QUANTITATIVE PORTFOLIO MANAGEMENT CONFERENCE

Linear Factor Models: Structure, Estimation, and Factor Selection

Cenk Ural
POINT Portfolio Modeling
Index, Portfolio and Risk Solutions Group

April 8, 2010

PLEASE SEE ANALYST CERTIFICATIONS AND IMPORTANT DISCLOSURES STARTING AFTER SLIDE 25.

BARCLAYS CAPITAL

Agenda

- Linear factor models
  - Motivation
  - Estimation
  - Factors and loadings
  - Equity vs. fixed income models
- Factor / model selection
  - Quantitative measures
    - In-sample analysis
    - Out-of-sample analysis
  - Qualitative measures

QUANTITATIVE PORTFOLIO
MANAGEMENT CONFERENCE

Linear Factor Models

BARCLAYS CAPITAL

Linear Factor Models – Motivation

- Portfolio managers want to
  - Estimate the total risk of their portfolios
  - Analyze portfolio risk in various dimensions: Sources of risk, contributions
- A simple idea: Estimate the risk of individual securities and then combine them
  - Individual securities are too noisy
  - Portfolios are generally composed of a large number of securities
- Factor model: Separate the return into systematic and idiosyncratic components
  - Idiosyncratic component – the noise – diversifies away in portfolios
  - Reduce dimensionality, a more robust structure to estimate future properties of the portfolio return distribution
- Two components in the estimation of portfolio risk (volatility vs. tail risk)
  - **Construction of the factor model** $r = L * F + \varepsilon$
  - Estimation of the factor correlations and volatilities (covariance matrix) $\sigma_r^2 = L^T * \sum * L + \Omega$

BARCLAYS CAPITAL

Linear Factor Models – Introduction

- Decompose security returns into a systematic and an idiosyncratic component
  - Systematic return: Generated by movements in common risk factors
    - Factors (*F*): Drivers of return
    - Loadings (*L*): Sensitivities to risk factors
  - Idiosyncratic return: Security-specific component unexplained by systematic factors
- An example: GM equity and bond systematic and idiosyncratic returns from June 2006 to June 2009
  - Systematic component is a bigger part of total return for the bond
  - Equity return is more volatile
  - Similar behavior in both issues moving toward bankruptcy

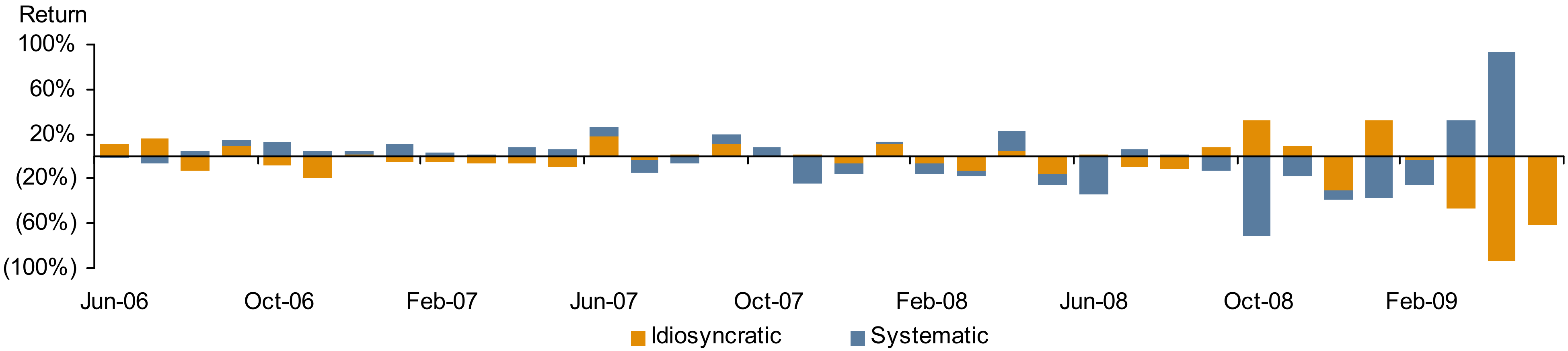
	Systematic Ret / Idiosyncratic Ret		
	Full History	**Before Sep-08**	**Monthly Volatility**
Equity	0.96	1.00	18.4%
Bond	1.24	1.74	9.1%

Note: The source for all figures in this presentation is Barclays Capital.

