LEHMAN BROTHERS

Fixed Income Research

Replication with Derivatives: The Global Aggregate Index and the Japanese Aggregate Index

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EXECUTIVE SUMMARY

- We replicate the returns of the Lehman Brothers Global Aggregate Index with derivatives, using several strategies. The replication is performed with combinations of treasury bond futures, money-market futures, and swaps in four currencies: U.S. dollar, euro, yen, and sterling.
- A simple replication strategy using only cash and treasury futures in the three major currencies achieved a tracking error of 22.4 bp/month over 23 months of history. More sophisticated variations of the strategy combine treasury futures with money market futures and swaps in four currencies to bring the tracking error as low as 10.0 bp/month.
- We show that it is sufficient to replicate only these four bond markets, which make up over 95% of the index. Full replication of all bond markets and currencies in the index would entail much higher transaction costs and might decrease tracking error by only 1 or 2 bp/month.
- Using conservative models to estimate transaction costs and market impact costs, we find the total cost of the strategy to be less than 3 bp at start-up and about 1 or 2 bp/month for rebalancing.
- Recent changes to the Global Aggregate Index have increased the contribution of the Asian market from 14% to 24%. This report includes a detailed study of replication strategies for the Japanese Aggregate Index, the one major market segment not covered in our previous publications.
- Once the core currencies are covered, the tracking errors obtainable by a replication strategy depend strongly on the selection of derivatives used to replicate each local market. We study the effect of combining different strategies based on the correlations among their tracking errors.

TABLE OF CONTENTS

Introduction 3
Outline 5

Replicating the Global Aggregate Index 6
Index Return Volatility Analyzed: Yield versus Currency Volatility 6
Current Index Composition and Historical Simulation 7
Forming a Replication Strategy 9
Performance Summary 11
Unhedged Global Aggregate 11
Hedged Global Aggregate 14
Replication Details 16
Liquidity and Transactions Cost 21
Liquidity in the Derivatives Markets 21
Estimation of Transaction Costs: Assumptions and Methodology 25
Results 28

Replicating the Japanese Aggregate Index 30
Methodology 31
Performance 32

Conclusions 37

Appendix A. Recent Changes to the Global Aggregate Index 38
Appendix B. Hedged Currency Returns and Index Replication 40
Appendix C. Derivation of the Replication Portfolio for the Japanese Treasury Index with Cash Securities 44

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INTRODUCTION

The risk diversification effect offered by the Lehman Brothers Global Aggregate Index makes it an attractive benchmark for investors. Its comprehensive coverage of global investment-grade fixed income markets, the broad acceptance of its underlying component indices, and the addition of Asian corporate bonds make it a benchmark of choice for asset-allocation funds and investors interested in global spread products.[1] Therefore, the Lehman Brothers Global Aggregate Index is increasingly becoming the benchmark of choice for global fixed income portfolios.

However, the management of a bond fund benchmarked to the Global Aggregate requires particular skills and resources. Investors must confront a portfolio management problem with many dimensions: 14 exchange rates, 15 yield curves, and often several credit quality and sector components for each underlying currency. In addition, the potential for currency hedging introduces another decision dimension, as it allows the manager to separate the currency view from the fixed-income investment decision.

Managing money against the Global Aggregate requires resources at the macro, yield curve, and credit levels, as well as institutional expertise in each individual market. Only very large asset-management organizations will be able to assemble the necessary resources. Other asset managers can subcontract some of the individual currency portfolios, but this can lead to a blurring of responsibilities and introduce informational inefficiencies between the plan sponsor, the main manager, and the specialist managers.

The Global Aggregate benchmark also raises issues regarding asset-allocation shifts and the investment of cash balances. Moving large balances from one currency's bond market to another can be time consuming and expensive, especially when moving into some of the smaller or less liquid markets. Also, if substantial cash balances accumulate in a bond portfolio (e.g., from cash inflows or coupon payments), there is a potential for large tracking errors to develop if duration, spread, or currency risks remain unhedged.

Many of these portfolio management challenges can be met by using futures and/or swaps to replicate index returns on all or part of a portfolio. The Global Aggregate Index is a particularly attractive target for such index replication techniques. Its diversity of exposures to currency, yield curve, and spread risks makes it a prime candidate for replication with derivatives.

When large cash inflows occur, derivatives can be used to match index risk sensitivities temporarily until suitable active investment opportunities are found.

[1] See the soon-to-be-published Lehman Brothers *Global Aggregate Index Primer*.

Similarly, if a fund has to keep a significant amount of cash on hand for redemptions, it may experience substantial deviations from index returns. Derivatives, which require essentially zero cash outlays up front, can be used to replicate the index returns for the amount of the cash holdings to achieve overall closer index tracking. Managing cash with derivatives through replication can be an especially useful approach for funds in the start-up phase, during which diversified cash investments in tradable sizes are not feasible or economical.

Derivatives can play a useful and important role in reducing the dimensions of the global investor's portfolio management problem. Using derivatives to replicate some of the indices composing the Global Aggregate provides an effective way for managers to track markets in which they have no expertise, allowing them to concentrate on those markets in which active management adds value. In addition, liquid exchange-traded derivatives can help facilitate rapid asset allocation shifts between currencies and yield curves.

Futures and swaps, coupled with some aggressive ways of investing cash, can also be used as part of active strategies to outperform a benchmark. The cash left over after initial margin and some reserve for future variation margin is invested in sectors in which the manager has strong expertise (e.g., short-term high yield product or ABS). The return on the cash investment, plus any over- or underperformance of the tracking derivatives portfolio, determines the manager's total performance versus the benchmark. This style of active management is called "portable alpha" as the replicating basket of futures or swaps allows the manager to carry his performance alpha from his area of expertise and apply it to an arbitrary benchmark.

Replication is closely related to hedging. The return on a replicating portfolio differs from the return of the equivalent hedge portfolio simply by the additional return that the replicating portfolio achieves from investing the cash. If this investment does not add significantly to the volatility of the hedge portfolio, then our results concerning tracking errors of the replicating portfolios offer important insights for the tracking errors of the equivalent hedging portfolios.

The focus of this study is tracking error, which measures the which measures the return deviation between the portfolio and the underlying benchmark. While we publish historical mean returns of the replication strategy versus the index, this is done only to give investors an indication of the cost of the replication strategies. The relatively short replication history for the Global Aggregate does not allow reliable conclusions to be drawn about the long-term mean outperformance of the replication strategies.

Since this study emphasizes tracking error and not mean returns, we have chosen to ignore some finer points of index replication that affect tracking error and mean performance only peripherally. For example, estimates of transaction costs are computed separately from the mean outperformance numbers. Also, the initial and variation margin for futures are ignored. Initial margin can be up to two percent of notional value for many futures markets, and variation margin payments occur as the value of the position changes. The net result of the margin rules is that a small portion of the portfolio earns a slightly different return than the rest of the portfolio. In this study, we assume that 100% of portfolio cash is available for investment as called for by the strategy.

In this paper, we study the efficacy of replication strategies that aim to track passively the returns of the entire Global Aggregate Index using only derivatives and cash. As we have discussed, though, we expect these techniques to be applied to a much broader set of portfolio management needs, often layered on top of traditional bond portfolios.

Outline

This publication builds on two previous studies on the replication of bond indices with derivatives[2] to complete our coverage of global fixed income markets. To replicate the Global Aggregate Index, we complement the replication of the U.S. Aggregate and Euro-Aggregate indices with replication portfolios for the Sterling Aggregate and Japanese Aggregate indices.

The report is divided into two main parts. The first discusses replication of the Global Aggregate Index by combining replicating portfolios for each of the dominant component markets. This section reviews the replication techniques used in each local market, but focuses on the new issues that arise due to the global nature of the index. We explore the replication of both hedged and unhedged returns, the number of markets that need to be replicated, and the treatment of yield curve and currency risk for markets not replicated. In addition, estimates of transaction cost and market impact cost by instrument type and currency show how practical the strategy is to implement.

The second part presents a detailed study of index replication in the Japanese market. This market deserves special attention for several reasons. First, changes to the Global Aggregate Index rules as of October 1, 2000, have increased the market share of the Asian-Pacific Aggregate Index to 24%, roughly equal to that of the euro-denominated portion. Second, replication in the Japanese market faces some unique challenges. Derivatives with sufficient liquidity have not been as broadly available there as in the U.S. dollar and the euro area. In addition, the Japanese government bond market has been well known for the relative illiquidity of many of its issues, given the size of the total market and the average issue size. Finally, withholding taxes on Japanese corporate bonds complicate investing in Japanese credit for many foreign investors. As derivatives are not subject to withholding tax rules, they can significantly simplify the investment process.

[2] See the Lehman Brothers publications: *Replicating Index Returns with Treasury Futures,* November 1997 and *Replication of Index Returns with Treasury Futures, Eurodollar (Euribor) Futures, and Swaps,* March 2000.

REPLICATING THE GLOBAL AGGREGATE INDEX

Index Return Volatility Analyzed: Yield versus Currency Volatility

Returns on the Global Aggregate Index can be stated in terms of any base currency, either on a hedged or an unhedged basis. Portfolios may include bonds denominated in several currencies, with their returns hedged back to the base currency, to diversify their exposures to interest rate risk without taking on currency risk. For such portfolios, the hedged Global Aggregate Index may be the appropriate benchmark. Multi-currency bond portfolios are typically benchmarked against a multi-currency index such as the Global Treasury Index. However, the increasing role of credit across global markets is prompting many such investors to switch to the unhedged Global Aggregate Index, which includes spread products.

An analysis of index return volatilities illustrates the diversification advantage offered by the hedged Global Aggregate Index relative to a single-currency index. As shown in Figure 1, the four largest local components of the Global Aggregate (U.S. Aggregate, Euro Aggregate, Japanese Aggregate, and Sterling Aggregate) had return volatilities ranging from 86.0 bp/month to 119.1 bp/month for the time period shown.[3] The return volatility of the hedged Global Aggregate Index was approximately 75 bp/month regardless of base currency. Comparing the volatility of the hedged Global Treasury with that of the hedged Global Aggregate shows that the inclusion of spread product adds only about 1 bp/month of volatility.

[3] The data shown in Figure 1 cover only 23 months, due to the recent inception dates of some of the indices shown. Also, the Japanese component of the Global Aggregate Index consisted only of the Japanese Treasury Index before October 2000. The yen data up to that point are therefore based on the Japanese Treasury Index. We do not report mean returns, as the time span is far too short for any meaningful evaluation of the risk/return tradeoff. The return volatilities, derived from second moments, can be expected to be more stable over time than mean returns. While it would certainly be advantageous to analyze returns over a longer time period, the qualitative relationships observed here are likely to hold true over any time period.

