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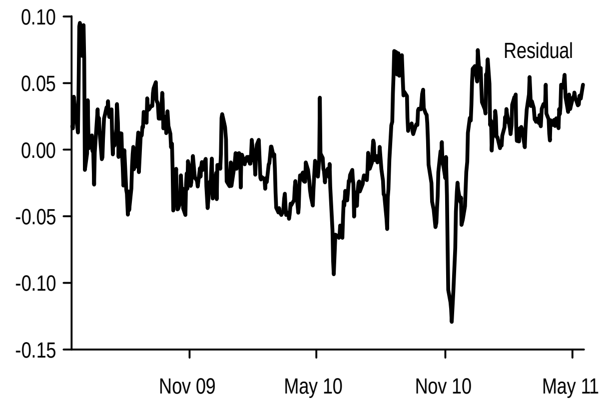
Research Note

Implied Weights in Yield Curve Relative Value Trades

- **We revisit the simplest of relative value trading concepts—that of hedging yield curve and butterfly trades against exposure to the level of yields and/or the slope of the curve by choosing appropriate risk weightings on the various legs of the trade**
- **Common practice is to rely on empirical estimation of the proper weightings needed to achieve level and/or curve neutrality, using backward-looking techniques such as linear regressions or Principal Component Analysis**
- **Such estimation techniques are often adequate, but fail in two important ways—when the yield curve is in transition between different behavioral regimes, and in periods where weightings may depend on the level of rates, as is often the case when front-end yields are near zero**
- **We seek to overcome these estimation problems by using market-implied forward-looking estimates for the proper weights. In particular, with the swaptions market providing rich forward-looking volatility information, and with the YCSO market providing forward-looking correlation information, it is straightforward to combine these to obtain implied weights for use in weighted curve or butterfly trades. We also discuss a hybrid approach that combines implied volatilities with empirical correlations that can be used when implied correlations are not available**
- **We also discuss the use of swaption skews to glean market expectations of nonlinear yield curve behavior**
- **The jury is still out on the long term performance characteristics of implied weights, but several benefits are already evident. In particular, implied weights may offer the most value when they swing the most, signaling regime shifts in yield curve behavior that empirical schemes will only slowly pick up. Also, they offer an alternate risk decomposition approach for risk management purposes.**
- **We back-test a relative value trading strategy using implied weights, and present comparative results**

Exhibit 1: The equal-weighted 2s/10s/30s swap butterfly spread, adjusted for the level of yields and the curve

In-sample residual from regressing the 2s/10s/30s swap yield butterfly against the level of 10-year swap yields and the 2s/30s yield curve; 05/2009 – 05/2011; %



Introduction

The purpose of this research note is to explore a novel scheme to arrive at risk weightings used in yield curve relative value trades. For instance, it is relatively commonplace for investors to hedge a curve trade (steepener or flattener) against its exposure to the level of rates, producing a weighted curve trade (assuming the hedge is constructed using one of the two legs in the original curve trade). Similarly, investors who trade 50:50 risk-weighted yield curve butterflies have long recognized that such trades carry implicit exposure with respect to the level of rates as well as the slope of the yield curve. Again, it is relatively commonplace to

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hedge an equal-weighted butterfly trade's embedded exposure to the level of yields and the slope of the curve; if one uses (say) the belly of the butterfly as the chosen instrument to hedge rate exposure, and if curve exposure is hedged using the spread between the two wings, then the hedged trade amounts to a unequally risk-weighted butterfly. Making matters more precise, if we assume that:

$$F \equiv y_M - (0.5*(y_L+y_R)) = \beta_l\, y_M + \beta_c(y_R - y_L) + \alpha + \varepsilon$$

where F denotes the equal-weighted butterfly spread, y_L, y_M and y_R denote the yields on the left wing, middle leg and the right leg respectively, and β_l and β_c denote level and curve betas respectively of the 50:50 butterfly, then a modified trade that is hedged against moves in rate levels and the slope of the curve would involve trading the weighted butterfly spread given by

$$F_w := y_M - ((0.5-\beta_c)/(1-\beta_l))\, y_L - ((0.5+\beta_c)/(1-\beta_l))\, y_R$$

