Bloomberg

Stress Matrix Pricing in PORT

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April 15, 2015

Version 1.1

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1 Definition of Stress Matrix Pricing (SMP)

Stress Matrix Pricing (SMP) is a method for speeding up pricing of portfolios containing derivatives over large numbers of scenarios. The most accurate approach to pricing such portfolios is to use what is called "full valuation", that is, pricing the derivative independently for each scenario, using an appropriate model with input parameters set exactly according to the scenario. Since derivatives' values are generally highly non-linear functions of their underlying asset prices and model parameters, only the full valuation approach can completely capture all of the pricing function's irregularities. However, valuation under such models generally involves use of lattice or Monte Carlo methods, which can be prohibitively time consuming if large numbers of scenarios are to be run.

To speed up the computation while faithfully capturing the value of nonlinear instruments, many methods have been developed. A consistent theme among these methods is recognizing that the number of times a portfolio has to be priced does not have to be equal to the numbers of times the securities prices have to be computed using the model. One such method is the Stress Matrix Pricing approach, a method used by many risk analytic vendors.

The basic idea of SMP is to perform full valuation computations on a low dimensional grid in the relevant model parameters, and then the P&L for a specific scenario is approximated by interpolation and extrapolation of the grid values. Generally, interpolation gives more reliable values than extrapolation.

The approach taken in PORT<GO> to portfolio risk calculations is based on a mixture of full valuation, SMP and linear approximation. This mixture of approaches seeks to optimize the tradeoff between speed and accuracy. Simulation-based Monte Carlo and historical VAR calculations, involving large numbers of scenarios, are processed using SMP. Individual scenarios under the Scenario tab of PORT<GO> are processed using either full valuation or a factor model linear approximation, depending on the whether the scenario is designated as a Full Valuation Scenario or a Factor Model Scenario.

In this document, we explain the SMP methodology in detail for all the models where it is used. Each model is described in its own section.

2 SMP for Equity and Currency Derivatives and Commodity Futures Options

Equity and currency derivatives in PORT are modeled using the Black-Scholes-Merton (BSM) model. Commodity futures options are modeled using the driftless sub-case of this model, usually called the Black model. While the shortcomings of the BSM model are widely known, we use the model to translate scenarios of the underlying variables into option prices, modeling the movement of volatility as well as market-traded variables; we do not assume volatility is constant as in the BSM world. Thus the model functions essentially as a quotation mechanism for translating underlying variable movements including volatility into derivative prices.

SMP for the BSM model prices the derivative over a range of underlying prices and volatilities, thus creating a two-dimensional grid of model values at stock prices S_i and volatilities σ_j. Therefore, for pricing under a particular scenario for any value of stock price and volatility (S, σ), we interpolate the grid points containing prices for values (S_i, σ_j) linearly in two dimensions to get the derivative price for that scenario.

The rest of this section provides details on the exact setup of the grid.

2.1 Model Inputs

We use the BSM model, specifically, for equities the BS-Discrete version of this model, found in the derivative pricing function OVME<GO>, and for currencies, the Black-Scholes model in OVML<GO>. These models compute the derivative price using the Black-Scholes formula where applicable, or else by solving the BSM partial differential equation using finite difference methods. The model inputs are:

- Underlying price
- Interest rate
- Dividends
- Volatility
- Option deal parameters (strike, time to expiry, etc.)

For simple vanilla options, the deal parameters consist simply of strike price and time to expiry; we allow for more general derivative structures and simply gather all deal parameters together as one multi-valued input. In the grid, the underlying stock price and volatility are allowed to vary over a range of values. While rates may actually vary in the scenarios, their effect for most derivatives is secondary to that of underlier price and volatility, and so we hold these values fixed. To do otherwise would require increasing the grid dimension from two, in which case the number of pricings required to build the grid quickly explodes.

2.2 Pricing Grid Construction

A unique price grid is set up for each option in the portfolio. For an option with deal parameters as observed at the current time T given by $D(T)$, the grid is a set of values where the (i, j) node value in the grid is the BSM model price return

$$R(S_i, \sigma_j, r_0, d_0, D(T + h))$$

Here, h is the horizon time, that is, the deal is priced at a time h after the current time. For example, in the case of a vanilla option, this simply requires reducing the time to expiry by h. We denote the current underlying spot price by S_0, and S_i represents the i-th grid point for the spot price. Similarly, we denote the base implied vol by σ_0, which is the midpoint on the volatility surface for the underlying stock or index at the maturity and strike price of the deal, and σ_j is the j-th grid point for the implied volatility. We use r_0 and d_0 to denote the current interest rate and dividends to the deal maturity.

