LEHMAN BROTHERS

Fixed Income Research

Quantitative Management of Bond Portfolios

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SUMMARY

- We describe our decade-long experience interacting with investors on issues spanning the entire process of quantitative fixed-income portfolio management: asset class analysis and asset allocation, benchmark selection and customization, formulation of investment guidelines and strategies, analysis of portfolio and benchmark composition and risk, and performance attribution.
- We present models and techniques developed to support decision making at every step in the investment process and illustrate them with examples.
- We discuss methods of constructing proxy portfolios that closely track broad indices but contain relatively few securities. We also present synthetic index replication strategies using futures contracts or swaps.
- We illustrate portfolio optimization strategies for central banks and other investors that operate within strict risk limits These strategies are designed to consistently outperform the benchmark while controlling risk.

The authors thank Jack Malvey for the motivation behind this overview and Ravi Mattu for detailed discussions of its content.

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I. INTRODUCTION

For more than a quarter century, Lehman Brothers has been the dominant provider of domestic and global bond market indices. The broad acceptance of Lehman indices and the company's focus on adding value for fixed-income portfolio managers create unique opportunities for a quantitative portfolio strategies group. At Lehman Brothers, we have at our disposal a broad array of tools for meaningful analysis of portfolios relative to indices: security databases and calculators, coupled with portfolio-level models and analytics. The broad penetration of Lehman indices adds to the credibility of our portfolio analysis tools with investors who appreciate the ability to evaluate their holdings in a framework consistent with their benchmark. The benefits of such consistent evaluation are self-evident: identical pricing of the portfolio and the benchmark eliminates noise in performance attribution, identical security-level analytics ensure that durations and convexities are comparable, and consistent yield curve assumptions allow meaningful comparisons of scenario analysis results.

Two decades ago, only pension funds were commonly using the concept of a broad-based market-capitalization-weighted benchmark for measuring portfolio performance. Consultants advised their clients on the appropriate choice of both benchmark and asset manager. Performance of other portfolios was evaluated relative to peer groups or compared with short-term yield targets such as LIBOR. Over the past decade, we have witnessed a complete transformation in investors' approach to performance benchmarking and risk measurement. Today, there is hardly an investment manager left without a market-observable benchmark, either externally mandated or self-imposed.

While many investors use standard published indices, some portfolios require customized benchmarks. Insurance companies increasingly look for benchmarks that reflect both their asset mix and their liability structure. Many supranational institutions have devised benchmarks that represent their actual "no view" investment strategy. This ensures continuity of the investment process through management changes. Central banks put elaborate effort into designing strategic and tactical benchmarks with appropriate duration targets, currency mixes, and, most recently, allocations to spread product. Their primary objectives are liquidity of reserves and capital preservation; return maximization comes second. Even issuers have begun to form benchmarks of secondary market debt, both their own and those of appropriate peer groups. Today, Lehman publishes more than a thousand customized indices. Each is produced on behalf of an institutional investor who sought our advice on benchmarking the investment process.

Bond portfolio management is the art of decision making. Rewards come to those who possess and successfully integrate experience from several fields. At different stages, the process may require knowledge of economics and political analysis, business and financial markets, or international relationships and domestic social trends. One thing is certain, though: the increasingly quantitative nature of fixed-income markets has made portfolio analytics indispensable.

We share with investors analytical models and portfolio tools that address the entire core set of activities present in practically any portfolio management context. At the outset of most investment programs, sponsors decide on the performance benchmark and strategic asset allocation, determine their tolerance for risk, and establish guidelines on security selection and investment style (e.g., yield curve timing versus sector allocation). The day-to-day management of benchmarked portfolios requires numerous tactical decisions that must be supported by ongoing analysis. The types of portfolio analysis involved include:

- Monitoring differences in structural composition ("profiling") of the portfolio and benchmark;
- Quantifying the risk arising from such differences;
- Projecting future performance based on yield curve and credit spread forecasts; and
- Attributing the achieved returns to views reflected in the portfolio.

We approach these issues from a unique perspective. Although we are a research group at a broker/dealer, we focus on the problems facing portfolio managers rather than those facing traders. Interaction with a diverse group of investment professionals and researchers from both sides of the Street has helped direct our attention to many interesting research topics and strategies.

This report represents a survey of quantitative portfolio management techniques that we have developed and applied successfully over the past ten years in servicing investors benchmarked against Lehman Brothers bond indices. The applications of these techniques range from top-level asset allocation decisions down to understanding how a particular security affects portfolio performance. Our activities span the entire process of fixed-income portfolio management and can be subdivided into three main categories. We describe our capabilities to carry out analysis at each step in quantitative portfolio management relative to a benchmark, as well as our experience in applying these capabilities for specific investors.

- **Empirical studies** of asset class and investment style characteristics, in which we utilize our extensive historical databases of index securities:
  - Justification of asset class composition of a benchmark
  - Benchmark customization
  - Investment policy constraints
  - Justification of investment style
- **Analytical models and software** for portfolio management relative to a benchmark:
  - Qualitative profiling of a portfolio relative to a benchmark
  - Quantifying structural mismatches into tracking error projections

- Measuring portfolio risk by sensitivity to pre-specified interest rate, spread, and volatility shocks
- Return (at individual security level) and performance (at the portfolio versus benchmark level) attribution

• **Quantitative portfolio strategies** (complete with historical simulation of their performance) designed and developed for specific groups of investors:
  - Index replication methodologies
  - Active portfolio strategies for various groups of investors

This report follows the above outline. At each step, we describe the requirements of modern portfolio management practice, as well as our current or expected ability to meet them.

We begin with some of the basic issues confronting an investment manager. How does one choose the appropriate investment set and asset allocation? What benchmark will give the fairest measurement of performance? How much freedom should a manager have to deviate from the benchmark, and in which directions?

Next, we turn our attention to the techniques we offer for portfolio management relative to a benchmark. A manager forms a view, positions the portfolio accordingly, and measures the results. Forward-looking techniques quantify portfolio-versus-benchmark risk based on historical volatilities and correlations, perform yield curve and spread scenario analyses, guide managers to rebalancing transactions that achieve desired positioning of the portfolio relative to the benchmark, and help design and maintain portfolios that closely track benchmarks. Backward-looking techniques analyze and compare portfolio and benchmark performance and attribute the achieved returns to the assumed risk exposures. We describe our approach to each of these tasks and give examples to illustrate each model.

In some cases, models play more than just a supporting role. Entire investment strategies, tailored to achieve a wide range of risk/return objectives, have been built around quantitative techniques. The final section of this report reviews several quantitative portfolio strategies that we developed working with different groups of investors. We discuss three approaches to index replication, in which the goal is to track index returns as closely as possible. We then present examples of portfolio optimization schemes that target two different sets of investment objectives.

Appendix I summarizes our research activities, including a list of publications. Appendix II provides details of Lehman's software platforms for portfolio and index analytics, both present and future. The current platform, Lehman Brothers PC Product, was used to implement many of the models and techniques described in this paper. The next-generation platform, POINT, currently in an advanced stage of development, will offer enhancements such as broader asset class coverage and performance attribution with transaction.

II. PERFORMANCE BENCHMARKS AND INVESTMENT GUIDELINES

Every investment portfolio has its own set of objectives and constraints that stem from investor preferences, risk tolerance, type and structure of liabilities, legal issues, etc. Risk/return considerations are central at the stage of investment policy formulation and benchmark selection. We work with various types of investors to help design policy guidelines and construct benchmarks that serve their needs. Our database of historical information on the thousands of individual bonds that compose the Lehman indices, combined with a well-developed portfolio analytics platform, enables us to provide unique input to this process.

Asset Allocation and Benchmark Selection

Any investment program begins by establishing the core composition of the portfolio and choosing an appropriate benchmark. Three fundamental decisions are closely interrelated: the benchmark selection, the choice of asset classes to include in the portfolio, and the strategic (default) allocation to each asset class.

Quantitative techniques can be an important part of this decision-making process. Historical analysis of asset-class returns elucidates their risk/return characteristics. A decision to include a particular spread sector can be supported by a comparison of its historical returns with those of duration-matched Treasuries. Asset allocation models can determine the optimal allocation for given risk constraints.

The priority of these decisions varies among investors. In some cases, the benchmark is mandated and determines the asset allocation. The manager, though, might choose to deviate from the benchmark allocations to reflect his market views or take advantage of a perceived opportunity. In other cases, a benchmark is self-imposed and reflects long-term asset allocation considerations. Such benchmarks typically reflect the accepted investment policy, and managers are less likely to deviate from the chosen allocation. In general, investors seek asset classes and allocations that best meet their particular goals and constraints. They may prefer to design customized benchmarks against which to measure portfolio performance.

It is important that the benchmark match the desired strategic allocation of portfolio assets. When portfolio performance is compared with the benchmark, it is important to know that any difference is due to the manager's decisions and not to externally imposed high-level allocation mismatches. The benchmark and investment policy should always allow portfolio managers to buy the benchmark when and if they so desire. When the primary consideration is selecting a widely used, broad benchmark, the number of constraints placed on the portfolio should be minimized. When the primary high-level decision is selecting a particular asset allocation, a customized benchmark should reflect this allocation. Mandates given to portfolio managers often include investment guidelines that specify permitted deviations from the benchmark.

Reflecting Investor Constraints

Investment policy provisions may be designed to meet return objectives, limit risk, or reflect political or social considerations (e.g., no "sin bonds"). Investment policy might place minimum and maximum constraints on allocations to individual asset classes. It might allow investment in certain types of out-of-benchmark securities and disallow others, or impose limits on the maximum exposure to any single issue or issuer. For example, an investment policy for a portfolio benchmarked against the Lehman Brothers Government/Credit Index could include some or all of the provisions shown in Figure 1.

Quantitative methods help evaluate the potential impact of a given constraint on portfolio performance. For example, to investigate the effect of excluding callable bonds from the portfolio, one could measure the historical tracking error of an all-bullet portfolio strategy that matches benchmark duration and sector allocations. Constraints may refer to quantitative models directly. For instance, an investment policy may quantify the permitted level of risk by specifying a maximum allowed tracking error as measured by the Lehman Brothers risk model (see the section on Risk Budgeting below).

Benchmark Customization

Changing Relative Sector Weights

When an investment policy requires specific allocations to selected sectors or imposes other restrictions, a standard market-weighted index may not be an appropriate benchmark. To measure the manager's performance fairly under such an investment policy, the benchmark should be subject to similar restrictions. We have designed custom benchmarks for a diverse set of circumstances in which explicit guidelines were issued. In each case, the goal was to make the benchmark as broad-based and well diversified as possible, at the same time meeting all requirements of the investment policy.

In a simple case, a customized index merely changes relative weights of a standard index's components. Such a benchmark consists solely of securities in

Figure 1. **Sample Constraints of a Gov/Credit Investment Policy**

- Portfolio duration must be within +/- 6 months of benchmark duration.
- Between 20% and 30% of the portfolio should be invested in corporate bonds.
- The corporate spread duration must be within +/- 1 year of the benchmark spread duration.
- Up to 10% of the portfolio may be invested in high-yield bonds with a minimum rating of B.
- No single issue may account for more than 3% of the portfolio.
- No single corporate issuer may constitute more than 5% of the portfolio.
- No futures, options, or other derivative securities may be purchased.
- No callable bonds are allowed.

the standard index, but weights them according to the preferred allocation scheme rather than by market capitalization. For instance, a money manager might receive a mandate specifying equal investments in government and corporate bonds. As corporates compose only about one-third of our Government/Credit Index, a more appropriate benchmark for this portfolio would be a customized index containing two equal parts representing the Government Index and the Credit Index.

A major Latin American corporation required a more complex benchmark. Its investment policy calls for a certain allocation to Latin American debt. The country weights are controlled to disallow sovereign debt rated lower than BB (the credit rating of the corporation's domicile) and to limit exposure to any single country. To stabilize returns while maintaining the regional character of the portfolio, the policy provides for an allocation to supranational issuers servicing Latin America. We helped this investor translate the investment policy constraints into a specific *market-observable* benchmark that meets all these criteria. This rules-based index contains securities from the Lehman Brothers Aggregate and Emerging Markets indices and is used by the corporation to measure risk and performance of its internal treasury portfolio. By applying the rules that define this benchmark to our historical index data, we retroactively created time series of benchmark returns and durations (in addition to studying current risk sensitivities). The ability to analyze the historical behavior of the benchmark, coupled with the ongoing publication of the benchmark's returns by Lehman Brothers, facilitated acceptance of the benchmark by the corporation's money managers.

Formalizing Accepted Practice

Some institutions do not follow any formal benchmark, but historically have maintained a certain asset mix. This has been the case, for instance, with many commercial banks that manage their assets against LIBOR. There is a growing realization that without a benchmark, one cannot truly tell whether the manager added any value in a particular market segment. One bank that traditionally had taken this approach decided to initiate a performance benchmark. The strategic asset allocation of the bank's existing portfolio did not match the market capitalization of any standard index, and a customized benchmark was necessary to measure performance fairly. Starting from an in-depth analysis of the asset mix of the existing portfolio, we helped design a broad-based benchmark consistent with the bank's investment objectives. We have since helped several other commercial banks create internal performance benchmarks using this "reverse-engineering" method.

Another reason to reverse-engineer existing holdings into a benchmark is to prove the effectiveness of policy changes or new trading strategies. The outperformance against such a benchmark becomes a measure of success of the implemented changes.

Duration Targeting

Many asset portfolios are managed against a specific duration target, which often reflects the structure of a corresponding set of liabilities. If this duration does not match any standard index, an appropriate benchmark may be constructed as a blend of a longer and a shorter index. For example, a government/credit portfolio with a target duration of eight years might use a benchmark with 70% in the Long Government/Credit Index and 30% in the Intermediate Government/Credit Index. The weights needed to achieve a desired duration remain fairly stable over time for indices that consist mainly of bullet bonds.

For investors in mortgage-backed securities (MBS), it is more difficult to maintain portfolio duration at a constant level. The duration of these securities can change dramatically in response to interest rate movements. Hedging techniques that must be applied to keep the duration of the MBS portfolio constant make it difficult to measure its performance against the Lehman Brothers MBS Index (or any other unhedged benchmark). We have designed and simulated a methodology for the construction of "constant duration" mortgage indices to meet the needs of such investors. A core investment in the MBS Index is hedged dynamically with a leveraged overlay of liquid securities or futures contracts. We have conducted a detailed historical study of the performance of this synthetic asset class.[1]

Figure 2 presents a summary of the simulated historical performance of the constant duration MBS strategies against the Lehman Brothers Treasury Index, which served as both the duration target and the basis for performance comparison. The choice of the hedging instrument affects the strategy return due to the combined effect of special financing and exposure to nonparallel yield curve movements. Over the five-year study period, all variants of the strategy per-

[1] Dynkin L.,Hyman J., Konstantinovsky V., Mattu R. "Constant Duration Mortgage Index," *Journal of Fixed Income,* Vol. 10, No. 1, 2000.

