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Assessing Interest Rate Risk Beyond Duration — Shift, Twist, Butterfly

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Summary

High fiscal deficits, a sharp rise in the issuance of sovereign debt from major developed economies, rising inflation expectations, and possible changes in central bank rates could cause the yield curves from around the world to change significantly. In the US, the yield curve has already experienced significant steepening in the past twelve months. This environment poses challenges for Fixed Income professionals who need to address possible non-parallel changes to the term structure.

This paper illustrates the capabilities of Shift-Twist-Butterfly (STB) factor models to help address these challenges. We provide a longer-term perspective on term structure changes in the Euro zone, US and Japan. Using four portfolio case studies, we show that the use of risk measures such as duration, convexity or key rate durations have some limitations and may not be very efficient. Therefore we show how these limitations can be efficiently overcome by the complementary use of advanced fixed income risk models based on STB interest rate risk factors.

The article is organized in six sections:

1. Underscores the arrival of a new interest rate risk environment
2. Reviews how term structure risk can be modeled and explained with a Shift-Twist-Butterfly model
3. Provides a long-term historical perspective of interest rates
4. Reviews the EUR, USD and JPY interest rates since January 1999 to illustrate the insightfulness of using STB factors
5. Discusses portfolio case studies to show that STB interest rate risk factors are relevant and complementary to risk measures like duration/convexity and key rate durations (KRDs)
6. Summarizes our findings

1. Arrival of a New Interest Rate Risk Environment?

Recently, the US term structure has experienced sharp steepening with the 2-10-year yield spread widening to a multi-decade high level of 291 bp in February 2010. This steepening has contributed negatively to returns in sovereign bond portfolios in a major way. As an example, we created a sample US sovereign bond portfolio by equally weighting the bonds from the Citi US Government Bond Index, which returned -0.60% during the 11 months from March 31, 2009 to February 28, 2010. Readers may recall that March 2009 was the month which saw the bottom of the 2008/2009 equity market crisis. We briefly show how this performance can be attributed to Shift-Twist-Butterfly factors. The formula for the STB performance attribution is as follows:

> *Explained portfolio return = risk_free_return + (shift_factor_exposure * shift_factor_return) + (twist_factor_exposure * twist_factor_return) + (butterfly_factor_exposure * butterfly_factor_return)*

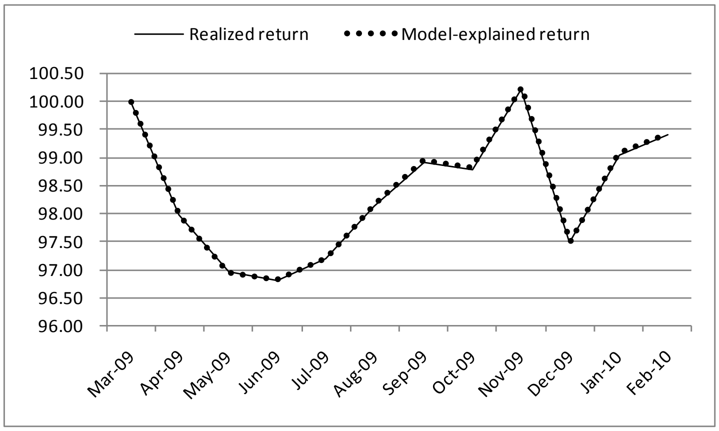
Put another way, using *factor_performance_contribution = factor_exposure * factor_return* and summing over the factors:

> *Explained portfolio return = risk_free_return + sum_for_i_over_S,T,B (factor_i_performance_contribution)*

During this period, the *risk-free return* was 0.01%, derived from 1-month T-bill rates. The equally weighted sample portfolio generated excess returns to STB factors (*factor_i_performance_contribution* in the formula above) of +1.07%, -1.29% and -0.33% respectively. By adding the risk-free return of +0.01%, to the three excess STB factor returns, the portfolio's cumulative performance amounted to -0.57%[1]. Note that the return explained by the Twist factor is the largest contributor to the portfolio's negative return, reflecting the steepening of the yield curve.

Exhibit 1 below shows the monthly portfolio return and the explained return using STB factors. Readers might note that on a monthly basis the performances of the Sample and Factor weighted portfolio are very close to each other. Typical differences between portfolio return and STB model-explained return are below 1 bp, ultimately resulting in a cumulative discrepancy of 3 bp over the 11-month period. The complete portfolio return attribution to STB, including monthly portfolio factor exposures and factor returns can be found in Appendix A.

Exhibit 1: Cumulative monthly realized and explained returns (= sum (exposures * factor_return)) for a sample US sovereign bond portfolio from March 31, 2009 to February 28, 2010.



Readers may recall that the financial markets crisis of 2008/2009 was triggered by turmoil in the credit supply and the securitized sub-prime mortgage markets. As a result of the credit squeeze, many investors fled from higher risk and leveraged positions to "safe haven" sovereign bonds. In response to the most severe financial crisis in decades, central banks around the world lowered rates dramatically to stimulate the recovery of the global economy. Some central banks, including the US Federal Reserve Bank, lowered their rates to unprecedented levels close to zero percent. This has created a unique environment of interest rate risk for fixed income investors for several reasons. First of all, in some cases, further rate reductions are not possible. Combined with a huge supply of government debt to finance historically high fiscal deficit, and a possible rise in inflation, this could result in a sharp change in the interest rate environment. This change in environment might affect the volatility and the shape of the term structure.

This situation brings up two central questions for fixed income professionals:

- How is the interest rate risk of a fixed income portfolio measured?
- How does one hedge against the effects of non-parallel yield curve changes?

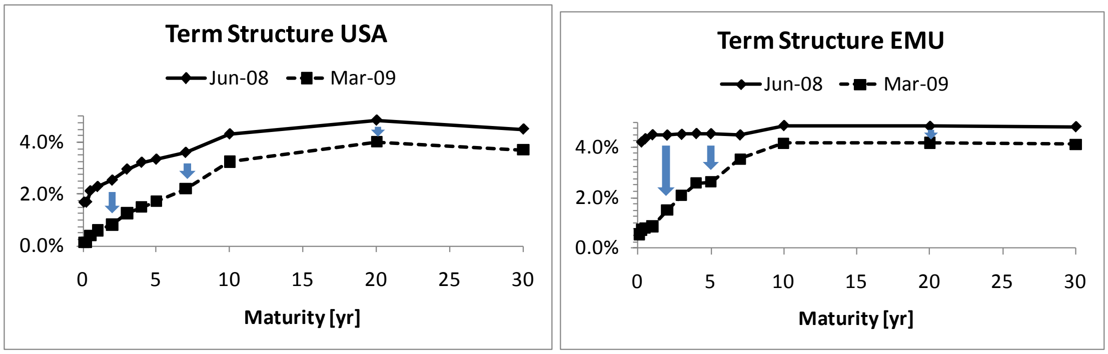
2. Term Structure Risk Explained with Shift-Twist-Butterfly

The local market term structure is the interest rate yield curve derived from local market sovereign debt, e.g., the US term structure is derived from the US Treasury bills, notes and bonds. We define term structure risk as the risk in bond portfolios due to changes in the term structure. A common practice among fundamental fixed income managers is to measure term structure risk in their portfolios using duration and/or convexity. However, we believe that these measures fail to account for complete yield curve dynamics and the resulting risk. Duration on a stand-alone basis has limited power in explaining term structure risk. Firstly, duration captures the risk associated with a parallel

[1] Note that the total explained return, i.e., the sum of the risk-free rate and the excess factor performance contributions, differs by a few basis points from the historical performance due to residual excess performance not explained by the common factors.

change in interest rates. Secondly, for a global bond portfolio, duration assumes that interest rates in different markets change in unison. These assumptions do not hold up very well. An example of the term structure change for the US and the Eurozone (EMU) from June 2008 to March 2009 is shown in Exhibit 2. This example illustrates that changes in the term structure are often non-parallel shifts that are specific to their respective domestic markets.

Exhibit 2: Term structure change for the US and Eurozone, from June 2008 to March 2009.