2.2.1 Equities and Commodities

The spot price grid node points S_i form an evenly spaced grid on the logarithmic scale, centered at S_0. The grid spacing is given by half of the standard deviation of the daily logarithmic historical return, calculated using the 90 most recent trading days' closing price of the underlier (available from the API field VOLATILITY_90D of the spot ticker). The range of the grid goes from -3 to +3 of these return standard deviations. To be specific, let v_S be the annualized underlier 90 day historical volatility, then the spot grid is given by

$$S_i = \ S_0(1 + v_S)^{i/2},\ i = -6, -5, \dots, -1, 0, 1, \dots, 5, 6$$

The implied volatility grid node points σ_j form an evenly spaced grid on the logarithmic scale, centered at σ_0. The grid spacing is given by half of the standard deviation of the daily logarithmic historical return, calculated using the daily Variance-Covariance Matrix entry of the corresponding VIX risk factor. Here we make the following assumption: the return on the option's implied vol is the same as the return of VIX, namely that

$$(1 + r_\sigma) = \left(1 + \frac{\delta\sigma}{\sigma}\right) = (1 + r_{\text{VIX}}) = \left(1 + \frac{\delta\text{VIX}}{\text{VIX}}\right)$$

The range of the grid goes from -3 to 3 VIX return standard deviations. To be specific, let v_{VIX} be the annualized VIX 90 day historical volatility, then the implied vol grid is given by

$$\sigma_j = \ \sigma_0(1 + v_{\text{VIX}})^{j/2}, \ j = -6, -5, \ldots, -1, 0, 1, \ldots, 5, 6$$

In implementation, for each option contract, the base implied vol σ_0 is interpolated from the OVDV volatility surface.

The Greeks Δ_0, Γ_0 and Λ_0 are obtained by calling the "BS Discrete" pricing model in OVME. For each of the grid points (S_i, σ_j), the security price $P(S_i, \sigma_j, r_0, d_0, D(T + h))$ is obtained from that same pricing model.

2.2.2 Currencies

For currencies, we change the grid dimensions from square, that is, the same number of nodes in the spot and volatility dimensions, to rectangular, with 27 spot price levels and 8 volatility levels, since most of the variability in option price comes from spot shifts rather than volatility shifts.

The spot price grid node points S_i form an evenly spaced grid on an arithmetic scale, centered at S_0. The grid spacing is given by half of the standard deviation of the daily logarithmic historical return, calculated using the 90 most recent trading days' closing price of the underlier (available from the API field VOLATILITY_90D of the spot ticker). The range of the grid goes from -6 to +6 of these return standard deviations. To be specific, let v_S be the annualized underlier 90 day historical volatility, then the spot grid is given by

$$S_i = \ S_0(1 + v_S * i/2), \ i = -12, -11, \ldots, -1, 0, 1, \ldots, 11, 12$$

This gives us 25 levels ranging from -6 to +6 standard deviations; to this we add two extreme levels at +10 and -10 standard deviations. All spot levels are of course floored at zero.

The volatility levels are constructed with two levels centered around the option's implied volatility and taking aritmetic steps two standard deviations of the 1M ATM implied for the given currency pair up and down from the two bracketing levels. This gives us 6 levels, and then we add two more extreme levels to eliminate the need to extrapolate the grid.

3 SMP for Callable Bonds

3.1 Overview

Callable bonds[1] in PORT are modeled using the Black-Karasinsky model, available in function OAS1<GO> as the R model. The model uses the short-term interest rate as its state variable, and assumes this short rate follows the SDE

$$d\log(r) = [\kappa(t) - \alpha\log(r)]dt + \sigma dW$$

The volatility term σ is assumed constant using the value that OAS1 would use for the security on startup. The value of the mean reversion parameter α is .03. The function $\kappa(t)$ is calibrated to match the model to the initial yield curve. For more detail on the R model, see the "Model Descriptions" section of the OAS1<GO> Help pages.

SMP for callable bonds prices and saves the returns on the bond over a range of underlying rates and volatilities, thus creating a two-dimensional grid of returns. Actually stored in the grid is a residual value, which is the difference between the full return at each grid point and the linear return on the bond at that grid point. Here the linear return includes returns due to shifts in selected yield curve term points using key rate durations.

