16 June 2010

Portfolios Under Construction
Volatility = 1/N

Key summary
This research studies the dynamic relationship between quantitative factors and volatility. We find that quant factors have an inherent and significant volatility exposure and that controlling for this exposure can lower risk, increase factor performance and provide diversification.

Quantitative factors and the effects of volatility

Quantitative factors have significant exposure to volatility
In this report, we find that quantitative factors can have inherent and significant exposure to volatility. In addition, we find that this exposure can significantly impact their performance during episodes of investor de-risking and re-risking.

Neutralizing volatility exposure
We show a simple technique that can be used to mitigate the impact of volatility in any quant factor. We find that this technique will safe-guard factors from sudden changes in investor risk-aversion, improve factor performance over time, and provide better factor diversification. In addition, we show how this method can be used as a substitute to mean-variance optimization with surprising efficacy.

Avoiding the "value trap"
We find that during episodes of high uncertainty, value factors are more likely to fall into the infamous "value trap". We show a simple method that can be used to safe-guard against this phenomenon.

May 2010 and what to expect for June
We report on the impact that volatility had in May 2010 factor performance, and show how some quant factors are positioned for uncertainty in June 2010.

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Condensed risk factors

Introduction

Impetus

Certain factors do well during market volatility, while others perform poorly

Given the recent episodes of volatility and the drastic movements in the market, we feel it imperative that we discuss the topic of risk and the relationship between traditional quant factors and volatility. We feel this is of great importance because we have found certain quant factors do well during volatile market episodes, while others perform poorly. We find that some of our factors have changing exposure to volatility and not adjusting for this exposure can lead to poor performance when investors suddenly change their risk aversion.

Two new measures of volatility

We outline two new risk measures that take into account asset volatility as well as correlation

To better understand the sensitivity to risk of alpha factors, we outline two new measures of risk that take into account both the volatility of an asset as well as its correlation with other assets. Both of the measures condense the information in a risk model into one factor and therefore we will call them *condensed* risk factors. The first factor is a statistical artifact derived from a risk model. This factor is the factor that best condenses the variability from the risk model into one factor. The second factor is more intuitive and is constructed by accounting for the contribution to volatility of each asset to the universe. These factors are related in that they score stocks using both their individual volatilities as well as their correlations with the rest of the stocks in the universe. We find both insightful and practical uses for these factors.

Applications of condensed volatility factors

Using our new volatility factors, we are able to measure and track the exposure to volatility of each of our alpha factors

Using our new volatility factors, we are able to measure and more importantly track the exposure to volatility of each of our alpha factors. We can also use these new risk factors to accurately measure the amount of alpha and skill (IC) that is attributable to volatility in each alpha factor. For example, we will be able to discern whether the factor performance at a particular point in time was due more to the underlying factor itself or as a result of a spike or drop in risk-aversion. In essence, we will be able to ascertain whether or not the factor has any added-value above-and-beyond its exposure to volatility. We culminate this section by developing a simple and useful technique which can be used to control for much of the risk in a risk model using our new volatility factor. We find that this technique can be used as an alternative to optimization and generates factor portfolios that are more aligned with the underlying factors, while still controlling for some of the volatility implied by a risk model.

Scaling volatility exposure

We also devise a simple technique to scale the volatility correlation to protect value from the infamous “value trap”. We also show how this method protected the value factor from the July 2007 quant crises as well as the 2008 financial crises.

Volatility and equally weighted portfolios

In our last section, we unravel the mysterious title by discussing the link between one of our volatility factors and the equally weighted portfolio strategy. Specifically, we show that the equally weighted portfolio strategy (1/N) is aligned with our volatility factor. We discuss some of the ramifications of this finding and its impact on the debate between naïve versus mean-variance portfolio construction. We also find an analytical expression to show that the equally weighted portfolio strategy converges to the market portfolio as the number of assets (N) becomes large.

Volatility underperformance in the US

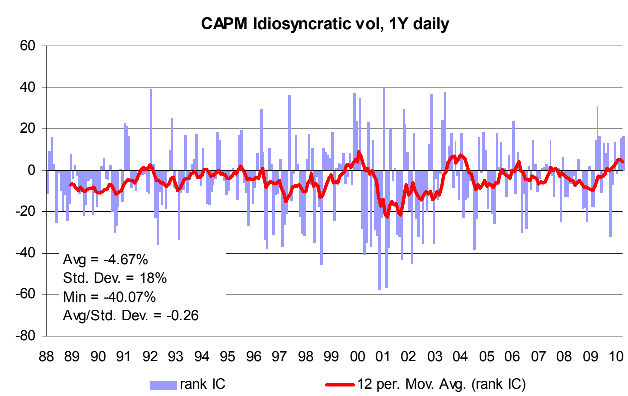
Volatility and other proxies for risk are negatively correlated with future returns

In the US, we find that stock volatility and risk, in general, is an attribute to steer away from. Our findings show that volatility and other proxies for risk are negatively correlated with future returns and are better used as defensive factors in a total alpha model. That is, we prefer our final alphas to have negative exposure to these factors. To see this, Figures 1-2 show the evolution of cross-sectional rank IC for two traditional volatility factors: CAPM idiosyncratic volatility and kurtosis. The idiosyncratic volatility factor scores assets according to their idiosyncratic volatility computed from one year daily returns. The kurtosis factor scores assets according to their one year daily return kurtosis. Note that the results in the graphs are based on taking negative exposure to each factor, which is consistent with our hypothesis that we prefer to negative exposure to assets with high volatility and positive exposure to assets with lower volatility. Figure 3, shows the statistics for all of our basic volatility measures.

These results contradict the notion that higher expected risk should produce higher realized returns

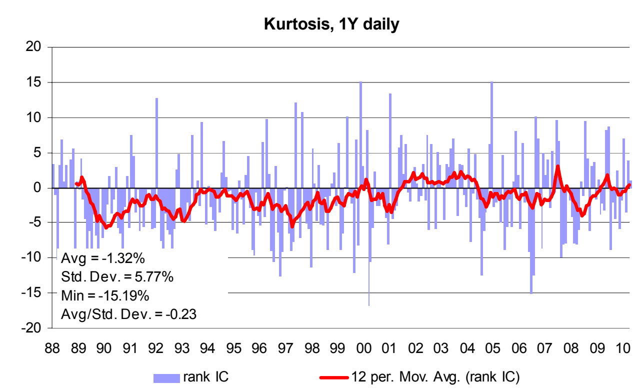
Looking to theory, we point out that these results contradict the notion that higher expected risk should produce higher realized returns. While these results may vary by market, the empirical evidence in the US equity market strongly supports our hypothesis that risk does not pay. Other studies have found similar results for international markets, see Ang *et. al* (2009). In addition, we can point to our second *Signal Processing* report, Cahan *et* al [2010], where we found that signals capturing shorter-term volatility implied by the options market to be strong negative predictors of return.

Figure 1: Idiosyncratic volatility rank IC



Source: Deutsche Bank

Figure 2: Kurtosis rank IC



Source: Deutsche Bank

Figure 3: Rank IC statistics for basic volatility related factors

Factor	IC Avg (%)	IC Std Dev (%)	IC Min (%)	IC Avg / IC Stdev
CAPM idosyncratic vol, 1Y daily	-4.67	18.00	-40.07	-0.26
CAPM beta, 5Y monthly	-0.59	15.91	-47.10	-0.04
Realized vol, 1Y daily	-4.58	18.52	-40.26	-0.25
Skewness, 1Y daily	-1.14	5.47	-14.21	-0.21
Kurtosis, 1Y daily	-1.32	5.77	-15.19	-0.23

Source: Compustat, IBES, Russell, S&P, Deutsche Bank

Two new volatility factors

We construct two new volatility factors that condense the volatility inherent in a multi-dimensional risk model into one factor

In this section, we construct two new volatility factors that condense the volatility inherent in a multi-dimensional risk model into one factor. Similar to the other risk measures in Figure 3, these factors tend to be negative predictors of returns. However, we will not treat them as alpha factors. Instead, we use them as risk factors and in this context they will serve several purposes. First, they will help us gain insight into the characteristics and attribution of our factors and portfolios to volatility. Second, they will serve as a tool that investors can use to control for volatility in factors and portfolios in a simple way. Third, in some instances they can serve to improve factors by modifying their composition to volatility.

Statistical risk factor (StatRisk)

Our first condensed volatility factor (StatRisk) is the loading on the first principal component

Our first condensed volatility factor is a statistical artifact that can be derived from any risk model. Specifically, given an asset-by-asset covariance matrix, we use the eigenvector corresponding to the first principal component of the matrix to score our stocks. This factor condenses the maximum amount of information in the covariance matrix into one factor. It uses the variance and correlation structure to arrive at a factor, which has the maximum expected volatility. In essence, it is the factor that will maximize our volatility exposure. In this report, we are using the Axioma US mid-horizon risk model, which uses a factor covariance matrix to capture the systematic portion of risk. Therefore, for computational ease and to minimize statistical error we compute our eigenvector from the factor covariance matrix and then scale up to the asset level using the factor portfolios implied by the risk model. Because of this construction technique, this factor will condense only the risk information which arises from common factors. We could correct for this by adding the risk model specific risk forecasts.

Contribution to risk factor (CT-Risk)

The second factor, CT-Risk, is the contribution to the variance of the asset to the entire universe

The second factor, the contribution to risk factor (CT-Risk), is constructed in a more intuitive way. The factor score for each asset is taken to be the contribution to the variance[1] of the asset to the entire universe. Therefore, the score of asset *i* for the CT-Risk factor is computed as follows:

$$score_i = \mathrm{var}(r_i) + \sum_{j \neq i} \mathrm{cov}(r_i, r_j) \tag{1}$$

The asset variance piece tends to be dominated by asset specific risk, while the covariance term is closely related to market beta

The first term of the asset score is the asset variance and the second term is the asset's covariance contribution to the rest of the universe. Therefore, all else equal, a stock with stronger positive correlation across the other assets will receive a higher score. In addition, we can use the two pieces of the factor separately. The asset variance piece tends to be dominated by asset specific risk, while the covariance term tends to be closely related to market beta. In the second section of the report, we show the latter result and find that increasing the number of assets in the universe makes this score more closely related to beta.

Both of these factors are risk-model specific

Finally, we want to point out that both of these factors are risk-model specific and can be constructed given any forecast of the variance-covariance matrix of asset returns. This measure could also be constructed across other volatility measures such as option implied stock volatility as well as option implied correlation. One advantage of this measure with respect to the StatRisk factor defined previously is that it allows us to add higher moments to the scores. For example, we could have added skewness, co-skewness and kurtosis to the scores in equation (1), which would attempt to account for any non-normal return qualities of the multivariate asset return distribution.

[1] The scores also aligned (up to a constant) with contribution to volatility of the assets to then entire universe.

Also, we note that as we do with all of our factors, we normalize them so that they are on equal footing with each other. In addition, the normalization process implicitly neutralizes some of the systematic risks by giving positive scores to assets in the top half of the scores and negative scores to the assets in the bottom half. Therefore, after normalization, the CT-Risk factor can be interpreted as a portfolio that is long stocks with high beta and high specific risk, while short stocks with low beta and low specific risk – i.e. the strategy takes exposure to high systematic risk as well as high idiosyncratic risk.

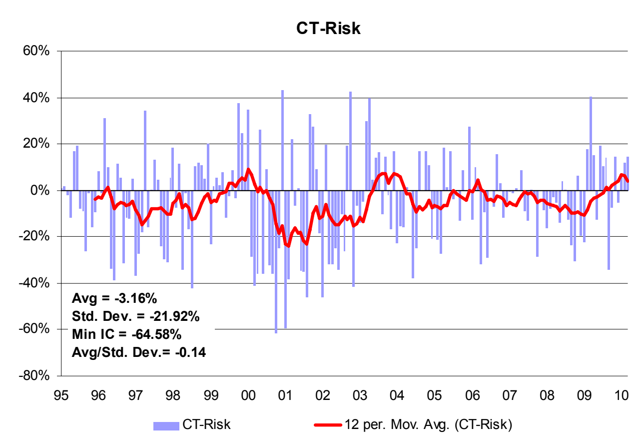
Factor properties

CT-Risk and StatRisk are decent predictors of negative return, but are highly correlated, especially as the number of assets increases. They have a negative Beta to risk aversion.

What are the properties of these factors? Can they be used as alpha factors to predict return? Are they closely related to each other and how do they compare with more naïve measures of volatility such as our idiosyncratic volatility and CAPM-beta factors? To answer the first question we point to Figure 4 and Figure 5, which show the factor level rank IC's for our two condensed volatility factors, CT-Risk and StatRisk. We note two observations. First, we find that they are decent predictors of negative return. This is expected from our earlier results and the argument that most measures of risk can serve as negative predictors of future returns. However, because these factors arise from the risk model, we expect any optimization procedure that uses a similar risk model to neutralize much of their return predictability. Second, we note that these factors have very similar performance over time. In Figure 7 we show that these factors are indeed highly correlated. In the second section of the report, we show that this is due to the large number of assets in our universe, which is roughly the Russell 3000. There we show that as the number of assets increase, these factors become more closely related. Third, note that these factors tend to outperform when investors' appetite for risk increases (1999-2000, 2003-2004 and early 2009). Therefore, we are confident that they can identify and will capture episodes of rapid changes in risk-aversion.

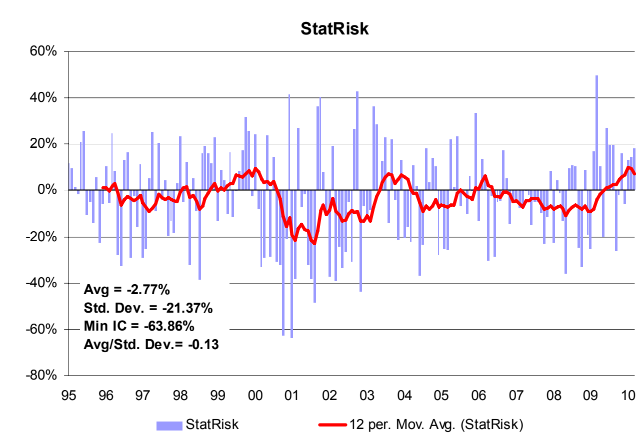
Figure 6 summarizes the IC stats for these two factors as well as the different components of the CT-Risk factor, AssetRisk and CT-Cov. In the table in Figure 6 we added a third factor, Specific-Risk, directly from the risk model. This is just the vector of specific variance of each asset. This factor is related to our idiosyncratic volatility factor in that it tries to capture the risk of each asset that is not related to systematic factors.

Figure 4: CT-Risk volatility factor rank IC (January 1995 – April 2010)



Source: Axioma, Compustat, IBES, Russell, S&P, Deutsche Bank

Figure 5: Statistical volatility factor rank IC (January 1995 – April 2010)



Source: Axioma, Compustat, IBES, Russell, S&P, Deutsche Bank

Figure 6: Rank IC statistics for condensed risk factors

Factor	IC Avg (%)	IC Std Dev (%)	IC Min (%)	IC Avg / IC Stdev
Stat-Risk	-2.73%	21.32%	-63.86%	-0.13
CT-Risk	-3.12%	21.86%	-64.58%	-0.14
Asset variance	-4.69%	20.83%	-61.77%	-0.22
Predicated Beta	-2.64%	20.06%	-65.85%	-0.13
Specific Risk	-4.78%	19.17%	-57.34%	-0.25

Source: Axioma, Compustat, IBES, Russell, S&P, Deutsche Bank

Another question to address is whether these factors have any information above-and-beyond our basic volatility measures. Figure 7 shows the average cross-sectional rank correlation over time between our condensed risk-model related factors (Stat-Risk, CT-Risk and Axioma specific risk forecast) along with our basic risk related alpha factors (CAPM beta, idiosyncratic volatility, realized volatility, skewness and kurtosis). We note a couple of quick things. First, the significant correlations indicate that there is a level of consistency across these factors. We expected this given that, in essence, they are all out to capture risk. Secondly, we find that our two condensed factors, Stat-Risk and CT-Risk are basically capturing the same information. Therefore, in the proceeding analysis we focus on the CT-Risk factor, which is the more intuitive of the two. In addition, this factor is easier to construct and as we stated above, can be easily augmented to capture other risks not captured by the risk model.

