BARCLAYS CAPITAL PORTFOLIO OPTIMIZER: SOFT CONSTRAINTS

An application in cash replication of EURO Credit index

A previous version had the incorrect index name in the title, apologies for the inconvenience

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The Barclays Capital portfolio optimizer, available in the POINT[1] system, supports soft constraints functionality. With this functionality, the Portfolio Optimizer chooses an optimal tradeoff between the objective function and the portfolio's (soft) constraints violation penalty. The penalty is proportional to the amount of constraint violation.

- Soft constraints significantly reduce the occurrence of infeasible problems – leading to greater flexibility and transparency to the portfolio managers.
- The realized trade-off between the soft constraint penalty and the objective function (e.g., risk-adjusted returns) gives users a clear picture of the effect of constraints on the optimal portfolio characteristics.
- Soft constraints allow managers to construct portfolios based on estimation-free analytics by minimizing the weighted sum of absolute deviations between portfolio analytics and benchmark analytics or given constant targets.
- Soft ticker/issuer/issue concentration constraints resolve the frequent infeasibility observed with their hard counterparts, thus facilitating model-free and flexible control of a portfolio's idiosyncratic risk.
- Soft constraints can be used to eliminate the undesirable boundary effect observed with hard constraints.

We illustrate this functionality using alternative optimization formulations of the (cash) index replication problem for the Barclays Capital European Credit index.

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Introduction

Flexible support for generic portfolio constraints is an important requirement in a practical portfolio optimizer. Portfolio constraints allow users to selectively eradicate the potentially unstable dependence of the optimal portfolios on the large numbers of estimated parameters in risk and alpha models. Oftentimes, an optimization problem with a (large) number of (hard) constraints is infeasible. Furthermore, it is typically difficult to decipher which constraints cause the infeasibility, and can be relaxed to achieve a feasible solution. This is because the effect of hard constraints on the set of feasible portfolios is highly intertwined.

Soft constraints are helpful in resolving both these difficulties. An optimization problem with only soft constraints is always feasible. Furthermore, users choose the per-unit penalty to control the relative importance (degree of softness) given to various constraints in the optimization process. Thus, the soft constraints functionality significantly enhances the flexibility available to the users to formulate and solve practical portfolio optimization problems.

How do they work?

We assume that the readers are familiar with the Barclays Capital Portfolio Optimizer available in POINT system based on the optimizer *white paper* (Kumar [2009]) and the *user guide* (Kumar and Lazanas [2009]).

Soft constraints allow the optimizer to present a portfolio that violates the (soft) constraints at the cost of paying a penalty proportional to the amount of violation. The total penalty for all soft constraints is subtracted (or added in case of minimization) from the problem objective function. The constant of proportionality (i.e., per-unit penalty) is specified by the user for each constraint. The constraint violations below the lower bound and above the upper bound are treated symmetrically. Users can achieve an asymmetric treatment by implementing the upper bound and the lower bound in two separate constraints. A mathematical description of the optimization model with soft constraints is provided in the appendix.

The user interface for generic constraints, available on the *generic constraint tab* in the *optimizer report*, has been enhanced to include two additional columns titled *Soft* and *Penalty*. Soft versions are supported with both the *generic bucket constraints* (Figure 1) and the *For-Each issue/issuer/ticker constraints* (Figure 2). Selecting the soft checkbox indicates that the constraints are soft. The *Penalty* column is used to input the corresponding per-unit-penalty. Users can change the default penalty provided by the interface to any non-negative number. Negative penalties are not allowed.

We do not allow soft versions of the combinatorial constraints (i.e., the constraints on number of securities, the number of trades, and the minimum trade size). We use custom heuristic algorithms to support the combinatorial constraints (Section II in the white paper (Kumar [2009])), which already allow the optimizer to present a solution violating[2] these constraints in case of infeasibility. We also do not allow soft risk constraints (the Tracking Error Volatility (TEV) constraints). First, the constraints with lower bound on the portfolio risk terms are not allowed due to non-convexity (see page 20 in the *user guide*). Second, the soft versions of upper bound risk constraints are equivalent to either using the corresponding risk term in the objective function if there is no feasible portfolio satisfying the (hard) risk constraints, or using the hard constraint if there are feasible portfolios satisfying the risk constraint.

[2] User guide (Kumar and Lazanas [2009]), pp 42-43.