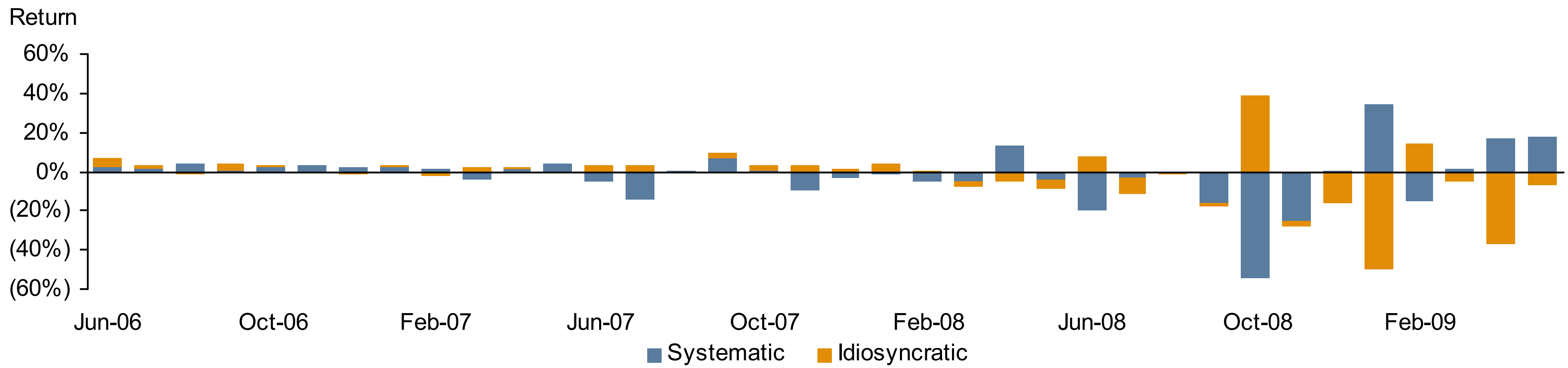
BARCLAYS CAPITAL

Linear Factor Models – Introduction

Equity



Bond



Linear Factor Models – Estimation

Choice of factors determines the characteristics of loadings and vice versa

$$r = L * F + \varepsilon$$

Model	Known	Estimated
Cross-sectional	r, L	F
Time series	r, F	L
Statistical	r	L, F

Heteroskedasticity in residual volatilities: Weighted regressions

	Cross Sectional	Time Series
Input set	Security-specific loadings – rich	Factor history – easier to access
# of parameters	(no. of factors) * (no. of months)	(no. of securities) * (no. of factors)
Interpretation	Cleaner interpretation of loadings and univariate factors	Straightforward interpretation of factors
Types of factors	Fundamental/Technical – Estimated, Classification – Estimated	Macroeconomic, Classification – Observed Fundamental/Technical – Constructed
Advantages	More dynamic in nature: loadings can adjust very fast	Direct implication with respect to time series characteristics of factors and residuals

Linear Factor Models – Factors

- Factors can be
  - Observed: Change in interest rates, equity market return – S&P500
  - Estimated: Through cross-sectional regressions $\hat{F} = (L'L)^{-1} L'r$
  - Constructed: Factor mimicking portfolios (Fama and French) $\sum_{\substack{i \in Small \\ Cap}} w_i r_i - \sum_{\substack{i \in Large \\ Cap}} w_i r_i$
- Underlying variables
  - Macroeconomic: Macro variables that represent the state of the economy
    - Change in interest rates, industrial production
  - Fundamental and technical: Investment themes for portfolio managers
    - Size, value, momentum
  - Classification: Widely used in various models
    - Industry, country, rating
  - Statistical: No direct economic interpretation; may be remapped to known factors
    - First three PCs for rates: Level, slope, curvature. First PC in equities: Market

Linear Factor Models – Loadings

- Loadings can be
  - Analytical: Coming out of a pricing model (bond durations, option greeks)
    - Used primarily in “structured” assets, e.g., IG credit bonds
      - The market translates the risk-free rate and the quoted spread into the bond price using a pricing model; analytics can be generated

$$OAD = -100 * \frac{1}{P} * \frac{\partial P}{\partial r} \approx \frac{P_{-25bps} - P_{+25bps}}{P_0^{Full} * 2 * .25}$$

