

# Quantitative Portfolio Strategy

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## **PORTFOLIO STRATEGIES IN TIMES OF UNCERTAINTY: BENCHMARK REPLICATION**

The potential for continuing dramatic movements in short-term interest rates and credit spreads in the U.S. and global bond markets makes close index tracking a reasonable tactic even for those fixed-income managers who normally pursue active strategies. An additional argument for switching to passive benchmarking at least for the rest of 2001 is that many managers, who correctly anticipated a series of Fed tightenings as well as spread product performance, may wish to preserve the healthy year-to-date outperformance they had produced by the end of September.

Over the last decade we have developed a number of strategies and tools for replicating most of the widely used Lehman indices. The two main categories in which all strategies fall are replication with cash securities (proxy portfolios), and replication with derivative instruments (futures and swaps). The variety of replication strategies makes it possible to reflect objectives and constraints of different investors. In markets with considerable idiosyncratic risk, bond portfolios may better match issuer distribution of the replicated benchmark. On the other hand, derivatives strategies help maintain index tracking in portfolios with dynamic cash inflows and outflows. Currently, when many mutual funds experience higher-than-usual activity, derivatives replication strategies may prove very useful. In the brief review that follows, we revisit our major index replication techniques and present their historically simulated performance.

### **Replication with Proxy Portfolios**

We have consistently demonstrated that relatively small proxy portfolios of liquid securities can replicate broad indices quite successfully. The replication portfolios can be built using one of two approaches – by an optimized sampling process, or via tracking error minimization using Lehman multi-factor risk models.<sup>1</sup>

#### ***Optimized Sampling Replication Techniques***

Sampling techniques represent the “common sense” approach. To replicate an index one breaks down the set of index securities into cells along one or more important risk dimensions. Then, all major cells are represented by a few, preferably liquid, securities. The total weight of securities in a particular cell is usually computed to match that cell’s contribution to overall index duration. The drawback of this approach is that a mismatch to the benchmark in any cell appears to be equally important. In reality, matching some cells is more critical than matching others because return (or spread) volatility associated with them is higher. Sampling techniques also ignore correlations among cells. On the other hand, sampling strategies are usually straightforward, intuitive, and model-independent.

Most recently, we have applied optimized sampling to replicate the Lehman High Yield and Global Aggregate indices.

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<sup>1</sup> For the U.S. risk model see *The Lehman Brothers Multi-Factor Risk Model*, Lehman Brothers, July 1999. A multi-factor risk model for euro-denominated assets was introduced earlier this year.

We developed three strategies to replicate the High Yield Index.<sup>2</sup> The first (“issuer strategy”) invests only in the largest issuers in the index and does not explicitly control for Treasury duration, thus assuming that idiosyncratic risk is a key component of returns volatility in the high-yield market. The second (“structure strategy”) matches market weights and contributions to spread duration within each sector/quality bucket of the index, matching the overall index spread duration. In addition, this strategy also matches both Treasury duration and convexity of the index. Finally, the third strategy (“structured-issuer strategy”) is similar to the second one but further reduces the list of eligible bonds to one (largest) security from every issuer in the index. This strategy combines the emphasis on issuer diversification of the first strategy with the index structure matching of the second. As a result, the tracking error it produced was the lowest of all three strategies. Figure 1 shows performance results of all three strategies.

The Lehman Brothers Global Aggregate Index offers comprehensive coverage of global investment-grade bond markets and is fast becoming an attractive benchmark for investors interested in global spread products. We propose methodologies for constructing relatively compact proxy portfolios of liquid securities, which reliably replicate this very diverse multi-currency index. The replication portfolios are built using an optimized sampling process that maximizes portfolio liquidity subject to a number of constraints. The techniques we propose do not pursue very close tracking, and make the same compromises that investors are likely to make in real life. The holdings are limited to the most liquid bonds in the four major currencies – USD, EUR, JPY, and GBP – that together account for over 95% of the index market value. All the other currencies are mapped onto one of these four.

