A FLEXIBLE BENCHMARK-RELATIVE METHOD OF ATTRIBUTING RETURNS FOR FIXED INCOME PORTFOLIOS

By Stanley J. Kwasniewski, CFA
Charles Williams, CFA, CAIA

A Flexible Benchmark-Relative Method of Attributing Returns for Fixed Income Portfolios

Contents

Introduction

The primary purpose of an attribution analysis is to explain a portfolio's performance over a period of time using factors that correspond to a portfolio manager's investment strategy and decision-making process. Fixed income performance attribution is the art of decomposing a fixed income portfolio's benchmark-relative performance into a series of attribution factors that sum to fully explain the variation in return over some period of time.

Attribution analysis can be used by investment management firms for both internal and external purposes. Internal consumers of attribution analysis are typically portfolio managers, performance analysts, risk analysts, and senior management. External consumers of attribution analysis are the clients of the investment management firm and consultants. The attribution model requirements for these audiences can vary.

Portfolio managers use attribution to validate investment strategy bets. As such, an important feature of an attribution model is that it uses factors consistent with a portfolio manager's primary investment decisions. The attribution analysis is most robust when it explains how much each individual investment decision contributed to benchmark-relative performance. If this criterion isn't met, the value of the attribution analysis can deteriorate and become less meaningful.

Performance, risk, and senior management teams use attribution to conduct portfolio performance reviews and risk audits. These individuals need to use attribution in the same way a portfolio manager does, but they may also require the flexibility to add additional attribution effects to reveal and quantify the impact of unintended bets. For example, is the portfolio consistently losing a few basis points (bps) relative to the benchmark because it is invested in securities that have a lower income return?

Consultants and clients of investment management firms have varying degrees of financial sophistication and therefore have different attribution requirements. Less is more for some, while others need to see an attribution analysis similar to what a portfolio manager would consume.

Given the requirements of the different consumers of attribution, in addition to the fact that the investment decision-making process can vary for fixed income portfolios within the same firm, a "one size fits all" fixed income attribution model with an inflexible set of attribution factors is likely to fall short of satisfying every consumer. A customizable attribution model, with factor flexibility built from a common framework, is the preferred way to meet the needs of all internal and external consumers of attribution.

Attribution Model Considerations

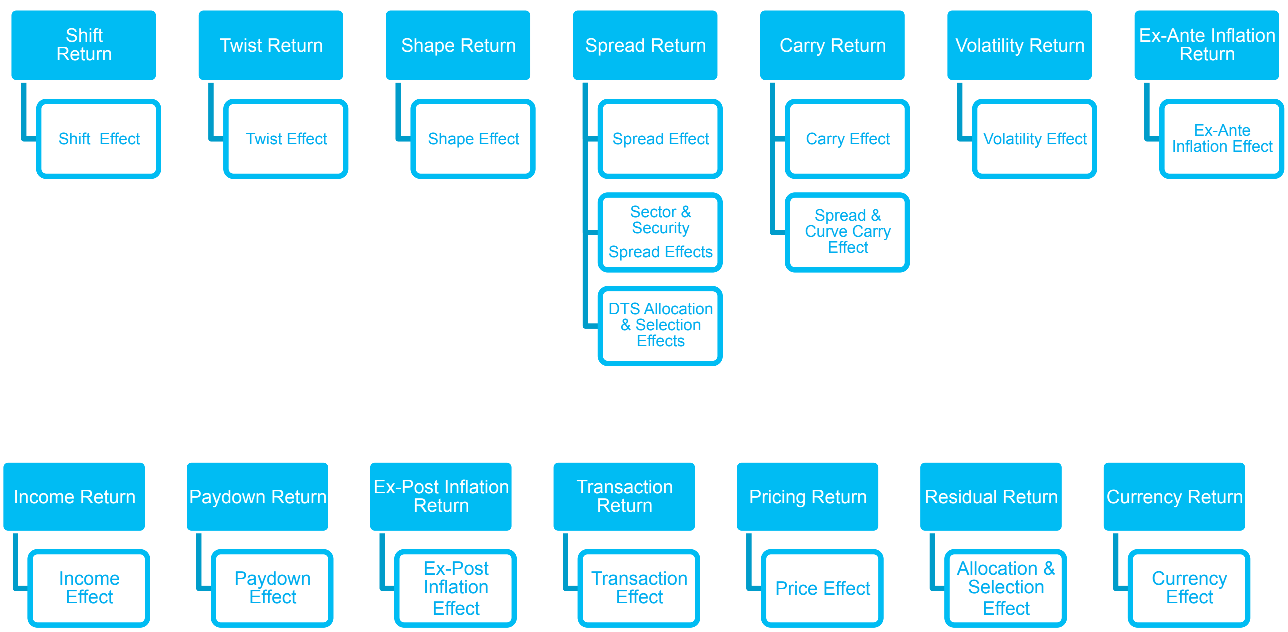
Some important considerations when running a fixed income attribution.

- **Data Maintenance**: Who bears the responsibility for maintaining the portfolio and benchmark holdings and returns? How are the attribution model inputs calculated, uploaded, reconciled, and cleaned? Maintenance is an important consideration because errors at the individual security level can erode the integrity of an attribution analysis.
- **Pricing**: Does the attribution model allow the user to choose the pricing and analytics sources for the portfolio and benchmark? If different pricing sources can be used, is it possible to separate pricing source noise from portfolio management skill using a pricing effect?
- **Model Flexibility**: Does the attribution model allow the user to choose which attribution effects are included in the analysis? Does the attribution model allow the user to define the average change in interest rates? Does the attribution model allow the user to specify the type of durations used as inputs (modified, effective, coupon curve, partial, etc.) for purposes of calculating shift effect?
- **Cash**: How is cash handled in the attribution analysis? Can it be treated either as a part of the fixed income strategy or as a bet on a separate asset class?
- **Derivatives**: How are derivative securities handled in the attribution model? Is the impact of holding derivative securities properly captured in the attribution analysis?
- **Transactions**: Does the attribution model allow the user to provide transaction-level detail? If so, can the impact of transactions on benchmark-relative performance be quantified using a transaction effect?
- **Currency**: Does the attribution model allow the user to quantify the impact of currency management?
- **Security Bucketing**: Does the attribution model allow for user-defined bucketing of portfolio and benchmark securities so that the resulting report is reflective of the investment process?
- **Transparency**: Is the attribution model fully transparent? Can it expose data to the security level? Can its calculations be audited?

Attribution analysis has the potential to become noisy or irrelevant if ample consideration isn’t given to these kinds of questions. FactSet’s fixed income attribution model addresses all of these issues.

Model Background

FactSet's fixed income attribution model was designed to explain the arithmetic difference between the portfolio and benchmark total return using additive attribution effects. It was built on the framework outlined in The Attribution of Portfolio and Index Returns in Fixed Income, by Timothy J. Lord. FactSet used this framework as the basis for its fixed income attribution model because it explained benchmark-relative performance using factors that correspond to the most common investment decisions made by fixed income portfolio managers of investment grade portfolios: the portfolio's duration bet, the portfolio's curve positioning bets, the portfolio's sector bets, and the portfolio's individual security bets. The Lord model could easily be extended to include additional attribution factors to meet the needs of investment managers with different investment processes. The model was also a logical extension of the Brinson Fachler method of attributing the performance of equity portfolios, building on the fundamental attribution concept of asset allocation and security selection.



Return Decomposition

The core concept in FactSet's fixed income attribution model is that a security's total return can be decomposed into additive subcomponent returns. Each subcomponent return corresponds to an investment decision and is subsequently used to calculate the attribution effect that quantifies the impact of that particular investment decision.

The "off the shelf" FactSet fixed income attribution model is designed to be parsimonious with respect to how it decomposes total return. It quantifies the impact of only the primary drivers of benchmark-relative performance and delivers the most relevant attribution analysis to the portfolio manager based on his or her likely investment decisions.

The most basic form of this total return decomposition is as follows:

$$Total\ Return = Shift\ Return + Twist\ Return + Currency\ Return + Residual\ Return$$

Return Component	Formula	Investment Decision Measured
Shift Return	$-D_B * \Delta_{ShftPt} + ½ * C_B * (\Delta_{ShftPt})^2$	Duration
Twist Return	$-D_B * (\Delta_{DMT} - \Delta_{ShftPt})$	Curve positioning
Currency Return	*Total Return – Total Return (Local)*	Currency management
Residual Return	*Total Return – (Shift Return + Twist Return + Currency Return)*	Group allocation and security selection

Effective duration is the default duration used to calculate shift and twist returns because it's the most commonly used measure of interest rate sensitivity for all security types, including those with embedded options. FactSet's fixed income attribution model also allows users to specify that modified, coupon curve, or partial durations be used instead of effective durations. The duration type selection has a direct impact on the shift and twist returns that the model produces. Users also have the ability to specify whether the durations are based on valuation vs. a government, LIBOR, or municipal AAA GO curve. The curve choice is a subjective decision that reflects the practitioner's view of which curve most accurately captures the risk-free rate that should be used to discount the security's cash flows.

