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Systematic Variations in Corporate Bond Excess Returns

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We propose a multi-factor model, the Credit Factor Model (CFM), that is useful to understand the systematic component of excess returns in global credit markets. We identify five credit market factors that explain a significant proportion of credit returns. These factors are: (i) an overall credit market factor, (ii) variation in credit quality premium, (iii) variation in the slope of credit spread curve, (iv) variation in the liquidity premium affecting less liquid corporate bonds, and (v) a factor capturing the cross-market momentum effect originating in equity markets. We also quantify the extent to which systematic factors driving equity market movements can add to the explanatory power of factors that we identify. Our statistical tests confirm the hypothesis that the factor models presented here explain the cross-sectional dispersion in average excess returns on corporate bonds. We also estimate a relation between factor exposures of sectors and issuers and the cross section of corporate spreads and introduce the CFM Alpha Score for sector and asset selection.

1. INTRODUCTION

One of the most important tasks facing portfolio managers is to estimate the risk premium that various assets command at different times. Fundamental principles of asset pricing imply that risk premia on financial securities are compensation for bearing systematic or undiversifiable risk. It is, therefore, necessary to understand the common sources of risk in financial markets in order to assess the risk premium that may be embedded in an asset's price. In this article, we develop a macro-factor model to capture systematic variations in credit market returns. This model aims to enhance the understanding of the undiversifiable or common sources of risk in credit markets and to quantify credit risk premia.

There are at least two ways in which our model is useful for managers of credit portfolios. First, our model provides a methodology and empirical evidence for assessing the macro-risks embedded in any credit portfolio. Second, given that investors would require a risk premium for bearing such systematic risk, our model can be used to examine whether individual assets or sectors do indeed provide a risk premium commensurate with their systematic risk. Evaluating the trade-off between risk and risk premium is fundamental to the formulation of an investment strategy: our model and its future versions aim to provide a framework for making such an evaluation.

The principle of linking risk premium to systematic risk is exemplified very simply by the celebrated Capital Asset Pricing Model (CAPM), which states that the risk premium on any risk asset is proportional to a measure of co-variation of that asset's return with the return on the aggregate market portfolio. Thus, in this model, risk premium compensates for the fact that a part of an asset's return varies with the return on the market portfolio and, hence, cannot be diversified away. More generally, the return on the market portfolio may not be the only systematic source of risk that carries a risk premium. Indeed, in a dynamic world, the Inter-temporal Capital Asset Pricing Model (ICAPM) would suggest that the risk premium on an asset would also compensate for the co-variation of an asset's return with changes in

[1] *We thank Mark Howard, Dev Joneja, Marco Naldi and Stuart Turnbull for their helpful comments and suggestions.*

macro-economic variables (such as interest rates) that represent changes in the investment opportunity sets that investors face. Empirical research (eg, Fama and French (1993)) in equity markets has identified a number of risk factors in addition to the return on aggregate equity portfolios that explain common variations in the value of equity stocks and the cross-sectional dispersion in average excess returns on these assets. These factors include factors related to the relative performance of small firms vs large firms and that of low book-to-market firms vs high book-to-market firms.

We investigate the systematic factors that explain the common variation in excess returns in the US dollar and euro markets. The excess returns that are the dependent variables in our study are the returns on corporate bonds over and above the returns on duration-matched treasuries. Thus, our study is intended to shed light on the common sources of risk in the movements of credit spreads. We identify five systematic factors that explain a significant portion of the time series variation in excess returns on sector-quality based portfolios of corporate bonds. These factors include: an overall market factor, a factor capturing the movements in the premium for credit quality, a factor capturing the variations in the slope of the credit curve, a factor capturing the liquidity premium commanded by less liquid bonds and a factor capturing the cross-market momentum effect on the corporate market emanating in the equity markets. We show that these factors explain 70–90% of the excess returns on all our industry sector portfolios. Analyses at issuer level and sector-quality level also confirm the usefulness of these factors in explaining the time-variation in the value of credit assets. We name our model the Credit Factor Model (CFM) to signify that it is a factor model for credit markets.

The systematic factors that we include in our analysis are returns on zero investment portfolios. If these are the only important factors in the valuation of credit assets, the risk premium on any corporate bond would be related linearly to the betas of the bond with respect to our factor-mimicking portfolios. We test and do not reject the hypothesis that the cross-sectional dispersion of average credit returns can be explained by such a relationship. This analysis leads to the construction of a measure, which we call the CFM-Alpha Score, of the excess spread of an issuer or sector over and above the spread necessary to compensate for the risk embedded in our factors. This extension of the model has clear applications in an asset-selection context. If the asset pricing framework underlying our analysis is complete, then such excess spreads would represent a possibility of super-normal returns. We will examine in future research whether or not investment strategies contingent on these excess spreads generate excess returns after adjusting for risk.

Our study is parallel to that of Fama and French (1993), who focus on common sources of variations in equity markets. Fama and French also consider ratings-based portfolios of corporate bonds. In our work, we extend this analysis to industry-based portfolios of corporate bonds. We also identify risk factors which are different from the ones considered in their study. Fama and French examine whether the systematic factors that prevail in equity markets are useful in explaining the behavior of ratings-based corporate portfolios. Naldi (2002) also conducts an analysis of this type. He shows how betas of corporate bonds with respect to equity market factors can be used to decompose credit spreads into their various components relating to default premium, risk premium and liquidity premium. Our findings are complementary to those of Fama and French (1993) and Naldi (2002).[2] We first show the

[2] *The CFM model is also related to Beta+, which decomposes changes in spread into a credit market, industry sector and idiosyncratic returns. The CFM factors link economically interpretable factors such as both the credit and equity markets, credit quality, slope of the spread curve, liquidity, bond equity momentum, financial distress and the business cycle.*

explanatory power that one can get by using credit market factors alone and how this changes by including equity market factors.

Our study is also related to the analysis that underlies the Lehman Brothers US dollar and euro Risk Models. A detailed description of our US Credit Risk Model can be found in Naldi, Chu and Wang (2002) and Berd and Naldi (2002). In the Lehman Brothers risk models, excess returns in credit markets are driven by systematic and unsystematic factors. The systematic factors that are considered are spread variation at the sector-quality level, the variation in the slope of the credit spread curve and variation in the premium for credit quality and liquidity. Thus, the risk models are based on a detailed, micro-level framework ideally suited to managing portfolios against a benchmark and allowing sector/quality concentrations to be included explicitly in the risk analysis. In addition, the risk model framework allows detailed modeling of idiosyncratic risk. As such, our risk models remain the standard tools for risk analysis and tracking error attribution.

Our analysis here takes a broader, multi-currency, macro-level approach to the analysis of risk factors. Although we include factors that capture some of the same sources of risk that are included in our risk models, we do not include credit spread variation at sector-quality level as an additional class of risk factors. In this way, our study can be thought of as an examination of how much of the sector-quality effect can be explained by these more macro-level risk factors. Our aim is to provide a macro-level understanding of credit risk factors. Whether such macro-level risk attribution should be included in portfolio risk management tools is a question for further research and discussion. It may also be noted that as we do not include sector-quality factors, the residual returns of the model presented here are likely to be larger than those in our risk models. Also, the present analysis does not consider the behavior of idiosyncratic risk.

Indeed, the analysis presented here can serve as a guide in a risk modeling exercise that we are currently engaged in, namely, the development of the Global Risk Model. Our global risk model is being developed to cover multiple currencies and to incorporate interest rate, credit and other risks in all these currencies. The task of measuring systematic risk in such a model will be tractable only by reducing the dimension of the risk factors that are included in the model. The analysis of this paper can be viewed as being useful in such dimension reduction, thereby facilitating a concise estimation of the required covariance matrix.

This paper is organized as follows. In section 2, we introduce our methodology. In section 3, we present the regression results for the model with credit market factors; in section 4, we incorporate the equity market factors. Section 5 presents the results of cross-sectional regressions to estimate risk premia and decompose sector spreads. In section 6, we introduce the CFM Alpha Score as a potential application of our model. We conclude in the last section.

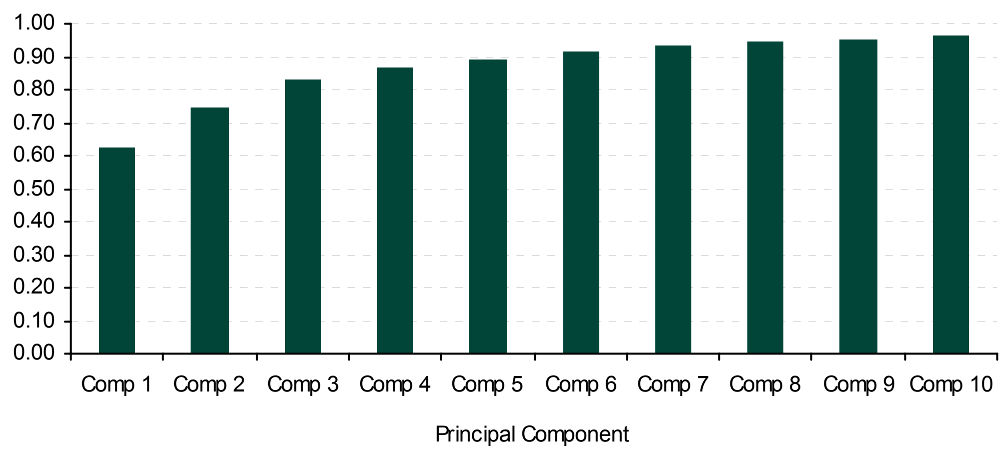
2. A MODEL WITH CREDIT MARKET FACTORS

2.1 Motivation

Credit risk premia across sectors and issuers are not independently distributed. For instance, when the credit market as a whole improves, all sectors tend to improve at the same time. That there are common sources of variation in credit returns can be seen by a principal component analysis (PCA) of returns on sector-quality portfolios. To illustrate this point, we carry out a PCA decomposition on a set of 24 time series of sector-quality excess returns[3] in the US corporate bond market. The sources of variation can be seen through these different components. The first component is able to explain around 63% of the variance of the series, the first two components 75%, and the first eight components around 95%.

It therefore seems reasonable that excess returns on corporate bonds can be explained by a small number of systematic factors. In our analysis, we show that a set of pre-specified economically intuitive factors can be chosen to account for this common variation.

Figure 1. PCA decomposition of sector/rating excess returns



2.2 Methodology: time-series regression

The simplest specification for a factor model is a linear factor model. In such a model, the excess return of a bond or an issuer is explained by a combination of factors, factor exposures and a random term as shown in the following equation:

$$R_{it} = a_i + \beta_i^{'} f_t + \varepsilon_{it,}\ t = 1,2,\ldots,T \tag{1}$$

Here R_{it} is the monthly excess return of an issuer or a sector i in period t, f_t is a vector of factors, and β_i is a vector of factor loadings. The random idiosyncratic shock ε_{it} is assumed to have mean zero and is assumed to be uncorrelated with the systematic factors.

The above model can be estimated by a time-series OLS regression. If the model is well specified, factors should be statistically significant in these regressions (as evidenced by the respective t-statistics).

The above equation implies that the expected excess return of a given asset i equals:

$$E(R_{it}) = a_i + \beta_i^{'} E(f_t) \tag{2}$$

Here $E(R_{it})$ is the expected excess return of an issuer or a sector i. The expected excess return is a linear combination of expected factor realizations. In our analysis, the factors are chosen to be returns on zero investment portfolios that capture the risk factors we are interested in modeling. In this way, the risk premia are linear combinations of risk premia on

[3] *The sectors we use are banking, basic industries, communications, consumer cyclicals, consumer non-cyclicals, energy, financials, and utilities. The rating categories used are AA, A and BBB.*

these factor-mimicking portfolios (with the weights given by the beta of the asset with respect to each factor). If our model captures the cross-sectional dispersion in average returns, then we would expect the intercept term a_i to be negligible, a hypothesis that we test. Our empirical methodology is identical to the one used in Fama and French (1993) and originates from the seminal work of Black, Jensen and Scholes (1972).

2.3 Data

We use monthly excess returns at the bond, issuer and sector level from the Lehman Brothers US and Euro Corporate Bond Indices. The excess return on a given corporate bond is computed by subtracting the total return on duration-matched treasuries from the total return on the bond. All the bonds we consider in the US are AA, A and BBB-rated US dollar corporate bonds with listed equity (same universe as the ESPRI model, ie, approximately 90% of the Lehman US Corporate Index). The time period is May 1994 to December 2002. For issuer and sector monthly excess returns, we use the par-weighted average of monthly bond excess returns. The sectors are chosen to be the sectors from the Lehman Brothers US Risk Model. They are banking, basic industries, communications, consumer cyclicals, consumer non-cyclicals, energy, financials and utilities. Figure 2 provides some descriptive statistics of the rating categories and sectors included in our analysis.

Figure 2. US summary statistics for the credit sectors and ratings: April 1994-December 2002

Sector	Average Duration (year)	Average OAS (bp)	Average Amount Outstanding ($ million)	Average Age (Months)
BANKING	4.6	115.0	362	34
BASIC INDUSTRIES	6.1	150.5	264	47
COMMUNICATIONS	6.4	208.2	425	40
CONSUMER CYCLICALS	6.2	158.3	348	40
CONSUMER NON-CYCLICALS	6.0	116.1	279	39
ENERGY	6.6	152.6	249	44
FINANCIALS	4.2	148.0	358	31
UTILITIES	5.3	201.7	267	30
Rating				
AA	5.1	89.2	338	43
A	5.5	123.9	313	39
BBB	6.1	202.6	285	36

In Europe, the bonds we consider are AA, A and BBB-rated euro corporate bonds with listed equity (same universe as the ESPRI model, ie, approximately 70% of the Lehman Euro Corporate Index but we do not keep the senior bank bonds). The time period is from January 1999 (inception date of the euro) to December 2002. Figure 3 provides some descriptive statistics of the rating categories and sectors included in our analysis.

Figure 3. Euro summary statistics for the credit sectors and ratings: January 1999-December 2002

Sector	Average Duration (year)	Average OAS (bp)	Average Amount Outstanding (€ million)	Average Age (Months)
BANKING	5.8	84.8	391	30
BASIC INDUSTRIES	4.6	111.0	495	18
COMMUNICATIONS	4.2	162.8	987	21
CONSUMER CYCLICALS	3.7	118.3	547	22
CONSUMER NON-CYCLICALS	4.5	91.2	541	20
ENERGY	4.8	128.1	447	21
FINANCIAL	4.3	119.7	595	21
UTILITIES	4.8	73.9	566	24
Rating				
AA	5.3	60.8	488	28.0
A	4.5	103.6	584	20.2
BBB	4.1	187.8	638	21.5

2.4 Credit factors

There are two approaches to selecting factors which can explain systematic variation in returns. We can either look directly at macroeconomic variables or we can use portfolios which are proxies for economic factors (ie, factor-mimicking portfolios). We use the second approach because the data are more readily available. We explore several factors from the credit market that can be justified on *a priori* economic grounds. We form factor-mimicking portfolios to represent the following economic factors in our regressions: (i) the overall market movement; (ii) variations in the premium for credit quality; (iii) changes in the slope of the credit spread curve; (iv) variations in the liquidity premium; and (v) an equity-bond momentum factor. We describe the methodology for constructing these factors below.

i) *The credit market factor*: This is taken to be the excess return on a portfolio denoted as the CMK portfolio. The CMK portfolio is the par-weighted[4] portfolio of all the bonds in the universe. It should be one important driving variable explaining excess returns similarly to the market portfolio in the Capital Asset Pricing Model (CAPM). The risk premium is then measured by the market risk premium and the beta exposure to the market. Simple one-factor models assume that the market is the only driving variable of credit spreads and excess returns.

ii) *The market-neutral quality premium factor*[5]: This is computed from the excess return on a portfolio denoted as the SHL portfolio. The SHL portfolio is a zero investment portfolio that is long high spread bonds and short low spread bonds. To construct this factor, we first control for the slope effect by sorting the universe into three duration buckets and each duration bucket into three spread buckets. We then compute the difference in excess returns between the high spread bucket portfolio

[4] *These are proxies for market weights.*

[5] *We refer to the portfolio as the quality premium factor or quality premium portfolio in the rest of the article.*

and the low spread bucket portfolio in each duration bucket. Finally, the equally weighted average of these spread-based long-short portfolios in each duration bucket is computed, controlling for the market effect. We control for the market effect by using the regression residual of the excess return against the excess return on the CMK portfolio. We use a market-neutral quality premium factor instead of a simple quality premium factor because of the high positive correlation of this factor with the credit market. This factor captures the difference in returns between firms with low quality and high credit quality. This factor is related to the default factor DEF (difference between yields-to-maturity of BBB and AA-rated corporate bonds) used in Fama and French (1993) as it also attempts to capture a quality premium effect. However, DEF is not controlled for the credit market factor.

iii) *The market-neutral slope factor*[6]: This is computed from the excess return on a portfolio denoted as the DHL portfolio. The DHL portfolio is a zero investment portfolio that is long high duration bonds and short low duration bonds. We construct the market-neutral slope factor by first computing the excess return on the DHL portfolio and then using the regression residual of the DHL portfolio's excess return against the CMK portfolio. Here also, we use a market-neutral factor instead of a simple slope factor because of its high correlation with the credit market. Since this factor is computed as the excess return on long-dated bonds less the returns on short-dated bonds (controlling for overall market movements), this factor should capture purely the effect of changes in the slope of the credit spread curve. This factor is related to the slope factor TERM used in Fama and French (1993) as it also attempts to capture a slope effect. TERM captures variations in the slope of the treasury yield curve; instead, our factor captures variation in the slope of the credit spread curve.

iv) *The liquidity factor*: This is computed from the excess returns on a portfolio denoted the LLH portfolio. The LLH portfolio is a zero investment portfolio that is long low liquidity bonds and short high liquidity bonds. It captures a liquidity premium. We control for slope and spread effects by sorting the universe into three duration buckets and each duration bucket into three spread buckets. Within a duration/spread bucket, we divide the bonds into four size buckets and four age buckets. We then compute the excess return difference between the bonds with the largest size and the most recent issue date and the bonds in the next-to-largest size bucket and next-to-youngest age bucket[7]. The LLH portfolio excess return is the negative of the average of these excess returns across the nine duration and quality premium portfolios.

v) *The equity-bond momentum factor:* This is computed from the excess return on a portfolio denoted the MHL portfolio. The MHL portfolio is a zero investment portfolio that is long bonds with high past three months' equity returns and short bonds with low past three months' equity returns. We control for slope and spread effects by sorting the universe into three duration buckets and each duration bucket into three spread buckets. We then compute the difference in excess returns between the bonds with high past equity returns and bonds with low past equity returns within each duration/spread bucket. The MHL portfolio excess return is the average of these excess returns across the nine duration and quality premium portfolios. MHL is a

[6] *We refer to the portfolio as the slope factor or slope portfolio in the rest of the article.*

[7] *The largest in term of amount outstanding and the most recently issued bonds are usually seen as the most liquid. Not all recently issued bonds nor all largely issued bonds are considered liquid.*

measure of cross-market equity momentum effect and captures the bond equity correlation. This factor is similar to the UMD factor in the finance literature that captures the equity momentum effect (see eg, Carhart (1997)).

Figure 4 and 5 present summary statistics for the US and euro credit factors.

Figure 4. Summary statistics for the US credit factors: June 1994-Dec 2002

	CMK	SHL	DHL	LLH	MHL
Average (% p.m.)	-0.02	0.00	-0.02	-0.03	0.21
Volatility (% p.m.)	0.81	0.62	0.47	0.53	0.57
Autocorrelation	0.02	0.10	0.05	0.01	-0.05

CMK: excess returns on the credit market portfolio, SHL: excess return differential on high spread and low quality premium portfolios, after controlling for the market effect, DHL: excess return differential on high duration and low duration portfolios, after controlling for the market effect:, LLH: excess return differential on portfolios of low and high liquidity bonds, MHL: excess return differential on bonds with positive and negative equity momentum.

Figure 5. Summary statistics for the euro credit factors: Feb 1999-Dec 2002

	CMK	SHL	DHL	LLH	MHL
Average (% p.m.)	-0.02	0.02	0.02	-0.07	0.23
Volatility (% p.m.)	0.52	0.19	0.20	0.49	0.54
Autocorrelation	0.01	0.45	0.09	0.09	0.07

CMK: excess returns on the credit market portfolio, SHL: excess return differential on high spread and low quality premium portfolios, after controlling for the market effect, DHL: excess return differential on high duration and low duration portfolios, after controlling for the market effect:, LLH: excess return differential on portfolios of low and high liquidity bonds, MHL: excess return differential on bonds with positive and negative equity momentum.

We note that only the MHL portfolio has an average positive excess return (21bp per month on average for the US and 23bp for euro) over our sample. The market-neutral slope factor and the liquidity factor have not earned a positive excess return during that period whereas the market-neutral quality premium factor has earned a slight positive return.

3. REGRESSION RESULTS

We have identified several interesting credit factors as potential explanatory variables in a multi-factor model. In this section, we examine two factor specifications. The first specification uses the credit market as the single explanatory variable. The second includes the credit market and several other factors that reflect the quality premium, the credit slope, liquidity and the momentum spillover from equity to bond market.

3.1 A single-factor model

The simplest specification of a factor model is a single-factor model: we use the credit market alone (the CMK factor) and try to explain all returns with a beta to the credit market. This simple model identifies high beta and low beta issuers. If the fit of the model is good then looking at one beta alone is a good indicator of expected returns and risk. If the fit is poor then selecting issuers simply based on their betas is not recommended because it misses other important factors.

We start with rating level regressions. We attempt to explain a ratings-based excess return with the market factor. As Figure 6 shows, the R^2 are relatively high. At this level of aggregation, the firm or sector-specific risks are averaged out and only remain systematic effects of which the market is a large component. This confirms that for rating buckets, the most important driver is the credit market.

Figure 6. Rating category level time-series regressions (US) June 1994-Dec 2002

Sector	R^2	const	β_{CMK}
AA	80%	0.04	0.54
		(1.86)	(20.31)
A	92%	0.02	0.82
		(0.86)	(33.58)
BBB	96%	0.00	1.35
		(-0.03)	(47.52)

The above results are from the following model:
$R_{it}=a_i+\beta_{CMK}CMK_t+\varepsilon_{it}$
i =AA, A, BBB
R_{it} is the monthly excess return of quality i in period t.
CMK_t : excess returns on the credit market portfolio in month t.

t-statistics appear in parenthesis.

Next, we examine sector-quality level regressions. We should expect to find lower R^2 because idiosyncratic risks are more prominent at the disaggregated level. The results (not reported) indicate that R^2 values are around 65-70%, which is consistent with the PCA decomposition that we have performed previously. The best fit is with the consumer-cyclical BBB sector. The poorest fit is with the banking BBB sector.

3.2 A multi-factor credit model

We now consider a multi-factor specification. The PCA results mentioned before indicate that we should be able to construct a better model by incorporating additional factors. The following analysis shows the extent of this improvement when we include excess returns on our factor-mimicking portfolios in addition to the market factor.

We expand our range of explanatory variables by using the quality premium factor (SHL), the slope factor (DHL), the liquidity factor (LLH), and the equity momentum factor (MHL). The construction of these factors is done as explained in the previous section.

The results for the regressions on rating-buckets are given in Figure 7.Not surprisingly, the rating category most sensitive to the credit market is the BBB quality bucket (beta of 1.34) followed by A (0.87) then AA (0.53). Lower quality sector buckets are more sensitive to the systematic risk captured by the credit market.

From Figure 2, we see that the BBB quality sector has the longest duration (6.1 years) and the highest level of spread (203bp) relative to the other rating sectors. It is thus more exposed to the slope factor and the quality premium factor. The BBB category has a positive factor exposure to the DHL and SHL portfolios. When the slope factor or quality premium factor rallies, BBB-rated issuers tend to rally at the same time, assuming all the other factors remain constant.

Figure 7. Rating category level time-series regressions (US) June 1994-Dec 2002

sector	R^2	Const	β_{CMK}	β_{DHL}	β_{SHL}	β_{LLH}	β_{MHL}
AA	93%	0.03	0.53	-0.03	-0.29	-0.07	0.03
		(1.97)	(23.30)	(-0.97)	(-9.73)	(-2.05)	(0.98)
A	95%	0.00	0.87	0.00	-0.12	0.10	0.08
		(0.26)	(31.13)	(-0.07)	(-3.13)	(2.45)	(1.81)
BBB	98%	-0.01	1.34	0.09	0.24	-0.11	0.04
		(-0.52)	(45.94)	(2.20)	(6.34)	(-2.79)	(0.94)

The above results are from the following model:
$R_{it}=a_i+\beta'_{CMK}CMK_t+\beta'_{DHL}DHL_t+\beta'_{SHL}SHL_t+\beta'_{LLH}LLH_t+\beta'_{MHL}MHL_t+\varepsilon_{it}$
i =AA, A, BBB
R_{it} is the monthly excess return of quality i in period t
CMK: excess returns on the credit market portfolio, SHL: excess return differential on high spread and low quality premium portfolios, after controlling for the market effect, DHL: excess return differential on high duration and low duration portfolios, after controlling for the market effect:, LLH: excess return differential on portfolios of low and high liquidity bonds, MHL: excess return differential on bonds with positive and negative equity momentum.
t-statistics appear in parenthesis.

In contrast to the BBB quality bucket, the A quality and AA quality sectors have relatively low spreads (89bp and 124bp respectively) which command negative factor exposures to the quality premium factor.

The A quality bucket is relatively less liquid than the AA and BBB quality buckets because of a smaller average amount outstanding and/or older age ($313 million, vs $338 million for AA and 39 months vs 36 months for BBB); as such, it receives a positive loading to the liquidity factor whereas AA and BBB quality buckets have a negative exposure to this factor.

We run the same regressions at the sector level. The results of these regressions are reported in Figure 8.

The most sensitive sectors to the credit market are the utility (beta of 1.44), consumer cyclicals (1.42) and energy sectors (1.24). The least sensitive ones are the banking (0.68), consumer non-cyclicals (0.68)and basic industries (0.74). The higher quality and the less cyclical sectors have on average a lower beta to the credit market.

Sectors differ by their average durations and average spreads (Figure 2). The sectors with the highest spreads are the telecom sector (OAS of 208bp) and utility sector (202bp). They are most exposed to the quality premium factor. High spread firms benefit from the carry as captured by the quality premium factor. On the other hand, the sectors with the lowest spreads such as banking and financials receive a negative exposure to the quality premium factor. When the quality premium portfolio performs, these sectors underperform, all the other factors remaining the same.

The sectors with the longest duration are the telecom sector (6.4 years) and energy sector (6.6 years). They are most exposed to the duration factor. Duration is shorter for financials (4.2 years) and banking (4.6 years), which implies that these sectors have a negative or null exposure to the slope factor.

