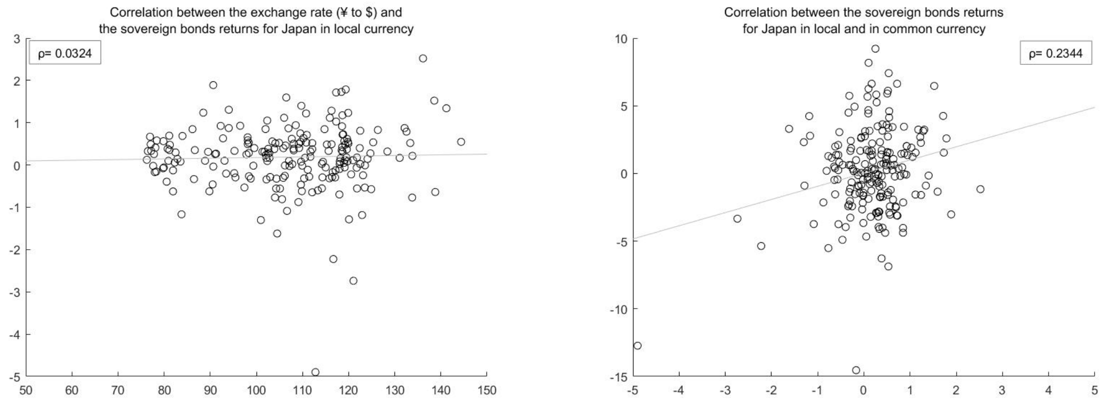


Figure 3.6: In this figure the correlation between the exchange rate and the data in local currency and the one between data in common and in local currencies are shown.

CORPORATE BONDS

Corporate bonds are securities issued by companies. In this way, the companies are able to borrow money from investors who buy their bonds. They pay back the bonds issued by reimbursing fixed or variable coupons at pre-determined intervals and the principal at the maturity. The difference between corporate bonds and stocks is that when investors buy a stock they have some ownership interest in the company, when they buy corporate bonds they buy a form of long-term debt.

The risk the investors face by buying corporate bonds is to lose some or even all the amount they have invested if the company becomes insolvent. For this reason, the corporate bonds are riskier and have higher returns than government bonds.

The Merrill Lynch Bank of America has provided the indexes used for the corporate bonds analysis. These indexes are the Bank of America Merrill Lynch United States Corporate Master for the US area and the Bank of America Merrill Lynch Euro Corporate for the Euro area. These two indexes are displayed together in **Figure 3.7**. It shows the time series concerning corporate bonds, expressed in the same common currency. Displaying them in a single figure allows seeing how the two lines move one in respect to the other. **Figure 3.7** gives evidence that the two time series move almost always in the same direction, with different intensity. In fact, it is rare that when one line moves in one direction the other goes into the opposite one. The US corporate bond returns are more concentrated near the average return value with respect to the European index. The European returns are more volatile and their trend seems to amplify the movements of the US corporate bonds returns.

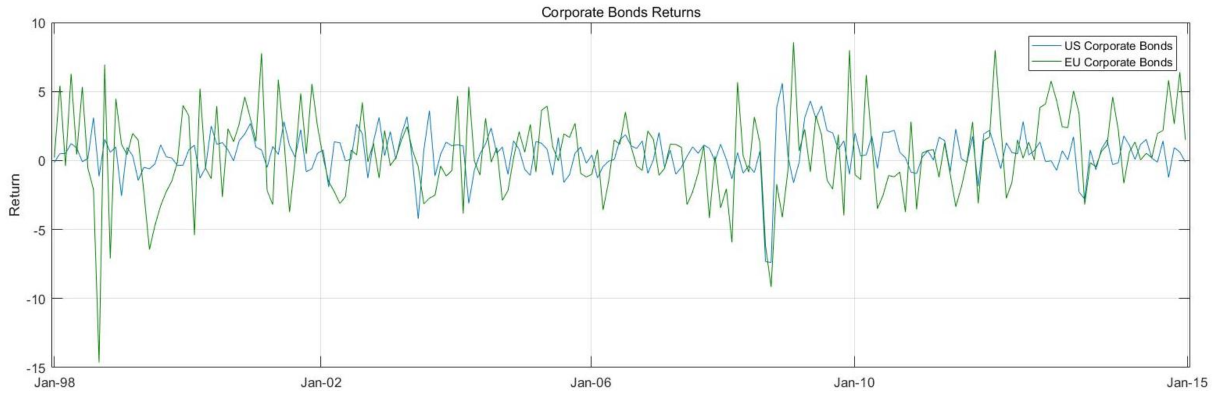


Figure 3.7: Corporate bonds returns calculated based on the common currency total return index of the two geographical areas considered for this category of bonds (US and Europe).

UNITED STATES

The returns calculated from the total return index of BofA ML US corporate bonds is shown in **Figure 3.8**. This time series exhibits that almost all return values are restricted between ±5%, with the only exception of the months of September and October 2008. In these months, the values of the returns reached their lowest levels. On the 30th of September 2008 the return is -7.32% and on the 31st of October 2008 it is -7.38%. This phenomenon confirms the effect of the financial crisis on the sector of the corporate bonds.

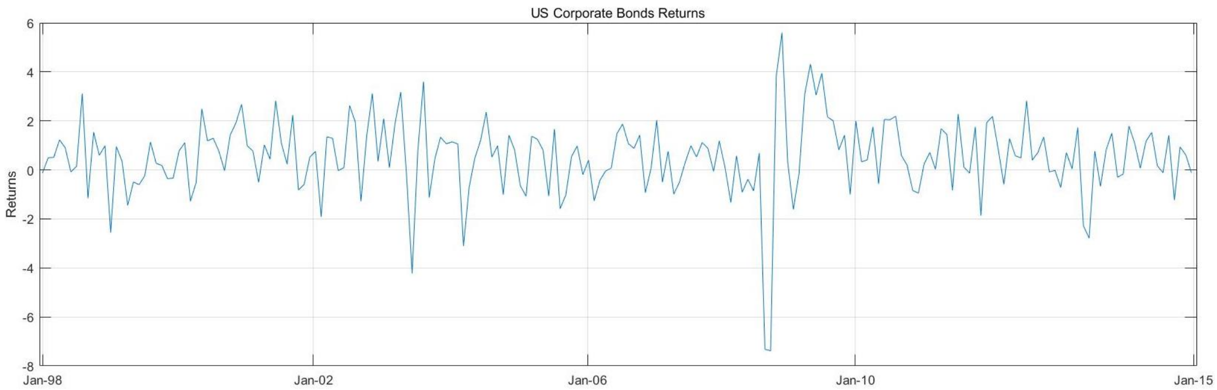


Figure 3.8: BofA ML US Corporate Master bonds returns, obtained from total return index in $.

The range in the returns levels not only verify the low returns that characterize this type of asset class, but also the low volatility.

EUROPE

The returns obtained by considering the BofA ML European corporate bond total return index are shown in **Figure 3.9**.

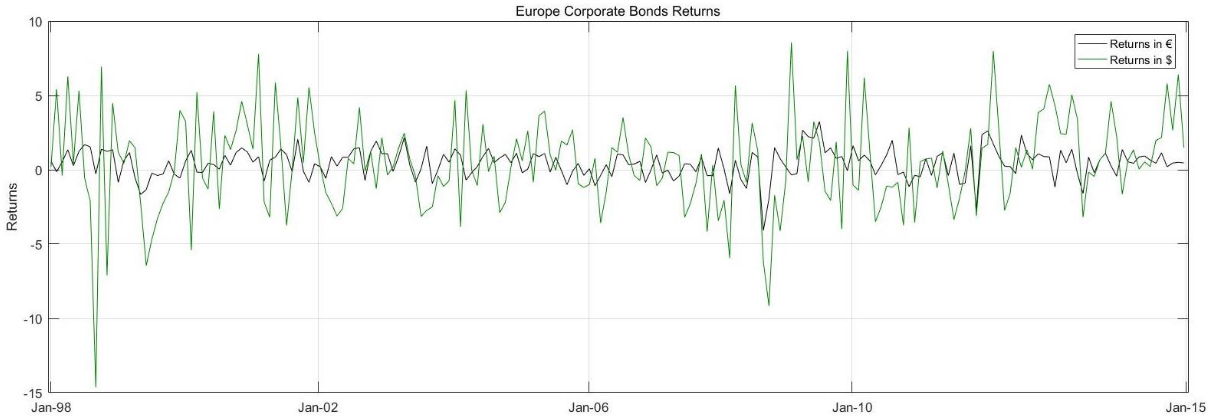


Figure 3.9: BofA ML Euro Corporate bond returns (obtained from total return index in local, €, and common, $, currency).

In **Figure 3.9**, the two time series show the returns calculated by the total returns in the common currency and the returns obtained from the total return index in the local currency. The black line exhibits a quite stable path with just one negative peak (-4.09%) that goes out of the range of ±2.5%, where all the other data are contained. On the opposite, the green line appears to be less stable and with frequent positive and negative peaks that are or not in the common range of ±5%.

Negative peaks appeared in the first years considered for the analysis and also in the period between the last trimester of 2008 and the second trimester of 2012, when the European companies suffered the most for the consequences of the financial crisis. The positive peaks are more frequent and they are quite equally spread in the whole period analysed. The highest value reached (10.37%) was on the 31st of October 2008.

What is interesting to highlight is the disparities between the two lines. The reason for this difference is noticeable in the currency risk. This risk is more evident during the periods of high and frequent changes of return levels. Both the correlation between the exchange rate and the data in local currency and the one between the data in common and local currency are positive and low. The variables do not move highly together. Then the impact of the currency risk is very strong in **Figure 3.9** and it is also proved by the negative difference between the volatility of the asset returns in local currency and the volatility of exchange rate.

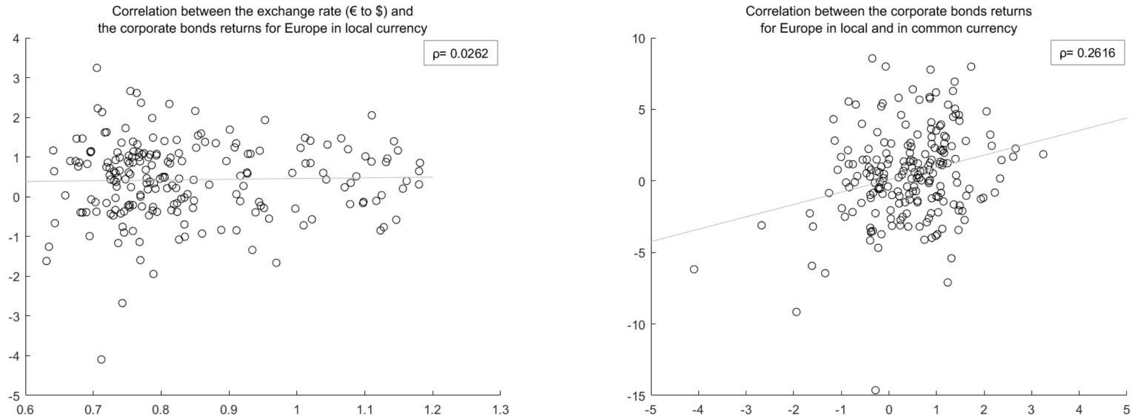


Figure 3.10: This figure shows the correlation between the exchange rate and the data in local currency and the correlation between data in common and local currencies.

3.1.3. Stocks

Stocks are also known as *equity securities* or simply *equities*. They are an ownership share of a company. In the stock market exists several type of stocks and they are classified by considering the rights they give to the investors (or shareholders) who have bought them. In this sense, *common stocks* and *preferred stocks* are the two most known typologies of stocks.

The *common stocks* give to their buyers the rights to take part in the company decisions. In fact, the shareholders may vote on corporate issues and have the opportunity to be the first to buy any new stock the company issues. The disadvantage of this privilege is that the investors are the last subjects to be paid by the company. After interests and taxes have been paid, the company managers may decide to pay dividends to the shareholders or to reinvest that part of the income in the company business. Moreover, if the company goes bankrupt, the shareholders are the last to be repaid: they come after the lenders, the employees and the lawyers.

Preferred stocks, instead, do not give any right to vote in corporate board meeting, but the investors receive pre-fixed dividends. For that reason, if the company goes bankrupt these shareholders will be paid back only after lenders. The employees, the lawyers and the common stockholders come after the preferred stockholders.

Other kinds of stocks are also adopted in investments. Some examples of these other categories are:

- *growth stocks*, they are issued by companies that are fast-growing and they are characterized by high levels of risk and high returns;
- *tech stocks*, technology companies issue these and they are high risk and returns stocks, like growth stocks;

- *small-cap*, *mid-cap* and *large-cap stocks*[28], these stocks classify the company by considering its capitalization level.

Whatever type of stocks are being considered they are very volatile in short periods, but the investors who choose to hold these assets for long periods will receive high returns and the stocks volatility is significantly reduced.

Merrill Lynch has provided the indexes used here for the equity markets' analysis. For each area, the indexes chosen to take into account both, large and mid-cap segments. The information concerning the following data has been retrieved from each index fact sheet, which contains data updated to the 31st of August 2015. The constituents included in each index are weighted by taking into account their market capitalization and they cover all the market sectors.

UNITED STATES

The MSCI USA is the index taken into account in order to obtain the historical data of the US equity market. 639 constituents compose this index which covers 85% of the free float-adjusted US market capitalization.

The figure analysed shows the past performance of the US equity market. It exhibits the high volatile path that characterised the stocks. In **Figure 3.11**, the values of the MSCI USA total return index are included in a very large range, its lowest value of -17.10% has been recorded in 2009 and its highest (10.97%) in 2011.

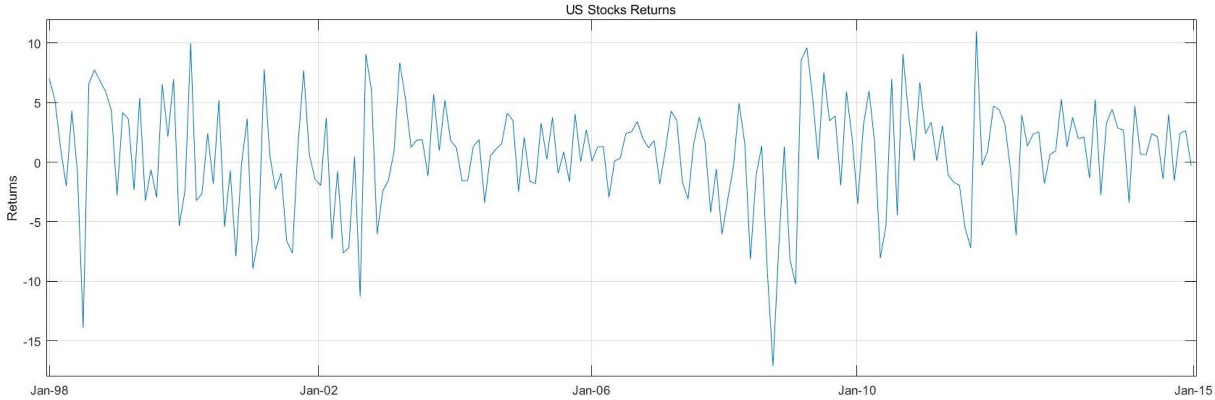


Figure 3.11: MSCI USA returns, obtained from total return index in $.

[28] Cap is the abbreviation used for capitalization.

EUROPE

The index analysed for the Euro area is the MSCI Europe. It contains 15 European developed countries[29] and covers the 85% of the free float-adjusted market capitalization across the European developed markets equity universe. The number of its constituents is 442.

The returns path calculated can be divided ideally in four parts: two parts with high levels of fluctuations and likewise parts of stable paths. The two periods in which the returns are characterised by stable paths occur during 2003 – 2007 and 2012 – until the end of the period analysed. On the opposite, in the first part (from 1998 to the end of the third trimester of 2003) the returns fluctuate inside values of ±13% which is not a really stable path. During 2007-2012, the fluctuations in returns are among the most extensive ever occurred in this figure.

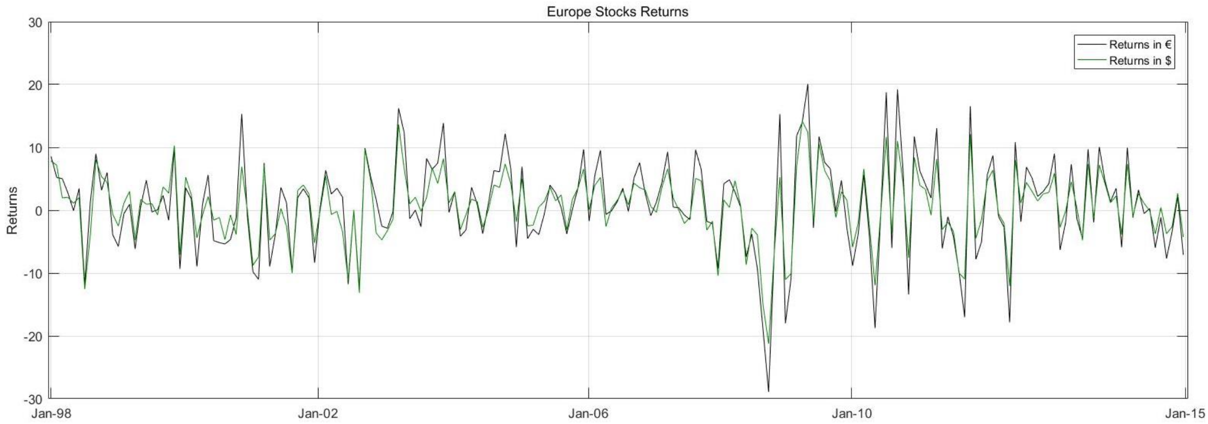


Figure 3.12: MSCI Europe returns (obtained from total return index in local, €, and common, $, currency).

The effect of the currency risk is exhibited in **Figure 3.12**, where both the returns calculated, by considering the total return index conveyed in euro and the one obtained in dollars, are displayed in the same figure. The two curves move almost always together without showing any particular differences in their paths. The co-movement is shown also in the correlation figure between the two variables. In fact, the value of the correlation coefficient is highly positive, it is very close to one; when one variable increases the other one increases with almost the same intensity. Hence, it is possible to say that the currency risk only feebly affects the returns. It is also visible by computing the variance of the returns of asset in local currency and that of the returns of the exchange rate. The first is higher than the latter meaning that the currency risk do not affect the asset returns.

[29] Austria, Belgium, Denmark, Finland, France, Germany, Ireland, Italy, the Netherlands, Norway, Portugal, Spain, Sweden, Switzerland and the UK.

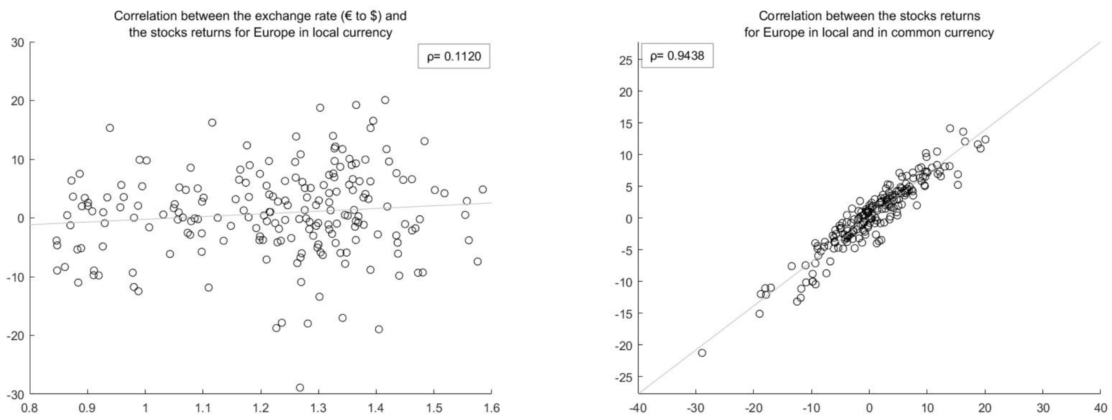


Figure 3.13: In this figure the correlation between the exchange rate and the data in local currency and the correlation between data in common and local currencies are shown.

JAPAN

The MSCI Japan Index is the index chosen to analyse the Japanese equity market. Considering 314 constituents it covers approximately 85% of the free float-adjusted Japanese market capitalization.

The returns obtained exhibit the non-stability of their path. In fact, they change in a wide range of values. Their highest value occurs on the 30th of October 1998 and the lowest one happens on the 31st of October 2008, then the extent to which the returns move is quite large.

Figure 3.14 shows the path of the returns calculated by considering both the total return index expressed in US dollars and the one expressed in the local currency, the yen. There are no huge differences between the two curves. This means that the changes in the values is affected only by the performance changes that the index suffered and not by the currency risk.

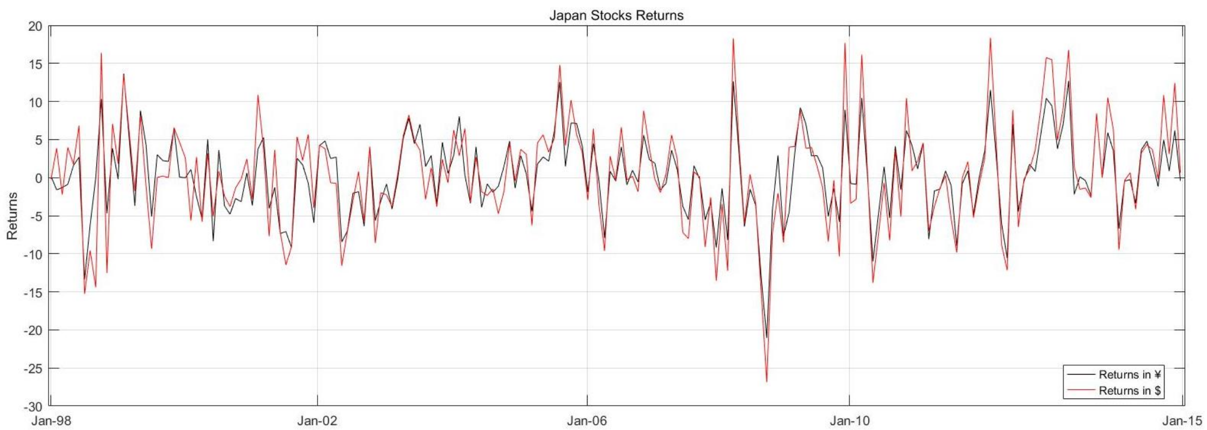


Figure 3.14: MSCI Japan returns (obtained from total return index in local, ¥, and common, $, currency).

The returns obtained in local and in common currency are almost overlapped and this behaviour is confirmed by the correlation coefficient of the two returns time series which is high and positive and also by the positive difference between the variance of the asset returns in local currency and the variation of the exchange rate. This behaviour of the correlation coefficients and of the two variances suggests the absence of the impact of the currency risk.

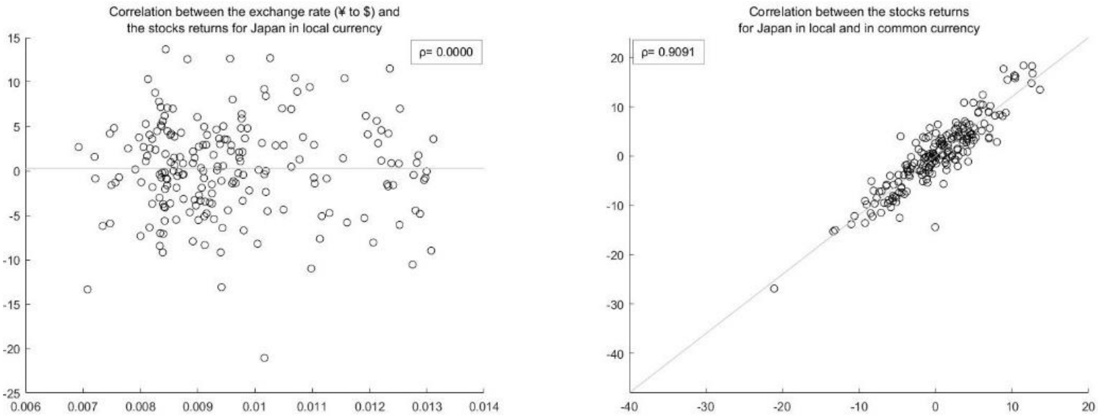


Figure 3.15: This figure represents the correlation between the exchange rate and the data in local currency and the correlation between data in common and local currencies.

By displaying together all the stocks returns time series obtained calculating the returns from the total return index expressed in common currency, it is possible to make evidence that the three lines are highly correlated one to the other. They move together, in fact, it is almost impossible to distinguish one time series curve from another.

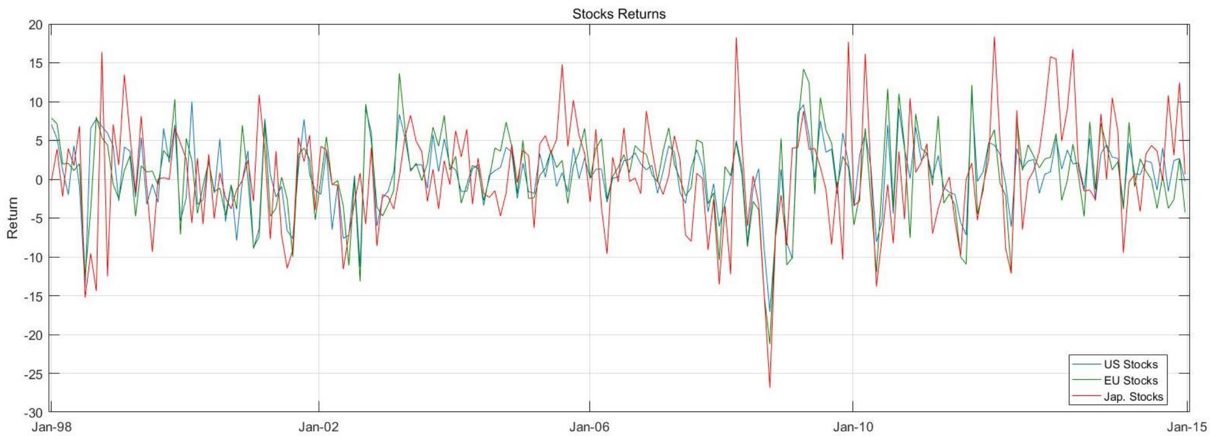


Figure 3.16: Stocks returns calculated based on the common currency total return index of the three geographical areas considered (US, Europe and Japan).

3.2. ALTERNATIVE ASSET CLASSES

The term alternative refers to all assets that can be seen as substitute with respect to common asset classes. Usually, the *alternative asset classes* are purchased in private markets and they can amplify the opportunity set of the investment. Moreover, alternative assets can be seen as long-term investments and their main characteristic could be a shortage of liquidity. This lack of liquidity implies a potentially higher return with a consequently higher level of risk.

In a global economy, traditional asset classes are increasingly linked. On the opposite, in many cases, the performance of an alternative asset is often strongly dependent on the characteristics of the individual investment instead of being highly correlated to an overall market. This lack of correlation is what makes this group of asset classes interesting because the low correlation level (sometimes closed to zero) within it and the other types of investments can help raise the diversification level of the portfolio and its returns.

In addition to the alternative asset classes that have been used and analysed in this work, there could be also other alternative asset classes such as: *natural resources*, *art*, *antiques*, *gems*, *collectibles*, *gold and precious metals* and *investments in frontier markets*. To be thorough, in what follows there is a brief definition of each.

In the category of *natural resources* there are some of that are doomed to run out, as oil, natural gas and so on, and others that can be refilled, as timber and so on. In addition, some resources may produce a profit from their normal use or they may also increase their value being used in a different way. For example, the timber produces a profit arising from the trees harvested or it may be used in real estate development creating a greater profit.

Investment in *arts, gems and collectibles* is often considered as an investment-grade because it is able to maintain its value over time, if not even increase it with the rise in inflation. Nevertheless, this type of investment generates a completely unpredictable return that can be a consequence of changes in supply and demand, market conditions and the conditions of a single piece or a whole collection.

Talking about *gold and precious metals* mean to refer to gold, silver, palladium and platinum. The choice to acquire these types of precious metals is, generally, because the investors believe those metals can be an insurance in periods of economic crisis. The idea that these metals do not change their value can be justified by the fact that in past times gold was used as a replacement for paper money and it is hedged by fluctuations in inflation. In recent periods, the gold prices have demonstrated that they are not stable and that they are very volatile with sudden rises and fallings in value.

In **Chapter 3.2.6**, the investments in emerging markets will be analysed. The *investments in frontier markets*[30] are characterized by lower levels of liquidity and market capitalizations than those in investments in emerging markets. In addition, investing in the frontier markets allows to obtain high returns with a long-term horizon and a low correlation level between investing in frontier markets and the other asset classes.

All the alternative asset classes indexes used in this work are expressed in the same currency, the US dollar. In order to create portfolios as global as possible the indexes used are the most international and complete.

3.2.1. HIGH-YIELD BONDS

The *high-yield bonds* are also known as *junk bonds*, *non-investment grade bonds* or *speculative-grade bonds*. They are bonds that are rated below the investment grades[31]. The investors who decide to invest in high-yield bonds are subject to high levels of risk of default or other adverse credit events that have to be compensated with high levels of possible returns.

Barclays Global High-Yield Index is the index analysed in order to obtain data concerning high-yield bonds. It is expressed in US dollars and it contains bonds issued by corporates and sovereigns, agencies and emerging markets local authorities rated Ba1 or BB+ or below. It represents the union of the US High Yield, the Pan-European High Yield and Emerging Markets Hard Currency High Yield Indices[32].

The high-yield returns are characterised by periods of stability and periods in which the changes in prices give back high variations among returns. In particular, a not stable returns pattern typifies the periods between 1998 and 2003. The value of returns in that period were enclosed in the range of -22.01% and 9.81%. At a later time, the returns of the high-yield bonds remained stable for a period of five years with a difference in values comprised between ±4%. The period of the financial crisis has a strong impact also in the high-yield bonds returns levels; it is during this period of frequent changes that the index recorded in just one semester its highest (13.87%) and lowest (-22.89%) returns. After 2010, the high-yield bond returns come back to their values of ±4% showing a quite stable path with just some periods of turbulences.

[30] The countries that belong to the frontier markets, according to Credit Suisse classification, are Bahrain, Bangladesh, Côte d'Ivoire, Croatia, Jordan, Kazakhstan, Kenya, Lebanon, Macedonia, Malta, Mauritius, Mongolia, Nigeria, Oman, Qatar, Serbia, Slovakia, Slovenia, Sri Lanka, Trinidad and Tobago, Tunisia and Vietnam.

[31] The investment grades are ratings that indicate the level of risk of default linked to an asset. Usually, these ratings are provided by Standard & Poor's or Moody's. In particular, the investment grades are assets that achieve a rating above the 'BBB' level, from Standard & Poor's, and above 'Baa', from Moody's.

[32] Information retrieved from Barclays Global High Yield Index fact sheet uploaded to September 2015.

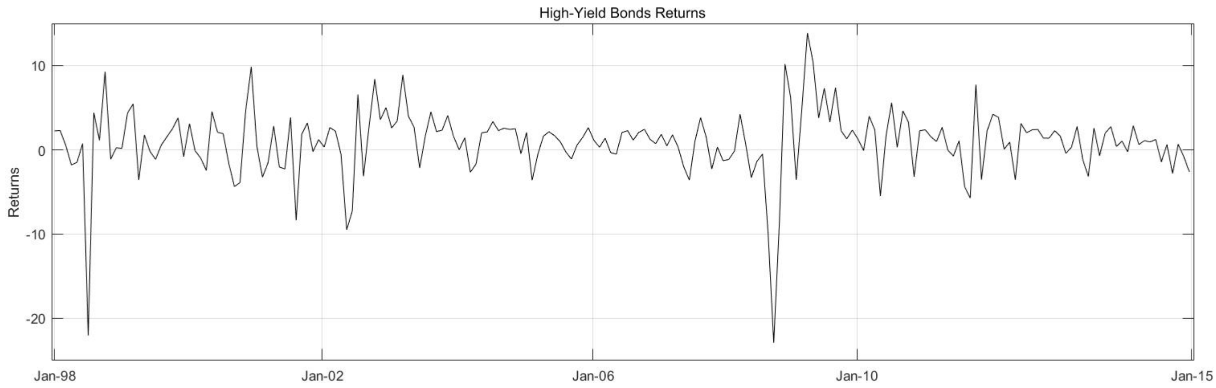


Figure 3.17: Barclays Global High Yield Index returns calculated by the total return in $ time series.

3.2.2. REAL ESTATE

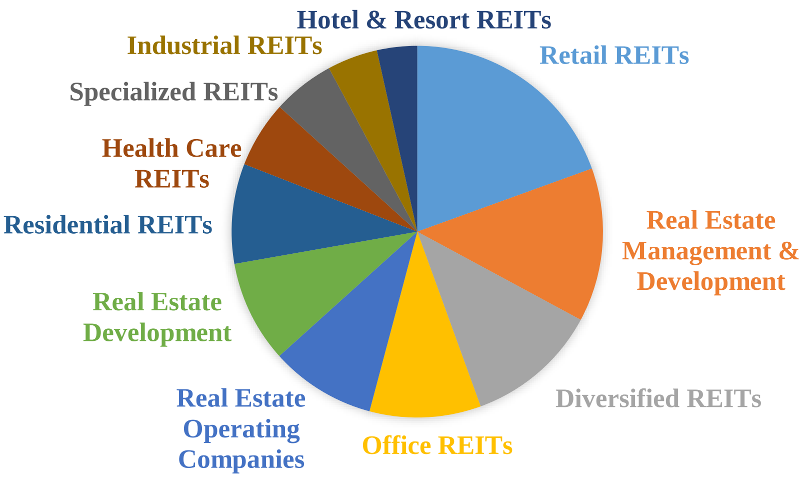
Investing in *real estate* means to consider the investments in properties. There are several types of properties and they can be defined by considering their use. In the real estate markets there can be housing or business properties, the latter can be divided in subcategories of commercial properties. In particular, these categories can be office, retail or industrial properties. It is important to clarify this point because each of the categories listed are characterised with different levels of risk and returns. The risk-return features among housing and commercial properties are the most evident and the behaviour in one of the two types of these markets does not directly affect the other. In other words, events that affect commercial property markets may not have implications in the housing markets.

In literature there are controversial ideas about including the real estate investments into the common or into the alternative asset classes. Although the majority of the authors[33] analysed agree with the opinion that real estate does not belong to alternative asset classes, in this work it has been included. The reason for this choice is that the asset classes included in the strategic portfolio, analysed as classical portfolio, are only stocks and bonds.

The Standard and Poor Corporation has provided the index used to analyse the real estate asset class. The S&P Global Property index defines and measures the total investable universe of publicly traded property companies. A number of 771 companies[34] constitutes the index considered for this alternative asset.

[33] Wilcox & Fabozzi (2013) and Anson (2006) are just some of several examples which are possible to find in the literature.

[34] This data is contained in the S&P Global Property index fact sheet, which is updated to 31st of August 2015.



Graph 3.1: A personal elaboration of S&P Global Property Index composition[35]

In the S&P Global Property index, all the companies' real estate sectors are included. In particular, the weight of each sector is shown in **Graph 3.1**, these data have been retrieved from the index fact sheet.

Figure 3.18: S&P Global Property Index returns calculated by the total return in $ time series.

The real estate trend is characterized by a stable period for most of the time. This period is followed by a slow but significant increase and then the index subsequently dropped during the years of the financial crisis. In the last part of the figure it is possible to see an increase in real estate total return index values.

The years in which the index recorded the highest decrease in its values (-27.27%) correspond with the global recession period. Real estate is one of the sectors that has been hit the most. At the beginning, the housing bubble affected only the US real estate markets but in a short time it involved the whole real estate industry.

[35] The data used in this graph have been retrieved in the S&P Global Property Index fact sheet, pp. 4.

3.2.3. HEDGE FUNDS

Hedge funds are, generally, private investment vehicles not publicly traded that manage money for institutions or wealthy subjects. Usually, they exploit several and complex tools and techniques in order to achieve returns for their investors. The main aim of a hedge fund is to create a positive return, irrespective of the market conditions. There is not a unique typology of hedge funds, they may have different risk and return performance characteristics, according to the strategy that is behind them. Some of these strategies are *equity hedged*, *event-driven*, *relative-value* and *trading*.

Equity hedged is the hedge fund strategy in which managers hold equities of a company that is considered undervalued. Hence, the equities prices are expected to rise.

Event-driven are funds that invest in companies that are going through an important corporate event (such as bankruptcy, merge, restructuring and so on). The managers of these funds are willing to take advantages of performance.

Relative value are funds that try to attain arbitrage opportunities, i.e. opportunities related to the differences in prices of assets with similar characteristics.

The last one hedge funds strategy is the *trading*. These are funds that use trading techniques to achieve excess returns.

The HFRX Global Hedge Fund Index is the index chosen to represent the hedge funds in this work. The Hedge Fund Research Inc. provides and designs this index in order to have an index representative of the overall composition in the hedge fund universe. In this index all the hedge fund strategies are comprised and they are asset weighted based on the distribution of assets in the hedge funds industry[36].

At the beginning of the period considered, after four years of instability, the hedge fund returns had a quite steady path with a huge decrease during 2008. This period is the one in which the financial crisis was triggered by the bankruptcy of Lehman Brothers and it caused a restriction on investors withdrawals and a decrease in their popularity. However, this period of depression did not last long time for the hedge funds returns level.

The highest (5.95 %) and lowest (-9.35 %) values exhibited in the hedge funds returns happened, respectively, on the 29th of February 2000 and on the 31st of October 2008.

[36] This information is contained in the HFRX Global Hedge Fund Index fact sheet (pp. 1), the version used herein is the one updated to November 2015

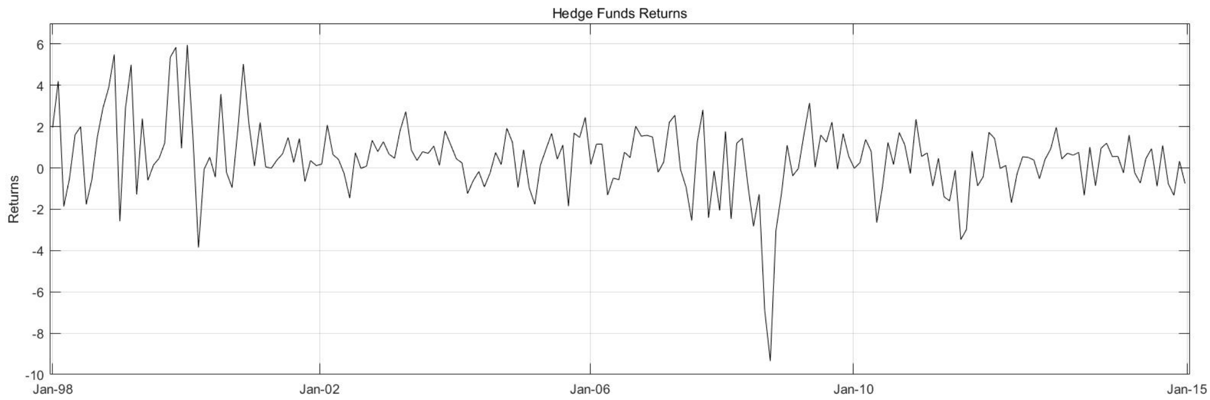


Figure 3.19: HFRX Global Hedge Fund Index returns calculated by the total return in $ time series.

3.2.4. PRIVATE EQUITIES

Private equities are investments similar to stocks because both represents the investors' ownership interest in a company. At the same time, they are different because stocks are listed and/or publicly traded and private equity assets are not. Generally, the investments in private equities require a long-term before providing rewards to their investors. Since this type of investment requires a large wealth to invest, the investors that buy these assets usually are qualified or institutional investors. Because of the huge amount of wealth necessary, investors have a significant effect on the management and have access to financial information about the company in which they invest.

There are several types of private equities which are linked to the stage of development of a company at the moment of investing. The private equity investments achieve different risk-return features according to the stage in which the company lays. In the private equity asset class, the returns take the form of a mix of dividends, capital gains on selling and exiting investments and increased business value. The major part of the risk is in the illiquid nature of private equities, they take time to become liquid. Private equity might be attractive for long-term investors. This type of investments is riskier and gives higher returns than stocks.

Some examples of the forms that private equities may adopt are: *angel investors*, *mezzanine financing capital*, *expansion or growth financing capital*, *venture capital funds* and *buyouts*.

Angel investors are investors who provide capital to start-up companies and may have a personal stake in the venture providing business expertise, industry experience and contacts as well as capital.

Mezzanine financing capital happens any time in which investors lend money to a company that is usually going through an expansions or acquisitions. Investors are rewarded with a high interest that the company pays on the loan.

If a company is expanding or growing their business, it usually needs more capital than has, so investors operate what is called an *expansion or growth financing*.

Venture capitals are investments in companies that are at the beginning of their development and are not yet producing a positive cash flow.

The last type of private equity is the *buyout*. It occurs when a private investor buys all or part of a company which is already established. Usually, the company produces a positive cash flow, but investors think that its market value is lower than its real or potential value. The potential value is the one that the company may achieve after some improvements.

The LPX50 is a global index that consists of the 50[37] largest liquid LPE companies covered by the LPX Group. It is the index chosen to represent the private equity and is mainly focused on the last three types of private equities: the buyouts (93.8%), the growth financing (5.8%) and the venture capital (0.4%).

The history of private equity is characterised by sequences of booms and decreases. The first boom happened from 1995 (even if the data analysed here start in 1998) to 2000. This increase in the return of private equities, in particular in venture capital and growth financing capital, was influenced by the arrival of internet and the new technologies. In this period, the highest level of return was reached in 2000 (31.34%). In the years from 2003 to 2007 a stable period is shown. The returns do not exhibit high levels of increases or decreases even if this period experienced a huge increase in the level of buyouts. In particular, the 87% of the largest leveraged buyout transactions in the history took place.

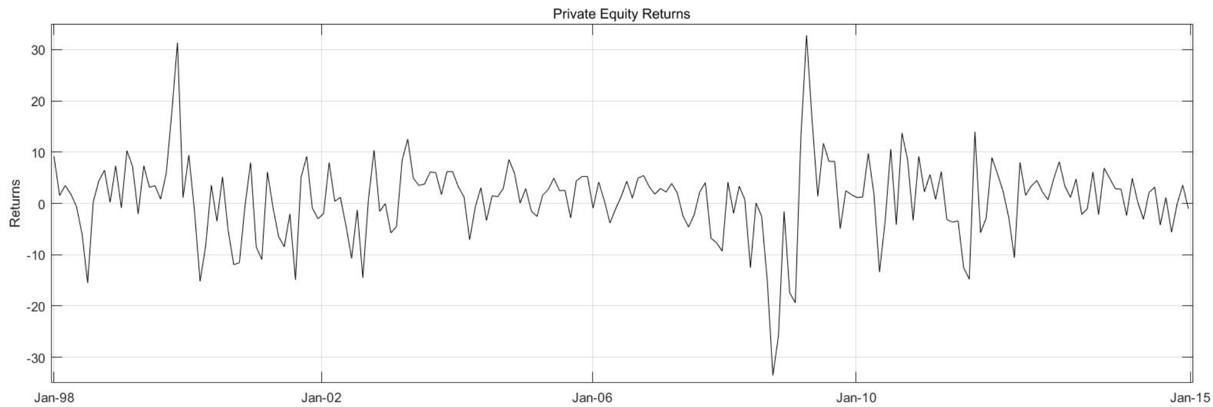


Figure 3.20: LPX50, Private Equity returns calculated by the total return in $ time series.

The lowest value (-33.57%) of the private equity returns appeared on the 31st of October 2008. This fall in values is a consequence of the subprime mortgage crisis that has strongly affected the entire financial markets.

[37] The information about the data mentioned in this paragraph has been retrieved in the LPX Group newsletter, pp. 4

3.2.5. Commodities

Commodities are physical and tangible raw materials which are used in the creation of other products creation. The main difference with respect to other type of products is that commodities are not distinguishable in any way one from the other. This type of asset class can be divided considering the sectors in which they are adopted (energy, industrial or base metals, precious metals, agricultural commodities and livestock). Basically, they can be divided into two categories: *hand* and *soft commodities*. The distinction between hand and soft commodities concerns the way in which they are obtained. In particular, the *soft commodities* are all the raw materials that are grown, such as fruits, soybeans, corn, wheat, etc. *Hand commodities*, instead, are those obtained from mines extraction.

Another important feature of the commodity asset class is that the factors that affect their prices are different from those that affect common asset classes. In fact, commodities changes in price depend on elements such as weather, geographical conditions and supply constraints. For this reason, commodities and common asset classes are not highly and frequently correlated one to another.

As long as commodities are tangible assets, it is not completely understood how to invest in this type of securities. Investors have several ways to make investments in commodity assets. One way is to buy the underlined commodity: this is the most obvious way, but also the most complicated one because of the issue concerning the storage and transportation costs. Other ways to invest in commodities can be to hold securities of a company which derives a significant portion of its revenues from the sale of commodities or to hold securities of a commodity Exchange Traded Fund which can invest in a commodity or in a group of commodities. Another more common way is to achieve commodity derivative contracts[38].

