

# Firm Volatility in Granular Networks\*

Bernard Herskovic

UCLA Anderson

Bryan Kelly

Chicago Booth and NBER

Hanno Lustig

Stanford GSB and NBER

Stijn Van Nieuwerburgh

NYU Stern, NBER, and CEPR

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## Abstract

Firm volatilities co-move strongly over time, and their common factor is the dispersion of the economy-wide firm size distribution. In the cross section, smaller firms and firms with a more concentrated customer base display higher volatility. Network effects are essential to explaining the joint evolution of the empirical firm size and firm volatility distributions. We propose and estimate a simple network model of firm volatility in which shocks to customers influence their suppliers. Larger suppliers have more customers and the strength of a customer-supplier link depends on the size of the customer. The model produces distributions of firm volatility, size, and customer concentration that are consistent with the data.

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# 1 Introduction

Recent research has explored how the network structure and firm size distribution in an economy can influence aggregate volatility. Acemoglu et al. (2012) and Carvalho (2010) show that sparsity of inter-sector linkages inhibits diversification in an economy and raises aggregate volatility. Gabaix (2011) points out that “granularity,” or extreme skewness in firm sizes, concentrates economic mass among a few very large firms, similarly stifling diversification and increasing aggregate volatility.

This research is silent about the impact of firm networks and firm size concentration on the volatility at the firm level. The volatility of firm-level stock returns and cash flows varies greatly over time (e.g., Lee and Engle (1993)) and across firms (e.g., Black (1976), Christie (1982)). Firm-level fluctuations in uncertainty have important implications for investment and hiring decisions as well as firm value, as highlighted by Bloom (2009).<sup>1</sup> But the underlying determinants of firm volatility are poorly understood. In much of the work on volatility in economics and finance, firms are modeled to have heteroscedastic shocks without specifying the source of heteroscedasticity. Our goal is to understand, both theoretically and empirically, how inter-firm linkages and size distributions interact to endogenously produce heteroscedasticity at the firm level.

We propose a simple model in which firms are connected to other firms in a customer-supplier network.<sup>2</sup> It has three assumptions. Firms’ idiosyncratic growth rate shocks, which are homoscedastic, are influenced by the growth rates of their customers. As a result, the firm-specific shocks propagate through the network via connected firms. The appendix provides a simple general equilibrium model with inter-connected sectors of production and consumer demand shocks that delivers a structural interpretation. Second, the probability of a customer-supplier link depends on the size of the supplier so that large firms typically

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<sup>1</sup>See also Leahy and Whited (1996), Bloom, Bond, and Van Reenen (2007), Stokey (2012), Bloom, Floetotto, Jaimovich, Saporta-Eksten, and Terry (2016) and the papers cited therein.

<sup>2</sup>A recent literature explores the role of production networks in macro-economics, trade, asset pricing, and banking. See Atalay, Hortaçsu, Roberts, and Syverson (2011), Foerster, Sarte, and Watson (2011), Acemoglu, Carvalho, Ozdaglar, and Tahbaz-Salehi (2012), Carvalho and Gabaix (2013), Ahern and Harford (2014), Acemoglu, Ozdaglar, and Tahbaz-Salehi (2015), Farboodi (2015), Herskovic (2017), Kramarz, Martin, and Mejean (2017), Stanton, Wallace, and Walden (2017). For reviews of the network literature, see Allen and Babus (2009), Jackson (2014), and the references therein. This paper circulated earlier as Kelly, Lustig, and Van Nieuwerburgh (2013).

supply to a higher number of customers. Third, the importance of a customer-supplier link depends on the size of the customer. Large customers have a stronger connection with their suppliers, presumably because they account for a large fraction of their suppliers' sales. We provide microeconomic evidence for all three assumptions based on the observed customer-supplier networks among Compustat firms. Differences in firms' network connections impart total firm volatility with cross-sectional heteroscedasticity.

Firms are aggregators of their own idiosyncratic shocks and the shocks to connected firms. The sparsity and granularity of a firm's customer network, which in turn depend on the firm size distribution, determine how well a firm diversifies the shocks that it is exposed to. The latter determines its volatility. By connecting the firm size distribution to network formation, our model generates a rich set of implications for volatility in the cross-section and the time-series, which we can test. We study data on firm-level sizes, volatilities, and customer-supplier linkages, establishing a new set of stylized facts about firm volatility and confirming the model's implications.

Firm-level volatilities exhibit a common factor structure where the factor is firm size dispersion in the economy. In the model, each supplier's network is a random draw from the entire population of firms, so that any firm's customer network inherits similar dispersion to that of the entire size distribution. An increase in size dispersion slows down every firm's shock diversification and increases their volatility. We find that firm volatilities possess a strong factor structure in the data.

The factor structure implies strong time series correlations between moments of the size and volatility distributions. An increase in the size dispersion translates into higher average volatility among firms. It also raises the cross section dispersion in volatilities. In the time series, size dispersion has a 63% correlation with mean firm volatility and 78% with the dispersion of firm volatility. Our paper is the first to document these facts and to provide an economic explanation for the factor structure in firm-level volatility by connecting it to firm size dispersion. A persistent widening in the firm size dispersion should lead to a persistent rise in mean firm volatility. We observe such a widening (increase in firm concentration) between the early 1960s and the late 1990s, providing a new explanation for the run-up in mean firm volatility studied by Campbell et al. (2001).

In the cross-section, differences in volatility across firms arise from two sources: differences in the number of a customers and differences in customer size dispersion. First, large firms are less volatile than small firms because they are connected to more customers, which improves diversification regardless of the size profile of its customer base. This effect also appears in the model’s volatility factor structure. Smaller firms have larger exposures to the common volatility factor, implying that small firms have both higher levels of volatility and higher volatility of volatility. In the data, we find a strong negative correlation between firm size and variance, and small firms indeed have higher volatility factor exposures.

Second, holding the number of connections fixed, a supplier’s customer network is less diversified if there is more dispersion in the size of its customers. Because customer size determines the strength of a link, severe customer size disparity means that shocks to the biggest customers exert an outsized influence on the supplier, raising the supplier’s volatility. Differences in customer size disparity arise from probabilistic network formation – some suppliers will link to a very large (or very small) customer by chance alone. The data indeed show a strong positive correlation between a firm’s “out-Herfindahl,” our measure of concentration in a firm’s customer network, and its volatility. Firm size and firm out-Herfindahl remain the leading determinants of firm volatility after the inclusion of other determinants of volatility previously proposed in the empirical literature. Collectively, this evidence supports a network-based explanation of firm volatility.

To gauge its quantitative plausibility, we estimate our network model using data on the customer-supplier network among US firms. Our estimation targets moments and cross-moments of the distributions of firm size, firm variance, and inter-firm business linkages. The model can account for the large dispersion in firm volatility while respecting the evidence on the number and concentration of customers. The estimation reveals that this requires strong network effects: a firm’s customer’s customer shocks are nearly as important as the shocks that directly hit the firm’s customer. Strong network effects are necessary but not sufficient for quantitatively matching observed firm volatilities. An internal diversification mechanism whereby larger firms have lower shock volatility complements the external diversification mechanism of the customer network. A statistical test fails to reject the null hypothesis that the moments in model and data are equal.

The rest of the paper is organized as follows. Section 2 presents new empirical evidence on the link between the firm size and volatility distributions. Section 3 explains these links with a simple network model. Section 4 estimates the model and Section 5 tests two additional model predictions in the micro-data. One is on the network determinants of firm-level volatility and the other establishes that the firm size dispersion is an important common factor driving firm volatilities. The structural model, the proofs of the theoretical results, and the auxiliary empirical evidence are relegated to the appendix.

## 2 Evidence on Firm Size Dispersion and Firm Variance

This section documents our main new empirical facts about the joint evolution of the firm size and firm variance distributions.

### 2.1 Data

We consider market-based and fundamentals-based measures of firm size and firm volatility. Both are calculated at the annual frequency. Our main measure of firm size is the equity market value at the end of the calendar year. The alternative fundamentals-based measure is total sales within the calendar year. All variables in our analysis are deflated by the consumer price index. Our main measure of firm variance is defined as the variance of daily stock returns during the calendar year. Fundamentals-based variance in year  $t$  is defined as the variance of quarterly sales growth (over the same quarter the previous year) within calendar years  $t$  to  $t + 4$ .<sup>3</sup> The sample is the universe of publicly-listed firms. Stock market data are from CRSP for the period 1926-2016 and sales data are from the merged CRSP/Compustat file for the period 1952-2016. The cross-sectional size and variance distributions are well approximated by a lognormal distribution. As a result, each distribution may be summarized by two moments: the cross-sectional mean and cross-sectional variance, or dispersion, of the log quantities.

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<sup>3</sup>We also consider fundamental volatility measured by the standard deviation of quarterly sales growth within a single calendar year. The one- and five-year fundamental volatility estimates are qualitatively identical, though the one-year measure is noisier because it uses only four rather than twenty observations.

## 2.2 Comovement of Firm Size and Volatility Distributions

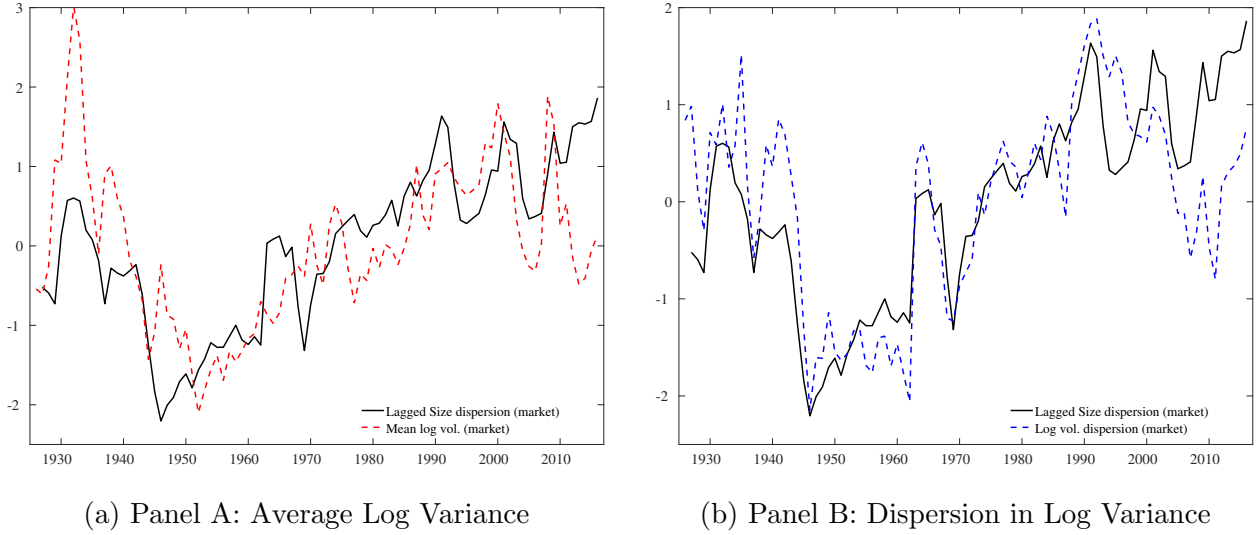
Panel A of Figure 1 plots the cross-sectional *average* of log firm variance against lagged firm size dispersion, using our market-based measures. For ease of readability, both series are standardized to have mean zero and standard deviation one. The correlation between average firm variance and firm size dispersion is 63.3%. Mean firm variance experienced several large swings in the past century, especially in the 1920s and 1930s and again in the last two decades of the sample. These changes are preceded by similar dynamics in the cross-sectional dispersion of firm size. The high positive correlation between mean firm volatility and firm size dispersion is our first main new fact. The second main stylized fact links the *dispersion* in firm variance to the dispersion in firm size. Panel B of Figure 1 shows a strong positive association between the cross-sectional dispersions of firm variance and firm size, based on the market measures. The correlation between the two time-series is 77.6%. Appendix C.1 shows that the positive correlation between size dispersion and the first and second moments of the firm variance distribution is also present at business cycle frequencies; it uses the HP-filter to decompose the time series in trend and cycle components.

The same relationship between the moments of the firm size and variance distributions exists for our fundamentals-based measure. First, the correlation between average firm variance and lagged firm size dispersion is 68.7%. The left panel of Figure 2 shows the strong positive association. Because the sales-based data only start in 1965, their dynamics are more affected by the persistent increase in firm size dispersion and variance that took place between the 1960s and the 1990s. The right panel shows a strong positive correlation between the dispersion of variance and the dispersion of size for the sales-based measure. The correlation is 81.7%. Both facts corroborate the market-based evidence. Any explanation of these facts must confront the high degree of similarity between market volatilities and its (more coarsely measured) fundamental counterpart.<sup>4</sup> This evidence suggests that financial explanations, such as discount rate or leverage effects, are incomplete.

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<sup>4</sup>Market- and fundamentals-based measures of average log firm variance have an annual time-series correlation of 68.9%, while the two volatility dispersion measures have a correlation of 73.5%.

Figure 1: MARKET-BASED DISPERSION IN FIRM SIZE AND FIRM VARIANCE



*Notes:* Panel A plots the lagged cross-sectional dispersion of log firm size (solid line) and the cross-sectional mean of the log variance distribution (dashed red line). Panel B plots the lagged cross-sectional dispersion of log firm size (solid line) and the cross-sectional standard deviation of the log variance distribution (dashed blue line). Firm size is measured as market value of equity; firm variance is measured as the variance of daily stock returns within the year. All series are rescaled for the figure to have mean zero and variance one. Sample is from 1926 to 2016 at annual frequency.

