What Makes Risk Parity Work?

A Performance Attribution Approach

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- The risk parity (RP) approach has been adopted recently by many investors to improve risk-adjusted portfolio performance.
- Across various markets, risk parity portfolios have often outperformed their conventional peers in the last two decades. This outperformance is typically attributed to better diversification. However, they have also benefited from superior performance of low-risk assets, notably including a long and steady decline of interest rates. Which of these is the primary effect?
- We introduce a performance attribution approach to separate the performance gains of risk parity portfolios into three effects: diversification, re-allocation, and weight dynamics.
- While better risk diversification is intended by portfolio construction and is thus expected to contribute most to outperformance of risk parity portfolios, the re-allocation and weight dynamics effects are typically unintended.
- The re-allocation effect occurs due to differences in realized risk-adjusted asset performance in conjunction with changes in portfolio weights. Risk parity portfolios typically overweight less volatile assets. When these have higher risk-adjusted returns than their riskier peers, the re-allocation effect is positive. The weight dynamics effect arises when changes in allocation weights are correlated with asset returns.
- We consider various portfolio examples, including macro bond-equity and Treasury allocations as well as equity-only portfolios and find that the unintended effects can be surprisingly large.
- In an equity-only case study, we find that the negative effect of weight dynamics due to market weighting is the main reason for risk parity portfolios to outperform market-weighted ones. Market-weighted portfolios overweight stocks that have just risen in price and underweight those that have declined. Almost any weighting scheme not related to price dynamics produces better performance; RP is not necessarily the best. For example, constant security weights can result in better performance than volatility-adjusted dynamic weights.
- Our findings suggest that risk parity should be expected to benefit the performance of relatively concentrated portfolios with low correlations among underlying assets. Additional improvements can be achieved when less risky assets generate a persistently higher performance than their riskier peers.

Introduction

The risk parity (RP) approach to portfolio construction has recently become popular among investors.[1] Designed to diversify risk across a set of assets or risk factors, it provides an effective way to build balanced allocations that do not rely on projected expected returns.

Historically, investors have used mean-variance optimization (MVO), pioneered by Markowitz, to build optimal portfolios with the highest expected return per unit of variance. This approach, however, requires estimates of expected asset returns, which are difficult to formulate. At the same time, small errors in estimated expected returns could result in dramatic changes of portfolio weights.[2] Various modifications of the mean-variance approach have been proposed to improve portfolio robustness and effectively diversify portfolio risk.[3] RP is one such modification.

Risk parity aims to allocate portfolio risk equally across assets or individual risk factors, while expected returns are disregarded in portfolio construction. A risk parity portfolio can be loosely defined as an allocation that balances risk across underlying assets. From this perspective, several well-known approaches to portfolio construction are conceptually close to risk parity. Equal risk contribution (ERC) equalizes percentage risk contributions of portfolio assets. It requires estimates of volatilities and correlations of asset returns. Other frameworks have been also discussed in the literature. Most diversified (MD)[4] portfolios maximize the 'diversification ratio' defined as the ratio of portfolio volatility under the perfect correlation assumption to its realized volatility. Equal volatility (EV) portfolios are constructed to make isolated volatility contributions of each asset the same. In contrast to ERC and MD, EV portfolios do not require correlation estimates. While correlations provide useful information for portfolio diversification, they can be unstable. As a result, the EV approach is less exposed to estimation errors in input parameters and produces more stable weights.

Risk parity approaches have been applied to a wide variety of markets. In many cases RP has led to improved risk-adjusted performance over traditional fixed-weight or market value-weighted allocations. While critics of the approach have highlighted dangers associated with leverage, negative risk premia, and volatility as a measure of portfolio risk[5], the approach remains very popular among investors.

In this report, we present a mechanism for attributing the performance gains of risk parity portfolios to three effects: diversification; re-allocation; and weight dynamics. This decomposition can help investors better understand the nature of the performance improvements associated with risk parity.

For example, RP has worked very well for allocating between equities and bonds over the last two decades. Why was this the case? If the key to this outperformance was superior risk diversification compared to other weighting schemes, it is likely to persist. If it was due to the secular rally in bonds – the low volatility asset class that RP overweighs – it may not be as successful in the future. While a potential improvement in diversification is the main motivation to change an allocation to RP, the scope for diversification varies from one investment universe to another depending on correlations and breadth. High correlations or a large number of securities in the portfolio can limit the potential benefit from improved diversification.

The re-allocation effect measures the isolated impact on portfolio performance from long-term changes in weights, so that improvements due to diversification are disregarded. It is

[1] For an overview, see Allen (2010) or Lee (2011).
[2] On the sensitivity of MVO portfolios to estimation errors see, for example, Dhingra (1980).
[3] See, for example, *A Case Study in Combining Active Strategies: Enhancements to the Mean-Variance Approach*.
[4] See Choueifaty and Coignard (2008).
[5] See, for example, Inker (2010).

often positive for risk parity portfolios when low volatility assets exhibit higher risk-adjusted returns. We observe this in bond-equity as well as in equity-only RP portfolios. In the latter case, it is related to the phenomenon called the 'low risk anomaly' - the propensity of less risky stocks to persistently generate higher risk-adjusted returns.[6]

We use a few case studies to illustrate the attribution framework and show that the re-allocation and weight dynamics effects can be significant.

The report is organised as follows. First, we focus on a simple example of bond-equity allocation to explain the basics of the risk parity approach. In this context, we show how we can measure the contributions of diversification and re-allocation effects to the outperformance of the risk parity portfolio over a naïve allocation. Next, we apply our attribution methodology to RP portfolios in different investment universes, including Treasuries, diversified bond-equity, and equity-only. We then briefly discuss the role of correlations in RP before concluding with a summary of lessons learned. A short technical appendix presents the details of our attribution methodology.

Measuring the Diversification and Re-allocation Effects

In its simplest form, risk parity allocates portfolio risk equally across individual assets. We can use a high level bond-equity example as an illustration, taking as a starting point an equally weighted (EW) allocation between equities and bonds frequently utilized by pension plans.[7] Equity and bond markets are represented by the S&P 500 and Barclays US Aggregate indices.

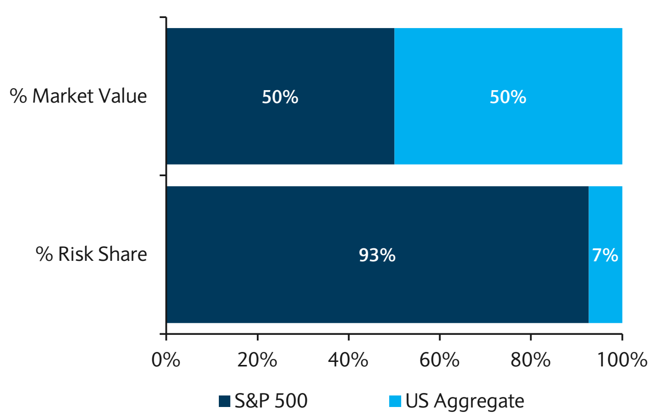
Between January 1991 and December 2014, the return volatilities of the equity and bond indices were 14.49 %/yr and 3.60 %/yr, respectively.[8] As a result, the risk allocation of the EW portfolio is strongly biased towards equities. Panel A of Figure 1 shows weights and risk distributions in the EW portfolio. Equities represent a 93% share of portfolio risk.

FIGURE 1

US bond-equity allocations, January 1991 – December 2014

PANEL A

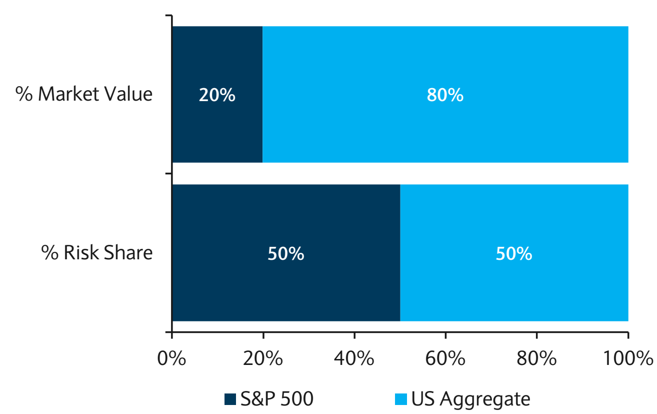
Equal-weighted US bond-equity portfolio



Source: Barclays Research

PANEL B

Risk parity US bond-equity portfolio



Source: Barclays Research

Panel B of Figure 1 shows weights and risk allocations in the RP portfolio. To ensure that both equities and bonds contribute equal shares of portfolio risk, RP increases the allocation to bonds from 50% to 80%, and so the weight of equities declines to 20%.

[6] See, for example, Haugen and Heins (1975) who documented superior performance of portfolios with low-volatility stocks relative to market portfolios.
[7] Typically a more aggressive 60/40 allocation between equities and bonds has been used by investors, but we felt that a more conservative equally-weighted (50/50) portfolio provides a qualitatively similar illustration.
[8] Returns are measured over 1-month USD Libor.

