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Who Shrunk My Tracking Error?

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Investors have struggled to reach their target levels of tracking error in the current low-risk environment. In a search for solutions, we put tracking error under the microscope, dissecting its components and their relationship with market risk. We also reveal the potential risk-related pitfalls of different portfolio construction methodologies. But the real moral to the story is that investors cannot rely on tracking error alone. The focus must be on the overall quality of the portfolio.

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Summary

Some of our clients have reported recently that their portfolio construction process is having trouble using their full target tracking budget—a potential cause for concern amongst both asset owners and consultants. In this report we discuss the reasons why tracking error will change over time and explore the impact of portfolio constraints. We show that targeting higher tracking error can reduce the quality of the portfolio and we warn investors against focusing too heavily on tracking error at the expense of other considerations, such as unwanted portfolio exposures. Active risk is closely tied to market risk, and in a low-risk environment, such as we currently see in the US, investors may have to live with lower tracking error than they have been used to in previous years. Investment managers should be focusing on the complete picture for their portfolios. This will include achieving the highest possible information ratio and transfer coefficient, having a well-balanced and diversified portfolio, and maximizing tilts as much as possible toward the bets for which the manager has a view, while avoiding those where there is no expected payoff. The single figure called "tracking error" does not tell the whole story of the portfolio.

Below, we examine some of the reasons behind the difficulty in reaching target levels of active risk and discuss the implications. But first, let's look at a simple backtest to illustrate the issue.

We ran a monthly backtest from the end of December 2001 through June 2013 (12.5 years) for an optimized active portfolio based on the FTSE Developed universe, tilted toward an average of Axioma's Value and Medium-Term Momentum factor exposures (henceforth we will just refer to this tilt as Momentum). Using Axioma's Worldwide fundamental medium-horizon model we included the following relatively loose constraints:

- Active stock, sector and country weights of +/- 3%
- Active factor exposures (excluding Value and Momentum) of +/- 0.2 standard deviations
- Two-way turnover of 20% per month
- Active risk limit of 5%[1]

Figure 1 shows the ex-ante tracking error for our test portfolio. We were only able to reach our tracking error goal about one third of the time. There were also long stretches of time when tracking error was well below the 5% target.

Figure 1. Base Case Scenario Ex-Ante Tracking Error

Source: FTSE, Axioma, Inc.

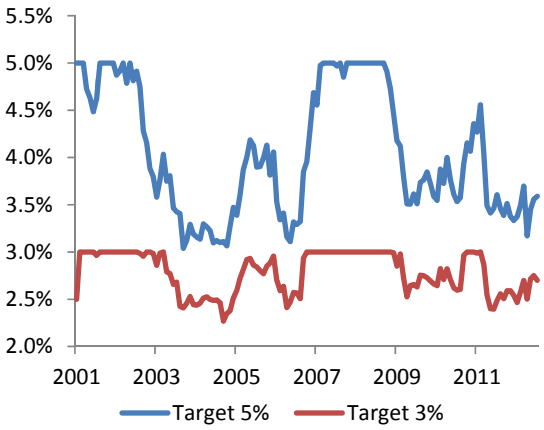
Our inability to reach the target active-risk level was not just a function of the relatively high target tracking error (5%). We ran the same test using 3% as the active risk maximum and setting the constraints at 60% of their levels in the 5% test. While we hit the target more frequently, and the minimum ex-ante tracking error over the period was closer to the target, the target tracking error was not reached for long stretches of time (Figure 2). Moving even farther down the active-risk scale resulted in smaller deviations from that target, but we still saw stretches of time where the target was not reached. The rest of our analysis will focus on tests using the 5% limit, but the conclusions remain

[1] Note that an alternative (and commonly used) formulation is to also include risk as a term in the objective function. This has the advantage that the risk model will still play a part in the solution even when the target risk level has not been reached. For illustration purposes we have not included risk in the objective term here.

the same regardless of one's target tracking budget. We also note that realized tracking error over the full test period was even lower than the ex-ante tracking error.[2]

Figure 2. Comparison of Results at Different Levels of Target Risk

	Target Tracking Error	
	5%	**3%**
Constraints		
Country. Sector, Asset	3%	1.80%
Factor	0.2	0.12
Turnover	20%	12%
Results		
% of months at target	23%	40%
Average ex-ante	4.04%	2.78%
Minimum TE	3.04%	2.27%
Minimum/Target	60.8%	75.6%
Realized TE	3.56%	2.48%



Source: FTSE, Axioma, Inc.

Fixed Exposure or Fixed Tracking Error? Some managers, including many quantitatively driven managers, build portfolios by targeting a fixed level of tracking error. By targeting a specific level of tracking error the portfolio's exposures will vary over time. Other managers will target a fixed level of exposure, such as "a one standard deviation exposure to Value". Although they may not be as explicit, fundamental-manager strategies are similar to fixed-exposure strategies, in that they will seek to have as much exposure as possible to the areas they view as adding value. Finally, some managers may include risk aversion in their objective function (rather than using a tracking error constraint). This risk aversion may vary with the overall level of market risk, or it may stay fixed.

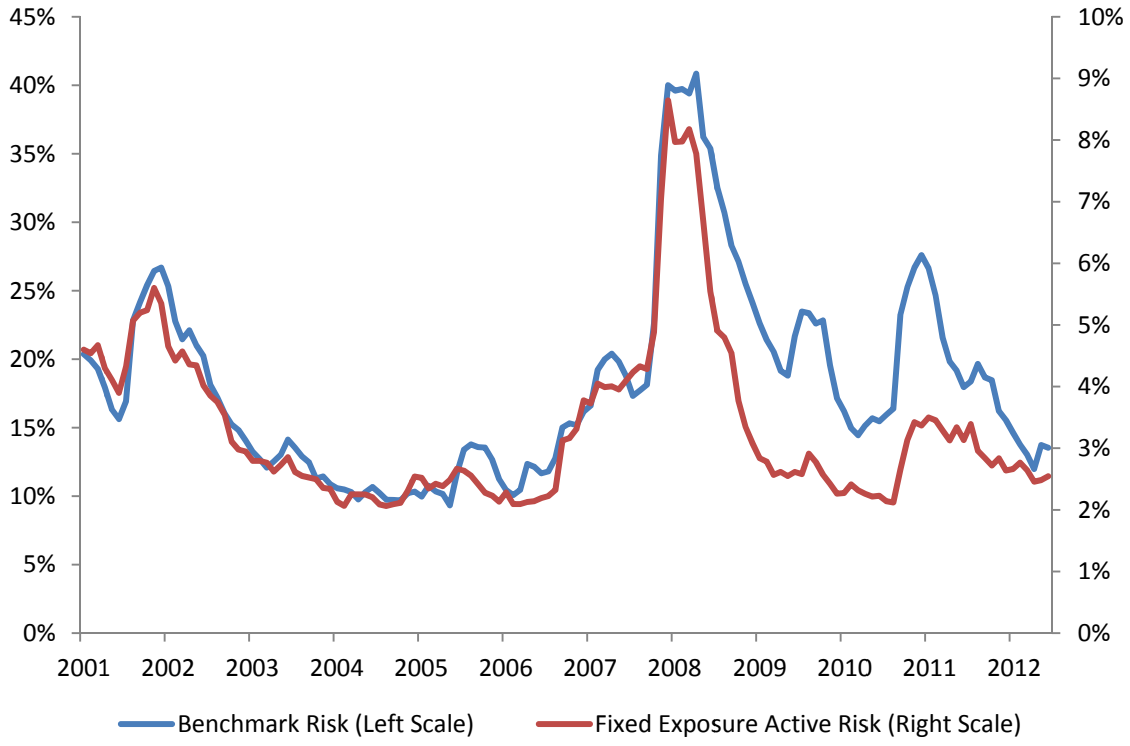
Portfolio managers who (1) target a fixed level of exposure to their investment signal, (2) use a constant "risk aversion" in their utility function if optimizing, or (3) adopt a fundamental approach to building their portfolio, will observe that the tracking error of their portfolio varies over time. The tracking error will be predominantly a function of overall market volatility, though it can also change according to stock selection and the volatility of particular investment styles, along with its correlation with other

