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Statistical Factor Modeling: Factor Hunting at an Indian Buffet

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From Genetics to Equities

We investigate a recently developed approach to statistical factor modeling known as 'Non-Parametric Sparse Factor Analysis' (NSFA) in respect to various equity markets. The probabilistic, Bayesian technique has previously been applied in fields such as gene-expression modeling. NSFA uses the 'Indian Buffet Process' (IBP), which incorporates the notion of sparsity. We see how the NSFA approach can uncover factors comprising distinct clusters of stocks.

Attribution: Countries vs Industries

Applying the NSFA technique to global, European, and US markets, we observe how the technique can identify distinct factors that are dominated by single-country or industry effects. The identification of these factors is intuitive and is particularly clear in periods of increased volatility. In turn we see how the country effect typically dominates the industry effect in global and regional markets.

Isolating Alpha

Using NSFA models in isolation or in combination with more traditional, fundamental factor models, we find that we can build portfolios that purify 'alpha' based on signals such as Momentum or Reversal. We see the resulting portfolios typically have less systematic risk and lower drawdowns but generally maintain the returns of the original alpha signals.

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Introduction

Factor-based modeling is central to many aspects of quantitative finance. Applications include the modeling and attribution of financial risk and returns, as well as portfolio construction. In this report we explore the use of '*Non-parametric Bayesian Sparse Factor Models*' (NSFA), which are a recent evolution in the field of statistical factor modeling. As we show throughout this report, 'sparsity' plays an important role in the development of advanced statistical factor models that aids the identification of market dynamics.

In the first part of the report we provide an overview of several factor-based asset pricing models. For readers who are already familiar with the topic, we would suggest reading Section: Bayesian Sparse Factor Analysis in which we introduce the key concepts associated with the NSFA model. We then present the main findings of this research, focusing on two key applications: a risk-based attribution analysis and using the new modeling approaches to isolate idiosyncratic returns to build portfolios based on momentum or reversal signals.

Single Factor Model - CAPM

One early theory, the Capital Asset Pricing Model (CAPM), states that only one force drives all stock returns – the *market portfolio*. The theory asserts the market has reached an equilibrium such that each stock is held in proportion to its market capitalization relative to the total market capitalization of all stocks (Sharpe, 1964). Despite its simplicity, the model provides important insights into the problem of asset pricing.

Separation of Risk

CAPM postulates that an individual stock is exposed to two types of risk: systematic (non-diversifiable) and unsystematic (diversifiable). Unsystematic risk is risk that is specific to a company, such as business risk and financial risk. Investors who hold many stocks largely reduce or eliminate this type of risk. By contrast, systematic risk represents the portion that links directly to movements in the general market or economy, often referred to as market risk, and which cannot be eliminated through portfolio diversification. In general, investors who bear this risk will be compensated with a return premium. We note that in the CAPM, the source of systematic risk is solely captured by the market portfolio.

Risk-return Trade-off

Under the CAPM framework, the systematic risk of an individual stock is measured as its sensitivity to the broader market, defined by the beta measure. While stocks with low betas show little movement with regard to fluctuations in the stock market, high-beta stocks generally exhibit large variations in response to a small change in the market. This relationship is consistent with the concept of the risk-return trade-off. That is, the expected return of a stock is positively related to the risk premium indicated by its beta.

However, a large number of empirical studies suggest that in addition to market beta, other variables, such as size, the ratio of book to market value, the price-to-earnings ratio, and macroeconomic variables, also help to explain cross-sectional variations of stock returns. This has led to the development of multi-factor models.

Multi-factor Models

Ross (1976) proposed the first multi-factor framework, namely the arbitrage pricing theory (APT). While CAPM has only one risk factor – the market portfolio – APT allows multiple sources of systematic risk. Each source captures a pervasive factor that impacts systematic movements of stock returns.

Using APT, a stock's excess return is expressed in terms of two components:

$$\underbrace{y_i}_{\substack{stock\\excess\\return}} = \underbrace{\sum_{j=1}^{K} x_{i,j} f_j}_{\substack{common\\factor\\return}} + \underbrace{e_i}_{\substack{specific\\return}} \quad (1)$$

where $\boldsymbol{y_i}$ is the stock excess return of stock $\boldsymbol{i}$, $\boldsymbol{x_{i,j}}$ is the exposure (factor loadings) of stock $\boldsymbol{i}$ to factor $\boldsymbol{j}$ and $\boldsymbol{f_j}$ denotes the factor return of factor $\boldsymbol{j}$. Furthermore, assuming some pre-specified risk factors, the expected return is expressed as a linear function of factor exposure and the factor forecast, such as:

$$E\{y_i\} = \sum_{j=1}^{K} x_{i,j} v_j \quad (2)$$

where $\boldsymbol{v_j}$ is the factor forecast for factor $\boldsymbol{j}$.

However, the original APT does not specify either the number or identities of these risk factors. Since its creation, many researchers have proposed different approaches in order to determine what these factors might be. As Connor (1995) concludes, multi-factor models may be divided into three types: macroeconomic, fundamental, and statistical models. The relationship among these three types of factor models is shown in Figure 1 . Each one is described here.

Figure 1: An overview of the procedures for the three types of factor models

Factor Model Type	Inputs	Estimation Technique	Outputs
Macroeconomic	Asset returns and macroeconomic variables	Time-series regression	Factor exposures
Fundamental	Asset returns and firm characteristics	Time-series or cross-sectional regression	Fundamental factor exposures or factor returns
Statistical	Asset returns	Principal Component Analysis or Factor Analysis	Both statistical factor exposures and their corresponding factor returns

Source : Deutsche Bank Quantitative Strategy, Connor (1995)

Macroeconomic Models

Macroeconomic factor models use observable economic time series to represent pervasive factors that explain stock returns. This approach typically involves the estimation of factor exposures via time-series regression of stocks returns on the time series of pre-specified macroeconomic factors. For instance, Chen et al. (1986) argues stock returns can be reasonably explained by five economic factors: inflation, industrial production, risk premium (as measured by the spread between low-grade and high-grade bonds), the term structure of the interest rate, and the market

The macroeconomic factor model uses a time-series regression to estimate unknown factor exposure from the pre-specified financial time series.

index.

In general, macroeconomic models produce a deeper and more intuitive economic explanation for return co-movements. However, the time-series regression often entails noisy estimates of factor exposures, and it requires a long and stable history of returns for an asset to accurately estimate factor exposures. Connor (1995) and Grinold and Kahn (2000) have reported their empirical explanatory power tends to be substantially lower than fundamental and statistical factor models.

Fundamental Models

Fundamental factor models make use of asset-specific attributes, such as firm size, standard accounting information, and industry classification, for modeling asset returns. In this case, two approaches are often used in practice.

1. Time-series regression

One type of fundamental model uses the times series regression to estimate unknown factor exposure from factor returns derived using firm characteristics.

The first approach, pioneered by Fama and French (1996), considers a time-series regression for building fundamental factor models. This method extends the original CAPM into a three-factor model in which the market portfolio and two additional firm characteristics – book-to-price (value) and market capitalization (size) – are included.

The model includes two stages. In the first stage, stocks are cross-sectionally sorted into fractile portfolios based on value and size characteristics. The differences between returns on the top and bottom fractile portfolios are then used to calculate factor returns. The model also includes the factor return of the market portfolio as defined in the CAPM. In the second stage, the factor exposures of each asset are estimated via a time-series regression of asset returns on the derived factor returns.

This 3-factor model has become very influential in the asset pricing literature. However, many researchers have argued that other factors such as momentum, low volatility, profitability, and investment are needed to accurately explain the common factor returns (Carhart, 1997; Fama and French, 2015).

2. Cross-sectional regression

Another type of fundamental model uses the cross-sectional regression to estimate unknown factor returns from a set of prespecified asset descriptors.

The second approach uses prespecified asset descriptors as the pervasive factors and employs a cross-sectional regression to estimate unknown factor returns. These descriptors can be accounting ratios as suggested by Rosenberg (1974) or Industry/Country dummies as proposed by Heston and Rouwenhorst (1994).

Similar to the macroeconomic models, this type of fundamental model often provides intuitive explanations of the asset returns. However, given the factor structure is defined *a priori*, it may fail to capture some transient factors that can emerge in different market regimes.

Statistical Models

In statistical factor models, both factor exposures and factor returns are not directly observable and must be estimated from a panel of asset returns using statistical techniques. Commonly used techniques include Principle Component Analysis (PCA) and Factor Analysis (FA). Each one is described here.

1. Principle Component Analysis

In PCA, we deem principal components and eigenvectors as factor returns and factor exposures, respectively.

Ordinary PCA aims to project the observable data onto a lower dimensional linear space, known as the *principal subspace*, while preserving as much information as possible. In other words, the objective is to find an orthogonal projection such that the variance of the projected data is maximized[1] (Hotelling,1933; Bishop, 2007). The solution to this problem turns out to be the eigenvectors of the sample covariance matrix. For each eigenvector, the corresponding projection of the data is known as the *principal component,* defined as: (Zivot, 2011)

$$\begin{aligned} f_{1n} &= e_1' Y_n = e_{11} Y_{1n} + \ldots + e_{1D} Y_{Dn} \\ f_{2n} &= e_2' Y_n = e_{21} Y_{1n} + \ldots + e_{2D} Y_{Dn} \\ &\vdots \\ f_{Dn} &= e_D' Y_n = e_{N1} Y_{1n} + \ldots + e_{DD} Y_{Dn} \end{aligned} \qquad for\ n = 1 \ldots N \qquad (3)$$

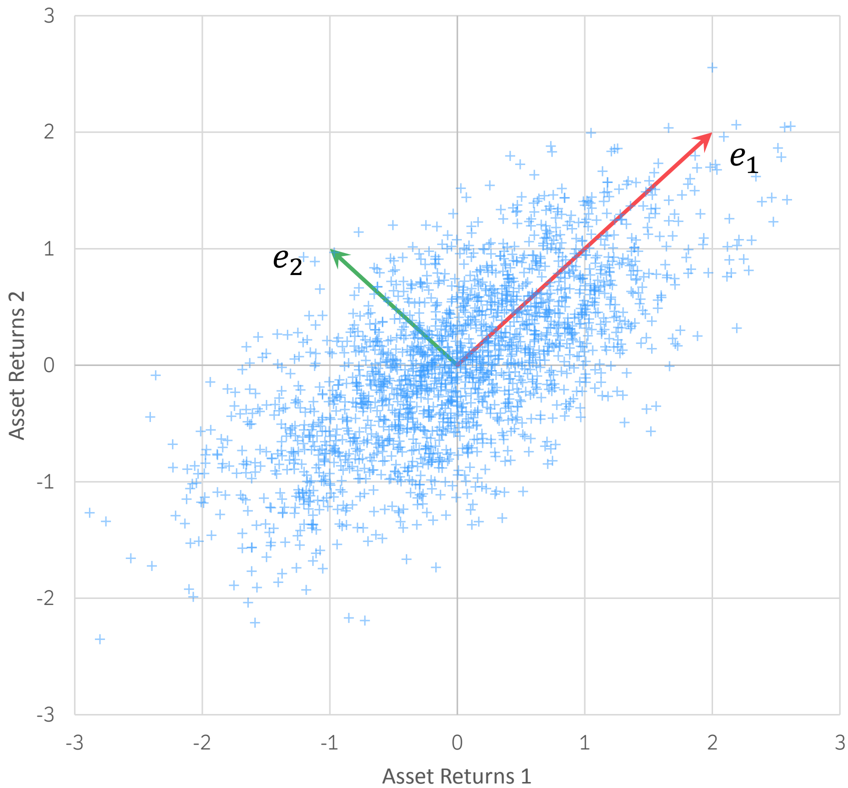
where $\boldsymbol{Y_n}$ denotes a vector of asset returns of size $\boldsymbol{D \times 1}$ at the time $\boldsymbol{n}$. $\boldsymbol{D}$ is the number of assets and $\boldsymbol{N}$ represents the time periods [2] . $\boldsymbol{e_1, e_2 \ldots e_D}$ denote the eigenvectors of the sample covariance matrix sorted by their eigenvalues. The principal components $\boldsymbol{f_{1n}, f_{2n} \ldots f_{Dn}}$ are those uncorrelated factor returns which are constructed and ordered such that $\boldsymbol{f_{1n}}$ explains the largest portion of variances in the sample covariance matrix, and $\boldsymbol{f_{2n}}$ explains the next largest portion, and so on.

An example of an eigenvector analysis on two simulated asset returns is illustrated in Figure 2 . It can be seen that the maximum variance of two asset returns is aligned with the first eigenvector which is represented with the red axis. The remaining variance is explained by the second eigenvector, which is illustrated with the green axis, and is orthogonal to the first one.

In terms of a multi-factor model, we can deem principal components and eigenvectors as factor returns and factor exposures, respectively. However, how many principal components are needed to capture the common factor returns remains a difficult problem. Typically, some model selection techniques such as Akaike information criterion (AIC) or Bayesian information criterion (BIC) are used to choose an appropriate number of factors.

1 Equivalently, the objective function can also be defined as minimizing the mean squared distance between the data points and their projections.
2 Suppose Y is the D × N matrix of observed returns. When D < N, ordinary PCA is typically performed based on the D x D sample covariance matrix. However, when N < D, asymptotic PCA is often used based on an eigenvector analysis of the N x N covariance matrix.

Figure 2: Eigenvector analysis on two synthetic asset returns



Source : Deutsche Bank Quantitative Strategy

2. Factor Analysis (FA)

In FA, both factor exposures and factor returns are determined jointly from the asset returns.

Factor analysis is a dimension reduction technique that finds a small set of latent variables to represent the observed data. Recall ***Y*** of size ***D x N*** represents the observed data. Traditional factor analysis aims to determine a time-invariant factor-loading matrix such that the observed data can be represented as:

$$\underbrace{Y}_{(D \text{ x } N)} = \underbrace{X}_{(D \text{ x } K)} \underbrace{F}_{(K \text{ x } N)} + \underbrace{e}_{D \text{ x } N} \quad (4)$$

where ***X*** denotes the factor-loading matrix, ***F*** represents the factor returns and ***e*** represents the noise components. Each column of ***X*** represents the factor exposure corresponding to each of ***K*** latent variables. Traditionally, we model factor analysis as a linear Gaussian latent variable model, such that all random variables follow a Gaussian distribution,

- ***X ~ N(0,I)***
- ***e ~ N(0,Ψ)***
- ***Y ~ N(0,XX' +Ψ)***

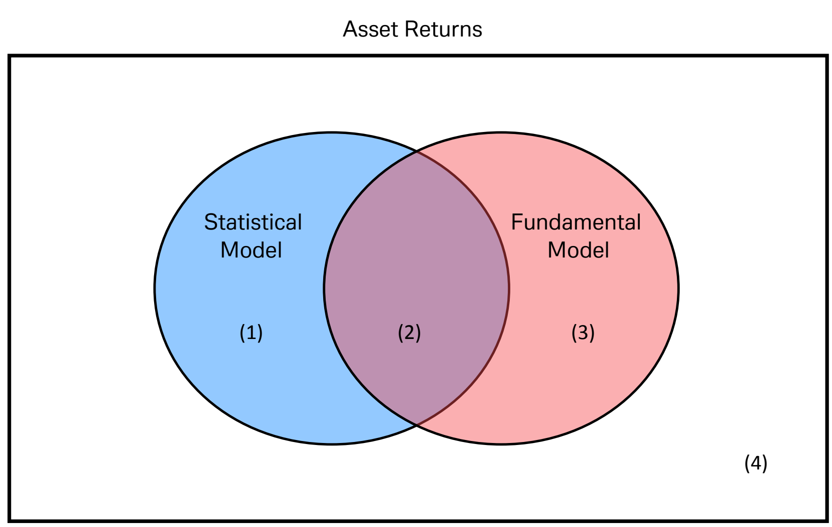
where ***Ψ*** represents the noise covariance matrix of size ***D x D***. In factor analysis, the columns of $\boldsymbol{X}$ capture the correlations between observed variables, and the diagonal elements of $\boldsymbol{\Psi}$ represent the independent variances for each of the variables (Bishop, 2007). Interestingly, Tipping and Bishop (1999) show that ordinary PCA could be derived from this probabilistic model when the noise covariance $\boldsymbol{\Psi} \rightarrow \boldsymbol{0}$. Because factor analysis is a latent variable model, both factor-loading matrix $\boldsymbol{X}$ and factor returns $\boldsymbol{F}$ can therefore be easily determined using an expectation–maximization (EM) algorithm (Roweis, 1997). Similar to PCA, model selection techniques are used to determine the number of latent factors $\boldsymbol{K}$ in factor analysis.

