Managing against the Lehman MBS Index: Evaluating Measures of Duration

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SUMMARY

- We examine the relative performance of model-based and empirical MBS duration measures since 2001. Generally, model durations have performed relatively well. While model durations generally outperform empiricals, the outperformance is greater for 30-year FNMAs than for GNMAs and 15-year FNMAs.
- The performance of model durations deteriorates rapidly for an MBS whose dollar price exceeds 104. Empirical durations also begin to deteriorate, but at a slower rate, and begin to outperform model durations. This finding suggests that investors should rely on empirical durations for such high-priced MBS.
- KRDs did not perform noticeably better than OADs, even during periods of significant curve reshaping.
- A notable development during 2002 and 2003 was the prevalence of high dollar-priced MBS with negative model durations. While those model durations performed poorly on an absolute basis, they performed better than the positive model durations for other high dollar-priced MBS. Nevertheless, the realized price-yield relationship for negative duration bonds was not fully consistent with a negative duration value. Consequently, their relative performance most likely had more to do with the low absolute value of the negative durations than with their negative sign.
- The Lehman Global Risk Model uses monthly historical data to estimate variances and covariances of changes in Treasury rates and other MBS risk factors. These data can be used to construct "risk model" empirical MBS duration measures. For all but very high dollar-priced MBS, risk model durations tend to perform better than other empirical measures.

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Introduction

Between May 2001 and February 2005, the 2-year U.S. Treasury yield declined from 4.24% to 3.60% while the 10-year yield declined from 5.29% to 4.38%. As MBS investors recall, these relatively modest declines masked more substantial changes during the intervening years. For example, between May 2001 and June 2003, the 2-year Treasury yield fell from 4.24% to 1.32% while the 10-year yield fell from 5.29% to 3.35%. These low yields produced large increases in mortgage prepayment forecasts and large declines in MBS option-adjusted (Treasury) durations. During the May 2001-June 2003 period, the Lehman MBS Index OAD decreased from 3.33 to 0.58—a record low. Over this period, despite the sharp decline in Treasury rates and acceleration in forecasted (and actual) prepayments, MBS performed well. For the 14-month period, the MBS Index had a total return of 16.23% and outperformed key-rate duration matched Treasuries by 132 bp. It was a good time to have an MBS overweight versus Treasuries.

The success of an MBS-UST basis trade, however, depends on the assumed Treasury duration of the MBS position. Suppose a manager decides to add a 5% MBS market value overweight (against Treasuries) in anticipation of tightening in MBS spreads. Assuming the manager does not wish to take a duration position versus the benchmark, Treasuries (or some other non-MBS asset) are sold, based on the assumed MBS duration, to make room for the MBS overweight on a duration-neutral basis. The performance of the MBS basis trade depends on whether the assumed MBS duration measure accurately captures MBS interest rate sensitivity. If the duration measure is too low, then the portfolio's actual duration will be greater than intended. Conversely, if the duration measure is too high, then the portfolio's actual duration will be less than intended. Either way, movements in Treasury rates will affect the success of the overweight strategy. This is why portfolio managers are concerned that MBS durations accurately measure Treasury rate sensitivity.

How well MBS durations measure Treasury rate sensitivity is also important for managers who hold only MBS securities in their portfolios and are supposed to be duration-neutral to the MBS Index. These managers may meaningfully deviate from the MBS Index by combining different MBS positions in such a way that their portfolios will have the same duration as the MBS benchmark. For example, the manager might overweight the discount and premium coupons and underweight the current coupons, while maintaining duration neutrality with respect to the benchmark. Since the Treasury manager may have overall portfolio-duration responsibility, the MBS manager wants to be sure there are no unintended duration bets in the MBS portfolio.

There are many possible MBS-duration measures. Our goal is to evaluate how well several common duration measures, including Lehman's own, have performed in recent years in explaining movements in MBS prices. Investors know that MBS price returns are driven by exposure to many risk factors. In fact, the Lehman Global Risk Model has 27 MBS risk factors consisting of six key-rate Treasury yield changes, the average yield change squared, six key-rate swap spread changes, two volatility factors, and 12 spread-risk factors depending on the MBS's program, price tier, and WALA.[1] However, in this

[1] Please see *The Lehman Brothers Global Risk Model: A Portfolio Manager's Guide*, April 2005, Lehman Brothers, for a description of the Global Risk Model and its uses.

report, we are interested in predicting MBS price returns solely on the basis of changes in Treasury yields. In effect, we are only considering a single-factor model of MBS price returns (or a six-factor model when we use key-rate durations). In addition, our focus is on *daily* price returns, not monthly returns, which are the basis of the Risk Model. Given that Treasury yields are more volatile than MBS spreads, we are assuming that over such a short time period, changes in Treasury yields are the primary driver of MBS price returns.[2] Our goal is to evaluate which duration measure, together with changes in the associated Treasury yield(s), best explains MBS price returns.

Our general formulation is as follows: For a given MBS, we select a duration measure and a daily UST yield change. Given the actual daily yield change, we calculate the predicted MBS percentage price return, $Ret_{predicted}$, by multiplying duration by the yield change. (In the case of key-rate durations, we multiply each key-rate duration by the change in the corresponding key rate and then add up the results for all key rates.) We then compare the predicted change with the actual percentage price change, Ret_{actual}. The difference, or "error," is our measure of the accuracy of the duration measure. For each duration measure, we calculate an average daily root mean squared error[3] (RMSE) over the past four years and over shorter sub-periods. We standardize the RMSEs by dividing each asset's RMSE over the given period by its corresponding price-change volatility. We evaluate eight different duration-yield change pairs and measure their performance according to their standardized RMSEs (StdRMSE). A duration measure with a lower StdRMSE value is a more accurate duration measure than one with a higher StdRMSE value.

MBS Duration Measures

There are two general categories of MBS durations: "model" durations and "empirical" durations. Model durations are typically calculated by shifting the UST par curve in a specific way, regenerating expected cash flows using a prepayment model in response to the rate shift, and then re-pricing the MBS security assuming unchanged spreads and volatilities. In contrast, empirical durations eschew prepayment models and estimate duration by regressing actual MBS percentage price changes on actual changes in UST yields. Empirical durations do not assume unchanged spreads and volatilities and, hence, reflect any correlation between UST yield movements and spread movements (or movements in any other MBS risk factor).

Both types of duration measures have their uses. Model durations are useful because they are forward-looking, incorporating the latest research in prepayment modeling, and are relatively insensitive to transitory technical influences in the market that may not persist, but are nevertheless "picked-up" by empirical duration. In addition, model durations can be used when interest rates move outside recent interest rate bands or

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2 For example, the volatility of the 10-year key-rate Treasury risk factor is 27.3 bp/month whereas the volatility of new production, current coupon MBS spreads is approximately 6.8 bp/month.

3 We do not calculate R^2, the usual measure of the success of a regression, because regression analysis is not appropriate here. Regressing $\%\Delta P_{actual}$ against $\%\Delta P_{predicted}$ would mean writing

$$Ret_{actual} = \alpha + \beta \times Ret_{predicted} + error$$

and then choosing the values of α and β that minimize the error term. We wish to write simply

$$Ret_{actual} = Ret_{predicted} + error$$

and measure the magnitude of the resulting error term.

when there are few relevant historical data available to estimate empirical durations (such as when a new index generic coupon enters the Index). Empirical durations are useful because they incorporate any historical correlation between UST yield movements and changes in other risk factors (e.g., mortgage spreads). The idea behind the use of empirical durations is that although such correlations tend to change over time, any correlation among risk factors over the most recent short estimation period will persist for at least some short time going forward. Therefore, empirical durations that exploit such correlations are likely to be more accurate than model durations that typically assume no correlation.

In calculating $Ret_{predicted}$ we not only have to select a duration measure but we also have to specify which UST yield change to apply. Thus each $Ret_{predicted}$ series is determined by a duration-yield change pair. It is not always obvious which yield change to use with a particular duration measure. For example, OAD measures sensitivity to a parallel shift in the fitted par UST curve. Since the UST curve does not typically move in a parallel fashion, when we calculate $Ret_{predicted}$ we must decide whether to multiply the OAD by the parallel shift component of the curve movement (however that may be defined) or by the movement in a particular point on the yield curve. KRD, on the other hand, measures the sensitivity to movements at a particular point on the par curve and, so, is paired with the yield change at that part of the curve.

Model Durations

Lehman produces two sets of MBS model durations: OAD and KRDs. To calculate OAD, the entire fitted par UST curve (often referred to as the “fitted spline curve”) is shifted up and down 15 basis points. For each shift, Lehman generates many paths of short-term interest rates, each with a prepayment vector from the Lehman prepayment model. Assuming a constant OAS, Lehman calculates a new price along each path. The average price is the assumed price change as a result of the shift in the UST curve. After a shift up and a shift down, the difference between the two calculated prices divided by the initial price level multiplied by 30 bp is the OAD measure.[4]

How can we evaluate the adequacy of OAD as a duration measure? If the par rate curve moved in a parallel fashion, and the actual percentage change in the price of the MBS equaled the OAD multiplied by the actual (parallel) change in the par curve, then OAD would be an accurate measure of the bond’s sensitivity to changes in UST rates. However, the actual percentage change in the MBS price may not equal OAD × ΔUST yield for a number of reasons. First, other MBS risk factors may change simultaneously with the change in the Treasury rate. Another reason is that the par Treasury curve may move in a non-parallel fashion, and the actual change in price is likely to be different from the price change that would be caused by a true parallel shift. In the results section below, we examine the performance of Lehman’s OAD value using two measures of yield change: changes in the 10-year on-the-run UST yield and changes in the “average” UST yield calculated as the average of yield changes along the yield curve.[5] This average yield change is our measure of the “parallel” shift in the yield curve.

4 Specifically, the latest available par curve is shifted up 15bp in a parallel fashion and shifted down 15bp in a parallel fashion. Lognormal trees are constructed from the two shifted curves using current volatility data, and the MBS is priced off each of the two lognormal trees using Lehman prepayment model. The difference between the two prices is divided by 30 bp times the current price of the MBS to obtain the OAD.

5 Specifically, we take the average of the changes in the six key-rate points (0.5-year, 2-year, 5-year, 10-year, 20-year and 30-year).

