A Complete and Robust Set of Backtesting Tools:

THE FIRST STEP IN OVERCOMING THE MANY CHALLENGES OF BACKTESTING

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I. Introduction

Practitioners and academics test many investment strategies over various time horizons. This paper will discuss how to overcome a wide array of mechanical difficulties that can occur while completing a backtest.

- In Section I, we'll summarize the most frequent challenges that occur during backtesting and how Axioma solves for them.
- In Section II, we'll look at a frontier backtest example that highlights Axioma's comprehensive set of backtesting functionalities.
- In Section III, we'll detail other useful features.

Many practitioners have experienced frustration in completing a backtest when they run into failures as the number of backtesting periods increases or as the strategy becomes more complex. In either case, the likely culprit is the optimizer failing to converge onto a solution during a rebalancing period. It can be time consuming to diagnose the source of the failure, apply the necessary adjustment(s), and then rerun the backtest. Often, the practitioner can become further frustrated when a modified backtest fails yet again during a different time period. In addition to this common occurrence, Table 1 outlines the most frequently occurring challenges during backtesting and Axioma's respective solutions.

	CHALLENGE	SOLUTION
Infeasibility Handling	1. Simple Infeasibilities	Infeasibility Diagnosis / Failed Workspace
	2. Competing Objective and Constraints	Soft Constraints
	3. Complex Infeasibilities	Constraint Hierarchy
	4. Interrupted Backtest	Backtest Resumption
Flexibility and Ease of Setup	5. Initial Backtest Setup	TimeSeries Workspace Converter
	6. Rebalancing on Various Frequencies	Calendar Recurrence Tool
	7. Testing through Changing Regimes	Dated Strategies
	8. Conditional Changes to Strategies	Dynamic Strategies
	9. Varying a Strategy Parameter	Frontier Backtesting
Understanding the Results	10. Analyzing Individual Periods	Period Summary / Per-Period Output
	11. Reviewing Overall Performance	Backtest Summary
	12. Analyzing the Impact of Constraints	Constraint Attribution
	13. Understanding Overall Performance	Feed into Risk Analysis and Performance Attribution by Axioma Portfolio Analytics

Table 1. A summary of frequently occurring difficulties in backtesting.

II. A Frontier Backtest Example

In recent years, institutional mandates have witnessed the rise of so-called smart beta funds. These funds attempt to bridge the gap between passive and active management by taking major indices and reweighting them according a specific factor in order to beat the market in the long term instead of simply hugging the index.[1] Since factors have different risk and return profiles, managers can take the smart beta framework one step further by combining different factors to achieve superior risk-adjusted returns. In such cases, constructing an efficient portfolio through understanding the risk-return trade-off relationship is not only required, but demands rigorous backtesting.

Figure A examines the factor return trends for four style factors from Axioma's US4 medium-horizon fundamental factor model. Historically, medium-term momentum and profitability trended positively, whereas size and volatility trended in the opposite direction. The diagram also illustrates their cumulative factor returns since 2006.

[1] Ian Webster (2016). *Multi-factor Investing: Practical Considerations for Portfolio Managers.*

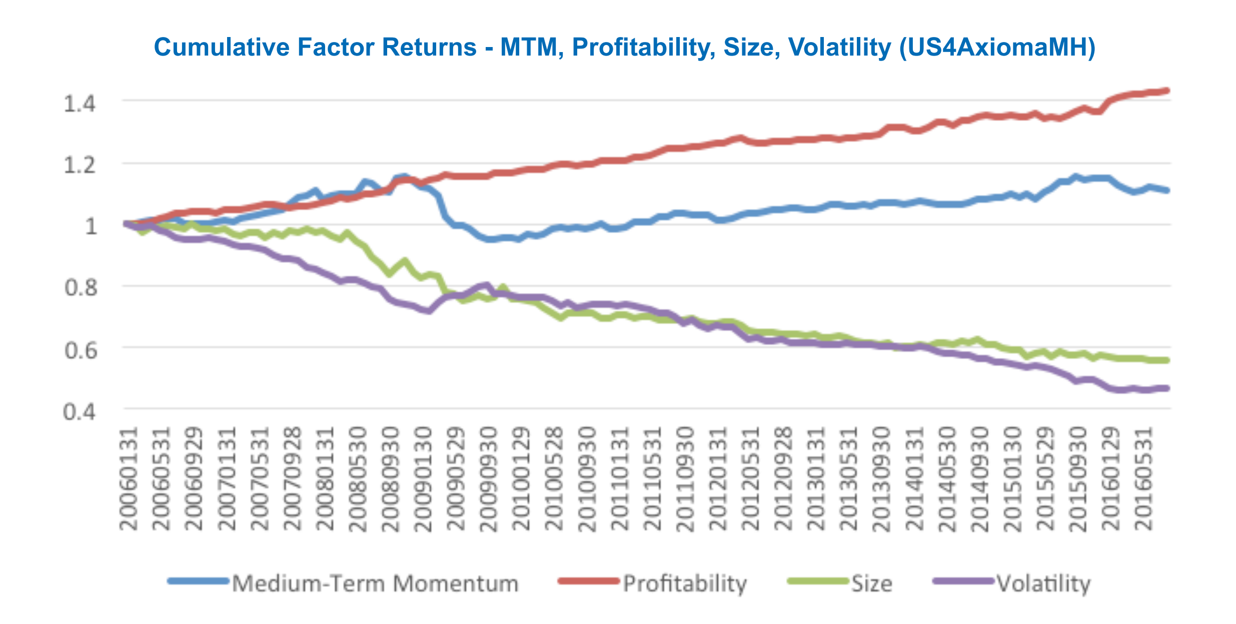

Figure A. Cumulative returns of 4 US4AxiomaMH style factors.