Figure 2 compares the historical performance of the EW and RP portfolios. The RP portfolio has average excess returns over Libor of 3.91%/yr, lower that the 5.27%/yr delivered by the EW portfolio. This difference in returns is a direct result of the re-allocation to bonds, an asset class with lower risk and lower returns. Indeed, the average return of the US Aggregate Index is only 3.01%/yr, less than half that of the S&P 500 index in the period considered. However, in terms of risk-adjusted performance, bonds outperformed stocks and RP outperformed EW: the Sharpe ratio of the RP portfolio was 0.92, higher than that of the EW portfolio (0.69). Even though RP earned a lower return than EW, the higher Sharpe ratio can be translated into a higher return for the same level of risk as long as one can use leverage. The rightmost column of Figure 2 shows the return statistics of the leveraged RP portfolio that matches the volatility of the EW portfolio, with borrowing costs assumed to be 1 month USD Libor. The average return of the leveraged RP portfolio becomes 7.02%/yr, which is 1.75%/yr higher than that of the EW portfolio for the same level of realized volatility. In figure 2, we choose to leverage up the RP portfolio to match the volatility of the baseline EW portfolio but we could, perhaps more realistically, also scale down the EW portfolio to match the risk of the unlevered RP portfolio. The return advantage of RP over EW would then be smaller in absolute terms.

FIGURE 2

Performance statistics of bond-equity allocations, January 1991 – December 2014

Period: 199101 – 201412	S&P 500	US Aggregate	Equally Weighted (EW)	Risk Parity (RP)	Leveraged Risk Parity (LRP)
Avg. Return over 1M Libor, %/yr	7.53	3.01	5.27	3.91	7.02
Volatility, %/yr	14.49	3.60	7.60	4.23	7.60
Sharpe Ratio	0.52	0.84	0.69	0.92	0.92
Skewness	-0.66	-0.28	-0.63	-0.53	-0.53
Excess Kurtosis	1.42	0.79	1.71	1.87	1.87
Max Drawdown, %	-57.5%	-8.2%	-29.9%	-12.0%	-21.0%

Source: Barclays Research

The relatively low volatility of the RP portfolio in the absence of leverage can be explained by improved diversification, but also by the fact that bonds are less risky than equities and so, as the weight of bonds increases, portfolio volatility declines. We would like to separate these two effects and quantify the contribution of diversification to the reduction in portfolio risk.

The effect of diversification can be measured by comparing portfolio volatility to the volatility calculated under a 'perfect correlation' scenario. Perfect correlations imply that diversification offers no reduction in volatility, so the difference between the volatility calculated under perfect correlations and the actual volatility is attributed to diversification. The diversification effect should be larger for a portfolio with a better balanced allocation of risk.

Under the assumption of perfect correlation between stocks and bonds, portfolio volatility can be calculated as a weighted average of the individual volatilities of the two asset classes. The 'perfect correlation' volatilities for the EW and RP portfolios are 9.04%/yr and 5.76%/yr, respectively. To measure the diversification effect, we compare actual volatilities, as reported in Figure 2, with the volatilities calculated under the perfect correlation assumption. For each portfolio, we calculate *the diversification ratio,* defined as the ratio of volatility under the perfect correlation assumption and the realized volatility.[9] The diversification ratios for the EW and RP portfolios are 1.19 (9.04/7.60) and 1.36 (5.76/4.23), respectively. That is, diversification reduces portfolio risk compared to the perfect

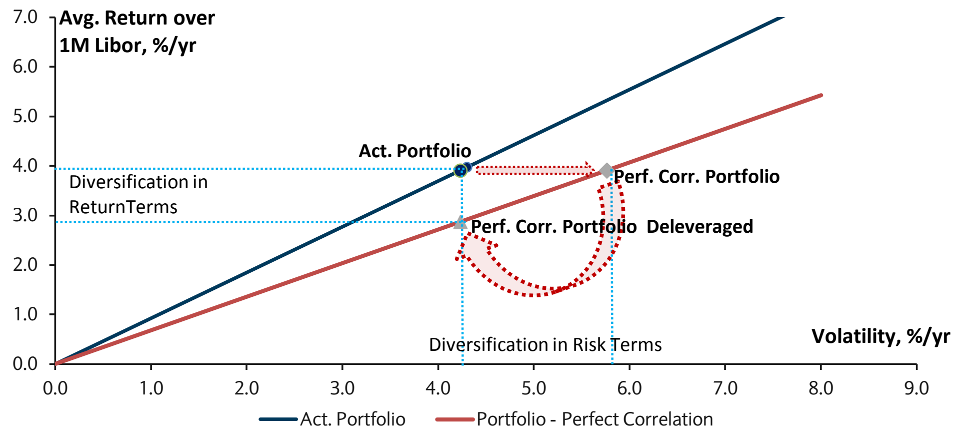
[9] For more details on the diversification ratio see Choueifaty and Coignard (2008).

correlation scenario by 19% for the EW portfolio and by 36% for the RP portfolio. The higher diversification ratio of the RP portfolio points to a better balanced allocation of risk.

The diversification effect can also be expressed in units of return. Figure 3 illustrates how we transform units of volatility into units of return according to the portfolio performance characteristics. The blue circle denotes the observed portfolio in terms of its average realised excess return over 1M Libor (3.91%/yr) and its volatility (4.23%/yr).[10] The dark blue line shows all possible return-volatility combinations that can be achieved if the portfolio is leveraged or de-leveraged using the same funding cost.[11]

FIGURE 3

Measuring portfolio diversification effect in risk and return terms



Source: Barclays Research

Under the assumption of perfect bond-equity correlation, portfolio volatility increases while return remains unchanged. This corresponds to the grey diamond (volatility of 5.76% and return of 3.91%/yr) on the flatter grey line, which represents the reduced Sharpe ratio associated with perfect correlations. Under the assumption of perfect correlation, portfolio volatility is significantly larger than its realized volatility. To measure the diversification effect in return terms, we de-leverage the portfolio such that its volatility calculated under the perfect correlation assumption matches the realized volatility of the original RP portfolio. The resulting de-leveraged portfolio (under the perfect correlation) is denoted by the grey triangle on Figure 3. The return difference between the original portfolio (blue circle) and the re-scaled portfolio derived under the perfect correlation assumption (grey triangle) is attributed to diversification.

In our bond-equity example, the performance gain of shifting from EW to a RP allocation is not only due to diversification but also to the fact that RP increases the weight of the asset class with the highest realized risk-adjusted return. We call this the re-allocation effect.

In many cases of successful implementation of RP, the re-allocation effect is positive and relates to the higher risk-adjusted performance of low volatility asset classes. Indeed, by balancing risk allocation, RP tends to increase weights of less volatile assets, which in many instances, delivered higher Sharpe ratios (SR) than riskier ones, especially in recent history.

As shown in Figure 2, the SR of bonds (0.84) was significantly higher than that of equities (0.52) over the last two decades. Switching from EW to RP shifts 30% of the portfolio market value from equities to bonds, and portfolio Sharpe ratio increases from 0.69 to 0.92.

[10] For this illustration, we use the statistics of the RP portfolio reported in Figure 2.
[11] We assume identical rates for borrowing and lending: 1-month USD Libor.

This gain can be measured in units of return and can be attributed to diversification and re-allocation effects.

Figure 4 quantifies diversification and re-allocation effects when switching from the EW to the RP bond-equity portfolio. The overall performance gain of 0.97%/yr is almost equally split between diversification and re-allocation:[12] the increase attributed to diversification is 0.49%/year, while the reallocation effect is 0.48%/yr.

FIGURE 4

Quantifying diversification and re-allocation effects from switching to risk parity, January 1991 – December 2014

Period 199101 - 201412	S&P 500	US Aggregate	Equally Weighted (EW)	Risk Parity (RP)
Avg. Return over 1M Libor, %/yr	7.53	3.01	5.27	3.91
Volatility, %/yr	14.49	3.60	7.60	4.23
Sharpe Ratio	0.52	0.84	0.69	0.92
Volatility Perf. Corr, %/yr	-	-	9.04	5.76
Avg. Returns at Target Vol. of 4.23 %/yr	2.20	3.54	2.93	3.91
Diversification Ratio	-	-	1.19	1.36
Performance Gain from Switching to RP, %/yr	**-**	**-**	**0.97**	**-**
Diversification Effect, %/yr	-	-	0.49	-
Re-allocation Effect, %/yr	-	-	0.48	-

Source: Barclays Research

To calculate the overall performance gain from switching to RP, the EW portfolio is de-leveraged to match the realized volatility of the RP portfolio (4.23%/yr). Excess cash is assumed to be invested into 1-month USD Libor. Row 5 of figure 4 reports portfolio average returns achieved at the target volatility level of 4.23%/yr. The overall performance gain can be calculated as the difference in average returns between the RP and the de-leveraged EW portfolios: 3.91 – 2.93 = 0.97%/yr.