In contrast to ordinary PCA, factor analysis is fundamentally different in two aspects. First, ordinary PCA does not consider the noise term $\boldsymbol{e}$, while factor analysis allows the noise to be modeled explicitly. Second, ordinary PCA concentrates on explaining the diagonal elements of the sample covariance matrix while factor analysis aims to explain the off-diagonal elements of the sample covariance matrix by a small number of factors (Jolliffe, 2002).

Comparison of Fundamental and Statistical Models

In practice, risk model providers such as Barra and Axioma have developed commercially available products using both fundamental and statistical models. In terms of the explanatory power, Connor (1995) shows that both fundamental and statistical models are similar for asset returns. However, each type of model offers different and alternative insights into market dynamics. As illustrated in Figure 3 , each approach can capture a different segment of the asset returns in the factor space. Applications of a combination of fundamental and statistical models are explored later in this report.

Figure 3: Asset returns attribution with different factor model approaches



(1) Returns captured by statistical factors, not by fundamental factors
(2) Returns captured by both statistical, and fundamental factors
(3) Returns captured by fundamental factors, not by statistical factors
(4) Specific returns not explained by either statistical or fundamental factors

Source : Deutsche Bank Quantitative Strategy

Bayesian Sparse Factor Analysis

To date, statistical factor models have been widely used to uncover latent sources of risk and returns in quantitative finance. It is also a standard approach found in quant cookbooks when fundamental factors are unobservable or not easily identifiable (e.g., when modeling alternative asset classes). However, two major drawbacks are known to be associated with the conventional statistical models. First, the conventional statistical models lack of sparsity, which could limit the ability of the model to capture signals with 'membership' type criteria (e.g., a stock either has exposure to a country factor, or it does not). Second, it is difficult to disentangle the idiosyncratic risk from the systematic risk without an explicit cut-off (e.g., specified number of features in a PCA model). Given these limitations, researchers have shown an increased interest in sparse factor models with an automatic determination in the number of factors.

Sparse Factor Models

Sparsity plays a critical role in modern statistics and machine learning. A sparse statistical model aims to recover the underlying signal using a small number of non-zero parameters or weights (Hastie et al., 2015). This often improves the tractability and generalization of learning algorithms. With the advent of the big-data era, the sparsity assumption allows us to extract useful and reproducible patterns from big datasets.

A sparse statistical model aims to recover the underlying signal using a small number of nonzero parameters or weights.

Sparse factor models are relatively new techniques in quantitative finance and their applications to asset pricing have not been widely explored. However, sparsity offers the potential to identify effects similar to country or industry factors in a fundamental model, where an asset would typically have zero or non-zero exposure. Sparsity could also aid factor attribution and interpretation.

Zou et al. (2006) proposed an extension to ordinary PCA in which Lasso[3] is used to produce modified principal components with sparse loadings. They argue that enforcing sparsity offers several advantages, such as computational efficiency and an ability to identify important variables.

In the case of factor analysis, a sparsity constraint is often enforced on the number of non-zero entries permitted in the factor-loading matrix. However, imposing sparsity constraints on the factor loadings is not a straightforward task. Recently, researchers have shown that Bayesian Statistics can be a powerful framework for solving this problem.

Bayesian Modeling

As we have seen in the previous subsections, factor analysis uses a probabilistic modeling approach to extract pervasive factors. This type of modeling approach is directly related to the Bayesian framework. At the core of Bayesian statistics is Bayes' theorem, which takes the form

$$p(\theta|Y) = \frac{p(Y|\theta)p(\theta)}{p(Y)} \qquad (5)$$

$$\text{posterior} = \frac{\text{likelihood} \times \text{prior}}{\text{marginal likelihood}}$$

where observations ***Y*** are generated from a model with parameters ***Θ***, ***p(Y|Θ)*** is the

3 short for 'Least Absolute Shrinkage and Selection Operator'

likelihood function, $p(\Theta)$ is the prior and $p(Y)$ is the marginal likelihood. The effect of the observed data Y is conditional on the model parameters Θ through the conditional probability $p(Y|\Theta)$, which can be viewed as the likelihood function. The likelihood function is often used to generate estimators such as the maximum likelihood estimator. Parameters Θ have a prior $p(\Theta)$, which summarizes all knowledge of the parameters prior to the observed data. $p(Y)$ is the normalization constant that ensures the sum of $p(\Theta|Y)$ over all values of Θ equaling one. Bayes' theorem allows us to evaluate the uncertainty in Θ after we have observed data in the form of the posterior probability $p(\Theta|Y)$ (Bishop, 2007).

The empirical success of Bayesian techniques has inspired the discovery of new sparse factor models such as Sparse Probabilistic PCA (Guan and Dy, 2009), Dense Message Passing (Sharp and Rattray, 2010), Empirical Bayes Matrix Factorization (Wang and Stephens, 2018), Bayesian group factor analysis (Virtanen et al., 2012) and Nonparametric Bayesian Sparse Factor Models (NSFA) (Knowles and Ghahramani, 2011).

NSFA is a recent development proposed by Knowles and Ghahramani (2011). State-of-the-art performances were previously achieved when applying this model to gene expression data. By drawing on the concept of the Indian Buffet Process (discussed below), NSFA automatically selects the latent dimensionality of the factor space. This property is useful as we do not need significant manual tuning or prior knowledge about the latent structures. Being a Bayesian model, NSFA is also robust against overfitting as it allows an efficient inference without explicit assumptions on the parameters.

Parametric and Non-parametric Models

To understand NSFA, it is important to distinguish the differences between parametric and non-parametric models (Ghahramani, 2012).

A parametric model has a finite set of parameters that are assumed to capture everything there is to know about the data. These parameters are bounded even when the amount of observed data becomes unbounded. This makes the parametric model not very flexible, in general.

In contrast, a non-parametric model assumes the data distribution can be defined in terms of infinitely many parameters that are represented by a function. The complexity of the model grows as the amount of observed data increases. Predictions from a non-parametric model are memory-based, meaning that it is required to store or remember a growing amount of information about training data.

NSFA is a non-parametric model because the number of factors varies as the amount of observed data increases. In practice, many influential methods in machine learning are non-parametric, such as k-Nearest Neighbors, Random Forests, RBF kernel Support Vector Machines and Gaussian Process.

Non-parametric Bayesian Sparse Factor Models (NSFA)

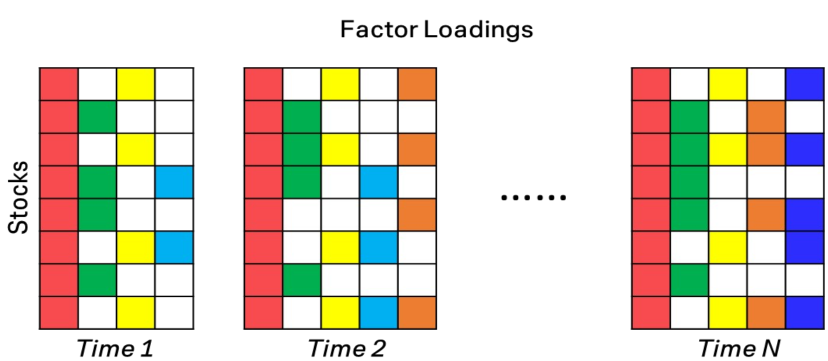
Note that in this research, we choose NSFA for building statistical factor models. We refer readers to Appendix A for a concrete review on the model specifications. Here, we summarize the rationale behind choosing this model in relation to asset pricing.

NSFA is a non-parametric Bayesian extension of factor analysis. The main idea comes from a stochastic process, called the Indian Buffet Process (IBP). As described by Griffiths and Ghahramani (2011), it defines a probabilistic distribution on sparse binary matrices with a finite number of rows (objects) and an infinite number of columns (features). In recent years, IBP has attracted considerable attention in the machine learning community and demonstrated a great number of practical applications such as sparse factor analysis, matrix factorization, feature extraction and graphical modeling. Without fixing the number of features, IBP can efficiently learn the latent representations as the observed data evolves over time.

In general, we can better understand IBP by considering the following scenarios. Imagine there is a finite number of customers entering an Indian buffet restaurant one after the other. Each customer can choose from infinitely many dishes and stop once the customer's plate becomes overburdened. Dishes are chosen in proportion to their historical popularity and each new customer may potentially try new dishes. We can use a binary matrix $\boldsymbol{Z}$ with $\boldsymbol{D}$ rows and an infinite number of columns to indicate which customers chose which dishes, where $\boldsymbol{Z}_{d,k} = 1$ if the d^{th} customer chose the k^{th} dish.

This conceptual framework provides great insights for understanding statistical factor models from a new perspective. For the purpose of asset pricing, we can assume that every stock is exposed to some systematic factors that are not fixed and are likely to evolve over time when new information about the market becomes available. This can be shown schematically in Figure 4 where each color represents a latent factor that captures a distinct co-movement among stocks. As such, we prefer a statistical model that is flexible enough to calibrate itself not only to the existing set of factors, but also to potential new sources of risk. For this reason, NSFA is suitable to capture the essence of market behaviour.

Figure 4: factor-loading matrices learned by NSFA



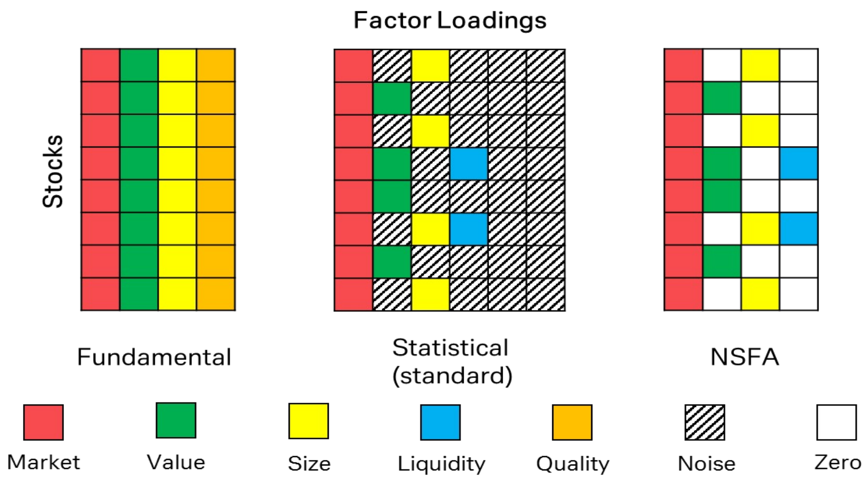
Source : Deutsche Bank Quantitative Strategy

Comparison with other factor models

A schematic representation of the factor-loading exposures obtained from different factor models is illustrated in Figure 5 . In this plot, each row corresponds to one stock and each column represents one determined systematic factor.

For fundamental factor models, the systematic factors are assumed to be market, value, size and quality. For standard statistical models, we assume that there are six latent factors to be extracted. For NSFA, we do not make any prior assumptions. Elements are color coded when stocks have exposures to certain factors.

Figure 5: Factor-loading attributions with different model approaches



Source : Deutsche Bank Quantitative Strategy

This plot demonstrates several key points:

- NSFA identifies fewer non-zero elements in the factor-loading matrix due to the sparsity constraints;
- In contrast to the standard statistical models, NSFA automatically identifies key factors while suppressing the noise to zero;
- NSFA factors might capture other co-movements (e.g., liquidity) while fundamental factors do not capture these;
- Similar to the standard statistical models, some factors (e.g., quality) are defined in the fundamental models but might not be captured by NSFA.

Contributions

We make two noteworthy contributions through this work. First, this work contributes to existing knowledge of statistical factor models by studying an advanced statistical algorithm, called the Non-parametric Bayesian Sparse Factor Models (NSFA), which proves useful in expanding our understanding of how statistical factors can be interpreted. Second, we propose two methods for constructing residual momentum and reversal strategies based on a combination of statistical and fundamental factors.

The rest of the report proceeds as follows: Section: Risk Attribution begins by analyzing a hypothetical portfolio, and looks at how NSFA factors can be attributed to traditional fundamental factors. Section: Residual Momentum and Reversal Strategies presents the methodology and results of our new residual momentum and reversal strategies. The final section summarises the main findings of this report.

Risk Attribution

In this section we perform an initial analysis with the NSFA approach on a carefully selected global universe of stocks. We first focus on the nature of factors uncovered by the model, attributing the resulting factor portfolios using a fundamental risk model.

Our analysis reveals that the NSFA factors are relatively distinct and easy to interpret over time. Our results are also in line with the finding of Heston and Rouwenhorst (1994) that country effects tend to dominate industry effects in explaining variations in stock returns. Futhermore, we show that the NSFA model identifies distinct stock co-movements in the United Kingdom.

Methodology

We first build a 'universe' of stocks in which around 400 stocks are selected from the BMI-World universe (approximately 8000 stocks). The stocks are then selected according to their country and sector classifications, ensuring that every country or sector has the same amount of stocks (i.e., 40 stocks). Our sample period covers the period November 1998 to December 2018.

This is achieved by solving a linear programming problem, where $\boldsymbol{n_{C_iS_j}}$ *denotes the number of stocks in Sector* ***j*** *of Country* ***i.***

$$\max \sum_{i=1}^{10} \sum_{j=1}^{10} n_{C_i S_j}$$

$$\text{subject to } \sum_{i=1}^{10} n_{C_i S_j} \leq 40, j = 1, 2, 3, \ldots, 10$$

$$\sum_{j=1}^{10} n_{C_i S_j} \leq 40, i = 1, 2, 3, \ldots, 10$$

Constructing a universe with stocks that are equally dispersed across sectors and countries helps to reduce excessive correlations across the universe around a single country or industry, and thus helps provide insight into the NSFA algorithm's performance in uncovering latent signals. Ten countries and sectors are used:

- **Countries (10)**: United States, United Kingdom, Japan, Hong Kong, Germany, France, Switzerland, Canada, Italy, Australia
- **Sectors (10)**: Industrials, Consumer Discretionary, Financials, Information Technology, Materials, Health Care, Telecommunication, Consumer Staples, Energy, Utilities

We note that the sector affiliations are defined based on the Global Industry Classification Standard (GICS). Figure 6 presents an example of how the selection criteria is used. However, in some cases in the deeper history, there may be less than 40 stocks available for a given bucket.

Figure 6: Number of selected stocks in each bucket (Dec, 2018)

	Consumer Staples	Industrials	Consumer Discretionary	Financials	Health Care	Materials	Information Technology	Telecommunication Services	Energy	Utilities	Total
United States	40										40
United Kingdom		40									40
Japan			22						2	16	40
Hong Kong			18	11			3	4		4	40
Germany				4	13	12	8			3	40
France					4	7	10	13		6	40
Switzerland				12	12	8	7	1			40
Canada							7	10	23		40
Italy				13	3	3	2	6	5	8	40
Australia					8	10	3	6	10	3	40
Total	40	40	40	40	40	40	40	40	40	40	

Source : Deutsche Bank Quantitative Strategy

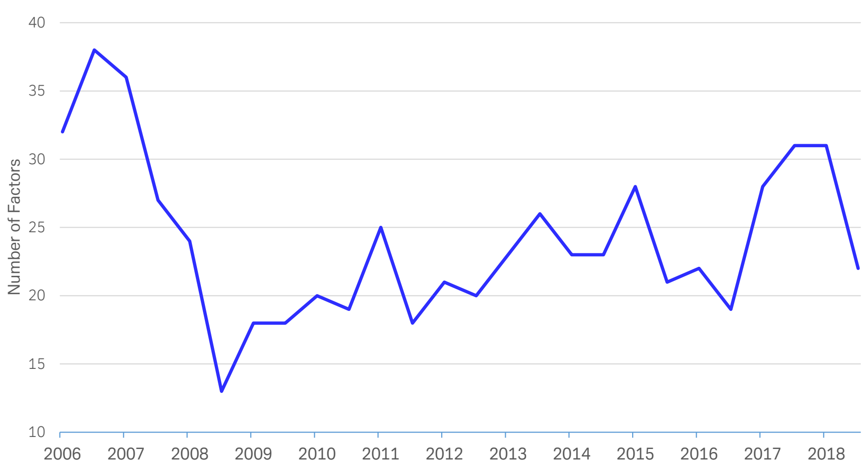
Factor Learning

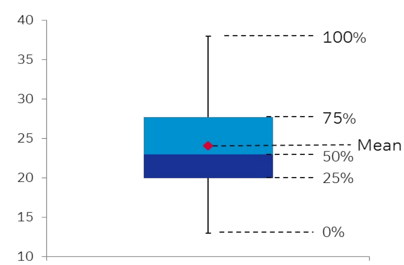
For every six months starting from June 2006, the following steps are taken to extract the statistical factors[4] :

1. Use linear programming to indicate how many stocks are required in each bucket;
2. Select stocks based on their market capitalization from high to low;
3. Compute two days' returns using the stock prices of the past 2000 days to account for the non-synchronous trading effects between different time zones;
4. Compute standardized scores for each selected return time series;
5. Stack standardized asset returns into a matrix and use the NSFA model to extract the factor loadings.

