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Quantitative Portfolio Strategy
Equity Research

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A Parameter-Free Forecast of the Equity Risk Premium Using Option Prices: Evidence from European Markets

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Introduction

- Though many theoretical models posit that equity risk premium is high when the market is volatile, little research is successful in finding predictability using various volatility measures such as VIX, realized return variance, etc. from predictive regressions*

$$r_{t+1} = \alpha + \beta x_t + \varepsilon_{t+1}$$

- Predictability evidence is, in general, scarce. Most predictors are highly persistent, leading to unstable estimates of α and β, hurting out-of-sample performances
- Recently, Martin (2017) argues that risk-neutral return variance computed from option prices can be used *directly* as a forecast, saving the need to estimate α, β
- We apply this approach in Europe and examine such forecast's predictability of European equity risk premium and its economic value in the context of asset allocations

* See for example Goyal and Welch 2008, Bollerslev, Tauchen and Zhou 2009
Goyal, A., Welch, I., 2008, A comprehensive look at the empirical performance of equity premium prediction, Review of Financial Studies 21(4)
Bollerslev,T., Tauchen, G., Zhou, H., 2009, Expected stock returns and variance risk premia, Review of Financial Studies 22(11)
Martin, I., 2017, What is the expected return on the market?, Quarterly Journal of Economics 132(1)

Outline

- Methodology Overview
- Data
- Statistical Evidence
- Economic Value of the Forecast
- Summary

Methodology Overview

Risk Premium Identity

- We define the following notations:
  - ✓ $R_T, R_{f,t}$: gross return on the asset and risk free asset from time t to time T
  - ✓ M_T: stochastic discount factor that prices time-T payoffs as of time t
- Martin (2017) provides an identity of risk premium (i.e., expected excess return) for any asset by relying just on no-arbitrage*

$$E_t(R_T) - R_{f,t} = \frac{1}{R_{f,t}} Var_t^*(R_T) - cov_t(M_T R_T, R_T)$$

where asterisks indicate risk-neutral quantities

- When underlying asset is the market, the covariance term is weakly negative in almost all quantitatively reasonable models of financial markets. Empirically, the term is very small in the US
- Hence $\frac{1}{R_{f,t}} Var_t^*(R_T)$ can be used as a forecast of the equity risk premium
- More importantly, the quantity can be observed from option prices and no coefficient estimation is needed

*: See appendix for derivation

Uncovering Risk-neutral Variance from Option Prices

- Risk-neutral variance can be computed using option prices (Martin 2017, see appendix for details)

$$\frac{1}{R_{f,t}} Var_t^*(R_T) = \frac{2}{S_t^2}\left\{\int_0^{F_{t,T}} put_{t,T}(K)dK + \int_{F_{t,T}}^{\infty} call_{t,T}(K)dK\right\}$$

where $F_{t,T}$ is the forward price of the underlying, S_t is the spot price, $put_{t,T}(K)$ and $call_{t,T}(K)$ are put/call option prices with strike price K

- The integral can be approximated by option prices from a set of discrete strike prices

From Risk-neutral Variance to the Equity Risk Premium

	U.S.	Europe
No arbitrage	$E_t(R_T) - R_{f,t} = \frac{1}{R_{f,t}} Var_t^*(R_T) - cov_t(M_T R_T, R_T)$	✓
When asset is the market	$cov_t(M_T R_T, R_T) \leq 0$ $E_t(R_T) - R_{f,t} \geq \frac{1}{R_{f,t}} Var_t^*(R_T)$	✓
Empirically, bound is tight	$E_t(R_T) - R_{f,t} \approx \frac{1}{R_{f,t}} Var_t^*(R_T)$	?

Data

Data

- We consider equity market indices of two major European currencies
  - ✓ Euro: STOXX 50 Index
  - ✓ Pound sterling: FTSE 100 Index

- Daily index option price data* from Optionmetrics' Ivy DB Euro database are used to compute discounted risk neutral variance. The sample starts from Jan 2002, due to Optionmetrics' data availability

- Risk-neutral variance is computed daily for two expiration dates, one right before t+1 month and one right after t+1 month. We report the risk neutral variance for t+1 month horizon as the interpolation of the two

*: We use settlement prices calculated by exchanges. These are preferred prices for options traded in Europe as suggested by Optionmetrics. This is different from the data provided by Ivy DB US database for US options, where best bid and ask quotes at market close are reported.

The Lower Bound Is Possibly Tight in European Markets

- Like in the US, discounted risk-neutral variance $\frac{1}{R_{f,t}} Var_t^*(R_T)$ in Europe is highly volatile, right-skewed and fat-tailed

Summary Statistics of Discounted Risk-neutral Variance (Ann., %)									
Index	**Mean**	**StdDev**	**Skew**	**Kurt**	**Min**	**Perc10**	**Perc50**	**Perc90**	**Max**
FTSE 100	4.15	4.64	4.06	24.91	0.74	1.28	2.61	8.48	61.35
STOXX 50	5.92	5.82	3.20	15.30	1.07	1.91	4.09	12.23	78.40

Note: the sample period is from January 2002 to December 2018. Source: Optionmetrics, Barclays Research

- In a series of annual surveys of academics, financial analysts and CFOs, Fernandez and his coauthors report the surveyed risk premium is 5-6% for western European countries for the years 2010-18*

- Given $\frac{1}{R_{f,t}} Var_t^*(R_T)$ is a lower bound on the equity risk premium, the small gap between its average magnitude and numbers reported by Fernandez's team show that the bound is likely to be tight in European markets