Strategy

For our example backtest, we will evaluate the following strategy from Dec. 31, 2005, to Dec. 31, 2015, using Axioma's US4 medium-horizon fundamental factor risk model and the Russell 3000 benchmark:

Maximize: αp – λ TEp	SOLUTION
(Objective Function)	A strategy aimed to maximize the balance between the portfolio's expected return and predicted active risk. Expected return is a combination of 50% medium-term momentum and 50% profitability.
S.T. (Constraints)	
wi >= 0	Long only
∑wi = 100%	100% invested
0.95 <= βp <= 1.05	Range for portfolio predicted beta
80 <= Number of Names <= 120	Range for number of names held
Active Volatility Factor Exposure <= 0	No active exposure to Volatility
Active Size factor Exposure <= +/- 0.1	Active exposure to Size within +/- 0.1 standard deviation
Active Style Factor Exposures <= +/- 1	Active style factor exposures within +/- 1 standard deviation
Active Sector weights <= +/- 2%, drift allowed to +/- 3%	Typical active weight constraint
Active Sector weights <= +/- 3%, hard bound	Typical active weight constraint
Active Ind.G. weights <= +/- 2%, drift allowed to +/- 3%	Typical active weight constraint
Active Industry Group weights <= +/- 3%, hard bound	Typical active weight constraint
Active Industry weights <= +/- 2%, drift allowed to +/- 3%	Typical active weight constraint
Active Industry weights <= +/- 3%, hard bound	Typical active weight constraint
Monthly Turnover <= 20%	20% two-way turnover per month

Execution

Using Axioma's frontier backtesting capability, we simultaneously ran 16 backtests, each with a different active risk aversion (λ). The active risk aversion (λ) ranged from -10 on the first backtest, which is less risk-averse, to -60 on the last backtest, which is more risk-averse. This frontier aims to depict the trade-off relationship between active risk and expected return. Each backtest on the frontier is rebalanced monthly.

We used the following backtesting features from Table 1 while completing the frontier backtest example:

a. TimeSeries Workspace Converter

Challenge – Initial Backtest Setup: The initial setup of a backtest can be complex and time consuming. This is due to a large amount of data dependencies, such as risk models, fundamental attributes, benchmarks, classifications, and other data items required to complete the backtest.

Solution: The TimeSeries Workspace Converter helps the user streamline the setup process by effectively converting a point-in-time workspace into a time-series workspace that is ready for backtesting. In this frontier backtest example, a point-in-time workspace was created first and converted seamlessly in this fashion. Figure B1 below shows the tool in action:

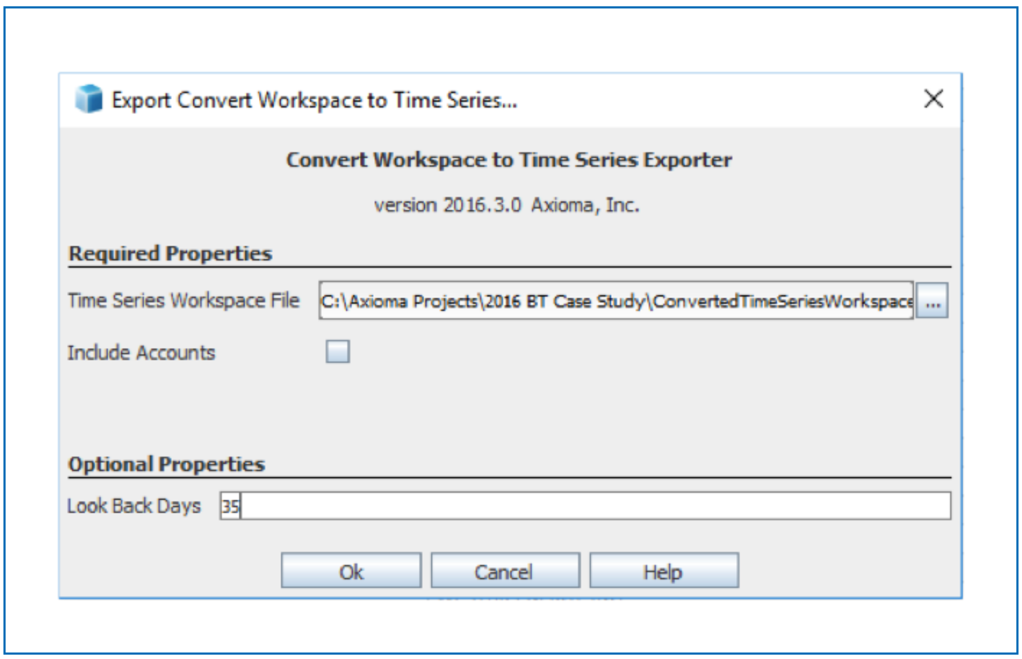

Figure B1. TimeSeries Converter creates a backtest with a single click.

b. Calendar Recurrence Tool

Challenge – Rebalancing on Various Frequencies: We needed to select a large number of rebalancing dates over the time horizon of a backtest.

Solution: With the Calendar Recurrence Tool, we were able to create rebalancing frequency combinations on daily, weekly, monthly, and yearly bases. Additional logic specifying particular week days, weeks, and months can also be employed to customize the dates. Figures B2 and B3 illustrate several possible combinations. Figure B2 details the rebalancing frequency the frontier backtest example used:

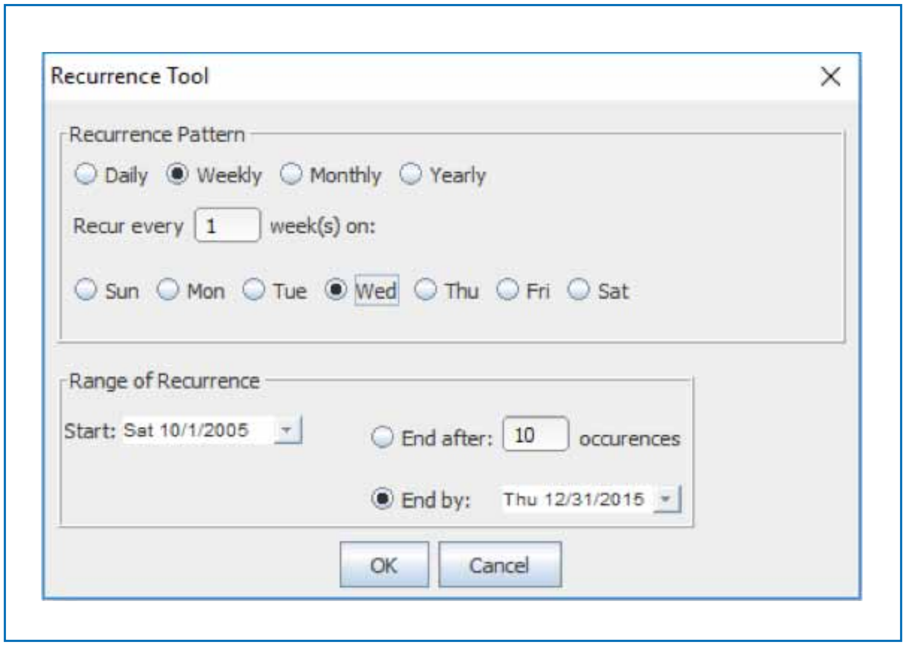

Figure B2. Weekly rebalancing frequency on very Wednesday.

