

Tail Risk Estimation

Radu C. Găbudean POINT Portfolio Modeling Index, Portfolio and Risk Solutions

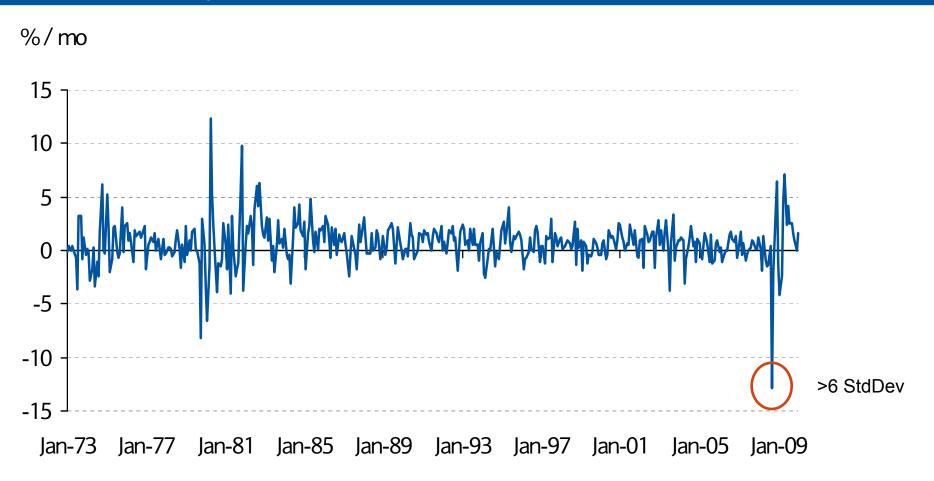
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Tail Risk Example

Barclays Capital US Financials IG Index – Total Return





Importance of Tail Risk

- Extreme losses
- Maximum allowed portfolio loss: Exceedance → fund / institution failure
- Magnitude of extreme losses: Loosely linked to volatility
- Mean-variance framework
 - Poor predictor under extreme scenarios
- Failures of the framework
 - Likelihood of extreme events is higher than predicted by normal distribution
 - Behavior of "mean-variance hedges" under stressed scenarios
 - Asymmetry of positive vs. negative extreme scenarios
- Maximum allowed loss may change under extreme macro events



Agenda

- Measurement
- Tail Risk Models
 - Historical
 - Marginal / Copula approach
- Marginal / Copula in detail
- Direct modeling of extreme events: EVT, CAViaR
- Testing
- VaR by horizon



Measures of Tail Risk

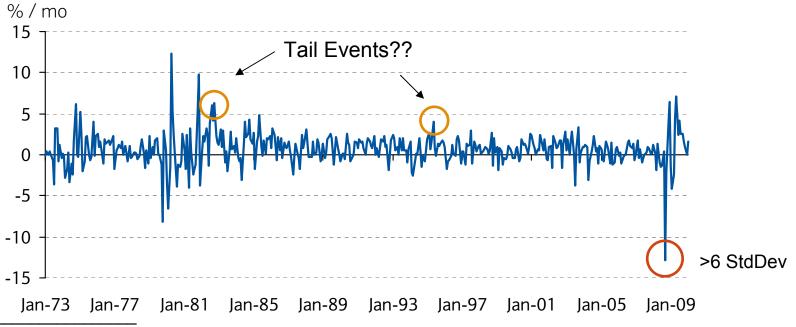
- VaR: Best of worst case scenarios
 - Use: When looking at a threshold, e.g., default
 - Problematic: No diversification (VaR₁+VaR₂<VaR_{1,2}), unless normal or T
- CVaR/ES: Average of worst case scenarios
 - Use: Understand the portfolio behavior under stressed scenarios
 - Coherent measure, i.e., monotonic, homogeneous, translational invariant, sub-additive
 - Sub-additive: CVaR₁+CVaR₂>CVaR_{1.2}
 - Better behaved in optimization problems
- Normal distribution → VaR/CVaR = simple volatility rescaling



Initial Models: Historical Option

- Historical option: Construct portfolio values using time-series of pricing inputs
 - No distributional assumption → more robust but less precise
 - Backward looking, in-sample
 - Cannot do weighting, otherwise lose older events
 - We can do better