[2] Although realized tracking error was lower than forecast, our tests indicate that the model was unbiased.

market factors. To illustrate this, we ran our backtest again, using *fixed* exposures to Value and Momentum[3].

Figure 3 shows a plot of the ex-ante tracking error for our fixed exposure portfolio and compares it to the risk of the FTSE Developed Index. Clearly, the active risk of a portfolio with fixed exposures will vary widely and the level is correlated with the overall level of market risk. In fact, when risk was highest the tracking error far exceeded our previous 5% limit.

Figure 3. Active Risk vs. Market Risk, Fixed Exposure Portfolio



Source: FTSE, Axioma, Inc.

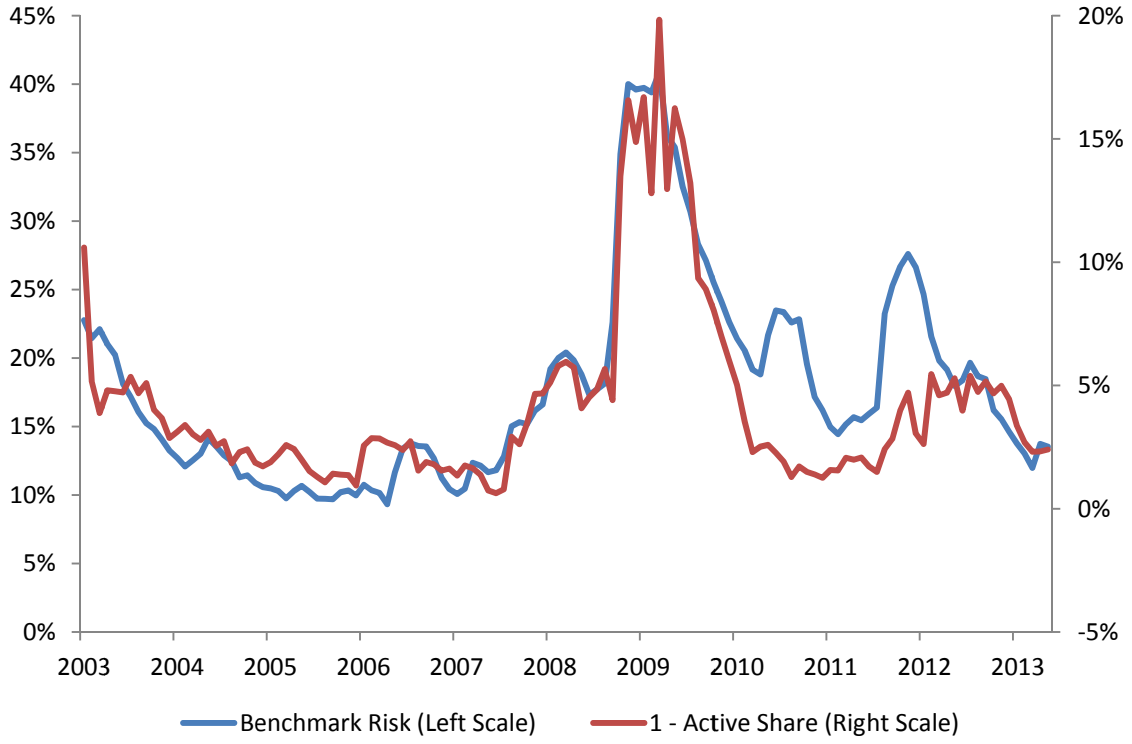
Alternatively, some investors prefer to target a fixed tracking error, as we did in our initial test, which attempts to keep the relative risk of their investment process constant through time. Since volatility changes over time, managers targeting a fixed tracking error are effectively varying the magnitude of their bets (along with the mix of assets) to achieve a constant tracking error.

The magnitude of active weights or "active share" (the sum of the long-active positions) of an active portfolio is a common measure of the size of a manager's bets. Active share will range from zero (in the case where the portfolio is equal to the benchmark) up to a maximum of 100% (in the case where none of the portfolio assets is in the benchmark). In Figure 4 we plot 1 minus the active share of an

[3] The level of exposure was set at the average exposure to the two "alpha" variables from our initial test.

unconstrained Value and Momentum portfolio with a constant 5% tracking error[4]. As expected, during the period of higher risk the active share was lower, as the portfolio needed to take smaller bets to avoid exceeding the tracking error target of 5%. When overall risk was lower the active share was higher.

Figure 4. (1 - Active Share) vs. Market Risk



Source: FTSE, Axioma, Inc.

The question of which of these approaches is "better" is a subject for another discussion. Having a constant tracking error is attractive from a risk-budgeting perspective, but offers no guarantee that the information ratio will remain constant. On the other hand, if there is information in the dispersion of your signal then there may be value in letting this (or some other external signal) dictate the level of risk allocated.

Components of Tracking Error. Examining the drivers of tracking error may help us better understand the issue. Tracking error, or "active risk", is calculated as the volatility of the active portfolio. For a long-only portfolio with no leverage, the active portfolio is effectively a long-short portfolio with weights that sum to zero. Tracking error is computed as:

[4] We use (1 – active share), rather than the active share, to better highlight the correlation between the size of the bets and benchmark risk.

$$tracking\ error = \sqrt{\underbrace{(w-b)^T X \Omega X^T (w-b)}_{\text{variance}} + \underbrace{(w-b)^T \Delta^2 (w-b)}}$$

