LEHMAN BROTHERS

Fixed Income Research

Replication with Derivatives: The Global Aggregate Index and the Japanese Aggregate Index

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EXECUTIVE SUMMARY

- We replicate the returns of the Lehman Brothers Global Aggregate Index with derivatives using several different strategies. The replication is performed with combinations of Treasury bond futures, money-market futures, and swaps in four currencies: U.S. dollar, euro, yen, and sterling. We achieved a tracking error of 14.8 bp/month versus the unhedged index return, and 13.9 bp/month against the currency-hedged version of the Global Aggregate, over the last 20 months.

- We show that it is sufficient to replicate just these four bond markets, which make up over 95% of the index. Full replication of all bond markets and currencies in the index would entail much higher transaction costs and might decrease tracking error by only 1 or 2 bp/month.

- Recent changes to the Global Aggregate Index have increased the contribution of the Asian market from 14% to 24%. This report includes a detailed study of replication strategies for the Japanese Aggregate Index, the one major market segment not covered in our previous publications.

- Once the core currencies are covered, the tracking errors obtainable by a replication strategy depend strongly on the selection of derivatives used to replicate each local market. We study the effect of combining different strategies based on the correlations among their tracking errors.

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INTRODUCTION

The risk diversification effect offered by the Lehman Brothers Global Aggregate Index makes it a very attractive benchmark for investors. Its comprehensive coverage of global investment-grade fixed income markets, the broad acceptance of its underlying component indices, and the addition of Asian corporate bonds make it a benchmark of choice for asset-allocation funds, and investors interested in global spread products.[1]

The management of a bond fund benchmarked to the Global Aggregate requires particular skills and resources. Decision making in the case of the Global Aggregate benchmark has many more dimensions than in the single-currency case: there are 10 exchange rates, 11 yield curves, and often several credit quality and sector components for each currency's index.[2] In addition, the potential for currency hedging opens up another decision dimension that allows the manager to separate the currency view from the fixed-income investment decision. Resources are needed at the macro level, as well as the yield curve, credit and institutional expertise at the individual market level, in order to be able to manage all these aspects of the decision process. Only very large asset-management organizations will be able to do this. Outside of such organizations, the possibility of subcontracting the management of some of the individual currency portfolios exists. This, though, can lead to a blurring of responsibilities and introduce informational inefficiencies between plan sponsors, the main manager, and the specialist managers acting in the individual markets.

Derivatives can play an important role in reducing the decision dimensions the global investor is facing. In addition, they are a tool that can be used in many more investment applications that are relevant to all benchmarked investors.

Other important issues raised by a Global Aggregate benchmark are the efficiency by which asset-allocation moves can be best accomplished and the investment of cash balances in such funds. Moving large balances from one currency's bond market to another bond market can be a time consuming and expensive task. Investment into some of the smaller or less liquid markets and sectors can be taxing. If substantial cash balances are allowed to accumulate (e.g. from cash inflows or coupon payments), there is a potential for large tracking errors to develop. Managing cash with derivatives through replication can be an especially useful approach for funds in the startup phase, where diversified cash investments in tradable sizes are not feasible in the less liquid markets.

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[1] See the soon-to-be-published Lehman Brothers *Global Aggregate Index Primer.*
[2] These numbers are as of 9/30/00. The replacement of the JGB index, and the Australian and the New Zealand Treasury indices in the global Aggregate with the new Asian-Pacific Aggregate index will add another four fixed income markets, denominated in four different currencies, to the index.

Building on our research on index replication with derivatives,[3] we will show how replication techniques using futures and swaps can provide an efficient way of supporting the management of such a portfolio. Replication with derivatives of some of the indices that are part of the Global Aggregate can provide an effective way for a manager to track very closely the markets where he has no expertise, and actively manage the markets where he has expertise, and where active management matters. Similarly, liquid exchange-traded derivatives can help facilitate rapid asset allocation shifts between currencies and yield curves. If a fund has to keep a significant amount of cash, it may experience substantial deviations from index returns. Derivatives, which require essentially zero cash outlays up front, can be used to replicate the index returns for the amount of the cash holdings to achieve overall closer index tracking.

Baskets of futures, or portfolios of swaps, can be used to outperform their respective indices, when coupled with some aggressive ways of investing cash. The amount left over after initial margin and some reserve for future variation margin is then invested in sectors where the manager has strong expertise, for example short-term High Yield product, or ABS. The return on the alternative investment chosen by the manager, plus any over- or underperformance of the tracking futures portfolio, determines the total outperformance of the manager versus the benchmark. The term “portable alpha” is often used to describe this style of portfolio management. In this case the replicating basket of futures allows the portfolio manager to concentrate fully on asset classes of choice, whether or not they are part of the benchmark.

Replication is closely related to hedging. The return on a replicating portfolio differs from the return of the equivalent hedge portfolio simply by the additional return that the replicating portfolio achieves from investing the cash. If this investment does not add significantly to the volatility of the hedge portfolio, then our results concerning tracking errors of the replicating portfolios offer important insights for the tracking errors of the equivalent hedging portfolios.

The focus of this study is tracking error, which measures the risk that investors are subject to when using replication strategies. We also publish the historically experienced mean outperformance versus the index for each strategy. This is done so that investors have an indication of the cost of the strategies in terms of their performance difference to the index. For the same reason, our results are before transactions costs. However, the comparatively short replication history for the Global Aggregate does not allow us to draw reliable conclusions about the long-term costs of the replication strategies.

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[3] See the Lehman Brothers publications: *Replicating Index Returns with Treasury Futures*, by Lev Dynkin, Jay Hyman, and Wei Wu, November 1997, and *Replication of Index Returns with Treasury Futures, Eurodollar (Euribor) Futures, and Swaps*, by Lev Dynkin and Peter Lindner, March 2000.

Given this de-emphasis of mean returns in this study, we have chosen not to model some of the finer points of index replication that affect the mean performance but not the tracking error. First, our results do not include any estimate of transaction costs.[4] Second, we do not model the initial and variation margin that is required of futures market participants. Initial margin can be up to two per cent of notional for many fixed income futures. Variation margin payments occur when a futures market participant receives or delivers cash due to a change in the value of his position. The net result of the margin rules is that this small portion of the portfolio earns interest differently from the rest of the portfolio. In this study, we assume that 100% of portfolio cash is available for investment as called for by the strategy.

Outline

This publication completes in terms of market coverage our two existing studies on the replication of bond indices with derivatives (see footnote 3). To replicate the Global Aggregate Index, we complement the replication of the U.S. Aggregate and of the Euro-Aggregate Indices with replication portfolios for the Sterling Aggregate and the Japanese Aggregate indices.

This paper is divided into two parts. The first addresses the replication of the Global Aggregate Index by combining replicating portfolios for the dominant component markets. In this section, we assume that an appropriate methodology has been selected for each component, and focus on the new issues that arise due to the global nature of this index. These include the replication of both hedged and unhedged returns, the number of markets that need to be replicated, and the treatment of yield curve and currency risk for the markets that are not fully replicated.

The second part of this paper presents a detailed study of index replication in the Japanese market. This market deserves special attention for several reasons. First, changes to the Global Aggregate Index as of October 1, 2000 have increased the market share of the Asia-Pacific Aggregate Index to 24%, roughly equal to that of the Euro-denominated portion. Second, replication in the Japanese market faces some unique challenges. Derivatives with sufficient liquidity have not been as broadly available there as in the U.S.-Dollar and the Euro area. In addition, the Japanese Government Bond market has been well-known for the relative illiquidity of many of its issues, given the size of the market as a whole and the average issue size.

Finally, withholding taxes on corporate bonds in Japan complicate investing in Japanese credit for many foreign investors. Derivatives are not subject to withholding tax rules, and therefore can simplify the investment process significantly.

[4] Our work on replication of the U.S. Aggregate Index indicates average transactions cost of about 1 bp/month. Since this number was conservatively estimated, it should provide a good estimate of the monthly cost of a Global Aggregate replication portfolio, when the investor concentrates the replication on the largest markets.

REPLICATING THE GLOBAL AGGREGATE INDEX

Index Composition and Return Volatility

In our studies of index replication with derivatives in a single currency (U.S. Aggregate Index, Euro-Aggregate Index), there were two main areas of focus: matching interest rate exposures across the term structure, and the level of spread exposure attainable by using swaps or LIBOR-based futures. As we extend the methodology to the Global Aggregate Index, another important consideration is how closely to match exposures to each global market, in terms of both interest rates and foreign exchange rates.

Fortunately, the Global Aggregate Index is highly concentrated in the largest three or four currencies. As illustrated in Figure 1, 91.5% of index market value is concentrated in the US, Euro and Japanese markets. With the addition of the UK market, the top four currencies comprise 95.5% of the index. All of these markets have liquid bond futures contracts. The strategies used in this paper to replicate the Global Aggregate are thus based on combinations of replicating portfolios for these top four markets.

