BARCLAYS CAPITAL PORTFOLIO OPTIMIZER

Applications of Sharpe ratio optimization in portfolio construction

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Sharpe and Information ratios are commonly used risk adjusted performance measures for total return portfolios and benchmarked portfolios, respectively. A Sharpe ratio optimal portfolio maximizes the expected excess (of the risk free rate) return per unit of total risk taken by the portfolio. An Information ratio optimal portfolio maximizes the expected active (i.e., net of benchmark) return per-unit of active risk taken by the portfolio. These ratios endogenize the risk aversion parameter, thus facilitating an automated trade-off between the risk and returns by choosing an optimal point on the mean-variance efficient frontier.

Barclays Capital Portfolio Optimizer supports both Sharpe and Information ratio optimization. The optimizer supports generic portfolio constraints (including combinatorial constraints such as maximum number of securities in the portfolio) in Sharpe and information ratio maximization problems. This functionality further enhances the flexibility of the portfolio optimization problems that can be solved using the Barclays Capital Portfolios Optimizer in POINT[1]. Clients can use their custom excess return forecasts or performance analytics available in POINT (e.g., yields) as a proxy to define the expected excess returns for optimization purposes. We demonstrate the use of this functionality using practical examples in the construction of long-short US equities portfolios, and the construction of risk-consistent carry-enhanced fixed income portfolios. The results from Sharpe ratio optimization may be sensitive to the inconsistencies between the expected return and the risk forecasts. The examples discuss how in practice flexible constraints can be used in conjunction with Sharpe ratio maximization to overcome some of these issues.

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Introduction

Sharpe ratio is a common investment performance measure used to evaluate (and forecast) the risk-adjusted performance of an investment strategy. It makes implicit assumptions originating in the mean-variance portfolio selection theory (Markovitz, 1952). It assumes that either the security returns are normally distributed or investors only consider the mean-variance statistics of the investment strategy. The mean and standard deviations of the strategy returns distribution are sufficient statistics to capture the strategies' returns and risk, respectively, under these assumptions. Since risk-averse investors prefer higher returns and lower risk, the Sharpe ratio is defined as the excess (of risk free rate) returns generated by the portfolio per unit of the portfolio volatility (Sharpe, 1966).

$$SR = \frac{E(R-R_f)}{\sqrt{Var(R-R_f)}} = \frac{E(R-R_f)}{\sigma(R)} \qquad (1)$$

R is the portfolio return, R_f is the risk free rate of return, $E(R-R_f)$ is the expected value of the excess of the portfolio return over the risk free rate, and σ is the standard deviation of the portfolio's excess returns. All metrics are defined for a standard investment horizon.

The second equality above assumes that the risk free rate is constant. In practice, the risk free rate changes with time, and the excess returns should be defined with respect to this time varying risk free rate (Sharpe, 1994). In particular, the risk free rate is time varying, but not stochastic as the rate of return is known at the time of investment. Sharpe ratio is the returns per unit of risk generated by the zero investment strategy, which borrow all investment outlays at the risk free rate and invest in the given strategy. The zero investment requirement ensures that the levering (up or down) a strategy (e.g., to satisfy the risk budget) by borrowing at the risk free rate does not change the Sharpe ratio. This is important to facilitate comparison of alternative strategies using this ratio (Sharpe, 1994).

In the above definition, if we replace the risk free returns with the returns of the given (risky) benchmark portfolio, then the resulting ratio is called the Information ratio (Goodwin, 1998).

$$IR = \frac{E(R-R_B)}{\sigma(R-R_B)} = \frac{E(R-R_B)}{\sqrt{Var(R-R_B)}} \qquad (2)$$

As is evident from the definition, Sharpe ratio and Information ratios are used for total return portfolios and benchmarked portfolios, respectively. Similar to any other performance measures, such as expected returns and volatility, these ratios can be defined as both (ex-ante) predicted and (ex-post) realized. For example, if the annualized excess (of risk free rate) expected return of the portfolio is estimated at 5% and the annualized predicted risk of the portfolio, defined as standard deviation of the portfolio's excess return, is 10%, then the predicted Sharpe ratio will be 0.5.

Similarly, we can compute the realized Sharpe ratios. For example, the annualized average monthly return of the S&P 500 total return index from January 1982 to December 2009 is 9.13%. The annualized average standard deviation of the monthly returns over the same time period is approximately 15.5%. Thus, assuming the risk-free return over this period is 3%, the average, long-term Sharpe ratio of the US equity market as represented by the S&P 500 total return index is approximately 0.40. Figure 1 shows the realized Sharpe ratios of the four benchmark stock and bond markets over the past 20 years. It uses (the time series of)

1m Libor deposit rate as the risk free rate. The following two observations warrant a discussion regarding these realized ratios.