Where:

w = portfolio weights

b = benchmark weights

X = factor exposures for each asset

Ω = factor covariance matrix

Δ^2 = stock-specific variance

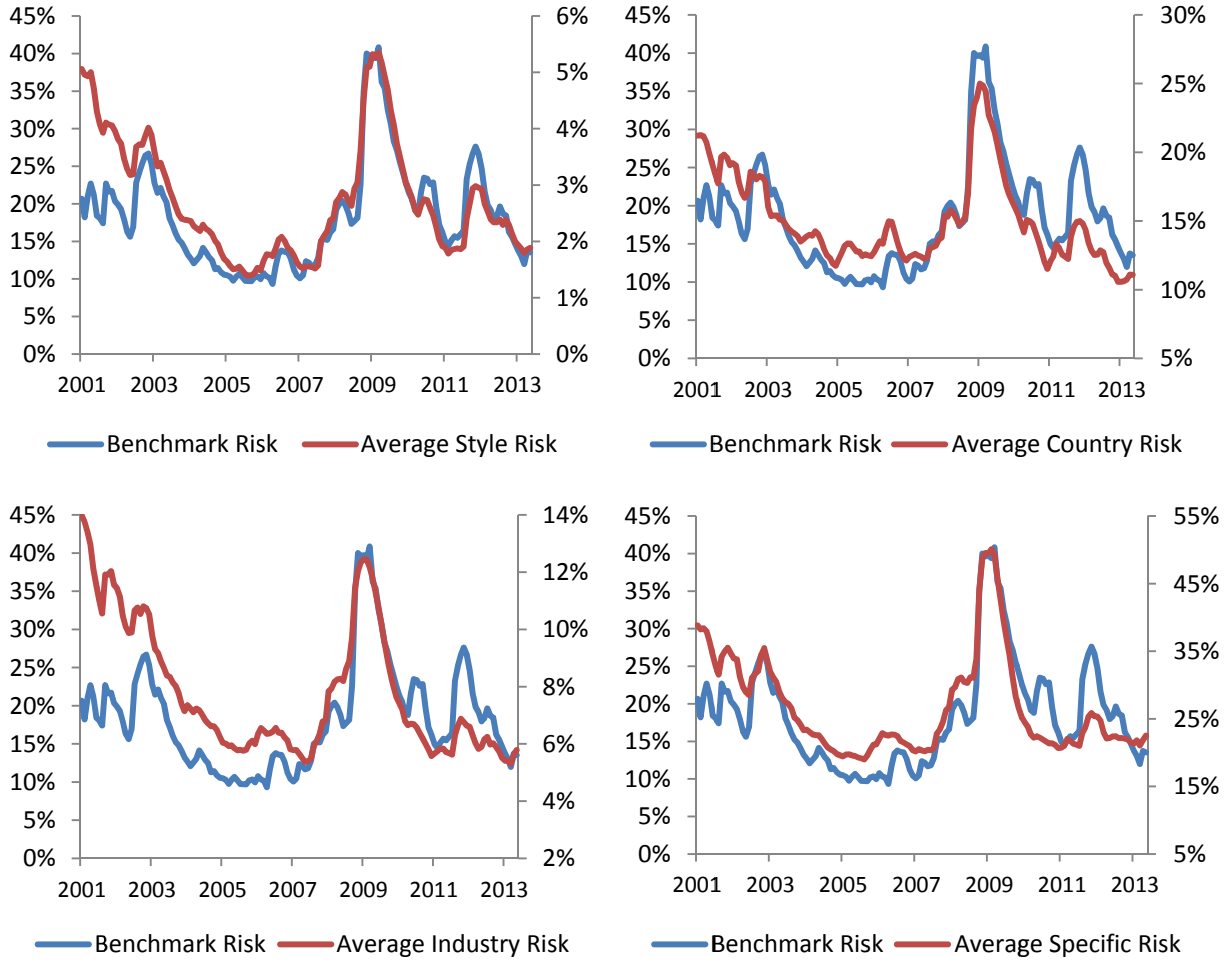
From this formula we can see that there are number of scenarios in which tracking error may fall (rise), all else being equal. Some of these include:

- If the magnitude of active weights decreases (increases)
- If the factor-specific or stock-specific volatilities decrease (increase)
- If the net factor exposures decrease (increase)
- If implied correlations between the long and short assets increase[5] (decrease). This may be due to a change in factor correlations or a decrease in relative contribution of stock-specific risk.

All Risk Model Components Are Impacted By Changes in Market Risk. Stock-specific risk is typically a larger proportion of active risk than it is of total risk because the exposures to common market factors can be hedged by the short positions to reduce the active factor risk. Although it is possible for active portfolios to have very little exposure to the market (i.e., a beta close to zero) this does not imply that their risk is immune to changes in market level volatility. In fact, decreases (increases) in market level volatility tend to be reflected in lower (higher) volatility across the entire factor structure, as well as in the level of stock-specific volatilities, as we show in Figure 5. Therefore changes in overall market volatility tend to be the primary driver of changes in tracking error.

[5] When we refer to "long and short assets" we mean the *active* portfolio, which is long and short around the benchmark.

Figure 5. FTSE Developed Risk (Left Scale) vs. Style, Country, Industry and Specific Risk (Right Scale)



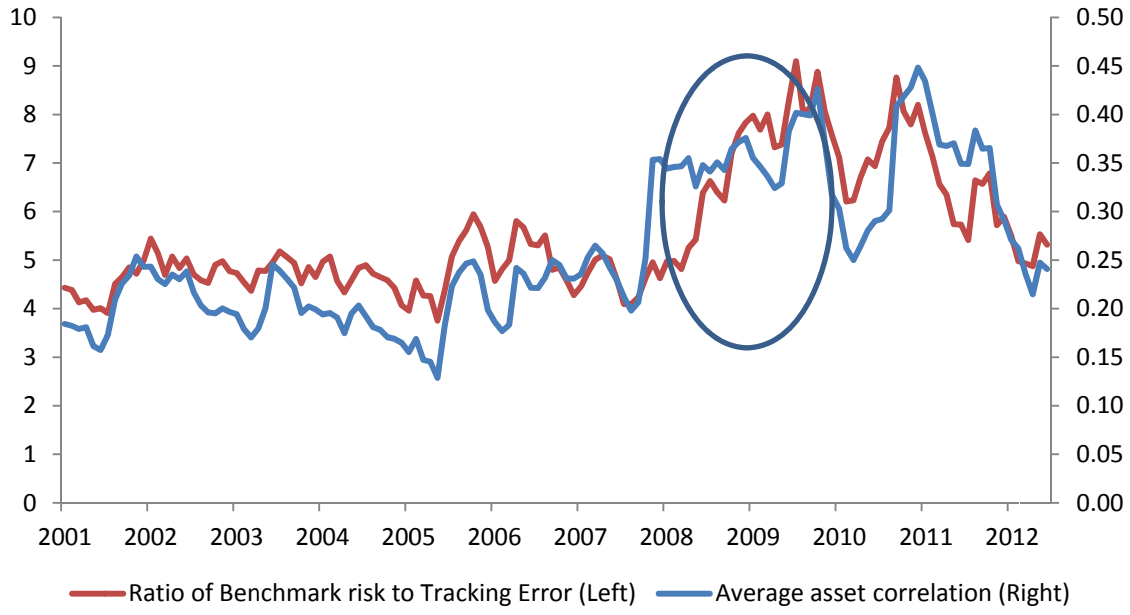
Source: FTSE, Axioma, Inc.

Higher Asset Correlations Mean Lower Tracking Error. Correlations, of course, do have a part to play in all this. Higher *asset-level* correlations imply that common factors are driving stock returns; hence there is a smaller proportion of stock-specific risk (relative to factor risk) in the market. This, in turn, may contribute to lower tracking error. Higher correlation *between factors* can also reduce the factor component of active risk as short exposures become a better hedge for long exposures. It is typically after high volatility events, while correlations remain high but volatility is falling, that we see the impact of this.