As of October 1, 2000, the Global Aggregate experienced a significant realignment. The share of Asian bonds in the Index has increased from 14 per cent to 24 per cent. As the details provided in Appendix 1 show, almost all of this increase in the Asian component is due to an increase in Yen-denominated bonds, in both the government and credit sectors. Our tracking error results should therefore carry over to the new Global Aggregate.

Figure 1. **Composition of the Global Aggregate Index**
Original and Current, by Currency, as of September 30, 2000

	Global Agg. Original		Global Agg. Current	
Currency	**% of Index**	**Cumulative**	**% of Index**	**Cumulative**
US Dollar	48.5	48.5	44.0	44.0
Euro	28.9	77.4	24.1	68.1
Japanese Yen	14.1	91.5	23.4	91.5
British Pound	4.8	96.3	4.0	95.5
Canadian Dollar	1.5	97.8	1.4	96.9
Swedish Krona, Danish Krone, Greek Drachma, Norwegian Krone	1.9	99.6	1.8	98.7
Taiwan Dollar, South Korean Won, Malaysian Ringgit, Singapore Dollar	0.0	99.6	0.9	99.6
Australian Dollar, New Zealand Dollar	0.4	100.0	0.4	100.0

Investors have diverse goals, and will use the Global Aggregate Index in different ways. Returns on the Global Aggregate Index, like any multi-currency index, can be stated in terms of any base currency, either on a hedged or unhedged basis. Single-currency portfolios might include bonds denominated in other currencies, hedged back to the base currency, to diversify their exposures to interest rate risk without taking currency risk. For such portfolios, the hedged returns on the Global Aggregate Index might be the appropriate benchmark. Multi-currency bond portfolios are typically benchmarked against the unhedged returns of a multi-currency index, such as the Global Treasury Index. The increasing role of credit across global markets is prompting many such investors to switch to the unhedged Global Aggregate Index, which includes spread products.

An analysis of index return volatilities illustrates the diversification advantage offered by the hedged Global Aggregate Index relative to a single-currency index. As shown in Figure 2, the four largest local components of the Global Aggregate (U.S. Aggregate, Euro Aggregate, Japan Aggregate, and Sterling Aggregate) had return volatilities ranging from 80.1 bp/month to 113.7 bp/month for the time period shown.[5] The return volatility of the hedged Global Aggregate Index was close to 70 bp/month, regardless of base currency. The inclusion of spread product in this index seems to add only about 1 bp/month of volatility relative to the hedged Global Treasury Index.

[5] The data shown in Figure 2 cover only 20 months, due to the recent inception dates of some of the indices shown. Also, historical data on Japanese spread products were unavailable, so the Yen data are based on the Japanese Treasury Index. We do not report mean returns, as the time span is far too short for any meaningful evaluation of the risk/return tradeoff. The return volatilities, derived from second moments, can be expected to be more stable over time than mean returns. While it would certainly be advantageous to analyze returns over a longer time period, the qualitative relationships observed here are likely to hold true over any time period.

Figure 2. **Index Return Volatilities and Tracking Errors,** by base currency, bp/month, February 1999-September 2000

	Dollar	Euro	Yen	Sterling
Index Return Volatilities				
Local Currency Component of Global Agg	91.2	80.1	100.2	113.7
Hedged Global Aggregate	69.9	70.9	67.1	68.7
Hedged Global Treasury	68.5	69.5	65.8	67.7
Unhedged Global Aggregate	141.0	154.0	241.3	176.1
Unhedged Global Treasury	173.2	145.2	215.7	186.4
Volatilities of Index Return Differences				
Hedged Global Agg vs. Local Component	40.7	35.7	113.7	97.3
Hedged Global Agg vs. Unhedged Global Agg	109.7	138.0	246.2	166.9
Hedged Global Agg vs. Hedged Global Treasury	19.8	19.9	19.5	20.0
Unhedged Global Agg vs. Unhedged Global Treasury	43.3	43.1	42.1	42.8

When currency exposures are not hedged, volatilities increase dramatically. The return volatility of the unhedged Global Aggregate Index is significantly higher than that of any single currency index, ranging from 141.0 bp/month in U.S. dollars to 241.3 bp/month when expressed in terms of Japanese Yen. This shows that the returns on the unhedged indices are primarily driven by currency movements. The differences in the volatilities of unhedged returns in different base currencies is due to the combination of two effects: the foreign exchange volatility of the base currency and the percentage of the index in the base currency. The smaller the part of the index in the base currency, the greater the part of the index that is exposed to currency risk. This latter effect also helps explain the differences in unhedged return volatilities between the Global Aggregate and the Global Treasury indices. The relatively large spread product components in the U.S. Aggregate and Sterling Aggregate indices give these markets a larger share in the Global Aggregate than they have in the Global Treasury Index, and hence the Global Aggregate has a lower return volatility when viewed in these base currencies.

To sharpen the focus on different components of return, Figure 2 also shows the volatilities of return differences (tracking errors) between different pairs of indices. For a U.S. dollar-based investor, the U.S. Aggregate index tracks the hedged Global Aggregate to within 40.7 bp/month; the differences between the hedged and unhedged returns on the Global Aggregate are much greater, at 109.7 bp/month. This pattern repeats itself across all currencies, driving home the point that the returns on the unhedged index are dominated by currency risk.

The return differences between the Global Aggregate and Global Treasury indices are largely due to the differences in currency allocation, as discussed above. This is evidenced by the fact that the volatilities of the differences in unhedged returns are more than double that of the hedged case.

Forming a Replication Strategy

Essentially, replicating the Global Aggregate Index is accomplished by separately replicating several of its components, and combining the replicating portfolios. All of the decisions involved in single currency index replication must be made for each such component. Additionally, the multi-currency aspect of the index opens up several new decision dimensions in the formulation of the replication strategy. How many of the component markets should be included in the replication scheme? Where is the cash invested? How do we account for the yield curve and currency exposures of the markets excluded from the replication scheme? This set of decisions is summarized in Figure 3.

For each local market to be replicated, one must choose a replication strategy to best match the term structure and spread exposures in that market, just as in the single-currency case. In any market, the best way for a passive investor to match index returns is to actually buy a portfolio of representative index securities. However, especially at the global level, this approach entails difficulties of several kinds, including lack of liquidity, necessity of local market expertise, and transaction costs. Withholding taxes on interest payments add to the complexity of investing in cash bonds in a number of markets. The goal of our studies of replication using derivatives has been to achieve the smallest tracking error without buying index bonds. In previous work, we have explored various combinations of bond futures, money market futures, and swaps in the U.S. and Euro markets. Later in this paper, we offer a detailed analysis of replication strategies for the Japanese market. In each case, decisions must be made of how many instruments should be used, and which ones. These decisions will determine how closely a strategy will track the term structure and spread exposures of each local market.

The basic assumption in this replication study is that a manager has been given funds to invest against the Global Aggregate Index. Derivatives like futures and swaps, which are used to replicate the interest rate exposures of the index, do not require a significant outlay of cash. A small amount may be required for initial margin, variation margin, and collateral payments; this leaves the majority to be invested in short term securities. In our replication of single currency indices, we have assumed that the cash is invested in short term securities in the same currency

Figure 3. **Decisions in Forming a Replication Strategy for the Global Aggregate Index**

Local Market (Term Structure and Spread)
- Cash Bonds
- Bond Futures
- Money Market Futures
- Swaps

Where to place cash?
- Local market
- Home market

How many currencies to replicate?
- Two
- Three
- Four
- All

How treat curve risk for currencies not included in replication?
- Ignore
- Proxy with replication of most correlated major currency

How treat currency risk for currencies not included in replication?
- Keep cash in base currency
- Spread cash proportionally among selected currencies
- Place cash in most correlated major currency (proxy approach)
- Hedge all currency risk

as the index. In the case of the Global Aggregate, the manager has the freedom to invest the cash in any short term market around the globe. These cash positions may be used to match the currency exposures of the index, or currency forwards may be layered above the cash position. For example, a Japanese investor replicating the Euro-Aggregate component of the index may choose to invest the cash in U.S. asset-backed floaters and cross-hedge the currency exposure back to Yen in the forward market.

For simplicity, we will limit our analysis to two cases. In the first case, for each market that is replicated, the proportional amount of cash will be invested in 1-month deposits in the local currency. The Euro-Aggregate will be replicated by Euro futures and swaps and Euro time deposits, the U.S. Aggregate will be replicated by U.S. futures and swaps and U.S. dollar deposits, and so on. This approach matches the full currency exposures of each index without requiring additional hedging with currency forwards; this is a very practical strategy for replicating the returns of the unhedged index. In the second case, all cash is assumed to be invested in the base currency. This is an appropriate strategy for managers who aim to replicate the hedged index returns.