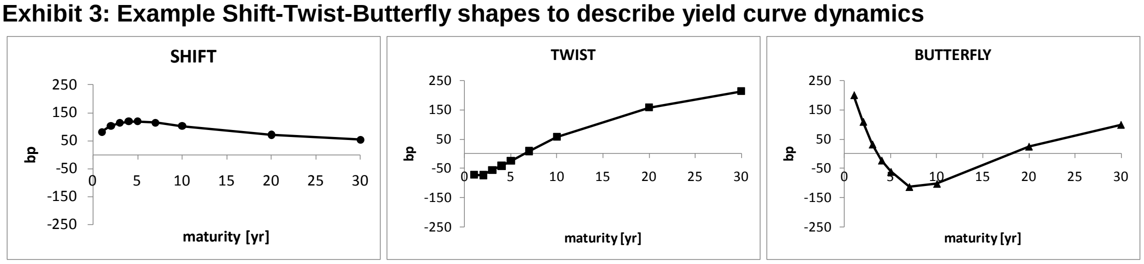
How can these interest rate changes and the corresponding risk be measured more exhaustively than by duration?

Textbook fixed income analysis (e.g., Fabozzi 2007) shows that yield curve dynamics can be described by STB movements. These three movements are the driving factors of the interest rates change across the term structure. They capture between 90-98%of interest rate variation in most developed markets (see the Barra Risk Model Handbook 2007). The STB movements are, in order of importance, as follows:

- Shift: captures the changes in the level of the yield curve
- Twist: captures the change in the slope of the yield curve
- Butterfly: captures the changes in the curvature of the yield curve

In the Barra fixed income risk models, the STB factors are derived mathematically in a rigorous way. Stable STB factor shapes[2] are calculated from the history of the term structure for each market. Examples of the shapes of these three factors are shown in Exhibit 3.

Exhibit 3: Example Shift-Twist-Butterfly shapes to describe yield curve dynamics



With the STB shapes, any term structure change can be described as the sum of three (3) partial STB movements. Each movement corresponds to a factor in the multi-factor model, which can go either up or down, resulting in a total of six (6) factor movements for the term structure. For an explicit graphical presentation of these six term structure movements, see Appendix B.

[2] Currently, STB shapes in Barra fixed income models have been fixed taking into account the results of a historical principal components analysis (PCA) analysis. This analysis has been done for each market where interest rate data allows and where STB factors provide statistical explanation power of the interest rate changes. Additional smoothing, flattening and normalizing techniques are used to generate stable and reasonable shapes. The overall performance of capturing interest rate movements with these shapes is similar to shapes fixed using other approaches, such as shape parameterization (e.g., Nelson, Siegel 1987).

Bond prices and yields are critically impacted by term structure changes. Exhibit 4 gives an overview of how the prices of bonds of different maturities change with the STB factors. The table shows the relation between the movement of the STB factors, the corresponding term structure movement, or yield curve change, and bond prices. The bond prices are subdivided in categories for short, medium-term and long maturity bonds.

Exhibit 4: Price changes for short, medium-term and long maturity bonds with Shift, Twist, and Butterfly factor movements. There are 6 movements, resulting from 3 independent factors each going up or down.

			Bond* price (for different maturity categories)		
Factor return		Term structure movement	Short (< 1-yr)	Medium-term (~3-yr)	Long (~20-yr)
Shift	Positive	Parallel shift down	+	+	+
	Negative	Parallel shift up	-	-	-
Twist	Positive	Flattening (clock-wise)	-	-/0/+	+
	Negative	Steepening (anti-clock wise)	+	+/0/-	-
Butterfly	Positive	More curvature	+	-	+
	Negative	Less curvature	-	+	-

** Bond without embedded options*

The table is explained below:

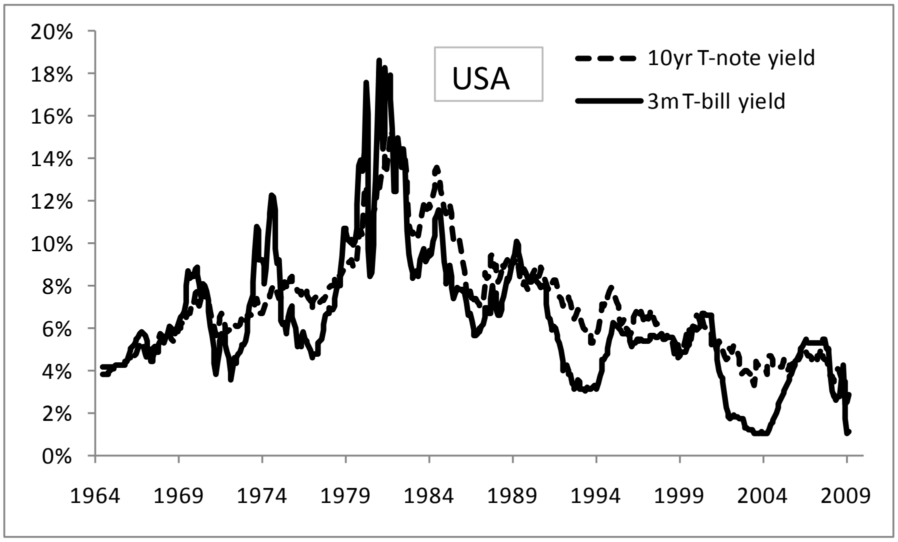
- A positive Shift factor return corresponds to the yield curve shifting down via short, medium-term, and long yields going down in parallel. This causes all bond prices to go up. Conversely, a negative Shift factor return corresponds to the yield curve shifting up via short, medium-term, and long yields going up in parallel. This causes all bond prices to go down.
- A positive Twist factor return corresponds to the yield curve flattening via short yields going up and long yields going down. This causes short maturity bond prices to go down and long maturity bond prices to go up. Conversely, a negative Twist factor return corresponds to the yield curve steepening via short yields going down and long yields going up. This causes short maturity bond prices to go up and long maturity bond prices to go down.
- A positive Butterfly factor return, corresponding to the yield curve getting more curvature via short yield going down, medium-term yield going up and long yield going down. This causes short and long maturity bond prices to go up, while medium-term maturity bond prices go down. Conversely, a negative Butterfly factor return corresponds to the yield curve getting less curvature via short yield going up, medium-term yield going down and long yield going up. This causes short and long maturity bond prices to go down, while medium-term maturity bond prices go up.

It should be noted that these simple relations are mainly valid for zero-coupon and low-coupon bonds. For high-coupon bonds, the exact exposure of the bonds to the STB factors as given by the shape factor durations should be considered to derive the corresponding bond price changes. These exposures directly relate the price change of the bond to the interest rate factor returns.

3. A Longer Historical Perspective

Today's post-subprime crisis interest rate environment should be put in historical perspective. Exhibit 5 shows the history of government bond yield in the US since 1964. A first observation is that current interest rate levels, both for short and long maturities, are at historical lows. Secondly, the Exhibit points out the inflationary periods of the 70s and the 80s, marked by significant spikes in the short-term yield. Therefore, significant interest rate spikes are not beyond the realm of possibilities.

