

Value of Security Selection versus Asset Allocation in Credit Markets: Part II—An “Imperfect Foresight” Study

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SUMMARY

- We simulate several styles of fixed-income portfolio management using a variation of the “perfect foresight” approach and explore performance at manager skill levels ranging from 0% (random selection) to 100% (perfect foresight).
 - We design portfolio strategies to isolate each of the following management styles: duration allocation, sector allocation, quality allocation, and security selection. Information ratios are reported for each strategy at various levels of manager skill.
 - At equivalent levels of skill, the strategy based on security selection outperforms duration allocation and sector/quality allocation techniques on an information ratio basis.
 - The number of independent decisions in the allocation process is a key determinant of strategy success.
 - The advantage of security selection lies in the diversification of risk provided by making many security selection decisions simultaneously. This does not contradict the importance of duration decision-making in fixed-income portfolio management.
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TABLE OF CONTENTS

Introduction	3
Strategy Design	6
Cell Definitions	6
Building Duration-Neutral Strategies	7
Bet Size	8
Asset Allocation Strategies	10
Setting the Allocation Probabilities	10
By Random Selection	11
By Skill at Choosing Any Winning Sector	12
By Skill at Choosing the Best Sector	13
Calculating Mean and Variance of Overall Portfolio Outperformance	14
Sector Allocation Results	16
Making Fewer Sector Decisions	17
Quality Allocation Results	19
Yield Curve Allocation	20
Security Selection Strategy	21
Number of Securities	21
Duration Matching	21
Selection Criterion: Excess Return	21
Skill Implementation	22
Equal Weighting versus Market Weighting	22
Results	23
Analysis of Results	26
Value of a Single Decision	28
Diversification of Risk Among Different Strategies	32
Conclusion	34
Appendix I. Independence of Information Ratio from Bet Size	36
Appendix II. Decomposing the Variance of Strategy Outperformance.	37
References	40

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INTRODUCTION

In the late 1990s, several new groups of investors started adding credit securities to their debt portfolios. First, the European Monetary Union served as a catalyst for increasing the size and liquidity of the European credit markets. This spurred greater demand for credit products from European portfolio managers. Second, a decrease in outstanding U.S. Treasury securities prompted central banks to look for alternative ways to invest their reserve portfolios. Especially in Europe, with the European Central Bank providing the first line of reserves in support of the euro, the national central banks switched to maximization of total return as an objective for their portfolios. Over a long investment horizon, this favors credit securities over government bonds.

As they begin the process of credit investing, portfolio managers have been asking some fundamental philosophical questions. If an investor's objective is maximization of risk-adjusted return, what style of portfolio management holds the most promise? Is it yield curve timing, sector rotation, or security selection? Can an intuition be developed to understand the relative merits of each style? Can this be quantified?

In a report published in March 1999, we evaluated the relative merits of different investment styles on a risk/return basis. Investment strategies were created to isolate the effects of security selection, yield curve allocation,¹ sector allocation, and quality allocation within the U.S. investment-grade corporate bond market. In each case, a "perfect foresight" approach was used to identify the best performance that could be achieved if allocation decisions were based on knowledge of future returns. While all such strategies turn in unrealistically high returns, these results can be viewed as an upper bound on the amount of return per unit of risk that can be achieved using each investment style. We found significant differences in the information ratios (return per unit of risk) of the different strategies, with security selection outperforming all of the asset allocation strategies.

The extreme nature of the perfect foresight assumption at the heart of this study left several questions. If a strategy outperforms every month under this unrealistically optimistic assumption, does the variance of this outperformance constitute a fair measure of the strategy's true risk of underperformance? To what extent can these results be applied to the more realistic situation in which expressing a view can lead to either gains or losses? What sort of performance can be expected from managers with a certain amount of skill in each investment style?

¹ In this report, the terms "duration allocation" and "yield curve allocation" are used interchangeably to refer to a single strategy in which the manager chooses to overweight one of three segments along the yield curve.

To address these questions, this report revisits the evaluation of investment styles using an “imperfect foresight” approach. Rather than choosing the single best allocation decision each month, we now incorporate the notion that even well-informed investment decisions will sometimes result in losses or underperformance. The simulated manager of this study will not be assumed to call the market correctly every month. He will position the portfolio to be neutral to the benchmark in every dimension but one. In this selected dimension, he will express a view, which may be right or wrong. This view leads to the risk of performance differences between the portfolio and benchmark, known as tracking error. If the position is chosen purely at random, there should be no mean outperformance of the benchmark to justify this risk. If the manager is skilled at this task, he will choose correctly more often, and the portfolio will outperform on average.

We simulate the performance of various investment strategies using historical data from the Lehman Brothers U. S. investment-grade Corporate Index, and we use information ratios² to evaluate performance. Managerial skill is modeled as follows. In the unskilled case (0% skill), each decision made by a manager will consist of a random selection from among a discrete set of possibilities, with equal probabilities assigned to each. In the perfect foresight case (100% skill), the manager always chooses a correct decision, which will lead to outperformance (as determined by future results). We investigate two different approaches to defining a “correct” decision in this context: one in which only the best decision is considered correct and another that includes any decision that outperforms the index. In either case, for skill levels between 0% and 100%, the selection probabilities for all choices are linearly interpolated between these two extremes.

A similar definition of skill was used by Fox [1999] to simulate manager performance at tactical allocation between stocks and bonds. This simulation-based approach was applied by Fjelstad [1999] to duration allocation and sector allocation in fixed-income portfolios. In both of these studies, the allocation along each dimension was limited to a binary decision (long or short duration, overweight or underweight corporates relative to governments). Security selection was not addressed. Sorensen et al. [1998] simulate manager skill at security selection for equity portfolios and address the implications for allocation of funds among managers of different classes.³

² The information ratio is the mean annual outperformance of an investment strategy divided by the annualized standard deviation of the outperformance. Both risk and return are measured versus the benchmark. The Sharpe ratio can be considered the special case of an information ratio with a riskless asset (cash) as the benchmark.

³ All of these studies consider the unskilled case to correspond to a skill of 50%, where skill is defined as the probability of a correct decision. A skill level of 60% according to this convention corresponds to 20% skill by our definition.

In this report, we explore a set of reasonable investment strategies that isolates one investment style at a time. As the outcome of a particular strategy in a given month is not deterministic, the risk and return of each strategy are evaluated on a probabilistic basis. The measurement of portfolio/benchmark performance deviation across all possible allocation decisions and over time allows an accurate assessment of the risk of a given strategy. Expected returns are evaluated as a function of manager skill for each investment style, using a combination of closed form calculations and simulation. All of the strategies in this report are based on a foresight horizon of one month with monthly rebalancing.

STRATEGY DESIGN

The investment strategies analyzed in this report are designed to focus on just one form of risk at a time. Thus, our sector allocation strategy is designed to take no risk versus the index in term structure allocation, quality allocation, or security selection. To control risk in all but the single dimension in which the strategy expresses a view, we begin with a detailed analysis of index composition.

Cell Definitions

The investment universe consists of all bonds in the Lehman Brothers Corporate Index. The index is divided into cells along three dimensions: duration, sector, and quality. As shown in Figure 1, we use three duration cells,⁴ four broadly defined sectors, and three quality cells, for a total of 36 cells overall. The index is characterized by the percentage of market capitalization within each of these cells, as well as the average duration of the bonds in each cell.

⁴ There are two minor differences between the cell definitions used in this report and those of our previous perfect foresight study. The term structure axis is now defined based on duration cells rather than average life. This change was made to improve the treatment of callable bonds and to avoid numerical difficulties in the duration-matching scheme described below. Also, the previous report used four quality cells for security selection and only three for asset allocation. In the current study, three quality cells are used throughout.

Figure 1. **Corporate Index Profile by Duration, Sector, and Quality**
% of Market Value, as of 7/1/99

	0 to 4	4 to 7	Over 7	Total
Aaa and Aa				
Industrials	1.6	1.1	2.5	5.2
Utilities	0.1	0.2	0.2	0.5
Finance	4.9	2.9	1.7	9.5
Yankees	<u>4.5</u>	<u>3.4</u>	<u>2.3</u>	<u>10.2</u>
Total Aaa-Aa	11.1	7.6	6.7	25.4
A				
Industrials	4.6	5.5	8.7	18.8
Utilities	0.7	0.8	1.0	2.5
Finance	6.6	5.7	3.1	15.4
Yankees	<u>1.4</u>	<u>2.2</u>	<u>2.8</u>	<u>6.4</u>
Total A	13.3	14.2	15.6	43.1
Baa				
Industrials	4.5	6.9	7.5	18.9
Utilities	1.2	1.4	1.2	3.8
Finance	1.8	1.3	0.3	3.4
Yankees	<u>1.7</u>	<u>2.9</u>	<u>0.8</u>	<u>5.4</u>
Total Baa	9.2	12.5	9.8	31.5
Corporate Index	33.6	34.3	32.1	100.0

The marginal sums of this three-dimensional market view can provide a similar two-dimensional view along any two of these axes. The rightmost column of Figure 1 gives the index composition by sector and quality. The subtotals at the bottom of each credit quality level give the breakdown by quality and duration. The two-dimensional profile by duration and sector is presented in Figure 2.

To isolate the effect of one type of investment decision, we constrain each portfolio to match the index exactly according to one of the views discussed above. Our security selection strategy is constrained to match the index weights and durations in each cell shown in Figure 1, but does so by selecting a small number of the bonds in each cell. Our asset allocation strategies each match the index along two out of three dimensions, but vary the allocations along the third. For instance, our quality allocation strategy matches the index view shown in Figure 2, but achieves the desired allocation to each duration x sector cell by adjusting the weights of the three qualities within the cell. This ensures that the returns of our quality allocation strategy are not colored by inadvertent secondary exposures to duration or sector.

Building Duration-Neutral Strategies

In our previous study, the term structure composition of the portfolio was constrained to match the index in percentage of market value assigned to each of the three cells along the term structure. As the duration of each sector within a given cell can vary, there remained a small duration exposure in each of our investment strategies. In the current study, we remove this duration bias entirely by matching cell durations, as well as percentages, to those of the index.

