CORRELATION FORECASTING

Capturing rich dynamics with higher-frequency data

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We improve the quality of correlation forecasts by using higher-frequency data, which capture dynamically and in more detail the various sources of risk in a dataset. We lay out a framework for modeling correlations that is described by a parsimonious set of parameters and it embeds some of the most advanced correlation models. The framework is centered on weighting schemes of historical data that are allowed to have a higher frequency than the forecasting horizon. Higher frequency data potentially improve the stability and the responsiveness of the forecasts. We identify challenges to the high-frequency data setting and propose a solution to tie the magnitude of low-frequency correlations to the potentially different magnitude of the higher-frequency forecast.

Empirical results from two datasets, US equity sector returns and US rates changes, validate the use of daily data to forecast monthly correlations. The correlation matrix is more responsive because it requires less history to be stable than pure monthly models do, and optimized portfolios constructed from the matrix inverse have better properties.

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Introduction[1]

Correlations are notoriously difficult to estimate because of the large number of parameters involved compared with the number of observations available. One avenue to improve the quality of the estimates relies on using observations from a higher frequency, a technique successfully applied when forecasting volatilities. However, that technique does not account for the difference in magnitude between correlations extracted from data at different horizons (a term structure of correlations), which may greatly limit its application. We propose an intuitive and efficient solution based on adjusting the correlation matrix eigenvalues. By mitigating significantly the magnitude of the difference, our solution helps realize the benefits of using higher frequency data for correlation estimation.

We start our study by taking stock of the main challenges surrounding correlation forecasting. We describe a general model for forecasting correlations centered on a weighted combination of historical data. The model encompasses the class of Dynamic Conditional Correlation (DCC) models pioneered by Engle (2002). This general model is extended to include data at frequencies higher than the forecast horizon. We describe and address the challenges related to the use of higher frequency data. We follow the theoretical part with examples, showing how the forecasting performance changes, in sample and out of sample, as we sequentially add more sophistication to the model.

Forecasting Correlations

The typical portfolio construction methods and risk forecasting models require the forecast of the covariance matrix of the underlying securities. The exercise may be separated into forecasting volatilities and correlations because these two objects have a different nature. Volatility is inherently a univariate object, a property associated solely with the univariate distribution of an asset, without any information about how that asset is connected to other assets. In contrast, correlations are a multivariate object, describing how various assets depend on each other without containing any information about the univariate distribution of the individual assets. The differing nature of the two quantities calls for potentially different forecasting techniques.

The typical statistical approach to forecasting any quantity relies on historical estimates. Correlation matrix estimates suffer from the well-known curse of dimensionality: the number of correlation coefficients is approximately of the order $N^2/2$, while the number of datapoints available is NxT, where N is the number of dataseries and T the number of observations per series. The more data series N we have, the fewer observations we have for each coefficient.

Almost all solutions to the curse of dimensionality seek to reduce the number of coefficients we need to estimate by imposing some structure on the correlation matrix. For example, the ubiquitous factor models (for an application see Lazanas et. al (2012)) reduce the number of parameters by describing the N dataseries with a smaller number of K factors. Each dataseries is a linear function of the same K factors and a residual assumed to be uncorrelated across dataseries. To get all the N(N-1)/2 correlation parameters, we need to estimate NxK exposures of dataseries to the factors and the K(K-1)/2 correlation parameters of the factor correlation matrix, which amounts to Kx(N+(K-1)/2) parameters. For example, explaining 400 securities with 10 factors, we have 4,045 parameters to estimate, compared with 79,800 correlation coefficients. If we have 200 observations for

[1] We thank Antonio Silva and Niels Schuehle for valuable comments and insights.

each security, we have 20 observations per parameter in the factor model, compared with one observation per correlation coefficient.

We can increase the precision of the estimates and keep the richness of the structure if we raise the number of observations per series instead of reducing the number of parameters. A technique successfully employed in volatility forecasting increases the number of observations by sampling the data at a higher frequency[2]. But raising the sampling frequency has no guarantee of improving the quality of any estimate: for example, the error around the mean estimate remains the same. The intuition about why this technique may work for correlations comes from the fact that the correlation ρ between two series can be expressed as a function of the individual volatilities (σ_x and σ_y) and the volatility of the series' sum σ_{x+y}: $\rho = \frac{\sigma_{x+y}^2 - \sigma_x^2 - \sigma_y^2}{2\sigma_x\sigma_y}$. Hence, improving the precision of volatility forecasts may improve the precision of correlation forecasts.

A forecasting model from historical data

In this section, we develop a forecasting framework for correlations using historical data, which we extend to a mixed frequency setting in the following sections.

Consider the returns *R* of N securities over T-1 periods. We want to estimate the next period correlation matrix given the data we have up to T:

$$\Omega_T = Corr\left(R_T \mid R_1 \ldots R_{T-1}\right)$$

Because volatility may vary over time, we must adjust historical data to have a constant volatility. The theoretical argument for this adjustment is presented by Engle and Sheppard (2001) in the context of the Dynamic Conditional Correlation model of Engle (2002). Intuitively, we must estimate volatilities and correlations at the same time, as the two may depend on each other. If we assume that changes in correlations do not directly cause changes in volatility (however, both may change at the same time because of a third cause), then we can estimate volatility disregarding correlations and use these estimates in the correlation estimation. The link between dynamic volatilities and correlations can be seen from:

$$Corr_{T-1}\left(x_T, y_T\right) \equiv \frac{Cov_{T-1}\left(x_T, y_T\right)}{Vol_{T-1}\left(x_T\right)Vol_{T-1}\left(y_T\right)} = E_{T-1}\left(\frac{x_T}{Vol_{T-1}\left(x_T\right)}\frac{y_T}{Vol_{T-1}\left(y_T\right)}\right)$$

Where *x* and *y* are two zero-mean variables.

Thus, correlations can be described as the expected value of the product between the two volatility-adjusted variables. The typical statistical method to obtain the expected value of a quantity with stable properties over time is to average its historical values. Therefore, the correlation estimate of *x* and *y* is

$$Corr_{T-1}\left(x_T, y_T\right) \equiv \frac{1}{T-1}\sum_{t=1}^{T-1}\frac{x_t}{Vol_{t-1}\left(x_t\right)}\frac{y_t}{Vol_{t-1}\left(y_t\right)} = \frac{1}{T-1}\sum_{t=1}^{T-1}\tilde{x}_t\tilde{y}_t$$

Where $\tilde{x}_t$ and $\tilde{y}_t$ are volatility-scaled versions of the original variables. Note that the observation at t must be scaled by the volatility forecast available as of previous period t-1. Moreover, the scaled variables have a constant volatility over time, which gives them more stable properties than the un-scaled ones.

[2] See the pioneering work of Ghysels et al. (2006) and for a general framework see Gabudean and Schuehle (2011).

Returning to the *N* securities let $\widetilde{R}$ be their zero-mean returns scaled by their forecast volatility. Then, following the argument above, a general correlation forecast model can be written as

(1) $$\Omega_T = \frac{1}{\sum_t w_t} \sum_{t=1}^{T-1} w_t \widetilde{R}_t \widetilde{R}_t'$$

The next-period correlation forecast is a weighted average of the previous-periods outer-product of $\widetilde{R}$ with itself. The resulting matrix does not have a diagonal of ones, as we use forecast (not realized) volatility in the scaling. Therefore, we treat the estimate as a covariance matrix and extract the correlations from it in the typical fashion.

Popular choices for the weighting scheme are: constant weights (w = 1/(T-1)), or moving-window weights (w = 1/M for the most recent M observations), and an exponentially decaying weighted scheme (EWMA). For a discussion on these schemes and how they apply to volatility, see Gabudean and Schuehle (2011).

The fixed-weight model is defined as:

M1 $$\overline{\Omega} = \frac{1}{T-1} \sum_{t=1}^{T-1} \widetilde{R}_t \widetilde{R}_t'$$

The moving window model is defined as

M2 $$\Omega_T^{MWin} = \frac{1}{M} \sum_{t=T-M}^{T-1} \widetilde{R}_t \widetilde{R}_t'$$

This model has one parameter, the size of the window *M*, which must be specified.

The EWMA model, with parameter *b*:

M3 $$\Omega_T^{EWMA} = (1-b)\widetilde{R}_{T-1}\widetilde{R}_{T-1}' + b\Omega_{T-1}^{EWMA}$$

The model parameter *b* is often transformed to represent the number of periods required for the weight to be half the weight associated with the current period, or the half-life: $b^{HalfLife} = 0.5$

The DCC (1,1) model of Engle (2002) with parameters *a* and *b* is specified as:

$$\Omega_T^{DCC} = (1-a-b)\overline{\Omega} + a\widetilde{R}_{T-1}\widetilde{R}_{T-1}' + b\Omega_{T-1}^{DCC}$$

Where $\overline{\Omega}$ is the unweighted correlation matrix from M1. If we let $\lambda = \frac{a}{1-b}$ then we can write the model as

M4 $$\Omega_T^{DCC} = (1-\lambda)\overline{\Omega} + \lambda\Omega_T^{EWMA}$$

The EWMA component is defined the same way as in M3 above. Therefore, the DCC model can be understood as an average between a long-term model (M1) and a EWMA model (M3). DCC shares this property with its related volatility model GARCH. As shown for GARCH(1,1) by Gabudean and Schuehle (2011), DCC(1,1) can be interpreted as a model using only one weighting scheme which decays exponentially over recent observations and gets flat, but not zero, over early observations.

Forecasting Correlations with High-Frequency Data

The models presented thus far assume that one period of historical data has the same length as the forecast horizon, eg, if we forecast monthly correlations, we use only monthly data. In this example, we may use daily data for the monthly forecast. To incorporate higher frequency data, we must alter the general specification (1). Let H be the number of sub-periods within each period and let $\widetilde{Z}_{t+h/H}$ be the volatility-adjusted return for the sub-period t+(h-1)/H to t+h/H. Note that to get $\widetilde{Z}$, we require the volatility forecast at the higher-frequency horizon, differing from the volatility forecast used to obtain $\widetilde{R}$. Even though the additional volatility model is a burden, volatility at high frequencies can be forecast well with a parsimonious model. The intuitive model extension of (1) that incorporates higher-frequency data $\widetilde{Z}$ is

(2) $$\Omega_T^{HighFq} = \frac{1}{\sum \omega_{t+h/H}} \sum_{t=0}^{T-2} \sum_{h=1}^{H} \omega_{t+h/H} \widetilde{Z}_{t+h/H} \widetilde{Z}'_{t+h/H}$$

Where ω is a weighting scheme applied to the high-frequency returns.

Colacito, Ghysels, and Engle (2011) present a model that combines (1) and (2) in a version of DCC, called DCC-MIDAS, or DCC-M. The DCC-M model can be written approximately as

M5 $$\Omega_T^{DCC-M} = (1-\lambda)\Omega_T^{HighFq} + \lambda\Omega_T^{EWMA}$$

The EWMA component is the same as M3 defined above and uses data at the same horizon. The high-frequency component, called MIDAS, is defined from equation (2). Its associated weighting scheme ω is constant within a month and follows a decay across months defined by a beta function. The beta function decay can be flat (parameter close to 1), linear (parameter close to 2), or exponential (for higher parameters). Its parameter can be treated as additional parameter of the model, along λ and *b*.