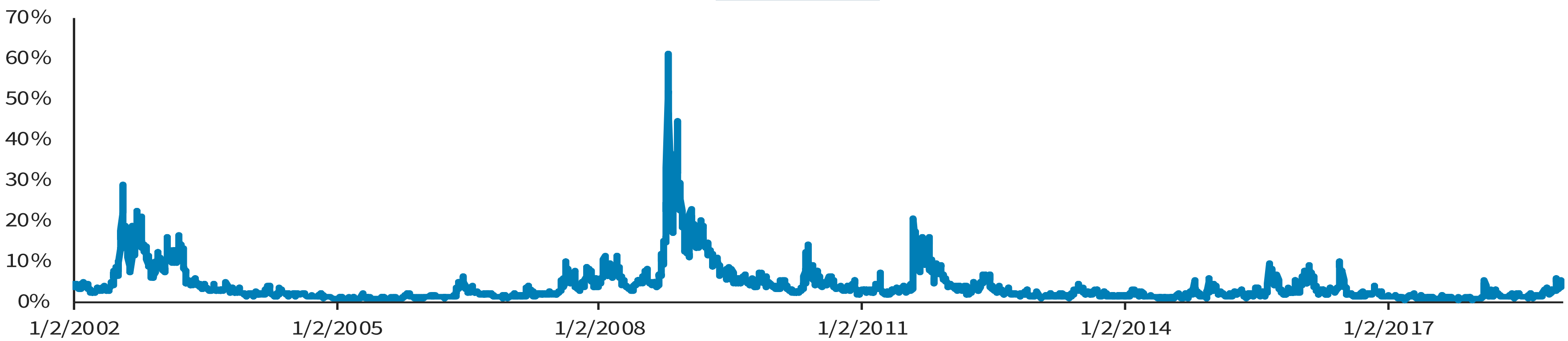
* See: https://papers.ssrn.com/sol3/papers.cfm?abstract_id=3155709 for the survey results of 2018. Page 8 in the report provides links of survey results from previous years.

Risk-Neutral Variances Spike in Volatile Episodes

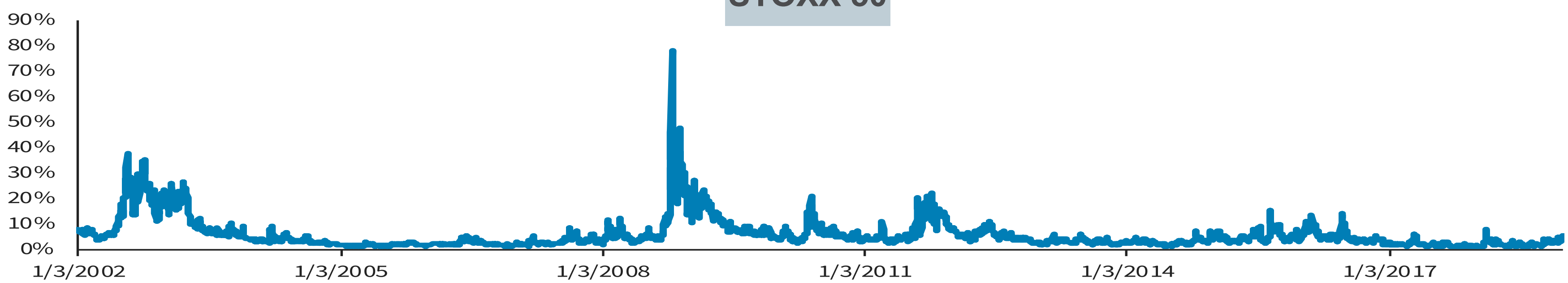
- The risk-neutral variance spiked at the end of the dotcom bubble and during the Global Financial Crisis, European sovereign debt crisis and Brexit referendum

Time Series of Discounted Risk-Neutral Variance of Return (t,t+1mon) (Ann., %)

FTSE 100



STOXX 50



Note: the sample period is from January 2002 to December 2018. Source: Optionmetrics, Barclays Research

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Statistical Evidence

Empirically $Cov\ (M_T R_T, R_T)$ is Small

- One way to show $E_t(R_T) - R_{f,t} \approx \frac{1}{R_{f,t}} Var_t^*(R_T)$ is to check whether $cov_t(M_T R_T, R_T)$ is close to zero
- We estimate M and $cov(M_T R_T, R_T)$ by GMM* using the following test assets: market, riskless asset, 5 times 5 portfolios sorted on size and B/M, 5 times 5 portfolios sorted on size and momentum
- Factors and test assets are monthly European data from Ken French's website
- Empirically, $cov(M_T R_T, R_T)$ is small and negative

Estimates of Coefficients in the four-factor model and of $Cov(M_T R_T, R_T)$

Market	Constant	$R_M - R_f$	SMB	HML	MOM	$\widehat{cov}(M_T R_T, R_T)$
Eurozone	1.055	-2.696	0.738	-4.749	-8.843	-0.0006
	(0.032)	(1.042)	(1.938)	(2.187)	(1.587)	(0.0018)
UK	1.059	-3.015	-0.745	-7.579	-5.690	-0.0012
	(0.030)	(1.076)	(1.954)	(2.290)	(1.449)	(0.0012)

Note: the sample is based on full sample on Ken French's website, November 1990 to July 2019. Numbers in parentheses are standard errors from GMM estimation. Eurozone market returns are returns on Euro Stoxx 50 index and are calculated using ECU before the official adoption of Euro. Source: Bloomberg, Ken French's website, STOXX.com, Barclays Research

* See Hansen, L. P., 1982, Large Sample Properties of Generalized Method of Moments Estimators, Econometrica, 50(4)

Predictive Regressions

- The second way is to test the null hypothesis $\alpha = 0$ and $\beta = 1$ in predictive regressions

- We estimate the following predictive regression with t running daily from Jan 2002 to Dec 2018 and $T = t + 30d$

$$R_T - R_{f,t} = \alpha + \beta \times \frac{1}{R_{f,t}} Var_t^*(R_T) + \varepsilon_T$$

- Robust Hansen-Hodrick standard errors* are reported to control for overlapping in observations. Cannot reject the null hypothesis in both the UK and the eurozone

