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Fixed Income

Government fair value yield curves

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Government fair value yield curves

A fair value government par yield curve is estimated from a universe ("in-sample" universe) of government securities generated daily based on a set of exclusion rules that programmatically filter out bonds that trade with either a large liquidity premium or concession. It is well known that variations across the term structure of a high-rated G7 sovereign issuer can be accounted for by difference in liquidity of its bonds. Factors that drive this liquidity differential can vary across markets. For example, in the U.S. Treasury market there is well-documented evidence (Federal Reserve Bank of New York, 2001) that shows that on-the run (OTR) securities are more liquid then their off-the-run (OFR) counterparts and command a liquidity premium reflected by a lower yield for the OTR versus a comparable-maturity OFR. In contrast, in the German Government bond market there is strong evidence (European Central Bank, 2009) that deliverability into note and bond futures contracts is the primary driver of liquidity as opposed to benchmark status. The complete set of exclusion rules for the programmatically generated U.S. Treasury securities in-sample universe is listed in Appendix 1. Once the fair value curve is fitted to this in-sample universe, the fitted spreads of securities that were previously filtered out are also calculated and the resulting fitted spreads are a measure of the bond's liquidity premium or concession.

Modelling assumptions & methodology

Bloomberg fits the underlying discount function using piecewise cubic and exponential spline methodologies (see Appendix 2 for mathematical specifications) rather than parametric functional forms such as Nelson Siegel (NS) or Nelson Siegel Svensson (NSS) for a number of reasons. First, both these parametric functional curve models suffer from insufficient localization. Specifically, small changes in the front end could drive spurious changes in the long end of the curve. Second, while the parametric methodology produces smooth curves that filter out any "kinks" in the issuer's curve (a desirable property, for instance, in curve estimation for the new issue market), these parametric models limit the possible shapes that the fitted curve can assume. This makes them insufficiently flexible for the purpose of bond-specific relative value applications.

The discount function is fitted using any one of the spline methodologies mentioned above. The errors between a bond's theoretical yields and its market yields are reduced by minimizing the Root Mean Square Error (RMSE):

$$RMSE = \sqrt{\frac{\sum[YTM(market) - YTM(model)]^2}{n}}$$

Where YTM(market) = YTM derived from market price.

YTM(model) = YTM derived from the model price implied by the fitted spline curve.

Summation taken across the in-sample universe described above and Appendix 1.

The RMSE is a measure of the average yield error across the fitting universe. The fitting algorithm seeks to minimize this error. The quantity defined as the YTM (market) less the YTM (model) is referred to as the fitted spread of the bond. Positive values for the fitted spreads represent bonds that are trading "cheap" to the fair value fitted curve and typically require a liquidity concession for investors to buy them. Well-seasoned U.S. Treasuries that have rolled down the curve and have to compete with more recently issued benchmark OTRs often trade at a liquidity concession.

Negative values for the fitted spread represent bonds that are trading "rich" to the fitted curve and typically represent bonds that trade at a liquidity premium because they are highly demanded collateral. The OTR and first OFR U.S. Treasury issues usually trade rich as they are used by the dealer community for hedging purposes. Similarly, the cheapest-to-deliver bond into the front Treasury futures contract often trades rich to the curve.