In the spirit of Gabudean and Schuehle (2011) MXF model for volatility, we propose an extension for correlations specified as

M6 $$\Omega_T^{MXF} = (1-\lambda)\Omega_T^{LongRun-HF} + \lambda\Omega_{T-1+1/H}^{EWMA-HF}$$

The weighting schemes of this model are smooth over daily data, ie, they do not make a distinction between weights within and across months, as DCC-M does. The distinction, while theoretically appealing, makes little practical difference. The long-run component uses daily data with a moving-window *M* days weighting scheme.

$$\Omega_T^{LongRun-HF} = \sum_{\tau=T-1-(M-1)/H}^{T-1} \widetilde{Z}_{\tau/H} \widetilde{Z}'_{\tau/H}$$

The EWMA uses high frequency data with an exponential decay weighting scheme:

$$\Omega_{T+h/H}^{EWMA-HF} = (1-b^{HF})\widetilde{Z}_{T+(h-1)/H} \widetilde{Z}'_{T+(h-1)/H} + b^{HF}\Omega_{T+(h-1)/H}^{EWMA-HF}$$

The biggest change from DCC-M is the fact that both components use high-frequency data, as opposed to only one.

On a closer inspection, the formulation in (2) is identical to a model of forecasting the correlations over the next sub-period T-1+1/H, using sub-period returns. Thus, our mixed frequency model simply makes the long-horizon forecast equal to the short-horizon forecast. Correlations typically vary across horizons; hence, our approach thus far is inadequate.

One issue not addressed yet is the term structure of correlations – the fact that correlations typically vary across horizons. This issue arises in many other financial applications, namely the estimation of volatility. For the latter, Gabudean and Schuehle (2011) propose to calibrate the model parameters to the longer-horizon data and add an extra scaling parameter. In the next two subsections, we explore how these techniques can be extended to correlations.

Calibrate parameters to lower-frequency data

The weighting schemes associated with the various estimators described before can be characterized as a function of one or more parameters (except for the constant-weighted scheme of M1 that has no parameters). Parameters are chosen such that the resulting estimates fit the data best[3]. In the mixed frequency case, to forecast volatility at the low-frequency level, we use the low-frequency data as the dependent variable (right-side variable) and high-frequency data as the independent one (left-side variable).

We can follow the same approach when forecasting correlations. In particular, we define the weighting scheme ω a function of a general set of parameters θ and calibrate the parameters such that the resulting correlation matrices Ω_t forecasts the correlation of monthly data $\widetilde{R}$, while using the daily data $\widetilde{Z}$.
Using a Maximum Likelihood Estimation (MLE) approach and assuming returns have a multivariate normal joint distribution, the optimum parameters θ satisfy

$$\theta_{ML} = \arg\max_{\theta} \sum_t -\left(\ln\left(\left|\Omega_t(\theta)\right|\right) + \widetilde{R}_t'\Omega_t^{-1}(\theta)\widetilde{R}_t\right)$$

Where $\Omega_t(\theta)$ is defined following the general specification (2) and the weighting scheme $\omega(\theta, t-1-\tau-h/H)$ is a function only of θ and how far the observations are from the forecast period *t* (note that *t* replaces *T* and *tau* replaces *t* because we apply (2) to each period *t*, not only to the last period *T*). This approach is followed by the DCC-M model of Colacito et. al (2011).

The intuition for MLE is an extension of the one provided for the volatility case in Gabudean and Schuehle (2011). We start from the observation that if the forecast is correct, then, on average, we have $\widetilde{R}_t\widetilde{R}_t' = \Omega_t$, which implies that $\Omega_t^{-1}\left(I_N - \Omega_t^{-1}\widetilde{R}_t\widetilde{R}_t'\right) = 0_N$. The left-hand side term enters the equations that the MLE estimators need to solve (i.e., the first-order conditions of the maximization). Note the double-weighting by the inverse of Ω_t in the equality, which shows that the MLE procedure puts an emphasis on estimating Ω_t^{-1} correctly, not only Ω_t.

Scale correlation forecasts across horizons

The set of parameters θ above changes the weights given to the higher-frequency data – that is, it changes the memory we want the process to have. Typically, the longer the horizon of forecast, the longer we want the memory to be. However, independent of this parameter set, weights end up summing to one. Therefore, the approach above does not change the "average" level of correlations away from those observed in the higher-frequency data. This poses a problem if the term structure of correlations is not flat. The second element we introduce in the analysis – a scaling parameter – addresses these concerns. For example, the magnitude of volatility at a given horizon typically depends on the square root of that horizon length and the autocorrelation properties. Therefore, the mixed-frequency volatility estimate must be scaled by a parameter and researchers employ various methods to calibrate this parameter.

[3] As an example for volatility models, the parameters are such that the ratio between the original data and the forecasted volatility has volatility equal to one.

The approach just described cannot be easily applied to correlations, and to our knowledge no one has addressed directly this issue in the literature. The issue resides in the key requirements for a correlation matrix: the diagonal elements to be one and the matrix to be positive definite. Scaling the entire matrix by a parameter creates a matrix with diagonals different from one. Using the usual trick of treating the scaled matrix as covariance and extract correlations from it takes us back to the same correlation matrix before scaling. Alternatively, scaling only the non-diagonal elements may lead to a non-positive definite matrix, not to mention the possibility of correlations larger than one.

To illustrate the importance of adjusting the correlations, Figure 1 shows how realized correlations change between the daily and monthly horizons for two separate datasets: returns on 25 US equity sectors and yield changes of 6 points on the US Treasury curve. Typically, daily correlations are higher than the monthly ones, perhaps because over the short term, all series are more driven by common factors (such as variations in risk appetite, or latest economic news) while over the longer horizon, more particular factors affect the various series (such as the performance of the technology versus utility sectors, or the long-term rates versus the short term ones). Moreover, the relation between monthly and daily correlations varies over time, which argues for a dynamic adjustment method. To have a sense of the confidence intervals around these point estimates, simulation results detailed in the appendix (see the sample correlation results in Figure 24) show approximately a +/-5% confidence interval at 90% confidence level.

Figure 1: Average historical correlations at daily and monthly frequencies for a set of US equity sector returns and a set of US rates yield changes, by five-year periods

	US Equity Sectors		US Rates	
	Daily	Monthly	Daily	Monthly
Jan 92 - Dec 96	47%	42%	86%	86%
Jan 97 - Dec 01	49%	51%	78%	72%
Jan 02 - Dec 06	59%	44%	83%	77%
Jan 07 - Dec 11	72%	63%	70%	62%

Source: Barclays Research

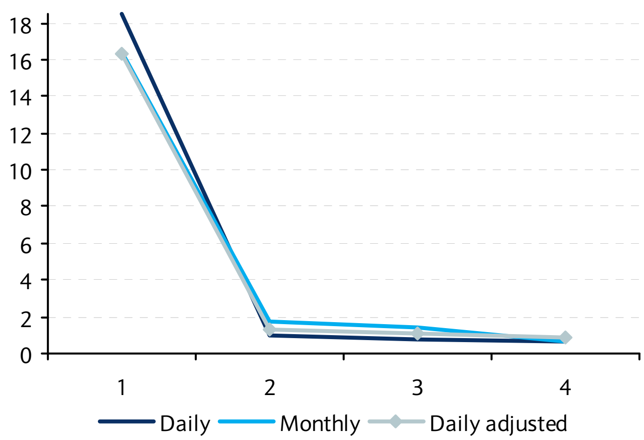
A solution for scaling correlations across horizons

To solve the scaling issue, we observe that the magnitude of correlations broadly relates to the first, or largest, eigenvalue of the correlation matrix. Empirically, we can see this relation in Figure 2 and Figure 3, where the first eigenvalue of the correlation from monthly data is smaller than the one from daily data, in line with the last row from Figure 1. To wit, the higher – positive or negative – the correlations among various series are, the more likely the data are driven by one source of risk. The first eigenvector statistically captures the most pervasive risk in the data and the importance of this risk source is given by its associated eigenvalue. Therefore, we can adjust the magnitude of the correlations by changing the first eigenvalue; our proposed solution[4] to the scaling issue relies on this insight.

However, adjusting the first eigenvalues creates secondary effects. Intuitively, the amount of risk in some dataseries is captured by volatilities and the allocation of risk among various components is captured by correlations. Therefore, the total risk captured by correlations should remain constant. This requirement translates statistically in the condition that the sum of all correlation matrix eigenvalues are fixed. Indeed, the sum of eigenvalues equals the sum of all

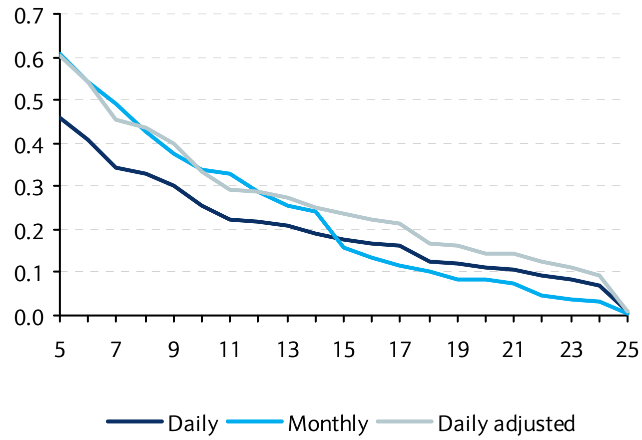
[4] The recently-popular random matrix theory works by decomposing the correlation matrix into eigenvectors and eigenvalues. Hautsch and Kyj (2010) take this decomposition a step forward and forecast eigenvalues and eigenvectors separately with data at possibly different horizons. Our method is in the spirit of their work, even though the exact modeling approach, the motivation, and the level of complexity differs.

Figure 2: Eigenvalues 1-4 of the correlation matrix of US Equity sectors, Jan 07 – Dec 11



Source: Compustat, Barclays Research

Figure 3: Eigenvalues 5-25 of the correlation matrix of US Equity sectors, Jan 07 – Dec 11



Source: Compustat, Barclays Research

elements on the matrix diagonal, which all must equal to one for a correlation matrix. We may see this effect in the figures above where for daily data eigenvalues 2-13 are lower to compensate for the higher first eigenvalue. The implication for our solution is that we must adjust the other eigenvalues to compensate for the change in the first eigenvalue. The additional benefit from keeping the eigenvalues sum to N is that the diagonal elements are close to one and even though we must still transform the eigenvalue-adjusted matrix into a correlation matrix, the transformation would preserve the increased importance of the first eigenvalue.