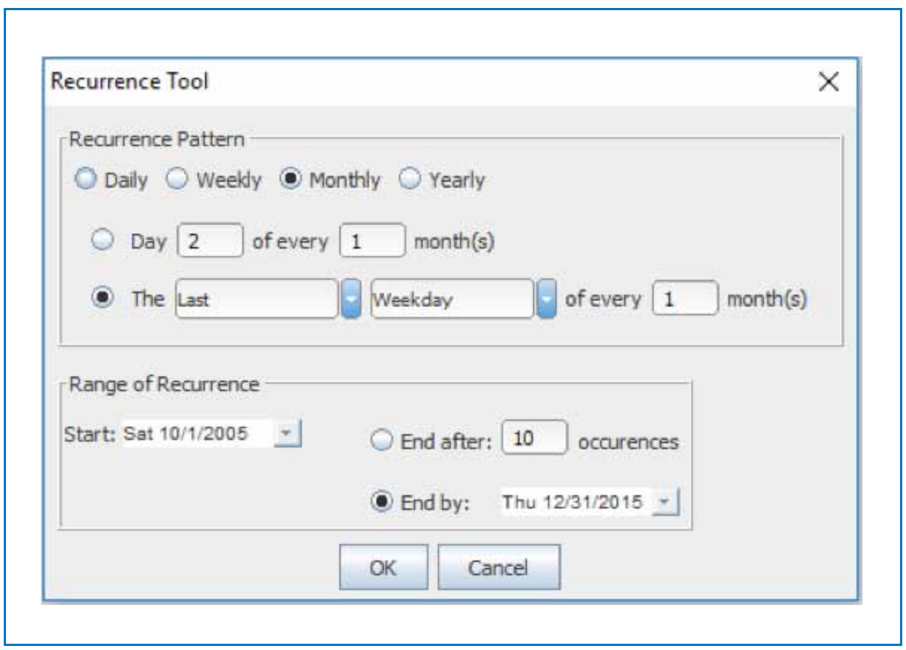

Figure B3. Monthly rebalancing frequency on the last weekday of each month.

c. Soft Constraints

Challenge – Competing Objective and Constraints: Competing constraints often lead to optimization infeasibilities during certain time periods. Suppose a strategy has a tracking error limit constraint at 1%, and at the same time limits the total number of names held to a restrictively low number. These competing constraints can cause the backtest to fail during certain periods. Additionally, a practitioner may occasionally desire to relax certain constraints in order to improve the overall solution value, such as by allowing for more turnover if he or she can achieve significantly more alpha.

Solution: Soft constraints present a possible solution to these challenges by allowing the user to establish priorities between the objectives and constraints. Another added benefit is that soft constraints often promote backtest continuity.

In the frontier backtest example, the turnover constraint with a total limit of 20% (two-way) was made into a soft constraint with a linear penalty of 2. Since the optimization objective function is {1 x α- λ x TE}, the linear penalty on turnover modifies the objective function to {1 x α-λ x TE- 2 x (turnover above 20%)}. This allows the optimizer to violate the turnover constraint when any amount of turnover beyond 20% results in twice as much of an improvement in the original objective function. Furthermore, a side benefit to relaxing the turnover constraint is that it ensures backtesting continuity in case the optimizer can't find feasible solutions within a 20% two-way turnover limit. Figure B4 shows how the turnover constraint was softened:

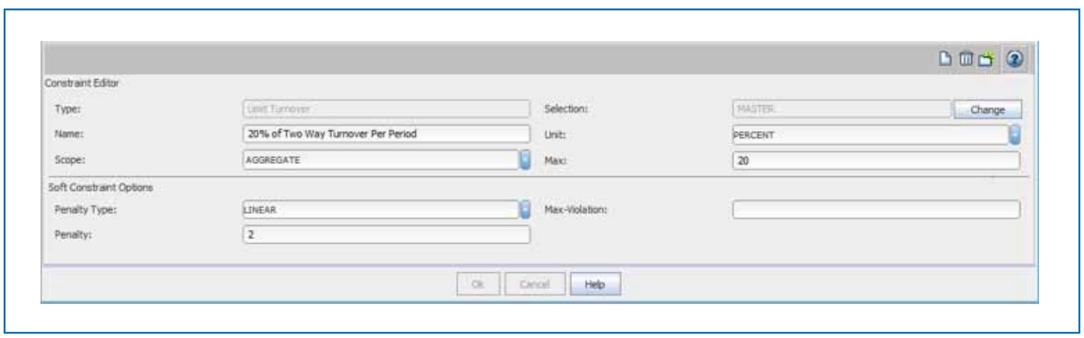

Figure B4. This turnover constraint was softened through a linear penalty of 2.

d. Frontier Backtesting

Challenge – Varying a Strategy Parameter: Practitioners frequently adjust a single parameter within a backtested strategy to see how the strategy would have performed differently throughout the backtest. The size of the adjustment is often arbitrary, the direction of the adjustment is often initially unknown, and it may take a lot of time to complete each adjustment. These mechanical difficulties make it extremely time consuming to alter a backtest strategy parameter. It is also a mundane administrative task, prone to human error, to track the effects of the changes.

Solution: Axioma addresses these challenges by giving users the ability to create and run frontier backtests. Frontiers can be run by varying many different types of parameters, including objective term coefficients, such as coefficients or multipliers, and objective terms, such as risk and return, constraint min and max bounds, soft constraint penalty values, etc. Figure B5 below details how we configured the example frontier backtest by varying the weight or coefficient on active risk aversion, λ, between a range of -10 to -60. We simultaneously ran a total of 16 backtests throughout this range.

