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PORTFOLIO & RISK ANALYTICS

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FIXED INCOME

Fundamental Factor Model

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Bloomberg

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> INTRODUCTION

Factor models have become an indispensable tool for modern portfolio and risk management in the last several years. They provide greater understanding of sources of portfolio risk, the ability to forecast absolute risk as well as risk relative to a given benchmark, attribute portfolio performance, and improve portfolio construction. Recent market volatility has highlighted the importance of controlling unwanted factor exposures in portfolios. While the factor models have been in use for at least two decades, the market collapse in 2008 attracted attention of traditional and quantitative portfolio managers alike and has dramatically increased client interest in factor models.

Factor models are based on the basic principle that security returns are driven by a set of common factors, therefore portfolio risk depends on how volatile and correlated these factors are and by the level of portfolio exposure to individual factors. Additionally, there are risks not captured by common factors, hereafter referred to as “non-factor” risks and factor models help estimate these as well.

There are in general three types of factor models: statistical, explicit and implicit factor models. They differ in their approach to defining factors and security factor exposures. We list these classes below and highlight their advantages and disadvantages.

Statistical factor models use methods similar to principal component analysis. Both factor returns and factor exposures are determined from asset returns. The advantage of such models is that they require little data: only security returns are needed. Their main disadvantage is interpretability. For example, it is not clear what a portfolio manager should do if the risk model ascribes a great deal of risk to the fourth principal component since there is no easy way to associate economic meaning with the principal components.

Explicit factor models start by specifying factor returns and then use techniques such as time series regression analysis to determine exposures to factors. In terms of data requirements, such models require asset returns as well as factor returns. Sometimes these models are also called exogenous factor models (since factors are specified outside of the model, i.e. exogenously) or time-series models (since security exposures are determined on a security-by-security basis by running a time- series regression). The advantage of such models is that they allow for inclusion of arbitrary factors, as long as factor time-series data is available. Therefore, it is easy to incorporate various macro- economical quantities, such as country/industry indices, changes in GDP growth. The main disadvantage is that security-level exposures in these models tend to be non-intuitive. For example, a utility stock can be strongly exposed to a technology factor or a small cap stock may have a strongly positive size factor exposure. Moreover, since many exposures are determined from historical time- series regressions, such models tend to perform well in-sample, but have poor fit out-of-sample.

Implicit factor models define security exposures to each factor and then impute factor returns from a regression of security returns on the exposures. Such models are typically the most data intensive. They require asset returns and several factor exposures for each security. Sometimes these models are also called endogenous factor models (since factors are derived from other returns and exposures), cross-sectional factor models (since factors are revealed through a cross-sectional regression) or fundamental factor models (since exposures are typically based on fundamental information unique to each security).

Bloomberg integrated fixed-income model is based on implicit factor approach. Some factors such as the yield curve changes are unambiguously observable in the market and therefore are extracted from the total returns of the securities. The remaining portion of the return, called *excess return*, reflects risk dimensions such as credit quality, industry, or capital structure effect, which lacks strong consensus in the market[2]. Therefore, we estimate these factor returns using implicit factor models.

The main motivation of choosing this methodology is its better interpretability by the user. It gives greater insight into the portfolio risk sources and leads to intuitive action items. Factors and exposures to factors are meaningful and intuitive for portfolio managers and risk managers. Furthermore, they are important measurements commonly used throughout the management process. For instance, the exposures to our curve factors are the key rate durations along a few points on the term structure. A portfolio manager can easily express her view on the curve movement by accurately setting up the curve exposures along these points and assess the risk accordingly. Finally, she can run performance attribution on the same set of key rate points, and adjust the exposures as needed. The consistency across portfolio reporting, risk analysis and performance attribution makes risk models and attribution models an integral part of the investment process.

The fixed income model covers sovereign, agency (quasi government) and corporate bonds in both investment and high yield grades denominated in 38 currencies[3]. Traditionally, most fixed income indices are organized primarily based on the currency denomination, rather than the country of the issuer. For fixed income risk model purpose, we group the whole world into two categories: the developed markets, and the emerging markets[4]. So we would have four different combinations as illustrated in the following chart: developed countries issue bonds in developed currencies (or hard currencies hereafter), emerging countries issue bonds in hard currencies, emerging countries issue bonds in local emerging currencies and developed countries issue bonds in emerging currencies. For risk model purpose, we group the last two together since there are very limited data for bonds in emerging market currency denominated bonds.

For the first case of developed countries issue bonds in developed currencies, we have developed what we will refer to as the “G7 models”: the USD, EUR, JPY, GBP, CAD, AUD and CHF. In this category, we will add other markets such as NOK, DKK soon. For the case of emerging countries issue bonds in developed currencies, we developed the EM hard currency model. For bonds denominated in emerging market currencies, we have the EM local currency model. In this document, we focus on the G7 model. For the EM models, we defer the details to the white paper on fixed income emerging market factor model.

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[2] Even though some sector and credit quality spread curves are available. The discrepancy from various providers can be rather large due to difference in pricing, constituents, etc. Therefore, we don’t use the changes of these curves as explicit factors.

[3] Only a few major markets have High Yield factors. For other markets, the high yield corporate bonds may have higher risk than an investment grade counterpart by having a larger exposure to the spread factors.

[4] Developed markets include Australia, Canada, US, Japan, UK, the Euro zone 17 nations, Denmark, New Zealand, Norway, Sweden, and Switzerland.

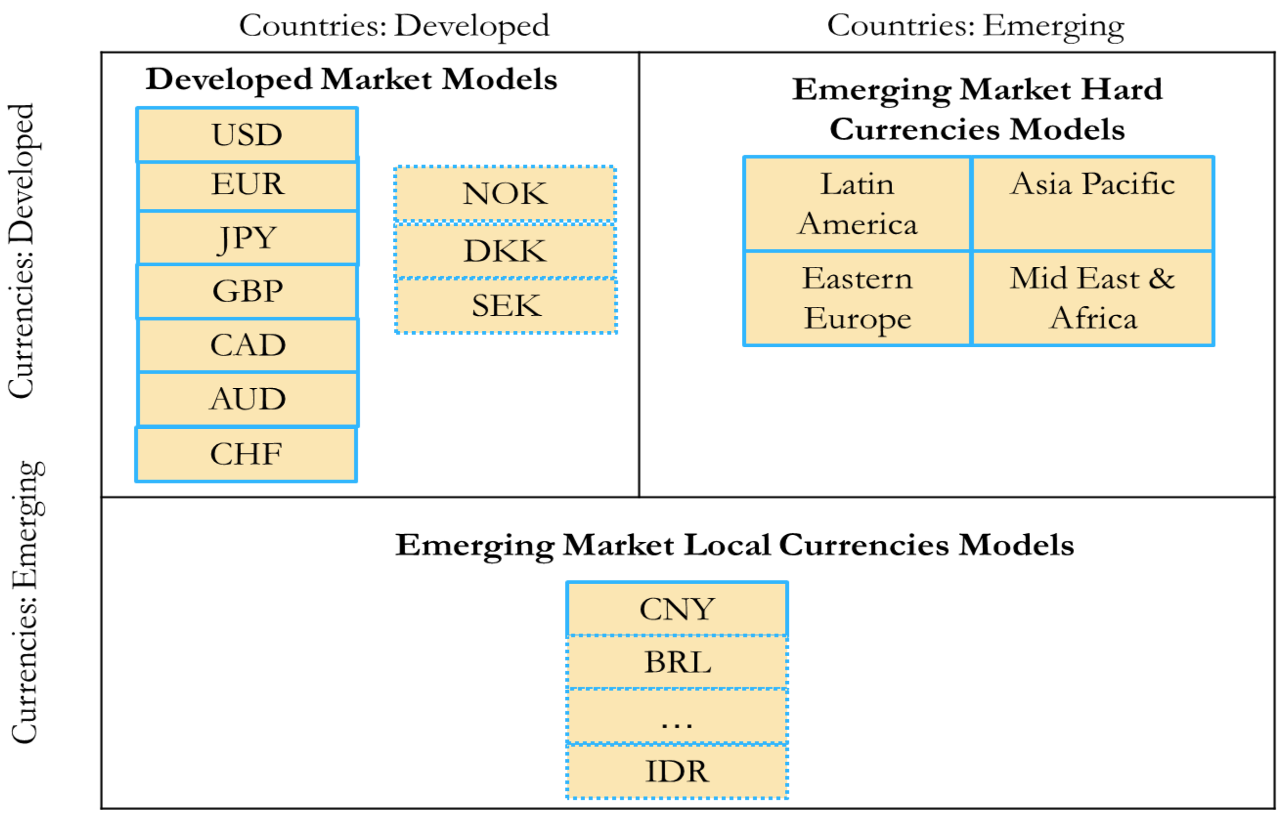


Figure 1: Fixed Income Factor Model Overview

The rest of the document is organized as follows. First, we describe the universe of securities the G7 model intends to cover and identify the subset of that universe used to construct the model. Next, we present the basic model structure and go over each group of factors in detail using the USD model as an example. In the following section, we discuss the covariance construction for individual currency model as well as integration of multiple models. Finally, we discuss model performance in various tests and conclude.

Estimation and Data Universes

All bonds in the G7 estimation universe are first classified into one of the seven currency blocks based on the currency of denomination. In each of these seven individual currency models, all bonds have exposures to common factors such as the currency factor, the curve factors and the volatility factor, as well as some specific credit spread factors depending upon the asset class they belong to.

Securities denominated in USD are classified into sovereign, agency, corporate and distressed asset classes[5]. The rest of the G7 securities, i.e. denominated in EUR, JPY, GBP, AUD and CAD, are classified only into sovereign, agency and corporate.

For the data universe, the minimum requirements for coverage include:

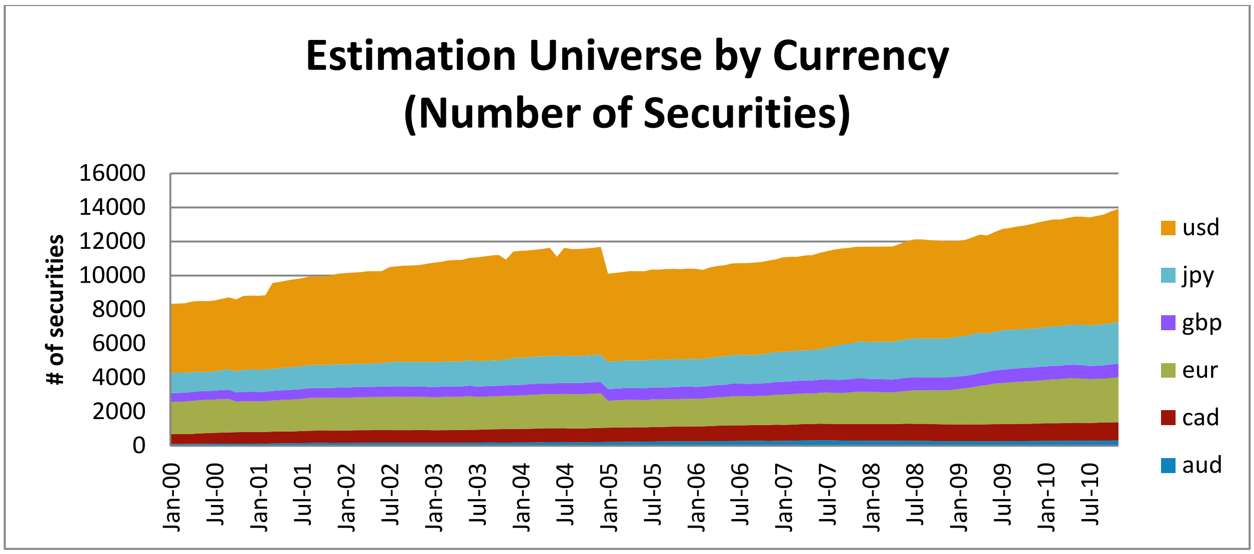
- Single security prices
- Risk exposures (e.g., key rate durations, vega, OAS, spread duration, etc.) based on the above prices

5 Mortgages and tax-exempt munis are part of the USD model, because they each have unique factors, the details are discussed in separate white papers.

- Descriptive information including country, sector and industry etc. which allow us to map securities to the right model factors.

The estimation universe is constructed from a few sources: Bloomberg-Barclays fixed income bond indices, Bloomberg security terms and conditions, Bval pricing, and Bloomberg analytics. In general, bonds in the estimation universe have at least one year remaining term to final maturity, and satisfy some minimum amount outstanding requirements. For instance, the current minimum amount outstanding for the U.S. corporate bonds is $250 million.

The evolution of the estimation universe of the G7 is shown in Figure 2. The drop in the number of bonds in the estimation universe in January 2005 comes mainly from the USD and EUR corporate sectors.



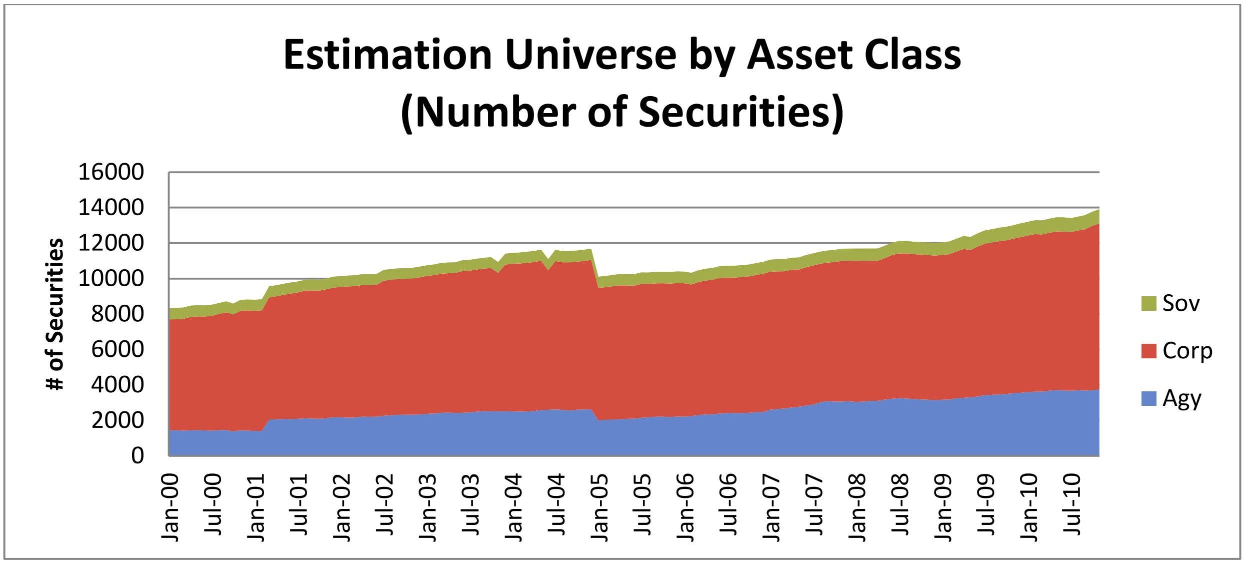
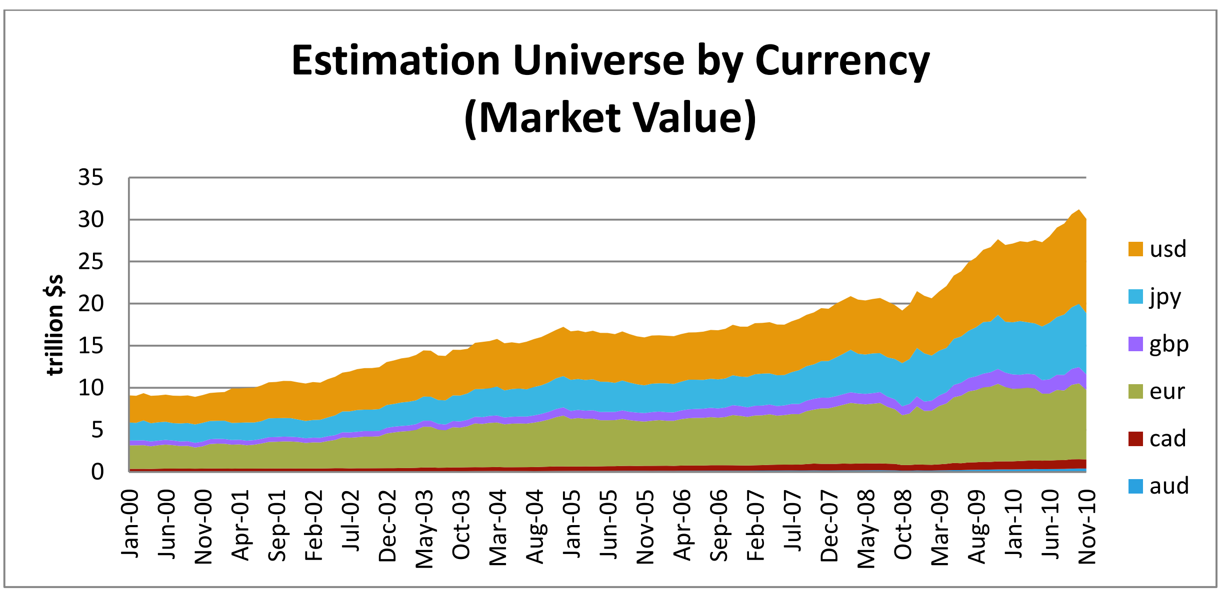


Figure 2: Estimation Universe by Currency in Number of Securities

Figure 3 is the same estimation universe evolution over time in terms of the market value by currency and asset class. A few observations can be made by comparing the two figures. First, the government sectors account for a much larger share of the universe because the government issues are in general much larger than corporate ones. Second, the market value dropped during the 2008 crisis and this is mainly due to the drop in prices of the securities, rather than the number of securities. Finally, the total market value went up rapidly in the last two years, which is primarily due to the increased activities in government issuance around the world.



