MOODY'S
INVESTORS SERVICE

SPECIAL COMMENT

Trading Prices as Predictors of Ultimate Corporate Recovery Rates

Table of Contents:

Analyst Contacts:

NEW YORK 1.212.553.1653

Albert Metz 1.212.553.4867
Managing Director-Credit Policy Research
albert.metz@moodys.com

Steffen Sorensen 1.212.553.6865
Vice President-Senior Analyst
steffen.sorensen@moodys.com

David Keisman 1.212.553.1487
Senior Vice President
david.keisman@moodys.com

David Chiu 1.212.553.1718
Associate Analyst
david.chiu@moodys.com

Summary

In this paper we use recovery rate databases available at Moody's to analyze two related questions. First, we explore the predictive content of 30 day post-default trading prices as measures of ultimate recovery. Second, we explore whether the trading price recorded at any other day close to the default date has any more predictive content relative to the 30 day trading price.

Post-default trading prices are often quoted as measures of recovery, since they are available near the time of default and presumably reflect a market expectation of ultimate recovery. Still, this begs the question of how good a proxy they really are. Do trading prices give the correct values on average, in a portfolio sense? Are they useful measures at the instrument level, meaning that if instrument A has a higher price than does B, then its ultimate recovery is likely to be higher as well? The use of trading prices also begs the question of whether there is an optimal time to measure those prices. Typical convention is to use 30 day post-default prices, but little justification is usually given for that choice.

This paper is not the first study to look at the predictive content of post-default trading prices. Using a sample of US recovery rates between 1982 and 2004, Cantor and Varma (2005) found that the 30 day trading price, at the issuer level, is closely aligned with the trading price at resolution discounted back to the default date using the Corporate single-B Index.

The current study has been updated to include more than twice as many observations including the most recent global recession and, taking advantage of this additional data, employs more robust econometric specifications. In addition to standard linear regression, we adopt a two-sided Tobit model with generalized heteroskedasticity, and introduce a new regression approach, a two-sided truncated Beta regression, also with generalized heteroskedasticity.

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The post-default trading price reflects the market's expectation of recovery at resolution discounted back to the day when the debt is traded. We do not know the market discount rate as it is not observable; hence we can only test whether the trading price is a good predictor of ultimate recovery rates conditional on some choice of discount rate. For our purposes we will discount ultimate recoveries by the coupon rate. This is almost surely not correct – the market presumably discounts by something else. Still, discounting by the coupon is a common enough market convention, and average coupon rates do not vary widely across seniority type.[1] This has been a subject of academic interest, such as Acharya et al (2003). In their study of the determinants of ultimate recovery rates, they find a close alignment between recovery rates discounted back to the default date using either a high yield bond index or the coupon of the bond. We will explore this topic in future research.

Independently of how the market may price recoveries, discounting ultimate recoveries by the instrument coupon is a reasonable measure of the loss given default (LGD) as of the default date. The reason is quite simple. If, at resolution, the noteholders receive the principal they are owed plus all accrued interest since default – in other words, if they receive "full recovery" – then that amount discounted at the coupon rate back to the default date will be 100%. Even if we do not expect that the market rate is the same as the coupon – if we expect to see a "bias" between post-default trading prices and discounted recoveries – it remains a useful exercise to determine how well post-default trading prices anticipate this measure of LGD.[2]

Our primary findings include:

» Trading prices are slightly biased measures of ultimate recovery. Put differently, trading prices somewhat understate ultimate recoveries when the latter are discounted back at the coupon rate. Ultimate recoveries are on average 3 percentage points higher than the trading price. This is a small difference but hides important distinctions across seniority classes.

  - The difference is most pronounced among loans and senior secured bonds. For loans one possible explanation is that they have lower coupon rates relative to bonds. The average coupon on senior secured bonds, however, is not markedly different from senior unsecured and subordinated bonds.

  - Among senior unsecured bonds there is equally a marked difference between the average 30 day trading rice and the ultimate recovery rate while in fact trading prices of subordinated bonds slightly overstate this measure of recovery on average.

» Despite this bias, 30 day trading prices are nevertheless good predictors of ultimate recovery rates.

  - Over time, the average trading price closely tracks average ultimate recoveries.

  - At the instrument level, much of the variation in ultimate recoveries is explained by the variation in trading prices. The predictive power of trading prices declines at lower levels of the capital structure.

» One possible explanation for the bias in trading prices is the presence of a risk and liquidity premium. Our analysis suggest, however, that such premium must be a function of instrument specific characteristics. We find that there is no common time-series variable which can correct for the bias in trading prices.

[1] While the average coupon for loans is less than that of bonds (7% versus 10% on average), there is little difference across bond seniority classes. Senior secured, senior unsecured and subordinated bonds all have average coupon rates of about 10% in our sample.

[2] Preliminary research, not explored further herein, suggests that the coupon rates do contain useful cross-sectional information.

» For defaulted bonds where the time to resolution is less than one year, trading prices are far less biased and have far more predictive content.

» The 30 day convention seems well founded. More observations are available 30 days after default than are immediately available, and the prices 30 days post default explain more of the variation in ultimate recoveries than do the prices closer to default.

1. Introduction

When a company defaults on its debt service obligations there can be considerable uncertainty about the magnitude of final investor losses. In cases of bankruptcy it may take years before the debt holder knows how much can ultimately be recovered. Post-default trading prices provide a useful and early source for gauging the market's expectation of the ultimate instrument recovery rate.[3]

Previous Moody's research has looked at the ability of the 30 day post-default trading price to predict ultimate recovery. Cantor and Varma (2005) use trading prices at the time of resolution as their measure of ultimate recovery. On a sample of over 300 bond issuers, they find that trading prices 30 days after default do a good job of explaining trading prices close to resolution discounted by the yield on the single-B corporate index.

In this paper we will look at the predictive content of post-default trading prices as a measure of ultimate recovery. We shall explore two basic questions: First, how good a predictor are those prices? In particular, do trading prices represent "biased" forecasts of ultimate recoveries? Second, is there any reason to prefer 30 day post-default prices (as is the market convention) to any other period? We use Moody's database on ultimate recovery rates developed subsequent to the Cantor and Varma study which provides us with more than twice as many observations.[4] We conduct our analysis on the instrument rather than the issuer level.

For convenience we discount the ultimate recovery rates by the coupon of the bond. There are any number of reasons why we would not expect the coupon to be equal to the market discount rate. Still, it is a common enough practice to use the coupon rate, and we do not expect our results will be too sensitive to this choice. Acharya et al (2003), for example, find little difference in the ultimate recovery rate whether discounting with the high yield bond index or the coupon of the bond. Nevertheless this remains an area of ongoing research.

The paper is organized as follows. Section 2 describes the data and presents summary statistics on ultimate recoveries and trading prices both over time and through the capital structure. Section 3 formalizes our interpretation of prices as expectations of ultimate recoveries and describes the econometric approaches we employ. Section 4.1 presents results on the absolute forecasting content of 30 day post-default trading prices, Section 4.2 and 4.3 explores whether there are any common factors that reconciles the 30 day trading price and the ultimate recovery rate, Section 4.4 presents the predictability of trading prices as a function of the time to emergence/resolution, and Section 4.5 presents results on the relative optimality of using 30 day (as opposed to 10 day or 90 day, for example) trading prices. Finally, Section 5 concludes.

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[3] Of course, to the investor who chooses to liquidate his position before the bankruptcy resolution, the post-default trading price *is* his recovery rate. We will distinguish between "trading prices" and "recovery rates" only as a matter of convenience.

[4] For further information on Moody's Ultimate Recovery data base, see Emery, Keisman and Ou (2007). The ultimate recovery rates is the recovery calculated at the resolution of the bankruptcy. Although we would not expect material differences between the ultimate recovery rate and the trading price close to resolution it nevertheless may provide a more correct estimate of recovery.

2. Data

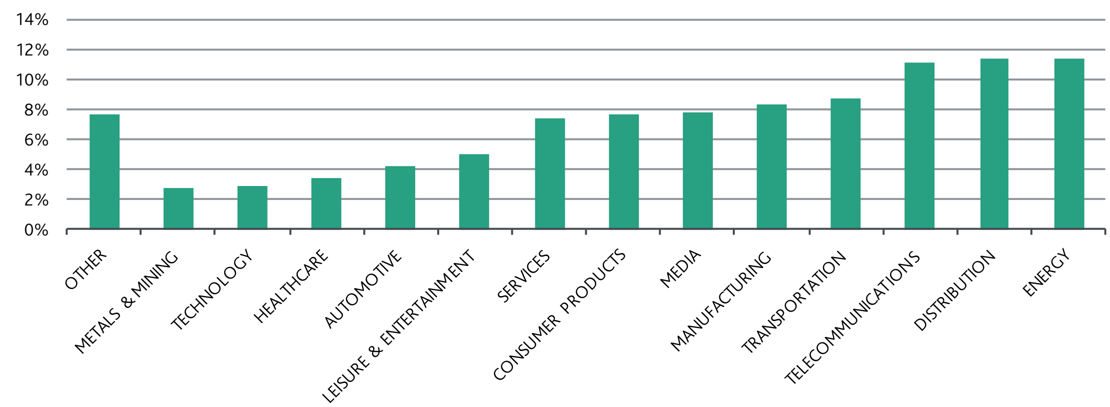
2.1 Available Databases

At Moody's we have two recovery rate databases available, the Ultimate Recovery Database and the 30 Day Trading Price (30 Day TP) database. In addition we collect the trading price time-series for bonds, when available, from the default date to resolution.[5]

The ultimate recovery rate is a realization of the recovery rate once a company emerges from bankruptcy.[6] The database has more than 3800 observations for non-financial North American corporate issuers. The recovery rates relate to debts of different seniorities including Term Loans, Senior Secured bonds, Senior Unsecured bonds and Subordinated bonds.[7] The recovery rates are discounted back to the last date when interest was paid on the instrument using the coupon of the bond or loan. Excluding distressed exchanges we have around 3100 observations at the end of December 2011. The majority of the observations are defaulted debt instruments issued by companies in the energy, distribution and telecommunication sectors, see Exhibit (1).

EXHIBIT 1

Distribution of Observations by Sector



The 30 Day TP database includes recovery rates of different seniorities including Term Loans, Senior Secured bonds, Senior Unsecured bonds, and Subordinated bonds. Between 1985 and 2011, the database has more than 5100 observations with about 92% of these being recovery rates on North American instrument defaults.[8]

In practice we do not always have a trading price available 30 days after default. In those cases we record the trading price as close to 30 days as possible. This means that the "30 Day" trading price is actually recorded anywhere between 14 and 46 days post default with the majority of observations close to 30 days after the default date. The only exceptions are distressed exchanges where the trading price is recorded on the day of the default as the default date, most often, coincides with the resolution date. The absence of a continuum of data post default is possibly explained by the lack of active trading which may equally be a problem in the period before default.

[5] Moody's usually publishes average statistics for recovery rates by seniority in our Annual Default Studies. See Moody's Special Comment: 'Corporate Default and Recovery Rates, 1920-2011', February 2012, S. Ou for an example.

[6] For additional information and research on ultimate corporate recoveries, please see Moody's Special Comment: 'Moody's Ultimate Recovery Rate Database', April 2007, K. Emery, D. Keisman and S. Ou.

[7] The database equally has a number of observations on revolving loans.

[8] Defaulted bond pricing data is derived from Bloomberg, Reuters, IDC, and TRACE. The majority of these market quotes represent an actual bid on the debt instrument, although no trade may have occurred at that price.

2.2 Data Exclusions

Our aim is to investigate the ability of trading prices to forecast ultimate recovery rates; we exclude the following observations from our analysis:

» Defaults which lack any trading price observed between 14 and 46 days post default;

» Defaults with a trading price recorded above par; and

» Distressed Exchanges (DE).

There are observations where the ultimate recovery is above 100%. A recovery rate above 100% is most often associated with the issue of preferred stock to debt holders. We choose to cap the ultimate recovery rates at 100%. Likewise there are a few observations with a trading price above 100%. We choose to treat these as an anomaly, as it is hard to justify a market price implied expected recovery rate above 100%, and exclude them from our analysis. We discuss this further in Section 3.

We do not otherwise apply any filters to the data. However, we would point out that given the ultimate recovery database is North American instrument defaulters, our dataset only has North American observations.

Out of a total of 3094 ultimate recovery rates we have a matching trading price meeting our criteria for 1296 bankruptcies. Of these 1296 observations, 1067 are bond and the remaining 229 are loan recovery rates. Note that when an issuer first misses an interest or principal payment, and only later files for bankruptcy, we record the trading price 30 days after the missed payment rather than 30 days after bankruptcy. Likewise, a defaulter may have a number of instruments defaulting but as our ultimate aim is to understand the predictive content of trading prices by instrument we do not aggregate defaulted instruments at the issuer level.

2.3 Descriptive Statistics

Trading prices are not always available 30 days after default. The corporate debt market is not all that liquid. Exhibit (2) includes summary statistics on the average number of days between default and recorded trading price for our sample. The average number of days is 28 with a median of precisely 30. The average number of days is lowest for Senior Unsecured and Subordinated bonds. Even if the 30 Day TP is less liquid it appears that our sample is reasonably representative of a trading price recovery rate recorded 30 days post default.

Descriptive statistics for ultimate recovery rates in our sample, as compared to the full dataset, can be found in Exhibit (3). Our sample has a lower mean than the full sample of ultimate recoveries and slightly lower volatility. The lower average level is largely explained by a thinner coverage of loans and Senior Secured bonds.

EXHIBIT 2

Number of Days between Default and Recorded Trading Price Recovery

	All Bonds and Loans	First Lien Term Loans	Senior Secured Bonds	Senior Unsecured Bonds	Subordinated Bonds
Mean	28.03	30.35	29.29	26.76	28.11
Median	30	30	30	29	29
Std	5.57	3.38	4.45	6.09	5.7
Min	14	14	14	14	14
Max	46	45	38	46	44
# Obs	**1296**	**168**	**149**	**520**	**398**

Exhibit (4) shows the descriptive statistics of the ultimate and 30 Day TP recoveries while Exhibit (5) plots the ultimate recovery rates against the trading price.[9] In Exhibit (A1) of the Appendix we show the number of observations available per year across seniority. Most observations are available for Senior Unsecured and Subordinated bonds. Other interesting findings include the following:

» Recall that we are comparing the trading price against the present value of the ultimate recovery. If they were equal on average, that would mean that investors in distressed debt had earned the instrument coupon rate on average – it would not mean that they earned no return on average. To the extent trading prices are below the present value of ultimate recoveries, investors are earning more than the coupon rate; to the extent they are above, investors are earning less than the coupon rate (but still, quite possibly, a positive average return).