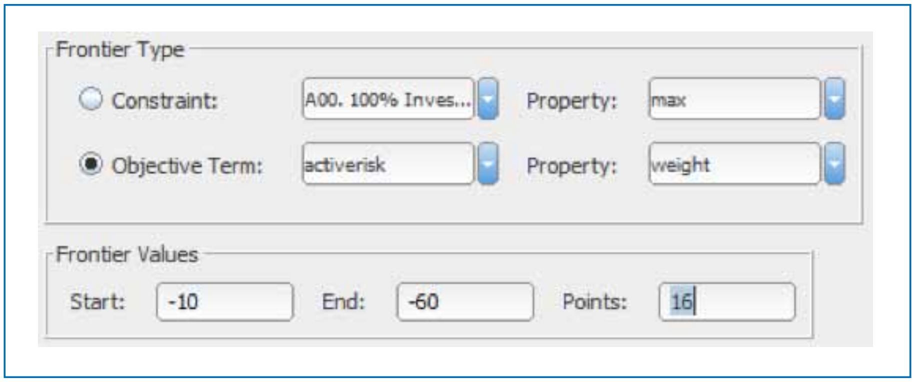

Figure B5. Example of a frontier backtest setup.

Challenge – Analyzing Individual Periods: It can be hard to understand the per-period outputs of a backtest.

Solution: Axioma provides granular details of per-period outputs generated during a backtest for analysis and record-keeping. The table below describes those used in the frontier example:

OUTPUT & DESCRIPTION			
Period Summary			
The period summary details the following metrics for every rebalancing period, in the case of the frontier backtest example, the metrics are reported on a monthly basis:			
Total Return	Transaction Cost	Total Active Risk - Alpha Factor	Tax Liability
Long Return	Turnover	Short Names	Number of Trades
Short Return	Leverage	Long Names	Optimization Objective Values
Cash Return	Expected Return	Tax Gains (LT)	Optimization Constraint Status (Binding/Non-Binding/Loose)
Benchmark Return	Expected Active Return	Tax Gains (ST)	
Active Return	Transfer Coefficient	Tax Losses (LT)	
Total Risk	Predicted Beta	Tax Losses (ST)	
Active Risk	Total Predicted Risk - Alpha Factor	Unrealized Gains	
Portfolio Size			
Risk Analysis			
The period summary details the following categories of metrics for every rebalancing period:			
Risk Summary	Factor Contribution to Risk	Active Factor Exposure Details	Factor MCAR
Factor Risk Details	Factor MCTR	Factor Contribution to Active Risk	Return Statistics
Factor Exposure Details	Active Factor Risk Details		Return Summary
Account Holdings			
Per-period account holdings were produced for possible further analysis or for importing into attribution systems, such as Axioma Portfolio Analytics.			
Period Workspace			
Per-period point in time workspaces were produced for possible further analysis.			

Table 2. A summary of per-period output types available through the backtester.

f. Backtest Summary

Challenge – Reviewing Overall Performance: It is hard to get one view of the performance of all backtests.

Solution: Axioma provides a Frontier Backtest Summary to detail the overall performance of each backtest on the frontier. Table 3 is a condensed snapshot detailing its format:

Cumulative Summary	Point-1	Point-2	…	Point-11
Frontier Constraint/ Objective	activerisk	activerisk	…	activerisk
Property Name	weight	weight	…	weight
Property Value	-30.00	-33.00	…	-60.00
Annualized Pre Tax Return:	8.86%	8.70%	…	7.94%
Annualized Post Tax Return:	8.86%	8.70%	…	7.94%
Annualized Liquidated Post Tax Return:	8.86%	8.70%	…	7.94%
Average Annualized Risk:	15.31%	15.35%	…	15.39%
Annualized Active Return:	1.38%	1.22%	…	0.46%
Average Annualized Active Risk:	3.04%	2.82%	…	1.59%
Average Turnover:	19.66%	19.51%	…	16.45%
Average Number of Assets per Period:	142	151	…	159
Average Number of Trades per Period:	99.66	111.13	…	155.92
Sharpe Ratio:	0.469	0.458	…	0.407
Risk Free Rate:	0.017	0.017	…	0.017
Information Ratio:	0.453	0.433	…	0.291
Compounded Tax Liability:	$0.00	$0.00	…	$0.00
Total Tax Gains (LT):	$0.00	$0.00	…	$0.00
Total Tax Gains (ST):	$0.00	$0.00	…	$0.00
Total Tax Losses (LT):	$0.00	$0.00	…	$0.00
Total Tax Losses (ST):	$0.00	$0.00	…	$0.00
Total Unrealized Gains:	$0.00	$0.00	…	$0.00

Table 3. Frontier backtest summary. Backtest summaries are also available for single backtests.

g. Feed into Axioma Portfolio Analytics for Performance Attribution

Challenge – Understanding Overall Performance: Running a backtest can help you uncover insight, but it can still be challenging to understand the results.

Solution: Axioma's more advanced attribution system, Axioma Portfolio Analytics, can provide more detailed analysis. In our example, the holdings of the frontier backtest were automatically fed into this system. Figures B9 through B13 demonstrate a small set of Axioma Portfolio Analytic's capabilities to decompose returns through factor-based performance attribution:

	Return	Risk	IR	T-Stat
Portfolio	9.32 %	15.26 %		
Benchmark	7.36 %	15.53 %		
Active	1.96 %	3.79 %	0.52	1.64

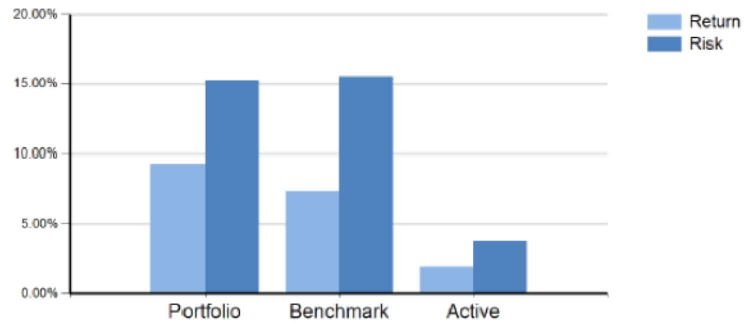

Figure B9. Overall backtest realized risk and return (annualized geometric).

Return Decomposition

	Return	Risk	IR	T-Stat
Active	1.96 %	3.79 %	0.52	1.64
Specific	-3.68 %	2.79 %	-1.32	-4.17
Factor	5.63 %	3.20 %	1.76	5.57
Style	5.08 %	3.02 %	1.68	5.32
Industry	0.56 %	0.99 %	0.56	1.79
Market	-0.01 %	0.06 %	-0.09	-0.29

Figure B10. Decomposition of active returns into specific and factor components.

