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J.P.Morgan

Global Quantitative & Derivatives Strategy
10 October 2017

Post-Modern Portfolio Construction

Examining Recent Innovations in Asset Allocation

Ever since Markowitz's publication of Mean Variance Optimisation theory in the 1950s the technique has drawn criticism, most of which centres on the sensitivity to errors in the covariance matrix. We investigate three techniques that have recently been proposed to create optimal portfolios.

- In 2016 Marcos Lopez de Prado introduced 'Hierarchical Risk Parity' (HRP) in his paper 'Building diversified portfolios that outperform out of sample'. We examined the technique for Cross-Asset Risk-Premia case earlier this year here, and in this report we test exclusively within equities.

- In the original implementation of HRP there is no scope for a source of expected returns; we have modified the HRP technique to support an alpha, which we call **Hierarchical Alpha Portfolio** (HAP). We can also combine the two approaches using a risk aversion parameter λ to allocate between HAP and HRP, a method we call **Hierarchical Alpha Risk Portfolio** HARP.

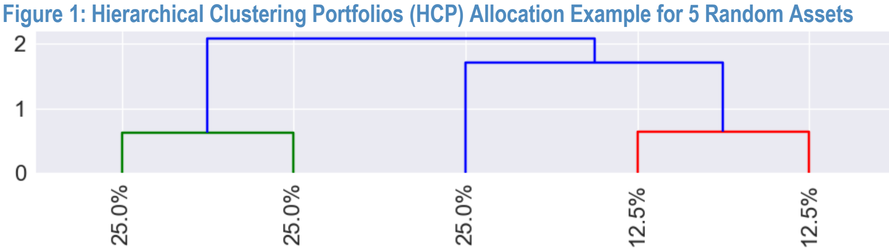
- Another method to create a diversified weighting is to distribute the capital across each cluster hierarchy such that many correlated assets receive the same total allocation as a single uncorrelated one. This idea was recently published by Thomas Raffinot in his paper "Hierarchical Clustering based Asset Allocation". We refer to this as **Hierarchical Clustering Portfolio** (HCP).

- **Across various MSCI universes we find the post-modern portfolio construction technique of HRP outperforms other risk-based methods while HARP often beats naïve mean variance optimisation on a risk adjusted basis. HCP is sensitive to cluster choice with Ward results best in general.**

- Covariance Shrinkage is a method to correct the covariance estimation problems and we investigate five alternatives: Ledoit-Wolf, Oracle Approximating Shrinkage Estimator, Random Matrix Theory, Nearest and Truncation.

- **In general the matrix shrinkage approaches improve the covariance estimation and produced stronger risk adjusted returns** especially when used without an alpha**.** The Random Matrix Theory filtered covariance matrix produced the most consistent results across different MSCI based universes.

Figure 1: Hierarchical Clustering Portfolios (HCP) Allocation Example for 5 Random Assets



Source: J.P. Morgan QDS, Original ideas: Thomas Raffinot "Hierarchical Clustering based Asset Allocation"

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Introduction

The most common methods for determining portfolio weights in a portfolio are based on the pioneering mean-variance portfolio theory of Markowitz, yet practical application of this approach is complicated with estimation errors of the asset covariance matrix, and forecast errors in the expected returns (if used).

It has been widely reported by numerous authors (such as; DeMiguel et al 2009, Duchin & Levy 2009, Tu & Zhou 2009, Kritzman et al. 2010) that an optimal portfolio sometimes cannot beat a naïve diversification strategy in which all asset weights are equal to 1/N (equal weighted).

These estimation errors can cause unconstrained optimisation to result in extreme long and short positions that may breach investors' mandates or preferences. For example it is not uncommon for an *unconstrained* MVO or GMV optimisation to allocate 100% to a single asset.

In this report we examine a few techniques designed to address these practical limitations.

With the recent publication of Marcos Lopez de Prado's paper on Hierarchical Risk Parity portfolio construction (which we discussed in a cross asset class paper, here) there has been a lot of interest in looking at alternative ways to build portfolios.

In this paper we extend HRP with the introduction of an alpha, Hierarchical Alpha Portfolio (HAP) which can be combined intuitively with de Prado's HRP approach, scaling the relative weights using a risk aversion parameter. The HARP model combines HRP and HRP with risk aversion λ as shown below:

$$\text{HARP} = \lambda\ \text{HRP} + (1-\lambda)\ \text{HAP}$$

There has been some confusion around the actual construction method used in HRP, and we feel that some might have thought the approach more akin to "Hierarchical Clustering based Asset Allocation" introduced by Thomas Raffinot, which we call Hierarchical Clustering Portfolio (HCP).

For completeness we will compare these approaches to classic optimization or portfolio construction methods such; as MVO, MDP, EW and others.

Finally we will also examine the effect of Covariance Shrinkage as alternative method to ensure that the covariance matrix is positive semi-definite. Methods discussed include; Ledoit-Wolf, Oracle Approximating Shrinkage estimator (OAS), Nearest (Highman's), Random Matrix Filtering (RMF) and Eigen vector Truncation.

Note, Shrinkage does not refer to a reduction in the size of the covariance matrix, rather a reduction in the extremes of the covariance, and/or a normalization to ensure the matrix is positive semi-definite, as assumed by most optimization routines. Matrix dimension reduction is usually achieved via PCA or Factor Exposure Matrices (such as the MSCI Barra risk model amongst others) and is beyond the scope of this report.

Big Data & Machine Learning

Big Data and AI Strategies: Machine Learning and Alternative Data Approach to Investing.

Global Lending Risk Premia: Alternative Data - Stock Signals Using Short Interest, Borrow Cost and Utilization Ratio

Dynamic Cluster Neutralization in Global Equity Markets: Using Unsupervised Machine Learning to Enhance Returns and Reduce Risk

Value Strategies based on Machine Learning: Incorporating Profitability Measure and Sentiment Signals to Identify Winners and Losers

Cross Asset Portfolios of Tradable Risk Premia Indices: Hierarchical Risk Parity.

European Equity Derivatives: Enhancing call overwriting returns with fundamental factors

FX Derivatives Research Note: Machine Learning approach to FX option trading: preliminary results

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Post-Modern Portfolio Construction

Hierarchical Risk Parity & Hierarchical Alpha Portfolios

Marcos Lopez de Prado's introduced 'Hierarchical Risk Parity' (HRP) in his paper 'Building diversified portfolios that outperform out of sample' and we examined the technique for the Cross-Asset Risk-Premia case earlier this year, here.

In the original implementation of HRP there is no scope for a source of expected returns; we have modified the HRP Optimisation technique to support an Alpha.

In HRP all individual stock weights are set to = 1.0 at the start, then for each loop they are scaled down recursively by relative risk until at the end the weights sum to 1.0. Note that in the original source code, "alpha" is used to represent the inverse volatility weighting of each bifurcation of the portfolio.

```
Alpha = 1. - cVar0 /(cVar0+cVar1)
w[cItems0] *= alpha    # weight group 1 (cItems0 = current assets)
w[cItems1] *= 1.-alpha # weight group 2 (cItems1 = alt assets)
```

We have adopted a similar approach for the expected returns, rescaling the z-scored alphas to range from 0.0 to 1.0. This way, stocks with negative alpha will have a scaling factor < 0.5 and if they are positive alpha, scaler is > 0.5.

We use the average expected returns (expReturns) for the group in each branch of the asset list, which ultimately will be just the pairs of stocks at the leaves.

```
eVar0 = xReturns[cItems0].mean()                # Expected Returns of group 1
eVar1 = xReturns[cItems1].mean()                # Expected Returns of group 2
alpha = 1. - cVar0 /(cVar0+cVar1)               # HRPi Relative Variance - HRP 'alpha'
xprtn = 0. + eVar0 /(eVar0+eVar1)               # HAPi Expected Return Relative
w[cItems0] *= riskaversion*alpha + 1.0*(xprtn*(1.-riskaversion))    # HARPi  weights Group 1
w[cItems1] *= riskaversion*(1.-alpha) + 1.0*(1.-xprtn)*((1.-riskaversion)) # HARPxi Wt2
```

The Hierarchical Risk Parity HRP weights for Portion "i" of the portfolio are calculated as 1- relative variance, per below. The other portion is labelled 'xi'.

$HRP_i = 1 - Var_i / (Var_i + Var_{xi})$

The Hierarchical Alpha Parity HAP weights for Portion "i" of the portfolio are calculated as relative expected returns, per below. The other portion is labelled 'xi'.

$HAP_i = E(Rtn_i)/ (E(Rtn_i) + E(Rtn_{xi}))$

We use risk aversion λ Lambda to change relative ratio between the two. The final Hierarchical Alpha Risk Parity HARP weights are then calculated by combining the two weights, as shown below;

$HARP = \lambda\ HRP + (1- \lambda)\ HAP$

The advantage of this method is that it provides a familiar trade-off between risk and return portfolio allocations.

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Figure 2: With RISK AVERSION = 0 (i.e. risk embracing)

Source: J.P. Morgan QDS

We can see the results of the inclusion of an expected return focus with some random data generated from de Prado's example code, for 10 assets. Using the random data in the examples above for HAP and below for HARP we note the HRP has an expected return of -0.1% (based on the mean return per asset), while the HAP increases expected returns to 0.6%. Combining the two with a risk aversion of λ=0.5, the HARP model has an expected return of 0.3%, neatly in-between the two extremes.

Further we note that generally the higher alpha stocks have a higher weight in HAP c.f. HRP models, while the negative alpha stock weights approach zero. These allocations are less extreme with HARP, with fewer stocks at zero weight, and the maximum weights also reduced.

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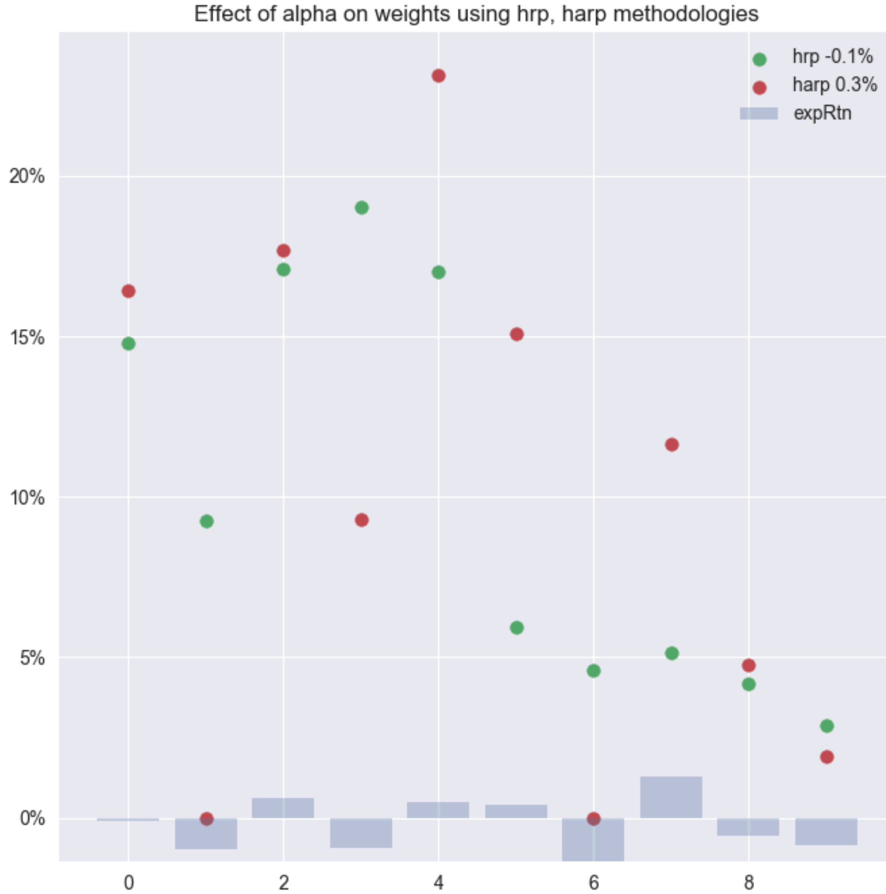
Figure 3: With Risk Aversion = 0.5 the HARP model improves expected returns.



