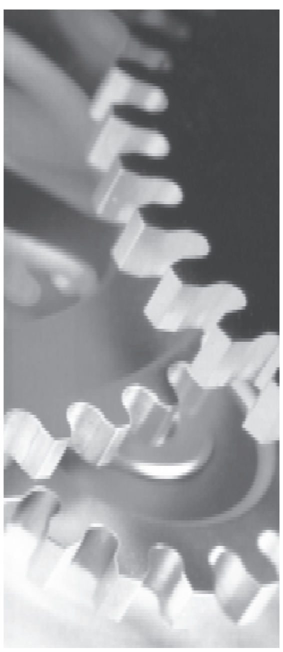
Fixed Income Research

Developments in Modeling Portfolio Risk: The Quest for Invariance

(Extracted from Global Relative Value, February 5, 2007)

Anthony Lazanas



LEHMAN BROTHERS

Global Capital Market Ideas

Developments in Modeling Portfolio Risk: The Quest for Invariance

Anthony Lazanas
212-526-3127
alazanas@lehman.com

We discuss the difficulty of predicting risk in a changing environment. Ideally, we would like to identify return models whose parameters remain stable over time. Since this is practically impossible, we introduce methodologies that address this issue. In particular, we discuss how two new additions to the Lehman Brothers Global Risk Model—the tail risk calculation model and the new credit model—attempt to deal with such issues.

PREDICTING RISK IS AN EXERCISE IN DISCOVERING INVARIANCE

All quantitative models that aspire to predict possible future states of the world must rely on past observations and on the assumption that some relationships remain unchanged over time. The success of a model depends critically on the stability of the relationship presumed invariant, as well as on the quality and quantity of available data.

To illustrate this principle, consider the task of estimating the volatility of returns on bonds of different issuers. Even the simplest model must recognize that the risk characteristics of bond returns are not the same for bonds of different maturities and different issuers. To address these issues, typically we would (a) transform returns into a function of a different variable—yield changes, and (b) group issuers of a similar nature together and perform a separate prediction for each group. Essentially, the yield change of the representative bond in each sector becomes a risk factor in this model. The return of any bond is expressed as a function of the risk factors (the yield change of each group). The hope is that the volatility of the yield change of bonds within each group is the same for all bonds and remains unchanged over time. Although it is understood that this is not quite true, this is the first big step toward invariance.

More complex models, such as the Lehman Brothers Global Risk Model available in POINT, use a more detailed transformation of bond returns by employing analytics such as key-rate durations, vega, and spread duration. The risk factors driving the return in this model—changes in risk-free yields, implied volatility changes, and option-adjusted spread moves—display much more stable properties, but are still not truly invariant.

Increasing model complexity in the quest of parameter invariance can go only so far. Data availability places a limit on the complexity of the models whose parameters can be estimated with a certain degree of confidence. An in-depth discussion of invariance properties in different markets, as well as tools to study invariance, can be found in [Meucci, 2005].

LOCAL INVARIANCE

Long-term parameter invariance is rarely achievable. One would have to look into physics to discover models with parameters that actually do not change over time (consider, for instance, the speed of light). In finance, we usually rely on the principle of "local invariance," which corresponds to parameters that remain stable over some window of time or parameters that change very slowly, such that recent history can be used to reasonably predict the (near) future.

"Regime switching" is one of the tricks sometimes employed to gain local invariance. Essentially, this method truncates the data available to predict the future into sets with a financial and economic environment similar to the current period, or "regime." Changes in Fed policy or exchange rate targets are favorite regime switching points. Even if there is agreement about the definition of the different regimes, this method usually suffers from the inability to predict the risk associated with switching to a new regime.