As a solution to the scaling issue, we propose to increase (decrease) the highest eigenvalue and reduce (increase) proportionally the other eigenvalues such that their sum remains constant. An illustration of this solution for the example above is shown in Figure 2 and Figure 3, where the "daily adjusted" represents the daily eigenvalues adjusted such that the first eigenvalue matches the monthly one. The next eigenvalues have daily levels after adjustment closer to the monthly ones, supporting our choice to adjust the other eigenvalues proportionally as opposed to, for example, adjusting only the second eigenvalue. For the smallest eigenvalues (note the change in scale between the two graphs), the adjustment moves the daily eigenvalues away from the target. However, these eigenvalues are clearly less important for the overall correlation matrix. One may think of a more complex redistribution among smaller eigenvalues, but that requires more than one parameter and is beyond the scope of this paper. This adjustment is simple and intuitive, as it allows us to shift the average levels of correlation in a parsimonious way.

Since we do not adjust the eigenvectors, the definition of the most pervasive risk source, as given by the first eigenvector, remains the same. Intuitively, we retain the relative relation among various correlation coefficients and scale them similarly. Other models may potentially adjust the relation among the eigenvectors themselves. However, those complications, while potentially useful, do not target directly the problem of variation in the magnitude of correlations as our solution does.

We formalize the model by adding an extra parameter γ to specification (2):

$$(\theta_{ML}, \gamma_{ML}) = \arg\max_{\theta,\gamma} \sum_t -\left(\ln\left(\left|\hat{\Omega}_t(\theta,\gamma)\right|\right) + \tilde{R}_t' \hat{\Omega}_t^{-1}(\theta,\gamma)\tilde{R}_t\right) \quad (3)$$

Where the adjusted correlation matrix is defined from the eigenvectors V of the original matrix and its adjusted eigenvalues $d_{1\ldots N}$, grouped in the diagonal matrix D:

$$\hat{\Omega}_t(\gamma,\theta) = V_t \hat{D}_t V_1'$$

We specify the eigenvalue adjustment function as $\hat{d}_{1,t} = N(1-\gamma) + d_{1,t}\gamma$ and $\hat{d}_{j,t} = d_{j,t}\gamma$ for all *j*>1. The parameter γ must be positive for eigenvalues 2 to N to remain positive and must be lower than $N/(N-d_1)$ for $\hat{d}_1$ to be positive. For $\gamma < 1$, we get $\hat{d}_{1,t} > d_{1,t}$, meaning that we are increasing correlations. The opposite happens for $\gamma > 1$. This specification insures that the matrix trace stays equal to *N* and the matrix is still symmetric and positive definite. The first eigenvalue is pulled toward *N* and the rest of the eigenvalues are reduced proportionally. Thus the higher the adjustment parameter, the lower the first eigenvalue is. Note that in this specification the eigenvalues vary over time, but we keep the parameters θ and γ constant. We may loosen this restriction and let them slowly vary.

In a simple bi-variate case, this specification translates into shrinking the correlation coefficient ρ_t toward 1. From the adjustment of the second eigenvalue, we have $1-\hat{\rho}_t = \gamma(1-\rho_t)$, or $\hat{\rho}_t = \gamma * \rho_t + (1-\gamma) * 1$. The adjustment parameter γ and the correlation $\hat{\rho}$ move in opposite directions. Note that this specification for the correlation scaling differs from one where the correlation is multiplied by a fixed parameter, ie, $\hat{\rho}_t = param * \rho_t$. In our model, lower correlations are scaled up more in absolute value, while in the proportional scaling model this happens with the higher correlations.

This result about the average correlation generalizes to a constant-correlation matrix of any dimension, as proven in the appendix, in part because scaling a constant-correlation matrix in this fashion results in another constant correlation matrix. The relationship between the adjusted and unadjusted correlation is the same as in the bi-variate case above. Therefore, scaling a constant correlation matrix can be interpreted as a scaling of the non-diagonal elements. In the extreme case in which all correlations are zero, then all eigenvalues are equal and the corresponding eigenvectors can be taken to be Cartesian basis vectors. Should we decide to still reallocate risk from one eigenvalue to another, the resulting matrix would still have zero correlations but the diagonal elements would differ from one.

If some correlations are negative, we can switch their sign without loss of generality by considering the short exposure to that asset. Thus, we may interpret "average correlation" as the average of absolute values of correlations. In the appendix, we further investigate in idealized settings the properties of the adjusted high-frequency correlations estimator.

To better understand the practical behavior of this adjustment, we look at the properties of the time series of average return across securities. The average return typically corresponds to the combination defined by the first eigenvector, called the first principal component. In this case the first eigenvalue, which is the variance of the first principal component, becomes a linear function of the average correlation:

$$d_1 = \frac{1}{N^2} i_N' \Omega i_N = \frac{N + N(N-1)\rho}{N^2} = \frac{1}{N} + \frac{N-1}{N}\rho \quad \text{where } i_N \text{ is a vector of ones}$$

This equation shows the typical relation between the first eigenvalue and the average correlation.

Examples

In this section, we look at two examples of forecasting monthly correlations of returns among two separate data sets: 25 US equity sector indices[5] and 6 US Treasury rates of various tenors[6]. The two sets comprise strongly correlated series, but with some differences. The equity returns are positively correlated, on average 53%, with some diversity among various sectors and no clear clustering. The rates series are smaller in number (matrix dimension is important for correlation forecasting); have higher correlations, 70% on average; and there is some clustering: tenors closer together are more correlated.

As forecasting models that include only monthly data, we consider the exponential weighted moving average EWMA (see M3) and DCC model of Engle (2002) (see M4). As models that include daily data to forecast at the monthly horizon, we select the DCC-M model of Colacito, et al. (2011) and the MXF model detailed in this paper.

In the DCC-M model, for robustness, we simplify the beta decay weighting scheme of the MIDAS component to be a moving window (effectively setting the beta function parameter to one). We investigate several choices for the window length, and even though the authors intended the MIDAS component to capture the long run, we found the most compelling version with a relatively short window of 126 days or half a year. We will detail the results only with this window length, even though we comment on unreported results (typically worse) from versions with longer windows.

Our MXF models construct the correlation forecasts solely from daily data, in contrast to DCC-M. Monthly data enter only in the calibration of parameters, as the dependent variable. The long-term component is defined as a 504-day moving–window estimate. We consider a version labeled “MXF Unadj” without the scaling parameter (more precisely, $\gamma = 1$), which calibrates two parameters: the weight of EWMA λ and the half-life of the EWMA. The second version, labeled MXF, adds the scaling parameter and fixes the EWMA parameter to 63 days, which comes from our extensive investigations of various data series.

We list all the models in Figure 4, with their associated parameters and parameter values. “C” means the parameter is calibrated each period from historical data using the maximum likelihood procedure. Given the re-formulation of the DCC and DCC-M models in terms of averages between a long-term component and an EWMA, we will present their parameters as the allocation to EWMA (λ) and the half-life of the EWMA weighting scheme, instead of the original parameters *a* and *b*.

Figure 4: Forecasting models and associated parameters

Models	Weight on EWMA	EWMA half life	Long-term window	Scaling parameter
EWMA	1	C	-	-
DCC	C	C	C	-
DCC-M	C	C	126 days	-
MXF Unadj	C	C	504 days	-
MXF	C	63 days	504 days	C

Note: C = Calibrated historically each period.
Source: Barclays Research

[5] These are value-weighted indices on sectors defined by GICS level 2 codes from a universe of top 2000 US securities. For more details see Silva, et al. (2009)
[6] We retain the 6mo, 2y, 5y, 10y, 20y, and 30y rates. The returns are defined as the negative of rates change over the period.

The four models evolve from simple to complex. The EWMA model captures the dynamic nature of correlations and their structure. The DCC model adds the feature of mean-reversion to the long-term mean. The DCC-M model replaces the fixed long-term mean with a time-varying one derived from high-frequency data. The MXF model has both components, long-term and short-term derived from high-frequency data. The eigenvalue correction bridges the gap between average magnitude of monthly and daily correlations.

Before forecasting correlations, we must scale the data so they have the same volatility over time and across data series. For that, we require volatility forecasts for daily and monthly horizons. For the monthly horizon, we use the MXF model proposed by Gabudean and Schuehle (2011), which is an average of a long-term and an EWMA component, each computed using daily data. The long-term component is a moving-window model using the last 504 days (two years). The EWMA component has a half-life parameter of 42 days (two months). Moreover, the model has a scaling parameter, the volatility counterpart of our γ.

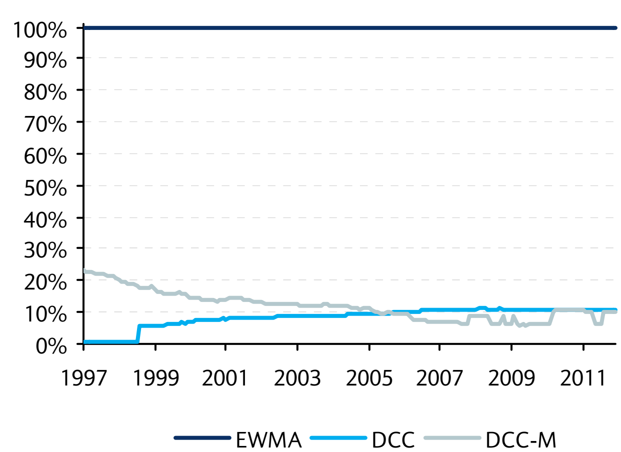
The weight to EWMA and the scaling parameter are calibrated each month and for each series using a Maximum Likelihood technique, with more focus on recent observations. The daily forecast model is a version of GARCH(1,1) that calibrates the parameters focusing on recent observations.

In-sample results

To showcase the models, we present the parameter results and in-sample dynamics for the two datasets, beginning with the US Equity sector returns.

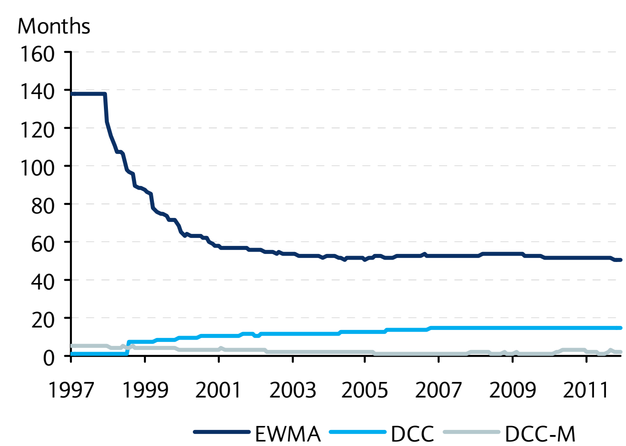
We start with the evolution of the two parameters common across all models: the allocation to EWMA and EWMA half-life. Figure 5 and Figure 6 show their value for models with the EWMA component constructed from monthly data: EWMA, DCC, and DCC-M. All models put a negligible weight on the EWMA, except the for EWMA model that has it at 100% by construction. This result can be interpreted as data preferring a stable correlation matrix that comes from an estimation using a significant amount of historical data. Supporting this result, the optimal EWMA model behaves like a long-term, fixed-weight model: the half-life parameter is between 50 and 140 months (see Figure 6). The half-life parameters for the other models are inconsequential because their allocation to EWMA is small. For them, the half-life often hits the lower-bound of one month. Similar results (not shown) are obtained even for longer windows on the DCC-M model.