Figure 2. **Constant Duration MBS: Performance versus Treasury Index**
December 1993–December 1998

Constant Duration Strategy	**Annualized Outperf.** (%)	**Tracking Error** (%)	**Information Ratio** (Outperf./TE)
Hedging Instrument:			
2-Year Treasury	0.19	1.18	0.16
5-Year Treasury	0.41	1.14	0.36
10-Year Treasury	0.50	1.10	0.46
30-Year Treasury	0.37	1.07	0.35
Current Coupon MBS	0.19	1.37	0.14
Lehman Brothers MBS Index	-0.01	1.88	0.00

formed better against the Treasury Index than the unhedged MBS Index, providing higher return with lower risk.

Issuer Benchmarks

Corporate issuers are non-traditional index users who have recently begun to use benchmarks to monitor the behavior of their secondary market debt relative to a peer group. The definition of a peer group is not always obvious. We have conducted historical correlation studies to define and support such groupings. Comparisons of secondary market performance among issuers can be revealing. These comparisons can help guide timing, size, and maturity distribution decisions for issuing debt.

Liability-Based Benchmarks

Sometimes the use of a customized benchmark is necessitated by a desired term structure exposure rather than by a specific sector allocation. Insurance companies typically divide their portfolios along product lines, with investment guidelines reflecting the expected liability stream of each insurance product. Portfolios may be managed against either a duration target or a detailed projection of liability cash flows. We have devised analytical methods of tailoring a benchmark to liability cash flows while maintaining the advantages of using a broad market-weighted index.[2]

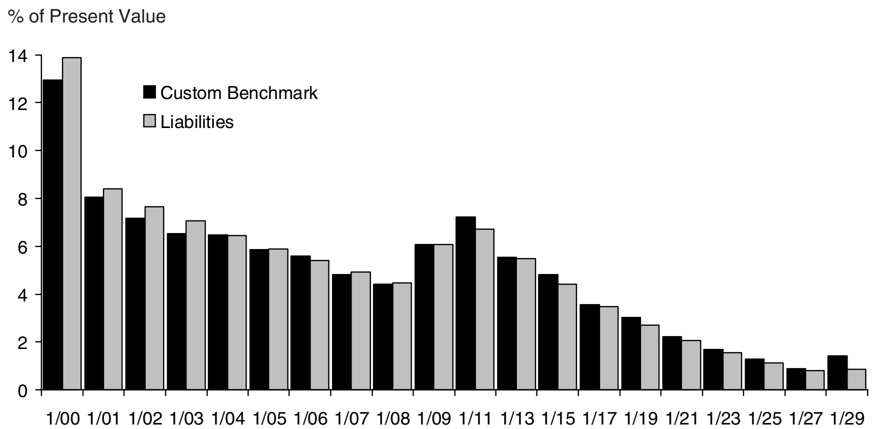
In one representative case, an insurance company portfolio, managed to fund a particular liability stream, had a targeted sector allocation of 70% Treasuries and 30% corporates. Using portfolio optimization, we developed a performance benchmark for this portfolio that satisfied both requirements while using a highly diversified set of securities. The benchmark is defined as the portfolio whose cash flows best match the liability stream, subject to a set of constraints on benchmark composition. In addition to specifying the relative weights of Treasuries and corporates, these constraints ensure that the corporate portion of the benchmark matches the sector and quality distribution of the broad Lehman Brothers Credit Index. Sufficient diversification is guaranteed by the constraint that no single bond may comprise more than 0.6% of benchmark market value. As shown in Figure 3, the cash flows of the resulting benchmark closely match the liability cash flows, while the sector and quality allocations of its corporate portion parallel the Credit Index.

A liability-based benchmark is a "neutral" benchmark that gives the manager a performance yardstick incorporating both the term structure constraints imposed by the liability schedule and any investment restrictions. Managers can be confident that if they hold the positions underlying the liability benchmark, they will meet their liability schedule while satisfying the investment restrictions. This makes the liability benchmark a neutral benchmark.

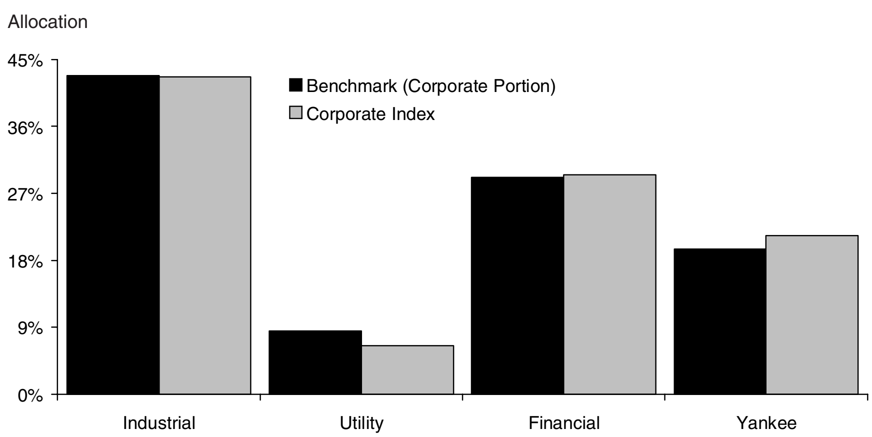
[2] "Liability-Based Benchmarks," Lehman Brothers, in *Global Relative Value*, February 2001.

Figure 3. **Liabilities-Based Benchmark**

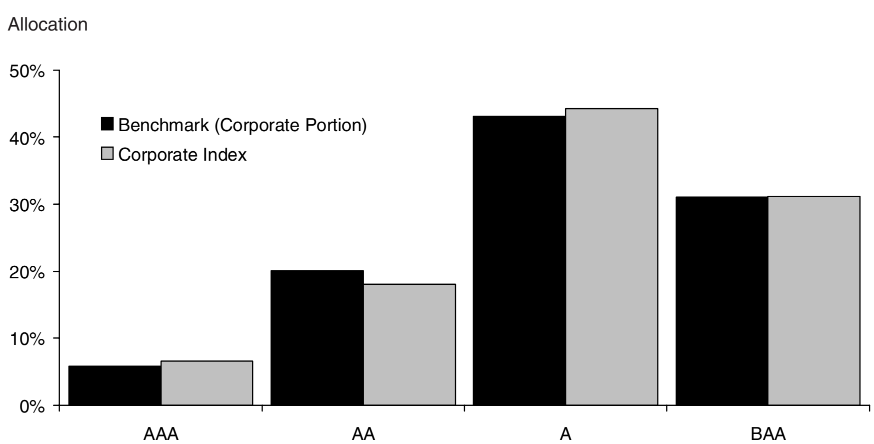
a. **Cash Flow Distribution**



b. **Corporate Sector Distribution**



c. **Quality Distribution**



A liability-based benchmark also retains many of the desirable attributes of a market-based index—benchmark returns are calculated using market prices, the investment manager can replicate the benchmark, and the benchmark is well defined—so that the manager can actively monitor and evaluate its risk and performance. Furthermore, if the liability benchmark contains marketable securities, its performance can be calculated and published by third-party index or market data providers.

Asset Class Analysis

An analysis of index data often yields valuable information to asset allocators. For example, we have studied the level and volatility of excess returns over duration-matched Treasuries achieved historically by various spread sectors (Figure 4). Some investors have used these results to map out risk/return tradeoffs across the fixed-income landscape.

Other investors have focused on the merits of expanding their existing investment guidelines to include additional asset classes. In particular, many seek to diversify into assets that offer both high spreads over Treasuries and low correlations with their core holdings, such as emerging markets debt and U.S. high yield corporate bonds. Figure 5 demonstrates that combining either of these asset classes with the U.S. Aggregate Index can increase the overall Sharpe ratio (return per unit of risk). It is interesting to note that in both cases, there is a clear maximum (optimal allocation) beyond which further additions are detrimental.

An analysis of historical Sharpe ratios along the yield curve represents a possible approach to yield curve positioning as well. Once again, the focus is on the extra return to be gained per unit of additional risk.

The steady decrease in the Treasury supply that began in the 1990s is changing the relative importance of asset classes within the U.S. Aggregate and other broad indices. By the beginning of 2000, MBS had overtaken Treasuries as the largest component of the Aggregate Index. This phenomenon has caused concern that the increasingly negative convexity of the Aggregate Index will make its duration

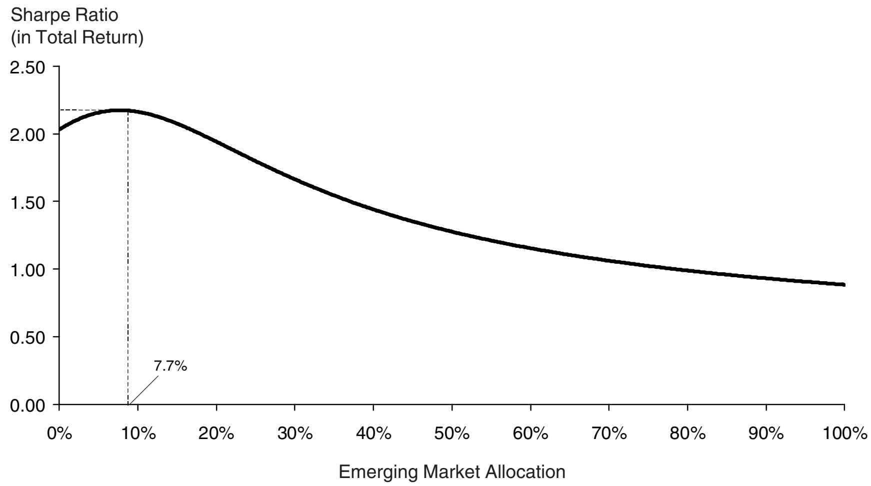
Figure 4. **Annualized Corporate Excess Returns** (bp/year) **over Duration-Matched Treasuries,** 1990–1999

	Industrial		Utility		Finance		Yankee		Total Corp.	
Quality	**Avg.**	**Vol.**	**Avg.**	**Vol.**	**Avg.**	**Vol.**	**Avg.**	**Vol.**	**Avg.**	**Vol.**
Aaa	11	104	-20	156	60	101	25	98	23	95
Aa	22	105	24	146	45	127	38	118	32	115
A	45	147	36	152	56	163	18	201	43	148
Baa	54	250	64	172	95	300	46	286	67	213
Total	45	169	43	151	65	158	30	161	45	150

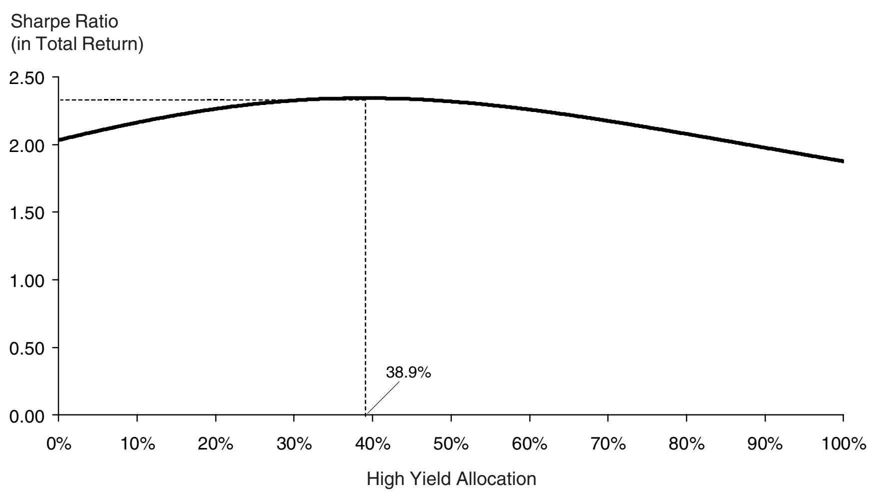
unacceptably volatile for many domestic fixed-income portfolios. (This effect was somewhat masked in 1999, as rising yield levels made mortgages less negatively convex than usual.) To address this issue, we simulated durations that the Aggregate Index would have had over the past nine years if the MBS market share was at one of several constant levels. In Figure 6, we plot the duration volatility of the index as a function of the allocation to MBS. The results show that a small allocation to MBS (up to about 21%) reduces the duration volatility of the resulting index below that of the Government/Credit Index (which is equivalent to the Aggregate Index

Figure 5. **Effect of Adding Emerging Markets and High Yield to the U.S. Aggregate Index,** 1993-1999

a. **Optimal Allocation Between Emerging Market and Aggregate Index**



b. **Optimal Allocation Between High Yield and Aggregate Index**



with 0% MBS). This is because the negative convexity of mortgages offsets the positive convexity of the government/credit component. Further increases in the MBS allocation reverse this trend. As the market share of MBS continues to grow beyond the 34% level (in 1999), the duration volatility of the Aggregate Index is likely to increase and may soon exceed that of the Government/Credit Index. This undesirable characteristic may enhance the appeal of the new macro indices such as the Universal and the Global Aggregate.

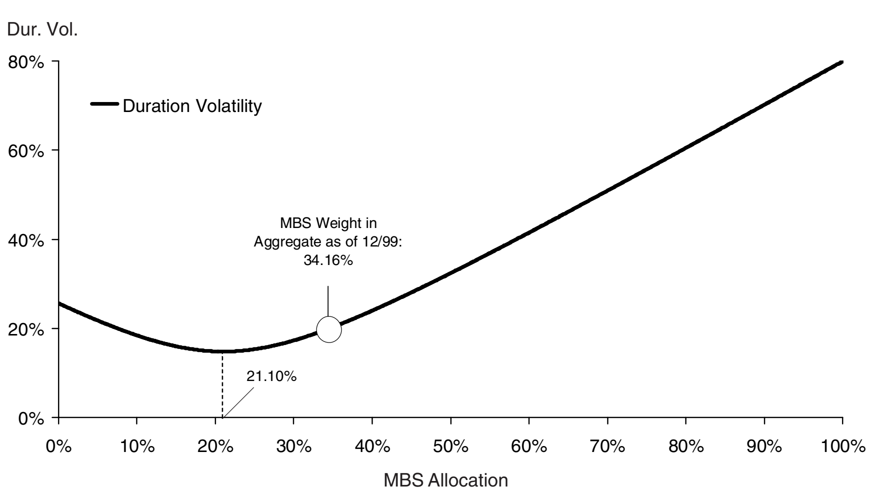
Investment Style Selection

The question of what portfolio management style is most effective is complex. To what extent can a skillful analyst predict future movements of yield curves, sector spreads, and security prices? Which of these predictions, if true, would lead to the highest outperformance per unit of risk relative to an index? Together with a large European investor, we have studied the relative merits of security selection versus asset allocation in managing corporate bond portfolios.

