PURE FACTOR RETURNS IN REAL TIME

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Introduction

The week of 6-Aug-2007 was a devastating period for many quantitative equity portfolio managers. Although the broad equity market was up slightly over the week, the *active returns* of these managers were strongly negative. This episode in the financial markets is known as the “Quant Meltdown.”

As the Quant Meltdown was unfolding, quantitative managers could observe in real time that their portfolios were severely underperforming, but they did not know *why*. We now know that many of these quantitative equity managers were making bets on the same *factors*, such as value and momentum. Moreover, some of these managers were highly levered, so that as their portfolios began to lose money, they were forced to unwind their positions to keep portfolio leverage in check. This selloff caused the factors to underperform even more, which in turn forced another round of deleveraging. This vicious cycle continued, effectively producing the financial equivalent of an “avalanche.”

What if these portfolio managers had been able to observe the intra-day factor returns during the Quant Meltdown? Armed with this information, they would have known why their portfolios were underperforming and may have been able to take quick preventative action. At a minimum, they would have been able to make better-informed investment decisions during the crisis period.

Now, for the first time, factor returns in real time will be available on the Bloomberg Professional Service. As explained in this document, factor returns are given by the returns of *pure factor portfolios*. By making these portfolios available on the Bloomberg Professional Service, clients can treat them as any other tickerized portfolio and observe their performance tick-by-tick.

Background

Factor models were pioneered by Barr Rosenberg (1974) in the 1970s for the purpose of predicting portfolio risk. He identified a set of return drivers, called factors, that explained equity return comovement. In doing so, he formalized the intuitive concept that stocks with similar characteristics (e.g., industry, country, or style) tend to also exhibit similar return patterns.

In the decades following Rosenberg, factor models have become increasingly ubiquitous. Nowadays, factor models are deployed in virtually every stage of the investment process, including: (a) return forecasting, (b) portfolio construction, (c) risk management, and (d) performance attribution.

Moreover, with the growing proliferation of "smart-beta" investing, factor models have now entered the financial mainstream. The rationale underlying factor-based investing is that certain asset characteristics (factors) appear to capture return premia that cannot be explained by the Capital Asset Pricing Model (CAPM). As discussed by Sharpe (1964), the CAPM asserts that the market portfolio is efficient and the expected return is fully explained by the asset's beta relative to the market, with the unexplained residual having zero expected return.

Nevertheless, many academic studies have found compelling evidence of "pricing anomalies" that appear to violate CAPM. For instance, Basu (1977) showed that stocks with high earnings-to-price ratio (i.e., value stocks) have outperformed on a risk-adjusted basis. Banz (1981) showed that small-cap stocks tend to outperform their large-cap counterparts, in what has become known as the "size effect." Another pricing anomaly was documented by Jegadeesh and Titman (1993), who identified the "momentum factor" in which recent "winners" tend to outperform recent "losers." More recently, Ang *et al.* (2009) showed that low-volatility stocks tend to outperform, in what has become known as the "low-volatility anomaly."

Factor Portfolios

A fundamental concept in factor modeling is that of the *factor-mimicking portfolio*. As described by Menchero and Lee (2015), factor portfolios can be classified into three broad categories. The first is the *simple* factor portfolio, which is constructed by *univariate* regression against the factor in isolation. Top-quintile minus bottom-quintile portfolios also fall into this category. The second category is known as the *pure* factor portfolio, which is constructed by *multivariate* cross-sectional regression. The third category is the *minimum-volatility* factor portfolio, which is constructed using mean-variance optimization. The technical appendix provides mathematical details on the construction techniques for these three categories of factor portfolios. For portfolio analytics purposes, the pure factor portfolios are by far the most important, and will be the main focus of this paper.