Figure 8. Sector level time-series regressions (US) June 1994-Dec 2002

Sector	R^2	Const	β_{CMK}	β_{DHL}	β_{SHL}	β_{LLH}	β_{MHL}
BANKING	72%	0.05	0.68	0.02	-0.23	0.14	0.10
		(1.43)	(11.51)	(0.24)	(-2.92)	(1.65)	(1.10)
BASIC INDUSTRIES	86%	0.03	0.74	0.10	-0.30	0.21	-0.11
		(1.04)	(16.06)	(1.45)	(-4.96)	(3.20)	(-1.63)
COMMUNICATIONS	90%	0.00	0.94	0.36	0.24	-1.07	-0.40
		(-0.07)	(11.61)	(3.09)	(2.22)	(-9.26)	(-3.30)
CONSUMER CYCLICALS	94%	-0.06	1.42	0.04	0.04	-0.08	0.36
		(-1.96)	(28.63)	(0.61)	(0.60)	(-1.16)	(4.96)
CONSUMER NON-CYCLICALS	88%	0.03	0.68	0.11	-0.25	0.20	0.02
		(1.23)	(18.99)	(2.11)	(-5.30)	(3.94)	(0.39)
ENERGY	91%	-0.03	1.24	0.12	0.13	0.37	0.32
		(-0.86)	(25.90)	(1.68)	(2.00)	(5.51)	(4.56)
FINANCIALS	74%	-0.01	1.02	-0.38	0.13	0.17	0.26
		(-0.30)	(12.88)	(-3.33)	(1.28)	(1.51)	(2.21)
UTILITIES	76%	0.00	1.44	-0.37	0.69	0.53	0.19
		(-0.06)	(13.00)	(-2.30)	(4.74)	(3.37)	(1.17)

The above results are from the following model:
$R_{it}=a_i+\beta'_{CMK}CMK_t+\beta'_{DHL}DHL_t+\beta'_{SHL}SHL_t+\beta'_{LLH}LLH_t+\beta'_{MHL}MHL_t+\varepsilon_{it}$
i = Risk model sector
R_{it} is the monthly excess return of industry sector i in period t
CMK: excess returns on the credit market portfolio, SHL: excess return differential on high spread and low quality premium portfolios, after controlling for the market effect, DHL: excess return differential on high duration and low duration portfolios, after controlling for the market effect:, LLH: excess return differential on portfolios of low and high liquidity bonds, MHL: excess return differential on bonds with positive and negative equity momentum.
t-statistics appear in parenthesis.

Most of the sectors have to pay a liquidity premium except for the telecom sector, which seems to receive a liquidity discount. The telecom sector is indeed the most liquid sector in terms of average amount outstanding ($425 million against an average of $309 million across all the sectors). The less liquid sectors such as energy and utilities (because of their relatively small average amount outstanding, $249 million and $267 million respectively) have the highest positive loading to the liquidity factor.

Finally, the sectors most sensitive to the equity momentum factor are the consumer cyclicals, the energy, utility and financial sectors. These sectors tend to do well when the equity momentum factor is doing well[8].

We also note from Figure 8 that a significant explanatory power is added by considering factors other than the overall market factor. The regression coefficients on most of these additional factors are statistically significant for most sector buckets. We also run the regressions for sector-quality portfolios. The results are available on request. The R^2 also improve markedly. For example, in the case of the telecom BBB sector, R^2 for the single-factor model is 58% while it improves to 83% for the multi-factor model.

To examine whether a single-factor model is sufficient in another way, we have also run an issuer level regression. Figure 9 reports the average across issuers of the estimated regression

[8] *The effect is negative on the telecom sector, probably due to the limited sample size.*

coefficients. The reported t-statistics are the mean of the estimates across all the issuers divided by their standard deviation. We see that on average the single factor model is only able to explain 33% of the time series of issuer excess returns using the market factor. This means that two-thirds of the time-series variation remains unexplained by the model. In the multi-factor model, on average we are able to explain 49% of the time series of issuer excess returns using these credit factors. The improvement is significant.

Figure 9. Issuer level time-series regressions (US): June 1994-Dec 2002

R^2	*const*	β_{CMK}	β_{DHL}	β_{SHL}	β_{LLH}	β_{MHL}
33%	0.02	0.83				
	(3.07)	(33.35)				
49%	0.00	0.97	-0.15	-0.03	0.22	-0.04
	(0.41)	(28.89)	(-6.06)	(-0.98)	(6.52)	(-0.89)

The above results are from the following models:
$R_{it}=a_i+\beta'_{CMK}CMK_t+\varepsilon_{it}$
$R_{it}=a_i+\beta'_{CMK}CMK_t+\beta'_{DHL}DHL_t+\beta'_{SHL}SHL_t+\beta'_{LLH}LLH_t+\beta'_{MHL}MHL_t+\varepsilon_{it}$
i =issuer
R_{it} is the monthly excess return of issuer i in period t
CMK: excess returns on the credit market portfolio, SHL: excess return differential on high spread and low quality premium portfolios, after controlling for the market effect, DHL: excess return differential on high duration and low duration portfolios, after controlling for the market effect:, LLH: excess return differential on portfolios of low and high liquidity bonds, MHL: excess return differential on bonds with positive and negative equity momentum.
Reported coefficients and R^2 values are averages of regression coefficients and R^2 of single issuer regressions. The t statistics are for these averages.

3.3 Testing the pricing effectiveness of the multi-factor model: GRS test

The Gibbons, Ross and Shanken (1989) or "GRS" statistic tests whether the intercepts or pricing errors a in the previous time-series regression across all the sector-quality portfolios are jointly statistically different from 0. It is an F-test that extends the t-test used for individual time-series regressions and can be used in finite-sample with OLS regressions. We fail to reject that the pricing errors are statistically different from 0. This is consistent with the hypothesis that risk premia on credit assets are determined by the co-variation of excess returns with the factors that we have introduced.

3.4 Results for the euro market

We have performed the same analysis on the euro market. We use the factors described in Figure 5, which are constructed from euro corporate bond excess returns.

We present the rating level regressions in Figure 10. Again, R^2 are relatively high but less so than for the US credit market, especially for the AA-rated sector. The explanation could be that the Lehman Brothers Euro Credit Market Index as a whole has been more volatile than the AA-rated sector. Still, the credit market remains an important driver for rating buckets.

We also run the same regressions with the multifactor model. Results are presented in Figure 11. We use as explanatory variables, the quality premium factor (SHL), the slope factor (DHL), the liquidity factor (LLH), and the equity momentum factor (MHL). These factors are constructed exactly like the US factors except that duration and spread bucketings are each done in two rather than three buckets.

Figure 10. Rating category level time-series regressions (euro) Feb 1999-Dec 2002

Sector	R^2	const	β_{CMK}
AA	67%	0.05	0.37
		(2.64)	(9.53)
A	90%	0.02	0.94
		(0.88)	(19.62)
BBB	80%	-0.05	1.38
		(-0.91)	(13.29)

The above results are from the following model:
$R_{it}=a_i+\beta'_{CMK}CMK_t+\varepsilon_{it}$
i =AA, A, BBB
R_{it} *is the monthly excess return of quality i in period t.*
CMK_t *: excess returns on the credit market portfolio in month t.*
t-statistics appear in parenthesis.

By including the additional factors, the R^2 improves significantly and the factors are mostly significant, especially DHL, SHL and LLH. Because the AA sector has the longest duration (5.3 years) and lowest spread level (61bp), its loading on the DHL (resp. SHL) is positive (resp. negative). In contrast, the BBB sector has a positive loading on the SHL factor because of its higher spread level (188bp) and an insignificant loading on the DHL factor. Compared with these sectors, the A sector seems the most liquid with a significant negative exposure to the LLH factor.

Figure 11. Rating category level time-series regressions (euro) Feb 1999-Dec 2002

Sector	R^2	Const	β_{CMK}	β_{DHL}	β_{SHL}	β_{LLH}	β_{MHL}
AA	90%	0.06	0.38	0.15	-0.49	0.00	0.02
		(4.06)	(13.28)	(2.23)	(-6.46)	(0.12)	(0.76)
A	91%	0.01	0.98	-0.12	-0.26	-0.11	0.04
		(0.46)	(15.99)	(-0.83)	(-1.61)	(-2.11)	(0.71)
BBB	83%	-0.11	1.48	0.18	0.92	-0.01	0.16
		(-1.76)	(11.22)	(0.62)	(2.67)	(-0.09)	(1.16)

The above results are from the following model:
$R_{it}=a_i+\beta'_{CMK}CMK_t+\beta'_{DHL}DHL_t+\beta'_{SHL}SHL_t+\beta'_{LLH}LLH_t+\beta'_{MHL}MHL_t+\varepsilon_{it}$
i =AA, A, BBB
R_{it} *is the monthly excess return of quality i in period t*
CMK: excess returns on the credit market portfolio, SHL: excess return differential on high spread and low quality premium portfolios, after controlling for the market effect, DHL: excess return differential on high duration and low duration portfolios, after controlling for the market effect:, LLH: excess return differential on portfolios of low and high liquidity bonds, MHL: excess return differential on bonds with positive and negative equity momentum.
t-statistics appear in parenthesis.

We run the same analysis at the sector level. The results are reported in Figure 12.

In the euro market, the most sensitive sectors to the credit market are the telecom (beta of 1.44) and financial sectors (1.21). The least sensitive are the banking (0.59) and utility sectors (0.50). The lower quality and the most cyclical sectors have on average a higher beta to the credit market.

Figure 12. Sector level time-series regressions (euro): Feb 1999-Dec 2002

Sector	R^2	Const	β CMK	β DHL	β SHL	β LLH	β MHL
BANKING	76%	0.11	0.59	0.09	-0.40	0.00	-0.07
		(3.22)	(7.94)	(0.52)	(-2.06)	(0.03)	(-0.88)
BASIC INDUSTRIES	77%	0.02	0.91	-0.36	-0.24	0.08	-0.02
		(0.33)	(8.59)	(-1.50)	(-0.86)	(0.85)	(-0.15)
COMMUNICATIONS	66%	-0.09	1.44	0.58	1.35	-0.32	0.17
		(-0.95)	(6.85)	(1.22)	(2.44)	(-1.74)	(0.76)
CONSUMER CYCLICALS	73%	-0.02	0.78	-0.38	-0.26	0.27	0.07
		(-0.31)	(7.51)	(-1.62)	(-0.94)	(2.99)	(0.67)
CONSUMER NON CYCLICALS	67%	0.00	0.72	0.21	-0.06	0.06	0.19
		(0.08)	(7.60)	(1.00)	(-0.26)	(0.76)	(1.91)
ENERGY	59%	0.01	0.75	0.65	-0.18	0.43	-0.17
		(0.07)	(4.40)	(1.68)	(-0.39)	(2.87)	(-0.97)
FINANCIALS	68%	0.07	1.21	-0.62	-0.72	-0.15	-0.14
		(0.81)	(6.59)	(-1.49)	(-1.49)	(-0.94)	(-0.71)
UTILITIES	76%	0.03	0.50	0.16	-0.35	-0.06	0.08
		(0.98)	(8.93)	(1.23)	(-2.34)	(-1.25)	(1.40)

The above results are from the following model:
$R_{it}=a_i+\beta'_{CMK}CMK_t+\beta'_{DHL}DHL_t+\beta'_{SHL}SHL_t+\beta'_{LLH}LLH_t+\beta'_{MHL}MHL_t+\varepsilon_{it}$
i = Risk model sector
R_{it} is the monthly excess return of industry sector i in period t.
CMK: excess returns on the credit market portfolio, SHL: excess return differential on high spread and low quality premium portfolios, after controlling for the market effect, DHL: excess return differential on high duration and low duration portfolios, after controlling for the market effect:, LLH: excess return differential on portfolios of low and high liquidity bonds, MHL: excess return differential on bonds with positive and negative equity momentum.
t-statistics appear in parenthesis.

The sectors with the highest spreads are the telecom sector (OAS of 163bp) and energy sector (128bp). They are most exposed to the quality premium factor. In contrast to the US utility sector, the euro utility sector is not as exposed to this factor reflecting a lower average spread.

The sectors with the longest duration are the banking sector (5.8 years), utility and energy sectors (4.8 years). They are most exposed to the slope factor. Duration is the shortest for the consumer cyclical sector (3.7 years), which causes a negative exposure to the slope factor.

Most of the sectors have positive exposure to the liquidity factor except for the telecom, financial and utility sectors. They appear to be more liquid than the other sectors.

In conclusion, we find similar patterns in the euro market. However, some differences remain, most likely caused by the different industrial and financial structure of bond issuers between the euro and US markets.

4. INCORPORATING EQUITY MARKET FACTORS

Multifactor models have been used extensively in the equity market. To address the poor empirical fit of the CAPM, Fama and French (1992, 1993) have suggested using a three-factor model. They use:

i) The equity market factor: this is proxied by the return on the MKT portfolio, which is a market-value weighted average of equity excess returns of stocks from the NYSE, AMEX and NASDAQ.

ii) The book-to-market factor: this is the return on the HML portfolio, which is the difference in monthly return between the high book-to-market and low book-to-market stock portfolios[9]; it is a proxy for relative distress since weak firms with persistently low earnings have high book-to-market and positive slope to HML; firms with low cash flow per share/ stock price, low earnings per share/ stock price and high sales growth have typically negative loading on HML factor.

iii) The size factor: this is the return on the SMB portfolio, which is the difference in monthly return between the small-cap and big-cap stock portfolios[10]; it serves as a proxy for business cycle risk: small firms are more sensitive to recession than large firms. Small firms tend to outperform in average over a long period.

Naldi (2002) has used these factors to decompose the corporate bond risk premia and found that they had explanatory power combined with some sectoral factors.

In this section, we analyze the incremental explanatory power added by incorporating equity market factors in our multi-factor model. As a starting point, Figure 13 presents summary statistics of the equity factors.

Figure 13. Summary statistics for the US equity factors: June 1994-Dec 2002

	MKT	SMB	HML
Average (%)	0.46	-0.39	0.60
Volatility (%)	4.83	4.53	4.25
Autocorrelation	0.01	0.15	0.13

The statistics are the mean, monthly volatility, autocorrelation (returns in %)
MKT: market-weighted excess returns of the equity market, SMB: excess return differential on small equity market capitalization and large equity market capitalization portfolios, HML: excess return differential on portfolios of high book-to-market and low book-to-market issuers.

All the portfolios have positive excess returns except for the size portfolio which has underperformed by 39bp per month over our sample period.

4.1 A mixed credit-equity factor model

As shown by Naldi (2002), it is possible to explain some of the bond excess return variation with some equity factors. However, on average we are only able to explain 23% of the time series of issuer excess returns using the equity factors. The corresponding numbers for the credit models are 33% for the single credit market and 49% for all the credit factors. This is consistent with the credit sector dummies being significant in Naldi's analysis. We also note that the market risk, the business cycle risk (size factor) and financial distress (book-to-market factor) have positive loadings. Thus these risks get rewarded on average.

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[9] *Fama and French sort all the stocks from the NYSE, AMEX and NASDAQ into two size groups (small and big, S and B) and three book-to-market groups (high, medium, low, H, M and L) and take the intersection of these groups to construct six size/book-t-market benchmark stock portfolios. The HML portfolio is constructed by going long the high book-to-market stock portfolios (value portfolios) and short the two low book-to-market portfolios (growth portfolios).*

[10] *The SMB portfolio is constructed by going long the three small stock portfolios and short the three large book-to-market portfolios.*

Figure 14 presents the regression results where equity factors are included in addition to credit factors.

Figure 14. Rating category time-series regressions (US) June 1994-Dec 2002

sector	R^2	const	β_{CMK}	β_{DHL}	β_{SHL}	β_{LLH}	β_{MHL}	β_{MKT}	β_{SMB}	β_{HML}
AA	93%	0.03	0.56	-0.02	-0.29	-0.06	0.04	0.00	0.00	0.00
		(1.78)	(19.56)	(-0.61)	(-9.66)	(-1.77)	(1.27)	(-0.76)	(-1.06)	(0.12)
A	95%	0.00	0.90	0.01	-0.11	0.11	0.09	0.00	-0.01	0.00
		(0.26)	(25.64)	(0.14)	(-3.04)	(2.46)	(2.02)	(-0.88)	(-1.12)	(-0.44)
BBB	98%	-0.01	1.31	0.08	0.24	-0.11	0.03	0.00	0.00	0.00
		(-0.33)	(36.29)	(1.78)	(6.38)	(-2.51)	(0.63)	(0.84)	(0.74)	(-0.57)

The above results are from the following model:
$R_{it}=a_i+\beta'_{CMK}CMK_t+\beta'_{DHL}DHL_t+\beta'_{SHL}SHL_t+\beta'_{LLH}LLH_t+\beta'_{MHL}MHL_t+\beta'_{MKT}MKT_t+\beta'_{SMB}SMB_t+\beta'_{HML}HML_t+\varepsilon_{it}$
i =A, AA, BBB
R_{it} is the monthly excess return of quality i in period t
CMK: excess returns on the credit market portfolio, SHL: excess return differential on high spread and low quality premium portfolios, after controlling for the market effect, DHL: excess return differential on high duration and low duration portfolios, after controlling for the market effect:, LLH: excess return differential on portfolios of low and high liquidity bonds, MHL: excess return differential on bonds with positive and negative equity momentum., MKT: market-weighted excess returns of the equity market, SMB: excess return differential on small equity market capitalization and large equity market capitalization portfolios, HML: excess return differential on portfolios of high book-to-market and low book-to-market issuers.
t-statistics appear in parenthesis.

At the rating category level, the equity factors are dominated by the credit factors. Not all the equity factors seem to be significant. They are significant for the telecom and energy sectors.

Figure 15 presents the results of sector-level regressions.

The sector most sensitive to the equity market factor is the energy (especially natural gas) sector. The telecom sector is negatively correlated with the equity market, the business cycle (SMB) and financial distress (HML) factors. SMB affects positively mostly the energy sector, whereas HML affects the energy and utility sectors. The improvement at the sector level is marginal when we include equity factors.

Figure 15. Sector time-series regressions (US) June 1994-Dec 2002

Sector	R^2	const	β_{CMK}	β_{DHL}	β_{SHL}	β_{LLH}	β_{MHL}	β_{MKT}	β_{SMB}	β_{HML}
BANKING	73%	0.04	0.68	0.03	-0.22	0.19	0.11	0.01	-0.01	0.00
		(1.10)	(9.23)	(0.38)	(-2.79)	(2.11)	(1.18)	(1.31)	(-0.87)	(0.19)
BASIC INDUSTRIES	87%	0.04	0.71	0.07	-0.29	0.24	-0.13	0.01	0.00	-0.01
		(1.23)	(12.49)	(1.06)	(-4.86)	(3.38)	(-1.82)	(0.73)	(-0.01)	(-1.17)
COMMUNICATIONS	90%	0.02	1.08	0.36	0.25	-1.00	-0.36	-0.03	-0.03	-0.04
		(0.39)	(10.86)	(3.07)	(2.41)	(-8.24)	(-2.97)	(-2.25)	(-2.04)	(-2.39)
CONSUMER CYCLICALS	94%	-0.06	1.39	0.02	0.04	-0.08	0.35	0.00	0.00	-0.01
		(-1.73)	(22.38)	(0.33)	(0.62)	(-1.01)	(4.65)	(0.36)	(0.42)	(-0.51)
CONSUMER NON-CYCLICALS	89%	0.04	0.68	0.10	-0.26	0.15	0.01	-0.01	0.01	0.00
		(1.69)	(15.78)	(1.88)	(-5.65)	(2.85)	(0.23)	(-1.94)	(1.42)	(-0.27)
ENERGY	92%	-0.04	1.14	0.11	0.11	0.33	0.29	0.02	0.02	0.03
		(-1.40)	(19.91)	(1.66)	(1.89)	(4.71)	(4.21)	(2.89)	(2.59)	(2.84)
FINANCIALS	74%	-0.02	1.02	-0.36	0.12	0.13	0.26	0.00	0.01	0.01
		(-0.41)	(10.29)	(-3.08)	(1.17)	(1.06)	(2.17)	(-0.13)	(0.44)	(0.86)
UTILITIES	77%	-0.03	1.33	-0.36	0.67	0.45	0.16	0.02	0.03	0.04
		(-0.37)	(9.66)	(-2.20)	(4.59)	(2.64)	(0.94)	(1.25)	(1.50)	(1.81)

The above results are from the following model:
$R_{it}=a_i+\beta'_{CMK}CMK_t+\beta'_{DHL}DHL_t+\beta'_{SHL}SHL_t+\beta'_{LLH}LLH_t+\beta'_{MHL}MHL_t+\beta'_{MKT}MKT_t+\beta'_{SMB}SMB_t+\beta'_{HML}HML_t+\varepsilon_{it}$
i =Risk model sector
R_{it} is the monthly excess return of industry sector i in period t
CMK: excess returns on the credit market portfolio, SHL: excess return differential on high spread and low quality premium portfolios, after controlling for the market effect, DHL: excess return differential on high duration and low duration portfolios, after controlling for the market effect:, LLH: excess return differential on portfolios of low and high liquidity bonds, MHL: excess return differential on bonds with positive and negative equity momentum., MKT: market-weighted excess returns of the equity market, SMB: excess return differential on small equity market capitalization and large equity market capitalization portfolios, HML: excess return differential on portfolios of high book-to-market and low book-to-market issuers.
t-statistics appear in parenthesis.

The improvement is more significant when we run time-series regression at the issuer level. Results are presented in Figure 16:

Figure 16. Issuer level time-series regressions (US) June 1994-Dec 2002

R^2	const	β_{CMK}	β_{DHL}	β_{SHL}	β_{LLH}	β_{MHL}	β_{MKT}	β_{SMB}	β_{HML}
53%	-0.01	0.95	-0.15	-0.03	0.19	-0.04	0.00	0.01	0.01
	(-0.45)	(25.46)	(-5.86)	(-0.94)	(5.55)	(-0.80)	(2.05)	(4.53)	(3.85)

The above results are from the following model:
$R_{it}=a_i+\beta'_{CMK}CMK_t+\beta'_{DHL}DHL_t+\beta'_{SHL}SHL_t+\beta'_{LLH}LLH_t+\beta'_{MHL}MHL_t+\beta'_{MKT}MKT_t+\beta'_{SMB}SMB_t+\beta'_{HML}HML_t+\varepsilon_{it}$
i =issuer
R_{it} is the monthly excess return of issuer i in period t
CMK: excess returns on the credit market portfolio, SHL: excess return differential on high spread and low quality premium portfolios, after controlling for the market effect, DHL: excess return differential on high duration and low duration portfolios, after controlling for the market effect:, LLH: excess return differential on portfolios of low and high liquidity bonds, MHL: excess return differential on bonds with positive and negative equity momentum., MKT: market-weighted excess returns of the equity market, SMB: excess return differential on small equity market capitalization and large equity market capitalization portfolios, HML: excess return differential on portfolios of high book-to-market and low book-to-market issuers.
t-statistics appear in parenthesis, they are computed as the average of the regression coefficients across the issuer regressions, divided by the standard deviation of these regression coefficients and multiplied by the square root of the numbers of issuers.

At the individual issuer level, on average we are able to explain 53% of the time series of issuer excess returns. When only credit factors are included, the corresponding number is 49%. Also the three equity factors tend to have statistically significant coefficients.

Thus our analysis indicates that while equity factors in themselves are not sufficient to explain large systematic variation in credit markets, they do add useful explanatory power, especially at the issuer level.

4.2 GRS test

The "GRS" test-statistic can also be applied for these regressions. The individual t-statistics are all below the critical level but the "GRS" statistic tests that the pricing errors α for sector-quality portfolios are jointly statistically different from 0. Again, we fail to reject that the pricing errors are statistically different from 0.

5. DECOMPOSING CREDIT SPREADS

Taking our lead from Naldi (2002), we now use the beta coefficients that are estimated using our multi-factor model to decompose credit spreads into risk premia coming from different sources of systematic risk included in the model.

5.1 Methodology

In our multi-factor model, the excess return of a bond or an issuer is explained by a combination of factors, factor exposures and a random idiosyncratic shock. Taking conditional expectation in (1), assuming that the shocks have a conditional mean of zero and also assuming that the intercepts are uniformly zero, the (conditional) expected excess return or risk premium of a given asset i equals:

$$E_t(R_{it+1}) = \beta_i^{'} E_t(f_{t+1})$$

The expected excess return is thus a linear combination of the expected returns of the factor portfolios. The expected excess returns are also the price of risk associated with the factors. Indeed, if a portfolio has only exposure to a given factor, its expected return should be exactly the price of risk associated with that factor. The idiosyncratic risk should not be priced because it can be diversified away.

Expected excess returns are associated with the level of spreads (OAS). However, these two quantities are not one and the same. The OAS alone does not fully take into account the risk of downgrade and the risk of default. The expected change in OAS should also compensate for these risks. However, if we assume that expected excess returns are a constant multiple of credit spreads,[11] we can write:

$$OAS_{it} = \beta'_i \, \pi_t \,, \text{ i=1,2,...,N} \tag{3}$$

where $\pi_t = \lambda E_t f_{t+1}$ are the factor risk premia derived from risk factors and the λ are constant.

We can then estimate these factor risk premia by running cross-sectional regressions of the full universe of issuer OAS on the factor betas estimated from the previous sections on each

[11] *This assumption is clearly not without loss of generality. However, it can be derived under a simplified model of credit spread movements.*

date. The multi-factor model used incorporates the credit and equity factors. The slope of the regressions should be the factor risk premia for the credit and equity factors. In the following section we discuss the level of these factor risk premia. Note that the estimated factor loadings have been estimated in full sample; they should ideally be estimated out-of-sample to use the model in forecasting.

5.2 Factor risk premia

In Figure 17, we present the time-series averages and volatilities of market prices of risk corresponding to various risk factors estimated using cross-sectional regressions described above. Figure 18 shows the time series of these prices of risk. We comment on these results in the following discussion.