Barclays Capital US Financials IG Index – Total Return





State-of-the-Art: Marginal-Copula

- Forecast the distribution of pricing inputs (factors) instead of portfolio
 - Limited types of distributions and better behaved
 - Fit a distribution: Can look as far out in the tail as a user wants
 - Fit a multivariate distribution
- Multivariate distributions
 - MV Normal: Simple, covariance matrix defines everything
 - MV Student-T: Add one parameter
 - All individual factors' (marginal) distributions are the same (link 1)
 - Marginal distributions linked to factors inter-dependency (link 2)
- Marginal-Copula: It breaks the links
 - Model each marginal distribution separately,
 - Then join them with a function (=copula), which defines factor dependency
 - POINT approach



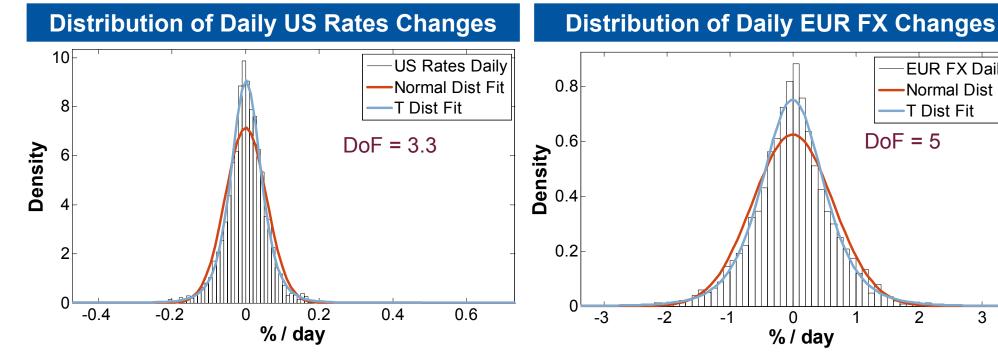


Marginal-Copula Models



Marginal Distributions

- Focus on only individual (univariate) distributions
- Use various types: Normal, Student-T, Skew-T, etc.
- Trade-off: Complexity vs. robustness
- Type varies by asset class: e.g., IR are skewed, EUR FX are less



Source: Barclays Capital.



EUR FX Daily

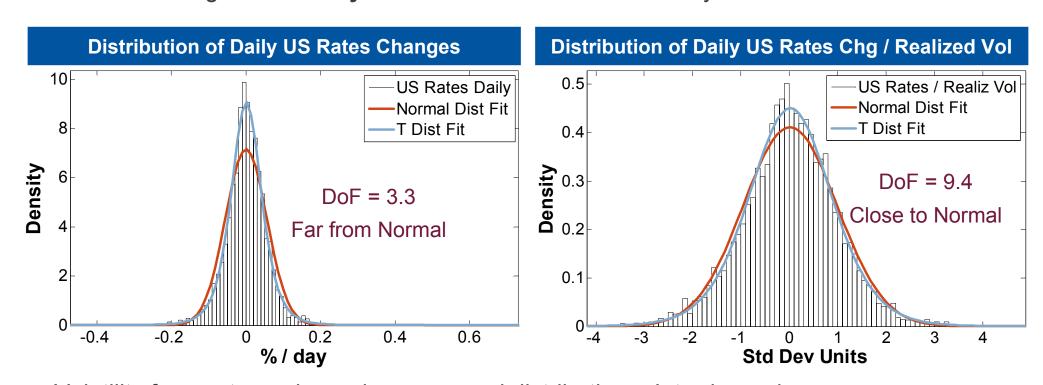
T Dist Fit

DoF = 5

Normal Dist Fit

Marginal Distributions: Conditioning

- Mixing simple distributions with varying volatility → complex unconditional distributions
 - Divide by volatility before defining distribution type → constant vol
 - Resulting distributions are closer to normal
 - Focus on a good volatility forecast model instead of fancy distribution