Coefficient Estimates of Predictive Regressions

Index	α	Std. err.	β	Std. err.	Adjusted R Square (%)
FTSE 100	-0.002	0.003	0.869	0.694	0.5
Stoxx 50	-0.002	0.004	0.447	0.813	0.1

Note: the sample is from 2002 to 2018. Robust Hansen-Hodrick standard errors are reported. Source: Bloomberg, Optionmetrics, Barclays Research

* See Hansen, L. P., Hodrick, R. J., 1980, Forward exchange rates as optimal predictor of future spot rates: an econometric analysis, Journal of Political Economy, 88

Out-of-sample R-squared

- The third way is to examine the out-of-sample R^2 (see Goyal and Welch 2008, Campbell and Thompson 2008)

$$R_{OS}^2 = 1 - \frac{\sum \epsilon_t^2}{\sum \varepsilon_t^2}$$

where ϵ_t is the forecast error when discounted risk-neutral variance is used as the forecast and ε_t is the forecast error when the historical average is used as the forecast*

- In both markets, discounted risk-neutral variance outperforms the prevailing mean benchmark

- Overall, statistical evidence is stronger in the UK market than the eurozone market, potentially due to Stoxx 50 being less representative of the eurozone market than FTSE 100 of the UK market

Out-of-sample R^2

Index	Out-of-sample R Square
FTSE 100	1.2%
Stoxx 50	0.8%

Note: the sample is from 2002 to 2018. The excess return is computed over ICE GBP 1 month Libor and 1 month Euribor respectively. The historical average is computed based on an expanding window. For FTSE 100, the expanding window starts from November 1989 due to the availability of GBP Libor data on Bloomberg. For Stoxx 50, the expanding window starts from December 1998 due to the availability of Euribor data on Bloomberg. Source: Bloomberg, Optionmetrics, Barclays Research

Campbell, J. Y., Thompson, S.B., 2008, Predicting excess stock returns out of sample: can anything beat the historical average? Review of Financial Studies 21(4)

Does the Forecast Have Economic Value?

Testing Economic Value of the Forecast in the Context of Equity-bond Allocations

- A daily rebalanced portfolio of equity index futures and 10 year government bond futures is formulated to maximize quadratic utility $E(R_p) - \gamma Var(R_p)$, using the equity risk premium forecast from option prices. Portfolio is denoted as MVO

- Volatilities and correlations forecasts use sample estimates from daily future returns over the previous month. The bond return forecast is based on most recent yields

- Alternative portfolios of futures are formulated as benchmarks

✓ Risk parity portfolio

✓ Equally weighted portfolio

✓ Equity- or bond-only portfolios

✓ A second portfolio maximizing quadratic utility with the historical average return as the equity return forecast (MVO-HA)

Performance Statistics in the UK

- MVO outperforms all benchmark portfolios, generating the highest Sharpe ratio. It delivers a 31% increase in the Sharpe ratio relative to the second highest
- It also has one of the least severe drawdowns and a tail leaning more toward right than left (mean exceedance is much greater than the mean shortfall compared with other portfolios)

Performance Statistics of Futures Portfolios in the UK, Jan 2002 - Dec 2018

Performance Stats	MVO	Risk Parity	Equally Weighted	MVO-HA	FTSE 100	Long Gilt
Avg Ret (%, Ann.)	7.84	6.07	3.96	4.45	2.05	5.06
Vol (%, Ann.)	8.38	8.38	8.38	8.38	8.38	8.38
Sharpe Ratio (Ann.)	**0.94**	**0.72**	**0.47**	**0.53**	**0.24**	**0.60**
Maximum Drawdown	-16.67	-24.17	-23.22	-13.53	-24.75	-15.45
Mean Shortfall	-1.20	-1.18	-1.21	-1.27	-1.25	-1.14
Mean Exceedance	1.29	1.13	1.21	1.17	1.22	1.16
Avg Equity Notional	36%	48%	48%	56%	45%	0%
Avg Bond Notional	91%	117%	48%	70%	0%	138%

Note: mean shortfall (exceedance) is the daily average of returns below (above) the 5-th (95-th) percentile. Average equity/bond notional are average of absolute notional exposures. MVO and MVO-HA sets $\gamma = 7$. Note Sharpe ratios are unaffected by the selection of γ. MVO-HA uses historical average from an expanding window that starts from February 1988. The starting date is determined by the data availability of equity futures. All returns are scaled up/down so that the ex-post volatility matches that of MVO. Annualized numbers are based on 257 days a year. Source: Bloomberg, Optionmetrics, Barclays Research

Performance Statistics in the Eurozone

- Results are very similar in the eurozone – MVO generated the highest Sharpe ratio (a 31% increase from the second highest), the least severe drawdown and a tail leaning more toward the right

Performance Statistics of Futures Portfolios in the Eurozone, Jan 2002 - Dec 2018