Referring back to Figure 3, we notice that the relationship between tracking error and market risk is not constant (tracking error falls faster than benchmark risk after 2008). Figure 6 illustrates that the majority of the difference can be explained by rising correlations over this period. *Increased asset correlations in*

recent years have contributed to the reduction in active risk. We note that recent levels of correlation in the market seem to have reverted towards the longer term average, which has reduced this effect.

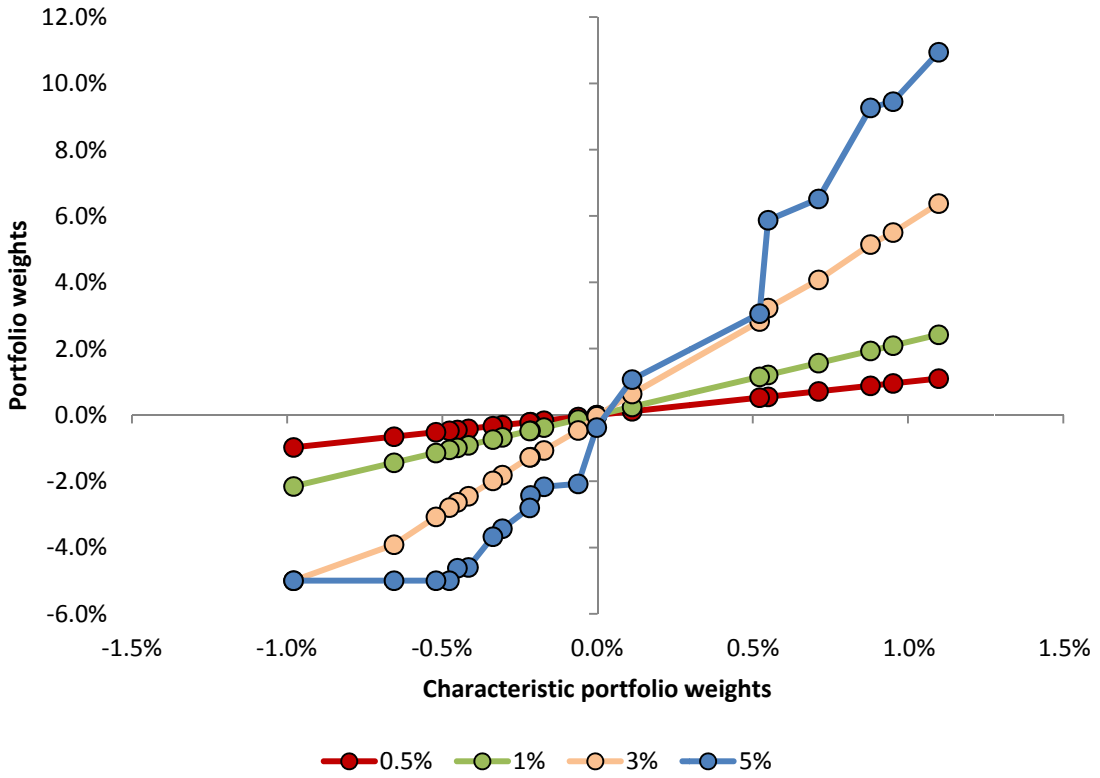
Figure 6. Impact of Asset-Asset Correlations



Source: FTSE, Axioma, Inc.

The Impact of Constraints. As previously mentioned, the primary way to control tracking error in the presence of changing levels of market risk is to scale the size of the bets that are being taken. When optimizing an active portfolio, there is actually a single optimal long-short combination of assets that will be scaled in exact proportions to achieve the required risk—up to a point. One implicit constraint on the active long-short portfolio is that the short positions can only be as large as the weights of stocks in the benchmark. This limits the extent to which active risk can be scaled by leverage alone. As the portfolio tries to take increasingly large positions, the "shape" of the characteristic active portfolio will bend as it tries to accommodate this constraint. Of course there are usually numerous other investment-related constraints—regulatory, client imposed or manager imposed—that compound this problem. A simple visualization of this scaling effect is shown for an active portfolio based on a 20-stock equally weighted benchmark in Figure 7. Assets weights increase in size proportionally up to the point where the constraints begin to bind. You will observe that weights for the 1% tracking portfolio are held in the same proportions as the 0.5% portfolio (just with a larger active share). Weights for the 5% tracking portfolio are distorted as the optimizer tries to increase exposure while accommodating the constraints. The more constraints that are imposed, the more significant this becomes.

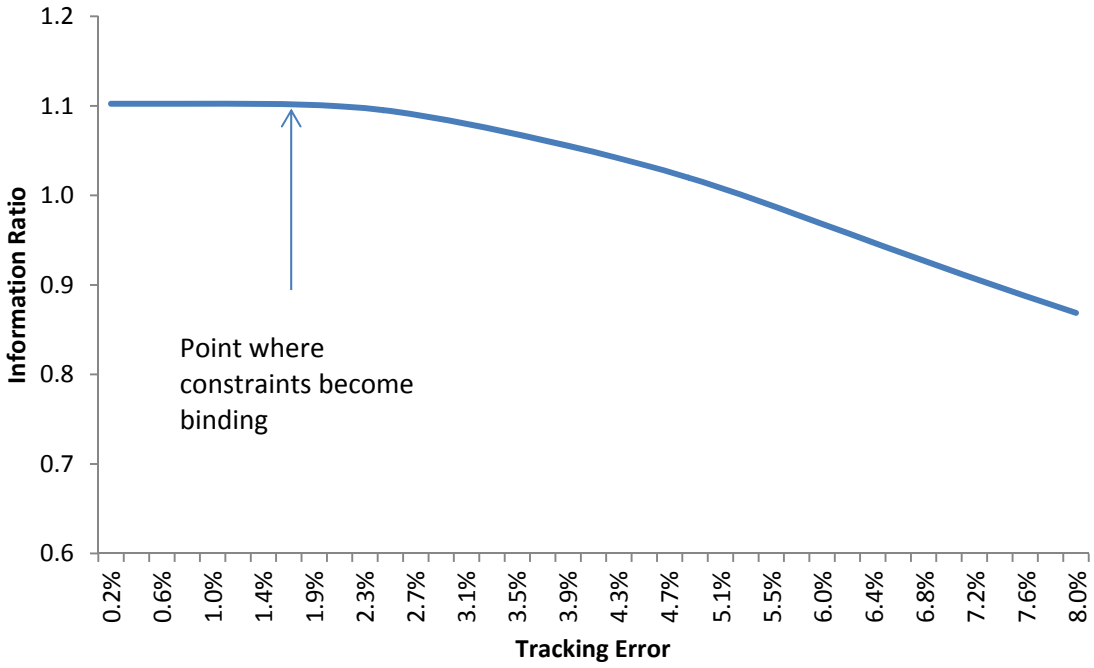
Figure 7. The No-Short Constraint Drives the Allocation to Overweight Positions



Source: Axioma, Inc.

