THE REPLICATION OF THE LEHMAN GLOBAL AGGREGATE INDEX WITH CASH INSTRUMENTS

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The Lehman Brothers Global Aggregate Index is increasingly becoming the benchmark of choice for global fixed income portfolios. The comprehensive coverage of global investment-grade bond markets, the broad acceptance of its underlying component indices, and the addition of Asian corporate bonds make it an attractive benchmark for asset-allocation funds and investors interested in global spread products.

Managing assets against the Global Aggregate requires skills and resources. Investors confront a portfolio management problem involving multiple yield curves, exchange rates, and several credit quality and sector components in some of the underlying currencies.

The goal of this study was to reduce the potentially overwhelming task of replicating this index to manageable proportions. We demonstrate that investors can successfully track the Global Aggregate Index with portfolios of relatively few bonds. While derivatives-only replication strategies remain attractive alternatives for many global asset managers,[1] many investors are precluded from holding significant derivatives positions. Besides, asset managers with strong expertise in a particular local market in the Global Aggregate Index may choose to exploit additional opportunities that replication with bonds provides. True, spread risk does not dominate in the Global Aggregate. A large government component and the overall high quality of the index reduce the importance of idiosyncratic risk as well. The main sources of unhedged return volatility in this index are exchange rates and curve movements in the major currencies. Nevertheless, such significant portions of the index as the Credit Index and U.S. mortgage-backed securities (20.0% and 16.5%, respectively, as of September 1, 2001) are likely to be better tracked with bond portfolios.

For the components of the Global Aggregate Index covered by the existing multi-factor risk models[2] we have developed replication strategies based on tracking error minimization. This approach has been successfully applied to replicate credit and mortgage[3] portions of the index. Yet, until there are risk models for *all* parts of the Global Aggregate, sampling remains the most viable approach to constructing proxy portfolios. The replication techniques we propose do not pursue very close tracking. Such a goal would lead to unacceptably high costs and unrealistically assume that the managers have equal expertise in many local markets. Instead, we looked for relatively simple strategies that would make the same compromises that investors are likely to make in real life. For example, we assumed that the holdings will be limited to only the four major currencies: USD, EUR, JPY, and GBP. We also took care to limit the investable set of securities to liquid issues.

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[1] *Replication with Derivatives: The Global Aggregate Index and the Japanese Aggregate Index*, Lehman Brothers, March 2001.

[2] For the U.S. risk model, see *The Lehman Brothers Multi-Factor Risk Model*, Lehman Brothers, July 1999. A multi-factor risk model for euro-denominated assets will be introduced this year.

[3] *Tradable Proxy Portfolios for the Lehman Brothers MBS Index*, Lehman Brothers, July 2001.

We present three strategies that differ in the breadth of the investable universe. The simplest strategy (Treasury only) is limited to treasury securities of the four major index currencies. This reduced set allows us to replicate index allocations only across currencies and term structure. Exposures to spread movements are ignored. The second strategy (Treasury plus) expands the available set to include agency and collateralized (except asset-backed) securities. Finally, the third strategy (All sectors) fully exploits the advantages of bond replication by adding all major credit sectors in the index.

Replication Methodology

All three strategies rely on the stratified sampling technique. In this study, stratified sampling is implemented as a linear optimization process that maximizes portfolio liquidity subject to a number of constraints. We consider liquidity a reasonable objective function, but in the case of constructing proxy portfolios, the constraints are the most important element. Besides the investable set, the choice of optimization constraints is the primary determinant of the strategy performance. This approach relies only on the knowledge of index composition at the time of replication. There are no attempts to reflect prior patterns of the index performance or to introduce any relative value considerations.

For each calendar month, the optimizer selected bonds and their weights in the replicating portfolio, based on the then-current index characteristics. We then measured the portfolio return for the month. The accumulated time series of monthly return differences between our strategies and the index were used to calculate the tracking errors.

As noted above, we limited the replicating portfolio to the four main currencies in the index: USD, EUR, JPY, and GBP. Together, these four currencies account for over 95% of the index market value. All remaining currencies are "wrapped" into one of the four. Canadian, Australian, and New Zealand dollars are consolidated with USD. All European currencies (except GBP) are represented by EUR. Finally, JPY covers all Asian currencies in the index. Aggregating smaller currencies in such a way essentially means assuming their perfect correlation with the large currency that represents them. Another way to distribute the remaining 5 percent was to allocate them proportionately to the four big pieces. The outcomes of the two methods were shown to be practically equivalent.