More frequently, though, the concept of local invariance allows for the usage of only "recent" history when estimating model parameters. What constitutes "recent" is open to interpretation, but it is primarily driven by the prediction horizon. The Lehman Brothers Global Risk Model, which has a monthly prediction horizon, offers two methods of parameter estimation. The "unweighted" method uses the entire sample of available data[1] to estimate the tracking error of a portfolio versus its benchmark.[2] The "weighted" method assigns less importance to calculations further in the past by assigning weights that decrease exponentially with the age of the observation. The decay rate is such that an observation that is one year old has half of the weight of the most recent observation. Under this method, what happened during the most recent year predominately determines the value of the estimated tracking error.[3] Recently, we have been studying ways to reduce the dependence of our tracking error estimation on the choice of the estimation window. Below, we describe our findings and explain how they are being incorporated in the framework of the Global Risk Model.

HANDLING INFREQUENT LARGE EVENTS

Practitioners prefer to use the "weighted" estimation in order to exploit local invariance and achieve better predictive ability. On the other hand, this method has difficulty handling infrequent "events" that cause big swings in model parameters. History shows that significant market-moving events occur every few years, and it is reasonable to expect this to continue to be the case. Such events seem to be separate from "regular" market risk. In the absence of a big event in recent history, the "weighted" method alone does not account for the risk of such an event's occurring. However, when such an event does occur, the "weighted" method overreacts and predicts a much higher volatility than is typically realized after big events.

Figure 1 presents the estimated volatility of the duration-adjusted monthly excess return[4] of the Lehman Brothers U.S. Credit Index. The unweighted estimation uses the full available sample and appears to react slowly to market events. After a steady decline through the 1990s it nudges up in 1998, but not sufficiently to accurately predict the increased volatility at the turn of the millennium. The weighted estimation uses a rolling window of 24 monthly observations; it is too sensitive to the big events of 1998 and 2001 and overpredicts the subsequent volatility.

To address this issue, we have introduced a new methodology in the POINT Global Risk Model that estimates normal volatility and big event risk separately. First, we look at the entire sample available to us (unweighted) to measure the frequency and magnitude of large events. Conditional on this information, we then estimate the "regular" volatility of our risk factors looking only at recent history (weighted sample). Even if a large event

[1] *Most time series exceed five years of monthly observations.*

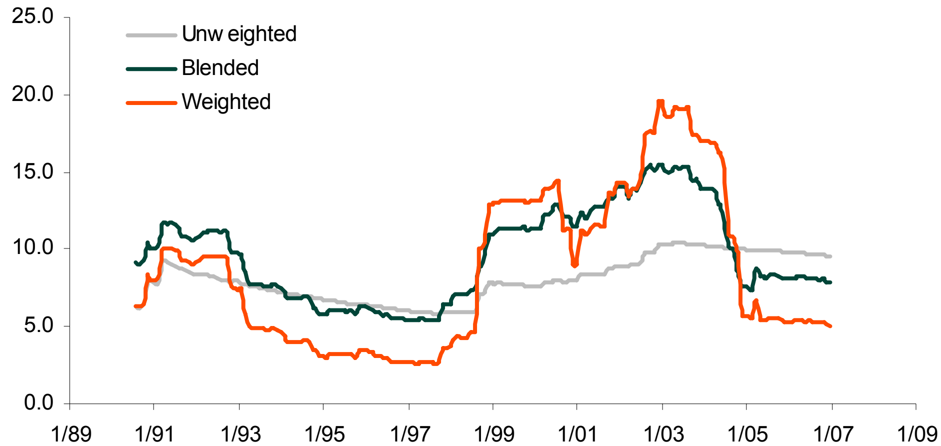
[2] *In technical terms, the tracking error is the square root of the second central moment of the distribution of the net return of the portfolio over the benchmark.*

[3] *Keep in mind that since the weights never become zero, an observation with sufficiently large magnitude can still significantly affect the estimated tracking error even if it is two or three years old.*

[4] *We define duration-adjusted excess return as the monthly excess return divided by the duration of the index at the beginning of the month. We use duration as a proxy for spread duration because the latter is not available for the entire sample period. Duration-adjusted return as defined here is a proxy for the negative of the index spread change and is stated in basis points.*

has occurred recently, our estimate of regular volatility will not be significantly affected as long as similar large events have occurred in the past. Details of the implementation of this methodology can be found in [Purzitsky, 2006]. The third line in Figure 1 represents a "blended" estimation. Big events do affect future volatility prediction, but the size of the effect is mitigated. On the other hand, in low-volatility periods, the predicted volatility is higher than the weighted estimate since big event risk is always present even in the absence of recent big events. This method seems to do a better job in predicting future volatility[5].