Figure 17. Average and volatility of US factor risk premia (bp) June 1994-Dec 2002

		R^2	CMKT	DHL	SHL	LLH	MHL	FFMKT	SMB	HML
Average	1994-2002	76%	111	-22	-53	4	-14	239	362	-205
	1994-1998	77%	65	-1	-49	10	-6	257	142	-89
	1999-2002	75%	163	-46	-58	-3	-23	217	613	-339
Volatility	1994-2002		59	37	29	20	25	206	323	204
	1994-1998		20	11	15	7	4	117	122	64
	1999-2002		44	42	38	28	35	274	298	228
t-statistic	1994-2002		19.01	-6.11	-18.87	2.03	-5.62	11.79	11.35	-10.19
	1994-1998		24.27	-0.94	-23.86	11.32	-11.45	16.30	8.65	-10.30
	1999-2002		25.44	-7.69	-10.48	-0.71	-4.55	5.49	14.23	-10.30

CMK: excess returns on the credit market portfolio, SHL: excess return differential on high spread and low quality premium portfolios, after controlling for the market effect, DHL: excess return differential on high duration and low duration portfolios, after controlling for the market effect:, LLH: excess return differential on portfolios of low and high liquidity bonds, MHL: excess return differential on bonds with positive and negative equity momentum., MKT: market-weighted excess returns of the equity market, SMB: excess return differential on small equity market capitalization and large equity market capitalization portfolios, HML: excess return differential on portfolios of high book-to-market and low book-to-market issuers.

5.2.1 US credit factor risk premia

As seen in Figure 17, the estimated price of risk of the credit market is positive. Exposure to the market has to be rewarded by a risk premium. The price of risk of illiquidity is also positive. Illiquid bonds have to earn more than liquid bonds on average. Similarly, liquid issues have a negative premium because, everything else being equal, investors would rather hold these bonds and sell the illiquid bonds (flight to quality and to liquidity)[12] during a liquidity crisis.

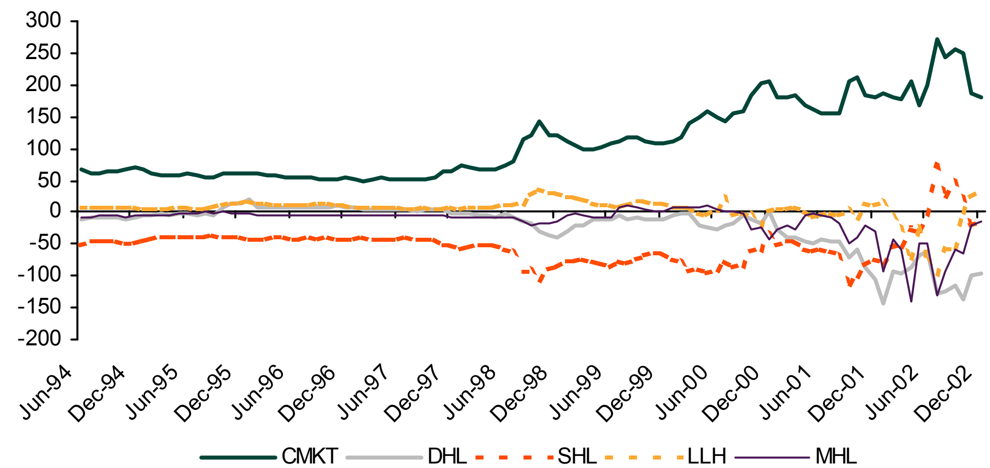
The price of risk of the slope factor is negative. One explanation may be that the risk premium embedded in long-maturity bonds does not increase proportionally to duration. As a result, the higher is the beta to a *market-neutral* spread factor, the lower is the OAS (holding other things constant). There is also a convexity advantage embedded in long-duration bonds. Everything else being equal, long-term bonds would provide a hedge against spread volatility. In equilibrium, investors are willing to pay some premium for convexity.

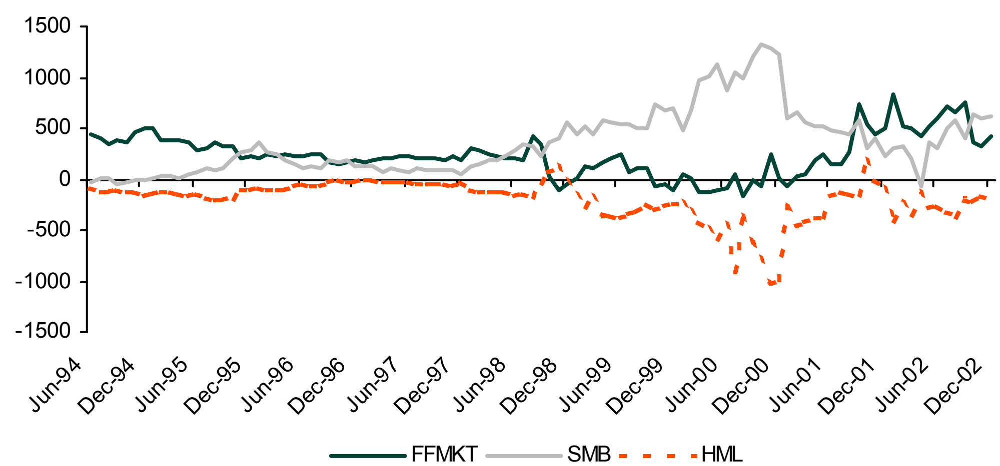
[12] In the recent period, there was an episode of negative liquidity premium, the investors had to be compensated for holding liquid bonds, we'll come back to this point in our spread decomposition sector analysis.

The price of risk of the quality premium factor is also negative. However, recall that the quality premium factor is by construction market neutral. If we add back the market effect, and recompute the price of risk for the quality premium factor, we get a positive price of risk as expected. Also, holding a market-neutral position in a zero investment portfolio that is long high spread bonds and short low spread bonds provides a carry advantage which is likely to be priced in equilibrium.

The cross equity-bond momentum factor MHL is interesting. It has a negative risk premium because it has a negative covariance with the credit market and serves as a hedge. In equilibrium, investors should pay to get exposure to high equity momentum bonds and avoid poor equity momentum bonds.

Figure 18. US credit and equity factor risk premia (bp) June 1994-Dec 2002





The first panel shows the estimated factor risk premia corresponding to credit market factors. The second panel shows the risk premia for equity market factors.

5.2.2 US equity factor risk premia

On the equity side, the prices of risk of the equity factors are all positive except for the financial distress factor HML. This is consistent with the findings of Naldi (2002). The market risk premium is positive because exposure to the equity market risk has to be compensated for. The risk premium for the business cycle risk (SMB) is also positive except in March 2002, when investors seemed to be willing to pay to get exposure to small firms and were betting on an economic recovery. The risk premium came up positive after this month.

The risk premium for distress risk (HML) is negative and was especially so after the Russian crisis until the end of 2000. Exposure to growth stocks (low book-to-market stocks) was perceived as risky by the corporate bond market and needed compensation in the form of a higher risk premium. Examples of value stocks include steel, oil and pulp & paper companies, whereas growth stocks include telecom, media and technology (TMT) companies. Another interpretation suggested by Naldi (2002) is that corporate bonds were regarded as a relatively safe asset class compared with equity and as a good hedge against distress risk. After the end of 2000, the hedging attribute of corporate bonds seems to have weakened and investors wanted to be compensated for this risk.

5.3 Spread decomposition

Once we have estimates of the prices of risk, equation (3) can be used to decompose the spreads of different sectors and rating groups into portions attributable to different risk factors. Before going into the spread decomposition, it is important to keep in mind the interpretation of the factor risk premia:

i) The credit market factor (CMK) measures the overall market view on credit as an asset class, against treasuries. A high credit market risk premium is a sign of greater aversion to credit.

ii) The market-neutral quality premium factor (SHL) measures the preference of credit investors for quality. A high premium means that investors are asking for a high compensation for going towards lower-rated credits.

iii) The market-neutral slope factor (DHL) measures the maturity preference of credit investors. A high premium means that investors are asking for a high compensation for going towards the long end of the credit curve.

iv) The liquidity factor (LLH) measures the liquidity preference of credit investors. A high premium means that the investors are asking for a high compensation for going towards illiquid issues. A negative liquidity premium means that investors are willing to give up spread to hold less liquid issues.

v) The equity momentum factor (MHL) measures the correlation between the credit and equity market. A negative premium means that investors are willing to pay to get exposure to issuers with relative positive equity momentum and/or avoid issuers with relative negative momentum.

vi) The equity market factor (MKT) measures the overall market view on equity as an asset class. A high equity market risk premium is a sign of greater aversion to equity.

vii) The size factor (SMB) measures the overall market view on the business cycle. The size or business cycle premium is expected to be high during a recession and low during an expansion.

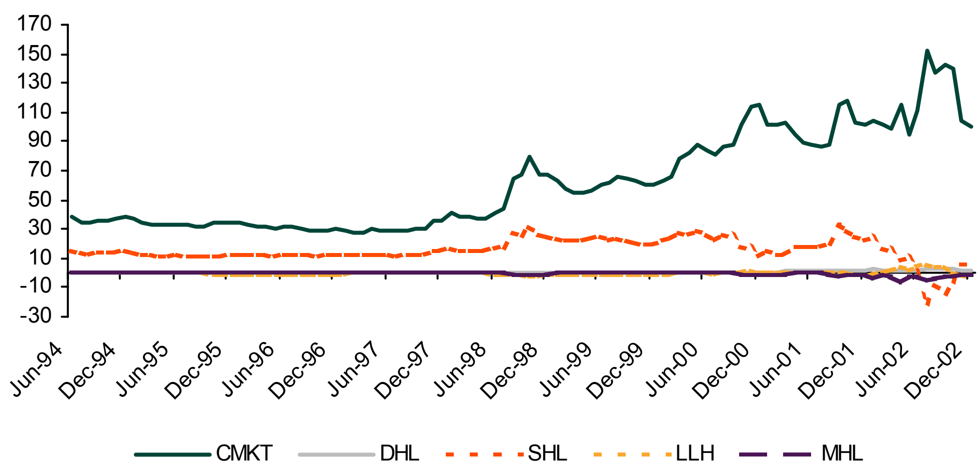
viii) The book-to-market factor (HML) measures the overall market view on financial distress. The book-to-market premium is expected to be high during episodes of high financial stress and high level of bankruptcy.

Below, we provide two examples of spread decomposition. The first example considers the AA sector in the US market and the second example considers the US telecom sector.

5.4 Spread decomposition of the US AA-rated sector

As seen in Figure 19, the aggregate credit market is the most influential component of the AA spread. During most of the sample period, the spread attributable to the quality premium factor is positive. This is explained by the fact that the AA-rated sector has a negative beta with respect to the quality premium factor and the price of risk for this factor is also negative. In the year 2002, the contribution from the quality premium factor drops with the general spread widening trend. Duration, liquidity and equity momentum spread contributions remain small.

Figure 19. US AA-rated sector spread decomposition (bp) June 1994-Dec 2002



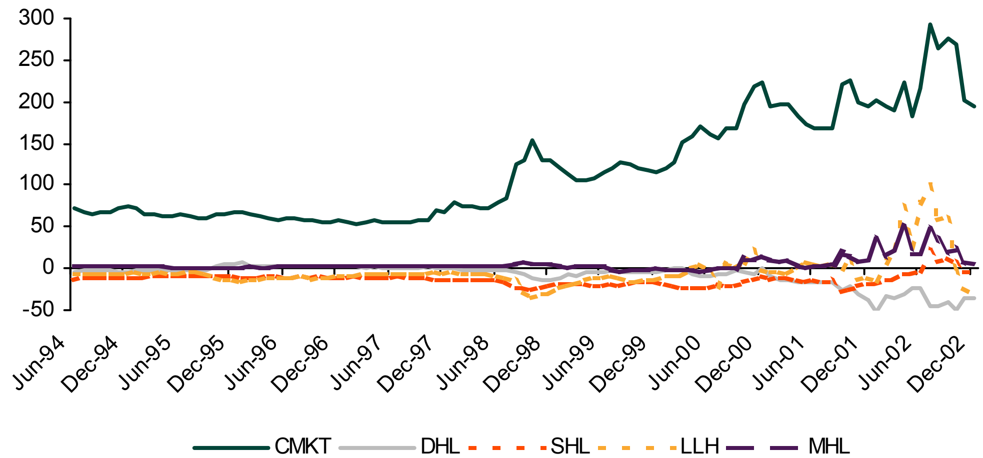
The spread contribution of equity factors is relatively small at this aggregated level. The equity factors do not seem to be the main direct drivers of the AA-rated corporate bond market.

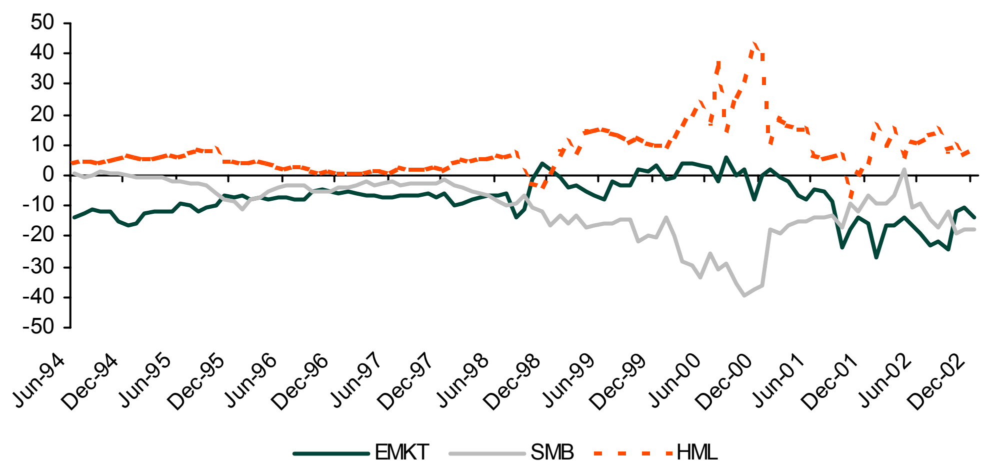
5.5 Spread decomposition of the US telecom sector

In Figure 20, we see that the US telecom sector is very sensitive to the credit market. In the past two years, the liquidity and equity momentum factors have contributed positively to telecom spreads. Liquidity seems to have been penalized during that period when investors moved away from large issues to avoid market volatility. Towards the end of 2002, the liquidity premium normalized and gave a negative contribution to the sector, as one would expect in normal market conditions.

Compared with other sectors, the telecom sector is very sensitive to the equity market. The negative sensitivity to equity markets, however, is a sample-specific result. The telecom sector receives a positive spread contribution from the financial distress factor (HML factor) but a negative contribution from the business cycle factor (SMB factor). It was perceived as a defensive sector in an economic downturn, but is sensitive to financial distress and the equity market.

Figure 20. US telecom sector spread decomposition (bp) June 1994-Dec 2002





6. POTENTIAL APPLICATION: THE CFM ALPHA SCORE

Applications of the framework we have discussed here are numerous. The spread decomposition discussed earlier provides the spread component explained by the systematic factors. For asset selection, it is possible to use the model to come up with an estimate of excess spreads not attributable to any systematic factors. This residual specific to an issuer (or sector) can be used as a relative indicator for the cheapness or richness of the spread.

In this computation, it is necessary to control for the systematic risk of the issuer. In a diversified portfolio, an appropriate measure is the CFM Alpha Score defined by a formula similar to the generalized Treynor ratio[13].

If the benchmark is the whole credit market (the CMK factor), the score that measures the risk-adjusted excess spread relative to the credit market is:

$$\text{CFMAlphaScore} = \alpha\left[\frac{\pi_{CMKt}}{\beta'\pi_t}\right]$$

where α is the excess spread of the issuer not explained by our systematic factors. β is the vector of factor exposures. π_t is the vector of the factor prices of risk. π_{CMKt} is the price of risk of the credit market (the CMK factor).

The ratio $\left[\frac{\pi_{CMKt}}{\beta'\pi_t}\right]$ is the risk-adjustment used to control for the systematic risk embedded in the issuer relative to the benchmark. If the issuer has the same level of systematic risk as the market then this ratio is exactly 1. If systematic risk is higher (lower), this ratio is lower (higher) than 1 and the excess spread is adjusted downward (upward).

We can then compute CFM Alpha Scores for individual issuers, sectors, rating categories, credit portfolios or segment of credit portfolios. The formula is flexible enough to be applied with a different benchmark as long as the factor exposures of the new benchmark are used.

In Figure 21, we report the CFM Alpha Scores for the end of December 2002. In that period, the best rating category seemed to be BBB and the best sectors seemed to be the telecoms, financials and consumer non-cyclicals.

Figure 21. US CFM Alpha Scores (bp) December 2002

Sector	**CFM Alpha**
A	16
AA	-15
BBB	38

Sector	**CFM Alpha**
BANKING	-23
BASIC INDUSTRIES	24
COMMUNICATIONS	267
CONSUMER CYCLICALS	-26
CONSUMER NON-CYCLICALS	36
ENERGY	1
FINANCIALS	45
UTILITIES	-29

[13] *See eg, Hubner (2003)*

In future research, we will examine if investment strategies contingent on the above CFM Alpha Score do indeed generate superior risk-adjusted returns.

7. CONCLUSION

We have proposed a credit factor model for bond excess returns that is useful to understand the drivers of corporate bond excess returns, the components of the risk premium, and the segmentation between the equity and the credit markets. Decomposing the risk premium is key to uncovering the source of outperformance in the credit market as well as the source of risk that the premium is compensating for. We have identified a quality premium factor, a slope factor, a liquidity factor, an equity correlation factor, an equity market factor, a business cycle factor and a financial distress factor as factors to be used in the CFM model. The understanding of the factor exposures to these economy-wide factors is useful both to assess the fair valuation of spreads and for asset and sector selection. We suggest a CFM Alpha Score that provides information regarding the risk-adjusted potential outperformance of any securities or portfolios in the corporate bond market against any benchmarks. We plan to investigate in more details the performance of investment strategies based on the CFM Alpha Score.

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Valuation of Credit Default Swaps

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We present the market standard pricing model for marking credit default swap positions to market. Our aim is first to explain why credit default swaps require a valuation model, and then to explain the standard model – the one most widely used in the market. In the process of setting out the model, we take care to explain and justify the various modeling assumptions made. We also provide examples.

1 INTRODUCTION [1]

The credit default swap is a simple derivative contract that has revolutionized the trading of credit risk. Over the past five years it has become the most widely used credit derivative product, representing about 72.5% of a total outstanding market notional currently estimated to be around $2.3 trillion[2]. The default swap market is truly global, with contracts linked to the credit risk of a wide array of US, European and Asian corporate names as well as to a number of sovereigns.

The point of this paper is to present a complete and practical exposition of the **market standard model** and so help those new to credit derivatives to be able to value default swap positions. We intend to publish a more complete study of the valuation and risk management of credit default swaps shortly and we refer the reader to that for many of the technical details omitted from this abridged paper.

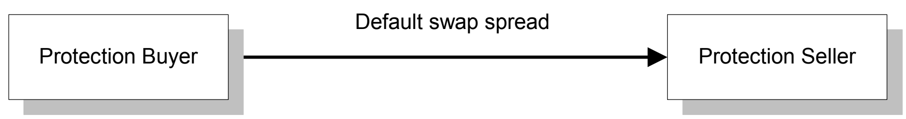
2 THE CREDIT DEFAULT SWAP

Credit default swaps (CDS) have been explained in detail elsewhere[3]. In brief, a CDS is used to transfer the credit risk of a reference entity (corporate or sovereign) from one party to another. In a standard CDS contract one party purchases credit protection from another party, to cover the loss of the face value of an asset following a **credit event**. A credit event is a legally defined event that typically includes bankruptcy, failure-to-pay and restructuring. This protection lasts until some specified maturity date. To pay for this protection, the protection buyer makes a regular stream of payments[4], known as the **premium leg**, to the protection seller as shown in Figure 1. This size of these premium payments is calculated from a quoted **default swap spread** which is paid on the face value of the protection. These payments are made until a credit event occurs or until maturity, whichever occurs first.

1 *We thank Arthur Berd, Jordan Mann, Marco Naldi, Lutz Schloegl and Minh Trinh for comments and suggestions.*
2 Risk Magazine Credit Derivatives Survey, *February 2003.*
3 *See* Credit Derivatives Explained, *Lehman Brothers Fixed Income Research, March 2001.*
4 *The normal frequency of payments is quarterly, although payments can be monthly or semi-annual.*

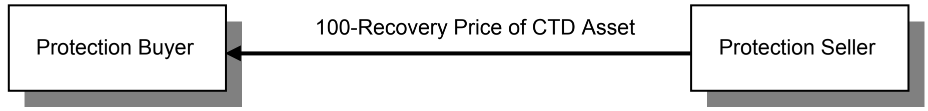
Figure 1. Mechanics of a default swap premium leg

Between trade initiation and default or maturity, protection buyer makes regular payments of default swap spread to protection seller



If a credit event does occur before the maturity date of the contract, there is a payment by the protection seller, known as the **protection leg**. This payment equals the difference between par and the price of the cheapest to deliver[5] (CTD) asset of the reference entity on the face value of the protection and compensates the protection buyer for the loss. It can be made in cash or physically settled format. This is shown in Figure 2.

Figure 2. The protection leg following a credit event



Example

Suppose a protection buyer purchases 5-year protection on a company at a default swap spread of 300bp. The face value of the protection is \$10 million. The protection buyer therefore makes quarterly payments approximately[6] equal to \$10 million × 0.03 × 0.25 = \$75,000. Assume that after a short period the reference entity suffers a credit event and that the CTD asset of the reference entity has a recovery price of \$45 per \$100 of face value. The payments are as follows:

- The protection seller compensates the protection buyer for the loss on the face value of the asset received by the protection buyer. This is equal to \$10 million × (100% – 45%) = \$5.5 million.
- The protection buyer pays the accrued premium from the previous premium payment date to time of the credit event. For example, if the credit event occurs after a month then the protection buyer pays approximately \$10 million × 0.03 × 1/12 = \$18,750 of premium accrued. Note that this is the standard for corporate reference entity linked default swaps. For sovereign-linked default swaps there may be no payment of premium accrued.

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[5] *The protection buyer in a CDS specified with Physical Delivery has the option to choose the cheapest asset to deliver into the protection in return for payment of the face value in cash. The cash settled default swap has the same economic value at a credit event but is settled in cash.*

[6] *The exact payment amount is a function of the calendar and basis convention used.*

3. COMPUTING THE MARK-TO-MARKET VALUE

Unlike bonds, the gain or loss from a CDS position cannot be computed simply by taking the difference between current market quoted price plus the coupons received and the purchase price. To value a CDS we need to use a term structure of default swap spreads, a recovery rate assumption and a model.

To see this, consider an investor who initially buys 5-year protection on a company at a default swap spread of 60bp and then wishes to value the position after one year. On that date the 4-year credit default swap spread quoted in the market is 170bp. What is the current value of the position? This is given by

MTM = Current Market Value of Remaining 4-year Protection

– Expected Present Value of 4-year Premium Leg at 60bp

The first observation is that the investor has a CDS contract that has increased in value since he is paying only 60bp for something for which the market is now willing to pay 170bp. As the mark-to-market value of a new default swap is zero, this implies that

Current Market Value of Remaining 4-year Protection

= Expected Present Value of Premium Leg at 170bp

Using this knowledge, we can write that the market-to-market value to the protection buyer is

MTM = Expected Present Value of 4-year Premium Leg at 170bp

– Expected Present Value of 4-year Premium Leg at 60bp

If we define the **Risky PV01** (RPV01) as the expected present value of 1bp paid on the premium leg until default or maturity, whichever is sooner, then we can rewrite the MTM as

$$\text{MTM} = 170\text{bp} \times \text{Risky PV01} - 60\text{bp} \times \text{Risky PV01} = 110\text{bp} \times \text{Risky PV01}. \quad (1a)$$

Hence we need to calculate the Risky PV01. The Risky PV01 is called "risky" because it is the expected present value of an uncertain stream of premia. The uncertainty is due to the fact that the premia payments terminate if there is a credit event.

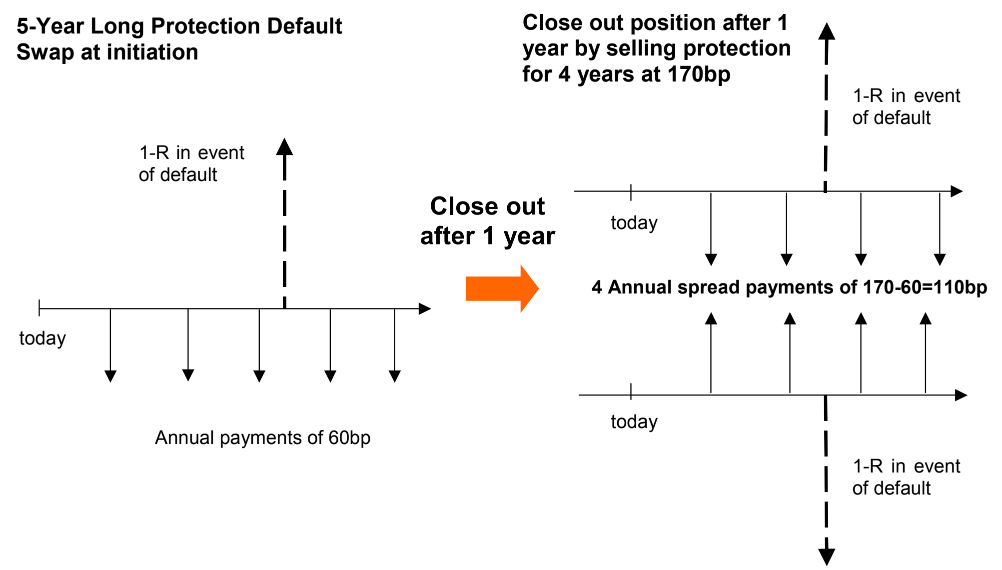
To realize this mark-to-market gain or loss, the investor has two choices :

i. Unwind it with the initial counterparty (or have it reassigned to another counterparty) for a cash unwind value. The cash unwind value should equal the MTM of the position.

ii. Enter into the offsetting position in which the investor sells protection on the same reference entity for the next four years at 170bp as shown in Figure 3. This creates a positive premium income of 170 – 60 = 110bp per annum until a credit event or maturity, whichever occurs sooner. If there is a credit event, the investor has no principal risk since the defaulted bond delivered on one side can be delivered into the protection bought and similarly the payments of face value are exchanged. While the investor has no principal risk, there is still a premium risk. The risk is that the reference entity does not survive until the maturity date of the contract and that the four years of 110bp of annual income are not received. These cash flows are risky, and this risk must be accounted for by the Risky PV01 which effectively discounts the cash flows at a spread over Libor.

Both choices have the same economic value today. However they are different. In case (i) the P&L is realised immediately and the position is terminated. In case (ii) the P&L is only realised over the remaining life of the swap and the investor is taking the risk that a credit event occurs and the realised P&L is less than that they would have achieved if they had unwound the position for a cash amount. On the other hand, if no credit event occurs, and the net spread income is positive, they will receive more than the cash unwind value.