- The realized Sharpe ratios vary significantly across various asset classes. The MBS index has the highest Sharpe ratios overall as well as in both the first and second half of this 20-year time period, followed by the US credit index. This is primarily because of lower volatility (due to shorter duration of the MBS index) compared to the rest of the bond indices (~3.5 years for the MBS index vs. 6 years for the US credit, 5 years for the US Treasury, and 6 years for the global treasury index). Of course, Sharpe ratio is not the only consideration investors should make when constructing their portfolios. From an asset allocation point of view, asset classes with diverse Sharpe ratios play an important diversification role in cross asset portfolios. In particular, Sharpe ratio is a marginal performance measure, and does not reflect the diversification benefits of the strategy to an existing portfolio. Furthermore, the non-normality of the return's distribution makes the Sharpe ratio an incomplete measure of the return/risk trade-off. Finally, leverage constraints on investors imply that they can not simply lever up (or down) the asset class with highest Sharpe ratio to achieve their risk-return targets.
- Realized Sharpe ratios can change significantly, even when measured over relatively long time periods (Figure 1). Even though long-term volatility tends to be relatively stable over such periods, the average realized returns can vary drastically due to bubble-bust cycles in financial markets. For example, the S&P 500 index realized Sharpe ratio is negative 0.25 during January 2001-January 2010 due to the large negative returns realized in the dot-com bust of 2001-2002 and the credit crisis of 2008.

Figure 1: Realized Sharpe ratios in benchmark stock and bond markets (January 1990 – January 2010)

	Jan 1990 - Jan 2010			Jan 1990 - Dec 1999			Jan 2001 - Jan 2010		
	Mean	Standard Deviation	Sharpe Ratio	Mean	Standard Deviation	Sharpe Ratio	Mean	Standard Deviation	Sharpe Ratio
U.S. Treasury	2.3%	4.6%	0.50	2.5%	4.1%	0.60	2.2%	5.2%	0.42
U.S. Credit	2.9%	5.4%	0.55	2.8%	4.6%	0.61	3.0%	6.1%	0.49
U.S. Mortgage Backed Securities	2.6%	3.0%	0.88	2.6%	3.1%	0.83	2.7%	2.9%	0.94
S&P 500 (TR)	1.5%	15.2%	0.10	7.4%	13.9%	0.53	-4.1%	16.4%	-0.25
Global Treasury	2.8%	6.7%	0.42	1.9%	5.9%	0.32	3.7%	7.6%	0.49

Note: The bond market returns are based on Barclays Capital benchmark bond indices. US equity market returns are based on Standard and Poor's S&P 500 Total return index. 1m Libor deposit rate is used as the risk free rate. The monthly return time series is generated using the time series plotter (TSP) tool available on Barclays Capital's online research delivery platform BarcapLive. Both monthly realized returns and risk are annualized using standard independent distribution assumption (i.e., times 12 for expected returns and times square root 12 for the standard deviation). Source: Barclays Capital

The Sharpe ratio was originally proposed as a metric for measuring mutual funds' performance (Sharpe, 1966). It does not distinguish between the systematic and idiosyncratic risk of the portfolio. Based on the Capital Asset Pricing model (CAPM), several performance measures were introduced, which only penalize the systematic risk based on the CAPM assertion that only systematic risk earns a risk premium. For example, Treynor ratio is defined as the ratio of portfolio's excess returns to portfolio's beta ($(R - R_f)/\beta_P$), and Jensen's alpha is defined as the return in excess of the Security Market Line (SML), $R_P - R_F - \beta \cdot (R_P - R_F)$. Generalization based on other risk metrics (e.g., downside risk, tails risk) are also proposed in literature (Farinelli et. al., 2008).

Sharpe/Information ratio maximization

In addition to measuring the realized risk adjusted performance (ex-post realized Sharpe ratios), we can construct portfolios that optimize the predicted Sharpe ratios using ex-ante risk and return forecasts. A particular advantage of this approach over the standard risk-adjusted return maximization approach is that it endogenizes the choice of the risk tolerance level based on the risk free rate and the risk premia available in the market.

The Sharpe ratio maximization problem is to choose a feasible portfolio x (i.e., a vector of position amounts in each security in the investable universe) that maximizes the predicted Sharpe ratio based on variance-covariance and expected return forecasts. A portfolio is feasible if it satisfies all the regulatory (e.g., long-only) and strategy constraints (e.g., sector neutral) required by the portfolio manager. The set of all feasible portfolios is said to be the investment set, represented by IS .

$$\max_{x \in IS} \frac{R - R_B}{\sigma(R - R_B)} = \max_{x \in IS} \frac{\sum_{j=1}^{n} \mu_j (x_j - x_j^B)}{\sqrt{\sum_{i,j=1}^{n} (x_i - x_i^B) \Sigma_{ij} (x_j - x_j^B)}} \tag{3}$$