Before describing pure factor portfolios, however, it is helpful to first understand simple factor portfolios. As shown in the technical appendix, simple factor portfolios are strictly dollar-neutral, holding long positions in stocks with positive exposure to the factor and short positions in stocks with negative exposure. Of course, a portfolio constructed by treating the factor in isolation will generally have many “incidental” exposures to other factors. For instance, if oil stocks have recently performed well, then the momentum simple factor portfolio will have an incidental overweight on the energy sector. These incidental exposures typically represent uncompensated sources of risk; that is, they increase portfolio risk without increasing expected return.

The pure factor portfolio solves this problem by disentangling the return sources via multivariate cross-sectional regression. For instance, factor returns in the Bloomberg Global Equity Model are estimated by regressing asset returns against four basic types of factors: market, countries, industries, and styles. In other words,

$$r_n = f_M + \sum_c X_{nc} f_c + \sum_i X_{ni} f_i + \sum_s X_{ns} f_s + u_n \quad , \tag{1}$$

where r_n is the return of stock n, f_M is return of the market factor, X_{nc} is the exposure of stock n to country factor c, and f_c is the return of the country factor. Similarly, X_{ni} and f_i are the exposures and return for industry factor i, while X_{ns} and f_s are the corresponding quantities for the style factors. The component of stock return u_n that is unexplained by the factors is termed idiosyncratic, or stock-specific. These stock-specific returns are assumed to be mutually uncorrelated, and also uncorrelated with the factors.

From Equation (1), we see that every stock has unit exposure to the market factor. By contrast, country and industry exposures are represented by (0,1) dummy variables. That is, a stock is assigned unit exposure to its country and industry of membership, and zero exposures to all other countries and industries. Exposures to style factors are expressed as standardized z-scores, which have a cap-weighted mean of zero and an equal-weighted standard deviation of 1. Hence, a stock with positive exposure can be interpreted as scoring “above average” for that particular attribute.

As a concrete example, we consider the Bloomberg Global Equity Model on analysis date 31-Jan-2016. In Table 1, we present the weights of several pure factor portfolios in various segments of the global equity markets. For reference, we also report the weights of the cap-weighted market portfolio in these same segments.

We first consider the pure market factor portfolio. From Table 1, we see that it is 100 percent net long, with net weights that exactly match the cap-weighted market portfolio in all segments that correspond to risk model factors (e.g., US, Japan, autos). In segments that do not correspond to risk model factors (e.g., Japan autos), the weights are similar but not identical. For instance, the market portfolio had a weight of 99 bps in Japanese autos, whereas the market factor had only a weight of 89 bps in this segment. Nevertheless, for all intents and purposes, the market factor represents the cap-weighted market portfolio.

Table 1: Bloomberg Global Equity Risk Model (31-Jan-2016).

Market Segment	Market Portfolio	Market Factor	US Factor	Japan Factor	Auto Factor	Value Factor
World (Net)	100.00%	100.00%	0.00%	0.00%	0.00%	0.00%
Long	100.00%	105.19%	61.25%	93.46%	108.69%	40.18%
Short	0.00%	-5.19%	-61.25%	-93.46%	-108.69%	-40.18%
US (Net)	39.55%	39.55%	60.45%	-39.55%	0.00%	0.00%
Long	39.55%	39.65%	60.45%	0.07%	15.75%	9.52%
Short	0.00%	-0.09%	0.00%	-39.62%	-15.75%	-9.52%
Japan (Net)	8.36%	8.36%	-8.36%	91.64%	0.00%	0.00%
Long	8.36%	9.32%	0.02%	91.64%	26.25%	4.42%
Short	0.00%	-0.97%	-8.38%	0.00%	-26.25%	-4.42%
Auto (Net)	2.93%	2.93%	0.00%	0.00%	97.07%	0.00%
Long	2.93%	3.11%	0.98%	4.94%	97.07%	1.24%
Short	0.00%	-0.18%	-0.98%	-4.94%	0.00%	-1.24%
US Auto (Net)	0.38%	0.43%	0.87%	-0.97%	11.98%	0.10%
Long	0.38%	0.43%	0.87%	0.00%	11.98%	0.18%
Short	0.00%	0.00%	0.00%	-0.97%	0.00%	-0.08%
Japan Auto (Net)	0.99%	0.89%	-0.19%	4.94%	26.25%	0.09%
Long	0.99%	0.94%	0.01%	4.94%	26.25%	0.33%
Short	0.00%	-0.04%	-0.20%	0.00%	0.00%	-0.23%