3.2 Model Inputs

The Black-Karasinsky R model inputs are:

- Volatility: The default source of the volatility cube used for analytics is defined in the Interest Rate Volatility Cube function VCUB<GO>. Each currency has its own volatility cube, and we use the ATM slice of the appropriate currency's cube. The volatility is obtained by matching the bond's early redemption schedule to the swaption term and remaining maturity at that early redemption date to the swaption tenor. This method for determining the bond volatility is the same as that used to populate the wakeup volatility in OAS1<GO>.
- Yield Curve: The yield curve used is the swap curve for the bond currency, except for Treasury and Agency bonds denominated in USD, EUR, JPY, GBP, CAD and AUD, in which case PORT uses the sovereign par curve for the currency.
- Bond Price: An OAS spread is implied from the bond price and current levels for the appropriate curve and volatility. This OAS spread is then held constant as we populate the price grid. The pricing source used for the bond price in the absence of a user-supplied price is "BVAL".

[1]The method described here also applies to Hybrid ARMs, which are fixed-to-adjustable rate mortgage backed securities.

- Bond deal parameters: This includes all the relevant bond descriptive data, such as cash flow schedule, call schedule, put schedule, etc.

3.3 Pricing Grid Construction

A unique price grid is set up for each bond with embedded options in the portfolio, for a given pricing horizon. For a bond priced at current time T with deal parameters $D(T)$, the grid is a set of values where the (i, j) node value in the grid is the R-model bond price return over linear factor returns from yield curve moves

$$R(D(T + h), r(\cdot), \sigma_{ATM}(\cdot))$$

Here, h is the horizon time, that is, the deal is priced at a time h after the current time. The pricing curve $r(\cdot)$ and the volatility cube $\sigma(\cdot)$ are each reduced to a single dimension for purposes of grid construction. The curve $r(\cdot)$ is based on the 5 year rate on the pricing curve, r_0^{5Y}. From the 5 year rate shock, the perturbed curve for the ith grid point is determined by propagating the 5 year shock through a series of betas. In particular, the ith grid rate for maturity t_m is given by

$$r_i(t_m) = r_0(t_m) + \beta_{t_m}(r_i^{5Y} - r_0^{5Y})$$

where $r_i(t_m)$ is the perturbed rate, $r_0(t_m)$ is the current curve level for maturity t_m, β_{t_m} is the beta of the rate shock at maturity t_m against the rate shock at the 5 year maturity, and $r_i^{5Y} - r_0^{5Y}$ is the shift in the 5 year rate at the ith rate level.

For volatility, the cube is first reduced to a surface $\sigma_{ATM}(\cdot)$ in dimensions expiry by tenor by considering only ATM points. This surface $\sigma_{ATM}(\cdot)$ is then based on the 2x10 ATM point of the volatility cube, and the other ATM points of the volatility cube are shifted using a beta against the change in the 2x10 ATM point. The points on the ATM surface are

$$\sigma_j(t_e, t_t) = \sigma_0^{ATM}(t_e, t_t) + \beta_{t_e,t_t}(\sigma_i^{2x10} - \sigma_0^{2x10})$$

where t_e, t_t are the time to expiry and tenor, respectively, $\sigma_j(t_e, t_t)$ is the jth volatility grid level for this expiry and tenor, $\sigma_0^{ATM}(t_e, t_t)$ is the current cube level for this expiry and tenor, β_{t_e,t_t} is its beta versus the 2x10 year ATM volatility, and $(\sigma_i^{2x10} - \sigma_0^{2x10})$ is the change in the 2x10 ATM point at the jth grid level.

The betas for both rates and volatilities are computed periodically, as needed.

The residual value $R(D(T + h), r(\cdot), \sigma_{ATM}(\cdot))$ over linear factor returns stored in the stress matrix at the (i, j) node is then defined as follows:

$$R(D(T+h), r(\cdot), \sigma_{ATM}(\cdot)) = P(D(T+h), r(\cdot), \sigma_{ATM}(\cdot)) - L$$

where $P(\cdot)$ represents the return on the bond based on the R model price, and L represents the linear return components computed from the K standard key rate durations, that is,

$$L = (r_i^{5Y} - r_0^{5Y}) \sum_{k=1}^{K} \beta_k \mathrm{KRD}_k$$

Here r_i^{5Y} is the 5 year rate at the (i, j) node of the grid.