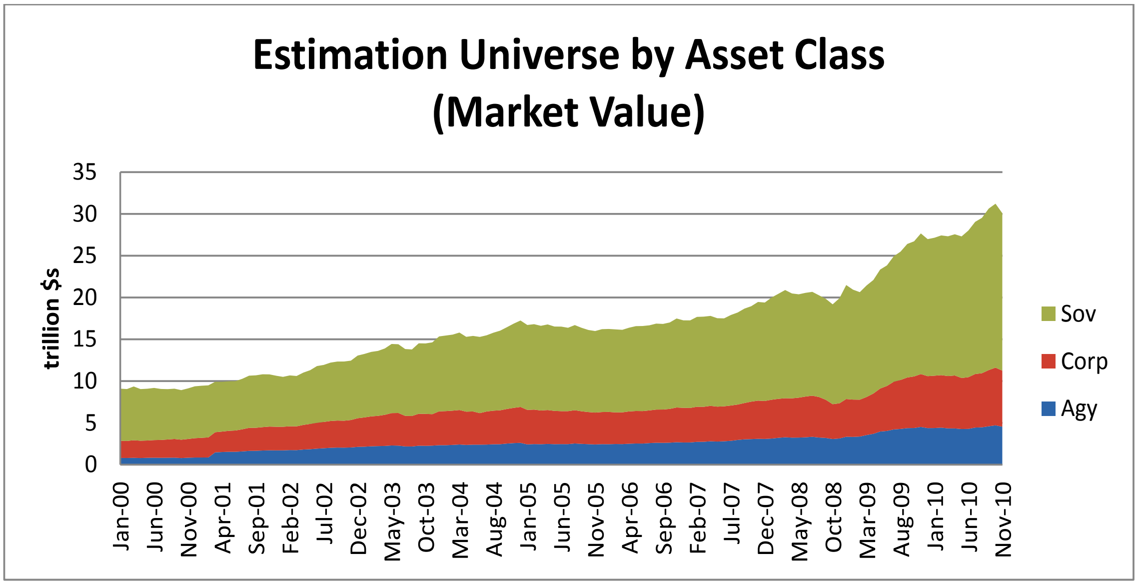


Figure 3: Estimation Universe by Currency and Asset Class in Trillion USD Dollars

> MODEL STRUCTURE

In general, the total return of a generic security n over time period t can be written as the sum of several return components, each in turn can be expressed as the product of a factor return and exposure to the factor. The risk (or volatility) of this return comes from the volatility of all these factors as well as their correlations. First note that one return component is of different nature than all other components: the time component. The time return is the return due to the passage of time with all other factors held constant. In other words, it is the return if the bond's maturity (and all its cash flows) were shortened by one period. This component is deterministic and therefore we explicitly subtract it from the total return. The difference between the total return and the time return, which will be referred to as the stochastic return, can therefore be written as:

$$R_n^t - R_n^{t,time} = \sum_{k=1}^{K} X_{nk}^t \cdot F_k^t + \varepsilon_n^t \qquad (1)$$

where

R_n^t is the total return for security *n* during time period *t*,

$R_n^{t,time}$ is the time return for security *n* during time period *t,*

X_{nk}^t is a pre-defined factor exposure of security *n* to a factor *k*, at time *t* with *K* factors in total,

F_k^t is the factor *k* at time *t* to be derived from running the above regression,

ε_n^t is residual return of security *n* at time *t*.

We drop the super-script t for ease of notation hereafter. Formula (1) is a generic specification for all asset classes. The factors are of two kinds. One type of factors (explicit factors) is observable in the market and therefore we simply use the observed change directly. Currency factors, curve factors and volatility factors are the explicit factors in our model. Other factors are obtained via cross-sectional regression (implicit factors). Spread factors for different asset classes are the implicit factors, and they are slightly different in each of the G7 models. We will discuss details on all of the factors for the USD model in subsequent sections. Details on factors for other models are provided in the appendices.

> SYSTEMATIC FACTORS

Currency Factors

When the currency of denomination for security n is different from the base currency of the portfolio, its total return expressed in base currency can be approximated by the sum of its local return and the currency return, where the currency return is computed as the linear return of the exchange rate. The base currency of the portfolio is often referred to as the numeraire currency. For instance, we chose US dollar as the numeraire, meaning that we would like to consider our risk and return from the perspective of a US investor. The return of security n in USD can be represented as:

$$(1 + R_n^{usd}) = (1 + R_{FX}^{loc:usd})(1 + R_n^{loc}) \approx 1 + R_{FX}^{loc:usd} + R_n^{loc} \tag{2}$$

Where

R_n^{usd} is the total return of security *n* in the numeraire currency (USD);

$R_{FX}^{loc:usd}$ is the return of currency *loc* relative to USD;

R_n^{loc} is the total return of security *n* in local currency *loc.*

The expected currency return can be computed as the difference in cash rates in the two currencies$\left(r_f^{usd} - r_f^{loc}\right)$. We extract the expected time return from the total return, equation (2) can be re-written as:

$$R_n^{usd} - \left(R_n^{loc,time} + \left(r_f^{usd} - r_f^{loc}\right)\right) = \left(R_n^{loc} - R_n^{loc,time}\right) + \left(R_{FX}^{loc:usd} - \left(r_f^{usd} - r_f^{loc}\right)\right) \tag{3}$$

In this equation, the left hand side is simply the stochastic return in numeraire currency, the first bracket on the right hand side is the stochastic return in local currency and the second bracket on the right hand side is the unexpected currency return. Equation (3) fits in the generic factor model structure in equation (1) with currency return as one of the factors. The advantage of this return decomposition is that local excess return is independent of the numeraire currency. Therefore the switch of the numeraire will only affect the portion of return and risk captured by the currency return. After isolating the currency return component, we apply the fundamental factor decomposition to local stochastic return.

Curve Factors

Most fixed-income securities' prices are largely impacted by the movement of the yield curve, especially bonds with investment grade ratings. There are generally two different approaches to construct the curve factors. One is to explicitly define them as the yield change on a set of pre-determined tenor points along the yield curve. The other one is to use the principal component approach to construct the three well known curve factors: the shift, the twist and the butterfly. These three factors have been proven to explain a large portion of the curve movements.

We have chosen the first approach mainly in favor of the transparency and interpretability. The curve factors are nine par rate changes along the 6M, 1Y, 2Y, 3Y, 5Y, 7Y, 10Y, 20Y, and 30Y tenors and the square of the average curve change to capture the second-order effect. The exposures to these factors are simply the key rate durations and the option-adjusted convexity, i.e.

$$R_{yc} = -\sum_{i=1}^{9} KRD_i \cdot \Delta y_i + \frac{1}{2} OAC \cdot (\overline{\Delta y})^2 \quad (4)$$

Where R_{yc} is the return due to curve change, Δy_i is the rate change of the par swap curve at i-th tenor point and $\overline{\Delta y}$ is the simple average of changes across all tenor points.

It is important to note that the base curve for sovereign and agency bonds is the sovereign curve, and the base curve for corporate bonds is the swap curve. All analytics in the models are defined relative to the specific base curve. The sovereign curves in different markets had served as the benchmark to all other fixed income securities in the past. However, the status of the swap curve has ascended rapidly over the last decade due to the explosive growth of the interest rate derivatives market in general and the swaps market in particular. Swap instruments have been increasingly employed to either directly express curve views or hedge out duration risks. In addition, there has been rapid growth in assets managed by investors who fund themselves at LIBOR (i.e., swap rate). For these reasons, we chose the swap curve as the base curve for corporate asset class.

Figure 4 shows the cumulative returns for the 9 explicit key rate change factors for the swap curve. All the tenor points follow a similar pattern over time, which implies high correlations among these factors. Over the last decade rates have grinded lower, therefore the negative cumulative factor returns mean positive returns from curve factors for investors.

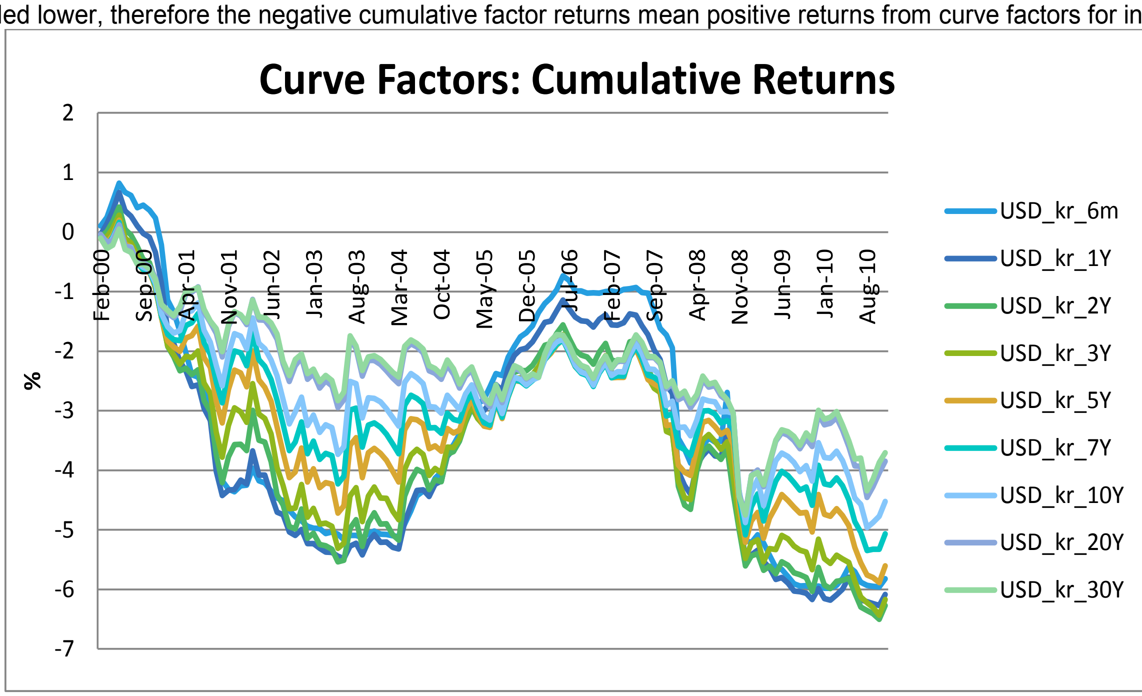


Figure 4: Cumulative Returns for Curve Factors

Volatility Factors

For bonds with embedded options (e.g., callable or puttable corporate), an important factor is changes in implied interest rate volatilities[1]. Bloomberg fixed income model uses a single volatility risk factor that is based on the average changes of a selected group of grid points on the implied at-the-money swaption volatility surface. The exposure to the volatility factor

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[1] The implied volatilities can be observed in the interest rate options market, i.e., caps, floors, and swaptions.

is measured by the bond's "volatility duration", which is computed as the bond's vega divided by its full price. Hence the volatility return can be expressed by

$$R_{vol} = \frac{vega}{P + AI} \cdot \Delta\sigma \tag{5}$$

Where R_{vol} is the return due to volatility change, P and AI are the clean price and accrued interest, and $\Delta\sigma$ is the average change in volatility.

The grid points chosen are provided in Table 1 for the USD model. Other models use the same grid as long as the points are available. The choice of one volatility factor is based on the PCA analysis on the volatility change of the 20 grid points. PCA is a mathematical procedure that uses an orthogonal transformation to convert a set of observations of possibly correlated variables into a set of uncorrelated variables called principal components. The principal components are orthogonal to each other and accounts for as much as the variability in the data as possible in a descending order. For the USD model, the first principal component, which is roughly the parallel shift of the volatility surface, accounts for 78% of the variance.

Term	2 YR	5 YR	10 YR	30 YR
6 MO	USSV0F2	USSV0F5	USSV0F10	USSV0F30
1 YR	USSV012	USSV015	USSV0110	USSV0130
5 YR	USSV052	USSV055	USSV0510	USSV0530
10 YR	USSV102	USSV105	USSV1010	USSV1030
30 YR	USSV302	USSV305	USSV3010	USSV3030

Table 1: Grid Points Used for Volatility Factor on the ATM Swaption Volatility Surface

Spread Factors

Most fixed income securities often trade at a spread to the base curve. The level of the spread reflects the additional premium an investor requires for taking the additional credit risk, liquidity risk, and other risks. The change of the spread, which reflects primarily the change in perceived risk of a security, comes from both the common forces, affecting all bonds with similar characteristics, and specific shock to that particular issuer. The common forces are captured by the systematic spread factors. Spread factors are constructed for each model and asset class separately. For example, USD model has four groups of spread factors (sovereign, agency, credit, distressed), other currencies have three groups (sovereign, agency, credit).

Sovereign

The sovereign sector consists of bonds issued by the government in its local currency, and it is generally a significant portion of the bond market in terms of amount outstanding. For instance, the United States government securities, often referred to as the treasury securities, have an amount outstanding of 9.7 trillion for the marketable portion as of October 31, 2011 according to Treasury Direct. Theoretically, sovereign bonds should have zero spread to the sovereign curve. In reality, since the curve is constructed using a small subset of the bonds, which in generally is the most liquid on the run bonds, most sovereign bonds in the estimation universe have non-trivial spread to the curve. To capture the impact of the spread change, we try to explain the implied spread change by an average spread change factor, a slope factor to capture the term structure difference in spread change and an OAS factor to capture the liquidity impact on the spread change.

Spread returns of the US sovereign sector are modeled with three factors:

$$\frac{R_n^{ex}}{-OASD_n} = F_{Sprd}^{Sov} + (OASD_n - medOASD) \cdot F_{Slope}^{Sov} + (OAS_n - medOAS) \cdot F_{OAS}^{Sov} + \varepsilon_n \quad (6)$$

F_{Sprd}^{Sov} is the sovereign spread factor, representing the average sovereign spread change;

F_{Slope}^{Sov} is the slope factor, and $medOASD$ denotes the median OASD of all the USD denominated sovereign bonds in the Estimation Universe. This factor measures the additional spread change for every year of spread duration increase from the median. This factor is positive when spread curve steepens, negative when it flattens. It essentially captures the term structure of the spread change in a simplified fashion. We limit the exposure to this factor to [max(-5, pct25_OASD-medOASD), min(5, pct75_OASD-medOASD)], where pct25_OASD and pct75_OASD are the 25th and 75th percentile of the OASD distribution for the sovereign estimation universe.

F_{OAS}^{Sov} is the OAS factor, and $medOAS$ denotes the median OAS of all the USD denominated government bonds in the Estimation Universe. This factor measures the additional spread change for 1% increase in OAS from the median. This factor is positive when low OAS securities tighten relative to high OAS securities. We limit the exposure to this factor to [max(-100bps, pct25_OAS-medOAS), min(100bps, pct75_OAS-medOAS)], where pct25_OAS and pct75_OAS are the 25th and 75th percentile of the OAS distribution for the sovereign estimation universe.

Figure 5 shows the explanatory power of the USD sovereign factors. To do that, we compute the R-squared as the ratio of sum of squared projected return components over the sum of squared stochastic return. A projected return component is computed as the exposure to a factor multiplied by the factor return. For instance, the projected explicit return is computed as the sum of products of key rate durations and rates change at each point, plus the convexity return and volatility return. Reported below is the 6 month rolling r-squared. Explicit curve factors explain most of the return volatility. However, the spread factor, the slope and the OAS factor do add additional explanatory power, especially at times when the curve factors do not perform very well.