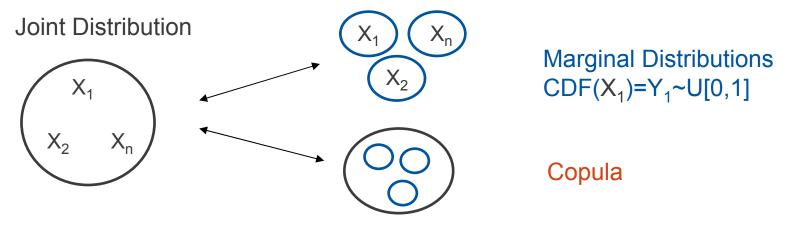


Volatility forecast can depend on assumed distribution – Interdependence



Copulae

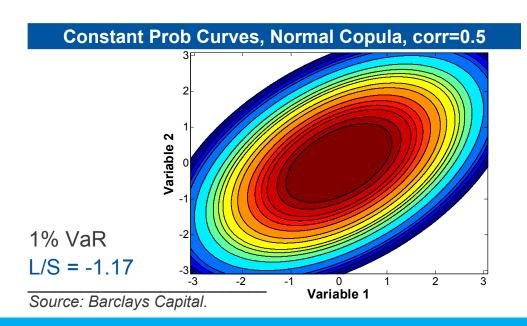
• Links marginal distributions $M_1(X_1),...M_n(X_n)$ into the joint distribution F $F(X_1,X_2...X_n) = C(M_1(X_1),M_2(X_2)...M_n(X_n),parms)$

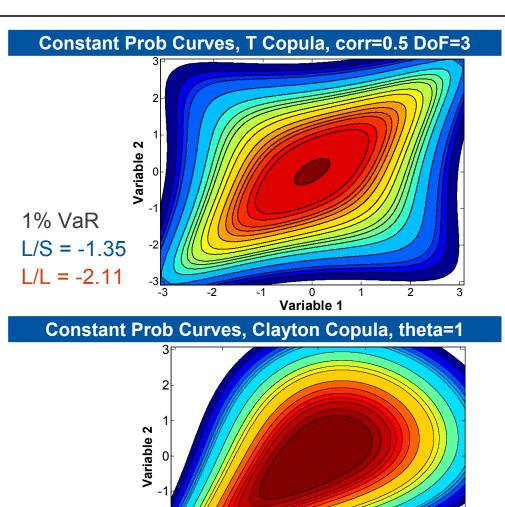


- Types: Normal Copula, Student-T Copula, Clayton, Gumbell
- Focus on only capturing factors' dependence
 - Incorporate correlations
 - Can go beyond correlations: Tail dependence, asymmetry
 - Measures: Pearson, Spearman, Kendall correlations; coeff of tail dependence
 - No perfect measure
- Harder to calibrate than marginals

Copulae – Examples

- A pair of variables, both have the same marginal distribution, N[0,1], same corr=.5
- Compute 1%VaR Long-Short and Long-Long Portfolios for various copulae





1% VaR

L/L = -2.23



Variable 1

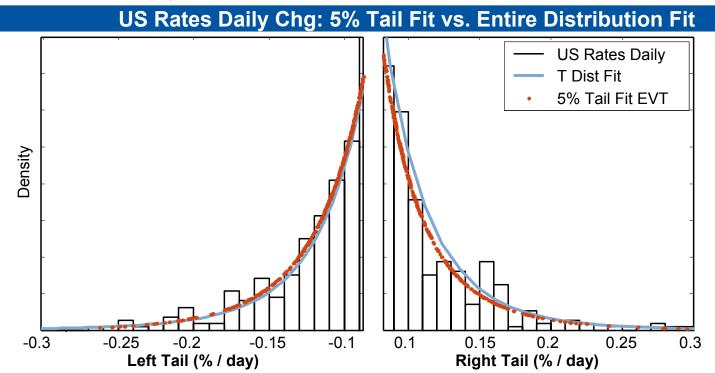


Alternative Modeling Frameworks



Direct Modeling of Tails: EVT

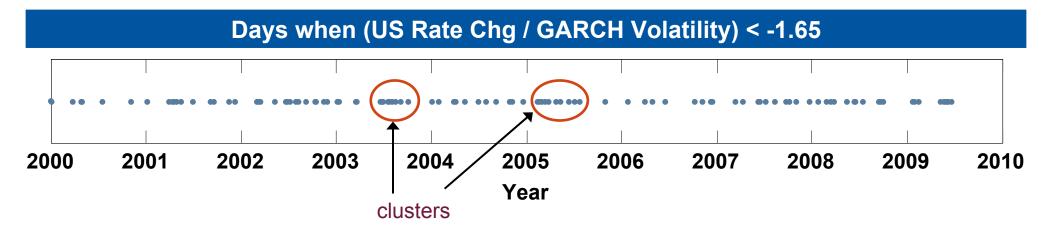
- Focus directly on tails, ignore the rest of the distribution
- Extreme Value Theory: The asymptotic distribution of min / max of a sample (=VaR)
 - Assumptions on tail distribution: What is "tail"?
 - Difficult to do multivariate distribution
 - Use: When hard to calibrate entire distribution
- E.g., fit the 5% tails using a power-law (EVT)