Figure 1. Weighted versus Unweighted Volatility Estimation: U.S. Credit Index Monthly Duration-Adjusted Excess Return Volatility, bp



Source: LehmanLive

This estimation method is not available as part of the standard Lehman Brothers Global Risk Model, but a version of it is used to produce the newly added tail-risk part of the risk model report. Despite the restrictive designation (tail risk), this module constitutes a full-fledged simulation-based risk model that produces tracking error, value-at-risk, and expected shortfall at any user-defined confidence level up to 99%.

IS TRACKING ERROR SUFFICIENT TO DESCRIBE RISK?

Asset managers have traditionally sought to describe portfolio risk versus a benchmark with a single number—tracking error—that is the volatility of the excess return of the portfolio over the index. Until recently, tracking error and its breakdown were the only outputs of the Lehman Brothers Global Risk Model available in POINT. Tracking error describes what happens in "normal" states of the world and, in general, should not be used to infer the probability of extreme losses. Only when the distribution of excess returns is the normal distribution is it possible to infer the probability of large losses or gains from the tracking error.

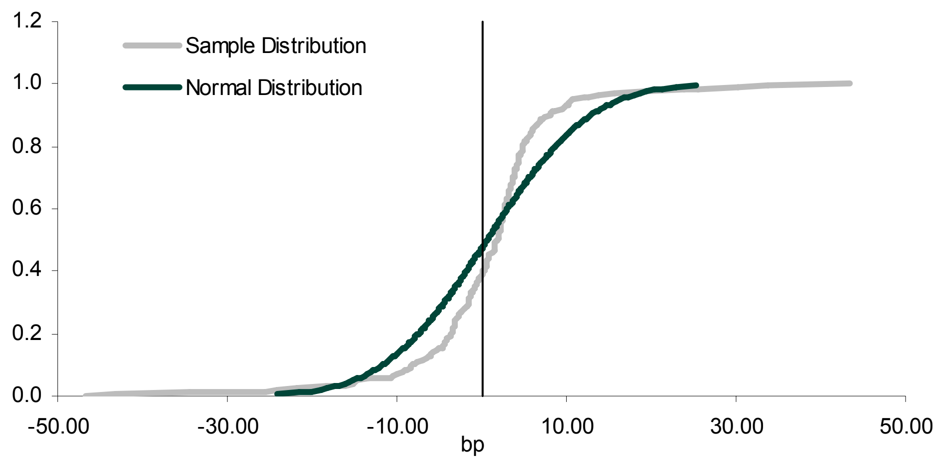
We tested the hypothesis of normal distribution of duration-adjusted excess returns for many portfolios and benchmarks and concluded that most of the time it does not hold. Most credit portfolios exhibit returns with significantly fat tails, such that extreme losses or gains are much more likely than a normal distribution would predict. Figure 2 depicts

[5] In Figure 1, we have assumed that big event risk is known at the beginning of the sample period. In practice big event risk is re-estimated continuously using information from the entire sample available at the time. Consequently, in practice, the blended estimate of volatility will react significantly to the first few big event realizations if their rate and size differ significantly from our prior assumptions about big event risk.

the cumulative distribution of the duration-adjusted monthly excess return of the U.S. Credit Index and compares it with the normal distribution. Clearly, the actual sample distribution has far more large-magnitude realizations than a normal distribution with the same sample mean and variance would have predicted. Another potential departure from normality occurs in portfolios with significant optionality, which induces asymmetric returns: the probability of large losses is very different from the probability of large gains.