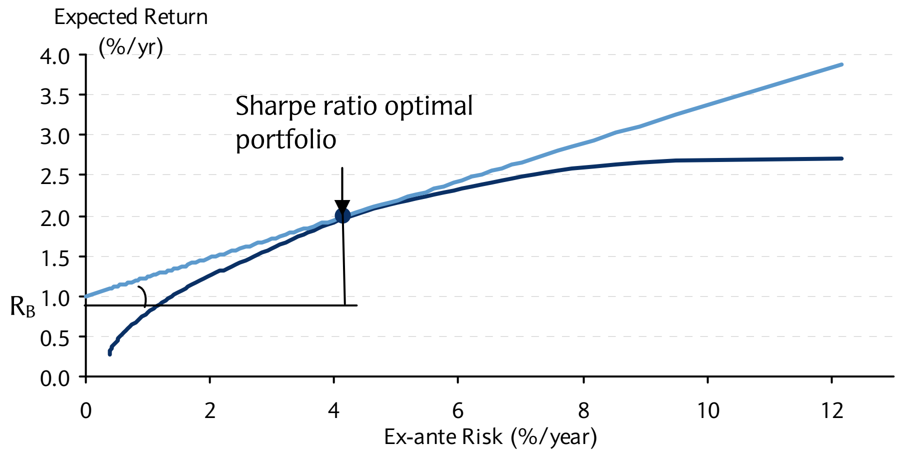
Where μ_j represents the expected returns of security j and Σ_{ij} represents the covariance between security i and j . x_j^B represents the weight of security j in the benchmark portfolios. This definition encompasses both Sharpe ratio and Information ratio by setting the benchmark portfolio to the risk free asset and generic benchmark, respectively. The covariance matrix is typically computed by combining the systematic covariance and idiosyncratic covariance in a linear factor model framework.

$$\Sigma = LSL' + I$$

L, S and I represent the matrix of loadings to the given set of factors, the factor covariance matrix, and the idiosyncratic covariance matrix, respectively.

Sharpe ratio and Information ratio optimal portfolios have similar properties. To simplify the discussion, we restrict our attention to Sharpe ratio to intuitively explain these properties. Figure 2 depicts the tradeoffs in a typical mean-variance portfolio selection problem. The x-axis represents estimated risk and the y-axis represents the estimated expected return of the feasible portfolios. Any feasible portfolio represents a point in this risk-return diagram. Given a portfolio $(\sigma(P), \mu(P))$, the Sharpe ratio of the portfolio is equal to the slope of the line connecting $(0, R_F)$ and $(\sigma(P), \mu(P))$ in the risk-return diagram. The curved line represents the set of feasible mean-variance efficient portfolios excluding the risk free security.

Figure 2: Sharpe ratio optimal portfolio as the tangency portfolio on the mean-variance efficient frontier



Source: Barclays Capital

A feasible portfolio is said to be mean variance efficient if it has the highest expected return among a set of portfolios having the same level of risk. In particular, there is no feasible portfolio (excluding the riskless asset) above the curved line. To choose a portfolio with the highest Sharpe ratio, we want to maximize the slope of the straight line. The following two results should be evident based on this analysis.

- The Sharpe ratio optimal portfolio is mean-variance efficient. This is because if the Sharpe ratio optimal portfolio is not efficient, then we can find a portfolio on the efficient frontier with strict higher returns and the same risk, hence a strictly larger Sharpe ratio, invalidating the optimality. Thus, there exists a risk aversion parameter that depends on the problem parameters such that any feasible portfolio maximizing the risk adjusted expected excess returns also maximizes the Sharpe ratio. This is also true of the Information ratio optimal portfolio in the active risk-active return space.

- The line connecting the portfolio to the risk free asset on the y-axis in the risk-return diagram is tangent to the mean-variance frontier. In other words, the Sharpe ratio optimal portfolio is the "tangency" portfolio (aka, market portfolio). Note that the efficient frontier is constructed with risky securities only. If we include the risk-free security, the mean-variance frontier would become a straight line connecting the tangency portfolio and the risk free rate.

There are two approaches for constructing the Sharpe ratio optimal portfolio.

Constructing the full efficient frontier – the brute force approach

Since the Sharpe ratio optimal portfolio lies on the efficient frontier, we can sequentially search along the efficient frontier. We can construct a large number of efficient portfolios representing the complete efficient frontier, and choose the one with the highest Sharpe ratio. The discretized version of the complete efficient frontier can be computed as follows. Compute the minimum risk portfolio and maximum return portfolio. Choose a set (say K) of equidistant expected returns on the expected return interval between the expected return of the minimum risk portfolio and the maximum expected return portfolio. Construct a portfolio by minimizing the risk subject to achieving an expected return equal to the set of equidistant points. Since we need to choose a large enough K for better accuracy, this procedure can be computationally intensive.

Homogenization

The homogenization is a direct approach that exploits the homogeneity[2] of the Sharpe ratio function. It translates the non-linear optimization problem into an equivalent mean-variance quadratic programming problem. The solution is obtained in a single step by translating the solution of the equivalent problem to the solution of the original problem. Appendix 1 gives the details of the transformation. This is the approach implemented in the portfolio optimizer in POINT.

Guidelines for constructing Sharpe ratio optimal portfolio in POINT

We assume that the readers are familiar with the Barclays Capital Portfolio Optimizer available in POINT system based on the optimizer white paper (Kumar, 2009), user guide (Kumar and Lazanas, 2009) and other optimizer documentation.