To compare the performance of different investment styles, we designed a set of idealized investment strategies, each based on a single type of investment decision. We then simulated the historical performance of each of these strategies executed with access to some information about future market performance. Our first study of this type took a "perfect foresight" approach,[3] in which the manager always

[3] Dynkin L., Ferket P., Hyman J., Van Leeuwen E. and Wu W. "Value of Security Selection versus Asset Allocation in Credit Markets: A Perfect Foresight Study." *Journal of Portfolio Management*, Vol. 25, No. 4 (1999), pp. 11-27.

Figure 6. **Duration Volatility of Aggregate Index as a Function of the MBS Share**



makes the best possible decision based on this knowledge of the future (over various horizons ranging from one month to a year). We measured the outperformance achieved by each strategy versus the Lehman Brothers Credit Index, as well as the amount of risk taken (measured by the monthly standard deviation of this outperformance). The nature of this scheme is such that none of the investigated investment strategies could actually be implemented. They are of interest because they constitute a limiting case—the best possible results that could be expected from each approach.

This study isolates one mode of investment decision (and underlying research) at a time. First, we investigated a strategy that uses cell matching to create a portfolio neutral to the Lehman Brothers Credit Index in term structure and sector and quality allocations. Outperformance comes from the choice of securities purchased to represent each market cell in the index. Securities were selected based on curve-adjusted return (excess return), given perfect foresight. In one case ("find winners"), we chose only superior performers, e.g., the top 5%. In the other ("avoid losers"), we avoided only the worst performers, e.g., by choosing the top 95%.

Then we simulated performance of strategies that do not involve security selection. We broke the index into market cells defined by term structure, sector, and quality. If the portfolio purchased a given market cell, it was assumed to earn the return of the index in that market cell. We investigated one strategy based solely on yield curve allocation, one on sector allocation, and one on quality allocation.

Strategies are compared by information ratios. Information ratio is defined as the ratio of the mean portfolio outperformance versus the benchmark to the standard deviation of this outperformance, or tracking error. The results show that the index-matched strategy based on security selection performs better on a risk-adjusted basis than any of the asset allocation schemes. This is achieved by accumulating small and steady return advantages, accompanied by low tracking errors relative to the index. Figure 7 summarizes the performance of various strategies investigated in this study.

We revisited the comparison of investment styles using an "imperfect foresight" approach.[4] Once again, we simulated the performance of idealized investment strategies using historical data from the Lehman Brothers Investment-Grade Credit Index, and we used information ratios to evaluate performance. Rather than choosing the single best allocation decision each month, we incorporated the notion that even well-informed investment decisions will sometimes result in losses or underperformance. Portfolio manager skill is modeled as follows. The totally unskilled (0% skill) case is represented by a random selection from a discrete set of possibilities, with equal probabilities assigned to each. In the perfect foresight case (100% skill), the manager always makes a correct

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[4] Dynkin, L., Hyman, J., Wu, W. "Value of Security Selection versus Asset Allocation in Credit Markets: Part II—An "Imperfect Foresight" Study," *Journal of Portfolio Management*, Vol. 27, No. 1, Fall 2000.

decision leading to outperformance (as determined by future results). For all intermediate skill levels the selection probabilities are linearly interpolated between these two extremes.

In this probabilistic framework, each strategy has many possible outcomes in a given month. The level of risk is determined by the combination of the variance of performance over all of these possible outcomes and the variance of strategy performance over time. This is a more realistic estimate of the true risk of a given strategy than the extreme best case of the perfect foresight approach. The mean outperformance is determined largely by skill, which increases the probabilities of favorable outcomes.

The results of this research, shown in Figure 8, support the conclusion of the perfect foresight paper that security selection is the most efficient route to outperforming the index. Comparing all the strategies at the same skill level, security selection achieves the highest information ratio. We trace the source of this advantage in risk-adjusted performance to the diversification of risk among many independent security decisions (thus, the more securities used, the better). If all strategies are limited to a single decision, duration allocation achieves the highest information ratio, confirming the commonly held notion that the duration exposure is the single most important portfolio management decision. Only by making many equally skillful decisions that lead to a highly diversified set does security selection outperform.

Mortgage investors are equally interested in evaluating different investment skills, and in a separate study[5] we applied the perfect foresight approach to the

[5] "Value of Risk Taking in a Mortgage Portfolio: A Perfect Foresight Study," Lehman Brothers, in *Global Relative Value*, February 2001.

Figure 7. **Value of Perfect Foresight in the U.S. Corporate Market**
Monthly Data, January 1990-May 1997

View Placed	Foresight Horizon	Portfolio Average Return (%)	Portfolio Average Duration	Return Diff. vs. Index (%)*	Tracking Error vs. Index(%)	Annualized Information Ratio
Duration	12 Months	0.90	6.53	0.14	0.69	0.70
	1 Month	1.33	5.80	0.57	0.48	4.06
Sector	12 Months	0.85	5.46	0.09	0.23	1.38
	1 Month	1.01	5.43	0.25	0.21	4.24
Credit Rating	12 Months	0.82	5.32	0.06	0.18	1.13
	1 Month	0.94	5.34	0.18	0.15	4.11
Pick Winners	12 Months	1.10	5.62	0.34	0.36	3.29
	1 Month	1.92	5.49	1.17	0.41	9.85
Avoid Losers	12 Months	0.78	5.41	0.02	0.04	1.71
	1 Month	0.84	5.36	0.08	0.06	5.03

*Lehman Brothers Corporate Index (Avg. Tot Ret: 0.76, Avg. Dur: 5.38).

Figure 8. **Investment Styles with Imperfect Foresight (Skill Level of 20%, 1-Month Foresight Horizon)**, August 1988–July 1999

Strategy	**Mean Outperformance** (bp/year)	**Tracking Error** (bp/year)	**Information Ratio**
Duration Allocation	119.8	226.4	0.53
Quality (any winner per cell)	36.8	33.2	1.11
Quality (one per sector)	31.1	44.8	0.69
Quality (one decision)	26.1	62.1	0.42
Sector (any winner per cell)	42.3	42.8	0.99
Sector (one per quality)	35.6	57.6	0.62
Sector (one decision)	31.6	80.0	0.40
Security (5% of bonds)	109.4	31.1	3.52
Security (10% of bonds)	105.3	22.5	4.69
Security (25% of bonds)	96.1	15.3	6.27

MBS market. The majority of portfolio decisions that mortgage managers make revolves around three risk factors: realized volatility, spread movements, and prepayments. Besides the ability to forecast changes in these risk factors, it is equally important to structure a portfolio correctly by appropriately adjusting its sensitivities to these risk factors. The link between correctly anticipated risk factor changes and the correct portfolio restructuring response is not as straightforward as it is in the corporate market.

In this study, we answered the following questions:

- Assuming perfect foresight, how much value (defined as outperformance of the Lehman MBS Index) can be gained from boosting a portfolio's exposure to each one of the three main risk factors while staying neutral in the other two?
- How much value can be gained from successful security selection when the portfolio is matched to the index in all three main risk dimensions?

Results of this study, presented in Figures 9 and 10, suggest that the most valuable skill for mortgage investing is security selection within a portfolio matching the benchmark in systematic risk. This conclusion mirrors that of the corporate perfect foresight study.

There are some caveats to these results. First, the difficulty of isolating exposure to one risk while being neutral to the other two indicates that in practice, managers might be taking a combination of bets at any given time. Second, the meaning of security selection is less clear in the mortgage market. Third, the study may have ignored other mortgage risk factors, e.g. changes in implied volatility that affect a portfolio according to its vega. The relative performance of the three risk factors may have reflected embedded vega positions.

Figure 9. **MBS Perfect Foresight Study: Portfolio Sensitivities**
February 1993-December 2000

	Spread Duration		Convexity		dP/dPSA	
	Bet Size (abs value)	**Outperf.** (bp)	**Bet Size** (abs value)	**Outperf.** (bp)	**Bet Size** (abs value)	**Outperf.** (bp)
Average:	0.21	4	0.38	4	0.15	3
Maximum:	0.40	39	0.60	36	0.35	26
Standard Deviation:		6		5		4
# of "No-bet" Months:	18		25		29	
# of Negative Bets:	43		28		31	
# of Positive Bets:	34		42		35	
(Total = 95 Months)						
Ann. Info. Ratio:		2.74		2.68		2.61

Figure 10. **MBS Perfect Foresight Study: Security Selection**
February 1993-December 2000

	10 Bonds Min		25 Bonds Min		50 Bonds Min	
Monthly:	**# Issues**	**Outperf.** (bp)	**# Issues**	**Outperf.** (bp)	**# Issues**	**Outperf.** (bp)
Average	14	16	29	13	54	10
Minimum	12	3	27	3	52	3
Maximum	16	50	31	40	56	27
Standard Deviation		8		6		4
Annualized Inf. Ratio:		6.88		7.09		7.46

III. PORTFOLIO ANALYSIS RELATIVE TO A BENCHMARK

The selection of investment guidelines and an appropriate benchmark marks the beginning of the portfolio management process. Once a portfolio is established, investors must continually monitor its positioning relative to the benchmark. Periodic transactions are needed to maintain desired exposures (e.g., six months longer in duration or overexposed to corporates by 10%) and to express changes in market outlook.

In a typical investment cycle, forward-looking analytics are applied at the start of each period to position the portfolio according to the investor's view. At the end of the period, *ex-post* analysis is used to review the achieved performance.

To illustrate these techniques, we will take as a case study the government/credit portfolio of the Invest-Rite Asset Management Company. This hypothetical portfolio is benchmarked against the Lehman Brothers Gov/Credit Index, but deviates from the benchmark in both term structure and sector allocations. We follow this portfolio through a full monthly investment cycle and show how these deviations are analyzed and how their effect on performance is quantified.

Analyzing Portfolio and Benchmark Composition: A Cell-Based Approach

Market structure analysis is a structural comparison of the portfolio and benchmark. The two sets of bonds are compared by partitioning them into a matrix of cells. Different choices of partition variables focus on different aspects of portfolio composition. Corporate portfolios, for example, can be divided along a hierarchy of industry categories (e.g., basic industry, consumer cyclical, or energy). Segmenting by duration alone highlights the yield curve exposures. Segmenting by duration and sector shows how yield curve views are implemented using various asset classes (e.g., a yield-curve barbell achieved by combining MBS at the short end with long corporates). The amount of information a portfolio manager derives from a market structure report depends on the proper selection of risk dimensions and output fields. More information can be derived by analyzing the projected returns under various interest rate and credit spread scenarios within each cell.

Figure 11 shows a market structure comparison of the Invest-Rite portfolio and the Gov/Credit Index. Contributions of a given cell to both risk and return of the overall portfolio or benchmark are functions of the weight of the cell. Contribution is defined as the percentage allocated to a cell times the cell average (e.g., contribution to duration equals percentage allocation times duration and is proportional to dollar duration). To be risk-neutral to the benchmark, a portfolio should match contributions to duration within each cell. Our example portfolio strongly underweights the long and short portions of the index, with over 95% of its market value in the 3- to 7-year duration range. The portfolio is short duration relative to the benchmark by 0.25 years. This difference can be broken down in terms of either sector (long by

Figure 11. **Invest-Rite Portfolio versus Gov/Credit Index: Market Structure Report,** October 31, 1999

Dur.		% of Market Value			Adjusted Duration			Contrib. to Duration		
Range		Govt.	Corp.	Total	Govt.	Corp.	Total	Govt.	Corp.	Total
0-3	Portfolio	0.8	0.0	0.8	2.71	0.00	2.71	0.02	0.00	0.02
	Index	26.4	7.1	33.5	1.80	2.06	1.85	0.47	0.15	0.62
	Diff.	-25.6	-7.1	-32.7	0.92	-2.06	0.86	-0.45	-0.15	-0.60
3-7	Portfolio	46.2	49.7	95.8	5.00	5.20	5.10	2.31	2.59	4.89
	Index	21.4	16.6	38.0	4.59	5.05	4.79	0.98	0.84	1.82
	Diff.	24.7	33.1	57.8	0.41	0.16	0.31	1.32	1.75	**3.07**
7-10	Portfolio	3.1	0.2	3.3	7.02	7.02	7.02	0.22	0.02	0.23
	Index	6.5	4.7	11.2	8.68	8.42	8.57	0.56	0.40	0.96
	Diff.	-3.4	-4.5	-7.9	-1.67	-1.39	-1.55	-0.34	-0.38	-0.72
10+	Portfolio	0.0	0.0	0.0	0.00	0.00	0.00	0.00	0.00	0.00
	Index	12.4	4.9	17.3	11.64	11.26	11.53	1.44	0.55	2.00
	Diff.	-12.4	-4.9	-17.3	-11.64	-11.26	-11.53	-1.44	-0.55	-2.00
Total	Portfolio	50.1	49.9	100.0	5.08	5.21	5.15	2.55	2.60	5.15
	Index	66.7	33.3	100.0	5.19	5.80	5.40	3.46	1.93	5.40
	Diff.	-16.6	16.6	0.0	-0.11	-0.59	-0.25	**-0.92**	**0.67**	**-0.25**

0.67 to corporates, short by –0.92 to governments) or term structure (long by 3.07 in the 3- to 7-year duration range, short in all the others).

Another analytical approach to comparing a portfolio with a benchmark is to generate and compare their cash flow profiles. Cash flows may be represented on a discounted or non-discounted basis, in absolute (dollar) terms or as percentages of the total present value. They can be projected by month, quarter, year, or another frequency. This analysis can be followed by a more detailed market structure analysis to identify the sources and implications of cash flow differences. For a portfolio of bullet securities, this is equivalent to key rate durations analysis. Figure 12 plots the differences in cash flow distribution (in percentages of present value) between the Invest-Rite portfolio and the Gov/Credit Index. The butterfly-type pattern in this graph reveals significant exposure to non-parallel yield curve movements. As will become apparent in scenario analysis, these exposures reflect a view that the yield curve will steepen.

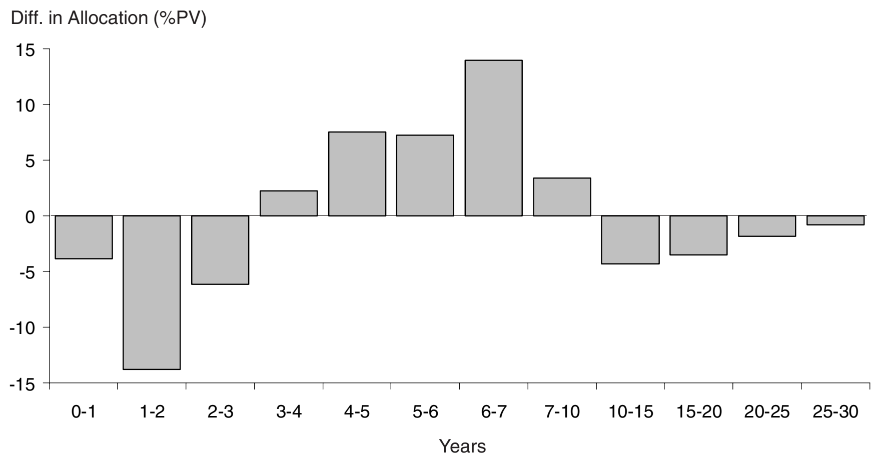
The portfolio profiling described above is useful alongside any sophisticated quantitative model of risk and return because it represents the basic intuitive view of portfolio exposures relative to the benchmark.

