LEHMAN BROTHERS | Fixed Income Research

Liquid Markets Research Quarterly

Volume 2005-Q3

As part of the continuing expansion of the universe of securities handled by our Global Risk Model and Hybrid Performance Attribution models, we have recently included inflation-linked securities in this universe. This article presents the details of our newly developed approach to the modeling of these securities.

We examine the phenomenon of global savings-investment imbalances through the lens of the life-cycle model of optimal savings choice. We ask how far the current savings rates in the world's large economies are from their optimum. Our analysis suggests that the US savings rate is not unwarrantedly low, nor the Japanese savings rate unwarrantedly high.

We evaluate the long-term performance of a buy-and-hold investment strategy in MBS versus comparable strategies in credit and agency bonds. We show that not only has an MBS strategy outperformed credit and agency strategies, but the distribution of MBS book income has displayed remarkable stability and diversification.

Following the launch of the Lehman Brothers/HFN Hedge Fund Index, we illustrate how Style Analysis can be used to gain insight into the risk exposures of hedge funds and the construction of investable indices. We also present a simple measure that can identify inconsistencies between a fund's actual and self-proclaimed style.

Constant maturity mortgage products are standardized derivative instruments that provide an explicit and isolated exposure to mortgage rates. This article examines this emerging family of rate-based mortgage products, including their market development, comparative advantages, contractual features, valuation techniques, risk characteristics.

CONTACTS

Editors

Lev Dynkin .... 1-212-526-6302 .... ldynkin@lehman.com

Prafulla Nabar .... 1-212-526-6108 .... pnabar@lehman.com

Vasant Naik .... 44-20-7260-2813 .... vnaik@lehman.com

Quantitative Market Strategies

Srivaths Balakrishnan .... 44-20-7102-2180 .... sbalakri@lehman.com

Mukundan Devarajan .... 44-20-7102-9033 .... mudevara@lehman.com

Adam Purzitsky .... 44-20-7102-9023 .... apurzits@lehman.com

Simon Polbennikov .... 44-20-7102-3883 .... sipolben@lehman.com

Alexei Jiltsov .... 44-20-7103-4360 .... ajiltsov@lehman.com

POINT Modeling

Cevdet Aydemir .... 1-212-526-2097 .... aaydemir@lehman.com

Anthony Lazanas .... 1-212-526-3127 .... alazanas@lehman.com

Attilio Meucci .... 1-212-526-5554 .... ameucci@lehman.com

Antonio Silva .... 1-212-526-8880 .... antonio.silva@lehman.com

Huarong Tang .... 1-212-526-3619 .... hutang@lehman.com

Quantitative Portfolio Strategies

Albert Desclee .... 44-20-7102-2474 .... adesclee@lehman.com

Anthony Gould .... 1-212-526-2821 .... agould@lehman.com

Bruce Phelps .... 1-212-526-9205 .... bphelps@lehman.com

Jeremy Rosten .... 44-20-7102-1020 .... jrosten@lehman.com

Liquid markets quantitative research

Zhengyun Hu .... 44-20-7102-3934 .... zhhu@lehman.com

Dev Joneja .... 1-212-526-8035 .... djoneja@lehman.com

Charlie Liu .... 81-3-6440-1435 .... chliu@lehman.com

Dominic O'Kane .... 44-207-102-2628 .... dokane@lehman.com

Bruce Tuckman .... 1-212-526-2252 .... btuckman@lehman.com

Fei Zhou .... 1-212-526-8886 .... feiz@lehman.com

FOREWORD

The current debate in the investment management community is focused around a few themes: global macro-economic imbalances, the growth of hedge fund and funds of hedge funds as an asset class, the threat of inflation and the risks caused by movements in mortgage rates and their potential impact on house prices. The Lehman Brothers research team has been busily working this autumn on a number of questions related to various aspects of this debate. Some of the output of this research is what is presented in this issue of the *Liquid Markets Research Quarterly (LMRQ).*

We are publishing five articles in this issue. The first article deals with risk modeling and performance attribution of inflation-linked securities. Inflation-linked bonds are becoming increasingly important globally as the demand for long-dated assets intensifies with the ageing of world populations. As more and more fixed income portfolios include these bonds, there is a greater need to understand their risk and return characteristics in a manner consistent with other asset classes in the portfolio. This article provides a framework aimed at achieving this objective.

The second article examines global savings and investment imbalances using a life-cycle model framework. The worsening US current account deficit is increasingly becoming a cause for concern. This paper focuses on US, Germany and Japan to analyze savings-investment choices and derives capital market implications for yields and exchange rates.

The third paper investigates the merits of MBS investing over long horizons. Many significant MBS investors use buy-and-hold strategies and measure their performance using the book accounting method. This paper evaluates MBS versus credit and agency investment strategies using these long-term performance metrics.

The fourth paper presents a methodology for utilizing Style Analysis to gain insights into the risk exposures of hedge funds. The paper also suggests an approach for using this methodology to construct fund portfolios with desired risk-return characteristics. Finally, the paper proposes a simple measure to identify discrepancies between actual and self-proclaimed styles.

With the tremendous growth of the mortgage market in the recent past, the need for hedging mortgage rate related risk has also grown. The fifth paper analyzes Constant Maturity Mortgage products that have proved effective for this purpose. A valuation framework is presented and risk exposures are analyzed. Finally, the paper discusses various applications of these products, including hedging of mortgage servicing.

Prafulla Nabar

Global Head of Fixed Income Quantitative Research

Lev Dynkin

Global Head of Quantitative Portfolio Strategies

Vasant Naik

Global Head of Quantitative Market Strategies

Risk Modelling and Performance Attribution for Inflation-linked Securities[1]

Anthony Lazanas
+1 212 526 3127
alazanas@lehman.com

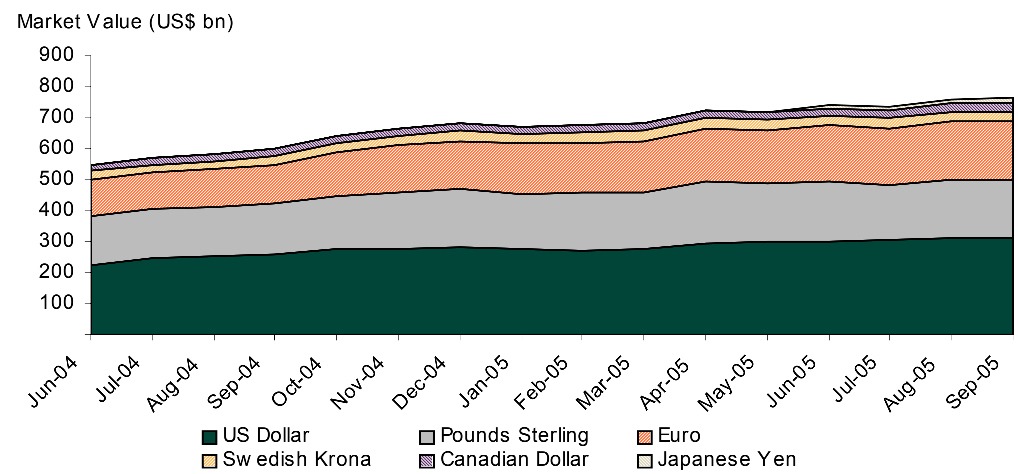
Jeremy Rosten
+44 (0)20 7102 1020
jrosten@lehman.com

Accessed through POINT[2], our Global Risk Model and Hybrid Performance Attribution Model allow a comprehensive analysis of the risk and return characteristics of global fixed income portfolios. As part of the continuing expansion of the universe of securities handled by these two models, we present an overview of our newly developed approach to the analytics of inflation-linked debt. We outline our methodology for the integration of this asset class within the existing frameworks of the models and arrive at what we consider to be a simple yet powerful formulation of the risk factors driving returns, leading to an intuitive treatment in terms of both risk and return.

1. INTRODUCTION

In October 1997, the global market for inflation-linked government bonds accounted for approximately 2% of the total government bond market. By the end of September 2005, the figure had risen to around 7.2%[3] totalling some $766 billion. Moreover, we think there are reasons to expect that the size of the inflation-linked market will continue to increase both in total size and as a proportion of the global government debt market. Figures 1 and 2 show the recent growth in the market as well as the current distribution of inflation-linked debt across the main sovereign issuers.

Figure 1. Recent growth of the inflation-linked market



Source: POINT.

Figure 2. Current distribution of market value across major sovereign inflation issuers[4]

	US Dollar	Euro	Pounds Sterling	Swedish Krona	Canadian Dollar	Japanese Yen	Total
Market Value ($Bn)	311.9	189.6	185.5	32.3	29.4	17.3	766

Source: POINT.

1 *The authors would like to thank Vasant Naik, Prafulla Nabar, Adam Purzitsky and Adam Shergold for their help, advice and comments.*
2 *POINT is the Lehman Brothers Portfolio and Index analysis Tool.*
3 *The figures represent market values in the Lehman Brothers Global Inflation-linked Index plus the Japan Inflation-linked Index as compared with that of the Lehman Brothers Global Treasury Index as of September 30, 2005.*
4 *All figures are as of September 30, 2005 – Source POINT*

Given the size and projected growth of this asset class, the need for analytics, risk measurement and performance-attribution tools has never been greater. As part of the growing asset-class coverage of POINT, the Lehman Brothers Global Risk and Hybrid Performance Attribution models have this year seen the inclusion of inflation-linked bonds. This article outlines the approaches that we have taken in modelling these securities.

In our approach, we consider the exposure of inflation-linked securities to nominal yields and their exposure to inflation-related factors separately. Although there is a strong correlation between nominal rates and inflation (of the order of 50%), we prefer to keep these two risk factors separate so as to remain consistent with the risk framework of other security classes. Inflation risk affects these securities in two ways: (a) the risk that realised inflation could be different from market expectations, thus affecting the size of the paid cash flows; and (b) the risk that the market expected inflation over the life of the bond may change. We introduce three new factors in the Global Risk Model in each market[5]: one for realised inflation and two (a short-term and a long-term factor) for expected inflation.

In the Hybrid Performance Attribution model, we follow a similar approach: Yield curve return is extracted to isolate inflation-driven return. The latter comes once again from two main sources: (a) the difference between monthly realised inflation and expected inflation; and (b) changes in expected inflation over the lifetime of the bond. We introduce a technique to smooth the discrete (monthly) nature of the first component and report this return as a daily "carry" (representing the difference between the short-term realised inflation and the long-term expected inflation implied by the bond price) plus smaller monthly "inflation surprise" return shocks. With respect to the expected inflation component of return, we allow the user to implicitly break it down into smaller components by partitioning a benchmark index, similar to the treatment of credit securities.

The structure of the article is as follows:

In **section 2** we review some essential analytical concepts as they apply to the inflation-linked market, together with inflation-specific analytical terms. We look at yield and sensitivity definitions and conventions and relate them to some widely used measures.

Section 3 describes the hurdles faced in seeking to model inflation risk and attribute returns. We then develop the formal mathematical framework and the pricing equation that provides the basis for the identification and specification of factors driving risk and return.

Section 4 builds on the results of this analysis to describe the risk factors for the inflation-linked market in the Global Risk Model. We comment on some sample output of the Risk Model.

Section 5 presents the handling of these securities in our Performance Attribution model.

In **section 6** we make our concluding remarks.

Appendix A lays out the cash flow calculations and conventions referred to throughout the body of the article.

Appendix B presents the details of how we convert the discrete month-to-month inflation accretion return (occurring on index announcement days) into a smooth daily return.

5 In some markets, only one expected inflation factor is employed. See section 4.2 for more details.

2. BASIC ANALYTICS OF INFLATION-LINKED BONDS: TERMINOLOGY AND CONVENTIONS

Inflation-linked bonds (also called "reals", "inflation linkers" or simply "linkers") have numerous specific concepts and definitions in terms of yields and sensitivities. We take a whistle-stop tour of some of the principal conventions and measures in the market. This gives us not only a better understanding of how the market operates, but also helps us in our modelling the risk and return characteristics of this asset class.

2.1. Cash flows of inflation-linked bonds

An inflation-linked bond is typically a coupon-paying instrument with cash flows linked to a specified price index.[6] A *real coupon*, *c*, is defined along with the mechanics of indexation: the price index to be used and the rules to calculate the delivered cash flow. For a cashflow occurring at time *t*, we first define the Index Ratio, IR_t, as the ratio of the price index level to be used at time *t* to the index level at the issue of the bond (see appendix A for details). The index level to be used at time *t* is typically a lagged value of the price index. Then, the coupon (and similarly the principal[7]) payable at time *t* is given by:

$$c_t = c \cdot IR_t$$

In this way, the cash flows grow with the index and provide protection against inflation.

2.2. Real vs. nominal yield

Once we have such an index and securities with cash flows defined with reference to it, we can proceed to define and derive the standard analytical measures and concepts.

2.2.1. Real Yield

We define the *real yield* as the yield of the inflation-linked bond in terms of index units. That is, let B_t be the nominal price of the inflation-linked bond at time *t*, IR_t be the index ratio at time *t* and *c* the real coupon of bond. Translated into index units $\frac{B_t}{IR_t}$ is the real value of the bond and *c* is that of the real coupon (since the coupon will grow with the index it remains a constant in terms of index units). We can then define the real yield as the value of *r* that solves the pricing equation:

$$\frac{B_t}{IR_t} = \sum_{n=1}^{N} \frac{c \cdot 100}{(1+r)^n} + \frac{100}{(1+r)^N} \qquad (1)$$

6 *Inflation-linked bonds in the US are linked to the Index of Consumer Prices for Urban Consumers (CPURNSA). The indices in the UK, France, Japan, Canada, and Sweden are respectively, the General Index of Retail Prices all-items non-seasonally adjusted (UKRPI), Consumer Price Index excluding tobacco for all households residing in mainland France (FRCPXTOB), Japan CPI Nationwide General Ex Fresh Food (JCPNGENF), all-items CPI non-seasonally adjusted, published monthly by Statistics Canada (CACPI) and Sweden CPI (SWCPI) respectively. For Eurozone inflation-linked securities the index is Eurostat Eurozone HICP Ex Tobacco Unrevised series NSA(CPTFEMU)*

7 *In some markets floors exist for principal payments. See Appendix A for details.*

2.2.2. Nominal Yield

The *nominal yield* can be defined in a number of ways. The most common method is that produced by making an explicit assumption, π, of future inflation that is constant across the life of the bond. Then, by inflating all future cash flows at this rate and computing an internal rate of return (IRR) for the resulting pricing equation, one arrives at the nominal yield.[8] Assuming a constant inflation rate of π, at time t, the n^{th} coupon, c_n, can be written:

$$c_n = IR_{t+n} \cdot c = \frac{IR_{t+n}}{IR_t} \cdot IR_t \cdot c = (1+\pi)^n \cdot IR_t \cdot c$$

To obtain the nominal yield of the inflation-linked bond we solve the following pricing equation for y:

$$B_t = IR_t \cdot \sum_{n=1}^{N} \frac{c \cdot 100 \cdot (1+\pi)^n}{(1+y)^n} + IR_t \cdot \frac{100 \cdot (1+\pi)^N}{(1+y)^N} \qquad (2)$$

On its inflation analytics page, Bloomberg uses the most recent realisation of annual CPI growth as the inflation assumption (index ratio for the current settlement date divided by the index ratio at the settlement date for one year in the past[9]). This is also the methodology currently implemented in POINT.[10]

Intuitively, the nominal yield differs from the corresponding real yield by some measure of expected[11] inflation across the life of the bond. This brings us to the concept of *Breakeven Inflation*.

2.2.3 Breakeven Inflation (BEI)

Just as yields are defined to be the single discount rate that, if applied to all cash flows, correctly prices the bond, we can define the inflation *breakeven* yield as the single inflation assumption which, if applied to inflate all cash flows, correctly prices the inflation-linked bond. To define breakeven inflation we first need to identify a benchmark nominal bond with respect to which the breakeven inflation is measured. The term “breakeven” inflation is indeed derived from the fact that if inflation over the life of the bond is constant and equal to the breakeven rate, then the yield to maturity of the real bond exactly equals that of the nominal benchmark bond. Hence the investor “breaks even”. We denote by y^{bench} the yield of the nominal government security benchmark and define, π^{bei}, the inflation breakeven by equation (3):

$$B_t = IR_t \cdot \sum_{n=1}^{N} \frac{c \cdot 100 \cdot (1+\pi^{\text{bei}})^n}{(1+y^{\text{bench}})^n} + IR_t \cdot \frac{100 \cdot (1+\pi^{\text{bei}})^N}{(1+y^{\text{bench}})^N} \qquad (3)$$

---

8 *A second methodology is to generate a forward inflation curve, derived from the prices of inflation-linked securities in the market. Then, using the resultant curve, the appropriate breakeven rate,* π_{t,t_n} *, is applied for the period from the pricing date to the date of the nth cash flow. This method gives a set of projected cash flows consistent with current market expectations of inflation. This security, consisting of the projected cash flows, can be correctly priced with one yield. Formally, we define the n-th cash flow as follows:*

$$cf_n = \begin{cases} c(1+\pi_{t,t_n})^n & n < N \\ (1+c)(1+\pi_{t,t_N})^N & n = N \end{cases}$$

This leads to the pricing equation defining the nominal yield, y, thus: $B_t = IR_t \cdot \sum_{n=1}^{N} \frac{100 \cdot cf_n}{(1+y)^n}$

9 *That is, take the index ratio that applies to the settlement date for a trade done exactly one year in the past. In fact this will in general need two historical values of CPI and an interpolation between them. In so far as this differs from the breakeven inflation, it results in a nominal yield that differs from that of the corresponding nominal Treasury bond.*

10 *The inflation assumption can be viewed in POINT in the index/portfolio contents view in the inflation field entitled ‘Assumed Inflation’.*

11 *Strictly speaking, this is an expectation under the risk-neutral measure.*

With this definition, π^{bei} represents a (risk-adjusted) expectation of average inflation over the life of the bond, and the nominal yield of the bond is equal to the yield of the equivalent nominal benchmark. It is straightforward to see that the real yield, *r*, the equivalent nominal benchmark yield[12], y^{bench}, and the corresponding breakeven inflation, π^{bei}, are related via the equation:

$$(1+r)(1+\pi^{\text{bei}}) = 1 + y^{\text{bench}}$$

While this is a consistent definition of breakeven inflation, it is not the only convention used when quoting breakevens. For example, the simplest and most common approach is to use the difference between the yield of the benchmark Treasury bond and the real yield of the inflation-linked security which, by the above formula, will in general be a good approximation.

2.3. Durations and measures of yield curve exposure

Inflation-linked bonds have a more complex relationship with the nominal yield curve than nominal government bonds or, indeed, corporates. This arises from the fact that breakeven and realised inflation are often highly correlated with movements in the underlying nominal curve. Therefore, a full picture of the reaction of the security's price to a shift or reshaping of the nominal yield curve necessarily requires a consideration of where inflation is likely to go given nominal rate movements. Despite this, some simple duration measures remain in use. We survey the most common measure of sensitivities used in the market.

2.3.1. Real modified duration

Just as we defined the real yield of an inflation-linked bond, so we may define the real modified duration as the sensitivity of price to a change in that yield. We simply reprice the bond in real terms according to equation (1), shifting the current yield and calculating the price sensitivity.

Formally, we define the real duration as (minus) the proportional change in the price of the linker per unit change in the real yield.

$$D_t^{real} = -\frac{1}{B_t}\left(\frac{\partial B_t}{\partial r}\right)$$

12 In practice, where there is no government benchmark bond of the same maturity as the inflation-linked bond interpolation is used between yields of adjacent government par bonds.

2.3.2. Nominal duration (OAD)

This measures the sensitivity of the inflation-linked bond with respect to a (small) parallel shift in the level of the nominal par curve. It is calculated by making a single inflation assumption[13], as in the definition of nominal yield, and then computing sensitivity to the shift of the nominal curve on present value of the newly projected cash flows. In that the inflation growth is fixed, it is a measure of the sensitivity of the security's price to movements of the nominal yield curve assuming no accompanying movement in average inflation rates across the life of the bond.[14]

2.3.3. Empirical duration

Empirical duration seeks to estimate the empirical sensitivity of an inflation-linked security to nominal yields with reference to their effect on real yields. We have noted above in 2.3.1 that, given a movement in the real yield of a linker, we can approximate the return implications via the Real Modified Duration. We couple this with an estimate of how that real yield reacts to a movement in nominal yields, for example by regressing changes in the observed real yield of the inflation-linked bond, Δy_t^{real}, against changes in the yield of the nominal benchmark, $\Delta y_t^{\text{nominal}}$. We can then translate a movement in the latter into a return for the inflation-linked security via the real duration.

Formally, we estimate the sensitivity, β, of real yields to nominal by regression, choosing β in the equation below, to minimise the sum of the squares of the error terms over the period considered:[15]

$$\Delta y_t^{\text{real}} = \beta \Delta y_t^{\text{nominal}} + \varepsilon_t \qquad (4)$$

This measure is principally designed to provide a hedge ratio for a position in a linker with respect to the benchmark nominal security.[16]

By way of comparison, Figure 3 shows the real, nominal and empirical durations of the OATi 2029 for the period May-04 to Sep-05.

---

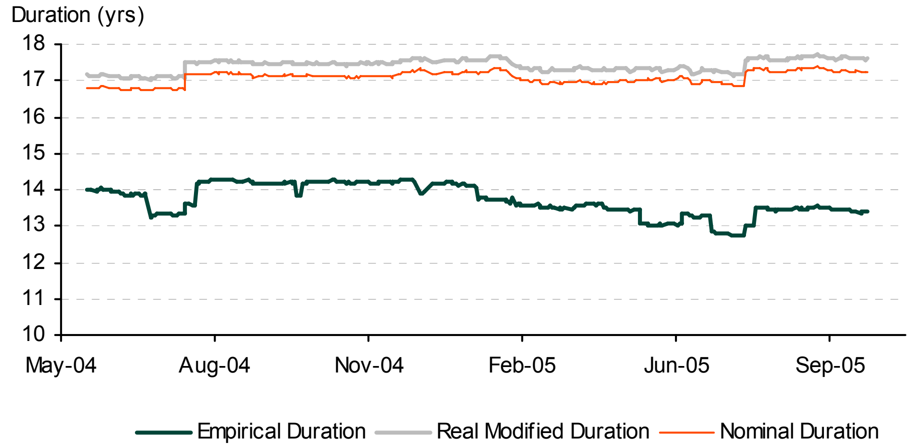
13 See section 4.5.2 for details on the inflation assumption in POINT for this calculation.

14 This assumption of zero correlation of inflation and nominal yields, whilst artificial and at odds with experience, is of value in two particular contexts: First, by way of comparison with spread markets where spreads, whilst known to have non-zero correlation with yields, are kept constant in the calculation of durations. Second, such a measure allows the user to define their own scenarios for nominal and real rates even if those scenarios are not in keeping with the historically observed relationship between the two sources of risk. The user is free, in other words, to choose her own betas for real versus nominal yields in a 'what-if' analysis.

15 Technically, we should guard against the term dependence of the correlation and note that the maturities of the linker change across time. To be precise, we should compute time series of constant maturity real and nominal yields at the current maturity of the bond in question and use the correlation at this point on the curve. Given, though, the short time window over which the correlation is estimated and that the interpolation will be required to construct constant maturity yields at the particular maturity of the bond we are seeking to model, this approximation seems reasonable. Also note, we consciously omit the estimation of the constant term so as to produce an empirical sensitivity that is independent of the magnitude of the move in nominal rates.

16 Denote by D^{real} the Real Modified Duration of the inflation-linked security and by D^{nominal} the nominal modified duration of the treasury benchmark. Then $1 Market Value of the real bond could be approximately hedged with respect to the nominal yield curve by a short position of $\beta \cdot \left(D^{\text{real}} \big/ D^{\text{nominal}} \right)$ in the nominal security.

Figure 3. Nominal and Empirical Correlations For OATi 2029



Source: LehmanLive.

As would be expected, the empirical duration lies below the other non-correlation based measures.

In section 4 we demonstrate another measure of empirical duration that is based on the historical monthly correlations of inflation and nominal risk factors.

3. MODELLING INFLATION RISK AND RETURN

3.1. Consistency with existing frameworks

The risk and performance attribution models for inflation-linked bonds is a part of our Global Risk and Hybrid Performance Attribution Models. It is therefore necessary that the analytical framework used for inflation-linked bonds be consistent with that used for other asset classes. In our models, we measure risk and return in nominal terms. As a result, inflation-linked securities whose cash flows depend on future inflation realizations are exposed to inflation risk in addition to their exposure to nominal risk sources.[17] We, therefore, seek to isolate inflation factors that reflect the exposure of linkers to inflation-related sources of nominal return. From the point of view of the Global Risk Model, we need to identify those market-wide drivers of risk and return that are associated with a given inflation index. These should also be specified so as to complement the existing systematic factors.

To identify these risk factors we need to perform an attribution of the monthly returns of inflation-linked securities as we do for all other securities in the Risk Model universe. This enables us to isolate a "residual" return; a component of return that is above and beyond that due to (nominal) yield curve movements, volatility changes and time-related effects such as rolldown and accretion. It is this term that represents the return arising from the inflation-linked nature of the security and that we need to model.[18]

17 One could argue that it is precisely inflation-linked products, protected as they are against the effects of the increase in prices, that are immune to inflation effects and that inflation risk is present only in nominal securities. This implies that the riskless asset is cash growing with inflation. Common convention, however, dictates that we consider unadjusted cash as the riskless asset. Under this convention, it is inflation-linked securities that are exposed to inflation risk.

18 In so far as the security has other components of return, such as corporate spreads for corporate inflation-linked securities, or country spreads for European sovereign issuers (e.g. Greece or Italy which trade at a spread to the French-German Euro Treasury curve that is the risk free EUR curve in POINT as a whole), these effects will be stripped out before calibration of the inflation risk factors. In practice, due to lack of liquidity, non-treasury linkers do not take part in the estimation of inflation risk factors.

To perform this attribution, however, we must first define a pricing equation that forms a basis for measuring the impact of the passage of time, changes in the yield curve and potential volatility effects. Such an equation would also be needed to compute analytical sensitivities to allow us to extract risk factors from the residual return described above.

As a first step towards achieving a consistency between our modelling of inflation-linked bonds and that of other asset classes, we establish below a parallel between the approaches we use for inflation-linked bonds and for corporate bullets.

3.2. The parallel between inflation linked bonds and corporate bullets

The difficulty in modelling an inflation-linked security is the uncertainty regarding its cash flows. This uncertainty is a different one from that encountered in the credit asset class, for example. In this asset class the uncertainty concerns whether the coupon will be paid at all. For inflation-linked securities issued by national treasuries, the uncertainty faced is with regard to the size of the payment, which depends on the announced level of a particular index at some point in the future. Nonetheless, despite these differences, the analytical approach taken in handling credit securities can be adapted to our purposes.

For a security with credit risk, we note that uncertainty regarding the cash flows is reflected in the market price of the bond. This correspondingly defines an option-adjusted spread, oas_t over the relevant discounting curve, described by the zero rates $\{i_{t,t_i}\}_{i\geq 1}$, as in (5):

$$B_t = \sum_{i=1}^{N} \frac{100.c}{(1+i_{t,t_i}+oas_t)^{t_i-t}} + \frac{100}{(1+i_{t,t_N}+oas_t)^{t_N-t}} \qquad (5)$$

or

$$B_t = B(c, YC_t, T-t, oas_t)$$

Effectively, we assume that the cash flows, as given by the cash flow schedule at the time of evaluation, are assured and we solve for the fixed spread, the OAS, over the discount curve that correctly prices those cash flows.

For inflation-linked securities we can adopt a similar approach that allows us to look at these bonds as simple variations on bullet bonds with an appropriate spread. The idea here is that whereas for a credit security the risk of default is *discounting* the expected value of a given cash flow leading to a positive spread, the effect of index linkage is to *increase* the expected value (from that of the real coupon) resulting in a *negative* spread. We develop the relevant mathematics in the following section.

3.3. The pricing equation

To aid intuition, we initially present a simplified analysis, where we assume that the inflation index is observed continuously and without lag. Relaxing this assumption, we develop a full pricing equation in the subsequent section.

3.3.1. A simplified analysis

We recall from section 3.1 that the coupon c_{t_i} at time t_i is given by:

$$c_{t_i} = c \cdot IR_{t_i}$$

Where c is the real coupon given at the issue of the bond and IR_t is the index ratio which applies to a cash flow at time t, as set out in the terms of the bond issue.

Our next step is to distinguish between that part of the Index Ratio, IR_{t_i}, that applies to time t_i at some time in the future, that is known at evaluation time t and that part that is unknown and represents the future effects of inflation. So, for a coupon, c_{t_i}, at a time t_i, we can re-write the index ratio for that cash flow, IR_{t_i} as follows:

$$IR_{t_i} = \left({IR_{t_i}} \middle/ {IR_t} \right) \cdot IR_t = (1+\pi_{t,t_i})^{t_i - t} \cdot IR_t \qquad (6)$$

The index ratio, IR_t, is known at time t and the quantity $\left({IR_{t_i}} \middle/ {IR_t} \right)$ represents the effect of inflation between time t and time t_i This last term has been rewritten in the form $(1+\pi_{t,t_i})^{t_i-t}$ where we define a variable π_{t,t_i} to account for the growth in the index ratio between time t and t_i.

This gives the following expression for the cash flow, c_{t_i}, at time t_i

$$c_{t_i} = c \cdot IR_t \cdot \left(1+\pi_{t,t_i}\right)^{t_i - t}$$

Standing at time t, and employing the above notation, we can write that the price of the inflation-linked bond at time t is the present value of this set of cash flows:

$$B_t = IR_t \cdot PV_t \left(\left\{ c.100.(1+\pi_{t,t_i})^{t_i - t} \right\}_{i=1 \ldots N-1}, (1+c).100.(1+\pi_{t,t_N})^{t_N - t} \right)$$

Our next step is to solve for a *single* inflation rate that prices the bond. In other words, we seek π_t that satisfies:

$$B_t = IR_t \cdot \sum_{i=1}^{N} \frac{c.100.(1+\pi_t)^{t_i - t}}{(1+i_{t,t_i})^{t_i - t}} + IR_t \cdot \frac{100.(1+\pi_t)^{t_N - t}}{(1+i_{t,t_N})^{t_N - t}} \qquad (7)$$

This should be compared with equation (2) for the nominal yield above. There we derive a single yield to discount cash flows, calculated with reference to a single inflation assumption; here we work with the observed nominal discount curve, deriving instead an inflation rate across the life of the bond. Thus we can write:

$$B_t = IR_t M(YC_t, \pi_t, T - t)$$

where M is the price, at time t, of a "model bond", whose cash flows are coupons of size c (the real coupon) and principal repayment at maturity $T = t_N$, and where YC_t is the nominal yield curve at t. In particular π_t represents a measure of break-even inflation expectations across the life of the bond.[19]

19 *Strictly speaking* π_t *will be a risk-neutral spread and will include risk premium, liquidity risk and other effects above and beyond inflation expectations.*

Now we are ready to establish the parallel with a credit security.[20] The two formulations are equivalent when we write $oas_t \approx -\pi_t$. In the limit, as we tend toward continuous compounding, the equivalence is exact.[21] Thus, our information is equivalent to a credit bond with spread *under* the nominal curve of magnitude π_t, the "breakeven inflation" derived in (7).[22]

Under this formulation, the sources of risk of such a security would therefore be movements in nominal rates, changes in the realised rate of inflation, IR_t and movements in this π_t or breakeven inflation.

This is the essence of the approach we adopt. We associate with the inflation-linked security a spread and proceed to treat it in much the same way as any other spread product. Nonetheless, this is not the final word. The idiosyncrasies of the inflation-linked market necessitate a more complex variation of the above formulation. We outline these in the next section.

3.4. The full pricing equation

There are two further issues that we must address and that result in a pricing equation that is a refinement of equation (7). We outline each in turn along with the approach that we have taken in resolving them. We then present the final pricing equation. Details of the method are shown in Appendix B

3.4.1. Lag effects

In section 3.3, we have assumed that the price index is observed continuously and without lag. Under this assumption, the index ratio applicable to cash flows arriving at t becomes known at t. In fact, this is not the case. Price indices are announced with a lag and at monthly intervals. As a result, the index ratio applicable to cash flows arriving at t is known before t.

Consider an example of a French Local Inflation-linked bond.[23] Suppose we wish to know the cash flow of an inflation-linked bond that is scheduled to occur on July 25, 2005. By the terms of the bond, this is determined by the level of the French Local Price index (FRCPXTOB) for April and May 2004. The latter is published at some time mid June 2005. This means that already in mid-June 2005 we know the coupon that we will receive some weeks later on July 25, 2005.

This "foreknowledge" of the index ratio has three implications for our pricing equation:

(a) At any given moment in time, the future cash flows of an inflation-linked security can be broken down into those that are known at the time of evaluation, CF_t^{known}, and those,

---

20 To express equation 7 in the form of equation 5, i.e. in classical credit spread terms, we need merely solve equations of the form:

$$\left(\frac{1+\pi_t}{1+i_t}\right)^n \equiv \frac{1}{(1+i_t+oas_t)^n} \qquad \Leftrightarrow \qquad \frac{1+\pi_t}{1+i_t} = \frac{1}{1+i_t+oas_t}$$

$$\Leftrightarrow \qquad oas_t = -\pi_t\left(1+\frac{i_t-\pi_t}{1+\pi_t}\right)$$

21 That is, we write $\exp\{-(t_n-t)\cdot(i_{t,t_n}-\pi_t)\}$ *for the inflation-linked securities 'discount factor' and* $\exp\{-(t_n-t)\cdot(i_{t,t_n}+oas_t)\}$ *for that of the credit bond. These are the same for all n iff* $oas_t = -\pi_t$

22 In section 2.2.3 we defined breakeven inflation with respect to the yield of the nominal benchmark security. Here we are defining it with respect to the entire nominal yield curve. This is more precise. Henceforth, we use the term break-even inflation in this sense.

23 The same will be true for a US TIPS or an HICP linked bond since they have the same conventions – see Appendix A.

CF_t^{unknown}, that are not. This means that we can write more comprehensively that the (inflated) price of an inflation-linked security is given by:

$$\begin{aligned} B_t &= PV(CF_t^{\text{known}}) + PV(CF_t^{\text{unknown}}) \\ &= PV(CF_t^{\text{known}}) + IR_t M_t(YC_t, \pi_t, T-t) \end{aligned} \qquad (8)$$

The first term represents the present value of a vanilla bullet security and the second is our familiar spread formulation of the security with inflation linkage. Note that this decomposition into known and unknown cash flows means that we allow inflation risk to apply only to those cash flows that are truly exposed to it and incorporate the latest information in the market regarding realised inflation.

(b) In equation (6), we should use the index ratio computed using the most recently announced price index level This ensures that we are modelling as stochastic only that part of inflation thus far unknown to the market.

(c) A consequence of (b) is that we must change the exponent in the spread terms $(1+\pi_t)^{t_i - t}$ to something akin to $(1+\pi_t)^{t_i - t - 1}$ (or $(1+\pi_t)^{t_i - t - 6}$ in the UK market where the lag is 8 months not 3 as for HICP above).

3.4.2. Smoothing the return

Because inflation-linked securities are linked to the level of a discretely published index – CPI or RPI – traditional methods of performance attribution lead to discontinuous and unintuitive results. This is because in our pricing equation, inflation effects take two different forms. One representation of inflation is through π_t, the spread. This term results in a spread carry as time passes as would be the case for a conventional credit security. The second way in which inflation effects are reflected in the pricing equation is via the index ratio term. As we have described in the previous section, this uses the *latest* index ratio corresponding to the most recent CPI announcement. This term is, therefore, static throughout the month until the day of publication of the next CPI. On that day, it jumps with a daily return equal to the entire growth of the index for that month.

For example, consider an inflation-linked security with a spread of -365bp as described by equation (7) with π_t=365bp.[24] All other things remaining equal, spread accretion takes place at the rate of approximately -1bp per day.[25] This large negative spread carry is typically countered by an equally large positive jump in the index ratio on the next CPI announcement day.[26] The result is a series of daily attributions with a large negative excess return, with the exception of a large discontinuity on one particular day, which is misleading and undesirable.

To deal with this problem, we use a projection for the next index level, replacing the latest index ratio by a moving update. This allows us to smooth the monthly growth of the index ratio. We define a date, t_{proj}, up to which all cash flows are assumed to be known and form the corresponding index ratio $IR_{t,t_{proj}}$. This moves forward each day until the new CPI is published, at which point the process of projection and updating begins again. We leave the details of this procedure for Appendix B, but the underlying idea is straightforward: we predict a month into the future and advance the latest index ratio daily.

24 So that 365bp represents a measure of inflation expectations across the life of the bond.

25 This is calculated by dividing the spread by the number of days used for accretion which we have assumed for illustration to be 365,

26 This discontinuity on the day of CPI announcement, subject to seasonal effects and shocks to underlying inflation, will largely offset the spread accretion term, in that it is the monthly return on the inflation index.

3.4.3. The full equation

With the modifications described in sections 3.4.1 and 3.4.2 in mind, we state the new pricing equation. If we now consider the index ratio as known up to the point t_{proj}, we can rewrite the pricing equation (7) as follows (we now switch to continuous notation for the discounting and spread effects):

$$B_t = \sum_{i}^{t_i \le t_{proj}} IR_{t,t_i} cf_i \cdot df_{t,t_i} + \sum_{i}^{t_i > t_{proj}} IR_{t,t_{proj}} cf_i \cdot df_{t,t_i} e^{-s_t \cdot (t_i - t_{proj})} \qquad (9)$$

where $cf_i = \begin{cases} 100 \cdot c & if \quad t_i < T \\ 100 \cdot (1+c) & if \quad t_i = T \end{cases}$

Here we explicitly include the known (and projected) cash flows under the first summation sign on the right hand side and write df_{t,t_i} for the discount factor between times t and t_i.

Further, we use s_t to denote the inflation spread which is equal to the negative of the breakeven inflation for the bond.

4. INFLATION RISK FACTORS IN THE GLOBAL RISK MODEL

Having arrived at a modelling framework, we now describe the application of our analysis in the risk modelling arena and thereafter to performance attribution.

4.1. Return splitting

Having cast the inflation-linked security into a familiar form, namely that of a bullet credit bond with the associated spread, we can apply the standard risk model machinery to identify risk factors specific to the inflation market. In this way we seek to model the sources of return of this asset class over and above those that they have in common with nominal securities.

4.1.1. Risk modelling for traditional securities

The calibration of the risk model factors begins, in all cases, with the process of *return splitting*. Here we decompose the total return of each security over the preceding month into various components:

$$\mathrm{Ret}_t^{\mathrm{Total}} = \mathrm{Ret}_t^{\mathrm{FX}} + \mathrm{Ret}_t^{\mathrm{Time}} + \mathrm{Ret}_t^{\mathrm{YC}} + \mathrm{Ret}_t^{\mathrm{Vol}} + \mathrm{Ret}_t^{\mathrm{Res}}$$

Having stripped out the return due to currency movements, the passage of time, the movements and reshaping of the yield curve, and volatility effects[27], we are left with the residual return, $\mathrm{Ret}_t^{\mathrm{Res}}$.

27 This term comes in to play for securities with embedded optionality such as callable bonds.

In the context of a credit security, this component of return would be associated with the movements in spread of an issue.[28] This spread return can be modelled in terms of systematic risk factors and an idiosyncratic residual. Once we have decided the structure of these market-wide sources of risk, we can use our monthly data to put numbers to these risk factors month by month, thereby arriving at time series of factors. We regress the spread return of all the bonds in the relevant calibration universe against both indicative and continuous variables to obtain monthly time series of systematic factors.[29]

4.1.2. Adaptation to Inflation-linked Securities

In the case of inflation-linked securities, residual return relates to inflation factors, both long-term expectations and short-term effects. We see from (9) that, in addition to those sources of return for a nominal security, we have to consider the effect of the realised index ratio. In addition, the interpretation of spread return is now in terms of movements in breakeven inflation rates. Our return splitting equation takes on the following modified form:

$$\mathrm{Ret}_t^{\mathrm{Total}} = \mathrm{Ret}_t^{\mathrm{FX}} + \mathrm{Ret}_t^{\mathrm{Time}} + \mathrm{Ret}_t^{\mathrm{YC}} + \mathrm{Ret}_t^{\mathrm{Vol}} + \mathrm{Ret}_t^{\mathrm{IR}} + \mathrm{Ret}_t^{\mathrm{BEI}}$$

where now $\mathrm{Ret}_t^{\mathrm{IR}}$ is the monthly return due to the change in the $IR_{t,t_{proj}}$ term in the second summation in equation (9) and $\mathrm{Ret}_t^{\mathrm{BEI}}$ is that return due to changes in breakeven inflation, the "spread" of the inflation-linked security over the reference curve.[30]

We see, therefore, that there are two new dimensions of risk for an inflation-linked security over and above those that apply to nominal bonds: changes in realised index ratios and changes in long-term inflation expectations. We consider each category of factor in turn.

4.1.3. The index ratio risk factor

With respect to the first of these risk factors, we note from (9) that the exposure of a security to this factor is simply the fraction of its market value that is represented by the unknown cash flows. The risk factor is, then, the returns of the index ratio. We note that we use the latest projected index ratio available at the time of calibration and not the official index ratio used for quotation of the security price in the market place.

The interpretation of this risk factor should be as a measure of the short-term movements of the inflation index, reflecting the seasonality that plays such a significant part in CPI behaviour.[31] It seeks to capture the difference between the expectations of inflation in the near term and those being released on a monthly basis.[32] Insofar as the CPI growth from one month to the next is not generally in line with that implied by the breakeven inflation rate for the security, a distinct risk factor is necessary for a full description of the risk profile of an inflation-linked security.

4.2. Breakeven factors

Having performed the return split detailed above and isolated the breakeven return term, $\mathrm{Ret}_t^{\mathrm{BEI}}$, we proceed to model the implied change in breakeven inflation for the bond in the following way:

---

28 For credit securities we further strip out the return due to movements of Swap spreads.

29 This process is described fully in out publication 'publication 'The Lehman Brothers Global Risk Model: A Portfolio Manager's Guide' (D.Joneja/L.Dynkin April 2005).

30 We ignore the second order cross terms between $\mathrm{Ret}_t^{\mathrm{IR}}$ and other sources of return for tractability in the same way we already do for FX return in general.

31 For an assessment of the significance of seasonality in inflation indices see 'Understanding Seasonality in Price Indices', Brondolo and Giani July 2005.

32 To be sure, such a difference may be due to a number of factors besides seasonality such as the slope of the breakeven curve, liquidity and inflation risk premia.

Using a linear approximation for spread return (in excess of carry) we write:

$$\text{Ret}_t^{\text{BEI}} \approx -OASD_t * \Delta s_t$$

$$\Rightarrow \quad \text{Ret}_t^{\text{BEI}} \Big/ OASD_t \approx -\Delta s_t = \Delta \pi_t$$

This means that our risk factors take the form of changes in breakeven inflation.

So, for each security, we can isolate a measure of the change in its breakeven inflation across the month. Having done this, we need to identify a parsimonious set of risk factors that capture, as far as possible, the movements of these expected inflation rates for all inflation-linked securities in the various markets.

The description of yield curve and swap spread risk factors currently employed in the model is one of modelling certain key tenors along the respective curves. Intuitively, it makes sense to consider risk factors capturing the movements at different points along the breakeven curve in the same way. In this way, we would be approximating the changes in level and shape of the breakeven inflation curve in a piecewise linear fashion. A one-factor model would correspond to a model that captured only changes in the overall level of the curve, the shift; two factors to one that reflected the shift and some measure of the change in slope; three to shift, slope and curvature effects, and so on. In this way, we avoid the difficult task of fitting an entire inflation curve across all maturities. This choice of factors must not only accurately describe the behaviour of inflation, but must also have an eye to the future. The factors must remain relevant going forward in that sufficient issuance should be expected at the chosen tenors. In addition, they must be chosen in such a way as to be mutually compatible, allowing comparison and integration of inflation risk across markets.

From a practical point of view, the decision of which tenors to use must also address several additional considerations. Firstly, we need to have sufficient issuance across the maturity spectrum to justify the factors. For example, in the Japanese government inflation-linked market, at the time of writing, all four of the distinct outstanding bonds have maturities in the 8-10 year range, making any more than one breakeven factor both unnecessary and unsuitable. Secondly, we seek a parsimonious model, keeping the number of risk factors to a minimum given a certain lower bound on explanatory power of the model. Finally, we want to make the model consistent across markets, choosing the same points on the curve where data permit.