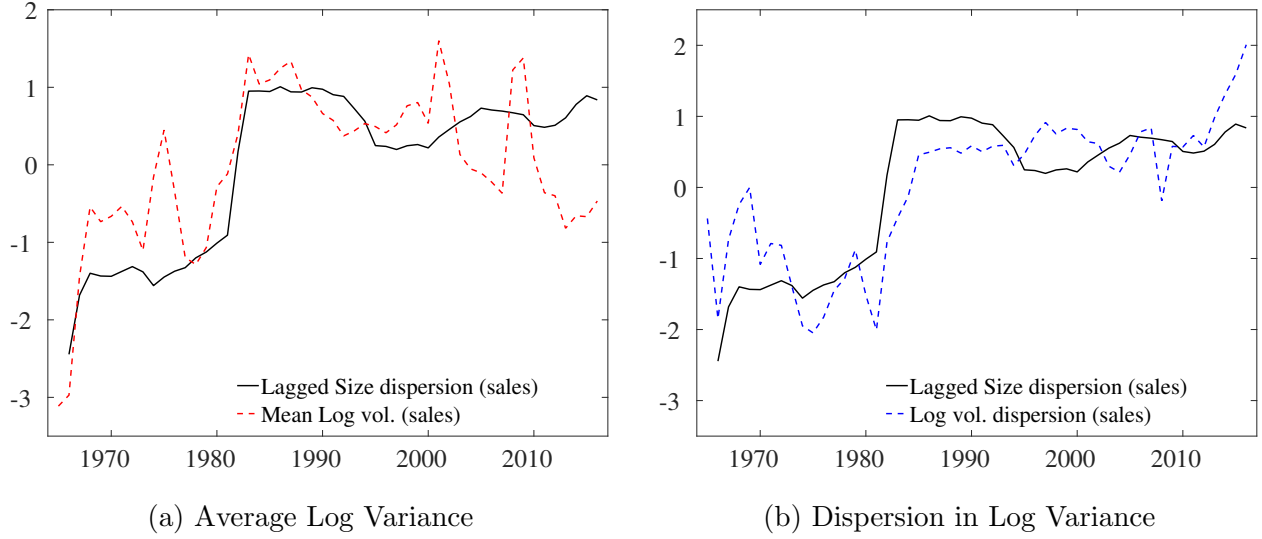
## 2.3 Subsample Results

Table 1 establishes that the two key correlations between firm size dispersion and the average (column 3) and dispersion of firm variance (column 4) hold for various subsets of our data (listed in column 1). The second column reports the number of firms in each subsample. The first panel splits the sample of firms into three size terciles based on the market value of equity. Firms are resorted each year. Our main correlations are large and positive in all size groups.

The second panel groups firms into nine industries. The two main correlations are large and positive in virtually all industries. The one exception is utilities, which only has a 17% correlation between size dispersion and average variance. For consumer durables, that same correlation is 35%. These two industries have the fewest firms. All correlations between size dispersion and variance dispersion are higher than 50%.

Third, we find virtually the same correlation for firms that have been public less than five years and firms that have been public for more than five years. Similar correlations are also

Figure 2: FUNDAMENTALS-BASED DISPERSION IN FIRM SIZE AND FIRM VARIANCE



*Notes:* Panel A plots the lagged cross-sectional dispersion of log firm size (solid line) and the cross-sectional mean of the log variance distribution (dashed red line). Panel B plots the lagged cross-sectional dispersion of log firm size (solid line) and the cross-sectional standard deviation of the log variance distribution (dashed blue line). Firm size is measured as firm sales; firm variance is measured based on 20 quarters of growth in firm sales. All series are rescaled for the figure to have mean zero and variance one. Sample is from 1965 to 2016 at annual frequency.

found for firms that have been public for at least 10 and at least 25 years. This shows that our results are not driven by firms that recently went public, and whose size and volatility characteristics may be different from older firms or may have changed over time.

Fourth, the results hold both in the first and in the second half of the sample. They are somewhat stronger in the first half of the sample.

Fifth, the correlations between size dispersion and variance mean and dispersion are larger, but in the same ballpark, for firms listed on the NYSE and firms not listed on the NYSE.

To summarize, we observe a strong positive association of firm size dispersion with average firm variance and dispersion in firm variance, throughout the distribution of firm size, industry, age, and over time. The next section presents a network model that generates these associations.



Table 1: COMPOSITION

	# Firms	Correl. of Size Dispersion with	
		Avg. Var.	Disp. Var
All Firms	3069.9	67.5	77.4
By size			
Smallest third	1023.4	65.8	59.9
Middle third	1023.0	59.3	67.2
Largest third	1023.5	55.0	66.4
By industry			
Consumer Non-Dur.	240.8	59.5	67.2
Consumer Durables	104.6	34.9	71.7
Manufacturing	516.4	51.7	77.3
Energy	150.2	67.7	57.4
Tech/Telecom	494.4	73.9	51.0
Retail	305.8	62.1	63.4
Healthcare	199.0	73.6	53.6
Utilities	111.5	16.9	58.9
Other	884.0	56.6	66.2
By Time Since Entry			
Less than 5 yrs.	905.0	74.0	79.3
At least 5 yrs.	2159.3	71.5	77.1
At least 10 yrs.	1447.5	72.9	76.4
At least 25 yrs.	489.0	46.9	77.9
By Subsample			
1926-1969	496.7	74.5	83.8
1970-2015	2529.5	57.0	42.3
By Exchange			
NYSE	1204.3	60.8	76.6
non-NYSE	1865.6	41.3	42.4

*Notes:* Annual data 1926-2016. We use the market-based volatility measure constructed from stock returns and the market-based measure of size (market equity). The second column reports the time series average of the cross section number of firms. Column 3 reports the correlation between average log volatility ( $\mu_{\sigma,t}$ ) and lagged log size dispersion ( $\sigma_{S,t-1}$ ). Column 4 reports the correlation between dispersion in log volatility ( $\sigma_{\sigma,t}$ ) and lagged log size dispersion.

### 3 A Network Model of Firm Growth and Volatility

This section develops a simple network model of connections between firms. It generates the positive correlations between firm size dispersion on the one hand and average firm volatility and the dispersion of firm volatility on the other hand. Section 4 uses size, volatility, and network moments to estimate the model presented here.

### 3.1 Firm Growth

Define  $S_i$  as the size of firm  $i = 1, \dots, N$  with growth rate as  $g_i$ , where

$$S'_i = S_i \exp(g_i) \quad (1)$$

denotes firm size at the end of the period. Firms are connected through a network. Firm  $i$ 's growth rate depends on its own idiosyncratic shock and a weighted average of the growth rates of the firms  $j$  it is connected to:

$$g_i = \mu_g + \gamma \sum_{j=1}^N w_{i,j} g_j + \varepsilon_i. \quad (2)$$

The parameter  $\gamma \in [0, 1)$  governs the rate of decay as a shock propagates through the network. The network weight  $w_{i,j}$  determines how strongly firm  $i$ 's growth rate is influenced by the growth rate of firm  $j$ . If  $i$  and  $j$  are not connected then  $w_{i,j} = 0$ . By convention, we set  $w_{i,i} = 0$ . We refer to the matrix of connection weights  $\mathbf{W} = [w_{i,j}]$  as the network matrix. We assume that all rows of  $\mathbf{W}$  sum to one. Thus, the largest eigenvalue of  $\mathbf{W}$  equals one. In this directed network, connections are not symmetric. We identify firm  $i$  as the supplier and firm  $j$  as the customer. Firm  $j$  can be a customer of firm  $i$  without  $i$  being a customer of  $j$ .

Let  $\mathbf{g}$  and  $\boldsymbol{\varepsilon} \sim N(0, \sigma_\varepsilon^2 \mathbf{I})$  be the  $N \times 1$  vectors of growth rates and shocks, respectively. For simplicity, set  $\mu_g = 0$ . The growth rate equation (2) for all firms can be written in vector form as:

$$\mathbf{g} = \gamma \mathbf{W} \mathbf{g} + \boldsymbol{\varepsilon} = (\mathbf{I} - \gamma \mathbf{W})^{-1} \boldsymbol{\varepsilon}. \quad (3)$$

The Leontieff inverse matrix  $(\mathbf{I} - \gamma \mathbf{W})^{-1}$  is the key object describing the effects of network structure on the behavior of growth rates.

In this section, we purposely impose stark assumptions on the nature of the underlying innovations: each firm  $i$  experiences i.i.d. growth rate shocks  $\varepsilon_i \sim N(0, \sigma_\varepsilon)$ . In the next section, we augment the model to consider heterogeneity in firms' idiosyncratic shock variances.

### 3.2 A Structural Interpretation

Appendix A derives equation (3) as the equilibrium outcome in a production economy. It is a constant returns-to-scale economy with  $N$  sectors of production in the tradition of Long and Plosser (1987). The output of one sector is used as input in the production of another sector. A representative agent has Cobb-Douglas preferences defined over the  $N$  goods that are produced, with preference weights collected in the vector  $\boldsymbol{\theta}$ . The reduced-form shocks  $\boldsymbol{\varepsilon}$  are consumer demand shocks for the various goods,  $d\boldsymbol{\theta}$ , scaled by the vector  $\boldsymbol{\psi}$  whose  $i$ th entry measures the ratio of output of sector  $i$  to economy-wide value added. These taste shocks travel upstream from final goods producers to their customers, the producers of intermediate goods, affecting the growth in their real output  $\mathbf{g} = dY/Y$ . The parameter  $\gamma$  measures the share of output used by intermediate good producers, averaged across sectors. The entries of the network matrix  $\mathbf{W}$ ,  $w_{ij}$ , measure the sales of good  $i$  to intermediate goods sector  $j$  divided by the total output of sector  $i$ , and multiplied by  $\gamma$ . These entries are linked to the cost shares, which are primitives of the production function. The following proposition, proved in the appendix, shows that the structural model delivers the same “network equation” (4) as the reduced-form model’s equation (3).

**Proposition 1.** *The responses of firm output to demand shocks  $d\boldsymbol{\theta}$  equals:*

$$\frac{d\mathbf{Y}}{\mathbf{Y}} = (\mathbf{I} - \gamma\mathbf{W})^{-1} \frac{d\boldsymbol{\theta}}{\boldsymbol{\psi}}. \quad (4)$$

Shea (2002) and Kramarz, Martin, and Mejean (2017) consider different models where shocks travel upstream from customers to suppliers, like in our model. Acemoglu et al. (2012) solve a similar model, but emphasize productivity shocks that are transmitted downstream from suppliers to customers.

### 3.3 Size Effects in Network Structure

The content of the spatial autoregression model (3) is in the specification of the weighting matrix  $\mathbf{W}$ . Since we cannot directly measure  $w_{ij}$  in the data, we make two plausible assumptions on the probability and strength of connections between suppliers and their customers.

These assumptions link the firm size distribution to the network structure of the economy. In the next section, we will confront the model with data on the production network in the US and validate these assumptions.

The linkage structure is determined by the firm size distribution at the start of the period. The existence of a link between supplier  $i$  and customer  $j$  is described by:

$$b_{i,j} = \begin{cases} 1 & \text{if } i \text{ connected to } j \\ 0 & \text{otherwise.} \end{cases}$$

Each element of the connections matrix,  $\mathbf{B} = [b_{i,j}]$ , is drawn from a Bernoulli distribution with probability  $P(b_{i,j} = 1)$ . This connection probability is assumed to be a function of the size of the supplier  $i$ :

$$P(b_{i,j} = 1) \equiv p_i = \frac{\tilde{S}_i}{Z} N^{-\zeta} \quad (\text{for } i \neq j), \quad (5)$$

where  $\tilde{S}_i = S_i/E[S_i]$  is the relative size of firm  $i$  versus the population mean and  $Z$  is a scalar. While the functional form matters quantitatively, the crucial qualitative assumption is that the probability of a connection depends on the (relative) size of firm  $i$ . That is, larger firms have more connections on average. This is the model's first size effect.

Equation (5) also builds sparsity into the network. The sparsity parameter  $\zeta \in (0, 1)$  governs the rate at which the likelihood of a connection decreases as the number of firms  $N$  grows large. It implies that the number of links (customers) in the system diverges as the number of firms goes to infinity, but that the probability of connecting to any single customer goes to zero. In a large economy, the expected number of customers for firm  $i$ , called the out-degree, is:

$$N_i^{out} \approx N p_i = \frac{\tilde{S}_i}{Z} N^{1-\zeta}. \quad (6)$$

The number of linkages grows with the number of firms in the economy, but the rate of growth is slower when  $\zeta$  is closer to 1.

The second key assumption on the network we make is that, conditional on a link existing

between firm  $i$  and firm  $j$ , the strength of that link depends on the size of the customer  $j$ :

$$w_{i,j} = \frac{b_{i,j}S_j}{\sum_{k=1}^N b_{i,k}S_k}, \quad \forall i, j. \quad (7)$$

One natural measure of size is sales. This weighting scheme then has the natural feature that customers  $j$  who represent a larger share of supplier  $i$ 's sales have a stronger impact on the growth rate of  $i$ . This is the model's second size effect.

While there is obvious value in understanding more deeply why firms forge connections, modeling the endogenous choice over network linkages is notoriously difficult.<sup>5</sup> Our approach is to model two key features of the observed supplier-customer networks. Any network choice model would have to generate these features as an outcome. Our simpler approach suffices to study how the network structure among firms affects the link between the firm size and volatility distribution. This simplicity allows us prove theoretical results on the relationship between firm volatility and the structure of its customer network. Our paper also contributes new facts on the properties of production networks that the literature on endogenous network formation can target as outcomes. This simplicity is also what enables us to estimate the model in the next section.

### 3.4 Firm Variance

Conditional on  $\mathbf{W}$ , the variance-covariance matrix of growth rates  $\mathbf{g}$  is given by

$$\mathbf{V}(\mathbf{g}) = \sigma_\varepsilon^2 (\mathbf{I} - \gamma \mathbf{W})^{-1} (\mathbf{I} - \gamma \mathbf{W}')^{-1}. \quad (8)$$

The vector of firm volatilities is the square root of the diagonal of the variance-covariance matrix. In standard network settings, the Leontieff inverse  $(\mathbf{I} - \gamma \mathbf{W})^{-1}$  is an obstacle to deriving a tractable analytic characterization of volatility. Our model, in contrast, lends itself to a convenient variance representation when the number of firms in the economy becomes large.

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<sup>5</sup>For some promising steps forward in this direction recently, see Acemoglu, Ozdaglar, and Tahbaz-Salehi (2015), Farboodi (2015), Oberfield (2017), Stanton, Wallace, and Walden (2017), and Herskovic and Ramos (2017).