Figure 3. A protection buyer who seeks to value a 5-year protection position after one year



The present value of a position initially traded at time t_0 at a contractual spread of $S(t_0, t_N)$, with maturity t_N and which has been offset at valuation time t_V with a position traded at a spread of $S(t_V, t_N)$ is given by

$$MTM(t_V, t_N) = \pm[S(t_V, t_N) - S(t_0, t_N)] \times RPV01(t_V, t_N) \tag{1b}$$

where the positive sign is used for a long protection position and a negative sign for a short protection position. $RPV01(t_V, t_N)$, known as the risky PV01, is the present value at time t_V of a 1bp premium stream which terminates at maturity time t_N or default, whichever sooner. This is identical to equation (1a) derived above and shows that the value of both investor choices (i) and (ii) are equal.

The calculation of the Risky PV01 requires a model because we need to take into account the riskiness of each premium payment by calculating the probability of the reference entity surviving to each premium payment date. These survival probabilities used in the valuation of the Risky PV01 must be the **arbitrage-free survival probabilities**. These are the survival probabilities that are implied by the market default swap spreads. A valuation model is therefore needed to calculate these survival probabilities from market default swap spreads. Such a model must:

i. Capture the risk of default of the reference entity;

ii. Model payment of the recovery rate as a percentage of the face value;

iii. Be able to model the timing of the default (especially important as the value of a default swap is the present value - all payments must be discounted to today),

iv. Be flexible enough to refit the term structure of quoted default swap spreads – the model should not generate any arbitrages;

v. Be as simple as possible. A model that makes fewer assumptions but requires greater implementation effort, and that produces spreads that differ by an amount well within the bid-offer spread should be rejected in favor of the simpler model.

A model that allows this to be done is described next.

4. MODELING CREDIT USING A REDUCED-FORM APPROACH

The world of credit modelling is divided into two main approaches, one called the structural and the other called the reduced-form. In the **structural** approach, the idea is to characterize the default as being the consequence of some company event such as its asset value being insufficient to cover a repayment of debt. Such models are usually extensions of Merton's 1974 firm-value model – see O'Kane and Schloegl (2001) for a more complete discussion – that used a contingent claims analysis for modeling default. Structural models are generally used to say at what spread corporate bonds *should* trade based on the internal structure of the firm. They therefore require information about the balance sheet of the firm and can be used to establish a link between pricing in the equity and debt markets. However, they are limited in at least three important ways: they are hard to calibrate because internal company data is only published at most four times a year; they generally lack the flexibility to fit exactly a given term structure of spreads; and they cannot be easily extended to price credit derivatives.

In the **reduced-form** approach, the credit event process is modeled directly by modeling the probability of the credit event itself. Using a security pricing model based on this approach, this probability of default can be extracted from market prices. Reduced form models also generally have the flexibility to refit the prices of a variety of credit instruments of different maturities. They can also be extended to price more exotic credit derivatives. It is for these reasons that they are used for credit derivative pricing.

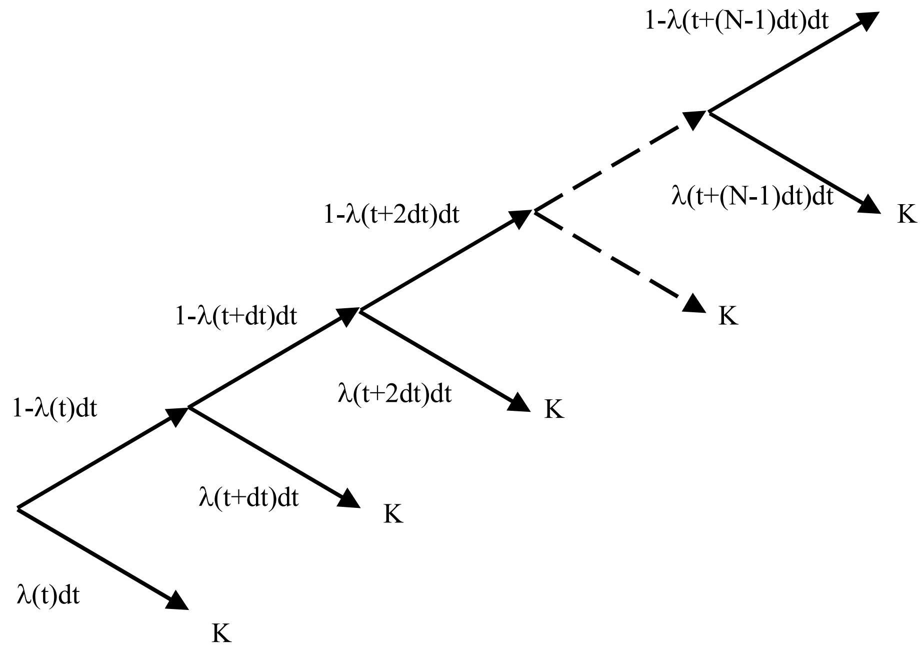
The most widely used reduced-form approach is based on the work of Jarrow and Turnbull (1995), who characterize a credit event as the first event of a Poisson counting process which occurs at some time τ with a probability defined as

$$\Pr[\tau < t + dt \mid \tau \geq t] = \lambda(t)dt \tag{2}$$

ie, the probability of a default occurring within the time interval [t,t+dt) conditional on surviving to time t, is proportional to some time dependent function $\lambda(t)$, known as the *hazard rate*, and the length of the time interval dt. We can therefore think of modelling default in a one-period setting as a simple binomial tree in which we survive with probability $1-\lambda(t)dt$ or default and receive a recovery value R with probability $\lambda(t)dt$.

We make the simplifying assumption *that the hazard rate process is deterministic*. By extension, this assumption also implies that the hazard rate is independent of interest rates and recovery rates. Towards the end of this paper we will discuss the validity of this assumption in the context of pricing. All we say now is that for almost all market participants, these assumptions are acceptable as their pricing impact is well within the typical bid-offer spread for credit default swaps.

Figure 4. The equivalent of a binomial tree in the modeling of default in which the tree terminates and makes a payment K at default



We can extend this model to multiple time periods, as shown pictorially in Figure 4, where K is the payoff at the time of a default. We can compute the **continuous time** survival probability to time T conditional on surviving to time t_V by considering the limit $dt \rightarrow 0$. It can be shown that the survival probability is given by

$$Q(t_V, T) = \exp\left(-\int_{t_V}^{T} \lambda(s)ds \right). \tag{3}$$

In the following sections we will first show how to use this model to value both the premium and protection legs, and hence the breakeven spread of a default swap. We can then use this model to imply the term structure of arbitrage-free survival probabilities from market spreads. These will then be used to mark our existing position to market.

5. VALUING THE PREMIUM LEG

The premium leg is the series of payments of the default swap spread made to maturity or to the time of the credit event, whichever occurs first. It also includes the payment of premium accrued from the previous premium payment date until the time of the credit event. Assume that there are n=1,... ,N contractual payment dates t_1,... ,t_N where t_N is the maturity date of the default swap. Denoting the t_N maturity contractual default swap spread by $S(t_0, t_N)$, and ignoring premium accrued, we can write the present value of the premium leg of an existing contract as

$$\text{Premium Leg PV}(t_V, t_N) = S(t_0, t_n) \sum_{n=1}^{N} \Delta(t_{n-1}, t_n, B) Z(t_V, t_n) Q(t_V, t_n)$$

where

- $\Delta(t_{n-1},t_n,B)$ is the day count fraction between premium dates t_{n-1} and t_n in the appropriate basis convention denoted by B.
- $Q(t_V\ ,t_n)$ is the arbitrage-free survival probability of the reference entity from valuation time t_V to premium payment time t_n. This factors into the pricing the risk that a reference entity will not survive to a premium payment time.
- $Z(t_V,t_n)$ is the Libor discount factor from valuation date to premium payment date n. In what follows, we have assumed that the user has been able to bootstrap a full term structure of Libor discount factors, Z(t,T) in the currency of the default swap being priced.

This equation ignores the effect of premium accrued – the fact that upon a credit event the contract will usually require the protection buyer to pay the fraction of premium that has accrued from the previous premium payment date to the time of credit event.

To include the effect of premium accrued, we have to work out the expected accrued premium payment by considering the probability of defaulting at each time between two premium dates, and calculating the probability weighted accrued premium payment. To do this, we have to

1. Consider each premium accrual period starting at t_{n-1} with the payment date at t_n.
2. Determine the probability of surviving from the valuation date t_V to each time s in the premium period and then defaulting in the next small time interval ds. The probability of this is given by $Q(t_V,s)\lambda(s)ds$.
3. Calculate the accrued payment since the previous premium date to each time.
4. Discount this payment back to the valuation date using the Libor discount factor.
5. Integrate over all times in the premium period. Strictly speaking this is a discrete daily integration since premium payments are only calculated on a daily basis. However for mathematical simplicity we tend to approximate this as a continuous integral. The difference is essentially negligible in this context,
6. Sum over all premium periods from n=1 to the final premium n=N.

The resulting expression for the premium accrued is given by

$$S(t_0,t_N)\sum_{n=1}^{N}\int_{t_{n-1}}^{t_n}\Delta(t_{n-1},s,B)Z(t_V,s)Q(t_V,s)\lambda(s)ds$$

The integral makes this is a complicated expression to evaluate exactly. However, as we demonstrate in O'Kane and Turnbull (2003), it is possible to approximate this equation with

$$\frac{S(t_0,t_N)}{2}\sum_{n=1}^{N}\Delta(t_{n-1},t_n,B)Z(t_V,t_n)(Q(t_V,t_{n-1})-Q(t_V,t_n))$$

by noting that if a default does occur between two premium dates, the average accrued premium is half the full premium due to be paid at the end of the premium period. The full value of the premium leg is then given by

$$S(t_0,t_N)\times RPV01 \tag{4}$$

where RPV01 is the risky PV01 defined as

$$RPV01 = \sum_{n=1}^{N} \Delta(t_{n-1}, t_n, B) Z(t_V, t_n) \left[Q(t_V, t_n) + \frac{1_{PA}}{2} \left(Q(t_V, t_{n-1}) - Q(t_V, t_n) \right) \right] \quad (5)$$

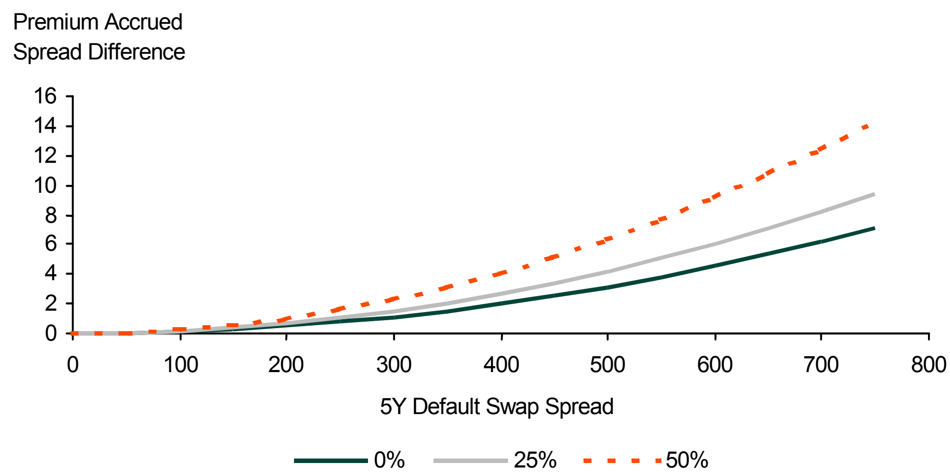
where $1_{PA}=1$ if the contract specifies premium accrued (PA) and 0 otherwise.

The effect of premium accrued on the breakeven spread is typically small, although not negligible, and we plot it in Figure 5 as a function of the default swap spread level for a recovery rate assumption of 40%. The effect of premium accrued on the spread can be very well approximated by

$$\frac{S^2}{2(1-R)f}$$

where f is the frequency of payments on the premium leg.

Figure 5. The effect of premium accrued shown by computing the difference between the breakeven default swap spread with and without premium accrued using a full model. We show results for a 0%, 25% and 50% recovery rate assumption.



The breakeven spread with premium accrued has to be less than that without premium accrued as a protection buyer will want to pay a lower spread in order to offset the possible extra accrued payment if there is a credit event. For a contract with a credit default swap spread of 200bp and an expected recovery rate of 40%, the change in spread due to premium accrued on a quarterly-paying default swap is approximately equal to 0.83bp. This is well inside typical bid-offers for names that trade at this spread level. However, for wide spread names, this difference, which is quadratic in the spread, cannot be ignored.

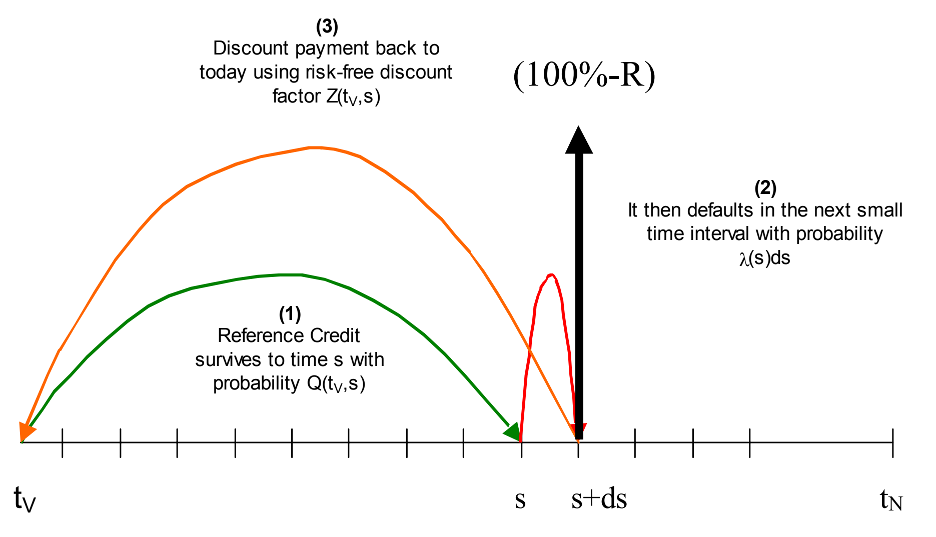
6. VALUING THE PROTECTION LEG

The protection leg is the contingent payment of (100% – R) on the face value of the protection made following the credit event. R is the expected recovery rate – to be precise, it is the expected price of the CTD obligation into the protection at the time of a credit event. There may be a delay of up to 72 calendar days between notification of the credit event and settlement of the protection leg payment, but we typically assume that this payment is made immediately.

In pricing the protection leg, it is important to take into account the timing of the credit event because this can have a significant effect on the present value of the protection leg – especially for longer maturity default swaps. Within the hazard rate approach we can solve this timing problem by conditioning on each small time interval [s,s+ds] between time t_V and time t_N at which the credit event can occur. The steps are described below.

1. Calculate the probability of surviving to some future time s which equals $Q(t_V,s)$
2. Compute the probability of a credit event in the next small time increment ds which is given by $\lambda(s).ds$.
3. At this point an amount (100% – R) is paid, and we discount this back to today at the risk-free rate $Z(t_V,s)$.
4. We then consider the probability of this happening at all times from $s = t_V$ to the maturity date t_N. Strictly speaking the timing of a credit event should not be resolved to less than a day. However, assuming that a credit event can occur intra-day has almost no effect on the valuation while simplifying the exposition.

Figure 6. Steps in the calculation of the expected present value of a recovery rate which is paid at the time of a credit event



These steps are also shown in Figure 6. As a result, we calculate the expected present value of the recovery payment as

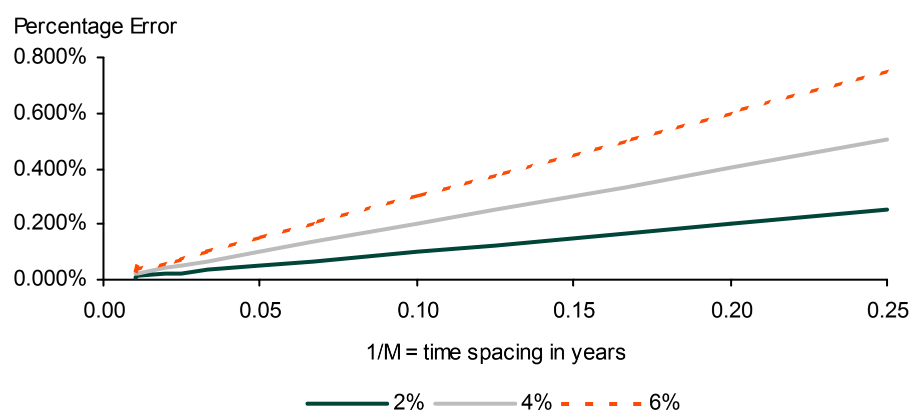
$$(1-R)\int_{t_V}^{t_N} Z(t_V,s)Q(t_V,s)\lambda(s)ds \qquad (6)$$

where R is the expected recovery price of the CTD asset at the time of the credit event. The integral, makes this expression tedious to evaluate. It is possible to show that we can, without any material loss of accuracy, simply assume that the credit event can only occur on a finite number M of discrete points *per year*. For a t_N maturity default swap, we have $M\times t_N$ discrete times which we label as m=1,... ,$M\times t_N$. We then have

$$(1-R)\sum_{m=1}^{M\times t_N} Z(t_V,t_m)(Q(t_V,t_{m-1})-Q(t_V,t_m)) \tag{7}$$

The lower the value of M, the fewer calculations we have to do. However it also means that the accuracy is reduced. In terms of spread change, we show in O'Kane and Turnbull (2003) that for a flat hazard rate structure, the percentage difference in the computed spread between the continuous and discrete case is given by r/2M where r is the continuously compounded default free interest rate. The quality of this approximation is demonstrated in Figure 7 for different values of M and r. For example, assuming that r=3% and M=12 (corresponding to monthly intervals) we have a percentage error in the spread of 0.125%, i.e., an absolute error of 1bp on a spread of 800bp compared with the continuous case. This level of accuracy is well inside the typical bid-offer spread. This is encouraging as it means that the model can be fast, simple *and* accurate.

Figure 7. Model-based calculation of the percentage discretization error on protection leg PV shown as a function of 1/M, the interval time spacing



7. CALIBRATING EXPECTED RECOVERY RATES

One of the required inputs which we have not yet discussed is the recovery rate R. Unlike the spread or interest rate term structure, this is not a market observable input. The expected recovery rate R is not the expected value of the asset following the post-default workout process. Instead it is the price of the CTD asset, expressed as a percentage of face value, within approximately 72 calendar days after notification of the credit event. This is similar to the definition used by rating agencies such as Moody's for their recovery rate statistics. However, there are a number of caveats with rating agency recovery statistics: (i) rating agencies do not view restructuring as a default while standard default swaps do; (ii) they are heavily biased towards US corporates, because that is where the greatest amount of default data originates – see Figure 8 – and so may not be appropriate for European names; (iii) they are historical, not forward looking, and so fail to take into account market expectations about the future; (iv) they are not name or sector specific. Despite these, for good quality investment grade credits, most dealers use the rating agency recovery rate data as a starting point.

These typically show the average recovery rate by seniority and type of credit instrument, and usually focus on a US corporate bond universe. Adjustments may be made for non-US corporate names and for certain industrial sectors.

Using a valuation model to extract information about the recovery value from bond prices may be one way to overcome this calibration problem. However, this is difficult for good quality names because the low default probability means that the recovery rate is only a small component of the bond price and of the same order of magnitude as any bid-offer spread. However, this is not the problem it may seem because as we will show in our longer paper (O'Kane and Turnbull 2003), the sensitivity of the mark-to-market of a default swap to the recovery rate assumption is very low for low spread levels. For much lower credit quality names the recovery rate sensitivity is much higher, and our hope is that the lower bond prices begin to reveal more information about market expectations for the future recovery rates.

Figure 8. Average defaulted debt recovery rate estimates Europe vs US, 1985-2001 (percentage of face)

	Europe		US	
Seniority	**Average**	**Count**	**Average**	**Count**
Bank Loan (Sr. Secured)	71.8%	4	66.8%	201
Sr. Secured	55.0%	1	56.9%	150
Sr. Unsecured	20.8%	28	50.1%	565
Sr. Sub	24.0%	4	32.9%	359
Sub	13.0%	1	31.3%	342
All Bonds	22.0%	34	42.8%	1,416

Source: Moody's Investor Services

8. CALCULATING THE BREAKEVEN DEFAULT SWAP SPREAD

We have now presented a model that values the protection and premium legs of a CDS. The next step is to work out the survival probabilities from the market quoted default swap spread. This is the breakeven spread, and is given by

PV of Premium Leg = PV of Protection Leg.

For a new contract we have $t_V = t_0$ so that substituting from equation (5) and equation (7) and rearranging, we get

$$S(t_V, t_N) = \frac{(1-R)\sum_{m=1}^{M \times t_N} Z(t_V, t_m)[Q(t_V, t_{m-1}) - Q(t_V, t_m)]}{RPV01} \qquad (9)$$

for the breakeven spread where the RPV01 has been defined in equation (5).

Now we have a direct relationship between the default swap spread quoted in the market and the survival probabilities it implies. However, this is still not sufficient to enable us to extract all of the required survival probabilities. To see this, consider the case of a 1Y CDS which has a quoted spread of 85bp. Assuming quarterly payments on the premium leg, a monthly discretization frequency (M=12), and premium accrued, we can rewrite equation (9) as

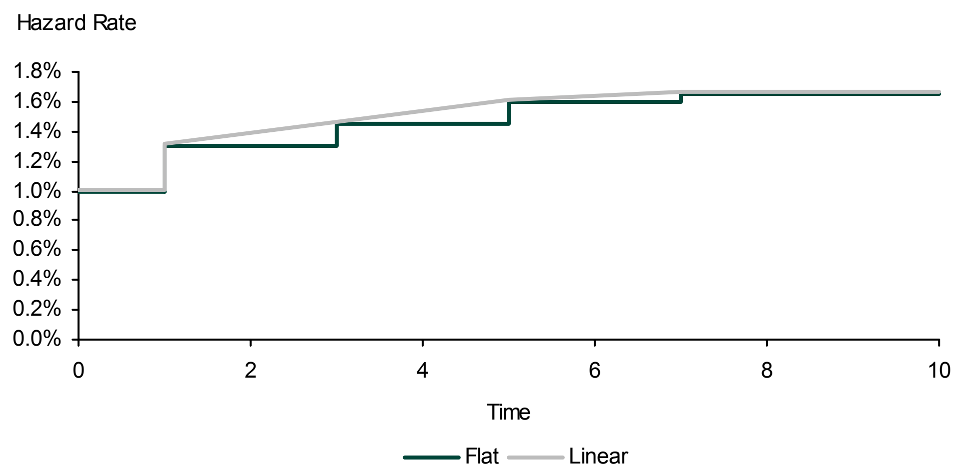
$$0.0085 = \frac{(1-R)\sum_{m=1}^{12} Z(t_V, t_m)(Q(t_V, t_{m-1}) - Q(t_V, t_m))}{(1/2)\sum_{n=3,6,9,12} \Delta(t_{n-3}, t_n, B) Z(t_V, t_n)(Q(t_V, t_{n-3}) + Q(t_V, t_n))}$$

In this equation we know all of the accrual factors, we can make an assumption about the recovery rate R, and we can calculate all of the Libor discount factors from the Libor discount curve. All we then need to know are a maximum[7] of 12+4=16 survival probabilities. Clearly this one equation cannot give all of these survival probabilities. We therefore need to make a simplifying assumption about the term structure of survival probabilities.

9. BUILDING A HAZARD RATE TERM STRUCTURE

The standard modeling assumption used in the credit default swap market is to assume that the hazard rate is a piecewise flat function of maturity time. This is an entirely reasonable assumption because, given only one data point, it is not possible to extract more than one piece of information about the term structure of hazard rates.

Figure 9. Given market default swap spreads at 1Y, 3Y, 5Y, 7Y and 10Y, the simplest assumption is of a piecewise flat hazard rate term structure. One can also assume a flat then linear term structure



One could go a stage further and assume that the curve is piecewise linear. However, this only makes a difference if (i) we do not have quoted spreads for many maturities *and* (ii) the curve is steeply sloped. This is generally not the case because most names only have liquidity at the 5-year default swap and the curve is therefore assumed to be flat. The main exception is when we have an inverted spread curve, usually associated with a distressed credit. In this case we usually have more market spreads, especially at short maturities. As a result we will keep the model simple by assuming a piecewise flat structure, although we note that the generalization to a linear scheme is actually fairly straightforward.

[7] *This is an upper bound because some of the premium dates may coincide with those used in the discretization for the calculation of the protection leg value.*

Given 1Y, 3Y, 5Y, 7Y and 10Y default swap spread values, we would assume that we have a hazard rate term structure with five sections $\lambda_{0,1}$, $\lambda_{1,3}$, $\lambda_{3,5}$, $\lambda_{5,7}$ and $\lambda_{7,10}$, as shown in Figure 9.

The process of constructing the term structure of hazard rates is an iterative one commonly known as **bootstrapping**. It starts with taking the shortest maturity contract and using it to calculate the first survival probability. In this case, the 1Y default swap has to be used to calculate the value of $\lambda_{0,1}$. Assuming a quarterly premium payment frequency, using a value of M=12, and assuming that premium accrued is not paid, this is achieved by solving

$$\frac{S(t_V, t_V + 1Y)}{1-R} \sum_{n=3,6,9,12} \Delta(t_{n-3}, t_n, B) Z(t_v, t_n) e^{-\lambda_{01}\tau_n} = \sum_{m=1}^{12} Z(t_v, t_m)(e^{-\lambda_{01}\tau_{m-1}} - e^{-\lambda_{01}\tau_m})$$

for the value of $\lambda_{0,1}$. Our (M=12) monthly discretization means that

$$\tau_0 = 0.0, \tau_1 = 0.0833, \tau_2 = 0.167, \tau_3 = 0.25, ..., \tau_{12} = 1.00$$

Such an equation can be solved using a one-dimensional root-searching algorithm[8]. This procedure is then repeated to solve for $\lambda_{1,3}$ and so on until the final maturity default swap is reached. Beyond this, it is often assumed that the hazard rate is flat. Defining $\tau = T - t_V$, we have

$$Q(t_V, T) = \begin{cases} \exp(-\lambda_{0,1}\tau) & \text{if } 0 < \tau \le 1 \\ \exp(-\lambda_{0,1} - \lambda_{1,3}(\tau - 1)) & \text{if } 1 < \tau \le 3 \\ \exp(-\lambda_{0,1} - 2\lambda_{1,3} - \lambda_{3,5}(\tau - 3)) & \text{if } 3 < \tau \le 5 \\ \exp(-\lambda_{0,1} - 2\lambda_{1,3} - 2\lambda_{3,5} - \lambda_{5,7}(\tau - 5)) & \text{if } 5 < \tau \le 7 \\ \exp(-\lambda_{0,1} - 2\lambda_{1,3} - 2\lambda_{3,5} - 2\lambda_{5,7} - \lambda_{7,10}(\tau - 7)) & \tau > 7 \end{cases}$$

We have assumed that the hazard rate remains flat beyond the 10Y maturity.