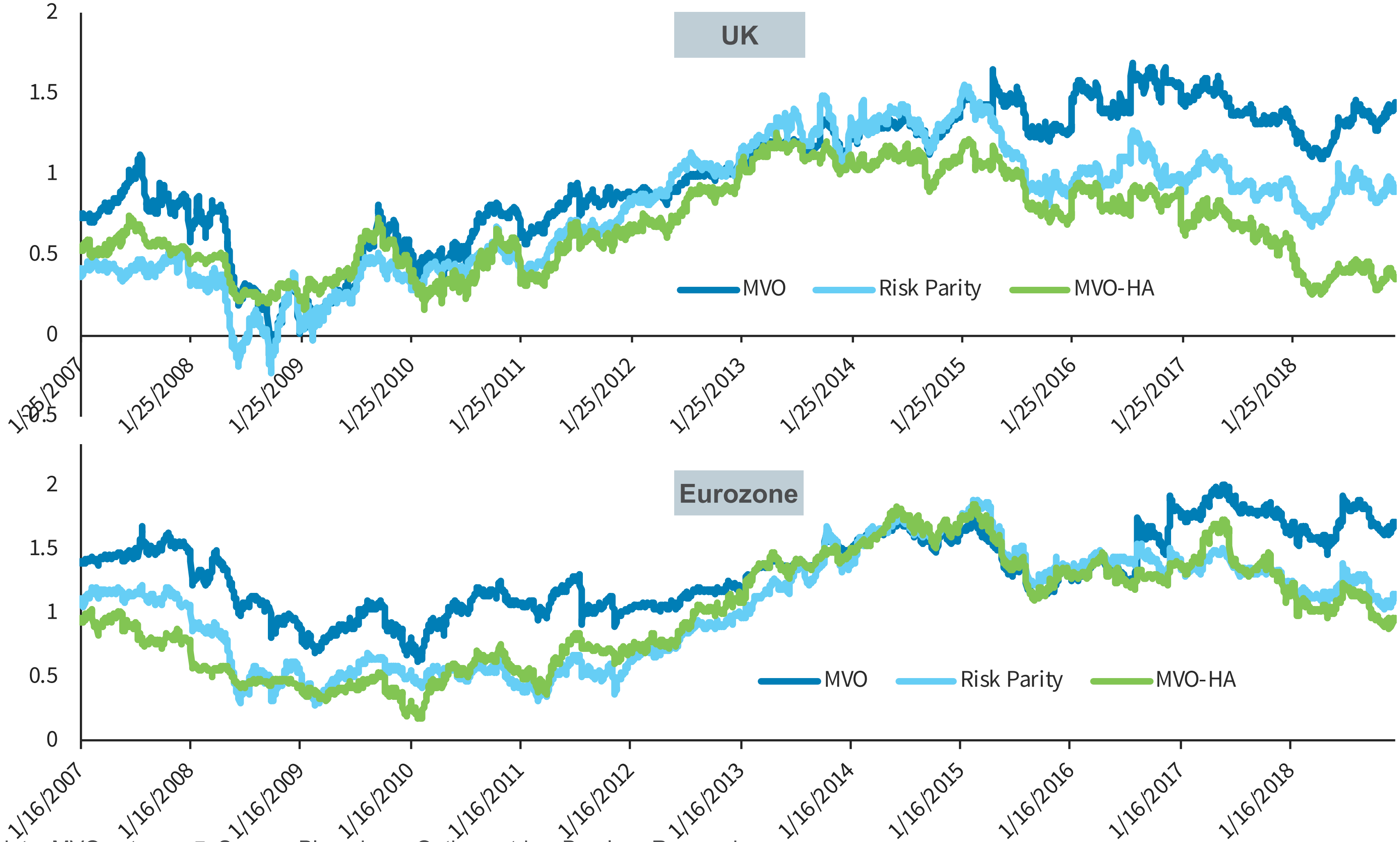
Performance Stats	MVO	Risk Parity	Equally Weighted	MVO-HA	Euro Stoxx 50	Euro Bund
Avg Ret (%, Ann.)	11.10	8.50	3.71	7.75	1.48	8.39
Vol (%, Ann.)	9.64	9.64	9.64	9.64	9.64	9.64
Sharpe Ratio (Ann.)	**1.15**	**0.88**	**0.39**	**0.80**	**0.15**	**0.87**
Maximum Drawdown	-16.22	-28.16	-28.15	-23.87	-30.22	-17.26
Mean Shortfall	-1.39	-1.38	-1.40	-1.42	-1.42	-1.34
Mean Exceedance	1.50	1.28	1.40	1.44	1.40	1.31
Avg Equity Notional	38%	46%	44%	47%	41%	0%
Avg Bond Notional	144%	164%	44%	130%	0%	177%

Note: mean shortfall (exceedance) is the daily average of returns below (above) the 5-th (95-th) percentile. Average equity/bond notional are average of absolute notional exposures. MVO and MVO-HA sets $\gamma = 7$. Note Sharpe ratios are unaffected by the selection of γ. MVO-HA uses historical average from an expanding window that starts from June 1998. The starting date is determined by the data availability of equity futures. All returns are scaled up/down so that the ex-post volatility matches that of MVO. Annualized numbers are based on 257 days a year.
Source: Bloomberg, Optionmetrics, Barclays Research

Outperformance Is Present throughout the Sample

- In both markets, the five-year rolling Sharpe ratio of MVO is almost always no below than that of risk parity and of MVO-HA