The diversification effect is measured as the difference between the diversification ratios of the RP and EW portfolios adjusted by the Sharpe ratio of the RP portfolio under the perfect correlation assumption: 4.23 x (1.36 – 1.19) x 0.92/1.36 = 0.49%/yr. The diversification effect is proportional to the difference in diversification ratios of the RP and EW portfolios.

The re-allocation effect can be measured as the difference between the Sharpe ratios of the RP and EW portfolios under the perfect correlation assumption, adjusted for the diversification ratio of the EW portfolio: 4.23 x (0.92/1.36 - 0.69/1.19) x 1.19 = 0.48%/yr. Intuitively, the expression measures the difference between portfolio returns after adjusting for differences in risk and removing the diversification effects by assuming a perfect correlation between bonds and equities. Calculation details are provided in the Appendix.

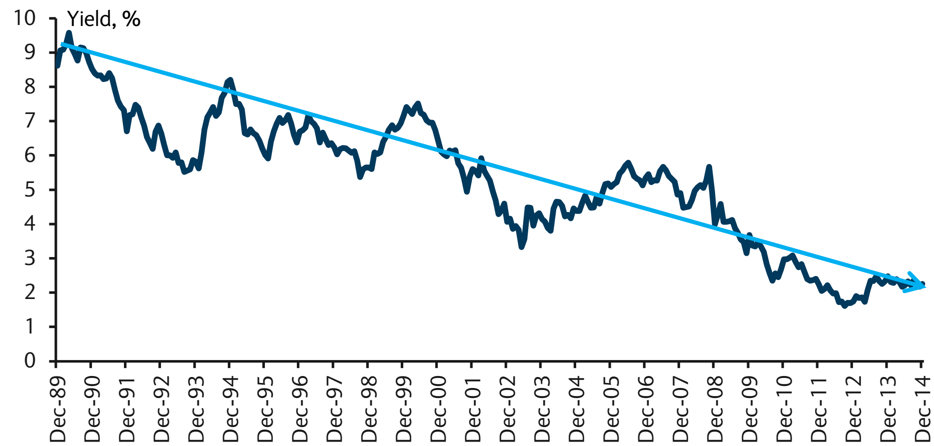
We see that in the bond-equity example, the RP portfolio outperformance is attributed to two separate effects. First, the RP portfolio achieves a better risk diversification due to the low correlation between bond and equity returns. Second, the allocation change improves portfolio risk-return characteristics by overweighting bonds, whose historical risk-adjusted returns were significantly higher than those of equities. Figure 4 suggests that roughly half of the performance gain is attributed to this re-allocation effect. The RP portfolio outperformance would be significantly lower without this effect.

[12] EW portfolios (both actual and the one calculated under perfect correlations) are de-leveraged to match the volatility of the RP portfolio (4.23 %yr).

Should investors expect that bonds will continue to outperform equities in risk-adjusted terms as they have done in the past? Bond yields have declined significantly over the last two decades, as shown in Figure 5. Indeed, the average yield of the Barclays US Aggregate Bond Index was only 2.25% in December 2014, down from 9.06% in January 1990. Can such a downward trend in yields continue? How would our example change if today's yield environment had prevailed in the period of our analysis?

FIGURE 5

Average yield of Barclays US Aggregate Bond Index



Source: Barclays Research

We can construct an approximate answer to this question by re-calculating the monthly returns of the US Aggregate index with effects from declining yields and historical carry removed. First, we subtract from the historical monthly returns of the US Aggregate index the portion of return associated with the realized yield decrease of about 2bp/month. We estimate this trend as the difference between index yields on 31 December 2014 and 31 January 1990 divided by the number of months between these two dates. The monthly yield contraction contributes approximately 10bp per month to index returns, assuming an average index duration of five years. Second, we replace historical carry, estimated as the accrued yield of the index, with the monthly yield carry of 0.19% (2.25/12) observed in December 2014. The equation below summarizes the return adjustment:

Adj. Return = Return + Trend x OAD – Hist Yield/12 + Current Yield/12

Figure 6 shows the performance of the EW and RP portfolios based on adjusted returns of the US Aggregate index. The average return of bonds drops from 3.01% /yr to 1.95%/yr and the Sharpe ratios of bond and equity indices become comparable. The performance improvement from switching to RP is now much smaller than in the previous case without the return adjustment; it is now predominantly attributed to the diversification effect.

FIGURE 6

Diversification and re-allocation effects from switching to risk parity with returns of the US Aggregate Bond Index adjusted for yield trend and carry, January 1991 – December 2014

Period 199101 – 201412	S&P 500	US Aggregate (Adjusted)	Equally Weighted (EW)	Risk Parity (RP)
Avg. Return over 1M Libor, %/yr	7.53	1.95	4.74	3.06
Volatility, %/yr	14.49	3.56	7.60	4.22
Sharpe Ratio	0.52	0.55	0.62	0.72
Volatility Perf. Corr, %/yr	-	-	9.02	5.73
Avg. Returns at Target Vol. of 4.22 %/yr	2.19	2.31	2.63	3.06
Diversification Ratio	-	-	1.19	1.36
Performance Gain from Switching to RP, %/yr	**-**	**-**	**0.43**	**-**
Diversification Effect, %/yr	-	-	0.39	-
Re-allocation Effect, %/yr	-	-	0.04	-

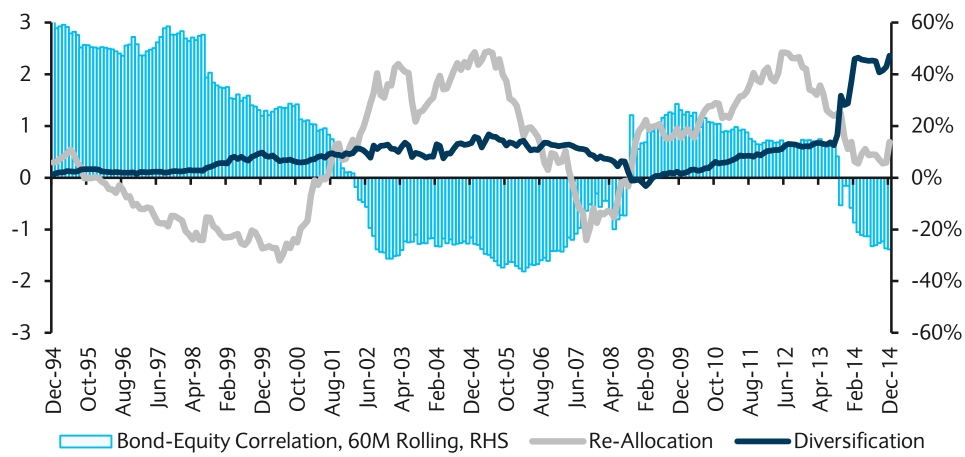
Source: Barclays Research

Indeed, the total performance gain from switching to RP is 0.43%/yr, with only 0.04%/yr attributed to the re-allocation effect. The contribution of the re-allocation effect becomes negligible when the underlying assets have similar Sharpe ratios.

The relative importance of the diversification and re-allocation effects can vary significantly over time even without adjusting historical returns for yield trends. Figure 7 shows a historical decomposition of performance gains resulting from the switch to RP in five-year rolling periods *without* adjusting returns of bonds for the historical trend in yields and carry. Portfolios are (de-)leveraged to match the target full-sample volatility of the original RP portfolio (4.23%/year). This ensures that the risk of the underlying portfolio remains stable over time.

FIGURE 7

Historical diversification and re-allocation effects expressed in return terms from switching to risk parity in rolling 5-year periods, January 1990 – December 2014



Source: Barclays Research

Note: The results shown at each point in time correspond to the trailing 5-year period.

The decomposition of performance gains in rolling five-year periods is very similar to the one implemented previously for the full sample. As an example, we can pick the recent five-year period that starts in January 2009 and ends in December 2014. Both equities and bonds performed very well in this period. Average return and volatility of equities were 15.07%/yr and 13.00%/yr, while those of bonds were respectively 4.18%/yr and

2.68%/yr,. Bonds outperformed equities in risk-adjusted terms with respective Sharpe ratios of 1.56 and 1.16. Finally, the correlation between equity and bond returns was -28%, implying a significant scope for diversification. The EW bond-equity portfolio had an average return of 9.62%/yr and volatility of 6.26%/yr (Sharpe ratio of 1.54). The RP portfolio had a significantly higher risk-adjusted performance in the same period, with an average return of 6.04%/yr and volatility of only 2.68%/yr (Sharpe ratio of 2.26).

The performance gain when switching from the EW to RP portfolio, calculated at the target volatility level of 4.23%/yr, is 3.05%/yr. The portions attributed to diversification and re-allocation effects are 2.36%/yr and 0.69%/yr, respectively. The diversification effect has been quite significant in the last five years, while the re-allocation effect has been smaller.