All strategies match two index cell attributes: market value and contribution to duration. The sampling is done along five dimensions: term structure (26 buckets), quality rating (4 buckets), sector (7 buckets), currency bloc (4 buckets), and country bloc (4 buckets).[4] Constraints are also applied to combinations of these dimensions, such as sector by country and quality rating by country (Figure 1).

The sampling scheme is intended to capture the main sources of return volatility while corresponding to the standard and intuitive index partitioning. Allocation by currency represents the risk of divergence among yield movements in the major

[4] Some countries in the Global Aggregate Index have large credit issues in several currencies. If only currency, but not country, allocations are constrained, a replicating portfolio may end up heavily overweighted in one country.

markets of the index. Sectors and quality rating partitioning describes the sources of spread movements. The term structure matching is done through a simplified key rate duration (KRD) mechanism (when the KRD calculations are implemented for all securities in the index, this scheme will be replaced by a full-fledged KRD profile matching). For each bond, we calculate sensitivities to two adjacent key rates out of the six (1-, 3-, 5-, 7-, 10-, and 30- years) and constrain the resulting portfolio KRD profile to be close to the index. The duration of the bond is linearly interpolated based on the average life. For example, the duration of a bond with exactly 2-year average life is split equally between the 1- and 3-year points. A bond with a 1.5-year average life will have 75% of its duration allocated to the 1-year point and 25% to the 3-year point. If a bond's average life is shorter than 1 year or longer than 30 years, its entire duration is allocated to the 1-year or the 30-year point, respectively. We use this six-point matching for the overall index. For each of the four currency blocs, we use only five points, combining the less-important 7-year point with the 10-year. Thus, overall, we have 26 constraints related to the key rate allocation.

We made the three strategies differ in the level of detail in the stratification scheme to test performance of different portfolio concentrations. The number of bonds in the replicating portfolio is usually close to the number of imposed constraints (lower when constraints overlap).

Some cells represent large portions of the index, while others, such as the asset-backed sector, are relatively small (see the broad market structure of the index in Figure 2). We defined these buckets to match published index breakdowns and to capture special behavior of most market segments. While only one or two bonds end up representing most buckets, the local tracking errors tend to be uncorrelated and, therefore, offset each other. The bias in favor of the most liquid securities in the index, introduced by all strategies, is a systematic risk that does not get diversified away. However, because the mean return of the replicating portfolios was observed to be very close to the index, we can reasonably assume that this liquidity bias is minimal.

Figure 1. **Optimization Constraints**

Term Structure Constraints (Matching Duration Contribution)	
6 KRD Points (the Overall Portfolio)	1, 3, 5, 7, 10, and 30 Years
5 KRD Points (4 Main Currencies)	1, 3, 5, 10, and 30 Years
One-Dimension Cell Constraints (Matching Market Value and Duration Contribution)	
4 Currency Blocs	USD, EUR, JPY, GBP
7 Sectors	Treasury, Agency, Finance, Utility, Industrial, MBS, ABS
4 Country Blocs	US, Europe, Japan, Other
4 Rating Categories	Aaa, Aa, A, Baa
Combination Constraints (Matching Duration Contribution)	
	Examples
Currencies x Curve Points (Max 20)	Duration Contribution to the 5-Year USD
Country Blocs x Sectors (Max 14)	European-Based Finance, Irrespective of Currency
Country Blocs x Rating (Max 9)	Japanese Baa and A (Together), Irrespective of Currency

To force issue-level diversification, we constrained the amount that can be invested in any single security. The magnitude of the single bond constraint is a function of the bond's sector and rating quality. This flexibility ensures higher diversification in less homogeneous segments of the index, where individual issues are more likely to display higher idiosyncratic volatility.

Every month, we started construction of the proxy portfolio "from scratch," discarding the previous month proxy. This may not be a realistic approach for real-life tracking portfolios that have to pay transaction costs. Nevertheless, for the purposes of comparing relative merits of different replication strategies, this approach makes results less dependent on the portfolio inception date or particular study period. In a practical implementation, we would certainly seek actively to limit turnover.