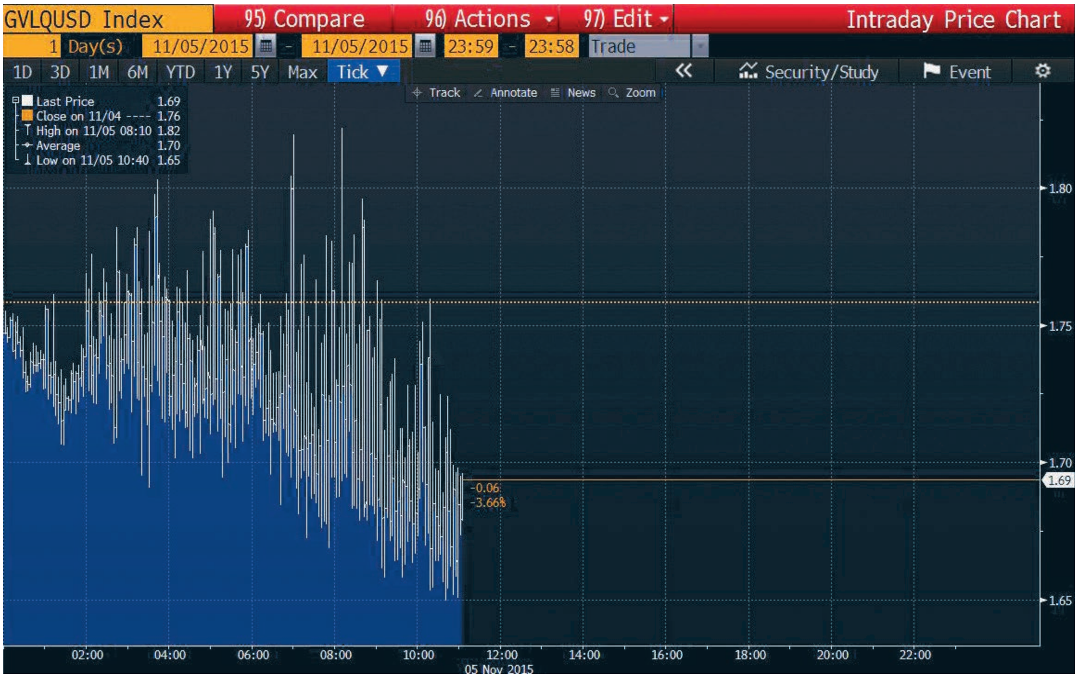
An alternative formulation minimizes the errors between the bonds' theoretical and market prices rather than yields. Bloomberg does not favor this approach because the lower price risk of shorter duration bonds implicitly assigns lower weight for equal yield errors relative to longer duration bonds, resulting in larger yield-fitting errors at the front end. Directly minimizing errors in yields as defined above ensures that the errors are weighted uniformly across the term structure.

To calculate the model price of the bond, the Bloomberg Cash Flow engine first lays out the bond's cash flows based on various settlement, coupon frequency, compounding frequency, currency and calendar conventions, to name a few of the key attributes. An initial guess is made for the discount function fit based off any one of the spline-fitting methodologies, and the RMSE is calculated as above. The Bloomberg proprietary optimizer will then perturb the hyper-parameters of the spline and look for any improvement or reduction in the RMSE. Once the RMSE converges to a particular value for some level of tolerance, the optimization is halted and the values of the fitted curve are stored.

Intra-day pricing, fitting & liquidity index (patent pending)

The fitter uses the Bloomberg CBBT price source. The fitter runs off a timer that initiates a refit approximately every 500 milliseconds. If at the time of a refit there is no price for more than 20% of the fitting universe, the fitted curve does not get created and the fitter attempts a refit based of the timer described above. A fitted spread is ticked for a bond if its last price is refreshed.

In addition to the in-sample RMSE (see "Modelling Assumptions and Methodology" section above) , Bloomberg also calculates a RMSE across the entire universe of U.S. Treasury notes and bonds with a remaining maturity one year or greater, based off the intra-day Bloomberg relative value curve fitter. Intra-day values of this RMSE are ticked through the GVLQUSD Index ticker every time a new tick comes in for any of the bonds in the universe. This index is a measure of prevailing intra-day liquidity conditions in the USD Treasury market.



Intra-day time series of U.S. Government Securities Liquidity Index. Smaller values of the index represent more favorable liquidity conditions.

When liquidity conditions are favorable, the index value, representing average yield errors, is small as any dislocation from fair value is normalized within a short time frame. Under stressed liquidity conditions and limited availability of risk capital, dislocations from fair value implied by the curve fitter can remain persistent, resulting in larger index value due to "liquidity tiering." It is widely known that there are many different ways to measure liquidity (Adrian, Fleming, Stackman and Vogt, 2015), however, Bloomberg's framework is one of the few aggregate measures of liquidity available to investors that can be monitored intra-day. The fitted spreads and liquidity index for U.S. Treasuries are ticked from the start of trading in Tokyo the previous day (7:00–8:00 PM NYC depending on daylight savings adjustment) to 5:00 PM of the current day in NYC. An end-of-day history for GVLQUSD Index is also available going back several years and spanning multiple liquidity events. That said, end-of-day history is limited to the extent that it cannot capture any transitory order book imbalances during the course of the trading session.

