Quantitative Portfolio Strategies

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ADJUSTING BID SPREADS OF CALLABLE BONDS FOR YIELD CURVE SHIFTS

When the Treasury curve shifts, bullet bonds will typically be repriced at a constant bid spread (spread to the benchmark Treasury security), assuming no change in credit outlook. This is not necessarily true for callables. OAS-neutral shifts in the Treasury curve may alter a callable bond's bid spread. This column develops a simple formula that estimates this change based on the difference between the callable bond's modified option-adjusted duration and its modified duration-to-worst.

A bond's yield is the rate at which all of its cash flows must be discounted so that their present value equals the bond's market price. For securities with uncertain cash flows, such as callable bonds, yield becomes an ambiguous measure: different yields may be calculated for the same price, depending on what assumptions are made about the future cash flows. The yield on callable bonds is typically quoted using a "to worst" convention, which takes the lower of the yields given by two different cash flow assumptions:

1) the bond survives to maturity and pays out the same as a bullet bond to maturity; or
2) the bond is called and pays out as a bullet to the call date.

The bid spread is the difference between this yield-to-worst and the yield on a liquid, bullet Treasury whose maturity most closely matches either the call date or the maturity date, whichever corresponds to the lower yield.

Investors and analysts have long recognized the limitations of this static representation. As Treasury yields move, the call option becomes more or less likely to be exercised, changing the amount of option value included in the security's price. To model this correctly, option-adjusted models have become widely accepted. Based on trees or lattices of interest rates, these models calculate option-adjusted spread (OAS) as a spread over the entire Treasury curve and produce an option-adjusted duration which gives the sensitivity of a bond's price to a parallel shift in Treasury yields.

One major drawback of option-adjusted models is that they are not standardized. Each firm may make slightly different assumptions in modeling the term structure of interest rates, the term structure of volatility, and the dynamics of the interest rate process. An OAS of 100 bp for a particular bond may, therefore, translate to different prices on different systems. For this reason, pricing levels are typically quoted as bid spreads, which allow unambiguous conversion into prices. But when rates change, the bid spread is likely to change as well, even with no change in the view on the credit.

Consider a shift in the Treasury curve that leaves option-adjusted spreads unchanged. What will be the effect of this Treasury curve shift on the bid spread for the callable bond?

The bond's modified option-adjusted duration D_{OA} can be used to approximate the return on the callable bond as follows:

$$\frac{\Delta P}{P} \approx -D_{OA}\,\Delta y_{Treasury} \qquad (1)$$

where $\Delta y_{Treasury}$ is the shift to the Treasury curve.

Alternatively, modified duration-to-worst can be used to approximate the bond return:

$$\frac{\Delta P}{P} \approx -D_{worst}\,\Delta y_{worst} \qquad (2)$$

where Δy_{worst} is the shift in the callable bond's yield-to-worst. The modified duration-to-worst approximation is not as accurate as the modified option-adjusted duration approximation, since the duration-to-worst approximation treats the callable bond as if it were a bullet to worst. It ignores the extent to which the Treasury curve shift affects the option value of the callable bond. Nonetheless, Equation (2) is typically a useful approximation for small yield changes.

Since the true bond return caused by the shift in the Treasury curve is the same regardless of how it is expressed, the right hand sides of Equations (1) and (2) will be approximately equal:

$$D_{OA}\,\Delta y_{Treasury} \approx D_{worst}\,\Delta y_{worst} \qquad (3)$$

Solving for the change in the callable bond's yield-to-worst, we have

$$\Delta y_{worst} \approx \frac{D_{OA}}{D_{worst}} \Delta y_{Treasury} \qquad (4)$$

The true duration of a callable bond is usually different from its duration-to-worst and is much more closely approximated by the bond's modified option-adjusted duration. Equation (4) corrects for this effect and solves for the change in yield-to-worst that results in the correct overall percentage change in bond value as provided by the modified option-adjusted duration approximation (Equation [1]).

If the change in Treasury yields is sufficiently small, the callable bond will be quoted to the same Treasury bond both before and after the curve shift. Therefore,

$$\Delta\text{Bid Spread} = \Delta y_{worst} - \Delta y_{Treasury}$$

From Equation (4), this implies

$$\Delta\text{Bid Spread} \approx \frac{D_{OA}}{D_{worst}} \Delta y_{Treasury} - \Delta y_{Treasury}$$

$$\Delta\text{Bid Spread} \approx \left(\frac{D_{OA}}{D_{worst}} - 1\right) \Delta y_{Treasury} \qquad (5)$$

Consider a parallel shift in Treasury rates that has no effect on option-adjusted spreads. Equation (5) shows that this shift in the Treasury curve will, nonetheless, change the bond's quoted spread off Treasury if the bond's option-adjusted duration differs from its duration-to-worst.

