Rethinking Risk

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Abstract

Volatility is the most widely-used measure of risk but its relevance is questionable in many settings. For long-term investors, short-term volatility is something they just have to live with and disregard as much as possible. Tail risks, however, are critical because, although rare by definition, they have a large impact on terminal wealth. Using a comprehensive sample that spans over 19 countries and 110 years, this article argues that when 1%, 5%, or 10% tail risks materialize, stocks offer long-term investors better downside protection than bonds in the form of a higher terminal wealth. In fact, stocks have both a higher upside potential and a more limited downside potential than bonds, even when tail risks strike. Hence, their higher volatility essentially is higher upside risk; that is, uncertainty about how much better, not how much worse, long-term investors are expected to fare with stocks rather than with bonds.

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1. Introduction

Consider an asset, look back a few years, and calculate the holding-period return it delivered; whatever number you came up with, other investors considering the same asset and period will agree with your figure. Then consider the same asset over the same period, assess the asset's risk instead, and many investors doing the same are likely to disagree with you. The reason is obvious: Different investors assess risk in different ways. Put differently, the return of a given asset over a given period is whatever the asset delivered; its risk instead is whatever investors perceive it to be.

Although the most common ways to assess the risk of an asset are volatility and beta, there is a wide variety of other risk measures that have been proposed and are used in practice. This implies that investors may, and often do, disagree on both the absolute risk of an asset and the relative risk of two assets. As is often said, risk, like beauty, is in the eyes of the beholder.

The different ways in which investors assess risk, and the implications for the evaluation of investment strategies, are at the heart of the issues discussed in this article. More precisely, the discussion is centered around two measures of risk, volatility and lower-tail terminal wealth; the conditions that may make the latter more relevant than the former; and the circumstances under which stocks can be plausibly argued to be less risky than bonds.

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The evidence discussed here, based on a comprehensive sample of 19 countries over 110 years, suggests that investors that focus on uncertainty are likely to view stocks as riskier than bonds, and those that focus on long-term terminal wealth are likely to view stocks as less risky than bonds *even if they are concerned with tail risks*. This is the case because, even when tail risks do materialize, investors are more likely to have a higher terminal wealth (that is, more capital at the end of the holding period) by investing in stocks than by investing in bonds.

The rest of the article is organized as follows. Section 2 discusses in more detail the issue at stake. Section 3 discusses the evidence based on a comprehensive sample covering 19 countries and the 11 full decades between 1900 and 2009. Finally, section 4 provides an assessment. An appendix with technical issues and tables concludes the article.

2. The Issue

This section starts with a brief digression on risk and the different ways to assess it, focusing on the two risk measures at the center of this article, namely, volatility and lower-tail terminal wealth. Then it discusses in more detail two types of volatility, which aim to assess uncertainty; and lower-tail terminal wealth, which aims to assess terminal wealth in the presence of tail risks.

2.1. A Brief Digression on Risk

Markowitz (1952, 1959) defined the risk of an asset as the standard deviation of the asset's returns. Sharpe (1964) and others subsequently argued that the risk of the same asset within a properly-diversified portfolio is given by beta. As of today, the standard deviation and beta remain the two most widely used risk measures. However, many others have been proposed and are used by investors; these include the semideviation, downside beta, shortfall probability, value-at-risk, the gain-loss spread, factors such as size and value, and many others. Estrada (2009) and the references therein provide an overview of this literature.

The fact that there is a wide variety of risk measures is further complicated by the fact that risk depends not only on the way investors perceive it but also on the holding period. Estrada (2013*a*), using the same comprehensive sample used in this article, suggests that investors that focus on the short term and assess risk with volatility are bound to conclude that stocks are both risky and riskier than bonds; on the other hand, investors that focus on the long term and assess risk with the probability of enhancing purchasing power, as Warren Buffett does, are bound to conclude that stocks are both not very risky and less risky than bonds.

This article offers a somewhat different, though related and complementary perspective. Consider an investor with a long holding period, that fully intends (hence creates the

conditions) to stick to that holding period.[1] For this investor, how relevant are the short-term fluctuations in the value of his portfolio that will inevitably occur along the way? How much should this investor worry about short-term volatility?

Obviously, it is for nobody but the investor himself to say what lets him sleep at night. That being said, Charlie Munger, Warren Buffett's longtime partner at Berkshire, has some advice for investors in the setting described. In fact, Munger (1994) argues that if "you're investing for 40 years in some pension fund, what difference does it make if the path from start to finish is a little more bumpy or a little different than everybody else's so long as it's all going to work out well in the end? So what if there's a little extra volatility."

To be sure, there may be long-term investors that simply cannot help being concerned with, and react to, the short-term fluctuations in the value of their portfolios. And if that is the case, short-term volatility is how they assess risk and little of what is discussed in this article may be relevant to them. That being said, would it not make sense to at least *also* worry about whether a conservative strategy will, by the end of the holding period, enhance purchasing power or underperform an aggressive strategy by a wide margin?

Granted, an investor may be fully aware that a conservative strategy is likely to underperform an aggressive one and still be happy with choosing the former if he is concerned with tails risks, such as a big loss close to the end of the holding period, or a holding period of very low stock returns, however unlikely they may be.[2] And yet, should not this investor also consider whether he will be better off (that is, end with a higher terminal wealth) by pursuing an aggressive strategy *even in the case that tail risks do materialize*?

That last part is critical. It is clear that the longer is the holding period the more likely is a riskier strategy to outperform a less risky one; that is, in fact, what theory suggests and what the evidence shows.[3] And yet some investors may stay away from an aggressive strategy simply out of fear of tail risks without grasping that, *even if these risks do materialize*, their terminal wealth is likely to be higher than with a conservative strategy. Assessing the empirical evidence on whether (and if so, to what extent) this is the case is what this article is largely about.

2.2. Two Measures of Risk

In July 2012 the yield on 10-year Treasury notes hovered around 1.5%. Some investors may have viewed this note as a riskless investment, simply because it had a (near) 0 probability

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[1] Creating the conditions to stick to a holding period amounts to taking actions that make it as certain as possible that the portfolio will not have to be liquidated before the end of the holding period due to unforeseen contingencies. Such actions include isolating the portfolio from having to deal with liquidity needs, emergencies, and the like.

[2] In fact, Barro (2009) and Bollerslev and Todorov (2011) argue that the observed equity risk premium is consistent with the compensation required by investors to bear tail risks.

[3] Estrada (2013*b*) discusses this issue, and the broader issue of time diversification, at length.

of default and 0 uncertainty about the timing and size of the cash flows to be paid out. Yet some others may have viewed it as very risky, simply because it had an extremely high probability of delivering a loss of purchasing power over the 10-year holding period. In fact, between 1900 and 2009 inflation as measured by the CPI run at the annual rate of 3.1%

Also in July 2012 the dividend yield on the S&P-500 stood around 2.3%. Hence, over the subsequent 10 years stocks were quite likely to pay more cash than 10-year Treasury Notes; and, if history is any guide, they were also quite likely to go up in price. Some investors may have viewed investing in the S&P-500 during 10 years as risky simply because there was a high uncertainly about its expected return; some others may have viewed this investment as much less risky simply because it was quite likely to both enhance purchasing power and beat an investment in the Treasury note. In fact, considering all possible overlapping 10-year holding periods between 1900 and 2009, the S&P-500 delivered a negative real return (underperformed bonds) only 13.9% (14.9%) of the time

This comparison between stocks and bonds, together with the discussion in the previous section, suggest two very different ways of assessing risk, namely, focusing on uncertainty and focusing on terminal wealth, particularly in the presence of tail risks. Again, there is no right or wrong, only whatever lets an investor sleep at night. That being said, the evidence discussed in this article mildly questions the excessive focus on volatility and the little focus on terminal wealth in the presence of tail risks.