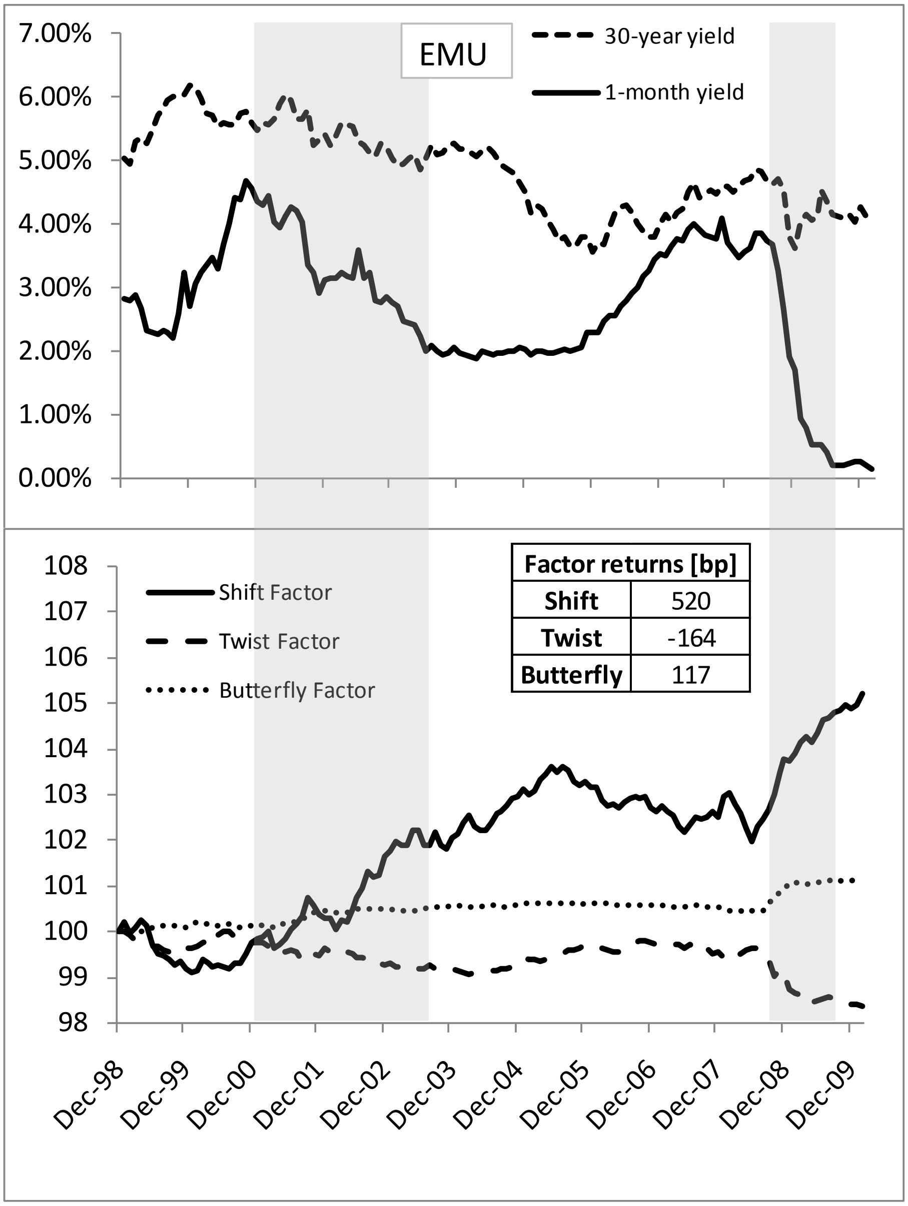
Exhibit 5: Short-term (3-month) and long-term (10-year) US government bond yields for the period 1964-2009 (monthly data).



4. Term Structure Movements and STB Factor Performance in the Last Decade

This section describes the interest rate movements for the last decade for three major developed fixed income markets using STB factors. More specifically, we look at short-term and long-term yield in relation to the STB factor performance for the Eurozone, US and Japan. The exhibits in the following pages show how the STB factors perform in rising and falling interest rate periods with corresponding term structure changes, as explained in the previous section on term structure risk. The cumulative factor performance figures highlight the relative importance of Shift, Twist and Butterfly. The movement and performance of the Shift factor is the highest of the three factors, while the Twist factor is second in terms of magnitude, and the Butterfly factor has the smallest magnitude.

Exhibit 6: Top: short-term (1-month) and long-term (30-year) term structure yields for the Eurozone in the period Dec 1998 – Feb 2009. Bottom: cumulative factor performance and returns. The shaded areas highlight two major declines in short-term yield, which are analyzed in the next section.



Factor returns [bp]	
Shift	520
Twist	-164
Butterfly	117

Exhibit 7: Top: short-term (1-month) and long-term (30-year) term structure yields for the US in the period December 1998 – February 2010. Bottom: cumulative factor performance and returns. The shaded areas highlight two major declines in short-term yield, which are analyzed in the next section

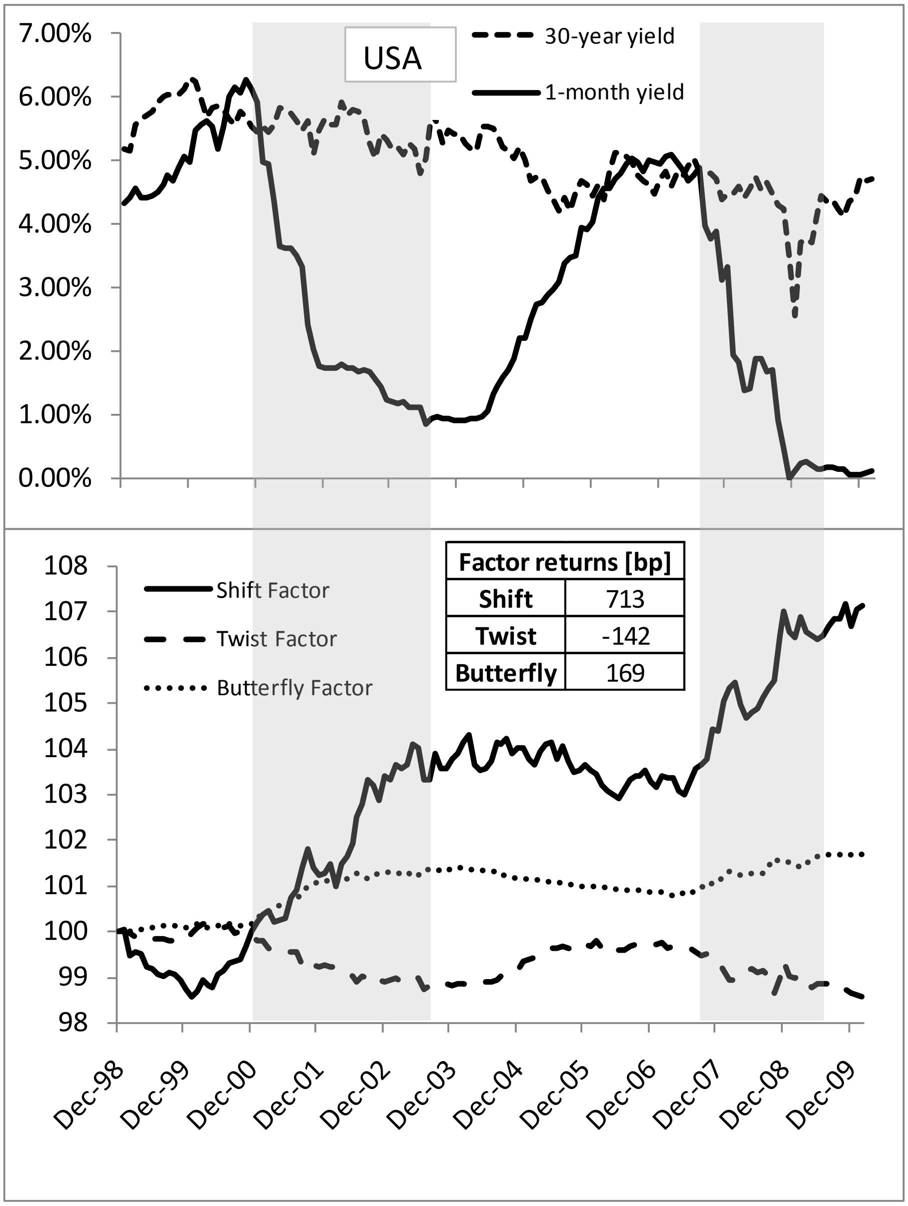
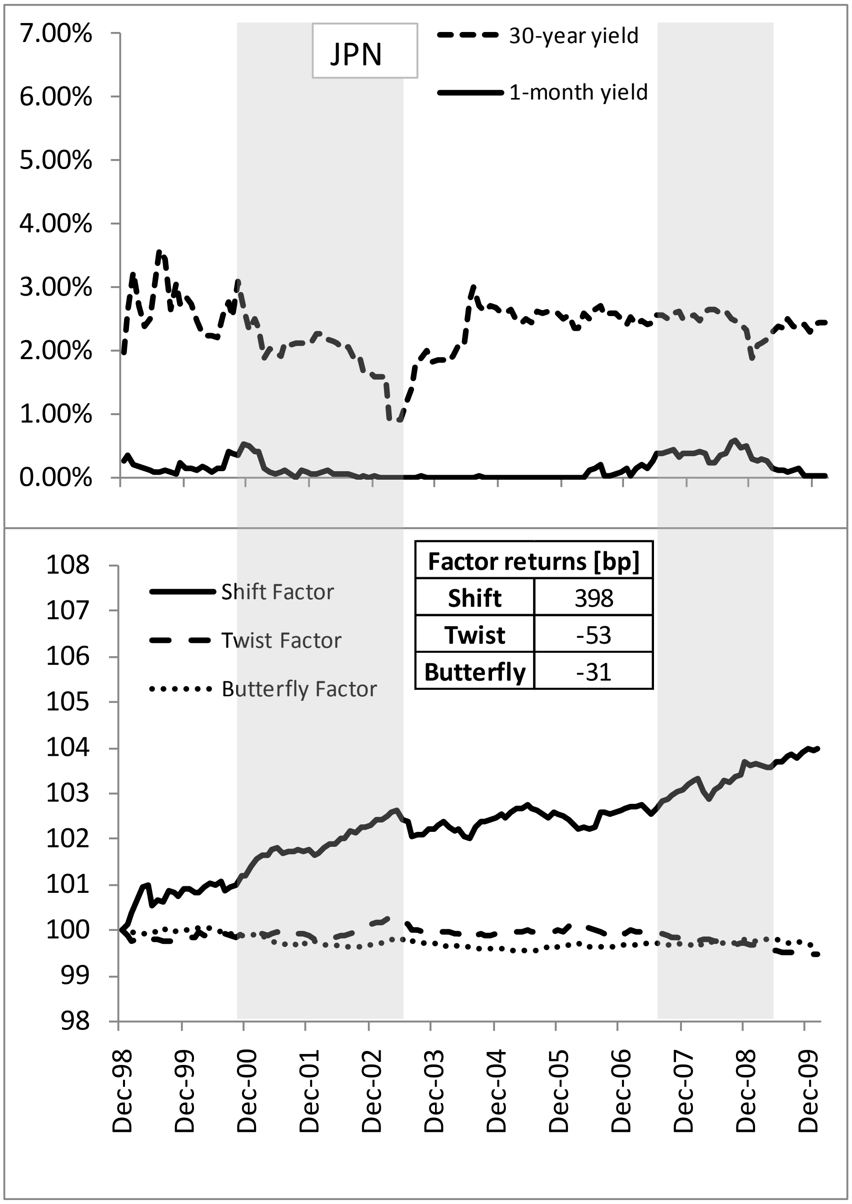