To accomplish this, each market cell shown in Figure 1 is further divided by duration. An appropriate blend of the long and the short half of any cell can then match the required duration. For example, Figure 3 shows a detailed view of short A-rated corporates. This cell, as shown in Figure 1, makes up 13.3% of the index and has an average duration of 2.61. If we chose to represent this cell in our portfolio by purchasing a single sector according to its market composition, we would be short duration had we chosen industrials and long if we had chosen any

Figure 2. **Corporate Index Composition by Duration and Sector**
% of Market Value, as of 7/1/99

	0 to 4	4 to 7	Over 7	Total
Industrials	10.7	13.5	18.7	42.9
Utilities	2.0	2.4	2.4	6.8
Finance	13.3	9.9	5.1	28.3
Yankees	<u>7.6</u>	<u>8.5</u>	<u>5.9</u>	<u>22.0</u>
Total	33.6	34.3	32.1	100.0

other sector. By adjusting the market weights to the long and short halves of the cell, we can create a set of single-sector investments that matches the 2.61 duration of the index for the cell. If the short and long halves of the cell have durations of D_S and D_L , respectively, then the weights needed to match a benchmark duration of D_B are obtained by solving the set of equations

$$(1) \quad \begin{aligned} x_S + x_L &= 1 \\ x_S D_S + x_L D_L &= D_B \end{aligned}$$

to obtain $x_L = (D_B - D_S) / (D_L - D_S)$. Figure 3 shows that for industrials, such a position would be composed by blending 44.2% of the 0-2.5 duration cell with 55.8% of the 2.5-4.0 duration cell, overweighting the longer cell relative to the index. A similar position in short single-A utilities would require 42.9% of the 0-2.5 duration cell and 57.1% of the 2.5-4.0 duration cell, overweighting the shorter cell.⁵ The sector allocation strategy explored in this report chooses one of these duration-neutral single-sector investments within each quality x duration cell, ensuring no incidental curve exposure due to duration differences between sectors. This technique is utilized for quality allocation as well. A similar approach is used to match cell duration in our security selection strategy, as explained below.

Bet Size

Any allocation strategy consists of two parts. First, a manager forms a view favoring one market segment over another; then, the portfolio is constructed by overweighting the selected segment. More or less risk (and potential for excess return) can be assumed by making larger or smaller deviations from the benchmark.

All the allocation strategies in the next section are presented in their purest form, with an extreme application of manager views to portfolio composition. Once a decision is made to favor a particular market segment (either on a cell-by-cell basis or for the portfolio as a whole), we shift the entire portfolio to reflect this view. We do not imply that this is a realistic approach to sector allocation. Rather, we assume that managers will take more moderate stances to implement their views; we can approximate their performance by blending the extreme approach with an investment in the benchmark.

To achieve more moderate levels of risk, the strategy can be applied to only part of the portfolio assets. Thus, for a bet size b , we can invest a percentage b in one

⁵ Other mechanisms could be used to match duration and market value within each cell. One alternative method that does not require sub-dividing the cell is to blend the selected portion of the cell (for example, short single-A utilities) with a cash position. Equations 1 could be re-interpreted to provide the necessary weights for bonds and cash, with the cash duration D_S set to zero. This procedure has the advantage of maintaining the relative weights of each security within a cell. However, when the duration of the selected sector in a given cell is shorter than the target duration, this method requires leveraging the portfolio with a negative cash position. The method used in our study never requires such leveraging.

of the above strategies, leaving a percentage $1 - b$ invested in the benchmark. Applying any of the above strategies in this way will reduce both the mean outperformance and the tracking error by the factor b , leaving the information ratio unchanged. (The proof of this may be found in Appendix I.) In this way, we can apply any of the strategies presented below at any desired level of risk.

Figure 3. **Construction of Duration-Neutral Sector Allocation Strategy**
Short Single-A Corporates, July 1999

	Short Single-A Corporate Index Composition			Duration-Matched Sector Selection Strategy		
	0.0-2.5	2.5-4.0	Total	0.0-2.5	2.5-4.0	Total
Duration						
Industrials						
Number of Bonds	81	86	167			
Market Value (\$ mn)	25,773	25,336	51,109			
Percent of Short A	17.5%	17.2%	34.7%			
Percent of A Corporates	5.4%	5.3%	10.7%			
Percent of Cell	50.4%	49.6%	100.0%	44.2%	55.8%	
Duration	1.70	3.33	2.51			2.61
Total Return (%)	0.24	-0.10	0.07			0.05
Utilities						
Number of Bonds	13	24	37			
Market Value (\$ mn)	2,759	5,079	7,838			
Percent of Short A	1.9%	3.4%	5.3%			
Percent of A Corporates	0.6%	1.1%	1.7%			
Percent of Cell	35.2%	64.8%	100.0%	42.9%	57.1%	
Duration	1.60	3.37	2.75			2.61
Total Return (%)	0.00	0.22	0.14			0.13
Finance						
Number of Bonds	105	130	235			
Market Value (\$ mn)	31,222	41,757	72,979			
Percent of Short A	21.2%	28.3%	49.5%			
Percent of A Corporates	6.6%	8.8%	15.3%			
Percent of Cell	42.8%	57.2%	100.0%	45.9%	54.1%	
Duration	1.82	3.28	2.66			2.61
Total Return (%)	0.07	-0.39	-0.19			-0.18
Yankees						
Number of Bonds	18	29	47			
Market Value (\$ mn)	5,304	10,193	15,497			
Percent of Short A	3.6%	6.9%	10.5%			
Percent of A Corporates	1.1%	2.1%	3.3%			
Percent of Cell	34.2%	65.8%	100.0%	39.7%	60.3%	
Duration	1.64	3.25	2.70			2.61
Total Return (%)	0.35	-0.16	0.01			0.04
Total						
Number of Bonds	217	269	486			
Market Value (\$ mn)	65,058	82,366	147,424			
Percent of Short A	44.1%	55.9%	100.0%			
Percent of A Corporates	13.7%	17.3%	31.0%			
Percent of Cell	44.1%	55.9%	100.0%			
Duration	1.75	3.29	2.61			
Total Return (%)	0.16	-0.23	-0.06			

ASSET ALLOCATION STRATEGIES

To define each asset allocation strategy, we first assign probabilities to each allocation decision. The probabilities are a function of a skill parameter that controls the likelihood of a correct decision. The probability distribution of strategy performance can then be evaluated directly from these decision probabilities. To illustrate the strategy formulation and the calculation of the performance statistics, we take the sector allocation strategy as an example. Starting with an explanation of how the strategy works in a single cell in a single month, we extend the calculation to cover the entire portfolio and its evolution over time.

Setting the Allocation Probabilities

The construction of a duration-neutral position in a single sector, as shown in Figure 3, forms the basis for our sector selection strategy. The index return within this cell, short single-A corporates, is -0.06%. This represents the benchmark for the strategy's performance within the cell. The rightmost column of the figure shows the returns that would have resulted from an implementation of this strategy in July 1999. We can see that had we placed our short single-A allocation entirely in the financial sector, the resulting return (-0.18%) would have underperformed by 0.12%. Had we selected any other sector, we would have outperformed this portion of the index. Because our sector allocation strategy matches index weights by quality and duration, overall strategy outperformance of the Corporate Index can be expressed as a weighted sum of such cell-by-cell outperformance numbers.

We view the strategy outperformance of the index within each cell as a random variable. Each month, the strategy will choose one of the four sectors within each cell. If we assume that many portfolio managers are carrying out the same strategy (by making one of the four possible sector choices), we find that the distribution of results consists of only four possible events, weighted by the probabilities of selection. The success of the strategy may be measured by the mean outperformance \bar{r} and the standard deviation of outperformance σ . If r_i represents the outperformance of the duration-neutral strategy using sector i , and p_i is the probability of a manager choosing sector i , then the mean and variance of the outperformance⁶ are given by

$$(2) \quad \begin{aligned} \bar{r} &= \sum_{i=1}^4 p_i r_i \\ \sigma^2 &= \sum_{i=1}^4 p_i (r_i - \bar{r})^2. \end{aligned}$$

⁶ The quantities defined in Equation 2 are actually the conditional mean and variance of the strategy, given a particular market outcome. A more formal treatment may be found in Appendix II.

Figure 4 illustrates this calculation under three different sets of sector selection probabilities, corresponding to different assumptions about manager skill.

By Random Selection

In the simplest case, we assume that the strategy chooses one sector at random, with equal probabilities for all sectors. If there are n possibilities, the selection probabilities are given simply by

$$(3) \quad p_i^{RANDOM} = 1/n.$$

For the sector allocation problem at hand, in which the strategy selects one of four sectors, this random selection rule gives $p_i = 25\%$. As shown in Figure 4, this “no skill” strategy outperforms the index by an average of 6.8 bp for the month, with a standard deviation of 11.4 bp.

The reason that this random selection outperforms the index on average is clear. The index return is heavily influenced by the negative return in the finance sector, which makes up 49.5% of the index in this cell. As the assumed selection probability for finance in our equally weighted strategy is much lower than this, the strategy outperforms on average. For months in which a single large sector outperforms the others by a great deal, thereby bringing up the index return, this strategy will tend to underperform. All in all, we expect that this strategy will

Figure 4. **Imperfect Foresight: Representing Skill by Manipulating Sector Selection Probabilities**
Single-Cell Example, Short Single-A Corporates, July 1999, in %

							Sector Selection Probabilities by Skill Level		
	Index			Strategy		Random Selection (No Skill)	20%		20%
	Percent	Dur. (yrs)	Return	Return	Outperf.		Choose Any Winner	Choose Best	
Industrials	34.7	2.51	0.07	0.05	0.11	25	27	20	
Utilities	5.3	2.75	0.14	0.13	0.18	25	27	40	
Finance	49.5	2.66	-0.19	-0.18	-0.12	25	20	20	
Yankees	<u>10.5</u>	<u>2.70</u>	<u>0.01</u>	0.04	0.10	25	27	20	
Index Totals	100.0	2.61	-0.06						
Mean Strategy Outperformance (%)						0.068	0.080	0.091	
Standard Deviation of Strategy Outperformance (%)						0.114	0.106	0.112	

outperform in some months and underperform in others, but, over time, it should perform roughly similar to the index.⁷

Note that even in a month in which the strategy outperforms on the whole, there is certainly a possibility of underperformance. The 25% of managers who choose to purchase only finance bonds in this cell would underperform the index by 12 bp. The 11.4 bp standard deviation shown here represents the variation across different managers implementing the same strategy. This measure provides a fair assessment of strategy risk, as it reflects the losses that the strategy would incur if the view that is implemented turns out to be incorrect.