S&P GSCI Commodity Index is the index exploited to analyse commodities. This index has been chosen because it is one of the most representative of the commodities market. In fact, in this index 24 commodity futures are included. They are based on physical commodities across five sectors (energy, livestock, industrial metals, agriculture and precious metals). The S&P GSCI Commodity index weights each commodity by the world production and adjusts them for futures trading volume[39]. The world production is calculated as the average quantity of production over the last five years of available data.

The monthly returns shown in **Figure 3.21** move in a very wide range of values that goes among ±15%. The highest and lowest returns are obtained during the years of the crisis,

[38] Wilcox & Fabozzi, (2013), pp. 285-286.

[39] Information obtained from the analysis of the S&P Dow Jones Indices, S&P GSCI Commodities Index factsheet, pp. 1. The data are updated to the 30th September 2015.

respectively, on the 29th of May 2009 the value was 19.67% and on the 31st of October 2008 the value was -28.20%.

The high risk and high return levels that characterise the commodities are well shown in **Figure 3.21**.

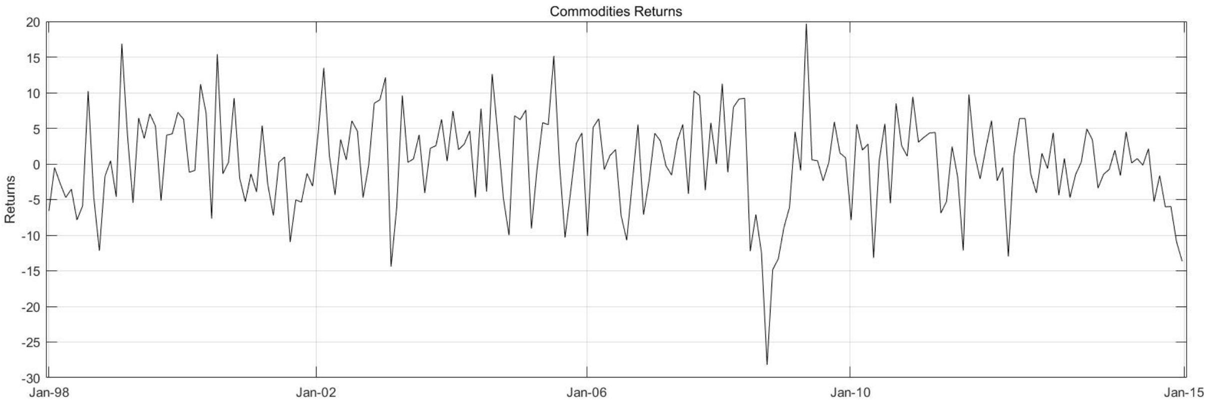


Figure 3.21: S&P GSCI Commodity Index, commodity index returns calculated by the total return in $ time series.

3.2.6. INVESTMENTS IN EMERGING MARKETS

Talking about the *emerging markets*[40] means the countries that cannot be considered developed markets, but are growing fast. Generally, the markets that are classified as emerging markets can be divided in two group: the *primary* and *secondary*. The classification of primary or secondary emerging markets is given by considering some criteria and how far or close the countries are respect than criteria. These criteria are made by taking into account the development level of the developed markets.

One of the features of emerging markets investments are high returns, usually higher than developed markets returns, and high levels of risks. This kind of asset class provides high returns because it has exposure to several risk factors such as illiquidity risk and institutional and political conditions. Moreover, the investments in emerging markets are usually low correlated with the common asset classes and so these higher returns are rewarded by the diversification benefits when the investments in emerging markets are included in a portfolio.

The index used to analyse the emerging markets is the MSCI Emerging Market Index. It is a free float-adjusted market capitalization index and it aims to measure the performance of the emerging markets equity market. It considers also large and mid-cap segments of the emerging

[40] Usually, the countries classified as emerging markets are: Argentina, Brazil, Bulgaria, Chile, China, Czech Republic, Estonia, Hungary, India, Indonesia, Latvia, Lithuania, Malaysia, Mexico, Pakistan, Peru, Philippines, Poland, Romania, Russia, South Africa, Thailand, Taiwan, Turkey, Ukraine and Venezuela.

countries. The index with its 23 emerging markets[41] and its number of constituents equal to 834[42], covers approximately 85% of the free float-adjusted market capitalization in each country considered for the index construction.

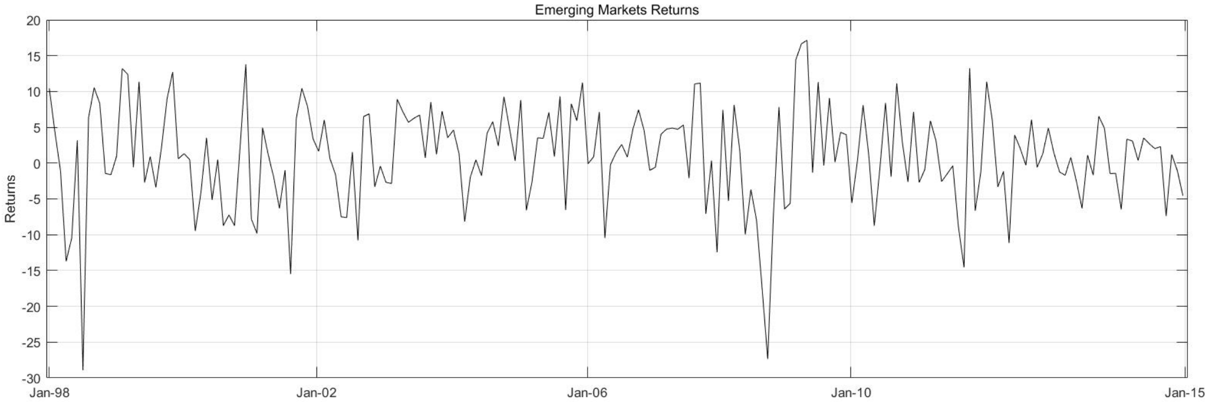


Figure 3.22: MSCI Emerging Markets Index, investments in emerging markets returns calculated by the total return in $ time series.

The returns that investors may achieve by investing in emerging markets are not stables and they are affected by frequent changes in the trend direction. The positive peaks can reach very high levels of remuneration, as happened in 2009 when the performance touched its highest peak (17.15%). In particular, **Figure 3.22** gives the impression that the two negative peaks dominate the entire return pattern. The first one (-28.91%) is the lowest return value that happened on the 31st of August 1998 and the latter one took place on the 31st of October 2008 with the value of -27.35%. The average return available for investors in emerging markets is the highest compared to those of all other asset classes considered in this work[43].

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[41] Brazil, Chile, China, Colombia, Czech Republic, Egypt, Greece, Hungary, India, Indonesia, Korea, Malaysia, Mexico, Peru, Philippines, Poland, Russia, Qatar, South Africa, Taiwan, Thailand, Turkey and United Arab Emirates

[42] Information retrieved from the MSCI Emerging Market Index factsheet. The data are updated to the 31st August 2015.

[43] The comparison between the several types of asset classes, considering their mean and standard deviation levels will be shown in **Figure 3.23**, in the following part of this chapter.

3.3. ESTIMATION OF THE MODELS INPUTS

In the previous sections (**Chapter 3.1** and **Chapter 3.2**), the different types of asset classes have been analysed separately. In the following part, a brief summary of the principal descriptive statistics that characterise each asset class has been provided and the description of the inputs for the models adopted.

The descriptive statistics analysed are shown in **Table 3.1** which will include the first four moments of the returns distribution, the Jarque-Bera statistic and the Sharpe ratio[44] of each asset class. Mean, variance, skewness and kurtosis are the analysed moments of the returns distribution.

The first moment (the *mean* or the *expected value*) measures the central tendency of the variable considered. It is obtained as $\mu_i = \sum_{j=1}^{n} \frac{R_j}{n}$, where n is the number of observations for each asset class i and the return compounded by the simple return formula is R_j.

The *variance* measures how far the data are spread out around the mean and it is obtained with the following formula: $\sigma_i^2 = \frac{1}{n} \sum_{j=1}^{n} \left(R_j - \mu_i\right)^2$. Here the variance is expressed as its square root ($\sigma_i = \sqrt{\sigma_i^2}$) that is called the *standard deviation*. The standard deviation is expressed in the same unit with respect to the mean and allows them to be compared.

The first two moments of the returns distribution are summarized in **Figure 3.23**. It exhibits the standard deviation values in the x-axis and the average returns obtained for each asset class in the y-axis. In this way, it is possible to have a look at the risk-return characteristics that each asset has in average for the whole period and to make a comparison between the positions of one asset class with respect to another one. What is expected is to find the asset classes with a low risk and low return (in particular bonds) close to the origin of the axes. On the other hand, moving away from the origin it is expected to find gradually increasing risk and return asset classes. In fact, almost all the alternative asset classes, with the exception of the hedge funds, are located in the part of the graph in which the level of risks and returns are high.

[44] To calculate the Sharpe ratio of each asset class, the risk-free asset index used is the Barclays US Treasury Index, already resented and analysed in **Chapter 3.1.1**.

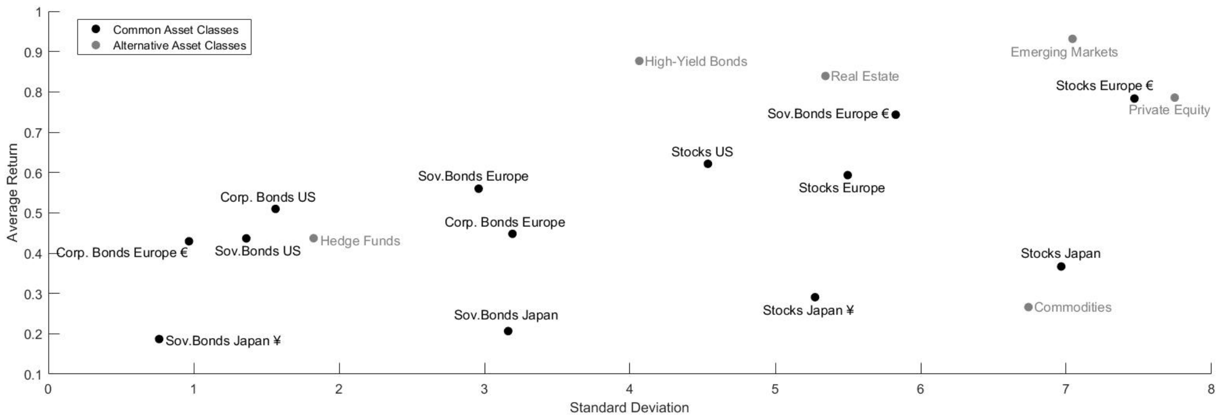


Figure 3.23: This figure summarizes the position of all asset classes considering their values in a standard deviation-average return plane.

The *skewness* is the third moment of a distribution and it is a measure of the symmetry of the data or more precisely the lack of symmetry of the data; $s_i = \frac{\frac{1}{n}\sum_{j=1}^{n}\left(R_j-\mu_i\right)^3}{(\sigma_i^2)^{\frac{3}{2}}} = \frac{\frac{1}{n}\sum_{j=1}^{n}\left(R_j-\mu_i\right)^3}{\sigma_i^3}$. In the normal distribution, the skewness value is expected to be equal to 0.

Kurtosis measures if the shape of the data distribution matches the normal distribution; $k_i = \frac{\frac{1}{n}\sum_{j=1}^{n}\left(R_j-\mu_i\right)^4}{(\sigma_i^2)^2} - 3$. If the distribution fits the Gaussian distribution, the value of the kurtosis is equal to 3. The ratio in the first part of the formula, in fact, should be equal to 0.

The *Jarque-Bera test* verifies whether the returns are distributed following the normal distribution or not. This test constructs a statistic, $jb_i = \frac{n}{6}\left(s_i^2 + \frac{(k_i-3)^2}{4}\right)$, whose value increases if the distribution considered does not fit well to the normal distribution. In particular, it sets the null hypothesis which is a joint hypothesis concerning the skewness and the excess kurtosis being equal to zero. If the null hypothesis is rejected this means that the distribution does not follow a normal distribution. The values obtained and shown in **Table 3.1** give evidence that, in most cases, the asset classes analysed do not follow a normal distribution. Since mean-variance theory and CAPM work well if the variables follow a Gaussian distribution, some problems may arise in using these kind of data. Nonetheless, since the data considered are variables described by a probability distribution which is characterized by location and scale parameters, the Meyer's study (1987) states that these two parameter conditions[45] are enough to apply the Markowitz's theorem framework.

The last parameter analysed is the *Sharpe ratio* ($sr_i = \frac{R_i-R_f}{\sigma_i^2}$). It measures the average return earned in excess of the risk-free rate per unit of volatility. Generally, by comparing two or more securities, higher the Sharpe ratio that refers to that security more preferable the security

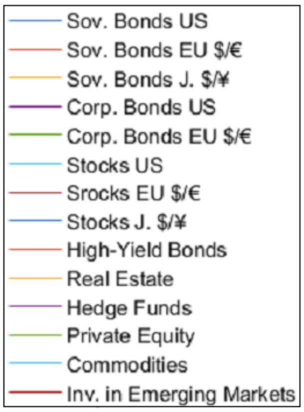
[45] Meyer J., (1987), pp. 422.

analysed. Some of the Sharpe ratios obtained are negatives, this means that these asset class returns are in average lower than the risk-free asset return.

	μ	***σ***	***sk***	***k***	***j-b***	***sr***
Sov.Bonds US	0,4365% (5,2380%)	1,3618% (4,7175%)	-0,1397	4,4599	18,6888	-0,0170 (-0,0590)
Sov.Bonds Europe	0,5600% (6,7197%)	2,9597% (10,2526%)	0,0116	3,8412	5,9894*	0,0340 (0,1177)
Sov.Bonds Europe €	0,7434% (8,9209%)	5,8278% (20,1882%)	0,1621	3,9061	7,4209*	0,0530 (0,1835)
Sov.Bonds Japan	0,2067% (2,4799%)	3,1625% (10,9553%)	-0,5066	5,8749	78,5921	-0,0802 (-0,2778)
Sov.Bonds Japan ¥	0,1869% (2,2425%)	0,7625% (2,6413%)	-1,6762	13,2045	975,8527	-0,3671 (-1,2716)
Corp.Bonds US	0,5097% (6,1166%)	1,5627% (5,4133%)	-1,0926	8,8080	325,7087	0,0321 (0,1113)
Corp.Bonds Europe	0,4480% (5,3755%)	3,1922% (11,0583%)	-0,3975	5,0973	42,5505	-0,0037 (-0,0127)
Corp.Bonds Europe €	0,4293% (5,1514%)	0,9676% (3,3519%)	-0,5787	5,0932	48,3916	-0,0064 (-0,0222)
Stocks US	0,6215% (7,4585%)	4,5364% (15,7147%)	-0,6405	3,9692	21,8258	0,0358 (0,1239)
Stocks Europe	0,5935% (7,1216%)	5,4979% (19,0453%)	-0,5397	4,1409	20,8666	0,0244 (0,0845)
Stocks Europe €	0,7835% (9,4020%)	7,4694% (25,8748%)	-0,3482	4,1210	14,7310	0,0467 (0,1618)
Stocks Japan	0,3669% (4,4029%)	6,9660% (24,1309%)	-0,0966	3,9749	8,3552*	-0,0133 (-0,0462)
Stocks Japan ¥	0,2907% (3,4881%)	5,2731% (18,2664%)	-0,2641	3,9128	9,4082*	-0,0333 (-0,1155)
High-Yield Bonds	0,8768% (10,5210%)	4,0644% (14,0794%)	-1,7612	13,2053	985,8619	0,1029 (0,3564)
Real Estate	0,8394% (10,0732%)	5,3447% (18,5144%)	-0,9281	7,2418	181,3312	0,0712 (0,2468)
Hedge Funds	0,4370% (5,2443%)	1,8248% (6,3212%)	-0,6548	8,2759	249,9476	-0,0124 (-0,0431)
Private Equity	0,7858% (9,4299%)	7,7463% (26,8339%)	-0,3057	7,0216	139,9623	0,0422 (0,1462)
Commodities	0,2661% (3,1935%)	6,7407% (23,3504%)	-0,3393	4,0730	13,6322	-0,0288 (-0,0997)
Emerging Markets	0,9315% (11,1785%)	7,0437% (24,3999%)	-0,7482	4,9242	50,2562	0,0672 (0,2327)

Table 3.1: The table contains the descriptive statistics obtained. The numbers in brackets under mean, standard deviation and Sharpe ratio are the annualized values. The annualized mean and standard deviation have been obtained by multiplying the monthly values for the number of months in one year (12), whereas, the annualized Sharpe ratio has been calculated by multiplying its value by the square root of 12. Jarque-Bera tests with * means that the test is accepted with a significance level of 1%, in all the other cases the normality distribution hypothesis is rejected.

So far, the analysis of the time series used in the portfolio construction has been done by considering simple descriptive statistics calculated for the entire available historical data period. From here, the *rolling sample moments* and the *exponential smoothing* will be calculated in order to obtain out-of-sample data and data which are purified from the so-called market noise and their trend. The following legend explains which colour is linked with each asset classes.



3.3.1. Moving Average

The *rolling sample moments*, also known as the *simple moving average* is the first method used in this work in order to forecast one step ahead data. This method also allows the cleaning of the time series from short-term fluctuations, considering only long-term trends. The peculiarity of this process is to choose a number of past observations (the so-called *rolling window*), to calculate their moments and use these estimated values as the one-step ahead values. The process is replicated from period to period, each time maintaining the same number of observations in the informative set. Every time a new observation comes into the informative set an old one drops out.

Although this method allows smoothing the time series from short-term oscillations, it may have some limits. More precisely, if the considered historical data are not centred on the mean, the obtained rolling mean lags behind the last data. In addition, the new observations that come in and the old ones that drop out the informative set can highly affect the rolling moments. Lastly, another limit of the simple moving average is that it does not consider some trends if they last less than the number of observations included into the rolling window.

In this work, the chosen rolling window is that of 5 years, 60 monthly observations. The first moment of the 61^{st} month is calculated as the average of the historical data from the 1^{st} to the 60^{th} observations; the mean of the 62^{nd} month is the average from the 2^{nd} and 61^{st} historical data, and so on. Hence, the number of components in this sample is reduced by the observations used in the rolling window[46].

Figure 3.24 shows the rolling mean calculated for each asset class. It is divided into two parts because in the previous one the asset classes considered are those expressed in the common currency and in the other part the rolling means are expressed in local currencies. In

[46] The number of observations in the initial sample was 203, then, with the rolling process, this number is changed to 143.

both situations, it appears almost clear that before January 2006 and after December 2009 (with the exception of the last year) the return trends look quite stable; on the contrary, in the period between 2006 and 2009, the pattern is characterized by increasing and decreasing periods. For this reason, in **Chapter 5** the sample will be split into three subsamples (from 2003 to 2006, from 2007 to 2009 and from 2010 to the end of the available data).

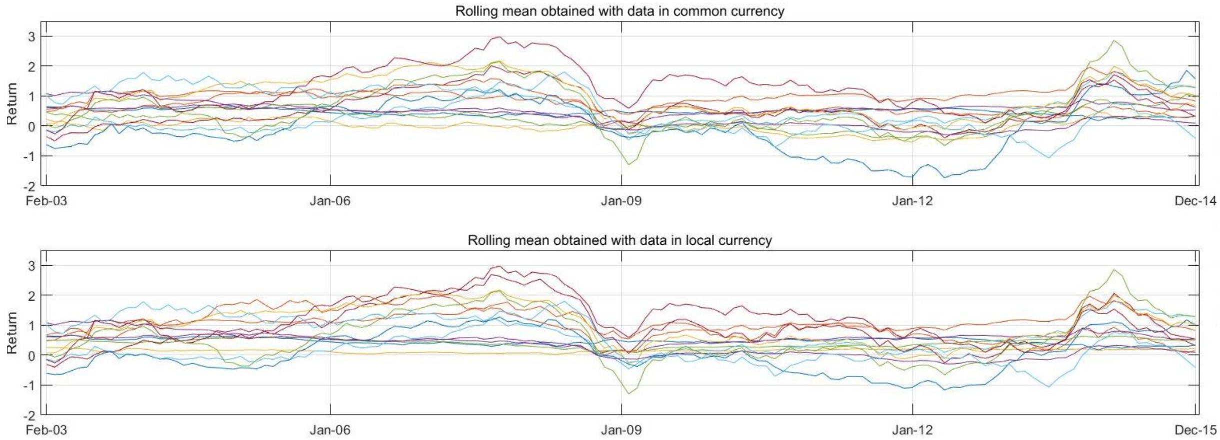


Figure 3.24: Rolling mean for the entire period considering separately common (the top part of the picture) and local currency (bottom part of the picture).

3.3.2. EXPONENTIALLY WEIGHTED MOVING AVERAGE

The *simple exponential smoothing* (or *exponentially weighted moving average*) is another method used to forecast purified data. This method differs from the simple moving average because in the latter the historical data are equally weighted, whereas, in the single exponential smoothing, the historical data are exponentially weighted. More precisely, the observations used to evaluate the one-step ahead value have exponential decreasing weights. The oldest observation considered in the informative set is exponentially weighted less than the younger one, the sum of all the weights is equal to one and the weights never reach zero. One of the peculiarities of this method is that parameter λ (the *smoothing factor* or the *decay factor*) is included in its calculation. This parameter is constant and takes values between 0 and 1. It represents the sensitivity of the EWMA statistic to the historical data. In fact, if the smoothing factor has a high value, this means that the EWMA statistic is not highly affected by the past information. Generally, the λ value is between 0.75 and 0.98, but choosing the best way to select its value is not simple because it is usually arbitrarily chosen, I have chosen to use a value of λ equal to 0.97.

The rolling window length is 5 years, as the one selected for the previous method.

This method requires a more complex calculation than the previous one. However, the result obtained gives a more precise and sensitive method to recent prices and to variations of them.

The figure below shows the first moment obtained with the exponentially weighted moving average method, calculated both for the data expressed in local and in common currency and it takes into consideration the entire sample data.

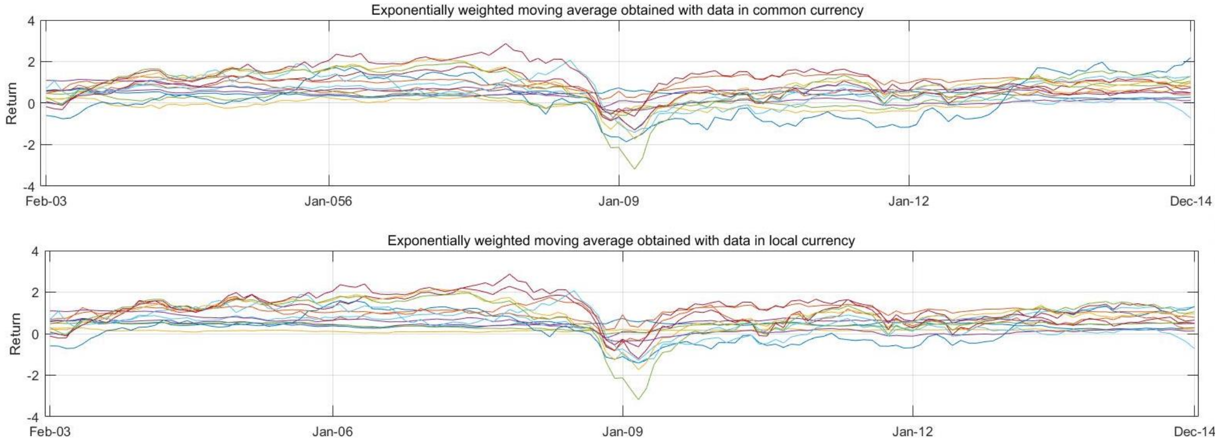


Figure 3.25: Exponentially weighted moving averages calculated in the entire period. The figure on the top shows the EWMA obtained by common currency; the part on the bottom exhibits the EWMA calculated on local currencies.

By comparing the two methods used to smooth the historical data, **Figure 3.25** exhibits a more stable behaviour than that of **Figure 3.24**. The exponential smoothing average, in fact, seems to be affected only by one shock, during the period between 2008 and 2010 (in both data, those expressed in common and those in local currency).

Even though the clearest differences between the exponential and simple moving averages have already been explained, a further clarification is needed. In particular, the rolling mean can be considered a true average value of the returns historical data, even if it reacts in a slower way than the exponential moving average.

The inputs of Markowitz's model are the mean and the variance calculated period by period, with both methods. They will be used in finding the weights of portfolios in the tactical asset allocation strategy, hence, all the necessary elements to construct the following chapters have already been easily obtained.

On the other hand, finding the Black and Litterman inputs means to obtain the expected equilibrium return from the CAPM and combining its variables with some qualitative observations about the assets (the views).

The inputs used to compute the Black-Litterman model are those mentioned in **Chapter 2.2**. More precisely, following step by step the construction of this model means to choose and combine together the quantitative and qualitative inputs.

In the formula of implied returns two ways can be followed. It is possible to calculate implied returns by using inputs that need to be calculated: the risk aversion coefficient (δ), the vectors of weights invested in each assets (ω) and the variance covariance matrix of the asset classes invested in the market portfolio Σ. Alternatively, another possible way to obtain the Black and Litterman implied returns is to use the CAPM results. In this work, the implied returns are calculated as the product between the expected market risk-premium (the excess returns between the market portfolio and the asset free-risk) and the beta of the market and the asset classes that compose the market. Since the expected market and risk-free asset returns are calculated with the rolling sample moments and with exponentially weighted moving average method, also the value of implied returns will be obtained following these two methods.

Hence, the achieved Π is exhibited in the figures below.

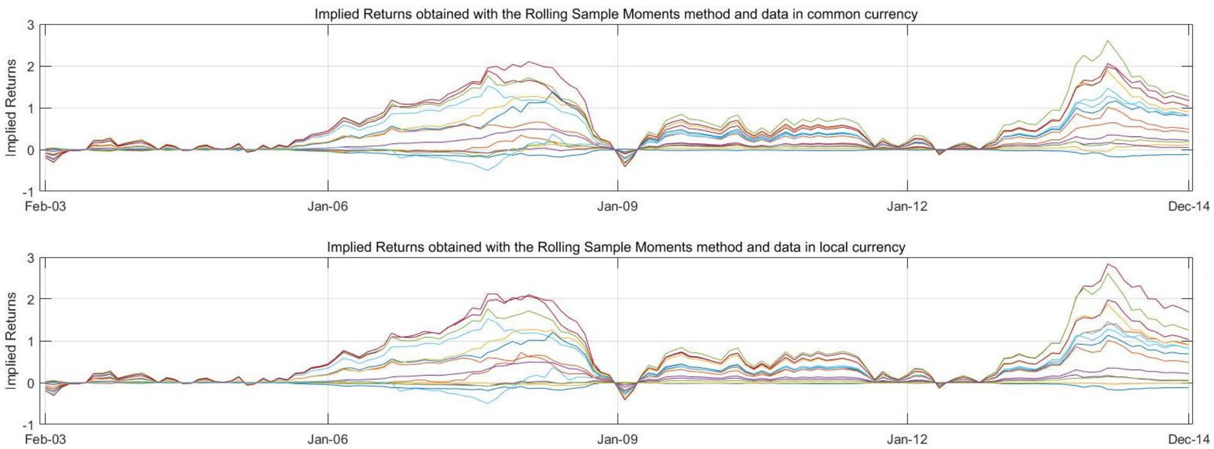


Figure 3.26: The figure exhibits the implied returns achieved by using the data obtained from the moving average method. The part of the figure on the top is where the data are expressed in common currency, the other part is the data in local currencies.

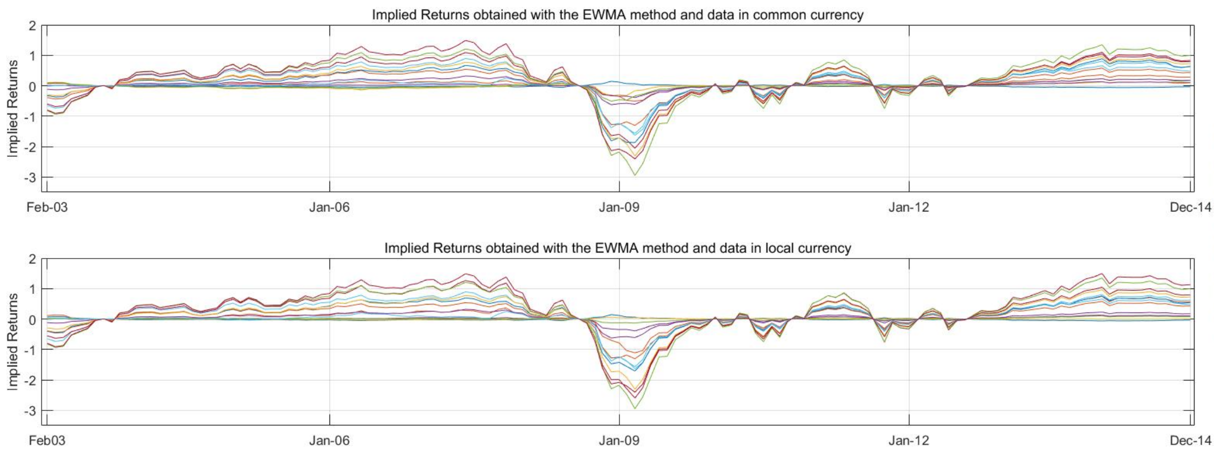


Figure 3.27: This figure shows the implied returns achieved with data calculated from the exponentially weighted moving average method. In the top part of the figure the data are expressed in common currency, in the bottom part, in local currencies.

The following step of the Black and Litterman model, in order to obtain the posterior distribution, is to achieve some qualitative observations regarding the asset classes. To construct the *P* matrix I have chosen to use a performance indicator to obtain some insight about asset classes. The indicator that I have used to construct the Black and Litterman views is the Modigliani Index. This index appeared in 1997 in a paper by Franco and Leah Modigliani, *Risk-Adjusted Performance*. The authors aim to create a performance indicator that can be easily calculated and compared from severals asset classes or portfolios. The value obtained may be used to confront portfolios or asset classes on the same unit of measure. In particular, it is the only index that allows to directly compare the value of the risk-adjusted return obtained from one asset class to the others.

In order to achieve the Black and Litterman views, I have estimated the Modigliani index each month for all the asset classes and I have calculated the difference between its maximum and its minimum monthly value. In this way I was able to state that the security with the maximum value of the Modigliani index has over performed the one with the minimum value of that index[47]. The views obtained following this method are relative views. In order to construct the *P* matrix the method followed is that chosen by Satchell S. and Scowcroft A., in their article *A Demystification of the Black-Litterman Model: Managing Quantitative and Traditional Portfolio Construction* (2000). More precisely, in the matrix of views, the asset with the maximum monthly Modigliani index takes value 1, the one with the minimum monthly value is -1 and all the others are 0. Hence, matrix *P* will be composed only by 0, 1 and -1.

The figure below shows which asset classes are more often chosen as preferred or penalized assets by following the Modigliani index criterion. In particular, the figure is divided considering the type of the data used. In other words, on the top left represents how many times each asset class has obtained the maximum Modigliani index value considering the data in common currency achieved with the simple moving average method. The figure below left exhibits how many times each asset reached the lowest value in the Modigliani index by taking into account the same type of data.

[47] For example, by considering the data presented in common currencies and calculated with the moving average method, the Modigliani index values obtained for the first monthly observation allow to state that the hedge funds (the value with the maximum Modigliani index, 1,6302) are expected to outperform the European stocks (the asset with the minimum level of Modigliani index, -0,3251) by a value of 1,9553.

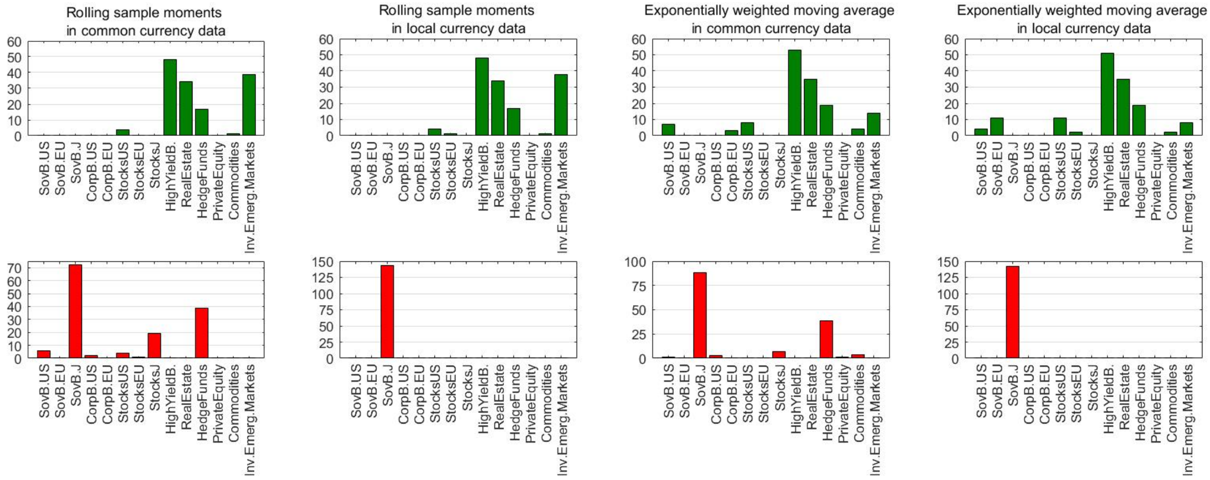


Figure 3.28: In the top part the preferred asset classes (those with the maximum value of Modigliani index) used on choosing the Black and Litterman views are shown. In the bottom part, the penalized asset classes (the ones with the minimum Modigliani index level) are exhibited.

By focalizing the attention only on two parts of the **Figure 3.28**, the penalized assets computed with local currencies by considering both the EWMA and the rolling sample moments, can be seen that in both cases the penalized asset chosen is always the Japanese sovereign bond.

Once P and Q are obtained, the last parameter which needs to be calculated to achieve the Black and Litterman mean and variance posterior distribution is Ω. It is the variance of the views and it is calculated as $\Omega = P(\tau\Sigma)P'$, where τ is arbitrarily chosen as 0,025.

Finally, the necessary parameters are specified and **Figure 3.29** and **Figure 3.30** show the first moment of the Black-Litterman posterior distribution.

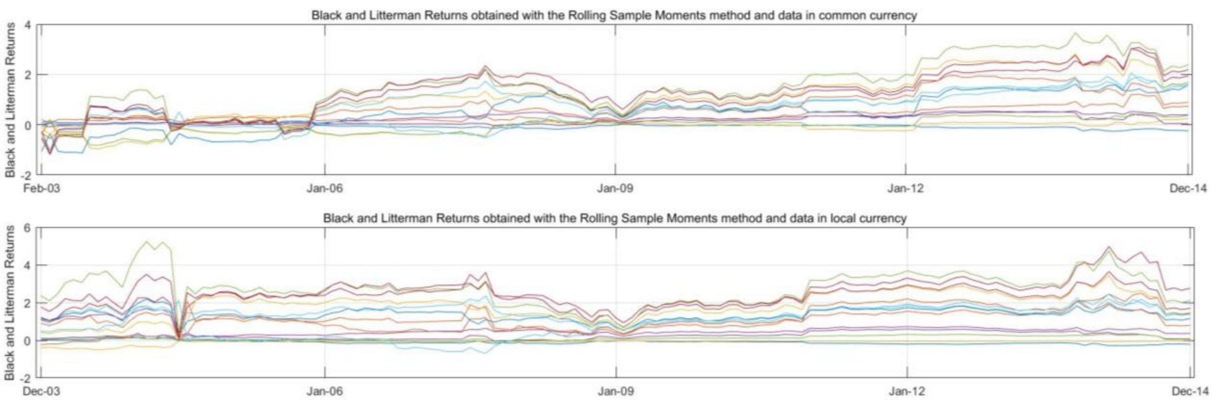


Figure 3.29: This figure exhibits the first moment of the Black and Litterman posterior distribution achieved with data calculated from the moving average method. In the top part of the figure the data are expressed in common currency and, in the bottom part, the data are in local currencies.

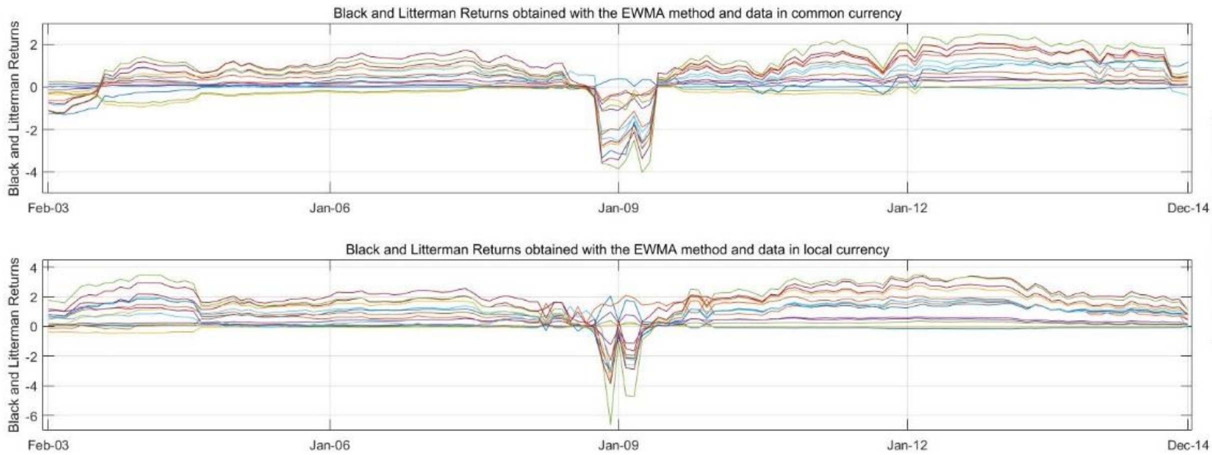


Figure 3.30: In this figure the mean of the Black and Litterman posterior distribution is shown. It has been obtained using data calculated from the exponentially weighted moving average method. In the top part of the figure the data are expressed in common currency and, in the bottom part, the data are in local currencies

In the following chapters, the Markowitz and Black and Litterman means and variances are adopted to construct portfolios considering the tactical asset allocation strategies.

4. EMPIRICAL ANALYSIS: ASSET ALLOCATION

The empirical analysis is one of the main chapters of this work, because it aims to clarify the way in which the several portfolios have been obtained. In particular, this chapter is organized by considering strategic and tactical asset allocation.

In this work, institutional investors have been chosen and, in particular, US institutional investors. This choice comes from the fact that it gives a more general view; the recent financial events, in fact, have taken origin from the US economy. Moreover, to consider US institutional investors makes sense also because the major part of the asset classes times series is expressed in US dollars.

In order to consider an active portfolio strategy, it is necessary to introduce a reference portfolio with whom the investors' portfolios have to be compared. It does not exist a benchmark constructed with the same elements chosen in this work, thus, the strategic portfolios will be used in **Chapter 5** as benchmark portfolios. Hence, the choice of the weights for each asset classes is a necessary decision in order to construct a benchmark. Generally, an institutional investor gives a higher weight to domestic securities than to non-domestic securities. By considering an US institutional investor means that US asset classes are domestic securities and all the others are non-domestic.

In the strategic asset allocation, the benchmark portfolios are constructed and they are the portfolios with which the tactical portfolios will be compared.

In each part of the following chapter, the graph for the realized returns, realized cumulative returns and realized risks calculated for each type of asset allocation will be shown. The realized return (R_p^{eff}) is obtained by computing the return of a portfolio considering the weights obtained one period before and the data return effectively achieved (provided by the historical time series data). The realized cumulative returns ($R_p^{eff\_cum}$) are a general indicator of portfolio performance. They take into account all the returns achieved by investing in the portfolio considered. This type of indicator differs from the portfolio realized returns because the realized returns do not take into account the previous and the later return obtained, each time they are reset. Whereas, the cumulative returns give evidence of the overall profit or loss that the investment has achieved. The cumulative return is a time independent measure. The last parameter analysed is the volatility of the portfolios obtained. This element is given by the realized standard deviation computed monthly for each portfolio.

In addition, in the tactical asset allocation part, the weights assigned to each asset are also shown for each portfolio calculated.

4.1. STRATEGIC ASSET ALLOCATION

Constructing a strategic portfolio implies to assign a defined weight to each asset included in it. In the strategic asset allocation part, I have constructed four types of portfolios each are calculated twice (once with data in common currency and once with data in local currencies):

1. a portfolio in which the investors' wealth is divided between only stocks and bonds (30% to stocks and 70% to bonds);
2. a portfolio in which also the hedge funds have been included (5% to hedge funds and the equity weights have been decreased for the same amount);
3. an equally-weighted portfolio in which the wealth is divided between the securities included only in stocks and in bonds;
4. another equally-weighted portfolio which differs from the previous one because in this case also the hedge funds are included.

The portfolios achieved in this part are those that will be used as benchmark portfolios.

STRATEGIC PORTFOLIOS WITH ONLY STOCKS AND BONDS

In this part, portfolios are constructed by including only common asset classes. Four types of portfolios have been calculated. In particular, two of them are portfolios calculated in common and in local currency and whose weights have been assigned to the asset classes in this proportion:

- sovereign bonds: 40% to US, 15% to Europe and 5% to Japan;
- corporate bonds: 5% to US and 5% to Europe;
- stocks: 15% to US, 10% to Europe and 5% to Japan.

The other two portfolios obtained in this part are common and local currency equally-weighted portfolios. Since in this part of the analysis it is assumed that only stocks and bonds are included, the weight assigned to each asset class is $\omega_i = \frac{1}{n}$, where n, in this case, is the number of common asset classes considered, which is equal to eight[48]. Therefore, the weight assigned to each asset in this portfolio is equal to 12,5%.

Figure 4.1 shows the results of the portfolio construction by considering only stocks and bonds. The left part of this figure exhibits the realized returns obtained in the equally-weighted portfolio and in the one whose weights are 70%-30% in bonds and stocks with both data in

[48] The number is obtained by considering all the common asset classes in each geographical area. In this work it has been considered: 3 sovereign bonds indexes, 2 corporate bonds indexes and 3 stocks indexes. Hence, the weight assign to the total sovereign, corporate bonds and stocks are, respectively equal to: 37,5%, 25% and 37,5%.

common and local currencies. The two curves do not show any peculiar differences in the returns for the portfolios calculated with the two methods to assign weights. The portfolios obtained by considering data in local currencies recorded high levels of returns when they are positive and no differences in values when the returns are negatives.

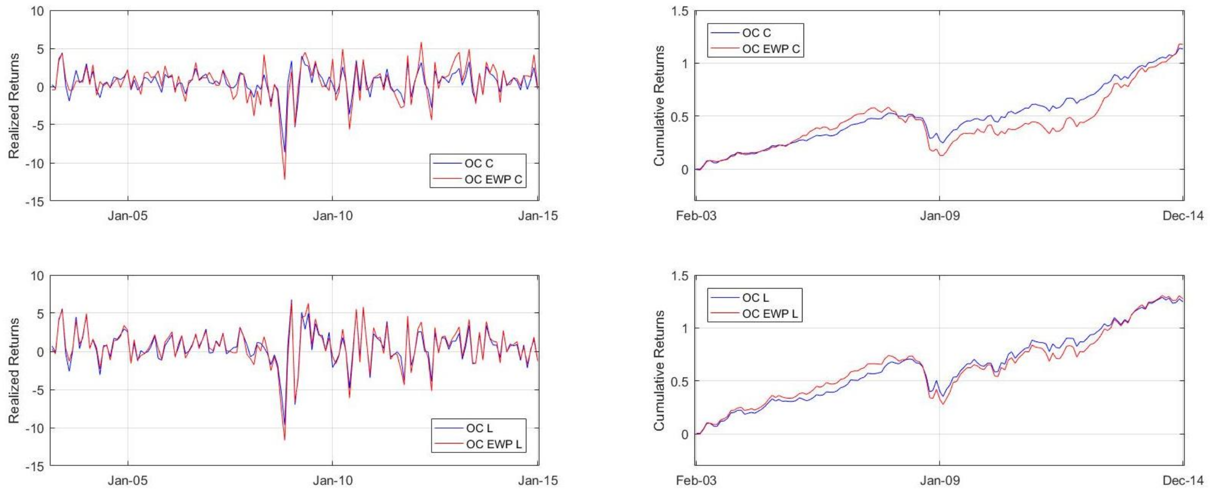


Figure 4.1: Realized returns and realized cumulative returns for the portfolios obtained by including only stocks and bonds.

On the right side of **Figure 4.1** the realized cumulative returns are depicted. In the entire figure, the returns suffer a strong decrease in their values during the period of the global financial crisis.

The realized standard deviation achieved for these portfolios is depicted in **Figure 4.2**. It shows a high increase in volatility during the years of the global financial crisis for both portfolios in common and in local currencies. Another high positive peak is exhibited in the last part of the year 2014.