In order to assess uncertainty, two measures of volatility are considered here, one focusing on the value of the portfolio *at the end* of the holding period (SD_E), and the other focusing on the value of the portfolio *during* the holding period (SD_D). The former captures the variability of *terminal* wealth across all holding periods of the same length; the latter captures the variability of the *periodic* (in our case, annual) value of the portfolio during a given holding period, subsequently averaged across all holding periods of the same length. In other words, the former reflects the uncertainty an investor bears about how much capital he will have at the end of the holding period; the latter reflects the uncertainty the investor bears about the value of his portfolio during the holding period. See section A1 of the appendix for the technical details.

In order to assess terminal wealth in the presence of tail risks, the measure proposed here is the *lower-tail terminal wealth* (LTW_x), defined as the average terminal wealth in the lower x% of the distribution of terminal wealth, with x being 1%, 5%, and 10%. (Again, see section A1 of the appendix for the technical details.) The LTW_x of any strategy is meant to be compared to that of another strategy in order to evaluate relative performance in the presence of tail risks. It can also be used to assess whether a strategy enhances or destroys purchasing power (or whether it does so more or less than another strategy) when tail risks strike.

For the sake of perspective, the upper-tail terminal wealth (UTW_x), defined as the average terminal wealth in the upper x% of the distribution of terminal wealth (with x again being 1%, 5%, and 10%) is also calculated and discussed in the following section. Again, see section A1 of the appendix for the technical details.

3. Evidence

The sample considered here is the Dimson-Marsh-Staunton (DMS) dataset, described in detail in Dimson, Marsh, and Staunton (2002). The sample contains annual returns for stocks and government bonds over the 1900-2009 period for 19 countries. Returns are real (adjusted by local inflation) and in local currency for individual markets, and real (adjusted by US inflation) and in dollars for the World market. In all cases returns account for capital gains/losses and cash flows (dividends or coupons). Exhibit A1 in the appendix summarizes some characteristics of all the series of stock and bond returns in the sample.

An initial $100 investment in the stock market and another $100 in the bond market are assumed to remain passively invested through the end of each holding period. The lengths of the holding period considered are 10, 20, and 30 years. Given that for each country and (stock and bond) market there are 110 annual returns, then 101, 91, and 81 overlapping holding periods of 10, 20, and 30 years are considered in the analysis. The discussion largely, but not exclusively, focuses on terminal wealth (the capital accumulated at the end of each holding period) and the distribution of terminal wealth that stems from aggregating all holding periods of the same length.

Exhibit 1 reports the results of the analysis for the US and the World market; it also reports cross-sectional averages for the 19 countries in the sample, which are referred to below as the average country in the sample. Exhibit A2 in the appendix reports similar results on a country-by-country basis.

3.1. The Upside

Although the focus of the analysis is on risk, a brief analysis of variables that characterize the upside is useful for perspective. It should be no surprise, and Exhibit 1 confirms, that portfolios invested in stocks accumulate more wealth over long holding periods than those invested in bonds. In the US, the mean terminal wealth from investing in stocks, relative to investing in bonds, is 59% ($207.2 versus $130.6), 146% ($419.6 versus $170.3), and 299% ($744.3 versus $186.8) higher over 10-year, 20-year, and 30-year holding periods.

These differences are similar, although somewhat lower, in the World market (49%, 116%, and 231%) and in the average country in the sample (51%, 112%, and 222%). In fact, as

Exhibit A2 shows, the mean terminal wealth from investing in stocks is higher than that from investing in bonds in every country in the sample and for all three holding periods considered.

Exhibit 1: The Distribution of Terminal Wealth

This exhibit characterizes the distribution of terminal wealth for an initial $100 investment in stocks and bonds over all possible 10-year, 20-year, and 30-year overlapping holding periods between 1900 and 2009 for the US, the World market, and the average country in the sample. The statistics reported are the mean, median, and standard deviation (SD_E), as well as the lower-tail terminal wealth (LTW_x) and upper-tail terminal wealth (LTW_x) for *x* equal to 1%, 5%, and 10%. SD_D is the volatility of the periodic value of the portfolio during the holding period, averaged across all holding periods of the same length. The data is described in Exhibit A1 in the appendix. All figures in dollars.

	10 Years		20 Years		30 Years	
	Stocks	Bonds	Stocks	Bonds	Stocks	Bonds
USA						
Mean	207.2	130.6	419.6	170.3	744.3	186.8
Median	187.5	111.1	347.1	120.6	628.6	160.8
UTW_1	474.0	289.5	1,082.0	521.6	2,022.9	597.6
UTW_5	430.0	247.3	1,037.2	466.2	1,780.3	506.2
UTW_{10}	400.8	231.0	949.0	428.9	1,620.9	441.7
SD_E	97.7	54.0	250.2	113.5	386.5	112.8
SD_D	45.8	17.7	113.3	33.3	219.2	41.1
LTW_1	66.7	57.7	119.1	53.7	229.4	54.6
LTW_5	70.7	64.3	129.4	62.4	257.6	57.1
LTW_{10}	75.2	69.4	140.7	66.4	308.0	60.4
World						
Mean	194.7	130.5	363.0	168.1	621.9	187.7
Median	168.6	112.8	315.1	121.0	605.6	133.7
UTW_1	530.9	291.0	1,263.4	552.7	1,573.4	599.6
UTW_5	477.6	273.5	974.1	481.4	1,280.9	567.6
UTW_{10}	422.5	254.3	841.7	433.8	1,148.9	526.9
SD_E	101.5	60.7	222.9	120.8	278.8	141.4
SD_D	40.6	19.2	93.6	37.3	174.6	46.6
LTW_1	51.2	38.0	88.0	43.8	187.3	53.0
LTW_5	66.7	45.5	101.3	50.2	216.1	57.7
LTW_{10}	75.1	49.9	112.7	51.9	243.9	59.8
Cross Section						
Mean	197.0	130.3	355.4	167.4	588.8	182.6
Median	168.7	113.2	263.2	115.7	441.3	135.3
UTW_1	728.6	325.3	1,787.6	564.1	2,699.1	629.5
UTW_5	573.7	285.5	1,337.2	490.6	2,112.4	575.6
UTW_{10}	474.7	264.6	1,105.6	454.0	1,736.7	517.0
SD_E	129.2	68.4	318.5	132.9	492.6	148.8
SD_D	47.5	21.4	102.0	40.9	176.7	51.1
LTW_1	41.4	31.3	65.5	32.1	130.8	38.0
LTW_5	49.2	37.5	79.0	38.2	146.1	40.6
LTW_{10}	57.4	42.8	89.3	41.9	164.6	43.2

Note from Exhibits 1 and A2 that, for both stock and bond markets and all three holding periods considered, the mean terminal wealth is higher than the median terminal wealth. This indicates a positive skewness in the distribution of terminal wealth, which in turn indicates two things. First, that these distributions have more upside potential than downside potential; and second, that the probability of obtaining the mean terminal wealth is lower than 50%.