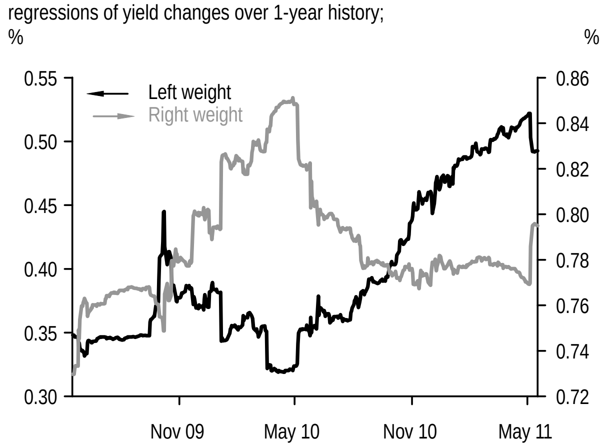
Such a trade would attempt to profit from the residual mispricing (in a statistical sense) of the butterfly, after adjusting for the current level of yields and the curve. For instance, the equal weighted 2s/10s/30s swap butterfly spread recently appeared too wide after adjusting for the level of 10-year yields and the 2s/30s curve, using a regression over one year of history (**Exhibit 1**). Should an investor expect mean reversion in this spread, he/she could position for it by receiving fixed in the belly of a weighted swaps butterfly trade. In particular, using the estimated beta of the equal-weighted fly with respect to 10-year yields (0.3) and curve (0.1), the investor would receive fixed in the belly of a -57/+100/-86 risk weighted butterfly, expecting to profit from a correction in fair value while being agnostic to any subsequent moves in the level of yields and/or the curve.

The many perils of backward-looking estimates of yield curve weightings

All this seems simple enough; unfortunately, the problem, all too common in the world of finance, is that history never perfectly repeats itself—i.e., in our context, risk weights estimated using inherently backward-looking regressions or PCA may not properly estimate the "correct" weightings that might characterize subsequent yield movements at different

Exhibit 2: Level and curve neutral weights on the 2-year and 30-year legs of a 2s/10s/30s swap butterfly can vary significantly even under stable conditions …

Weights on left and right wings of a 2s/10s/30s butterfly calculated using rolling regressions of yield changes over 1-year history;



points on the curve. This is evident simply by looking at a time series of backward-looking left and right wing weights on a level and curve neutral 2s/10s/30s swap butterfly (**Exhibit 2**).

The success of weighted yield curve strategies hinges on the proper estimation of *forward-looking* weights, since the investor is only exposed to the empirical exposures that unfold after the trade is initiated. Regression or PCA-based weighting schemes are by nature backward-looking, so as a practical matter the stability of weights over recent history is viewed as a loose indicator of ongoing stability. However, empirical observation suggests that such stability is difficult to find, leaving relative value trading strategies routinely exposed to moves in yield levels and curve due to shifting weights. Moreover, under certain market conditions, stability of weights can be conceptually unreasonable to expect, leaving a relative value trader with no good way to estimate the appropriate weights going forward. For instance, in periods of transition between Fed regimes, such as the current period (where we may be in the early stages of transitioning from a Fed on hold regime to a tightening regime), it is a given that historical estimates of relative volatilities will prove unreliable going forward, if Fed policy expectations begin to shift (**Exhibit 3**).

Nonlinearities can offer another reason for non-stable weights. For instance, in low yield regimes such as the

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Exhibit 3: … let alone when the market is transitioning between Fed regimes …

Estimation error (defined as rolling 6M forward looking level and curve neutral weight on the 2Y leg of a 2s/10s/30s swap butterfly, minus corresponding 6M backward looking weight), versus the fed funds target rate;

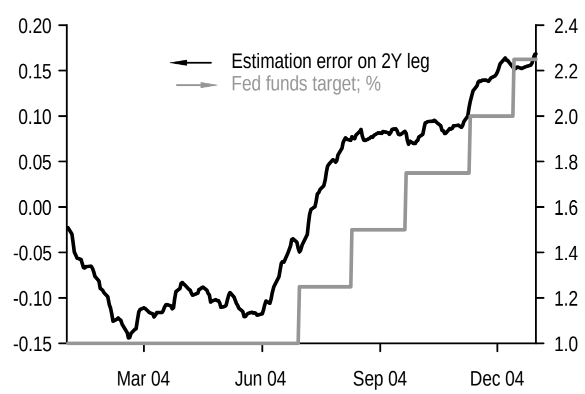
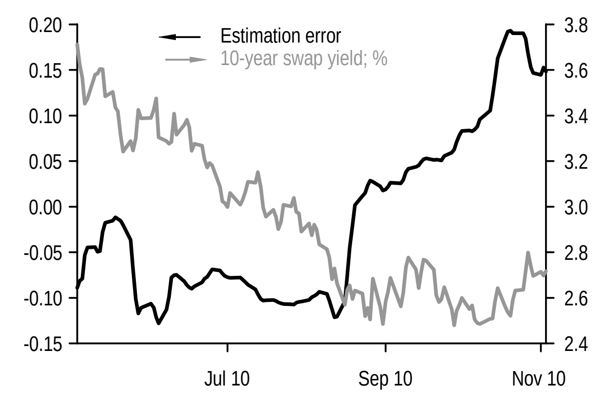