No bond in the index was explicitly excluded from the investable set. Rather, the optimizer's preferences were influenced through a system of incentives, implemented as a liquidity score[5] computed for every bond in the index. This score is an inverse function of the bond's age, and also reflects the bond's relative liquidity within its peer group. For this purpose, we define four peer groups: treasury, agency, collateralized, and credit securities. A strong disincentive is built into the liquidity score for certain

[5] (((1 + Age x Age Multiplier) x AvePeerSize) / Outstanding) + 100 x IneligibleFlag,

where: Age Multiplier is 0 for treasuries, 0.5 for agencies, and 1 for all other bonds.
AvePeerSize = average amount outstanding across the four major sectors (treasuries, agencies and supranational, collateralized, credit).
IneligibleFlag is 1 for illiquid, callable (putable) bonds, very old issues, and currencies other than the main four; 0 for all other bonds.

Figure 2. **The Global Aggregate Index: Currency by Sector Distribution**
September 1, 2001

	USD	EUR	JPY	GBP	Others	Total
Government						
Issues	417	423	338	30	293	1,501
Mkt. Value	2,214,166	2,492,416	2,664,229	306,736	534,546	8,212,093
Percent	15.67%	17.64%	18.85%	2.18%	3.78%	58.12%
Contr. to Dur.	0.89	0.91	1.04	0.16	0.18	3.19
Credit						
Issues	2,401	654	756	235	195	4,241
Mkt. Value	1,661,463	543,154	464,435	176,314	121,858	2,967,224
Percent	11.74%	3.83%	3.29%	1.25%	0.86%	20.97%
Contr. to Dur.	0.62	0.18	0.15	0.09	0.03	1.07
Collateralized						
Issues	720	337	1	11	28	1,097
Mkt. Value	2,508,603	402,597	308	6,314	31,154	2,948,976
Percent	17.75%	2.85%	0.00%	0.05%	0.22%	20.87%
Contr. to Dur..	0.53	0.12	0.00	0.01	0.00	0.67
Total						
Issues	3,538	1,414	1,095	276	516	6,839
Mkt. Value	6,384,232	3,438,167	3,128,972	489,364	687,558	14,128,293
Percent	45.16%	24.32%	22.14%	3.48%	4.86%	100.00%
Contr. to Dur.	2.04	1.21	1.19	0.26	0.22	4.93

groups of bonds considered not investable: old issues (with "old" defined separately for different sectors); callable, putable, and sinking fund securities; bonds denominated in small currency sectors of the index; etc. In fact, at any point in time, no more than a quarter of the index holdings is typically considered investable. Given the sufficient availability of investable bonds across all major index sectors, the tracking portfolio never invested outside of this narrow universe.

In the all-sectors strategy, we introduced one exception to this liquidity score mechanism to reduce turnover: a special treatment of bonds already in the portfolio. For such bonds, the score is set to a very small number irrespective of the bond's outstanding amount or age. If a bond in the portfolio drops out of the investable universe, we set the maximum allowed holding to the current position. As a result, we can continue to hold this bond (or sell it) but cannot buy more of it. A sell is forced only if the bond leaves the index.

Historical Simulation Results

All historical simulations were performed over a 28-month period from February 1999 through May 2001. Structural changes that happened in the European markets when the European Monetary Union went into effect in 1999 make historical analysis prior to that year largely irrelevant. To obtain more pertinent conclusions from simulations results, we defined the index over the entire study period using the current minimum outstanding amount of USD 300 million.

Figure 3 presents the summary performance results for the three replication strategies. The treasury-only strategy resulted, of course, in the smallest portfolios, averaging 22 holdings over the study period. As more sectors are made available for replication, the average portfolio size increases to 28 bonds for the treasury-plus strategy and to 51 for the all-sectors strategy. Predictably, tracking errors decline as the index composition is matched more closely. The currency risk contributes significantly to tracking error, increasing it by 14% for the treasury-only strategy and by 40% for the all-sectors strategy. The long-term success of the replication is illustrated in Figure 4. Even for the simplest strategy variant, treasury-only, cumulative returns of the index and the replicating portfolios stay remarkably close most of the time.