How many markets need to be included in the replication strategy? As discussed above, the index is highly concentrated in the largest currencies, with more than 90% of the index market value in the top three and over 95% in the top four. Matching the index exposures in these markets should produce a sufficiently close tracking of index returns. Additionally, most of the smaller currencies are not well represented by liquid futures markets. Accordingly, our primary focus is on replication strategies based on the U.S. Dollar, Euro and Yen markets; we examine the incremental improvement in tracking that can be achieved by replicating the Sterling market as well.

How do we treat the smaller markets, for which we do not explicitly replicate the bond market risk? One possibility is to leave the smaller markets completely out of the replication. The percentage of exposure that the index allocates to those markets is then proportionally allocated to the largest markets according to their market capitalization in the index.

Another possibility, based on the correlations among interest rates, is to replicate an index interest rate exposure in a small market by taking a corresponding exposure to rates in a larger country with similar economic characteristics. For example, the replication strategy can treat the Canadian dollar component of the Global Aggregate as increasing the contribution of the U.S. Aggregate, and the Malaysian Ringgit as an increase to the Yen component. We denote this procedure as *proxy replication.*

Similarly, for the case of the unhedged index, how do we handle the currency exposures of these smaller markets? One approach would be to match all currency exposures via the cash portfolio. That is, even if the replicating portfolio of derivatives is limited to three or four currencies, one could distribute the cash among all index currencies in the right proportions. A somewhat simpler scheme, which might make sense when proxy replication is used for the interest rate risk, is to extend this approach to the currency profile as well. The cash position is then distributed among the replicating currencies in the same proportions that were used for the bond market weights. If proxy replication is not used, then the excess cash can be left all in the base currency, or it may be distributed among the replicating currencies proportionally to their weights.

In the proxy replication scheme used here, the returns of all smaller markets are divided into three blocs, and each bloc is represented by the replication portfolio of a single reference market. Denmark, Norway, Sweden, and Greece are replicated with the Euro-Aggregate replication portfolio; Australia, Canada, and New Zealand are replicated with the U.S. Aggregate replication portfolio; for Taiwan, South Korea, Malaysia and Singapore we use the Japanese Yen replication portfolio. This means that the smaller country's currency returns are replicated by the reference markets' currency returns, too.

Performance Summary

Unhedged Global Aggregate

Figure 4 shows the results achieved in replicating the unhedged returns of the Global Aggregate Index. The U.S. Aggregate Index is replicated by using either a portfolio of four Treasury futures contracts, or by a hybrid strategy consisting of Eurodollar futures contracts up to five years along with 10-year and 30-year swaps.

Figure 4. **Tracking Errors for the Unhedged Global Aggregate Index Dollar based Returns**, bp/month, February 1999-September 2000*

U.S. Aggregate	**Euro Aggregate**	**Yen Treasury Index**	**Sterling Aggregate**	**Tracking Error** (bp/m)	**Mean Outperf.** (bp/m)
Treasury Futures	Treasury Futures	None	None	42.5	-1.0
Eurodollar Futures and Swaps	Euribor and Treasury Futures	None	None	40.6	-11.7
Treasury Futures	Treasury Futures	10-yr Tsy Futures	None	24.5	1.3
Eurodollar Futures and Swaps	Euribor and Treasury Futures	10-yr Tsy Futures and Swaps	None	16.2	-7.9
Treasury Futures	Treasury Futures	10-yr Tsy Futures	10-yr Tsy Futures	21.4	4.7
Eurodollar Futures and Swaps	Euribor and Treasury Futures	10-yr Tsy Futures and Swaps	10-yr Tsy Futures	14.8	-4.4

*Using proxy replication.

The Euro Aggregate Index is replicated either with a portfolio of German Treasury futures contracts or a hybrid strategy of Euribor contracts up to four years with long German Treasury futures. All strategies shown include replication of these two markets. We then see how the tracking is improved by the addition of replicating portfolios for the Yen and Sterling markets. The Japanese Treasury Index is replicated either using the 10-year Treasury bond futures contract alone, or with a hybrid strategy that combines this contract with 2-, 5- and 20-year swaps, as detailed in the next section of this paper. The Sterling Aggregate is replicated with the Sterling 10-year Treasury futures contract. In each case, cash is assumed to be invested in each of the currencies used in the replication strategy. All other currencies and bond markets are represented using the proxy replication technique described above. (When the Yen Index is not included in the replication, it is proxied by the U.S. Aggregate; the Euro Aggregate serves as the proxy for the Sterling market.) The numbers shown were calculated using a base currency of U.S. dollars, but the results change only slightly when other base currencies are used, as shown in Appendix 2.

The results for different strategies show how the tracking error declines as more markets are replicated, and as more precise tracking methodologies are used to replicate each market. The large reduction in tracking error as the Japanese Treasury index is added to the replication shows that proxy replication is not sufficient to cover this large component of the market. Matching the exposure to the Japanese Yen interest rate and currency exposures offers a much bigger improvement than fine tuning the replication within the U.S. and Euro markets. Once these three markets are covered, however, this is no longer the case. To improve upon the 24.5 bp/month tracking error achievable using Treasury futures in these three markets, we can change the methods used within each of these markets to the more effective hybrid strategies, or we can add a replication portfolio for the Sterling Aggregate. In this case, the improvement in the replication of the local markets is more effective, reducing the tracking error to 16.2 bp/month. The all-Treasury futures replication in the four markets gives a tracking error of 21.4 bp/month. (The small reduction in tracking error obtained by adding the Sterling replication is due to the small size of the Sterling Aggregate, which comprises only about 4 percent of the Global Aggregate.) When all of these enhancements are included, our replication strategy achieves a tracking error of 14.8 bp/month.[6]

The mean outperformance numbers shown in Figure 4 seem to indicate that the replication strategies using Treasury futures turn in steadily higher returns than those using swaps and Libor-based futures. We do not feel that this should be interpreted

[6]The Sterling Aggregate was only replicated with Treasury futures contracts. Any additional contribution towards a reduced tracking error for the Global Aggregate from using other derivatives can be expected to be small.

as a projection of the relative performance of these strategies in the future, for several reasons. First, the period that our simulations cover was only 20 months. This is too short a period to draw definite conclusions for future performance.

Second, market particulars can have a substantial influence on the relative performance of different replication strategies from a mean tracking error perspective. Since about 1994, the 10-year Treasury futures contract in the U.S. has traded cheap compared to the prices predicted by many models. This resulted in extra return for an investor who was consistently long in the futures contract, compared to an investor who invested in similar maturity cash Treasury bonds. In addition, volatility drifted consistently lower over this period, resulting in additional returns for the futures buyer. Such market conditions can change and result in the opposite direction of the outperformance of particular derivatives portfolios.

Thirdly, from about 1994 onward, swap spreads have increased consistently. That lead to an underperformance of swaps compared to treasury futures and portfolios that included large chunks of Government bonds. But even if swap spreads were to increase at the same rate in the future, this would not lead to more underperformance of swaps. Swap yields are currently so high that the additional income would approximately neutralize such a spread widening. The mean outperformance numbers in Figure 5 therefore represent only a picture of what happened in the past.

How do the tracking errors from the replication of the Global Aggregate compare to the tracking errors that the local currency replication portfolios exhibit versus the local currency components of the Global Aggregate? Figure 5 displays the volatilities and means of the outperformance of the replication portfolios set up to replicate the four largest local component markets in the Global Aggregate. Comparing the local market tracking errors with the tracking errors in Figure 4, we find that the 14.8 bp/month that we were able to achieve for the Global Aggregate is less than the tracking errors of most of the single-currency index replications. Only the Euro-

Figure 5. **Means and Tracking Errors of Outperformance of Replication Portfolios versus Corresponding Indices**, **Local Currency Returns,** bp/month
February 1999-September 2000

Index	**U.S. Aggregate**			**Euro-Aggregate**		**Japanese Treasury**			**Sterling Agg.**
Replication Port.	**Tsy Futures**	**ED-Futures and Tsy Fut.**	**ED-Futures and Swaps**	**Tsy Futures**	**EUR-Futures and Tsy Futures**	**10-yr. Tsy Futures**	**10-yr. Tsy Futures and Swaps**	**Swaps**	**Long Gilt Futures**
Tracking Error	33.4	25.2	21.9	15.5	12.9	46.5	28.1	18.9	57.3
Mean	5.4	4.5	-11.7	3.8	2.9	7.3	6.1	2.3	-10.3

Aggregate Index was replicated with a lower tracking error (12.9 bp/month). This is due to the diversification benefit that the Global Aggregate enjoys. Its outperformance is essentially a combination of local market outperformance values, which are less than perfectly correlated.