Exhibit 4: … or if yield curve relationships become nonlinear due to reasons such as low yield levels, producing greater estimation as yields move

Estimation error (defined as rolling 6M forward looking level and curve neutral weight on the 30Y leg of a 2s/10s/30s swap butterfly, minus corresponding 6M backward looking weight), shown versus the level of 3Mx10Y forward swap yield;



present, the proximity of yield levels to absolute barriers (such as the zero rate boundary) or even empirical barriers (such as the funds rate in periods where the Fed is not expected to ease) can cause weights to depend on yield levels themselves, producing inherent instability in yield curve weightings. We have discussed at length nonlinearities in yield curve behavior in very low yield regimes, in the US as well as in Japan (see Treasuries, *US Fixed Income Markets 2011 Outlook*, 11/24/2010). This can result in unstable regression-based weights. For instance, in Fall 2010, estimation errors (defined as the difference between the rolling 6-month "forward-looking" empirical weight on the 30-year leg of a 2s/10s/30s butterfly and the corresponding backward-looking weight) rose as yields settled into a new low yield regime. In other words, even though yields appeared to stabilize, creating good overall conditions for trading relative value, such trades would have experienced greater risk due to higher weight estimation errors (**Exhibit 4**), itself a result of the inherent nonlinearity in yield curve relationships in low-yield regimes.

A novel scheme for generating forward-looking weightings

Having argued that backward looking techniques (such as regressions) for estimating the proper weightings for use in relative value trades can suffer from significant estimation errors under some conditions, we turn our attention to the next logical question—can we somehow generate forward-looking weightings? And if so, does such a weighting scheme offer superior performance relative to baseline strategies?

We can indeed generate forward-looking weights for use in yield curve relative value trades, by using "implied" information from the options markets. For instance, taking the example of a 2s/10s/30s butterfly, if we assume that:

$$\Delta y_{10} = \beta_2 \, \Delta y_2 + \beta_{30} \, \Delta y_{30} + \varepsilon$$

it is then mathematically the case that:

$$\beta_2 = (\sigma_{10} / \sigma_2)\,(c_{2,10} - c_{10,30}\, c_{2,30}) / (1 - (c_{2,30})^2)\ ,$$

and

$$\beta_{30} = (\sigma_{10} / \sigma_{30})\,(c_{10,30} - c_{2,10}\, c_{2,30}) / (1 - (c_{2,30})^2)\ ,$$

where $c_{2,10}$ denotes the correlation coefficient between 2- and 10-year yield changes; $c_{10,30}$ and $c_{2,30}$ respectively denote the correlation coefficients between 10- and 30-year yields and 2- and 30-year yields; and σ_2, σ_{10} and σ_{30} denote the normal volatilities of 2-, 10- and 30-year yields. Similar formulas for the univariate and tri-variate cases are included in the Appendix. We

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Exhibit 5: Forward-looking implied weights can differ significantly from empirical weights …

Implied as well as empirical weights* on the left and right wings of various forward-swap benchmark butterflies (3-, 6- and 9-month forward terms);

Butterfly	Mat	Left Weight (%)		Right Weight (%)	
		Empirical	Implied	Empirical	Implied
2s/5s/7s	3M	35.9	22.4	76.1	83.1
	6M	34.3	-0.5	75.1	100.1
	9M	33.0	2.3	74.2	98.9
2s/5s/10s	3M	63.7	35.2	59.7	74.2
	6M	59.4	27.8	58.5	74.5
	9M	56.1	33.1	57.2	70.4
2s/10s/30s	3M	45.3	57.9	77.6	79.7
	6M	41.4	48.4	76.3	79.2
	9M	38.7	42.2	75.2	79.7
5s/10s/30s	3M	53.2	47.6	53.1	60.3
	6M	52.4	47.0	53.0	62.1
	9M	52.0	46.3	52.7	62.3