Figure 10. An example of hazard rates fitted to a market curve consisting of 6M, 1Y, 2Y, 3Y, 5Y, 7Y and 10Y spreads. We show the value of the hazard rates for which the model calculated spread equals the market default swap spreads. A recovery rate of 40% was assumed.

Term	Hazard Rate	Market Spread (bp)	Model Spread (bp)	Protection Leg	Risky PV01
6M	1.6832%	100	100	0.4941%	0.4941
1Y	2.0203%	110	110	1.0825%	0.9841
2Y	2.1950%	120	120	2.3061%	1.9218
3Y	3.0838%	140	140	3.9264%	2.8046
5Y	2.8126%	150	150	6.6329%	4.4219
7Y	3.2054%	160	160	9.3720%	5.8575
10Y	3.0386%	165	165	12.7162%	7.7068

An example fitting is presented in Figure 10. This shows the term of each default swap spread, the market spread and the value of the hazard rate for which the model spread and market spread are equal. We also show the present value of the protection leg. Finally, we show that the model spread is the value of the protection leg divided by the Risky PV01.

[8] *Bisection or gradient-based methods such as Newton-Raphson.*

The hazard rates calculated here are the **arbitrage-free** ones. This means that they are the level of hazard rate required by our choice of model in order to fit the market. The arbitrage-free hazard rate is typically much larger than the hazard rate implied by historical data because it includes other non-default factors such as liquidity risk premia, spread risk premia and market supply-demand effects.

When the spread curve is inverted, it is sometimes found that this implies negative hazard rates. This is clearly incorrect as a probability can never be negative. If we believe the model specification and the recovery rate assumption, then the negative probability implies an arbitrage, which can be either model dependent or model independent.

Figure 11. An example of hazard rates fitted to a market curve consisting of 6M, 1Y, 2Y, 3Y, 5Y, 7Y and 10Y spreads. For an inverted curve it is possible to find that the hazard rate is negative. This may reflect an arbitrage in the term structure of credit default swap spreads

Term	Hazard Rates	Market Spread (bp)	Model Spread (bp)	Protection Leg	Risky PV01
6M	13.2731%	800	800	3.7869%	0.4734
1Y	6.5547%	600	600	5.5692%	0.9282
2Y	4.8028%	450	450	8.0081%	1.7796
3Y	**-0.4883%**	300	300	7.7733%	2.5911
5Y	0.5448%	200	200	8.2728%	4.1364
7Y	3.3592%	200	200	11.0672%	5.5336
10Y	3.3593%	200	200	14.6438%	7.3219

An example of a model-independent arbitrage can be seen by considering the calibration in Figure 11, where the market default swap spread term structure is steeply inverted, dropping from 800bp at a 6-month maturity to 200bp at five years. We see that the hazard rate for the 2-3 year period is negative at -0.4883%. This negative hazard rate has been caused by a 2-year spread level at 450bp followed by a 3-year spread level at 300bp.

This is a clear arbitrage – we can buy 3Y protection at 300bp and sell 2Y protection at 450bp. As long as there is no credit event in the first two years, we receive 150bp in both years. We use these to fund the payment of 300bp in the final year. A credit event in the first two years results in a netting of positions – we also keep the net carry to the date of the credit event. A credit event between years two and three results in a windfall gain of (100% − R). As long as interest rates are positive we have a strategy which costs nothing to enter into but which has a payoff greater than or equal to zero in all scenarios (we ignore counter-party risk). Identification of this arbitrage does not require a model.

In other cases a negative hazard rate may imply a **model-dependent arbitrage**. In the above example, the hazard rate only becomes positive once the 3Y default swap spread is greater than 310bp, not 300bp. The model is saying is that this extra 10bp per year is the value of the gain from (i) the positive carry if there is a credit event at any time, and (ii) payment of (100% − R) if there is a credit event between years two and three. This threshold of 310bp therefore depends on the valuation model, the assumed recovery rate of 40% and the term structure of interest rates.

10. COMPUTING THE MARK TO MARKET

In this section we compute the mark-to-market of a default swap position when the valuation date is on a premium payment date. This simplifies things because it avoids a discussion of accrued interest and the treatment of premium accrued. We refer the reader to the longer version for the discussion of these issues. We remind ourselves that the full mark-to-market of a long protection default swap position is given by

$$MTM(t_V, t_N) = \pm[S(t_V, t_N) - S(t_0, t_N)] \times RPV01(t_V, t_N).$$

where

$$RPV01 = \sum_{n=1}^{N} \Delta(t_{n-1}, t_n, B) Z(t_V, t_n) \left[Q(t_V, t_n) + \frac{1_{PA}}{2}(Q(t_V, t_{n-1}) - Q(t_V, t_n)) \right].$$

$1_{PA}=1$ if premium accrued is part of the contract and 0 otherwise. The value of the current market spread to maturity $S(t_V,t_N)$ is given by solving

$$(1-R) \sum_{m=1}^{M \times t_N} Z(t_V, t_m)(Q(t_V, t_{m-1}) - Q(t_V, t_m)) = S(t_V, t_N) \times RPV01(t_V, t_N)$$

A full example calculation of a default swap mark-to-market is shown in Figure 12 for a \$10M long protection position initially transacted at a contractual spread of 200bp. The survival probabilities and discount factors are also shown. The risky PV01 is also easy to compute given that we now have all the discount factors, survival probabilities and accrual factors.

The protection was bought at 200bp, spreads have tightened and the current market spread for the remaining term is now worth 142.7bp, according to the model. Note that the current breakeven spread is a model-based interpolation between the 4Y and 5Y CDS spreads of 140bp and 150bp, but closer to the former given that the existing contract has a remaining maturity of four years and three months. The mark-to-market is negative at -\$223K.

Figure 12. Results of model calculation of the CDS mark-to-market.

LIBOR Rates	6M	1.35	Default Swaps	1Y	110bp
	1Y	1.43		2Y	120bp
	2Y	1.90		3Y	130bp
	3Y	2.47		4Y	140bp
	4Y	2.936		5Y	150bp
	5Y	3.311		Recovery Rate R	40%

Notional	$10,000,000	**Frequency**	Quarterly
Contractual Spread	200bp	**Basis**	Actual 360
Effective Date	20 June 2002	**Calendar**	USD
Maturity Date	20 Sep 2007	**Premium Accrued**	Yes

Valuation Date: 19 June 2003

Payment Dates	Day Count	Actual Flows	Survival Probability	Libor discount factor
Mon 22 Sep 2003	0.261111	52,222.22	99.567%	0.99649
Mon 22 Dec 2003	0.252778	50,555.56	99.150%	0.99311
Mon 22 Mar 2004	0.252778	50,555.56	98.657%	0.98953
Mon 21 Jun 2004	0.252778	50,555.56	98.164%	0.98583
Mon 20 Sep 2004	0.252778	50,555.56	97.628%	0.98084
Mon 20 Dec 2004	0.252778	50,555.56	97.092%	0.97523
Mon 21 Mar 2005	0.252778	50,555.56	96.559%	0.96899
Mon 20 Jun 2005	0.252778	50,555.56	96.030%	0.96218
Tue 20 Sep 2005	0.255556	51,111.11	95.420%	0.95450
Tue 20 Dec 2005	0.252778	50,555.56	94.815%	0.94630
Mon 20 Mar 2006	0.250000	50,000.00	94.220%	0.93754
Tue 20 Jun 2006	0.255556	51,111.11	93.616%	0.92800
Wed 20 Sep 2006	0.255556	51,111.11	92.934%	0.91879
Wed 20 Dec 2006	0.252778	50,555.56	92.259%	0.90931
Tue 20 Mar 2007	0.250000	50,000.00	91.597%	0.89946
Wed 20 Jun 2007	0.255556	51,111.11	90.924%	0.88899
Thu 20 Sep 2007	0.255556	51,111.11	90.173%	0.87902

Risky PV01	3.899	**Breakeven Spread**	142.7
PV of Protection	$557,872	**Full Mark-to-Market**	-$223,516

11. CONCLUSIONS

This paper has set out the standard valuation model adopted by the credit derivatives market for the valuation of credit default swaps. Despite being quite simple, the model manages to capture all of the market risks of the credit default swap.

A number of simplifying assumptions have been made. For example, we have ignored interest rate and credit correlations. However, the extremely low interest rate sensitivity of a credit default swap means that the effect of this correlation is at most second-order and we can reasonably ignore it. The model we have set out is therefore capable of doing the specified task.

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The Restructuring Clause in Credit Default Swap Contracts

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The new 2003 ISDA Credit Derivatives Definitions allow for four different clauses for handling restructurings as credit events that trigger default swaps: Old Restructuring, Modified-Restructuring, Modified-Modified-Restructuring and No Restructuring. We describe a model to price these different contracts that explicitly addresses the valuation of the protection buyer's cheapest-to-deliver option. As introduction, we first describe the contents of the restructuring clause and then discuss three known cases where the restructuring clause was triggered. In all three cases, the firms were forced to restructure to survive the refinancing of significant amounts of maturing bank debt.

1. INTRODUCTION

In the current standard credit default swap contract linked to a corporate (non-sovereign) reference credit, there are three credit events that can trigger the payment of protection. They are bankruptcy, failure to pay, and restructuring. Two years ago the mechanism for settling a default swap following a credit event was the same for all of these types of credit events. If a credit event occurred, protection buyers with physically settled contracts would settle them by delivering a face value amount of deliverable obligations in return for the face value amount paid in cash. The market standard was to allow the delivery of obligations with a maximum maturity of 30 years.

Today we have four different types of default swaps differentiated by their handling of the settlement of the default swap following a restructuring credit event: the old restructuring clause (Old-R), the deletion of restructuring as a credit event (No-R), the (American) modified restructuring (Mod-R), and the proposed[1] (European) modified-modified restructuring (Mod-Mod-R). The catalyst for this change was the restructuring of the US insurer Conseco, Inc. in September 2000.

The public announcement that Conseco had reached an agreement with a bank consortium to extend maturing bank loans for 15 months, was used by Conseco default swap protection holders as evidence that a credit event had occurred. The default swap holders used the broad 1999 ISDA definitions on deliverable obligations (Old-R) and settled the contracts with longer maturity deep discount bonds, trading in the 65-80 range. The event caused considerable controversy as the default swap sellers argued that protection buyers had purchased protection to reduce their credit exposure to Conseco's short-term loans and that the restructuring had not impaired these short maturity loans – they argued that this was a "technical default". Banks who had bought protection to hedge these loan exposures, and who were party to the restructuring, were able to take advantage of the delivery option by purchasing and delivering these cheaper long-dated bonds, thereby receiving a windfall gain, at the expense of protection sellers. It could be argued that this event highlighted a weakness in the credit derivative documentation. However, the aim of permitting a broad range of deliverables in a credit default swap is not to create a cheapest-to-deliver option but to enable

[1] *This new mechanism has been recently proposed and is expected to be adopted by the European market on 6 May 2003.*

a standard contract that can be used to hedge the credit risk in a wide range of *pari passu* assets issued by a reference entity. While these assets should trade at the same price following a bankruptcy or failure to pay, this is not the case for a restructuring event. For this reason, restructuring needs to be treated as a separate case.

This was achieved in the US through the adoption of the Mod-R contract in May 2001. This standard was not adopted within Europe, where bank regulators were not ready to accept as a hedge a contract which severely constrained the basket of deliverables. After long consultations with market participants, ISDA introduced the Mod-Mod-R contract in March 2003, which gives the protection buyer a larger set of deliverable obligations than specified in the Mod-R contract.

The ISDA documentation specifies the conditions that must be satisfied for an agreement between a company and its lenders to qualify as a restructuring credit event that triggers default swaps. To estimate the likelihood that a particular company will restructure, it is necessary to understand the specific composition and terms of the company's debt. In the next section, we discuss the contents of the restructuring clause options incorporated into the new 2003 ISDA definitions. We explain the restrictions on deliverable obligations that must be taken into account when determining which obligation is the cheapest to deliver to settle the default swap.

In the third section we discuss three cases where the restructuring clause has been triggered, and a fourth case where a restructuring was a possibility. These cases indicate that if a company restructures it is likely to find itself in a situation where its credit has deteriorated, it is cash constrained, and it is having problems rolling over maturing bank debt. In the restructuring, the banks agree to extend the maturity of the debt or otherwise modify its terms to prevent forcing the company to seek protection in bankruptcy. Once a restructuring agreement has been reached, the likelihood that the company will default in the short term is diminished, and as such a restructuring is good-news for holders of short-term debt.

Finally, in section 4, we present a simple model to price differences in the restructuring clause. We modify the Jarrow-Turnbull credit pricing framework to explicitly incorporate a curve of restructuring probabilities as well as a curve of default probabilities after a restructuring.

We conclude in section 5.

2. THE RESTRUCTURING CLAUSE

We start by defining what qualifies as a restructuring, then we describe the different types of contracts.

2.1 What qualifies as a restructuring?

According to the 2003 ISDA definitions, a debt obligation is considered restructured if there is:

1) interest rate reduction,

2) reduction in principal or premium,

3) postponement or deferral (maturity extension),

4) change in the priority ranking of payments, or

5) change in currency or composition of payment of principal or interest.

The restructured obligation must be held by more than three unaffiliated holders and at least two-thirds of the holders must have consented to the restructuring.

It is also important to note that *the occurrence of, agreement to or announcement of any of the five triggers in circumstances where such trigger does not directly or indirectly result from a deterioration in the creditworthiness or financial condition of the reference entity shall not constitute a restructuring.*

2.2 Old-R

Before ISDA published the *restructuring supplement* to the 1999 definitions on 11 May 2001, default swaps traded with restructuring included as a credit event and with a straight 30-year maturity limitation on deliverable obligations. As discussed in the introduction, the Conseco case revealed the problems with this combination. Today, in the US, Old-R default swap trades are rare and confined to unwindings of existing positions. In Asia and Europe Old-R contracts are still the standard.

2.3 No-R

The No-R clause eliminates restructuring as a credit event and is a simple solution to the problems identified by the Conseco case. No-R has especially been advocated by JPMorganChase and a number of insurance companies.

2.4 Mod-R

The 11 May 2001 ISDA restructuring supplement introduced a number of changes. In particular, the documentation allowed parties to specify *restructuring maturity limitation applicable*. This is the Mod-R clause. It states that if a restructuring is the only credit event specified in the credit event notice, then the swap may only be settled with delivery of an obligation that matures before the *restructuring maturity limitation date* (RMLD).

To determine RMLD it is necessary to first find the following three dates:

1) Let LMD be the latest (possibly extended) maturity date of any restructured obligation.

2) Let RD be the restructuring date, ie, the date the restructuring is legally effective.

3) Let STD be the scheduled termination date on the default swap.

Now let $M = \min\{LMD, RD + 30 \text{ months}\}$. If $M < STD$ then $RMLD = STD$. If $M > STD + 30$ months then $RMLD = STD + 30$ months. Otherwise, if $STD \leq M \leq (STD + 30 \text{ months})$ then $RMLD = M$. This implies that RMLD is bounded between STD and STD + 30 months.

The formula for finding RMLD can seem confusing. An approximation that is often used is

$$RMLD = \max\{STD, RD + 30 \text{ months}\}$$

We examine the accuracy of this approximation in the example below.

Under Mod-R, an obligation must be *fully transferable*, meaning that it must be either a transferable bond or a loan that can be transferred to an *eligible transferee* without consent. In particular, a loan is not fully transferable if its transfer can be blocked by the borrower (the restructured firm). Under Mod-R, eligible transferee is broadly defined and does not take into account whether the transferee is prevented from owning the obligation for regulatory reasons.

2.5 Mod-Mod-R

The Mod-Mod-R clause was introduced with the new 2003 ISDA definitions and defines the restructuring maturity limitation date by the formula

$$\mathrm{RMLD} = \max\{\mathrm{STD}, \mathrm{RD} + \mathrm{A}\}$$

where STD and RD have the same definitions as section 2.4 and A = 60 months for restructured obligations and A = 30 months for all other obligations. Note that, unlike Mod-R, this definition is independent of the maturity of the restructured debt. It only depends upon the maturity date (STD) of the default swap and whether a restructured obligation is being delivered.

Under Mod-Mod-R, a restructured obligation must be *conditionally transferable*, which differs from fully transferable (as it applies to Mod-R) with respect to the definition of eligible transferee. The Mod-Mod-R definitions only require a loan to be transferable without consent to an entity engaged in the loan business.

In Figure 1 we compute the restructuring maturity limitation date for Mod-R and Mod-Mod-R contracts.

Figure 1. Comparison of restructuring maturity limitation dates for Mod-R and Mod-Mod-R contracts

				Mod-R		Mod-Mod-R	
Case	**RD**	**LMD**	**STD**	**Exact**	**Approx.**	**Restructured**	**Non-restructured**
1	0	24	12	24	30	60	30
2	0	36	12	30	30	60	30
3	0	72	12	30	30	60	30
4	0	24	48	48	48	60	48
5	0	72	48	48	48	60	48

All numbers are in months from today's date.

In Figure 1, under Mod-Mod-R for restructured obligations, the restructuring maturity limitation date is 60 months. For cases 3 and 5 the maturity of the restructured obligations is greater than 60 months and, hence, the constraint is binding. Under Mod-R, the situation is more complex, because the restructuring maturity limitation date depends on the maturity of the restructured obligations. In cases 2, 3 and 5 the constraint is binding.

2.6 Other provisions in the restructuring clause

1) The restructuring maturity limitation only applies when the credit event has been triggered by the protection buyer.

2) The deliverable obligation must be *pari passu* with the reference obligation. *Pari passu* is determined at the trade date (the date the swap contract was entered into). In particular, if after a restructuring an obligation that used to be a *pari passu* is subordinated, the obligation may still be deliverable.

3. RESTRUCTURING CASES

We discuss three cases where the restructuring clause was triggered and a fourth case where restructuring may soon occur. There are three particular issues on which we focus.

1) The shape of the curve of bond prices after the restructuring.

2) In all three restructuring cases, only bank loans were restructured.

3) The problem of distinguishing a restructuring and a refinancing: is a restructuring a modification of an existing loan or a roll-over into a new loan?

3.1 Recent restructurings

Identifying restructurings is an industry-wide problem. Moody's, for example, only classifies some restructurings as defaults to be included in their ratings and default data. In particular, Moody's only considers *involuntary distressed exchanges* as defaults[2]. Unfortunately, we have not been able to obtain reliable historical data that specifically identify restructurings. The main problem is that restructurings received little attention prior to the development of the default swap market, and in particular before the Conseco restructuring in autumn 2000. We manually examined a number of major default swap triggering events occurring after the Conseco case and only identified two additional restructurings: Xerox and Solutia.

3.2 Conseco

On 22 September 2000, Conseco announced an agreement with a 25-bank consortium to resolve issues surrounding $1.4 billion of bank debt coming due on that day. Over the preceding four years, Conseco's bank and public debt had increased to $5.9 billon and the agreement called for a reduction of $1.52 billon of bank debt and $1.56 billon of public debt over the following three years. To begin the debt reduction Conseco identified $2 billon worth of assets to be sold, of which $700 million had already been realized. The agreement included immediate repayment of $650 million to the banks, an extension of $571 million of bank debt until year-end 2001 to be financed by asset sales, and an acceleration of $300 million on $1.5 billon of bank debt due in September 2003 but with the option to extend the remaining $1.2 billon of bank debt until 2005. The rating agencies interpreted this financial restructuring plan positively and focused mainly on the fact that the plan improved liquidity and resolved concerns about the maturing bank debt[3].

The extension of the $571 million of bank debt until year-end 2001 clearly qualifies as a restructuring. The fact that $1.2 billion of the bank debt maturing in September 2003 was modified to include an option to extend until 2005 may also qualify as a restructuring. The latest maturity of restructured debt would then be 2005. However, protection sellers may argue that the option to extend was compensated by the $300 million acceleration, and therefore does not qualify as a restructuring. The legal issues are not clear.

Conseco, Inc. had ten bonds trading during September 2000, according to Lehman's index database. The reaction of the bond market to the restructuring announcement can be seen from the prices in Figure 2, where the two dates shown are before and after the restructuring announcement.

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[2] *For discussion see "Understanding The Risks In Credit Default Swaps", Moody's Special Report, 16 March 2001.*

[3] *See announcements made by A.M. Best and Fitch on 25 September 2000, and by S&P on 26 September 2000.*

Figure 2. Conseco Inc. bonds outstanding when the restructuring was announced on 22 September 2000

Maturity	15 Dec 2000	15 Jun 2001	21 Jun 2001	15 Oct 2002	10 Feb 2003	15 Feb 2003	9 Feb 2004	15 Dec 2004	15 Jun 2005	15 Oct 2006
Coupon	7.875	6.40	7.60	8.50	6.40	8.125	8.750	10.50	6.80	9.00
Outstanding (m)	150	550	118.9	450	250	95.3	800	24.55	250	500
Currency	USD	USD	USD	USD	USD	USD	USD	USD	USD	USD
Options / Sinking fund	None	Call/Put	None	Call	Call	None	Call	None	Call	Call
Price (31 Aug 2000)	94.20	90.86	91.50	66	62	65.19	62	55.98	62	61
Price (30 Sep 2000)	97.44	92.32	92.88	82	72	78.35	71	73.18	66	69
Excess return[4](%)	3.45	1.59	1.60	23.36	16.05	20.28	14.65	30.13	6.23	12.62

Assume that only the extension until 12/31/2001 qualifies as a restructuring and consider a default swap that matures in one year. Under Mod-R, the restructuring maturity limitation date would be 12/31/2001 and the cheapest-to-deliver (among the bonds listed) are the two June 2001 bonds priced around 92-93. Under Mod-Mod-R, the restructuring maturity limitation date would be 3/22/2003 for non-restructured obligations and 9/22/2005 for restructured obligations. None of the listed bonds were restructured, so the cheapest-to-deliver is the February 2003 bond priced at 72. When considering Old-R contracts, all listed bonds can be delivered and the cheapest-to-deliver is the June 2005 bond priced at 66.

3.3 Xerox

Market participants had long feared the October 2002 maturity of Xerox's $7 billion revolving credit facility[5]. On 21 June 2002, Xerox announced a renegotiation of the facility, paying down $2.8 billion and refinancing the remaining $4.2 billion. The financing consisted of three loans totaling $2.7 billion (maturing on 9/15/2002 to 4/30/2005) and a $1.5 billion revolver[6].

According to articles in *Derivatives Week* on 8 September and 22 December 2002, default swap sellers were disputing the occurrence of the credit event both for lack of publicly available information and by arguing that the refinancing did not *"directly or indirectly result from a deterioration in Xerox's creditworthiness or financial condition"*. On the other hand, default swap buyers argue that *"Xerox had no chance of repaying the loan and was forced into a restructuring"*.

The experience highlights the problem of determining whether a refinancing is a modification of an existing loan or roll-over into a new loan. The borrower may be indifferent and may easily be persuaded by the banks to classify the refinancing as a restructuring that can trigger default swaps held by the banks.

Using the Lehman index database we extracted prices for seven Xerox Corp. bonds trading during June 2002. Prices are shown in Figure 3.

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4 *Excess return is excess return over what can be attributed to changes in Treasury rates. The calculation method used is described in "A New Method of Excess Returns Computation", Index Report, September 2000.*

5 *See, for example, the 12 March 2001 issue of* Distressed Digest *published by our distressed research team.*

6 *For details see, for example, equity research analyst Caroline Sabbagha's 24 June 2002 report on Xerox.*

Figure 3. Xerox Corp. bonds outstanding when the restructuring was announced on 21 June 2002.

Maturity	15 Nov 2003	15 Dec 2003	4 Feb 2004	1 Aug 2004	3 Dec 2004	15 Jan 2009	15 Jan 2009	1 Apr 2016	15 Nov 2026
Coupon	5.5	5.25	3.5	7.15	5.25	9.75	9.75	7.20	6.25
Outstanding (m)	600	250	300	200	750	600	250	250	350
Currency	USD	USD	EUR	USD	EUR	USD	EUR	USD	USD
Options / Sinking fund	None	None	None	None	None	Call	None	None	Put
Price (31 May 2002)	92.5	91.5	85	91	83	93	87	75	92
Price (28 Jun 2002)	85.5	85	76	83	74	82	77	73	89
Excess return (%)	-7.67	-7.10	-10.93	-8.90	-11.10	-12.10	-11.82	-3.67	-3.3

Outstanding amounts and prices are in the currency indicated. The USD/EUR exchange rate was 0.9321, 0.9700 and 0.9914 on 31 May 2002, 21 Jun 2002 and 28 Jun 2002 respectively.

In Figure 3, there are bonds denominated in euros as well as US dollars. ISDA documentation contains an option to specify the currency of deliverable obligations. It is common not to specify any particular currency, in which case the default is the currencies of the G7 countries and Switzerland. It is interesting to note the lower prices (but also lower coupons) on the two shorter term euro bonds, which raises the question of whether US dollar bonds have been bid up by default swap holders looking to purchase obligations to settle contracts that require US dollar deliverables. On the other hand, there is also the possibility that the euro bond prices are stale or that they may not be delivered for other reasons.

Assume that the latest maturity of a restructured obligation is 4/30/2005, that is about 34 months from the restructuring date. In this case the Mod-R maturity limitation approximation is exact and the maturity limitation under Mod-R is the same as the Mod-Mod-R maturity limit on non-restructured obligations. Ignoring the euro bonds, the cheapest-to-deliver under Mod-R is the August 2004 bond priced at 83. This is also the cheapest non-restructured bond to deliver under Mod-Mod-R. Under Old-R, the cheapest to-deliver is the April 2016 bond priced at 73.

3.4 Solutia

Solutia announced on 19 July 2002 that it had reached an agreement with its banks to extend the maturity of a 5-year maturing revolving loan facility for two years, and to reduce the facility from $800 million to $600 million. The $200 million paid to the banks came from a bond issued two weeks earlier. This was viewed as being detrimental to the wealth of existing obligation holders.

At the end of June 2002, there were five Solutia bonds in Lehman's index database and the new bond mentioned above. Prices are given in Figure 4. The 2005 bond is issued by Solutia Europe. The remaining four bonds are issued by Solutia Inc.