  - Empirical: Estimated using statistical techniques (equity market beta)
    - Used when market dynamics (which are hard to model) are more important than fundamental relationships
    - Asset and factor returns must be sufficiently “invariant”

$$\hat{L} = F^T r \big/ F^T F$$

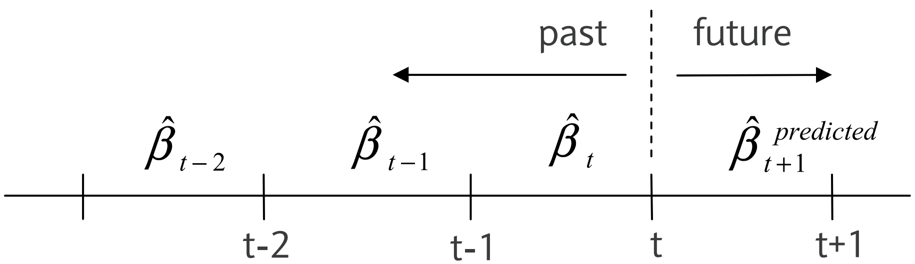
  - Observed: Security characteristics used in cross-sectional models (E/P, size)
  - Dummy: Used with classification factors (industry, country dummies)

Linear Factor Models – Loadings

- Standardized loadings
  - Implies a relative view, de-trends the loading (e.g., size factor)
  - Helps produce a more invariant factor

$$L^{STD} = \frac{L-\mu}{\sigma}$$

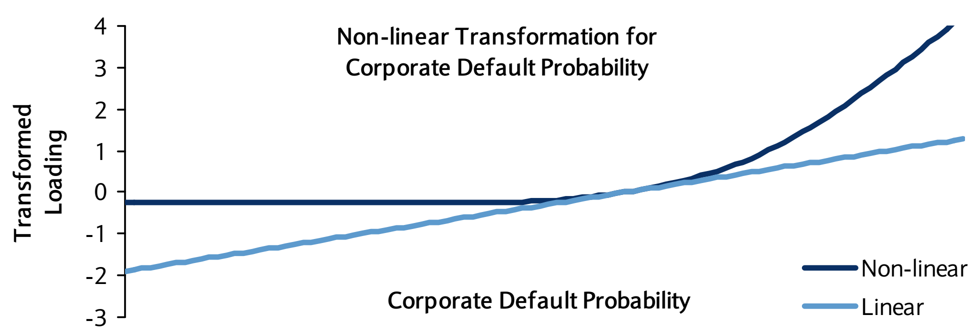
- Use of mixed frequency data to estimate the empirical loadings
  - High frequency data to produce dynamic sensitivities: More robust, yet responsive



$$\hat{\beta}_{t+1}^{predicted} = \sum_{l=1}^{L} w_{t-l}\hat{\beta}_{t-l,t-l+1}$$

- Linear factor models do not restrict non-linear relationships
  - Non-linear transformations for factor loadings

$$r = g(L) * F + \varepsilon$$



BARCLAYS CAPITAL

Linear Factor Models – Fixed Income vs. Equity

- Fixed income
  - Generally structured assets; pricing model determines the factor structure
    - Variables in the pricing model: Interest rates, credit spread, implied volatility
  - Analytical loadings are widely used: e.g., Durations

Credit Bond

$$r_{BOND} = \underbrace{r_{carry}}_{\text{Carry}} + \underbrace{\sum_{i=1}^{N} KRD_i * (-\Delta KR_i)}_{\text{Interest rate}} + \underbrace{Vega * F^{\sigma}}_{\text{Volatility}} + \underbrace{OASD * F^{OASD}}_{\text{Credit Spread}} + \varepsilon$$

- Equity
  - Pricing models are not widely used in practice: High uncertainty about dividends, companies with no dividends
  - Empirical sensitivities are more common
  - Relatively smaller explanatory power

Common Stock

$$r_{EQ} = \underbrace{\beta * F^{IND}}_{\text{Industry}} + \underbrace{\sum_{i=1}^{N} \gamma_i * F_i^{FUND}}_{\text{Fund \& Tech}} + \varepsilon$$

QUANTITATIVE PORTFOLIO
MANAGEMENT CONFERENCE

Factor / Model Selection

BARCLAYS
CAPITAL

Factor / Model Selection – Motivation

- Goal: From a large set of n potential factors, select a subset of k factors in an optimal way to
  - Produce accurate estimates of portfolio risk
    - Across different market conditions and for a diverse set of portfolios
  - Facilitate intuitive analysis of total risk: Decomposition, exposures
- Choice of a factor cannot be disassociated from the characteristics of its loading
- How to achieve the goal?
  - Quantitative analysis
    - In-sample testing
    - Out-of-sample testing
  - Qualitative analysis

