Active Portfolio Management with Conditional Tracking Error

Winfried G. Hallerbach[1] & Igor Pouchkarev[2]

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Abstract

Institutional investment decisions are generally centered around mandates, where a manager's deviation from the benchmark is controlled by means of a tracking error volatility (TEV) constraint. This constraint is of absolute nature: once imposed, it should be honored irrespective of market developments. In this paper, we introduce the concept of a dynamic or conditional TEV constraint. In this set-up, a manager's active risk budget is tied in a relative sense to the benchmark volatility level and hence relative to the cross-sectional dispersion in the returns on the underlying securities. Such a budgeting of risk allows for controlling a manager's active risk exposure *vis à vis* changing market conditions. When the opportunities in the market are widening, a conditional TEV constraint offers a manager the additional room to "hunt" for value and to outperform. Also when there is a surprise or shock in the volatility of the benchmark, a conditional TEV constraint will not hold the manager responsible for the increase in overall volatility. Likewise, a conditional TEV constraint will prevent a manager to deviate too much from his benchmark in a stable (i.e. dull) market, thus mitigating the risk of blow-ups.

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[1] Quantitative Strategies, Robeco Asset Management, w.hallerbach@robeco.com.
[2] GIS – Global Investment Risk Solutions, Global Asset Management, UBS AG, igor.pouchkarev@ubs.com.
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Active Portfolio Management with Conditional Tracking Error

Institutional investment decisions are generally centered around mandates, provided to professional investment managers. In a typical investment mandate, a benchmark portfolio is identified and the portfolio manager is assigned the task to beat that benchmark over a specified horizon. In order to control the risk of the active portfolio, the investment mandate includes a constraint on the *ex ante* tracking error volatility (TEV). This TEV is the standard deviation of the current portfolio's return in excess of the benchmark and it is monitored periodically in order to check whether the portfolio manager does not exceed the specified TEV limit. This TEV constraint is of absolute nature: once imposed, it should be honored irrespective of financial market dynamics. Over recent years, the financial markets experienced quite large swings in volatility. This raises concerns, not only about the ability of portfolio managers to obey the *ex ante* imposed TEV limit, but also on how to incorporate a TEV constraint in the decision process.[3]

In this paper, we introduce the concept of a mandate subject to a dynamic or conditional TEV constraint where a manager's risk budget is tied in a relative sense to the benchmark volatility. Such a relative budgeting of active risk allows controlling a manager's risk exposure *vis à vis* changing market conditions. After all, in his active portfolio a portfolio manager deviates from the benchmark. This exposes the portfolio manager to changes in the cross-sectional dispersion (XSD) of the returns on the underlying securities.[4] We show that there is a strong link between the benchmark volatility and the XSD. When the volatility of the benchmark increases, we expect the XSD to widen too and consequently the active positions will generate a higher TEV. A conditional TEV constraint moves in line with the benchmark volatility and hence will not hold the manager responsible for the increase in overall volatility levels. This facilitates the active investment decision process since the manager does not have to worry about the impact of general market dynamics on *ex ante* TEV. Conversely, a conditional TEV constraint will prevent

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[3] See for example Burmeister, Mausser and Mendoza [2005].

[4] Lillo et al [2002] denote the XSD as the "variety" of an opportunity set of securities. Grant and Satchell [2014] analyze the mathematical properties of XSD.

a manager to deviate too much from his benchmark in a stable (i.e. dull) market, thus mitigating the risk of blow-ups when volatility surges unexpectedly. We investigate the link between the XSD and the benchmark volatility and corroborate our theoretical results with an empirical illustration for the EURO STOXX 50 Index. Finally, we show how a conditional TEV constraint can be incorporated into investment mandates. An Appendix provides technical details.