Five Year Rolling Annualized Sharpe Ratios, Jan 2002 - Dec 2018



Note: MVO sets $\gamma = 7$. Source: Bloomberg, Optionmetrics, Barclays Research

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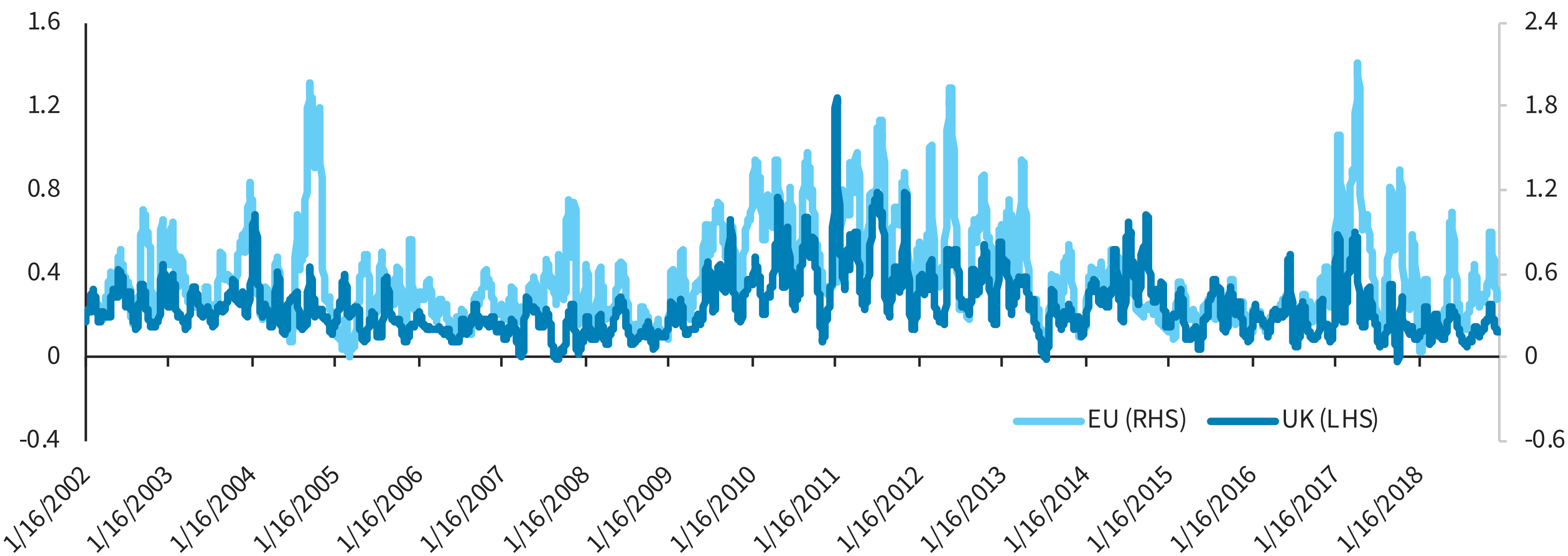
Portfolio Is Not Buying the Dip

- The risk premium forecast is high when the market is volatile
- Is MVO just buying the dip? No, the invested position is a trade-off between risk premium and the risk. Risk is also high when the market is volatile
- In both markets, equity exposures in the MVO portfolios were low during the Financial Crisis. The correlation between equity exposure and volatility index is 0.04 in the UK and 0.07 in the eurozone

Equity Exposures of MVO Portfolios, Jan 2002 - Dec 2018



Note: MVO sets $\gamma = 7$. Figure shows 10-day moving averages. Source: Bloomberg, Optionmetrics, Barclays Research

Portfolio Turnover and Net Cost Performance

- Turnover is defined as half of the mean absolute change in daily exposure (ann.)
- Turnover becomes higher when mean-variance optimization is used in allocation. Nonetheless, MVO still outperforms all the benchmarks after transaction costs
- Net cost performance uses the following transaction cost estimates, expressed as a fraction of the notional value traded: 1.0 bp for FTSE 100, 3.0 bp for Euro STOXX 50, 1.0 bp for Gilt and Bund futures

Turnover and Net Cost Performance of Futures Portfolios, Jan 2002 - Dec 2018

Turnover Stats		MVO	Risk Parity	Equally Weighted	MVO-HA	Equity Future Only	Bond Future Only
UK	Equity Turnover	5.60	2.05	0.00	5.99	0.00	0.00
	Bond Turnover	12.29	2.05	0.00	11.65	0.00	0.00
	Avg Gross Ret	7.84	6.07	3.96	4.45	2.05	5.06
	Avg Net Ret	**7.66**	**6.03**	**3.96**	**4.28**	**2.05**	**5.06**
Eurozone	Equity Turnover	5.71	2.10	0.00	5.62	0.00	0.00
	Bond Turnover	18.56	2.10	0.00	18.22	0.00	0.00
	Avg Gross Ret	11.10	8.50	3.71	7.75	1.48	8.39
	Avg Net Cost	**10.75**	**8.41**	**3.71**	**7.40**	**1.48**	**8.39**

Note: MVO and MVO-HA sets $\gamma = 7$. MVO-HA uses historical average from an expanding window. The starting date is determined by the data availability of equity futures. All returns are scaled up/down so that the ex-post volatility matches that of MVO. Annualized numbers are based on 257 days a year. Source: Bloomberg, Optionmetrics, Barclays Research