Shift Return

$$-D_B * \Delta_{ShftPt} + ½ * C_B * (\Delta_{ShftPt})^2$$

where:

- DB = Beginning Duration
- ∆_ShftPt = Change in Shift Point Yield
- CB = Beginning Convexity

Shift return measures the portion of price return resulting from the average change in interest rates. The aggregate shift return of the securities in the portfolio and benchmark are subtracted to determine whether the portfolio manager's duration bet had a positive or negative effect on relative performance.

Defining the parallel shift, or average, change in interest rates is a subjective exercise. Most practitioners will set the shift point as the point on the yield curve closest to the overall effective duration of the benchmark. FactSet's fixed income attribution model allows the shift point to be defined in a variety of ways, enabling the user to specify what the average change in interest rates is based on: the portfolio or benchmark duration, a specific yield curve point, or the average change of multiple yield curve points. Shift yield changes are calculated for each security based on the observed movement of its respective local currency par yield curve.

Par yield curve movements are used in favor of spot yield curve movements for two reasons. First, portfolio managers generally observe par curve changes, not spot curve changes. Second, for coupon-bearing securities, the par rate is a better approximation of the change in the yield curve relative to that security. In contrast, spot rate changes tend to be too volatile for coupon-bearing securities. A convexity adjustment is applied to the shift return to account for the fact that a security's price change is not a linear function of its duration.

Twist Return

$$-D_B \, * \left(\Delta_{DMT} - \Delta_{ShftPt}\right)$$

$$or$$

$$\left(-1 \, * \, D_{P1} \, * \, \left(\Delta_{PP1} - \Delta_{ShftPt}\right)\right) +$$
$$\left(-1 \, * \, D_{P2} \, * \, \left(\Delta_{PPt2} - \Delta_{ShftPt}\right)\right) +$$
$$\left(-1 \, * D_{Pn} \, * \, \left(\Delta_{PPtN} - \Delta_{ShftPt}\right)\right)$$

where:

- D_B = Beginning Duration
- Δ_{DMT} = Change in Duration-Matched Treasury Yield
- Δ_{ShftPt} = Change in Shift Point Yield
- D_P = Beginning Partial Duration
- Δ_{PPt} = Change in Partial Point Yield

Twist return measures the portion of price return resulting from a non-parallel shift in the yield curve. The aggregate twist return of the securities in the portfolio and benchmark are subtracted to determine whether the portfolio manager's yield curve positioning bet had a positive or negative effect on relative performance.

The partial duration-based method of calculating twist return is discussed in the "Attribution Model Options" section of this paper. A basic way to define the non-parallel change in interest rates is to observe the yield changes of duration-matched treasury (DMT) securities. Each security in both the portfolio and benchmark is assigned a DMT, which is a par-priced, synthetically created government security, denominated in the same currency as that of the security under observation. The DMT represents a risk-free investment with the same effective duration as each security in the portfolio and benchmark. The daily change in the yield of this DMT is used to approximate the impact of the non-parallel yield curve change on each security's price return. The change in yield at the shift point on the yield curve is subtracted from the change in yield of this DMT to determine twist return.

Currency Return

$$Total\ Return - Total\ Return\ (local)$$

Currency return measures the portion of total return resulting from changes in currency exchange rates. The aggregate currency return of the securities in the portfolio and benchmark are subtracted to determine whether the portfolio manager's ability to manage currency had a positive or negative effect on relative performance.

Residual Return

$$Total\ Return\ -\ (Shift\ Return\ +\ Twist\ Return\ +\ Currency\ Return)$$

Residual return quantifies the unexplained portion of the security's total return. Residual returns are used to determine the impact of the portfolio manager's ability to effectively allocate the portfolio's assets to different sectors (or countries) as well as the portfolio manager's ability to select superior performing securities within each sector (or country), adjusted for the already quantified impact of duration, curve, and currency.

Shift, twist, currency, and residual returns are calculated daily for each security in the report. They are compounded over time by multiplying prior period returns by subsequent period security-level total returns. Group-level returns are market value weighted averages.

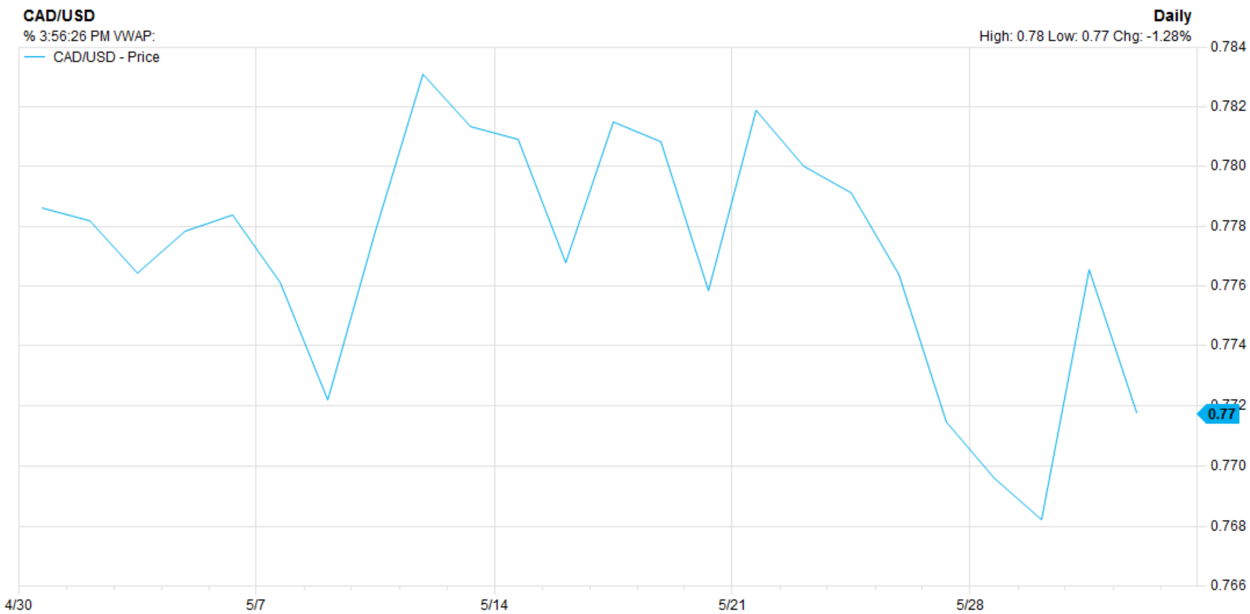
<u>Example</u>

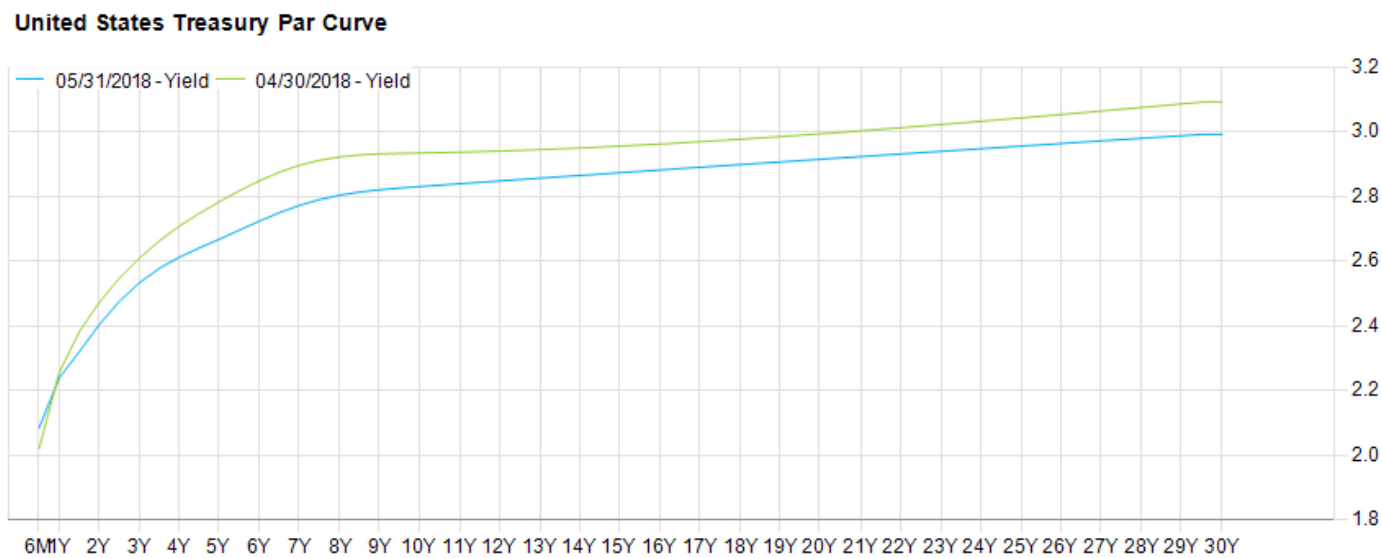
This example illustrates a basic return decomposition for a demo portfolio over the course of a month:

Portfolio Returns

04/30/2018 - 05/31/2018 Level3

		Portfolio Analytics			Portfolio Returns				
	Avg Weight	Avg Effective Duration	Shft Pt Yield Chg	DMT Yield Chg	Shift Return	Twist Return	Residual Return	Currency Return	Total Return
Total	**100.00**	**7.19**	**-0.12**	**-0.10**	**0.85**	**-0.09**	**-0.11**	**1.20**	**1.84**
Agency	0.02	10.34	-0.12	-0.10	1.28	-0.15	-0.33	1.20	2.00
Automotive	2.34	4.77	-0.12	-0.09	0.56	-0.08	0.21	1.20	1.88
Banking	20.20	4.90	-0.12	-0.10	0.57	-0.07	0.04	1.20	1.73
Basic Industry	3.65	7.78	-0.12	-0.10	0.93	-0.10	-0.60	1.19	1.43
Capital Goods	4.93	7.23	-0.12	-0.10	0.86	-0.10	0.03	1.20	1.99
Consumer Goods	6.58	7.57	-0.12	-0.10	0.91	-0.09	-0.17	1.20	1.84
Energy	10.53	7.72	-0.12	-0.10	0.91	-0.10	0.14	1.20	2.16
Financial Services	1.91	5.20	-0.12	-0.10	0.61	-0.06	0.00	1.20	1.74
Healthcare	10.06	7.93	-0.12	-0.10	0.94	-0.10	-0.00	1.20	2.04
Insurance	3.06	8.19	-0.12	-0.10	0.97	-0.10	-0.36	1.20	1.71
Leisure	0.34	4.60	-0.12	-0.11	0.54	-0.05	0.07	1.20	1.76
Media	3.77	8.36	-0.12	-0.10	0.99	-0.11	-1.26	1.19	0.81
Real Estate	2.75	5.75	-0.12	-0.11	0.67	-0.04	0.08	1.20	1.90
Retail	4.94	8.12	-0.12	-0.10	0.96	-0.10	0.10	1.20	2.17
Services	1.03	9.22	-0.12	-0.10	1.13	-0.13	-0.04	1.20	2.16
Technology & Electronics	9.44	7.02	-0.12	-0.10	0.83	-0.09	0.09	1.20	2.03
Telecommunications	4.80	8.72	-0.12	-0.10	1.04	-0.12	-0.80	1.19	1.31
Transportation	2.40	9.06	-0.12	-0.11	1.08	-0.12	-0.23	1.20	1.93
Utility	7.24	9.75	-0.12	-0.10	1.17	-0.13	-0.27	1.20	1.96
[Unassigned]	0.01	1.43	-0.12	-0.06	0.16	-0.09	0.52	1.20	1.78





The demo portfolio represents a U.S. strategy managed by a Canadian investor. The discount curve used for all the securities in this portfolio was the U.S. government yield curve, and the shift point used was the five-year point. The portfolio reporting currency was Canadian dollars.

Because the shift point decreased by 12bps, the average change in interest rates implied an increase in security prices, resulting in an overall portfolio shift return of 85bps. The short end of the yield curve remained relatively flat over the course of this month, while the long end of the yield curve fell considerably. The demo portfolio had stronger exposure to movements in the short end of the yield curve and therefore lost 10bps in twist return. Canadian dollars depreciated relative to U.S. dollars, causing the portfolio to gain 1.20% in currency return. Of the total return, 10bps remained unexplained by the return decomposition, as can be seen in the residual return.

Attribution Model Calculations

FactSet's basic fixed income attribution model uses the following factors to quantify benchmark-relative performance:

$$Total\ Effect\ =\ Shift\ Effect\ +\ Twist\ Effect\ +\ Allocation\ Effect\ +\ Selection\ Effect + Currency\ Effect$$

Return Component	Formula	Investment Decision Measured
Shift Effect	$(Wt_P * R_{PShft}) - (Wt_B * R_{BShft})$	Duration
Twist Effect	$(Wt_P * R_{PTwst}) - (Wt_B * R_{BTwst})$	Curve positioning
Allocation Effect	$(Wt_P - Wt_B) * (R_{BRes} - \bar{R}_{BRes})$	Group allocation
Selection Effect	$Wt_P * (R_{PRes} - R_{BRes})$	Security selection
Currency Effect	*Total effect – Total effect (Local)*	Currency management
Total Effect	*Shift effect + Twist effect + Allocation effect + Selection effect + Currency effect*	Summation of effects described above

Shift Effect

Shift effect quantifies the impact of the portfolio manager's duration bet. It is calculated as follows:

$$(Wt_P * R_{PShft}) - (Wt_B * R_{BShft})$$

where:

- Wt_P = Portfolio Weight
- R_{PShft} = Portfolio Shift Return
- Wt_B = Benchmark Weight
- R_{BShft} = Benchmark Shift Return

Twist Effect

Twist effect quantifies the impact of the portfolio manager's yield curve positioning bet. It is calculated as follows:

$$(Wt_P * R_{PTwst}) - (Wt_B * R_{BTwst})$$

where:

- Wt_P = Portfolio Weight
- R_{PTwst} = Portfolio Twist Return
- Wt_B = Benchmark Weight

- R_{BTwst} = Benchmark Twist Return

Allocation Effect

Allocation effect quantifies the portion of benchmark-relative return that can be attributed to group allocation decisions after adjusting for duration, curve positioning, and currency. It is calculated as follows:

$$(Wt_P - Wt_B) * (R_{BRes} - \bar{R}_{BRes})$$

where:

- Wt_P = Portfolio Weight
- Wt_B = Benchmark Weight
- R_{BRes} = Benchmark Residual Return
- $\bar{R}_{BRes}$ = Overall Benchmark Residual Return

Selection Effect

Selection effect quantifies the portion of benchmark-relative return that can be attributed to security selection decisions after adjusting for duration, curve positioning, and currency. It is calculated as follows:

$$Wt_P * (R_{PRes} - R_{BRes})$$

where:

- Wt_P = Portfolio Weight
- R_{PRes} = Portfolio Residual Return
- R_{BRes} = Benchmark Residual Return

Currency Effect

Currency effect quantifies the portion of benchmark-relative return that can be attributed to currency management. It is calculated as follows:

$$(TE_R - TE_L)$$

where:

- TE_R = Total Effect in Reporting Currency
- TE_L = Total Effect in Local Currency

All attribution effects are calculated daily at the security level. Security-level shift and twist effects are summed to arrive at totals. Allocation, selection, and currency effects are calculated independently at each report level. The allocation, selection, and currency effects at the highest report grouping level are summed to arrive at totals. All daily attribution effects are combined over time using a compounding algorithm (see Appendix).

Example

This example illustrates a basic attribution for the demo portfolio relative to its benchmark over the course of a month:

IG Corporate Attribution

04/30/2018 - 05/31/2018 Level3

	Portfolio			Benchmark			Variation		Attribution Effects					
	Avg Weight	Average Effective Duration	Residual Return	Avg Weight	Average Effective Duration	Residual Return	Shift Point Yld Chg	Var. in Total Return	Shift Effect	Twist Effect	Allocation Effect	Selection Effect	Currency Effect	Total Effect
Total	**100.00**	**7.19**	**-0.11**	**100.00**	**6.94**	**-0.26**	**-0.12**	**0.18**	**0.03**	**-0.00**	**0.01**	**0.14**	**0.00**	**0.18**
Agency	0.02	10.34	-0.33	--	--	--	-0.12	2.00	0.00	-0.00	-0.00	--	0.00	0.00
Automotive	2.34	4.77	0.21	2.76	4.26	0.19	-0.12	0.08	-0.00	0.00	-0.00	0.00	0.00	-0.00
Banking	20.20	4.90	0.04	22.99	5.00	-0.20	-0.12	0.23	-0.02	0.00	-0.00	0.05	-0.00	0.03
Basic Industry	3.65	7.78	-0.60	4.36	7.23	-0.63	-0.12	0.11	-0.00	0.00	0.00	0.00	0.00	0.00
Capital Goods	4.93	7.23	0.03	4.71	7.07	-0.09	-0.12	0.14	0.00	-0.00	0.00	0.01	0.00	0.01
Consumer Goods	6.58	7.57	-0.17	6.24	7.29	-0.24	-0.12	0.11	0.01	-0.00	0.00	0.00	0.00	0.01
Energy	10.53	7.72	0.14	11.55	7.39	-0.27	-0.12	0.46	-0.00	0.00	0.00	0.04	0.00	0.04
Financial Services	1.91	5.20	0.00	2.03	5.33	0.02	-0.12	-0.02	-0.00	0.00	-0.00	-0.00	0.00	-0.00
Healthcare	10.06	7.93	-0.00	8.19	7.89	-0.06	-0.12	0.07	0.02	-0.00	0.00	0.01	0.00	0.03
Insurance	3.06	8.19	-0.36	4.30	7.66	-0.41	-0.12	0.11	-0.01	0.00	0.00	0.00	-0.00	-0.00
Leisure	0.34	4.60	0.07	0.21	4.82	0.11	-0.12	-0.06	0.00	-0.00	0.00	-0.00	0.00	0.00
Media	3.77	8.36	-1.26	3.28	8.21	-1.22	-0.12	-0.02	0.01	-0.00	-0.00	-0.00	0.00	-0.00
Real Estate	2.75	5.75	0.08	2.37	5.72	0.16	-0.12	-0.08	0.00	-0.00	0.00	-0.00	0.00	0.00
Retail	4.94	8.12	0.10	4.07	8.07	-0.06	-0.12	0.17	0.01	-0.00	0.00	0.01	0.00	0.02
Services	1.03	9.22	-0.04	1.01	8.93	-0.10	-0.12	0.12	0.00	-0.00	0.00	0.00	0.00	0.00
Technology & Electronics	9.44	7.02	0.09	7.68	6.98	0.08	-0.12	0.01	0.02	-0.00	0.01	0.00	0.00	0.02
Telecommunications	4.80	8.72	-0.80	4.24	8.54	-0.87	-0.12	0.08	0.01	-0.00	-0.00	0.00	0.00	0.01
Transportation	2.41	9.03	-0.23	2.19	8.96	-0.57	-0.12	0.36	0.00	-0.00	-0.00	0.01	-0.00	0.01
Utility	7.24	9.75	-0.27	7.82	9.06	-0.51	-0.12	0.32	0.00	-0.00	0.00	0.02	-0.00	0.02

Overall

The portfolio manager outperformed the benchmark by 18bps over the month, due largely to security selection.