By Skill at Choosing Any Winning Sector

What is skill? It is not our purpose here to philosophize on what abilities, personality traits, or organizational factors contribute to the success of a particular manager. A manager who consistently outperforms the index will be considered skillful. From this result-oriented viewpoint, skill can be defined as the ability to make correct decisions more frequently than incorrect ones. The views of a successful manager are not always borne out to be correct, but they are correct more often than random selection.

With perfect foresight, as defined in our previous paper, a manager would always choose correctly. Our “imperfect foresight” technique similarly uses knowledge of future returns to determine which sector allocation decisions are the right ones, but does not assume that the manager always chooses the best possible sector. Rather, we simulate the effect of skill by shifting the selection probabilities between the two extremes of random selection and perfect foresight. We have explored two slightly different interpretations of manager skill. A particular decision may be deemed “correct” as long as it outperforms the index, or only if it is the best of the available choices. By leaving the number of correct decisions as a variable, the same set of equations can be used to define the selection probabilities for both of these approaches.

For a selection among n choices, in which n_w turn out to be correct decisions (winners) and $n_L = n - n_w$ are incorrect (losers), the probabilities under perfect foresight are

$$(4) \quad p_i^{PERFECT} = \begin{cases} 1/n_w & \text{if } i \text{ is a correct decision} \\ 0 & \text{otherwise.} \end{cases}$$

⁷ An alternative to the base case (no skill) assumption would be to use index weights as the sector selection probabilities. This would have an advantage in that the mean outperformance of the strategy would be close to zero every month. However, this would imply a connection between sector views and market weights. We prefer to carefully match the index along two dimensions, but to leave the manager free from indexation constraints in the dimension in which a view is to be expressed.

If more than one decision is deemed correct in a given month, then the strategy assigns equal probabilities to each of the correct decisions.

For a manager with skill s , we assume that the selection probabilities $p_i(s)$ are scaled between random selection and perfect foresight and are given by

$$(5) \quad p_i(s) = (1-s)p_i^{RANDOM} + sp_i^{PERFECT} = \begin{cases} (n_W + sn_L) / n_W(n_W + n_L) & \text{if } i \text{ is a correct decision} \\ (1-s) / (n_W + n_L) & \text{otherwise.} \end{cases}$$

As there are n_W correct decisions, the overall probability of selecting a winning sector is $(n_W + sn_L) / (n_W + n_L)$, which converges to $n_W / (n_W + n_L)$ for the unskilled case ($s = 0$) and to 1 for the perfect foresight case ($s = 100\%$).

In the first approach, skill represents the ability to select a sector that outperforms the index, but not necessarily the best one. At 100% skill, this approach assumes a weakened form of perfect foresight, in which the manager has equal probabilities of choosing from among all the outperforming sectors. For lower skill levels, the selection probabilities are scaled between random selection and this weakened form of perfect foresight, with increased probabilities for all sectors that outperform the index and decreased probabilities for all underperforming sectors.

In the sector allocation example in Figure 4, there are three sectors that outperform the index (industrials, utilities, and Yankees); only the financial sector underperforms. Evaluating Equation 5 at 20% skill with $n_W = 3$ and $n_L = 1$ gives a probability $p_i(20\%) = 3.2 / 12 \approx 27\%$ of choosing any of the winning sectors and a probability of $0.8 / 4 = 20\%$ of choosing the underperforming financial sector. The mean and standard deviation of the strategy results within this cell for this month are calculated according to Equation 2. Figure 4 shows that under this more favorable set of selection probabilities, the standard deviation of strategy performance is almost identical to that under purely random selection, but that the mean return has increased from 6.8 bp to 8.0 bp.

By Skill at Choosing the Best Sector

The second approach interprets skill as the ability to choose the best-performing sector. 100% skill according to this interpretation corresponds to perfect foresight as defined in our previous paper. In the example in Figure 4, a manager with perfect foresight would choose utilities and outperform the index by 18 bp. Our imperfect foresight technique similarly uses knowledge of future returns to determine which sector allocation decisions are the right ones, but it does not assume that the manager always chooses the best possible sector. Rather, we simulate the effect of skill by shifting the selection probabilities between the two extremes of random

selection and perfect foresight. These probabilities are given by Equations 4 and 5 in the special case that $n_w = 1$. Only the single best sector is considered “correct.”

The probability of choosing the best sector is 25% with no skill and 100% with perfect foresight. For a manager with 20% skill, the linear interpolation rule of Equation 5 gives a 40% probability of choosing the best sector ($n_w = 1$). The probability of choosing any of the other sectors is reduced to 20%. This set of probabilities leads to even better performance. Once again, the standard deviation of strategy performance changes little, but mean outperformance is increased to 9.1 bp.

For all of the strategies considered, the performance numbers shown are for the extreme case in which the portfolio is invested entirely in the selected sector within each cell. At a bet size of 25%, both the mean outperformance and the standard deviation would be scaled down accordingly. Within the cell shown in Figure 4, the standard deviation of outperformance would be about 2.8 bp, with the mean outperformance ranging from 1.7 bp in the random case to 2.3 bp for 20% skill at choosing the best sector.

Calculating Mean and Variance of Overall Portfolio Outperformance

The portfolio is constructed by investing in each cell a percentage w_j corresponding to the percentage of the market capitalization of the index in that cell. The above sector allocation scheme is applied independently in each quality/duration cell. The overall portfolio performance is then the weighted sum of the cell-by-cell results. That is, if the random variable r_j represents the strategy outperformance of the index within a particular cell j , characterized by a mean \bar{r}_j and a standard deviation σ_j , then the index outperformance of the overall portfolio is given by

$$(6) \quad r = \sum_j w_j r_j ,$$

and the mean and standard deviation of r are given by

$$(7) \quad \begin{aligned} \bar{r} &= \sum_j w_j \bar{r}_j \\ \sigma^2 &= \sum_j w_j^2 \sigma_j^2 \end{aligned} .$$

This calculation is illustrated in Figure 5 for the sector allocation strategy with 20% skill at choosing any winning sector in July 1999. As we saw in Figure 4, this strategy achieved a mean return of about 0.08%, with a standard deviation of

0.11% in the short (0-4 years duration) single-A cell. This cell makes up 13.3% of the index and, hence, the portfolio. In other cells (such as Baa over seven years), this strategy gives a mean return below that of the index for the particular month. On the whole, the strategy produces a mean outperformance of 0.02%, with a standard deviation of 0.05%. This represents the distribution across a population of managers of equivalent skill, all pursuing the same strategy for the month. (If we take the strategy outlined above and simulate the results obtained for the month at this skill level many times, the mean and standard deviation will converge to these values. Simulation is not necessary for this case, as the calculation shown in Figures 4 and 5 is both more precise and computationally more efficient.)

Of course, strategy results will vary over time. The mean outperformance in a given month might be more or less than the 2 bp observed in Figure 5 for this strategy (we will see that the long-term average is 5 bp/month), and the standard deviation across sectors (and, hence, across managers) will be larger in more volatile months and smaller during calm periods. After calculating the mean and variance of strategy performance as in Figure 5 for each month of available data, overall strategy performance is obtained by analyzing the time series of results. The mean outperformance is given by the average of the monthly means. The variance of strategy outperformance is measured in two ways. First, we calculate the time average of the variance across managers in a given month (as in Figure 5). This represents the risk of choosing wrongly and is related to the magnitude of the performance difference between the best and worst sectors. Second, we measure the variance over time of the mean strategy outperformance. This represents the risk due to the fact that changing market conditions make the strategy more effective in some months than in others. The sum of these two

Figure 5. **Sector Allocation Example: Calculating the Mean and Variance of Strategy Outperformance**
July 1999, 20% Skill, Choose Winning Sectors In Each Cell

Quality Cell	Duration Cell	Percent	Index Return	Strategy Return	Strategy Outperf.	Variance Outperf.	St. Dev. Outperf.
Aaa and Aa	0 to 4	11.0	-0.10	-0.05	0.05	0.01	0.09
Aaa and Aa	4 to 7	7.6	-0.75	-0.73	0.02	0.00	0.06
Aaa and Aa	over 7	6.8	-1.17	-1.09	0.08	0.03	0.18
A	0 to 4	13.3	-0.06	0.02	0.08	0.01	0.11
A	4 to 7	14.2	-0.56	-0.49	0.07	0.02	0.13
A	over 7	15.4	-1.06	-1.12	-0.06	0.03	0.17
Baa	0 to 4	9.3	-0.04	-0.01	0.03	0.00	0.07
Baa	4 to 7	12.6	-0.40	-0.41	-0.01	0.03	0.17
Baa	over 7	<u>9.9</u>	<u>-1.04</u>	<u>-1.27</u>	<u>-0.13</u>	<u>0.03</u>	<u>0.18</u>
Total		100.0	-0.55	-0.53	0.02	0.0023	0.05

variance terms gives the overall variance of strategy outperformance. A proof of this assertion, as well as a more precise formulation of this calculation in terms of conditional probabilities, is given in Appendix II.

Sector Allocation Results

Figure 6 shows the results of the sector allocation strategy over time for different levels of manager skill. For the 20% skill case, the strategy outperforms the index by an average of 60.6 bp/year, with a standard deviation (or tracking error) of 45.3 bp/year. Dividing the mean outperformance by the tracking error, we obtain an information ratio of 1.34. Of the 45.3 bp of tracking error, we find that 40.7 bp is due to the variance across managers (or the risk of choosing the wrong sector for a given month) and 19.9 bp is due to the volatility of the spread markets over time.

We see that for reasonable levels of skill, the tracking error is fairly stable, at about 40-50 bp/year. Mean outperformance improves steadily with increasing skill, from near zero for the random selection case (0% skill) to 329.7 bp/year for 100% skill. The results shown here are for the “choosing the best sector” variant of the strategy. This strategy, when applied at the 100% skill level, corresponds most closely to the perfect foresight case considered in our previous report. The results obtained in that report were a mean monthly outperformance of 25 bp with a standard deviation of 21 bp/month, corresponding to an annualized outperformance of 305 bp/year with a tracking error of 72 bp, for an information ratio of 4.24. The information ratio of 4.81 shown for this case is somewhat better than the perfect foresight result, mainly because the duration-matching technique introduced in this report reduces the residual yield curve risk of the sector allocation strategy. Other differences between the two sets of results may be due to the small differences in the definition of the cells and in the time periods considered.