Before deriving our main result, it is useful to build intuition for the behavior of variance by considering a simplified version of the network model. Suppose that growth rates follow the process:

$$\mathbf{g} = (\mathbf{I} + \gamma \mathbf{W}) (\boldsymbol{\mu}_g + \boldsymbol{\varepsilon}). \quad (9)$$

In our full network model (2), a firm's growth rate is influenced by the growth rates of each of its connections. The latter are in turn influenced by their connections' growth rates, and so on. The simplification in (9) differs from the full network in that idiosyncratic shocks only propagate one step in the supply chain and then die out. In fact, (9) is a first order approximation to (2), because

$$(\mathbf{I} - \gamma \mathbf{W})^{-1} = \mathbf{I} + \gamma \mathbf{W} + \gamma^2 \mathbf{W}^2 + \gamma^3 \mathbf{W}^3 + \dots \approx \mathbf{I} + \gamma \mathbf{W},$$

under our maintained assumption that  $\gamma \in [0, 1)$ . In this system, the variance of firm  $i$ 's growth rate simplifies to

$$V(g_i) = V\left(\gamma \sum_j w_{i,j} \varepsilon_j + \varepsilon_i\right) = \sigma_\varepsilon^2 (1 + \gamma^2 H_i^{out}), \quad (10)$$

where

$$H_i^{out} \equiv \sum_{j=1}^N w_{i,j}^2 \quad (11)$$

is the Herfindahl index of firm  $i$ 's network of customers. We refer to  $H_i^{out}$  as the out-Herfindahl. Equation (10) shows that, to a first order approximation, the variance of a firm's growth rate is determined by its out-Herfindahl, the volatility of the underlying innovations  $\sigma_\varepsilon^2$ , and the strength of shock transmission in the network  $\gamma$ .

The higher firm  $i$ 's out-Herfindahl, the more concentrated is its network of connections. Standard diversification logic applies: a low degree of diversification in a firm's customer base raises its variance. The out-Herfindahl is driven by two characteristics: the number of customers and the size dispersion among its customers. The supplier is more diversified and has lower volatility when it has more customers and when the size dispersion of its customers is small.

Because all firms, large and small, draw their connections from the same economy-wide firm size distribution, all firms' customer networks have equal firm size dispersion *in expectation*. The expected customer size dispersion of any firm  $i$  is given by the *economy-wide* firm size dispersion. Thus, the economy-wide out-Herfindahl is a key determinant of the volatility of each firm. The economy-wide out-Herfindahl moves all firms' volatilities up and down together; it is a common factor in firms' volatilities. Differences between firms' volatilities arise from how many connections they draw from this economy-wide firm size distribution. Small firms draw fewer links on average than large firms and therefore are more volatile.

Our main theoretical proposition below formalizes the above intuition. It connects the variance of a firm to its size and to the concentration of firm sizes throughout the economy. It does so for the full network model, i.e., without the first-order approximation in (9). The proof is relegated to Appendix A as are the lemmas the proposition relies on.

**Proposition 2.** *Consider a sequence of economies indexed by the number of firms  $N$ . If  $S_i$  has finite variance, then the Leontief inverse has limiting behavior described by.<sup>6</sup>*

$$(I - \gamma W)^{-1} \sim I + \gamma W + \frac{\gamma^2}{1 - \gamma} \bar{W},$$

and firm volatility has limiting behavior described by

$$V(g_i) \sim \sigma_\varepsilon^2 \left[ 1 + \left( \frac{\gamma^2}{N^{1-\zeta}} \frac{Z}{\tilde{S}_i} + \frac{2\gamma^3 - \gamma^4}{N(1 - \gamma)^2} \right) \frac{E[S^2]}{E[S]^2} + \frac{2\gamma^2}{1 - \gamma} \frac{\tilde{S}_i}{N} \right]. \quad (12)$$

This proposition highlights the determinants of firm-level growth rate variance in a large  $N$  economy. A firm's variance depends on its own shock variance  $\sigma_\varepsilon^2$  plus a term that reflects the network effect of interest. The latter consists of three terms.

First, firm variance depends on economy-wide firm size dispersion given by the ratio of the second non-central moment of the size distribution to the squared first moment  $E[S^2]/E[S]^2$ . In the special case where the firm size distribution is log-normal with variance  $\sigma_s^2$ , this ratio simplifies to  $\exp(\sigma_s^2)$ . This establishes a theoretical connection between firm size dispersion and the firm volatility distribution (and its mean and dispersion), just like we found em-

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<sup>6</sup>We use the asymptotic equivalence notation  $x \sim y$  to denote that  $x/y \rightarrow 1$  as  $N \rightarrow \infty$ .

pirically in Section 2. Economy-wide size dispersion is a common factor affecting all firms' volatilities, since more size dispersion makes every supplier less diversified. The first-round reduction in customer network diversification has ripple effects through the higher order terms: customers' customer networks are also less diversified, etc. As  $\gamma$  approaches one, these higher order terms become quantitatively important as firms idiosyncratic shocks have far reaching effects throughout the network.

Second, relative firm size,  $\tilde{S}_i$ , affects volatility in two ways. Its primary role is captured by the first coefficient on  $E[S^2]/E[S]^2$ . Larger firms have a lower exposure to the common factor in volatility, economy-wide size dispersion, than do smaller firms because they typically connect to more customers and achieve better shock diversification. This diversification channel lowers the volatility of large firms. Smaller factor exposure also makes large firms less sensitive to fluctuations in the firm size dispersion. That is, they display lower volatility of volatility.

Third, the firm's relative size also appears in the numerator of the last term. Shocks to the largest firms are the most strongly propagated shocks in the model since these firms have the largest influence on their suppliers. Shocks to large firms ultimately "reflect" back, raising large firms' own volatility. In contrast, small firms' shocks die out relatively quickly in the network. Thus, the last term in (12) captures a countervailing increase in volatility for larger firms. In all of our numerical results, we find that the diversification channel of the first size term dominates the reflection channel of the last term.

Finally, we note that the strength of the diversification channel depends on the term  $N^{1-\zeta}$ . A high  $\zeta$  means that there are relatively few linkages compared to the size of the economy. This network sparsity effect slows down the diversification benefits for all firms. Because the other two network terms in (2) decline at a faster rate  $N$ , the diversification channel becomes the dominant effect in an economy with many firms.

### 3.5 Additional Theoretical Results

We derive several more theoretical results which we relegate to Appendix B. Proposition 3 characterizes firm variance when the firm size distribution follows a power law. Gabaix (2011) emphasizes that extreme right skewness of firm sizes can also slow down volatility decay in



large economies. We show that the firm-level network structure adds a complementary mechanism that slows down the volatility decay beyond Gabaix’s granularity mechanism. In the absence of network effects, firm variance decays at rate  $N^{2-2/\eta}$ , where  $\eta$  is the power law coefficient of the firm size distribution. With network sparsity, the rate of decay is  $N^{(2-2/\eta)(1-\zeta)}$ , where  $\zeta$  is our network sparsity parameter.

Proposition 4 characterizes the covariance across firms’ volatilities. Proposition 4 shows that comovement among firm variances decays at rate  $N^{1+2(1-\zeta)}$  in a large economy. Intuitively, covariance among a pair of firms is lowest when both firms are large, since large firms have low exposure to overall size concentration.

Proposition 5 studies the behavior of aggregate volatility, measured as the volatility of the average growth rate  $g_a = \boldsymbol{\nu}'\mathbf{g}$ , where the weights are proportional to firm size,  $\boldsymbol{\nu} = \mathbf{S}/\iota'\mathbf{S}$  and  $\iota$  a vector of ones. The variance of the aggregate growth rate  $V(g_a)$  has several interesting properties. First, changes in the network matrix  $\mathbf{W}$  and the size distribution  $\mathbf{S}$  induce changes in aggregate variance, even though all underlying shocks are i.i.d. across firms. Interpreting this comparative statics from a time-series perspective, the network mechanism generates endogenous heteroscedasticity. Second, our sparse network model drives a wedge between the rates of decay of aggregate and firm-level variance. Aggregate variance decays at rate  $1/N$ , which is generally faster than the rate of decay of firm variance.

## 4 Simulated Method of Moments Estimation

Section 2 documents strong statistical relationships between the distributions of firm size and firm volatility. Section 3 provides a network-based foundation for these relationships. In this section we go one step further and ask whether the production network model can quantitatively account for the observed joint distribution of firm size, firm volatility, and inter-firm production network linkages. We estimate the key network parameters in a Simulated Method of Moments (SMM) framework. By insisting on matching data on customer-supplier relationships we provide evidence for the network assumptions made in the model.

## 4.1 Model Extensions

The stylized model of the previous section implicitly assumed that when two firms of equal size merge, the new firm is as volatile as each of the two original firms. A large literature on mergers suggests that many firms seek diversification benefits from mergers resulting in lower firm volatility of the combined entity. We label these gains from *internal diversification* to distinguish them from external diversification gains that accrue through the firm’s network of customers. To capture internal diversification benefits in a simple way, we assume that the volatility of a firm’s own fundamental shock depends negatively on its size. Specifically,

$$\sigma_{\varepsilon,i} = \sigma_{\varepsilon} + \lambda \log \left( 1 + \frac{S_{median}}{S_i} \right), \quad (13)$$

where  $\lambda$  governs the sensitivity of fundamental volatility to firm size. The parameter  $\sigma_{\varepsilon}$  is the minimum shock volatility, after all possible internal diversification benefits have been exhausted. The larger the firm, the closer its shock volatility is to  $\sigma_{\varepsilon}$ . For the median firm,  $S_i = S_{median}$ , and the shock volatility equals  $\sigma_{\varepsilon} + \lambda \log(2)$ . This functional form keeps all firm variances positive.

We also generalize the weighting function  $w_{i,j}$ , which governs the importance of customer  $j$  in supplier  $i$ ’s network:

$$w_{i,j} = \frac{b_{i,j} S_j^{\psi}}{\sum_{k=1}^N b_{i,k} S_k^{\psi}}. \quad (14)$$

The sensitivity of the importance of the link to the size of the customer is governed by the curvature parameter  $\psi$ . When  $\lambda = 0$  (and therefore  $\sigma_{\varepsilon,i}^2 = \sigma_{\varepsilon}^2$ ) and  $\psi = 1$ , we recover the simple model of Section 3.

## 4.2 Network Data, Selection, and Truncation

We use data on customer-supplier networks from the Compustat segment dataset. The sample is 1980-2012. If a customer represents more than 10% of its sellers’ revenue, then the customer’s name and sales amount are reported. Combining this information with the total sales, available in Compustat, we obtain the sales shares  $w_{i,j}$ . In a typical sample year, we have about 1,330 firms (suppliers) with non-missing customer information. We compute the

cross-sectional distribution of the number of customers or out-degree ( $N_i^{out}$ ), the dispersion of the customer network or out-Herfindahl ( $H_i^{out}$ ), the number of suppliers or in-degree ( $N_i^{in}$ ), and the dispersion of the supplier network ( $H_i^{in}$ ). Key moments of these four distributions are reported in the first column of Table 3 and discussed further below. For consistency, the moments of the firm size and variance distributions, reported in that same table, are computed over the same sample period.

There are two data issues we address as part of our estimation algorithm. First, our network data as well as our firm size and volatility data cover a subset of firms. Only Compustat firms present in the customer segment data are included in the network statistics. Customer-supplier links between public and private companies are unobserved, as well as links of public firms with missing customer data. To capture this selection effect, we simulate the model for  $N$  firms, and allow all these firms to forge links with each other according to the network formation rules outlined above. However, we select only the largest  $N_{pub}$  firms to compute model-implied moments. Implicit is the assumption that large, listed firms are the most likely to have non-missing customer data. We set  $N = 5,000$  for computational reasons;  $N_{pub}$  is a parameter estimated to match the average number of firms with non-missing network data.

Second, firms in Compustat are only required to report customers that represent at least 10% of their sales. Despite some voluntary reporting of smaller customers, the vast majority of weights  $w_{i,j}$  we observe exceed 10%. Our model accounts for all customers, with weights larger and smaller than 10%. To capture this truncation effect, we treat a link as unobserved whenever  $w_{i,j}$  is below 10%. For consistency, we delete the few  $w_{i,j}$  observations below 10% in the data as well. The procedure allows us to compare truncated moments in the model to truncated moments in the data. Since both truncated and untruncated moments are available in the model, we can make indirect inferences about the full network structure.

### 4.3 Parameters

Our empirical approach is to estimate the key parameters that govern the network by SMM. These parameter are listed in Panel A of Table 2. While all parameters jointly determine all moments, we nevertheless provide some intuition for which moments most directly identify

which parameters. First,  $\gamma$  governs the strength of the network. When  $\gamma = 0$ , there are no network effects. A value of  $\gamma$  close to 1 implies strong higher-order effects, or equivalently, slow spatial decay in the network. Second, the fundamental volatility of the innovations  $\sigma_\varepsilon$  governs the average level of firm volatility. Third, the probability of forming a supplier-customer connection in (5) depends on the parameter  $Z$ . This parameter affects the average number of customers, the average out-degree. Fourth, as just discussed,  $\lambda$  affects the relative volatility of large and small firms (over and above the endogenous differences generated by the network) and  $\psi$  affects the relative importance of large and small customers in determining a supplier's growth rate. The average number of firms with network data pins down  $N_{pub}$ . Finally, the mean and variance of the observed log firm size distribution identify the first two moments of the log-normally distributed firm size distribution,  $\mu_s$  and  $\sigma_s$ , in the model. We collect the parameters to be estimated in the vector  $\Theta$ .

Two parameters are determined outside the estimation. Mean firm growth rate  $\mu_g$  is set equal to zero, matching the observed full-sample growth rate in real market capitalization. Second,  $1 - \zeta$  is the elasticity of the average number of customers to the number of firms in the economy; see equation (6). We set  $\zeta = 0.87$  to match the estimated elasticity in our network data.<sup>7</sup>

## 4.4 Simulated Method of Moments

Our SMM estimation is standard; see Gouriéroux, Monfort, and Renault (1993) and Duffie and Singleton (1993). The estimation chooses the parameter vector  $\Theta$  which minimizes the distance between the data moments, collected in the  $1 \times K$  vector  $\mathcal{G}$ , and the corresponding moments obtained from a simulation of the network model, collected in  $\hat{\mathcal{G}}(\Theta)$ :

$$\mathcal{F} = \min_{\Theta} \mathbb{E} \left[ \left( \mathcal{G} - \hat{\mathcal{G}}(\Theta) \right) \mathcal{W} \left( \mathcal{G} - \hat{\mathcal{G}}(\Theta) \right)' \right].$$

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<sup>7</sup>We estimate a time-series regression of the log average number of connections on a constant and the log number of firms. The slope of this regression, which corresponds to  $1 - \zeta$ , is estimated at 0.13 with  $t$ -statistic of 2.85. The estimation sample is the same sample of Compustat firms for which we have non-missing customer information (1980-2012).

