

More than Just a Second Risk Number:

UNDERSTANDING AND USING STATISTICAL RISK MODELS

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Executive Summary

Although fundamental factor risk models are more commonly used and understood by portfolio managers, statistical factor risk models provide an important alternative and adaptable view on risk. In times of unusual market movements and trends that are not well modelled or captured by traditional fundamental factors, statistical risk models can be leveraged to identify these unexpected sources of risk. This paper describes how a combination of fundamental and statistical factor risk models can be exploited in any investment process.



1. Introduction

There are strategic benefits to incorporating different kinds of risk models – fundamental, statistical, and macroeconomic factor risk models – into an investment process.

Fundamental factor risk models decompose risk using well-understood and intuitive factors. The factors have been heavily researched and are known to give highly reliable risk predictions. However, the factors used by a fundamental factor risk model are fixed¹. As a result, such models may have trouble modeling unusual market trends. When such trends are not well modeled by a fundamental model's fixed set of factors, the risk associated with those trends is modeled as asset-specific, idiosyncratic risk.

In contrast, statistical factor risk models do not impose or assume a fixed factor structure but instead use asset returns directly to mathematically construct an optimal set of factors explaining the current risk environment, regardless of whether the factors represent short- or long-term phenomena or are associated with intuitive, well-known factors. The factors of a statistical risk model evolve to fit the current market conditions. This adaptability means that statistical factors model risk extremely well. However, the lack of intuitive meaning to these evolving factors makes risk decomposition and performance attribution difficult.

Macroeconomic factor risk models constitute a third kind of factor risk model. In these risk models, estimates are computed for the sensitivity (beta) of an asset's time series of returns to historical changes in a set of broad macroeconomic variables such as economic growth and interest rates. These factors are intuitive and are particularly helpful for stress-testing a portfolio for market events and surprises. In fact, stress testing normally motivates the choice of macroeconomic factors. However, macroeconomic factors generally have less explanatory power than either fundamental or statistical factors. If they were as predictive, they would be included in fundamental factor models. As a result, fundamental and statistical risk models are generally considered more reliable than macroeconomic risk models.

A comparison of assumptions, strengths, and weaknesses of these three kinds of factor risk models is shown in Table 1.

	Fundamental Risk Models	Statistical Risk Models	Macroeconomic Risk Models
Assumed Inputs	Factor exposures	None	Factor returns
Estimated Outputs	Factor returns	Factor exposures and returns	Factor exposures
Strengths	Intuitive & widely used Consistent framework for: - Risk Decomposition - Perf. Attribution - Portfolio Construction	Factors are not fixed Responsive Captures short term phenomena Effective Portfolio Construction	Stress testing of macro events & surprises
Weaknesses	May miss short term trends	Lacks intuition Difficult to interpret - Risk Decomp. - Perf. Attribution	Broader, less predictive factors Less explanatory power

Table 1. A summary comparison of fundamental, statistical, and macroeconomic factor risk models.

¹ The exposures change from day to day, but the factor itself and underlying descriptors – Value, Industry, etc. – are fixed and do not change.

In the present paper, we describe how a statistical factor risk model can be used in conjunction with a fundamental factor risk model to improve an investment process. Even though statistical factors have no predefined meaning, there are a number of techniques that leverage the information in these models to help manage risk, construct portfolios, and explain performance. While fundamental factor risk models may be better understood and widely used in investment processes, statistical risk models uniquely capture and quantify unexpected market trends as well as aid in portfolio construction to account for these trends.

The outline of the paper is as follows. First, we provide an overview of statistical factor risk models, review how they are constructed, and contrast them with fundamental factor risk models. Next, we describe a number of approaches for comparing fundamental and statistical risk model predictions on a side-by-side basis. We use a detailed analysis of a case-study portfolio for illustrating these approaches. Finally, we offer suggestions for how these approaches can be applied in risk management, portfolio analysis, and portfolio construction.

2. An Overview of Statistical Factor Risk Models

A statistical factor risk model is a risk model whose factors are constructed by mathematically processing asset return time series, so that the set of factors chosen has the maximum possible explanatory power. The mathematical technique used is Principal Components Analysis (PCA), Asymptotic Principal Components Analysis (Asymptotic PCA), or a variant of these.

Because these mathematical techniques maximize the commonality among the asset returns, the techniques are free to find factors not found in fundamental factor risk models. Statistical factors frequently capture short-term market trends that are important over short periods of time even if they do not persist. Identifying and reacting to relevant market trends is, of course, an essential part of any investment process even if the trends do not last long enough to be included in a fundamental factor risk model.