In the search for the optimal factor specification, analysis was carried out to gauge the relative effectiveness of using different number of factors to capture more and more of the inflation curve behaviour. Figure 4 shows the effect on explanatory power, in the sense of monthly cross-sectional (weighted) R^2 for the Sterling index-linked Gilts market[33] using one, two and three terms to model the behaviour of breakeven inflation.[34] Each chart shows the increase in R^2 together with the level of the R^2 for the two-factor model.[35]

Figure 4 demonstrates that there is little evidence that the introduction of a third factor is justified. With the exception of a handful of months, it adds relatively little in terms of explanatory power. Changes in the slope and level of the inflation curve appear to be capturing most of the behaviour of the curve as a whole.

With this is mind, and for markets with a good number and spread of issues, we specify a two-factor model with a short and long factor as described in the equation below:

33 This experiment focused on the Sterling market as it was felt most likely to add explanatory power here given the downward pressures on the end of the Sterling curve peculiar to this market.

34 NOTE: The 3-factor model uses tenors of 5, 15 and 30 years as compared with 5 and 20 years for the 2-factor model. As such, since the 3 factors are not a super-set of those in the 2- factor model, risk factors will not necessarily give a higher R-squared, as is the case in a few months.

35 Note that because the R2is both adjusted and weighted negative values are possible.

$$\Delta\pi_{i,t} = \alpha_{i,t} f_t^{short} + (1-\alpha_{i,t}) f_t^{long} + \varepsilon_{i,t}$$

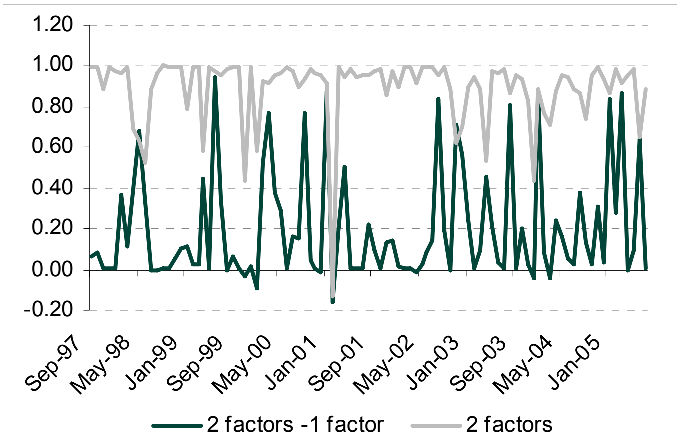
Here, $\pi_{i,t}$ denotes the break-even inflation of bond i at time t and $\alpha_{i,t}$ is a bond-specific interpolation factor determining the degree to which bond i at time t loads on the short and long factors.

Here, the change in the breakeven inflation of security *i* from time *t-1* to time *t* is modelled as an interpolated value between a short and a long breakeven factor. Once we have decided on the tenors of these two factors, we can then apply standard regression techniques (as we do across the entire Risk Model universe of asset classes). We use our cross-sectional data each month to estimate the time series of factor values that best describe the observed movements in breakeven inflation across all securities in that market during the course of the previous month.

The resultant time series allow us to measure the historical relationships between these new sources of risk in our new asset class and those already present in our model and so incorporate them into the Risk Model covariance matrix.

Figure 4. Increase in explanatory power by adding breakeven risk factors (UK market)

Figure 4a. Comparing one- and two-factor specifications for the breakeven spreads



Source: Lehman Brothers.

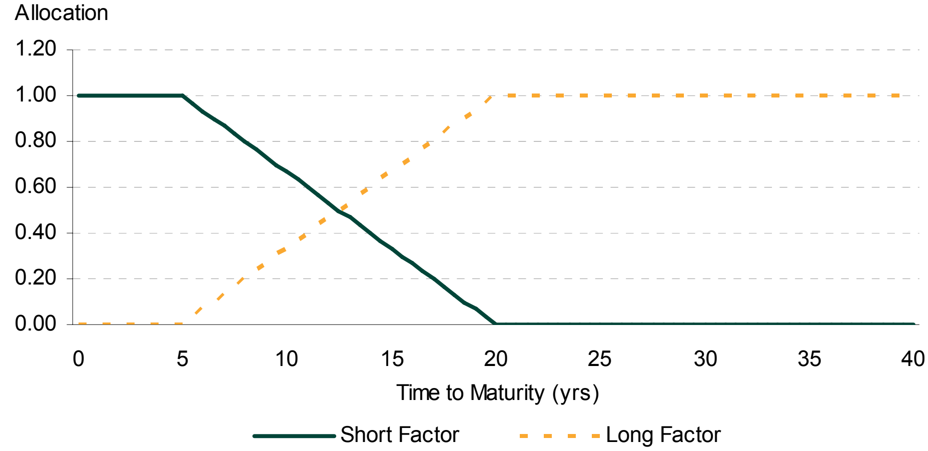
Figure 4b. Comparing Two- and Three-Factor Specifications for the Break-even Spreads

Source: Lehman Brothers.

Having considered the current structure of the markets and indicators as to the likely future points of issuance of the major inflation issuers across the globe, we have opted for 5-year and 20-year factors to capture the short and long end of the inflation curve respectively. In markets where the use of two factors is felt to be excessive, one parallel shift factor is employed.[36] Thus, the loading of bonds on these two factors is determined by their time to maturity in accordance with Figure 5 below:

36 This is the case for Canadian, Japanese and French local inflation markets.

Figure 5. Loadings on breakeven inflation factors as a function of time to maturity



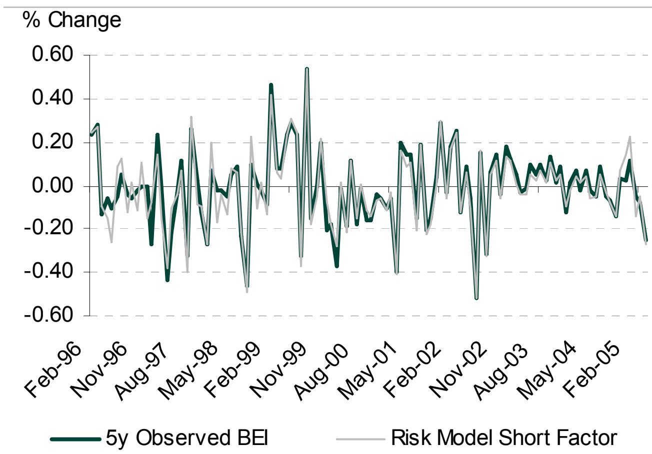
Source: Lehman Brothers.

So, an inflation-linked security of maturity 5-years or less loads only on the short inflation factor, and similarly, one of maturity 20-years or above only on the long inflation factor. Any bond of intermediate tenor loads in a natural fashion on both of the long and short factors.

Figures 6a and 6b show a comparison of the resultant long and short risk model breakeven factors with the observed changes in corresponding fitted constant maturity breakevens in the sterling market (these are interpolated from observed breakeven data).

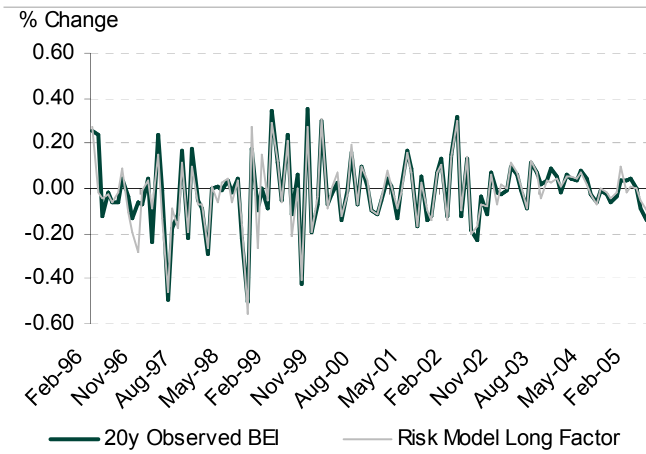
Figure 6. Breakeven inflation risk factors vs. observed (constant maturity) breakevens (UK market)

Figure 6a. Short (5y) factor



Source: Lehman Brothers.

Figure 6b. Long (20y) factor



Source: Lehman Brothers.

The above charts demonstrate that the factors that are produced by our model are very closely related to observed breakevens, with correlations with the constant maturity breakevens in excess of 90% and with similar volatilities.

Figure 7 presents some key correlations and volatilities of the inflation factors in various markets, together with correlations with some nominal factors of particular interest.

Figure 7 Key correlations and volatilities of inflation risk factors in the Global Risk Model[37]

	Volatility (Bp/M)	Correlation				
		Realised Inflation	Expected Inflation		Nominal Par Rate[38]	
			5y	20y	5y	30y
EUR (HICP)						
Realised Inflation	33.2	1.00	-0.26	-0.25	-0.14	-0.25
Expected Inflation						
Short (5y)	11.8	-0.26	1.00	0.45	0.50	0.50
Long (20y)	10.7	-0.25	0.45	1.00	0.19	0.26
UK						
Realised Inflation	30.6	1.00	-0.30	-0.26	-0.08	-0.05
Expected Inflation						
Short (5y)	14.6	-0.30	1.00	0.61	0.47	0.38
Long (20y)	8.7	-0.26	0.61	1.00	0.37	0.46
USD						
Realised Inflation	32.4	1.00	-0.32	-0.30	-0.06	-0.10
Expected Inflation						
Short (5y)	18.0	-0.32	1.00	0.69	0.52	0.53
Long (20y)	14.3	-0.30	0.69	1.00	0.43	0.54

Source: Lehman Brothers.

As we would have expected, correlations of inflation expectations and nominal rates are generally high, at about 50%.

4.3. Risk factor loadings: a worked example

We consider the 2015 $1\frac{5}{8}$ % US TIPS, trading with a closing (real) price on 12 September of \$99.547. If we price this bond using our methodology above, we find a breakeven inflation of 239bp. (The traditional breakeven inflation, defined as the difference between the maturity-matched par yield from the local-market fitted curve and the real yield[39], is 253bp[40]). Using this as the spread under the treasury curve and applying the standard procedure for spread duration calculation (namely moving the spread a small amount while keeping all else constant and repricing), we obtain an OASD of 8.67 years. Similarly, holding the spread constant and shifting the treasury par curve we obtain an OAD of 8.56 years, with exposures to the movement of key rate points along the curve of -0.02, -0.14, 0.70, 8.03, 0.00 and 0.00 years at the 6m, 2y, 5y, 10y, 20y and 30y points, respectively.[41]

Figure 8 shows the loadings of the bond on the relevant risk factors in the Risk Model. Note, the loading on the long-term (breakeven) inflation expectation factors is determined by the time to maturity of the security. In this case, the bond has a time to maturity of 9.34 years. This implies a loading of $\frac{20-9.34}{20-5}=0.71$ on the 5-year factor and $1-0.71=0.29$ on the

37 All volatilities and correlations are as of the end of May 2005 and are calculated using weighting with a half life of 1 year.
38 Note: Cross asset-class correlations are modelled via smaller sets of factors known as core factors.
39 Note: the fitted par yield used is a linear interpolated value between the nearest available fitted par yields (these are available at half-year intervals as explained in footnote 12).
40 All figures are taken from POINT.
41 Due to the heavy weighting of the later cash flows for a linker, in particular the principal repayment, analytically inflation-linked bonds bear some resemblance to nominal zero coupon bonds. This accounts for some negative key rate durations which are also observed in zeroes.

20-year factor. This must then be multiplied by the spread duration, 8.67, to obtain the exposure contributions to each factor.

Figure 8. Loadings of 2015 $1\frac{5}{8}$ US TIPS on risk factors in the Global Risk Model

Risk Factor Name	Unit	Loading
Nominal Yield Curve Factors		
USD 6M key rate	Years of Duration	-0.02
USD 2Y key rate	Years of Duration	-0.14
USD 5Y key rate	Years of Duration	0.70
USD 10Y key rate	Years of Duration	8.03
USD 20Y key rate	Years of Duration	0.00
USD 30Y key rate	Years of Duration	0.00
USD Convexity	Years^2 of Convexity	0.81
Inflation Factors		
US Realised Inflation	% Market Value	100%
US Expected Inflation Short (5y)	Years of Spread Duration	0.71*8.67=6.2
US Expected Inflation Long (20y)	Years of Spread Duration	0.29*8.67=2.5

Source: Lehman Brothers.

4.4. Extending the model to other asset classes

This approach can be applied to the modelling of other asset classes and we briefly describe two.

(a) **Corporate inflation-linked securities**

The above framework naturally lends itself to the integration of credit securities whose cash flows are linked to an inflation index. Our approach is simply to consider the spread of such a security as having two distinct components: an inflation breakeven, as given by that of the nearest government benchmark, and a credit-related spread whose risk characteristics are governed, as they would be for any nominal credit security, by reference to its sector and quality together with the other spread risk factors. In terms of its exposure to risk factors, it simply has exposure to both inflation factors and nominal credit risk factors (in addition to yield curve, swap spread and volatility as per any bond).

(b) **Inflation swaps**

The inflation swap is, in theory, little more than a position in a nominal zero coupon bond with an equal and opposite position in a zero coupon inflation-linked bond.[42] In addition, there will be an exposure to the "inflation swap spread", the spread of the inflation swap curve over the inflation bond curve as for interest rate swaps. An approach that models this inflation swap spread, the characteristics of whichmay differ from their nominal counterpart[43], and combines the risk factors already calibrated for the cash bond inflation market, provides an immediate method for the inclusion of inflation swaps into the risk and performance attribution models.

42 Here we have in mind the zero coupon inflation swap being the prevalent form of the instrument at present.
43 A thorough empirical study to assess the nature of this risk factor is warranted.

4.5. Risk model implied analytics

We are now in a position to outline two new analytical measures made possible by the framework developed in the preceding sections.

4.5.1. Risk model empirical OAD

Equipped with our new inflation risk factors, we are in a position to calculate additional measures of exposure to nominal yield curve movements. We now have use of the historical relationships between changes in key rates on the nominal curve and those of the CPI and breakeven factors, as summarised in the risk model covariance matrix. This allows us to produce empirical sensitivities to parallel shifts in the nominal curve; an OAD that incorporates the correlations of inflation and nominal yields.

One particular difference exists between the approaches we have already outlined in section 2.3.3 and this new risk model based approach. The previous methodologies are based, in practice at least, on daily regressions, whereas risk model covariances use data at monthly intervals and so monthly correlations are the result. They differ, often substantially. Noise that is prominent in daily data should be smoothed by using monthly frequency data. All other things being equal, this should, lead to higher correlations. Patterns of high or low correlations that persist for short time periods are not captured by methodologies based on monthly data, whereas if data at daily intervals are used these will be reflected in correlation estimates.

In short, it is a case of "horses for courses". Daily data-driven betas of real yields against one nominal benchmark yield suit those who seek to establish hedge ratios for the real bond using the closest nominal and whose intuition is that betas change significantly even on a very short-term basis.[44] Monthly-based correlations, across multiple points on the nominal curve with a wider set of inflation risk factors, are possibly more relevant to managers of real and hybrid real-nominal portfolios rebalancing at monthly intervals who believe in longer term underlying betas between real and nominal yields. They may wish to compare sensitivities of their real bonds with the classical OAD of nominal securities, defined as the sensitivity to a parallel shift of the entire yield curve.[45]

4.5.2. Nominal OAD

Finally, we note that our pricing equation (9) allows us to compute a nominal OAD where we choose the single inflation assumption to be that breakeven spread that is the result of pricing the security. In POINT, it is this "Model Bond OAD" that is reported as simply "OAD" for real securities.

4.6. Risk reports for portfolios with inflation-linked bonds

This section briefly samples the implementation of the model that we have described above as seen in POINT. We begin with an analysis of a specific portfolio against the Lehman Brothers Global Treasury Index (nominal). We choose a portfolio that is a composite of inflation-linked and nominal treasury indices in US dollar, sterling and euro.[46]

We begin by looking at Figure 9 which shows the Tracking Error Report. This report summarises the estimated risk of the portfolio relative to the benchmark by high level asset

---

44 Or more accurately, who believe that daily historical relationships are more relevant in predicting likely future behaviour than a longer term outlook.

45 It should be noted that the sensitivity of a real yield to the benchmark yield will not in general give the same results since the parallel shift corresponds to a more restrictive subset of yield curve scenarios than the more general movements of one particular point on that curve. To do this in a similar framework, changes in real yields could be simultaneously regressed against several (preferably constant maturity) points on the nominal curve such as key rates. The sum of the resulting estimated coefficients would yield the required sensitivity to changes in the level of the nominal curve.

46 We use a composite of 70% nominal Treasuries (40% US dollar, 20% euro and 10% sterling) and 30% inflation-linked treasuries (10% US TIPS, 10% euro inflation-linked and 10% inflation-linked Gilts).

class components. The first column, labelled "Isolated TEV" is a measure of risk adjusted exposure that combines exposures and covariances that relate to yield curve factors exclusively, ignoring cross terms with other components of risk. In our example, we note that the exposure to yield curve factors is approximately 58bp of return volatility per month. This compares with 23bp of risk arising from inflation-related sources. These two are the principal sources of risk in the portfolio and combine, taking cross correlations into account, to 49bp per month, as seen in the second row of column 2 ('Cumulative TEV'). It is immediately apparent therefore, that the inflation exposure serves to reduce the overall risk implied by the yield curve exposure. This effect is also observable from the fourth column, "Percentage of tracking error variance". In this column, on the line corresponding to inflation risk, we see -7.68% as the estimate for the contribution to risk arising from the inflation-linked sources of risk as seen in a portfolio context. This implies that the positive correlation between inflation expectations and nominal yields in effect reduces exposure to the yield curve. Indeed, the difference between the analytical nominal OAD of the portfolio and benchmark is calculated to be 0.99 years, whereas when the correlation between inflation factors and nominal yields is taken into account this drops to zero.[47]

Figure 9. Sample tracking error volatility report

LEHMAN BROTHERS | POINT Global Risk Model

Tracking Error 10/7/2005

Portfolio : composite_LMRQ
Benchmark : Global Treasury

Global Risk Factor	Isolated TEV (bps)	Cumulative TEV (bps)	Contribution to TEV (bps)	Percentage of tracking error variance (%)	Systematic beta
Global					
Yield Curve	57.89	57.89	53.25	107.75	1.54
Inflation	22.72	49.56	-3.8	-7.68	
Swap Spreads	0.64	49.58	0.03	0.06	0.81
Volatility	0.01	49.58	-0.0	-0.01	2.04
Investment-Grade Spreads	1.34	49.33	-0.23	-0.46	0.72
Treasury Spreads	1.34	49.33	-0.23	-0.46	0.72
Systematic risk	49.33	49.33	49.25	99.66	1.38
Idiosyncratic risk	2.88	49.42	0.17	0.34	
Credit default risk	0.0	49.42	0.0	0.0	
Total risk		**49.42**	**49.42**	**100.0**	
Portfolio volatility (bps/month)					114.7
Benchmark volatility (bps/month)					78.24

Source: POINT.

Figure 10 shows the details of the active exposures to inflation related risk factors.

The portfolio holds only inflation-linked bonds, and so it is only in the first of the two columns that we see non-zero exposures.

As explained in section 4.3 the loading on the realised inflation factor in each market will be the total market value represented by the inflation-linked securities in that market.[48] Since our portfolio is in fact 30% by market value in inflation linkers, 10% in each of euro, US dollar and sterling the four inflation expectation exposures (US, HICP, French Local and UK) sum to 30%. If we move along the first row of this section of the report we see that the volatility of this factor in the US is approximately 32bp. The fifth column labelled "TE impact of an isolated 1 std. dev. up change" brings together the exposure of 10% and the

47 However, it should be noted that the bulk of the yield curve risk remains as it is spread across several curves of different sovereigns and is only neutral to a simultaneous parallel shift of equal amount in all currencies.

48 To be precise, it will be the market value of the unknown cash flows considering the index projection as known for this purpose.

volatility of 32bp to imply an outperformance of 3.2bp for a 1 standard deviation upward move in this factor, holding all else constant. The next column presents the result of the same movement but now on a correlated basis, moving all other risk factors by their conditional expectation given the 32bp move in US realised inflation. On this basis the outperformance is expected to be rather less, at approximately 1.6bp. This is due to the positive correlation between CPI and nominal rates with respect to which we are long duration.

If we now consider the expected inflation, or breakeven, factors we note that the overall exposure of an inflation-linked bond to this factor will be its option-adjusted spread duration (OASD) multiplied by the interpolation factor that determined how much it loads on the short factor and how much on the long. For US short expected inflation for example, on row 2 of the report, we see that the total exposure to this factor is 0.337 years of spread duration. The volatility of the factor is estimated at 18.5bp/m implying an outperformance of 6.2bp for an upward move of 1 standard deviation. On a correlated basis, however, reflecting the likely upward move in nominal yields that would accompany an increase in inflation expectations, this changes to an underperformance of about 12.6bp. For further explanation of the report see our publication *The Lehman Brothers Global Risk Model: A Portfolio Manager's Guide* (D.Joneja/L.Dynkin April 2005).

Figure 10. Inflation risk factors in a sample factor exposure report

Factor name	Sensitivity/Exposure	Portfolio exposure	Benchmark exposure	Net exposure	Factor volatility	TE impact of an isolated 1 std. dev. up change (bps)	TE impact of a correlated 1 std. dev. up change (bps)	Marginal contribution to TEV (bps)	Percentage of tracking error variance (%)	Contribution to TEV (bps)
INFLATION										
US Realized Inflation	MW%	10.0	0.0	10.0	0.32	3.2	1.63	0.011	0.21	0.11
US Expected Inflation Short	OASD (Yr)	0.337	0.0	0.337	18.46	6.22	-12.62	-4.715	-3.21	-1.59
US Expected Inflation Long	OASD (Yr)	0.497	0.0	0.497	14.54	7.22	-10.61	-3.12	-3.13	-1.55
EuroZone Realized Inflation	MW%	6.49	0.0	6.49	0.34	2.21	8.21	0.057	0.74	0.37
EuroZone Expected Inflation Short	OASD (Yr)	0.242	0.0	0.242	11.96	2.89	-7.05	-1.707	-0.83	-0.41
EuroZone Expected Inflation Long	OASD (Yr)	0.389	0.0	0.389	10.6	4.12	5.85	1.256	0.99	0.49
French Realized Inflation	MW%	3.51	0.0	3.51	0.29	1.03	7.4	0.044	0.31	0.15
French Expected Inflation	OASD (Yr)	0.275	0.0	0.275	9.44	2.6	0.17	0.033	0.02	0.01
UK Realized Inflation	MW%	10.0	0.0	10.0	0.31	3.14	2.94	0.019	0.38	0.19
UK Expected Inflation Short	OASD (Yr)	0.338	0.0	0.338	14.95	5.05	-4.52	-1.368	-0.93	-0.46
UK Expected Inflation Long	OASD (Yr)	0.788	0.0	0.788	8.87	6.99	-7.76	-1.393	-2.22	-1.1

Source: POINT.

The next section deals with the return attribution for inflation-linked securities.

5. Performance attribution

Having adopted a risk view for inflation linkers that accounts for the exposure to the nominal government curve and to inflation growth expectations separately, we follow the same methodology for the attribution of daily returns, making the analysis of inflation-linked bonds similar to the analysis of credit securities. We calculate the following return components:

(a) *Return due to the change of the government curve*. The sensitivities that drive this return are the key-rate durations of the security (computed by holding the inflation spread constant, i.e. ignoring any correlation between nominal rates and inflation).

(b) *Carry return earned because of the exposure to the government curve*. Here, the sensitivities are the weights of a portfolio of hypothetical par government securities that have the same key-rate durations as the linker (curve-matched portfolio). This carry return is partially offset by the carry return lost because of the negative inflation spread.

(c) *Return due to changes of implied volatility.* The exposure of inflation bonds to implied volatility is negligible as their only optionality stems from the principal guarantee of most bonds, which is unlikely to have any value as inflation is generally positive.

(d) *Return due to changes of inflation spread.* Return due to changes in long-term expected inflation growth, liquidity and risk premia etc as expressed by the change of the spread of the issue. The sensitivity for this component is the spread duration of the security.

(e) *A negative spread carry*. This arises because of the negative spread that represents the expected growth of inflation. Here, the sensitivity is the percentage of the market value weight of the unknown cash flows.

5.1. The inflation accretion return

All the above return components are similar to those of any other credit issue. However, inflation-linked securities have an additional component of return, namely that due to inflation accretion. This return, which is driven by the monthly realisation of the inflation index, counterbalances the return lost because of the negative spread carry. In a risk neutral and prescient world, the return lost because of spread carry would be exactly equal to the return gained from inflation accretion, such that the total return of the linker would be equal to the return of a standard nominal security of the same term. In practice, of course, these returns do not cancel each other, although they do have opposite signs most of the time. In the short term, seasonality effects and differences between short-term and long-term inflation growth expectations may generate significant discrepancies between these two returns. In the long term, their difference should reflect the risk and liquidity premium (or penalty) of inflation linkers.

The exact amount of inflation accretion becomes known discretely once a month, on the announcement day. As discussed before, if we updated our pricing equation monthly, any return analysis performed between two announcement days would show an accretion return of zero, something less than desirable. For this reason, we have decided to project the inflation for the first unknown month on a daily basis – one day at a time – following the inflation announcement day. We use this estimate (which may change from day to day) to compute accretion for each passing day, thus countering the daily carry lost to spread until the next announcement day. On that day, when the true inflation becomes known, we treat it as a final (accurate this time) estimate and we correct our monthly accretion. Thus on each day we have two accretion components of return:

(f) *Inflation accretion.* One-day accretion using the inflation projection.

(g) *Inflation surprise.* A correction term for all days elapsed since the last announcement day reflecting the difference between yesterday's and today's inflation projection (or realisation).

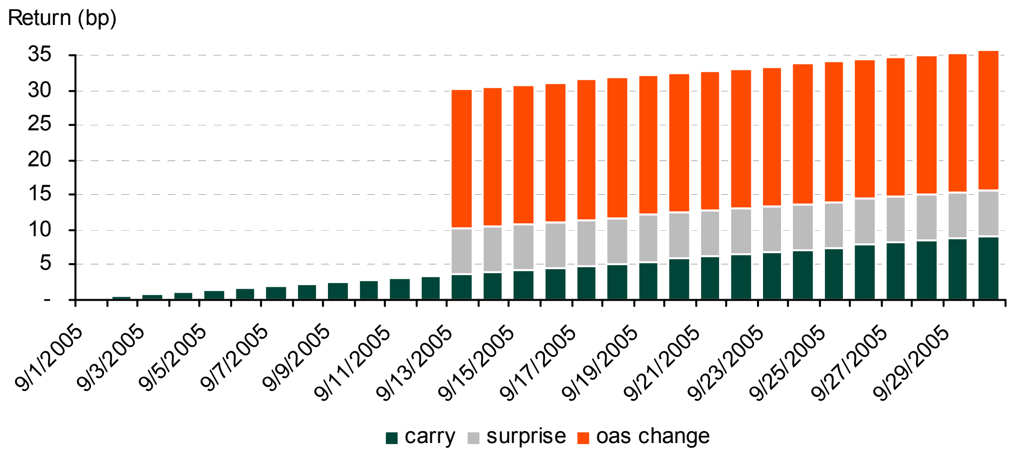
5.2. An example

Consider an inflation-linked bond which trades at a spread of -200bp relative to the nominal curve. This means that the average inflation expectation over the life of the security is approximately 2% per annum. Let us also assume that because of seasonality the short-term inflation for the next month is projected to come in higher at 3%. The security begins earning a daily carry return of (300 – 200)/365 = 0.27bp per day.

Let us assume that nothing changes until the index announcement day, at which point the announced inflation comes in higher than the expectation at a 3.5% annual rate. This means that the annual carry rate used since the previous announcement day (a period of approximately one month) was lower by 50bp. We correct for that by adding a one-time "inflation surprise" return component of approximately 50/12 = 4.1bp.

After the realised inflation is announced, longer term inflation expectations are affected and the spread of the bond grows in magnitude by falling to -210bp. At the same time we form a projection for the inflation rate of the subsequent month, let us assume 2.5%. The rate at which carry is earned now is (250 – 210)/265 = 0.11bp per day. Finally, the change of long-term inflation expectations generates a return proportional to the spread duration of the bond. Even if we assume that the spread duration is small, say 2 years, this return will be quite significant at – (–210 + 200)×2 = +20bp. Figure 11 below illustrates the inflation return growth of a linker over the course of a month.

Figure 11. Return growth of an inflation-linked security



Source: *POINT.*

5.3. Return components – detail

Let us now examine how the pricing equation (9) is being used to calculate the various return components of inflation-linked securities. Recall that equation (9) sets the inflated price of the security equal to a function of time, nominal interest rates, the inflation spread and the currently projected index ratio, as follows:

$$B_t = \sum_i^{t_i \le t_{proj}} IR_{t,t_i} cf_i \cdot df_{t,t_i} + IR_{t,t_{proj}} \sum_i^{t_i > t_{proj}} cf_i \cdot df_{t,t_i} e^{-s_t \cdot (t_i - t_{proj})}$$

(a) Curve change return

The exact effect of the change of the discounting curve is computed by repricing the security using the last business day's nominal closing curve but keeping today's inflation spread unchanged, following the methodology established for standard credit securities.

$$\hat{B}_t = \sum_i^{t_i \le t_{proj}} IR_{t,t_i} cf_i \cdot df_{t,t_i}^{prev} + IR_{t,t_{proj}} \sum_i^{t_i > t_{proj}} cf_i \cdot df_{t,t_i}^{prev} e^{-s_t \cdot (t_i - t_{proj})}$$

Assuming no cash flows[49] at time t, the return due to curve change is: $\dfrac{B_t - \hat{B}_t}{\hat{B}_t}$.

In addition, we compute the key-rate durations in a similar manner to any other security, by perturbing the nominal par curve but keeping the inflation spread over the resultant discount curve unchanged. We can then split the above total return due to curve change into the contribution of each key-rate separately, as well as a residual term that includes the incremental effect of all other curve points plus convexity.

(b) Curve carry

Curve carry is also computed in the same way as for standard securities. A key-rate duration-matching portfolio of hypothetical par zero-spread bonds and cash is constructed, and its carry return is theoretically calculated. The curve carry return of the security is set equal to the carry return of the curve-matching portfolio.

(c) Volatility return

Volatility return is computed in a similar fashion to the curve change calculation, by repricing the security using today's curve and spread but yesterday's volatility parameters.

(d) Spread change return

As mentioned before, for all practical purposes we treat the inflation spread as equivalent to the option-adjusted spread of the security. We thus define the return that is induced by changes in inflation spread to be the spread change return of linkers. We also define the price sensitivity to changes of the inflation spread as the option-adjusted spread duration of the security. Note however, that unlike regular securities where the spread discounting is applied to all cash flows after the settlement day, the inflation spread discounting (actually accretion since the spread is negative) term is only applied to cash flows after the last projected ratio date.

(e) Spread carry return

Since spread discounting is only applied to cash flows after the last projected ratio date, we cannot use the simple $s_t \times \Delta t$ formula to compute spread carry. Instead, we have to carefully compute the sensitivity of the inflated price to the spread discounting term.

$$R_{\text{spread\_carry}} = \frac{1}{B_t} \cdot IR_{t,t_{\text{proj}}} \sum_{i}^{t_i > t_{\text{proj}}} cf_i \cdot df_{t,t_i} e^{-s_t \cdot (t_i - t_{\text{proj}})} \cdot s_t \cdot \Delta t_{\text{proj}}$$

If we now define the "modified price" as the present value of the cash flows beyond the projected date t_{proj}:

$$B_t^{MOD} = IR_{t,t_{\text{proj}}} \sum_{i}^{t_i > t_{\text{proj}}} cf_i \cdot df_{t,t_i} e^{-s_t \cdot (t_i - t_{\text{proj}})}$$

we can re-write:

$$R_{\text{spread\_carry}} = \frac{B_t^{MOD}}{B_t} \cdot s_t \cdot \Delta t_{\text{proj}}$$

49 On cash flow days the formula is slightly more complicated.

We have to remember here that while generally the last projection date follows the movement of the pricing date, on inflation announcement days the projection date may move in a discontinuous fashion, as explained in footnote 57 to Appendix B.

(f) Inflation accretion

Each day, the projected index ratio grows by a rate implied by our next month index projection. The index ratio may also change if the CPI projection changes, or on the announcement day, when the actual CPI becomes known. The total change of the index ratio from day t to day t^{next} is derived from equation (6) in Appendix B as:

$$IR_{t^{\text{next}},t_{\text{proj}}^{\text{next}}} - IR_{t,t_{\text{proj}}} = \Delta^{\text{time}}\left(IR_{t,t_{\text{proj}}}\right) + \Delta^{CPI}\left(IR_{t,t_{\text{proj}}}\right)$$

$$\Delta^{\text{time}} IR_{t,t_{\text{proj}}} = \frac{CPI_{t^{\text{next}}}^{\text{proj}} - CPI^{\text{known}}}{DIR_0} \cdot \frac{\text{Days}\left(t_{\text{proj}}^{\text{next}} - t_{\text{proj}}\right)}{\text{DaysOfMonth}\left(t_{\text{known}}\right)}$$

$$\Delta^{CPI} IR_{t,t_{\text{proj}}} = \frac{CPI_{t^{\text{next}}}^{\text{proj}} - CPI_{t}^{\text{proj}}}{DIR_0} \cdot \frac{\text{Day}\left(t_{\text{proj}}\right) - \text{Day}\left(t_{\text{known}}\right)}{\text{DaysOfMonth}\left(t_{\text{known}}\right)}$$

While the part of the index ratio change that is driven by changes in the projected CPI is addressed separately, the time component of the index ratio change generates an inflation accretion return which is given by the following formula:

$$R_{\text{accretion}} = \frac{B_t^{MOD}}{B_t} \cdot \left(\frac{\Delta^{time} IR_{t,t_{proj}}}{IR_{t,t_{proj}}} \right)$$

On the announcement day we use the announced actual inflation index level instead of the projected level in the above formula.

(g) Inflation surprise

On the announcement day, the announced rate of inflation is in general different from our projection as of the previous day. This causes a "surprise" return driven by the change of the projected index ratio $IR_{t,t_{proj}}$. In addition, any cashflows that fall between the last known ratio date and the last projected ratio date, which have been estimated using the projected inflation rate, must be adjusted to use the correct index ratio computed using the announced value of the index. To simplify the computation of the surprise return, we compute, on a daily basis, an "inflation index duration", $IndexDur$, that measures the sensitivity of the return of the linker to changes in the projected CPI, such that:

$$B_t \cdot IndexDur \equiv \frac{\partial B_t}{\partial CPI_t^{\text{proj}}}$$

Then, to calculate the impact of a revision in the CPI projection (or the effect of the announced CPI differing from the projected level we have been using to date), we simply compute the inflation surprise return as:

$$R_{\text{surprise}} = IndexDur \cdot \Delta CPI_t^{\text{proj}}$$

On the announcement day, the projected index change is set equal to the difference between the realised inflation index and the last projection. On any other day, to the extent that our projection does not change, this term is zero. If the projection is revised, then we classify the return due to this change as a "surprise" return since it is presumably the result of new information.[50]

5.4. Incorporation in the HPA framework

The curve and volatility components of return, as well as the spread change return fall seamlessly into the attribution framework, like the corresponding return of any other security. The spread carry and the inflation accretion components are added together since they jointly represent the liquidity and risk premium return of the security.[51] This combined number is attributed as spread carry return. Finally, the inflation surprise return is reported as a separate component of return in the fashion of a prepayment surprise return in the realm of securitised products such as MBS. Ultimately, it should be accounted together with the inflation accretion component since it is essentially the residual between projected and realised inflation accretion.

Figure 12 below, displays the "Security Return Splits" report that the Lehman Hybrid Attribution Model generates when run on a portfolio of US TIPS for the month of September. We can see that all securities had a large negative (nominal) curve change return which was partly compensated by (inflation) spread change gains.

Figure 12. Return Splits Report

LEHMAN BROTHERS | POINT

USD : Security Return Splits

Portfolio: INFL : TIPS Index **8/31/2005 to 9/30/2005**

Benchmark: Inflation Linked US TIPS **Base Currency: USD**

	Coupon %	Maturity	Ticker	Curve Carry bps	Curve Change bps	Volatility bps	Spread Carry bps	Spread Change bps	Inflation Surprise	Residual bps	Total Return bps
0 - 2 years											
9128272M	3.375	1/15/2007	US/TII	32.7	-43.3	0.0	40.3	16.0	10.5	-4.5	51.7
2 - 4 years											
9128273T	3.625	1/15/2008	US/TII	33.5	-77.6	0.0	41.6	55.2	10.6	-4.6	58.7
9128275W	4.25	1/15/2010	US/TII	33.9	-139.3	0.0	42.7	104.3	10.6	-4.3	47.9
4 - 6 years											
912828CZ	0.875	4/15/2010	US/TII	33.9	-156.4	0.0	38.0	109.0	10.5	1.0	36.0
9128276R	3.5	1/15/2011	US/TII	34.0	-170.5	0.0	41.8	116.5	10.6	-1.9	30.5
9128277J	3.375	1/15/2012	US/TII	35.1	-197.0	0.0	41.2	131.0	10.6	-0.5	20.4
6 - 8 years											
912828AF	3	7/15/2012	US/TII	35.7	-210.3	0.0	40.9	135.2	10.5	0.2	12.2
912828BD	1.875	7/15/2013	US/TII	37.3	-239.2	0.0	39.7	129.9	10.5	1.1	-20.7
8 - 10 years											
912828DH	1.625	1/15/2015	US/TII	40.0	-272.0	0.0	37.7	140.1	10.5	1.6	-42.1
912828CP	2	7/15/2014	US/TII	25.7	-183.3	0.0	20.4	97.6	10.5	2.6	-26.5
912828EA	1.875	7/15/2015	US/TII	40.8	-280.6	0.0	36.8	140.2	10.5	3.2	-49.1
Over 10 years											
912810FD	3.625	4/15/2028	US/TII	40.3	-474.9	0.0	43.0	315.5	10.4	-11.2	-76.9
912810FQ	3.375	4/15/2032	US/TII	39.4	-513.1	0.0	38.9	326.8	10.3	-9.1	-106.8

The magnitude of the negative returns due to rising interest rates is about double the magnitude of the positive returns due to increasing inflation expectations. We also see that all securities exhibit a positive inflation (surprise) return, which means that the August index (which was announced on September 15) was higher than the expectation we used in our

50 *Note though that on the day after the announcement day this term is by definition zero, since no projection is being used to price the security on the announcement day itself.*

51 *Plus the effects of seasonality and the existence of non-zero inflation slope.*

pricing model. The spread carry return that combines inflation accretion with return loss due to the negative spread is positive, indicating that the short-term expected inflation was much higher than the long-term expected inflation implied from the spread of these securities. Indeed, CPI grew in August by about 50bp, while inflation spreads imply a long-term inflation expectation of about 2.4% per annum generating a loss of about 20bp per month.

In Figure 13 we can see the "Asset Allocation" report from the same run. The portfolio is being compared with the Lehman "Inflation Linked US TIPS" index. A duration partition has been used to identify positions on the term structure of inflation. As we see in the report, the portfolio has overweighted short-term inflation and underweighted long-term inflation. In the column "Benchmark OAS Change" we see how inflation spreads have moved in September for different duration buckets. Short-term inflation expectations increased by about 13.3bp (as indicated by the negative spread change), whereas medium- and long-term inflation expectations rose much more, 27bp and 20bp respectively. The large overweight of short-term inflation was a poor position since short-term inflation increased less than the average for the benchmark. The large underweight in long-term inflation was less important as the move of long-term inflation was similar to the one of the benchmark as a whole.

Figure 13. Asset Allocation Report

LEHMAN BROTHERS | POINT

USD : Asset Allocation - Duration: OA (2 year)

Portfolio: INFL : TIPS Index **8/31/2005 to 9/30/2005**

Benchmark: Inflation Linked US TIPS **Base Currency: USD**

Level 1: Duration: OA (2 year)

Portfolio Market Weight: 100.00

Contribution to Portfolio Spread Duration: 6.41

	Benchmark OAS (bps)		Market Weight (%)					Outperformance (bps) Explained by Allocation		
			Average		Port Minus Bench					
	Init Level	Change	Port	Bench	Mean	Min	Max	Market Weight	Spread Duration	Total
0 - 2 years	-288.9	-13.3	18.0	5.8	12.2	12.2	12.3	-1.1	-0.9	-2.0
2 - 4 years	-252.6	-27.1	13.4	16.1	-2.7	-2.7	-2.7	0.1	-0.2	-0.1
4 - 6 years	-232.6	-24.4	17.1	13.2	3.8	3.8	3.8	0.1	1.5	1.6
6 - 8 years	-231.2	-18.8	22.2	24.3	-2.2	-2.9	-0.9	-0.1	0.1	0.0
8 - 10 years	-236.6	-16.2	17.1	13.3	3.8	2.6	4.6	0.1	-0.5	-0.4
Over 10 yea	-246.1	-19.5	12.3	27.4	-15.0	-15.1	-15.0	0.1	0.1	0.2
Total	-243.1	-19.7	100	100	0	0	0	-0.8	0.1	-0.7

Extending this algorithm to corporate inflation-linked securities is straightforward once we split the spread of the security into a credit and an inflation component. Having done that, the inflation component will be allocated and reported separately from the credit spread components.

6. Conclusion

We have presented in this paper the approach that we have developed in modelling the risk and return attributes of securities whose cash flows are linked to inflation indices in various markets. We have shown that a simple framework that treats such securities as a variation on the more familiar credit asset class serves us well in enabling analytical properties to be calculated and thus to define and estimate risk factors and performance components. We define a spread-based pricing equation and find that the spread associated with these securities is negative, broadly representing a measure of breakeven inflation, and an expectation of future inflation across the life of the bond.

We defined long and short inflation expectation (or breakeven) factors in the major markets (US TIPS, UK inflation-linked Gilts, HICP linked euro-denominated government debt and Swedish government inflation-linked securities.). In smaller markets or those concentrated about one particular maturity, namely those of Canada, Japan and French local inflation, we use a single-factor to capture the shift in the inflation curve. For each market we use the returns on the (projected) index level as an additional risk factor.

In seeking to attribute the return of such securities we are able to identify and quantify several inflation-related components of return such as inflation surprise due to shocks to underlying inflation as well as a combined term that captures the inflation risk premium, liquidity and seasonal effects.

Our methodology further allows analytical quantities to be defined and calculated such as empirical OAD as a measure of sensitivity to a parallel shift across the nominal yield curve as a whole, as compared with the prevalent beta-based approach which measures sensitivities to a single benchmark nominal yield.

Finally, the application of this work to the asset classes of corporate inflation-linked securities and inflation swaps is straightforward, with the former already being processed by the Risk Model and the latter envisaged as following in the near future.

REFERENCES

Brondolo A. and Giana G. (2005): *Understanding Seasonality and Its Impact on Inflation Assets*, Fixed Income Strategies, Lehman Brothers.

Desclée A, Klaeffling M. and Mendes-Vives D. (2003): “A Guide to Inflation-linked Government Bonds in the Euro Area” *Liquid Markets Research Quarterly,* Lehman Brothers, Vol. 2003-Q3.

Sack B. and Elasser R. (2002): Treasury Inflation-Indexed Debt: A Review of the U.S. Experience.

