#### Deutsche Bank Research







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# Dodge the Sting in the Tail: Integrating Tail Risk into Portfolio Construction

#### **Building Tail Risk Averse Portfolios**

We describe a practical approach to better measure and account for 'left tail' risk in portfolio construction. We show how 'Tail Risk-Adjusted' (TRA) models can be used in asset allocation, and to build risk premia and single stock equity portfolios that typically have lower drawdowns, and higher aggregate and risk-adjusted returns.

#### **Practical Applicability for Practitioners**

The TRA models can be used in conjunction with portfolio construction techniques such as mean variance, inverse volatility, and risk parity optimization, and can also be used to modify commercial risk models. Moreover, the TRA models can be used in standard (quadratic) portfolio construction frameworks in conjunction with most associated constraints. The approach satisfies the conflicting requirements of reliable estimates of tail characteristics, and a risk estimate that can adapt to changing market conditions.

#### Strong Performance

Through extensive backtests with inverse volatility, minimum volatility, risk parity and mean variance optimization we observe that in most cases for asset allocation, risk premia and single stock equity portfolios we attain improved performance measured in terms of both risk and returns. For example, in 600 backtests building cross asset premia portfolios we see improvement in Sharpe Ratios in 86%-96% of cases, and improved CVaR in 82%-92% of cases.

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## Introduction

Many investors evaluate their performance with risk-adjusted returns. However, what constitutes risk and how to measure it is evolving continuously. In light of the many instances of sharp drawdowns observed across financial markets since 2007, and recently in 2018, investors are increasingly turning to risk measures that specifically account for drawdowns and the left tails of return distributions.

Markowitz' mean-variance framework remains a mantra for quantitative portfolio construction. The process maximizes returns with respect to a given level of risk. A significant limitation with this framework is that the measure of risk is the standard deviation, which entails the implicit assumption that asset returns follow a multivariate normal distribution. However, it is known that the returns of financial assets do not follow a normal distribution. The volatility varies, or 'clusters' across time, resulting in non-stationary, heteroskedastic time-series. The high volatility clusters are associated with the fat tails.

A considerable amount of academic and practitioner literature has been dedicated to tail risk measures, as well as ways to incorporate them into portfolio construction. While there are many approaches, with varying degrees of academic rigor, it is often not straightforward to implement these approaches in practice, particularly when considering a broad range of constraints that can be required by portfolio managers.

The primary challenges associated with this are threefold: having a reliable measure of tail risk, retaining a risk measure that is able to adapt to different market conditions, and a practical approach to building portfolios with consideration to these measures.

In this paper we present a straightforward approach to adjust a standard risk model (i.e. a covariance matrix) to incorporate a measure of tail risk. By directly adjusting the covariance matrix we can retain the use of commercial optimization frameworks such as those offered by Axioma or MSCI (Barra), or quadratic solvers typically found in statistical software packages such as R, Python, or Matlab. Moreover, we can make use of risk-based portfolio construction techniques such as risk parity or maximum diversification with our adjusted risk measure.

The next section of this report reviews some of the more commonly adopted measures of tail risk and methods to build portfolios based on these methods, along with advantages and disadvantages associated with the approaches. The third section describes the approach we have developed to incorporate tail risk into a risk model. The fourth section discusses the results of applying our approach to asset allocation, and to building cross-asset risk premia, and single stock equity portfolios. The final section concludes.

Mean variance optimization does not explicitly account for tail risk

The Tail Risk Adjustment (TRA) can be used in standard (quadratic) portfolio construction frameworks



#### **Existing Approaches to Tail Risk Management**

In this section we briefly review the tail risk measures most widely used in finance. We consider their use as risk and tail risk measurements, the typical estimation process, the ability to reliably estimate parameters in different market conditions, and the practical implications of their use in portfolio construction. For all approaches discussed there are numerous variations described in the literature by which to estimate the risk measures. However, we focus on the main concepts.

### Value at Risk (VaR) and Conditional Value at Risk (CVaR, AKA Expected Shortfall)

Value at Risk (VaR), is perhaps the most commonly used tail risk measure. Here we refer to the nonparametric measure. It is simply the change in wealth associated with a point on the returns probability distribution. Formally it is defined as:

$$VaR_{\alpha}(x) := -\inf\{x \in \mathbb{R}: F_X(x) > \alpha\}$$
 (1)

The VaR threshold is most commonly set at 5%, although 1% and 10% are also used. These are respectively referred to as VaR<sub>95</sub>, VaR<sub>99</sub>, and VaR<sub>90</sub>.

VaR was formally defined by RiskMetrics (then part of JP Morgan) in 1994. It is now widely accepted in institutional finance, and is required in many reporting regulations. However, a significant shortcoming of VaR is that it is not a 'coherent' measure of risk (Artzner et al., 1999). Artzner et al. defined four criteria that a risk measure must satisfy to be considered 'coherent':

VaR is not a 'coherent' risk measure; CVaR is a coherent risk measure.

- Subadditivity
- Monotonicity
- Positive Homogeneity
- Translation Invariance

Of these, perhaps the most important is subadditivity. This condition stipulates that the risk of a portfolio of two assets cannot be greater than the sum of the risks of the two individual assets, which is also known as the principle of diversification. However, it is theoretically possible to have a portfolio of two assets with a VaR that is greater than that of the constituent assets.

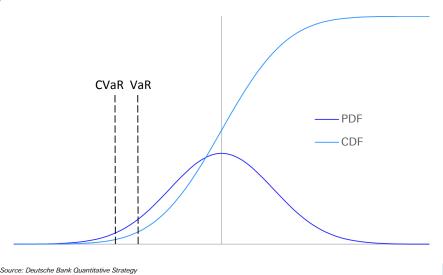
A variation on VaR is conditional value at risk (CVaR), which is also known as expected shortfall. This is the expected change in wealth beyond a given loss (VaR) threshold:

$$\text{CVaR}_{\alpha}(x) := -\mathbb{E}[x|x \le -VaR_{\alpha}(x)]$$
 (2)

In contrast to VaR, CVaR is a coherent risk measure. CVaR is gradually gaining more traction in risk management and reporting. However, building a portfolio based on asset-level, nonparametric VaR and CVaR measures does not take any co-dependency between the asset returns into account.







A summary of strengths and weaknesses of VaR and CVaR is given in Figure 2.

Figure 2: Advantages and Disadvantages of VaR and CVaR as Measures of Tail Risk

Advantage	Disadvantage				
<ul> <li>Simple and Intuitive</li> </ul>	<ul> <li>VaR is not a 'coherent' risk measure</li> </ul>				
<ul> <li>VaR Measures often required</li> </ul>	<ul> <li>VaR and CVaR don't account for co-dependency</li> </ul>				
for regulatory reporting	between assets				
<ul> <li>CVaR is a 'coherent' risk measure</li> </ul>					
Source: Deutsche Bank Quantitative Strategy					

#### Sample Covariance Matrix

A sample covariance matrix is the cornerstone of the most risk management tools. The covariance matrix describes the second moment of a probability distribution. If asset returns follow a (multivariate) normal distribution, then the first and second moments would be termed 'sufficient statistics', and would completely capture all characteristics of the returns. In these conditions, tail risk measures are easily determined parametrically based on the means and covariances of the asset returns.

Many commercial risk models (e.g. Axioma, Barra (MSCI), Sungard APT) have factor covariance matrices at their core. Accordingly, a vast amount of research has been done in the field in regard to enhancing the accuracy of volatility forecasting for various portfolio classes. A common approach is to use exponential, time-series weighting to separately estimate variances and correlations. However, more responsive risk models can be susceptible to outliers and noise.

It is known that asset returns do not follow a normal distribution, with high levels of skewness and kurtosis commonly observed. Nevertheless, the covariance matrix is still a very powerful and useful tool for risk management. Indeed, meanvariance, minimum volatility, and other risk-based optimization approaches are based on the use of a covariance matrix in quadratic programming problems. As a result of this, many commercial financial optimizers have been developed around

Figure 3: Exponential Weights for Correlation and Variance Estimation



A sample covariance matrix can be estimated with an EWMA, and often generates the most accurate volatility forecasts.



quadratic solvers or related variants such as second order cone programming. Many commercial financial optimizers now have the option to incorporate a plethora of constraints with varying degrees of complexity through efficient numerical algorithms (Kolm et al., 2014). Retaining the use of such frameworks for portfolio construction is very attractive.