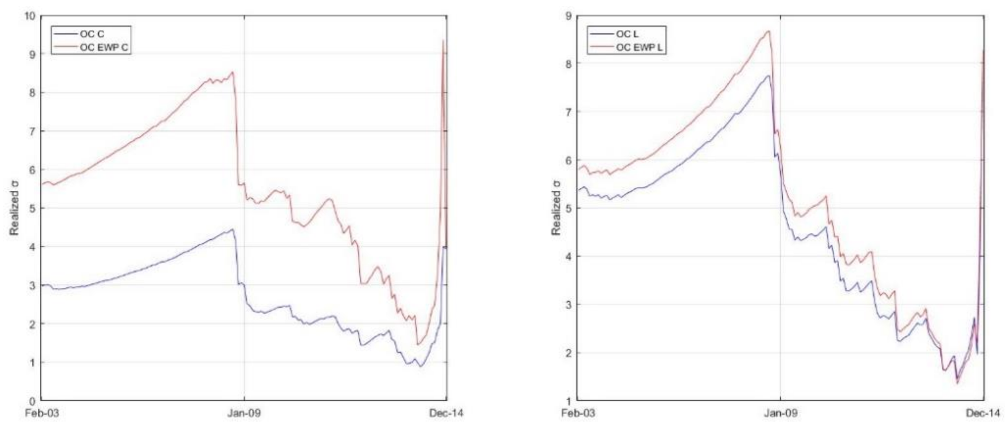


Figure 4.2: In this figure, the realized standard deviation for the portfolios constructed with only stocks and bonds are presented and the figure is divided by considering the data in common currency on the left and the data in local currencies on the right.

STRATEGIC PORTFOLIOS WITH STOCKS, BONDS AND HEDGE FUNDS

This group of portfolios has been constructed by considering the way in which pension funds usually construct their portfolios. Generally, this type of institutional investors includes hedge funds in their portfolio.

In order to insert this asset class, the equity weight has been reduced by 5% which is the amount of hedge funds included. The percentage of wealth put into sovereign and corporate bonds does not change, but the stocks weights become 13% on US, 8% on European, 4% on Japanese equities and the remaining 5% is invested in hedge funds. The other way in which the portfolios have been built in this part was by considering the investors wealth equally divided in each asset examined. Since the assets taken into account now is nine, the percentage amount of wealth that has to be invested in each asset is $11,\overline{11}\%$[49].

The results obtained in the realized cumulative returns figure show in almost all the cumulative return pattern the predominance of the portfolio constructed by giving a predetermined weight to each asset with respect to the equally-weighted portfolio. This tendency is presented by considering the data in common currency as well as considering those in local currencies.

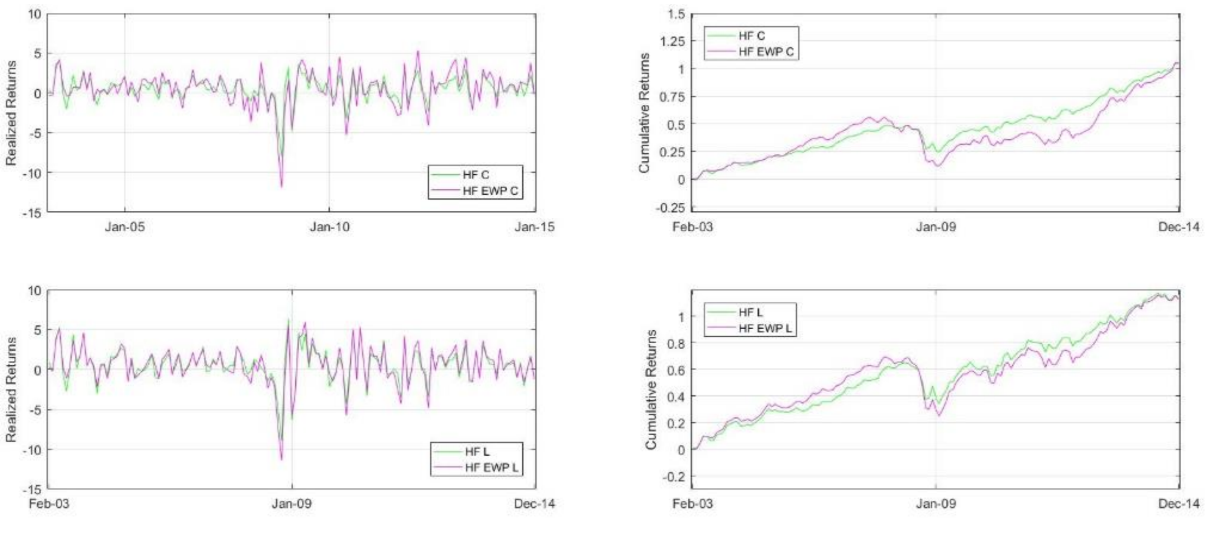


Figure 4.3: Realized returns and realized cumulative returns for portfolios obtained by considering stocks, bonds and hedge funds.

The realized returns are close to each other without showing any strong difference between the two types of portfolios.

[49] In this part of the work, in order to build the equally-weighted portfolio, the weights assigned to the sovereign bonds, corporate bonds, stocks and hedge funds are, respectively equal to: $33,\overline{3}\%$, 22,2%, $33,\overline{3}\%$ and $11,\overline{1}\%$.

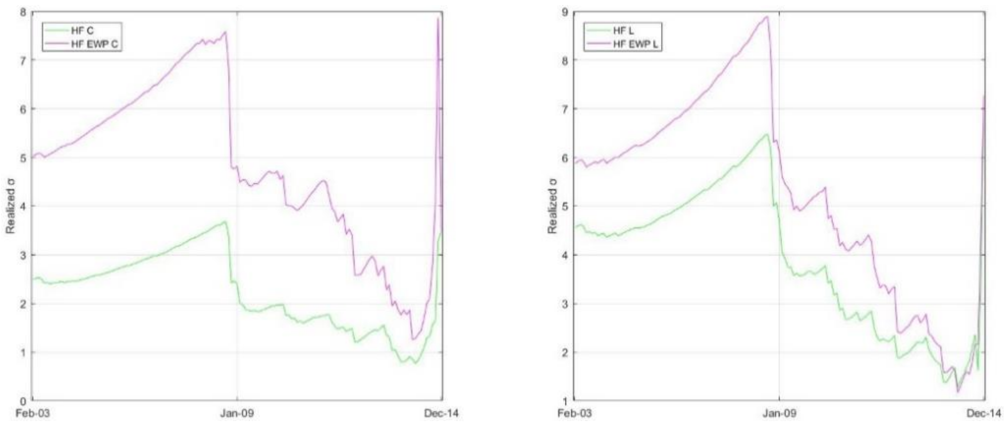


Figure 4.4: In this figure, the realized standard deviation for the portfolios constructed with stocks, bonds and hedge funds are presented. The figure is divided horizontally in two parts: on the left the data in common currency and on the right the data in local currencies.

The realized volatility depicted in **Figure 4.4** shows lower values of the standard deviation for non-equally-weighted portfolios than the volatility values of equally-weighted portfolios. This is true for both portfolios in local and in common currencies. There are no particular differences in the trends exhibited in **Figure 4.4** with respect to those in **Figure 4.2**: high increasing peaks from the beginning of the period analysed until the year 2009 and at the end of the year 2014.

So far the strategic portfolios have been built and now they have to be managed in a tactical way. In the following part of this chapter, the eight portfolios obtained will be managed by considering two ranges for making tactical choices.

4.2. TACTICAL ASSET ALLOCATION

In the previous part of this chapter, only some asset classes have been considered, instead, in the tactical asset allocation all the asset classes described in **Chapter 3** have been taken into account. In order to manage the strategic portfolios with the tactical asset allocation, some decisions about the width in which the strategic weights may vary have to be considered. Since any global regulation concerning constraints adopted by institutional investors does not exist, these constraints have been chosen arbitrarily. I have tried to take some reasonable and coherent decisions considering the way in which the institutional investors usually allocate their portfolios. In particular, I have decided that some types of asset classes (sovereign bonds and equities) can never reach a too low value (they never go under a weight of 10%) and in addition, I have decided to calculate two types of ranges: a deep and a narrow range.

In the narrow type, I have assumed that the asset weights may vary in a range of 10% with respect to the strategic asset allocation weights. In this way, I have taken into account the point of view of investors that do not like to change too much with respect to the strategic asset allocation portfolio composition. Probably, the investors that do not want too many transaction costs may prefer this strategy than the following one because the first may require to adopt less frequent variations in the assets weights than the second strategy. The frequent changes in the portfolio composition causes higher transaction costs than maintaining almost the same asset in almost the same proportion for the entire time horizon considered.

The choices made to construct the narrow range take into account that usually institutional investors do not make their decisions by considering the asset classes in their totalities. In other words, by looking at sovereign bonds, US institutional investors would never want that their domestic sovereign bonds arrive at zero. Hence, the division between the securities that compose asset classes such as sovereign bonds and stocks has been considered. The corporate bonds, instead, have been considered in their totality because they already have little values. When the equally-weighted portfolios have been constructed, the weights assigned to each asset class are considered without dividing the asset classes in the securities in which they are composed. The minimum and maximum weights inside which each asset can vary by considering this first type of tactical choices is shown in **Table 4.1**.

<table>
<tr><th></th><th></th><th>$\boldsymbol{\omega^{OC}_{str.}}$
$\boldsymbol{(\omega^{HF}_{str.})}$</th><th>$\boldsymbol{\omega^{OC}_{tac.1}}$
$\boldsymbol{\omega^{HF}_{tac.1}}$</th><th></th><th>$\boldsymbol{equal.\,\omega^{OC}_{str.}}$
$\boldsymbol{(equal.\,\omega^{HF}_{str.})}$</th><th>$\boldsymbol{(equal.\,\omega^{OC}_{tac.1})}$
$\boldsymbol{(equal.\,\omega^{HF}_{tac.1})}$</th></tr>
<tr><td rowspan="6">Sov.Bonds</td><td rowspan="2">US</td><td>40%</td><td>35%-45%</td><td rowspan="6">Sov.Bonds</td><td rowspan="3">37,5%</td><td rowspan="3">32,5%-42,5%</td></tr>
<tr><td>(40%)</td><td>(35%-45%)</td></tr>
<tr><td rowspan="2">Europe</td><td>15%</td><td>10%-20%</td></tr>
<tr><td>(15%)</td><td>(10%-20%)</td><td rowspan="3">$(33,\bar{3}\%)$</td><td rowspan="3">$(28,\bar{3}\%$-$38,\bar{3}\%)$</td></tr>
<tr><td rowspan="2">Japan</td><td>5%</td><td>0%-10%</td></tr>
<tr><td>(5%)</td><td>(0%-10%)</td></tr>
<tr><td colspan="2" rowspan="2">Corp.Bonds</td><td>10%</td><td>5%-15%</td><td rowspan="2">Corp.Bonds</td><td>25%</td><td>20%-30%</td></tr>
<tr><td>(10%)</td><td>(5%-15%)</td><td>(22,2%)</td><td>(17,2%-27,2%)</td></tr>
<tr><td rowspan="6">Stocks</td><td rowspan="2">US</td><td>15%</td><td>10%-20%</td><td rowspan="6">Stocks</td><td rowspan="3">37,5%</td><td rowspan="3">32,5%-42,5%</td></tr>
<tr><td>(13%)</td><td>(8%-18%)</td></tr>
<tr><td rowspan="2">Europe</td><td>10%</td><td>5%-15%</td></tr>
<tr><td>(8%)</td><td>(3%-13%)</td><td rowspan="3">$(33,\bar{3}\%)$</td><td rowspan="3">$(28,\bar{3}\%$-$38,\bar{3}\%)$</td></tr>
<tr><td rowspan="2">Japan</td><td>5%</td><td>0%-10%</td></tr>
<tr><td>(4%)</td><td>(0%-9%)</td></tr>
<tr><td colspan="2" rowspan="2">Hedge Funds</td><td></td><td></td><td rowspan="2">Hedge Funds</td><td></td><td></td></tr>
<tr><td>(5%)</td><td>(0%-10%)</td><td>$(11,\bar{1}\%)$</td><td>$(6,\bar{1}\%$-$16,\bar{1}\%)$</td></tr>
</table>

Table 4.1: The table shows the weights adopted in the strategic asset allocation part with ($\omega^{HF}_{str.}$) and without ($\omega^{OC}_{str.}$) the inclusion of the hedge funds and the maximum and minimum feasible weights chosen in the first range of the tactical asset allocation ($\omega^{OC}_{tac.1}$ and $\omega^{HF}_{tac.1}$ are the weights used in the composition for, respectively, the portfolio with only common asset classes and the one with also the hedge funds). The $equal.\,\omega$ indicates the weights used in the equally-weighted portfolio construction.

The second range was considered for those investors that are more willing to follow a better return without taking into account too much the transaction costs and the level of changes in their portfolio composition. In this range, a level of 30% has been considered. To construct this deep range, the only asset class in which each security was considered is the sovereign bonds. This is because, since the values of equities cannot drop to zero, to distinguish the weights of one geographical area to the others is not possible. For this reason, the equities have to be considered in their totality. In addition, the condition that the weights of the equities have never to have a value lower than 10% is satisfied. In the construction of this range the equally-weighted portfolios have been obtained by looking at the asset classes in their totalities.

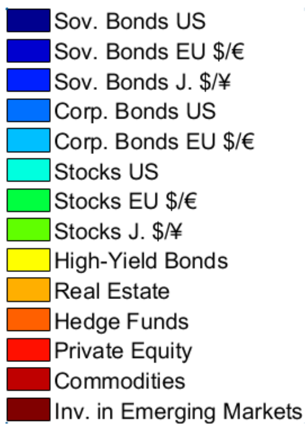
		$\boldsymbol{\omega^{OC}_{str.}}$ $\boldsymbol{(\omega^{HF}_{str.})}$	$\boldsymbol{\omega^{OC}_{tac.2}}$ $\boldsymbol{\omega^{HF}_{tac.2}}$		$\boldsymbol{equal.\omega^{OC}_{str.}}$ $\boldsymbol{(equal.\omega^{HF}_{str.})}$	$\boldsymbol{(equal.\omega^{OC}_{tac.2})}$ $\boldsymbol{(equal.\omega^{HF}_{tac.2})}$
Sov.Bonds	***US***	40% (40%)	25%-55% (25%-55%)	***Sov.Bonds***	37,5%	22,5%-52,5%
	Europe	15% (15%)	0%-30% (0%-30%)			
	Japan	5% (5%)	0%-20% (0%-20%)		$(33,\bar{3}\%)$	$(18,\bar{3}\%\text{-}48,\bar{3}\%)$
Corp.Bonds		10% (10%)	0%-25% (0-25%)	***Corp.Bonds***	25% (22,2%)	10%-40% (7,2%-37,2%)
Stocks		30% (25%)	15%-45% (10%-40%)	***Stocks***	37,5% $(33,\bar{3}\%)$	22,5%-52,5% $(18,\bar{3}\%\text{-}48,\bar{3}\%)$
Hedge Funds		(5%)	(0%-20%)	***Hedge Funds***	$(11,\bar{1}\%)$	$(0\%\text{-}26,\bar{1}\%)$

Table 4.2: The table shows the weights adopted in the strategic asset allocation part with ($\omega^{HF}_{str.}$) and without ($\omega^{OC}_{str.}$) the inclusion of the hedge funds and the maximum and minimum feasible weights chosen in this second range variation for the tactical part ($\omega^{OC}_{tac.}$ and $\omega^{HF}_{tac.}$ are the weights used in the composition for, respectively, the portfolio with only common asset classes and the one with also the hedge funds). The $equal.\omega$ indicates the weights used in the equally-weighted portfolio construction.

The optimal portfolio constructed in this tactical part results from the optimization process which tries to maintain the risk on the same level with respect to the strategic portfolio which has the same characteristics (in terms of method to create the inputs, data in local or in common currency and so on). This decision has been made in order to maintain as similar as possible in terms of risk the strategic and the tactical portfolios. Specifically, after having found all the feasible portfolios with the Markowitz and the Black and Litterman models, the one that satisfies the condition to have about the same level of risk as the strategic portfolio was chosen.

Furthermore, the choice of considering Markowitz and the Black and Litterman models to construct the portfolios comes from the fact that it is easier to understand the real effect of the tactical choices without being influenced by the limits that one model may have. In other words, if by comparing two or more portfolios which are obtained with these two models result in the same preferred portfolio, this issue can highlight the prevalence of the real portfolio performance with respect to everything else. Hence, the following part of this chapter will be divided by considering the portfolios constructed with the Markowitz and the Black and Litterman model. In each of these two models, portfolios are constructed on the basis of the strategic portfolios previously obtained and also basing on the fact that the deepest or the narrowest range has been considered. Therefore, 64 portfolios have been obtained, 32 for Markowitz and the same amount for Black and Litterman model. This high number of resulting portfolios is due to the fact that each strategic portfolio has been calculated twice with the two methods analysed to achieve the data (the moving average and the exponentially-weighted moving average method) and twice again with the two variation ranges taken into account.

The realized returns, the realized cumulative returns, the realized volatility and the different allocation of the assets are presented for each portfolio constructed. The asset allocation explains how each asset class has been included in each portfolio. It is the amount of wealth (in a percentage point of view) that has been included in the portfolio considered in order to obtain the returns and the cumulative returns. The assets included in the following portfolios are those examined in **Chapter 3** and the following legend explains which colour is linked with each asset class.



4.2.1. MARKOWITZ PORTFOLIOS

The portfolios obtained in this part of the chapter are those in which the inputs of mean and variance have been found in **Chapter 3.3** and have been used and combined with the standard Markowitz optimisation process. These resulting portfolios take their origin from previous portfolios and the weights that compose them are monthly rebalanced by considering not only the asset classes included in the strategic portfolios but all the asset classes.

Figure 4.5 and **Figure 4.7** exhibit the results obtained from strategic portfolios with only common asset classes in common and local currency and combined by considering the narrow range variation. The part on the left of **Figure 4.5** shows higher negative returns than those of the strategic portfolios constructed in the same way. This seems to mean that strategic portfolios are preferable than those achieved in this part. On the other hand, by looking at the realized cumulative returns, on the right of the same figure, the tactical portfolios reach higher values than strategic portfolios. The firsts have cumulative return values which are almost double with respect to the latter by considering both the data expressed in common currency and those in local currencies. The only exceptions for the tactical choices made with respect to portfolio obtained by considering EWMA data and a non-trivially approach to assign weights. This portfolio with both data in local and in common currencies reach the same value in the cumulative returns as the strategic portfolio not equally-weighted.

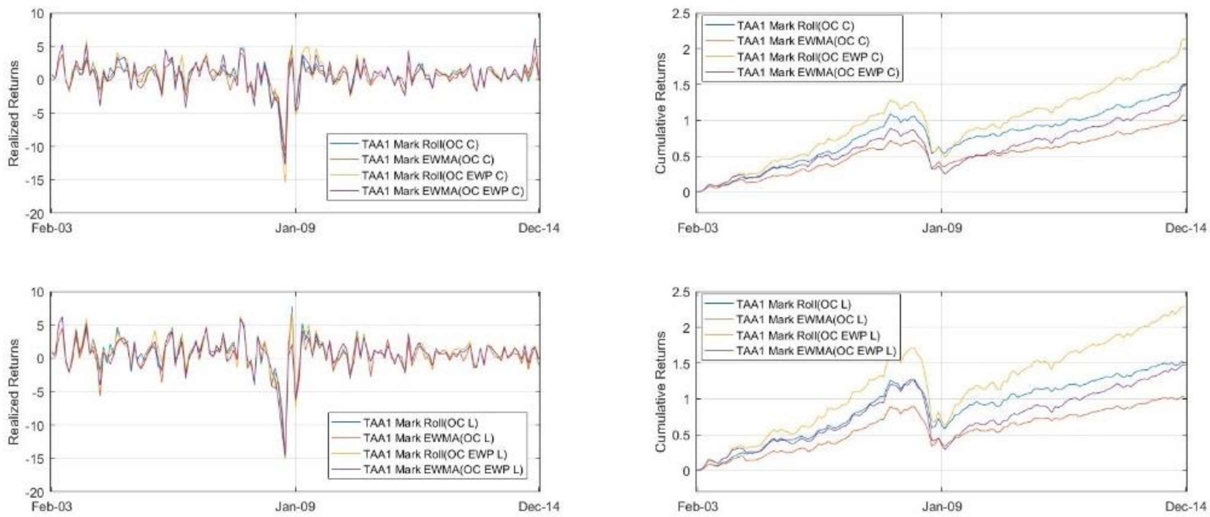


Figure 4.5: The realized returns (on the left) and the realized cumulative returns (on the right) are shown. The tactical asset allocation is based on the narrow range considered and on the strategic portfolios in which have been made tactical decisions. These strategic portfolios are those constructed with only stocks and bonds. The data examined are in common currency (on the top) and in local currencies (on the bottom).

Since the portfolios obtained in this part have levels of risk close to those of the strategic portfolios, it seems reasonable to not expect a lot of differences. However, it is not always possible that the two types of asset allocation strategies may replicate the same risk levels.

The realized volatility pattern for the portfolios achieved by considering only stocks and bonds strategic portfolios does not differ too much from the strategic portfolios on which the tactical choices are made. Despite that, the level of the standard deviation realized by portfolios managed in the tactical way is lower than that of the strategic portfolios. This happens both for portfolios calculated with data in common currency and for those with data in local currencies.

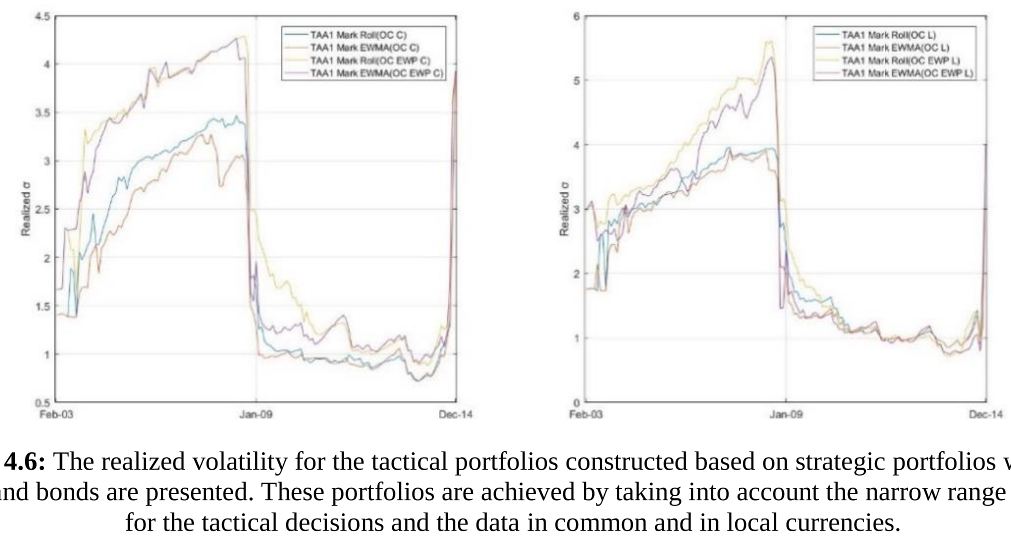


Figure 4.6: The realized volatility for the tactical portfolios constructed based on strategic portfolios with only stocks and bonds are presented. These portfolios are achieved by taking into account the narrow range variation for the tactical decisions and the data in common and in local currencies.

Figure 4.7 explains which asset classes have been included in the tactical decisions concerning the rebalancing of the strategic portfolios. Almost all the asset allocations shown in this figure do not combine different types of alternative asset classes at the same time. By looking at the kinds of portfolios achieved, it is evident that the portfolios constructed by using the EWMA and those equally-weighted (with both rolling sample and simple exponential smoothing methods) decrease the inclusion of the alternative asset classes during and, in some cases, after the global financial crisis.

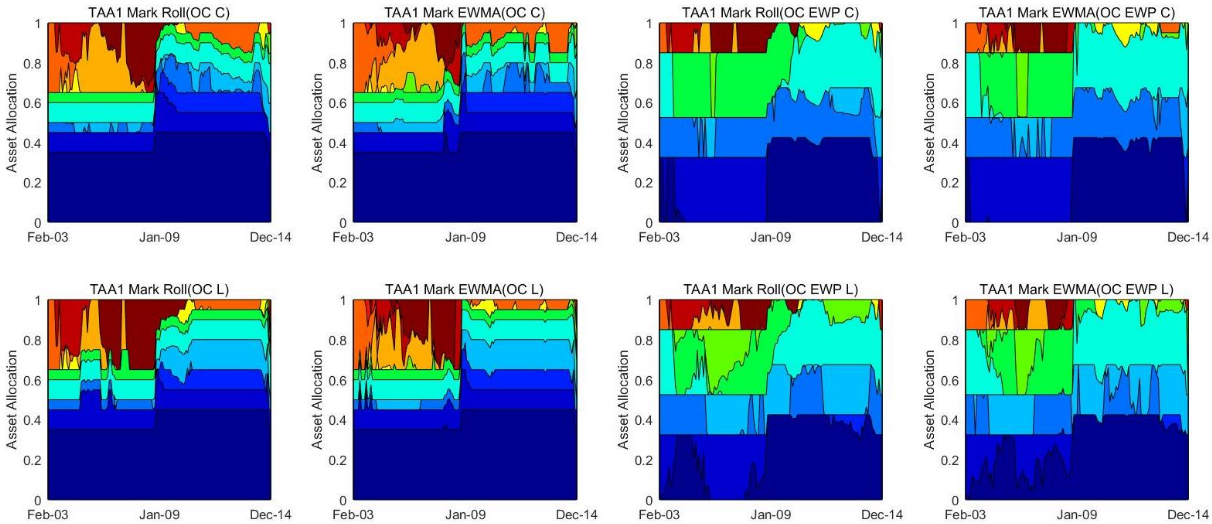


Figure 4.7: In this figure, the composition of the tactical portfolio is depicted.

The next portfolios examined are those that take their origins from the strategic portfolio with stocks, bonds and hedge funds. The figures from **Figure 4.8** to **Figure 4.10** show the results of the tactical choices made from the narrow range variation of the weights and from the forecasted inputs with the rolling sample moments and simple exponential smoothing methods. The realized cumulative returns obtained in these portfolios are higher than those of the strategic portfolios from which they take their origin. The only portfolios which has a cumulative return close to that of the strategic portfolio is the tactical portfolio achieved with data calculated by the EWMA and whose weights are not equally assigned. This happens to portfolios obtained with both data in common and in local currencies. The realized returns exhibited on the left of **Figure 4.8** have similar values than those of the strategic portfolios.

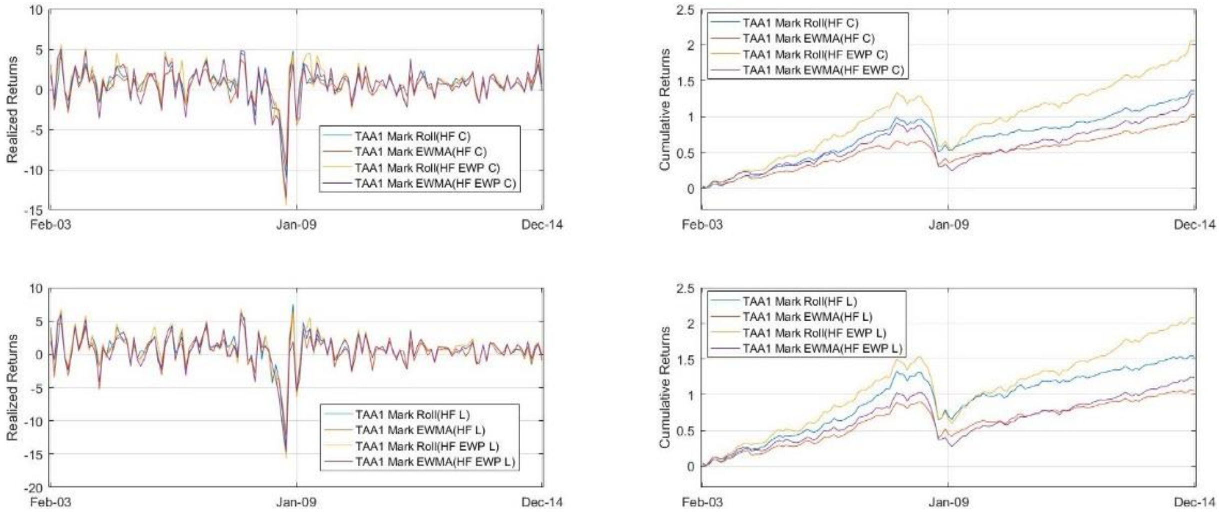


Figure 4.8: The realized returns (on the left) and the realized cumulative returns (on the right) are shown. The tactical asset allocation is based on the narrow range considered and on the strategic portfolios in which have been made tactical decisions. These strategic portfolios are those constructed with stocks, bonds and hedge funds. The data examined are in common currency (on the top) and in local currencies (on the bottom).

The standard deviation realized for the tactical decisions on strategic portfolios with stock, bonds and hedge funds has a lower value than the strategic portfolios from which the tactical asset allocation takes its origin. This is true for portfolios obtained with both data in local and in common currencies. Only at the end of the period considered, the portfolios with data in common currency has a higher volatility than the strategic portfolios.

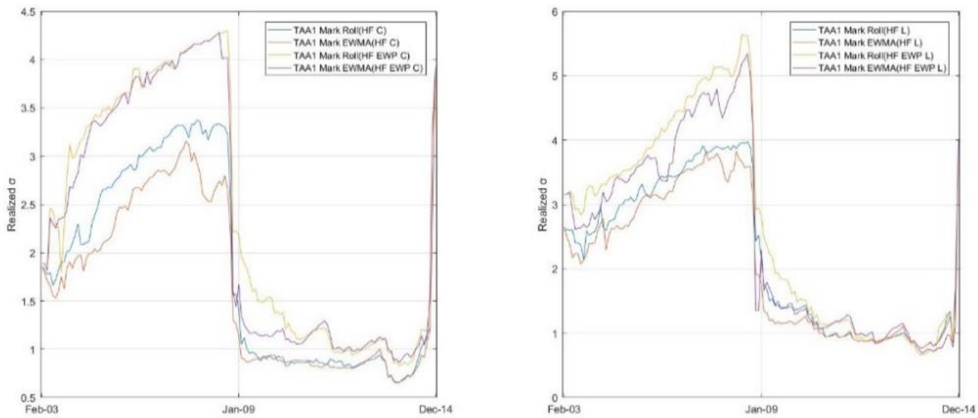


Figure 4.9: The realized volatility for the tactical portfolios constructed based on strategic portfolios with stocks, bonds and hedge funds are presented. These portfolios are achieved by taking into account the narrow range variation for the tactical decisions and the data in common and in local currencies.

The asset allocations shown in **Figure 4.10** highlight high variations in the weights of the asset classes during the entire period analysed and a constant presence of the alternative asset classes in each portfolio and in almost all the period.

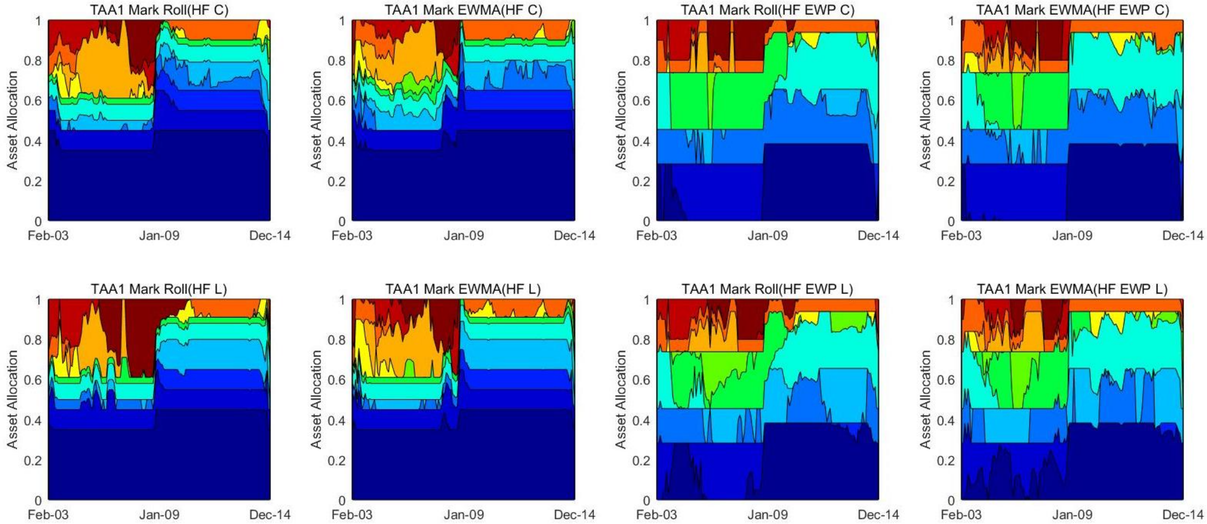


Figure 4.10: In this figure, the composition of the tactical portfolio is depicted.

From here on, the deep range variation for the weights of the asset classes has been considered. What is expected to be found is a higher number of rebalancing in the asset allocation and higher returns and volatility than in the previous part.

The portfolios analysed in **Figure 4.11**, **Figure 4.12** and **Figure 4.13** are those in which tactical choices have been made considering strategic portfolios in which only stocks and bonds were included. The realized cumulative returns of the portfolios in common currency obtained in the deep range do not achieve different values one to the other, all of these have a final value higher than 1.8. The final cumulative values for portfolios obtained with the same conditions in the narrow range were more spread out and, apart from the one calculated with rolling sample moments and obtained based on the equally-weighted strategic portfolio, were lower than these portfolios. The cumulative returns of portfolios in local currencies result in a higher value than the cumulative returns of the portfolios in local currencies achieved in the narrow range part. The realized returns in common currency of tactical portfolios do not differ in terms of values and of pattern with respect to the portfolios achieved by adopting weights that vary among the narrow range. The realized returns in local currencies, instead, have in average higher values during the end of the year 2008.

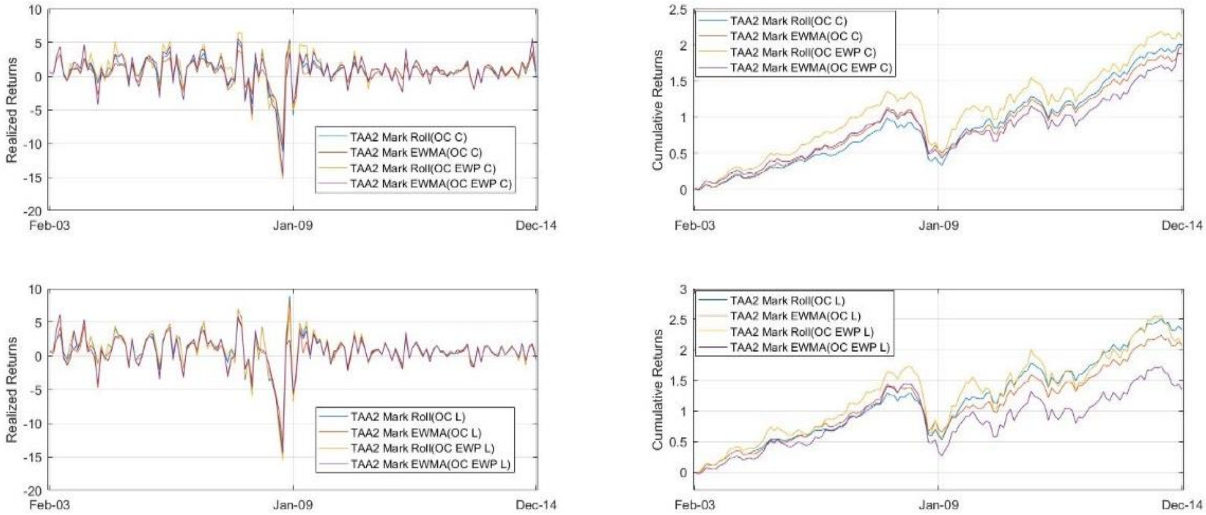


Figure 4.11: The realized returns (on the left) and the realized cumulative returns (on the right) are shown. The tactical asset allocation is based on the deep range considered and on the strategic portfolios in which have been made tactical decisions. These strategic portfolios are those constructed with only stocks and bonds. The data examined are in common currency (on the top) and on local currencies (on the bottom).

The volatility realized for the portfolios with data expressed in common currency is lower than the one of the strategic portfolios to which these portfolios refer. But also, higher in the period of the financial crisis than the same portfolios found by considering narrower variations range. On the other hand, the portfolios composed with assets in local currencies have almost the same volatility as those achieved in the narrow range. The standard deviation of the strategic portfolios is higher again than the portfolios obtained in this part.

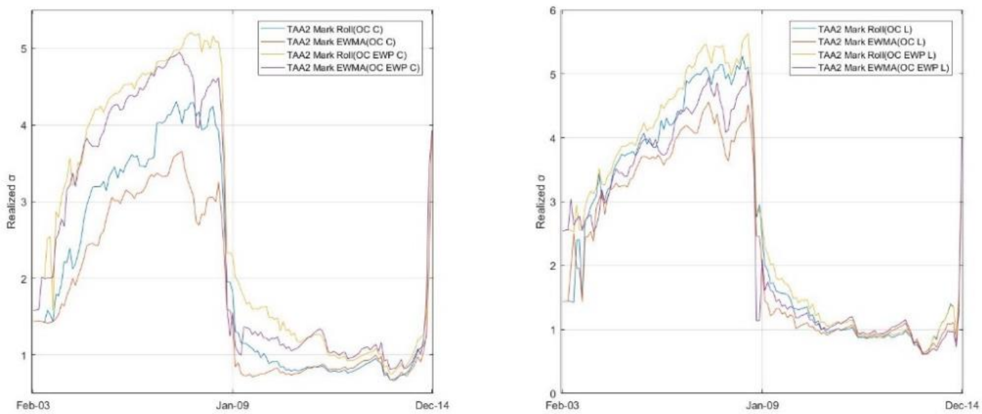


Figure 4.12: The realized volatility for the tactical portfolios constructed based on strategic portfolios with stocks, bonds and hedge funds are presented. These portfolios are achieved by taking into account the deep range variation for the tactical decisions and the data in common and in local currencies

The asset allocations exhibited in **Figure 4.13** show a high and persistent presence of the alternative asset classes. The only case in which the alternative asset classes are not always included is those portfolios in which the data are forecasted with the EWMA method. However, the non-inclusion of these types of assets happen only during the year 2009.

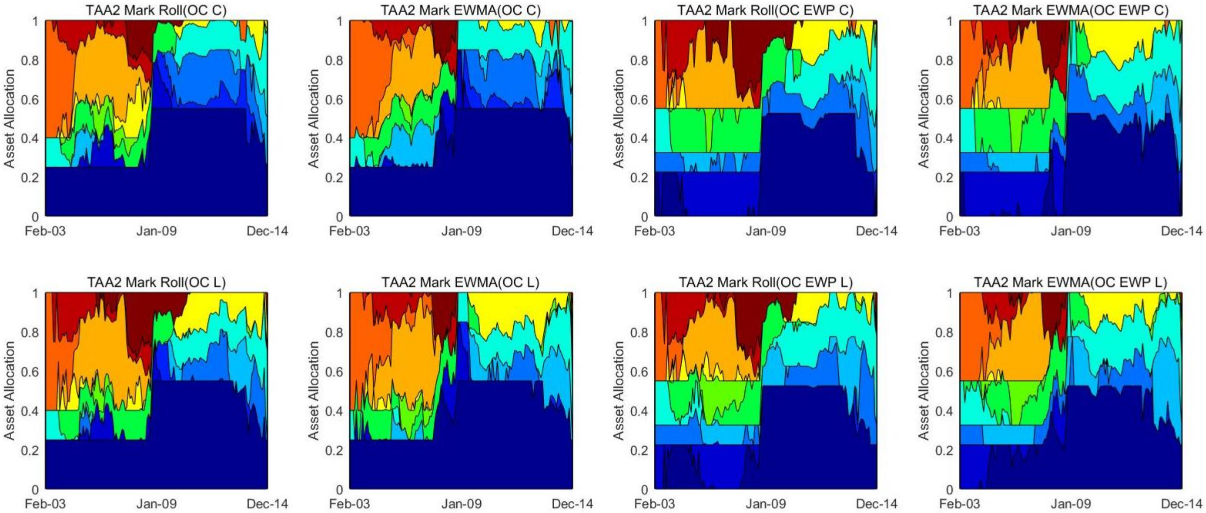


Figure 4.13: In this figure, the composition of the tactical portfolio is depicted.

The portfolios obtained in **Figure 4.14** are those in which the asset classes included in the strategic portfolios are stocks, bonds and hedge funds and the tactical choices are made by referring to the deep range variation considered for the asset classes weights. The realized returns depicted in this figure do not differ from those obtained considering the narrow range variation. The realized cumulative returns shown in **Figure 4.14** have higher values than those in **Figure 4.8**.

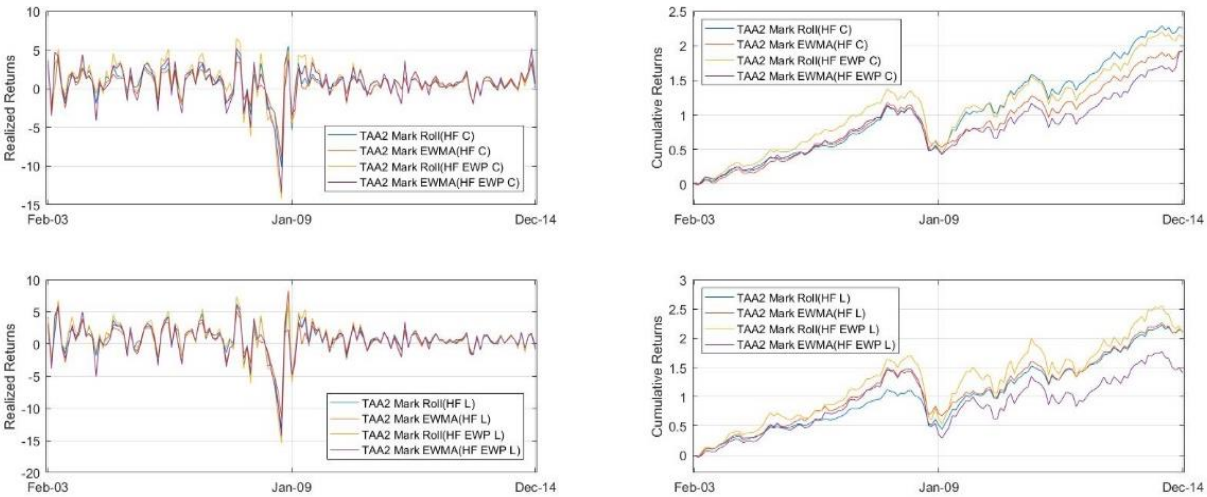


Figure 4.14: The realized returns (on the left) and the realized cumulative returns (on the right) are shown. The tactical asset allocation is based on the deep range considered and on the strategic portfolios in which have been make tactical decision. These strategic portfolios are those constructed with stocks, bonds and hedge funds. The data examined are in common currency (on the top) and on local currencies (on the bottom).

The realized standard deviation achieved by considering data in common currency is presented in **Figure 4.15**. It exhibits a high and increasing level before the last part of the year 2008. This period is followed by a very low level of volatility from 2009 until almost the end of the interval of time analysed. In this last period the standard deviation levels suffer from a high increase. The same pattern is followed by the standard deviation of data in local currencies, but the decreasing levels during the end of the year 2008 has a lower intensity than those of portfolios expressed in common currency.

By comparing these volatility levels with respect to those of the strategic portfolios on which the tactical decisions are made, it is clear again that the standard deviation of the strategic portfolios is higher than their tactical managed portfolios. This statement holds for portfolios produced by considering both data in local and in common currencies. On the other hand, the tactical decisions that derive from the narrow and deep range of the weights do not produce any relevant differences between the standard deviation levels in local currencies. Though, the volatility created by tactical portfolios in common currencies whose weights belong to the deep variation range is higher than that which is generated by portfolios whose weights variation is lower.

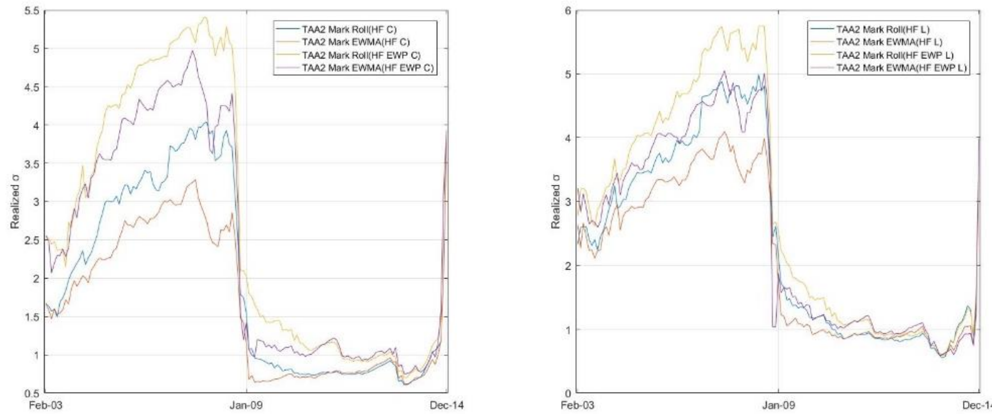


Figure 4.15: The realized volatility for the tactical portfolios constructed based on strategic portfolios with stocks, bonds and hedge funds are presented. These portfolios are achieved by taking into account the deep range variation for the tactical decisions and the data in common and in local currencies

The asset allocation generated by these types of portfolios is characterized by frequent changes in the weights assigned to each asset class. This is particularly evident in **Figure 4.16**.