As shown in Figure 7 , the number of latent factors identified by the NSFA model has varied over time, with an average of 24 factors in this equity universe.

Figure 7: Number of identified NSFA factors over time





Source : Deutsche Bank Quantitative Strategy, Factset, Axioma

Fundamental Risk Model

In this study, we have defined a set of fundamental factors based on the Axioma global risk model in which factor exposures, covariance matrices and asset-specific risks are provided. As shown in Figure 8 , this fundamental risk model includes one market factor and nine style factors, in addition to the country and sector classifications. Note that the market factor represents the regression intercept term that captures the marketwise co-movements. In terms of how the model is built, please refer to its model document (Axioma, 2011) for more detailed information.

4 Note that we have only included stocks with a complete price history so that we do not make any prior assumptions for the missing values. In doing so, survivorship bias is likely to be present in the observed data. However, since we are only interested in assessing the risk contributions in this study, we believe this bias has little impact on our findings.

Figure 8: Market and style factors included in the Axioma fundamental model

Type	Factors	Descriptions
Market	Market	Regression intercept term; all assets have unit exposure. Allows the model to better distinguish between country and industry risk contribution effects
Style	Exchange Rate	Sensitivity 6 month beta to returns of currency basket containing USD, EUR, GBP, JPY
	Growth	Sustainable growth rate, historical earnings growth, historical sales growth
	Leverage	Debt-to-assets ratio
	Liquidity	1 month average daily volume over market capitalization
	Medium-Term Momentum	Cumulative return over past year excluding most recent month
	Short-Term Momentum	Cumulative return over past month
	Size	Natural logarithm of total issuer market capitalization
	Value	Book-to-price ratio, earnings-to-price ratio
	Volatility	3 month average of absolute return over cross-sectional standard deviation

Source : Axioma, Deutsche Bank Quantitative Strategy

Risk Contribution

As illustrated earlier in Figure 3 , the common factor returns explained by the statistical factor model can be broken down into two components:

- Returns captured by statistical factors but not by fundamental factors
- Returns captured by both statistical and fundamental factors

For returns captured by both statistical and fundamental factors, it is feasible to perform an attribution analysis such that the portfolio risk captured by the statistical factors is represented linearly by a set of pre-specified, fundamental factors. To quantify the corresponding risk contribution, we adopt a very useful framework - the '*x-sigma-rho*' method proposed by Davis and Menchero (2010). In this framework, the factor risk contribution is quantified as:

$$\sigma(R) = \underbrace{\sum_k x_k \sigma(f_k)\rho(f_k, R)}_{\substack{factor\ risk\\ contribution}} \quad (6)$$

x-sigma-rho formula identifies three intuitive drivers of risk: source exposure, source volatility and source correlation with the portfolio.

where:

- $\boldsymbol{x_k}$ is the portfolio exposure to fundamental factor ***k.***
- $\boldsymbol{\sigma(f_k)}$ is the volatility of factor ***k.***
- $\boldsymbol{\rho(f_k, R)}$ is the correlation between the factor return and the active portfolio return.

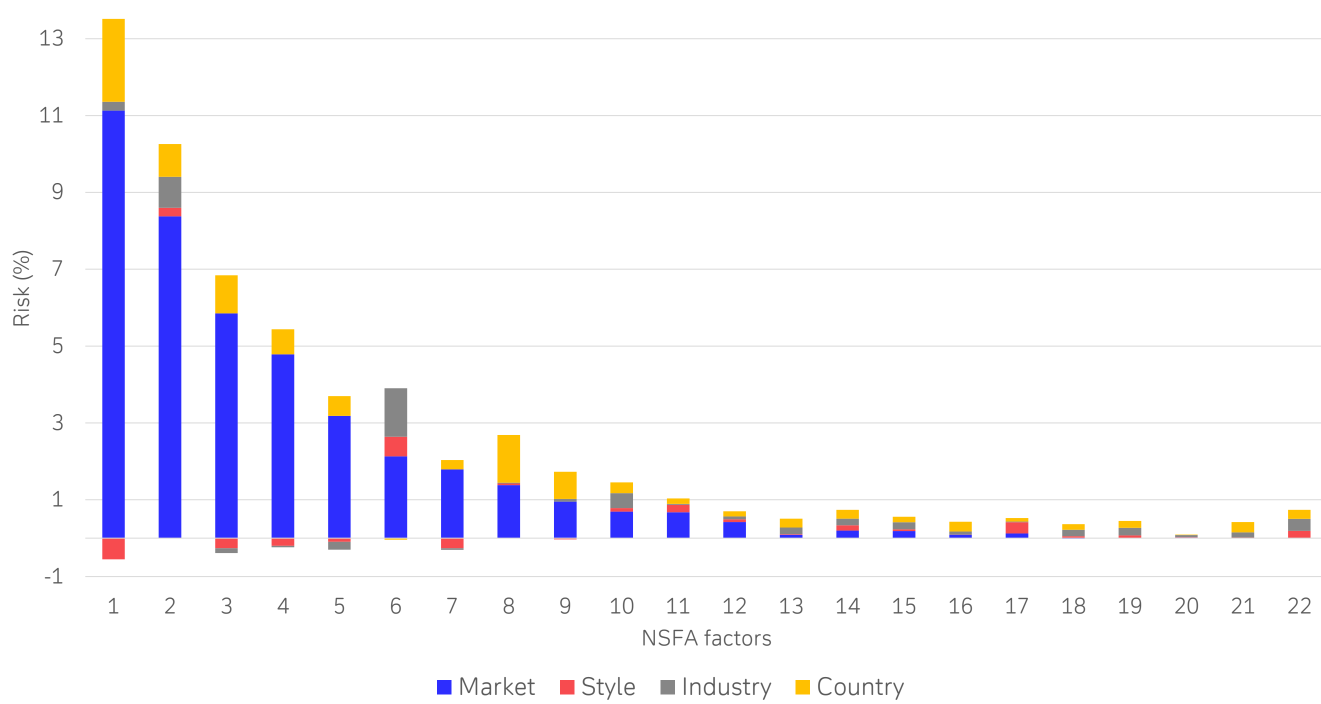
Results

In this subsection, we use the '*x-sigma-rho*' method to analyze the risk profile of NSFA factors. More specifically, we treat each NSFA factor as an active portfolio where factor exposures are the portfolio weights. Based on the '*x-sigma-rho*' formula, we can thus show what portion of the NSFA factors is explained by the fundamental factor exposures.

Hypothetical Portfolio

For example, the NSFA model learns 22 latent factors as of December 2018. For each of the 22 NSFA factors, the total risk contribution is broken down by different types of fundamental factors as shown in Figure 9 . By sorting the NSFA factors using their corresponding factor variances, we see a rapid decay in the total risk contribution of each NSFA factor. It is apparent that this decay is driven by a large decline in the risk contributions of the 'Market' factor.

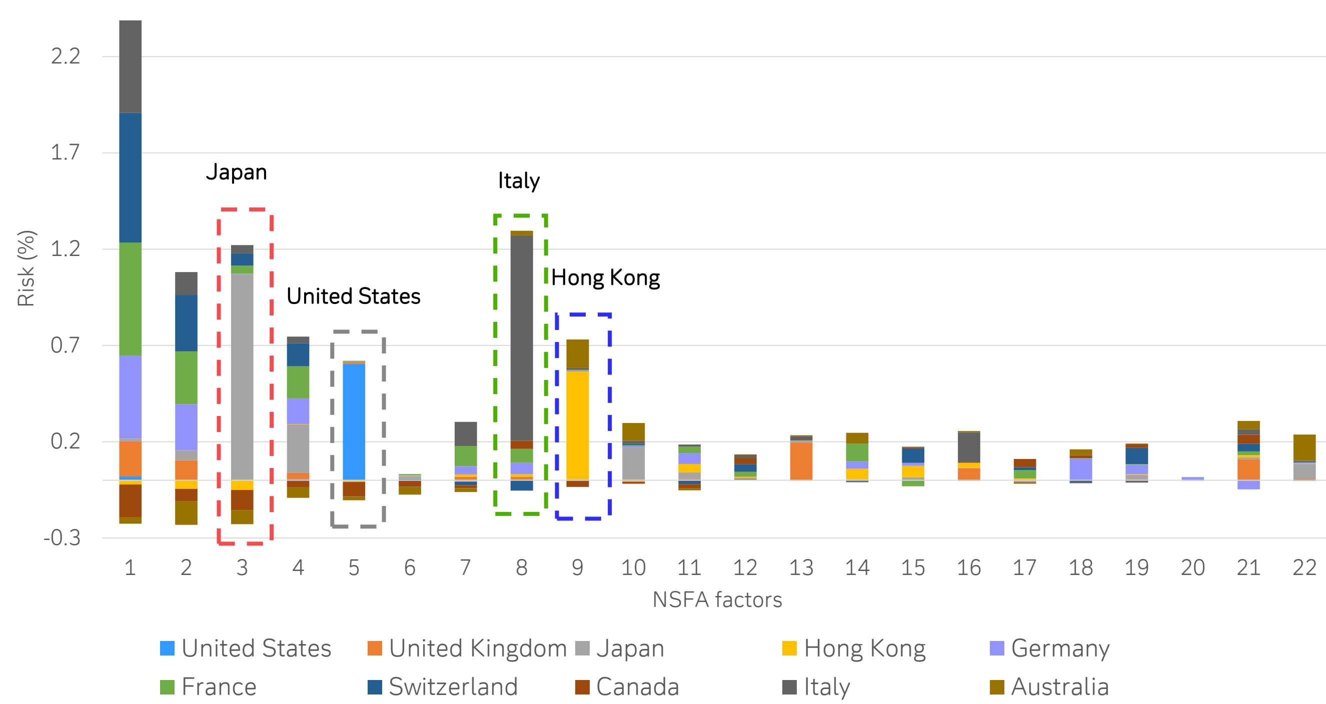
Figure 9: Risk contributions by different categories of fundamental factors (Dec, 2018)



Source : Deutsche Bank Quantitative Strategy, FactSet, Axioma

In Figure 10 , we focus on the risk contributions by country. As we can see from the chart, some of the NSFA factors have a high proportion of risk coming from a single country. For instance, factor 3, factor 5, factor 8 and factor 9 have a high percentage of risk contribution from Japan, the United States, Italy, and Hong Kong, respectively.

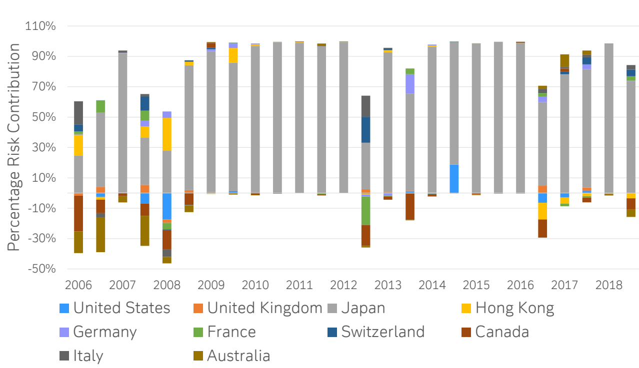
Figure 10: Risk contributions by country classification (Dec, 2018)

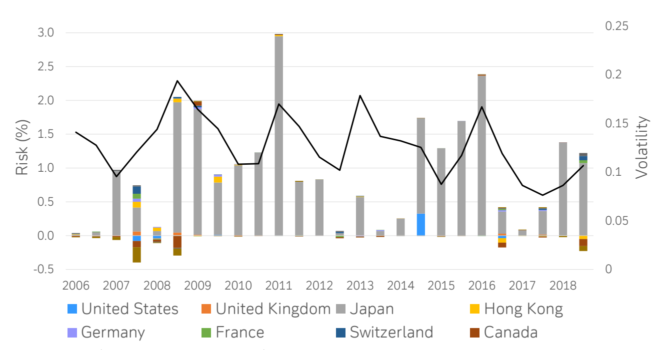


Source : Deutsche Bank Quantitative Strategy, FactSet, Axioma

To investigate whether this phenomenon is consistent over time, we select one NSFA factor at each rebalance date where the 'Japan' factor has the highest percentage risk contribution of all countries, as shown in Figure 11 . It is apparent from this chart that the NSFA model consistently identifies an individual factor where the 'Japan' factor dominates the risk contribution in respect to the countries. Interestingly, the same conclusion can also be drawn for the 'Hong Kong' factor as shown in Figure 12 . In both cases, we can see that the risk contribution from the respective country is consistent with the factor volatility as measured with a fundamental factor model (right side).

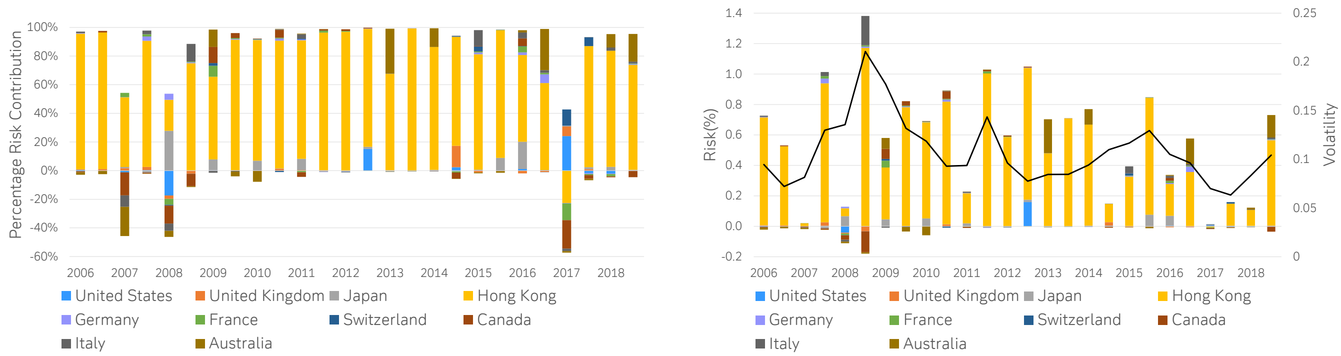
Figure 11: NSFA factors with Japan being the dominant risk contribution





Source : Deutsche Bank Quantitative Strategy, FactSet, Axioma

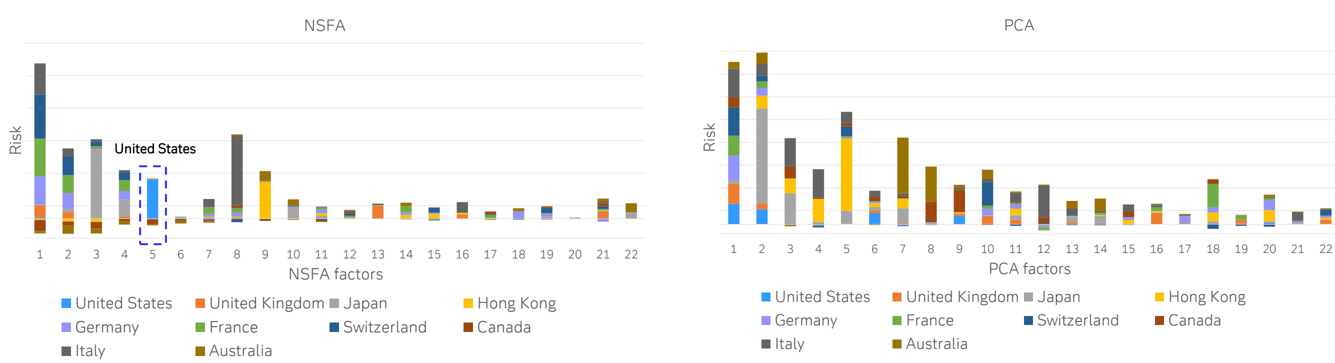
Figure 12: NSFA factors with Hong Kong being the dominant risk contribution



Source : Deutsche Bank Quantitative Strategy, FactSet, Axioma

Figure 13 shows the comparison between PCA and NSFA factors in terms of the risk contributions by country classification as of December 2018. We see the NSFA factors offer clearer interpretability when compared to the PCA factors. For instance, ordinary PCA fails to identify the United States as a distinct factor, whereas NSFA identifies a factor (factor 5) which has dominant risk contributions from the United States.