Figure 4: Advantages and Disadvantages of Sample Covariance Matrices as Measures of Tail Risk

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Advantage	Disadvantage
<ul> <li>Often most accurate volatility forecasts</li> </ul>	<ul> <li>Does not specifically account for 'fat tails'</li> </ul>
<ul> <li>Results are typically intuitive</li> </ul>	<ul> <li>Can be susceptible to outliers</li> </ul>
Source: Deutsche Bank Quantitative Strategy	·
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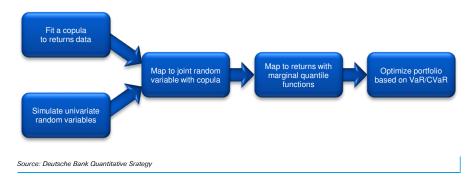
#### Copulas

Copulas have received increased attention in quantitative finance in recent years. They are used to capture the co-dependence between marginal distributions of returns. There are numerous copulas, e.g. Student's t, Clayton, Gumbel, and Frank Copulas. Copulas are parametric, and each captures (tail) co-dependence in accordance with an underlying model; for example, the Student's t-distribution underlies the t-copula. Each copulas requires a different approach to 'calibrate' it in respect to the data. Once a given copula has been calibrated, returns are typically simulated via a Monte Carlo approach, and a measure such as VaR or CVaR is then optimized in determining a portfolio.

A significant limitation with the practical use of copulas can be the computational expense involved with calibration. Fitting a copula to 15 years of daily single-stock equity data for a universe of 100 stocks or more can take days or weeks of computational time.

Copulas are parametric (model-dependent), and can take prohibitive amounts of computational time to calibrate.

Figure 5: Process for Portfolio VaR or CVaR Optimization Based on a t-Copula



#### **Higher Order Moments**

It is possible to directly incorporate higher order moments into the portfolio construction process. One approach to do this is known as 'Polynomial Goal Programming', and is described by Fabozzi et al. (2007), and we have reported on the practical application of this approach (Yuo et al., 2011).

A schematic representation of polynomial goal programming is given in Figure 6. In essence the process is done in two steps; in the first, the portfolios to optimize with respect to the mean, variance, co-skewness and co-kurtosis are estimated. The relative weights of the importance of these moment-specific portfolios are embedded in the polynomial coefficients, p<sub>1</sub>, p<sub>2</sub>, p<sub>3</sub> and p<sub>4</sub>. These are estimated in the second step, and can be determined empirically, or by optimizing some

Build portfolios based directly on higher order moments (e.g. skewness, kurtosis) using polynomial goal programming.



objective function. It is common to scale the portfolio holdings by a uniform value (leverage) such that the portfolio variance in the second term is equal to 1, and can be dropped from the polynomial estimation step.

Figure 6:Polynomial Goal Programming Objective Function



In practice this process is limited by the amount of data needed to reliably estimate the higher order moments. For covariance there are O2 parameters, while for co-skewness and co-kurtosis there are O3 and O4 parameters respectively. Accordingly, for an equity universe of 1,000 assets there are of the order of 1,000,000,000,000 parameters to be estimated in a co-kurtosis tensor. In such scenarios an impossibly large amount of data is required to reliably estimate these parameters, thus robust processes such as Bayesian techniques should be used in practice (Luo et al., 2011).

Figure 7: Advantages and Disadvantages of Polynomial Goal Programming

Figure 7: Advantages and Disadvantages of Polynomial Goal Programming					
Advantage	Disadvantage				
<ul> <li>Directly incorporates higher order moments into</li> </ul>	<ul> <li>Typically requires a huge amount of data</li> </ul>				
portfolio construction	Cannot be implemented in a standard portfolio				
<ul> <li>Use of higher order moments is based on core</li> </ul>	construction framework				
statistical principles	Large data requirement makes it difficult to				
	adjust to different volatility regimes				
·					
Source: Deutsche Bank Quantitative Strategy					

#### **Cornish Fisher Expansion**

Cornish Fischer expansions are a set of equations to parametrically estimate the quantiles of a given distribution based on the 'cumulants' (Cornish & Fisher, 1938). In practice these are based on derivations of the central moments (Maillard, 2012). Accordingly, Cornish Fisher expansions are not considered here due to the inherent limitations in estimating higher order moments discussed above.

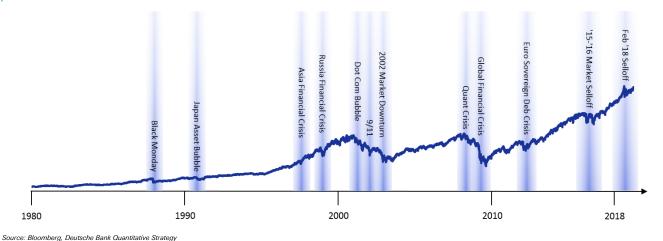
#### Stress Periods and Scenario Analysis

A more empirical approach to tail risk aversion is to calibrate risk management models using data from historical stressed periods (IAA, 2013). Such an approach is attractive by virtue of it's simplicity, and forms the basis of many approaches to historically based VaR, or CVaR estimation, or stress-testing. Data can also be aggregated together to determine a 'stressed' covariance matrix that can be used for portfolio construction with a conservative perspective.

Using stress periods requires a selection of a historical data window or regimes. The data may not be relevant to contemporaneous market conditions.







An issue using historical stressed market data is that while volatilities are generally higher across the market cross-section, the correlations can vary significantly across the different periods. This can be seen by reviewing the circumstances of many of the stress periods seen in Figure 8; for example, the Russian financial crisis stemmed from a default on Russian sovereign debt, the collapse of the 'dot-com' bubble in 2000 was driven by the sell-off in technology stocks, the 2008 financial crisis was primarily driven by banks and financial companies, and the European sovereign debt crisis was instigated by government debt problems in Greece. Accordingly, the origination, contagion and correlation dynamics associated with each of these periods was different. Using each period for stress testing would clearly result in the generation of differing portfolios, and combining data form different stressed periods my generate portfolios with holdings that are difficult to interpret.

Figure 9: Advantages and	Disadvantages of Using Historical Stress Periods
Advantage	Disast sentence

Advantage	Disadvantage
<ul> <li>Easy to select the historical stress periods</li> <li>Can be used to generate risk-averse, counter-cyclical portfolios</li> </ul>	Each historical period has it's own correlations that may not be applicable at another time     Splicing together data from disconnected perdiods can yield unpredictable results     Can 'polarize' the results, resulting in very concentrated and defensive portfolios
Source:Deutsche Bank Quantitative Strategy	



# DB Tail Risk Adjustment

In reviewing several approaches to tail risk management, some common themes have emerged:

- Reliable estimation of statistical parameters associated with tail risk requires a large amount of historical data.
- Relying on a large amount of historical data to calibrate a model for portfolio construction belies the need for a risk model that evolves with changing market conditions (Ward et al., 2016).
- Methods to build portfolios based on measures of tail risk can require bespoke portfolio construction frameworks.

With consideration to this, our approach involves four separate processes to estimate the volatilities and correlations of a set of assets to generate a covariance matrix that can be used in a quadratic¹ optimization framework such as those developed by Axioma, MSCI (Barra), or available in statistical programming packages such as Python, R, or Matlab.

The Tail Risk Adjustment (TRA) generates a covariance matrix. The process combines long- and short-horizon volatility and correlation estimates.

Calculating the tail risk-adjusted covariance matrix is done in 4 separate steps:

- Calculate a contemporaneous estimate of univariate asset volatility
- Calculate a univariate, CVaR-based adjustment coefficient
- Generate a contemporary correlation estimate
- Calculate tail risk weighted correlations

The contemporaneous volatility estimate is combined with the CVaR adjustment coefficient to generate a univariate volatility estimate, while the contemporaneous and tail risk weighted correlations are combined to generate the correlation matrix. The volatility and correlation estimates are then combined to form the final covariance matrix as in Equation 3.