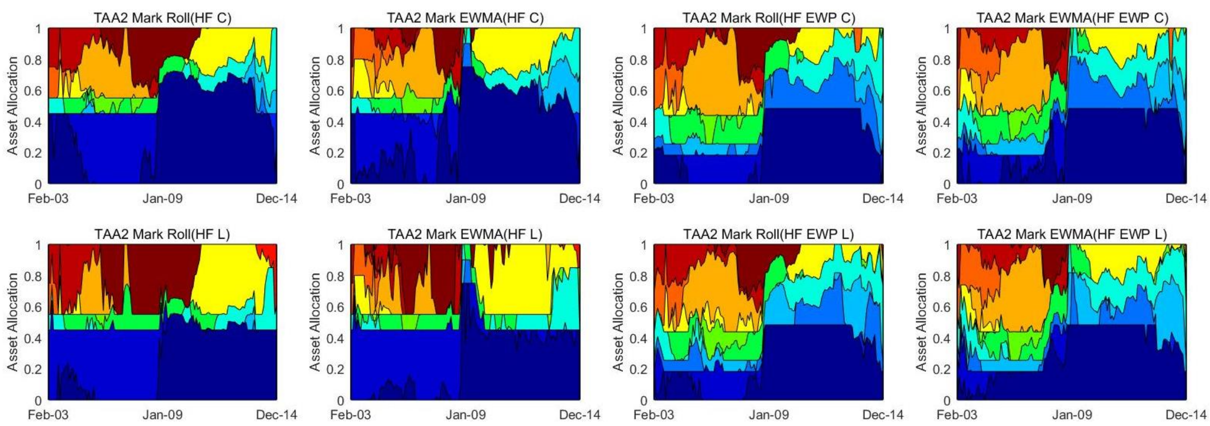


Figure 4.16: In this figure, the composition of the tactical portfolio is depicted.

4.2.2. BLACK AND LITTERMAN PORTFOLIOS

Portfolios obtained by making tactical decisions concerning the strategic portfolios and by using the Black and Litterman inputs will be achieved in this part. The results shown have to be analysed, keeping in mind that a large part of these is affected by the choices made by considering the Black and Litterman views considered. In particular, in **Chapter 3.3** it has been explained the method followed to construct the views of Black and Litterman. The resulting preferred and penalized securities are those that are expected to be found in large part or in little part in the asset allocation composition. Because the preferred assets are those which are more attractive than the penalized securities. For example, in the portfolios constructed by taking into account data in local currencies achieved with both EWMA and rolling sample methods it is expected that the weight assigned to Japanese sovereign bonds (the only penalized security) would be very low. In all the other types of portfolios considered, it is supposed to find higher weights in the preferred asset classes and lower in the penalize asset classes.

The first types of portfolios analysed are those calculated in the narrow variation range for the weights assigned to each portfolio and by making the tactical decisions on the strategic portfolios. These portfolios provide resulting returns that are higher than the same portfolios obtained with the Markowitz model. The portfolios achieved from the data in common currency, in fact, produce, in average, high levels of returns during the first semester of the year 2009. The data in local currencies exhibit a positive return higher than the one in the Markowitz tactical portfolios and the same levels of negative returns. The realized cumulative returns in common currency do not show any strong difference, apart from the final realized cumulative return level resulting from tactical decisions made with data obtained with rolling sample moments. This portfolio shows a higher value than those obtained with Markowitz model. The cumulative returns in local currencies, instead, are higher in Black-Litterman portfolios tactically managed than those in Markowitz portfolios.

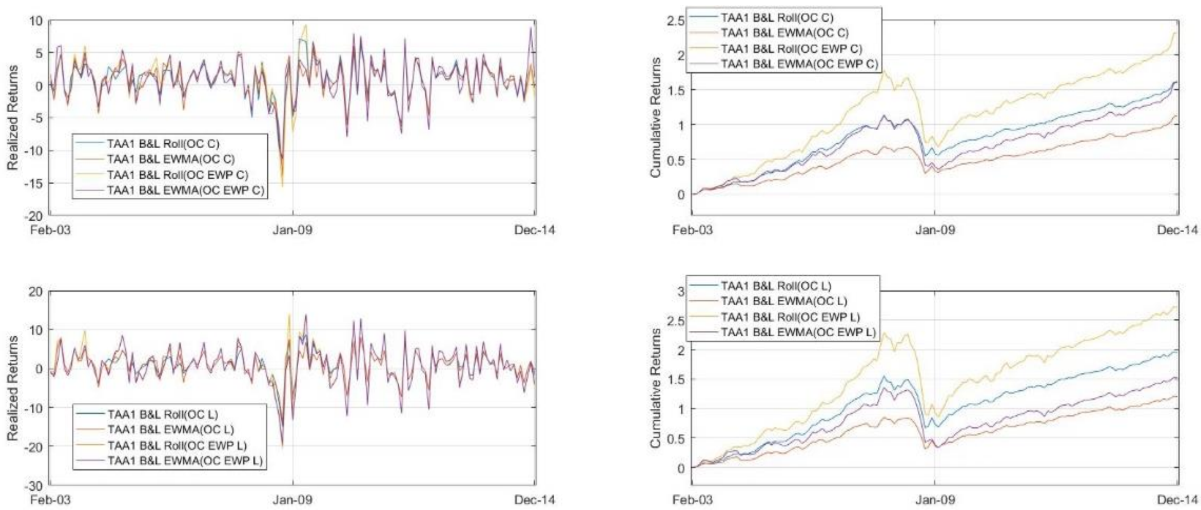


Figure 4.17: The realized returns (on the left) and the realized cumulative returns (on the right) are shown. The tactical asset allocation is based on the narrow range considered and on the strategic portfolios in which have been made tactical decisions. These strategic portfolios are those constructed with only stocks and bonds. The data examined are in common currency (on the top) and on local currencies (on the bottom).

The volatility values realized for the Black and Litterman portfolios in the narrow range of the weights variation are higher than Markowitz portfolios obtained in the same conditions.

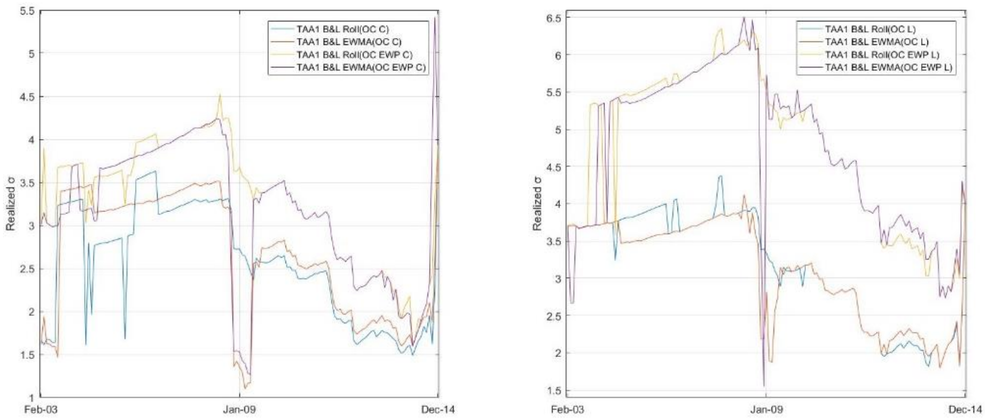


Figure 4.18: The realized volatility for the tactical portfolios constructed based on strategic portfolios with only stocks and bonds are presented. These portfolios are achieved by taking into account the narrow range variation for the tactical decisions and the data in common and in local currencies.

The asset allocation in the portfolios obtained in this part is characterized by few changes in the weights assigned to each asset class.

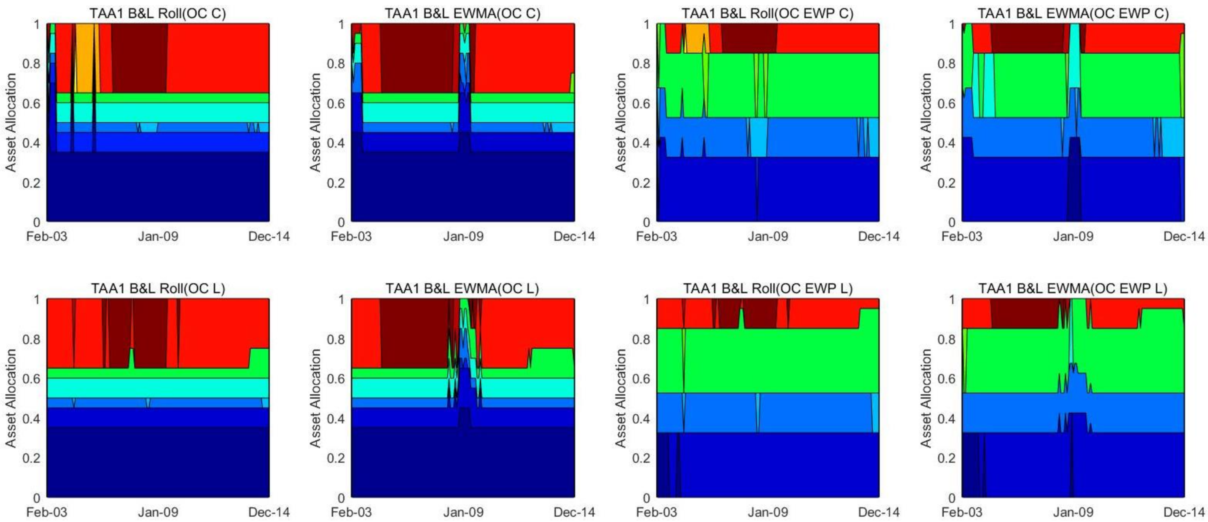


Figure 4.19: In this figure, the composition of the tactical portfolio is depicted.

The realized returns and realized cumulative returns depicted in **Figure 4.20** are those obtained by the tactical choices carried out from strategic portfolios composed with stocks, bonds and hedge funds and by adopting the narrow range variation. The realized returns obtained with data in common currencies are higher than the Markowitz portfolios calculated in the same conditions during the first part of year 2009. In the part of the realized returns in which the data are expressed in local currencies, the realized returns during the years of the financial crisis present a higher reward but also a higher loss of values than the portfolios analysed in the Markowitz's part. The cumulative returns found with the EWMA method and the data in local currencies do not differ too much one to the other. Portfolios obtained with the Black and Litterman model result in higher realized cumulative returns than Markowitz portfolios, by considering data both in common and in local currencies.

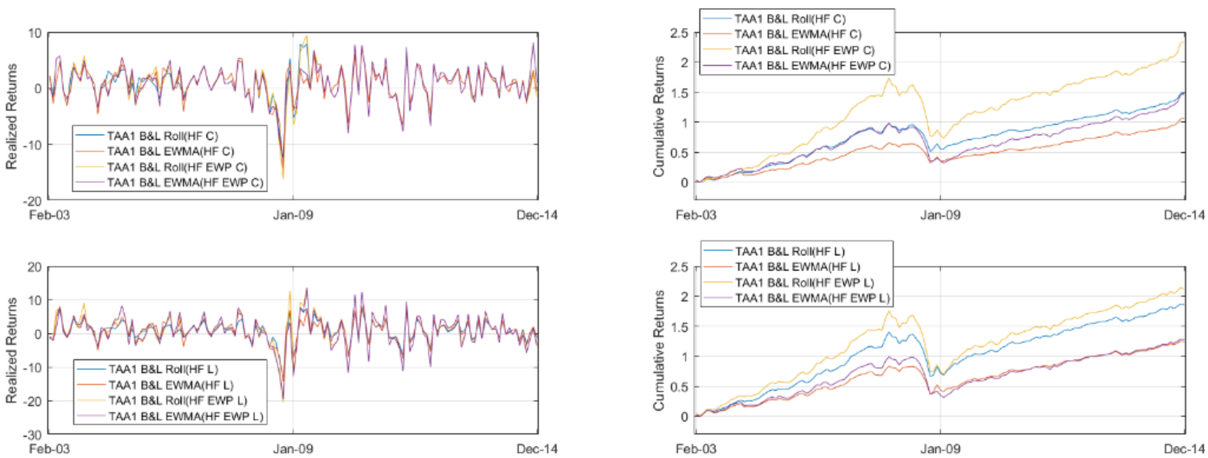


Figure 4.20: The realized returns (on the left) and the realized cumulative returns (on the right) are shown. The tactical asset allocation is based on the narrow range considered and on the strategic portfolios in which have been made tactical decisions. These strategic portfolios are those constructed with stocks, bonds and hedge funds. The data examined are in common currency (on the top) and on local currencies (on the bottom).

The realized volatilities shown in **Figure 4.21** exhibit high variations in their values, in particular during the first and the last part of the entire period analysed, the end part of the year 2008 and the beginning of the year 2009. Their levels are lower than those shown in **Figure 4.9** for the Markowitz tactical portfolios.

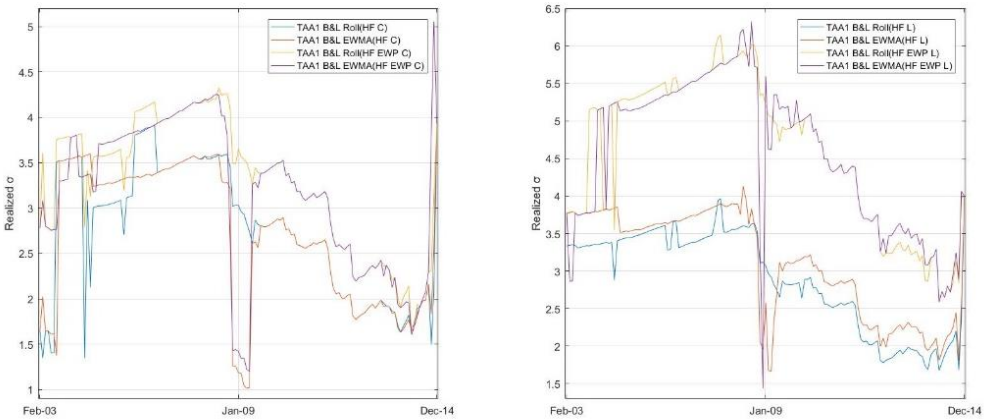


Figure 4.21: The realized volatility for the tactical portfolios constructed based on strategic portfolios with stocks, bonds and hedge funds are presented. These portfolios are achieved by taking into account the narrow range variation for the tactical decisions and the data in common and in local currencies.

The asset allocation depicted in **Figure 4.22** shows the stability in the choice of the portfolio composition. No frequent variations occur in almost any of the types of portfolios considered in this part. The only type of portfolio in which there are frequent reallocations in the asset classes are the two portfolios obtained with EWMA inputs whose securities are non-equally assigned. These portfolios with both local and common currencies are frequently rebalanced during the year 2009.

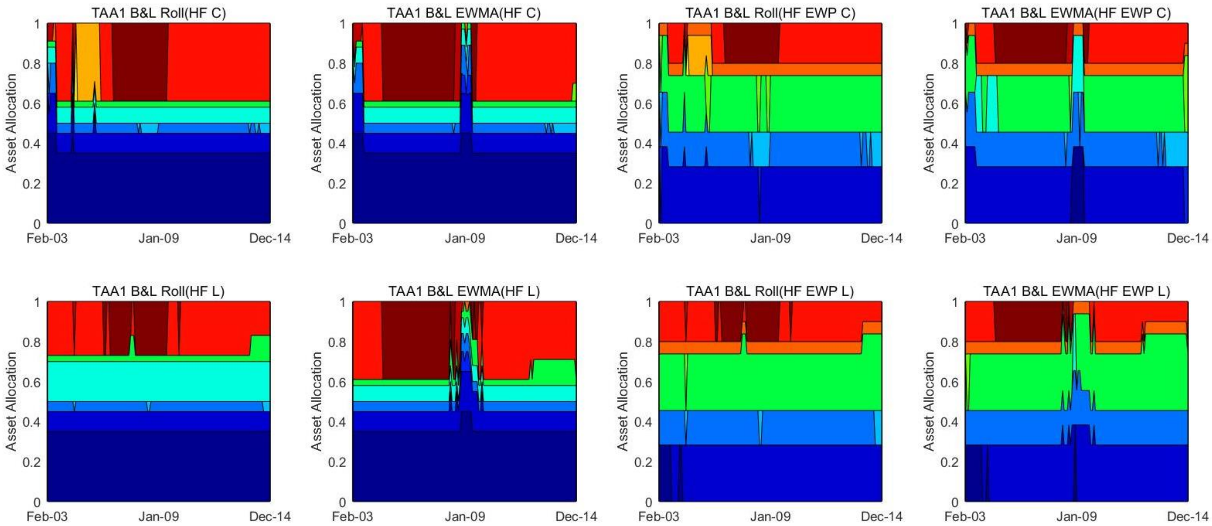


Figure 4.22: In this figure, the composition of the tactical portfolio is depicted.

The portfolios in which the tactical decisions are made by considering the deep range are considered and analysed afterwards. In particular, the first types of tactical portfolios taken into account are those in which the strategic portfolios are composed only with stocks and bonds. The realized returns exhibited in **Figure 4.23** provide higher positive and higher negative returns than those obtained in the part of Markowitz's portfolios. The realized cumulative returns are also higher. These higher returns are shown in the same figure and they are valid for both the data expressed in common and in local currencies.

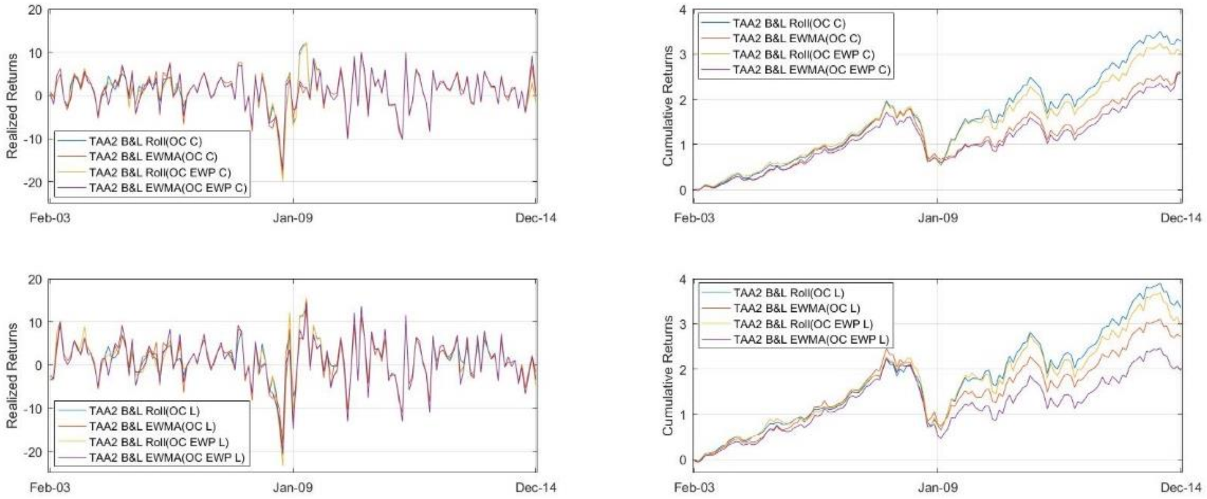


Figure 4.23: The realized returns (on the left) and the realized cumulative returns (on the right) are shown. The tactical asset allocation is based on the deep range considered and on the strategic portfolios in which have been made tactical decisions. These strategic portfolios are those constructed with only stocks and bonds. The data examined are in common currency (on the top) and on local currencies (on the bottom).

The volatility realized for portfolios obtained by tactical choices made from strategic portfolios constructed with only stocks and bonds and by considering the weights that belong to the deep range is depicted in **Figure 4.24**. This figure shows high variations in almost all of the period depicted. Generally, portfolios whose inputs are obtained with the simple moving average method are less volatile than those whose inputs have been provided with the EWMA method. The standard deviation exhibited here is higher than that seen in **Figure 4.12** for almost all the portfolios considered.

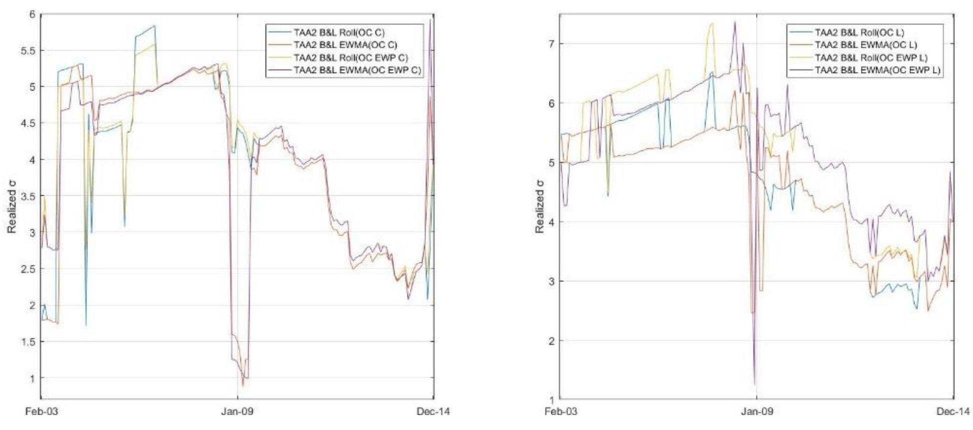


Figure 4.24: The realized volatility for the tactical portfolios constructed based on strategic portfolios with only stocks and bonds are presented. These portfolios are achieved by taking into account the deep range variation for the tactical decisions and the data in common and in local currencies.

The portfolios obtained in this part do not have frequent changes in their asset allocation. The period of time in which the asset composition tends to change more frequently, in almost all these portfolios, is during the year 2009.

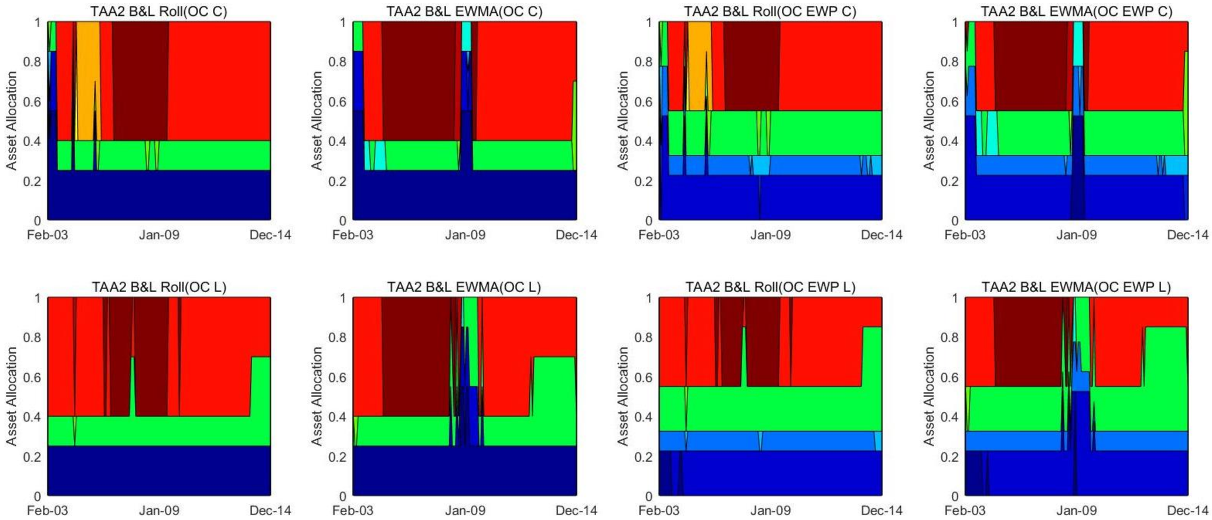


Figure 4.25: In this figure, the composition of the tactical portfolio is depicted.

The realized cumulative returns achieved for tactical decisions concerning the strategic portfolios composed with data in common currency and with the inclusion of stocks, bonds and hedge funds obtained in this part are higher than those in the previous part (**Chapter 4.2.1**).

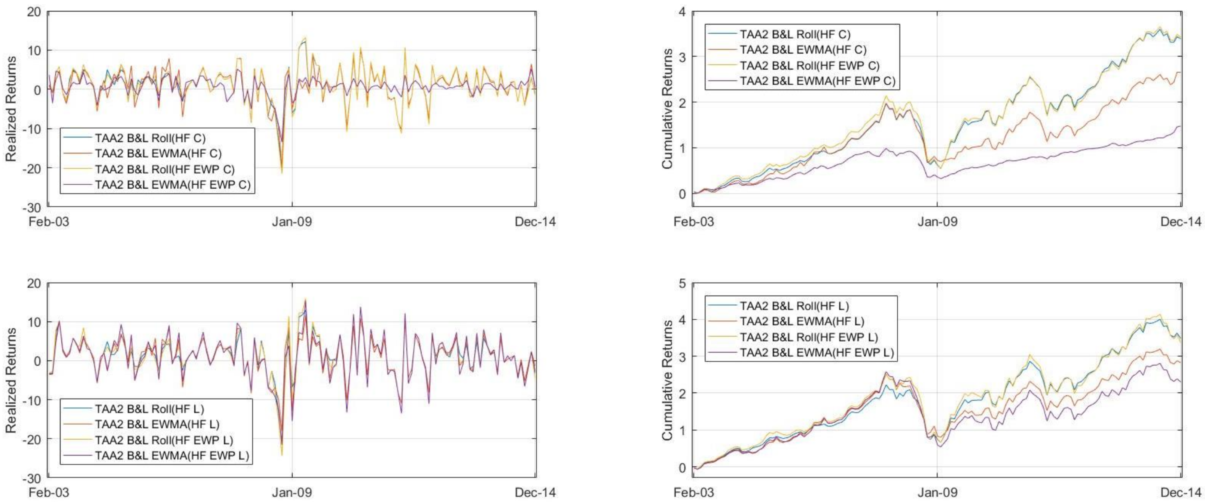


Figure 4.26: The realized returns (on the left) and the realized cumulative returns (on the right) are shown. The tactical asset allocation is based on the deep range considered and on the strategic portfolios in which have been made tactical decisions. These strategic portfolios are those constructed with stocks, bonds and hedge funds. The data examined are in common currency (on the top) and on local currencies (on the bottom).

In **Figure 4.27** the realized volatility for these portfolios are higher than those obtained from the Markowitz analysis. The higher standard deviation is presented for each type of portfolio constructed and depicted.

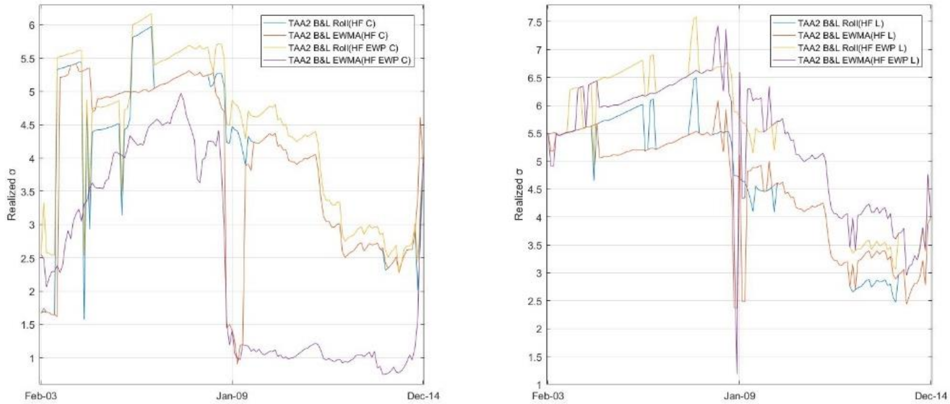


Figure 4.27: The realized volatility for the tactical portfolios constructed based on strategic portfolios with stocks, bonds and hedge funds are presented. These portfolios are achieved by taking into account the deep range variation for the tactical decisions and the data in common and in local currencies.

The **Figure 4.28** shows the weights assign to each asset classes and it shows quite infrequent rebalances on the asset allocation. Tactical choices made on strategic portfolios constructed with equally-weighted composition between the securities in stocks, bonds and hedge funds are assigned and whose data are obtained with the EWMA method and common currency show frequent changes. This is the only portfolio in which the weights assigned to each asset class are frequently rebalanced. In particular, this portfolio is composed by almost all the asset classes analysed in this work. Generally, the others portfolios are composed by considering only one type of security for each asset classes.

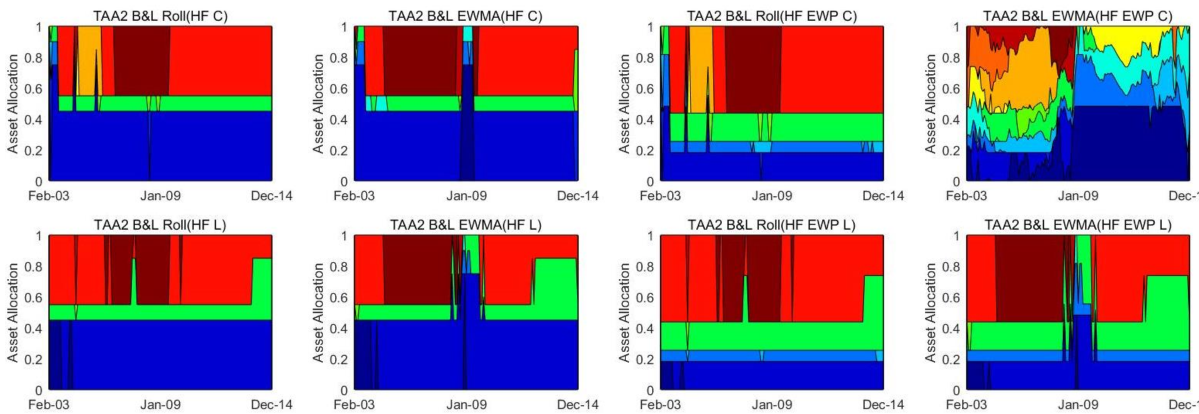


Figure 4.28: In this figure, the composition of the tactical portfolio is depicted.

In concluding this chapter, two types of portfolios have been presented: the strategic portfolios and those portfolios tactically managed. Their realized returns, realized cumulative returns and realized volatility are the parameters which have been shown in order to understand their performances. Since these parameters are calculated separately they cannot give an exact idea of the real portfolio performance. It is for this reason that **Chapter 5** presents some performance measures which take into account the risks and the returns together.

5. PERFORMANCE EVALUATION

In the previous chapter, the return and risk performances have been given for each type of constructed portfolio. Nonetheless, returns and risks are only the starting point on evaluating portfolio performances. The reason behind this statement is that they may give a misrepresented opinion about the real performance of a portfolio. In fact, the correct way to understand portfolio performance is looking at risk-adjusted returns. Therefore, this chapter aims to give an advice regarding which strategy performs in the best way. In particular, in this work, the Sharpe ratio, the alpha of Jensen, the Treynor ratio, the Sortino index and the information ratio have been chosen as risk-adjusted returns. They are briefly presented and calculated in the following part of this chapter.

Irrespective of the performance indicator adopted in comparing one portfolio to the others, the higher the value of the performance indicator, the better the efficiency of the portfolio. In order to be able to say that constructing portfolios with tactical asset allocation and alternative asset classes provides better returns than strategic portfolios with the classical composition (stocks, bonds or stocks, bonds and hedge funds), it is expected that tactical portfolios have higher performance indicators than the strategic portfolios.

The first four performance indicators considered take into account the portfolio returns and compare them with returns of risk-free assets considering also some possible measures of risk. Only the information ratio directly compares the strategic portfolio with the tactical portfolio performances. In this work, all the portfolios constructed by considering the strategic asset allocation are those that are considered as benchmark portfolios. If the information ratio is expected to be positive, the tactical asset allocation performs better than the strategic asset allocation. Otherwise, if the information ratio is negative, the strategic portfolio performs better than the tactical one, then the tactical portfolios performance is poor.

The performance indicators listed above look at measures of risk-adjusted returns, but it is also reasonable to take into account some other measures of risk different from the standard deviation seen in **Chapter 4**. The measures of risk considered in this chapter focus the attention on the losses that the investors may suffer by investing in the portfolio analysed. In order to give an additional idea about the risk, the other indicators analysed are the Value at Risk and the Expected Shortfall.

Each indicator will be calculated in the entire period considered and it will be split also into three subsamples. The first sub-sample takes into account the period from February 2003 to December 2006, the second one, from January 2007 to December 2009 and the last one goes from January 2010 until the end of the available data (December 2014). The choice of splitting the entire sample derives from the fact that until January 2007 the market has experienced a positive trend of returns and a low volatility interval of time. After that period the global financial crisis produced its strongest effects which are low levels of returns and high volatility. The last period considered has been chosen in order to see if the portfolios constructed would have suffered from the sovereign debt crisis which occurred in those years.

5.1. SHARPE RATIO

The *Sharpe ratio* is one of the most common performance indicators. It identifies an excess portfolio return adjusted by its risk. In particular, the excess return is the difference between the portfolio return and the return of the risk-free asset, or the return on a benchmark portfolio that is usually the market portfolio. There are several types of risk measures and the one used by Sharpe is the standard deviation, which measures the portfolio diversification.

The Sharpe ratio formula is:

$$S_p = \frac{R_p - R_f}{\sigma_p},$$

where R_p and σ_p are, respectively, the realized return and standard deviation of the portfolio considered and R_f is the rate of return of the risk-free asset. The riskless asset has not been taken into account in the Sharpe ratio calculation for the portfolios analysed, because the risk-free asset has not been included in the construction of these portfolios. Hence, the previous formula can be seen as only the ratio between the realized return and the realized standard deviation of the portfolios.

The Sharpe ratio is useful to understand how well the investors are rewarded for the risk they have taken by the return obtained investing in the portfolio they have chosen. For this reason, the portfolio that achieves the highest Sharpe ratio is the best portfolio because it identifies a portfolio with higher return and lower or equal risk level.

The table below summarizes the Sharpe ratios obtained for portfolios in **Chapter 4** by considering the entire sample analysed and three sub-samples defined in the introductory part of this chapter. In this table only the maximum and the minimum values of the Sharpe ratios obtained in each period analysed is presented.

Sharpe Ratio	Minimum value	Maximum value
Entire sample	0,1451 *TAA1_B&L_EWMA(OC_EWP_L)*	0,3447 *TAA2_Mark_Roll(OC_C)*
2003-2006	0,4606 *TAA1_B&L_EWMA(OC_EWP_L)*	1,0273 *TAA2_Mark_Roll(OC_C)*
2007-2009	-0,0692 *TAA2_Mark_EWMA(HF_EWP_C)* *TAA2_B&L_EWMA(HF_EWP_C)*	0,1300 *TAA1_B&L_Roll(HF_C)*
2010-2014	0,0948 *TAA1_B&L_EWMA(OC_EWP_L)*	0,7596 *TAA2_Mark_Roll(OC_C)*

Table 5.1: Maximum and minimum values of the Sharpe ratio achieved in the periods considered.

TAA2_Mark_Roll(OC_C) is the portfolio obtained by making deep range variation tactical decisions on a strategic portfolio constructed with only stocks and bonds, by using moving average moments and by achieving the data in common currency. It is the portfolio that reaches the maximum Sharpe ratio in the highest number of periods. Only in the period of high volatility, the one in which the global financial crisis took place, the portfolio *TAA1_B&L_Roll(HF_C)* is the best. The portfolios that achieved the lowest Sharpe ratio levels are those in which the Black and Litterman model has been applied. In particular, the worst portfolio is that that takes its origin from the strategic portfolio achieved with only stocks and bonds combined in an equally-weighted manner and with data in local currencies. During the period of the financial crisis the lowest value of the Sharpe ratio is -0,0692 and it has been achieved by both portfolios constructed with Markowitz and Black and Litterman models with data obtained with EWMA method and which are based on *HF_EWP_C* strategic.

By separately considering the types of portfolios obtained, three tables have been constructed in order to focalize the attention on the Sharpe ratio values that are generated from the strategic portfolios and the tactical portfolios achieved by the narrow range and those built with the deep range. In **Table 5.2** the Sharpe ratios values obtained by the strategic portfolios are shown by maintaining the division among the periods analysed.

Sharpe Ratio	Entire sample	2003-2006	2007-2009	2010-2014
OC_C	0,3179	0,6492	0,0606	0,4055
OC_L	0,2575	0,5091	0,1031	0,2488
OC_EWP_C	0,2431	0,6951	-0,0412	0,3418
OC_EWP_L	0,2374	0,6065	0,0447	0,2359
HF_C	0,3243	0,6244	0,0721	0,4175
HF_L	0,2607	0,4933	0,1117	0,2532
HF_EWP_C	0,2363	0,7009	-0,0450	0,3317
HF_EWP_L	0,2317	0,6116	0,0356	0,2299

Table 5.2: This table summarizes the values obtained for the strategic portfolios' Sharpe ratios.

Table 5.3 shows the Sharpe ratios for the portfolios that have been obtained by making tactical decisions on the portfolios previously seen and by considering the narrow range variation. In this situation the portfolios constructed with the Black and Litterman model have very low Sharpe ratio levels and the highest levels of the Sharpe ratios are concentrated in the portfolios constructed with the Markowitz model. This is true for all the period analysed apart from the high volatility period. In that period, the Black and Litterman portfolios have higher performance indicator levels than Markowitz portfolios which reach the lowest values. During the global financial crisis, in fact, the situation seems to be reversed.

Sharpe Ratio	Entire sample	2003-2006	2007-2009	2010-2014
TAA1_Mark_Roll(OC_C)	0,3254	0,8574	0,0379	0,5576
TAA1_Mark_Roll(OC_L)	0,2650	0,6929	0,0726	0,3275
TAA1_Mark_EWMA(OC_C)	0,2867	0,6154	0,0087	0,5313
TAA1_Mark_EWMA(OC_L)	0,2145	0,4863	0,0239	0,3401
TAA1_Mark_Roll(OC_EWP_C)	0,3230	0,7687	0,0134	0,5914
TAA1_Mark_Roll(OC_EWP_L)	0,2998	0,7172	0,0211	0,5625
TAA1_Mark_EWMA(OC_EWP_C)	0,2752	0,5128	-0,0279	0,5757
TAA1_Mark_EWMA(OC_EWP_L)	0,2402	0,5136	-0,0265	0,5531
TAA1_Mark_Roll(HF_C)	0,3234	0,7451	0,0342	0,5696
TAA1_Mark_Roll(HF_L)	0,2729	0,6675	0,0754	0,3523
TAA1_Mark_EWMA(HF_C)	0,2933	0,5655	0,0071	0,5497
TAA1_Mark_EWMA(HF_L)	0,2313	0,4887	0,0238	0,3626
TAA1_Mark_Roll(HF_EWP_C)	0,3252	0,7784	0,0168	0,6085
TAA1_Mark_Roll(HF_EWP_L)	0,2966	0,7258	0,0212	0,5527
TAA1_Mark_EWMA(HF_EWP_C)	0,2559	0,5242	-0,0414	0,5708
TAA1_Mark_EWMA(HF_EWP_L)	0,2299	0,5120	-0,0290	0,5133
TAA1_B&L_Roll(OC_C)	0,2819	0,6280	0,1077	0,3269
TAA1_B&L_Roll(OC_L)	0,2656	0,6358	0,1142	0,2359
TAA1_B&L_EWMA(OC_C)	0,2675	0,5481	0,0432	0,3042
TAA1_B&L_EWMA(OC_L)	0,2534	0,5864	0,0532	0,2349
TAA1_B&L_Roll(OC_EWP_C)	0,2452	0,6982	0,0668	0,2042
TAA1_B&L_Roll(OC_EWP_L)	0,1777	0,5119	0,0927	0,0963
TAA1_B&L_EWMA(OC_EWP_C)	0,2529	0,5620	0,0358	0,2561
TAA1_B&L_EWMA(OC_EWP_L)	0,1451	0,4606	0,0273	0,0948
TAA1_B&L_Roll(HF_C)	0,2804	0,6146	0,1300	0,2936
TAA1_B&L_Roll(HF_L)	0,2725	0,6345	0,1039	0,2646
TAA1_B&L_EWMA(HF_C)	0,2636	0,5316	0,0405	0,3013
TAA1_B&L_EWMA(HF_L)	0,2540	0,5840	0,0446	0,2391
TAA1_B&L_Roll(HF_EWP_C)	0,2428	0,7162	0,0681	0,2051
TAA1_B&L_Roll(HF_EWP_L)	0,1845	0,5449	0,0891	0,1062
TAA1_B&L_EWMA(HF_EWP_C)	0,2463	0,5609	0,0272	0,2520
TAA1_B&L_EWMA(HF_EWP_L)	0,1523	0,4912	0,0203	0,1048

Table 5.3: This table summarizes the Sharpe ratios obtained for the tactical portfolios considering the narrow range variation. The terms in brackets specify the strategic portfolios on which the tactical decisions have been made.

Table 5.4 shows the portfolios obtained by applying tactical decisions on strategic portfolios and by considering that the weights assigned to each asset class may vary in the deep range. The levels of the Sharpe ratios that result from this type of portfolio construction do not show particular differences with respect to the previous types of portfolios achieved with the narrow range. In this case, in fact, the portfolios constructed with the Markowitz model have higher Sharpe ratio levels than the Black and Litterman model, with the only exception of the period in which the financial crisis took place.

Sharpe Ratio	Entire sample	2003-2006	2007-2009	2010-2014
TAA2_Mark_Roll(OC_C)	0,3447	1,0273	-0,0111	0,7596
TAA2_Mark_Roll(OC_L)	0,3010	0,9530	0,0257	0,5693
TAA2_Mark_EWMA(OC_C)	0,2992	0,6813	-0,0338	0,7385
TAA2_Mark_EWMA(OC_L)	0,2421	0,5636	-0,0120	0,5773
TAA2_Mark_Roll(OC_EWP_C)	0,3147	0,9290	0,0104	0,5466
TAA2_Mark_Roll(OC_EWP_L)	0,2937	0,8969	-0,0018	0,5465
TAA2_Mark_EWMA(OC_EWP_C)	0,2733	0,6348	-0,0509	0,5934
TAA2_Mark_EWMA(OC_EWP_L)	0,2376	0,5655	-0,0494	0,5545
TAA2_Mark_Roll(HF_C)	0,3423	0,8406	-0,0036	0,7513
TAA2_Mark_Roll(HF_L)	0,3094	0,8210	0,0307	0,6122
TAA2_Mark_EWMA(HF_C)	0,3041	0,5754	-0,0274	0,7321
TAA2_Mark_EWMA(HF_L)	0,2590	0,5126	0,0109	0,5990
TAA2_Mark_Roll(HF_EWP_C)	0,3287	0,9134	0,0092	0,5921
TAA2_Mark_Roll(HF_EWP_L)	0,2956	0,8543	0,0030	0,5362
TAA2_Mark_EWMA(HF_EWP_C)	0,2705	0,5944	-0,0692	0,6357
TAA2_Mark_EWMA(HF_EWP_L)	0,2363	0,5295	-0,0409	0,5487
TAA2_B&L_Roll(OC_C)	0,2483	0,7169	0,1041	0,2257
TAA2_B&L_Roll(OC_L)	0,2346	0,6800	0,0913	0,1828
TAA2_B&L_EWMA(OC_C)	0,2276	0,5477	-0,0015	0,2509
TAA2_B&L_EWMA(OC_L)	0,2106	0,5783	0,0167	0,1809
TAA2_B&L_Roll(OC_EWP_C)	0,2406	0,7320	0,0865	0,2061
TAA2_B&L_Roll(OC_EWP_L)	0,2006	0,6254	0,0898	0,1268
TAA2_B&L_EWMA(OC_EWP_C)	0,2327	0,5462	0,0063	0,2455
TAA2_B&L_EWMA(OC_EWP_L)	0,1671	0,5460	-0,0033	0,1244
TAA2_B&L_Roll(HF_C)	0,2486	0,6866	0,1103	0,2314
TAA2_B&L_Roll(HF_L)	0,2404	0,6794	0,0942	0,1947
TAA2_B&L_EWMA(HF_C)	0,2264	0,5310	0,0004	0,2529
TAA2_B&L_EWMA(HF_L)	0,2169	0,5761	0,0132	0,1930
TAA2_B&L_Roll(HF_EWP_C)	0,2382	0,7266	0,0912	0,2077
TAA2_B&L_Roll(HF_EWP_L)	0,2091	0,6524	0,0880	0,1421
TAA2_B&L_EWMA(HF_EWP_C)	0,2705	0,5944	-0,0692	0,6357
TAA2_B&L_EWMA(HF_EWP_L)	0,1760	0,5589	-0,0085	0,1401

Table 5.4: This table summarizes the Sharpe ratios obtained for the tactical portfolios considering the deep range variation. The terms in brackets specify the strategic portfolios on which the tactical decisions have been made.

5.2. JENSEN'S ALPHA

The *alpha of Jensen* (or the *Jensen's performance index*) identifies if the portfolio has achieved an abnormal return with respect to its expected return computed by means of the CAPM.

The formula used to obtain the Jensen's alpha is:

$$\alpha_p = R_p - \overline{R_{p_{CAPM}}},$$

where, as usual, R_p is the realized portfolio return and $\overline{R_{p_{CAPM}}}$ is the expected return of the portfolio which is calculated with the CAPM. In fact, $\overline{R_{p_{CAPM}}} = R_f + \left(\overline{R_M} - R_f\right) * \beta_{p,M}$, where $\overline{R_M}$ is the expected market return and $\beta_{p,M}$ is the part of the risk that cannot be eliminated because it is the risk that comes from the market. This last parameter comes from the regression of the risk-premium (the expected excess returns between the risky assets and the risk-free asset) on the excess returns between the expected return of the market and the risk-free asset.

The concept behind the alpha of Jensen is that if an asset provides a higher return than the risk-free asset (or any other security chosen), the risk is expected to be higher as well. When this is not true, the analysed asset has an abnormal return.