Figure 7: Average cross-sectional rank correlation (Jan 1995 – April 2010)

Factor	Stat-Risk	CT-Risk	Specific Risk[2]	CAPM Beta 5Y	Idiosyncratic Volatility 1Y	Realized Volatility 1Y	Skewness 1Y	Kurtosis 1Y
Stat-Risk	--	99%	62%	46%	62%	68%	8%	4%
CT-Risk		--	67%	46%	67%	72%	9%	6%
Specific Risk[2]			--	34%	90%	87%	15%	33%
CAPM Beta 5Y				--	36%	40%	4%	4%
Idiosyncratic Volatility (1Y)					--	98%	17%	35%
Realized Volatility (1Y)						--	18%	32%
Skewness (1Y)							--	16%
Kurtosis (1Y)								--

Source: Axioma, Compustat, IBES, Russell, S&P, Deutsche Bank

Monitoring factor alignment and correlation to volatility

One question that arises frequently when analyzing factor characteristics is how much volatility alignment or exposure is inherent in a factor. This is important for many reasons. First, it provides an indication as to whether or not a factor will be hurt by sudden changes in risk-aversion. For example, a factor with positive exposure to volatility will tend to underperform when risk-aversion suddenly rises and investors sell off stocks that are heavy contributors to risk. Conversely, when risk appetite increases a factor that is positively aligned or exposed to volatility will tend to outperform, while factors negatively aligned with volatility tend to underperform. The analysis presented in this section shows a few ways to capture the relationship between any factor and any volatility proxy.

[2] Z-scores based on Axioma US Medium Horizon model specific risk forecasts.

Value and quality have long-term negative volatility exposure, while momentum's is a highly cyclical pattern

In particular we find that value and quality have a long-term negative exposure to volatility, while momentum experiences a highly cyclical pattern of correlation with volatility. In addition, we find that correlations between volatility and value have changed significantly since the quant crises in July of 2007, but have started to revert to their longer-term value.

A short note on measuring correlation between factors

To quantify the relationship between any two factors we measure their correlation. Correlation can be measured across a time-series of returns or at the cross-sectional level. The time-series measure works well for looking back and analyzing what happened. However, a major problem with the time-series measure is that it is heavily dependent on past returns and does not adjust quickly to capture dynamic changes in correlation. The cross-sectional correlation is an instantaneous measure that allows the investor to *monitor* the correlation between factors in real-time. I.e. cross-sectional correlation can answer the question: what is my volatility exposure right now? In the end, cross-sectional correlation and time-series correlation should be consistent when compared over time.

We can measure cross-sectional correlation along factor alignment, factor-score correlation, and factor-portfolio correlation

In this report we are concerned with monitoring volatility exposure so we focus our analysis on cross-sectional correlation. We can measure cross-sectional correlation along three different dimensions: factor alignment, factor-score correlation, and factor-portfolio correlation.

factor alignment captures how closely assets are aligned by factor scores

factor-score correlation is the correlation between the raw factor-scores implied by the variance-covariance matrix

MVO factor-portfolio correlation, is the correlation of the optimized factor-portfolios implied by the variance-covariance matrix of asset returns

In the appendix, we show how to compute each measure and explain some of the differences between them. Here we only provide a short summary of each measure. The first measure, *factor alignment* is computed using the Spearman rank correlation across the two factor scores. This captures how closely the assets are aligned by the two series of factor scores. Typically, this measure gives us a lower bound on the magnitude of future return correlation because it does not account for common factor drivers of correlation (see appendix for more details). The second measure, *factor-score* correlation, is the correlation between the raw factor-scores that is implied by the variance-covariance matrix of returns. The third measure, *MVO factor-portfolio* correlation, is the correlation of the optimized factor-portfolios implied by the variance-covariance matrix of asset returns. The first two measures are closely related given that the correlation is measured across the factor scores prior to optimization. We find that the rank correlation (factor alignment) and the factor-score measures of correlation are consistent in the direction of the correlation. However, as we hinted above, the factor-score correlations tend to be larger in magnitude – lower negative and higher positive values than what we find with rank correlations. When we look at a time-series analysis of returns and IC, we find that both measures are biased and the right answer is somewhere in middle. For consistency we will report most of the following results in rank correlation space and we can provide the summary results for the factor-score correlations by demand. The purpose of the rest of this analysis is to showcase the techniques and note that much of our analysis is consistent across both measures. In the section on neutralization we compare the factor level correlation measures with the MVO factor-portfolio correlation and report on the differences.

Correlation dynamics

We look at the level and dynamics of cross-sectional correlation between alpha factors and CT-Risk

In the following we look at the level and dynamics of cross-sectional correlation between a few basic alpha factors and our condensed volatility factor CT-Risk. This will give us insight into the volatility exposure of alpha factors on two levels:

1. Current volatility exposure. This is important so that we can monitor the volatility exposure of a factor or portfolio. It will provide us with a guide for what to expect if risk-aversion suddenly changes.
2. Natural longer-term volatility exposure and deviations from this natural average. Changes in correlation to volatility may provide important insight into the composition of the factor and its natural relationship with volatility.

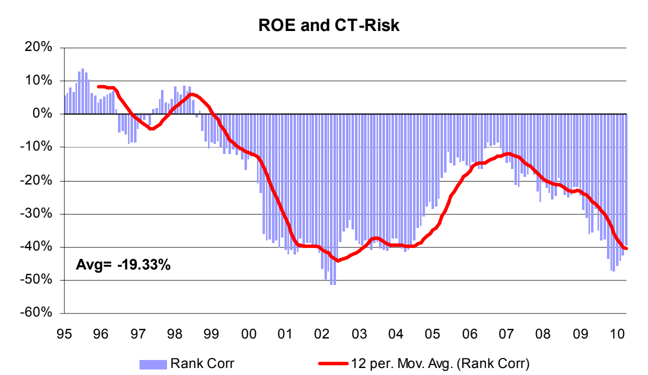
Lastly, we can also view the correlation results from a volatility perspective, which can provide interesting insight into investor risk behavior. For example, if our volatility factor is positively correlated with momentum then we can suggest that investors are bullish risk, while a negative correlation implies that investors are bearish risk. In the case of value, we can get a sense of how cheap or expensive investors are pricing risk.

Quality

Quality factors tend to have negative exposure to volatility given they are considered "safer"

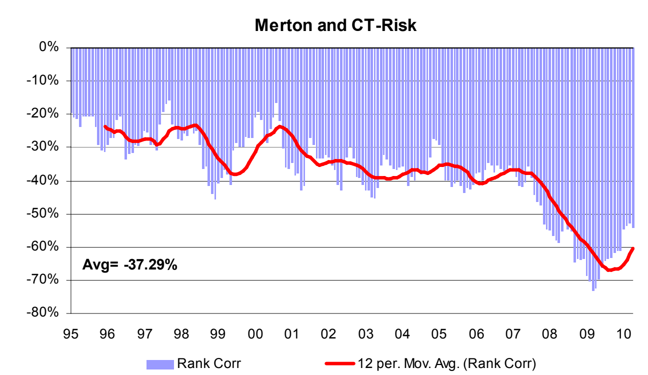
To start things off, we analyze the correlation between our CT-Risk factor and two factors from the *quality* theme. Specifically we look at our ROE and Merton factors. The ROE factor ranks stocks according to their return to equity, while the Merton default factor, scores stocks by their distance to default. As the name implies these measures tend to give higher scores to safer stocks. Therefore, we expect both factors to be negatively correlated to volatility. In general, quality factors tend to have negative exposure to volatility given that quality assets are considered "safer" and therefore we expect them to have less risk relative to other assets. Figure 8 and Figure 9 show the cross-sectional Spearman rank correlation of the CT-Risk factor with our ROE and Merton factors across time. Notice that with exception to the earlier period for ROE, they are consistently negatively correlated to volatility. This negative correlation implies that we should expect these factors to perform well when risk-aversion spikes upwards (flight to quality) and underperform when investors become less risk-adverse. Another observation to note is that both factors have taken on more negative volatility exposure since 2007, which helps explain why these factors did so well during the crises in the fall 2008 and were hurt by the subsequent drop in risk-aversion that started in March 2009. We also note that the Merton factor has become more negatively correlated with volatility than at any time since 1995. This observation tells us two things. First, that stocks which are less likely to default have now shifted to a lower level of volatility than we have observed historically and vice-versa. Second this factor is more susceptible than at any point in its history to a sudden increase in risk appetite. In a later section, we demonstrate a technique that can be used to mitigate underperformance due to a sudden increase in risk appetite.

Figure 8: Rank correlation: ROE and CT-Risk



Source: Axioma, Compustat, IBES, Russell, S&P,Deutsche Bank

Figure 9: Rank correlation: Merton and CT-Risk



Source: Axioma, Compustat, IBES, Russell, S&P, Deutsche Bank

Both value factors are currently negatively correlated with volatility and experienced a significant increase in correlation with volatility starting in the middle of 2007 (quant crises) to the end of 2008

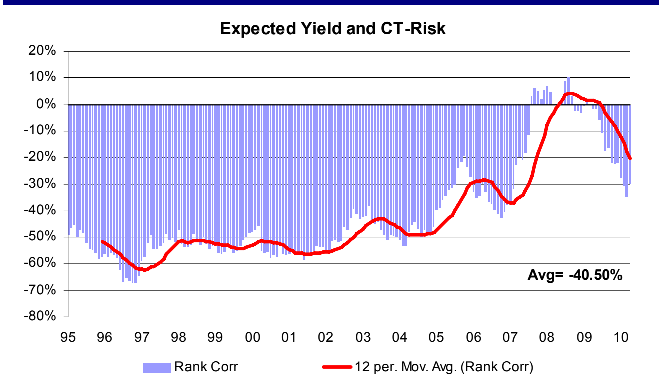
Value

To get a sense of the relationship between value and volatility we perform the correlation analysis on two basic factors from our value bucket: expected dividend yield and FY1 earnings yield. Figure 10 and Figure 11 show the rank correlation across time of the CT-Risk factor with our expected dividend yield and FY1 earnings yield factors. We note a few things with respect to these graphs. First, we find that both factors are currently negatively correlated with volatility going into May 2010. Therefore, all else equal, a sudden rise in risk aversion (as we saw in May 2010) should generate positive returns for value. Second, both graphs tell us that these factors tend to have a long-term negative correlation with volatility. Third, we find that both factors experienced a significant increase in correlation with volatility starting in the middle of 2007 (quant crises) to the end of 2008. This change in trend seems very interesting given that we know that both of these factors have lost much of their predictive power from 2007 to the beginning of 2009. Also we note that the increase in correlation during the crises in fall 2008 is most likely due to distressed stocks that became very cheap as investors fled vigorously towards safety.

Value does better when it is more negatively correlated with volatility

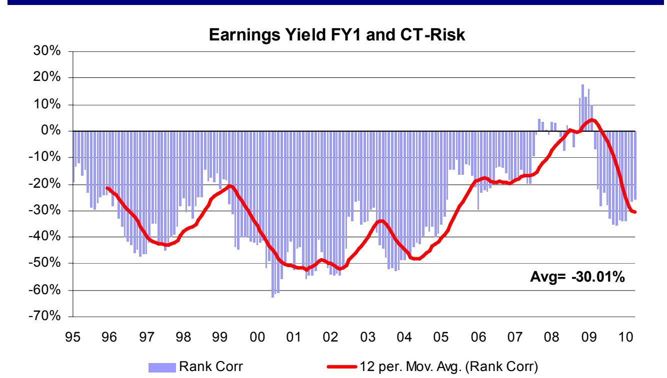
Another way to interpret this analysis is to look at it from a volatility perspective. From this angle the correlation analysis can indicate how cheap or expensive risk is being priced by investors. For example in Figure 11, the decrease in correlation from the beginning of 1999 to the middle of 2000 suggests that assets with less contribution to volatility became cheaper, while assets with higher contribution to volatility became more expensive; i.e. risk became more expensive, while safety became cheaper. This is in line with narrative that investors bought up risky technology and internet stocks, while selling off stocks with less risk. In the sections below, we will elaborate on the relationship between value and volatility. We find that value does better when it is more negatively correlated with volatility (safe value) and we show how we can improve value by aligning it closer to its longer-term volatility exposure.

Figure 10: Rank correlation: Exp Dividend yld and CT-Risk



Source: Axioma, Compustat, IBES, Russell, S&P, Deutsche Bank

Figure 11: Rank correlation: FY1 Earn yld and CT-Risk



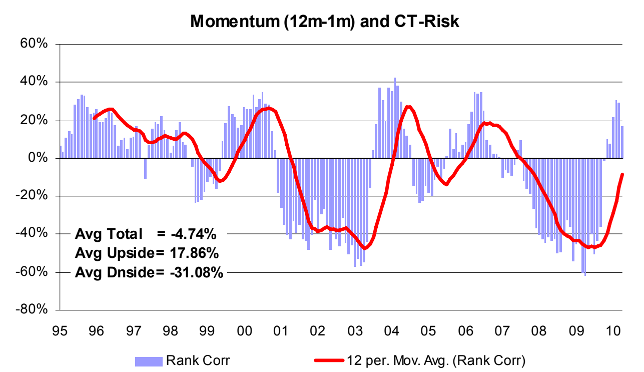
Source: Axioma, Compustat, IBES, Russell, S&P, Deutsche Bank

All else equal, momentum will chase investor risk preference

Momentum and the lottery factor

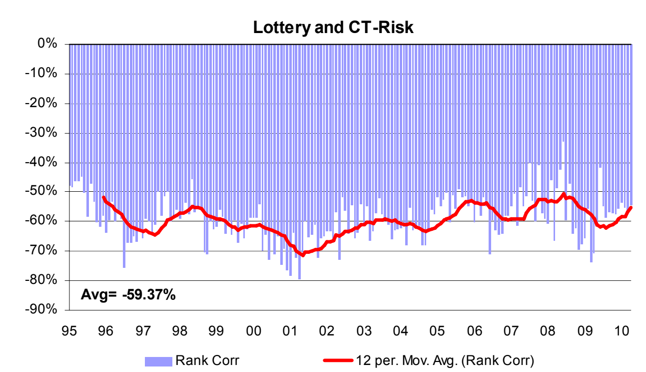
Figure 12 shows the correlation analysis between our traditional price return momentum (12m-1m) factor and the CT-Risk volatility factor. As we can see in Figure 12, the correlation between momentum and volatility has a different behavior to what we found for value and quality. First, we find that the correlation between momentum and volatility oscillates dramatically. There are periods when the correlation is significantly positive and periods when it is significantly negative. This can be explained by noting that all else equal, momentum will chase investor risk preference. That is, when investors are risk averse and trade off stocks with high contribution to risk for stocks with lower contribution to risk, momentum begins to load up on negative volatility. Conversely, when investors are bullish risk then all else equal, momentum will chase higher volatility contributing stocks. To see this note that during the technology bubble (1999 to early 2000) momentum was positively correlated with volatility as investors piled into risk. Subsequently, as the technology bubble bursts and investors de-risk, the correlation between momentum and volatility drops and subsequently remains negative during the bear market of 2001-2003. Interestingly, we see the same behavior between momentum and volatility starting in early 2007 as investors began de-risking during the quant crises in the summer of 2007 marching into to the full-blown market crises of 2008. Another interesting observation is that leading into 2008 we find that the correlation between momentum and volatility were at levels last seen during the bear market of 2001-2002.

Figure 12: Rank correlation: Momentum and CT-Risk



Source: Axioma, Compustat, IBES, Russell, S&P,Deutsche Bank

Figure 13: Rank correlation: Lottery and CT-Risk



Source: Axioma, Compustat, IBES, Russell, S&P, Deutsche Bank

The lottery factor has a strong and consistent negative correlation with volatility

Last, but not least, Figure 13 shows the evolution of the correlation between our lottery factor and the CT-Risk factor. The lottery factor, ranks stocks in negative relation to their highest one-day return over the prior month. The idea is to take negative exposure to stocks with very high and sudden upswings in price returns. The expectation is that these quick run-up stocks will reverse because investors tend to chase winners especially those that have recent extraordinary returns. The reason we chose to analyze this factor is because it is one of the best performing in our factor library. Secondly, and as is quite obvious from looking at Figure 13, this factor has a strong and consistent negative correlation with volatility. This strong correlation is intuitive given that, all else equal, stocks with the highest 1-day return over a given month tend to be stocks with higher volatility.