» The average ultimate recovery rate is 3 percentage points higher than the 30 Day TP. While this may seem relatively small, the average hides some important differences across seniority. For loans, the average 30 Day TP is substantially lower than the ultimate recovery. The difference between the 30 Day TP and the ultimate recovery rate falls as we get further down the capital structure. For Subordinated bonds the average 30 Day TP is 0.5 percentage point higher than the ultimate recovery rate.

» While for the sample as a whole the average trading price is below the ultimate recovery rate, the median trading price is around 0.5 percentage points above the ultimate recovery rate. That means that even if on average the trading price is lower than the ultimate recovery rate, the typical trading price is higher than the present value of what is subsequently realized. The largest difference in the median is recorded for Subordinated debt.

» The ultimate recovery rates are nearly 25% more volatile than the trading price. This observation appears consistent across different levels of seniority. This is unsurprising when we view the 30 Day TP as an expectation of the ultimate recovery rate.

» Ultimate recovery rates tend to have a significant mass at 0% and 100%, while trading prices in our sample do not take the value of 0% or100%.[10]

[9] Note, as discussed in Section 3 we discount the 30 day trading price back to the date of last interest payment using the coupon of the bond. This is necessary to make a fair comparison between the two.

[10] Recall that we exclude the handful of cases with a trading price recorded above 100%.

EXHIBIT 3

Comparing Restricted Sample to Sample of All Ultimate Recovery Rates

	All B&L		All Term Loans		First Lien (FL) Term Loans		SS Bonds		SU Bonds		Sub Bonds	
	All Data	Our Sample	All Data	Our Sample	All Data	Our Sample	All Data	Our Sample	All Data	Our Sample	All Data	Our Sample
Mean	47.32	39.67	71.97	66.75	76.78	71.54	61.59	51.64	41.98	37.80	20.71	22.05
Median	39.34	29.37	93.53	74.15	100	78.33	59.7	43.91	32.84	28.78	8.61	12.16
Std	38.45	34.02	34.63	32.60	30.15	29.98	33.69	31.41	35.63	31.33	28.04	26.86
Min	0	0	0	0	0	0	0.86	1.28	0	0	0	0
Max	100	100	100	100	100	100	100	100	100	100	100	100
#Obs	3094	1296	754	229	520	168	539	149	1018	520	783	398

Abbreviations: All B&L: All bonds and loans, SS: Senior Secured, SU: Senior Unsecured, Sub: Subordinated

EXHIBIT 4

Comparing Ultimate Rates to 30 Day TP Recovery Rates

	All B&L		All Term Loans		First Lien(FL) Term Loans		SS Bonds		SU Bonds		Sub Bonds	
	30D	Ult	30D	Ult	30D	Ult	30D	Ult	30D	Ult	30D	Ult
Mean	36.59	39.67	62.36	66.75	64.45	71.54	47.15	51.64	32.99	37.80	22.53	22.05
Median	29.82	29.37	66.24	74.15	67.65	78.33	42.37	43.91	24.72	28.78	18.84	12.16
Std	27.34	34.02	25.13	32.60	23.90	29.98	26.52	31.41	25.19	31.33	18.54	26.86
Min	0.01	0	2.12	0	11.82	0	1.00	1.28	0.29	0	0.01	0
Max	99.95	100	99.95	100	99.42	100	98.92	100	97.87	100	90.33	100
#Obs	1296		229		168		149		520		398	

Abbreviations: All B&L: All bonds and loans, SS: Senior Secured, SU: Senior Unsecured, Sub: Subordinated, Ult: Ultimate recovery rates, 30D: 30 day trading price recovery rates. Exhibit (A1) in the Appendix has the number of observations per seniority level broken out by year.

In Exhibit (6) we plot the average recovery rate by default year for different seniority classes. In Exhibit (A2) in the Appendix we compare the yearly averages by emergence rather than default year. While the average differences between ultimate recovery rates and 30 Day TPs are similar to those reported in Exhibit (4), Exhibit (6) suggests that recovery rates do change over time and that the average 30 Day TP recovery rates does a good job at explaining the time variation in ultimate recovery rates. This is true whether we plot the averages by default or emergence year. Recovery rates appear to be strongly cyclical, falling in recessions and years with a large number of defaults such as the early 1990s, early 2000s and latest in 2008-2009.[11] This is the case for trading prices as well as ultimate recovery rates.

The data suggest that ultimate recovery rates are quite volatile and a natural question is whether that volatility is different depending on the level of the trading price recorded 30 days after default. In Exhibit (7) we group trading prices into ten equally spaced intervals between 0 and 100% and compute the standard deviation of ultimate recovery rates conditional on trading prices belonging to a given interval. For higher seniority bonds and loans we notice that there appears to be a quadratic relation. For very low or very high trading prices the dispersion in ultimate recovery rates is lower than when trading prices are in the middle range. For Senior Unsecured and Subordinated bonds we note

[11] See Keisman (2011), 'Moody's Ultimate Recovery Database, Lessons from 1000 Corporate Defaults' for a discussion of ultimate recovery rates in the 2008-2009 recession

that the standard deviation appears to increase with the trading price. We will return to this point below in our discussion of econometric specifications.

EXHIBIT 5

Comparing Ultimate to 30 Day TP Recovery Rates

EXHIBIT 5A

All Bond and Loans

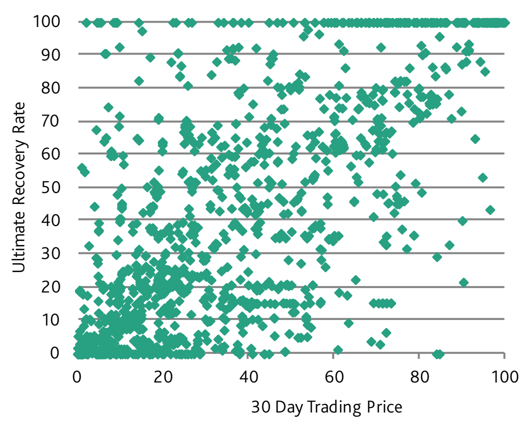


EXHIBIT 5B

Term Loans, All Liens

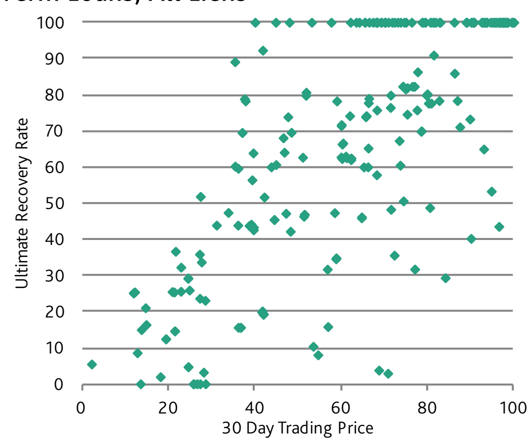


EXHIBIT 5C

Term Loans, First Lien

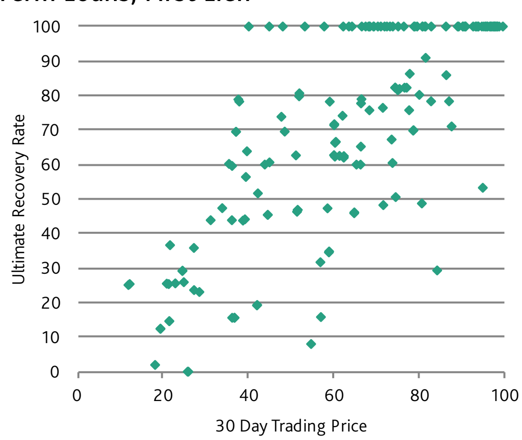


EXHIBIT 5D

Senior Secured Bonds

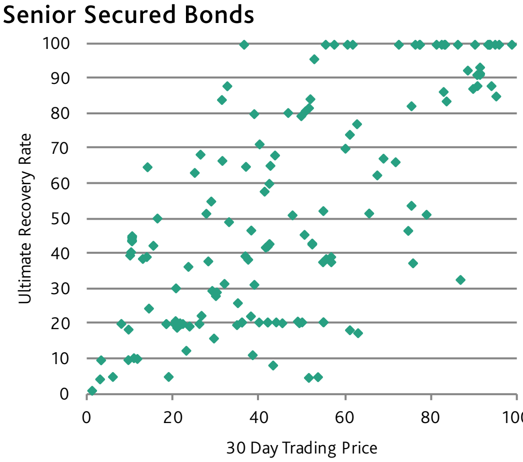


EXHIBIT 5E
Senior Unsecured Bonds

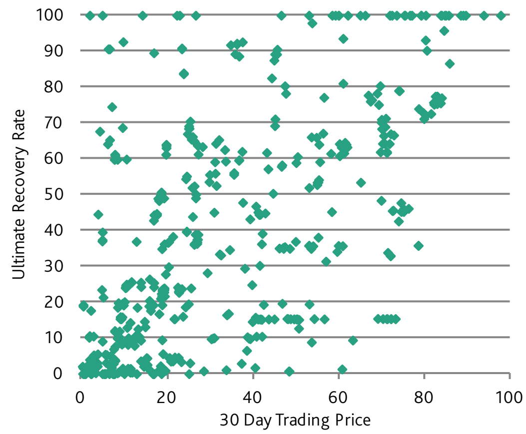


EXHIBIT 5F
Subordinated Bonds

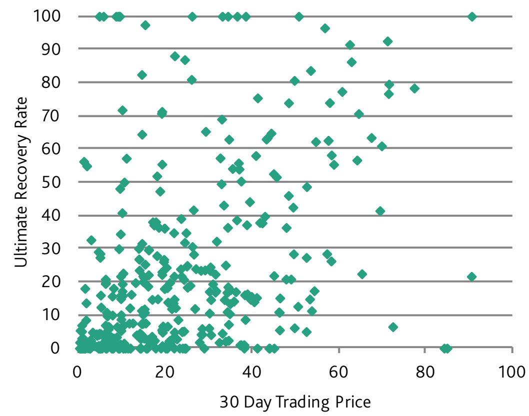


EXHIBIT 6

Comparing Ultimate and 30 Day TP Recovery Rates by Default Year

EXHIBIT 6A

All Bond and Loans

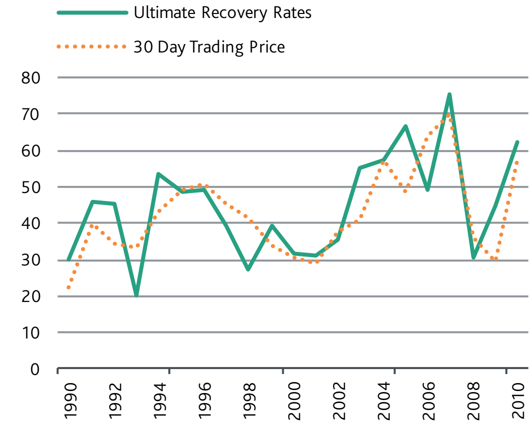


EXHIBIT 6B

Term Loans, All Liens

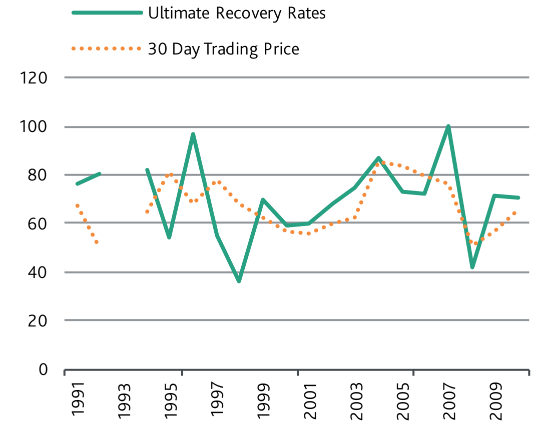


EXHIBIT 6C

Term Loans, First Lien

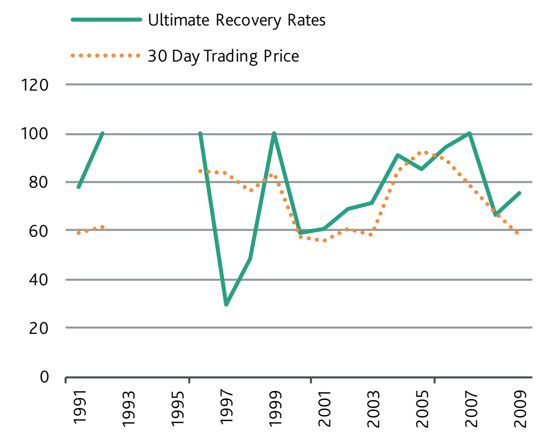


EXHIBIT 6D

Senior Secured Bonds

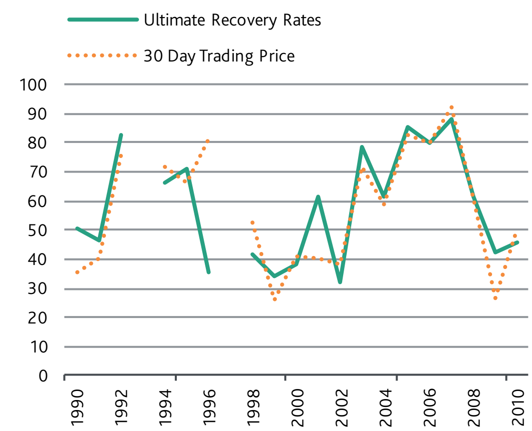


EXHIBIT 6E

Senior Unsecured Bonds

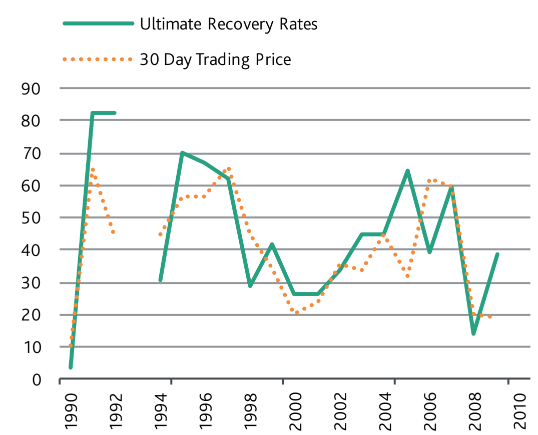


EXHIBIT 6F

Subordinated Bonds

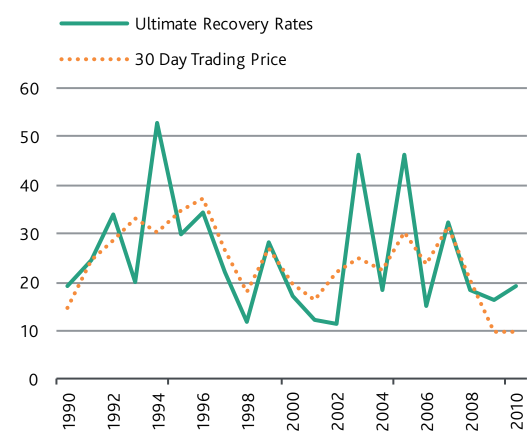


EXHIBIT 7

Empirical Standard Deviation of Ultimate Recovery Rates Conditional on Trading Price Level