APPENDIX A: CASH FLOW CALCULATION

In this appendix we present the mechanics of cash flow calculation in the various inflation-linked markets.[52]

The cash flows for inflation-linked securities are computed as follows. For the coupons, we adjust the stated fixed coupon of the bond (the "real coupon") for inflation, to obtain the actual coupon paid. The formula is:

$$c_{\text{Paid}} = c \cdot IR_t \cdot N \tag{1}$$

Where c and c_{Paid} represent the real and actual coupons paid, respectively, N denotes the par amount, and IR_t the price index ratio. The index ratio is computed by dividing a daily inflation reference index, DIR_t, by the value of the daily index at the "base date"[53] that does not change over the life of the bond. That is:

$$IR_t = \frac{DIR_t}{DIR_0} \tag{2}$$

The daily inflation reference index is necessary in order to compute accrued interest on a daily basis, and is computed by interpolating between two observations of the officially published inflation index. The fact that there is a delay in publishing the inflation figures means that the indexation is with respect to lagged inflation and is, hence, imperfect. Because the price index of a given month m-2 is always published before the end of the following month m-1, it is always available to calculate the daily inflation reference for each day of month m. Let CPI_m denote the index level as of the end of month m. For any day t of a given month m, the inflation reference is obtained by interpolating between CPI_{m-3} and CPI_{m-2} according to the formula:

$$DIR_t = CPI_{m-3} + \frac{nbd(t)-1}{ND_m} \cdot \left(CPI_{m-2} - CPI_{m-3}\right) \tag{3}$$

where $nbd(t)$ denotes the number of business days between date t and the start of the month m, ND_m the number of days in month m, and CPI_{m-i} the price index of month *m-i*. The result is rounded to the fifth decimal place. Note that all indexation is done with respect to the first estimates for inflation and even if these are later updated, the updated numbers are never used for the purpose of recalculating the cash flows..

The bond is redeemed at maturity at the nominal, adjusted for the inflation ratio. If this is lower than the base date, the bond is redeemed at par:

$$PRINCIPAL_{\text{maturity}} = \max\left(100, \frac{IR_T}{IR_0} 100\right) \tag{4}$$

In the US, Canada and the UK, the coupons are paid semi-annually. The US inflation reference is the non-seasonally adjusted US city average all-items CPI for all urban consumers, published by the Bureau of Labor Statistics. The daily reference index is

52 *This appendix is drawn from Desclée, Klaeffling and Mendes-Vives "A Guide to Inflation-linked Government Bonds in the Euro Area" (LMRQ October 2003).*

53 *Typically the issue date of the bond.*

computed as in the French case, with a three-month lag. The US also guarantees redemption at par in case of deflation over the life of the bond.

The Canadian "real return bond" cash flows are computed as in the US, with the inflation index being the all-items CPI non-seasonally adjusted, published monthly by Statistics Canada (Bloomberg ticker CACPI <Index >). Contrary to the US, Canadian inflation-linked bonds do not have a guaranteed redemption at par.

In the UK, the coupons are semi-annual and the index is the General Index of Retail Prices or RPI all-items non-seasonally adjusted (Bloomberg ticker UKRPI <Index>). An important feature of the UK market is that the inflation used is lagged eight months[54] (two months to allow for the compilation and publication of the RPI, and six months to ensure that the nominal size of the next coupon payment is known at the start of each coupon payment for accrued interest calculations). In order to compute the daily inflation reference index, we use the formula:

$$DIR_t = RPI_{m-8} + \frac{nbd(t)-1}{ND_m} \cdot \left(RPI_{m-7} - RPI_{m-8}\right) \tag{5}$$

Note that there is no redemption floor at par in the case of UK index-linked bonds.

In Sweden, apart from the usual inflation-linked coupon bonds (maturing in 2008, 2015, 2020 and 2028), the Riksgalds Kontoret (Swedish National Debt office) has also issued two inflation-linked zero coupons (one already matured in 2004 and one maturing in 2014). Cash flows are computed similarly to the French case (three-month lag). The coupon is annual and paid on December 1. One peculiarity is that only two of the coupon bonds have redemption at par guaranteed in case of deflation (2015 and 2028); all other coupon bonds including the zero coupons, do not.

54 As of June 30, 2005 the DMO announced that it would be issuing inflation-linked gilts with a three-month lag going forward. These will be processed in the Risk Model in the appropriate way.

APPENDIX B: THE PROJECTED INDEX RATIO $IR_{t,t_{proj}}$

As explained in section 3.4.2, we face the problem that observation of the index ratio occurs as a jump process with an announcement at some time during the month. This consists of a statement of the CPI level for a month in the past with reference to which we determine the baseline for all future growth of inflation.

On the index announcement day, t_{ann}, all cash flows occurring on or before a particular future day, t_{known}, are fully specified. The size of cash flows occurring after t_{known} is still uncertain since the appropriate index ratio is unknown. As we move forward in time no new information becomes available until the next announcement day (after about one month), on which date the index ratio for the period of approximately 30 days following t_{known} will become specified.

To remove this discontinuity we introduce an "inflation estimate" for the first unknown month. On each announcement day we project the index value, CPI^{proj}, in each market for the next month. On the next day, $t_{\text{ann}}+1$, we use this projection to calculate a projected value for the index ratio corresponding to the day following t_{known}. We do this by interpolating between the last known index CPI^{known} and the projected index CPI^{proj}. The following day, $t_{\text{ann}}+2$, we interpolate the index level for another day, advancing the *projected index ratio* one further day and so on. This continues throughout the month, using CPI^{proj} to extend our daily index ratio calculations day by day. When we reach the day of the next CPI announcement (that we have been projecting throughout the course of the month) we begin again, now setting the new CPI to the latest known index level and projecting next month's announcement.

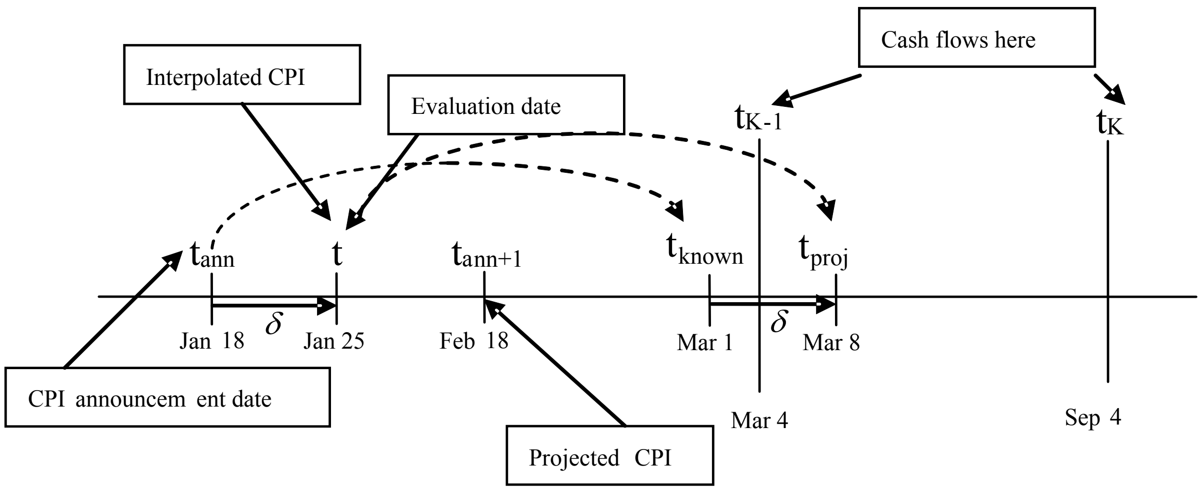
The projected index ratio that advances daily is denoted $IR_{t,t_{\text{proj}}}$, where t_{proj} is the (also daily advancing) date to which it applies. $IR_{t,t_{proj}}$ moves forward each day, and pertains to cash flows and accruals up to a time t_{proj}, thereby smoothing the return under certain projection assumptions. This allows us to approximate a smooth growth in the latest index ratio subject to the accuracy of our CPI prediction.[55] We treat all cash flows up to and including t_{proj} as known and we now use $IR_{t,t_{proj}}$ as the latest "known" index ratio.

Suppose, for example, that on January 18, 2005 the December 2004 HICP index level is announced. The latest index ratio, given by the ratio of the announced CPI level to a base level, determines the potential coupon and accrued that applies to the date of March 1, 2005. This is now both the *latest index ratio date*, t_{known}, and the *projected index ratio date,* t_{proj}. On January 19, the *projected index ratio,* namely that applying to March 2, 2005, is calculated using a projection for January's CPI figure. In this way, the date to which the *projected index ratio* pertains, t_{proj}, moves forward every day and the ratio itself grows daily.

Figure 14 presents is a pictorial version of this procedure.

55 The effects of the discrepancy between the projected next index value and the later announced value are described in section 5.2 (g) and in Appendix B.

Figure 14. The mechanics of the CPI projection methodology



Source: Lehman Brothers.

At time t we are in possession of the CPI level as announced a few days previously at time t_{ann}. This CPI, due to the lag effect mentioned above, pertains to the calculation of a cash flow at time $t_{known.}$ (The next actual cash flow for our security in this example is a few days after t_{known} at time t_{k-1}.) As explained above, at time t, we have a projected level for the next, as yet unpublished, CPI (CPI_{proj}). The *official* CPI for this month is published at time t_{ann}+1m, one month later. This means that we are projecting the CPI in such a way as to be able to calculate (as an estimate) the index ratio that applies to a cash flow at time t_{known}+1m and, by the definition of the index ratio, for any date in between t_{known} and t_{known}+1m. At time t, δ days after t_{ann}, we are thus in a position to project the index ratio for time t_{proj} (which is in turn δ days after t_{known}). Thus, as t advances one day at a time, we use the latest[56] value for CPI_{proj} to project the index ratio that applies to the time t_{proj} which, in turn is advancing with t, the evaluation date. Eventually, the new CPI is published and we begin the cycle again, projecting the next CPI and so on.[57]

This approach enables us to smooth the behaviour of the index ratio factor. It further allows us to quantify an inflation shock factor that reflects the genuine jump effect of the variation of the announced value for CPI from the expected one. This estimate should ideally reflect both seasonality and prevailing economic conditions. Once we have made this projection of the CPI, CPI_t^{proj}, the definition of the corresponding projected index ratio $IR_{t,t_{proj}}$ is as for all other index ratio calculations.

56 The CPI projection could, in theory, be updated midmonth.

57 This methodology can result in jumps in the projection date as the interval between CPI announcements is not totally predictable. For example, let us assume that the January '05 CPI was announced on February 23. On this date, the last date for which the index ratio is known for US securities (which have a three-month lag) is April 1. On this day, we also set the last projected ratio date to April 1.The next day, February 24, the last day for which the index is known is still April 1. However, the last projected ratio date is now April 2, and the projected ratio for April 2 is derived by our projection for the February CPI.

On March 22 (after 27 days have elapsed since the announcement day) the last projected ratio date has moved to April 28. Let us now assume that the February '05 CPI is announced on the next day, March 23. On this day the last projected ratio date is set to the new last day with a known ratio, May 1. So, between March 22 and March 23, although the pricing date moves by only one day, the projection date moves by three.

Indeed, the projection date can move backwards. Consider another case where the November CPI is announced on December 17, and the December CPI is announced on January 19. On December 17 the projection date is set to February 1. On January 18 it has moved 32 days forward to March 5. The next day it moves back four days to March 1. In such eventualities, spread carry and accretion on the jump day will be computed using the jump size.

Formally, given the CPI projection CPI_t^{proj}, the index ratio is given by:

$$IR_{t,t_{\text{proj}}} = \frac{CPI^{\text{known}}}{DIR_0} + \frac{CPI_t^{\text{proj}} - CPI^{\text{known}}}{DIR_0} \cdot \frac{\text{Day}(t_{\text{proj}}) - \text{Day}(t_{\text{known}})}{\text{DaysOfMonth}(t_{\text{known}})} \tag{6}$$

Where t_{known} denotes the last known ratio date and t_{proj} is the last projected ratio date as described above. This equation is derived from equation (3) in Appendix A by dividing by the base date daily index DIR_0. In addition, here we make no assumption about what day of the month t_{known} is, while equation (3) assumes that it is the first of the month as is typically the case.

Global Savings-Investment Imbalances: A Look through the Life-Cycle Model[1]

Vasant Naik
+44 (0)20 7102 2813
vnaik@lehman.com

Amitabh Arora
+1 212 526 6566
aarora@lehman.com

Srivaths Balakrishnan
+44 (0)20 7102 2180
sbalakri@lehman.com

Alexei Jiltsov
+44 (0)20 7103 4360
ajiltsov@lehman.com

We examine the phenomenon of global savings-investment imbalances through the lens of the life-cycle model of optimal savings choice. We ask how far the currently observed savings rates in the world's large economies are from their optimal savings rates. Our analysis suggests that the US savings rate is not unwarrantedly low, nor the Japanese savings rate unwarrantedly high. For Germany, our model implies a savings rate that is higher than the actual savings rate. The model includes return on capital, labour income growth, demographic profile and wealth-to-income ratios as determinants of the optimal level of savings. We show that differences in these variables between countries and across time help explain the cross-country and temporal behaviour of savings rates. We argue that these variables are likely to change only slowly over the medium term. Hence, in our view, a sudden resolution of global savings-investment imbalances is unlikely.

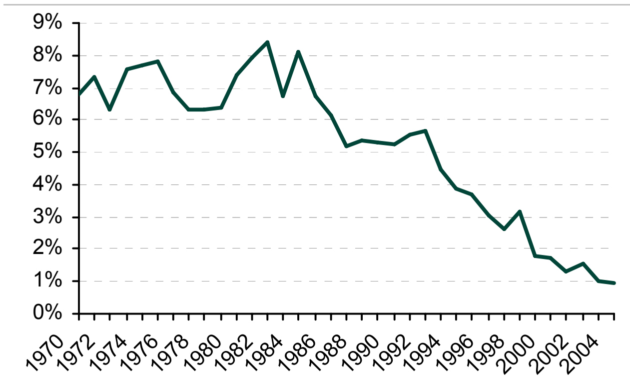
1. INTRODUCTION

Over the past few years, the steady worsening of the US current account balance (Figure 1) has become a cause for concern among international economists as well as policymakers. The fact that this has coincided, at least recently, with a decline in the US personal savings rate and a rapid accumulation of foreign exchange reserves by Asian central banks (Figure 2) has led many to argue the following:

- US consumers are living beyond their means and the day of reckoning is being postponed by the willingness of Asian countries to finance this consumption (Figure 1).
- Aggressive currency management by Asian central banks – which keep their currencies undervalued relative to the dollar – is the mechanism by which this is accomplished. Incorrect currency valuations lead to an adverse trade balance, which is passively financed by these countries (Figure 2).
- The system will unravel as US indebtedness to the rest of the world becomes unsustainable, leading to a flight from dollar-denominated assets; the natural consequence is a sharp decline in the dollar and a rise in US interest rates relative to the rest of the world.

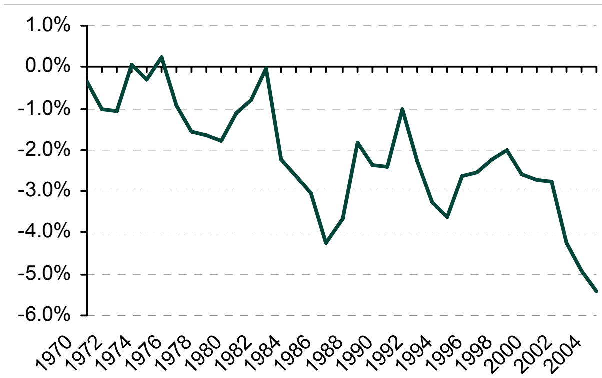
Figure 1. Falling savings rates and increasing deficits[2]

Net personal saving as % of GDP



Source: OECD Annual National Accounts, Bureau of Economic Analysis.

Current A/C balance as % of GDP



Source: OECD Annual National Accounts, Bureau of Economic Analysis.

[1] *We wish to thank Joseph Abate, Jeffrey Biby, John Butler, Mukundan Devarajan, Ethan Harris, John Llewellyn, Ravi Mattu, James McCormick and Vaidyanathan Venkateswaran for their helpful comments and suggestions.*

[2] *Current account balance is defined as the difference between gross saving and gross capital formation.*

Figure 2. Surge in Asian central bank reserves[3]

	Changes in Central Bank Reserves						Amount Outstanding
($, billion)	**1999**	**2000**	**2001**	**2002**	**2003**	**2004**	**Feb-05**
Asia (Total)	154	123	117	238	465	535	2445
Japan	75	70	41	64	201	172	821
China	10	11	47	74	117	207	643
NIE Asia	46	38	21	65	99	100	685
Rest of Asia	23	4	8	35	48	57	297

Source: BIS Annual Report June 2005.

A core component of this reasoning is a direct association between the low US savings rate and the historically extreme US current account deficit. Such an association has at least three shortcomings. First, domestic savings alone do not determine a country's current account balance: domestic investments must also be taken into account. Second, the current account deficit of one country is the surplus of another, so if the savings rate is low in one country (relative to its investments), then it must be high somewhere else. Thus, one cannot look at current account imbalances or savings-investment gaps for a single country in isolation. Finally, whether or not savings rates are too low can be judged only relative to a benchmark specifying what the warranted or optimal level of savings is.

To understand the drivers of the international configuration of current account imbalances, one needs to analyze the actual level of savings in the context of a model of optimal savings. Moreover, such an analysis needs to be carried out in an international context. This article aims to provide such an analysis. We ask how far the observed savings rates in major economies are from what a simple model of optimal savings would suggest. Our analysis suggests that:

(i) The US private savings rate is not unwarrantedly low, just as the Japanese savings rate is not unwarrantedly high. The savings rate in Germany, in contrast, could be lower than optimal. A rapid increase in wealth and the surge in productivity can explain a large part of the decline in the US private savings rate in the 1990s. Comparing countries, private savings rates in the US are lower than those for the other main OECD regions for similar reasons: the US has better returns on capital, higher productivity growth and a more benign demographic profile.

(ii) The current pattern of global current account imbalances is caused by variables that are either secular or change at business cycle frequencies. As such, a significant resolution of these imbalances is unlikely to occur in the short term. In our view, a persistent resolution will most likely be associated either with a US-led global slowdown or with a significant improvement in investments in countries such as Germany and Japan. It could possibly also be associated with a marked reduction in oil prices and a material increase in capital flows to developing economies such as China, India, Brazil and Russia.

The arguments above are not completely new. Bernanke (2004) and Levey and Brown (2005) among others have recently made similar arguments. Our contribution is in evaluating these arguments against a simple life-cycle model of savings choice to demonstrate that economic factors can quantitatively explain savings behaviour across time and across countries.

Our analysis is based on the premise that the level of savings and investments is a private sector decision. Therefore, we cannot capture the contribution made by the managed exchange rate policies of China and other emerging Asian economies to global imbalances. The length of time for which such policies persist and the suddenness with which they are

[3] *NIE Asia includes Hong Kong, Korea, Singapore and Taiwan.*

removed will affect the resolution of these imbalances. It is a limitation of our analysis that it does not account for these factors. However, it should be noted that these countries collectively account for about 30% of the present US current account deficit.

2. SAVINGS, INVESTMENTS AND THE CURRENT ACCOUNT BALANCE

While current account balances are typically thought of as the difference between exports and imports (plus net factor income from abroad), an alternative perspective is useful. A basic macroeconomic identity states that:

Current account deficits equal the difference between savings and investments; savings rates alone cannot determine these deficits

Current Account Surplus = Savings – Domestic Investment

To understand this relation, consider a country which has no imports but \$30 worth of exports. Suppose further that it produces \$100 worth of goods for domestic use, of which \$80 are consumption goods (wine and salmon), and \$20 are capital goods (wine presses and fishing boats). The country's total output is \$130, of which it consumes \$80. It saves \$50, but only \$20 is invested domestically; the balance is exported. Thus, the country has a current account surplus of \$30 and that is precisely the excess of its domestic savings over its domestic investment. The amount exported also results in an investment abroad by acquisition of claims on foreign countries. Figure 3 illustrates the idea that the export surplus must necessarily equal the excess of savings over domestic investment. This basic intuition does not change if the situation is complicated to account for imports or government spending and taxes.

Figure 3. Export surplus equals foreign asset accumulation

Total Output		Total Usage		
Consumer Goods for Domestic Use	\$ 80	Consumption	\$ 80	
Capital Goods for Domestic Use	\$ 20	Domestic Investment	\$ 20	Total Saving = \$ 50
Exports	\$ 30	Foreign Investment	\$ 30	
Total	**\$ 130**	**Total**	**\$ 130**	

Source: Lehman Brothers.

While the equality of the current account balance with the savings-investment gap is merely an identity, it focuses one's attention on the determinants of savings and investments to understand the dynamics of current accounts. It shows that the relation between a country's domestic savings rate and its current account balance is not simple. In particular, a country could balance its current account with a low savings rate if its domestic investment rate is also low (as is the case for many developing economies). In contrast, Japan in the 1970s is an example of a country that had a very high domestic savings rate and yet it ran a current account deficit because its domestic investment rate was even higher.

Does the US current account reflect a rational allocation of global capital or an irrational profligacy of the US consumer?

The question we consider is whether factors such as return on capital, income growth, and demographic profile can justify the observed savings rates in major economies in the world. If fundamental economic factors can explain the observed pattern of savings, then the current configuration of current account imbalances reflects a rational allocation of global capital and is likely to adjust over the medium term as the underlying factors adjust. On the other hand, if the US savings rate, say, is unjustifiably low and reflects an irrational consumption boom, then concerns about the lack of sustainability of the current imbalances may be justified. Which alternative is more plausible has important implications for asset prices. If US households have turned profligate, then the current account deficit is financing an unsustainable consumption boom, which may end in tears (similar to the Asian crisis in 1997). If, on the other hand, the current account balance reflects the relative attractiveness of

US assets, then the situation may not change unless investment opportunities emerge elsewhere, most importantly in economies with weak domestic demand such as Japan and Germany.

In sections 3 and 4 below, we review the recent history of global capital flows. In section 5, we use a life-cycle model to understand how much a country should save given reasonable parameters describing its economic environment; this is a prerequisite to judging whether a particular savings rate is adequate. In sections 6 and 7, we compare three major economies in the world – the US, Germany and Japan – in terms of how far each might be from the optimal savings rate predicted by our simple model. In section 8, we discuss the capital market implications of our analysis.

3. A BRIEF HISTORY OF INTERNATIONAL CURRENT ACCOUNT BALANCES

Before we delve into an analysis of optimal savings and investment rates, it is instructive to review the post-Bretton Woods history of capital flows in the world. The cast of surplus and deficit countries has undergone considerable change over the past three decades (Figure 4).

Figure 4. The changing cast of deficit and surplus countries

Largest Current A/c Deficits ($ billion)		Largest Current A/c Surpluses ($, billion)	
1980		**1980**	
Germany	-14	Saudi Arabia	42
Brazil	-13	Kuwait	15
Mexico	-11	UAE	10
Japan	-11	Qatar	8
Italy	-11	Libya	8
Poland	-9	Nigeria	6
1990		**1990**	
US	-79	Germany	45
UK	-40	Japan	44
Canada	-20	China	12
Spain	-18	Taiwan	11
Italy	-16	Switzerland	9
Australia	-16	Venezuela	8
2000		**2000**	
US	-413	Japan	120
UK	-36	Russia	45
Germany	-26	Switzerland	31
Brazil	-24	Norway	26
Spain	-19	China	21
Mexico	-18	Canada	20
2004		**2004**	
US	**-666**	Japan	172
Spain	-49	Germany	96
UK	-47	China	70
Australia	-39	Russia	60
Italy	-24	Saudi Arabia	49
Turkey	-16	Switzerland	43

Source: IMF World Economic Outlook Database, April 2005.
http://www.imf.org/external/pubs/ft/weo/2005/01/data/index.htm

In 1980, the world had just experienced an oil price shock, with the effect that oil producers became the dominant providers of capital through their current account surpluses. This capital was destined for western Europe and Asia (including Japan). By 1990, Germany, Japan and the so-called newly industrialised Asia were no longer net investors but net savers (exporting capital rather than importing it). On the other hand, oil prices were on their way down and the oil surplus had shrunk considerably. The net investing countries were the US, other Anglo-Saxon economies and continental Europe (excluding Germany). The current account configuration in 2004 combines the effects seen in 1980 and in 1990. Oil prices have risen again significantly and oil exporters are consequently running a large surplus as their domestic investment opportunities have not grown to the same extent. At the same time, Japan and Germany continue to save well in excess of domestic investment needs and export large sums of capital. China is an important addition to the set of countries exporting capital in 2004.

This discussion suggests that we can group countries into certain blocks based on whether they are net importers or exporters of capital as follows:

Figure 5. Global current account balances by country group ($ bn)[4]

Investors		Savers	
1980		**1980**	
Germany, Japan, Switzerland	-46	Oil Producers	174
France, Italy and Spain	-37	US	4
NIE Asia	-17		
UK, Canada, Australia	-11		
Developing Asia	-7		
1990		**1990**	
US	-97	Germany, Japan, Switzerland	120
UK, Canada, Australia	-92	Oil Producers	25
France, Italy, Spain	-54	NIE Asia	21
Developing Asia	-11		
2000		**2000**	
US	-413	Germany, Japan, Switzerland	124
UK, Canada, Australia	-32	Oil Producers (incl Russia)	156
France, Italy, Spain	-7	Developing Asia	48
		NIE Asia	40
2004		**2004**	
US	**-615**	Germany, Japan, Switzerland	287
France, Italy, Spain	-73	Oil Producers (incl Russia)	206
UK, Canada, Australia	-56	Developing Asia	98
		NIE Asia	83

Source: IMF World Economic Outlook Database, April 2005, Lehman Brothers.
http://www.imf.org/external/pubs/ft/weo/2005/01/data/index.htm

(i) **"Low real return" countries:** We classify Japan, Germany and Switzerland in this group. Our hypothesis is that these countries have relatively inferior investment opportunities at home and, in an open economy, some savings flow into higher yielding areas such as the US. These are currently contributing about 40% of the surplus of all current-account surplus

[4] *These are measured in 2000 dollars. Note that Figure 4 reports various quantities measured in current dollars.*
Low real return countries include Germany, Japan and Switzerland.
Oil producers include Bahrain, Iran, Kuwait, Libya, Nigeria, Norway, Oman, Qatar , Saudi Arabia, UAE and Venezuela.
NIE Asia includes Hong Kong, Korea, Singapore and Taiwan.
Developing Asia includes China, India, Indonesia, Malaysia, Philippines and Thailand.

countries. Note also that the situation was very different a few decades ago when both Japan and Germany were experiencing a post-war growth boom; then these countries were importing capital.

Low growth countries and oil exporters are financing much of the global current account deficits

(ii) **Oil producers:** The recent spike in oil prices is mainly responsible for the large surpluses of oil-producing countries. Because oil prices are volatile around a long-term mean of about $26/bbl in real terms, much of the recent increase in oil revenues is seen as a windfall gain for oil producers and a loss for oil consumers. Economic theory would argue that any windfall gain (like winning a lottery ticket) should be consumed over a person's lifetime and that is exactly what the oil producers might be doing by saving much of the windfall. Consistent with this argument, oil-producing countries ran large surpluses during the oil price boom in the 1970s and deficits during the low oil price regime in the 1990s.

(iii) **NIE Asia (Newly Industrialised and Emerging Asian countries):** These countries have become net savings contributors over the past few years. This is a puzzling development and not warranted by traditional economic theory: because these are developing economies, they should normally be expected to be net capital importers rather than exporters.

This article uses a model of optimal savings that is tailored for an analysis of the behaviour of the private sector: it assumes that savings decisions are taken to maximise lifetime utility and respond to variables such as return on capital, income growth and demographic profile. Our analysis is best suited to understand the contribution made by the first group of "low real return" countries to the current state of imbalances because over the past 20 years or so, most of the capital flow from these countries has been driven by the private sector. To the extent that the surplus of oil-exporting countries is also related to a lack of domestic investment opportunities, our analysis applies to them as well. However, our set-up does not adequately address the impact of the third group of countries. In these countries, current account surpluses are intimately linked to a managed exchange rate policy: it could be argued that a model of decision-making in the private sector is not suitable for public sector intermediaries (mainly central banks).

4. GLOBAL TRENDS IN SAVINGS AND INVESTMENTS

We examine the dynamics of current account balances from the savings-investment perspective. The previous section looked at the trends in these balances internationally. Here we consider the trends in the two components that define the current account balance (ie, savings and investments).

Box 1: What measure of savings should we consider?

There are several measures of savings one could consider. One popular measure of national savings is net household savings. Net household savings are defined as household disposable income less household consumption and depreciation (attributable to households). While this measure of saving has been the focus of much literature on current account imbalances, we focus on a broader measure of national savings: gross private savings. Gross private savings are defined as the sum of household and corporate savings, without subtracting depreciation. The reason we focus on private savings instead of personal savings is to capture the savings flow in the private sector as a whole. Since corporations are ultimately owned by individuals, we consolidate these two sectors. Further, because depreciation accounting can change over time and can differ across countries, we prefer to analyze gross savings rather than net, especially when making international comparisons. Moreover, depreciation is ultimately an imputed cost and an estimate. It is best not to let the inaccuracies in this estimate vitiate a cross-country analysis of savings and investments.

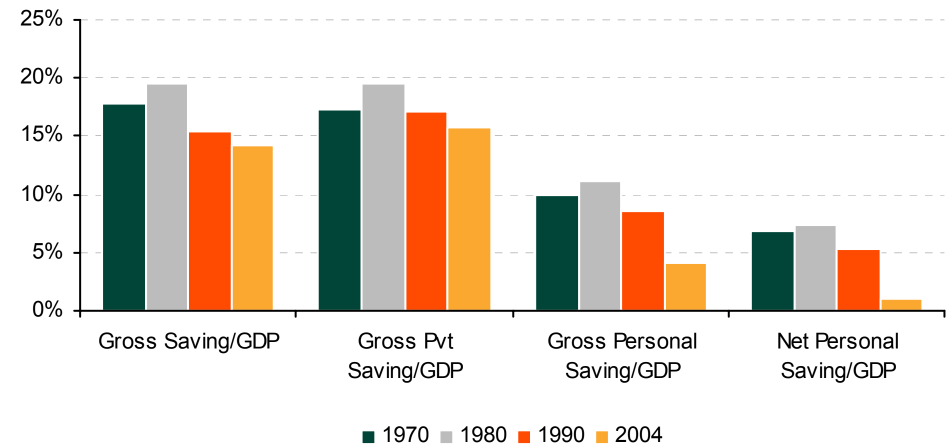
4.1. Savings

As explained in Box 1, we focus on gross private savings as a measure of savings. Two trends are relevant for our analysis:

(i) Since 1980, gross savings in the US have fallen more than gross private savings as government finances have deteriorated from being in balance in 1980 to -1.6% of GDP in 2004. Partly, this is a business cycle phenomenon since government balances turn negative during recessions.

(ii) While gross private savings have declined in the US since the mid-1980s, the drop is less marked than that for the net personal savings. For instance, while the net personal savings rate has fallen by about 6.5% points from 1980, the gross private savings rate has fallen by only 3.8% points (Figure 6). This is due to differences in private versus personal savings trends: corporate savings have fallen less than personal savings. In fact, over the period 2000-2004, gross private savings rose from 13.6% to 15.8% as the increase in corporate savings more than offset the decline in personal savings.

Figure 6. Savings/GDP trends



Source: Lehman Brothers.

Figure 7. Trends in savings and investment

Gross Private Saving/GDP (Gross Govt. Saving/GDP in parenthesis)

Year	US	Japan	Germany
1970	17.2% *(0.6%)*	33.8% *(7.0%)*	-
1980	19.6% *(-0.1%)*	29.5% *(2.9%)*	19.2% *(2.1%)*
1990	17.1% *(-1.7%)*	26.1% *(7.7%)*	24.0% *(1.2%)*
2000	13.6% *(3.9%)*	28.0% *(-0.2%)*	18.5% *(1.6%)*
2004	15.8% *(-1.6%)*	28.4% *(-1.8%)*	21.8% *(-1.2%)*

Source: OECD Annual and Quarterly National Accounts Database.

Gross Capital Formation/GDP

Year	US	Japan	Germany
1970	18.1%	39.4%	30.2%
1980	20.6%	32.6%	25.4%
1990	17.9%	32.9%	23.5%
2000	20.2%	26.3%	21.8%
2004	19.7%	23.9%	17.3%

Source: OECD Annual and Quarterly National Accounts Database.

4.2. Investments

The following trends in the level of investment activity and in the composition of investments (in terms of which sectors have invested and where investments have happened) are of interest:

(i) While savings rates have remained roughly constant in Japan and Germany since the 1980s, investment activity has fallen markedly (Figure 7). In particular, since 1980, gross capital formation in Japan has dropped from just under 40% of GDP in 1980 to 24% of GDP in 2004. The trend in Germany is similar. In contrast, capital formation in the US has remained steady, despite the drop in the savings rate.

The US current account deficit has increased as the US private savings rate has declined, while the investment rate has remained stable. In contrast, savings rates have been stable in Germany while investment rates have declined

(ii) The decline in investments in Japan and Germany is the result of a significant fall in both government and private fixed capital formation (Figure 8a). In Japan, while government investments have halved as a percentage of GDP, from 9.4% in 1980 to 4.9% in 2004, private investments have fallen by almost 4 percentage points over the same period. The large drop in private investments in Japan in the 1990s was a major contributor to the recession during this period. Also, the relatively high level of government investment in Japan reflects expansionary public works projects undertaken during the last decade.

(iii) The two sectors in which a large percentage of investments have been made are housing and other construction (excluding housing). The investment in housing has declined in Japan and Germany since 1980 (Figure 8b), while it has increased in the US from 4.5% in 1980 to 5.7% in 2004. On the other hand, the investment in other construction (excluding housing) has been historically high in Japan compared with both US and Germany. The increased investment in housing in the US in the past five years has been interpreted by some observers as being a reflection of a bubble in the US housing market.

Figure 8a. Trends in public vs private capital formation

		Government Gross Fixed Capital Formation/GDP			Private Gross Fixed Capital Formation/GDP	
	US	Germany	Japan	US	Germany	Japan
1980	3.7%		9.4%	17.8%		22.5%
1990	3.8%	2.6%	6.5%	14.9%	20.6%	25.8%
2000	3.1%	1.8%	6.8%	17.0%	19.7%	19.6%
2004	3.2%	1.4%	4.9%	16.1%	15.8%	18.9%

Source: OECD Annual and Quarterly National Accounts.

Figure 8b. Investment in housing and other construction

		Housing/GDP			Other Construction/GDP	
	US	Germany	Japan	US	Germany	Japan
1980	4.5%	6.6%	6.4%	7.2%	7.6%	13.1%
1990	4.0%	6.0%	5.6%	5.4%	6.4%	12.1%
2000	4.5%	6.8%	4.0%	5.0%	4.9%	9.7%
2004	5.7%	5.5%	3.6%	4.3%	3.9%	n/a

Source: OECD Annual and Quarterly National Accounts.

This analysis suggests that the current configuration of current account balances is driven not only by a decline in the US savings rate, but also by the drop in domestic investments in Germany and Japan. In the following sections, we examine if the savings rates in the US, Germany and Japan are justified by the investment opportunities, demographics and wealth of these countries.

5. THE OPTIMAL SAVINGS RATE: A SIMPLE LIFE-CYCLE MODEL

To assess the appropriate level of savings, we analyze the optimal savings rate for a representative economic agent facing an economic environment similar to that faced by US, German and Japanese consumer-investors. We frame the question of optimal savings choice within the framework of the so-called life-cycle model, which explicitly incorporates an individual's age (together with a number of other factors) in savings decisions. We then compare the optimal savings rates predicted by the model with the observed gross private savings rates. Our approach of thinking of the private sector of a country as an individual trying to save for retirement is admittedly simplistic. It is nonetheless useful because it is a framework that can incorporate the many factors that influence savings rates.

The private saving rate is a blend of individual decisions, based on each individual's objective economic circumstances

To get a feel for the life-cycle model of savings choice, we first illustrate it through a few simple examples, which assume that there is no uncertainty (Figure 9). Our benchmark case is a person who is saving for 20 years of retirement after 30 years of working life; he earns $1/year, has accumulated wealth of $5, gets a zero return on savings, and weights future consumption as highly as current consumption (his rate of time preference is zero). This person's lifetime total income is $35. Therefore, he can have a stable consumption rate of $0.70/year over his lifetime, and therefore needs to save 30% of his income. In the absence of discounting, this is the optimal savings rate of the person considered.

If this person is older and has a shorter working life of say 20 years, his lifetime income is only $25 and he can afford a stable consumption rate of only $0.50/year. His savings rate climbs to 37% of income. The reason is that he has fewer working years in which to accumulate savings, but the same number of retirement years to amortize the savings. The wealth accumulation argument easily leads to the conclusion that if he had higher accumulated wealth, he would need to save less. For example, if his accumulated wealth were $10 rather than $5, his lifetime income would be $40, affording an annual consumption rate of $0.80 per year: the savings rate drops to 20%.

Figure 9. Savings in the life-cycle model without uncertainty: illustration

		Base Case	Age Effect	Wealth Effect	With Labour Income Growth
Time to Retirement	**(T)**	30 yrs	**20 yrs**	30 yrs	30 yrs
Post-Retirement Period	**(R)**	20 yrs	20 yrs	20 yrs	20 yrs
Wealth	**(W)**	$5	$5	**$10**	$5
Labour Income/year	**(I)**	$1	$1	$1	$1
Return on capital		0%	0%	0%	0%
Labour Income Growth		0%	0%	0%	**2%**
Lifetime	**(L = T + R)**	50 yrs	40 yrs	50 yrs	50 yrs
Total Consumption	**(C = W + T * I)**	$35	$25	$40	$46
Consumption/year	**(c = C / L)**	$0.7	$0.63	$0.8	$0.91
Savings/Labour Income		**30%**	**37%**	**20%**	**9%**

Source: Lehman Brothers.

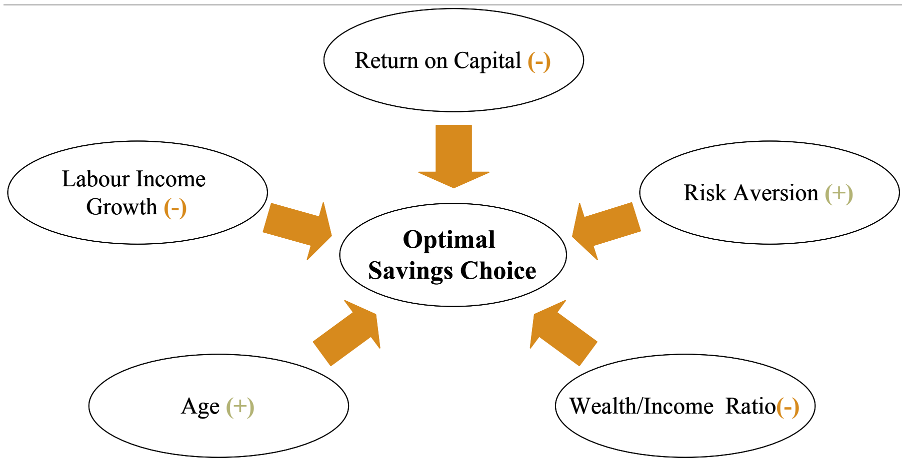
Age, wealth, income growth and the return on savings determine individual saving rates

These examples illustrate that the required savings rate is sensitive to the demographics and the accumulated wealth (relative to current income) of the agent under consideration. By aggregation, these factors must matter for the whole economy. So far, the analysis has assumed that there is no growth in income and no return on savings. If we assume a more realistic scenario, where income is expected to grow at 2% per year, the savings rate falls from 30% in the benchmark case to 9% (Figure 9). The intuition is that a growing income increases a person's lifetime income and consumption possibilities and lowers the required savings rate. A positive return on savings lowers the required savings rate for the same reason.

The model we use in our calibration (reported in the next two sections) is an extension of the model described above. The details of the model are available in the appendix. Here we note that the main difference in the extended model from the simple model considered above is that it explicitly accounts for the uncertainty about income growth and return on capital.

Figure 10 summarizes the key drivers of saving decisions according to the life-cycle model and the direction in which these drivers impact the optimal level of savings:

Figure 10. Determinants of the savings rate in a simple life-cycle model[5]



Source: Lehman Brothers.

6. THE DRIVERS OF THE SAVINGS RATE: THE US VERSUS GERMANY AND JAPAN

Having identified the factors that should influence savings choices, we now investigate how the US compares with Germany and Japan on these variables. We also quantify the impact of the variables on the savings/income ratio based on our life-cycle model. For this, we start with a set of parameters for the base case (ie, the US, see Figure 11) and then change one parameter at a time to illustrate its effect on the model's implied savings ratio. The assumptions on the parameters of the model for the base case are given in Figure 11. The wealth levels and average population age are available from national accounts and census data. Wage growth rates are set to reflect the historical experience in the US.[6] The return on capital is set at 4% for the US. Since there is only one asset in this model, this should be thought of as the real return on a blend of all assets in an economy – corporate as well as residential real estate. We begin with the first parameter: the wealth-to-income ratio.

[5] *The inverse relation between optimal savings rate and return on capital is contingent on the coefficient of relative risk aversion of the representative investor being greater than one (the log case). Most empirical estimates of this parameter imply this property. The result that saving rates rise with age assumes that we are considering individuals who are not too close to retirement.*

[6] *The average real wage growth in the US over 1980-2004 is estimated at 1.1% p.a. while this estimate over 1995-2004 is 2.0%. We use an average of these estimates.*

Figure 11. Base case parameters for the life-cycle model

	Base Case
Wealth/Income	5.5
Labour Income Growth	1.5%
Volatility of Labour Income	5%
Risky Asset Mean Return	4%
Risky Asset Volatility	10%
Correlation between labour income and risky asset	35%
Productive Life	25yrs to 65 yrs
Retirement Period	15yrs+Bequest[7]
Average Age	45 yrs
Coefficient of Relative Risk Aversion	5
Rate of Time Preference	2%

Source: Lehman Brothers.

6.1. Wealth-to-income ratio

The lower wealth-to-income ratio in the US requires higher savings rates…

Wealth-to-income ratio has increased significantly in the US since the mid-1980s due to the rise in house prices as well as the stock price boom (Figure 12). However, this ratio is still smaller in the US than it is in Japan. This may seem puzzling since the Japanese stock market and housing markets imploded in the 1990s. The reason is that Japan has been saving (and investing) a much higher proportion of GDP over the past several decades, and therefore the accumulated wealth is high relative to current income. If the three countries differed only in their wealth/income ratios, the US savings rate ought to be about 6% points higher than in Japan (Figure 13). However, the other determinants are not the same, as we show below.

Figure 12. Japan has the highest wealth-to-income ratio[8]

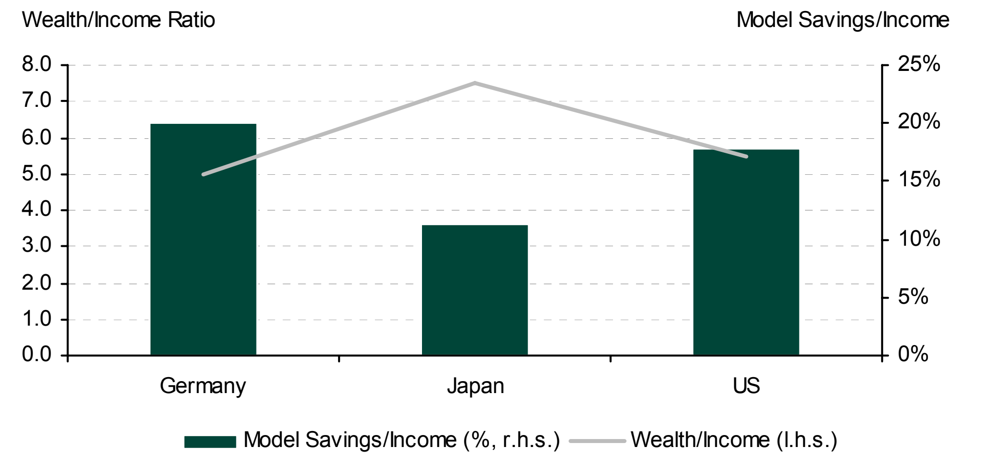
	Net Wealth of Households/Household Disposable Income			
	1992-94	1995-97	1998-00	2001-03
Germany	4.8	5	5.2	5
Japan	7.8	7.6	7.6	7.6
US	4.8	5.3	6	5.3

Source: OECD Economic Outlook Statistical Annex Table 58, Dec 2004.