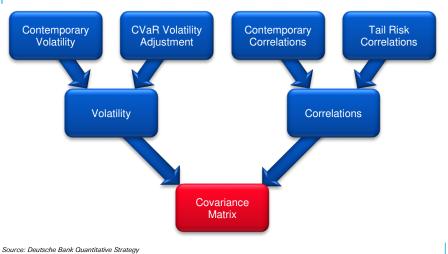
$$COV(x,y) = \sigma_x \sigma_y \rho_{xy} \tag{3}$$

In estimating both the volatilities and correlations, a long-term measure is used to determine the tail characteristics, and a more responsive measure is used to allow the model to respond to different market conditions. Figure 10 schematically illustrates the construction of the complete model.

<sup>1</sup> or second order cone programming (Nesterov & Nemirovsky, 1994; Lobo et al., 1998)



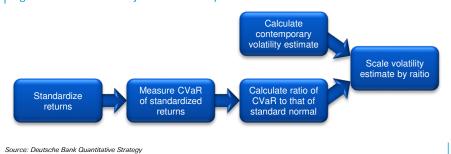
Figure 10: Schematic Representation of DB Tail Risk Covariance Matrix Construction



#### Tail Risk-Adjusted Volatility Estimation

The estimation of the tail risk-adjusted volatility is done separately for each asset in turn. The process involves 5 steps, as represented in Figure 11.

Figure 11: Tail Risk-Adjusted Volatility Estimation



#### Standardizing Returns

In standardizing the returns, the full history of returns of each asset up to the given time point is used. The standardization generates a z-score, but uses a robust measure:

$$Z_{t} = \frac{r_{t} - med(r)}{1.4826 \times MAD(r)} \quad \{r_{t}: t = 1, 2, ..., T\}$$
 (4)

The robust measure uses the median rather than the mean as the average measure, and uses the median absolute deviation from the median (*MAD*) multiplied by a constant 1.4826 as a robust proxy of a standard deviation. For a normally distributed random variable the robust proxy of 1.4826 x MAD(*r*) is equal to the standard deviation (Huber & Ronchetti, 2009). However, it is less susceptible to the presence of outliers, which would otherwise lead to a volatility

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estimate that is too large, in turn leading to Z-scores with a dispersion that is too small.

#### Measure the CVaR of the Standardized Returns

Here we simply calculate the conditional value at risk (expected shortfall) at the 95% level of the standardized returns:

$$\text{CVaR}_{95}(Z) = -\mathbb{E}[Z|Z \le -VaR_{95}(Z)]$$
 (5)

#### Calculate Tail Risk Adjustment Coefficient

In the third step we determine the ratio of the CVaR measure determined above to that of a standard normal distribution:

$$\lambda = log\left(\frac{CVaR_{95}(Z)}{CVaR_{95}(\mathcal{N}(0,1))}\right) - 1 \tag{6}$$

We take the natural log in order to winsorize large ratios that can be observed when dealing with very left-skewed or kurtotic returns such as those crude oil returns.

#### Calculate Contemporary Volatility Estimate

We calculate a responsive estimate of the asset's volatility. The ability of a volatility forecast to adapt to changing market conditions is typically incorporated through a weighting scheme such as exponentially weighted moving average (EWMA) or generalized autoregressive conditional heteroskedasticity (GARCH), or through the amount of data history used (window length), or a combination of both. Here we have already explicitly accounted for the presence of 'fat tails' through the scalar in Equation 6. Accordingly, we seek a robust but responsive volatility measure, and again make use of the MAD. We thus incorporate responsiveness into the estimate with the window length:

$$\sigma_0 = 1.4826 \times MAD(r) \quad \{r_t : t = T - w + 1, T - w + 2, ..., T\}$$
 (7)

where w is the number of returns periods used for the estimate (the length of the window).

#### Scaling Volatility Estimate

The final, tail risk-adjusted volatility estimate is determined simply by multiplying the contemporary volatility estimate by the tail risk adjustment coefficient:

$$\hat{\sigma} = \lambda \sigma_0 \tag{8}$$

It is important to note that if the value of lambda does not equal 1, the tail risk-adjusted volatility will not be the same as the standard deviation.



#### Tail Risk-Adjusted Correlation Estimation

Calculation of the tail risk-adjusted correlation matrix involves the simple average of two correlation matrices. Akin to the volatility measure, the approach is designed to incorporate a contemporaneous measure of the correlations with a longer-term measure that incorporates the (co-)tail dependence.

The contemporaneous correlation matrix is calculated with an exponentially weighted moving average (EWMA) approach as described by Ward et al. (2016). For this measure we use a window with a length equal to four 'half-lives'. For example, when calculating volatility using EWMA with a half-life of 125 days, the length of the data window would be equal to  $4 \times 125 = 500$  days.

The tail risk-adjusted correlation matrix again makes use of all available data in order to best capture the tail characteristics. Here we make use of a simple weighting (or basis) function that is conditioned on the returns:

$$\omega_t \propto e^{-r_t}$$
  $\{r_t: t = 1, 2, ..., T\}$  (9)

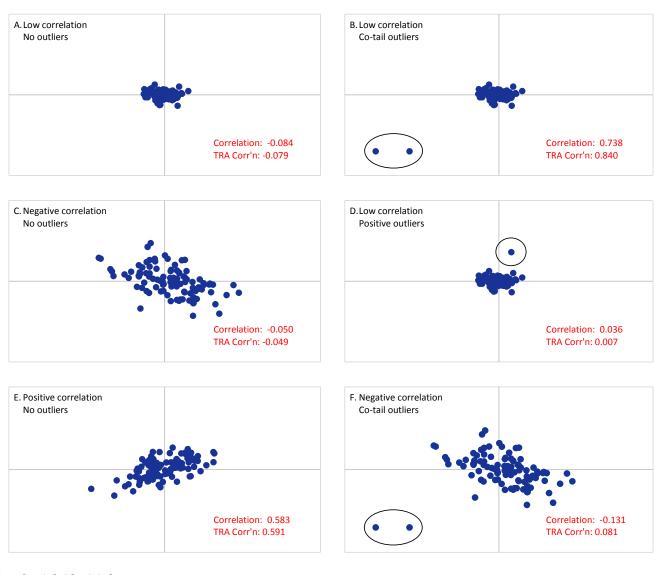
TRA adjusted correlation estimates are designed to be higher when there is co-occurrence of large, negative returns (correlated drawdowns).

The total weight vector for each asset is scaled to sum to the number of observations.

The weighting function in Equation 9 results in greater weights being associated with more negative returns. Some illustrations on the effects of this on simulated pairs of returns are given in Figure 12. In Figure 12A, C and D we see synthetic returns with respective low, negative and positive correlations, but no 'co-tail' outliers, and that the difference between a normal Pearson correlation and the tail risk-adjusted correlation is very small. In Figure 12B and E we see two instances of co-tail outlier events, and that in both cases the tail risk-adjusted correlation estimate is higher than the Pearson correlation. Finally, in Figure 12D we see a positive outlier in one asset that has a relatively small effect on the correlation estimate.



Figure 12: Correlation Estimates With and Without Tail Risk Adjustment



Source: Deutsche Bank Quantitative Strategy

The final correlation matrix is then calculated as a simple average of the contemporaneous and tail risk-adjusted correlation matrices:

$$\hat{\rho}_{xy} = \frac{\rho_{0,xy} + \hat{\rho}_{0,xy}}{2} \tag{10}$$

where  $\rho_0$  is the contemporaneous correlation and  $\hat{\rho}_0$  is the base tail risk-adjusted correlation matrix.

Under the assumption that there are more sample points than assets, and that there is not perfect collinearity between the returns of any assets, both the

15 January 2019

Portfolios Under Construction



contemporaneous and base tail risk-adjusted matrices are assured to be positive definite; it follows that  $\hat{\rho}$  will also be positive definite (Gentle, 2007).

#### Tail Risk-Adjusted Covariance Matrix

Calculating the final tail risk-adjusted covariance for each element is akin to Equation 3, and determined with the standard combination of the individual tail risk-adjusted elements:

$$\widehat{COV}(x,y) = \hat{\sigma}_x \hat{\sigma}_y \hat{\rho}_{xy} \tag{11}$$

#### Specific Risk

The risk of equities is often modeled with factor risk models that separate factor risk from idiosyncratic, stock specific risk. These models are used by commercial risk model vendors such as Axioma, MSCI (Barra), and Sungard APT.