<sup>2</sup> *High Yield Index Replication*, Lehman Brothers, in *Global Relative Value*, March 2001.

Figure 1. **Replication of the Lehman High Yield Index with Proxy Portfolios**  
January 1993 – December 2000

<b>Number of Issues</b>	<b>Mean Outperform. Error (bp/month)</b>	<b>Tracking (bp/month)</b>	<b>% Of Variance Explained</b>
<b>Issuer Strategy</b>			
20	18.8	90.4	63.4
40	11.4	68.6	78.9
60	8.3	58.0	84.9
80	8.5	50.9	88.4
100	8.1	46.2	90.4
<b>Structure Strategy</b>			
46	4.6	67.4	79.6
78	2.3	52.6	87.6
<b>Structured-Issuer Strategy</b>			
46	4.7	46.1	90.5
78	0.8	37.9	93.6

We developed three strategies that differ in the breadth of the investable universe.<sup>3</sup> The simplest (Treasury-only) is limited to treasury securities of the four currencies and replicates index allocations across currencies and term structure. The second strategy (Treasury-plus) begins to address spread exposure by including agency and collateralized securities. Finally, the third strategy (All-sectors) includes replication of all major credit sectors in the index.

All strategies match two index cell attributes: market value and contribution to duration. The sampling is done along five dimensions: term structure, quality rating, sector, currency bloc, and country bloc. Constraints are also applied to combinations of these dimensions, such as sector by country, and quality rating by country. Figure 2 presents a performance summary for the three replication strategies.

### Tracking Error Minimization Techniques

Multi-factor risk models use historical variances and correlations of risk factors (covariance matrix) to translate structural differences between the proxy portfolio and benchmark into an expected tracking error. While sampling techniques attempt to match the index risk parameters explicitly, optimization capabilities of risk models can be employed to create *minimum tracking error* portfolios without requiring any explicit decisions about relative importance of various risk dimensions. Such optimizers may ignore a significant structural mismatch that historically did not result in large return volatility and instead deploy more securities where achieving a good match matters the most from the historical risk perspective.

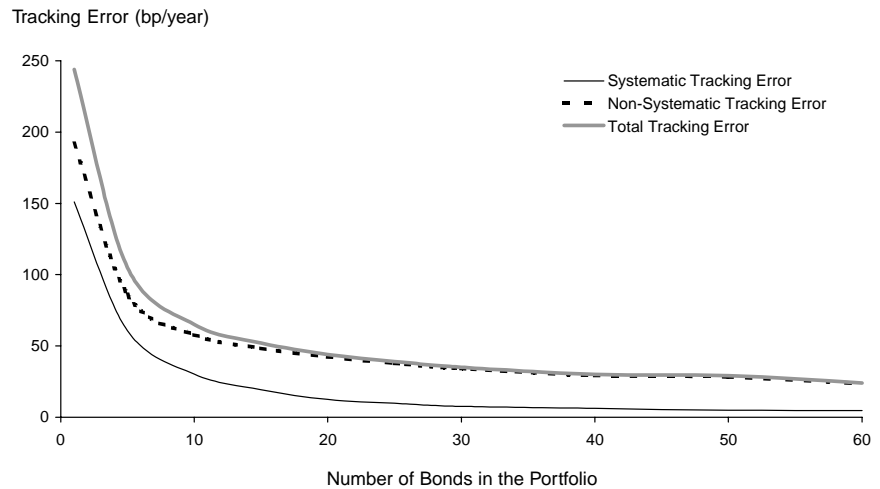
The U.S. multi-factor risk model has been extensively used over many years to construct replicating portfolios for investment-grade credit portfolios. A graph in Figure 3 shows the dynamics of tracking error decline as the number of bonds in the proxy portfolio grows. It is evident from this graph that it takes only 25-30 securities to essentially eliminate systematic risk, leaving only the special or idiosyncratic component.