Mathematically, both fundamental and statistical risk models begin with the same linear factor model of asset returns:

$$R = Bf + u$$

R is a vector of asset returns, B is a matrix of factor exposures or factor loadings, f is a vector of factor returns, and u is a vector of asset-specific, idiosyncratic returns.

While R is known, fundamental and statistical risk models approach the solution of the rest of the terms in this equation differently.

With fundamental models, the factors and their exposures, B , are given, and the equation is solved for the factor return, f using regression. This permits risk modelers to select factors that are intuitive, well researched, and predictive. The factors used in a fundamental factor risk model on one day are the same factors used on the next day, although the factor exposures are updated daily.

For statistical risk models, both the matrix of factor exposures, B , and the vector of factor returns, f , are solved for simultaneously so as to maximize the predictive power of the above equation. Statistical factors, factor

exposures and returns are re-estimated independently for each risk model update. As a result, the factors and factor exposures may change substantially from one day to the next as they adapt to market conditions.

When compared with fundamental factor risk models, the adaptability of statistical factor risk models has two key drawbacks. First, the factors have no obvious economic or investment meaning. They are simply numerical exposures that best explain the observed asset returns. Second, the factors change from one day to the next. This makes statistical factor exposures difficult to incorporate into a portfolio construction strategy or use in creating a meaningful performance attribution over time.

The advantage of the statistical approach, however, is precisely the adaptability of the factors. During time periods when the factors in a fundamental risk model include all the key factors driving risks in the market, fundamental risk models work well. However, suppose that the market starts to be driven by a new and unexpected factor that is not included or well represented by the fixed set of fundamental factors. In this situation, the explanatory power of the fundamental risk model decreases.

A statistical factor risk model, however, adapts to the changing market, and the factors and the risks associated with them would be properly reported by the statistical risk model. In other words, the chances of being hurt by an unintentional exposure to new market forces are significantly less when using a statistical factor risk model because its factors are able to change and adapt over time.

3. Case Study: Using Statistical Models For An Additional Risk Perspective

Next, we present a case study on a representative quantamental portfolio, in order to illustrate some of the most useful and insightful practices that have emerged since Axioma first introduced its suite of fundamental and statistical risk models.

The case study portfolio is an actual, real-world Large Cap Core strategy benchmarked to the Russell 1000 and typically aims to target around 3% to 4% annualized realized active risk while holding 50-100 names. We use Axioma's latest US Risk Model suite, US4, for analysis.

3a. Risk Differences

Fig. 1 shows a time series plot of the predicted active risk using Axioma's US4 Fundamental Medium Horizon risk model. The portfolio had an active risk of more than 4%, starting in January 2010, but the tracking error quickly dropped to almost 2% by January 2011. Since then, the tracking error of the portfolio has been steadily rising, with tracking error hovering around 3.5% since mid-2014.



Fig. 1. The predicted active risk of the Large Cap Core portfolio using Axioma's US4 Fundamental Medium Horizon risk model. The quarterly spikes indicate portfolio rebalancing, not an abrupt change in predicted risk.

Fig. 1 gives only one risk model's prediction – that is, only one view on risk. However, Axioma's risk model suite includes four different risk models:

- A fundamental, medium horizon risk model (MH – already shown in Fig. 1)
- A fundamental, short horizon risk model (SH)
- A statistical, medium horizon risk model (MH-S)
- A statistical, short horizon risk model (SH-S)

Fig. 2 shows the tracking error of the same portfolio for all four risk models. Overall, the trends are similar, and the four different predictions of tracking error are consistent. However, there are trends in Fig. 2 that suggest whether or not the statistical risk model is picking up a factor that is missing from the fundamental model.