The values obtained by the alpha of Jensen performance indicator are summarized in the following tables. In the first table the maximum and the minimum level of the Jensen's alpha are shown and in the other tables all the values achieved by this indicator are presented by considering the type of portfolios analysed. The strategic portfolios are examined separately and the tactical portfolios are divided by looking at the range among which the weights can vary.

Table 5.5 exhibits that the portfolio *TAA2_Mark_Roll(HF_EWP_C)* reaches the highest Jensen's alpha value in the entire period and in the low volatility period. Instead, the portfolio constructed with the Black and Litterman model in the deep range variation and which takes its origin from the *HF_EWP_L* strategic portfolio is the best portfolio in the high volatility period. This portfolio is the only one constructed with data in local currencies which reaches the highest value of the Jensen's alpha that has ever been obtained. *TAA1_Mark_Roll(OC_EWP_C)* is the portfolio on which the tactical decisions have been made and it achieves the highest value of the alpha of Jensen in the period of the sovereign debt crisis. In addition, it is the only portfolio that has been obtained by starting from a strategic portfolio composed with only stocks and bonds which, by considering this performance indicator, is the best.

Jensen's Alpha	Minimum value	Maximum value
Entire sample	-0,1088 *TAA1_B&L_EWMA(OC_EWP_L)*	0,4871 *TAA2_Mark_Roll(HF_EWP_C)*
2003-2006	0,0537 *TAA1_Mark_EWMA(OC_EWP_C)*	1,1203 *TAA2_Mark_Roll(HF_EWP_C)*
2007-2009	-0,2336 *TAA2_Mark_EWMA(HF_EWP_C)* *TAA2_B&L_EWMA(HF_EWP_C)*	0,8560 *TAA2_B&L_Roll(HF_EWP_L)*
2010-2014	-0,5587 *TAA1_B&L_EWMA(OC_EWP_L)*	0,5993 *TAA1_Mark_Roll(OC_EWP_C)*

Table 5.5: The minimum and the maximum values of the alpha of Jensen achieved in the periods considered.

All the minimum values of the alpha of Jensen have been achieved in the portfolios whose data inputs have been obtained with the EWMA method and in the portfolios constructed with the equally-weighted approach to assign weights to the asset classes. The period of 2007-2009 is a unique one which has two minimum values of the alpha of Jensen, those achieved by *TAA2_Mark_EWMA(HF_EWP_C)* and by *TAA2_B&L_EWMA(HF_EWP_C)*. The worst portfolio in the entire period and in the last one considered is *TAA1_B&L_EWMA(OC_EWP_L)*, whereas portfolio *TAA1_Mark_EWMA(OC_EWP_C)* realizes the minimum Jensen index value during the period of the low volatility that had preceded the global financial crisis.

In **Table 5.6** the Jensen's alpha values of the strategic portfolios are summarized. The evidence that comes from this table is that the values with the maximum level of the index are those obtained by the portfolios in which only stocks and bonds have been included and whose data are in common currency. Only the best portfolio in the period of 2003-2006 has data in local currencies. The worst portfolios, instead, are those in which stocks, bonds and hedge funds have been equally-weighted into the portfolio composition.

Jensen's Alpha	Entire sample	2003-2006	2007-2009	2010-2014
OC_C	0,2478	0,2383	0,1944	0,3132
OC_L	0,2169	0,1754	0,4072	0,0968
OC_EWP_C	0,1789	0,3182	-0,1043	0,3603
OC_EWP_L	0,1506	0,2007	0,2397	0,0637
HF_C	0,2436	0,2296	0,2047	0,3008
HF_L	0,2154	0,1728	0,4022	0,1012
HF_EWP_C	0,1486	0,2814	-0,1128	0,3063
HF_EWP_L	0,1235	0,1769	0,1929	0,0428

Table 5.6: This table summarizes the values obtained for the strategic portfolios' alphas of Jensen.

Table 5.7 summarizes the alpha of Jensen obtained by the strategic portfolios combined by following the narrow range in which the weights assigned to the asset classes may vary. For the entire sample and the first and the last sub-samples, the highest values of the Jensen's alpha are concentrated in the portfolios built with the Markowitz method. In the period characterized by the high-volatility, instead, the best portfolios are those in which the Black and Litterman model has been adopted. In addition, the portfolio with the highest alpha of Jensen obtained in the period 2007-2009 is the best portfolio with the highest value of Jensen index by looking at entire **Table 5.7**.

Jensen's Alpha	Entire sample	2003-2006	2007-2009	2010-2014
TAA1_Mark_Roll(OC_C)	0,3618	0,6599	0,1818	0,4039
TAA1_Mark_Roll(OC_L)	0,2875	0,5965	0,3804	0,1942
TAA1_Mark_EWMA(OC_C)	0,2673	0,2999	0,0612	0,3691
TAA1_Mark_EWMA(OC_L)	0,1619	0,1226	0,1598	0,2059
TAA1_Mark_Roll(OC_EWP_C)	0,4108	0,5528	0,1407	0,5993
TAA1_Mark_Roll(OC_EWP_L)	0,3912	0,5970	0,1884	0,4772
TAA1_Mark_EWMA(OC_EWP_C)	0,2776	0,0537	-0,0357	0,5917
TAA1_Mark_EWMA(OC_EWP_L)	0,2170	0,0862	-0,0287	0,4324
TAA1_Mark_Roll(HF_C)	0,3631	0,7006	0,1491	0,3879
TAA1_Mark_Roll(HF_L)	0,3266	0,7300	0,3779	0,2107
TAA1_Mark_EWMA(HF_C)	0,2886	0,3423	0,0498	0,3763
TAA1_Mark_EWMA(HF_L)	0,2158	0,2389	0,1467	0,2178
TAA1_Mark_Roll(HF_EWP_C)	0,4139	0,6352	0,1417	0,5654
TAA1_Mark_Roll(HF_EWP_L)	0,3926	0,6670	0,1915	0,4467
TAA1_Mark_EWMA(HF_EWP_C)	0,2366	0,0851	-0,0936	0,5359
TAA1_Mark_EWMA(HF_EWP_L)	0,1965	0,0962	-0,0435	0,3810
TAA1_B&L_Roll(OC_C)	0,2954	0,4395	0,5283	0,2915
TAA1_B&L_Roll(OC_L)	0,2933	0,3974	0,6478	0,0722
TAA1_B&L_EWMA(OC_C)	0,2889	0,2032	0,2212	0,2501
TAA1_B&L_EWMA(OC_L)	0,2733	0,1458	0,3056	0,0700
TAA1_B&L_Roll(OC_EWP_C)	0,2232	0,4893	0,4281	-0,0184
TAA1_B&L_Roll(OC_EWP_L)	0,0469	0,2194	0,7984	-0,5497
TAA1_B&L_EWMA(OC_EWP_C)	0,2623	0,0772	0,2028	0,1918
TAA1_B&L_EWMA(OC_EWP_L)	-0,1088	0,1073	0,3201	-0,5587
TAA1_B&L_Roll(HF_C)	0,3290	0,5600	0,6850	0,2283
TAA1_B&L_Roll(HF_L)	0,2792	0,3167	0,5545	0,1410
TAA1_B&L_EWMA(HF_C)	0,2983	0,2265	0,2107	0,2506
TAA1_B&L_EWMA(HF_L)	0,2807	0,1550	0,2635	0,0852
TAA1_B&L_Roll(HF_EWP_C)	0,2116	0,5004	0,4401	-0,0235
TAA1_B&L_Roll(HF_EWP_L)	0,0591	0,2469	0,7446	-0,4902
TAA1_B&L_EWMA(HF_EWP_C)	0,2381	0,0639	0,1691	0,1624
TAA1_B&L_EWMA(HF_EWP_L)	-0,0862	0,0986	0,2611	-0,4978

Table 5.7: This table summarizes the alphas of Jensen obtained for the tactical portfolios considering the narrow range variation. The terms in brackets specify the strategic portfolios on which the tactical decisions have been made.

The Jensen's alpha exhibited in **Table 5.8** are those achieved by making tactical decisions considering the deep range in which the weights assigned to each asset class may vary. The portfolios obtained with the Markowitz model are the best portfolios during the entire period and in the two sub-periods: the first from 2003 to 2006 and the last from 2010 to 2014. Only during the last time period considered and the high volatility period, the portfolios obtained with the model of Black and Litterman have been classified as the best portfolios by using the alpha of Jensen criterion. In both cases, the portfolios that recorded the maximum value of the performance indicator examined here are those constructed by taking into account the weights assigned equally in the strategic portfolios and by using the data achieved by the EWMA method.

Jensen's Alpha	Entire sample	2003-2006	2007-2009	2010-2014
TAA2_Mark_Roll(OC_C)	0,4194	0,7946	0,0044	0,5507
TAA2_Mark_Roll(OC_L)	0,4062	0,8913	0,1909	0,4247
TAA2_Mark_EWMA(OC_C)	0,3089	0,3110	-0,0770	0,5545
TAA2_Mark_EWMA(OC_L)	0,2531	0,2318	-0,0017	0,4274
TAA2_Mark_Roll(OC_EWP_C)	0,4446	0,9313	0,1190	0,5222
TAA2_Mark_Roll(OC_EWP_L)	0,3894	0,8778	0,0788	0,4250
TAA2_Mark_EWMA(OC_EWP_C)	0,3092	0,3499	-0,1651	0,5838
TAA2_Mark_EWMA(OC_EWP_L)	0,2241	0,2130	-0,1568	0,4200
TAA2_Mark_Roll(HF_C)	0,4258	0,8287	0,0267	0,5342
TAA2_Mark_Roll(HF_L)	0,4257	0,8804	0,2063	0,4458
TAA2_Mark_EWMA(HF_C)	0,3307	0,3418	-0,0554	0,5381
TAA2_Mark_EWMA(HF_L)	0,3069	0,2538	0,0805	0,4326
TAA2_Mark_Roll(HF_EWP_C)	0,4871	1,1203	0,0961	0,5406
TAA2_Mark_Roll(HF_EWP_L)	0,4127	0,9922	0,1035	0,4120
TAA2_Mark_EWMA(HF_EWP_C)	0,3076	0,3400	-0,2336	0,5934
TAA2_Mark_EWMA(HF_EWP_L)	0,2353	0,2183	-0,1190	0,4142
TAA2_B&L_Roll(OC_C)	0,3097	0,8073	0,7979	0,0498
TAA2_B&L_Roll(OC_L)	0,2550	0,5509	0,7482	-0,1439
TAA2_B&L_EWMA(OC_C)	0,2683	0,2177	0,0361	0,1686
TAA2_B&L_EWMA(OC_L)	0,1871	0,1441	0,1839	-0,1534
TAA2_B&L_Roll(OC_EWP_C)	0,2598	0,6947	0,6770	-0,0376
TAA2_B&L_Roll(OC_EWP_L)	0,1191	0,4217	0,8458	-0,4657
TAA2_B&L_EWMA(OC_EWP_C)	0,2655	0,0846	0,0902	0,1582
TAA2_B&L_EWMA(OC_EWP_L)	-0,0321	0,0941	0,0964	-0,4799
TAA2_B&L_Roll(HF_C)	0,3256	0,8279	0,8552	0,0783
TAA2_B&L_Roll(HF_L)	0,2863	0,5968	0,7646	-0,0848
TAA2_B&L_EWMA(HF_C)	0,2803	0,2231	0,0446	0,1802
TAA2_B&L_EWMA(HF_L)	0,2242	0,1454	0,1518	-0,0936
TAA2_B&L_Roll(HF_EWP_C)	0,2745	0,7727	0,7684	-0,0333
TAA2_B&L_Roll(HF_EWP_L)	0,1598	0,4999	0,8560	-0,3977
TAA2_B&L_EWMA(HF_EWP_C)	0,3076	0,3400	-0,2336	0,5934
TAA2_B&L_EWMA(HF_EWP_L)	0,0105	0,0921	0,0497	-0,4095

Table 5.8: This table summarizes the alphas of Jensen obtained for the tactical portfolios considering the deep range variation. The terms in brackets specify the strategic portfolios on which the tactical decisions have been made.

5.3. TREYNOR RATIO

The *Treynor ratio* (or the *reward to volatility ratio*) is a measure of the attractiveness of the investment. This index differs from the Sharpe ratio only for the denominator. The Treynor ratio uses the beta as the risk measure and the formula for this performance indicator is:

$$\text{Treynor} = \frac{R_p - R_f}{\beta_p},$$

where, as usual, R_p is the realized portfolio return, R_f is the return of the risk-free asset and β_p is the portfolio beta. As it has already been said in the Sharpe ratio description, the presence of the riskless asset makes no sense in this index calculation because no risk-free assets have been included in the portfolios construction. Therefore, the risk-free asset is assumed to take value equal to zero.

The beta coefficient is the systematic risk, the risk that arises from the market. For this reason, to calculate the Treynor ratio implies that the analysed portfolio is already well diversified. When the value of this ratio is high the exposure to the market risk in compensated by the portfolio performance.

Table 5.9 contains the maximum and minimum values achieved by the Treynor ratio. What is interesting to notice is that in the whole period considered and in the years from 2003 to 2006 the best portfolio is the same, it is the *TAA2_Mark_Roll(HF_C)*. The Treynor ratio is the first performance indicator that classifies, as the best portfolio, one of the strategic portfolios. In fact, during the high-volatility period, the portfolio constructed with stocks, bonds and hedge funds, whose data are expressed in local currencies and whose asset allocation is the same from the beginning to the end of the investment time horizon, reaches the highest Treynor ratio value. Another particularity of this index is that all of the best portfolios are those in which the portfolio on which they are based is composed with stocks, bonds and hedge funds. In fact, also the highest Treynor ratio in the period of 2010-2014 is reached by the portfolio *TAA2_Mark_EWMA(HF_C)*. Almost all of the maximum values of this ratio are achieved by portfolios constructed with Markowitz, the minimum values, instead, are obtained by Black and Litterman ones. There was one Markowitz portfolio which had the lowest value of the Treynor ratio during the period between 2007 and 2009.

All these maximum and minimum values of the Treynor index are summarised in **Table 5.9**.

Treynor Ratio	Minimum value	Maximum value
Entire sample	0,7619 *TAA1_B&L_EWMA(OC_EWP_L)*	2,5093 *TAA2_Mark_Roll(HF_C)*
2003-2006	1,7061 *TAA2_B&L_EWMA(HF_EWP_L)*	4,7798 *TAA2_Mark_Roll(HF_C)*
2007-2009	-0,6519 *TAA2_Mark_EWMA(HF_EWP_C)* *TAA2_B&L_EWMA(HF_EWP_C)*	0,8884 *HF_L*
2010-2014	0,4331 *TAA1_B&L_EWMA(OC_EWP_L)*	9,2124 *TAA2_Mark_EWMA(HF_C)*

Table 5.9: The minimum and the maximum values of the Treynor ratio achieved in the periods considered

The strategic portfolios that have obtained the highest values of the Treynor ratio, in almost all of the periods considered, are those constructed with stocks, bonds and hedge funds. Only in the low-volatility period the best portfolio is the one obtained with the inclusion of only stocks and bonds, this portfolio is *OC_EWP_C*.

Treynor Ratio	Entire sample	2003-2006	2007-2009	2010-2014
OC_C	1,6037	2,3758	0,3827	1,8375
OC_L	1,3798	2,0313	0,7992	1,1304
OC_EWP_C	1,2729	2,5528	-0,4108	1,6910
OC_EWP_L	1,1639	2,0367	0,2877	1,0342
HF_C	1,6755	2,4232	0,4774	1,9188
HF_L	1,4322	2,0652	0,8884	1,1670
HF_EWP_C	1,2195	2,4462	-0,4388	1,6135
HF_EWP_L	1,1244	2,0012	0,2135	1,0016

Table 5.10: This table summarizes the values obtained for the strategic portfolios' Treynor ratios.

The values achieved by the Treynor index when the narrow variation range is considered are shown in the **Table 5.11**. The highest values of this index are concentrated in the portfolios constructed with the Markowitz model in the first, in the last and in the whole period, and then with the Black and Litterman model in the period between 2007-2009.

By looking at the periods characterized by high and low volatility, the maximum and minimum values of the Treynor ratio are not concentrated in portfolios constructed with one model with respect to another. In these periods, in fact, the Markowitz portfolios have both high and low values of the Treynor index and this is true also for the Black and Litterman portfolios.

Treynor Ratio	Entire sample	2003-2006	2007-2009	2010-2014
TAA1_Mark_Roll(OC_C)	1,9347	3,6224	0,2286	3,0920
TAA1_Mark_Roll(OC_L)	1,5271	3,0539	0,4882	1,6510
TAA1_Mark_EWMA(OC_C)	1,7738	2,4083	-0,0090	3,0021
TAA1_Mark_EWMA(OC_L)	1,2706	1,8675	0,1412	1,7306
TAA1_Mark_Roll(OC_EWP_C)	1,7193	2,6702	0,0505	2,9957
TAA1_Mark_Roll(OC_EWP_L)	1,6699	2,7081	0,1248	2,5956
TAA1_Mark_EWMA(OC_EWP_C)	1,4982	1,7197	-0,2504	3,0513
TAA1_Mark_EWMA(OC_EWP_L)	1,3480	1,7677	-0,2339	2,5603
TAA1_Mark_Roll(HF_C)	2,1139	4,1694	0,2073	3,4481
TAA1_Mark_Roll(HF_L)	1,6791	3,5895	0,5174	1,8822
TAA1_Mark_EWMA(HF_C)	2,0086	2,6105	-0,0172	3,5499
TAA1_Mark_EWMA(HF_L)	1,4758	2,1175	0,1578	1,9676
TAA1_Mark_Roll(HF_EWP_C)	1,7715	2,9253	0,0604	3,0832
TAA1_Mark_Roll(HF_EWP_L)	1,6660	2,8387	0,1225	2,5510
TAA1_Mark_EWMA(HF_EWP_C)	1,4161	1,7755	-0,3598	3,0535
TAA1_Mark_EWMA(HF_EWP_L)	1,3012	1,7810	-0,2600	2,3885
TAA1_B&L_Roll(OC_C)	1,3749	2,7406	0,6441	1,4048
TAA1_B&L_Roll(OC_L)	1,2981	2,2743	0,7403	1,0191
TAA1_B&L_EWMA(OC_C)	1,3870	1,9573	0,2591	1,3128
TAA1_B&L_EWMA(OC_L)	1,2925	1,8098	0,3548	1,0154
TAA1_B&L_Roll(OC_EWP_C)	1,1828	2,4162	0,4119	0,8954
TAA1_B&L_Roll(OC_EWP_L)	0,9235	1,8786	0,6747	0,4389
TAA1_B&L_EWMA(OC_EWP_C)	1,2882	1,7388	0,2081	1,1669
TAA1_B&L_EWMA(OC_EWP_L)	0,7619	1,7636	0,1950	0,4331
TAA1_B&L_Roll(HF_C)	1,4011	3,0753	0,8277	1,2728
TAA1_B&L_Roll(HF_L)	1,3089	2,1710	0,6676	1,1311
TAA1_B&L_EWMA(HF_C)	1,4017	1,9933	0,2430	1,3067
TAA1_B&L_EWMA(HF_L)	1,3039	1,8177	0,2858	1,0363
TAA1_B&L_Roll(HF_EWP_C)	1,1617	2,4515	0,4098	0,8892
TAA1_B&L_Roll(HF_EWP_L)	0,9368	1,9154	0,6225	0,4766
TAA1_B&L_EWMA(HF_EWP_C)	1,2416	1,7180	0,1334	1,1261
TAA1_B&L_EWMA(HF_EWP_L)	0,7844	1,7484	0,1290	0,4718

Table 5.11: This table summarizes the Treynor ratios obtained for the tactical portfolios considering the narrow range variation. The terms in brackets specify the strategic portfolios on which the tactical decisions have been made.

Table 5.12 contains the Treynor ratios achieved by the portfolios whose weights assigned to each asset classes may vary in the deep range variation. This table does not show peculiar behaviours in the Treynor index with respect to the other performance indicators. In fact, only during the period in which the global financial crisis took place the portfolios made with the Black and Litterman model achieved the highest values in the Treynor index. In all other periods the best portfolios are concentrated into the Markowitz portfolios values.

Treynor Ratio	Entire sample	2003-2006	2007-2009	2010-2014
TAA2_Mark_Roll(OC_C)	2,2050	3,9667	-0,1717	6,2066
TAA2_Mark_Roll(OC_L)	1,7906	3,6944	0,1303	3,1816
TAA2_Mark_EWMA(OC_C)	2,0150	2,4317	-0,4005	6,4654
TAA2_Mark_EWMA(OC_L)	1,5496	2,0779	-0,1853	3,2705
TAA2_Mark_Roll(OC_EWP_C)	1,7641	3,5512	-0,0012	2,9695
TAA2_Mark_Roll(OC_EWP_L)	1,6698	3,3638	-0,0602	2,7290
TAA2_Mark_EWMA(OC_EWP_C)	1,5571	2,2215	-0,4815	3,2734
TAA2_Mark_EWMA(OC_EWP_L)	1,3883	1,9703	-0,4785	2,7786
TAA2_Mark_Roll(HF_C)	2,5093	4,7798	-0,1106	8,5303
TAA2_Mark_Roll(HF_L)	1,9697	3,8744	0,1933	4,1533
TAA2_Mark_EWMA(HF_C)	2,3651	2,6003	-0,3734	9,2124
TAA2_Mark_EWMA(HF_L)	1,8214	2,1313	0,0158	4,1255
TAA2_Mark_Roll(HF_EWP_C)	1,9531	4,4751	-0,0251	3,4919
TAA2_Mark_Roll(HF_EWP_L)	1,7216	3,6874	-0,0236	2,7542
TAA2_Mark_EWMA(HF_EWP_C)	1,6281	2,2238	-0,6519	3,8286
TAA2_Mark_EWMA(HF_EWP_L)	1,4266	1,9815	-0,4111	2,8456
TAA2_B&L_Roll(OC_C)	1,2079	3,0379	0,6216	0,9699
TAA2_B&L_Roll(OC_L)	1,1239	2,2503	0,5418	0,7864
TAA2_B&L_EWMA(OC_C)	1,1984	1,8655	-0,1302	1,0909
TAA2_B&L_EWMA(OC_L)	1,0670	1,7540	0,0343	0,7791
TAA2_B&L_Roll(OC_EWP_C)	1,1515	2,6679	0,5165	0,8819
TAA2_B&L_Roll(OC_EWP_L)	0,9800	2,0599	0,5776	0,5545
TAA2_B&L_EWMA(OC_EWP_C)	1,1983	1,7207	-0,0454	1,0760
TAA2_B&L_EWMA(OC_EWP_L)	0,8467	1,7151	-0,0777	0,5468
TAA2_B&L_Roll(HF_C)	1,2233	3,1439	0,6676	0,9987
TAA2_B&L_Roll(HF_L)	1,1583	2,3255	0,5608	0,8392
TAA2_B&L_EWMA(HF_C)	1,2120	1,8698	-0,1178	1,1019
TAA2_B&L_EWMA(HF_L)	1,1099	1,7534	0,0030	0,8318
TAA2_B&L_Roll(HF_EWP_C)	1,1458	2,7752	0,5443	0,8887
TAA2_B&L_Roll(HF_EWP_L)	1,0101	2,1263	0,5488	0,6155
TAA2_B&L_EWMA(HF_EWP_C)	1,6281	2,2238	-0,6519	3,8286
TAA2_B&L_EWMA(HF_EWP_L)	0,8856	1,7061	-0,1308	0,6092

Table 5.12: This table summarizes the Treynor ratios obtained for the tactical portfolios considering the deep range variation. The terms in brackets specify the strategic portfolios on which the tactical decisions have been made.

5.4. SORTINO INDEX

The *Sortino index* or *Sortino ratio* is a performance measure which adjusts the excess returns by considering a measure of risk that is the *downside risk*. It differs from the Sharpe ratio only for the use of a different measure of risk with respect to the standard deviation. The formula adopted to calculate the Sortino index is:

$$\mathrm{Sort.}_{\mathrm{p}} = \frac{\mathrm{R_p} - \mathrm{R_f}}{\mathrm{DSR}}$$

where R_p is the realized return of the portfolio analysed, R_f is the return of the risk-free asset which is considered equal to 0 for the same reasons explained in the Sharpe ratio part and DSR is the downside risk.

The downside risk does not take into account the entire variations of the investment levels with respect to their average level as the standard deviation used by Sharpe, but it considers only the negative part of the volatility. In particular, the idea under which the Sortino index is built is that the investors are afraid of the values which are not higher or at least equal to a minimum level of returns that they consider acceptable. Hence, the downside risk takes into account only those returns whose level is lower than the minimum level decided by the investors. If the portfolio returns are higher than the minimum value of returns the downside risk is equal to zero. Generally, the minimum level is represented by the return of the risk-free asset, which in the case analysed in this work is zero.

Higher the Sortino index value, higher the performance achieved by the portfolio analysed. This is because when the Sortino index is high, there are more returns that exceed their minimum acceptable level than those under their minimum level considered. Thus, the variability of the returns is concentrated on values which are higher than the minimum level chosen.

Table 5.13 summarizes the maximum and minimum values achieved by the Sortino index. This table does not show any differences with respect to the other performance indicators. In the Sortino index values, the best portfolios in almost all of the periods analysed are those in which the Markowitz model has been applied. In particular, in the first and in the last periods the highest value of the index has been obtained by the same portfolio, it is the *TAA2_Mark_Roll(OC_C)*. Only during the period of 2007-2009 the portfolio constructed with the Black and Litterman model is the best portfolio.

The worst portfolios achieved by considering the Sortino index are those in which the EWMA method has been used in order to calculate the inputs of the models. In particular, in almost all of the periods examined the portfolios in which the asset classes have been allocated by following the equally-weighted approach are the worst portfolios. Only in the low-volatility period the worst portfolio is that whose assets are not equally-weighted. Another fact that is important to be noticed is that the tactical choices based on the strategic portfolios in which stocks, bonds and hedge funds have been included are considered the worst portfolio only during the period of 2007-2009.

Sortino Index	Minimum value	Maximum value
Entire sample	0,2137 *TAA1_B&L_EWMA(OC_EWP_L)*	0,4925 *TAA2_Mark_Roll(HF_C)*
2003-2006	0,8462 *TAA1_Mark_EWMA(OC_L)*	3,4634 *TAA2_Mark_Roll(OC_C)*
2007-2009	-0,1000 *TAA2_Mark_EWMA(HF_EWP_C)* *TAA2_B&L_EWMA(HF_EWP_C)*	0,1737 *TAA1_B&L_Roll(HF_C)*
2010-2014	0,1438 *TAA1_B&L_EWMA(OC_EWP_L)*	2,0525 *TAA2_Mark_Roll(OC_C)*

Table 5.13: Minimum and the maximum values of the Sortino index achieved in the periods considered.

In **Table 5.14** the Sortino values based on the strategic portfolios are shown. The highest values exhibited in this table are those achieved by the portfolios obtained with the inclusion of stocks, bonds and hedge funds. But also the worst portfolios have been obtained by combining the same asset classes. The highest and lowest values of the Sortino index are concentrated on the bottom part of the figure, the part in which the portfolios composed with stocks, bonds and hedge funds can be found.

Sortino Index	Entire sample	2003-2006	2007-2009	2010-2014
OC_C	0,4833	1,8218	0,0680	0,7742
OC_L	0,4056	1,2579	0,1430	0,4240
OC_EWP_C	0,3536	2,1413	-0,0684	0,6136
OC_EWP_L	0,3603	1,7881	0,0541	0,3913
HF_C	0,4921	1,6534	0,0832	0,8007
HF_L	0,4104	1,1732	0,1561	0,4306
HF_EWP_C	0,3384	2,1817	-0,0727	0,5859
HF_EWP_L	0,3466	1,7609	0,0399	0,3780

Table 5.14: This table summarizes the values obtained for the strategic portfolios' Sortino indexes.

Table 5.15 shows the Sortino index values obtained by portfolios in which the tactical choices have been made by looking at the narrow range variation for the asset classes weights. The values contained in this table do not behave in a different way with respect to the tables in which the previous performance indicators are summarized. In particular, by considering the entire time period analysed and the one from 2010 to 2014, the highest Sortino index values have been reached by the portfolios in which the Markowitz model has been adopted. In the period of the global financial crisis, instead, the situation is reversed: the Markowitz model produces the worst portfolios and the Black and Litterman model the best ones. In the first time period examined (2003-2006) the portfolios obtained with the Markowitz model reach the highest and the lowest Sortino levels.

Sortino Index	Entire sample	2003-2006	2007-2009	2010-2014
TAA1_Mark_Roll(OC_C)	0,4541	2,0696	0,0366	1,2688
TAA1_Mark_Roll(OC_L)	0,3816	1,9098	0,0842	0,5663
TAA1_Mark_EWMA(OC_C)	0,3943	1,1181	-0,0013	1,1723
TAA1_Mark_EWMA(OC_L)	0,2939	0,8462	0,0222	0,5948
TAA1_Mark_Roll(OC_EWP_C)	0,4605	2,6826	0,0088	1,4373
TAA1_Mark_Roll(OC_EWP_L)	0,4311	2,4795	0,0209	1,2776
TAA1_Mark_EWMA(OC_EWP_C)	0,3936	1,0795	-0,0420	1,4131
TAA1_Mark_EWMA(OC_EWP_L)	0,3294	1,1831	-0,0367	1,1953
TAA1_Mark_Roll(HF_C)	0,4578	1,6106	0,0315	1,3124
TAA1_Mark_Roll(HF_L)	0,3976	1,6595	0,0879	0,6117
TAA1_Mark_EWMA(HF_C)	0,4172	1,0381	-0,0024	1,2619
TAA1_Mark_EWMA(HF_L)	0,3233	0,8564	0,0239	0,6364
TAA1_Mark_Roll(HF_EWP_C)	0,4616	2,5805	0,0104	1,4425
TAA1_Mark_Roll(HF_EWP_L)	0,4258	2,6610	0,0206	1,2059
TAA1_Mark_EWMA(HF_EWP_C)	0,3558	1,0989	-0,0588	1,3701
TAA1_Mark_EWMA(HF_EWP_L)	0,3146	1,1358	-0,0407	1,0364
TAA1_B&L_Roll(OC_C)	0,4279	1,2617	0,1323	0,5636
TAA1_B&L_Roll(OC_L)	0,4173	1,6036	0,1524	0,3903
TAA1_B&L_EWMA(OC_C)	0,3974	1,0107	0,0427	0,5181
TAA1_B&L_EWMA(OC_L)	0,3898	1,2755	0,0605	0,3911
TAA1_B&L_Roll(OC_EWP_C)	0,3724	2,1266	0,0822	0,3261
TAA1_B&L_Roll(OC_EWP_L)	0,2722	1,4245	0,1290	0,1461
TAA1_B&L_EWMA(OC_EWP_C)	0,3962	1,4094	0,0370	0,4279
TAA1_B&L_EWMA(OC_EWP_L)	0,2137	1,1349	0,0342	0,1438
TAA1_B&L_Roll(HF_C)	0,4332	1,2020	0,1737	0,4947
TAA1_B&L_Roll(HF_L)	0,4268	1,6085	0,1374	0,4491
TAA1_B&L_EWMA(HF_C)	0,3912	0,9653	0,0388	0,5100
TAA1_B&L_EWMA(HF_L)	0,3895	1,2386	0,0481	0,3972
TAA1_B&L_Roll(HF_EWP_C)	0,3658	2,0563	0,0822	0,3266
TAA1_B&L_Roll(HF_EWP_L)	0,2815	1,5469	0,1208	0,1619
TAA1_B&L_EWMA(HF_EWP_C)	0,3783	1,3165	0,0233	0,4161
TAA1_B&L_EWMA(HF_EWP_L)	0,2241	1,2150	0,0227	0,1600

Table 5.15: This table summarizes the Sortino indexes obtained for the tactical portfolios considering the narrow range variation. The terms in brackets specify the strategic portfolios on which the tactical decisions have been made.

The Sortino ratio values calculated for the portfolios in which tactical choices are made by considering the deep range in which the weights assigned to each asset class may vary are summarized in **Table 5.16**. This table exhibits the same disposition of high and low Sortino values as the previous table. Only one thing really stands out and this is that in the last period considered, all of the portfolios obtained with Black and Litterman model are the worst portfolios. The lowest values of the Sortino index are concentrated in the part of the table in which the Black and Litterman portfolios can be found.

Sortino Index	Entire sample	2003-2006	2007-2009	2010-2014
TAA2_Mark_Roll(OC_C)	0,4841	3,4634	-0,0269	2,0525
TAA2_Mark_Roll(OC_L)	0,4266	3,2740	0,0220	1,2214
TAA2_Mark_EWMA(OC_C)	0,4185	1,4215	-0,0575	1,9952
TAA2_Mark_EWMA(OC_L)	0,3358	1,0703	-0,0273	1,2139
TAA2_Mark_Roll(OC_EWP_C)	0,4441	3,1228	-0,0002	1,2761
TAA2_Mark_Roll(OC_EWP_L)	0,4129	3,4596	-0,0101	1,2121
TAA2_Mark_EWMA(OC_EWP_C)	0,3755	1,3114	-0,0759	1,4019
TAA2_Mark_EWMA(OC_EWP_L)	0,3235	1,2161	-0,0725	1,1669
TAA2_Mark_Roll(HF_C)	0,4925	1,9551	-0,0165	1,9178
TAA2_Mark_Roll(HF_L)	0,4503	2,2559	0,0326	1,3025
TAA2_Mark_EWMA(HF_C)	0,4362	1,0346	-0,0506	1,8915
TAA2_Mark_EWMA(HF_L)	0,3717	0,8994	0,0023	1,2706
TAA2_Mark_Roll(HF_EWP_C)	0,4713	2,5811	-0,0043	1,4721
TAA2_Mark_Roll(HF_EWP_L)	0,4224	2,9474	-0,0040	1,1398
TAA2_Mark_EWMA(HF_EWP_C)	0,3760	1,1703	-0,1000	1,5600
TAA2_Mark_EWMA(HF_EWP_L)	0,3220	1,0101	-0,0621	1,1298
TAA2_B&L_Roll(OC_C)	0,3760	1,5288	0,1301	0,3617
TAA2_B&L_Roll(OC_L)	0,3599	1,6638	0,1126	0,2892
TAA2_B&L_EWMA(OC_C)	0,3326	1,0346	-0,0200	0,4078
TAA2_B&L_EWMA(OC_L)	0,3167	1,1702	0,0058	0,2901
TAA2_B&L_Roll(OC_EWP_C)	0,3628	1,8096	0,1066	0,3269
TAA2_B&L_Roll(OC_EWP_L)	0,3066	1,7519	0,1174	0,1954
TAA2_B&L_EWMA(OC_EWP_C)	0,3494	1,1464	-0,0073	0,4005
TAA2_B&L_EWMA(OC_EWP_L)	0,2479	1,2516	-0,0134	0,1930
TAA2_B&L_Roll(HF_C)	0,3772	1,4190	0,1398	0,3706
TAA2_B&L_Roll(HF_L)	0,3688	1,6115	0,1165	0,3086
TAA2_B&L_EWMA(HF_C)	0,3305	0,9891	-0,0177	0,4097
TAA2_B&L_EWMA(HF_L)	0,3254	1,1417	0,0005	0,3102
TAA2_B&L_Roll(HF_EWP_C)	0,3583	1,6956	0,1128	0,3282
TAA2_B&L_Roll(HF_EWP_L)	0,3187	1,7496	0,1125	0,2199
TAA2_B&L_EWMA(HF_EWP_C)	0,3760	1,1703	-0,1000	1,5600
TAA2_B&L_EWMA(HF_EWP_L)	0,2606	1,2197	-0,0223	0,2188

Table 5.16: This table summarizes the Sortino ratios obtained for the tactical portfolios considering the deep range variation. The terms in brackets specify the strategic portfolios on which the tactical decisions have been made.

5.5. INFORMATION RATIO

The *information ratio* or the *appraisal ratio* is a performance measure which allows to directly compare the two asset allocation strategies used in this work: the strategic and the tactical asset allocation. This index analyses the investors' skills on trying to beat the benchmark which they have considered in order to construct their portfolio. In particular, the information ratio is a ratio between the excess return of the portfolio considered with respect to the benchmark portfolio and the standard deviation of this excess return. The information ratio formula, in fact, is given by:

$$\mathrm{IR} = \frac{\mathrm{R_p} - \mathrm{R_B}}{\sigma_{\mathrm{R_p} - \mathrm{R_B}}},$$

where R_p and R_B are, respectively, the portfolio and the benchmark returns and $\sigma_{R_p - R_B}$ is the *tracking error*, or the risk that arises by considering the difference between the return of the portfolio and that of the benchmark. In this work, the strategic portfolios are considered as the benchmark portfolios. Hence, the higher the information ratio, the better the performance of the tactical portfolio. If the information ratio is positive, the portfolio tactically managed performs better than the strategic portfolio, if it is negative the opposite is true.

The notation in the following tables indicates that if the information ratio of *TAA1_Mark_Roll(OC_C)* is calculated, it means that it has been achieved by considering the performance of the tactical portfolio (*TAA1_Mark_Roll(OC_C)*) with respect to its benchmark which is the strategic portfolio whose name is contained in the brackets (*OC_C*).

The tactical portfolios that perform better than their strategic portfolios and also with respect to all the others tactical portfolios are those in which the Black and Litterman model has been used. This is true for all periods analysed in **Table 5.17**, apart from the period from 2003 to 2006. On the opposite, the worst portfolios are those obtained with the Markowitz model.

Information Ratio	Minimum value	Maximum value
Entire sample	-0,0616 *TAA1_Mark_EWMA(OC_L)*	0,2154 *TAA1_B&L_Roll(HF_L)*
2003-2006	0,0817 *TAA1_Mark_EWMA(OC_L)*	0,6959 *TAA2_Mark_Roll(OC_EWP_L)*
2007-2009	-0,2287 *TAA2_Mark_EWMA(OC_L)*	0,1984 *TAA1_B&L_Roll(OC_EWP_C)*
2010-2014	-0,1268 *TAA1_Mark_EWMA(OC_C)*	0,2317 *TAA1_B&L_Roll(HF_L)*

Table 5.17: Minimum and the maximum values of the information ratio achieved in the periods considered

Table 5.18 summarises the information ratios obtained by considering the narrow range in which the asset classes weights may vary. Almost all of the portfolios tactically managed, that result inefficient, are those in which the Markowitz model has been used. Whereas, the portfolios in which the Black and Litterman model has been used to obtain the tactical portfolios have a positive and high levels of the information ratio. In particular, in the first sub-period analysed, all the portfolios tactically managed give back an efficient portfolio, in fact, the information ratio is always positive. In the second and the last periods, a high number of tactical portfolios, in which the Markowitz model has been adopted, have a negative value of the information ratio. This means that the strategic portfolios on which the tactical portfolios are based are more efficient than those tactically managed.

Information Ratio	Entire sample	2003-2006	2007-2009	2010-2014
TAA1_Mark_Roll(OC_C)	0,1116	0,5438	-0,0199	-0,0748
TAA1_Mark_Roll(OC_L)	0,0660	0,3762	-0,0313	-0,0918
TAA1_Mark_EWMA(OC_C)	-0,0192	0,2100	-0,1034	-0,1268
TAA1_Mark_EWMA(OC_L)	-0,0616	0,0817	-0,1567	-0,0871
TAA1_Mark_Roll(OC_EWP_C)	0,1607	0,3810	0,1057	0,0507
TAA1_Mark_Roll(OC_EWP_L)	0,1649	0,5885	-0,0461	0,1187
TAA1_Mark_EWMA(OC_EWP_C)	0,0576	0,1151	0,0296	0,0364
TAA1_Mark_EWMA(OC_EWP_L)	0,0076	0,1354	-0,1624	0,0671
TAA1_Mark_Roll(HF_C)	0,1070	0,4675	-0,0483	-0,0747
TAA1_Mark_Roll(HF_L)	0,0986	0,4485	-0,0290	-0,0759
TAA1_Mark_EWMA(HF_C)	0,0012	0,2420	-0,1262	-0,0958
TAA1_Mark_EWMA(HF_L)	-0,0208	0,1967	-0,1825	-0,0764
TAA1_Mark_Roll(HF_EWP_C)	0,1876	0,4320	0,1158	0,0692
TAA1_Mark_Roll(HF_EWP_L)	0,1998	0,6230	-0,0159	0,1388
TAA1_Mark_EWMA(HF_EWP_C)	0,0593	0,1483	0,0018	0,0376
TAA1_Mark_EWMA(HF_EWP_L)	0,0309	0,2023	-0,1434	0,0725
TAA1_B&L_Roll(OC_C)	0,1786	0,3286	0,1269	0,1670
TAA1_B&L_Roll(OC_L)	0,2069	0,5063	0,0870	0,1715
TAA1_B&L_EWMA(OC_C)	0,1453	0,3110	-0,0009	0,1424
TAA1_B&L_EWMA(OC_L)	0,1705	0,4317	-0,0704	0,1728
TAA1_B&L_Roll(OC_EWP_C)	0,1386	0,3684	0,1984	-0,0470
TAA1_B&L_Roll(OC_EWP_L)	0,1109	0,3601	0,1410	-0,0242
TAA1_B&L_EWMA(OC_EWP_C)	0,1230	0,2188	0,1369	0,0545
TAA1_B&L_EWMA(OC_EWP_L)	0,0418	0,2326	0,0059	-0,0253
TAA1_B&L_Roll(HF_C)	0,1976	0,3902	0,1552	0,1479
TAA1_B&L_Roll(HF_L)	0,2154	0,5204	0,0524	0,2317
TAA1_B&L_EWMA(HF_C)	0,1547	0,3297	-0,0122	0,1621
TAA1_B&L_EWMA(HF_L)	0,1770	0,4487	-0,0835	0,1871
TAA1_B&L_Roll(HF_EWP_C)	0,1616	0,4070	0,1963	-0,0069
TAA1_B&L_Roll(HF_EWP_L)	0,1316	0,4182	0,1393	0,0029
TAA1_B&L_EWMA(HF_EWP_C)	0,1427	0,2555	0,1249	0,0831
TAA1_B&L_EWMA(HF_EWP_L)	0,0656	0,2964	0,0013	0,0018

Table 5.18: This table summarizes the information ratios obtained by considering the narrow range variation. The terms in brackets specify the passive portfolio on which the active portfolio refers.

In **Table 5.19** there are the information ratios achieved by considering the deep range variation in which the tactical portfolios have been obtained are shown. The values of this ratio exhibit, in almost all of the periods analysed (apart from the period of 2007-2009), a more efficient tactical management than the strategic one. In fact, in these periods, the information ratio is positive in most cases; thus, the performance of the tactical choices made on strategic portfolios is better than their underlined portfolios. Only during the period of the global financial crisis the portfolios tactically managed perform in a poor way with respect to their benchmarks. In particular, the information ratio is negative for almost all of the portfolios obtained with the Markowitz model and for half of those achieved with the Black and Litterman. Furthermore, in this period the strategic portfolios over-perform their tactical management.

Information Ratio	Entire sample	2003-2006	2007-2009	2010-2014
TAA2_Mark_Roll(OC_C)	0,1190	0,6415	-0,1270	0,0175
TAA2_Mark_Roll(OC_L)	0,1189	0,5195	-0,1102	0,0572
TAA2_Mark_EWMA(OC_C)	0,0002	0,2063	-0,1911	0,0171
TAA2_Mark_EWMA(OC_L)	-0,0077	0,1417	-0,2287	0,0549
TAA2_Mark_Roll(OC_EWP_C)	0,1642	0,5895	0,0752	-0,0196
TAA2_Mark_Roll(OC_EWP_L)	0,1331	0,6959	-0,0974	0,0415
TAA2_Mark_EWMA(OC_EWP_C)	0,0754	0,2834	-0,0360	0,0138
TAA2_Mark_EWMA(OC_EWP_L)	-0,0013	0,2080	-0,2189	0,0333
TAA2_Mark_Roll(HF_C)	0,1166	0,5297	-0,1239	0,0199
TAA2_Mark_Roll(HF_L)	0,1336	0,5497	-0,1085	0,0716
TAA2_Mark_EWMA(HF_C)	0,0130	0,2251	-0,1972	0,0186
TAA2_Mark_EWMA(HF_L)	0,0254	0,2004	-0,1973	0,0594
TAA2_Mark_Roll(HF_EWP_C)	0,1942	0,5987	0,0779	0,0134
TAA2_Mark_Roll(HF_EWP_L)	0,1747	0,6793	-0,0600	0,0654
TAA2_Mark_EWMA(HF_EWP_C)	0,0800	0,2718	-0,0645	0,0435
TAA2_Mark_EWMA(HF_EWP_L)	0,0323	0,2076	-0,1735	0,0609
TAA2_B&L_Roll(OC_C)	0,1831	0,5541	0,1050	0,1111
TAA2_B&L_Roll(OC_L)	0,1804	0,5739	0,0512	0,1178
TAA2_B&L_EWMA(OC_C)	0,1447	0,3887	-0,0573	0,1501
TAA2_B&L_EWMA(OC_L)	0,1429	0,4436	-0,0764	0,1170
TAA2_B&L_Roll(OC_EWP_C)	0,1763	0,4499	0,1772	0,0362
TAA2_B&L_Roll(OC_EWP_L)	0,1601	0,5545	0,1156	0,0411
TAA2_B&L_EWMA(OC_EWP_C)	0,1477	0,3135	0,0495	0,1053
TAA2_B&L_EWMA(OC_EWP_L)	0,0946	0,4105	-0,0556	0,0390
TAA2_B&L_Roll(HF_C)	0,1884	0,5407	0,1061	0,1307
TAA2_B&L_Roll(HF_L)	0,1890	0,5686	0,0517	0,1394
TAA2_B&L_EWMA(HF_C)	0,1506	0,3906	-0,0555	0,1621
TAA2_B&L_EWMA(HF_L)	0,1544	0,4507	-0,0812	0,1390
TAA2_B&L_Roll(HF_EWP_C)	0,1942	0,4809	0,1702	0,0819
TAA2_B&L_Roll(HF_EWP_L)	0,1830	0,5920	0,1123	0,0811
TAA2_B&L_EWMA(HF_EWP_C)	0,0800	0,2718	-0,0645	0,0435
TAA2_B&L_EWMA(HF_EWP_L)	0,1237	0,4499	-0,0520	0,0792

Table 5.19: This table summarizes the information ratios obtained by considering the deep range variation. The terms in brackets specify the passive portfolio on which the active portfolio refers.