Shift Effect

The portfolio had, on average, an effective duration of 7.19, while the benchmark had, on average, an effective duration of 6.94. This long duration bet coupled with a decrease in rates of 12bps led to the fund outperforming the benchmark by 3bps.

Twist Effect

The portfolio had a similar composition to the benchmark in terms of distribution along the yield curve, causing twist effect to be negligible.

Allocation Effect

Better overall sector allocation decisions attributed 1bp of outperformance. The portfolio overweighted sectors that outperformed relative to the benchmark and/or underweighted sectors that underperformed relative to the benchmark. The performance used to calculate allocation effect is the residual return, i.e., duration- and curve-adjusted returns.

Selection Effect

Better security selection within the sectors attributed 14bps of outperformance. The manager-selected securities outperformed relative to their respective sector-level performance in the benchmark. Specifically, the manager-selected securities within the Banking, Energy, and Utility groups outperformed. Like allocation effect, the performance used to calculate selection effect is the residual return, i.e., duration- and curve-adjusted returns.

Attribution Model Calculations

Partial Duration-Based Twist Return and Twist Effect

It is possible to use partial durations to calculate twist returns instead of DMTs. This is accomplished by multiplying each partial duration by the difference in the change in yield at the partial duration point relative to the change in yield at the shift point:

$$\left(-1 * D_{P1} * \left(\Delta_{PPt1} - \Delta_{ShftPt}\right)\right) +$$
$$\left(-1 * D_{P2} * \left(\Delta_{PPt2} - \Delta_{ShftPt}\right)\right) +$$
$$\left(-1 * D_{Pn} * \left(\Delta_{PPtN} - \Delta_{ShftPt}\right)\right)$$

where:

- D_P = Beginning Partial Duration
- Δ_{PPt} = Change in Partial Point Yield
- Δ_{ShftPt} = Change in Shift Point Yield

Using the partial twist return calculation above, we can derive the twist effect at each partial point. This is accomplished by taking the difference of the portfolio weight multiplied by the portfolio twist return and the benchmark weight multiplied by the benchmark twist return, at each partial point:

$$(Wt_P * R_{PTwst1}) - (Wt_B * R_{BTwst1}) +$$

$$(Wt_P * R_{PTwst2}) - (Wt_B * R_{BTwst2}) +$$

$$(Wt_P * R_{PTwstN}) - (Wt_B * R_{BTwstN})$$

where:

- Wt_P = Portfolio Weight
- R_{PTwst} = Portfolio Partial Twist Return
- Wt_B = Benchmark Weight
- R_{BTwst} = Benchmark Partial Twist Return

Using partial durations to calculate twist return and twist effect produces more precise results for securities whose cash flow profiles are significantly different from government securities. For example, mortgage-backed securities (MBS) repay principal monthly and therefore can have different interest rate sensitivities compared to government securities with similar effective durations. Partial durations will account for this difference when calculating twist returns, while changes in DMT yields will not.

Example

This example highlights the differences in twist return when using DMT yield vs. partial duration yield changes. The portfolio is comprised solely of 30yr MBS, and the report is run over one year. Income return (see the "Expanded Return

Decomposition" section) is also included to remove noise from the residual. Coupon curve partial duration is used as FactSet measures curve changes using the par curve.

Partial Duration Yield Change with Coupon Curve Partial Duration

08/31/2017 - 08/31/2018 Sector Subgroup

		Portfolio Analytics			Portfolio Returns				
	Avg Weight	Avg Effective Duration	Shft Pt Yield Chg	DMT Yield Chg	Shift Return	Twist Return	Income Return	Residual Return	Total Return
Total	**100.00**	**5.21**	**1.03**	**0.89**	**-5.22**	**1.23**	**3.60**	**-0.13**	**-0.53**
FHLMC Single Family 30yr	21.63	5.38	1.03	0.88	-5.37	1.32	3.66	-0.09	-0.49
FNMA Single Family 30yr	44.33	5.49	1.03	0.87	-5.53	1.42	3.63	-0.06	-0.54
GNMA I Single Family 30yr	3.25	4.27	1.03	0.98	-4.32	0.70	3.96	-0.68	-0.34
GNMA II Single Family 30yr	30.78	4.80	1.03	0.92	-4.78	0.95	3.47	-0.21	-0.58

DMT Yield Change with Coupon Curve Duration

08/31/2017 - 08/31/2018 Sector Subgroup

		Portfolio Analytics			Portfolio Returns				
	Avg Weight	Avg Effective Duration	Shft Pt Yield Chg	DMT Yield Chg	Shift Return	Twist Return	Income Return	Residual Return	Total Return
Total	**100.00**	**5.21**	**1.03**	**0.89**	**-5.22**	**0.76**	**3.60**	**0.34**	**-0.53**
FHLMC Single Family 30yr	21.63	5.38	1.03	0.88	-5.37	0.85	3.66	0.38	-0.49
FNMA Single Family 30yr	44.33	5.49	1.03	0.87	-5.53	0.91	3.63	0.45	-0.54
GNMA I Single Family 30yr	3.25	4.27	1.03	0.98	-4.32	0.25	3.96	-0.23	-0.34
GNMA II Single Family 30yr	30.78	4.80	1.03	0.92	-4.78	0.54	3.47	0.20	-0.58

Notice the twist return calculated with partial durations is significantly higher, resulting in a reduction of residual return when compared to the twist return using DMT yield changes (13bps vs. 34bps). Clients can customize the number of partial points used when calculating twist return as well as modify the duration type. Additionally, clients can break out the twist return per partial point to identify which part of the curve drove performance.

08/31/2017 - 08/31/2018 Sector Subgroup

		Twist Return																	
	Average Weight	1 M...	3 M...	6 M...	1 Year	2 Year	3 Year	4 Year	5 Year	6 Year	7 Year	8 Year	9 Year	10 Y...	15 Y...	20 Y...	25 Y...	30 Y...	Total
Total	**100.00**	--	**-0.00**	**-0.00**	**-0.01**	**-0.05**	**-0.04**	**-0.03**	--	**0.03**	**0.02**	**0.08**	**0.12**	**0.38**	**0.27**	**0.28**	**0.16**	**0.02**	**1.23**
FHLMC Single Family 30yr	21.63	--	-0.00	0.00	-0.01	-0.04	-0.03	-0.03	--	0.03	0.01	0.09	0.12	0.41	0.28	0.30	0.17	0.02	1.32
FNMA Single Family 30yr	44.33	--	-0.00	-0.00	-0.01	-0.04	-0.03	-0.03	--	0.03	0.00	0.07	0.13	0.45	0.30	0.32	0.20	0.03	1.42
GNMA I Single Family 30yr	3.25	--	-0.00	-0.00	-0.02	-0.06	-0.06	-0.04	--	0.03	0.04	0.08	0.10	0.20	0.23	0.16	0.03	0.00	0.70
GNMA II Single Family 30yr	30.78	--	-0.00	-0.00	-0.01	-0.05	-0.05	-0.04	--	0.03	0.03	0.08	0.11	0.27	0.23	0.21	0.11	0.01	0.95

The most meaningful results are achieved when partial duration points corresponding to the yield curve points employed during the investment decision-making process are used in the attribution analysis.

Expanded Return Decomposition

FactSet's fixed income attribution model also allows for a further decomposition of residual return into additional subcomponents for practitioners who require more granularity:

Return Component	Formula	Investment Decision Measured
Shift Return	$-D_B * \Delta_{ShftPt} + ½ * C_B * (\Delta_{ShftPt})^2$	Duration
Twist Return	$-D_B * (\Delta_{DMT} - \Delta_{ShftPt})$	Curve positioning
Shape Return	Calculated by repricing the bond using the end-of-day yield curve and subtracting shift return and twist return.	Curve residual
Spread Return[1]	$-1 * D_{BSprd} * \Delta_{OAS}$	Spread management
Carry Return	Calculated by holding the yield and spread constant, moving settlement date forward to the ending date, and repricing the bond.	Time management
Volatility Return	Calculated as the price percent change caused by holding the option-adjusted spread (OAS) of the bond constant and moving the settlement date forward, while changing the term structure of volatility to the ending yield curve's term structure of volatility.	Interest rate volatility
Ex-Ante Inflation Return	Calculated as the price percent change caused by holding the OAS of the bond constant and moving the settlement date forward, while applying the ending date's inflation projection.	Inflation expectations
Income Return	$((AI_E + CPN_E) - AI_B) / (P_B + AI_B) * 100$	Income/coupon management
Paydown Return	$(PRIN_E – P_E * PRIN_E) / (P_B + AI_B) * 100$	Principal repayment
Ex-Post Inflation Return	$-(P_E + AI_E) * ((FACTOR_B - FACTOR_E) / FACTOR_B) / (P_B + AI_B) * 100$	Inflation changes
Residual Return	The remaining return after subtracting the included return components (shift, twist, shape, etc.) from the total return.	Group allocation and security selection
Currency Return	*Total Return – Total Return (Local)*	Currency management

Variables used in the table above are defined in the subsequent sections.