Sharpe ratio is available in the optimizer as objective function terms similar to any other objective function term (Figure 3). Choosing “Net vs. Bmark” in the measure column with Sharpe ratio attribute creates the Information ratio. The Sharpe ratio maximization technology is fully integrated with the rest of the functionality available in the portfolio optimizer. In particular, users can construct portfolios maximizing Sharpe ratio while enforcing the combinatorial constraints, bucket constraints, and the issue/issuer/ticker constraints with the following restrictions[3].

- Only maximization of the Sharpe/Information ratio is permitted. Portfolios with minimum Sharpe/Information ratio are of little practical interest, and are difficult to construct due to the non-convex nature of the underlying optimization problem.
- Risk constraints (the constraints on Total TEV, Systematic TEV, and Idiosyncratic TEV) or risk objective function terms are not allowed with Sharpe ratio optimization.
- If the users combine the Sharpe ratio maximization with any other objective function term, then such terms are simply added to the numerator of the Sharpe ratio expression to define the overall objective function[4]. For example, having a transaction cost term in the objective function leads to the following objective function:

$$\frac{w_1 \sum_{j=1}^{n} \mu_j (x_j - x_j^B) - w_2 TC(x - x_0)}{\sqrt{\sum_{i,j=1}^{n} (x_i - x_i^B) \Sigma_{ij} (x_j - x_j^B)}} \tag{4}$$

  where w_1 and w_2 are the user provided weights of the Information ratio and Transaction Costs terms, respectively. Note that the system would only allow a negative weight for the transaction cost term because of the convexity requirement. This normalization by risk is helpful in allowing users to combine the objective function terms with the Sharpe ratio term in a unit-consistent fashion.
- The soft constraints (Kumar, 2010) create shadow objective function terms to model the portfolio penalty for constraints violation. Hence, using soft constraints in a Sharpe

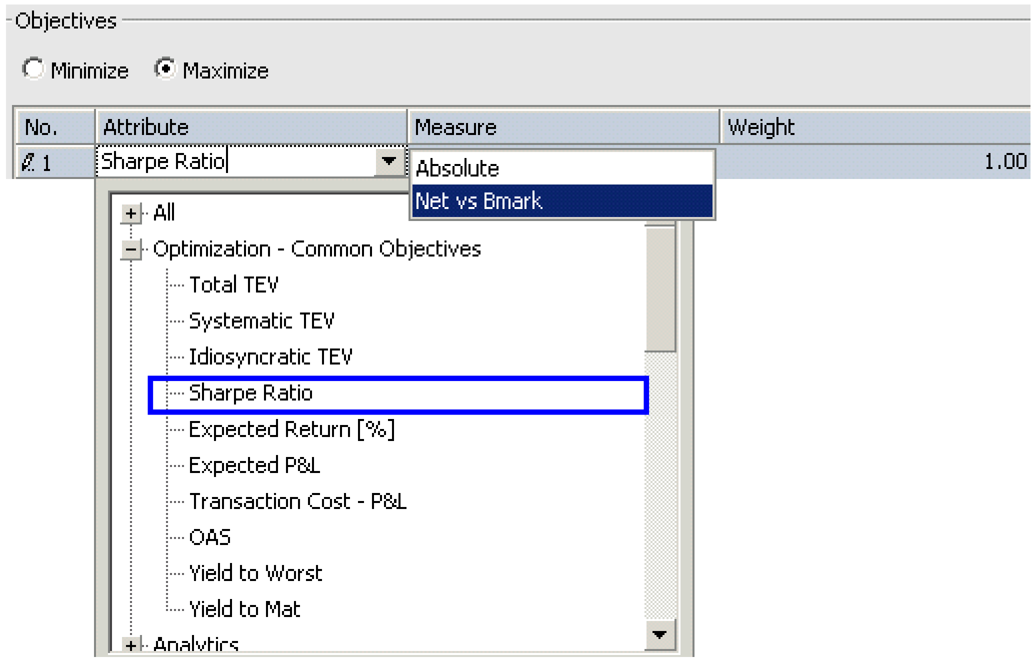
[2] A function f(x) is called homogeneous of degree 0 if $f(tx) = f(x)$ for any scalar t.

[3] The system invalidates optimization requests which do not satisfy these restrictions with a user message containing the notification of the validation requirement.

[4] The alternative behavior where we take the weight average of unit-less Sharpe ratio and the transaction costs to define the overall objective function leads to non-convexity. In particular, the homogenization approach discussed previously can not be used to solve such problems.

ratio optimization problem is similar to using additional objective function terms. Regarding hard constraints, there is no restriction or change. In particular, any number of hard constraints (including issue/issuer/ticker constraint) can be used in a Sharpe ratio maximization problem.

Figure 3: Accessing Sharpe ratio term in the objective function



Source: Barclays Capital POINT

Imported expected returns

Users should ensure that the per-security expected returns are populated for each tradable security. POINT has extensive support for user-provided custom security-specific numeric (expected returns, transaction costs, etc.) and text (indicatives, internal identifiers, etc.) data. Users can either simply paste the custom data using the *paste wizard*, or import it from the CSV file. For details on this feature, see the optimizer user guide (Kumar and Lazanas, 2009) pp 34-38, and the POINT's documentation on *User Defined Data Fields*.