Figure 1: Soft generic bucket constraints

Setup | **Generic Constraints** | No Trade List | Trade List | Final Portfolio | Warnings/Exclusions

Constraints on values aggregated by Buckets

No.	Soft	Penalty	Attribute	Universe	Measure	Lower B...	Upper B...	Unit	Initial V...	Realized V...
2	✓	100.00	KRD 2yr	Final Portfolio	Net vs B...	0.00	0.00	yrs	-0.85	0.02
3	✓	100.00	KRD 5yr	Final Portfolio	Net vs B...	0.00	0.00	yrs	-1.91	0.00
4	✓	100.00	KRD 10yr	Final Portfolio	Net vs B...	0.00	0.00	yrs	-1.20	0.00
5	✓	100.00	KRD 20yr	Final Portfolio	Net vs B...	0.00	0.00	yrs	-0.23	0.00
6	✓	100.00	KRD 30yr	Final Portfolio	Net vs B...	0.00	0.00	yrs	-0.08	0.00
7	✓	10,000.00000	Market Value [%]	Government-Related-Gov-Related Agencie...	Net vs B...	0.00000	0.00000	%	-3.76981	0.00000
8	✓	10,000.00000	Market Value [%]	Government-Related-Gov-Related Local Au...	Net vs B...	0.00000	0.00000	%	-1.08508	0.00000
9	✓	10,000.00000	Market Value [%]	Government-Related-Gov-Related Sovereig...	Net vs B...	0.00000	0.00000	%	-4.36243	0.00000
10	✓	10,000.00000	Market Value [%]	Government-Related-Gov-Related Suprana...	Net vs B...	0.00000	0.00000	%	-7.87561	0.00000
11	✓	10,000.00000	Market Value [%]	Corporate-Banking-1a:Class 1 by Credit Se...	Net vs B...	0.00000	0.00000	%	-2.00571	0.00000
12	✓	10,000.00000	Market Value [%]	Corporate-Banking-1b:Class 1 by Credit Se...	Net vs B...	0.00000	0.00000	%	-13.22232	-0.00002
13	✓	10,000.00000	Market Value [%]	Corporate-Banking-2:Class 1 by Credit Sect...	Net vs B...	0.00000	0.00000	%	-16.79813	-0.00003
14	✓	10,000.00000	Market Value [%]	Corporate-Banking-3:Class 1 by Credit Sect...	Net vs B...	0.00000	0.00000	%	-2.79346	0.00000
15	✓	10,000.00000	Market Value [%]	Corporate-Finance:Class 1 by Credit Sector...	Net vs B...	0.00000	0.00000	%	-7.55978	0.00000
16	✓	10,000.00000	Market Value [%]	Corporate-Basic:Class 1 by Credit Sectors/...	Net vs B...	0.00000	0.00000	%	-7.73603	0.00000
17	✓	10,000.00000	Market Value [%]	Corporate-ConsumerCycl:Class 1 by Credit ...	Net vs B...	0.00000	0.00000	%	-5.39398	0.00000
18	✓	10,000.00000	Market Value [%]	Corporate-ConsumerNonCycl:Class 1 by Cr...	Net vs B...	0.00000	0.00000	%	-7.27411	0.00000
19	✓	10,000.00000	Market Value [%]	Corporate-EnergyTransport:Class 1 by Cre...	Net vs B...	0.00000	0.00000	%	-4.20808	0.00000
20	✓	10,000.00000	Market Value [%]	Corporate-TechnologyCommun:Class 1 by ...	Net vs B...	0.00000	0.00000	%	-8.47613	0.00000
21	✓	10,000.00000	Market Value [%]	Corporate-Utility:Class 1 by Credit Sectors/...	Net vs B...	0.00000	0.00000	%	-7.43933	0.00000
22	✓	1,000.00	OASD	Government-Related-Gov-Related Agencie...	Net vs B...	0.00	0.00	yrs	-3.69	0.13
23	✓	1,000.00	OASD	Government-Related-Gov-Related Local Au...	Net vs B...	0.00	0.00	yrs	-5.90	1.69
24	✓	1,000.00	OASD	Government-Related-Gov-Related Sovereig...	Net vs B...	0.00	0.00	yrs	-4.64	0.00

Source: Barclays Capital POINT

How to set the penalties?

It is convenient to first think of the two boundary cases. A soft constraint with zero penalty is redundant. On the other end, a soft constraint with very large penalty is equivalent to a hard constraint. As users increase the penalty values, the optimizer should present a solution with a decreasing amount of violation in the corresponding constraint, if it is possible to do so.

The system provides a default penalty value of one, which is independent of the constraints attribute. Users can change the default penalty values based on the constraint attribute, and expected cost of constraint violation. For example, it is expected that users would not find it equivalent to violate a duration (e.g., OASD) constraint by one unit (in years) and violate a spread (e.g., OAS) constraint by one unit (in basis points). To avoid such a default trade-off, users can use a set of penalties to convert the constraints penalties expressions to comparable units across the set of constraints. For example, a penalty value of 100 for the duration constraints and 1 for the spread constraints would assign equal penalties to a portfolio violating the duration constraint by one year and a spread constraint by 100bp.