By default, only shift, twist, and currency return are stripped out of residual return. Shape, spread, carry, volatility, ex-ante inflation, income, paydown, and ex-post inflation return can be stripped out of residual return by including the relevant column in the report. In general, the more subcomponents used in the total return decomposition, the smaller the residual return. A small residual will almost always exist as the FactSet performance attribution model uses an exposures and factors approach instead of full repricing. Full repricing is offered via the FactSet return attribution model, although this model is not benchmark-relative. An exposures and factors approach is preferred for the benchmark-relative model, as it allows more flexibility in choosing return components, duration type, and curve measurement.

[1] Spread Return can also be analyzed using a Duration Times Spread (DTS) approach.

In this U.S. investment grade portfolio, there is11bps of residual return when isolating only shift, twist, and currency return.

04/30/2018 - 05/31/2018 Level2

		Portfolio Analytics			Portfolio Returns				
	Avg Weight	Avg Effective Duration	Shft Pt Yield Chg	DMT Yield Chg	Shift Return	Twist Return	Residual Return	Currency Return	Total Return
Total	**100.00**	**7.19**	**-0.12**	**-0.10**	**0.85**	**-0.09**	**-0.11**	**1.20**	**1.84**
Financial	25.18	5.32	-0.12	-0.10	0.62	-0.08	-0.01	1.20	1.73
Industrials	67.33	7.61	-0.12	-0.10	0.90	-0.10	-0.13	1.20	1.87
Quasi & Foreign Government	0.02	10.34	-0.12	-0.10	1.28	-0.15	-0.33	1.20	2.00
Utility	7.48	9.71	-0.12	-0.10	1.16	-0.13	-0.27	1.20	1.96

By adding spread return and income return to the report, we can reduce residual by 8bps:

04/30/2018 - 05/31/2018 Level2

		Portfolio Analytics			Portfolio Returns					
	Avg Weight	Avg Effective Duration	Shft Pt Yield Chg	DMT Yield Chg	Shift Return	Twist Return	Spread Return	Income Return	Residual Return	Total Return
Total	**100.00**	**7.19**	**-0.12**	**-0.10**	**0.85**	**-0.09**	**-0.43**	**0.35**	**-0.03**	**1.84**
Financial	25.18	5.32	-0.12	-0.10	0.62	-0.08	-0.34	0.32	0.01	1.73
Industrials	67.33	7.61	-0.12	-0.10	0.90	-0.10	-0.45	0.36	-0.04	1.87
Quasi & Foreign Government	0.02	10.34	-0.12	-0.10	1.28	-0.15	-0.53	0.38	-0.19	2.00
Utility	7.48	9.71	-0.12	-0.10	1.16	-0.13	-0.56	0.35	-0.07	1.96

Shape Return

Shape return measures the change in bond price due to changes in the yield curve after accounting for shift and twist return. Using FactSet's enterprise calculation engine, each bond is repriced nightly using the end-of-day yield curve, holding all other variables constant. Comparing the new bond price to the original bond price reflects the return due to changes in the yield curve. FactSet then subtracts the shift and twist return to isolate the shape return. The sum of these three represent the overall price change due to yield curve movements.

Spread Return

$$-1 * D_{BSprd} * \Delta_{OAS}$$

where:

- D_{BSprd} = Beginning Spread Duration
- Δ_{OAS} = Change in Option Adjusted Spread

Spread return measures the portion of price return resulting from a change in the security's spread. This is measured by multiplying the bond's spread change by its sensitivity to spread changes, i.e., spread duration. This is multiplied by -1 to reflect that as spread's increase, bond prices decrease. Spread changes can be a major driver of return for high-yield portfolios which derive the majority of their portfolio performance through credit quality.

Spread return can also be measured with duration times spread (DTS) return. DTS return is calculated as:

$$(-1 * D_{BSprd} * OAS_B) * \%\Delta_{OAS}$$

where:

- D_{BSprd} = Beginning Spread Duration
- OAS_B = Beginning Option Adjusted Spread
- $\%\Delta_{OAS}$ = Percent Change in OAS

$D_{BSprd} * OAS_B$ is floored at 1bps for corporate, municipal, and government related bonds as well as credit default swaps (CDS). The floor is enforced to prevent bonds with near zero or slightly negative spreads from showing extreme or unintuitive spread return results.

Note: With the exception of the floor described above, the DTS return formula is identical to the standard spread return formula.

Carry Return

Carry return measures the portion of price return resulting from the passage of time. It is the combined impact of accretion and rolldown returns; both calculations require the use of FactSet's security calculation engine. Accretion return is calculated by holding the yield of a security constant, moving the settlement date forward to the ending date, then repricing the security. Rolldown return is calculated by holding the spread of a security constant, moving the settlement date forward to the ending date, then repricing the security.

The performance attribution model allows carry return to be viewed as a single return or to be decomposed into curve carry and spread carry.

Curve Carry

Curve carry reflects the return earned due to the passage of time for government debt and is calculated as:

$$\boldsymbol{Rolldown\ Return + 1\ day\ DMT\ Return}$$

Rolldown return reflects the yield a security collects as it rolls down (or up) the yield curve. When the yield curve has a positive slope, rolldown return will be positive, as the yield will decrease each day, thus increasing the price of the bond. If the yield curve is negatively sloping, yield will increase as the bond moves closer to maturity, thus decreasing the price of the bond. FactSet assumes the slope of the credit curve is similar to the slope of the sovereign curve, therefore attributing the rolldown return to curve carry. DMT return is security-specific and represents the daily yield on a synthetic government bond with the same duration as the security in question. The one-day DMT return combined with the rolldown return represents the curve carry portion of carry return.

Spread Carry

Spread carry is the return earned due to the passage of time in excess of the government curve. Spread carry is calculated as:

$$\boldsymbol{Accretion\ Return - 1\ day\ DMT\ Return}$$

Accretion return reflects a bond's pull to par, thus bonds trading at a premium will have a negative accretion return and bonds trading at a discount will have a positive accretion return. FactSet subtracts one day's DMT return to remove the carry return due to government debt, with the remainder being attributed to spread carry. Notice the summation of spread carry and curve carry represents the overall carry return. Consequently, decomposing carry return into the separate effects won't change the overall carry return.

Volatility Return

Volatility return measures the portion of a bond's price return that can be attributed to the rising or falling of interest rate volatility. Volatility return is calculated by holding OAS constant, changing the term structure of volatility to the ending yield curve's term structure of volatility, moving the settlement date forward, and then repricing the bond. Volatility return is applicable for securities that have uncertain cash flows due to prepayments, optionality, or floating rates, such as MBS, callable bonds, and floating rate notes. Volatility return is calculated nightly in FactSet's enterprise calculation engine due to the full repricing requirement.

Ex-Ante Inflation Return

Ex-ante inflation return reflects the portion of a security's price return that can be attributed to movement in inflation expectations. Ex-ante inflation is calculated by holding the bond OAS constant and moving the settlement date forward, while applying the ending date's inflation projection. Similar to volatility return, ex-ante inflation return is calculated nightly and databased due to the full repricing requirement.

Income Return

$$((AI_E + CPN_E) - AI_B) / (P_B + AI_B) * 100$$

where:

- AI_E = Ending Accrued Interest
- CPN_E = Ending Coupon Payment
- AI_B = Beginning Accrued Interest
- P_B = Beginning Price

Income return measures the portion of total return resulting from coupon payments and changes in the security's accrued interest.

Paydown Return

$$(PRIN_E - P_E * PRIN_E) / (P_B + AI_B) * 100$$

where:

- $PRIN_E$ = Ending Principal Repayment
- P_E = Ending Price
- P_B = Beginning Price
- AI_B = Beginning Accrued Interest

Paydown return measures the return generated when a security pays back a portion or all of its principal early. Paydown return is most common for MBS and structured products. While slightly counterintuitive, a principal repayment can generate a negative return if the security is trading at a premium. To better understand this, consider a bond trading at $105 with a face value of $100. If the issuer pays 20% of the face in a principal repayment, the investor will now have a bond worth $84 (1.05 * 80) and a cash flow of $20, or $104 total. This reflects a loss of $1 due to the principal repayment. Bonds trading at a discount see the opposite effect and record a positive paydown return.

Ex-Post Inflation Return

$$-(P_E + AI_E) * ((FACTOR_B - FACTOR_E) / FACTOR_B) / (P_B + AI_B) * 100$$

where:

- P_E = Ending Price
- AI_E = Ending Accrued Interest
- $FACTOR_B$ = Beginning Inflation Factor
- $FACTOR_E$ = Ending Inflation Factor
- P_B = Beginning Price
- AI_B = Beginning Accrued Interest

Ex-post inflation return measures the portion of an inflation-linked bond's total return resulting from a principal adjustment due to rising or falling inflation. Ex-post inflation differs from ex-ante inflation in that it is realized inflation – e.g., it reflects actual changes to the inflation index since the previous period.