Quantifying Portfolio Risk Relative to a Benchmark

The Multi-Factor Approach

Managers must monitor portfolio exposures versus the benchmark along many dimensions simultaneously. They may choose to express a market view by overweighting a particular part of the yield curve, sector, or quality. They also need to

Figure 12. **Invest-Rite Portfolio versus Gov/Credit Index: Cash Flow Distribution,** October 31, 1999



be aware of any unintentional exposures. Any difference between portfolio and benchmark composition represents a potential for return deviations.

The qualitative cell-based comparison by allocations to different market segments has two shortcomings. First, it does not indicate which mismatches are likely to contribute the most to return deviations between portfolio and benchmark. The biggest mismatches do not necessarily carry the most risk. For example, a spread duration mismatch in the AA-industrial cell is not nearly as risky as a mismatch of the same magnitude in the BAA-Yankee cell. Second, it ignores correlations that might exist among different risk axes.

One approach to quantifying the expected return deviation between portfolio and benchmark is to attribute most of the individual security return variance to a small set of systematic market influences, or risk factors. This method, known as multi-factor risk analysis, is based on a comparison of sensitivities of the portfolio and benchmark to each of the risk factors. The analysis of positions consisting of thousands of bonds is reduced to a comparison of a few risk exposures. Based on the historical volatilities and correlations of the risk factors, the model quantifies the net risk due to these exposures.

The Lehman Brothers multi-factor risk model[6] covers all securities in our U.S. Aggregate Index. The model uses risk factors representing exposures to term

[6] *The Lehman Brothers Multi-Factor Risk Model*, Lehman Brothers, July 1999.

structure, agency and corporate sectors, credit quality, coupon, optionality, MBS prepayments, MBS volatility, and MBS spreads. The risk factors are explanatory variables in a statistical analysis of historical returns for individual securities. The result of this analysis is a covariance matrix of the historical volatilities and correlations of all the risk factors since 1987 for Treasury, agency, and corporate bonds, and since the second half of 1993 for mortgage-backed securities. The covariance matrix is updated monthly.

Common market risk factors do not always explain the entire return variance of a security. Credit events associated with individual issuers can pose significant risk to a portfolio that is not sufficiently diversified. Because our model is based on historical returns of individual bonds, it is able to quantify non-systematic (diversifiable) risk. Parts of each bond's return unexplained by the systematic risk factors are used to model non-systematic risk.

To measure risk, the model computes sensitivities of each bond in the portfolio and in the index to all risk factors and aggregates them by percentage of market value. The difference between the resulting aggregate sensitivities of the portfolio and benchmark is then multiplied by the covariance matrix to produce the magnitude of systematic risk. In addition, the model measures the differences in concentration of the portfolio and benchmark by issue and issuer to estimate non-systematic risk. The two measures of risk are then combined to compute the total tracking error, the projected standard deviation of the difference between the portfolio and benchmark returns.

The systematic tracking error can be reduced by restructuring the portfolio so that its risk characteristics match the index more closely. The non-systematic component can be reduced through diversification. The optimal path of reducing the tracking error is rarely obvious because each security contributes to many dimensions of risk. The current tracking error minimization method employed by the model guides portfolio managers through a sequence of optimal transactions using the "gradient-descent" method. This method ranks each security from a specified investable set (and the portfolio) by the sensitivity of tracking error to a change in the security holding. The manager then chooses the most practical transaction from a list ranked by potential tracking error reduction.

The risk model, relying on historical volatilities and correlations, interprets the structural differences between the portfolio and benchmark to produce a more accurate picture of portfolio risk. Simple "eyeballing" of the market structure report shown in Figure 11 might leave some doubt as to the relative importance of the different types of risk the Invest-Rite portfolio faces. The magnitude of the overweight to corporates (measured by difference in contribution to duration) is 0.67 years, while the overall duration difference is only –0.25 years. This might lead one to overemphasize the sector risk and underestimate the term structure risk. As shown in Figure 13, the tracking error for this portfolio is due predominantly to the

Figure 13. **Invest-Rite Portfolio versus Gov/Credit Index: Risk Analysis Report,** October 31, 1999

	Tracking Error		Incremental
	Isolated	**Cumulative**	**Change**
Systematic Tracking Error Breakdown (% per year)			
Due to:			
Term Structure	0.663	0.663	0.663
Non-Term Structure	0.300		
Sector	0.165	0.673	0.010
Quality	0.136	0.694	0.021
Optionality	0.075	0.707	0.013
Coupon	0.041	0.721	0.014
MBS Sector	0.000	0.721	0.000
MBS Volatility	0.000	0.721	0.000
MBS Prepayments	0.000	0.721	0.000
Total Systematic			0.721
Non-Systematic Tracking Error Breakdown (% per year)			
Issuer-Specific			0.108
Issue-Specific			0.108
Total Non-Systematic			0.108
Total Tracking Error			0.729

term structure exposure. This is because yields are more volatile than credit spreads and because the overall duration difference is not a sufficient measure of yield curve risk for this portfolio. The extent to which the long and short exposures to different parts of the curve offset each other is closely related to the correlation between the movement of long and short Treasury yields. The risk model evaluates all these effects simultaneously when projecting tracking error.

Our next generation of risk models, currently under development, will improve upon the present model. The portfolio characteristics that determine a portfolio's risk profile will include more sophisticated option-adjusted sensitivities to market changes. For example, term structure exposures will be measured by key-rate durations or principal component durations, and sensitivities to changes in implied volatility (vegas) will enhance the treatment of optionality risk.

Risk Budgeting

One of the benefits of our multi-factor risk model is that it makes various types of risk directly comparable by expressing them in the same units. This has brought into existence an entirely new approach to setting risk limits for investment portfolios. In the past, virtually all investment policies explicitly limited specific types of deviation from the benchmark (duration range, maximum and minimum allocation to sectors, maximum issuer/issue exposure, etc.). Now we are witnessing a tendency to specify the overall risk level (tracking error) and to leave the choice of the type of risk to the manager. This unties a manager's hands. With such an arrangement, a manager can assume duration exposures in one month, sector overweights in another, and a large exposure to one or two "bargain" securities in a third. In the process, the manager may venture outside the bounds that a classical investment mandate would have imposed on him. Yet at all times, the overall risk stays within the prescribed limits. This flexibility makes risk management more dynamic, leading to a more efficient allocation of the main investment "resource"—risk.

Figure 14 illustrates risk budgeting. Using the Lehman Brothers risk model, we construct three 200-bond portfolios benchmarked against the U.S. Aggregate Index and consisting of essentially the same securities. The first portfolio passively replicates the index with a very low total tracking error of just 6 bp per year. This fully indexed portfolio becomes the base from which we deviate; first, by significantly increasing the portfolio duration (term structure exposure) and second, by overweighting corporates at the expense of governments (sector exposure). The 0.35 years duration mismatch gives rise to 27 bp of isolated tracking error due to term structure. A roughly 25% reallocation between governments and corporates increases the sector tracking error from 1 bp to 24.8 bp. However, the total tracking error in both cases is the same (30 bp).

Figure 14. **Risk Budgeting: Putting Different Types of Risk in Common Terms,** November 1999

	U.S. Aggregate Index	Passive Replication	Term Structure Exposure	Sector Exposure
Duration	4.94	4.95	5.29	4.91
% in Governments	41.9	41.0	41.0	15.6
% in Corporates	21.3	22.0	22.0	44.5
% in Mortgages	34.1	34.5	34.5	35.5
Isolated Annualized Tracking Error (bp) due to:				
Term Structure		1.3	**27.0**	3.2
Sector		1.0	1.0	**24.8**
Quality		0.7	0.7	2.8
Optionality		0.6	0.6	2.2
Coupon		1.0	4.8	5.4
MBS Sector		0.5	0.5	1.9
MBS Volatility		0.4	0.4	0.6
MBS Prepayments		0.5	0.5	0.8
Total Tracking Error		6.0	30.0	30.0

The Asset Allocation Approach

An alternative approach to modeling risk divides the market into a few asset classes and derives their return variances from historical observations, without addressing the market causes of these variances. This method calculates portfolio risk relative to the benchmark based on the differences in weights assigned to each asset class. It can be used to construct a mean-variance efficient frontier, which specifies the asset allocation providing the highest expected return for any level of risk. Required inputs to the model include time series of historical returns and return forecasts for each asset class.

The Lehman Brothers global asset allocation model uses this approach to quantify risk in multi-currency portfolios. For each currency, the model analyzes time series of historical returns within several maturity ranges. A covariance matrix is formed directly from the return volatilities of each asset class and the correlations among them. Tracking error is calculated based on differences between portfolio and benchmark allocations to currencies and maturity ranges.

A noteworthy feature of the Lehman model is that expected returns for each asset class are *not* assumed equal to their historical means. Rather, they are projected based on user forecasts of yield curve movements. If desired, a market-implied equilibrium forecast can be backed out of asset class capitalizations in the benchmark. The equilibrium forecast can be modified to reflect multiple investor views with varying levels of confidence. An optimizer generates an efficient frontier of optimal portfolio allocations for different degrees of risk tolerance. The model includes a flexible set of constraints and hedging modes and can be customized to include any additional asset classes for which historical returns are available.

Both asset allocation and multi-factor models have strengths and weaknesses. Asset allocation models work best for assets with low correlations. In a single-currency market, returns on most fixed-income assets are typically highly correlated because of the common impact of interest rate changes. High correlations make the composition of minimum-risk portfolios produced by asset allocation models very unstable. Small changes in the inputs of projected or historical returns may result in significant changes in the optimal portfolios. Consequently, the asset allocation approach has been applied primarily to asset classes with low correlations, such as equity versus fixed-income allocation or multi-currency portfolios. Also, an asset class may itself change systematically over time. Within a single-currency fixed-income market, a carefully designed multi-factor model can provide a more precise assessment of risk. In addition, a multi-factor model built from individual security return data can measure both systematic and non-systematic risk. When portfolios are represented solely by asset-class holdings, there is no notion of non-systematic risk.

The multi-factor approach requires much more data and analysis to produce a risk model for a given sector. The asset allocation approach, which requires historical return data only at the asset-class level, is more easily adaptable to new asset classes, such as emerging markets.

Projecting Returns: Scenario Analysis

Another perspective on risk involves comparing projected portfolio and benchmark returns under various yield curve and spread scenarios. A manager can focus on the set of scenarios that he considers most likely or those he considers most dangerous. This type of analysis complements the multi-factor risk model in that it allows a manager to stress-test benchmarked portfolios by subjecting both portfolio and benchmark to extreme scenarios not consistent with the history underlying the risk model. While extreme scenarios are unlikely, they often highlight potential sources of return deviations that may not manifest themselves under normal circumstances. Figure 15 shows several scenario performance projections for the Invest-Rite portfolio, some of them likely and some extreme. The portfolio's duration view (shorter than the benchmark) is apparent in the performance under moderate parallel shift scenarios; the steepening and flattening scenarios highlight its bullet-versus-barbell term structure exposure. As this figure shows, extreme scenarios can lead to one-month performance differences as far as eight tracking errors apart and produce surprising results. For instance, the portfolio underperforms in the "up 250 bp" parallel shift scenario, despite being short duration. This is the effect of the portfolio's much lower convexity relative to the benchmark.

Our analytics platform provides several mechanisms for pricing bonds at the horizon: constant yield spread to the off-the-run Treasury par curve, constant zero

Figure 15. **Invest-Rite Portfolio versus Gov/Credit Index: Scenario Stress-Testing,** Horizon Date: November 30, 1999

	Duration	Convexity
Portfolio:	5.15	0.26
Index:	5.40	0.56
Total Systematic Tracking Error (bp/mo):		20.8
Term Structure		19.1
Non-Term Structure		8.7

Scenario	Portfolio Return (%)	Index Return (%)	Difference (bp)	Multiple of Tracking Error
No Change	0.58	0.55	3	0.1
Parallel Shift				
Up 25 bp	-0.66	-0.76	9	0.5
Down 25 bp	1.85	1.89	-4	0.2
Up 250 bp	-11.15	-10.83	-32	1.5
Down 250 bp	13.76	15.46	-170	8.2
Twisting Movement				
10 bp Flattening	0.45	0.55	-10	0.5
10 bp Steepening	0.71	0.55	16	0.8
100 bp Flattening	-0.79	0.49	-129	6.2
100 bp Steepening	1.99	0.66	133	6.4

volatility (ZV) spread over the Treasury forward curve, or constant option-adjusted spread (OAS) over a lognormal tree of short rates. Mortgage-backed securities are priced at a constant OAS over a set of stochastic (Monte Carlo) forward rate paths. The spline models and volatility calibration methods used to create the trees and interest rate paths for these analyses incorporate the latest innovations from Lehman Brothers Quantitative Research. The methodology for generation of Monte Carlo rate paths employs an exponential variant of the Heath-Jarrow-Morton model. The term structure of volatility is calibrated to the swaptions market.[7]

For multi-currency portfolios, scenario specifications include horizon yield curves in the relevant currencies, as well as horizon exchange rates and the cost of hedging (forward premium). The implied cost of hedging can also be computed dynamically from interest rate differentials between any two currencies.

Analyzing Performance

Regardless of the methods used to construct a portfolio, the ultimate result is portfolio performance relative to the benchmark. The identification of sources of relative performance helps measure the effects of various allocation decisions within the overall portfolio management process. Careful analysis of return surprises may point out unintended portfolio characteristics that should be corrected in the future. All types of investors can benefit from the analysis of past performance.

The first level of return analysis is qualitative. Market structure reports compare portfolio and benchmark returns on a partitioning grid of choice. Within each cell, one can examine allocations, returns, and contributions to return and draw conclusions about the impact of *ex ante* allocation decisions on *ex post* returns.

The next level of performance analysis quantifies the outperformance achieved due to specific exposures in the portfolio. Currently, we attribute achieved returns using two different models. First, we use our *return attribution* model to calculate subcomponents of return for individual securities and publish the aggregated results for all major Lehman Brothers indices. This framework is somewhat similar in approach to our multi-factor risk model, in that it deals with *market sources* of individual securities' returns. The second methodology attributes achieved outperformance to portfolio-level *allocation differences* relative to the benchmark. This *performance attribution* model is closely related to the asset allocation approach to modeling risk.

For all spread securities, we also report *excess return* over duration-equivalent Treasuries. Excess return excludes yield curve effects to allow performance comparisons across asset classes on a curve-adjusted basis.

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[7] *The Lehman Brothers Yield Curve Model: Implications for MBS*, Lehman Brothers, November 17, 1998.