The optimizer uses the POINT field called "Expected Return [%]" as the security specific excess expected return to define the numerator term[5] of the Sharpe ratio. The "Expected Return [%]" is a mapped field. It defaults to the user defined data field named "User_Expected_Return". If the "User_Expected_Return" is missing, then it defaults to the Yield to Worst for fixed income securities. For equities, the default "Expected Return [%]" value is blank. It is expected that users would import their expected return estimates in the "User_Expected_Return" field before running a Sharpe ratio optimization problem for equities.

In POINT, we provide a set of examples of Sharpe/Information ratio optimization problems for illustration purposes. Figure 4 lists a number of these reports available through the *system* portfolio named "***Optimizer demo: Sharpe ratio.***" The system portfolios are

[5] The default risk free rate is assumed to be zero. Since typical assumptions of risk free rate are not really risk free (e.g., money market deposit rate has credit risk of financial institutions and short-term Treasury rates have sovereign risk), and it can change from investor to investor based on their funding costs, we outsource this assumption to POINT clients by letting them directly import the excess expected returns.

accessible to any POINT user by changing the portfolio author to "system." Even though all these sample reports demonstrate portfolio construction examples, this functionality can be used for portfolio rebalancing problems similarly.

Figure 4: The sample Sharpe/Information ratio optimization reports in POINT

Name	Asset Class	Universe	Description
SR1-EQ	Equities	S&P 500	Long only. Maximize Sharpe ratio.
SR2-EQ	Equities	S&P 500	Long only. Maximize Sharpe ratio. Match GICS L1 sector weights to S&P 500.
SR3-EQ	Equities	S&P 500	Long only. Maximize Sharpe ratio. Match GICS L1 sector weights to S&P 500. Limit ticker exposure to 2%.
SR4-EQ	Equities	S&P 500	100 positions Long-Short. Maximize Sharpe ratio. Limit leverage to 130-30. Limit GICS L1 sector exposure to 20%, and individual ticker exposure to 3% on the long and 2% on the shorts.
IR1-EQ	Equities	S&P 500	Long-only. Maximize Information ratio.
IR2-EQ	Equities	S&P 500	Long-only. Maximize Information ratio. Match GICS L1 sector weights to S&P 500.
IR3-EQ	Equities	S&P 500	Long-only. Maximize Information ratio. Match GICS L1 sector weights to S&P 500. Limit individual ticker exposure to 2%.
SR-FI	Fixed Income	US High Yield	Long-only. Maximize Sharpe ratio. Limit Class 2 credit sector exposure to 20%. Limit individual issuer to 2%.
IR-FI	Fixed Income	US Credit	100 positions Long only. Maximize Information ratio. Match Class 2 credit sector exposure to the benchmark. Limit individual issuer's active exposure to +/- 3%.

Source: Barclays Capital

Applications in portfolio construction: Practical examples

Carry maximization in fixed income portfolios

Earning carry to enhance performance is a common theme in many fixed income portfolio management strategies. Typical asset allocation carry enhancement strategies include moving up the yield curve (duration overweight) and down the credit quality (allocating to the high yield securities). Carry enhancement can also be achieved in the security selection step by picking relatively higher yielding securities while controlling for the risk exposure of the portfolio at the target level. We demonstrate performing such an exercise using the Sharpe ratio maximization functionality in the portfolio optimizer.

Suppose we want to construct a yield enhanced 100mn USD corporate credit portfolio at the month-end of February 2010. The tradable universe is restricted to the investment grade universe of corporate bonds represented by the Barclays Capital US Credit Corp index. Additionally, the portfolio is required to satisfy the following constraints:

1. We constrain the portfolio's sector allocations to match the index in banking (24.5%), finance (11%), basic Industries (10%), consumer cyclical (5%), consumer non-cyclical (12%), energy and transport (9.5%), technology and communications (16%), and utility (11%). The numbers in parenthesis represent the sector weights in the benchmark index.

2. We control the concentration risk of the portfolio by limiting the dollar exposure to any single issuer by 3%.

3. We constrain the spread duration (OASD) of the portfolio to match the index to avoid duration overweight or underweight.

4. To control the liquidity risk, we constrain the portfolio to achieve an average Liquidity Cost Score[6] at least equal to the index.

5. Each position should be at least 100,000 USD.

Since the objective of the portfolio is to earn enhanced carry, we use the yield-to-worst of the bonds in the universe to define the numerator term of the Sharpe ratio. The denominator is defined using the Global Risk Model (GRM) in POINT. We construct two versions. Version 1 (V1) maximizes the Sharpe ratio[7], and Version 2 (V2) maximizes the Information ratio with respect to the index. Both versions satisfy all the constraints mentioned previously. Figure 5 shows the analytical profile of the two optimal portfolios. We see that average credit quality profile of both portfolios is below the index as indicated by the average index ratings (last column). This quality difference is the primary source of active risk and the enhanced carry in both these portfolios.