While the market factor portfolio is 100% net long, all other pure factor portfolios are strictly dollar neutral. For instance, the Japan country factor is 100% long the Japanese market, and 100% short the market portfolio. However, since the market portfolio itself has a weight of 8.36% in Japanese equities, it follows that the Japan factor portfolio is 91.64% net long Japanese equities and 91.64% net short all other equity markets. Also note that the Japan factor portfolio is industry neutral (i.e., it has zero net weight within every industry). Finally, although not apparent from Table 1, the Japan factor portfolio is also style neutral, meaning that the exposure to all style factors is exactly zero.

Industry pure factor portfolios are 100% long the industry in question, and 100% short the market portfolio. For the auto pure factor portfolio, this results in a net long weight of 97.07% in the auto industry and a net short weight of 97.07% in all other industries. Note that industry factor portfolios are country neutral and have zero exposure to all style factors.

Style pure factor portfolios (e.g., value) have unit exposure to their own style factor, and zero exposure to all other style factors. Moreover, pure style factor portfolios are dollar neutral within each country and within each industry.

To see how pure factor portfolios can be applied in practice, consider a portfolio manager who has a bullish view on Japan, but no particular view on industries or styles. One way to express that view would be to buy the cap-weighted Japanese market. However, this would result in a portfolio that is strongly overweight the auto industry. For instance, from Table 1 we see that 8.36% of the market portfolio consists of Japanese stocks, versus 0.99% for Japanese auto stocks. Hence, the Japanese equity market has a weight of 11.8% in the auto industry, versus 2.93% for the market portfolio. Pure factor portfolios solve this problem by cleanly disentangling the sources of return. More specifically, by overlaying the market portfolio with the Japan pure factor portfolio, we obtain a portfolio with the following characteristics: (a) 100% net long Japan, (b) zero net weight in every country, (c) industry neutral, meaning that the industry weights match those of the market portfolio, and (d) style neutral. In other words, we obtain a portfolio that cleanly reflects the manager's views, without taking incidental bets on other factors.

The interpretation of pure factor portfolios in a single-country model such as the Bloomberg US Equity Model is very similar to the global case. For instance, the US market factor essentially represents the cap-weighted US market portfolio. Similarly, the industry factor portfolios are style neutral and dollar neutral. Finally, style pure factor portfolios are dollar neutral with zero exposure to all other styles and industries.

Example

To illustrate the functionality of real-time factor returns, we consider the US pure value factor portfolio (PVALUEUS) and the US pure momentum factor portfolio (PMOMENUS) from the Bloomberg US Equity Model. In Figure 1, we present the intra-day returns of US value (white) and US momentum (red) on analysis date 29-Nov-2017. The predicted daily volatilities of these portfolios on this day were 18.1 bps and 17.8 bps, respectively. From the price chart, we see that the intra-day price moves were roughly 51 bps for value and -70 bps for momentum. Hence, these correspond to large intra-day price moves of about 2.8 and 3.9 standard deviations, respectively.

Figure 1: Intra-day factor returns for Bloomberg US Pure Value and Momentum Portfolios on 29-Nov-2017.

From Figure 1, we see that most of the price movements occurred in the first part of the trading day. This type of intra-day information enables portfolio managers to make more informed investment decisions throughout the trading day.

Summary

Pure factor portfolios provide a powerful framework for analyzing portfolios. They cleanly disentangle the otherwise confounding effects of multiple factors acting simultaneously. This allows for a clear explanation of the impact of individual factors.