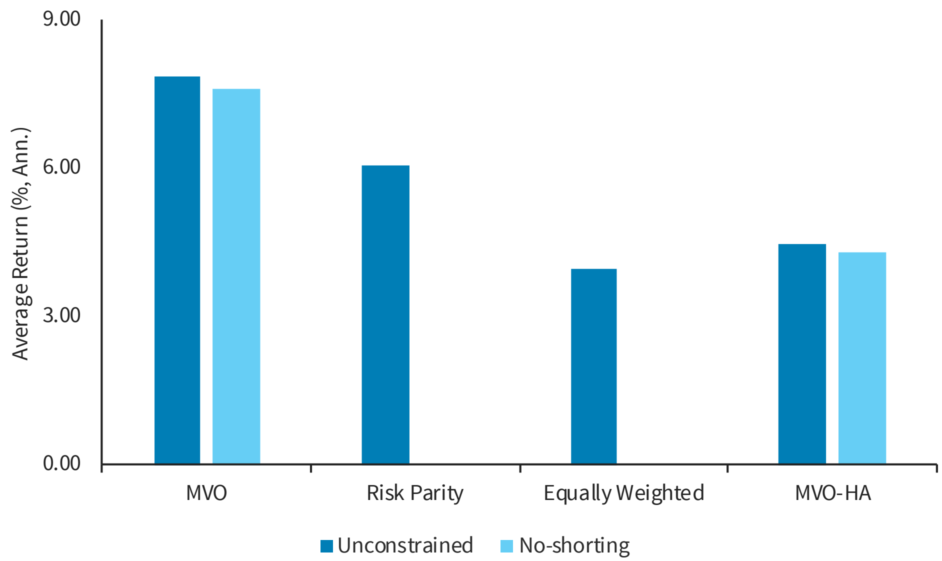
Robustness Analysis

Results Are Robust to No-shorting Constraints

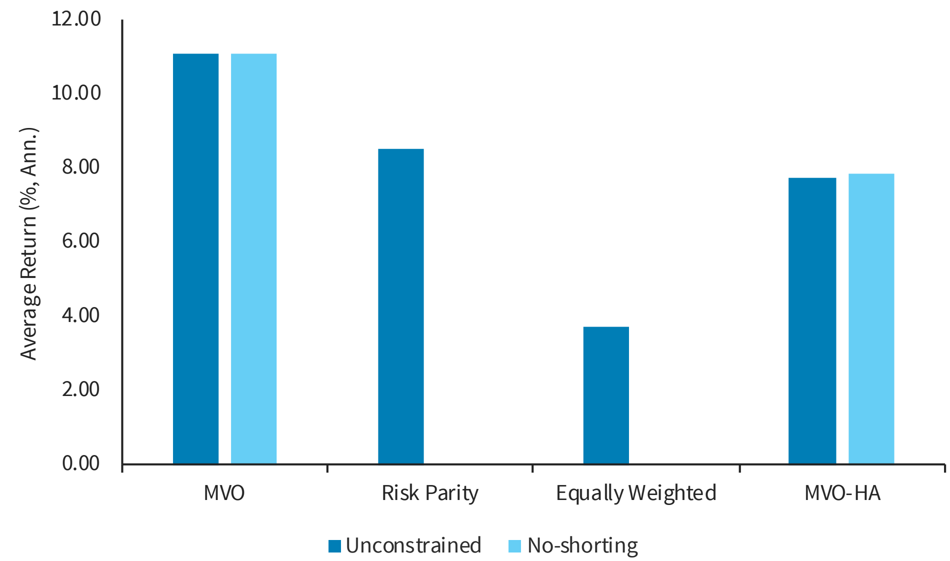
- Risk parity and equally weighted portfolios by construction do not allow shorting. Are outperformances driven by shorting?
- No, imposing no-shorting constraints generated similar performance. Unreported results show the imposition of no-shorting constraints lead to reduction in turnovers

Portfolio Performances with/without Shorting Constraints, Jan 2002 - Dec 2018

UK



Eurozone

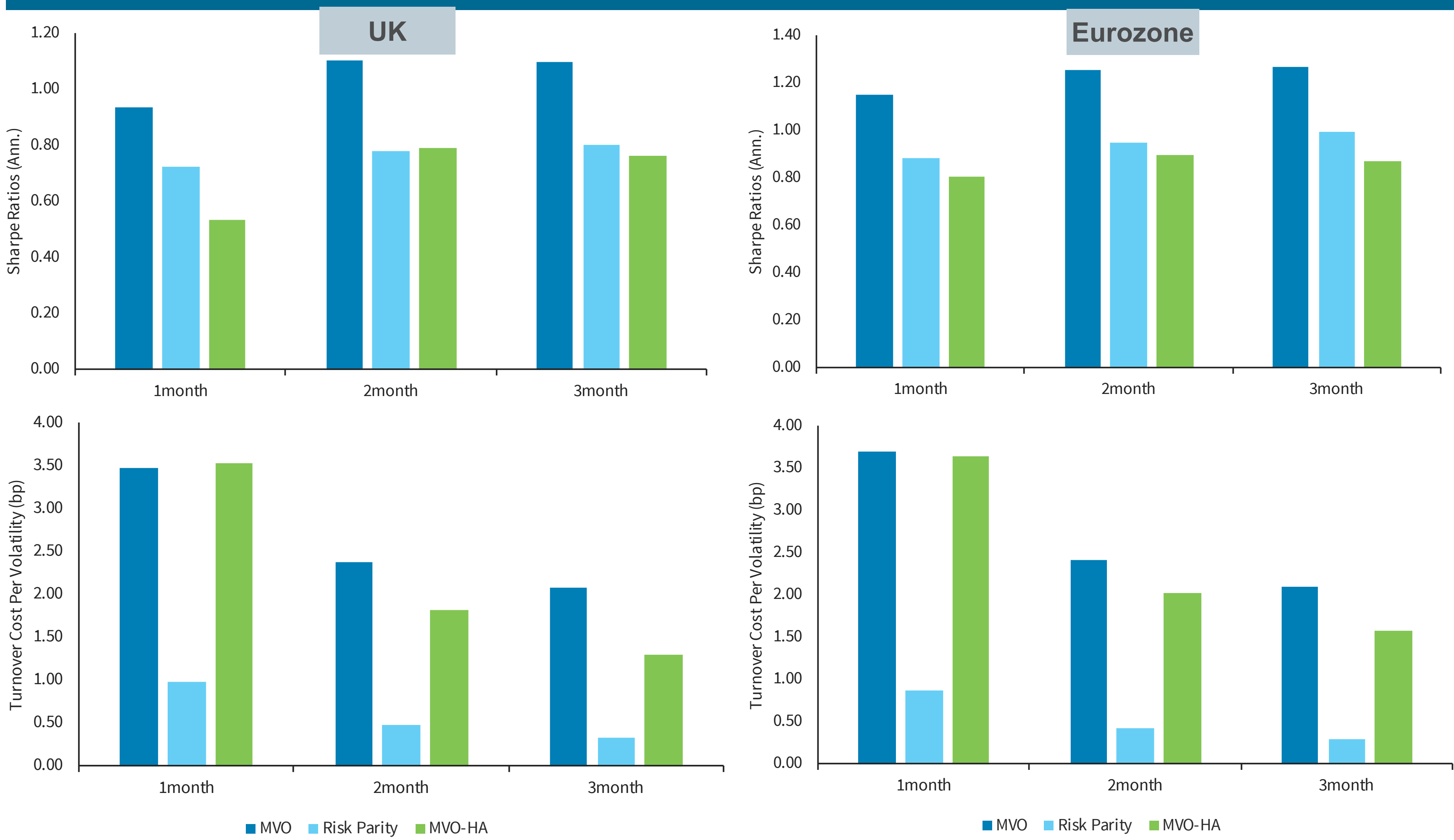


Note: MVO and MVO-HA sets $\gamma = 7$. MVO-HA uses historical average from an expanding window. The starting date is determined by the data availability of equity futures. All returns are scaled up/down so that the ex-post volatility matches that of MVO. Annualized numbers are based on 257 days a year. Source: Bloomberg, Optionmetrics, Barclays Research