One must note that the lack of distribution invariance—the fact that the distribution of returns changes over time—makes it unfair to compare the actual sample with a normal distribution that always remains the same. To account for this, we performed several statistical tests that investigate the possibility of a normal distribution whose volatility changes over time. Even in this case, the normal distribution hypothesis is usually rejected.

Figure 2. Fat Tails in the U.S. Credit Index Monthly Duration-Adjusted Excess Returns: Empirical Distribution of the Duration-Adjusted Excess Return of the U.S. Credit Index versus the Normal Distribution



Source: LehmanLive

The view of the entire probability distribution of portfolio returns, as provided by the tail-risk addition to the Global Risk Model, is much more powerful that just the tracking error. The model can use this rich set of information to produce any specific measure of risk, including tracking error, but also value at risk (VaR) and expected shortfall (or conditional VaR) at any confidence level. The model is also capable of breaking down total risk measures into the contributions of exposures to specific risk factors. A short description of the model can be found in [Meucci, 2007]. Details of the model can be found in an upcoming publication.

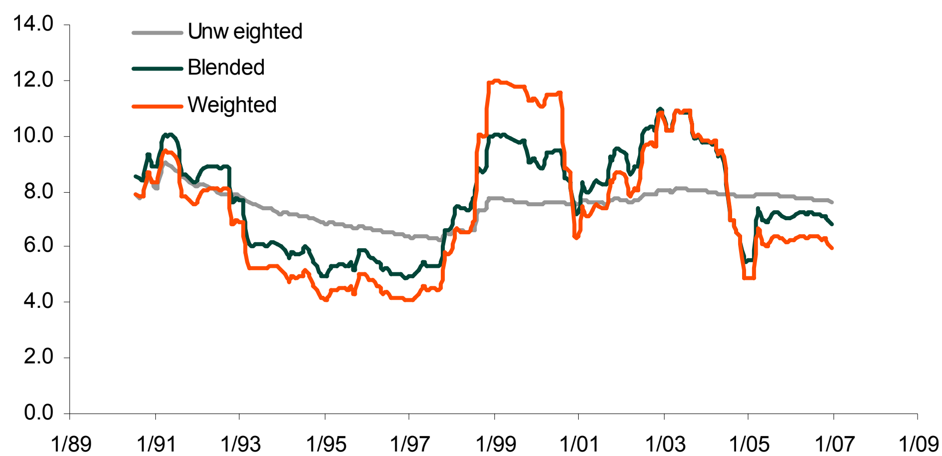
SPREAD LEVEL AS A PREDICTOR OF SPREAD CHANGE VOLATILITY

Even after accounting for the effect of infrequent large events, it is clear that credit excess return volatility fluctuates significantly over time. In our quest for invariance, we investigated whether contemporaneous observable variables could help predict the future volatility of credit returns. It turns out that this is indeed possible.

Many studies have established a relationship between the volatility of credit spread changes and the level of credit spreads [Ben Dor et al., 2005, 2006]. This relationship is almost linear when spreads are high, and it flattens for low spreads. One theory consistent with this observation is that the volatility of credit spreads is the result of two risk factors, one that has to do with pure credit concerns and another that has to do with liquidity. It is reasonable to assume that the risk associated with pure credit would increase monotonically with the spread level of an issue. On the other hand, it is not obvious that the liquidity risk factor should be linked to the spread level. The combination of the two factors results in a relationship between spread risk and spread level that is flat when spreads are low and increases linearly with spreads for high-spread securities.

Clearly, for high-spread securities, the volatility of proportional spread changes (ΔSpread/Spread) should be much more stable over time than the volatility of absolute spread changes (ΔSpread). By switching from an absolute spread change risk factor to a proportional change risk factor, we are gaining invariance. To highlight this, we adjust excess returns not by the index duration, but by the product of the index duration times its spread (DTS_{SM}). The time series of the DTS-adjusted excess returns volatility of the U.S. Credit Index using unweighted, weighted, and blended estimation methods is shown in Figure 3. It is important to note that although volatility does vary over time, its fluctuations are significantly lower than when using just duration to adjust excess returns. Indeed, Figure 2 shows that the ratio of the maximum-to-minimum 2-year rolling monthly volatility of duration-adjusted excess returns is about 7.5. The same ratio for DTS-adjusted returns is 3. This is a remarkable improvement in volatility stability.