Source: J.P. Morgan QDS

Scaling Expected Returns by Z-Scores & Logistic Curves

Before we can feed the expected returns to the model we need to scale them. Due to the way the algorithm works, specifically the “exprt” line in the code snippet above, we find that a scaling method is required.

Without the scaling alpha modification our prior testing showed that the alpha model was giving portfolios that were close to equally weighed.

This occurred because the alphas were originally just rescaled from zero to one and the relative weighting by the mean of the group resulted in alpha dilution.

To counter this we recommend the use of a rescaled alpha using the Logistic curve as shown below. The Y-Axis is the alpha score, while the X-Axis corresponds to the stocks rank order, scaled between -10 and +10. This curve applies a harsh penalty to low/moderate alphas and boosts high alphas.

Further we should be aware that because of the scaling mechanism used by these algorithms, the expected returns need to be scaled between 0 and 1. A stock with 0.0 expected return will ultimately have its weight multiplied by 0 and so will be unheld.

Below we show the Raw Alpha in blue, pre- and post- Logistic scaling.

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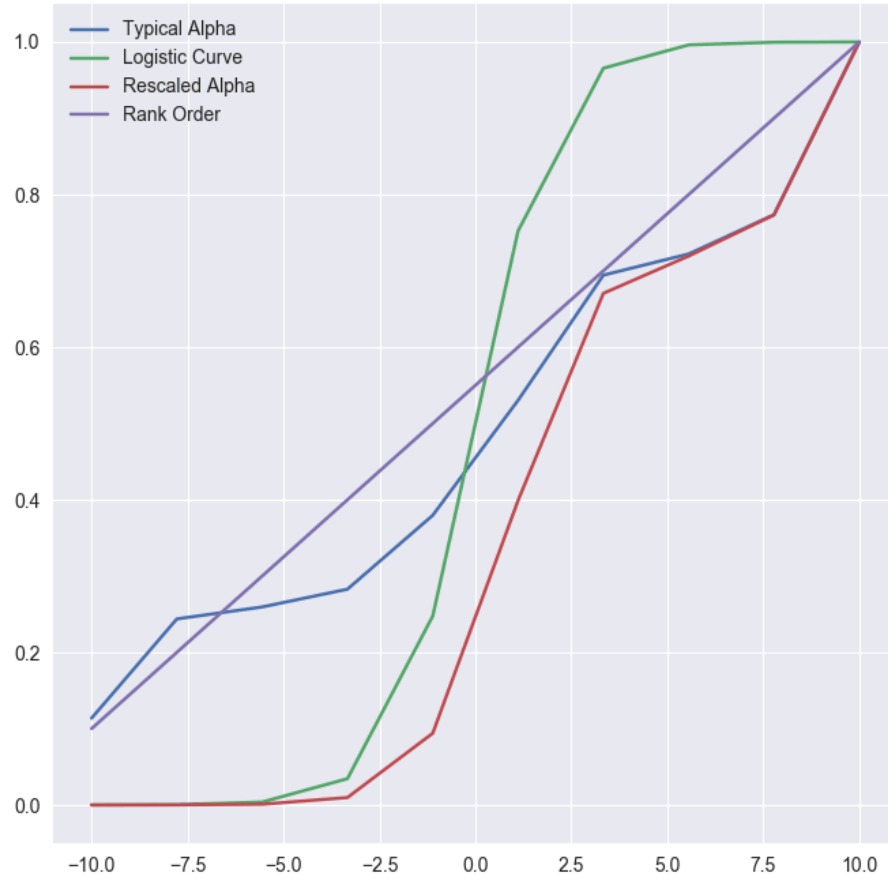
Figure 4: Alpha scaling methods; Typical Alpha (c.f. Z-Score), Rank and Logistic scaling



Source: J.P. Morgan QDS

MSCI Global Developed Markets Backtest Results

The global results show MDP achieving the highest returns and lowest risk for a risk adjusted return of 0.96, thanks also to the lowest risk of all models tested. HRP was the second best model with lower returns of 8%, for a similar risk level as MDP giving a risk adjusted return ratio of 0.72.

Adding an Alpha to the optimistation problem gives more varied results, with MVO giving strong returns of 17.3% and Sharpe of 0.64. The HAP model (ignoring risk) actually produced diversified portfolios with half the risk of MVO at 14.5%, but also lower returns of 10.5% giving an acceptable ratio of 0.72. It is worth nothing the HARP model proved sensitive to alpha scaling, with the Raw Alpha performing very poorly (5.2% p.a. Long only returns), while logistic scaling improved this to 9.3%.

Figure 5: Portfolio Formation – MSCI GDM Universe

MSCI GDM Backtest Description	IC	Return	Volatility	Sharpe	Drawdown	Ratio	t-Stat
EW GDM Ops {Eql Wts}	*3.07%*	*8.4%*	*15.1%*	*0.56*	*54%*	*62%*	*2.72*
EMV GDM Ops {Eql Marginal Vol}	3.07%	8.5%	13.4%	0.63	51%	65%	3.00
GMV L-O GDM Ops	3.07%	4.8%	11.9%	0.41	47%	62%	2.03
MDP L-O GDM Ops	**3.07%**	**10.4%**	**10.9%**	**0.96**	**35%**	**67%**	**4.30**
HRP L-O GDM Ops	**3.07%**	**8.0%**	**11.0%**	**0.72**	**46%**	**66%**	**3.33**
MVO Ops Alpha	*3.07%*	***17.3%***	*26.8%*	*0.64*	*65%*	*60%*	*3.25*
HARP Ops Alpha	3.07%	5.2%	16.7%	0.31	69%	64%	1.73
HARP Ops Logistic Curved Alpha	**3.07%**	**9.3%**	**12.6%**	**0.74**	**49%**	**67%**	**3.42**
HAP Ops No Risk Logistic Curved Alpha	**3.07%**	**10.5%**	**14.5%**	**0.72**	**52%**	**65%**	**3.38**

Figure 6: Risk Weighting MSCI GDM

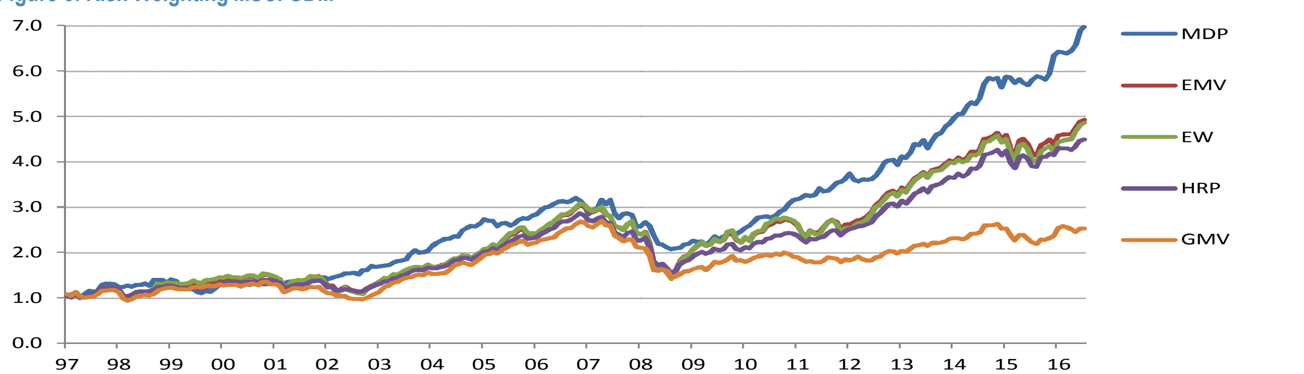
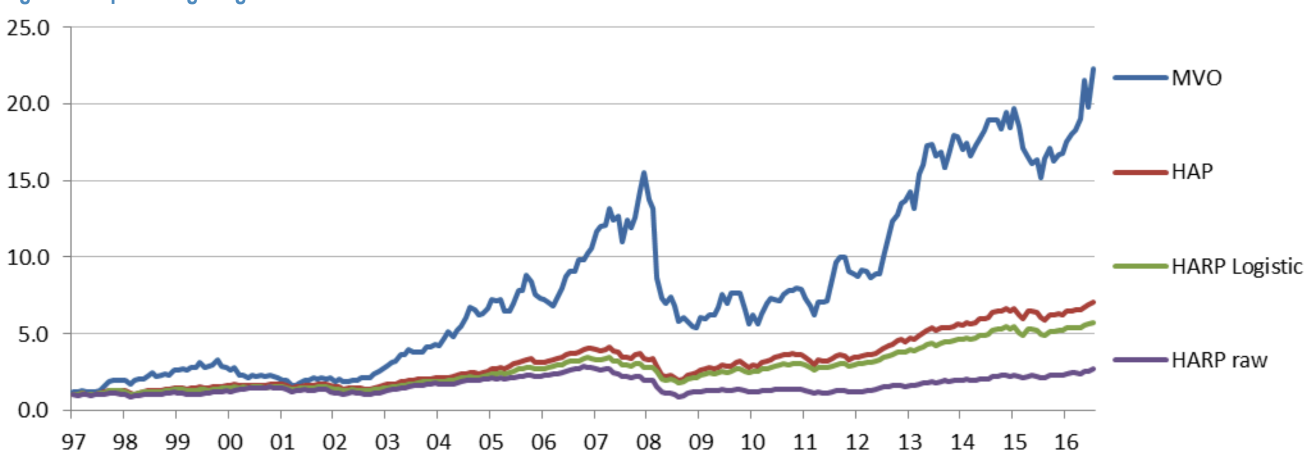


Figure 7: Alpha Weighting– MSCI GDM



Source: J.P. Morgan QDS

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MSCI Global Emerging Markets Backtest Results

For the GEM universe we note that HRP has the best Sharpe Ratio (0.71) of the 5 methods tested without a source of Alpha, again with a slightly lower return, but greatly reduced risk c.f. Equal Weighting. When we include an Alpha, the MVO method again performs admirably with risk adjusted returns just above 1 while HARP can achieve ratios above 0.8 but is sensitive to the parameters chosen (logistic Alpha scaling recommended). Interestingly the HAP model produced well diversified portfolios with realised risk of under 20%, and strong performance of 17.7% resulting in a high Sharpe of 0.9.