Historical correlations

The changes in the fitted yields, estimated by the spline model, track changes in market yields. We computed the coefficient of determination or R-square (a measure of how well the model explains the variability of outcomes) between market yield changes and fitted yield changes over the past five years (analysis date 10/19/2015) for seven representative bonds across the yield curve. The results for daily, weekly and monthly yield changes are displayed below and indicate a high R-square between actual and fitted yield changes. Similar results were obtained for other bonds in the universe.

Changes	T 4.25% 11/15/17	T 4.25% 11/15/17	T 6.25% 8/15/23	T 6.375% 8/15/27	T 5.375% 2/15/31	T 4.5% 02/15/36	T 4.375% 05/15/40
1d	0.994	0.996	0.998	0.998	0.997	0.997	0.999
5d	0.995	0.997	0.998	0.999	0.998	0.998	0.997
10d	0.996	0.998	0.999	0.999	0.998	0.998	1.00
20d	0.996	0.998	0.999	0.999	0.998	0.997	0.999

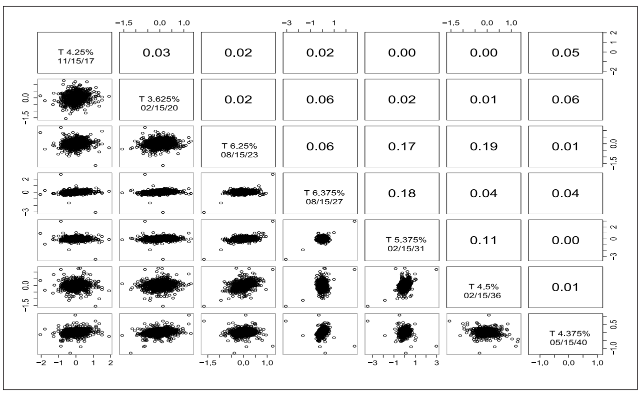
R-square of changes in fitted yields versus actual yields over the past five years (analysis date: 10/19/15).

Changes between fitted spread and yields in the table below display low R-squares, confirming that fitted spreads generally tend not to be market directional.

Changes	T 4.25% 11/15/17	T 4.25% 11/15/17	T 6.25% 8/15/23	T 6.375% 8/15/27	T 5.375% 2/15/31	T 4.5% 02/15/36	T 4.375% 05/15/40
1d	0.007	0.006	0.050	0.031	0.035	0.028	0.003
5d	0.009	0.011	0.047	0.041	0.018	0.009	0.004
10d	0.007	0.019	0.072	0.103	0.053	0.024	0.005
20d	0.000	0.057	0.089	0.117	0.049	0.039	0.003

R-square of changes between fitted spreads and fitted yields over the past five years (analysis date: 10/19/15).

The chart shows that, generally, the R-squares are close to 0.0 in most cases, underscoring that the fitted spreads represent idiosyncratic risk of each bond rather than any factor risk common to all bonds. The scatter plots confirm the weak to non-existent linear relationship between changes in fitted spreads. When the R-square is larger than zero, it is clear from the scatter plots that these modestly elevated levels are due to the presence of outliers. This is to be expected; large changes in fitted spreads are usually the result of a macroeconomic shock, and fitted spreads of seasoned bonds are likely to track each other more closely as their respective liquidity premiums are driven by a common risk factor under these special circumstances. Similar results were obtained across most of the remaining bonds in the universe.



Corrgram: Upper diagonal displays R-squares of daily changes (past five years, analysis date: 10/19/15) of fitted spreads of select securities across the curve. Lower diagonal displays scatter plot of changes in fitted spreads.

GOVY function

Fitted spreads from the Bloomberg U.S. Treasury fitted curve model can be accessed through GOVY <GO>. This section describes the function briefly. More detailed information is available on the function's help page. Users have the ability to select fitted spreads from both a piecewise cubic as well as piecewise exponential spline using the "Spread" drop down in the top left portion of the function. The most recent value of the fitted spread is displayed under the spread column alongside its historical range and z-score. The selected spread's z-score is a measure of its dislocation from its rolling average measured in historical standard deviations of this spread over the same period. The window used to calculate a single z-spread value can be selected through the "Range" drop down. From a relative value perspective it is important to analyze the z-score of the fitted spread in addition to its absolute value, as some bonds will trade systematically rich or cheap to the fitted curve. The GOVY function also displays each bond's yield curve roll-down over various horizons. The roll-down is calculated off the cubic spline assuming a constant fitted spread of the bond to the spline.