Note that the linear returns here include only those due to perturbations to the yield curve. Other linear factor returns are not subtracted out since the other factors are often spread-related and the spread factors as proxied by OAS are held constant in the grid. That is, returns from these spread factors are effectively fixed at zero throughout the grid since OAS is fixed at the current market implied OAS value. For factors which are not spread related, the grid still does not capture their effect, since the grid just provides better information on returns due to rate curve and volatility shifts.

We now describe the construction of the node points in the grid's two dimensions. The 5 year rate grid dimension is evenly spaced, centered at the current 5 year rate level. The grid spacing is given by half of the standard deviation of the daily historical return, calculated using the 90 most recent trading days closing price of the 5 year rate of the pricing curve (available from the API field VOLATILITY_90D of the rate's ticker). The range of the grid goes from -3 to 3 of these return standard deviations. To be specific, let v_5 be the annualized 90 day historical volatility, then the 5 year rate grid is given by

$$r_i = r_0^{5Y} + \frac{i}{2} v_5, i = -6, -5, \ldots, 0, \ldots, 5, 6$$

The 2x10 ATM swaption volatility grid is an evenly spaced grid, centered at the current level of this volatility. The grid spacing is given by the standard deviation of the daily historical return of the 2x10 ATM swaption volatility of the currency of the bond. For example, if a callable bond is denominated in USD, then the ticker of the 2x10 ATM swaption vol is USSV0210 index, and the grid spacing is calculated using the 90 most recent trading days closing level of USSV0210 index (available from the API field VOLATILITY_90D of the VIX Index). The range of the grid goes from -3 to 3 of these return standard deviations. To be specific, let v_{2x10} be the annualized 90 day historical volatility, then the swaption vol grid is given by

$$\sigma_j = \sigma_0^{2x10} + \frac{j}{2} v_{2x10}, j = -6, -5, \ldots, 0, \ldots, 5, 6$$

3.4 Scenario P&L Calculations

The main use of SMP for Callable Bonds is to compute the P&L under scenarios for VAR calculations. For the currency relevant to a given security, each scenario specifies

- $\Delta R_k, k = 1, \ldots, K$: Yield curve shocks at maturities $T_1, T_2, \ldots, T_K$ over time horizon h.
- $\Delta\sigma$: The shock to the 2x10 ATM swaption volatility over horizon h. The ATM swaption volatility after the shock is $\sigma_{scn} = \Delta\sigma + \sigma_0^{2x10}$.
- $r_{F_m}, m = 1, \ldots, M$: Implicit factor returns over horizon h. Note that the r_{F_m} are estimated from the excess returns, after the explicit factor returns have been removed from the total return.[2] The excess returns represent additional compensation required by investors for bearing risks other than those of the base yield curves and swaption volatilities built into the OAS model. The major source of excess risks for corporate bonds is generally default risk.
- ϵ: Simulated residual return, provided by the scenario generator.

With these scenario values, the return under a scenario is computed as

$$\left[\sum_{m=1}^{M} \beta_m r_{F_m}\right] + \left[\sum_{k=1}^{K} \mathrm{KRD}_k\ \Delta R_k + \hat{R}(t+h, \{\Delta R_{5Y}\}, \sigma_{scn})\right] + \epsilon$$

The first bracketed expression is the sum of returns due to factors other than curve shifts, typically spread factors but including other factors, and the betas there refer to their factor loadings. The second bracketed expression is the sum of returns due to curve shifts, including first the linear estimate of those returns, and second, an estimate of the non-linear part of those returns, $\hat{R}(t+h, \{\Delta R_{5Y}\}, \sigma_{scn})$, computed by linear interpolation in the dimensions r^{5Y}, σ^{2x10} from the grid. Finally, there is the residual return.

To compute the interpolated value $\hat{R}(t+h, \{\Delta R_{5Y}\}, \sigma_{scn})$, we need to know ΔR_{5Y} and σ_{scn}. The latter value comes directly from the scenario. Recalling that the rate dimension of the stress matrix is the 5Y rate from which other rates are perturbed using betas, we might naively take ΔR_{5Y} directly from the scenario. However, the change to the 5Y point under any particular scenario may not be a likely change for that point given the changes to other points that have a larger impact on the security. Therefore, we compute a maximum likelihood estimate of the 5Y change ΔR_{5Y} given the changes throughout the curve and the relative importance of those changes to this security. This maximum

[2]For details regarding return splitting and factor return estimation in the factor models, please refer to the document "Fixed Income Fundamental Factor Models", found under Documents in HELP PORT<GO>.

likelihood estimate of ΔR_{5Y} is then used in a two-dimensional linear interpolation of the stress matrix.