Factor / Model Selection – In-Sample Analysis

- In-sample analysis
  - Factors that have high explanatory power $R^2 = 1 - \sum \varepsilon_i^2 \Big/ \sum r_i^2$
    - Issues with overfitting
  - Statistically significant: t-test, p-values, F-test $t_{score} = \hat{L} \Big/ SE_{\hat{L}}$
  - Factors should be systematic in nature, well populated
  - Residuals should be uncorrelated with each other and with systematic factors

$$\rho(\varepsilon_i, \varepsilon_j) = 0 \quad \forall i, j \qquad \rho(\varepsilon_i, F_k) = 0 \quad \forall i, k$$

- Statistical factor selection procedures
  - Discrete search
  - Continuous search

Factor / Model Selection – In-Sample Analysis

- Discrete search: Subset selection
  - + Directly attacks the problem, produces more interpretable results
  - - Involves heavy combinatorial search; results are very sensitive to inputs
- Continuous search: OLS with constraints (e.g., ridge regression)

$$\hat{F} = \arg\min_{F} \sum_{i} \left(r_i - \sum_{j} L_{ij} * F_j \right)^2 \qquad s.t. \quad \sum_{j} F_j^2 \leq \lambda$$

  - + Computationally feasible, produces more stable results
  - - Only shrinks the coefficients to zero, requires further analysis for selection
- Lasso estimates: A hybrid methodology

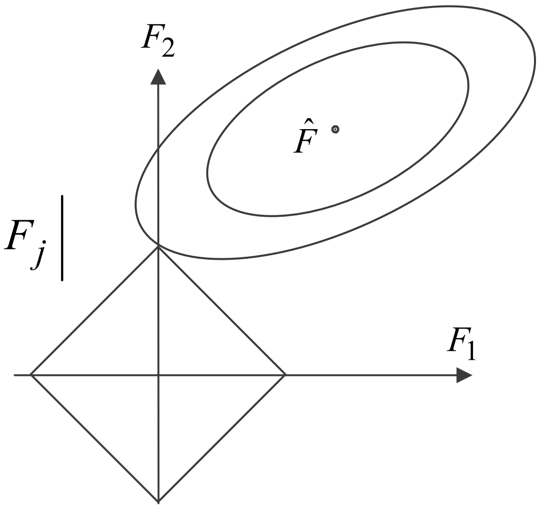
$$\hat{F} = \arg\min_{F} \sum_{i} \left(r_i - \sum_{j} L_{ij} * F_j \right)^2 \qquad s.t. \quad \sum_{j} \left| F_j \right| \leq \lambda$$

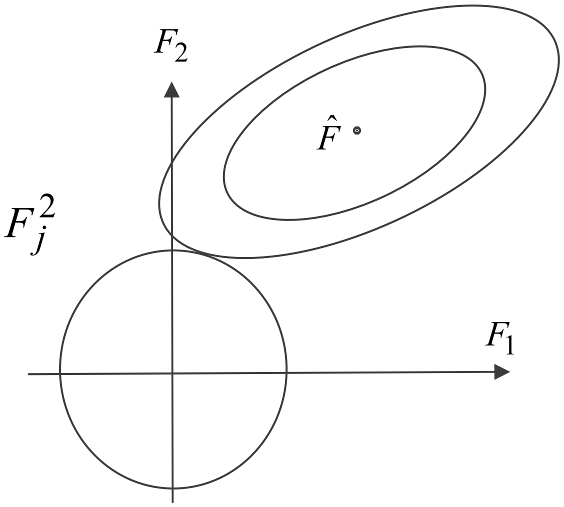
  - Sets some coefficients to zero (controlled by λ) and shrinks the others
  - Enjoys the benefits of both discrete and continuous search

Factor / Model Selection – In-Sample Analysis

- Comparing lasso and ridge regression in the case of two factors

Lasso	**Ridge Regression**
$g(F) = \sum_j \lvert F_j \rvert$	$g(F) = \sum_j F_j^2$





- Lasso selects corner solutions

$$F_1 = 0 \quad F_2 = \lambda$$

Factor / Model Selection – In-Sample Analysis

- Stepwise variable selection is a popular heuristic for subset selection
  - At each step of the procedure
    - Find the factor with the largest additional explanatory power
    - Check if the factor is significant in the model
    - Drop any factor if it becomes insignificant
  - Stop when no new factor can be added with the desired significance
  - US equities, % of months each fundamental / technical factor is selected