Factor skill (IC) and risk attribution to volatility

We investigate how much skill can be attributed to factor exposure to volatility

In the last section, we found that certain factors may have a significant and at times inherent correlation to volatility. In this section, we use a simple technique to investigate how much of a factor's performance or skill (IC) can be attributed to its exposure to volatility. This analysis will help the investor answer the question: Did my factor do well because of an inherent tilt towards or against volatility?

To attribute factor IC to our CT-Risk factor we use the cross-sectional correlation between the two factors and the following decomposition formula[3]:

[3] This decomposition can be obtained as a result of Euler's decomposition of positive homogeneous functions. A more general version can be found explicitly in Grinold [2006]. It can also be derived as a variant of Litterman's [1996] decomposition of volatility.

$$IC_{factor} = \rho_{factor,vol} \times IC_{vol} + IC_{res} \tag{2}$$

where IC_{factor} is the skill of the alpha factor, IC_{vol} is the skill of the volatility proxy (CT-Risk), $\rho_{factor,vol}$ is the cross-sectional correlation between the alpha factor and our CT-Risk, and IC_{res} is the factor skill that is residual to the skill implied by the factor's volatility exposure. What is nice about this attribution technique is that it can be applied cross-sectionally at every point in time. This cross-sectional technique allows for a clearer attribution because we are not prisoner to stability of the correlation across time.

In the following analysis, we use formula (2) to attribute the IC for both the FY1 earnings yield factor and the lottery factor[4].

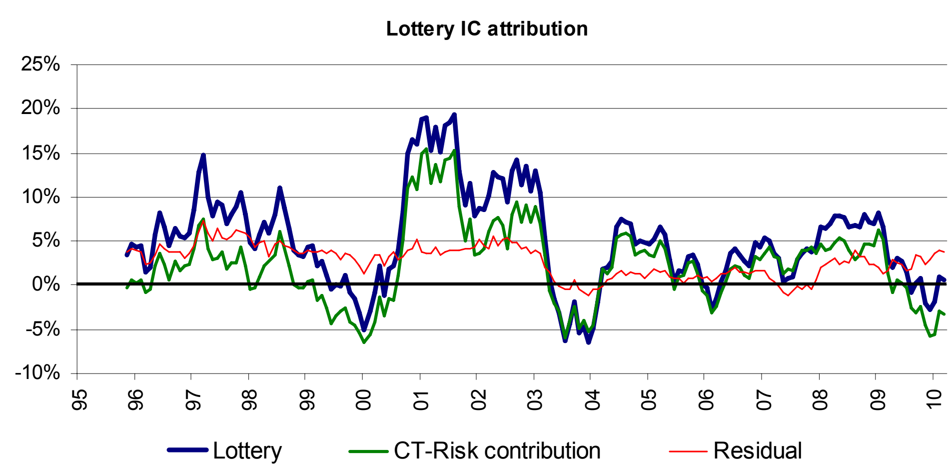
Lottery factor

We ask whether the lottery factor has any skill (IC) above and beyond its exposure to volatility

We first use the attribution methodology to analyze the lottery factor. We begin with this factor in particular, because we found that it has a very strong and consistent negative correlation to our volatility factor over time (see Figure 13). Knowing that negative exposure to volatility can produce superior returns, we ask whether the lottery factor has any skill (IC) above and beyond its exposure to volatility. Figure 14 shows the attribution analysis for the IC lottery factor. The blue line is the 12-month rolling average of the cross-sectional IC of the actual factor. The green line denotes the portion that is attributable to CT-Risk factor – the portion of the IC that is due to the lottery factor's exposure to volatility. The red line represents the residual IC of the lottery factor after accounting for its exposure to the CT-Risk factor – the portion that is orthogonal to our CT-Risk factor. We note that the portion of IC arising from the CT-Risk factor is very strong and on the surface seems to account for a major portion of the IC performance in the lottery factor. However, if we look a little closer at the residual portion, we find that it has some very appealing properties. First, it has higher consistent positive performance and shows greater stability than the original factor. Second, the residual portion avoids the dips in performance experienced by the original factor during periods of re-risking. Notice how the residual portion holds up after 2009 when investors increased their risk appetite. This last observation is important given the recent and significant fluctuations in investor risk-aversion.

The residual portion has some very appealing properties. First, it has higher consistent positive performance and shows greater stability than the original factor

Figure 14: Lottery factor IC attribution to CT-Risk (12m rolling ave)



Source: Axioma, Compustat, IBES, Russell, S&P, Deutsche Bank

[4] A summary of the attribution for other basic quant factors in our library can be provided by demand.

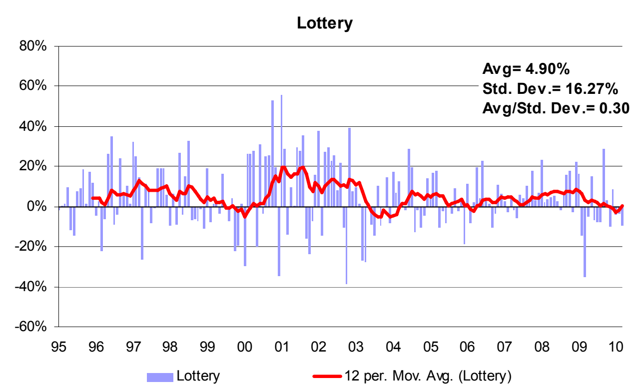
The residual portion has nearly double the risk-adjusted performance of the original factor

To more rigorously compare the residual portion to the original factor, Figure 15 and Figure 16 show the total factor IC and the residual IC from CT-Risk over time. While the residual portion is lower in magnitude to the original factor, the true test is whether the IR of the residual portion is greater than that of the original factor. Qian, Hua, and Sorensen [2007] show that IR can be closely approximated by the ratio of the IC and its dispersion std(IC). Using this measure to compare IR, we find that the residual portion has nearly double the risk-adjusted performance of the original factor. This tells us two things:

1. The lottery factor adds risk-adjusted value above and beyond its volatility exposure
2. Controlling for volatility improves the factor's risk-adjusted performance

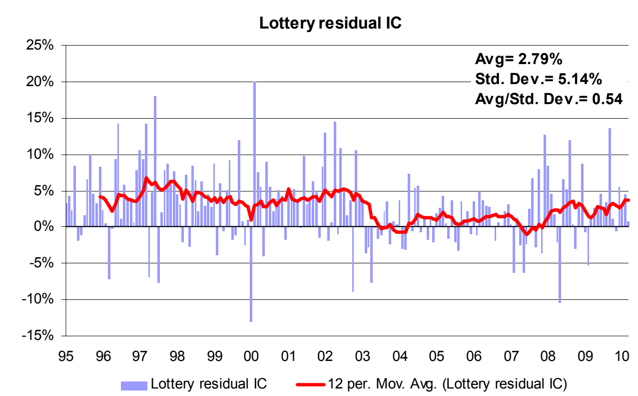
Finally, we show the complete IC statistics for the three components in the attribution analysis in Figure 17.

Figure 15: Lottery factor IC



Source: Axioma, Compustat, IBES, Russell, S&P, Deutsche Bank

Figure 16: IC of Lottery factor residual to CT-Risk



Source: Axioma, Compustat, IBES, Russell, S&P, Deutsche Bank

Figure 17: Lottery factor attribution summary (Jan 1995 – April 2010)

Component	Average IC	Std Dev of IC	Min IC	Avg/Std ~ IR
Lottery factor (total)	4.82%	16.26%	-38.55%	0.30
CT-Risk	2.03%	13.91%	-36.52%	0.15
Residual	2.79%	5.14%	-12.98%	0.54

Source: Axioma, Compustat, IBES, Russell, S&P, Deutsche Bank

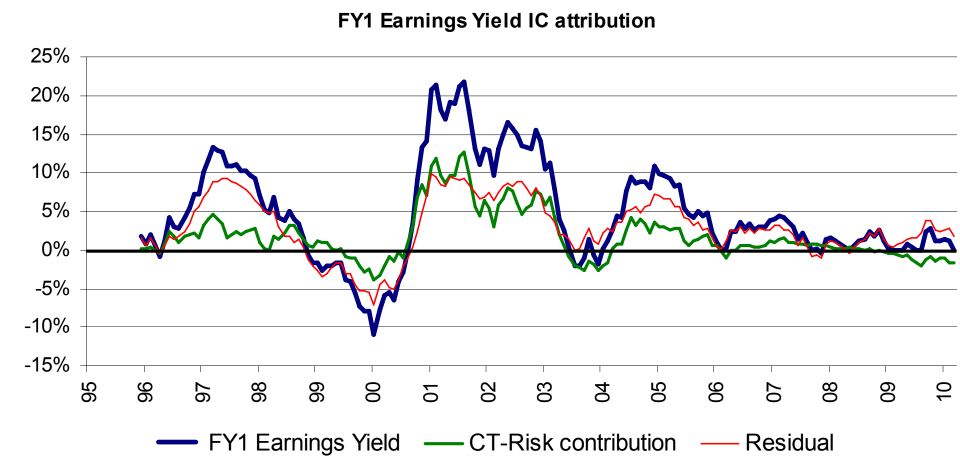
There is potential to improve factors by neutralizing their volatility exposure

The analysis above hints that there is potential to improve factors by neutralizing their volatility exposure as measured by the correlation to our CT-Risk factor. In the following section "Neutralizing volatility exposure", we show a simple technique to achieve this and show the differences between this technique and full mean-variance optimization.

Value

We now turn our attribution methodology to analyze the FY1 earnings yield factor. In Figure 11 we saw that there are periods when this factor has a significant negative correlation with the CT-Risk factor. Therefore, as we did with the lottery factor above, we can use the IC attribution equation in (2) to identify the portion of skill in our value factor that we can attribute to volatility and a residual component that is orthogonal to our volatility factor. Similarly to the lottery factor we find that the CT-Risk factor component is significant and accounts for roughly half of the IC of the value factor across time. However, we can also see that the FY1 earnings yield factor does seem to have skill above and beyond its exposure to volatility. To further investigate this observation, we again look at both the IC of the original factor and that of the residual portion to discriminate any added value in risk-adjusted performance.

Figure 18: FY1 Earnings yield IC attribution to CT-Risk (12-month rolling ave.)



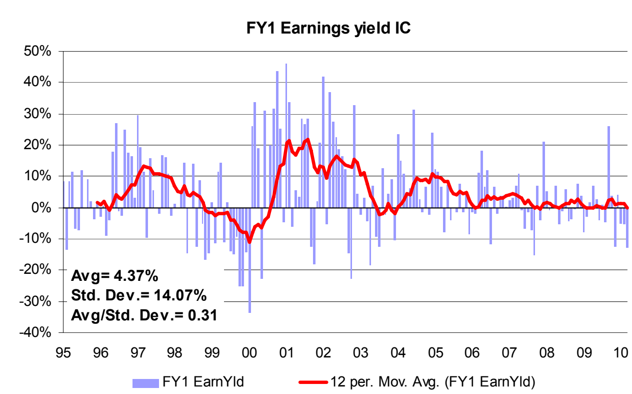
Source: Axioma, Compustat, IBES, Russell, S&P, Deutsche Bank

As opposed to the lottery factor, we do not observe a significant increase in risk-adjusted performance in the residual

This result suggests that value may work best when negatively correlated with volatility

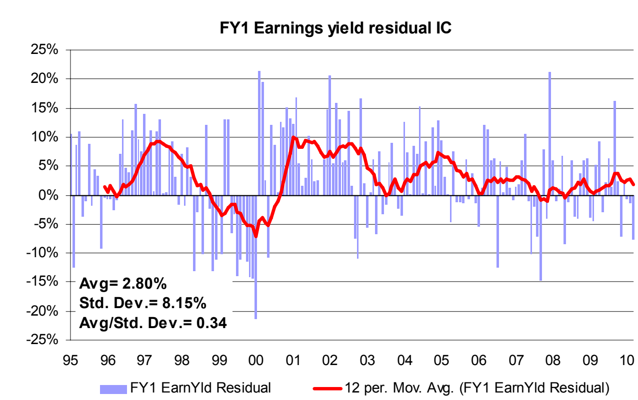
As opposed to the lottery factor, we do not observe a significant increase in risk-adjusted performance in the residual portion. However, we do note that the residual performance improves subsequent to 2006 around the time when value begins to lose its historical level of skill. In Figure 21, we summarize the performance of each component in the attribution analysis from January 2006 to April 2010. Note that performance does improve slightly around this time, but an interesting fact is that the portion of IC attributable to volatility through our CT-Risk factor is actually negative. This seems to be out of line with the consistent negative correlation between value and quality and the underperformance of volatility over this period. However as we saw in Figure 11 this is consistent with the upswing in correlation between value and volatility after summer 2007 and the persistent negative performance of our CT-Risk factor. This result suggests that value may work best when it is negatively correlated with volatility. We will expand on this in a later section.

Figure 19: FY1 Earn yld IC (Jan 1995 – Apr 2010)



Source: Axioma, Compustat, IBES, Russell, S&P, Deutsche Bank

Figure 20: FY1 Earn yld residual IC (Jan 1995 – Apr 2010)



Source: Axioma, Compustat, IBES, Russell, S&P, Deutsche Bank

Figure 21: FY1 Earnings yield factor attribution summary (Jan 2006 – April 2010)

Component	Average IC	Std Dev of IC	Min IC	Avg/Std ~ IR
FY1 Earnings yield factor (total)	1.29%	8.11%	-15.23%	0.16
CT-Risk	-0.19%	3.26%	-6.29%	-0.06
Residual	1.47%	6.99%	-14.71%	0.21

Source: Axioma, Compustat, IBES, Russell, S&P, Deutsche Ban

Neutralizing volatility exposure

The analysis in our last section was informative, but can we use that insight to improve our factors? The answer is yes. To do so we implement a technique called *neutralization*, which as among others has the following benefits:

1) improving factor performance
2) neutralizing exposure and correlation to volatility
3) factor diversification – less correlated factors

We show a technique which neutralizes the exposure to any attribute from a factor

In this section we show how to extract the factor scores that correspond to the residual performance observed in Figure 16 and Figure 20. To do this we use a technique called neutralization[5]. As the name implies, this technique neutralizes or removes the exposure to any attribute (such as our CT-Risk factor scores) from a factor. As we will see later, this technique can also be applied to portfolios.

The math

The general technique (see appendix) can be used to neutralize a factor against many different attributes. However, in this analysis we only use it to neutralize against our CT-Risk factor. In the one-factor case the neutralization technique takes on a very intuitive and simple form[6]:

$$f_{neut,i} = f_i - \left(\rho_{f,risk}\frac{\sigma_f}{\sigma_{risk}}\right) f_{risk,i} \tag{3}$$

where f_i corresponds to the original factor score of asset *i*, $f_{risk,i}$ is the risk factor (CT-Risk) score for asset *i*, $f_{neut,i}$ is the neutralized factor score for asset *i*, $\rho_{f,risk}$ is the correlation forecast between the original factor and the CT-risk factor, and σ_f and σ_{risk} are the volatility forecast of the original factor and the CT-Risk factor, respectively. It is quite easy to show from equation (3) that the IC arising from the neutralized factor f_{neut}, is equal to the residual portion of the IC in equation (2). In the appendix, we describe how to use the asset-by-asset covariance matrix to compute the correlation and volatility forecasts. In the final part of this section, we show how to use a vector we call the *risk-cancellation* factor to easily neutralize a factor and extract the residual factor scores.