EXHIBIT 7A

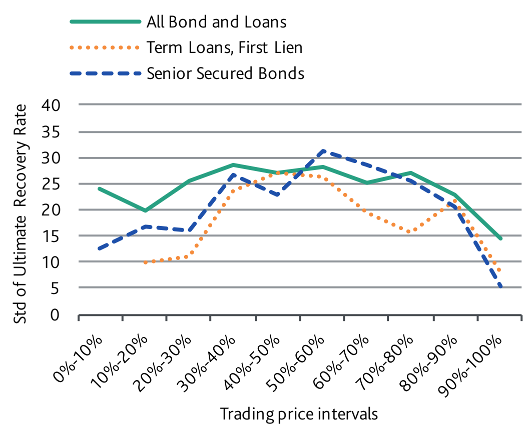
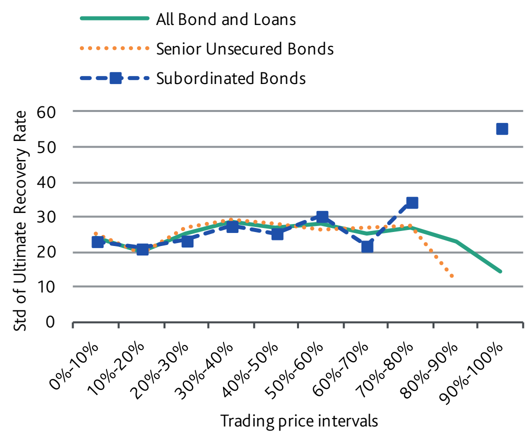


EXHIBIT 7B



3. Theory and Models

3.1 The Expectation Hypothesis

We denote the realized ultimate recovery for instrument i, realized at resolution date T, by $Y_{i,T}$. At time $t < T$ we form an expectation of $Y_{i,T}$ conditional on the information set (Ω)available at t. Let $E_t(x)$ $E(x\,|\Omega_t)$ for any x. Then we can express the realized ultimate recovery rate as a deviation from that expectation:

$$Y_{i,T} = E_t(Y_{i,T}) + \Psi_{i,t\to T}, \quad t \le T \tag{1}$$

where $\Psi_{i,t\to T}$ represents the cumulative impact of new information obtained between the date of the forecast t and the date of ultimate resolution T. The closer we get to the resolution date the smaller the variation in ultimate recovery rates is unexpected. Put differently, as t approaches T, $\Psi_{i,t\to T}$ approaches 0.

Denote the trading price for instrument i at time t by $X_{i,t}$. Under general conditions we know that the trading price at t is the expected discounted future payoff:

$$X_{i,t} = E_t\left[m_{t,T}Y_{i,T}\right] = \frac{E_t\left[Y_{i,T}\right]}{R_{i,t\to T}} \equiv E_t\left[y_{i,T}\right] \tag{2}$$

where $R_{i,\,t\to T}$ is an asset-specific discount factor and $m_{t,T}$ denotes the pricing kernel between period t and T. Given knowledge of $R_{i,\,t\to T}$, and applying it to both sides of Equation (1), this would support the following regression model:

$$y_{i,T} = a_{t\to T} + b_{t\to T}X_{i,t} + \frac{\Psi_{i,t\to T}}{R_{i,t\to T}} \equiv a_{t\to T} + b_{t\to T}X_{i,t} + \varepsilon_{i,t\to T} \tag{3}$$

We discount the ultimate recovery rate back to the last date when interest was paid on the debt instrument. This means that ultimate recovery rates can only take values between 0% and 100%

except in the case where preferred stock was issued to the debt holder at emergence. To make sure we compare like for like we discount the trading price back to the last day of interest payment using the coupon of the bond.[12] If the day of last interest payment is *t-k* this simply means we are testing whether

$$\frac{X_{i,t}}{R_{i,t-k\to t}} = \frac{E_t\left[Y_{i,T}\right]}{R_{i,t-k\to T}}$$

From Equation (3) a test of the Expectation Hypothesis (EH) is simply the strict (joint) hypothesis that a = 1-b = 0 for all periods t. A less strict version implies that b is equal to 1 once we make a constant adjustment, a, to the trading price. This is the weak form of the EH.

There could be a number of reasons why the trading price may differ from the discounted ultimate recovery rate. First, the trading price is an expectation formed before resolution, possibly years before. New information may arrive after the day on which the expectation is formed as markets learn more about the assets and liabilities of the company. We do not expect that $\Psi_{i,t\to T}$ is always equal to 0; we just expect it to be 0 on average.

Second, the post-default trading price may reflect other things besides the physical expected recovery. The pricing may reflect an ex-ante risk premium and an ex-ante liquidity premium. All of this is to say that we do not actually observe the appropriate discount rate $R_{i,t\to T}$. Instead we will use the instrument coupon rate as our proxy, but nothing guarantees that it is correct. The risk premium may be positive or may be negative, i.e. the trading price may be above or below the ultimate recovery rate, depending on the correlation between the pricing kernel and ultimate recovery rates. If ultimate recovery rates are low in bad states of the world, such as recessions with high unemployment and low consumption, we would expect a positive risk premium.

A rejection that the intercept a equals zero in Equation (3) may simply suggest the presence of a constant risk and/or liquidity premium. If the intercept changes over time, the risk premium and/or liquidity premium may be time-varying with changing economic and financial market conditions. If b is less than 1 this may suggest the presence of a risk and/or liquidity premium negatively correlated with the trading price expected recovery and the slope greater than 1 that risk and/or liquidity premia are positively correlated with the trading price recovery rate.

3.2 Regression Models

Recall that we cap the ultimate recovery rates at 100% such that all ultimate recovery rates take values in the closed interval from 0% to 100%. We also exclude observations where the 30 Day TP is greater than 100%. Our initial specification assumes the discounted ultimate recovery rate is linear in the trading price:

$$y_{i,T} = a_{t\to \mathrm{T}} + b_{t\to \mathrm{T}} X_{i,t} + \varepsilon_{\mathrm{i},t\to \mathrm{T}} \tag{4}$$

We further assume that the standard deviation of the ultimate recovery rate is a function of the trading price:

$$\sqrt{V_t(y_{i,T})} = \exp(c_{t\to \mathrm{T}} + d_{t\to \mathrm{T}} X_{i,t} + e_{t\to \mathrm{T}} X_{i,t}^2) \tag{5}$$

[12] Note, throughout when we write $X_{i,t}$ or use the term trading price we mean the trading price discounted back to the day of last interest payment using the coupon of the bond.

We include the square of the 30 Day TP to allow the conditional volatility of ultimate recovery rates to be lower at the extremes of trading prices, as the data suggested above in Exhibit (7).

As we will discuss below, we may modify Equation (4) to account for the truncation of ultimate recoveries at 0% and 100%. Such censored regressions require distributional assumptions. We discuss these in turn.

Linear Regression

In the linear regression we adopt Equation (4) as our model of the observed ultimate recovery rate. We estimate the model parameters by Feasible Generalized Least Squares (FGLS), using Equation (5) as our model of the conditional variance of ultimate recoveries. No special allowances are made for observations of 0% or 100% which is hard to justify economically.

While this is a straightforward model, it does present some statistical problems. Since ultimate recoveries cannot fall below 0, or rise above 1, we have a problem of heteroskedastic errors and biased parameter estimates. It would seem to be prudent to use methods which appropriately account for this censoring.

Censored Regressions

We may alternatively think that there is some underlying unobserved recovery variable (y*) which is linear in the 30 Day TP. In practice we observe the ultimate recovery rate (y) which is driven by, and often equal to, this unobserved process (y*). In this respect it is a standard two-sided censored regression where the observable ultimate recovery rate is non-linear in the 30 Day TP. Specifically:

$$
\begin{aligned}
y^{*}_{i,T} &= a_{t\to T} + b_{t\to T} X_{i,t} + \varepsilon_{i,t\to T}\,, \\
y_{i,T} &= y^{*}_{i,T} \text{ if } 0\% < y^{*}_{i,T} < 100\% \\
y_{i,T} &= 0\% \quad \text{if} \quad y^{*}_{i,T} \le 0\%; \\
y_{i,T} &= 100\% \quad \text{if} \quad y^{*}_{i,T} \ge 100\%
\end{aligned}
\tag{6}
$$

If we make an assumption on the distribution of the residual, $i, t\ T$, and by extension the unobserved and observed ultimate recovery rate, the model can be estimated using Maximum Likelihood (ML). We will explore two distributional assumptions. First, we estimate the model assuming that the residual is normally distributed. This will be recognized as a Tobit model, though with a modification for conditional heteroskedasticity as per Equation (5). Second, we adopt a Beta distribution, again with the model for heteroskedasticity. To our knowledge this is a new regression approach, at least in this context.

Tobit Regression

In the Tobit regression we assume that the error term of the unobservable recovery variable is normally distributed. While the unobserved recovery rate is linear in the 30 Day TP, the expected observed ultimate recovery rate is non-linear in the 30 Day TP given by:

$$
\begin{aligned}
&E_t\left(y_{i,T} \mid X_{i,t}\right) = \\
&\Phi(-\delta_{U,t,T}) + \left(\Phi(\delta_{U,t,T}) - \Phi(\delta_{L,t,T})\right)\left(a_{t\to T} + b_{t\to T} X_{i,t} + \sqrt{V_t(y^{*}_{i,T})}\,\frac{\phi(\delta_{L,t,T}) - \phi(\delta_{U,t,T})}{\Phi(\delta_{U,t,T}) - \Phi(\delta_{L,t,T})} \right)
\end{aligned}
$$

(7)

where

$$\delta_{L,t,T} = \frac{-a_{t\to T} - b_{t\to T} X_{i,t}}{\sqrt{V_t(y^*_{i,T})}}, \delta_{U,t,T} = \frac{1 - a_{t\to T} - b_{t\to T} X_{i,t}}{\sqrt{V_t(y^*_{i,T})}}$$

(8)

Φ is the Cumulative Distribution Function (CDF) and φ is the Probability Density Function (PDF) of the standard normal distribution. The log likelihood function is standard and given by:

$$\ln(L_{i,t,T}) = \sum_{y_{i,T}=0} \ln\Phi(\delta_{L,t,T}) + \sum_{y_{i,T}=1} \ln\Phi(-\delta_{U,t,T}) + \sum_{0<y_{i,T}<1} \ln\frac{1}{\sqrt{V_t(y^*_{i,T})}}\phi\left(\frac{\varepsilon_{i,t\to T}}{\sqrt{V_t(y^*_{i,T})}}\right)$$

(9)

Beta Regression

The censored beta regression is estimated replacing the normal CDF and PDF with those of the Beta distribution. As the standard Beta distribution restricts the unobserved recovery process to take values in the open interval between 0% and 100% we make one slight modification to allow some probability that the ultimate recovery rate will take values of exactly 0% and 100%. This is achieved by changing the support of the distribution to range from f to g, with $f < 0$ and $g > 1$. Specifically, we transform the unobserved ultimate recovery rate:

$$z^*_{i,T} = \frac{y^*_{i,T} - f}{g - f} \tag{10}$$

where f and g are parameters to be estimated. The distribution of the unobserved recovery rate process is thus modified beta distributed:

$$z^*_{i,T} \sim \mathrm{B}(\alpha_{t\to T}, \beta_{t\to T}) \tag{11}$$

where the two shape parameters of the beta distribution are given by:

$$\alpha_{t\to T} = E_t(z^*_{i,T})\left(\frac{E_t(z^*_{i,T})(1-E_t(z^*_{i,T}))}{V_t(z^*_{i,T})}\right) - 1 \quad \text{and} \quad \beta_{t\to T} = \left(1 - E_t(z^*_{i,T})\right)\left(\frac{E_t(z^*_{i,T})(1-E_t(z^*_{i,T}))}{V_t(z^*_{i,T})} - 1\right)$$

The expected value and the variance of the unobservable recovery rate process are given by:

$$E_t(z^*_{i,T}) = \frac{E_t(y^*_{i,T}) - f}{g - f} \qquad \text{and} \qquad V_t(z^*_{i,T}) = \frac{V_t(y^*_{i,T})}{(g-f)^2}$$

The expected observable recovery rate is given by:

$$E_t\left(y_{i,T} \mid X_{i,t}\right) = P\left(z^*_{i,T} \geq \frac{1-f}{g-f}; \alpha_{i,t\to T}, \beta_{i,t\to T}\right) + \int_0^1 X f_\beta\left(\frac{X-f}{g-f}; \alpha_{i,t\to T}, \beta_{i,t\to T}\right) dX \tag{12}$$

which can be obtained numerically. The likelihood function is equally straightforward to derive. It is given by:

$$\ln(L_{i,t,T}) = \sum_{y_{i,T}=0} \ln F_\beta\left(\frac{-f}{g-f}; \alpha_{i,t\to T}, \beta_{i,t\to T}\right) + \sum_{y_{i,T}=1} \ln\left(1 - F_\beta\left(\frac{1-f}{g-f}; \alpha_{i,t\to T}, \beta_{i,t\to T}\right)\right)$$
$$+ \sum_{0<y_{i,T}<1} \ln\left(\frac{1}{g-f} f_\beta\left(\frac{y_{i,T}-f}{g-f}; \alpha_{i,t\to T}, \beta_{i,t\to T}\right)\right)$$
(13)

F_β and f_β are the CDF and PDF, respectively, of the beta distribution.

4. Results

4.1 Testing the Expectation Hypothesis with Constant Parameters

First we test the Expectation Hypothesis (EH) assuming the parameters in the mean and variance processes are constant, i.e.

$$a_{t\to T} = a, \qquad b_{t\to T} = b, \qquad c_{t\to T} = c, \qquad d_{t\to T} = d, \qquad e_{t\to T} = \mathrm{e}$$

We test the null hypothesis that the strong form of the EH holds:

$$H_0 : a = 1 - b = 0$$

and the null hypothesis that the weak form of the EH holds, i.e.

$$H_0 : b = 1$$

Results can be found in Exhibit (8).

Linear Regression

The following stand out:

» We accept the weak form of the EH only for loans at a 5% significance level.

» The variation in ultimate recovery rates, as measured by the R^2, is explained better by the 30 Day TP the higher the seniority level of the debt. The R^2 is lowest for Subordinated bonds and highest for First Lien Loans.

» While there does not seem to be any systematic pattern in the intercept as a function of seniority, the slope coefficient is closer to 1 the higher the seniority of the bond. In other words, we find stronger support for the EH the higher the debt is placed in the capital structure. A slope coefficient significantly lower than one would be consistent with a presence of a risk/liquidity premium that is negatively correlated with the trading price.