To perform meaningful analysis of return difference between the portfolio and benchmark, investors must be aware of conventions and assumptions that might be used in computing benchmark returns. While an all-Treasury benchmark may not present any difficulties, other asset classes (mortgage passthroughs, for example) often have specific trading practices and market conventions that complicate index return calculations.[8] Investors need to understand how their portfolio return calculations might differ from those of the benchmark.

Return Attribution

The return attribution model[9] explains returns by ascribing portions of each security's price return to passage of time (accretion, rolldown), changes in the yield curve (shift, twist, butterfly, and a residual), and changes in volatilities and spreads. As illustrated in Figure 16, the model analyzes the month-over-month changes in the yield curve in terms of the yield changes of on-the-run Treasuries. In November 1999, this model determines that the yield curve exhibited an upward parallel shift of 17.6 bp, a twist (flattening) of 7.1 bp, and a butterfly movement of 2.8 bp. Security-level results are aggregated to analyze the portfolio and benchmark performance in isolation and in comparison with each other. Return subcomponents correspond to exposures commonly taken by portfolio managers. For example, if a portfolio is barbelled to express a yield curve flattening view, the anticipated outperformance should result from twist return.

Figure 17 presents the attribution of the price return earned by the Invest-Rite portfolio versus its benchmark in November 1999. The portfolio price return was 9 bp higher than the benchmark. The overweight to corporates led to a 7 bp

[8] Dynkin, L., Hyman J., Konstantinovsky V., and Roth N., "MBS Index Returns: A Detailed Look." *Journal of Fixed Income*, Vol. 8, No. 4 (1999), pp. 9-23.

[9] Dynkin, L., Hyman J., Konstantinovsky V., "A Return Attribution Model for Fixed-Income Securities," in Frank J. Fabozzi (ed.), *Handbook of Portfolio Management*, 1998

Figure 16. **Return Attribution: Breakdown of Yield Curve Movement**
November 1999

On-the-Run	**Yield** (%)		**Change**	**Shift**	**Twist**	**Butterfly**
Treasury	**Begin**	**End**	(bp)	(bp)	(bp)	(bp)
2-Year	5.795	6.003	20.8	17.6	3.6	-0.9
5-Year	5.931	6.131	20.0	17.6	0.0	1.9
10-Year	6.013	6.172	15.9	17.6	-1.9	0.5
30-Year	6.258	6.395	13.7	17.6	-3.6	-0.9

Shift: average of movements of 2-, 5-, 10-, and 30-year = 17.6 bp.
Twist: change in spread between 2- and 30-year = -7.1 bp flattening.
Butterfly: average movement of 2- and 30-year minus 5-year = -2.8 bp.

advantage in spread return, and another 2 bp was due to yield curve positioning. The beneficial effect of the shorter duration on shift return (+6 bp) in a month of rising interest rates was partially offset by an underperformance in return due to non-parallel curve movement (-3 bp). This is due to the more concentrated (bullet) cash flows of the portfolio relative to the benchmark, as seen in Figure 12. In terms of total return, the portfolio gains another 2 bp in coupon return, which is not part of the attribution of price return. This is, in large part, another outcome of the overweight to corporate bonds.

This approach provides both intuitive clarity and computational accuracy. For clarity, we define yield curve movements based on changes in easily observable bellwether yields; for accuracy, a "full valuation" approach is used at each attribution stage. Rather than base the analysis on beginning-of-month sensitivities to various effects, the model calculates each component of return using a full option-adjusted spread (OAS) valuation.

To extend this model to another market, specifics of that market must be reflected in the definitions of yield curve movements and the selection of bellwether points.

Attribution of Portfolio Performance Relative to a Benchmark

The performance attribution model,[10] by contrast, explains only return *differences* between a portfolio and a benchmark rather than the returns themselves. Instead of dividing the total return from each security into components, the model

[10] *Attribution of Portfolio Performance Relative to an Index*, Lehman Brothers, March 1998.

Figure 17. **Invest-Rite Portfolio versus Gov/Credit Index: Return Attribution,** in %, November 1999

	Portfolio	Index	Difference
Accretion	0.00	0.00	0.00
Rolldown	0.01	0.02	0.00
Yield Curve			
Shift	-0.87	-0.93	0.06
Twist	0.03	0.06	-0.04
Butterfly	-0.06	-0.01	-0.05
Shape	0.19	0.14	0.05
Volatility	0.00	-0.01	0.00
Spread	0.20	0.13	0.07
Price	-0.50	-0.60	0.09
Coupon	0.56	0.54	0.02
Total	0.06	-0.06	0.11
Time Passage	0.02	0.02	0.00
Nonparallel Yield Curve	0.16	0.19	-0.03
Overall Yield Curve	-0.72	-0.74	0.02
Spread + Volatility	0.19	0.13	0.07

attributes portfolio performance relative to the benchmark to allocation differences between the two. Performance attribution can be performed only in the context of comparison to a benchmark. This method cannot be applied to a portfolio in isolation.

The model attributes performance using a two-dimensional grid. Typically, the grid consists of a term structure axis (duration, maturity, or average life) and an arbitrary second axis that reflects allocations to any non-yield-curve-related market segments. The allocation dimensions should be selected and prioritized to reflect the portfolio manager's decision process. One may analyze a mortgage portfolio by allocations to seasoning levels and a corporate portfolio by quality. The outperformance of the portfolio versus its benchmark is explained in terms of the explicit decisions made by the portfolio manager: curve positioning (allocation along the term structure axis), sector (or quality) allocation, and all other factors, which fall under the broad rubric of security selection.

In order to attribute performance to portfolio management decisions, the model answers two separate questions. First, by how much would the portfolio's performance have differed from the index if it had earned the index return within each market segment? This difference, due solely to the mismatches in segment weights, is attributed to the high-level allocation decision. Second, how much extra return was achieved due to differences in returns *within* each market segment? This portion of return difference is attributed to security selection. In our example, this technique is used first to evaluate the portfolio's positioning along the term structure, described by allocations to narrow duration ranges, and then to analyze the sector composition within each duration range.

Figure 18 shows a summary performance attribution report for the Invest-Rite portfolio. This method attributes 9.5 bp to the yield curve allocation differences and 1.8 bp to the combined effects of sector and security selection.

The two attribution models offer complementary views of the sources of outperformance. In the return attribution model, there is no distinction between sector allocation and security selection, and all returns related to spread changes contribute to "spread return." In addition, the coupon advantage associated with spread products does not appear in the spread component of price return but remains part of the coupon return, which is not partitioned by the model.

Performance due to term structure is also analyzed somewhat differently by the two models. In the return attribution model, the yield curve movement is defined by the movements of on-the-run Treasury yields, while in the performance attribution model, it is determined by a duration-cell breakdown of the benchmark. Term-dependent spread changes (e.g., when long corporate spreads widen but short spreads remain stable) might thus be reflected in performance attribution under the category of yield curve allocation rather than sector allocation.

Figure 18. **Invest-Rite Portfolio versus Gov/Credit Index: Performance Attribution,** November 1999

Outperformance (bp) due to:	Curve Positioning	9.5
	Sector Selection	1.4
	Security Selection	0.4
	Total	11.3

The attribution reports for the Invest-Rite portfolio demonstrate this point. While return attribution analysis (Figure 17) ascribed 2 bp of price return to yield curve effects and 7 bp to spread effects (or 9 bp, including coupon return), performance attribution (Figure 18) shows 9.5 bp of the total return due to curve positioning and 1.8 bp due to combined spread effects. Figure 19 shows that in November 1999, the magnitude of spread return (as measured by return attribution) was very uneven along the term structure. As we have shown above, the sample portfolio has a markedly different term structure allocation from the benchmark. In particular, it is overweighted in the 6- to 7-year duration range, which experienced the largest returns due to spread. This led to the 7 bp spread return advantage captured by the return attribution model. The performance attribution model, on the other hand, looks at the performance of each duration cell without asking what caused that performance. Thus, a higher portfolio allocation to a better-performing duration cell leads to a reported outperformance due to curve allocation.

Excess Return

The goal of the excess return methodology is to strip out the effects of yield curve positioning from total return. It measures the outperformance of a given portfolio or index over a term-structure-matched position in U.S. Treasuries. This allows for a more meaningful comparison of spread sector portfolios or indices with different durations. For instance, direct comparison of returns on the Credit Index and the MBS Index reveals little more than the overall direction of yield curve movement. After subtracting an appropriate Treasury return from each, a comparison of their excess returns provides a meaningful measure of the relative performance of these two sectors.

It is well known that a security's duration does not fully reflect its yield curve exposures, particularly for securities with embedded optionality, such as callable bonds or MBS. A more precise method is to characterize each security's exposure along the curve by a set of key rate durations (KRD).[11] Then its return can be compared with that of an all-Treasury portfolio with the same KRD profile. The movements of the par yields at the selected points (0.5-, 2-, 5-, 10-, 20- and 30-years)

11 "A New Method of Excess Return Computation; Index Conversion Probable in 2001," Lehman Brothers, in *Global Relative Value*, September 2000.

Figure 19. **Invest-Rite Portfolio versus Gov/Credit Index: Term Structure of Spread Returns (Effect on Performance Attribution)**
November 1999

Duration		Spread Return (%)	% of Mkt. Value	Contrib. to Spread Return	Total Return (%)	Contrib. to Total Return
0- to 2-Year	Portfolio	0.00	0.00	0.00	0.00	0.00
	GovCorp Index	0.01	0.21	0.00	0.24	0.05
	Difference	-0.01	-0.21	0.00	-0.24	-0.05
2- to 3-Year	Portfolio	-0.17	0.01	0.00	-0.06	0.00
	GovCorp Index	0.08	0.13	0.01	0.17	0.02
	Difference	-0.25	-0.12	-0.01	-0.23	-0.02
3- to 4-Year	Portfolio	-0.03	0.17	-0.01	0.05	0.01
	GovCorp Index	0.00	0.13	0.00	0.07	0.01
	Difference	-0.03	0.04	-0.01	-0.02	0.00
4- to 5-Year	Portfolio	0.04	0.25	0.01	0.17	0.05
	GovCorp Index	0.01	0.08	0.00	0.08	0.01
	Difference	0.03	0.17	0.01	0.09	0.04
5- to 6-Year	Portfolio	0.09	0.29	0.03	-0.08	-0.02
	GovCorp Index	0.19	0.07	0.01	0.04	0.00
	Difference	-0.10	0.21	0.01	-0.13	-0.03
6- to 7-Year	Portfolio	0.63	0.24	0.15	0.15	0.04
	GovCorp Index	0.53	0.09	0.05	0.01	0.00
	Difference	0.10	0.16	0.11	0.14	0.04
7+ Year	Portfolio	0.45	0.03	0.02	-0.29	-0.01
	GovCorp Index	0.21	0.28	0.06	-0.52	-0.15
	Difference	0.24	-0.25	-0.05	0.22	0.14
Total	Portfolio	0.20	1.00	0.20	0.06	0.06
	GovCorp Index	0.13	1.00	0.13	-0.06	-0.06
	Difference	0.07	0.00	0.07	0.11	0.11

are assumed to capture the overall movement of the yield curve. Sensitivities of a bond to these six yields summarize its exposure to yield curve movements. To compute these sensitivities, the yield curve is perturbed by applying a change in the par yield curve centered on each of these points one at a time and the bond is repriced at a constant OAS. The distribution of the bond's duration among the six KRDs gives a more detailed view of how it will respond to different types of yield curve movements.

To calculate excess returns, at the start of each month, we construct a set of six hypothetical par-coupon Treasuries corresponding exactly to the maturities of the six KRDs. Each of these bonds is priced exactly off the curve (at zero OAS). A combination of these bonds and a riskless one-month cash position is used to match the market value and KRD profile of any security at the beginning of the month. At the end of the month, each of the hypothetical securities is repriced at zero OAS off the end-of-month Treasury curve and its total return for the month is calculated. An excess return for each security is then calculated as the difference between its total return and that of the equivalent Treasury position.

IV. QUANTITATIVE STRATEGIES FOR BENCHMARKED PORTFOLIOS

Index Replication

Many investors manage portfolios (or parts of portfolios) to match index returns. Even active managers may fall back to passive index tracking in times when they have no definite views. The simplest way to replicate an index is to buy most of the securities in the index in the proper proportions. However, this method is practical only for the largest index funds. For smaller portfolios, maintaining the necessary proportions of a large number of bonds would necessitate buying odd lots and lead to overwhelming transaction costs. Furthermore, this strategy is appropriate only for portfolios that are to remain neutral to the index over a long period of time. Investors with smaller portfolios often build index proxies, i.e. portfolios that contain only a small number of securities yet deviate minimally from the returns of much larger target indices.

Investors sometimes use proxy portfolios for modeling purposes, rather than for direct investment. Any mathematical model used in a portfolio management context becomes more valuable if it can be applied consistently to both the portfolio and the benchmark. In some cases, though, it is not feasible to include the entire benchmark in an analysis, due to either constraints on processing time or data availability. Proxy portfolios are often used in such settings to stand in for the actual benchmark and speed up the analysis.

We have used several different techniques for constructing proxy portfolios. In addition, we have investigated other methods of tracking index returns, such as replication with futures contracts or swaps. These techniques may be more appropriate for investors who wish to maintain a passive position for only a limited time or for those who seek index returns on a fund with frequent inflows and outflows.

There are two basic approaches to index replication, presented below in this order: cell matching (stratified sampling) and tracking error minimization with a multi-factor risk model. We view them as complementary.

Sampling techniques are the "common sense" approach. To replicate an index, one has to represent its every important component with a few securities. The holdings of securities in a particular cell are usually computed to match that cell's contribution to overall duration. The problem with this approach is that a mismatch to the benchmark in any cell appears to be equally important. In reality, matching some cells is more critical than matching others because the return (or spread) volatility associated with them is higher. Sampling techniques also ignore correlations among cells that sometimes cause risk from an overweight in one cell to be canceled with an overweight in another.

Risk models allow us to replicate indices by creating minimum tracking error portfolios. These models rely on historical volatilities and correlations between returns of different asset classes or different risk factors in the market. So the factor model's "knowledge" is limited to the historical experience observed over the calibration period. Such model may ignore a significant structural mismatch that historically did not result in returns volatility. For example, in replicating an index of 3,000 corporate bonds with a portfolio of 30 bonds, the risk model may ignore allocations to AAA/AA credit qualities because, historically, the volatility of their spreads was comparatively low. The model will deploy these 30 bonds where it believes it matters the most from a historical risk perspective. At the same time, experienced portfolio managers may be alerted to this mismatch by stratified sampling analysis and may wish to take corrective measures based on their expectations that are not necessarily reflective of history.