Stock-specific risk is typically a univariate volatility estimate that is calculated using a time-series of idiosyncratic returns. Accordingly, we can estimate a volatility scaling (lambda) based on specific returns to adjust a contemporaneous specific risk estimate.

For factor models the TRA methodology to adjust volatilities can be used to calculate and adjust stock-specific risk



# Applications of Tail Risk Adjustment

We investigate using tail risk-adjusted covariance matrices for portfolio construction in three frameworks: asset allocation, risk premia index portfolios, and single-stock equity portfolios. Each is described here.

#### **Asset Allocation**

For the studies in asset allocation we used a universe of 7 indices, incorporating equities, fixed income, foreign exchange, commodities, and hedge funds, as listed in Figure 13.

Figure 13: Indices for Asset Allocation

Asset Class	Index	
Equities - Developed Markets	MSCI World	
Equities - Emerging Markets	MSCI Emerging Markets	
Fixed Income - Government Bonds	BAML Global Government Bond	
Fixed Income - High Yield	BAML Developed High Yield Bond	
FX	Bloomberg Dollar Index Spot	
Commodities	S&P Goldman Sachs Commodity	
Hedge Funds	HFRX Global Hedge Fund	

Source: Deutsche Bank Quantitative Strategy

We used 3 portfolio construction techniques to build asset allocation portfolios from January 2000 to August 2018. The techniques are described in Figure 14.

Figure 14: Portfolio Construction Techniques for Asset Allocation

Portfolio Description		Methodology	
Inverse Volatility	Weight in each stock inversely proportional to the inverse of the stock volatility	$W \propto diag(C)^{-1/2}$	
Minimum Volatility	Portfolio weights optimized to achieve the minimum possible portfolio volatility	$\min_{W} (U(W)) = W/CW$	
Risk Parity	Portfolio weights such that the risk contribution of each stock is equal	$\min_{W} (U(W)) = W/CW$ s.t. $w_i \sigma_i \rho_{i,p} = w_j \sigma_j \rho_{j,p}  \forall i, j$	

Source: Deutsche Bank Quantitative Strategy

For each portfolio construction approach we ran two versions, using covariance matrices of index returns built with and without the tail risk-adjustment, with parameters listed in Figure 15. The backtests were run from January 2000 to August 2018, rebalancing at the end of the last business day of each month, and using a long only constraint.



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	Base	Tail Risk-Adjusted
Volatilities	EWMA 60 Day Half-Life	120 Day MAD
voiatilities	EVVIVIA 00 Day Hall-Lile	CVaR Adjustment
Correlations	FIAMA 2EO Doy Holf Life	EWMA 250 Day Half Life
	EWMA 250 Day Half-Life	TRA Correlation Matrix

Source: Deutsche Bank Quantitative Strategy

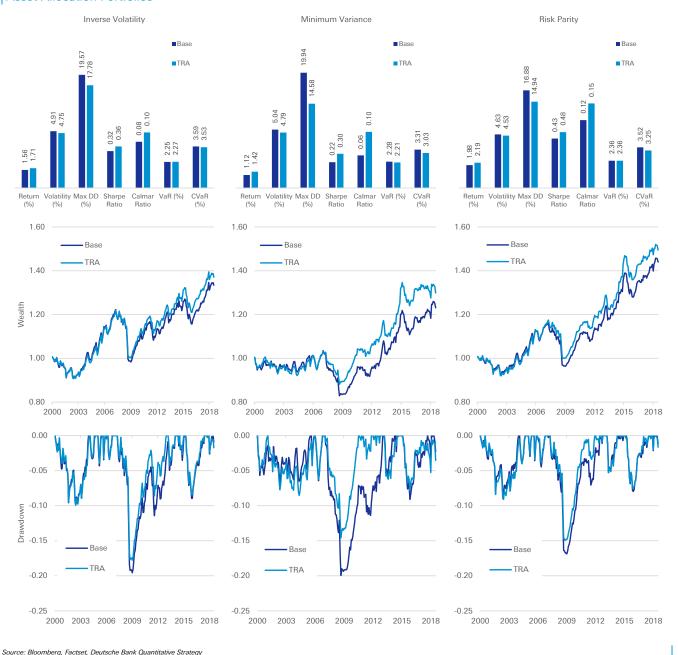
#### **Asset Allocation Results**

Figure 16 displays performance metrics for the three portfolio construction methods with and without use of the tail risk adjustment. We see, on aggregate, an improvement in almost all metrics<sup>2</sup> for all three approaches. Observing the wealth and drawdown curves we see that the largest divergence between the base and tail risk-adjusted models happened around the financial crisis for minimum volatility and risk parity, while for inverse volatility there was a gradual divergence in the performance from 2007. In general we also see a larger difference in the performance metrics when looking at the results from the minimum variance and risk parity backtests. Given that inverse volatility is the only one of the approaches not to account for correlations, this indicates that the tail risk adjustments to the volatilities and correlations are both effective.

<sup>2</sup> VaR under inverse volatility was the only metric that worsened with the tail risk adjustment, by 2bps.



Figure 16:Performance Statistics of Base vs. Tail Risk-Adjusted Inverse Volatility, Minimum Variance and Risk Parity Asset Allocation Portfolios



To delve deeper into the results, we look at the statistics for each calendar year, or part thereof. Figure 18 displays box plots of the range of differences in statistics, as well as 'hit rates' of the statistics for the tail risk-adjusted method when compared with the base method<sup>3</sup>. The differences are calculated between the tail risk-adjusted and base models, for each portfolio construction approach, for each statistic, for each year. The differences are aligned such that a value above zero indicates the tail risk-adjusted method has out-performed the base method, i.e.

Figure 17: Box Plot Anatomy

100%

75%

Mean
25%

3 I.e. the number of calendar years the TRA model out-performed the base model

Source: Deutsche Bank Quantitative Strategy

0%



the returns or Sharpe ratio are higher, or the volatily, drawdowns, VaR or CVaR are lower. The results indicate that each metric is typically improved in around 60% or more of the years from 2000 to 2018. The distributions of differences also indicate that in the years where the tail risk-adjusted method returns an inferior value for a given metric, the difference is typically small.

Figure 18: Annual Statistics of Base vs. Tail Risk-Adjusted Inverse Volatility, Minimum Variance and Risk Parity Asset Allocation Portfolios

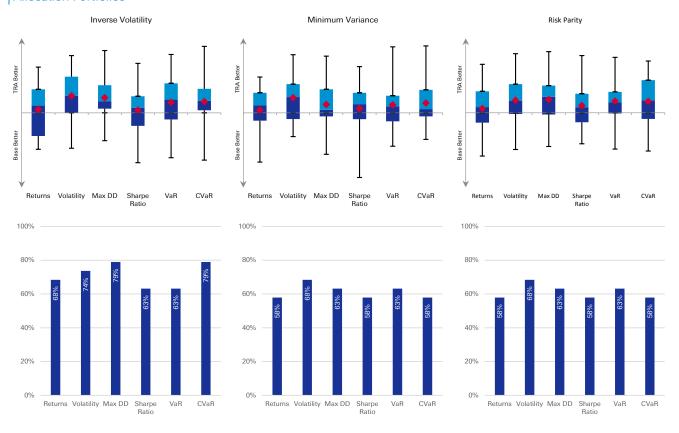


Figure 19 displays the evolution of the volatility adjustment coefficients for the seven indices since January 2000. The coefficients are estimated with an expanding window, and so would typically become more stable over time. However, the values for all indices except for government bonds exhibited a jump around the financial crisis in 2008. We also see that this government bond index has exhibited less left-tail risk since 2000, with the associated value of lambda decreasing from approximately 1.6 to around 1.2.