Figure 5: Allocation to EWMA component for EWMA, DCC, and DCC-M models, US Equities, 1997-2011



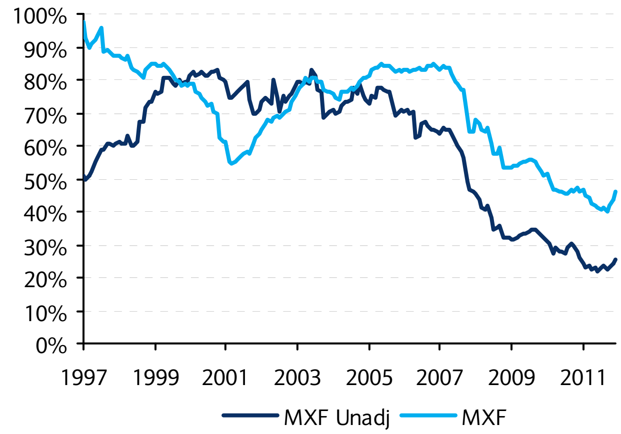
Source: Compustat, Barclays Research

Figure 6: Half-life of the EWMA component for EWMA, DCC, and DCC-M models, US Equities, 1997-2011



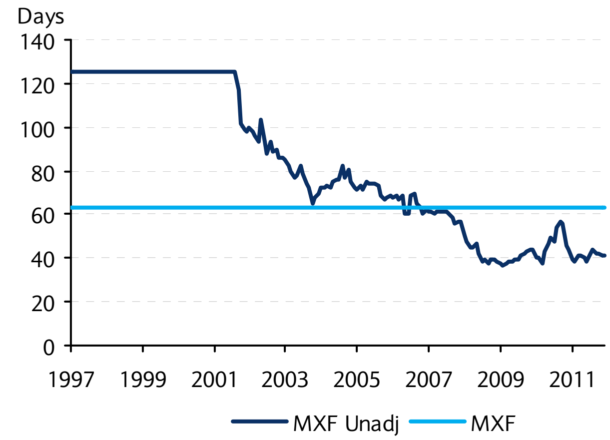
Source: Compustat, Barclays Research

Figure 7: Allocation to EWMA component for MXF models, US Equities, 1997-2011



Source: Compustat, Barclays Research

Figure 8: Half-life of the EWMA component for MXF models, US Equities, 1997-2011



Source: Compustat, Barclays Research

Figure 7 and Figure 8 present the same two parameters for the two versions of the MXF model. In contrast to the other models, they allocate significantly and dynamically to the EWMA component[7]. Interestingly, while the DCC-M results establish that daily data for the long-term component improve the model in sample, this result suggests that using daily data for both components, EWMA and long-term, further improves in-sample performance.

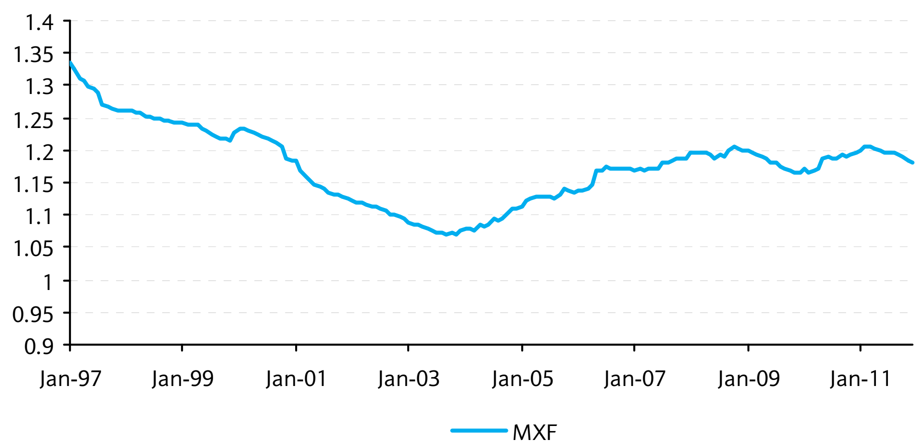
For the MXF Unadj model, the allocation to EWMA and the half-life move in lock-step almost every period, except for earlier in the sample when the half-life hits the 126–day cap we impose to keep the EWMA component different from the long-term one. In other words, we allocate more to EWMA when EWMA looks more like the long term. This dynamic was typical, even when we analyzed other data sets or when we used other model specifications, thus it is worthwhile to understand it better. The two effects counter each other: an increase of weight to EWMA prompts the model to extract more information from more recent data while an increase in half-life shifts the weight toward data further in the past. We may interpret this result as a sign that often one free parameter provides enough dynamic in the model to accommodate the data features. Hence, if we fix one of these two parameters, the other parameter may be more stable without damaging the in-sample performance and possibly providing some robustness out of sample against overfitting. The MXF model, which fixes half-life at 63 days, incorporates this insight (i.e., has a smoother allocation to EWMA over time, compared with a version of the MXF that calibrates all three parameters - not shown).

Lastly, given that the EWMA allocation has a similar dynamic between the two models, we may conclude that its effect is more powerful than the counter-dynamic we see in the half-life, with the implication being that we rather fix the half-life than the allocation between the two components. For example, the MXF drops its allocation to EWMA from 60-80% to less than 50% after 2007 on because during the crisis correlations varied widely and the model prefers a stable longer-term forecast to an aggressive one. The MXF Unadj model, which calibrates both parameters, shows simultaneously a decrease in the EWMA allocation and a more aggressive EWMA to capture the same dynamics of the correlation matrix.

[7] The MXF models are calibrated each month with focus on most recent observations. DCC models are calibrated from the entire dataset evenly up to the corresponding month. However, the more dynamic calibration of MXF does not account for the more dynamic nature of the parameters. In unreported results, when we calibrate the DCC models only from the most recent 48 months, we get virtually the same small weight on EWMA as in the case we report.

Figure 9 shows the results for the scaling parameter of the MXF model. The parameter decreases initially as monthly correlations increase toward the daily ones (see Figure 1), but after 2004, the difference between monthly and daily correlations widens again. Note that a parameter larger than one implies monthly correlations lower than daily ones.

Figure 9: Correlation scaling parameter for MXF model, US Equities, 1997-2011

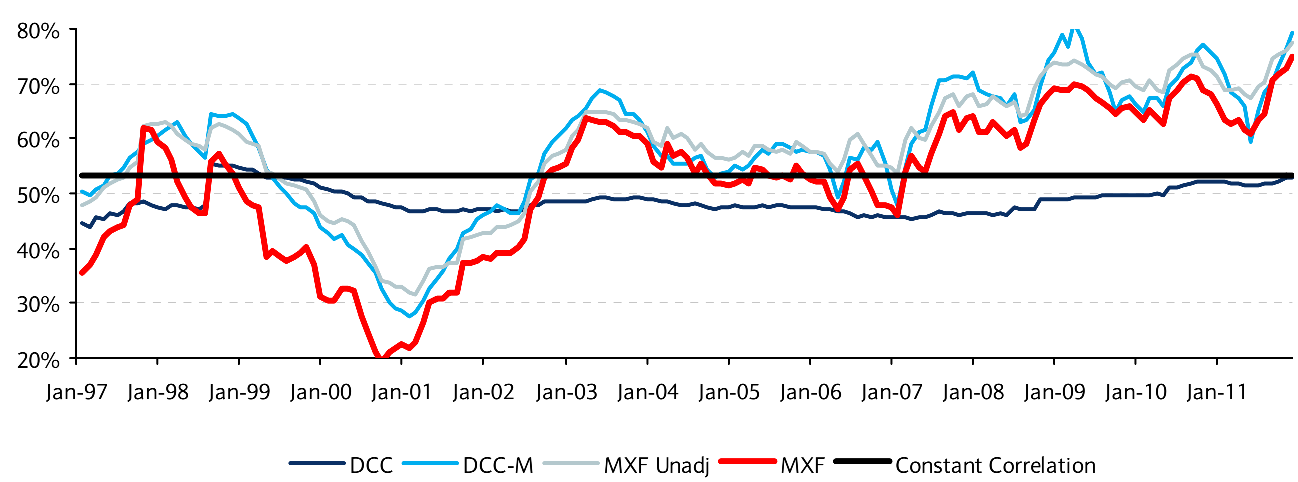


Source: Compustat, Barclays Research

From unreported results, we conclude that fixing the EWMA half-life parameter has little effect on the scaling parameter, which implies that the difference between daily and monthly correlations cannot be captured simply by adjusting the weighting scheme on daily data. The two horizons are fundamentally different and justify the presence of the adjustment parameter.

To understand how various models forecast the average correlations, Figure 10 shows their dynamics over time.

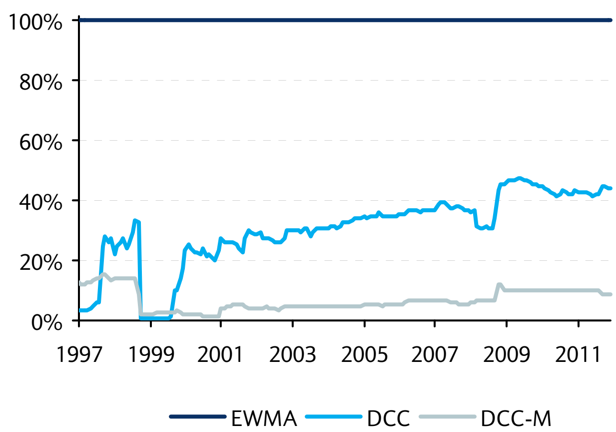
Figure 10: Time variation of average forecast correlations, US Equities



Source: Compustat, Barclays Research

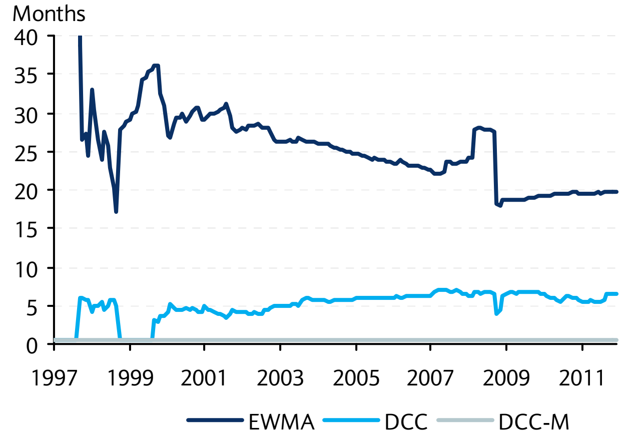
The average level of correlation over the entire sample is 53%. The DCC model correlations move slowly, as expected due to its focus on the long-term monthly horizon data. The DCC-M model is virtually a 126-day moving window model, which makes for more dynamic average correlations. MXF Unadj mirrors the DCC-M results as both use mainly daily data,

Figure 11: Allocation to EWMA component for EWMA, DCC, and DCC-M models, US Rates, 1997-2011



Source: Barclays Research

Figure 12: Half-life of the EWMA component for EWMA, DCC, and DCC-M models, US Rates, 1997-2011



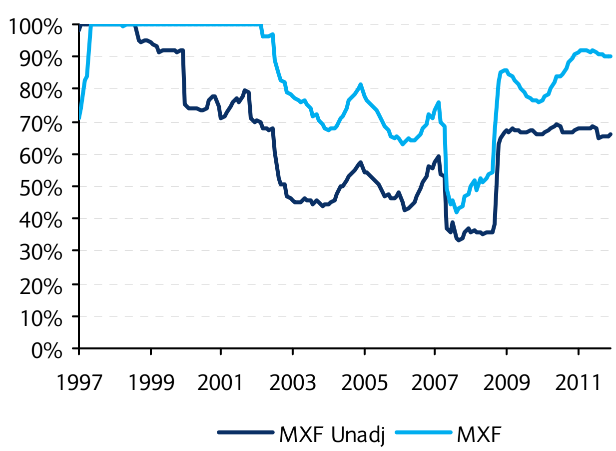
Source: Barclays Research

but shows a somewhat smoother behavior over 2008-11 due to its long-term component. The adjustment parameter typically reduces the average level of correlation compared with MXF Unadj, particularly at the beginning and end of the sample. This was expected from Figure 9 (a higher parameter means lower average correlation). For example, in 2000, the average forecast correlation is 20%, or 10 percentage points lower than the MXF Unadj forecast and well below the sample average.