For a bullet bond, option-adjusted duration and duration-to-worst will be identical. As predicted by Equation (5), a shift in the Treasury curve that leaves OAS unchanged will not affect the quoted bid spread on a bullet. Similarly, the quoted bid spread on a callable bond will be affected by OAS-neutral shifts in the Treasury curve only to the extent that the bond's option-adjusted duration differs from its duration-to-worst. Consider a callable bond that we are almost certain will be redeemed at call or one that we are almost certain will be held to maturity. In each of these cases, duration-to-worst and option-adjusted duration will be very similar, leading to negligible changes in quoted spread in response to Treasury curve shifts.

However, a callable bond whose effective maturity is close to midway between its potential call date and its maturity date will generally have a large disparity between its option-adjusted duration and its duration-to-worst. Here, noticeable changes in bid spreads can be expected as the Treasury curve shifts.

Example

Consider the following New York Times bond with a 8 1/4% coupon and 3/15/2025 maturity. This bond becomes callable on 3/15/2005. PC Product reports the following statistics for the bond as of May 1, 2000:

Analysis	**Modified Duration**[1]	**Yield (%)**	**Spread**
To call	3.91	8.762	
To maturity (= to worst)	10.42	8.205	210 bp over on-the-run 30-yr (bid spread)
Option-adjusted	6.84		105 bp over Treasury curve (OAS)

Yield to maturity is the lower of the two calculated yields, and, thus, the quoted bid spread expresses this yield as a spread over the on-the-run 30-year Treasury. The strong role played by optionality in the pricing of this bond is reflected both in the option-adjusted duration, which is roughly midway between the two static durations, and in the fact that the bid spread is much larger than the OAS, reflecting the option value. Equation (5) predicts that the bid spread on this bond should contract by approximately 34.4 bp for every 100 bp increase in the Treasury curve.

We can use PC Product to see how well this approximation holds. The Embedded Option Calculator listed under the PC Product Calculator menu allows the user to generate a new pricing tree for this bond corresponding to a 5 bp increase in Treasury rates. Holding OAS constant and pricing the bond with respect to this new curve returns a yield-to-worst of 8.238%, an increase of 3.3 bp. Given that Treasury yields increased by 5 bps, this corresponds to a spread contraction of 1.7 bp. A 1.7 bp spread contraction on a 5 bp yield curve increase amounts to a contraction of 34 bp per hundred.

[1] PC Product reports duration-to-call (4.08) and duration-to-maturity (10.85) in Macaulay format. Converting these numbers to the modified duration format yields the quantities listed above.

Alternatively, we can generate a new pricing tree that corresponds to a 5 bp decrease in Treasury rates. Repeating the previous steps gives us a yield spread widening of 1.7 bp or 34 bp per 100 bp increase in Treasury yields. Both positive and negative shifts to the yield curve produced the same result: spread contraction of 34 bp per 100 bp increase in Treasury yields, which is quite close to the approximation coming from Equation (5), 34.4 bp per hundred.

Now let's consider what would have happened if we had ignored the effect that OAS-neutral shifts in the Treasury curve have on bid spreads and decided to price the bond at the same spread over Treasury. First, consider the case in which Treasury yields have increased by 5 bp. Holding the bond's bid spread constant moves the bond's yield-to-worst up from 8.205 to 8.255. Repricing the bond at this yield and recalculating the OAS over the shifted yield curve gives an OAS of 1.078. OAS has increased by 2.8 bp in response to a 5 bp increase in Treasury yields. For this bond, holding bid spread constant results in OAS errors on the order of 56 bp per 100 bp change in Treasury yields. If we repeat this exercise for a 5 bp decrease in yields, we find that holding bid spread constant causes the OAS calculation to be off by 2.9 bp, or 58 bp per 100 bp change in Treasury yields.

Conclusion

This analysis shows that bid spreads on callable bonds need to change in response to Treasury yield shifts in order to maintain a stable credit outlook. A simple equation based on modified option-adjusted duration and modified duration-to-worst approximates the required change in bid spread.

Publications—L. Pindyck, A. DiTizio, B. Davenport, W. Lee, D. Kramer, J. Threadgill, R. Madison, A. Acevedo, K. Kim, C. Rial, J. Batstone