Figure 2: Soft issue/issuer/ticker constraints

Constraints on each Issue/Issuer/Ticker

Universe Class 1 by Credit Sectors

No.	Soft	Penalty	Attribute	Universe	For Each	Measure	Lower B...	Upper B...	...	Initial Value	Realized Value
1	+ ✓	0.01000	Market Value [%]	Government-Related-Gov-Related ...	Issuer	Net vs Bmark	0.00000	0.00000	%	-0.33763	1.66099
2	+ ✓	0.01000	Market Value [%]	Government-Related-Gov-Related ...	Issuer	Net vs Bmark	0.00000	0.00000	%	-0.55893	0.13449
3	+ ✓	0.01000	Market Value [%]	Government-Related-Gov-Related ...	Issuer	Net vs Bmark	0.00000	0.00000	%	-1.00180	1.62433
▸4	− ✓	0.01000	Market Value [%]	Government-Related-Gov-Related ...	Issuer	Net vs Bmark	0.00000	0.00000	%	-6.65530	1.22030

Government-Related-Gov-Related Supranational:Class 1 by Credit Sectors/ANUKUMAR(Issuer)

Name	Initial Value	Realized Value
EUROPEAN INVESTMENT BANK	-6.65530	1.22030
EUROPEAN BANK FOR RECONSTRUCTI	-0.01806	-0.01806
INTER-AMERICAN DEVELOPMENT BAN	-0.02422	-0.02422
COUNCIL OF EUROPE DEVELOPMENT	-0.02684	-0.02684
NORDIC INVESTMENT BANK	-0.06568	-0.06568
EUROFIMA	-0.08669	-0.08669
INTERNATIONAL BANK FOR RECONST	-0.39309	-0.39309
EUROPEAN COMMUNITY	-0.60572	-0.60572

No.	Soft	Penalty	Attribute	Universe	For Each	Measure	Lower B...	Upper B...	...	Initial Value	Realized Value
5	+ ✓	0.01500	Market Value [%]	Corporate-Banking-AAA:Class 1 by...	Issuer	Net vs Bmark	0.00000	0.00000	%	-1.93459	0.07112
6	+ ✓	0.02000	Market Value [%]	Corporate-Banking-AA:Class 1 by ...	Issuer	Net vs Bmark	0.00000	0.00000	%	-1.13952	4.35122
7	+ ✓	0.02000	Market Value [%]	Corporate-Banking-A:Class 1 by Cr...	Issuer	Net vs Bmark	0.00000	0.00000	%	-1.16236	5.66812
8	+ ✓	0.03000	Market Value [%]	Corporate-Banking-BBB:Class 1 by ...	Issuer	Net vs Bmark	0.00000	0.00000	%	-0.26483	1.13400
9	+ ✓	0.01000	Market Value [%]	Corporate-Finance:Class 1 by Cred...	Issuer	Net vs Bmark	0.00000	0.00000	%	-1.70734	1.70016
10	+ ✓	0.01000	Market Value [%]	Corporate-Basic:Class 1 by Credit ...	Issuer	Net vs Bmark	0.00000	0.00000	%	-0.76952	2.37073
11	+ ✓	0.01000	Market Value [%]	Corporate-ConsumerCycl:Class 1 b...	Issuer	Net vs Bmark	0.00000	0.00000	%	-0.67281	0.99506
12	+ ✓	0.01000	Market Value [%]	Corporate-ConsumerNonCycl:Class...	Issuer	Net vs Bmark	0.00000	0.00000	%	-0.70518	3.00359
13	+ ✓	0.01000	Market Value [%]	Corporate-EnergyTransport:Class ...	Issuer	Net vs Bmark	0.00000	0.00000	%	-0.64350	2.20959
14	+ ✓	0.01000	Market Value [%]	Corporate-TechnologyCommun:Cla...	Issuer	Net vs Bmark	0.00000	0.00000	%	-1.17959	4.28083
15	+ ✓	0.01000	Market Value [%]	Corporate-Utility:Class 1 by Credit ...	Issuer	Net vs Bmark	0.00000	0.00000	%	-1.12802	1.65342

Source: Barclays Capital POINT

The issue/issuer/ticker constraints (Figure 2) expand to a (large) number of constraints in the solver. The number of these *child constraints* is equal to the number of unique issue, issuer, or ticker spanned by the set of securities in the constraint universe (a partition bucket of the union of *all* tradable universes provided in the problem) depending on the user's choice in the "For-Each" column (i.e., issue, issuer, or ticker). A soft version is created for each of the child constraints with a uniform penalty value provided by the user (similar to the constraint bounds) in the parent constraint for the soft issue/issuer/ticker constraints. This amplifies the effect of soft constraint penalty with these constraints. For example, the "For-Each" issuer exposure (market value [%]) constraints with an upper bound of 2% with 1,000 tradable issuers leads to the following penalty function:

$$p \cdot \sum_{i=1}^{1000} \max(0, (MV_i - 0.02 * MV_P))$$

where p is the penalty value. The sum in the expression above is over the set of 1,000 issuers. Suppose the optimal portfolio has positions in 100 issuers, out of which 10 issuers exceed the 2% bound by 1% on average. The average constraint violation is 0.1%, leading to a total realized penalty of 100*.1*p, where p is the user provided per-unit penalty. As a rule of thumb, users can scale down the penalty value of these constraints by the number of issue, issuer, or tickers expected to violate the constraint. For example, for "absolute", "Net-vs-Benchmark", or "trading" For-Each issuer constraints, we can scale the intended penalty by the number of issuers expected to have non-zero holding (active holding and trading, respectively) exposure at the optimal portfolio.