Targeting Higher Tracking Error May Reduce the Information Ratio (Due to Constraints). One consequence of the impact of constraints is that the expected return per unit of risk (i.e., the ex-ante information ratio) will start to degrade after the point the constraints become binding. In other words the potential reward for taking on more risk diminishes in the presence of constraints as more aggressive risk targets are pursued. This implies that there is an optimal level of tracking error for a given strategy, which is illustrated in Figure 8. The contribution of the long-only constraint to this effect gave rise to the popularity of the 130/30 strategy, which relaxes the implied index weight constraint for long-only portfolios and allows managers to more fully reflect their negative views.

Figure 8. Information Ratio as Tracking Error Increases

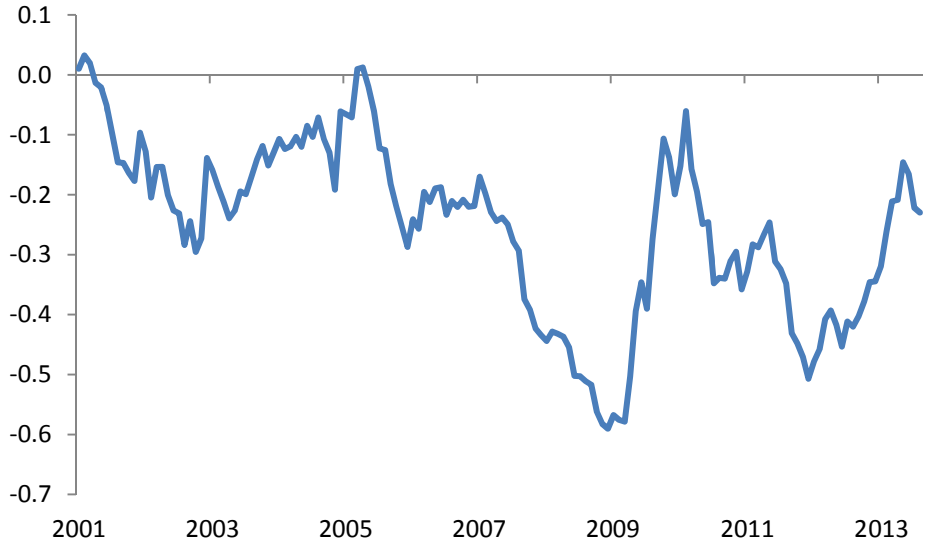


Source: FTSE, Axioma, Inc.

The Interaction Between Constraints and Correlations. Constraining factor exposures can problematic when the alpha signal becomes correlated with risk factors. These correlations can vary over time and therefore need to be monitored so the impact of style constraints can be understood. As we see in Figure 9, the exposure correlation between our alpha components, Value and Momentum, has ranged from roughly zero to -0.56 over our test period. Zero correlation is a desirable characteristic as it means the two signals are independent and do not simply represent a bet on the same stocks. While the median correlation between both Value and Momentum and the other factors hovered around zero, there were wide variations among some of the factors. For example, Momentum's correlation with Size was generally positive, but ranged from no correlation to a high of about 0.4 (Figure 10). And Momentum's correlation with Size generally moved opposite from its correlation with Volatility (and also opposite to Value's correlation with Size). Momentum's correlation with Volatility showed an even wider range, from -0.65 to +0.53, with changes occurring quite rapidly. The magnitude of Value's correlations with Size and Volatility was somewhat smaller, but also quite changeable through time. So why does this matter? If you are constraining these risk factors then there will be times when your alpha exposure will be reduced because of this correlation (e.g. when value is highly correlated with volatility you will get less value exposure if you are constraining volatility). Because the magnitude of your alpha tilt is reduced then the tracking error is also likely to fall and the transfer coefficient will almost certainly go down as well. On the other hand if you are not constraining these factors then you are inheriting biases in your portfolio that may not be deliberate. At Axioma we have devoted a large research effort to better understanding the interaction between alpha factors, risk factors and constraints in the

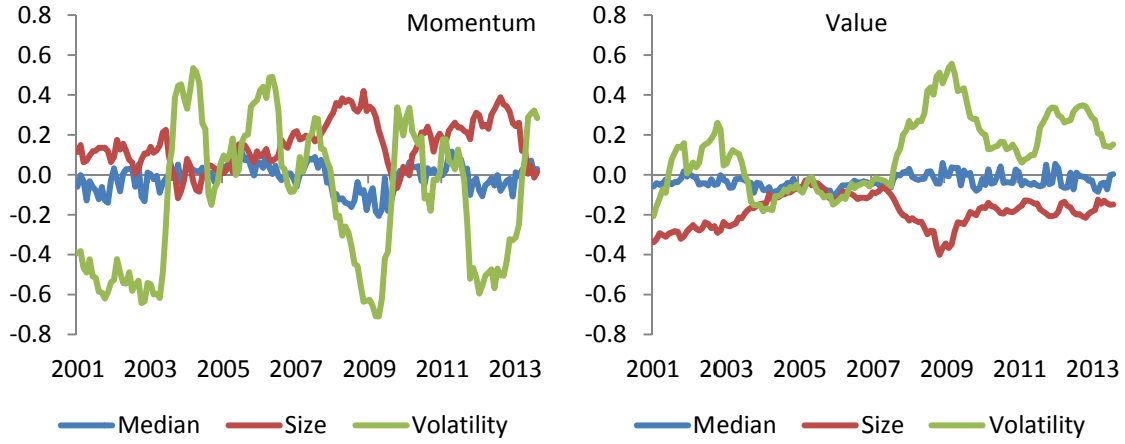
portfolio construction process. We refer you to our latest research paper[6] on this topic, which describes a consistent approach to portfolio construction where the interaction between these three components is taken into account from the outset when constructing and combining the alpha signals. This results in more intuitive portfolios with fewer "unexpected" side effects from constraints or the risk model..

Figure 9. Median Exposure Correlation of Value and Momentum



Source: Axioma, Inc.

Figure 10. Value and Momentum Exposure Correlations with Select Other Factors



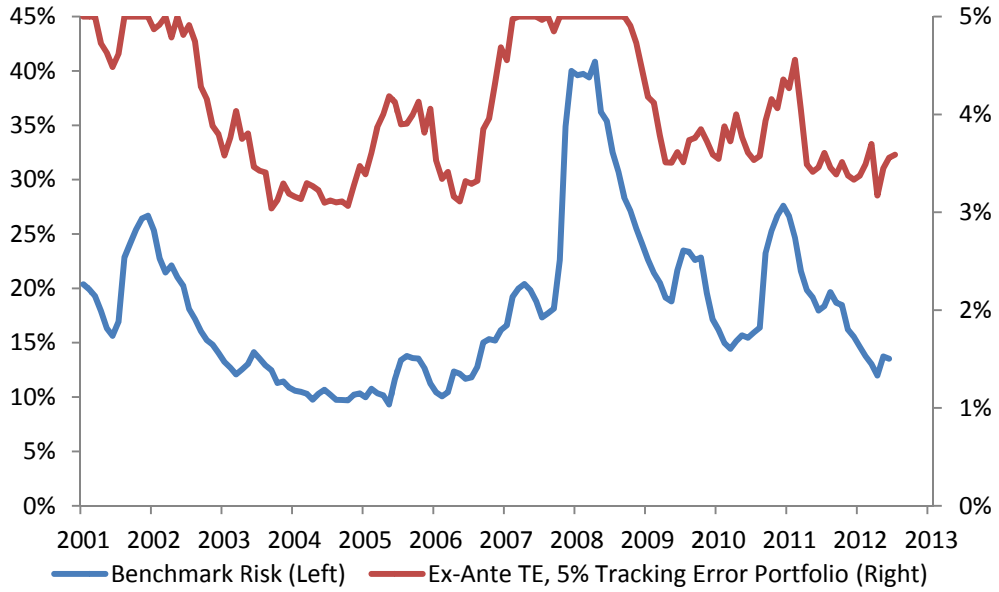
Source: Axioma, Inc.