Figure 8: Portfolio Formation – MSCI GEM Universe

MSCI GEM Backtest Description	IC	Return	Volatility	Sharpe	Drawdown	Ratio	t-Stat
EW GEM Ops {Eql Wts}	*6.27%*	*10.7%*	*20.3%*	*0.53*	*52%*	*61%*	*2.68*
EMV GEM Ops {Eql Marginal Vol}	6.27%	10.3%	17.6%	0.59	50%	61%	2.87
GMV L-O GEM Ops	6.27%	7.9%	15.6%	0.50	51%	61%	2.50
MDP L-O GEM Ops	6.27%	6.6%	13.0%	0.50	44%	60%	2.46
HRP L-O GEM Ops	**6.27%**	**9.7%**	**13.6%**	**0.71**	**48%**	**63%**	**3.33**
MVO L-O Q-Scores GEM Ops	*6.27%*	*33.3%*	*31.7%*	*1.05*	*56%*	*66%*	*4.76*
HARP Ops Raw Alpha	6.27%	7.3%	22.6%	0.32	57%	57%	1.88
HARP Ops Logistic Curved Alpha	6.27%	14.0%	16.4%	0.85	45%	65%	3.92
HAP Ops No Risk Logistic Curved Alpha	**6.27%**	**17.7%**	**19.6%**	**0.90**	**49%**	**66%**	**4.13**

Figure 9: Risk Weighting MSCI GEM

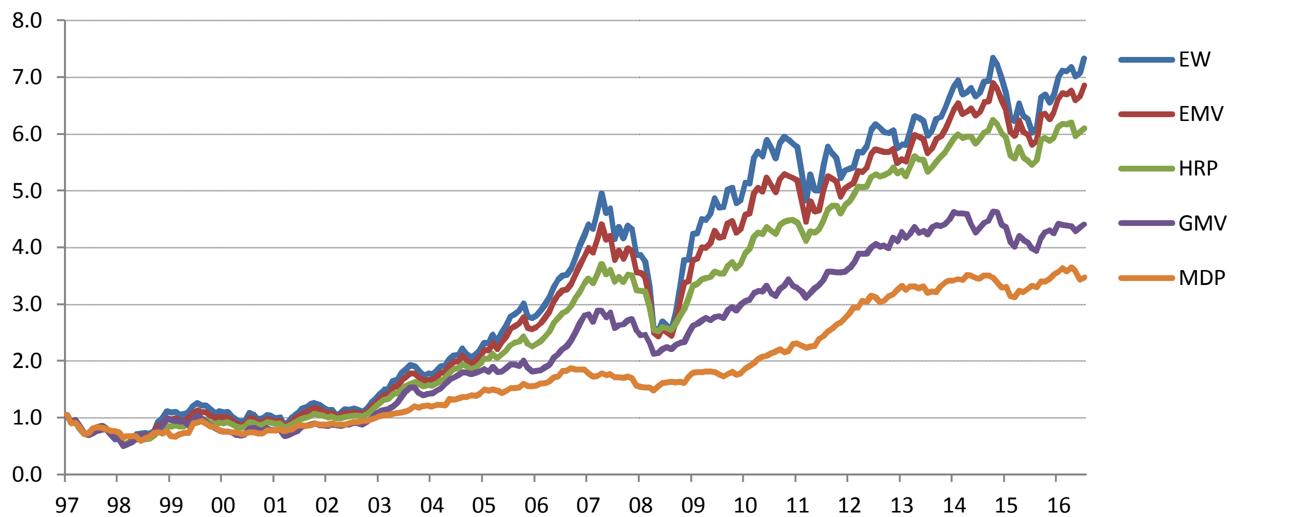
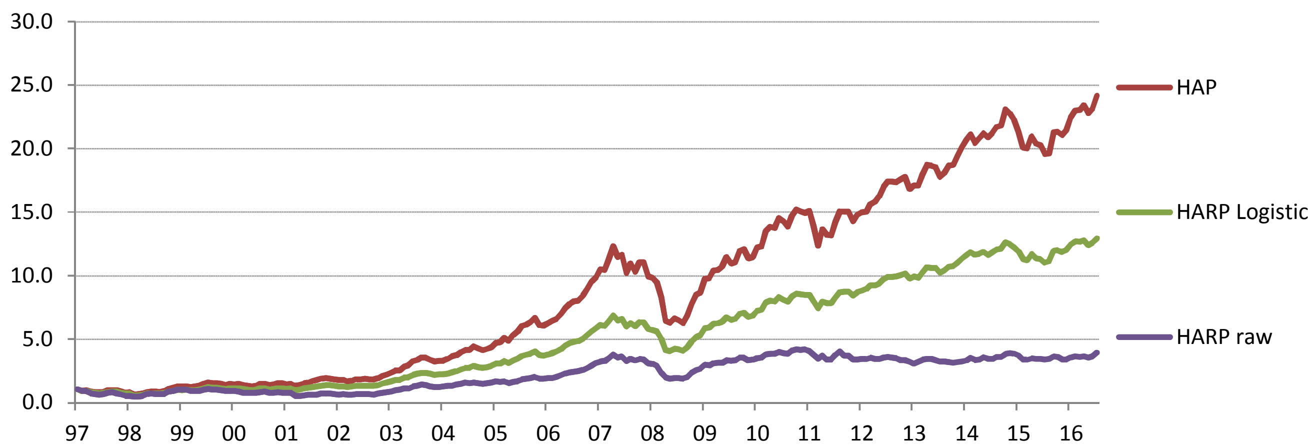


Figure 10: Alpha Weighting– MSCI GEM {MVO removed for clarity}



Source: J.P. Morgan QDS

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MSCI AU Backtest Results

Without an alpha, the HRP method had an impressive Sharpe of 0.81 in the MSCI Australia universe, closely followed by MDP at 0.79, this time outperforming EW in both risk and return. We also note the GMV portfolio managed the highest returns of 10.2% p.a., but the higher ex-post risk dropping the Sharpe to 0.75.

When we introduced the Q-Scores model alpha, the MVO technique produced strong results with peak returns of 15.4%, but at the cost of the highest volatility (19.1%) and draw-down of 62%. We found the most stable portfolio was formed by the HARP model with 70% HRP and 30% HAP, producing a risk adjusted return of 0.75, preserving most of the low volatility from the HRP model (11.9%) with a low draw-down of 48%.

Figure 11: Portfolio Formation – MSCI Australia Universe

Description	IC	Return	Volatility	Sharpe	DrawDown	Ratio	t-Stat
EW AU Ops {Eql Wts}	*4.26%*	*7.9%*	*14.3%*	*0.55*	*53%*	*62%*	*2.67*
EMV AU Ops {Eql Marginal Vol}	4.26%	8.6%	12.9%	0.66	50%	63%	3.11
GMV L-O AU Ops	4.26%	10.2%	13.7%	0.75	46%	66%	3.46
MDP L-O AU Ops	4.26%	9.8%	12.4%	0.79	48%	63%	3.63
HRP L-O AU Ops	**4.26%**	**9.6%**	**11.9%**	**0.81**	**46%**	**65%**	**3.68**
MVO L-O Q-Scores AU Ops	*4.26%*	*15.4%*	*19.1%*	*0.81*	*62%*	*63%*	*3.77*
HAP L-O Q-Scores AU Ops lambda 0.0 logistic	**4.26%**	**9.9%**	**14.3%**	**0.69**	**54%**	**63%**	**3.25**
HARP L-O Q-Scores AU Ops lambda 0.3 logistic	4.26%	9.5%	13.3%	0.71	51%	64%	3.32
HARP L-O Q-Scores AU Ops lambda 0.5 logistic	4.26%	9.3%	12.8%	0.73	49%	64%	3.38
HARP L-O Q-Scores AU Ops lambda 0.7 logistic	**4.26%**	**9.3%**	**12.3%**	**0.75**	**48%**	**64%**	**3.47**
HARP L-O Q-Scores AU Ops lambda 0.5 zscore	4.26%	8.4%	16.2%	0.51	51%	62%	2.55
HARP L-O Q-Scores AU Ops lambda 0.5 rank	4.26%	8.8%	12.9%	0.68	50%	63%	3.17

Figure 12: Risk Weighting MSCI Australia

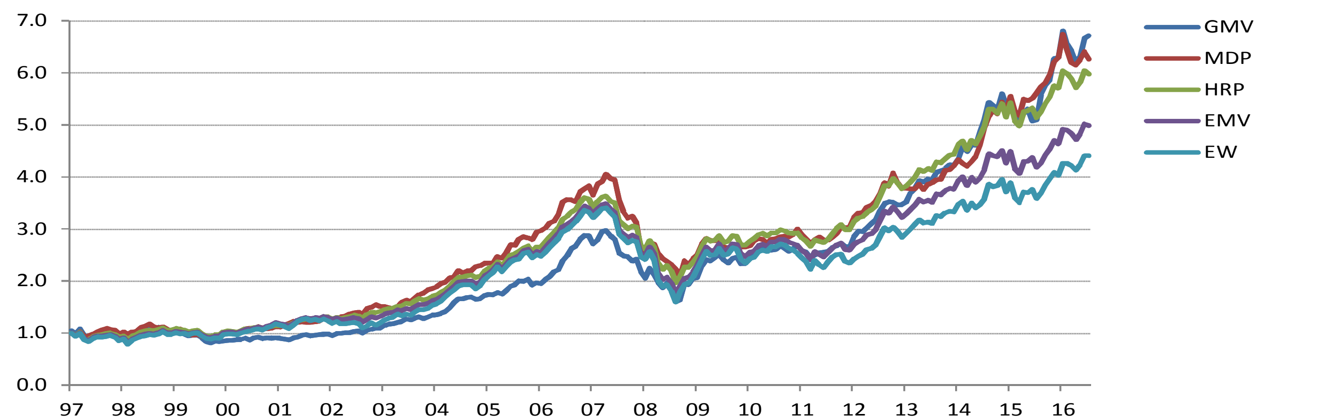
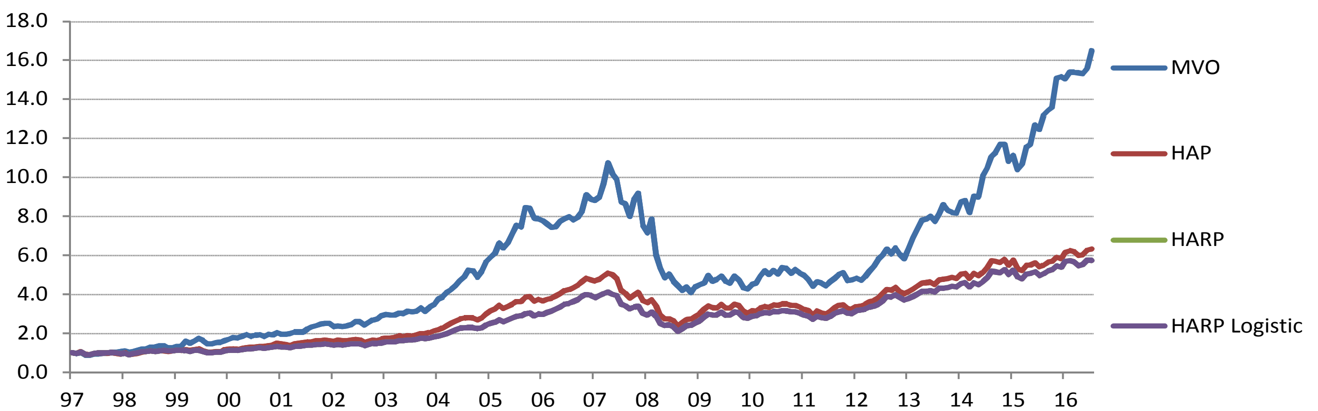


Figure 13: Alpha Weighting MSCI Australia



Source: J.P. Morgan QDS λ = 0.5 shown for HARP model.

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MSCI Asia Ex Japan Backtest Results

Our Asia Ex Japan backtests showed that HRP, HAP and HARP had strong risk adjusted returns of 0.70 to 0.73 c.f. EMV & MVO at 0.5 and Equal Weighted portfolios at 0.4.

However it is worth mentioning that the MVO portfolio delivered extreme backtest performance with long-only return of 17.5%, and volatility of 35% (p.a.) the highest within this batch of tests.

