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REPLICATING INDEX RETURNS WITH TREASURY FUTURES: DURATION CELLS VERSUS KEY-RATE DURATIONS

Since the rediscovery of duration in the late 1970s, investors have been looking for better ways to measure interest-rate sensitivity. Duration proved to be a useful measure of price sensitivity to parallel shifts in the yield curve, though managers recognized that for non-parallel shifts, additional information was needed to gauge interest-rate risk properly. Many managers sliced their portfolios and indices into maturity buckets and used duration distribution across these buckets. As managers switched from government/credit benchmarks to aggregate benchmarks, with a high percentage of callable securities, duration buckets replaced maturity buckets.

In recent years, partial durations have become increasingly popular as a measure of yield curve sensitivity. Instead of a single duration number, a vector of partial durations describes the sensitivity to yield curve twists. The sensitivity of a given bond to a non-parallel yield curve movement is a function of the distribution of its cash flows. If a portfolio is constructed from bullet bonds, the present values of whose cash flows are largely distributed within a narrow maturity "window" (e.g., bullet securities), duration bucketing should give a reasonable view of yield curve risk. However, where the present value contributions from bonds' cash flows are distributed more evenly across the curve (e.g., amortizing securities such as MBS), duration bucketing is likely to be less satisfactory.

Consider the durations and key-rate durations[1] (KRD) of a Treasury security and an MBS security in Figure 1. Both securities have near-identical option-adjusted durations (OAD), but very different interest rate profiles. Accordingly, a long position in one security, offset by a short position in the other, will be sensitive to non-parallel interest-rate movements. The Lehman Brothers global risk model can quantify the yield curve risk arising from this KRD mismatch.[2] Examining just the term structure risk due to the KRD mismatch (excluding risk due to convexity or sector mismatches), we find this to be 7.8 bp of return volatility per month.

It is often argued that duration bucketing should provide a reasonable picture of interest rate exposure for diversified portfolios and indices. The reasoning is that while some securities may indeed be placed into duration buckets that do not reflect their true

[1] KRD is a "flavor" of a partial duration. Certain points on the par curve are selected as key rates. For maturities between the key rates, it is assumed that rates move according to linear interpolation. For example, in Lehman's model, we use six key rates—6-month , 2-year, 5-year, 10-year, 20-year, and 30-year. A 5-year KR shift assumes no shift in the 2- or 10- year rate, and interpolated shifts between the 5- and 2-year and the 5- and 10-year, a so-called tent shift. The 5-year KRD of any bond is then the sensitivity of the bond price to a 100 bp shift in the 5-year key rate with an appropriate tent shift in the term structure between two and ten years.

[2] Risk is a function of the exposure (the key rate duration mismatch) and the historical volatility of that exposure.

Figure 1. **Option-Adjusted Duration and Key-Rate Durations of U.S. Treasury and Mortgage Security**

		Key Rate Duration					
	OAD	**6-Mo**	**2-Yr**	**5-Yr**	**10-Yr**	**20-Yr**	**30-Yr**
UST 6.5% 2/10	4.74	0.02	0.10	4.05	0.57	0.00	0.00
FNMA 5.5% 2003	4.73	0.15	0.57	0.99	1.77	1.15	0.11

interest-rate sensitivities, perhaps these errors are reduced in large portfolios. To examine this assertion, in Figure 2, we compare the duration profiles of the Lehman Brothers Intermediate Treasury Index and the U.S. MBS Index.

A comparison of the duration contributions with the KRDs confirms our intuition that for bullet bonds, duration bucketing provides a reasonable view of yield curve exposure. For the Intermediate Treasury Index, the buckets' duration contributions provide a view of yield curve exposure not too different from the KRD profile. However, for the MBS Index, duration buckets present a somewhat misleading picture. If we were to view the Treasury Index as a portfolio and the MBS Index as its benchmark, a duration-bucketing view would suggest that the portfolio has a large yield-curve mismatch compared with the index. In particular, the portfolio would seem to have a substantial underweight in the 4- to 6-year duration bucket, almost fully offset by an overweight in the 6- to 8-year duration bucket. Accordingly, a hypothetical portfolio manager might conclude that the portfolio was exposed to yield curve flattening and choose to reduce risk by increasing exposure to the 6- to 8-year bucket. However, the KRD exposures tell a very different story. The portfolio is overweighted to 5- and 10-year yield curve points and underweighted to the 20-year point. Therefore, our hypothetical manager is actually exposed to a yield-curve steepening.

Some of our more skeptical readers may ask: yes, but are the KRDs necessarily right? As a simple exercise, we can examine the return effect of a particular yield curve shift. In Figure 3, we apply instantaneous shifts of plus and minus 25 bp to the 5-year key rate and revalue every security in each index. By the definition of key-rate shifts, the move in the 5-year will not affect the par rates shorter than two and longer than ten years, but will affect intervening maturities at a declining linear rate.

