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LIABILITY-BASED BENCHMARKS

Introduction

Plan sponsors and investment managers are well acquainted with market-based fixed-income indices (e.g., the Lehman Government/Credit Index). These indices are defined as a set of well-publicized rules that govern which bonds are added and deleted. When a market-based index reflects the risk preferences of the plan sponsor and the investment opportunities facing the investment manager, the index serves as a useful tool for performance evaluation and risk analysis. In other words, the index is a "neutral" benchmark, and the manager is evaluated based on performance versus the index. While the sponsor may impose some additional investment constraints (e.g., credit and issuer concentration and limits on deviations from the index), the sponsor otherwise wants the manager to be unfettered within the confines of the index in the search for added returns.

However, some investment managers must operate in a more constrained environment. A plan's assets may be "dedicated" to satisfying a well-defined liability schedule, and assets must be managed to satisfy those liabilities.[1] In these cases, the sponsor specifies, based on risk preferences, the universe of bonds in which the manager may invest and the liability schedule that must be satisfied. Often the investable universe is defined as a market-based index. However, the index will usually have a term structure that is very different from the liability schedule (e.g., the liability schedule may have a longer duration than the market index).

The manager now has two goals: produce added returns to help the plan achieve its long-term investment goals and, simultaneously, keep the portfolio's term structure aligned with the liability schedule. How does the sponsor evaluate the manager's performance? If the manager underperforms the market index, was it due to the manager's poor sector and security selection or the manager's correctly structuring the portfolio to satisfy the liability term structure? What is needed is a "neutral" benchmark that reflects both goals of the plan sponsor. The manager's performance can then be properly compared with the return on the "neutral" benchmark.

The Usefulness of Liability-Based Benchmarks

A liability-based benchmark is a "neutral" benchmark that gives the sponsor and manager a performance yardstick incorporating both the term structure constraints imposed by the liability schedule and the investment restrictions imposed by the sponsor's risk preferences. Sponsors can be confident that if they hold the positions underlying the liability benchmark, they will meet their liability schedules while satisfying their investment restrictions. This makes the liability benchmark a "neutral" benchmark.

A liability-based benchmark can also retain many of the desirable attributes of a market-based index: benchmark returns are calculated using market prices, the investment manager can replicate the benchmark, and the benchmark is well defined so that the sponsor and manager can actively monitor and evaluate its risk and performance. Furthermore, if the liability benchmark contains published market-based indices or marketable securities, its performance can be calculated and published by third-party index or market data providers.

Since the liability benchmark reflects the sponsor's liability schedule and investment restrictions, a manager can directly evaluate an investment portfolio against the benchmark. Using standard portfolio analytics, the manager can estimate tracking error, perform scenario analysis, and evaluate individual security swaps. Also, since the liability benchmark is a "neutral" benchmark, its performance can be compared directly with the manager's performance. This greatly facilitates sponsor-manager communication.

Types of Liability-Based Benchmarks

A liability-based benchmark reflects the term structure of the liability schedule and the investment restrictions of the plan sponsor. Two possible ways to construct a liability benchmark are:

[1] A dedicated portfolio refers to a portfolio of marketable securities that services a prescribed set of liabilities. There are various ways to construct a dedicated portfolio: cash-matched, immunization, horizon matched, and contingent immunization. For an analysis of these various approaches see "Duration, Immunization and Dedication," Part III D in *Investing: The Collected Works of Martin L. Leibowitz*, Probus, 1992.