Figure 14: Portfolio Formation – MSCI Asia Ex Japan Universe

Asia Ex Japan Backtest Description	IC	Return	Volatility	Sharpe	Drawdown	Ratio	t-Stat
EW Ops	*5.47%*	*9.1%*	*20.9%*	*0.43*	*57%*	*62%*	*2.31*
EMV Ops	5.47%	8.8%	17.2%	0.51	52%	63%	2.56
GMV Ops	5.47%	2.3%	15.0%	0.15	66%	57%	1.00
MDP Ops	5.47%	9.0%	9.7%	0.93	29%	67%	4.16
HRP Ops	**5.47%**	**8.6%**	**12.3%**	**0.70**	**45%**	**70%**	**3.27**
MVO Ops Alpha	*5.47%*	***17.5%***	*34.7%*	*0.50*	*61%*	*61%*	*2.85*
HARP Ops Alpha	5.47%	5.8%	21.0%	0.28	64%	58%	1.66
HARP Ops Logistic Curved Alpha	**5.47%**	**11.4%**	**16.1%**	**0.71**	**47%**	**69%**	**3.36**
HAP Ops No Risk Logistic Curved Alpha	**5.47%**	**14.8%**	**20.3%**	**0.73**	**52%**	**66%**	**3.49**

Figure 15: Risk Weighting MSCI Asia Ex Japan

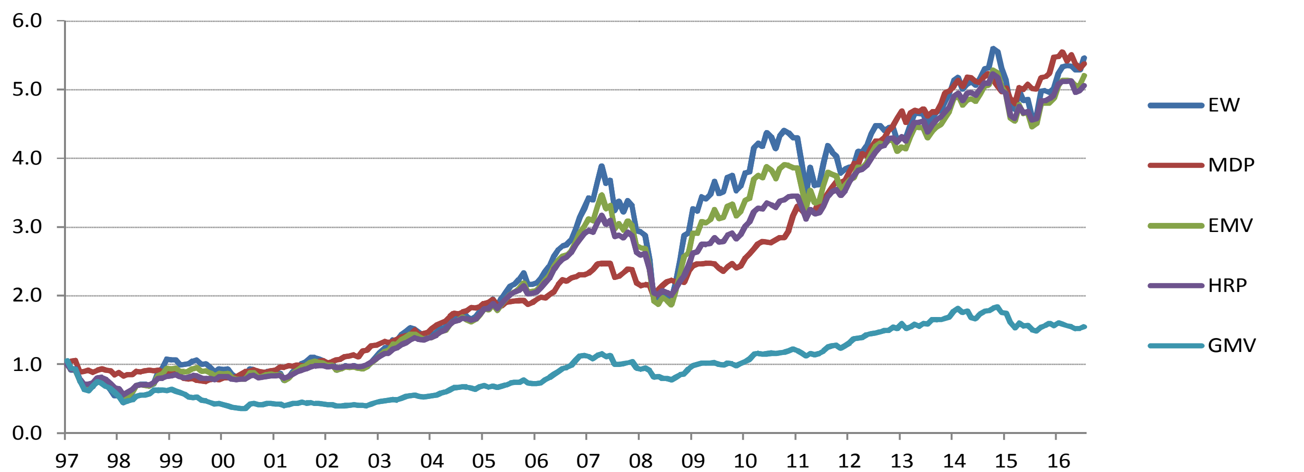
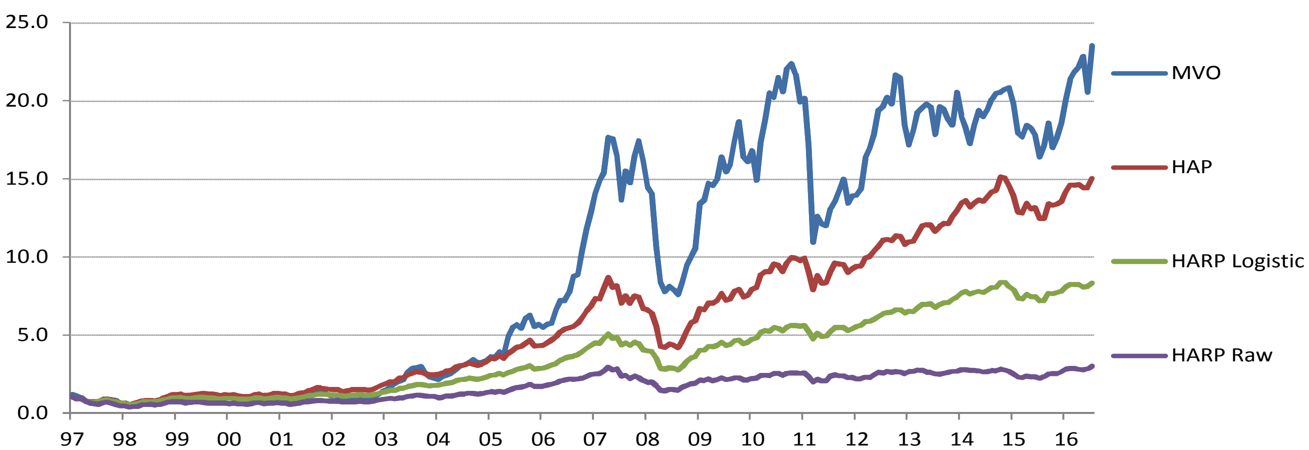


Figure 16: Alpha Weighting MSCI Asia Ex Japan



Source: J.P. Morgan QDS

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Hierarchical Clustering Portfolio (HCP)

There is another recently proposed method to create a diversified weighting which is to distribute the capital across each cluster hierarchy, such that many correlated assets receive the same total allocation as a single uncorrelated one. Then, within a cluster, an equal-weighted allocation is assigned.

This idea was recently published by Thomas Raffinot in his paper "Hierarchical Clustering based Asset Allocation". We call this allocation method Hierarchical Clustering Portfolio (HCP) to match our nomenclature.

In his paper, Thomas highlights that complex systems, such as financial markets, have a structure and are usually organized in a hierarchical manner, with separate and separable sub-structures (Simon 1962). The hierarchical structure of interactions among elements strongly affects the dynamics of complex systems.

He (and Lopez de Prado) notes that correlation matrices lack the notion of hierarchy, which allows weights to vary freely in unintended ways, and describes an example "…for deciding the allocation to a large publicly-traded U.S. Financial stock like J.P. Morgan, we will consider adding or reducing the allocation to another large publicly-traded U.S. bank like Goldman Sachs, rather than a small community bank in Switzerland, or a real estate holding in the Caribbean. To sum up, a correlation matrix makes no difference between assets. Yet, some assets seem closer substitutes of one another, while others seem complementary to one another."

In the contrived example below a small dendrogram is shown with five assets and three clusters. The first cluster is made up of assets 1 and 2, asset 3 constitutes the second cluster and the third cluster consists of assets 4 and 5.

Based on the hierarchical clustering, making a split at vertical distance measure of 1.5 (say), the weights for cluster number one is 0.5 (simply a 1/2 = 0.5) and weights for clusters 2 and 3 are 0.25 (0.5/2 = 0.25) each.

Since there are two assets in the cluster number one, the final weights for assets 1 and 2 are 25%. Asset 3 has been assigned a weight of 25% while, assets 4 and 5 would get a weight of 0.25 divided equally between them (12.5%)

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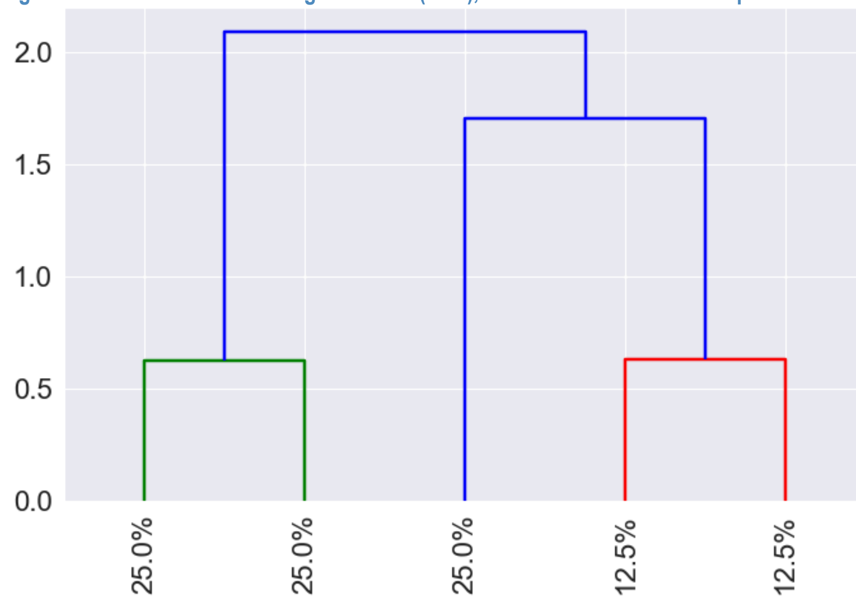
Figure 17: Hierarchical Clustering Portfolios (HCP), an alternative to HRP: Sample Asset Weights



Source: J.P. Morgan QDS, Original ideas: Thomas Raffinot "Hierarchical Clustering based Asset Allocation"

We are using the Scikit-Learn Hierarchical Clustering 'Shrinkage' algorithm to form our clusters, with the distances constructed from the correlation matrix. Clusters can be formed from various measures, including: Single {Default}, Average, Complete, Ward, Median, and Centroid. The shape and cluster formation can vary greatly depending on the clustering algorithm chosen.

For more details on clustering algorithms see our report: "Dynamic Cluster Neutralisation in Global Equity Markets", 29/Aug/2017, Berowne Hlavaty

Extending to the 10 asset sample from our earlier example shows the weights for lower branched leaves rapidly diminish.

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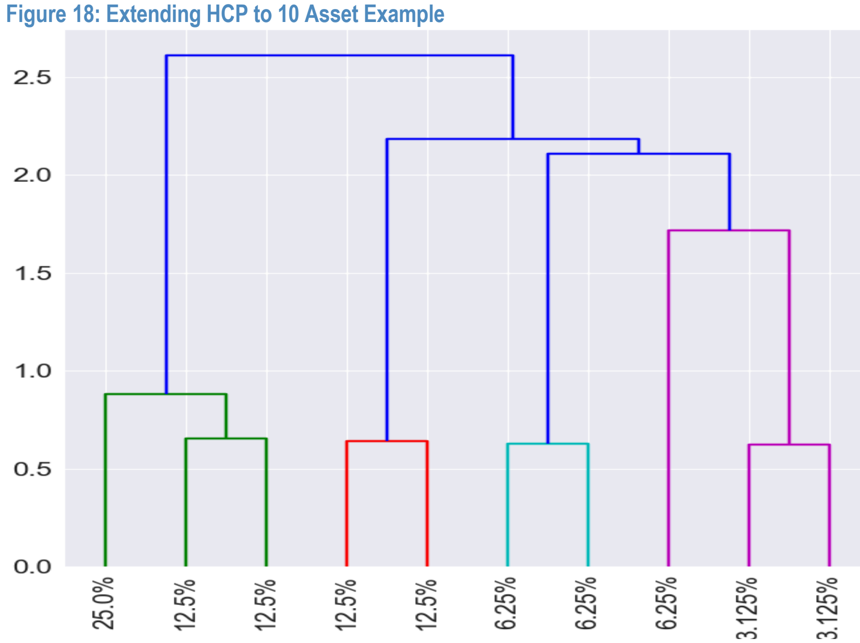
Figure 18: Extending HCP to 10 Asset Example



Source: J.P. Morgan QDS.

Figure 19: 10 Asset Example with Allocations from HRP, HAP, HARP and HCP.