KRDs address the problem of non-parallel movements in yields. KRDs allow the manager to measure MBS price sensitivity to six particular par Treasury rates.[6] Lehman calculates each KRD by shifting the corresponding rate up and down 15 basis points and shifting the part of the spline curve between the adjoining key rate points in a "hat" shaped pattern. For each shift, Lehman generates multiple paths of short rates, each with a prepayment vector from the Lehman prepayment model. Again, assuming a constant OAS, Lehman calculates a new price which is the average price across all paths. After a shift up and a shift down, the difference in the two calculated prices divided by the initial price level multiplied by 30 bp is the KRD measure.

Each of the six KRDs is multiplied by the change in the corresponding key rate. The sum of the six products produces a $Ret_{predicted}$ value that is not dependent on the assumption of a parallel shift in the par rate curve. For this reason, many investors use KRDs to measure MBS interest rate sensitivity. We might expect KRDs to do a better job of explaining MBS returns, especially when the yield curve moves in a significantly non-parallel fashion.

Empirical Durations

Both OAD and KRD are "model-dependent" duration measures since they rely on Lehman's term structure and prepayment models. However, it is possible to generate interest rate sensitivity measures that are model-independent. One approach is to calculate "empirical" durations by measuring the historical price sensitivity of a particular MBS to changes in a particular UST yield. For example, we can regress past MBS percentage price changes on past changes in UST yields and use the resulting regression coefficient as an empirical duration measure.

There are many possible empirical duration measures.[7] For our analysis, we calculate empirical duration by regressing daily MBS percentage price changes on daily changes for the on-the-run UST 10-year yield using either 10 or 20 business days of historical price data.[8]

An argument in favor of empirical durations is that the marketplace reacts to changes in MBS prepayment behavior faster than modelers can update their models. As a result, if the market senses that prepayment speeds are faster or slower than model forecasts, MBS prices will react to yield changes differently from what is predicted by a model-generated OAD. In addition, if rates are at levels where prepayment models have not been tested, empirical durations may offer more reliable guidance regarding sensitivity to yield changes.

Another empirical measure uses "relative coupons." For example, suppose a portfolio manager wishes to measure the duration of a GNMA 6%. Currently trading in the market are GNMA 5.5% and GNMA 6.5%, with similar seasoning profiles. The manager can then look at the prices of those two securities to estimate the price change

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[6] For each MBS Index generic Lehman reports KRDs for six key rate points (0.5-year, 2-year, 5-year, 10-year, 20-year and 30-year). These KRDs are available *via* POINT.

[7] We discuss only a handful of empirical duration measures commonly used by portfolio managers. There are many others. For example, see "Special Topic: A Study of Empirical Durations," *Global Relative Value*, Lehman Brothers, May 21, 2001 by S. Modukuri and J. Ryu.

[8] We chose the 10-year UST yield because it is highly correlated with the mortgage rate that drives prepayments.

for an up and down 50-basis-point change in “interest rates” from the 6% level. The total difference in prices between the outer coupons is then used as the basis for the empirical duration for GNMA 6%s.

Definitions and Data Requirements of MBS Duration Measures

Data Set

Our data set contains daily index price changes for 17 annual aggregates (or “generics”) in the Lehman MBS Index, and for the Index itself, for the period from June 25, 2001 through February 28, 2005. These generics were selected because they represented new production at the beginning of the data period and because their coupons span the range of coupons available in the marketplace. For each generic and the Index, we calculate a daily percentage (full) index price change.[9] The 17 generics are:

30-year FNMAs: 5%, 5.5%, 6%, 6.5%, 7%, 7.5% and 8% coupons, all of 2001 vintage;

15-year FNMAs: 5.5%, 6%, 6.5%, 7% and 7.5% coupons, all of 2001 vintage; and

30-year GNMAs: 6%, 6.5%, 7%, 7.5% and 8% coupons, all of 2001 vintage.

MBS Duration Measures

We examine eight duration measures (three model-based and five empirical) and associated UST yield changes to calculate predicted MBS percentage price changes. Each duration-yield change pair generates a predicted daily percentage price change for each generic listed above.[10] We let P_t denote the full index price of an MBS generic at the close of day t. We use $\Delta_{10}yield_t$ to denote the daily change in the 10-year on-the-run UST yield at the close of day t and we use $\Delta_p yield_t$ to denote the daily parallel shift, i.e., the average of daily yield changes at six points on the par curve. The eight duration-yield change pairs are as follows:

Model Duration – Yield Change Pairs

i. OAD & ΔUST 10-year yield (“OAD(10)”)

$$Ret_{predicted,t} = OAD_{t-1} \times \Delta_{10}yield_t$$

ii. OAD & ΔUST parallel yield (“OAD(p)”)

$$Ret_{predicted,t} = OAD_{t-1} \times \Delta_p yield_t.$$ [11]

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9 We remove several daily observations each month from the data set. On these dates, (*i.e.,* the first day of the month, the pricing PSA-switch date, and the index factor date) the index price can change due to the mechanics of the index price calculation and not due to any movements in UST yields or other risk factors. For a discussion of Lehman MBS index prices and returns see: *Managing against the Lehman MBS Index: Prices and Returns,* Jordan I. Mann and Bruce D. Phelps, Lehman Brothers, November 21, 2003.

10 Except for the relative coupon measure, which could not be used on the highest (lowest) coupon security in each program because there was no security in the program with a higher (lower) coupon. The relative coupon measure also could not be calculated for the MBS Index.

11 The “parallel” ΔUST yield, $\Delta_p yield_t$, is defined as the arithmetic average change in the 6-month, 2-year, 5-year, 10-year, 20-year, and 30-year fitted par UST yields.

iii. KRD & ΔUST KRD yield ("KRD")

$$Ret_{predicted,t} = \Sigma_i \, (KRD_{i,\,t-1} \times \Delta_{KRD(i)} yield_t)$$

Empirical Duration – Yield Change Pairs

iv. "10-day" & ΔUST 10-year yield ("Emp(10,10)")

Using 10 consecutive observations of percentage changes in the MBS price and changes in the on-the-run UST 10-year yield, ending with the observation on day t-1, we regress percentage MBS price change against the yield change to get

$$Ret_{actual,t} = \alpha + \beta_{t-1} \times \Delta_{10} yield_t + \varepsilon_t$$

The estimated regression coefficient β_{t-1} is the empirical duration measure. Using this measure and the daily change in the on-the-run UST 10-year yield at time t, we derive the predicted daily MBS percentage price change:

$$Ret_{predicted,t} = \beta_{t-1} \times \Delta_{10} yield_t$$

The predicted return is calculated ignoring the constant term—an assumption that has negligible effect on the results.

v. "10-day" & ΔUST parallel yield ("Emp(10,p)")

Same as iv. except that the independent variable in the regression is the change in the average UST yield, $\Delta_p yield_t$. The estimated regression coefficient is multiplied by $\Delta_p yield_t$ to get the predicted percentage price change:

$$Ret_{predicted,t} = \beta_{t-1} \times \Delta_p yield_t$$

vi. "20-day" & ΔUST 10-year yield ("Emp(20,10)")

Same as iv. above except that the regression uses the last 20 business days of data to generate the regression coefficient.

vii. "20-day" & ΔUST parallel yield ("Emp(20,p)")

Same as v. above except that the regression uses the last 20 business days of data to generate the regression coefficient.

viii. "Relative Coupon" & ΔUST 10-year yield ("RelC(10)")

This measure is best explained by example. To calculate the relative coupon duration for FNA06001,[12] let $P6.5_t$ = price of FNA06401, $P6.0_t$ = price of FNA06001 and $P5.5_t$ = price of FNA05401, all at time t. We define

[12] The abbreviations for MBS Index generics follow the usual index conventions: FNA06401 represents 30-year FNMA 6.5% of 2001. Vintage year is determined by the WALA of the annual aggregate. For details on the construction of index MBS generics, please see "Managing against the Lehman MBS Index: Pools vs. Annual Aggregates," Lehman Brothers, *Global Relative Value*, June 7, 2004.

relative coupon duration measure $\equiv RDM_{FNA06001,t-1}$
$\equiv [P6.5_{t-1} - P5.5_{t-1}]/[P6.0_{t-1}]$

$Ret_{predicted,t} = RDM_{FNA06001,t-1} \times \Delta_{10}yield_t$

Results

For the 17 generics and the MBS Index, we calculate daily actual percentage price changes over a given period. We also calculate daily predicted percentage price changes using the eight duration measures generated as of the end of the previous day multiplied by the indicated change in Treasury yield. Then, for the given period consisting of n consecutive daily observations, we calculate the RMSE as follows:

$$RMSE \equiv \sqrt{[\Sigma_{i=0,\ n-1} (Ret_{predicted,t(i)} - Ret_{actual,t(i)})^2/n]}$$

The RMSE measures an "average" return error, in basis points, between the actual return and the predicted return.

Finally, because different generics can have very different price return volatilities, we standardize the RMSE by dividing a generic's RMSE by the volatility (i.e., standard deviation) of its actual price return for the same period:

$$StdRMSE \equiv RMSE/stdev(Ret_{actual}).$$

StdRMSEs can be interpreted as the percentage of price variance that is not explained by the duration measure. We compare the eight duration measures using the StdRMSE. Figure 1 presents StdRMSEs for the generics and Index for the period from June 25, 2001 through February 28, 2005. Figure 1 also shows the average dollar price for each generic for the period. This information allows us to examine a commonly held view that the effectiveness of model durations degrades as the dollar price of the MBS increases. To put the performance of the various MBS duration measures into perspective, Figure 1 also supplies results for some non-MBS securities and indices. Specifically, we show StdRMSE values for four agency indices (0-3 year duration bullets; 0-3 year callables; 3-8 year bullets; and 3-8 year callables), a AA-rated corporate bond (WFC 7.55% of 6/10), and a high-coupon Treasury (9.875% of 11/15). We selected agencies because many MBS investors often compare the relative value of MBS and agencies on a duration-neutral basis. The high-quality corporate and Treasury, selected at random, are shown because most investors would expect them to have very low StdRMSEs.