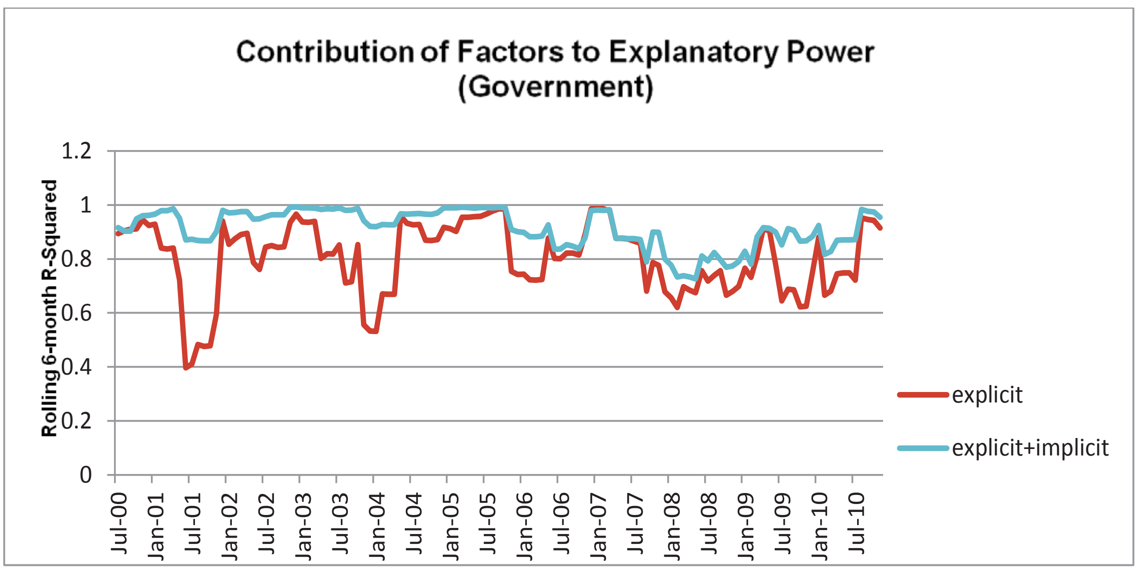

Figure 5: Contribution of Factors to Explanatory Power for Sovereign Sector

Agency

The agency sector consists of bonds issued by government agencies and foreign government and supranational. Bonds issued by government agencies are in general not fully guaranteed in the same way as the sovereign bonds, therefore they are viewed as a separate asset class. The structures of the agency factors are different for G7 models. We try to classify bonds in this estimation universe into groups based on the BICS classification, the country of risk, and we also include the slope factor to account for the term structure difference in spread change and the OAS factor to account for liquidity difference.

The USD agency sector is modeled as:

$$\frac{R_n^{ex}}{-OASD_n} = F_{I(n)}^{Agcy} + \left(OASD_n - medOASD_{I(n)}\right) \cdot F_{Slope}^{Agcy} + \left(OAS_n - medOAS_{I(n)}\right) \cdot F_{OAS}^{Agcy} + \varepsilon_n \quad (7)$$

$F_{I(n)}^{Agcy}$ is the spread factor for group $I(i)$, representing the average spread changes of bonds belonging to a particular group, $I(n)$. We have six groups in the USD model, Fannie Mae (FNMA), Freddie Mac (FHLMC), Federal Home Loan Bank (FHLB), Farmer Mac (FARM), Supranational and Others;

F_{Slope}^{Agcy} and F_{OAS}^{Agcy} are similarly defined as in the sovereign asset class. The only difference is that the medians used are the medians of the group that a specific bond belongs to, rather than the median of the whole USD agency estimation universe. We limit the exposure to this factor to [max(-5, pct25_OASD-medOASD), min(5, pct75_OASD-medOASD)], where pct25_OASD and pct75_OASD are the 25th and 75th percentile of the OASD distribution for the agency estimation universe. We limit the exposure to F_{OAS}^{Agcy} to [max(-100bps, pct25_OAS-medOAS), min(100bps, pct75_OAS-medOAS)], where pct25_OAS and pct75_OAS are the 25th and 75th percentile of the OAS distribution for the agency estimation universe.

Figure 6 shows the explanatory power of the USD agency factors. Performance for this sector is very good and the explicit curve factors explain most of the return volatility: curve and volatility factors account for 51% of the variance. The implicit factor, which includes agency spreads, slope and the OAS factors, on average adds another 10% to the explanatory power.

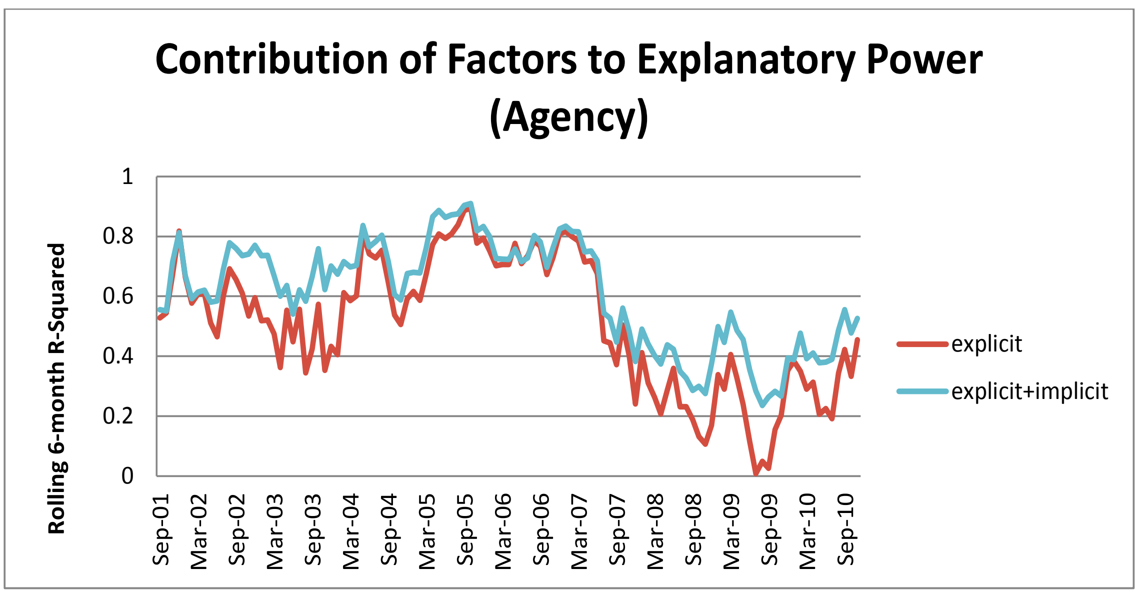

Figure 6: Contribution of Factors to Explanatory Power for Agency Sector

IG and HY Corporate

Corporate sector is the largest fixed-income sector by number of issues for most markets. The basic factor structure for this sector is similar across G7 models with different level of granularity. The most prominent dimension for classifying bonds into groups is industry sectors. Issuers in the same industry are affected by the same business cycles, supply demand change, etc. Therefore, we use relative spread change for different industry groups as systematic factors. In addition, high yield bonds are traditionally viewed as a different sub asset class than investment grade bonds, so we include a factor to capture the additional spread change for high yield bonds. We also include a factor accounts for the difference between senior and subordinated bonds. Finally, we include slope factors to capture the term structure effect for spread change. One important difference between this sector and the sovereign and agency sector is that the spread factors are modeled as relative spread change, rather than the absolute change of spread. This choice is motivated by the empirical finding that the relative spread change, rather than absolute spread change better explains the historical credit spread behavior.

The USD investment grade and high yield corporate sector' spread factors are modeled as follows:

$$\begin{aligned}\frac{R_n^{ex}}{-OASD_n} &= F_{Basic}^{Crd} + OAS_n^+ \cdot \left(F_{I(n)}^{Crd} + 1_{Frgn} \cdot F_{Frgn}^{Crd} + 1_{HY} \cdot F_{HY}^{Crd} + 1_{Sub} \cdot F_{Sub}^{Crd}\right) \\ &+ \left(OASD_n - medOASD_{I(n)}\right)^+ \cdot F_{Long}^{Crd} + \left(OASD_n - medOASD_{I(n)}\right)^- \cdot F_{Short}^{Crd} + \varepsilon_n\end{aligned} \tag{8}$$

F_{Basic}^{Crd} is essentially the regression intercept. Most bonds other than those with very low initial spreads will load on an additional industry spread factor.

$F_{I(n)}^{Crd}$ is the industry-specific spread factor, representing the average *proportional* changes in spreads of bonds belonging to the same industry, $I(i)$. The exposure to the industry-specific factor is the initial spread level, where OAS_i^+ denotes max(0, OAS). We consider 23 industry groups based on the Bloomberg Industry Codes, or BICS. The list of industry groups and the mapping between BICS sector and group to our industry factors are given in Table 2. Here the exposures are discussed in the context of the regression. When the risk of a portfolio is the context, the exposures to all the spread factors should be pre-multiplied by the negative spread duration.

F_{Frgn}^{Crd} is the foreign factor, representing the average incremental *proportional* changes in spreads for all bonds that are issued by foreign entities[2]. The factor is positive when spreads of bonds issued by foreign entities widen relative to bonds issued by domestic entities, holding everything else the same.

F_{HY}^{Crd} is the high yield factor, representing the average incremental *proportional* changes in spreads of high yield (HY) bonds[3]. It is positive when HY bonds' spreads widen on average relative to IG bonds, holding everything else the same.

F_{Sub}^{Crd} is the seniority factor, representing the average incremental *proportional* changes in spreads of bonds in the subordinated class[4]. It's positive when spreads of subordinated debts widen on average relative to senior debts, holding everything else the same.

F_{Long}^{Crd} is the long-duration factor, and $mmedOASD_{I(n)}$ denotes the median OASD of the bonds that belong to the same industry group I(i) as bond i in the estimation universe. We use the notation $X^+ = \max(0, X)$. This factor is positive when spreads of long-duration bonds widen relative to that of short-duration bonds, negative when they tighten. We cap the exposure to this factor by min(5, pct75_OASD- medOASD), where pct75_OASD is the 75th percentile of the OASD distribution of the group a bond belongs to in the estimation universe.

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2 Foreign bonds are identified by Bloomberg fields Country_risk_iso_code (VM101) and Country_of_risk (DX129) when the first field is not available.

3 Bonds rated BB+ and below are considered high yield (HY). Rating assignment is from BB Rating Composite, Moody's, S&P and Fitch, in that order based on availability. If no rating information is available, or rated NR, then we assign a broad rating category (i.e., IG, HY, or Distressed) based on OAS.

4 Subordinated bonds are identified by Bloomberg field IS_SUBORDINATED (DX825).

F_{Short}^{Crd} is the short-duration factor, and $medOASD$ denotes the median OASD of the bonds that belong to the same industry group I(i) as bond i in the estimation universe. We use the notation $X^{-} = \min(0, X)$. This factor is positive when spreads of short-duration bonds tighten, negative when they widen. We cap the exposure to this factor by max(5, medOASD-pct25_OASD), where pct25_OASD is the 25^{th} percentile of the OASD distribution of the group a bond belongs to in the estimation universe.

Industry_sector	Industry_group	Industry_factor
Basic Materials	all groups	Basic Materials
Communications	Media	Media
	all groups except for Media	Telecommunications
Consumer Cyclical	Airlines	Airlines
	Auto Manufacturers Auto Parts & Equipment	Auto
	Retail	Retail
	all groups in the sector except for those covered by factors above	Consumer Cyclical
Consumer Non-cyclical	Healthcare-Products Healthcare-Services	Healthcare
	Pharmaceuticals	Pharmaceuticals
	all groups in the sector except for those covered by factors above	Consumer Non-cyclical
Diversified	all groups	Industrial
Industrial	Aerospace/Defense	Aerospace/Defense
	Transportation	Transportation
	all groups in the sector except for those covered by factors above	Industrial
Energy	Oil &Gas Oil &Gas Services	Oil &Gas
	all groups in the sector except for those covered by factors above	Energy
Financial	Banks	Banks
	Insurance	Insurance
	REITS Real Estate	REITS
	all groups in the sector except for those covered by factors above	Diversified Finan Serv
Technology	Computers	Computers
	all groups in the sector except for those covered by factors above	Technology

Utilities	Gas	Gas Utilities
	all groups in the sector except for those covered by factors above	Electric

Table 2: BICS Sector and Group Mapping to Industry Factors

The industry factors for the USD model are chosen as combinations of the BICS sectors and groups. The choice was made based on several considerations: bonds that belong to the same factor should be as close as possible in their spread movements and bonds belonging to different factors should be as different as possible in their spread movements. Cluster analysis is used to achieve this goal. Furthermore, each industry factor should be well represented in the universe. Finally, sensibility is taken into consideration in making the final decision. Figure 7 shows the number of securities in each factor industry over time.

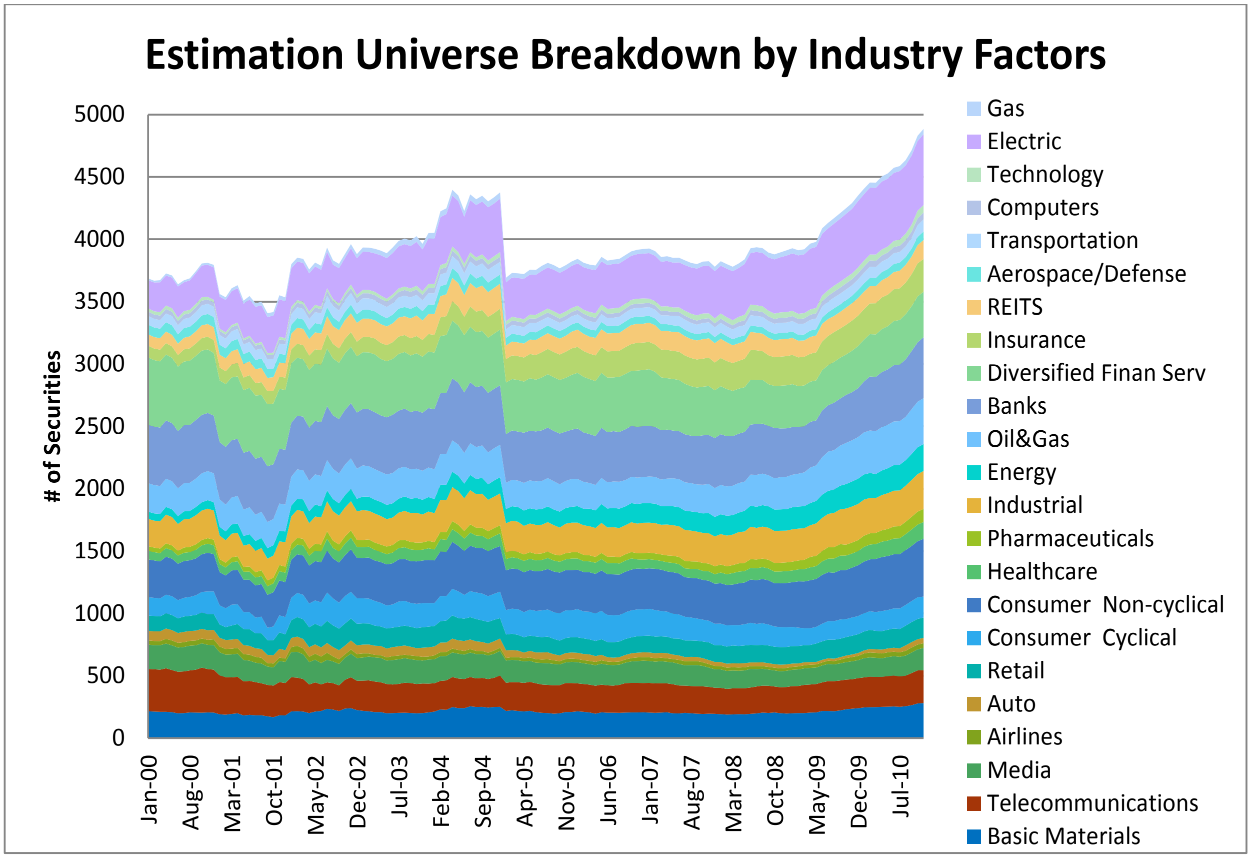


Figure 7: Number of Securities in USD Corporate Ex-Distressed Estimation Universe by Industry Factor

Figure 8 shows the explanatory power of the USD corporate sector:

- the explicit factors explain on average only 12.5% of the monthly return volatility for the credit non-distressed universe,
- base corporate spread factor adds 5.5%,
- the industry factors add another 24.8%,
- Slope, HY, sub and foreign combined add 4.6%.