Figure 3. **Performance Summary**, February 1999-May 2001

	Treasury Only	Treasury Plus	All Sectors
Average Number of Bonds in the Replicating Portfolio	22	28	51
USD Tracking Error (bp/month):			
Hedged	11.7	6.3	4.2
Unhedged	13.3	7.7	5.9
USD Average Monthly Return Difference (bp):			
Hedged	-2.7	-0.8	0.0
Unhedged	-2.4	-0.4	0.5

The overall tracking errors observed in all three simulations depend strongly on correlation among tracking errors within the four major currency blocs. To the extent that the correlations are not perfect (and frequently negative), imbalances in different currencies cancel each other. To demonstrate and quantify the effect of low inter-currency correlation on the reduction of the overall tracking error, we looked at the portfolio versus the index return differentials in the four main currency groups: USD, EUR, GBP, and JPY. We observed the correlation among these differentials and, for all replication strategies, calculated the overall tracking errors that would have resulted if the correlation were perfect (Figure 5). The ratio between these hypothetical tracking errors and the real ones measures the effect of diversification. As Figure 5 shows, the portfolio versus the index deviations are indeed uncorrelated among the four currencies. The tracking error reduction ranges from 37% for the treasury-only strategy to 47% for the most diversified all-sectors strategy.

Figure 4. **Treasury-Only Strategy: Cumulative Performance**
February 1999-May 2001

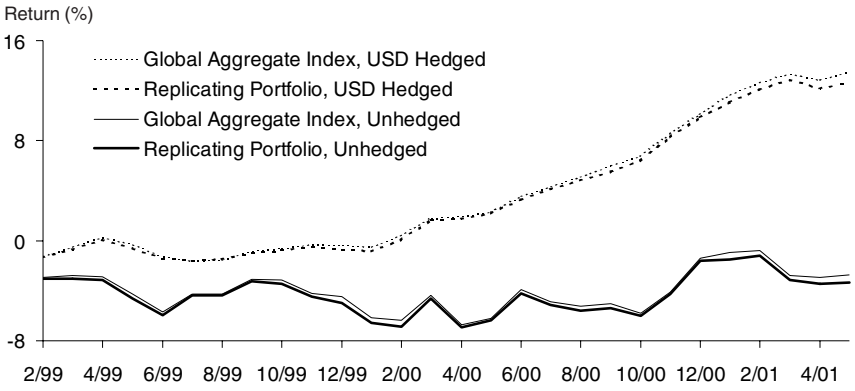

Figure 5. **Effect of Low Tracking Error Correlation Among the Major Currency Blocs,** Local Currency, February 1999-May 2001

Diversification Effect			**Return Differentials Correlation**		
			EUR	**GBP**	**JPY**
Treasury-Only Strategy					
Actual Local-Currency TE (bp/mo)	11.7				
Perfect-Correlation TE (bp/mo)	18.7	**USD**	-0.15	0.19	-0.18
TE Reduction	37%	**EUR**		-0.06	-0.08
		GBP			-0.29
Treasury-Plus Strategy					
Actual Local-Currency TE (bp/mo)	6.3				
Perfect-Correlation TE (bp/mo)	11.0	**USD**	-0.08	0.44	-0.08
TE Reduction	43%	**EUR**		-0.26	-0.10
		GBP			-0.17
All-Sectors Strategy					
Actual Local-Currency TE (bp/mo)	4.3				
Perfect-Correlation TE (bp/mo)	8.0	**USD**	-0.25	0.04	0.21
TE Reduction	47%	**EUR**		0.16	-0.06
		GBP			0.35

Conclusion

The inherent cross-market diversification in the Global Aggregate Index makes it a viable low-volatility benchmark alternative. This index is much less affected by changes in individual markets (e.g. reduction in the share of government debt, leading to substantial duration changes) than single-market indices. The breadth of market coverage makes the risk properties of the Global Aggregate Index more stable.

Currently, about 95% of the Global Aggregate's market capitalization is made up of issues denominated in the four major currencies, USD, EUR, JPY, and GBP. We demonstrate that by replicating these four largest markets, one can achieve a close overall index tracking.

We show that in spite of the great diversity of assets in the Global Aggregate Index—with multiple yield curves, exchange rates, and sectors—one can track this index with portfolios of relatively few bonds. The historical simulations of the cash replication strategies we propose demonstrate that this index can be replicated with reasonable accuracy while limiting investments to the most liquid securities and without making any relative value judgements. The tracking error in the all-sectors strategy is only about 4% of the index return volatility. Figure 6 provides a comparison between the cash strategies (represented by the all-sectors) and replication with futures and swaps (see footnote 1).