Table 2: PARAMETER ESTIMATES

		(1)	(2)	(3)
		Bench	No Network	No ID
$\gamma$	<i>Network propogation effect</i>	0.918 (0.026)	0.000 –	0.918 –
$\sigma_\varepsilon$	<i>Fundamental shock volatility</i>	0.298 (0.035)	0.298 –	0.388 –
$\lambda$	<i>Internal diversification effect</i>	0.131 (0.025)	0.131 –	0.000 –
$Z$	<i>New connections</i>	0.003 (0.001)	0.003 –	0.003 –
$\psi$	<i>Sensitivity of connection weight to customer size</i>	0.184 (0.003)	0.184 –	0.184 –
$\mu_s$	<i>Mean of initial log size distribution</i>	13.412 (0.243)	13.412 –	13.412 –
$\sigma_s$	<i>Standard deviation of initial log size distribution</i>	5.104 (0.102)	5.104 –	5.104 –
$N_{pub}$	<i>Number of Public Firms</i>	1332 (47.6)	1332 –	1332 –

We target  $K = 28$  moments in the estimation, listed in Table 3. They consist of four moments of the firm size distribution, four moments of the firm volatility distribution, twelve moments of the network distribution, seven correlation moments between size, variance, and network moments, and one moment related to the number of firms with network data. All moments are expressed as logs or log differences and are of similar magnitude. For simplicity and robustness we use the  $K \times K$  identity weighting matrix for  $\mathcal{W}$ . We draw an initial size distribution for  $N = 5,000$  firms and then simulate the network links (the  $b_{i,j}$  shocks) and fundamental shock volatilities (the  $\varepsilon$  shocks) 100 times. The moment function  $\hat{\mathcal{G}}(\Theta)$  is the average over the 100 draws. The calculation of the standard errors is detailed in Appendix C.2. We also provide a Wald test of the null hypothesis that all targeted moments are the same in model and data. The  $p$ -value calculation is also in Appendix C.2.

## 4.5 Estimation Results

The first column of Table 2 reports the SMM point estimates and standard errors for the parameters in  $\Theta$ . The corresponding moments are in the second column of Table 3. To

disentangle the separate roles of network effects and internal diversification, we consider two restricted versions of the benchmark model. Model “No Network” in column 2 of Table 2 is a model without network effects; it has the same parameters as the benchmark except that it sets  $\gamma = 0$ . Model “No ID” in column 3 of Table 2 shuts down internal diversification by setting  $\lambda = 0$  and  $\sigma_\varepsilon$  equal to the median volatility of the benchmark model in (13). All other parameters are held at benchmark values. The resulting model fits are reported in the last two columns of Table 3.

**Firm Size** Panel A shows that the benchmark model closely matches the moments of the firm size distribution. It generates not only the correct average firm size and firms size dispersion, but also generates large enough differences between the median firm and the smallest firms and between the median firm and the largest firms. This is mostly by virtue of the calibration. The log normality assumption on firm size fits the data very well. Since firm size is pre-determined, the restricted models fit the size distribution equally well.

**Firm Variance** The network effects reveal themselves in the distribution of firm variance, reported in Panel B of Table 3. The benchmark model matches the mean firm variance. Fundamental shock volatility is estimated just below 30% for the least volatile firms. The median firm has fundamental shock volatility of 38.9% ( $.298 + .131 \log(2)$ ). More importantly, the model generates most of the observed dispersion in firm variance (86% vs 105% in the data). The cross-sectional differences in firm variance are driven by differences in firms’ sizes and customer networks. Some firms have a poorly diversified customer base resulting in high variance, while others achieve a high degree of external diversification resulting in low variance. The point estimate for  $\gamma$  is 0.918 and is precisely estimated. A  $\gamma$  close to 1 points to strong network effects. The null hypothesis of no network effects ( $H_0 : \gamma = 0$ ) is strongly rejected.

The “No Network” model in column 3 of Table 3, generated under the null of no network effects, shows that the dispersion in firm volatility is very small. The model with no network effects still has an internal diversification mechanism which creates a negative cross-sectional correlation between size and variance. The first row of Panel D shows that the No Network

model matches that correlation closely. But the internal diversification mechanism generates far too little volatility dispersion.<sup>8</sup> Network effects are necessary. However, they are not sufficient. The “No ID” model in column 4 of Table 3 generates about half of the observed dispersion in volatility of the data and of the benchmark model that has both network effects and diversification.

**Network Moments** A key question is whether the network effects that are necessary to deliver the dispersion in firm variance are consistent with the observed properties of the network. Panel C of Table 3 shows that this is indeed the case. The median firm in the model has 1.12 customers whose sales represent at least 10% of that firm’s total sales, compared to 1.0 in the data. There are large differences in the cross-section. The firms with the most customers (at the 90th percentile) have 83% more customers than the median firm. The model produces a similar difference at 102%.<sup>9</sup>

Customer network concentration (out-Herfindahl), constructed from network weights as in equation (11), is also similar in model and data. We recall that the out-Herfindahl directly affects firm variance per equation (10). Dispersion in the out-Herfindahl directly maps into dispersion in firm variance. The model generates the same amount of dispersion in out-Herfindahls as in the data. The firms with the most sparse customer networks (90th percentile) have 185% higher out-Herfindahls than the median firm in the data and 171% in the model.

As Panel D shows, these high- $H^{out}$  firms tend to have higher variance. The cross-sectional correlation is 51% in the model, compared to 22% in the data. Firms with under-diversified (more concentrated) customer networks also tend to be smaller. The correlation is -88% in the model and -28% in the data. The reason these model correlations are not perfect is, first, because higher-order network effects affect variance as well (equation 12 versus equation 10), and second, because internal diversification provides an independent source of dispersion in variance. The model without internal diversification produces a correlation between out-

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<sup>8</sup>While internal diversification generates substantial variation in firm variance, most of this variation occurs between very small firms. The dispersion among the largest  $N_{pub}$  firms, reported in the table, is very small.

<sup>9</sup>The restricted models have the same network moments as the benchmark model since they have the same size distribution.

Table 3: SIZE, VARIANCE, AND NETWORK MOMENTS

	(1)	(2)	(3)	(4)
	Data	Model Bench	Model No Network	Model No ID
log Size				
Average	19.83	19.83	19.83	19.83
Stadard Deviation	2.56	2.53	2.53	2.53
prc 50 - prc 10	2.91	2.37	2.37	2.37
prc 90 - prc 50	3.88	3.91	3.91	3.91
log Variance				
Average	-1.40	-1.40	-2.42	-1.39
Stadard Deviation	1.05	0.86	0.01	0.46
prc 50 - prc 10	1.38	0.40	0.00	0.24
prc 90 - prc 50	1.31	1.69	0.02	0.93
$N^{out}$				
Median	1.00	1.12	1.12	1.12
log prc 50 - log prc 10	0.00	0.08	0.08	0.08
log prc 90 - log prc 50	0.83	1.02	1.02	1.02
$H^{out}$				
Median	0.05	0.18	0.18	0.18
log prc 50 - log prc 10	1.27	1.88	1.88	1.88
log prc 90 - log prc 50	1.85	1.71	1.71	1.71
$N^{in}$				
Median	1.00	1.00	1.00	1.00
log prc 50 - log prc 10	0.00	0.00	0.00	0.00
log prc 90 - log prc 50	1.69	1.09	1.09	1.09
$H^{in}$				
Median	1.00	1.00	1.00	1.00
log prc 50 - log prc 10	0.95	0.89	0.89	0.89
log prc 90 - log prc 50	0.00	0.00	0.00	0.00
Corr(log Size, log Var)	-0.64	-0.61	-0.67	-0.69
Corr(log $N^{out}$ , log Size)	-0.00	0.31	0.31	0.31
Corr(log $H^{out}$ , log Size)	-0.28	-0.88	-0.88	-0.88
Corr(log $H^{out}$ , log Var)	0.22	0.51	0.68	0.75
Corr(log $N^{in}$ , log Size)	0.53	0.34	0.34	0.34
Corr(log $H^{in}$ , log Size)	-0.47	-0.34	-0.34	-0.34
Corr(log $H^{in}$ , log Var)	0.13	0.19	0.19	0.21
log num of Net firms	7.15	6.95	6.95	6.95
FVAL	—	2.88	7.53	3.72
Wald	—	24.76	—	—
p-value	—	0.64	—	—

Herfindahl and variance that is indeed higher at 75%. The random nature of the network accounts for the less than perfect correlation between size and out-Herfindahl.

Our network model assumes that larger firms have more connections, on average. Panel



D bears this out with a 31% correlation between size and number of customers  $N^{out}$ . In the data the correlation is 0%, suggesting little empirical support for the assumption. However, that correlation is subject to a large truncation bias. The correlation between untruncated  $N^{out}$  and size is 69% in the model, implying a 38% bias. Applying that bias to the data would suggest a 38% correlation, indicating stronger support for the assumption. Appendix C.3 discusses biases arising from truncation in more detail. It also uses industry-level input-output data from the Bureau of Economic Analysis (BEA) which do not suffer from truncation. The BEA data show a 58% correlation between size and untruncated  $N^{out}$ . The BEA data imply an even larger truncation bias. Appendix C.3 also provides evidence for the second size assumption underlying the model, that larger customers represent a larger share of a firm’s sales.

We also study the implications of our model for two in-degree moments: the number of suppliers a firm has ( $N^{in}$ ) and the concentration of the supplier network ( $H^{in}$ ). Because the Compustat segment data do not contain information on a firm’s suppliers, we must use the customer data to infer supplier links. Since we measure suppliers the exact same way in the model, the selection issues that this data structure creates are reproduced in the model. Panel C of Table 3 shows that the model matches the median number of (truncated) suppliers and its dispersion, as well as the (truncated) supplier concentration and its dispersion. The model also fits the cross-sectional correlations between in-degree moments and firm size and variance quite well. The positive correlation between size and number of suppliers is entirely caused by truncation, since the model assumes no relationship. Subtracting the 34% upward bias from the 53% correlation in the data reduces the empirical estimate to below 20%.

We conclude that our simple model is consistent with the key moments of the cross-sectional distribution of firm size, firm variance, and the number and concentration of customer-supplier relationships.

Panel F of Table 3 reports the SMM function value, which is the sum of the squared distances between model and data moments. The restricted versions of the model in the last two columns have a substantially worse fit. Removing internal diversification increases the  $L_2$  norm from 2.88 to 3.72. Removing the network mechanism makes for a far greater deterioration in fit, from 2.88 to 7.53. The panel also reports a Wald statistic for the null

hypothesis that all moments (in Table 3) are equal in model and data. We fail to reject the null hypothesis, providing further support for the network model.

## 5 Additional Testable Implications

The network model suggests two additional testable predictions. First, a firm’s volatility should depend on its size and on its out-Herfindahl. Second, all firms’ volatilities should comove because they are driven by a common factor: the economy-wide dispersion in firm size. We find empirical support for both predictions.

### 5.1 Determinants of Firm-level Volatility

A large literature has examined the determinants of firm-level volatility on the basis of firm characteristics. Black (1976) proposed that differences in leverage drive heterogeneity in firm volatility, Comin and Philippon (2005) study the role of industry competition and R&D intensity, Davis et al. (2006) emphasize age effects, and Brandt et al. (2010) argue that institutional ownership is related to volatility. Our model predicts a negative correlation between volatility and firm size and a positive correlation between volatility and customer network concentration (out-Herfindahl), controlling for other firm characteristics.

Table 4 reports panel regressions of firm-level log annual return variance on log size and log out-Herfindahl, controlling for a range of firm characteristics including log age, leverage, industry concentration, as well as industry and cohort fixed effects.<sup>10</sup> Consistent with our model, we find that size and out-Herfindahl are important determinants of differences in firm variance. The elasticity of firm variance to firm size is around -0.15 and precisely estimated in all specifications. Decreasing the log firm size from the 90th percentile to the median increases firm variance by 58% ( $-3.88 \times -0.15$ ), which is more than one-half of a standard deviation in variance. The elasticity of firm variance to customer concentration is 0.05 and also significant in all specifications. Increasing the log out-Herfindahl from the median to the 90th percentile increases firm variance by 9.3% ( $1.85 \times 0.05$ ). As we showed in Table 3, log size and log out-Herfindahl are negatively correlated in both network model and data.

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<sup>10</sup>Cohorts are defined by the year in which a firm first appears in the CRSP/Compustat data set.

Network concentration conveys similar information since size determines network structure. Given that concentration in the customer network is measured with noise, size likely captures an important part of the true network concentration effect. Nevertheless, the table provides strong evidence that network concentration matters separately for firm variance and survives the inclusion of other well-known volatility determinants such as age and leverage.

## 5.2 Comovement in Firm Volatilities

Recent research has documented a puzzling degree of common variation in the panel of firm-level volatilities. Herskovic, Kelly, Lustig, and Nieuwerburgh (2016) show that firm-level stock return volatilities share a single common factor that explains roughly one-third of the variation in log volatilities for the entire panel of CRSP stocks.<sup>11</sup> They also show that this strong factor structure is not only a feature of return volatilities, but also of sales growth volatilities. The puzzling aspect of this result is that the factor structure remains nearly completely intact after removing all common variation in returns or sales growth rates. Hence, common volatility dynamics are unlikely to be driven by an omitted common return or sales growth factor. That paper raises the question of what the common factor in (idiosyncratic) firm volatility might be.