Figure 6. **Sector Allocation:
Historical Performance at Different Skill Levels**
August 1988-July 1999, by Skill at Choosing Best Sectors in Each Cell

Skill	Mean Outperf. (bp/yr.)	St. Dev. over Managers (bp/yr.)	St. Dev. over Time (bp/yr.)	Overall Tracking Error (bp/yr.)	Information Ratio
0%	-6.6	39.8	15.9	42.9	-0.15
10%	27.0	40.6	16.7	43.9	0.62
20%	60.6	40.7	19.9	45.3	1.34
40%	127.9	39.2	30.2	49.5	2.58
60%	195.2	34.9	42.5	55.0	3.55
80%	262.5	26.6	55.4	61.4	4.27
100%	329.7	0.0	68.6	68.6	4.81

The distinction between the two types of variance displayed in Figure 6 is a subtle one. For a single manager with a single time series of returns, an information ratio will be calculated based on the mean and standard deviation of this return series. While both sources of volatility come into play (better decisions are made in some months than in others, and market volatility levels change over time), it is not easy to separate the two effects. The attribution of volatility to these two sources is shown in Figure 6 to illustrate that skill has two distinct and opposing effects on the volatility of strategy performance. As skill increases, the risk of incorrect decisions decreases, but the exposure to market volatility increases.

One concern regarding our “perfect foresight” results was that the true risk of the strategy might be understated due to the one-sided nature of the results. At high skill levels, the strategy outperforms its benchmark every month, and the tracking error is merely the standard deviation of this outperformance. This measure does not reflect the risk of underperformance due to wrong decisions. The use of these numbers to calculate information ratios implied that this standard deviation of outperformance could be used as a rough estimate of the risk such a strategy would entail without perfect foresight. Figure 6 demonstrates that this does not, in fact, cause risk to be underestimated. It is true that in the case of 100% skill, the risk due to variance of results across managers is reduced to zero (from a maximum level of 40.7 bp/year), but this effect is more than counterbalanced by an increase in the variance over time (from 15.9 bp/year to 68.6). The increased skill level leads to extreme results in months with large market swings, thus causing a far greater variance of outperformance than would be observed at more realistic skill levels. The net result is that if the risk estimates of the perfect foresight report are inaccurate, they are more likely to be too high than too low.

Figure 7a compares the results achieved by our two definitions of skill. At all positive skill levels, choosing the best sector in each cell predictably gives higher mean returns. Choosing any of the outperforming cells produces lower variance of outperformance, but significantly lower mean outperformance as well, for a lower information ratio. However, it is misleading to compare these two approaches at equal skill levels, since choosing any outperforming sector is an easier task than choosing the best. A manager who is capable of choosing the best sector with 10% skill is likely to have higher skill at choosing any winning sector.

Making Fewer Sector Decisions

The strategy outlined above makes nine independent sector decisions, one for each duration/quality cell. This allows the portfolio to add value when different sectors outperform in different quality groups. It also leads to diversification of the portfolio sector exposures, helping to keep down the tracking errors versus the index. However, we do not know of anyone who manages a portfolio this way. Sector views for long single-A corporates and short single-A corporates are rarely, if ever, different and certainly not independent.

Consider a strategy in which the entire portfolio is placed in a single sector, across all quality and duration cells. We construct four single-sector duration-matched portfolios within each quality/duration cell, as before. These are then combined with index weights to form four single-sector portfolios that match the index in quality/duration composition. The skill setting determines the probability of choosing a sector for which this portfolio outperforms the index. In an intermediate version of the strategy, independent sector allocation decisions are made for each of the three quality groups and enforced across all duration cells.

Results for these constrained versions of the strategy are shown in Figure 7b. The definition of skill in each case involves choosing any outperforming sector—either for the portfolio overall (one decision), within each quality (three decisions), or within each quality/duration cell (nine decisions). We see that when we limit the strategy to a single overall sector allocation decision, the mean outperformance decreases somewhat and the risk increases significantly. At a skill level of 20%, for example, the tracking error is nearly twice as large as for the cell-by-cell allocation. As a result, the information ratio for choosing a single

Figure 7a. **Sector Allocation Results, Choosing One Sector per Quality/Duration Cell, by Skill Level**

Skill	Sector, Any Winner Per Cell			Sector, Best Per Cell		
	Mean	Std. Dev.	I R	Mean	Std. Dev.	I R
0%	-6.4	42.9	-0.15	-6.6	42.9	-0.15
10%	17.9	42.7	0.42	27.0	43.9	0.62
20%	42.3	42.8	0.99	60.6	45.3	1.34
40%	91.0	44.1	2.07	127.9	49.5	2.58
60%	139.7	46.6	3.00	195.2	55.0	3.55
80%	188.4	50.2	3.76	262.5	61.4	4.27
100%	237.0	54.6	4.34	329.7	68.6	4.81

Figure 7b. **Sector Allocation Results for Different Number of Decisions, by Skill Level, Choosing Any Outperforming Sector**

Skill	One Decision Per Cell			One Decision Per Quality			One Decision Overall		
	Mean	Std. Dev.	I R	Mean	Std. Dev.	I R	Mean	Std. Dev.	I R
0%	-6.4	42.9	-0.15	-6.6	58.0	-0.11	-6.6	79.9	-0.08
10%	17.9	42.7	0.42	14.5	57.8	0.25	12.5	80.1	0.16
20%	42.3	42.8	0.99	35.6	57.6	0.62	31.6	80.0	0.40
40%	91.0	44.1	2.07	77.8	57.3	1.36	69.8	78.5	0.89
60%	139.7	46.6	3.00	120.0	57.0	2.10	108.1	75.5	1.43
80%	188.4	50.2	3.76	162.2	56.9	2.85	146.3	70.6	2.07
100%	237.0	54.6	4.34	204.3	56.9	3.59	184.5	63.4	2.91

sector with 20% skill is only 0.40, similar to the results for choosing a winning sector within each cell at a skill level of 10%. The three-decision scheme, in which we choose one sector within each quality group, gives results that fall between those of the cell-by-cell strategy and the single decision strategy. Of course, it is harder to maintain a high level of skill when making a greater number of finer-grained sector calls.

Quality Allocation Results

The quality allocation strategy is analogous to that used for sector allocation. Within each of the twelve sector/duration cells, the portfolio is concentrated into a single credit quality level, matching index weights, and cell durations. The skill setting determines the probability of choosing any winning quality, or the best quality, within the cell.

The results, shown in Figure 8, are largely similar to those obtained for sector allocation. For the most part, mean outperformance and tracking error are somewhat smaller than for sector allocation at similar skill levels. As the differences in tracking error are more pronounced than the differences in mean outperformance, the information ratios are generally better for quality allocation than for sector allocation.

We also consider the single-decision case, in which a single quality level is chosen for the entire portfolio. Once again, risk is nearly double that of the cell-by-cell allocation scheme, and returns are lower, leading to much lower information ratios. Compared with the single-sector case, both mean outperformance and tracking error are lower, by about the same amounts. The resulting information ratios are roughly equivalent at similar levels of skill.

Figure 8. **Quality Allocation Results, by Skill Level**
Choosing Any Outperforming Sector

Skill	One Decision Per Cell			One Decision Per Sector			One Decision Overall		
	Mean	Std. Dev.	I R	Mean	Std. Dev.	I R	Mean	Std. Dev.	I R
0%	-2.5	33.1	-0.08	-2.8	45.0	-0.06	-2.0	62.0	-0.03
10%	17.1	33.0	0.52	14.2	44.9	0.32	12.1	62.2	0.19
20%	36.8	33.2	1.11	31.1	44.8	0.69	26.1	62.1	0.42
40%	76.1	34.6	2.20	65.0	44.9	1.45	54.2	61.2	0.89
60%	115.4	37.1	3.12	98.9	45.0	2.20	82.2	59.1	1.39
80%	154.8	40.4	3.83	132.8	45.3	2.93	110.3	55.8	1.98
100%	194.1	44.4	4.37	166.7	45.8	3.64	138.4	51.1	2.71

Yield Curve Allocation

The third type of allocation decision considered is placement along the yield curve. The entire portfolio is placed in one of the three duration cells. Each sector/quality cell is divided into three by duration, and the appropriate portion of each of these twelve cells is combined with index weights to obtain three possible portfolios (short, medium, and long duration), each matching the sector/quality composition of the index by market value.

In simulating this strategy, the choice of duration cell is assumed to be based on projections of Treasury yield curve movement. When adjusting the probability of selecting a given cell based on the skill level, the definition of an outperforming duration cell is based on the analysis of the Treasury Index. Nonetheless, the portfolio is assumed to remain all in corporates and to implement the duration view as an overweight to the appropriate duration cells relative to the Corporate Index.⁸

The performance achieved by this strategy at different skill levels is shown in Figure 9. Compared with the duration-neutral strategies considered above, this strategy entails much more risk but promises greater potential for returns. At 20% skill, the strategy achieves a mean annual outperformance of 119.8 bp and a tracking error of 226.4 bp/year, for an information ratio of 0.53. This information ratio is not as good as those obtained for the cell-by-cell versions of sector and quality allocation at this skill level but is better than the results for the strategies that commit the portfolio to a single sector or quality.⁹

⁸ This implementation of a duration view in an all-corporate portfolio carries with it an implicit spread view as well. While the portfolio matches benchmark allocations to each sector by percent of market value, a position that is long duration in this way will be long spread duration as well. This does not bias the results, however, since the implementation of skill is based solely on Treasury Index returns, offering no information on the direction of spread movement.

⁹ The information ratio of 0.29 shown in Figure 9 for duration allocation at 10% skill agrees perfectly with the results of Fjelstad [1999]. For the task of choosing one of two duration cells to overweight, with a 55% probability of choosing correctly, that paper reports a mean outperformance and tracking error that correspond to an information ratio of 0.29.