Pure factor portfolios are now uploaded and available on the Bloomberg Professional Service. The holdings are updated on a nightly basis. The returns to these portfolios can be viewed in real time, thus providing greater insight and more timely information on the drivers of equity returns.

Technical Appendix

This appendix describes construction techniques for simple, pure, and minimum-volatility factor portfolios. It also describes the interpretation of these factor portfolios. More detailed explanations can be found in Menchero (2010).

Simple Factor Portfolios. These portfolios are constructed by univariate cross-sectional regression,

$$r_n = f_M + X_{nk} f_k + u_n \quad , \tag{A1}$$

where r_n is the return of stock n, f_M is the intercept, X_{nk} is the exposure of stock n to factor k, f_k is the return of the factor, and u_n is the unexplained residual. We assume that the exposures have been standardized to be mean zero,

$$\sum_n v_n X_{nk} = 0 \ , \tag{A2}$$

and unit standard deviation

$$\sum_n v_n X_{nk}^2 = 1 \ , \tag{A3}$$

where v_n is the regression weight (summing to unity). The return to the intercept term is given by

$$f_M = \sum_n v_n r_n \quad , \tag{A4}$$

which represents the return of the market factor. Note that if we use market capitalization as the regression weight, then the simple market factor portfolio is exactly the cap-weighted market portfolio. The return to style factor k is given by

$$f_k = \sum_n \left(v_n X_{nk} \right) r_n \quad . \tag{A5}$$

The simple factor portfolio has weights $v_n X_{nk}$. Hence, the simple factor portfolio is dollar neutral, with long positions in all stocks with positive exposure, and short positions in all stocks with negative exposure.

Another popular way of constructing simple factor portfolios is to go long the top quintile (equally weighted) and go short the bottom quintile (equally weighted). This portfolio construction scheme is also of the form given by Equation (A5). More specifically, stocks in the middle three quintiles receive zero regression weight, whereas the regression weights are inversely proportional to the factor exposures for the top and bottom quintiles.

Pure Factor Portfolios. It proves convenient to express the factor model in matrix form,

$$\mathbf{r} = \mathbf{X}\mathbf{f} + \mathbf{u} \ , \tag{A6}$$

Where $\mathbf{r}$ is the $N \times 1$ vector of stock returns, $\mathbf{X}$ is the $N \times K$ matrix of factor exposures, $\mathbf{f}$ is the $K \times 1$ vector of factor returns, and $\mathbf{u}$ is the $N \times 1$ vector of specific returns. We solve for the factor returns by cross-sectional regression.

Note that in a single-country model there is one exact collinearity, since the sum of industry factor exposures is exactly 1 (i.e., identical to the market factor exposure). Hence, only $K-1$ variables are truly independent. As a result, we must impose one constraint to obtain a unique regression solution. The regression fit is independent of the constraint in the sense that the estimated specific returns $\mathbf{u}$ do not depend on the choice of constraint. Nevertheless, the constraint must be chosen carefully to ensure an intuitive interpretation of the pure factor portfolios. For the case of a single-country model, we set the cap-weighted industry factor returns to zero,

$$\sum_i w_i f_i = 0 \ . \tag{A7}$$

The constraint can be expressed in matrix form as

$$\mathbf{f} = \mathbf{R}\mathbf{g} \ , \tag{A8}$$

Where $\mathbf{R}$ is $K \times (K-1)$ constraint matrix, and $\mathbf{g}$ is a $(K-1) \times 1$ vector of auxiliary factor returns. For instance, suppose that there are three factors and one constraint in the form of Equation (A7). In this case, the constraint equation becomes

$$\begin{bmatrix} f_1 \\ f_2 \\ f_3 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ -(w_1/w_3) & -(w_2/w_3) \end{bmatrix} \begin{bmatrix} g_1 \\ g_2 \end{bmatrix} . \tag{A9}$$

The reader may easily verify that Equation (A9) satisfies the constraint in Equation (A7).