1) Use market-based indices that reflect the sponsor's investment restrictions to construct a *composite benchmark* that reflects the liability term structure. For example, if the liability schedule is longer duration than the Lehman Aggregate Index, a composite index of the Credit and Aggregate indices and a custom long Treasury strips index could be created matching the duration of the liability schedule and the sponsor's investment restrictions.[2]

   More complicated composite indices may contain several indices weighted so as to achieve various diversification goals and duration, convexity, and yield targets. Despite matching a targeted duration, however, composite benchmarks may still have cash flow distributions that differ significantly from the liability schedule. Consequently, the composite benchmark and the liability schedule may diverge due to non-parallel shifts in the yield curve. In addition, as the underlying market indices are a set of rules and not a fixed set of bonds, the characteristics of the indices change over time, which may make frequent rebalancing necessary.[3]

   Care must be taken in using composite benchmarks. Suppose the liability schedule is concentrated in the near-term years. The temptation may be to use a short credit index as one of the indices in the composite. However, the short credit index may introduce an unintended bias into the composite benchmark. For example, the industrial sector accounts for 37% of the 0-4 duration bucket of the Credit Index, whereas it accounts for 45.5% in the overall index. Consequently, using the 0-4 duration credit sub-index in the composite may inadvertently underweight industrial paper in the composite benchmark.

2) Create a *portfolio benchmark* by selecting bonds from the investable universe such that the portfolio's cash flows closely match the liability schedule and the overall portfolio satisfies the sponsor's investment restrictions.

   Since bonds in a portfolio benchmark are selected so that their overall characteristics match the investment restrictions, the risk described above of unintended biases with composite benchmarks is eliminated.

   Unlike a composite benchmark that consists of indices and their set of rules, a portfolio benchmark consists of a set of bonds. By design, a portfolio benchmark is explicitly structured to track a given liability schedule over time, reducing the need for rebalancing. However, the relatively few bonds in the portfolio benchmark (compared with the many bonds in the indices underlying a composite benchmark) make the portfolio benchmark susceptible to idiosyncratic risk. Consequently, in sectors in which there is significant event risk (e.g., corporates), great care must be taken to reduce idiosyncratic risk by holding many different issuers.[4]

   Below, we present our methodology for constructing liability-based portfolio benchmarks.

Building a Liability-Based Portfolio Benchmark

The traditional dedication approach is to minimize the cost of a portfolio funding a liability schedule subject to constraints such as requiring that the duration and convexity of the portfolio match those of the liabilities. Other constraints, such as sector weights and a sufficient number of issuers in the portfolio, ensure portfolio diversification. Overall, these optimization constraints help keep the portfolio's cash flows "matched" with the liabilities, while also adhering to the sponsor's investment guidelines. This traditional dedication approach is a linear optimization problem, as the objective function and constraints are linear equations.

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[2] For more on the construction of liability-based composite benchmarks see "Market-Oriented Benchmarks for Immunized Portfolios," by Boyce I. Greer, *Journal of Portfolio Management,* Vol. 18, Issue 3, Spring 1992, pp. 26-35.

[3] Rebalancing composite benchmarks, however, is straightforward. *POINT*, the Lehman Brothers portfolio analytics system, can calculate and update the desired index weights automatically to keep duration and sector exposures on target.

[4] A forthcoming Lehman Brothers publication, *Sufficient Diversification in Credit Portfolios* by L. Dynkin, J. Hyman and P. Weissman discusses a methodology for constructing replicating credit portfolios that minimizes event risk. For example, the Lehman Credit Index can be replicated with a 100-bond portfolio such that it will not underperform the index by more than 35 bp with 95% confidence. The key is to hold most of the 100 issues in the BBB category: using 62 BBB bonds to replicate the BBB quality sector having a 31% market weight in the Credit Index.

A different approach is used to construct a liability-based portfolio benchmark. The idea is to construct a portfolio such that its cash flows mimic as closely as possible the cash flows of the liability schedule subject to the portfolio investment constraints. In other words, the objective is to minimize the absolute value of the difference between each liability cash flow and the available cash flow from the portfolio at the time of each liability cash flow.

Since portfolio benchmark cash flows are unlikely to fall on the exact date of the liability cash flows, portfolio cash flows are either reinvested forward or, if permitted, discounted back to a liability cash flow date. Consequently, a portfolio's available cash flow at each liability cash flow date is the amount of portfolio cash that can be delivered to that date. To illustrate, consider a liability cash flow L_t that occurs at time t (Figure 1). There are several cash flows (assume, for simplicity, that they are zero coupon bonds) available that could possibly meet this liability cash flow. Two of these cash flows (P_1 and P_2) occur before and another one (P_3) occurs after the liability cash flow. However, depending on the assumptions allowed in the portfolio construction process, all three cash flows (if purchased in sufficient quantity) could satisfy the liability cash flow L_t.