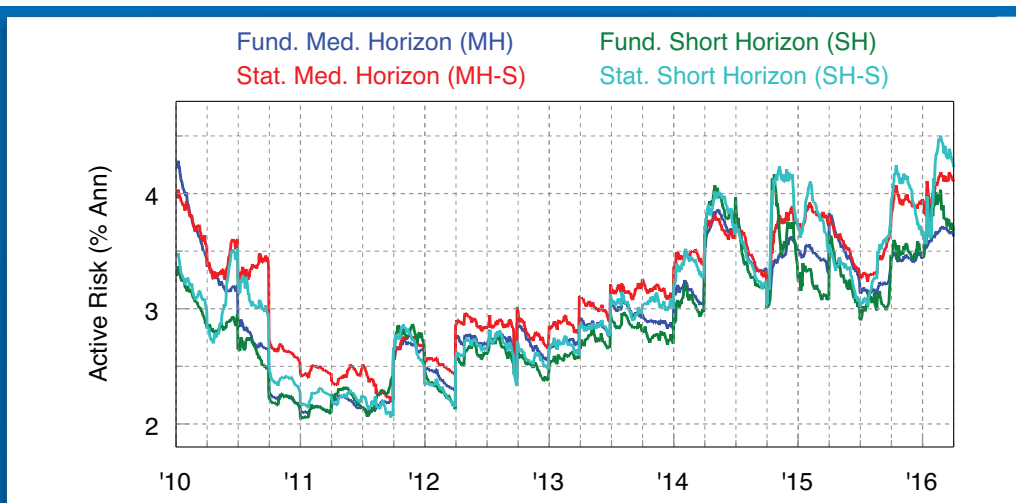
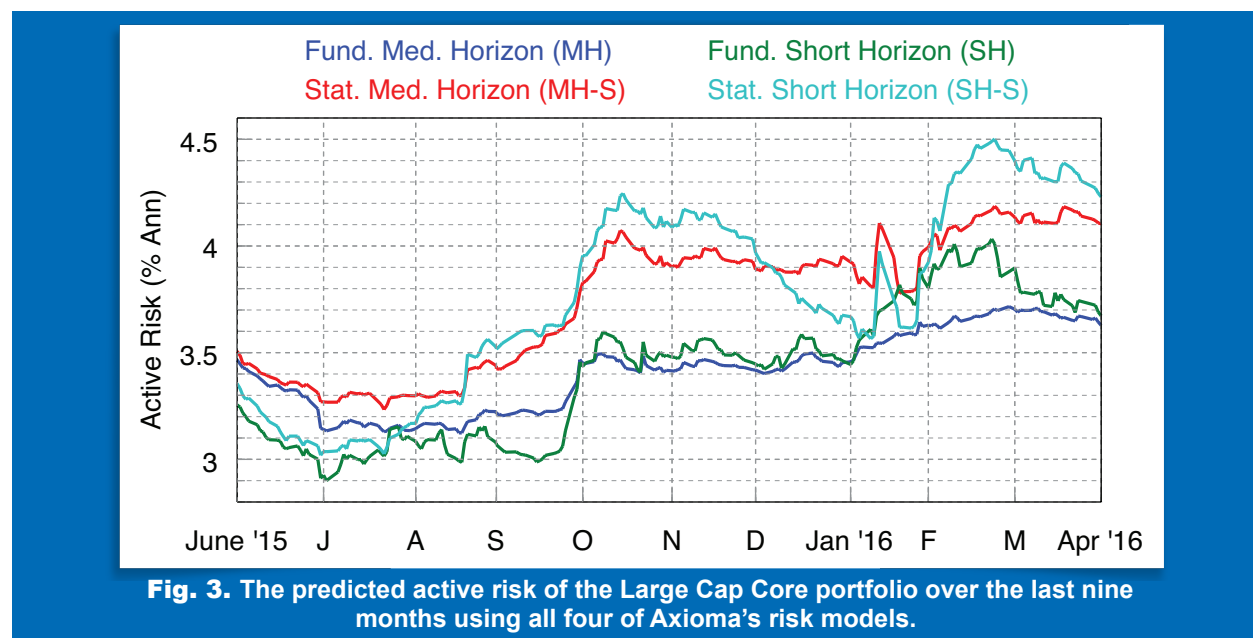


Fig. 2. The predicted active risk of the Large Cap Core portfolio using all four of Axioma's risk models. Models colors are shown above.

In January 2010, the two medium horizon models (MH (blue) and MH-S (red)) predict almost identical tracking error, while the two short horizon models (SH (green) and SH-S (turquoise)) also agree with each other, although they both predict somewhat smaller tracking error than the medium horizon models. The agreement between fundamental and statistical risk models with the same horizon suggests that there are no missing factors in the fundamental risk model.