5.6. VALUE AT RISK

The *Value at Risk*, also known as *VaR*, represents the maximum loss that the investors may suffer, with a given level of confidence equal to α, if they hold their portfolio without operating any changes. The VaR is a measure of risk that takes into account the market risks.

In order to obtain this measure of risk, the realized returns have been sorted and the quantile of this distribution that corresponds to the α level has been found. In this work α is equal to 5%. The VaR answers the question about which is the minimum loss that the investors can suffer in the 5% of the worst cases. Or in other words, which is the maximum loss that an investor can suffer in the 95% of the best cases. Since the VaR measures the losses, the higher the value at risk, the worse the investment performance. Hence, by analysing the tables in which the values of the VaR are summarized, the best portfolio, in this case, is the one with the lowest value of VaR.

The minimum and maximum values contained in **Table 5.20** identify the best and the worst portfolios in terms of the value at risk. The best portfolios are those whose VaR value is the lowest and in the most part of the periods analysed it is *HF_C*. This portfolio is the strategic portfolio composed with stocks, bonds and hedge funds and whose data are expressed in common currency. Only during the last period, 2010-2014, the portfolios that provide the lowest minimum loss, in the 5% of the worst cases, are tactical portfolios *TAA1_B&L_Roll(OC_EWP_L)* and *TAA1_B&L_EWMA(OC_EWP_L)*. On the other hand, in all of the cases in which the VaR levels are very high, the worst portfolios are represented by portfolios in which the tactical decisions by considering the deep range variation are made. In particular, *TAA2_B&L_EWMA(HF_EWP_L)* is the worst portfolio in three of four periods and *TAA2_Mark_Roll(OC_C)* achieves the maximum loss in the last period.

Value at Risk	Minimum value	Maximum value
Entire sample	1,8746 *HF_C*	8,5774 *TAA2_B&L_EWMA(HF_EWP_L)*
2003-2006	0,9387 *HF_C*	5,1753 *TAA2_B&L_EWMA(HF_EWP_L)*
2007-2009	4,7084 *HF_C*	14,5232 *TAA2_B&L_EWMA(HF_EWP_L)*
2010-2014	9,9958 *TAA1_B&L_Roll(OC_EWP_L)* *TAA1_B&L_EWMA(OC_EWP_L)*	2,0525 *TAA2_Mark_Roll(OC_C)*

Table 5.20: Minimum and the maximum values of the VaR achieved in the periods considered.

The strategic portfolios that generate the highest losses are those in which the data in local currencies have been used to build these portfolios, irrespectively of the periods considered. By looking at the data in local currencies, the portfolios in which only stocks and bonds have been included in the asset allocation are those whose losses are the highest. On the other hand, the portfolios in which the lowest VaR has been achieved are those in which common currency and in particular, stocks, bonds and hedge funds have been used.

Value at Risk	Entire sample	2003-2006	2006-2009	2009-2014
OC_C	1,9462	1,0468	5,1618	2,0490
OC_L	3,4149	1,6915	6,5028	3,6856
OC_EWP_C	2,7242	1,0703	6,6416	2,6808
OC_EWP_L	3,2750	1,3241	7,1960	3,7329
HF_C	1,8746	0,9387	4,7084	1,9064
HF_L	3,1263	1,5877	5,9404	3,3700
HF_EWP_C	2,7030	1,0213	6,4040	2,7414
HF_EWP_L	2,9456	1,1859	6,8967	3,4991

Table 5.21: This table summarizes the values obtained for the strategic portfolios' VaR.

Table 5.22 shows that in all the periods analysed to calculate the VaR the highest losses are reached by the portfolios in which the Black and Litterman model has been applied. Whereas, portfolios which are constructed with the Markowitz model result in the lowest values of this index. This situation happens in almost all of the periods taken into account and only in some cases the Black and Litterman portfolios have low VaR values[50] and those built with the Markowitz model achieved high levels of VaR[51].

[50] *TAA1_B&L_Roll(OC_L)* and *TAA1_B&L_Roll(HF_L)* in the low volatility period and *TAA1_B&L_EWMA(OC_C)* and *TAA1_B&L_EWMA(HF_C)* in the period of high-volatility.
[51] *TAA1_Mark_EWMA(HF_L)* in the period of 2003-2006.

Value at Risk	Entire sample	2003-2006	2006-2009	2009-2014
TAA1_Mark_Roll(OC_C)	2,1384	1,7211	7,0051	1,1754
TAA1_Mark_Roll(OC_L)	2,8744	1,9914	8,0210	1,9657
TAA1_Mark_EWMA(OC_C)	2,4947	2,4085	6,0843	1,0600
TAA1_Mark_EWMA(OC_L)	3,5929	2,7062	6,8603	2,0010
TAA1_Mark_Roll(OC_EWP_C)	3,0812	1,6332	8,7278	1,5139
TAA1_Mark_Roll(OC_EWP_L)	2,9323	1,5996	9,7863	1,3995
TAA1_Mark_EWMA(OC_EWP_C)	3,8063	2,3919	8,0177	1,5444
TAA1_Mark_EWMA(OC_EWP_L)	3,3562	2,2682	9,3015	1,2258
TAA1_Mark_Roll(HF_C)	2,4073	2,1728	6,4674	0,8610
TAA1_Mark_Roll(HF_L)	2,6713	2,2962	7,8584	1,7186
TAA1_Mark_EWMA(HF_C)	2,4463	2,5111	5,4053	0,8607
TAA1_Mark_EWMA(HF_L)	3,3201	3,2758	7,0851	1,7067
TAA1_Mark_Roll(HF_EWP_C)	2,8676	1,4167	8,7433	1,5158
TAA1_Mark_Roll(HF_EWP_L)	2,9591	1,4820	9,8169	1,1895
TAA1_Mark_EWMA(HF_EWP_C)	3,5786	2,7932	8,0135	1,4997
TAA1_Mark_EWMA(HF_EWP_L)	3,7080	2,4798	9,4019	1,2980
TAA1_B&L_Roll(OC_C)	4,4218	2,3471	7,1783	4,4431
TAA1_B&L_Roll(OC_L)	4,4638	1,6023	8,1962	4,8453
TAA1_B&L_EWMA(OC_C)	4,3878	3,2468	5,9295	4,4416
TAA1_B&L_EWMA(OC_L)	4,5925	3,2349	7,7234	4,8453
TAA1_B&L_Roll(OC_EWP_C)	4,8630	1,9583	9,1378	6,1991
TAA1_B&L_Roll(OC_EWP_L)	8,1486	3,2157	12,4005	9,9958
TAA1_B&L_EWMA(OC_EWP_C)	4,3209	2,5371	7,9231	6,1991
TAA1_B&L_EWMA(OC_EWP_L)	8,3514	3,6519	12,8524	9,9958
TAA1_B&L_Roll(HF_C)	4,5476	2,1260	7,5545	4,5671
TAA1_B&L_Roll(HF_L)	4,3074	1,3838	7,5758	4,2263
TAA1_B&L_EWMA(HF_C)	4,5476	3,4131	6,0187	4,5671
TAA1_B&L_EWMA(HF_L)	4,9123	3,4067	7,3621	4,9147
TAA1_B&L_Roll(HF_EWP_C)	4,9172	1,9826	9,2187	5,9466
TAA1_B&L_Roll(HF_EWP_L)	7,5976	2,7653	12,0581	9,2526
TAA1_B&L_EWMA(HF_EWP_C)	4,6692	2,7366	8,0203	5,9466
TAA1_B&L_EWMA(HF_EWP_L)	7,8450	3,3391	12,3797	9,2526

Table 5.22: This table summarizes the VaR obtained for the tactical portfolios considering the narrow range variation. The terms in brackets specify the strategic portfolios on which the tactical decisions have been made.

Table 5.23 exhibits the values of the VaR achieved for portfolios in which the tactical decisions are made by considering that the weights assigned to each asset class may vary in the deep range. In this table, nothing different from the previous one happens: the portfolios in which the model of Markowitz has been used are those that have low values of the VaR and those constructed with the adoption of the Black and Litterman model have high maximum losses levels.

Value at Risk	Entire sample	2003-2006	2006-2009	2009-2014
TAA2_Mark_Roll(OC_C)	2,8471	1,0905	7,2838	0,7048
TAA2_Mark_Roll(OC_L)	3,4193	1,2239	9,1487	1,3598
TAA2_Mark_EWMA(OC_C)	2,4993	2,1882	6,0378	0,7372
TAA2_Mark_EWMA(OC_L)	3,3416	2,9879	7,8453	1,5115
TAA2_Mark_Roll(OC_EWP_C)	3,4454	1,5911	10,1538	1,8291
TAA2_Mark_Roll(OC_EWP_L)	3,2826	1,3329	10,0082	1,3510
TAA2_Mark_EWMA(OC_EWP_C)	3,8526	3,2323	8,6379	1,5886
TAA2_Mark_EWMA(OC_EWP_L)	3,7227	3,2447	8,7565	1,4762
TAA2_Mark_Roll(HF_C)	2,6274	1,9174	6,4645	0,6705
TAA2_Mark_Roll(HF_L)	3,2008	1,8928	8,1700	1,1751
TAA2_Mark_EWMA(HF_C)	2,4671	2,4639	5,3159	0,6851
TAA2_Mark_EWMA(HF_L)	3,0500	3,1366	6,9267	1,1733
TAA2_Mark_Roll(HF_EWP_C)	3,7691	1,3584	9,4486	1,3900
TAA2_Mark_Roll(HF_EWP_L)	3,7942	1,7904	9,8304	1,3465
TAA2_Mark_EWMA(HF_EWP_C)	3,4484	3,2018	7,8358	1,2980
TAA2_Mark_EWMA(HF_EWP_L)	3,8639	3,3787	8,5883	1,4900
TAA2_B&L_Roll(OC_C)	7,0327	2,8210	11,3364	8,0157
TAA2_B&L_Roll(OC_L)	7,6381	2,9482	11,6948	8,4367
TAA2_B&L_EWMA(OC_C)	7,6767	4,7124	9,2072	8,0157
TAA2_B&L_EWMA(OC_L)	8,2429	4,8461	12,5100	8,4367
TAA2_B&L_Roll(OC_EWP_C)	7,1334	2,5652	11,0679	7,8589
TAA2_B&L_Roll(OC_EWP_L)	7,7471	2,6062	13,2959	9,3237
TAA2_B&L_EWMA(OC_EWP_C)	6,6913	4,1836	9,5172	7,8589
TAA2_B&L_EWMA(OC_EWP_L)	8,0746	4,2080	14,0667	9,3237
TAA2_B&L_Roll(HF_C)	7,1345	2,9770	11,4517	8,0427
TAA2_B&L_Roll(HF_L)	7,4271	3,1527	11,6906	8,3233
TAA2_B&L_EWMA(HF_C)	7,8041	4,8981	9,1488	8,0427
TAA2_B&L_EWMA(HF_L)	8,2757	4,9762	11,8922	8,3233
TAA2_B&L_Roll(HF_EWP_C)	7,5012	2,8588	12,0746	8,6637
TAA2_B&L_Roll(HF_EWP_L)	8,1194	2,7520	13,5440	9,6593
TAA2_B&L_EWMA(HF_EWP_C)	3,4484	3,2018	7,8358	1,2980
TAA2_B&L_EWMA(HF_EWP_L)	8,5774	5,1753	14,5232	9,6593

Table 5.23: This table summarizes the VaR obtained for the tactical portfolios considering the deep range variation. The terms in brackets specify the strategic portfolios on which the tactical decisions have been made.

5.7. EXPECTED SHORTFALL

The *Expected Shortfall* (*ES*) is the average loss that exceeds the VaR. In particular, it is calculated as the mean value of all the values that belong to the quantiles lower than the one considered by the level of α. The expected shortfall is higher in absolute values than the VaR. The ES answers the question about which is the average loss that the investor can suffer in the 95% of the best cases, or which is the average minimum loss that exceeds the level of VaR. In this case, as it has been considered for the VaR, the higher the ES, the worse the investment performance. Hence, the best portfolio is that whose shortfall level is the lowest.

The minimum levels of the expected shortfall have been achieved by the portfolios in which stocks, bonds and hedge funds have been included, or by portfolios in which those portfolios have been tactically managed. Only in the last period considered the best portfolio in terms of the expected shortfall is a tactical portfolio. In all the other periods the portfolios linked to the lowest average loss that may exceed the VaR are strategic portfolios. On the other hand, the portfolios that record the highest expected shortfall are those constructed with the model of Black and Litterman in which the tactical choices have been made by considering that the weights of each asset class may vary on the deep range.

Expected Shortfall	Minimum value	Maximum value
Entire sample	3,8546 *HF_C*	13,7511 *TAA2_B&L_EWMA(HF_EWP_L)*
2003-2006	1,5152 *HF_EWP_C*	6,3242 *TAA2_B&L_EWMA(HF_C)*
2007-2009	6,4331 *HF_C*	19,7696 *TAA2_B&L_Roll(HF_EWP_L)*
2010-2014	1,0914 *TAA2_Mark_Roll(HF_C)*	12,5960 *TAA2_B&L_Roll(HF_EWP_L)* *TAA2_B&L_EWMA(HF_EWP_L)*

Table 5.24: Minimum and the maximum values of the expected shortfall achieved in the periods considered.

In **Table 5.25** the expected shortfalls achieved by the strategic portfolios are shown.

Expected Shortfall	Entire sample	2003-2006	2006-2009	2009-2014
OC_C	4,1956	1,6878	6,9790	2,8834
OC_L	5,4485	2,8455	8,3050	4,2219
OC_EWP_C	5,9016	1,5640	9,7509	4,2756
OC_EWP_L	6,4429	1,9461	9,5673	5,2419
HF_C	3,8546	1,7826	6,4331	2,5430
HF_L	4,9893	2,8631	7,6464	3,7692
HF_EWP_C	5,6429	1,5152	9,5702	4,0893
HF_EWP_L	6,0897	1,8509	9,4070	4,9304

Table 5.25: This table summarizes the values obtained for the strategic portfolios' expected shortfalls.

The expected shortfalls calculated on the strategic portfolios highlight high values when the portfolios have been constructed with only stocks and bonds (except during the period from 2003 and 2006 for the portfolio *HF_L*) and low values for portfolios achieved with stocks, bonds and hedge funds.

In **Table 5.26**, the portfolios made by applying tactical strategies based on strategic portfolios and on the narrow range variation can be divided in two groups. In one group there are those portfolios composed considering the Markowitz model and they have low expected shortfall levels (apart from *TAA1_Mark_EWMA(OC_L)* and *TAA1_Mark_EWMA(HF_L)*). In the other group the Black and Litterman portfolios are characterised by high values on this index.

Expected Shortfall	Entire sample	2003-2006	2006-2009	2009-2014
TAA1_Mark_Roll(OC_C)	5,3973	2,1225	9,8979	1,6049
TAA1_Mark_Roll(OC_L)	6,3693	2,1510	11,6258	2,6949
TAA1_Mark_EWMA(OC_C)	4,8765	3,2667	8,7477	1,6138
TAA1_Mark_EWMA(OC_L)	6,4402	4,6234	10,5413	2,5591
TAA1_Mark_Roll(OC_EWP_C)	6,7231	1,6445	12,7840	2,1770
TAA1_Mark_Roll(OC_EWP_L)	7,1421	2,1570	12,9999	1,9620
TAA1_Mark_EWMA(OC_EWP_C)	6,1811	3,8992	11,0702	2,1711
TAA1_Mark_EWMA(OC_EWP_L)	6,9969	3,5132	12,6413	1,9813
TAA1_Mark_Roll(HF_C)	4,8246	2,5123	9,1630	1,3768
TAA1_Mark_Roll(HF_L)	6,2194	2,6655	11,2801	2,3190
TAA1_Mark_EWMA(HF_C)	4,4416	3,2360	7,7724	1,3968
TAA1_Mark_EWMA(HF_L)	5,9791	4,3603	10,0194	2,1779
TAA1_Mark_Roll(HF_EWP_C)	6,5950	1,8929	12,4898	2,0428
TAA1_Mark_Roll(HF_EWP_L)	7,2546	1,8485	13,4148	1,9553
TAA1_Mark_EWMA(HF_EWP_C)	6,1827	3,5380	11,5483	2,0341
TAA1_Mark_EWMA(HF_EWP_L)	7,0685	3,6763	12,7454	2,0980
TAA1_B&L_Roll(OC_C)	6,7930	3,1180	10,8522	5,3498
TAA1_B&L_Roll(OC_L)	7,8708	3,2886	11,8954	6,6468
TAA1_B&L_EWMA(OC_C)	6,6602	4,1010	10,2671	5,7253
TAA1_B&L_EWMA(OC_L)	7,7038	4,1961	11,0440	6,6468
TAA1_B&L_Roll(OC_EWP_C)	8,6692	2,5319	12,8478	7,4162
TAA1_B&L_Roll(OC_EWP_L)	12,6715	3,7769	16,6249	11,4214
TAA1_B&L_EWMA(OC_EWP_C)	7,5725	3,0320	10,3837	7,4162
TAA1_B&L_EWMA(OC_EWP_L)	12,6371	3,7579	16,4332	11,4214
TAA1_B&L_Roll(HF_C)	7,1654	3,9072	11,2895	5,8660
TAA1_B&L_Roll(HF_L)	7,2322	2,9813	10,9367	5,9696
TAA1_B&L_EWMA(HF_C)	6,8325	4,3737	10,4541	5,8660
TAA1_B&L_EWMA(HF_L)	7,6886	4,4637	10,9039	6,6632
TAA1_B&L_Roll(HF_EWP_C)	8,6796	2,6724	13,2432	7,5143
TAA1_B&L_Roll(HF_EWP_L)	12,1648	3,5993	16,5329	11,0019
TAA1_B&L_EWMA(HF_EWP_C)	7,7666	3,2083	10,9506	7,5143
TAA1_B&L_EWMA(HF_EWP_L)	12,1816	3,7798	16,0818	11,0019

Table 5.26: This table summarizes the expected shortfalls obtained for the tactical portfolios considering the narrow range variation. The terms in brackets specify the strategic portfolios on which the tactical decisions have been made.

Table 5.27 confirms that the behaviour of the expected shortfalls shown follows the same distribution between high and low expected shortfall values than **Table 5.26**. The portfolios obtained with the Markowitz model, in fact, have low average losses that exceed the VaR value and the Black and Litterman portfolios model are the worst.

Expected Shortfall	Entire sample	2003-2006	2006-2009	2009-2014
TAA2_Mark_Roll(OC_C)	5,5450	1,6042	9,5640	1,1240
TAA2_Mark_Roll(OC_L)	7,1357	1,9183	12,4752	1,6472
TAA2_Mark_EWMA(OC_C)	4,8176	2,5876	8,3813	1,2206
TAA2_Mark_EWMA(OC_L)	6,5640	3,9358	10,7541	1,6387
TAA2_Mark_Roll(OC_EWP_C)	7,4827	2,2336	13,5140	2,1020
TAA2_Mark_Roll(OC_EWP_L)	7,5136	1,6302	13,5080	1,8317
TAA2_Mark_EWMA(OC_EWP_C)	6,8549	3,8672	12,3414	2,1471
TAA2_Mark_EWMA(OC_EWP_L)	6,9750	3,8176	12,1462	1,9191
TAA2_Mark_Roll(HF_C)	4,9200	2,2686	8,5477	1,0914
TAA2_Mark_Roll(HF_L)	6,4109	2,3641	11,2474	1,4675
TAA2_Mark_EWMA(HF_C)	4,4429	3,1569	7,3268	1,1227
TAA2_Mark_EWMA(HF_L)	5,9681	4,4449	9,4591	1,4610
TAA2_Mark_Roll(HF_EWP_C)	7,0532	2,8582	12,5482	1,8712
TAA2_Mark_Roll(HF_EWP_L)	7,4971	2,1247	13,4299	1,9016
TAA2_Mark_EWMA(HF_EWP_C)	6,3120	3,8156	11,2315	1,8898
TAA2_Mark_EWMA(HF_EWP_L)	6,8891	4,3071	11,9733	1,9121
TAA2_B&L_Roll(OC_C)	10,8788	4,2730	16,1294	9,2701
TAA2_B&L_Roll(OC_L)	11,4980	4,4853	16,9964	9,9931
TAA2_B&L_EWMA(OC_C)	10,4280	5,9889	14,5518	9,2701
TAA2_B&L_EWMA(OC_L)	11,4269	5,9656	15,7731	9,9931
TAA2_B&L_Roll(OC_EWP_C)	10,8616	3,6963	16,3608	9,6027
TAA2_B&L_Roll(OC_EWP_L)	13,3478	3,7844	18,9880	12,3755
TAA2_B&L_EWMA(OC_EWP_C)	10,1571	4,6992	13,9399	9,6027
TAA2_B&L_EWMA(OC_EWP_L)	13,3784	4,8582	17,6823	12,3755
TAA2_B&L_Roll(HF_C)	10,9994	4,5890	16,3429	9,4000
TAA2_B&L_Roll(HF_L)	11,4122	4,6555	16,9208	9,8228
TAA2_B&L_EWMA(HF_C)	10,6208	6,3242	14,7658	9,4000
TAA2_B&L_EWMA(HF_L)	11,2055	6,3087	15,3842	9,8228
TAA2_B&L_Roll(HF_EWP_C)	11,8454	4,1210	17,6298	10,3408
TAA2_B&L_Roll(HF_EWP_L)	13,6167	4,3279	19,7696	12,5960
TAA2_B&L_EWMA(HF_EWP_C)	6,3120	3,8156	11,2315	1,8898
TAA2_B&L_EWMA(HF_EWP_L)	13,7511	5,3494	18,4979	12,5960

Table 5.27: This table summarizes the expected shortfalls obtained for the tactical portfolios considering the deep range variation. The terms in brackets specify the strategic portfolios on which the tactical decisions have been made.

EMPIRICAL RESULTS - CONCLUSIONS

This thesis aims to understand if the construction of portfolios with frequent rebalancing on the asset allocation and the inclusion of alternative asset classes can generate a better result than the classic stocks and bonds portfolios. In order to find an adequate answer, several types of portfolios have been obtained, more precisely the total amount of portfolios calculated in this work is 72.

First of all, 8 strategic portfolios have been built. These portfolios have been achieved by considering the inclusion of only stocks and bonds in one group and of stocks, bonds and hedge funds in the other. Both groups have been constructed in common currency and also in local currencies and by assigning the weights of each asset class by considering two different approaches (the equally-weighted approach and the one in which the weights are assign 70% - 30% to stocks and bonds and 70% - 25% - 5% to stocks, bonds and hedge funds).

Secondly, these portfolios have been tactically managed by considering not only the asset classes included in the strategic portfolios but the entire available set of asset classes examined in **Chapter 3**. In order to find the weights of each asset class to construct these portfolios, it has been necessary the use of out-of-sample estimations and some models to combine the inputs obtained. In particular, the simple moving average and the exponentially-weighted moving average are the methods used to calculate the out-of-sample data estimation. The Black and Litterman and Markowitz models are those examined to achieve the feasible portfolios. From those the one with the level of risk as similar as the risk of the underlined strategic portfolio was chosen as optimal portfolio. These tactical decisions result in a total of 64 portfolios.

Then, the strategic and tactical portfolios have been analysed by calculating some risk-adjusted returns, measures of risks and their information ratio. These estimations allow to have a more global and precise idea about how well the portfolios were performing and which one may compensate the oscillations concerning the market in the time period considered. The portfolios achieved with the tactical asset allocation strategy are those that have obtained the highest performance values during all the periods and for almost all the performance indicators used in **Chapter 5**. In particular, by considering the distinction about the period in which the performance indicators have been calculated, some clarifications are needed. In addition, it seems to be important to understand which role the currency risk had on producing the resulting portfolios values. It is interesting to analyse in which way, if there is any, the currency risk has contributed in affecting the performance obtained by the portfolios in common currency. To do so a linear regression has been made, namely a linear regression of the portfolios' realized returns in common currency on their realized returns in local currencies and on the two exchange rates examined (the euro-dollar and the yen-dollar exchange rates). The regression coefficients are shown in **Appendix A**, but the results and the meaning of the values of the coefficients are briefly examined in what follows.

In the whole period analysed, the Sharpe ratio, the Jensen's alpha, the Treynor index and the Sortino ratio result in the fact that the best portfolios are those obtained by applying tactical choices. In particular, these portfolios are those whose weights assigned to each asset class vary in the deep range and whose construction arises from the use of the Markowitz model and the rolling sample moments method.

The indicators which measure the level of risk achieved by the portfolios constructed do not move in the same direction as the risk-adjusted returns. Both the value at risk and the expected shortfall, in fact, record the highest expected losses for the portfolios that perform better with respect to the other indicators and the lowest expected losses for the strategic portfolios.

These are the results obtained by analysing separately each risk-adjusted returns and each measure of risks. To be sure it is necessary to compare at least one indicator of each category. In particular, the figure below shows together the Sharpe ratio, the VaR and the average realized return of each portfolio.

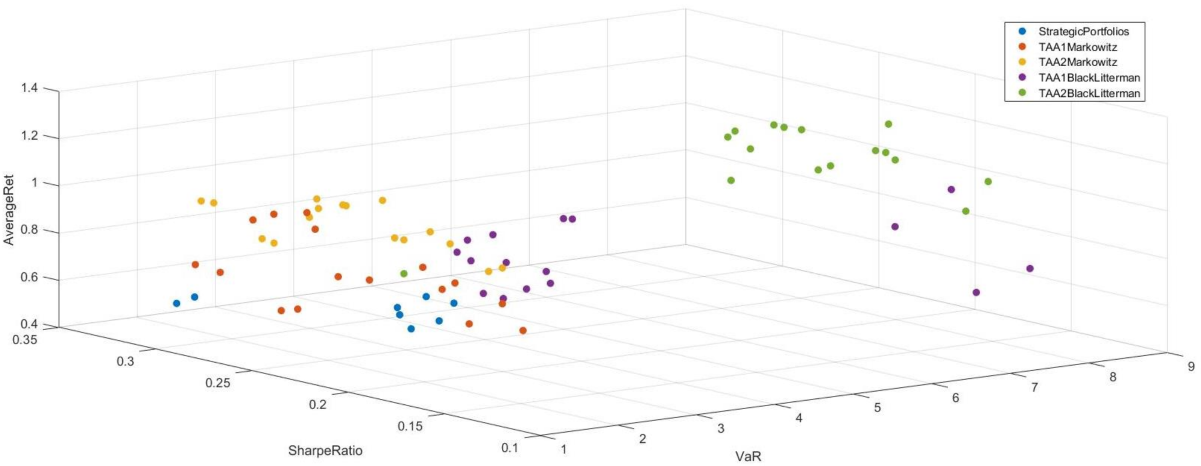


Figure C. 1: The average realized returns, the Sharpe ratios and the VaRs obtained in the whole period.

Figure C. 1 makes evident that the portfolios which can be considered the best by taking into consideration all the three parameters are those that are set on the bottom left part. In particular, those whose VaR is the lowest and whose average return and Sharpe ratio are the highest. In this picture, the distinction between one type of portfolio construction and another has been made. The strategic portfolios have high Sharpe ratios and low VaR, but also very low levels of average returns. The tactical portfolios achieved with the Markowitz model do not differ too much from the strategic portfolios on which they are based. They have low VaR and medium-high Sharpe ratios values, but higher average return levels than the strategic portfolios. The tactical portfolios obtained with the Black and Litterman model and the deep range variation of the asset allocation, instead, are concentrated on the right part of the picture. In this part there are very high levels of VaR, medium values of the average return and medium-high Sharpe ratios. The other range on which the Black and Litterman portfolios have been constructed generates values which are closer to the tactical Markowitz portfolios, with only some exceptions.

Moreover, by directly comparing the strategic portfolios with the tactical decisions made, it is not possible to compare the information ratio with the other performance indicators because they take into account different aspect of the performance of a portfolio. What is important to verify for this index is that the portfolios obtained by applying the tactical asset allocation are efficient with respect to the strategic portfolio on which they take origin. In fact, almost all of the information ratios achieved indicate a better performance of the active portfolios than their passive portfolios.

It is hard to understand which could be the effect, if there is any, of the currency risk and/or that of the tactical management by looking at the results obtained by the portfolios achieved in the whole period. However, some insight can be found by taking into account the results of the regression coefficients. In general terms, in this period the returns of the portfolios in common currency have gained some benefits from the tactical management and from the change from one currency to another. The possible losses have been minimized by these two elements.

In the period with the low-volatility, from 2003 to 2006 that precedes the global financial crisis, the risk-adjusted returns indicators and the measures of risks do not differ from those seen in the entire period analysed. The best portfolios, in fact, for every performance indicator are those in which the tactical decisions have been made by considering the Markowitz model, the rolling sample moments method and the deep range variation. The VaR and the ES, instead, reach their lowest values in the strategic portfolios.

By looking at **Figure C. 2** it is possible to see that the portfolios are more scattered than those in the previous figure analysed. In particular, the strategic portfolios are those whose values are the lowest for all the parameters considered. The tactical choices made with the Markowitz model have medium VaRs and average returns and medium-high Sharpe ratios. Their values reach the best positions in this figure, as a confirmation of what it has been previously said. Although the tactical portfolios are in the part of the graph in which the average returns and the Sharpe ratios are medium-high, their possible losses are the highest with respect to all of the other portfolios.

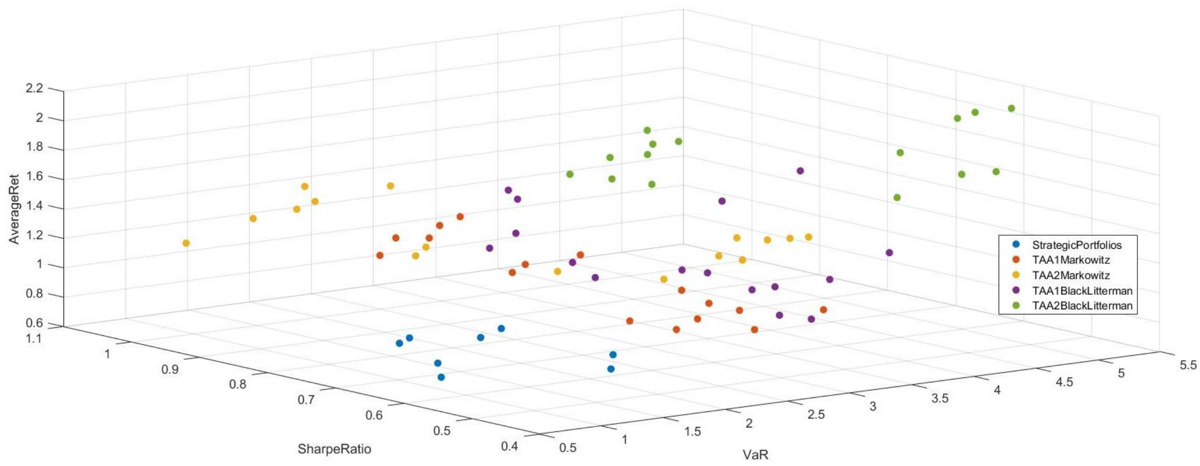


Figure C. 2: The average realized returns, the Sharpe ratios and the VaRs obtained in the whole period.

The information ratio calculated in this period does not suggest any peculiarity. All the tactical portfolios achieved in this period, in fact, are more efficient than the strategic portfolios from which they are originated.

As all the other indicators, the effect of the currency risk has not experienced any differences with respect to the entire period analysed. During the years from 2003 to 2006, in fact, the portfolios returns have contained their possible losses by including securities expressed in different currencies in the asset allocation.

The period from 2007 to 2009 is characterised by the predominance of the tactical portfolios achieved with the adoption of the Black and Litterman model. This is the only period in which these portfolios have high performance indicators. Only in one of the four risk-adjusted returns indicators, the Treynor ratio, a strategic portfolio is classified as the best portfolio.

The information ratio highlights the inefficiency of the tactical decisions made with the Markowitz model both by considering the narrow and the deep asset allocation range variation. However, the same portfolios obtained with the Black and Litterman model result almost always as efficient portfolios.

The two measures of risks calculated in the previous chapter do not move in the same direction as the other indicators. In this case, as well as in the previously analysed periods, the portfolios on which the tactical asset allocation has been applied have high levels of expected losses by considering the value at risk and the expected shortfall.

Therefore, in this period, the tactical portfolios achieve higher performance, when they have been constructed with the Black and Litterman model, but also higher possible losses than the strategic portfolios. This statement is also confirmed in **Figure C. 3** where the strategic portfolios are placed in three areas. Two of eight of these portfolios have low VaR and very low, even negative, Sharpe ratios and average returns. Other two strategic portfolios have medium VaR, negative average returns, but high Sharpe ratios. The remaining four portfolios, instead, have high Sharpe ratios, medium high average returns and low value at risk. The tactical decisions made with the Markowitz model are concentrated almost in the same part of the graph. These portfolios have positive, but not very high, Sharpe ratios, very low average returns and medium high VaR values. The last type of portfolios that need to be examined is the Black and Litterman tactical portfolios. These can be analysed by looking at the two range in which the weights of the asset classes may vary. The portfolios achieved by considering the narrow range are characterised by medium-high VaR values, low average returns and low but positive Sharpe ratios. The portfolios whose asset allocation variation follow the deep range, instead, are placed where the VaR is high and the average returns and the Sharpe ratios are also high.

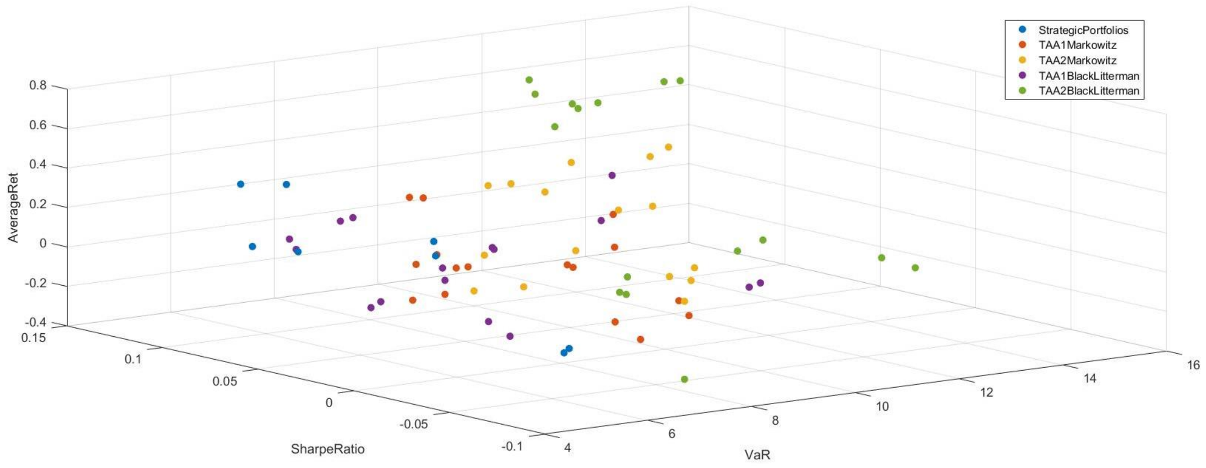


Figure C. 3: The average realized returns, the Sharpe ratios and the VaRs obtained in the whole period.

In the period of high-volatility and the high-instability of the financial markets, the portfolios achieved in this work have not recorded worsening returns caused by the presence of the currency risk. On the opposite, these portfolios have minimized the possible losses and sometimes they have also amplified their gains.

The last period analysed, from 2010 to 2014, is the one in which the European sovereign debt crisis took place. By considering the risk-adjusted returns indicators, only the Treynor ratio takes the highest value when it has been calculated for a strategic portfolio. All the others performance indicators have high values for the tactical portfolios. In particular, the Markowitz portfolios have the highest levels of performances.

The tactical decisions made have created efficient portfolios, with only some exception when the Markowitz model is used in the asset allocation in which the weights vary in the narrow range. In the deep range variation, instead, almost all the tactical portfolios over performed the strategic portfolios from which they were originated.

This is the only period in which the value at risk and the expected shortfall have the lowest losses recorded by portfolios achieved by tactically managing the strategic portfolios. In this period, the tactical asset allocation highlights its benefits not only in terms of performance levels but also in terms of risks.

In addition, this period is the only one in which the different types of portfolios are so clearly grouped in three parts, as **Figure C. 4** shown. In this figure, in fact, the portfolios achieved with the Markowitz model are characterised by low values of VaR, medium-high Sharpe ratios and medium-low average returns levels. The strategic portfolios are located in the central part of the figure. These portfolios have low values of average return, medium value at risk and medium-high Sharpe ratios. The Black and Litterman tactical portfolios are the most extreme ones. They have very high VaR, medium-low Sharpe ratios but medium-high average returns.

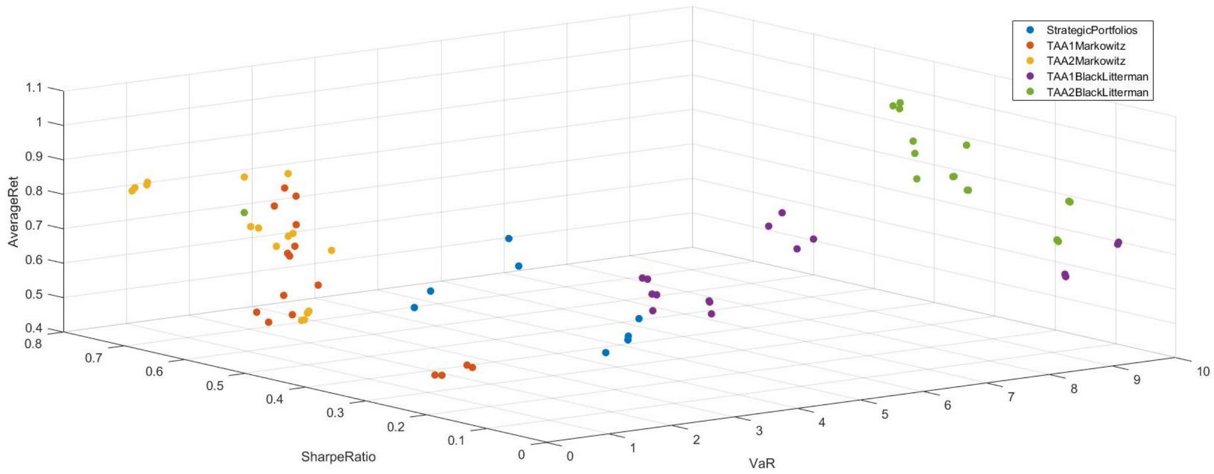


Figure C. 4: The average realized returns, the Sharpe ratios and the VaRs obtained in the whole period.

In this period, there are several portfolios that have experienced a poor performance due to the presence of securities expressed in different currencies. In all the periods analysed the portfolios that achieve the best performance in terms of risks and returns values are those whose data have usually been expressed in common currency. It is rare that the best portfolios have been constructed with local currencies data. This is probably due to the effect of the currency risk. In almost all the periods, in fact, the presence of different currencies has provided some small benefit by contrasting the possible losses or by rarely amplifying the gains. During the last period analysed, when the European sovereign debt crisis took place, a high number of portfolios have suffered from the presence of currency risk.

By summarizing the results obtained in this work, there is not a single answer to the initial question. The answer depends on several aspects, such as the situation that the financial markets are experiencing and the preferences of the institutional investors about how frequently they want to rebalance their portfolios weights. However, in the entire out-of-sample analysis the tactical portfolios, sometimes by using the Markowitz and others with the Black and Litterman models, have created better performances than the strategic portfolios on which they are based. Also during the periods of high-volatility levels or those in which the market is more affected by fluctuations and uncertainty, the tactical asset allocation has provided positive effects on portfolio performances. In addition, in almost all of these portfolios, all or at least some of the alternative asset classes have been used to obtain these results. Hence, by including the alternative asset classes in a frequently rebalanced asset allocation the results could be a better portfolios performance than the classical strategic portfolios made with only stocks and bonds or with the inclusion of hedge funds.

APPENDICES

Appendix A REGRESSION COEFFICIENTS

In order to understand the effect, if any, of the currency risk on defining the portfolios returns achieved in common currency, the following regression coefficients have been calculated.

The regressions whose results are shown in this appendix have been obtained by following this formula:

$$\mathrm{RealRet}_{\mathrm{common\ currency}} = \beta_0 + \beta_1 \mathrm{RealRet}_{\mathrm{local\ currencies}} + \beta_2 \mathrm{ExcRate}_{€/\$} + \beta_3 \mathrm{ExcRate}_{¥/\$},$$

where $RealRet_{common\ currency}$ is the realized returns obtained with the data expressed in common currency, β_0 is the intercept of the regression line (it is the value of the regression line when the parameters of β_1, β_2 and β_3 are zeros), β_1 is the coefficient of the realized returns in local currencies ($RealRet_{local\ currencies}$), β_2 and β_3 are the parameters of the returns of the two exchange rates (the euro-dollar exchange rate, $ExcRate_{€/\$}$, and the yen-dollar exchange rate, $ExcRate_{¥/\$}$). The parameters β_1, β_2 and β_3 represent the relation between the variable to which each are associated and the dependent variable ($RealRet_{common\ currency}$). In particular, the value of β_1 shows the variation of the dependent variable as a result of a unit change in the independent variable associated to the coefficient of β_1 when all the other independent variables are equal to zero.

The effect of the currency risk on the return of the portfolios in common currency can be analysed by looking at the behaviour of the $ExcRate_{€/\$}$ and the $ExcRate_{¥/\$}$ and their coefficient β_2 and β_3. In particular, if the β_2[52] has a higher value than the real weights assigned to the asset classes in euro there are two different situations that may arise: if the currency appreciates, the losses will be amplified; on the other hand, the currency and the tactical management effects will generate some benefits. Whereas, if the beta is lower than the weights of the euro asset classes, a contraction of the possible losses will happen, irrespectively of the values of the currency returns.

In what follows, the regression coefficients are presented, they have been calculated by considering the whole period and the three sub-periods analysed.

[52] The equation of the regression can be seen as the formula to obtain the returns and then the betas can be interpreted as the weights assigned to the asset classes. In particular, the β_2 which is the coefficient of $ExcRate_{€/\$}$ can be interpreted as the weights of the assets expressed in euro (the currency associated to the return of the exchange rate considered as parameter).

Regression coefficient obtained by considering the whole period, from 2003 to 2014:

	Estimate	SE	TStat	pValue
	-0,0401***	0,0145	-2,7712	0,0063
RLocal	0,9921***	0,0120	82,9434	2,88E-120
€/$	0,2386***	0,0092	25,9770	3,37E-55
¥/$	0,1598***	0,0047	33,7102	8,95E-69

Table A.1: Regression coefficients of *OC_C* on *OC_L* and on the two exchange rates.

	Estimate	SE	tStat	pValue
	-0,0399***	0,0124	-3,2174	0,0016
RLocal	0,9993***	0,0085	117,3832	6,21E-141
€/$	0,2451***	0,0073	33,5683	1,51E-68
¥/$	0,3858***	0,0045	86,5728	8,36E-123

Table A.2: Regression coefficients of *OC_EWP_C* on *OC_EWP_L* and on the two exchange rates.

	Estimate	SE	tStat	pValue
	-0,0360**	0,0141	-2,5519	0,0118
RLocal	0,9905***	0,0126	78,4793	5,36E-117
€/$	0,2185***	0,0089	24,4937	2,79E-52
¥/$	0,1487***	0,0046	32,3750	1,31E-66

Table A.3: Regression coefficients of *HF_C* on *HF_L* and on the two exchange rates.

	Estimate	SE	tStat	pValue
	-0,0362***	0,0109	-3,3046	0,0012
RLocal	1,0006***	0,0078	127,7845	5,18E-146
€/$	0,2187***	0,0063	34,6421	3,02E-70
¥/$	0,3427***	0,0040	86,2190	1,46E-122

Table A.4: Regression coefficients of *HF_EWP_C* on *HF_EWP_L* and on the two exchange rates.

	Estimate	SE	tStat	pValue
	0,0740**	0,0373	1,9822	0,0494
RLocal	0,8713***	0,0207	42,0911	5,54E-81
€/$	0,1185***	0,0176	6,7240	4,22E-10
¥/$	0,1078***	0,0130	8,2664	9,90E-14

Table A.5: Regression coefficients of *TAA1_Mark_Roll(OC_C)* on *TAA1_Mark_Roll(OC_L)* and on the two exchange rates.