<sup>3</sup> *The Replication of the Lehman Global Aggregate Index with Cash Instruments*, in *Global Relative Value*, September 2001.

Figure 2. **Replication of the Global Aggregate Index with Proxy Portfolios**  
February 1999–May 2001

	Treasury-Only	Treasury-Plus	All-Sectors
Average Number of Bonds in the Replicating Portfolio	22	28	51
USD Tracking Error (bp/month):			
Hedged	11.7	6.3	4.2
Unhedged	13.3	7.7	5.9
USD Average Monthly Return Difference (bp):			
Hedged	-2.7	-0.8	0.0
Unhedged	-2.4	-0.4	0.5

Figure 3. Portfolio Tracking Error as a Function of Number of Bonds



The tracking error minimization approach has also been successfully applied to replicate the Lehman MBS Index.<sup>4</sup> Unlike most bond indices, the MBS index contains only non-traded “generic” securities. As a result, the replication process has to start with forming the tracking proxy portfolio out of these generics and then proceed to select actual mortgage pools for each generic. The added layer of decision-making has a potential for additional tracking error.

The replication of the MBS Index by stratified sampling techniques described above would direct investors to buy certain amounts of seasoned product at the outset of the portfolio. The obvious benefit is the close replication of the benchmark from the very beginning. However, seasoned pools may be difficult to obtain. More importantly, investors buying seasoned products might be delivered small pools that create a real possibility for the added tracking error. Ensuring that all seasoned holdings are implemented with large pools is difficult, if not impossible, in practice. Finally, MBS replication by sampling requires the investor to have full back-office capabilities for MBS transaction processing from day one. To address this problem we developed two practical strategies that rely on the Lehman risk model and let the investor build MBS transaction processing capabilities gradually while being fully invested in the market. Both strategies create an optimized mortgage proxy portfolio at the end of each calendar quarter. The two strategies differ in terms of their definition of the investable set. The first strategy (TBA-only) uses only TBA contracts on

<sup>4</sup> *Tradable Proxy Portfolios for the Lehman Brothers MBS Index*, Lehman Brothers, July 2001.

actively traded, recently originated mortgage coupons. As the composition of the new-issue mortgage market changes over time, this strategy adjusts its holdings of TBA contracts so as to always reflect the most recent and active portion of the mortgage market. The second strategy (Large Pools-only) buys large MBS pools of current production but allows some holdings to remain in the portfolio and season over time. In both cases, the resulting proxy portfolio is rebalanced quarterly. The results for both strategies are summarized in Figure 4.

### Replication with Derivatives

Derivatives can effectively reduce the number of dimensions in the portfolio management problem. Their use also simplifies asset allocation shifts and deployment of cash inflows. Managing cash with derivatives that replicate the benchmark risk parameters can be especially useful in the startup phase, when diversified cash investments in tradable sizes are not feasible. Over the last years we conducted a number of studies on the replication of bond indices with derivatives.

The earliest and simplest technique developed was a variation of the cell-matching approach applied to the replication of the term structure exposure of any fixed-income index with Treasury futures<sup>5</sup>. We have developed and tested a methodology, implemented by a number of investors, that employs a proper mix of four Treasury futures contracts (2-, 5-, 10-, and 30-year) to replicate the curve allocation of an index. The index is divided into four duration cells, and market value allocations and dollar duration of each are matched with a combination of cash and an appropriate futures contract. The cash can be invested in Treasury bills or other, higher-yielding, short-term alternatives such as commercial paper or short-term asset-backed securities.