Figure 13: Comparison between NSFA and PCA factors (Dec, 2018)

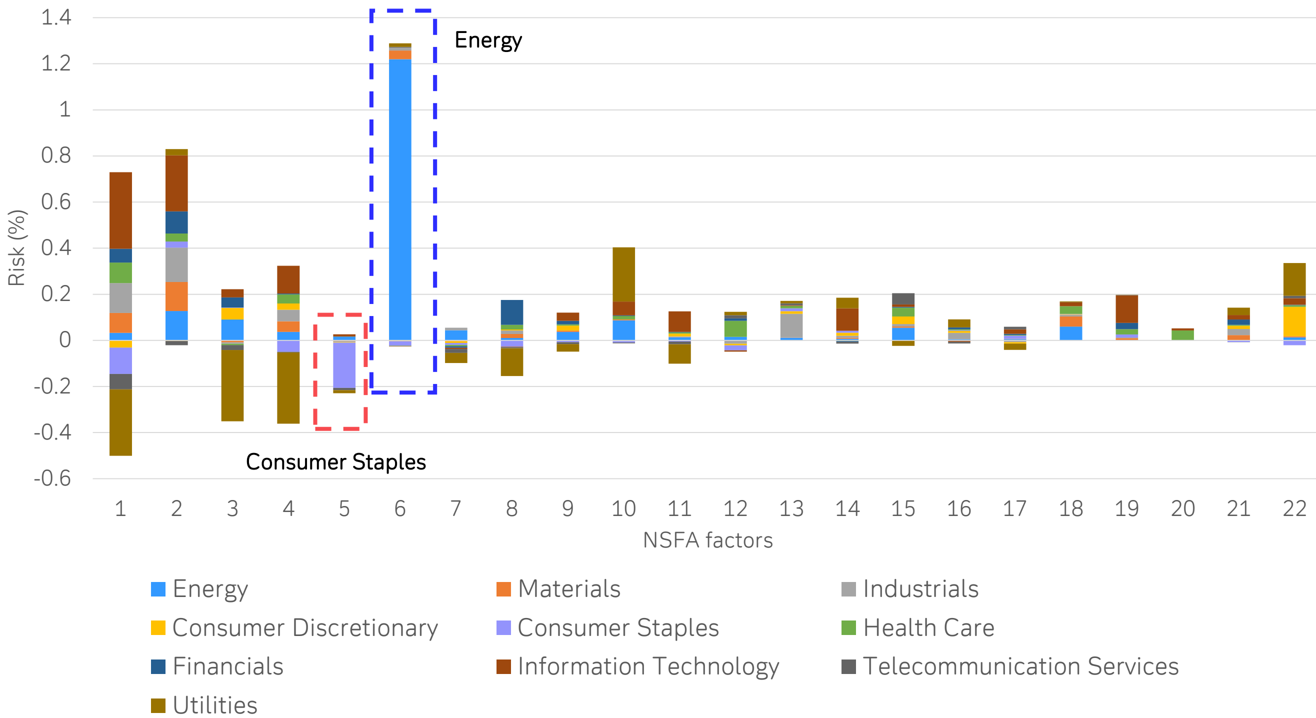


Source :Deutsche Bank Quantitative Strategy, FactSet, Axioma

In Figure 14 we see the risk contribution in terms of sector classification. We observe that some NSFA factors have a high percentage of risk contributions from a single sector. For example, factor 5 and factor 6 have dominant risk contributions from the Consumer Staples and Energy sectors, respectively. Combining with the risk contributions by country classification (Figure 10), we see factor 5 is exclusively exposed to the Consumer Staples sector in the United States. This is particularly interesting as it uncovers the fact that only the Consumer Staples sector is chosen for the United States in this dataset, as shown in Figure 6 .

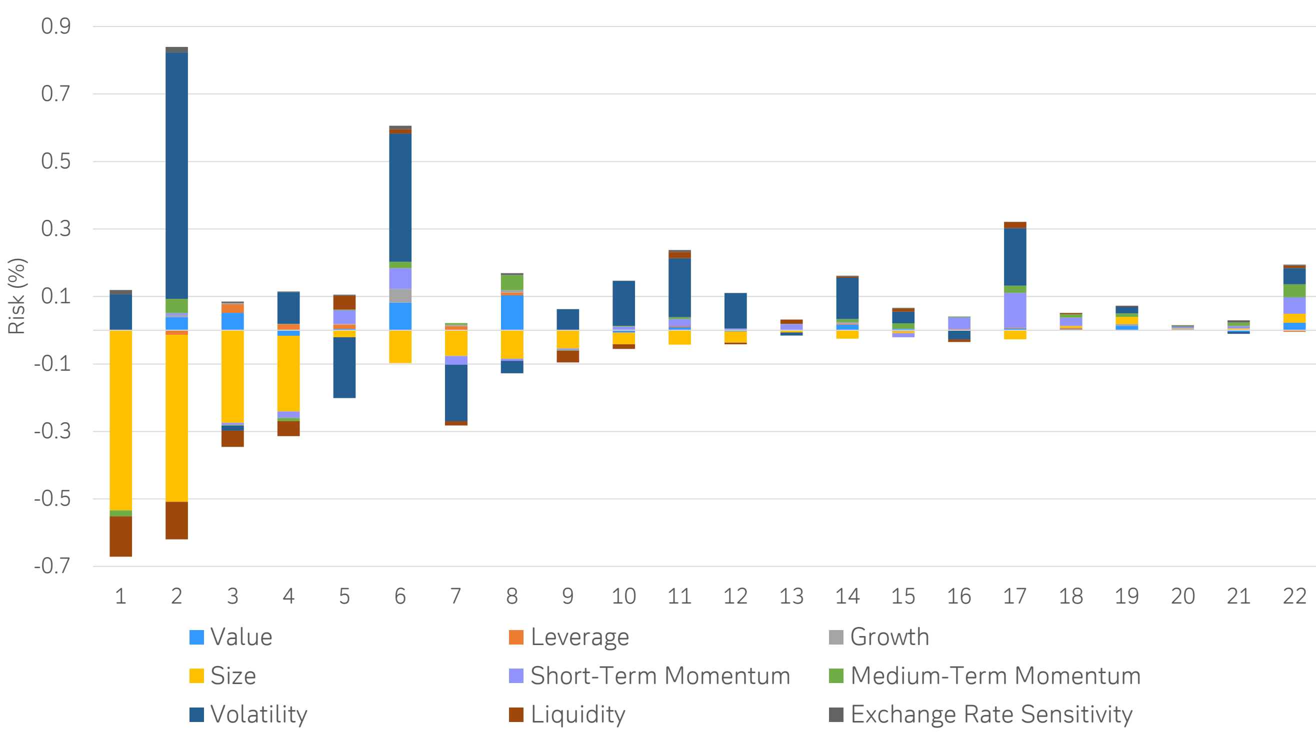
Figure 15 illustrates the risk contributions broken down by different style factors. We see that most NSFA factors are positively exposed to the 'Volatility' factor, but negatively exposed to the 'Size' factor for this dataset.

Figure 14: Risk contributions by sector classification (Dec, 2018)



Source : Deutsche Bank Quantitative Strategy, FactSet, Axioma

Figure 15: Risk contributions by style factors (Dec, 2018)

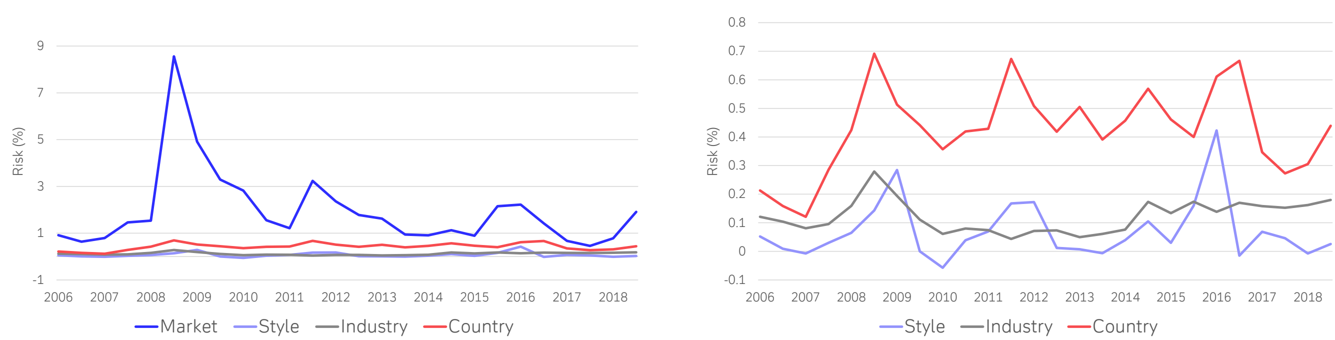


Source : Deutsche Bank Quantitative Strategy, FactSet, Axioma

Country effects dominate industry effects in explaining variations in stock returns.

Proceeding further, Figure 16 shows the average risk contribution within each of the fundamental categories over time. It can be seen that the 'Market' factor has the highest risk contribution, and that the 'Country' factor has dominated both 'Industry' and 'Style' factors in this balanced portfolio. The importance of country versus industry on asset returns has been the subject of intense debate within the scholarly community. Our results support the argument that country effects dominate industry effects in explaining variations in stock returns, as suggested by Heston and Rouwenhorst (1994).

Figure 16: Average risk contributions over time (hypothetical portfolio)



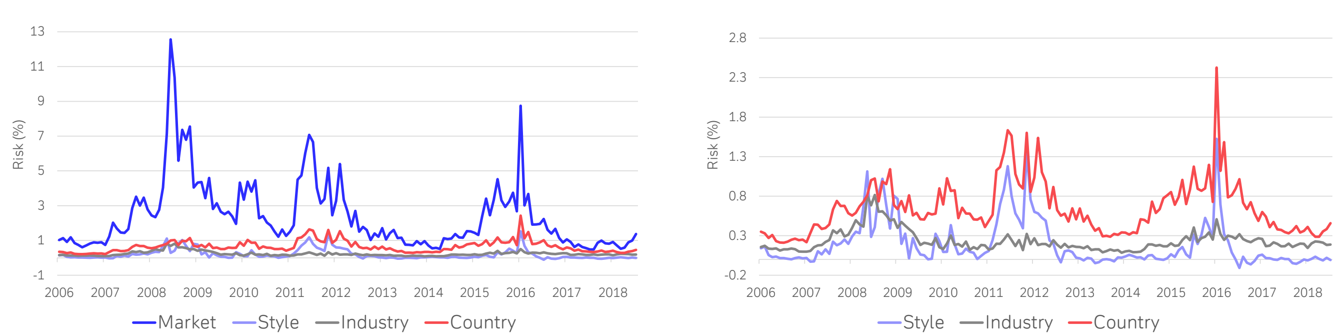
Source : Deutsche Bank Quantitative Strategy, FactSet, Axioma

MSCI-Europe Universe

Following the attribution analysis on this hypothetical universe, we now draw our attention to one commonly traded universe, determined by the 'MSCI Europe' index. To extract the NSFA factors, we use the stock returns of the past 1000 days and adopt a similar learning procedure as before. However, we avoid survivorship bias by including stocks that have missing values.

Figure 17 shows the average risk contributions for the 'MSCI Europe' portfolio over time. Similar to the results for the hypothetical portfolio, we observe that the 'Market' factor has the highest risk contribution, particularly during periods of economic stress (e.g., 2008 Financial Crisis, 2011 European Sovereign Debt Crisis), and that the 'Country' factor has dominated both 'Industry' and 'Style' factors over time in terms of risk.

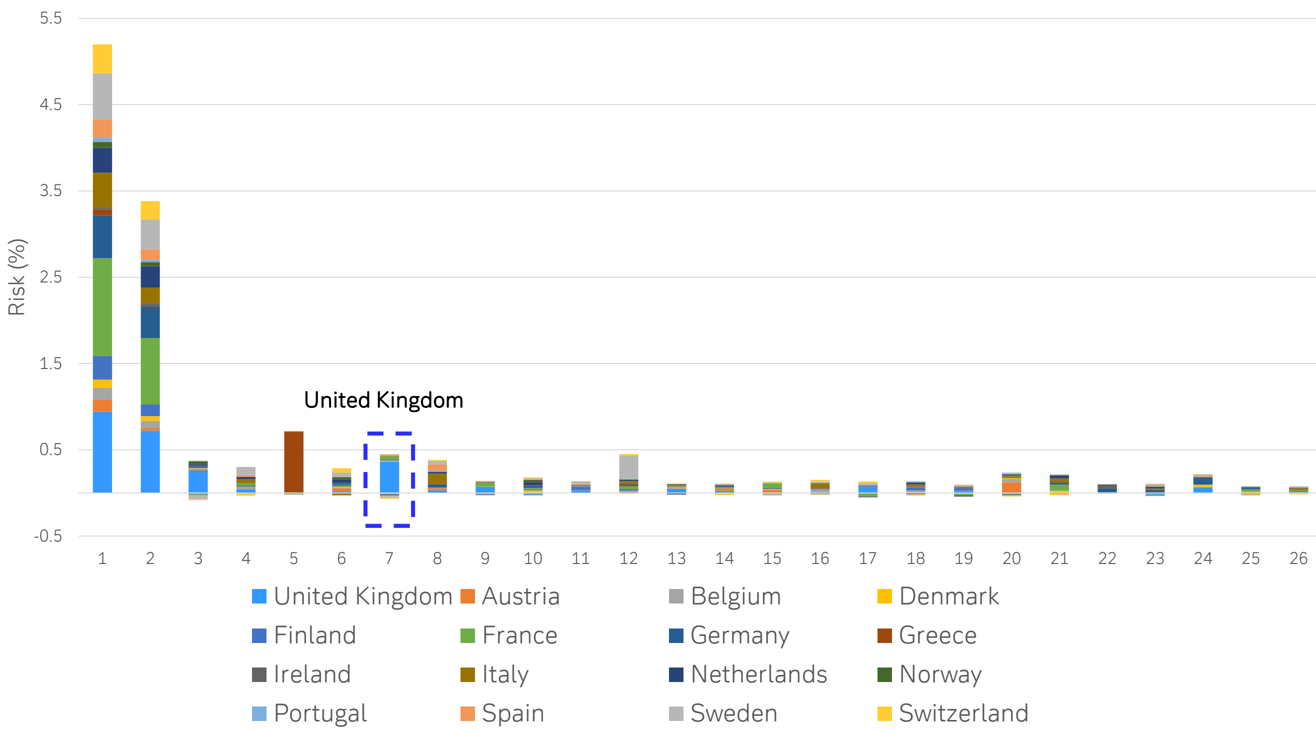
Figure 17: Average risk contributions over time (MSCI-Europe)



Source : Deutsche Bank Quantitative Strategy, FactSet, Axioma

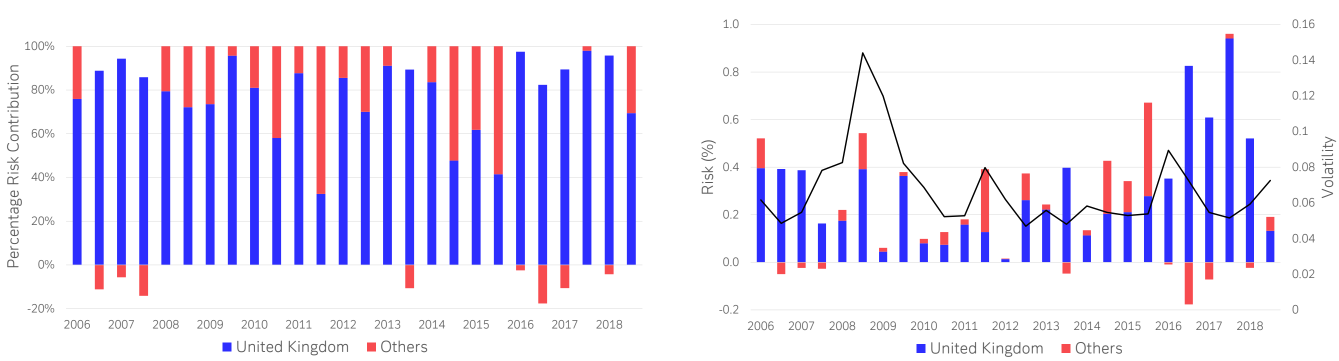
Figure 18 shows the risk contributions broken down by individual countries (as of December 2009). We see that the NSFA model identifies a latent factor (highlighted) that possesses a dominant risk contribution from the 'United Kingdom' . Interestingly, this pattern is found to be consistent over time as shown in Figure 19 , which indicates that stock co-movements in the United Kingdom are distinct from other European countries.

Figure 18: Risk contributions by country classification (Dec 2009, MSCI-Europe)



Source : Deutsche Bank Quantitative Strategy, FactSet, Axioma

Figure 19: NSFA factors with the United Kingdom being the dominant risk contribution (MSCI-Europe)



Source :Deutsche Bank Quantitative Strategy, FactSet, Axioma

Residual Momentum and Reversal Strategies

In the previous section, we saw that sparse factor models can provide a good basis to gain insight into market dynamics. Next, we focus our attention on potential applications of sparse factor models in relation to building momentum and reversal strategies.