[7] *Utility from bequest is assumed to equal the utility of 10 years of consumption.*

[8] *Households include non-profit institutions serving households. Ratios are calculated as net wealth over nominal disposable income. Net wealth is defined as non-financial and financial assets minus liabilities; net financial wealth is financial assets minus liabilities. Non-financial assets include stock of durable goods and dwellings, at replacement cost and at market value, respectively. Not included are assets with regard to social security pension insurance schemes.*

Figure 13. Life-cycle model: impact of wealth/income ratio on savings

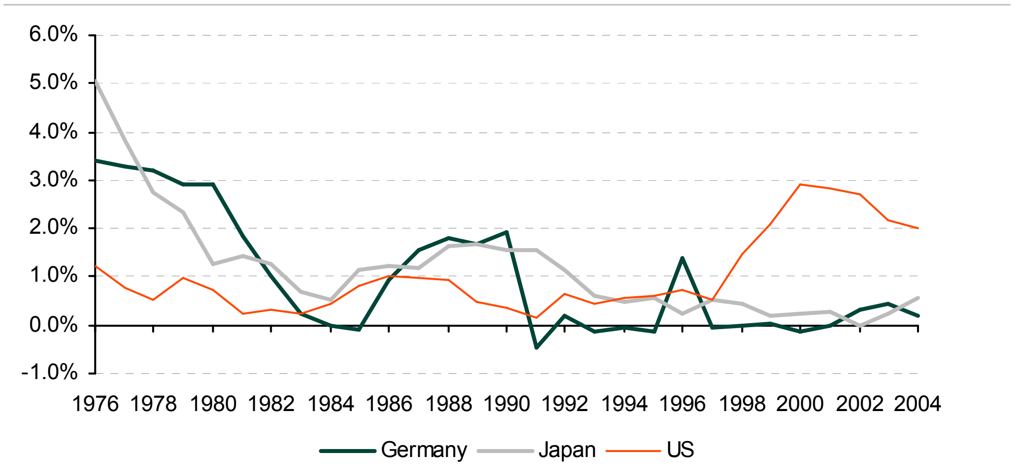


Source: Lehman Brothers.

6.2. Income growth

A remarkable change in relative wage growth patterns occurred in the past decade. In the two decades prior to 1985, wages grew faster in Japan and Europe than in the US (Figure 14a). Much of this was due to the rapid capital formation in the post-war period, after much of the countries' industrial infrastructure had been destroyed. This meant a sharp increase in labour productivity as the capital available per person increased (Figure 14b). The situation has changed in favour of the US in the past decade, where growth in labour productivity has been 2.2% compared with 1.2% in Germany and 1.3% in Japan. In the US, in the past two decades, wage growth has averaged 1.5% per annum; about 0.5pp more than in Japan and Germany. If we apply these growth rate differentials in our savings model (holding other parameters fixed at the base-case level), we find that the US can afford to save about 2pp less than Japan and Germany (Figure 15).

Figure 14a. Recent wage growth in the US dominates OECD partners
5-year average growth in real total compensation per employee[9]



Source: OECD Economic Outlook, Dec 2004.

[9] *This is defined as all payments by resident producers of wages and salaries to their employees, in kind and in cash, and of contributions, paid or imputed, in respect of their employees to social security schemes and to private pension, life insurance and similar schemes. Source: OECD Economics Glossary.*

Figure 14b. Labour productivity growth is higher in the US
Average annual growth in labour productivity per person engaged

	Germany	Japan	US
1976-85	2.0%	2.8%	1.2%
1986-95	2.3%	2.0%	1.3%
1996-05	1.3%	1.7%	2.2%

Source: Groningen Growth and Development Centre and the Conference Board, Total Economy Database, August 2005. http://www.ggdc.net.

Figure 15. Life-Cycle Model: impact of labour income growth on savings

	Labour Income Growth	Model Savings/Income
Germany	1%	20.9%
Japan	1%	20.9%
US	1.5%	17.8%

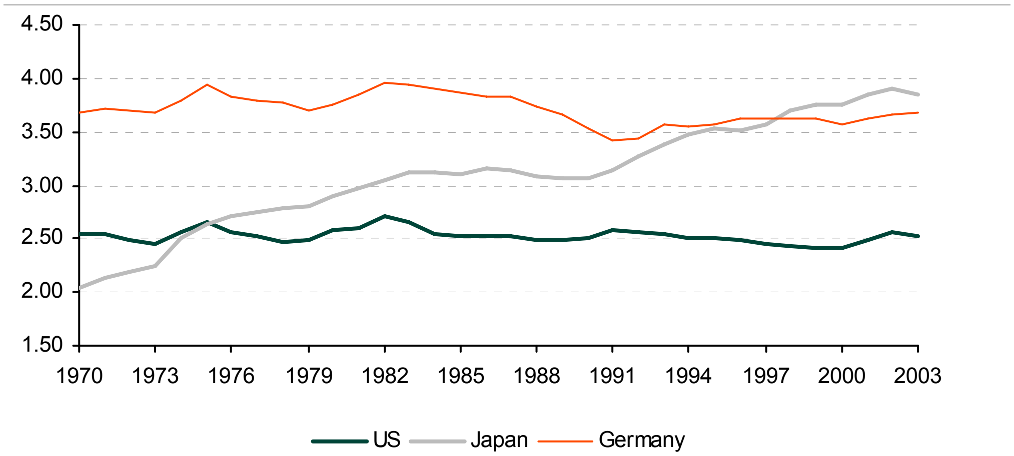
Source: Lehman Brothers.

6.3. Return on capital

…higher capital productivity and…

One of the principal ways in which the US differs from Japan and Germany is the efficiency with which capital is deployed. There are two ways to see this. First, at the macro level, the capital-output ratio, ie, the amount of capital required to produce a unit of output, is significantly higher in Germany and Japan than in the US (Figure 16). Since the composition of output is not materially different in the developed economies of the world, one possible reason that has been advanced for the greater use of capital per unit of labour (especially in Germany) is the lack of flexible labour markets. This can encourage capital-intensive production and reduce the efficiency of capital usage. Second, firm-level accounting data shows that US corporations have tended to have greater returns on assets than their counterparts in Japan and Germany (Figure 17). Financial market valuations seem to recognize the superior returns in the US: this can be seen in the ratio of estimated market value of the firm (enterprise value) to book value of capital employed (Figure 17) for the three countries. Again, in our model, if we assume that the US's return on accumulated wealth exceeds Japan's by 2pp (holding other parameters fixed at the base-case level), the optimal savings rate in the US is 13pp lower than in Japan (Figure 18).

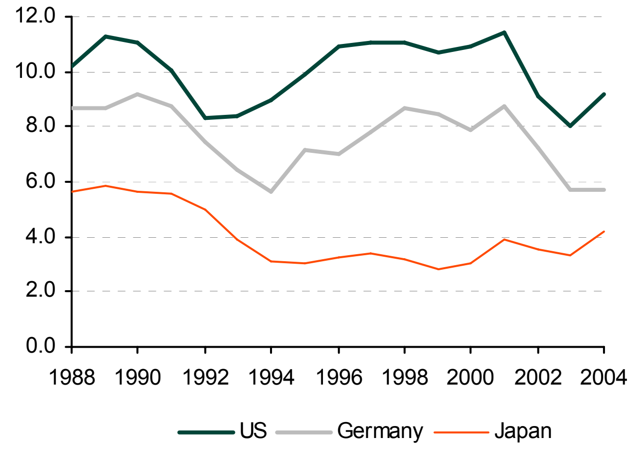
Figure 16. Capital intensity of output is the lowest in the US
Net capital stock to GDP ratio



Source: Christophe Kamps, Kiel Institute for World Economics, April 2004, Lehman Brothers http://www.ifw-kiel.de/staff/kampsc.htm

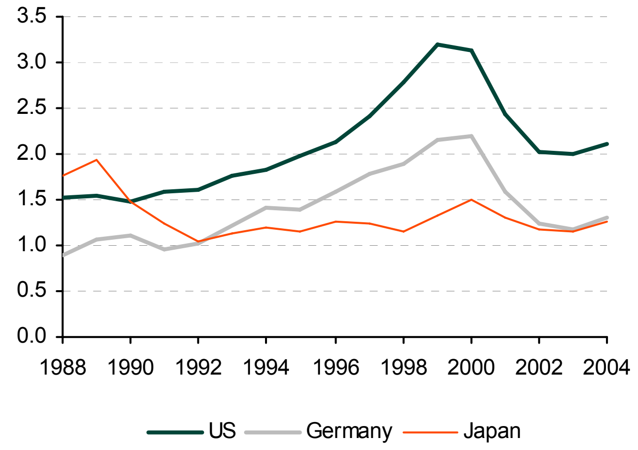
Figure 17. The US has better efficiency of capital employed[10]

Return on Capital Employed (%)



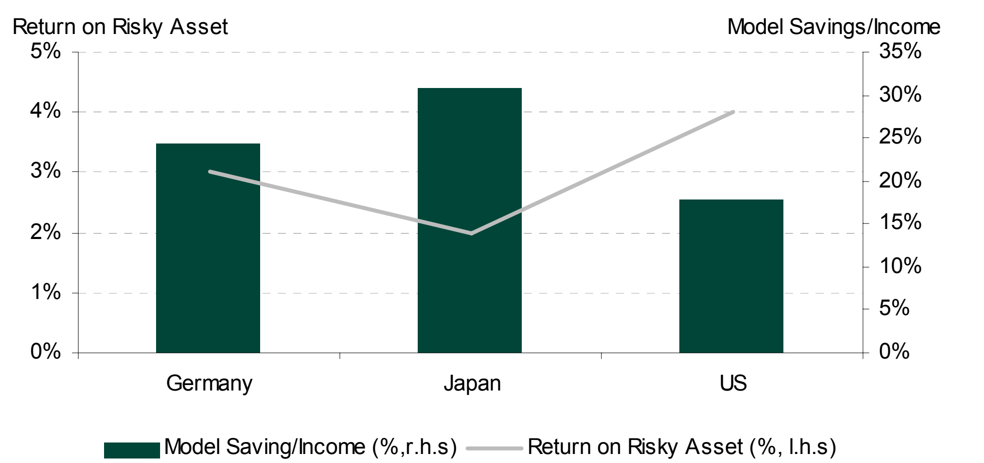
Source: Lehman Brothers Equity Strategy.

Enterprise Value/Capital Employed



Source: Lehman Brothers Equity Strategy.

Figure 18. Life-cycle model: impact of return on capital on savings rates



Source: Lehman Brothers.

6.4. Demographics

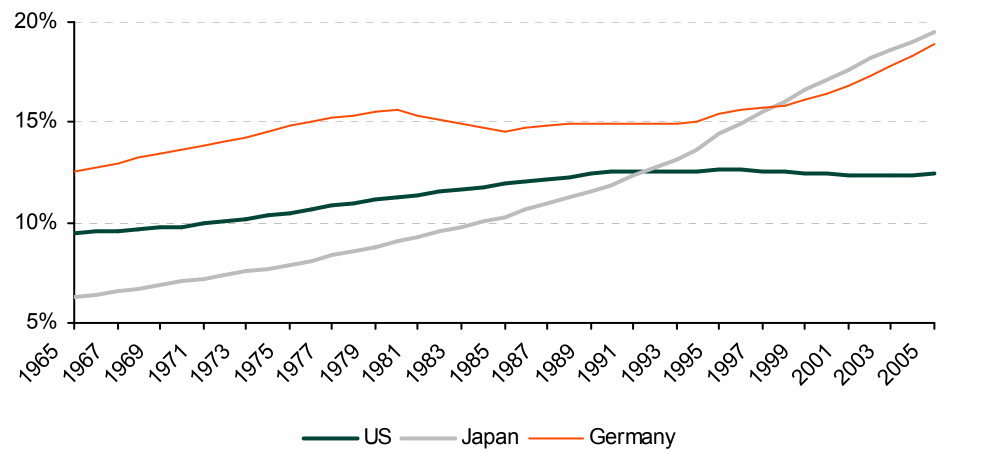
… a younger population profile

Finally, as is well known, the US has a younger population and a faster population growth rate than Germany or Japan. More revealing than the differences in average age (44 for the US versus 48 for Japan) is the difference in the age distribution. The old age dependency ratio – defined as the percentage of population over the age of 65 (Figure 19) – has risen significantly in Japan over the past four decades compared with the US. Currently, the proportion of the population over 50 years (empirically the peak saving age) is 47% in Japan compared with 37% in the US. A younger population, as discussed in the previous section, needs to save less as a fraction of current income since the person has more years to

[10] *Return on Capital Employed is defined as the profitability ratio of net income before goodwill divided by capital employed. Capital Employed is the sum of Total Capital (common equity, preferred stock, minority interest, long-term debt) and Short-Term Debt.Enterprise Value refers to the sum of Market Value of Common Equity and Net Total Debt.The universe considered for the above calculations is the FTSE World Index restricted to the specific country.*

accumulate savings for retirement. Applying the average age difference of four years in our model (holding other parameters fixed), we find that the optimal saving rate in the US is 6.5pp lower than in Japan (Figure 20).

Figure 19. The US has the lowest old age dependency ratio



Source: 1965-1994: The World Saving Database, The World Bank
http://www.worldbank.org/research/projects/savings/data.htm
1995-2005: US Census Bureau, International Database.
Notes: the dependency ratio is the proportion of the population above 65 years of age.

Figure 20. Life-cycle model: impact of age on saving

	Average Age (yrs)	Model Savings/Income
Germany	47	21.5%
Japan	48	23.1%
US	44	16.4%

Source: Lehman Brothers.

In summary, while the lower wealth-to-GDP ratio in the US argues for a higher savings rate in the US relative to Japan, the higher wage growth rate, the greater efficiency of capital deployment, as well as a younger population, point in the opposite direction.

7. THE OPTIMAL SAVINGS RATE IN A LIFE-CYCLE MODEL WITH UNCERTAINTY

In the previous section we illustrated the effect of changing one parameter at a time on the optimal savings rate. We now consolidate the impact of all these variables collectively using our model and compute the model-implied optimal savings rate for the US, Japan and Germany. For this, we need to set the appropriate levels for the variables considered: average population age, wage growth rate, return on capital and wealth levels. Figure 21 summarizes our assumptions. The parameters for the US are the same as those assumed for the base case in the previous section. The levels for Germany and Japan reflect the historical difference between the US and these countries as highlighted in the previous section.

Figure 21. Parameter assumptions for the life-cycle model

Variable	Description	US	Japan	Germany	Justification for parameter assumptions
Wealth/Income	Ratio of individual's wealth to initial wage income	5.5	7.5	5	Current ratio of household wealth to gross household disposable income
Labour Income Growth	Real Wage Growth	1.5%	1%	1%	Our assumption broadly reflects the historical differences over the last decade.
Volatility of Labour Income	Volatility of wage growth	5%	4%	4%	Assumed to be lower in Germany and Japan to account for greater labour protection.
Risky Asset Mean Return	Real return on all assets in the economy	4%	2%	3%	Assumed lower in Germany than US and lowest in Japan
Risky Asset Volatility	Volatility of return on all assets	10%	10%	10%	Reflects volatility of returns on a composite risky asset (hence lower than typical volatility estimates for equities). Assumed to be the same in all 3 countries
Correlation	Correlation between labour income and risky asset	35%	35%	35%	Assumed to be the same in all 3 countries

Source: Lehman Brothers.

Using the above three parameter sets, we compute the model-implied savings rates and obtain two important results (Figure 22a):

The private savings rate in the US and Japan is not far from the optimal savings rate, while Germany may be saving too little

(i) The private savings rate in the US and Japan is not far from the optimal savings rate, while Germany may be saving too little for its requirements. Note that the model predicts that the required savings rate for the US should be lower than in Japan and Germany (Figure 22a): the higher wage growth, better return on capital and the younger population outweigh the lower wealth-to-GDP ratio.

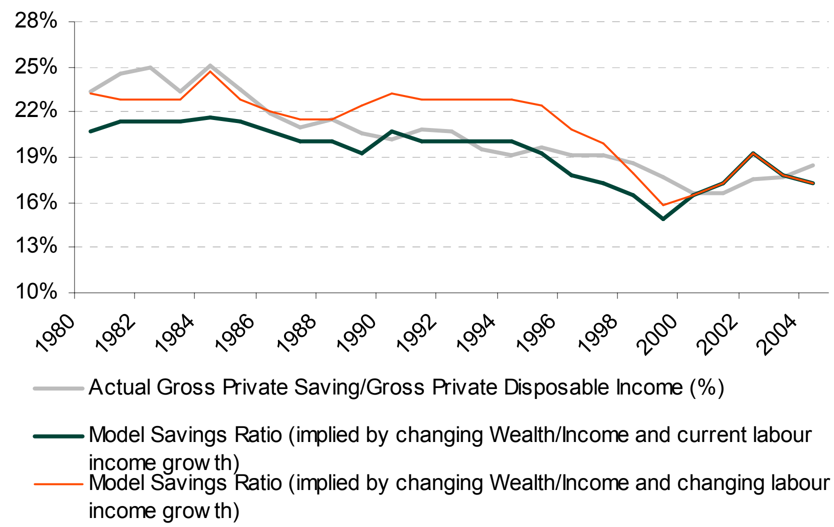
(ii) The increase in the wealth-to-GDP ratio explains much of the decline in the US savings rate since the mid-1980s. Some of the decline is also due to the improved outlook on wage growth today relative to the experience in the 1970s and 1980s (Figure 22b).

**Figure 22a. Model-implied optimal savings rates differ markedly
Comparison of actual savings rates and savings rates implied by the life-cycle model using parameters in Figure 21**

	Average Age (yrs)	Model Saving Rates for Average Age	Actual Private Saving Rate	Difference (Actual – Model)
US	44	16.4%	18.4%	2.0%
Japan	48	32.1%	33.3%	1.1%
Germany	47	30.8%	27.2%	-3.6%

Source: Lehman Brothers.

Figure 22b. Increasing wealth and labour productivity explain the decrease in the US private savings rate[11]



Source: Lehman Brothers.

We note that the results reported above are not obtained from a hard calibration of parameters to the data. We have chosen plausible values for various parameters and shown that the optimal savings rate depends on the underlying factors in an intuitive way. Indeed, the fact that the observed savings rate is reasonably well calibrated by the model for the US could be interpreted to mean that the parameters chosen are reasonable. Then, what our analysis shows is that a perturbation of these parameters to account for the German and Japanese environments, produces a savings rate that is higher than the US and (in the Japanese case) as high as what is observed.

8. CAPITAL MARKET IMPLICATIONS

In conclusion, the above analysis suggests that the US savings rate has not been unwarrantedly low compared with other developed countries or with historical benchmarks. The wealth effect from stock and housing assets and the rosier outlook for growth explain much of the decrease in the savings rate since the mid-1980s. This would be true even if the US were a closed economy, independent of any currency effects. Comparing the US with Germany and Japan, our analysis suggests that the life-cycle model can help explain the high savings rate in Japan while the savings rate in Germany is lower than that implied by the model. An interpretation of this analysis is that the difference in return on capital in the US relative to Japan and Germany is an important driver of US deficits and Japanese and German surpluses. One could say that savings seeking higher returns are flowing from these regions into the US.

The variables that drive the optimal savings decisions in our model, most notably return on capital, demographic profile and income growth, change slowly over time. The wealth-to-income ratio is probably the only exception to this observation given that wealth shocks can occur relatively suddenly. As such, a sudden readjustment of the global configuration of

[11] *The model savings ratio has been calculated using the actual values of the ratio of net wealth of households to household disposable Income (OECD Statistical Annexe) for the wealth/income ratio and mapping the observed 5-year real wage growth (OECD Economic Outlook) to three possible values for labour income growth: 1%, 1.25% and 1.5% as follows:*
If observed 5yr average real wage growth less than 1% - assumed to be 1% in the model.
If observed 5yr average real wage growth between 1% and 1.5% - assumed to be 1.25% in the model.
If observed 5yr average real wage growth more than 1.5% - assumed to be 1.5% in the model.

savings-investment imbalances seems unlikely unless it is caused by the sudden decline in the value of real assets.

We can see two ways in which the current account imbalances could be resolved:

i) *Investment opportunities revive in Japan and Germany* and compete for the savings that are currently going to the US. In this case, interest rates would rise globally and enough in the US to choke off the investment that was being financed from Japan and Germany. However, global yield differences would compress: currently US real interest rates are higher than in Japan or the euro area, which should induce capital imports.

ii) *The investment climate in the US worsens*: A plausible scenario is a real estate correction, which depresses new residential investment. Under this scenario the US leads the world into a general recession. The US current account moves towards balance; not because alternative investment opportunities have emerged, but because the decline in global economic activity reduces the pool of savings available to finance investments in the US. Global yields fall, but more so in the US.

In neither of these two scenarios a sudden catastrophic drop in the value of the dollar against other major currencies (yen and euro) is implied. US yields may be higher or lower depending on the mechanism by which the resolution happens, but yield spreads should narrow rather than diverge as the doomsday scenario implies. However, as stated earlier, our analysis does not cover the behaviour of central banks in emerging Asia who are actively intervening in currency markets, and therefore has no strong implications for the value of the dollar against their currencies.

REFERENCES

Benzoni, L,P. Collin-Dufresne, R.S. Goldstein (2004), *Portfolio Choice over the Life-Cycle in the Presence of 'Trickle Down' Labour Income*, Working Paper, University of Minnesota.

Bernanke, B.S (2005), *The Global Saving Glut and the U.S. Current Account Deficit*, Sandridge Lecture, Virginia Association of Economics, Virginia.

Cocco, J.F, F.J. Gomes, P.J. Maenhout (2002), *Consumption and Portfolio Choice over the Life-Cycle*, Working Paper, London Business School.

He, H. (1990), "Convergence from Discrete- to Continuous Time Contingent Claims Prices," *Review of Financial Studies*, 3, 4, pp. 523-546.

Hayashi, F (2004), *The Over-investment Hypothesis*, Empirical Analysis of Economic Institutions Discussion Paper Series.

Kamps, C (2004), *New Estimates of Government Net Capital Stocks for 22 OECD countries 1960-2001*, IMF Working Paper 04/67, Washington, DC.

Levey, D.H, S.S. Brown (2005), "The Overstretch Myth", *Foreign Affairs*, vol. 84, number 2.

Loayza, N, K. Schmidt-Hebbel and L. Serven (2000), "What Drives Private Saving Across the World", *The Review of Economics and Statistics*, vol. 82, issue 2, pp 165-181.

APPENDIX

Here we present the details of the life-cycle model of savings choice used in our analysis. We consider an investor with the following characteristics:

- The investor has a fixed horizon of T periods (years).
- He receives a labour income at each period up to period $T_0<T$. This labour income is subject to uncertainty[12] but is constrained to be non-zero and positive. The period from T_0 to T corresponds to retirement plus the period of time that captures the utility of bequest.
- He can make decisions at discrete time intervals from 0 to T-1 (T is the last period).
- At each decision point, the investor decides the proportion of his wealth to consume and the proportion to save. The saving is invested in a composite risky asset.
- The investor is not allowed to borrow against future income. However, his consumption in some periods can easily be larger than his income if he has accumulated positive wealth and is willing to spend it.
- Each unit of consumption brings a certain amount of utility to the investor. We use the Constant Relative Risk Aversion (CRRA) functional form of the utility which is a standard modelling tool in financial economics. This form implies that an investor's level of risk aversion to changes in his wealth is constant with respect to wealth. In other words, an investor with \$100 net worth would be as averse to losing \$10 as an investor with \$50 net wealth would be to losing \$5. The degree of this aversion is calibrated below.

We assume that the investor represents the savings-consumption choice of the entire economy. We concentrate on the choice of this investor to describe what happens to the economy. This does not assume that there is only one person in the economy. Nor does this assume that all the investors have the same utility function; they just behave like our investor in aggregate.

Optimization problem and notation

The agent solves the following optimization problem:

$$V\left(W_0, 0\right) = \max_{C_t} \; E_0\left[\sum_{t=1}^{T} \phi^t U\left(C_t\right) + \phi^T V\left(W_T\right)\right]$$

where maximization is done by optimally selecting the consumption C_t at each point in time t. In the equation above, $U(C_t)$ is the utility function, ϕ is the discount factor, $V(W_t)$ is the value function, W_t is the accumulated consumer wealth at time t and E_0 is the expectation operator conditional on the information at time 0.

We define the utility function in the standard CRRA form:

$$U\left(C_t\right) = \frac{1}{\gamma} C_t^{\gamma}$$

The optimization is done subject to a budget constraint such that:

[12] *We use the words uncertainty and risk interchangeably.*

$$W_{t+1} = (W_t + y_t - C_t)R_{t+1}$$

where:

- y_t is the labour income t
- R_{t+1} is the realization of the returns on the risky asset

The additional constraint is that consumption cannot be negative and the investor is not allowed to consume more than his current wealth W and labour income (in other words, he cannot borrow against his future income).

The utility of bequest is given by $V(W(T))$. We set this to equal the indirect utility of wealth for the representative investor assuming that he had 10 years of remaining life and no labour income.

Labour income and the return on the risky asset evolve as follows:

$$\begin{aligned} y_{t+1} &= y_t \eta_{t+1} \\ \eta_{t+1} &= \exp\left(\mu_y - .5\sigma_y^2 + \sigma_y Z_{t+1,y}\right) \\ R_{t+1} &= \frac{S_{t+1}}{S_t} = \exp\left(\mu_S - .5\sigma_S^2 + \sigma_S Z_{t+1,S}\right) \\ \rho &= corr\left(Z_{t+1,S}, Z_{t+1,y}\right) \end{aligned}$$

where μ_y is the growth rate of labour income, μ_S is the growth rate of risky asset income, σ_y is the volatility of labour income, σ_S is the volatility of risky asset, ρ is the correlation between risky asset and labour income shocks. The random variables $Z_{t,y}$ and $Z_{t,S}$ represent shocks to labour income and asset returns respectively. We assume that their probability distribution is a discrete approximation to the standard bivariate normal distribution with correlation ρ .

Solution approach

Dynamic programming approach

At each decision point the investor has to decide how much to consume every period. What is not consumed is invested in the risky asset. Thus, there is one decision variable.

The decision depends on his current wealth and labour income. These are the state variables that fully describe the current financial position of the investor. Equivalently (and more conveniently), the two state variables of the problem are labour ncome y and total wealth of the investor x = W+y.

We use a dynamic programming technique to solve the above problem. According to the Bellman optimality principle, the optimal consumption policy can be determined by recursively solving the following problem:

$$\begin{aligned} V_t(x_t, y_t) &= \max_{0 \le C_t \le x_t} \left[\frac{1}{\gamma} C_t^{\gamma} + \phi E_t\left[V_{t+1}(x_{t+1}, y_{t+1})\right] \right] \\ (A): x_{t+1} &= y_{t+1} + (x_t - C_t)\left[R_{t+1}\right] \\ (B): y_{t+1} &= y_t \eta_{t+1} \end{aligned}$$

The maximization is done with respect to decision variable subject to two conditions (A) and (B). The function $V_t(.)$ denotes the so-called value function at time t. Note that the problem of the investor changes after retirement as he no longer receives the labour income. From T_0 onwards, the problem is simply a consumption-investment problem with a single risky asset whose solution can also be computed by dynamic programming and is available in the literature. The recursion defining function $V_t(.)$ is therefore subject to the boundary condition that $V_{T_0}(.)$ is given by the value function of the investor solving the single asset consumption-investment problem with a remaining life of $T-T_0$ years. (By assumption, the utility of consumption over this period encompasses the utility of consumption over the retirement period plus the utility of bequest.)

Redefinition of the state variables and decision variable for interpretation and computational efficiency

To get a better interpretation of labour income as a state variable, we redefine it as a proportion of the total wealth x. We define this variable as z. This variable captures the importance of the current labour relative to the wealth accumulated over time. Because labour income y is included in total wealth x, and financial wealth W is not allowed to be negative, z does not exceed 1.

We also redefine consumption as a fraction of total wealth, hence:

$$\alpha_t = \frac{C_t}{x_t} \qquad z_t = \frac{y_t}{x_t}$$

Similarly, it is much easier to think about the consumption decision as a fraction of wealth rather than just the consumption decision (ie, consumption as a fraction of income).

It turns out that we can describe the optimisation problem of the investor with just one state variable because:

$$V_t(x_t, y_t) = x_t^{\gamma} G(z_t, t)$$

where

$$G_t(z_t) = \max_{0 \le \alpha_t \le 1} \left[\frac{1}{\gamma} \alpha_t^{\gamma} + \phi E_t \left[G_{t+1}(z_{t+1}) \right] \right]$$
$$(A): x_{t+1} = x_t \left[z_t \eta_{t+1} + (1-\alpha_t) R_{t+1} \right]$$
$$(B): z_{t+1} = \frac{z_t \eta_{t+1}}{z_t \eta_{t+1} + (1-\alpha_t) R_{t+1}}$$

The boundary condition for $G_t(.)$ is derived from the boundary condition for $V_t(.)$.

Numerical solution procedure

An analytical solution to the above problem is not available, and hence we resort to numerical methods. We use the discrete approximation for shocks to labour income and return on the risky asset as suggested in He (1990). Once the discrete approximation is adopted, the expectation operator is just a probability-weighted sum of future outcomes and can be easily evaluated.

Model Parameters

The following are the parameters used in the calculations for the base case (US).

	Base Case
Wealth/Income	5.5
Labour Income Growth	1.5%
Volatility of Labour Income	5%
Risky Asset Mean Return	4%
Risky Asset Volatility	10%
Correlation between labour income and risky asset	35%
Productive Life	25yrs to 65 yrs
Retirement Period	15yrs+Bequest (10yrs)
Average Age	45 yrs
Coefficient of Relative Risk Aversion	5
Rate of Time Preference	2%

Source: Lehman Brothers.

Comparison of model and actual savings rates

In our analysis we compare the model implied savings rate with the gross private savings as a percentage of the gross private disposable income. Gross private savings refers to the combined savings of the household sector and the corporate sector, calculated before depreciation. The reason for considering private savings as opposed to personal (or household) savings is that we are interested in understanding the saving behaviour of the household and the corporate sectors as a whole. Since corporations are ultimately owned by individuals, this consolidation of corporations with households is intuitive and necessary. The reason for considering gross private disposable income (before depreciation) as opposed to net private disposable income (after depreciation) is that wealth should reflect the depreciation for the household and the corporate sectors. Wealth includes the market value of all physical assets that households own (including real estate) and the market value of corporate equities (ie, household ownership of the corporate sector). The market value of physical assets includes the fall in the value of the assets due to depreciation corresponding to the household sector. Similarly, the market value of corporate equities includes the after-depreciation retained earnings (earnings after depreciation, dividends, interest and tax) that are ploughed back into the corporate sector (while dividends and interest which are paid out to shareholders and bondholders are already included in household disposable income). In our life-cycle model, the return on the risky asset is assumed to be a net return (ie, after depreciation). This incorporates the depreciation in the value of the assets (ie, a decrease in wealth) held by households and corporations. Since depreciation is already considered in wealth, income as defined in the model should be gross, or before depreciation. Hence, our use of gross private savings for comparison with the model is reasonable.

MBS Investing over Long Horizons

Yang Chen
212 526-7091
ychen@lehman.com

Michael Ng
212 526-6685
mikeng@lehman.com

Bruce D. Phelps
212 526-9205
bphelps@lehman.com

We evaluate the long-term performance (since year-end 1993) of a buy-and-hold investment strategy in MBS versus comparable strategies in credit and agency bonds. As many significant MBS participants follow such a strategy and account for their performance using book accounting, we analyze the performance of each strategy in the same framework. Not only has an MBS strategy outperformed credit and agency strategies, but the distribution of MBS book income has displayed remarkable stability and diversification (volatility and shortfall), properties warranting substantial allocations of MBS to buy-and-hold portfolios. We also analyze MBS buy-and-hold performance and risk profile over shorter (5-year) holding periods (beginning each quarter since year-end 1993) and find similar results. Finally, we show that relative MBS-credit yields and option-adjusted spreads contain information regarding the subsequent relative book income performance of the MBS and credit buy-and-hold strategies.

1. INTRODUCTION

Is it profitable to buy and hold MBS over long horizons, or is MBS an "opportunistic" asset class for which the only profits come from correctly timing spread widenings and tightenings? Essentially, does the spread offered by MBS more than adequately compensate investors who follow a strategy of maintaining a long-term holding of MBS?

To analyze this question, we consider the following MBS buy-and-hold strategy: invest in the most recently issued 30-year FNMA MBS index generic with a price closest to par from below. Then, as cash flow is generated, it is reinvested in the new 30-year par coupon FNMA. Over time, due to coupon and paydowns received and market movements, the portfolio will contain a range of coupons and vintages.

How should we measure the performance of such an investment strategy? For total return investors, absolute and relative performance is typically measured using mark-to-market returns. For these investors, we would calculate MBS cumulative market returns and compare them with market returns for other asset classes such as intermediate credit and agency bonds. We would then compute MBS monthly total (or excess) return volatility and compare these risk-adjusted returns with those for other asset classes.[1] However, total return investors are unlikely ever to follow a buy-and-hold strategy. Their performance is measured monthly using market prices versus an index whose performance is calculated similarly. For total return investors, buying and selling in anticipation of changes in spreads is their *raison d'etre*. Consequently, what is an appropriate performance metric for total return investors may not be appropriate for buy-and-hold investors.

[1] *For such analysis in a total return framework, please see Risk and Return in the Mortgage Market, by Amitabh Arora, David Heike, and Ravi Mattu, Lehman Brothers, January 13, 2000. This study found that MBS market excess returns, adjusted by the standard deviation of excess returns, compared unfavorably with credit and, in particular, with agency bonds. In this paper, we do not compute "excess returns" (to Treasuries) for several reasons. First, excess returns imply that the investor wants to compare performance with a constantly shifting mix of Treasuries. For buy-and-hold investors, this is typically not the case. Assets are purchased and held (versus liabilities), and the question for them is "should I buy MBS, credit or agency bonds that are roughly similar in maturity (duration) and hold them?" Second, excess returns are very sensitive to the calculation method and to the quality of the duration measure. Instead, we propose to analyze performance independent, as much as possible, of analytical sensitivity measures. Also, over long periods, income return is the dominant component of the total return performance of fixed-income securities.*

Investors most likely interested in a buy-and-hold MBS strategy are banks, insurance companies, official institutions (foreign and domestic) and individuals who, for various regulatory, organizational and business reasons, do not typically sell bonds after purchase. These investors seek income, not capital gains.[2] For these buy-and-hold investors, monthly book income, not monthly total return, is the relevant return measure, and the variability of book income is the relevant risk measure. As we will show, bond performance in book income space has different risk and return properties compared with performance measured in total (or excess) return space. Given that the MBS market is heavily influenced by investors who use book accounting, this may have ramifications for interpreting the relative value of various asset classes.

A bond held to maturity will generate total income equal to its coupon, the difference between its purchase price and par, and any reinvestment income earned on cash flows received prior to maturity. The "promised" annual return on the bond will equal its yield at purchase assuming it does not default, principal is received as anticipated at purchase and interim cash flows are reinvested at this yield. However, a bond's book yield (and income) is not assured. Defaults, downgrades that force selling before maturity, unanticipated changes in the bond's amortization, and reinvestment at rates other than the bond's initial yield will cause the bond's realized book income to differ from what was promised. This variability of a bond's book income is the risk faced by a buy-and-hold investor.

Over the life of a bond, its cumulative market return should equal its book return. However, monthly market returns are typically much more volatile than book returns. Forecasting market returns requires different investment skills than forecasting book returns. A total return investor asks: "What is the bond's likely total market return over the next month." In contrast, the buy-and-hold investor who is unlikely ever to sell the bond asks: "What is the bond's likely book income over its life?" While there is some linkage between the two questions, they reflect different risk and return assessments. As we will show, assets can have market return volatilities and correlations that differ from their book return counterparts. Consequently, asset allocation conclusions reached in one risk-return framework may differ from those drawn from another.

The purpose of this paper is to measure the long-term performance of MBS for buy-and-hold investors. Specifically, we address the following questions:

- What has been the long-term book income of MBS? How does MBS book income compare with credit and agency bonds?
- What has been the variability of MBS monthly book income? How does the distribution of MBS monthly book income compare with that for other asset classes?
- What are the relationships between credit and MBS book income? Are they highly correlated? Does the presence of MBS help to reduce the volatility of book income? If so, what is the role of MBS in a credit-MBS portfolio? Does MBS have book income diversification potential beyond helping to reduce a portfolio's shortfall risk? Or does MBS just help reduce shortfall risk?
- Do market spread (OAS) and yield levels contain information regarding MBS future relative book income performance? For example, does a wider MBS OAS level relative to credit signal an opportunity to earn additional book income, or does the spread simply compensate for greater convexity risk that is typically realized?

[2] *There are several reasons why investors follow such a portfolio strategy. For example, insurance companies and banks have regulatory and market constraints that prevent them from recognizing gains or losses that may arise from selling bonds before maturity. Some official institutions may be reluctant to sell assets because there is the potential of sending an implied signal to the marketplace. Other investors (e.g., individuals or small pension plans) may not have the infrastructure to monitor assets for a more active management style.*

2. MEASURING LONG-TERM PERFORMANCE: BOOK INCOME RETURN AND RISK

A buy-and-hold portfolio manager typically strives to identify assets that will produce relatively high book income (book yield) with a high degree of confidence (i.e., low default or prepayment risk) rather than anticipating monthly spread changes. This focus on book yield can often work to the advantage of the book manager. To the extent that a portion of a bond's yield reflects a risk premium to compensate total return managers for systematic spread volatility, the book manager can garner that additional spread because spread volatility does not affect the buy-and-hold manager's performance.[3]

Book accounting calculates a bond's book value based on its historical cost and periodically adjusts this value to amortize fully any premium or discount by the bond's anticipated maturity date. The bond's book yield is based on its yield at purchase (calculated using the bond's purchase price and expected amortization schedule) and remains relatively static until maturity, irrespective of changes in market yields. Book income is calculated by multiplying the bond's current book value by its book yield and including any discrete adjustments due to unanticipated prepayments or credit impairment. For MBS, as prepayments occur, the manager replaces expected with actual prepayments, updates the prepayment forecast and recalculates the bond's book yield and income. Any adjustment to book value is recognized as a book gain or loss this period, which is reflected in current book income.[4] Although book income is based on a prepayment model, over time book income is adjusted to reflect actual prepayments and updated prepayment forecasts.[5]

Due to the negative convexity of MBS, prepayments tend to accelerate when interest rates decline. Consequently, the MBS manager must reinvest principal received prematurely at lower interest rates, lowering the portfolio's book income. The portfolio's book income will now start to lag that of a portfolio that did not have negative convexity. The MBS manager may also receive paydowns when interest rates rise, offering an opportunity to increase portfolio book income. However, higher rates usually delay scheduled paydowns, causing the portfolio's book income to remain relatively static while other less negatively convex portfolios will be able to reinvest more cash flow at higher yields. If rates are steady, the MBS portfolio's book income will gradually increase over time, reflecting the growth in the portfolio's book value.

Depending on the movement in interest rates, MBS book income will fluctuate. Since MBS are not vulnerable to default and downgrade risk, which could produce large negative shocks to book income, we would not expect large negative shocks to book income, especially since we only purchase MBS which trade close to par. If we adjust for the growth of the portfolio's book value over time, the distribution of monthly book income for an MBS portfolio should be spread around the initial book income with a bit of a negative skew. However, there should be no significant part of the distribution with large negative observations. Given the limited tail risk for MBS, the risk of long-horizon MBS investing is best measured by the volatility of book income.

[3] *A focus on book yield can also work to the portfolio's disadvantage. Bonds that trade at wider spreads than their peer group may do so because the market is assuming relatively higher default or prepayment risk. Buying bonds simply based on yield may work for a short time until the bond's higher risk reveals itself through a downgrade, default, or prepayment and lower future book income. For a discussion of adjusting a buy-and-hold portfolio's book yield for embedded default risk please see Book Accounting Indices: Design and Uses, Lev Dynkin, Yang Chen, Michael Ng, and Bruce Phelps, Lehman Brothers, June 21, 2005. For an empirical study on the subsequent performance of distressed investment-grade bonds, please see Portfolio and Index Strategies During Stressful Credit Markets, by Lev Dynkin, Jordan Mann, Sandeep Mody and Bruce Phelps, Lehman Brothers, January 23, 2004, Chapter IV.*

[4] *If prepayments are faster than expected (and/or prepayment forecasts are speeded up), investors in premium MBS would have to mark down the book value of the holding (reducing book income) and report a lower book yield, which will also reduce book income. If prepayments are slower than expected, investors in premium MBS would have a book income gain and an increase in book yield. The opposite pattern will occur for holdings of discount MBS.*

[5] *If different investors use different prepayment models then their MBS book income will likely differ. Also, a large change to a prepayment model could produce a large change to MBS book income in the month of the model change. In this study, we update prepayment information (realizations and forecasts) each month.*

The book income for a credit (bullet) bond is calculated similarly. Given the absence of prepayment risk, the credit manager has less reinvestment risk and may have more confidence of locking in the yield over the duration of the bond. However, instead of MBS prepayment risk, the credit manager must contend with default risk and its effect on book income. The credit manager also has downgrade risk because many managers are required to sell credit bonds if their rating falls below some threshold (e.g., investment grade).[6] If a default or downgrade occurs, the investor will no longer receive the promised income and will likely recognize a book loss (which reduces current and future book income). The investor will then reinvest the recovery proceeds at what may be higher or lower book yields than the initial bond. The buy-and-hold credit investor must worry not only whether overall defaults and downgrades are greater than expected, but also whether issuer defaults and downgrades in the portfolio are correlated. While the realized overall default rate in the market may equal the expected rate, the portfolio's default rate may exceed the market default rate if particular names in the portfolio default together.[7]

The credit portfolio manager faces a very asymmetrical portfolio book income profile. Credit assets either produce their promised book income each period with some distribution around the initial book income value as cash flow is reinvested at higher and lower book yields, or they suffer a large decline in book income due to defaults and downgrades. Given this tail risk, the risk of buy-and-hold credit investing is often measured by shortfall risk (i.e., expected shortfall) as well as by the volatility (standard deviation) of book income.

The risk to book income from agency (bullet) bonds is minimal because default and downgrade risks are very low and there is no convexity risk. The distribution of book income from agency bonds should have very little variability. As discussed below, the absence of credit and convexity risk makes agency bonds a useful baseline to compare the other two asset classes.

To measure the historical performance of these three asset types, we constructed separate book accounting indices for each of the three asset classes beginning in December 1993.[8] To make the performance measures relevant to investors, the indices reflect the investment strategy of a typical buy-and-hold investor if he or she had an investment inflow. The indices have comparable durations and broadly reflect the asset choices facing a buy-and-hold portfolio manager. We do not make any attempt to match durations (or key rate durations) exactly of the three strategies, for two reasons. First, matching durations would be very important if the goal were to compare monthly market returns because relative performance would be heavily influenced by any duration differences in addition to monthly spread performance. However, this is not of much interest for buy-and-hold investors. Second, there is healthy skepticism about the quality of the duration number for MBS calculated long ago.[9] In general, buy-and-hold investors seek assets that will match somewhat coarse maturity or duration liability buckets.

For MBS, we construct an MBS book index with an initial $1 billion investment on December 31, 1993, in a single MBS issue: the most recently issued 30-year FNMA MBS index generic with a price closest to par from below. (Note: If such a discount generic is not

---

[6] *Even if the manager is not required to sell a downgraded bond, the manager may have to mark the bond to market (i.e., recognize a book loss) and record book income only when coupon payments are received.*

[7] *Please see "Optimal Credit Portfolios for Buy-and-Hold Investors," Lev Dynkin, Jay Hyman and Bruce Phelps,* Journal of Portfolio Management*, Summer 2004.*

[8] *Monthly book income is generated using Lehman's book index (BOOKIN) software. For details please see "Book Accounting Indices: Design and Uses." For the three book indices we assume bonds do not leave the book index if they have less than one year left to maturity or if they violate a future liquidity constraint (unlike Lehman Brothers' standard total return indices). Bonds leave a book index only if they are downgraded below investment grade, mature, or prepay (for MBS).*

[9] *We have recently analyzed the quality of Lehman Brothers' MBS analytical durations (since 2001) and have found them to compare favorably with many empirical duration measures. See, "Managing against the Lehman MBS Index: Evaluating Measures of Duration," by Lev Dynkin, Michael Ng, and Bruce Phelps, Lehman Brothers, August 5, 2005.*

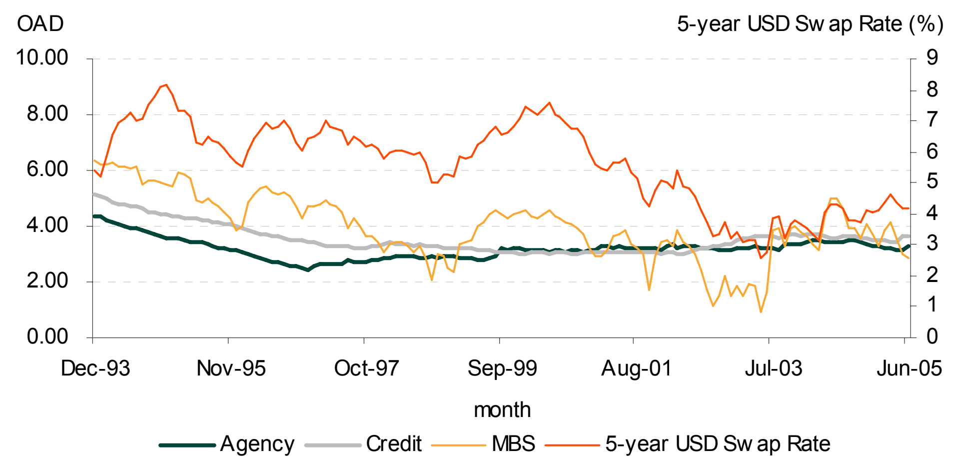
available, the index buys the most recently issued 30-year FNMA whose price is closest to par.) In other words, the index buys only newly issued, slightly discounted, MBS, which is typical of many buy-and-hold investors. As cash flow is generated by this portfolio, it is reinvested in the then current 30-year FNMA index generic. While the MBS book index begins as a single MBS generic, it gradually adds more generics as interest rates fluctuate and the current coupon MBS changes.