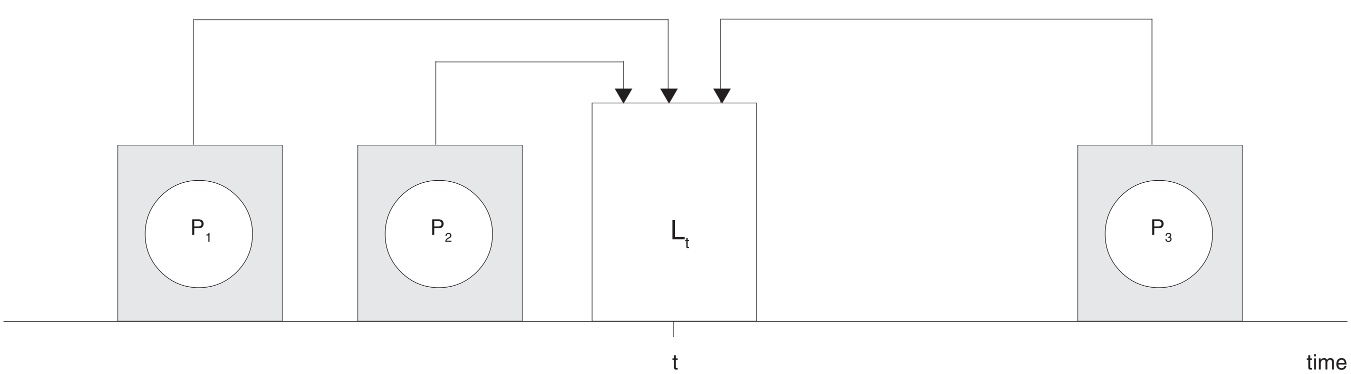
Consider cash flow P_1, which occurs before L_t. If the reinvestment rate, r, is assumed to equal zero, then a face amount of cash flow P_1 equal to L_t could be purchased today. When P_1 is received at maturity, it could be held until time t and would be sufficient to satisfy L_t. If the reinvestment rate were greater than zero, then less P_1 would be needed today to satisfy L_t. However, both P_1 and P_2 could each be carried forward to time t to satisfy completely the liability requirement.

Now consider cash flow P_3 that is received after the liability cash flow requirement. If borrowing is not allowed, then P_3 cannot satisfy L_t. However, if borrowing is permitted, then, at time t, cash could be borrowed against P_3 (at the assumed borrowing rate) in order to satisfy the liability cash flow L_t.

In the more general case, there are many liability cash flows of varying amounts and many feasible bonds, each with its many cash flows comprising periodic coupon payments and return of principal at maturity (Figure 2). To create a portfolio benchmark, our job is to select a set of bonds whose combined available cash flows at each liability payment date (given the reinvestment and borrowing rate assumptions) will most closely match the liability cash flows. The portfolio benchmark is the solution to this optimization problem.

To set up the optimization problem, the liability schedule is first defined according to the amount of cash flow required at each time period. A feasible set of bonds is then identified as a candidate for the benchmark. For example, if bonds must be rated AA or better, then the feasible set would be constrained to

Figure 1. **Borrowing and Lending to Fund a Liability**



contain bonds rated only AA or better. Then a series of investment restrictions are specified that further constrains bonds selected from the feasible set for the benchmark. For example, the benchmark may be required to have an asset mix of 60% governments and 40% corporates, with no single corporate issuer with more than 2% weight in the benchmark portfolio. Finally, a reinvestment rate (r) is specified (it may be zero), and borrowing is either denied or permitted at a specified rate (b).[5]

The goal of the optimization program is to select bonds for a portfolio benchmark such that the cash flows are as "close as possible" to the liability cash flow. In other words, the program[6]

$$\text{minimizes} \sum_{t=1}^{n} \left\{ \frac{|\,CF_t(L) - CF_t(P)\,|}{(1 + IRR)^t} \right\},$$

subject to the specified constraints. $CF_t(L)$ represents the nominal liability cash flow at time t. $CF_t(P)$ represents the nominal amount of portfolio cash flow that can be made available at time t either from a portfolio cash flow that occurs exactly at time t or earlier cash flows reinvested forward to time t and, if permitted, later cash flows discounted back to time t.