	Bond Pricing Information					Spline Cub			Description		
	Security	Dur	Price	Yield	Yld Chg	Spread	Spr Chg	Z-Score	OI	Amt Iss	SOMA%
21)	T 5 1/8 05/15/16	0.5	102-16	0.350	+3.4	2.6	+1.2	1.85	10yr	23.2	36.0
22)	T 7 1/4 05/15/16	0.5	103-19 5/8	0.348	+3.0	3.5	-2.0	0.91	30yr	18.8	29.9
23)	T 0 1/4 05/15/16	0.5	99-30 5/8	0.332	+0.8	-1.6	+0.0	-1.87	3yr	31.9	0.0
24)	T 0 3/8 05/31/16	0.6	100-00 1/4	0.361	+0.7	-0.6	-0.2	0.09	2yr	31.0	0.0
25)	T 1 3/4 05/31/16	0.6	100-25 1/8	0.359	+0.0	-0.1	-0.9	0.24	5yr	37.9	38.7
26)	T 3 1/4 05/31/16	0.6	101-19 7/8	0.381	+3.1	2.8	+1.2	2.31	7yr	26.8	39.3
27)	T 0 1/2 06/15/16	0.6	100-02 1/4	0.387	-0.4	0.4	-1.0	1.26	3yr	32.0	0.0
28)	T 3 1/4 06/30/16	0.6	101-26 7/8	0.410	+1.8	2.2	-0.1	1.95	7yr	27.7	35.0
29)	T 0 1/2 06/30/16	0.6	100-02	0.403	+1.2	0.4	-0.5	1.83	2yr	29.9	0.0
30)	T 1 1/2 06/30/16	0.6	100-22+	0.415	+3.1	1.9	+1.5	2.59	5yr	36.1	10.7

The Rates Trader function, GOVY <GO>, displays both current and historical fitted spreads from the spline.

Appendix 1

Daily Exclusion Rules for systematic generation of fitting Universe for U.S. Treasuries

- Drop all bills
- Drop all callable bonds (for historical fits) and floaters
- Drop all TIPS
- Drop all strips
- Drop all bonds with remaining maturities less than 0.5 years
- Drop the on-the-run and first off-the-run
- Drop original issue 30-year coupon bonds with remaining maturity less than 10-years
- Drop original issue 10-year notes with remaining maturity less than 5-years
- Drop original issue 7-year notes with remaining maturity less than 5-years
- Drop original issue 5-year notes with remaining maturity less than 2-years

Daily Exclusion Rules for systematic generation of fitting universe for Federal Republic of Germany notes & bonds

- Drop all notes and bonds with remaining maturity less than 0.5 years
- Drop all Bubills
- Drop all inflation linked notes and bonds
- Drop the cheapest-to-deliver for the front Euro-Schatz, Euro-Bobl, Euro-, Euro-Bund and Euro-Buxl futures contracts
- Drop all notes and bonds if yield to maturity differs from coupon by 500 bps

Daily Exclusion Rules for systematic generation of fitting universe for Japan JGBs

- Drop all bills
- Drop all floaters
- Drop all STRIPS
- Drop all bonds with remaining maturities less than 0.5 years
- Prior to September 2008, drop the on-the-run and first off-the-run (excluding 40-yr and 30-yr bonds)
- Drop original issue 40-year coupon bonds with remaining maturity less than 30-years
- Drop original issue 30-year coupon bonds with remaining maturity less than 20-years
- Drop original issue 20-year coupon bonds with remaining maturity less than 10-years
- Drop original issue 10-year notes with remaining maturity less than 5-years
- Drop original issue 5-year notes with remaining maturity less than 2-years

Daily Exclusion Rules for systematic generation of fitting universe for France notes (BTANs) and bonds (OATs)

- Drop all notes and bonds with remaining maturity less than 0.5 year
- Drop all BTFs
- Drop all inflation linked bonds
- Drop all strips
- Drop all floaters
- Drop all notes and bonds if yield to maturity differs from coupon by 500 bps and issue has not been tapped for more than 1 billion Euros over the past 2 years