However, starting in 2015, there have been three time periods during which both statistical predictions were significantly larger than both fundamental predictions. The first period started in January 2015 and lasted about three months. The second period starting in Q4 2015 and lasted three months. At the close of 2015, the risk predictions briefly came together, but as 2016 started, both statistical risk predictions shot up again. This is illustrated in closer detail in Fig. 3 which shows all four active risk predictions for just the last nine months. Interestingly, these last two time periods – September 2015 to January 2016, and February to April 2016 – coincide with two relatively challenging periods for active and long-short managers.



These changes can be conveniently captured by two different risk spreads:

- **Factor Risk Spread** = Highest predicted factor risk minus the lowest predicted factor risk across all risk models.
- **Stat Minus Fund Risk Spread** = Predicted risk from the statistical model minus the predicted risk from the fundamental model with the same estimation horizon.

Fig. 4 shows these two spreads since June 2015. Starting in August 2015, there was a notable increase in the spread that peaked near early October 2015 at nearly 100 bps of difference between the risk models. This spread contracted through year end, and then surged again in February of 2016. As of April 2016, both spreads were at historically large values.

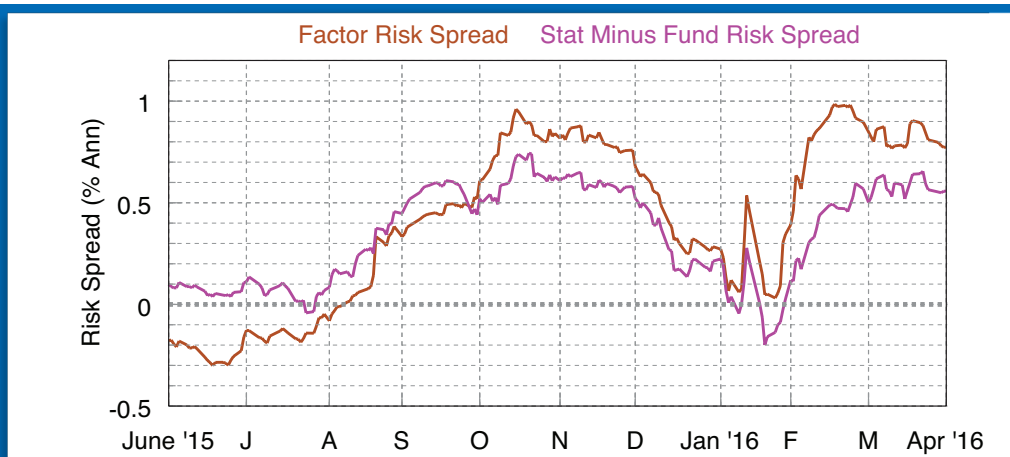


Fig. 4. The Factor Risk Spread and the Stat Minus Fund Risk Spread.

3b. Factor vs. Specific Risk

In addition to considering risk differences, as was done in the previous section, it is also important to recognize the changing proportions of risk coming from common factor risk and specific risk. As a general rule of thumb, stock pickers would expect more specific risk than factor risk, since their skill is picking individual stocks. Market timers would expect more factor risk than specific risk, since the factors of any risk model represent market trends.

Fig. 5 shows the common factor percentage of the total active variance (e.g., the proportion of risk associated with the risk model factors) for the medium horizon fundamental and statistical risk models since Q3 2015. The percentage predicted by the fundamental risk model varies between 48% and 60%, but has been steady at 55% since November 2015. The percentage predicted by the statistical risk model tracked the fundamental prediction until mid-August 2015, and then surged to more than 70%. Since then, this has remained greater than 60% except for January 2016. The implication is, of course, that the statistical risk model has found a factor (or set of factors) that is missing from the fundamental factor risk model, and that this missing factor impacts the portfolio and drives higher predicted factor risk. This corroborates what was observed in the previous section on risk differences.

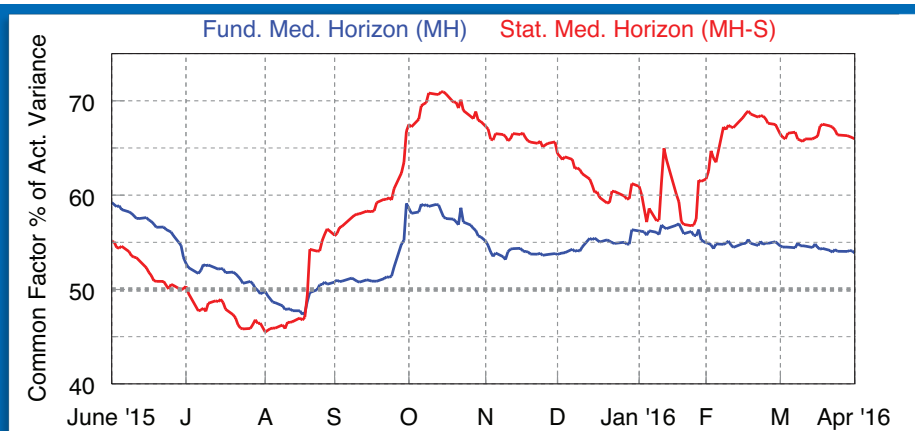


Fig. 5. The proportion of active common factor variance for the statistical and fundamental risk models.