Overall, the various MBS duration measures perform reasonably well. For the MBS Index, the seven duration measures (excluding the relative coupon measure) produce a StdRMSE value between 0.34 (using KRDs) and 0.38 (OAD(p)). In particular, even though the average dollar price for the MBS Index was 102.6 for the period, these error measures compare favorably with those for the agency bullet and callable indices and for the single corporate and Treasury bonds. For the 0-3 year duration bullet agency index, the OAD(10) duration measure produced a StdRMSE of 0.62, which was significantly greater than that for the MBS index (0.35). The short callable agency index performed similarly (0.60). The OAD(10) error values for the WFC and Treasury were 0.44 and 0.27, respectively. All of the duration error measures for the corporate bond ranged between 0.44 (KRDs) and 0.49 (Emp(10,10) and Emp(10,p) which is

Figure 1. **Predicted vs. Actual Percentage Price Change StdRMSE**
June 25, 2001 through February 28, 2005

	$Price	OAD (10)	OAD (p)	KRD	Emp (10,10)	Emp (10,p)	Emp (20,10)	Emp (20,p)	RelC (10)
MBS Index	102.60	0.35	0.38	0.34	0.42	0.41	0.39	0.37	
FNA05401	99.89	0.37	0.39	0.35	0.42	0.41	0.38	0.37	
FNA06001	101.87	0.38	0.38	0.35	0.44	0.43	0.42	0.40	0.99
FNA06401	103.33	0.41	0.40	0.39	0.46	0.45	0.44	0.43	0.55
FNA07001	104.71	0.48	0.46	0.47	0.53	0.51	0.51	0.49	0.89
FNA07401	105.83	0.79	0.66	0.69	0.66	0.64	0.61	0.60	1.61
FNA08001	106.98	1.66	1.37	1.48	0.82	0.80	0.79	0.77	
FNC05401	102.12	0.39	0.40	0.37	0.46	0.43	0.43	0.40	0.51
FNC06001	103.52	0.43	0.43	0.42	0.49	0.46	0.46	0.44	0.58
FNC06401	104.81	0.49	0.46	0.51	0.54	0.52	0.51	0.50	0.81
FNC07001	105.69	0.90	0.74	0.81	0.65	0.63	0.61	0.59	0.97
FNC07401	106.37	1.69	1.38	1.55	0.76	0.74	0.73	0.71	
GNA06001	102.42	0.40	0.38	0.38	0.44	0.43	0.41	0.41	0.51
GNA06401	103.92	0.45	0.40	0.42	0.47	0.46	0.45	0.44	0.53
GNA07001	105.22	0.55	0.49	0.54	0.57	0.56	0.53	0.52	0.80
GNA07401	106.31	0.76	0.69	0.79	0.71	0.71	0.67	0.67	1.32
GNA08001	107.20	0.99	0.93	1.05	0.91	0.90	0.88	0.87	
0-3 Agency bullet	103.41	0.62	0.57	0.47	0.69	0.64	0.66	0.60	
0-3 Agency callables	100.52	0.60	0.57	0.56	0.65	0.61	0.62	0.59	
3-8 Agency bullet	106.87	0.34	0.38	0.29	0.39	0.37	0.36	0.33	
3-8 Agency callables	100.13	0.59	0.57	0.57	0.65	0.63	0.60	0.59	
WFC 7.55% 6/10	115.57	0.44	0.45	0.44	0.49	0.49	0.47	0.47	
T 9.875% 11/15	148.34	0.27	0.31	0.23	0.34	0.36	0.30	0.33	

uniformly higher than for the MBS Index. Not surprisingly, the duration measures for the Treasury note performed best, ranging from 0.23 (KRDs) to 0.36 (Emp(10,p)).

As anticipated, KRDs usually performed better than the two OADs. However, somewhat unexpectedly, the improvement of KRDs over each of the two OADs was relatively small (0.34 versus 0.35 and 0.38) for the MBS Index. Among the individual index generics, KRDs were usually, but not always, the best model-based duration measure. KRDs also generally outperformed OADs for the non-MBS issues. For the bullet agency indices and the Treasury and corporate issues, KRDs were the best model duration measure by a considerable margin, which is somewhat unexpected given the bullet nature of their cash flows. In contrast, for agency callable indices, whose constituents have significant key rate exposure along the curve, KRDs remained the best measure, but by a much smaller margin.

For the MBS Index, empirical duration measures performed modestly worse in comparison to model durations. Empiricals produced StdRMSEs ranging from 0.37 (Emp(20,p)) to 0.42 (Emp(10,10)), which compares to the range of 0.34 to 0.38 for the model durations. Empirical durations using 20 days of data performed slightly better than their counterpart measures using only 10 days.

At the individual generic level, for coupons 7% and lower model durations outperformed empirical durations. As an example, for FNMA 6% of 2001 (*i.e.,* FNA06001), the eight duration measures had a StdRMSE between 0.35 (KRD) and 0.99 (RelC(10)). Model durations (OAD and KRD) handily outperformed the four empirical durations. In addition, KRDs outperformed both OAD(10) and OAD(p) and the empirical durations that used 20 days of history slightly outperformed the 10-day measures. For GNMAs and 15-year FNMAs, model durations also generally performed better than empirical durations, but to a lesser extent than for 30-year FNMAs.

Results for GNA06001 (GNMA 6% 2001) and FNC05401 (15-year FNMA 5.5% 2001) were somewhat similar to those for FNA06001. However, while model durations outperformed empirical durations, the performance gap for GNA06001 and FNC05401 was smaller than for FNA06001. In addition, KRDs performed worse for 15-year FNMAs and GNMAs than for similarly priced FNMAs.

For all duration measures, the StdRMSEs were relatively stable across price levels until the MBS price exceeded 104 – at which point StdRMSEs increased rapidly. This is not surprising as higher-priced MBS typically have greater prepayment uncertainty, less liquidity, and Treasury durations shorter than their spread durations.[13] Notably, however, the increase in StdRMSE was less extreme for empirical measures than for model measures. For the individual generics, the relative performance of empirical versus model durations depended strongly on the generic's coupon (or price level). Overall, empiricals (except RelC(10)) held up better than model durations as price increased.[14] For generics with average prices less than 104, empiricals performed somewhat worse than model durations. However, for generics with prices above 104, empiricals began to perform much better. For example, for FNA07401 (average dollar price of 105.83), empirical durations (excluding RelC(10)) performed better then model durations.

Despite its popularity with investors, the relative coupon duration (RelC(10)) performed slightly worse than the other empirical measures for the lower-priced generics and performed much worse for higher-priced generics (price > 104) across all programs. For FNA07001, RelC(10) had a StdRMSE of 0.89 whereas the next largest was Emp(10,10) with a StdRMSE value of 0.60.

For the non-MBS indices and issues, the empirical measures also performed reasonably well compared to OADs. For the corporate issue, the range of its four empirical duration measures was 0.47 (Emp(20,p)) to 0.49 (Emp(10,p)) compared with 0.45 and 0.44 for OAD(p) and OAD(10), respectively. The same relative pattern between the empirical and OAD measures also held for the agency indices and the single Treasury issue. In addition, for the entire period, the empirical measures using 20 business days of data performed better than their respective counterparts using only 10 days.

Model durations performed relatively poorly for the two callable agency indices. Surprisingly, empiricals also performed poorly. For example, the 3-8 year agency callable index (average dollar price of 100.13) had an StdRMSE of 0.59 for OAD(10) and a value of 0.60 for Emp(20,10). The 0-3 agency callable index (average dollar price of 100.52) displayed a similar pattern.

Figure 2 shows the relative performance, over the data period, of some duration measures as a function of a generic's average price. The duration measures displayed

[13] This increases the relative influence of spread changes compared to yield changes on price movements. To the extent that empirical durations reflect all historical influences on price changes, it is reasonable to expect that empiricals for such securities would have a tendency to outperform model durations.

[14] This finding has been reported previously using monthly data. See *Risk & Return in the Mortgage Market: Review and Outlook,* A. Arora, D. Heike and R. Mattu, Lehman Brothers, January 13, 2000.

are: OAD(10), KRD, Emp(20,10) and RelC(10). The figure shows that Emp(20,10) performed best across most price levels whereas the relative coupon measure generally performed worst. The performance of all duration measures began to degrade once the dollar price exceeded 104. However, the model and relative coupon duration degraded more severely than the Emp(20,10) measure.

To highlight the relative performance of MBS duration measures, as a function of the MBS price, we calculate the ratio of StdRMSEs for various duration measure pairs. For the overall period, we saw that model duration performance deteriorated more rapidly than empirical performance as the MBS price increased. This pattern is clearly shown in Figure 3. This figure shows the relative StdRMSE performance of KRD model duration versus Emp(20,10) empirical duration as a function of the MBS price. A value

Figure 2. **Predicted vs. Actual Percentage Price Change StdRMSE**
June 25, 2001 through February 28, 2005

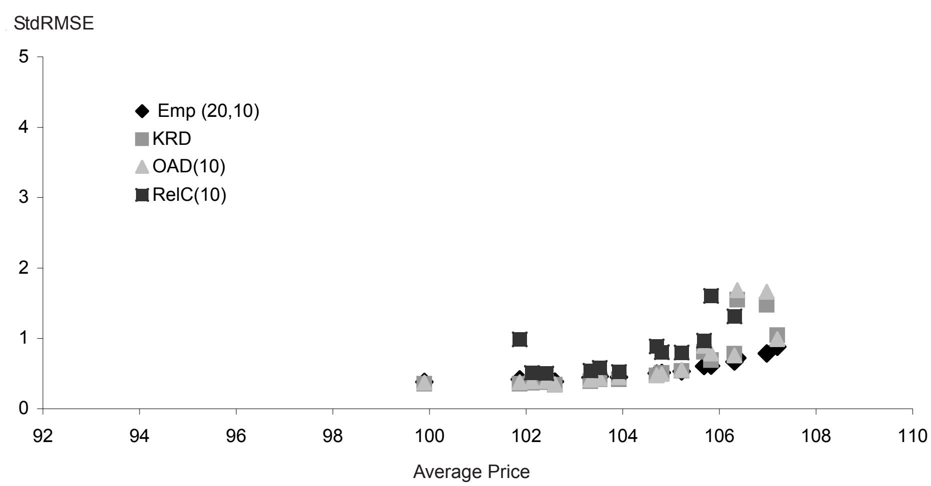
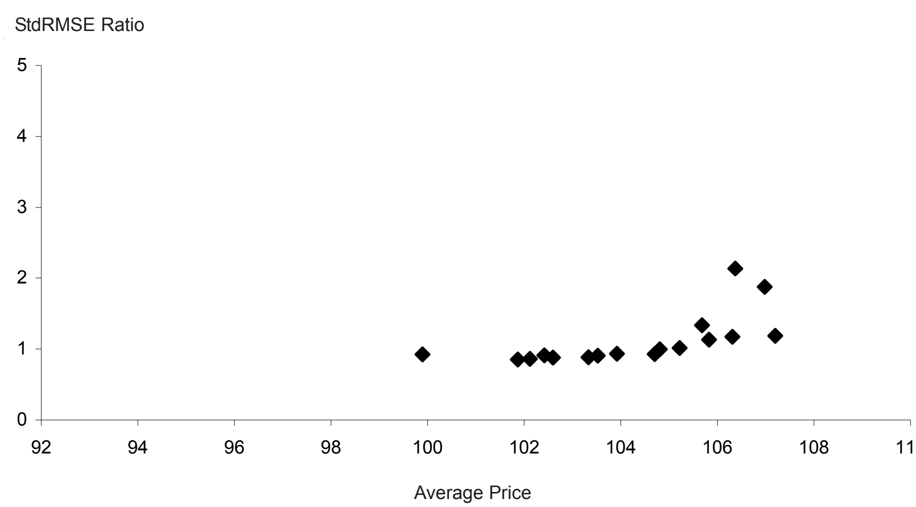