Furthermore, the interactive nature of the optimizer allows users to fine-tune the penalty values by successively vetting the current solution and changing the penalty values appropriately. For example, if the amount of violation in a particular constraint at the current solution is unacceptably large, users can increase the penalty value of the constraint and re-run.

The examples presented below should further illustrate the various points mentioned here.

Using soft constraints in portfolio construction

Stratified sampling approach is a commonly used technique in addition to the risk minimization approach (e.g., risk forecasts based on the Global Risk Model (GRM) in POINT) for portfolio construction. This is particularly true for bond portfolios, where the pricing model based sensitivity analytics (e.g., OASD of a corporate bond) for fixed-income securities are relatively more reliable measures compared with the empirical (i.e., regression-based) sensitivities (e.g., industry beta of a stock) for equity portfolios. Hence, it is important for a practical fixed-income portfolio optimizer to provide a flexible support for this approach. However, infeasibility of the underlying optimization problems is frequently observed when implementing a stratified sampling approach using hard constraints in the portfolio optimizer.

Furthermore, the cell exposure constraints eliminate the dependence of the optimal portfolios on the potentially noisy correlation estimates in the risk model. Hence, it is desirable to be able to combine the two approaches: minimizing risk forecasts, while constraining cell exposures. Implementing the cell exposure constraints as soft constraints is effective in resolving both these issues.

We illustrate this using the following index replication example. We construct four 75[3] bonds 100mn EUR portfolios to replicate the Barclays European Credit index as of November-end 2009. Euro Credit index is the credit component of the Euro Aggregate index. The monthly rebalanced index constitute of approximately 1,500 EUR-denominated credit bonds with a liquidity requirement of 300mn in amount outstanding and an investment grade rating (Baa3/BBB- and above) from at least two of the three rating agencies (Moody's, Standard and Poor's, and Fitch).

The index sector exposure can be broken down into corporate (financial, industrial, and utility issues) and non-corporate (supranational issues, sovereigns, foreign agencies, and foreign local authorities) sectors. We use a more detailed classification for the corporate component in this example based on the underlying industry (banking, finance, basic, consumer cyclical, consumer non-cyclical, energy and transport, technology and communications, and utility) to define the risk cells to be used in the stratified sampling portfolio construction formulation. Since the index is concentrated (>30% in market weight) in the banking sector, it is further partitioned in to four cells (AAA, AA, A and BBB, respectively) based on index ratings[4].

[3] The appropriate number of bonds needed in the cash replication of a beta index should be based on the tradeoff between the tracking error risk, and trading and management costs of implementing large portfolios. The management and trading costs depend on the liquidity profiles of index constituents and turnover in this liquidity profile. We chose 75 bonds as an example based on the number of systematic risk factors underlying the index and an acceptable level of diversification (idiosyncratic risk).

[4] Barclays Capital index ratings are a function of bond ratings from three difference agencies (Moody's, Standard and Poor's, and Fitch).

Figure 3: Euro Credit index: Defining the stratified sampling cells

Sector No.	Sectors Name	Number of bonds	Market Value [%]	Coupon (%)	OAS (bp)	OASD (yrs)	Index Rating	TEV Contribution (bp/mo)
	Total	**1546**	**100**	**4.902**	**167.5**	**4.34**	**A1/A2**	**118.5**
1	Government-Related-Agencies	63	3.77	4.437	139.3	3.72	AA2/AA3	4.1
2	Government-Related-Local Authority	16	1.09	4.474	98.0	5.95	AA2/AA3	1.3
3	Government-Related-Sovereign	56	4.36	4.695	165.4	4.67	A2/A3	4.6
4	Government-Related-Supranational	42	7.88	3.677	36.5	5.81	AAA	6.9
5	Corporate-Banking-AAA	15	2.01	4.199	70.4	4.97	AAA	1.7
6	Corporate-Banking-AA	172	13.22	4.549	124.5	3.59	AA2/AA3	10.7
7	Corporate-Banking-A	304	16.80	4.955	248.1	3.98	A1/A2	23.5
8	Corporate-Banking-BBB	76	2.79	5.469	632.7	4.26	BAA1/BAA2	8.9
9	Corporate-Finance	142	7.56	5.101	314.3	4.32	A1/A2	14.3
10	Corporate-Basic	147	7.74	5.382	152.4	4.07	A3/BAA1	8.8
11	Corporate-Consumer Cyclical	89	5.39	5.47	137.9	3.17	A3/BAA1	5.2
12	Corporate-Consumer Non-Cyclical	110	7.27	5.146	109.4	4.39	A2/A3	6.6
13	Corporate-Energy & Transport	63	4.21	4.745	110.7	5.24	A1/A2	4.8
14	Corporate-Technology & Communications	130	8.48	5.519	125.0	4.33	A3/BAA1	8.6
15	Corporate-Utility	121	7.44	5.191	102.7	5.31	A2/A3	8.4

As-of 11/30/2009. TEV Contributions based on the POINT's Global Risk Model (GRM). Source: Barclays Capital POINT

Figure 3 displays the bucket's exposures corresponding to the partition scheme described above. We see that risk is reasonably well distributed across various buckets as indicated by the distribution of market weights, OASD, and the contribution to the tracking error volatility[5]. Note that the banking-BBBs and banking-As have significantly higher Treasury spreads compared with the index average, hence leading to significantly larger contribution to the TEV. We refer to this scheme as the "risk partition."[6] The sector scheme defined above is for demonstration purposes only. Users can use their own sector definitions in this example.