* Implied as well as empirical weights as of 5/24/2011. Empirical weights calculated using 1-year history, based on a regression of forward swap yield changes

may think of regression-based weightings as simply using historical volatilities and historical correlation estimates in the above equations, thereby producing historical estimates for the betas (and thus weightings). Of course, we can also plug in *implied volatilities*, easily available from the swaptions market, and *implied correlations* (available from the Yield Curve Spread Options market) into the equations above; doing so produces implied betas or forward-looking weightings.

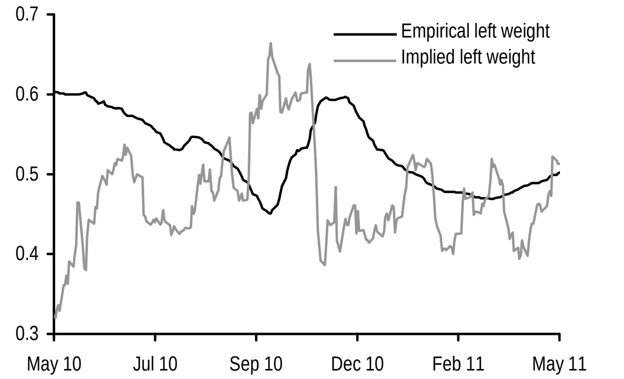
That is the essence of our approach—combining *implied volatilities* from the swaptions market and *implied correlations* from the curve options market (or any other bivariate options market) to calculate *implied betas* for use in relative value trades.

Comparing forward looking and historical weightings

Before going into the results of back-testing both strategies to measure their performance, we first look at the raw differences between the historical and forward-looking schemes by inspecting the recent weightings obtained for different benchmark butterflies. As seen in **Exhibit 5**, there are currently significant differences between implied and empirical weights, particularly for butterflies anchored at the front end of the yield curve. Differences are smaller for butterflies that are further out the curve, falling to less than 15% for the outmost one considered above (the 5s/10s/30s). It is also worth

Exhibit 6: … not just currently, but over time

Implied and empirical weight* on the 2-year leg of a 2s/10s/30s butterfly;



* Implied weight based on implied correlations and implied vols on 3-month expiry options. Empirical weights based on rolling 1-year regressions of yield changes in 10s versus 2s and 30s.

examining the evolution of the empirical versus implied weights over time. **Exhibit 6** shows a time series of the weight on the left leg of a 2s/10s/30s butterfly, estimated using the implied as well as empirical approaches. As can be seen, implied weights are more volatile—unsurprising, since empirical regressions are less reactive to recent data (unless of course one uses some weighting scheme to give more importance to recent data).

At first, the greater volatility in implied weights might seem like a disadvantage. However, to the extent that it merely reflects noise, it is easily smoothed (say, by using the moving average of the weight over the past one or two weeks). More importantly, using a forward-looking approach might have a clear advantage in periods of changing in market conditions. We examine this point further later in this paper, in our discussion of the results from backtesting our approach.

A final point—applying our approach in its purest form requires implied volatilities, which are generally easily available, as well as implied correlations, which may not always be available. For instance, good implied correlation data is unavailable for periods prior to 2007 in the US markets. As a workaround, we also evaluate a third "hybrid" approach, which uses implied volatilities in conjunction with empirical correlations. The premise here is that correlations are more stable than relative volatilities, and thus most of the "forward-looking" weighting information will be preserved by

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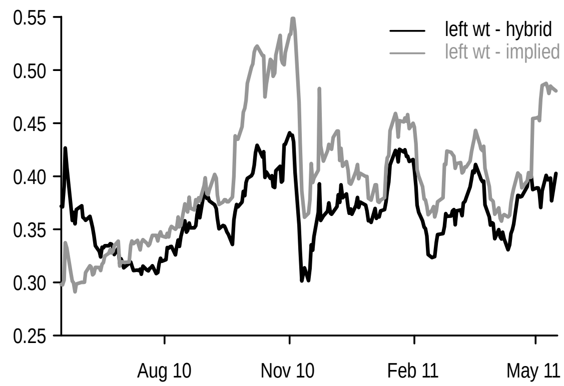
Exhibit 7: A hybrid approach that combines implied volatilities with empirical correlations appears to offer a viable alternate approach when implied correlations are not available

Implied and hybrid estimates* for the weight on the 2-year leg of a 2s/10s/30s butterfly



* Hybrid weights calculated using rolling 6-month correlations and 6-month expiry implied volatilities.