Figure 1. **Index Return Volatilities and Tracking Errors,** by base currency, bp/month, February 1999-December 2000

	Dollar	Euro	Yen	Sterling
Index Return Volatilities				
Local Currency Component of Global Agg	95.4	86.0	95.2	119.1
Hedged Global Aggregate	75.1	76.2	73.4	73.3
Hedged Global Treasury	74.0	75.3	71.3	72.6
Unhedged Global Aggregate	152.2	196.0	273.3	192.6
Unhedged Global Treasury	187.2	177.6	265.5	199.6
Volatilities of Index Return Differences				
Hedged Global Agg vs. Local Component	38.9	34.0	109.8	97.3
Hedged Global Agg vs. Unhedged Global Agg	112.9	206.3	259.8	182.2
Hedged Global Agg vs. Hedged Global Treasury	18.8	18.8	18.5	18.9
Unhedged Global Agg vs. Unhedged Global Treasury	44.4	44.2	44.0	44.0

Volatilities increase dramatically when currency exposures are not hedged. The return volatility of the unhedged Global Aggregate ranges from 152.2 bp/month in U.S. dollars to 273.3 bp/month in Japanese yen. Compare this with the volatilities of the single-currency indices, which range from 86.0 to 119.1 bp/month. This volatility difference shows that returns on unhedged indices are driven primarily by currency movements. Differences in volatilities of unhedged returns in different base currencies are due to the combination of two effects: the foreign exchange volatility of the base currency and the percentage of the index in the base currency. The smaller the part of the index in the base currency, the greater the part of the index that is exposed to currency risk. This latter effect helps explain the differences in unhedged return volatilities between the Global Aggregate and the Global Treasury indices. The relatively large spread product components in the U.S. Aggregate and Sterling Aggregate indices give these markets and currencies a larger share in the Global Aggregate than they have in the Global Treasury Index. Hence, the Global Aggregate has a lower return volatility when expressed in these base currencies.

To highlight the different components of return, Figure 1 also presents the volatilities of return differences (i.e., tracking errors) between different pairs of indices. For a U.S. dollar-based investor, the U.S. Aggregate Index tracks the hedged Global Aggregate to within 38.9 bp/month; the differences between the hedged and unhedged returns (in U.S. dollars) on the Global Aggregate are much greater, at 112.9 bp/month. This pattern repeats across all currencies, demonstrating that unhedged index returns are dominated by currency risk.

The return differences between the Global Aggregate and Global Treasury indices are largely due to the differences in currency allocation, as discussed above. This is evidenced by the fact that the volatilities of the differences between the two indices in unhedged returns are more than double that of the hedged case.

Current Index Composition and Historical Simulation

In our earlier replication studies of single currency indices using derivatives, there were two main areas of focus: matching interest rate exposures across the term structure using treasury futures and matching spread exposure using swaps or LIBOR-based futures. As we extend this methodology to the Global Aggregate Index, an important new consideration is how closely to match exposures to each global market, in terms of both interest rates and foreign exchange rates.

Fortunately, the Global Aggregate Index is highly concentrated in three to four currencies. As illustrated in Figure 2, 91.5% of index market value is concentrated in the U.S., euro and Japanese markets. With the addition of the U.K. market, the top four currencies make up 95.5% of the index. All of these markets have liquid bond futures contracts. The strategies used in this report to replicate the Global Aggregate are thus based on combinations of replicating portfolios for these top four markets.

Figure 2. **Composition of the Global Aggregate Index**
by Currency, as of September 30, 2000

	Old Constraints		New Constraints	
	% of Index	**Cum.**	**% of Index**	**Cum.**
U.S. Dollar	48.5	48.5	44.0	44.0
Euro	28.9	77.4	24.1	68.1
Japanese Yen	14.1	91.5	23.4	91.5
British Pound	4.8	96.3	4.0	95.5
Canadian Dollar	1.5	97.8	1.4	96.9
Danish and Norwegian Krone, Swedish Krona, Greek Drachma	1.9	99.6	1.8	98.7
Taiwan and Singapore Dollar, South Korean Won, Malaysian Ringgit	0.0	99.6	0.9	99.6
Australian and New Zealand Dollar	0.4	100.0	0.4	100.0

On October 1, 2000, the Global Aggregate experienced a significant realignment due to a number of changes in the criteria for index inclusion. The Asian component of the index now includes several additional markets besides Japan, as well as Asian credit product including euro-yen securities. In addition, the index now includes 144A and Eurodollar securities and has adopted a uniform minimum outstanding requirement of US$300 million across all markets. This $300 million minimum, translated for each non-dollar market into a rounded local currency-equivalent minimum,[4] is lower than the previous minimum outstanding for the yen and Canadian dollar markets, but is now higher for all other currencies. Due to the changes in the inclusion criteria, the share of Asian bonds in the index has increased from 14% to 24%. Appendix A shows that almost all of this increase in the Asian component is due to an increase in yen-denominated bonds caused by the lower minimum outstanding and the inclusion of the credit sector.

This study covers the period from February 1999 to December 2000, including October 1, 2000, the date on which the new index inclusion criteria went into effect. In order for our results to be as relevant as possible to the current composition of the Global Aggregate Index, all of our historical simulations of different replication strategies were carried out against a customized version of the index. The new minimum threshold of $300 million outstanding per issue was imposed across all markets for the entire time period. Furthermore, since the Asian component increased to 24% under the new index rules, we assume that the market share of Japan was a constant 24% before the rule change. This was accomplished

[4] The minimum outstanding numbers required for inclusion will be re-adjusted periodically to accomodate longer-term changes in exchange rates. These changes will lead the minimum outstanding values required for inclusion in the Global Aggregate Index to change in small increments. The latest change took place on December 31, 2000.

by increasing Japan's market capitalization share both in the index and in the replication portfolios to the 24% level. While this is a substantial increase from the 8%-14% market share that Japan occupied over that period, it will make the results more forward looking. Beginning with October 2000, the actual market shares were used, with yen-denominated Eurobonds allocated to Japan and dollar-denominated Eurobonds and 144A-securities allocated to the U.S. component.

Forming a Replication Strategy

Replicating the Global Aggregate Index is accomplished by separately replicating several of its components and then combining the replicating portfolios. All of the decisions involved in single currency index replication must be made for each such component. In addition, the multi-currency aspect of the index opens up several new decision dimensions in the formulation of the replication strategy. How many component markets to include in the replication scheme? Where to invest the cash? How to account for the yield curve and currency exposures of the markets excluded from the replication scheme? This set of decisions is summarized in Figure 3.

Figure 3. **Decisions in Forming a Replication Strategy for the Global Aggregate Index**

Local market (term structure and spread)
- Cash bonds
- Bond futures
- Money market futures
- Swaps

Where to place cash?
- Local market
- Home market

How many currencies to replicate?
- Two
- Three
- Four
- All

How to treat curve risk for currencies not included in replication?
- Ignore
- Proxy with replication of most correlated major currency

How to treat currency risk for currencies not included in replication?
- Keep cash in base currency
- Spread cash proportionally among selected currencies
- Place cash in most correlated major currency (proxy approach)
- Hedge all currency risk

To replicate each local market, the replication strategy must closely match the term structure and spread exposures in that market. One of the best ways to match a market's index returns is actually to buy a portfolio of representative index securities. However, especially at the global level, this approach entails several difficulties: lack of liquidity, necessity of local market expertise, and transaction costs. Withholding taxes on interest payments from cash bonds are an additional complexity in a number of markets. Our earlier studies of replication using derivatives developed a replication methodology to achieve minimum tracking error using various combinations of bond futures, money market futures, and swaps instead of cash index bonds. Replication using derivatives requires decisions regarding the number and kind of instruments to use. These decisions will determine how closely a replicating portfolio will track each local market.

The basic assumption in this replication study is that a manager has assets whose returns are benchmarked to those of the Global Aggregate Index. If the manager uses derivatives such as futures and swaps, which require only a small outlay of cash, the majority of the manager's assets will be invested in short-term securities. When replicating single currency indices, we assume that the cash is invested in short-term securities in the same currency as the index. In the case of the Global Aggregate, the manager has the freedom to invest cash in any short-term market around the globe. These cash positions may be used to match the currency exposures of the index, or currency forwards may be layered on top of the cash position. For example, a Japanese investor replicating the Euro-Aggregate component of the Global Aggregate may choose to invest some cash in U.S. asset-backed floaters and cross-hedge the currency exposure back to yen in the forward market.

For simplicity, we will limit our analysis to two cases. In the first case, for each market replicated, the proportional amount of cash will be invested in 1-month deposits in the local currency. For example, the Euro-Aggregate is replicated by euro-denominated futures and swaps and euro time deposits, and the U.S. Aggregate will be replicated by U.S. dollar-denominated futures, swaps, and time deposits. This approach matches the full currency exposures of the local index components and is a practical strategy for replicating the returns of the unhedged index. In the second case, all cash is assumed to be invested in the base currency. For example, a U.S. investor will invest all cash in U.S. dollar deposits. This is a practical strategy for managers who aim to replicate the hedged index returns without requiring additional hedging with currency forwards.

How many markets represented in the Global Aggregate need to be included in the replication strategy? As discussed above, the index is highly concentrated in the largest three to four currencies. Matching index exposures in these markets should produce a sufficiently close tracking of index returns. Also, as a practical matter, most of the smaller currencies are not well represented by liquid futures markets. Accordingly, replication strategies in this study will use the U.S. dollar, euro, and yen markets and will examine the incremental improvement in tracking error that can be achieved by replicating the sterling market as well.

Regarding the smaller markets in the Index, one possibility is to omit them from the replication. The percentage exposure allocated to those markets is then proportionally reallocated to the largest markets according to the largest markets' market capitalization. Another possibility, based on the correlations among interest rates, is to replicate a small market by taking a corresponding exposure in a larger country with similar economic characteristics, increasing the larger country's share of the index by an amount equal to the small country's share. For example, the replication strategy can treat the Canadian dollar component of the Global Aggregate as increasing the contribution of the U.S. Aggregate, and the Malaysian ringgit as an increase to the yen component. We refer to this procedure as *proxy replication*.

In the case of the unhedged index, the currency exposures of the smaller markets could be matched through the cash portfolio. So even though the replicating derivatives portfolio is limited to three or four currencies, the cash can be distributed among all index currencies in the right proportions. A somewhat simpler scheme distributes the cash among the replicating currencies in the same proportions that were used for the bond market weights, whether or not proxy replication is used to represent the smaller markets.

In our implementation of the proxy replication methodology, the smaller markets are divided into three blocs, with each bloc represented in the replicating portfolio by a single reference market. Specifically, Denmark, Norway, Sweden, and Greece are represented by the Euro-Aggregate replicating portfolio; Australia, Canada, and New Zealand by the U.S. Aggregate replication portfolio; and Taiwan, South Korea, Malaysia, and Singapore by the yen portfolio. The smaller countries' currency returns are also replicated by their corresponding reference markets' currency returns. When the sterling market is not included in the replication strategy, it is proxied by the Euro-Aggregate.

Performance Summary

Unhedged Global Aggregate

Figure 4 presents replication results for the unhedged returns of the Global Aggregate Index. Two distinct replication portfolios are considered: one is an all-treasury futures portfolio and the other uses a combination of futures and swaps. Each of the two is presented with and without replication of the U.K. portion of the index.

In the all-treasury portfolio, the U.S. Aggregate portion of the index is replicated with a portfolio of four treasury futures contracts. The Euro-Aggregate replication uses a portfolio of 2-, 5-, and 10-year German treasury futures contracts, and replication of the Japanese yen and sterling components each use a single 10-year treasury futures contract.

The second portfolio has the lowest tracking error of all possible combinations of the different local market replication portfolios. In the U.S., a combination of the four treasury contracts is used to replicate the treasury component and a portfolio of 2-, 5-, 10-, and 30-year swaps is used to replicate the spread component of the U.S. Aggregate.[5] Replication of the Euro-Aggregate is accomplished with a combination of Euribor and treasury futures contracts, but using a different technique. The portion of the index with up to four years to maturity is replicated with a strip of Euribor futures contracts, and the German 10-year treasury futures contract is used for all longer-maturity bonds. The Asian portion of the index is replicated with a portfolio of 2-, 5-, 10-, and 20-year yen swaps, and the replication of the Sterling Aggregate uses 2-, 5-, 10-, and 30-year swaps.