All Data		High VIX		Low VIX		First Half		Second Half	
SIZE	69%	SIZE	67%	SIZE	54%	SIZE	62%	SIZE	58%
MOMENTUM	54%	MOMENTUM	55%	MOMENTUM	49%	MOMENTUM	48%	MOMENTUM	54%
TURNOVER	53%	REALIZED VOL	52%	TURNOVER	39%	REALIZED VOL	44%	DEFAULT PROB	47%
REALIZED VOL	45%	DEFAULT PROB	50%	REALIZED VOL	36%	TURNOVER	41%	REALIZED VOL	42%
DEFAULT PROB	29%	TURNOVER	43%	DEFAULT PROB	31%	DEFAULT PROB	31%	TURNOVER	38%
TOTAL YIELD	27%	B/P	30%	TOTAL YIELD	23%	B/P	28%	FORWARD E/P	30%
FORWARD E/P	27%	FORWARD E/P	29%	B/P	20%	TOTAL YIELD	23%	E/P	24%
E/P	25%	E/P	27%	E/P	19%	E/P	21%	TOTAL YIELD	24%
B/P	23%	TOTAL YIELD	26%	FORWARD E/P	15%	ACCRUALS	10%	B/P	22%
ACCRUALS	20%	ACCRUALS	17%	ACCRUALS	12%	FORWARD E/P	8%	ACCRUALS	17%

- Consistent performance of the top five factors across different sub-periods

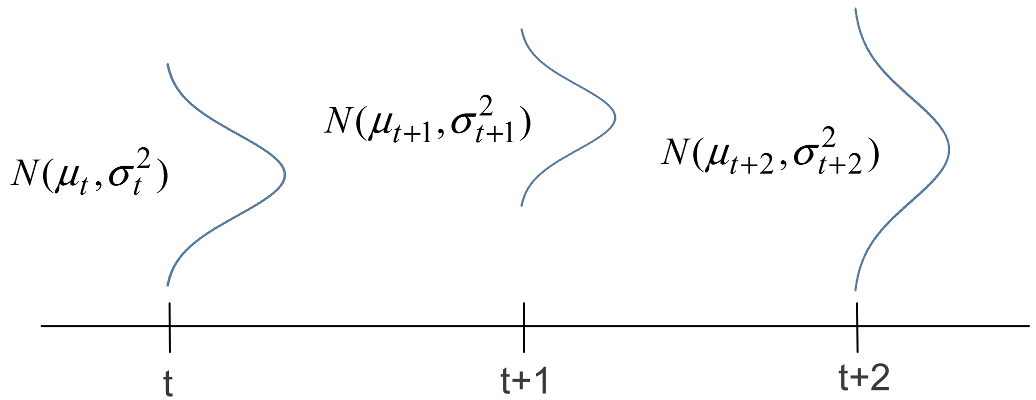
Factor / Model Selection – In-Sample Analysis

- Prevent excess correlations: Multicollinearity
  - Why is it a problem?
    - Instable coefficient estimates, wide confidence intervals
  - How to eliminate?
    - Remove some factors or combine factors into risk indices
    - Residualize one factor to the other
- Multicollinearity also brings interpretation issues
  - Estimating factors in a cross-sectional regression
    $r = L * F + \varepsilon$ where $L = [L_1 \; L_2 \; . \; . \; L_n]$
  - Univariate regression: The case of one factor $\hat{F}_1 = \frac{L_1^T r}{L_1^T L_1}$
  - Multivariate regression
    $\hat{F} = (L^T L)^{-1} L^T r$
    - If loadings (regressors) are uncorrelated, then $L^T L$ is diagonal and the estimated factors have the following form $\hat{F}_i = \frac{L_i^T r}{L_i^T L_i}$
    - Multivariate factors can keep a univariate interpretation

BARCLAYS CAPITAL

Factor / Model Selection – Out-of-Sample Analysis

- Out-of-sample analysis: Parsimony is the key
  - Testing the accuracy of portfolio volatility estimates in an out-of-sample setting
  - Need factors that facilitate the estimation of the properties of future multivariate distribution of factors
    - Invariance: Relatively stable distribution of factor returns across time; thus, history can be used to predict future distribution



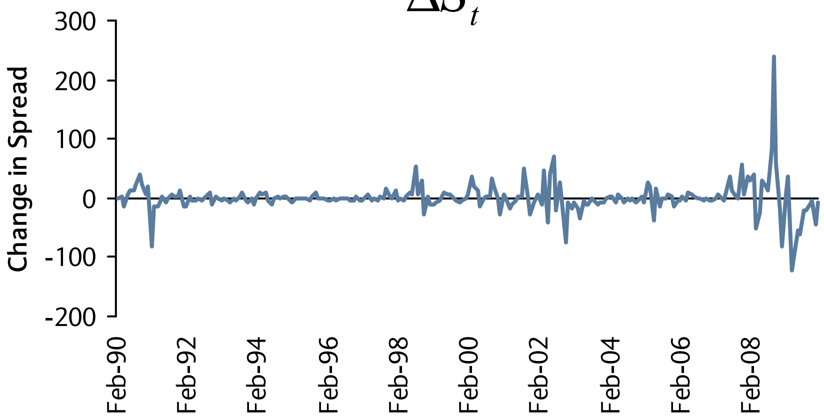
    - Stability of interactions between factors: Correlations
  - Need loadings that are forward looking and responsive
    - Also need some stability to avoid excessive rebalancing