Exhibit 8: The ex-post stability of butterfly spreads based upon implied and/or hybrid weights has been similar to or better than those based on empirical weights in over the past four years

Mean and standard deviation of subsequent 1-, 3- and 6-month changes in various level and curve neutral butterfly spreads*; bp

	1M horizon		3M horizon		6M horizon	
	Average	Std. dev	Average	Std. dev	Average	Std. dev
2s/5s/10s						
Implied	0.9	4.4	2.4	7.4	4.2	10.1
Empirical	0.9	4.6	2.6	8.3	4.3	11.2
Hybrid	0.5	4.4	1.8	7.6	3.1	10.5
2s/10s/30s						
Implied	0.6	8.8	2.6	13.4	5.8	16.7
Empirical	0.8	8.9	3.0	13.5	6.8	16.6
Hybrid	0.6	9.0	2.7	13.5	6.3	16.8

* Implied weights estimated using 6-month options. All butterflies are 6-months forward. Empirical weights based on backward looking 6-month regression of yield changes.

using implied vols, even if we are forced to use empirical correlations. This view is not entirely correct, and there can be significant deviations between implied weights and hybrid weights as seen in **Exhibit 7**; nonetheless, Exhibit 7 also shows that hybrid weights are sufficiently similar to implied weights to validate the choice of a hybrid approach in cases where implied correlations are unavailable.

Are implied weightings the better choice?

We approach this question in two different ways. First and foremost, more accurate weights should produce a butterfly spread that is better hedged going forward. Thus, the standard deviation of subsequent changes over varying horizons (say, 1- to 6-months) in the weighted butterfly spread is one measure of the quality of the weights. On this basis, implied weights generally appear to perform at least as well as empirical weights—standard deviations of changes in the weighted butterfly spread using implied weights have been similar to or better than when using empirical weights. Indeed, the same can be said for a hybrid weighting scheme as well, as seen in **Exhibit 8.** This suggests that for risk management applications, such as decomposing the risk in fixed income portfolios into benchmark sectors, implied weights can play a useful role, complementing the use of empirical decomposition schemes.

Another way to assess the quality of implied weights is to back-test all three weighting schemes using a trading strategy over the past two years. Our trading rule, applied to a 2s/10s/30s butterfly over the past two years, is as follows: for every day in the two year back-testing period, we compute left and right leg weights on a 2s/10s/30s butterfly using the different approaches. Using the calculated weights, we then calculate the 1-year historical Z-score of the weighted butterfly yield spread at that point in time, and a trade is initiated if the absolute value of the Z-score is above 1.5. A positive Z-score above 1.5 results in a long-belly trade, while a Z-score below -1.5 triggers a short-belly position. Trades are exited at a profit when the butterfly spread converges to the mean, stopped out at a loss if the butterfly spread moves an additional 1.5 standard deviation away from the mean and unwound at a 3-month horizon if neither of the previous conditions is met.

Results from such a trading strategy are presented in **Exhibit 9**, and several points are worth noting. First, over the past two years, a pure implied weighting approach appears to have modestly outperformed an empirical weighting scheme, with a higher average return as well as a higher information ratio. Interestingly, the hybrid approach over this period underperformed both approaches, suggesting that

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Exhibit 9: Over the past two years, a trading strategy based on implied weights modestly outperformed strategies based on empirical and hybrid weights

Number of trades, average P/L, standard deviation and information ratio per trade started in the 502 day period 5/04/2009-5/04/2011

	Empirical	Implied	Hybrid
# of trades	57	56	62
% days trade initiated	11%	11%	12%
Hit Ratio	89%	91%	74%
Average (bp)	6.2	8.7	6.7
Std Dev (bp)	4.7	5.5	7.9
Inf Ratio	1.3	1.6	0.8
5th percentile (bp)	-5.6	-2.2	-8.9
Median (bp)	6.4	7.9	9.3
95th percentile (bp)	12.9	16.5	15.7

implied correlations are crucial to the success of a forward-looking weightings scheme, for the purpose of predicting subsequent mean reversion, something we had not anticipated prior to our analysis. To be sure, two years of history is insufficient to draw firm conclusions regarding the various approaches, and these results bear watching going forward.