Exhibit 8: Top: short-term (1-month) and long-term (30-year) term structure yields for Japan in the period Dec 1998 – Feb 2010. Bottom: cumulative factor performance and returns. The shaded areas highlight two major declines in short-term yield, which are analyzed in the next section.



Factor returns [bp]	
Shift	398
Twist	-53
Butterfly	-31

Looking at the yield time series in Exhibits 6-8, we observe two major declines in short-term interest rates (shaded areas). These are related to major discount rate cuts by the central banks in Europe and the US, and smaller cuts in Japan. The declines were during the following periods:

1. "Post dot-com bubble" period from December 2000 to June 2003: a long period of short-term yield drop, with a smaller long-term yield drop in the US and the Eurozone. In Japan the short-term yields drop quickly almost to zero, while the long-term yield declines over the complete period.
2. "Post sub-prime credit crisis" period from August 2007 to March 2009 for the US and from June 2008 to March 2009 for the Eurozone and Japan. For the Eurozone and the US, we observe a steep drop in the short-term interest rate, driven by fast discount rate cuts. In the same period the long-term interest rate dropped much less. Note that in this crisis period, the US was leading in decreased short-term yield, and the Eurozone followed about a year later. Japan has only relatively small changes in the yields during the same period.

For each of these periods, we review in detail the changes in the term structure and the corresponding performance of the STB factors:

1. "Post dot-com bubble" (see Exhibits 9 and 10): the term structures for both Eurozone and US experienced a significant shift down. This corresponds to a positive factor return for the Shift factor. Japan experienced a much smaller shift. The size of the shift was larger in the US than in Eurozone. Correspondingly, the Shift factor return was higher in the US than in Eurozone. At the same time the Eurozone and US term structures steepened and got significant more curvature. These movements correspond to a negative Twist factor return and a positive Butterfly factor return respectively. For the Japan term structure, there was a flattening and almost no curvature change, corresponding to a positive Twist factor return and a small Butterfly factor return, respectively, as highlighted in Exhibit 10.

Exhibit 9: Significant term structure change for the Eurozone, US and Japan in the post dot-com bubble period from December 2000 to June 2003.

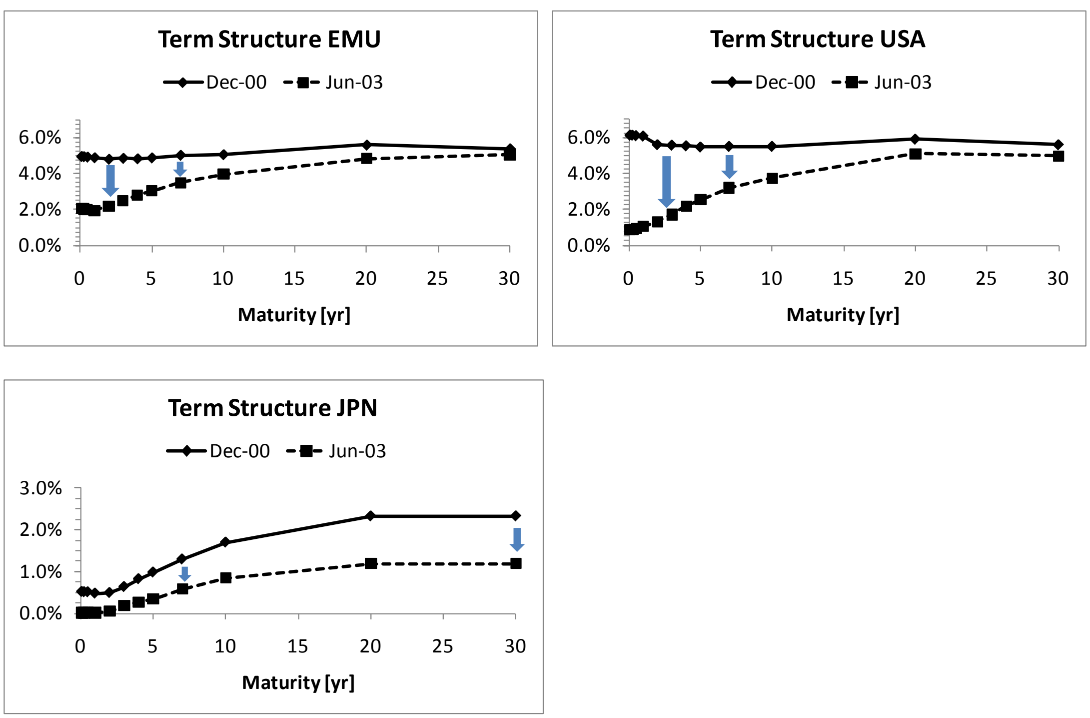
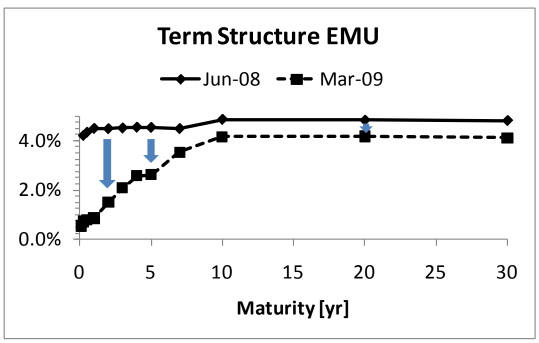


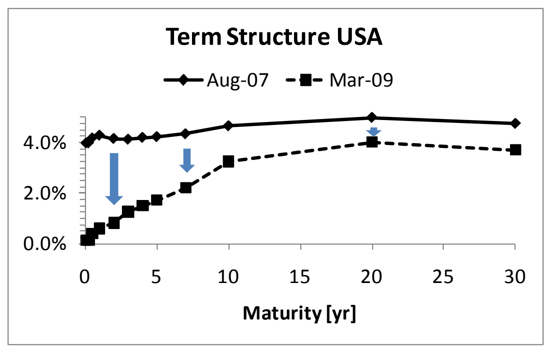
Exhibit 10: Shift-Twist-Butterfly factor returns explaining the term structure change in the post dot-com bubble period from December 2000 to June 2003.