Hedged Global Aggregate

When the goal of the replication strategy is to match the returns of the hedged index, we use a slightly different version of the strategy. Local market returns are replicated, as before, by buying futures or other derivatives contracts. However, the available cash is now invested in the home currency, rather than in the local currency. As the derivatives require zero investment up front, they do not incur the currency risk to principal that would come with a cash bond purchase. Only cash flows or a change in value of the interest rate derivatives are affected by currency risk. This second-order effect is very similar to the small currency returns experienced by a currency-hedged foreign bond market investment. We will refer to this method of replication, in which the currency risk is hedged indirectly by keeping cash in the base currency, as *implicit currency hedging*. Appendix 2 offers a formal treatment of this approach, and shows that it is approximately equivalent to conventional currency hedging.

Figure 6 shows the replication results for this strategy for replicating the returns of the Global Aggregate Index hedged to U.S. dollars. The tracking errors are consistently smaller than for the unhedged case shown in Figure 4. This effect is not very significant when all of the four major markets are being replicated. But,

Figure 6. **Tracking Errors for Global Aggregate Index Based on Implicit Currency Hedging Dollar based Returns,** bp/month, February 1999-September 2000*

U.S. Aggregate	Euro Aggregate	Yen Treasury Index	Sterling Aggregate	Tracking Error	Mean Outperf.
Tsy Futures	Tsy Futures	None	None	28.9	1.4
Eurodollar Futures and Swaps	Euribor and Tsy Futures	None	None	23.6	-9.3
Tsy Futures	Tsy Futures	10-yr Tsy Futures	None	19.8	6.0
Eurodollar Futures and Swaps	Euribor and Tsy Futures	10-yr Tsy Futures and Swaps	None	14.5	-3.2
Tsy Futures	Tsy Futures	10-yr Tsy Futures	10-yr Tsy Futures	20.5	4.9
Eurodollar Futures and Swaps	Euribor and Tsy Futures	10-yr Tsy Futures and Swaps	10-yr Tsy Futures	13.9	-4.2

*Proxy Replication used.

for example, when only the U.S. Aggregate and the Euro Aggregate are replicated, the hedged tracking errors shown here are much smaller than for the unhedged case. Why is that?

As we discussed above, the risk/return characteristics of the Global Aggregate Index are very different when viewed on a currency hedged or unhedged basis. The unhedged returns are dominated by currency risk, making them much more volatile. Thus, the first two rows of Figure 6 show tracking errors of 28.9 and 23.6 bp/month when only the US and Euro markets are replicated. The comparable tracking errors in Figure 4 are 42.5 and 40.6 bp/month, respectively. They incorporate currency volatility, whereas the ones in the first two rows of Figure 6 are essentially free of currency risk.

On the other hand, the tracking errors that can be achieved by a replication strategy that encompasses almost all markets, are approximately the same for the two cases. This is true because whichever currency allocation is desired (distributed with index weights for the unhedged case, or all in base currency for the fully hedged returns), it can easily be achieved in the forward markets. The primary source of the replication errors are the different ways in which they achieve their local market yield and spread exposures. In fact, in Appendix 2 we prove the equivalence between the replication errors in the hedged and unhedged cases, based on the assumption that the portfolio hedging transactions can be executed at the same levels as used in the index calculations.

Of course, this equivalence only holds in the ideal case, in which the replicating portfolio takes exposures to all currencies in the index, using the same hedging levels. The results shown in Figures 4 and 6 do not meet these criteria. The proxy replication method used in these simulations matches index currency exposures only in the replicated markets. This leaves a significant currency exposure when the Yen market is omitted from the replication in the unhedged case.

Our most detailed replication strategy, using four markets, produced a tracking error of 14.8 bp/month in the unhedged case, and 13.9 bp/month in the hedged case. How much of an improvement could we expect from replicating the smaller markets in addition to the larger markets? We know from the discussion of Global Aggregate Index return volatility, that currency volatility accounts for most of the index volatility, and therefore also of tracking error volatility. The difference of the above two tracking errors is 0.9 bp/month. This difference is due to currency volatility. Thus, any interest rate volatility can be expected to produce less than 1 bp/month in incremental tracking error.

Thisis supported by another way of estimating the potential maximum improvement due to adding the remaining seven markets left out of the replication. For this, we compare the returns of our replication portfolio based on the replication of four markets to the returns of a reference index consisting of only the four largest markets. These two portfolios exhibit a tracking error of 13.1 bp/month against each other. This tracking error is solely due to the error in tracking yield curve and spread movements. Comparing this to the 14.8 bp/month in the last row of Figure 4, we find the improvement in tracking error limited to less than two bp/month.

Replication Details

What does a replication portfolio look like? How does it perform? Figure 7 provides detailed information for a derivatives portfolio that was designed to track the Lehman Brothers Global Aggregate Index during the month of June 2000.

The results are for a portfolio with a market value of $ 1 Billion as of May 31, 2000. The replication strategy illustrated here used Eurodollar futures and swaps for the U.S. Aggregate, a Euribor/Treasury futures portfolio for the Euro-Aggregate, a portfolio combining the Japanese 10-year Treasury futures contract with swaps, and Sterling 10-year Treasury futures. When considering the number of contracts in the replication, it should be kept in mind that the notionals of the 10-year and the bond contract in the U.S. are $ 100,000, the notional of the German 10-year futures is Euro 100,000, and the notional of the UK 10-year contract is Sterling 100,000. The Japanese 10-year contract has a notional of Yen 100,000,000, which is at current exchange rates about 10 times larger than the notional of the U.S. 10-year contract. The swaps notional is

Figure 7. **Results of Global Aggregate Index Replication for June 2000**

		No. Futures Contracts/Swaps Used						**Local Curr. Rets** (bp)		
	% of Index (as of 5/31/00)	**Eurodollar /Euribor Futures**	**10-yr. Treasury Futures**	**Swaps 2-yr.**	**Swaps 5-yr.**	**Swaps 10-yr.**	**Swaps 30-yr.**	**Replicat. Port.**	**Index**	**Out-perf.**
U.S. Dollar	47.0	1472				169	67	225.4	208.0	17.4
Euro	30.5	1160	1125					30.6	39.0	-8.4
Yen	13.9		51	36	72			-18.8	-29.2	10.4
Sterling	4.8		253					47.1	21.4	25.7
Danish, Norwegian Krone, Swedish Krona, Greek Drachma	2.0							30.6	10.3*	20.3
Australian, Canadian, New Zealand Dollar	1.9							225.4	336.9**	-111.5
Global Aggregate Return (in $)								**258.4**	**253.5**	**4.9**

* Return in euros.
** Return in $.

assumed to be $ 1,000,000 for the U.S., and Yen 100,000,000 for Japan. For the U.S., yen, sterling, and the other non-euro currency European markets, the replication portfolios outperform their indices. For the remaining two markets they underperform their indices. This exemplifies the diversification effect that often occurs in the replication of the Global Aggregate Index.

Figure 8 contains tracking errors vs. the unhedged index (in U.S. dollars) for a larger number of replication strategies than presented in Figure 4. Five different combinations of methods are used to track the U.S. and Euro Aggregate Indices; these are used either alone or together with replications for the Yen and Sterling markets. In this set of results, we do not use proxy replication. Comparing these results with those in Figure 4, we see that proxy replication does not make too much of a difference. If only the two or three largest markets are replicated, proxy replication leads to an increase in tracking error. It leads to lower tracking errors when all four major markets are replicated.

Looking across any row of Figure 8 shows the effect of adding more currencies to the replication; comparisons within any column show the importance of the set of instruments chosen for the replication of the U.S. Aggregate and the Euro-Aggregate. Once the Japanese Treasury Index replication portfolio has been added to the replication, it becomes clear that replicating the U.S. Aggregate with Eurodollar futures and swaps and the Euro-Aggregate with Euribor and Treasury futures results in the lowest tracking errors.

Figure 8. **Tracking Errors for the Global Aggregate Index, Ignoring Small Markets Dollar-based Returns,** bp/month, February 1999-September 2000*

U.S. Aggregate	**Euro Aggregate**	**Yen Treasury Index:** None / **Sterling Agg.:** None	**Yen Treasury Index:** 10-yr Tsy Futures and Swaps / **Sterling Agg.:** None	**Yen Treasury Index:** 10-yr Tsy Futures and Swaps / **Sterling Agg.:** 10-yr Tsy Futures
Treasury Futures	Treasury Futures	41.5	22.4	20.9
Treasury Futures	Euribor and Treasury Futures	41.1	21.6	20.2
Eurodollar Futures and Treasury Futures	Euribor and Treasury Futures	38.9	18.3	16.9
Eurodollar Futures and Swaps	Treasury Futures	39.4	15.9	15.1
Eurodollar Futures and Swaps	Euribor and Treasury Futures	39.4	15.8	14.9

*No Proxy replication used.

The base currency in which returns are reported has only a minor impact on the performance of the replication strategy, as shown in Appendix 2. To illustrate the point, Figure 9 shows replication results for one strategy expressed in four base currencies. When we compare the results, we see that the differences are always less than 0.1 basis points.