Contribution to Active Return: Factor Group vs Stock Specific

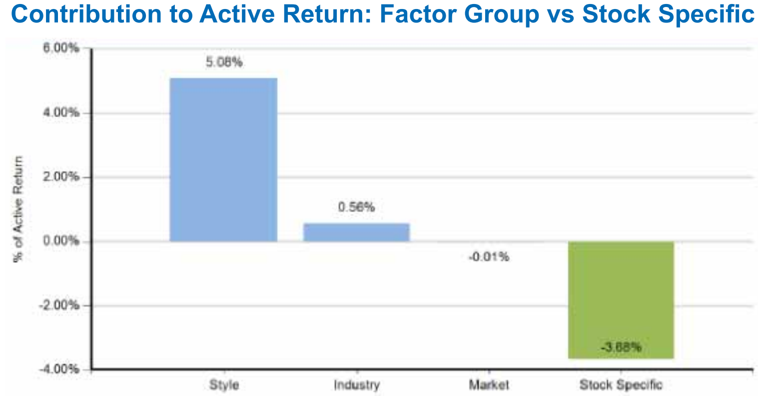

Figure B11. Contributions to returns are in line with expectations of positive sources being predominantly systematic factors (Medium-Term Momentum and Profitability).

Top/Bottom 5 Factors: Contributors to Active Return

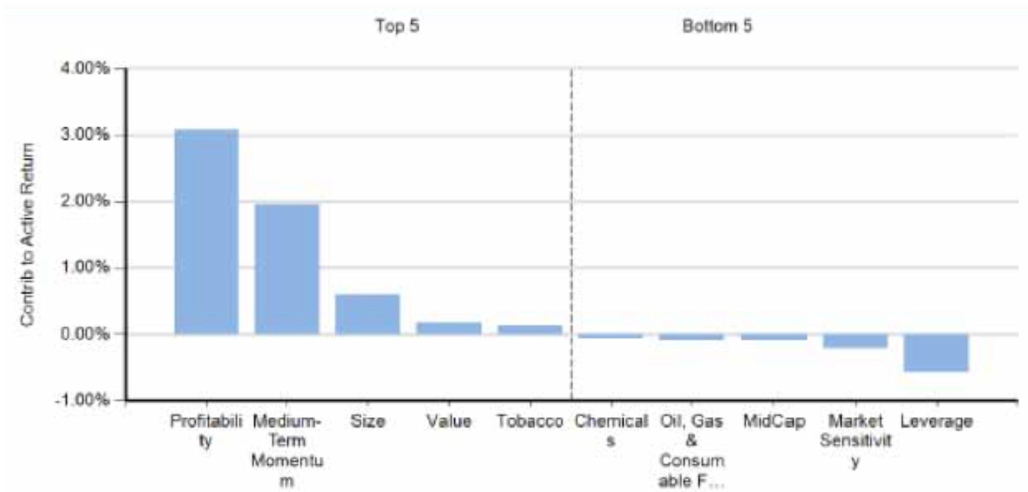

Figure B12. Further decomposition shows Profitability and Medium-Term Momentum as the highest-returning factors.

Contributors to Active Return by Style

Style	Contribution	Avg Wtd Exp	HR	IR
Profitability	3.08 %	0.8819	73.33 %	1.89
Medium-Term Momentum	1.96 %	0.7174	60.83 %	0.75
Size	0.60 %	-0.0996	60.83 %	1.04
Value	0.18 %	-0.4003	51.67 %	0.17
Earnings Yield	0.11 %	0.0414	59.17 %	0.64
Dividend Yield	0.08 %	-0.0818	59.17 %	0.54
Exchange Rate Sensitivity	0.06 %	0.0407	59.17 %	0.51
Volatility	-0.02 %	0.0005	65.83 %	-0.54
Growth	-0.02 %	0.1701	49.17 %	-0.08
Liquidity	-0.06 %	0.1048	50.00 %	-0.25
MidCap	-0.09 %	0.0517	43.33 %	-0.62
Market Sensitivity	-0.21 %	0.1013	42.50 %	-0.37
Leverage	-0.58 %	0.2997	45.83 %	-0.82
Total	**5.08 %**			

Figure B13. Further decomposition shows that Size and Volatility returns are also in line with expectations set forth by the backtested strategy.

Results

We ran a 16-point frontier on active-risk aversion from -10 to -60 in attempt to locate its optimal value. Figure C1 below plots active risk on the x-axis and active return on the y-axis:

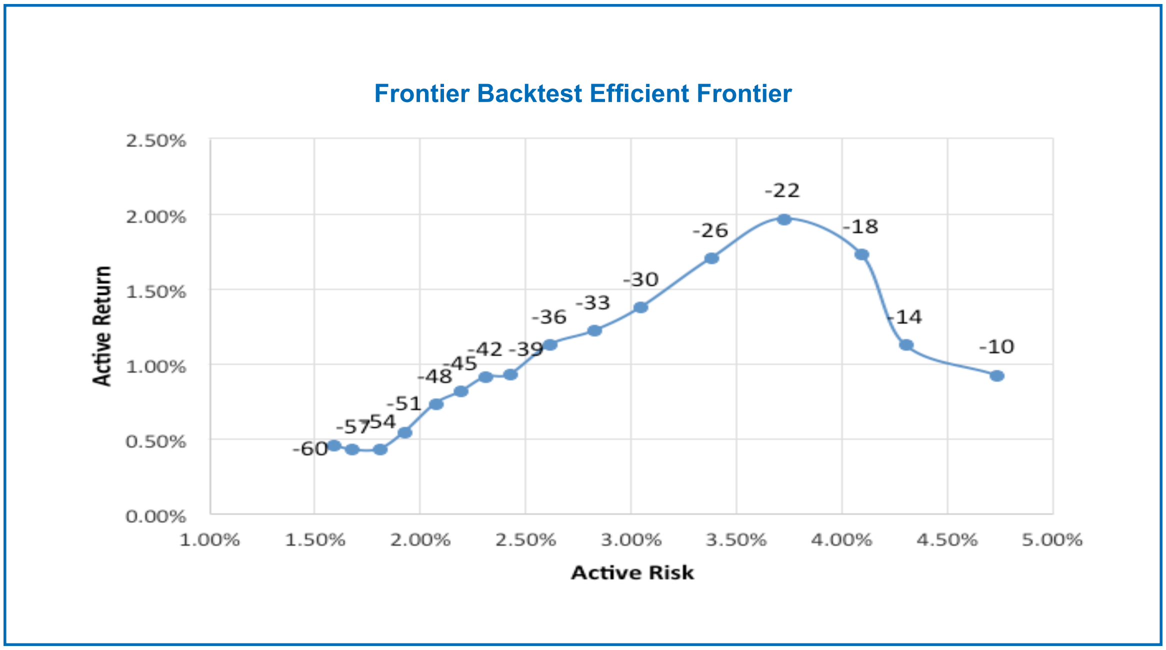

Figure C1. Realized efficient frontier of the example backtest.