Overall the 30 Day TP recovery rate does a good job at explaining the variation in the ultimate recovery rates as indicated by a relatively high R^2.

EXHIBIT 8

Regression of Ultimate Recovery Rate on 30 Day TP

	Model	a	b	c	d	e	f	g	R^2	Weak	Strong
All Bond and loans	Linear	0.076	0.887	-1.704	1.952	-2.129			48.77%	29.03**	58.56**
		(0.010)	(0.021)	(0.047)	(0.276)	(0.306)					
	Tobit	0.033	1.033	-1.437	0.535	-0.056			48.10%	0.96	28.60**
		(0.013)	(0.034)	(0.057)	(0.341)	(0.405)					
	Beta	0.089	0.923	-1.734	1.953	-1.403	**-0.009**	**1.382**	48.06%	5.80*	115.80**
		(0.009)	(0.033)	(0.069)	(0.353)	(0.366)					
Loans, First Lien	Linear	0.069	0.965	-3.215	7.023	-6.562			58.96%	0.76	15.10**
		(0.033)	(0.040)	(0.373)	(1.360)	(1.114)					
	Tobit	-0.072	1.412	-2.795	4.882	-3.518			60.32%	18.19**	38.39**
		(0.046)	(0.097)	(0.388)	(1.501)	(1.324)					
	Beta	-0.084	1.464	-2.805	4.925	-3.362	-0.678	42.575	59.88%	17.58**	31.20**
		(0.048)	(0.111)	(0.369)	(1.427)	(1.248)					
Senior Secured Bonds	Linear	0.129	0.860	-2.367	4.618	-4.748			47.25%	10.48**	23.94**
		(0.030)	(0.043)	(0.189)	(0.906)	(0.913)					
	Tobit	0.118	0.936	-2.297	3.916	-3.336			46.45%	0.982	26.98**
		(0.029)	(0.065)	(0.202)	(1.018)	(1.089)					
	Beta	0.106	0.972	-2.470	4.621	-3.815	-0.000	1.923	45.79%	0.12	30.78**
		(0.026)	(0.080)	(0.214)	(1.100)	(1.016)					
Senior Unsecured Bonds	Linear	0.120	0.785	-1.622	1.661	-1.894			35.88%	27.62**	51.12**
		(0.017)	(0.041)	(0.075)	(0.499)	(0.609)					
	Tobit	0.100	0.853	-1.415	0.474	-0.191			35.53%	7.99**	29.96**
		(0.019)	(0.052)	(0.085)	(0.557)	(0.698)					
	Beta	0.117	0.820	-1.614	1.406	-1.079	**-0.005**	1.464	35.54%	10.88**	52.48**
		(0.018)	(0.054)	(0.110)	(0.591)	(0.649)					
Subordinated Bonds	Linear	0.075	0.645	-1.633	0.930	-0.243			18.14%	23.81**	24.86**
		(0.017)	(0.073)	(0.078)	(0.558)	(0.756)					
	Tobit	0.034	0.737	-1.343	-0.169	1.067			18.16%	9.10**	11.03**
		(0.022)	(0.087)	(0.094)	(0.656)	(0.907)					
	Beta	0.097	0.582	-1.587	0.888	0.521	**-0.006**	3.103	17.11%	25.44**	32.29**
		(0.018)	(0.083)	(0.149)	(0.762)	(0.779)					

Note: We use the F test to test weak and strong form of the Expectation Hypothesis, * means that we reject the null hypothesis at the 0.05 level and ** that we reject the null at the 0.01 level. We report the R^2 adjusted for the number of parameters estimated in the model. For the beta model we report the estimated transformed lower (f) and upper (g) bound and indicate bold if statistically significant at the 0.05 level.

Tobit Regression

Estimating the censored Tobit regression suggests that:

- » By contrast to the linear regression we reject the EH for loans but accept the weak form for Senior Secured Bonds. Once we estimate the model across all bonds and loans we cannot reject that the slope coefficient is equal to 1 reflecting the mix of high/low intercepts and high/low slope coefficients across seniority. While this is an interesting finding it is perhaps of little practical use as we reject the EH for most seniority levels once we split the data by the instrument's place in the capital structure.
- » Relative to the linear regression, the slope coefficient is higher. For loans this means that the slope moves further away from theory while it gets closer to theory for all other seniority levels.
- » Similar to the linear regression, the R^2 increases with seniority.

Beta Regression

Estimates of the censored Beta regression are qualitatively similar to the estimates obtained from the Tobit model. The main points to notice are:

- » The R^2 is slightly lower for most seniority levels. Relative to the Tobit model, the slope coefficient is higher for loans and Senior Secured bonds while lower for Senior Unsecured and Subordinated bonds. When estimating the model across all bonds and loans this amounts to a slightly smaller slope coefficient relative to the Tobit model. In all cases the slope is further away from 1 although not in a statistical sense.
- » We cannot accept the null hypothesis that the strong version of the EH is true for any seniority level whereas we accept the weak form of the hypothesis only for Senior Secured bonds. This is consistent with the findings of the Tobit estimations.

A Comparison of the Expected Recovery Rate for Different Regression Models

Exhibit (9) plots the expected mean of the ultimate recovery rates as a function of the 30 Day TP. We include the estimated relationship between the standard deviation of the ultimate recovery rate and the 30 Day TP in Exhibit (A3) and (A4) of the Appendix.

We find that the models do a good job fitting the standard deviation observed in the actual data. The Beta model seems to fit the standard deviation better than the other models at trading prices where the volatility is higher. The exhibits also highlight the difference in the standard deviation of the observable and unobservable recovery rate process.

We compare the regression functions from each of the estimated models with that based on an Isotonic, or Monotonic, regression. The Isotonic regression is the best (in a minimum squared error sense) possible regression in the family of (weakly) monotonic functions, hence it serves as a benchmark for the best possible regression function we can obtain whether using a linear or non-linear monotonic regression model.

The main difference between the estimated regressions arises for higher levels of the 30 Day TP recovery rate. While the censored regressions do a relatively good job of fitting ultimate recoveries conditional on high levels of the 30 Day TP for instruments higher in the capital structure, the models find it difficult to fit very high recovery rates for Senior Unsecured and Subordinated bonds. This may in part be explained by the paucity of observations of high 30 Day TPs for these instruments. The censored regressions appear to be slightly closer to the isotonic regression for lower and medium trading prices at the cost of fitting the few observations at higher trading prices.

EXHIBIT 9
Expected Recovery Rate as a Function of the 30 Day TP

EXHIBIT 9A
All Bond and Loans

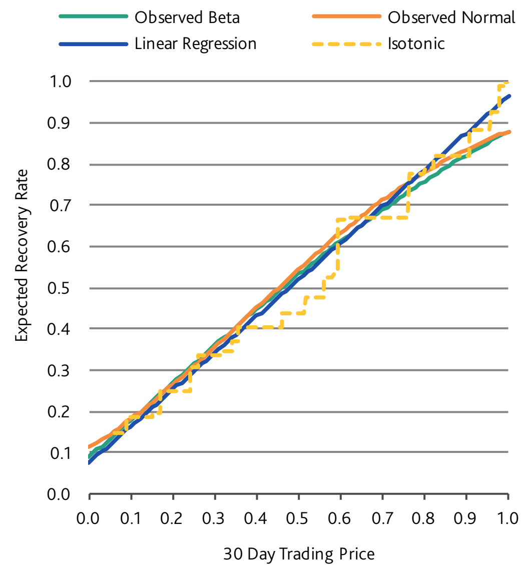


EXHIBIT 9B
Term Loans, First Lien

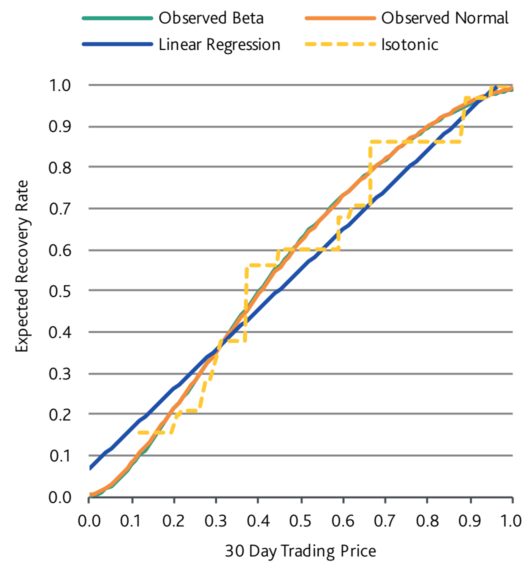


EXHIBIT 9C
Senior Secured Bonds

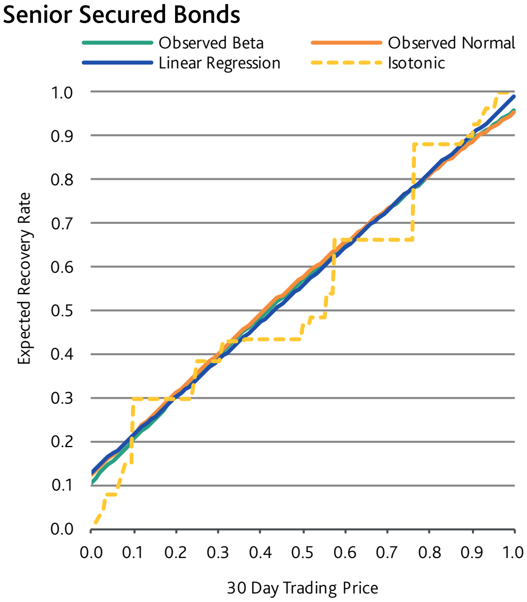


EXHIBIT 9D
Senior Unsecured Bonds

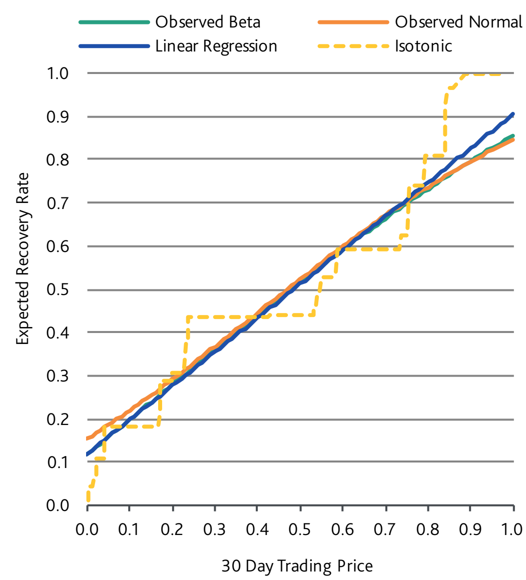
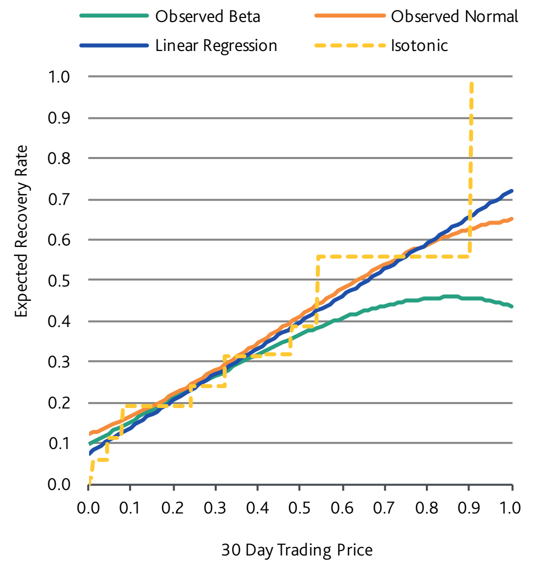


EXHIBIT 9E
Subordinated Bonds



4.2 Testing the Expectation Hypothesis with a Time-varying Intercept

Next we allow the intercept ($a_{t\rightarrow T}$) in the regression to vary over time, changing by year. All other parameters are assumed constant.[13] Including a time-varying intercept in the regression allows us to test the hypothesis that the slope is equal to 1 when conditioning on a time-varying variable in the regression perhaps as a proxy for risk and liquidity premia. If it turns out to be the case, we could then search for an appropriate observable time-varying variable. However, by using time-varying indicator variables we already have the best available time-series variable. If we fail to accept that the slope is equal to 1 with a time-varying intercept, this would obviate the need to search for any observable (or even set of observable) time-series variables.

To avoid over-fitting the model to the data by including too many parameters, we assume that the intercept only changes over time if at least eight observations are available in a given year.

Exhibit (10) shows the estimated parameters of the models and Exhibit (11) plots the intercept by seniority over time.

EXHIBIT 10
Regression Estimates, Time-varying Intercept By Default Year

	Model	b	c	d	e	f	g	R^2	Weak	Test for cst. Intercept
	Linear	0.877 (0.02)	-1.732 (0.048)	1.633 (0.278)	-1.656 (0.311)			52.92%	120.29**	29.22
All Bond and loans (21)	Tobit	1.040 (0.035)	-1.496 (0.057)	0.330 (0.338)	0.335 (0.401)			52.95%	1.35	145.39**
	Beta	0.913 (0.034)	-1.708 (0.069)	1.744 (0.355)	-1.174 (0.371)	**-0.009**	**1.388**	49.53%	6.58*	64.05**

13 Recall Exhibit (A1) in the Appendix which shows the number of observations per year for each level of seniority.

EXHIBIT 10

Regression Estimates, Time-varying Intercept By Default Year

	Model	b	c	d	e	f	g	R^2	Weak	Test for cst. Intercept
	Linear	0.989 (0.042)	-3.528 (0.451)	7.945 (1.602)	-7.208 (1.275)			58.97%	0.07	9.05
Term Loans, First Lien (6)	Tobit	1.516 (0.101)	-3.787 (0.498)	8.102 (1.891)	-5.498 (1.603)			60.91%	24.46**	31.27**
	Beta	1.643 (0.121)	-3.929 (0.489)	8.584 (1.816)	-6.055 (1.502)	-0.182	**676.3**	59.17%	28.26**	43.92**
	Linear	0.892 (0.051)	-2.427 (0.202)	4.664 (0.960)	-4.848 (0.953)			50.34%	4.59*	20.35**
Senior Secured Bonds (7)	Tobit	0.983 (0.069)	-2.363 (0.215)	4.099 (1.054)	-3.670 (1.100)			49.22%	0.06	18.68**
	Beta	0.945 (0.065)	-2.356 (0.205)	3.814 (0.954)	-3.363 (1.016)	-0.011	1.152	47.09%	0.73	8.94
	Linear	0.789 (0.038)	-1.825 (0.076)	2.312 (0.504)	-2.658 (0.626)			47.29%	29.94**	141.62**
Senior Unsecured Bonds (12)	Tobit	0.860 (0.048)	-1.669 (0.086)	1.511 (0.576)	-1.452 (0.734)			47.11%	8.46**	134.73**
	Beta	0.789 (0.059)	-1.654 (0.110)	1.374 (0.579)	-1.115 (0.626)	-0.005	1.264	43.71%	12.96**	70.43**
	Linear	0.637 (0.076)	-1.689 (0.080)	0.612 (0.575)	0.497 (0.784)			22.21%	22.55**	55.69**
Subordinated Bonds (17)	Tobit	0.711 (0.091)	-1.380 (0.100)	-0.648 (0.696)	2.007 (0.960)			23.38%	10.22**	51.23**
	Beta	0.505 (0.089)	-1.455 (0.223)	0.035 (0.988)	1.325 (1.066)	-0.005	3.129	14.48%	30.73**	23.60**

Note:We use the F test to test whether intercept is constant through time.* means that we reject the null hypothesis at the 0.05 level and ** that we reject the null at the 0.01 level. Standard errors in parenthesis. We report the R^2adjusted for the number of parameters in the model. Degrees of Freedom (DF) in parenthesis at seniority level label i.e. for loans there are 6 DF. For the beta model we report the estimated transformed lower (f) and upper (g) bound and indicate bold if statistically significant at the 0.05 level.