There is another approach to index replication (not described here)—simulation of portfolio and benchmark returns across a sufficiently complete set of stochastic scenarios. This is an alternative to the history-based approach employed by the risk model. The portfolio is optimized to have an average return that is as close as possible to the benchmark, as well as a similar distribution of returns over the set of scenarios. The number of bonds in such portfolios will be proportional to the tightness of the imposed constraints. Scenarios have to include all likely interest rate, spread, and volatility changes to capture all risk dimensions (and align the respective sensitivities) relevant to the particular portfolio and the benchmark.

The replicating portfolio does not necessarily consist of securities sampled out of the index being replicated. A very practical alternative is using futures and swaps—liquid market instruments with return characteristics similar to many of the index securities. We provide a brief account of our methodology of index replication with these derivatives. This is popular with investors engaged in "portable alpha" strategies (structuring liquid derivatives baskets to replicate index returns and investing cash outside of the index to gain alpha).

Replication by Stratified Sampling

The market structure analysis described above profiles indices or portfolios to show allocations and exposures along two arbitrary risk dimensions. Some useful grids include sector by quality, sector by duration, WAC by WAM, etc.

This form of profiling a benchmark can be applied to establish and maintain a proxy portfolio via a cell-matching technique. A portfolio manager can map a benchmark onto an arbitrary grid and then set portfolio allocations to each cell that match those of the benchmark. To improve tracking further, the manager may target characteristics of each individual cell such as duration, convexity, or quality when selecting and purchasing securities to represent the cell. The more securities are selected in each cell, the more closely the resulting portfolio tracks the index.

We have used this approach successfully to replicate the Lehman Brothers MBS index on a monthly basis.[12] The index is broken down along three dimensions: program (GNMA 30-year, conventional 30-year, all 15-year, and balloons), seasoning (TBA, moderately seasoned, seasoned) and price (premium, cusp coupon, discount).[13] The proxy portfolio is the result of an optimization that is run using constraints at two levels. Several statistical averages of portfolio attributes (price, coupon, duration, etc.) are constrained to match the overall index. Contributions to spread duration are constrained to match the index on a cell-by-cell basis. Because any portfolio that meets all these constraints is likely to track the index very closely, the choice of objective function is not particularly important. The process we have adopted maximizes liquidity (represented by amount outstanding). One possible alternative is to maximize the OAS of the resulting portfolio.

A similar optimization framework was used to replicate a far more diversified benchmark—the Lehman Brothers High Yield Index. As plan sponsors are increasingly choosing the U.S. Universal Index as a benchmark, many managers, some of whom are just beginning to build in-house expertise in high yield, need a safe way to track this sector with an acceptable tracking error. This need is important because the high yield market both represents a significant part of the Universal Index and exhibits volatile returns.

We performed a study[14] that resulted in three strategies for replicating the High Yield Index. The first (issuer strategy) selects the largest (by % market value) securities from the list of the largest issuers in the index. The issuer strategy assumes that idiosyncratic risk is a key component of returns in the high-yield market and does not explicitly control for Treasury duration. This assumption is based on the fact that only a small part of high yield total return volatility is explained by term structure volatility.

The second approach (structure strategy) divides the High Yield Index into industry and credit "buckets" and then selects eligible bonds to populate each bucket. The replicating proxy portfolio is constructed so that in each bucket, the market weights and contributions to spread duration match those of the index. This procedure ensures that the proxy portfolio's overall spread duration matches that of the index. In addition, this strategy also matches both Treasury duration and convexity of the index. Because idiosyncratic risk is an important factor in a high yield bond's return variability, the strategy imposes a set of eligibility criteria to avoid bonds with a

[12] *Replicating the MBS Index Risk and Return Characteristics Using Proxy Portfolios*, Lehman Brothers, March 1997.

[13] Partitioning the MBS universe by price is essentially equivalent to partitioning by coupon. The advantage of using price is that the cutoff levels defining the boundaries do not need to be changed over time.

[14] "High Yield Index Replication," Lehman Brothers, in *Global Relative Value*, March 2001.

potential for high returns volatility. The number of bonds placed in each bucket (a diversification constraint) is also based on the bucket's historical behavior. This requirement helps to diversify idiosyncratic risk further.

Finally, the third replication strategy (structured-issuer strategy) is similar to the second one except that this strategy filters the list of eligible bonds further. The final list contains only the largest market value security from every issuer in the index to force issuer diversification. Otherwise, this strategy follows the same methodology as the structure strategy. This strategy combines the emphasis on issuer diversification of the first strategy with the emphasis on index structure matching of the second strategy. As a result, the tracking error it produced was the lowest of all three strategies. Figure 20 shows performance results of all three strategies.

We have also developed a simple methodology that effectively replicates the returns of any single-currency government bond index using a six-bond proxy portfolio. The index is partitioned into three market-specific maturity segments selected to achieve the best tracking. The maturity breakdown for any particular country reflects market characteristics such as auction cycles, maturity distribution, refunding policies, etc. Within each segment, the bonds are divided into two groups: one with durations above the segment's average and one below. One liquid bond is selected from each half segment. These two bonds are weighted so that the total duration of the pair matches the duration of the segment they represent. The three pairs of bonds are then weighted to match the contributions of their segments to the index. This procedure ensures sufficiently close matching of the term structure allocation. Figure 21 presents the results of a historical simulation of this strategy using ten years of monthly data. When used to replicate the government bond

Figure 20. **Performance of the Three Replication Strategies versus the Lehman High Yield Index,** January 1993-December 2000

# of Issues	**Monthly Mean Outperformance** (bp/month)	**Monthly Tracking Error** (bp/month)	**% of Variance Explained**
Issuer Strategy			
20	18.8	90.4	63.40
40	11.4	68.6	78.90
60	8.3	58.0	84.90
80	8.5	50.9	88.40
100	8.1	46.2	90.40
Structure Strategy			
46	4.6	67.4	79.60
78	2.3	52.6	87.60
Structured-Issuer Strategy			
46	4.7	46.1	90.50
78	0.8	37.9	93.60

indices of the United States, Japan, the United Kingdom, Germany, and France, this methodology consistently explains over 98.9% of the total return variability. The last column in Figure 21 shows how diversification reduces the variability of return differences for the combined index below that of most of its components.

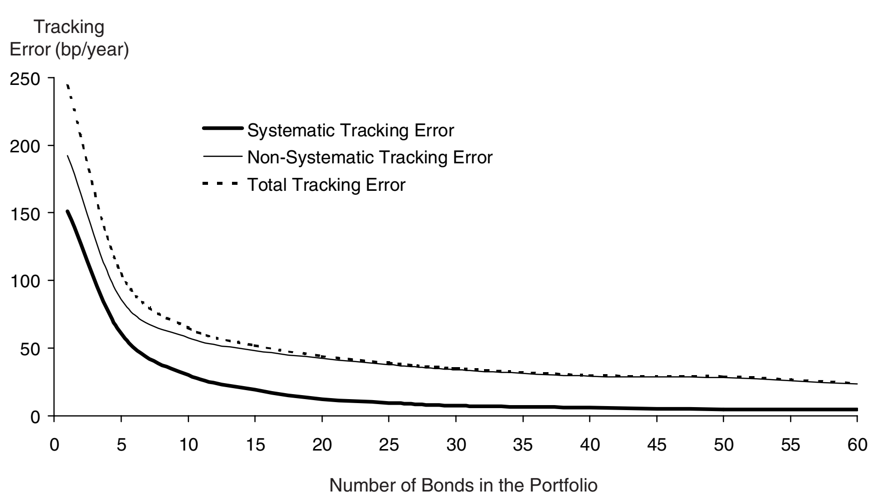
Tracking Error Minimization

Investors often look for “optimal” index proxies: portfolios that best track index returns using a given number of securities. The Lehman Brothers risk model has been successfully applied (using its tracking error minimization feature) to construct proxies for several popular Lehman Brothers indices: Government, Credit, and MBS. This approach has been particularly successful in the homogeneous MBS passthrough market, in which systematic risks (interest rates, volatility, and prepayment) dominate. Yet even for the highly diverse Credit Index, a sufficiently high level of tracking can be achieved with relatively few securities. Figure 22 shows the annualized tracking errors of a Credit Index proxy as a function of the

Figure 21. **Replicating Government Indices with Six-Bond Proxy Portfolios**
Annualized Results in %, No Rebalancing Costs, April 1989–April 1999

	U.S.	Japan	France	U.K.	Germany	Combined
Index	8.54	8.11	10.58	13.34	9.12	9.27
Proxy	8.57	8.04	10.57	13.14	9.13	9.30
Difference	0.03	-0.07	-0.02	-0.20	0.01	0.03
Standard Deviation	0.11	0.25	0.26	0.32	0.30	0.13

Figure 22. **Effect of Diversification on Tracking the Lehman Brothers Credit Index**



number of bonds. Figure 23 summarizes the observed performance of the Treasury, credit, and MBS proxies over twenty months and shows that the observed return deviations are well within the predicted ranges.

1998 was a challenging year for quantitative models in general and risk models in particular. Predictive power of many risk models suffered because of their use of a "time-decay" method that assigns lower weights to earlier observations. However the Lehman Brothers risk model held up well. This was due in part to our decision to give old and recent historical observations the same weight when computing risk factor variances and correlations and to go back in history as far as was reasonable. During the spread widening in August–September 1998, the actual deviations from the benchmark realized by most portfolios were not completely unanticipated by the Lehman Brothers risk model, falling within two to three model-predicted standard deviations. For many other risk models, actual deviations from the benchmark appeared completely unanticipated, as they were more than seven standard deviations away from expected deviations. While our approach may overstate risk during uneventful periods, it deals better with the "fat tails" of historical return distributions. Extreme market returns occur more often than suggested by the normal distribution. Consequently, a distribution with a higher standard deviation tends to match the tails of empirical distributions better. When an extreme event did occur in 1998, investors who relied on our model were better served by our conservative measure of risk.

Tradable Proxy Portfolios for the MBS Index

Unlike most bond indices, the MBS Index contains only non-traded "generic" securities. As a result, the replication process has to start with forming the tracking proxy portfolio out of these generics and then proceed to selecting actual mortgage pools for each generic. Yet there is no assurance that a particular pool will perform identically to its generic. This added layer of decision-making has a potential for significant additional tracking error.

The replication of the MBS Index by the stratified sampling described above would direct investors to buy certain amounts of seasoned product at the outset of

Figure 23. **Effectiveness of the Risk Model— Observed Performance of Proxy Portfolios**
January 1997–December 1998, bp/month

	Index		
	Treasury	**Corporate**	**MBS**
Predicted Tracking Error	6.6	13.5	4.3
Observed Tracking Error	3.2	12.5	1.9
Average Return Difference	0.0	6.6	-0.3
Minimum Return Difference	-6.1	-11.8	-3.4
Maximum Return Difference	6.0	38.0	3.4

the portfolio. The obvious benefit is the close replication of the benchmark from the very beginning. However, seasoned pools may be difficult to obtain. More important, investors buying seasoned products might be delivered small pools that create a real possibility for the added tracking error. Ensuring that all seasoned holdings are implemented with large pools is difficult, if not impossible, in practice. Finally, MBS replication by sampling requires the investor to have full back-office capabilities for MBS transaction processing from day one.

To address this problem, we conducted a study[15] that resulted in two practical strategies for replicating the MBS Index with liquid instruments. These strategies deal with the issue of pool-specific risk and let the investor build MBS transaction processing capabilities gradually while being fully invested in the market. For both strategies, the mechanics of constructing proxy portfolios are the same. At the end of each calendar quarter, an optimized mortgage proxy portfolio is created. The two replication strategies differ in terms of their definition of the "available set." The first strategy (TBA Only) uses only TBA contracts on actively traded, recently originated mortgage coupons. As the composition of the new-issue mortgage market changes over time, this strategy adjusts its holdings of TBA contracts so as to always reflect the most recent and active portion of the mortgage market. The second strategy (Large Pools Only) buys large MBS pools of current production but allows some holdings to remain in the portfolio and season over time. In both cases, the resulting proxy portfolio is rebalanced quarterly.

Figure 24 presents performance results for both replication strategies. One potential drawback of the Large Pools Only strategy is the steadily increasing number of pools in the proxy portfolio. However, by limiting the number of issues to roughly match that of the TBA Only strategy, we show that the improved tracking of the Large Pools strategy is due primarily to the presence of seasoned issues in the proxy and not to a larger number of securities.

[15] *Tradable Proxy Portfolios fot the Lehman MBS Index*, Lehman Brothers, May 2001.

Figure 24. **Performance Summary** (bp) **for the MBS Index Replication Strategies,** April 1994-February 2001

	TBA Only	Large Pools Only	Large Pools Only (12 Issues Max)
Monthly Return Difference (Portfolio vs. the Index)			
Average	-1.7	-0.2	0.2
Standard Deviation	5.6	3.0	2.9
Minimum	-17.4	-8.1	-7.8
Maximum	11.9	7.6	8.8
Annualized Tracking Error (Realized)	19.4	10.4	10.2

Both strategies provide investors with an effective way to replicate the MBS Index without detailed pool-level knowledge of the mortgage market. The relative simplicity of the two strategies may encourage some investors to attempt the MBS Index replication on their own.

Replication with Derivatives

A variation of the cell-matching technique can be applied to replicate the term structure exposure of any fixed-income index with Treasury futures.[16] Futures are widely used as a duration adjustment tool because of advantages such as no portfolio disruption, ease of establishing and unwinding positions, and low transaction costs. For funds with frequent and significant cash inflows and outflows, replication of benchmark returns with exchange-traded futures is often a strategy of choice. Similarly, a large asset allocation shift might be carried out initially using futures, due to the liquidity of the futures markets. Less liquid cash assets can then be redeployed over a longer period of time as opportunities arise.

By taking a long or short position in a single contract, investors can match the duration of any benchmark. However, meaningfully replicating the performance of a broad-based market index requires matching its exposures to all segments of the yield curve.

We have developed and tested a methodology, currently implemented by a number of investors, that employs four Treasury futures contracts (2-, 5-, 10-, and 30-year) to replicate the curve allocation of an index. By analyzing the distribution of security durations in the index, we determine the required mix of contracts. We divide the index into four duration cells. We then compute the allocation and dollar duration within each cell of a perfectly indexed investment of the desired size. Each cell's market value and dollar duration are then matched with a combination of a cash investment and a position in the appropriate futures contract. The cash is usually invested in Treasury bills, though portfolio managers are free to choose other alternatives, such as commercial paper or short-term asset-backed securities, as a source of extra return.

Term structure exposure can be hedged effectively with Treasury futures. Spread risk, inherent in the Credit and Mortgage indices, needs to be hedged separately. Eurodollar futures and swaps were used in a similar methodology to replicate spread indices. Since credit spreads are positively correlated with the TED spread and swap spreads, replication strategies based on these instruments[17] can better track the Credit and Mortgage indices, especially in times of "liquidity events." Because Eurodollar futures are less liquid beyond the five-year point, the Eurodollar strategy replicates only the shorter portion of index cash flows with Eurodollar

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[16] *Replicating Index Returns with Treasury Futures,* Lehman Brothers, November 1997.