Conventional momentum and reversal strategies are based on asset total returns, and exhibit strong time-varying dynamic exposures to systematic factors. Alvarez et al. (2011) find that beta is a major driver of risk and performance for momentum strategies over time. Grundy and Martin (2001) document that a total return momentum strategy has strong dynamic exposures to the three factors of the Fama-French model. As a result, it experiences losses when the sign of those factor returns over the holding period is opposite to the sign over the formation period (Blitz et al., 2011).

In recent years, several researchers report that the use of asset residual returns, rather than total returns, can boost the risk-adjusted profits of momentum and reversal strategies. For instance, Blitz et al. (2011, 2013) show that by hedging out dynamic Fama-French factor exposures, a residual momentum or reversal strategy can earn average Sharpe ratios that are roughly twice as large as that based on a total return-based strategy. Huij and Lansdorp (2017) confirm that their results are robust across different global stock universes and out-of-sample periods. Taken together, this combination of findings heightens the case for constructing signals using asset residual returns.

We hypothesize that by using both fundamental and statistical modeling we can better identify the market structure and correlations, and thus build a 'cleaner' signal (alpha) based on the residuals from both models.

Building on these ideas, we propose to construct residual momentum and reversal strategies that hedge out the dynamic factor exposures inferred from a combination of statistical and fundamental risk models. As mentioned earlier, a major advantage of using more than one modeling approach is that we can uncover underlying market dynamics through different perspectives. Although this study focuses on equity, we believe our method can be potentially applied to cross asset strategies. This is an interesting area for future research.

In the following subsections, we introduce two different approaches – '**double projection**' and '**constrained minimization**' – to construct residual momentum and reversal strategies.

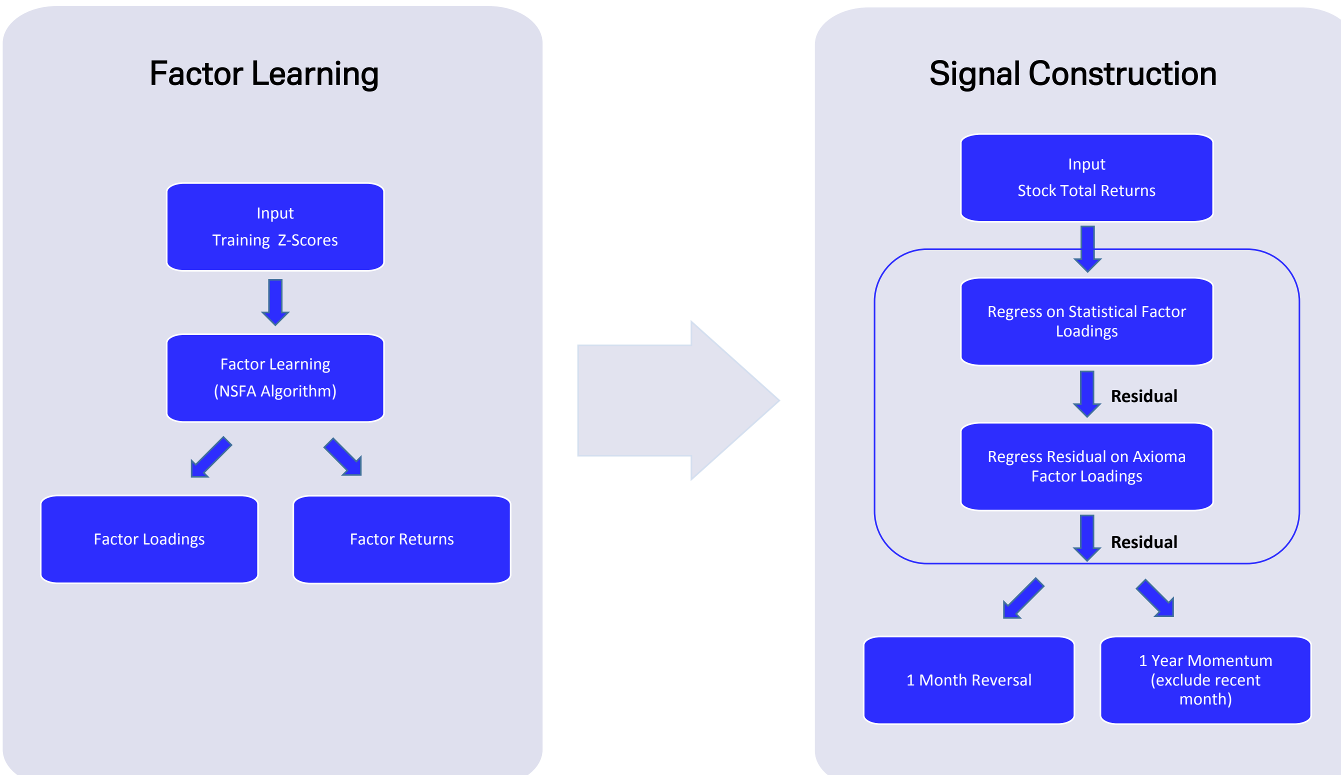
Both methods involve a factor learning stage that uses NSFA to extract statistical factors. To train the NSFA model, we employ daily data from the period between September 2002 and December 2018 from three major universes: MSCI-Europe (Europe), MSCI-Japan (Japan) and S&P 500 (United States). For every month-end starting June 2006, we follow a same learning procedure as described in Section: MSCI-Europe Universe.

Double Projection

The objective of a 'double projection' approach is to construct residual momentum and reversal strategies using asset residual returns obtained from a regression analysis, as illustrated in Figure 20.

'Double Projection': construct residual momentum and reversal strategies using asset residual returns.

Figure 20: 'Double Projection' to construct residual momentum and reversal strategies



Source : Deutsche Bank Quantitative Strategy

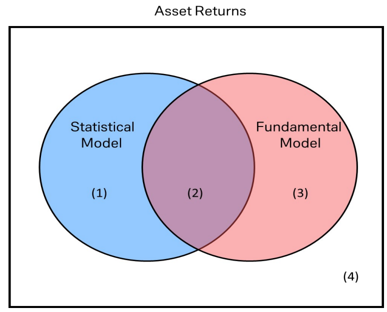
Methodology

As shown in this flow chart, we use a two-step regression to compute asset residual returns under the 'double projection' approach. In the first step, we use an ordinary least squares regression to obtain statistical residual returns based on the NSFA factor model. In the second step, we regress statistical residual returns on a set of fundamental factor exposures using a constrained least squares method[5] . The resulting asset residuals are then used for signal construction.

The 'double projection' approach uses asset residual returns that are not explained by either statistical or fundamental factors, which correspond to area [4] in Figure 3 . For comparison, we also consider different ways to compute asset residual returns during the back-testing. Each one represents one area in Figure 3 .

5 We use a constrained least squares regression to control the effects of multicollinearity for country and sector factors.

- **'Total Return'**: asset total returns [(1) + (2) + (3) + (4)]
- **'Statistical'** (NSFA): asset specific returns in the NSFA factor models [(3) + (4)]
- **'Fundamental'** (Axioma): asset specific returns in the Axioma fundamental factor models [(1) + (4)]
- **'Double Projection':** asset specific returns that are projected by both NSFA and Axioma factor models [(4)]



Note that a number of caveats need to be taken into account when choosing a fundamental risk model under the 'double projection' approach. In the Axioma global risk model, both 'short-term momentum' and 'medium-term momentum' are included as market-based factors (see Figure 8). Since the purpose is to investigate the efficacy of these factors in the residual returns space, we excluded them from the Axioma risk model when forming the fundamental factors.

Signal Construction

To construct the momentum and reversal signals, we follow a similar approach to those found in empirical studies, which include *ex ante* formulation of portfolio weights, based on *ex post* estimation of stock returns. Specifically, we divide all stocks into five equal groups (quintile) and construct a long-short portfolio based on a standardization score this is computed as follows.

Momentum

For a momentum strategy, we compute a standardization score based on trailing 1 year stock residual returns excluding the most recent month. More specifically, this score is measured by average residual return scaled by its standard deviation over the same period. As Blitz et al. (2011) pointed out, this is regarded as an improved measure because the raw average return might be a noisy estimate. We go long on the stocks in the group with the highest scores and we short the stocks in the group with the lowest scores.

Short-term Reversal

By contrast, for a short-term reversal strategy, we compute a standardization score at the end of each month based on their estimated residual returns during that month scaled by the standard deviation over the prior 36 months. And as we expect a price overreaction in the case of a short-term reversal strategy, we short the stocks in the group with the highest scores and go long on the stocks in the group with the lowest scores. Note that the signal construction employed here is similar to Blitz et al. (2013).

During the back-testing stage, we assign equal weightings to both long and short portfolios, which are rebalanced every month.

Constrained Minimization

In this subsection, we introduce an equality constrained quadratic minimization method to construct residual momentum and reversal strategies.

'Constrained Minimization': construct residual momentum and reversal strategies using constrained quadratic minimization techniques.

Methodology

As mentioned earlier, our objective is to isolate momentum and reversal risk premia while mitigating other systematic factor exposures. In other words, we want to find orthogonalized momentum and reversal factors such that they have zero exposure to other systematic factors.

This objective is similar to a recently developed 'Fast Factor' framework, where a constrained quadratic minimization is used to ensure unit exposure to the target factor and zero exposure to other systematic factors (Ward et al., 2018). The utility function adopted in this framework can be shown as:

The objective is to have unit exposure to the target factor and zero exposure to other systematic factors.

$$\min_{w} \frac{1}{2} w' C w \quad \text{subject to } X'w = h \tag{7}$$

where $\boldsymbol{w}$ is the portfolio weight, $\boldsymbol{C}$ is the return covariance matrix, $\boldsymbol{X}$ is the factor-loading matrix and $\boldsymbol{h}$ is a binary vector that contains either 1 or 0.

Inspired by this framework, we argue that residual momentum and reversal strategies can be efficiently constructed through a similar minimization process.

As shown in Equation 7, this quadratic program requires a robust estimate of the return covariance matrix, $\boldsymbol{C}$, that captures an inherent risk structure. Using the multi-factor model specified in Equation 1, this risk structure is often decomposed as: (Grinold and Kahn, 2000)

$$C_{n,m} = \sum_{k_1,k_2=1}^{K} X_{n,k_1} \cdot C^F_{k_1,k_2} \cdot X_{m,k_2} + \Delta_{n,m} \tag{8}$$

where

- $C_{n,m}$: covariance of asset n with asset m. If $n = m$, this gives the variance of asset n.
- X_{n,k_1}: exposure of asset n to factor k_1.
- $C^F_{k_1,k_2}$: covariance of factor k_1 with factor k_2. If $k_1 = k_2$, this gives the variance of factor k_1.
- $\Delta_{n,m}$: specific covariance of asset n with asset m. If $n = m$, this gives the specific variance of asset n. In practice, the specific covariance of the two assets is often assumed to be zero.

In this study, we estimate the return covariance matrix using the following steps:

1. Construct the factor-loading matrix $\boldsymbol{X}$ using both fundamental and statistical factors (see Figure 21);

2. A constrained least squares regression is employed to obtain the factor returns;
3. Compute the factor covariance matrix using factor returns;
4. Compute the specific variance for each asset;
5. Use Equation 8 to compute the return covariance matrix.

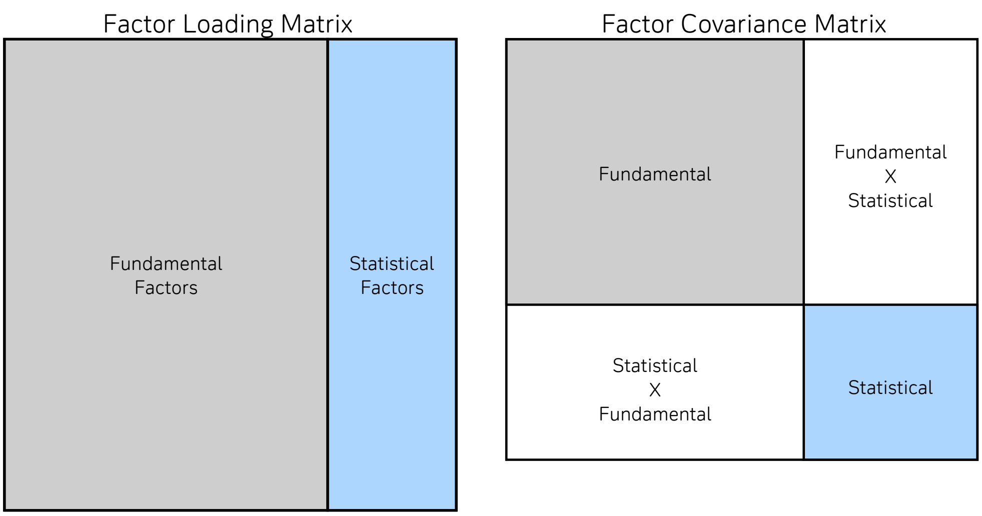
Figure 21: Systematic factors used in the experiments

Type	Factors
Statistical	Statistical factors obtained from the NSFA model
Fundamental	Axioma factors: Market Intercept, Exchange Rate, Growth, Leverage, Liquidity, Medium-Term Momentum, Short-Term Momentum, Size, Value, Volatility Country factors (only for MSCI-Europe) Sector factors: GICS sector classifications

Source : Deutsche Bank Quantitative Strategy

For a combination of fundamental and statistical factors, a schematic representation of the factor-loading and covariance matrices is given in Figure 22 .

Figure 22: A schematic representation of the factor-loading and covariance matrices



Source : Deutsche Bank Quantitative Strategy

Under the 'constrained minimization' approach, we no longer need to exclude 'short-term momentum' and 'medium-term momentum' from the Axioma fundamental risk model, since both are the target factors in this study. This implies that the construction of residual momentum and reversal signals relies on the factor definitions specified in the fundamental risk model. This is a key difference from the 'double projection' approach.

Signal Construction

To construct residual momentum and reversal strategies, the following steps are taken:

1. For a given target factor (e.g., momentum or reversal), we divide the stocks into five equal groups (quintile) based on their factor exposures;
2. Perform a constrained minimization on asset returns (1000 days) from the first and last groups;
3. Constraints are applied to ensure unit exposure to the target factor and zero exposure to other systematic factors;
4. Optimal portfolio weights are determined via a quadratic programming solver;
5. The portfolio is levered to have 100% notional weight on the long and short legs;
6. Portfolio holdings are rebalanced every month.

Momentum

For a momentum strategy, we set 'medium-term momentum' as our target factor and constrain other systematic factors to have zero risk exposures. We go long on stocks with positive portfolio weights and we short those stocks with negative portfolio weights.

Short-term Reversal

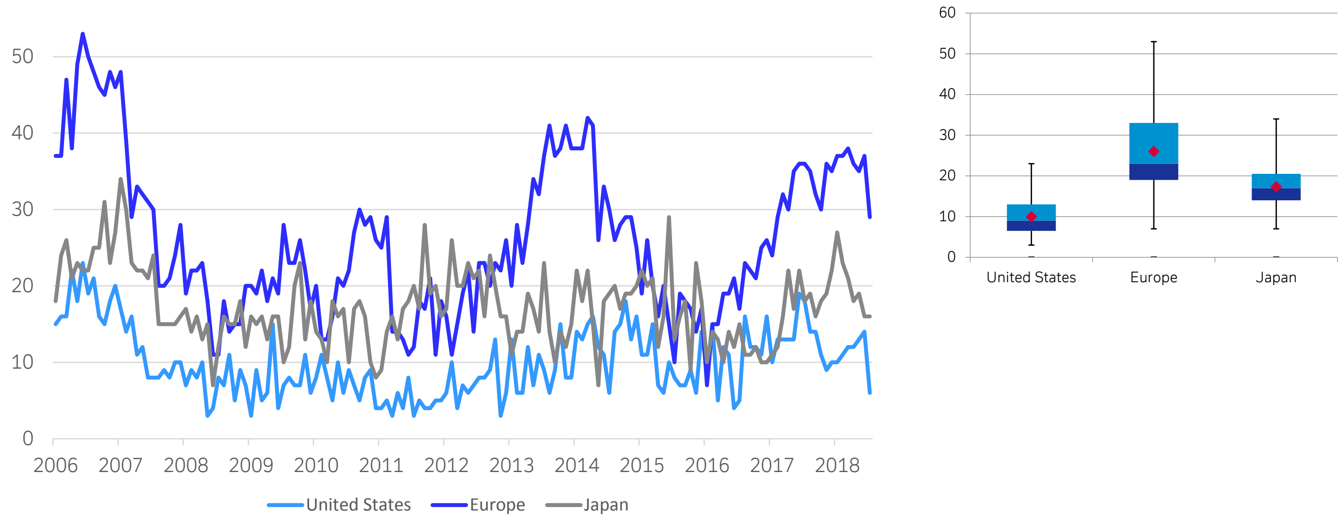
By contrast, for a short-term reversal strategy, we set 'short-term momentum' as our target factor and constrain other systematic factors to have zero risk exposures. We go long on stocks with negative portfolio weights and we short those stocks with positive portfolio weights.