Figure 3. **Ratio of KRD to Emp(20,10) StdRMSEs**
All Generics and MBS Index
June 25, 2001 through February 28, 2005



less than 1.0 indicates that the KRD measure performs better than the empirical measure. As the figure shows, up until a price of 104, KRD is superior. However, once the dollar price rises above 104, the KRD's relative performance deteriorates and becomes inferior to the empirical measure.

In contrast, note that there is no clear indication of a price level influence on the relative performance of KRD and OAD(10) model duration measures. This is shown in Figure 4.

To examine the relative performance of various durations in different interest rate environments, we divide the data period into four sub-periods (Figure 5). The first sub-period, June 25, 2001 to March 4, 2002, was characterized by slightly lower than average MBS prices (average dollar price equaled 102.08), relatively unchanged 10-year Treasury yield (although a heretofore new low was touched), and a 59 bp steepening of the 2-10 yield spread. The sharp reshaping of the yield curve gave KRDs an opportunity to outperform OADs during this sub-period.

The second sub-period, from March 4, 2002 through September 30, 2002, was a period of sharply and persistently declining 10-year Treasury yields (including heretofore generational lows) and little change in the 2-10 spread. Overall, the 10-year yield fell 138 bp while the 2-10 spread increased 9 bp. Such an extreme prepayment environment was a severe test for model durations. How well did they perform compared to empiricals?

The third period runs from October 7, 2002 through March 25, 2004. Although both the 10-year and the yield curve ended the period at levels with which they began the period, there was a sharp market reversal (July 2003) shortly after Treasury yields reached new lows (the 10-year reached an all-time low of 3.07% on 16 June 2003) as fears about lower yields began to attenuate. The movement in the 10-year yield during July 2003 period was particularly large – 95 bp, more than a three standard deviation move. However, rates remained generally low for the entire period, providing ample

Figure 4. **Ratio of KRD to OAD StdRMSEs**
All Generics and MBS Index
June 25, 2001 through February 28, 2005

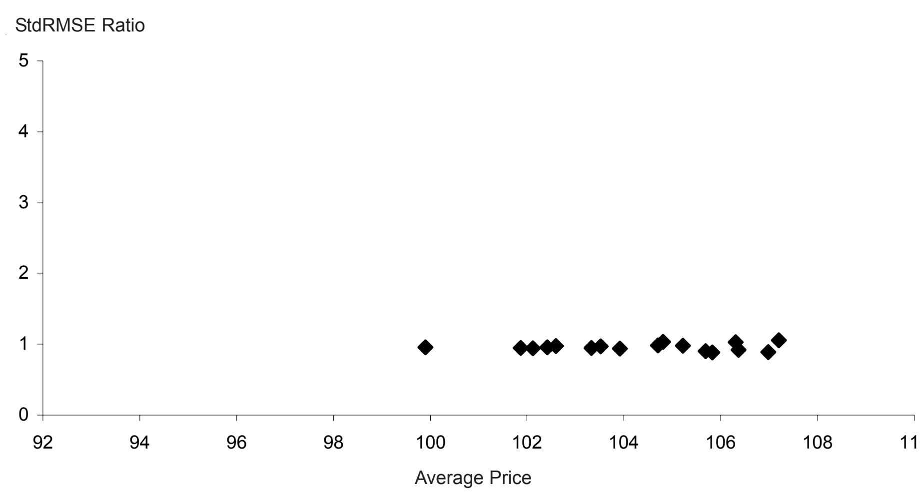
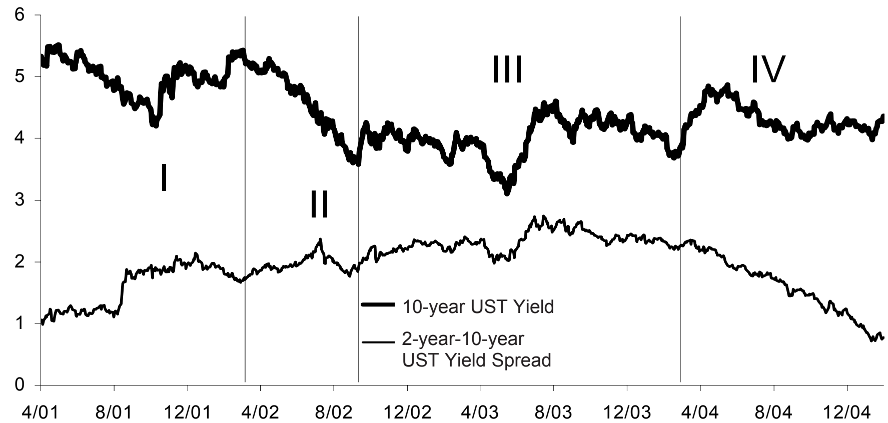


Figure 5. **10-year UST Yield and 2-year-10-year UST Yield Spread**
April 30, 2001 through February 28, 2005



Summary Statistics for the Four Sub-Periods

Period	Average MBS Price	Chg 10-yr Yield	Range 10-yr Yield	Chg 2-10 Spread
6/25/01 – 3/4/02	102.08	- 13 bp	123 bp	+ 59 bp
3/4/02 – 9/30/02	102.84	-138 bp	183 bp	+ 9 bp
10/7/02 – 3/25/04	103.46	+12 bp	150 bp	+ 35 bp
3/25/04 – 2/28/05	101.44	+64 bp	113 bp	-146 bp

time for mortgage prepayment speeds to explode. The sharp market reversal may have caused empiricals to underperform model durations during the period.

The final period covers March 25, 2004 through February 28, 2005. During this period, the 10-year increased 64 bp and the yield curve flattened 146 bp. Generally, this was an environment of lessened worries about prepayment risk. The considerable curve reshaping (a flattening compared to the first period's steepening) may reveal that key-rate durations sharply outperformed single duration measures.

What was the relative performance of the various duration measures during these four very different market environments? To simplify the presentation, we focus on two model durations (KRD and OAD(10)) and two empirical durations (Emp(20,10) and RelC(10)). Figure 6 shows the performance of these four duration measures for all the generics and the Index, across the overall period and the four sub-periods.

Across the sub-periods, all four duration measures shared a tendency to deteriorate once the MBS dollar price exceeded 104. However, for prices above 104, while the Emp(20,10) measure performs poorly in absolute terms, it greatly outperformed the RelC(10) measure. The two model durations (OAS(10) and KRD) perform well for prices below 104, but their performance deteriorated as prices moved beyond 104.

To get a clearer picture of relative duration performance, Figure 7 shows the relative StdRMSE ratios across the overall period and the four sub-periods for various duration pairs across all the generics and the Index.

Figure 7 reveals that the patterns for the individual sup-periods resemble the pattern for the overall period despite the very different market environments. Across all sub-periods, Emp(20,10) underperforms KRD and OAD(10) if the MBS dollar price is less than 104. This result is shown in Figure 7 as the StdRMSE ratio for KRD/Emp(20,10) is usually less than one and the ratio for Emp(20,10) is usually greater than one as long as the MBS dollar price is less than 104. However, across all four sub-periods, Emp(20,10) outperforms KRD and OAD(10) when the MBS dollar price exceeds 104. This can be seen by the KRD/StdRMSE ratio rising and the Emp(20,10)/OAD(10) ratio falling after a 104 price is reached.

Figure 6. **Performance of Four Duration Measures Across Four Sub-Periods and Overall Period**
April 30, 2001 through February 28, 2005