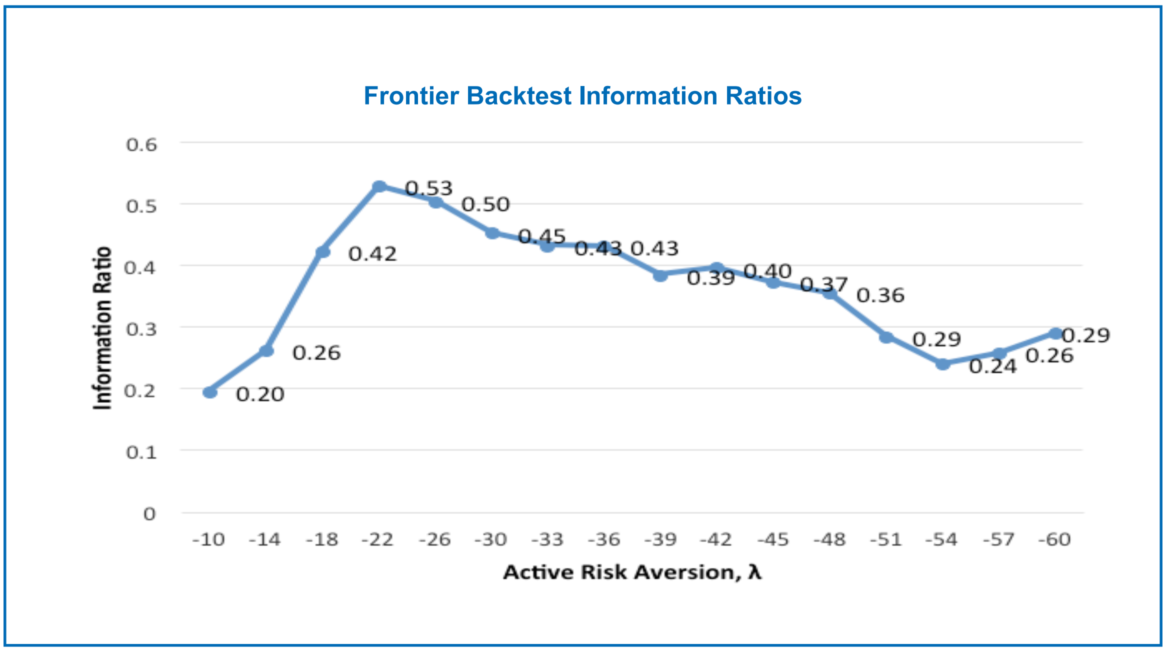

Figure C2. Information ratios of the frontier backtest example.

III. An Overview of Axioma Backtester's Other Differentiating Features

1. Infeasibility Diagnosis / Failed Workspace

During a failure during backtesting, Axioma's backtester automatically generates a failed workspace for the time period during which the failure occurred. This failed workspace can be opened in the Axioma Portfolio graphical user interface (GUI) to pinpoint the culprit(s) through infeasibility diagnostics. The example below illustrates a simple conflict arising from two constraints:

1. **Active Industry Group Weights against Russell 3000 within +/- 1%**
2. **Do Not Hold "Software & Services" Industry Group**

Together, these conditions cause an infeasibility and Axioma diagnoses it with the following output, based on a $100 mil. portfolio:

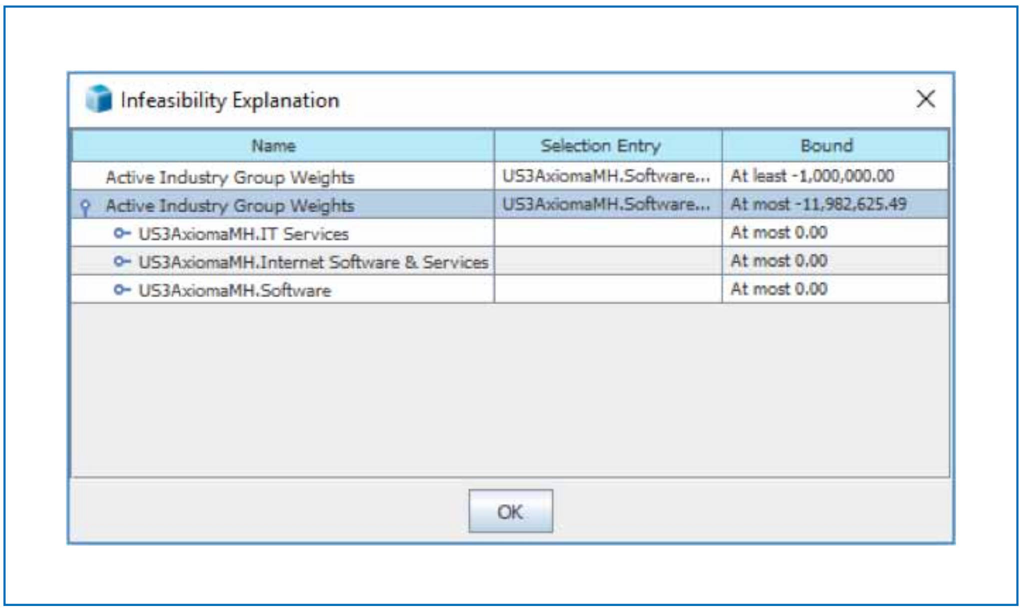

Figure D1. An example of an infeasibility diagnostic tree.

The output shows that Constraint 1 requires the optimizer to hold at least -$1 mil. (or -1%) of active holdings for each Industry Group against the Russell 3000. However, due to Constraint 2 requiring it to also not hold the "Software & Services" Industry Group at all, the most active weight the optimizer can reach for this Industry Group is -$11,982,625 (or -11.98%), creating an infeasibility.