While term structure exposure can be hedged effectively with Treasury futures, spread risk needs to be hedged separately. The next step in the development of derivatives replication techniques was to introduce Eurodollar futures and swaps in a similar methodology to

<sup>5</sup> *Replicating Index Returns with Treasury Futures*, Lehman Brothers, November 1997

Figure 4. **Performance Summary (bp) for the MBS Index Replication Strategies**  
January 1994–May 2001

Monthly Return Difference (Portfolio vs. the Index)	TBAs-Only	Large Pools-Only
Average	-1.2	0.2
Minimum	-18.7	-9.2
Maximum	17.6	23.6
Standard Deviation (Realized Tracking Error)	6.0	4.4
Annualized Realized Tracking Error	20.9	15.2

replicate spread indices.<sup>6</sup> Credit spreads are positively correlated with the TED spread and swap spreads, so replication strategies based on these instruments are better suited to such spread benchmarks as the Lehman Credit and Mortgage indices.

The replication of the Global Aggregate Index extended our derivatives replication capabilities to cover essentially all global fixed-income markets.<sup>7</sup> The Global Aggregate Index presents investors with a portfolio management problem involving multiple yield curves, exchange rates, as well as credit quality and sector components. The currency hedging issue adds another decision making dimension. This diversity of exposures makes the Global Aggregate a particularly good candidate for replication with derivatives.

The replication technique handles index components separately and then combines the resulting portfolios. We showed that the replication of the four largest markets (USD, EUR, JPY, and GBP) is sufficient for a close overall index tracking. All the decisions involved in single currency index replication must be made for each index component. Each local market is replicated when both the term structure and spread exposures are matched.

The multi-currency aspect of this index complicates the issue of cash investing. One alternative is to invest proportional amounts of cash in 1-month deposits in the respective local currencies. This approach matches the currency exposures of the local index components and is a practical strategy for replicating the unhedged index. Another option is to put all cash in the base currency. This is a realistic strategy for managers who replicate the hedged index returns.

Figure 5 presents the replication results for both unhedged and hedged Global Aggregate Index. Two distinct replication strategies are shown: one uses all-Treasury futures portfolio; the other adds swaps. In the all-Treasury portfolio, the U.S. Aggregate portion of the index is replicated with a portfolio of four Treasury futures contracts. The Euro-Aggregate replication uses a portfolio of 2-, 5-, and 10-year German Treasury futures contracts. The replications of the Japanese Yen and Sterling components both use a single 10-year Treasury futures contract.

## Conclusion

We have developed and tested a number of techniques that can be used to replicate most of the widely used Lehman indices, from the single-currency single-exposure Treasury Index to the High Yield Index to the extremely diverse multi-currency multi-sector Global Aggregate Index. The methods we studied and tested historically allow index replication with either liquid proxy portfolios or with derivative instruments such as futures and swaps in various currencies.

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<sup>6</sup> *Replication of Index Returns with Treasury Futures, Eurodollar (Euribor) Futures and Swaps*, Lehman Brothers, March 2000.

<sup>7</sup> *Replication with Derivatives: The Global Aggregate Index and the Japanese Aggregate Index*, Lehman Brothers, March 2001.

Figure 5. **Replication of the Global Aggregate Index with Derivatives**  
February 1999–May 2001

	U.S. Aggregate Treasury Futures	Euro Aggregate Treasury Futures	Yen Aggregate 10-year Treasury Futures	Sterling Aggregate 10-year Treasury Futures	Unhedged Index		Hedged Index	
					Tracking Error	Mean Outperform	Tracking Error	Mean Outperform
All- Treasury Strategy					20.8	4.2	20.5	1.5
Treasuries and Swaps Strategy	Treasury Futures and Swaps	Swaps	Swaps	Swaps	10.9	2.0	11.5	-0.8

We demonstrated that with either approach we can replicate the broadest market indices with tracking errors that are acceptable to the majority of investors. All strategies are flexible and can be easily customized along multiple dimensions to suit particular needs of individual portfolios.

In the weeks and months after the disaster that sent the global financial markets into turmoil, we expect that passive benchmark replication may prove to be the strategy chosen by many managers to control their risks until the markets stabilize.