Source: Bloomberg, Factset, Deutsche Bank Quantitative Strategy





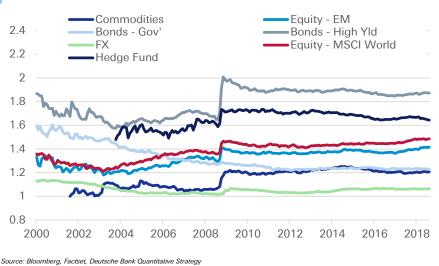
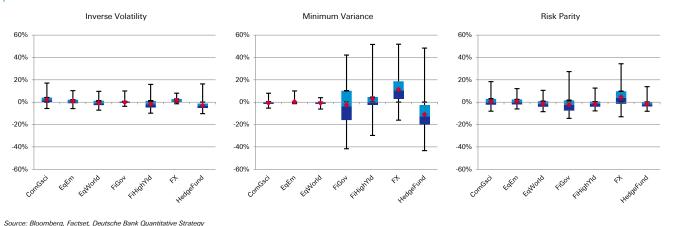


Figure 20 displays the range of the difference in the holdings in each of the indices for the tail risk-adjusted versus the base models for each of the portfolio construction approaches. We see the differences in holdings in the commodity and equity indices is relatively small, while there are larger differences in the holdings in fixed income, FX and hedge fund indices. When using the tail risk-adjusted model there was generally a larger allocation to the FX index and lower allocation to the hedge fund index. This is intuitive given their respectively low and high tail risk adjustment coefficients. The difference in holdings in the fixed income indices was less consistent, differing according to the portfolio construction technique, and across time.

Figure 20: Difference in Portfolio Holdings: Tail Risk-Adjusted (TRA) - Base



Page 18 Deutsche Bank AG/London



#### **Building Risk Premia Portfolios**

In the second part of the study we investigated building portfolios from a universe of risk premia indices. We use a relatively large universe of 31 indices, including equity, commodity, FX, fixed income (rates), credit, and cross-asset strategies. The indices chosen cover a broad spectrum of principle investable risk premia, are all live strategies offered by Deutsche Bank, and are listed in Figure 21.

re 21: Risk Premia Univers		
Class	Index	
	Momentum	
	Low Beta	
	Quality	
	Value	
Equities	Size	
	Merger Arbitrage	
	Dividends	
	Volatility Carry	
	Mean Reversion	
	Curve	
	Carry (Backwardation)	
Commodities	Trend	
Commodities	Momentum	
	Value	
	Volatility	
	Balanced Carry	
	Carry	
FX	Value	
	Momentum	
	Volatility	
	Implied vs Realised Volatility	
	Long Volatility	
Rates	Curve	
nates	Carry	
	Muni/Libor	
	Momentum	
	Carry	
Credit	Curve	
	Momentum	
Cross Asset	Trend Carry	

Source: Deutsche Bank Quantitative Strategy

With the larger universe of indices we adopted a broader study, randomly drawing 100 sets of 8 indices from the universe of 31. As with the study on asset allocation, we ran backtests with inverse volatility, minimum volatility, and risk parity allocations (see Figure 14), each with and without the tail risk adjustment, i.e. 600 backtests in total. Owing to the slightly shorter availably history for some of the indices, the backtests were run from January 2004 to August 2018.

#### Risk Premia Results

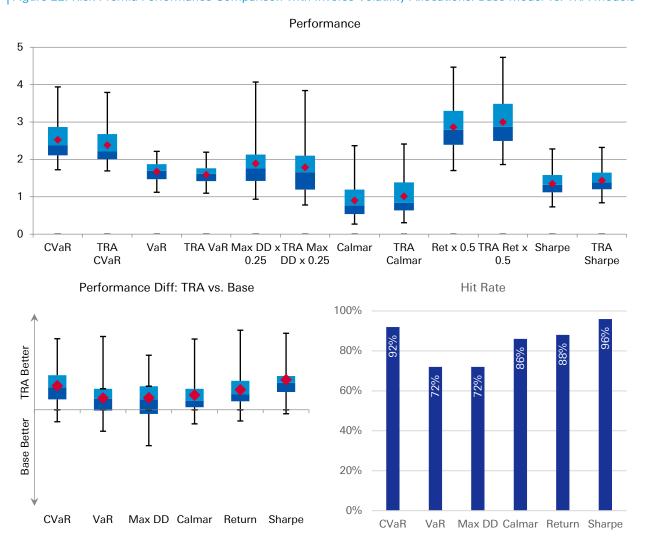
Figure 22 displays the results from the inverse volatility backtests. In the top panel we see that on average the portfolios generated with the tail risk-adjusted model had lower CVaR, VaR, and maximum drawdowns, while also having higher returns, and Sharpe and Calmar ratios. From the plot we see that these observations are valid for all recorded points on the distribution of results (the 0%, 25%, 50%, 75% and 100% points, as well as the mean values - see Figure 17). The box plot in the lower left corner of Figure 22 illustrates the distributions of differences in the same manner as Figure 18. Again, this plot is oriented such that a difference above zero indicates a preferable score in a given metric for the portfolio built with the tail risk-adjusted model. We see that in all cases the majority of the difference statistics are above the 'zero line'. The 'hit rates' displayed in the lower right-hand panel of Figure 22 indicate the percentage of the backtests resulting in improved scores for a given metric using the tail risk-



adjusted model. The hit rates show that in 96% and 92% of cases we observed improved Sharpe Ratios and CVaR respectively. In 72% of cases there was reduced VaR and maximum drawdown.

These results indicate the efficacy of the volatility adjustments for risk premia portfolios given that the allocation using inverse volatility does not consider correlations. This is consistent with the results seen previously when considering asset allocation.

Figure 22: Risk Premia Performance Comparison with Inverse Volatility Allocations: Base Model vs. TRA Models



Source: Bloomberg, Factset, Deutsche Bank Quantitative Strategy

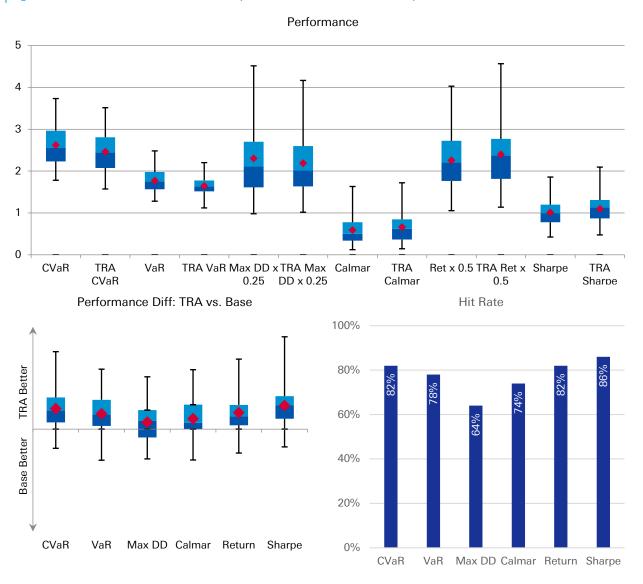
Figure 23 displays the equivalent results for risk premia portfolios built with minimum volatility. We see similar results to those observed with inverse volatility. That is, we observed reduced VaR, CVaR and maximum drawdowns as well as increased returns and Sharpe and Calmar ratios. The hit rates and distribution of differences in performance again indicated an improvement for all metrics in



the majority of cases, although the improvements were not quite as extensive as observed with inverse volatility.

The minimum volatility approach to portfolio construction is particularly challenging, as it typically results in the most concentrated portfolios, often requiring position constraints when used by practitioners. Nevertheless we see improved performance using the tail risk-adjusted model that makes use of the volatility and correlation adjustments.

Figure 23: Risk Premia Performance Comparison with Minimum Volatility Allocations: Base Model vs. TRA Models



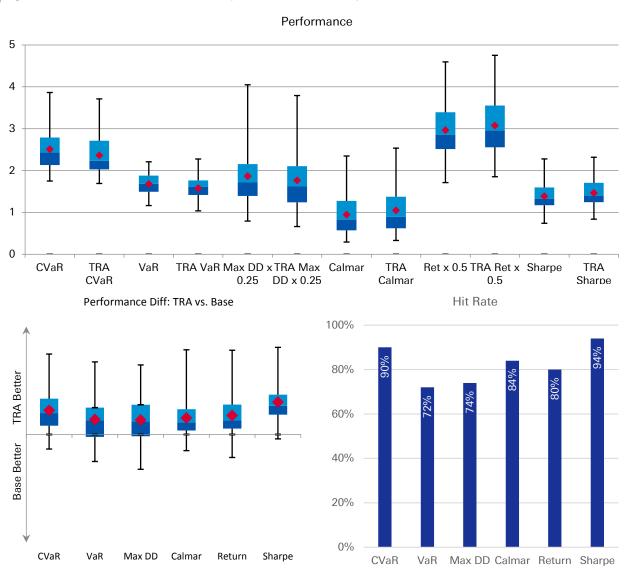
Source: Bloomberg, Factset, Deutsche Bank Quantitative Strategy

Figure 24 displays the results when using risk parity. Again we see an improvement in the results for all metrics. The distribution of differences for all metrics coupled with the hit rates indicate a greater majority of cases are improved than seen when using minimum volatility. We see that in 94% and 90%



of cases there were respective improvements in Sharpe Ratios and CVaR, while the lowest hit rate of improved statistics of 72% was observed for VaR.

