CREDIT INVESTING IN BUY-AND-HOLD PORTFOLIOS, PART II: THE EFFECT OF CORRELATIONS

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Two months ago in this space, we looked at credit portfolios versus Treasuries from a buy-and-hold perspective.[1] Using a simple binomial model of default, we demonstrated that even under some very pessimistic assumptions about default and recovery rates, credit portfolios should outperform Treasuries over a ten-year horizon with very high probability. Among the caveats that we pointed out at that time was the fact that our model did not consider default correlations among different credits. This month, we revisit this study to investigate how the consideration of correlations affects our results.

First, let us review the basic elements of our study. We compare holding period returns on portfolios of non-callable 10-year credits to those of 10-year Treasuries. We consider two rating categories: single-A and Baa. We use a very simple, stylized analysis that ignores the effects of reinvestment and inflation. Our interpretation of the buy-and-hold assumption is that, in the absence of defaults, we expect the annualized total return to equal yield to maturity. Default is modeled as a one-period problem—each bond either defaults during the next ten years or survives to maturity. Bonds that default within that period do not contribute anything to the cumulative performance beyond their recovery value. If we let y denote the average portfolio yield (in decimal), D the realized portfolio default rate, and R the average recovery rate on defaulted bonds, then our simple model for the terminal value of the portfolio after 10 years is

$$term\_value = (1-D)(1+y)^{10} + D \cdot R = (1+ann\_ret)^{10}$$

In the above equation, we tie the realized terminal value of the portfolio to the realized cumulative portfolio default rate—the proportion of bonds in the portfolio that go into default over the next ten years. This is related to the market-wide cumulative default rate, which we denote by p. This is the proportion of bonds in the marketplace that will default over the next 10 years, or the cohort default rate. Neither of these quantities is known yet, and they must be treated as random variables.

In our previous study, we did not attempt to model the distribution of p. We simply modeled the distribution of the portfolio default rate conditioned on an assumed value for p using the binomial distribution.

The use of the binomial model with a constant default rate is equivalent to assuming that whether or not a given issuer will default is independent of the outcome for any other issuer. However, the big risk for buyers of a diversified corporate bond portfolio versus Treasuries is that difficult economic conditions could produce a wave of defaults

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[1] "A Case for Credit Investing in Buy-and-Hold Portfolios," Lehman Brothers, November 2002.

throughout the sector. This common dependence of all credits on overall market conditions gives rise to a systematic risk that causes correlation of default events among the various issuers.

In our prior work, we addressed this issue by using high values for the market default rate. The rating agencies publish cohort default rates, as well as long-term historical averages. To be conservative, we set *p* equal to the highest observed cohort default rate over the period for which data was available.

This month, we repeat the analysis using a firm-value model in which correlations among issuers are used to derive a distribution for the market default rate *p*. The model we will use is a default-mode CreditMetrics™ portfolio credit model discussed by our Quantitative Credit Research colleagues in an earlier publication.[2]

The model, based on the work of Merton, assumes that an issuer goes into default when the value of its equity goes below that of its outstanding debt. Each firm's return is assumed to have a certain correlation β with the "market return." With the simplifying assumptions of a homogeneous market (all issuers have the same levels of debt and equity, the same β, and the same probability of default) and equally weighted portfolios, this model essentially boils down to a modified version of the binomial model. Conditioned on the market return, the probability of portfolio defaults follows the binomial distribution. If the market return is very negative, and β is positive, then most firms will have negative equity returns, and the probability of default is increased for all issuers simultaneously. If the market return is positive, then all issuers will have smaller default probabilities. Assuming a normal distribution for the market return, and integrating over all possible outcomes, we obtain the unconditional distribution of the number of portfolio defaults. The model can be calibrated to match a specified value for the expected probability of default.