For credit, we construct a credit book index by making an initial investment of $1 billion on December 31, 1993 in a modified 3-10 year maturity Baa-A credit index (bullets only).[10] We use this index to reflect the performance of a purely passive buy-and-hold credit portfolio manager. Bonds that default or are downgraded below investment grade (using the Lehman index quality rating) are sold from the index, with consequences for the index's book income. As the initial index generates cash (coupons, maturities and recoveries), we assume that the index/portfolio buys more of the current index.

Finally, we construct an agency book index by making a similar initial investment of $1 billion on December 31, 1993, in the 3-10 year agency index (bullets only). As the initial index generates cash (coupons and maturities), we assume that the portfolio buys more of the current index.

The three buy-and-hold strategies receive no additional investment inflows. For each strategy (as represented by its corresponding book index), we calculate its monthly book income, book value, cash flow, book yield, market value, and market value-weighted OAD. (For reference, the time series of each strategy's monthly OAD and the 5-year USD swap rate are shown in Figure 1.)

Figure 1. Monthly OAD for MBS, Credit and Agency Buy-and-Hold Strategy Portfolios (December 1993 – June 2005)



Source: Lehman Brothers.

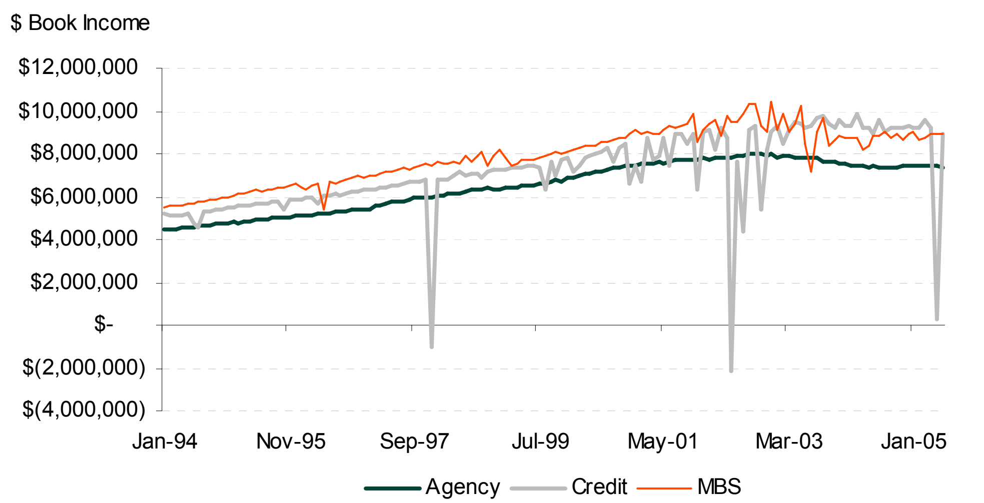
[10] *We modify the Lehman index to include only credit assets issued within the past five years and with a minimum amount outstanding that gradually increases over time. This restriction makes this exercise more realistic by limiting the index to buying only relatively large issues that are easily available. We also assume, unlike the standard Lehman Brothers total return indices, that bonds with less than one year remaining until maturity remain in the strategy portfolio until maturity. In future reports, we plan to analyze other credit strategies with various issuer caps.*

3. LONG-TERM PERFORMANCE OF MBS

Figure 2 shows the time series of monthly book income for the MBS, credit, and agency buy-and-hold investment strategies from December 1993-June 2005. For all three strategies, book income gradually increases over time, reflecting the reinvestment of coupon income. As expected, the book income pattern for the agency strategy is very smooth because there are no defaults, downgrades, or unanticipated amortizations to cause significant monthly fluctuations in book income. Book income gradually increases due to reinvestment and gradually higher yields and then begins to level off and decline slightly as market yields began to fall steadily starting in 2000.

The book income pattern for the credit strategy also displays extended periods of stability but is occasionally interrupted by some very sharp declines in book income due to defaults and downgrades. Not surprisingly, 1997-1998, 2002-2003, and 2005 produced some large negative shocks to book income. However, note how strongly book income recovered after these credit shocks as credit spreads widened, presenting an opportunity for cash flow generated by the strategy to be reinvested at higher book yields.

Figure 2. Monthly Book Income for MBS, Credit, and Agency Buy-and-Hold Strategy Portfolios (December 1993-June 2005, $)



Source: Lehman Brothers.

The mortgage strategy also displays relatively stable book income. There is an occasional drop in book income, due to unanticipated changes in prepayments (either faster or slower), but these are not nearly as severe as for credit. As interest rates fell sharply after 2000, MBS book income became more variable as prepayments surged and cash flow was unexpectedly reinvested at lower yields.

The stability of MBS book income is reflected in the monthly book return for the strategy, where book return is defined as book income this period divided by book value at the end of the prior period. Figure 3 plots the MBS strategy's monthly book return versus its monthly market return. Monthly book returns fell in a narrow band of approximately 0.40%-0.60%, whereas monthly market returns fluctuated at approximately -4.25% and +4.50%. A buy-and-hold investor may perhaps draw very different conclusions than a total return investor regarding the risk of investing in MBS.

Figure 3. Monthly Book Return versus Monthly Market Return for MBS Buy-and-Hold Strategy Portfolio (December 1993-June 2005)

Source: Lehman Brothers.

Figure 4 presents some summary information on monthly book income for the three buy-and-hold strategies. Over eleven and a half years, the MBS strategy produced an average monthly book income of $7.9 million, with a standard deviation of $1.3 million. The lowest monthly book income was $5.4 million. The MBS strategy produced a range of monthly book income of $5.0 million, and the average monthly book income in the worst 5% of months (i.e., seven months) was $5.6 million.

Figure 4. Monthly Book Income Summary Information: MBS, Credit, and Agency Buy-and-Hold Portfolios (December 1993-June 2005)

	MBS	Credit	Agency
mean	$7,907,649	$7,189,929	$6,525,134
stdev	$1,260,784	$1,902,635	$1,145,248
max	$10,395,557	$9,840,480	$7,995,244
min	$5,352,440	-$2,105,668	$4,459,097
range	$5,043,117	$11,946,148	$3,536,147
shortfall (5%)	$5,573,724	$2,282,657	$4,533,466

Source: Lehman Brothers.

For the credit strategy, the average monthly book income was $7.2 million with a standard deviation of $1.9 million. In sharp contrast to the MBS strategy, the lowest monthly book income was -$2.1 million. The credit strategy also produce a wide range of monthly book income ($11.9 million) and the average monthly book income in the 5% tail was $2.3 million.

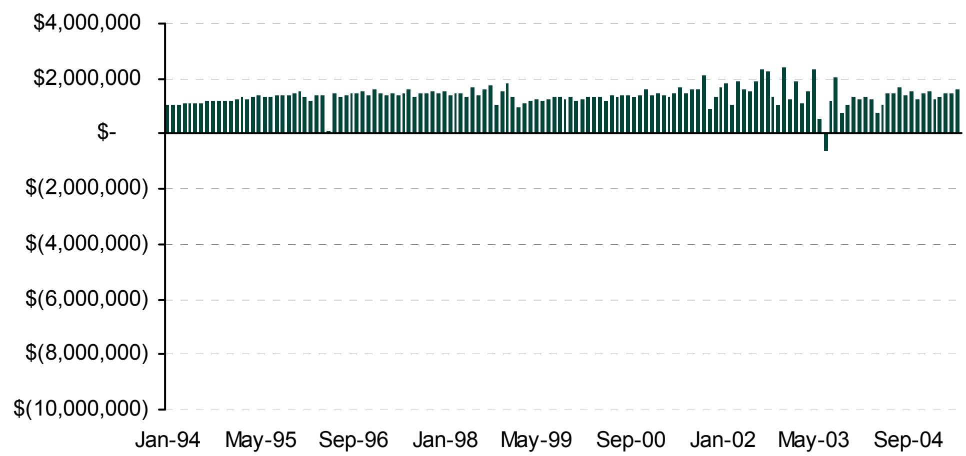
The agency strategy performed as we expected. While average monthly book income ($6.5 million) was less than that for the MBS strategy, its standard deviation of monthly book income was also less, at $1.1 million. The agency strategy produced a relatively narrow range of monthly book income ($3.5 million) and its 5% shortfall value, $4.5 million, suggests very little tail risk.

How is monthly book income correlated between MBS and credit? Is there a tendency for the book income for both strategies to fluctuate together? To highlight the monthly variability of book income, we de-trend MBS and credit monthly book income by subtracting the monthly book income for the agency strategy. Subtracting the book income of the agency strategy is akin to looking at excess book income (à la excess market returns). These net book income values can also be interpreted as net book income for a high-quality financial institution that funds its asset purchases at levels comparable to that of the agencies.

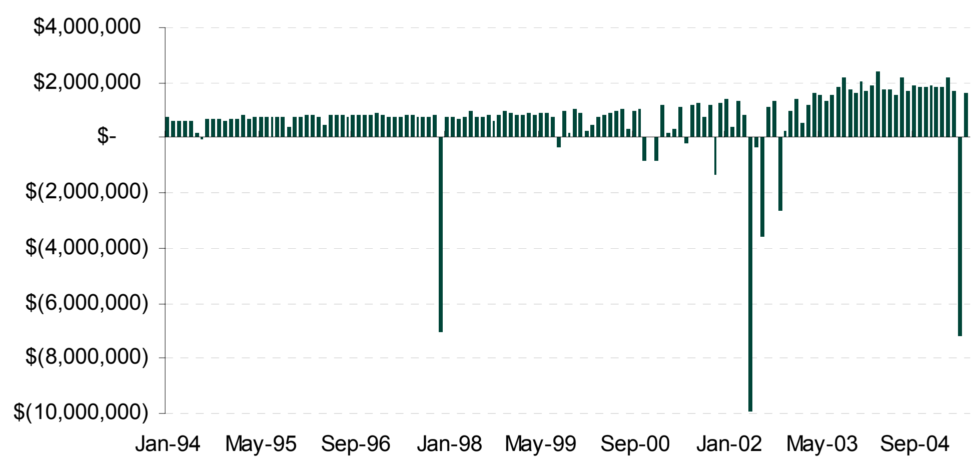
The time series of "net" monthly book income for the MBS and credit strategies are shown in Figure 5, and summary information is presented in Figure 6.

Figure 5. Monthly Net Book Income: MBS and Credit (net of Agency) Buy-and-Hold Strategies (December 1993-June 2005, $)

MBS – Agency Monthly Book Income



Credit – Agency Monthly Book Income



Source: Lehman Brothers.

Given the infrequent occurrence of negative net book income months for the MBS strategy (compared with the credit strategy), adding MBS to a credit portfolio can help reduce the portfolio's overall shortfall risk. For institutions sensitive to the potential for negative book income (net of funding), an MBS allocation can help the institution target a shortfall level that meets their risk profile.

By inspection, the fluctuations in net monthly book income seem uncorrelated, and, in fact, the sample correlation coefficient shows the two series are slightly negatively correlated (-

0.08). Out of curiosity, we calculate monthly market total returns for the MBS and credit strategies and subtract from each the corresponding monthly agency strategy market return to make the returns comparable to the net book income returns. Over the same period, the correlation of MBS and credit net market returns was 0.41.

Figure 6. Monthly Net Book Income and Market Returns Summary Information: MBS and Credit (net of Agency) Buy-and-Hold Strategies (December 1993-June 2005)

	MBS Strategy	Credit Strategy
Monthly Net Book Income		
mean	$1,382,515	$664,795
standard deviation	$349,450	$1,522,513
max	$2,400,313	$2,398,497
min	-$623,739	-$9,945,674
range	$3,024,052	$12,344,170
shortfall (5%)	$503,741	-$4,648,138
correlation	-0.08	
Monthly Net Market Returns		
mean	5 bp	4 bp
standard deviation	49 bp	41 bp
correlation	+0.41	

Source: Lehman Brothers.

Note how the two asset classes behave differently in a book accounting world compared with total return. Credit net market returns are less volatile than MBS (standard deviation of monthly net total returns of 41bp and 49bp, respectively) which is quite contrary to the book income world. Credit still underperforms MBS and we should not expect much of a difference with the book income world as eventually book income and market returns should converge over the life of the investment. However, the monthly volatility of book income and market returns can be significantly different.

Why is credit less volatile than MBS when expressed in terms of market returns? Monthly mark-to-market requires credit investors to recognize the market effect of gradual credit deterioration as it occurs each month. In contrast, the book investor reports the cumulative effect of credit deterioration only in the month when the bond is declared credit impaired.[11] Again, cumulative book return should approximately equal cumulative market return over a bond's life. However, in a book accounting framework, credit securities can experience more extreme monthly negative tails, which increases the relative portfolio benefit of including MBS to control the portfolio's shortfall risk.

The vulnerability of the credit strategy to shocks is most apparent if we remove the worst seven months (5% of all months) from the 138 months of the strategy's history. As shown in Figure 7, without these months, the average monthly net book income for credit would have been $0.9 million with a standard deviation of $0.5 million. Most notable is the reduction of tail risk because the worst book income month would have been only -$0.8 million (compared with -$9.9 million before) and a range of $3.2 million. Both of these values are somewhat more comparable to the MBS strategy. However, some tail risk remains for the

[11] *For the MBS book income calculation we update the prepayment forecast each month. Some book accounting investors may not do this; they may update only quarterly, etc. If so, the MBS mortgage book income values may appear smoother in our presentation than those experienced by some buy-and-hold investors.*

credit strategy because the (5%) shortfall value is -$0.2 million, which, while much less than before (-$4.6 million), is still considerably lower than the shortfall for the MBS strategy.

Also notable in Figure 7 is that the low correlation of monthly net book income remains even after removing the worst 5% of book income months for the credit strategy. This supports the idea that MBS have a diversification potential in buy-and-hold portfolios beyond their ability to reduce a portfolio's overall tail risk.

Figure 7. Monthly Net Book Income and Market Returns Summary Information: MBS and Credit (net of Agency) Buy-and-Hold Strategies excluding worst seven months for credit strategy (December 1993 – June 2005)

	MBS Strategy	Credit Strategy
Monthly Net Book Income		
mean	$1,378,920	$948,693
standard deviation	$352,308	$544,895
max	$2,400,313	$2,398,497
min	-$623,739	-$810,487
range	$3,024,052	$3,208,984
shortfall (5%)	$521,059	-$210,512
correlation	-0.01	
Monthly Net Market Returns		
mean	5bp	4bp
standard deviation	51bp	39bp
correlation	+0.41	

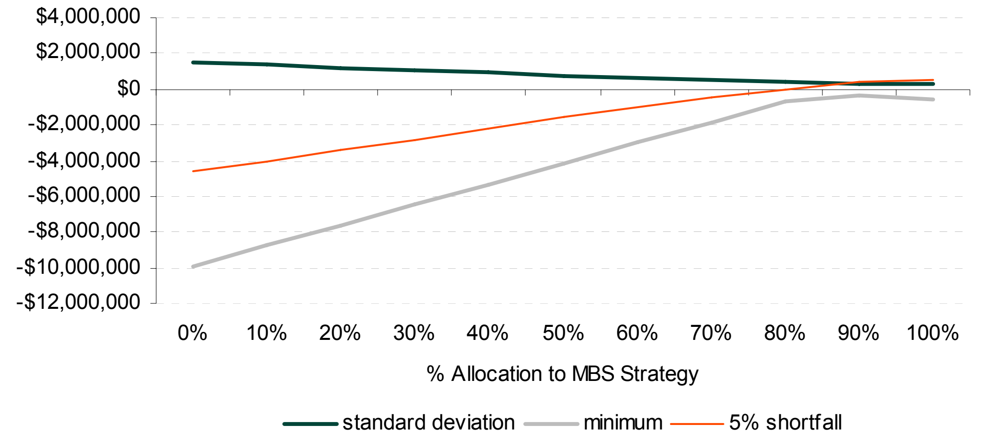
Source: Lehman Brothers.

4. CONSTRUCTING BUY-AND-HOLD PORTFOLIOS: ALLOCATION TO MBS AND CREDIT

The historical record suggests that MBS has performed well, the asset class has little tail risk, and its monthly book income has low correlation with that for credit, even after removing the negative credit tail event months. Given this diversification potential for MBS, what was the "optimal" historical allocation for an MBS-credit buy-and-hold portfolio over 1993-2005? While the record shows that the MBS strategy outperformed credit and agency strategies from a book income perspective, there are no assurances that this income outperformance will continue. So instead, we focus on the risk attributes of MBS in a buy-and-hold portfolio that are likely to be longer lasting. In other words, how did the book income risk of the portfolio vary depending on the portfolio's percentage allocation to MBS?

For various asset allocations to MBS and credit, Figure 8 shows the average and standard deviation of net book income, as well as the minimum monthly book income and shortfall (5% tail, i.e., the average book income in the worst seven months). For example, with a 0% allocation to the MBS strategy, Figure 8 shows the same results as in Figure 6. As the allocation to MBS is increased (in 10% increments), the standard deviation, minimum, and (5%) shortfall of monthly book income steadily improve. While the standard deviation declines until the MBS allocation equals 100%, the minimum and (5%) shortfall reach a minimum at a 90% allocation to MBS. If the goal is to obtain a (5%) shortfall greater than zero (i.e., always generate more income than funding cost), this is achieved with an 80%allocation to the MBS strategy. Figure 8 highlights the significant risk reduction potential offered by MBS in a buy-and-hold portfolio.

Figure 8. Monthly Net Book Income Summary Information: MBS and Credit (Net of Agency) Buy and Hold Strategy Portfolios - Risk Measures for Various Allocations to MBS and Credit Strategies (December 1993-June 2005)

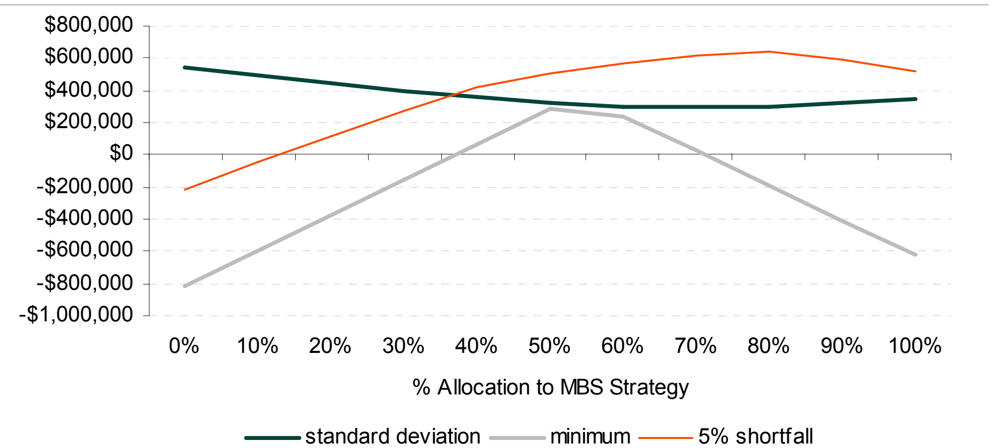


Source: Lehman Brothers.

Even after excluding the 5% of months when the credit strategy had the worst net book income, MBS continues to offer substantial risk reduction potential. Figure 9 is the same as Figure 8 except that the months associated with the worst seven (5%) net book income months for the credit strategy have been removed from the time series.

Figure 9 shows that MBS continues to offer substantial risk reduction potential even after much of the tail risk has been removed from the credit strategy. The standard deviation of portfolio net book income is minimized with a 70% allocation to the MBS strategy, and the minimum monthly book income is maximized with a 50% allocation. The (5%) shortfall now achieves a level greater than zero with a 20% allocation to the MBS strategy.

Figure 9. Monthly Net Book Income Summary Information: MBS and Credit (net of Agency) Buy-and-Hold Strategy Portfolios Excluding Worst Seven Months for Credit Strategy (December 1993-June 2005)



Source: Lehman Brothers.

Even without introducing the superior book income performance of MBS for buy-and-hold portfolios, we see that MBS offers meaningful diversification benefits. For institutions worried about shortfall risk (especially net book income less than zero), MBS can play an important role in reducing the shortfall risk to book income of a credit portfolio. We find it somewhat surprising that even without the negative credit tail event months, MBS continue to offer a risk reduction benefit to portfolios.

5. SENSITIVITY TO INITIAL INVESTMENT MONTH FOR CREDIT AND MBS BUY-AND-HOLD STRATEGIES

The previous section discussed the long-term performance of the MBS and credit strategies assuming a buy-and-hold investment was made at the end of 1993. However, the performance of each strategy may have been sensitive to the choice of the initial starting month as the portfolio's book income was, of course, influenced by the market yield at the time of the initial investment. How sensitive are our results to the initial investment month? Also, it would be informative to explore whether there are more opportune times to invest in the MBS or credit strategies. For example, does investing in the credit strategy when the MBS-credit OAS spread (or yield ratio) is below average produce relatively higher net book income, or is its relative OAS advantage offset by subsequent higher defaults and downgrades? Conversely, does investing in the MBS strategy when the MBS-credit OAS spread is above average produce higher net book income, or does its relative OAS advantage fully reflect the higher likelihood of prepayment surprises?

To answer these questions, we construct new buy-and-hold MBS and credit strategy portfolios at three-month intervals beginning in December 1993. We then examine each strategy's subsequent five-year book income performance. Although these five-year periods are overlapping and do not contain independent observations, they do give an indication of what the long run performance of an investment in MBS and credit would have been at quarterly intervals over the past eleven and a half years. We can also see if the relative performance of MBS versus credit was related to the ratio of MBS yields to credit yields, or relative OAS spreads, at the beginning of each quarterly period.

First, what is the relationship between a strategy's initial book yield and its subsequent average book income performance? For each starting calendar quarter, Figure 10 shows the initial book yield for each strategy and the subsequent average monthly book income over the following five years. For all three strategies, there is a strong linear relationship between the initial yield and the subsequent average monthly book income. In particular, for the agency strategy, the relationship is very strong with a sample correlation = 0.98. Not surprisingly, the relationship is less strong for MBS and credit because unexpected prepayments and credit impairments cause the realized book income to deviate from what was "promised" by the initial book yield. The correlation coefficient for the credit strategy was 0.89, which was greater than that for the MBS strategy at 0.82.

However, while the observed strong linear relationship between initial yield and subsequent income is reassuring to buy-and-hold investors, it does not address whether the level of book income is commensurate with each strategy's book yield. For example, does the level of book income reflect the strategy's initial yield, or is there a persistent underproduction of book income from a particular strategy? Also, what is the distribution of monthly book yield during the subsequent five years?

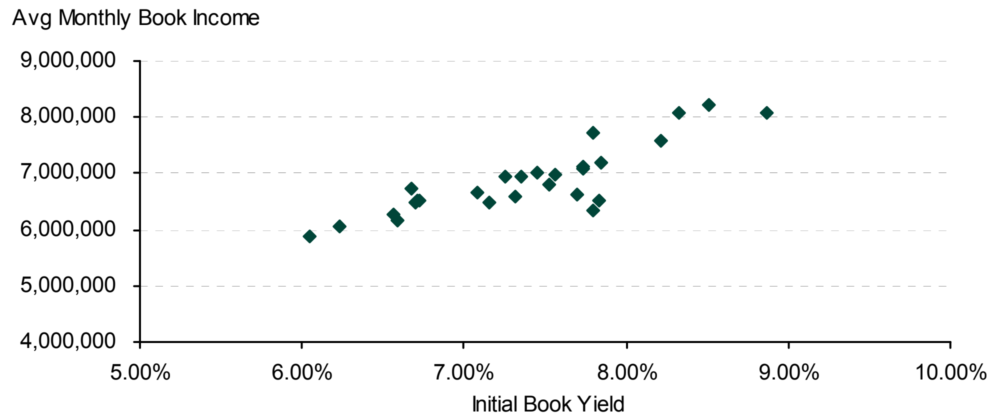
To answer these questions, we calculate the net (versus agency) book income performance over the following five years and report (Figure 11) the mean, standard deviation, minimum, and range for both the MBS and credit strategies. (Again, net book income can be interpreted as book income net of funding costs for highly-rated institutions.) We also report the shortfall of each strategy measured by the net book income for the worst three months (i.e., 5% of the months) over the following five years.[12] For example, assuming an investment at the end of June 1998, the average monthly net book income for the MBS and credit strategies was \$938,002 and \$950, respectively. The standard deviation of net book income for the MBS and credit strategies was \$191,207 and \$1,473,199, respectively. In addition, MBS had

[12] *Given that we are assuming 5-year investment periods, our last observation is for strategies that commence at the end of June 2000.*

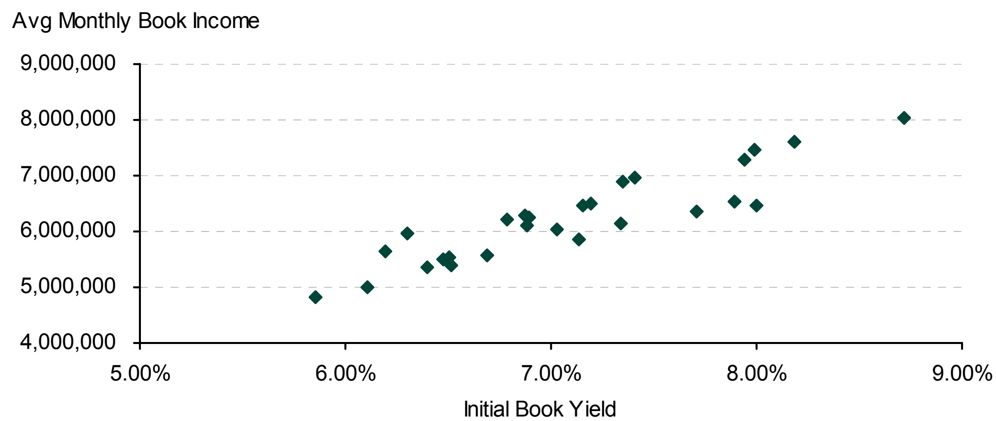
a minimum monthly net book income of $682,127 and a range of $803,493, compared with credit's minimum of -$8,771,326 and range of $10,040,564.

Figure 10. Initial Book Yield and Subsequent 5-year Average Monthly Book Income Performance: MBS, Credit, and Agency Buy-and-Hold Strategy Portfolios (Quarterly Starting Periods, December 1993-June 2005)

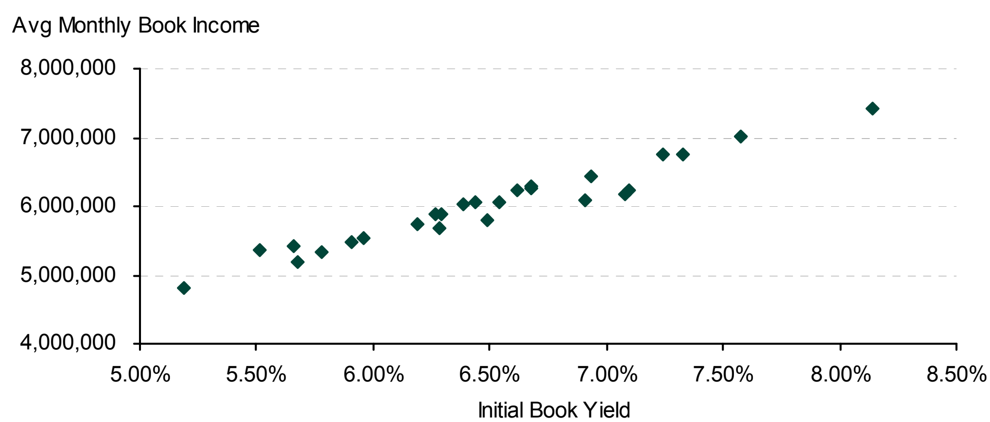
MBS Strategy, $



Credit Strategy, $



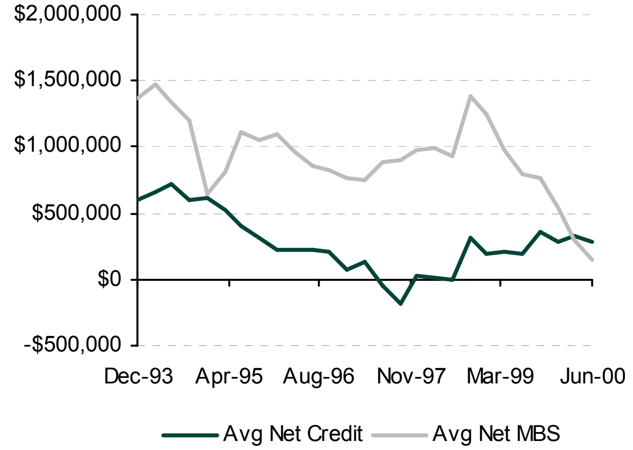
Agency Strategy, $



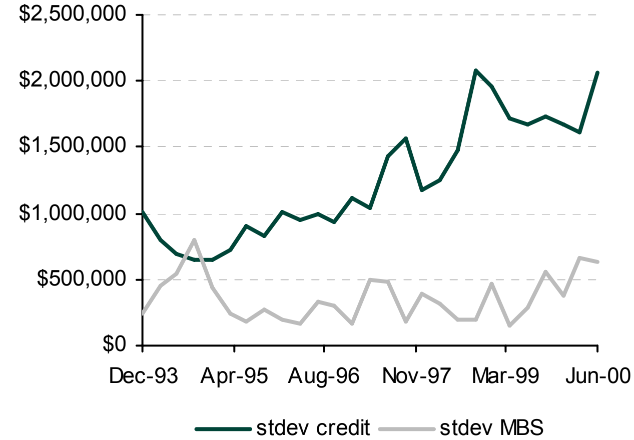
Source: Lehman Brothers.

**Figure 11. Five-Year Horizon MBS and Credit Buy-and-Hold Strategy Portfolios
Average, Standard Deviation, Minimum, Range, and (5%) Shortfall of Net Monthly Book Income
(Quarterly Starting Periods, December 1993-June 2005)**

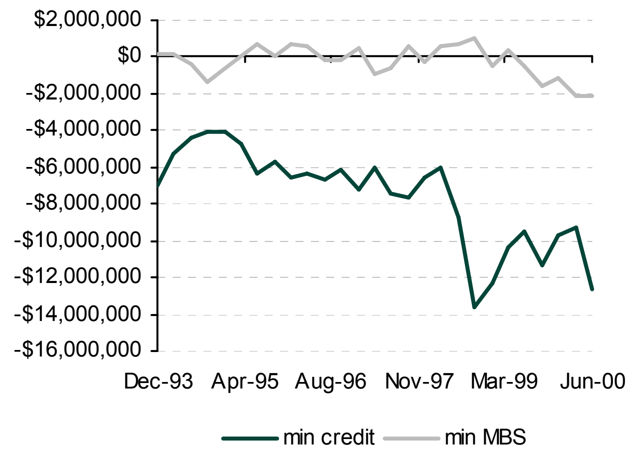
Average Monthly Net Book Income



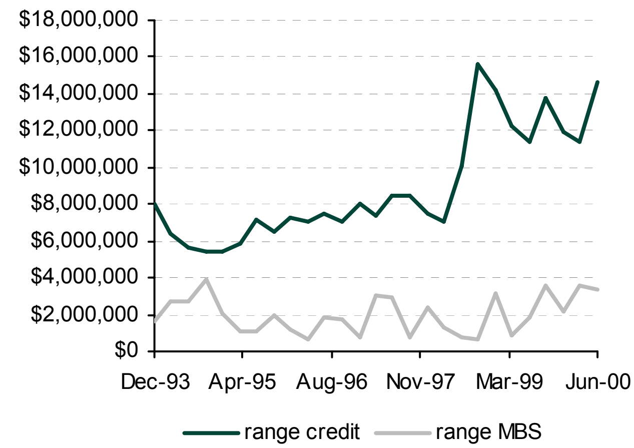
Standard Deviation Monthly Net Book Income



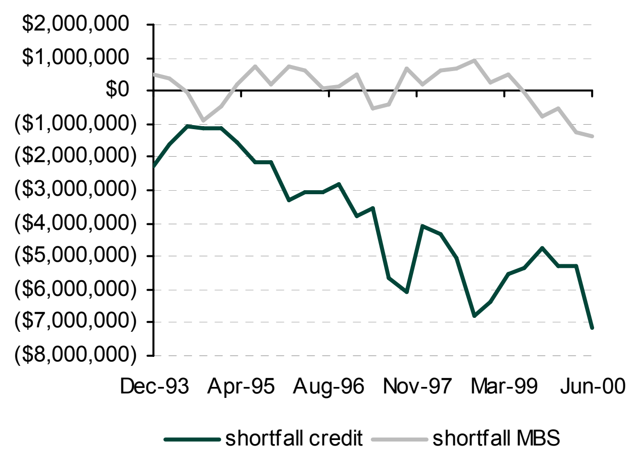
Minimum Monthly Net Book Income



Range of Monthly Net Book Income



Shortfall (5%) of Monthly Net Book Income



Source: Lehman Brothers.

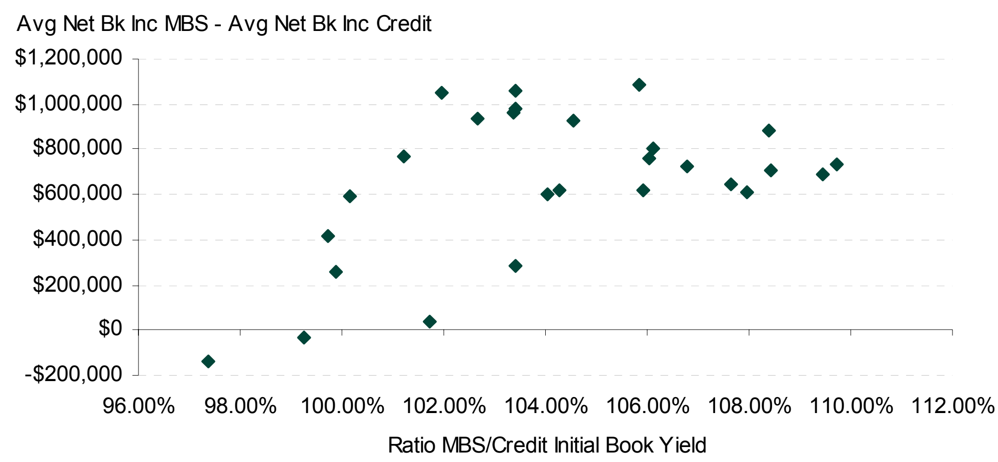
Finally, the worst three months (5% of all 60 months in a five-year period) produced an average net book income of $518,487 and -$7,421,970 for the MBS and credit strategies, respectively.

Figure 11 shows that the MBS strategy enjoyed greater average monthly net book income with a lower standard deviation compared with the credit strategy for just about any five-year period since December 1993. The MBS strategy started to underperform the credit strategy in 2000 as the subsequent unexpected very fast prepayment speeds and low market yields caused MBS book income to drop sharply. Most noticeably, however, the minimum, range and (5%) shortfall of monthly book income for the MBS strategy are relatively stable over time and reflect very little tail risk compared with the credit strategy.

Figure 11 shows that while the MBS strategy generally produces greater net book income than the credit strategy, there are months where MBS relative performance is stronger or weaker. Are there any indications at the beginning of the strategy, such as MBS-credit relative yield or OAS, that would help forecast the relative performance of the MBS and credit strategies and help buy-and-hold investors make asset allocation decisions for new investment inflows.

To investigate, at the beginning of each quarterly period we calculate the ratio of the MBS strategy book yield to that for the credit strategy and then plot the subsequent difference between the average net book incomes for the two strategies. Figure 12 suggests that there is only a moderate positive relationship (sample correlation = 50%) between the MBS yield advantage at the beginning of the five-year holding period and its subsequent net book income performance relative to the credit strategy. However, there does seem to be some basis for buy-and-hold investors to use relative yields as a basis for allocating new cash between MBS and credit. In other words, a sector's relative yield advantage does not appear to have been completely squandered by subsequent credit or prepayment events.

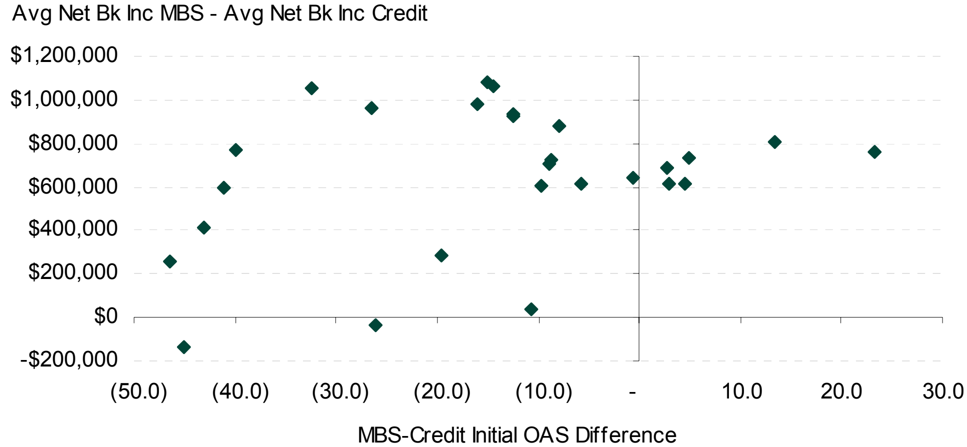
Figure 12. Initial Relative Yields and Subsequent 5-Year Average Relative Net Book Income Performance: MBS and Credit Buy-and-Hold Strategy Portfolios (Quarterly Starting Periods, December 1993-June 2005)



Source: Lehman Brothers.

Another potential relative performance indicator is the difference in OAS between MBS and credit. We calculate the OAS difference of the two strategies at the beginning of each quarter. We then plot (Figure 13) the subsequent difference between the average net book incomes for the two strategies. While the relationship is positive (sample correlation = 36%), the relationship is weaker than for relative yields.

Figure 13. Initial OAS Difference and Subsequent 5-Year Average Relative Net Book Income Performance MBS and Credit Buy-and-Hold Strategy Portfolios (Quarterly Starting Periods, December 1993-June 2005)



Source: Lehman Brothers.

6. CONCLUSION

This paper uses book accounting measures (book income and book return) to evaluate the long-term performance of a buy-and-hold investment in MBS. These are the same measures used by significant participants in the MBS market. By constructing buy-and-hold portfolios over 1994-2005, the paper compares the long-term book income performance of various asset classes (MBS, credit, and agency). We show that in comparison to an investment in credit, an investment in MBS offers superior book income with lower volatility and tail risk. In addition, MBS monthly book income has low correlation with credit book income, making it a good portfolio diversifier and warranting significant MBS allocations. These results also hold when we analyze the MBS buy-and-hold performance and risk profile over shorter five-year holding periods. Finally, there seems to be moderate correlation between relative MBS-credit market yields (and OAS differences) and subsequent relative MBS book income performance.

Style Analysis of Hedge Fund Returns: Actual versus Self-Proclaimed

Arik Ben Dor
1-212-526-7713
abendor@lehman.com

Lev Dynkin
1-212-526-6302
ldynkin@lehman.com

Tony Gould, CFA
1-212-526-2821
agould@lehman.com

Following the launch of the Lehman Brothers/HFN Hedge Fund Index, we illustrate how Style Analysis can be used to gain insight into the risk exposures of hedge funds and the construction of investable indices. We also present a simple measure that can identify inconsistencies between a fund's actual and self-proclaimed style.

1. INTRODUCTION

Institutional interest in hedge funds as an alternative to investments in traditional asset classes has increased substantially in recent years. Reflecting this trend, Lehman Brothers, in cooperation with HedgeFund.net (HFN), has recently launched a Global Hedge Fund Index.

As with all benchmarks in the Lehman Brothers Global Family of Indices, the Lehman Brothers/HFN Hedge Fund Index is constructed using an objective rules-based set of criteria to determine index eligibility.[1] However, while other indices represent composite returns of individual securities, the Hedge Fund Index reflects the performance of multiple investment strategies (or a single strategy in the case of style sub-indices) employed by the underlying funds.

The minimal disclosure requirements hedge funds face, coupled with an investment mandate that typically allows them to use leverage, short selling, derivatives and highly illiquid securities, present serious challenges for investors. How should they assess the risk/return characteristics of a certain hedge fund strategy in the context of their overall asset allocation? What is the correct approach for comparing the performance (or alpha) of individual hedge funds within the same style?

This article suggests one possible solution through the use of Return-based Style Analysis, introduced in the early 1990s by William Sharpe and used primarily for analyzing mutual funds. This technique provides a way of identifying the asset mix style of a manager and comparing it with the asset mix style of a specified performance benchmark.

We provide a short overview of style analysis and discuss how it may be extended to hedge funds with some modifications. We demonstrate how investors can use its results to better understand the nature of risks and exposures of various strategies, and the extent to which investments in various individual funds are correlated. The alpha or added value of a certain investment style or individual fund can also be measured using style analysis in a straight forward fashion.

Another application of style analysis is the construction of hedge funds portfolios. Lehman Brothers is planning to launch an investable index designed to closely track the broader index. Estimating the sensitivities of individual funds to common market factors can be used to construct such investable indices in an efficient manner. Alternatively, it may be used to find the composition of portfolios with a minimum volatility for a pre-specified level of expected return.

We also examine the common practice of classifying hedge funds into styles based on their self-reported investment strategy. The actual style of a fund may differ from its self-proclaimed style because it may not be uniquely defined or because the information reported by the fund may be inaccurate. We present a simple technique that can help identify

[1] *To be included in the index, funds must have at least $25 million in assets, a minimum one-year track record, an uninterrupted monthly return time series and an annual audit of fund returns. For more information see Lehman Brothers/HFN Global Hedge Fund Index Rules, Lehman Brothers, September 19, 2005.*

inconsistencies between the actual and self-proclaimed style of a fund based on a measure of distance between the return time-series of the two.

2. RETURN-BASED STYLE ANALYSIS

2.1. Methodology

Factor models are commonly used to characterize how industry and economy-wide factors affect the return on individual securities and portfolios of securities. Sharpe's (1988, 1992) return-based style analysis can be considered a special case of a generic factor model.[2] In this model, we replicate the performance of a managed portfolio over a specified period by the return on a passively managed portfolio of style benchmark index portfolios:

$$(1) \qquad \tilde{R}_{p,t} = [\delta_{1,p}x_{1,t} + \delta_{2,p}x_{2,t} + \dots + \delta_{n,p}x_{n,t}] + \tilde{\varepsilon}_{t,p} \qquad t = 1,2,3,\dots T$$

where $\tilde{R}_{p,t}$ represents the managed portfolio return at time *t* and $x_1,\ x_2 \dots x_n$ are the return on style benchmark index portfolios. The coefficients $\delta_1,\ \delta_2 \dots \delta_n$ represent the managed portfolio average allocation among the different style benchmark index portfolios, or asset classes, during the relevant period. The sum of the terms in the square brackets is that part of the managed portfolio return that can be explained by its exposure to the different style benchmarks and is termed the style of the manager. The residual component of the portfolio return - $\tilde{\varepsilon}_{t,p}$ is the part of the return attributable to the manager security selection ability. It reflects the manager's decision to deviate from the benchmark composition within each style benchmark class.