Improving factor performance

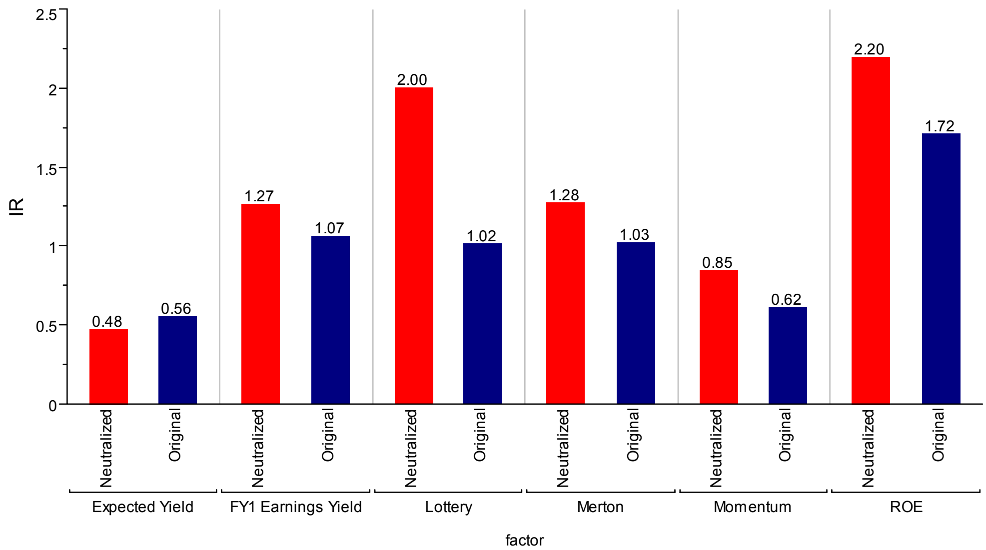
In Figure 16 we saw that the residual IC of the lottery factor showed better risk-adjusted performance than the original factor. As noted above, we can capture this residual IC using the neutralized factor from equation (3). The next question is whether neutralizing volatility (CT-Risk) exposure improves the risk-adjusted performance of our other factors. To investigate this, we apply the neutralization scheme to all of our factors and compare the risk-

[5] This technique is a variation of a set of exercises in the technical appendix of Chapter 14 in Grinold and Kahn [2000].

[6] Note that in the one-factor case, the formula can be manipulated to take on a similar expression to a CAPM model, except with the CT-Risk factor as the market factor. However, note that CAPM is usually presented in a returns-based framework involving returns. The formula in equation (3) is cross-sectional and involves asset scores.

adjusted performance of the original factor versus the neutralized factor. Figure 22 shows the approximated IR results for the six factors we have studied thus far[7].

Figure 22: Original factor vs. neutralized factor IR (Jan 1995 – Apr 2010) (IR is approximated by ratio of average IC to standard deviation of IC)



Source: Axioma, Compustat, IBES, Russell, S&P, Deutsche Bank

Neutralizing volatility exposure and correlation

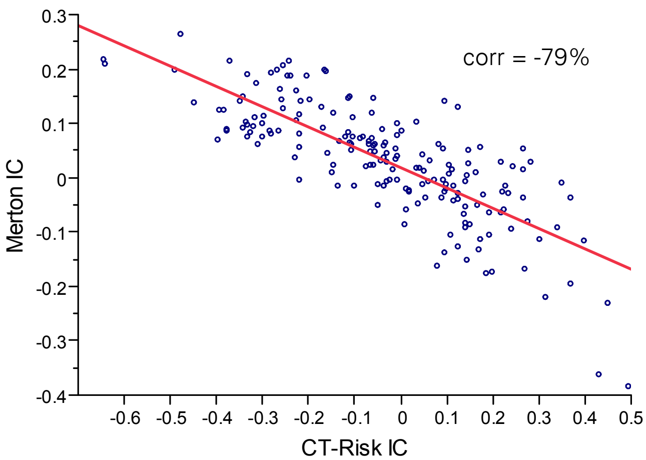
We investigate if this cross-sectional correction actually works across the time-series performance of the factor

Does this simple algebraic technique actually work to hedge or mitigate the impact of volatility and sudden changes in risk aversion in factor performance? Note that the neutralization scheme outlined in equation (3) is performed cross-sectionally at every point in time. Therefore, to be thorough, we must investigate if this cross-sectional correction actually works across the time-series performance of the factor. In essence, we must check if our point-in-time correction works out-of-sample over time. To investigate, we look at the realized time-series IC's of some of the factors studied above before and after neutralization to our CT-Risk factor[8]. For example, Figure 23 shows a scatter plot of the original Merton factor IC against our CT-Risk factor IC. Note the strong negative correlation of -79%, which indicates that the Merton factor will do well during periods of de-risking and will underperform when risk appetite increases. To check if the neutralization scheme worked we look at Figure 24, which shows the scatter plot and correlation between the IC's of the neutralized Merton factor and the CT-Risk over our test period. We note that the correlation is now negligible, which demonstrates that the neutralization technique worked effectively. This result together with those in Figure 22, suggest that we can improve risk-adjusted performance and eliminate exposure to changes in investor risk-aversion simultaneously. Figure 25 through Figure 30 show similar results for our FY1 earnings yield, lottery and momentum factors.

[7] Results for all of our other basic quant factors can be provided on demand.

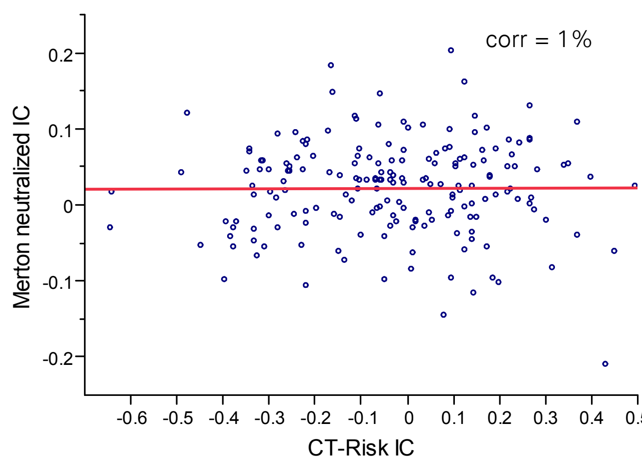
[8] Results of the correlation of realized returns of naïve portfolios built around these factors can be provided on demand.

Figure 23: Merton IC vs. CT-Risk IC



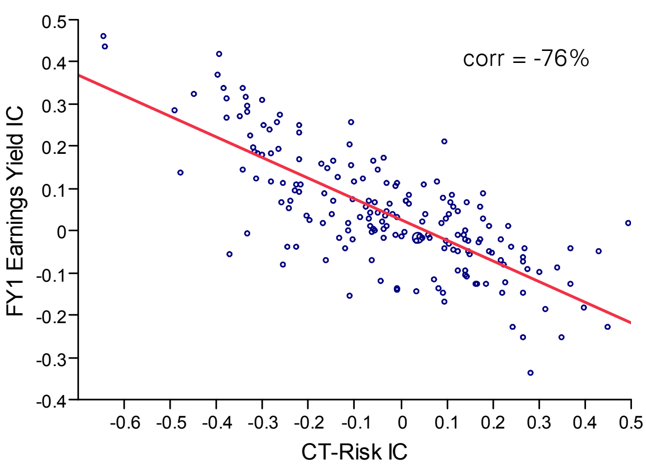
Source: Axioma, Compustat, IBES, Russell, S&P, Deutsche Bank

Figure 24: Merton neutralized IC vs. CT-Risk IC



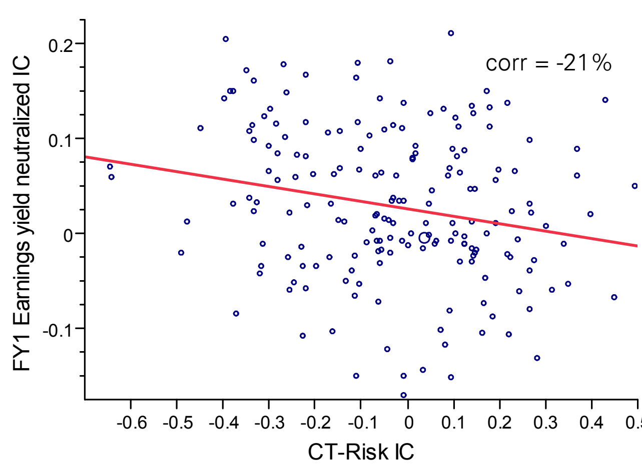
Source: Axioma, Compustat, IBES, Russell, S&P, Deutsche Bank

Figure 25: FY1 Earnings yield IC vs. CT-Risk IC



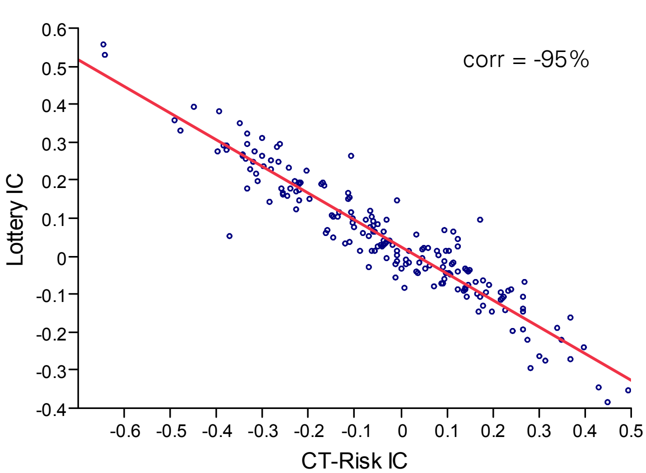
Source: Axioma, Compustat, IBES, Russell, S&P, Deutsche Bank

Figure 26: FY1 Earnings yld neutralized IC vs. CT-Risk IC



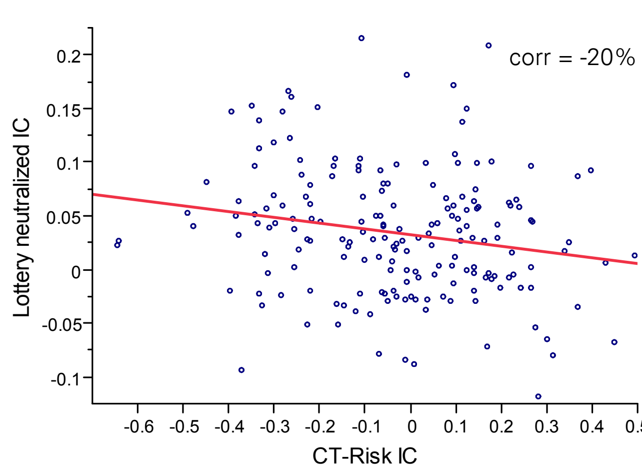
Source: Axioma, Compustat, IBES, Russell, S&P, Deutsche Bank

Figure 27: Lottery IC vs. CT-Risk IC



Source: Axioma, Compustat, IBES, Russell, S&P, Deutsche Bank

Figure 28: Lottery residual IC vs. CT-Risk IC

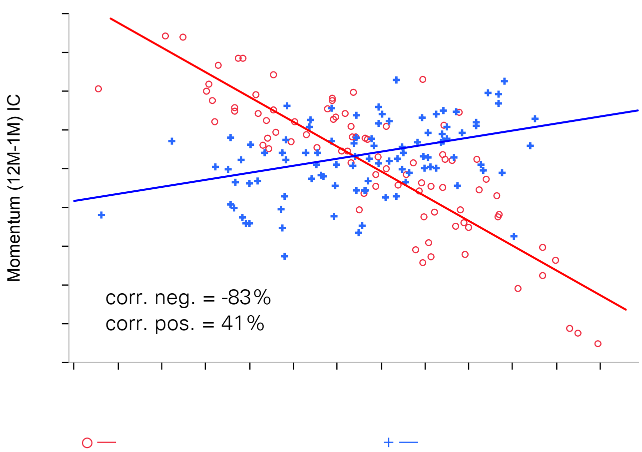


Source: Axioma, Compustat, IBES, Russell, S&P, Deutsche Bank

To rigorously check the efficacy of the neutralization scheme on momentum we apply the prior analysis, but discriminate between positively and negatively correlated periods

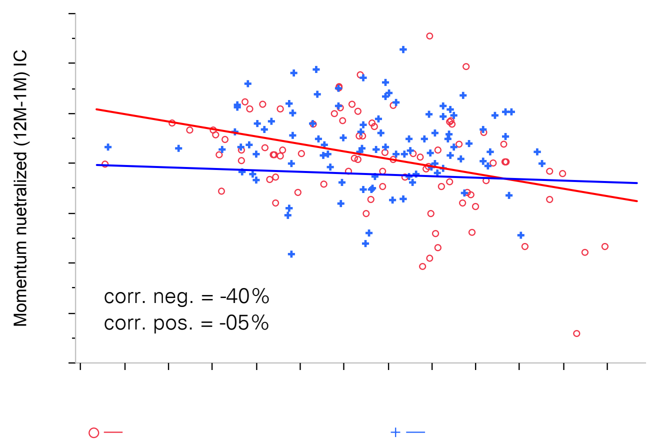
How do we check if neutralization works in the case of momentum? As we saw in Figure 12 the correlation between momentum and CT-Risk oscillates between positive and negative values and the average over time is very close to zero (slightly negative). Therefore, to rigorously check the efficacy of the neutralization scheme we apply the prior analysis, but discriminate between positively and negatively correlated periods. Figure 29 plots the realized ICs of the original momentum factor and the CT-Risk factor as we did for our other factors. However, we discriminate between periods when the cross-sectional correlation was negative versus those that were positive. Similarly, Figure 30 plots the realized IC of the neutralized momentum factor vs. the ICs of the CT-Risk factor over positively and negatively correlated periods. As we saw with the prior factors, the neutralization scheme did very well in mitigating the correlation and exposure to volatility as proxied by our CT-Risk factor.

Figure 29: Momentum IC vs. CT-Risk IC (conditioned on sign of cross-sectional section)



Source: Axioma, Compustat, IBES, Russell, S&P,Deutsche Bank

Figure 30: Momentum neutralized IC vs. CT-Risk IC (conditioned on original factor sign of x-sec correlation)



Source: Axioma, Compustat, IBES, Russell, S&P, Deutsche Bank

The efficacy of the neutralization scheme depends on:

1) ***The strength of the risk model, and***
2) ***The stability of the correlation between the original factor and CT-Risk***

We want to note a few important observations about the prior analysis. First, the efficacy of the neutralization scheme depends on two things: 1) the ability of the risk model to accurately capture and forecast volatility and correlation and 2) the stability of the correlation between the original factor and the CT-Risk factor (volatility). We can see the first condition explicitly in equation (3). Note that the term in the parenthesis depends on the forecast of the correlation and the volatilities. Therefore, a portion of the accuracy to with which the CT-Risk is able to forecast volatility comes directly from the risk model. This implies that had we constructed our CT-Risk factor with a deficient risk model, the efficacy of the neutralization would have been inferior. However, no risk model is perfect so we will always experience error in our forecasts, which will weaken the efficacy of the neutralization technique. The second condition, the stability of the correlation, involves the dynamic nature of the factor relationship with volatility. For example, Figure 12 shows that momentum has an inherent oscillating relationship with volatility as measured by our CT-Risk factor. In general, we would not expect to capture any significant and sudden dynamic changes in this correlation. This lack in predictability will also lower the efficacy of our risk factor as can be seen for the results in the case of momentum.

Last, we note that the IC correlations in Figure 23, Figure 25, Figure 27 and Figure 29 show higher realized correlations than reported in Figure 8 through Figure 13. The reason for this is two-fold. First, we are computing the IC correlation, which is scaled for risk. If we had just computed the return to the factor-scores directly then the correlation would be more in line with the rank correlations. Second, those figures reported the rank factor-score correlation (factor alignment), which typically underestimates the magnitude of the correlation (see discussion in the appendix on factor correlation). However, these correlations tend to be more stable than the risk-model implied correlations over time.

Factor diversification

We investigate whether or not we can achieve factor diversification through neutralization

The latter showed the efficacy of the neutralization technique in mitigating exposure to volatility as proxied by our CT-Risk factor. In the following we investigate whether or not we can achieve factor diversification through neutralization. The impetus for this analysis comes from observing that many of the factors studied above have a strong negative correlation with volatility as measured by our CT-Risk factor. Therefore, we can think of the CT-Risk factor as a *common factor*[9].

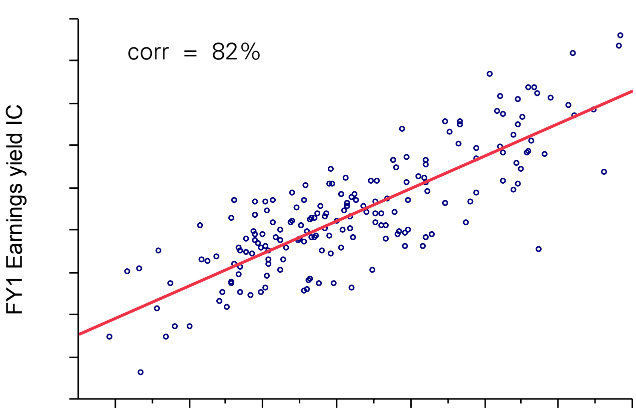
There is indeed diversification benefit from neutralizing factors - performance does not suffer, so the risk-adjusted benefit is very large

Can neutralization really diversify our signals without sacrificing performance? To investigate this question, we can look at the IC correlation between the original and neutralized factors for our two value factors: FY1 earnings yield factor and expected dividend yield. Figure 31 shows the IC plots of the two original value factors, which have a correlation of 82%. Figure 32 shows the IC of the factors after neutralizing them for the CT-Risk factor. Note the sharp drop in correlation to 42%, suggesting that there is indeed diversification benefit from neutralizing factors. In addition, Figure 22 shows that neutralized factors tend to outperform the original factors leading to even greater risk-adjusted benefit.