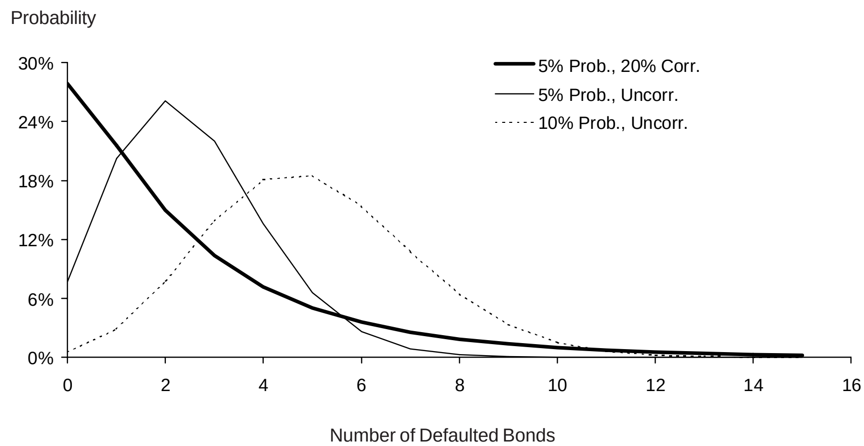
In Figure 1, we plot this distribution for a 50-bond portfolio with a expected market default rate of 5% and a correlation[3] assumption of 20%. We compare this distribution with those produced by the uncorrelated case (the plain binomial distribution) using market default rates of 5% and 10%. First let us compare the correlated and uncorrelated cases using the same 5% value for the expected default rate. In this case, for a 50-bond portfolio, the expected number of defaults is 2.5 for both the correlated and uncorrelated cases. The binomial distribution with no correlations has its peak near this value, and a relatively short tail. In the correlated case, the distribution shows a decreased probability of realizing the average default rate, and increased probabilities of either extremely high or extremely low defaults.

If we increase the market default rate to 10% in the uncorrelated binomial distribution, the whole distribution shifts to the right, and the tail of the distribution includes

[2] For a detailed description of this model, see "Modelling Credit: Theory and Practice" by Dominic O'Kane and Lutz Schlögl, Lehman Brothers, February 2001, pages 35-37.

[3] In O'Kane and Schloegl (2001), it is shown that if all assets are assumed to have the same correlation b with the market return, this results in a correlation of b^2 between any two assets. The correlation levels reported here are the pairwise correlations between the values of any two firms, and correspond to b^2

Figure 1. **Distribution of Number of Defaulted Bonds in a 50-Bond Portfolio**
5% Expected Default Probability with 20% Correlation,
Uncorrelated Model with Market Default Rates of 5% and 10%



high probabilities that 8, 9, or 10 bonds may default over the period. This comes much closer to the tail of the correlated distribution with a 5% expected default rate. However, even in this case, the probability of 11 or more bonds defaulting is higher in the correlated model.

Historical data on 10-year cumulative defaults on Baa securities indicate a long-term average default rate of about 5%, with the worst observed cohort experiencing a default rate just under 10%. In the correlated model, we reflect this by using an expected default rate of 5%, and the fat tails of the distribution are generated by the correlation assumption. In our previous article, the worst case assumption was obtained by using the tails of the binomial distribution with the highest observed default rate of 10%. Figure 1 compares these two sets of assumptions.

The same mathematical model can be understood from a slightly different viewpoint as well. The market default rate realized over the next ten years can be viewed as a random variable. Given a particular outcome of the market default rate over the next ten years, we use the binomial model for the distribution of the portfolio default rate. Conditioned on the market default rate, all bonds are assumed to default independently with the same probability. However, if we look at the unconditional default probabilities, we will find that the defaults of any two bonds are highly correlated events. Both bonds will be likely to default if the market default rate is high, and unlikely to default if it is low. As a result, bond A will be more likely to default when bond B does than when it does not. The distribution of the realized market default rate shown in Figure 2 corresponds to the correlated firm value model described above, with a default probability of 5% and a correlation of 20%. Note that the correlation model considers the possibility of market default rates as high as 25%. According to this assumption, the overall probability of a market default rate worse than 10% is 13.6%. When we previously assumed a "worst case" market default rate of 10%, the tail of the portfolio default distribution was entirely due to the portfolio underperforming the market due to poor security selection in a small portfolio. We see now that the reason for the increased tail probabilities shown in Figure 1 is that the correlation model considers the possibility of much higher market default rates as well. This is a systematic risk that cannot be diversified away.