Given a set of monthly returns for a managed fund, along with comparable returns for a selected set of style benchmark index portfolios, the portfolio weights, $\delta_1,\ \delta_2 \dots \delta_n$, in Equation (1) can be estimated. However, in order to get coefficients' estimates that closely reflect the fund's actual investment policy, it is important to incorporate restrictions on the style benchmark weights. The following two restrictions are often imposed:

$$(2) \qquad \delta_{j,p} \geq 0 \quad \forall\ j \in \{1,2,\dots n\}$$

$$(3) \qquad \delta_{1,p} + \delta_{2,p} + \dots + \delta_{n,p} = 1$$

The first restriction corresponds to the constraint that the fund manager is not allowed to take short positions in securities. The second restriction imposes the requirement that we are interested in approximating the managed fund return as closely as possible by the return on a portfolio of passive style benchmarks. For funds known to employ some leverage or short selling, such as hedge funds, other bounds may be imposed.

The goal of return-based style analysis is to find the set of non-negative style-asset class exposures $\delta_1,\ \delta_2 \dots \delta_n$ that sum to 1 and minimize the "unexplained" variation in returns (i.e., the variance of $\tilde{\varepsilon}_{t,p}$) referred to as the fund's tracking error over the style benchmark. The exposures are estimated through the use of quadratic programming since the presence of inequality constraints in (2) does not allow the use of regression analysis. The objective is not to choose style benchmarks that make the fund "look good" or "bad"; rather, the goal is to infer as much as possible about a fund's exposures to variations in the returns of the given style benchmarks during the period of interest.

[2] *Sharpe, William, 1992, "Asset Allocation: Management Style and Performance Measurement,"* Journal of Portfolio Management, *18, 7-19.*
---- *"Determining a Fund's Effective Asset Mix."* Investment Management Review*, December 1988, pp. 59-69.*

In the context of style analysis, R^2 provides a natural way to distinguish active from passive managers. An active manager is looking for ways to improve performance by investing in asset classes, as well as individual securities within each asset class, that the manager considers underpriced. Hence, the manager will typically have different exposure to the style benchmark asset classes compared with the performance benchmark. The manager will also be holding a different portfolio of securities within each style benchmark asset class. As a result, the selection component will be closer to zero for passively managed funds when compared with actively managed funds.

$$(4) \qquad R^2 = 1 - \frac{Var(\tilde{\varepsilon}_p)}{Var(\tilde{R}_p)}$$

The right side of Equation (4) equals 1 minus the proportion of variance "unexplained". The resulting R^2 value thus indicates the proportion of the portfolio return variance "explained" by the *n* asset classes.[3]

2.2. Application to Hedge Funds

Traditional return-based style analysis is unsuitable for determining the effective style of hedge funds for several reasons. First, unlike mutual funds that follow a defined investment strategy and are limited to investing in specific asset classes, hedge funds have a substantial amount of freedom to choose from among a variety of investment strategies. Second, hedge funds can employ leverage and take short positions in securities whereas most mutual funds cannot do so. Third, many hedge funds exhibit an option-like return pattern that is hard to capture with a linear factor model. This arises from the use of derivatives, either explicitly or implicitly, through the use of dynamic trading.[4]

To address these challenges, we modify return-based style analysis in several ways. To capture the investment universe available to hedge funds, we use an extensive set of more than 100 factors reflecting the returns to various asset classes, sectors, geographical regions, and currencies (for a complete description of the list of factors, see the appendix). In addition, we alleviate the no short-selling constraints and the requirement that the sum of the estimated coefficients be equal to one.

Although lifting the constraints enables the use of multivariate regressions, simply using ordinary least squares is inadequate, given the extended list of factors. Since *a priori* we have no exact knowledge of which factors should be included in the regressions, estimation with the full list of factors would require a long time-series of returns that is not available for many strategies and in particular for individual funds. The need to estimate the regressions over a relatively short timeframe is also necessitated by the flexibility hedge funds enjoy in their investment mandate. As a result, their style coefficients may be unstable and exhibit large variations over time.

We employ an estimation technique known as stepwise regression in which the variables are entered or removed from the model depending on the significance of the *F*-value (a 5% significance level is used for both inclusion and exclusion). The single best variable is chosen first; the initial variable is then paired with each of the other independent variables, one at a time, a second variable is chosen, and so on until no further variables are included or

[3] *Since the vector of residuals is not necessarily orthogonal to the matrix of benchmark returns, as is the case in multivariate regression, the alternative definition* $R^2 = Var(\delta_{1,p}x_{1,t} + \ldots + \delta_{n,p}x_{n,t}) / Var(R_p)$ *is not in general equivalent to the definition given in (4).*

[4] *The 15%-20% performance-based fee charged by fund managers also contributes to an option-like return profile.*

excluded from the estimation. Stepwise regression thus allows us to examine the importance of a large set of variables even when we have a relatively small number of observations.

When a manager's return is related to the benchmark returns in a nonlinear manner, it would be difficult to identify his performance using linear factor models, of which return-based style analysis is a special case. For example, if investors were to evaluate the performance of a manager selling call options on a standard benchmark by measures such as Jensen's alpha or the Treynor-Black appraisal ratio, such a manager would be falsely classified as a superior performer.[5]

A suggested remedy to this problem that we use in our analysis is to augment the returns on style benchmark indexes with returns on selected options on the style benchmark. We include the returns to six strategies that involve buying put and call options on the S&P 500 and holding them to expiration (the exact nature of the strategies is described in the appendix).

2.2.1. Empirical Results

To illustrate the use of return-based style analysis for hedge funds in practice, we use data collected by HedgeFund.net. The dataset includes a total of 2,712 distinct funds and a total of 147,261 monthly return observations for January 1991-June 2004. There are a total of 30 self-reported investment styles. Each fund reports the strategies it employs, and based on this information it is classified into the appropriate style.

Figure 1 presents the style analysis results for each strategy with available return history of at least three years. Monthly returns to a strategy are the equal weighted returns of all funds in that strategy, if at least 30 individual funds' returns are available.[6] Panel A reports the degrees of freedom, number of significant factors, explanatory power (R^2), and the variance of the selection components (regression residuals).

The number of significant factors and the explanatory power of the regression for each strategy vary substantially. For example, roughly 45% of the variation in returns to convertible arbitrage can be explained using five factors, whereas we are able to explain 88% of the return variation in long-short equity using 10 factors. Not surprisingly, the return profile of equity market-neutral turns out to be the most difficult to explain because in principle it should have no exposure to systematic factors (the R^2 is only 20%). In general, directional strategies are more sensitive to market movements and, therefore, are better explained than arbitrage/market neutral strategies.

Two more points are worth mentioning. The first five factors in order of significance (e.g., order of entry into the estimation) account for almost all the explanatory power of the regression. Hence, the systematic portion of the returns to most strategies can be captured by only five factors (and sometime fewer). Second, strategies with high explanatory power can still have a higher volatility of the regression residuals (the selection volatility shown in the last column) relative to other strategies with lower explanatory power. For example, the selection volatility of equity market neutral and small/micro cap is 0.77% and 2.07%, respectively, although the R^2 of the latter is more than four times larger (87% versus 20%). This is simply due to the fact that the overall risk (or return variation) of the two is very different. The market neutral strategy exhibits relatively stable returns and low market exposure, whereas the opposite is true for the small/micro cap strategy.

[5] *For a discussion of this issue see Ben Dor, Jagannathan and Meier, 2003, "Understanding Mutual Fund and Hedge Fund Styles Using Return-based Style Analysis,"* Journal of Investment Management*, 1, 94-134.,*

[6] *We use equal weighting since data on assets under management is missing for many of the funds. In addition, we do not want the results to be affected by the highly skewed size distribution of hedge funds.*

Figure 1. Style Analysis for Hedge Funds' Strategies

The number of observation used in the estimation varies by strategy. Monthly returns to a strategy are calculated as the equal weighted returns of all funds in that strategy, if at least 30 individual funds' returns are available. A description of the factors is provided in the appendix.

Panel A: Main statistics

			Adj R2		
Strategy	**DF**	**No. of Significant loadings**	**All factors**	**First 5 factors**	**Selection volatility (monthly)**
Convertible arbitrage	78	5	45.6%	43.8%	0.77%
Market neutral equity	63	3	20.6%	20.6%	0.72%
Merger arbitrage	67	3	57.2%	57.2%	0.82%
Event driven	76	7	79.6%	77.4%	0.96%
Distressed	59	6	79.5%	78.4%	0.82%
Value	65	5	86.6%	86.6%	1.41%
Finance	62	9	84.6%	78.2%	1.70%
Small/Micro cap	40	7	87.3%	84.1%	2.07%
Long/short equity	73	10	87.6%	82.4%	1.16%
Long-only	31	8	98.3%	97.3%	0.67%
FI arbitrage	70	7	66.4%	61.9%	0.79%
FI non arbitrage	65	4	55.1%	55.1%	0.89%
Macro	77	6	51.7%	49.8%	1.42%
Emerging markets	68	12	86.2%	75.3%	2.77%
CTA	79	4	30.4%	30.4%	2.35%
Multi strategy.	74	9	61.1%	46.4%	0.98%

Source: HedgeFund.net

Panel B: First 5 factors in order of significance

Strategy	**1**		**2**		**3**		**4**		**5**	
Convertible arbitrage	US Industrial	-0.07	US NonCyc	-0.09	US Credit	0.70	EM FI	0.08	Call OTM	2.2E-03
Market neutral Eq.	Slope 10-30	-0.12	Slope 2-10	0.20	HML	-0.05				
Merger arbitrage	EM FI	0.15	Slope 2-10	0.17	Put OTM deep	-4.9E-04				
Event driven	Wilshire 5000	0.15	EM FI	0.17	SMB	0.13	VIX	-0.08	US HY	0.14
Distressed	EM Healthcare	0.03	EM FI	0.15	SMB	0.09	VIX	-0.07	US HY	0.21
Value	Wilshire 5000	0.65	EM Tech	0.05	SMB	0.16	HML	0.30	US HY	0.17
Finance	Wilshire 5000	0.27	EM FI	0.32	SMB	0.39	Ausy Dollar	-0.27	Call OTM deep	5.0E-03
Small / Micro cap	Wilshire 5000	0.46	US Healthcare	0.20	SMB	0.56	Developed NonCyc	-0.31	EM Telecom	0.19
Long/short equity	Wilshire 5000	0.23	VIX	-0.10	SMB	0.22	HML	-0.14	EM Telecom	0.06
Long-only	Wilshire 5000	0.54	Developed Cyc	0.14	SMB	0.20	US Treasury	0.15	Slope 10-30	0.21
FI arbitrage	Far East HML	0.07	Developed Cyc	0.10	Global Treas	0.15	EM FI	0.12	JPY	-0.18
FI non arbitrage	MBS	0.99	US Credit	0.37	JPY	-0.13	VIX	-0.09		
Macro	Slope 10-30	-0.33	Developed Cyc	0.18	JPY	-0.20	EM FI	0.17	GS commodity	0.06
Emerging markets	EM NonCyc	0.38	Far East HML	-0.20	Global Treas	-0.43	EM FI	0.55	VIX	-0.26
CTA	US Reits	0.16	Latin America	0.06	US HY	-0.42	MLM commodities	0.56		
Multi strategy	Developed Tech	0.05	EM FI	0.12	VIX	-0.08	Developed Finance	-0.06	Slope 2-10	0.19

Source: HedgeFund.net, Lehman Brothers, MSCI, Wilshire, Goldman Sachs

Panel B of Figure 1 displays the first five factors in order of significance for each strategy and the exposure to them. Convertible Arbitrage, for example, has a beta of -0.07 and -0.09 with respect to U.S. stocks in the Industrial and Non-cyclical sectors, respectively. In contrast, the strategy has a positive loading on U.S. corporate bonds, emerging markets bonds, and the out-of-the-money call strategy. Indeed, convertible arbitrage often involves owning a convertible security and shorting the underlying stock to hedge the equity component. The exposure to the Industrials and Non-cyclical sectors probably reflects the fact that the companies issuing the convertibles belong primarily to these sectors. The long position in the convertible security is mimicked by the positive exposure to the returns of corporate bonds and call-option strategy.[7]

Figure 2 presents the results of style analysis for individual hedge funds assigned to one of eight investment styles (arbitrage, event-driven, sector, directional, fixed income, global, Commodity Trading Advisors (CTA), and other).[8] The estimation is based on the period January 1999-June 2004. Only funds that have at least 36 consecutive monthly returns were analyzed. With the exception of CTA, the median number of significant factors for individual funds is 4-5. As before, the explanatory power is higher for funds that employ directional strategies than for funds that use non-directional strategies.

Panel B presents the five factors that are most frequently found to be significant in each style. The exposures to each factor represent the average exposures across all funds within a style. For example, funds in the event-driven style have on average a beta of 0.3 against the returns of the Lehman HY index. This reflects the fact that issuers comprising this index may be in distress or takeover candidates. Notice also that the positive coefficient may reflect an exposure not only to bonds but also to the issuers' stocks as the returns of the two are highly correlated.[9]

[7] *The weight of −0.22% on the call option appears to be small. However, it can still have a significant amount of sensitivity to tail events. For example, consider investing \$100 in cash and writing 1.2 index put options with an exercise price of \$90 and 3 months to maturity when the current index value is \$100. If the interest rate is 5% and index volatility is 20% per year, the put option value will be \$0.55 based on Black-Scholes. The portfolio will have \$100 in T-bills and −\$0.66 in index put options, i.e. 100.7% of the funds invested in T-bills and −0.7% invested in out-of-the-money index put options. Suppose the index value drops steeply by 20% to \$80 right after forming the position then the position will lose \$12, or a 12%. Hence, the position can lose a significant amount in severely depreciating markets even though most of the money is in T-bills.*

[8] *The investment styles are defined as follows: Arbitrage: Statistical arbitrage, Capital structure arbitrage, Convertible arbitrage, Market neutral equity and other arbitrage. Event driven: Special situations, Reg D, Merger/risk arbitrage, Event driven, Distressed. Sector funds: Value, Technology, Energy, Healthcare, Finance, Small and Micro cap. Directional: Short bias, Long/short equity, Long-only, Market timing. Fixed Income: Fixed income non arbitrage, Fixed income arbitrage, MBS. Global: Macro, Emerging markets, Country specific. CTA: Managed futures. Other: Short term trading, Options strategies, Multi strategy.*

[9] *For further evidence on the relation between stocks and bonds of issuers in the HY index see "Empirical Duration of High-Yield Credit,"* Global Relative Value, *Lehman Brothers, November 8, 2004.*

Figure 2. Style Analysis Results for Individual Hedge Funds

All figures in the table represent medians. Only funds with at least 36 consecutive monthly returns are analyzed. The estimation is based on the period January 1997-June 2004.

Panel A: Main statistics

Style	# of funds	# of Significant loadings	DF	Adj. R^2
Arbitrage	149	4	54	39.9%
Event driven	164	4	53	44.7%
Sector funds	137	5	52	65.9%
Directional	402	5	51	61.0%
Fixed Income	95	4	54	41.2%
Global	132	4	53	54.2%
CTA	168	3	54	36.3%
Other	91	4	51	50.8%

Source: HedgeFund.net

Panel B: 5 most frequent significant factors

	1		2		3		4		5	
Arbitrage	LEH Vol. Index	-0.11	US Credit	0.18	US HY	0.42	EUROPE HY	0.14	HML	0.003
Event driven	US HY	0.30	LEH Vol. Index	-0.05	SMB	0.34	Slope 2-10	0.18	EUROPE HY	0.24
Sector funds	HML	0.00	SMB	0.66	US Financial	0.33	LEH Vol. Index	-0.46	US Tech	0.41
Directional	SMB	0.41	HML	-0.05	LEH Vol. Index	-0.25	EM Telecom	0.21	Wilshire 5000	-0.11
FI	MBS	1.52	JPU	-0.50	VIX	-0.08	Far East HML	0.20	Global Treas	1.16
Global	EM Telecom	0.42	EM FI	0.46	LEH Vol. Index	-0.19	MLM Commodity	0.30	US Healthcare	-0.45
CTA	GS Commodity	1.11	EUROPE AGG	0.83	US Healthcare	-0.41	US Cyc	-0.38	GS_commodity	0.35
Other	EM Telecom	0.22	US Healthcare	-0.13	EUROPE HY	0.24	SMB	0.17	Slope 2-10	0.30

Source: HedgeFund.net, Lehman Brothers, MSCI, Wilshire, Goldman Sachs

2.3. Hedge Funds Portfolios

A byproduct of the tremendous growth in the hedge funds industry was the creation of so-called funds of funds (FOF), which are essentially portfolios of individual hedge funds. Many institutional investors (and individuals) are interested in getting exposure to hedge funds but do not posses the knowledge to identify funds that suit their investment objectives and evaluate them. FOF can be attractive to such investors since they can offer exposure to a certain style with increased liquidity and reduced risk. In addition, they may have expertise that allows them to identify the better-performing funds in each style.

Several providers of hedge funds indices offer similar products known as investable indices. These are designed to track a broader index closely and provide investors with a practical way of getting exposure to hedge funds.[10]

In what follows, we demonstrate how the sensitivities of individual funds to common market factors can be used in constructing investable indices or portfolios with a minimum volatility for a pre-specified level of expected return. We compare the results with those of two other approaches that are commonly used: stratification and maximum correlation.

---

10 A hedge funds index may be composed of many funds that are closed to new investments or have liquidity provisions that preclude many institutions from investing in them. Lehman Brothers is planning to launch in the near future an 'Investable Index' based on the new Global Hedge Funds Index.

2.3.1. Constructing Investable Indices

We start by defining a reference benchmark (an index) that we are interested in replicating. For the purpose of our analysis, the index includes all funds in HedgeFunds.Net database, irrespective of style, size (AUM), available track record, or investment capacity (open or close to additional investments). The index returns are calculated monthly from January 1999-June 2004 as the equal-weighted returns of all funds.

The replicating portfolio is constructed of hedge funds that are part of the eligible universe. The composition of the eligible universe is updated quarterly. A fund is included in the eligible universe in a certain quarter only if it satisfies the following requirements (as of the end of the previous quarter):

1) It is open for additional investments.

2) Has a track record of at least three years (return data must be consecutive).

3) It is larger than the 75th percentile of the size distribution (in terms of AUM).

This last condition attempts to insure that the funds that ultimately comprise the replicating portfolio have adequate capacity in terms of new investments. As of June 2004, the eligible universe included 155 hedge funds with a minimum size threshold of $115 million.

To reflect real practices, the replicating portfolio is composed of at most 40 funds, with no fund having a weight larger than 5%. In order to reflect at least partially hedge funds' liquidity constraints, the replicating portfolio is re-balanced only quarterly.[11] In each re-balancing date, the composition of the replicating portfolio is determined based on one of the following approaches:

i) **Stratification**: The composition of the replicating portfolio mimics the style composition of the broad index. The weight of each style in the index is assigned to the two largest funds in that style that are part of the eligible universe. If a style has no representation in the eligible universe, its weight is divided proportionally among the other styles.

ii) **Maximum correlation**: The objective is to identify the set of funds that minimize the in-sample tracking error relative to the broader index. First, a variance-covariance matrix based on excess returns of all the funds in the eligible universe is constructed using the previous 36 months of data (excess return is defined as the fund return less the broader index return).

The weights assigned to each fund are determined in a two-stage process. Initially the optimization is performed using all the funds in the eligible universe. In the second stage, the 40 funds with the largest weights are selected and their weights are re-optimized (which can lead to some of them ending with a zero weight). In essence, this approach is equivalent to treating each fund as a separate factor.

iii) **Style-based optimization**: This approach identifies the optimal replicating portfolio based on a set of risk factors. The risk exposures of both the broad index and the individual funds comprising the eligible universe are estimated as described in the previous section.

The replicating portfolio is found by minimizing overall risk, systematic and idiosyncratic. Systematic risk is measured as the product of a variance-covariance matrix of the risk factors and the respective loadings' estimates. Idiosyncratic risk is represented by a variance-covariance matrix that is calculated using the entire set of residuals of each fund obtained from the regressions. Both matrices are based on the previous 36 months of data used in the estimation. Similar to the maximum correlation approach, weights are determined in a two-

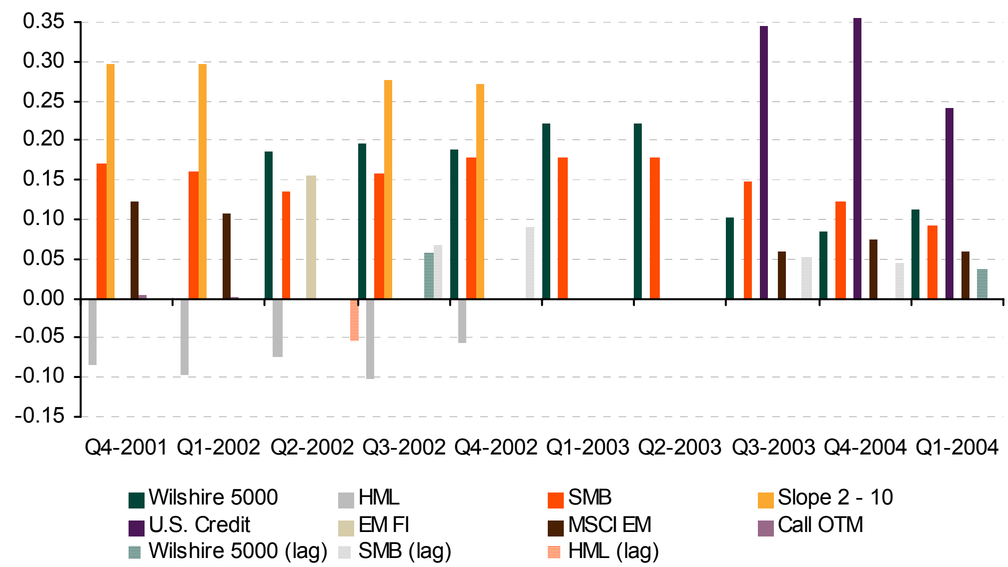
[11] *For a discussion of the effects of liquidity in hedge funds portfolios see Lo, Andrew, W., 2005,* The Dynamics of the Hedge Funds Industry*, CFA Institute, 61-84.*

stage process: first identifying the set of 40 funds that minimize the overall risk and then recalculating their weights to minimize the tracking error volatility.

Figures 3 and 4 illustrate the use of style analysis for the broad index. Each quarter beginning with December 2001, the index style exposures are estimated using the previous 36 months of data. Figure 3 plots the in-sample estimated style exposures quarter over quarter. In order to make the estimation robust and more tractable the list of factors used in the analysis is reduced, but it still spans the investment universe.[12] In fact, we find that the reduction in the number of factors improves the regression out-of-sample predictive power.

For example, at the end of 2001, the index had positive exposures to the medium term of the yield curve (10 years), emerging markets, and small-growth stocks and negative exposures to large-value stocks and the short end of the yield curve (two years). The in-sample explanatory power of the model was 80%-90% and was generally higher in the second half of the period.

Figure 3. Estimated Style Exposures for Broad Index
Estimated quarterly starting in December 2001 using the previous 36 months

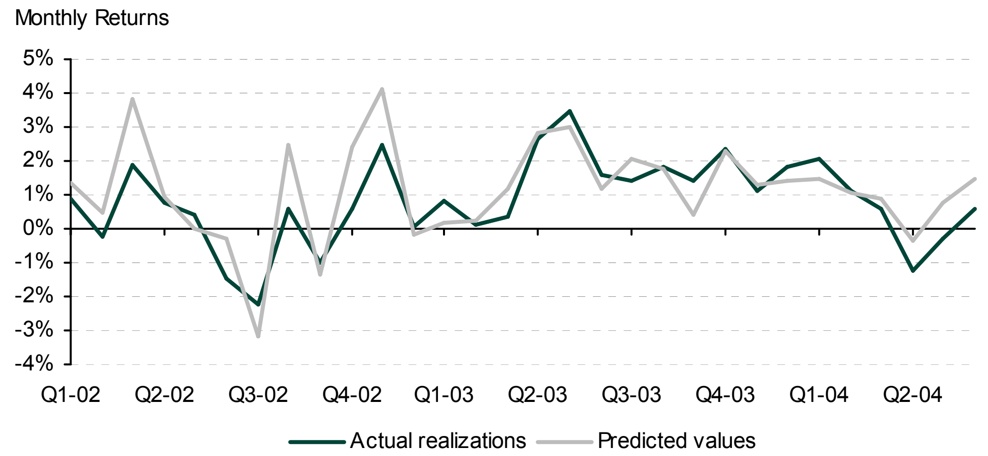


Source: HedgeFund.net, Lehman Brothers, MSCI, Wilshire, Goldman Sachs.

Figure 4 plots the actual index realizations and respective predicted values using the estimated factor loadings in the three months following the estimation date. The out-of sample tracking error over the entire period (January 2002-June 2004) was 83 bp/month. The improvement in the in-sample explanatory power (e.g. R^2) since 1Q03 is also reflected in smaller tracking errors out-of-sample (the far right part of Figure 4).

[12] *The reduced list includes 5 equity factors (Wilshire 5000, SMB, HML, MSCI EAFE+CANADA and MSCI EM), 6 Fixed Income factors (Aggregate Treasury, two yield curve slopes: 2-10 and 10-30, US credit, Global Treasury ex US and EM), two currencies (JPY, EUR), the GS Commodity Index and two options strategies (described in the appendix). In addition lagged realizations of the following four factors are included: Wilshire 5000, SMB, HML and MSCI EM. Lagged factors were introduced after an analysis of the residuals from regressing the broad index returns against the contemporaneous factors detected signs of autocorrelation. After the lagged variables were introduced, no autocorrelation was detected using the Durbin-Watson test.*

Figure 4. Realized Returns versus Model Predictions, based on broad index returns during January 2002-June 2004; results are computed out-of-sample.



Source: HedgeFund.net, Lehman Brothers, MSCI, Wilshire, Goldman Sachs.

2.3.2.. Out-of-Sample Performance

Figure 5 presents the mean and standard deviation of the tracking errors when the replicating portfolio is constructed according to each of the methods described earlier. The figures in the table are based on the out-of-sample realized tracking errors calculated in the three months following the formation date of the replication portfolio.

Using the stratification approach results in a tracking error volatility of 69 bp/month, whereas the maximum correlation approach fared much better with a T.E.V of 48 bp/month. The style-based optimization achieved the lowest tracking error (44 bp/month), which implies an annual tracking error of 152 bp.

The latter result may seem impressive given the fact the broad index includes over 2,000 funds, while the replicating portfolio is composed of no more than 40 funds. However, the results in Figure 5 should in no way be taken to represent actual replication results. Hedge funds data are notoriously affected by survivorship bias: if a fund stops reporting for any reason, its entire return history may be eliminated.[13] Hence, we are ex-ante guaranteed that all funds in the replicating portfolio do not "blow up" in the quarter following their inclusion date. However, since the effect of the survivorship bias is independent of the replication approach, the analysis still provides a good comparison of the three approaches.

The last column in the table (Modified Style Factors) contains results for a variant of the style-based optimization approach. Systematic risk is completely hedged using the actual factors (i.e., the portfolio allocation to the factors corresponds to the estimates of the loadings; typically, this implies positions in 4-5 factors). In contrast, idiosyncratic risk is minimized using the subset of hedge funds that have no exposure to any of the factors (i.e., none of the loadings in the individual regressions is significant, and the R^2 is zero).

The approach generates a higher tracking error (51 bp/month) than the “regular” style-based optimization. It seems that the cost involved in reducing the set of funds in the eligible universe (e.g., only funds with no systematic exposures) outweighs the benefit of using fewer loadings’ estimates in the optimization (i.e., only for the index and not for the individual funds). However, from a liquidity standpoint, the use of this approach may still prove

[13] *For a comprehensive discussion of the survivorship bias and other biases in hedge funds’ data see Malkiel Burton, G. and Saha, Atanu., 2005, “*Hedge Funds: Risk and Return,*” working paper, Princeton University.*

beneficial since it requires a smaller number of hedge funds (on average, only 20-25 funds comprise the replicating portfolio).

Interestingly, the average tracking error is negative (about 20 bp/month) regardless of the replication technique. One explanation is that the index includes many funds that were recently launched, whereas the replicating portfolio mandates a minimum three year's track record. Since many studies argue that hedge funds' performance declines as they mature, the difference in composition can account for this result.[14] Repeating the analysis with the same track record requirement applied to both the index and the replicating portfolio reduces the average negative tracking error by half (about 10 bp/month). Another way to reduce the negative tracking error may be to incorporate explicitly a no-underperformance constraint in the optimization (e.g., require the index alpha to equal that of the replicating portfolio in-sample).

Figure 5. Replication Results by Approach

January 2002-June 2004. Based on out-of-sample return realizations of the broad index and replicating portfolio.

Approach	Stratification	Maximum Correlation	Style Factors	Modified Style Factors*
T.E.V	0.69%	0.48%	0.44%	0.51%
Mean TE	-0.19%	-0.21%	-0.17%	-0.23%

** systematic risk is minimized using actual factors rather than individual hedge funds.*
Source: HedgeFund.net.

3. ACTUAL VERSUS SELF-PROCLAIMED INVESTMENT STYLE

The classification of hedge funds by investment style has important implications. Because of the low level of disclosure required from hedge funds, investors use style classification in asset allocation to characterize investment strategies with different risk-return profiles. Similarly, when evaluating the performance of a specific fund, it is typically carried out against a peer group of funds in the same style category.

Style classification is usually performed based on the self-reported investment strategy of the fund. However, a fund's actual style may differ from its self-proclaimed. This can happen because in some cases, there is no clear definition of what constitutes a certain style. A fund may also report it employs a different strategy than it does in practice to attract investors.[15]

Although style analysis can be used to compare the actual and self-proclaimed style of a given fund, it has several shortcomings. First, the time-series of returns to the "pure" style itself is typically unavailable. As a result, the appropriate set of factors and exposures that define a certain style is often unclear. Second, even if a style is clearly defined, determining if a fund belongs to it requires specifying acceptable ranges for the exposures to multiple factors. Third, style analysis cannot be applied to funds with relatively short histories, which constitute a substantial fraction of the hedge funds universe.

This section presents a different approach to evaluating the accuracy of a self-reported style. The main idea is that the proximity between any two funds (or styles) can be represented using a distance metric that measures the similarity in their performance. Similarly, the distance between a certain fund and a group of peer funds (identified based on the funds'

14 It is not clear if this finding reflects capacity constraints (older funds tend to also be larger) or deterioration of skill among 'mature' managers or simply survivorship bias (e.g. "young" hedge funds that prove unsuccessful are closed and subsequently their return data is excluded).

15 For example, if a certain investment style performed well in the past and enjoys positive inflows, a fund employing a different style may claim to be employing that style.

self-declared strategy) can be measured using the return time-series of the two. The advantage of this one-dimensional metric lies in its simplicity and the fact it does not require specifying a set of factors (benchmarks).

3.1. Method and Data

Let $D^{T}_{i,J}$ denote the distance of fund *i* from strategy *J* such that,

$$(5) \qquad D^{T}_{i,J} = (1 - \rho^{T}_{i,J})^{2}$$

where $\rho^{T}_{i,J}$ is the correlation between the returns of fund *i* and strategy *J* over the last *T* months. The returns to strategy *J* are calculated as the time-series of average monthly returns of all funds that report themselves as employing strategy *J*. Depending on the correlation, the value of $D^{T}_{i,J}$ can be 0-4. When the correlation is negative, the distance will be larger than 1, whereas if the correlation is positive the distance is below 1.

Defining the distance based on correlation of returns has several appealing features. First, the correlation measure is intuitive and does not require a long time series to compute. Second, since the degree of leverage is unobservable (but exhibits large cross-sectional variation) the distance should be invariant to the amount of leverage. The division by the standard deviation controls for the effect of leverage. Third, the distance function is convex, which will help to identify funds that are very dissimilar to their self-reported style.

The distance measure can be used to examine the classification of hedge funds using a two-step approach. First, the distance between a fund and its self-reported strategy, which we term 'self-distance' (that is, $D^{T}_{i,J}$ where $i \in J$), is calculated. Funds with self-distances exceeding a given threshold will be candidates for reclassification. The threshold can be specified in absolute terms (e.g., 0.25, which corresponds to a correlation higher or equal to 0.5) or in relative terms (e.g., the top 5% of the funds ranked by self-distance). Second, we calculate the distance between each fund and the rest of the strategies. A fund *i* will be reclassified from its self-reported strategy *J* to another strategy *K* if $D^{T}_{i,J}$ exceeds the threshold and $D^{T}_{i,K}$ indicates it is relatively close to strategy *K* (in a fashion which will be made precise in the next section).

Notice that if a fund's self-distance is below the specified threshold, it would not be re-classified even if its self-distance were much larger than its distance to a certain strategy *K*. Hence, a fund is required to be both dissimilar to its self-reported style and sufficiently similar ("close") to another strategy in order to be re-classified.

To illustrate how the distance measure is employed in practice, we use data from January 1999-June 2004. The first classification is performed in December 1999 and then in six-month intervals until June 2004 (a total of 10 classifications). The distance is computed based on the previous 12 monthly returns. At each classification, only funds that have a complete history over the past year are examined. Monthly returns to a strategy are simply the equal-weighted returns of all funds reporting to be in that strategy. In order to insure meaningful results, only strategies represented by at least 10 funds are included in the analysis.

3.2. Classification Results

3.2.1. The Distance between Actual and Self-Reported Style

Figure 6 presents the upper part of the distance distribution by strategy (median, 75 percentile, 90 percentile, 95 percentile, and maximum) based on 10 classification periods and illustrates that the distribution varies significantly from one strategy to another and that certain strategies tend to be more cohesive then others. For example, the median distance for convertible arbitrage is 0.08, whereas it is 0.56 for equity market neutral (these figures correspond to correlations of 0.71 and 0.2, respectively). The difference becomes even greater if we look at the 95 percentile (0.75 versus 1.98). In general, the table demonstrates that we should not use a uniform threshold but rather that the threshold should vary by strategy.

Figure 6. Distribution of Distance by Strategy

Based on 10 consecutive classifications with six-month intervals starting on December 1999. Each classification is based on the past 12 monthly returns.

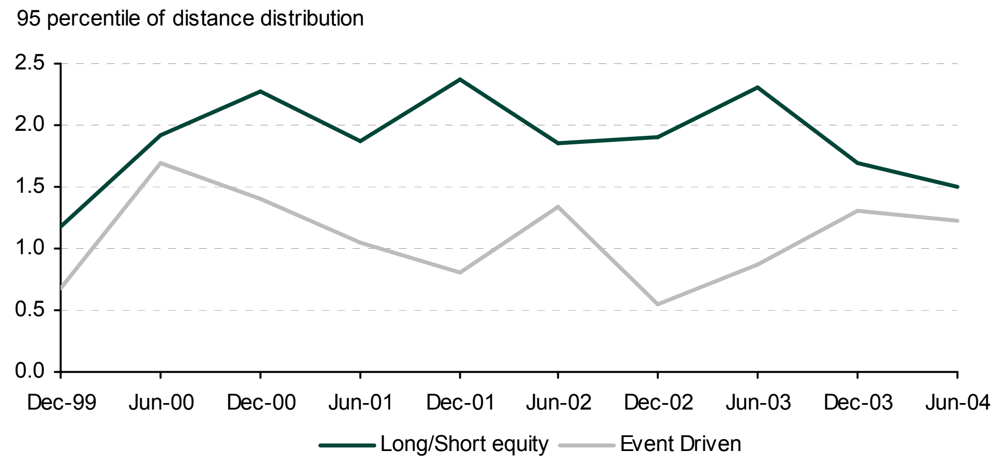
Strategy	# of funds	P_{50}	P_{75}	P_{90}	P_{95}	Max
Stat. Arb	172	0.42	0.74	1.11	1.41	1.91
Convert. Arb	712	0.08	0.23	0.50	0.75	2.34
Market Neutral	625	0.56	1.04	1.65	1.98	2.97
Other arbitrage	103	0.56	1.08	1.42	1.58	2.32
Reg D	43	0.35	0.79	1.45	1.67	1.75
Risk Arb.	340	0.11	0.33	0.61	0.87	1.59
Event driven	702	0.12	0.40	0.83	1.22	2.48
Distressed	429	0.11	0.34	0.83	1.02	2.79
Value	392	0.10	0.32	0.73	1.13	2.12
Tech	179	0.05	0.27	0.61	1.44	3.40
Finance	410	0.14	0.46	1.31	1.83	3.07
Small cap	282	0.08	0.28	0.62	1.04	2.07
Short bias	142	0.03	0.11	0.34	0.66	2.02
Long/short Eq.	3610	0.18	0.55	1.26	1.86	3.67
Long only	221	0.04	0.15	0.30	0.51	1.31
FI non Arb.	384	0.21	0.48	1.00	1.45	3.38
FI Arb.	542	0.42	0.79	1.16	1.79	3.03
Macro	561	0.41	0.77	1.17	1.47	3.00
EM	599	0.07	0.22	0.68	0.95	2.85

Source: HedgeFund.net.

The distance distribution is also not stable across time for the same strategy. Figure 7 plots the 95 percentile of the distribution for each of the 10 classification periods for two popular strategies: long/short equity and event-driven. It is clear that the 95 percentile (as well as the rest of the distance distribution, which is not shown) varies significantly from period to period. In general, we may expect some time variation due to changes in the composition of strategies. In addition, for strategies with a relatively small population of funds, the 95 percentile may be heavily affected by only a few funds.

Figure 7. 95 Percentile of Distance Distribution by Classification Period

For funds with self-reported styles of event-driven and long/short equity. Based on 10 consecutive classifications with a six-month interval starting on December 1999.



Source: HedgeFund.net.

Clearly, there are many possible ways to determine the potential candidates for reclassification. We use the 95 percentile of the distance distribution determined separately for each strategy and period as the threshold. Any fund with a self-distance that exceeds the 95 percentile will be further examined relative to other strategies.

3.2.2. Robustness of the Reclassification Criterion

An effective criterion for selecting candidates for reclassification should identify funds that are very different than their peers (based on their self-reported style). Another desirable feature is that funds that are not selected (e.g., have self-distances below the threshold) are on average "closer" to their strategy than to any other strategy.

Figure 8 reports average distances between the subset of funds not selected for potential reclassification and each of the strategies. If the threshold criterion is effective, then for each row of the matrix the figures on the diagonal should be the smallest. Indeed, this is what we observe in practice. For example, the average self-distance of funds in the statistical arbitrage strategy is 0.38, whereas the distance of these funds to the second nearest strategy is 0.73 (long/short equity). This result can be expected because statistical arbitrage is based on taking offsetting positions in closely related securities (often stocks) that trade at very different prices.

Despite the results in Figure 8, some funds are closer to a different strategy than the one they report. To see this, we calculate for each fund its distance to a certain strategy divided by its self-distance. Figure 9 reports the 25 percentile of the distribution of this ratio. If the self-distance is always smaller than any other distance, the 25 percentile of the distribution would be above 1.

However, as Figure 9 illustrates, in many cases more than 25% of the funds in a certain strategy are actually closer to another strategy (such cases are marked by a grey background). This gives us a good sense of the cohesiveness of each strategy or level of diversification it offers. For example, looking at risk-arbitrage or short-bias funds, we see that over 75% of the funds are closer to their strategy than to any other strategy. In contrast for the case of market neutral, there are always at least 25% of the funds that are closer to any of the other strategies.

Figure 8. Average Distance of "non-candidate" Funds to Each Strategy

Using only the subset of funds with self-distance below the 95 percentile of the distribution. Figures in the table are based on 10 consecutive classifications with six-month intervals during December 1999-June 2004.

Strategy	Stat. Arb.	Convert. Arb.	Market Neutral	Other Arbitrage	Reg D	Risk Arb.	Event Driven	Distressed	Value	Tech	Finance	Small cap	Short bias	Long/ short Eq	Long only	FI non Arb.	FI Arb.	Macro	EM
Stat. Arb	0.38	0.94	0.77	0.93	0.58	1.01	0.88	0.91	0.89	0.85	0.82	0.87	1.12	0.73	0.87	0.89	0.96	0.87	1.03
Convert. Arb	0.80	0.07	0.83	0.35	0.43	0.56	0.52	0.37	0.65	0.71	0.74	0.62	1.32	0.70	0.78	0.39	0.40	0.61	0.66
Market Neutral	0.76	0.88	0.52	0.91	0.80	0.71	0.81	0.83	0.96	0.98	0.95	0.96	1.05	0.85	0.95	0.78	0.75	0.85	0.81
Other arbitrage	0.82	0.76	0.75	0.51	0.83	0.93	0.92	0.79	1.01	0.88	0.82	0.92	1.00	1.01	1.04	0.77	0.88	0.78	1.06
Reg D	0.42	0.64	0.57	0.70	0.30	0.79	0.49	0.51	0.59	0.55	0.58	0.56	1.39	0.40	0.55	0.59	0.53	0.48	0.50
Risk Arb.	0.97	0.58	0.62	0.96	0.83	0.09	0.28	0.40	0.37	0.50	0.44	0.42	1.85	0.34	0.41	0.36	0.47	0.46	0.33
Event driven	0.85	0.38	0.78	0.80	0.45	0.21	0.11	0.14	0.24	0.28	0.26	0.21	2.25	0.19	0.25	0.24	0.41	0.37	0.22
Distressed	0.92	0.32	0.71	0.54	0.38	0.35	0.15	0.09	0.34	0.36	0.35	0.31	1.85	0.28	0.40	0.24	0.38	0.41	0.31
Value	1.02	0.60	0.91	1.14	0.55	0.32	0.15	0.25	0.09	0.21	0.22	0.16	2.67	0.14	0.12	0.38	0.70	0.51	0.22
Tech	0.68	0.72	0.75	1.34	0.56	0.39	0.20	0.32	0.18	0.03	0.15	0.10	2.85	0.08	0.08	0.47	0.65	0.51	0.26
Finance	0.86	0.73	0.95	1.01	0.52	0.34	0.27	0.35	0.22	0.24	0.13	0.22	2.31	0.16	0.18	0.46	0.81	0.56	0.30
Small cap	0.84	0.56	0.74	1.18	0.41	0.34	0.12	0.19	0.15	0.13	0.17	0.07	2.54	0.09	0.10	0.32	0.53	0.37	0.23
Short bias	1.08	1.59	1.10	0.84	1.42	2.11	2.60	2.24	3.05	2.94	2.64	2.82	0.03	3.02	3.16	1.99	1.43	1.79	2.62
Long/short Eq.	0.83	0.68	0.76	1.15	0.55	0.45	0.24	0.34	0.27	0.23	0.27	0.19	2.31	0.16	0.18	0.45	0.65	0.45	0.30
Long only	0.84	0.66	0.77	1.27	0.56	0.31	0.14	0.26	0.09	0.06	0.11	0.07	3.10	0.05	0.04	0.37	0.71	0.40	0.14
FI non Arb.	0.81	0.44	0.69	0.55	0.41	0.43	0.29	0.36	0.44	0.55	0.55	0.48	1.75	0.44	0.55	0.19	0.44	0.40	0.37
FI Arb.	0.92	0.70	0.73	0.78	0.72	0.68	0.68	0.73	0.86	0.87	0.91	0.87	1.10	0.87	0.92	0.57	0.39	0.70	0.70
Macro	0.83	0.75	0.66	0.70	0.61	0.61	0.51	0.64	0.73	0.76	0.67	0.62	1.38	0.58	0.67	0.47	0.62	0.36	0.53
EM	0.84	0.51	0.65	0.96	0.55	0.28	0.15	0.22	0.18	0.27	0.26	0.19	2.48	0.15	0.17	0.25	0.45	0.33	0.06

Source: HedgeFund.net.

Figure 9. Ratio of Each Distance to Self-Distance by Strategy

The 25 percentile of the distribution of the ratio is reported for each combination of two strategies. Using only the subset of funds with self-distance below the 95 percentile of the distribution. Figures are based on 10 consecutive classifications with six-month intervals during December 1999-June 2004.