	Period		Factor returns [bp]		
	Start	End	Shift	Twist	Butterfly
EMU	Dec-00	Jun-03	245	-57	34
US	Dec-00	Jun-03	402	-98	107
Japan	Dec-00	Jun-03	125	31	-8

2. "Post sub-prime credit crisis" (see Exhibit 11 and 12): both US and Eurozone term structures experienced a significant shift down. This corresponds to a positive Shift factor return. Especially in the Eurozone term structure this was accompanied by a very significant steepening and increase of curvature. These movements correspond to a negative Twist factor return and a positive Butterfly factor return. The Japan term structure went through similar STB changes, but an order of magnitude smaller than for the Eurozone and US. The STB factor returns are shown in Exhibit 12.

Exhibit 11: Significant term structure change post subprime credit crisis from June 2008 to March 2009 for the Eurozone (EMU) and Japan, and from August 2007 to March 2009 in the US.





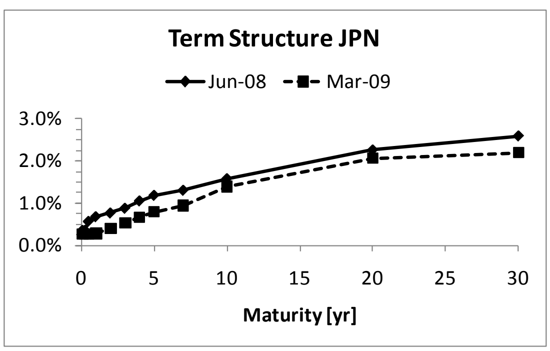


Exhibit 12: Shift-Twist-Butterfly factor returns explaining the term structure change in the post sub-prime credit crisis period, August 2007 to March 2009 for US, and June 2008 to March 2009 for the Eurozone and Japan.

	Period		Factor returns [bps]		
	Start	**End**	**Shift**	**Twist**	**Butterfly**
EMU	Jun-08	Mar-09	211	-102	63
US	Aug-07	Mar-09	318	-48	52
Japan	Jun-08	Mar-09	50	-13	6

When comparing the term structure change for the post sub-prime credit crisis period to the change of the post dot-com bubble period (see exhibits 10 and 12), several interesting observations can be made. It is striking that a similar term structure change happened in the two periods, but with two important differences. First, the term structure change post sub-prime crisis was larger, especially at the short end of the curve. Secondly, the change happened only over a period of nine months in the Eurozone and about 18 months in US. During the post dot-com bubble crisis a similar change took about two-and-a-half years to occur. In Japan, term structure movements were much smaller due to the particular interest rate environment of that market.

To conclude, we review the STB factor returns of the last 12 months, up until the end of February 2010 (Exhibit 13). Several STB factor returns stand out:

- In the Eurozone, there was a significant Shift factor return of 102 bp. This is mainly a result of the European Central Bank lowering interest rates about 12 months earlier, thus steepening the term structure, which is reflected here as a negative Twist factor return of -28 bp.
- In the US, the Twist factor return had the largest change, with a negative factor return of -51 bp. This reflects a strong steepening of the yield curve. In this particular period the magnitude of the twist and Butterfly factor returns were larger than the Shift factor return. This is explained by the fact that no further lowering of the central bank interest rate happened, with the rate being close to zero percent.
- In Japan, the factor returns have been smaller, but show a similar picture as for the Eurozone and the US. The yield curve shifted down, with a positive shift factor return of 36 bp. In addition, the yield curve steepened, with a negative Twist factor return of -19 bp and showed less curvature, with a negative Butterfly factor of -11 bp.

Exhibit 13: Shift-Twist-Butterfly factor returns for the 12 months up to and including February 2010 for the Eurozone, US and Japan.

	Period		Factor returns [bp]		
	Start	End	Shift	Twist	Butterfly
EMU	Mar-09	Feb-10	102	-28	8
US	Mar-09	Feb-10	24	-51	27
Japan	Mar-09	Feb-10	36	-19	-11

Fixed income portfolios are impacted in a major way by these non-parallel term structure changes. Portfolio case studies in the next section shows how portfolio losses can result from these large interest rate movements and how to avoid them using complementary interest rate risk measures.

5. STB Factor Models Help Fixed Income Managers in Addressing Interest Rate Risk in their Portfolios

In this section, we offer four case studies to highlight how STB factor models can add value to fixed income portfolio and risk management. For illustrative purposes, we selected the 2008/2009 timeframe to demonstrate the capabilities of STB factor models in a sharply changing interest rate and a non-parallel term structure change environment.

In the first case study, we compare single duration matching with STB exposure matching. In the second case study, we compare duration+convexity matching with STB exposure matching. In the third case study, we compare KRD matching and STB [exposure?] matching. In the fourth case study, we analyze risk for a global sovereign bond portfolio using duration and the Shift factor of the STB model.

For the first three case studies, portfolios are constructed in order to track a benchmark portfolio. This is equivalent to funding a future liability. The portfolio construction takes place in 2008. The benchmark/liability chosen is a single bond portfolio maturing in 2014. This translates into a time to maturity of about six years from the portfolio construction date.

Case Study 1: Single Duration-matching versus STB-matching performance in the recent markets turmoil

In the first case study, we illustrate that twist and butterfly movements of the term structure can lead to unexpectedly large losses in portfolios, if only duration is taken into account. Nowadays, probably very few portfolio managers would consider duration as a solitary risk measure. Still, we include this naïve case to point out the complexity of interest rate risk.

For this purpose we use the classical method of duration matching in fixed income portfolios. The investment goal is to track a benchmark represented by a single bond; this is equivalent to funding a specific liability represented by the same bond. In the duration matching approach, a portfolio is constructed that matches the duration of this benchmark or liability. As such the portfolio is said to be immunized for interest rate risk. We compare this duration-matched portfolio with a STB-matched portfolio. The STB-matched portfolio is constructed to match Shift, Twist and Butterfly exposures of the liability. To illustrate the impact of changing interest rates, we review the performance of a duration-matched portfolio through the "post sub-prime credit crisis" period that was reviewed in the previous section. The example discussed here is for a European institutional investor investing in EUR bonds. However, the results are not specific to the currency. A similar case can be constructed for any domestic or international bond investor with any base currency.

We start with a benchmark or liability maturing in 2014, modeled by a EUR 10m German government bond maturing October 1, 2014. To track this benchmark (or fund this liability), we construct two asset portfolios, shown in Exhibit 14:

- Duration-matched portfolio: we construct a classical barbell portfolio by mixing 2010 and 2030 German government bonds to have the same duration as the liability. The motivation to use 2030 bonds in the portfolio is to profit from their significantly higher yield. This portfolio has a yield-to-maturity (YTM) of 4.24% at construction time.
- STB-matched portfolio: we construct a portfolio that selects bonds from the Citi German Government Bond Index*. The portfolio is constructed by maximizing the YTM, while minimizing the total term structure risk. The total term structure risk is determined by the portfolio exposure to the Shift, Twist and Butterfly factors and the factor volatilities.* At the time of construction, the portfolio has an YTM of 3.81%, lower than the duration-matched portfolio. This was driven by the risk hedging constraints.