Residual Return

$$Total\ Return - Sum\ of\ all\ other\ returns\ included\ in\ the\ decomposition$$

Residual return is calculated by subtracting both the basic and optional model inputs from the security's total return.

All optional returns are calculated daily for each security in the report. They are combined over time by multiplying prior period returns by subsequent period security-level total returns. Group-level returns are market value-weighted averages.

<u>Example</u>

This example illustrates an expanded return decomposition for a demo portfolio over the course of eight months:

12/29/2017 - 08/31/2018 Level2

		Portfolio Analytics					Portfolio Returns								
	Avg Weight	Avg Effective Duration	Shft Pt Yield Chg	DMT Yield Chg	Avg Spread Duration	Chg in OAS	Shift Return	Twist Return	Shape Return	Spread Return	Carry Return	Income Return	Residual Return	Currency Return	Total Return
Total	**100.00**	**6.58**	**0.34**	**0.29**	**6.57**	**21**	**-2.45**	**0.58**	**0.01**	**-1.62**	**-0.06**	**2.16**	**0.10**	**-1.13**	**-2.42**
⊞ Covered	0.01	2.19	0.53	0.65	2.19	6	-0.15	-0.20	0.03	-0.42	1.01	1.21	-0.04	-1.88	-0.44
⊞ Financial	29.55	4.96	0.32	0.30	4.96	24	-1.68	0.16	0.00	-1.49	-0.08	2.05	0.08	-1.20	-2.15
⊞ Industrials	62.81	7.15	0.36	0.30	7.15	19	-2.83	0.72	0.01	-1.62	-0.02	2.20	0.10	-0.98	-2.41
⊞ Quasi & Foreign Government	0.33	4.81	0.22	0.20	4.81	14	-1.06	0.04	0.04	-0.66	-0.27	2.04	0.03	-1.68	-1.52
⊞ Securitized	0.46	9.15	0.28	0.13	9.24	29	-2.68	1.61	0.01	-2.35	-0.56	2.72	0.14	-3.84	-4.94
⊞ Utility	6.83	8.17	0.22	0.13	8.16	23	-2.31	0.98	0.00	-2.26	-0.38	2.18	0.16	-1.94	-3.57

By including shape, spread, carry, and income return we're able to reduce residual return from 58bps to 10bps, thus increasing the explanatory power of the model.

Advanced Attribution Model Calculations

FactSet's performance attribution model uses a "variation in contribution" approach when measuring over/underperformance for each additional effect. With a few exceptions, each effect will be calculated as $(Wt_P * R_P) - (Wt_B * R_B)$.

Shape Effect

The shape effect quantifies the portion of benchmark-relative return that can be attributed to the residual of the portfolio's yield curve positioning bet not incorporated in shift and twist. It is calculated as follows:

$$(Wt_P * R_{PShp}) - (Wt_B * R_{BShp})$$

where:

- Wt_P = Portfolio Weight
- R_{PShp} = Portfolio Shape Return
- Wt_B = Benchmark Weight
- R_{BShp} = Benchmark Shape Return

Spread Effect

Spread effect quantifies the impact of the portfolio manager's ability to manage spreads. It is calculated as follows:

$$(Wt_P * R_{PSprd}) - (Wt_B * R_{BSprd})$$

where:

- Wt_P = Portfolio Weight
- R_{PSprd} = Portfolio Spread Return
- Wt_B = Benchmark Weight
- R_{BSprd} = Benchmark Spread Return

The overall spread effect can be decomposed and replaced with two additive component effects: sector spread effect and security spread effect.

Sector Spread Effect

Sector spread effect quantifies the impact of the portfolio manager's ability to manage spreads at the lowest level of report grouping, such as sectors or countries. It is calculated as follows:

$$(Wt_P * -D_{PSprd} * \Delta_{\underline{B}OAS}) - (Wt_B * -D_{BSprd} * \Delta_{\underline{B}OAS})$$

where:

- Wt_P = Portfolio Weight
- D_{PSprd} = Portfolio Spread Duration

- $\Delta_{\underline{B}OAS}$ = Group-Level Benchmark Change in OAS
- Wt_B = Benchmark Weight
- D_{BSprd} = Benchmark Spread Duration

Security Spread Effect

Security spread effect quantifies the impact of the portfolio manager's ability to manage spreads at the security level. When added to sector spread effect, it equals the overall spread effect. It is calculated as follows:

$$\left(Wt_P * -D_{PSprd} * (\Delta_{POAS} - \Delta_{\underline{B}OAS})\right) -$$

$$\left(Wt_B * \ -D_{BSprd} * (\Delta_{BOAS} - \Delta_{\underline{B}OAS})\right)$$

where:

- Wt_P = Portfolio Weight
- D_{PSprd} = Portfolio Spread Duration
- Δ_{POAS} = Portfolio Change in OAS
- $\Delta_{\underline{B}OAS}$ = Group-Level Benchmark Change in OAS
- Wt_B = Benchmark Weight
- D_{BSprd} = Benchmark Spread Duration
- Δ_{BOAS} = Benchmark Change in OAS

DTS Selection and Allocation Effects

The benchmark-relative spread effect can also be broken out into DTS allocation and DTS selection effects. DTS is calculated as spread multiplied by spread duration and has become a popular tool for managers looking to better compare securities across the credit spectrum. It has been observed that spreads tend to change on a relative basis rather than an absolute basis, with larger spreads widening at faster rates than tighter spreads. Using DTS, a security with a spread of 400bps and spread duration of 1 is comparable to a security with a spread of 200bps and a spread duration of 2.

$$DTS\ Allocation = ((Wt_P * \ DTS_P) - (Wt_B * DTS_B)) * \ \ \%\Delta_{BOAS}$$

$$DTS\ Selection = \ -(\%\Delta_{POAS} - \%\Delta_{BOAS}) * (Wt_P * DTS_P)$$

where:

- Wt_P = Portfolio Weight
- DTS_P = Portfolio DTS
- Wt_B = Benchmark Weight
- DTS_B = Benchmark DTS
- $\%\Delta_{BOAS}$ = % Percent Benchmark Change in OAS

- $\%\Delta_{POAS}$ = Percent Portfolio Change in OAS

Evaluating DTS allocation and DTS selection is helpful when managing the risk of DTS through weighting decisions at the sector/security level. DTS allocation represents the relative exposure to spread at the group level when compared against a general barometer of spread changes, in this case benchmark percent OAS change. DTS selection measures the manager's ability to select securities with relatively better changes in credit quality within a specific sector or group. In the example below, the manager had 6bps of outperformance in the telecommunications sector due to a relatively lower spread bet in an increasing spread environment. The manager lost 3bps in DTS selection as the portfolio telecommunication securities had a relatively higher percent change in OAS when compared to the benchmark. Thus, the decision to take smaller spread bets in the sector paid off, but the securities selected performed worse than the benchmark securities.

US HY DTS Attribution

04/30/2018 - 05/31/2018 Level3

	Portfolio			Benchmark			Variation	Attribution Effects						
	Average Weight	Port % Chg OAS	Port Beg DTS	Average Weight	Bench. % Chg OAS	Bench Beg DTS	Var. in Total Return	Shift Effect	Twist Effect	DTS Allocation Effect	DTS Selection Effect	Allocation Effect	Selection Effect	Total Effect
Total	**100.00**	**8.86**	**11.464**	**100.00**	**6.49**	**14.420**	**-0.11**	**0.04**	**0.00**	**0.13**	**-0.22**	**-0.01**	**-0.06**	**-0.11**
Automotive	1.53	10.83	12.465	1.77	13.35	12.931	0.39	0.00	0.00	0.00	0.01	-0.00	-0.00	0.01
Banking	3.03	9.43	10.255	3.21	20.80	14.125	1.48	-0.00	-0.00	0.02	0.03	0.00	-0.00	0.05
Basic Industry	14.79	10.97	12.375	11.74	8.55	12.640	-0.18	0.03	-0.00	-0.03	-0.04	-0.00	-0.00	-0.05
Capital Goods	4.03	14.37	8.193	5.47	13.16	9.757	0.12	-0.00	0.00	0.03	-0.01	0.00	-0.00	0.02
Consumer Goods	3.45	2.34	12.314	2.82	7.64	15.607	0.73	0.00	-0.00	0.00	0.02	0.00	-0.00	0.02
Energy	19.76	4.27	13.913	15.37	2.76	18.002	-0.11	0.03	-0.00	-0.00	-0.05	-0.00	0.01	-0.02
Financial Services	3.05	7.78	8.320	4.02	5.97	10.092	-0.08	-0.00	0.00	0.01	0.00	0.00	-0.01	0.00
Healthcare	6.31	3.86	9.970	10.30	0.28	14.576	-0.47	-0.02	0.00	-0.00	-0.02	-0.00	-0.00	-0.05
Insurance	0.41	7.31	8.990	1.06	1.07	19.192	-0.74	-0.00	0.00	0.00	-0.00	-0.00	-0.00	-0.01
Leisure	4.84	16.67	9.928	4.21	13.32	10.572	-0.23	0.01	0.00	-0.01	-0.02	-0.00	0.00	-0.02
Media	10.19	8.87	11.590	11.00	6.78	14.128	-0.07	0.00	0.00	0.02	-0.02	-0.00	-0.00	-0.00
Real Estate	1.32	7.46	9.985	0.94	5.93	12.315	-0.01	0.00	-0.00	-0.00	-0.00	-0.00	-0.00	-0.00
Retail	3.88	12.01	15.096	4.29	7.52	20.091	-0.47	-0.00	0.00	0.02	-0.03	-0.00	-0.01	-0.02
Services	6.37	14.63	11.260	5.48	12.45	13.350	-0.01	0.01	-0.00	0.00	-0.02	0.00	-0.00	-0.01
Technology & Electronics	7.00	10.56	7.807	5.66	4.90	9.069	-0.25	0.01	-0.00	-0.00	-0.02	-0.00	-0.00	-0.02
Telecommunications	5.70	11.69	11.806	9.26	6.35	19.244	-0.38	-0.02	0.00	0.06	-0.03	-0.00	-0.01	-0.00
Transportation	1.11	20.10	4.544	0.96	1.08	13.214	-0.69	0.00	-0.00	0.00	-0.01	0.00	-0.00	-0.01
Utility	3.21	7.66	10.760	2.45	3.21	11.649	-0.41	0.01	-0.00	-0.00	-0.01	-0.00	-0.00	-0.01