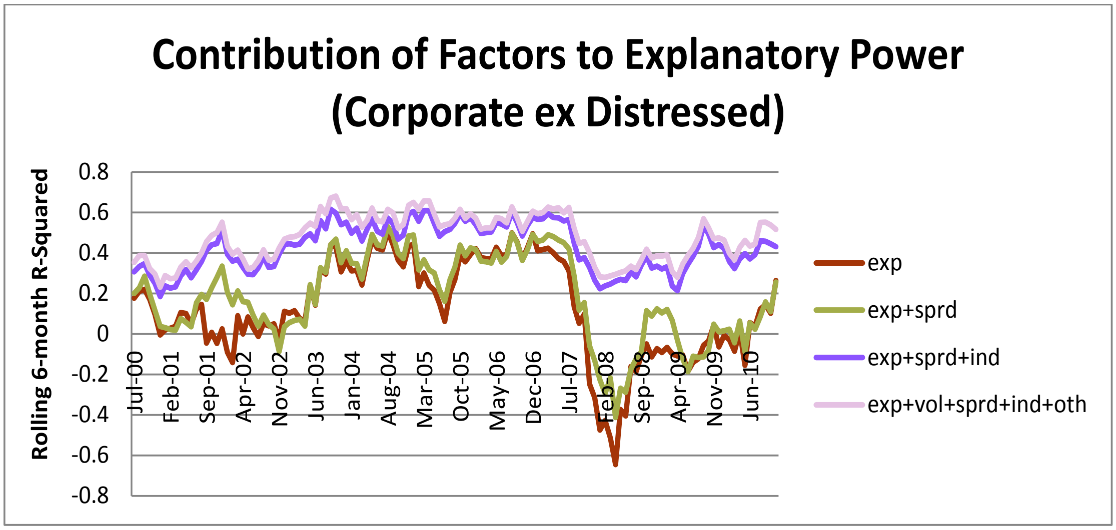


Figure 8: Contribution of Factors to Explanatory Power for Corporate Ex-Distressed Sector

Distressed Corporate

In the USD model, we separate a portion of the corporate universe with lower credit quality as the distressed asset class. Bonds with a Bloomberg composite rating of CCC+ and below in the USD universe are classified as distressed corporate[5]. Since distressed debts are often traded on prices rather than spreads, we model the excess returns of distressed corporate bonds directly with four factors:

$$R_n^{ex} = F_{Level}^{Dis} + 1_{Sub} \cdot F_{Sub}^{Dis} + (P_n - medP) \cdot F_{Price}^{Dis} + [\ln(AMT_n) - \ln(medAMT)] \cdot F_{Size}^{Dis} + \varepsilon_n \quad (9)$$

F_{Level}^{Dis} is the distressed level factor, representing the average excess returns of distressed debts;

F_{Sub}^{Dis} is the seniority factor, representing the average incremental excess returns of the distressed debts of the subordinated class. It's positive when subordinated debts outperform relative to senior debts, negative otherwise.

F_{Price}^{Dis} is the price factor, and $medP$ denotes the median price of bonds in the USD distressed corporate estimation universe. This factor is positive when excess return of premium bonds is higher that of the discount bonds, negative otherwise[6];

F_{Size}^{Dis} is the size factor, where size is measured using the current amount of the issue outstanding[7]. $medAMT$ denotes the median size of bonds in the USD distressed corporate estimation universe. This factor is positive when excess returns of high-balance bonds outperform the low-balance bonds, negative otherwise.

Figure 9 below shows explanatory power of the distressed factors. Since distressed bonds trade more like equities, the explanatory power is not as high as for other sectors. The explicit factors can't explain any variance of the returns. However, the implicit factors explain on average 19% of the variance.

---

[5] If a bond has no rating information, we use the spread level to decide whether it is in the corporate IG and HY or the distressed universes. The cut-off between the high yield and the distressed bonds is calculated as the average of the 25th percentile of the spread for the distressed universe and the 75th percentile of the high yield universe.

[6] Note that here the price premium or discount is relative to the median price as opposed to par.

[7] Current amount outstanding is identified by Bloomberg field AMT_OUTSTANDING (DS021).

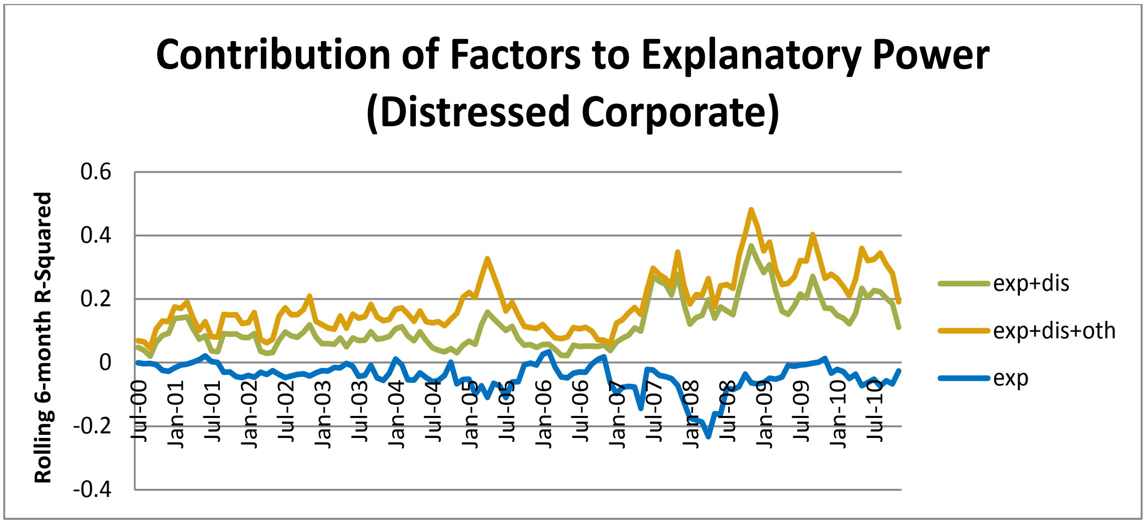


Figure 9: Contribution of Factors to Explanatory Power for Corporate Distressed Sector

> NON-FACTOR RISK AND CORRELATIONS

The systematic factors explain a significant portion of the returns variability. However, for a long-short portfolio or a portfolio that is not well diversified, non-systematic risk can represent a significant portion of total risk. We build a model for the non-systematic risk utilizing the error term ε_n^t in (1).

Using the panel of residuals ε_n^t from the factor regression (1), we estimate a simple model to describe dependence of asset-specific variance on bond characteristics. To explain a bond's specific variance, we resort to the same peer groups used in the systematic regression. More specifically, after we run the regression to obtain the systematic factors, we subsequently fit a model for its absolute residuals ($|\varepsilon_n^t|$), using the aforementioned groups for each type of bonds:

$$|\varepsilon_n^t| = \sum_{I(n)=1}^{I} Y_{I(n)}^t G_{I(n)}^t + \varphi_n^t \tag{10}$$

where

- $Y_{I(n)}^t$ is an indicator function of group $I(n)$ (i.e., 1 if a member of that group, 0 otherwise) for all asset classes except for the non-distressed corporate sector;
- for non-distressed corporate sector, $Y_{I(n)}^t$ is the indicator function multiplied by max(OAS, 2bps);
- $G_{I(n)}^t$ are coefficients determined by the regression (10). We refer to them as residual factors --they represent the average cross-sectional volatility of non-systematic spread in each group.

Similar to the systematic factor regression that is run on a monthly basis, the residual factor regression (10) is run each month to obtain non-factor coefficients $G_{I(n)}^t$ history. A forecast of the non-factor volatility will then be based on the historical coefficients using Exponentially Weighted Moving Average (EWMA) with a half-life of six months. To be more specific, the residual volatility for a non-distressed corporate bond or sovereign, agency bond is given as follows, the $\sqrt{\frac{\pi}{2}}$ at the end is the kurtosis correction factor[8].

$$OASD_n Y_{I(n)}^t ewma\left(\widehat{G_{I(n)}^t}\right)\sqrt{\frac{\pi}{2}} \tag{11}$$

In addition to the magnitude of specific risk, for corporate bonds we also consider the correlation between bonds issued by the same company. The correlations between two bonds of the same issuer are determined as a function of their duration proximity. Our empirical studies showed that the closer between the duration, the higher the correlation.

More specifically, we choose to describe the correlation structure as:

$$\rho(SD_i; SD_j) = \alpha + \gamma \cdot exp(-|SD_i - SD_j|) \tag{12}$$

where D_i and D_j are spread durations of two bonds. The coefficients in equation (12) are calibrated based on empirical correlations between each pair of factor regressions residuals ε_n^t from (1), where the data of the entire Corporate Estimation Universe for the last ten years is used. Each pair of eligible securities used in the calibration must be issued by the same company and have a minimum joint observation length of 24 months[9]. The signs of the alpha and gamma coefficient are positive. The positive gamma coefficient suggests that securities with larger difference in durations have lower correlation than securities that are close in durations.

> CONSTRUCTION OF FACTOR COVARIANCE MATRICES

In this section construction of covariance matrices is presented. We begin with a discussion of individual single currency factor covariance matrix construction. Methods like Exponentially Weighted Moving Average, Random Matrix theory and shrinkage are utilized. Next, we discuss Bloomberg methodology for aggregating multiple individually estimated covariance matrices into one multi-currency integrated covariance matrix. This allows us to construct an integrated multi-currency model to manage the risk of portfolios of assets issued by various countries and in different currencies.

GENERAL APPROACH AND ESTIMATION

Given time series of factor returns, factor covariance matrix is calculated in the following fashion.

The factor covariance matrix is represented as $\Sigma_{factors} = V \times C \times V$, where correlation matrix C and the diagonal matrix V containing factor volatilities are computed separately.

Individual factor variances are estimated using the EWMA (GARCH) model:

$$\sigma_{t+1}^2 = (1-\lambda)\sigma_t^2 + \lambda f_{t+1}^2 = \Sigma_{i=0}^{t+1} w_i f_i^2, \tag{13}$$

where $w_i = \lambda(1-\lambda)^{t+1-i}$ and $\lambda = 1 - 2^{\frac{-1}{half\ life}}$.

Correlations are estimated using a similar technique, but with a different half life.

[8] The correction factor serves to convert the forecast from mean absolute return to standard deviation units under the assumption of normally distributed non-factor returns.

The estimation of GARCH processes is usually carried out using Maximum Likelihood Estimation with parameters chosen to maximize the collective likelihood of the observations:

$$\max \Pi_{i=0}^{t}\left[\frac{1}{\sqrt{2\pi\sigma_i^2}}\exp^{-\left(\frac{f_i^2}{2\sigma_i^2}\right)}\right], \tag{14}$$

where f_t represents the factor return for the period $t-1 \rightarrow t$ and model assumptions being:

$$\begin{gathered}\mu_t = \mathbb{E}(f_t|\mathcal{F}_t) = 0 \\ \sigma_{t+1}^2 = (1-\lambda)\sigma_t^2 + \lambda f_{t+1}^2 \\ f_t = \sigma_t\varepsilon_{t,}\ \ \varepsilon_t \sim \mathcal{N}(0,1),\ \ Law(f_t|\mathcal{F}_t) \sim \mathcal{N}(0,\sigma_t^2),\end{gathered} \tag{15}$$

where $\mathcal{F}_t$ denotes the information available up to time t.

Empirical testing indicated that optimal values for half life parameters are 12 months for correlations and 6 months for variances.

Once correlations matrix C_t and volatilities matrix $V_t = diag(\sigma_t)$ are estimated, the final covariance matrix becomes $\Sigma_{factors} = V_t \times C_t \times V_t$.

STABILIZING CORRELATION MATRIX ESTIMATION

The procedure described above contains measurement errors due to the fact that limited number of factor realizations is available for correlations estimation. For example, in tests presented below, monthly data starting from 2000 until 2011 was used. Bloomberg uses classical techniques from Random Matrix Theory, briefly described below, to mitigate the errors in the correlations matrix estimation.

Suppose a true covariance matrix of N-dimensional vector X is $\sigma^2 \times I_{N\times N}$, then the sample covariance matrix of X given the sample size of T has eigenvalues distributed according to the Marcenko-Pastur law:

$$\rho(\lambda) = \frac{Q}{2\pi\sigma^2}\frac{\sqrt{(\lambda_+-\lambda)(\lambda-\lambda_-)}}{\lambda}, \tag{16}$$

where $Q = \frac{T}{N}$, and $\lambda_\pm = \sigma^2\left(1 \pm \sqrt{\frac{1}{Q}}\right)^2$.

If the estimated covariance matrix were indeed random, its eigenvalues would be distributed according to Marcenko-Pastur law. In order to determine which eigenvalues are significant, one can define σ^2 as the average variance of all factors and compute maximum eigenvalue $\lambda_+ = \sigma^2\left(1 + \sqrt{\frac{1}{Q}}\right)^2$. The eigenvalues of the covariance matrix that are above the threshold λ_+ are significant, the ones below λ_+ can be averaged as to preserve the total variance.

To stabilize estimation of the correlation matrix further, shrinkage towards some well-behaved target covariance matrix is applied. We use shrinkage of 10% towards identity matrix:

$$\Sigma_{shrunk} = (1-w)\times\Sigma + \mathrm{w}\times\mathrm{Identity},\quad \mathrm{w} = 10\% \tag{17}$$

Shrinkage technique is applied after RMT cleaning resulting in a more stable and better conditioned correlation matrix.

AGGREGATION OF COVARIANCE MATRICES ACROSS TWO DIFFERENT CURRENCY MODELS

In order to aggregate multiple individual covariance matrices into one, a multi-layer covariance matrix methodology is used. An example below illustrates the technique on two models.

Assume two factor models have been built in the usual fashion

$$\begin{cases} r_1 = B_1 f_1 + \varepsilon_1 \\ r_2 = B_2 f_2 + \varepsilon_2 \end{cases} \tag{18}$$

where $r_{1,2}$ represent the asset returns, $f_{1,2}$ -- detailed factors, $B_{1,2}$ -- factor loadings and $\varepsilon_{1,2}$ – the residuals. The time subscript has been dropped for clarity.

Time series of factor returns $f_{1,2}$ are used for the construction of individual detailed covariance matrices $\Sigma_{11}$and Σ_{22}. Frequently the joint number of factors exceeds the length of the time series available, making it numerically unstable to compute the joint weighted sample covariance matrix from series $\binom{f_1}{f_2}$. In order to construct the combined covariance matrix Bloomberg model uses more coarse, **core** factors $c_{1,2}$, obtained from a cross-sectional regression:

$$\begin{cases} r_1 = L_1 c_1 + \varrho_1 \\ r_2 = L_2 c_2 + \varrho_2 \end{cases} \tag{19}$$

There are significantly fewer core factors than detailed ones which allows a modeller to compute sample covariance matrix Σ_{core} from time series $\binom{c_1}{c_2}$, and apply cleaning and shrinkage techniques described in the previous section.

In order to connect detailed factors covariance matrix $\Sigma_{detailed}$ with core factors' Σ_{core} , a regression is run in time dimension, where time series of **core** factors $c_{1,2}$ are already known and loadings on **core** factors $G_{1,2}$ are computed:

$$\begin{cases} f_1 = G_1 c_1 + \tau_1 \\ f_2 = G_2 c_2 + \tau_2 \end{cases} \tag{20}$$

Finally, multi-layer technique leads to the following estimator for a full factor covariance matrix:

$$\begin{aligned} Covariance\left(\begin{bmatrix} \mathrm{f}_1 \\ \mathrm{f}_2 \end{bmatrix}\right) &= \begin{pmatrix} G_1 & 0 \\ 0 & G_2 \end{pmatrix} \times Covariance\left(\begin{bmatrix} \mathrm{c}_1 \\ \mathrm{c}_2 \end{bmatrix}\right) \times \begin{pmatrix} G_1' & 0 \\ 0 & G_2' \end{pmatrix} + Covariance(\tau) = \\ &= \Omega_{detailed} = \begin{pmatrix} \Omega_{11} & \Omega_{12} \\ \Omega_{21} & \Omega_{22} \end{pmatrix} = Vol(\Omega) \times \ Corr(\Omega) \times \ Vol(\Omega) \end{aligned} \tag{21}$$

The aggregated covariance matrix needs to be consistent with the individual detailed currency covariance matrices, which is accomplished via the following transformation:

$$\Sigma_{detailed} = \begin{pmatrix} U_1 & 0 \\ 0 & U_2 \end{pmatrix} \times Corr(\Omega) \times \begin{pmatrix} U_1' & 0 \\ 0 & U_2' \end{pmatrix} = \begin{pmatrix} \Sigma_{11} & \Sigma_{12} \\ \Sigma_{21} & \Sigma_{22} \end{pmatrix}, \tag{22}$$

where Σ_{ii} are the covariance matrices of the individual currency models and the transformation matrices are $U_i = Vol(\Sigma_{ii})\big(Corr(\Sigma_{ii})\big)^{1/2}\big(Corr(\Omega_{ii})\big)^{-1/2}$. Note that $Corr(\Sigma_{ii})$ and $Corr(\Omega_{ii})$ are positive definite matrices, which makes the above operations of inversion and square root extraction possible. One can verify that the diagonal blocks of the final factor covariance matrix $\Sigma_{detailed}$ will match the individually estimated matrices Σ_{ii}.