We use the following four distinct formulations to construct the optimal replicating portfolios. All four of these *optimization reports* are accessible in POINT in the *central portfolios* named "*Optimizer Demo: Soft Constraints*". All formulations use the index constituents as the tradable universe and the benchmark universe.

Risk minimization (PCS-1)

PCS-1 minimizes the TEV forecast of the portfolio subject to the budget constraints, and the constraint on the number of bonds in the portfolio. The optimizer report in POINT also outputs the KRD profile, risk partition sector weights, sector OASD, and sector OAS of the optimal portfolio net of the benchmark using redundant bucket generic constraints with *blank* bounds in the *realized values* columns. Similarly, it outputs the active issuer weights using the "For-Each" issuer constraint[7]. The TEV forecasts are based on the weighted calibration of the POINT's Global Risk Model (GRM) constituting a systematic, idiosyncratic, and default risk component. The systematic risk model covering the index is based on 49 risk factors,

[5] Based on the Global Risk Model (GRM) in POINT (weighted model calibrations).
[6] User can access it as a system partition titled "Euro Credit Sectors".
[7] The redundant For-Each constraints use a penalty value of zero with upper and lower bound set to zero. The bounds are used so that the reported realized values are worst case active issuer exposures for each constraint.

including the 7 EUR curve (6 KRDs + 1 convexity), 4 swap spreads, 26 credit factors, and 8 EUR-denominated emerging market debt credit factors.

Stratified sampling (PCS-2)

PCS-2 matches the market weights, OASD, and OAS of the portfolio to the benchmark index in all risk partition sectors, and limits the active issuer exposure to +/- 2% for all issuers spanning the index constituents using hard constraints. Additionally, it matches the six portfolio KRDs to the benchmark index to mitigate the active curve risk. The objective function is blank.

Stratified sampling using soft constraints (PCS-3)

PCS-3 implements all the bucket and KRDs constraints in PCS-2 using soft constraints. It uses a penalty of 100, 1000, 200, and 10 for the set of the KRDs, market weights, OASD, and OAS constraints, respectively. This creates an effective penalty cost (shadow objective function) expression as follows:

$$100\sum_{i=1}^{6}\left|KRD_i^P MV^P - KRD_i^B MV^B\right| + 1000\sum_{j=1}^{15}\left|MV_j^P - MV_j^B\right| \\ + 200\sum_{j=1}^{15}\left|OASD_j^P MV_j^P - OASD_j^B MV_j^B\right| + 10\sum_{i=1}^{15}\left|OAS_j^P MV_j^P - OAS_j^B MV_j^B\right| \tag{1}$$

The soft version of the "For-Each" constraint on active issuer exposure creates an effective penalty cost (additional shadow objective function term) as follows:

$$\sum_{j=1}^{15} p_j \sum_{k=1}^{N_j}\left|MV_k^P - MV_k^B\right| \tag{2}$$

where i, j, and k indexes the six (6m, 2y, 5y, 10y, 20y, and 30y) KRDs, the 15 risk partition buckets, and the 540 issuers spanning the set of 1546 issues constituting the index. The "For-Each" constraints are also divided using the risk partition, thus allowing 15 distinct penalty values (p_j above). We use a penalty of 0.01 is used for all government-related sectors, and a penalty of 0.015 for all corporate sectors except banking. We use penalties of 0.01, 0.015, 0.02, and 0.03 for AAAs, AAs, As, and BBBs, respectively, in the corporate banking sector. Similar to PCS-2, the objective function is left blank.

The relative penalty values used across various constraints in this example are motivated by level of risk captured by the corresponding constraints. For example, a high penalty value of 0.03 in the corporate-BBBs bucket of the "For-Each" issuer constraints is trying to capture a relatively high level of idiosyncratic risk compared with other buckets. Similarly, the penalties for the bucket constraints (equation (1) above) imply that the optimizer gives equal weight to violating the OASD constraint by 5 (exposure) units, the OAS constraint by 100 units, and the KRD constraint by 10 units.

All hard constraints must be satisfied at a feasible solution irrespective of the size of the constraints bucket in the portfolio/benchmark. Tracking portfolios, containing small number of positions, do not typically have any positions in the small buckets. This situation frequently leads to infeasibility unless the user is careful to remove hard constraints on insignificant buckets or use partitioning schemes which does not lead to small buckets. Soft constraints resolve this difficulty by giving less importance to small buckets using the implicit market value weighting in the penalty function, and allowing the optimizer to leave such small buckets empty.

The soft constraints operate on the exposure variables. For example, a soft OASD bucket constraint implies that the penalty is equal to absolute value of the active OASD exposures (OASD times the market value or duration basis for unfunded exposure) in the bucket. This implies that market value weighting across constraints is **implicit** in the penalty expression.