Daily Exclusion Rules for systematic generation of fitting universe for Italy BTPs

- Drop all BTPs with remaining maturity less than 0.5 year
- Drop all bills (BOTs and CTZ)
- Drop all floaters (CCTs)
- Drop all inflation linked bonds
- Drop all strips
- Drop all notes and bonds if yield to maturity differs from coupon by 500 bps and issue has not been tapped for more than 1 billion Euros over the past 2 years

Daily Exclusion Rules for systematic generation of fitting universe for UK Conventional Gilts

- Drop all notes and bonds with remaining maturity less than 0.5 year
- Drop all bills
- Drop all inflation linked bonds
- Drop all strips
- Drop all floaters
- Drop all callable bonds
- Drop all notes and bonds if yield to maturity differs from coupon by 500 bps
- Drop all notes and bonds if issue amount less than 1.5 billion GBP

Daily Exclusion Rules for systematic generation of fitting universe for Spain (Bonos and Obligaciones)

- Drop all notes and bonds with remaining maturity less than 0.5 year
- Drop all bills
- Drop all floaters
- Drop all inflation linked bonds
- Drop all notes and bonds if yield to maturity differs from coupon by 500 bps and issue has not been tapped for more than 1 billion Euros over the past 2 years

Daily Exclusion Rules for systematic generation of fitting universe for Canada bonds

- Drop all bonds with remaining maturity less than 0.5 year
- Drop all bills
- Drop all real return bonds
- Drop all strips
- Drop Canada Notes
- Drop Canada Premium Bonds
- Drop Canada Savings Bonds
- Drop all fixed-coupon marketable bonds if issue amount less than 1.0 billion CAD

Appendix 2

Piecewise cubic spline

The functional form of this curve is the piecewise quadratic continuously compounded forward rate. The first derivative of the rate is continuous at knot points. Also, the first derivative of the forward rate is zero at the beginning and at the last knot point:

$$\frac{dr(0)}{dt} = 0$$
$$\frac{dr(T_N)}{dt} = 0$$

The discount factor of the smooth curve is defined as:

$$D(t) = \exp(-L(t))$$

where the log discount factor L(t) is:

$$L(t) = \lambda_0 \cdot t + \tfrac{1}{6}\sum_{T_n<t} \lambda_n (t - T_n)^3 - \tfrac{1}{6} P \cdot t^3$$

Where T_n is n^{th} knot point; the summation is done for all n=1 .. N-1 such that $T_n < t$, which leads to a curve L(t) being piecewise cubic. At each knot point $t=T_n$ a new additional cubic polynomial starts to the right from the point $t=T_n$ in a manner that assures that first and second derivatives of L(t) are continuous. The coefficients λ_N (where n=0 .. N-1) are the independent parameters. The last λ_N is not an independent parameter, but defined

as: $\lambda_N = -\frac{1}{T_N}\sum_{n=1}^{N-1} \lambda_n T_n$

The parameter P is defined as:

$$P = \sum_{n=1}^{N} \lambda_n$$

This choice of P assures that for $t>T_n$ the cubic term in L (and quadratic in forward rate) is zero. The choice of λ_N similarly assures that for $t>T_n$ the quadratic term in L (and linear in forward rate) is zero. The curve is "smooth," e.g., the forward rate has a continuous first derivative. The curve described above satisfies all the boundary conditions, e.g., continuous first derivative of the forward rate at knot points or zero derivatives at the ends of the curve by construction, and it has N degrees of freedom. However, it is easy to see that change in any parameter λ_N, where n=0 .. N-1, leads to changes in λ_N and P and thus affects the value of the function everywhere.

Piecewise exponential spline

The log discount factor L(T) is defined to be linear between knot points. Alternatively, the continuously compounded forward rate is piecewise constant between knot points:

$$L(t) = \lambda_{n+1}(t - T_{n+1}) + \sum_{T_n<t} \lambda_n (T_n - T_{n-1})$$

This model has a continuous discount function, but the first derivative is discontinuous. The utility of this model is that it is highly localized to the degree that changing the value of any one of the parameters has localized impact to adjacent knot points during the curve-fitting/optimization process.

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