Strategy Performance

In the following subsections, we summarize the performance analytics of our newly proposed residual momentum and reversal strategies. Our findings indicate that removing unintended systematic factor exposures significantly reduces volatility, and thus improves the risk-adjusted returns of momentum and reversal strategies. In particular, we show that removing statistical factor exposures is important when forming the residual momentum and reversal strategies.

Figure 23 shows the number of statistical factors discovered in these three markets over time. We see that on average Europe has a much higher number of factors than Japan or the United States. This is intuitive, given that Europe has a more complex market structure, covering multiple countries and industries.

Figure 23: Number of identified NSFA factors in three regions over time

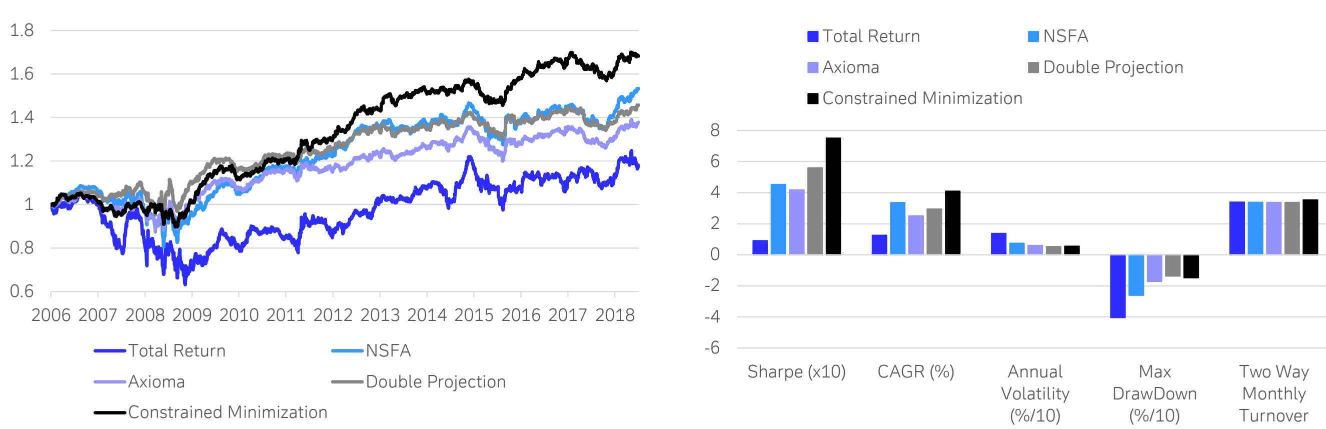


Source : Deutsche Bank Quantitative Strategy, FactSet, Axioma

Short-term Reversal

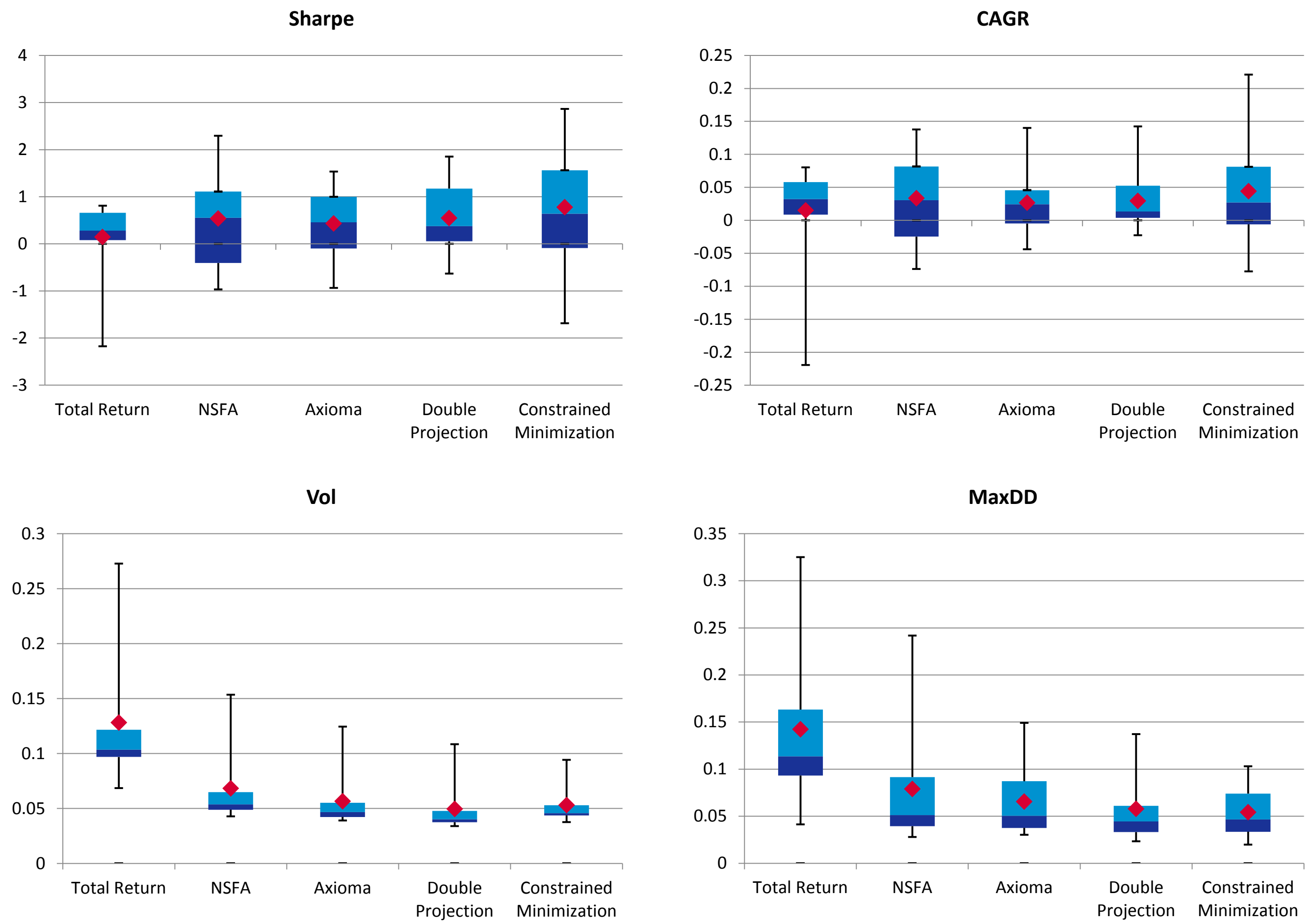
We begin our analysis with the short-term reversal strategies. Figure 24 shows a comparison between residual reversal strategies and total return reversal in the United States. Looking at the performance statistics, we see all four residual reversal strategies exhibiting higher returns, lower volatility, and lower maximum drawdowns than in total return reversal. In particular, the results indicate that after removing both statistical and fundamental factor exposures, both 'double projection' and 'constrained minimization' approaches significantly dampen the volatility of the reversal strategy and result in a lower maximum drawdown. This can be further justified by looking at the box plot of annual performance statistics since 2007, as illustrated in Figure 25 . From the chart, we see that both methods have suffered lower drawdowns over the years.

Figure 24: One month short-term reversal strategies in the US



Source : Deutsche Bank Quantitative Strategy, FactSet, Axioma

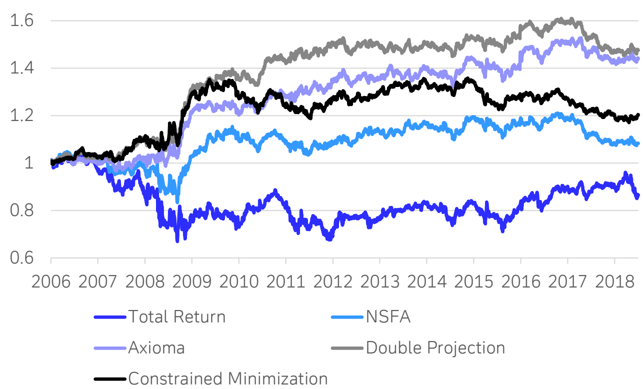
Figure 25: Box plot of annual performance statistics (US Reversal)

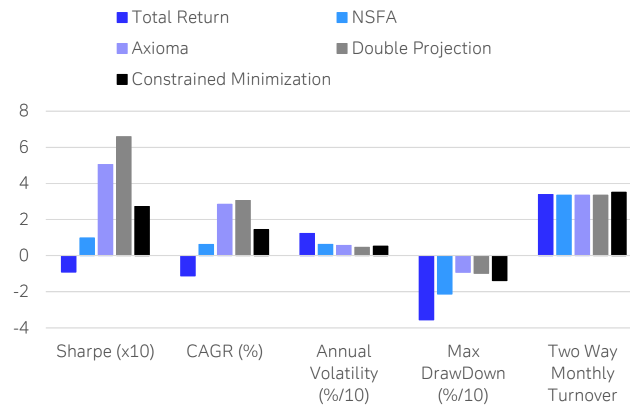


Source :Deutsche Bank Quantitative Strategy, FactSet, Axioma

Figure 26 shows the performance of our residual reversal strategies in Europe. Whilst total return reversal generates negative returns over the back-testing period, we see a significant improvement in risk-adjusted returns having removed the dynamic factor exposures. In particular, when we look at the box plot of annual performance statistics in Figure 27 , we see that on average the 'double projection' approach has the best performance over the years. This demonstrates the efficacy of removing factor exposures from the statistical factor model.

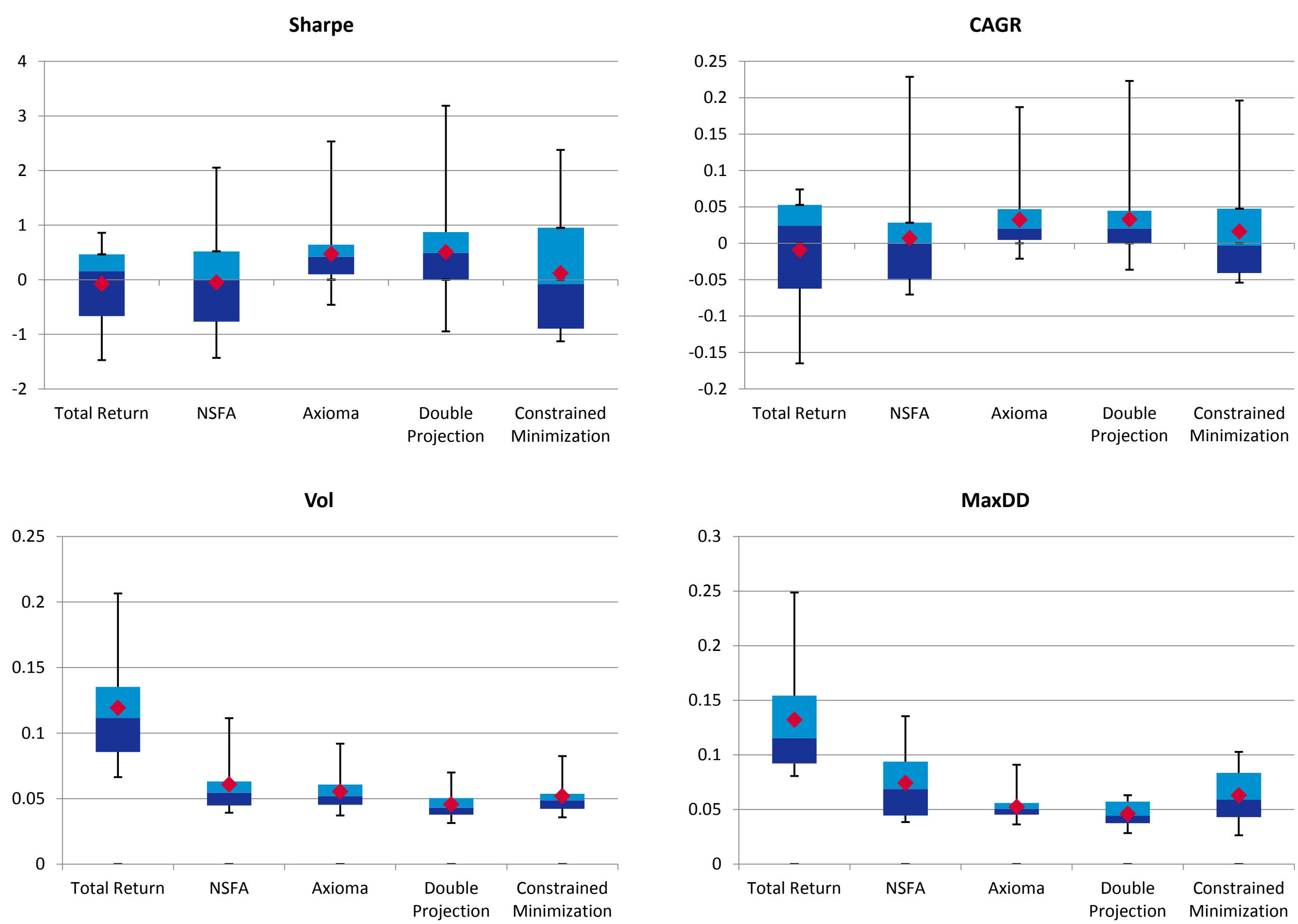
Figure 26: One month short-term reversal strategies in Europe





Source : Deutsche Bank Quantitative Strategy, FactSet, Axioma

Figure 27: Box plot of annual performance statistics (Europe Reversal)

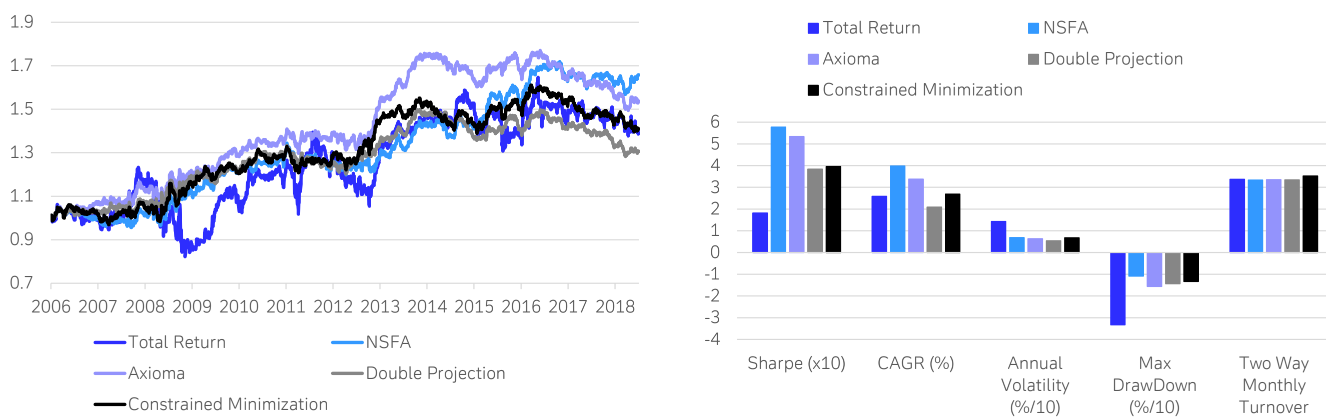


Source :Deutsche Bank Quantitative Strategy, FactSet, Axioma

In Figure 28 and Figure 29 , we observe consistent results in the Japanese market, in which our residual reversal strategies have attained a high Shape Ratio with lower maximum drawdowns. Interestingly, we observe that the realized short-term reversal premium has been on a downward trend since 2016.