Figure 9. **Duration Allocation Results, by Skill Level**
Choosing Any Winning Cell

Skill	Mean	Std Dev	I R
0%	11.0	226.1	0.05
10%	65.4	226.8	0.29
20%	119.8	226.4	0.53
40%	228.6	222.4	1.03
60%	337.4	213.7	1.58
80%	446.1	199.8	2.23
100%	554.9	179.4	3.09

SECURITY SELECTION STRATEGY

In our security selection strategy, the portfolio allocates funds along the three-dimensional grid described above to match the percentage of index capitalization and the average index duration in each cell exactly. There is no attempt to outperform the index based on systematic duration differences or sector exposures. Rather, the manager's skill at security selection within each cell is the key to strategy performance.

Unlike the allocation strategies, for which we were able to calculate exact statistics by summing across the entire distribution of possible results each month, the performance of the security selection strategy requires simulation. The simulation procedure detailed below was used to generate 10,000 portfolios each month for each set of parameters.

Number of Securities

The most important determinant of the risk of this strategy is the number of bonds in the portfolio. The larger the exposure of the portfolio to any single security or issuer, the greater the nonsystematic risk. As more securities are purchased, diversification reduces this risk, and the portfolio will behave more like the index. In our simulations, we express the size of the portfolio as a percentage of the number of bonds in the index. Within a given cell, the number of bonds that the portfolio will purchase is computed by taking this percentage of the number of index bonds in the cell.

Duration Matching

To ensure that the bonds selected for the portfolio in a given cell match the duration of the index in that cell, we split each cell in two before selecting bonds. We choose one set of bonds from those with duration above the average and another from the set of bonds below the average cell duration. An appropriate mix of these two portfolios can always be found to match the index duration for the cell as a whole. To make this possible, we choose a minimum of one bond from each half-cell, regardless of the targeted number of bonds based on the percentage of the index.

Selection Criterion: Excess Return

In our perfect foresight report, the performance measure used to select bonds, sectors, and qualities was excess return¹⁰ over duration-matched Treasuries, over a foresight horizon that varied from one to twelve months. In each strategy,

¹⁰ Excess return is a commonly used mechanism for comparing the performance of different spread products on a duration-adjusted basis. There are several different methods of computing excess returns. Our procedure is as follows. We segment the Treasury universe into ½-year duration cells and calculate the market weighted average total return and average duration for each cell. To calculate the excess return of a given bond, we look up our bond's duration in this array of Treasury durations and returns and interpolate between the nearest two entries to find the appropriate Treasury return. This Treasury return is then subtracted from the bond's return to obtain the excess return.

we selected the candidates with the highest excess returns. We used excess returns, not total returns, to avoid a duration bias in the selection process. That is, in the month of a yield curve rally, we do not want to favor sectors or securities with a longer duration.

In the current study, measures of future performance (foresight) are used to shift the relevant selection probabilities away from the purely random. For sector and quality allocation, we choose to use total returns, with duration neutrality ensured by the method described above. For security selection, however, the selection process occurs before the duration correction. We use our skill to select the best performing bonds within each half-cell and then blend the results. Security selection based on total returns during a yield curve rally would then show a bias toward the longer securities in each half-cell, which would need to be corrected during the weighting phase by more heavily weighting the shorter half-cell. To avoid this anomaly, we once again use excess returns as the basis for security selection.

Skill Implementation

Within each half-cell, the following procedure is used to simulate the selection of securities at a certain level of skill. The number of bonds we need to select is determined in advance, based on the desired percentage of index bonds. Using our foresight of excess returns, we calculate the market-weighted average performance of all index bonds in the cell and divide the bonds into those that perform better than the average (winners) and those that perform worse (losers). The probability of selecting each security is calculated according to Equation 5 (page 13), based on manager skill and the numbers of winners and losers available. Bonds are selected in a sequential fashion to avoid selecting the same bond twice in a given month. After each bond is selected, it is removed from the pool of available securities. The numbers of winners and losers remaining in the pool are updated, and the selection probabilities are once again interpolated between random selection and perfect foresight according to Equation 5. This procedure is repeated until the desired number of securities has been selected.

Equal Weighting versus Market Weighting

Once the required number of bonds is chosen within each half-cell, we need to set the amounts of each security to be purchased in the portfolio. We consider two weighting schemes. In market weighting, the selected securities are weighted by the ratios of their overall market capitalizations. Larger issues are given a bigger share of the portfolio. This helps make the portfolio more like the index, which is similarly market-weighted, especially when many securities are selected. In equal weighting, we purchase the same market value of each security selected within a half-cell. This method avoids overly large exposures to any single issuer. Unless otherwise noted, the results reported for our security selection strategies use market weighting within each cell to generate portfolios.

Results

Figure 10 shows the results of the security selection strategy, selecting 5% of the bonds in the index. With a tracking error of about 30 bp/year, this strategy generates a mean outperformance of 56.3 bp/year, for an information ratio of 1.93, at a skill level of only 10%. At 20% skill, the information ratio rises to 3.52. As seen in the perfect foresight study, the information ratios that can be achieved by security selection greatly exceed those obtained by any of the asset allocation strategies.

These results are not directly comparable with those of our perfect foresight study. In that study, our strategy deterministically chose the top performing bonds in the index. Here, we choose bonds at random. Consider a cell with 100 bonds. The “top 5%” strategy, using perfect foresight, would deterministically choose the five best performing bonds in the cell. With imperfect foresight, our 5% case chooses five bonds at random. Even with a 100% skill setting, the strategy would randomly choose five bonds from among the 50 or so “winners.” The average performance of the strategy will thus be closer to that of the “top 50%” case using perfect foresight, but with greater variance.

Figure 11a shows the dependence of these results on the size of the portfolio at 10% skill. We see that as more securities are selected in the portfolio, there is a continued decrease in tracking error because of increased diversification. While mean outperformance declines slightly as more securities are chosen,¹¹ the information ratio increases steadily with the number of bonds. In Figure 11b, we

¹¹ This effect may be understood based on the fact that several bonds are chosen sequentially from each cell without replacement. At each stage, as described above, the probability of choosing a winner at a skill level s is $(n_w + sn_L)/(n_w + n_L)$. When choosing with skill has increased the chances of picking winners in the early rounds, the remaining pool of securities has a higher concentration of losers, limiting the potential outperformance. Consider a cell with four bonds, two winners and two losers. On the first pick, the unskilled manager will have a 50% chance of selecting a winner. If he has picked a winner, his probability of choosing another on the next pick will have decreased to $1/3$. If the first pick was a loser, the probability of selecting a winner on the second try is $2/3$. So the four possible outcomes of choosing two bonds from the four are given by:

$$p_{ww} = \frac{1}{2} \frac{1}{3}, \quad p_{wl} = \frac{1}{2} \frac{2}{3}, \quad p_{lw} = \frac{1}{2} \frac{2}{3}, \quad p_{ll} = \frac{1}{2} \frac{1}{3}$$

The resulting probability of choosing two winners is $1/6$, and the probability of choosing one winner sums to $2/3$. The number of winners out of the two bonds is, thus, exactly one, and the mean performance of the strategy in the unskilled case is exactly the same for choosing two bonds as it is for choosing one. The probability of choosing a winner at the second pick is never the same as it was on the first, but the lack of skill on the first pick makes it equally likely that the probability of picking a winner on the second pick is higher or lower. In the skilled case, because the probability of choosing a winner on the first pick is higher, the overall probability of choosing a winner on the second pick will be lower. For example, for 20% skill, we have

$$p_{ww} = \left(\frac{2 + 0.2 * 2}{2 + 2} \right) \left(\frac{1 + 0.2 * 2}{1 + 2} \right) = \frac{2.4}{4} \frac{1.4}{3}, \quad p_{wl} = \frac{2.4}{4} \frac{1.6}{3}, \quad p_{lw} = \frac{1.6}{4} \frac{2.2}{3}, \quad p_{ll} = \frac{1.6}{4} \frac{0.8}{3}.$$

In this case, the probability of choosing two winners is 28%, the probability of choosing one winner is 61%, and the mean number of winners selected out of the two bonds is 1.17. The mean performance will be somewhat lower than that of a strategy that picks one bond at 20% skill, with 0.6 winners on average.

Figure 10. **Security Selection:**
Historical Performance at Different Skill Levels
August 1988-July 1999, Choosing 5% of Index Bonds

Skill	Mean Outperformance (bp/yr)	Overall Tracking Error (bp/yr)	Information Ratio (annualized)
0%	3.4	28.4	0.12
10%	56.3	29.2	1.93
20%	109.4	31.1	3.52
40%	215.2	37.4	5.75
60%	320.8	46.0	6.98
80%	426.4	55.8	7.64
100%	532.0	66.2	8.03

Figure 11a. **Security Selection: Historical Performance for**
Different Portfolio Sizes, August 1988-July 1999
Skill 10%, by Portfolio Size, as a % of Index Bonds

% of Index Bonds	Mean Outperformance (bp/yr)	Overall Tracking Error (bp/yr)	Information Ratio
2.5%	57.7	39.3	1.47
5.0%	56.3	29.2	1.93
7.5%	54.5	23.3	2.34
10.0%	53.4	19.8	2.70
15.0%	51.3	15.9	3.22
20.0%	49.5	13.5	3.68
25.0%	47.9	11.9	4.03

Figure 11b. **Security Selection: Market Weighted versus Equal Weighted**
within Each Cell, Skill 20%, by Portfolio Size, as a % of Index Bonds

% of Index Bonds	Market Weighted			Equal Weighted		
	Mean Outperf. (bp/yr)	Overall Tracking Error (bp/yr)	Inform. Ratio	Mean Outperf. (bp/yr)	Overall Tracking Error (bp/yr)	Inform. Ratio
2.5%	111.0	40.6	2.73	111.1	40.9	2.72
5.0%	109.4	31.1	3.52	109.9	31.3	3.52
7.5%	107.0	25.7	4.16	108.2	26.6	4.07
10.0%	105.3	22.5	4.69	106.9	23.9	4.47
15.0%	102.1	18.9	5.39	104.2	21.1	4.95
20.0%	99.1	16.8	5.91	101.4	19.3	5.25
25.0%	96.1	15.3	6.27	98.5	18.3	5.38

see that the same effect holds true at 20% skill, regardless of whether the bonds selected within each cell are purchased in equal market values or weighted according to their market capitalization. When using a small number of bonds, there is little performance difference between these two schemes. (In cells in which only a single bond is chosen, the two are identical.) As a greater number of bonds is included in the strategy, the information ratio of the market-weighted scheme increases faster due to lower tracking error relative to the market-weighted index. For this reason, we have chosen to concentrate on the market-weighted version of the strategy; all subsequent results use this approach.