The magnitude of the correlations across factors decreases significantly after neutralization

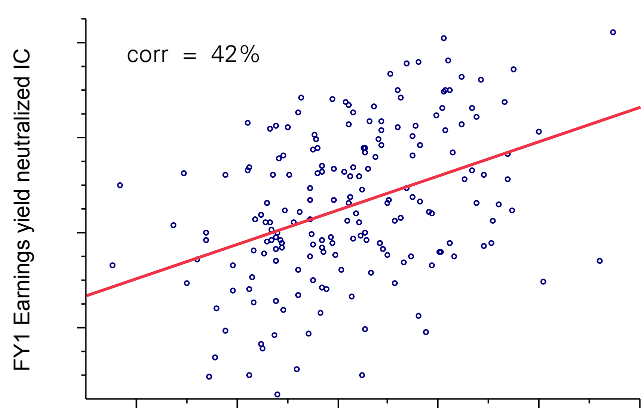
We chose this example because value factors tend to be highly correlated. Therefore, if we can find diversification benefit for two value signals by neutralizing their exposure to volatility then we can expect the diversification to extend across our entire factor universe. In Figure 33 and Figure 34 we show the correlation heat maps of our factor ICs before and after neutralization for a larger sample of our factor library. First, we note that the general intensity (magnitude) of the correlations across factors decreases significantly after neutralization. In particular, we see how some of the value factors have become less correlated (the decrease in the correlation across value is more pronounced along the lines of earnings vs. dividends).

Figure 31: FY1 Earnings yield IC vs. Expected yield IC



Source: Axioma, Compustat, IBES, Russell, S&P, Deutsche Bank

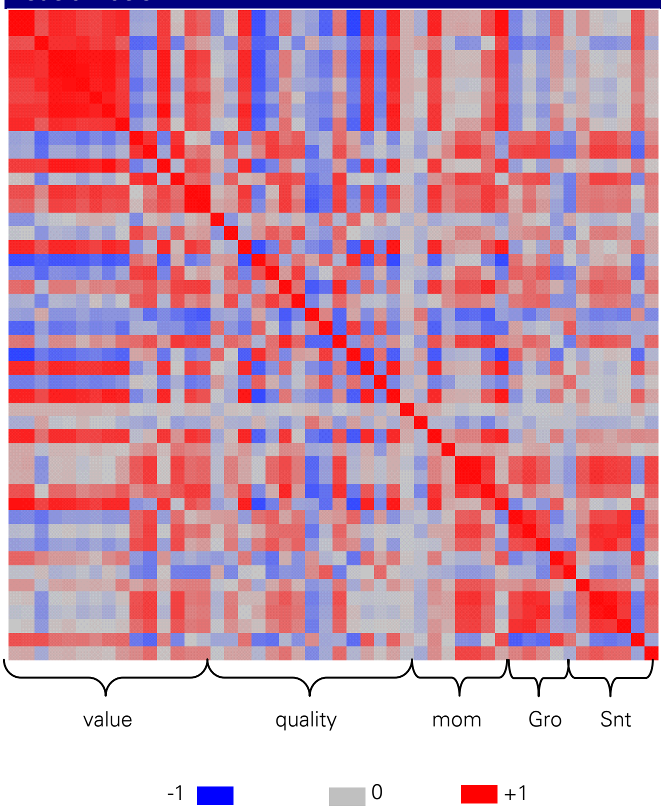
Figure 32: Earnings yield neutralized IC vs. expected yield residual IC



Source: Axioma, Compustat, IBES, Russell, S&P, Deutsche Bank

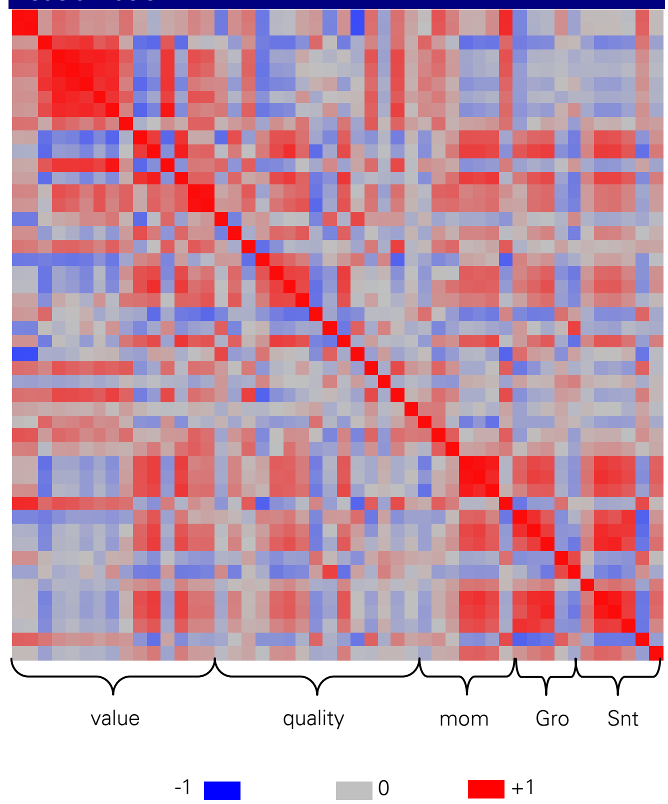
[9] Indeed, we have constructed our CT-Risk factor from the vendor covariance matrix and so we expect that it will capture a significant amount of common factor return.

Figure 33: Factor IC correlation heat map before neutralization



Source: Axioma, Compustat, IBES, Russell, S&P, Deutsche Bank

Figure 34: Factor IC correlation heat map after neutralization



Source: Axioma, Compustat, IBES, Russell, S&P, Deutsche Bank

When neutralizing with respect to one factor the neutralization procedure becomes as simple as multiplying two sets of scores

Neutralization made easy

How simple is this technique to implement? In the case of neutralizing to only one factor (e.g. CT-Risk) it can be made very simple. As we show in the appendix, we can simplify the neutralization scheme by using the *risk-cancellation* factor in equation (*A*6). Once this factor is formed we no longer have to worry about computing the correlations and volatilities in equation (3), which can be computationally intensive and require matrix operations using the full asset-by-asset covariance matrix. Therefore, when neutralizing with respect to one factor the neutralization procedure becomes as simple as multiplying two sets of scores.

The procedure involves:

1. computing a simple dot product between the original factor and the risk-cancellation factor
2. multiply the factor scores of the CT-Risk factor by the product from step 1
3. subtract the original factor scores from the scaled factors scores computed in step 2

Given a risk-cancellation factor $\hat{\boldsymbol{p}}_{risk}$, we can follow the three steps above to neutralize our original factor scores. The steps can be summarized via the following equation:

$$f_{neut,i} = f_i - \left(\hat{\boldsymbol{p}}'_{risk} \cdot \boldsymbol{f}\right) \times f_{risk,i} \tag{4}$$

This technique can be applied to neutralize against any factor

where $\boldsymbol{f}$ is the original vector of factor scores, f_i is the original factor score for asset *i*, $\hat{\boldsymbol{p}}_{risk}$ is the risk-cancellation factor, and $f_{risk,i}$ is the volatility factor score for asset *i*. Note that $\hat{\boldsymbol{p}}_{risk}$ is dependent on f_{risk}, which is the factor we wish to neutralize (CT-Risk in our case). It is important to note two things. First, $\hat{\boldsymbol{p}}_{risk}$ is dependent on the neutralization factor, and the asset variance-covariance matrix. Therefore, it only needs to be computed once for any given universe. However, it is important to note that this technique can be applied to neutralize against any factor such as an industry, implied volatility, skewness, etc.

How different is neutralization to mean-variance optimization?

Can we get better risk-adjusted performance using neutralization as a substitute to mean-variance optimization?

In the last section we found that neutralization provides a way to eliminate a portion of the risk inherent in the factors. Indeed, this is the premise for mean-variance optimization. So the natural question is whether neutralizing to a risk factor, such as our CT-Risk factor, can provide the risk benefits of optimization. The theoretical answer of course is no. However, we know that mean-variance optimization poses many problems. Most of the issues surround estimation error in the alphas and the risk forecasts. However, even a perfect model can at times hurt performance by transforming a well-performing factor into noise or worse. This phenomenon has been explored by Stefek and Lee [2008], which show that risk models and basic mean-variance optimization can misalign the portfolio into taking unintended factor exposure. In essence, they found that under mean-variance optimization, the interaction between the asset-by-asset covariance matrix and alphas can position the investor away from their information. Checking all the scenarios can be an arduous task, which we leave to a future report. In this report, we are only interested as to whether we can get better risk-adjusted performance using neutralization as a substitute to mean-variance optimization.

To investigate this question, we construct two basic portfolios.

1. **Mean-variance optimal factor portfolio**. The holdings of this portfolio are the ideal mean-variance optimal holdings (no t-costs, constraints, etc.), where risk is controlled using the full asset-by-asset covariance matrix, $\boldsymbol{V}$. The portfolio holdings take on the form:

$$\boldsymbol{h}_{MVO} = \left(1/\lambda_{MVO}\right) \boldsymbol{V}^{-1}\, \boldsymbol{f}_{raw} \tag{5}$$

2. **Neutralized factor portfolio**. This is the portfolio that uses the neutralized factor from equation (3). Note that by neutralizing to the CT-Risk factor, we are providing a certain level of risk control. The holdings for the neutralized factor portfolio are:

$$\boldsymbol{h}_{neut} = \left(1/\lambda_{neut}\right)\, \boldsymbol{f}_{neut} \tag{6}$$

3. **Neutralized factor portfolio corrected for specific risk**. This is a middle of the road portfolio construction method, which is not as binding as MVO, but more sensible than only neutralizing. The impetus for this is that we have seen that neutralization corrects for common factor risks through the CT-Risk factor. However, it may not control fully for specific risks. Therefore, a middle of the road strategy between (5) and (6) is:

$$\boldsymbol{h}_{neut+sprsk} = \left(1/\lambda_{neut}\right) \boldsymbol{\Delta}^{-1}\, \boldsymbol{f}_{neut} \tag{7}$$

In each case, $\boldsymbol{h}$ is the holding vector, $\boldsymbol{V}$ is the asset-by-asset covariance matrix, $\boldsymbol{\Delta}$ is the diagonal matrix of specific variance, $\boldsymbol{f}_{raw}$ is the vector of the original factor scores, $\boldsymbol{f}_{neut}$ are the neutralized factor scores and the λ's are the risk aversion parameters. We note that the two portfolios will be of different scale in volatility terms. To remedy this, we can adjust the

λ's to set the ex-ante risk of each of the portfolios to be equal[10]. However, in the following analysis, we compare risk-adjusted return (IR) so this adjustment for portfolio volatility is not necessary (λ is of no consequence to our results because it scales away).

The MVO portfolio only outperforms in the case of the value signals and ROE. In the case of momentum, it severely underperforms

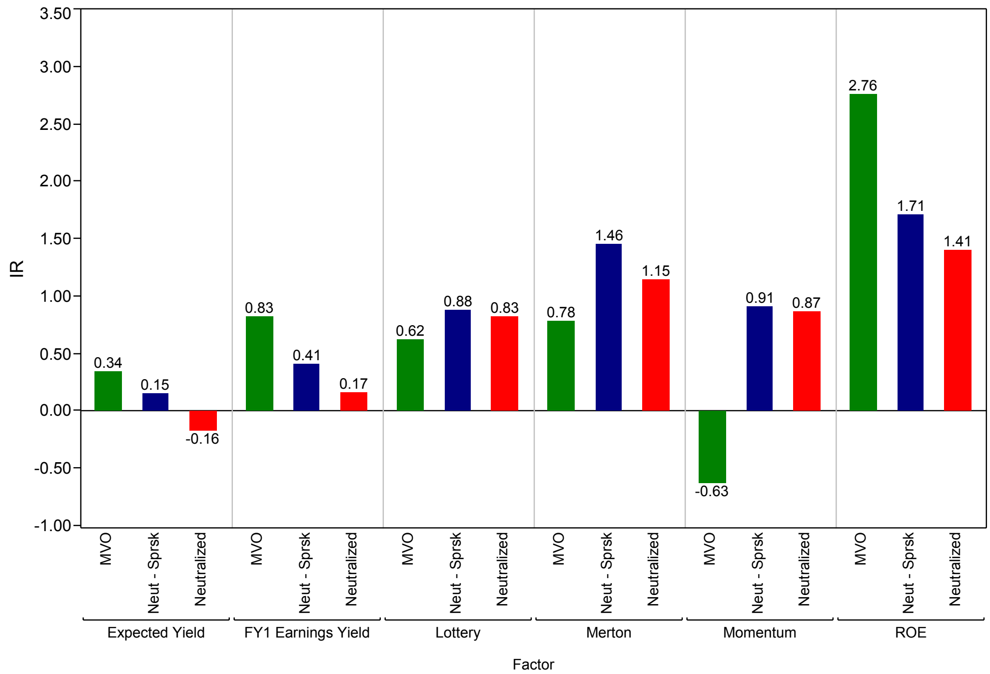
Figure 35 compares the IR for each of the portfolio schemes for each factor that we have studied above. The backtest period starts in January 1995 and ends in April 2010. We note that the MVO portfolio – that with the most risk control – only outperforms the neutralized portfolio in the case of the value signals and ROE. In the case of momentum, it severely underperforms and actually seems to be flipping the skill of the factor from positive to negative. More importantly, we note that in each case the neutralized factor optimized with respect to specific risk (neut-risk case), is equal or superior to the neutralized factor portfolios. This points to the fact that controlling for specific risk indeed a crucial process of the portfolio construction process. In addition, we point out that the results for the specific risk corrected neutralized portfolios are very sensitive to the quant crises of July 2007. If we take that data point out of the computations the results for the case of value look equivalent to the MVO portfolios. Without July 2007, the specific risk corrected expected yield portfolio has IR of 0.40, which is equivalent to the MVO portfolio. Similarly, the specific risk corrected FY1 earnings yield portfolio has an IR of 0.75, which while lower is not significantly different than the 0.83 IR for the MVO portfolio. This tells us two things in the case of value: 1) the results are not robust over time; and most importantly 2) the full risk model did indeed add value by protecting the value factor. We admit that this result calls for more investigation and we leave that to a follow up note or report.

We find that for the quant crises in July 2007, the full risk model optimization did indeed add protection to the value factors

Another question we ask ourselves is whether these results change if we look at the recent period of quant factor degradation. Figure 36 shows the same IR results, except that we focus on the period January 2006 to April 2010. Notice that the MVO portfolios now only outperform in two of the six cases. In addition, the outperformance is only significant in the case of the expected dividend yield factor. Similar to the results over the entire backtest in Figure 35, we find that the MVO portfolio for momentum is severely altering the factor for the worse. In addition, we find that the Merton factor has also experienced the same reversal in performance. In the next section we investigate a possible reason for these surprising results.