[17] *Replication of Index Returns with Treasury Futures, Eurodollar (Euribor) Futures, and Swaps,* Lehman Brothers, March 2000.

Figure 25. **Replicating Lehman Brothers Indices with Derivatives: Performance of Four Replicating Strategies versus Major Lehman Brothers Indices**
January 1994–December 1999, bp/month

	Treasury Futures		Eurodollar and Treasury Futures		Eurodollar Futures and Swaps		Swaps	
Index	**Avg. Ret. Diff.**	**Std. Dev.**	**Avg. Ret. Diff.**	**Std. Dev.**	**Avg. Ret. Diff.**	**Std. Dev.**	**Avg. Ret. Diff.**	**Std. Dev.**
Treasury	3.2	11.3	2.6	9.2	-1.6	25.0	-2.8	25.8
Corporate	1.2	50.2	0.9	48.3	-8.8	34.3	-8.7	33.8
Gov/Credit	2.7	20.0	2.2	17.2	-3.8	18.3	-4.8	18.6
Mortgage	0.2	33.1	-2.3	27.9	-5.0	28.0	-0.9	30.4
Aggregate	2.3	21.1	1.7	18.4	-7.3	15.6	-7.7	15.7

contracts and uses the 10- and 30-year Treasury futures for the longer portion. Figure 25 summarizes the historical performance of these replication strategies versus several Lehman Brothers indices. Results are shown for the three variants of the strategy: using Treasury futures alone, Eurodollar and Treasury futures together, and swaps.

The same technique has been applied to the Lehman Global Aggregate Index.[18] Its diversity of exposures to currency, yield curve, and spread risks makes replication with derivatives a natural choice for this index. We developed several replication strategies using combinations of treasury bond futures, money-market futures, and swaps in four currencies: U.S. dollar, euro, yen, and sterling. These four markets make up over 95% of the Global Aggregate Index. Attempts to fully replicate all the other currencies will meet with diminishing returns, as tracking error declines by no more than 1-2 bp/month while transaction costs increase dramatically. Figures 26 and 27 show the derivative instruments used in the replication, as well as the tracking errors achieved in the four major markets and for the overall index.

Portfolio Optimization

"No View" Yield Curve Optimization

We have designed a Treasury portfolio optimization program tailored to the needs of central banks. The two guiding principles of central bank investment policies are liquidity and preservation of capital. These usually translate into investments in short-term risk-free instruments such as U.S. Treasury bills and notes, managed against a relatively short duration benchmark. The potential for picking up extra return is limited because of the low or zero loss tolerance.

[18] *Replication with Derivatives: The Global Aggregate Index and the Japanese Aggregate Index*, Lehman Brothers, March 2001.

Figure 26. **Instruments Used in Lehman Brothers Global Aggregate Index Replication**
February 1999-December 2000

	U.S.	Euro	Japan	Sterling
Treasury Futures Replication	2-, 5-, 10-yr, and bond futures contracts	Schatz, Bobl, & Bund futures contracts	10-yr. futures contract	10-yr. futures contract
Optimal Replication	Treasuries: 2-, 5-, 10-yr, & bond futures; Spread product: 2-, 5-, 10-, & 30-yr. swaps	Short end: Euribor futures; Long end: Schatz, Bobl, & Bund futures contracts	2-, 5-, 10-, & 30-yr. swaps	2-, 5-, 10-, & 30-yr. swaps

Figure 27. **Tracking Errors of Lehman Brothers Global Aggregate Index Replication**
February 1999-December 2000, bp/month

	Global Aggregate Index		Local Market Components of LBGAI			
	Unhedged	Hedged	U.S.	Euro	Japan	Sterling
Treasury Futures Replication	19.4	17.6	29.2	15.7	43.3	52.7
Optimal Replication	10	8.7	14.8	14.1	21	43.7

We have shown that even under these restrictive conditions, there is potential for outperformance, based on optimal selection and dynamic adjustment of the target duration. The objective of our optimization process is to maximize expected return. Constraints maintain portfolio duration within some range around the benchmark and guarantee (at some confidence level) that the realized return will not be less than a specified minimum over a given review period. At each trade date, we determine the risk/return characteristics of all securities in the investable set. Expected returns are determined through total return analysis under a "no curve view" assumption, using the current yield curve. Risk is projected based on long-term historical volatilities of on-the-run Treasuries. Worst case return is defined as some number of standard deviations below the expected return for each asset. This strategy formulation allows for easy adjustment of risk tolerance through appropriate choice of the review period, minimum return threshold, and confidence level. Figure 28 presents a performance example of such a strategy with a 6-month review period, two percent minimum annualized return target, and the worst case defined as one standard deviation below the expected return. This low-risk strategy achieved a relatively high information ratio of 0.74. Central banks are not the only potential users of this technique. Any asset manager constrained to low-risk, liquid instruments may find this strategy useful.

Figure 28. **Performance of the “No-View” Treasury Optimization Program**
October 1996–May 1999

	Optimization Program Return (%)	**2-Year Treasury Benchmark Return** (%)	**Outperformance** (%)
Monthly Average	0.49	0.47	0.03
Monthly Standard Deviation	0.43	0.46	0.12
Cumulative	15.75	14.90	0.84
Annualized Information Ratio			0.74

The strategy can be implemented with a dynamic self-adjustment feature that modifies optimization parameters based on the cumulative achieved performance. For example, if the first-quarter return falls short of the threshold, the minimum return target for the next month can be boosted to compensate for the shortfall.

Scenario-Based Optimization

Horizon returns for individual securities projected under different scenarios are often used in portfolio optimization. We have worked with a major life insurance company to construct and rebalance an MBS portfolio optimized to outperform the Lehman Brothers MBS Index. At the start of each month, the possible evolution of the yield curve over the month is modeled by a set of 500 randomized scenarios, based on a principal components analysis of one year of daily yield curve movements. The optimizer selects the portfolio of MBS passthroughs with the maximum expected outperformance averaged across all scenarios while limiting underperformance of the index return under each scenario.

In other cases, we have worked with investors to incorporate a specific yield curve outlook into the optimization. The objective is to maximize the expected return under a probability-weighted set of scenarios reflecting investor views, subject to constraints on underperformance in the worst case scenarios.

Sufficient Diversification in Credit Portfolios

The portfolio optimization methods described above deal primarily with systematic risk exposures (e.g., interest rates and sector spreads) of a portfolio relative to a benchmark. However, even after controlling for systematic risk factors, portfolios remain vulnerable to non-systematic risks due to issuer-specific events. Such events tend to affect a portfolio much more dramatically than a much broader benchmark. The credit bombs of the year 2000 painfully taught many credit portfolio managers to ask a key question: How much diversification is necessary and sufficient to protect a portfolio tracking a benchmark from the risks of downgrades and defaults?

We have established a quantitative framework[19] for estimating the risk of downgrades in a credit portfolio and for structuring a portfolio to minimize this exposure. Our model uses rating transition probabilities published by the rating agencies, combined with the historical return data for the Lehman Credit Index, to estimate the downgrade risk of a single bond, a portfolio, and, finally, the tracking error of a credit portfolio to its benchmark.

The model answers the following question: for a portfolio of a given number of bonds, how many bonds of each credit quality should be held to achieve the lowest tracking error due to downgrade risk? For a 100-bond portfolio, we show (in Figure 29) that the optimal allocation holds twice as many Baa-rated bonds than a portfolio in which all holdings are equally weighted. In other words, to minimize downgrade risk, the size of each Baa-rated position must be half of what it is in the equally weighted portfolio. In contrast, the holdings of higher-rated securities become necessarily more concentrated, with Aa-rated positions averaging 2.5% of the portfolio. Overall, the tracking error due to downgrades is 21 bp. The lesson is that to lower the overall tracking error due to downgrades, it is most important to diversify exposures to lower-rated issuers. This conclusion has implications for plan sponsors as well: guidelines should direct investment managers to diversify lower-rated holdings but allow greater concentrations for higher-rated holdings.

While a more diversified portfolio is less exposed to default risk, there are costs incurred by increasing the number of issuers in a portfolio. First, transactions costs increase as portfolio holdings become too small. Second, there is the overhead of monitoring a larger number of issuers. Finally, increasing the number of issuers in a portfolio dilutes the value of credit research, as managers must add issuers beyond those most highly rated by the credit analyst team. Consequently, the "right" amount of diversification is determined by the tradeoff of its two main effects: downgrade risk reduction and dilution of outperformance.

[19] *Sufficient Diversification in Credit Portfolios,* Lehman Brothers, forthcoming in 2001

Figure 29. **Optimal Allocation of Bonds to Qualities in a Credit Portfolio**

Quality	**Percent MV of Index**	**Number of Bonds**	**Track Err due to Downgrades** (bp/yr)	**Position Size** (% MV)
Aa-Aaa	26.3	8	25.8	3.3
A	44.1	30	30.1	1.5
Baa	29.6	62	62.3	0.5
Total	100.0	100	23.7	

We developed a model for the value of credit research to determine how much diversification will maximize the information ratio, i.e., the ratio of expected outperformance from credit research to tracking error due to downgrade risk. This ratio depends on the size of the portfolio and on the assumed credit research function. This model also shows that maximizing the information ratio requires the portfolio to remain well diversified in the lower-rated issues while taking concentrated bets in the higher-rated issues.

V. CONCLUSION

We stand today on the threshold of explosive growth in the utilization of quantitative techniques by fixed-income investors. Several ongoing developments point in this direction.

Emergence of the European corporate bond market has generated unprecedented interest on the part of European investors in expanding their asset mix from traditional government securities to credit products, both investment grade and high yield. Accustomed to a highly quantitative style of multi-currency portfolio management, European investors are looking for quantitative tools for credit investing as well.

The creation of the European Central Bank (ECB) with a separate reserve portfolio freed the national central banks from the immediate need to provide on-demand liquidity. As a consequence, national central banks in Euroland are starting to act more like conservative, yet opportunistic, total return asset managers, with an eye on spread products and even prepayment risk.

The declining issuance of U.S. Treasuries is raising interest on the part of plan sponsors and mutual funds in macro indices such as the Lehman Brothers U.S. Universal Index[20] and the Global Aggregate Index. The inclusion of the additional markets covered by these indices (U.S. dollar high yield corporates and emerging markets and non-dollar corporate and government bonds) can help compensate for the loss of convexity in the U.S. market due to the rising share of MBS. The diminishing supply of U.S. Treasury securities is also forcing reserve managers of central banks around the world to consider alternative investments, such as U.S. agency securities, sovereign and supranational debt, ABS, and investment grade corporates.

These developments increase the demand for portfolio analysis in new markets, generalization of models for global investing, mapping of risk/return characteristics of macro indices, and empirical justification of changes in asset class composition.

We are excited to be working with investors during this time of rapid change in the landscape of fixed income investing. We will continue to offer value-added models, tools, and methodologies for every step in quantitative portfolio management relative to indices.

[20] *The U.S. Dollar-Denominated Universal Index*, Lehman Brothers, January 2000.

APPENDIX I.
QUANTITATIVE PORTFOLIO STRATEGIES GROUP

The diagram in Figure A-1 presents a concise summary of the research activities carried on by the Lehman Brothers Quantitative Portfolio Strategies Group. These include direct participation in creating and producing Lehman bond indices, empirical research based on historical index data, portfolio analysis, development and justification of investment strategies, benchmark selection and construction, and analytics development. As the diagram shows, all the above activities are interconnected and support each other. The results of major studies and developments are reflected in the "shelf" Lehman publications listed below.

Library of Publications on Portfolio Management Techniques and Tools

Portfolio Management

- "Quantitative Analysis of Fixed Income Portfolios Relative to Indices," in *Handbook of Portfolio Management* (Frank Fabozzi), 1998.
- "Liabilities-Based Benchmarks," Lehman Brothers, in *Global Relative Value*, March 2001.

Investment Strategies

- *Value of Security Selection versus Asset Allocation in Credit Markets: A Perfect Foresight Study*, Lehman Brothers, March 1999, and in *The Journal of Portfolio Management,* Vol. 25 No. 4, Summer 1999.
- *Value of Security Selection versus Asset Allocation in Credit Markets: Part II—An "Imperfect Foresight" Study*, Lehman Brothers, March 2000, and in *The Journal of Portfolio Management*, Vol. 27 No. 1, Fall 2000.
- "Value of Risk Taking in a Mortgage Portfolio: A Perfect Foresight Study," in Lehman Brothers *Global Relative Value*, February 2001.
- *Sufficient Diversification in Credit Portfolios*, Lehman Brothers, forthcoming in 2001.

Risk Modeling

- *The Lehman Brothers Multi-Factor Risk Model*, Lehman Brothers, July 1999.

Performance Attribution

- *A Return Attribution Model for Fixed Income Securities*, Lehman Brothers, May 1996, and in *Handbook of Portfolio Management* (Frank Fabozzi), 1998
- *Attribution of Portfolio Performance Relative to an Index*, Lehman Brothers, March 1998.

Index Replication

- *Replicating Index Returns with Treasury Futures*, Lehman Brothers, November 1997.
- *Constant Duration MBS Index*, Lehman Brothers, October 1999, and in *The Journal of Fixed Income,* Vol. 10, No. 1, 2000.
- *Replication of Index Returns with Treasury Futures, Eurodollar (Euribor) Futures and Swaps*, Lehman Brothers, March 2000.
- *Tradable Proxy Portfolios for the Lehman MBS Index*, Lehman Brothers, May 2001.
- *Replication with Derivatives: The Global Aggregate Index and the Japanese Aggregate Index*, Lehman Brothers, March 2001.
- "High Yield Index Replication," in Lehman Brothers *Global Relative Value*, March 2001.