Figure 5: Carry enhanced portfolios: Average analytics

	Number of positions	Yield to Worst (%)	OASD (yrs)	OAS (bp)	Coupon (%, Par weighted)	Average Index Rating
V1: Total Return	38	8.07	6.23	324	6.39	A3/BAA1
V2: Active	41	7.88	6.23	306	6.29	A3/BAA1
Index	3389	4.47	6.23	173	6.03	A2/A3

Source: Barclays Capital POINT

The portfolios have a comparable total risk of 2.9%/mo (Figure 6). The active risk of the Information ratio maximizing portfolio (0.7%/mo) is lower than the Sharpe ratio optimizing portfolio (1%/mo) as we would expect. Figure 6 also displays the realized returns of both portfolios in March 2010. We see that the two portfolios outperformed the index in March 2010 by 1.07% and 0.93%, respectively. In terms of the total returns, both portfolios realized an excess return of more than 2%. The curve returns are negative as the Treasury yields increased in March 2010. The average yields of the US Treasury index increased by 22bp from 2.15% to 2.375%, constituting the major component of the 1% curve loss for portfolio duration of 6.23 years. Not surprisingly, most of this outperformance is attributable[8] to the security selection in each of the sectors relative to the index. The security selection outperformance is mostly due to the OAS carry (not shown).

Figure 6: Carry enhanced portfolios: risk and returns

	Total Risk			Active risk			Total Returns			Active returns			
Portfolio	Total	Systematic	Idiosyncratic risk	Total	Systematic	Idiosyncratic risk	Total	Curve	Excess	Total	Curve	Asset allocation	Security selection
V1: Total Return	293	289	52	95	82	47	135	-104	239	107	3	-1	104
V2: Active	292	288	48	65	48	44	123	-104	227	93	2	-1	92

Source: Barclays Capital POINT. All number in bp/mo.

[6] See *Introducing LCS: Liquidity Score for US Credit bonds*, Quantitative Portfolio Strategies, 6 October 2009.
[7] Risk free rate is assumed to be zero.
[8] The return decomposition is performed using the Hybrid performance attribution model in POINT, which allocate the curve returns at the portfolio level, and then allocate the excess returns in to asset allocation and security selection in user specified sectors. See *Barclays Capital Hybrid Performance Attribution: User Guide*, 11 January 2010.

This example illustrates how the optimizer and other tools in POINT can be used to construct such carry enhanced portfolios. In particular, investigating whether such a strategy is profitable in the long run on a risk adjusted basis is beyond the scope of this paper. The additional carry compensates for additional risks – in particular, the tail risk associated with the relative spread in credit quality within the investment grade universe.

Equity long-short portfolio construction

Leveraged long-short equity portfolios, typically targeting total returns, are of interest to active quantitative portfolio managers. The basic idea is to go long the stocks that are expected to outperform, and at the same time short stocks that are expected to underperform. To keep the leverage low, the short side of the portfolio is typically limited to a small fraction of the long side (e.g., 130-30 portfolios constitute 30% of net market value in the shorts). Sharpe/Information ratio maximization can be used to construct such portfolios based on available risk and return forecasts.

As an example, suppose we rely on proprietary models to forecast the excess returns (over the risk free rate) of the US large cap stocks represented by the S&P 500 index. We are interested in constructing a 100mn (net investment) long-short portfolio that implements these expected return views, and does not take unwarranted risks as of March 31, 2010. For demonstration purposes, we use a simple estimate of expected excess return as a constant (8%/year) times the market beta for each stock (i.e., the CAPM expected excess returns). We use the POINT's monthly equity risk model to define the risk forecast of the universe.

We start the exercise by constructing a long-only Sharpe ratio optimal portfolio. The optimal portfolio consists of 17 stocks with the top two positions having market value weights of 12% and 19%, respectively. This portfolio achieves an expected excess return of 12.8%/year and a risk[9] of 3.61%/mo with a Sharpe ratio of 1.04[10]. Due to large concentrated positions, it is expected that this portfolio would not be deemed diversified in practice. Even though the (idiosyncratic) volatility of these two large positions could be low, the tail risk (not being taken into consideration here) can be unacceptably high. To control against such high name risk, we add a ticker exposure limit of 3%. The constrained optimal portfolio achieved a marginally lower Sharpe ratio of 0.94 with 44 positions, and an expected excess return of 11.0%/mo and a risk of 3.45%/mo.