We now turn to the analysis of the six US key rate series. The analysis follows the same sequence used to describe the US equity sectors. The results present a similar pattern, although quantitatively they are different.

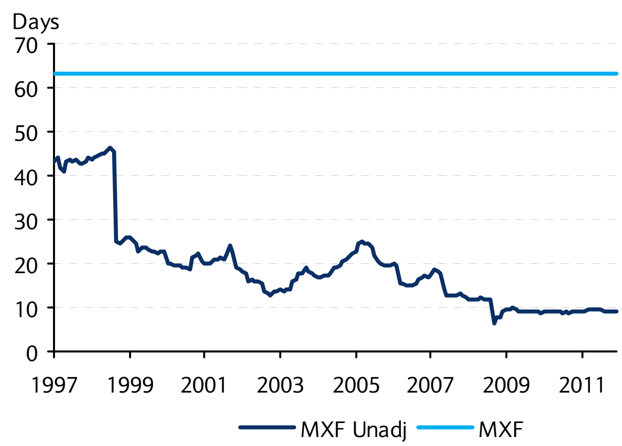
Figure 11 and Figure 12 show the dynamics of the two parameters for EWMA, DCC, and DCC-M. The allocation to EWMA remains small. When the allocation is higher than 20%, the half-life is very aggressive at five to eight months. However, when we force a 100% allocation to EWMA (as in the EWMA model), the optimal half-life is much higher, at 20-35 months. The US rates models are more aggressive than in the US equities models possibly

Figure 13: Allocation to EWMA component for MXF models, US Rates, 1997 - 2011



Source: Barclays Research

Figure 14: Half-life of the EWMA component for MXF models, US Rates, 1997 - 2011



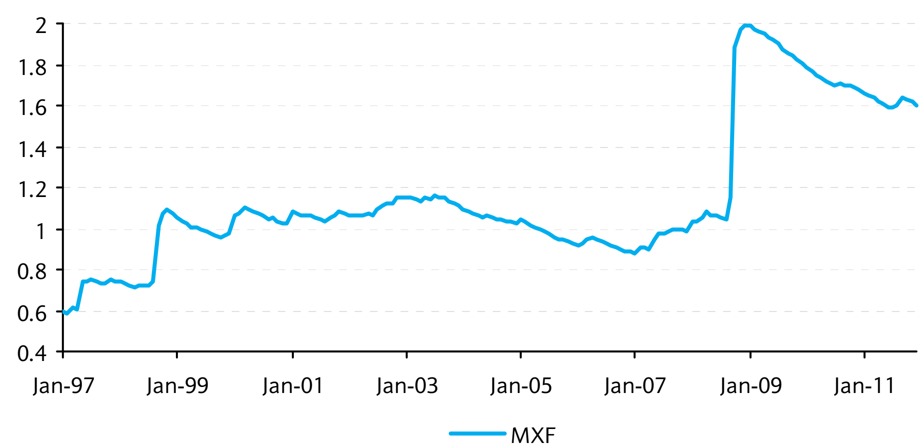
Source: Barclays Research

because of the smaller number of data series, six versus 25, which do not require as long of a history to produce a stable matrix. In unreported results, a longer window for DCC-M increases over some periods the allocation to EWMA at levels close to the allocation in the DCC model. The result may be due to the poor fit of a model with very long window, even when that window has daily data.

Figure 13 and Figure 14present the evolution for MXF models of the same two parameters. The previously noted positive relation between half-life and EWMA allocation can be observed here as well. Moreover, the optimal half-life is much lower than the 63 days we choose for the MXF model and that result remains unchanged when we calibrate all three parameters (scaling, EWMA allocation, and half life). The question, addressed later, becomes whether setting this parameter significantly higher than the optimum level will affect the forecast accuracy out of sample.

In Figure 15, we report the scaling parameter of MXF for the US Rates data. Its dynamic differs from that of US equities (Figure 9), with two significant jumps in 1998 (Russian default) and 2008 (Lehman Brothers collapse). Extreme events tend to push daily correlations higher than the monthly ones (understandable, given that these events may look historically more disproportionate in daily data than in monthly data). Moreover, they tend to affect all data series the same way, pushing correlations higher at all horizons.

Figure 15: Correlation scaling parameter for MXF model, US Rates, 1997-2011

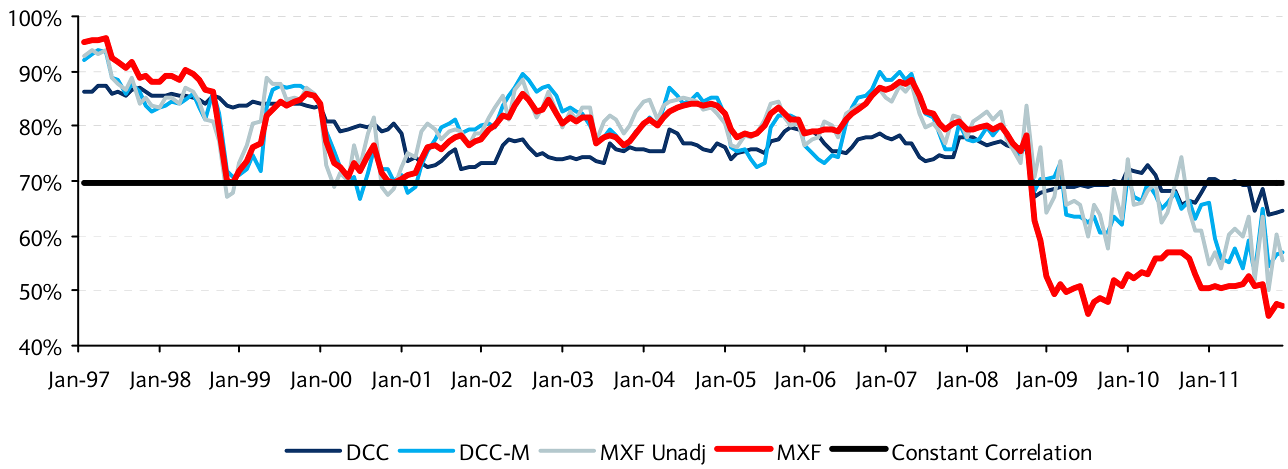


Source: Barclays Research

When we calibrate all three parameters (scaling, EWMA allocation, and half-life) and compare the results (not shown) with MXF Unadj and MXF, we draw the same conclusions as for US equities. The scaling parameter is not significantly affected by fixing the half-life, but fixing the scaling at one (as done in MXF Unadj) decreases the allocation to EWMA.

Figure 16 shows the average correlations for US rates. As was the case for US Equity, the average correlation forecast by DCC changes slowly, while DCC-M and MXF Unadj ones are dynamic and track each other. The adjustment parameter reduces significantly the level of forecast correlations after 2008, from 60% to 50%, in line with the drop we observed in Figure 1. Moreover, the MXF Unadj model has very volatile correlations over 2009-11. This is due to the aggressive level of the half-life parameter. In contrast, the MXF model has a much smoother level, validating our choice of a longer, fixed half-life parameter.

Figure 16: Time variation of average forecast correlations, US Rates



Source: Barclays Research

From the parameter dynamics observed for the two examples we conclude the following:

- Models that use only monthly data require many periods to construct the matrix because shorter periods create unstable matrices and the benefit of more responsiveness is not enough to counterbalance this instability.
- Models prefer daily data over monthly when offered the choice because the resulting matrix is both stable and responsive.
- Often there is redundancy between the allocation to EWMA short-term component and its aggressiveness. Fixing one of the two parameters does not affect the model fit in sample, but potentially reduces the risk of overfitting out of sample. The dynamic of the half-life parameter seems to be overshadowed by the dynamic of EWMA allocation; hence, a reasonable choice is to fix the half-life.
- The scaling parameter between daily and monthly correlations improves the model in sample because its introduction affects noticeably the optimal values of the other parameters. Without scaling, the model allocates more to the long-term component, which may diverge less from the monthly levels than the recent daily correlations do, but forgoes the benefits of capturing the recent dynamics from daily data.

Out of sample results

The multivariate nature of correlations makes it difficult to test the forecast quality of a model. Tests on individual series cannot be easily described parsimoniously; thus, we have to rely on tests that address the entire matrix simultaneously. Over the next two sections, we present several tests and comment on the performance of the different models based on these tests.

General tests

The statisticians' test of choice for a model's overall performance is the likelihood test, shown by Patton (2010) to give an unbiased result in testing volatility models. We employ[8] a straightforward multivariate extension of this test to gauge the overall performance of the model. For each model we compute, after dropping some irrelevant constants,

[8] We are not aware whether an extension of the proof has been explicitly constructed for the multivariate case.

$$LL = \sum_t -\left(\ln\left(\left|\Omega_{t+1}\right|\right) + \widetilde{R}'_{t+1}\Omega^{-1}_{t+1}(\theta,\gamma)\widetilde{R}_{t+1}\right) \text{ where } \Omega_{t+1} = Corr\left(R_{t+1} \mid R_1 \ldots R_t\right)$$

We note that if correlations are zero, the series have a unit volatility, and we use the identity matrix as the correlation forecast, then $LL = -T * N$. For easier interpretation across models and datasets, we display a scaled version of this statistic:

$$\text{LL Test} = \frac{LL}{T * N} + 1$$

Under this normalization, the test statistic is not sensitive to the number of variables or observations used.

Under the above assumptions of zero correlations, we expect the test value to be close to 0. Any deviations may come from a mis-forecast of the variance of *R* or its correlations. This underscores an important point about these tests: they are joint tests of both the volatility forecast model and the correlation model Therefore, prior to testing the correlation forecasts, we must ensure that the volatility was forecast adequately and that $\widetilde{R}$ has a volatility of one.

General tests such as LL provide a good general indication of model performance, but may hide poor performance episodes. For example, if we have sustained periods of over-prediction followed by periods of under-prediction, this statistic averages the results, hiding systematic biases in the forecasting model. This poor performance may occur not only between various periods in the sample, as is the case with volatility, but also among various correlation pairs, adding another dimension to consider in our model testing.