Strategy	Stat. Arb.	Convert. Arb.	Market Neutral	Other Arbitrage	Reg D	Risk Arb.	Event Driven	Distressed	Value	Tech	Finance	Small cap	Short bias	Long/ short Eq	Long only	FI non Arb.	FI Arb.	Macro	EM
Stat. Arb	1.00	1.15	1.10	1.39	1.02	1.02	1.08	1.04	0.95	0.73	0.71	0.85	1.37	0.90	0.94	1.20	1.08	1.08	1.05
Convert. Arb	3.63	1.00	4.41	3.06	4.09	3.01	2.49	2.26	3.10	3.74	3.55	3.05	6.35	3.51	3.88	2.84	2.80	3.81	4.06
Market Neutral	0.75	0.76	1.00	0.95	0.83	0.67	0.69	0.74	0.68	0.76	0.77	0.77	0.82	0.77	0.77	0.74	0.76	0.84	0.70
Other arbitrage	0.75	0.76	0.67	1.00	0.72	1.05	0.72	0.88	0.75	0.65	0.69	0.78	1.01	0.67	0.95	0.75	0.81	0.76	1.02
Reg D	1.11	1.33	1.08	1.02	1.00	0.73	0.67	0.85	1.19	1.28	0.64	1.02	1.66	0.99	1.10	1.04	1.00	0.77	0.83
Risk Arb.	3.15	2.25	2.12	2.69	1.90	1.00	1.38	1.93	1.85	2.11	1.98	1.92	4.95	1.92	2.19	1.76	2.12	2.00	1.44
Event driven	2.13	1.40	1.57	2.26	1.73	0.79	1.00	0.68	1.00	1.06	1.09	0.98	4.93	0.94	1.07	1.07	1.40	1.34	1.11
Distressed	2.87	1.30	2.13	2.69	2.34	1.52	1.06	1.00	1.48	1.70	1.64	1.47	4.76	1.36	1.61	1.18	1.70	1.84	1.47
Value	3.25	2.15	2.62	4.37	1.93	1.31	0.92	1.19	1.00	1.12	1.04	0.84	8.26	0.78	0.86	1.64	2.53	1.87	1.07
Tech	3.57	3.09	3.72	5.33	4.17	2.58	1.97	2.11	1.61	1.00	1.50	1.23	12.49	0.95	0.99	4.10	3.86	2.78	2.23
Finance	1.92	1.88	1.96	2.80	1.69	0.99	0.91	1.13	0.80	1.18	1.00	0.99	5.97	0.88	0.93	1.38	2.21	1.67	0.91
Small cap	3.28	2.45	2.88	4.20	2.51	1.54	0.78	1.19	0.95	0.98	1.00	1.00	8.88	0.78	0.87	1.78	2.23	1.53	1.31
Short bias	14.77	19.11	14.22	8.71	9.24	17.14	27.22	22.30	27.07	27.92	22.93	26.92	1.00	27.45	30.38	23.34	15.10	20.85	25.06
Long/short Eq.	1.66	1.49	1.41	2.07	1.31	1.02	0.83	1.04	0.85	0.87	0.95	0.88	3.44	1.00	0.83	1.15	1.39	1.09	0.95
Long only	7.30	6.47	5.96	10.52	6.83	3.58	1.62	2.45	1.14	1.10	1.51	1.12	27.40	0.82	1.00	4.04	7.71	2.86	1.86
FI non Arb.	1.55	1.17	1.40	1.15	1.25	1.21	0.61	0.74	0.85	1.04	1.01	0.76	2.86	0.74	0.80	1.00	1.17	0.97	0.88
FI Arb.	1.03	0.97	1.01	0.96	0.83	0.92	0.95	0.96	1.18	1.11	1.02	1.06	1.14	1.13	1.19	0.88	1.00	1.00	1.00
Macro	1.12	1.11	1.00	1.12	1.16	1.00	0.92	0.97	1.06	1.13	0.98	0.81	1.23	0.76	0.82	0.83	0.99	1.00	0.95
EM	3.94	2.67	2.62	3.40	2.47	1.34	0.98	1.47	1.35	1.61	1.48	1.35	11.58	1.13	1.13	1.20	2.56	1.60	1.00

Source: HedgeFund.net.

3.2.3. Re-classifying funds

For a fund to be reclassified, we require not only that its self-distance is greater than the specified threshold. It should also be relatively "close" to at least one other strategy to which it will be reclassified. For the set of funds identified as candidates for reclassification, Figure 10 reports (by strategy) the distance to the closest strategy, the self-distance, and the ratio of the two (calculated individually for each fund; the figures in the table represent medians).

Except for short bias, the figures in the last column are all substantially smaller than 1. This indicates that for funds identified as candidates for reclassification, we can find strategies that their return histories resemble much more than their self-reported strategy. Based on Figure 10, we decided to reclassify a fund to a different strategy if it is the closest to the fund (relative to all other strategies) and the distance to it is smaller than the self-distance.

Figure 10. Summary Statistics for Reclassified Funds

All figures represent medians. Based on 10 consecutive classifications with six-months intervals during December 1999-June 2004.

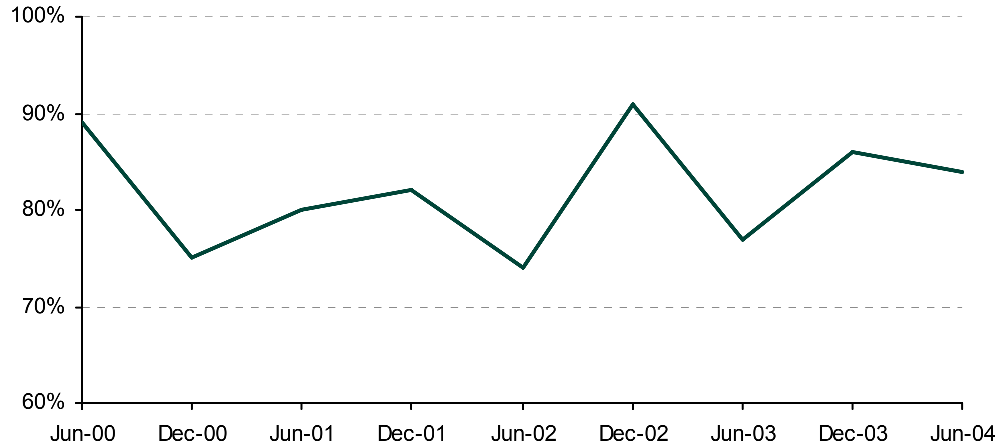
Strategy	# of reclassified funds	Distance to closest strategy	Self - Distance	Ratio of Distance to Closest Strategy over Self Distance
Stat. Arb	12	0.25	1.44	0.18
Convert. Arb	41	0.24	0.84	0.30
Market Neutral	36	0.37	2.25	0.19
Other arbitrage	9	0.25	1.61	0.18
Reg D	3	0.42	1.67	0.25
Risk Arb.	24	0.16	0.79	0.23
Event driven	43	0.27	1.34	0.25
Distressed	29	0.29	1.09	0.25
Value	24	0.23	1.15	0.24
Tech	13	0.15	1.65	0.08
Finance	26	0.23	1.83	0.15
Small cap	18	0.15	1.01	0.18
Short bias	11	0.32	0.38	1.00
Long/short Eq.	184	0.16	2.36	0.07
Long only	16	0.16	0.56	0.30
FI non Arb.	24	0.18	1.57	0.16
FI Arb.	30	0.46	1.94	0.20
Macro	32	0.40	1.83	0.20
EM	34	0.32	1.43	0.30

Source: HedgeFund.net.

A desirable feature of any classification scheme is stability. In our context, we think of stability in the following sense: if a fund is re-classified at time t to a certain strategy, then the results of the next classification (at $t+6$) should point to the same strategy. Figure 11, plots by period the percentage of funds that were reclassified at time t but were not reclassified again at $t+6$ (i.e., these funds remained in their new assigned strategy). The results suggest that the classification scheme is stable because, on average, 80% of the funds that are reclassified at time t remain in their newly assigned strategy following the next classification.

Figure 11. Stability of Classification

% of reclassified funds at time *t* remaining in their new assigned strategy a time *t*+6; based on 10 consecutive classifications with six-months intervals during January 1999-June 2004.



Source: HedgeFund.net.

4. CONCLUSION

Over the past decade, hedge funds have probably been the fastest growing sector in the financial services industry, with over $1 trillion in assets currently under management. Their growth has been largely fueled by the view that they are consistently able to generate positive alphas and provide diversification benefits due to the nature of the strategies they employ.

Hedge funds' investment mandates typically allow them to use leverage, short selling, derivatives, and highly illiquid securities. These characteristics can generate return profiles that exhibit auto-correlation, fat tails, options-like payoffs, and unstable correlations with other asset classes. The lack of almost any investment constrains, coupled with the minimum disclosure hedge funds offer (due to the lax of regulatory oversight), presents serious challenges for investors. Are the returns to a certain strategy indeed uncorrelated with the returns to other assets in their portfolios? How should the left-tail risk of a strategy with a relatively short return history be measured? What is the correct approach for comparing the performance (or alpha) of individual hedge funds within the same style?

This article discusses several quantitative tools that can help investors address some of these issues. We start by showing how return-based style analysis, originally applied to mutual funds, may be extended to hedge funds with some modifications. We demonstrate how investors can use the factor exposures estimates to better understand the nature of risks and exposures of various strategies and the extent to which investments in various individual funds are correlated.

Style analysis can also be used in constructing investable indices or portfolios of hedge funds. We construct an investible index using three distinct approaches: stratification, maximizing in-sample correlation, and based on factor exposures estimated using style analysis. We find that under realistic conditions and using actual hedge funds data, the style-based optimization achieves the lowest out-of-sample tracking error volatility.

Another important issue we examine is the investment style classification of hedge funds. The style assigned to a fund carries major implications for asset allocation and performance evaluation against peers. Yet in practice, style classification commonly relies on hedge funds' self-reported investment strategies. We present a simple technique that can be used to identify inconsistencies between a fund's actual and self-reported strategies and reclassify it to another strategy that it more closely resembles (based on its return history). Despite the short data history we use in the analysis, which does not allow to us to perform traditional statistical tests, we find that the technique generates stable and meaningful results.

APPENDIX

Figure 12 presents the list of factors used in the style analysis of hedge funds, separately by asset class. The equity component is broken down by geography (U.S., developed countries excluding U.S. and emerging markets) and sector (ten basic industries) using the Dow Jones indices. The Wilshire 5000 and MSCI EAFE+Canada serve as proxies for the aggregate U.S. market and developed countries (ex-U.S.), respectively. Emerging markets are broken down further to three regions: Latin America, Europe and Middle East, and the Far East. We also construct value-growth and size factors for the U.S. using the six Wilshire sub-indices (Small-Value, Large-Growth, etc.). For the other developed countries and emerging markets, we use the respective MSCI-value index less the MSCI-growth index.

The fixed-income component is similarly broken into three regions but has less detailed coverage. All factors, except the 1-month LIBOR rate, which serves as the risk-free rate, are based on Lehman indices. The U.S. market is represented by the Lehman Treasury, Credit, MBS, and HY indices (the return to all spread asset classes are in excess of the duration-matched treasury returns). In addition, variations in the slope of the yield curve are modeled using two factors that are actually returns to long-short strategies: 10-year less 2-year and 30-year less 10-year (both strategies are duration neutral). For the non-U.S. markets, we use as factors the Global Treasury (ex-U.S.), Euro Aggregate, Euro HY, and EM aggregate indices.

Additional factors are the returns to four currencies (JPY, EUR, Swiss Franc, and Australian Dollar), a commodity index, two measures of implied volatility (VIX and LEH volatility index) and the returns to six options strategies that involve buying a one-month put or a call on the S&P500 and holding it until expiration.[16] We also include lagged factor realizations to control for possible serial correlation in funds' returns.[17]

[16] We compute the returns to holding at-the-money (ATM), out-of-the-money (OTM) and deep OTM Calls and Puts. ATM, OTM and deep OTM are defined based on the strike price being equal to the index price at the time of purchase, index price + 0.5 std and index price + 1 std respectively.

[17] See for example Asness, C., Krail, R. and J. Liew, 2001, "Do Hedge Funds Hedge?", The Journal of portfolio Management" 28, 6-19.

Figure 12. List of Factors for Return-based Style Analysis

Equities

	U.S	Developed countries exc. U.S.	EM
Agg. Market	Wilshire 5000	MSCI EAFE+Canada	MSCI EM
Value-Growth (HML)	Wilshire	MSCI EAFE+Canada	MSCI EM
Small-Big (SMB)	Wilshire		
Sector			
Basic Materials	DJ	DJ	DJ
Industrials	DJ	DJ	DJ
Cyclical	DJ	DJ	DJ
Non Cyclical	DJ	DJ	DJ
Financials	DJ	DJ	
Energy	DJ	DJ	DJ
Healthcare	DJ	DJ	DJ
Technology	DJ	DJ	DJ
Telecom	DJ	DJ	DJ
Utilities	DJ	DJ	DJ
Reits	DJ		

Separately for: Latin America, Europe and Middle East, Far East

1/3[(large value-large growth)+ (mid value-mid growth)+ (small value-small growth)]

1/2[(large value - small value) + (large growth -small growth)]

Fixed Income (all Lehman indices)

	U.S	Developed countries exc. U.S.	EM
Risk-free rate	1-Month Libor		
Treasury	US Treasury	Global Tresury ex. U.S.	
Credit (excess returns)	US Credit	Euro. Agg.	EM Agg.
MBS (excess returns)	MBS		
HY (excess returns)	US_HY	Euro. HY	
Yield slope	2 - 10, 10 - 30		

Other

Currencies	JPY, EUR, Swiss Franc, Australian Dollar
Commodities	Gold, Oil, GS commodity index
Volatility	VIX (equity), Lehman vol index (swaptions)
Options	S&P500 Call and Put (ATM, Out of the money, deep out of the money)

Source: Lehman Brothers, MSCI, Wilshire, Goldman Sachs

Constant Maturity Mortgages

Fei Zhou
Interest Rates Modeling
212-526-8886
feiz@lehman.com

Li Chen
Interest Rates Modeling
212-526-8097
lichen@lehman.com

Prasanth Subramanian
Mortgage Strategies
212-526-8311
psubrama@lehman.com

Constant maturity mortgage products are standardized derivative instruments that appeal broadly to the fixed income market because of their explicit and isolated exposure to mortgage rates, the fundamental measure of the mortgage market. This article examines this emerging family of rate-based mortgage products, including their market development, comparative advantages, contractual features, valuation techniques, risk characteristics, and practical applications.

1. OVERVIEW

Constant Maturity Mortgage (CMM) products include mainly forward rate agreements and swaps. Compared with traditional mortgage products, CMMs are relatively simple, with payoffs directly referenced to mortgage rates. They have been structured by dealers to accommodate the market's desire to transact directly in mortgage rates, without the mortgage basis and negative convexity complications.

The CMM index, on which all CMM products are based, tracks the prevailing mortgage rate in the secondary market. Deduced from to-be-announced (TBA) mortgage prices and calculated based on a linear par-price interpolation, it is a proxy for the coupon on a TBA 30-year mortgage passthrough priced at par. This conversion from observed TBA prices to current coupon rates avoids reliance on complex mortgage models or prepayment assumptions; hence, it is a clean and objective mechanism for referencing mortgage rates in the market.

In its entirety, the family of CMM products helps formulate an explicit curve view of the mortgage market and facilitates a direct comparison with other fixed income sectors. These simplifying and convenient features appeal to many market participants, not just from mortgage community. Since the products were formalized last year, the CMM market has already enjoyed considerable growth.

Despite the attractiveness of CMM products, their valuation has been challenging. Forward CMM rates are either derived from replicating TBAs in the case of short maturities or referenced off chosen benchmark rates for long maturities. Although there appears to be some agreement on how to model short-dated instruments, there has been much less on how to deal with long-dated ones. As a result, activities in the CMM market have thus far been mostly at short maturities.

CMM instruments share, to varying degrees, the complex risk characteristics of TBA passthroughs. Short-dated CMM risks are similar to those of replicating TBA portfolios, except for the lack of negative convexity. Long-dated CMMs, hedged with either liquid TBAs or comparable swaps, have roll or basis risk. In addition, the swap hedge leaves residual curve and volatility risks in the position.

There is a repertoire of CMM applications that makes them much more effective than traditional instruments. Specifically, CMMs are ideal for those whose primary concern is to hedge exposure to mortgage rates. Conversely, they can be used as an alternative to traditional mortgages to gain exposure to the mortgage market. Because of the lack of convexity in CMMs, they are better than passthroughs for structuring mortgage basis positions. For the same reason, they can be employed for mortgage carry trades to decouple curve gain from convexity compensation.

2. CMM MARKET DEVELOPMENT

Regular bonds, such as Treasuries, have moderate convexity and, hence, can be viewed as rate instruments. In contrast, mortgage securities are considerably negatively convex due to the embedded prepayment option. Mortgage prices vary in a very nonlinear fashion with rates and can hardly be considered as rate instruments. Due to this negative convexity, it is inconvenient to express a view on mortgage rates through mortgage bonds. This is problematic because current and future mortgage rates drive the value of all mortgage securities due to their effect on mortgage prepayments and cash flows. There has, therefore, always been a need in the market for rate-based mortgage products. It is in response to this need that current coupon mortgage products were conceived.

Prior to the standardization of CMM instruments, current coupon mortgage products were customized by dealers to meet clients' specific needs to manage exposure to mortgage rates. In the early days, such transactions were all privately negotiated between counterparties. There was no consensus, even on the calculation of current coupon rates. The one-on-one nature of deal-making was not flexible enough to satisfy the market's growing appetite for current coupon products. In particular, due to often opaque pricing and large transaction costs, it was painfully inconvenient for investors to unwind or roll trades, preventing current coupon products from growing into an efficient market. Dealers came to realize that market standardization was essential for the continuing growth of the product. They eventually standardized the calculation of the daily CMM index[1] and the structures for benchmark CMM forward and swap contracts. Investors are now able to compare pricing of current coupon instruments across multiple dealers, so their willingness to transact has increased dramatically. As a result, the volume in CMM trades has multiplied quickly.

CMM transactions are now mostly short dated. The liquidity in the long end is considerably lower. As will be explained later, the hedging risk of the long-dated current coupon rates is much greater due to the lack of proper matching instruments, and the confidence in dealing with CMMs of longer maturities is therefore lower. Meanwhile, there are not yet enough inter-dealer transactions at the long end to foster an active long-dated CMM market.

The establishment of the current CMM market has been gradual. While still nascent, it has already enjoyed strong volume growth, albeit mostly in short-dated contracts. There has been participation so far from mortgage hedgers, bank portfolios, and hedge funds. It is expected that with broader and deeper participation over time, the CMM instruments will be traded more efficiently further along maturity.

3. CMM ADVANTAGES

3.1. Traditional Hedging Instruments

The mortgage community has been relying considerably on two types of hedging instruments: interest rate swaps and mortgage passthroughs. Swaps have been popular because of their structural simplicity, almost unlimited liquidity, and good correlation with mortgage rates. Passthroughs, though by no means simple, are still the most straightforward instruments in the mortgage market. Although both swaps and passthroughs are well accepted hedging instruments, neither choice is satisfactory.

Despite the better correlation between mortgage and swap rates, the mortgage basis, defined as the difference between mortgage and swap rates, is fairly volatile. The basis fluctuates significantly over time (Figure 1), and the related basis risk is considerable, given the size of the markets. Although the interest rate risk in mortgages can be well covered with the use of

[1] *See the Appendix for a detailed description of the standardized CMM index calculation.*

swaps, no liquid instruments exist to hedge the remaining basis risk. The current practice of handling mortgage basis is often haphazard and approximate at best.

On the other hand, mortgage passthroughs, TBAs in particular, are inherently free of basis risk. They have been the natural choice for obtaining and hedging the mortgage market exposure. However, TBAs do possess other risk factors. As Figure 2 shows, the price of a TBA is negatively convex, so TBA passthroughs can be used to track mortgage rates only within a small range of rate moves. In addition, TBAs can be used only as far out as there is liquidity in the TBA roll markets.

3.2. Benefit of CMMs

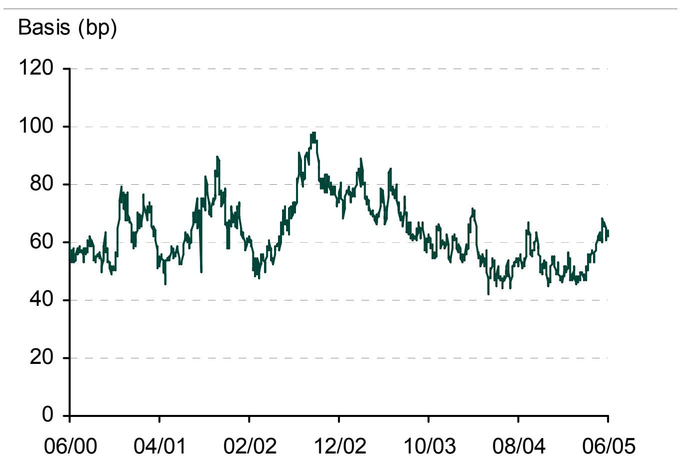
Despite the prevalent use of these two traditional hedging instruments, the mortgage community has constantly faced the dilemma of either dealing with the basis risk in the case of swaps or coping with the negative convexity and the roll risks in the case of TBAs. By contrast, CMM instruments avoid these problems and simplify hedging decisions. Highlighted below are the main advantages of utilizing CMM products:

- Minimization of replication effort: Replicating current coupon mortgage rates with TBAs requires continuous monitoring. CMM products shift the replication effort to dealers and other investors who are better positioned to manage the related risks.
- Elimination of negative convexity**:** Replicating current coupon mortgage rates with TBAs introduces negative convexity. This is not a concern in CMM-based products.
- No roll and basis risks**:** Replicating long-dated mortgage rates can be done through rolling short-dated TBAs or using forward starting interest rate swaps. The first strategy carries the risk that forward TBA prices move differently from spot; the second carries basis risk. CMMs avoid both.

Not only can CMMs make life easier for mortgage investors, they can also expose the vast opportunity of the mortgage market to investors in other sectors who are attracted to higher yields in mortgages but accustomed to making decisions based mainly on rate behavior. Even more broadly, the simplicity offered by CMM products serves to increase investors' comfort level with the mortgage market and ease the apprehension of those who normally shun mortgages because of the perceived complexity.

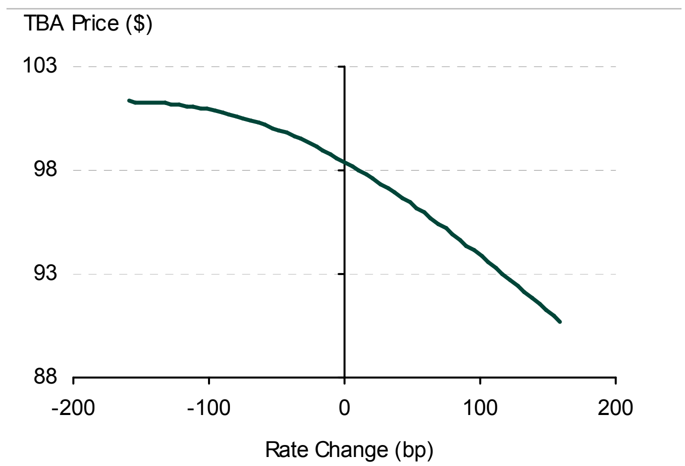
Figures 1-2. Mortgage Basis Fluctuation over the Last Five Years and Price Profile with Changing Rates for a TBA Mortgage

Basis Risk



Source: Lehman Brothers.

Negative Convexity



Source: Lehman Brothers.

4. STANDARD CMM INSTRUMENTS

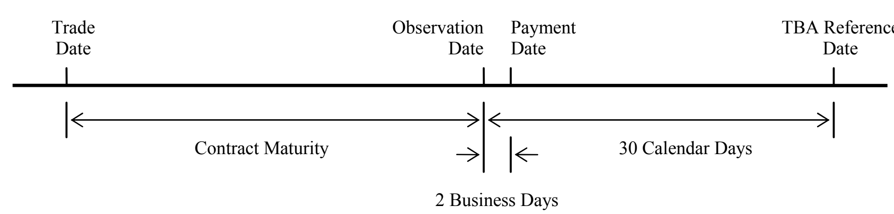
4.1. CMM Forwards

A CMM forward rate agreement is a contract written on a CMM index to be observed on a forward maturity date and is quoted in terms of a forward breakeven rate. At the maturity date, the contract exchanges a cash flow that equals the difference between the quoted breakeven rate and the prevailing CMM index, multiplied by a notional. Maturities of CMM forwards range from a few weeks to as long as five years.

The markets have standardized two sets of benchmark CMM forward contracts. One is based on fixed forward dates that correspond to the next three consecutive PSA settlements for the 30-year FN TBAs. For this set of forwards, the CMM index is observed 30 days prior to the PSA settlement date, and the cash flow is exchanged two business days after the observation. For example, the forward for the April 13 PSA settlement observes its CMM index on March 14 and pays off its cash flow on March 16. The other set of forwards is rolling and re-establishes itself every day for ten benchmark maturities of 1, 2, 3, 6, and 9 months and 1, 2, 3, 4, and 5 years. The index for this set is observed on the benchmark maturity date for a reference TBA that settles 30 days after and the related cash flow is delivered in two business days. For example, a 1-year CMM forward contract traded on May 3, 2005, has its index observed on May 3, 2006; its reference TBA settled on June 3, 2006; and its payoff paid on May 5, 2006.

Figure 3 depicts the relationship among various dates of a CMM forward contract. For a PSA-based contract, the TBA reference date coincides with a PSA settlement date, while for a roll-based contract, the reference settlement date may land anywhere between two consecutive PSA dates.

Figure 3. Various Dates and Periods in a CMM Forward Contract



4.1.1. CMM Swaps

A CMM swap, on the other hand, is a contract that exchanges periodic cash flows of a fixed rate with reset CMM index rates. The benchmark maturities of CMM swaps include 1, 2, 3, 4, and 5 years, with reset, accrual, and payment dates following the standard swap date rules. Structurally, a CMM swap resembles a constant maturity interest rate swap.

A CMM swap differs from a standard swap in that it resets and exchanges payments according to a monthly schedule and both legs accrue with the 30/360 day count. These features are intended to mimic mortgage cash flows. For example, a 1-year 5.5% CMM swap resets its floating rate to the CMM index every month and exchanges the difference of 5.5% with the observed CMM index on the monthly payment dates for 12 months. Note, though, that a one-period CMM swap is not exactly a CMM forward due to the swap's end-of-period payment. A variation of a standard CMM swap is a CMM-CMS basis swap that locks in a fixed spread rate at initiation between the CMM index and a benchmark CMS swap rate.

Through the life of the basis swap, on reset dates, this fixed spread is exchanged for the actual level of the spread.

For illustration, Figure 4 displays a snapshot of the CMM instruments standardized in the market, including the CMM swaps described above and the two sets of CMM forwards specified before.

Figure 4. Sample Quotes on May 3, 2005, for CMM Forwards of Rolling and PSA Settlements and for CMM Swaps

Rolling Forward	Observation Date	Mid Level	Spread To Spot
Spot	05/03/05	5.254	
1M	06/03/05	5.271	1.7
2M	07/05/05	5.296	4.2
3M	08/03/05	5.316	6.2
6M	11/03/05	5.371	11.7
9M	02/03/06	5.421	16.7
1Y	05/03/06	5.466	21.2
2Y	05/03/07	5.601	34.7
3Y	05/06/08	5.711	45.7
4Y	05/05/09	5.791	53.7
5Y	05/04/10	5.876	62.2

PSA Forward	Observation Date	Mid Level	Spread To Spot
Spot	05/03/05	5.254	
June	05/13/05	5.256	0.2
July	06/14/05	5.281	2.7
August	07/12/05	5.301	4.7

Swap	Maturity	Mid	Spread
1Y	05/03/06	5.356	10.2
2Y	05/05/07	5.441	18.7
3Y	05/03/08	5.506	25.2
4Y	05/04/09	5.561	30.7
5Y	05/05/10	5.611	35.7

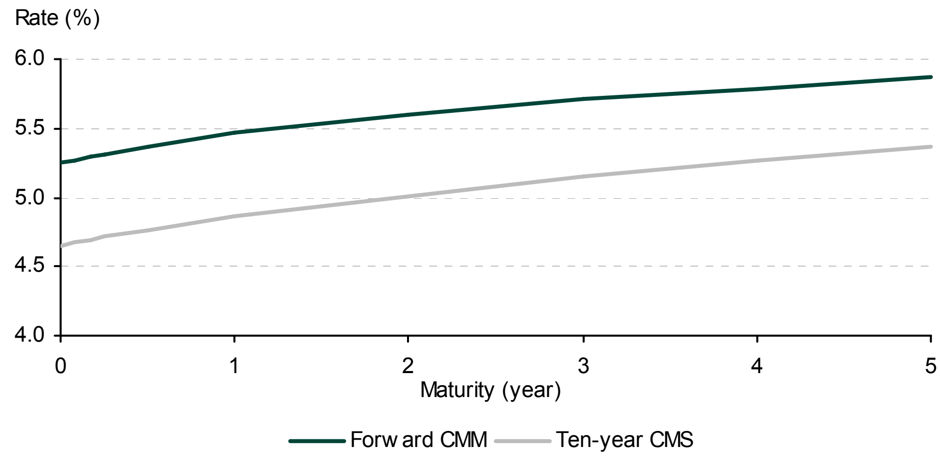
Source: Lehman Brothers

5. TERM STRUCTURE OF MORTGAGE RATES

In contrast to the Treasury market, which has long established a term structure of forward rates, the mortgage market has always lacked a similar curve establishment. Even though there are enough mortgage bonds traded in the market, the static curve method used in the construction of the Treasury curve is not applicable to mortgages because of their embedded optionality. Neither is a dynamic curve model because mortgage bonds are highly individualized in terms of their pool characteristics and prepayment histories. It is unrealistic to expect a unified curve model that reprices most mortgage bonds with a single curve. In the absence of such a curve, the mortgage market has been relying on Treasuries or swaps curves as benchmarks, and individual mortgage bonds are usually valued at an OAS to benchmark curves.

CMM products, on the other hand, are insensitive to individual characteristics of mortgage securities. They provide a pure and explicit view on mortgage rates, the most fundamental measure of mortgage investments. The current coupon rates referenced by CMMs at various forward maturities represent a uniform measurement of the mortgage market. Consecutive CMM forward and swap contracts along the maturity ladder reveal a term structure of mortgage rates, i.e., the sought-after mortgage curve. Thus, CMM instruments establish a convenient linkage between the mortgage and other markets, enabling mortgage rates to be compared directly with other ones. As an example, Figure 5 compares a sample constant maturity mortgage curve with its constant maturity swap counterpart.

Figure 5. The Constant Maturity Mortgage Curve versus the Constant Maturity Swap Curve, May 3, 2005



Source: The curves are constructed with Lehman Brothers' methods.

6. MODELING OF CMM PRODUCTS

For short-dated CMM rates, TBA passthroughs of comparable forward settlement can be used as replicating instruments. In fact, the spot CMM index itself is determined via a par-price interpolation of TBAs with nearest settlements. Similarly, short-dated forward CMM rates can be determined from TBAs with settlements further out. Their valuation relies on the availability of matching TBAs. The TBA-based valuation for short-dated CMMs is relatively straightforward and has already gained consensus in practice.

Unfortunately, TBAs are actively quoted only for settlements in the next few months. CMM contracts with maturities longer than those of available TBAs have to be analyzed alternatively. A common approach is to borrow existing mortgage rate propagation models that are universally employed in mortgage prepayment and OAS analyses. Most of those models propagate mortgage rates well into the future based on the propagation of certain benchmark rates. The future mortgage rates are then used as inputs to mortgage prepayment models to project future mortgage cash flows. The future current coupon rate referenced by long-dated CMMs can be modelled in the same fashion. Not only is this approach consistent with the industry practice, it is also beneficial from leveraging mature mortgage rate models.

Despite the fact that mortgage rate models—a prerequisite in prepayment analysis—are ubiquitous in mortgage modeling, there is still a lack of consensus on practical details. The disparate and proprietary nature of most mortgage rate models in actual applications has resulted in a wide range of pricing levels for long-dated CMM rates. Compared with the relatively transparent TBA-based approach for the short end, the alternative method for the long end is less certain, in spite of the linkage to the traditional approach. For this reason, there has been reluctance thus far to transact CMM contracts at the long end.

Elaborated in the sections below are both the TBA-based valuation approach for short-dated CMM products and the benchmark-rate method for long-dated ones.

7. TBA-BASED VALUATION

7.1. Breakeven CMM Rate

The standard CMM index calculation (described in the Appendix) can be extended to determine the fair forward rate of a CMM contract. However, a direct application of the static index calculation to the forward CMM rate is inadequate. The actual payoff of a CMM payment depends on the realized CMM index, which in turn depends on the realized TBA prices used in the index interpolation. TBA prices are volatile; so is the CMM index. The value of a CMM forward payment is normally computed by the weighted average of all likely CMM indices on the observation date. This weighted average is called the breakeven CMM rate. As long as the price information on TBAs, including their distribution, is attainable, the average over the corresponding index distribution can be computed, and the breakeven CMM rate can be determined.

Due to negative convexity, TBA prices do not vary linearly with the underlying rate: they appreciate more slowly in a rally and depreciate faster in a selloff. The resultant CMM index distribution is inevitably influenced by this skewed price movement, and the effect of the price bias has to be taken into account in the breakeven rate computation. As will be shown below, the breakeven CMM rate is usually lower than the forward CMM index interpolated directly from the quoted TBA prices. This difference is often referred to as the convexity adjustment.

7.2. The Convexity Adjustment

7.2.1. A Simple Explanation

In order to dissect the cause of the convexity, it is instructive to examine a hypothetical yet simple case in which there is only one TBA with settlement coinciding with that of a CMM forward and the TBA happens to be priced at par. With this simplification, the par-price conversion of the CMM index is trivial, and the interpolated index is just the TBA coupon.

Assume that the price of this fictitious TBA is a function of the current coupon rate *cc* and its duration and convexity with respect to *cc* are *D* and *C*. Then, the price variation of this TBA from par may be approximated via the following expression:

$$P - par \cong D(cc - cpn) + \frac{1}{2}C(cc - cpn)^2,$$

where *cpn* represents the TBA coupon. In the special case of zero convexity, i.e., $C = 0$, the price change, i.e., $D(cc - cpn)$, would be linear with the change of current coupon. It is then straightforward to argue that the breakeven CMM rate is none other than the coupon rate *cpn* itself since the CMM payoff, if adopting *cpn* as the breakeven rate, is $cc - cpn$ and this payoff, if hedged with $1/D$ unit of the TBA, is immune to any current coupon variation.

In the more realistic case in which considerable convexity exists in the TBA, such a hedge becomes imperfect due to the contribution associated with the convexity *C*. Because the TBA is negatively convex, i.e., $C < 0$, the CMM payoff, if still based on the breakeven rate *cpn*, would always be under-hedged by the TBA. This means that the combined position would always become negative in value with changing rates. To compensate for this potential loss, the CMM needs to pay off more on average than the zero-convexity amount $cc - cpn$, which requires that the breakeven CMM rate *cpn* be adjusted downwards.

It can be shown that the amount of the adjustment in this controlled analysis may be approximated analytically and the breakeven CMM rate, denoted by CMM_{be}, may be expressed as:

$$CMM_{be} \cong cpn + \frac{1}{2}\frac{C}{D}\sigma_{bp}^{2}T,$$

where σ_{bp} is the basis point volatility of the current coupon rate and T is the contract maturity.[2] This expression reveals that the adjustment is proportional to the ratio of convexity over duration. Similar analysis is routinely applied to the equivalent of Treasuries, for example, the convexity adjustment in a Treasury rate lock whose payoff is the difference between the Treasury yield and a breakeven lock yield. However, the convexity for a TBA bond is substantially larger than that for a Treasury bond, and the TBA duration is usually much shorter than the Treasury's. These differences between a TBA and a Treasury lead to a much bigger convexity adjustment for the CMM rate than for the Treasury par yield, not to mention in the opposite directions.

Although the analysis given above is simple and even allows for the quantification of the convexity effect, excessive reliance should not be placed on its numerical significance. After all, the analysis is carried out for an ideal setting with the assumptions that the CMM index could be "interpolated" from only one TBA price, the price variation with respect to the current coupon rate would be attainable, and the corresponding duration and convexity would remain unchanged with changing rates. None of these assumptions is realistic. Thus, the simple expression derived above, though helpful in understanding the qualitative aspect of the convexity adjustment, may not be used numerically for the actual breakeven CMM rate calculation.

7.2.2. Actual Convexity Profile

Due to the random TBA price fluctuation, it is possible that the par price on the CMM observation date can be straddled by any pair of adjacent coupons. In a rally, for instance, all TBA prices move higher. The initial pair of coupons bracketing the par price, say 5s and 5.5s, could both move above par, and the discount pair 4.5s and 5s would become the reference TBAs for the reset CMM index calculation. Similarly, in a selloff, the premium pair 5.5s and 6s could become the interpolation pair.

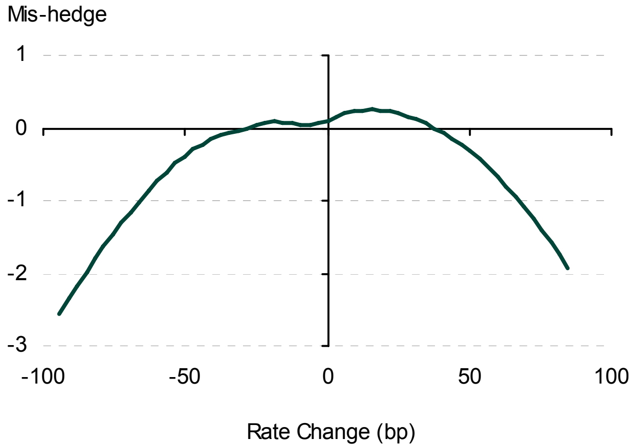
Because of these potential switches in the par-price interpolation of the CMM index, the actual convexity profile is rather complex, and any static convexity adjustment based on a single TBA is inadequate. In reality, a forward CMM rate is hedged with a portfolio of TBAs, as opposed to just a single one illustrated earlier. Given the price profiles of these TBAs, the corresponding convexity profile from the mis-hedge can be studied. Figure 6 compares the convexity profiles resulting from both portfolio and single TBA hedges.

Compared with the clean quadratic profile studied in the hypothetical case, the actual convexity profile appears rather irregular. This is expected because of the unavoidable pair switches in interpolating scenario CMM indices, as well as the considerable TBA convexity variation with rates. In addition, there is only a handful of TBA coupons for a particular settlement, which may not cover a wide enough range of rate scenarios to pin down the wings of the convexity profile. The integration of this profile has to be performed numerically to arrive at the actual convexity adjustment. It is also important to note that in practice, the convexity adjustment is estimated with respect to the underlying rate *r*, as opposed to the current coupon rate assumed in the simple analysis.

[2] *In arriving at this formula, one assumes a normal distribution for the current coupon rate; integrates over the quadratic amount,* $C(cc\text{-}cpn)^2/2$*, for the mis-hedge; and then converts this integrated amount to an equivalent rate contribution with the hedge ratio* $1/D$.

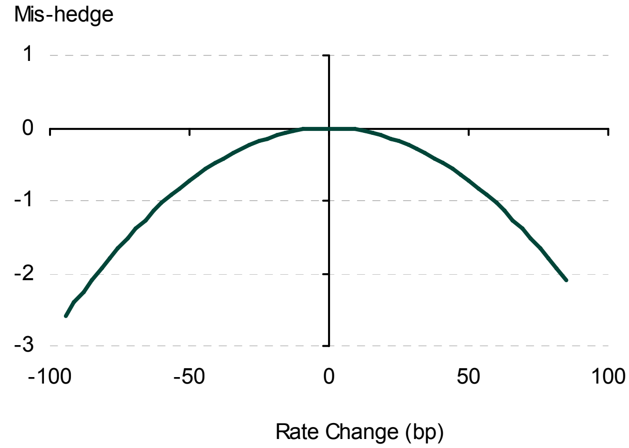
Figure 6. Convexity Profile Resulting from the Full TBA Hedge; for Comparison, the Quadratic Convexity Profile from Simple Hedge Also Shown

Actual Convexity Profile



Source: Based on Lehman Brothers' CMM model.

Simple Convexity Profile



Source: Based on a single TBA hedge.

7.3. CMM Rate Computation

7.3.1. Scenario CMM index

In reality, the determination of a breakeven CMM rate starts with the selection of two sets of TBAs whose settlements bracket the CMM settlement. Next, the settlement interpolation and delay accrual are applied to arrive at an adjusted set of TBA prices. Then, the calculation proceeds with the identification of interpolating coupon pairs under different rate scenarios.

In order to capture all possible realizations of TBA pairs, the price profiles for the entire TBA coupon stack with respect to some underlying rate are required. For example, a combination of swap rates can be used as the underlying rate over which TBA prices can be profiled. In general, the TBA price of a given coupon can be represented by $P^c = P^c(r)$, where r represents the underlying rate and the superscript c indicates a particular coupon in the coupon stack. For a particular scenario rate r, the adjusted TBA prices $P_a^c(r)$ can then be derived.

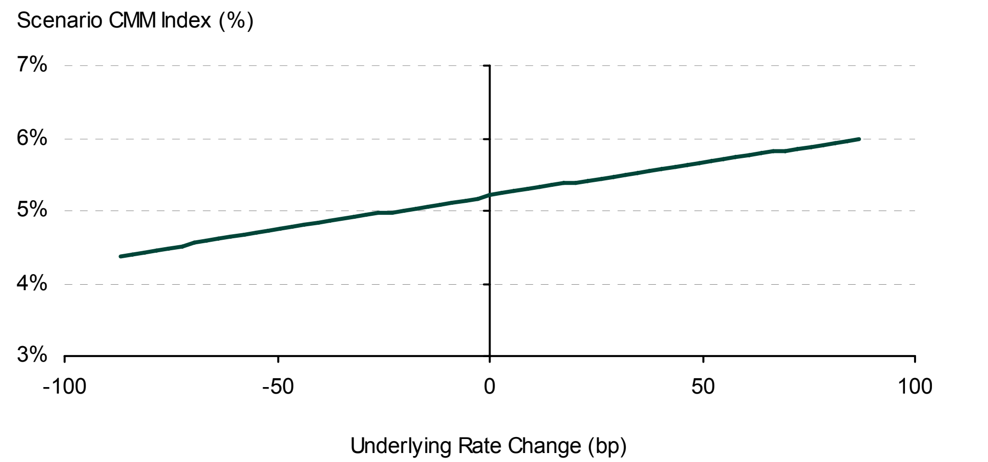
The CMM index for this scenario can, in turn, be obtained by first identifying the two coupons with their prices straddling the par and then interpolating the par coupon. Thus, the realized CMM index depends on the relative price relationship of all the TBAs involved. Conceptually, this dependency can be encapsulated by an interpolation function that takes the adjusted TBA prices of all coupons as inputs, i.e.,

$$cmm(r) = Interp(P_a^{c_1}, P_a^{c_2}, P_a^{c_3}, \ldots),$$

where the lower case symbol *cmm* is used here to denote a scenario CMM index. Though not explicit, it is understood that the dependency of *cmm* on the underlying rate r stems from the adjusted TBA price $P_a^c(r)$. The price profiles of all TBAs involved have to be provided in order to calculate the *cmm* rate for all scenarios from the *Interp* function. Those TBA price profiles can be generated separately and have to be calibrated to either user inputs for TBA price variation or scenario outputs of a quality mortgage prepayment model.[3]

The *interp* function itself may be of a complex relationship between the scenario CMM index and the TBA prices; however, the behavior of the scenario index with changing underlying rate is relatively simple (Figure 7). Although the $cmm(r)$ does not vary exactly linearly with r, the non-linearity is barely discernable.

Figure 7. Scenario CMM Index as a Function of Underlying Rate Change



Source: From Lehman Brothers' CMM model.

7.3.2. Average over Scenario Rates

As already established, the breakeven CMM rate depends on how the TBAs of different coupons reprice under various rate scenarios. Once the rate distribution is known, the average value over scenario CMM indices can be computed accordingly. Explicitly, the breakeven forward CMM rate can be integrated as follows:

$$CMM_{be} = \int_0^\infty cmm(r)g(r)dr,$$

where the scenario index $cmm(r)$ has been discussed earlier and $g(r)$ is the distribution function for the underlying rate.[4]

In order to apply the rate distribution to the integration of CMM_{be}, the volatility of rate r needs to be specified. Since the underlying rate is closely tied to a combination of forward swap rates, its volatility can be estimated from the volatilities of corresponding swaptions. Alternatively, the implied volatility from the mortgage options market can also be used. In fact, the underlying TBA in a mortgage option belongs to the TBA family used for calculating the CMM index.