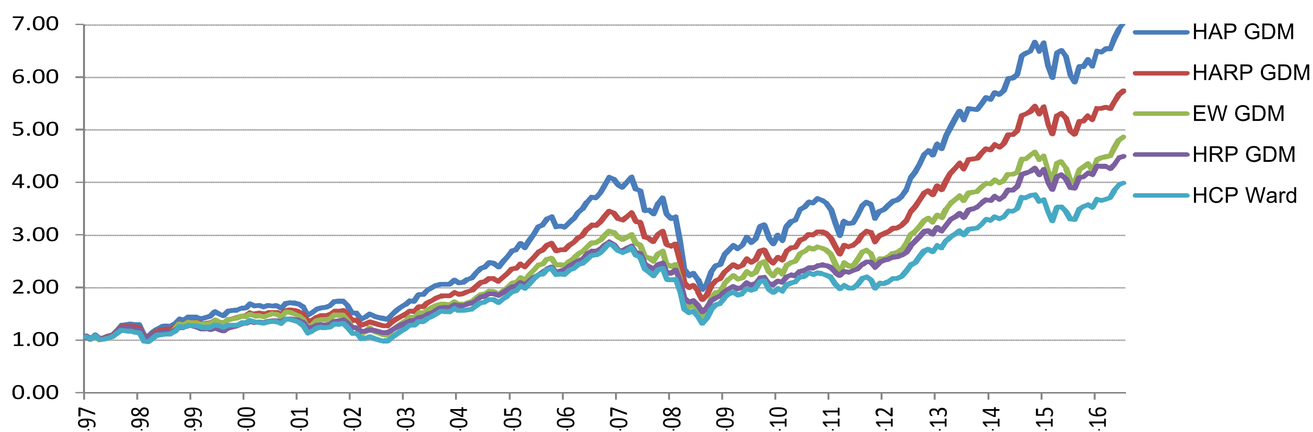
Source: J.P. Morgan QDS.

Results from HCP Model in Developed Markets

The technique is also sensitive to parameter selection in large universes such as MSCI GDM. The single linkage clustering algorithm resulting in zero average returns, while the best method (Ward) returning 7.3% p.a. and a Sharpe of 0.53, close to the 0.56 of an Equal weight portfolio.

Figure 20: Portfolio Formation – MSCI GDM Universe

MSCI GDM Backtest Description	IC	Return	Volatility	Sharpe	Drawdown	Ratio	t-Stat
EW GDM Ops {Eql Wts}	*3.07%*	*8.4%*	*15.1%*	*0.56*	*54%*	*62%*	*2.72*
EMV GDM Ops {Eql Marginal Vol}	3.07%	8.5%	13.4%	0.63	51%	65%	3.00
GMV L-O GDM Ops	3.07%	4.8%	11.9%	0.41	47%	62%	2.03
MDP L-O GDM Ops	3.07%	10.4%	10.9%	0.96	35%	67%	4.30
HRP L-O GDM Ops	3.07%	8.0%	11.0%	0.72	46%	66%	3.33
MVO Ops Alpha	3.07%	17.3%	26.8%	0.64	65%	60%	3.25
HARP Ops Alpha	3.07%	5.2%	16.7%	0.31	69%	64%	1.73
HARP Ops Logistic Curved Alpha	3.07%	9.3%	12.6%	0.74	49%	67%	3.42
HAP Ops No Risk Logistic Curved Alpha	3.07%	10.5%	14.5%	0.72	52%	65%	3.38
HCP GDM Ops Single	**3.07%**	**0.0%**	**19.9%**	**0.00**	**68%**	**56%**	**0.46**
HCP GDM Ops Average	**3.07%**	**5.9%**	**13.0%**	**0.46**	**55%**	**61%**	**2.26**
HCP GDM Ops Complete	**3.07%**	**5.9%**	**13.2%**	**0.44**	**58%**	**62%**	**2.22**
HCP GDM Ops Ward	**3.07%**	**7.3%**	**13.7%**	**0.53**	**53%**	**61%**	**2.59**
HCP GDM Ops Median	**3.07%**	**2.6%**	**15.9%**	**0.17**	**59%**	**56%**	**1.09**
HCP GDM Ops Centroid	**3.07%**	**4.0%**	**17.4%**	**0.23**	**68%**	**57%**	**1.40**



Source: J.P. Morgan QDS.

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HCP Results in Global Emerging Markets

In GEM we note the HCP returns are still volatile but somewhat better than the developed markets. Again we note that the Ward clusters come to the fore with the best return of 11.0% and Sharpe of 0.57.

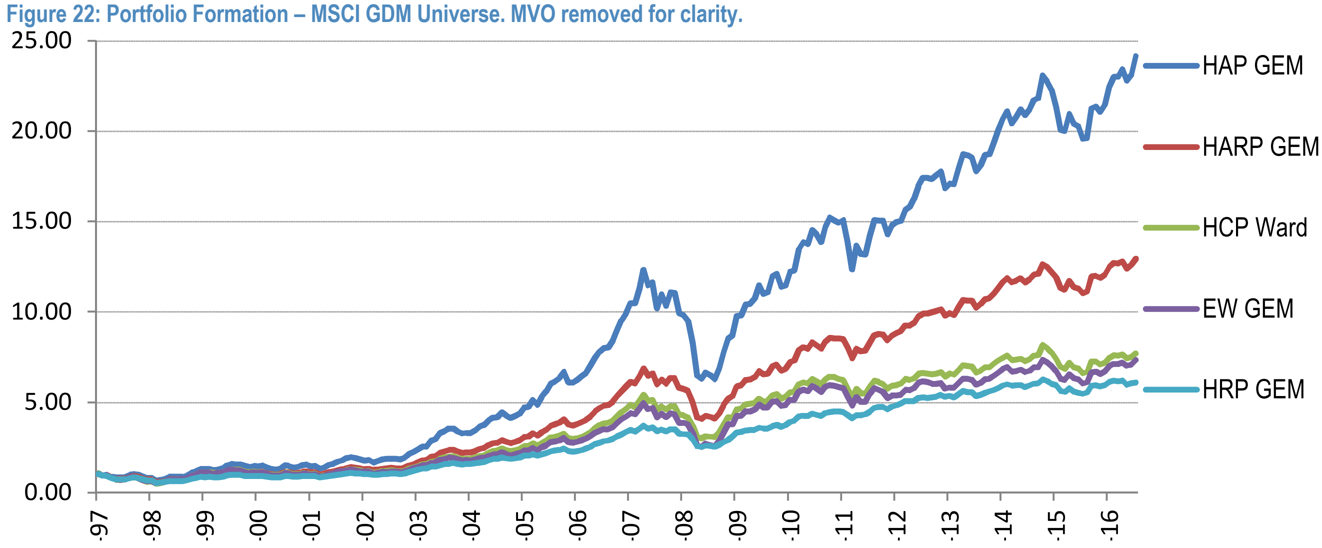
The best HCP method (Ward) is only slightly better than Equal Weighting in terms of return, risk and Sharpe, but fails to beat HRP.

Figure 21: Portfolio Formation – MSCI GEM Universe

MSCI GEM Backtest Description	IC	Return	Volatility	Sharpe	Drawdown	Ratio	t-Stat
EW GEM Ops {Eql Wts}	*6.27%*	*10.7%*	*20.3%*	*0.53*	*52%*	*61%*	*2.68*
EMV GEM Ops {Eql Marginal Vol}	6.27%	10.3%	17.6%	0.59	50%	61%	2.87
GMV L-O GEM Ops	6.27%	7.9%	15.6%	0.50	51%	61%	2.50
MDP L-O GEM Ops	6.27%	6.6%	13.0%	0.50	44%	60%	2.46
HRP L-O GEM Ops	6.27%	9.7%	13.6%	0.71	48%	63%	3.33
MVO L-O Q-Scores GEM Ops	6.27%	33.3%	31.7%	1.05	56%	66%	4.76
HARP Ops Raw Alpha	6.27%	7.3%	22.6%	0.32	57%	57%	1.88
HARP Ops Logistic Curved Alpha	6.27%	14.0%	16.4%	0.85	45%	65%	3.92
HAP Ops No Risk Logistic Curved Alpha	6.27%	17.7%	19.6%	0.90	49%	66%	4.13
HCP GEM Ops Single	**6.27%**	**8.9%**	**27.8%**	**0.32**	**68%**	**57%**	**1.97**
HCP GEM Ops Average	**6.27%**	**10.1%**	**17.0%**	**0.59**	**59%**	**64%**	**2.90**
HCP GEM Ops Complete	**6.27%**	**8.1%**	**17.6%**	**0.46**	**59%**	**61%**	**2.37**
HCP GEM Ops Ward	**6.27%**	**11.0%**	**19.4%**	**0.57**	**55%**	**61%**	**2.82**
HCP GEM Ops Median	**6.27%**	**5.8%**	**29.2%**	**0.20**	**69%**	**57%**	**1.44**
HCP GEM Ops Centroid	**6.27%**	**5.1%**	**21.5%**	**0.24**	**62%**	**57%**	**1.52**

Source: J.P. Morgan QDS.

Figure 22: Portfolio Formation – MSCI GDM Universe. MVO removed for clarity.



Source: J.P. Morgan.

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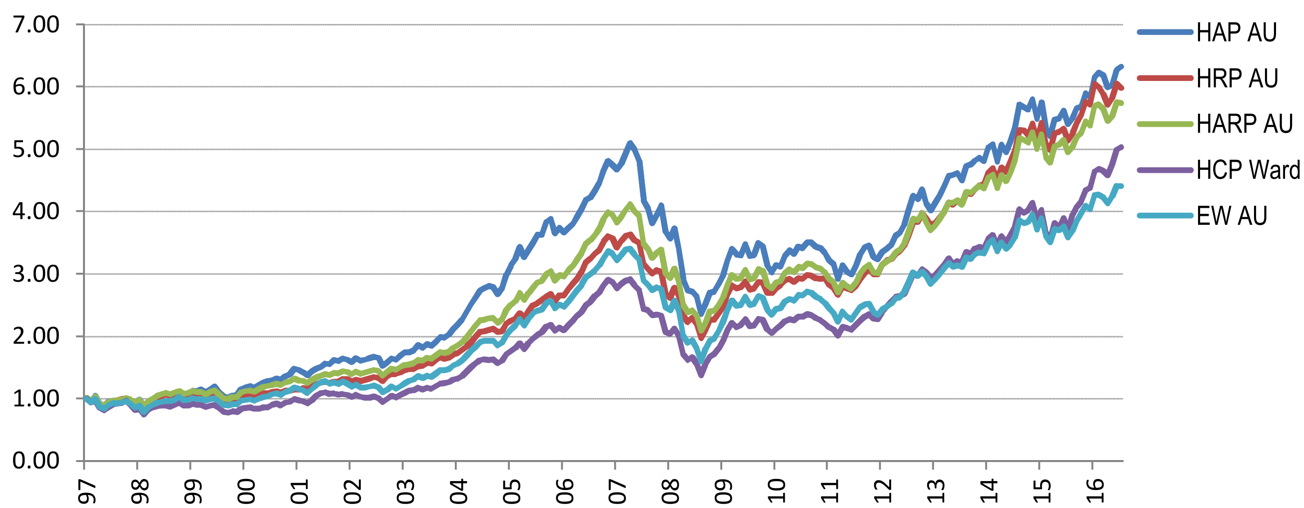
HCP Results in Global Emerging Markets

Testing HCP in the MSCI Australia universe we find the technique to be quite sensitive to clustering algorithm chosen…the returns doubling from 5.5% to 11.7% (Single c.f. Centroid) as shown. The variation in risk is more muted (14.1% to 18.8%) giving Sharpe's from 0.3 to 0.65.

Overall, HCP struggles to outperform Equal Weighting in a small market such as MSCI AU.