3c. Risk Decomposition Using Projection

We can corroborate this observation in yet a third way by using the Risk Decomposition features in Axioma Portfolio. In particular, we can take advantage of Axioma Portfolio's ability to project a first risk model's predictions onto the factors of a second risk model. The factor risk that can be explained by the second set of factors will be reported in terms of those factors. Any risk that cannot be explained by the second set of factors will be reported as "unexplained" risk.

Table 2 shows the risk of the portfolio as of 3/31/2016 decomposed using the fundamental, medium horizon risk model. The predicted active risk is 3.63% annual volatility. Of the total active variance, 39% is specific risk, while factor risk accounts for the other 61%, which, using US4, can be further decomposed into Style, Industry, and Market factors.

	Pred Risk (% Ann)	% of Variance
Total Risk	18.02%	100%
Benchmark Risk	17.13%	100%
Total Active Risk	3.63%	100%
Specific Active Risk	2.25%	39%
Factor Active Risk (Fund)	2.84%	61%
Style (Fund)	1.95%	34%
Industry (Fund)	1.77%	29%
Market (Fund)	0.25%	-1%

Table 2. The risk decomposition of the portfolio as of 3/31/2016 using the fundamental, medium horizon risk model.

Table 3 shows the decomposition of the same 3/31/2016 portfolio using the statistical, short horizon risk model. Two decompositions are shown. On the left, the decomposition is done directly on the statistical risk factors. On the right, the decomposition is done using the fundamental factors, with the **missing** risk reported as unexplained.

	Pred Risk (% Ann)	% of Variance
Total Risk	20.23%	100%
Benchmark Risk	18.29%	100%
Total Active Risk	4.42%	100%
Specific Active Risk	2.48%	31%
Factor Active Risk (Fund)	3.66%	69%

	Pred Risk (% Ann)	% of Variance
Total Risk	20.23%	100%
Benchmark Risk	18.29%	100%
Total Active Risk	4.42%	100%
Specific Active Risk	2.48%	31%
Factor Active Risk (Fund)	3.66%	69%
Unexplained (Stat)	1.27%	15%
Common Factors (Fund)	3.04%	54%

Table 3. The risk decomposition of the portfolio as of 3/31/2016 using the statistical, short horizon risk model. On the right, the risk has been projected onto the fundamental factors: 15% of the active variance is unexplained by the fundamental factors.

Clearly, the first five lines are identical. The statistical risk model predicted 4.42% annual volatility (higher than the fundamental risk model): 31% specific risk and 69% common factor risk. However, when the common factor risk of the statistical risk model is projected onto the fundamental factors (Style, Industry, Market), a full 15% of the risk is unexplained. This 15% corresponds to an annual volatility of 1.27% — a substantial fraction of the overall risk budget.

At this stage, after having compared the active risk predictions using different models, various risk spreads, the proportion of factor risk, and performed high level risk decompositions, the typical next step – at least for a fundamental factor risk model – would be to drill down into each of the factors, identify meaningful active exposures, and the active risk associated with them. This was partially performed already shown in Table 2, where the factors were separated into Style, Industry, and Market factors.

For statistical risk models, we recommend skipping this step, as it is difficult to interpret the results and even harder to take action based on them. Table 4 shows this decomposition. The first five lines are the same as in Table 3, but an additional column has been added for the factor exposures, which are blank for these first five lines.

The additional information is shown in the last 16 lines, which lists the active exposure, percent annual volatility, and proportion of variance for each of the 15 statistical factors and then the covariance among the factors. For this particular decomposition, the largest contributions are Factors 2, 1, and 6. However, this information is not helpful. Knowing the portfolio is underweight -0.00128% to Statistical Factor 6 does not provide immediate insight, at least not without substantial analysis of which other interpretable factors may be similar to Statistical Factor 6.