To test the correlations in the cross-section, we note that the volatility forecast of any portfolio depends on correlations across assets, as well as their individual volatility. Assuming that the latter is controlled – or at least similar across forecasting models – a test on the volatility of portfolios is a test on the correlation model. While there are many volatility tests, we limit ourselves to the ubiquitous ratio test:

$$\text{Portf Vol Ratio} = \sqrt{\frac{1}{T}\sum_t \frac{r^2_{p,t}}{\sigma^2_{p,t}}} - 1 \text{ where } r_{p,t} = w'_{p,t}\widetilde{R}_t \text{ and } \sigma^2_{p,t} = w'_{p,t}\Omega_t w'_{p,t}$$

Two popular portfolios we will analyze are the equal-weight and the global minimum variance (GMV). Because, in our case, all returns are scaled by volatility, these two portfolios can be interpreted as two important risk-centered portfolios: the equal-weight of $\widetilde{R}$ is equivalent to the equal-volatility weighted on un-scaled returns *R*, and the GMV of $\widetilde{R}$ is mean-variance optimal if the Sharpe ratios of *R* are the same. These portfolios explore the estimation of the correlation matrix differently: the equal-weight relies on the average accuracy of the correlation matrix while the GMV relies on the average accuracy of the inverse of the correlation matrix. The equal-weight portfolio shows how the model performs when applied to a market-wide index. The GMV portfolio shows how the model performs for an optimized portfolio because most optimized portfolios are constructed with the inverse of the correlation matrix. Formally, we specialize the ratio test by defining the portfolio weights as:

$$w^{EqualWeight}_{p,t} = 1/N \text{ and } w^{GMV}_{p,t} = \frac{1}{i'_N\Omega^{-1}_t i_N}\Omega^{-1}_t i_N$$

Both portfolios are scaled such that weights sum to one.

For US rates we add another relevant portfolio (or combination of rates) that captures the dynamic of the rates curve slope by investing in the 2y rate and shorting an equal amount of the 10y rate. Because this portfolio is typically similar to the one defined by the second eigenvector, its variance is close to the second eigenvalue, which makes it of particular interest for us because we manipulate the eigenvalues. Practitioners view this combination of rates as an important economic signal, which makes it a major driver of the term structure of interest rates, second only to the overall level of rates. Formally,

$$w_{p,t}^{Slope} = \begin{bmatrix} 0 & -.5 & 0 & 0 & .5 & 0 \end{bmatrix}$$

We display the LL and ratio test statistics in Figure17 for the previously described US equities and US rates datasets. For the volatility ratio tests, we show in brackets bootstrapped 5%/95% confidence values. Regarding the models compared, we drop the EWMA model from the analysis because its results are very similar to DCC, of which EWMA is a special case. Furthermore, we add a constant correlation model in which all correlations, across pairs and time, are the same. This model is useful as a reference benchmark. For this model, we set the forecast level of correlation to the sample level: 53% for equities and 70% for rates.

Figure 17: Likelihood and portfolio volatility tests of various models, Jan 1997-Dec 2011; bootstrapped 5%/95% confidence values in brackets

	US Equity Sectors			US Rates			
Model	**LL**	**GMV Vol Ratio**	**EqWgt Vol Ratio**	**LL**	**GMV Vol Ratio**	**EqWgt Vol Ratio**	**Slope Vol Ratio**
Constant Correlation	0.72	-5% [-16%, 5%]	-5% [-16%, 5%]	0.72	-0.5% [-11%, 9%]	-0.5% [-11%, 9%]	-19% [-30%, -9%]
DCC	0.882	25% [11%, 38%]	-1% [-11%, 9%]	1.777	6.4% [-5%, 17%]	-3.8% [-13%, 6%]	12% [-2%, 25%]
DCC-M	1.013	24% [8%, 38%]	-9% [-19%, 2%]	1.813	4.4% [-6%, 15%]	-4.3% [-14%, 5%]	22% [7%, 34%]
MXF Unadj	1.088	4% [-8%, 15%]	-10% [-19%, 0%]	1.815	3.0% [-9%, 13%]	-4.6% [-13%, 5%]	23% [7%, 38%]
MXF	1.092	1% [-10%, 12%]	-3% [-14%, 7%]	1.785	3.0% [-8%, 13%]	-3.2% [-14%, 6%]	15% [0%, 28%]

Source: Compustat, Barclays Research

The LL test value for the identity matrix (not shown) is 0.1 for US equities, which we interpret as the forecast of the monthly variance is on average 10% higher than realizations and translates into 5% higher in volatility terms. This value sets a benchmark for the influence of volatilities on the LL values of the other models. The better a model is compared with the indentity matrix, the higher LL is versus the identity matrix benchmark. Since we keep the same volatility model for all correlation models, the effect of volatilities on the LL test remains unchanged from the identity matrix case. The value for US rates is close to zero, showing little bias in forecasting rates volatility.

The Constant Correlation (CC) brings a major improvement over the identity matrix as the LL increases to 0.72 for both series, underscoring the importance of getting the average level of correlations right[9].

Capturing the richness of the dynamics of the correlation matrix, as shown by the DCC model, increases the LL value even further, particularly for US rates, because the correlation of the 6mo rate is significantly lower than the correlations among all other rates. The DCC-M increases the LL function even further, showing the added value of daily data. For equities, the MXF Unadj model is another step up from DCC-M, showing the value of choosing the optimal combination of daily data. Last, the MXF adds little improvement for equities and none for rates relative to MXF Unadj from the perspective of the LL test. We may test statistically whether a model is significantly better than another using a paired t-test on the time-series terms of LL, as done in Gabudean and Schuehle (2011). The t-tests results (not shown) broadly follow these conclusions.

The next two tests look at the forecasting ability of each model for the volatility of the GMV and equal-weights portfolios. Starting with equities, the CC performs well, as expected given its in-sample nature. Its 5% miss comes solely from the over-forecast of volatilities, matching the LL test with zero correlations. The DCC and DCC-M models underestimate significantly the volatility of their associated GMV, by around 25%. These models are too optimistic about what they can achieve in terms of minimizing portfolio volatility. The MXF models are much closer to the target, with almost spot-on forecasts. Regarding the confidence around these tests, the reported bootstrapped values show an approximate +/- 10 percentage point confidence range around the point estimates for most tests, statistically supporting our conclusions.

The equal-weight portfolio has its volatility forecast well by the monthly-only DCC model. However, the models that use mainly or solely daily data, DCC-M and MXF Unadj, overestimate its volatility (borderline statistically significant) because daily correlations are on average higher than the monthly ones for the equity dataset. But this situation is what the adjustment was designed for and indeed, the adjustment delivers: the volatility forecasting of the equal-weight portfolio is improved, from a 10% overestimation to only a 3% overestimation, which is statistically indistinguishable from zero.

The ratio tests results for the rates dataset are presented in the right-hand section of Figure17 and they show that most models perform well, in line with the LL results. There are some minor improvements as we go to more complex models, and the adjustment seems to make a positive but insignificant difference for the MXF model.

For the special slope portfolio (shown in the last column of Figure 17), the CC model significantly overestimates its volatility, which means the realized correlation between the changes in 2y and 10y rates is lower than the average forecast correlation. To wit, correlations of rates with more similar tenors are higher than rates with tenors further apart, such as the 2y and 10y. This detail cannot be captured by a constant correlation matrix. In contrast, the DCC model captures some of this detail, but this time it understates somewhat the correlation level, as shown by the borderline insignificant 12% value of our test. Using daily data makes things worse, as we understate the correlation level by as much as the average correlation overstates it. However, the adjustment alleviates this issue, though not

[9] In that vein, Engle and Kelly (2011) propose the DECO model to capture the dynamic of average correlations. DECO sets all correlations to be the same across pairs, but allows them to change over time. While we do not include it in our tests, we expect it to perform at least as well in the LL test as the constant correlation model.

completely, showcasing how the adjustment helps not only with the average level of correlations but also with other important details of the matrix.

The tests presented in this section show that while monthly models have a good performance at the aggregate level, they fare poorly in optimization problems and daily data may help in these situations. However, models based on daily data may underperform the monthly models in average correlation tests. The eigenvalue adjustment we propose mitigates this underperformance, retaining the benefits of daily data while removing its major pitfalls.

Tests of the structure of the GMV portfolio and of uncorrelated portfolios

Next, we turn to other tests of the correlation matrix. For the GMV portfolio, the correlation forecast does not determine only its volatility forecast but its structure as well. In particular, we expect the best correlation model to be able to produce GMV portfolios with lowest ex-post realized volatility.[10] Therefore we test the models by computing

$$\text{GMV Vol} = \sqrt{\frac{1}{T}\sum_t r_{GMV,t}^2}$$

Another GMV aspect we can test is the amount of leverage required to achieve it. The GMV portfolio may have both positive and negative weights. It is well known that optimized portfolios exploit mis-estimation errors in the covariance matrix, leading to large gross exposures. We can therefore look at these as an indication of potential lack of regularity in the covariance estimate. To this end, we compute

$$\text{GMV Wgt} = \frac{1}{T}\sum_t \sum_{i=1:N} Abs\left(w_{t,i}^{GMV}\right)$$

Apart from these general portfolios, there are some statistically important sets of portfolios and we analyze two of them. First, the portfolios defined by eigenvectors are often employed to parsimoniously capture the risk properties of the underlying securities. If the correlation forecast is accurate, these portfolios should be uncorrelated and their variance should be equal to the associated eigenvalues. To test the entire covariance matrix of the eigenvector portfolios, we employ a mean-squared error criterion, which can be interpreted as the Frobenius norm.

$$\text{EigenV Covar Test} = \sqrt{\frac{1}{N^2} NormFrob\left(\frac{1}{T}\sum_t R_{EigenV,t} R'_{EigenV,t} - D_t\right)}$$ where

$NormFrob(A) = \sum_{i,j} a_{i,j}^2$ and $R_{EigenV,t} = V_t' \widetilde{R}_t$

In the following subsection, we investigate in more detail these eigenvector portfolios, looking at tests on each of them individually.

[10] If all assets have the same Sharpe ratios then our GMV portfolio is optimal and its Sharpe ratio equals the assets' Sharpe divided by the portfolio volatility. For this case the GMV volatility equals the ratio of assets' Sharpe to the portfolio Sharpe. Thus lower GMV volatility implies a higher portfolio Sharpe ratio, and a better correlation forecast model.
Engle and Colacito (2006) develop a test for covariance forecasts using an economic criterion, which can be interpreted as the Sharpe ratio loss of the mean-variance optimum (MVO) portfolio caused by the wrong covariance matrix. Between two MVO portfolios obtained from different covariance matrices, the one with a higher Sharpe ratio is deemed better. A critical ingredient in such an exercise is the expected returns of underlying assets, which can be poorly forecasted, thus affecting the accuracy of the test. To address the issue, they investigate the average loss assuming a distribution over expected returns. Our GMV test is in the same spirit, but we assume all Sharpe ratios are identical as opposed to constructing a distribution over them, which is a valid technique, but beyond our scope.