Practical implementation of these strategies may raise some additional issues common to all replication strategies, such as trade execution at index prices or controlling for portfolio turnover. The proposed strategies can be easily modified to incorporate these additional features.

Figure 6. **Comparing Replication Strategies: Bonds versus Derivatives**
February 1999-May 2001

	Cash Replication (All-Sectors Strategy)	**Futures and Swaps**
Tracking Error	4 bp/month	10 bp/month
Liquidity	Medium in Corporate Sectors	High
Leverage	No	Yes
Turnover	40%-80% per Year Depending on Fund Size	400%+ per year
Setup Cost	Approx. 9 bp	2.9 bp
Rebalancing Cost	1 bp/month	1.2 bp/month

Appendix. **Global Aggregate Replicating Portfolio, All-Sectors Strategy,** May 2001

Cusip	Issuer	Coupon	Avg. Life	Country	Curr.	Sector	Qual.	Outst. ($ mn)	%
135087WZ	GOVERNMENT OF CANADA	6.38	3.6	Canada	USD	Sovereign	AA2	2,000	0.38
169428AA	CHINA TELECOM HK LTD	7.88	3.5	Hong Kong	USD	Industrial	BAA2	600	0.11
465410AR	REPUBLIC OF ITALY	7.25	3.8	Italy	USD	Sovereign	AA3	2,000	0.38
91086QAD	UNITED MEXICAN STATES	9.88	8.8	Mexico	USD	Sovereign	BAA3	2,000	0.39
SG323061	SINGAPORE POWER	7.25	4.0	Singapore	USD	Other Utilities	AAA	300	0.06
111021AD	BRITISH TELECOMMUNICATIONS	8.13	9.6	U.K.	USD	Industrial	A2	3,000	0.58
060505AG	BANKAMERICA CORPN	7.40	9.7	U.S.A.	USD	Finance	AA3	3,000	0.56
20033TAF	COMED98-1 A-6	5.63	5.2	U.S.A.	USD	ABS	AAA	761	0.14
25746UAB	DOMINION RESOURCES INC.	7.63	4.2	U.S.A.	USD	Other Utilities	BAA1	700	0.13
264399DK	DUKE ENERGY CORP	6.00	27.6	U.S.A.	USD	Other Utilities	A1	300	0.05
3133MBER	FEDERAL HOME LN BKS	6.75	1.0	U.S.A.	USD	US Agencies	AAA+	6,500	1.19
3134A32S	FEDERAL HOME LN MTG CORP	6.88	3.7	U.S.A.	USD	US Agencies	AAA+	8,000	1.53
3134A35H	FEDERAL HOME LN MTG CORP	6.88	9.4	U.S.A.	USD	US Agencies	AAA+	10,000	1.90
FNA07400	FNMA	7.50	6.5	U.S.A.	USD	MBS	AAA+	55,765	10.20
FNA06098	FNMA	6.00	8.1	U.S.A.	USD	MBS	AAA+	57,289	9.94
FNA08000	FNMA	8.00	5.3	U.S.A.	USD	MBS	AAA+	28,129	5.21
345397TR	FORD MOTOR CREDIT COMPANY	6.88	4.8	U.S.A.	USD	Finance	A2	6,500	1.20
345397SJ	FORD MOTOR CREDIT COMPANY	6.70	3.2	U.S.A.	USD	Finance	A2	4,000	0.74
GNA07098	GNMA	7.00	8.2	U.S.A.	USD	MBS	AAA+	25,646	4.65
617446GL	MORGAN STANLEY DEAN WITTER	6.10	5.0	U.S.A.	USD	Finance	AA3	3,500	0.62
852060AD	SPRINT CAPITAL CORP	6.88	27.5	U.S.A.	USD	Industrial	BAA1	2,500	0.39
852060AE	SPRINT CAPITAL CORP	5.88	3.0	U.S.A.	USD	Industrial	BAA1	1,000	0.17