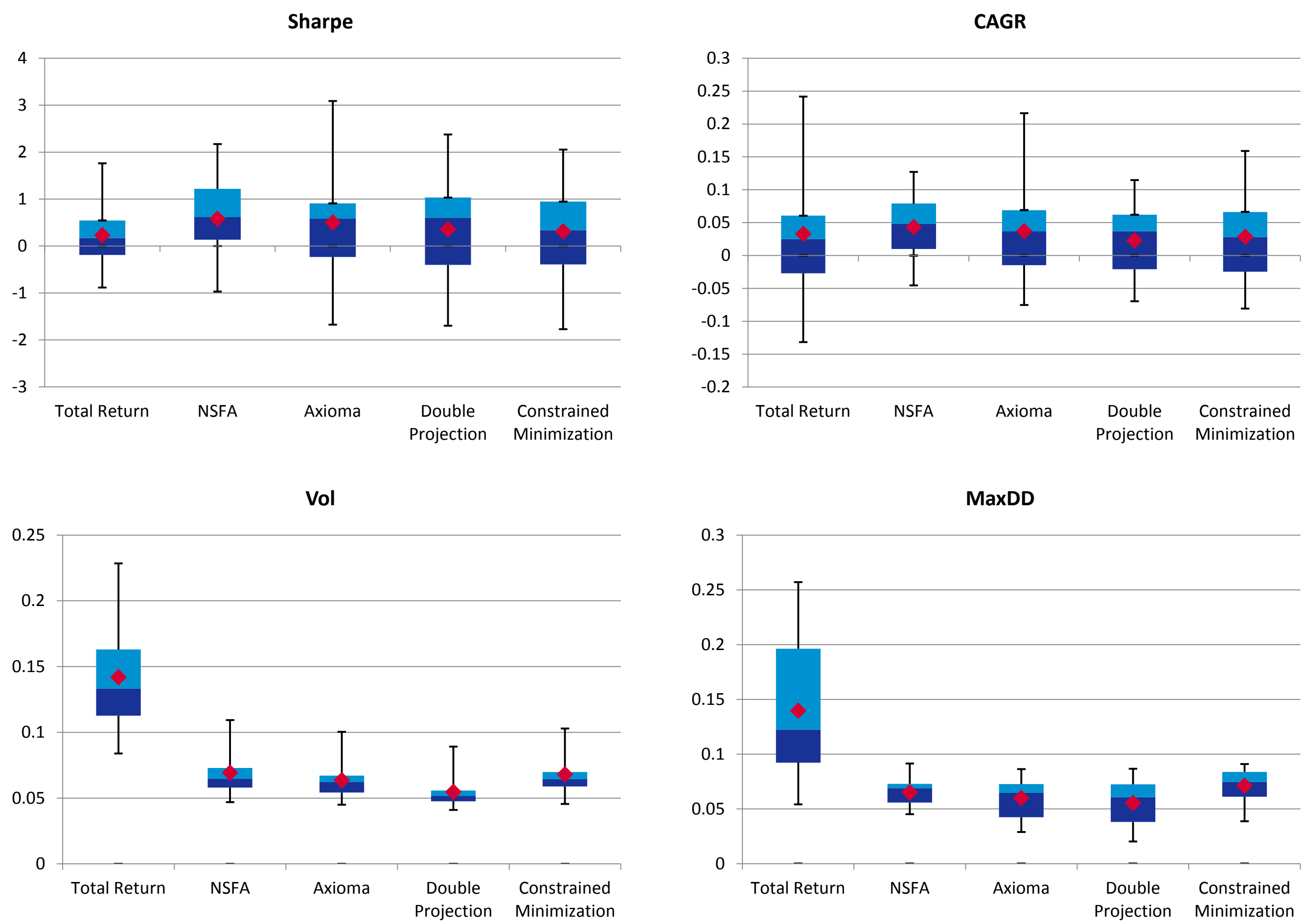
In all three regions, we see that the two-way monthly turnover is largely unchanged after replacing total return with stock residual return in constructing the short-term reversal strategy. Only the 'constrained minimization' approach results in a slight increase in the portfolio turnover.

Figure 28: One month short-term reversal strategies in Japan



Source : Deutsche Bank Quantitative Strategy, FactSet, Axioma

Figure 29: Box plot of annual performance statistics (Japan Reversal)



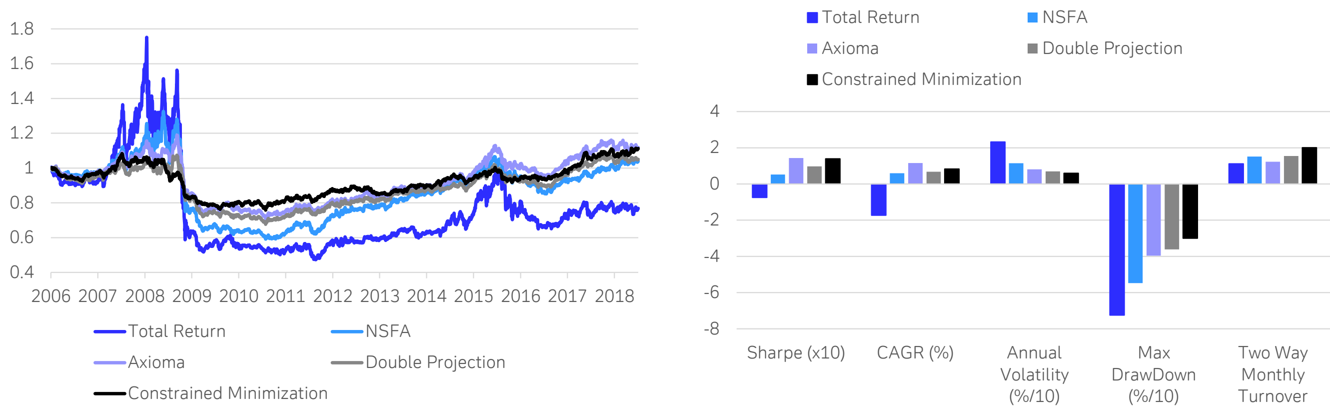
Source :Deutsche Bank Quantitative Strategy, FactSet, Axioma

Momentum

Next, we turn our attention to the experimental results on momentum strategies. Overall we show that our residual momentum strategies are profitable across different regions with reduced volatility and maximum drawdowns, with only a small increase in the portfolio turnover.

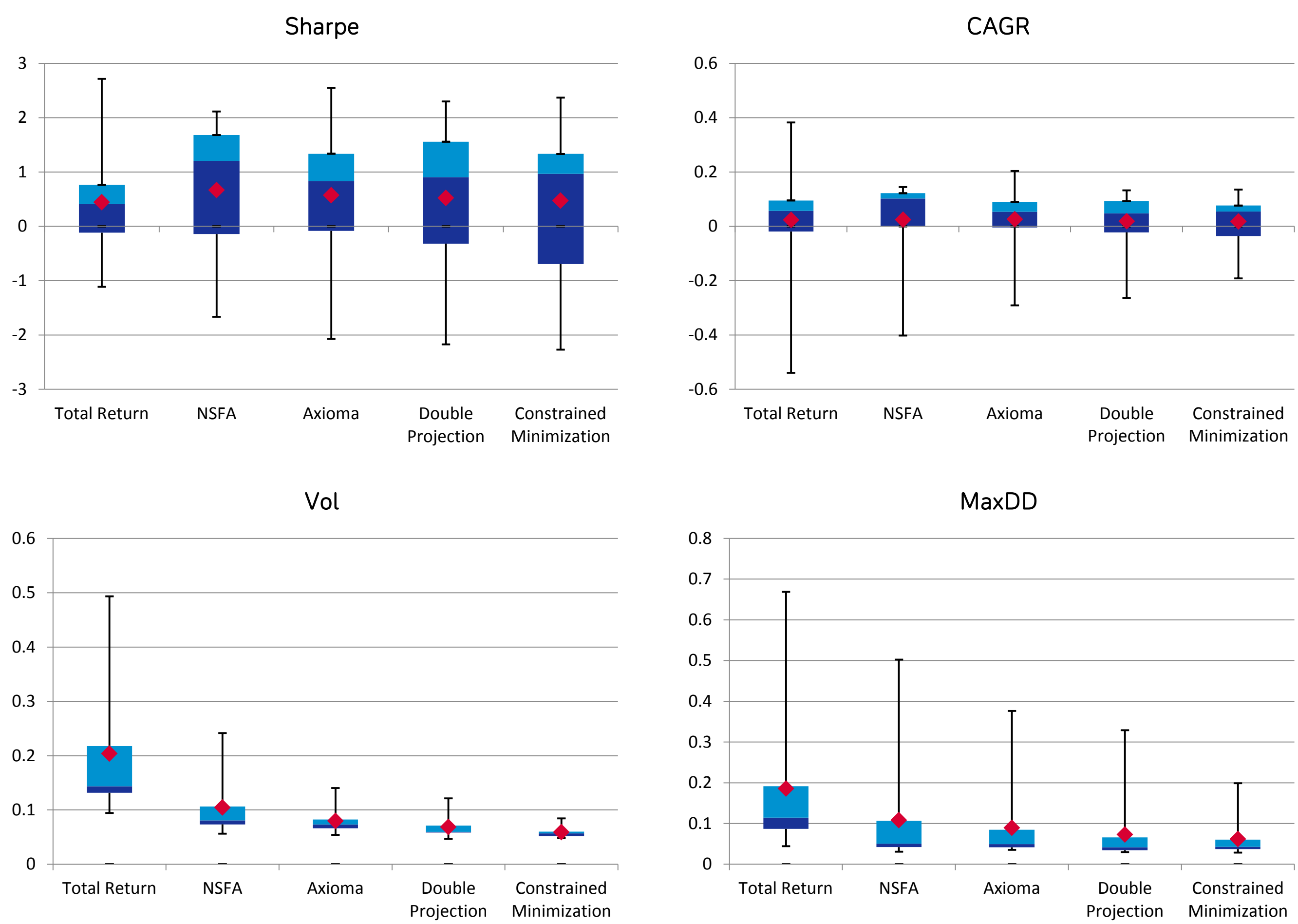
In Figure 30 and Figure 31 , we see that in the US market our residual momentum strategies have achieved a significant reduction in terms of maximum drawdown and volatility. For example, a total momentum strategy has experienced a 'crash' over the three-month period from March to May of 2009. As explained by Daniel and Moskowitz (2016), momentum crashes occur during periods of strong market reversals when the short leg of the portfolio — the losers — are moving substantially higher. By contrast, looking at the left-hand side of Figure 30 , we see that our residual momentum strategies are more resilient to the market reversals in early 2009. Although both 'double projection' and 'constrained minimization' approaches have performed well over time, we note that there is an increase in the average portfolio turnover.

Figure 30: 11-month momentum strategies in the US



Source : Deutsche Bank Quantitative Strategy, FactSet, Axioma

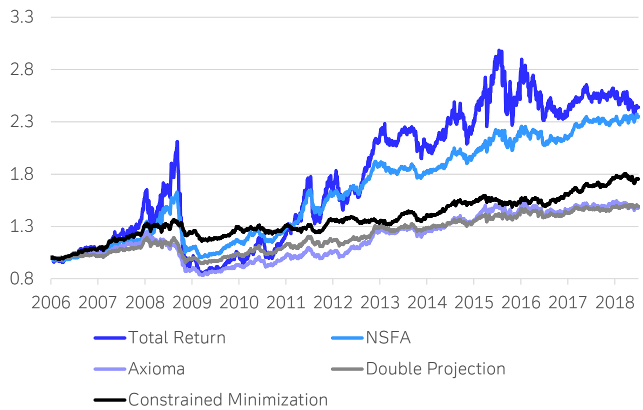
Figure 31: Box plot of annual performance statistics (US Momentum)

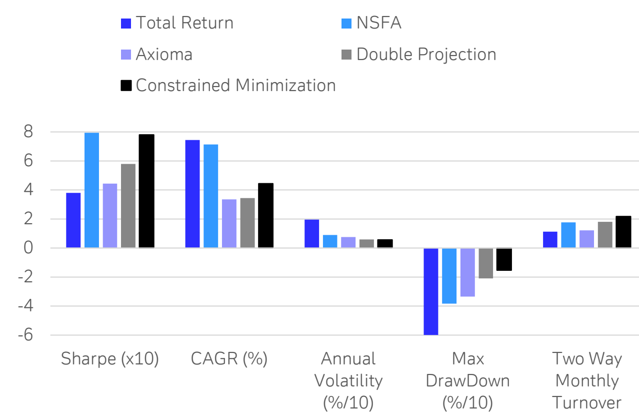


Source :Deutsche Bank Quantitative Strategy, FactSet, Axioma

Figure 32 and Figure 33 further highlight the efficacy of the residual momentum strategies in Europe. The residual strategies exhibit significantly lower volatility and reduced maximum drawdowns in Europe. Compared to the 'fundamental' approach, hedging out statistical factor exposures improves the risk adjusted returns of the momentum strategy in Europe.

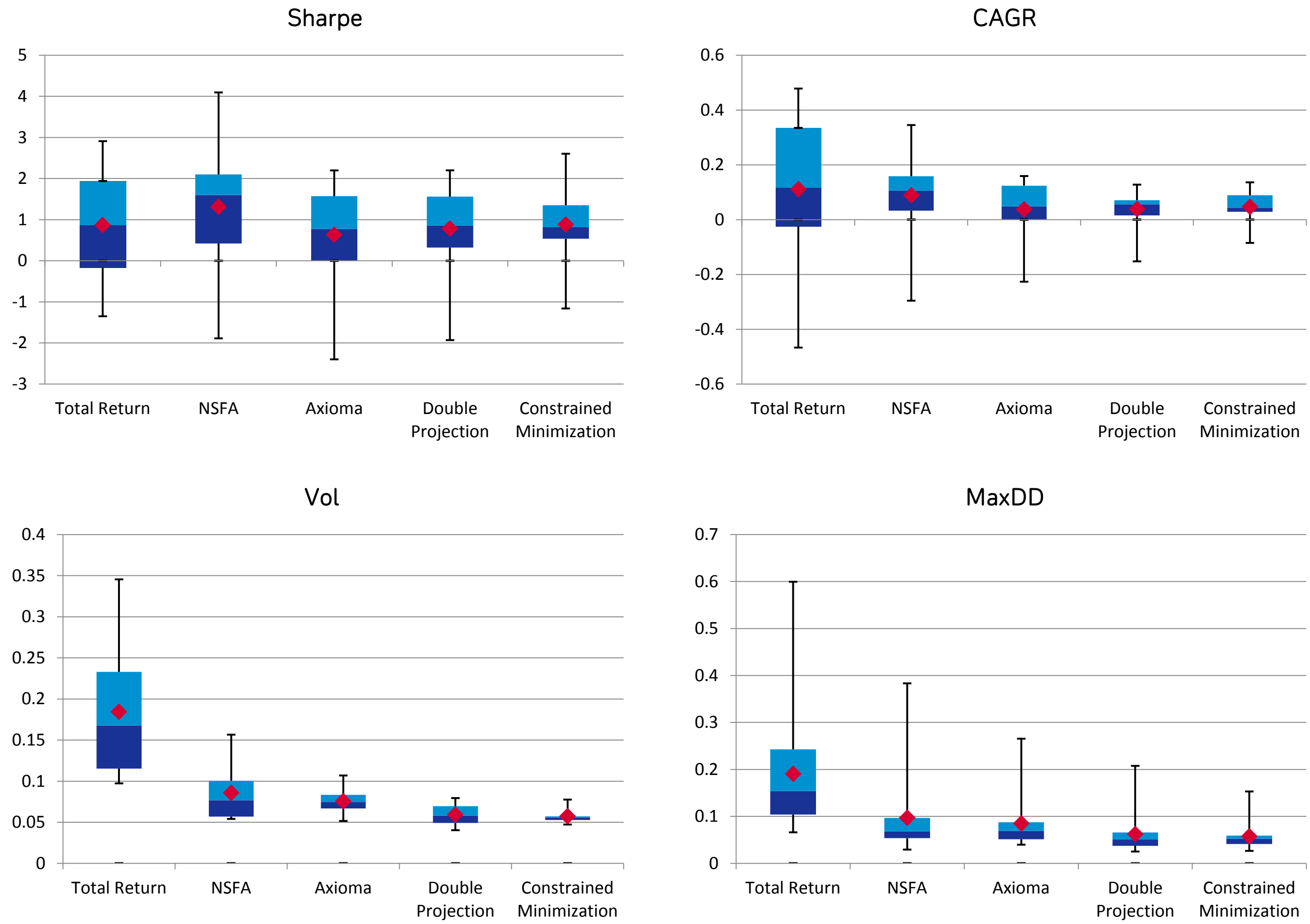
Figure 32: 11-month momentum strategies in Europe





Source :Deutsche Bank Quantitative Strategy, FactSet, Axioma

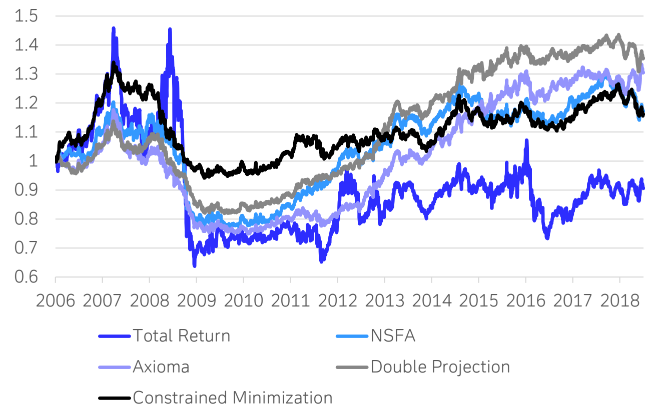
Figure 33: Box plot of annual performance statistics (EU Momentum)

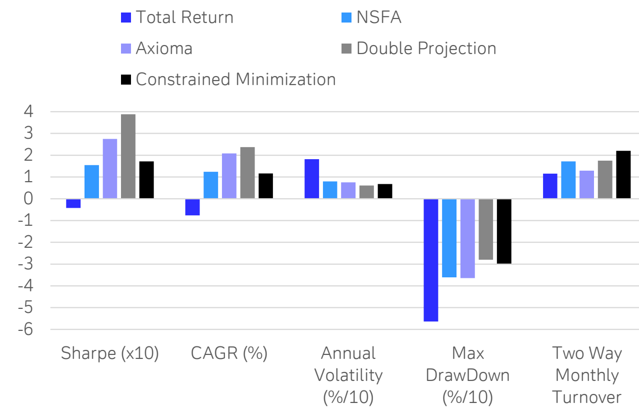


Source :Deutsche Bank Quantitative Strategy, FactSet, Axioma

Now let us focus on the Japanese market, where the traditional momentum premium was previously found to be nonexistent due to the dominance of the value risk premium in Japan. The reasons for this exception have been widely discussed in literature (see Asness, 2011, for discussion). As shown in Figure 34 and Figure 35 , the 'double projection' approach produces a higher CAGR, lower volatility and lower maximum drawdowns consistently over time. Interestingly, our results are in agreement with the findings of Chang et al. (2018) that residual momentum profits are significant in Japan.