In all cases, it is assumed that cash is invested in each of the currencies at 1-month LIBOR or equivalent 1-month rates. Proxy replication is not used here; the market-weighted positions in the three or four major currencies used are scaled up to represent the full market value of the omitted portion of the index. The numbers

[5] 10- and 5-year agency futures contracts have been trading since late 1999. The 10-year contract traded on the CBOT has traded most actively from the beginning of that period, and trading volume has been increasing consistently compared with volume in the 10-year Treasury note contract. It could constitute an alternative to swaps. We did not employ this contract in this study because of its limited price history. The correlation of the returns on the nearest 10-year agency contracts and 10-year swaps is 0.98 (data from January 2000 to December 2000).

Figure 4. **Tracking Errors for the Unhedged Global Aggregate Index Dollar-Based Returns**, bp/month, February 1999-December 2000*

U.S. Aggregate	Euro Aggregate	Japanese Aggregate Index	Sterling Aggregate	Tracking Error	Mean Outperf.
Treasury Futures	Treasury Futures	10-year Tsy Futures	None	22.4	10.5
Treasury Component: Treasury Futures; Spread Component: Swaps	Short End: Euribor Futures; Long End: Treasury Futures	Swaps	None	12.4	2.7
Treasury Futures	Treasury Futures	10-year Tsy Futures	10-year Tsy Futures	19.4	8.6
Treasury Component: Treasury Futures; Spread Component: Swaps	Short End: Euribor Futures; Long End: Treasury Futures	Swaps	Swaps	10.0	2.0

*No proxy replication used.

shown were calculated using a base currency of U.S. dollars, but the results change only slightly when other base currencies are used, as shown in Appendix B.

The results for different strategies show how tracking error declines as the Sterling Aggregate is added to the replication and as additional derivative instruments are used to replicate each market more precisely. For example, the 22.4 bp/month tracking error achieved using only treasury futures in the three largest markets can be improved to 19.4 bp/month if a replication portfolio for the Sterling Aggregate is added or reduced to 12.4 bp/month using a combination of futures and swaps in the three largest markets. The small reduction in tracking error obtained by adding the sterling replication is due to the small capitalization of the Sterling Aggregate, which composes only about four percent of the Global Aggregate. When both of these enhancements are included, our replication strategy achieves a tracking error of 10.0 bp/month. We simulated replication strategies using many different combinations of futures contracts and swaps in these four markets. The hybrid strategy illustrated in Figure 4 is the one that achieved the lowest tracking error against the Global Aggregate Index for this time period.[6]

Figure 4 also indicates that replication strategies using treasury futures alone have considerably higher mean returns than strategies that also use swaps and LIBOR-based futures. However, these results should not be used to predict relative performance of these strategies in the future. The period that our simulations cover was only 23 months long, too short to draw definite conclusions for future performance. Also, market particulars can have a substantial influence on the relative mean returns of different replication strategies. Since 1994, the 10-year U.S. Treasury futures contract has traded cheap compared with the prices predicted by many models. This phenomenon has produced extra return for the long futures investor compared with the long cash bond investor. In addition, volatility has drifted consistently lower over this period, resulting in additional returns for the futures buyer. Such market conditions can change and thereby alter the mean outperformance of particular derivatives portfolios.

Finally, swap spreads have increased consistently since 1994, causing swaps to underperform treasury futures and portfolios with large holdings of treasury bonds. Even if swap spreads continued to increase at the same rate, this would not necessarily lead to underperformance of swaps, as high swap yields provide sufficient income to offset such spread widening. Consequently, the mean outperformance numbers in Figure 4 likely reflect a unique historical period.

[6] In December 2000, U.S. dollar swaps outperformed the U.S. Aggregate by a considerable margin across the curve. For example, the 30-year swap returned almost 6%, whereas the highest-duration cell of the U.S. Aggregate returned about 2.5%. December's returns increased tracking error for the Global Aggregate minimum tracking error portfolio by more than 0.5 bp/month compared to results without this observation.

It is instructive to compare the tracking errors from the replication of the Global Aggregate Index with those for the replications of the individual local currency components of the Global Aggregate. Figure 5 shows tracking errors and mean outperformance for replication portfolios for the four largest components markets in the Global Aggregate. Comparing Figure 5 with Figure 4 shows that the 10.0 bp/month tracking error achieved for the Global Aggregate is less than that achieved by any single-currency replication portfolio. This result reflects the diversification benefit offered by the Global Aggregate Index compared with the local markets. Its outperformance is essentially a combination of local market outperformance values, which are less than perfectly correlated.

Hedged Global Aggregate

Matching returns of the hedged index requires a slightly different replication methodology. Replication of local market returns is the same as discussed above, but the available cash is now invested in the home currency rather than in the local currency. As derivatives require zero initial investment, they avoid the currency risk to principal that accompanies a cash bond purchase. Only cash flows and changes in value of the derivatives are affected by currency risk. This is a second-order effect similar in magnitude to the small currency returns experienced by a currency-hedged foreign bond investment. We will refer to this method of replication, in which the currency risk is hedged indirectly by keeping cash in the base currency, as *implicit currency hedging*. Appendix B offers a formal treatment of this approach and shows that it is approximately equivalent to conventional currency hedging.

Figure 6 presents replication results for the Global Aggregate Index hedged to U.S. dollars. The tracking errors are consistently smaller than for the unhedged case shown in Figure 4 due to the volatility-dampening effect of currency hedging. This effect is not very significant when all of the four major markets are replicated. But, for example, when the Sterling Aggregate is omitted, the hedged tracking errors shown here are much smaller than for the unhedged case. For instance, the first two

Figure 5. **Means and Tracking Errors of Outperformance of Replication Portfolios versus Selected Local Markets' Indices,** February 1999-December 2000, bp/month,

Index	U.S. Agg			Euro-Agg			Japanese Agg		Sterling Agg	
Replication Port.	Tsy Futures	Swaps	Tsy Comp: Tsy Futures; Spread: Swaps	Tsy Futures	Swaps	Short End: Euribor Futures Long End Tsy. Futures	10-yr. Tsy Futures	Swaps	Long Gilt Futures	Swaps
Tracking Error	29.2	27.1	14.8	15.7	13.3	14.1	43.3	21.0	52.7	43.7
Mean	6.2	-8.6	-4.0	4.2	2.4	3.0	13.4	2.1	-9.7	9.0

rows of Figure 6 show tracking errors of 18.8 and 9.9 bp/month when only the U.S., the euro, and the Japanese markets are replicated. The comparable tracking errors in Figure 4 are 22.4 and 12.4 bp/month, respectively. Why is that?

The risk and return characteristics of the Global Aggregate Index are very different when viewed on a currency hedged basis versus an unhedged basis. The unhedged returns are dominated by currency risk, which makes them much more volatile. Tracking errors in Figure 4 incorporate the currency volatility associated with sterling's being unhedged, whereas those in Figure 6 are essentially free of the currency risk associated with pound sterling. Therefore, the improvement achieved by covering another currency, the pound sterling, is greater in Figure 4, in which it reduces currency risk, as well as curve risk, than in Figure 6, in which it reduces only curve risk.

How much of an improvement could we expect from replicating the smaller markets in addition to the larger markets? We note that our most detailed replication strategy, using four markets, produced a tracking error of 10.0 bp/month in the unhedged case and 8.7 bp/month in the hedged case, for a difference in tracking error of 1.3 bp/month. This difference is due to currency volatility. We know from the discussion of Global Aggregate Index return volatility that currency volatility accounts for most of the index volatility, and therefore also of tracking error. Thus, interest rate volatility, the other component of tracking error, should be substantially less than currency volatility. As the incremental currency volatility incurred by leaving out the smaller markets is 1.3 bp, we would expect the incremental interest rate volatility to be no greater than 1 bp/month.

Figure 6. **Tracking Errors for the Hedged Global Aggregate Index Based on Implicit Currency Hedging,** Dollar-based returns, bp/month, February 1999-December 2000*

U.S. Aggregate	Euro Aggregate	Japanese Aggregate Index	Sterling Aggregate	Tracking Error	Mean Outperf.
Treasury Futures	Treasury Futures	10-year Tsy Futures	None	18.8	9.8
Treasury Component: Treasury Futures; Spread Component: Swaps	Short End: Euribor Futures; Long End: Treasury Futures	Swaps	None	9.9	2.0
Treasury Futures	Treasury Futures	10-year Tsy Futures	10-year Tsy Futures	17.6	8.1
Treasury Component: Treasury Futures; Spread Component: Swaps	Short End: Euribor Futures; Long End: Treasury Futures	Swaps	Swaps	8.7	1.4

*No proxy replication used.

To support this estimate of the potential improvement in tracking error from hedging every currency and bond market in the index, we can ask the question another way. How much smaller would the tracking error be if our four-currency strategy were used to replicate an index containing only these four currencies? This exercise yields a tracking error of 8.9 bp/month, due almost exclusively to the error in tracking yield curve and spread movements. Comparing this with the 10.0 bp/month in the last row of Figure 4, we find that the additional tracking error versus the Global Aggregate Index from omitting exposures to the remaining eleven markets (seven before October 1, 2000) is limited to considerably less than two bp/month over the period studied.

The Global Aggregate represents a good benchmark for an enhanced indexing or portable alpha investor, as it can be replicated with small tracking error. Using our replication methodology, we achieved a tracking error of 8.7 bp/month. Not only is this smaller than the replication errors for any of our single-currency aggregate indices, but it is even lower than the tracking errors that we observed when replicating the U.S. Treasury Index solely with treasury futures (Figure 19).

Replication Details

What does a replication portfolio look like? How does it perform? Figure 7a details the composition of the replicating portfolio for the hybrid strategy described above as of May 31, 2000, for a portfolio with a market value of $1 billion. The replication strategy uses treasury futures and swaps for the U.S. Aggregate, a Euribor/treasury futures portfolio for the Euro-Aggregate, and replication portfolios solely consisting of swaps for the Japanese and the sterling markets. When considering the number of contracts in the replication, it should be kept in mind that the notional value represented by a single futures contract or swap is different for every market, as shown in Figure 7c. For example, the Japanese 10-year futures contract has a notional of JPY100 million, which at current exchange rates is about 10 times as large as the $100,000 notional of the U.S. 10-year contract.

Figure 7b shows the currency allocation and June 2000 performance of the index and the replicating portfolio. The differences between the currency allocations represent the use of proxy replication. For example, the replicating portfolio adds an additional 1.9% to the U.S. dollar portion to represent the index allocations to Canada, Australia, and New Zealand. The analysis of the strategy's performance by currency exemplifies the diversification effect that often occurs in the replication of the Global Aggregate Index. The strategy outperforms the index in some markets and underperforms in others. These gains and losses partially offset each other, leaving the strategy with a modest outperformance relative to the index.