The breakeven CMM rate presented here takes into account the non-linear relationship between the underlying rate r and the TBA prices of all coupons. The aggregate profile of all TBA prices is captured properly, and the negative convexity effect is incorporated automatically. The TBA-based valuation is robust and reliable. A forward CMM rate can be

[3] *See, for example, Zhou and Subramanian [2004] for a detailed description of TBA price functions in the context of modeling mortgage options.*
[4] *For convenience, the rate is usually assumed to have a normal distribution. Other distributions may be employed to address the concern that basis point volatility is not invariant, as evident from observed volatility skews and smiles.*

modelled as long as the price information on the corresponding TBAs is available. As noted earlier, however, TBA prices are actively quoted for settlements of only a few months. With some effort, fair TBA prices may be derived further out by estimating TBA dollar rolls under certain financing and prepayment assumptions. For even longer maturities, though, there is hardly any reliable TBA price estimation.

Because of this limitation, the TBA-based valuation is applicable only to short-dated CMM contracts. For long-dated CMMs, alternative methods, such as the one discussed below, need to be considered.

8. BENCHMARK RATE METHOD

8.1. Determinants of Current Coupon Rate

For CMM contracts of relatively long maturities, i.e., CMM swaps and long-dated CMM forwards, the breakeven CMM rates have to be evaluated for forward maturities up to five years. Since current coupon rates over those long maturities become decoupled from available TBAs, their dependency on other observables has to be analyzed.

The current coupon rate measures the potential rate of return for a representative mortgage bond. Similar to a regular coupon bearing bond, a mortgage bond is sensitive to interest rates but in a much different way. This is because a mortgage bond has a scheduled principal amortization, which shortens its duration. Moreover, mortgage borrowers can repay the principal anytime before the stated maturity. This prepayment option makes a mortgage mature even faster, further shortening its duration.

There are two sources of mortgage prepayments: housing turnover and loan refinancing. The former occurs when home owners decide to sell for reasons other than the current mortgage rate. The amount of prepayment due to turnover is usually independent of interest rate volatility. The prepayment due to refinancing behaves much like an interest rate option and is sensitive to the volatility of interest rates.

A realistic current coupon model would take into account these essential features in a mortgage bond, that is, a baseline principal amortization overlaid with turnover prepayment and refinancing options.

8.2. Mortgage Rate Models

8.2.1. Current Coupon Rate Dependency

The price of a mortgage bond can then be viewed as consisting of two parts: one is sensitive only to interest rates and the other is also volatility dependent. Thus, it is convenient to treat a mortgage as an option-free amortizing bond plus an embedded short option. Furthermore, it is reasonable to assume that this decomposition carries over to the measure of mortgage rates. That is, the mortgage rate dependency can also be decomposed into two components: rates and volatility.

In the model considered below, the mortgage rate, and by extension the current coupon rate, is assumed to be a function of three benchmark swap rates plus one representative volatility. Specifically, the following dependency is considered:

$$cc = B(S_2, S_5, S_{10}) + cV$$

where S_2, S_5, and S_{10} are benchmark swap rates for 2-, 5-, and 10-year maturities, and V is a benchmark swaption volatility. In above expression, B is a deterministic function of three swap rates, and c is the volatility coefficient.

The three benchmark rates are chosen to take into account the belief that the current coupon rate depends on the entire term structure of interest rates and that the entire term structure can be explained to a satisfactory degree of accuracy by a small number of factors. The separate dependency on volatility is used to capture the effect of the refinancing option on the current coupon rate. Here, the linear dependence is chosen for illustration. The actual volatility dependence, of course, relies on one's understanding of the interplay between the TBA mortgage and the volatility surface.

8.2.2. Benchmark Rate Functions

In practice, there are different ways to model and calibrate the benchmark rate function *B*, ranging from the naive linear regression analysis to the sophisticated approach that recognizes notional amortization of mortgage bonds.

The regression approach, though usually with satisfactory fitting errors, is not always stable. The fitted coefficients can be rather sensitive to the period of data used in the linear estimation. It is not hard to argue that this instability results from an oversimplification of the linear model. A more realistic specification of the rate function *B* takes into account the fact that a TBA mortgage is an amortizing coupon bond with rate-sensitive turnover prepayments. The scheduled mortgage amortization is known deterministically, and, as in most prepayment models, the overall turnover prepayment can be modeled with a seasoning curve. For instance, the benchmark rate function can be modelled as the yield of an artificial bond with the standard mortgage principal amortization plus an additional PSA-based seasoning curve for turnover related notional reduction. Compared with the linear model, the relationship between the current coupon rate and the yield of a bond resembling an actual mortgage is far more stable and reliable.

To compare the stability of the linear and amortizing models described above, the model coefficients of these two approaches across three sample periods are generated with the regression of mortgage rates against the three swap rates directly and the yield of an amortizing bond. The 3- by 10-year swaption volatility is used in the analysis to account for the volatility dependency of the current coupon rate. Figure 8 displays the comparison between the two models. While the linear model fits well, the coefficients are highly unstable. In contrast, the coefficients in the amortizing model are much more stable, at only a slight deterioration of fitting errors.

Figure 8. Stability Comparison Between the Linear and the Amortizing Models

Model	Sample Period	Yield	2y Swap	5y Swap	10y Swap	3x10 Swaption	R-square	Error (bp)
Linear	2000-02		-0.05	0.49	0.41	0.0025	0.996	4.3
	2001-02		0.01	0.44	0.30	0.0048	0.994	3.8
	2002-04		0.23	-0.06	0.71	0.0108	0.989	6.0
Amortizing	2000-02	0.86				0.0065	0.994	5.6
	2001-02	0.85				0.0063	0.991	4.7
	2002-04	0.91				0.0105	0.988	6.2

Source: Based on Lehman Brothers mortgage rate analyses.

8.2.3. Forward Volatility

In the current coupon method described above, it is assumed that the volatility dynamics are available to capture the volatility dependency of the current coupon rate. It is not always convenient, however, to incorporate random volatility into the current coupon model, and, even if possible, it may be unreliable. Fortunately, for the purpose of capturing the variability of volatility in the current coupon relationship proposed above, it is not necessary to have the full dynamics of volatility. A static forward volatility curve is usually sufficient and is actually more stable than a dynamic volatility process.

The forward volatility represents the expected volatility for a forward date. This expectation is usually different from the spot volatility level and is also different for different forward dates. The variation in forward volatility is normally represented by a forward volatility curve. It reflects the non-stationary nature of interest rate volatility. Though not directly observable, the non-stationary information is embedded in the spot volatility surface. The forward volatility as a function of forward date can be extracted from the spot volatility surface through various techniques.

8.3. CMM Valuation

The models proposed above establish the dependency of the current coupon rate on benchmark swap rates and swaption volatility, which can then be used to price a forward CMM payment within a dynamic interest rate model. For each rate and volatility scenario generated by the interest rate model, this relationship with the three swap rates and the volatility is applied to project the scenario current coupon rate. The present value of the CMM payment is then computed by taking the discounted average of the current coupon CMM payment over the distribution of interest rates and volatility. For a forward contract, the breakeven rate is just the present value of the current coupon payment net of the forward discount; for a swap, the present value calculation is repeated for each CMM payment, and the swap rate is determined as the breakeven rate of all the current coupon payments.

9. RISK EXPOSURES OF CMM INSTRUMENTS

CMM forwards are the most liquid current coupon instruments. Though similar to forward contracts on swap or Treasury rates, they share some of the complex risk characteristics of TBA passthroughs. For short-dated forwards, their risks can be hedged and understood in terms of a replicating portfolio of TBAs. Such hedged positions would be negatively convex, but with minimal exposure to other risks. Long-dated CMM forwards are usually hedged with either short-dated TBAs or other forward instruments such as forward-starting swaps. The former carries roll risk, in addition to the negative convexity of TBA hedging portfolios. The latter carries basis risk and also has implicit curve and volatility sensitivities.

9.1. Risks of Short-Dated Forwards

9.1.1. A Simplistic Two-Coupon Hedge

In order to understand and manage the risks of a short-dated CMM forward, it is necessary to analyze its TBA replicating portfolio. The CMM index is a simple linear combination of two interpolating TBA coupons, and these two TBAs can be thought as the hedge for the CMM forward. Determining the notionals of the two TBAs in the replicating portfolio is relatively easy: they are just the partial differentials of the CMM index with respect to the two TBA prices.[5] Such a hedge, though straightforward, may be too simplistic and lead to misleading results. As underlying rates move, the hedge ratios also change. Across a coupon boundary,

---

[5] *It can be shown, for example, that the spot CMM index is*

$$C^{par} = C^{disc} + 0.5\% \times \frac{par - P^{disc}}{P^{prem} - P^{disc}},$$

where prem and disc denote the two TBA coupons, one premium and the other discount that straddle par. Differentiating the index with respect to the two prices leads to the following hedge ratios of the replicating TBAs.

$$h_{disc} = 0.5\% \times \frac{par - P^{prem}}{\left(P^{disc} - P^{prem}\right)^2}, \qquad h_{prem} = 0.5\% \times \frac{P^{disc} - par}{\left(P^{disc} - P^{prem}\right)^2}.$$

Even for the two-coupon hedge, these ratios are simplistic, ignoring certain aspects of the CMM index calculation. They are presented here for illustration only.

the two-TBA hedge switches to a different pair of coupons, causing discontinuities in hedge ratios. For instance, Figure 9a shows the notional amount of TBA 5s to hedge the CMM index. As the index moves through the 5% boundary, there is a sudden jump in the hedge notional.

9.1.2. Basket TBA Hedges

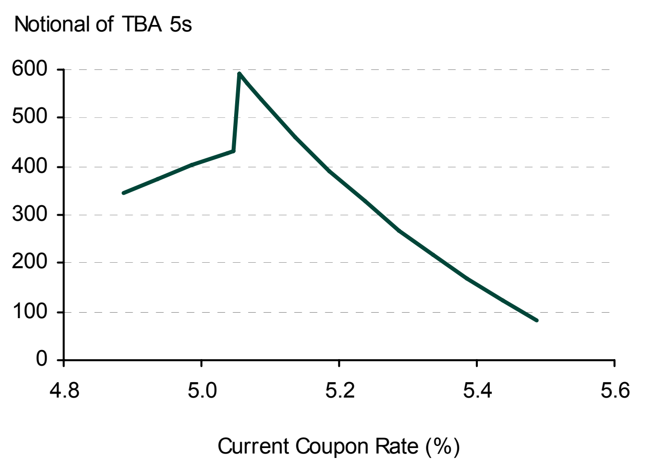
The jump in hedge ratios at coupon boundaries is due to the constraint that only two TBA passthroughs are used in the hedge. As already explained in the section of the TBA-based valuation, the forward CMM rate does not depend on just two spot interpolating TBA coupons. Rather, it is derived from the price distribution of all TBAs under various rate scenarios. The replicating portfolio for a CMM forward should, therefore, consist of all the TBAs used in the valuation of the CMM forward since they all play a role in determining the levels of the CMM rate. Unlike the simple two-TBA hedge, this basket hedge does not have jumps in the notionals of replicating TBAs, as shown in Figure 9b.

9.1.3. Negative Convexity of Replicating Portfolios

The difference between a short-dated CMM forward and its replicating portfolio is that the CMM forward rate by construction has almost no convexity but the TBA replicating portfolio is negatively convex. For illustration, consider replicating the spot CMM index with two TBAs. Figure 10 shows the difference between the changes in the CMM index and the changes in its replicating portfolio. For instance, in a 100 bp selloff, the CMM rate moves up 100 bp, but the TBA portfolio moves close to 125 bp. In a 50 bp rally, the replicating portfolio moves only 35 bp.

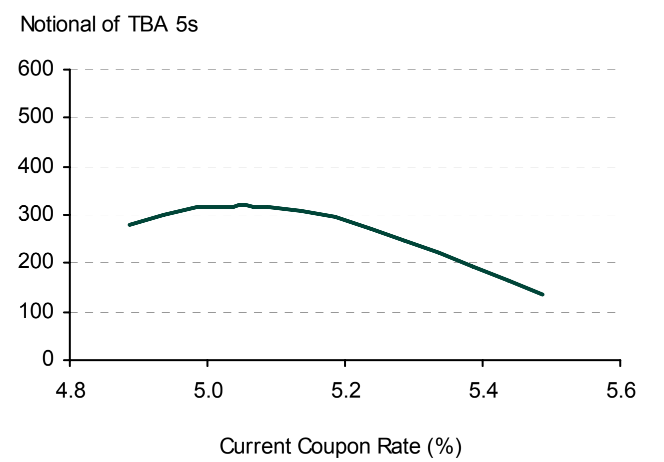
Figure 9. Notional of TBA 5s for Hedging the CMM Rate

9a. Simple Hedge



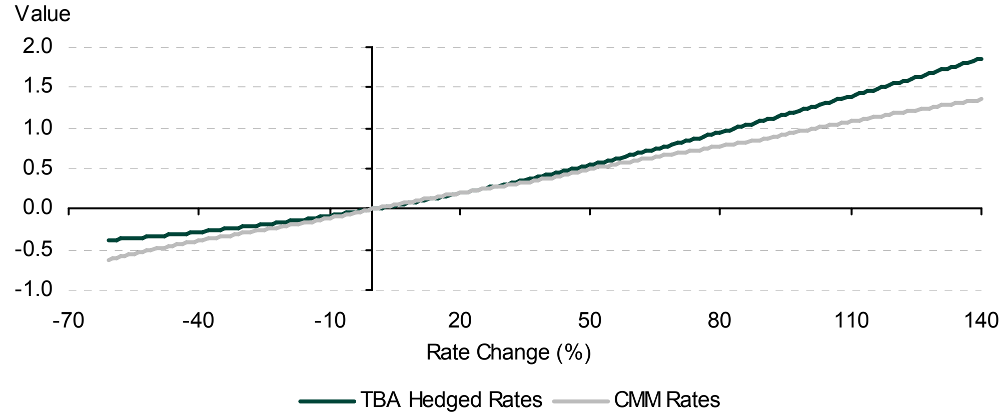
Source: Calculated from a two-TBA hedge.

9b. Basket Hedge



Source: Calculated from Lehman Brothers CMM model.

Figure 10. Induced Convexity with TBA Hedges



Source: Based on Lehman Brothers analysis.

9.4. Risks of Long-Dated Forwards

Unlike for short-dated forwards, the risks for long-dated forwards are harder to categorize and quantify. This is primarily because there are no liquid TBA instruments of comparable maturity available for replication. Long-dated CMM forwards are replicated, instead, with swaps, and their risks are usually analyzed in terms of duration and vega.

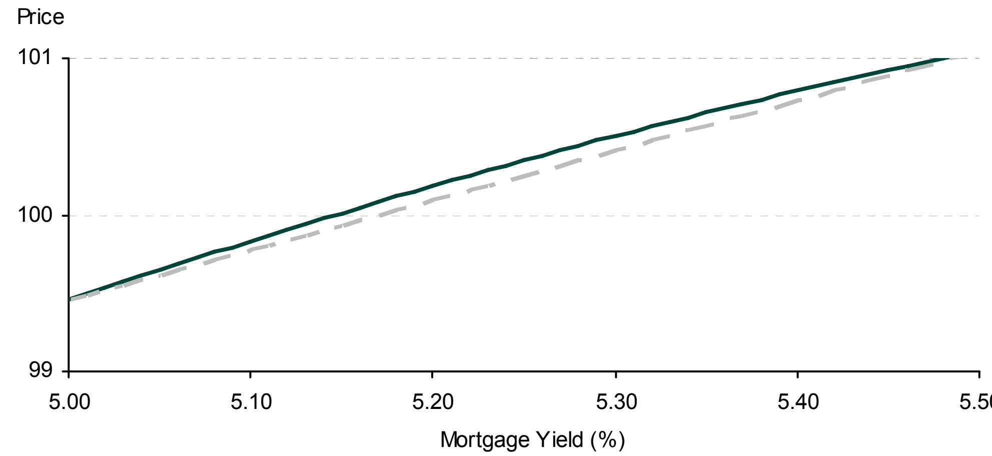
9.4.1. Effect of Interpolation Error on Duration and Convexity

The value of a CMM forward moves one-for-one with changes in the forward CMM rate; hence, its duration with respect to the CMM rate is one. However, because of the error induced by linear interpolation, the CMM rate does not move by the same amount for a parallel move in all rates. Therefore, a CMM forward does not have a duration of exactly one with respect to underlying rates. In Figure 11, for example, TBA 5s and 5.5s are priced below and above par, respectively. It shows the price profiles of synthetic coupons between these two TBAs based on both a prepayment model and a linear interpolation. The model-based price is slightly negatively convex. This small difference in shape leads to a difference in the determination of the current coupon rate. The model-based calculation gives the current coupon rate at 5.14%, compared with the linearly interpolated rate at 5.17%. If rates were to rally 14 bp, all else unchanged, this would make 5s the current coupon, and the CMM index would rally 17 bp (since there is no interpolation error at the 5% current coupon rate). Compared with the convexity of a current-coupon TBA mortgage, the convexity introduced by the linear interpolation is minor and is usually ignored, except for extremely short-dated positions.

9.4.2. Partial Durations

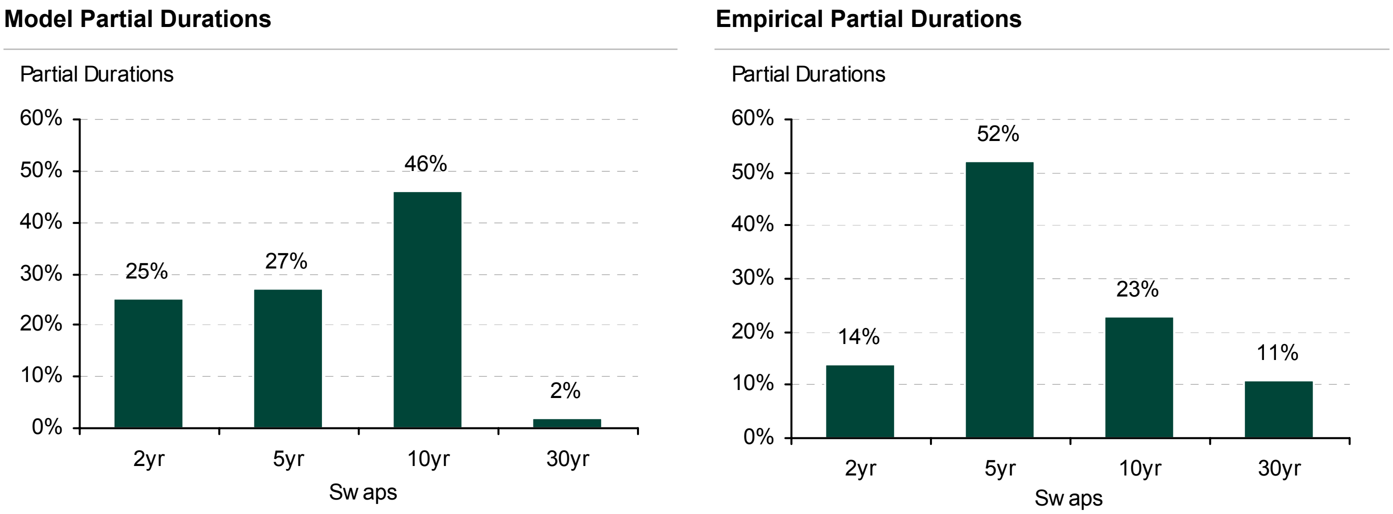
Similar to a TBA mortgage, a CMM forward has sensitivities to rates across the curve. In order to hedge against non-parallel moves in the curve, it is essential to know the partial rate exposure of the current coupon rate. This partial duration can be calculated based on a prepayment model. As shown in Figure 12a, changes in the CMM rate are predominantly (close to 50%) driven by changes in the 10-year swap rate, with the rest driven roughly evenly by changes in 2- and 5-year swaps. A CMM forward can, therefore, be replicated by forward rate agreements on these swaps with similar weightings to those derived from the model. On the other hand, the partial durations can also be calculated empirically. For comparison, the empirical relationship between current coupon rates and swap rates is analyzed here, as well. As shown in Figure 12b, the distribution of empirical durations appears quite different from that of the model ones. Empirical fits tend to be unstable and are not suitable for hedging CMM positions.

Figure 11. Effect of Interpolation Error on Current Coupon Rate



Source: Obtained from the Lehman Brothers' prepayment model.

Figure 12. Model and Empirical Partial Duration Exposures of Current Coupon Rates



Source: A multiple regression based on data from 7/15/2004 through 7/15/2005, of changes in CMM rates to changes 2-, 5-, 10-year swap rates and 3- by 10-year swaption volatility, is used to estimate the empirical partial durations.

9.4.3. Vega Exposure

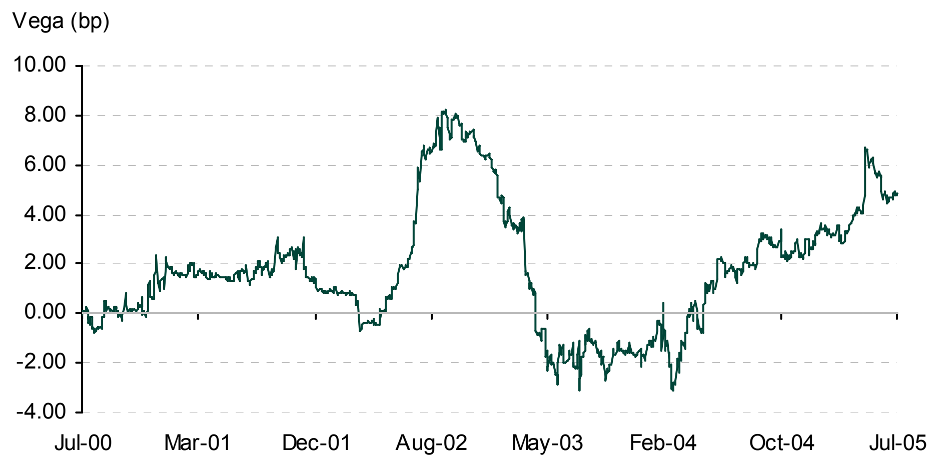
TBAs are sensitive to changes in implied volatility; so are CMM rates. Figure 13 displays typical price variations of two TBA coupons and the corresponding variation of the current coupon rate in response to a 1% change in percentage volatility across volatility surface. In this example, about a 5-6 bp of change in the current coupon rate is expected for a 1% change in volatility. The interplay between the volatility and mortgage markets is largely influenced by mortgage hedgers, primarily the servicers and the agencies. They routinely transact in the interest rate options products to hedge convexity or volatility exposure. The level of their activity is a key driver of the empirical relationship between implied volatility and mortgage rate changes. This relationship varies over time, as shown in Figure 14. There are extended periods when volatility is priced in more strongly into mortgage rates.

Figure 13. Sample Model Vega for Current Coupon Rate

Volatility	Price for 30-year 5.0s	Price for 30-year 5.5s	Current coupon
Up 1%	99-20	101-07	5.11
Base	99-26	101-11	5.06
Down 1%	100-00	101-15	5.00

Source: Based on Lehman Brothers' mortgage prepayment model.

Figure 14. Empirical Vega Exposure for Current Coupon Rate



Source: Based on a rolling 6-month regression of changes in CMM rates versus changes in 2-, 5-, 10-, and 30-year swap rates and 3- by 10-year swaption volatility. The figure shows the coefficient of the volatility term in the regression. If volatility is directional with rates, much of the volatility repricing in mortgage rates is captured in the duration term itself, which explains why empirical vegas estimated by this method are much lower on average than the model vegas.

10. CMM APPLICATIONS

10.1. Hedging Servicing Portfolios

With over $4 trillion outstanding in capitalized servicing and with top 10 servicers owning almost 60% of this universe, the hedging of mortgage servicing rights has gained increasing importance over time.[6] Mortgage servicing portfolios, as well as the holders of interest-rate only (IO) mortgages, are active hedgers of mortgage rate exposure. The valuation of an IO is sensitive to prepayments, which are driven primarily by mortgage rates. For similar reasons, mortgage servicing portfolios are sensitive to mortgage rates, as well. The principal contributor to the value of servicing portfolios is the stream of servicing income for mortgage loans. The income stream ebbs and flows with mortgage prepayment activities, much like the interest stream of IOs. Indeed, an IO is a reasonable proxy for a servicing portfolio.

Prior to the 1998 financial crisis, when mortgage spread volatility was low and mortgage rates were fairly correlated with Treasury rates, servicers relied on just Treasury bonds to hedge servicing portfolios. Until 2000, when mortgages moved in sync with other spread products, servicers used swaps as the primary hedging instrument. Since 2001, the volatility in mortgage spreads has increased substantially, which has made it necessary for servicers to

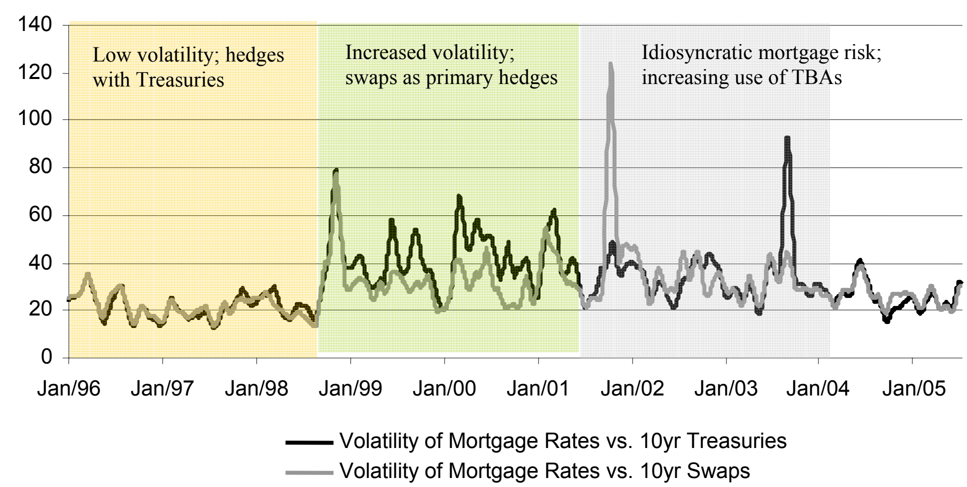
[6] *See Modukuri [2000] for a description of the risk exposures of servicing portfolios and the evolution of hedging practices, and see Shilpiekandula and Miller [2004] for constructing optimal hedges for servicing portfolios.*

consider mortgage products in the hedging mix and resulted in the increased use of TBA passthroughs. Figure 15 recounts the timelines of servicers' hedging practice over the last decade.

While the passthrough hedge helps minimize the spread exposure of a servicing portfolio, it adds other risks. The use of passthroughs as a duration hedge exacerbates the negative convexity and curve bias of servicing portfolios. Using passthroughs also requires frequent rebalancing of the coupons used.

Paying the CMM index in return for locking in a fixed rate is similar to a long position in TBAs. Yet, the CMM position does not introduce those TBA-related risks mentioned above. As an example, a comparison of using TBA passthroughs and CMM forwards to hedge a 5.5% trust IO is shown in Figure 16. Hedged with TBAs, the hedged portfolio has significant negative convexity. In a 50 bp rally, the convexity loss for the combined position is about 8% of the portfolio's value; with CMM forwards, the loss is much less. Moreover, if rates were to rally 50 bp, in addition to experiencing convexity losses, servicers would also need to shift duration hedges from 5s to 4.5s to maintain the current coupon rate exposure. Such a realignment of coupon is not necessary with CMM forwards since the CMM rate dynamically tracks the current coupon rate.

Figure 15. Changes in Hedging Practice of Mortgage Servicers in Response to Spread Volatility, One-Month Basis Point Spread Volatility between Mortgage and 10-Year Treasury or Swap Rates



Source: Based on Lehman Brothers' historical analysis.

Figure 16. Lower Negative Convexity with CMM Forward

Security	-75 bp	-50 bp	-25 bp	Base	+25 bp	+50 bp
FNT-346 5% IO	11.69	14.14	16.83	19.44	21.58	23.24
FNCL 5.0%	101.79	101.33	100.68	99.85	98.84	97.72
CMM Forward	4.38	4.63	4.88	5.10	5.35	5.59
P/L hedged with TBA	-14%	-8%	-3%	0%	-3%	-9%
P/L hedged with CMM	-3%	-3%	-2%	0%	-2%	-6%

Source: Based on a sample analysis.

10.2. Managing Exposure to the Mortgage Market

CMM instruments can be used by fixed income investors to add or reduce mortgage exposure selectively. They can be used as hedges for mortgage portfolios or as vehicles to replicate the return of a mortgage portfolio synthetically. For example, instead of buying the securities in an MBS Index to gain broad exposure to the mortgage market, one can obtain similar exposure by selling CMM forwards. The benefit of utilizing CMM instruments is that they are structurally simpler, easier to understand, and in many ways cleaner to adjust. CMMs are particularly attractive for those who do not primarily deal with mortgage securities and who may not have the expertise or back office capability to handle the complexities and intricacies of the mortgage market.

Considered here, as an example, are two strategies: investing cash in the MBS Index versus investing the same cash in LIBOR and selling CMM forwards to match the duration exposure of the MBS Index. For illustration, Figure 17 compares the returns on these two investments for June 2005, and Figure 18 compares the monthly and cumulative returns of the two investments since January 2003.

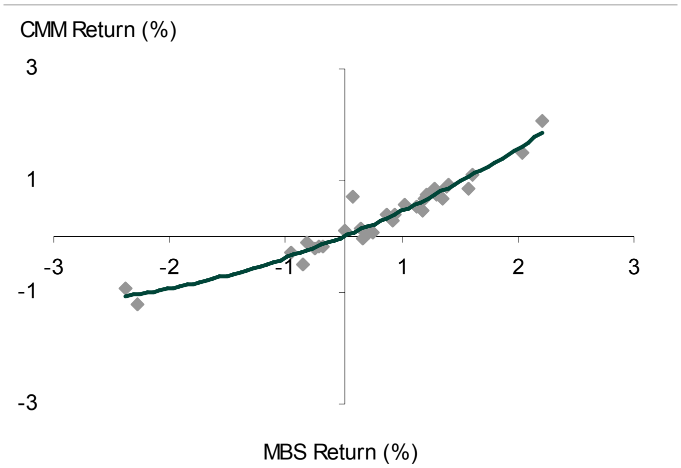
Figure 17. Return on CMM Forwards versus Return on MBS Index, June 2005

May 31, 2005	**May 31, 2005**
1. Invested $100 cash at LIBOR at 3.13% 2. Sell $238 worth of CMM forward contracts with settlement on June 30, 2005 and strike at 5.06%	Buy $100 worth of the MBS Index with beginning duration of index, 2.38
June 30, 2005	**June 30, 2005**
1. Cash earns $0.26 for the month 2. CMM settles at 5.0% and the contract earns $0.21	Ending duration, 2.23
Total return: $0.47	Total return: $0.36

Source: Sample analysis.

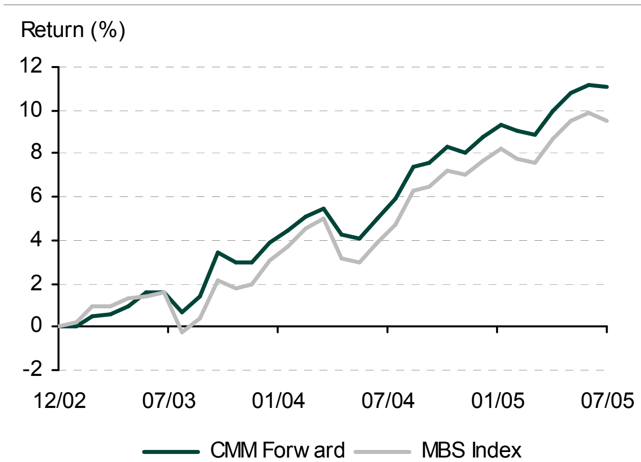
Figure 18. Performance of Rolling CMM Forwards versus Buying the MBS Index

Monthly Returns



Source: Lehman Brothers.

Cumulative Returns



Source: Lehman Brothers.

Because of the absence of negative convexity in CMMs, it is not surprising to observe the positively convex monthly returns of the CMM-based strategy with respect to the index returns. While not a perfect match, it does appear that, historically, the returns on selling CMM forwards have been a good proxy for that of the MBS Index.[7]

10.3. Mortgage Basis Trades

Buying or selling mortgage passthroughs against benchmark Treasuries or swaps has been the traditional choice for relative value investors to express views on the mortgage basis. There is, however, uncertainty in the traditional approach. As pointed out earlier, while both Treasuries and swaps can be largely considered as rate instruments, it is hardly true for mortgages because of their considerable negative convexity. With passthroughs in a mortgage basis trade, the additional convexity exposure is inevitable, which may obfuscate the view expressed on the mortgage basis. In contrast, CMM instruments are convexity free and can substitute for passthroughs in basis trades to eliminate this concern.

The benefit of such a substitution is illustrated in an example. On May 31, 2005, there was a 79 bp spread observed between the current coupon mortgage rate and a mix of swaps with comparable curve exposure, e.g., 25% 2-year, 25% 5-year, and 50% 10-year swaps. Historically, this spread has averaged 100 bp. The traditional strategy is to buy a FN 5s mortgage and hedge its duration with a mix of swaps. A newer strategy to take advantage of what appeared to be a rich level on the basis would be to buy a one-month CMM forward versus a portfolio of corresponding CMS forwards.

The returns on these two basis trades are compared in Figure 19. While the spot spread between CMM and swaps was 79 bp, the one-month forward spread was 83 bp; i.e., mortgages were projected to widen 4 bp along the forwards. The basis turned out flat over the month, and the position lost 4 bp in returns. The CMM basis trade resulted in a net P&L of 13 bp on a 360 notional. For the FN 5s basis trade, however, it is hard to make such a conclusive statement, and the return attribution depends to a large extent on prepayment models. Moreover, the FN 5s position needs frequent duration rebalancing, while the CMM trade has the natural duration match with no rebalancing need.

Figure 19. Basis Trades of CMMs versus TBAs, May 31, 2005

	Trade Description	**Spot**	**Initial**	**Final**	**P/L (bp)**
Trade 1	Buy 360 one month CMM forward	5.03	5.09	5.01	-29
	Sell 90 one month forward on 2 yr CMS	3.95	3.98	4.00	-1
	Sell 90 one month forward on 5 yr CMS	4.17	4.19	4.15	3
	Sell 180 one month forward 10 yr CMS	4.42	4.44	4.36	14
	Mortgage basis	79	83	79	
	Net P/L				-13
Trade 2	Net P/L from selling 100 5s with swap hedge				-21

Source: Sample analysis.

[7] *Despite the convenience of using CMM products, there are many differences between returns on CMM forwards and on the MBS Index. The former have no convexity while latter is negatively convex. Also, while the MBS Index is composed of a variety of passthroughs, a CMM forward mimics the returns of only the synthetic current coupon 30-year TBA. Moreover, the performance of CMM forward is affected by roll specialness or mortgage financing levels, while the MBS Index returns are not at all affected by the funding specialness.*

10.4. Mortgage Carry Trades

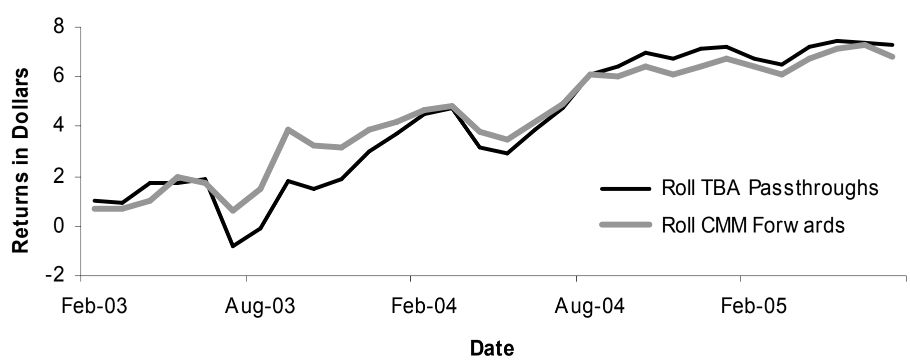
Mortgages have long been the favorite instruments for investors to accumulate carry in a range-bound market. Like other duration instruments, mortgages pick up carry with an upward sloping curve because of their higher yields than short-dated funding rates. In addition, mortgages may benefit from special financing. The roll market sometimes provides better financing opportunities than other funding sources, such as LIBOR, and a mortgage trading special earns the incremental carry from favorable funding opportunities. Furthermore, a significant portion of carry in mortgages results from the compensation for the inherent negative convexity.

There are two different applications of CMMs to carry trades. First, they can be added to reduce mortgage spread risk in a convexity carry trade. A portfolio manager may want to enhance yield from the exposure to the negative convexity of mortgages, but may not want to subject the portfolio to the mortgage spread variation. This manager could include CMM forward contracts in the portfolio to minimize the mortgage spread risk.

The second is to own CMM instruments outright for direct carry from steep curves. For instance, according to the term structure of forward CMM rates displayed in Figure 4, selling a 9-month CMM forward on May 3, 2005, is a position that would realize positive P&L as long as the CMM index at the 9-month maturity does not reset 16.7 bp higher than today's spot. Similar to a carry position with passthroughs, a carry position in CMMs also benefits from any funding specialness of current coupon mortgages. It differs from the passthough carry, though, in that the CMM carry is on average lower because the passthroughs pick up additional carry from negative convexity.

For an example of the relative performance of passthrough versus CMM carries, the returns are compared for the following two trades: buying TBA 6s in January 2001 and rolling the security every month on the TBA notification date versus receiving a one-month CMM forward in January 2001 and reestablishing the forward CMM position at each contract expiry. The notional of CMM forward is adjusted to reflect the duration match with TBA 6s at the beginning of each month. Somewhat surprisingly, the returns on the two carry trades are rather comparable from a historical perspective (Figure 20). While both positions earn carry from an upward sloping curve, the TBA position earns additional carry from negative convexity. In periods of low realized volatility, the CMM trade generally underperforms.

Figure 20. Returns from Rolling CMM Forwards versus Rolling TBA Passthroughs



Source: Based on Lehman Brothers' historical analysis.

10.5. Hedging Pipeline Negative Convexity

Options on CMM rates would be a natural extension to the standard CMM forward contracts. Once they become readily available, they could be used to hedge negative convexity in traditional mortgages. For example, the pull-through risk of origination pipelines is associated with taking applications from prospective mortgage borrowers who may decline a quoted mortgage rate within a certain grace period. The number of borrowers who accept a mortgage rate depends on how rates vary in the interim. This is the lock-in option that originators are short, which results in significant negative convexity. Originators have typically used swaptions or mortgage options to hedge this convexity risk. While a better hedge than swaptions, mortgage options carry the additional complexity that they are options on negatively convex instruments. CMM options are written directly on mortgage rates and, therefore, are an even better choice to hedge the pipeline convexity.

REFERENCES

F. Zhou and P. Subramanian, *Mortgage Options: A Primer*, Lehman Brothers, 2004.

S. Modukuri, Mortgage Servicing Rights: Risk Exposures and Hedging Strategies, Lehman Brothers, 2000.

V. Shilpiekandula and A. Miller, *Designing the Optimal Hedge for Servicing Portfolios*, Lehman Brothers, 2004.

ACKNOWLEDGEMENTS

The authors are grateful to Dick Kazarian and Huarong Tang for insightful discussions on modeling mortgage rates and would also like to thank their colleagues in Fixed Income Research—Srinivas Modukuri, Vikas Shilpiekandula, Dev Joneja, Prafulla Nabar, and Venkateswaran Vaidyanathan—for their suggestions as well as patience in reading the drafts of this article.

APPENDIX: CMM INDEX COMPUTATION

The CMM index is designed to track the current coupon rate that represents the prevailing secondary mortgage rate of new mortgage issuance. It is determined by a standardized procedure based on linear interpolations of actively quoted prices for FNMA 30-year TBAs. They are the most liquid segment of the mortgage market, and their price transparency is ideal for establishing a reference on the mortgage rate that is otherwise not directly observable. The interpolation-based conversion from the quoted prices to the par coupon rate is straightforward and objective, avoiding dependence on any model assumptions. Described below are the steps for calculating the CMM index:

1) Obtain the TBA prices of all coupons for the two consecutive PSA settlement dates that bracket the reference TBA settlement of the CMM under consideration, with the following notation denoting the two sets of TBAs.

T: 30-day forward date of the reference TBA;

T_1: PSA settlement date before T;

T_2: PSA settlement date after T;

P_1: TBA prices for settlement date T_1;

P_2: TBA prices for settlement date T_2;

C: TBA coupon.

2) For each pair of PSA settlements dates, interpolate linearly between the two corresponding TBA prices, which gives the 30-day forward price P of the reference TBA settled on date T,

$$P = \frac{T_2 - T}{T_2 - T_1} P_1 + \frac{T - T_1}{T_2 - T_1} P_2 .$$

3) Recognize the 24-day payment delay for the FNMA 30-year TBAs and append to the forward price the accrued interest for the delay. The delay-adjusted price P_a for each TBA coupon is then given by

$$P_a = P + \frac{24}{360} C .$$

4) Identify the two consecutive coupons whose adjusted prices straddle the par price and interpolate linearly between the two coupons to arrive at the par coupon rate:

$$C^{par} = \frac{P_a^{prem} - par}{P_a^{prem} - P_a^{disc}} C^{disc} + \frac{par - P_a^{disc}}{P_a^{prem} - P_a^{disc}} C^{prem} ,$$

where the superscripts *prem* and *disc* represent TBAs whose prices are immediately above and below the par, respectively. The par rate C^{par} is considered a monthly compound rate since TBAs are instruments with monthly coupon payments.

5) Finally, convert the monthly par rate C^{par} to its semi-annual equivalent as below and define the transformed rate as the CMM index:

$$CMM = 2\left[\left(1 + \frac{C^{par}}{12}\right)^6 - 1\right].$$

This final conversion enables the CMM index to be compared conveniently with rates in other fixed-income markets that usually accrue on a semi-annual basis.

As described above, the CMM index is computed explicitly from TBA prices via a two-dimensional linear interpolation along both TBA settlements and coupon stacks. It appears a bit involved; nevertheless, the intention is clear and the steps are transparent. For illustration, Figure 21 shows a sample CMM index calculation according to these five steps.

Figure 21. CMM Index Calculation on May 3, 2005, with 30-Day Forward Reference Settlement on June 3, 2006, Bracketed by the PSA Settlements of May 12, 2005, and June 13, 2005; Highlighted Is the Actual Pair of Premium-Discount TBAs Identified in the Interpolation

Step	**1**			**2**	**3**	**4**	**5**
Variable	C	P_1	P_2	P	P_a	C^{par}	CMM
Date		5/12/2005	6/13/2005	6/3/2005	6/3/2005	5/3/2005	5/3/2005
CMM Index Interpolation	4.5	96.56	96.37	96.43	96.73		
	5.0	99.03	98.80	98.87	99.20	5.198%	5.254%
	5.5	101.00	100.78	100.85	101.22		
	6.0	102.69	102.47	102.54	102.94		
	6.5	104.02	103.88	103.92	104.35		

Source: Lehman Brothers.

The views expressed in this report accurately reflect the personal views of Jeremy Rosten, Anthony Gould, Arik Ben Dor, Vasanttilak Naik, Fei Zhou, Amitabh Arora, Bruce Phelps and Lev Dynkin, the primary analysts responsible for this report, about the subject securities or issuers referred to herein, and no part of such analysts' compensation was, is or will be directly or indirectly related to the specific recommendations or views expressed herein.

Any reports referenced herein published after 14 April 2003 have been certified in accordance with Regulation AC. To obtain copies of these reports and their certifications, please contact Valerie Monchi (vmonchi@lehman.com; 212-526-3173) or Larry Pindyck (lpindyck@lehman.com; 212-526-6268).

Lehman Brothers Inc. and any affiliate may have a position in the instruments or the Company discussed in this report. The Firm's interests may conflict with the interests of an investor in those instruments.

The research analysts responsible for preparing this report receive compensation based upon various factors, including, among other things, the quality of their work, firm revenues, including trading, competitive factors and client feedback.