Our granular network model suggests an answer. It predicts a high degree of comovement in firm volatility arising from concentration in the firm size distribution. Proposition 2 implies that the dispersion of the firm size distribution is the common factor. Proposition 4 further clarifies that large firms have low exposure to this size concentration factor. In other words, fluctuations in firm size dispersion are an important determinant of fluctuations in firm-level volatility, and more so for small than for large firms. Furthermore, if the true data generating process is a network model, then factor model residual volatilities will possess a similar degree of comovement as total volatilities, despite residual growth rates themselves being nearly uncorrelated. To understand this, consider that in the literature, idiosyncratic volatility

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<sup>11</sup>Similarly, Engle and Figlewski (2015) document a common factor in option-implied volatilities since 1996, and Barigozzi, Brownlees, Gallo, and Veredas (2014) examine the factor structure in realized volatilities of intra-daily returns since 2001. Bloom, Floetotto, Jaimovich, Saporta-Eksten, and Terry (2016) show that firm-specific sales growth and productivity exhibit cross-sectional dispersion that fluctuates with the macroeconomy.

Table 4: DETERMINANTS OF FIRM-LEVEL VOLATILITY

	Dependent Variable: Log Firm Volatility											
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)
Log Size	-0.16		-0.15	-0.15		-0.15	-0.14		-0.15	-0.15		-0.15
	-16.98		-15.67	-28.68		-18.38	-19.45		-19.02	-20.15		-19.60
Log $H^{out}$		0.06	0.03		0.05	0.02		0.05	0.02		0.05	0.02
		4.22	2.65		4.46	2.17		4.44	1.96		18.67	6.29
Log Age				-0.09	-0.20	-0.14						
				-2.89	-9.93	-8.15						
Leverage				0.14	0.09	0.02	0.16	0.10	0.04			
				1.49	1.24	0.40	1.73	1.43	0.79			
Ind. Conc.&			0.53	0.02	0.13	0.59	0.04	0.16				
				1.70	0.04	0.42	2.02	0.11	0.50			
Constant	-1.00	-0.37	-0.80	-0.92	0.00	-0.55	-1.15	-0.46	-0.89	-0.98	-0.41	-0.84
	-19.70	-8.82	-11.81	-5.88	0.05	-4.56	-10.89	-4.46	-8.48	-26.85	-10.66	-18.94
FE	None	None	None	None	None	None	Cohort	Cohort	Cohort	Ind.	Ind.	Ind.
Obs.	167,440	28,718	28,232	133,026	22,970	22,949	136,879	23,084	23,063	167,440	28,718	28,232
Adj. $R^2$	0.283	0.028	0.299	0.338	0.133	0.373	0.358	0.145	0.396	0.397	0.137	0.379

Notes: This table reports panel regressions of firms' return volatility on a range of characteristics including log size, log customer network Herfindahl ( $H^{out}$ ), log age, book leverage, competition measured by industry size concentration, as well as cohort and industry fixed effects in certain specifications. Size is defined as the lagged market equity relative to the cross-sectional average market equity. Book leverage is defined as debt in current liabilities plus total long-term debt divided by total assets. Industries are defined as 4-digit SIC codes and industry concentration is measured by the Herfindahl index of sales. Cohorts are defined by the year in which a firm first appears in the CRSP/Compustat data set. Standard errors are clustered at the industry level. Sample is at annual frequency from 1980 to 2012.

is typically constructed by first removing the aggregate component of growth rates with a statistical procedure such as principal component analysis, then calculating the volatilities of the residuals. In a granular network model like ours, such a factor regression approach is misspecified. There is no dimension-reducing common factor that fully captures growth rate comovement since, by virtue of the network, *every* firm’s shock may be systematic. A sign of the misspecification of the factor model is that the residuals exhibit a volatility factor structure that looks very similar to the factor structure for total firm volatility.

Table 5 provides empirical support for the fact that the common factor in firm volatilities is the firm size dispersion. It contains panel regressions for firm variance using three different factor models. In column 1, the factor is the lagged dispersion in log firm size (market-based). In column 2 the factor is the lagged cross-sectional average volatility. Since this is essentially the first principal component of firm volatilities, it is a natural yardstick for any one-factor model. The third column instead uses the contemporaneous average volatility as a factor. Because it uses finer conditioning information, it can be considered an upper bound on the explanatory power of a single factor. The main point of Table 5 is that the explanatory power of firm size dispersion ( $R^2$  of 15.65%) is nearly as high as that of mean volatility ( $R^2$  of 20.94%), and about half as large as the one-factor upper bound ( $R^2$  of 33.65%). Firm size dispersion is a powerful factor.

Columns 4-6 repeat the exercise on model-generated data. For the model calculations, we match the observed cross-sectional average and standard deviation of the log size distribution, year by year, from 1980 to 2012. The model implies a firm variance distribution in each year. We use the model-implied time series for firm size dispersion and mean firm variance to estimate the factor model. Similar to the data, lagged firm size dispersion is about as strong a predictor as lagged average firm variance. The  $R^2$  is about half that of contemporaneous mean variance.<sup>12</sup>

Panel B confirms the model prediction that large firms, in quintile 5 (Q5) of the firm size distribution have lower exposure to the common size dispersion factor than small firms in quintile 1 (Q1). Finally, the data in Panel A show that the size dispersion factor explains

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<sup>12</sup>In unreported results, we confirmed these results for fundamental variance, as well as for residual market and fundamental variances. Residual variances are obtained by orthogonalizing firm returns or sales growth rates to a common factor in returns or sales growth rates, and then taking a variance.

Table 5: COMOVEMENT IN FIRM VARIANCES

	Data			Model		
	(1)	(2)	(3)	(4)	(5)	(6)
	Factors			Factors		
	$\sigma_{S,t-1}$	$\mu_{\sigma,t-1}$	$\mu_{\sigma,t}$	$\sigma_{S,t-1}$	$\mu_{\sigma,t-1}$	$\mu_{\sigma,t}$
<b>Panel A: Return Volatility <math>R^2</math> by Size Quintile</b>						
All firms	15.65	20.94	33.65	52.36	56.19	93.23
Q1	20.96	25.17	31.82	52.28	55.34	84.17
Q5	10.53	16.54	35.81	52.14	56.46	98.44
<b>Panel B: Return Volatility Loadings by Size Quintile</b>						
All firms	0.55	0.48	0.73	1.31	0.67	1.06
Q1	0.76	0.60	0.75	1.12	0.57	1.15
Q5	0.36	0.42	0.76	0.31	0.16	0.24

*Notes:* Each factor model regression is a time-series regression of log total volatility on a factor. Total volatility is measured as variance of daily returns within the calendar year, and size is measured as market equity. All volatility factor regressions take the form  $\log \sigma_{i,t} = a_i + b_i \text{factor}_t + e_{i,t}$ . For each stock  $i$ , we estimated the factor model and report the cross-sectional average of the slopes in Panel A and cross-sectional average of  $R^2$  in Panel C. We require a minimum of 25 observation to run the regression. In each panel, we report the full sample average and the average within the first and last quintile of the time-series average size distribution. We deflate size by CPI when constructing size quintiles. We consider three different volatility factors. The first, motivated by our network model, is the lagged cross section standard deviation of log market equity,  $\sigma_{S,t-1}$  (Columns 1 and 4). The second and third factors we consider are the lagged and contemporaneous cross-sectional average log volatility,  $\mu_{\sigma,t-1}$  (Columns 2 and 5) and  $\mu_{\sigma,t}$  (Columns 3 and 6). All three factors use the full sample in the cross section. Columns 1-3 report the estimation results for the data at annual frequency from 1926 to 2016. We estimate the same factor structure for the model as well in Columns 4-6.

a smaller fraction of the variation for large firms than for small firms, as predicted by Proposition 4.

## 6 Conclusion

We document new features of the joint evolution of the firm size and firm volatility distribution and propose a new model to account for these features. In the model, shocks are transmitted from customers to suppliers in a production network. The larger the supplier, the more customer connections it has, the better diversified its customer base, and the lower its volatility. Large customers have a strong influence on their suppliers, so shocks to large firms have an important effect throughout the economy. A equilibrium model with consumer

demand shocks and multiple inter-connected sectors of production delivers a structural interpretation of the reduced-form network model.

When the firm size dispersion increases in this economy, large firms become more important, and many customer networks become less diversified. In those times, average firm volatility is higher as is the cross-sectional dispersion of volatility. Because the underlying innovations are i.i.d. over time, the model endogenously generates “uncertainty shocks.” The model quantitatively replicates the most salient features of the firm size and the volatility distributions, while being consistent with data on customer network linkages. The estimation reveals the importance of strong network effects, without which the model cannot account for the large dispersion in firm volatilities.

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# A Structural Model of Upstream Shock Transmission

This appendix sets up a simple structural model that delivers upstream transmission of shocks, and that delivers the network equation (2). It is a simple version of the canonical multi-firm Long and Plosser (1987) model. Our derivation relies heavily on teaching notes that Xavier Gabaix graciously shared with us (Gabaix (2016)). The result generalizes Acemoglu, Akcigit, and Kerr (2015).

## A.1 Setup

Households supply labor  $\bar{L}$  inelastically. They derive utility from consuming each of the  $N$  goods:

$$U = \sum_{i=1}^N \theta_i \log c_i,$$

where  $\theta_i$  is a taste shifter for good  $i$ . The household chooses its consumption basket to maximize total utility.

Good  $i$  is produced using labor as well as all of the other commodities:

$$Y_i = \exp(z_i) L_i^{b_i} \Pi_j X_{ij}^{a_{ij}},$$

where the coefficient satisfy  $\sum_{j=1}^N a_{ij} + b_i = 1, i = 1, 2, \dots, N$ , implying constant returns to scale.

## A.2 Characterizing Equilibrium

Market clearing for goods implies that, for each good  $j$ :

$$C_j + \sum_{i=1}^N X_{ij} = Y_j \tag{A1}$$

Market clearing in the labor market implies that  $\sum_{i=1}^N L_i = \bar{L}$ .

Each firm maximizes profits. The first order conditions for profit maximization dictate that, for each good  $i$ , the demand for labor inputs and other goods satisfy:

$$p_j X_{ij} = a_{ij} p_i Y_i, \tag{A2}$$

$$w L_i = b_i p_i Y_i. \tag{A3}$$

Define economy-wide value added  $G$  as  $G = \sum_{i=1}^N p_i C_i$ . Define the ratio of firm  $i$ 's output to total value added as  $\psi_i$ :

$$\psi_i = \frac{p_i Y_i}{G}.$$

We can restate equation (A2) as:

$$a_{ij} = \frac{p_j X_{ij}}{p_i Y_i} = \frac{p_j X_{ij}}{\psi_i G}. \tag{A4}$$

This shows that  $a_{ij}$  is the cost of input  $j$  in the value of the output  $i$ .

The household's first order condition for good  $j$  is given by:

$$p_j C_j = \theta_j G$$

Substituting this equation and equation (A4) into the market clearing condition (A1), we obtain:

$$\begin{aligned} p_j Y_j &= p_j C_j + \sum_i p_j X_{ij}, \\ \psi_j G &= \left( \theta_j + \sum_i a_{ij} \psi_i \right) G, \\ \psi_j &= \theta_j + \sum_i a_{ij} \psi_i. \end{aligned}$$

If sector/firm  $i$  is the supplier of good  $i$  while firm/sector  $j$  denotes a customer who uses good  $i$  as an input, then we get:

$$\psi_i = \theta_i + \sum_{j=1}^N a_{ji} \psi_j \quad (\text{A5})$$

We use  $\mathbf{A}$  to denote an  $N \times N$  matrix with  $a_{ij}$  as the  $(i, j)$ th element of the matrix. We use  $\boldsymbol{\psi}$  to denote the vector of sectoral shares with  $\psi_i$  as the  $i$ th element, and we use  $\boldsymbol{\theta}$  to denote the vector of preference parameters  $\theta_i$ . Using matrix notation, equation (A5) can be stated as follows:

$$\boldsymbol{\psi} = \boldsymbol{\theta} + \mathbf{A}' \boldsymbol{\psi}, \quad (\text{A6})$$

which in turn implies that the sectoral shares can be stated as follows:

$$\boldsymbol{\psi} = (\mathbf{I} - \mathbf{A}')^{-1} \boldsymbol{\theta}.$$

The sectoral shares do not depend on the productivity shocks, but are only determined by preferences  $\boldsymbol{\theta}$  and costs shares  $\mathbf{A}$ .

### A.3 Link with W Matrix

Define  $\tilde{w}_{ij}$  as the ratio of sales of supplier  $i$  to customer  $j$ , divided by the output of good  $i$ :

$$\tilde{w}_{ij} = \frac{p_i X_{ji}}{p_i Y_i} = \frac{p_i X_{ji}}{p_j Y_j} \frac{p_j Y_j}{p_i Y_i} = a_{ji} \frac{\psi_j}{\psi_i} \quad (\text{A7})$$

Let  $\boldsymbol{\Psi} = \text{diag}(\boldsymbol{\psi})$  be the  $N \times N$  matrix with the vector  $\boldsymbol{\psi}$  on its diagonal. Then (A7) can be written in matrix notation as:

$$\tilde{\mathbf{W}} = \boldsymbol{\Psi}^{-1} \mathbf{A}' \boldsymbol{\Psi}.$$

Note that the rows of the  $\tilde{\mathbf{W}}$  sum to less than one:

$$\begin{aligned} \sum_{j=1}^N \tilde{w}_{ij} &= \sum_{j=1}^N a_{ji} \frac{\psi_j}{\psi_i}, \\ &= \sum_{j=1}^N \frac{p_i X_{ji}}{p_i Y_i}, \\ &= \frac{p_i Y_i - p_i C_i}{p_i Y_i} \equiv \gamma_i < 1, \end{aligned}$$

unless good  $i$  is only used as an intermediary good but not used for final consumption. We call the fraction of total output of good  $i$  used as inputs for other sectors,  $\gamma_i$ .

In our empirical implementation, we cannot directly measure the expenditure share matrix  $\mathbf{A}$  for the U.S economy. However, we make two plausible assumptions. First, we assume that  $\tilde{w}_{ij}$  is more likely to be non-zero when firm  $i$  is larger. Second, we assume that  $\tilde{w}_{ij}$  is more likely to be larger when firm  $j$  is larger. Sales is our proxy for size.