[6] See, Sebastian Ceria, Kartik Sivaramakrishnan, and Robert A. Stubbs, "**Alpha Construction in a Consistent Investment Process",** Axioma Research Paper number 45, June 12, 2013.

Returning to our sample portfolio, we see that with all constraints in place the portfolio only reaches the risk limit of 5% in periods of high volatility (Figure 11). Clearly, constraints are preventing the portfolio from taking the magnitude of exposures it needs to achieve the required tracking error. Relaxing each of the constraints in turn shows that the most binding constraint is the factor-exposure constraint, followed by the active-stock-weight constraint (Figure 12)[7].

We see a similar impact when we look at the transfer coefficient—a measure of how well our alpha is reflected in the portfolio. Relaxing the asset and factor constraints leads to an improvement in the transfer coefficient most of the time, whereas relaxing the turnover constraint during the high volatility period of 2008 to 2011 also resulted in a higher transfer coefficient (Figure 13).

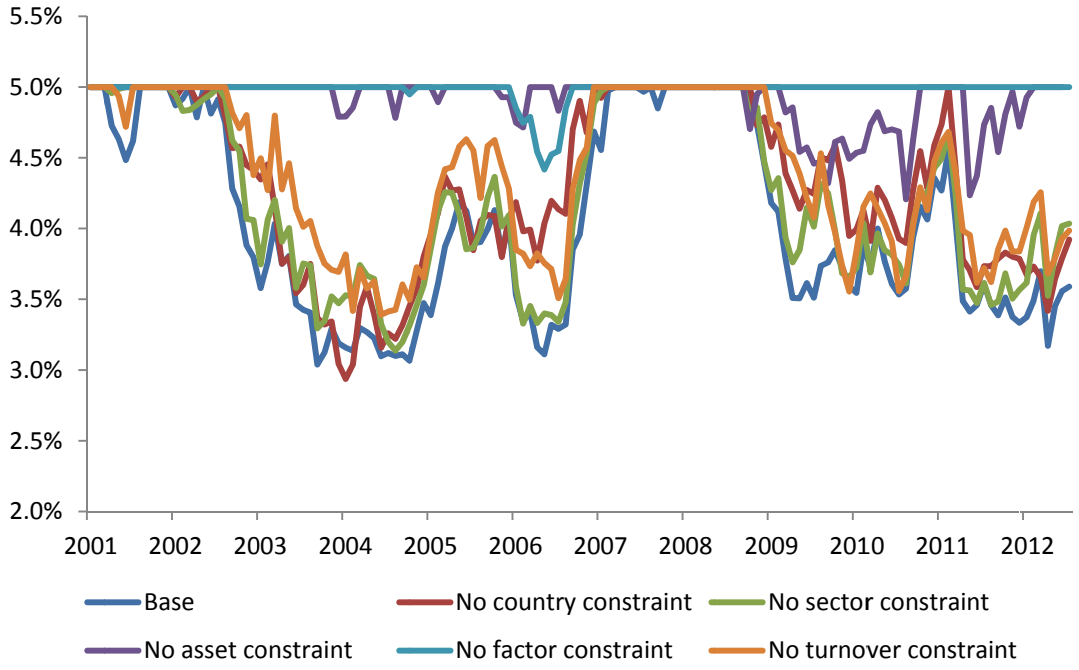
Figure 11. Base Portfolio Tracking Error vs. Market Risk



Source: FTSE, Axioma, Inc.

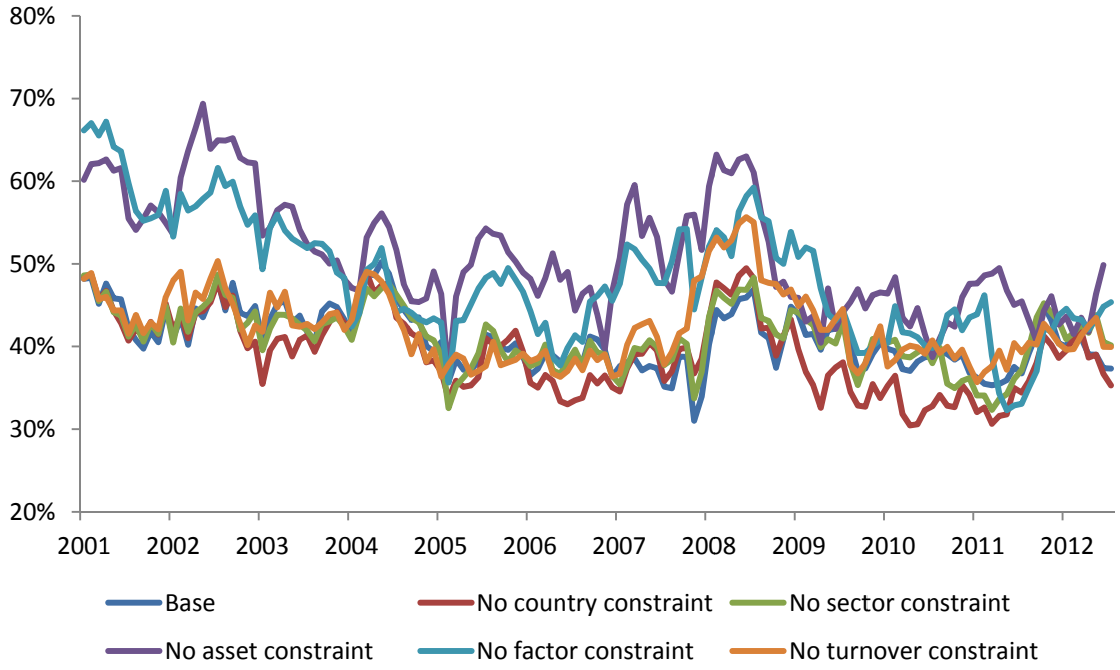
[7] While these tests were run for a global portfolio, results were quite similar for various regional portfolios