A second conclusion is that in episodes characterized by significant "regime change" (by which we mean a shift in market expectations that is large enough to trigger a significant change in the implied weights), forward-looking schemes have a clear advantage. For instance, in late 2010, growing expectations of QE2 after Bernanke's Jackson Hole speech in late August resulted in arguably a different yield curve regime. This was quickly reflected in implied weights but naturally not in empirical weights (**Exhibit 10**). As a result, in the following months, back-testing a strategy using implied weights produced superior return-to-risk characteristics versus a trading strategy based on empirical weights (**Exhibit 11**)—fewer trades resulted from both forward-looking weighting schemes (the "pure implied" as well as hybrid approaches), but hit ratios, average P/L and the information ratio were all better than the strategy based on empirical weights. Interestingly, the hybrid approach performed the best over this time period. The fact that the hybrid strategy performed the best might be coincidence over this period, but it does suggest that in periods of regime shifts, the value of using new information regarding relative volatilities is sufficient to outperform purely empirical strategies, even if correlation estimates remain backward-looking.

Exhibit 10: Implied weights reacted swiftly to growing QE2 expectations in the aftermath of Fed Chairman Bernanke's Jackson Hole speech in late August 2010

Implied weights on the 2- and 30-year legs of a 2s/10s/30s swap butterfly

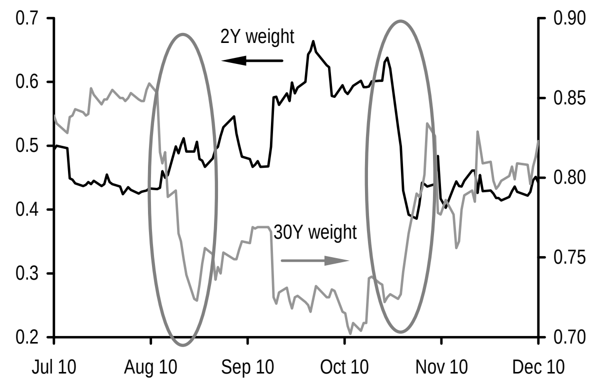

Exhibit 11: Implied and hybrid weighting schemes both outperformed in Fall '10, hinting at the value of forward-looking information in periods of regime shifts

Number of trades, average P/L, standard deviation and information ratio per trade started in the period 8/09/2010-11/15/2010

	Empirical	Implied	Hybrid
# of trades	25	13	15
% days trade initiated	37%	19%	22%
Hit Ratio	92%	100%	100%
Average (bp)	7.7	11.3	11.2
Std Dev (bp)	4.2	4.5	3.7
Inf Ratio	1.8	2.5	3.0
5th percentile (bp)	-3.2	6.2	6.6
Median (bp)	8.2	10.2	10.0
95th percentile (bp)	12.5	18.8	17.0

Extracting nonlinearity information from implied volatility skews

It is well understood that when yields are sufficiently close to zero (or even a non-zero "effective" barrier), volatility falls and the distribution of yield changes becomes more lognormal rather than normal. This can pose significant challenges to regression-based estimation approaches when yields are low. In addition, as discussed earlier, this also raises the risk to relative value trades, as weightings implemented at trade

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Exhibit 12: Implied volatility skews can be used to infer likely shifts in ATM implied volatility as yields shift …

Statistics regarding 6-month expiry implied volatility skews on 2-, 10- and 30-year tails and skew-implied change in ATM implied volatility under various rate shifts; bp/day

		Implied vol			Estimated ATM under rate shifts		
	ATMF	A-25	A	A+25	-25 bp	0 bp	+25 bp
2Y	1.111	4.25	4.82	5.40	3.69	4.82	5.98
10Y	3.466	6.31	6.42	6.57	6.20	6.42	6.71
30Y	4.126	5.84	5.84	5.87	5.84	5.84	5.90

* As of COB 5/17/2011

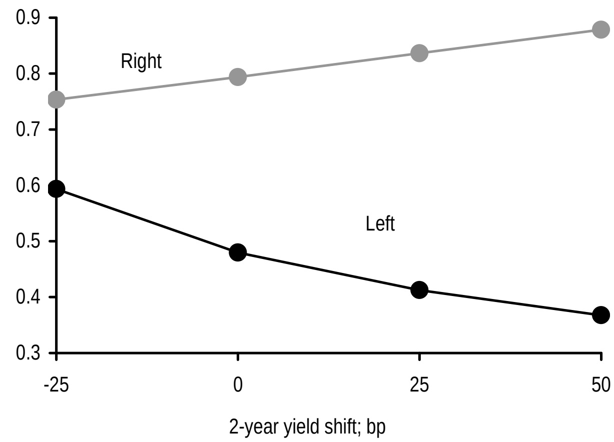
inception prove inaccurate as yields subsequently drift significantly lower or higher.