Switching the focus from the mean to the median, then, Exhibit 1 shows that the median terminal wealth from investing in US stocks, relative to investing in US bonds, is 69% ($187.5

versus \$111.1), 188% (\$347.1 versus \$120.6), and 291% (\$628.6 versus \$160.8) higher over 10-year, 20-year, and 30-year holding periods. These differences are 49%, 160%, and 353% in the World market; and 49%, 127%, and 226% in the average country in the sample. In fact, as Exhibit A2 shows, the median terminal wealth from investing in stocks is higher than that from investing in bonds in every country in the sample and for all three holding periods considered.[4]

Unsurprisingly, the rest of the upside potential variables point in the same direction. Regardless of whether the focus is on the upper 1%, 5%, or 10% of the distribution (hence on UTW_1, UTW_5, and UTW_{10}), Exhibit 1 shows that in all cases the terminal wealth from investing in stocks is substantially higher than that from investing in bonds in the US, in the World market, and in the average country in the sample, for all three holding periods considered. In fact, as Exhibit A2 shows, this is the case in every country in the sample and for all three holding periods considered.[5]

3.2. Risk – Focus on Volatility

As already discussed, two measures of uncertainty are considered in the analysis: The volatility of *terminal* wealth across all the holding periods of the same length (SD_E), and the volatility of the periodic value of the portfolio *during* the holding period, averaged across all the holding periods of the same length (SD_D). The evidence in Exhibits 1 and A2 is hardly surprising.

SD_E is higher for stocks than for bonds in the US, in the World market, and in the average country in the sample, for all three holding periods considered. This is in fact the case in every country in the sample and for all three holding periods considered.[6] A higher SD_E indicates more uncertainty about how much capital an investor will have at the end of a holding period and, therefore, the evidence unsurprisingly suggests that stocks are riskier than bonds in this regard. That being said, a more insightful interpretation will be discussed later.

Furthermore, SD_D is higher for stocks than for bonds in the US, in the World market, and in the average country in the sample, for all three holding periods considered. In fact, this is the case in every country in the sample and for all three holding periods considered. A higher SD_D indicates that an investor faces more variability in the value of his portfolio during the holding period; hence, it is again hardly surprising that the evidence suggests that stocks are riskier than bonds in this regard.

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[4] The only exception is Germany, where the median terminal wealth for bonds is 1.3% higher than that for stocks over 10-year holding periods.

[5] The only exception is Norway, where UTW_1, UTW_5, and UTW_{10} are higher for bonds than for stocks over 10-year holding periods.

[6] The exceptions are Denmark, where SD_E is higher for bonds over 10-year and 20-year holding periods; New Zealand, where SD_E is higher for bonds over 20-year holding periods; and Norway, where SD_E is higher for bonds over 10-year holding periods; see Exhibit A2 in the appendix.

In short, Exhibits 1 and A2 clearly suggest that stocks are riskier than bonds when risk is assessed with volatility. And this is the case regardless of whether volatility is measured during or at the end of the holding period. That being said, volatility is just *one* way of thinking about risk; another way, perhaps more plausible under some conditions, is discussed immediately below.

3.3. Risk – Focus on Terminal Wealth and Tail Risks

Most investors are aware of the higher compounding power of stocks relative to bonds, and yet may underweight (or simply stay away from) stocks for fear of tail risks. However, although risk is in the eyes of the beholder, the evidence should at least inform how plausible such fear is. Put differently, the evidence should be useful for an investor to assess how badly he may fare when tail risks strike by investing in stocks and bonds.

Consider an investor that is concerned about 1% tail risks; that is, scenarios in the lower 1% of the distribution of terminal wealth. Exhibit 1 shows that in the US, in the World market, and in the average country in the sample LTW_1 is higher for stocks than for bonds, for all three holding periods considered. In fact, as Exhibit A2 shows, this is the case in every country in the sample and for all three holding periods considered.[7] In other words, *when 1% tail risks strike, investors have more capital at the end of the holding period by investing in stocks than by investing in bonds*.

Furthermore, recall that in all cases the initial investment is $100 and that the returns considered are real (adjusted by inflation). Then, Exhibit 1 also suggest that when 1% tail risks materialize, investing in bonds may lead to a loss of purchasing power *even after 30 years*; such a loss of purchasing power, as Exhibit A2 shows, has occurred in every country in the sample (with the sole exception of Switzerland) and been as high as 45.4% in the US, 47.0% in the World market, and 62.0% in the average country in the sample.[8] Investing in stocks in the presence of 1% tail risks, however, led to a loss of purchasing power over 30 years in less than half (in fact, eight) of the 19 countries in the sample, although as Exhibit 1 shows, such a loss did not occur in the US, in the World market, or in the average country in the sample. Over 10 and 20 years, losses of purchasing power in the presence of 1% tail risks have occurred in many countries *but the losses have always been lower (less pronounced) for stocks than for bonds*.[9]

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[7] The only exceptions to this statement are Spain and Sweden over 10-year and 20-year holding periods, and Switzerland over all three holding periods.

[8] Exhibit 1 shows that over 30-year holding periods LTW_1 is $54.6 in the US, $53.0 in the World market, and $38.0 in the average country in the sample. The losses mentioned in the text follow from subtracting these figures from the initial investment of $100.

[9] The only exceptions to this statement are Spain, Sweden, and Switzerland over 10-year and 20-year holding periods; see Exhibit A2 in the appendix.

When tail risks are less extreme, such as in the lower 5% of the distribution, Exhibit 1 shows that in the US, in the World market, and in the average country in the sample LTW_5 is higher for stocks than for bonds, for all three holding periods considered. Therefore, it remains the case that when tail risks (now defined as the lower 5% of the distribution of terminal wealth) strike, investors have more capital at the end of the holding period by investing in stocks than by investing in bonds. In fact, as Exhibit A2 shows, this is the case in every country in the sample and for all three holding periods considered.[10]

Furthermore, when 5% tail risks strike, investing in bonds may lead to a loss of purchasing power even after 30 years, which has occurred in every country (with the sole exception of Switzerland). However, in the US and in the World market, purchasing power more than doubled by investing in stocks (and increased by over 46% in the average country in the sample) even in the presence of 5% tail risks. Over 10 and 20 years, losses of purchasing power in the presence of 5% tail risks have occurred in many countries *but the losses have always been lower (less pronounced) for stocks than for bonds*.[11]

As Exhibits 1 and A2 show, the results just discussed for 5% tail risks are similar to those in the presence of 10% tail risks; that is, when considering the lower 10% of the distribution of terminal wealth. Thus, the overall evidence clearly suggests that, *even when 1%, 5%, or 10% tail risks strike*, stocks, relative to bonds, 1) enhance more, or destroy less, purchasing power, and 2) deliver a higher terminal wealth. Should not investors consider these facts when assessing the relative risk of stocks and bonds?

3.4. Risk – Is Volatility Necessarily Bad?

A brief recap is in order. Section 3.1 highlights that stocks have a higher upside potential (mean, median, and UTW_x) than bonds. Section 3.2 highlights that stocks are more volatile than bonds, regardless of whether volatility is evaluated during (SD_D) or at the end (SD_E) of the holding period. And section 3.3 highlights that stocks deliver a higher terminal wealth than bonds (thus enhancing more, or destroying less, purchasing power) when tail risks strike (LTW_x). Under these circumstances, is volatility necessarily bad?

Notice that, relative to bonds, stocks have both a higher upside potential *and a more limited downside potential*. This indicates that the higher volatility of terminal wealth of stocks is mostly upside risk; that is, uncertainty about *how much better, not how much worse*, stocks are expected to perform relative to bonds. This consideration is typically ignored when comparing the volatility of stocks and bonds but it is critical for investors to notice. There is in fact little

[10] The only exceptions to this statement are Spain and Sweden over 10-year holding periods, and Switzerland over 10-year and 20-year holding periods.