Results Are Robust to Volatility Estimation Windows

- Results are robust to volatility estimation windows. In fact, MVO's Sharpe ratios increase and turnovers reduce when a longer window is used

Sharpe Ratios and Turnover Cost Per Portfolio Volatility by Different Volatility Estimation Windows, Jan 2002 - Dec 2018



Note: MVO and MVO-HA sets $\gamma = 7$. MVO-HA uses historical average from an expanding window. The starting date is determined by the data availability of equity futures. Annualized numbers are based on 257 days a year. Source: Bloomberg, Optionmetrics, Barclays Research

Results Are Robust with Caps on Exposures

- Notional exposures on bond futures can be high. They become less extreme when a longer volatility estimation window is used

95-th Percentile of MVO's Bond Exposures by Different Volatility Estimation Windows, Jan 2002 - Dec 2018

UK			Eurozone		
1 month	2 month	3 month	1 month	2 month	3 month
247%	210%	201%	374%	293%	271%

Note: MVO sets $\gamma = 7$. Source: Bloomberg, Optionmetrics, Barclays Research

- Performances are similar after imposing a maximum cap of 250% on bond and equity exposures

- Results are reported without any scaling, given caps are imposed on exposures

Performance of Capped Versus Uncapped Portfolios, Jan 2002 - Dec 2018

Performance Stats	UK				Eurozone			
	MVO	MVO-Capped	MOV-HA	MVO-HA-Capped	MVO	MVO-Capped	MOV-HA	MVO-HA-Capped
Avg Ret (%, Ann.)	7.84	7.89	6.79	6.99	11.10	9.81	7.57	6.59
Vol (%, Ann.)	8.38	7.85	12.78	11.80	9.64	8.49	9.42	8.66
Sharpe Ratio (Ann.)	**0.94**	**1.00**	**0.53**	**0.59**	**1.15**	**1.15**	**0.80**	**0.76**
Maximum Drawdown	-16.67	-16.67	-20.15	-18.36	-16.22	-16.22	-23.37	-23.18

Note: MVO and MVO-HA sets $\gamma = 7$. MVO-HA uses historical average from an expanding window that starts from February 1988 for FTSE 100 and June 1998 for Euro Stoxx 50. The starting dates are determined by the data availability of equity futures. Exposures are capped at 250%. Annualized numbers are based on 257 days a year. Source: Bloomberg, Optionmetrics, Barclays Research

Results Are Robust with Volatility-Targeting Constraints

- Another constraint in which we are interested is the volatility constraint, given the popularity of volatility-targeting portfolios
- To match previous results, set volatility target at 8.38% and 9.64% in the UK and EU. In each market, all portfolios are subject to the same volatility ex-ante (see appendix for details). The ex-post targeting errors vary across portfolios
- Improvement is still present, though becomes weaker. The exposures on bond futures are less extreme than the unconstrained MVO, despite the higher ex-post volatility

Performance of Volatility Targeting Portfolios, Jan 2002 - Dec 2018

Performance Stats	UK				Eurozone			
	MVO	Risk Parity	EW	MVO-HA	MVO	Risk Parity	EW	MVO-HA
Avg Ret (%, Ann.)	7.87	6.20	3.39	6.58	11.25	10.12	3.98	9.19
Vol (%, Ann.)	9.51	9.24	8.85	9.51	11.12	10.87	10.35	11.04
Sharpe Ratio (Ann.)	**0.83**	**0.67**	**0.38**	**0.69**	**1.01**	**0.93**	**0.38**	**0.83**
Maximum Drawdown	-26.55	-27.55	-24.57	-21.71	-31.51	-32.51	-32.14	-32.18
95%-tile of Bond Notional	2.00	2.25	1.08	1.90	3.16	3.15	1.06	3.10

Note: MVO-HA uses historical average from an expanding window that starts from February 1988 for FTSE 100 and June 1998 for Euro Stoxx 50. The starting dates are determined by the data availability of equity futures. Ex-ante, all portfolios in the UK targets an annualized volatility of 8.38% whereas the target in the eurozone is 9.64%. The ex-ante volatility target level does not change Sharpe ratios of portfolios. Source: Bloomberg, Optionmetrics, Barclays Research

Results Are Robust When Rebalanced at Monthly Horizons

- Unlike other traditional forecasts, $\frac{1}{R_{f,t}} Var_t^*(R_T)$ is a model-free function of option prices. Therefore, it has a nice feature of being available at very high frequencies
- Our main analysis is based on a daily frequency. However, outperformances are robust when portfolios are rebalanced monthly

Performance Statistics of Monthly Futures Portfolios, Feb 2002 - Dec 2018

Performance Stats	UK				Eurozone			
	MVO	Risk Parity	EW	MVO-HA	MVO	Risk Parity	EW	MVO-HA
Avg Ret (%, Ann.)	7.81	5.95	4.14	6.15	10.98	8.96	4.12	7.99
Vol (%, Ann.)	7.72	7.72	7.72	7.72	9.51	9.51	9.51	9.51
Sharpe Ratio (Ann.)	**1.01**	**0.77**	**0.54**	**0.80**	**1.16**	**0.94**	**0.43**	**0.84**
Maximum Drawdown	-11.69	-15.51	-22.54	-10.10	-12.02	-22.07	-32.70	-18.35
Mean Shortfall	-3.50	-4.27	-5.28	-3.76	-4.62	-5.47	-6.42	-5.15
Mean Exceedance	6.69	5.41	4.40	6.06	7.87	6.01	5.78	7.72
Avg Equity Notional	36%	42%	56%	54%	38%	46%	56%	43%
Avg Bond Notional	89%	104%	56%	68%	141%	162%	56%	119%
Equity Turnover	1.02	0.50	0.00	1.61	1.17	0.65	0.00	1.40
Bond Turnover	2.61	0.50	0.00	2.74	4.42	0.65	0.00	4.33