	Active Exposure	Pred Risk (% Ann)	% of Variance
Total Risk		20.23%	100%
Benchmark Risk		18.29%	100%
Total Active Risk		4.42%	100%
Specific Active Risk		2.48%	31%
Factor Active Risk		3.66%	69%
Statistical Factor 2	0.0191%	2.30%	27.0%
Statistical Factor 1	-0.0132%	1.55%	12.3%
Statistical Factor 6	-0.0128%	1.42%	10.3%
Statistical Factor 10	0.0096%	1.08%	5.91%
Statistical Factor 8	-0.0091%	1.07%	5.91%
Statistical Factor 13	0.0070%	0.80%	3.30%
Statistical Factor 15	0.0049%	0.51%	1.31%
Statistical Factor 14	0.0045%	0.47%	1.15%
Statistical Factor 7	-0.0041%	0.39%	0.76%
Statistical Factor 11	-0.0029%	0.31%	0.51%
Statistical Factor 9	-0.0025%	0.29%	0.43%
Statistical Factor 12	-0.0023%	0.29%	0.43%
Statistical Factor 3	-0.0027%	0.27%	0.38%
Statistical Factor 4	0.0006%	0.06%	0.02%
Statistical Factor 5	-0.0004%	0.05%	0.01%
Covariance			-1.22%

Table 4. The risk decomposition of the portfolio as of 3/31/2016, using the statistical, short horizon risk model, drilling down into individual factors.

3d. Asset Level Decomposition — % of Active Risk

Instead of decomposing the portfolio along factors, we recommend decomposing risk at the asset level contribution to risk, termed “% of Active Risk” in Axioma Portfolio. This is a decomposition of the total tracking error into separate contributions from each asset, based on analyzing the asset’s active weight and its riskiness (as quantified by the marginal contribution to active risk, MCAR). This metric is intuitive, sums to 100% for all the assets in the portfolio and the benchmark, and spans all sources of risk present in any risk model (e.g., style, industry, statistical and specific).

Table 5 shows five select names from the portfolio, their active weight, and their % of Active Risk as computed with the fundamental, medium horizon risk model. This table is taken directly from Axioma Portfolio, which automatically computes the % of Active Risk. The sum of % of Active Risk of just these five names – out of the 1,000 in the portfolio and benchmark – is 24.71%. That is, these five positions take up almost a quarter of the full tracking error budget for this portfolio. Since these five over-weights are so risky, a portfolio manager should be highly confident in these particular positions. If not, he or she should consider down-weighting the ones in which he or she has less confidence. This is exactly analogous to managing Style and Industry factor exposures – they should not be large unless the portfolio manager intends them to be large. Notice also that the ordering of Active Weight and % of Active Risk is not the same. The largest active weight shown – 2.35% for Foot Locker – does not have the largest % of Active Risk.

Ticker	Company Name	Active Weight (%)	% of Active Risk
DAL	DELTA AIR LINES INC DEL	2.14%	5.61%
SWKS	SKYWORKS SOLUTIONS INC	2.00%	7.56%
LNC	LINCOLN NATL CORP IND	2.05%	3.07%
MGA	MAGNA INTL INC	2.18%	4.60%
FL	FOOT LOCKER INC	2.35%	3.88%
		SUM	24.71%

Table 5. The active weight and % of Active Risk for five portfolio names. The sum of just these five names – out of 1,000 in the portfolio and benchmark – uses almost 25% of the full active risk budget.

In Table 6, we extend the previous analysis to include the statistical, medium horizon risk model². We also include five more names, each of which has a negative % of Active Risk; that is, these positions, all underweights, are diversifying positions that reduce the total tracking error of the portfolio. Also included in the Table is a column labeled DELTA with the difference between the statistical % of Active Risk and the fundamental % of Active Risk. We have sorted each set of names using this difference.

Of the 1,000 names in the portfolio and benchmark, these 10 names represent the names with the largest differences in % of Active Risk.

Whereas the five overweight names consume almost 25% of the risk budget according to the fundamental risk model, they consume almost 40% of the risk budget according to the statistical risk model. This is a large

² We could, of course, do the analysis for all four risk models. We use two risk models solely to make the results more legible.

difference and is expected, in that these are the five names with the largest difference in % of Active Risk (e.g., the differences for all the other names will be considerably less). Similarly, for the five names with the most diversifying (negative) % of Active Risk, the fundamental risk model predicts that these positions reduce the risk by 3.05%, whereas the statistical risk model predicts that they reduce risk by 10.68%.