[11] The only exceptions to this statement are Spain and Sweden over 10-year holding periods and Switzerland over 10-year and 20-year holding periods; see Exhibit A2 in the appendix.

doubt that stocks keep investors more uncertain than bonds about the return they will get and the capital they will have at the end of their holding period. But this uncertainty is largely one sided; it is uncertainty about how much more, not how much less, capital an investor expects to have at the end of the holding period by investing in stocks rather than in bonds.

It is important to keep in mind the type of investor for whom the arguments discussed are most relevant, namely, those with a long holding period, that create the conditions to stick to that holding period. If these investors were able to choose whether or not to react to short-term volatility, why would they choose to do so? Why not ignoring short-term volatility and focus on terminal wealth instead?

To be sure, in the same way that there may be smokers that would like to quit but cannot, there may be investors that would like to ignore short-term volatility but simply cannot. Everything else equal, all investors prefer a higher expected terminal wealth but, more often than not, everything is not equal. The more efficient markets are, the more it is the case that higher expected returns are a compensation for bearing more risk; and the cost of bearing risk may be dramatically different across investors. As Samuelson (1971) points out, maximizing expected terminal wealth is not the same as maximizing expected utility. In other words, a very aggressive investor may largely ignore short-term risk and attempt to maximize his expected terminal wealth; a very risk averse investor may be willing to forgo a substantial upside potential in exchange for a small reduction in risk instead.[12] *De gustibus non est disputandum*.

A mutual fund manager is an interesting case in point. He may want to invest for the long term, he may want to ignore short-term volatility, but if the fund's investors do not, then he cannot either. In fact, if the fund's investors are expected to exit the fund whenever its returns become very volatile and short-term losses accumulate, the fund manager will be forced, whether he likes it or not, to factor short-term volatility into his investment decisions. For this reason, Kritzman and Rich (2002) advocate to focus on losses *throughout*, not just at the end, of the holding period.

All that being said, it does remain the case that if an investor 1) has a long holding period; 2) creates the conditions to stick to that long holding period; and 3) is able to focus on the endgame and ignore short-term fluctuations in the value of his portfolio, then the evidence suggests that in the long term this investor should view stocks as being less risky than bonds. This is because, when focusing on terminal wealth, stocks would provide him with a higher upside potential and a more limited downside potential, even when tail risks strike.

[12] Estrada (2010) and De Santiago and Estrada (2013) show that geometric mean maximization, or the maximization of expected terminal wealth, is a very aggressive criterion, with little diversification and high volatility, and therefore not appropriate for short-term or very risk averse investors.

3.5. Risk-Adjusted Returns

The most widely-accepted way of comparing the performance of investment strategies is by taking into account both risk and return in a measure of risk-adjusted return. And because there are many ways to assess risk, and more than one way to put together return with a given definition of risk, there are many definitions of risk-adjusted return, such as the Jensen alpha, the Treynor ratio, the Sharpe ratio, or the Sortino ratio, to name but a few.

Two definitions of risk-adjusted return are considered here, one for each of the two definitions of risk already discussed; the first (RAR-SD_E) is defined as the ratio between mean terminal wealth and SD_E, and the second (RAR-SD_D) as the ratio between mean terminal wealth and SD_D. Thus, RAR-SD_E and RAR-SD_D relate mean terminal wealth to the risk borne by investors at the end of and during the holding period. Exhibit 2 reports both measures of risk-adjusted return for the US, the World market, and the average country in the sample; Exhibit A3 in the appendix does the same on a country-by-country basis.

Exhibit 2: Risk-Adjusted Returns

This exhibit shows two measures of risk-adjusted return, RAR-SD_D = Mean/SD_D and RAR-SD_E = Mean/SD_E, both based on the figures in Exhibit 1. The data is described in Exhibit A1 in the appendix.

	10 Years		20 Years		30 Years	
	Stocks	Bonds	Stocks	Bonds	Stocks	Bonds
USA						
RAR-SD_D	4.52	7.39	3.70	5.11	3.40	4.54
RAR-SD_E	2.12	2.42	1.68	1.50	1.93	1.66
World						
RAR-SD_D	4.79	6.78	3.88	4.50	3.56	4.03
RAR-SD_E	1.92	2.15	1.63	1.39	2.23	1.33
Cross Section						
RAR-SD_D	4.25	6.20	3.52	4.18	3.33	3.67
RAR-SD_E	1.77	1.95	1.31	1.29	1.33	1.32

Consider RAR-SD_D first. Given this definition of risk-adjusted return, in the US, in the World market, and in the average country in the sample, and for all three holding periods, Exhibit 2 shows that stocks underperform bonds. In fact, with a few exceptions, Exhibit A3 shows that the same is the case for every country in the sample and for all three holding periods considered. Therefore, when investors focus on volatility *during* the holding period, the bulk of the evidence shows that stocks underperform bonds in terms of risk-adjusted return.

When considering RAR-SD_E, however, the evidence is less clear. Exhibit 2 shows that in the US, in the World market, and in the average country in the sample, stocks outperform bonds over 20-year and 30-year holding periods, though the opposite is the case over 10-year holding periods. Furthermore, Exhibit A3 shows that in some countries (such as Australia and Canada) stocks outperform bonds for all three holding periods, whereas in some others (such as Finland and Germany) stocks underperform bonds for all three holding periods. Therefore, when

investors focus on volatility *at the end* of the holding period, the evidence on whether stocks outperform bonds is at best mixed.

Importantly, the conclusions drawn from the figures in Exhibits 2 and A3 need to be put into perspective with the discussion in the previous sections, and in particular with the type of investor that has been the focus of the analysis; that is, one that has a long holding period, creates the conditions to stick to that long holding period, and is able to ignore the short-term fluctuations in the value of his portfolio. This type of investor cares little about volatility during the holding period and therefore about the fact that RAR-SD_D is higher for bonds than for stocks. In other words, how relevant is the fact that stocks underperform bonds in terms of RAR-SD_D if by the end of the holding period stocks have a higher upside potential *and* a more limited downside potential, even in the presence of tail risks?

For this type of investor, the volatility of terminal wealth is far more important than volatility during the holding period. This is because he does care about what happens at the end of the holding period and, therefore, uncertainty about his terminal wealth at that point in time is detrimental. However, this uncertainty is far less detrimental if the higher volatility of stocks is, as already discussed, upside risk; that is, uncertainty about how much better, not how much worse, an investor is expected to fare by investing in stocks rather than in bonds, even in the presence of tail risks. Under these conditions, how relevant is the fact that stocks may underperform bonds in terms of RAR-SD_E?

Note that the two measures of risk-adjusted return discussed here essentially are slight variations of the Sharpe ratio, without a risk-free rate in the numerator, and two different definitions of volatility in the denominator. Hence, the previous two paragraphs question not only the relevance of RAR-SD_D and RAR-SD_E but also that of the Sharpe ratio for the type of investor considered here. Put differently, if this investor evaluates two investment strategies, X and Y; X offers a better upside potential and a more limited downside potential, *even in the presence of tail risks*; and Y offers a higher risk-adjusted return; why would he conclude that Y is the better choice if X provides a higher terminal wealth in both good *and bad* scenarios?