Figure 12 provides the results of the security selection strategy across a wide range of skill levels and portfolio sizes.

Figure 12. **Security Selection Results, by Skill Level and Portfolio Size**

Skill	2.5% of Index			5% of Index			7.5% of Index			10% of Index		
	Mean	T E	I R	Mean	T E	I R	Mean	T E	I R	Mean	T E	I R
0%	4.6	38.7	0.12	3.4	28.4	0.12	2.0	22.4	0.09	1.7	18.7	0.09
10%	57.7	39.3	1.47	56.3	29.2	1.93	54.5	23.3	2.34	53.4	19.8	2.70
20%	111.0	40.6	2.73	109.4	31.1	3.52	107.0	25.7	4.16	105.3	22.5	4.69
40%	217.2	45.3	4.80	215.2	37.4	5.75	212.1	33.3	6.37	209.3	30.7	6.82
60%	323.2	52.1	6.21	320.8	46.0	6.98	317.3	42.8	7.41	313.8	41.0	7.66
80%	429.0	60.1	7.14	426.4	55.8	7.64	422.6	53.5	7.90	418.9	52.2	8.03
100%	534.9	69.1	7.74	532.0	66.2	8.03	528.3	64.8	8.15	524.8	64.0	8.20

	15% of Index			20% of Index			25% of Index		
	Mean	T E	I R	Mean	T E	I R	Mean	T E	I R
0%	0.9	14.7	0.06	0.5	12.2	0.04	0.3	10.5	0.03
10%	51.3	15.9	3.22	49.5	13.5	3.68	47.9	11.9	4.03
20%	102.1	18.9	5.39	99.1	16.8	5.91	96.1	15.3	6.27
40%	204.5	28.1	7.27	199.6	26.3	7.58	195.0	25.2	7.75
60%	308.3	39.1	7.88	302.6	37.6	8.05	296.8	36.5	8.13
80%	413.9	51.1	8.10	408.0	49.7	8.21	401.5	48.6	8.26
100%	521.5	63.6	8.20	515.7	62.4	8.26	508.3	60.9	8.35

ANALYSIS OF RESULTS

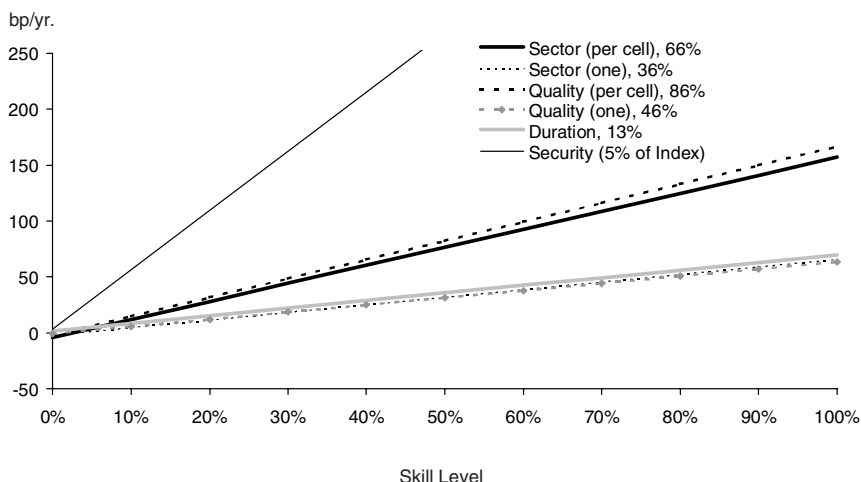
What is the most appropriate way to compare the results of these very different investment strategies? One approach is simply to examine the results of all strategies at the same skill level. Figure 13 shows that at 20% skill, the strategies span a wide range of mean outperformance, tracking error, and information ratios. The information ratios of the security selection strategies far surpass all of the allocation strategies. Cell-by-cell sector and quality allocations outperform the duration allocation scheme, which in turn surpasses the single-sector and single-quality strategies.

A slightly different way of looking at the relative performance of the strategies is to compare the mean outperformance that can be achieved at a given level of risk. Let us define the intrinsic risk of a given strategy as the tracking error achieved by that strategy at the 0% skill level. For the security selection strategy using 5% of index securities, the intrinsic risk is 28.4 bp/year. By choosing an appropriate bet size, as described above, any of the allocation strategies can be implemented so as to have the same level of intrinsic risk. For example, the intrinsic risk of the duration allocation strategy, which is 226.1 bp at a bet size of 100%, can be reduced to 28.4 bp by using a bet size of 13%. This strategy, at a skill level of 20%, will achieve a mean outperformance of 13% x 119.8 bp, or about 15 bp. In Figure 14, we compare the strategies (only the “any winner” variants are plotted), with all bet sizes adjusted to achieve an intrinsic risk of 28.4 bp/year. Mean outperformance is plotted as a function of skill for each strategy.

Figure 13. **Performance of Different Investment Styles
with Imperfect Foresight**, August 1988-July 1999, Skill 20%

Strategy	Mean Outperf. (bp/yr.)	Overall Tracking Error (bp/yr.)	Information Ratio
Duration	119.8	226.4	0.53
Quality (best per cell)	43.5	34.2	1.27
Quality (any winner per cell)	36.8	33.2	1.11
Quality (one per sector)	31.1	44.8	0.69
Quality (one decision)	26.1	62.1	0.42
Sector (best per cell)	60.6	45.3	1.34
Sector (any winner per cell)	42.3	42.8	0.99
Sector (one per quality)	35.6	57.6	0.62
Sector (one decision)	31.6	80.0	0.40
Security (5% of bonds)	109.4	31.1	3.52
Security (10% of bonds)	105.3	22.5	4.69
Security (25% of bonds)	96.1	15.3	6.27

Figure 14. **Mean Outperformance as a Function of Skill for Different Investment Styles, with Bet Sizes Chosen to Achieve Equivalent Levels of Risk**



The results of Figure 14 are divided into three tiers. Security selection earns far and away the greatest return for a given skill level, followed by sector and quality allocation within each cell, followed by the three allocation schemes that make a single decision (duration cell, sector, or quality) for the entire portfolio.

To properly interpret this graph, we must recognize that the skill levels of these different strategies are not directly comparable. One can achieve the unlikely result of 50 bp of mean annual outperformance for this amount of risk by applying security selection with 10% skill, sector or quality allocation per cell with about 35% skill, or one of the single-decision allocation methods with 75% skill. However, it is not clear which of these is the hardest to achieve.

The clear tiering effect shown in Figure 14 suggests that the number of independent decisions that are required to implement a strategy is a major determinant of risk-adjusted performance. The security selection method, in which the number of decisions is equal to the number of securities in the portfolio (for 5% of the index, this averaged 178 securities), is by far the best performer. The quality allocation strategy, with twelve decisions (one per sector/duration cell), and the sector allocation strategy, with nine decisions (one per quality/duration cell), form the next performance tier. The three single-decision allocation strategies have the lowest information ratios for a given skill level when viewed in this manner.

The reason for this is clear. In the performance comparison of Figure 14, the bet sizes have been chosen to achieve the same level of risk for each strategy. When an investment strategy is the result of many independent decisions, each month's performance will be a combination of some successful bets and some unsuccessful ones. The diversification of the risks decreases the overall risk of the strategy without reducing the expected return. This allows the strategies with better diversification of risk to take larger positions and achieve greater outperformance.

Value of a Single Decision

To form a simplified model of this effect, let us assume that each strategy (at any given skill level) can be viewed as an equally weighted sum of n subcomponents reflecting the individual decisions taken. Let r_i be the outperformance due to a single decision i taken alone, and let the overall portfolio outperformance be the average of n such terms,

$$(8) \quad r = \frac{1}{n} \sum_{i=1}^n r_i ,$$

where all of the r_i are independent and identically distributed random variables with mean $\mu_{decision}$ and standard deviation $\sigma_{decision}$. The outperformance of the overall strategy will then have a mean $\mu_{strategy} = \mu_{decision}$ and a standard deviation

$$\sigma_{strategy} = \sigma_{decision} / \sqrt{n} .$$

For example, in the sector allocation strategy, the r_i could represent the return difference between the portfolio and index components within each of the $n = 9$ quality/duration cells. This model would be precise if all the cells had equal weights in the index, the distribution of strategy outperformance at a given skill was the same in each cell, and the results in each cell were independent of each other. While these conditions do not necessarily hold, it is interesting to look at the per-decision tracking errors $\sigma_{decision}$ implied by this model for each strategy. While not directly observable, we can back them out of our observations by multiplying the tracking error $\sigma_{strategy}$ by \sqrt{n} . Information ratios at the per-decision level can be computed as

$$\mu_{decision} / \sigma_{decision} .$$

Figure 15 revisits the performance data of Figure 13 in light of this analysis. We calculate the implied per-decision tracking errors as described above to reflect the number of independent decisions involved in each strategy. Comparing the

resulting per-decision information ratios, we find that the results for the different strategies are much closer in magnitude than before, with the relative rankings almost exactly the opposite of our previous results. The highest per-decision information ratio is achieved by the duration decision; next are the single-decision versions of sector and quality allocation. The cell-by-cell sector and quality decisions come next, closely followed by security selection.

How do we reconcile these two diametrically opposed points of view? Which better represents the truth, Figure 13 or Figure 15? Is security selection the most important or the least important portfolio strategy?

The answer, of course, is both. Figure 15 confirms the commonly held notion that the most important single decision is the duration call. However, Figure 13 emphasizes the power of diversification in reducing risk. When portfolio managers attempt to enhance portfolio return by taking several independent risk exposures instead of one large one, tracking error is reduced. The information ratio is increased as a result, provided that the same level of skill (and hence outperformance) can be maintained across the greater number of decisions.