The results displayed so far clearly show that the tracking error of a given replication portfolio depends heavily on the local market replication portfolios it consists of, and their weights in the index. The correlations among the local market replication portfolios are the second most important factor for the magnitude of the tracking error. Figure 10 displays the correlations of the local market outperformance of the replication portfolios we are using. Between the replication portfolios for the U.S. Aggregate and the Euro-Aggregate, the lowest correlations are found when Eurodollar futures and swaps are used in the US, and a hybrid of Euribor and Treasury futures are used in Europe. A combination of these two is part of the Global Aggregate replication portfolio exhibiting the lowest tracking error. This replication portfolio for the U.S. Aggregate also exhibits the lowest correlation with the Sterling Aggregate replication portfolio, resulting in the portfolio with the lowest overall tracking error.

Figure 9. **Tracking Errors for the Global Aggregate Index Replication, Minimum Tracking Error Strategy by Base Currency,** bp/month
February 1999-September 2000*

	Dollar	**Euro**	**Yen**	**Sterling**
Tracking Error	14.8	14.9	14.8	14.8

*Eurodollar/Euribor and Treasury Futures used for the U.S. and the Euro zone, respectively; 10-year Treasury Futures and Swaps used for Japan; 10-year Treasury futures used for the replication of the Sterling Aggregate. No Proxy replication used.

Figure 10. **Correlations of Local Markets' Outperformance,** February 1999–September 2000

U.S. Aggregate			**Euro-Aggregate**		**Japanese Treasury Index**			**Sterling Aggregate**		
Treasury Futures	**ED-Futures and Treasury Futures**	**ED-Futures and Swaps**	**Treasury Futures**	**ED-Futures and Treasury Futures**	**10-yr. Treasury Futures**	**10-yr. Treasury Futures and Swaps**	**Swaps**	**Long Gilt Futures**		
1.00	0.98	-0.70	0.41	0.36	-0.15	-0.30	-0.04	0.42	Treasury Futures	U.S. Aggregate
	1.00	-0.63	0.44	0.41	-0.21	-0.34	-0.05	0.42	ED-Futures and Treasury Futures	
		1.00	-0.05	0.01	0.13	0.26	0.23	-0.11	ED-Futures and Swaps	
			1.00	0.99	-0.24	-0.16	0.14	0.51	Treasury Futures	Euro-Aggregate
				1.00	-0.22	-0.13	0.11	0.53	ED-Futures andTreasury Futures	
					1.00	0.82	0.17	0.10	10-yr. Treasury Futures	Japanese Treasury Index
						1.00	0.52	0.11	10-yr. Treasury Futures and Swaps	
							1.00	0.20	Swaps	
								1.00	Long Gilt Futures	Sterling Aggregate

REPLICATING THE LEHMAN BROTHERS JAPANESE AGGREGATE INDEX

The recent reformulation of the Global Aggregate Index (see Appendix 1) has increased the market share of Yen-denominated bonds, and has also expanded the coverage of the Japanese market to include spread products as well as Treasuries. As a result, there is a lot of interest in replication strategies for this market. As in our previous work with the U.S. and Euro Aggregate Indices, we have investigated replication strategies based on combinations of Treasury futures, money market futures, and swaps. We compare the results of these efforts to a replication strategy based on a proxy portfolio of a few cash bonds.

Our first attempt at replication of the U.S. market with futures used Treasury futures only. We found this approach, using four contracts along the yield curve, to be very successful in replicating the U.S. Treasury Index, but less effective at tracking indices containing spread products. We then found that the incorporation of swaps and Eurodollar futures in the replicating portfolio could help improve performance by adding some spread exposure.

In this historical study of index replication in the Japanese market, we are presented with somewhat of the opposite situation. Liquidity constraints have limited our use of Treasury futures to the 10-year contract. Swaps of four maturities are used to better match index exposures along the yield curve. Nevertheless, due to the recent inception of the Japanese Aggregate Index (October 1, 2 000), all historical testing of the replication strategies are carried out against the Japanese Treasury Index. We are thus faced with the situation that when swaps are used in the replication, there is exposure to spreads only in the replicating portfolio and not in the index. Although the tracking error numbers presented here show performance in replicating only the Japanese Treasury Index, we are confident that going forward, the replication methodologies incorporating swaps will prove to be effective against the Japanese Aggregate Index as well.

The Japanese Treasury market has been harder to trade and to invest in than the government bond markets of other developed nations. One reason is the concentration of liquidity in a few issues, particularly in the 10-year benchmark bond. Although down from earlier years, still a substantial part of Treasury bond trading volume is concentrated in these securities.

The Treasury futures market consists of a 5-, 10-, and 20-year futures contract. Again, liquidity is primarily concentrated in a single contract, the 10-year futures. This contract will be used in our simulations. Clearly, the use of the 10-year contract alone for futures replication should lead to additional yield curve risk compared with what was experienced in the U.S. or the euro zone.

The Japanese 10-year Treasury futures contract is used by many investors as a vehicle for hedging their bond positions. Therefore, there exists a substantial natural short base in the contract. This leads to the contract trading cheap. At the same time, the relatively high probability of a squeeze on the cheapest-to-deliver bond against the contract presents a significant risk to anyone short this bond. This leads to a lack of arbitrage activity. Consequently, the basis of the closest contract usually trades expensive, and with high volatility. We therefore expect tracking errors in the replication of the Japanese Treasury Index with the 10-year futures contract to be higher than that experienced for the Euro-Aggregate or the U.S. Aggregate. Another issue that makes the Japanese futures market somewhat less efficient is the fact that execution and clearing of futures orders on the Tokyo Stock Exchange must take place through the same broker.

There are also Euro-Yen futures contracts traded on the Tokyo Financial Futures Exchange (TIFFE). The closest six contracts exhibit acceptable levels of liquidity. We use them to replicate the returns on the Japanese Treasury bonds that have maturities of less than 18 months. Due to the historically limited share of bonds with such low maturities in the index, the potentially positive effects of using Euro-Yen together with Treasury futures in the replication are limited in our simulations.[7]

Another class of derivatives whose index-tracking properties we investigate are swaps. Since swaps might not track the Japanese Treasury Index very closely due to the spread exposure that swaps are subject to, we also investigate the replication of the Japanese Treasury Index with a few liquid cash bonds. Given the transactions cost, cash replication can only be recommended for longer-term replicating portfolios. We found that the cash portfolios improve on replication with derivatives by a wide margin.

Methodology

Replication with Treasury futures is very simple in this case, since we are using just a single contract. The strategy buys the right number of 10-year futures contracts to hedge the duration of the entire Japanese Treasury Index. The durations we use for the 10-year futures contract are CTD-durations forward to delivery. We use the closest contract[8] and roll it at the month-end preceding delivery at the closing prices.

The replication of the bonds with up to 1.5 years in maturity with Euro-Yen contracts is conducted using the “stripped hedge” methodology, in which a sequence of contracts covering consecutive calendar quarters is used to constitute

[7] In other words, the positive effect from using different kinds of instruments for different parts of an index—a strategy we call *hybrid replication*—is limited.

[8] There was one exception to this rule of using the closest contract. From August 1999 through the end of the year, we used the March 2000 contract, rather than the nearest December 1999 contract. This contract was avoided by investors due to the risks that were associated with the Y2K problem, and therefore suffered from low liquidity.

the required cash flow exposure.[9] This technique, when practical, offers very good matching of yield curve risk. Unfortunately, the limited liquidity of longer-dated Euro-Yen contracts has so far restricted the use of this strategy further out along the curve.

For the replication with swaps, we use the same methodology as for the U.S. and Euro markets. We divide the Japanese Treasury Index into four cells according to their modified adjusted duration. Each cell is replicated separately with the corresponding instrument on a PVBP-equivalent basis. The outperformance numbers for each cell are then added up on a market-capitalization basis to arrive at the outperformance for the full index. To accommodate the lower yield levels that prevailed in Japan over the historical period, with the resulting high durations, we choose somewhat wider duration cells than for the U.S. and the Euro zone. These boundaries are 0-3 ½ years, 3 ½ -7 years, 7-10 years, and more than 10 years in modified adjusted duration.[10] We use 2-, 5-, 10-, and 20-year swaps to replicate each duration cell, respectively. To save on transaction costs, we turn over the swaps portfolio only once every six months.

For the cash portfolios, we divide the index into the same four cells as for the swaps-based replication. One liquid bond is selected to represent each cell. For the two high duration cells, the most recently issued bond in each cell is considered the most liquid. For the two low-duration cells, the bonds with the maximum amount outstanding in each cell were selected. In addition, in these two cells, we imposed the additional restrictions that the bonds selected for the replication have to be within 1 ½-3 years or 4 ½-6 ½ years in duration, respectively. This leads to a closer match with the durations of the index cells that are being replicated. The details of the construction of the cash portfolios is given in Appendix 3.