Figure 24:Risk Premia Performance Comparison with Risk Parity Allocations: Base Model vs. TRA Models



Source:Bloomberg, Factset, Deutsche Bank Quantitative Strategy

Running 100 pair-wise backtests for each portfolio construction technique enables us to use the paired t-statistic to assess the statistical significance of the improvements in each metric. Figure 25 displays the absolute values of the paired t-statistics, which vary from 2.4 to 11.1, indicating that the improvements in each metric are highly (statistically) significant.



Figure 25: Absolute Paired t-Statistics for Improved Porfolio Performance Metrics (TRA vs. Base) with Risk Premia Portfolios Grouped by Portfolio Construction Technique



The results in this section have indicated that the tail risk adjustment improves performance, as measured by CVaR, VaR, maximum drawdown, returns, Calmar or Sharpe Ratios in 72% - 96% of cases when using inverse volatility or risk parity, and that the improvements are highly statistically significant.



#### Single Stock Equity Portfolios

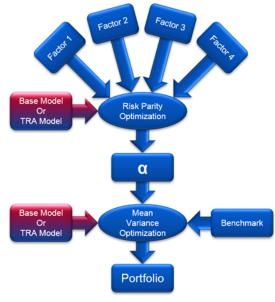
We used tail risk adjustment for building single stock equity portfolios. We used a set of widely adopted equity alpha signals, listed in Figure 26. Most of the signals are defined as 'generic' signals from Deutsche Quantitative Strategy, and by Axioma. The generic and Azioma factors can have the same name, the definitions can be somewhat different (e.g. Book/Price vs. EV/EBITDA for value). Use of multiple definitions helps to overcome potential issues associated with the alignment of 'alpha' factors with those in the risk model (Lee et al., 2007).

Class	Factor	Class	Factor
	Growth		Growth
Axioma	Medium-Term Momentum	Generic	Momentum
	Size		Size
	Value		Value
	Volatility		Low-Volatility
	Liquidity		Quality

Source: Axioma. Deutsche Bank Quantitative Strategy

To build the portfolios we used the technique presented by Osiol et al (2017), where multiple signals are combined to a single alpha with risk parity blending weights. The alpha is then combined with the benchmark using mean-variance optimization.

Figure 27: Combining Multiple Alpha Signals in Equity Portfolio Construction



Source: Deutsche Bank Quantitative Strategy

In order to effectively utilize the tail risk adjustments to the risk model, an alternative approach to standard benchmark-tracking mean variance optimization

Page 24 Deutsche Bank AG/London



was required. To illustrate the need for this we consider a standard mean-variance optimization utility function with respect to a benchmark:

$$\min_{W} (U(W)) = \frac{\lambda}{2} (W - B) / \hat{C}(W - B) - (W - B) / A \qquad \text{s.t. } X/W = G$$
 (12)

where W, B, and A are vectors of portfolio weights, benchmark weights, and alpha scores respectively, C is the asset-by-asset covariance matrix, and X, and G, are matrices that define linear constraints on the portfolio holdings, that may, for example and as considered here, include the requirement that the portfolio be fully invested (i.e. 100% net long).

The analytic solution to Equation 12 is given by:

$$W = \underbrace{B}_{Benchmark} + \underbrace{\frac{1}{\lambda}\hat{C}^{-1}(A - X\gamma)}_{Active} \qquad \gamma = f(X, C, A, G, \lambda)$$
(13)

From Equation 13 we see that the solution to the portfolio weights is separated into two parts; the benchmark is 'isolated' from the other optimization parameters. The other parameters are used in the part of the solution that determines the active portfolio weights. We can see that the optimization is generating a portfolio of active positions that are simply added to the benchmark. If the portfolio is required to be fully invested (100% net long), then the portfolio of active positions is necessarily long-short and dollar-neutral. This is significant with regard to the tail risk adjustment, as the volatility measure is quadratic.

With a quadratic risk measure, the risk of a unit position is the same if the unit is positive or negative. Accordingly, the risk of a single asset in an active portfolio is the same whether the position is long or short. However, tail risk is one-sided, as it only considers the left tail, for which there is a distinction between long and short positions. In the studies earlier in this report on asset allocation and risk premia portfolios the problems were all long-only, and thus the optimization was only concerned with the magnitude of positive positions in the different assets. This imparts directionality on the risk measure that is aligned with (left) tail risk. It follows that here we must orientate the problem such that the optimizer is again considering a (net) long portfolio. We do this with the following objective problem:

$$\min_{W} (U(W)) = \frac{\lambda}{2} W' \hat{C} W - W' A \qquad \text{s.t. } \sigma_P \le \sigma_B, \ X' W = G$$
 (14)

Here we only consider the total portfolio risk, but add the constraint that the portfolio risk,  $\sigma_P$ , is not greater than the risk of the benchmark,  $\sigma_B$ , which should be some diversified, long-only equity portfolio. For reasonable values of  $\lambda$  this would results in a portfolio with volatility equal to that of the benchmark. Given that the volatility of long equity portfolios is dominated by the 'market', it follows that the solution will have a high correlation with the benchmark. The analytic solution is given by the second part of equation 13:



$$W = \frac{1}{\lambda} \hat{C}^{-1} (A - X\gamma) \qquad \text{s.t. } \sigma_P \le \sigma_B$$
 (15)

#### Single Stock Equity Study

In this part of the study we used the utility function as defined in Equation 15, and set the constraints that the portfolio should be fully invested, not have short positions, and have a maximum risk equal to that of the (ex-ante) risk of the MSCI World index. We randomly drew 100 sets of 4 signals from those listed in Figure 26 to form the composite alpha signals, and thus performed backtests with 100 different alphas.

We used the Axioma global medium horizon factor model as the 'base' model for this study. For the tail risk adjustments we calculated the covariance matrix as described above. We also calculated the stock-specific risk with the tail risk volatility adjustment as described earlier. In order to make an 'apples-to-apples' comparison, we also re-calculated the 'base' model, as Axioma incorporate a number of proprietary numerical adjustments to their covariance and specific risk estimates. The parameters used for the base and tail risk-adjusted equity risk factor models are given in Figure 28. The backtests were run from 1st January 2000, and the portfolios were rebalanced at the end of the last trading day of each month.

	Base	Tail Risk-Adjusted
Volatilities & Specific Risk	EWMA 125 Day Half-Life	125 Day MAD
		CVaR Adjustment
Correlations	EWMA 250 Day Half-Life	EWMA 250 Day Half Life
		TRA Correlation Matrix

Figure 29 displays the aggregate performance statistics related to the 200 backtests (100 base and 100 TRA) in the same format as in the previous section. We see that on average we have lower VaR, CVaR, and maximum drawdowns, and increased returns, Calmar, and Sharpe ratios. The metrics are improved in 78%-89% of the backtests. Again we see that these include the returns as well as the risk metrics. This is related to the nature of the increased drawdown protection; that is, on aggregate the benefits of protecting against drawdowns exceed any reduction to returns observed in more bullish regimes.