3.6. Some Further Considerations

Before making an assessment and concluding with some final thoughts, a few considerations are in order. First, note that lower-tail terminal wealth has been defined as the *average* of terminal wealth levels in the lower *x*% of the distribution of terminal wealth.[13] But it could have also been defined as the *cutoff point* for *x*% of the distribution; that is, as the terminal

[13] More precisely, it is calculated by first ranking from the lowest to the highest all terminal wealth levels for holding periods of the same length; then determining the number of terminal wealth levels that make up *x*% of the observations; and finally calculating the average of those observations. See also section A1 of the appendix.

wealth level that leaves x% of the observations to its left. There does not seem to be a very compelling reason to choose one over the other. The main reason for using the average here is because it uses all the terminal wealth levels observed in the lower x% of tail; the cutoff point, in turn, uses only one value and disregards all terminal wealth levels to its left.

Second, note that lower-tail terminal wealth is different from a shortfall probability, as those calculated in Estrada (2013*a*,*b*) for the same sample used here. The latter are historical frequencies that show how often an asset did not beat a benchmark in terms of return, over holding periods of a given length. The main drawback of shortfall probabilities is that they do not account for the magnitude of the shortfall; that is, being short of the benchmark by 1% or by 50% counts in the same way, as two observations that did not beat the benchmark.[14] The lower-tail terminal wealth takes into account both observed levels of terminal wealth and the probability with which they have been observed.

Third, the analysis proposed here may be not be as compact as calculating a mean and a standard deviation and combining them into a Sharpe ratio or similar measure of risk-adjusted return. It requires a more detailed analysis of the distribution of terminal wealth, particularly in the lower tail, but it does provide a more insightful perspective of the tail risks that investors seem to be most concerned about.

Finally, notice that if the distribution of terminal wealth of any given strategy is relatively stable over time, then the theoretical distribution and its relevant parameters could be estimated from the observed data. Such theoretical distribution could then be used to perform an analysis even more exhaustive than that performed here based on empirical distributions.

4. Assessment

Most investors tolerate moderate levels of volatility and losses, which are typically seen as inherent to investing. What investors fear the most, and what has a big influence on their investment decisions, are tail risks, which as the discussion in this article highlights, are not properly reflected in volatility figures. Hence the approach discussed here that emphasizes tail risks.

The framework proposed in this article focuses not only on tail risks but also on terminal wealth. The focus on the latter may be questionable. Some would argue that what happens during the holding period is just as relevant as what happens at the end of it. No argument there. As already stressed, investors differ widely and the approach and discussion in this article is not, and does not aim to be, relevant to all of them. Short-term investors, those that

[14] For this reason, Estrada (2013*b*) combines the shortfall probability with the shortfall magnitude and calculates expected shortfalls as the product of the two.

aim to invest for the long term but are likely to bail out in the short term, and those that tend to react to short-term volatility may find the discussion here of limited value. On the other hand, long-term investors that create the conditions to stick to their long holding period, and that focus on the endgame largely ignoring short-term volatility, may find the approach proposed here insightful and useful.

In the same way that some people like smoking but refrain from it after learning about its potential consequences, investors should learn about the detrimental impact of reacting to short-term volatility and focus on the endgame instead. After all, the evidence against investors' ability to successfully and consistently time the market is massive. In the words of Bogle (2000) when referring to market timing, "[a]fter nearly fifty years in this business, I do not know of anybody who has done it successfully and consistently. I don't even know anybody who *knows* anybody who has done it successfully and consistently." That being the case, why not clearly define the goal of the portfolio and its implied holding period, and focus on the capital accumulated at the end of that holding period instead?

The approach proposed here does just that. By focusing on terminal wealth and accounting for the presence of tail risks, it helps investors put together upside potential variables, such as the mean and median terminal wealth, with the terminal wealth expected in the presence of tail risks. This approach distinguishes strategies that are simply very volatile from those that are truly risky in terms of exposing investors to big losses when tail risks strike.

In fact, as the evidence from the comprehensive sample discussed in this article suggests, it is clear that stocks are more volatile than bonds, but it is far from clear that they are riskier than bonds for the type of investor considered here. This is because, even when tail risks strike, stocks enable investors to accumulate more wealth by the end of the holding period than bonds. Hence, in what sense are stocks riskier than bonds for a long-term investor that focuses on the endgame? If stocks offer both a higher upside potential and a more limited downside potential even in the presence of tail risks, why are they typically thought of as riskier than bonds? Perhaps the pervasive use of volatility and the excessive focus on the short term are part of the answer.

Although the focus here was on a comparison between stocks and bonds, the same approach can obviously be used to compare any other investment strategies. The questions to be asked and the variables to be used are the same as those considered in this article. When comparing the long-term performance of any two strategies, the type of investor considered here would focus on long-term terminal wealth; assess the upside with the mean and the median, and perhaps with upper-tail terminal wealth; and assess the downside in the presence of tail risks with lower-tail terminal wealth. This type of analysis may be somewhat less

straightforward than comparing strategies with the Sharpe ratio, but it yields a more insightful perspective about the relative performance of the strategies considered.

In short, then, this article advocates that investors should 1) clearly determine the goal of the portfolio and the implied holding period; 2) create the conditions to stick to that holding period; 3) largely ignore the short-term volatility that will inevitably affect the portfolio during the holding period; 4) and focus on the endgame carefully considering the impact of tail risks on their terminal wealth. And, as Charlie Munger would say, "so what if there's a little extra volatility."

Appendix

A1. Formal Definitions of Risk

An initial $100 investment in the stock market and another $100 in the bond market are assumed to remain passively invested through the end of each holding period. The lengths of the holding period considered are 10, 20, and 30 years. Given that for each country and (stock or bond) market there are 110 annual returns, then 101, 91, and 81 overlapping holding periods of 10 years (1900-1909, 1901-1910, …), 20 years (1900-2019, 1901-2020, …), and 30 years (1900-2029, 1901-2030, …) are considered in the analysis.

Formally, the volatility of terminal wealth across all holding periods of the same length (SD_E) is given by

$$SD_E = \left\{(1/S) \cdot \sum_{s=1}^{S} (W_{Ts} - EW_T)^2\right\}^{1/2}, \quad (1)$$

where S is the number of holding periods; T is the length of the holding period; W_T is the capital accumulated at the end of each holding period of length T; EW_T is the mean terminal wealth across all holding periods of length T; and s indexes holding periods. When T=10, S=101; when T=20, S=91; and when T=30, S=81.

The volatility of the periodic value of the portfolio during a given holding period, averaged across all holding periods of the same length (SD_D), is given by

$$SD_D = (1/S) \cdot \sum_{s=1}^{S} \sigma_s, \quad (2)$$

where σ_s is in turn given by

$$\sigma_s = \{(1/T) \cdot \sum_{t=1}^{T} (W_t - EW_s)\}^{1/2}, \quad (3)$$

where W is the periodic (in our case, annual) value of the portfolio during each holding period; EW_s is the mean wealth over each holding period; and t indexes years.

Lower-tail terminal wealth (LTW_x) is defined as the average terminal wealth in the lower x% of the distribution of terminal wealth; that is,

$$LTW_x = (1/N_L) \cdot \sum_{n=1}^{N_L} W_{Tn}, \quad (4)$$

where N_L is the number of observations in the lower x% of the distribution; n indexes observations (terminal wealth levels); and x takes a value of 1, 5, and 10.

For the sake of perspective, the upper-tail terminal wealth (UTW_x), defined as the average terminal wealth in the upper x% of the distribution of terminal wealth is also calculated and is given by

$$UTW_x = (1/N_U) \cdot \sum_{n=1}^{N_U} W_{Tn}, \quad (5)$$

where N_U is the number of observations in the upper x% of the distribution.