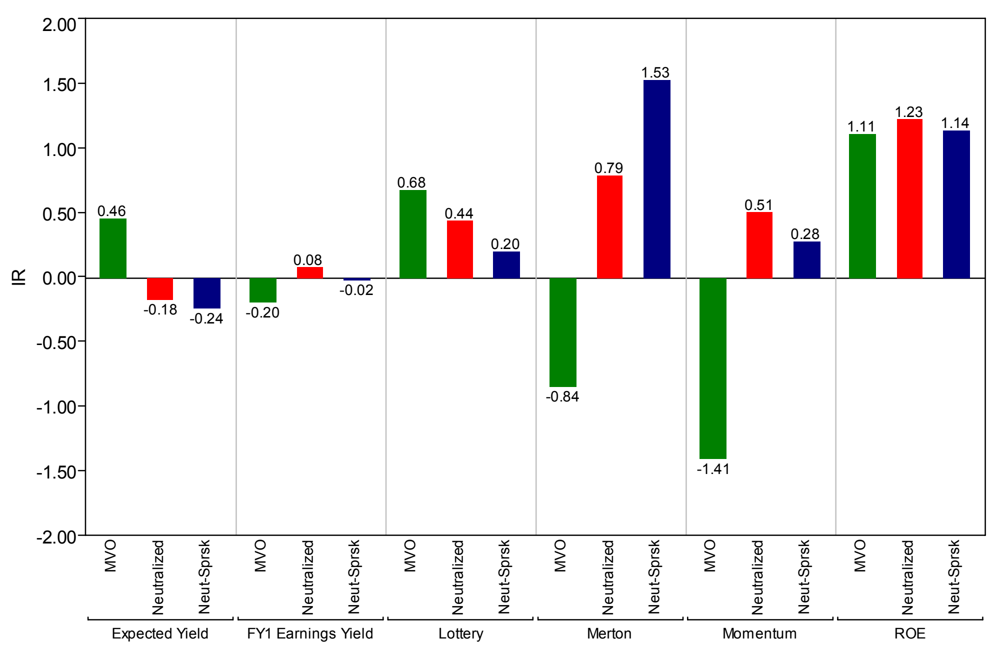
[10] Here we are using the risk-aversion parameter loosely in more of a portfolio engineering context to adjust for our desired ex-ante portfolio risk.

Figure 35: MVO, neutralized factor portfolio return IR (January 1995 – April 2010)



Source: Axioma, Compustat, IBES, Russell, S&P, Deutsche Bank

Figure 36: MVO, neutralized factor portfolio return IR (January 2006 – April 2010)



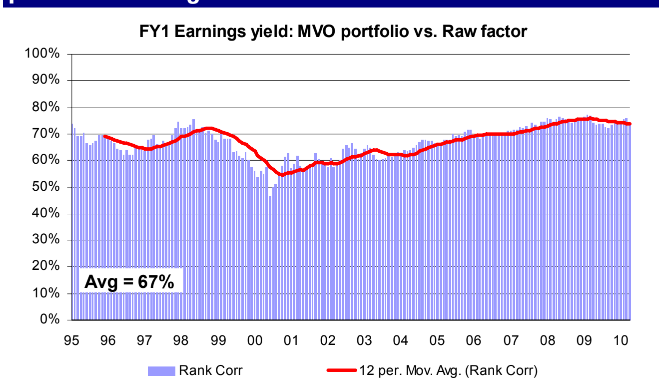
Source: Axioma, Compustat, IBES, Russell, S&P, Deutsche Bank

A risk model can re-direct our information and generate portfolios that are misaligned with our intended exposure

Basic mean-variance optimization (no t-costs or constraints) works by risk-adjusting the alphas via a forecast of the variance-covariance matrix of asset returns. It is also well known that the efficacy of the procedure is highly dependent on the covariance matrix. In addition, as we mentioned above, the risk model can re-direct our information and generate portfolios that are misaligned with our intended exposure. This effect depends on many attributes, in particular, the extent to which our alphas are aligned with risk model factors. To gauge this effect, we can compare the raw factor score stock rankings to the final portfolio stock holding rankings. In the case, that the risk model does not perturb our original factor rankings the rank correlation will be high. On the contrary, if the risk model significantly perturbs our original factor rankings then we would expect a rank correlation to be lower.

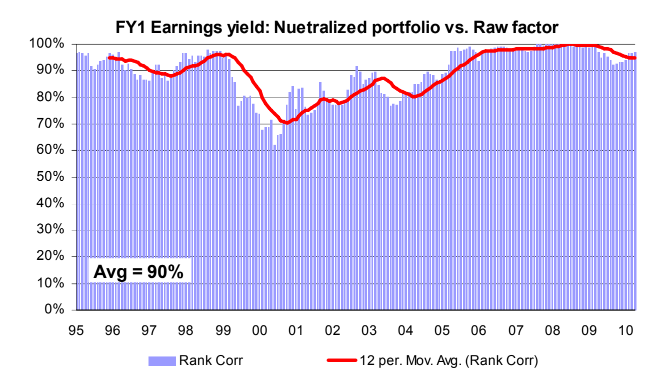
To see this effect, Figure 37 and Figure 38 show the cross-sectional rank correlation over-time for our FY1 earnings yield factor with the mean-variance optimal factor-portfolio and the neutralized factor portfolio, respectively. Note that the correlation with the mean-variance portfolio is consistently lower, which makes sense in that it is controlling for risk across many more factors than the neutralized version. In Figure 39 and Figure 40, we show the same two correlations for the momentum factor. Note that in this case, the mean-variance portfolio holdings are ranking stocks very differently than the original factor. This tells us that the risk model and mean-variance optimization are altering the alignment of the factor's intended exposure. Indeed this is the classic example in Lee and Stefek (2008), where they showed that momentum becomes misaligned once it is optimized with a risk model constructed with a similar factor. To adjust for this we took out the momentum factor from the risk model and re-computed the mean-variance factor portfolio. Figure 41 shows the new rank correlation between this new MVO factor portfolio and our original momentum factor. Note the increase in rank correlation over time between Figure 39 and Figure 41. This result is due, solely, to taking out the momentum risk factor from the factor covariance matrix before computing the full asset-by-asset covariance matrix. However, we find that even in the case when we extract the momentum risk factor from the model, the MVO momentum factor portfolio still underperforms relative to the neutralized factor portfolio. The reason we showed this example, is to underline the fact that mean-variance optimization, in its quest to control for risk, can actually misalign our intended exposure.

Figure 37: Rank correlation FY1 Earnings yield MVO portfolio holdings vs. raw factor scores



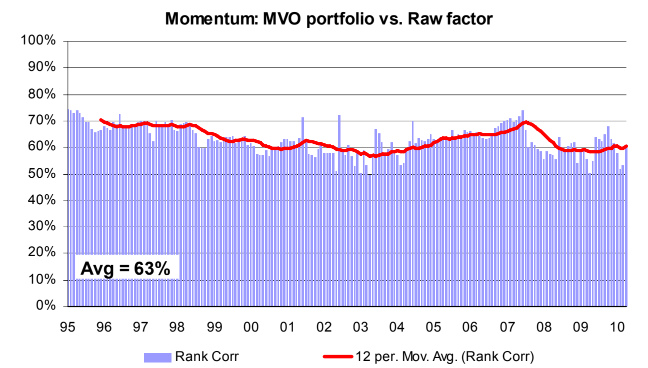
Source: Axioma, Compustat, IBES, Russell, S&P, Deutsche Bank

Figure 38: Rank correlation FY1 Earnings yield neutralized portfolio holdings vs. raw factor scores



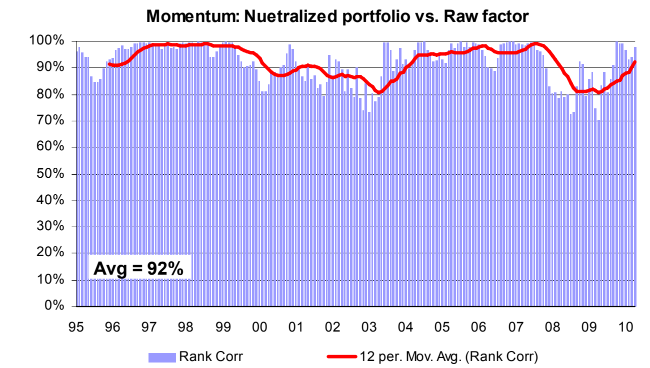
Source: Axioma, Compustat, IBES, Russell, S&P, Deutsche Bank

Figure 39: Rank correlation momentum MVO portfolio holdings vs. raw factor scores



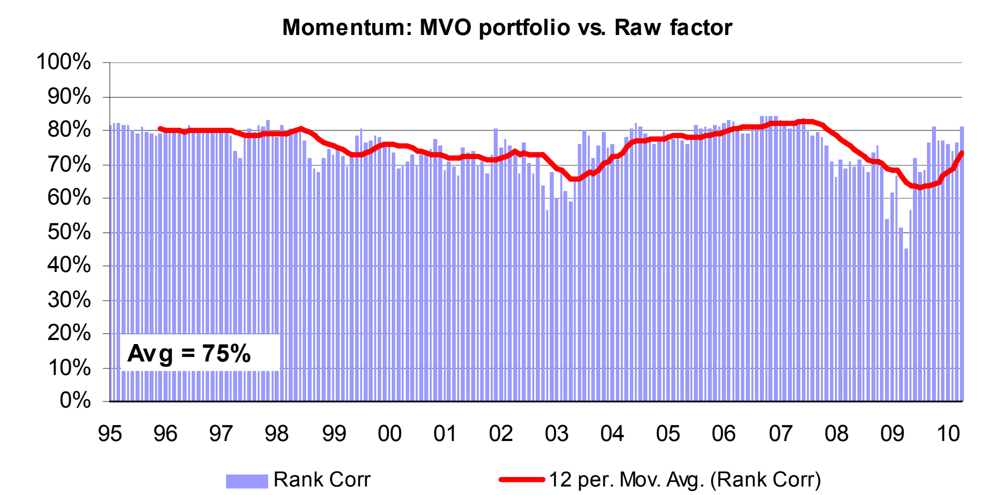
Source: Axioma, Compustat, IBES, Russell, S&P, Deutsche Bank

Figure 40: Rank correlation momentum neutralized portfolio holdings vs. raw factor scores



Source: Axioma, Compustat, IBES, Russell, S&P, Deutsche Bank

Figure 41: Rank correlation momentum MVO portfolio holdings vs. raw factor scores, MVO portfolio computed using the risk model without the momentum factor



Source: Axioma, Compustat, IBES, Russell, S&P, Deutsche Bank

Multiplying the neutralized factors by the covariance matrix will mitigate the effects of the risk model for cases such as momentum

Tricking the optimizer

Many investors must use optimizers not only to control for risk, but also to adhere to certain mandates (e.g. long only constraint). It would be naïve to think that these investors could actually substitute their optimization process by using only neutralization. However, in the case when an investor would prefer to align their portfolios to a neutralized factor, there is a simple trick that will mitigate the potential negative effects of the risk model for cases such as momentum or the Merton factor. The trick involves simply multiplying the neutralized factors by the asset-by-asset covariance matrix, $\boldsymbol{V}$, before combining them with other factors into a final alpha forecast. Specifically, suppose we wanted to mitigate any adverse risk-model effects to our momentum or Merton factors and preferred to control risk through pure neutralization. Then we could use the following simple scheme:

$$\boldsymbol{f}_{neut}^{opt} = \left(1/\lambda_{neut}\right)\boldsymbol{V}\,\boldsymbol{f}_{neut} \qquad (8)$$

where $\boldsymbol{V}$ is the asset-by-asset covariance matrix, $\boldsymbol{f}_{\boldsymbol{neut}}$ is the neutralized factor, and $\boldsymbol{f}_{neut}^{opt}$ is the factor that would be combined with other factors into a final alpha before optimization.

Protecting factors by scaling their volatility correlation

We can protect our value factor by forcing it to be more negatively correlated to volatility and avoid "value traps"

In this section, we take the neutralization scheme one-step further and use it to adjust the exposure to volatility of a factor. Why do this? One reason is that we have seen that there are times when negative exposure to volatility can enhance performance. Another reason is that we find that some factors tend to do better when they are negatively correlated to volatility. In the case of value there is a sensible fundamental reason. As we saw in Figure 11, value may load up on volatile or distressed assets as investors flee from risk and momentarily make risky stocks cheaper relative to everything else. This refers to the commonly phrased "value trap". Therefore, we could think of protecting our value factor by forcing it to be more negatively correlated to volatility – call it *safe value*.

To scale the correlation to volatility, we use a variant of the neutralization scheme outlined in equation (3). The scheme consists of specifying the target correlation coefficient $\rho_{new,risk}$ and constructing the scaled factor, f_i^{new}, as (see appendix for details):

$$f_i^{new} = f_{neut,i} + \beta_{new,risk} f_{risk,i} \tag{9}$$

where,

$$\beta_{new,risk} = \frac{\sigma_{neut}}{\sigma_{risk}} \frac{\rho_{new,risk}}{\left(1-\rho_{new,risk}^2\right)^{1/2}} \tag{10}$$

where σ_{neut} is the volatility of the neutralized factor, and σ_{risk} is the volatility of the risk factor we are neutralizing against.

A simple way to approximate the potential improvement in performance from the correlation scaling in equation (7) is to use equation (2) with the new correlation, $\rho_{new,risk}$. To see this, note that the top line in equation (7) just combines the neutralized factor with the risk factor using a new target correlation. Therefore, we can approximate the IC change using equation (2) with $\rho_{new,risk}$ instead of $\rho_{factor,vol}$.

Case study – forward earnings

Instead of scaling the correlation to our CT-Risk factor, we use the specific risk factor

We can apply the prior methodology to protect our FY1 earnings yield factor from the infamous "value trap". However, we use a slight twist as opposed to our previous analysis in this report. Instead of scaling the correlation to our CT-Risk factor, we use the specific risk factor. The reason we do this is because the CT-Risk factor has a very large element of common factor risk. Therefore, negative exposure to this factor will have a significant negative exposure to common factor risk. This risk usually gets heavily controlled by the optimization under a full risk model. Specific risk, on the other hand, has an asset specific component to it. Therefore, negative exposure to this factor will be more than just negative correlation with systematic factors (they are nevertheless correlated as can be seen in Figure 7). Also, as we will see below, the correlation between value and specific risk has been more stable over time than that with CT-Risk.

Figure 42 shows us the rank correlation of our FY1 Earning yield factor with the specific risk factor from the Axioma US medium horizon model. Notice that since the quant crises in the summer of 2007, the correlation between the factor and specific risk has risen significantly. This indicates that the factor is loading up on more volatile assets. To apply the scaling we use a very simple algorithm. Basically, we use the fact that prior to 2007 the average value of the correlation was roughly -40%. Therefore, we apply the following very simple algorithm:

1) if the correlation is less than -40% then use the original factor

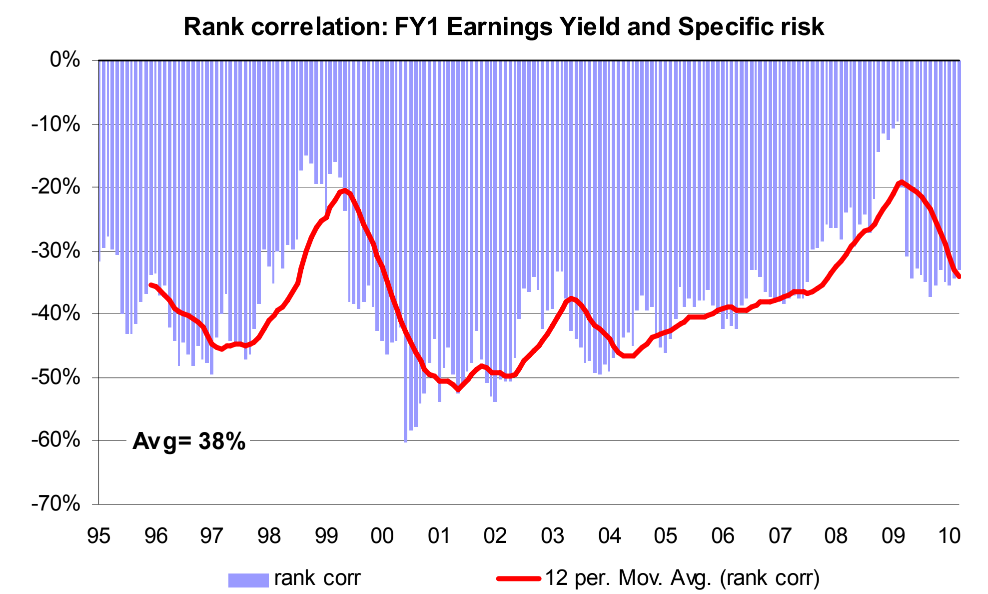
2) if the correlation is greater than -40% then scale the factor using equation (7) with a target correlation of -40%.

We can test the new scheme by specifying the new correlation to volatility to be:

$$\rho_{new,risk} = \min(\rho_{f,risk}, -0.40) \tag{11}$$

Figure 43, shows the difference in rank IC between the original FY1 Earnings yield and the scaled FY1 Earnings yield factor for the entire backtest period. Note that the simple procedure outlined in equation (11) significantly helped the factor. Also note that the episodes when the correction helped were those when there was heavy de-risking (Russian default and LTCM crises in 1998, technology crises in 2001, quant crises of 2007 and financial crises of 2008, European debt scare of May 2010).