EXHIBIT 11

Intercept from Regression Models by Default Year

EXHIBIT 11A

All Bond and Loans

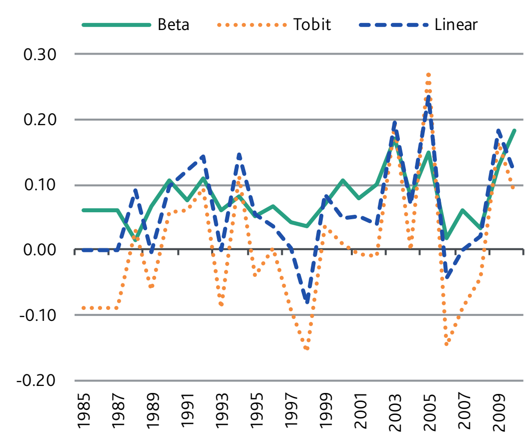


EXHIBIT 11B

Term Loans, First Lien

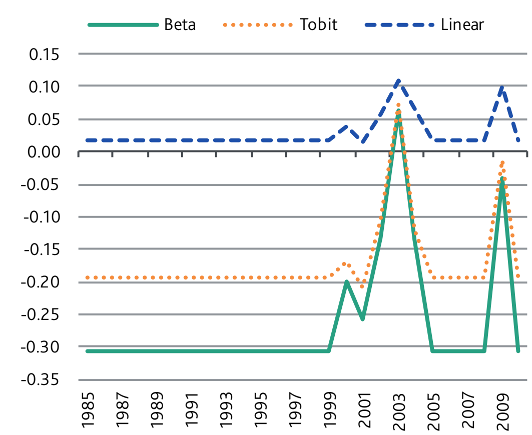


EXHIBIT 11C
Senior Secured Bonds

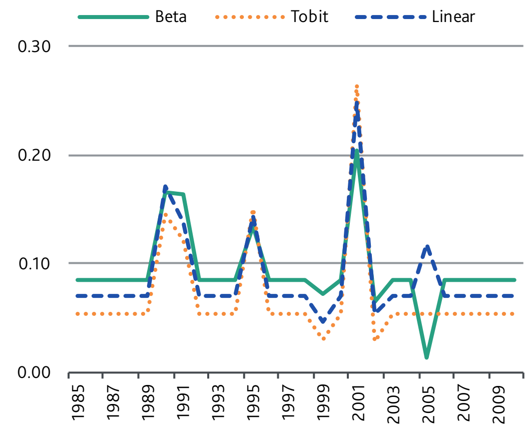


EXHIBIT 11D
Senior Unsecured Bonds

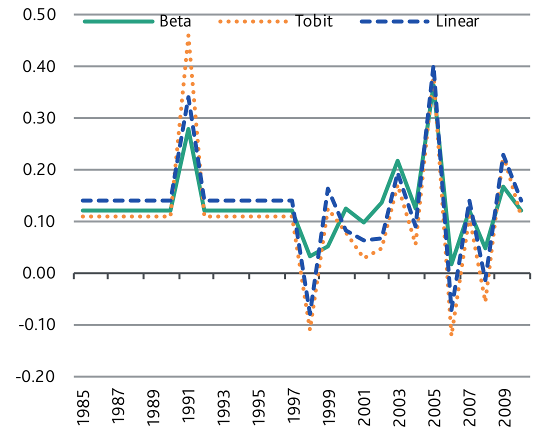
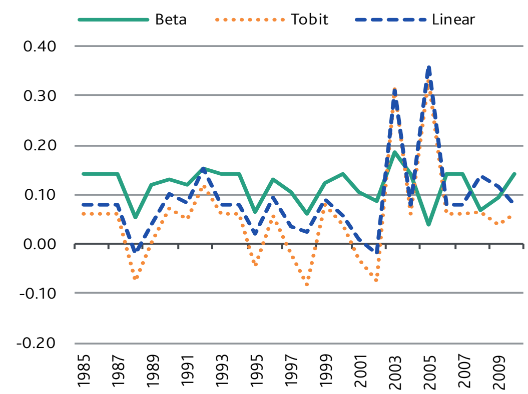


EXHIBIT 11E
Subordinated Bonds



Overall, our results suggest that no time-varying variable exists that can be included in the regression such that we accept weak version of the EH for bonds and loans consistently across all levels in the capital structure. If the weak version of the EH is ever true it could only be the case if we have omitted relevant cross-sectional variables. In other words, trading prices alone fail to explain the cross sectional variation in recovery rates within the same seniority group while it does a good job at explaining changes in recovery rates over the economic cycle.

In Exhibits (A5) and (A6) of the Appendix we report the estimates and plot the intercept from models where we allow the intercept to change by emergence rather than default year. The conclusions reached above are virtually the same although the slope tends to be estimated lower once we allow the intercept to change by emergence year. The estimated intercept is different in the two models and tend to be more variable for Senior Secured bonds and loans once we do the estimation by emergence year.

4.3 Testing the Expectation Hypothesis with Time-varying Intercept and Slope

Next we estimate the models allowing the intercept as well as the slope to change by year, i.e. $a_{t\to T}$ and $b_{t\to T}$ are no longer restricted to be constant.

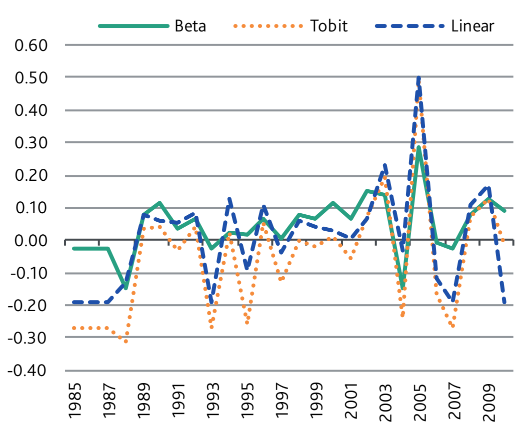
Exhibit 12
Other Parameter Estimates for Different Models

	Model	c	d	e	f	g	R^2	Test of constant intercept	Test of constant slope	Test of constant intercept and slope
All Bond and loans (21,42)	Linear	-1.767 (0.047)	1.693 (0.280)	-1.780 (0.312)			55.58%	144.21**	99.83**	234.68**
	Tobit	-1.505 (0.057)	0.272 (0.345)	0.298 (0.411)			55.30%	123.91**	81.12**	235.84**
	Beta	-1.576 (0.289)	1.284 (0.49)	-0.448 (0.216)	**-0.028**	8.901	51.24%	69.18**	103.14**	177.44**
Term Loans, First Lien (6,12)	Linear	-3.857 (0.453)	8.843 (1.638)	-7.811 (1.316)			59.58%	20.21**	12.83*	25.68*
	Tobit	-3.959 (0.499)	8.693 (1.902)	-6.489 (1.616)			61.32%	13.92*	9.67	42.11**
	Beta	-4.169 (0.493)	9.490 (1.894)	-6.878 (1.590)	-0.261	139.67	60.81%	15.13*	7.59	52.48**
Senior Secured Bonds (7,14)	Linear	-2.469 (0.197)	4.516 (0.975)	-4.537 (1.011)			50.72%	10.74	6.29	26.46*
	Tobit	-2.507 (0.223)	4.090 (1.155)	-3.241 (1.248)			50.79%	9.85	13.75	29.96**
	Beta	-2.459 (0.254)	4.047 (1.218)	-3.093 (1.189)	-0.000	223.21	48.12%	20.99**	39.30**	46.48**
Senior Unsecured Bonds (12,24)	Linear	-1.819 (0.080)	1.927 (0.558)	-2.240 (0.691)			51.62%	104.69**	47.27**	206.92**
	Tobit	-1.608 (0.093)	0.674 (0.660)	-0.486 (0.843)			51.61%	93.69**	48.34**	194.40**
	Beta	-1.652 (0.098)	0.544 (0.659)	-0.050 (0.752)	**-0.041**	1.527	48.55%	111.28**	142.96**	196.67**
Subordinated Bonds (17,34)	Linear	-1.670 (0.082)	0.278 (0.632)	0.674 (0.911)			27.16%	49.63**	32.77*	89.96**
	Tobit	-1.360 (0.099)	-1.025 (0.727)	2.273 (1.063)			27.75%	45.09**	32.38*	85.18**
	Beta	-1.554 (0.195)	0.032 (1.007)	1.945 (1.166)	**-0.004**	70.855	13.34%	35.58**	57.60**	92.35**

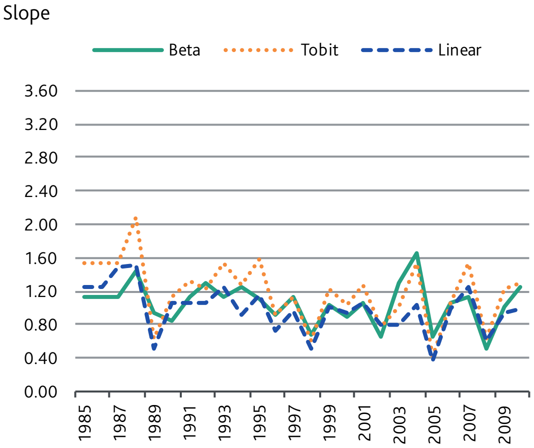
Note: We use F test whether the slope and intercept coefficients are constant through time. * rejecting the null at the 0.05 level, ** rejecting the null at the 0.01 level. Degrees of Freedom (DF) in parenthesis at seniority level label i.e. for loans there are 12 DF. Standard error in parenthesis. R^2 is adjusted for the number of parameters in the model. For the beta model we report the estimated transformed lower (f) and upper (g) bound and indicate bold if statistically significant at the 0.05 level.

EXHIBIT 13A
Time-varying Intercept and Slope
All Bonds and Loans

Intercept

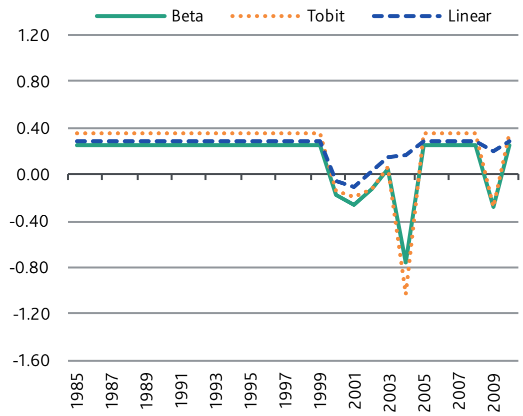


Slope

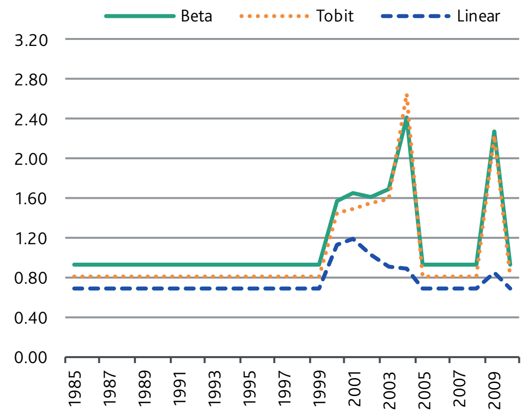


Term Loans, First Lien

Intercept

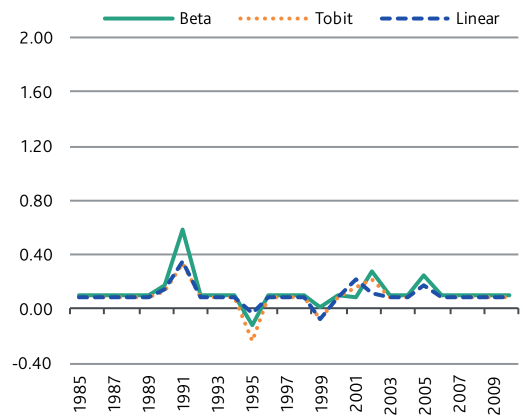


Slope

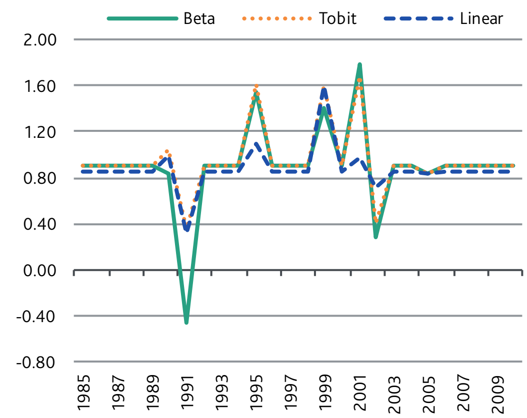


Senior Secured Bonds

Intercept



Slope



Like above we assume that the intercept and slope change over time only if eight or more observations are available in that given year. Parameter estimates for the Linear, Tobit and Beta regressions can be found in Exhibit (12) while we plot the time-varying slope and intercepts in Exhibits (13a) and (13b). The following stands out:

» Allowing the intercept and slope to change over time increases the R^2, least so for the Beta regression. The difference in the R^2 from the Tobit and Beta model is largest for Subordinated bonds.

» The estimated slope has been very volatile over the years ranging between negative and relatively large positive values for Senior Unsecured and Subordinated bonds in the years after 2000. The Beta regression has more stable parameters, the intercept in particular, relative to the Tobit and Linear model which may be an indication that recovery rates are better aligned with a censored Beta rather than normal or censored normal distribution.