2. Constraint Hierarchy

Highly constrained strategies often create conflicts within a larger group of constraints that are difficult to diagnose. The severity of these conflicts is further compounded when it involves combinatorial constraints imposing discrete conditions. Some examples of these constraints include threshold holdings, threshold trades, and max names constraints. Although it is possible to convert many of these constraints into soft constraints with varying penalty values according to the user's order of priority, assigning penalty values can often become extremely challenging—mathematical analysis is required in order to come up with the correct penalty values between competing constraints, while keeping the objective function itself under consideration. Not only is this approach outside of the core competency of most managers, its robustness is also precarious. When dealing with infeasibilities stemming from such complex conflicts, Axioma allows the user to enable a constraint hierarchy in a specified order. The optimizer then relaxes each constraint in this order on an as-needed basis until it reaches a solution. The screenshot below displays an example of how multiple constraints can be added to a hierarchy to ensure feasibilities through time.

Constraints in Hierarchy	Active	Constraint Priority
Asset Weights	☑	1
Sector Weights	☑	2
Industry Weights	☑	3
Overall Predicated Beta	☑	4
Region Weights	☑	5
Max Number of Names	☑	6
Total Turnover	☑	7
Threshold Holding Size	☑	8

Figure D2. A constraint-hierarchy example with "Asset Weights" assigned the highest priority and "Threshold Holding Size" assigned the lowest priority.

3. Backtest Resumption

In addition to infeasibilities, other unforeseen circumstances, such as data problems, corrupt files, systems crashes, and network problems, can also cause failures during backtesting. In such scenarios, the user will be forced to rerun entire backtests. With Axioma's Backtester, a failed backtest can be resumed from the point of failure after addressing the root cause(s). This potentially saves the user a tremendous amount of time because they do not have to restart a backtest from the beginning.

Furthermore, after a strategy or a backtest is deployed in a research or production environment, the passage of time requires additional rebalancings to be added to the tail end of the backtest. Axioma's backtest-resumption feature allows the user to add these additional rebalancings while generating updated reports that detail risk and performance.

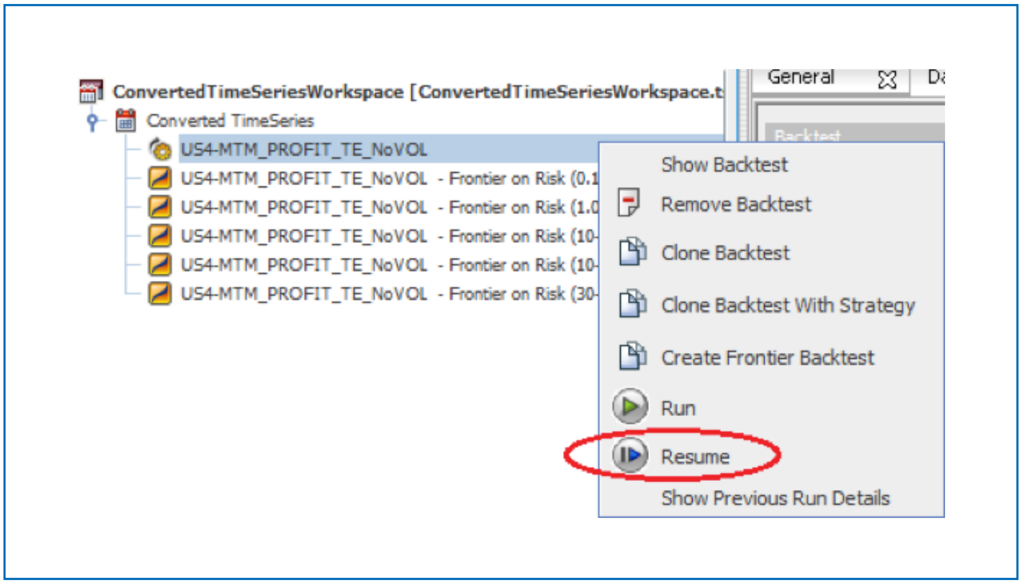

Figure D3. Backtest resumption feature.

4. Dated Strategies

Many practitioners want the ability to vary parameters within a given strategy throughout a backtest. Axioma's Backtester allows dated strategies within certain parts, and even allows an entire strategy to change for predefined dates. This is accomplished by staggering predefined strategies as a dated series. For example, managers who construct portfolios using certain Russell benchmarks may wish to increase a strategy's turnover limit around the time of FTSE Russell's annual reconstitution. A dated strategy series inside Axioma's backtester allows the user to easily implement this adjustment. Figure D4 below details a graphical representation on how a strategy would alter on such a timeline

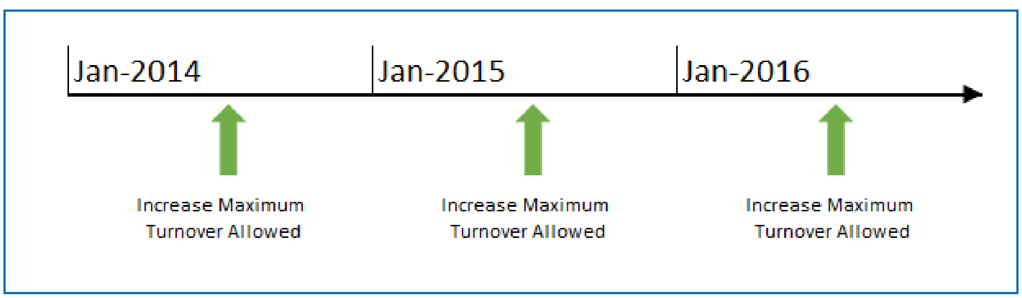

Figure D4. Temporarily increasing maximum turnover during the June Russell reconstitutions.

5. Dynamic Strategies

One further dated strategies step is the ability to deploy dynamic strategies, also known as "call backs," within Axioma's platform. While a dated strategy series allows the user to pre-specify changes within a strategy according to specific dates, one may argue that there are obvious concerns associated with this approach. One such concern is the introduction of forward-looking bias. Call backs allow the backtest to change a strategy's parameter(s) according to a set of predefined logic.

At a high level, call backs work as follows:

- During a backtest, a workspace is created for each rebalancing period, within which an optimization is performed.
- The final portfolio is then carried over to the next rebalancing period, during which another workspace is created and used as its initial portfolio.
- This process is repeated as a loop until the entire backtest is completed.