According to Figure 7, both diversification and re-allocation effects tend to vary over time. Re-allocation contributions become negative on several occasions as stocks underperformed bonds in risk-adjusted terms. Diversification contributions remained positive throughout, as expected, but were relatively low during periods of high bond-equity correlations. Diversification contributions increased when bond-equity correlations declined. In the last five years, the re-allocation effect was low as both asset classes had similar Sharpe ratios, while low bond-equity correlation helped inflate the diversification effect.

Applying Risk Parity to Different Markets

We continue our analysis with three case studies: macro multi-asset; equity-only; and macro Treasury-only.

In our RP portfolios, all assets have equal volatility contributions on an isolated basis. This equal volatility (EV) approach does not utilize correlations. The role of correlations is discussed in the next section.

We now compare risk parity (RP) portfolios with market-weighted (MW) and equally-weighted (EW) allocations. In contrast to the bond-equity example discussed in the previous section, we now allow the weights of RP portfolios to change over time. Weights of MW portfolios are also dynamic due to valuation changes and new issuance. Portfolio weights are calculated at the beginning of each month based on realised return volatilities over the previous 24 months. Portfolio returns are measured at the end of month, when weights are re-balanced again. This ensures that performance measurement is separated from portfolio construction.[13]

Dynamic weights can have positive or negative contributions to portfolio performance depending on their relationship with asset returns. Changes in market weights are known to be positively correlated with performance. In equity portfolios this occurs due to valuation errors, as the market value of overpriced stocks increases, and has a detrimental effect on portfolio performance.[14] In bond portfolios, the positive relationship between weights and returns is also detrimental to portfolio performance and is associated with the pro-cyclicality of bond issuance.[15] To account for the possible effects of dynamic weights in our decomposition, we add an additional element to our attribution: *the dynamic weight effect*.

Performance attribution of Risk Parity now includes three elements: diversification, re-allocation, and dynamic weights. The last two are often unintended effects.

[13] The introduction of dynamic asset weights raises the issue of estimation risk. The results in the previous section were based on an in-sample analysis, in which we choose a single set of fixed weights for the period after having observed the realized volatilities and correlations. In reality, these must be estimated; and if future asset returns display very different statistics, portfolios whose construction depends on them can underperform in unanticipated ways. RP approach requires confidence in one's ability to accurately estimate these parameters. The choice of an EV approach rather than ERC could reflect a view that volatilities can be estimated with greater accuracy than correlations. This is discussed briefly in our later treatment of correlations.
[14] See Hsu (2006) and Arnott, Hsu, Kalesnik and Tindall (2013).
[15] See *Issuance Dynamics and Performance of Corporate Bonds*.

Macro example: equity, treasuries, and TIPS

In our first example, we combine three asset classes: US equities, Treasuries, and TIPS.[16] These assets are represented by the following indices: S&P500, Barclays US Treasury, and Barclays US TIPS.

Figure 8 reports the historical performance and correlations of the three asset classes. TIPS delivered the highest average return, while their Sharpe ratio was close to that of Treasuries. The Sharpe ratio for equities was much lower. Correlations between equities and fixed income assets (Treasuries and TIPS) are low, which potentially helps the case for improving diversification. The correlation between TIPS and Treasuries is 70%.

FIGURE 8

Performance characteristics of S&P 500, Barclays US Treasuries, and Barclays US TIPS indices, October 1999 – December 2014

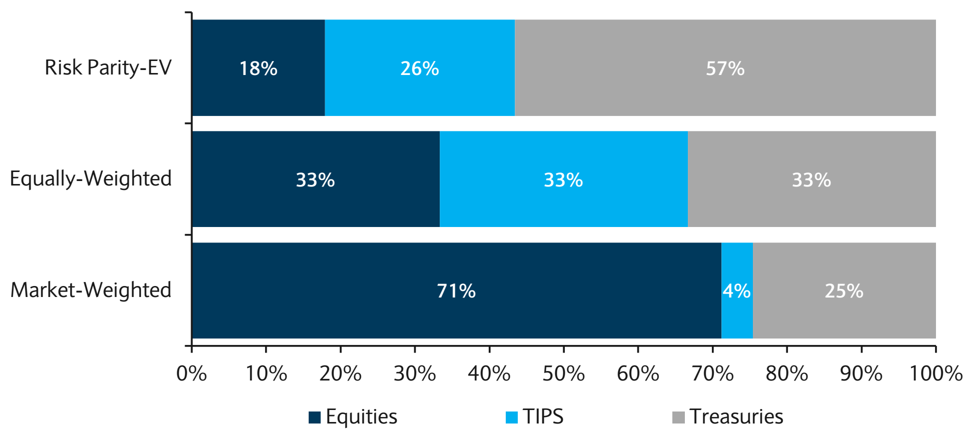
	S&P500	TIPS	TSY
Avg. Return (%/yr)	3.93	4.21	2.98
Volatility (%/yr)	15.37	6.27	4.49
Sharpe Ratio	0.26	0.67	0.66
Correlation Matrix			
S&P500	100%	3%	-31%
TIPS	3%	100%	70%
TSY	-31%	70%	100%

Source: Barclays Research

Figure 9 shows asset weights in all three portfolios as of 31 December 2014. Equities have the largest weight in the MW portfolio, while TIPS represent only 4%. Asset weights in the RP allocation are very different from MW, with Treasuries and TIPS representing more than 80% of portfolio value. The EW portfolio is between these extremes, except that the weight of TIPS is highest in the EW portfolio.

FIGURE 9

Asset weights in macro portfolios, December 2014



Source: Barclays Research

Did RP portfolio outperform its MW and EW counterparts? How much performance gain is attributed to diversification, re-allocation and weight dynamics? Using the attribution methodology adopted for our bond-equity example we measure the performance effects resulting from the shift to RP and report the results in Figure 10.

[16] This example is motivated by Qian (2013), who argues that the particular choice of economic exposures is important for the success of the RP approach.

FIGURE 10

Performance gains from switching to risk parity, October 1999 – December 2014

Period 199910 - 201412	Market Weighted (MW)	Equally Weighted (EW)	Risk Parity (RP)
Dynamic Portfolio Weights			
Avg. Return over 1M Libor, %/yr	2.76	3.71	3.84
Volatility, %/yr	11.64	5.75	4.39
Sharpe Ratio	0.24	0.64	0.88
Avg. Returns at Target Vol. of 4.39 %/yr	1.04	2.83	3.84
Constant Portfolio Weights			
Avg. Return over 1M Libor, %/yr	3.75	3.71	3.59
Volatility, %/yr	11.59	5.75	4.45
Sharpe Ratio	0.32	0.64	0.81
Volatility Perf. Corr, %/yr	12.91	8.71	6.83
Diversification Ratio	1.11	1.51	1.53
Avg. Returns at Target Vol. of 4.39 %/yr	1.42	2.83	3.54
Contribution of Dynamic Weights, %/yr	**-0.38**	**0.00**	**0.30**
Performance Gain from Switching to RP, %/yr	**2.80**	**1.01**	**-**
Dynamic Weight Effect, %/yr	0.68	0.30	-
Diversification Effect, %/yr	0.97	0.05	-
Re-allocation Effect, %/yr	1.15	0.67	-

Source: Barclays Research

The RP portfolio allocates substantially to fixed income assets which also have a higher risk-adjusted performance. The Sharpe ratio of the RP portfolio is therefore much higher than those of the MW and EW portfolios. The average return of the MW portfolio rescaled to the target volatility level of 4.39%/yr is only 1.04%/yr, as opposed to 3.84%/yr for the RP portfolio. The 2.80%/yr return difference is the gain of switching from MW to RP.

The effect of dynamic weights is measured by constructing portfolios with constant weights equal to the time-average weights in the original portfolios. The second section of Figure 10 illustrates this. The average return of the original MW portfolio at the target volatility of 4.39%/yr is 1.04%/yr. This increases to 1.42%/yr if we use constant instead of dynamic weights. So, the contribution of dynamic weights to the return of the MW portfolio (at 4.39% vol) is -0.38%/yr. This performance drag reflects the fact that MW bond-equity portfolios tend to overweight equities when they are expensive and underweight them when they are cheap.

The contribution of dynamic weights to the performance of the RP portfolio is calculated in the same way. The effect is now positive (0.30%/yr) which reflects the fact that, in our sample, increases in realised S&P500 index volatility tended to be followed by poor performance relative to bonds. The dynamic weight effect in the EW portfolio is zero by definition as asset weights are constant.

The net effect of dynamic weights when switching from MW to RP portfolio is the difference in the respective contributions of the two portfolios. It is reported in the first column of Figure 10: 0.68%/yr (= 0.30 +0.38).