BARCLAYS CAPITAL

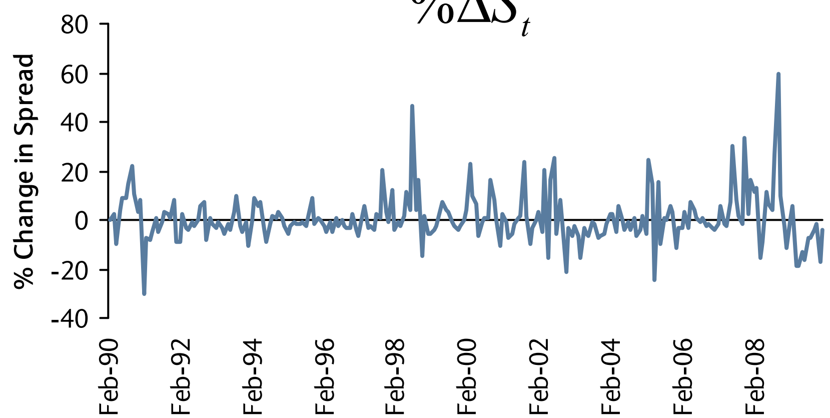
Factor / Model Selection – Out-of-Sample Analysis

- Suppose we identify change in credit spreads as a source of returns for corporate bonds. How can we operationalize this source? $R = f(\Delta S)$
- Monthly changes in spread for the Barclays Capital USD Credit BAA Index

ΔS_t



$\%\Delta S_t$



Statistics	ΔS_t	$\%\Delta S_t$
Skewness	2.34	1.63
Kurtosis	28.64	7.44
5% Percentile	(1.10)	(1.25)
95% Percentile	1.33	1.66

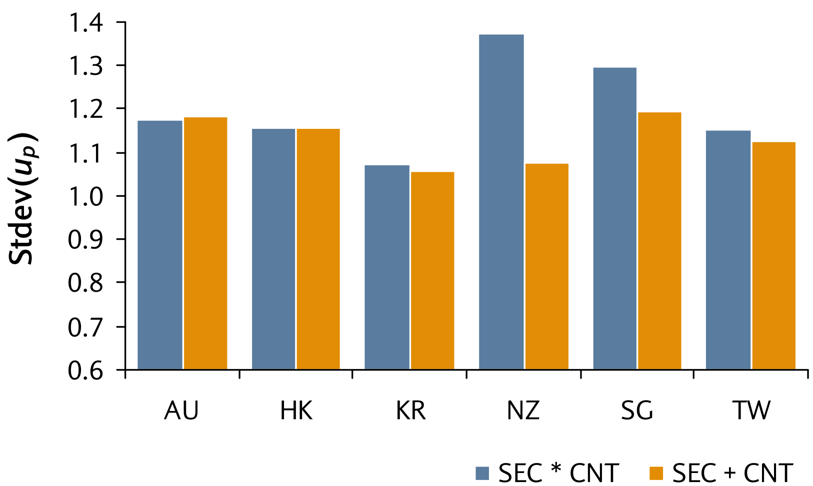
The evidence seems to suggest that $\%\Delta S_t$ is somewhat better behaved

Factor	***Loading***	
ΔS_t	$OASD$	***More Stable Loading***
$\%\Delta S_t$	$OASD * S$	***Conditional Loading***

BARCLAYS CAPITAL

Factor / Model Selection – Out-of-Sample Analysis

- Ratio test: The goal is to gauge how well the model predicts the volatility of portfolios
  - Standardize realized return with its estimated volatility: u_p^t
  - If the forecast is accurate, standard deviation of u_p should be close to 1
    - Distribution of u_p is important; if far from normal, then need a large sample
  - Two competing models for equity country portfolios in ex-JP Asia
    - Sector for each country (10*6 factors)
      - *Better in-sample fit, worse performance out-of-sample*
    - Regional sector and country factors (10+6 factors)
      - *Worse in-sample, better out-of-sample*



$$u_p^t = \frac{r_p^t}{\sigma_{p,forecast}^t}$$

BARCLAYS CAPITAL

Factor / Model Selection – Qualitative Measures

- Intuitive factors and loadings that have clean interpretation
  - Economic sense, easy to understand and communicate
    - Statistical factors are harder to interpret
- Ability to capture major investment themes in the specific asset class
  - Interest rates, industry, fundamentals
- Target investor base
  - Short- vs. long-horizon investors
    - Market dynamics are more important for short horizon
    - Macroeconomic variables are more relevant for long horizons
  - Local investors vs. global investors
    - Local models: Good fit for local portfolios, need consistency for effective combination
    - Global models: More parsimonious, more stable estimates, may underestimate diversification benefits