Each workspace contains market data that an optimizer users to create portfolios in accordance with the strategy. This data can include risk models, benchmarks, and market data such as prices and ADV, as well as data items that the user provides. A call back can modify the strategy based on any data item it observes within the workspace prior to performing the optimization.

Some examples include:

a. If the total predicted risk of a benchmark is greater than 30%, then the total risk of the portfolio is limited to no more than 20%.
b. If the return of the momentum factor has been positive for the previous two periods, the active exposure to momentum is allowed to grow by 25%.
c. If the specific risk of a stock is above a certain threshold, then it is added or removed from the investible universe.

One can infer from the above examples that there are potentially infinite approaches with which a user can alter a backtested strategy based on data inputs and creative logic.

6. Period Summary / Per Period Output

Axioma's Backtester makes the following output available for every rebalancing period:

- Period Summary detailing granular metrics pertaining to each period in between rebalancings
- Risk Analysis Summary detailing granular metrics pertaining to factor risks, factor exposures, etc.
- Account Holdings in Shares or Currency denominations (Available in any chosen identifier, e.g., CUSIP, SEDOL, ISIN, Ticker)
- Period Workspaces that can be opened for in the GUI for in-depth point in-time analysis

7. Constraint Attribution

Constraints are now an integral part of the portfolio construction process. With constraints come the challenge of understanding how they cause the optimal portfolios to deviate from a tradeoff, dictated by the forecasts of risk and return. Axioma offers a technique called constraint attribution to quantify the impact of individual constraints in several different ways. This includes decomposing the difference between the optimal constrained and unconstrained portfolios and the difference between alphas and implied alphas as described in earlier work by Grinold and others. Figure D5 below illustrates an example of how an asset-bound constraint and a sector constraint impacted two assets' raw alpha scores to arrive at each asset's respective implied alphas:

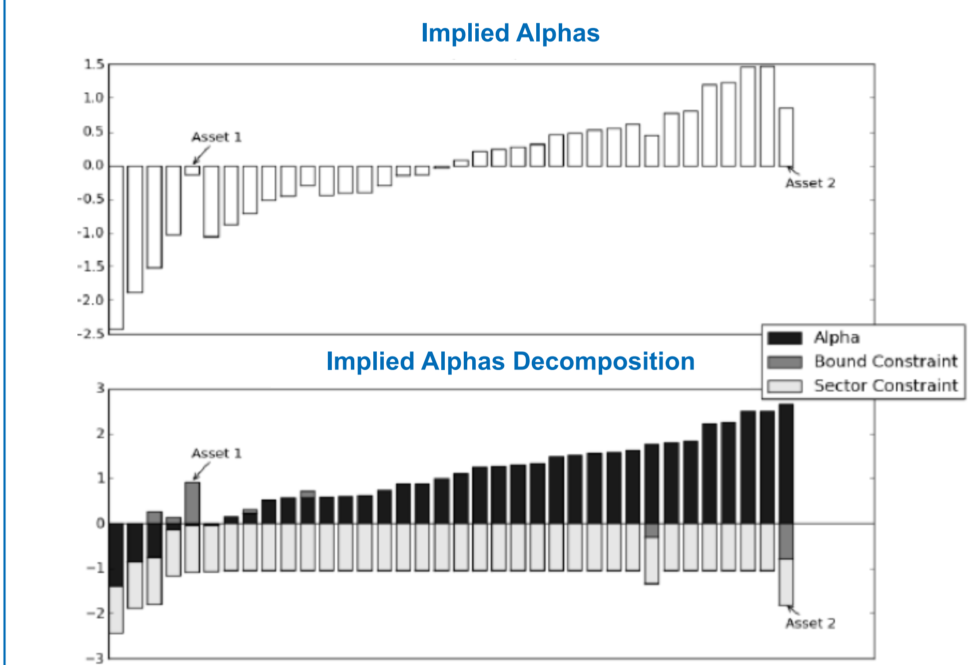

Figure D5. Alphas, impact to alphas, and implied alphas of Asset 1 and Asset 2.
(Axioma Research Paper No. 014, Exhibit 2)

Furthermore, constraint attribution by Axioma goes one step further, with decompositions on an ex-post basis, to provide an understanding of how constraints actually impacted realized risk and return. Figure D6 illustrates an example of how constraints impacted the realized risk and return of a 130/30 portfolio with 3% tracking error against the Russell 1000:

	Annualized Return Attribution	Risk Attribution
Portfolio	15.21%	
Benchmark	11.35%	
Active	3.86%	2.72%
MVO	7.05%	3.03%
Max Long	-2.53%	-0.12%
Industry Bounds	-0.15%	-0.08%
Budget	0.76%	-0.02%
Max Turnover	-1.54%	0.08%
Asset Bounds	0.17%	-0.14%
Other	0.08%	-0.02%

Figure D6. Realized risk and return decomposition by constraint attribution.
(Axioma Research Paper No. 014, Exhibit 8)

[2] Robert A. Stubbs and Dieter Vandenbussche (2009). Axioma Research Paper No. 014: Constraint Attribution

For additional information, please refer to Axioma Research Paper No. 014 – Constraint Attribution, or contact Axioma Support at support@axioma.com.

8. Feed into Axioma Portfolio Analytics for Risk Analysis and Performance Attribution

We can't overstate the convenience of quickly understanding the performance of a backtest. Axioma provides its users with the ability to generate in-depth time-series risk analysis, returns-based, and factor-based performance attribution reports for completed backtests. The backtester creates a direct feed into Axioma's more advanced reporting platform, Axioma Portfolio Analytics, to generate more sophisticated reports.

Please refer to Section II. A Frontier Backtest Example above for additional details.

For questions related to Axioma Portfolio Analytics, please contact Axioma Support at support@axioma.com.

IV. Conclusion

In recent years, a steady increase in complexity of backtested strategies further demands greater flexibility by the backtester. It is of paramount importance to have comprehensive backtesting features and functionalities. Axioma's backtester is a complete toolset that is capable of evaluating strategies at all levels of sophistication. Its ease of setup and advanced diagnostics are coupled with powerful features to customize, mass produce, and report in-depth analytics.

To have an Axioma expert look at your strategy, contact: sales@axioma.com.

Flexible is better.®

Contact us to learn more about how Axioma can bring more information and insights to your investment process.

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