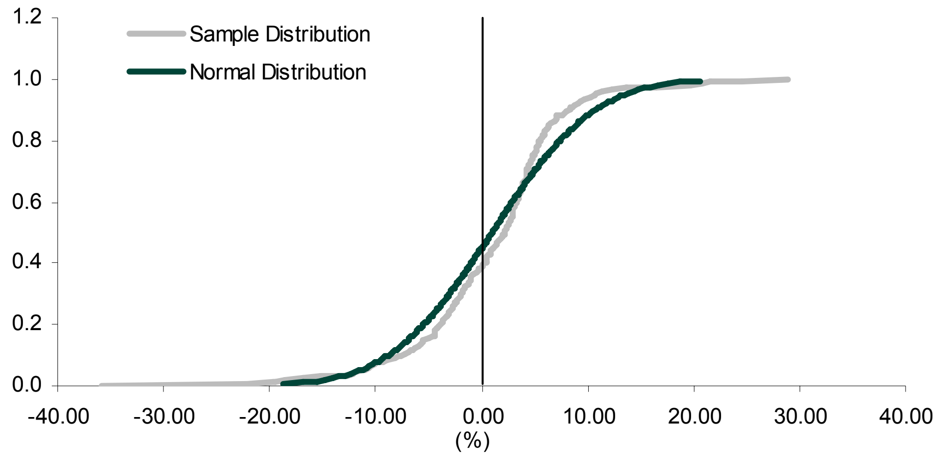
Figure 3. U.S. Credit Index Monthly DTS-Adjusted Excess Return Volatility, %



Source: LehmanLive

Figure 4 shows the empirical distribution of DTS-adjusted excess returns versus the normal distribution. Note how much closer to normal it is compared with the duration-adjusted returns distribution shown in Figure 2. Still, several extreme realizations are not consistent with the normality hypothesis. Therefore, a blended estimation that accounts for the occurrence of large events separately still has value. In Figure 3, we can see that the blended estimation did account for the risk of large events during quiet periods and did not overreact to the events of 1998, successfully predicting the high volatility levels of 2000-2003.

Figure 4. Fat Tails in the U.S. Credit Index Monthly DTS-Adjusted Excess Returns: Empirical Distribution of the DTS-Adjusted Excess Return of the U.S. Credit Index versus the Normal Distribution



Source: LehmanLive

Our newly released version of the Global Risk Model uses both absolute spread change factors (the primary drivers of risk for very low-spread securities) and proportional spread change factors (the primary drivers of risk for high-spread securities), as described in more detail in [Rosten & Silva, 2007]. In the previous model, the weighted estimation of credit spread change volatility was capturing, to a large extent, the spread level effect. Since spreads can move a lot from month to month, even the weighted model was slow to react to spread level changes. The current model immediately changes its volatility estimate as spread levels change, resulting in improved predictive ability.

TERM STRUCTURE OF RISK

As the above discussion indicates, asset return risk is highly variable over time. Intuitively, it must also have mean-reverting properties, since risk cannot become arbitrarily large. As a result, a prediction for short-term risk (e.g., monthly, as in the case of the Global Risk Model) cannot be used for long-term portfolio optimization using simple time scaling. Consider the finding in the previous section—that credit return volatility is proportional to the level of credit spreads. Because credit spreads are currently at historical lows, our one-month risk estimate for credit portfolios will also be low. It is likely that, over time, credit spreads will fluctuate around their historical means, and so will credit return volatility. Therefore, for long-term analysis, one must take into consideration that monthly volatility will fluctuate at a level higher than the current estimation. For an excellent discussion on the subject, see [Campbell & Viceira, 2002].