Note: There may be slight differences where the DTS allocation and selection effects do not sum to the overall spread effect due to smoothing.

Carry Effect

Carry effect quantifies the impact of the portfolio manager's ability to manage the passage of time. It is calculated as follows:

$$(Wt_P * R_{PCar}) - (Wt_B * R_{BCar})$$

where:

- Wt_P = Portfolio Weight
- R_{PCar} = Portfolio Carry Return
- Wt_B = Benchmark Weight
- R_{BCar} = Benchmark Carry Return

Curve and Spread Carry Effect

The overall carry effect can be decomposed into two subcomponent effects: curve carry effect and spread carry effect. Curve carry and spread carry will sum to the original carry effect.

Note: There may be slight differences where the curve and spread carry effects do not sum to the overall carry effect due to smoothing.

Volatility Effect

The volatility effect quantifies the portion of benchmark-relative return that can be attributed to the rising or falling of interest rate volatility. It is calculated as follows:

$$(Wt_P * R_{PVol}) - (Wt_B * R_{BVol})$$

where:

- Wt_P = Portfolio Weight
- R_{PVol} = Portfolio Volatility Return
- Wt_B = Benchmark Weight
- R_{BVol} = Benchmark Volatility Return

Income Effect

Income effect quantifies the impact of the portfolio manager's ability to manage coupon payments. It is calculated as follows:

$$(Wt_P * R_{PInc}) - (Wt_B * R_{BInc})$$

where:

- Wt_P = Portfolio Weight
- R_{PInc} = Portfolio Income Return
- Wt_B = Benchmark Weight
- R_{BInc} = Benchmark Income Return

Paydown Effect

Paydown effect quantifies the impact of the portfolio manager's ability to manage principal repayments. It is calculated as follows:

$$(Wt_P * R_{PPdwn}) - (Wt_B * R_{BPdwn})$$

where:

- Wt_P = Portfolio Weight
- R_{PPdwn} = Portfolio Paydown Return
- Wt_B = Benchmark Weight

- R_{BPdwn} = Benchmark Paydown Return

Ex-Ante Inflation Effect

Ex-ante inflation effect quantifies the portion of benchmark-relative return due to the manager's ability to manage inflation expectations.

$$(Wt_P * R_{PExAInfl}) - (Wt_B * R_{BExAInfl})$$

where:

- Wt_P = Portfolio Weight
- $R_{PExAInfl}$ = Portfolio Ex-Ante Inflation Return
- Wt_B = Benchmark Weight
- $R_{BExAInfl}$ = Benchmark Ex-Ante Inflation Return

Ex-Post Inflation Effect

Ex-post inflation effect quantifies the impact of the portfolio manager's ability to manage inflation. It is calculated as follows:

$$(Wt_P * R_{PExPInfl}) - (Wt_B * R_{BExPInfl})$$

where:

- Wt_P = Portfolio Weight
- $R_{PExPInfl}$ = Portfolio Ex-Post Inflation Return
- Wt_B = Benchmark Weight
- $R_{BExPInfl}$ = Benchmark Ex-Post Inflation Return

Below is a one-month example of an inflation-linked EUR-denominated portfolio compared against a benchmark with a similar mandate. The manager picked up 7bps of outperformance by choosing securities with relatively better inflation expectations, most noticeably foregoing exposure to Italian inflation-linked bonds. The lack of investment in Italian linkers hurt the portfolio when looking at the ex-post inflation return, or the realized return due to inflation changes over the course of the month.

06/29/2018 - 07/31/2018 Country

	Portfolio		Benchmark		Variation	Attribution Effects							
	Average Weight	Average Effective Duration	Average Weight	Average Effective Duration	Var. in Total Return	Shift Effect	Twist Effect	Spread Effect	Ex-Ante Inflation Effect	Ex-Post Inflation Effect	Allocation Effect	Selection Effect	Total Effect
Total	**100.00**	**4.79**	**100.00**	**7.79**	**0.02**	**0.36**	**-0.10**	**-0.26**	**0.07**	**-0.00**	**-0.03**	**0.00**	**0.02**
France	63.51	4.78	47.29	8.28	0.13	0.10	-0.05	-0.16	0.02	0.08	-0.01	0.01	-0.01
Germany	20.14	4.56	14.01	8.63	0.21	0.03	-0.01	-0.05	0.00	0.03	0.00	-0.01	0.00
Italy	--	--	29.04	7.09	0.04	0.25	-0.05	-0.06	0.05	-0.15	-0.02	--	0.02
Spain	16.35	5.12	9.66	6.28	0.06	-0.03	0.00	0.01	-0.00	0.03	0.00	-0.00	0.01

All attribution effects are calculated daily at the security level. Security-level effects are then summed to arrive at totals. All daily attribution effects are combined over time using a compounding algorithm (see Appendix).

Example

This example illustrates an expanded attribution for a U.S. high-yield portfolio for a one-month time period:

03/30/2018 - 04/30/2018 Level3

	Portfolio		Benchmark		Variation	Attribution Effects								
	Average Weight	Average Effective Duration	Average Weight	Average Effective Duration	Var. in Total Return	Shift Effect	Twist Effect	Shape Effect	Carry Effect	Spread Effect	Income Effect	Allocation Effect	Selection Effect	Total Effect
Total	**100.00**	**4.30**	**100.00**	**4.04**	**-0.60**	**-0.05**	**0.01**	**-0.01**	**-0.01**	**-0.25**	**-0.28**	**-0.00**	**-0.00**	**-0.60**
Automotive	1.54	4.71	1.83	4.11	0.51	0.00	-0.00	0.00	-0.00	0.04	-0.01	-0.00	-0.01	0.01
Banking	3.02	4.70	3.23	5.56	0.17	0.01	-0.00	-0.00	0.00	0.00	-0.00	-0.00	-0.00	0.00
Basic Industry	14.97	4.88	11.78	4.26	-0.38	-0.05	0.01	0.00	0.00	0.01	-0.02	0.00	0.00	-0.05
Capital Goods	3.92	3.98	5.47	3.50	-0.26	0.01	-0.00	0.00	0.00	-0.01	-0.02	0.00	0.00	-0.02
Consumer Goods	3.39	4.04	2.82	4.23	0.26	-0.00	0.00	-0.00	-0.00	0.02	-0.00	-0.00	-0.00	0.01
Energy	19.43	4.60	15.05	4.36	-1.00	-0.05	0.00	-0.00	-0.00	-0.04	-0.04	-0.00	-0.00	-0.14
Financial Services	3.11	3.26	3.94	3.18	-0.32	0.01	-0.00	0.00	0.00	-0.01	-0.01	0.00	-0.00	-0.02
Healthcare	6.13	3.69	10.23	3.81	-0.52	0.04	-0.00	0.00	-0.00	-0.04	-0.05	0.00	0.00	-0.05
Insurance	0.41	4.07	1.04	4.32	-0.87	0.01	-0.00	-0.00	-0.00	0.00	-0.01	0.00	0.00	-0.00
Leisure	4.45	4.36	4.11	4.00	-0.59	-0.01	0.00	-0.00	0.00	-0.01	-0.01	-0.00	0.00	-0.03
Media	10.06	4.18	10.75	3.96	-0.33	0.00	-0.00	0.00	0.00	-0.02	-0.03	0.00	0.01	-0.04
Pfd-Banking	--	--	0.18	6.86	0.24	0.00	-0.00	-0.00	-0.00	0.00	-0.00	-0.00	--	0.00
Real Estate	1.30	3.46	0.98	3.69	-1.01	-0.00	0.00	-0.00	-0.00	-0.00	-0.01	-0.00	0.00	-0.01
Retail	3.87	4.95	4.38	4.54	-1.28	0.00	-0.00	0.00	-0.00	-0.04	-0.01	-0.00	-0.00	-0.05
Services	6.89	4.43	5.57	3.81	-0.25	-0.02	0.00	-0.00	-0.00	0.02	-0.02	-0.00	0.00	-0.02
Technology & Electronics	7.25	3.55	5.71	3.25	0.04	-0.02	0.00	-0.00	0.00	0.03	-0.01	-0.00	0.00	0.01
Telecommunications	5.81	4.19	9.46	4.30	-1.94	0.04	-0.00	0.00	-0.00	-0.20	-0.03	-0.00	-0.01	-0.21
Transportation	1.09	2.15	0.95	2.72	-0.49	0.00	-0.00	-0.00	-0.00	0.00	-0.01	-0.00	-0.00	-0.00
Utility	3.36	4.20	2.54	3.67	-0.42	-0.01	0.00	0.00	-0.00	0.00	0.00	0.00	-0.00	-0.01

Here, the portfolio manager underperformed the benchmark by 60bps over the month. The portfolio had a slightly higher duration bet, which coupled with rising rates contributed to a negative shift effect. Both twist and shape effect were negligible at approximately 1bps. Additionally, the portfolio earned slightly less than the benchmark due to the pull to par and yield curve rolldown, both of which are reflected in the carry return.