912810EL	UNITED STATES TREASURY	8.00	20.5	U.S.A.	USD	US Treasury	AAA+	28,084	6.42
912827G5	UNITED STATES TREASURY	6.38	1.3	U.S.A.	USD	US Treasury	AAA+	19,983	3.71
92344GAB	VERIZON GLOBAL FDG CORP	7.25	9.6	U.S.A.	USD	Industrial	A1	2,000	0.38
931142BE	WAL-MART STORES	6.88	8.3	U.S.A.	USD	Industrial	AA2	3,500	0.66
JPY02215	DEUTSCHE AUSGLEICHSBANK	1.85	9.4	Germany	JPY	For. Agencies	AAA	809	0.15
JPY01724	DAIWA SECURITIES CO LTD	1.40	4.1	Japan	JPY	Euro Finance	BAA2	809	0.15
JP740067	GOVERNMENT OF JAPAN	4.60	3.4	Japan	JPY	Euro Treasury	AA2	24,285	4.98
JP141W77	GOVERNMENT OF JAPAN	1.80	8.4	Japan	JPY	Euro Treasury	AA1	26,956	5.08
JP891S93	GOVERNMENT OF JAPAN	3.10	5.4	Japan	JPY	Euro Treasury	AA1	16,914	3.42
JPY01541	JAPAN FINANCE CORP	1.80	9.0	Japan	JPY	Euro Agencies	AA2	2,428	0.45
JPY02203	SONY CORP	2.04	9.4	Japan	JPY	Euro Industrial	A1	405	0.08
JPY02174	THE INDUSTRIAL BANK OF JAPAN	1.25	4.2	Japan	JPY	Euro Finance	A3	1,155	0.21
JPY02207	CITIGROUP INC	0.80	1.4	U.S.A.	JPY	Euro Finance	AA3	607	0.11
GB998302	UNITED KINGDOM TREASURY	7.50	5.6	U.K.	GBP	Euro Treasury	AAA	16,740	3.40
GB404191	UNITED KINGDOM TREASURY	6.00	27.6	U.K.	GBP	Euro Treasury	AAA	16,471	3.48
FR015959	FRENCH REPUBLIC	4.50	1.2	France	EUR	Euro Treasury	AAA	19,236	3.54
FR802273	FRENCH REPUBLIC	3.50	3.2	France	EUR	Euro Treasury	AAA	15,937	2.82
FR208654	FRENCH REPUBLIC	5.25	7.0	France	EUR	Euro Treasury	AAA	21,795	3.95
EUR00053	DEUTSCHE FINANCE NETHERLANDS	4.25	8.2	Germany	EUR	Euro Finance	AA3	1,995	0.34
DE135069	FEDERAL REPUBLIC OF GERMANY	5.63	26.7	Germany	EUR	Euro Treasury	AAA	12,693	2.30
EUR03352	FEDERAL REPUBLIC OF GERMANY	5.25	9.7	Germany	EUR	Euro Treasury	AAA	20,392	3.79
GRD00032	HELLENIC REPUBLIC GOVT BOND	8.60	6.9	Greece	EUR	Euro Treasury	A2	3,348	0.71
XEU00226	ENTE NAZIONALE PER L ENERGIA	4.50	7.4	Italy	EUR	Euro Utility	AA3	887	0.15
IT224309	REPUBLIC OF ITALY	5.00	7.0	Italy	EUR	Euro Treasury	AA3	20,251	3.58
IT278511	REPUBLIC OF ITALY	5.25	28.5	Italy	EUR	Euro Treasury	AA3	20,611	3.33
EUR01415	TECNOST INTERNATIONAL NV	5.83	3.2	Italy	EUR	Euro Industrial	BAA2	3,990	0.74
EUR00828	TOKYO ELECTRIC POWER CO INC	4.38	8.0	Japan	EUR	Euro Utility	AA2	887	0.15
EUR04152	SOGERIM SA	7.00	10.0	Luxembourg	EUR	Euro Industrial	BAA1	1,773	0.32
DEM03104	DSM NV	4.75	4.6	Netherlands	EUR	Euro Industrial	A2	340	0.06
XEU00253	EUROPEAN INVESTMENT BANK	4.00	8.0	Supranat.	EUR	Supranational	AAA	4,024	0.66
EC537797	KINGDOM OF SWEDEN	3.50	5.0	Sweden	EUR	Sovereign	AA1	2,217	0.37

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