Figure 23: Portfolio Formation – MSCI Australia Universe

Description	IC	Return	Volatility	Sharpe	DrawDown	Ratio	t-Stat
EW AU Ops {Eql Wts}	*4.26%*	*7.9%*	*14.3%*	*0.55*	*53%*	*62%*	*2.67*
EMV AU Ops {Eql Marginal Vol}	4.26%	8.6%	12.9%	0.66	50%	63%	3.11
GMV L-O AU Ops	4.26%	10.2%	13.7%	0.75	46%	66%	3.46
MDP L-O AU Ops	4.26%	9.8%	12.4%	0.79	48%	63%	3.63
HRP L-O AU Ops	4.26%	9.6%	11.9%	0.81	46%	65%	3.68
MVO L-O Q-Scores AU Ops	4.26%	15.4%	19.1%	0.81	62%	63%	3.77
HAP L-O Q-Scores AU Ops lambda 0.0 logistic	4.26%	9.9%	14.3%	0.69	54%	63%	3.25
HARP L-O Q-Scores AU Ops lambda 0.3 logistic	4.26%	9.5%	13.3%	0.71	51%	64%	3.32
HARP L-O Q-Scores AU Ops lambda 0.5 logistic	4.26%	9.3%	12.8%	0.73	49%	64%	3.38
HARP L-O Q-Scores AU Ops lambda 0.7 logistic	4.26%	9.3%	12.3%	0.75	48%	64%	3.47
HARP L-O Q-Scores AU Ops lambda 0.5 zscore	4.26%	8.4%	16.2%	0.51	51%	62%	2.55
HARP L-O Q-Scores AU Ops lambda 0.5 rank	4.26%	8.8%	12.9%	0.68	50%	63%	3.17
HCP AU Ops Single	**4.26%**	**5.5%**	**18.3%**	**0.30**	**61%**	**58%**	**1.69**
HCP AU Ops Average	**4.26%**	**9.7%**	**14.9%**	**0.65**	**40%**	**60%**	**3.09**
HCP AU Ops Complete	**4.26%**	**7.5%**	**14.1%**	**0.53**	**51%**	**60%**	**2.59**
HCP AU Ops Ward	**4.26%**	**8.6%**	**14.3%**	**0.60**	**53%**	**62%**	**2.89**
HCP AU Ops Median	**4.26%**	**8.7%**	**18.8%**	**0.46**	**57%**	**61%**	**2.41**
HCP AU Ops Centroid	**4.26%**	**11.7%**	**18.5%**	**0.64**	**46%**	**58%**	**3.08**



Source: J.P. Morgan.

Shrinkage

One of the key arguments put forth for developing the HRP approach was the instability (and occasional non-positive semi-definite covariance matrix) making regular optimization problematic. Another range of improvements have been investigated that help improve the existing optimization framework is commonly referred to as covariance 'shrinkage'. This area of research can become quite analytically involved; however, in this short report we keep our analysis high level.

Covariance matrices often have large, potentially outlaying values which can cause havoc with inversion. A potential solution is to 'shrink' the covariance matrix using a technique such as Ledoit-Wolf's approach.

These shrinkage techniques consist of reducing the ratio between the smallest and the largest eigenvalue of the empirical (traditional) covariance matrix. This can be achieved by simply shifting every eigenvalue according to a given offset, which is equivalent of finding the l2-penalized Maximum Likelihood Estimator of the covariance matrix. See sckit-learn for a visual example here.

In practice, shrinkage boils down to a convex transformation, combining the empirical covariance matrix Σ and a transformation Ψ, using a shrinkage constant δ which takes values between zero and one.

$$\Sigma_{shrunk} = (1-\delta)\Sigma_{epmirical} + \delta\Psi$$

Some of the techniques available to shrink the covariance matrix are discussed in turn below.

Ledoit-Wolf (LW)

The central message of Ledoit and Wolf' paper "Honey, I Shrunk the Sample Covariance Matrix" was that nobody should be using the sample covariance matrix for the purpose of portfolio optimization.

This technique is called shrinkage, since the sample covariance matrix is 'shrunk' towards the structured estimator. The number δ is referred to as the shrinkage constant. The beauty of the principle is that by properly combining two 'extreme' estimators one can obtain a 'compromise' estimator that performs better than either extreme. Intuitively, there is an 'optimal' shrinkage constant. It is the one that minimizes the expected distance between the shrinkage estimator and the true covariance matrix. The sklearn function returns both the shrunk covar matrix and optimal shrinkage coefficient.

Oracle Approximating Shrinkage Estimator (OAS)

The Oracle Shrinkage Approximating estimator attempts to find a better covariance matrix than the one given by Ledoit and Wolf's formula by minimizing the Mean Squared Error by selecting a better shrinkage coefficient as described by Chen et al.

Random Matrix Filtering (RMF)

The latest attempt to reduce the noise in covariance estimates is a branch from physics that uses Random Matrix Theory (RMT) prediction. The prediction is that when the number of securities is large relative to the number of observations, the eigenvalues of the covariance matrix within a predicted band closely resemble the

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distribution as if they were generated from purely random returns. These studies believe that by modifying the eigenvalues within the predicted band, the "filtered" covariance matrix would contain better information than the raw sample matrix.

Nearest (Near)

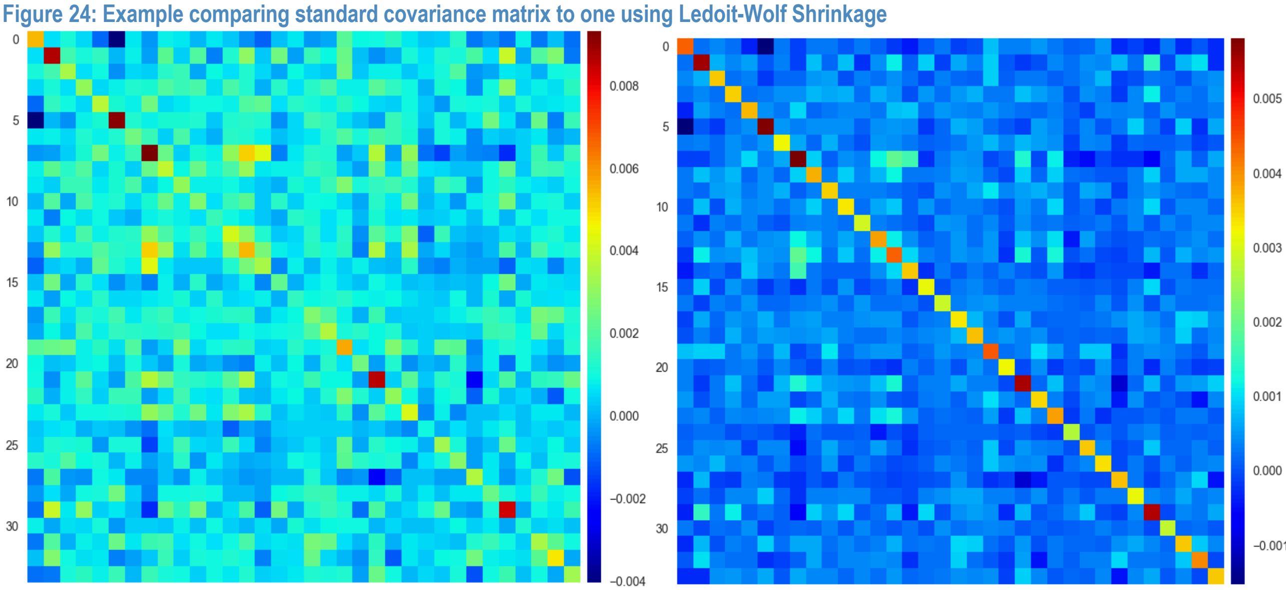
While technically not a shrinkage method, the Basic Nearest Correlation matrix method finds a true correlation matrix Σ that is closest to the approximate input matrix, Ψ, in the Frobenius norm. That is, we find the minimum of $\|\Psi - \Sigma\|_F$

Further details of the technique are described in a paper by Borsdorf and Higham as well as another by Qi and Sun.

Truncate Eigen Values (Repair)

Simply compute the Eigen Values for the covariance matrix, and truncate any negative values at zero. This is a brute force approach to ensure positive semi-definite matrix, however it does lose full representation of the original data.

Figure 24: Example comparing standard covariance matrix to one using Ledoit-Wolf Shrinkage



Source: J.P. Morgan.

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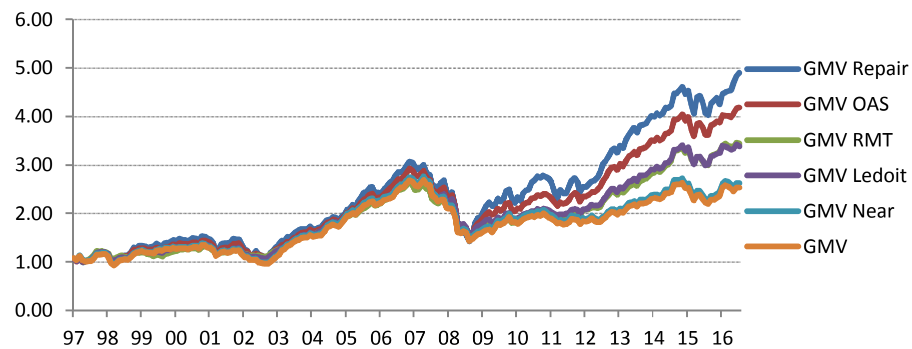
Empirical Results – GMV with Shrinkage

Below we show some results of a Global Minimum Variance (GMV) optimization performed without an alpha, so the impact of the covariance shrinkage can be seen.

In our MSCI GDM tests we note that the shrinkage methods have a small yet generally beneficial effect, boosting returns and cutting risk. We found in this case the RMF approach had the best outcome for a risk based portfolio optimisation, raising the risk adjusted returns to 0.68 from 0.41 for the empirical covariance.

Figure 25: Impact of Covariance Shrinkage on Risk based Optimisation – MSCI GDM

Description	IC	Return	Volatility	Sharpe	Drawdown	+ve	t-Stat
GMV MSCI GDM Ops	**3.07%**	**4.8%**	**11.9%**	**0.41**	**47%**	**62%**	**2.03**
GMV MSCI GDM Ledoit	3.07%	6.4%	10.4%	0.62	43%	66%	2.89
GMV MSCI GDM OAS	3.07%	7.6%	11.9%	0.64	49%	65%	2.99
GMV MSCI GDM RMF	**3.07%**	**6.5%**	**9.5%**	**0.68**	**41%**	**67%**	**3.15**
GMV MSCI GDM Near	3.07%	5.0%	11.9%	0.42	46%	62%	2.11
GMV MSCI GDM Repair	3.07%	8.5%	15.1%	0.56	54%	62%	2.73



Source: J.P. Morgan QDS

Interestingly for the GEM universe the shrinkage techniques proved (almost) consistently better ex-post. The usual 'Empirical' covariance matrix delivered returns of 7.9%pa. and a Sharpe of 0.5, which was beaten by all models except the "Near" covariance matrix which almost matched these results.

The strongest returns were from the truncated covariance ("Repair") while the RMF method produced the strongest Sharpe ratio.