Note: mean shortfall (exceedance) is the monthly average of returns below (above) the 5-th (95-th) percentile. Average equity/bond notional are average of absolute notional exposures. $\gamma = 7$. MVO-HA uses historical average from an expanding window that starts from February 1988 for FTSE 100 and June 1998 for Euro Stoxx 50. The starting dates are determined by the data availability of equity futures. All returns are scaled up/down so that the ex-post volatility matches that of the corresponding MVO. Source: Bloomberg, Optionmetrics, Barclays Research

Summary

Key Takeaways

- We find statistical evidences that discounted risk-neutral variances computed from European equity index options can be used directly as a forecast of the equity risk premium in Europe

- The forecasts have similar time-series dynamics as their US counterpart: highly volatile, right-skewed and fat-tailed

- A mean-variance investor that uses discounted risk-neutral variances as the equity risk premium forecast to allocate between bond and equity futures can earn a 31% increase in Sharpe ratios over alternative allocations in both the UK and the eurozone markets

- Portfolio performances are robust to shorting restrictions, the volatility estimation window, caps on exposures, volatility targeting and a lower rebalancing frequency

Appendix

Appendix I: Risk Premium Identity

- Under no-arbitrage, R_T can be priced either via the SDF or the risk-neutral probabilities

$$1 = E_t(M_T R_T)$$
$$R_{f,t} = E_t^*(R_T)$$

- Rewrite the risk premium as

$$\begin{aligned} E_t(R_T) - R_{f,t} &= \left[E_t(M_T R_T^2) - R_{f,t}\right] - \left[E_t(M_T R_T^2) - E_t(R_T)\right] \\ &= \frac{1}{R_{f,t}} Var_t^*(R_T) - \left[E_t(M_T R_T^2) - E_t(R_T)\right] \\ &= \frac{1}{R_{f,t}} Var_t^*(R_T) - cov_t(M_T R_T, R_T) \end{aligned}$$

- The first equality adds and subtracts $E_t(M_T R_T^2)$. The equality in the second line follows from

$$\frac{1}{R_{f,t}} Var_t^*(R_T) = \frac{1}{R_{f,t}} E_t^*(R_T^2) - \frac{1}{R_{f,t}} \left[E_t^*(R_T)\right]^2 = E_t(M_T R_T^2) - R_{f,t},$$

and the equality in the third line follows from the fact $E_t(M_T R_T) = 1$

Appendix II: Discounted Risk-neutral Variance

- $\frac{1}{R_{f,t}} Var_t^*(R_T) = \frac{1}{R_{f,t}} E_t^* R_T^2 - \frac{1}{R_{f,t}} (E_t^* R_T)^2$
- Since $E_t^* R_T = R_{f,t}$, only need to calculate $\frac{1}{R_{f,t}} E_t^* R_T^2$, which equals to $\frac{1}{R_{f,t} S_t^2} E_t^* S_T^2$
- $E_t^* S_T^2$ is the price of a squared contract on time T's stock price S_T
- Note $x^2 = 2\int_0^\infty \max\{0, x - K\}\, dK$, then

$$\frac{1}{R_{f,t}} E_t^* S_T^2 = 2\int_0^\infty \frac{1}{R_{f,t}} E_t^* \max\{0, S_T - K\}\, dK = 2\int_0^\infty call_{t,T}(K)\, dK$$

- Using put-call parity, risk-neutral variance equals to

$$\frac{1}{R_{f,t}} Var_t^*(R_T) = \frac{2}{S_t^2}\left\{\int_0^{F_{t,T}} put_{t,T}(K) dK + \int_{F_{t,T}}^\infty call_{t,T}(K) dK\right\}$$

where $F_{t,T}$ is the forward price

Appendix III: Volatility Targeting Portfolios

- Target σ^g; volatility, correlation and expected return estimates: $\sigma_{t-1}, \rho_{t-1}, \mu_{t-1}$; notional w_t; superscript e, b for equity and bond respectively

- Risk parity portfolio $w_t^e = \frac{\sigma^g}{\sqrt{2+\rho_{t-1}}} \frac{1}{\sigma_{t-1}^e}$; $w_t^b = \frac{\sigma^g}{\sqrt{2+\rho_{t-1}}} \frac{1}{\sigma_{t-1}^b}$

- Equally weighted portfolio $w_t^e = \frac{\sigma^g}{\sqrt{\left(\sigma_{t-1}^e\right)^2+\left(\sigma_{t-1}^b\right)^2+2\rho_{t-1}\sigma_{t-1}^e\sigma_{t-1}^b}}$; $w_t^b = \frac{\sigma^g}{\sqrt{\left(\sigma_{t-1}^e\right)^2+\left(\sigma_{t-1}^b\right)^2+2\rho_{t-1}\sigma_{t-1}^e\sigma_{t-1}^b}}$

- Equity only portfolio $w_t^e = \frac{\sigma^g}{\sigma_{t-1}^e}$

- Bond only portfolio $w_t^b = \frac{\sigma^g}{\sigma_{t-1}^b}$

- Other portfolios: solve the constrained optimization problem

$$\max_{w_t^e, w_t^b} w_t^e \mu_{t-1}^e + w_t^b \mu_{t-1}^b$$

$$s.t. \sqrt{(w_t^e \sigma_{t-1}^e)^2 + \left(w_t^b \sigma_{t-1}^b\right)^2 + 2\rho_{t-1} w_t^e w_t^b \sigma_{t-1}^e \sigma_{t-1}^b} \leq \sigma^g$$

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