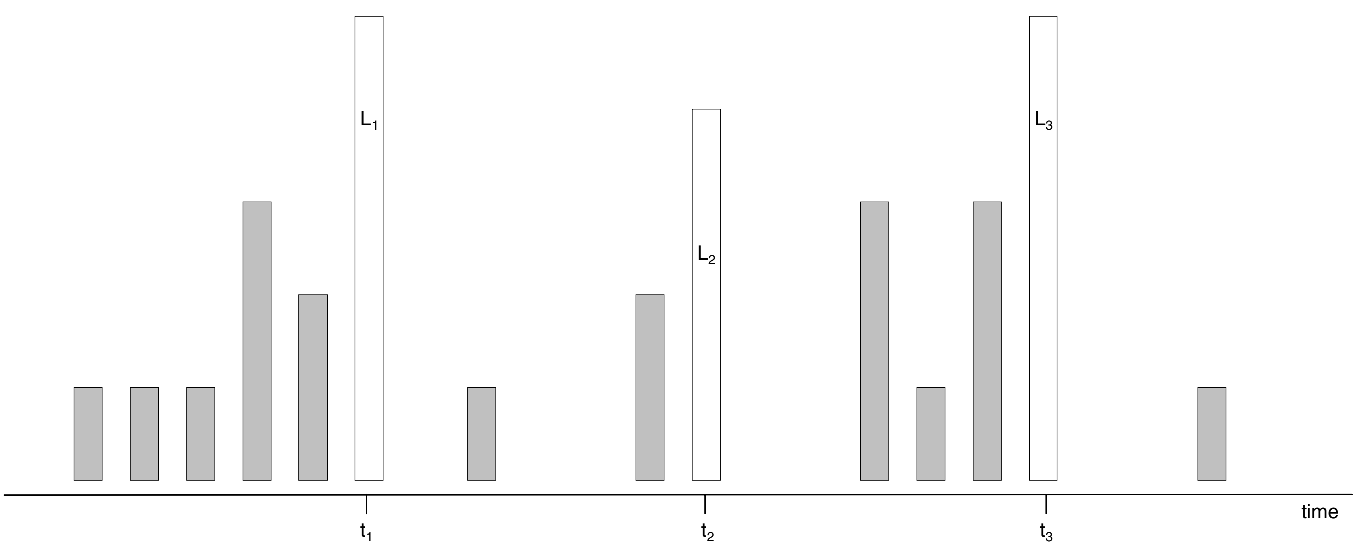
Mechanically, the program works as follows. All available portfolio cash flows that occur before each liability cash flow at time t are reinvested forward to time t at rate r. If borrowing is allowed, then all available portfolio cash flows that occur after time t are discounted back to time t. The program then selects the portfolio of bonds whose cash flows minimize the sum of the absolute values of the cash flow differences across all time periods in which a liability cash flow occurs.

To build intuition for the optimization program, consider the case of two equal liability cash flows, L_1 and L_2. There are two possible bonds, P_1 and P_2. Bond P_1 has one cash flow that occurs before L_1 and whose nominal value equals $(L_1 + L_2)$. Bond P_2 has

[5] To be conservative and insure that all liability payments have sufficient cash, the sponsor could assume that the reinvestment rate equals zero and prohibit borrowing.