Figure 4. Solutia Inc. bonds outstanding when the restructuring was announced on 19 July 2002

Maturity	15 Oct 2002	14 Feb 2005	15 Jul 2009	15 Oct 2027	15 Oct 2037
Coupon	6.50	6.25	11.25	7.375	6.72
Outstanding (m)	150	200	223	300	150
Currency	USD	EUR	USD	USD	USD
Options / Sinking fund	None	None	None	Call	Call/Put
Price (28 Jun 2002)	100.0	86	NA	62.3	91.99
Price (31 Jul 2002)	99.59	78	88.5	60	80
Excess Return (%)	0.00	-9.54	NA	-5.38	-13.79

Outstanding amounts and prices are in the currency indicated. The USD/EUR exchange rate was 0.9914, 1.0136 and 0.9778 on 28 Jun 2002, 19 Jul 2002 and 31 Jul 2002 respectively.

The loan facility was extended until 7/19/2004. For default swaps maturing before that date, this is the restructuring maturity limitation date under Mod-R. Under the Mod-Mod-R, the date is 1/19/2005 for non-restructured obligations. However, this six-month difference is not enough to make the February 2005 deliverable. Of the bonds in the table, only the October 2002 bond may settle Mod-R and Mod-Mod-R contracts. Old-R contracts, on the other hand, can be settled with the October 2027 bond priced at 60.

3.5 Goodyear

On 5 March 2003, it was reported that Goodyear Tire and Rubber had obtained a $1.3 billion conditional asset-backed credit facility that would come into effect if/when Goodyear finishes negotiating changes in existing loan agreements. The reports were not specific about the required changes but the possibility of a restructuring event was apparent.

On 1 April 2003, it was reported that Goodyear had reached an agreement to restructure and refinance its loans. $2.9 billion of existing facilities were replaced by a $750 million secured revolving credit facility due in 2005, a $645 million secured US term facility due in 2005, a $650 million secured European facility due in 2005, and a $1.3 billion asset-backed facility due in 2006. According to a Reuters report *"the company said its restructured credit agreements replace facilities that generally have shorter maturities"*.

According to a default swap trader, the company paid down existing maturing facilities and replaced them with longer dates ones. Consequently a default swap restructuring event was deemed not to have occurred.

Prices of Goodyear bonds taken from our index database are shown in Figure 5.

Figure 5. Goodyear bonds outstanding when the possibility of a restructuring was announced on 5 March 2003

Maturity	15 Mar 2003	6 Jun 2005	17 Mar 2006	1 Dec 2006	15 Mar 2007	15 Mar 2008	15 Aug 2011	15 Mar 2028
Coupon	8.125	6.375	5.375	6.625	8.50	6.375	7.857	7.00
Outstanding (m)	300	400	200	250	300	100	650	150
Currency	USD	EUR	CHF	USD	USD	USD	USD	USD
Options / Sinking fund	Call	None	None	Call	Call	Call	Call	Call
Price (28 Feb 2003)	99.728	74	62.1	73	75	67.212	67	60
Price (31 Mar 2003)	100	78	73	75	77	69.459	71	65
Excess Return (%)	0.47	5.71	16.70	3.34	3.38	4.04	7.18	9.52

Outstanding amounts and prices are in the currency indicated. The USD/EUR exchange rate was 1.0820 and 1.0973 on 28 Feb 2003 and 5 Mar 2003 respectively. The CHF/USD exchange rate was 1.3521 and 1.3294 on those dates.

3.6 Lessons

Even from this small sample we see that there can be considerable value in the delivery option. Figure 6 shows the recovery under the different clauses.

Figure 6. Recovery on default swaps with three different restructuring clauses

	Mod-R	Mod-Mod-R	Old-R
Conseco	93%	72%	66%
Xerox	83%	83%	73%
Solutia	99.4%	99.4%	60%

Recovery is the cost (as a percentage of par) of the cheapest-to-deliver obligation under the particular contract. Only bonds included in a Lehman Brothers index were considered.

Unfortunately, the sample is too small to discern any general pattern. This is not surprising, as restructuring can occur for many reasons. A company, while basically healthy, may have liquidity issues forcing it to restructure or a company may be in serious financial trouble when it restructures. A restructuring may reduce the default risk for short-term claim holders, although the terms of the restructuring may alter possible recovery if default does occur.

4. PRICING

In this section we present a simple framework to analyze the pricing implications of the four different restructuring clauses. Before describing the pricing model, we consider the relative rankings of the different spreads.

4.1 Basics

Let S_{NR}, S_{MR}, S_{MMR}, and S_{OR} be the default swap spreads under the four different restructuring clauses, No-R, Mod-R, Mod-Mod-R, and Old-R. A simple argument implies that

$$S_{NR} \le S_{MR} \le S_{MMR} \le S_{OR}$$

The spread for No-R, S_{NR}, will be the lowest of the four spreads, since in all the other three clauses, the protection buyer has the option but not the obligation to settle the contract if a restructuring occurs. The spread for Old-R, S_{OR} will be the largest of the four spreads, because any obligation that can be delivered under Mod-R and Mod-Mod-R can also be delivered under Old-R. Finally, we saw in the previous section that any obligation that may be delivered under Mod-R may also be delivered under Mod-Mod-R, which implies that the spread for Mod-Mod-R should be greater than or equal to the spread for Mod-R.

4.2 A simple model

In the absence of restructuring as a credit event, we could price a default swap using the term structure of "risk-neutral" default probabilities. Default would then be the usual definition and include failure to pay and bankruptcy but not restructuring. The article "Valuation of Credit Default Swaps" included elsewhere in this publication describes how this can be done.

When a company files for Chapter 11 or fails to make debt payments it is usually in dire financial trouble and the company's debt will trade on the expected recovery value. In particular, two debt obligations with the same seniority will trade at roughly the same price even if one obligation matures in two years and the other obligation matures 20 years later. All obligations that may be delivered to settle a default swap contract will trade at roughly the same price, and there is little reason to specifically model which obligation is the cheapest-to-deliver. This is not the case for restructurings. As we have seen in section 3, although a company that restructures its debt is unlikely to be entirely financially healthy, it may well avoid bankruptcy (at least for a few years). For this reason, maturity matters when pricing the company's debt and the deliverable obligations will usually not trade at the same price. It is therefore important to explicitly model which obligation is the cheapest-to-deliver.

We modify the well-known Jarrow-Turnbull credit pricing framework and directly incorporate the possibility of restructuring as well as default. We model the occurrence of a restructuring the same way as the occurrence of a default, by specifying a hazard rate for a jump process.

The model takes as input:

1) A curve of default-free interest rates: $\{r_t\}$.

2) A curve of risk-neutral one-year conditional probabilities of default: $\{\lambda_t\}$.

3) A curve of risk-neutral one-year conditional probabilities of restructuring: $\{q_t\}$.

4) A default recovery rate for the *pari passu* obligations: R.

5) A curve of risk-neutral one-year conditional probabilities of default after a restructuring: $\{\theta_t\}$.

6) A minimum coupon on debt outstanding after a restructuring: c_{min}.

7) A maturity limit on debt outstanding after a restructuring: T_{max}.

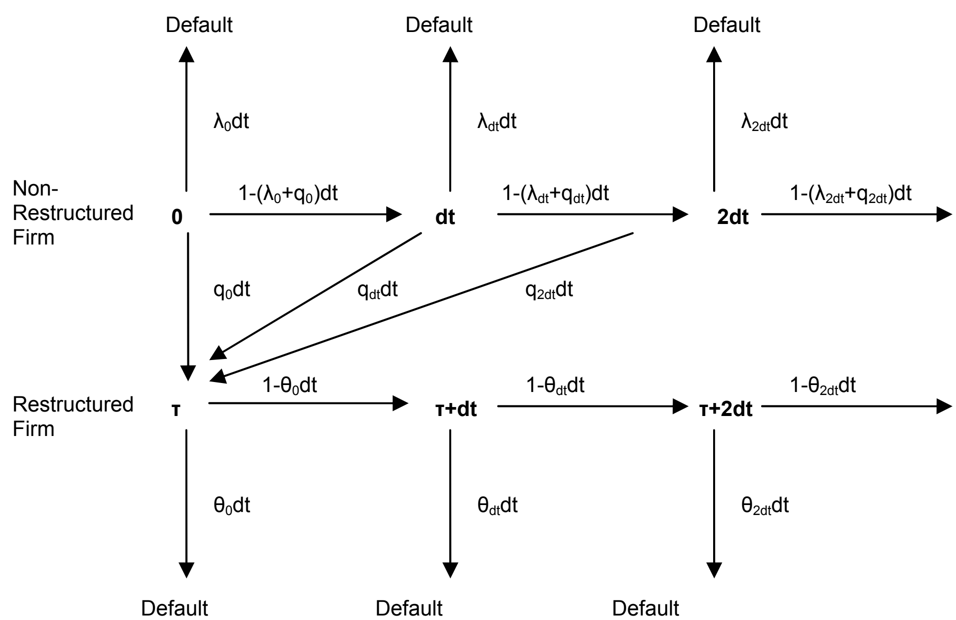
The model is based on the following simplifying assumptions:

1) A restructuring can only occur once in the life of a default swap.

2) After a restructuring, the cheapest-to-deliver is a bond maturing with a coupon of c_{min} and a maturity of $\max\{T_{max},T\}$, where T is the remaining time to maturity of the default swap.

3) Under Mod-R we use T_{max} = 30 months, under Mod-Mod-R we use T_{max} = 60 months, and under Old-R we use T_{max} = 360 months.

Figure 7. Pricing model with both default and restructuring included as credit events



The firm starts as a non-restructured firm. Next period the firm will default, restructure or survive as a non-restructured firm. τ is the time of restructuring. Each arrow represents the passing of a time period of length dt. The model assumes that a firm can only restructure once and that the default probabilities after a restructuring are the same no matter when the firm restructured.

Figure 7 illustrates how the model works. It is straightforward to solve the model numerically using standard backwards-induction option pricing techniques[7]. To find the spread on a default swap with a particular restructuring clause take the following steps:

1) For a swap with a 5-year maturity, divide the 5-year period into a number of discrete time points, say of length dt. Then calculate the probabilities of restructuring and default over each time period as illustrated in Figure 7.

2) For every time point, find the price of the cheapest-to-deliver bond. This is done using

   i) the curve of default-free interest rates starting from the (time) point of the restructuring,

   ii) the curve of probabilities of default after a restructuring,

[7] *Because all hazard rates are constant, solution of the model amounts to nothing more than calculation of a few sums. With stochastic hazard rates the model becomes more complicated and we would need to build a lattice.*

iii) the default recovery rate, and

iv) the scheduled payments for the set of deliverable obligations.

3) Price the swap by backwards induction using the probabilities from step 1, the default recovery rate and the cheapest-to-deliver bond from step 2. It must be taken into account that the protection buyer has the option not to settle the contract at restructuring and instead wait for a possible future default that may give a higher protection payment.

4.3 Calibration

Determining the input parameters is the final step before reporting results. This is also the most difficult step. We need to determine:

1) the probabilities of default before a restructuring, $\{\lambda_t\}$,

2) the probabilities of restructuring, $\{q_t\}$, and

3) the probabilities of default after a restructuring, $\{\theta_t\}$.

We suggest thinking about the probabilities of restructuring as a multiple of the probabilities of default. That is, the conditional probabilities of restructuring are $q_t = c * \lambda_t$, for all t, where c is a constant. In Figure 8, below, we consider three different values for c, c = 2, 1 and 0.5. We use this specification because factors that increase the probability of default will usually also increase the probability of restructuring.

When picking the multiple c, the analyst should focus on factors that mainly affect the probability of restructuring. For example, whether the firm has large amounts of bank debt and how complex its capital structure is (both are important determinants of how easy it will be to reach a restructuring agreement).

For choosing the probabilities of default after a restructuring, $\{\theta_t\}$, we refer to the curve of bond prices observed in the three restructuring cases. For reasons discussed in section 3, we suggest using an upward sloping curve of probabilities.

4.4 Results

In Figure 8 we report the default swap spreads for a number of different parameter cases. In all cases, we have fixed the curve of default-free interest rates to be upward sloping with a short rate of 3% and a long rate of 6%[8]. We use a default recovery rate of R = 50% and a minimum coupon of c_{min} = 6%. The results are also based on constant probabilities of default ($\lambda_t = \lambda_0$, for all t) and a curve of probabilities of default after restructuring where the long-run probability is the double of the short-run probability[9].

Suppose that for No-R the spread is 100bp and the restructuring/default probability ratio is 2. We then compute the spreads for the Mod-R, Mod-Mod-R and Old-R. Next, we alter the ratio to 1. We keep the spread for No-R fixed at 100bp and we alter the default probability, λ_0, so that No-R remains at 100bp, and then we compute the new level of spreads for the Mod-R, Mod-Mod-R and Old-R contracts.

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[8] *We use $r_t = 3\%\cdot(2\text{-}\exp(-0.1\cdot t))$, for all $t \geqslant 0$.*

[9] *We use the $\theta_t = \theta_0\cdot(2 - \exp(-0.1\cdot t))$, for all t>0.*

Figure 8. Default swap spreads (in bp) under different restructuring clauses

1st year default probability after restructuring[10]	5%				10%				15%			
Restructuring/default probability ratio	**2**	**1**	**0.5**		**2**	**1**	**0.5**		**2**	**1**	**0.5**	
Restructuring Clause	Spread	Spread	Spread	Recovery	Spread	Spread	Spread	Recovery	Spread	Spread	Spread	Recovery
No-R	100	100	100	NA	100	100	100	NA	100	100	100	NA
Mod-R	100	100	100	99%	104	102	101	93%	107	104	102	88%
Mod-Mod-R	103	102	101	97%	115	109	105	86%	122	114	108	78%
Old-R	150	127	114	82%	159	134	119	71%	155	134	119	65%
No-R	200	200	200	NA	200	200	200	NA	200	200	200	NA
Mod-R	201	201	200	99%	209	206	203	93%	215	209	205	88%
Mod-Mod-R	211	206	203	97%	239	223	212	86%	252	232	218	78%
Old-R	321	264	233	82%	334	277	241	71%	321	273	241	65%
No-R	300	300	300	NA	300	300	300	NA	300	300	300	NA
Mod-R	321	310	305	99%	330	317	309	93%	336	322	312	88%
Mod-Mod-R	344	322	311	97%	379	345	324	86%	393	356	331	78%
Old-R	521	412	356	82%	528	428	368	71%	501	420	367	65%

Recovery is the cost (as a percentage of par) of the cheapest-to-deliver obligation under the particular contract.

The spread between Old-R and Mod-Mod-R is larger than the spread between Mod-R and Mod-Mod-R[11]. The spread between No-R and Mod-R is relatively small, but the spread between Mod-Mod-R and Old-R can be substantial. The market rule of thumb of adjusting the spread between the different contracts by a fixed percentage seems to be inappropriate. Given the difficulty in calibrating the model, we have arbitrarily chosen the parameter values and consequently the spread differences presented in figure 8 may not be realistic.

4.5 Back-of-the-envelope calculations

To gain some intuition into the numbers in Figure 8, consider the case where the spread on the No-R contract is 300bp, the restructuring/default probability ratio is 2, and first year default probability after restructuring is 15%. Let us go through a very rough back-of-the-envelope type of calculation to explain the order of magnitude of the Old-R spread, which is 501bp in Figure 8, compared to the No-R spread.

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[10] *To be precise the numbers in this row are θ_0 and the instantaneous (risk-neutral) hazard rates are $\theta_t = \theta_0 \cdot (2 - \exp(-0.1 \cdot t))$, for $t>0$. That is, we are using an upward sloping curve of hazard rates where the long-run hazard rate is the double the short-run hazard rate. The risk-neutral probability of default within the first year is 1-exp(-θ'), where θ' is the integral from 0 to t of θ_t.*

[11] *It is important to remember that the Old-R spreads in Figure 8 are based on the assumption that the firm will have 30-year debt outstanding at the time of restructuring. It is important to evaluate in each particular case whether this is a reasonable assumption.*

First consider the No-R contract. For this contract restructuring is not a defined credit event but does affect the probabilities of default. There are two default events to consider.

1) No restructuring and then a default with a loss of 0.5. This occurs with a probability of (1–2p)·p, where p is the probability of default and (1–2p) is the probability of no restructuring. The expected loss is (1–2p)·p·0.5.

2) Restructuring and then a default. After a restructuring the new probability of default is approximately 18% (remember we use an upward sloping curve of hazard rates, see footnote 10 for details; the average hazard rate over the first five years following a restructuring is approximately 18%) and the expected loss is 2p·18%·0.5 for this event, where the term 2p is the probability of restructuring.

The total expected loss should equal the spread of 300bp, ie,

$$(1-2p)\cdot p\cdot 0.5+2p\cdot 18\%\cdot 0.5=3\%$$

The equation gives an implied probability of default p = 4.74%[12].

Now consider the Old-R contract. We either have default and no restructuring with probability p·(1–2p) and a loss of 0.5 or no default and restructuring with probability (1- p)·2p and a loss of 0.35. Note that we ignore the joint event of restructuring and default occurring together. The spread is

$$p\cdot(1-2p)\cdot 0.5+(1-p)\cdot 2p\cdot 0.35=5.27\%$$

This compares with a spread for Old–R in the Figure 8 of 501bp.

Other types of back-of-the-envelope calculations can be made. Consider the same case as above. Assume that only two events can occur: 1) The firm goes straight to default (without restructuring first) with a hazard rate of 4.2%, or 2) it restructures with a hazard rate of 8.4%. If we know what the protection buyer's position is worth in each event under the two contracts we can approximate the spreads on the contracts. For the Old-R contract, the payment at default is 0.5 and the payment at restructuring is 0.35, and the spread should be approximately 4.2%·0.5 + 8.4%·0.35 = 5.04%, which is close to the 501bp in Figure 8. For the No-R contract the payment at default is also 0.5 but what is the value of the position to the protection buyer right after a restructuring has occurred? If this value is 0.105, the spread on No-R contracts would be 4.2%·0.5 + 8.4%·0.105 = 2.98%, which is close to the 300bp in Figure 8. Can we justify a value of 0.105? Here is an attempt: Given that a restructuring has occurred during the five years of the contract, it will on average have occurred with 2.5 years remaining (because of the flat hazard rate of restructuring). After a restructuring, the hazard rate of default over the remaining 2.5 years is about 17% and the loss given default is still 0.5, so the spread after a restructuring on a new 2.5 year swap (with a market value of zero) is approximately 17%·0.5 = 850bp or 550bp more than the protection buyer is currently paying. In other words, entering into an offsetting position will provide 550bp for about 2.5 years or until default, whichever comes first. To value this stream of payments we must know the PV01 of the stream, which in this case is about 1.9[13]. The No-R protection buyer would have

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[12] *Note this is a quadratic with two roots. We have taken the root that is close to the range of acceptable default probabilities.*

[13] *It is a bit more cumbersome to justify that the PV01 is 1.9. The PV01 can be approximated by $(1+x)^{-1}+(1+x)^{-2}+(1+x)^{-3}+(1+x)^{-4}+(1+x)^{-5}$, where x = 0.5·(17% + 3%) and 3% is the default free rate. This calculation produces 1.90 but is not exact. See the article "Valuation of Default Swaps" elsewhere in this publication for details on how to calculate PV01 and in general for details on how a default swap should be marked-to-market.*

a profit at restructuring of approximately $0.055 \cdot 1.9 = 0.1045$, which is close to the profit of 0.105 we wanted to justify.

There are many reasons why the back-of-the-envelope numbers differ from the numbers in Figure 8. We ignore the maturity of the contract, the term structure of interest rates and the random timing of the events.

5. CONCLUSION

In this paper we describe the differences between Old-R, Mod-R and Mod-Mod-R and present a simple model that can used to price these different types of contracts. The major difficulty is the calibration of the model. To apply the model to individual firms we need to specifically model the maturity of the bonds the firm is expected to have outstanding after a restructuring, and we need to have a view on both the likelihood of a restructuring vs a default and the probabilities of default after a restructuring.

Leverage and Correlation Risk of Synthetic Loss Tranches

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Securitization is generally praised because of its ability to offer inexpensive, non-recourse financing to junior investors, as well as relatively high spreads to senior note holders. In this article, we focus on the application of well-developed securitization technologies to the world of synthetic credit. We first observe that slicing synthetic default risk introduces exposure to changes in default correlations, and that tranches with different seniorities, beyond offering a different leverage, exhibit very different sensitivities to correlation movements. We then show how an investor can select either an appropriately positioned mezzanine tranche or an appropriately weighted combination of different tranches to obtain a customized exposure that: 1) is hedged against parallel movements of the correlation matrix; and 2) achieves the desired leverage. [1]

1. INTRODUCTION

Recent surveys have shown a dramatic increase in the popularity of multi-name credit derivatives such as synthetic CDOs and portfolio loss tranches.[2] The rapid expansion of the default swap market and the increasing demand for structured credit products has induced a number of dealers to adapt established securitization technologies to synthetic credit products. The practice of sourcing correlated default risks through single-name contracts, pooling them together, slicing up the losses, and placing the resulting tranches with different types of investors has become a popular way of redistributing credit risk to a wide range of end-users.

Securitization is generally praised because of its ability to offer inexpensive, non-recourse financing to junior investors, as well as relatively high spreads to senior note holders. Essentially, securitization allows junior investors to finance a leveraged position with limited downside at a cost equal to the spread paid to senior investors. It follows immediately that such a technology is able to generate economic surplus if and only if senior spreads can be set at a level that is: 1) higher than other comparable investments; and 2) lower than alternative borrowing costs for junior investors. The lasting success of securitization shows that these two conditions can often be met.

In this article, we focus on the application of well-developed securitization technologies to the world of synthetic credit. We first observe that slicing synthetic default risk into tranches introduces exposure to changes in default correlations, and that tranches with different seniorities, beyond offering a different leverage, exhibit very different sensitivities to correlation movements. We then show how an investor can select either an appropriately positioned mezzanine tranche or an appropriately weighted combination of different tranches ("combo") to obtain a customized exposure that:

[1] *We would like to thank Jin Chang, Sunita Ganapati, Mark Howard, Dominic O'Kane, Lutz Schloegl and Stuart Turnbull for comments and suggestions.*

[2] *See the comprehensive survey in the February 2003 issue of* Risk.

1. is hedged against parallel movements of the correlation matrix; and
2. achieves the desired leverage (and therefore the desired yield).

Monitoring the exposure to correlation movements should be of particular interest to investors who need to mark-to-model their synthetic investments before maturity, and are concerned about the volatility of such marks. According to FAS 133, all public companies in the US are responsible for periodically marking the value of their open derivative contracts using models that are reasonable and appropriate. This obligation introduces the possibility of an increase in systematic earnings volatility, and thus in the risk premium that equity investors will charge in order to hold a company's stock.

The simple technique we are going to describe in this paper can be applied during the initial selection of a reference portfolio, as well as at the time of a restructuring, to manage the correlation risk of a portfolio of multi-name credit exposures, and thus possibly mitigate this undesirable by-product of FAS 133.

In Section 2, we introduce a simple homogeneous portfolio of reference credits that we use throughout the article, and show how the loss distribution of the entire portfolio and its first two moments change as we vary the correlation of the constituent names. In Section 3, we slice the loss distribution of the reference portfolio into loss tranches; here we show that tranches of different seniorities differ not only in terms of their expected losses, but also in terms of their sensitivities to correlation movements. We use this result in Section 4 and Section 5 to show that it is possible to construct correlation-hedged tranches and combos with a variety of expected losses. Section 6 briefly concludes.

6. THE REFERENCE PORTFOLIO

6.1 A simple homogeneous portfolio

Throughout this article, we work with a homogenous synthetic portfolio of default-risky instruments. The reader can think of a portfolio of default swaps written on a number of different names. We make the following assumptions:

1. The portfolio is composed of 100 names, each with the same notional.
2. The portfolio is exposed to losses occurring over a 5-year horizon.
3. For each name in the portfolio, the loss given default (LGD) is 60%. Therefore, the portfolio experiences a loss of 0.6% of the total notional for each default that occurs over the 5-year horizon.
4. All names in the portfolio are equally risky: each default time is exponentially distributed with an annual hazard rate of 1% under the risk-neutral measure, which implies that the cumulative default probability over the 5-year horizon is equal to $1\text{-}\exp(-0.01\cdot 5)=4.88\%$.
5. The 5-year default correlation[3] of any two names in the portfolio is denoted as ρ_{DC}. All the calculations in this article are based on a copula-based model of joint defaults. In this framework, each name is associated with an asset value process, and default occurs when the asset value falls below a given threshold. Correlated defaults are therefore modeled through dependent asset returns. For all exercises in this article, we assume that the

[3] *The 5-year default correlation between name i and name j is the correlation between the random variables x_i and x_j, defined by x_i =1 if firm i defaults within the 5 years, and x_i =0 if firm i does not default.*

underlying asset returns have a Gaussian copula, so that their dependence structure is fully determined by their correlations. Therefore, default correlations over a specified horizon are completely determined by the cumulative default probabilities and the asset correlations. In the remainder of this article, when we talk about "correlations" (and "correlation sensitivities") we refer to asset correlations, unless otherwise specified. The asset correlation between any two names in our homogeneous portfolio will be denoted by ρ. We will let ρ take on different values to show the sensitivity of alternative synthetic investments to correlation changes.

Every distribution, expectation and standard deviation we refer to in this article should be interpreted as computed under risk-neutral probabilities. As we explain more precisely in a later section, this allows us to relate variations in the underlying parameters to variations in the value of an open position.

6.2 Expected loss and standard deviation of the loss

If L denotes the total percentage loss on the portfolio – which in our simple example is equal to the number of defaults multiplied by the loss given default (LGD) divided by 100 (the number of names) – then the expected percentage loss over the 5-year horizon is given by 60%·4.88%=2.93%.

The standard deviation of the percentage loss can be calculated as:

$$Std\ \ Dev\left[L\right] = 60\%\sqrt{4.88\% \cdot (1-4.88\%) \cdot \left(1+99\,\rho_{DC}\right)\%}\ .$$

Two things should be noted. First, the expected loss does not depend on default correlations. The expected loss for the entire portfolio is simply the sum of the expected losses on each name, which only depends on the individual default probabilities and the loss given default. Second, the standard deviation of the loss is an increasing concave function of default correlation. In other words: 1) a higher default correlation implies a higher standard deviation; and 2) the lower the default correlation, the higher the sensitivity of the standard deviation to changes in default correlation.

6.3 Portfolio loss distribution

The portfolio loss is clearly not symmetrically distributed. It is therefore informative to look at the entire loss distribution rather than summarizing it in terms of expected value and standard deviation. Over a range of plausible correlation values, we can expect to observe one of the two shapes shown in Figure 1:

1. a skewed bell curve;
2. a monotonically decreasing curve.