Performance

Figure 11 summarizes the return volatilities and mean returns for the Japanese Treasury Index. To facilitate comparisons with our results for the U.S. and Europe, our simulations start in January 1994, and end in September 2000. We display the results for the full time period, and for two sub-periods, January 1994 to June 1998, and July 1998 to September 2000. We choose this division because the world credit crisis of the summer of 1998 led to higher corporate bond and swap spreads and higher spread volatility. These changes affect the relationship between the derivatives and the cash markets, and therefore the tracking errors of the replication portfolios. It is also of interest to see how the replication performed during the period of increased spread volatility since July 1998.

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[9] Details concerning the replication methodologies used with the different derivatives can be found in our November 1997 and March 2000 studies on replication (see footnote 3).

[10] It could be argued that these duration cells were chosen with the benefit of hindsight. In reality, though, an investor knows the market structure at any given point in time, and can adjust the cells according to changes in the composition of the index. This should lead to an improvement in terms of tracking error compared to the numbers we found.

Figure 12a contains the tracking errors for the five replication portfolios for the full period and two sub-periods. Figure 12b shows the R-squared (R^2) values for the same periods, and Figure 12c the means. The R^2 tells us what part of the variance of the index returns is explained by the replication portfolio's returns.

We consider three replication portfolios that use the same kind of instruments across all four duration cells, and two hybrid replication portfolios: Euroyen futures

Figure 11. **Japanese Treasury Index, Mean and Volatility**
bp/month, January 1994–September 2000

	1/94–9/00	1/94–6/98	7/98–9/00
Volatility	141.0	139.1	144.4
Mean	37.2	49.0	13.6

Figure 12a. **Tracking Errors for the Replication of the Japanese Treasury Index,** bp/month, January 1994–September 2000

	1/94–9/00	1/94–6/98	7/98–9/00
10-yr. Treasury Futures	35.6	32.0	42.5
Euro Yen Futures and 10-yr. Treasury Futures	35.1	31.5	42.0
10-yr. Treasury Futures and Swaps	29.7	28.5	32.1
Swaps	32.6	31.0	35.6
Cash Bonds	8.4	8.5	8.2

Figure 12b. **R^2 for the Replication of the Japanese Treasury Index,** in %
January 1994–September 2000

	1/94–9/00	1/94–6/98	7/98–9/00
10-yr. Treasury Futures	93.6	94.9	90.9
Euro Yen Futures and 10-yr. Treasury Futures	93.8	95.0	91.1
10-yr. Treasury Futures and Swaps	95.6	95.8	95.1
Swaps	94.7	95.0	93.9
Cash Bonds	99.6	99.6	99.7

Figure 12c. **Mean Outperformance for the Replication of the Japanese Treasury Index,** bp/month, January 1994–September 2000

	1/94–9/00	1/94–6/98	7/98–9/00
10-yr. Treasury Futures	11.6	13.0	8.8
Euro Yen Futures and 10-yr. Treasury Futures	11.6	13.0	8.8
10-yr. Treasury Futures and Swaps	6.9	5.5	9.8
Swaps	2.1	-0.2	6.7
Cash Bonds	-0.7	-1.0	-0.2

combined with the 10-year Treasury futures contract, and the 10-year Treasury futures contract combined with 2-, 5-, and 20-year swaps.

First, we find that the 10-year Treasury futures portfolio results in only slightly higher tracking errors than the hybrid Eurodollar futures/Treasury futures portfolio. The reason for this is that we use Euro-Yen futures out to 18 months to expiration to replicate bonds with cash flows of up to 18 months. The mean share of bonds in the index with maturity below 18 months is 2.6 per cent over the full period. The tracking errors from the hybrid Euroyen/Treasury futures replication portfolio is therefore not affected very much by the inclusion of Euroyen futures. The tracking errors are lower than the ones from the pure Treasury futures replication. If the share of shorter maturity bonds in the Treasury Index increases, or the liquidity of longer dated Euroyen futures improves, so that they cover more index bonds, they can become a valuable addition to the set of instruments useful in the replication of the Japanese Treasury index.

We will first discuss the derivatives-based replication strategies only. For the full period and the first sub-period, the hybrid replication portfolio consisting of swaps and 10-year treasury futures delivers the lowest tracking errors. These results are consistent with the results for the United States and the Euro zone bond markets. Figure 13 displays the tracking errors for Treasury indices of the U.S., the Euro zone, and Japan. During the period starting with July 1998, futures returns differed on a few occasions significantly from treasury returns in Japan. This leads to a substantially higher tracking error for the Japanese treasury futures replication portfolio for that period.

The tracking errors from replicating parts of an index with different kinds of instruments are often negatively correlated. The reason for this is that credit spreads have historically been negatively correlated with rate movements. At the same time, when spreads are under upward pressure, Treasury futures trade richer

[11]Additionally, in times of a credit crisis, the lower settlement risk of futures contracts will contribute to their richness.

Figure 13. **Tracking Errors for Different Replication Strategies for the U.S., the Euro-, and the Japanese Treasury Indices,** bp/month
January 1994-September 2000.

	Euro*	U.S.			Japan		
	2/99-9/00	1/94-9/00	1/94-6/98	7/98-9/00	1/94-9/00	1/94-6/98	7/98-9/00
Treasury Futures	13.9	11.6	8.3	16.5	35.6	32.0	42.5
Hybrid Replication**	11.7	9.2	6.1	13.4	29.7	28.5	32.1
Swaps		32.4	12.7	53.6	32.6	31.0	35.6

*The replicatio simulations for the euro zone start with February 1999 due to data limitations.
**Eurodollar/Euribor and Treasury Futures for the U.S. and the Euro zone, Swaps and 10-year Treasury Futures for Japan.

compared to cash, due to their higher liquidity.[11] Replicating a uniform index, like a Treasury index, with swaps and Treasury futures, will therefore lead to reduced tracking errors compared to only using one set of instruments.

Replication with swaps leads to lower tracking errors than replication with the 10-year Treasury futures contract alone, or with swaps only. The gain from fitting the curve risk better with swaps outweighs the additional tracking error due to the movement in swap spreads.

The lowest tracking errors overall are achieved with cash bonds. For the full period and for each of the two sub-periods, it is slightly above 8 basis points per month. Clearly, to anyone interested in the long-term replication of the Japanese market, cash instruments are far superior to derivatives.

The R^2-values in Figure 12b confirm these results. They also show that during the second sub-period the 10-year Treasury futures contract did not replicate the index returns very well. To allow a comparison with other results, we display in Figure 14 tracking errors and R^2-values for the replication of the U.S. Treasury index. For the replication with the Eurodollar futures/swaps portfolio over the full period, the U.S. tracking errors and R^2-values are comparable to the ones encountered in the Japanese case. For the replication with bond futures and the hybrid Eurodollar/Treasury futures portfolio, they are much lower. Two reasons for this are the availability of more Treasury futures contracts along the U.S. curve and the lower basis volatility in the U.S.

Figure 14a. **Tracking Errors for the Replication of the U.S. Treasury Index,** bp/month, January 1994–September 2000

	1/94–9/00	1/94–6/98	6/98–9/00
2-, 5-, 10-year, and Bond Futures Contract	11.6	8.3	16.5
Hybrid Eurodollar/Treasury Futures	9.2	6.1	13.4
Hybrid Eurodollar Futures/Swaps	32.1	12.1	53.2

Figure 14b. **R^2-Values for Replication of the U.S. Treasury Index**
bp/month, January 1994–September 2000

	1/94–9/00	1/94–6/98	6/98–9/00
2-, 5-, 10-year, and Bond Futures Contract	99.1	99.6	97.8
Hybrid Eurodollar/Treasury Futures	99.4	99.8	98.6
Hybrid Eurodollar Futures/Swaps	92.9	99.1	77.3

The U.S. experience shows that swaps do not always improve index tracking relative to futures, particularly against an all-government index. Note the high tracking errors and low R^2 that we encounter for the replication of the U.S. Treasury index when using the hybrid Eurodollar/Swaps replication portfolio. This is due to the high spread volatility encountered since July 1998.

Let us also consider the mean outperformance numbers in Figure 12c. The 10-year Treasury futures replication portfolio leads to the highest mean outperformance. The outperformance of the swaps portfolios is lower, particularly for the first sub-period. The replication portfolio made up of cash bonds exhibits the lowest mean outperformance.

As we can see, futures vastly outperformed cash and swaps in the replication of the Japanese Treasury Index.[12] In Japan, the long in the Treasury futures contract has the advantage that many market participants consider it risky to short the futures basis. Shorting bonds against the futures carries the risk of a squeeze, and the high variability of the basis makes basis trades subject to a lot of mark-to-market risk. Therefore, the Japanese Treasury futures contract trades very cheap to cash compared to other markets. Although the tracking error to the index is almost 150 basis points per year, the additional return that a Treasury futures strategy can entail is a factor to consider in terms of what replication strategy to choose.