Since both OASD and "*OASD exposure*" are available as constraint attributes in the optimizer, it is important to note the distinction between the penalty expressions they create. The POINT attribute OASD Exposure of a position is defined as the negative of OASD multiplied by the position size (aka *Duration Basis* in POINT) and scaled by 10^-4 (i.e., the PVO1 for 1 basis point change in the spreads). Hence, a soft constraint on OASD and OASD-Exposure imply the same penalty function except for constant (10^-4). The behavior is similar for other duration type attributes. See optimizer user guide (page 37-38 and the appendix therein) for more discussion on this issue.

Pseudo risk model using soft constraints

The expression (equation (1) above) for the total penalty function corresponding to the soft bucket constraints can be interpreted as a simple Mean-Absolute-Deviation (MAD) type systematic risk model (Konno and Yamazaki [1991]) with zero correlations, and volatilities in proportion to the per-unit-penalty coefficient. In other words, the total penalty is equal to the *absolute sum* total of *active* portfolio loadings to 51 (=6 KRDs+15 (Sectors) times 3 (MV%, OASD, and OAS)) pseudo factors. Similarly, the soft For-Each constraint penalty (equation (2) above) can be interpreted as a simple MAD type idiosyncratic risk model with sector specific issuer volatilities.

Such a measure of risk is not meaningful in a total return optimization setting, where we want to trade off the portfolio risk with an expected return measure because it is neither based on consistent units across pseudo risk factors, nor calibrated to forecast risk. But for low tracking error portfolio (e.g., index beta replication), such a risk penalty is expected to be small for low TEV portfolios. Hence, if minimizing this penalty gives low TEV portfolios, then it is a convenient mechanism to add robustness to the portfolio construction process. This motivates our next formulation, where we minimize a weighted combination of the portfolio's TEV (based on the risk model calibration) and the pseudo risk model used in PCS-3.

Risk minimization + stratified sampling (using soft constraints) (PCS-4)

This version combines the risk minimization and the stratified sampling approach (PCS-1 and PCS-3). It uses a *Total TEV* term with a weight of 100 in the objective function in addition to all of the soft constraints in PCS-3.

Figure 4 present the total TEV achieved and its components in the four optimal portfolios (PCS-1:4). We see that the TEV achieved is lowest for the two portfolios where the optimizer was explicitly asked to optimize TEV (PCS-1 and PCS-4). Furthermore, the increase in the TEV in PCS-4 compared with PCS-1 is small (3.2bp). This marginal increase in TEV can potentially be preferable, due to the benefit of achieving a better analytics profile relative to the benchmark.

Figure 4: Realized TEV under various optimizer formulations

	PCS-1	PCS-2	PCS-3	PCS-4
Total TEV	10.4	28.6	26.9	13.6
Systematic TEV	2.4	5.4	11.1	2.9
Idiosyncratic TEV	10.2	28.1	24.5	12.3
Number of Bonds	75	76	72	75

TEV is basis points/month. Source: Barclays Capital POINT

Figure 5: Top index issuers (MV% larger than 1%)

Issuer	Index Exposure (MV[%])	Number of issues
EUROPEAN INVESTMENT BANK	6.66	25
RABOBANK NEDERLAND NV	2.05	16
GE CAPITAL EUROPEAN FUNDING	1.71	21
INTESA SANPAOLO SPA	1.66	23
DEUTSCHE TELEKOM INTERNATIONAL	1.18	14
CITIGROUP INC	1.16	14
BNP PARIBAS	1.13	14
E.ON INTERNATIONAL FINANCE BV	1.13	13
GOLDMAN SACHS GROUP INC/THE	1.11	15
DEUTSCHE BANK AG	1.07	9
BARCLAYS BANK PLC	1.05	11
POLAND GOVERNMENT INTERNATIONAL	1.00	7

Source: Barclays Capital POINT

The top index issuers are listed in Figure 5. The maximum issuer exposure is achieved for "European Investment Bank" in PCS -1, for "Intesa Sanpaolo SPA" in PCS-2, PCS-3, and for "Deutsche Telekom International" in PCS-4 (Figure 6). The negative issuer exposure in PCS-1 indicates that the portfolio is underweight the "European Investment Bank". Even though the worst active issuer exposure is lowest in PCS-2 (the hard constraint version), it has the highest idiosyncratic risk. On the other hand, the PCS-1 and PCS-4 have relatively lower idiosyncratic risk with large worst active issuer exposures. This is because a large number (25 out of 72) of issuers in PCS-2 are hitting the 2% limit in PCS-2, leading to larger active exposure on average (Figure 6). Furthermore, the worst exposure is occurring for the issuers "European National Bank" (large super-nationals) and the "Intesa Sanpaolo SPA" in PCS-1 and PCS-4, which have relatively lower idiosyncratic risk.

Figure 6 displays the minimum and maximum (across issuers) issuer exposures across various sectors in the four optimal portfolios. A large number of issuers (25 out of 72) are hitting the 2% limit in PCS-2. This is what we refer to the boundary effect with hard constraints. In PCS-3 with the help of soft constraints, we are able to eliminate this effect by changing the bounds from +/-2% to +/-0% and using a penalty function instead. The active issuer exposures (i.e., the amount of violation) in PCS-3 vary across issuers depending on their weight in the index.