The skewed bell curve applies to the low correlation cases. When correlation is zero, the mode[4] is only slightly less than the expected loss. As correlation increases, the mode falls and the high quantiles increase: the curves become monotonically decreasing.

For very high levels of asset correlations (hardly ever observed in practice), the distribution becomes U-shaped: when correlation is equal to 1, all the probability mass is located at the two ends of the distribution.

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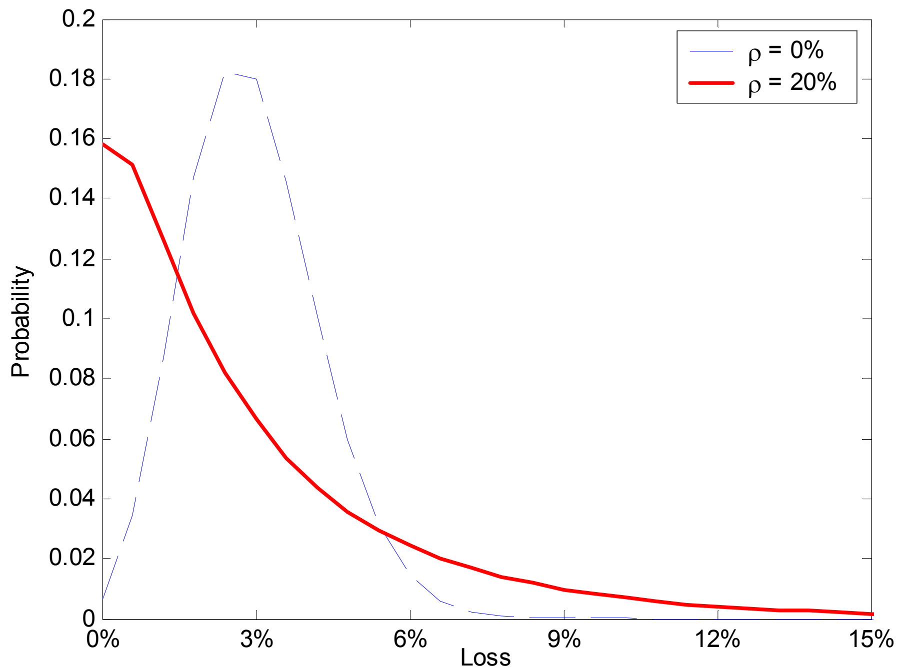
[4] *The mode of a discrete distribution is the outcome that has the highest probability of occurring.*

Figure 1. Risk-neutral loss distribution for different asset correlations



5-year horizon, homogeneous portfolio (1% risk-neutral default probability, 60% LGD)

6.4 Prices, premia and expected losses

No-arbitrage pricing theory suggests that the fair price paid to a protection seller is equal to the expected loss under the risk-neutral probabilities, properly discounted by the default-free interest rates. If the protection seller is compensated by a recurring premium instead of an initial payment, the fair premium is equal to the fair price divided by the PV01[5]. Therefore, the fair premium must be proportional to the expected discounted loss under the risk-neutral probabilities.

In the case of our homogeneous portfolio, and assuming for simplicity that the term-structure of risk-free discount factors is flat and equal to 1, the fair price of 5-year protection is simply equal to the loss given default times the 5-year (risk-neutral) default probability of each reference name, ie, $60\% \cdot 4.88\% = 2.93\%$.

Because the value of protection on the whole portfolio does not depend on correlation, the buyer and the seller of this protection are not exposed to correlation risk. We will see in the next section that this is not the case when we consider slices of the portfolio loss distribution.

[5] *The PV01 is a term commonly used when discussing default swap premia. It is simply the present value of receiving one cent (annualized) until the (random) termination of the swap.*

7. LEVERAGE AND CORRELATION RISK OF LOSS TRANCHES

We now slice our simple portfolio into loss tranches, and consider a simple absolute priority structure where each tranche is determined by two loss levels: its lower bound (also called its "attachment point") and its upper bound. The attachment point of a tranche is the level of total portfolio losses required before the tranche experiences a loss. The upper bound of a tranche caps the loss to which the tranche is exposed, and is equal to the attachment point of the tranche just above it in the capital structure.

An investor can gain exposure to, and compensation for, the losses on a particular tranche by selling loss protection on the tranche. The investor, or protection seller, must reimburse the protection buyer for any loss incurred on the tranche during the life of the contract. In return, the protection buyer either makes an initial lump-sum payment or pays a recurring premium.

7.1 Leverage

For the same reasons mentioned in Section 2.4, the fair premium paid for protection on a given tranche is proportional to the expected loss on the tranche under the risk-neutral probabilities. In particular, a larger expected loss implies a higher fair premium. In this sense, the expected loss on a tranche is a measure of the leverage of the tranche; in what follows, we define the leverage of a tranche as the ratio between the expected loss on the tranche and the expected loss on the reference portfolio.

Figure 2 shows the expected loss and leverage of several different tranches as we vary the correlation parameter.

Figure 2. Expected loss and leverage of five tranches for different asset correlations

	Expected Loss (Leverage)				
Tranche	0%-6%	6%-12%	12%-18%	18%-24%	36%-42%
Asset Correlation					
0%	48.62% (16.6)	0.15% (0.1)	0.00% (0.0)	0.00% (0.0)	0.00% (0.0)
10%	45.03% (15.4)	3.52% (1.2)	0.20% (0.1)	0.01% (0.0)	0.00% (0.0)
20%	40.59% (13.9)	6.60% (2.3)	1.26% (0.4)	0.24% (0.1)	0.00% (0.0)
30%	36.33% (12.4)	8.49% (2.9)	2.68% (0.9)	0.88% (0.3)	0.05% (0.0)
40%	32.26% (11.1)	9.62% (3.3)	3.95% (1.3)	1.66% (0.6)	0.23% (0.1)
50%	28.30% (9.7)	10.34% (3.5)	4.67% (1.6)	2.90% (1.0)	0.71% (0.2)
60%	24.32% (8.3)	10.78% (3.7)	5.08% (1.7)	3.89% (1.3)	1.20% (0.4)
70%	20.25% (6.9)	11.03% (3.8)	5.36% (1.8)	4.12% (1.4)	1.18% (0.4)
80%	16.22% (5.5)	10.96% (3.7)	5.29% (1.8)	4.12% (1.4)	3.82% (1.3)
90%	12.97% (4.4)	9.46% (3.2)	4.48% (1.5)	4.12% (1.4)	4.18% (1.4)
100%	4.88% (1.7)	4.88% (1.7)	4.88% (1.7)	4.88% (1.7)	4.88% (1.7)

5-year horizon, homogeneous portfolio (1% risk-neutral default probability, 60% LGD)

7.2 Correlation risk

In addition to the risk of rising default probabilities, tranche protection sellers are faced with correlation risk. Unlike the expected loss on the entire portfolio, the expected loss on a tranche changes with changes in correlation, even when default probabilities stay constant. Given the direct relationship between the (risk-neutral) expected loss on a tranche and the fair value of protection, we find it useful to think of correlation risk as the sensitivity of the expected loss to changes in correlation. Whatever precise definition we use, the correlation risk of a particular tranche depends not only on its relative seniority, but also on the exact location of its bounds and on the present level of correlation.

Note how correlation risk changes along the seniority spectrum. In Section 2.3, we have seen that, as correlation increases, the portfolio loss distribution changes by relocating probability mass towards the end-points of the distribution. An increase in correlation therefore decreases the expected loss on the most junior tranche and increases the expected loss on the most senior tranche, as shown in Figure 2. In other words, sellers of equity protection and buyers of senior protection profit from an increase in correlation and incur a loss when correlation decreases. As for a mezzanine tranche, its sensitivity to changes in correlations depends on the location of its bounds and the current level of correlations.

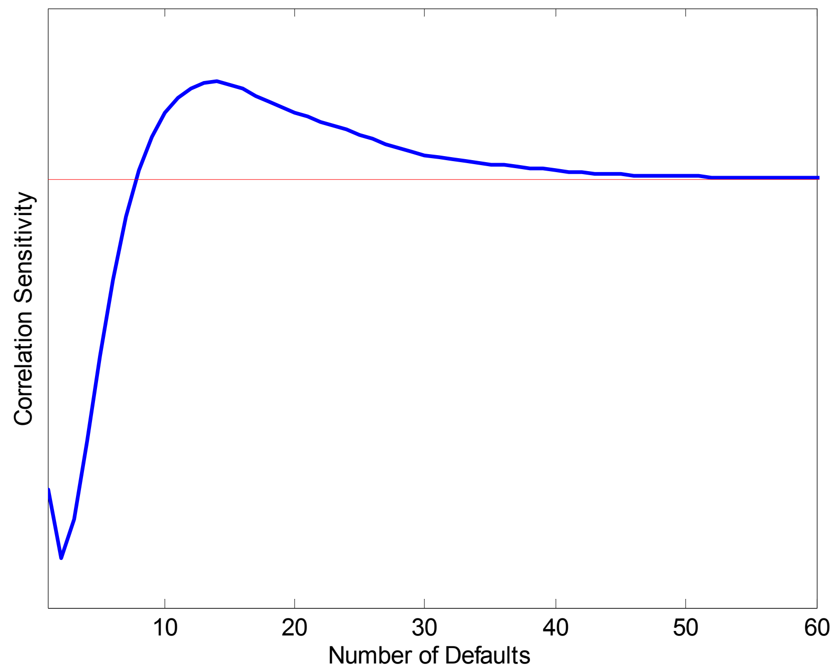
In summary, correlation risk changes sign from negative to positive as we move along the seniority spectrum of a given structure. This simple observation has an immediate consequence: it should be possible to obtain a desired correlation exposure by selecting an appropriately positioned (combination of) tranche(s).

We now show how to measure correlation risk across the seniority range of a given capital structure. Consider the homogeneous portfolio we defined earlier, and set each pairwise asset correlation ρ equal to 20%. We first compute the probabilities of having at least n defaults in the portfolio, for n=1, 2, …, 100. By definition, these are simply the probabilities of triggering n^{th}-to-default basket swaps. To see that these probabilities are directly related to our problem, we need to recognize that, with deterministic and homogeneous recovery rates, one can always express a loss tranche as a portfolio of n^{th}-to-default basket swaps referring to the same names. For example, the 6%-12% loss tranche on a 100-name portfolio where each name has 60% LGD, is equivalent to an equally-weighted portfolio of 10 n^{th}-to-default basket swaps, where n=11, 12,…, 20.

Next, we locally perturb the correlation matrix and compute the changes in these basket-triggering probabilities for a 1% uniform change of the correlation surface. Figure 3 plots these sensitivities, and gives us all the information we need to select a slice of the loss distribution with a desired correlation exposure.

The curve depicted in Figure 3 has only one “zero”, ie, it crosses the x-axis only once. The probability of having a number of defaults greater than or equal to any given value *below* this point *decreases* with an increase in correlation. Viceversa, the probability of having a number of defaults greater than or equal to any given value *above* this point *increases* with an increase in correlation. In the next two sections, we show how to use the information contained in Figure 3 to select exposures which are hedged against a uniform (local) movement of the correlation matrix.

Figure 3. Correlation sensitivity of the probability of exceeding *n* defaults



5-year horizon, homogenous portfolio (1% risk-neutral default probability, 60% LGD, 20% asset correlation)

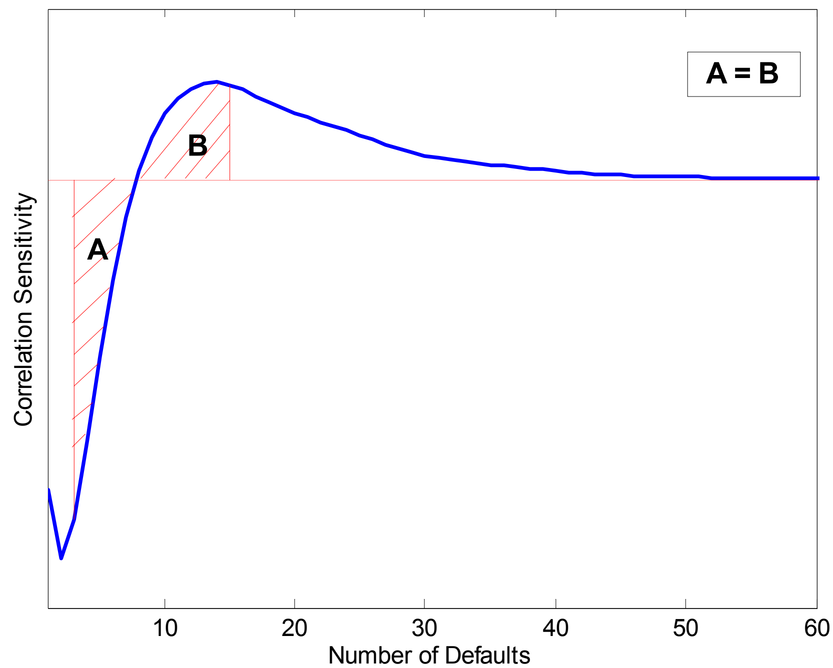
8. CORRELATION-HEDGED TRANCHES

Every correlation-hedged tranche can be created by picking a lower and an upper bound around the only zero of the function in Figure 3, in such a way that the integral between the selected bounds is equal to zero.

In particular, Figure 4 shows a correlation-hedged tranche that is exposed to defaults higher than the third and up to the fifteenth. By construction, the shaded area on the left of the zero (A) is equal to the shaded area on its right (B). Since the bounds are expressed in terms of number of defaults, one can translate them back into loss percentages using the known LGD. The tranche shown in Figure 4 covers portfolio losses in the range 1.80%-9%.[6]

[6] *With heterogeneous and/or stochastic recovery rates, we can do the same exercise by computing the correlation sensitivities of the attachment probabilities (i.e., the probabilities of exceeding different levels of portfolio loss). We lose the intuitive duality with basket-triggering probabilities, but we can still locate the correlation-neutral tranches in exactly the same way.*

Figure 4. Correlation sensitivity of the probability of exceeding *n* defaults: locating a correlation-hedged tranche



5-year horizon, homogeneous portfolio (1% risk-neutral default probability, 60% LGD, 20% asset correlation)

Figure 5 shows several correlation-hedged tranches and their leverage for different values of asset correlations, while keeping all other parameters at the previously defined levels (1% flat hazard rates, 60% LGD, 5-year horizon).

The last row says that the whole portfolio is, of course, a correlation-hedged tranche. The next -to-last row shows that the default-risky portion of the portfolio is also correlation-neutral, and has a higher leverage than the whole portfolio by a factor equal to 1/LGD. The remaining cells show that, for a given correlation level, the range of leverage achievable with correlation-neutral tranches is somewhat limited. The reason is that, as soon as one raises the lower bound to eliminate the exposure to the first default, correlation-neutrality requires moving down the upper bound of the tranche from 60% to a much lower level (eg, 16.2% with 20% correlation); this significantly increases the expected loss of the new tranche, leaving a whole range of leverages unattainable.

One way to attain a wider range of expected losses while keeping the local neutrality to correlation changes is to create a combination ("combo") of tranches referencing the same portfolio.

Figure 5. Correlation-hedged bounds and leverage for different asset correlations

	Upper Bound (Leverage)				
Asset Correlation	10%	20%	30%	40%	50%
Lower Bound					
6.6%					7.8% (4.2)
6.0%				7.2% (4.6)	8.4% (4.2)
5.4%				7.8% (4.6)	9.6% (4.2)
4.8%			6.0% (5.8)	8.4% (4.7)	10.2% (4.2)
4.2%			6.6% (5.9)	9.0% (4.8)	11.4% (4.1)
3.6%		5.4% (7.5)	7.2% (6.0)	10.2% (4.7)	13.2% (3.9)
3.0%	4.2% (10.4)	6.0% (7.6)	8.4% (5.9)	10.8% (4.9)	16.2% (3.6)
2.4%	5.4% (9.6)	7.2% (7.4)	9.6% (5.8)	13.2% (4.5)	19.2% (3.4)
1.8%	6.6% (9.3)	9.0% (6.9)	12.0% (5.3)	16.2% (4.2)	21.0% (3.4)
1.2%	8.4% (8.6)	11.4% (6.3)	15.6% (4.7)	19.8% (3.8)	24.0% (3.3)
0.6%	12.0% (7.1)	16.2% (5.2)	20.4% (4.2)	25.2% (3.4)	31.2% (2.8)
0.0%	60.0% (1.7)	60.0% (1.7)	60.0% (1.7)	60.0% (1.7)	60.0% (1.7)
0.0%	100.0% (1)	100.0% (1)	100.0% (1)	100.0% (1)	100.0% (1)

5-year horizon, homogeneous portfolio (1% risk-neutral default probability, 60% LGD)

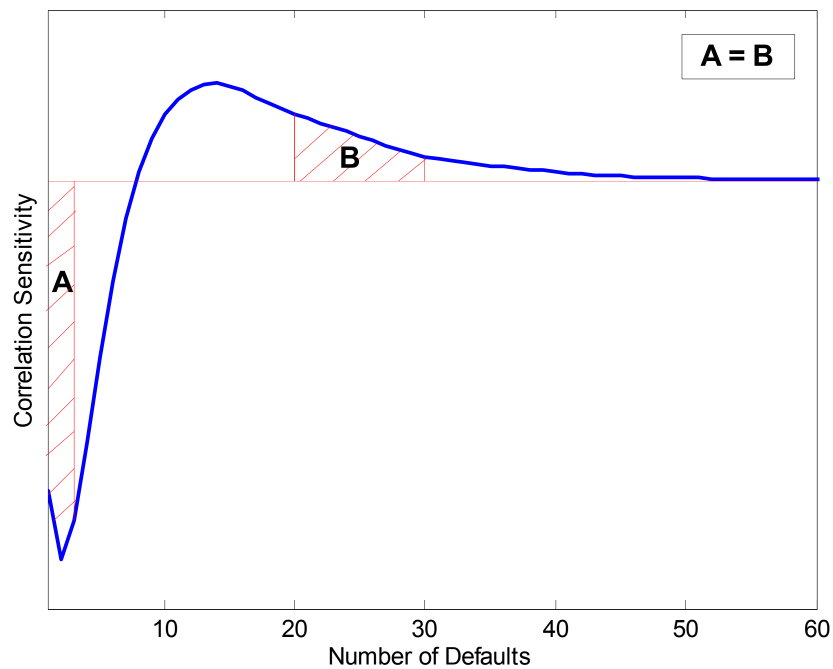
9. CORRELATION-HEDGED COMBOS

For a given set of names in the reference portfolio, we can view a single tranche as a portfolio of tranches that:

1. are adjacent to each other; and
2. refer to the same notional.

The flexibility of a synthetic structure allows for both of these constraints to be relaxed; the advantage of doing so is that we can maintain local neutrality to correlation risk while improving our ability to achieve a desired leverage. Figure 6 shows that we can zero out our correlation exposure by selecting a combination of different tranches across the seniority spectrum. It is easy to verify that this allows us to achieve a wider variety of expected losses while preserving neutrality to correlation movements.

Figure 6. Correlation sensitivity of the probability of exceeding *n* defaults: locating a correlation-hedged combo

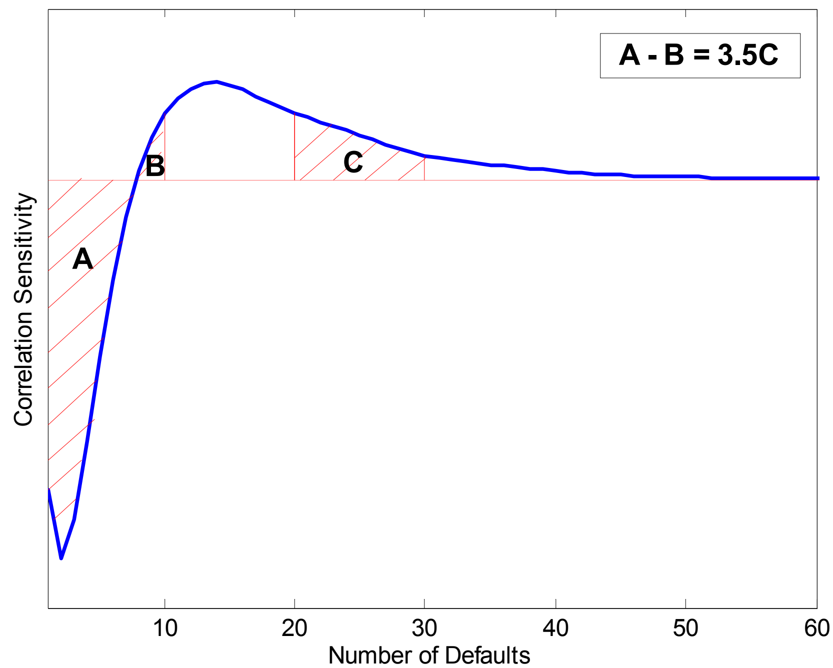


5-year horizon, homogeneous portfolio (1% risk-neutral default probability, 60% LGD, 20% asset correlation)

It is also possible to focus on specific portions of the capital structure, and set up correlation-neutral positions by having the different components of the combo refer to different notionals. Of course, the chosen tranches must have correlation exposures of both positive and negative signs for this to be possible.

Figure 7 shows that we can achieve a correlation-hedged position by selling protection on the 0%-6% and 12%-18% tranches, as long as the mezzanine slice refers to a notional that is 3.5 times higher than the one for the equity piece. This is because the difference between area A and area B in Figure 7, which represents the correlation exposure of the 0%-6% tranche, is about 3.5 times larger than area C, which is the correlation exposure of the 12%-18% tranche.

Figure 7. Correlation sensitivity of the probability of exceeding *n* defaults: locating a correlation-hedged combo with variable notionals



5-year horizon, homogeneous portfolio (1% risk-neutral default probability, 60% LGD, 20% asset correlation)

The possibility of creating a correlation-neutral position while focusing on specific portions of the capital structure offers an extra degree of freedom that can be used to implement one's views. An investor may feel that a senior tranche is trading "cheap" relative to the rest of the capital structure, and may prefer to achieve a mezzanine-like leverage by holding a combo of equity and senior tranches. If neutrality to correlation is also a goal, then the two objectives can be obtained simultaneously by constructing a correlation-hedged combo with variable notionals.

Figure 8 shows the loss distributions for five different tranches referring to the same portfolio described above, while Figure 9 shows delta,[7] expected loss and leverage of four correlation-hedged combos obtained by appropriately combining the junior loss tranche with each of the remaining four.[8] These results show that it is indeed possible to build correlation-hedged combos with a variety of leverage ratios.

[7] *The delta of a combo is the dollar notional of a combo's senior tranche per dollar notional of the combo's junior tranche.*

[8] *Any risk more senior than the 36%-42% loss tranche cannot be used to form a correlation-hedged combo, because its expected loss is flat at zero and has virtually no sensitivity to local correlation movements.*

Figure 8. Loss distributions and expected losses of five tranches

Loss	Tranche	0%-6%	6%-12%	12%-18%	18%-24%	36%-42%
0%		15.81%	2.43%	0.42%	0.09%	0.00%
10%		15.16%	2.01%	0.36%	0.07%	0.00%
20%		12.62%	1.68%	0.31%	0.06%	0.00%
30%		10.21%	1.41%	0.26%	0.05%	0.00%
40%		8.22%	1.18%	0.22%	0.04%	0.00%
50%		6.63%	0.99%	0.18%	0.04%	0.00%
60%		5.37%	0.82%	0.15%	0.03%	0.00%
70%		4.38%	0.69%	0.13%	0.03%	0.00%
80%		3.59%	0.58%	0.11%	0.02%	0.00%
90%		2.95%	0.49%	0.10%	0.02%	0.00%
100%		15.07%	2.79%	0.55%	0.10%	0.00%
	Exp Loss	40.59%	6.60%	1.26%	0.24%	0.00%

5-year horizon, homogeneous portfolio (1% risk-neutral default probability, 60% LGD, 20% asset correlation)

Figure 9. Delta, expected loss and leverage of four correlation-hedged combos

Combo	0%-6% 6%-12%	0%-6% 12%-18%	0%-6% 18%-24%	0%-6% 36%-42%
Delta	1.75	3.51	10.02	172.64
Exp Loss	18.6%	10.14%	4.09%	0.26%
Leverage	6.36	3.47	1.40	0.09

5-year horizon, homogeneous portfolio (1% risk-neutral default probability, 60% LGD, 20% asset correlation)

10. SUMMARY

We have presented a simple methodology to locate correlation-neutral credit exposures along the seniority spectrum of a synthetic structure. After showing that an investor can always obtain a correlation-hedged exposure by choosing an appropriately positioned mezzanine tranche, we have argued that the same objective can be reached without constraining the leverage of the resulting investment. In particular, we have shown that synthetic combos with variable notionals offer investors the possibility of virtually separating the choice of yield and correlation risk.

Our results confirm that the flexibility of synthetic investments can offer significant advantages to the credit investor. The recent increase in popularity of synthetic loss tranches is largely due to the fact that these instruments allow for a high level of customization. What we have shown in this article is that the extra degrees of freedom can be used to monitor and manage correlation risk in a portfolio of multi-name credit exposures. We have also pointed out that this should be of particular interest for US public companies, since they are required by FAS 133 to periodically recognize the changes in value of their open derivative contracts.

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Credit spreads compensate investors for various types of risk associated with credit instruments. In addition to the fundamental risk of default, these include market risks such as sector-wide spread volatility, credit deterioration and ratings downgrades of specific issuers. For buy-and-hold investors, short-term spread volatility and liquidity squeezes are of little importance. The only risk that affects them directly is the default risk. We conducted a simple analysis of the trade-off between spread and default risk for buy-and-hold investors in a portfolio of corporates and in CDO tranches. As the payout of a cash flow or synthetic CDO tranche is determined by the realized default and recovery rate on the underlying portfolio, the analysis of a CDO is closely related to the analysis of a buy-and-hold portfolio. [1]

1. INTRODUCTION

Credit spreads compensate investors for various types of risk associated with credit instruments. In addition to the fundamental risk of default, these include market risks such as sector-wide spread volatility, credit deterioration and ratings downgrades of specific issuers. Spreads also include a liquidity premium, reflecting the risk that in a crisis, the issue may be very difficult to sell. Every investment decision is based on the investor's perception of this risk and reward trade-off. Yet, **not all investors are necessarily subject to all risks. To the extent that an investor is protected from a particular risk that is built into the "compensation" part**, there is potential for superior risk-adjusted returns. The volatility and liquidity risks, for example, are relevant only to total-return investors who need to mark-to-market or trade the security. This is certainly the case for active managers and enhanced indexers. Buy-and-hold investors, on the other hand, could view credit spreads from a very different perspective. For such investors, short-term spread volatility and liquidity squeezes are of little importance. The only risk that affects them directly is the default risk[2]. Yet, they still stand to benefit from spreads that presumably compensate active managers for their exposures to short-term market risk.