Figure 12. Ex-Ante Tracking Error, Portfolios with Constraints Relaxed



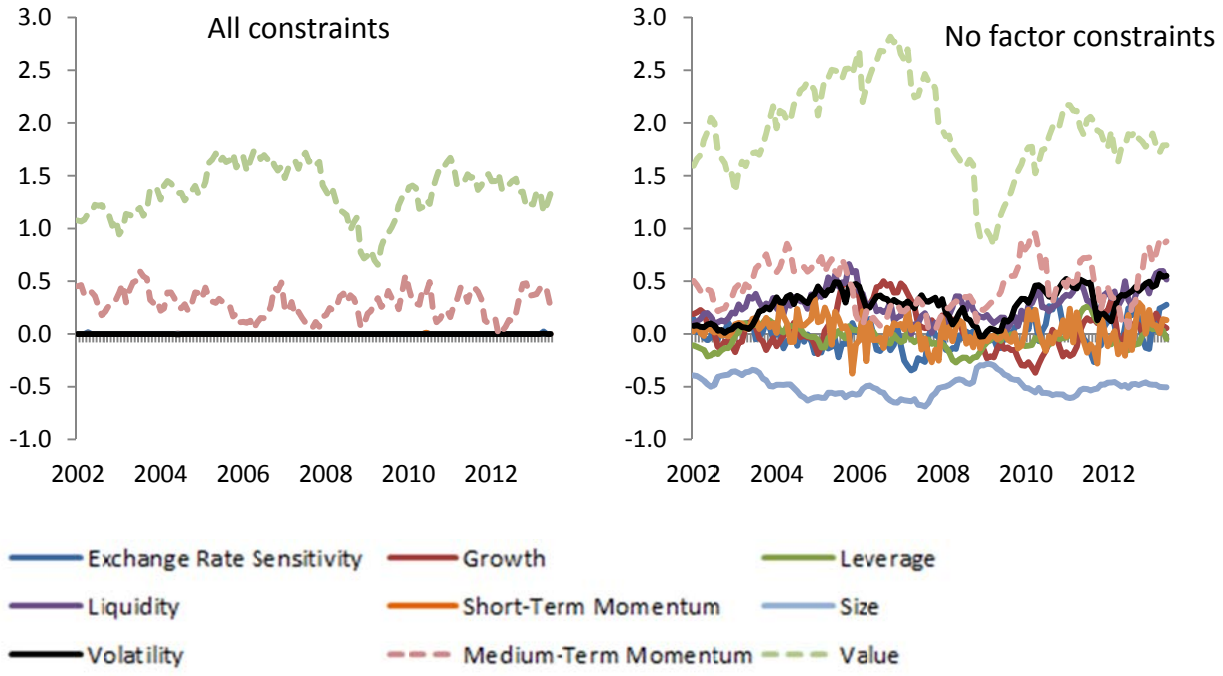
Source: FTSE, Axioma, Inc.

Figure 13. Transfer Coefficient with Constraints Relaxed



Relaxing Constraints to Boost Active Risk Can Lead to Unwanted Exposures. While the wisdom of relaxing constraints purely for the sake of increasing tracking error is questionable, like all parts of the investment process, any constraints should be well researched and intuitive. Factor exposures (both with and without the active-factor constraint) are shown in Figure 14. Constraining factor exposures is a sensible thing to do for the manager who is taking deliberate exposure to certain factors and wants to protect the portfolio from the influence of factors on which they have no view. As we see in the chart, unconstrained factor exposures can be quite significant (note the small cap and high volatility bias).

Figure 14. Factor Exposures, Factor Constrained vs. Unconstrained



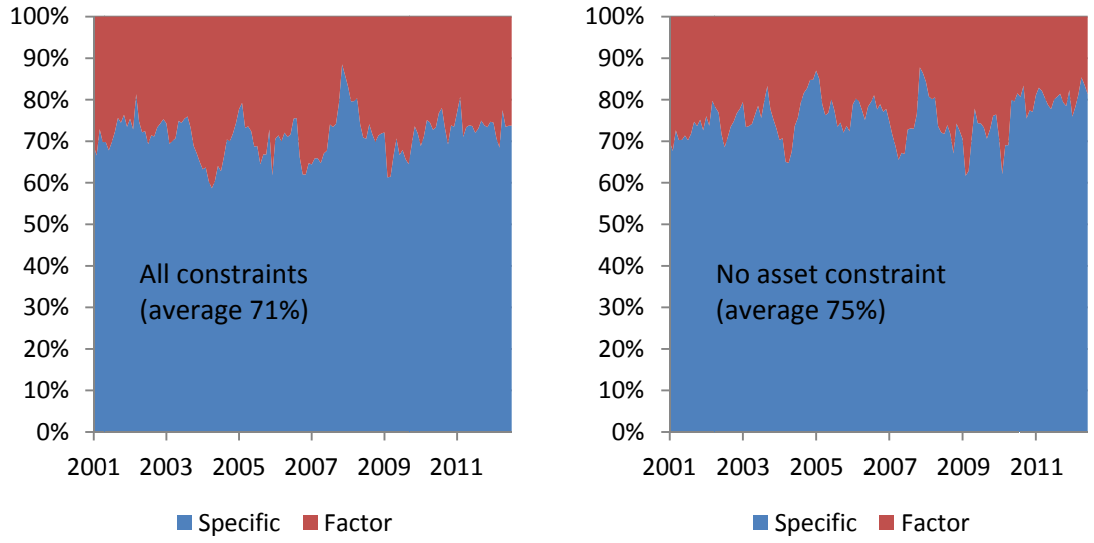
Source: FTSE, Axioma, Inc.

As an aside you will also notice that, despite the fact that value and momentum were weighted equally in the alpha signal, the resulting exposures were not equal (Value having considerably higher exposure than Momentum). This is partly a result of the risk model penalizing Momentum more highly than Value because its volatility is higher. This is not necessarily a problem but ideally it should be deliberate rather than just a side effect of the risk model and optimization process. Once again, the "Consistent Portfolio Management" approach we referred to earlier addresses this by giving investors more direct control over their target investment weightings.[8]

[8] See, Sebastian Ceria, Kartik Sivaramakrishnan, and Robert A. Stubbs, "**Alpha Construction in a Consistent Investment Process",** Axioma Research Paper number 45, June 12, 2013.