A2. Exhibits

Exhibit A1: The Data – Summary Statistics

This exhibit shows, for the series of annual returns, the arithmetic (AM) and geometric (GM) mean return, standard deviation (SD), semideviation for a 0% benchmark (SSD), and lowest (Min) and highest (Max) return for all the stock and government bond markets in the Dimson-Marsh-Staunton (DMS) dataset over the 1900-2009 period. Individual country returns are real (adjusted by local inflation) and in local currency; returns for the World market are real (adjusted by US inflation) and in dollars. In all cases returns account for both capital gains/losses and cash flows (dividends or coupons). All figures in %.

	AM	GM	SD	SSD	Min	Max
Stocks						
Australia	9.1	7.5	18.2	9.3	−42.5	51.5
Belgium	5.2	2.5	23.6	12.6	−57.1	109.5
Canada	7.2	5.8	17.2	8.5	−33.8	55.2
Denmark	6.7	4.9	20.7	8.9	−49.2	107.8
Finland	9.1	5.1	30.3	14.1	−60.8	161.7
France	5.7	3.1	23.5	12.6	−42.7	66.1
Germany	8.1	3.0	32.2	15.1	−90.8	154.6
Ireland	6.5	3.8	23.1	12.2	−65.4	68.4
Italy	6.2	2.1	29.0	15.8	−72.9	120.7
Japan	8.6	3.8	29.8	15.5	−85.5	121.1
Netherlands	7.1	4.9	21.8	10.4	−50.4	101.6
New Zealand	7.6	5.9	19.7	9.2	−54.7	105.3
Norway	7.2	4.1	27.4	11.9	−53.6	166.9
South Africa	9.5	7.2	22.5	9.2	−52.2	102.9
Spain	6.0	3.8	22.1	11.1	−43.3	99.4
Sweden	8.6	6.2	22.8	10.9	−43.6	89.8
Switzerland	6.1	4.3	19.8	10.3	−37.8	59.4
UK	7.2	5.3	20.0	9.9	−57.1	96.7
USA	8.2	6.2	20.3	10.6	−38.0	56.5
World	6.9	5.4	17.7	9.4	−40.4	70.1
Bonds						
Australia	2.3	1.4	13.2	7.7	−26.6	62.2
Belgium	0.6	−0.1	12.0	8.3	−30.6	40.5
Canada	2.5	2.0	10.4	5.5	−25.9	41.7
Denmark	3.6	3.0	11.6	5.1	−18.2	50.1
Finland	1.0	−0.3	13.7	11.1	−69.5	30.2
France	0.7	−0.2	13.0	9.7	−43.5	35.9
Germany	0.7	−2.0	15.6	12.7	−95.0	62.5
Ireland	2.1	1.1	14.6	7.9	−34.1	61.2
Italy	−0.4	−1.6	14.1	11.9	−64.3	28.7
Japan	1.5	−1.2	20.1	15.0	−77.5	69.8
Netherlands	1.8	1.4	9.4	5.2	−18.1	32.8
New Zealand	2.4	2.0	9.0	4.9	−23.7	34.1
Norway	2.4	1.7	12.2	7.0	−48.0	62.1
South Africa	2.2	1.7	10.4	5.9	−32.6	37.1
Spain	2.0	1.4	11.7	7.0	−30.2	53.2
Sweden	3.2	2.5	12.4	6.1	−36.7	68.2
Switzerland	2.5	2.1	9.3	4.3	−21.4	56.1
UK	2.2	1.3	13.6	7.2	−30.7	59.0
USA	2.4	1.9	10.1	5.3	−19.4	35.1
World	2.2	1.7	10.3	5.6	−27.1	31.7

Exhibit A2: The Distributions of Terminal Wealth

This exhibit characterizes the distribution of terminal wealth for an initial $100 investment in stocks and bonds over all possible 10-year, 20-year, and 30-year overlapping holding periods between 1900 and 2009 for all 19 countries in the sample. The statistics reported are the mean, median, and standard deviation (SD_E), as well as the lower-tail terminal wealth (LTW_x) and upper-tail terminal wealth (LTW_x) for *x* equal to 1%, 5%, and 10%. SD_D is the volatility of the periodic value of the portfolio during the holding period, averaged across all holding periods of the same length. The data is described in Exhibit A1. All figures in dollars.

	10 Years		20 Years		30 Years	
	Stocks	Bonds	Stocks	Bonds	Stocks	Bonds
Australia						
Mean	223.4	137.0	471.3	189.4	966.8	191.3
Median	224.7	111.9	431.9	95.4	779.6	140.9
UTW_1	486.8	374.5	1,352.6	755.2	2,239.7	614.9
UTW_5	412.0	327.2	1,196.8	553.2	2,179.5	511.6
UTW_{10}	379.3	298.0	1,029.7	505.3	2,113.1	478.4
SD_E	85.9	80.7	253.2	161.3	595.9	143.6
SD_D	48.2	24.8	125.3	50.4	275.7	61.5
LTW_1	56.0	41.5	138.0	43.8	233.7	29.0
LTW_5	66.2	42.9	150.4	44.6	256.6	30.5
LTW_{10}	85.2	46.2	163.7	47.8	296.9	32.9
Belgium						
Mean	164.3	116.5	260.9	137.8	340.0	153.8
Median	128.3	101.3	142.8	95.2	177.8	76.5
UTW_1	445.6	263.2	1,298.4	458.6	1,926.4	618.3
UTW_5	416.6	241.9	1,151.9	445.0	1,626.6	595.3
UTW_{10}	374.9	230.4	954.5	425.9	1,303.8	553.3
SD_E	103.6	63.2	275.7	124.5	386.7	167.2
SD_D	41.5	20.4	75.5	36.1	103.2	44.8
LTW_1	30.3	17.6	21.0	13.4	39.4	18.8
LTW_5	37.7	22.2	38.0	14.9	57.1	22.9
LTW_{10}	41.8	26.4	44.8	17.2	64.9	24.9
Canada						
Mean	191.7	136.9	360.5	192.0	666.8	213.4
Median	172.2	109.6	356.4	116.7	565.9	176.7
UTW_1	469.3	285.1	883.2	627.5	1,574.4	745.1
UTW_5	390.6	274.4	782.9	535.0	1,455.2	661.1
UTW_{10}	354.4	257.0	724.0	507.0	1,340.0	569.5
SD_E	75.8	65.1	172.4	146.0	322.5	150.1
SD_D	38.8	20.5	89.1	41.4	181.5	49.4
LTW_1	77.8	39.6	120.1	37.9	240.1	65.3
LTW_5	85.3	48.4	137.6	51.3	271.4	67.8
LTW_{10}	90.8	55.6	147.3	61.2	288.8	71.5
Denmark						
Mean	175.2	150.4	293.9	234.3	476.1	322.6
Median	160.1	116.2	236.5	141.5	351.1	202.1
UTW_1	485.9	362.9	1,043.5	764.3	1,765.1	1,140.2
UTW_5	379.7	329.0	875.1	736.2	1,521.5	1,100.7
UTW_{10}	333.5	304.9	755.7	678.7	1,321.9	1,033.8
SD_E	71.5	74.4	183.7	187.7	326.1	296.3
SD_D	32.8	23.1	68.3	51.3	120.1	78.8
LTW_1	81.5	43.9	111.3	63.2	200.7	81.5
LTW_5	86.8	57.4	129.2	73.3	215.9	82.5
LTW_{10}	92.3	65.8	135.8	76.2	230.3	85.2

Continues

Exhibit A2: The Distributions of Terminal Wealth (Cont.)