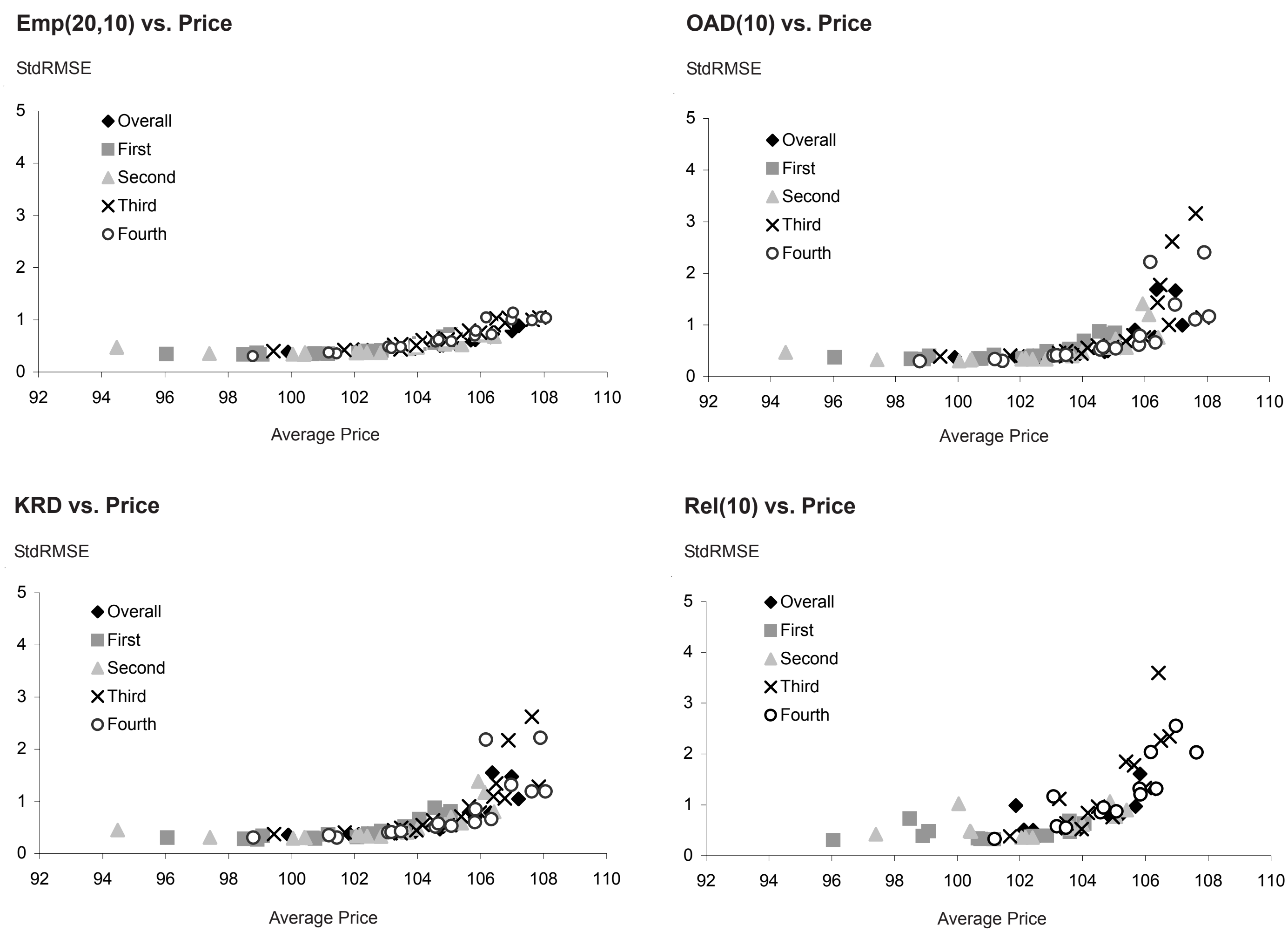
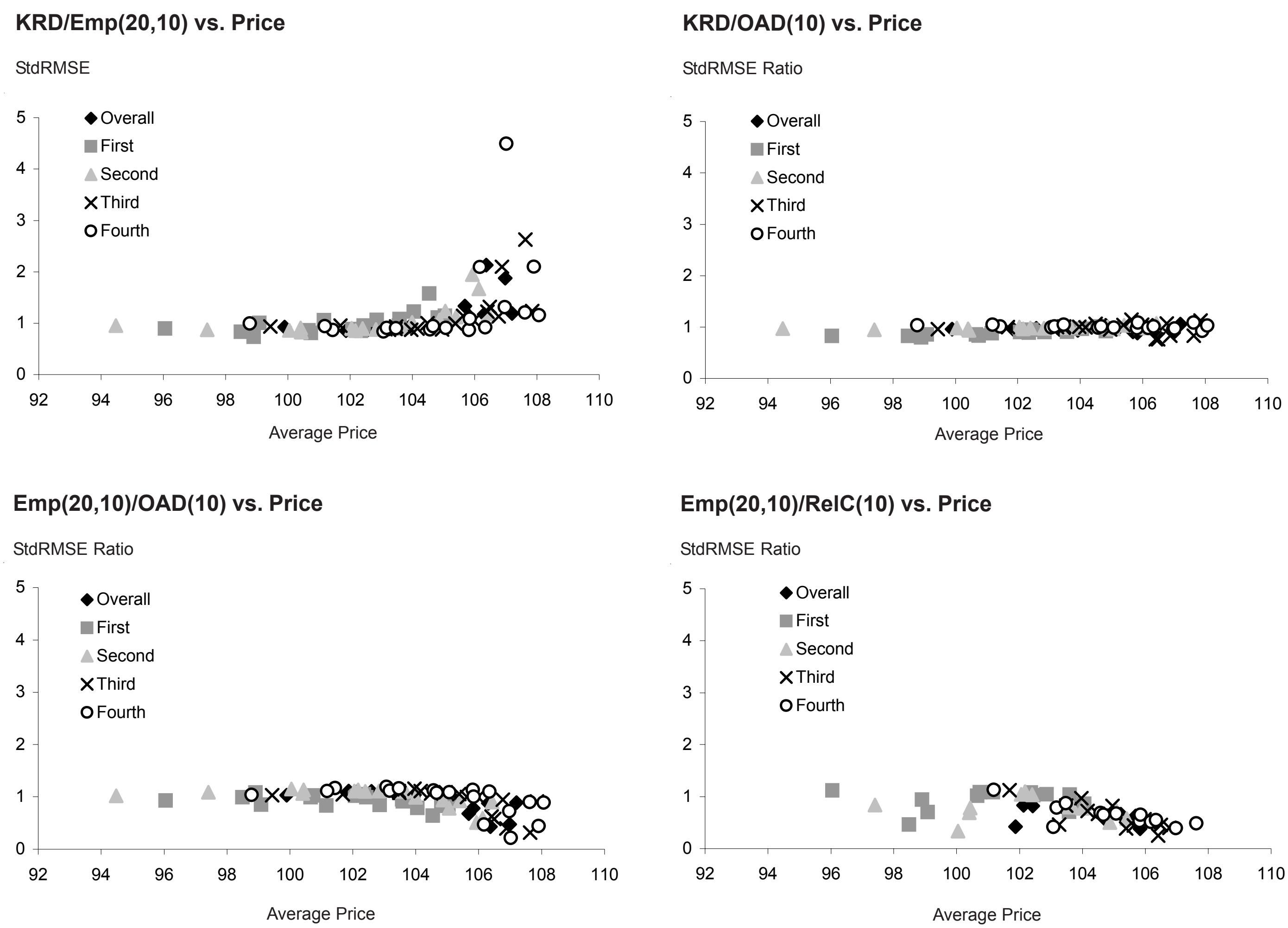


Figure 7 also shows that the Emp(20,10) empirical measure regularly outperforms the RelC(10) measure (i.e., StdRMSE ratio is usually less than one). Rarely is the RelC(10) duration measure the better empirical duration. Between the two model durations, KRD typically outperforms OAD. In fact, there is no sub-period in which OAD is the better model duration.

However, there were some notable differences across the four sub-periods. In the first sub-period, a period of declining yields with the 10-year reaching a heretofore record low, empiricals performed particularly well versus model durations as the values of the StdRMSE ratio for KRD/Emp(20,10) and Emp(20,10)/OAD(10) are noticeably above and below, respectively, the patterns for the rest of the period. In the fourth sub-period where the 10-year yield was relatively unchanged but the curve steepened considerably, the empirical duration performed poorly compared to model durations. The sharp

Figure 7. **Relative Performance of Four Duration Measures Across Four Sub-Periods and Overall Period**
April 30, 2001 through February 28, 2005



reshaping of the yield curve during the fourth sub-period gave KRDs an opportunity to outperform OADs. Interestingly, however, there is no perceptible KRD advantage versus OAD during this period.

To check to see if the results above differed depending on the MBS program, Figure 8 breaks down Figure 6 into two MBS groups: 30-year FNMA (labeled "FNMA") and 30-year GNMA & 15-year FNMA.

Figure 8 shows that the pattern of duration measure performance is broadly similar across the two sets of MBS programs. However, there are some notable exceptions. First, model durations tended to perform slightly better for the FNMA set than for the GNMA & 15-yr FNMA set. This can be seen in the StdRMSE value using both OAD (10) and KRD. For a given price level, the StdRMSE value is slightly lower for the FNMA than for the GNMA & 15-yr FNMA set. Second, as the price increases above 104, the deterioration in StdRMSE is greater for the GNMA & 15-yr FNMA set than for the FNMA set. Finally, we see that both empirical measures did better for the GNMA & 15-yr FNMA set than for the FNMA set.

Figure 9 separates Figure 7 into the two MBS groups and shows the relative performance of the various duration measures.

Figure 9 clearly reveals that for the GNMA & 15-yr FNMA set, empirical durations outperform model durations beginning with MBS price levels slightly above par. Note the StdRMSE ratio for KRD/Emp(20,10). For FNMAs, KRDs clearly outperform Emp(20,10) until the MBS price reaches above 104. In contrast, for the GNMA & 15-yr FNMA set, KRDs only slightly outperform Emp(20,10) for lower price levels and begin to underperform Emp(20,10) at MBS prices of less than 104. This pattern is also visible in the Emp(20,10)/OAD(10) ratio as it begins to fall below 1.0 at a lower price level for the GNMA & 15yr FNMA set compared to the 30-year FNMA set.

Negative Duration

A notable development during 2002 and 2003 was the prevalence of negative OADs for high coupon MBS.[15] Negative durations imply that the MBS generic price will tend to *increase (decrease)* if UST yields *increase (decrease)*. While negative durations are common among certain MBS derivatives, they are unusual for passthroughs. For some investors, it is a leap of faith to add securities with negative durations to their portfolios. The existence of negative durations also led some investors to question the relevance of any MBS model duration. How well did negative OADs explain the price behavior of MBS?

We examine several 2000 vintage generics that began to have negative OADs toward year-end 2002 and which persisted through June 2003. For comparison, we also examined the corresponding 2001 vintage generic having the same coupon and program but which had positive OADs.[16] How well did the negative model durations perform? Did they make any sense?

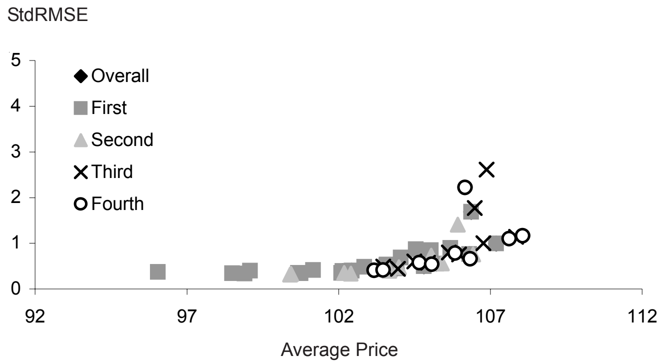
[15] As of June 30, 2003, more than 9% of the MBS Index's market value consisted of generics with negative OADs.