[6] Essentially, to achieve the closest match, we would like to minimize the "distance" between the portfolio cash flows and those of the liabilities. This could be expressed using the "least mean squares" approach, in which we minimize the sum of the squared differences, or by the absolute value approach shown above. Neither of these objective functions is linear. We have chosen to work with the formulation based on absolute values because it can be converted to a linear program. To accomplish this, the problem variables, which represent the cash flow carry-overs from one vertex to the next (which can be positive or negative), are each split into two non-negative variables, one representing a reinvestment and the other a loan. A linear program is used to minimize the weighted sum of all of these variables, using weights that make the problem equivalent to the absolute value minimization shown above.

Figure 2.



two equal cash flows with one occurring before L_1 and the other occurring after L_1 but before L_2. Each cash flow's nominal value equals L_1 (and L_2). (Figure 3 illustrates the cash flows.) Further, assume that the reinvestment rate is zero and that borrowing is not allowed. Finally, the market value of bond P_1 is 95, whereas the market value of bond P_2 equals 100.

Both bonds would fully satisfy the liability schedule. However, bond P_1 would do so at lower cost than bond P_2. Which bond does the optimizer select? The sum of the differences in cash flow between each liability cash flow and the available cash flow is less for bond P_2 than for bond P_1. Why is this? Both P_1 and P_2 exactly fund the liability cash flow at t_2, but P_1 must do this by overfunding the liability cash flow at time t_1. In other words, bond P_2 matches the liability schedule more closely than does bond P_1. Consequently, the optimizer will select bond P_2 over bond P_1. This example highlights that the portfolio benchmark approach selects the best matching portfolio, not necessarily the least expensive portfolio, even if it also satisfies the liability schedule.

The term $(1 + IRR)^t$ in the denominator above is an additional discount factor for which IRR is the internal rate of return on the benchmark portfolio. This discount term essentially says that the optimization program cares more about minimizing near-term cash flow mismatches than more distant mismatches.

The solution of this optimization program is a liability-based benchmark portfolio of marketable securities whose cash flows are as close as possible to the liability cash flows. Note that this approach does not minimize the cost of the benchmark portfolio, as is the case for other dedication programs. Here, the goal is to create a portfolio benchmark whose cash flows closely mimic the liability schedule and meet investment constraints: a "neutral" benchmark.

Example:
Creating Composite and Portfolio Benchmarks

Recently, a fund manager, working with the plan sponsor, decided to create a benchmark for a fixed liability stream (Figure 4) with a duration of 12.5. The sponsor's investment restrictions required the benchmark to have an asset mix of 50% government, 40% corporate, and 10% CMBS. The minimum credit quality allowed was A3.

To create a *composite benchmark,* at least three different sub-indices are needed, one for each asset class. A fourth sub-index is needed in order for the composite index to match the duration