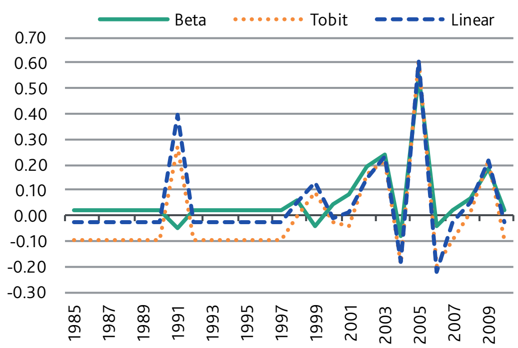
Although there are some differences in the R^2 across the Beta and Tobit regressions, the estimated coefficients in the mean equation are quite similar for all but Subordinated debt.

EXHIBIT 13B

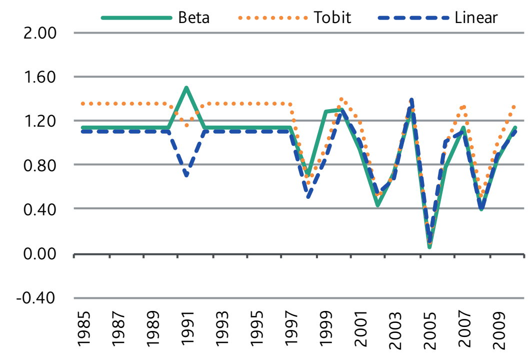
Time-varying Intercept and Slope

Senior Unsecured Bonds

Intercept

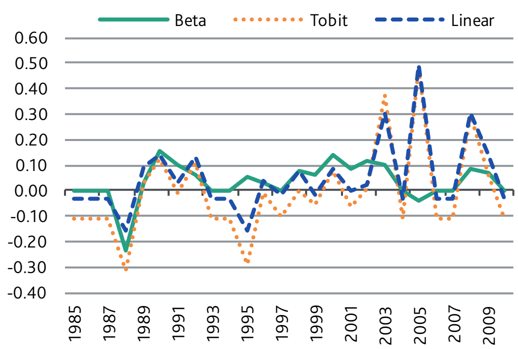


Slope

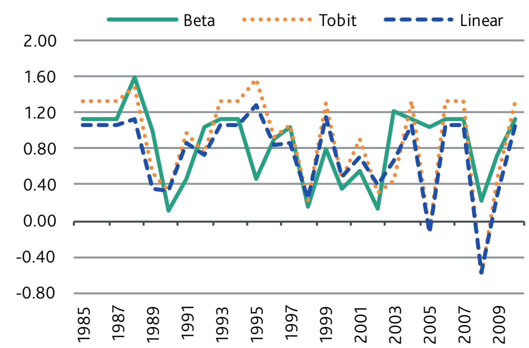


Subordinated Bonds

Intercept



Slope



4.4 Testing the Expectation Hypothesis by Time To Emergence

The number of days between the default and resolution varies considerably across instruments as can be seen in Exhibit (14). On average the number of days to resolution is 593 days while the median is 474 days. Intuitively we may expect the trading price to be more informative about the ultimate recovery rate the closer the resolution is to the default date. For long and complex bankruptcies one may expect larger surprises to the realized recovery rate relative to what was expected close to the default date.

To test this we consider the following regression where the intercept and slope becomes a function of the number days between default and emergence.

$$y_{i,T} = (a + hD_{i,T-t>1\,year}) + (b + iD_{i,T-t>1\,year})X_{i,t} + \varepsilon_{i,t\rightarrow T} \quad (14)$$

D_i is an indicator variable taking the value of 1 if the time to resolution is less than 365 and zero otherwise. We can re-arrange further:

$$y_{i,T} = a + bX_{i,t} + hD_{i,T-t>1\,year} + iD_{i,T-t>1\,year}X_{i,t} + \varepsilon_{i,t\rightarrow T} \quad (15)$$

Estimation of Equation (15) can be found in Exhibit (15). In Exhibit (16) we show test statistics of the weak and strong form of the EH. The regression results and tests suggest the following:

» Including the one year indicator variable the adjusted R^2 increases for most seniority levels across all models.

» For loans we are close to accepting the strong form of the EH once the number of days between default and emergence is more than 365 days. Once the number of days is less than equal 365 days we reject the strong as well as weak form of the hypothesis.

» For bonds it goes mostly in the other direction. When the number of days to emergence is more than 365 we consistently reject the weak and strong form of the EH across all models. The exception is Senior Secured bonds where we cannot reject the weak form in the censored models. When the number of days is less than one year, however, there are more acceptances of the weak as well as the strong form of the EH. It is now only in the Beta model where it is difficult to accept the weak and strong form of the EH. Interestingly we cannot reject the weak form of the EH for Subordinated bonds across all models for bankruptcies where the time to resolution is less than 1 year. When the time to emergence is less than one year the trading price is considerably more informative about the ultimate recovery rate for Subordinated debt.

EXHIBIT 14

Descriptive Statistics, Number of Days between Default and Emergence

	Days from Default to Resolution
Mean	593.3
Median	474
Standard Dev	445.5
Min	54
Max	3416

EXHIBIT 15

Allowing Parameters to Be a Function of Days to Resolution Indicator

	Model	a	b	c	d	e	f	g	h	i	R^2
All Bond and loans	Linear	0.042	0.980	-1.711	2.015	-2.230			0.053	-0.157	49.10%
		(0.017)	(0.032)	(0.047)	(0.274)	(0.305)			(0.021)	(0.042)	
	Tobit	-0.012	1.190	-1.452	0.694	-0.304			0.068	-0.245	48.48%
		(0.022)	(0.054)	(0.056)	(0.339)	(0.401)			(0.027)	(0.068)	
	Beta	0.092	1.022	-1.774	2.174	-1.617	**-0.009**	**1.459**	-0.013	-0.147	47.67%
		(0.012)	(0.048)	(0.072)	(0.380)	(0.383)			(0.011)	(0.048)	
Term Loans, First Lien	Linear	0.144	0.903	-3.088	6.571	-6.261			-0.094	0.037	60.36%
		(0.060)	(0.067)	(0.383)	(1.382)	(1.124)			(0.072)	(0.087)	
	Tobit	-0.147	1.686	-2.876	5.053	-3.663			0.135	-0.529	62.45%
		(0.077)	(0.162)	(0.400)	(1.556)	(1.379)			(0.091)	(0.182)	
	Beta	-0.162	1.745	-2.839	4.886	-3.331	-0.531	60.027	0.142	-0.550	62.02%
		(0.079)	(0.171)	(0.386)	(1.483)	(1.311)			(0.091)	(0.180)	
Senior Secured Bonds	Linear	0.020	0.963	-2.401	4.458	-4.500			0.169	-0.162	50.98%
		(0.044)	(0.070)	(0.193)	(0.932)	(0.939)			(0.056)	(0.088)	
	Tobit	0.019	0.981	-2.310	3.564	-2.838			0.154	-0.067	50.46%
		(0.049)	(0.112)	(0.209)	(1.079)	(1.172)			(0.061)	(0.144)	
	Beta	0.084	0.904	-2.397	3.972	-3.224	-0.000	1.455	0.050	0.062	48.48%
		(0.033)	(0.096)	(0.205)	(0.991)	(1.000)			(0.042)	(0.115)	
Senior Unsecured Bonds	Linear	0.048	0.873	-1.616	1.540	-1.744			0.102	-0.127	36.94%
		(0.031)	(0.076)	(0.076)	(0.512)	(0.630)			(0.036)	(0.089)	
	Tobit	0.037	0.887	-1.399	0.236	0.133			0.091	-0.051	36.54%
		(0.037)	(0.103)	(0.086)	(0.572)	(0.725)			(0.044)	(0.121)	
	Beta	0.128	0.774	-1.611	1.353	-0.998	**-0.005**	1.455	-0.018	0.071	35.23%
		(0.022)	(0.082)	(0.113)	(0.614)	(0.677)			(0.019)	(0.082)	
Subordinated Bonds	Linear	0.070	0.904	-1.624	0.730	-0.025			0.011	-0.404	20.90%
		(0.030)	(0.121)	(0.078)	(0.562)	(0.768)			(0.037)	(0.150)	
	Tobit	0.043	0.998	-1.333	-0.387	1.307			-0.012	-0.405	21.04%
		(0.037)	(0.145)	(0.094)	(0.662)	(0.921)			(0.047)	(0.181)	
	Beta	0.096	0.938	-1.571	0.529	1.047	**-0.005**	18.796	-0.007	-0.509	20.09%
		(0.022)	(0.116)	(0.169)	(0.860)	(0.900)			(0.016)	(0.098)	

Note: Standard errors in parenthesis. * means that we reject the null hypothesis at the 0.05 level and ** that we reject the null at the 0.01 level. We report the R^2 adjusted for the number of parameters estimated in the model. For the beta model we report the estimated transformed lower (f) and upper (g) bound and indicate bold if statistically significant at the 0.05 level.

EXHIBIT 16

Test of the Weak and Strong Form of Expectation Hypothesis

		Strong		Weak		# Observations	
	Model	Less than 1 year	More than 1 year	Less than 1 year	More than 1 year	Total	<1 year
All Bond and loans	Linear	10.46**	62.49**	**0.37**	43.72**	1296	460
	Tobit	23.82**	18.48**	11.94**	**1.84**		
	Beta	119.17**	66.74**	**0.20**	11.65**		
Term Loans, First Lien	Linear	21.67**	**1.49**	**2.07**	**1.13**	168	87
	Tobit	37.06**	6.33*	17.97**	**2.21**		
	Beta	37.09**	6.97*	18.98**	**2.86**		
Senior Secured Bonds	Linear	**0.28**	37.61**	**0.27**	13.42**	149	58
	Tobit	**0.20**	42.13**	**0.03**	**1.01**		
	Beta	7.80*	33.12**	**0.99**	**0.14**		
Senior Unsecured Bonds	Linear	**2.99**	58.21**	**2.79**	28.60**	520	155
	Tobit	**1.25**	37.99**	**1.22**	6.91**		
	Beta	39.91**	39.93**	7.61**	6.88**		
Subordinated Bonds	Linear	7.50*	32.16**	**0.63**	31.61**	398	134
	Tobit	**3.23**	24.18**	**0.01**	14.46**		
	Beta	30.08**	50.18**	**0.29**	50.16**		

Note: We use the F test. * means that we reject the null hypothesis at the 0.05 level and ** that we reject the null at the 0.01 level.

4.5 Why Choose the 30 Day Trading Price?

Above analysis has been conducted assuming that the 30 day trading price is the best choice for predicting ultimate recovery rates; the analysis did suggest that it does a reasonable job at explaining the level of, and variation in, ultimate recovery rates discounted back using the coupon of the bond.

We could obviously have chosen to record the trading price at any point in the weeks soon after the default has occurred. At Moody's, we choose as one measure of "recovery rate" the trading price recorded 30 days post-default. But why not choose the trading price at the default date, one week or two weeks after default, or later?

For our data sample it turns out that more defaulters have a trading price available around four weeks after default.[14] Exhibit (17) shows that the number of instrument trading prices available has its peak at 4 weeks increasing from 977 at default to 1059 in the fourth week and then slowly falling over the subsequent months from the 8th week onwards. While it may be counterintuitive that you would see more trading prices in the weeks after default the explanation is likely to be a change of investors from large institutional investors to investors with expertise in distressed debt. Moreover, resolutions slowly start to occur as we get past the first few months after default.

To compare the predictions of trading prices taken at different dates we need to make some additional restrictions to ensure we compare trading price predictions from the same sample. We add the following requirement for a defaulter to be included in the analysis. 1) a trading price has to be available every week starting from the default date until 6 weeks after default, and 2) a defaulter cannot emerge from default until at least 6 weeks after default. All loans are excluded from our analysis as we do not have a time-series of loan trading prices available. This leaves us with a total of 756 observations.

Exhibit (18) shows empirical characteristics on the difference between the ultimate recovery rate and the trading price recorded at different dates post default. We find that:

» The average difference between the ultimate and trading price recovery rates, across all seniority levels of bonds, is higher 3-6 weeks after default relative to 1-2 weeks after default.

» This result, however, is unique to Senior Unsecured bonds. The average difference between the ultimate recovery rate and trading price 4-6 weeks after default is close to zero for Subordinated bonds. Only for Senior Unsecured bonds do we see a statistically significant difference between the ultimate recovery rate and the trading price 3-6 weeks after the default date.

» The median trading price implied recovery rate, however, is better aligned with the median ultimate recovery rate when we pool across bonds of all seniorities. Once again, pooling masks some important differences across seniority. While for Senior Secured bonds the median difference between ultimate and trading price recovery is close to zero, the median difference is significantly positive for Senior Unsecured bonds and negative for Subordinated bonds.

» The standard deviation of the forecast error decreases with the number of weeks the trading price is recorded after default.

So while the average difference between the ultimate recovery rate and the trading price recorded 3-5 weeks after default is significantly positive, the median difference is close to zero. With the standard

[14] We record the trading price x weeks after the Friday in the week of the default. That means, for example, that the 4 week TP is recorded between 28 and 32 days after the default date.

deviation of the difference falling as we get closer to the resolution date it seems a reasonable choice to record the trading price 3-5 weeks after default.

We can support this conclusion further by running the Tobit and Beta regressions outlined in the previous section. Results can be found in Exhibits (19) and (20). The estimations reveal the following in support for choosing the trading price 3-6 weeks post default:

» For both sets of regressions, the R^2 increases almost monotonically in the number of weeks the trading price is recorded after default.

» Once we reach 4 to 6 weeks beyond the default date, it is more difficult to reject the weak EH. In other words the slope is increasing with the number of weeks the trading price is taken after default. We also find that the intercept is falling marginally by horizon.

Admittedly the above analysis is conducted on a relatively small, but liquid, sample. But the analysis does suggest that the variation in the prediction errors and bias of trading prices decreases the longer we wait to record the trading price. There is little in the analysis that suggest that choosing the trading price in any other week after default would improve the predictability of the ultimate recovery conditional on the trading price. Obviously if we could wait for a year or two the market price would be considerably better aligned with the realized recovery rate.