	Estimate	SE	tStat	pValue
	0,1141***	0,0420	2,7157	0,0075
RLocal	0,7913***	0,0244	32,4937	8,37E-67
€/$	0,0757***	0,0198	3,8280	1,95E-04
¥/$	0,1140***	0,0149	7,6340	3,30E-12

Table A.6: Regression coefficients of *TAA1_Mark_EWMA(OC_C)* on *TAA1_Mark_EWMA(OC_L)* and on the two exchange rates.

	Estimate	SE	tStat	pValue
	0,0612	0,0691	0,8852	0,3776
RLocal	0,9436***	0,0340	27,7686	1,41E-58
€/$	0,0555***	0,0312	1,7806	0,0772
¥/$	0,0544**	0,0240	2,2635	0,0252

Table A.7: Regression coefficients of *TAA1_Mark_Roll(OC_EWP_C)* on *TAA1_Mark_Roll(OC_EWP_L)* and on the two exchange rates.

	Estimate	SE	tStat	pValue
	0,0903	0,0621	1,4551	0,1479
RLocal	0,9317***	0,0316	29,4549	1,27E-61
€/$	0,0428	0,0273	1,5686	0,1190
¥/$	0,0644***	0,0221	2,9100	0,0042

Table A.8: Regression coefficients of *TAA1_Mark_EWMA(OC_EWP_C)* on *TAA1_Mark_EWMA(OC_EWP_L)* and on the two exchange rates.

	Estimate	SE	tStat	pValue
	0,0773*	0,0418	1,8510	0,0663
RLocal	0,7925***	0,0222	35,7437	6,02E-72
€/$	0,0699***	0,0185	3,7683	2,42E-04
¥/$	0,1128***	0,0146	7,7440	1,81E-12

Table A.9: Regression coefficients of *TAA1_Mark_Roll(HF_C)* on *TAA1_Mark_Roll(HF_L)* and on the two exchange rates.

	Estimate	SE	tStat	pValue
	0,0770**	0,0311	2,4772	0,0144
RLocal	0,8158***	0,0183	44,4876	4,26E-84
€/$	0,0885***	0,0140	6,3084	3,52E-09
¥/$	0,1348***	0,0110	12,2823	6,12E-24

Table A.10: Regression coefficients of *TAA1_Mark_EWMA(HF_C)* on *TAA1_Mark_EWMA(HF_L)* and on the two exchange rates.

	Estimate	SE	tStat	pValue
	0,0540	0,0600	0,8990	0,3702
RLocal	0,9249***	0,0284	32,5369	7,11E-67
€/$	0,0798***	0,0265	3,0067	0,0031
¥/$	0,0614***	0,0209	2,9306	0,0040

Table A.11: Regression coefficients of *TAA1_Mark_Roll(HF_EWP_C)* on *TAA1_Mark_Roll(HF_EWP_L)* and on the two exchange rates.

	Estimate	SE	tStat	pValue
	0,0584	0,0580	1,0062	0,3161
RLocal	0,9334***	0,0289	32,3535	1,42E-66
€/$	0,0615**	0,0252	2,4351	0,0162
¥/$	0,0819***	0,0208	3,9455	1,26E-04

Table A.12: Regression coefficients of *TAA1_Mark_EWMA(HF_EWP_C)* on *TAA1_Mark_EWMA(HF_EWP_L)* and on the two exchange rates.

	Estimate	SE	tStat	pValue
	0,0071	0,0570	0,1251	0,9006
RLocal	0,9445***	0,0253	37,3481	2,38E-74
€/$	0,0999***	0,0287	3,4763	6,79E-04
¥/$	0,0108	0,0198	0,5463	0,5857

Table A.13: Regression coefficients of *TAA1_B&L_Roll(OC_C)* on *TAA1_B&L_Roll(OC_L)* and on the two exchange rates.

	Estimate	SE	tStat	pValue
	-0,0300	0,0643	-0,4668	0,6414
RLocal	0,9651***	0,0299	32,2622	2,01E-66
€/$	0,1670***	0,0335	4,9857	1,81E-06
¥/$	0,0488**	0,0223	2,1833	0,0307

Table A.14: Regression coefficients of *TAA1_B&L_EWMA(OC_C)* on *TAA1_B&L_EWMA(OC_L)* and on the two exchange rates.

	Estimate	SE	tStat	pValue
	0,0422	0,0797	0,5287	0,5978
RLocal	0,9081***	0,0362	25,1142	1,63E-53
€/$	0,4851***	0,0624	7,7787	1,49E-12
¥/$	0,0902***	0,0277	3,2510	0,0014

Table A.15: Regression coefficients of *TAA1_B&L_Roll(OC_EWP_C)* on *TAA1_B&L_Roll(OC_EWP_L)* and on the two exchange rates.

	Estimate	SE	tStat	pValue
	0,2990***	0,1114	2,6847	0,0081
RLocal	0,7287***	0,0493	14,7668	2,88E-30
€/$	0,2450***	0,0822	2,9811	0,0034
¥/$	0,0932**	0,0402	2,3155	0,0220

Table A.16: Regression coefficients of *TAA1_B&L_EWMA(OC_EWP_C)* on *TAA1_B&L_EWMA(OC_EWP_L)* and on the two exchange rates.

	Estimate	SE	tStat	pValue
	-0,0064	0,0675	-0,0953	0,9242
RLocal	1,0513***	0,0324	32,4330	1,05E-66
€/$	0,0934***	0,0336	2,7800	0,0062
¥/$	0,0078	0,0235	0,3309	0,7412

Table A.17: Regression coefficients of *TAA1_B&L_Roll(HF_C)* on *TAA1_B&L_Roll(HF_L)* and on the two exchange rates.

	Estimate	SE	tStat	pValue
	-0,0269	0,0724	-0,3710	0,7112
RLocal	0,9660***	0,0326	29,6325	6,16E-62
€/$	0,1519***	0,0367	4,1375	6,05E-05
¥/$	0,0425*	0,0253	1,6802	0,0952

Table A.18: Regression coefficients of *TAA1_B&L_EWMA(HF_C)* on *TAA1_B&L_EWMA(HF_L)* and on the two exchange rates.

	Estimate	SE	tStat	pValue
	0,0277	0,0719	0,3854	0,7005
RLocal	0,9263***	0,0313	29,5722	7,86E-62
€/$	0,4383***	0,0519	8,4427	3,66E-14
¥/$	0,0768***	0,0253	3,0359	0,0029

Table A.19: Regression coefficients of *TAA1_B&L_Roll(HF_EWP_C)* on *TAA1_B&L_Roll(HF_EWP_L)* and on the two exchange rates.

	Estimate	SE	tStat	pValue
	0,2480**	0,1056	2,3499	0,0202
RLocal	0,7731***	0,0454	17,0178	8,41E-36
€/$	0,2611***	0,0732	3,5667	4,96E-04
¥/$	0,0913**	0,0383	2,3844	0,0185

Table A.20: Regression coefficients of *TAA1_B&L_EWMA(HF_EWP_C)* on *TAA1_B&L_EWMA(HF_EWP_L)* and on the two exchange rates.

	Estimate	SE	tStat	pValue
	0,0727*	0,0396	1,8358	0,0685
RLocal	0,7835***	0,0180	43,6345	5,26E-83
€/$	0,0675***	0,0159	4,2551	3,82E-05
¥/$	0,0705***	0,0139	5,0712	1,24E-06

Table A.21: Regression coefficients of *TAA2_Mark_Roll(OC_C)* on *TAA1_Mark_Roll(OC_L)* and on the two exchange rates.

	Estimate	SE	tStat	pValue
	0,0793**	0,0338	2,3452	0,0204
RLocal	0,8034***	0,0176	45,7563	1,10E-85
€/$	0,0868***	0,0141	6,1471	7,87E-09
¥/$	0,0664***	0,0120	5,5218	1,60E-07

Table A.22: Regression coefficients of *TAA2_Mark_EWMA(OC_C)* on *TAA1_Mark_EWMA(OC_L)* and on the two exchange rates.

	Estimate	SE	tStat	pValue
	0,0513	0,0584	0,8786	0,3811
RLocal	1,0149***	0,0261	38,9305	1,23E-76
€/$	0,0622**	0,0243	2,5596	0,0115
¥/$	0,0780***	0,0206	3,7947	2,20E-04

Table A.23: Regression coefficients of *TAA2_Mark_Roll(OC_EWP_C)* on *TAA1_Mark_Roll(OC_EWP_L)* and on the two exchange rates.

	Estimate	SE	tStat	pValue
	0,0958*	0,0536	1,7870	0,0761
RLocal	1,0025***	0,0263	38,0728	2,09E-75
€/$	0,0362	0,0225	1,6070	0,1103
¥/$	0,1008***	0,0192	5,2431	5,75E-07

Table A.24: Regression coefficients of *TAA2_Mark_EWMA(OC_EWP_C)* on *TAA1_Mark_EWMA(OC_EWP_L)* and on the two exchange rates.

	Estimate	SE	tStat	pValue
	0,0504	0,0423	1,1897	0,2362
RLocal	0,7871***	0,0200	39,3879	2,77E-77
€/$	0,0660***	0,0166	3,9886	1,07E-04
¥/$	0,0762***	0,0148	5,1665	8,11E-07

Table A.25: Regression coefficients of *TAA2_Mark_Roll(HF_C)* on *TAA1_Mark_Roll(HF_L)* and on the two exchange rates.

	Estimate	SE	tStat	pValue
	0,0541	0,0303	1,7877	0,0760
RLocal	0,7961***	0,0160	49,8972	1,28E-90
€/$	0,0756***	0,0122	6,2179	5,53E-09
¥/$	0,0879***	0,0107	8,2324	1,20E-13

Table A.26: Regression coefficients of *TAA2_Mark_EWMA(HF_C)* on *TAA1_Mark_EWMA(HF_L)* and on the two exchange rates.

	Estimate	SE	tStat	pValue
	0,0638	0,0490	1,3036	0,1945
RLocal	0,9763***	0,0210	46,5988	1,01E-86
€/$	0,0882***	0,0199	4,4298	1,90E-05
¥/$	0,0644***	0,0172	3,7375	2,71E-04

Table A.27: Regression coefficients of *TAA2_Mark_Roll(HF_EWP_C)* on *TAA1_Mark_Roll(HF_EWP_L)* and on the two exchange rates.

	Estimate	SE	tStat	pValue
	0,0784*	0,0441	1,7803	0,0772
RLocal	0,9615***	0,0207	46,3620	1,97E-86
€/$	0,0476***	0,0179	2,6589	0,0088
¥/$	0,0800***	0,0158	5,0635	1,28E-06

Table A.28: Regression coefficients of *TAA2_Mark_EWMA(HF_EWP_C)* on *TAA1_Mark_EWMA(HF_EWP_L)* and on the two exchange rates.

	Estimate	SE	tStat	pValue
	0,0599	0,0931	0,6441	0,5206
RLocal	0,9241***	0,0268	34,4651	5,71E-70
€/$	0,0655	0,0442	1,4835	0,1402
¥/$	0,0223	0,0338	0,6586	0,5113

Table A.29: Regression coefficients of *TAA2_B&L_Roll(OC_C)* on *TAA1_B&L_Roll(OC_L)* and on the two exchange rates.

	Estimate	SE	tStat	pValue
	0,0785	0,1371	0,5725	0,5679
RLocal	0,8783***	0,0408	21,5044	4,69E-46
€/$	0,1311*	0,0679	1,9321	0,0554
¥/$	0,1523***	0,0495	3,0796	0,0025

Table A.30: Regression coefficients of *TAA2_B&L_EWMA(OC_C)* on *TAA1_B&L_EWMA(OC_L)* and on the two exchange rates.

	Estimate	SE	tStat	pValue
	0,0573	0,0836	0,6854	0,4942
RLocal	0,9232***	0,0266	34,7375	2,14E-70
€/$	0,3158***	0,0499	6,3302	3,15E-09
¥/$	0,0567*	0,0301	1,8823	0,0619

Table A.31: Regression coefficients of *TAA2_B&L_Roll(OC_EWP_C)* on *TAA1_B&L_Roll(OC_EWP_L)* and on the two exchange rates.

	Estimate	SE	tStat	pValue
	0,2309*	0,1382	1,6707	0,0970
RLocal	0,8313***	0,0451	18,4497	3,42E-39
€/$	0,2632***	0,0827	3,1836	0,0018
¥/$	0,1295**	0,0505	2,5644	0,0114

Table A.32: Regression coefficients of *TAA2_B&L_EWMA(OC_EWP_C)* on *TAA1_B&L_EWMA(OC_EWP_L)* and on the two exchange rates.

	Estimate	SE	tStat	pValue
	0,0644	0,0992	0,6494	0,5172
RLocal	0,9212***	0,0279	33,0285	1,12E-67
€/$	0,0135	0,0455	0,2970	0,7669
¥/$	0,0162	0,0360	0,4495	0,6538

Table A.33: Regression coefficients of *TAA2_B&L_Roll(HF_C)* on *TAA1_B&L_Roll(HF_L)* and on the two exchange rates.

	Estimate	SE	tStat	pValue
	0,0574	0,1375	0,4170	0,6773
RLocal	0,8953***	0,0400	22,3540	7,14E-48
€/$	0,1080	0,0658	1,6429	0,1027
¥/$	0,1424***	0,0496	2,8711	0,0047

Table A.34: Regression coefficients of *TAA2_B&L_EWMA(HF_C)* on *TAA1_B&L_EWMA(HF_L)* and on the two exchange rates.

	Estimate	SE	tStat	pValue
	0,0578	0,0914	0,6325	0,5281
RLocal	0,9253***	0,0260	35,6350	8,81E-72
€/$	0,2374***	0,0501	4,7398	5,23E-06
¥/$	0,0451	0,0332	1,3577	0,1768

Table A.35: Regression coefficients of *TAA2_B&L_Roll(HF_EWP_C)* on *TAA1_B&L_Roll(HF_EWP_L)* and on the two exchange rates.

	Estimate	SE	tStat	pValue
	0,3102**	0,1453	2,1347	0,0345
RLocal	0,3554***	0,0422	8,4209	4,14E-14
€/$	0,0914	0,0805	1,1353	0,2582
¥/$	0,0704	0,0532	1,3249	0,1874

Table A.36: Regression coefficients of *TAA2_B&L_EWMA(HF_EWP_C)* on *TAA1_B&L_EWMA(HF_EWP_L)* and on the two exchange rates.

When the coefficients betas are statistically significant at the confidence level of 99%, 95% or 90% they are marked as, respectively, ***, **, *; otherwise they are not statistically significant.

Regression coefficient obtained by considering the first period analysed, from 2003 to 2006:

	Estimate	SE	tStat	pValue
	-0,0047	0,0260	-0,1799	0,8581
RLocal	0,9659***	0,0222	43,5641	3,27E-37
€/$	0,2396***	0,0166	14,4523	4,13E-18
¥/$	0,1608***	0,0109	14,8031	1,74E-18

Table A.37: Regression coefficients of *OC_C* on *OC_L* and on the two exchange rates.

	Estimate	SE	tStat	pValue
	-0,0166	0,0241	-0,6885	0,4949
RLocal	0,9877***	0,0189	52,2887	1,47E-40
€/$	0,2528***	0,0138	18,3041	6,46E-22
¥/$	0,3879***	0,0095	40,8210	5,03E-36

Table A.38: Regression coefficients of *OC_EWP_C* on *OC_EWP_L* and on the two exchange rates.

	Estimate	SE	tStat	pValue
	-0,0025	0,0254	-0,0999	0,9209
RLocal	0,9625***	0,0229	42,0868	1,40E-36
€/$	0,2179***	0,0162	13,4482	5,27E-17
¥/$	0,1500***	0,0108	13,9203	1,57E-17

Table A.39: Regression coefficients of *HF_C* on *HF_L* and on the two exchange rates.

	Estimate	SE	tStat	pValue
	-0,0172	0,0214	-0,8067	0,4243
RLocal	0,9918***	0,0174	57,0325	3,71E-42
€/$	0,2266***	0,0120	18,8616	2,04E-22
¥/$	0,3448***	0,0085	40,7344	5,50E-36

Table A.40: Regression coefficients of *HF_EWP_C* on *HF_EWP_L* and on the two exchange rates.

	Estimate	SE	tStat	pValue
	0,1508*	0,0883	1,7065	0,0951
RLocal	0,8743***	0,0531	16,4641	3,52E-20
€/$	0,1726***	0,0444	3,8851	3,48E-04
¥/$	0,0419	0,0343	1,2215	0,2285

Table A.41: Regression coefficients of *TAA1_Mark_Roll(OC_C)* on *TAA1_Mark_Roll(OC_L)* and on the two exchange rates.

	Estimate	SE	tStat	pValue
	0,1603**	0,0620	2,5848	0,0132
RLocal	0,8449***	0,0365	23,1598	6,52E-26
€/$	0,1295***	0,0323	4,0137	2,35E-04
¥/$	0,0553*	0,0278	1,9855	0,0535

Table A.42: Regression coefficients of *TAA1_Mark_EWMA(OC_C)* on *TAA1_Mark_EWMA(OC_L)* and on the two exchange rates.

	Estimate	SE	tStat	pValue
	0,1685	0,1762	0,9565	0,3441
RLocal	0,8518***	0,0994	8,5693	7,52E-11
€/$	0,0655	0,0951	0,6890	0,4945
¥/$	0,0243	0,0654	0,3708	0,7126

Table A.43: Regression coefficients of *TAA1_Mark_Roll(OC_EWP_C)* on *TAA1_Mark_Roll(OC_EWP_L)* and on the two exchange rates.

	Estimate	SE	tStat	pValue
	-0,0264	0,1159	-0,2274	0,8212
RLocal	1,0082***	0,0688	14,6553	2,50E-18
€/$	0,1317*	0,0682	1,9329	0,0598
¥/$	0,0568	0,0504	1,1255	0,2666

Table A.44: Regression coefficients of *TAA1_Mark_EWMA(OC_EWP_C)* on *TAA1_Mark_EWMA(OC_EWP_L)* and on the two exchange rates.

	Estimate	SE	tStat	pValue
	0,0847	0,0881	0,9613	0,3418
RLocal	0,8354***	0,0463	18,0535	1,09E-21
€/$	0,1276***	0,0417	3,0591	0,0038
¥/$	0,0321	0,0356	0,9013	0,3724

Table A.45: Regression coefficients of *TAA1_Mark_Roll(HF_C)* on *TAA1_Mark_Roll(HF_L)* and on the two exchange rates.

	Estimate	SE	tStat	pValue
	0,0781*	0,0442	1,7680	0,0842
RLocal	0,8302***	0,0230	36,1178	8,40E-34
€/$	0,0766***	0,0216	3,5406	9,74E-04
¥/$	0,1174***	0,0200	5,8698	5,65E-07

Table A.46: Regression coefficients of *TAA1_Mark_EWMA(HF_C)* on *TAA1_Mark_EWMA(HF_L)* and on the two exchange rates.

	Estimate	SE	tStat	pValue
	0,0571	0,1615	0,3537	0,7253
RLocal	0,9231***	0,0869	10,6241	1,33E-13
€/$	0,1695**	0,0857	1,9781	0,0543
¥/$	0,0415	0,0598	0,6949	0,4909

Table A.47: Regression coefficients of *TAA1_Mark_Roll(HF_EWP_C)* on *TAA1_Mark_Roll(HF_EWP_L)* and on the two exchange rates.

	Estimate	SE	tStat	pValue
	-0,0273	0,0933	-0,2932	0,7708
RLocal	0,9908***	0,0525	18,8819	1,96E-22
€/$	0,1448***	0,0529	2,7375	0,0090
¥/$	0,0771*	0,0407	1,8964	0,0646

Table A.48: Regression coefficients of *TAA1_Mark_EWMA(HF_EWP_C)* on *TAA1_Mark_EWMA(HF_EWP_L)* and on the two exchange rates.

	Estimate	SE	tStat	pValue
	0,1736	0,1641	1,0580	0,2960
RLocal	0,7184***	0,0792	9,0666	1,55E-11
€/$	0,0164	0,0774	0,2122	0,8329
¥/$	-0,0814	0,0678	-1,2003	0,2366

Table A.49: Regression coefficients of *TAA1_B&L_Roll(OC_C)* on *TAA1_B&L_Roll(OC_L)* and on the two exchange rates.

	Estimate	SE	tStat	pValue
	-0,0164	0,1474	-0,1115	0,9118
RLocal	0,8595***	0,0621	13,8340	1,95E-17
€/$	0,0896	0,0701	1,2779	0,2081
¥/$	-0,0348	0,0631	-0,5510	0,5845

Table A.50: Regression coefficients of *TAA1_B&L_EWMA(OC_C)* on *TAA1_B&L_EWMA(OC_L)* and on the two exchange rates.

	Estimate	SE	tStat	pValue
	0,4124**	0,1957	2,1071	0,0410
RLocal	0,6809***	0,1046	6,5078	6,67E-08
€/$	0,1084	0,1447	0,7496	0,4576
¥/$	0,0746	0,0761	0,9804	0,3324

Table A.51: Regression coefficients of *TAA1_B&L_Roll(OC_EWP_C)* on *TAA1_B&L_Roll(OC_EWP_L)* and on the two exchange rates.

	Estimate	SE	tStat	pValue
	0,2542	0,1762	1,4428	0,1563
RLocal	0,7173***	0,0929	7,7211	1,18E-09
€/$	0,0686	0,1224	0,5601	0,5783
¥/$	0,0836	0,0755	1,1072	0,2744

Table A.52: Regression coefficients of *TAA1_B&L_EWMA(OC_EWP_C)* on *TAA1_B&L_EWMA(OC_EWP_L)* and on the two exchange rates.

	Estimate	SE	tStat	pValue
	0,1786	0,1875	0,9527	0,3460
RLocal	0,8100***	0,1000	8,0971	3,46E-10
€/$	0,0432	0,0889	0,4858	0,6296
¥/$	-0,1108	0,0774	-1,4318	0,1594

Table A.53: Regression coefficients of *TAA1_B&L_Roll(HF_C)* on *TAA1_B&L_Roll(HF_L)* and on the two exchange rates.

	Estimate	SE	tStat	pValue
	-0,0165	0,1691	-0,0976	0,9227
RLocal	0,8546***	0,0680	12,5702	5,40E-16
€/$	0,0754	0,0790	0,9551	0,3449
¥/$	-0,0459	0,0728	-0,6305	0,5317

Table A.54: Regression coefficients of *TAA1_B&L_EWMA(HF_C)* on *TAA1_B&L_EWMA(HF_L)* and on the two exchange rates.

	Estimate	SE	tStat	pValue
	0,3677**	0,1806	2,0357	0,0480
RLocal	0,7019***	0,0933	7,5207	2,29E-09
€/$	0,1053	0,1217	0,8654	0,3916
¥/$	0,0528	0,0702	0,7519	0,4562

Table A.55: Regression coefficients of *TAA1_B&L_Roll(HF_EWP_C)* on *TAA1_B&L_Roll(HF_EWP_L)* and on the two exchange rates.

	Estimate	SE	tStat	pValue
	0,1306	0,1570	0,8320	0,4100
RLocal	0,8151***	0,0772	10,5594	1,61E-13
€/$	0,1476	0,0993	1,4869	0,1443
¥/$	0,0738	0,0670	1,1010	0,2770

Table A.56: Regression coefficients of *TAA1_B&L_EWMA(HF_EWP_C)* on *TAA1_B&L_EWMA(HF_EWP_L)* and on the two exchange rates.

	Estimate	SE	tStat	pValue
	0,0914	0,0934	0,9781	0,3335
RLocal	0,8060***	0,0466	17,3012	5,49E-21
€/$	0,0717*	0,0364	1,9728	0,0550
¥/$	0,0886**	0,0329	2,6954	0,0100

Table A.57: Regression coefficients of *TAA2_Mark_Roll(OC_C)* on *TAA1_Mark_Roll(OC_L)* and on the two exchange rates.

	Estimate	SE	tStat	pValue
	0,1555**	0,0587	2,6493	0,0112
RLocal	0,7663***	0,0314	24,4028	8,05E-27
€/$	0,0940***	0,0280	3,3562	0,0017
¥/$	0,0463*	0,0258	1,7960	0,0795

Table A.58: Regression coefficients of *TAA2_Mark_EWMA(OC_C)* on *TAA1_Mark_EWMA(OC_L)* and on the two exchange rates.

	Estimate	SE	tStat	pValue
	0,0185	0,1629	0,1134	0,9102
RLocal	1,0371***	0,0812	12,7681	3,17E-16
€/$	0,1578**	0,0716	2,2037	0,0329
¥/$	0,0847	0,0564	1,5014	0,1405

Table A.59: Regression coefficients of *TAA2_Mark_Roll(OC_EWP_C)* on *TAA1_Mark_Roll(OC_EWP_L)* and on the two exchange rates.

	Estimate	SE	tStat	pValue
	0,1237	0,0954	1,2964	0,2017
RLocal	1,0092***	0,0485	20,7910	4,65E-24
€/$	0,1415***	0,0495	2,8601	0,0065
¥/$	0,0196	0,0415	0,4709	0,6401

Table A.60: Regression coefficients of *TAA2_Mark_EWMA(OC_EWP_C)* on *TAA1_Mark_EWMA(OC_EWP_L)* and on the two exchange rates.

	Estimate	SE	tStat	pValue
	-0,0083	0,0847	-0,0983	0,9222
RLocal	0,8656***	0,0410	21,1064	2,58E-24
€/$	0,1244***	0,0348	3,5700	8,94E-04
¥/$	0,0546*	0,0321	1,7030	0,0958

Table A.61: Regression coefficients of *TAA2_Mark_Roll(HF_C)* on *TAA1_Mark_Roll(HF_L)* and on the two exchange rates.

	Estimate	SE	tStat	pValue
	0,0841*	0,0449	1,8745	0,0677
RLocal	0,7939***	0,0219	36,2940	6,86E-34
€/$	0,0669***	0,0212	3,1519	0,0030
¥/$	0,0619***	0,0201	3,0760	0,0036

Table A.62: Regression coefficients of *TAA2_Mark_EWMA(HF_C)* on *TAA1_Mark_EWMA(HF_L)* and on the two exchange rates.

	Estimate	SE	tStat	pValue
	0,0045	0,1259	0,0356	0,9718
RLocal	1,0366***	0,0568	18,2381	7,42E-22
€/$	0,2139***	0,0545	3,9268	3,07E-04
¥/$	0,0867*	0,0454	1,9095	0,0629

Table A.63: Regression coefficients of *TAA2_Mark_Roll(HF_EWP_C)* on *TAA1_Mark_Roll(HF_EWP_L)* and on the two exchange rates.

	Estimate	SE	tStat	pValue
	0,1139**	0,0559	2,0379	0,0477
RLocal	0,9736***	0,0252	38,5840	5,33E-35
€/$	0,1034***	0,0273	3,7897	4,65E-04
¥/$	0,0559**	0,0249	2,2470	0,0298

Table A.64: Regression coefficients of *TAA2_Mark_EWMA(HF_EWP_C)* on *TAA1_Mark_EWMA(HF_EWP_L)* and on the two exchange rates.

	Estimate	SE	tStat	pValue
	0,4694*	0,2622	1,7904	0,0804
RLocal	0,6229***	0,0854	7,2936	4,85E-09
€/$	-0,0657	0,1150	-0,5711	0,5709
¥/$	-0,0900	0,1079	-0,8347	0,4085

Table A.65: Regression coefficients of *TAA2_B&L_Roll(OC_C)* on *TAA1_B&L_Roll(OC_L)* and on the two exchange rates.

	Estimate	SE	tStat	pValue
	0,0578	0,2294	0,2519	0,8023
RLocal	0,8140***	0,0628	12,9614	1,89E-16
€/$	0,0144	0,1034	0,1394	0,8898
¥/$	-0,0044	0,1000	-0,0445	0,9647

Table A.66: Regression coefficients of *TAA2_B&L_EWMA(OC_C)* on *TAA1_B&L_EWMA(OC_L)* and on the two exchange rates.

	Estimate	SE	tStat	pValue
	0,4431*	0,2222	1,9939	0,0525
RLocal	0,6616***	0,0837	7,9035	6,50E-10
€/$	-0,0071	0,1186	-0,0601	0,9524
¥/$	-0,0046	0,0883	-0,0525	0,9584

Table A.67: Regression coefficients of *TAA2_B&Ll_Roll(OC_EWP_C)* on *TAA1_B&L_Roll(OC_EWP_L)* and on the two exchange rates.

	Estimate	SE	tStat	pValue
	0,0165	0,1791	0,0919	0,9272
RLocal	0,8588***	0,0586	14,6464	2,56E-18
€/$	0,1109	0,0923	1,2010	0,2363
¥/$	0,0621	0,0774	0,8023	0,4268

Table A.68: Regression coefficients of *TAA2_B&L_EWMA(OC_EWP_C)* on *TAA1_B&L_EWMA(OC_EWP_L)* and on the two exchange rates.

	Estimate	SE	tStat	pValue
	0,4626	0,2797	1,6538	0,1054
RLocal	0,6096***	0,0886	6,8789	1,93E-08
€/$	-0,0982	0,1197	-0,8204	0,4165
¥/$	-0,1143	0,1159	-0,9862	0,3295

Table A.69: Regression coefficients of *TAA2_B&L_Roll(HF_C)* on *TAA1_B&L_Roll(HF_L)* and on the two exchange rates.

	Estimate	SE	tStat	pValue
	0,0374	0,2458	0,1522	0,8797
RLocal	0,8131***	0,0647	12,5633	5,51E-16
€/$	-0,0199	0,1089	-0,1828	0,8558
¥/$	-0,0214	0,1075	-0,1989	0,8433

Table A.70: Regression coefficients of *TAA2_B&L_EWMA(HF_C)* on *TAA1_B&L_EWMA(HF_L)* and on the two exchange rates.

	Estimate	SE	tStat	pValue
	0,4884*	0,2513	1,9437	0,0585
RLocal	0,6340***	0,0843	7,5239	2,26E-09
€/$	-0,0594	0,1230	-0,4828	0,6317
¥/$	-0,0432	0,1012	-0,4266	0,6718

Table A.71: Regression coefficients of *TAA2_B&L_Roll(HF_EWP_C)* on *TAA1_B&L_Roll(HF_EWP_L)* and on the two exchange rates.

	Estimate	SE	tStat	pValue
	0,3735	0,2604	1,4343	0,1587
RLocal	0,4133***	0,0739	5,5905	1,44E-06
€/$	-0,0604	0,1262	-0,4783	0,6349
¥/$	0,1255	0,1132	1,1087	0,2737

Table A.72: Regression coefficients of *TAA2_B&L_EWMA(HF_EWP_C)* on *TAA1_B&L_EWMA(HF_EWP_L)* and on the two exchange rates.

When the coefficients of the betas are statistically significant at the confidence level of 99%, 95% or 90% they are marked as, respectively, ***, **, *; otherwise they are not statistically significant.

Regression coefficient obtained by considering the first period analysed, from 2007 to 2009:

	Estimate	SE	tStat	pValue
	-0,0575*	0,0314	-1,8342	0,0759
RLocal	0,9976***	0,0223	44,7345	2,00E-30
€/$	0,2359***	0,0192	12,3070	1,12E-13
¥/$	0,1627***	0,0093	17,5592	5,47E-18

Table A.73: Regression coefficients of *OC_C* on *OC_L* and on the two exchange rates.

	Estimate	SE	tStat	pValue
	-0,0503*	0,0285	-1,7667	0,0868
RLocal	0,9939***	0,0149	66,5073	7,05E-36
€/$	0,2365***	0,0145	16,2720	4,93E-17
¥/$	0,3874***	0,0090	43,2210	5,91E-30

Table A.74: Regression coefficients of *OC_EWP_C* on *OC_EWP_L* and on the two exchange rates.

	Estimate	SE	tStat	pValue
	-0,0536*	0,0302	-1,7738	0,0856
RLocal	0,9983***	0,0241	41,3465	2,38E-29
€/$	0,2175***	0,0191	11,3600	9,16E-13
¥/$	0,1513***	0,0089	17,0465	1,29E-17

Table A.75: Regression coefficients of *HF_C* on *HF_L* and on the two exchange rates.

	Estimate	SE	tStat	pValue
	-0,0449*	0,0253	-1,7745	0,0855
RLocal	0,9951***	0,0138	72,2718	5,01E-37
€/$	0,2109***	0,0127	16,6595	2,51E-17
¥/$	0,3443***	0,0081	42,6018	9,31E-30

Table A.76: Regression coefficients of *HF_EWP_C* on *HF_EWP_L* and on the two exchange rates.

	Estimate	SE	tStat	pValue
	-0,0760	0,0854	-0,8903	0,3799
RLocal	0,8693***	0,0375	23,1681	1,45E-21
€/$	0,1181**	0,0435	2,7162	0,0106
¥/$	0,0698**	0,0277	2,5234	0,0168

Table A.77: Regression coefficients of *TAA1_Mark_Roll(OC_C)* on *TAA1_Mark_Roll(OC_L)* and on the two exchange rates.

	Estimate	SE	tStat	pValue
	0,0275	0,1267	0,2173	0,8294
RLocal	0,6844***	0,0596	11,4755	7,05E-13
€/$	-0,0292	0,0640	-0,4569	0,6508
¥/$	0,1330***	0,0405	3,2813	0,0025

Table A.78: Regression coefficients of *TAA1_Mark_EWMA(OC_C)* on *TAA1_Mark_EWMA(OC_L)* and on the two exchange rates.

	Estimate	SE	tStat	pValue
	-0,0186	0,1235	-0,1506	0,8813
RLocal	1,0663***	0,0543	19,6188	2,11E-19
€/$	0,2178***	0,0676	3,2216	0,0029
¥/$	-0,0064	0,0400	-0,1605	0,8735

Table A.79: Regression coefficients of *TAA1_Mark_Roll(OC_EWP_C)* on *TAA1_Mark_Roll(OC_EWP_L)* and on the two exchange rates.

	Estimate	SE	tStat	pValue
	0,0112	0,1378	0,0813	0,9357
RLocal	0,9355***	0,0579	16,1590	6,02E-17
€/$	0,0899	0,0658	1,3665	0,1813
¥/$	0,0094	0,0439	0,2131	0,8326

Table A.80: Regression coefficients of *TAA1_Mark_EWMA(OC_EWP_C)* on *TAA1_Mark_EWMA(OC_EWP_L)* and on the two exchange rates.

	Estimate	SE	tStat	pValue
	-0,0569	0,1081	-0,5263	0,6023
RLocal	0,7298***	0,0474	15,4071	2,34E-16
€/$	0,0065	0,0532	0,1217	0,9039
¥/$	0,1063***	0,0350	3,0364	0,0047

Table A.81: Regression coefficients of *TAA1_Mark_Roll(HF_C)* on *TAA1_Mark_Roll(HF_L)* and on the two exchange rates.

	Estimate	SE	tStat	pValue
	0,0377	0,0873	0,4318	0,6688
RLocal	0,7527***	0,0484	15,5637	1,75E-16
€/$	0,0491	0,0470	1,0440	0,3043
¥/$	0,1370***	0,0279	4,9076	2,60E-05

Table A.82: Regression coefficients of *TAA1_Mark_EWMA(HF_C)* on *TAA1_Mark_EWMA(HF_L)* and on the two exchange rates.

	Estimate	SE	tStat	pValue
	-0,0196	0,0980	-0,2005	0,8424
RLocal	1,0305***	0,0402	25,6380	6,66E-23
€/$	0,2259***	0,0508	4,4480	9,80E-05
¥/$	-0,0215	0,0321	-0,6689	0,5084

Table A.83: Regression coefficients of *TAA1_Mark_Roll(HF_EWP_C)* on *TAA1_Mark_Roll(HF_EWP_L)* and on the two exchange rates.

	Estimate	SE	tStat	pValue
	-0,0092	0,1563	-0,0591	0,9532
RLocal	0,9455***	0,0636	14,8546	6,54E-16
€/$	0,1025	0,0730	1,4032	0,1702
¥/$	0,0418	0,0501	0,8332	0,4109

Table A.84: Regression coefficients of *TAA1_Mark_EWMA(HF_EWP_C)* on *TAA1_Mark_EWMA(HF_EWP_L)* and on the two exchange rates.

	Estimate	SE	tStat	pValue
	0,0330	0,0766	0,4313	0,6691
RLocal	0,9973***	0,0259	38,5282	2,19E-28
€/$	0,1266***	0,0342	3,7038	7,98E-04
¥/$	0,0211	0,0235	0,8954	0,3773

Table A.85: Regression coefficients of *TAA1_B&L_Roll(OC_C)* on *TAA1_B&L_Roll(OC_L)* and on the two exchange rates.

	Estimate	SE	tStat	pValue
	0,0184	0,1534	0,1199	0,9053
RLocal	1,0418***	0,0654	15,9258	9,11E-17
€/$	0,2847***	0,0786	3,6211	0,0010
¥/$	0,0814*	0,0472	1,7232	0,0945

Table A.86: Regression coefficients of *TAA1_B&L_EWMA(OC_C)* on *TAA1_B&L_EWMA(OC_L)* and on the two exchange rates.

	Estimate	SE	tStat	pValue
	-0,1315	0,1838	-0,7157	0,4794
RLocal	0,9728***	0,0612	15,8907	9,70E-17
€/$	0,6276***	0,1148	5,4650	5,13E-06
¥/$	0,1241**	0,0558	2,2253	0,0332

Table A.87: Regression coefficients of *TAA1_B&L_Roll(OC_EWP_C)* on *TAA1_B&L_Roll(OC_EWP_L)* and on the two exchange rates.

	Estimate	SE	tStat	pValue
	0,0576	0,2670	0,2158	0,8305
RLocal	0,6800***	0,0867	7,8408	6,05E-09
€/$	0,2823*	0,1503	1,8782	0,0695
¥/$	0,0233	0,0822	0,2837	0,7785

Table A.88: Regression coefficients of *TAA1_B&L_EWMA(OC_EWP_C)* on *TAA1_B&L_EWMA(OC_EWP_L)* and on the two exchange rates.

	Estimate	SE	tStat	pValue
	0,1279	0,1071	1,1947	0,2410
RLocal	1,1002***	0,0392	28,0744	4,11E-24
€/$	0,1043**	0,0474	2,2015	0,0350
¥/$	0,0452	0,0328	1,3759	0,1784

Table A.89: Regression coefficients of *TAA1_B&L_Roll(HF_C)* on *TAA1_B&L_Roll(HF_L)* and on the two exchange rates.

	Estimate	SE	tStat	pValue
	0,0405	0,1764	0,2296	0,8199
RLocal	1,0546***	0,0738	14,2882	1,94E-15
€/$	0,2847***	0,0884	3,2187	0,0029
¥/$	0,0670	0,0549	1,2205	0,2312

Table A.90: Regression coefficients of *TAA1_B&L_EWMA(HF_C)* on *TAA1_B&L_EWMA(HF_L)* and on the two exchange rates.

	Estimate	SE	tStat	pValue
	-0,1023	0,1621	-0,6313	0,5323
RLocal	0,9847***	0,0513	19,1917	4,04E-19
€/$	0,5571***	0,0936	5,9503	1,25E-06
¥/$	0,1065**	0,0498	2,1364	0,0404

Table A.91: Regression coefficients of *TAA1_B&L_Roll(HF_EWP_C)* on *TAA1_B&L_Roll(HF_EWP_L)* and on the two exchange rates.

	Estimate	SE	tStat	pValue
	0,0748	0,2754	0,2715	0,7877
RLocal	0,7116***	0,0877	8,1109	2,91E-09
€/$	0,2846*	0,1492	1,9079	0,0654
¥/$	0,0519	0,0852	0,6086	0,5471

Table A.92: Regression coefficients of *TAA1_B&L_EWMA(HF_EWP_C)* on *TAA1_B&L_EWMA(HF_EWP_L)* and on the two exchange rates.

	Estimate	SE	tStat	pValue
	-0,0920	0,0947	-0,9711	0,3388
RLocal	0,7528***	0,0357	21,0674	2,53E-20
€/$	0,0383	0,0449	0,8534	0,3998
¥/$	0,0588*	0,0312	1,8891	0,0680

Table A.93: Regression coefficients of *TAA2_Mark_Roll(OC_C)* on *TAA1_Mark_Roll(OC_L)* and on the two exchange rates.

	Estimate	SE	tStat	pValue
	-0,0280	0,0758	-0,3693	0,7143
RLocal	0,8124***	0,0363	22,3539	4,27E-21
€/$	0,0996**	0,0399	2,4966	0,0179
¥/$	0,0405	0,0240	1,6861	0,1015

Table A.94: Regression coefficients of *TAA2_Mark_EWMA(OC_C)* on *TAA1_Mark_EWMA(OC_L)* and on the two exchange rates.

	Estimate	SE	tStat	pValue
	0,0699	0,0970	0,7213	0,4760
RLocal	1,0950***	0,0355	30,8045	2,34E-25
€/$	0,1820***	0,0460	3,9571	3,95E-04
¥/$	-0,0040	0,0320	-0,1257	0,9008

Table A.95: Regression coefficients of *TAA2_Mark_Roll(OC_EWP_C)* on *TAA1_Mark_Roll(OC_EWP_L)* and on the two exchange rates.

	Estimate	SE	tStat	pValue
	0,0538	0,1348	0,3988	0,6927
RLocal	1,0039***	0,0555	18,0843	2,32E-18
€/$	0,0406	0,0619	0,6560	0,5165
¥/$	0,0929**	0,0431	2,1576	0,0386

Table A.96: Regression coefficients of *TAA2_Mark_EWMA(OC_EWP_C)* on *TAA1_Mark_EWMA(OC_EWP_L)* and on the two exchange rates.

	Estimate	SE	tStat	pValue
	-0,0679	0,1138	-0,5968	0,5548
RLocal	0,7185***	0,0465	15,4454	2,18E-16
€/$	-0,0004	0,0531	-0,0075	0,9940
¥/$	0,0797**	0,0373	2,1372	0,0403

Table A.97: Regression coefficients of *TAA2_Mark_Roll(HF_C)* on *TAA1_Mark_Roll(HF_L)* and on the two exchange rates.

	Estimate	SE	tStat	pValue
	-0,0553	0,0693	-0,7975	0,4310
RLocal	0,7747***	0,0364	21,2875	1,85E-20
€/$	0,0705*	0,0361	1,9531	0,0596
¥/$	0,0732***	0,0218	3,3568	0,0020

Table A.98: Regression coefficients of *TAA2_Mark_EWMA(HF_C)* on *TAA1_Mark_EWMA(HF_L)* and on the two exchange rates.

	Estimate	SE	tStat	pValue
	0,0158	0,0573	0,2761	0,7843
RLocal	1,0206***	0,0201	50,8273	3,55E-32
€/$	0,1554***	0,0260	5,9761	1,16E-06
¥/$	-0,0169	0,0189	-0,8934	0,3783

Table A.99: Regression coefficients of *TAA2_Mark_Roll(HF_EWP_C)* on *TAA1_Mark_Roll(HF_EWP_L)* and on the two exchange rates.

	Estimate	SE	tStat	pValue
	-0,0870	0,1132	-0,7687	0,4477
RLocal	0,9400***	0,0451	20,8308	3,55E-20
€/$	0,0350	0,0499	0,7018	0,4879
¥/$	0,0487	0,0362	1,3460	0,1878

Table A.100: Regression coefficients of *TAA2_Mark_EWMA(HF_EWP_C)* on *TAA1_Mark_EWMA(HF_EWP_L)* and on the two exchange rates.

	Estimate	SE	tStat	pValue
	0,0907	0,1150	0,7890	0,4359
RLocal	0,9972***	0,0242	41,2037	2,66E-29
€/$	0,1227**	0,0460	2,6653	0,0120
¥/$	0,0182	0,0362	0,5018	0,6193

Table A.101: Regression coefficients of *TAA2_B&L_Roll(OC_C)* on *TAA1_B&L_Roll(OC_L)* and on the two exchange rates.

	Estimate	SE	tStat	pValue
	0,0965	0,4014	0,2403	0,8116
RLocal	0,8810***	0,1032	8,5361	9,37E-10
€/$	0,2563	0,1852	1,3837	0,1760
¥/$	0,2709**	0,1232	2,1993	0,0352

Table A.102: Regression coefficients of *TAA2_B&L_EWMA(OC_C)* on *TAA1_B&L_EWMA(OC_L)* and on the two exchange rates.

	Estimate	SE	tStat	pValue
	-0,0223	0,1431	-0,1560	0,8770
RLocal	0,9916***	0,0327	30,3042	3,88E-25
€/$	0,4314***	0,0690	6,2534	5,22E-07
¥/$	0,0677	0,0448	1,5115	0,1405

Table A.103: Regression coefficients of *TAA2_B&Ll_Roll(OC_EWP_C)* on *TAA1_B&L_Roll(OC_EWP_L)* and on the two exchange rates.

	Estimate	SE	tStat	pValue
	0,1804	0,4090	0,4410	0,6622
RLocal	0,7825***	0,1075	7,2824	2,82E-08
€/$	0,3192	0,2047	1,5596	0,1287
¥/$	0,1601	0,1269	1,2621	0,2160

Table A.104: Regression coefficients of *TAA2_B&L_EWMA(OC_EWP_C)* on *TAA1_B&L_EWMA(OC_EWP_L)* and on the two exchange rates.