We also cannot accurately measure  $\gamma_i$  for different goods produced by different firms. In our empirical work, we impose that all the rows of  $\mathbf{W}$  sum to a constant  $\gamma$ , where  $1 - \gamma$  denotes the average share of output that accrues to final consumption. That is,  $1 - \gamma$  measures the leakage in the production network due to final goods consumption. Define  $w_{ij} = \tilde{w}_{ij}/\gamma$  and thus the matrix  $\mathbf{W}$  as:

$$\mathbf{W} = \frac{1}{\gamma} \tilde{\mathbf{W}}$$

We note that the rows of  $\mathbf{W}$  sum to one:  $\sum_{j=1}^N w_{ij} = 1$ .

## A.4 Upstream Transmission of Preference Shocks

We are now ready to state the main result, Proposition 1, which delivers the network equation of the reduced-form model in the main text from the structural model described in this appendix.

**Proposition.** *The responses of firm output to taste shocks  $d\theta$  approximately equals:*

$$\frac{d\mathbf{Y}}{\mathbf{Y}} = (\mathbf{I} - \gamma\mathbf{W})^{-1} \frac{d\theta}{\psi}. \quad (\text{A8})$$

*Proof.* By differentiation of equation (A5), we obtain the following system of equations:

$$d\psi_i = d\theta_i + \sum_{j=1}^N a_{ji} d\psi_j, i = 1, \dots, N.$$

This system of equations implies the following system for growth rates in sectoral shares:

$$\begin{aligned} \frac{d\psi_i}{\psi_i} &= \frac{d\theta_i}{\psi_i} + \sum_{j=1}^N a_{ji} \frac{d\psi_j}{\psi_i}, \\ \frac{d\psi_i}{\psi_i} &= \frac{d\theta_i}{\psi_i} + \sum_{j=1}^N \gamma w_{ij} \frac{d\psi_j}{\psi_j}, \end{aligned}$$

where we have used the mapping from the cost shares to the expenditure shares in (A7) and the link between  $w_{ij}$  and  $\tilde{w}_{ij}$ .

Hence, in matrix notation, we get the following result:

$$\frac{d\psi}{\psi} = \frac{d\theta}{\psi} + \gamma\mathbf{W} \frac{d\psi}{\psi}.$$

This in turn implies that:

$$\frac{d\psi}{\psi} = (\mathbf{I} - \gamma\mathbf{W})^{-1} \frac{d\theta}{\psi}.$$

Let  $\hat{x}$  denote  $dx/x$ . To complete the proof, we need to show that  $\hat{\mathbf{Y}} = \hat{\psi}$ .

We start with the goods market. First, note that the firm share  $\psi_i = \frac{p_i Y_i}{G}$ . Hence, the percentage changes satisfy:  $\hat{p}_i + \hat{Y}_i = \hat{\psi}_i$ , because taste shocks do not change total value added  $G$ . Similarly, for sector  $j$ :  $\hat{p}_j + \hat{Y}_j = \hat{\psi}_j$ . Second, equation (A4) implies that:

$$\hat{\psi}_i = \hat{p}_j + \hat{X}_{ij}.$$

By combining the two preceding equations, we get the following expression for the growth rate of intermediate inputs:

$$\hat{X}_{ij} = \hat{\psi}_i - \hat{\psi}_j + \hat{Y}_j. \quad (\text{A9})$$

Next, we turn to the labor market where:

$$wL_i = b_i p_i Y_i = \psi_i b_i G.$$

In growth rates, this implies that:

$$\widehat{L}_i = \widehat{\psi}_i. \quad (\text{A10})$$

Finally, we consider the change in output in firm  $i$ , which is given by:

$$\widehat{Y}_i = \widehat{z}_i + b_i \widehat{L}_i + \sum_j a_{ij} \widehat{X}_{ij}$$

Because there are only preference shocks and no productivity shocks,  $\widehat{z} = 0$ . After substituting expressions (A9) and (A10), we obtain:

$$\widehat{Y}_i = b_i \widehat{\psi}_i + \sum_j a_{ij} (\widehat{\psi}_i - \widehat{\psi}_j + \widehat{Y}_j).$$

This can be simplified to yield:

$$\widehat{Y}_i = \widehat{\psi}_i + \sum_j a_{ij} (-\widehat{\psi}_j + \widehat{Y}_j),$$

where we have used the constant returns to scale assumption  $b_i + \sum_j a_{ij} = 1$ . In matrix notation, this implies that:

$$\widehat{\mathbf{Y}} - \mathbf{A}\widehat{\mathbf{Y}} = \widehat{\boldsymbol{\psi}} - \mathbf{A}\widehat{\boldsymbol{\psi}} \Rightarrow \widehat{\mathbf{Y}} = \widehat{\boldsymbol{\psi}}.$$

□

By setting the reduced-form shocks equal to the scaled taste shocks  $\boldsymbol{\varepsilon} = d\boldsymbol{\theta}/\boldsymbol{\psi}$  and  $\mu_g = 0$  (an assumption we make in our estimation exercise), equation (A8) is identical to the network equation (3) in the main text.

The structural model provides an interpretation of the shocks as consumer preference shocks. It also provides an interpretation of the network coefficient  $\gamma$ , which modulates the strength of the network effects. In the structural model,  $\gamma$  is the sectoral average of the share of output used as inputs in other sectors. Intuitively, shocks to one sector do not propagate if other sectors do not use that sector's good as an input. Finally, the structural model makes clear that demand shocks propagate upstream. A simple example where good 1 is the intermediate good and good 2 is the final good illustrates this:

$$\mathbf{A} = \begin{bmatrix} 0 & 0 \\ \alpha & 0. \end{bmatrix}$$

Then

$$\tilde{\mathbf{W}} = \gamma \mathbf{W} = \begin{bmatrix} 0 & \alpha \frac{\psi_2}{\psi_1} \\ 0 & 0. \end{bmatrix}$$

and

$$(\mathbf{I} - \gamma \mathbf{W})^{-1} = \begin{bmatrix} 1 & \alpha \frac{\psi_2}{\psi_1} \\ 0 & 1. \end{bmatrix}$$

Using our expression, this implies that the growth rates can be expressed as:

$$\frac{d\psi_1}{\psi_1} = \frac{d\theta_1}{\psi_1} + \alpha \frac{\psi_2}{\psi_1} \frac{d\theta_2}{\psi_2}. \quad (\text{A11})$$

$$\frac{d\psi_2}{\psi_2} = \frac{d\theta_2}{\psi_2}. \quad (\text{A12})$$

Generally, if  $\mathbf{A}$  is lower triangular, which means that  $i$  uses only inputs from  $j < i$ , then  $(\mathbf{I} - \gamma \mathbf{W})^{-1}$  is upper triangular. Hence, demand shocks are transmitted upstream, from the final goods sector (sector 2) to the intermediate goods sector (sector 1).

## B Proofs of Propositions in Main Text

Returning to the full network specification in (2), the following results formalize the preceding intuition in a large  $N$  economy. First, we provide a limiting description of each supplier's customer Herfindahl. Throughout, we use asymptotic equivalence notation  $x \sim y$  to denote that  $x/y \rightarrow 1$  as  $N \rightarrow \infty$ . Where there is no ambiguity, we suppress time and/or firm subscripts. We start by stating and proving two helpful lemmas.

### Lemma 1

**Lemma 1.** *Consider a sequence of economies indexed by the number of firms  $N$ . If  $S_i$  has finite variance, then*

$$H_i \sim \frac{1}{N^{(1-\zeta)}} \frac{Z}{\tilde{S}_i} \frac{E[S^2]}{E[S]^2}.$$

*In particular, if  $\log(S_i)$  is normal with variance  $\sigma_s^2$ , then*

$$H_i \sim \frac{1}{N^{1-\zeta}} \frac{Z}{\tilde{S}_i} \exp(\sigma_s^2).$$

*Proof.* Note that sample moments of  $\{B_{i,k}S_k\}_{k=1}^N$  are equivalent to the sample moments of a random sample of  $N_i$  elements from the size distribution, where  $N_i = \sum_k B_{i,k}$ . If  $S_i$  has finite variance, then the LLN implies that

$$N_i H_i = \frac{N_i^{-1} \sum_k B_{i,k} S_k^2}{(N_i^{-1} \sum_k B_{i,k} S_k)^2} \xrightarrow{a.s.} \frac{E[S^2]}{E[S]^2}.$$

Because  $N_i$  is a sum of *iid* bernoulli variables,  $N_i \sim Np_i$ . This implies that  $N_i/N^{(1-\zeta)} \xrightarrow{a.s.} \tilde{S}_i/Z$ , which delivers the result. Under lognormality,  $E[S^2]/E[S]^2 = e^{\sigma_s^2}$ .  $\square$

This lemma highlights the common structure in customer Herfindahls across suppliers. The ratio of the second non-central moment of the size distribution to the squared first moment captures the degree of concentration in the entire firm size distribution. Economy-wide firm size concentration affects the customer network concentration of all firms. Differences in customer Herfindahl across suppliers are inversely related to supplier size, capturing the model feature that larger firms are connected to more firms on average. Under lognormality,  $E[S^2]/E[S]^2$  equals the (exponentiated) cross-sectional standard deviation of log firm size. The lemma applies more generally to the case where the size distribution has finite variance, and below we analyze firm volatility decay in the case of power law size distributions with infinite variance but finite mean.

**Lemma 2** As described above,  $H_i$  is the first order determinant of firm volatility in our model. The following intermediate lemma allows us to capture higher order network effects that contribute to a firm's variance.

**Lemma 2.** *Consider a sequence of economies indexed by the number of firms  $N$ . Define the matrix  $\bar{W}$  as  $[\bar{W}]_{i,j} = \tilde{S}_j/N$ . If  $S_i$  has finite variance, then for  $q = 2, 3, \dots$ ,*

$$[W^q]_{i,j} \sim \frac{\tilde{S}_j}{N}.$$

*Furthermore,*

$$[W\bar{W}']_{i,j} \sim \frac{E[S^2]}{NE[S]^2} \quad \text{and} \quad [\bar{W}\bar{W}'] \sim \frac{E[S^2]}{NE[S]^2}.$$

*Proof.* We begin with the case  $q = 2$ .

$$[W^2]_{i,j} = \sum_k w_{i,k} w_{k,j} = \sum_k \left( \frac{B_{i,k} S_k}{\sum_m B_{i,m} S_m} \frac{B_{k,j} S_j}{\sum_l B_{k,l} S_l} \right) = \left( \frac{S_j}{\sum_m B_{i,m} S_m} \right) \left( \sum_k \frac{B_{i,k} B_{k,j} S_k}{\sum_l B_{k,l} S_l} \right).$$

As in the previous lemma, the LLN implies  $N_i^{-1} \sum_k B_{i,k} S_k \xrightarrow{a.s.} E[S]$ . This implies

$$\frac{S_j}{\sum_m B_{i,m} S_m} \sim \frac{\tilde{S}_j}{\tilde{S}_i} \frac{Z}{N^{1-\zeta}}.$$

To characterize the asymptotic behavior of the second term, first note that Markov's LLN for heterogeneously distributed variables implies that  $N^{-(1-2\zeta)} \sum_k B_{i,k} B_{k,j} \xrightarrow{a.s.} \tilde{S}_i / Z^2$ .<sup>13</sup> This in turn implies that

$$\sum_k \frac{B_{i,k} B_{k,j} S_k}{\sum_l B_{k,l} S_l} \sim \sum_k \frac{B_{i,k} B_{k,j} S_k}{\frac{\tilde{S}_k}{Z} N^{1-\zeta}} \sim \frac{Z}{N^{1-\zeta}} N p_i E[p_k] = N^{-\zeta} \frac{\tilde{S}_i}{Z}.$$

Together with the asymptotic behavior of the first term we have

$$[W^2]_{i,j} \sim \frac{\tilde{S}_j}{N}.$$

For induction, assume true for  $q > 2$ , so that

$$[WW^q]_{i,j} \sim \sum_k \frac{B_{i,k} S_k \tilde{S}_j}{N (\sum_l B_{i,l} S_l)} \sim \frac{\tilde{S}_j}{N} \frac{\sum_k B_{i,k} S_k}{\sum_l B_{i,l} S_l} \sim \frac{\tilde{S}_j}{N}.$$

This quantity is of the same form as  $[W^2]_{i,j}$ , and the same rationale applied in that case gives  $[W^{q+1}]_{i,j} \sim \tilde{S}_j / N$ .

We also have that

$$[W\bar{W}']_{i,j} = \sum_k \frac{B_{i,k} S_k^2}{(\sum_l B_{i,l} S_l) N E[S]} \sim \frac{E[S^2]}{N E[S]^2}$$

because the finite variance of  $S_k$  and the LLN imply  $N_i^{-1} \sum_k B_{i,k} S_k \xrightarrow{a.s.} E[S]$  and  $N_i^{-1} \sum_k B_{i,k} S_k^2 \xrightarrow{a.s.} E[S^2]$ . Similarly,

$$[\bar{W}\bar{W}']_{i,j} = \sum_k \frac{\tilde{S}_k^2}{N^2} \sim \frac{E[S^2]}{N E[S]^2}.$$

Note that  $H_i = [WW']_{ii}$  and Lemma 1 applies. For off-diagonal elements of  $WW'$ ,

$$[WW']_{i,j} = \frac{\sum_k B_{i,k} B_{j,k} S_k^2}{(\sum_l B_{i,l} S_l) (\sum_l B_{j,l} S_l)} \sim \frac{E[S^2]}{N E[S]^2}.$$

□

**Main Result on Firm Volatility: Proposition 2** Our main theoretical proposition connects the variance of a firm to its size and to the concentration of firm sizes throughout the economy.

**Proposition.** *Consider a sequence of economies indexed by the number of firms  $N$ . If  $S_i$  has finite variance, then the Leontief inverse has limiting behavior described by*

$$(I - \gamma W)^{-1} \sim I + \gamma W + \frac{\gamma^2}{1 - \gamma} \bar{W},$$

and firm volatility has limiting behavior described by

$$V(g_i) \sim \sigma_\varepsilon^2 \left[ 1 + \left( \frac{\gamma^2}{N^{1-\zeta}} \frac{Z}{\tilde{S}_i} + \frac{2\gamma^3 - \gamma^4}{N(1-\gamma)^2} \right) \frac{E[S^2]}{E[S]^2} + \frac{2\gamma^2}{1-\gamma} \frac{\tilde{S}_i}{N} \right].$$

<sup>13</sup>See Theorem 3.7 of White (2001). To Markov's LLN, the so-called Markov condition must hold. Applied to the current setting, this requires  $\sum_{k=1}^\infty E|B_{k,j} - p_k|^{1+\delta} / k^{1+\delta} < \infty$ . Because  $B_{k,j}$  are independent Bernoulli draws, this condition is satisfied.