	10 Years		20 Years		30 Years	
	Stocks	Bonds	Stocks	Bonds	Stocks	Bonds
Finland						
Mean	231.9	122.9	448.9	131.9	720.9	117.2
Median	198.1	122.1	213.3	98.6	375.7	60.8
UTW_1	1,173.6	272.9	3,817.8	424.3	5,050.6	432.9
UTW_5	875.4	262.7	2,507.1	386.0	4,019.0	426.0
UTW_{10}	688.1	252.1	1,855.2	367.7	3,082.9	376.9
SD_E	195.9	71.3	621.2	115.3	922.7	108.4
SD_D	75.0	22.6	153.0	41.2	242.9	45.4
LTW_1	20.7	9.7	49.4	12.3	95.6	17.7
LTW_5	26.6	12.9	54.3	16.1	106.7	23.1
LTW_{10}	31.9	15.1	62.2	17.5	137.3	26.2
France						
Mean	174.4	126.3	269.1	170.0	344.3	226.8
Median	136.6	118.7	152.5	113.7	188.0	45.2
UTW_1	575.6	291.6	1,275.1	670.5	2,244.4	1,055.9
UTW_5	482.7	263.3	1,135.8	591.3	1,717.0	1,014.0
UTW_{10}	445.9	254.1	966.7	563.0	1,396.9	939.9
SD_E	121.3	75.1	277.1	179.8	410.2	305.4
SD_D	46.4	23.0	83.8	47.9	112.3	72.9
LTW_1	19.1	8.1	30.7	7.8	62.4	9.8
LTW_5	29.4	9.2	40.7	8.6	69.2	10.2
LTW_{10}	41.8	13.9	49.2	9.4	77.5	10.8
Germany						
Mean	227.4	121.1	357.6	140.3	529.2	135.1
Median	136.3	138.0	245.0	164.0	389.1	25.3
UTW_1	2,448.1	272.1	5,373.9	406.7	5,341.9	445.3
UTW_5	1,654.0	261.1	2,326.8	378.5	2,689.2	432.1
UTW_{10}	1,027.8	246.6	1,632.2	349.4	1,882.1	410.7
SD_E	372.2	72.2	634.9	121.7	678.8	148.9
SD_D	63.3	23.4	119.3	43.9	172.9	59.0
LTW_1	11.7	5.5	11.6	4.7	13.7	5.5
LTW_5	20.5	6.2	19.3	5.0	26.9	7.1
LTW_{10}	25.5	6.5	24.2	6.1	32.6	9.2
Ireland						
Mean	190.9	132.9	353.8	181.3	565.1	183.9
Median	150.3	98.2	257.1	89.6	391.3	138.0
UTW_1	495.3	344.4	1,434.3	672.0	3,563.3	759.0
UTW_5	456.7	320.5	1,237.6	585.9	2,413.7	721.5
UTW_{10}	424.4	294.7	1,143.3	551.9	1,889.3	605.1
SD_E	115.8	79.7	313.8	163.7	558.5	167.5
SD_D	42.6	24.7	98.8	48.8	160.7	54.7
LTW_1	37.2	30.0	42.2	29.6	135.9	28.8
LTW_5	47.6	40.5	66.2	39.7	143.7	34.2
LTW_{10}	54.2	44.8	79.0	43.9	153.1	38.4
Italy						
Mean	170.0	114.0	209.3	114.5	229.4	87.6
Median	151.5	113.8	127.9	72.2	178.9	68.2
UTW_1	691.3	284.4	762.7	447.9	1,167.2	465.7
UTW_5	561.3	259.4	707.5	439.4	868.5	421.2
UTW_{10}	475.1	246.6	661.7	410.9	704.3	355.6
SD_E	137.3	71.3	186.8	123.1	187.3	106.2
SD_D	47.5	21.9	75.5	39.6	94.1	42.3
LTW_1	22.7	3.6	29.2	3.6	50.6	4.3
LTW_5	27.1	4.1	34.0	4.0	55.9	4.5
LTW_{10}	30.2	6.1	39.5	4.3	67.8	4.8

Continues

Exhibit A2: The Distributions of Terminal Wealth (Cont.)

	10 Years		20 Years		30 Years	
	Stocks	Bonds	Stocks	Bonds	Stocks	Bonds
Japan						
Mean	238.7	136.6	421.5	169.7	749.2	182.0
Median	195.8	132.8	277.6	156.9	391.6	205.9
UTW_1	1,503.6	376.6	2,812.5	422.4	5,534.3	540.4
UTW_5	1,144.6	325.2	2,299.8	414.4	4,449.6	516.6
UTW_{10}	820.4	299.2	1,822.1	394.4	3,454.5	484.7
SD_E	246.7	81.5	544.1	125.5	1,071.7	165.8
SD_D	67.9	27.4	144.8	49.7	270.3	70.6
LTW_1	2.5	0.6	6.7	0.6	6.7	0.9
LTW_5	4.3	1.1	10.4	0.7	11.0	1.1
LTW_{10}	11.3	1.9	14.0	0.9	30.5	1.2
Netherlands						
Mean	202.8	125.6	407.2	161.2	642.9	180.8
Median	156.3	102.8	288.7	125.5	458.6	158.2
UTW_1	567.8	303.7	2,446.4	419.6	2,372.8	486.3
UTW_5	525.3	247.0	1,994.0	388.3	2,198.0	478.5
UTW_{10}	472.9	230.1	1,567.0	363.0	2,012.1	453.0
SD_E	129.3	55.5	446.3	106.4	562.6	124.2
SD_D	48.4	17.8	118.7	34.7	202.7	46.0
LTW_1	57.0	47.3	58.3	46.6	106.3	35.6
LTW_5	59.5	57.6	73.8	49.2	116.9	38.7
LTW_{10}	64.4	62.2	87.1	50.9	133.1	41.1
New Zealand						
Mean	192.4	135.5	331.0	181.4	608.7	188.1
Median	178.6	112.7	319.9	120.2	533.8	167.4
UTW_1	678.6	318.6	695.5	557.3	1,407.1	575.6
UTW_5	411.8	305.1	603.6	512.2	1,309.8	486.2
UTW_{10}	355.4	278.6	566.9	474.4	1,178.7	429.6
SD_E	79.9	66.5	119.4	132.5	273.1	113.4
SD_D	41.5	19.4	84.0	38.7	166.6	43.8
LTW_1	68.8	48.1	142.7	44.4	185.0	46.7
LTW_5	79.0	49.5	151.1	49.1	219.0	49.2
LTW_{10}	91.3	54.6	160.4	52.5	243.0	52.1
Norway						
Mean	167.2	135.8	264.5	176.1	390.5	187.8
Median	165.5	108.7	179.1	101.7	245.5	151.9
UTW_1	432.0	459.7	1,179.1	650.8	3,121.1	730.3
UTW_5	342.3	345.0	940.3	506.7	2,026.3	609.4
UTW_{10}	310.1	314.1	786.8	475.1	1,438.5	515.0
SD_E	79.1	80.1	224.6	139.6	443.7	137.8
SD_D	36.7	21.5	70.0	40.5	101.9	45.1
LTW_1	33.5	24.3	62.3	33.3	69.8	48.2
LTW_5	41.9	35.5	70.9	44.1	91.6	50.7
LTW_{10}	49.8	47.0	76.4	49.3	106.8	53.7
South Africa						
Mean	224.2	127.9	480.5	150.4	988.8	154.7
Median	200.0	107.5	412.4	108.8	928.1	142.0
UTW_1	542.1	301.3	1,522.6	472.1	2,265.3	408.5
UTW_5	522.4	272.7	1,293.5	414.8	2,127.2	331.4
UTW_{10}	477.4	250.9	1,145.9	384.6	1,965.8	305.7
SD_E	114.5	60.0	287.9	102.8	467.6	87.7
SD_D	49.9	18.5	131.2	30.7	284.0	35.6
LTW_1	59.4	50.8	101.9	44.8	320.6	52.7
LTW_5	79.3	59.1	118.0	48.0	337.4	54.7
LTW_{10}	90.0	63.0	142.1	51.7	370.0	56.5

Continues

Exhibit A2: The Distributions of Terminal Wealth (Cont.)