It is clear from Figure 15 that the model of n independent sources of risk as implied by Equation 8 does not provide a perfect adjustment for the number of decisions. In particular, once we have made our adjustment, we would expect the information ratio per security selection decision to be independent of the number

Figure 15. **Performance of Different Investment Styles, with Risk Adjusted for Number of Independent Decisions**

August 1988-July 1999, Skill 20%

Strategy	Mean Outperf. (bp/yr.)	Overall Track. Err. (bp/yr.)	No. of Independent Decisions	Tracking Error per Decision	Information Ratio per Decision
Duration	119.8	226.4	1	226.4	0.53
Quality (best per cell)	43.5	34.2	12	118.6	0.37
Quality (any winner per cell)	36.8	33.2	12	115.1	0.32
Quality (one per sector)	31.1	44.8	4	89.7	0.35
Quality (one decision)	26.1	62.1	1	62.1	0.42
Sector (best per cell)	60.6	45.3	9	136.0	0.45
Sector (any winner per cell)	42.3	42.8	9	128.4	0.33
Sector (one per quality)	35.6	57.6	3	99.7	0.36
Sector (one decision)	31.6	80.0	1	80.0	0.40
Security (5% of bonds)	109.4	31.1	178	414.9	0.26
Security (10% of bonds)	105.3	22.5	369	432.2	0.24
Security (25% of bonds)	96.1	15.3	961	474.3	0.20

of bonds selected. Instead, we seem to have adjusted by too much. This is consistent with the situation in which there are positive correlations among the various decisions (e.g., correlations between bonds of the same issuer or industry group). When dividing risk among n positively correlated decisions, the risk is decreased by less than \sqrt{n} , and our adjustment overstates the benefit of diversification. This effect can also explain why the information ratios per decision seem to be lower for the sector and quality allocation strategies that make separate decisions in each cell. The best sector allocation for single-A bonds may not be the same as that for Baa-rated bonds, but there is certainly a positive correlation between the two. The adjusted numbers in Figure 15 should thus be viewed as only a crude approximation.

The presentation of results according to a constant skill level is possibly misleading in another way as well. The strategies requiring many decisions (e.g., sector allocation in twelve cells, selection of 961 bonds) are compared with similar strategies requiring fewer decisions (e.g., single-decision quality allocation, selection of 178 bonds) at the same skill level. This is where the greatest challenge lies. A sector rotation specialist may always have a view favoring one sector or another on a macro basis. But if asked to choose a favorite sector separately in each of nine quality/duration cells, will the analyst be equally confident of each of these views? It would seem to be much harder to maintain the same skill level across this expanded set of decisions. A similar argument can be made regarding security selection. While an analyst may have an excellent track record concerning the performance results of top picks, it is difficult to maintain the same skill level when it becomes necessary to select a greater number of securities. Clearly, for a fixed number of bonds, even a very small increase in the skill level of the security selection process can have a marked effect on overall performance. To help decide how to allocate a fixed research budget, it might be more interesting to compare the tradeoff between skill and the number of decisions. For instance, we see in Figure 12 that security selection using 5% of the index with 20% skill achieves an information ratio of 3.52, while using 20% of the index with 10% skill achieves an information ratio of 3.68.

Are there specific sectors in which security selection is most important? Figure 16 gives a detailed breakdown by sector/quality/duration cells of our results for the security selection strategy using 5% of the bonds in the index at a 10% skill level. Comparing the tracking errors achieved by the strategy in different cells, we find that certain trends hold in general, but not in every case. Within a given sector and quality cell, longer-duration cells tend to have greater risk than their shorter-duration counterparts. Similarly, lower-quality cells tend to have greater tracking errors than higher qualities. Also, cells with greater tracking errors tend to offer a skilled manager more opportunity for outperformance. Nevertheless, the information ratios cover a fairly wide range, from 0.17 for Baa short Yankees to 0.56 for A short financials. It is noteworthy that these are also the smallest and largest cells in the index, respectively. Strategy risk in the larger

cells is reduced by the additional diversification provided by choosing more securities. Once again, we divide the information ratio by the square root of the average number of bonds in the portfolio to obtain an information ratio per decision. These numbers have a much tighter distribution, ranging from 0.09 to 0.24.

Figure 16. Performance of Security Selection Strategy by Cell, Choosing 5% of Bonds, 10% Skill

Quality	Duration		Sector	Avg. No. Bonds in Cell	Avg. % of Index Market Value	Avg. Cell Duration	Avg. Cell Return (%/mo.)	Avg. No. Bonds in Portfolio	Mean Outperf. (bp/yr.)	Tracking Error (bp/yr.)	Inform. Ratio	Inform. Ratio per Decision
	Cell	Cell										
Aaa-Aa	Short	Industrial	62.2	1.8	2.44	0.62	2.4	29.4	106.7	0.28	0.18	
Aaa-Aa	Med	Industrial	57.7	1.8	5.49	0.74	2.1	49.7	151.4	0.33	0.23	
Aaa-Aa	Long	Industrial	48.5	1.7	9.45	0.82	2.1	119.7	369.5	0.32	0.22	
Aaa-Aa	Short	Utility	78.7	1.3	2.51	0.59	3.4	55.3	153.7	0.36	0.19	
Aaa-Aa	Med	Utility	174.5	2.7	5.67	0.75	7.8	68.9	154.2	0.45	0.16	
Aaa-Aa	Long	Utility	121.5	2.4	8.82	0.83	5.4	68.5	161.3	0.42	0.18	
Aaa-Aa	Short	Finance	156.3	4.2	2.29	0.64	7.0	30.5	65.0	0.47	0.18	
Aaa-Aa	Med	Finance	78.1	2.1	5.38	0.74	3.3	56.1	129.3	0.43	0.24	
Aaa-Aa	Long	Finance	43.1	0.9	9.46	0.82	2.1	109.0	357.2	0.31	0.21	
Aaa-Aa	Short	Yankees	91.1	3.1	2.56	0.63	3.7	27.2	88.0	0.31	0.16	
Aaa-Aa	Med	Yankees	94.7	4.3	5.54	0.76	3.8	43.4	163.2	0.27	0.14	
Aaa-Aa	Long	Yankees	69.6	2.0	9.46	0.85	2.7	66.4	214.9	0.31	0.19	
A	Short	Industrial	187.1	4.9	2.49	0.64	8.3	27.4	71.1	0.39	0.13	
A	Med	Industrial	216.7	5.8	5.49	0.75	9.9	50.8	90.6	0.56	0.18	
A	Long	Industrial	207.3	6.2	9.32	0.84	9.4	69.2	123.7	0.56	0.18	
A	Short	Utility	110.5	1.8	2.45	0.61	4.8	45.0	122.7	0.37	0.17	
A	Med	Utility	214.1	3.3	5.62	0.76	9.8	64.5	120.8	0.53	0.17	
A	Long	Utility	103.3	2.3	8.44	0.84	4.4	74.2	265.5	0.28	0.13	
A	Short	Finance	304.7	7.6	2.41	0.65	14.3	22.5	40.4	0.56	0.15	
A	Med	Finance	217.6	5.8	5.40	0.75	9.9	44.9	101.3	0.44	0.14	
A	Long	Finance	75.6	2.2	8.79	0.82	3.3	96.3	248.2	0.39	0.21	
A	Short	Yankees	38.3	1.1	2.56	0.62	2.0	52.6	196.0	0.27	0.19	
A	Med	Yankees	60.3	2.5	5.62	0.73	2.6	60.4	200.5	0.30	0.19	
A	Long	Yankees	53.0	2.5	9.71	0.85	2.7	47.5	164.4	0.29	0.17	
Baa	Short	Industrial	130.0	3.6	2.53	0.67	5.6	47.7	227.3	0.21	0.09	
Baa	Med	Industrial	170.8	4.9	5.49	0.73	7.6	98.8	264.5	0.37	0.14	
Baa	Long	Industrial	128.1	4.2	8.89	0.82	5.7	87.3	380.3	0.23	0.10	
Baa	Short	Utility	118.4	2.2	2.46	0.64	5.0	48.2	140.6	0.34	0.15	
Baa	Med	Utility	196.9	3.2	5.58	0.79	8.9	72.8	175.5	0.41	0.14	
Baa	Long	Utility	79.0	1.9	8.54	0.86	3.3	59.4	282.4	0.21	0.12	
Baa	Short	Finance	85.7	1.9	2.40	0.68	3.4	48.2	196.6	0.24	0.13	
Baa	Med	Finance	86.3	1.9	5.41	0.75	3.3	106.0	355.1	0.30	0.16	
Baa	Long	Finance	20.8	0.5	8.66	0.81	2.0	108.2	456.8	0.24	0.17	
Baa	Short	Yankees	13.5	0.4	2.49	0.68	2.0	58.4	353.0	0.17	0.12	
Baa	Med	Yankees	28.4	0.9	5.64	0.71	2.2	115.3	345.0	0.33	0.23	
Baa	Long	Yankees	20.4	0.5	9.24	0.85	1.9	89.0	394.7	0.23	0.16	

Diversification of Risk Among Different Strategies

We have emphasized the role played by the number of independent decisions within a given strategy in reducing risk and improving risk-adjusted return. The same effect is achieved by combining strategies that express independent views in different dimensions.

Consider a strategy that takes risk in four dimensions simultaneously, at approximately equal levels of tracking error. Specifically, we allocate 13% of the portfolio to the duration allocation strategy, 36% to the sector allocation strategy, and 46% to the quality allocation strategy (one decision each). (These weights correspond to the bet sizes used in Figure 14 to obtain equivalent risk levels.) The remaining 5% of the portfolio is neutral to the benchmark. The combination of these strategies will be used to set the portfolio allocations to sector/quality/duration cells. We further assume that the portfolio is composed of only 5% of index securities and is thus subject to the non-systematic tracking error that we observed in our security selection strategy. Assuming independence of the results for the different strategies, the mean outperformance and tracking error of this blended strategy can be calculated using Equation 7. This blended strategy has a tracking error of 55.7 bp/year.