Figure 8 contains tracking errors versus the unhedged index (in $) for a larger number of replication strategies than presented in Figure 4. Six different combinations of methods are used to track the U.S. and Euro-Aggregate indices, which are then combined with different replication strategies for the yen and

Figure 7a. **Global Aggregate Index Replication Portfolio Composition for June 2000**
Based on Proxy Replication

	Euribor Futures	Treasury Futures (No. of Contracts) 2-yr	5-yr	10-yr	30-yr	Swaps (Multiples of Lot Size) 2-yr	5-yr	10-yr	30-yr***
U.S. Dollar*		242	160	272	469	52	23	126	21
Euro**	1005			1657					
Yen						75	112	65	11
Sterling						4	6	6	9

*Includes Australian, Canadian, New Zealand Dollar.
**Includes Swedish Krona, Danish Krone, Norwegian Krone, Greek Drachma.
***20 Years Maturity for Japan.

Figure 7b. **Global Aggregate Index Replication Portfolio, Weights and Returns for June 2000,** Based on Proxy Replication

	% of Index	% of Portfolio	Local Curr. Returns (bp) Replication	Index	Outperf.
U.S. Dollar	41.7	43.6	201.9	205.4	-3.6
Euro	26.3	28.3	30.4	37.8	-7.4
Yen	24.0	24.0	-26.3	-29.0	2.7
Sterling	4.1	4.1	-9.0	26.5	-35.5
Australian, Canadian, New Zealand Dollar*	1.9	0.0	201.9	336.9	-135.1
Swedish Krona, Danish Krone, Norwegian Krone, Greek Drachma**	1.9	0.0	30.4	10.2	20.2
Global Aggregate Return (in $)			**232.9**	**239.7**	**-6.8**

*Replicated using U.S. dollar replication portfolio as proxy.
**Replicated using euro replication portfolio as proxy.

Figure 7c. **Notional Values and Standard Lot Sizes for Futures Contracts and Swaps***

	U.S. Dollar	Euro	Yen	Sterling
Money Market Futures	1,000,000	1,000,000	100,000,000	500,000
Treasury 2-year Futures	200,000	100,000		
Treasury 5-year Futures	100,000	100,000		
Treasury 10-year Futures	100,000	100,000	100,000,000	100,000
Treasury 30-year Futures	100,000			
Swaps (all maturities)	1,000,000	1,000,000	100,000,000	1,000,000

* Futures notionals set by exchanges. Standard swaps size assumed for purposes of this study.

sterling markets. For the three cases in Figure 8 (shown in bold) that correspond to combinations included in Figure 4, the numbers differ slightly because of the effects of proxy replication. The results in Figure 8 were obtained using the proxy replication methodology described above, in which the weight of each omitted market is assigned to the major currency considered most relevant. In Figure 4, by contrast, all omitted markets are replicated using a market-weighted blend of the major currencies. These two approaches give similar results, because the weights obtained from the proxy replication procedure are not very different from the market weights. The maximum difference in tracking error occurs when treasury futures contracts are used in the replication of all four markets, producing a difference of 0.6 bp/month (19.4 - 18.8 bp/month).

Looking across any row of Figure 8 shows the effect of adding sterling to the replication and the impact of using swaps instead of treasury futures for the Japanese and the Sterling Aggregate. Comparisons within any column show the importance of the set of instruments chosen for the replication of the U.S. Aggregate and the Euro-Aggregate. Once the Sterling Aggregate Index replication portfolio has been added to the replication, the lowest tracking errors are

Figure 8. **Tracking Errors for the Unhedged Global Aggregate Index, Ignoring Small Markets**
Dollar-based returns, bp/month, February 1999-December 2000*

U.S. Aggregate	**Euro-Aggregate**	**Jpn. Agg.** Treasury Futures / **Sterling Agg.** None	**Jpn. Agg.** Treasury Futures / **Sterling Agg.** Treasury Futures	**Jpn. Agg.** Swaps / **Sterling Agg.** Swaps
Treasury Futures	Treasury Futures	**22.7**	**18.8**	16.1
Treasury Futures	Euribor and Treasury Futures	22.4	18.7	15.8
Eurodollar and Treasury Futures	Euribor and Treasury Futures	20.8	17.1	14.0
Treasury Component: Treasury Futures; Spread Component: Swaps	Short End: Euribor Futures; Long End: Treasury Futures	18.2	14.2	**10.2**
Treasury Component: Treasury Futures; Spread Component: Treasury Futures and Swaps	Short End: Euribor Futures; Long End: Treasury Futures	18.6	14.5	10.2
Treasury Component: Treasury Futures; Spread Component: Swaps	Treasury Component: Treasury Futures; Spread Component: Swaps	18.1	14.0	10.4

* Proxy Replication used.

obtained by the strategy that replicates the U.S. Aggregate with treasury futures and swaps and the Euro-Aggregate with Euribor futures at the short end and treasury futures at the long end.

The base currency in which returns are reported has only a minor impact on the performance of the replication strategy, as shown in Appendix B. To illustrate the point, Figure 9 presents replication results for one strategy expressed in four base currencies. In every case, the results are within 0.1 basis points of those reported in terms of U.S. dollars.

For markets in which the replication of the local currency component of the Global Aggregate Index with derivatives is possible only with relatively large tracking error, replication with cash securities is an option. Although this study deals primarily with replication with derivatives, we replicated the Japanese index with four liquid bonds. Each bond replicated duration-based cells identical to the ones used in the replication of the Japanese Treasury Index with swaps. Leaving the other three replication portfolios the same as in the lowest tracking error strategy, the use of cash bonds to replicate the Japanese index reduces the tracking error of the unhedged index from 10.0 to 7.4 bp/month.

We have seen that the tracking error of a given strategy versus the Global Aggregate depends heavily on the weights of each currency in the index and on the composition of the replication portfolio in each local market. In particular, we focused in Figure 5 on the tracking errors achievable by different strategies in each local market. It is also important to consider the correlations among the outperformance histories of the local market replication portfolios. Figure 10 displays the correlations of the local market outperformances of a number of replication portfolios. The correlations between the replication portfolios for the U.S. Aggregate and the Euro-Aggregate that go into the lowest tracking error portfolio are a low 0.03. The fact that these two replication portfolios exhibit correlations close to zero with the Sterling Aggregate and the Japanese Aggregate replication portfolios, together with their low tracking errors as they pertain to the local markets, results in the portfolio with the lowest overall tracking error.

Figure 9. **Tracking Errors for the Global Aggregate Index Replication, Minimum Tracking Error Strategy by Base Currency,** bp/month, February 1999-December 2000

	Dollar	Euro	Yen	Sterling
Tracking Error	10.0	10.0	10.0	9.9

Figure 10. **Correlations of Local Markets' Outperformance,** February 1999-December 2000

U.S. Aggregate			Euro-Aggregate		Jpn. Aggregate Index		Sterling Aggregate			
Tsy. Fut.	**Swaps**	**Tsy. Comp.: Tsy. Fut.; Spread: Swaps**	**Tsy. Fut.**	**Short End: Euribor Fut.; Long End: Tsy. Fut.**	**10-yr. Tsy. Fut.**	**Swaps**	**Long Gilt Fut.**	**Swaps**		
1.00	-0.55	0.23	0.46	0.44	-0.18	-0.10	0.40	0.11	Treasury Futures	
	1.00	0.65	-0.22	-0.24	0.09	0.19	-0.09	-0.11	Swaps	
		1.00	0.09	0.03	-0.15	0.04	0.18	-0.11	Treasury Component: Treasury Futures; Spread Swaps	U.S. Aggregate
			1.00	0.98	-0.21	0.13	0.49	-0.06	Treasury Futures	
				1.00	-0.18	0.16	0.48	-0.01	Short End: Euribor Futures; Long End: Treasury Futures	Euro-Aggregate
					1.00	0.38	0.17	-0.30	10-yr. Tsy. Futures	
						1.00	0.26	-0.07	Swaps	Japanese Tsy. Index
							1.00	-0.20	Long Gilt Futures	
								1.00	Swaps	Sterling Aggregate

Figure 11. **Tracking Error and Replication Methodology,** bp/month, February 1999-December 2000

	Replication Portfolio			
	4 Major Markets, Best Derivatives Portfolio	4 Major Markets, Best Derivatives Portfolio and Currencies of Small Markets	4 Major Markets, Best Derivatives Portfolio and Jap. Aggregate with Cash Bonds	4 Major Markets, Best Derivatives Portfolio and Jap. Aggregate with Cash Bonds, and Currencies of Small Markets
Index Being Replicated				
Global Aggregate Index	10.0	8.7	7.4	6.9
4 Major Markets Index	8.9	N/A	7.4	N/A

The replication results for the hedged Global Aggregate Index are summarized in Figure 11. For a pure derivatives portfolio, using only cash deposits and derivatives in the four major markets, the lowest tracking error achieved was 10.0 bp/month. By matching index exposures to all currencies, leaving the derivatives positions intact, tracking error was reduced to 8.7 bp/month. If, in addition, the Japanese Aggregate Index is replicated with four cash bonds, the overall tracking error can be reduced to 6.9 bp/month.

Liquidity and Transaction Costs

Liquidity in the Derivatives Markets

Replication of the Global Aggregate Index with derivatives may require large positions in a number of derivatives markets. Although derivatives markets are often more liquid than the underlying cash markets, both markets will react if large transactions require execution over a short period of time.

This section provides estimates for bid-offer spreads, tradable sizes, and market impact costs for the swaps and futures markets in the four largest components of the Global Aggregate (U.S., euro, U.K. and Japan). These estimates are derived from our swaps traders and from our FCM personnel working on the relevant futures exchanges.

Figure 12 presents estimated bid-ask spreads and sizes tradable at these spreads for the swaps markets. The bid-ask spreads are for fully collateralized trades, with ISDA master agreement and credit approval in place. Assuming durations of 2, 4 ¼, 7, and 12 years for the four maturity sectors, the bid-ask spreads for 2-, 5-, and 10-year swaps are competitive with the bid-asks in the local government bond markets except for the most liquid bonds, such as the on-the-runs in the U.S.

These bid-ask spreads are too wide to allow rolling an entire swaps portfolio into new at-the-market swaps on a monthly basis. One alternative would be to adjust the swaps replication portfolios by adding new at-the-market payer or receiver

Figure 12a. **Bid-Ask Spreads for Swaps,** bp
Normal Market Conditions, Fall 2000*

	2-Year	5-Year	10-Year	30-Year
U.S.	1.0	1.0	1.0	1.5
Euro	1.5	1.5	1.5	2.0
Yen	1.0	1.0	1.0	1.5
Sterling	1.5	1.5	1.5	2.25

Figure 12b. **Tradable Sizes for Swaps,** Million $-Equivalents
Normal Market Conditions, Fall 2000*

	2-Year	5-Year	10-Year	30-Year
U.S.	250	250	250	100
Euro	250	250	250	100
Yen	200	200	200	80
Sterling	75	75	75	60

*For fully collateralized trades, with master agreement and credit approval in place.

swaps that help rebalance the replication portfolio. However, this procedure would substantially increase the number of positions over time, which many investors would consider unacceptable.

There is a more convenient alternative for rebalancing the swaps replication portfolio. Swaps that are 3-6 months old can be replaced by new at-the-market swaps at a cost of 0.25 basis points under normal market conditions. This bid-ask applies to the most liquid points on the curve and for all four markets. This execution approach makes swaps cost competitive with exchange-traded derivatives and removes the potential of a rapidly growing swaps book. Our tracking error and transactions costs estimates are based on quarterly rebalancing of the swaps replication portfolios.

So far, all spreads and sizes hold for normal market conditions. As swap transactions occur between counterparties on a bilateral basis, a swap contains market and counterparty credit exposures over the life of the swap. Although these risks are mitigated when evaluated in a portfolio context and further reduced through the use of netting agreements and collateralization, periods of heightened risk aversion can lead to a noticeable decrease in swaps market liquidity.