Figure 3. **Portfolio Benchmark Approach Selects Bond P_2 over Bond P_1**

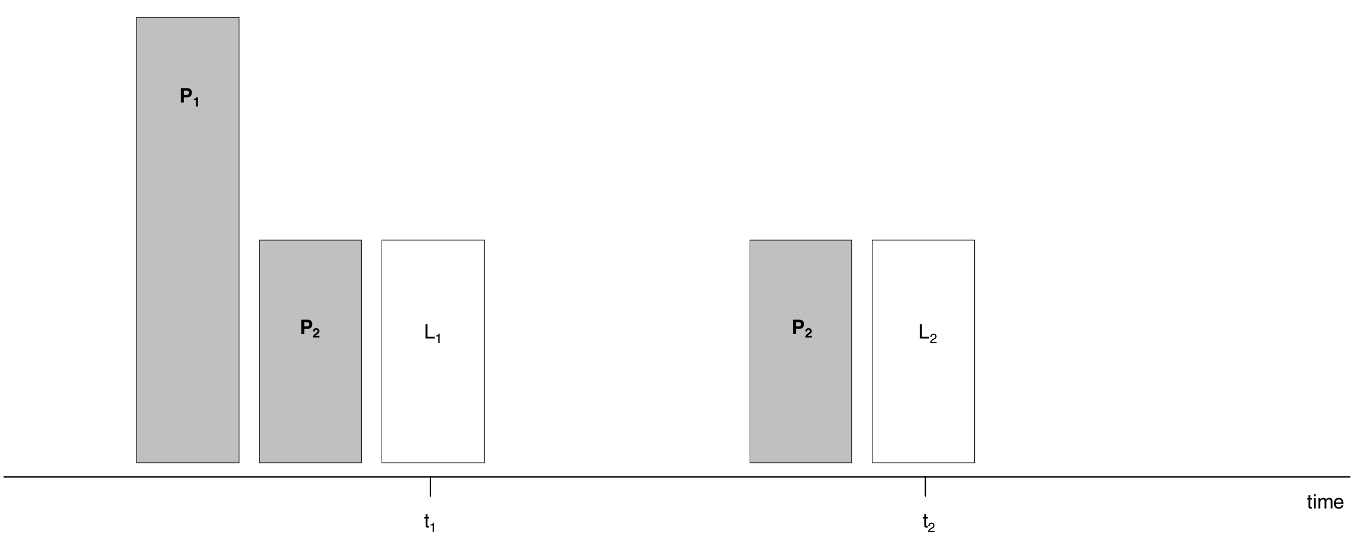


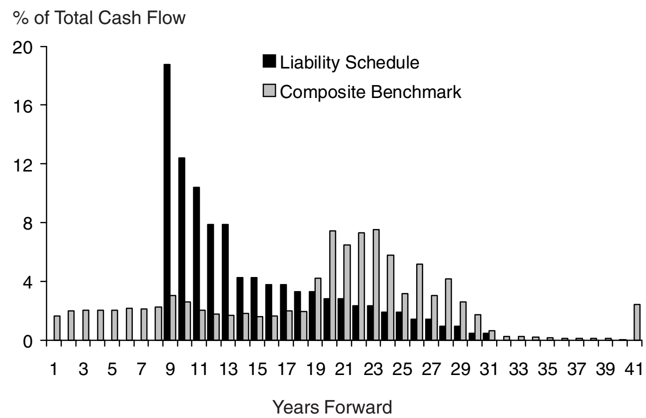
Figure 4. **Liability Schedule,** $

Year	Amount	Year	Amount
1	0	17	7,400,380
2	0	18	6,475,332
3	0	19	6,475,332
4	0	20	5,550,285
5	0	21	5,550,285
6	0	22	4,625,237
7	0	23	4,625,237
8	0	24	3,700,190
9	36,631,879	25	3,700,190
10	24,236,243	26	2,775,142
11	20,351,044	27	2,775,142
12	15,355,787	28	1,850,095
13	15,355,787	29	1,850,095
14	8,325,427	30	925,047
15	8,325,427	31	925,047
16	7,400,380		

Figure 5. **Composite Benchmark Weights,** %

Index	Weight
Long Corporate (A3 and higher)	40.0
CMBS	10.0
Long Government	23.1
Treasury Strip (18 years +)	26.9

Figure 6. **Cash Flow Comparison:Composite Benchmark versus Liability Schedule**



target of 12.5. A long corporate index containing only quality A3 and higher is 40% of the composite benchmark, the CMBS index is 10%, and the remaining 50% is split between the Long Government Index and a custom Treasury strips index containing strips of 18 years and longer. The weights of these two government indices, 23.1% and 26.9%, are such that they add up to 50% and produce an overall composite benchmark duration of 12.5. The weights are presented in Figure 5.

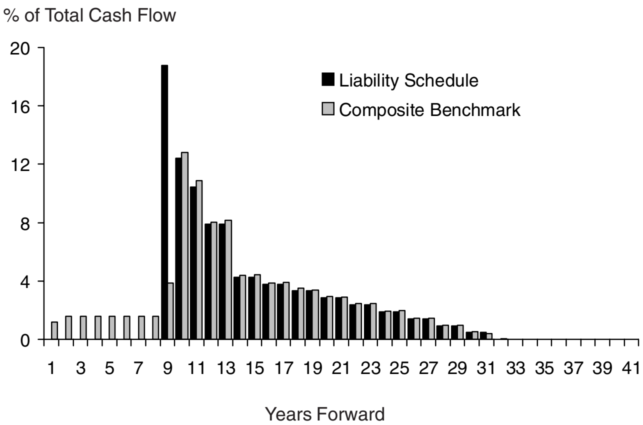
Figure 6 compares the cash flows of the composite benchmark with those of the liability schedule. Note that while the duration of the composite benchmark matches that of the liability schedule, there are considerable mismatches in the timing of cash flows. It is likely that cash flows could be more closely matched if additional sub-indices, appropriately weighted, were added to the composite benchmark.