[16] An important reason for the OAD difference is that new mortgagors paying such high coupons in 2001, when mortgage rates were much lower, were likely to be credit constrained and had limited prepayment possibilities.

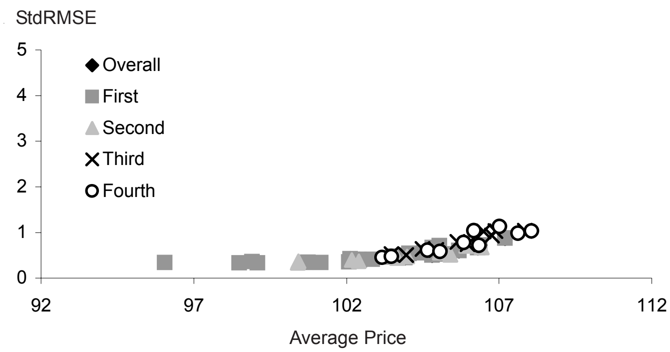
Figure 8. **Performance of Four Duration Measures Across Four Sub-Periods and Overall Period**
April 30, 2001 through February 28, 2005

GNMA & 15-year FNMA

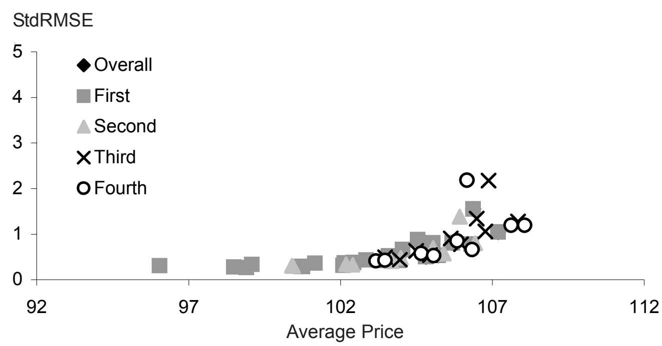
OAD(10) vs. Price



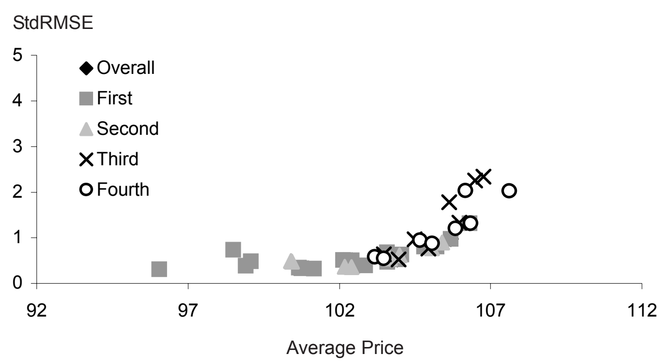
Emp(20,10) vs. Price



KRD vs. Price

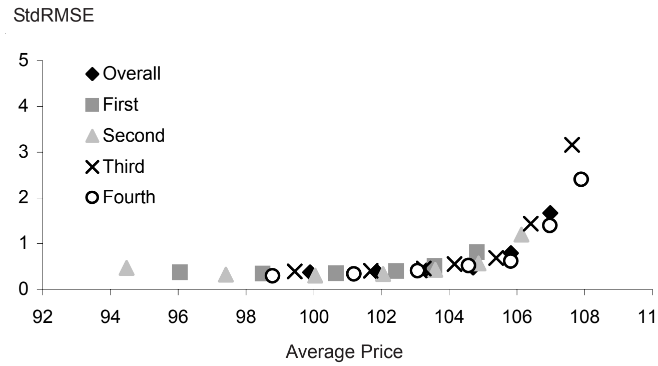


RelC(10) vs. Price



FNMA

OAD(10) vs. Price



Emp(20,10) vs. Price

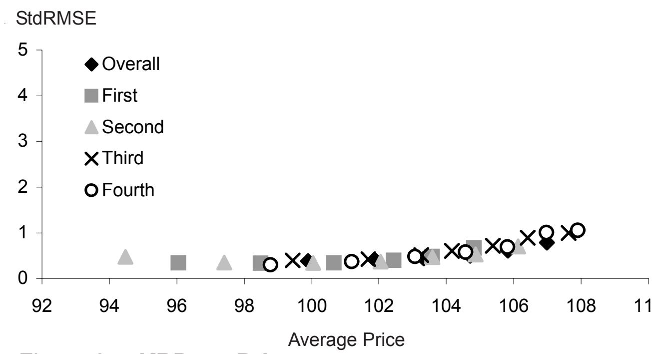
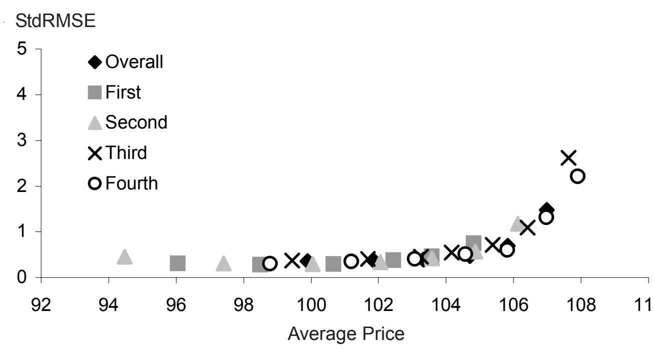


Figure 8. **KRD vs. Price**



RelC(10) vs. Price

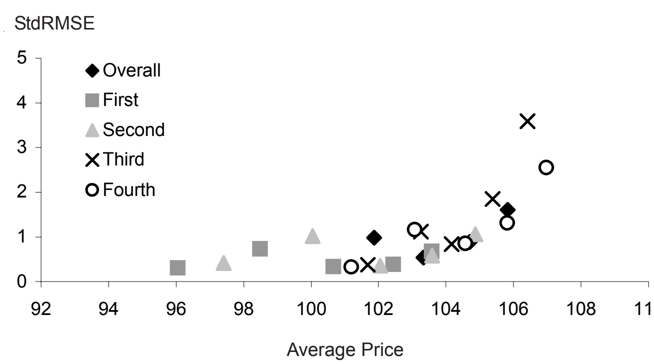
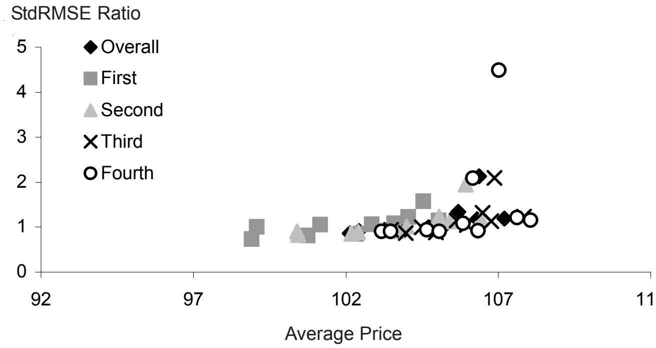


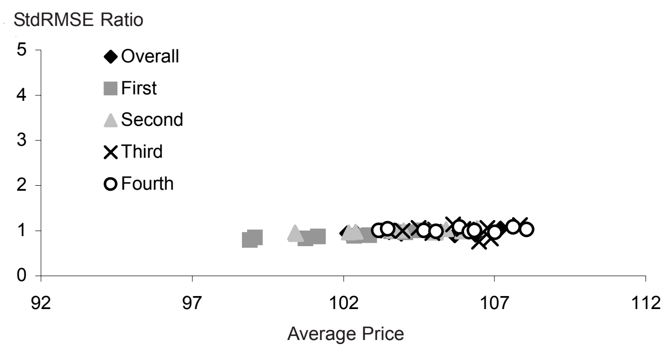
Figure 9. **Relative Performance of Four Duration Measures Across Four Sub-Periods and Overall Period**
April 30, 2001 through February 28, 2005