To account for the evolution of risk over time, we need to describe risk factors with evolution models rather than static probability distributions. A particular class of such models are GARCH models, which attempt to estimate the properties of time-varying volatility. In fact, the weighted estimation of volatility is equivalent to modeling factors with a particular, simple GARCH process.

In our current "weighted" estimation method, we have somewhat arbitrarily determined that the observation weights should be halved every year. We use the same decay rate for all estimated factors. However, the invariance properties of each factor may be different, and the same weighting scheme may not be optimal for all factors. Using GARCH models that allow for time-varying volatility of factors, we have discovered that each factor has its own volatility persistence properties. This implies that we should be using different decay intensities for each factor when using the "weighted" estimation method. This methodology has been developed but is not yet available in the POINT Global Risk Model, as we are investigating more complex evolution models.

CONCLUSION

To predict risk successfully based on historical data, we must rely on the assumption that the future will be "like" the past. We thus need to identify the market invariants—variables that can be described by models with stable parameters. Since this is a practical impossibility, we use various techniques to help us maximize the predictive ability of our models.

REFERENCES

Ben Dor, Dynkin, Houweling, Hyman, van Leeuwen, and Penninga (2005), "A New Measure of Spread Exposure in Credit Portfolios," *Lehman Brothers Fixed Income Quantitative Portfolio Strategies*, June 2005.

Ben Dor, Polbennikov & Rosten (2006), "DTS_{SM} (Duration Times Spread) for CDS—A New Measure of Spread Sensitivity," *Global Relative Value Weekly*, October 13, 2006.

Campbell & Viceira (2004), "Long-Horizon Mean Variance Analysis: A User Guide (Manuscript, Harvard University).

Meucci A. (2005), Risk and Asset Allocation (Springer).

Meucci A. (2007), "Tail Risk in POINT," *Global Relative Value Weekly*, January 22, 2007.

Purzitski A. (2006), "Tail behavior of systematic risk factors Lehman Brothers Global Risk Model," *Fixed Income Liquid Markets Research Quarterly*, *2006-Q1/Q2*, May 2006.

Rosten & Silva (2007), "A New Approach to Credit in the Lehman Brothers Global Risk Model," *Global Relative Value Weekly*, January 16, 2007.

Analyst Certification

The views expressed in this report accurately reflect the personal views of Anthony Lazanas, the primary analyst responsible for this report, about the subject securities or issuers referred to herein, and no part of such analyst's compensation was, is or will be directly or indirectly related to the specific recommendations or views expressed herein.

Important Disclosures

Lehman Brothers Inc. and/or an affiliate thereof (the "firm") regularly trades, generally deals as principal and generally provides liquidity (as market maker or otherwise) in the debt securities that are the subject of this research report (and related derivatives thereof). The firm's proprietary trading accounts may have either a long and / or short position in such securities and / or derivative instruments, which may pose a conflict with the interests of investing customers.

Where permitted and subject to appropriate information barrier restrictions, the firm's fixed income research analysts regularly interact with its trading desk personnel to determine current prices of fixed income securities.

The firm's fixed income research analyst(s) receive compensation based on various factors including, but not limited to, the quality of their work, the overall performance of the firm (including the profitability of the investment banking department), the profitability and revenues of the Fixed Income Division and the outstanding principal amount and trading value of, the profitability of, and the potential interest of the firms investing clients in research with respect to, the asset class covered by the analyst.

Lehman Brothers generally does and seeks to do investment banking and other business with the companies discussed in its research reports. As a result, investors should be aware that the firm may have a conflict of interest.

To the extent that any historical pricing information was obtained from Lehman Brothers trading desks, the firm makes no representation that it is accurate or complete. All levels, prices and spreads are historical and do not represent current market levels, prices or spreads, some or all of which may have changed since the publication of this document.

Lehman Brothers' global policy for managing conflicts of interest in connection with investment research is available at www.lehman.com/researchconflictspolicy.

To obtain copies of fixed income research reports published by Lehman Brothers please contact Valerie Monchi (vmonchi@lehman.com; 212-526-3173) or clients may go to https://live.lehman.com/.

Legal Disclaimer