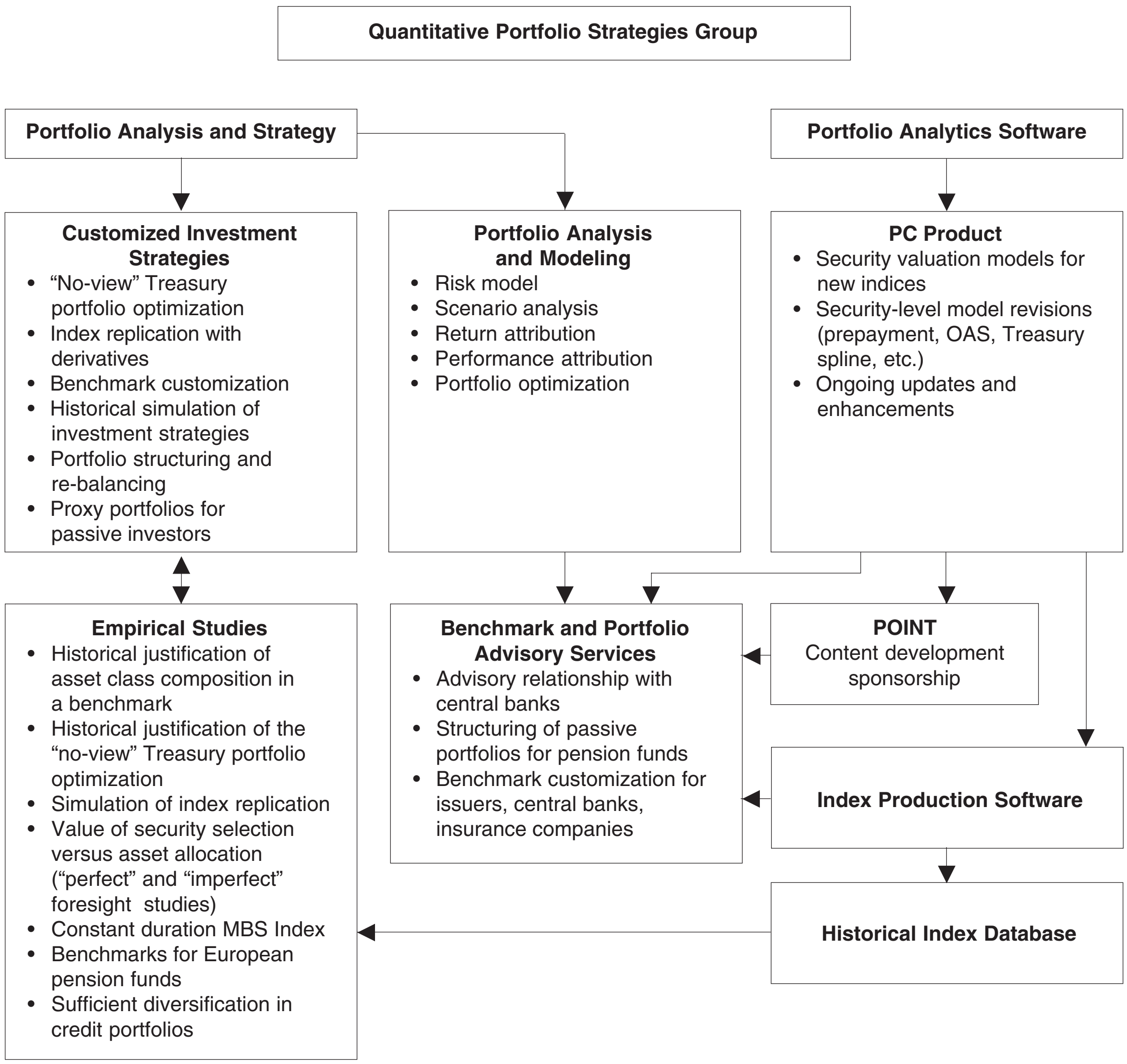
Index and Portfolio Analytics

- *Analyzing Portfolios Relative to the Euro-Aggregate Index, the U.S. Dollar Denominated Universal Index, and the Global Aggregate Index*, in the respective index primers.
- *MBS Index Returns: A Detailed Look*, Lehman Brothers, August 1998 and in *The Journal of Fixed Income*, Vol. 8 No. 4, 1999.
- "A New Method of Excess Returns Computation," in Lehman Brothers *Global Relative Value*, September 2000.
- "Building an MBS Index: Conventions and Calculations," in the *Handbook of MBS Securities* (Frank Fabozzi), to be published in 2001.

Research Presentations

- LB Index Advisory Council
- LB Analytics Advisory Council
- LB Relative Value, Liquid Market, and Credit conferences
- LB PC Product and POINT Seminars in New York, Europe, and Asia
- LB Central Bank Seminars
- LB Institute of Finance and Associates Programs
- Industry conferences (AIMR, INQUIRE, etc.)

Figure A-1



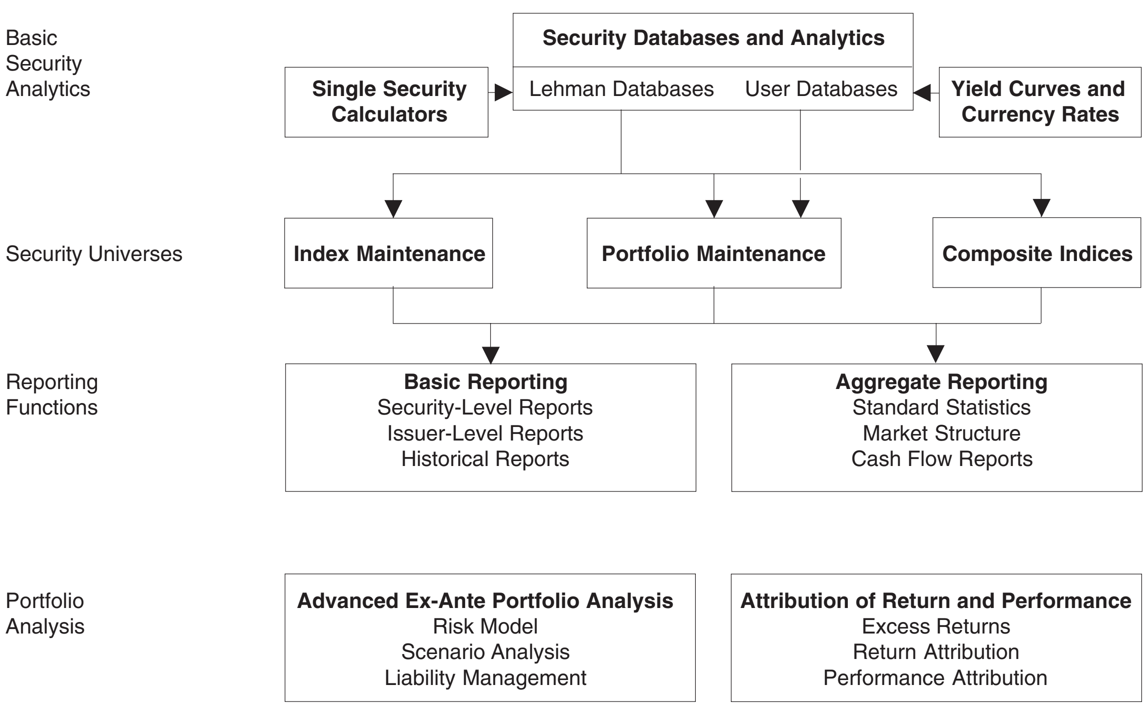
Quantitative Portfolio Strategies Group
Portfolio Analysis and Strategy
Portfolio Analytics Software
Customized Investment Strategies
• "No-view" Treasury portfolio optimization
• Index replication with derivatives
• Benchmark customization
• Historical simulation of investment strategies
• Portfolio structuring and re-balancing
• Proxy portfolios for passive investors
Portfolio Analysis and Modeling
• Risk model
• Scenario analysis
• Return attribution
• Performance attribution
• Portfolio optimization
PC Product
• Security valuation models for new indices
• Security-level model revisions (prepayment, OAS, Treasury spline, etc.)
• Ongoing updates and enhancements
Empirical Studies
• Historical justification of asset class composition in a benchmark
• Historical justification of the "no-view" Treasury portfolio optimization
• Simulation of index replication
• Value of security selection versus asset allocation ("perfect" and "imperfect" foresight studies)
• Constant duration MBS Index
• Benchmarks for European pension funds
• Sufficient diversification in credit portfolios
Benchmark and Portfolio Advisory Services
• Advisory relationship with central banks
• Structuring of passive portfolios for pension funds
• Benchmark customization for issuers, central banks, insurance companies
POINT
Content development sponsorship
Index Production Software
Historical Index Database

APPENDIX II. PC PRODUCT AND POINT: SOFTWARE FOR QUANTITATIVE PORTFOLIO ANALYSIS

Figure A-2 charts the main capabilities of PC Product, the software platform used by the Quantitative Portfolio Strategies Group, as well as by many investors, as a research and portfolio analysis tool. The same system also supports production of all Lehman Brothers fixed-income indices. The four broad functional levels of PC Product are databases and basic security analytics; creation and maintenance of bond "universes," i.e., portfolios and indices; multiple reporting engines, both current and historical; and, building on all the above, portfolio analysis tools.

Individual security databases and associated security calculators provide the foundation of the software platform. The daily index production at Lehman Brothers updates a database containing all securities in the Lehman Global family of indices, as well as thousands of out-of-index bonds. All securities are priced based on trader marks, either directly or via matrix pricing. Besides this main database, the software provides tools for creating additional user databases. These can contain user-modeled securities missing from the main database, as well as

Figure A-2. **Portfolio/Index Software (PC Product)**



price overrides for bonds already in the database. The individual bond calculators are security type specific and allow multiple quoting mechanisms. Corporate and government bonds with embedded optionality are valued using the Lehman Brothers proprietary lognormal option pricing model. Mortgage passthrough calculators use the Lehman Brothers prepayment and interest-rate models and employ a multi-path Monte-Carlo valuation process. An important part of the foundation is a set of tools to define and change the interest rate environment and foreign exchange rates.

PC Product provides three methods of defining sets of bonds or bond "universes." *Portfolios* are represented as lists of holdings (bond identifiers[21] and par amounts). *Indices* are defined by sets of rules or logical filters that identify bonds matching certain criteria at a particular time. While the rules are constant, the set of bonds in the index changes over time. Within an index, bonds are weighted by their market values. As the market moves, relative allocations to market sectors and individual securities fluctuate. Finally, a *composite index* is a combination of indices with fixed weights (e.g., 50% of the Corporate Index, 20% of the Government Index, and 30% of the Mortgage Index). While the component-level weights remain constant, each component is fluid in terms of its sector allocations and detailed composition.

Reports are essential for day-to-day portfolio management. PC Product supports multiple ways of looking at portfolios and indices. Individual security and issuer-level reports show a set of specified attributes for each security or issuer. Market structure reports break down the bond universe into cells defined along any two dimensions specified by the user. Cash flow reports profile allocations along the term structure. Statistical reports compute weighted average duration, yield, etc. for portfolios and indices; and, finally, single security- and index-level historical reports provide time series of relevant statistics and returns.

At the top level, PC Product is a comprehensive set of portfolio management tools that allow *ex-ante* analysis and modification of portfolios together with *ex-post* attribution of achieved performance. The centerpiece of this set is the *multi-factor risk model*. A flexible *scenario analysis* engine provides a complementary perspective on risk. A set of *dedication and immunization* analytics is widely used in asset-liability management. Two models explain and attribute achieved performance. The *return attribution* model associates portions of price return of each bond to such factors as time passage, yield curve shift and twist, and change in volatility and spreads. The *performance attribution* model attributes the difference in portfolio and benchmark performance to allocation mismatches along flexible risk dimensions.

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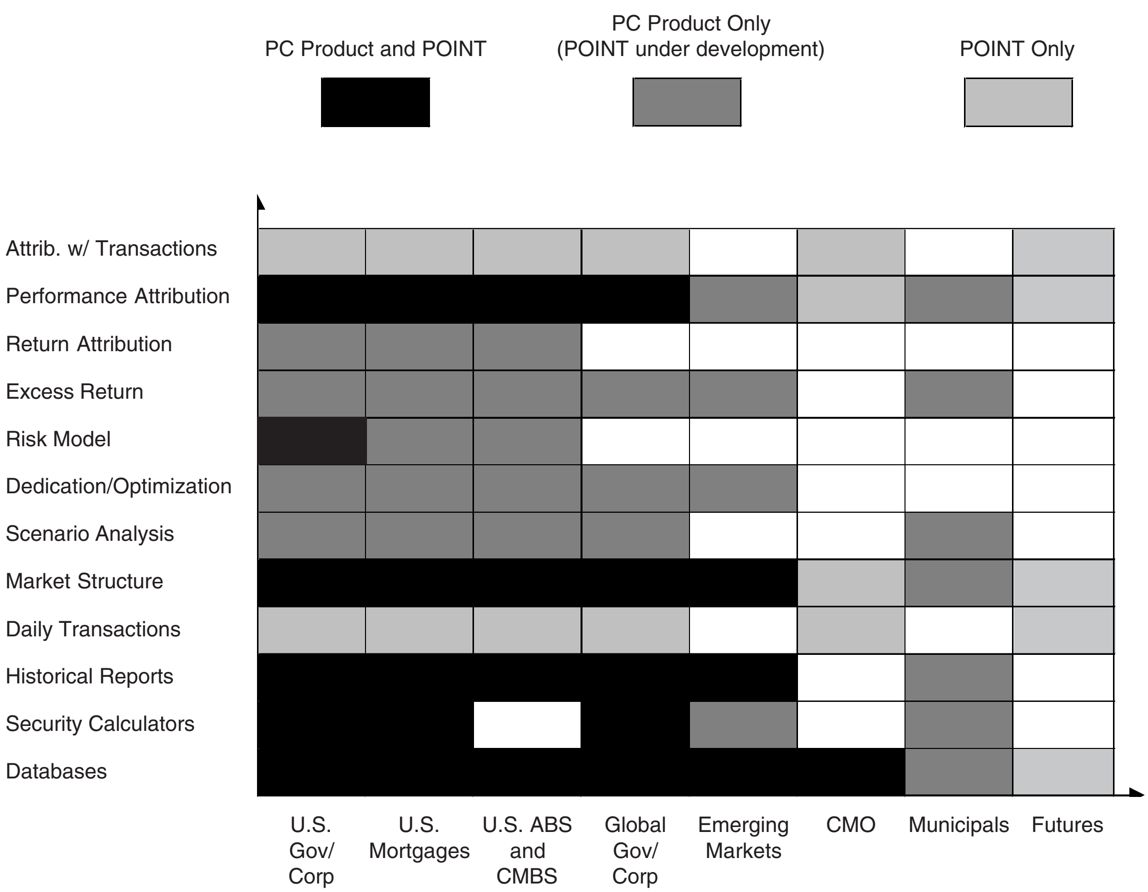
[21] Supported identifiers include CUSIP and ISIN numbers for bonds. MBS holdings may be entered as generic issues using a proprietary CUSIP convention, or as pools using either the pool CUSIP or the pool number.

A major effort is under way at Lehman Brothers to develop the next-generation software platform for portfolio and index analytics. The new system, known as POINT (*PO*rtfolio *IN*dex *T*ool), will extend PC Product functionality in both asset class coverage and analytical models. Figure A-3 plots along these two dimensions the capabilities of PC Product and the early-2000 release of POINT. While this POINT release does not include all of the analytical capabilities of PC Product, it offers several distinct advantages. It covers on a single platform most asset classes typically found in fixed-income portfolios. Its dynamic treatment of portfolios enables a more precise attribution of performance for portfolios with mid-month transactions. Our eventual goal for POINT is to provide full analytical coverage, with all of the models shown in Figure A-3 enhanced to support each of the asset classes.

Figure A-3. **Capabilities of PC Product and POINT**



Publications: L. Pindyck, A. DiTizio, B. Davenport, W. Lee, D. Kramer, J. Threadgill, R. Madison

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