References

- A. Asvanunt and A. Staal (2009), *The Corporate Default Probability model in the Barclays Capital POINT platform (POINT CDP)*, Barclays Capital, April 2009
- A.B. Silva (2009), *A Note on the New Approach to Credit in the Barclays Capital Global Risk Model*, Barclays Capital, September 2009
- A.B. Silva, A.D. Staal, and C. Ural (2009), *The US Equity Risk Model*, Barclays Capital, March 2009
- R. Tibshirani (1996), *Regression Shrinkage and Selection via the Lasso*, Journal of the Royal Statistical Society B., Vol. 58, No. 1, pp. 267-288

Appendix

Linear Factor Models – Estimation

- Single-step
  - Allows for a natural decomposition of total variance
  - Loss of interpretation in jointly estimated factors / loadings
  - Multicollinearity issues elevated
- Multi-step
  - Need to take a stand on the order of factors: Important factors come first
    - Order of factors makes a significant difference in their interpretation
      - First set has clean interpretation (can be univariate)
      - Others are excess / residual factors
  - Provides the setup to combine different types of factors
    - E.g., observed and estimated factors in the credit model

$$r_{BOND} = r_{carry} + \sum_{i=1}^{N} KRD_i * (-\Delta KR_i) + Vega * F^{\sigma} + OASD * F^{OASD} + \varepsilon$$

Analyst Certifications and Important Disclosures

Analyst Certification(s)

I, Cenk Ural, hereby certify (1) that the views expressed in this research report accurately reflect my personal views about any or all of the subject securities or issuers referred to in this research report and (2) no part of my compensation was, is or will be directly or indirectly related to the specific recommendations or views expressed in this research report.

Important Disclosures

For current important disclosures regarding companies that are the subject of this research report, please send a written request to: Barclays Capital Research Compliance, 745 Seventh Avenue, 17th Floor, New York, NY 10019 or refer to https://ecommerce.barcap.com/research/cgi-bin/all/disclosuresSearch.pl or call 212-526-1072.

Barclays Capital does and seeks to do business with companies covered in its research reports. As a result, investors should be aware that Barclays Capital may have a conflict of interest that could affect the objectivity of this report. Any reference to Barclays Capital includes its affiliates. Barclays Capital and/or an affiliate thereof (the "firm") regularly trades, generally deals as principal and generally provides liquidity (as market maker or otherwise) in the debt securities that are the subject of this research report (and related derivatives thereof). The firm's proprietary trading accounts may have either a long and/or short position in such securities and/or derivative instruments, which may pose a conflict with the interests of investing customers. Where permitted and subject to appropriate information barrier restrictions, the firm's fixed income research analysts regularly interact with its trading desk personnel to determine current prices of fixed income securities. The firm's fixed income research analyst(s) receive compensation based on various factors including, but not limited to, the quality of their work, the overall performance of the firm (including the profitability of the investment banking department), the profitability and revenues of the Fixed Income Division and the outstanding principal amount and trading value of, the profitability of, and the potential interest of the firm’s investing clients in research with respect to, the asset class covered by the analyst. To the extent that any historical pricing information was obtained from Barclays Capital trading desks, the firm makes no representation that it is accurate or complete. All levels, prices and spreads are historical and do not represent current market levels, prices or spreads, some or all of which may have changed since the publication of this document. Barclays Capital produces a variety of research products including, but not limited to, fundamental analysis, equity-linked analysis, quantitative analysis, and trade ideas. Recommendations contained in one type of research product may differ from recommendations contained in other types of research products, whether as a result of differing time horizons, methodologies, or otherwise.