The underperformance of the cash replication portfolio is due to this strategy's selection of the most liquid bonds in the market. This is particularly true in the two higher duration buckets. Increased liquidity of an asset usually is accompanied by lower long-term returns. The bulk of the bonds in the index are off-the-run, lower liquidity bonds. The index should therefore outperform a selection of the most liquid bonds. Our returns do not include any specialness premium that might accrue to such bonds in the repurchase markets. The most liquid bonds trade rich to comparable off-the-run bonds. This liquidity premium is not fixed, but varies over time. It therefore represents an additional source of risk. Using older, less liquid bonds would probably result in a somewhat lower tracking error. In our study, we want to use securities with liquidity comparable to that of the derivatives we use. The already low tracking errors of the cash replication portfolio could probably experience a modest improvement.

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[12] This is similar to what occurred in the U.S. and Europe. Over the past eight years, U.S. Treasury futures, as well as German Treasury futures before and after the introduction of the euro, have provided the buyer of a futures contract with higher returns than cash positions in a similar maturity range. There are two main reasons for this. Both are based on the fact that the buyer of a futures contract is short an option. For one, over many cycles, these options traded rich as indicated by many pricing models. Furthermore, yield volatilities underwent a seminal decline, leading to the option experiencing a more rapid price decline than under unchanged volatilities.

CONCLUSION

The cross-market diversification that is built into the Global Aggregate Index makes it a low-volatility benchmark alternative to many local market indices. In addition, it is not as much affected by changes in individual local markets, for example, reduction in Government debt. This can lead to substantial changes in important characteristics of an index, like duration or convexity. The Global Aggregate will therefore be more stable in terms of the risk properties of an index.

Given the more than 10 bond markets and currencies represented in the Global Aggregate, the use of derivatives can help to reduce the dimensionality of the management problem, and support the investment of cash and asset allocation shifts.

Replication of the Global Aggregate is found to work very well, usually with tracking errors that are lower than the ones found for local market replication portfolios. The hedged Global Aggregate can therefore represent an attractive benchmark for enhanced index funds. We found that the performance of our strategy at replicating the returns of the Global Aggregate depends strongly on the choice of instruments that are used to replicate each of the local components. A portfolio that combines Eurodollar futures and swaps for the US, Euribor and Treasury futures for the Euro zone, 10-year Treasury futures and swaps for Japan, and 10-year Treasury futures for the replication of the Sterling Aggregate delivered the lowest tracking error.

Currently, about 95 % of the Global Aggregate's market capitalization is made up by issues denominated in four currencies. We find that it suffices to replicate the four largest markets.

The replication of the Japanese Treasury Index shows relatively high tracking errors for the derivatives based replication portfolios. The lowest tracking errors are achieved by a hybrid replication portfolio combining swaps and the 10-year Treasury futures contract. The Japanese Treasury Index is most effectively replicated with a portfolio of cash bonds.

Appendix 1. **Recent Changes to the Global Aggregate Index**

The Lehman Brothers Global Aggregate Index experienced two significant changes as of October 1, 2000:

1) The liquidity constraint was changed to $300 million equivalent outstanding for all currencies.[13]
2) The Asian-Pacific Aggregate Index was incorporated into the Global Aggregate.

Both of these changes contribute to an increased role of the Asian component of the index. For the U.S. Aggregate and Euro Aggregate components of the index, the new liquidity constraint represents an increase in the threshold,[14] causing the total market value included to decline slightly. For the Japanese Treasury Index, where the prior liquidity constraint had been set at 1 trillion Yen, the new unified constraint is lower, allowing more bonds to be included. In addition, the coverage

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[13]To avoid fluctuations in index composition due to exchange rate volatility, the official thresholds were fixed in local currency based on exchange rates as of July 31, 2000. The resulting local currency numbers where rounded somewhat to achieve reasonable thresholds in local currency. A complete listing of the precise thresholds in each currency is available in the soon-to-be-published *Global Aggregate Primer*.

[14]The minimum until September 30, 2000, was $ 150 million for the U.S. Aggregate, Euro 150 million for the Euro-Aggregate, Yen 1 trillion for the Yen Treasury Index, and Pound Sterling 100 million for the Sterling Aggregate.

Figure A-1. **Composition of the Global Aggregate Index by Currency,** as of 9/29/00

	Old Constraints			**New Constraints**		
	Market Value ($ mn)	**% of Index**	**Number of Issues**	**Market Value** ($ mn)	**% of Index**	**Number of Issues**
U.S. Aggregate	5,781,440	48.5	6044	5,160,145	41.5	2645
Eurodollar and 144A, ex-U.S.Aggregate				304,958	2.5	550
Euro-Aggregate	3,439,201	28.9	3925	2,999,566	24.1	1527
Sterling Aggregate	569,961	4.8	562	492,432	4.0	236
Danish, Norwegian Krone, Swedish Krona, Greek Drachma	223,836	1.9	76	221,179	1.8	62
Yen Government	1,682,760	14.1	85	2,310,787	18.6	218
Yen Credit, incl. Euroyen				600,546	4.8	721
Other Asian Issues				110,085	0.9	125
Australia, New Zealand	42,320	0.4	19	49,895	0.4	31
Canadian	176,458	1.5	40	176,458	1.4	40
Total	**11,915,976**	**100**		**12,426,051**	**100**	

of the Asia-Pacific region has been expanded in several dimensions. Besides Japanese Treasury bonds, the Asian-Pacific Aggregate contains Yen-denominated corporate and sovereign issues, and government and credit securities denominated in other Asian currencies. Figure A-1 shows the composition of the Global Aggregate Index as of September 29, 2000, according to both the new set of index rules and the previous one. The net effect of these changes is that the share of the Asian-Pacific component in the Global Aggregate, previously represented by the Japanese, Australian and New Zealand Treasury Indices, has increased from about 14 % to about 24 %.

These changes in the index must be taken into consideration when evaluating the results of this replication study. The market shares under the old set of rules, which were in place through September 30, 2000, are representative of the period for which our simulations were performed. The new rules lead to an increased share for the Asian-Pacific Aggregate, which is replacing the Japanese Treasury Index.

Nonetheless, we are confident that our results should give a very good indication of what to expect with respect to the tracking errors of replication portfolios going forward under the revised rules. In the new Asian-Pacific Aggregate, Japanese Treasuries still comprise more than 75% of index market value. Of the increase in the Asian component of the Global Aggregate, 4.5 percentage points are due to an increase in share of the Japanese Government component, 4.8 percentage points to the Yen-denominated non-government issues, and 0.9 percentage points to the introduction of bonds denominated in Asian currencies other than Yen. Our results for the Japanese Treasury component will be applicable to the future. The small share of the non-Yen denominated Asian issues can be left un-replicated, in line with our results concerning markets with small capitalizations in the Global Aggregate. The replication results using swaps, in particular, should continue to perform well with the addition of spread products to this index.

Appendix 2. **Hedged Currency Returns and Index Replication**

Property 1. The tracking error of a replication strategy is the same for hedged and unhedged returns.

The outperformance of a replicating portfolio, versus the index it was constructed to replicate, in any period t is given by

$$(1) \qquad r^{R,G}_{t,u,j} - r^{I,G}_{t,u,j}$$

where r_t denotes period t return, u stands for unhedged, and j for currency j. Superscripts R, I, and G denote replication portfolio, index, and global, respectively. All returns are expressed as fractions on a per-period (i.e., un-annualized) basis.

$$(2) \qquad r^{I,G}_{t,u,j} = \sum_{i=1}^{N} w_{t,i} \left[\left(1 + r^{I}_{t,i}\right)\left(1 + r^{c}_{t,i}\right) - 1\right].$$

$w_{t,i}$ is market i's portfolio share applicable to period t (for the Global Aggregate Index, this is the beginning-of-month market value share of market i). $r^{I}_{t,i}$ is local market's i index return, and $r^{c}_{t,i}$ is the currency return[15] attributable to that market. $r^{c}_{t,j}=0$. Similarly, the return of the replication portfolio is given by

$$(3) \qquad r^{R,G}_{t,u,j} = \sum_{i=1}^{N} w_{t,i} \left[\left(1 + r^{R}_{t,i}\right)\left(1 + r^{c}_{t,i}\right) - 1\right].$$

Returns for derivatives portfolios are always expressed as the marked-to-market change in value plus the cash flow of the derivatives position divided by the amount invested in the market at the beginning of the period.