After a year, we evaluated the value and the performance of the portfolios (Exhibit 14). The value of the liability rose more than the value of the duration-matched bond portfolio, thus resulting in a negative surplus or funding gap. The value of the liability exceeded the value of the assets, both in absolute and in percentage terms. For the duration-matched portfolio, the funding gap amounts to EUR 369,000, or a funding gap of 3.40% relative to the liability. At the same time, the STB-matched portfolio has a lower funding gap of EUR 93,000, or 0.86%.

Exhibit 14: A Liability, a Duration-Matched and a STB-Matched portfolio invested through the post Sub-Prime Credit Crisis Period.

	Liability Portfolio	Duration-matched portfolio				STB-matched portfolio			
	Value (m EUR)	Value (m EUR)	YTM	Funding gap (m EUR)	Funding gap (percentage)	Value (m EUR)	YTM	Funding gap (m EUR)	Funding gap (percentage)
4/1/2008	10.000	10.000	4.24%	0.000	0.00%	10.000	3.81%	0.000	0.00%
4/1/2009	10.848	10.478	3.48%	0.369	3.40%	10.755	2.40%	0.093	0.86%

Where does this funding gap come from? Wasn't the interest rate risk hedged according to our duration-matching approach? The explanation is straightforward.

First, during the investment year, short-term interest rates fell, as reviewed in the previous section. Falling interest rates caused the liability, the duration-matched portfolio, and the STB-matched portfolio to go up in value. Duration-matching accounted for that parallel shift in interest rates. However, the term structure steepened and its curvature changed. These secondary movements of the term structure had a different implication for the liability than for the duration-matched portfolio, thus resulting in the funding gap.

In summary, the duration matching ignored the Twist and Butterfly movements. The STB-matched portfolio accounted for the secondary term structure movements, thereby minimizing the funding gap.

Case Study 2: Duration- and Convexity-Matching versus STB-Matching Performance in the Recent Markets Turmoil

In the second case study, we compare duration- and convexity-matching to STB-matching. The goal and setup of the case is identical to Case Study 1. The difference is that in the construction of the portfolio we also require the convexity of the benchmark/liability to be matched with the portfolio.

We start with a benchmark or liability maturing in 2014, modeled by a EUR 10m German government bond maturing October 1, 2014, with a duration of 5.8 and a convexity of 39.8. To track this benchmark (or fund this liability), we construct two asset portfolios (shown in Exhibit 15):

- A duration- and convexity-matched portfolio: we construct a portfolio by optimizing weights for six German government bonds with different maturities till the portfolio has the same duration and convexity as the liability. This portfolio has a yield-to-maturity (YTM) of 3.80% at construction time.
- An STB-matched portfolio: as in Case Study 1.

After a one-year holding period, we examine the value and performance of the various portfolios (Exhibit 15). Here again, the value of the liability rose more than the value of the duration- and convexity-matched bond portfolio resulting in a funding gap of EUR 121,000 or 1.12%. The STB-matched portfolio, on the other hand, has a lower funding gap of EUR 93,000 or 0.86%.

Exhibit 15: A Liability, a Duration-and-Convexity-Matched and a STB-Matched portfolio invested through the post Sub-Prime Credit Crisis Period.

	Liability Portfolio	Duration- and convexity-matched portfolio				STB-matched portfolio			
	Value (m EUR)	Value (m EUR)	YTM	Funding gap (m EUR)	Funding gap (percentage)	Value (m EUR)	YTM	Funding gap (m EUR)	Funding gap (percentage)
4/1/2008	10.000	10.000	3.80%	0.000	0.00%	10.000	3.81%	0.000	0.00%
4/1/2009	10.848	10.726	2.42%	0.121	1.12%	10.755	2.40%	0.093	0.86%

Compared to the first Case Study, the second Case Study improved the tracking of the benchmark/liability by adding convexity-matching to the portfolio construction process. However, the funding gap was still greater than the STB-matched portfolio by 26 bp. Using the STB model (see MSCI Barra Risk Model Handbook 2007), we determined that this gap was due to a larger exposure to term structure risk for the duration- and convexity-matched portfolio.

Case Study 3: Key Rate Durations and STB-Matching

The issues of the simple duration-matching case (Case Study 1) and the duration-convexity-matching (Case Study 2) can be addressed by decomposing duration across the yield curve. This method is called KRD matching. In this study we review the performance of a KRD-matched portfolio and a STB-matched portfolio.

The method of Key Rate Matching is first to divide the term structure into distinct maturity segments or key rates. Secondly, one can hedge against changes of these key interest rates in each segment by matching the KRDs. A complete description of the term structure may require that 8 to 15 key rates be matched to their respective durations. A more complete description of KRD can be found in Ho (1992).

From a theoretical standpoint, the Key Rate Matching approach is expected to address interest rate risk issues with greater precision. However, the drawback of this method is that it does not account for the correlation of movement in key rates. To put it another way, the rates across the term structure rarely move independently, and as a result describing movements with eight to 15 changing key rates is much less efficient than with just the three factors of Shift, Twist, Butterfly. This theoretical argument has important implications for a model that describes co-movement of interest rates across different domestic markets. The inter-market correlations when using 8 to 15 key rate factors cannot be easily measured, requiring a very large amount of data. In addition, spurious temporal correlations might be picked up by the model. In contrast, the efficiency of a description with only three STB factors describes inter-market correlations in a more reliable way.

For practical evaluation, we consider a fixed income investment through the 2008/2009 turmoil period as in the previous case study. The portfolios are constructed in April 2008. The liability is maturing in 2014; which is about six years to maturity from the portfolio construction date. It is by construction a EUR 10m portfolio, consisting of a German 2014 sovereign bond with 3.75% coupon. The goal of our investment exercise is first to match the liability, secondly to hedge interest rate risk, and thirdly to have a high-yielding investment. The investment universe at our disposal is the German sovereign bonds that are constituents of the Citi German Government Bond Index. In this portfolio case study transaction costs are ignored. Two asset portfolios are constructed:

1. KRD-matched portfolio: the portfolio is constructed by matching eight key rate durations of the liability. This way, the active KRDs are targeted to be zero. The key rate matching provides the interest rate hedging.
2. STB-matched hedged portfolio: the portfolio is constructed by minimizing active risk versus liability. The active STB risk exposures to their respective term structure STB factors are minimized. At the same time, the optimizer targets an enhanced portfolio yield.

We define active risk as the difference between portfolio duration and liability duration. In both cases the active risk measure versus the liability (or benchmark) is minimized. For example: consider a case with a liability that has a one-year key rate duration of 5.0 and with the asset portfolio that has a one-year key rate duration of 5.5. By definition, the active duration of the liability is zero. The active duration of the asset portfolio is 0.5 (= 5.5 – 5.0). We review the characteristics of both portfolios at the time of construction in Exhibit 16. The main observations are:

- Overall, both portfolios have low active KRDs and low active Shift, Twist and Butterfly risk exposures.
- The KRD-matched portfolio has low active duration at every key of the eight key rates by construction. It contains two bonds, because the benchmark or liability consists of only one bond position, resulting in two key rate durations (5-year and 7-year KRD) that need to be addressed by the hedge. It has slightly higher active shift risk, because this risk exposure is not explicitly constrained in key rate matching.

- The STB-matched portfolio has low active risk for the three interest rate risk factors of Shift, Twist and Butterfly by construction. It has non-zero active duration for 20-year and 30-year maturity. The portfolio contains six bonds, needed to provide a more complete hedge to the shift risk exposure than the two bonds of the KRD-matched exposure.
- The STB-matched portfolio has an effective duration that is closer to the effective duration of the liability than the KRD-matched portfolio.

Exhibit 16: Liability, KRD-matched portfolio, STB-matched portfolio return and risk characteristics at construction time (April 1, 2008).