The structuring of a CDO, with either a cash flow or synthetic structure, is designed to monetize spread components not attributable to default risk, or the difference between the spread implied default rate and the realized default rate. As the payout of a CDO tranche will be determined by the realized default and recovery rate on the underlying portfolio, the analysis of a CDO is closely related to the analysis of a buy-and-hold portfolio. In both cases, the analysis must center on long-term considerations of default and recovery rather than shorter-term issues like spread volatility and liquidity.

We conducted a simple analysis of the trade-off between spread and default risk for buy-and-hold investors. Our analysis shows that spread levels at different points in time can provide a significant cushion against default risk, and that in order for a diversified corporate portfolio to underperform Treasuries over a 10-year holding period, the default rate would need to be

[1] *The authors would like to thank Dominic O'Kane, Abhishek Kalra and Vadim Konstantinovsky for their extensive help on this report. This report extends the analysis that we had undertaken on buy-and-hold portfolios in the Quantitative Portfolio Strategy section of* Global Relative Value*, 9 December 2002 and 10 February 2003.*

[2] *We realize that this is a simplification. Certain book value investors, for example, may be subject to downgrade risk (eg, insurance companies with risk-based capital requirements).*

significantly higher than historical averages. When we include the effect of issuer correlations in our model, we find that higher correlations make such extreme occurrences more likely, but that diversified credit portfolios still offer a very favorable risk/return profile.

2. METHODOLOGY

To make the case that the credit market can be attractive relative to Treasuries for a buy-and-hold investor at various spread levels, we first compare holding period returns on portfolios of non-callable 10-year credits with those of 10-year Treasuries. We then apply the conclusions to a CDO context and make the case for investing in CDO tranches tailored to one's risk appetite.

The two rating categories considered are single-A and Baa. We use a very simple, stylized analysis that ignores the effects of reinvestment and inflation. Our interpretation of the buy-and-hold assumption is that, in the absence of defaults, we expect the annualized total return to equal yield to maturity. Default is modeled as a one-period problem – each bond either defaults during the next ten years or survives to maturity. Bonds that default within that period do not contribute anything to the cumulative performance beyond their recovery value.

Essentially, we assume that all bonds that default do so at the very beginning. This makes our analysis more conservative because the coupons received prior to default are ignored[3].

We assume a homogeneous equally-weighted portfolio of n bonds, each from a different issuer, and each sharing the same credit quality and the same default and recovery assumptions. Let y denote the average portfolio yield, and R the average recovery rate on defaulted bonds. The number of bonds $n_{default}$ that default before reaching maturity is considered as a random variable. Conditioned on $n_{default}$, our simple model for the terminal value of the portfolio after ten years is

$$(1) \qquad term\_value = \left(1 - \frac{n_{default}}{n}\right)(1+y)^{10} + \frac{n_{default}}{n} \cdot R = (1 + ann\_ret)^{10}$$

The terminal value of a \$1 investment in such a credit portfolio (or the equivalent annualized return) is expressed in equation (1) as a function of the realized portfolio default rate $D = \frac{n_{default}}{n}$. In Figure 1, we use our simple model of portfolio return to show what default assumptions would be needed to break even with Treasuries at a given spread level. The figure is based on a Treasury yield of 4%, but will change only slightly with small changes in Treasury yield. At this level, using simple annual compounding, a 10-year Treasury investment of \$1 will have a terminal value of 1.48 $\left(=1.04^{10}\right)$, while a credit investment with a spread of 200bp will have a terminal value of 1.79 (if it does not default). Assuming a 20% recovery rate, equation (1) tells us that a realized portfolio default rate of 19.5% would make the return on the credit portfolio equal to the Treasury return. This breakeven default rate demonstrates just how much cushion can be generated by credit spreads – with a spread of 200bp, we can experience nine defaults in a 50-bond portfolio and still outperform Treasuries!

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[3] *Equation (1) assumes that today's yields for both credit and Treasuries will be compounded throughout the 10-year period. This neglects the role of reinvestment risk. While both the credit portfolio and its equivalent Treasury counterpart are subject to reinvestment risk, the possibility that spreads may tighten over the horizon may reduce the overall return advantage over the period. We do not believe that this simplification has a significant impact on our results.*

Figure 13. Breakeven cumulative 10-yr portfolio default rates, 20% recovery[4]

Corporate Spread (bp)	Corporate Yield (%)	Corporate Terminal Value	Breakeven Default Rate (to Treasuries)
100	5	1.63	10.4%
125	5.25	1.67	12.8%
150	5.5	1.71	15.1%
175	5.75	1.75	17.4%
200	6	1.79	19.5%
225	6.25	1.83	21.6%
250	6.5	1.88	23.7%
275	6.75	1.92	25.6%
300	7	1.97	27.6%
325	7.25	2.01	29.4%
350	7.5	2.06	31.2%
375	7.75	2.11	33.0%
400	8	2.16	34.6%
Treasury yield			4%
Treasury terminal value			1.48
Recovery Rate			20%

See Appendix for historical average default rates.

From a CDO perspective, we can carry out a similar type of analysis, but the breakeven calculation will be somewhat different. Depending on its place in the capital structure, each tranche of a CDO will break even at a different realized rate of defaults on the underlying portfolio. In Figure 2, we apply this analysis to tranches of a cash flow investment-grade CDO issued in autumn 2002. Underlying this CDO is a portfolio of 100+ bonds with average quality Baa and an average life of 8.5 years. The three tranches shown in Figure 2, like most CDO tranches, pay a floating rate coupon based on a spread to LIBOR. We therefore computed breakevens to LIBOR rather than Treasuries, by varying the assumed default rate on the collateral until we obtained a discount margin equal to zero for each tranche.[5] . At that default rate, the portion of the default losses absorbed by the tranche exactly offset the tranche spread, and the investment effectively earns no spread over LIBOR. Unlike in Figure 1, in this calculation, defaults are not assumed to all occur at the beginning of the period, but are uniformly spaced out in time under the assumption of a constant annual default rate (CADR). We assumed a conservative recovery rate of 20% to be consistent with the rest of the analysis.

[4] *The 20% recovery assumption is conservative compared with the historical average of 41% for senior unsecured credits.*

[5] *The calculation of the cash flows of each tranche as a function of the collateral default rate depends on the details of the CDO structure, and therefore cannot be easily approximated using the simple form of equation (1). The data in Figure 2 were obtained by changing the assumptions fed into a calculator for valuing the example CDO.*

Figure 2. CDO default rate breakeven analysis

Tranche Rating	Tranche Spread Over Libor (bp)	Breakeven Realized Default Rate
Aaa / AAA	55	28.0%
Aa2 / AA	125	18.8%
A2 / A	250	13.4%

Assumptions described in text.

A hasty comparison between Figures 1 and 2 might well raise some eyebrows. The difference between the two tables is far greater than can be explained by the difference between Treasury and LIBOR breakevens. Why is it that the breakeven portfolio default rates increase with rising spread in Figure 1, and decrease with rising spread in Figure 2? The answer is that we are comparing two very different investment propositions. In Figure 1, as we increase the spread, we are changing the characteristics of the bond portfolio. Presumably, higher spreads will correspond to groups of bonds with higher expected default rates and lower quality ratings. For bonds trading at a spread of 125bp, the market expectations of default will be such that a realized default rate of 12.8% over ten years is deemed very unlikely. In Figure 2, however, all tranches depend on the default experience of the same underlying portfolio of Baa-rated bonds. The increased ratings and lower spreads as we move up the table are not due to a decreased expectation of the portfolio default rate, but rather to the increased protection offered by the structure of the CDO and correspondingly the decreased loss probability of the CDO tranche.

In addition, the breakeven default rates were computed under different assumptions on the timing of default. If we recalculate Figure 1 under the CADR assumption rather than with all the defaults up front, we would find that to break even at a spread of 125bp, cumulative defaults could be 16.3% rather than 12.8%. In our example, for the same 125bp of spread, the AA CDO tranche can tolerate a higher portfolio default rate of 19.0% to LIBOR (and somewhat higher to break even with Treasuries) than a portfolio of corporates that earn the same spread (16.3% breakeven default rate). It must also be kept in mind that as the CDO collateral is composed of lower-rated debt than the bond portfolio, higher default rates can be expected.

Of course, the breakeven default rate tells only a small part of the story. A bond portfolio with an average spread of 125bp over LIBOR will have a very different return distribution than the CDO tranche shown here at that spread. The portfolio return will degrade smoothly as the realized default rate increases, while the CDO tranche will have a much more abrupt change in returns as the realized portfolio losses cross some threshold. For any low to moderate realization of portfolio defaults, the CDO tranche should be expected to outperform, but in an extreme crisis, the losses on the CDO tranche are likely to be much greater.

3. USING THE BINOMIAL MODEL FOR PORTFOLIO DEFAULT RATES

In equation (1) above, we tied the realized terminal value of the portfolio to the realized cumulative portfolio default rate – the proportion of bonds in the portfolio that go into default over the next ten years. This is related to the market-wide cumulative default rate, which we denote by p. This is the proportion of bonds in the marketplace that will default over the next ten years, or the cohort default rate. Neither of these quantities is known yet, and they must be treated as random variables.

In the simplest approach, we do not attempt to model the distribution of *p*. We simply model the distribution of the portfolio default rate conditioned on an assumed value for *p* using the binomial distribution. The number of defaults $n_{default}$ in a portfolio follows the binomial distribution with parameters *n* and *p*,

$$P(n_{default} = k) = \binom{n}{k} p^k (1-p)^{n-k} \tag{2}$$

where *n* is the number of bonds in the portfolio and *p* is the assumed market-wide default rate.

Conditioned on a given market default rate *p*, the distribution of $n_{default}$ given in equation (2) can be translated to a distribution of portfolio returns via equation (1).

This simple model can be applied in various ways. We can choose a conservative assumption for the market default rate on a particular market segment, and determine how much worse the realized defaults (and hence the returns) could be on the portfolio due to poor security selection on an n-bond portfolio. If we target a particular number for the worst case portfolio return with 95% (or 99%) confidence, or the confidence level with which we will break even with Treasuries, we can back out the portfolio size required. Alternatively, we can fix the portfolio size, and back out what market-wide default rates can be tolerated while still maintaining a desired level of confidence that our portfolio will outperform Treasuries.

To be conservative, we have applied this model using values of *p* that exceed the highest observed cohort default rates for a given credit rating. As shown in the Appendix, the values chosen for the 10-year cumulative default probabilities are 5% for A-rated bonds and 10% for Baa. We will look at some numeric results of this analysis later in this article.

All the same types of questions would also be relevant in a CDO context. The question of how many bonds to include would be of interest to the CDO manager at the time of structuring the deal. Investors could use the binomial model to compute the distributions of realized defaults and tranche returns under different assumptions for the overall market default rate.

4. ADDRESSING DEFAULT CORRELATION

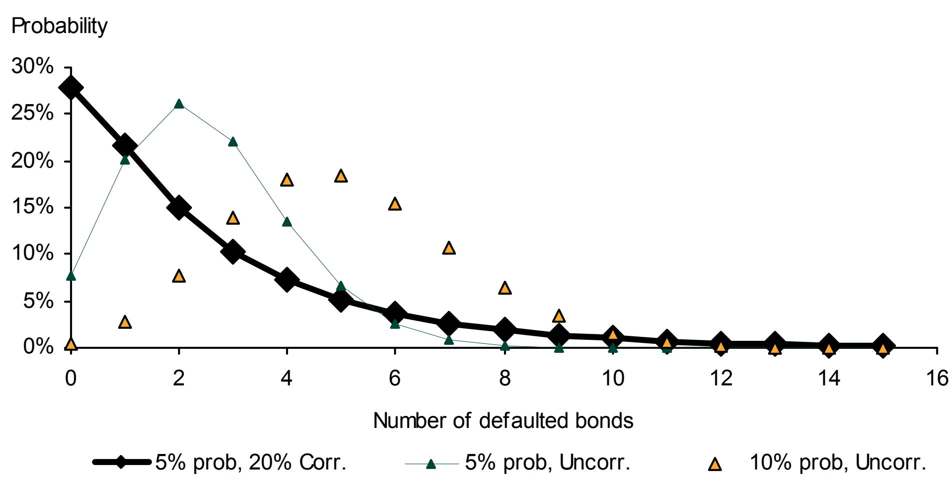
The use of the binomial model with a constant default rate is equivalent to assuming that whether a given issuer will default is independent of the outcome for any other issuer. However, the big risk for buyers of a diversified corporate bond portfolio versus Treasuries is that difficult economic conditions could produce a wave of defaults throughout the sector. This common dependence of all credits on overall market conditions gives rise to a systematic risk that causes correlation of default events among the various issuers.

A simple way to address the issue of correlation is to use the binomial model with a value of n that is smaller than the actual number of bonds in the portfolio. Moody's uses this approach in evaluating CDOs. They analyze the portfolio underlying the CDO based on allocations to issuers and industries, and use intra-industry correlations to calculate a diversity score, which represents the effective number of independent bonds in the portfolio. They then use the binomial model with *n* equal to the diversity score and *p* set to the historical default probability corresponding to the WARF (weighted average rating factor) of the portfolio.

A more comprehensive consideration of correlations is achieved using a firm-value model in which correlations among issuers are used to derive a distribution for the number of defaults in an n-bond portfolio. The model we will use is a default-mode CreditMetrics™ portfolio credit model discussed by our colleagues in an earlier publication. [6]

In Figure 3, we plot this distribution for a 50-bond portfolio with an expected market default rate of 5% and a correlation assumption of 20%. We compare this distribution with those produced by the uncorrelated case (the plain binomial distribution) using market default rates of 5% and 10% (corresponding to the worst observed cohort default rates for an A and a Baa rated portfolio respectively). First let us compare the correlated and uncorrelated cases using the same 5% value for the expected default rate. In this case, for a 50-bond portfolio, the expected number of defaults is 2.5 for both the correlated and uncorrelated cases. The binomial distribution with no correlations has its peak near this value, and a relatively short tail. In the correlated case, the distribution shows a decreased probability of realizing the average default rate, and increased probabilities of either extremely high or extremely low defaults.

Figure 3. Distribution of number of defaulted bonds in a 50-bond portfolio. 5% expected default probability with 20% correlation, uncorrelated model with market default rates of 5% and 10%



If we increase the market default rate to 10% in the uncorrelated binomial distribution, the whole distribution shifts to the right, and the tail of the distribution includes high probabilities that eight, nine, or ten bonds may default over the period. This comes much closer to the tail of the correlated distribution with a 5% expected default rate. However, even in this case, the probability of 11 or more bonds defaulting is higher in the correlated model.

The correlation model used here takes advantage of the simplifying assumption that any two issuers are related by the same correlation coefficient. In reality, the correlations among different issuers reflect two types of factors: general macro-economic trends that affect all issuers, and industry-specific circumstances that can affect a particular sector of the market. A generally accepted market practice is to assume 30% correlation among issuers within the same industry, and 15% correlation between issuers from different industries. As the model

[6] *For a detailed description of this model, see O'Kane and Schloegl [2001], pages 35-37. The model is originally due to Vasicek [1987].*

uses just a single coefficient, 20% seems like a reasonable value. Although our model cannot account for industry-specific correlations[7], these can be avoided in large part by diversification of industry exposures in the portfolio. If lack of liquidity in the market makes such diversification impossible, our breakeven default rates would have to be adjusted upwards for industry correlations. Nevertheless, we believe it is feasible under most market conditions to construct a corporate portfolio of 20 or 50 names well-diversified across industries.

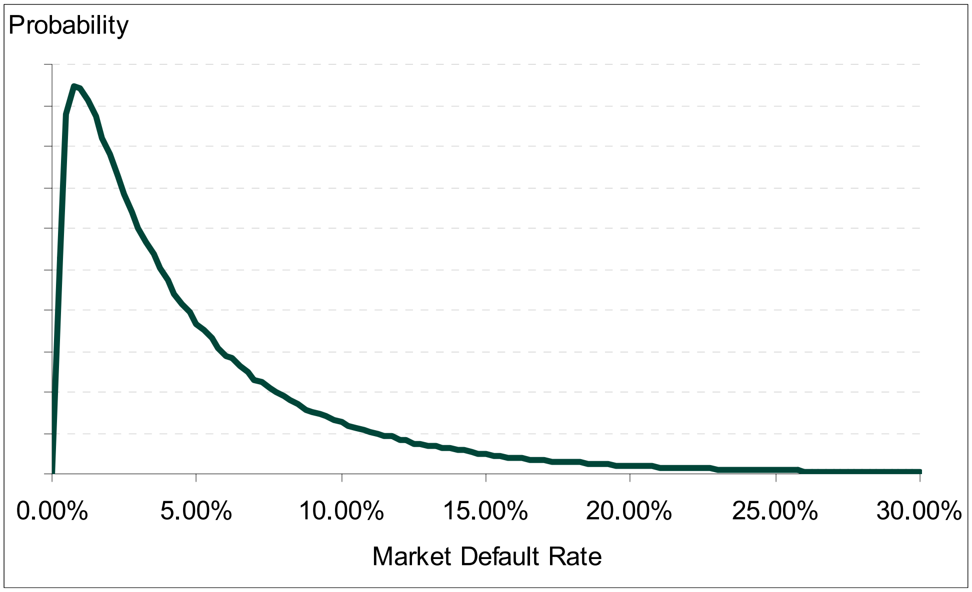
Figure 3 illustrates two methods for identifying "worst case" Baa portfolio default rates. Historical data on 10-year cumulative defaults on Baa securities indicate a long-term average default rate of about 5%, with the worst observed cohort experiencing a default rate just under 10%. Using the simple binomial model, a worst case assumption for realized portfolio defaults can be obtained by using the tails of the binomial distribution with the highest observed default rate of 10%. In the correlated model, we use an expected default rate of 5%, and the fat tails of the distribution are generated by the 20% correlation assumption.

To develop a bit more insight into the results of the correlated model, let us look at the model in slightly more detail. It is based on the idea that the asset returns of issuer firms, which can trigger defaults when they go below some threshold, are all correlated to a single variable Z, which represents the asset returns due to a common market factor. The correlation of each issuer's asset return to that of the market is β, resulting in a correlation of β^2 between any two issuers. The market default rate *p(Z)* is then shown to be a function of this normally distributed market factor. The model follows equation *(2)* for the distribution of the number of defaults conditioned on a given market default rate *p(Z)*. The advantage of this model is that p is no longer considered a constant whose value we must posit, but is now modeled as a random variable with a known distribution. The shape of this distribution ultimately depends on two parameters: the expected default probability and the correlation.

This distribution of the realized market default rate *p* is shown in Figure 4 for a default probability of 5% and a correlation of 20%. Note that the correlation model considers the possibility of market default rates as high as 25%. According to this assumption, the overall probability of a market default rate worse than 10% is 13.6%. When we used the binomial model with a "worst case" market default rate of 10%, the tail of the portfolio default distribution was entirely due to the portfolio underperforming the market due to poor security selection in a small portfolio. We see now that the reason for the increased tail probabilities shown in Figure 3 is that **the correlation model considers the possibility of much higher market default rates** as well. This is a systematic risk that cannot be diversified away.

[7] *The model can compute portfolio loss distributions assuming a different beta for each asset. However, this would complicate the analysis without necessarily changing any of the main results.*

Figure 4. Distribution of market default rate implied by correlation model. Expected cumulative default rate 5%, correlation 20%



5. MEASURING TAIL RISK (OR WORST CASE RISK) IN INVESTMENT GRADE PORTFOLIOS

Using our above methodology for default distributions and correlations, we compare the worst case realized portfolio default rates at 95% confidence levels using two different assumptions. The first is the worst case assumption that we used in the uncorrelated case, with the market default rate assumed to take on its worst observed historical value (5%, and 10%) but with no correlations. The second assumes asset correlations of 20%, with the expected value of the market default rate set to the long-term historical average (2% and 5%)[8]. (The default distributions corresponding to these two sets of assumptions were shown in Figure 3 to have fairly similar tails.) The results are shown for portfolios of 20, 50, and 100 bonds, using default probabilities characteristic of A and Baa ratings. **The assumption of 20% correlation reduces the advantage of increasing the portfolio size from 20 to 50 or 100 bonds.** Also, while the results in both cases are similar for the 20-bond case, differences do arise for the larger size portfolios.

[8] *It should be noted that the confidence intervals shown here are consistent with the Vasicek model described above. Models with tail dependence can produce higher estimates of the probability of extreme events, and will lead to greater estimates of worst case losses. See O'Kane and Shloegl [2002] for details.*

Figure 5. Comparing worst case realized portfolio default rates using historical average 10-year default rates with 20% correlation and worst case historical default rates with 0% correlation

A-rated portfolios		95% Confidence Number of bonds			99% Confidence Number of bonds		
	Correlation	20	50	100	20	50	100
Historic Worst Case: p = 5%	0%	15%	10%	9%	20%	**14%**	11%
Historic Mean: $E[p]$ = 2%	20%	10%	8%	8%	20%	16%	**14%**

BBB-rated portfolios		95% Confidence Number of bonds			99% Confidence Number of bonds		
	Correlation	20	50	100	20	50	100
Historic Worst Case: p = 10%	0%	20%	18%	15%	30%	**20%**	18%
Historic Mean: $E[p]$ = 5%	20%	20%	18%	16%	30%	28%	**26%**

CDOs are generally structured with a portfolio size of about 100 and can have average qualities of both A and BBB. The table in Figure 5 shows that with 99% confidence we estimate that stylized A-rated CDO portfolios will experience 14% or less default rates, and BBB portfolios will experience 26% or less default rates respectively, assuming that the portfolio default rate follows the historical average and a 20% correlation. Another way to interpret the results is that for a portfolio with a diversity score of 50 (or 50 independent bonds), even with the market default rate assumed to equal the worst observed cohort results, we can say with 99% confidence that the portfolio default rate will be 14% or less for an A-rated portfolio and 20% or less for a BBB-rated portfolio (Figure 5).

6. SPREADS ON THE PORTFOLIO COMPENSATE FOR DEFAULT RISK

Using the breakeven spread methodology illustrated in Figure 1, we revisit the comparison of the two models shown in Figure 5. For a 50-bond Baa portfolio, the uncorrelated model with an assumed 10% market default rate shows 99% confidence that 20% or less of the portfolio will default. Figure 1 shows that, using this assumption, a spread of 200-225bp would be sufficient to ensure out-performance with 99% confidence. However, for a 100-bond portfolio to achieve this level of confidence using the correlated model, one would need to allow for a realized portfolio default rate of up to 26%, which would require a spread greater than 275-300bp to break even with Treasuries.

We now combine the breakeven analysis on the example CDO that was introduced in Figure 2 with the default analysis in Figure 5. A CDO tranche performance depends directly on the default rate of the portfolio. A portfolio that has bought collateral at a higher spread clearly has a higher breakeven default rate to Treasuries. In our example we used a portfolio that had ramped up at 320bp, and therefore can support a higher default rate to breakeven to Treasuries or other risk free rates (Figure 6). Today a similar portfolio would be at a spread of about 200-250bp. As discussed above, the bond portfolio consists of over 100 bonds of Baa quality. We compared the breakeven 10-year portfolio default rates to the probability distribution of a 100-bond portfolio with 20% correlation and a 5% expected default rate.

Figure 6. Probability of CDO Default Rate Breakeven Scenarios

Tranche Rating	Tranche Spread Over Libor (bp)	Breakeven Realized Default Rate	Prob. Of Portfolio Default Rate < Breakeven Rate
Aaa / AAA	55	28.0%	>99%
Aa2 / AA	125	18.8%	>97%
A2 / A	250	13.4%	>92%

The results presented in Figure 6 show that senior CDO tranches are structured such that the breakeven default scenarios have very low probabilities of occurrence. There is less than a 1% chance that the AAA tranche will underperform LIBOR and similarly less than a 3% chance that the AA tranche will underperform LIBOR. The probability of the A-rated tranche underperforming is a relatively high 8%. This number is significantly affected by our very conservative recovery rate assumption. For example, if we increase the recovery rate to 30%, the probability of underperformance goes down to <5%

7. CONCLUSIONS

The average index OAS for A and Baa credit is 144bp and 221bp over Treasuries respectively, implying a 15% and 22% breakeven default rate to 10-year Treasuries respectively (from Figure 1). These default rates are much higher than even the worst historical cohort, which experienced 5% and 10% default rates respectively. Even with a correlation assumption that allows for much higher default rates, the breakeven probabilities for reasonably sized portfolios are well above 95%, despite the significant tightening in the corporate market over the past five months. Based on this analysis, buy and hold investors in general, and CDO investors/ sponsors in particular, can take advantage of this spread differential to execute a mean reversion assumption in default rates.

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APPENDIX

An historical perspective

To put our findings into an historical perspective, we obtained 10-year cumulative default rates from Moody's. Figure 1 shows such rates for two investment-grade rating categories, A and Baa, issued from 1970 through 1992 (so that the last 10-year frame is 1992-2001). Although for some groups the realized default rates could be as high as 9.7% (1982 Baa cohort), the average annual rates are relatively modest at 1.57% for single-A and 5.09% for Baa[9]. We will proceed to show that these rates would not be nearly enough to justify the current spread levels if defaults were the only source of risk in the market. Another critical assumption, besides the default rate, is the recovery rate. According to Moody's, although historical recovery rates for defaulted bonds span the range from 0% to 100%, the average historical recovery rate was 41%, with a standard deviation of 28%; the median recovery rate was 35%. In our analysis, we will use a fixed recovery rate assumption; to be conservative, we have used a value of 20% throughout this article.

Figure 1. Average 10-yr cumulative default rates for yearly cohorts

	10-year	
Annual Cohort	**A**	**Baa**
1970	0.45%	2.86%
1971	0.84%	2.74%
1972	0.00%	3.10%
1973	0.40%	3.55%
1974	0.80%	3.73%
1975	0.39%	3.77%
1976	0.67%	4.90%
1977	1.30%	5.04%
1978	1.33%	4.84%
1979	2.78%	4.79%
1980	3.26%	5.82%
1981	3.08%	8.56%
1982	3.82%	9.70%
1983	3.55%	7.99%
1984	4.21%	6.61%
1985	4.85%	6.70%
1986	2.40%	8.62%
1987	1.75%	7.20%
1988	1.45%	5.25%
1989	0.58%	3.50%
1990	0.00%	1.17%
1991	0.28%	1.18%
1992	0.49%	3.37%
Average	**1.57%**	**5.09%**
Worst case	**4.85%**	**9.70%**

Source: Moody's Investors Service, February 2002.

[9] *Note that Moody's weights the cohort default rates by cohort size (number of issuers) when calculating the long-term average default rates, so that a simple average will give slightly different numbers.*

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