The total return of a portfolio that is hedged for the risk in currency changes has the following form

$$(4) \qquad r^{I,G}_{t,h,j} = r^{I,G}_{t,u,j} + r^{c}_{t,h,j},$$

with subscript h indicating hedged, and superscript c denoting currency. The currency hedging portfolio return has the following form:

$$(5) \qquad r^{c}_{t,h,j} = \sum_{i=1, i\neq j}^{N} w_{t,i} \left[\left(1 + r^{e,I}_{t-1,t;i}\right)\left(e^{F}_{t-1,t;i} - e_{t,i}\right) \big/ e_{t-1,i}\right].$$

[15] Note that this expression of total return as the product of local return and currency return does not follow the conventions used in reporting currency returns on Lehman Brothers Indices. Index currency returns are reported using an additive convention rather than this multiplicative one. Details may be found in the soon-to-be-published *Global Aggregate Index Primer*. However, the two conventions are mathematically equivalent.

$r^{e,I}_{t-1,t;i}$ is a term representing the return expected at the end of period *t-1* for the *i*th local market bond index return over period *t*. For the Lehman Brothers International bond indices, the index yield is used for this. $e_{t,i}$ is country *i*'s exchange rate, in terms of the number of home currency units per country *i* currency unit. $e^{F}_{t-1,t;i}$ stands for the forward or futures exchange rate of the same currency, agreed upon at the end of *t-1* for delivery at *t*.

If $e^{F}_{t-1,t;i} = e_{t-1;i}$ and $r^{e,I}_{t-1,t;i} = r^{I}_{t,i}$, then all influence of currency movements is eliminated, and the currency hedge would be perfect. If the currency hedge used for the investment in each local market's replication portfolio is the same as the one employed for each local market itself, the return of the currency hedged replication portfolio consists of two terms:

$$r^{R,G}_{t,h,j} = r^{R,G}_{t,u,j} + r^{c}_{t,h,j} \ . \tag{6}$$

Therefore, using equation 1,

$$r^{R,G}_{t,h,j} - r^{I,G}_{t,h,j} = r^{R,G}_{t,u,j} - r^{I,G}_{t,u,j} \ . \tag{7}$$

Property 2. Replication tracking errors are nearly independent of base currency.

From equations 2 and 3, we can write the outperformance in period *t* of a derivatives portfolio constructed to replicate the Global Aggregate index as

$$\begin{aligned} r^{R,G}_{t,u,j} - r^{I,G}_{t,u,j} &= \sum_{i=1}^{N} w_{t,i}\left[\left(r^{R}_{t,i} - r^{I}_{t,i}\right)\left(1 + r^{c}_{t,i}\right)\right] \\ &\approx \sum_{i=1}^{N} w_{t,i}\left(r^{R}_{t,i} - r^{I}_{t,i}\right) \ . \end{aligned} \tag{8}$$

$r^{c}_{t,i}$ is often in the -3 to +3 per cent range, so that the approximation holds. $1+ r^{c}_{t,i}$ will not differ much from 1. Overall therefore, for each country *i*, the effect from currency conversion is usually of second order. Hence, although portfolio statistics are heavily influenced by currency conversion, outperformance is not.

Property 3. Implicit currency hedging (by keeping cash in the base currency) is approximately equivalent to conventional hedging.

In the text we discuss the results of a strategy we call implicit currency hedging. For implicit currency hedging, a foreign index return is replicated by purchasing zero-cost foreign derivatives. The investor then earns approximately the index return minus the cost of financing the equivalent cash position. Since the investor puts up essentially no cash, the only cash flow subject to currency risk are gains or losses on the position. The investor is therefore virtually protected against changes in the exchange rate.

In this appendix, we will show the approximate equivalence of conventional currency hedging and implicit hedging.

(9) $$r_{t,h\_imp,j}^{I,G} = \sum_{i=1}^{N} w_{t,i}\left[r_{t,i}^{R,F}\left(1+r_{t,i}^{c}\right)\right] + r_{t,j}^{s}\,.$$

$r_{t,j}^{s}$ denotes the home currency short-term rate, in which the cash is invested. $r_{t,i}^{R,F}$ is the return on the derivatives portfolio in market *i* that the investor is long (*F*=Futures or forward, *R*=replication). $r_{t,i}^{R,F}$ is defined as the cash flow plus change in value of the derivatives contract, divided by the index share of market *i* in the Global Aggregate multiplied by the portfolio value. *h_imp* denotes implicit hedge. Now,

(10)
$$\begin{aligned}
r_{t,h_imp,j}^{I,G} &= \sum_{i=1}^{N} w_{t,i}\left[r_{t,i}^{R,F}\left(1+r_{t,i}^{c}\right)\right] + r_{t,j}^{s} \\
&= \sum_{i=1}^{N} w_{t,i}\left[\left(1+r_{t,i}^{R}-\left(1+r_{t,i}^{s}\right)\right)\left(1+r_{t,i}^{c}\right)-1+1\right] + r_{t,j}^{s} \\
&= r_{t,u,j}^{R,G} + r_{t,j}^{s} - \sum_{i=1}^{N} w_{t,i}\left[\left(1+r_{t,i}^{s}\right)\left(1+r_{t,i}^{c}\right)-1\right] \\
&\approx r_{t,u,j}^{R,G} + r_{t,j}^{s} - \sum_{i=1}^{N} w_{t,i}\left(r_{t,i}^{s}+r_{t,i}^{c}\right).
\end{aligned}$$

Since interest-rate parity has to hold in the foreign-exchange market,

(11) $$e_{t-1,t;i}^{F} = e_{t-1,i}\left(1+r_{t,j}^{s}\right)\Big/\left(1+r_{t,i}^{s}\right).$$

From equation (5), we have

$$
\begin{aligned}
r^{c}_{t,h,j} &= \sum_{i=1,i\neq j}^{N} w_{t,i}\left[\left(1+r^{e,I}_{t-1,t;i}\right)\left(e^{F}_{t-1,t;i}-e_{t,i}\right)\big/ e_{t-1,i}\right] \\
&= \sum_{i=1,i\neq j}^{N} w_{t,i}\left[\left(1+r^{e,I}_{t-1,t;i}\right)\left(\left(1+r^{s}_{t,j}\right)\big/\left(1+r^{s}_{t,i}\right)-e_{t,i}\big/ e_{t-1,i}\right)\right] \\
&\approx \sum_{i=1,i\neq j}^{N} w_{t,i}\left[\left(1+r^{e,I}_{t-1,t;i}\right)\left(\left(1+r^{s}_{t,j}-r^{s}_{t,i}\right)-\left(1+r^{c}_{t,i}\right)\right)\right] \\
&= \sum_{i=1,i\neq j}^{N} w_{t,i}\left[\left(1+r^{e,I}_{t-1,t;i}\right)\left(r^{s}_{t,j}-r^{s}_{t,i}-r^{c}_{t,i}\right)\right] \\
&\approx \sum_{i=1,i\neq j}^{N} w_{t,i}\left(r^{s}_{t,j}-r^{s}_{t,i}-r^{c}_{t,i}\right) \\
&= r^{s}_{t,j}-\sum_{i=1}^{N} w_{t,i}\left(r^{s}_{t,i}+r^{c}_{t,i}\right)
\end{aligned} \tag{12}
$$

using (11) to establish the first equality. The fifth line is based on $r^{e,I}_{t-1,t;i}$ being usually small (for hedged Lehman Brothers' Indices it is the beginning-of-month yield for the month).

The last line in (12) equals the last two terms in the last line of (10). Therefore, the implicit replication and conventional currency hedging are equivalent to the first order.

Appendix 3. **Derivation of the Replication Portfolio for the Japanese Treasury Index with Cash Securities**

The index is divided into four duration-based cells. Each cell is replicated by one cash bond. The duration of each of these cash bonds is bracketed by the duration of the cell it represents. We select four conditions to solve uniquely for the four components:

(1) $\sum_{i=1}^{4} N_{t,i} PVBP_{t,i} = PVBP_t^P$ ($PVBP$-equivalence for the whole portfolio)

(2) $\sum_{i=1}^{4} N_{t,i} P_{t,i} = CAP_t^P$ (capital-equivalence for the whole portfolio)

(3) $N_{t,3} PVBP_{t,3} = PVBP_{t,3}^P$ ($PVBP$-equivalence of third cell)

(4) $N_{t,4} PVBP_{t,4} = PVBP_{t,4}^P$ ($PVBP$-equivalence of fourth cell)

$PVBP_{t,i}$, $i = 1, \ldots, 4$, is the present value of a basis point of bond i at the start of period t. $N_{t,i}$ is the number of bond i in the portfolio, $P_{t,i}$ is its price, and CAP_t^P is the amount invested in the portfolio. The superscript P indicates portfolio. Conditions (3) and (4) help to achieve a good Zfit at the long end. (1)-(4) represent a system of equations, that can be uniquely solved for the $N_{t,i}$, $i = 1, \ldots, 4$.

The cash portfolios are constructed so that the value of the securities in the replicating portfolio is less or equal to the amount invested in the index. Any residuals are invested at 1-month Yen LIBOR.

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