Figure 6: Active Issuer exposures (Market Value [%]) in various sectors

	PCS-1		PCS-2		PCS-3		PCS-4	
	Min	Max	Min	Max	Min	Max	Min	Max
Total	**-3.61**	**2.21**	**-1.27**	**2.00**	**-3.53**	**7.22**	**-1.71**	**4.88**
Government-Related-Agencies	-0.34	1.47	-0.34	1.70	-0.32	3.43	-0.32	2.35
Government-Related-Local Authority	-0.32	0.71	-0.56	0.52	-0.07	0.13	-0.32	0.53
Government-Related-Sovereign	-1.00	1.91	-1.00	**2.00**	-1.00	1.68	-1.00	1.82
Government-Related-Supranational	**-3.61**	-0.02	**-1.27**	**2.00**	**-3.53**	4.15	-0.61	1.22
Corporate-Banking-AAA	-1.93	-0.07	-0.07	0.07	-0.07	0.07	-0.07	0.07
Corporate-Banking-AA	-1.14	1.02	-1.13	2.52[8]	-0.96	5.83	-1.13	3.52
Corporate-Banking-A	-0.78	**2.21**	-1.16	**2.00**	-0.72	**7.27**	-1.16	1.81
Corporate-Banking-BBB	-0.26	-0.01	-0.26	1.45	-0.17	1.62	-0.16	0.98
Corporate-Finance	-1.71	1.76	-1.15	**2.00**	-0.64	4.10	**-1.71**	1.39
Corporate-Basic	-0.41	0.75	-0.77	**2.00**	-0.32	3.96	-0.37	1.38
Corporate-Consumer Cyclical	-0.67	1.67	-0.67	1.83	-0.43	1.35	-0.63	1.64
Corporate-Consumer Non-Cyclical	-0.71	1.14	-0.71	**2.00**	-0.71	2.03	-0.53	2.73
Corporate-Energy & Transport	-0.64	1.63	-0.55	0.97	-0.55	1.43	-0.64	1.33
Corporate-Technology & Communications	-0.98	1.13	-1.18	**2.00**	-0.71	2.92	-0.98	**4.88**
Corporate-Utility	0.71	1.39	-0.76	**2.00**	-0.34	2.29	-0.37	1.68

Source: Barclays Capital POINT

An upper bound of 75 bonds is included in all formulations described above except PCS-2. The following two remarks are warranted regarding the interaction between the large number of generic constraints and the combinatorial constraint on the maximum number of bonds in the portfolio.

- We use custom iterative heuristic algorithms (refer to the optimizer white paper (Kumar [2009], Section II)) to support various combinatorial constraints (e.g., maximum number of positions in the portfolio). These heuristics are less effective for problems involving a large number of (hard) constraints (e.g., the stratified sampling formulation above). In particular, feasible problems can be declared infeasible if the cardinality constraints are so tight that the heuristic algorithms do not have enough leeway to satisfy all the generic constraints. Furthermore, the problem can be truly infeasible. For example, the optimizer declares the PCS-2 formulation infeasible if the number of bonds is limited to 75. Without the constraint on the number of bonds, the optimal portfolio consists of 76 positions. Since the underlying optimization model is a linear feasibility problem, leading to a corner point solution with minimum number of positions[9], it is expected that the formulation requiring 75 bonds is truly infeasible. Soft constraints are useful in both these situations as they allow the heuristic algorithms to present a reasonable solution in both these situations (PCS-3). In particular, at least as many positions are required in the portfolio as the number of hard constraints, but any number soft constraints can be implemented irrespective of the number of positions allowed.

- Without the soft constraint functionality, it is difficult to combine the stratified sampling (with hard constraints) and the risk minimization approach in problems involving combinatorial constraints. PCS-4 demonstrates that soft constraints bridge this gap. This is a significant enhancement because the portfolio constraints add robustness and

[8] The INTESA SANPAOLO SPA issues falls in both Corporate-Banking-AA and Corporate-Banking-A buckets. Even though its active exposure is 2% by construction, its sector exposures are 2.5% and -0.5% in the AA and A sectors, respectively.

[9] Assuming the problem is not degenerate.

transparency to the portfolio construction process by eliminating dependence on detailed risk model estimates. Furthermore, insightful constraints can be complimentary to the combinatorial heuristics occasionally leading to an almost similar level of realized objective function as the unconstrained problem. For example, the constraints to match broad sector weights in a TEV minimization problem are complementary to the problem objective function as the minimum TEV portfolio is expected to match the sector weights.

Figure 7 displays the active weights, OAS, and OASD of the four optimal portfolios. For PCS-3 and PCS-4, only non-zero active analytics are displayed. PCS-2 matches all these analytics by construction, and hence is not displayed. Note that the lowest TEV PCS-1 overweights the single A's sector by more than 18%, and underweights the AAs by more than 9%, while overweighting the overall corporate sector by more than 4%. Furthermore, banking-AAAs and BBBs are missing from the portfolio. Contrast this with PCS-4, which matches all these sector weights at the cost of increasing the TEV marginally. The optimizer is able to achieve a low TEV in PCS-1 even with such large sector weights using the beta substitution that the risk model allows based on correlations.