Many combinations of the underlying securities should result in uncorrelated portfolios, not only the eigenvector ones. Another interesting set of portfolios is defined by the square root of the inverse of the correlation matrix. These portfolios should be uncorrelated and have a volatility of one. We use a test statistic similar to that defined for the eigenvector portfolios:

$$\text{Ortho Covar Test} = \sqrt{\frac{1}{N^2} NormFrob\left(\frac{1}{T}\sum_t R_{Ortho,t} R'_{Ortho,t} - I_N\right)} \quad \text{where}$$

$$R_{Ortho,t} = \Omega_t^{-1/2}\widetilde{R}_t = V_t D_t^{-1/2} V_t' \widetilde{R}_t$$

For a spatial interpretation of the two sets of portfolios, the eigenvectors represent a basis for the space spanned by the N assets, where the basis is defined by choosing the first coordinate vector to have the maximum length possible, then the second coordinate vector to have the maximum length given the already chosen first coordinate, etc. The portfolio defined by the inverse of the matrix square root defines an orthonormal basis, where each coordinate vector has the same length.

Figure 18 presents the results of these additional tests. Because their statistical significance is either not illuminating or difficult to compute, we do not show it and focus instead on their economic interpretation. For the equity dataset we see the big advantage of getting the dynamics of the correlation structure versus just the static average level of correlations. While the CC model produces a portfolio with 70% volatility, which is the equal-weighted portfolio[11], the DCC produces a portfolio with 48% volatility. To understand the economic impact of these numbers, note the interpretation of GMV volatility as the ratio of assets' Sharpe to portfolio Sharpe, assuming all assets have the same Sharpe ratio. The 70% volatility of the equal-weighted portfolio shows the benefits of simple diversification: potential portfolio Sharpe is 1/70% = 1.4 of the individual assets' Sharpe ratio. The perceived benefits of diversification are much higher when we take the dynamic nature of correlations into account. For instance, for the DCC model, the potential portfolio Sharpe ratio increases to 1/48%=2.1. The addition of daily data in DCC-M improves the potentially optimum portfolio significantly, as volatility drops from 48% to 33%. The MXF Unadj improves only a little over DCC-M. However, the MXF adjustment helps the optimum portfolio notably, as volatility drops to 27%, resulting in a potential portfolio Sharpe ratio of 1/27%=3.7. The results for the rates series are more ambiguous: all models present very similar outcomes.

Figure 18: Covariance and other tests of various models, Jan 1997-Dec 2011

	US Equity sectors				US Rates			
Model	GMV Vol	GMV Abs Wgt	EigenV Covar	Ortho Covar	GMV Vol	GMV Abs Wgt	EigenV Covar	Ortho Covar
Constant Correlation	70%	1	15%	19%	86%	1	20%	64%
DCC	48%	4.2	7%	11%	86%	4.2	8%	35%
DCC-M	33%	7.5	12%	13%	84%	4.3	10%	30%
MXF Unadj	31%	7.2	12%	10%	84%	3.9	11%	33%
MXF	27%	6.2	7%	9%	83%	3.7	8%	28%

Source: Barclays Research

[11] When all correlations and Sharpe ratios are the same, as in the CC model, all assets are similar and the optimized portfolio (i.e., the GMV) is the equal-weighted one. Note that the GMV using the DECO model is the same equal-weight portfolio, thus the CC results apply to it as well.

The weight test sheds more light on the structure of the GMV portfolio under various models. To gauge how big of an exposure it takes in absolute values, we show the sum of the absolute weights. Since the GMV for the CC model is long-only, the total exposure is 100%. Using monthly data, the total exposure increases to more than 400%. Daily data increase the exposure even further for equities, to more than 700%, and the MXF model brings it back down to 620%. For rates, daily data do not increase the exposure, but the MXF model does reduce it slightly. These large exposures make the portfolio very sensitive to forecast errors: small errors in correlation forecasts may affect portfolio realized volatility markedly.

The last two tests show how well we forecast the covariance of two sets of portfolios that should have zero correlations. These tests can be interpreted as our ability to use the correlation matrix to construct portfolios with stable multivariate properties over time, such as zero correlations. The magnitude of the numbers has a root-mean squared error interpretation, or the typical magnitude of realizations' deviation from the expected value. The results, consistent across the two datasets, show a great improvement when we add simple correlation matrix dynamics to our model, i.e., the DCC model. Adding daily data (the DCC-M model) has a mixed effect, worsening the tests for the equities example and improving them for rates. However, the MXF eigenvalue adjustment significantly improves the performance in all tests and we end up with values similar to or better than the DCC model.

This section's tests show that the eigenvalue adjustment may mitigate any shortfalls the daily data models have over the monthly ones while keeping the daily data benefits, such as a lower volatility of the GMV portfolio.

Eigenvector portfolios

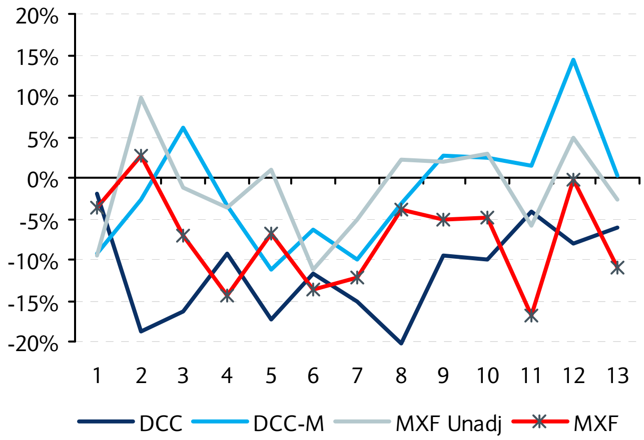
One key contribution of the paper is the adjustment to the eigenvalues as we go from high to low frequencies. Therefore, it warrants investigating how well we forecast the volatility of each eigenvector portfolio separately. However, we caution that, as in the GMV test, the correlation model defines the portfolio as well, not only forecasts its variance. For each eigenvector portfolio we test

$$\text{EigenV Ratio Test}_{\mathrm{k}} = \sqrt{\frac{1}{T}\sum_t \frac{r^2_{EgV,k,t}}{d_{k,t}}} - 1 \text{ where } R_{EigenV,t} = V_t'\widetilde{R}_t = \begin{bmatrix} r_{EgV,1,t} \\ \dots \\ r_{EgV,K,t} \end{bmatrix}$$

Where *d* is the associated eigenvalue and the forecast for the eigenvector portfolio variance.

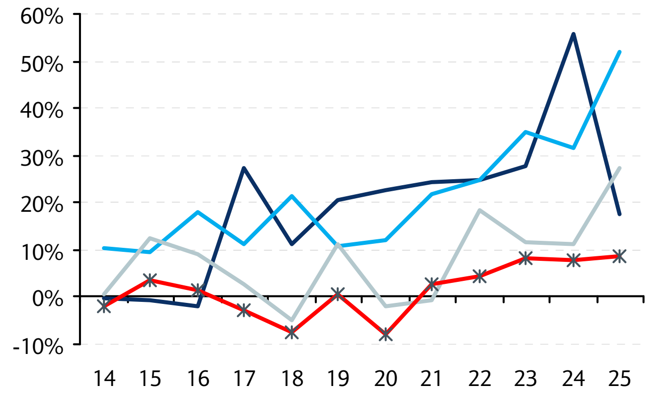
The results, applied to US equities, are shown in Figure 19 and Figure 20. Notice the good performance regarding the first eigenvector for the DCC model, with the ratio test close to zero, so the first eigenvalue correctly forecasts the volatility of the first eigenvector portfolio, a result in line with what we saw for the equal-weight portfolio in Figure 17. However, the other eigenvectors are poorly forecast by the monthly model. We have an overestimation of 15-20% for the higher ones, followed by an underestimation of the same order of magnitude for the smaller eigenvalues. In unreported results, other models using only monthly data have a similar performance, good forecast of the first eigenvector, but a poor one for the following eigenvectors. Moreover, models that choose to include data from longer periods have less extreme underestimation of smaller eigenvalues, validating our conclusions from the in-sample discussion that models with monthly data are forced to choose a long horizon to create a matrix with stable properties.

Figure 19: Volatility Ratio test for eigenvector portfolios, US Equity, eigenvalues 1-13



Source: Compustat, Barclays Research

Figure 20: Volatility Ratio test for eigenvector portfolios, US Equity (cont), eigenvalues 14-25



Source: Compustat, Barclays Research

The models with daily data DCC-M and MXF have weaker performance with the first eigenvector portfolio than DCC has, but they perform much better with other portfolios associated with other large eigenvalues (Figure 19). However, for DCC-M the performance deteriorates for portfolios associated with smaller eigenvalues (Figure 20), similar to what happens with the DCC model. This weak performance may be due to the small sample of 126 daily observations used by DCC-M. Indeed, if we choose a longer window the performance improves markedly (not shown). The MXF Unadj, whose optimal weighting scheme is similar to a window of approximately 250-300 observations, performs much better for small eigenvalues than DCC-M. The MXF portfolio performs better than the other daily models regarding the first eigenvalue, thus justifying the adjustment. The performance is weaker for other large eigenvalue portfolios than the other daily-data models, but typically better than monthly-data ones. However, for small eigenvalues, its performance is remarkable, significantly better than any other model. Thus, the MXF model has the most consistent performance across all eigenvalues, with the test ratio between -17% and 10%. Other models tend to perform more irregularly across the different eigenvalues.

We do not show the eigenvector portfolio tests for the rates datasets because we already capture the results for the first two eigenvalues in the equal-weight and slope portfolios. The conclusions for small eigenvalues are similar to the equities case.

Sub-period performance

So far we have investigated the model performance over the entire period only, but we want to test whether performance varies across sub-periods. To that end, we report the values for some of the tests above for the US Equities dataset for three sub-periods: 1997-2006, 2007-2008, and 2009-2011. As tests, we selected the accuracy of the volatility forecast of the equal-weight and GMV portfolios, the realized volatility of the GMV portfolio, and the test of the covariance matrix of the eigenvector portfolios.

Figure 21: Select tests of various models, US Equities, by sub-period

Model	GMV Vol Ratio	EqWgt Vol Ratio	GMV Vol	EigenV Covar
2009-2011				
DCC	6%	7%	47%	14%
DCC-M	48%	-8%	41%	20%
MXF Unadj	-6%	-9%	36%	17%
MXF	-25%	-6%	28%	13%
2007-2008				
DCC	41%	13%	56%	31%
DCC-M	37%	-3%	36%	20%
MXF Unadj	-11%	-4%	27%	21%
MXF	-21%	0%	23%	23%
1997-2006				
DCC	26%	-7%	46%	13%
DCC-M	11%	-11%	27%	14%
MXF Unadj	8%	-11%	30%	14%
MXF	12%	-4%	27%	9%

Source: Compustat, Barclays Research

The relative performance of various models does vary by sub-period. The performance of models with monthly data varies more than that of models with daily data when we analyze the equal-weight and GMV portfolios. For example, the equal-weight portfolio ratio test for DCC is 7%, 13% and -7% for the three sub-periods, while MXF Unadj has -9%, -4%, and -11%. Moreover, for this test the MXF improves the performance of the MXF Unadj model for all sub-periods regarding the equal-weight portfolio, showing that the adjustment works also by sub-periods. The numbers for the GMV portfolio are more dramatic, but they still show that the MXF model improves upon DCC-M, although MXF Unadj performs even better.