	Characteristics		STB-matching		
	Number of bonds	Effective Duration	Active Shift Risk	Active Twist Risk	Active Bfly Risk
Liability Portfolio	1	5.84	0.00	0.00	0.00
KRD hedged portfolio	2	5.72	0.09	0.01	0.01
STB hedged portfolio	6	5.86	0.03	0.06	0.01

	Key Rate Duration matching							
	Active KRD 1-yr	Active KRD 2-yr	Active KRD 3-yr	Active KRD 5-yr	Active KRD 7-yr	Active KRD 10-yr	Active KRD 20-yr	Active KRD 30-yr
Liability Portfolio	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
KRD hedged portfolio	0.01	0.01	0.00	-0.08	-0.06	0.00	0.00	0.00
STB hedged portfolio	0.01	0.42	0.01	0.00	0.00	0.00	-0.18	-0.24

We now review these portfolios after investing through the turmoil period of 2008/2009 with its large non-parallel yield curve changes, and significant term structure Twist and Butterfly movements. The results are shown in Exhibit 17, below. We find that all three portfolios rose in value, and that both the KRD-matched and STB-matched portfolio have a small funding gap, of 1.04% and 0.86% respectively, with the STB and KRD portfolios producing similar results.

Exhibit 17: Liability, KRD-matched portfolio, STB-matched portfolio performance.

	Liability Portfolio	KRD-matched asset portfolio				STB-matched asset portfolio			
	Value (m EUR)	Value (m EUR)	YTM	Funding gap (m EUR)	Funding gap (percent)	Value (m EUR)	YTM	Funding gap (m EUR)	Funding gap (percent)
4/1/2008	10.000	10.000	3.79%	0.000	0.00%	10.000	3.81%	0.000	0.00%
4/1/2009	10.848	10.735	2.35%	0.113	1.04%	10.754	2.40%	0.093	0.86%

In conclusion:

- Both KRD-matching and STB-matching address the shortcomings of single duration-matching during a period of large non-parallel yield curve shift.
- In this particular “bullet” liability case, STB is slightly more efficient in addressing the interest rate hedging than KRD-matching as shown by its effective duration of 5.86 versus 5.72 (see Exhibit 16).
- On the downside, the STB portfolio required six bonds versus two for the KRD portfolio.

For more complex liability cash flows, such as for pension funds, we expect STB-based matching to provide similar efficiency versus KRD-matching. However, we want to highlight the additional efficiency of using only three factors with low correlation instead of using more than 10 key rates with high correlation. This efficiency in number of risk factors reduces the complexity of the portfolio or risk management task, especially in international portfolios.

It can also be shown that for an STB factor engine, risk views can be translated to equivalent KRD representations (MSCI Barra 2009). This way, analysts can use either the KRD representation or the STB representation, according to their own preference.

Case Study 4: A Global Sovereign Bond Portfolio

In the fourth and last case study, we review the use of duration to evaluate interest rate risk in a global sovereign bond portfolio. The purpose of this evaluation is to help risk managers in their monitoring focus, and to help portfolio managers decide on risk-hedging measures.

We start from an investment in April 2008 in a global G7 sovereign bond portfolio. The portfolio was constructed by selecting all sovereign bonds of G7 countries (US, Japan, UK, Germany, France, Italy, and Canada) that are constituents of the Merrill Lynch Global Government Bond Index. The portfolio has the same holdings and relative country weights as the index. By construction, the portfolio is fully hedged for currency effects, as is often the case in international bond portfolio mandates. Next, the interest rate risk for the various country bond holdings needs to be analyzed.

Two methods can be considered:

1. Duration risk analysis: here sovereign bonds are ranked per country according to the contribution to the duration of the portfolio.

2. STB risk analysis: in this approach, sovereign bonds are ranked per country according to their contribution to risk, based on the Shift factor in each sovereign market.

First, we review the duration risk analysis, as shown in exhibit 18:

- Japan is ranked first: it is the largest contributor to portfolio duration due to its large bond weight of 36% and duration of 6.33. The duration of Japanese bonds is higher than the total portfolio duration of 6.24
- The EUR group is second and the US third, due to lower relative duration
- If one were to not group EUR countries together, then,
  - Italy and Germany would rank before the UK and France, and
  - US would be ranked second

Exhibit 18: Duration risk analysis for a global sovereign bond portfolio.

Duration risk analysis				
Market	**Bond weight**	**Duration**	**Contribution to Duration**	**Rank according to Cont. to Duration**
Japan	36%	6.33	2.29	1
EUR group	31%	6.21	1.91	2
Italy	11%	6.59	0.70	4
Germany	11%	5.82	0.64	5
France	9%	6.24	0.57	7
US	25%	5.28	1.32	3
UK	6%	9.57	0.58	6
Canada	2%	7.28	0.15	8
Total	100%		6.24	

The question is, do the Japanese sovereign bond holdings pose the main risk in this portfolio, as the duration analysis suggests? Next, we analyze the risk for the same portfolio with the STB model. To have an equivalent comparison with duration as above, the analysis focuses on the risk from shifts in the term structure, ignoring the Twist and Butterfly term structure effects in this case.

We first clarify some definitions according to common multi-factor model terminology. In general, portfolio risk can be decomposed into a part driven by the common factors, and a part specific to the assets. In our case, the specific part of the risk can be neglected, since for developed market sovereign bonds the risk beyond the interest rate risk is usually small. The risk due to the Shift factor can be calculated as: Risk = Factor Volatility x Factor Exposure. For the more general case of multiple factors and specific risk contributing to the risk, risk definitions and calculation methodology can be found in the Barra Risk Model Handbook (2007).

The results from the STB risk analysis are shown in Exhibit 19. Please note Risk (Column C) = Factor Volatility (Column A) * Exposure (Column B):

- US Shift factor exposure is the largest contributor to risk, driven by high factor volatility.
- The EUR group is second and the Japan Shift factor third due to low factor volatility.
- If EUR countries are not grouped together:
  - Italy and Germany rank before France and the UK; however the contributions are very similar (0.25 versus 0.26)
  - Japan shift is second

Exhibit 19: Shift factor risk analysis for a global sovereign bond portfolio.

Factor	Factor Volatility	Factor Exposure	Risk	Rank According to Risk
US Shift	0.88	1.26	1.10	1
EUR group			1.05*	2
Italy Shift	0.60	0.66	0.40	4
Germany Shift	0.59	0.60	0.35	5
France Shift	0.58	0.54	0.32	6
Japan Shift	0.36	2.23	0.81	3
UK Shift	0.58	0.47	0.27	7
Canada Shift	0.66	0.12	0.08	8

* Risk for the EUR group sub-portfolio is the square root of the sum of first the sum of the squared risks coming from individual shift factors plus secondly, the sum of interaction terms due to correlations between the individual factor risks

Let us evaluate both risk analysis methods, side-by-side:

- The duration analysis suggests that Japanese bonds are most risky. This is based on the assumption that "interest rates" move in unison in all markets. Duration is a measure that only gives the exposure to this uniform risk. As such, it neglects different term structures changing in different ways and in different magnitudes across markets (as discussed earlier in this paper).
- The STB risk analysis suggests that US bonds are most risky; this is based on incorporating the riskiness of each market through factor volatilities. In particular, STB breaks down risk in risk drivers (risk factors) and the exposure to these drivers (factor exposures), incorporating all local term structure risk factors.

In conclusion, for a global bond portfolio, risk exposure to all local market term structure changes need to be considered. STB-based risk modeling provides more insight for portfolio construction and risk monitoring than simple duration analysis.

6. Summary

The current interest rate risk environment poses challenges for Fixed Income professionals, who need to address possible changes to the term structure of interest rates around the globe. In this paper we illustrate the capabilities of Shift, Twist, Butterfly factor models to help address this and related challenges.