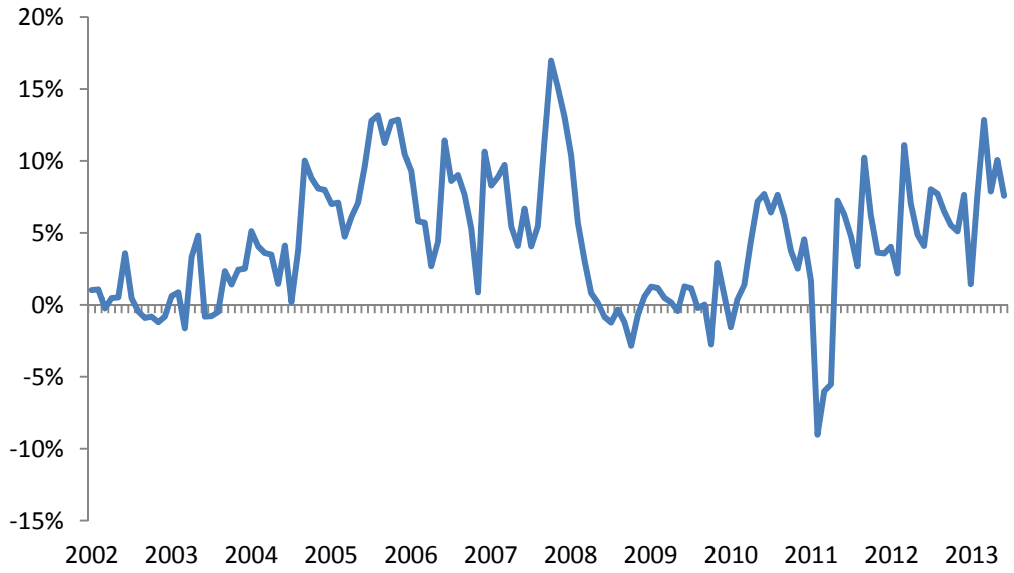
Looser Asset Constraint Means Higher Proportion of Stock-Specific Risk. Diversification of stock-specific risk applies in the calculation of active risk just as it does for total risk because the same assumption of zero correlation across stock-specific returns still applies. The most diversified active portfolio (in terms of stock-specific risk) will have a large number of small active weights. Active portfolios often have a large number of positions since every stock that is not held at benchmark weight becomes an active position. However, merely having a large number of active positions does not guarantee diversification, i.e., the size of the positions is also important. As the active variance is a function of the square of the active weights, any large active weights will dominate the overall variance, reducing the diversification benefit. Removing the active stock limit can therefore have the effect of increasing the proportion of stock-specific risk (Figure 15). Although the average difference in the proportion of stock specific risk looks relatively small, there were periods of our test history when the difference was quite large. For example, with no asset constraint the proportion of stock-specific risk was almost 17% higher than in the constrained portfolio in September 2007 (Figure 16).

Figure 15. Proportion of Stock-Specific Risk, With and Without an Asset Constraint



Source: FTSE, Axioma, Inc.

Figure 16. Difference in Percent of Stock Specific Risk, "No Asset Constraint" Portfolio Minus Base



Source: FTSE, Axioma, Inc.

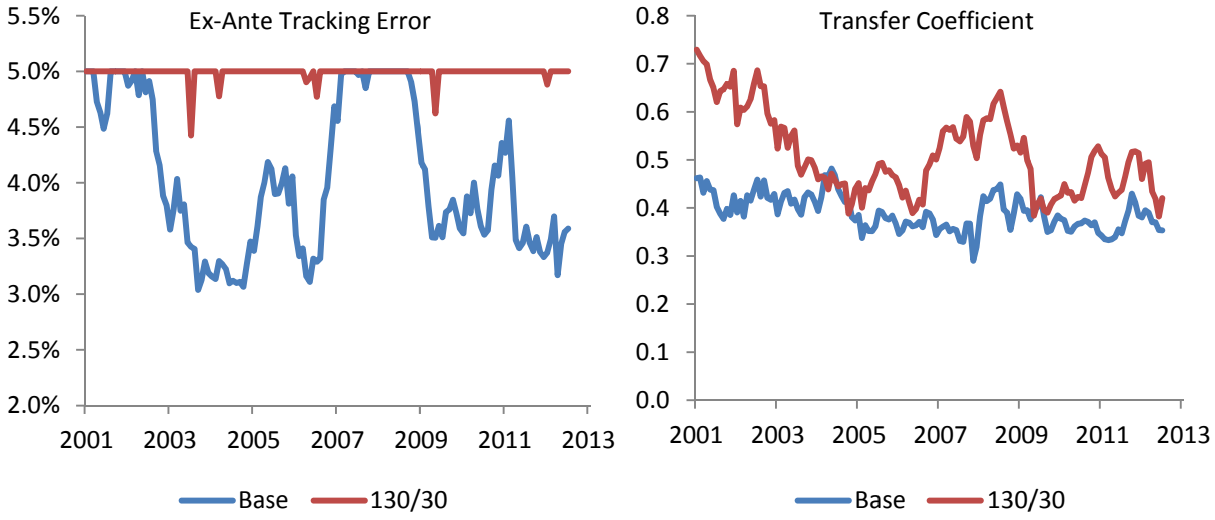
Managers May Have to Live With Lower Tracking Error In Low Risk Periods. So, what conclusions can we draw? As market volatility shrinks so will tracking error, and managers may find themselves having trouble reaching their targets. Constraints will also impact the outcome and the more constraints or tighter they are, the more difficult it will be to attain the desired tracking error. Lastly, the correlation of risk factors will also have an impact. Managers cannot do anything about the level of market volatility or factor correlations. Loosening constraints can go a long way to alleviating the problem of reaching active risk targets, but there is a tradeoff.

Looser asset constraints can lead to more concentrated portfolios, which may increase stock-specific risk. Looser factor constraints may result in unwanted factor bets that potentially have a big impact on returns, yet have nothing to do with the manager's process. Other constraints may not even lead to higher tracking error when relaxed, but have potential consequences as well. In our tests relaxing country, sector and turnover constraints had little impact on tracking error but led to potentially very large active weights (Figure 17) or unacceptably high turnover. However, when we ran the base-case scenario allowing up to 30% of the portfolio to be short (a 130/30 strategy), we reached, or just about reached, the 5% target in every period, while also generally achieving a higher transfer coefficient than we did in the base case (Figure 18).

Figure 17. Maximum Active Sector and Country Weights When Constraints Were Removed

Source: FTSE, Axioma, Inc.

Figure 18. Ex-Ante Tracking Error and Transfer Coefficient, Base Portfolio vs. 130/30



Source: FTSE, Axioma, Inc.

For managers who are able to hold short positions, a short extension is the obvious way to scale up risk in a balanced way without degrading information ratio. Other constraints will still impact the ability to do this, though constraining active short positions to benchmark weight is often one of the most binding constraints.

While a constant level of tracking error may be desirable, it may not always be achievable, or even always appropriate. It is probably unrealistic to expect to scale your bets to be twice as large because prevailing levels of risk in the market have halved. In the presence of constraints, scaling tracking error

can also degrade the information ratio because the constraints cause the portfolio to become unbalanced.

It could be that some level of education is required on this topic amongst asset owners. In many cases it is the asset owners (or asset consultants) who make demands on active managers to "be more active". This is not to say active managers should be closet index huggers (far from it) but it needs to be recognized that the volatility of various investment styles will vary throughout the market cycle. Just because a manager's tracking error is half what it used to be does not necessarily mean that manager is only half as active now, or that active returns will be only half of what they used to be. Asset owners and consultants using past returns and information ratios as guides to hiring and firing managers should also be aware of the risk environment in which those returns were generated, and how the manager has adopted his or her process to the changes in risk.

The interaction between investment constraints and correlations should be monitored. If your alpha signal becomes correlated with other factors that you are constraining, then this will potentially have an impact on both your alpha exposure and active risk. The investment environment is not static—relationships that might have held true during a backtest may not persist going forward—and it is natural that an investment process should evolve as market conditions change.

The bottom line is that rather than trying to boost tracking error purely for the sake of taking more risk, a better strategy is to focus on the overall quality of the solution. The expected information ratio, the alpha exposure, the balance and diversification within the portfolio, the appropriate risk level for the prevailing market environment, the interaction between alpha and risk factors, the level of dispersion in the alpha signal—all are important and not adequately characterized by a single active risk estimate.