To create a *portfolio benchmark,* a set of about one thousand bonds was chosen as the feasible set from which the optimizer can select bonds for the portfolio. Only bullet corporate and agency bonds were considered (so the cash flows would not fluctuate with interest rates) and only strips represented the Treasury sector. The bulk of the feasible set is corporate bonds, with good representation in all corporate sectors. This is desirable, as the portfolio benchmark must contain many corporate names for appropriate diversification.

The optimization problem was set up with constraints that reflect the investment restrictions: an asset mix of 50% government, 40% corporate, and 10% CMBS and a minimum credit quality of A3 for all issues. In addition, the 40% of the portfolio in corporates was further constrained to have the same proportional industry and quality breakdown as the Credit Index. No credit sector and no issuer was allowed to make up more than 22% and 1%, respectively, of the overall benchmark. (If desired, separate diversification constraints can be imposed by sector or by quality, to reflect varying levels of protection from event risk.) As a result, the resulting portfolio benchmark consisted of approximately 100 securities.

Figure 7 compares the cash flows of the portfolio benchmark with those of the liability schedule. Overall, the portfolio benchmark cash flows closely match the liability cash flows. Note, however, that the first liability cash flow (year 9) is mostly pre-

Figure 7. **Cash Flow Comparison: Portfolio Benchmark vs. Liability Schedule**



funded by the portfolio. This is to be expected, given the investment constraints, as 40% of the portfolio must be invested in corporates that predominantly pay a coupon. Consequently, the portfolio benchmark receives coupon payments in the first eight years that must be reinvested to meet the first liability cash flow in year 9.

Because the portfolio benchmark reflects the liability structure and the investment constraints, the sponsor and investment manager can use the portfolio benchmark as a "neutral" benchmark: the manager can construct a portfolio using the benchmark as his bogey, and the sponsor can appropriately evaluate the manager's performance relative to the benchmark. The manager can also use the portfolio benchmark to identify the sources of risk in the investment portfolio relative to the benchmark and, therefore, relative to the liability structure. This is accomplished using the Lehman Brothers Risk Model. The risk model will identify sources of risk (i.e., tracking error) and suggest trades from a manager-selected list of bonds in order to reduce both systematic and security-specific risk. The risk model will also suggest trades to move the portfolio toward matching the portfolio benchmark in yield curve, sector, and quality exposures. In general, if the manager wishes to deviate from the "neutral" benchmark, the risk model can estimate his potential tracking error.

Conclusion

A liability-based benchmark retains many of the desirable attributes of a market-based index while simultaneously more closely matching the sponsor's liability term structure. A liability benchmark is a "neutral" benchmark, allowing the sponsor to evaluate appropriately the manager's performance and allowing the manager actively to monitor investment risk and opportunities.

Two types of liability benchmarks are *composite benchmarks* (using market-based indices) and *portfolio benchmarks* (using a fixed portfolio of bonds). Portfolio benchmarks have two advantages: less frequent rebalancing and reduced risk of introducing unintended biases into the benchmark. Care must be taken, however, to minimize idiosyncratic risk in a portfolio benchmark by holding many different issuer names in the portfolio. We discuss our methodology for constructing portfolio benchmarks and present an example.

Plan sponsors and managers are adopting the portfolio benchmark approach to liability benchmark construction and are utilizing Lehman's fixed-income quantitative portfolio management tools to implement this strategy. Also, given Lehman's expertise in constructing and maintaining indices, sponsors and managers have turned to Lehman Brothers to serve as a third-party provider of portfolio benchmark performance measurement and analysis.