How might the transaction costs in Figure 12 change during periods of market stress, which are usually periods of heightened credit and liquidity concerns, particularly in the over-the-counter swaps market? Based on the extraordinary experience of the fall of 1998, our swaps traders estimate that tradable notional sizes would decline by 50%-75% and bid-ask spreads would at least double. Although this represents a substantial reduction in liquidity, it should be compared with the reduction in liquidity that occurred in the cash markets during the credit crisis of 1998, when bid-ask spreads for off-the-run U.S. Treasuries increased significantly as well. Sector credit risk also increases substantially in importance during such stressful periods. During the fall of 1998, bid-ask spreads on individual names in the financial sector reached many tens of basis points at times. For many investors, such issues became untradable.

Swap spreads reflect the credit spreads inherent in cash LIBOR on a forward basis. Since LIBOR over T-bill spreads represent the average spread paid by some of the highest rated banks, swap spreads generally widen less during times of crisis than other spreads in the corporate bond market. This is what happened in 1998, indicating that swaps offer relative liquidity advantages to some cash market sectors during stressful periods.

Figures 13 and 14 contain estimates of bid-ask spreads and market impact costs for treasury and LIBOR or LIBOR-equivalent money-market futures for the four

Figure 13a. **Bid-Ask Spreads for Liquid Treasury Futures Contracts**
in 1/100s of 1 Percent of the Futures Price
Normal Market Conditions, Fall 2000

	2-Year	5-Year	10-Year	30-Year
U.S.	1.563	1.563	1.563	3.125
Euro (Eurex)	1.5	1.5	1.5	
Yen			1.0	
Sterling			2.5	

Figure 13b. **Tradable Sizes for Liquid Treasury Futures,** Number of Contracts
Normal Market Conditions, Fall 2000*

	2-Year	5-Year	10-Year	30-Year
U.S.	600	1000	2000	2000
Euro	700	700	1000	
Yen			250	
Sterling			500	

Figure 13c. **Market Impact Cost for Treasury Futures**
in 1/100s of 1 Per Cent of the Futures Price
Normal Market Conditions, Fall 2000*

	2-Year	5-Year	10-Year	30-Year
U.S.	3.125	3.125	3.125	3.125
Euro	2.0	3.0	6.0	
Yen			6.0	
Sterling			12.0	

*Trading period of about 5 minutes assumed.

largest markets of the Global Aggregate. The tradable sizes assumed in these cost estimates are sufficiently large for a manager replicating a $1 billion Global Aggregate portfolio. To be conservative, the purchase or sale of the position is assumed to take place over a period of about five minutes.

The highest bid-ask spreads are found again in the smallest market, the U.K. The estimates of market impact cost show the highest numbers for the smaller markets,

Figure 14a. **Bid-Ask Spreads for Money Market Futures**
in 1/100s of 1 Per Cent of the Futures Price
Normal Market Conditions, Fall 2000

U.S.	Years 0-2 1/2	Years 3-5 3/4-1	Beyond 5 years 1-1 1/2
Euro (LIFFE)	Years 0-2 1/2	Year 3 1	Year 4 1 1/2
Yen	Up to 18 months 1/2		
Sterling	Years 0-2 3/4	Year 3 1-1 1/2	

Figure 14b. **Tradable Sizes for Money Market Futures**
Number of Contracts for each Expiration Date
Normal Market Conditions, Fall 2000*

U.S.	Years 0-2 1000	Years 3-5 1000	Beyond 5 years 1000
Euro (LIFFE)	Years 0-2 600	Year 3 400	Year 4 200
Yen	Up to 18 months 100		
Sterling	Years 0-2 50	Year 3 25	

Figure 14c. **Market Impact Cost for Money Market Futures**
in 1/100s of 1 Percent of the Futures Price
Normal Market Conditions, Fall 2000*

U.S.	Years 0-2 0.50	Years 3-5 1.00	Beyond 5 years 1 1/2
Euro (LIFFE)	Years 0-2 1	Year 3 1 1/2	Year 4 2
Yen	up to 18 months 1/4		
Sterling	Years 0-2 1/2	Year 3 1	

*Trading period of about 5 minutes assumed.

except for the German 2-year (Schatz) treasury futures, which are known to be very liquid. As for swaps, these numbers are indicative under normal market conditions. They cannot be expected to prevail at all times, or under all market conditions.

Liquidity-related concerns are not as acute for exchange-traded derivatives. The short settlement periods in these markets and the role of the clearing corporation in collecting and paying variation margin significantly reduce counterparty credit risk. Consequently, liquidity does not dry up in stressful markets as it does in the cash or OTC-derivatives markets. The size of the bid-ask spread in futures markets is more a function of market risk than of changes in liquidity. If market risk is perceived to be high (e.g., before announcement of an important economic number), the bid-ask spread will widen to compensate market makers for their risk as they provide liquidity to the market. In general, the ability to get a trade done within a particular time-frame during periods of market stress is less of an issue in the futures markets than in the OTC-derivatives markets.

All the derivatives markets are undergoing a period of immense change. Futures markets, in particular, are moving rapidly toward electronic trading. Eurex and LIFFE have already done so, and the CBOT and CME are moving in that direction. Electronic trading has implications for the way that orders are executed and their costs. The exchanges are making trading more flexible and anonymous. In particular, more and more transactions are allowed to take place outside the confines of the exchange. The CME recently extended "exchange-for-physical" transactions from the first two Eurodollar contracts to all contracts. In an exchange-for-physical transaction, a broker trades a fixed income security in exchange for a strip of Eurodollar contracts that is designed to hedge that bond. Similar to block trades in the treasury futures markets, the trades take place off the floor, but are reported to the exchange. All futures contracts traded in this way are counted toward open interest and daily volume. In a block trade, the bid or offer does not have to be posted with the exchange. Instead, the futures buyer or seller trades directly with a broker such as Lehman Brothers, which makes a market for block trades. The dealer has an obligation to find the best execution for the customer by contacting other brokers. Block trades also allow for greater trading anonymity. Similarly to the liberalization in the rules for exchange-for-physical transactions in the CME, the LIFFE recently reduced the minimum size for block trades in the Long Gilt futures from 750 to 500 contracts.

Overall, derivatives markets are changing rapidly. New products and new ways of trading must be monitored by the investor when deciding which derivatives products to use and how to use them.

Estimation of Transaction Costs: Assumptions and Methodology

Figure 15 provides estimates of transaction costs for two different Global Aggregate Index replication strategies: replication of the four major currencies (dollars, euros, yen and sterling) using liquid treasury futures and the lowest-tracking error replication method described in the last row of Figure 4. In both cases, the U.S. dollar was used as a proxy for the Canadian, Australian, and New Zealand dollars, and the euro was used as a proxy for the Danish and Norwegian krone, the Swedish krona, and the Greek drachma. The Japanese Aggregate replication portfolio is used to proxy the other Asian markets (Hong Kong, Singapore, Korea, and Malaysia).

For each strategy, the costs are first broken down by currency and then by startup and average monthly maintenance costs. Costs for each currency are expressed as a fraction of the market value of that currency's portfolio. For each currency, transaction costs are expressed in basis points. The first month's transaction costs (in bp) are reported separately as startup costs. The transaction costs for all months thereafter are averaged to obtain average monthly maintenance costs.

In addition to expressing the transaction costs for each currency as a fraction of the market value of that currency's portfolio, we report the total transaction costs for all the currencies as a fraction (expressed in basis points) of the market value of the entire portfolio.

The following assumptions underlie the transaction cost estimates:

Swaps. Bid-ask spreads for swaps are given in terms of basis points of the fixed leg. For example, if the bid-ask is given as 2 bp, this means the trader demands 2 basis points more when receiving the fixed leg than when he is paying. The value of the bid-ask for a given notional is, therefore, the present value of the spread for the life of the swap. Figure 12a presents bid-ask spreads (in bp) for various currencies and maturities. The investor's one-way transaction cost is assumed to be one-half of the bid-ask spread.

Note that the present value of the bid-ask spread over the life of the swap equals the mark-to-market of the swap if the swap rate changes by the amount of the spread. We therefore estimate the present value of the spread for the life of the swap by multiplying the PVBP of the swap by the spread.

When an investor rolls over a swap position (i.e., unwinds an existing swap position created within the last few months and simultaneously enters into a new swap position of the same notional and original maturity) the transaction cost is assumed to be ¼ basis points of fixed leg, regardless of currency or maturity.

The swaps in the replicating portfolio are rolled over every three months, when the replicating portfolio is rebalanced. When the rollover is performed, the difference in the required notional for a particular maturity and currency is regarded as the establishment or unwinding of a position with the remainder of the position regarded as a rollover. For example, if a rollover is performed in

June and the replicating portfolio requires the fixed leg of a 10-year swap on a $104 million notional (an increase from December's $100 million notional), then $100 million will be regarded as the rollover amount (bid-ask = ¼ bp) and the remaining $4 million will be regarded as a new position (cost = half of the bid-ask given in Figure 12a). If we needed $100 million last December and now need $98 million, then $98 million is regarded as the rollover amount, and the remaining $2 million is the unwound amount.

Tradable sizes in the swap market are sufficiently small that the impact of our swaps trades on bid-ask spreads is negligible.

Treasury Futures. Bid-ask spreads are given in terms of basis points of the face value of the contract, i.e., in hundredths of a currency unit for every 100 currency units of face value. So, for example, the bid-ask for 2-year U.S. Treasury futures is 1.563 bp (=1/64), or 1.563 cents for every $100 of face value. Bid-ask spreads under normal market conditions for treasury futures of various currencies and maturities are given in Figure 13a. The investor's transaction cost for a one-way trade is assumed to be one-half of the bid-ask spread.

The market impact of the treasury futures transactions needed for a $1 billion total replication portfolio is not negligible. If the number of contracts in a transaction in treasury futures of a particular currency and maturity is less than or equal to a quarter of the corresponding tradable size given in Figure 13b, no market impact is assumed. However, if the number of contracts is greater than a quarter of the tradable size given in Figure 13b, then the market impact cost for the trade is scaled as a linear function of the ratio of transaction size to tradable size:

Market-impact cost per trade = (transaction size/tradable size) * market impact cost.

For example, the tradable size for the 10-year gilt contract is 500 contracts, and the market impact cost is 12 bp. Suppose we wish to enter a long position in 250 contracts. Since 250 is more than 500/4, the market-impact cost for this transaction will be

$$(250/500) * 12 \text{ bp} = 6 \text{ bp}$$

The market impact costs for various maturities and currencies are given in Figure 13c.

In addition to the bid-ask spread, there is a fixed transaction fee per contract regardless of the maturity of the contract, representing the cost of clearing and execution. That fee is $7 for any U.S. Treasury contract, EUR7 per German treasury contract, JPY6000 per JGB contract, and GBP10 per gilt contract.

Treasury futures positions in the replicating portfolio are initially established in the nearest contract in whichever maturities are used (e.g., the nearest 2-year, 5-year, and 10-year German treasury contracts; the nearest 10-year JGB contract). At the end of the month preceding the expiration of the contracts, the position in each contract is rolled over into the next contract. There is no discount for rollovers. In other months, the positions are simply rebalanced to accommodate the changing replication conditions.