Figure 42: Rank correlation between FY1 Earnings yield factor and specific risk factor

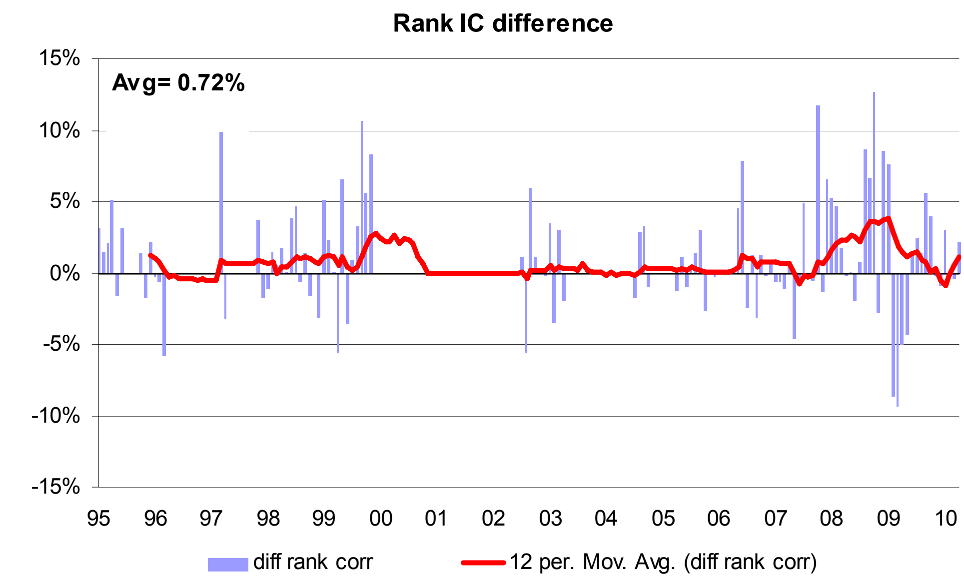


Source: Axioma, Compustat, IBES, Russell, S&P, Deutsche Bank

In Figure 44, we analyze the period between January 2007 and April 2010. Note the improvement in performance since the quant crises in July 2007 and continuing into 2008 as we headed into the full financial crises in the fall of 2008. Of course, the scaled factor suffers during the volatility (junk) rally in the spring of 2009, but overall we find that the scaled factor does indeed improve the performance of the factor during this period.

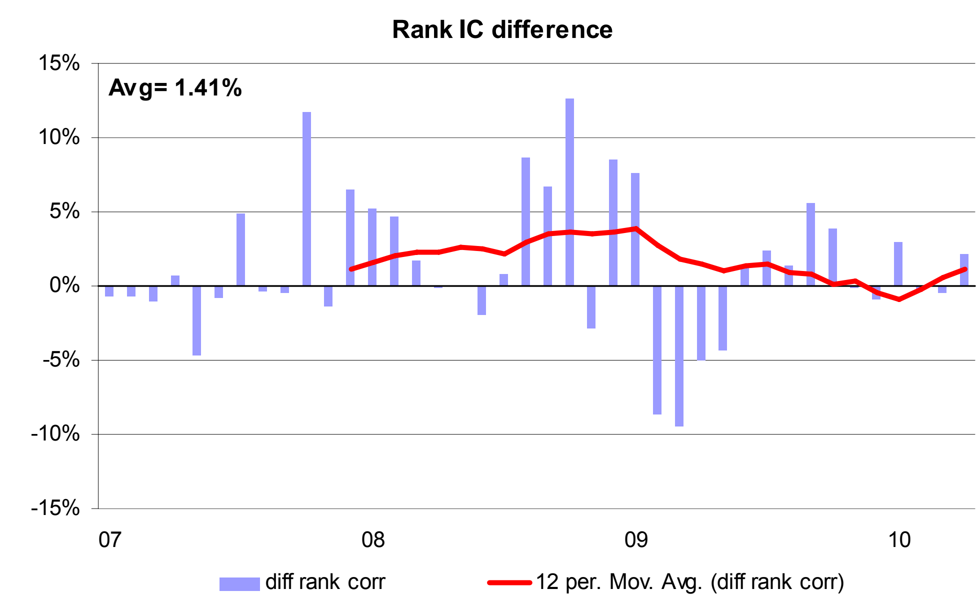
Note that the scheme above, injects value with more and more negative specific risk, which may ultimately transform the factor away from its original form. The argument can be made that this factor is no longer a value play. In our opinion, we feel that staying away from a poisonous factor is much better than suffering underperformance by adhering to dogma. Another way to think about this is that it is similar to turning the factor off (or tuning it down) when we believe it will not work and playing it safe by loading up on less volatility. This can also be thought as a sort of a timing strategy, but here we are using a fundamental argument – basically value has turned dangerous so we prefer to alter its composition even when the final factor no longer represents "true value".

Figure 43: Rank IC difference between original FY1 Earnings yield and specific-risk corrected FY1 Earnings yield (Jan 1995 – May 2010)



Source: Axioma, Compustat, IBES, Russell, S&P, Deutsche Bank

Figure 44: Rank IC difference between original FY1 Earnings yield and specific-risk corrected FY1 Earnings yield (Jan 2007 – May 2010)



Source: Axioma, Compustat, IBES, Russell, S&P, Deutsche Bank

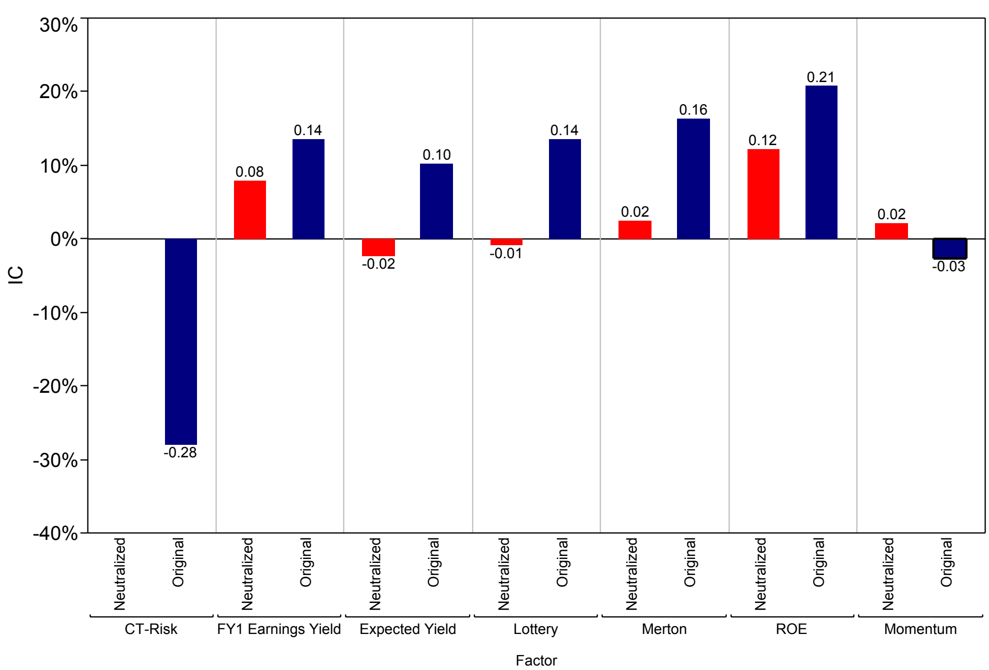
Last we want to note that the procedure in equation (11) is rather simplistic and we used it purely to show how it can be used to guard the factor. We could have used equation (11) in a factor-timing context and changing correlation dynamically over time.

What happened in May 2010

We find that after accounting for volatility only a few factors outperformed

May 2010 brought on an extreme level of de-risking as investor risk aversion spiked as a consequence of European debt fears. Did the sudden spike in risk-aversion show up through our CT-Risk factor? If so how much of the quantitative factor performance was due to volatility? To answer these questions, we point to Figure 45, which shows the performance over May of our volatility factor (CT-Risk) and each of the factors we have looked at above. We note a few obvious points. Volatility as proxied by our CT-Risk factor did very poorly, which validates the fact that this factor can capture sudden and significant episodes of risk-aversion changes. Second, we note that all the original factors with negative volatility exposure leading into May significantly outperformed. However, for three of the factors in our study, lottery, Merton, and expected yield, this performance seemed to be driven purely by its exposure to negative volatility. Conversely, FY1 earnings yield and ROE performed well above-and-beyond their volatility exposure. Last, we note that momentum, which leading into May 2010 had positive correlation to volatility (investors had been bullish risk until April), underperformed due to its volatility exposure. However, the neutralized momentum factor, which is corrected for volatility exposure seemed to have slightly positive, albeit insignificant, performance.

Figure 45: Rank IC for original factor and neutralized factor over May 2010



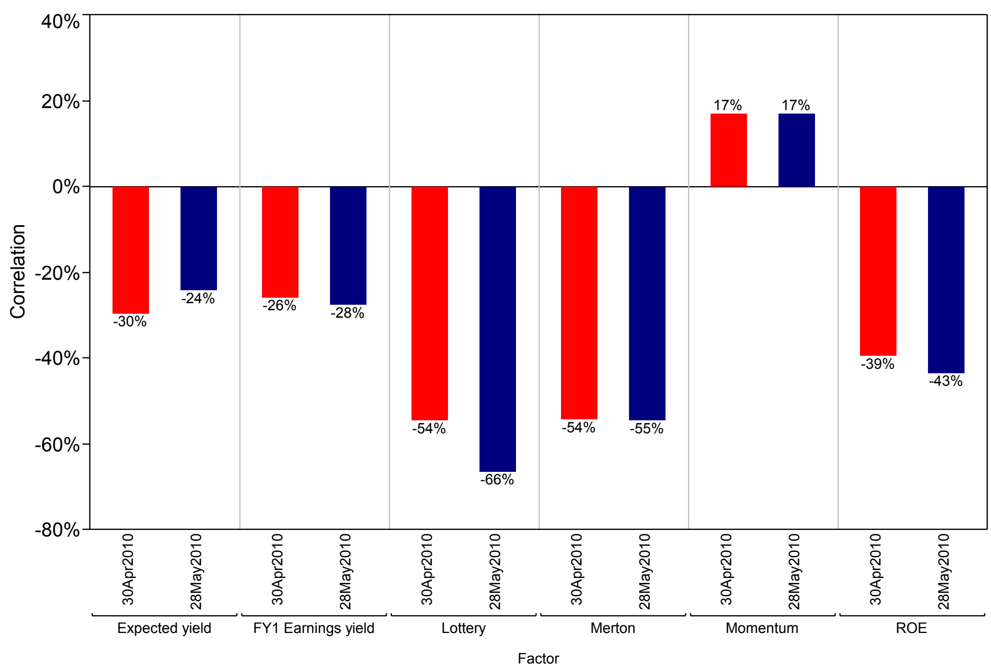
Source: Axioma, Compustat, IBES, Russell, S&P, Deutsche Bank

How are factors positioned for June?

Factor correlations with volatility have not changed much since April; however we feel value factors should be aligned towards safer stocks

What were the correlations leading into June and how have they changed since May? This question is important given what we've seen in the prior analysis. Specifically, we saw how value can become tainted with stocks with high contribution to volatility after significant episodes of de-risking. Is value in a "value trap" today? Figure 46 shows us the correlations of each of the factors we have studied with volatility for end of April and end of May 2010. We note that the May 2010 de-risking has not significantly changed the factor alignment of any factor. The only increase in volatility alignment was observed for the expected dividend yield factor. Even though we find that value has not changed its alignment with volatility over May 2010, we still feel that its level of alignment with volatility is dangerously high. Therefore, we suggest using the volatility scaling method described previously to better align value with safer, less volatile stocks. In addition, we find that momentum is still positively aligned with volatility. Therefore, if an investors want to play it safe and are expecting more de-risking we advise either staying away from momentum or neutralizing it to our CT-Risk factor. Please note that alignment for all our other basic quant factors can be provided on demand.

Figure 46: Rank correlation between factors and volatility (CT-Risk factor) for end of April and May 2010



Source: Axioma, Compustat, IBES, Russell, S&P, Deutsche Bank

1/N = Volatility

Equal weights and volatility

With help from our CT-Risk factor, we find that 1/N is purely a volatility play

In our last report in this series, see Luo, *et* al [2010], we compared mean-variance optimization (MVO) to a naïve top-minus-bottom decile portfolio construction strategy. Our main finding was that signal characteristics were important in determining which portfolio construction method worked best. In this report, we add to the debate on naïve vs. mean-variance optimization by looking at a very important property of the equally weighted portfolio construction strategy (1/N). Specifically, we help from our CT-Risk factor, we find that 1/N is purely a volatility play.

Equal weighting is an implied tilt toward volatility

To encourage the discussion, suppose that we had an equally weighted portfolio consisting of two assets that were uncorrelated, but had different volatilities. If we had a choice to which asset outperformed over the next year, which would we choose? Of course, the answer would be the most volatile asset given that its returns would dominate the portfolio performance over time. This simple example hints that equal weighting is an implied tilt toward volatility since, all else equal, our portfolio will do better if the more volatile asset outperforms.

Next, we provide another example to further elucidate the point. In this case, we assume that we want to construct a portfolio from a universe of asset returns that are completely uncorrelated and have different volatilities. The latter implies that the asset-by-asset covariance matrix is a diagonal matrix where the i^{th} element of the diagonal is the forecast of the return variance of the i^{th} asset. If we use the traditional quadratic utility function to maximize expected return and minimize expected risk our holdings (ignoring risk-aversion which scales all the holdings) take the following form:

$$h_i = \frac{\alpha_i}{\sigma_i^2}$$

where h_i is our holding in asset *i*, α_i is the forecast return to asset *i*, and σ_i^2 is the variance of asset *i*. Now suppose we have a decent forecast of the asset variances and we hold the equally weighted portfolio. Then we can use the equation above to solve for the implied expected returns (alphas) as:

$$h_i = \frac{1}{N} \quad \Rightarrow \quad \alpha_i = \frac{\sigma_i^2}{N}$$

Our expected returns are perfectly aligned with contribution to variance and volatility

The last expression states that our expected returns are perfectly aligned with the variance (or volatility) of each asset. This is another way of saying that our scores are aligned with risk. The analysis in the prior sections of this report and specifically the results in Figures 1-6, suggest that this is a perilous strategy to pursue. In the following section we show that equal weights imply volatility in the general case.

Suppose that we have a decent forecast of the variance for each asset and its covariance with every other asset in the universe, i.e. we have a forecast of the asset-by-asset variance-covariance matrix. Then the CT-Risk volatility factor defined in equation (1) can be written in matrix form as:

$$\boldsymbol{s} = \boldsymbol{V}\,\boldsymbol{1}$$

where $\mathbf{s}$ is the vector of factor scores, $\mathbf{V}$ is the forecast of the asset-by-asset covariance matrix, and $\mathbf{1}$ is a vector of ones.

Now suppose we wanted to construct an optimal mean-variance risk-adjusted portfolio using our CT-Risk factor as a proxy for expected returns. Then using the same forecast of the asset-by-asset covariance matrix that was used to construct the CT-Risk factor, our holdings under traditional mean-variance quadratic utility with the best risk-adjusted return forecast is:

$$\boldsymbol{h} = (1/\lambda)\boldsymbol{V}^{-1}\boldsymbol{s} = (1/\lambda)\boldsymbol{V}^{-1}(\boldsymbol{V}\ \boldsymbol{1}) = (1/\lambda)\ \boldsymbol{1}$$

The most efficient portfolio that captures our contribution to risk exposure is the equally weighted portfolio

where λ is the risk-aversion parameter, and $\boldsymbol{h}$ is our vector of portfolio holdings. ***The last expression says that the most efficient portfolio that captures the desired volatility exposure is the equally weighted portfolio.*** We believe this result is quite compelling. While it is intuitive that equally weighted portfolios have a volatility bias, it is not completely obvious that the equally weighted portfolio is that which captures the return to assets with the highest contribution to variance in the universe. Looking at the results from Figure 1 through Figure 6, we find that positive exposure to volatility has negative skill. Therefore, all else equal, we would expect such a strategy to underperform.