To summarize, core factor methodology allows computing covariances of detailed factors via estimating significantly smaller number of pair wise core factor covariances. For example, if we have 100 detailed factors, then $4950 = \frac{100(100-1)}{2}$ pair wise covariances need to be estimated. If we have 10 core factors, then $45 = \frac{10(10-1)}{2}$ pair wise core factor covariances are estimated in a stable fashion and then core factor covariance matrix is expanded into detailed factor covariance matrix via expression (20). In this example, the methodology allows to estimate only 45 covariances instead of 4950, which is a significant reduction of complexity.

For example, in the USD model the core factors are:

Sovereign:

$$\frac{R_n^{ex}}{-OASD_{n,t-1}} = F^{cor\_tsy} + \varepsilon_{nt} \qquad (23)$$

The average spread change factor with one as loading

Agency:

$$\frac{R_n^{ex}}{-OASD_{n,t-1}} = F^{cor\_agy} + \varepsilon_{nt} \qquad (24)$$

The average spread change factor with one as loading

Corporate:

$$\frac{R_n^{ex}}{-OASD_{n,t-1}} = OAS_{n,t-1}^{+} \times F^{cor\_crd} + OAS_{n,t-1}^{+} \times 1_{HY} \times F^{cor\_HY} + \varepsilon_{nt} \qquad (25)$$

- The average percentage spread change factor with $OAS_{n,t-1}^{+} = \max(\mathrm{OAS}_{t-1}, 0)$ as loading
- The HY factor with $OAS_{n,t-1}^{+} = \max(\mathrm{OAS}_{t-1}, 0)$ multiplied by HY dummy as loading.

Distressed:

$$\frac{R_n^{ex}}{-OASD_{n,t-1}} = F^{cor\_dis} + \varepsilon_{nt} \qquad (26)$$

The average spread change factor with one as loading.

> MODEL PERFORMANCE

The Bloomberg Fixed Income factor model is thoroughly back-tested for various characteristics and on numerous portfolios. The following performance measurements were considered:

- Bias tests
- Spearman rank correlations: ability to predict the ranking of the asset and portfolio variances
- Comparisons of predicted risks and realized portfolio returns

In the following sections the results on model performance are presented and discussed in more detail.

First, results for the USD model are presented in detail, and then performance of the integrated model is demonstrated. For the tests below, four currencies were considered: USD, GBP, EUR, JPY.

The types of portfolios are specific to an asset class, i.e. sovereign, agency, credit, distressed and are the same across currencies.

Types of Portfolios Tested

Below we describe the types of long only and long-short portfolios used for testing. Table below summarizes the types of portfolios tested in each asset class with the detailed explanations followed.

TYPE/ASSET	TREASURY	AGENCY	CORPORATE	DISTRESSED
MARKET	X	X	X	X
SIZE: TOP/BOTTOM 25%	X	X	X	X
OAS: TOP/BOTTOM 25%	X	X	X	X
OASD: DURATION SHORTER /LONGER THAN 5 YEARS	X	X	X	X
INVESTMENT GRADE /HIGH YIELD		X		
ISSUERS		X		
SECTORS			X	X
ZERO VEGA	X	X	X	X
NON-ZERO VEGA		X	X	X
PRICE: TOP/BOTTOM 25%				X

Table 3: Type of portfolios used in back test

Market portfolio:

- Securities from the entire estimation universe are taken equally weighted

Size or OAS portfolios:

- Top (bottom) 25% (in terms of size or OAS) assets of the estimation universe are taken equally weighted

OASD portfolios:

- Assets of the estimation universe with duration longer (shorter) than 5 years are taken equally weighted

Investment Grade / High Yield portfolios:

- Assets of the corporate estimation universe are chosen according to their credit quality: investment grade or high yield

Issuer portfolios:

- Assets of the agency estimation universe are chosen according to their issuers

Sectors portfolios:

- Assets of the corporate estimation universe are chosen according to their BICS classification

Zero Vega Non-zero Vega portfolios:

- Non-Zero Vega: assets of the corresponding estimation universe which have non-zero volatility exposure, for example have callability features
- Zero Vega portfolios consist of securities with no exposure to volatility, i.e. straight bonds

A number of long-short portfolios are considered as well:

Various long-short portfolios:

- Long top 25% in OAS, short bottom 25% in OAS
- Long OASD longer than 5 years, short OASD shorter than 5 years
- Long Investment Grade, short High Yield

Bias Testing

In order to measure the bias of the model let us introduce *normalized portfolio returns*.

Given portfolio weights w_{t-1}, at time t portfolio return is equal to $P_t = w_{t-1} \times R_{t-1\to t}$, where R is securities' return over time period $t-1 \to t$. The risk model produces a prediction for the portfolio volatility $\hat{\sigma}_t|\mathcal{F}_{t-1}$ for the period of $t-1 \to t$.

At each time t variable

$$q_t = \frac{P_t}{\hat{\sigma}_t} \tag{27}$$

is called *normalized portfolio return.* If we were to predict the volatility of the portfolio perfectly, then q_t has expected standard deviation 1. A quantitative way to judge the performance of the model is to look at the time series of normalized portfolio returns and verify that $StDev(q_t)$ is close to 1.

Note that even perfect risk forecast would not produce $StDev(q_t) = 1$ due to the estimation and sampling errors. Let us introduce a bias statistic for a given portfolio:

$$b_t = \sqrt{\frac{\sum_{i=t}^{t+T} q_t^2}{T}} = St\widehat{Dev(q_t)} \tag{28}$$

where T is the number of periods in the observation window used to estimate $StDev(q_t)$. Under the assumption of normality of returns, perfect risk forecast and sufficiently large T, bias statistic b_t is approximately normally distributed with mean 1 and standard deviation $\sqrt{\frac{1}{2T}}$.

In the figures below we report the percentage of observations the estimate $St\widehat{Dev(q_t)}$ falls within the 95% confidence interval:

$$1 - \sqrt{\frac{2}{T}} < St\widehat{Dev(q_t)} < 1 + \sqrt{\frac{2}{T}} \tag{29}$$

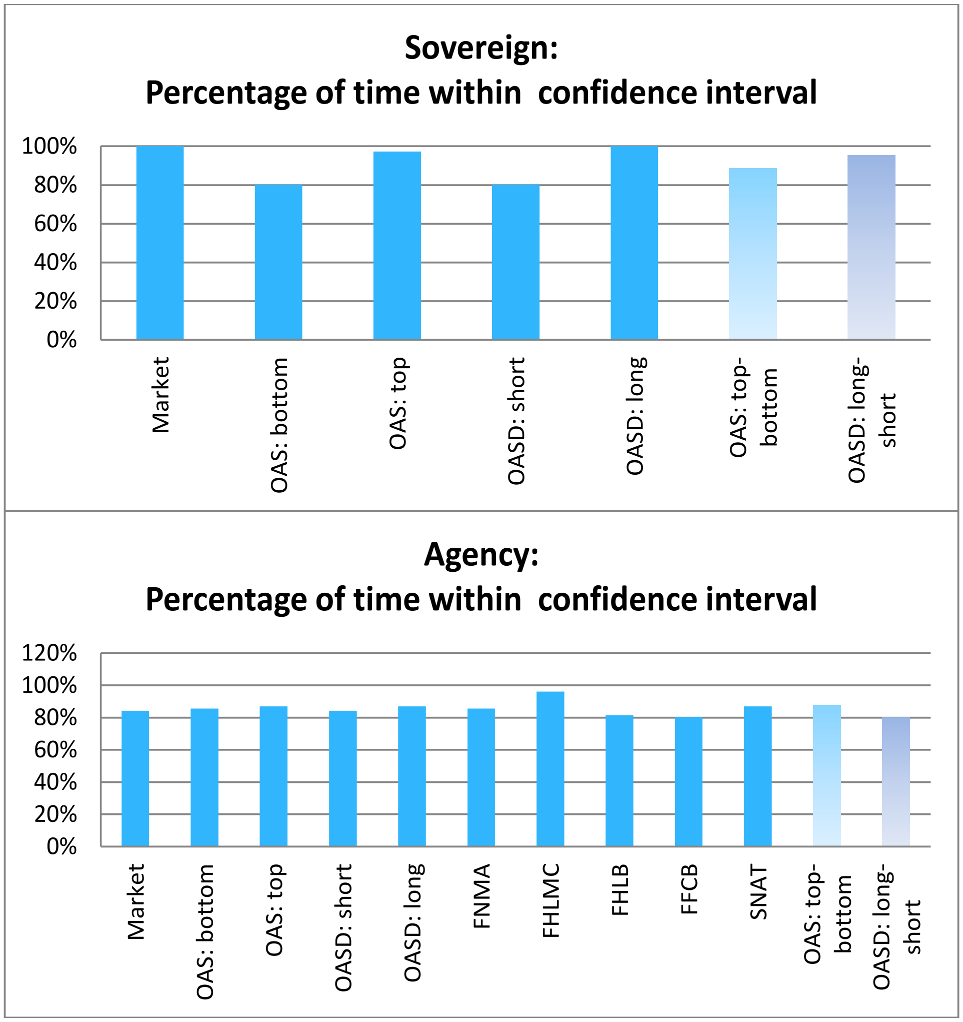
where $T = 10\ months$ is the size of the rolling window used for estimation of the standard deviation $StDev(q_t)$.

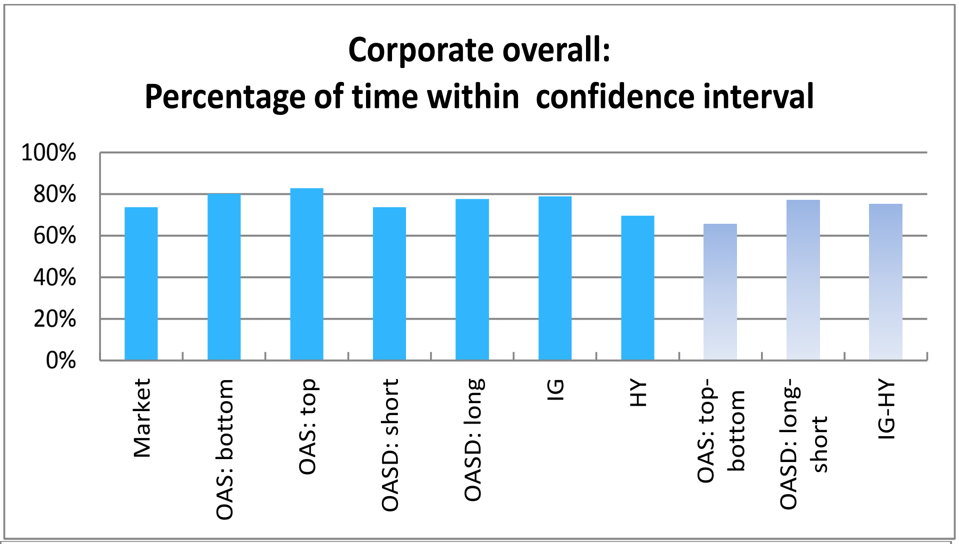
In short,

$$Bias = Std\ Dev\left(\frac{Realized\ Return}{Forecast\ Risk}\right) \tag{30}$$

- Bias is simply a measure of how well the portfolio forecast risk normalizes portfolio returns, i.e. if the normalized portfolio returns have variance equal to 1.0.
- A good model should have a distribution of bias statistic close to 1.0 with high confidence.
- Plots below give percentages of observations for each portfolio type that fall within two standard deviation bounds. The blue bars represent long only portfolios, gradient filled represent long/short portfolios

- Model forecast risk is consistently within confidence intervals, which allows portfolio and risk manager to have confidence in the model predictive abilities





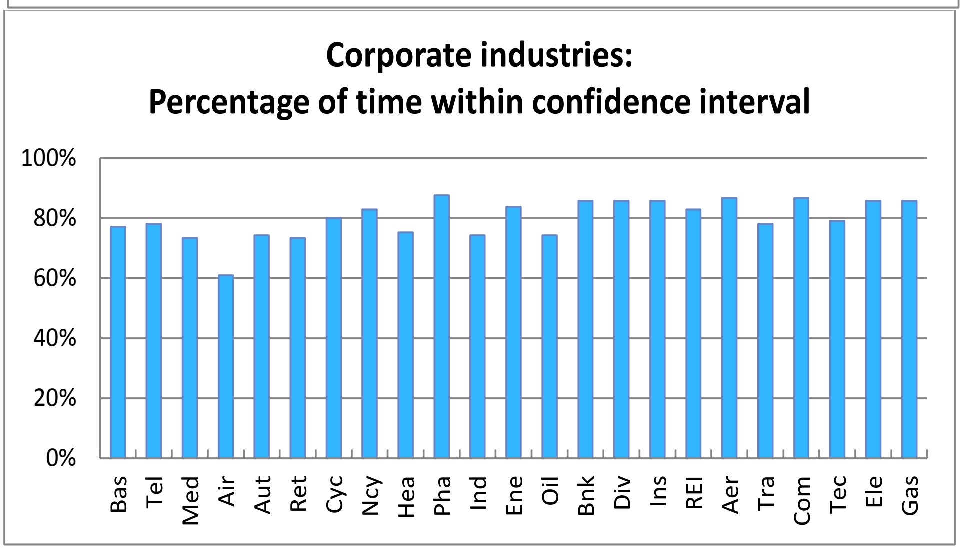


Figure 10: The percentage of time the risk estimate was within confidence interval (USD)

Spearman Rank Correlation

An important metric of the model is its ability to predict not only the risk of portfolios but the relative order of the security risks as well. For example, if the model consistently predicts that asset A will have higher volatility than asset B and this is what realizes on the market, this indicates model's ability to rank assets' variance correctly and is useful to portfolio and risk manager.

To test model's ranking ability, an estimator of realized variance of a security is needed. In tests below, at each time point we approximate security realized variance by taking the variance of the security returns over the future 10 month. We compare this estimator of the realized variance to the model predicted variance and compute Spearman rank correlation for assets in the estimation universe.

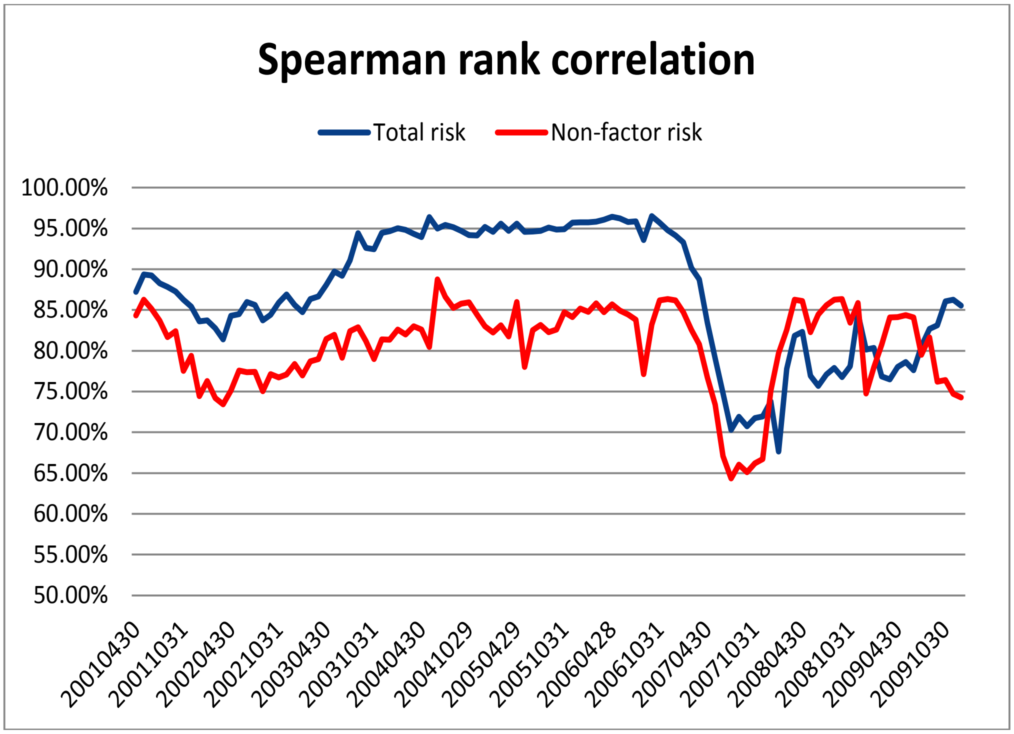


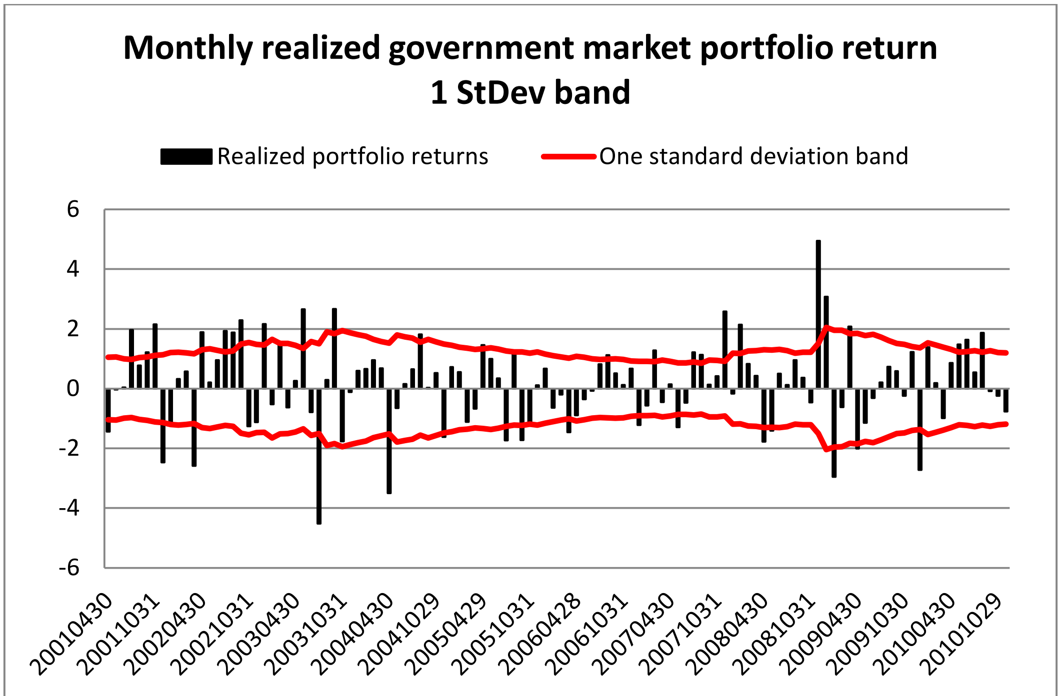
Figure 11: Rank correlation of the predicted risk at t and the realized risk over t to $t + 10\ month$