Figure 17 shows the performance of this strategy as a function of the skill levels at each management task. In the unskilled case (all skill levels at 0%), the strategy does no better than the index on average. When the skill level for any one of the

Figure 17. **Performance of a Blended Investment Strategy at Different Skill Levels for Each Style**

Skill Levels (%)				Performance		
Duration	Sector	Quality	Security	Mean Outperf.	Tracking Error	Information Ratio
0	0	0	0	1.5	55.7	0.03
10	0	0	0	8.1	55.7	0.14
0	10	0	0	8.2	55.7	0.15
0	0	10	0	7.8	55.7	0.14
0	0	0	2	12.1	55.8	0.22
0	0	0	4	22.7	55.9	0.41
10	10	10	0	21.1	55.8	0.38
20	10	10	0	27.6	55.8	0.49
10	20	10	0	27.7	55.8	0.50
10	10	20	0	27.4	55.8	0.49
10	10	10	2	31.6	55.9	0.57
10	10	10	4	42.2	56.0	0.75
20	20	20	0	40.6	55.8	0.73

allocation strategies is raised to 10%, we see a modest gain of about 8 bp/year for any of the three, with information ratios of about 0.14. If the skill at all three allocation tasks is raised to 10%, the gains combine for an expected outperformance of 21.1 bp/year and an information ratio of 0.38. Comparing these results with those in Figures 7-9, we see that even though we have increased our risk estimate to include the effect of security risk, we achieve a higher information ratio than with 10% skill at any of these three single-decision allocation strategies alone. The effect of a small increase in skill at security selection is even more striking. Increasing the security selection skill from 0% to 2% provides more outperformance than 10% skill at any single allocation dimension; at 4% it outperforms 10% skill at each of the three allocation strategies. A similar effect is observed if we look at the incremental effect of raising allocation skills from 10% to 20%.

CONCLUSION

At equivalent skill levels, the security selection strategy gives the highest information ratios of the strategies considered. This is true, in large part, because of the diversification of risk among the many independent decisions involved in selecting each security in the portfolio. This observation provides a clear message for all portfolio managers, including the purest of asset allocators: the single most important element in achieving a high information ratio is diversification of risk among several independent return-enhancing strategies.

Concerning security selection, we are left with the question of what skill level is reasonable to expect across a wide range of securities. Nevertheless, we have unequivocally demonstrated the importance of security selection skill. Any systematic improvement in the selection process will undoubtedly give a significant boost to portfolio performance.

This study was conducted on a single asset class (corporate bonds) in a single market (U.S. fixed income) over a single decade. Care should be used when generalizing these results to other asset classes (such as mortgages), other markets (e.g., Europe), or other time periods. In our previous report, we did consider different parts of the economic cycle and different credit markets. We repeated our perfect foresight analysis for another market (Eurobonds) and for a recessionary time period and found our conclusions to be quite robust. Several interesting issues remain for further study. This paper includes neither a model for transaction costs nor a mechanism for reducing turnover. What levels of skill are required to produce steady outperformance once transaction costs are considered? How will the performance achievable at a given skill level be affected by constraints on portfolio turnover? Our previous work under the perfect foresight assumption underscored the importance of matching the foresight horizon to the average holding period. In future research to explore these issues, we will apply the imperfect foresight approach to foresight horizons longer than one month.

The conclusions of this paper should be of particular interest to new investors in the credit markets, such as European credit portfolio managers and central banks, who are in the process of establishing their investment style.

This paper does not offer a quantitative model for building views on market sectors or individual credits. As such, none of the strategies studied are directly implementable. However, interpretation of these results can influence portfolio management practice in several ways. First, it can help guide the formation of an investment style and an associated research program. In particular, our results underscore the importance of skill at security selection. More generally, they highlight the importance of diversifying portfolio views among several independent sources of risk. This should encourage risk-conscious managers to pursue multiple avenues of research simultaneously.

Second, the results of such simulation studies can be used to help evaluate manager performance. Fox [1999] and Fjelstad [1999] analyze the observed performance distributions for managers with known investment styles. The skill and bet size parameters that produce a simulated distribution most similar to observed performance may be said to characterize the skill of the manager.

The comparison of information ratios between simulated strategies and actual manager track records provides another interesting interpretation. Goodwin [1998] reports empirically observed information ratios for institutional money managers benchmarked against the Lehman Brothers Aggregate Index. Only 20.5% of the managers in the sample achieve information ratios over 0.5, and only 2.6% reach 1.0 or better. Our results show that, at least when transaction costs are neglected, such results can be achieved even at fairly low levels of skill, where correct views are established only slightly more often than with random selection.

APPENDIX I. INDEPENDENCE OF INFORMATION RATIO FROM BET SIZE

Denote the return of a given strategy by R_S and that of the benchmark by R_B . The mean μ_S and variance σ_S^2 of the strategy outperformance are given by

$$\begin{aligned}\mu_S &= E(R_S - R_B) \\ \sigma_S^2 &= \text{Var}(R_S - R_B)\end{aligned}$$

Now consider an investment scheme in which only a portion b of portfolio assets is committed to the strategy, with the remainder invested in the benchmark. The return on this investment is given by

$$R_{S,b} = bR_S + (1-b)R_B = R_B + b(R_S - R_B).$$

We can easily see that the mean and variance of outperformance of the strategy at bet size b are given by

$$\begin{aligned}\mu_{S,b} &= E(R_{S,b} - R_B) = E(b(R_S - R_B)) = bE(R_S - R_B) = b\mu_S \\ \sigma_{S,b}^2 &= \text{Var}(R_{S,b} - R_B) = \text{Var}(b(R_S - R_B)) = b^2 \text{Var}(R_S - R_B) = b^2 \sigma_S^2\end{aligned}$$

Turning our attention to the strategy information ratio

$$IR_S = \frac{\mu_S}{\sigma_S},$$

we find it to be independent of bet size:

$$IR_{S,b} = \frac{\mu_{S,b}}{\sigma_{S,b}} = \frac{b\mu_S}{b\sigma_S} = \frac{\mu_S}{\sigma_S} = IR_S.$$

APPENDIX II. DECOMPOSING THE VARIANCE OF STRATEGY OUTPERFORMANCE

In this appendix, we characterize the strategy outperformance by a conditional random distribution. We develop expressions for the unconditional mean and variance of the distribution and show that the variance of outperformance can be viewed as the sum of two terms: the variance of performance across managers and the variance of performance over time.

Let us represent the outperformance of each deterministic strategy considered by a set of N random variables x_1, x_2, \dots, x_N . Using sector allocation as an example, we have $N = 4$, and the x_i are the outperformances of the four different sectors in our duration-neutral and quality-neutral strategy. We use a vector x to represent them.

We assume that the vector random variable x has a probability distribution function (pdf) $f(x)$ and that each month of historical observation corresponds to a single outcome of this random variable.

Let the random variable y denote the outperformance of the portfolio strategy. The process of using imperfect foresight to alter the probabilities of choosing the different sectors makes the distribution of the strategy outperformance y conditional on the outcome of the sector return vector x . The strategy outperformance y is thus a Bayesian process and follows a conditional random distribution.

Let us denote by $p(y/x)$ the conditional pdf of y for given x . This represents the probability of any particular outcome of the strategy, given our weighted probabilities for choosing each sector. Figure 4 shows explicitly the conditional pdf for one particular cell for a given month.

The distributions $p(y/x)$ are in discrete form, due to the finite number of sectors to choose from. Nevertheless, we will use a continuous representation in this discussion. By using the Dirac function $\delta(x)$, which has the properties that $\delta(x) = 0$ when $x \neq 0$ and $\int \delta(x) dx = 1$, we can use the continuous form of pdf to express a discrete distribution $p(y/x)$ in terms of the sum of different Dirac functions centered at different points weighted by the appropriate weights. We can thus use the continuous representation with no loss of generality.

Bayesian statistics and conditional probability theory state that if event(s) x has an unconditional distribution $f(x)$ and event y has a conditional pdf $p(y/x)$, then the joint pdf of x and y is $p(y/x) \cdot f(x)$ and the unconditional pdf for y is then given by

$$g(y) = \int p(y/x) \cdot f(x) dx$$

The conditional mean and variance are defined as

$$E(y | x) = \int y p(y | x) dy$$

$$Var(y | x) = E((y - E(y | x))^2 | x) = E(y^2 | x) - [E(y | x)]^2 = \int y^2 p(y | x) dy - (\int y p(y | x) dy)^2$$

where the notation E_x denotes the expectation of a random variable x under its pdf. The conditional mean and variance of strategy outperformance is exactly what we have calculated for each month of historical data considered in our study. Figure 5 (on page 15) shows the details of this calculation conditioned on x_{July99} , the performance vector of individual sectors in July 1999.

When we consider the overall performance of a strategy over time, two different measures of variance are of interest. The first considers the variance of the expected strategy performance over time, $Var(E(y | x))$. If a strategy has a small positive expected return every month, it is less risky in some sense than one that has a large positive expected return under some market outcomes and a large negative expected return in others. The second considers the conditional variance of the strategy performance within each month. This is the variance that we will see across a population of managers implementing the same strategy independently under a given market outcome. Taking the average $E(Var(y | x))$ of this conditional variance gives us another (very different) measure of the long-term variance of strategy outperformance. We conjecture that the overall variance of strategy outperformance is equal to the sum of these two terms and that the unconditional mean is equal to the expectation of the conditional means:

$$(9) \quad E(y) = E(E(y | x))$$

$$(10) \quad Var(y) = Var(E(y | x)) + E(Var(y | x))$$

Equation 9 is almost obvious:

$$E(y) = \int y g(y) dy = \iint y p(y | x) f(x) dx dy = \int (\int y p(y | x) dy) f(x) dx = \int E(y | x) f(x) dx = E(E(y | x))$$

To prove Equation 10, we will first expand each term separately:

$$(11) \quad Var(y) = E(y^2) - (E(y))^2 = \int y^2 g(y) dy - (\int y g(y) dy)^2$$

$$= \iint y^2 p(y | x) f(x) dx dy - (\iint y p(y | x) f(x) dx dy)^2$$

$$(12) \quad Var(E(y | x)) = \int (E(y | x))^2 f(x) dx - (\int E(y | x) f(x) dx)^2$$

$$= \int (\int y p(y | x) dy)^2 f(x) dx - (\int \int y p(y | x) f(x) dy dx)^2$$

$$(13) \quad E(Var(y | x)) = \int Var(y | x) f(x) dx = \int (\int y^2 p(y | x) dy - (\int y p(y | x) dy)^2) f(x) dx$$

$$= \iint y^2 p(y | x) f(x) dy dx - \int (\int y p(y | x) dy)^2 f(x) dx$$

We can see that the first item of Equation 12 and the second item of Equation 13 are the same. So when we add Equations 12 and 13, these terms cancel out, and we have

$$(14) \quad \text{Var}(E(y | x)) + E(\text{Var}(y | x)) = \iint y^2 p(y | x) f(x) dy dx - \left(\iint y p(y | x) f(x) dy dx \right)^2$$

The right hand sides of Equations 11 and 14 are the same, and we have proved our conjecture (Equation 10).

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