Direct Modeling of Tails: CAViaR

- Use time-series dependence: Current tails are a function of past realizations
 - Plot only realizations (scaled by vol) above forecasted VaR



Same intuition as GARCH: Extreme observations clustering

$$VaR_{t}^{95\%} = \omega + \beta \cdot VaR_{t-1}^{95\%} + \alpha \cdot f(r_{t-1})$$

- Estimate with quantile regressions
- CAViaR (Engle Manganelli 2002)



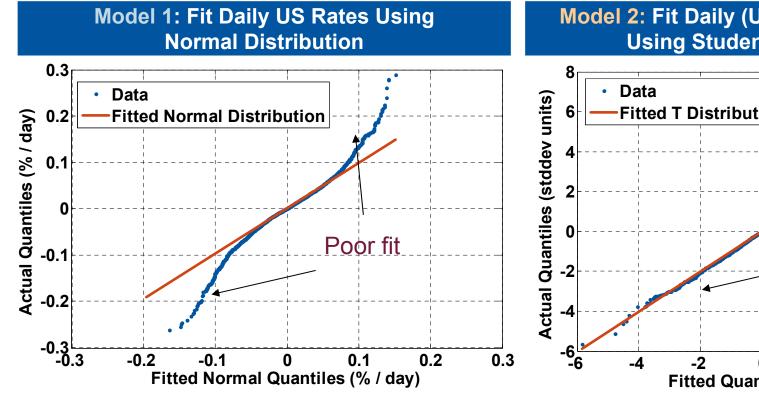


Testing Models

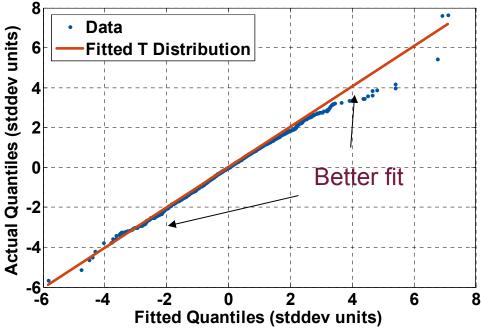


Testing: QQ Plots

- In-sample test
- Plot quantiles of fitted distribution vs. real data (QQ Plot), see if they match
- Cannot do out-of-sample because fitted distribution varies over time
- QQ plots for two tail-risk models



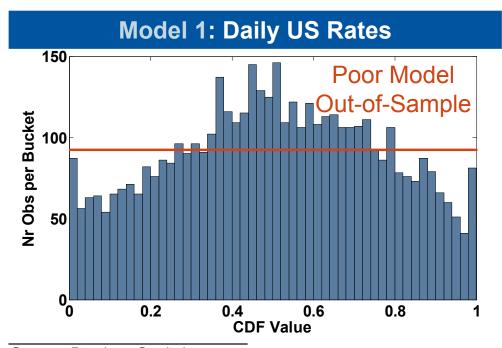
Model 2: Fit Daily (US Rates / GARCH Vol)
Using Student-T Distribution



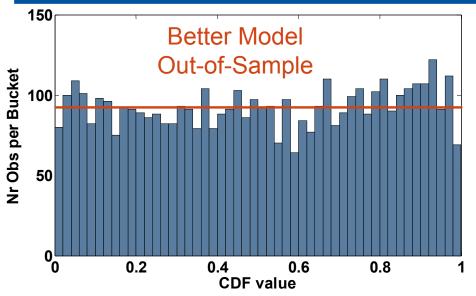


Testing: CDF of Data

- Compute Y = forecasted CDF at realized values
 - Y should be IID Uniform [0,1]
 - Testing: Plot the histogram of Y; formally, use Kolmogorov-Smirnov and Chi-sq tests
- Out-of-Sample
- Allows for changing distribution
- Histogram of forecasted CDF using previous two tail-risk models