Figure 11 demonstrates the sorting ability of total and non-factor risk through time. We calculate the Spearman (rank) correlation of this statistic with the risk forecast on the month before the 10-month period. The sorting ability is rather high, with rank correlation between realized and forecast non-factor risk averaging 80%, total risk averaging 87% over the last several years. In a portfolio of a disciplined bond picker who is neutral to the benchmark with respect to systematic factors, the only active risk remaining is non-factor. This implies that a portfolio manager using this model is more likely to allocate risk budget appropriately, helping size the bets correctly. Notably, the Bloomberg model does not lose predictive ability in market downturns like that of Lehman's collapse or credit crisis of 2008.

Portfolio Returns vs. Predicted Risk

Figure 12 demonstrates equally weighted market portfolios for each asset class (sovereign, agency, credit) versus one standard deviation bands. Also, a long-short portfolio (long Investment Grade, short High Yield) is presented. The results indicate the following:

- the risk prediction responding promptly to the volatility in the realized portfolio returns, which is a highly desirable feature of the risk model
- the responsiveness in the credit model is due to the "duration times spread" DTS technique. Our methodology models the percentage change in spread rather than the absolute spread level, hence yielding to factors with more explanatory power and a model with improved and timely risk prediction.


Monthly realized government market portfolio return
1 StDev band
Realized portfolio returns
One standard deviation band
6
4
2
0
-2
-4
-6
20010430
20011031
20020430
20021031
20030430
20031031
20040430
20041029
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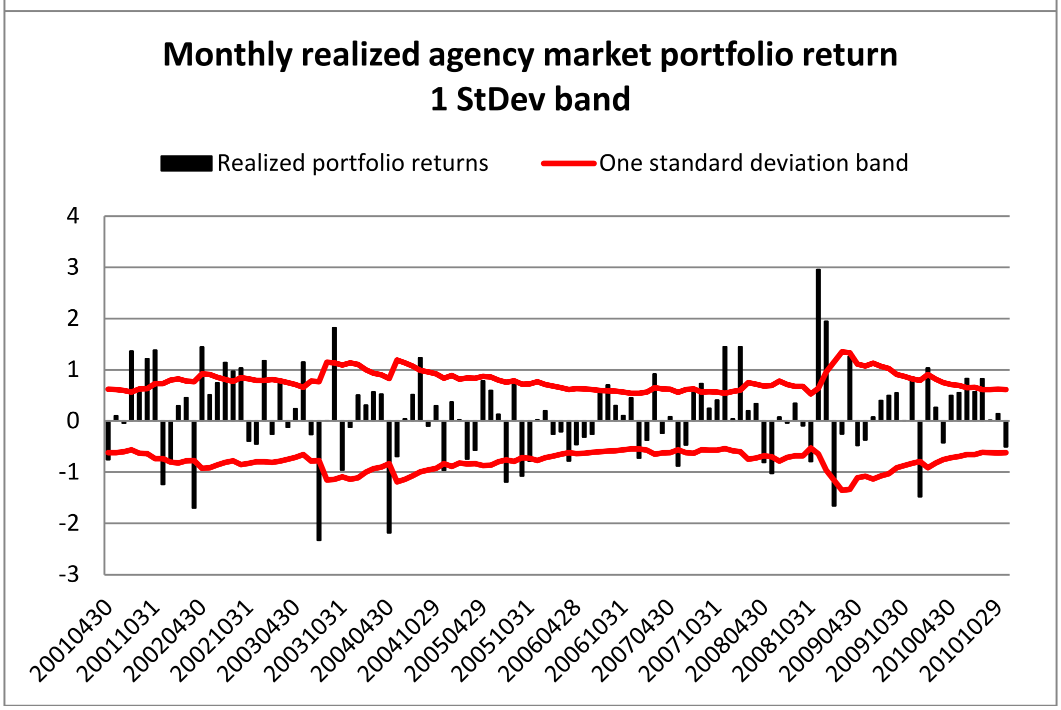
Monthly realized agency market portfolio return
1 StDev band
Realized portfolio returns
One standard deviation band
4
3
2
1
0
-1
-2
-3
20010430
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20020430
20021031
20030430
20031031
20040430
20041029
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20100430
20101029

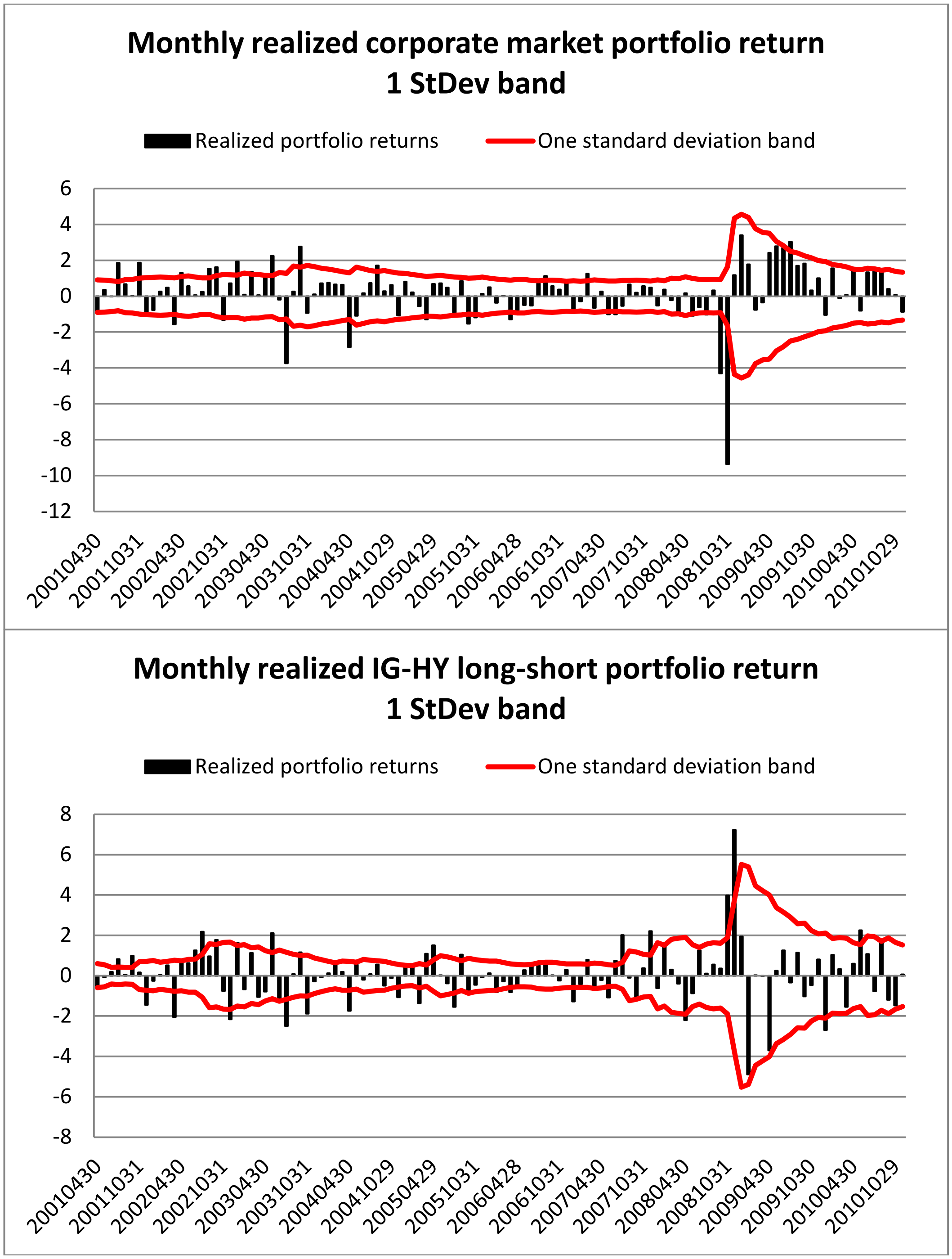


Figure 12: Realized Return vs. Predicted Risk (USD)

Integrated Multi-Currency Model

In this section we demonstrate the performance of the integrated model of four currencies: USD, GBP, EUR, JPY.

Across currencies, the following portfolios are considered:

Long only:

- Sovereign, Agency and Corporate portfolios constructed from USD, GBP, EUR, JPY with weights USD = 46%, GBP = 6%, EUR = 27%, JPY = 21% The weights are chosen based on relative market capitalization of the countries
- OAS portfolios: where sovereign, agency and corporate OAS top and bottom portfolios are taken with country weights
- OASD portfolios: where sovereign, agency and corporate OASD long and short duration portfolios are taken with country weights
- IG/HY portfolios: where corporate portfolios are taken with country weights

Long short portfolios:

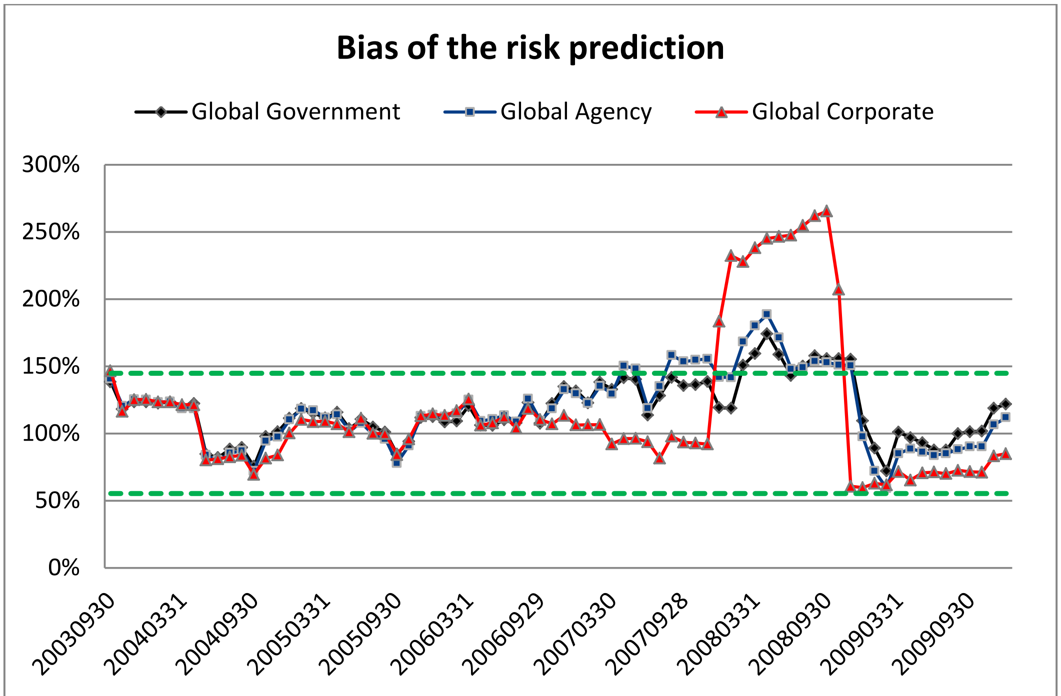
- Long top OAS and short bottom OAS for all countries
- Long OASD longer than 5 years, short OASD shorter than 5 years for all countries
- Long IG and short HY
- Industries and sectors

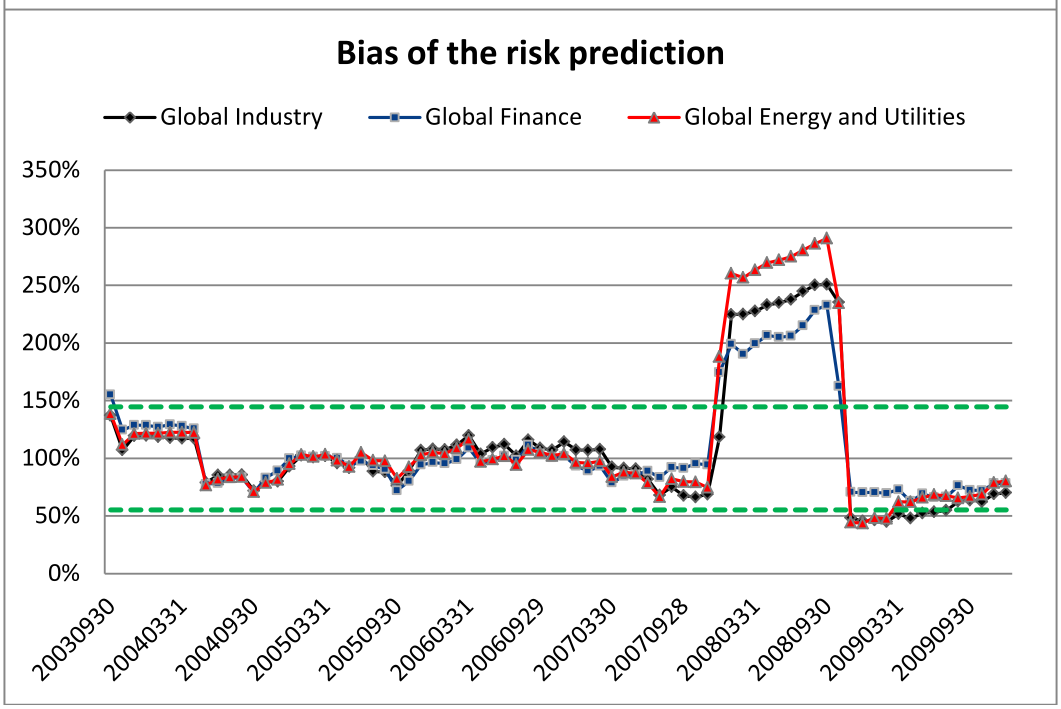
Integrated Model: Bias Tests

In this section we investigate in more detail the bias of the common strategies portfolio managers might be especially interested in. In the tests below it is assumed that the currency exposure has been hedged.

Figure 13 demonstrates the evolution of the model bias over time for various portfolios. Commonly to all portfolios, during the crisis of 2008 the model tends to underestimate the risk which is an artifact of historical averaging. The green dotted line in the panels below indicates the confidence bounds. One can observe that most of the time the model predicted risk is within confidence interval.

In the first panel displayed is the bias of the Global Market portfolio consisting of weighted securities from Sovereign, Agency and Corporate sectors in USD, GBP, EUR, JPY currency models. In the second panel, displayed is the bias for Global Sovereign, Agency and Corporate portfolios. The third panel demonstrates the evolution of the model bias for Global Finance, Industry and Energy & Utilities indices. The bottom panel demonstrates the evolution of the model bias for Global IG, HY indices as well as a long-short market neutral portfolio with long IG and short HY securities.