GNMA & 15-year FNMA

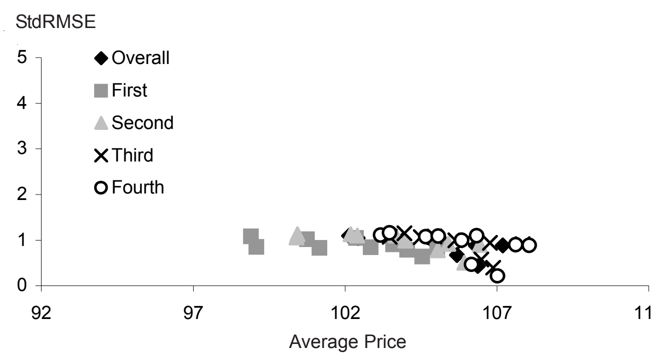
KRD/Emp(20,10) vs. Price



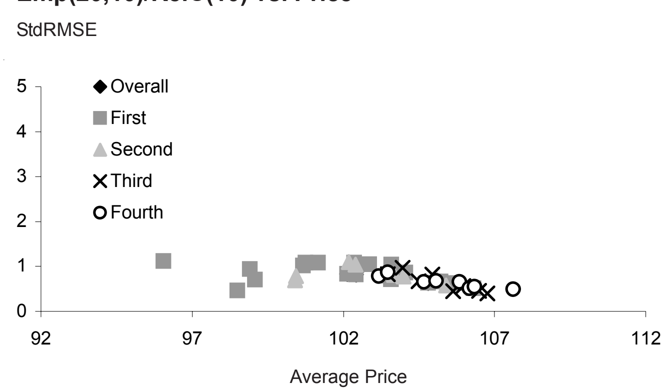
KRD/OAD(10) vs. Price



Emp(20,10)/OAD(10) vs. Price

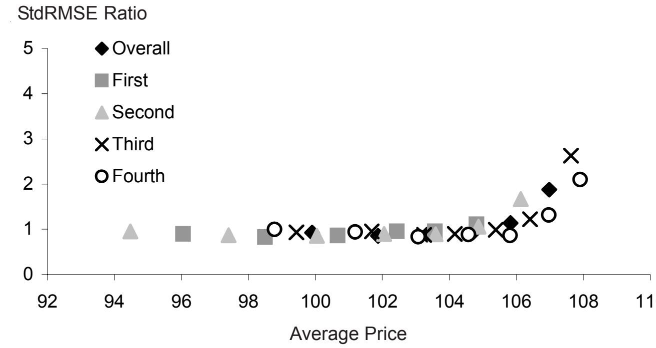


Emp(20,10)/RelC(10) vs. Price

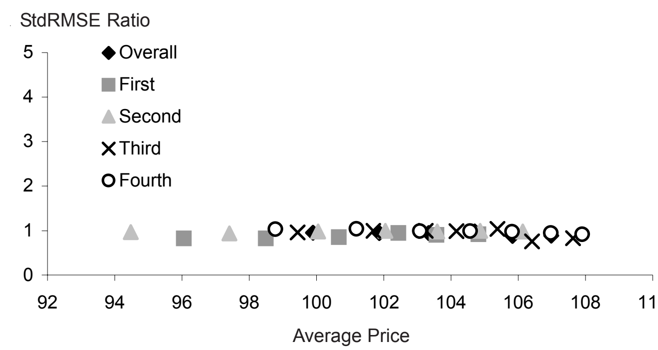


FNMA

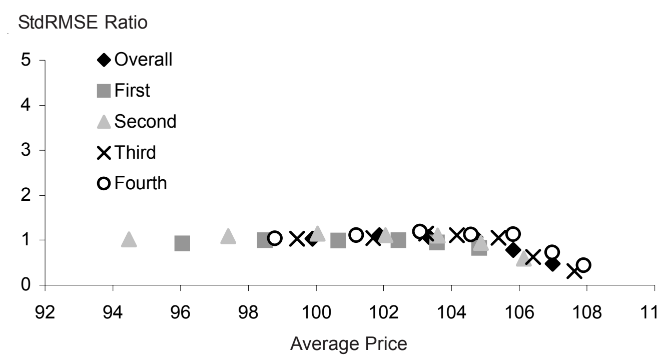
KRD/Emp(20,10) vs. Price



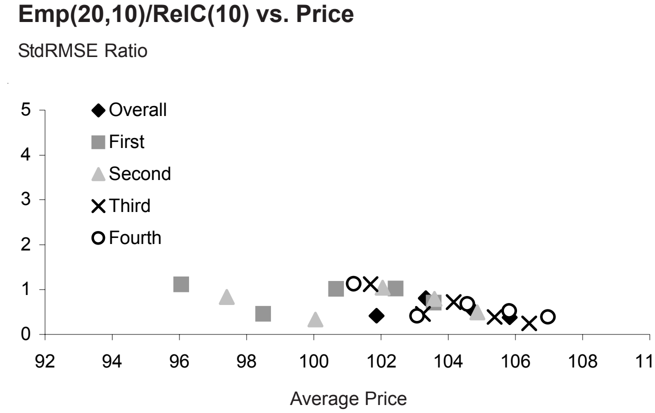
KRD/OAD(10) vs. Price



Emp(20,10)/OAD(10) vs. Price



Emp(20,10)/RelC(10) vs. Price



FNMA 8% of 2000 (FNA08000) had negative OADs from October 2002 through June 2003. While the OADs were negative, they were only slightly so (e.g., -0.31 on October 1, 2002 and -0.29 on June 30, 2003). In contrast, the OAD for FNA08001 was significantly positive throughout the same period (*e.g.,* 1.48 on October 1, 2002 and 1.30 on June 30, 2003). [17] FNA07400 and FNA07401 displayed a similar pattern, although the OAD for FNA07400 did become slightly positive in late 2002. FNC06400 spent most of 2003 with a negative OAD (again, only slightly negative as the OAD on June 30, 2003 was -0.16) while FNC06401 remained positive throughout, but also very close to zero. In contrast to the FNMA 7.5%s and 8%s, there was little OAD difference between the 2000 and 2001 vintages for GNMA 7.5%s and 8%s most of the time between October 2002 and June 2003.

Figure 10 presents the relative duration performance for these 10 generics. Specifically, we compare the performance of the 2000 vintages (with very low or zero model durations) with their 2001 counterparts. Generally, for these high-coupon generics all durations, model or empirical, performed poorly. Most StdRMSE values were close to (or greater than!) one, indicating that the duration error was almost equal to the price volatility itself. In other words, assuming a duration value of zero performed almost as well (if not better) than the empirical or model duration value. Notably, the StdRMSE (OAD(10)) for FNA08001 (positive model duration) greatly exceeded that for FNA08000 (negative model duration) while their KRD StdRMSE's were comparable. For both generics, their empirical durations performed similarly and were much better than any of their model durations. However, given that the StdRMSEs were greater than one, assuming a duration value of zero would have performed better. The fact that

[17] While the FNA08000 and FNA08001 generics had very different OADs there was some similarity in their respective KRD profiles. Despite the negative OAD for FNA08000, the security had a positive KRDs for the 0.5-year and 2-year key rate points. In fact, these positive KRDs were slightly higher than the corresponding KRDs for FNA08001. The source of the negative OAD for FNA08000, and the source of the difference with FNA08001, lies in the longer maturity KRD points: the 5-year, 10-year, and 20-year. FNA08000 had significantly negative KRDs at all three points. In contrast, the 5-year and the 10-year KRDs for FNA08001 were modestly positive and 20-year KRD was close to zero. Given the large variations in price sensitivity along the curve, especially for FNA08000, OAD(10) is not likely a particularly good duration measure for either security. The StdRMSEs (using KRD) for the two annual aggregates were much closer. Nevertheless, empiricals remained the best performers.

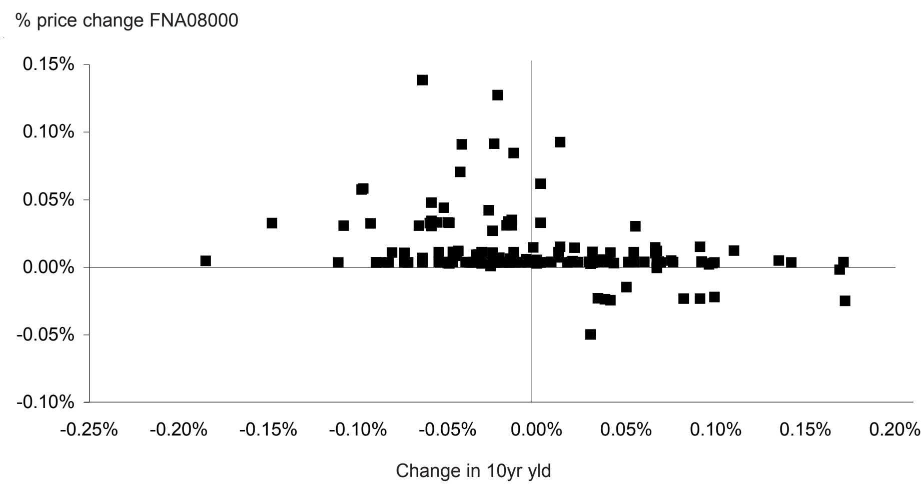
Figure 10. **Predicted vs. Actual Percentage Price Change StdRMSE**
October 2, 2002 through June 30, 2003

	$Price	OAD10	OADp	KRD	Emp (10,10)	Emp (10,p)	Emp (20,10)	Emp (20,p)	RelC10
FNA07400	106.10	1.13	1.10	1.61	0.97	0.96	0.95	0.95	
FNA07401	106.19	1.40	1.15	1.09	0.97	0.96	0.95	0.94	4.13
FNA08000	107.51	1.69	1.54	2.46	1.03	1.04	1.04	1.05	
FNA08001	107.55	2.96	2.40	2.43	1.03	1.03	1.04	1.05	
FNC06400	105.42	0.91	0.93	1.23	0.81	0.81	0.79	0.79	
FNC06401	105.51	0.74	0.73	0.89	0.81	0.81	0.79	0.79	1.55
GNA07400	106.60	0.98	0.99	1.15	0.99	0.99	0.98	0.98	
GNA07401	106.63	0.98	0.99	1.16	0.99	0.99	0.98	0.98	2.47
GNA08000	107.82	1.20	1.15	1.49	0.99	0.99	0.99	0.98	
GNA08001	107.81	1.18	1.14	1.47	0.99	0.99	0.99	0.98	

the negative model duration for FNA08000 produced a lower StdRMSE value than the positive model duration for FNA08001 most likely reflects the small absolute value of the negative duration for FNA08000 compared to the positive duration for FNA08001. The negative model durations made sense largely to the extent that their absolute values were close to zero.

If durations were truly negative, then we would expect to observe a positive relationship between changes in yields and percentage changes in prices. Figure 11 presents a plot of daily changes in the 10-year UST yield (horizontal axis) versus daily percentage changes in the price of FNA08000 (with negative model OADs) for the period from October 1, 2002 through June 30, 2003. The scatter plot does show that there are many instances of a rise in 10-year yields associated with positive percentage price changes and relatively few instances of a rise in yields associated with negative price change. However, there are no instances of a fall in yields associated with negative price changes. This asymmetric relationship between yield and price changes is not fully consistent for a security with negative model duration. While negative model durations performed reasonable well during this period, their performance owed more to their low absolute values (which closely matched empirical durations) than the fact that the duration value was negative.

Figure 11. **Actual Percentage FNA08000 Price Change vs. Change in UST 10-year Yield**
October 1, 2002 through June 30, 2003



RISK MODEL EMPIRICAL DURATIONS

MBS returns are driven by many risk factors, besides changes in UST yields. These other MBS risk factors (e.g., spreads and volatilities) are likely to be correlated with changes in UST yields. In fact, the Lehman Brothers Global Risk Model uses monthly historical data to estimate the variances and covariances of changes in Treasury key rates and MBS risk factors to produce estimates of portfolio tracking error volatility versus a benchmark.[18] These monthly data could be used to construct MBS duration measures. Although the risk model uses monthly data to estimate the variance-covariance matrix, it would be interesting to examine how well "risk model" duration measures perform on a daily basis.