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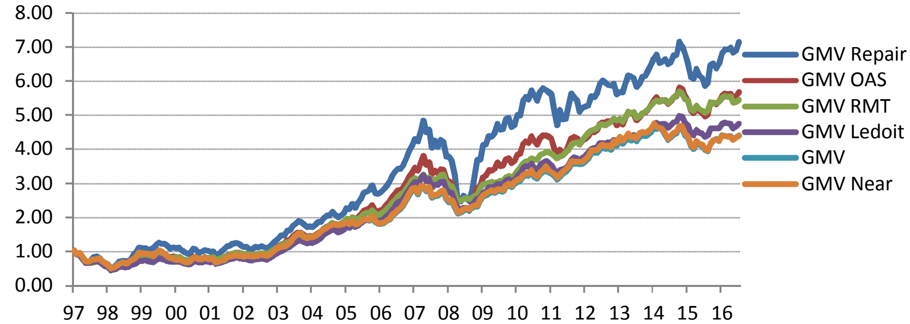
Figure 26: Impact of Covariance Shrinkage on Risk based Optimisation - MSCI GEM

Description	IC	Return	Volatility	Sharpe	Drawdown	Ratio	t-Stat
GMV MSCI GEM Empirical	**6.27%**	**7.9%**	**15.6%**	**0.50**	**51%**	**61%**	**2.50**
GMV MSCI GEM Ledoit	6.27%	8.3%	14.2%	0.58	56%	63%	2.81
GMV MSCI GEM OAS	6.27%	9.3%	16.2%	0.57	53%	61%	2.80
GMV MSCI GEM RMF	**6.27%**	**9.0%**	**12.7%**	**0.71**	**53%**	**65%**	**3.32**
GMV MSCI GEM Near	6.27%	7.8%	15.6%	0.50	51%	61%	2.49
GMV MSCI GEM Repair	6.27%	10.6%	20.2%	0.52	52%	61%	2.66

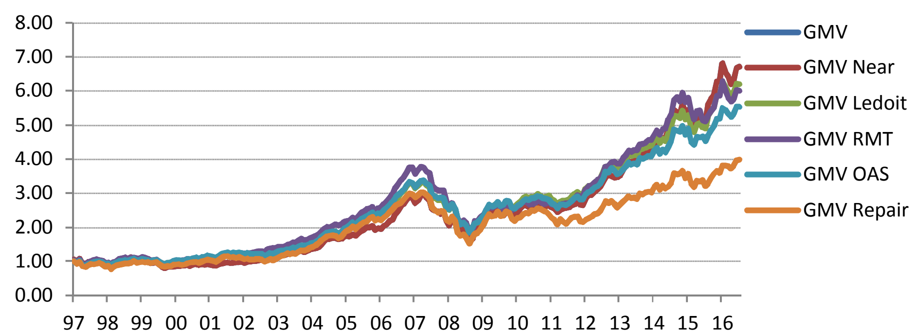


Source: J.P. Morgan QDS

We also examined a smaller sample using the MSCI Australia universe with as few as 40 assets some months. We note that the impact of shrinkage on Sharpe Ratio and other statistics is minimal. However these raw statistics can mask the benefit of the shrinkage method to improve robustness of backtests and reduce warnings from optimization tools about solution intractability etc.

Figure 27: Impact of Covariance Shrinkage on Risk based Optimisation - MSCI AUS

Description	IC	Return	Volatility	Sharpe	Drawdown	Ratio	t-Stat
GMV AU Regular Covariance	**4.26%**	**9.6%**	**13.2%**	**0.72**	**47%**	**63%**	**3.37**
GMV AU Ledoit	4.26%	9.2%	12.3%	0.74	48%	63%	3.43
GMV AU OAS	4.26%	8.9%	12.3%	0.73	47%	61%	3.36
GMV AU RMF	**4.26%**	**9.2%**	**12.8%**	**0.72**	**53%**	**67%**	**3.35**
GMV AU Near	4.26%	9.6%	13.2%	0.72	47%	63%	3.37
GMV AU Repair	4.26%	7.3%	14.2%	0.51	50%	63%	2.52



Source: J.P. Morgan QDS

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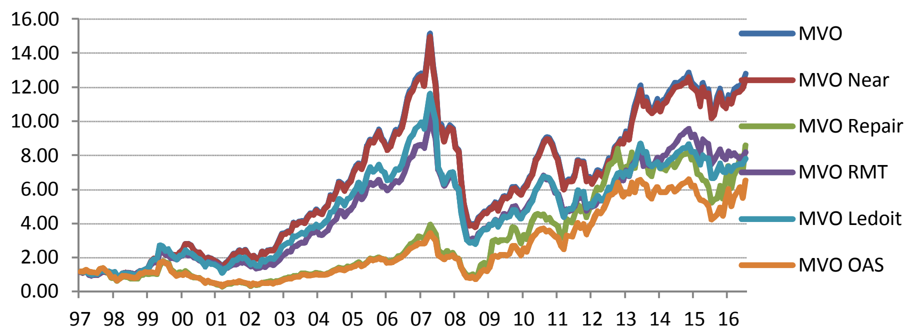
Empirical Results – MVO with Shrinkage

Examining the Mean Variance Optimisation with our Q-Scores model as the alpha we found that shrinkage generally hurt the results. Looking at the results a little closer we note that the RMF method was consistently the closest to results from our standard covariance matrix.

Figure 28: Impact of Covariance Shrinkage on Alpha based Optimisation in GDM

Description	IC	Return	Volatility	Sharpe	Drawdown	+ve	t-Stat
MVO Q-Scores MSCI GDM	**3.07%**	**13.9%**	**26.2%**	**0.53**	**74%**	**59%**	**2.79**
MVO Q-Scores MSCI GDM Ledoit	3.07%	11.1%	29.4%	0.38	76%	59%	2.23
MVO Q-Scores MSCI GDM OAS	3.07%	10.1%	40.0%	0.25	81%	56%	1.92
MVO Q-Scores MSCI GDM RMF	**3.07%**	**11.3%**	**24.7%**	**0.46**	**71%**	**61%**	**2.48**
MVO Q-Scores MSCI GDM Near	3.07%	13.7%	26.5%	0.52	75%	59%	2.74
MVO Q-Scores MSCI GDM Repair	3.07%	11.6%	43.0%	0.27	83%	54%	2.03

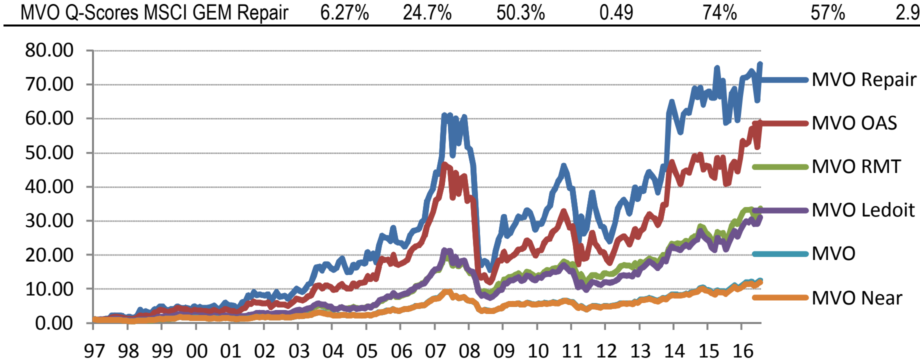


Source: J.P. Morgan QDS

In the GEM markets, matrix shrinking can benefit the results, with RMF delivering the best Sharpe of 0.7, thanks to low risk and reasonable returns of 19.6%, with the lowest draw-down.

Figure 29: Impact of Covariance Shrinkage on Alpha based Optimisation in GEM

Description	IC	Return	Volatility	Sharpe	Drawdown	Ratio	t-Stat
MVO Q-Scores MSCI GEM	**6.27%**	**13.8%**	**26.6%**	**0.52**	**63%**	**59%**	**2.75**
MVO Q-Scores MSCI GEM Ledoit	6.27%	19.2%	29.7%	0.65	65%	61%	3.29
MVO Q-Scores MSCI GEM OAS	6.27%	23.1%	41.3%	0.56	74%	60%	3.13
MVO Q-Scores MSCI GEM RMF	**6.27%**	**19.6%**	**26.8%**	**0.73**	**57%**	**62%**	**3.57**
MVO Q-Scores MSCI GEM Near	6.27%	13.5%	26.8%	0.50	63%	59%	2.70
MVO Q-Scores MSCI GEM Repair	6.27%	24.7%	50.3%	0.49	74%	57%	2.97



Source: J.P. Morgan QDS

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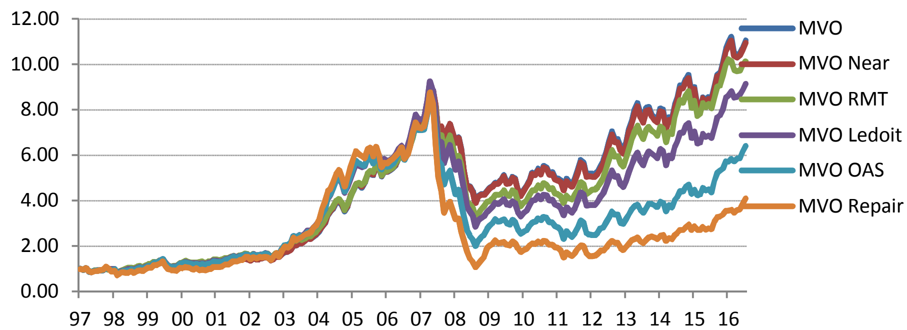
The effect of shrinkage for MVO results in a small market such as the MSCI Australia set is quite pronounced. In this case the regular MVO worked well, delving 13.1% and a Sharpe of 0.71.

The brutal method of truncating the Eigen vectors causes the most damage to the results, reducing returns to just 7.5% and the Sharpe to 0.27, while increasing the drawdown to 88%.

Figure 30: Impact of Covariance Shrinkage on Alpha based Optimisation in Australia

Description	IC	Return	Volatility	Sharpe	Drawdown	Ratio	t-Stat
MVO AU Regular Covariance	**4.26%**	**13.1%**	**18.4%**	**0.71**	**56%**	**60%**	**3.37**
MVO AU Ledoit	4.26%	12.0%	20.5%	0.58	69%	63%	2.92
MVO AU OAS	4.26%	9.9%	22.8%	0.44	77%	60%	2.36
MVO AU RMF	**4.26%**	**12.6%**	**18.1%**	**0.69**	**63%**	**62%**	**3.32**
MVO AU Near	4.26%	13.0%	18.6%	0.70	57%	60%	3.34
MVO AU Truncate	4.26%	7.5%	27.2%	0.27	88%	59%	1.79



Source: J.P. Morgan QDS

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Conclusions

We have tested a number of post-modern portfolio construction techniques that are designed to address some of the concerns with Mean Variance Optimisation theory. For risk only based allocation methods we find the Marcos Lopez de Prado approach of forming 'Hierarchical Risk Parity' (HRP) portfolios to be attractive on a risk adjusted basis, often producing equivalent or superior Sharpe ratios than GMV, MDP and Equal Weighted portfolios.

In the original implementation of HRP there is no scope for a source of expected returns, so we have modified the HRP technique to support an alpha, which we call Hierarchical Alpha Portfolio (HAP). We can also combine the two approaches using a risk aversion parameter λ to allocate between HAP and HRP, a method we call Hierarchical Alpha Risk Portfolio HARP. We find these methods can produce well diversified portfolios with attractive risk-adjusted returns, however the performance can often be inferior (in absolute terms) to Mean Variance Optimisation.

Another method to create a diversified weighting which is to distribute the capital across each cluster hierarchy, as recently published by Thomas Raffinot in his paper "Hierarchical Clustering based Asset Allocation". We refer to this as Hierarchical Clustering Portfolio (HCP). We found that HCP is extremely sensitive to cluster choice with poor results in general. Of the clustering techniques tested the best results were close to the return profile of equal weighting, and the Ward method appeared to be the most consistent.

Covariance Shrinkage is a method to correct the covariance estimation problems and we investigated five alternatives; Ledoit-Wolf, Oracle Approximating Shrinkage Estimator, Random Matrix Theory, Nearest and Truncation. In general the matrix shrinkage approaches improved the covariance estimation and produced stronger risk adjusted returns than empirical MVO. The Random Matrix Theory filtered covariance matrix produced some of the most consistent results across different MSCI based universes tested (especially the Global Minimum Variance optimisation tests, without an alpha).

In conclusion: HRP and HARP produce portfolios with attractive risk adjusted returns, but with lower absolute returns, while HCP results approach Equal Weighted portfolios at best. Finally MVO can be improved with covariance shrinkage, and we find the RMF technique offered the most consistent risk adjusted returns.