03/30/2018 - 04/30/2018 Level3

	Portfolio		Benchmark	
	Average Weight	Coupon Rate	Average Weight	Coupon Rate
Total	**100.00**	**5.586**	**100.00**	**6.314**
Automotive	1.54	5.370	1.83	5.440
Banking	3.02	5.762	3.23	6.233
Basic Industry	14.97	5.485	11.78	6.189
Capital Goods	3.92	5.236	5.47	6.230
Consumer Goods	3.39	5.484	2.82	5.901
Energy	19.43	5.940	15.05	6.473
Financial Services	3.11	5.766	3.94	6.439
Healthcare	6.13	5.363	10.23	6.190
Insurance	0.41	5.234	1.04	6.690
Leisure	4.45	5.508	4.11	6.118
Media	10.06	5.393	10.75	6.206
Pfd-Banking	--	--	0.18	--
Real Estate	1.30	5.182	0.98	5.604
Retail	3.87	5.750	4.38	6.144
Services	6.89	5.262	5.57	6.530
Technology & Electronics	7.25	5.409	5.71	6.114
Telecommunications	5.81	5.915	9.46	6.950
Transportation	1.09	5.862	0.95	6.966
Utility	3.36	5.786	2.54	6.226

The two biggest contributors to underperformance were spread and income effect. The portfolio managed spread relatively worse when compared to the benchmark and lost 25bps on spread decisions. Specifically, the Telecommunications sector contributed 20bps of underperformance. The portfolio lagged the benchmark across all sectors in income effect, suggesting the portfolio was invested in lower coupon securities. This is supported when comparing weighted average coupon rates at the sector level.

Note: The securities in this portfolio had no sensitivity to inflation and paydown effect was negligible, as the majority of the securities did not experience a principal repayment during the measurement period. Similarly, volatility return was insignificant as the majority of securities lacked optionality. For portfolios that have a significant exposure to these factors, it's important to include the associated effects in the attribution report for better explanatory power.

Price Effect

It is fairly common to have securities held in both the portfolio and benchmark that are valued differently by the investment management firm and the benchmark vendor. While pricing differences create noise in the attribution analysis, it is possible to eliminate this noise by introducing a price effect. The price effect quantifies the impact of using different pricing sources for securities that are held in common between the portfolio and benchmark. It is measured by calculating the contribution to return of the portfolio using the portfolio's prices, recalculating the contribution to return of the portfolio using the benchmark's prices, then taking the difference between the two values. All subsequent attribution effects are calculated using the portfolio weight/return calculated with the benchmark's prices, thereby ensuring that the remainder of the attribution effects are reflective of the manager's skill and are not influenced by differences in portfolio and benchmark valuations.

$$(Wt_{PPS} * R_{PPS}) - (Wt_{BPS} * R_{BPS})$$

where:

- Wt_{PPS} = Portfolio Weight using Portfolio Pricing Sources
- R_{PPS} = Portfolio Return using portfolio Pricing Sources
- Wt_{BPS} = Portfolio Weight using Benchmark Pricing Sources
- R_{BPS} = Portfolio Return using Benchmark Pricing Sources

Because the price effect calculation adjusts security weights, securities not held in the benchmark will also show minimal security-level price effects. This adjustment in weight is important to correctly attribute over/underperformance to price effect when applicable. Consider a two-security portfolio relative to an identical two-security benchmark. The portfolio and benchmark hold the same face value of each bond, with the only difference being the pricing sources used.

	Portfolio						Benchmark						Attribution	
	Face Value	B. Price	BMV	E. Price	EMV	Return	Face Value	B. Price	BMV	E. Price	EMV	Return	Var in Return	Price Effect
Bond A	5000	1.01	5050	1.02	5100	0.99%	5000	1.005	5025	1.018	5090	1.29%	-0.30%	-0.15%
Bond B	5000	1.01	5050	1.015	5075	0.50%	5000	1.02	5100	1.021	5105	0.10%	0.40%	0.20%
Total			10100		10175	0.74%			10125		10195	0.69%	0.05%	0.05%

In this example, it is apparent the portfolio outperformance is strictly due to pricing source differences. If the portfolio wasn't reweighted using the benchmark pricing sources, the price effect wouldn't fully capture the outperformance, thus causing noise in other attribution effects. FactSet suggests focusing on price effect at the portfolio level rather than the security or group level.

Note: When price effect is used in conjunction with a portfolio that sends daily transactional data, transaction effect (see below) will be included in the price effect unless it is specifically isolated as its own effect. This is because the benchmark does not include transaction data, thus repricing the portfolio using benchmark pricing sources will inherently strip out the impact of transactions.

Example

This example illustrates a basic attribution for the demo portfolio over the course of a month that includes a price effect:

04/30/2018 - 05/31/2018 Level2

	Portfolio		Benchmark		Variation	Attribution Effects							
	Average Weight	Average Effective Duration	Average Weight	Average Effective Duration	Var. in Total Return	Shift Effect	Twist Effect	Spread Effect	Income Effect	Price Effect	Allocation Effect	Selection Effect	Total Effect
Total	100.00	4.35	100.00	4.07	-0.11	0.04	0.00	-0.09	-0.07	0.01	0.00	-0.00	-0.11
Financial	6.50	3.99	8.30	4.31	0.42	-0.01	0.00	0.06	-0.01	-0.00	0.00	-0.01	0.03
Industrials	90.29	4.37	89.26	4.06	-0.15	0.04	0.00	-0.13	-0.06	0.01	0.00	0.00	-0.14
Utility	3.21	4.34	2.45	3.73	-0.41	0.01	-0.00	-0.02	0.00	-0.00	0.00	-0.00	-0.01

The price effect removes 1bps of noise caused by pricing differences between the portfolio and benchmark from the attribution analysis. The impact of including the pricing effect is mostly seen in the allocation and selection effects.

Transaction Effect

FactSet supports three holdings methodologies for calculating returns within Portfolio Analysis: buy and hold, order management system (OMS), and transactions-based returns (TBR). For clients who provide daily transactional data via OMS or TBR, the performance attribution model can calculate a transaction effect to isolate the effect intraday trades had on the over/underperformance of the portfolio. A transaction effect results when a security is purchased or sold at a price that is different from the security's closing price on the day of the transaction. This can have a positive or negative effect on the portfolio's total return. It is quantified by calculating the return of the portfolio using intraday transactions, recalculating the return of the portfolio without using intraday transactions, and then taking the difference between the two returns. FactSet offers four TBR methodologies (see Appendix).

Summary

FactSet's fixed income attribution model explains the benchmark-relative total return of a fixed income portfolio in a manner that relates the primary investment decisions made by a portfolio manager to changes in the market environment over the measurement period. It acknowledges that a "one size fits all" model is not appropriate for all consumers of fixed income attribution. The model overcomes this challenge by allowing a high degree of user-defined flexibility that can be leveraged to tailor an attribution analysis to the specific purpose and audience for which it is intended.

Appendix

Attribution effects are combined over time using one of four compounding algorithms:

- Basic – Forward Looking
- Basic – Backward Looking
- Residual Free – Portfolio Cumulative
- Residual Free – Benchmark Cumulative

The first two compounding algorithms require the use of a residual smoothing algorithm to ensure the attribution model's total effect equals the difference between the portfolio and benchmark total return. The latter two compounding algorithms do not leave a residual and require no residual smoothing algorithm. The details of these compounding and smoothing algorithms are outside of the scope of this paper.

Transactions-based returns can be calculated using one of four methodologies:

- Daily Valuation
- Cash Flows at Start of Day
- Purchase at Start of Day
- Cash Flows at Middle of Day (resembles "Mid-Point Dietz")

The details of these transactions-based return methodologies are outside of the scope of this paper.

ABOUT FACTSET

FactSet creates flexible, open data and software solutions for tens of thousands of investment professionals around the world, providing instant access to financial data and analytics that investors use to make crucial decisions.

For 40 years, through market changes and technological progress, our focus has always been to provide exceptional client service. From more than 60 offices in 24 countries, we're all working together toward the goal of creating value for our clients, and we're proud that 95% of asset managers who use FactSet continue to use FactSet, year after year. Subscribe to our thought-leadership blog to gain fresh insight delivered daily at insight.factset.com.

Learn more at www.factset.com, and follow on Twitter at www.twitter.com/factset

IMPORTANT NOTICE