Figure 2. **Comparative Duration Exposures for the Intermediate Treasury and Mortgage Indices,** As of May 31, 2004

	Duration					
	0-2 Yr	**2-4 Yr**	**4-6 Yr**	**6-8 Yr**	**8-10Yr**	**10+Yr**
Market Value (%)						
Intermediate Treasury	35.07	29.13	19.84	12.94	0.00	0.00
MBS	5.30	40.04	50.60	4.07	0.00	0.00
OAD Contributions						
Intermediate Treasury	0.54	0.86	0.90	0.93	0.00	0.00
MBS	0.09	1.28	2.52	0.26	0.00	0.00
Key-Rate Durations	**6-Mo**	**2-Yr**	**5-Yr**	**10-Yr**	**20-Yr**	**30-Yr**
Intermediate Treasury	0.14	0.96	1.37	0.98	0.00	0.00
MBS	0.16	0.56	1.02	1.53	0.78	0.09

Figure 3. **Projected Total Returns under Instantaneous Yield Curve Shifts**
Total Return (bp) under Scenario

Index	5-Year KR Down 25 bp	5-Year KR Up 25 bp
Intermediate Treasury	34.6	-34.3
MBS	26.8	-30.1

Figure 3 shows that the Treasury Index is more sensitive to a shift in the 5-year rate than the MBS Index. This is consistent with the sensitivities indicated by the KRD profile in Figure 2, but is not consistent at all with the duration bucketing pattern.

Empirical test is perhaps the most effective way of gauging whether KRDs are indeed superior as a measure of yield curve exposure. In particular, we can test whether a strategy that seeks to replicate a given index by matching KRDs is superior (i.e., results in a lower tracking error) to one that matches the index by duration bucketing.

In a series of studies dating back to 1997, we examined techniques for replicating returns of popular Lehman Brothers indices with baskets of Treasury futures. These techniques are popular with asset managers engaged in portable alpha strategies or in active tactical asset allocation.[3] As part of these studies, we examined the tracking errors associated with replicating various indices using a duration-bucketing approach. Typically, the relevant index is divided into four duration cells: 0-3 year, 3-5 year, 5-7.5 year, and 7.5 years and higher, with the exception of mortgages.[4] For a given target portfolio size, we calculate the number of 2-year, 5-year, 10-year, and long bond futures contracts required to match the dollar duration of each cell. At the end of each month, this calculation is performed on the forward-looking ("statistics") universe of the index, and the numbers of futures contracts are adjusted as appropriate. Once a quarter, the contracts are rolled to avoid the possibility of the exercise of the delivery option.

An alternative to the duration bucketing approach is KRD-matching, which minimizes the differences between the KRD profiles of a given index and the replicating futures position. Because there are six KRDs in Lehman's term structure model and only four futures contracts, it is not possible to achieve a perfect match. Therefore, we set up an optimization that minimizes the sum of the squared differences between the respective index and replicating portfolio KRDs, subject to the constraint that the sum of the KRDs must be identical. The cash is assumed to be invested in 1-month LIBOR.

Figure 4 shows the results of replications of the U.S. Treasury Index, the MBS Index, and the Credit Index, using the duration bucketing approach and the KRD-matching approach.

[3] *Replicating Index Returns with Treasury Futures*, Lehman Brothers, November 1997; *Replication with Derivatives—The Global Aggregate Index and the Japanese Aggregate Index*, Lehman Brothers, March 2001; "Hedging and Replication of Fixed-Income Portfolios," Dynkin, Hyman, and Lindner, *The Journal of Fixed Income*, March 2002.

[4] For the MBS Index, given the lack of long-duration securities, we eliminate the 7.5 + duration bucket so that the third bucket becomes 5+ year duration, which is replicated with 10-year note futures contracts.

Figure 4. **Comparison of the KRD Replication Approach with Duration Bucketing, Monthly Rebalancing,** June 2000-April 2004

	Monthly Tracking Error Volatility (bp)			
Index	**Duration Buckets**	**KRD Matching**	**Difference**	**Percent**
Treasury	10.7	8.6	-2.1	-19.10
MBS	38.3	36.9	-1.4	-3.60
Credit	87.9	86.7	-1.2	-1.40

We find that the KRD-matching approach does improve tracking in the replication of all three indices. As we see, the biggest improvement, in both absolute and percentage terms, is achieved in replicating the Treasury Index. This is not entirely unexpected, since yield curve exposure is the only important source of risk, where the advantage of KRD matching matters most. On the other hand, the Credit Index shows the smallest improvement because of the magnitude of other risk exposures.

Conclusion

We developed our replication strategy using duration buckets in 1997. In mid-2000, we began to generate key-rate durations for U.S. fixed income securities and for bond futures Only now do we have sufficient historical data to make reasonable inferences from an empirical comparison of a duration-bucketing strategy and a KRD-matching strategy. Our analysis suggests that using key-rate durations to replicate indices leads to only a small improvement in the performance of replication strategies using futures. The improvement is not perhaps as great as might have been expected, for which we offer two explanations. First, in using four futures, we can only *minimize* key rate duration exposures, not *eliminate* them. Because of this constraint in our replication study, we found that an optimization that minimized key-rate duration dispersions was not able to reduce them substantially. Second, diversified exposures in broad indices help diminish the additional replication errors introduced by the duration-bucketing approach.

In previous replication studies, we examined the use of interest-rate swaps in isolation and in combination with Treasury futures ("hybrid" replication). The present study suggests that the previously demonstrated advantages of hybrid replication (swaps are clearly superior to futures in replicating spread indices) may also extend to improved yield curve matching. This is an area worthy of further study, and we look forward to reporting our results in the future.

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