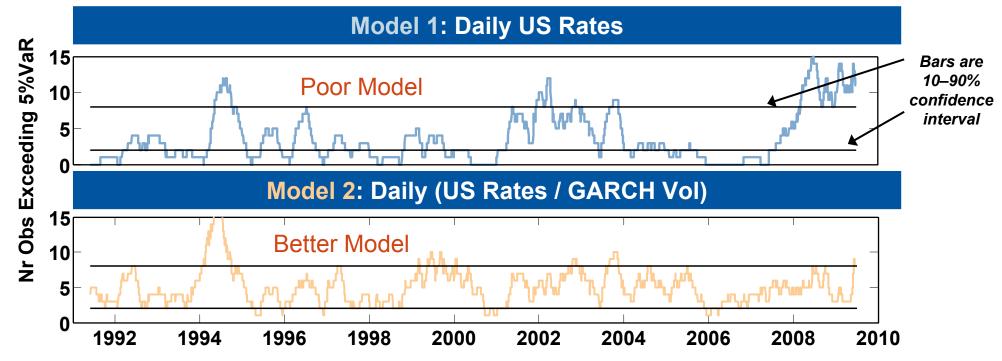
Model 2: Daily (US Rates / GARCH Vol)





Testing: VaR Exceedances

- Analyze N = % of realizations above Q%VaR
 - Used also when forecast quantile Q directly (EVT, CAViaR)
 - N should = Q on average, not vary too much, and be unpredictable
- Plot N over a 100-day period, rolling, using previous two tail-risk models



Conditional tests: Regress indicators on predictors; test N·R-sq~X²_N



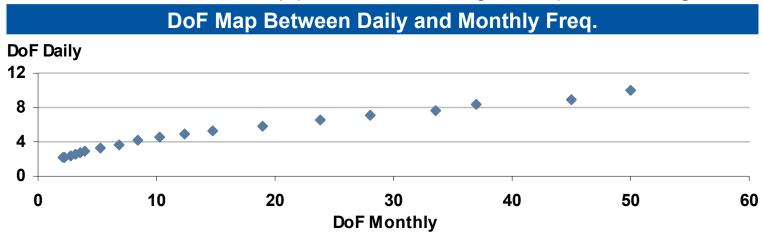


VaR by Horizon



VaR by Horizon

- VaR has similar horizon issues as volatility / correlation
 - Autocorrelation affects sqrt rule of volatility scaling, affecting VaR
 - May use higher-freq data to better forecast lower-freq moments
- Harder to test as horizon increases less observations
- Tails become more normal as horizon increases (CLT; need close-to-IID)
 - Distributions may not aggregate to the same type, e.g., Sum of Student-T is not a Student-T
 - But we may assume so and derive aggregation rules for DoF parameter
- Use
 - For intermediate horizons, map parameters from higher frequencies using MC results



• For very long horizons, assume that conditional on volatility, distribution is normal



0.15

0.1

0.05

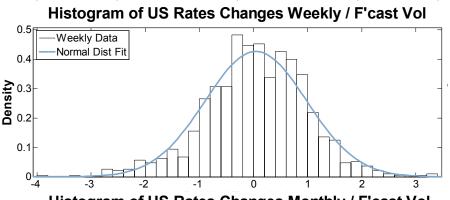
VaR by Horizon: Getting Close to N

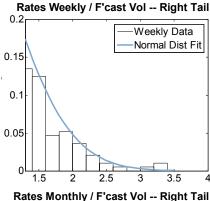
Daily / weekly /monthly histograms 0.5 fitted by Normal distribution

 As frequency decreases they get close to N

Jarque-Berra Test for Normality

| Frequency | Nr. Obs | Stat | p-val |
|-----------|---------|---------|----------|
| Daily | 4,620 | 1,485.0 | 1.00E-20 |
| Weekly | 963 | 32.1 | 1.00E-04 |
| Monthly | 222 | 3.0 | 0.18 |





Rates Daily / F'cast Vol -- Right Tail

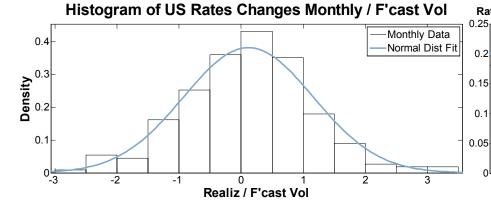
Daily Data

Normal Dist Fit

Monthly Data

Normal Dist Fit

3.5



Source: Barclays Capital.



Realiz / F'cast Vol

Conclusion

- Marginal-copula is better than historical: More flexibility, invariance
- Condition on volatility before modeling
 - Need a good volatility model
- Copula defines factor dependence
- Testing is complicated, but not impossible
 - Focus on exceedances and on CDF of realized values
 - Test conditionally if enough data
- As frequency decreases, get close to normal





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