Money Market Futures. Bid-ask spreads for money market futures are also given in terms of basis points of the face value of the contract, i.e., in hundredths of a currency unit for every 100 currency units of face value. So, for example, the bid-ask for the nearest eight euribor contracts is 0.5 bp, or 0.005 euros for every EUR100 of face value. Bid-ask spreads under normal market conditions for money market futures of various currencies and expirations are given in Figure 14a. Again, transaction costs are assumed to be one-half of the bid-ask spread for a one-way trade.

The increase, if any, in the bid-ask due to the market impact of the transaction is calculated in the same way as that for treasury futures. Tradable sizes for money market futures are given in Figure 14b, and market impact costs for money market futures are given in Figure 14c.

In addition to the bid-ask spread, there is a fixed fee per contract representing the cost of clearing and execution. This cost does not vary with the expiration of the contract. The fixed fee for the Euribor contract, the only money market contract used in our lowest tracking error replication strategy, is EUR7.

Results

Figures 15 summarizes the transaction costs results for the two replication portfolios considered. These numbers are conservative estimates of the transaction costs an investor would have likely encountered. Actual costs would have been much lower in most cases. Nonetheless, the cost estimates and the assumptions

Figure 15. **Transaction Costs: Results,** February 1999-December 2000

	Treasury Futures Replication Portfolio					Minimum Tracking Error Replication Portfolio				
	U.S. Agg.	**Euro-Agg.**	**Yen Agg.**	**Sterling Agg.**	**Global Agg.**	**U.S. Agg.**	**Euro-Agg.**	**Yen Agg.**	**Sterling Agg.**	**Global Agg.**
Market Impact Costs Not Included										
Setup Cost (bp)	1.2	1.3	0.8	1.8	**1.2**	1.8	1.5	3.1	6.7	**2.2**
Rebalancing Cost (bp/month)	1.0	1.0	0.6	1.4	**0.9**	0.7	0.6	0.6	0.7	**0.6**
Market Impact Costs Included										
Setup Cost (bp)	2.1	3.7	2.7	4.0	**2.8**	1.9	3.5	3.1	6.7	**2.9**
Rebalancing Cost (bp/month)	1.7	3.1	1.7	3.1	**2.2**	0.8	2.4	0.6	0.7	**1.2**

underlying them are indicative in nature. Bid-ask spreads, and market impact cost could exceed the numbers found in Figures 12 to 14 at times.

We find that for replication programs of $1 billion or more, the market impact of trades can potentially have a greater impact on performance than other transaction costs. However, these results represent a rather aggressive assumption concerning trading time in the futures market (five minutes). These costs can be reduced by carefully spreading execution out over time. In any case, the total transaction costs, even under these rather conservative assumptions, are quite reasonable: less than 3 bp to establish the portfolio and about 1 to 2 bp/month for rebalancing.

We find that based on these transaction cost assumptions, the minimum tracking error portfolio has lower overall cost than the treasury futures portfolio, due to the fact that it relies heavily on the use of swaps. Swaps are rolled over only once every three months and then for only ¼ basis points. When used in this way, swaps-based replication portfolios have a fairly low long-term cost.

REPLICATING THE JAPANESE AGGREGATE INDEX

The recent reformulation of the Global Aggregate Index (Appendix A) increased the market share of yen-denominated bonds and has also expanded the coverage of the Japanese market to include spread products, as well as treasuries. These changes have increased interest in replication strategies for this market. This section investigates replication strategies based on combinations of treasury futures, money market futures, and swaps and compares the results to a replication strategy based on a proxy portfolio of a few cash bonds.

Our first attempt at replication of the U.S. market with futures used treasury futures only. We found this approach, using four contracts along the yield curve, to be very successful in replicating the U.S. Treasury Index, but less effective at tracking indices containing spread products. We then found that the incorporation of swaps and Eurodollar futures in the replicating portfolio could help improve performance by adding some spread exposure.

In this historical study of index replication in the Japanese market, we are presented with the opposite situation, to some extent. Liquidity constraints have limited our use of treasury futures to the 10-year contract. Swaps of four maturities are used to match index exposures better along the yield curve. Nevertheless, because the Japanese Aggregate Index was introduced only recently (October 1, 2000), most historical testing of the replication strategies is carried out against the Japanese Treasury Index. Consequently, if swaps are used in the replication, there is exposure to spreads only in the replicating portfolio and not in the index for most of the historical period.

We are, therefore, doubly confident that the methodology presented here will provide a good replication of the Japanese Aggregate Index, although the tracking errors in this study were mainly measured against the Japanese Treasury Index. First, treasuries make up about 75% of the Japanese Aggregate Index, making the two indices very similar. Second, to the extent that the use of swaps in the replication strategy introduces a spread exposure versus the Treasury Index, there is reason to believe that this strategy might be even more effective against the Japanese Aggregate Index.

The Japanese treasury market has been harder to trade and invest in than the government bond markets of other developed nations. One reason is the concentration of liquidity in a few issues, particularly in the 10-year benchmark bond. Although down from earlier years, a still substantial part of treasury bond trading volume is concentrated in these securities. The Japanese treasury futures market consists of a 5-, a 10-, and a 20-year futures contract. However, liquidity is also concentrated in a single contract, the 10-year futures. This contract will be used in the replication simulations. Clearly, the use of the 10-year contract alone for futures replication should lead to additional yield curve risk compared with what was experienced in the U.S. or the euro zone. The 10-year futures contract also has some peculiarities. It is used by many investors as a vehicle for hedging their bond positions. Therefore, there is a substantial natural short base in the contract. This leads to the contract's trading cheap. At the same time, the relatively high probability of a squeeze on the cheapest-to-deliver bond into the contract presents a significant risk to anyone short this bond. This leads to a lack of arbitrage activity. Consequently, the basis of the closest contract usually trades expensive and with high volatility. This implies that replication of the Japanese Treasury Index with the 10-year futures contract produces higher tracking errors than those experienced for the Euro-Aggregate or the U.S. Aggregate. Another issue that makes the Japanese futures market somewhat less efficient is that execution and clearing of futures orders on the Tokyo Stock Exchange must take place through the same broker.

There are also euro-yen futures contracts trading on the Tokyo Financial Futures Exchange (TIFFE). The closest six contracts exhibit acceptable levels of liquidity, and they are used to replicate the returns on Japanese treasury bonds with maturities of less than 18 months. However, due to the limited share of bonds in the index with such short maturities, the potentially positive effect of combining euro-yen with treasury futures in the replication is limited.

Swaps are another class of derivatives whose index-tracking properties we investigate. Since swaps might not track the Japanese Treasury Index very closely due to the spread exposure embedded in swaps, we also explore the replication of the Japanese Treasury Index with a few liquid cash treasury bonds. Given the transactions costs, cash replication can be recommended only for longer-term replicating portfolios. We found that the cash portfolios improve on replication with derivatives by a wide margin.

Methodology

Replication with treasury futures is straightforward, since we are using just a single contract. The strategy buys the appropriate number of 10-year futures contracts to hedge the duration of the entire Japanese Treasury Index. The durations used for the 10-year futures contract are the CTD-durations forward to delivery. We use the closest contract[7] and roll it at the month-end preceding delivery at the closing prices.

[7] There was one exception to this rule of using the closest contract. From August 1999 through November 1999, we used the March 2000 contract, rather than the nearest December 1999 contract. This contract was avoided by investors due to the risks that were associated with the Y2K problem, and therefore suffered from low liquidity.

Euro-yen contracts are employed to replicate bonds with up to 1.5 years in maturity using the "stripped hedge" methodology, which involves a sequence of contracts covering consecutive calendar quarters to constitute the required cash flow exposure.[8] This technique, when practical, offers very good matching of yield curve risk. Unfortunately, the limited liquidity of longer-dated euro-yen contracts restricts use of this strategy further out along the curve.

Using swaps in replication follows the same methodology we use for the U.S. and euro markets. The Japanese Treasury Index is divided into four cells according to their modified adjusted duration. Each cell is replicated separately with the corresponding instrument on a PVBP-equivalent basis. The outperformance numbers for each cell are then added up on a market-capitalization basis to arrive at outperformance for the full index. To accommodate the low yield levels that prevailed in Japan over the historical period, with the resulting high durations, we choose somewhat wider duration cells than for the U.S. and the euro zone. These boundaries are 0-3 ½ years, 3 ½ -7 years, 7-10 years, and more than 10 years in modified adjusted duration.[9] We use 2-, 5-, 10-, and 20-year swaps to replicate each duration cell, respectively. To save on transaction costs, we turn over the swaps portfolio only once every three months.

For replicating portfolios using cash instruments, we divide the index into the same four duration cells as for the swaps-based replication. One liquid bond is selected to represent each cell. For the two high-duration cells, the most recently issued bond in each cell is considered the most liquid. For the two low-duration cells, the bond with the maximum amount outstanding in each cell is selected, and we impose the additional restriction that the bonds selected for the replication have to be 1 ½-3 years or 4 ½-6 ½ years in duration, respectively. This leads to a closer match with the durations of the index cells being replicated. The details of the construction of the cash portfolios are given in Appendix C.

Performance

Figure 16 summarizes the return volatilities and mean returns for the Japanese Treasury Index. To facilitate comparisons with our results for the U.S. and Europe, our simulations start in January 1994 and end in December 2000. We display the results for the full time period and for two sub-periods, January 1994 to June 1998 and July 1998 to December 2000. We choose this division because the world credit

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[8] Details concerning the replication methodologies used with the different derivatives can be found in our November 1997 and March 2000 studies on replication (see Footnote 2).

[9] It could be argued that these duration cells were chosen with the benefit of hindsight. In reality, though, an investor knows the market structure at any given point in time, and can adjust the cells according to changes in the composition of the index. This should lead to an improvement in terms of tracking error compared to the numbers we found.

crisis of the summer of 1998 led to higher corporate bond and swap spreads and higher spread volatility. These changes affect the relationship between the derivatives and the cash markets and, therefore, the tracking errors of the replication portfolios. It is also of interest to see how the replication performed during the period of increased spread volatility since July 1998.

Figure 16. **Japanese Treasury Index, Mean and Volatility**
January 1994-December 2000, bp/month

	1/94-12/00	1/94-6/98	7/98-12/00
Volatility	138.7	139.1	137.9
Mean	37.8	49.0	17.6

Figure 17a. **Tracking Errors for the Replication of the Japanese Treasury Index,** January 1994-December 2000, bp/month

	1/94-12/00	1/94-6/98	6/98-12/00
10-yr Treasury Futures	35.1	32.0	40.6
Euro-yen Futures and 10-yr Treasury Futures	34.8	31.4	42.1
10-yr Treasury Futures and Swaps	29.5	29.0	30.7
Swaps	31.2	31.0	31.9
Cash Bonds	8.3	8.5	8.1

Figure 17b. **R-Squared Values for the Replication of the Japanese Treasury Index,** January 1994-December 2000

	1/94-12/00	1/94-6/98	6/98-12/00
10-yr Treasury Futures	93.6	94.7	91.2
Euro-yen Futures and 10-yr Treasury Futures	93.7	94.9	90.6
10-yr Treasury Futures and Swaps	95.5	95.7	95.0
Swaps	94.9	95.0	94.6
Cash Bonds	99.6	99.6	99.7

Figure 17c. **Mean Outperformance for the Replication of the Japanese Treasury Index,** January 1994-December 2000, bp/month

	1/94-12/00	1/94-6/98	6/98-12/00
10-yr Treasury Futures	11.9	13.0	10.1
Euro-yen Futures and 10-yr Treasury Futures	10.4	13.0	5.7
10-yr Treasury Futures and Swaps	7.0	5.5	9.7
Swaps	3.0	1.1	6.3
Cash Bonds	-0.8	-1.0	-0.5

Figure 17a contains tracking errors for five replication portfolios for the full period and the two sub-periods. Figure 17b shows the R-squared (R^2) values for the same periods, and Figure 17c presents the means. The R^2 tells us what part of the variance of the index returns is explained by the replication portfolio's returns.