Diversification and re-allocation effects are both sizable for the MW portfolio. The diversification effect contributes 0.97%/yr, which is 35% of the overall performance gain in percentage terms. The re-allocation effect is even larger: 1.15%/yr, or 41% in percentage terms. This effect is associated with the higher risk-adjusted performance of less volatile fixed income assets relative to equities.

The second column of numbers in Figure 10 measures performance gains when the allocation is changed from EW to RP. The overall performance gain of 1.01%/yr is much smaller than when switching from a MW portfolio. The re-allocation effect (0.67%/yr) constitutes the largest part of the improvement, while the diversification effect (0.05%/yr) is very small.

This example illustrates the potential importance of re-allocation and weight dynamics in explaining the historical outperformance of RP portfolios. But RP was not designed with an intention to capture these two effects. In particular, overcoming the negative drag from dynamic market weights does not necessarily require the RP approach. Adopting any allocation with less pro-cyclical weights should be sufficient to improve the performance of the MW portfolio. Also, the re-allocation effect might be not sustainable as past history does not guarantee a superior risk-adjusted performance of fixed income assets going forward. In addition, the RP portfolio is not optimal when the higher Sharpe ratios of less risky assets are explicitly taken into account.

Risk parity to individual stocks in an equity portfolio

Next, we consider a case study involving individual stocks. Alternative weighting schemes of equity portfolios, often based on versions of risk-parity, have been studied in the literature. The risk-based allocations proposed include maximum diversification (MD), equal volatility (EV), equal risk contribution (ERC),[17] and minimum volatility (MV).[18] All these approaches try to achieve a better diversification and hence better risk-adjusted performance.

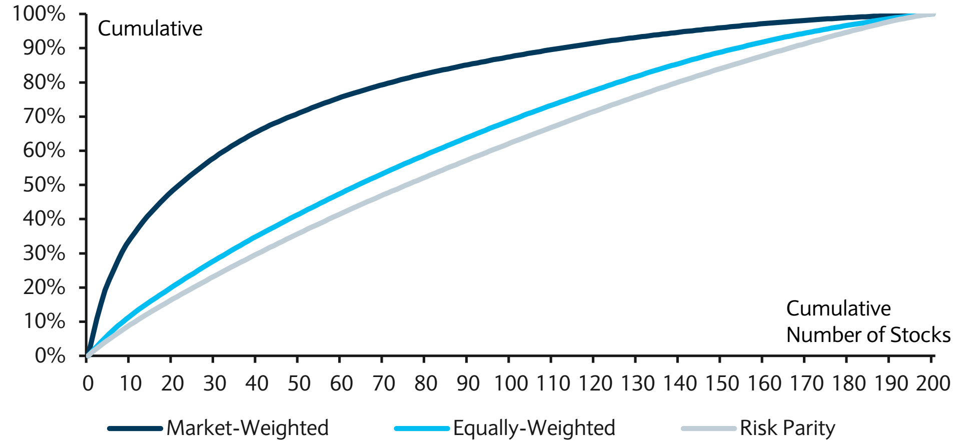
We consider the RP approach based on equal volatility weighting. This scheme equalizes the *isolated* volatility contributions of individual stocks.[19] We use a sample of the 200 largest stocks by market capitalization in the SP500 index as of 30 September 1999 that also had pricing information as of 31 December 2014.[20] As in the previous case study, we compare risk-parity (RP) with market-weighted (MW) and equally-weighted (EW) portfolios.

First, we can show that the MW and EW portfolios are less diversified than the RP one. We sort individual stocks in the portfolio by their percentage risk contributions, starting with the largest contributor. Volatile stocks with large weights contribute more to portfolio risk. Figure 11 shows the cumulative percentage contributions to portfolio variance as individual stocks are added in descending order of risk contribution.[21]

[17] For details on the maximum diversification portfolio, see Choueifaty and Coignard (2008). Other risk-based equity weighting schemes are compared in Demey, Maillard and Roncalli (2010).
[18] See, for example, the 'MSCI Global Minimum Volatility Indices Methodology'.
[19] By choosing this scheme we ignore correlations, implicitly assuming them to be similar across individual stocks. The resulting allocations are robust with respect to estimation errors in correlations. Other risk-based allocations, such as equal risk contributions and maximum diversification, lead to qualitatively similar results.
[20] Selecting companies with continuous history over the last 15 years likely creates a survivorship bias, meaning that portfolio returns might be overstated. Comparing performances of the MW portfolio of the 200 selected stocks and S&P 500 we can roughly assess the survivorship bias. The average return of the MW portfolio was 4.27%/yr during the sample period. This compares with the average return of S&P 500 of 3.93%/yr. Thus, we could roughly estimate the survivorship bias as 0.34%/yr (=4.27-3.93). This bias, however, applies equally to both market-weighted and risk parity portfolios, allowing us to analyse and attribute performance differences.
[21] For this, portfolio weights are taken as time-averages over the period from October 1999 to December 2014. The covariance matrix is estimated from the same sample of stock returns.

FIGURE 11

Cumulative percentage risk contribution of individual stocks; October 1999 - December 2014



Source: Barclays Research

In a perfectly diversified portfolio, the risk contributions of individual stocks would be the same, and the cumulative curve would form a straight line. We see that the RP portfolio (equal volatility) is reasonably close to the ideal case.[22] The equally weighted portfolio also seems reasonably well diversified. Still, the first 64 (out of 200) stocks in the EW portfolio account for over 50% of portfolio risk. For the MW portfolio, the cumulative distribution of risk contributions is significantly skewed, with the first 22 stocks (out of 200) contributing more than 50% of portfolio risk. Given these results, we expect the RP allocation to improve diversification relative to either the MW or EW portfolio, with stronger results against MW.

Changing portfolio allocation to RP typically increases the weights of low-risk assets. Low-volatility stocks have historically outperformed higher-risk ones in terms of the Sharpe ratio. This has been called the 'low risk anomaly' (LRA).[23] We illustrate this phenomenon by sorting our universe or 200 stocks into deciles based on their beta to the S&P 500 index.[24]

Figure 12 shows average returns, volatilities and Sharpe ratios for each decile. We confirm previous empirical findings that the Sharpe ratio of equity portfolios declines as market beta increases. This should translate into a positive re-allocation effect as, in our case study, the RP portfolio systematically allocates higher weights to less risky stocks.

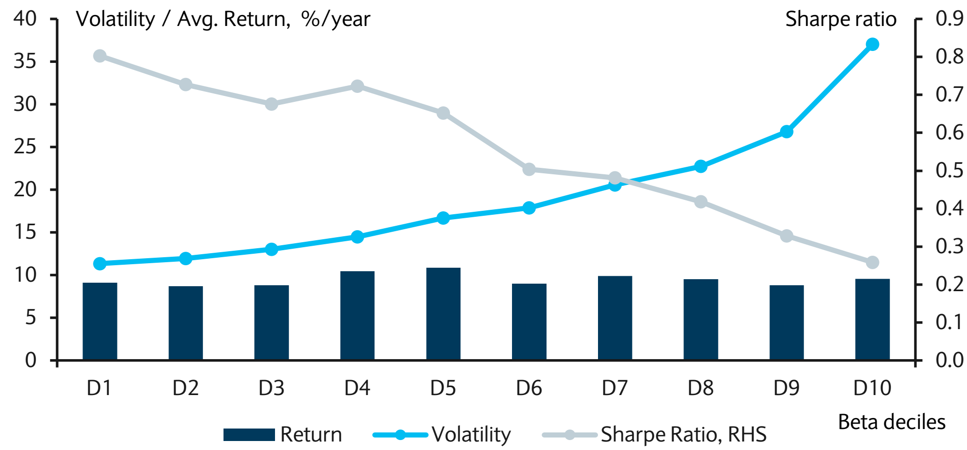
[22] The "ideal" case would correspond to the weighting scheme based on equal risk contributions, taking correlations into account.

[23] For details and possible explanations of the phenomenon see Blitz, Falkenstein and van Vliet (2013) or *Low Volatility Equity Strategies: Anomaly or Capital Structure Effect?*

[24] Higher beta stocks have typically higher market risk. Sorting on volatilities would produce qualitatively similar results.

FIGURE 12

Performance of equity portfolios by market beta, October 1999 - December 2014



Source: Barclays Research

Figure 13 reports the performance gains obtained by switching to RP from either the MW or EW portfolio, and attributes these gains to diversification, re-allocation and weight dynamics.