Figure 7: Active sector weights, OASD, and OAS under various formulations

	Market Value [%]	OASD (yrs)			OAS (bp)		
	PCS-1	PCS-1	PCS-3	PCS-4	PCS-1	PCS-3	PCS-4
Total	0.00	-0.02		-0.01	31.90	-0.1	0.1
Government-Related-Agencies	-0.37	0.84		0.13	-37.20		
Government-Related-Local Authority	0.19	-1.21	0.1	-1.21	-12.00	-12.0	-12.0
Government-Related-Sovereign	0.45	1.99			-19.50		
Government-Related-Supranational	-4.83	5.49			-13.90		
Corporate-Banking-AAA	-2.01	-4.92			-70.40		1.2
Corporate-Banking-AA	-9.50	2.46			15.90		
Corporate-Banking-A	18.34	-0.62			34.40		
Corporate-Banking-BBB	-2.79	-4.23			-632.70		
Corporate-Finance	1.48	-0.40			13.30		
Corporate-Basic	-5.52	0.69			-56.10		
Corporate-Consumer Cyclical	2.23	-0.26			-7.30		
Corporate-Consumer Non-Cyclical	-1.06	0.13			18.00		
Corporate-Energy & Transport	1.91	-1.07			76.00		
Corporate-Technology & Communications	1.69	0.39			12.80		2.4
Corporate-Utility	-0.22	-0.74			2.50		

All blank numbers are assumed to be zero. Source: Barclays Capital POINT.

Finally, the total outperformance in December 2009 of the four portfolios is 3.0, 2.9, -5.2, and -9.7bp for PCS-1, PCS-2, PCS-3, and PCS-4, respectively. This outperformance is reasonably small in magnitude relative to the TEV forecasts. But this single sample can not be taken as a test for either good tracking performance or to rank the alternative formulations. We would need a back-test over a longer period to be able to compare the quality of these tracking portfolios across the four strategies. Such a back-test can be performed in POINT and is a topic of a forthcoming report.

Conclusion

Soft constraints supplement and complement the generic portfolio constraints functionality available in the Portfolio Optimizer and are very useful in many practical portfolio optimization problems. They circumvent optimization model infeasibility, thus increasing the flexibility and transparency available to portfolio managers.

REFERENCES

Kumar, A. and Lazanas, A. (2009), *Barclays Capital Portfolio Optimizer: User Guide*, Portfolio Modeling, Barclays Capital Research publications.

Kumar, A. (2009), *Portfolio Optimization: Barclays Capital Portfolio Optimizer in POINT*, Portfolio Modeling, Barclays Capital Research publications.

Konno, H. and Yamazaki, H. (1991), "Mean-Absolute-Deviation Portfolio Optimization Model and Its Application to Tokyo Stock Market", *Management Science*, 37, pp 519-531.

Appendix: Modelling soft constraints

A canonical mathematical representation of a portfolio optimization problem is given in (3). The constraint's set is partitioned in to SOFT and HARD constraints.

$$
\begin{aligned}
\underset{x \in \mathrm{X} \subseteq \Re^n}{Minimize} \quad & f(x, \theta \mid \mathrm{M}) \\
\text{Such that} \quad & g_i(x \mid \mathrm{M}) \leq \delta_i \ \text{ for all } \ i \in SOFT \\
& g_i(x \mid \mathrm{M}) \leq \delta_i \ \text{ for all } \ i \in HARD
\end{aligned}
\tag{3}
$$

The notation in (1) is as follows.

x : The decision variables (i.e., the position amount in each security)

θ : The user provided trade-off parameters

M : The estimated market parameters (analytics, risk models, etc.)

$f(x, \theta \mid \mathrm{M})$: The objective function (e.g., risk adjusted returns)

$g_i(x \mid \mathrm{M})$: The constraints functions

δ_i : The constraint bounds

The soft constraints formulations corresponding to (3) is given in (4) below. All the soft constraints are removed from the constraint's set, and a new objective function term is created for each such constraint. The objective function term is equal to the soft constraint's penalty functions. Note that p_i represent the per-unit penalty for the *i*-th soft constraint.

$$
\begin{aligned}
\underset{x \in \mathrm{X} \subseteq \Re^n}{Minimize} \quad & f(x, \theta \mid \mathrm{M}) + \sum_{i \in SOFT} p_i \cdot \left(g_i(x \mid \mathrm{M}) - \delta_i \right)^+ \\
\text{Such that (1)} \quad & g_i(x \mid \mathrm{M}) \leq \delta_i \ \text{ for all } \ i \in HARD
\end{aligned}
\tag{4}
$$

where $z^+ = \max(z, 0)$. Note the nonlinear (or piecewise linear) structure in the definition of the penalty functions. It ensures that the penalty is zero for feasible portfolios.

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