We consider three replication portfolios that use the same type of instrument for all four duration cells and two hybrid replication portfolios: euro-yen futures combined with the 10-year treasury futures contract and the 10-year treasury futures contract combined with 2-, 5-, and 20-year swaps.

First, we find that the hybrid euro-yen futures and treasury futures portfolio gives tracking errors very similar to those obtained using only 10-year treasury futures. These results are not surprising, as the first six euro-yen futures replicate bonds with cash flows up to 18 months only. As the mean share of bonds in the index with maturity less than 18 months is 2.8%, the inclusion of euro-yen futures does little to reduce tracking error. If the share of shorter-maturity bonds in the Treasury Index increases, or the liquidity of longer-dated euro-yen futures improves so that they cover more index bonds, they can become a more valuable replication instrument for the Japanese Treasury Index.

Of all of the derivatives-based replication strategies, the hybrid replication portfolio consisting of swaps and 10-year treasury futures delivers the lowest tracking errors. These results are consistent with the results for the U.S. and the euro zone (Figure 18). During the second sub-period (beginning July 1998), futures returns differed significantly on a few occasions from treasury returns. This produces a substantially higher tracking error for the Japanese treasury futures replication portfolio for that period.

The tracking errors from replicating parts of an index with different kinds of instruments are often negatively correlated. This is due to credit spreads' historically having a negative correlation with rate movements. Furthermore, when credit

Figure 18. **Tracking Errors for Different Replication Strategies for the U.S., the Euro, the Japanese, and the Sterling Treasury Indices,** bp/month, January 1994-December 2000

	U.S.			Euro*	Japanese			Sterling
	1/94-12/00	**1/94-6/98**	**7/98-12/00**	**2/99-12/00**	**1/94-12/00**	**1/94-6/98**	**7/98-12/00**	**2/99-12/00**
Treasury Futures	10.8	8.3	14.4	14.1	35.1	32.0	40.6	49.8
Hybrid Replication**	9.1	6.1	12.9	12.9	29.5	29.0	30.7	42.0
Swaps	34.3	12.2	55.5	16.3	31.2	31.0	31.9	50.8

* The replication simulations for the euro zone start in February 1999 because of data limitations.
** Eurodollar/Euribor and Treasury Futures for the U.S. and the euro zone, swaps and 10-year treasury futures for Japan and the U.K.

spreads are widening, treasury futures tend to trade richer compared with cash due to their higher liquidity.[10] Replication with swaps leads to lower tracking errors than replication with the 10-year treasury futures contract alone. The gain from fitting the curve risk better with swaps outweighs the additional tracking error due to the movement in swap spreads. Overall, replicating the Treasury Index with swaps and treasury futures is likely to have lower tracking error than using only one set of instruments.

The lowest tracking errors overall are achieved with cash bonds. For the full period and for each of the two sub-periods, tracking error is slightly above eight basis points per month. Clearly, for long-term replication of the Japanese market, cash instruments are superior to derivatives. The R^2 values in Figure 17b confirm these results. They also show that during the second sub-period, the 10-year treasury futures contract did not replicate index returns very well. To allow a comparison with other results, we display in Figure 19 tracking errors and R^2 values for the replication of the U.S. Treasury Index. For the replication with the Eurodollar futures and swaps portfolio over the full period, the U.S. tracking error and R^2 values are comparable to the ones encountered for the swaps replication portfolios in the Japanese case. For the replication with bond futures and the hybrid Eurodollar and treasury futures portfolio, the U.S. tracking errors are much lower. Two reasons for this are the availability of more Treasury futures contracts along the U.S. curve and the lower basis volatility in the U.S.

[10] In addition, in times of a credit crisis, the lower settlement risk of futures contracts will contribute to their richness.

Figure 19a. **Tracking Errors for Replication of the U.S. Treasury Index**
January 1994-December 2000, bp/month

	1/94-12/00	**1/94-6/98**	**6/98-12/00**
2-, 5-, 10-year, and Bond Futures Contract	11.6	8.3	16.5
Hybrid Eurodollar/Treasury Futures	9.2	6.1	13.4
Hybrid Eurodollar Futures/Swaps	32.1	12.1	53.2

Figure 19b. **R-Squared Values for Replication of the U.S. Treasury Index**
January 1994-December 2000, bp/month

	1/94-12/00	**1/94-6/98**	**6/98-12/00**
2-, 5-, 10-year, and Bond Futures Contract	99.1	99.6	97.8
Hybrid Eurodollar/Treasury Futures	99.4	99.8	98.6
Hybrid Eurodollar Futures/Swaps	92.9	99.1	77.3

The U.S. experience shows that swaps do not always improve index tracking relative to futures, particularly against an all-government index. Note the high tracking errors and low R^2 that we encounter for the replication of the U.S. Treasury Index when using the hybrid Eurodollar and swaps replication portfolio. This is due to the high spread volatility encountered since July 1998.

The mean outperformance numbers in Figure 17c are also worth considering. The 10-year treasury futures replication portfolio leads to the highest mean outperformance. The outperformance of the swaps portfolios is lower, particularly for the first sub-period. The replication portfolio made up of cash bonds exhibits the lowest mean outperformance. Futures vastly outperformed cash and swaps in the replication of the Japanese Treasury Index.[11] In Japan, long positions in the treasury futures contract have this returns advantage, as many market participants consider it risky to short the futures basis. Shorting bonds against the futures contracts carries the risk of a squeeze, and the high variability of the basis makes basis trades subject to a lot of mark-to-market risk. Therefore, the Japanese treasury futures contract trades very cheap to cash compared with other markets. Although the tracking error to the index exceeds 120 bp per year, the additional return that a treasury futures strategy can produce is a factor to consider when choosing a replication strategy.

The underperformance of the cash replication portfolio is due to this strategy's selection of the most liquid bonds in the market. This is particularly true in the two higher-duration buckets. Increased liquidity of an asset usually is accompanied by lower long-term returns. The bulk of the bonds in the index are off-the-run, lower liquidity bonds. The index should therefore outperform a selection of the most liquid bonds. Our returns do not include any specialness premium that might accrue to such bonds in the repurchase markets.

The most liquid bonds trade rich to comparable off-the-run bonds, and this liquidity premium varies over time. It therefore represents an additional source of risk. In our study, we wished to use securities with liquidity comparable to that of the derivatives we used, but using older, less liquid bonds would probably result in a somewhat lower tracking error. The already-low tracking errors of the cash replication portfolio could probably be improved modestly.

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[11] This is similar to what occurred in the U.S. and Europe. Over the past 8 years, U.S. treasury futures, as well as German treasury futures before and after the introduction of the euro, have provided higher returns than cash, for two reasons. First, they traded cheap compared to many pricing models. Second, yield volatilities underwent a seminal decline, reducing the value of the option that is part of any treasury futures short position.

CONCLUSIONS

The cross-market diversification embedded in the Global Aggregate Index makes it a low-volatility benchmark alternative to many local market indices. In addition, it is not affected as much by changes in individual local markets, such as a reduction in a market's share of government debt, which may lead to substantial changes in the local index's duration or convexity. The Global Aggregate will therefore be more stable in terms of its underlying risk properties.

Given the more than ten bond markets and currencies represented in the Global Aggregate, the use of derivatives can help reduce the dimensionality of the management problem, support the investment of cash, and quickly implement asset allocation shifts.

Replication of the Global Aggregate is found to work very well, usually with tracking errors that are lower than the ones found for local market replication portfolios. The hedged Global Aggregate can therefore represent an attractive benchmark for enhanced index funds. We find that the performance of our strategy at replicating the returns of the Global Aggregate depends strongly on the choice of instruments that are used to replicate each of the local components. A portfolio that combines treasury futures and swaps for the U.S., Euribor and treasury futures for the euro zone, and swaps for both the Japanese and the Sterling Aggregate delivers the lowest tracking error.

Currently, about 96 % of the Global Aggregate's market capitalization is made up by issues denominated in four currencies. We find that it suffices to replicate these four largest markets.

The replication of the Japanese Treasury Index shows relatively high tracking errors for the derivatives-based replication portfolios. The lowest tracking errors are achieved by a hybrid replication portfolio combining swaps and the 10-year treasury futures contract. The Japanese Treasury Index is most effectively replicated with a portfolio of cash bonds.

Appendix A. **Recent Changes to the Global Aggregate Index**

The Lehman Brothers Global Aggregate Index experienced two significant changes as of October 1, 2000:

1) The liquidity constraint was changed to $300 million equivalent outstanding[12] for all currencies.
2) The Asian-Pacific Aggregate Index was incorporated into the Global Aggregate.

Both of these changes contribute to an increased role for the Asian component of the index. For the U.S. Aggregate and Euro Aggregate components of the index, the new liquidity constraint represents an increase in the threshold,[13] causing the total included market value to decline slightly. For the Japanese Treasury Index, where the prior liquidity constraint had been set at 1 trillion yen, the new unified constraint is lower, allowing more bonds to be included. In addition, the coverage

[12] To avoid fluctuations in index composition due to exchange rate volatility, the official thresholds were fixed in local currency based on exchange rates as of July 31, 2000. The resulting local currency numbers were rounded somewhat to achieve reasonable thresholds in local currency. A complete listing of the precise thresholds in each currency is available in the soon-to-be-published Lehman Brothers *Global Aggregate Index Primer*.

[13] The minimum until September 30, 2000, was $ 150 million for the U.S. Aggregate, EUR 150 million or the Euro-Aggregate, JPY 1 trillion for the Japanese Treasury Index, and GBP 100 million for the Sterling Aggregate.

Figure A-1. **Composition of the Global Aggregate Index by Currency,** as of 9/29/00

	Old Constraints			**New Constraints**		
	Market Value ($ mn)	**% of Index**	**Number of Issues**	**Market Value** ($ mn)	**% of Index**	**Number of Issues**
U.S. Aggregate	5,781,440	48.5	6044	5,160,145	41.5	2645
Eurodollar and 144A, ex-U.S.Aggregate				304,958	2.5	550
Euro-Aggregate	3,439,201	28.9	3925	2,999,566	24.1	1527
Sterling Aggregate	569,961	4.8	562	492,432	4.0	236
Danish, Norwegian Krone, Swedish Krona, Greek Drachma	223,836	1.9	76	221,179	1.8	62
Yen Government	1,682,760	14.1	85	2,310,787	18.6	218
Yen Credit, incl. Euroyen				600,546	4.8	814
Other Asian Issues				110,085	0.9	125
Australia, New Zealand	42,320	0.4	19	49,895	0.4	31
Canadian	176,458	1.5	40	176,458	1.4	40
Total	**11,915,976**	**100**		**12,426,051**	**100**	

of the Asia-Pacific region has been expanded in several dimensions. Besides Japanese Treasury bonds, the Asian-Pacific Aggregate contains yen-denominated corporate and sovereign issues, and government and credit securities denominated in other Asian currencies. Figure A-1 shows the composition of the Global Aggregate Index as of September 29, 2000, according to both the new set of index rules and the previous one. The net effect of these changes is that the share of the Asian-Pacific component in the Global Aggregate, previously represented by the Japanese, Australian and New Zealand Treasury Indices, has increased from about 14% to about 24%.