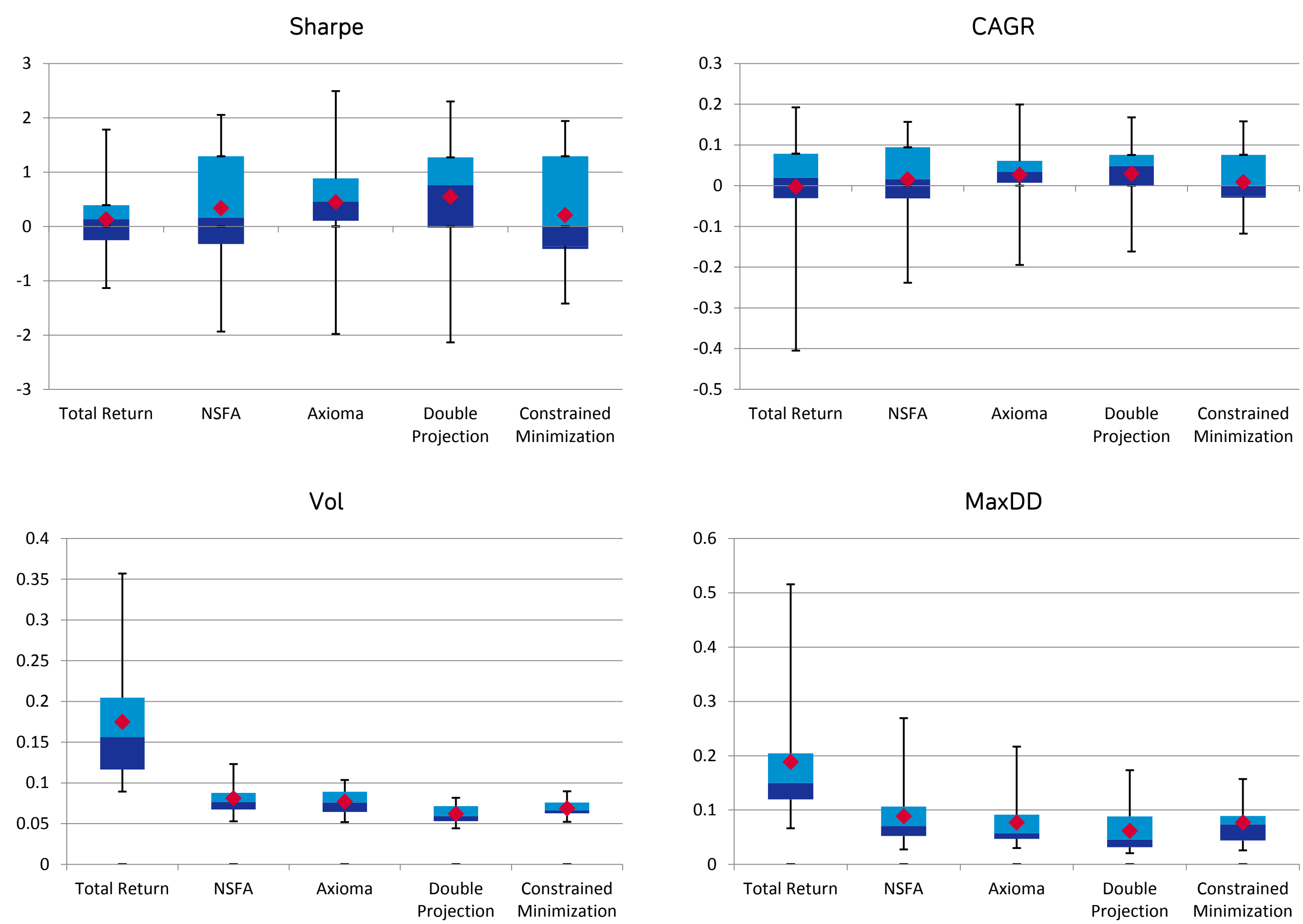
Figure 34: 11-month momentum strategies in Japan





Source : Deutsche Bank Quantitative Strategy, FactSet, Axioma

Figure 35: Box plot of annual performance statistics (JP Momentum)



Source :Deutsche Bank Quantitative Strategy, FactSet, Axioma

Conclusions

In this report, we reviewed a recent approach to statistical factor modeling, known as Non-Parametric Bayesian Sparse Factor Analysis (NSFA). The algorithm is governed by a stochastic process known as the 'Indian Buffet Process' (IBP), which models a factor-loading matrix with infinite dimensions. Together with the sparsity constraints, we have seen that this approach can automatically determine systematic factors that can be readily interpreted in fundamental terms, and capture the essence of market structures.

In reviewing commonly used statistical factor models, we found that conventional techniques generally suffered from a lack of interpretability, and it was difficult to isolate idiosyncratic components. Regarding these issues, we have designed two experiments to assess the efficacy of the NSFA model.

First, we conducted a risk attribution exercise. Analysis of factor contributions revealed that the NSFA factors were relatively distinct and easy to interpret over time. Our results showed that country effects dominated industry effects in explaining variations in stock returns. Futhermore, we showed that the NSFA model identified distinct stock co-movements in the United Kingdom.

Second, we developed two approaches for building residual momentum and reversal strategies. By neutralizing both NSFA and Axioma factor exposures, we showed that momentum and reversal premiums can be effectively isolated. We investigated the performance of our strategies in three commonly-traded markets. The following points were observed:

- Short-term Reversal
  - Both 'double projection' and 'constrained minimization' approaches significantly dampened the volatility of the reversal strategy and resulted in lower maximum drawdowns.
  - Annualized returns remained high after removing unintended factor exposures.
  - Two-way monthly turnover remained largely unchanged using our residual reversal strategies.

- Momentum
  - The residual momentum strategies were typically profitable across different regions, with reduced volatility and maximum drawdowns, with only a small increase in the portfolio turnover.
  - Contrary to the nonexistence of the traditional momentum premium in Japan, we saw a strong residual momentum premium using our 'double projection' approach.

Based on the findings presented above, we believe that this study should, therefore, be of value to investors wishing to explore sophisticated statistical factor models that provide new insights for risk management and factor investing.

Appendix

Appendix A: Nonparametric Bayesian Sparse Factor Models (NSFA)

Here we provide a concrete review on the model specifications of the NSFA model. For more detailed discussions, we would like to refer interested readers to [Knowles and Ghahramani, 2011].

Model

Recall factor analysis models the observed data ***Y*** of size ***D x N***, as a linear combination of independent hidden variables ***F*** of size ***K x N***:

$$\underbrace{Y}_{(D\,x\,N)} = \underbrace{X}_{(D\,x\,K)}\underbrace{F}_{(K\,x\,N)} + \underbrace{e}_{D\,x\,N}$$

where ***X*** denotes the factor-loading matrix, ***F*** represents the factor returns and ***e*** represents the noise components. Each column of ***X*** represents the factor exposure corresponding to each of ***K*** latent variables.

In the NSFA model, the factor-loading matrix is modeled as a 'spike and slab' distribution, which either samples from a Gaussian distribution or creates a point mass ($\boldsymbol{\delta_0}$) at 0 depending on a binary indicator $\boldsymbol{Z_{dk}}$, formulated as

$$p(X_{dk}|Z_{dk},\lambda_k) = Z_{dk}N\left(X_{dk}\middle|0,{\lambda_k}^{-1}\right) + (1 - Z_{dk})\delta_0(X_{dk})$$

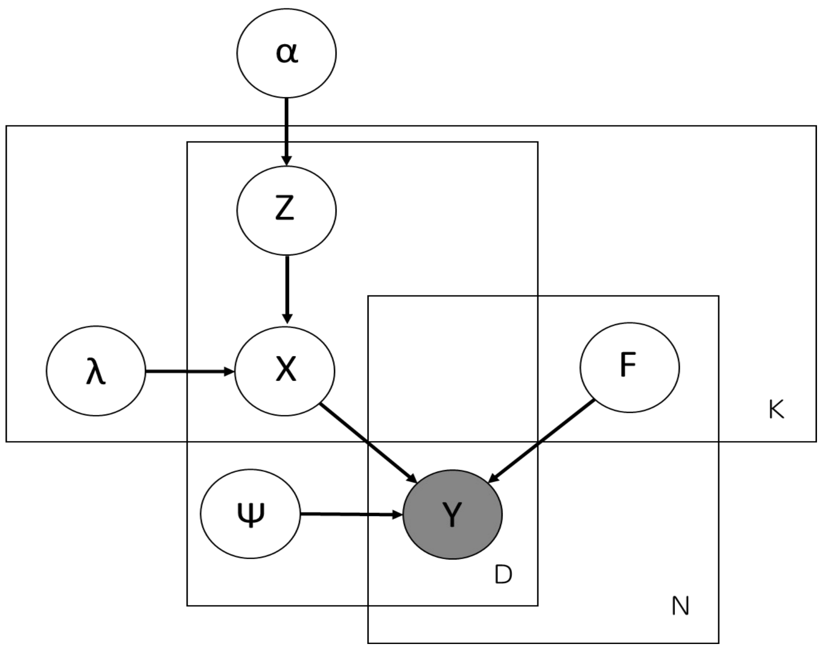
where $\boldsymbol{X_{dk}}$ denotes the elements of the factor-loading matrix followed by a Gaussian distribution, and $\boldsymbol{\lambda_k}$ represents the variance of factor ***k***. This type of priors is known to encourage great sparsity in Bayesian statistics. To potentially allow an infinite number of columns, the binary indicator $\boldsymbol{Z_{dk}}$ is then modeled as an IBP, which is typically formulated as a Beta-Bernoulli process with the total number of features ***K*** taken to infinity, such as

$$\begin{array}{c} \pi_k|\alpha \sim Beta(\frac{\alpha}{K},1) \\ Z_{dk}|\pi_k \sim Bernoulli(\pi_k) \end{array} \qquad K \rightarrow \infty$$

We note that each row of ***Z*** is expected to have Poisson($\boldsymbol{\alpha}$) active features (non-zero columns), and the total number of columns is distributed as Poisson($\boldsymbol{\alpha}$+$\boldsymbol{\alpha}$/2+...+$\boldsymbol{\alpha}$/N). The probability density of any particular matrix being produced by this process is discussed by Griffiths and Ghahramani (2006).

The complete graphical model that expresses the conditional dependence structure is shown in Figure 36. The columns $\boldsymbol{f_n}$ of the latent variable matrix ***F*** are distributed as $\boldsymbol{N(f_n|\mu_n,\theta)}$ where $\boldsymbol{\theta = X^T\Psi^{-1}X + I}$ and $\boldsymbol{\mu_n = \theta^{-1}X^T\Psi^{-1}y_n}$, which is a standard posterior distribution in factor analysis.

Figure 36: A graphical representation of the NSFA model



Source : Deutsche Bank Quantitative Strategy, Knowles and Ghahramani (2011)

In Machine Learning, a graphical model illustrates the conditional dependence structure between random variables without enumerating all settings of all variables in the model. For instance, 'A' -> 'B' means 'B' is conditionally dependent on 'A'. A text block indicates the dimension of that random variable. For instance, Observation Y is of dimension D x N. Hence, the variable Y is superposed by text blocks D and N.

The joint distribution defined by a graph is given by the product over all of the nodes of the graph, of a conditional distribution for each node conditioned on the variables corresponding to the parents of that node in the graph (see Chapter 8 in [Bishop 2007]). In the NSFA model, we can write this joint distribution as:

$$p(\lambda,\Psi,Y,X,Z,F,\alpha) \;=\; p(\alpha)p(\lambda)p(\Psi)p(F)p(Z|\alpha)p(X|\lambda,Z)p(Y|\Psi,X,F)$$

Inference

Once we have a probabilistic model that represents the input data ***Y***, the next thing to do in Bayesian statistics is inference. One method that achieves this is Gibbs sampling, a simple but widely-used Markov chain Monte Carlo (MCMC) algorithm. Gibbs sampling aims to sequentially sample from a multivariate distribution by sampling each of its variables conditioning on all the others. In the case of NSFA, we are interested in inferring latent variables ***Z, X, F*** and hyperparameters ***Ψ, λ, α*** via the following procedures:

for iteration t = 1:T:

- Sample $p(Z^{t+1}|F^t, X^t, \Psi^t, \lambda^t, \alpha^t, Y)$
- Sample $p(F^{t+1}|Z^{t+1}, X^t, \Psi^t, \lambda^t, \alpha^t, Y)$
- Sample $p(X^{t+1}|F^{t+1}, Z^{t+1}, \Psi^t, \lambda^t, \alpha^t, Y)$
- Sample $p(\Psi^{t+1}|X^{t+1}, F^{t+1}, Z^{t+1}, \lambda^t, \alpha^t, Y)$
- Sample $p(\lambda^{t+1}|\Psi^{t+1}, X^{t+1}, F^{t+1}, Z^{t+1}, \alpha^t, Y)$
- Sample $p(\alpha^{t+1}|\lambda^{t+1},\Psi^{t+1}, X^{t+1}, F^{t+1}, Z^{t+1}, Y)$

To sample individual elements of IBP matrix $\boldsymbol{Z_{dk}}$, NSFA computes a ratio of posterior probabilities between $\boldsymbol{Z_{dk}}$ = 1 and $\boldsymbol{Z_{dk}}$= 0 to indicate whether factor $\boldsymbol{k}$ is active for dimension $\boldsymbol{d}$. $\boldsymbol{X_{dk}}$ is then sampled when this ratio is large. Besides, NSFA also considers the option of adding new features since $\boldsymbol{Z}$ can have infinitely many columns. This is achieved by sampling the number of new features $\boldsymbol{k_d}$ which are active only for dimension $\boldsymbol{d}$, via a Metropolis-Hastings step with a Poisson prior as its proposal distribution. Columns of $\boldsymbol{F}$ are drawn from Gaussian distributions and hyperparameters $\boldsymbol{\Psi}$, $\boldsymbol{\lambda}$, $\boldsymbol{\alpha}$ are drawn from Gamma distributions.

Metropolis-Hastings (MH) MCMC

The inference of the NSFA model involves an intensive use of the Metropolis-Hastings (MH) MCMC algorithm. Given observation $\boldsymbol{y}$ and random variable $\boldsymbol{X}$, the MH MCMC algorithm samples from the posterior distribution $\boldsymbol{p(X|y)}$ via:

1. Pick a proposal distribution $\boldsymbol{q(X^t|X^{t-1})}$
2. Initialize $\boldsymbol{X^t}$ and set $\boldsymbol{t = 1}$
3. For $\boldsymbol{t = 1{:}T}$, generate samples $\boldsymbol{X'}$ from $\boldsymbol{q(X^t|X^{t-1})}$ and accept $\boldsymbol{X^t = X'}$ with probability $\boldsymbol{\beta}$, otherwise set $\boldsymbol{X^t = X^{t-1}}$

$$\beta = \min\{1, \frac{p(y|X')p(X')q(X^{t-1}|X')}{p(y|X^{t-1})p(X^{t-1})q(X'|X^{t-1})}\}$$

To reach appropriate equilibrium state, the MH MCMC algorithm satisfies the detailed rebalance (reversibility) condition as

$$q(X^t|X^{t-1})p(X^{t-1}|y) = q(X^{t-1}|X^t)p(X^t|y)$$

Mathematically, it can be shown that Gibbs Sampling is a special case of Metropolis-Hastings where the proposed moves are always accepted (the acceptance probability is 1).

Computation Cost

For each iteration, the computation cost of the NSFA model takes order $O(NKD)$ for sampling $\boldsymbol{Z}$ and $\boldsymbol{X}$, and $O(K^2 + K^3 + ND)$ for sampling $\boldsymbol{F}$. Thus the computational performance is largely driven by the total number of active features $\boldsymbol{K}$. A large input $\boldsymbol{Y}$ or a complex latent structure might result in a slow convergence.

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Appendix 1

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