Figure 29: Optimized Equity Portfolio Performance Comparison: Base Model vs. TRA Models

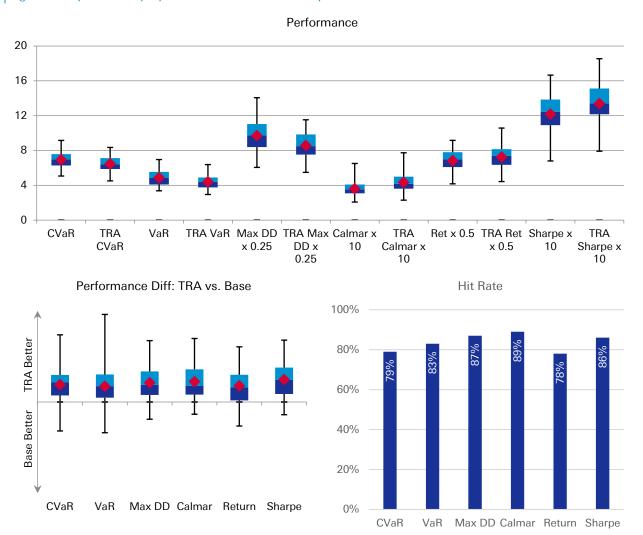


Figure 30 displays the absolute paired t-statistics corresponding to the difference in performance metric for each portfolio when built with the tail risk-adjusted or base models. We see the t-statistics vary from 8.6 to 12.3, which correspond to p-values ranging from 1.4x10<sup>-13</sup> to 1x10<sup>-21</sup>, indicating very high statistical significance (less than 1 in 1,000,000,000,000 chance of a type 1 error in the least significant case).



Figure 30: Absolute Paired t-Statistics for Improved Porfolio Performance Metrics (TRA vs. Base) with Equity Portfolios Grouped by Portfolio Construction Technique

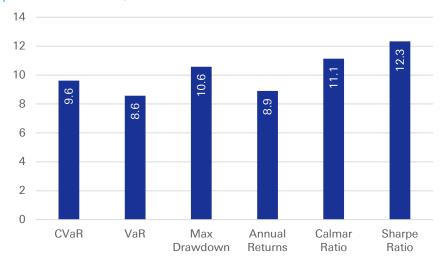


Figure 31 digs deeper into the performance in different market conditions by breaking the backtests into calendar years. We see that on average the portfolios built with the tail risk-adjusted models had improved Sharpe Ratios, CVaR and maximum drawdowns in 12 of the 19 years. We generally see a greater improvement when using the TRA model in years with large market corrections such as 2008, 2011 and 2016, while the base model generally yielded better performance in periods when the markets rallied, such as 2009. It is of note that the difference in performance in volatile (crash) periods was generally far greater than the difference in more placid periods.



Figure 31:Optimized Equity Portfolio Performance Comparison by Year: Base Model vs. TRA Models



#### Where is the Tail Risk Adjustment Most Effective?

Having looked at the distribution and time-varying nature of the performance statistics, we investigated the circumstances that lead to the tail risk-adjusted model yielded the greatest improvement. Figure 32 displays the average wealth curves using the base and tail risk-adjusted models in the top and bottom 10% of portfolios in terms of improvement in Sharp Ratio (when using the TRA model). We see that in the top 10% of cases there was a marked improvement in wealth when using the TRA model, with an average cumulated return of approximately 1,900%, compared to around 1,200% over the full backtest when using the base model. Looking at the bottom 10% of improvements in Sharpe Ratios we see that on average the TRA model yielded lower returns than the base model, with aggregated returns of approximately 790% and 970% with the TRA and base models respectively. We see that in the instances with the greatest improvements in Sharpe Ratios, the aggregate returns when using both the TRA and the base models was far higher than the cases where the base model fared best. The difference in performance was also far greater when the TRA model outperformed the base model than the reverse situation.

Portfolio performance was far stronger in instances where the TRA model outperformed the base model than when the base model out-performed the TRA model.



Figure 32: Average Wealth Curves in Cases with Top and Bottom 10% of Differences in Sharpe Ratios When using TRA Model



Investigating further, we looked at the differences in the risk contribution from factors when using the tail risk-adjusted versus base risk models. We look at risk contribution as it is the most relevant and complete metric for this, accounting for exposure (beta), factor volatility, and correlations, which are the metrics embedded into the portfolio construction. As the risk contribution is a forward looking (ex-ante) measure, it could be interpreted as the risk allocation, or risk 'budget'.

Figure 33 displays the differences in contribution to risk from the style factors (i.e. the risk coming from a given style for a portfolio built with the TRA model minus the risk contribution from the same style factor for the equivalent portfolio built with the Base model). We see that the portfolios built with the tail risk-adjusted models on average have less risk contribution from Value, short-term and medium-term Momentum, and Volatility. In contrast there is greater risk contribution from Size and Liquidity, inferring that the model 'prefers' larger cap<sup>4</sup> and more liquid stocks. This is intuitive, given that Value and Volatility are known to be highly cyclical factors, and that momentum signals are susceptible to sharp drawdowns. By contrast, large cap and more liquid stocks are typically more robust 'safe havens' for investors, and so would exhibit less tail risk.

The TRA model allocates less risk to the Volatility, Value, and Momentum factors, and more risk to the Size (large cap) and Liquidity factors.

<sup>4</sup> The Axioma Global Equity Factor Risk Model defines the Size factor to be aligned with the log of market cap. That is, large cap stocks have more positive values than small cap stocks.





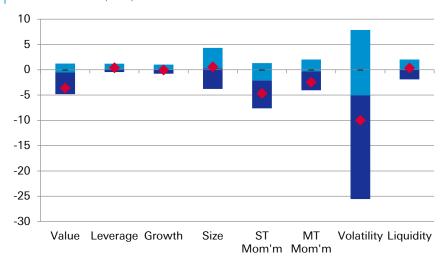
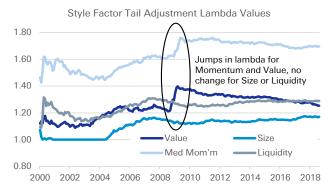


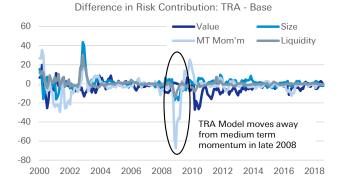
Figure 34 digs deeper into these results, looking at the evolution of tail risk adjustment lambda values for Value, Size, Medium Term Momentum, and Liquidity. When looking at the Lambda values (the values reflecting the degree of CVaR associated with a signal), we see that Medium-Term Momentum consistently has the highest values of the four factors, while Size has the lowest values. We also see that the lambda values for Medium Term Momentum and Value increased dramatically around the financial crisis in 2008, while the values for Liquidity and Size were relatively unaffected. This reflects the 'left tail' events associated with Momentum and Value in 2008.

Looking at the right-hand plot in Figure 34, we see a particularly large difference in the risk allocation to Medium Term Momentum in 2008, with the TRA Model assigning lower volatility to the factor. This follows from the high values of lambda for the Medium Term Momentum factor coupled with the high volatility observed in the factor in 2008.

The volatility adjustment coefficients (lambdas) for Momentum and Value were particularly affected in 2008, whereas the values for Size and Liquidity were relatively unaffected.

Figure 34:Tail Risk Adjustment Lambdas and Time Series Difference in Risk Contribution for Selected Style Factors: TRA Model - Base Model (BPS)





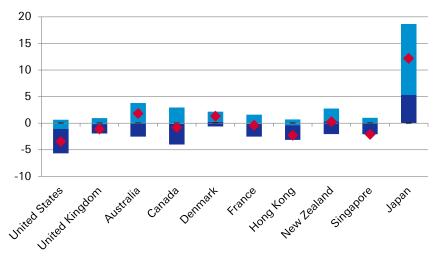
Source: Axioma, Bloomberg, Factset, Deutsche Bank Quantitative Strategy



Figure 35 displays the difference in risk contribution from countries between the tail risk-adjusted and base models. We see that the portfolios built with the tail risk-adjusted models generally had greater allocations to Australia, Denmark and Japan, and less exposure to the US, UK, Hong Kong, and Singapore. The largest difference is the allocation to Japan, which has been known for idiosyncratic, and relatively placid market conditions.

The TRA model typically allocated more risk to Japan and Australia, and less risk to the US, UK, Hong Kong, and Singapore.