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Appendix: Other Alternatives

The research into covariance estimation is deep and we have only brushed the surface in this report. The interested reader could look into some alternative areas of research such as partial covariance's or higher dimensional and frequency methods.

Partial Covariance

In their report on "Portfolio Theory in Terms of Partial Covariance", Dadler and Schmidt describe a Partial Pearson Correlation matrix approach that strips market returns from correlations, and is available on SSRN, here.

Abstract:
It is found that partial correlations between 12 major US equity sector ETFs conditioned on the state of economy (mimicked here by the S&P 500 index) are significantly lower than the Pearson's correlations. The Markowitz mean-variance portfolio theory is modified in terms of partial covariance. The maximum Sharpe portfolios formed by 12 equity sector ETFs in 2007 – 2015 are examined. With the exclusion of the bear market of 2008, the partial correlation based portfolios (PaCP) are much more diversified than the Pearson's correlation based portfolios (PeCP). Out-of-sample performance of the maximum Sharpe PeCP and PaCP, and the equal-weight portfolio (EWP) are compared. The results are very sensitive to the model parameters (portfolio calibration window and frequency of portfolio rebalancing). While the PeCP weights change significantly from month to month, the PaCP weights outside the bear market effects are almost constant. PaCP outperforms both EWP and PeCP when the 36-month calibration window and one-month rebalancing frequency are used. We conclude that partial covariance is a promising concept for constructing optimal portfolios.

In this paper the authors highlight some other approaches to resolve the problems with estimating covariance matrix, such as DeMiguel et al (2013) who used implied volatility instead of historical Pearson's correlations and Gerber et al (2015) who replaced Pearson's correlations in the Markowitz theory with a robust co-movement measure (Gerber statistic).

They note that Pearson's correlation between two asset prices may be affected by relationships of these prices with some other, common variable. Shapira et al (2009) and Kenett et al (2010, 2012, 2014) suggested using partial correlations to filter out such relationships.

The Partial correlation coefficient, $\rho ij|k$, between variables Xi and Xj that is conditioned on variable Xk measures correlation between residuals of linear regressions of Xi on Xk , and Xj on Xk (Johnston and DiNardo 1999) which the authors (Dadler and Schmidt) use in their paper.

Partial Pearson Correlations:

$$\rho_{ij|k} = \frac{\rho_{ij} - \rho_{ik}\rho_{jk}}{\sqrt{1-\rho_{ik}^2}\sqrt{1-\rho_{jk}^2}}$$

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Multi-Scale Spectral Components Framework

The Merit of High-Frequency Data in Portfolio Allocation Nikolaus Hautsch, Lada M. Kyj, and Peter Malec No. 2011/24 available here

Abstract:

This paper addresses the open debate about the usefulness of high-frequency (HF) data in large-scale portfolio allocation. Daily covariances are estimated based on HF data of the S&P 500 universe employing a blocked realized kernel estimator. We propose forecasting covariance matrices using a multi-scale spectral decomposition where volatilities, correlation eigenvalues and eigenvectors evolve on different frequencies. In an extensive out-of-sample forecasting study, we show that the proposed approach yields less risky and more diversified portfolio allocations as prevailing methods employing daily data. These performance gains hold over longer horizons than previous studies have shown.

The Multi-Scale Spectral Components Framework We introduce the Multi-Scale Spectral Components (MSSC) model as a flexible framework for providing forecasts based on time series of high-dimensional daily covariance matrices. The approach is motivated by the idea of: (i) separately modeling variances, correlation eigenvalues and correlation eigenvectors, (ii) conditioning the correlation matrix by imposing a factor structure, (iii) projecting eigenvalues on the underlying eigenvector basis, and (iv) allowing the individual covariance components to be averaged over different frequencies.

Market Condition Similarity Weighted

Estimating correlation and covariance matrices by weighting of market similarity Michael C. Munnix, Rudi Schafer, and Oliver Grothe, 2010, available here.

Abstract:

We discuss a weighted estimation of correlation and covariance matrices from historical financial data. To this end, we introduce a weighting scheme that accounts for similarity of previous market conditions to the present one. The resulting estimators are less biased and show lower variance than either unweighted or exponentially weighted estimators. The weighting scheme is based on a similarity measure which compares the current correlation structure of the market to the structures at past times. Similarity is then measured by the matrix 2-norm of the difference of probe correlation matrices estimated for two different times. The method is validated in a simulation study and tested empirically in the context of mean-variance portfolio optimization. In the latter case we find an enhanced realized portfolio return as well as a reduced portfolio volatility compared to alternative approaches based on different strategies and estimators.

Exponentially Weighted Moving Average (EWMA)

This is commonly referred to as the original Risk Metrics approach. Instead of using equal weighted variances, the method uses exponentially weighted returns to calculate the covariance matrix, giving a model that is much more responsive to recent shocks than traditional covariance models. Section 5 of the RiskMetrics Technical Document discusses the model in great detail. EWMA is commonly used in commercial risk packages (such as MSCIBarra, Northfield etc) to improve timeliness of the model to shocks.

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Minimum Covariance Determinant (MCD)

Real world data is often subjects to measurement and/or recording errors. Regular (repeated) but uncommon observations may also appear for a variety of reason. Every observation which is very uncommon (and often extreme) is called an outlier. The empirical covariance estimator and the shrunk covariance estimators are typically very sensitive to the presence of outlying observations in the data.

Therefore, one should use robust covariance estimators to estimate the covariance of its real data sets. Alternatively, robust covariance estimators can be used to perform outlier detection and discard/down weight some observations according to further processing of the data.

The sklearn.covariance package implements a robust estimator of covariance, the Minimum Covariance Determinant and is described below and on their site, here.

"The Minimum Covariance Determinant is a two-step robust estimator of a data set's covariance which works by finding a given proportion of "good" observations (outliers excluded) and compute their empirical covariance matrix. This empirical covariance matrix is then rescaled to compensate for the performed selection of observations ("consistency step"). From the Minimum Covariance Determinant estimator, weights to all observations are given according to their Mahalanobis distance, leading to a reweighted estimate of the covariance matrix ("reweighting step")."

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Code Samples

HARP

```
def getHARP(cov, sortIx, xReturns=None, riskaversion=0.5, minWt = 0.001, maxWt = 1.0):
    # Compute HAP, HRP and HARP asset allocation
    # xReturns Asset Returns must be rescaled from zero to 1.0! getxprtnnZeroOne(expReturns)
    # riskaversion = Lambda relative importance of risk vs expected returns
    # Based on de Prado's "getRecBipart" function

    w=pd.Series(1.0/sortIx.__len__() ,index=sortIx)
    wts=pd.DataFrame(data=0,index=sortIx, columns=[0])
    cItems=[sortIx] # initialize all items in one cluster
    wti=0
    while len(cItems)>0:
        cItems=[i[j:k] for i in cItems for j,k in /
                     ((0,len(i)/2), (len(i)/2,len(i))) if len(i)>1] # bi-section
        for i in xrange(0,len(cItems),2):      # parse in pairs  i=0
            cItems0=cItems[i]                  # cluster 1
            cItems1=cItems[i+1]                # cluster 2
            cVar0=getClusterVar(cov,cItems0)   # Single variance number for portfolio 0
            cVar1=getClusterVar(cov,cItems1)
            if xReturns is None:
                alpha=1-cVar0/(cVar0+cVar1)
                w[cItems0]*=alpha                  # weight 1
                w[cItems1]*=1-alpha                # weight 2
            else:
                eVar0 = xReturns[cItems0].mean()   # Expected Returns of group 1
                eVar1 = xReturns[cItems1].mean()   # Expected Returns of group 2
                alpha = 1. - cVar0 / (cVar0+cVar1) # HRPi Relative Variance - HRP 'alpha'
                xprtn = 0. + eVar0 / (eVar0+eVar1) # HAPi Expected Return Relative

                # High Risk Aversion results in more stock weight from variance vs. returns.
                w[cItems0] *= riskaversion*alpha + (xprtn*(1.-riskaversion))# HARPi Wt1
                w[cItems1] *= riskaversion*(1.-alpha) + (1.-xprtn)*((1.-riskaversion))#Wt2

        wts[wti] = w  #store incremental weights for debugging
        wti +=1
        w[w<minWt] = 0.0
        w = w.clip(0.0, maxWt)
        w[w<maxWt] = w[w<maxWt] / sum(w)   # fix rounding errors

    return w
```

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HCP

```
def getClusters(corr = pd.DataFrame, method='ward', bPlot=False):
    corr = corr.replace([np.inf, -np.inf], np.nan).fillna(0, inplace=False)
    clusters = []
    w=pd.Series(1.0,index=corr.index)
    Z = linkage(corr, method)            # 'ward', 'single', etc...)

    # Combine raw leaf nodes with linkage matrix
    raw  = pd.DataFrame(corr.index, index=corr.index, columns=['cItems0'])
    raw['cItems1'] = np.nan
    raw['Dist']    = 0
    raw['Count']   = 1
    link = pd.DataFrame(Z, columns=['cItems0','cItems1','Dist','Count'])
    linked = pd.concat([raw, link],axis=0, ignore_index=True)
    # Initial Weights
    linked['wt']=1.

    # Traverse linkage in reverse order
    w = linked.__len__()-1
    while w>Z.__len__():
        inWt = linked.loc[w,'wt']/2.0  # weight split
        cItems0, cItems1 = linked.iloc[w,0:2].astype(int)
        linked.loc[[cItems0, cItems1],'wt'] *= inWt
        w-=1

    if bPlot:  # Do we need a chart
        fig = PlotDendo(Z, labels=linked.loc[0:Z.__len__(),'wt'].values)  # corr.index)

    return linked.loc[0:Z.__len__(),'wt']
```

Support Functions

```
def PlotDendo(Z, labels):
    from scipy.cluster.hierarchy import dendrogram

    plt.figure(figsize=(8, 8))
    plt.title('Hierarchical Clustering Dendrogram on Correlations')
    # plt.xlabel('sample index')
    # plt.ylabel('distance')
    R = dendrogram(
        Z,
        leaf_rotation=90.,    # rotates the x axis labels
        leaf_font_size=16.,   # font size for the x axis labels
        labels=[str(word*100.0) + '%' for word in labels],  # label for plot
        color_threshold=None  # .5
    )
    ax = plt.gca()
    ax.tick_params(axis='y', labelsize=16)
    plt.show()
    return R
```

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```python
def getxprtnnZeroOne(expReturns=pd.Series):
    # Rescale expected returns from 0 to 1
    if expReturns is None:
        pass
    else:
        idx=expReturns.index
        expReturns = zscore(expReturns)
        expReturns = expReturns/max(abs(expReturns))/2.0+0.5
        expReturns = expReturns / max(abs(expReturns))
         # in case when max(abs()) was -ve, then new max  will be < 1.0
        expReturns = pd.Series(expReturns, index=idx)
    return expReturns


#-------------------------------------------------------------------------------



def zscore(a, axis=0, ddof=0, keepNaN=False):
    """ NAN Stable Z-Scores"""
    try:
        idx = a.index
    except:
        idx = None
    a = np.asanyarray(a)
    mns = np.nanmean(a, axis=axis)
    sstd = np.nanstd(a=a, axis=axis, ddof=ddof)
    if axis and mns.ndim < a.ndim:
        res = ((a - np.expand_dims(mns, axis=axis)) /
               np.expand_dims(sstd, axis=axis))
    else:
        res = (a - mns) / sstd  # RESult

    if not keepNaN:
        res = np.nan_to_num(res)  # Default set to zero where was NaN

    res = pd.Series(res, index=idx)
    return res
```

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