We now provide the specifics of the computation of ΔR_{5Y} and of the interpolation. To find ΔR_{5Y}, we first compute a single number summarizing the yield curve changes to the various points by their relative importance to this bond. This gives us the curve change at the duration of the bond. We then back this into a 5Y change using the beta of the duration point on the curve to the 5Y point.

In greater detail, first let

$$D = \sum_{k=1}^{K} \mathrm{KRD}_k$$

We recall that a bond's duration is closely approximated by D, the sum of its KRDs. With this in mind, we first find the maturity T_j such that $T_{j-1} \leq D < T_j$. We then compute our maximum likelihood 5Y change

$$\Delta R_{5Y} = \frac{\sum_{k=1}^{K} \mathrm{KRD}_k \Delta R_k}{(w_1 \beta_{j-1} + w_2 \beta_j) \sum_{k=1}^{K} \mathrm{KRD}_k}$$

where ΔR_k is the yield curve shock at maturity T_k under the scenario in question, β_j is the beta of the T_j swap rate against the 5Y swap rate, and the weights are

$$w_1 = \frac{D - T_{j-1}}{T_j - T_{j-1}} \qquad w_2 = 1 - w_1$$

Now, using our value for ΔR_{5Y} and σ_{scn}, we can interpolate $\hat{R}(t+h, \{\Delta R_{5Y}\}, \sigma_{scn})$ from the stress matrix using bilinear interpolation. Let $R_{scn} = \Delta R_{5Y} + r_0^{5Y}$, that is, the 5Y point after shifting the base 5Y level by our maximum likelihood shift. Then for a point (R_{scn}, σ_{scn}) within the grid extremes, the point is contained in some rectangle with corner points

$$\{(r_{i-1}^{5Y}, \sigma_{j-1}^{2x10}), (r_{i-1}^{5Y}, \sigma_j^{2x10}), (r_i^{5Y}, \sigma_{j-1}^{2x10}), (r_i^{5Y}, \sigma_j^{2x10})\}$$

Then the interpolated residual using bilinear interpolation can be expressed as the weighted sum of the four corners

$$\hat{R}(t+h, \{\Delta R_{5Y}\}, \sigma_{scn}) = w_1\, R(r_{i-1}^{5Y}, \sigma_{j-1}^{2x10}) + w_2\, R(r_{i-1}^{5Y}, \sigma_j^{2x10}) + w_3\, R(r_i^{5Y}, \sigma_{j-1}^{2x10}) + w_4\, R(r_i^{5Y}, \sigma_j^{2x10})$$

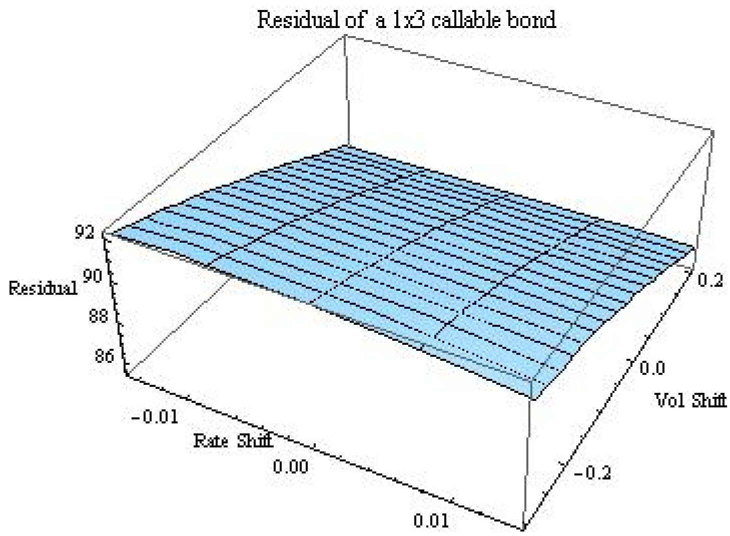
Let $w_r = \dfrac{R_{scn} - r_{i-1}}{r_i - r_{i-1}}$, and $w_\sigma = \dfrac{\sigma_{scn} - \sigma_{i-1}}{\sigma_i - \sigma_{i-1}}$. Then the weights are:

$$w_1 = (1 - w_r)(1 - w_\sigma) \quad w_2 = (1 - w_r)\sigma \quad w_3 = w_r(1 - w_\sigma) \quad w_4 = w_r w_\sigma$$