Regarding the realized volatility of the GMV, for all subsamples models with daily data produce a portfolio with lower volatility than models with monthly data. The adjustment to MXF brings a significant improvement to the GMV in all periods. The test for the eigenvectors covariance matrix again reveals a more consistent performance for daily models across sub-periods compared to the monthly ones: error ranges between 13 and 21% for the DCC-M and MXF Unadj models and 13 and 31% for the DCC model. Last, the MXF model has the best or close to the best performance in every sub-period, showing yet again the value of the eigenvalue adjustment.

We can construct other out-of-sample tests for the correlation forecasts models along the same lines we have used so far: eg, constructing a relevant portfolio and analyzing how well we forecast its volatility, or constructing several portfolios that must have a certain correlation and analyzing how close the realized correlations are to the target. Ultimately, each researcher must employ the tests that speak best to his or her model's purpose and focus on how the model performs under those conditions. So even if a model may perform well for some purposes, it may not do so well for others. Moreover, the benefits of additional complexity may be present in some datasets and less so in others, so a researcher should not reach for more complexity than is called for by the particular dataset or problem.

Conclusion

We have presented a method of forecasting correlations with data at a higher frequency than the forecast horizon. We embedded this method in a larger framework that encompasses many recent correlation models, some of which also make use of higher-frequency data. Our framework centers on the particular weighting scheme applied to past data, similar to the volatility framework proposed by Gabudean and Schuehle (2011).

Average correlation may vary across horizons, e.g., daily data have higher cross-sectional correlation than monthly data. Even though volatility has a similar behavior, it presents a particular challenge to correlation forecasting because scaling across horizons cannot be done directly on the correlation matrix. We have identified this challenge and addressed it by decomposing the correlation matrix in eigenvalues and eigenvectors, an approach similar to the one proposed independently by Hautsch and Kyj (2010) for more general purposes. Noting that the first eigenvector typically captures the average magnitude of correlations, we calibrate its magnitude from the longer-horizon data and adjust the other eigenvalues proportionally to keep the total amount of risk constant.

We calibrate the models in and out of sample to provide monthly horizon forecasts on two US datasets: one with equity sector returns and one with changes in Treasury rates. We further present a multitude of economically motivated statistical tests to compare the performance of these models. Results show that the dynamic correlation models using only monthly data optimally choose to make use of virtually entire sample – losing their dynamic nature – because they require many periods to construct a well-behaved matrix. Models that incorporate daily data tend to do better in many aspects, including the volatility of the global minimum variance portfolio, the covariance of the eigenvector portfolios and their individual volatilities. Models that choose between a daily data component and dynamic monthly data component optimally select the former. These models can replicate the quality of DCC model estimates, but with more responsiveness.

Results also suggest there is additional value in letting the model choose the appropriate weighting scheme over past data, as in the MXF model we introduced. Both components of the MXF model, the long and short term, contribute to the final forecast and the short-term one is optimally chosen to be relatively aggressive. However, daily data models seem to forecast less well the average correlation level at a different frequency, underscoring the need for the eigenvalue adjustment introduced in this paper. Once we add the adjustment, daily data models forecast the average monthly correlations as well as the monthly ones. Importantly, the MXF model with the proposed adjustment still retains the advantages of daily data models over monthly ones, stemming from their more dynamic nature. Moreover, the eigenvalue adjustment helps with the forecast of other eigenvector portfolios, particularly those associated with lower eigenvalues.

Appendix

Theoretical considerations of the Adjusted MXF model in a constant correlations setup

We investigate the theoretical relationship between the eigenvalue adjustment parameter and the correlation level for an equi-correlation (i.e., all correlations are the same) matrix.

Let $\Omega(\rho) = (1-\rho)I_N + \rho J_N$ where $J_N = i_N i'_N$

The eigenvalues are $d_1 = 1 + (N-1)\rho$ and $d_j = \frac{1}{N-1}(N - d_1) = 1 - \rho$ for all *j*>1.

Let the daily correlation matrix be $\Omega(\rho_{day})$ and the monthly one be $\Omega(\rho_{mo})$. Given that N-1 eigenvalues are identical, the ratio of daily to monthly eigenvalues is the same except for the first eigenvalue. Therefore our eigenvalue adjustment parameter is

$$\gamma = \frac{d_{j,mo}}{d_{j,day}} = \frac{1-\rho_{mo}}{1-\rho_{day}}$$

From this relation we have that $\rho_{mo} = \gamma * \rho_{day} + (1-\gamma) * 1$ or the monthly correlation is a weighted average of daily correlations and one, with the weight given by the scaling parameter. If $\gamma < 1$ then $\rho_{mo} > \rho_{day}$ and vice versa.

Simulations-based properties of the MXF model estimates

We investigate the properties of the MXF model numerically in an idealized setting.

We simulate returns on N data series from a standard normal distribution, where returns have a constant correlation. Data are assumed to have a volatility of 1 and no autocorrelation. Two separate data sets are simulated, one representing daily realizations and the other monthly realizations. The correlation level differs between daily and monthly horizons. The daily horizon has 2520 observations and the monthly one has 120, the equivalent of 10 years of data.

We estimate two correlation matrices using the simulated data:

- Unweighted from monthly data, or the sample correlation
- The MXF matrix, with the EWMA half-life fixed at 63 days. The maximum likelihood estimation puts more weight on recent observation, using a weighting scheme that decays exponentially with a half-life of 48 months. This is the MXF model we used in the examples section. The first 504 daily observations (two years) are required to compute the first correlation estimate, so only 100 monthly observations (eight years) are used in the maximum likelihood estimation. Note that because of the weighting scheme of the maximum likelihood, we use much less information than 100 months, in contrast with the unweighted model, which uses 120 months.

We look at two types of datasets, a large one (20 series) and a small one (six series), roughly corresponding to our two examples in the text. Moreover, we show how the model behaves when the monthly correlations are bigger than the daily, and vice versa. An additional dimension is investigating how the model behaves at low vs high levels of correlation.

Figure 22 sets out the parameters associated with the various setups.

Figure 22: Parameters associated with various simulation setups

Setup	Monthly correlation	Daily Correlation	Nr of Data Series
1	60%	50%	6
2	60%	50%	20
3	75%	85%	6
4	75%	85%	20
5	50%	50%	6
6	50%	50%	20

Source: Barclays Research

For each setup we repeat the exercise 400 times and investigate the distribution of various metrics, reporting the mean and the 5^{th} and 95^{th} percentiles.

First, we look at the distribution of the estimated adjustment parameter in Figure 23. We note that the average estimate is close to the theoretical value, with a small bias toward the upside. The bias is more evident for setups with larger datasets, the setups 2, 4, and 6. The 5^{th} and 95^{th} percentiles (P5 and P95) of the observed parameter estimates have a fairly narrow range, with a bigger range for larger theoretical values. Moreover, the upside bias is noticeable here as well, as the P95 is further away from the theoretical value than P5, particularly for setups with large datasets.

Figure 23: Calibrated Estimates of the MXF scaling parameter for various simulation setups

Setup	Theoretical Value	Average of estimate	P5 of estimates	P95 of estimates
1	0.8	0.81	0.72	0.91
2	0.8	0.84	0.78	0.89
3	1.67	1.69	1.49	1.92
4	1.67	1.74	1.61	1.88
5	1.00	1.01	0.89	1.14
6	1.00	1.04	0.98	1.11

Source: Barclays Research

Next, we look at the effect of parameter adjustment on average correlation estimates. For each setup we show, on average – across both pairs and simulations - how much the sample correlation and the MXF estimated correlation deviates from the true value, as well as the P5 and P95 of this deviation across simulations. The average sample estimate is almost to zero, as we would expect. The MXF is close to zero, but further apart than the sample estimate. Moreover, there seems to be a bias in the negative direction, matching the upside bias of the parameter estimate. However, the numbers are not material. Looking at the possible deviations of the sample estimate across various simulations, the sample correlation can be off by 4-7% in either direction. The error is smaller for larger datasets, as expected, because we have more data for an essentially one-parameter model. Moreover, for larger correlations we have smaller errors, the result of getting close to the maximum value of the correlation coefficient.

Interestingly, the MXF model has significantly smaller deviations from the true values, even though it uses data with different correlation levels. In particular, the errors on the upside are more limited than in the sample case (P95 is lower), again showing the bias we saw in the parameter estimate. This improvement is even more surprising because we use the full 100 monthly observations to get the sample estimate, while the MXF model focuses more on recent data.

Figure 24: The distribution of average correlation estimates, by model and simulation setup

	Sample correlation			MXF		
Setup	Average of estimate - theoretical	P5 of estimates - theoretical	P95 of estimates - theoretical	Average of estimate - theoretical	P5 of estimates - theoretical	P95 of estimates - theoretical
1	0.2%	-6.1%	6.2%	-0.7%	-6.1%	4.6%
2	-0.2%	-5.9%	5.4%	-1.8%	-5.1%	1.2%
3	-0.3%	-5.8%	4.2%	-0.4%	-4.9%	3.2%
4	-0.2%	-4.3%	4.0%	-1.2%	-3.7%	1.6%
5	-0.2%	-7.9%	7.0%	-0.7%	-7.8%	6.0%
6	0.1%	-5.7%	6.0%	-2.0%	-6.4%	2.3%

Source: Barclays Research

In the previous exercise, we investigated how well MXF forecasts the average correlation across all pairs. However, the correlation estimates may vary across pairs even though in theory they should all be the same. Another angle we may test is how close the correlations are to each other in the same matrix. So, for each simulation we retain the difference between the highest and lowest forecast across the matrix. The closer this dispersion measure is to zero, the better the model. In Figure 25 we show the average dispersion across all simulations and the 5^{th} and 95^{th} percentiles. We note the significantly lower values across all setups of the MXF model compared with the sample estimate.

Figure 25: The distribution of dispersion (maximum – minimum) in correlation estimates, by model and simulation setup

	Sample correlation			MXF		
Setup	Average dispersion	P5 of dispersion	P95 of dispersion	Average dispersion	P5 of dispersion	P95 of dispersion
1	16%	10%	23%	9%	5%	14%
2	26%	20%	33%	14%	11%	18%
3	10%	7%	16%	5%	3%	8%
4	17%	13%	21%	8%	7%	10%
5	20%	13%	28%	10%	7%	15%
6	32%	26%	38%	16%	13%	19%

Source: Barclays Research

The simulation results establish the validity of our adjustment approach, at least for simple setups, showing that the adjustment can bring the monthly and daily correlations in line, even outperforming a monthly-data model that should do particularly well in this setup of constant correlations. There is a small bias in the average correlations obtained from the MXF model, but the bias is not material and is much smaller than the difference between daily and monthly correlations.

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