In Figure 3, we compare the worst case realized portfolio default rates at 95% and 99% confidence levels using two different assumptions. The first is the worst case assumption that we used in the uncorrelated case, with the market default rate assumed to take on its worst observed historical value but with no correlations. The second assumes asset correlations of 20%, with the expected default rate set to the

Figure 2. **Distribution of Market Default Rate Implied by Correlation Model**
Expected Cumulative Default Rate 5%, Correlation 20%

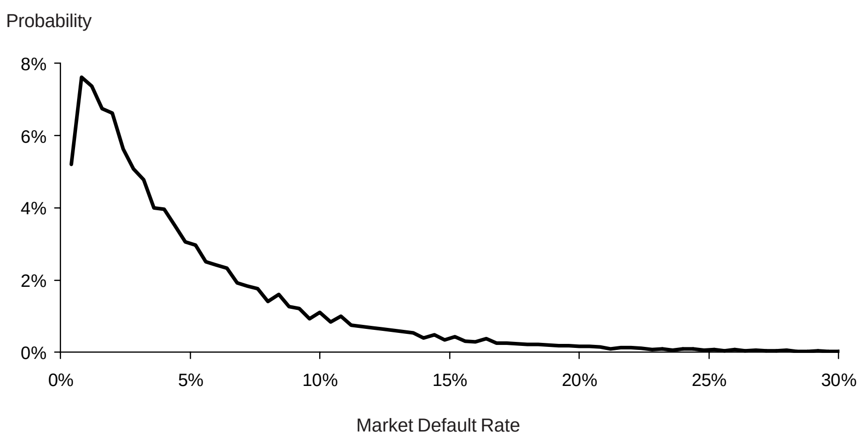


Figure 3. **Comparing Worst Case Realized Portfolio Default Rates Using Historical Average 10-Year Default Rates with 20% Correlation and Worst Case Historical Default Rates with 0% Correlation**

	Confidence	20 Bonds	50 Bonds
Worst Case Losses for A Portfolios			
Uncorrelated, 5% default prob.	95%	15%	10%
20% correlation, 2% exp. default prob.	95%	10%	8%
Uncorrelated, 5% default prob.	99%	20%	14%
20% correlation, 2% exp. default prob.	99%	20%	16%
Worst Case Losses for Baa Portfolios			
Uncorrelated, 10% default prob.	95%	20%	18%
20% correlation, 5% exp. default prob.	95%	20%	18%
Uncorrelated, 10% default prob.	99%	30%	20%
20% correlation, 5% exp. default prob.	99%	30%	28%

long-term historical average. (Recall that the default distributions corresponding to these two sets of assumptions were shown in Figure 1 to have fairly similar tails.) The results are shown for portfolios of 20 and 50 bonds, using default probabilities characteristic of A and Baa ratings. We find that the two sets of assumptions give quite similar results, particularly at the 95% confidence level. The most striking difference is that the assumption of 20% correlation reduces the advantage of increasing the portfolio size from 20 to 50 bonds. For a portfolio of Baa bonds, the worst case realized portfolio default rate at a 99% confidence level improves from 30% to 20% as we go from 20 to 50 bonds under the uncorrelated assumption. With the 20% correlation assumption, even a 50-bond portfolio may have a 28% default rate. Using the break-even spread methodology discussed in our previous article, it would require a spread greater than 325 bp to break even with Treasuries despite a 30% portfolio default rate. We thus conclude that under the correlated model with these assumptions, we cannot be 99% sure that our Baa portfolio will outperform Treasuries even if we diversify among 50 bonds.

With different assumptions, the correlation model allows for even more extreme predictions of portfolio default rates. Figure 4 shows the risk/return characteristics of a 50-bond Baa portfolio using the correlation model with expected default probability assumptions of 5%, 7.5% and 10%, and correlation assumptions of 20% and 30%. A recovery rate of 20% is assumed throughout. Whereas our assumption of 10% defaults with no correlations showed that we could have greater than 99% confidence of outperforming Treasuries over a 10-year period, the correlation model shows that in the worst case corporates can underperform by a substantial amount. The most pessimistic assumptions shown combine a 10-year expected default probability of 10% with a firm value correlation of 30%. According to these assumptions, there is a 1% chance that more than half the portfolio will default. With our assumption of only 20% recovery, the resulting underperformance can be 4.09% per year or worse. Yet even according to these most pessimistic assumptions, the compensation for taking these risks is clear. The probability of outperformance is 87.8%, and the mean outperformance is 1.30% per year. The distribution of returns can be used to calculate the standard deviation of return as well as the mean, and we find that the ex ante information ratio is 0.95. Under more benevolent assumptions, the information ratio can be as high as

Figure 4. **Risk/Return Characteristics of a 50-Bond Baa Portfolio Under Various Assumptions for Expected Default Rate and Correlation**
Corporate Yield 6.59%, Treasury Yield 4.22%, Recovery Rate 20%