EXHIBIT 17

Number of Instruments with Trading Prices Available in our Databases

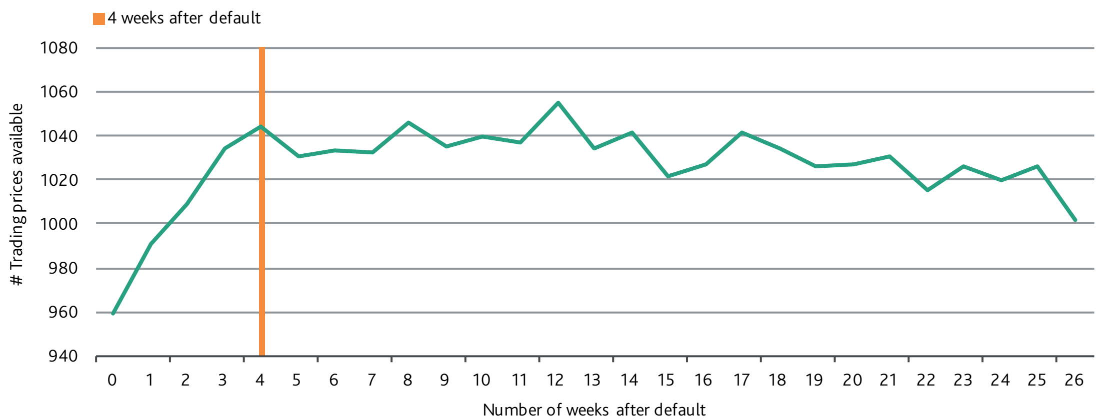


EXHIBIT 18

Difference between Ultimate and Trading Price Recovery Rates for All Bond Defaults with Continuum of Trading Prices Available by Weeks Post Default

	Weeks After Default Where Trading Price Is Recorded						
All (756 obs)	**Default**	**+1 week**	**+2 weeks**	**+3 weeks**	**+4 weeks**	**+5 weeks**	**+6 weeks**
Mean	1.17	1.49	1.76	2.79	3.31	3.30	4.05
Median	-0.55	-0.27	0.11	0.53	0.53	0.57	0.77
Std	25.95	25.96	25.18	25.20	25.15	24.74	24.55
Senior Secured (89obs)							
Mean	-1.52	-2.04	0.26	1.00	0.75	1.13	3.32
Median	0.24	0.26	-0.02	-0.41	0.06	0.10	0.28
Std	26.67	25.26	22.02	21.13	21.09	20.95	18.56
Senior Unsecured (399 obs)							
Mean	3.84	4.19	3.94	5.25	5.83	5.53	6.21
Median	1.32	2.00	2.36	3.83	3.78	2.54	4.54
Std	25.48	25.51	25.00	25.13	25.29	24.87	25.19
Subordinated (268 obs)							
Mean	-1.91	-1.37	-0.99	-0.27	0.40	0.71	1.09
Median	-2.66	-2.67	-2.31	-1.85	-1.98	-1.97	-1.99
Std	26.05	26.48	26.18	26.22	25.84	25.46	25.08

EXHIBIT 19

Censored Tobit Regression of Ultimate Recovery Rates on Trading Prices, All Bond and Loans

	Weeks After Default Where Trading Price Is Recorded						
	Default	**1 week**	**2 weeks**	**3 weeks**	**4 weeks**	**5 weeks**	**6 weeks**
a	0.059 (0.018)	0.068 (0.017)	0.056 (0.016)	0.060 (0.016)	0.061 (0.017)	0.054 (0.016)	0.055 (0.02)
b	0.834 (0.050)	0.822 (0.048)	0.864 (0.048)	0.885 (0.049)	0.896 (0.050)	0.917 (0.049)	0.942 (0.05)
c	-1.271 (0.073)	-1.338 (0.069)	-1.340 (0.070)	-1.317 (0.070)	-1.302 (0.069)	-1.308 (0.069)	-1.333 (0.07)
d	-0.674 (0.457)	-0.188 (0.441)	-0.301 (0.455)	-0.517 (0.460)	-0.541 (0.452)	-0.584 (0.456)	-0.444 (0.46)
e	1.365 (0.569)	0.756 (0.544)	0.877 (0.572)	1.206 (0.581)	1.194 (0.583)	1.231 (0.589)	1.086 (0.59)
R^2	35.00%	35.81%	38.53%	38.10%	38.02%	39.70%	40.45%
Test(b=1)	10.99**	13.98**	8.04**	5.49*	4.37*	2.88	1.40
Test(a=0, b=1)	12.49**	17.66**	11.72**	13.75**	14.98**	12.99**	17.92**

Note: Standard errors in parenthesis. The test is the standard F-test. * means that we reject the null hypothesis at the 0.05 level and ** that we reject the null at the 0.01 level. We report the R^2 adjusted for the number of parameters estimated in the model.

EXHIBIT 20

Censored Beta Regression of Ultimate Recovery Rates on Trading Prices, All Bond and Loans

	Weeks After Default Where Trading Price Is Recorded						
	Default	1 week	2 weeks	3 weeks	4 weeks	5 weeks	6 weeks
a	0.096 (0.014)	0.096 (0.014)	0.089 (0.013)	0.100 (0.013)	0.099 (0.013)	0.096 (0.013)	0.097 (0.013)
b	0.775 (0.048)	0.784 (0.050)	0.814 (0.049)	0.799 (0.047)	0.818 (0.048)	0.830 (0.046)	0.850 (0.047)
c	-1.650 (0.093)	-1.689 (0.094)	-1.718 (0.094)	-1.624 (0.086)	-1.623 (0.087)	-1.621 (0.088)	-1.604 (0.089)
d	1.396 (0.476)	1.633 (0.495)	1.701 (0.507)	1.118 (0.440)	1.172 (0.456)	1.077 (0.457)	0.973 (0.468)
e	-0.723 (0.492)	-0.951 (0.501)	-1.032 (0.524)	-0.382 (0.456)	-0.484 (0.484)	-0.372 (0.488)	-0.266 (0.500)
f	**-0.006**	**-0.007**	**-0.006**	**-0.006**	**-0.006**	**-0.005**	**-0.005**
g	1.506	1.563	1.547	**1.483**	1.495	1.534	1.528
R^2	35.03%	35.94%	38.64%	38.00%	38.08%	39.64%	40.42%
Test(b=1)	22.26**	18.80**	14.65**	18.67**	14.62**	13.42**	10.27**
Test(a=0, b=1)	50.53**	53.03**	51.76**	64.11**	65.07**	64.24**	68.88**

Note: Standard errors in parenthesis. We use the F-test. * means that we reject the null hypothesis at the 0.05 level and ** that we reject the null at the 0.01 level. We report the R^2 adjusted for the number of parameters estimated in the model. For the beta model we report the estimated transformed lower (f) and upper (g) bound and indicate bold if statistically significant at the 0.05 level.

5. Conclusions

We explore the predictive content of 30 day post-default trading prices as a measure of ultimate recovery discounted back to the day when interest was last paid on the debt instrument using the coupon of the bond. We adopt a variety of econometric specifications which allow us to fit the conditional empirical distribution of ultimate recovery rates as a function of the 30 day trading price. Among these specifications is a new censored regression based on the generalized beta distribution.

On average the ultimate recovery rate is around 3 percentage points higher than the 30 day trading price. While this may seem relatively small, the average hides some important differences across seniority. For loans and Senior Secured bonds, the average trading price is substantially lower than the ultimate recovery while for Subordinated bonds the averages are better aligned. This may suggest a larger risk or liquidity premium in the more senior debt.

The 30 day trading price, however, is a better predictor of the variation in ultimate recovery rates the higher the debt is placed in the capital structure. Trading prices have less predictive content for Subordinated bonds. While we find that the trading price does a good job at explaining the time-variation in, and cyclicality of, ultimate recovery rates, we reject the existence of a common systematic economic variable capturing potential risk and liquidity premia in trading prices. Our finding of an average gap between the ultimate and trading price recovery must be explained by cross-sectional differences between instruments of the same seniority not captured by the trading price.

The time from default to resolution is often more than two years. For defaulted bonds where the time to resolution is less than one year we find that the trading price is a better and frequently unbiased predictor of ultimate recoveries.

Finally, our analysis suggests that the trading price three to five weeks after default explains more of the variation in recovery rates than do earlier trading prices. Albeit evidence of a small bias, the trading price recorded 30 days post default provides a powerful prediction of the mean and variability of ultimate recovery rates, particularly for debt higher in the capital structure.

Our analysis has been conducted using the coupon of the bond as the appropriate rate to discount the ultimate recovery rate back to the default date. This may be different from the discount rate used by the market. Other candidate discount rates used in the literature are the yield on a high yield corporate index and a single B-index. Although other research (Acharya et al, 2003) has found a high correlation between the ultimate recovery rate discounted by the coupon of the bond or the high yield bond index, the choice of discount rate nevertheless remains an interesting topic for future research.

Moody's Related Research

Special Comments:

» Corporate Default and Recovery Rates, 1920-2011, February 2011 (140015)

» Moody's Ultimate Recovery Rate Database, Lessons from 1000 Corporate Defaults, December 2011 (137405)

» Moody's Ultimate Recovery Rate Database, April 2007 (102664)

» Hard Data for Hard Times II, The Crisis that Wasn't, February 2011 (131330)

» Hard Data for Hard Times, July 2010 (126338)

» Bond Prices at Default and at Emergence from Bankruptcy for US Corporate Issuers, June 2005 (92891)

» Debt Recoveries for Corporate Bankruptcies, June 1999 (SF7677)

To access any of these reports, click on the entry above. Note that these references are current as of the date of publication of this report and that more recent reports may be available. All research may not be available to all clients.

Other Research:

» Understanding the Recovery Rates on Defaulted Securities, CEPR Working paper 4098, October 2003, V. Acharya, S. Bharath and A. Srinivasan.

Appendix

EXHIBIT A1

Number of Observations by Year and Seniority

Year	All Loans	Loans, Fl	Senior Secured Bonds	Senior Unsecured Bonds	Subordinated Bonds	Total
1985	0	0	0	0	2	2
1986	0	0	0	0	0	0
1987	0	0	0	0	1	1
1988	0	0	2	4	10	16
1989	0	0	6	0	16	22
1990	0	0	21	4	30	55
1991	2	1	10	18	34	64
1992	4	3	1	1	19	25
1993	0	0	0	0	7	7
1994	2	2	1	3	6	12
1995	2	0	8	5	16	31
1996	3	2	2	1	9	15
1997	2	1	0	7	11	20
1998	3	2	5	22	9	39
1999	9	3	8	29	29	75
2000	29	23	7	32	44	112
2001	37	32	15	105	53	210
2002	38	37	28	139	35	240
2003	12	8	7	21	21	61
2004	12	11	3	20	3	38
2005	14	6	8	37	8	67
2006	7	4	2	12	3	24
2007	2	1	2	2	1	7
2008	14	7	6	11	12	43
2009	30	20	6	45	18	99
2010	7	5	1	0	1	9
2011	0	0	0	2	0	2
Total	**229**	**168**	**149**	**520**	**398**	**1296**

EXHIBIT A2

Comparing Ultimate and 30 Day TP Recovery Rates by Emergence Year

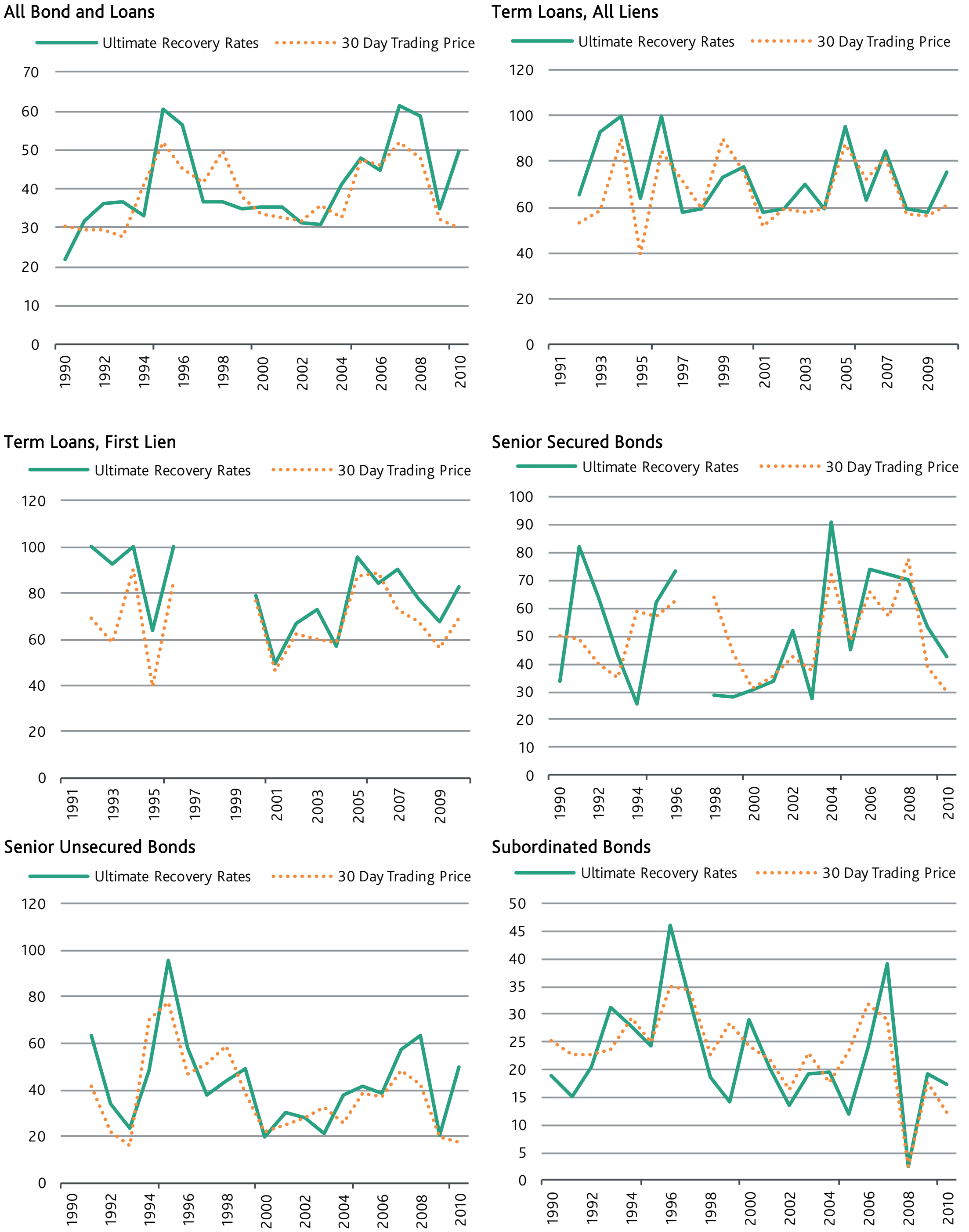
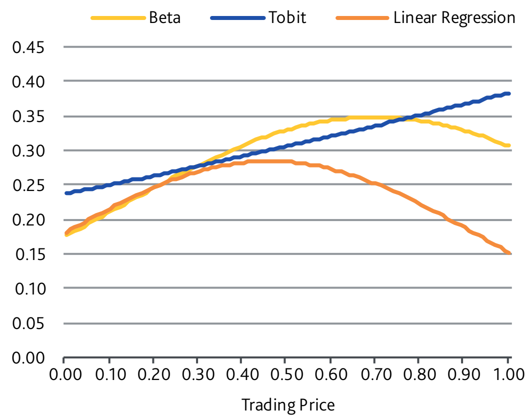