	10 Years		20 Years		30 Years	
	Stocks	Bonds	Stocks	Bonds	Stocks	Bonds
Spain						
Mean	176.5	121.8	286.2	139.0	296.5	133.6
Median	148.7	106.5	171.9	101.6	200.5	112.1
UTW_1	544.0	269.6	1,714.0	468.6	2,381.3	523.6
UTW_5	450.0	241.3	1,341.9	432.2	1,658.4	417.5
UTW_{10}	409.0	225.2	1,170.4	390.9	1,074.2	345.8
SD_E	110.6	54.0	330.3	101.6	347.9	91.4
SD_D	44.7	18.7	85.2	30.0	96.8	30.9
LTW_1	15.9	45.5	33.7	36.9	77.5	33.9
LTW_5	20.5	48.1	40.8	38.7	82.1	38.6
LTW_{10}	32.7	54.5	47.8	43.5	89.3	41.0
Sweden						
Mean	211.4	143.6	441.2	203.7	831.6	232.3
Median	186.4	111.9	303.4	139.2	566.9	200.6
UTW_1	793.8	429.4	3,218.2	892.8	4,005.7	640.4
UTW_5	586.4	335.9	2,159.6	604.5	3,476.1	633.4
UTW_{10}	492.0	306.3	1,613.8	558.8	2,943.8	588.2
SD_E	124.3	77.2	491.2	164.1	834.0	167.3
SD_D	50.9	23.3	122.9	47.5	242.9	60.5
LTW_1	38.1	49.1	49.0	49.6	102.0	49.4
LTW_5	42.3	58.7	66.4	59.3	107.3	52.0
LTW_{10}	54.7	63.8	81.6	62.9	121.7	55.0
Switzerland						
Mean	177.5	128.6	294.5	159.9	442.0	194.0
Median	157.3	118.0	232.3	135.4	405.9	184.2
UTW_1	530.2	331.7	787.7	533.6	1,100.8	520.5
UTW_5	443.1	272.1	741.2	426.3	1,015.7	431.9
UTW_{10}	396.7	236.8	693.7	343.8	927.6	359.5
SD_E	96.9	47.1	193.4	78.2	245.5	73.2
SD_D	39.0	14.8	79.5	26.0	124.2	34.3
LTW_1	30.8	41.6	46.3	53.3	108.3	111.3
LTW_5	42.8	53.7	58.6	73.2	123.1	112.0
LTW_{10}	54.5	69.2	67.7	85.2	130.0	115.1
UK						
Mean	195.6	132.5	381.7	177.5	653.3	197.2
Median	171.1	108.4	305.7	101.9	627.7	154.3
UTW_1	505.0	348.8	1,265.7	552.2	2,198.0	660.5
UTW_5	415.7	293.9	1,074.2	504.8	1,603.5	642.4
UTW_{10}	381.8	270.3	967.6	453.2	1,346.5	576.5
SD_E	96.6	70.0	245.6	138.1	338.2	159.3
SD_D	41.2	23.1	100.7	44.4	186.1	54.6
LTW_1	56.2	30.5	71.5	30.1	206.6	28.8
LTW_5	66.6	41.9	112.5	43.2	227.1	34.6
LTW_{10}	73.6	46.4	134.1	49.1	245.4	40.5

Exhibit A3: Risk-Adjusted Returns

This exhibit shows two measures of risk-adjusted return, RAR-SD_D = Mean/SD_D and RAR-SD_E = Mean/SD_E, both based on the figures in Exhibit A2. The data is described in Exhibit A1.

	10 Years		20 Years		30 Years	
	Stocks	Bonds	Stocks	Bonds	Stocks	Bonds
Australia						
RAR-SD_D	4.64	5.53	3.76	3.76	3.51	3.11
RAR-SD_E	2.60	1.70	1.86	1.17	1.62	1.33
Belgium						
RAR-SD_D	3.95	5.70	3.46	3.81	3.30	3.43
RAR-SD_E	1.59	1.84	0.95	1.11	0.88	0.92
Canada						
RAR-SD_D	4.94	6.69	4.05	4.64	3.67	4.32
RAR-SD_E	2.53	2.10	2.09	1.31	2.07	1.42
Denmark						
RAR-SD_D	5.34	6.53	4.31	4.57	3.96	4.10
RAR-SD_E	2.45	2.02	1.60	1.25	1.46	1.09
Finland						
RAR-SD_D	3.09	5.43	2.93	3.20	2.97	2.58
RAR-SD_E	1.18	1.72	0.72	1.14	0.78	1.08
France						
RAR-SD_D	3.76	5.48	3.21	3.55	3.07	3.11
RAR-SD_E	1.44	1.68	0.97	0.95	0.84	0.74
Germany						
RAR-SD_D	3.59	5.17	3.00	3.20	3.06	2.29
RAR-SD_E	0.61	1.68	0.56	1.15	0.78	0.91
Ireland						
RAR-SD_D	4.49	5.37	3.58	3.72	3.52	3.36
RAR-SD_E	1.65	1.67	1.13	1.11	1.01	1.10
Italy						
RAR-SD_D	3.58	5.21	2.77	2.89	2.44	2.07
RAR-SD_E	1.24	1.60	1.12	0.93	1.22	0.82
Japan						
RAR-SD_D	3.52	4.99	2.91	3.42	2.77	2.58
RAR-SD_E	0.97	1.68	0.77	1.35	0.70	1.10
Netherlands						
RAR-SD_D	4.19	7.06	3.43	4.64	3.17	3.93
RAR-SD_E	1.57	2.26	0.91	1.52	1.14	1.46
New Zealand						
RAR-SD_D	4.64	6.99	3.94	4.68	3.65	4.30
RAR-SD_E	2.41	2.04	2.77	1.37	2.23	1.66
Norway						
RAR-SD_D	4.55	6.31	3.78	4.35	3.83	4.17
RAR-SD_E	2.11	1.70	1.18	1.26	0.88	1.36
South Africa						
RAR-SD_D	4.49	6.90	3.66	4.89	3.48	4.34
RAR-SD_E	1.96	2.13	1.67	1.46	2.11	1.76
Spain						
RAR-SD_D	3.94	6.50	3.36	4.63	3.06	4.32
RAR-SD_E	1.60	2.26	0.87	1.37	0.85	1.46
Sweden						
RAR-SD_D	4.15	6.17	3.59	4.29	3.42	3.84
RAR-SD_E	1.70	1.86	0.90	1.24	1.00	1.39
Switzerland						
RAR-SD_D	4.56	8.71	3.70	6.16	3.56	5.65
RAR-SD_E	1.83	2.73	1.52	2.05	1.80	2.65
UK						
RAR-SD_D	4.74	5.73	3.79	4.00	3.51	3.61
RAR-SD_E	2.02	1.89	1.55	1.29	1.93	1.24

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