Figure 12 shows the estimated correlations from the Global Risk Model (as of February 28, 2005) between changes in the 5- and 10-year par Treasury rates and some select MBS risk factors. Note that changes in most of these MBS risk factors are correlated with changes in Treasury rates—sometimes negatively correlated. For example, when the 5-year Treasury rate increases, spreads on new premium MBS tend to narrow. The Global Risk Model's variance-covariance matrix was constructed as of November 2002, using MBS data back to May 1995, and is updated each month thereafter. It is constructed two ways. The first, called "un-weighted" or "equal-weighted," assigns equal weights to all historical observations and is appropriate for investors who believe that factor variances and covariances for the future are best represented by the average experience since 1995. Other investors may feel that more recently observed factor variances and covariances are more relevant for estimating factor behavior in the near future. For them, the Risk Model offers a "weighted" calculation of the covariance matrix that uses an exponential time decay to weight the historical data.[19]

We can use the risk model estimated variance-covariance matrix to estimate the full response of MBS percentage price changes to changes in UST yields. For example, suppose the true model of MBS returns were as follows:

$$\%\Delta P_{mbs,t} = S_{yield} \times \Delta yield_t + S_{F1} \times \Delta F1_t + S_{F2} \times \Delta F2_t + \varepsilon_t \qquad (1)$$

[18] For details on the risk model, see *The Lehman Brothers Global Risk Model: A Portfolio Manager's Guide,* April 2005.

[19] The speed of the time decay is fixed at a one-year half life, which implies that an observation of one year ago receives half the weight in the estimation as the most recent observations. For more details on this weighted estimation option, please see "New Estimation Options for the Lehman Brothers Risk Model: Adjusting to Recent Credit Events," A. Berd and M. Naldi, *Quantitative Credit Research*, Lehman Brothers, vol. 2002-Q3, September 2002.

Figure 12. **Estimated Correlations among Selected MBS Risk Factors**
Unweighted Variance-Covariance Matrix, February 28, 2005

MBS Risk Factor	5-Year par UST Rate	10-Year par UST Rate
Discount coupon/low WALA spreads	0.27	0.28
Current coupon/low WALA spreads	0.06	0.10
Premium coupon/low WALA spreads	-0.39	-0.39
Volatility (short):	-0.17	-0.16
Volatility (long):	-0.33	-0.32

where $yield_t$ is a Treasury rate and $F1_t$ and $F2_t$ are two MBS risk factors that affect MBS returns. S_j refers to the sensitivity of the MBS price return to changes in the risk factor j. For example, S_{yield} refers to the duration for the Treasury yield and S_{F1} represents the sensitivity (i.e., spread duration) of the MBS price return to changes in the spread risk factor F1.

For this study, we are concerned with how MBS price returns are related to changes in Treasury yields, either a single Treasury yield or several Treasury key rates. If changes in yield are correlated with changes in MBS risk factors, then we can generate a revised duration measure, called "risk model duration," that incorporates the changes in MBS risk factors associated with changes in yield.

To calculate the risk model duration measure, we proceed as follows. We now assume that there is a relationship between Δyield and ΔF1 and ΔF2, where F1 and F2 are risk factors that influence MBS price returns. Specifically,

$$\Delta F1_t = \gamma_{F1} \times \Delta yield_t + \xi_t$$

$$\Delta F2_t = \gamma_{F2} \times \Delta yield_t + \varphi_t$$

where ξ_t and φ_t are error terms. Using ordinary least squares,[20] we get

$$\gamma_{F1}^* = cov_{\Delta F1,\Delta yield}/var_{\Delta yield}$$

$$\gamma_{F2}^* = cov_{\Delta F2,\Delta yield}/var_{\Delta yield}$$

Substituting into equation (1), we get the following predicted value for the MBS percentage price change:

$$\begin{aligned} \%\Delta P_{predicted,t} &= S_{yield} \times \Delta yield_t \\ &\quad + S_{F1} \times (\chi_{F1}^* \times \Delta yield_t) + S_{F2} \times (\chi_{F2}^* \times \Delta yield_t) \qquad (2) \\ &= (S_{yield} + S_{F1} \times \chi_{F1}^* + S_{F2} \times \chi_{F2}^*) \times \Delta yield_t \\ &= (S_{yield} + S_{F1} \times cov_{\Delta F1,\Delta yield}/var_{\Delta yield} + S_{F2} \times cov_{\Delta F2,\Delta yield}/var_{\Delta yield}) \times \Delta yield_t \end{aligned}$$

The term in parentheses is our "risk-model duration" measure, S_{yield}^*,

$$\%\Delta P_{mbs,t} = S_{yield}^* \times \Delta yield_t$$

This example assumes that MBS price returns are a function of a single Treasury rate. However, the MBS risk model and its covariance matrix assume MBS price returns are a function of six Treasury key rates. Consequently, to use the Risk Model's factor covariance matrix, equation (2) should be rewritten in terms of the six key rates.

$$\%\Delta P_{predicted,t} = \sum_i (KRD_{i,\,t-1} \times \Delta y_{KRD(i),t}) + \text{effect of } \Delta y_{KRD(i),t} \text{ on other MBS risk factors}$$

[20] For simplicity, we are ignoring the constant term. As in the prior section, this assumption has a negligible effect on the results.

This complicates the calculation of risk model key rate durations as each non-key rate risk factor is now modeled as a function of the six key rates. For example, for spread risk factor F1 we assume:

$$\Delta F1_t = \sum_i (\gamma_i \times \Delta y_{KRD(i),t}) + \tau_t$$

Consequently, the estimators for the γ_i values will involve the correlations among the six key rate yield changes.

More generally, the risk model durations are generated as follows. If we let $\mathbf{X}$ represent the (6 x 6) covariance matrix for the six key rates, $\mathbf{C}$ represent the (6 x 21) matrix of the covariances of each of the six key rates with the twenty-one non-key rate MBS risk factors,[21] and $\mathbf{S}$ represent the (21 x 1) vector of risk factor loadings, then it can be shown that the (6 x 1) vector of risk model durations (KRD_{RM}) is given by

$$KRD_{RM} = KRD + \mathbf{X}^{-1}\,\mathbf{C}\,\mathbf{S} \qquad (3)$$

We construct risk model durations using both the unweighted and weighted factor covariance matrices. The performance of these durations, for December 3, 2002-February 28, 2005, is presented in Figure 13. The figure also presents results for model durations, OAD and KRD, and the empirical duration measure, Emp(20,10), for the same period.

For generics with prices less than 106, both sets of risk model durations perform similarly to OAD and KRD, and both tend to outperform ${}_{20}Emp_{10.}$ As prices rise above 106, both risk model durations begin to deteriorate along with OAD and KRD and underperformed Emp(20,10). This result is a bit unexpected because the risk model

21 Details of the MBS risk factors can be found in *The Lehman Brothers Global Risk Model: A Portfolio Manager's Guide,* April 2005.

Figure 13. **Predicted vs. Actual Percentage Price Change, Standardized RMSE**
Risk Model Durations – Unweighted and Weighted
December 3, 2002 - February 28, 2005

	$Price	OAD10	OADp	KRD	Emp(20,10)	$RM_{unweighted}$	$RM_{weighted}$
MBS Index	102.57	0.35	0.39	0.35	0.41	n.a.	n.a.
FNA06001	103.22	0.44	0.47	0.44	0.52	0.44	0.45
FNA06401	104.38	0.54	0.53	0.54	0.61	0.55	0.56
FNA07001	105.63	0.65	0.61	0.67	0.71	0.70	0.71
FNA07401	106.70	1.40	1.18	1.20	0.91	1.34	1.33
FNA08001	107.79	2.73	2.27	2.39	1.01	2.52	2.54
FNC05401	103.39	0.46	0.49	0.47	0.52	0.48	0.48
FNC06001	104.60	0.59	0.58	0.62	0.64	0.63	0.64
FNC06401	105.77	0.82	0.77	0.92	0.81	0.94	0.96
FNC07001	106.40	1.90	1.61	1.66	1.01	1.77	1.77
FNC07401	106.97	2.89	2.43	2.57	1.04	2.68	2.69
GNA06001	103.80	0.43	0.46	0.45	0.51	0.44	0.45
GNA06401	105.05	0.57	0.54	0.55	0.63	0.58	0.60
GNA07001	106.19	0.72	0.67	0.74	0.74	0.78	0.79
GNA07401	107.16	1.04	0.98	1.12	0.94	1.19	1.19
GNA08001	107.99	1.15	1.11	1.23	1.03	1.27	1.25

durations are, in essence, empirical durations and empirical measures tend to do better than analytical measures as the generic price rises. Nevertheless, for very high dollar-priced generics, the Emp(20,10) empirical measure, calculated using daily price and yield data, does much better than the risk model durations. This result highlights a limitation of risk model durations, with their monthly sampling of price and yield data, to serve as good empirical duration measures for daily percentage price changes of high dollar-priced generics.

Conclusion

The manager of an MBS portfolio, or a portfolio containing some MBS securities, needs confidence in MBS duration measures. For MBS, duration is relatively difficult to measure as the security's cash flows will change in response to changes in rates. Managers use a number of MBS duration measures. Some, such as OAD and KRDs, are model-based and rely on term structure and prepayment models. Empirical duration measures, on the other hand, rely on historical statistical relationships between mortgage prices and selected Treasury bond yields.

In order to examine the relative accuracy of various duration measures, one must choose the Treasury yield whose change will be multiplied by the duration measure to produce the predicted price return. We chose to evaluate eight duration-yield change pairs: Three model durations and five empirical ones. We examine the effectiveness of these pairs in predicting the daily price movements of 17 MBS generics and the MBS Index from June 2001 through February 2005. For the sake of comparison, we also applied these pairs to four agency indices, one Treasury, and one high-grade corporate bond.

We found that both model and empirical durations generally performed as well or better for MBS as they did for agency and corporate bonds though, not surprisingly, they performed best for the Treasury. Empirical durations generally underperformed model durations. However, as the MBS price increased, empiricals performed better than model durations. Empiricals also tended to perform better for GNMAs and 15-year FNMAs than for 30-year FNMAs. Surprising, KRDs only slightly outperformed OAD measures, even during periods of significant curve reshaping. Finally, we found that negative model durations for very high dollar-price MBS performed slightly better (although overall performance was poor) than durations for other high dollar-priced MBS. However, their relative performance was likely due to the fact that their absolute values were close to zero rather than because their durations were actually negative.

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