FIGURE 13

Performance gains from switching to risk parity in a portfolio of 200 stocks, October 1999 – December 2014

Period 199910 - 201412	Market Weighted (MW)	Equally Weighted (EW)	Risk Parity (RP)
Dynamic Portfolio Weights			
Avg. Return over 1M Libor, %/yr	4.27	9.46	8.57
Volatility, %/yr	14.96	16.93	14.54
Sharpe Ratio	0.29	0.56	0.59
Avg. Returns at Target Vol. of 14.54 %/yr	4.15	8.12	8.57
Constant Portfolio Weights			
Avg. Return over 1M Libor, %/yr	8.43	9.46	9.44
Volatility, %/yr	15.95	16.93	15.46
Sharpe Ratio	0.53	0.56	0.61
Volatility Perf. Corr, %/yr	30.75	32.96	30.42
Diversification Ratio	1.93	1.95	1.97
Avg. Returns at Target Vol. of 14.54 %/yr	7.69	8.12	8.88
Contribution of Dynamic Weights, %/yr	**-3.54**	**0.00**	**-0.30**
Performance Gain from Switching to RP, %/yr	**4.42**	**0.45**	**-**
Dynamic Weight Effect, %/yr	3.23	-0.30	-
Diversification Effect, %/yr	0.18	0.09	-
Re-allocation Effect, %/yr	1.01	0.66	-

Source: Barclays Research

Shifting the portfolio allocation from MW to RP increases average return from 4.27%/yr to 8.57%/yr. This gain, however, is mainly attributed to limiting the detrimental effects of weight dynamics in the MW portfolio. Indeed, the difference in average returns between MW portfolios with dynamic and constant weights, calculated at the target volatility level of

14.54%/yr, is -3.54%/yr (4.15 – 7.69).[25] The contribution of dynamic weights to the performance of the RP portfolio is also negative, but much smaller: -0.30%/yr. Indeed, volatility spikes and corresponding reductions in weights can make a stock be underweighted precisely when its valuation becomes attractive and this is likely to hurt portfolio performance.

The overall performance gain when switching from MW to RP is the difference in return calculated at the target volatility level, which we take as 14.54%/yr, the realised volatility of the unlevered RP portfolio. The bottom-left section of Figure 13 shows that the performance gain is 4.42%/yr (8.57 – 4.15). The effect of dynamic weights (3.23%/yr) dominates and the diversification effect of 0.18%/yr looks very small in comparison. The re-allocation effect, associated with the low risk anomaly, is quite significant: 1.01%/yr.

When taking the EW portfolio as a starting point, the overall performance gain from switching to RP is only 0.45%/yr. The dynamic weight effect, associated with the negative contribution of dynamic weights to the performance of the RP portfolio, is negative (-0.30%/yr) while the performance gain attributed to improved diversification is only 0.09%/yr. The low risk anomaly is in fact the most significant contributor to the outperformance of the RP portfolio: 0.66%/yr.

In this case study, the performance gain attributed to diversification, the main intended objective of RP, is low when switching from either MW or EW portfolios. This might be related to the large number of stocks included in the portfolios. Indeed, the scope for further diversification of name-specific risk in a portfolio of many stocks is likely to be limited.

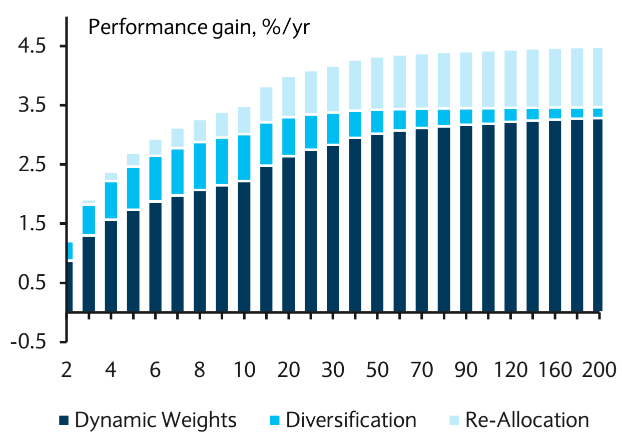
To verify this, we repeat our attribution exercise for portfolios with different number of stocks. We randomly select a given number of stocks from the original set of 200 names and construct MW, EW, and RP portfolios with a volatility target of 15%/yr. Then, we attribute the performance gains from switching to RP. This exercise is repeated 10,000 times. The simulated portfolios are used to calculate average contributions from dynamic weights, diversification, and re-allocation. Figure 14 shows the average attributed performance gains from switching to RP versus the number of stocks in the portfolio.

FIGURE 14

Performance gains from switching to risk parity and number of stocks in portfolio, October 1999 – December 2014

PANEL A

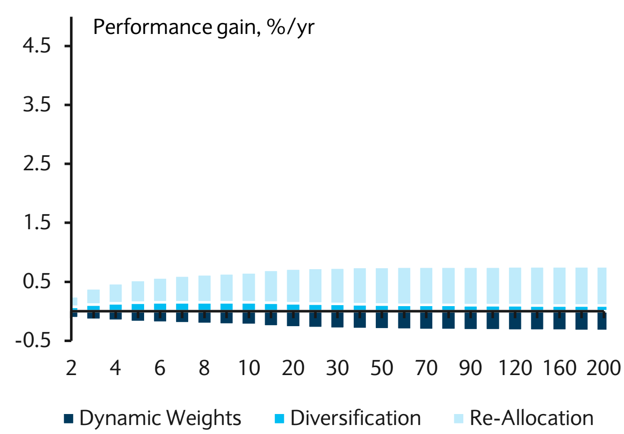
Switching from market-weighted portfolio



Source: Barclays Research

PANEL B

Switching from equally-weighted portfolio



Source: Barclays Research

[25] The negative effect of market weight dynamics on the performance of equity portfolios has been discussed in the literature. See, for example, Hsu (2006) or Arnott, Hsu, Kalesnik and Tindall (2013).

Panels A and B plot the attributed gains from switching to RP from MW and EW portfolios respectively. The dynamic weights effect constitutes most of the performance gain for MW portfolios. The size of this effect increases with the number of stocks. For the EW portfolio, the effect of dynamic weights is negative and relatively small.

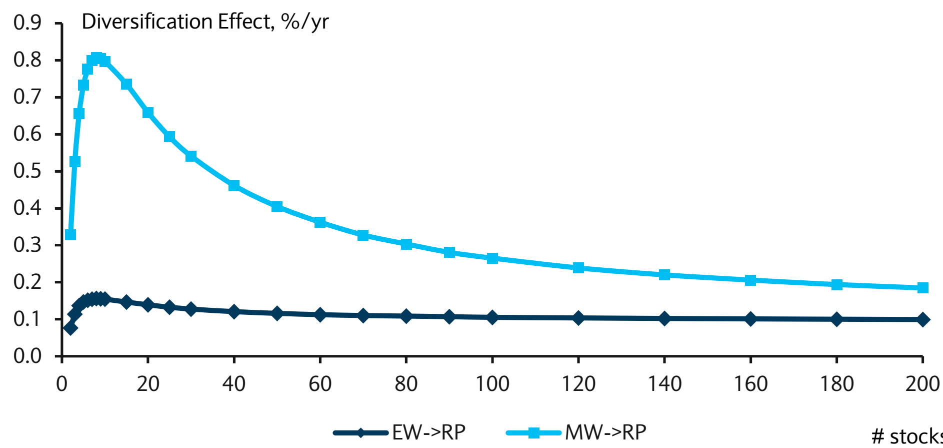
The contribution of the re-allocation effect, associated with the low risk anomaly, increases with the number of stocks and then flattens out when the number of stocks exceeds 30. The magnitude of the effect is roughly comparable in MW and EW portfolios.

Figure 15 provides a detailed analysis of the diversification effect, which is much larger for MW portfolios than for EW. This confirms our earlier intuition that an EW portfolio of individual stocks has a relatively balanced distribution of risk compared to a MW portfolio. This is also consistent with Figure 11, which showed that the risk of the MW portfolio is heavily concentrated in a few stocks.

We also find that the diversification effect is relatively small when implementing RP in portfolios with a large number of stocks. Indeed, idiosyncratic risk is already well diversified in a broad portfolio irrespective of the weighting scheme and it might be difficult to improve risk diversification further. The size of the diversification effect reaches a maximum for portfolios of 8-10 stocks and then gradually declines as the number of portfolio holdings increases further.

FIGURE 15

Performance gains due to diversification effect and number of stocks in portfolio, October 1999 - December 2014



Source: Barclays Research

We find that diversification plays a relatively small role in the outperformance of RP over MW or EW in broad portfolios of individual stocks. The re-allocation effect attributed to the low risk anomaly is much larger. We also confirm that dynamic weights of individual stocks in MW portfolios have a very negative impact on portfolio performance.[26]

US Treasury bonds

In our final example, we compare allocations across maturity sectors of the Barclays US Treasury Index. We take five maturity buckets (1-3yr, 3-5yr, 5-7yr, 7-10yr, 10+yr) as building blocks for portfolio allocation. Historical performance is summarized by maturity in

[26] To be fair, we should point out that the current study does not account for a number of factors that would favor market-weighted portfolios, such as transaction costs and liquidity. The dynamic weight changes in a market-weighted portfolio tend to occur automatically as prices fluctuate, and do not require any transactions. To dynamically adjust asset weights according to any other rule requires transactions, whose cost would somewhat reduce the performance advantages of the other schemes. Furthermore, MW portfolios are by definition concentrated in the largest and most liquid securities in the market; studies of risk-based approaches in some markets require controls to ensure that the portfolio positions and transactions are realistically achievable in practice.