A corollary of the result above is that these implied alphas are aligned with the contribution to volatility of each asset to the volatility of the total universe (as well as the marginal contribution to volatility). In addition, we provide further insight into the two pieces of the volatility measure – the variance piece versus the covariance piece. We also find an intuitive characterization of this factor by simplifying the risk model to a one factor CAPM-type model. It is quite easy to show that under a one-factor risk model:

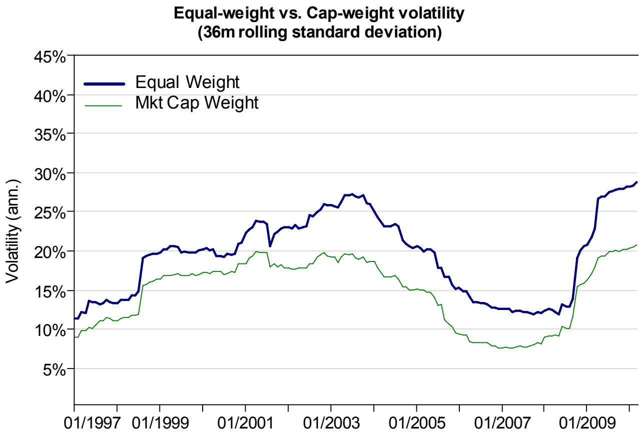
$$score_i = \left(\bar{\beta}\sigma_M^2\right)\beta_i + \frac{\sigma_{idio,i}^2}{N}$$

where β_i is the market beta of asset *i*, $\bar{\beta}$ is the average beta of the entire universe, σ_M is the volatility of the market factor, and $\sigma_{idio,i}$ is the specific risk of asset *i*.

As N grows the scores become more aligned with beta and as N becomes smaller the factor scores become more aligned with idiosyncratic risk

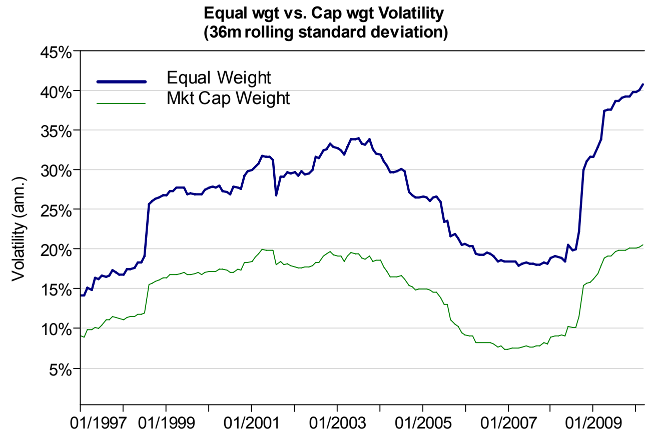
We note two observations about this approximation. First, as N grows the scores become more aligned with beta and therefore the characteristics of the 1/N portfolio begin to resemble those of market cap-weighted portfolio. Conversely, as N becomes smaller the factor scores become more aligned with idiosyncratic (asset specific) risk, which is unrelated to systematic factors and results in divergence between the cap-weighted and 1/N portfolio. In addition, it is easy to see from the formula that the 1/N alignment with specific risk results in higher volatility than the market portfolio and this volatility increases as N gets smaller. To see this result in action we point to Figure 47, Figure 48, Figure 49 and Figure 50, which show the 36-month rolling standard deviations of the equally weighted portfolio vs. cap-weighted portfolios for different universes: 1) our DB universe, which is roughly the Russell 3000; 2) the top 1000 assets by market cap; 3) the top 500 assets by market cap; and 4) the top 250 assets by market cap.

Figure 47: DB Quant Universe ~ Russell 3000



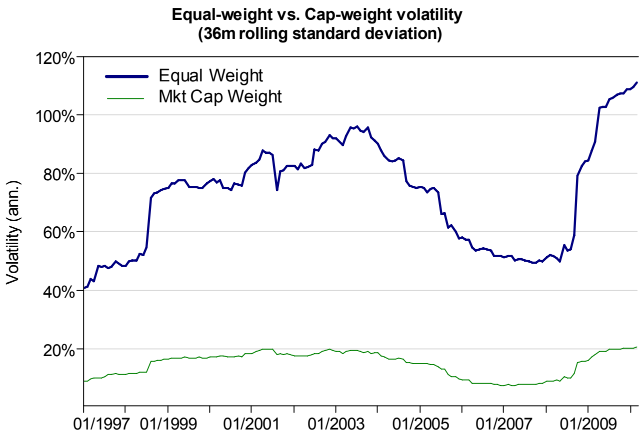
Source: Axioma, Compustat, IBES, Russell, S&P, Deutsche Bank

Figure 48: Top 1000 Assets by Market Cap



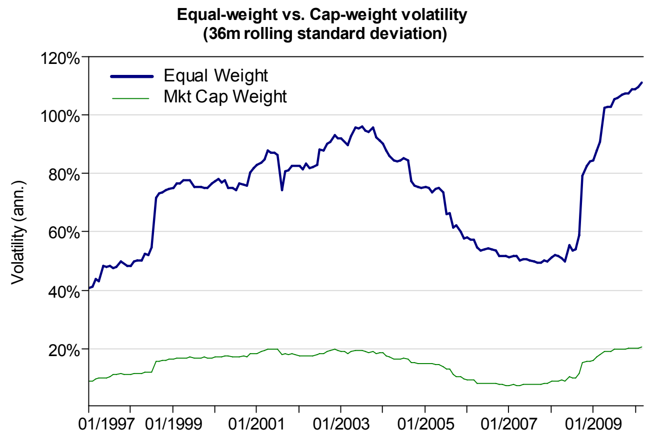
Source: Axioma, Compustat, IBES, Russell, S&P, Deutsche Bank

Figure 49: Top 500 Assets by Market Cap



Source: Axioma, Compustat, IBES, Russell, S&P, Deutsche Bank

Figure 50: Top 250 Assets by Market Cap



Source: Axioma, Compustat, IBES, Russell, S&P, Deutsche Bank

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Appendix A

Different measures of factor correlation

In this section, we provide the details behind the different correlation measures we used in the report.

Rank correlation (factor alignment)
Usually, we look at the correlation with volatility in both factor space and portfolio space. In pure factor space, we typically use the Spearman rank correlation to determine the degree of alignment between the two factor scores. If both factors rank stocks in a similar way then, we expect a high level of rank correlation. This correlation is computed as follows:

1) rank the scores in each factor

2) compute the correlation coefficient of the ranks

Factor-score correlation
This measure captures the correlation between the factor scores, but uses the variance-covariance structure of returns by correlating the factors using the asset-by-asset covariance matrix. The correlation is computed as follows:

$$\rho_{1,2} = \frac{\boldsymbol{f}_1' \boldsymbol{V} \boldsymbol{f}_2}{\left(\boldsymbol{f}_1' \ \boldsymbol{V} \boldsymbol{f}_1\right)^{1/2} \left(\boldsymbol{f}_2' \boldsymbol{V} \boldsymbol{f}_2\right)^{1/2}} \qquad (A.1)$$

where $\boldsymbol{f}_1$ are the factor scores for factor *k,* and $\boldsymbol{V}$ is the asset-by-asset covariance matrix of returns. The covariance matrix provides the forecast given the asset return structure.

Portfolio correlation
In portfolio space, we look at the correlation of portfolio holdings using the variance-covariance matrix of asset returns as our norm. In essence, given two portfolio holding vectors, $\boldsymbol{h}_1$ and $\boldsymbol{h}_2$ of the two factor-portfolios we compute the cross-sectional correlation (at time *t*) as:

$$\rho_{1,2} = \frac{\boldsymbol{h}_1 \boldsymbol{V} \boldsymbol{h}_2'}{\left(\boldsymbol{h}_1' \boldsymbol{V} \boldsymbol{h}_1\right)^{1/2} \left(\boldsymbol{h}_2' \boldsymbol{V} \boldsymbol{h}_2\right)^{1/2}} \qquad (A.2)$$

where $\boldsymbol{h}_1$ and $\boldsymbol{h}_2$ are the holding-vectors of the two portfolios, $\boldsymbol{V}$ is the *forecast* of the asset-by-asset covariance matrix of returns, and $\rho_{1,2}$ is the correlation (forecast) between the two portfolios.

There are accuracy issues regarding all three methodologies. When using the rank correlation, we are not accounting for the correlation in the assets due to their exposure to common factors. Therefore, rank correlation tends to underestimate the magnitude of the true correlation when we compare out-of-sample. However we find that when using the asset-by-asset covariance matrix to measure cross-sectional correlation, the correlation estimates tend to be quite noisy. We find that the out-of-sample correlations over time tend to be in between both methods. Therefore, we suggest using both measures to get a true sense of the correlation.

Neutralization

Assume, we have a factor and wish to neutralize its exposure (or correlation) to a set of other factors (e.g. CT-Risk, industry factors, etc). Another way to say this is that we want the correlation between the factor and the set of neutralization factors to be zero. The scheme is as follows.

Let $\boldsymbol{f}$ be the vector of asset scores that we wish to neutralize against a set of factors $\boldsymbol{g_1},\ldots,\boldsymbol{g}_k$. Let $\boldsymbol{V}$ be the asset-by-asset covariance matrix of asset returns and let $\boldsymbol{G}$ be the matrix where the i^{th} column corresponds to the scores of factor $\boldsymbol{g_i}$. Then our first equation arises from writing the factor scores $\boldsymbol{f}$ as a linear combination of the factors, $\boldsymbol{g_1},\ldots,\boldsymbol{g}_k$ and a portion that is residual:

$$\boldsymbol{f} = \boldsymbol{G}\,\boldsymbol{\beta} + \boldsymbol{f_{neut}} \qquad (A.3)$$

where $\boldsymbol{f}$ is the original factor, $\boldsymbol{G}$ is the matrix of the neutralization factors, $\boldsymbol{\beta}$ is an unknown vector of loadings of the original factor to the neutralization factors, and $\boldsymbol{f_{neut}}$ is the neutralized (residual) factor we want to extract. Our next equation arises from the condition that we want our neutralized factor to be uncorrelated to our original factors. This condition takes on the following equation (this neutralizes the covariance which suffices):

$$\boldsymbol{G'}\,\boldsymbol{V}\,\boldsymbol{f}_{neut} = \boldsymbol{0} \qquad (A.4)$$

Now we have two equations and we can solve uniquely for our neutralized factor $\boldsymbol{f_{neut}}$. To solve this system, we use equation (*A*3) to write $\boldsymbol{f}_{neut} = \boldsymbol{f} - \boldsymbol{G}\,\boldsymbol{\beta}$ and plug this value into (*A*4). This gives:

$$\boldsymbol{G'}\,\boldsymbol{V}\,(\boldsymbol{f} - \boldsymbol{G}\,\boldsymbol{\beta}) = \boldsymbol{0} \;\Rightarrow\; \boldsymbol{\beta} = (\boldsymbol{G'}\,\boldsymbol{V}\,\boldsymbol{G})^{-1}\,\boldsymbol{G'}\,\boldsymbol{V}\,\boldsymbol{f}\,.$$

Then plugging in this expression for $\boldsymbol{\beta}$ in (*A*3) and solving for $\boldsymbol{f_{neut}}$, we get our result:

$$\boldsymbol{f_{neut}} = \boldsymbol{f} - \boldsymbol{G}\,(\boldsymbol{G'}\,\boldsymbol{V}\,\boldsymbol{G})^{-1}\,\boldsymbol{G'}\,\boldsymbol{V}\,\boldsymbol{f}\,. \qquad (A.5)$$

When neutralizing against one factor, i.e. the case that $\boldsymbol{G}$ consists of one vector of scores, we get our result from equation (3) in the report.

In addition, if we are neutralizing to only one variable, we can simplify the computation in equation (*A*5) so that we only have to do the algebra once. To see this, let $\boldsymbol{g}$ be the vector of factor scores for which we wish to neutralize. Then we can set a projection vector $\hat{\boldsymbol{p}}$ as:

$$\hat{\boldsymbol{p}} = (\boldsymbol{g'}\,\boldsymbol{V}\,\boldsymbol{g})^{-1}\boldsymbol{g'}\,\boldsymbol{V}\,,$$

which is just a 1x*n* vector. When neutralizing to our CT-Risk factor, we call this factor the *risk-cancellation* factor. From equation (*A*5) we can solve for $\boldsymbol{f_{neut}}$ by taking a dot product of $\hat{\boldsymbol{p}}$ and $\boldsymbol{f}$ and then multiplying this scalar product by the scores of the factor $\boldsymbol{g}$. This leads to the formula in equation (4) in the report:

$$\boldsymbol{f_{neut}} = \boldsymbol{f} - (\hat{\boldsymbol{p}}\cdot\boldsymbol{f})\boldsymbol{g}\,. \qquad (A.6)$$

An aside on factor portfolio neutralization

We also note that equation (*A*5) resembles a very commonly seen equation in the portfolio construction literature when constructing minimum risk factor portfolios (see Grinold and Kahn 2000). The difference is that the formula uses $\boldsymbol{V}^{-1}$ instead of $\boldsymbol{V}$. This is because the literature usually solves for the MVO portfolios. However, we can arrive to their equation quite simply by noting that the MVO portfolios for $\boldsymbol{G}$ can be written up to a constant (the risk aversion parameter) as $\boldsymbol{H_G} = \boldsymbol{V^{-1}}\ \boldsymbol{G}$. Then if we multiply both sides of equation (A5) by $\boldsymbol{V}^{-1}$ we get:

$$\boldsymbol{h_{neut}} = \boldsymbol{V^{-1}}\ \boldsymbol{f} - \boldsymbol{H_G}\left(\boldsymbol{H'_G}\ \boldsymbol{V}^{-1}\ \boldsymbol{H_G}\right)^{-1}\ \boldsymbol{H'_G}\ \boldsymbol{f}$$

Setting $\boldsymbol{h_f} = \boldsymbol{V}^{-1}\boldsymbol{f}$ as our MVO factor portfolio we get:

$$\boldsymbol{h_{neut}} = \boldsymbol{h_f} - \boldsymbol{H_G}\left(\boldsymbol{H'_G}\ \boldsymbol{V}^{-1}\ \boldsymbol{H_G}\right)^{-1}\ \boldsymbol{H'_G}\ \boldsymbol{V}^{-1}\boldsymbol{h_f}\,, \qquad (A.7)$$

which is the result for neutralizing a portfolio to a set of minimum-variance factor portfolios. In addition, we can use equation (A7) to neutralize any portfolio against a set of minimum-variance factor portfolios $\boldsymbol{H_G}$. Last, we note that using the Woodbury matrix identity, it is easy to show that in the case where our factors are the risk factors that are used to build the variance-covariance matrix $\boldsymbol{V}$, equation (A6) becomes:

$$\boldsymbol{h_{neut}} = \boldsymbol{h_f} - \boldsymbol{H_G}\left(\boldsymbol{H'_G}\ \boldsymbol{\Delta}^{-1}\ \boldsymbol{H_G}\right)^{-1}\ \boldsymbol{H'_G}\ \boldsymbol{\Delta}^{-1}\boldsymbol{h_f} \qquad (A.8)$$

where $\boldsymbol{\Delta}$ is the diagonal matrix of asset specific risks.

Volatility correlation scaling

In this section we derive equations (9) and (10) in the report that show how to set the correlation to risk factor.

Say that we want the correlation between the new correlation-scaled factor to be $\rho_{new,risk}$. Then following equation (3) we can write:

$$\boldsymbol{f_{new}} = \boldsymbol{f_{neut}} + \beta_{new,risk}\ \boldsymbol{f_{risk}}\,,$$

where,

$$\beta_{new,risk} = \rho_{new,risk}\frac{\sigma_{new}}{\sigma_{risk}}$$

Noting that $\boldsymbol{f_{neut}}$ and $\boldsymbol{f_{risk}}$ are orthogonal we can write the variance of $\boldsymbol{f_{new}}$ as

$$\sigma^2_{new} = \sigma^2_{neut} + \beta^2_{new,risk}\ \sigma^2_{risk}\,.$$

Then simple algebra leads to the result in equation (9) and (10).

$$\beta_{new,risk} = \frac{\sigma_{neut}}{\sigma_{risk}}\frac{\rho_{new,risk}}{\left(1-\rho^2_{new,risk}\right)^{1/2}}\,.$$

Appendix 1

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