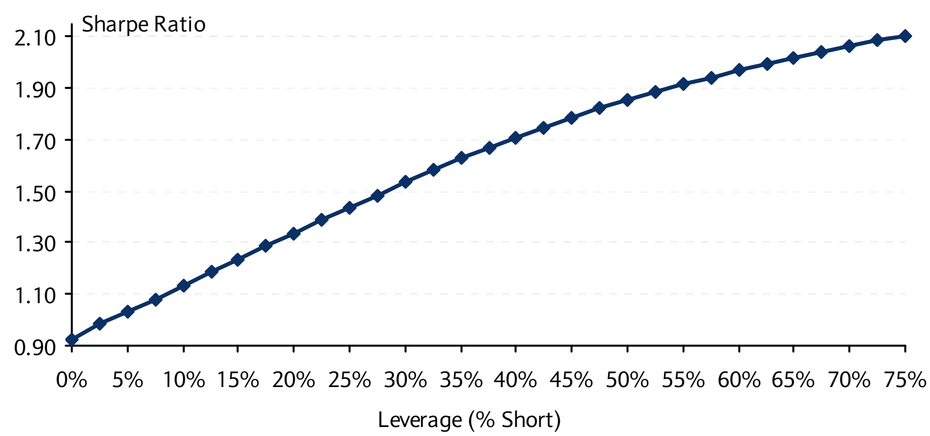
Next, we relax the long-only constraint and limit the leverage to 130-30. The 3% ticker exposure limit is only enforced for the longs. The optimal portfolio achieves a Sharpe ratio of 1.56 with 118 (29 shorts and 89 longs) positions. The top two positions on the short side have weights of 4% and 5%, respectively. Since the short side of the portfolio is smaller (only 30% of the net value), the optimizer tends to pick volatile names on the short side to offset the market risk of the long side as much as possible. We observe that the two largest short positions (5% and 4%, respectively) have relatively high industry beta and high idiosyncratic risk. To control the name risk on the short side, next we include a ticker exposure limit of 2% on the short side. The constrained long-short optimal portfolio in this case has 71 positions (13 shorts and 58 longs) with a Sharpe ratio of 1.25. The statistics associated with this portfolio are interesting. The risk *increased* significantly from 1.77%/mo (1.4%/mo systematic and 1.09%/mo idiosyncratic) to 2.32%/mo (2.11%/mo systematic and 0.98%/mo idiosyncratic) when we include this constraint. This result is due

[9] We define the percentage risk and return of the portfolio as the P&L risk and expected P&L normalized by the net value of the portfolio, respectively. Alternatively, the gross value of the portfolio can be used as the denominator. The optimizer operates in the P&L space itself, and the choice of the denominator has no impact on the optimal portfolio.
[10] The Sharpe ratio is quoted on an annualized risk return basis by convention. We multiply by square root of 12 to annualize the monthly risk numbers.

to the fact that the optimizer is now constrained in its ability to choose large short positions in high volatility names to hedge the systematic risk of the long portion of the portfolio. The increase in risk is coming from the systematic part. The idiosyncratic risk, on the other hand, decreased slightly as the optimizer is able to further diversify using the short side exposure also.

Next, we study the effect of increasing leverage on the optimal Sharpe ratios and corresponding expected excess returns and risks. Figure 7 plots the optimal Sharpe ratio achieved as a function of the leverage used. The leverage is quoted as the size of the short side of the portfolio as a percentage of the net value of the portfolio. Thus, 50% leverage means that a 100mn (net) USD portfolio can short 50mn USD with a gross value of 200mn. All these problems enforce the 3% ticker exposure limit on the long side, and a 2% ticker exposure limits on the short side discussed above. Additionally, the number of positions in the portfolio is now limited to 100. The long-only and the low leveraged portfolio satisfy the number of positions limit automatically. As the leverage becomes significantly high, the number of positions in the portfolio becomes large. Many of these positions are very small. Enforcing the 100 positions limit cleans these small positions without having a material impact on the optimal Sharpe ratio.

Figure 7: Sharpe ratio maximizing long-short US equity portfolios with increasing leverage: Sharpe ratio



Source: Barclays Capital

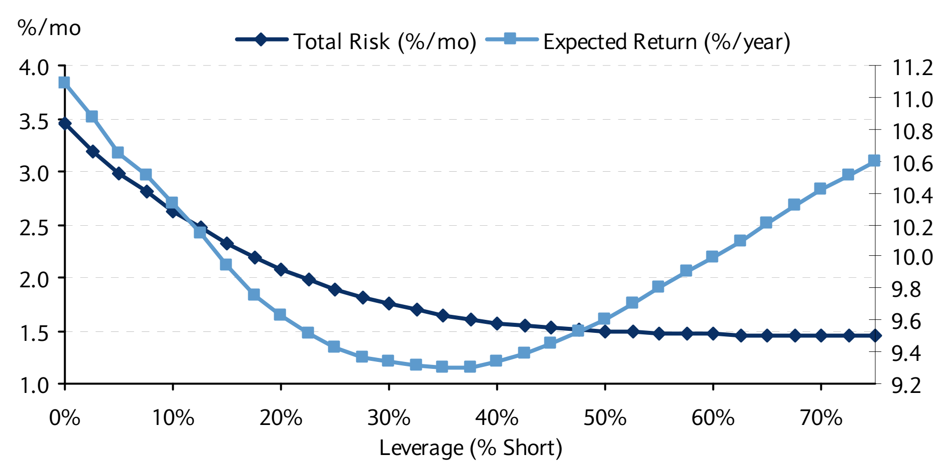
We see that the Sharpe ratio increases monotonically from 0.94 for the long-only portfolio to 2.08 for the 75% leveraged portfolio (Figure 7). Note that the gain in Sharpe ratio is decreasing as we reach higher and higher levels of leverage. Assuming[11] that there does not exist a zero market value portfolio with zero risk and positive expected excess returns, the Sharpe ratio can not increase indefinitely as we increase the leverage. Figure 8 plots the risk and expected excess return of the corresponding portfolios as a function of the portfolio leverage. Note that the risk and excess expected returns are plotted on the left hand and right hand axis, respectively, in Figure 8, and have different scale. We see the risk of the optimal portfolios decreases rapidly as we increase the leverage in the beginning (i.e., at low levels of leverage), but the decrease is much smaller at higher levels of leverage. This is because in the beginning the additional leverage allows the optimizer to use the shorts to