Bias of the risk prediction
Global Government
Global Agency
Global Corporate
300%
250%
200%
150%
100%
50%
0%
20030930
20040331
20040930
20050331
20050930
20060331
20060929
20070330
20070928
20080331
20080930
20090331
20090930


Bias of the risk prediction
Global Industry
Global Finance
Global Energy and Utilities
350%
300%
250%
200%
150%
100%
50%
0%
20030930
20040331
20040930
20050331
20050930
20060331
20060929
20070330
20070928
20080331
20080930
20090331
20090930

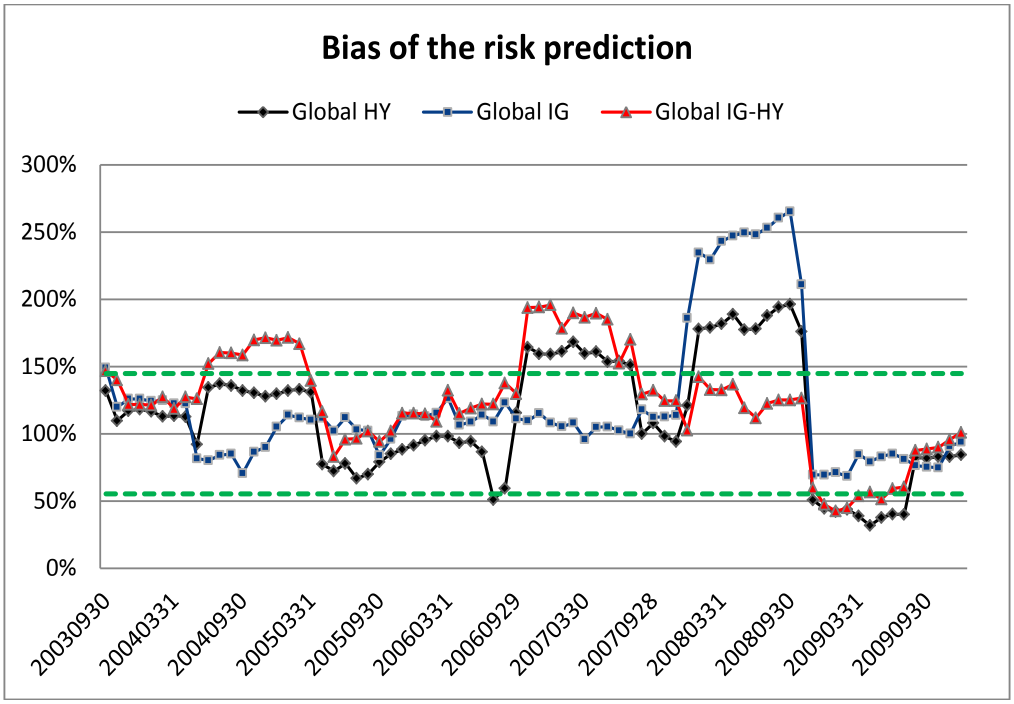
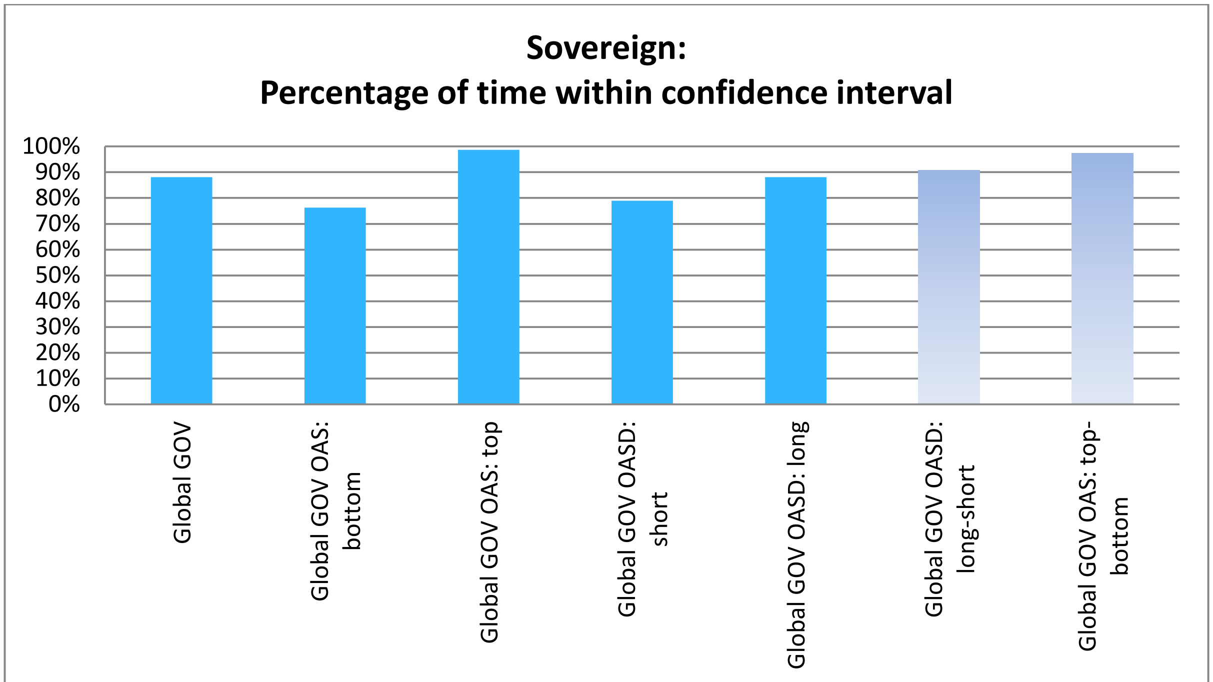
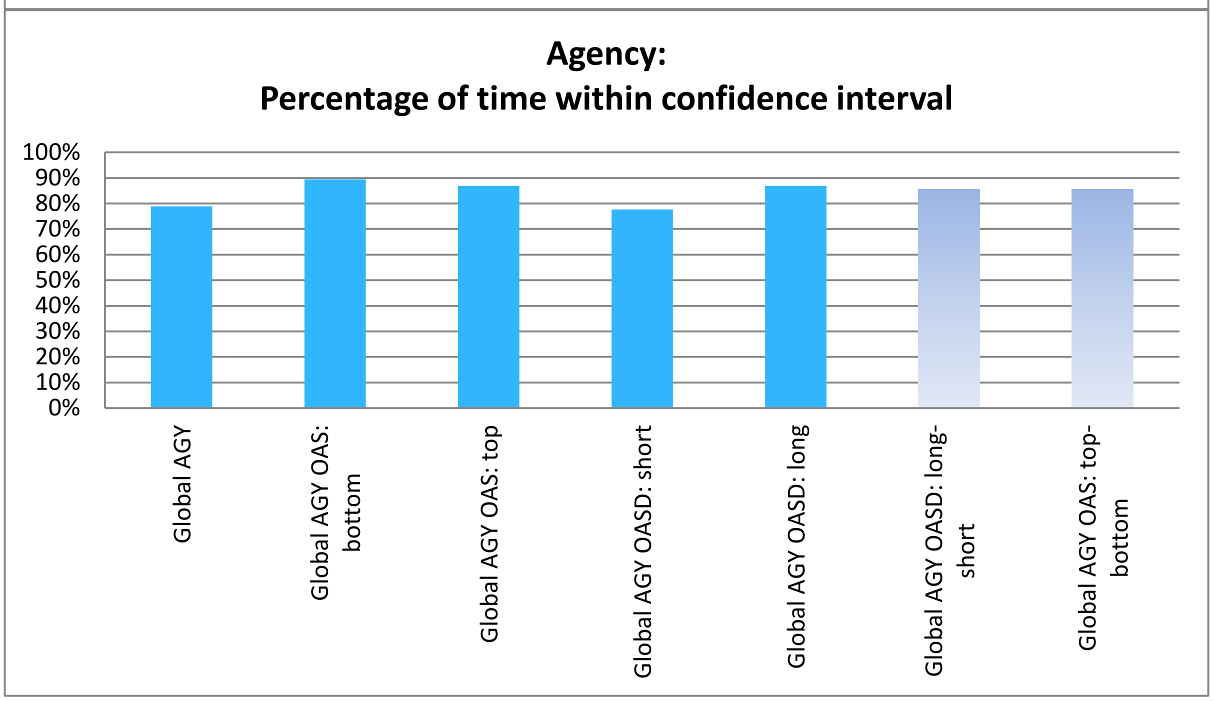


Figure 13: Evolution of bias in time (Integrated model)

One can observe from Figure 13 below that model forecast risk is consistently within confidence intervals, which allows portfolio and risk manager to have confidence in the model predictive abilities.


Sovereign:
Percentage of time within confidence interval
100%
90%
80%
70%
60%
50%
40%
30%
20%
10%
0%
Global GOV
Global GOV OAS: bottom
Global GOV OAS: top
Global GOV OASD: short
Global GOV OASD: long
Global GOV OASD: long-short
Global GOV OAS: top-bottom


Agency:
Percentage of time within confidence interval
100%
90%
80%
70%
60%
50%
40%
30%
20%
10%
0%
Global AGY
Global AGY OAS: bottom
Global AGY OAS: top
Global AGY OASD: short
Global AGY OASD: long
Global AGY OASD: long-short
Global AGY OAS: top-bottom

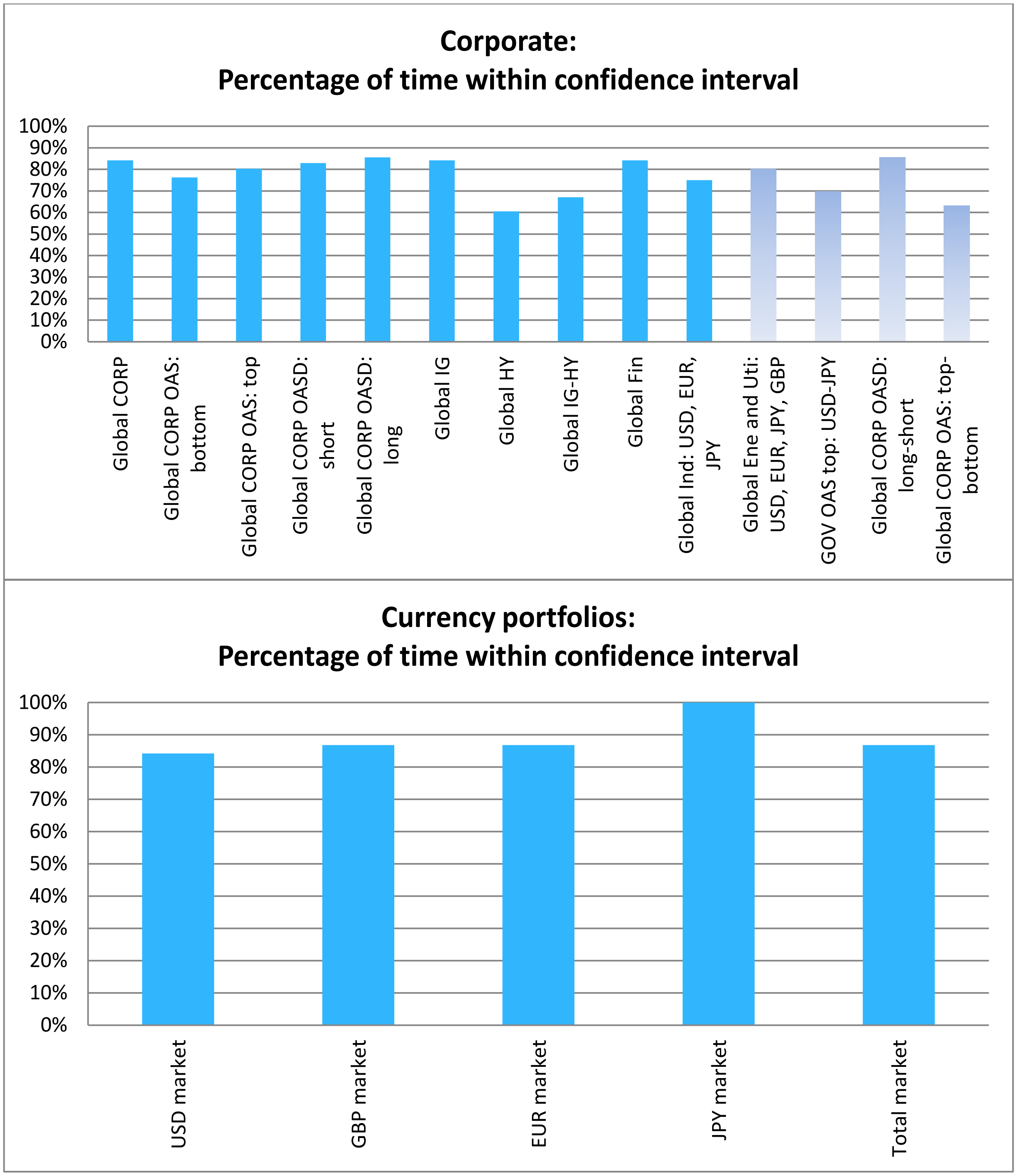
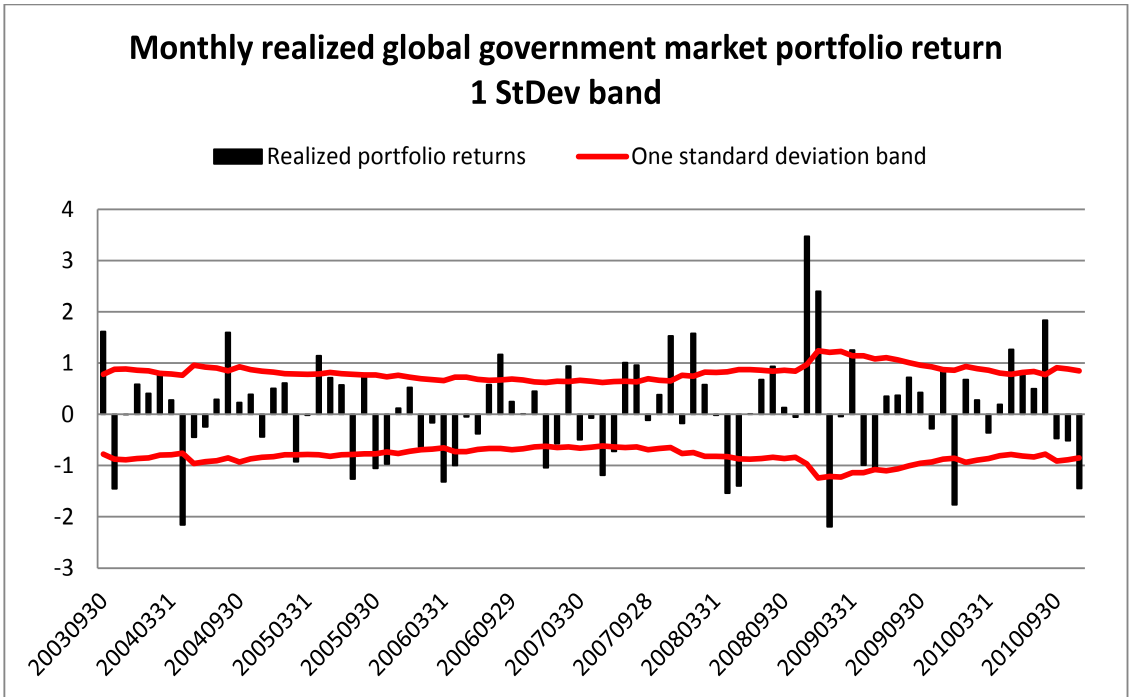


Figure 14: The percentage of time the risk estimate was within confidence interval (Integrated model)

Integrated Model: Portfolio Returns vs. Predicted Risk

Figure 15 demonstrate the realized portfolio returns vs the model predicted risk for Global Sovereign, Agency and Credit market portfolios.

One can observe stability of our risk prediction as well as responsiveness during the crisis.