To the extent that volatility is systematically related to yield levels, which would in turn cause implied weightings to shift with rates, we may extract that information *ex-ante* from swaption implied volatility skews. A generally accepted rule of thumb is that ATM implied volatility should rise with yields at a rate given by twice the skew—i.e., suppose 6Mx2Y A+25 strike implied volatility is 0.6bp/day higher than the ATMF strike implied volatility, which is currently the case. This suggests that if the underlying forward yield were to rise by 25bp, the then-at-the-money implied volatility would be 1.2bp/day higher than the current at-the-money implied vol. Thus, swaption skews on 2-, 10- and 30-year tails yield information about the yield dependency of volatility in those respective sectors. We may combine that with some reasonable assumption regarding the relative movement of 2-, 10- and 30-year rates to calculate a profile of how implied weights are expected to change as a function of yields.

This is further detailed below, sticking with our example of the 2s/10s/30s butterfly. **Exhibit 12** presents statistics on implied volatilities at various strikes, and an estimate of what then-ATM implied volatilities would be under various rate shifts in each sector (calculated as mentioned above). Now suppose 2-year yields fall by 25bp. As an approximation, we may assume that 30-year yields fall by 1.2 times that amount (the ratio of current ATM implied volatilities on 30-year tails to 2-year tails), or 30bp. Furthermore, we may weight the assumed moves in 2- and 30-year rates by their implied betas (as used in the earlier sections of this paper) to estimate the change in 10-year yields—currently, that amount is about 36bp. Also, as already noted, Exhibit 12 allows us to project what the

Exhibit 13: … which, when combined with implied correlations, can be used to estimate *a priori* the likely shifts in risk weights as yields move

Projected evolution of left and right leg weights for a 2s/10s/30s butterfly under various 2-year swap yield shifts*



* Our methodology is as follows. For each 2-year rate shift, we calculate the 30-year rate shift based on the current ratio of ATM vols. We then combine the 2- and 30-year rate shifts with the current implied weights to calculate a 10-year rate shift for that scenario. Third, we use the current implied vol skews to estimate the change in then-ATM implied vols for those rate shifts in each leg. Last, armed with estimates for then-ATM implieds in all three legs, we calculate implied left and right weights as discussed previously in this paper.

ATM implied volatilities would be if such yield shifts were indeed to occur, by interpolating or extrapolating for suitable yield shifts. Last, armed with estimates for the then-ATM implied vols in all three sectors if 2-year yields were to fall 25bp, we can—as before—calculate new implied weights on the left and right wings of the butterfly. In order to do so, we assume that implied correlations remain constant as yields shift, an assumption that is necessitated by the lack of good correlation skew data.

In sum, by carefully combining information from swaption implied volatilities, skews and implied correlations, we may estimate at trade inception how the left and right weights are likely to change with yields—for our chosen example, this is shown in **Exhibit 13**.

How can we utilize this in a trading strategy? Consider a long-the-belly butterfly trade that has been constructed to be level and curve neutral at inception. Now suppose front end yields decline 25bp; Exhibit 13 tells us that the "new" level and curve neutral weights would be higher at the front end and slightly lower at the back end. This leaves the trade—as constructed—

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long the market in an environment where yields are falling. (The slight decline in right-leg weight is a modest negative, but not enough to overcome the effect of falling left-leg weights). Similarly, the fall in left-leg weights in a sell-off implies the trade progressively becomes net short in a sell-off. Such positive convexity with respect to front-end yields is a desirable characteristic; thus, all else equal, relative value trades that involve being long the belly are to be favored over short-belly relative value trades of similar statistical attractiveness.