Figure 35: Difference in Country Risk Contributions: TRA Model - Base Model Portfolios (BPS)



Source: Axioma, Bloomberg, Factset, Deutsche Bank Quantitative Strategy

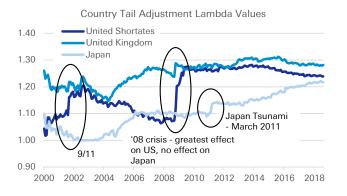
In Figure 36 we again dig deeper into these results, displaying the evolution of both the tail adjustment lambda values, and the risk contributions over time. Looking at the values of tail-adjustment lambdas, we see a dramatic increase in the values for the US, which was most dramatically affected by the financial crisis, a lesser effect for the UK, and no discernable effect for Japan. It is important to note that in the context of this global model, which has a global 'market' factor, a lack of change in the values of lambda for Japan in 2008 does not mean Japanese equities were not affected by the global financial crisis, but rather that they were not affected beyond the average global level.

In the left hand plot in Figure 36 we also see a rise in the lambda values for Japan in March 2011. This is associated with the earthquake and resulting tsunami that hit the east coast of Japan on 11 March 2011, resulting in dramatic destruction and a leak in a major nuclear power station, and negatively affecting the Japanese equity market. The change in the values of lambda and subsequent change in the tail risk-adjusted model illustrates how the technique can naturally incorporate shocks from sources beyond the financial markets. In the right hand side of Figure 36 we see the evolution in difference in risk contribution from selected countries. We see the risk allocation to the United States is lower in 2008, and that the allocation to Japan is typically higher, in alignment with the lambda values.

The volatility adjustment coefficients for the US and UK were affected by the events of 9/11 in 2002 and the financial crisis in 2008, while the coefficient for Japan was increased by the tsunami that struck Japan in March 2011.



Figure 36: Tail Risk Adjustment Lambdas and Time Series Difference in Risk Contribution for Selected Countries: TRA Model - Base Model (BPS)



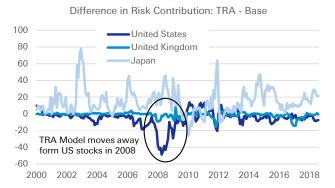
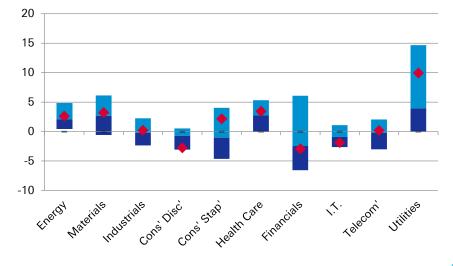


Figure 37 displays the difference in risk contributions for the sectors. We see persistently higher allocation of risk to the Utilities, Energy, Materials, Consumer Staples, and Health Care sectors, and lower allocations to the Consumer Discretionary, Financials, and I.T. sectors. The allocation is intuitive in that the sectors that are traditionally more cyclical have less risk allocated by the TRA model.

The TRA model generally allocates more risk to the Utilities, Materials, and Consumer Staples sectors, and less risk to Financials, I.T., and Consumer Discretionary stocks.

Figure 37:Difference in Sector Risk Contributions: TRA Model - Base Model Portfolios (BPS)



Source: Axioma, Bloomberg, Factset, Deutsche Bank Quantitative Strategy



Figure 38:Tail Risk Adjustment Lambdas and Time Series Difference in Risk Contribution for Selected Sectors: TRA Model - Base Model (BPS)

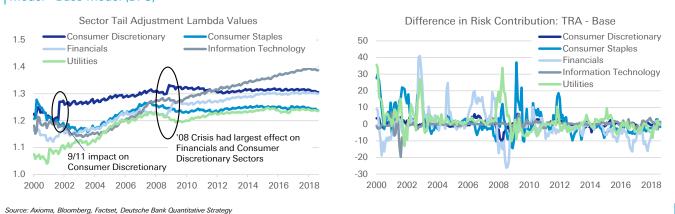


Figure 38 shows the evolution of the tail risk lambdas for 5 of the sectors, as well as the difference in associated risk contribution between the tail risk-adjusted and base models. We see jumps in the lambda values in 2001 for the Consumer Discretionary sector associated with 9/11, and in 2008 for the Financials and Consumer Discretionary sectors associated with the global financial crisis, while the less cyclical Consumer Staples, Utilities, and I.T. sectors exhibited little discernable change. From the risk contributions we observed lower risk allocation to the Financials sector in 2008, followed by increased risk budget for Consumer Staples in 2009. In late 2011 we see greater risk allocation to the Consumer Staples and Utilities sector associated with the European sovereign debt crisis.



## **Conclusions**

In this report we have developed a new, simple approach to building covariance matrices to better capture the left tail risk in equities and indices. In developing the approach, we have reviewed and assessed commonly adopted techniques for tail risk management and portfolio construction with regard to practical applicability. We have seen that our approach is effective in improving the performance, measured in terms of returns, risk, or risk-adjusted returns, in asset allocation, and when building cross asset risk premia, and single stock equity portfolios.

In reviewing commonly adopted approaches to account for tail risk, we have considered value at risk (VaR) and conditional value at risk (CVaR), Copulas, optimizing with respect to higher order moments (Polynomial Goal Programming), Cornish Fisher Expansions, and making use of historical Stress Periods. We have seen that many of the approaches can either neglect correlations, are not 'coherent' risk measures, require prohibitive computational time or an excessively large volume of data, or the development of a substantial coding infrastructure, or some combination of these and other issues. Another issue is that tail events are by definition rare, and thus a large amount of data is required to reliably calibrate a given model. This can be at odds with requirements for a risk model that can adapt to different regimes.

The approach we have developed is referred to as tail risk adjustment (TRA), and:

- Adjusts volatility using a CVaR base coefficient
- Adjusts correlations using a weighting (basis) function that assigns greater weight to negative returns
- Combines long-term estimates with more responsive measures when determining both volatilities and correlations
- Results in a covariance matrix, and thus can be used in standard (financial) optimizers, or with quadratic or second order cone programming libraries, allowing the use of any associated constraint mechanisms
- Can be used with many widely adopted portfolio construction techniques, including mean variance, inverse volatility, minimum volatility, maximum diversification, and risk parity optimization

We investigated the use of the tail risk-adjusted models for asset allocation, and to build cross asset risk premia and single stock equity portfolios with extensive backtests. We observed the following points:

- Building portfolios with tail risk-adjusted risk models typically leads to improved performance as measured by VaR, CVaR, maximum drawdown, returns, Sharpe ratios, or Calmar ratios
- When inspecting the performance by calendar year in the case of equity and asset allocation portfolios, we typically observed improved performance metrics in around 60%-70% of years. The improvement were particularly evident in years in which there were market corrections, such as 2008, 2011, and 2016

#### 15 January 2019

#### Portfolios Under Construction



- The difference in allocations to indices or factors when using the tail risk adjustment was generally intuitive, allocating less (risk) to more cyclical assets, indices, or factors, e.g.:
  - Greater allocation to FX, and lower allocations to equity and hedge fund indices in the context of assets allocation
  - Reduced risk allocation to more cyclical factors such as Momentum,
     Value, and Volatility, and Financial and Consumer Discretionary
     sectors in the context of building single stock equity portfolios
- In equity portfolios we observed the impact of the TRA was more effective on portfolios built with more cyclical, higher risk alpha signals such as Momentum and Value.

We believe that tail risk-adjusted models can be of use to investors looking to build portfolios with a more defensive stance, with greater protection from left tail events and drawdowns, and improved performance.



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15 January 2019
Portfolios Under Construction



# Appendix 1

#### **Important Disclosures**

#### \*Other information available upon request

\*Prices are current as of the end of the previous trading session unless otherwise indicated and are sourced from local exchanges via Reuters, Bloomberg and other vendors. Other information is sourced from Deutsche Bank, subject companies, and other sources. For disclosures pertaining to recommendations or estimates made on securities other than the primary subject of this research, please see the most recently published company report or visit our global disclosure look-up page on our website at <a href="https://research.db.com/Research/Disclosures/CompanySearch">https://research.db.com/Research/Disclosures/CompanySearch</a>. Aside from within this report, important risk and conflict disclosures can also be found at <a href="https://research.db.com/Research/Topics/Equities?topicld=RB0002">https://research.db.com/Research/Topics/Equities?topicld=RB0002</a>. Investors are strongly encouraged to review this information before investing.

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15 January 2019 Portfolios Under Construction



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