An historical review shows that the term structures for the Eurozone and US have experienced significant Twist and Butterfly movements, as was the case in the 2008/2009 market turmoil. Recently, US term structure has experienced sharp steepening with a 2-10-year yield spread for Treasuries widening to a multi-decade high level of 291 bp in February 2010. Since the end of March 2009, the month which saw the bottom of the 2008/2009 equity market crisis, 2-10-year yield spreads have widened by about 120 bp from 171 bp to a high of 291 bp on February 22, 2010, then declining recently to the current levels. The impact of the sharp steepening is evidenced by the large negative return of the Twist factor over the Shift and Butterfly factor returns during the same period.

In this paper we have reviewed several case studies to highlight how STB factor models can add value to fixed income portfolio and risk management. For illustrative purposes we have selected the 2008/2009 timeframe in order to demonstrate the capabilities of STB factor models, during a period of sharp non-parallel change in the term structure. In the first case study, we demonstrated how single duration-matching can lead to funding gaps for liabilities due to Twist and Butterfly effects. We also showed how this can be addressed by taking STB exposures into account at the time of portfolio construction. In the second case study, we showed that convexity-matching can partly address the shortcomings of duration-matching, but is not as exhaustive in addressing risk as STB-matching. In the third case study, we showed how STB analysis provides similar risk description and hedging capabilities as matching KRDs across the yield curve. At the same time, STB is more efficient in certain cases, because it needs fewer factors compared to KRD to describe interest rate risk. Finally, for a global sovereign bond portfolio, we illustrated how STB factor risk analysis takes into account interest rate risks from various local market term structures, while a single duration based analysis neglects this market complexity.

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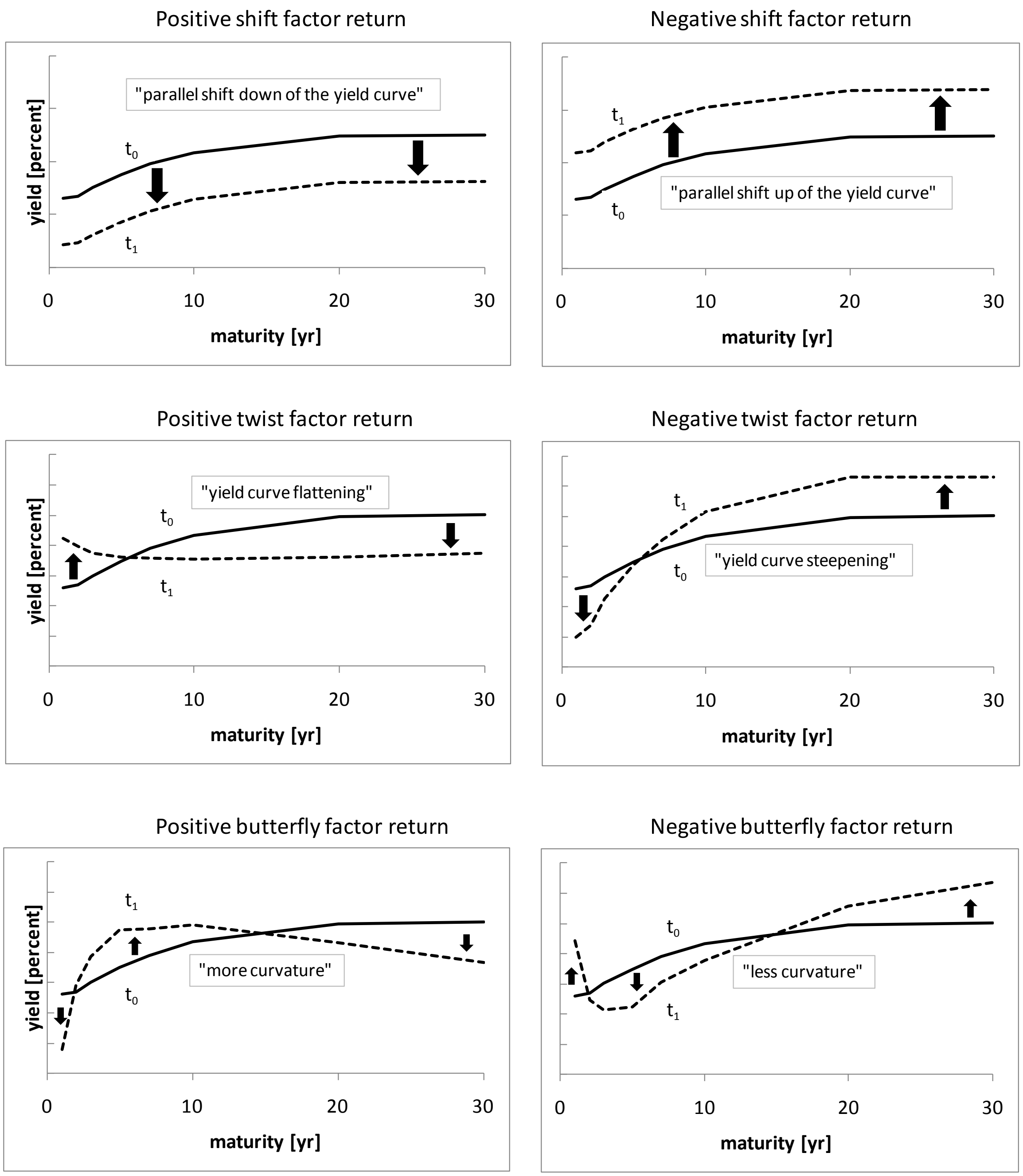
Appendix A

Exhibit 20: Monthly performance attribution of a sample US sovereign bond portfolio to Shift, Twist, Butterfly factors from March 31, 2009 to February 28, 2010. Realized return is explained by the STB model and decomposed in contributions from the STB factors.

	Portfolio	Model Explained Return				Factor Return			Portflolio Factor Exposure		
Month	**Realized Return**	**Total Explained by S-T-B**	Explained by Shift	Explained by Twist	Explained by Butterfly	Shift	Twist	Butterfly	Shift	Twist	Butterfly
Apr-09	**-2.03%**	**-2.03%**	-1.45%	-0.50%	-0.08%	-0.28%	-0.19%	0.07%	5.24	2.68	-1.16
May-09	**-1.04%**	**-1.03%**	-0.56%	-0.32%	-0.14%	-0.11%	-0.13%	0.12%	5.21	2.55	-1.21
Jun-09	**-0.14%**	**-0.14%**	-0.35%	0.27%	-0.06%	-0.07%	0.10%	0.05%	5.22	2.59	-1.20
Jul-09	**0.40%**	**0.41%**	0.43%	-0.04%	0.01%	0.08%	-0.01%	-0.01%	5.17	2.64	-1.10
Aug-09	**0.97%**	**0.98%**	1.00%	0.03%	-0.05%	0.19%	0.01%	0.05%	5.16	2.49	-1.12
Sep-09	**0.79%**	**0.79%**	0.69%	0.09%	0.01%	0.13%	0.03%	-0.01%	5.15	2.52	-1.09
Oct-09	**-0.13%**	**-0.12%**	0.15%	-0.28%	0.00%	0.03%	-0.11%	0.00%	5.23	2.55	-1.16
Nov-09	**1.45%**	**1.45%**	1.59%	-0.18%	0.04%	0.31%	-0.07%	-0.04%	5.16	2.45	-1.11
Dec-09	**-2.73%**	**-2.74%**	-2.52%	-0.18%	-0.04%	-0.48%	-0.07%	0.04%	5.20	2.52	-1.13
Jan-10	**1.60%**	**1.60%**	1.72%	-0.08%	-0.04%	0.34%	-0.04%	0.04%	5.07	2.32	-1.08
Feb-10	**0.36%**	**0.36%**	0.43%	-0.09%	0.02%	0.08%	-0.04%	-0.02%	5.13	2.42	-1.09
Total period	**-0.60%**	**-0.57%**	**1.07%**	**-1.29%**	**-0.33%**	**0.24%**	**-0.51%**	**0.28%**			

Appendix B

Exhibit 21: Shift, Twist, Butterfly movements for the yield curve and their corresponding risk factor movements. In each example movement below, the yield curve changes from the solid line at t_0 to the dotted line at t_1.



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