Monthly realized global government market portfolio return
1 StDev band
Realized portfolio returns
One standard deviation band
4
3
2
1
0
-1
-2
-3
20030930
20040331
20040930
20050331
20050930
20060331
20060929
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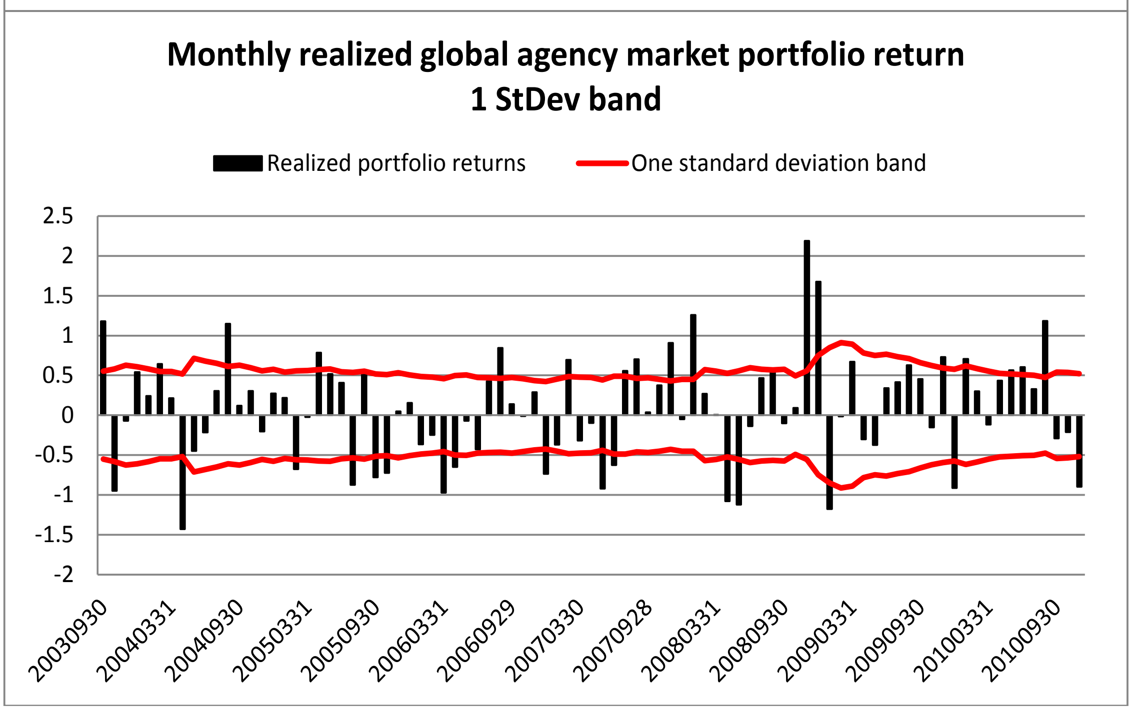
20090331
20090930
20100331
20100930


Monthly realized global agency market portfolio return
1 StDev band
Realized portfolio returns
One standard deviation band
2.5
2
1.5
1
0.5
0
-0.5
-1
-1.5
-2
20030930
20040331
20040930
20050331
20050930
20060331
20060929
20070330
20070928
20080331
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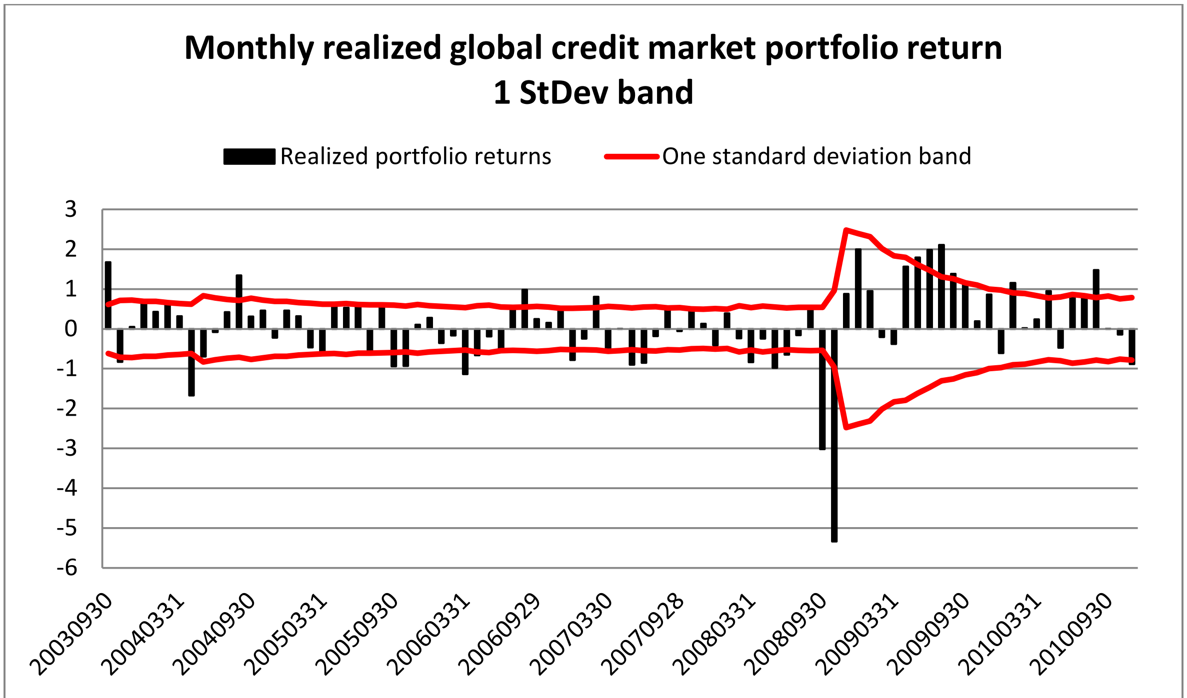
20090930
20100331
20100930



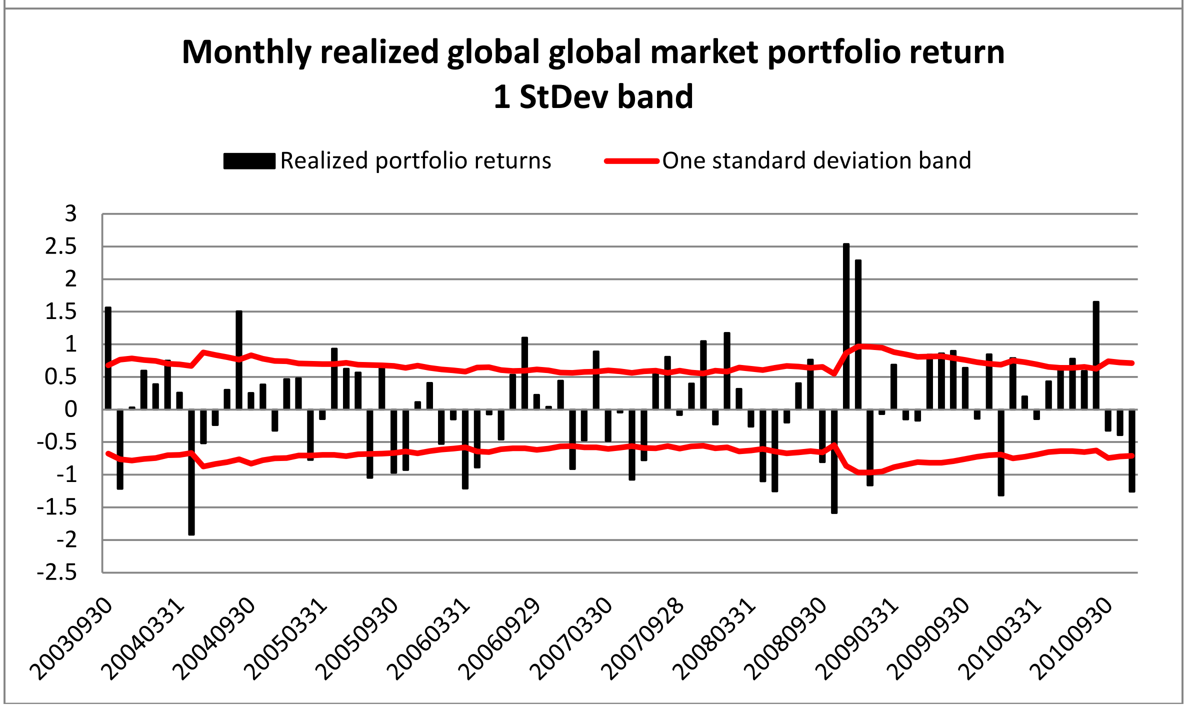


Figure 15: Realized Return vs. Predicted Risk (Integrated Model)

> SUMMARY AND CONCLUSIONS

- We have presented a new Bloomberg Fixed Income factor model based on fundamental factors.

- A single fixed income model providing risk forecast of a wide variety of portfolios ranging from those concentrated in a single market to large, international multi-asset class portfolios.
- The model is carefully constructed from a wide variety of data such as OAS, volatility, convexity, yield curves, market data on countries, industries and currencies.
- The structure of the model is chosen to be intuitive and with fully transparent methodology.
- At the same time, the model has high explanatory power for contemporaneous returns, maintains high forecasting ability in the high and low volatility environments and stays unbiased with no significant under- or over-forecasting of risk for a broad variety of portfolios.

Appendix I: Details of EUR Model

EUR model is designed to manage a portfolio of fixed income securities denominated in EUR currency. General structure of the EUR currency block is similar to the USD block as there are three sectors: Sovereign, Agency and Corporate. However, due to unique geo-political nature of the euro region, Bloomberg EUR currency block incorporates factors that are common to all three sectors mentioned above.

There are ten common country factors:

- eight individual country factors corresponding to Austria, Belgium, France, Germany, Italy, Netherlands, Portugal and Spain
- one factor corresponding to the European union not included in the list above
- one factor corresponding to the rest of the world countries that are not part of the European union

The structure of the EUR currency block is as follows:

- Spread returns of the EUR sovereign sector are modeled as:

$$\frac{R_n^{ex}}{-OASD_n} = F_{Country(n)}^{Common} + (OASD_n - medOASD) \cdot F_{Slope}^{Sov} + (OAS_n - medOAS) \cdot F_{OAS}^{Sov}$$

- Spread returns of the EUR agency sector are modeled as:

$$\frac{R_n^{ex}}{-OASD_n} = F_{Country(n)}^{Common} + (OASD_n - medOASD) \cdot F_{Slope}^{Agcy} + (OAS_n - medOAS) \cdot F_{OAS}^{Agcy} + F_{SNAT}^{Agcy}$$

  Supranational securities are identified based on country ISO code.

- Spread returns of the EUR credit sector are modeled as:

$$\frac{R_n^{ex}}{-OASD_n} = F_{Country(n)}^{Common} + (OASD_n - medOASD) \cdot F_{Slope}^{Crd} + OAS_{i,t-1}^{+} \cdot \left(F_{I(n)}^{Crd} + 1_{HY} F_{HY}^{Crd} + 1_{SUB}\, F_{SUB}^{Crd}\right)$$

  There are eight industry groups ($F_{I(n)}^{Crd}$): basic material; communications and technology; consumer (cyclical and non-cyclical); energy; financial; industrial; utilities.
  The distinction is made based on the BICS classification of industry sectors and groups. The foreign securities are identified based on country code.

Appendix II: Details of GBP Model

The structure of the GBP currency block is as follows:

- Spread returns of the GBP sovereign sector are modeled as:

$$\frac{R_n^{ex}}{-OASD_n} = F_{Sprd}^{Sov} + (OASD_n - medOASD) \cdot F_{Slope}^{Sov} + (OAS_n - medOAS) \cdot F_{OAS}^{Sov}$$

- Spread returns of the GBP agency sector are modeled as:

$$\frac{R_n^{ex}}{-OASD_n} = F_{I(n)}^{Agcy} + \left(OASD_n - medOASD_{I(n)}\right) \cdot F_{Slope}^{Agcy} + \left(OAS_n - medOAS_{I(n)}\right) \cdot F_{OAS}^{Agcy}$$

There are three agency groups: supranational, government and non-government. Government and non-government entities are identified based on BICS industry classification. Supranational securities are identified based on country ISO code.

- Spread returns of the GBP credit sector are modeled as:

$$\frac{R_n^{ex}}{-OASD_n} = F_{Basic}^{Crd} + OAS_n^{+} \cdot \left(F_{I(n)}^{Crd} + 1_{Frgn} \cdot F_{Frgn}^{Crd} + 1_{HY} \cdot F_{HY}^{Crd}\right) + \left(OASD_n - medOASD_{I(n)}\right) \cdot F_{Slope}^{Agcy} + OAS_n^{+} \times 1_{SUB}\, F^{SUB}$$

There are five industry groups: telecom and technology; consumer (cyclical and non-cyclical); basic material and industrial; energy and utilities; financial. The identification is made based on the BICS classification of industry sectors and groups. The securities issued by non-UK entities are identified based on country code. Note that non UK securities will load on one of the industry factors as well as the foreign factor.

Appendix III: Details of JPY Model

The structure of the JPY currency block is as follows:

- Spread returns of the JPY sovereign sector are modeled as:

$$\frac{R_n^{ex}}{-OASD_n} = F_{Sprd}^{Sov} + (OASD_n - medOASD) \cdot F_{Slope}^{Sov} + (OAS_n - medOAS) \cdot F_{OAS}^{Sov}$$

- Spread returns of the JPY agency sector are modeled as:

$$\frac{R_n^{ex}}{-OASD_n} = F_{I(n)}^{Agcy} + \left(OASD_n - medOASD_{I(n)}\right) \cdot F_{Slope}^{Agcy} + \left(OAS_n - medOAS_{I(n)}\right) \cdot F_{OAS}^{Agcy}$$

There are three agency groups: government, non-government and foreign. Government and non-government entities are identified based on BICS industry classification. Foreign securities are identified based on country code.

- Spread returns of the JPY credit sector are modeled as:

$$\frac{R_n^{ex}}{-OASD_n} = F_{Basic}^{Crd} + OAS_n^{+} \cdot \left(F_{I(n)}^{Crd} + 1_{Frgn} \cdot F_{Frgn}^{Crd}\right) + \left(OASD_n - medOASD_{I(n)}\right) \cdot F_{Slope}^{Agcy}$$

There are four industry groups: financial, industry, utilities and everything else, where the distinction is based on the BICS classification of industry sectors and groups. The foreign securities are identified based on country ISO. Note that non Japanese securities will load on one of the industry factors as well as the foreign factor.

Appendix IV: Details of AUD Model

The structure of the AUD currency block is as follows:

- Spread returns of the AUD sovereign sector are modeled as:

$$\frac{R_n^{ex}}{-OASD_n} = F_{Sprd}^{Sov}$$

- Spread returns of the AUD agency sector are modeled as:

$$\frac{R_n^{ex}}{-OASD_n} = F_{I(n)}^{Agcy} + \left(OASD_n - medOASD_{I(n)}\right) \cdot F_{Slope}^{Agcy} + \left(OAS_n - medOAS_{I(n)}\right) \cdot F_{OAS}^{Agcy}$$

  There are two agency groups: Australian and non-Australian, where securities are identified based on country code.

- Spread returns of the AUD credit sector are modeled as:

$$\frac{R_n^{ex}}{-OASD_n} = F_{Basic}^{Crd} + OAS_n^{+} \cdot \left(F_{I(n)}^{Crd} + 1_{Frgn} \cdot F_{Frgn}^{Crd}\right) + \left(OASD_n - medOASD_{I(n)}\right) \cdot F_{Slope}^{Crd}$$

  There are two industry groups: financial and non-financial, where the distinction is based on the BICS classification of industry sectors and groups. The foreign securities are identified based on country ISO.

Appendix V: Details of CAD Model

The structure of the CAD currency block is as follows:

- Spread returns of the CAD sovereign sector are modeled as:

$$\frac{R_n^{ex}}{-OASD_n} = F_{Sprd}^{Sov} + (OASD_n - medOASD) \cdot F_{Slope}^{Sov} + (OAS_n - medOAS) \cdot F_{OAS}^{Sov}$$

- Spread returns of the CAD agency sector are modeled as:

$$\frac{R_n^{ex}}{-OASD_n} = F_{I(n)}^{Agcy} + \left(OASD_n - medOASD_{I(n)}\right) \cdot F_{Slope}^{Agcy} + \left(OAS_n - medOAS_{I(n)}\right) \cdot F_{OAS}^{Agcy}$$

There are two agency groups: government, non-government. Government and non-government entities are identified based on BICS industry classification.

- Spread returns of the CAD credit sector are modeled as:

$$\frac{R_n^{ex}}{-OASD_n} = F_{Basic}^{Crd} + OAS_n^{+} \cdot \left(F_{I(n)}^{Crd} + 1_{Frgn} \cdot F_{Frgn}^{Crd}\right) + \left(OASD_n - medOASD_{I(n)}\right) \cdot F_{Slope}^{Crd}$$

There are four industry groups: financial, utilities, consumer (cyclical, non-cyclical); all others. The groups are identified based on the BICS classification of industry sectors and groups.

Appendix VI: Derivation of Bias Testing Statistic

If we assume q_t is of standard normal distribution, then $\sum_{i=t}^{t+T} q_t^2 \sim \chi^2(T+1)$.

Since $b_t = \sqrt{\frac{\sum_{i=t}^{t+T} q_t^2}{T}}$, we have $\sqrt{2Tb_t^2} \sim N(\sqrt{2T+1}, 1)$.

So that $b_t \sim N(\sqrt{1+\frac{1}{2T}}, \sqrt{\frac{1}{2T}})$.

When T is sufficiently large, b_t is approximately normally distributed with mean 1 and standard deviation $\sqrt{\frac{1}{2T}}$. Therefore, the 95% confidence interval for $\widehat{StDev(q_t)}$ is

$$1 - 2 * \sqrt{\frac{1}{2T}} < \widehat{StDev(q_t)} < 1 + 2 * \sqrt{\frac{1}{2T}}$$

i.e.

$$1 - \sqrt{\frac{2}{T}} < \widehat{StDev(q_t)} < 1 + \sqrt{\frac{2}{T}}$$

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