*Proof.* Because  $V(g) = \sigma_\varepsilon^2(I - \gamma W)^{-1}(I - \gamma W')^{-1}$ , we study the behavior of  $(I - \gamma W)^{-1}$  as the number of firms  $N$  becomes large. Noting that  $(I - \gamma W)^{-1} = I + \gamma W + \gamma^2 W^2 + \dots$ , Lemma 2 establishes the asymptotic equivalence

$$(I - \gamma W)^{-1} \sim I + \gamma W + \frac{\gamma^2}{1 - \gamma} \bar{W}.$$

The outer product of  $I + \gamma W + \frac{\gamma^2}{1 - \gamma} \bar{W}$  is

$$I + \gamma W + \gamma W' + \gamma^2 W W' + \frac{\gamma^2}{1 - \gamma} \bar{W} + \frac{\gamma^2}{1 - \gamma} \bar{W}' + \frac{\gamma^3}{1 - \gamma} W \bar{W}' + \frac{\gamma^3}{1 - \gamma} \bar{W} W' + \frac{\gamma^4}{(1 - \gamma)^2} \bar{W} \bar{W}'. \quad (\text{B1})$$

From Lemmas 1 and 2, the behavior of the  $i^{\text{th}}$  diagonal element of  $V(g)$  in a large economy is described by the stated asymptotic equivalence. In the lognormal special case,  $E[S^2]/E[S]^2 = \exp(\sigma_s^2)$ .  $\square$

**Firm Size Follows Power Law: Proposition 3** Thus far we have assumed that the firm size distribution has finite variance, thus the slow volatility decay in Proposition 2 arises only due to network sparsity. Gabaix (2011) emphasizes that extreme right skewness of firm sizes can also slow down volatility decay in large economies. In the next result, we show that the firm-level network structure adds a mechanism to further slow down volatility decay beyond Gabaix's (2011) granularity mechanism, which depends on the power law behavior of the size distribution. In the absence of network effects, power law-based firm sizes would imply that firm variance decays at rate  $N^{2-2/\eta}$ . For any given rate of decay determined by the power law, network sparsity further lowers the decay rate by  $\zeta$ .

**Proposition 3.** *Consider a sequence of economies indexed by the number of firms  $N$ . If  $S_i$  is distributed as a power law with exponent  $\eta \in (1, 2]$ , then firm variance decays at rate  $N^{(1-\zeta)(2-2/\eta)}$ .*

*Proof.* As in the finite variance case,  $H_i$  determines the rate of convergence for firm variance. Recall the expression for a firm's Herfindahl:

$$H_i = \sum_k \frac{B_{i,k} S_k^2}{(\sum_l B_{i,l} S_l)^2} = \frac{N_i^{2/\eta} N_i^{-2/\eta} \sum_k B_{i,k} S_k^2}{N_i^2 (N_i^{-1} \sum_l B_{i,l} S_l)^2}.$$

From Gabaix (2011, Proposition 2) we have that

$$N_i^{-2/\eta} \sum_k B_{i,k} S_k^2 \xrightarrow{d} u,$$

where  $u$  is a Levy-distributed random variable. Because  $\eta > 1$ , mean size is finite and  $N_i^{-1} \sum_l B_{i,l} S_l \xrightarrow{a.s.} E[S]$ . Therefore,

$$N_i^{2(1-1/\eta)} H_i = \frac{N_i^{-2/\eta} \sum_k B_{i,k} S_k^2}{N_i^{-1} \sum_k B_{i,k} S_k} \xrightarrow{d} \frac{u}{E[S]}.$$

Finally, recall that

$$N_i \sim N^{1-\zeta} \frac{\tilde{S}_i}{\bar{Z}}.$$

Combining gives us the result:

$$N^{(1-\zeta)(2-2/\eta)} H_i \xrightarrow{d} \frac{u}{E[S]} \left( \frac{Z}{\bar{S}_i} \right)^{2-2/\eta}.$$

$\square$

**Comovement in Firm Volatilities: Proposition 4** We have referred to the volatility structure described in Proposition 2 as a factor model. The next result characterizes comovement among firms' volatilities when  $N$  is large. Because  $H_i$  determines the rate of convergence for firm variance, we may understand how firm variances covary in a large economy by studying the asymptotic covariance among  $H_i$  and  $H_j$ . As the number of firms grows, not only does the level of volatility decay, but so does its variance and covariance between the volatilities of different firms. Proposition 4 shows that comovement among firm variances decays at rate  $N^{1+2(1-\zeta)}$ . Intuitively, covariance among a pair of firms is lowest when both firms are large, since large firms have low exposure to overall size concentration.

**Proposition 4.** *Consider a sequence of economies indexed by the number of firms  $N$ . If  $S_i$  has finite fourth moment (e.g., if  $S_i$  is a lognormal variable), then the covariance between  $H_i$  and  $H_j$  has limiting behavior described by*

$$\text{Cov}(H_i, H_j) \sim \frac{1}{N^{1+2(1-\zeta)}} \frac{V(S^2)}{S_i S_j} \frac{Z^2}{E[S]^2}.$$

The covariance between  $V(g_i)$  and  $V(g_j)$  decays at the same rate.

*Proof.* Because  $H_i$  determines the rate of convergence for firm variance, we may understand how firm variances covary in a large economy by studying the asymptotic covariance among  $H_i$  and  $H_j$ .

Define  $\hat{E}_i[S_k^2] = N_i^{-1} \sum_k B_{i,k} S_k^2$ . We first characterize the asymptotic behavior of

$$\text{Cov}(\hat{E}_i[S_k^2], \hat{E}_j[S_k^2]) = E \left[ N_i^{-1} N_j^{-1} \sum_k \sum_l B_{i,k} B_{j,l} S_k^2 S_l^2 \right] - E \left[ N_i^{-1} \sum_k B_{i,k} S_k^2 \right] E \left[ N_j^{-1} \sum_k B_{j,k} S_k^2 \right].$$

Because size has finite fourth moment, the LLN implies that

$$N^{-(1-2\zeta)} \sum_k B_{i,k} B_{j,k} S_k^4 \xrightarrow{a.s.} E[S^4] \tilde{S}_i \tilde{S}_j / Z^2,$$

$$N^{-(1-2\zeta)} (N-1)^{-1} \sum_{k \neq l} B_{i,k} B_{j,l} S_k^2 S_l^2 \xrightarrow{a.s.} E[S^2]^2 \tilde{S}_i \tilde{S}_j / Z^2,$$

and

$$N^{-(1-\zeta)} \sum_k B_{i,k} S_k^2 \xrightarrow{a.s.} E[S^2] \tilde{S}_i / Z.$$

These imply that

$$E \left[ N_i^{-1} N_j^{-1} \sum_k \sum_l B_{i,k} B_{j,l} S_k^2 S_l^2 \right] \sim N^{1-2\zeta} \frac{E[S^4] \tilde{S}_i \tilde{S}_j}{N^2 p_i p_j Z^2} + (N-1) N^{1-2\zeta} \frac{E[S^2]^2 \tilde{S}_i \tilde{S}_j}{N^2 p_i p_j Z^2} = \frac{1}{N} E[S^4] + \frac{N-1}{N} E[S^2]^2$$

and

$$E \left[ N_i^{-1} \sum_k B_{i,k} S_k^2 \right] E \left[ N_j^{-1} \sum_k B_{j,k} S_k^2 \right] \sim N^{2(1-\zeta)} \frac{E[S^2]^2 \tilde{S}_i \tilde{S}_j}{N^2 p_i p_j Z^2} = E[S^2]^2$$

so that

$$\text{Cov}(\hat{E}_i[S_k^2], \hat{E}_j[S_k^2]) \sim N^{-1} V(S^2).$$

Since  $H_i = N_i^{-1} \hat{E}_i[S_k^2] / (N_i^{-1} \sum_k B_{i,k} S_k^2)$ , we have

$$\text{Cov}(H_i, H_j) \sim \frac{1}{N^{2(1-\zeta)}} \frac{Z^2}{\tilde{S}_i \tilde{S}_j E[S]^4} \text{Cov}(\hat{E}_i[S_k^2], \hat{E}_j[S_k^2]),$$

which delivers the stated asymptotic equivalence.  $\square$

**Aggregate Volatility: Proposition 5** We next characterize the behavior of aggregate volatility. Let  $\mathbf{S}_t$  be the vector of sizes,  $\iota$  an  $N$ -vector of ones, and  $\boldsymbol{\nu} = \mathbf{S}/\iota'\mathbf{S}$  the vector of size weights. We define the aggregate growth rate of the economy as

$$g_a = \boldsymbol{\nu}'\mathbf{g} = \boldsymbol{\nu}'(\mathbf{I} - \gamma\mathbf{W})^{-1}(\boldsymbol{\mu}_g + \boldsymbol{\varepsilon}).$$

The variance of the aggregate growth rate is given by

$$V(g_a) = \sigma_\varepsilon^2 \boldsymbol{\nu}'(\mathbf{I} - \gamma\mathbf{W})^{-1}(\mathbf{I} - \gamma\mathbf{W}')^{-1}\boldsymbol{\nu}.$$

A first observation is that variation in  $\mathbf{W}$  and  $\mathbf{S}$  induces heteroscedasticity in aggregate growth rates, even though all underlying innovations are i.i.d. across firms and over time. The next proposition shows that our network model drives a wedge between the rates of decay for aggregate versus firm-level variance. When the variance of firm size is finite, aggregate variance decays at rate  $1/N$ , which is generally faster than the rate of decay of firm variance. That is, even if the firm size distribution is lognormal, there is slow volatility decay at the firm level but not at the aggregate level. For size distributions with infinite variance (but finite mean), aggregate variance decays more slowly than  $1/N$ , but remains unaffected by  $\zeta$ . As shown by Acemoglu et al. (2012), network sparsity may impact aggregate volatility. Our model, by focusing on size as the key determinant of network formation, combines the aggregate volatility effects of network sparsity and granularity into a single mechanism that acts through the power law parameter  $\eta$ .

**Proposition 5.** *Consider a sequence of economies indexed by the number of firms  $N$ . If  $S_i$  has finite variance, then firm volatility has limiting behavior described by*

$$V(g_a) \sim \frac{\sigma_\varepsilon^2}{(1-\gamma)^2} \frac{E[S^2]}{NE[S]^2}.$$

If  $S_i$  is distributed as a power law with exponent  $\eta \in (1, 2]$ , then aggregate variance decays at rate  $N^{2-2/\eta}$ .

*Proof.* Let  $\mathbf{S}$  be the vector of sizes, and  $\boldsymbol{\nu} = \mathbf{S}/\iota'\mathbf{S}$  the vector of size weights, then

$$g_a = \boldsymbol{\nu}'\mathbf{g} = \boldsymbol{\nu}'(\mathbf{I} - \gamma\mathbf{W})^{-1}(\boldsymbol{\mu}_g + \boldsymbol{\varepsilon}).$$

The variance of the aggregate growth rate is given by

$$\begin{aligned} V(g_a) &= \sigma_\varepsilon^2 \boldsymbol{\nu}'(\mathbf{I} - \gamma\mathbf{W})^{-1}(\mathbf{I} - \gamma\mathbf{W}')^{-1}\boldsymbol{\nu}, \\ &\sim \sigma_\varepsilon^2 \left[ \boldsymbol{\nu}'\boldsymbol{\nu} + \gamma\boldsymbol{\nu}'\mathbf{W}\boldsymbol{\nu} + \gamma\boldsymbol{\nu}'\mathbf{W}'\boldsymbol{\nu} + \gamma^2\boldsymbol{\nu}'\mathbf{W}\mathbf{W}'\boldsymbol{\nu} + \frac{\gamma^2}{1-\gamma}\boldsymbol{\nu}'\bar{\mathbf{W}}\boldsymbol{\nu} + \frac{\gamma^2}{1-\gamma}\boldsymbol{\nu}'\bar{\mathbf{W}}'\boldsymbol{\nu} \right. \\ &\quad \left. + \frac{\gamma^3}{1-\gamma}\boldsymbol{\nu}'\mathbf{W}\bar{\mathbf{W}}'\boldsymbol{\nu} + \frac{\gamma^3}{1-\gamma}\boldsymbol{\nu}'\bar{\mathbf{W}}\mathbf{W}'\boldsymbol{\nu} + \frac{\gamma^4}{(1-\gamma)^2}\boldsymbol{\nu}'\bar{\mathbf{W}}\bar{\mathbf{W}}'\boldsymbol{\nu} \right], \end{aligned} \quad (\text{B2})$$

where the second expression is asymptotically equivalent, as explained in Proposition 1. Define  $\bar{H} = E[S^2]/(NE[S]^2)$ . From the previous derivations, the following equivalencies hold:  $\boldsymbol{\nu}'\iota = 1$ ,  $\boldsymbol{\nu}'\boldsymbol{\nu} \sim \bar{H}$ ,  $\boldsymbol{\nu}'\mathbf{W} \sim \boldsymbol{\nu}'$ ,  $\boldsymbol{\nu}'\mathbf{W}' \sim \bar{H}\iota'$ ,  $\bar{\mathbf{W}}'\boldsymbol{\nu} \sim \boldsymbol{\nu}$ , and  $\bar{\mathbf{W}}\boldsymbol{\nu} \sim \iota\bar{H}$ . Applying these to (B2) and combining terms, we find that the aggregate variance is asymptotically equivalent to

$$V(g_a) \sim \sigma_\varepsilon^2 \left[ 1 + 2\gamma + \gamma^2 + 2\frac{\gamma^2}{1-\gamma} + \frac{2\gamma^3}{1-\gamma} + \frac{\gamma^4}{(1-\gamma)^2} \right] \bar{H} = \frac{\sigma_\varepsilon^2}{(1-\gamma)^2} \bar{H},$$