	Estimate	SE	tStat	pValue
	0,1144	0,1201	0,9524	0,3481
RLocal	0,9971***	0,0248	40,2602	5,50E-29
€/$	0,0672	0,0470	1,4298	0,1625
¥/$	0,0106	0,0379	0,2783	0,7825

Table A.105: Regression coefficients of *TAA2_B&L_Roll(HF_C)* on *TAA1_B&L_Roll(HF_L)* and on the two exchange rates.

	Estimate	SE	tStat	pValue
	0,1247	0,3927	0,3176	0,7528
RLocal	0,9260***	0,1006	9,2079	1,64E-10
€/$	0,2730	0,1773	1,5395	0,1335
¥/$	0,2603**	0,1209	2,1530	0,0390

Table A.106: Regression coefficients of *TAA2_B&L_EWMA(HF_C)* on *TAA1_B&L_EWMA(HF_L)* and on the two exchange rates.

	Estimate	SE	tStat	pValue
	0,0262	0,1369	0,1912	0,8496
RLocal	0,9944***	0,0280	35,5178	2,79E-27
€/$	0,3398***	0,0613	5,5391	4,14E-06
¥/$	0,0518	0,0431	1,2005	0,2388

Table A.107: Regression coefficients of *TAA2_B&L_Roll(HF_EWP_C)* on *TAA1_B&L_Roll(HF_EWP_L)* and on the two exchange rates.

	Estimate	SE	tStat	pValue
	-0,1831	0,3158	-0,5800	0,5660
RLocal	0,3917***	0,0745	5,2614	9,29E-06
€/$	-0,1397	0,1496	-0,9333	0,3577
¥/$	0,1824*	0,0978	1,8645	0,0715

Table A.108: Regression coefficients of *TAA2_B&L_EWMA(HF_EWP_C)* on *TAA1_B&L_EWMA(HF_EWP_L)* and on the two exchange rates.

When the coefficients of the betas are statistically significant at the confidence level of 99%, 95% or 90% they are marked as, respectively, ***, **, *; otherwise they are not statistically significant.

Regression coefficient obtained by considering the first period analysed, from 2010 to 2014:

	Estimate	SE	tStat	pValue
	-0,0364	0,0239	-1,5244	0,1330
RLocal	0,9958***	0,0218	45,5746	5,87E-46
€/$	0,2379***	0,0150	15,8616	2,26E-22
¥/$	0,1505***	0,0069	21,6939	6,26E-29

Table A.109: Regression coefficients of *OC_C* on *OC_L* and on the two exchange rates.

	Estimate	SE	tStat	pValue
	-0,0429**	0,0200	-2,1464	0,0362
RLocal	1,0087***	0,0157	64,2919	3,61E-54
€/$	0,2478***	0,0123	20,1728	2,33E-27
¥/$	0,3782***	0,0068	55,8564	8,44E-51

Table A.110: Regression coefficients of *OC_EWP_C* on *OC_EWP_L* and on the two exchange rates.

	Estimate	SE	tStat	pValue
	-0,0333	0,0231	-1,4401	0,1554
RLocal	0,9967***	0,0229	43,5705	6,81E-45
€/$	0,2191***	0,0142	15,4113	8,43E-22
¥/$	0,1395***	0,0067	20,9093	3,94E-28

Table A.111: Regression coefficients of *HF_C* on *HF_L* and on the two exchange rates.

	Estimate	SE	tStat	pValue
	-0,0386**	0,0174	-2,2133	0,0310
RLocal	1,0092***	0,0145	69,7163	4,09E-56
€/$	0,2208***	0,0105	20,9346	3,71E-28
¥/$	0,3360***	0,0060	56,1943	6,06E-51

Table A.112: Regression coefficients of *HF_EWP_C* on *HF_EWP_L* and on the two exchange rates.

	Estimate	SE	tStat	pValue
	0,0320	0,0416	0,7698	0,4446
RLocal	0,9595***	0,0452	21,2303	1,85E-28
€/$	0,1464***	0,0200	7,3141	1,05E-09
¥/$	0,1624***	0,0123	13,2232	7,24E-19

Table A.113: Regression coefficients of *TAA1_Mark_Roll(OC_C)* on *TAA1_Mark_Roll(OC_L)* and on the two exchange rates.

	Estimate	SE	tStat	pValue
	-0,0480	0,0385	-1,2485	0,2170
RLocal	1,0265***	0,0427	24,0507	3,37E-31
€/$	0,1741***	0,0182	9,5468	2,39E-13
¥/$	0,1578***	0,0113	14,0151	5,86E-20

Table A.114: Regression coefficients of *TAA1_Mark_EWMA(OC_C)* on *TAA1_Mark_EWMA(OC_L)* and on the two exchange rates.

	Estimate	SE	tStat	pValue
	0,0928	0,1108	0,8377	0,4058
RLocal	1,0016***	0,0889	11,2639	5,14E-16
€/$	0,0166	0,0378	0,4383	0,6629
¥/$	0,0577*	0,0330	1,7453	0,0864

Table A.115: Regression coefficients of *TAA1_Mark_Roll(OC_EWP_C)* on *TAA1_Mark_Roll(OC_EWP_L)* and on the two exchange rates.

	Estimate	SE	tStat	pValue
	0,0266	0,1041	0,2554	0,7994
RLocal	1,1373***	0,0868	13,1087	1,05E-18
€/$	0,0454	0,0348	1,3031	0,1979
¥/$	0,0833***	0,0309	2,6932	0,0093

Table A.116: Regression coefficients of *TAA1_Mark_EWMA(OC_EWP_C)* on *TAA1_Mark_EWMA(OC_EWP_L)* and on the two exchange rates.

	Estimate	SE	tStat	pValue
	0,0010	0,0348	0,0283	0,9775
RLocal	0,9780***	0,0401	24,3949	1,63E-31
€/$	0,1376***	0,0155	8,8884	2,73E-12
¥/$	0,1750***	0,0105	16,7312	1,90E-23

Table A.117: Regression coefficients of *TAA1_Mark_Roll(HF_C)* on *TAA1_Mark_Roll(HF_L)* and on the two exchange rates.

	Estimate	SE	tStat	pValue
	-0,0596	0,0408	-1,4632	0,1490
RLocal	1,0634***	0,0480	22,1563	2,17E-29
€/$	0,1678***	0,0179	9,3939	4,19E-13
¥/$	0,1699***	0,0122	13,9295	7,66E-20

Table A.118: Regression coefficients of *TAA1_Mark_EWMA(HF_C)* on *TAA1_Mark_EWMA(HF_L)* and on the two exchange rates.

	Estimate	SE	tStat	pValue
	0,0767	0,0882	0,8702	0,3879
RLocal	0,9802***	0,0728	13,4731	3,25E-19
€/$	0,0307	0,0301	1,0212	0,3116
¥/$	0,0750***	0,0262	2,8674	0,0058

Table A.119: Regression coefficients of *TAA1_Mark_Roll(HF_EWP_C)* on *TAA1_Mark_Roll(HF_EWP_L)* and on the two exchange rates.

	Estimate	SE	tStat	pValue
	0,0516	0,0908	0,5681	0,5722
RLocal	1,0627***	0,0776	13,6932	1,61E-19
€/$	0,0491	0,0309	1,5888	0,1177
¥/$	0,0897***	0,0276	3,2550	0,0019

Table A.120: Regression coefficients of *TAA1_Mark_EWMA(HF_EWP_C)* on *TAA1_Mark_EWMA(HF_EWP_L)* and on the two exchange rates.

	Estimate	SE	tStat	pValue
	0,0080	0,0346	0,2317	0,8176
RLocal	1,0047***	0,0185	54,2183	4,34E-50
€/$	0,1635***	0,0191	8,5772	8,76E-12
¥/$	0,0135	0,0111	1,2162	0,2290

Table A.121: Regression coefficients of *TAA1_B&L_Roll(OC_C)* on *TAA1_B&L_Roll(OC_L)* and on the two exchange rates.

	Estimate	SE	tStat	pValue
	0,0125	0,0477	0,2621	0,7942
RLocal	1,0121***	0,0257	39,3960	1,61E-42
€/$	0,1825***	0,0266	6,8621	5,85E-09
¥/$	0,0305*	0,0153	1,9942	0,0510

Table A.122: Regression coefficients of *TAA1_B&L_EWMA(OC_C)* on *TAA1_B&L_EWMA(OC_L)* and on the two exchange rates.

	Estimate	SE	tStat	pValue
	0,0069	0,0737	0,0940	0,9254
RLocal	0,9483***	0,0394	24,0759	3,19E-31
€/$	0,5664***	0,0678	8,3559	2,01E-11
¥/$	0,0522**	0,0235	2,2168	0,0307

Table A.123: Regression coefficients of *TAA1_B&L_Roll(OC_EWP_C)* on *TAA1_B&L_Roll(OC_EWP_L)* and on the two exchange rates.

	Estimate	SE	tStat	pValue
	0,1777	0,1817	0,9779	0,3323
RLocal	0,9274***	0,0970	9,5620	2,26E-13
€/$	0,5669***	0,1681	3,3723	0,0014
¥/$	0,1702***	0,0581	2,9302	0,0049

Table A.124: Regression coefficients of *TAA1_B&L_EWMA(OC_EWP_C)* on *TAA1_B&L_EWMA(OC_EWP_L)* and on the two exchange rates.

	Estimate	SE	tStat	pValue
	-0,0607	0,0536	-1,1327	0,2622
RLocal	1,1067***	0,0303	36,5152	9,66E-41
€/$	0,1454***	0,0284	5,1280	3,78E-06
¥/$	0,0044	0,0170	0,2572	0,7980

Table A.125: Regression coefficients of *TAA1_B&L_Roll(HF_C)* on *TAA1_B&L_Roll(HF_L)* and on the two exchange rates.

	Estimate	SE	tStat	pValue
	0,0128	0,0460	0,2790	0,7813
RLocal	1,0146***	0,0241	42,1016	4,39E-44
€/$	0,1650***	0,0250	6,5961	1,61E-08
¥/$	0,0285*	0,0148	1,9206	0,0599

Table A.126: Regression coefficients of *TAA1_B&L_EWMA(HF_C)* on *TAA1_B&L_EWMA(HF_L)* and on the two exchange rates.

	Estimate	SE	tStat	pValue
	-0,0008	0,0656	-0,0128	0,9899
RLocal	0,9643***	0,0344	28,0539	1,16E-34
€/$	0,5133***	0,0563	9,1103	1,20E-12
¥/$	0,0453**	0,0213	2,1277	0,0378

Table A.127: Regression coefficients of *TAA1_B&L_Roll(HF_EWP_C)* on *TAA1_B&L_Roll(HF_EWP_L)* and on the two exchange rates.

	Estimate	SE	tStat	pValue
	0,1456	0,1600	0,9096	0,3669
RLocal	0,9492***	0,0839	11,3102	4,38E-16
€/$	0,5218***	0,1385	3,7673	3,98E-04
¥/$	0,1496***	0,0519	2,8817	0,0056

Table A.128: Regression coefficients of *TAA1_B&L_EWMA(HF_EWP_C)* on *TAA1_B&L_EWMA(HF_EWP_L)* and on the two exchange rates.

	Estimate	SE	tStat	pValue
	0,0790	0,0601	1,3137	0,1943
RLocal	0,8497***	0,0548	15,5196	6,13E-22
€/$	0,0791***	0,0190	4,1717	1,06E-04
¥/$	0,0742***	0,0178	4,1590	1,11E-04

Table A.129: Regression coefficients of *TAA2_Mark_Roll(OC_C)* on *TAA1_Mark_Roll(OC_L)* and on the two exchange rates.

	Estimate	SE	tStat	pValue
	0,0438	0,0628	0,6966	0,4889
RLocal	0,8909***	0,0578	15,4201	8,22E-22
€/$	0,0968***	0,0197	4,9238	7,86E-06
¥/$	0,0902***	0,0186	4,8441	1,04E-05

Table A.130: Regression coefficients of *TAA2_Mark_EWMA(OC_C)* on *TAA1_Mark_EWMA(OC_L)* and on the two exchange rates.

	Estimate	SE	tStat	pValue
	-0,0031	0,0917	-0,0340	0,9730
RLocal	1,1078***	0,0781	14,1851	3,45E-20
€/$	0,0202	0,0303	0,6659	0,5082
¥/$	0,0939***	0,0270	3,4740	9,97E-04

Table A.131: Regression coefficients of *TAA2_Mark_Roll(OC_EWP_C)* on *TAA1_Mark_Roll(OC_EWP_L)* and on the two exchange rates.

	Estimate	SE	tStat	pValue
	0,0258	0,0865	0,2978	0,7670
RLocal	1,1467***	0,0755	15,1890	1,63E-21
€/$	0,0357	0,0283	1,2610	0,2125
¥/$	0,1198***	0,0256	4,6827	1,84E-05

Table A.132: Regression coefficients of *TAA2_Mark_EWMA(OC_EWP_C)* on *TAA1_Mark_EWMA(OC_EWP_L)* and on the two exchange rates.

	Estimate	SE	tStat	pValue
	0,0186	0,0543	0,3424	0,7333
RLocal	0,8881***	0,0522	17,0123	8,68E-24
€/$	0,0795***	0,0160	4,9823	6,38E-06
¥/$	0,0998***	0,0162	6,1429	8,89E-08

Table A.133: Regression coefficients of *TAA2_Mark_Roll(HF_C)* on *TAA1_Mark_Roll(HF_L)* and on the two exchange rates.

	Estimate	SE	tStat	pValue
	0,0180	0,0568	0,3175	0,7520
RLocal	0,8907***	0,0550	16,1964	8,62E-23
€/$	0,0902***	0,0168	5,3824	1,50E-06
¥/$	0,1189***	0,0172	6,9084	4,90E-09

Table A.134: Regression coefficients of *TAA2_Mark_EWMA(HF_C)* on *TAA1_Mark_EWMA(HF_L)* and on the two exchange rates.

	Estimate	SE	tStat	pValue
	0,0348	0,0850	0,4089	0,6842
RLocal	1,0465***	0,0732	14,2917	2,48E-20
€/$	0,0466	0,0280	1,6666	0,1012
¥/$	0,0801***	0,0254	3,1525	0,0026

Table A.135: Regression coefficients of *TAA2_Mark_Roll(HF_EWP_C)* on *TAA1_Mark_Roll(HF_EWP_L)* and on the two exchange rates.

	Estimate	SE	tStat	pValue
	0,0587	0,0818	0,7175	0,4760
RLocal	1,0737***	0,0719	14,9226	3,62E-21
€/$	0,0527*	0,0266	1,9837	0,0522
¥/$	0,1050***	0,0244	4,3077	6,72E-05

Table A.136: Regression coefficients of *TAA2_Mark_EWMA(HF_EWP_C)* on *TAA1_Mark_EWMA(HF_EWP_L)* and on the two exchange rates.

	Estimate	SE	tStat	pValue
	0,0515	0,0723	0,7125	0,4791
RLocal	0,9830***	0,0260	37,8489	1,40E-41
€/$	0,1600***	0,0397	4,0307	1,70E-04
¥/$	0,0218	0,0250	0,8738	0,3860

Table A.137: Regression coefficients of *TAA2_B&L_Roll(OC_C)* on *TAA1_B&L_Roll(OC_L)* and on the two exchange rates.

	Estimate	SE	tStat	pValue
	0,1198	0,1361	0,8801	0,3826
RLocal	0,9915***	0,0497	19,9580	3,95E-27
€/$	0,2122***	0,0768	2,7651	0,0077
¥/$	0,0794*	0,0468	1,6941	0,0958

Table A.138: Regression coefficients of *TAA2_B&L_EWMA(OC_C)* on *TAA1_B&L_EWMA(OC_L)* and on the two exchange rates.

	Estimate	SE	tStat	pValue
	0,0548	0,0895	0,6122	0,5429
RLocal	0,9604***	0,0352	27,3203	4,65E-34
€/$	0,4088***	0,0646	6,3308	4,38E-08
¥/$	0,0437	0,0303	1,4431	0,1546

Table A.139: Regression coefficients of *TAA2_B&Ll_Roll(OC_EWP_C)* on *TAA1_B&L_Roll(OC_EWP_L)* and on the two exchange rates.

	Estimate	SE	tStat	pValue
	0,2094	0,1887	1,1094	0,2720
RLocal	0,9451***	0,0747	12,6489	4,71E-18
€/$	0,4331***	0,1392	3,1121	0,0029
¥/$	0,1489**	0,0638	2,3357	0,0231

Table A.140: Regression coefficients of *TAA2_B&L_EWMA(OC_EWP_C)* on *TAA1_B&L_EWMA(OC_EWP_L)* and on the two exchange rates.

	Estimate	SE	tStat	pValue
	0,0576	0,0719	0,8010	0,4265
RLocal	0,9883***	0,0252	39,1592	2,23E-42
€/$	0,1188***	0,0378	3,1444	0,0027
¥/$	0,0196	0,0249	0,7881	0,4340

Table A.141: Regression coefficients of *TAA2_B&L_Roll(HF_C)* on *TAA1_B&L_Roll(HF_L)* and on the two exchange rates.

	Estimate	SE	tStat	pValue
	0,1086	0,1276	0,8511	0,3983
RLocal	0,9999***	0,0456	21,9336	3,61E-29
€/$	0,1726**	0,0689	2,5048	0,0152
¥/$	0,0686	0,0440	1,5583	0,1248

Table A.142: Regression coefficients of *TAA2_B&L_EWMA(HF_C)* on *TAA1_B&L_EWMA(HF_L)* and on the two exchange rates.

	Estimate	SE	tStat	pValue
	0,0451	0,0844	0,5342	0,5953
RLocal	0,9783***	0,0299	32,7182	3,45E-38
€/$	0,3565***	0,0557	6,4022	3,34E-08
¥/$	0,0350	0,0290	1,2086	0,2319

Table A.143: Regression coefficients of *TAA2_B&L_Roll(HF_EWP_C)* on *TAA1_B&L_Roll(HF_EWP_L)* and on the two exchange rates.

	Estimate	SE	tStat	pValue
	0,5028***	0,1359	3,6993	0,0005
RLocal	0,2372***	0,0487	4,8714	9,48E-06
€/$	0,2132**	0,0918	2,3232	0,0238
¥/$	0,0373	0,0465	0,8026	0,4256

Table A.144: Regression coefficients of *TAA2_B&L_EWMA(HF_EWP_C)* on *TAA1_B&L_EWMA(HF_EWP_L)* and on the two exchange rates.

When the coefficients of the betas are statistically significant at the confidence level of 99%, 95% or 90% they are marked as, respectively, ***, **, *; otherwise they are not statistically significant.

Appendix A MATLAB CODES

This appendix shows the majority of the MATLAB codes applied in this work. MATLAB is the software used in most part of the computations carried out in this writing and only to compute the realized cumulative returns Excel has been adopted.

```
%% DATA
%load the data that have to be used for the analysis
[data,text]=xlsread('TESI.xlsx','Data');
Assets=data(:,1:51);
Date=datenum(text(2:size(text,1),1),'dd/mm/yyyy');
Date=datestr(Date);
% I need to restrict the entire sample to a common starting point (Jan-
% 1998)
R=Assets(year(datenum(Date))>1997,:);      % price total return indices
d=Date(year(datenum(Date))>1997,:);        % dates
% To compute returns it exists the  Matlab function price2ret, or I can
% calculate it analitically with the following formula:
% (R_daily=((R(2:(size(R,1)),:)./R(1:(size(R,1)-1),:))-1)*100);
% the two methods give the same results.
Rd=(price2ret(R,'','Periodic'))*100; %daily percentage return vector I need
% to convert daily into monthly, and then into yearly, and do again the
% calculations for returns
pM=[];
dM=[];
d1=d(1,:);
for j=2:size(R,1);
    if month(d1)==month(d(j,:));
    else
        pM=R(j-1,:);
        dM=d(j-1,:);
        break;
    end;
end;
d1=dM;
for i=j:size(R,1);
    if month(d1)==month(d(i,:));
    else
        pM=[pM;R(i-1,:)];
        dM=[dM;d(i-1,:)];
        d1=d(i,:);
    end;
end;
rM=(price2ret(pM,'','Periodic'))*100;
rM=rM(2:size(rM,1),:);
dM=dM((end-size(rM,1)+1):end,:);
%% Rolling Sample Moments
w=60; % The rolling window I have set up is at 5 years, then 60 months.
% To compute the rolling mean I have used the MATLAB function tsmovavg.
% On the other hand, to obtain the rolling variance and covariance matrix,
% the used function is:
function [RollVarCovarMatrix,Std]=rollingVar(data,rollingWindow)
RollVarCovarMatrix=zeros(size(data,2),size(data,2),size(data,1)-
rollingWindow);
Std=zeros(size(data,1)-rollingWindow,size(data,2));
for j=rollingWindow:(size(data,1)-1);
    RollVarCovarMatrix(:,:,j-rollingWindow+1)=cov(data(j-
rollingWindow+1:j,1:size(data,2)));
    Std(j-rollingWindow+1,:)=sqrt(diag(RollVarCovarMatrix(:,:,j-
rollingWindow+1)));
```

```
end;
end
%% Exponential smoothing moments (EWMA)
function [EWMAverage, EWMVarCovar,Std]=EWMA(data,w)
EWMAverage=zeros(size(data,1)-w,size(data,2));
EWMVarCovar=zeros(size(data,2),size(data,2),size(data,1)-w);
Std=zeros(size(data,1)-w,size(data,2));
lambda=1-2/(w+1);
EWMAverage(1,:)=mean(data(1:w,:));
EWMVarCovar(:,:,1)=cov(data(1:w,:));
for i=(w+1):(size(data,1)-1);
    EWMAverage(i-w+1,:)=lambda*EWMAverage(i-w,:)+(1-lambda)*data(i,:);
    EWMVarCovar(:,:,i-w+1)=lambda*(EWMVarCovar(:,:,i-w))+...
        (1-lambda)*((data(i,:))'*data(i,:));
end
for j=1:(size(EWMVarCovar,3));
    Std(j,:)=sqrt(diag(EWMVarCovar(:,:,j)));
end
end
%% Strategic Asset Allocation
% The asset allocation for one of the strategic portfolios, from an US
% institutional  investor's point of view, is: 70% in bonds - 60% in
% sovereign  bonds (40% in JPM United States Government Bond, 15% in JPM
% Europe Government Bond and 5% in JPM Japan Government Bond), 10% in
% corporate bonds (5% in BofA ML US Corporate Bond and 5% in BofA ML Euro
% Corporate Bond)-; 40% in stocks (15% in MSCI Europe, 10% in MSCI USA, 5%
% in MSCI Japan).
st_w=[0.4 0.15 0.05 0.05 0.05 0.15 0.1 0.05];
for i=1:size(OCassetsC,1);
    OC_RetPortC(i,:)=OCassetsC(i,:)*(st_w');
    OC_RiskPortC(i,:)=st_w*OCassetsV_C(:,:,i)*(st_w');
    OC_StdPortC(i,:)=sqrt(OC_RiskPortC(i,:));
%
    OC_RetPortL(i,:)=OCassetsL(i,:)*(st_w');
    OC_RiskPortL(i,:)=st_w*OCassetsV_L(:,:,i)*(st_w');
    OC_StdPortL(i,:)=sqrt(OC_RiskPortL(i,:));
end
% % Strategic Portfolio with the introduction of Hedge Funds
st_wHF=[0.4 0.15 0.05 0.05 0.05 0.13 0.08 0.04 0.05];
for i=1:size(HFassetsC,1);
    HF_RetPortC(i,:)=HFassetsC(i,:)*(st_wHF');
    HF_RiskPortC(i,:)=st_wHF*HFassetsV_C(:,:,i)*(st_wHF');
    HF_StdPortC(i,:)=sqrt(HF_RiskPortC(i,:));
%
    HF_RetPortL(i,:)=HFassetsL(i,:)*(st_wHF');
    HF_RiskPortL(i,:)=st_wHF*HFassetsV_L(:,:,i)*(st_wHF');
    HF_StdPortL(i,:)=sqrt(HF_RiskPortL(i,:));
end
% % Equally-weighted Strategic Portfolios with only stocks and bonds
wEWP=(1/size(OCassetsC,2))*ones(1,size(OCassetsC,2));
for i=1:size(OCassetsC,1);
    EWP_OC_RetPortC(i,:)=OCassetsC(i,:)*(wEWP');
    EWP_OC_RiskPortC(i,:)=wEWP*OCassetsV_C(:,:,i)*(wEWP');
    EWP_OC_StdPortC(i,:)=sqrt(EWP_OC_RiskPortC(i,:));
%
    EWP_OC_RetPortL(i,:)=OCassetsL(i,:)*(wEWP');
    EWP_OC_RiskPortL(i,:)=st_w*OCassetsV_L(:,:,i)*(wEWP');
    EWP_OC_StdPortL(i,:)=sqrt(EWP_OC_RiskPortL(i,:));
end
```

```
% Equally-weighted Strategic Portfolios with stocks, bonds and hedge
% funds
HFwEWP=(1/size(HFassetsC,2))*ones(1,size(HFassetsC,2));
for i=1:size(HFassetsC,1);
% % % Portfolios with the MA method
    HF_EWP_RetPortC(i,:)=HFassetsC(i,:)*(HFwEWP');
    HF_EWP_RiskPortC(i,:)=HFwEWP*HFassetsV_C(:,:,i)*(HFwEWP');
    HF_EWP_StdPortC(i,:)=sqrt(HF_EWP_RiskPortC(i,:));
%
    HF_EWP_RetPortL(i,:)=HFassetsL(i,:)*(HFwEWP');
    HF_EWP_RiskPortL(i,:)=HFwEWP*HFassetsV_L(:,:,i)*(HFwEWP');
    HF_EWP_StdPortL(i,:)=sqrt(HF_EWP_RiskPortL(i,:));
end
%% Black and Litterman
% The function InverseOptimiz is the one used to find the B&L implied
% returns.
function [Beta,ImpliedReturns]=InverseOptimiz...
     (ExcessRetMarket,VarCovarMarketAssets,VarMarket)
Beta=zeros(size(ExcessRetMarket,1),size(VarCovarMarketAssets,1)-1);
ImpliedReturns=zeros(size(ExcessRetMarket,1),size(VarCovarMarketAssets,1)-
1);
for i=1:size(ExcessRetMarket,1);
    Beta(i,:)=VarCovarMarketAssets(15,1:14,i)/VarMarket(i,:);
    ImpliedReturns(i,:)=(ExcessRetMarket(i,:)')*Beta(i,:);
end
end
% B&L views:
function [Modigliani]=ModiglianiIndex...
    (RetAssets,RetRiskFree,DevStdAssets,DevStdMarket)
Modigliani=zeros(size(RetAssets,1),size(RetAssets,2));
for i=1:size(RetAssets,1);
Modigliani(i,:)=RetRiskFree(i,:)+((DevStdMarket(i,:)/DevStdAssets(i,:))*(Re
tAssets(i,:)-RetRiskFree(i,:)));
end
end
tau=0.025;
function [P,Q,Omega]=BLviews(ModiglianiIndex,VarCovarMatrix,tau)
P=zeros(size(ModiglianiIndex));
Q=zeros(size(ModiglianiIndex,1),1);
Omega=zeros(size(ModiglianiIndex,1),1);
for r=1:size(ModiglianiIndex,1);
    [~,indexOfMax]=max(ModiglianiIndex(r,:));
    P(r,indexOfMax)=1;
    [~,indexOfMin]=min(ModiglianiIndex(r,:));
    P(r,indexOfMin)=-1;
    Q(r,:)=max(ModiglianiIndex(r,:))-min(ModiglianiIndex(r,:));
    Omega(r,:)=diag(P(r,:)*(tau*VarCovarMatrix(:,:,r))*(P(r,:)'));
end
end
% Posterior Returns and Variance Covariance Matrix:
function [PostReturns,PostVarCovarMatrix]=BlackANDLitterman...
    (EqReturns,Sigma,tau,P,Q,Omega)
PostReturns=zeros(size(EqReturns,1),size(Sigma,1));
PostVarCovarMatrix=zeros(size(Sigma,1),size(Sigma,1),size(EqReturns,1));
txS=zeros(size(Sigma));
for i=1:size(P,1);
    txS(:,:,i)=tau*Sigma(:,:,i);
    Post(i,:)=((txS(:,:,i)*(P(i,:)'))*...
        inv(P(i,:)*txS(:,:,i)*(P(i,:)')+Omega(i,:))*(Q(i,:)-
(P(i,:)*(EqReturns(i,:)'))));
    PostReturns(i,:)=EqReturns(i,:)+Post(i,:);
PostVarCovarMatrix(:,:,i)=inv((inv(txS(:,:,i)))+((P(i,:)')* ))*...
        (inv(Omega(i,:)))*P(i,:)));
end
```

```
end
%% Tactical Asset Allocation 1
% In this part I impose that the weights of each asset classes considered
% in the strategic portfolio cannot vary more than 5% (the variation range
% is about 10%).
G1=[1 0 0 0 0 0 0 0 0 0 0 0 0 0];
G2=[0 1 0 0 0 0 0 0 0 0 0 0 0 0];
G3=[0 0 1 0 0 0 0 0 0 0 0 0 0 0];
G4=[0 0 0 1 1 0 0 0 0 0 0 0 0 0];
G5=[0 0 0 0 0 1 0 0 0 0 0 0 0 0];
G6=[0 0 0 0 0 0 1 0 0 0 0 0 0 0];
G7=[0 0 0 0 0 0 0 1 0 0 0 0 0 0];
G8=[0 0 0 0 0 0 0 0 0 0 1 0 0 0];
G9=[1 1 1 0 0 0 0 0 0 0 0 0 0 0];
G10=[0 0 0 0 0 1 1 1 0 0 0 0 0 0];

for i=1:size(rollM_C,1);
% % Tactical asset allocation with MA method (strategic asset allocation
% % with only stocks and bonds)
    TAA1_pRollC(i,:)=Portfolio('Name','RollTAA1.C.Portfolios',...
        AssetList',names_common,'AssetMean',rollM_C(i,:),...
        'AssetCovar',rollV_C(:,:,i));
    TAA1_pRollC(i,:)=TAA1_pRollC(i,:).setDefaultConstraints;
    TAA1_pRollC(i,:)=TAA1_pRollC(i,:).setGroups(G1,0.35,0.45);
    TAA1_pRollC(i,:)=TAA1_pRollC(i,:).addGroups(G2,0.10,0.20);
    TAA1_pRollC(i,:)=TAA1_pRollC(i,:).addGroups(G3,[],0.10);
    TAA1_pRollC(i,:)=TAA1_pRollC(i,:).addGroups(G4,0.05,0.15);
    TAA1_pRollC(i,:)=TAA1_pRollC(i,:).addGroups(G5,0.10,0.20);
    TAA1_pRollC(i,:)=TAA1_pRollC(i,:).addGroups(G6,0.05,0.15);
    TAA1_pRollC(i,:)=TAA1_pRollC(i,:).addGroups(G7,[],0.10);
    TAA1_pwvtPortRollC(:,i)=...
        TAA1_pRollC(i,:).estimateFrontierByRisk(OC_StdPortC(i,:));
    TAA1_pretPortRollC(i,:)=(assetsC(i,:)*TAA1_pwvtPortRollC(:,i));
    TAA1_priskPortRollC(i,:)=sqrt((TAA1_pwvtPortRollC(:,i)')*...
        assetsV_C(:,:,i)*(TAA1_pwvtPortRollC(:,i)));
% (strategic asset allocation with only stocks and bonds and EW criterion)
    TAA1_EWPpRollC(i,:)=Portfolio('Name','RollTAA1.EWsa.C1.Portfolios',...
        'AssetList',names_common,'AssetMean',rollM_C(i,:),'AssetCovar',...
        rollV_C(:,:,i));
    TAA1_EWPpRollC(i,:)=TAA1_EWPpRollC(i,:).setDefaultConstraints;
    TAA1_EWPpRollC(i,:)=TAA1_EWPpRollC(i,:).setGroups(G9,0.325,0.425);
    TAA1_EWPpRollC(i,:)=TAA1_EWPpRollC(i,:).addGroups(G4,0.20,0.30);
    TAA1_EWPpRollC(i,:)=TAA1_EWPpRollC(i,:).addGroups(G10,0.325,0.425);
    TAA1_EWPpwvtPortRollC(:,i)=...
        TAA1_EWPpRollC(i,:).estimateFrontierByRisk(EWP_OC_StdPortC(i,:));
    TAA1_EWPpriskPortRollC(i,:)=sqrt((TAA1_EWPpwvtPortRollC(:,i)')*...
        assetsV_C(:,:,i)*TAA1_EWPpwvtPortRollC(:,i));
    TAA1_EWPpretPortRollC(i,:)=assetsC(i,:)*TAA1_EWPpwvtPortRollC(:,i);
% % Tactical asset allocation with MA method (strategic asset allocation
% % with stocks, bonds and hedge funds)
    TAA1_HFpRollC(i,:)=Portfolio('Name','RollTAA1.HF.C.Portfolios',...
        'AssetList',names_common,'AssetMean',rollM_C(i,:),'AssetCovar',...
        rollV_C(:,:,i));
    TAA1_HFpRollC(i,:)=TAA1_HFpRollC(i,:).setDefaultConstraints;
    TAA1_HFpRollC(i,:)=TAA1_HFpRollC(i,:).setGroups(G1,0.35,0.45);
    TAA1_HFpRollC(i,:)=TAA1_HFpRollC(i,:).addGroups(G2,0.10,0.20);
    TAA1_HFpRollC(i,:)=TAA1_HFpRollC(i,:).addGroups(G3,[],0.10);
    TAA1_HFpRollC(i,:)=TAA1_HFpRollC(i,:).addGroups(G4,0.05,0.15);
    TAA1_HFpRollC(i,:)=TAA1_HFpRollC(i,:).addGroups(G5,0.08,0.18);
    TAA1_HFpRollC(i,:)=TAA1_HFpRollC(i,:).addGroups(G6,0.03,0.13);
    TAA1_HFpRollC(i,:)=TAA1_HFpRollC(i,:).addGroups(G7,[],0.09);
    TAA1_HFpRollC(i,:)=TAA1_HFpRollC(i,:).addGroups(G8,[],0.10);
    TAA1_HFpwvtPortRollC(:,i)=...
        TAA1_HFpRollC(i,:).estimateFrontierByRisk(HF_StdPortC(i,:));
```

```
    TAA1_HFpriskPortRollC(i,:)=sqrt((TAA1_HFpwvtPortRollC(:,i)')*...
        assetsV_C(:,:,i)*TAA1_HFpwvtPortRollC(:,i));
    TAA1_HFpretPortRollC(i,:)=assetsC(i,:)*TAA1_HFpwvtPortRollC(:,i);
% % (strategic asset allocation with stocks, bonds and hedge funds and
% % EW criterion)
    TAA1_HF_EWPp_RollC(i,:)=Portfolio('Name',...
        'RollTAA1.HF.EWsa.C.Portfolios','AssetList',names_common,...
        'AssetMean',rollM_C(i,:),'AssetCovar',rollV_C(:,:,i));
    TAA1_HF_EWPp_RollC(i,:)=TAA1_HF_EWPp_RollC(i,:).setDefaultConstraints;
    TAA1_HF_EWPp_RollC(i,:)=...
        TAA1_HF_EWPp_RollC(i,:).setGroups(G9,0.283,0.383);
    TAA1_HF_EWPp_RollC(i,:)=...
        TAA1_HF_EWPp_RollC(i,:).addGroups(G4,0.172,0.272);
    TAA1_HF_EWPp_RollC(i,:)=...
        TAA1_HF_EWPp_RollC(i,:).addGroups(G10,0.283,0.383);
    TAA1_HF_EWPp_RollC(i,:)=...
        TAA1_HF_EWPp_RollC(i,:).addGroups(G8,0.0611,0.161);
    TAA1_HF_EWPpwvtPortRollC(:,i)=...
        TAA1_HF_EWPp_RollC(i,:).estimateFrontierByRisk...
        (HF_EWP_StdPortC(i,:));
    TAA1_HF_EWPpriskPortRollC(i,:)=...
        sqrt((TAA1_HF_EWPpwvtPortRollC(:,i)')*assetsV_C(:,:,i)...
        *TAA1_HF_EWPpwvtPortRollC(:,i));
    TAA1_HF_EWPpretPortRollC(i,:)=assetsC(i,:)...
        *TAA1_HF_EWPpwvtPortRollC(:,i);
end
%% Tactical Asset Allocation 2
% In this part I impose that the weights of each asset classes considered
% in the strategic portfolio cannot vary more than 15% (the variation range
% is about 30%).

for i=1:size(rollM_C,1);
% % Tactical asset allocation with MA method (strategic asset allocation
% % with only stocks and bonds)
    TAA2_pRollC(i,:)=Portfolio('Name','RollTAA2.C.Portfolios',...
        'AssetList',names_common,'AssetMean',rollM_C(i,:),'AssetCovar',...
        rollV_C(:,:,i));
    TAA2_pRollC(i,:)=TAA2_pRollC(i,:).setDefaultConstraints;
    TAA2_pRollC(i,:)=TAA2_pRollC(i,:).setGroups(G1,0.25,0.55);
    TAA2_pRollC(i,:)=TAA2_pRollC(i,:).addGroups(G2,[],0.30);
    TAA2_pRollC(i,:)=TAA2_pRollC(i,:).addGroups(G3,[],0.20);
    TAA2_pRollC(i,:)=TAA2_pRollC(i,:).addGroups(G4,[],0.25);
    TAA2_pRollC(i,:)=TAA2_pRollC(i,:).addGroups(G10,0.15,0.45);
    TAA2_pwvtPortRollC(:,i)=...
        TAA2_pRollC(i,:).estimateFrontierByRisk(OC_StdPortC(i,:));
    TAA2_priskPortRollC(i,:)=sqrt((TAA2_pwvtPortRollC(:,i)')*...
        assetsV_C(:,:,i)*TAA2_pwvtPortRollC(:,i));
    TAA2_pretPortRollC(i,:)=assetsC(i,:)*TAA2_pwvtPortRollC(:,i);
% % (strategic asset allocation with only stocks and bonds and EW
% % criterion)
    TAA2_EWPpRollC(i,:)=Portfolio('Name',...
        'RollTAA2.EWsa.C.Portfolios','AssetList',names_common,...
        'AssetMean',rollM_C(i,:),'AssetCovar',rollV_C(:,:,i));
    TAA2_EWPpRollC(i,:)=TAA2_EWPpRollC(i,:).setDefaultConstraints;
    TAA2_EWPpRollC(i,:)=TAA2_EWPpRollC(i,:).setGroups(G9,0.225,0.525);
    TAA2_EWPpRollC(i,:)=TAA2_EWPpRollC(i,:).addGroups(G4,0.10,0.40);
    TAA2_EWPpRollC(i,:)=TAA2_EWPpRollC(i,:).addGroups(G10,0.225,0.525);
    TAA2_EWPpwvtPortRollC(:,i)=...
        TAA2_EWPpRollC(i,:).estimateFrontierByRisk(EWP_OC_StdPortC(i,:));
    TAA2_EWPpriskPortRollC(i,:)=sqrt((TAA2_EWPpwvtPortRollC(:,i)')*...
        assetsV_C(:,:,i)*TAA2_EWPpwvtPortRollC(:,i));
    TAA2_EWPpretPortRollC(i,:)=assetsC(i,:)*TAA2_EWPpwvtPortRollC(:,i);
% % Tactical asset allocation with MA method (strategic asset allocation
% % with stocks, bonds and hedge funds)
```

```
    TAA2_HFp_RollC(i,:)=Portfolio('Name','RollTAA2.HF.C.Portfolios',...
        'AssetList',names_common,'AssetMean',rollM_C(i,:),'AssetCovar',...
        rollV_C(:,:,i));
    TAA2_HFp_RollC(i,:)=TAA2_HFp_RollC(i,:).setDefaultConstraints;
    TAA2_HFp_RollC(i,:)=TAA2_HFp_RollC(i,:).setGroups(G1,0.25,0.55);
    TAA2_HFp_RollC(i,:)=TAA2_HFp_RollC(i,:).addGroups(G2,[],0.30);
    TAA2_HFp_RollC(i,:)=TAA2_HFp_RollC(i,:).addGroups(G3,[],0.20);
    TAA2_HFp_RollC(i,:)=TAA2_HFp_RollC(i,:).addGroups(G4,[],0.25);
    TAA2_HFp_RollC(i,:)=TAA2_HFp_RollC(i,:).addGroups(G10,0.10,0.40);
    TAA2_HFp_RollC(i,:)=TAA2_HFp_RollC(i,:).addGroups(G8,[],0.20);
    TAA2_HFpwvtPortRollC(:,i)=...
        TAA2_HFp_RollC(i,:).estimateFrontierByRisk(HF_StdPortC(i,:));
    TAA2_HFpriskPortRollC(i,:)=sqrt((TAA2_HFpwvtPortRollC(:,i)')*...
        assetsV_C(:,:,i)*TAA2_HFpwvtPortRollC(:,i));
    TAA2_HFpretPortRollC(i,:)=assetsC(i,:)*TAA2_HFpwvtPortRollC(:,i);
% % (strategic asset allocation with stocks, bonds and hedge funds and EW
% % criterion)
    TAA2_HF_EWPp_RollC(i,:)=Portfolio('Name',...
        'RollTAA2.HF.EWsa.C.Portfolios','AssetList',names_common,...
        'AssetMean',rollM_C(i,:),'AssetCovar',rollV_C(:,:,i));
    TAA2_HF_EWPp_RollC(i,:)=...
        TAA2_HF_EWPp_RollC(i,:).setDefaultConstraints;
    TAA2_HF_EWPp_RollC(i,:)=...
        TAA2_HF_EWPp_RollC(i,:).setGroups(G9,0.183,0.483);
    TAA2_HF_EWPp_RollC(i,:)=...
        TAA2_HF_EWPp_RollC(i,:).addGroups(G4,0.072,0.372);
    TAA2_HF_EWPp_RollC(i,:)=...
        TAA2_HF_EWPp_RollC(i,:).addGroups(G10,0.183,0.483);
    TAA2_HF_EWPp_RollC(i,:)=TAA2_HF_EWPp_RollC(i,:).addGroups(G8,[],0.261);
    TAA2_HF_EWPpwvtPortRollC(:,i)=...
        TAA2_HF_EWPp_RollC(i,:).estimateFrontierByRisk...
        (HF_EWP_StdPortC(i,:));
    TAA2_HF_EWPpriskPortRollC(i,:)=...
        sqrt((TAA2_HF_EWPpwvtPortRollC(:,i)')*assetsV_C(:,:,i)*...
        TAA2_HF_EWPpwvtPortRollC(:,i));
    TAA2_HF_EWPpretPortRollC(i,:)=...
        assetsC(i,:)*TAA2_HF_EWPpwvtPortRollC(:,i);
end
%% Performance indicators
% Cumulative returns
CumulativeReturns=xlsread('CumulativeReturns.xls','CumulativeReturns');
% Sharpe ratio
Sharpe_Ratio(1,:)=sharpe(OC_RetPortC);
...
Sharpe_Ratio(72,:)=sharpe(TAA2_HF_EWP_BLpretPortEsL);
% Jensen's alpha
Jensen_Alpha(:,1)=portalpha(OC_RetPortC,MarkPMr,0,'capm');
...
Jensen_Alpha(:,72)=portalpha(TAA2_HF_EWP_BLpretPortEsL,MarkPMr,0,'capm');
% Treynor ratio
function [Treynor_Ratio]=TreynorRatio(RealRetPortfolio,RealRetMarket)
zero=zeros(size(RealRetPortfolio));
for i=1:size(RealRetPortfolio,1);
    ExcPort(i,:)=RealRetPortfolio(i,:)-zero(i,:);
    ExcMarket(i,:)=RealRetMarket(i,:)-zero(i,:);
end
VarMarket=var(ExcMarket);
VarCovarMatrix=cov(ExcPort,ExcMarket);
Beta=VarCovarMatrix(1,2)/VarMarket;
Treynor_Ratio=(mean(RealRetPortfolio-zero))./Beta;
end
% Sortino index
function [SortinoIndex]=SortinoIndex(RealRetPortfolio)
SortinoIndex=(mean(RealRetPortfolio))/sqrt(lpm(RealRetPortfolio,2));
```

```
end
% Information ratio
Information_Ratio(1,:)=inforatio(TAA1_pretPortRollC,OC_RetPortC);
...
Information_Ratio(64,:)= ...
     inforatio(TAA2_HF_EWP_BLpretPortEsL,HF_EWP_RetPortL);
% Value at Risk & Expected Shortfall
alpha=0.05;
function [ValueAtRisk,ExpectedShortfall]=VaR_ES(RealRetPortfolio,alpha)
distributionPortRet=sort(RealRetPortfolio);
VaR=quantile(distributionPortRet,alpha);
ValueAtRisk=abs(VaR);
diff=zeros(size(RealRetPortfolio));
VarRealRet=zeros(size(RealRetPortfolio));
for i=1:size(RealRetPortfolio);
    if RealRetPortfolio(i,:)<VaR;
        diff(i,:)=1;
    else
        diff(i,:)=0;
    end
    VarRealRet(i,:)=RealRetPortfolio(i,:)*diff(i,:);
end
NonZerosElements=nonzeros(VarRealRet);
ExpectedShortfall=abs((sum(VarRealRet))/length(NonZerosElements));
end
```

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