In order for the tracking error results of this study to present valid indications of future tracking errors, we adjusted the market weights that were in effect before October 2000 so that they more closely reflect the index composition in effect during 4Q00. This means a substantial increase in the Asian component in the simulation, and will therefore reflect the index composition that can be expected to prevail in the near-to-intermediate future. For this purpose, we increased the share of the Japanese Aggregate in the simulation to a constant 24% for all months before October 2000. Although this index consists only of JGBs for that period, JGBs make up about 75% of the Asian-Pacific component of the Global Aggregate. This modification will therefore make our results much more forward-looking.

Appendix B. **Hedged Currency Returns and Index Replication**

Property 1. The tracking error of a replication strategy is the same for hedged and unhedged returns.

The outperformance of a replicating portfolio, versus the index it was constructed to replicate, in any period *t* is given by

$$(1) \qquad r_{t,u,j}^{R,G} - r_{t,u,j}^{I,G}$$

where r_t denotes period *t* return, *u* stands for unhedged, and *j* for currency *j*. Currency j is the base currency. Superscripts *R*, *I*, and *G* denote replication portfolio, index, and global, respectively. All returns are expressed as fractions on a per-period (i.e., un-annualized) basis.

$$(2) \qquad r_{t,u,j}^{I,G} = \sum_{i=1}^{N} w_{t,i}\left[\left(1 + r_{t,i}^{I}\right)\left(1 + r_{t,i}^{c}\right) - 1\right].$$

$w_{t,i}$ is market *j*'s portfolio share applicable to period t (for the Global Aggregate Index, this is the beginning-of-month market value share of market i). $r_{t,i}^{I}$ is local market *i*'s index return, and $r_{t,i}^{c}$ is the currency return[14] attributable to that market. $r_{t,j}^{c}=0$. Similarly, the return of the replication portfolio is given by

$$(3) \qquad r_{t,u,j}^{R,G} = \sum_{i=1}^{N} w_{t,i}\left[\left(1 + r_{t,i}^{R}\right)\left(1 + r_{t,i}^{c}\right) - 1\right].$$

Returns for derivatives portfolios are always expressed as the marked-to-market change in value plus the cash flow of the derivatives position divided by the amount invested in the market at the beginning of the period.

The total return of a portfolio that is hedged for the risk in currency changes has the following form

$$(4) \qquad r_{t,h,j}^{I,G} = r_{t,u,j}^{I,G} + r_{t,h,j}^{c},$$

with subscript *h* indicating hedged, and superscript *c* denoting currency. The currency hedging portfolio return has the following form:

[14] Note that this expression of total return as the product of local return and currency return does not follow the conventions used in reporting currency returns on Lehman Brothers indices. Index currency returns are reported using an additive convention rather than this multiplicative one. Details may be found in the *Global Aggregate Index Primer*. However, the two conventions are mathematically equivalent.

$$r^{c}_{t,h,j} = \sum_{i=1,i\neq j}^{N} w_{t,i}\left[\left(1 + r^{e,I}_{t-1,t;i}\right)\left(e^{F}_{t-1,t;i} - e_{t,i}\right)/e_{t-1,i}\right]. \tag{5}$$

$r^{e,I}_{t-1,t;i}$ is a term representing the return expected at the end of period *t-1* for the *i*th local market bond index return over period *t*. For the Lehman Brothers International bond indices, the index yield is used for this. $e_{t,i}$ is country *i*'s exchange rate, in terms of the number of home currency units per country *i* currency unit. $e^{F}_{t-1,t;i}$ stands for the forward or futures exchange rate of the same currency, agreed upon at the end of *t-1* for delivery at *t*.

If $e^{F}_{t-1,t;i} = e_{t-1;i}$ and $r^{e,I}_{t-1,t;i} = r^{I}_{t,i}$, then all influence of currency movements is eliminated, and the currency hedge would be perfect. If the currency hedge used for the investment in each local market's replication portfolio is the same as the one employed for each local market itself, the return of the currency hedged replication portfolio consists of two terms:

$$r^{R,G}_{t,h,j} = r^{R,G}_{t,u,j} + r^{c}_{t,h,j}. \tag{6}$$

Therefore, using equation 1,

$$r^{R,G}_{t,h,j} - r^{I,G}_{t,h,j} = r^{R,G}_{t,u,j} - r^{I,G}_{t,u,j}. \tag{7}$$

Property 2. Replication tracking errors are nearly independent of base currency.

From equations 2 and 3 in Property 1, we can write the outperformance in period *t* of a derivatives portfolio constructed to replicate the Global Aggregate index as

$$\begin{aligned} r^{R,G}_{t,u,j} - r^{I,G}_{t,u,j} &= \sum_{i=1}^{N} w_{t,i}\left[\left(r^{R}_{t,i} - r^{I}_{t,i}\right)\left(1 + r^{c}_{t,i}\right)\right] \\ &\approx \sum_{i=1}^{N} w_{t,i}\left(r^{R}_{t,i} - r^{I}_{t,i}\right). \end{aligned} \tag{8}$$

$r^{c}_{t,i}$ is often in the -3 to +3 per cent range, so that the approximation holds. $1 + r^{c}_{t,i}$ will not differ much from 1. Overall therefore, for each country *i*, the effect from currency conversion is usually of second order. Hence, although portfolio statistics are heavily influenced by currency conversion, outperformance is not.

Property 3. Implicit currency hedging (by keeping cash in the base currency) is approximately equivalent to conventional hedging.

In the text we discuss the results of a strategy we call implicit currency hedging. For implicit currency hedging, a foreign index return is replicated by purchasing zero-cost foreign derivatives. The investor then earns approximately the index return minus the cost of financing the equivalent cash position. Since the investor puts up essentially no cash, currency risk only affects gains and losses on the position. The investor is therefore virtually protected against changes in the exchange rate.

In this appendix, we will show the approximate equivalence of conventional currency hedging and implicit hedging.

(9)
$$r^{R,G}_{t,h\_imp,j} = \sum_{i=1}^{N} w_{t,i}\left[r^{R,F}_{t,i}\left(1+r^{c}_{t,i}\right)\right] + r^{s}_{t,j}\ .$$

$r^{s}_{t,j}$ denotes the home currency short-term rate, in which the cash is invested in currency j, the home currency. $r^{R,F}_{t,i}$ is the return on the investor's derivatives portfolio in market i (F=Futures or forward, R=replication). $r^{R,F}_{t,i}$ is defined as the cash flow plus change in value of the derivatives contract, divided by the index share of market i in the Global Aggregate multiplied by the portfolio value. *h_imp* denotes implied hedge. Now,

(10)
$$\begin{aligned}
r^{R,G}_{t,h_imp,j} &= \sum_{i=1}^{N} w_{t,i}\left[r^{R,F}_{t,i}\left(1+r^{c}_{t,i}\right)\right] + r^{s}_{t,j} \\
&= \sum_{i=1}^{N} w_{t,i}\left[\left(1+r^{R}_{t,i}-\left(1+r^{s}_{t,i}\right)\right)\left(1+r^{c}_{t,i}\right)-1+1\right] + r^{s}_{t,j} \\
&= r^{R,G}_{t,u,j} + r^{s}_{t,j} - \sum_{i=1}^{N} w_{t,i}\left[\left(1+r^{s}_{t,i}\right)\left(1+r^{c}_{t,i}\right)-1\right] \\
&\approx r^{R,G}_{t,u,j} + r^{s}_{t,j} - \sum_{i=1}^{N} w_{t,i}\left[\left(r^{s}_{t,i}+r^{c}_{t,i}\right)\right].
\end{aligned}$$

Since interest-rate parity has to hold in the foreign-exchange market,

(11)
$$e^{F}_{t-1,t;i} = e_{t-1,i}\left(1+r^{s}_{t,j}\right)\Big/\left(1+r^{s}_{t,i}\right).$$

From equation (5), we have

$$
\begin{aligned}
r^{c}_{t,h,j} &= \sum_{i=1,i\neq j}^{N} w_{t,i}\left[\left(1+r^{e,I}_{t-1,t;i}\right)\left(e^{F}_{t-1,t;i}-e_{t,i}\right)/e_{t-1,i}\right] \\
&= \sum_{i=1,i\neq j}^{N} w_{t,i}\left[\left(1+r^{e,I}_{t-1,t;i}\right)\left(\left(1+r^{s}_{t,i}\right)/\left(1+r^{s}_{t,i}\right)-e_{t,i}/e_{t-1,i}\right)\right] \\
&\approx \sum_{i=1,i\neq j}^{N} w_{t,i}\left[\left(1+r^{e,I}_{t-1,t;i}\right)\left(\left(1+r^{s}_{t,j}-r^{s}_{t,i}\right)-\left(1+r^{c}_{t,i}\right)\right)\right] \\
&= \sum_{i=1,i\neq j}^{N} w_{t,i}\left[\left(1+r^{e,I}_{t-1,t;i}\right)\left(r^{s}_{t,j}-r^{s}_{t,i}-r^{c}_{t,i}\right)\right] \\
&\approx \sum_{i=1,i\neq j}^{N} w_{t,i}\left(r^{s}_{t,j}-r^{s}_{t,i}-r^{c}_{t,i}\right) \\
&= r^{s}_{t,j}-\sum_{i=1}^{N} w_{t,i}\left(r^{s}_{t,i}-r^{c}_{t,i}\right)
\end{aligned}
\tag{12}
$$

using (11) to establish the first equality. The fifth line is based on $r^{e,I}_{t-1,t;i}$ being usually small (for hedged Lehman Brothers' Indices it is the beginning-of-month yield for the month).

The last line of (12) equals the last two terms in the last line of (10). Therefore, the implicit replication and conventional currency hedging are equivalent to the first order.

Appendix C. **Derivation of the Replication Portfolio for the Japanese Treasury Index with Cash Securities**

The index is divided into four duration-based cells. Each cell is replicated by one cash bond. The duration of each of these cash bonds is bracketed by the duration of the cell it represents. We select four conditions to solve uniquely for the par amount needed for each bond:

(1) $$\sum_{i=1}^{4} N_{t,i} PVBP_{t,i} = PVBP_t^P$$ (PVBP-equivalence for the whole portfolio)

(2) $$\sum_{i=1}^{4} N_{t,i} P_{t,i} = CAP_t^P$$ (capital-equivalence for the whole portfolio)

(3) $$N_{t,3} PVBP_{t,3} = PVBP_{t,3}^P$$ (PVBP-equivalence of third cell)

(4) $$N_{t,4} PVBP_{t,4} = PVBP_{t,4}^P$$ (PVBP-equivalence of fourth cell)

$PVBP_{t,i}$, i = 1, . . ,4, is the present value of a basis point of bond *i* per yen face at the start of period *t*. $N_{t,i}$ is the par amount of bond *i* in the portfolio, $P_{t,i}$ is its price per yen face, and CAP_t^P is the amount invested in the portfolio. The superscript *P* indicates portfolio. Conditions (3) and (4) help to achieve a good fit at the long end. (1)-(4) represent a system of equations that can be uniquely solved for the $N_{t,i}$, i = 1, . . ,4.

The cash portfolios are constructed so that the value of the securities in the replicating portfolio is less than or equal to the amount invested in the index. Any residuals are invested at 1-month yen LIBOR.

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