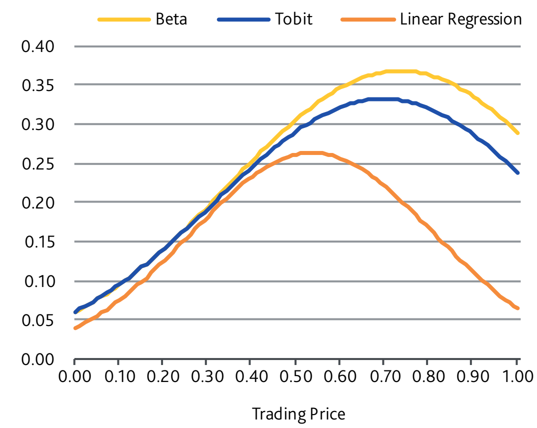
EXHIBIT A3

Standard Deviation of Unobservable Ultimate Recovery Process as Functions of the Trading Price[15]

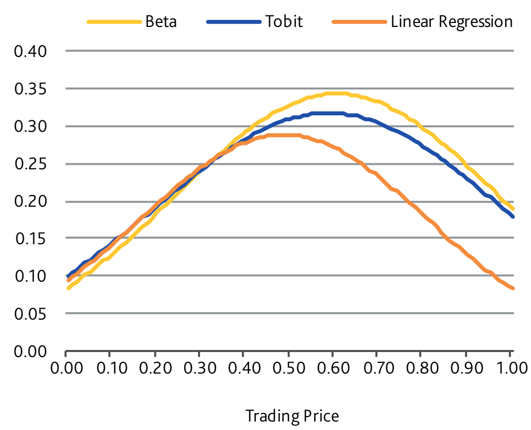
All Bond and Loans



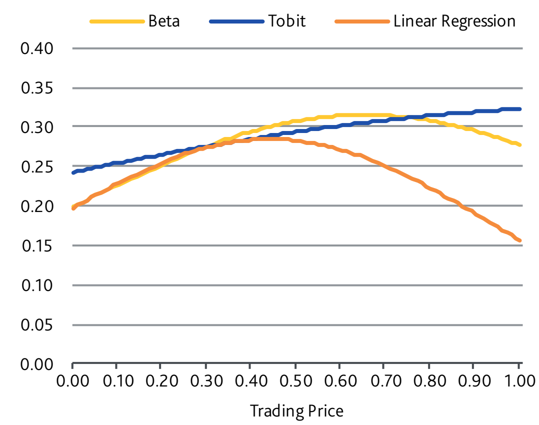
Term Loans, First Lien



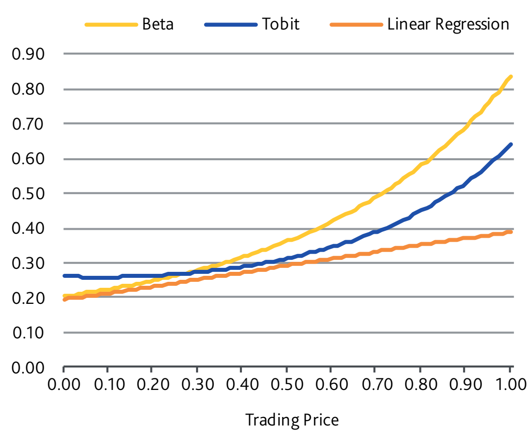
Senior Secured Bonds



Senior Unsecured Bonds



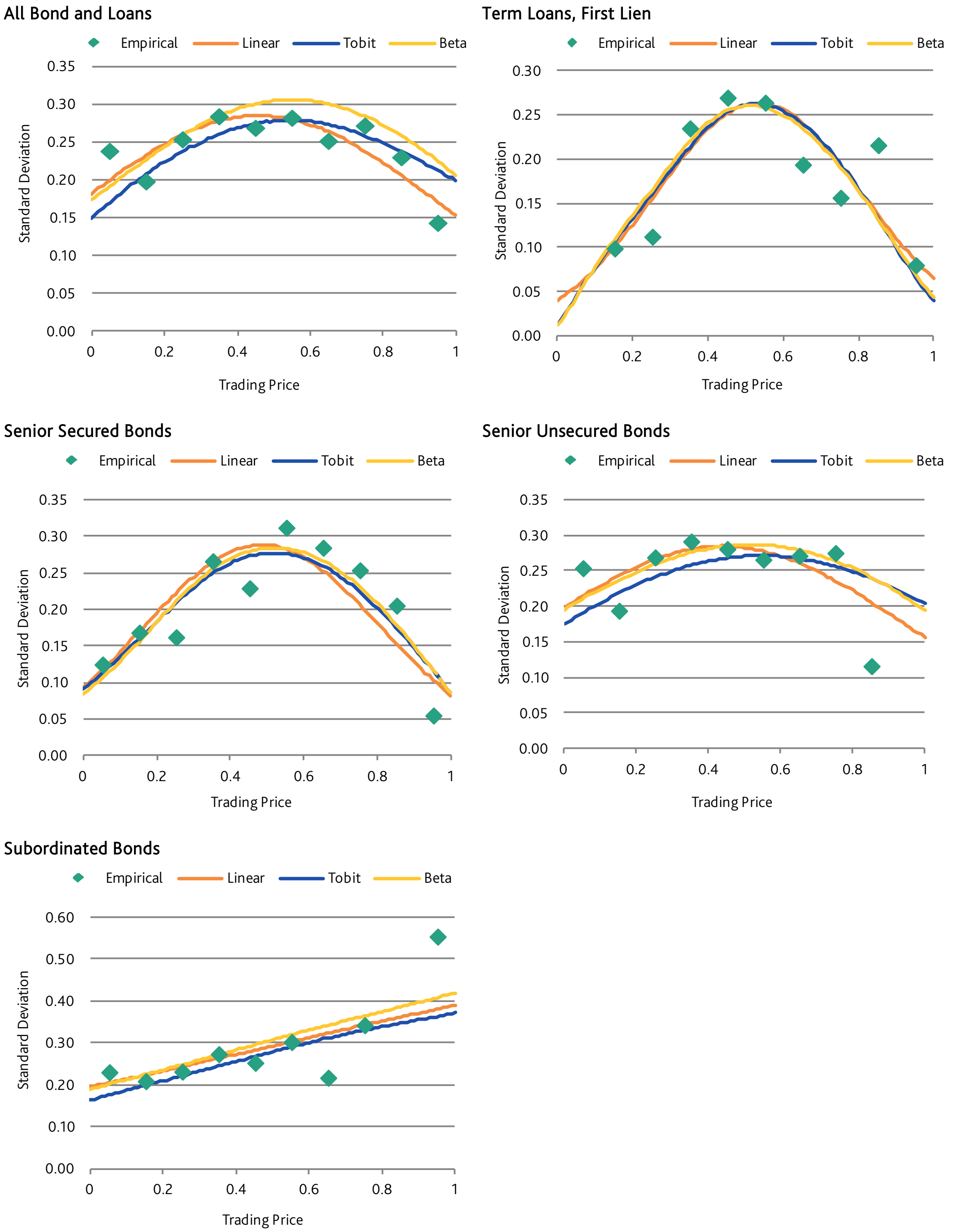
Subordinated Bonds



[15] Note, in the linear model the variance is the same for the unobservable and observable recovery rate.

EXHIBIT A4

Standard Deviation of Observable Ultimate Recovery Rate as Functions of the Trading Price Against Empirical Standard Deviation[16]



[16] Note, in the linear model the variance is the same for the unobservable and observable recovery rate.

EXHIBIT A5

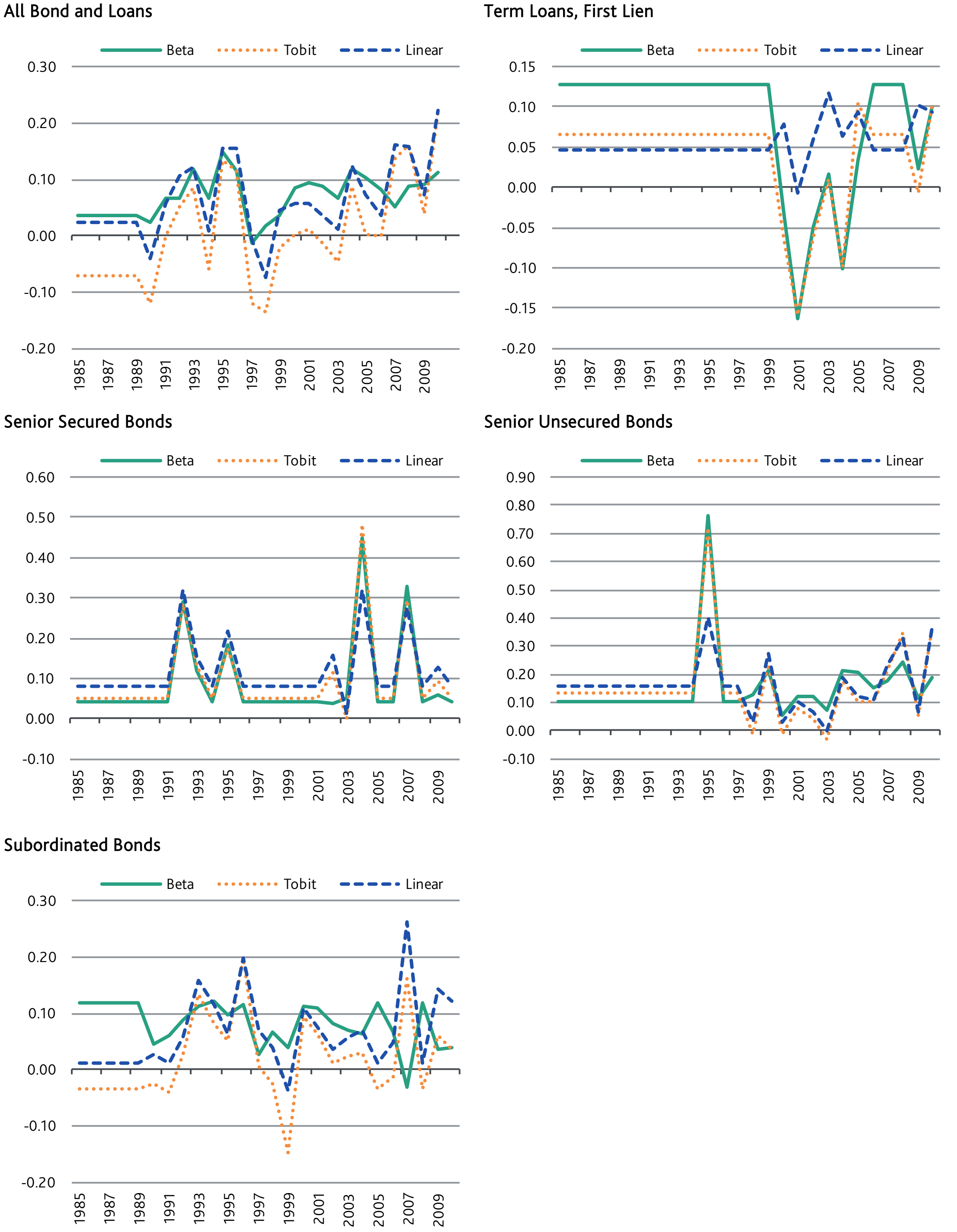
Regression Estimates, Time-varying Intercept By Emergence Year

	Model	b	c	d	e	f	g	R^2	Weak	Test for cst. Intercept
All Bond and loans (21)	Linear	0.877	-1.734	1.830	-1.956			52.04%	32.34**	104.93**
		(0.022)	(0.048)	(0.278)	(0.309)					
	Tobit	1.036	-1.476	0.423	0.122			51.70%	1.09	107.15**
		(0.034)	(0.057)	(0.347)	(0.415)					
	Beta	0.924	-1.717	1.778	-1.322	**-0.008**	**1.254**	48.56%	6.08*	50.99**
		(0.031)	(0.068)	(0.339)	(0.363)					
Term Loans, First Lien (6)	Linear	0.961	-3.411	7.605	-6.981			57.82%	0.82	7.57
		(0.041)	(0.391)	(1.446)	(1.193)					
	Tobit	1.334	-2.647	3.666	-2.183			62.96%	11.06**	12.24
		(0.101)	(0.475)	(1.936)	(1.745)					
	Beta	1.298	-2.817	4.411	-3.133	**-0.121**	1.485	62.78%	8.22**	18.31**
		(0.104)	(0.420)	(1.779)	(1.789)					
Senior Secured Bonds (7)	Linear	0.778	-2.608	4.627	-4.478			59.20%	26.16**	52.47**
		(0.041)	(0.247)	(1.053)	(0.966)					
	Tobit	0.875	-2.740	4.828	-3.936			58.92%	4.44	67.20**
		(0.059)	(0.261)	(1.171)	(1.123)					
	Beta	0.903	-3.326	7.340	-5.751	-0.000	1373.3	55.58%	2.17	133.32**
		(0.066)	(0.234)	(1.028)	(0.984)					
Senior Unsecured Bonds (12)	Linear	0.720	-1.743	1.486	-1.521			47.60%	41.62**	130.75**
		(0.043)	(0.081)	(0.513)	(0.617)					
	Tobit	0.775	-1.561	0.553	-0.213			47.75%	18.83**	132.02**
		(0.052)	(0.091)	(0.571)	(0.701)					
	Beta	0.738	-1.481	1.331	-1.111	**-0.009**	89.292	42.48%	17.36**	74.10**
		(0.063)	(0.155)	(0.742)	(0.780)					
Subordinated Bonds (17)	Linear	0.631	-1.695	1.171	-0.491			18.16%	24.33**	23.97**
		(0.075)	(0.082)	(0.593)	(0.789)					
	Tobit	0.710	-1.365	-0.189	1.081			18.04%	10.68**	19.41**
		(0.089)	(0.105)	(0.728)	(0.974)					
	Beta	0.585	-1.809	1.201	0.796	**-0.024**	102.12	12.24%	35.71**	52.69**
		(0.07)	(0.134)	(0.768)	(0.910)					

F test statistics for the null hypothesis that the intercept is constant through time.* means that we reject the null hypothesis at the 0.05 level and ** that we reject the null at the 0.01 level. Standard errors in parenthesis. We report the R^2 adjusted for the number of parameters in the model. Degrees of Freedom (DF) in parenthesis at seniority level label i.e. for loans there are 6 DF. For the beta model we report the estimated transformed lower (f) and upper (g) bound and indicate bold if statistically significant at the 0.05 level.

EXHIBIT A6

Intercept from Regression Models by Emergence Year



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Authors
Steffen Sorensen
Albert Metz

Production Associates
Janice Brown-Morris
Sarah Warburton
Judy Torre

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