Figure 16. As we move to longer maturities, risk increases faster than return, and so Sharpe ratios decline. Very high correlations across maturity buckets suggest that the scope for improving diversification might be limited. The lowest correlation of 65% is found for maturity sectors that are most remote from one another.

As in previous examples, we compare RP with MW and EW portfolios. RP portfolios are rebalanced every month based on volatility estimated from the previous 24 monthly returns. Figure 17 compares all three allocations and attributes the effect of switching from MW or EW to RP. The attribution is reported in units of return for risk-matched portfolios which are de-leveraged to match the volatility of the RP portfolio (3.38%/yr). RP outperforms both MW and EW, and this outperformance comes entirely through the re-allocation effect as RP tends to overweight higher Sharpe ratio, shorter-maturity buckets. Diversification effects are very small, consistent with the high correlations among the underlying assets. The effect of dynamic weights is very small as well.

FIGURE 16

Performance of US Treasury bonds by maturity bucket, October 1991 – December 2014

Period: 199110 - 201412	1-3yr	3-5yr	5-7yr	7-10yr	10+yr	MW	EW	RP
Avg. Return over T-Bills, %/yr	1.50	2.83	3.56	4.13	5.85	*3.20*	*3.58*	*2.66*
Volatility, %/yr	1.50	3.53	4.78	6.28	9.75	*4.43*	*4.95*	*3.38*
Sharpe Ratio	1.00	0.80	0.75	0.66	0.60	*0.72*	*0.72*	*0.79*
Correlations								
1-3yr		95%	88%	81%	64%	*84%*	*83%*	*91%*
3-5yr			98%	93%	78%	*94%*	*93%*	*97%*
5-7yr				98%	87%	*97%*	*98%*	*98%*
7-10yr					94%	*99%*	*99%*	*97%*
10+yr						*94%*	*95%*	*87%*

Source: Barclays Research

One interesting observation, although anecdotal given the very small size of the effects, is that RP does not improve diversification at all when compared to MW. This is because the market structure of the US Treasury index resembles a barbell allocation, with relatively small weights in intermediate maturity sectors, a consequence of US Treasury issuance patterns. This allows the MW portfolio to benefit from the relatively low correlation that prevails between the shortest and longest-maturity sectors.

FIGURE 17

Attributing performance gains from switching to risk parity in portfolios of US Treasuries, October 1991 – December 2014

Period 199110 - 201412	Market Weighted (MW)	Equally Weighted (EW)	Risk Parity (RP)
Dynamic Portfolio Weights			
Avg. Return over 1M T-Bills, %/yr	3.20	3.58	2.66
Volatility, %/yr	4.43	4.95	3.38
Sharpe Ratio	0.72	0.72	0.79
Avg. Returns at Target Vol. of 3.38 %/yr	2.44	2.45	2.66
Constant Portfolio Weights			
Avg. Return over 1M T-Bills, %/yr	3.30	3.58	2.65
Volatility, %/yr	4.47	4.95	3.29
Sharpe Ratio	0.74	0.72	0.81
Volatility Perf. Corr, %/yr	4.73	5.17	3.47
Diversification Ratio	1.059	1.04	1.055
Avg. Returns at Target Vol. of 3.38 %/yr	2.50	2.45	2.72
Contribution of Dynamic Weights, %/yr	**-0.06**	**0.00**	**-0.07**
Performance Gain from Switching to RP, %/yr	**0.22**	**0.21**	**-**
Dynamic Weight Effect, %/yr	-0.01	-0.07	-
Diversification Effect, %/yr	-0.01	0.03	-
Re-allocation Effect, %/yr	0.24	0.25	-

Source: Barclays Research

Role of Correlations in Risk Parity

As mentioned earlier, there are various techniques that implement RP in portfolio allocation across assets or risk dimensions. Examples include maximum diversification (MD), equal risk contribution (ERC) and equal volatility (EV) portfolios.

The examples presented so far use equal volatility. In this approach, portfolio weights are set to make isolated volatilities of individual assets equal. This makes portfolio construction simple and intuitive, as correlations between asset returns are not needed. In contrast, both MD and ERC exploit correlations.

Correlations can have a profound effect on portfolio risk, but including them in portfolio constructions can be challenging. This is particularly true when the risk properties of underlying assets vary over time. Indeed, correlations can be unstable and hard to predict from historical data. Dynamic correlations are also likely to increase portfolio turnover. So, how important is it to take correlations into account when building a RP portfolio?

We re-visit our macro example, with the RP portfolio now constructed using ERC. As before, the investment universe includes three assets: equities, Treasury bonds, and TIPS. Portfolio weights based on ERC ensure equal percentage risk contributions of all three assets to the portfolio variance.

Figure 18 shows that the correlation between Treasuries and TIPS is 70%, while the correlation between Equities and Treasuries is -31%. The correlation between TIPS and equities is 3%. Correlations among assets are heterogeneous, which makes this example particularly interesting. Considering correlations will tend to decrease allocations to assets that are highly correlated with each other, shifting the weight to less correlated assets. Indeed, Figure 18 shows notable differences in weights between EV and ERC-based RP portfolios. The ERC-based RP portfolio has slightly higher allocations to equities and Treasuries, and lower allocations to TIPS than the EV-based portfolio.

FIGURE 18

Risk and performance statistics of US Treasuries, US TIPS, and S&P 500, October 1999 – December 2014

	S&P500	TIPS	TSY
Avg. Return (%/yr)	3.93	4.21	2.98
Volatility (%/yr)	15.37	6.27	4.49
Sharpe Ratio	0.26	0.67	0.66
Correlation Matrix			
S&P500	100%	3%	-31%
TIPS	3%	100%	70%
TSY	-31%	70%	100%
Portfolio Weights			
RP - Equal Volatilities	15%	36%	50%
RP - Equal Risk Contributions	19%	29%	52%

Source: Barclays Research

Figure 19 compares the performance gains from switching to RP implemented with ERC and EV. The ERC-based RP portfolio return is smaller than the one of the EV-based RP (3.44%/yr vs. 3.84%/yr) for identical volatilities of 4.39%/yr.[27] Accordingly, RP-ERC outperforms MW by a smaller margin than RP-EV (2.40 vs. 2.80%/yr). The decline in the performance gain comes from smaller dynamic weights and re-allocation effects. But the diversification effect increases from 0.97%/yr to 1.05%/yr as the ERC-RP better accounts for correlations.

Column 2 of Figure 19 reports a similar attribution for switching from the EW portfolio. Again, the overall performance gain drops from 1.01%/yr to 0.61%/yr due to smaller dynamic weight and re-allocation effects while the diversification effect improves significantly: from 0.05 to 0.20%/yr. The decrease in the contribution of dynamic weights to the performance of the RP portfolio might result from the instability of asset correlations estimated out-of-sample.

[27] We used leverage to target a volatility of 4.39%/yr for RP-ERC, as for all the portfolios in this example.

FIGURE 19

Performance gains from switching to risk parity based on equal volatility (EV) and equal risk contributions (ERC), October 1999 – December 2014

	Market Weighted (MW)	Equally Weighted (EW)	Risk Parity -ERC (RP-ERC)	Risk Parity - EV (RP-EV)
Dynamic Portfolio Weights				
Avg. Return over 1M Libor, %/yr	2.76	3.71	3.44	3.84
Volatility, %/yr	11.64	5.75	4.39	4.39
Sharpe Ratio	0.24	0.64	0.78	0.88
Avg. Returns at Target Vol. of 4.39 %/yr	1.04	2.83	3.44	3.84
Constant Portfolio Weights				
Avg. Return over 1M Libor, %/yr	3.75	3.71	3.57	3.59
Volatility, %/yr	11.59	5.75	4.57	4.45
Sharpe Ratio	0.32	0.64	0.78	0.81
Volatility Perf. Corr, %/yr	12.91	8.71	7.35	6.83
Diversification Ratio	1.11	1.51	1.61	1.53
Avg. Returns at Target Vol. of 4.39 %/yr	1.42	2.83	3.42	3.54
Contribution of Dynamic Weights, %/yr	**-0.38**	**0.00**	**0.02**	**0.30**
Performance Gain from Switching to RP-ERC, %/yr	**2.40**	**0.61**	**-**	**-**
Dynamic Weight Effect, %/yr	0.40	0.02	-	-
Diversification Effect, %/yr	1.05	0.20	-	-
Re-allocation Effect, %/yr	0.95	0.39	-	-
Performance Gain from Switching to RP-EV, %/yr	**2.80**	**1.01**	**-**	**-**
Dynamic Weight Effect, %/yr	0.68	0.30	-	-
Diversification Effect, %/yr	0.97	0.05	-	-
Re-allocation Effect, %/yr	1.15	0.67	-	-

Source: Barclays Research

Overall, taking correlations into account changes results quantitatively, but leaves qualitative conclusions similar. The re-allocation effect remains a significant source of outperformance when switching to RP-ERC. However, the diversification effect is larger when using correlations.