			% of Active Risk		
Ticker	Company Name	Active Weight	Fund	Stat	DELTA
DAL	DELTA AIR LINES INC DEL	2.14%	5.61%	9.56%	3.96%
SWKS	SKYWORKS SOLUTIONS INC	2.00%	7.56%	10.54%	2.98%
LNC	LINCOLN NATL CORP IND	2.05%	3.07%	5.53%	2.46%
MGA	MAGNA INTL INC	2.18%	4.60%	6.94%	2.35%
FL	FOOT LOCKER INC	2.35%	3.88%	6.11%	2.23%
		SUM	24.71%	38.69%	

			% of Active Risk		
Ticker	Company Name	Active Weight	Fund	Stat	DELTA
BRK/B	BERKSHIRE HATHAWAY INC DEL	-1.44%	-0.16%	-1.45%	-1.29%
AAPL	APPLE INC	-1.33%	-0.73%	-2.19%	-1.46%
BAC	BANK AMER CORP	-0.96%	-0.59%	-2.12%	-1.53%
WFC	WELLS FARGO & CO NEW	-1.42%	-0.69%	-2.23%	-1.54%
FB	FACEBOOK INC	-1.20%	-0.88%	-2.70%	-1.82%
		SUM	-3.05%	-10.68%	

Table 6. The active weight and % of Active Risk computed with the fundamental and statistical risk models for 10 portfolio names.

For the top five names, we see that these names are both inherently risky (they consume a disproportionate fraction of the risk budget) and that the prediction of just how risky they are is uncertain. If a portfolio manager does not have confidence in these positions, he should consider reducing them.

Similarly, the five diversifying names also have uncertainty about how much they diversify the risk.

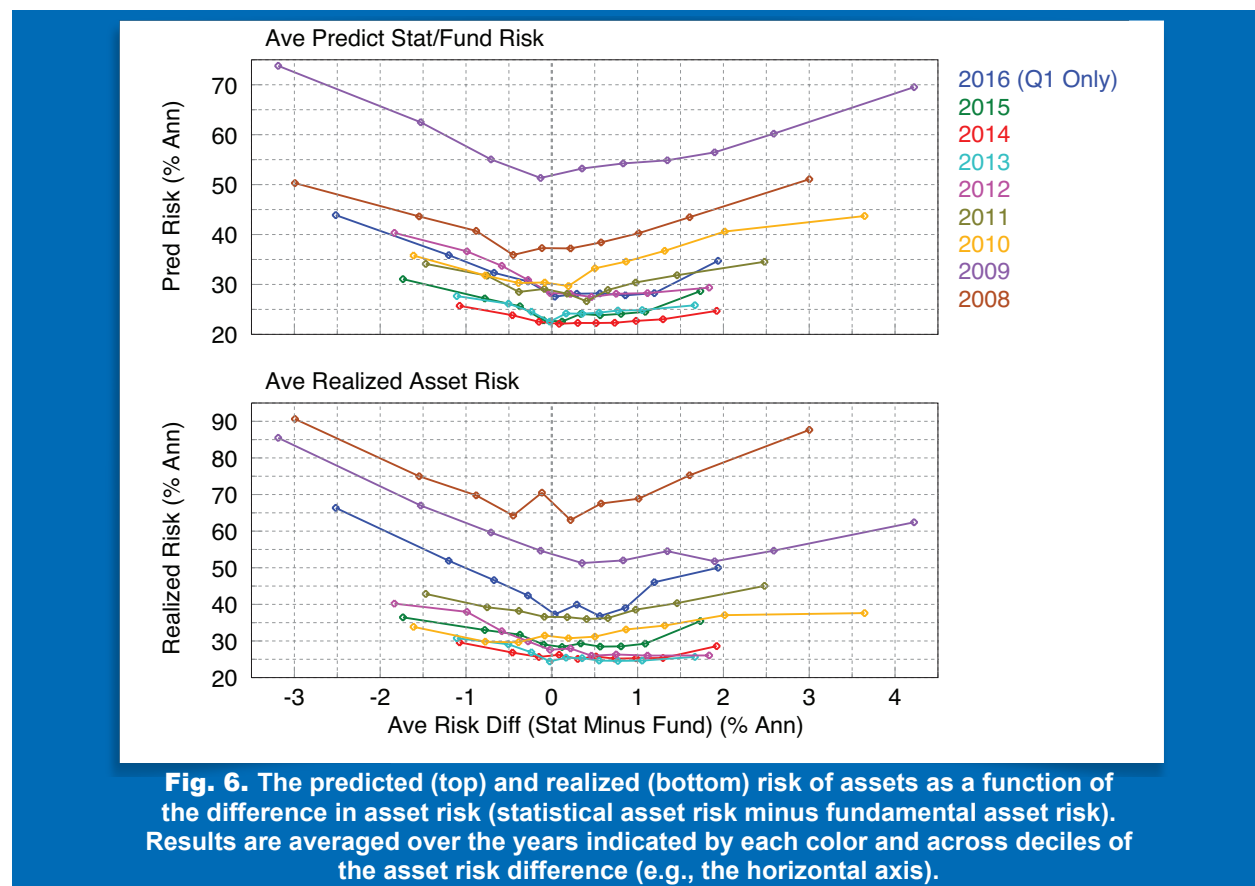
This kind of analysis can be performed across other risk models as well as using % of Active Factor Risk instead of % of Active Risk.