[11] The presence of idiosyncratic risk in the definition of Sharpe ratio guarantees this.

neutralize the marker exposure of the long side, but beyond a point, additional leverage can only be used to reduce the idiosyncratic risk. The gain in Sharpe ratio due to reduction in systematic risk dominates the loss due to the reduction in the expected excess returns in the beginning. At higher leverage, the optimizer focuses on the expected excess returns instead as the risk reduction is not significant at higher leverages. This explains the decrease in the expected excess returns at lower level of leverage, and increase at higher level of leverage (Figure 8).

If we relax the leverage constraint, the optimal Sharpe ratio achieved is 2.11 at the realized leverage of approximately 76%. In theory, this indicates there is no advantage of increasing leverage beyond 76% given the risk and return forecasts. In practice, managers may want to limit the leverage much below 76% because of explicit investment constraints or concerns regarding the uncertainly of aggregate risk and return forecasts based on linear models at such high levels of leverage.

Figure 8: Sharpe ratio maximizing long-short US equity portfolios with increasing leverage: Risk and expected returns



Source: Barclays Capital

In the example above, we discussed many practical issues in equity long-short portfolio construction using Sharpe ratio optimization. Our goal is to study the behaviour of an optimal portfolio as we change various constraints, highlighting the optimizer flexibility. In particular, the question[12] of whether the particular simple choice of excess return estimates efficiently contributes to the ex-post performance is beyond the scope of this paper.

Conclusion

The Sharpe and Information ratio maximization endogenize the risk-return trade-off required in the mean-variance optimization. But the optimal portfolios can be unintuitive (e.g., concentrated, not well diversified) if the estimates of expected returns are inconsistent with the risk (covariance) estimates of the underlying securities. POINT users can overcome this by using explicit portfolio constraints to control concentration risk while optimizing these ratios. This functionality significantly enhances the Portfolio Optimizer in allowing POINT clients to solve many practical portfolio optimization problems.

[12] There are number of publications which address this. For example see, Grinold, R. C. and Kahn, R. N. (2000) "The efficiency gains of long-short investing", *Financial Analyst Journal*, 56(6), pp 40-53.

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Appendix 1: Homogenization approach to Sharpe ratio maximization

To simplify the presentation, we discuss Sharpe ratio maximization. Similar results can be generally derived for Information ratio maximization. A canonical mathematical representation of a Sharpe ratio optimization problem is given in equation (5).

$$\begin{aligned} \underset{x \in X \subseteq \Re^n}{Maximize} \quad & \frac{\mu' x}{\sqrt{x' \Sigma x}} \\ \text{Such that} \quad & a_i' x \le b_i \ \text{ for all } i \in C \\ & e_i' x = 1 \end{aligned} \tag{5}$$

The notation in (1) is as follows.

- x : The decision variables (i.e., the $n \times 1$ vector of position amount in each security)
- Σ : The $n \times n$ covariance matrix
- μ : The expected excess returns (i.e., the $n \times 1$ vector of position amount in each security)
- a_i : The $n \times 1$ vector defining i^{th} linear constraint

b_i: The user provided bound for i^{th} constraint

C: The set representing the investment set (linear constraints)

e: A vector of all 1s.

To transform the non-linear programming problem in equation (5) above to an equivalent quadratic constrained quadratic programming problem (QCQP) given in equation (6) below, all linear constraints are *homogenized* using an additional scalar variable t. The objective function is changed to maximizing the expected excess returns, and a risk constraint is created with a nominal upper bound of 1.

$$\begin{aligned} \underset{y \in X \subseteq \Re^n, t \in \Re}{Maximize} \quad & \mu' y \\ \text{Such that} \quad & a_i' y - b_i \cdot t \le 0 \text{ for all } i \in C \\ & e_i' y - t \le 0 \\ & y' \Sigma y \le 1 \end{aligned} \tag{6}$$

Main result:

If there exists a feasible portfolio with positive expected return, then if (y^*, t^*) is a solution to (6) then $x^* = \frac{y^*}{t^*}$ is a solution (5) (Claim 1 in Iyengar and Wang [2005]).

We can solve the simple QCQP given in equation (6) using standard algorithms, and map the optimal solution using the above result to obtain the optimal solution to (5).

The above formulation only works if all constraints are convex. However, the Sharpe ratio optimization model in POINT supports a number of combinatorial constraints that are neither continuous nor convex. We use proprietary iterative heuristic algorithms to solve such Sharpe ratio optimization problems, which use the above reformulation technique at the intermediate iterations. This optimization approach extends to other performance ratios defined as the ratio of a convex expected return measure and a unit consistent risk measure (e.g. conditional value at risk (CVaR)). Allowing such measures in the optimization process remains part of our future research agenda.

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