Global factor models exhibit a second exact collinearity, since the sum of country factor exposures is also exactly 1 (i.e., again equal to the market factor exposure). In this case, we impose a second constraint

$$\sum_c w_c f_c = 0 \ , \tag{A10}$$

where w_c is the cap-weight in country c. That is, we set the cap-weighted country factor returns to zero.

Suppose that the specific returns are heteroscedastic (i.e., unequal volatilities). Let $\mathbf{V}$ be the diagonal matrix whose diagonal elements are given by the inverse specific variance

$$\mathbf{V} = \mathrm{var}^{-1}(\mathbf{u}) \ . \tag{A11}$$

In practice, we use square root of market capitalization as a proxy for inverse specific variance. Next, substitute Equation (A8) into Equation (A6) and multiply by $\sqrt{\mathbf{V}}$,

$$\sqrt{\mathbf{V}}\mathbf{r} = \sqrt{\mathbf{V}}\mathbf{XRg} + \sqrt{\mathbf{V}}\mathbf{u} \ . \tag{A12}$$

Upon doing a change of variables $\tilde{\mathbf{r}} = \sqrt{\mathbf{V}}\mathbf{r}$, $\mathbf{Y} = \sqrt{\mathbf{V}}\mathbf{XR}$, and $\tilde{\mathbf{u}} = \sqrt{\mathbf{V}}\mathbf{u}$, Equation (A12) becomes

$$\tilde{\mathbf{r}} = \mathbf{Yg} + \tilde{\mathbf{u}} \ . \tag{A13}$$

The regression has now been transformed into homoscedastic form. In other words, $\mathrm{var}(\tilde{\mathbf{u}})$ is proportional to the identity matrix. The ordinary least squares (OLS) solution is given by

$$\mathbf{g} = \left(\mathbf{Y}'\mathbf{Y}\right)^{-1}\mathbf{Y}'\tilde{\mathbf{r}} \ . \tag{A14}$$

Transforming back to our original variables, we obtain

$$\mathbf{f} = \mathbf{R}\left(\mathbf{R}'\mathbf{X}'\mathbf{VXR}\right)^{-1}\mathbf{R}'\mathbf{X}'\mathbf{Vr} \ . \tag{A15}$$

Let $\mathbf{W}$ denote the $K \times N$ matrix given by

$$\mathbf{W} = \mathbf{R}\left(\mathbf{R}'\mathbf{X}'\mathbf{V}\mathbf{X}\mathbf{R}\right)^{-1}\mathbf{R}'\mathbf{X}'\mathbf{V} \; . \tag{A16}$$

It follows that the rows of $\mathbf{W}$ provide the stock weights in the pure factor portfolios.

Minimum-Volatility Factor Portfolios. These portfolios are constructed by mean-variance optimization. Let $\mathbf{x}_k$ denote an $N \times 1$ column vector of exposures to factor k. As shown by Grinold and Kahn (2000), the minimum-volatility portfolio with unit exposure to the factor is given by

$$\mathbf{w}_k = \frac{\mathbf{\Omega}^{-1}\mathbf{x}_k}{\mathbf{x}_k'\mathbf{\Omega}^{-1}\mathbf{x}_k} \; , \tag{A17}$$

where $\mathbf{\Omega}$ is the $N \times N$ asset covariance matrix. If $\mathbf{x}_k$ represents the expected asset returns, then the portfolio in Equation (A17) has the minimum volatility of all portfolios with fixed expected return. Hence, it represents the maximum Sharpe ratio portfolio.

As with the case of simple factor portfolios, minimum-volatility factor portfolios in general have non-zero exposures to all risk factors. Unlike simple factor portfolios, however, these are not “incidental” exposures. On the contrary, these exposures are designed to minimize overall portfolio risk.

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