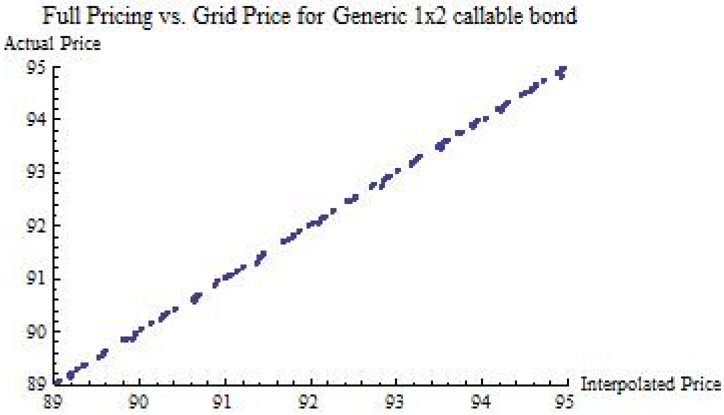
When the required point does not lie in the interior of the grid, we use flat extrapolation, that is, we find the closest point within the grid to the required point, and use the return value at that closest point.

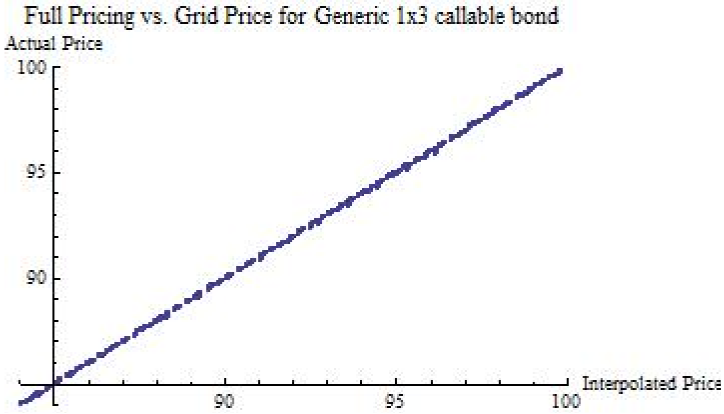
3.5 Numerical Tests

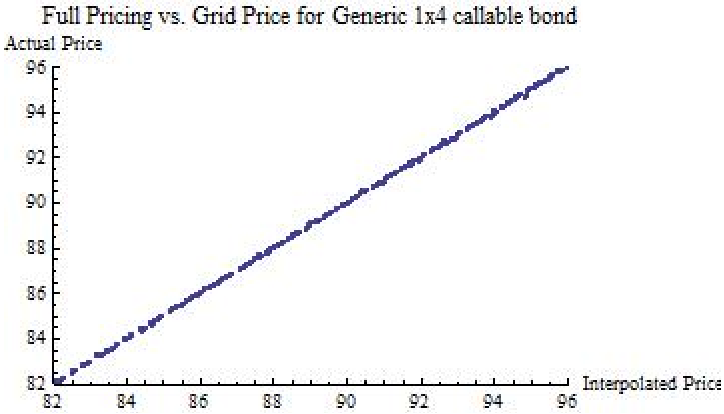
As a test case, we set up three callable bonds with maturities of 5yr, 4yr and 3yr; each is one year from the first call date, thereafter callable annually at par. Following our procedure, the residual of the bonds can be calculated. As an example, we show the residual surface of the 1x3 bond as a function of rate and vol shift:



To estimate the error committed by grid pricing, we price a uniform 2-D grid of swaption vol and swap rate over a width of 3 stdev in each of these directions, using the SMP method and compare with the OAS prices.







The average pricing error is 0.022% and the maximum error is less than 0.030%.

A Exception Handling

Under certain conditions, part or all of a stress matrix will be flagged as invalid. The conditions are:

- If a pricing error occurs in the interior of the stress matrix, the entire matrix is flagged as invalid.
- If the interior is good but a pricing error occurs at an edge, then that entire edge will be invalidated, but the remaining stress matrix will still be used for interpolation.
- If a pricing error occurs at a corner of the grid, the corresponding row and column will be invalidated, but the remaining stress matrix will still be used for interpolation.