Important Disclosures (cont'd)

This publication has been prepared by Barclays Capital, the investment banking division of Barclays Bank PLC, and/or one or more of its affiliates as provided below. This publication is provided to you for information purposes only. Prices shown in this publication are indicative and Barclays Capital is not offering to buy or sell or soliciting offers to buy or sell any financial instrument. Other than disclosures relating to Barclays Capital, the information contained in this publication has been obtained from sources that Barclays Capital believes to be reliable, but Barclays Capital does not represent or warrant that it is accurate or complete. The views in this publication are those of Barclays Capital and are subject to change, and Barclays Capital has no obligation to update its opinions or the information in this publication. Barclays Capital and its affiliates and their respective officers, directors, partners and employees, including persons involved in the preparation or issuance of this document, may from time to time act as manager, co-manager or underwriter of a public offering or otherwise, in the capacity of principal or agent, deal in, hold or act as market-makers or advisors, brokers or commercial and/or investment bankers in relation to the securities or related derivatives which are the subject of this publication.

The analyst recommendations in this report reflect solely and exclusively those of the author(s), and such opinions were prepared independently of any other interests, including those of Barclays Capital and/or its affiliates.

Neither Barclays Capital, nor any affiliate, nor any of their respective officers, directors, partners, or employees accepts any liability whatsoever for any direct or consequential loss arising from any use of this publication or its contents. The securities discussed in this publication may not be suitable for all investors. Barclays Capital recommends that investors independently evaluate each issuer, security or instrument discussed in this publication and consult any independent advisors they believe necessary. The value of and income from any investment may fluctuate from day to day as a result of changes in relevant economic markets (including changes in market liquidity). The information in this publication is not intended to predict actual results, which may differ substantially from those reflected. Past performance is not necessarily indicative of future results.

This communication is being made available in the UK and Europe to persons who are investment professionals as that term is defined in Article 19 of the Financial Services and Markets Act 2000 (Financial Promotion Order) 2005. It is directed at, and therefore should only be relied upon by, persons who have professional experience in matters relating to investments. The investments to which it relates are available only to such persons and will be entered into only with such persons. Barclays Capital is authorized and regulated by the Financial Services Authority ('FSA') and member of the London Stock Exchange.

Barclays Capital Inc., US registered broker/dealer and member of FINRA (www.finra.org), is distributing this material in the United States and, in connection therewith accepts responsibility for its contents. Any U.S. person wishing to effect a transaction in any security discussed herein should do so only by contacting a representative of Barclays Capital Inc. in the U.S. at 745 Seventh Avenue, New York, New York 10019.

Subject to the conditions of this publication as set out above, Absa Capital, the Investment Banking Division of Absa Bank Limited, an authorised financial services provider (Registration No.: 1986/004794/06), is distributing this material in South Africa. Absa Bank Limited is regulated by the South African Reserve Bank. This publication is not, nor is it intended to be, advice as defined and/or contemplated in the (South African) Financial Advisory and Intermediary Services Act, 37 of 2002, or any other financial, investment, trading, tax, legal, accounting, retirement, actuarial or other professional advice or service whatsoever. Any South African person or entity wishing to effect a transaction in any security discussed herein should do so only by contacting a representative of Absa Capital in South Africa, 15 Alice Lane, Sandton, Johannesburg, Gauteng 2196. Absa Capital is an affiliate of Barclays Capital.

Non-U.S. persons should contact and execute transactions through a Barclays Bank PLC branch or affiliate in their home jurisdiction unless local regulations permit otherwise.

In Japan, foreign exchange research reports are prepared and distributed by Barclays Bank PLC Tokyo Branch. Other research reports are distributed to institutional investors in Japan by Barclays Capital Japan Limited. Barclays Capital Japan Limited is a joint-stock company incorporated in Japan with registered office of 6-10-1 Roppongi, Minato-ku, Tokyo 106-6131, Japan. It is a subsidiary of Barclays Bank PLC and a registered financial instruments firm regulated by the Financial Services Agency of Japan. Registered Number: Kanto Zaimukyokucho (kinsho) No. 143.

Barclays Bank PLC Frankfurt Branch is distributing this material in Germany under the supervision of Bundesanstalt fuer Finanzdienstleistungsaufsicht (BaFin). This material is distributed in Malaysia by Barclays Capital Markets Malaysia Sdn Bhd.

IRS Circular 230 Prepared Materials Disclaimer: Barclays Capital and its affiliates do not provide tax advice and nothing contained herein should be construed to be tax advice. Please be advised that any discussion of U.S. tax matters contained herein (including any attachments) (i) is not intended or written to be used, and cannot be used, by you for the purpose of avoiding U.S. tax-related penalties; and (ii) was written to support the promotion or marketing of the transactions or other matters addressed herein. Accordingly, you should seek advice based on your particular circumstances from an independent tax advisor.

BARCLAYS CAPITAL

QUANTITATIVE PORTFOLIO MANAGEMENT CONFERENCE

BARCLAYS CAPITAL