Conclusions

In this paper, we have proposed a novel scheme to calculate the appropriate weights to hedge longs/shorts in one point on the curve with one or more other points on the curve. Our scheme, in its purest form, relies on *implied volatilities* and *implied correlations* from the options market and uses no backward looking information; as such, we may think of this as an *implied weighting* scheme. We also discuss a hybrid scheme that combines commonly available implied volatilities with empirical correlations; such a hybrid scheme would be more widely applicable, extending to cases where implied correlations are not available.

The jury is still out on whether implied weightings work better in the long run, and more research is needed to fully understand the pros and cons of implied weights vis-à-vis traditional backward-looking weights. However, implied weights offer sufficient benefits to warrant further consideration, and several points are already noteworthy. First, the implied and/or hybrid schemes offer an alternate (and perhaps superior) risk decomposition strategy for use in portfolio risk management; over the past four years, we found that in the case of the 2s/5s/10s as well as 2s/10s/30s butterflies, weightings based on implied/hybrid schemes produced similar or better subsequent stability in level/curve neutral butterflies. Second, implied weights may be most useful when they swing considerably over a short period of time, signaling a regime shift in yield curve behavior. Indeed, back-testing the use of implied weights in a butterfly trading strategy, we find that implied weights were superior in such regime shift episodes. Third, our scheme for computing implied weights can be extended to incorporate swaption skew information to provide an understanding of nonlinear yield curve behavior. Such nonlinearity can lend an additional edge in relative value trading strategies.

Appendix

Our formulas for implied betas can be derived from a general case. If we assume that yield changes Δy_a are correlated to N others, $\Delta y_1, \dots, \Delta y_N$, such that:

$$\Delta y_a = \beta_1 \Delta y_1 + \beta_2 \Delta y_2 + \dots + \beta_N \Delta y_N + \varepsilon$$

then, the betas are related to the volatilities and correlations via the following equation

$$\mathbf{v} = \sigma_a \mathbf{c}^{-1} \mathbf{w} \,,$$

where σ_a is the volatility of Δy_a and we introduced the vectors

$$\mathbf{v} = (\beta_1 \sigma_1, \beta_2 \sigma_2, \dots, \beta_N \sigma_N) \,,$$

and

$$\mathbf{w} = (c_{a,1}, c_{a,2}, \dots, c_{a,N}) \,,$$

while $\mathbf{c}$ is the NxN matrix of correlations with entries $c_{i,j}$ - the correlation coefficient between Δy_i and Δy_j. The entries in the diagonal of $\mathbf{c}$ are equal to 1, det $\mathbf{c}$ stands for its determinant and $\mathbf{c}^{-1}$ for its inverse. Similarly, $c_{a,1}, c_{a,2}, \dots, c_{a,N}$ represent the correlation coefficients between Δy_a and $\Delta y_1, \Delta y_2, \dots, \Delta y_N$, and σ_1, σ_2 and σ_N are the volatilities of Δy_1, Δy_2 and Δy_N respectively.

As a further example, in the trivariate case (N = 3) the formulas above yield

$$\beta_1 = (\sigma_a / \sigma_1) \left[c_{a,1} (1 - (c_{2,3})^2) - c_{a,2} (c_{1,2} - c_{1,3} c_{2,3}) + \right.$$
$$\left. + c_{a,3} (c_{2,3} c_{1,2} - c_{1,3})\right] / (\det \mathbf{c}) \,,$$

$$\beta_2 = (\sigma_a / \sigma_2) \left[- c_{a,1} (c_{1,2} - c_{1,3} c_{2,3}) + c_{a,2} (1 - (c_{1,3})^2) - \right.$$
$$\left. - c_{a,3} (c_{2,3} - c_{1,2} c_{1,3})\right] / (\det \mathbf{c}) \,,$$

and

$$\beta_3 = (\sigma_a / \sigma_3) \left[c_{a,1} (c_{1,2} c_{2,3} - c_{1,3}) - c_{a,2} (c_{2,3} - c_{1,2} c_{1,3}) + \right.$$
$$\left. + c_{a,3} (1 - (c_{1,2})^2)\right] / (\det \mathbf{c}) \,,$$

where

$$\det \mathbf{c} = 1 - (c_{1,2})^2 - (c_{1,3})^2 - (c_{2,3})^2 + c_{1,2} c_{1,3} c_{2,3} \,.$$

Trivially, for the univariate case (N = 1), in which there is a single beta we have

$$\beta_1 = (\sigma_a / \sigma_1) c_{a,1} \,.$$

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