This procedure identifies individual assets that have the largest contributions (positive and negative) to the risk budget as well as the largest differences (positive and negative) between the various models. Both of these characteristics are potential warning signals coming from the risk models.

3e. How Reliable Are These Signals?

We have described a number of techniques using a statistical risk model in conjunction with a fundamental risk model to identify missing factor risk and asset level differences in risk and risk contribution. It is reasonable to ask how reliable this information is.

The graphs in Fig. 6 give results indicating that the differences in risk between a statistical and fundamental risk model are meaningful and reliable. In both charts in Fig. 6, the horizontal axis is the asset total risk predicted by the statistical, medium horizon risk model minus the asset total risk predicted by the fundamental, medium horizon risk model. We compute these asset-level differences for all assets in the Russell 1000 index, on each trading day since January 2000. Then for each trading day in each year (Q1 only for 2016), we group the asset differences into 10 deciles. These correspond to the diamond points on the graphs. For each decile of differences, we compute the average predicted asset risk (average of the statistical and fundamental risk models). This data is shown in the top chart. We also computed the realized risk for the decile over the year, which is reported in the bottom chart.



For both the top and bottom chart, each color line is nominally U-shaped with its minimum value occurring at approximately no difference between the statistical and fundamental asset risk predictions. That is, assets with large positive or negative risk differences are riskier, both in predicted risk as well as realized risk. While the overall level of risk varies from year to year, the pattern of increased risk with increased difference in the risk models persists.

4. Implementation

Different investment processes have different priorities. Here we list some of the possible steps investment managers may consider using to exploit having both fundamental and statistical factor risk models available.

Quantitative Active Managers

- Introduce a second risk constraint or objective term that penalizes risk coming from the statistical model (in general, or when spreads suggest it necessary)
- Adjust asset-level constraints to reduce exposure to assets with high stat/fund differences
- Prescreen for risk differences

Fundamental/Quantamental Active Managers

- Adjust position sizes for problematic assets to ensure conviction is properly implemented

Long-Short Managers

- Explicitly hedge systematic risk as estimated by the statistical model in addition to the fundamental model
- You are not factor neutral if you are optimizing with only fundamental models
 - There is a better “best hedge”
 - Constrain assets with increased risk coming from the statistical model
- Early warning signal on potential problem areas

Passive/ETF/Tax-Efficient Managers

- Constrain tracking error using multiple risk models
- Tighten asset bounds for assets with larger differences in risk estimates

5. Conclusions

No risk model is perfect – fundamental models and statistical models each have their pros and cons. Given their intuitive factors, fundamental models are generally used for factor exposure management and performance attribution, neither of which can be done well with statistical risk models because of their adaptive factor structure. However, statistical risk models are useful precisely because their factors adapt and pick up ‘hidden’ or transitional risks in the market that are missed by fundamental factor risk models.

Axioma offers both risk model variants because we feel it is important for portfolio managers and risk managers to exploit the strengths of each kind of risk model. Different risk models will have different risk predictions, and it is useful to understand which model is predicting higher risk and whether that risk is factor or specific. The high level tracking error comparisons, differences in % of factor and specific tracking error, and asset level % of tracking error analytics help explain where differences in risk may arise.



Contact us to have an Axioma expert look at your portfolio through the lens of multiple risk models.

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