We have chosen the EV-based version of RP for the ease of exposition. There might be examples where accounting for asset correlations could have had a significantly stronger impact on allocation weights and attribution results. However, the logic of performance gain attribution remains the same.

Conclusion

The risk parity (RP) approach to portfolio construction aims to improve risk-adjusted performance by maximizing the benefits of diversification. Back-tests over the last two decades often show excellent risk-adjusted performance of RP portfolios compared to their peers.

We introduce a performance attribution methodology to evaluate the outperformance of RP portfolios relative to traditional allocations, such as market-weighted (MW) and equally weighted (EW) portfolios. Applying our methodology in various case studies, we find that a large part of RP performance gains is not necessarily due to better diversification, but rather due to two often unintended effects.

One is the effect of dynamic weights; this is less an advantage of RP and more a disadvantage of MW. In this respect, any portfolio weighting scheme that departs from dependence on market weights can enjoy this benefit. This includes equal weights and various forms of 'fundamental indexing' as well as risk parity. It is somewhat misleading to attribute such gains to a particular type of risk parity allocation scheme.

The other is the re-allocation effect. Risk parity schemes typically bias portfolio allocations to overweight lower-volatility assets. Risk parity should perform well if these lower-volatility assets generate persistently high risk-adjusted returns (i.e., in cases where the 'low risk anomaly' is observed). However, one of the key motivations for the risk parity approach is that it is optimal under the assumption that all assets are expected to have similar risk-adjusted returns. If this is not the case, then other explicit 'betting against beta' strategies could be designed to better exploit differences in expected Sharpe ratios.

We also show that risk parity is better suited to investment universes that are relatively concentrated, with low correlations among asset returns.

The risk parity approach has been hyped by some market participants as offering a near-guarantee of superior risk-adjusted returns; others have dismissed it as worthless. The truth lies somewhere between these extremes. The excellent performance seen in the historical evidence is the result of multiple effects mixed together. Our attribution analysis helps to disentangle these effects and demonstrate how much of the outperformance is due specifically to the risk diversification that lies at the core of RP strategies. After removing the effects of weight dynamics and re-allocation, we find that RP adds value in situations where the risk profile of the original portfolio was not well-diversified. However, investors should keep in mind that by construction, RP is primarily a risk-centric strategy, not a return-maximizing one. The re-allocation effect that has helped to boost RP strategy performance over the past two decades may not persist in the future. Certainly, interest rates cannot continue to descend too much further from their current near-zero levels. The open question, perhaps, is the persistence and prevalence of the "low-beta anomaly". In settings such as equity markets, where this phenomenon has been demonstrably persistent, investors may have reason to believe that RP will continue to provide them with a return advantage in addition to risk diversification.

We believe that the key principle driving RP portfolios – that no single risk should comprise too large a share of overall portfolio risk – is a fundamental element of prudent portfolio design. However, this purely risk-centric view should not necessarily be the main driver of portfolio overweights and underweights. Many different types of information can help project expected returns over various horizons; properly incorporating return forecasts within a risk-controlled framework can potentially offer greater promise of returns than a pure RP strategy alone. The view that low-risk assets will continue to outperform higher-risk assets is one possible view that can help guide portfolio construction; it is certainly not the only one.

Appendix: Attribution of Risk Parity Gains

We consider the example where we switch an allocation from market-weighted (MW) to risk parity (RP) and aim to attribute the associated performance gain to diversification and re-allocation effects. This attribution is provided in units of returns for a given volatility target.

Let us denote

μ_{RP} - average excess return of the RP portfolio over funding rate;

σ_{RP} - volatility of the RP portfolio excess returns;

μ_{MW} - average excess return of the MW portfolio over funding rate;

σ_{MW} - volatility of the MW portfolio excess returns;

σ^* - target volatility at which the attribution is implemented;

In order to measure the diversification and re-allocation effects, we calculate volatilities of the RP and MW portfolios under the assumption of perfect correlations but keep returns unchanged.

$\bar{\sigma}_{RP}$ - volatility of the RP portfolio returns under the perfect correlation assumption;

$\bar{\sigma}_{MW}$ - volatility of the MW portfolio under the perfect correlation assumption;

We further introduce the diversification ratio that characterizes each portfolio. This is the ratio of its volatility under the perfect correlation assumption to its realized volatility including correlations:

$$D_{RP} = \bar{\sigma}_{RP}/\sigma_{RP}$$

$$D_{MW} = \bar{\sigma}_{MW}/\sigma_{MW}$$

The diversification ratio, defined by Choueifaty and Coignard (2008), is always greater than or equal to one, and measures the amount by which diversification helps to decrease risk in a portfolio. The Sharpe ratios of the different portfolios can then be written as:

$$SR_{MW} = \mu_{MW}/\sigma_{MW}$$

$$\overline{SR}_{MW} = \mu_{MW}/\bar{\sigma}_{MW} = SR_{MW}/D_{MW}$$

$$SR_{RP} = \mu_{RP}/\sigma_{RP}$$

$$\overline{SR}_{RP} = \mu_{RP}/\bar{\sigma}_{RP} = SR_{RP}/D_{RP}$$

Let us also denote:

$\mu^*_{MW} = \sigma^* \times SR_{MW}$ - return of the leveraged MW portfolio;

$\mu^*_{RP} = \sigma^* \times SR_{RP}$ - return of the leveraged RP portfolio;

$\bar{\mu}^*_{MW} = \sigma^* \times \overline{SR}_{MW}$ - return of the leveraged “perfect correlation” MW portfolio;

$\bar{\mu}^*_{RP} = \sigma^* \times \overline{SR}_{RP}$ - return of the leveraged “perfect correlation” RP portfolio;

The overall performance gain of the RP portfolio can then be measured units of return for a given volatility target:

$$\mu_{RP}^* - \mu_{MW}^* = \sigma^*(SR_{RP} - SR_{MW})$$

$$\frac{\mu_{RP}^* - \mu_{MW}^*}{\sigma^*} = SR_{RP} - SR_{MW}$$

$$= [(SR_{RP} - \overline{SR}_{RP}) - (SR_{MW} - \overline{SR}_{MW})] + [\overline{SR}_{RP} - \overline{SR}_{MW}]$$

$$= \left[SR_{RP}\left(1 - \frac{1}{D_{RP}}\right) - SR_{MW}\left(1 - \frac{1}{D_{MW}}\right)\right] + [\overline{SR}_{RP} - \overline{SR}_{MW}]$$

$$= SR_{RP}\left(1 - \frac{1}{D_{RP}}\right) - SR_{RP}\left(1 - \frac{1}{D_{MW}}\right) + SR_{RP}\left(1 - \frac{1}{D_{MW}}\right) - SR_{MW}\left(1 - \frac{1}{D_{MW}}\right) + [\overline{SR}_{RP} - \overline{SR}_{MW}]$$

$$SR_{RP} - SR_{MW} = SR_{RP}\left(\frac{1}{D_{MW}} - \frac{1}{D_{RP}}\right) + (SR_{RP} - SR_{MW})\left(1 - \frac{1}{D_{MW}}\right) + [\overline{SR}_{RP} - \overline{SR}_{MW}]$$

$$(SR_{RP} - SR_{MW})\left[1 - \left(1 - \frac{1}{D_{MW}}\right)\right] = SR_{RP}\left(\frac{1}{D_{MW}} - \frac{1}{D_{RP}}\right) + [\overline{SR}_{RP} - \overline{SR}_{MW}]$$

$$SR_{RP} - SR_{MW} = SR_{RP}\left(1 - \frac{D_{MW}}{D_{RP}}\right) + D_{MW}(\overline{SR}_{RP} - \overline{SR}_{MW})$$

$$SR_{RP} - SR_{MW} = \frac{SR_{RP}}{D_{RP}}(D_{RP} - D_{MW}) + D_{MW}(\overline{SR}_{RP} - \overline{SR}_{MW})$$

$$SR_{RP} - SR_{MW} = \overline{SR}_{RP}(D_{RP} - D_{MW}) + D_{MW}(\overline{SR}_{RP} - \overline{SR}_{MW})$$

The improvement in Sharpe ratio achieved by switching from market weights to risk parity is thus shown to consist of two terms. The first term is called the *diversification effect* and is proportional to the extent to which the risk parity portfolio offers greater diversification than the MW portfolio. We call the second term the *re-allocation effect*. It is essentially the difference between the Sharpe ratios of the two portfolios obtained under the perfect correlation assumption. This term is multiplied by the diversification ratio of the MW portfolio to reflect the fact that even in the base case of market weights the portfolio has a diversification advantage over the perfect correlation case.

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