Prob. of Default	**10%**	**5%**	**7.5%**	**10%**	**5%**	**7.5%**	**10%**
Correlation	**0%**	**20%**	**20%**	**20%**	**30%**	**30%**	**30%**
Mean Outperformance (%/yr)	1.37%	1.87%	1.60%	1.33%	1.86%	1.58%	1.30%
Stdev of Outperformance (%/yr)	0.44%	0.64%	0.86%	1.06%	0.82%	1.11%	1.37%
Information Ratio	3.08	2.91	1.86	1.25	2.26	1.43	0.95
Probability of Outperformance	99.7%	98.0%	94.8%	90.3%	96.4%	92.5%	87.8%
Worst Case Num. Defaults, 95% conf.	9	9	12	14	10	14	17
Worst Case Outperformance, 95% conf.	0.52%	0.52%	-0.17%	-0.66%	0.29%	-0.66%	-1.42%
Worst Case Num. Defaults, 99% conf.	10	14	18	21	17	22	26
Worst Case Outperformance, 99% conf.	0.29%	-0.66%	-1.69%	-2.53%	-1.42%	-2.83%	-4.09%

2 or 3. If we use the historical average default rate of 5% as the expected value of the market default rate (keeping in mind that this reflects the possibility of much higher cohort default rates, as shown in Figure 2), then even under an assumption of 30% correlation, the portfolio will outperform Treasuries with more than 95% confidence.

The correlation model used here takes advantage of the simplifying assumption that any two issuers are related by the same correlation coefficient. In reality, the correlations among different issuers reflect two types of factors: general macro-economic trends that affect all issuers, and industry-specific circumstances that can affect a particular sector of the market. A generally accepted market practice is to assume 30% correlation among issuers within the same industry, and 15% correlation between issuers from different industries. As the model uses just a single coefficient, 20% seems like a reasonable value. While our model cannot account for industry-specific correlations,[4] these can be in large part avoided by diversification of industry exposures in the portfolio. If lack of liquidity in the market makes such diversification impossible, our break-even default rates would have to be adjusted upwards for industry correlations. Nevertheless, we believe it is feasible under most market conditions to construct a corporate portfolio of twenty or fifty names well-diversified across industries.

Conclusion

Credit by its very nature offers an asymmetric return profile. A corporate bond rewards investors with a small gain (spread) over the course of its lifetime in compensation for bearing the risk of a large loss (default) with a small probability. To a certain extent, default risk is an issuer-specific, or non-systematic risk, and can be diminished via diversification. However, correlations among issuers make it impossible to entirely eliminate default risk through diversification. The common dependence of all issuers on general economic conditions, and the common exposures of all issuers within a given industry, give default risk a systematic component which cannot be diversified away.

Our previous article on this topic, by ignoring correlations, may have given the impression that for a buy-and-hold investor, current spread levels are such that a diversified credit portfolio will outperform Treasuries with virtually no risk at all. This is clearly not the case. There is always the possibility that market default rates will skyrocket, causing all credit investments to underperform.

Please note that the analysis in our previous article remains correct. If we assume that the current cohort of Baa credits have a cumulative market default rate of 10% over the next ten years, then a 50-bond portfolio should outperform Treasuries with 99% probability. It is also true that by increasing diversification, one can always increase the probability that the realized portfolio default rate will approach that of the overall market. The one aspect of our previous paper that has been called into question by the correlation model is a subjective one, relating to the way we used historical data to model the future. We made the assumption that the worst observed 10-year cumulative default rate for Baa debt, which was slightly under 10%, represents a

[4] The model can compute portfolio loss distributions assuming a different beta for each asset. However, this would complicate the analysis without necessarily changing any of the main results.

pessimistic worst case limit for the market default rate. The correlation model described here considers the unsettling possibility that things could get even worse, and offers one way to quantify the probabilities of market default rates much higher than ever before observed.

Models that consider correlations among issuers generate fatter tails in the distribution of portfolio default losses. We found that an assumption of 20% correlations among issuers, combined with historical average default rates, estimates the tail risk for 95% confidence at levels similar to those achieved using worst case defaults with no correlations. The correlation model shows that we cannot achieve a 99% confidence of outperforming Treasuries with a portfolio of 50 Baa bonds. However, it still shows an excellent risk/return tradeoff for investors who can take a long-term view on credit.