which delivers the stated result.  $\square$

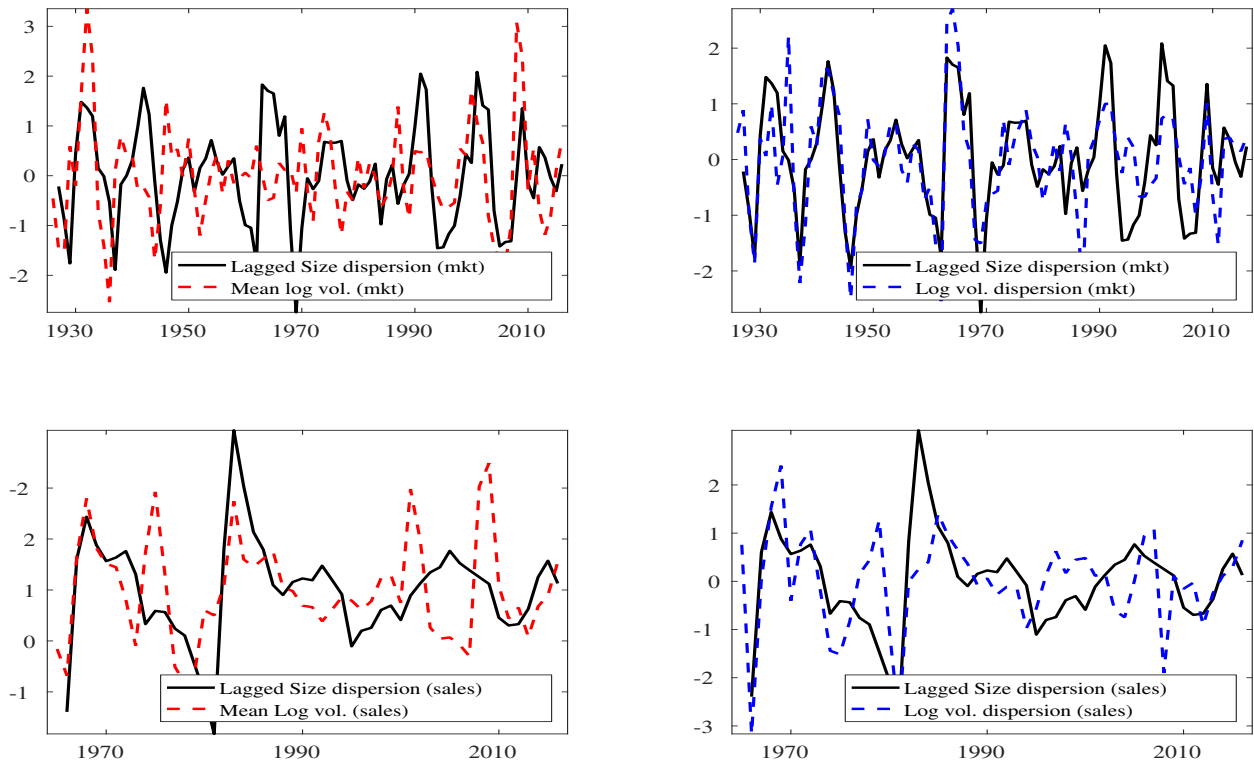
## C Empirical Appendix

This appendix discusses several additional empirical results.

### C.1 Frequency Decomposition

To study the trend and cycle in the size and volatility moments, we apply the Hodrick-Prescott filter with a smoothing parameter of 50. Figure A1 reports HP-detrended moments. The top-left panel shows firm size dispersion and mean firm variance based on market capitalization and return volatility. The top right panel reports size dispersion and variance dispersion, also based on market data. The bottom two panels are the counter-parts where size and variance are based on sales data. The correlations between the cyclical component in average variance and size dispersion are 25.8% for the market-based measure and 42.0% for the fundamentals-based measure. The correlations between the cyclical component in variance dispersion and size dispersion are 72.9% for the market-based measure and 54.1% for the sales-based measure. These results suggest that comovement between the dispersion in the firm size distribution and moments of the firm variance distribution occur both at cyclical and at low frequencies.

Figure A1: DETRENDED SIZE AND VARIANCE MOMENTS



Notes: The figure plots HP-detrended time series moments of size and volatility distributions using smoothing parameter of 50.

### C.2 Simulated Method of Moments Estimation Details

**Objective Function** Our estimation chooses the parameter vector  $\Theta$  which minimizes the distance between the data moments, collected in the  $1 \times K$  vector  $\mathcal{G}$ , and the corresponding moments obtained from

a simulation of the network model, collected in  $\hat{\mathcal{G}}(\Theta)$ :

$$\mathcal{F} = \min_{\Theta} \mathbb{E} \left[ \left( \mathcal{G} - \hat{\mathcal{G}}(\Theta) \right) \mathcal{W} \left( \mathcal{G} - \hat{\mathcal{G}}(\Theta) \right)' \right],$$

where the moment function  $\hat{\mathcal{G}}(\Theta)$  is the average over the 100 draws, while  $\mathcal{G}$  is a vector with the time-series average of the cross-sectional moments from the data.

Formally, these two moments functions are defined as

$$\begin{aligned} \mathcal{G} &= \frac{1}{T} \sum_{t=1}^T g_t \\ \hat{\mathcal{G}}(\Theta) &= \frac{1}{\mathcal{T}(T)} \sum_{t=1}^{\mathcal{T}(T)} \hat{g}_t(\Theta), \end{aligned}$$

where  $T$  is the number of time-series observations (33 for our sample),  $\mathcal{T}(T)$  is the number of simulations (100 for our model). The function  $g_t$  is a  $K \times 1$  vector of cross-sectional moments from the data for year  $t$ , while the function  $\hat{g}_t(\Theta)$  is a  $K \times 1$  vector of cross-sectional moments from the simulation  $t$  and parameter vector  $\Theta$ . The weighting matrix  $\mathcal{W}$  is the identity matrix. All moments are expressed in logs or are log differences so that they have the same order of magnitude.

**Standard Error Calculation** To derive the estimator asymptotics, we assume that  $\frac{T}{\mathcal{T}(T)} \rightarrow \tau$  with  $\tau$  being a finite positive number. Under sufficient regularity conditions, the asymptotic distribution of  $\Theta$  is given by:

$$\sqrt{T} \left( \hat{\Theta} - \Theta_0 \right) \rightarrow \mathcal{N}(0, V_0),$$

where

$$\begin{aligned} V_0 &= (1 + \tau) (G'_0 \mathcal{W} G_0)^{-1} G'_0 \mathcal{W} \Omega_0 \mathcal{W} G_0 (G'_0 \mathcal{W} G_0)^{-1}, \\ G_0 &= \mathbb{E} \left[ \frac{\partial}{\partial \Theta'} \hat{g}_t(\Theta) \Big|_{\Theta=\Theta_0} \right], \\ \Omega_0 &= \sum_{j=-\infty}^{\infty} \mathbb{E} \left[ (g_t - \mathbb{E}(g_t)) (g_t - \mathbb{E}(g_t))' \right]. \end{aligned}$$

To compute standard errors of the estimated parameters, we estimate  $G_0$  as the numerical derivative of  $\hat{\mathcal{G}}(\Theta)$  evaluated at the estimated parameters. We compute numerical derivatives by central difference approximation and by changing parameters by one percent. We estimate  $\Omega_0$  as the variance-covariance matrix of the moments from the data. For the estimated terms  $\hat{G}_0$  and  $\hat{\Omega}_0$  and assuming  $\tau = \frac{33}{100}$ , the asymptotic standard errors are given by the square root of the diagonal elements of the following variance covariance matrix:

$$\frac{1}{T} \hat{V}_0 = \left( 1 + \frac{T}{\mathcal{T}(T)} \right) \frac{1}{T} \left( \hat{G}'_0 \mathcal{W} \hat{G}_0 \right)^{-1} \hat{G}'_0 \mathcal{W} \hat{\Omega}_0 \mathcal{W} \hat{G}_0 \left( \hat{G}'_0 \mathcal{W} \hat{G}_0 \right)^{-1}.$$

**Wald statistic** To test whether some moments are collectively statistically equal to zero, we conduct the following hypothesis testing:

$$H_0 : h(\Theta_0) = 0 \quad \text{v.s.} \quad H_1 : h(\Theta_0) \neq 0,$$

where  $h(\Theta)$  is a vector of moments to be tested. In our case,  $h(\Theta)$  includes all the estimation moments.

Let  $H(\Theta)$  be a matrix of the partial derivatives of  $h(\Theta)$  with respect to  $\Theta$ , then the Wald statistic is

given by

$$W = Th(\hat{\Theta})' \left[ H(\hat{\Theta}) \hat{V}_0 H(\hat{\Theta})' \right]^{-1} h(\hat{\Theta}) \sim \chi_J^2$$

where  $J$  is the number of moments being tested.

### C.3 Network Data and Truncation

**Compustat Segment Data** Our data for annual firm-level linkages come from Compustat. It includes the fraction of a firm’s dollar sales to each of its major customers. Firms are required to supply customer information in accordance with Financial Accounting Standards Rule No. 131, in which a major customer is defined as any firm that is responsible for more than 10% of the reporting seller’s revenue. Firms have discretion in reporting relationships with customers that account for less than 10% of their sales, and this is occasionally observed. In our data, 23% of firms report at least one customer that accounts for less than 10% of its sales. The Compustat data have been carefully linked to CRSP market equity data by Cohen and Frazzini (2008). This link allows us to associate information on firms’ network connectivity with their market equity value and return volatility. We update the Cohen and Frazzini data to 2012. Cohen and Frazzini (2008) used the data to show that news about business partners does not immediately get reflected into stock prices. Atalay et al. (2011) and Herskovic (2017) also use Compustat sales linkage data to develop a model of customer-supplier networks.

**BEA Data** To gather additional evidence, we study a second data source which is at the industry level. Industry input-output data are from the Bureau of Economic Analysis (BEA). Because industry definitions vary quite dramatically over time, we focus on a set of 65 industries we can track consistently over time between 1997 and 2015. These are the input-output use tables, after redefinitions at producers’ prices. In related work, Ahern and Harford (2014) use the network topography implied by the BEA industry data to show that the properties of these networks have a bearing on the incidence of cross-industry mergers. Herskovic (2017) also uses the BEA data to study the asset pricing effects of aggregate shocks to network moments.

**Truncation** To evaluate the effect of the 10% customer share truncation as well as the effect of selection, we conduct two exercises. First, we use the model. Since the model specifies the full network before imposing truncation, we can compute both truncated and untruncated moments in the model. If the model is the true data generating process, the difference between untruncated and truncated moments is the truncation bias. Second, we use the BEA data, which do not suffer from truncation. Again, we can artificially impose truncation on these data to study the effect.

Table A1 reports our findings for several key correlations of interest. Panel A reports truncated moments, including the Compustat moments reported in the main text, Panel B reports untruncated moments. The model and BEA data both show large truncation biases. This is especially true for moments that involve the number of customers ( $N_i^{out}$ ) or the number of suppliers ( $N_i^{in}$ ). The second row shows that  $Corr(N_i^{out}, \log S_i)$  has a massive downward bias of 38 percentage points in the model and 65 percentage points in the BEA data. The fourth row shows that  $Corr(N_i^{in}, \log S_i)$  has an upward bias of 35 percentage points in the model and 17 percentage points in the BEA data. Since Herfindahl moments are based on network weights squared, they are dominated by the larger weights. Large customers or suppliers are less likely to be missing or truncated. The Herfindahl correlations in rows 3 and 5 are nearly unbiased in the BEA data. The model shows a modest bias for row 3 but a sizeable bias for row 5.

**Empirical Evidence for Model Assumptions** The model makes two key assumptions on the network. First, large suppliers are more likely to have more customers:  $Corr(S_i, N_i^{out}) > 0$ . Second, the importance of a customer is higher the larger that customer is:  $Corr(w_{ij}, S_j) > 0$ . These are assumptions on the full untruncated network of firms. The untruncated BEA data in column (4) of Table A1 provide empirical support for both assumptions in rows 1 and 2. The former correlation is 58% and the latter correlation is 40%. The former correlation is severely downward biased, as we discussed above. Adding either the model-implied bias or the BEA data-implied bias to the Compustat estimate raises the “untruncated”

Table A1: TRUNCATION ANALYSIS

	Panel A: Truncated			Panel B: Untruncated	
	(1)	(2)	(3)	(4)	(5)
	Compustat	BEA	Model	BEA	Model
$Corr(w_{i,j}, \log S_j)$	0.19	0.55	1.00	0.40	1.00
$Corr(N_i^{out}, \log S_i)$	-0.00	-0.07	0.31	0.58	0.69
$Corr(H_i^{out}, \log S_i)$	-0.28	-0.39	-0.88	-0.38	-0.67
$Corr(N_i^{in}, \log S_i)$	0.53	0.34	0.34	0.17	-0.01
$Corr(H_i^{in}, \log S_i)$	-0.47	0.10	-0.34	0.10	0.14

*Notes:* The table reports time-series averages of cross-sectional correlations between various features of customer-supplier networks with size and volatility. We report the following correlation in rows 1-5: correlation between the log network weight and log customer size, correlation between the out-degree (number of customers) and log supplier size, correlation between the out-Herfindahl and log supplier size, correlation between the in-degree (number of suppliers) and log supplier size, correlation between the in-Herfindahl and log supplier size, and correlation between the network weight and log size. Column (1) reports the correlation using Compustat sample for firms with network data available. Compustat sample is at firm-level and annual frequency for the period 1980-2012. Columns (2) and (4) are based on annual industry-level Bureau of Economic Analysis data for a set of 65 consistently measured industries for the period 1997-2015. Column (2) reports the correlation using BEA data assuming and artificially imposing 10% truncation on the network weights, which implies that we discard all customer-supplier pairs that represent less than 10% of supplier sales. Column (4) is not truncated. Columns (3) and (5) report the correlations for the truncated and untruncated model, respectively.

Compustat to a substantial positive number. The latter correlation is already positive in the Compustat data and suffers much less from bias. We conclude that the empirical evidence provides support for the two main assumptions in the model.