Active relative investing and tracking error

The return on the portfolio P and the benchmark B over some future period *t* are defined as $\tilde{r}_{Bt}$ and $\tilde{r}_{Pt}$, where a tilde denotes a stochastic variable. The differential *ex ante* tracking error TE is simply the future return difference on the portfolio and the benchmark:

$$\tilde{r}_{Pt} - \tilde{r}_{Bt} = \alpha_P + \tilde{e}_{Pt} \tag{1}$$

where α_P is the mean TE and $\tilde{e}_P$ is the zero-mean stochastic part of the TE. The TE volatility TEV is the standard deviation of the TE, denoted as σ_{eP}. The conventional TEV constraint used to restrict the active manager takes the form:

$$\sigma_{eP} \leq TEV^* \tag{2}$$

where TEV^* is the maximum allowed *ex ante* TEV. Note that the differential TE variance can be written as:

$$\sigma_{eP}^2 = \mathrm{var}(\tilde{r}_P - \tilde{r}_B) = \sigma_P^2 + \sigma_B^2 - 2\rho_{P,B}\sigma_P\sigma_B \geq \left(\sigma_P - \sigma_B\right)^2 \tag{3}$$

where var(.) denotes the variance and where $\rho_{P,B}$ is the correlation between the portfolio and the benchmark. Hence, $TEV \geq \left|\sigma_P - \sigma_B\right|$, so a TEV constraint restricts total portfolio risk even though we do not know the correlation between the portfolio and the benchmark beforehand.

Introducing the conditional TEV

In tracking error based investment plans the TEV* is related to average volatility of the underlying asset class or the underlying benchmark. For example, the target TEV* for fixed income mandates is typically set from 25 to 150 bps whereas the usual target level of TEV for active equity strategies lies in the range of 200 to 600 bps per annum. So the larger the volatility of an asset class, the higher the TEV* is set and hence the more deviations are allowed from the corresponding benchmark. When the level of TEV* is obviously linked to differences in volatilities *over* asset classes, this raises the question why a TEV* is not allowed to fluctuate with the volatility *within* an asset class. After all, the volatility of an asset class can fluctuate substantially over time. For example, taking the S&P500 as a benchmark and the VIX as a proxy for its *ex ante* volatility, a volatility range of 10%-30% can be observed.[5] For government bond portfolios, the *ex ante* volatility as proxied by the MOVE index ranges normally between 75 bps and 150 bps.[6]

We now formally introduce the concept of a dynamic or conditional TEV constraint $CTEV^*$ where a manager's risk budget in terms of TEV is tied to the benchmark volatility by the proportionality parameter γ^*:

$$CTEV^* \leq \gamma^* \cdot \sigma_B \qquad \text{with } \gamma^* > 0 \tag{4}$$

This implies that when benchmark volatility is high (low), also a higher (lower) TEV is allowed. For example, when we set γ^* = 5%, then an *ex ante* benchmark volatility of 20% implies CTEV* = 1%. When the benchmark volatility is instead 25%, this implies CTEV* = 1.25%. Alternatively, setting γ^* = 10% implies that TEV* equals 2% and 2.5%, respectively.

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[5] On November 20, 2008, the VIX reached a daily high of 80.86. The lowest daily value was 9.31, realized on December 22, 1993. See http://www.cboe.com/micro/vix/historical.aspx . Since inception (January 2, 1990) to February 2015, the median value was just over 18.

[6] The Bank of America Merrill Lynch's Option Volatility Estimate (MOVE) Index is a yield curve weighted index of the normalized implied volatility on 1-month Treasury options which are weighted on the 2, 5, 10, and 30 year contracts. See http://www.rimes.com/merrill-option-volatility-estimate-indices .

Whereas the conventional TEV constraint controls total portfolio volatility (see eq.(3)), the conditional TEV constraint controls relative total portfolio risk. The basic identity:

$$\sigma_P^2 = \sigma_B^2 + \sigma_{eP}^2 + 2\rho_{P,B}\sigma_P\sigma_B \tag{5}$$

implies that when the TEV constraint is met, we have:

$$\sigma_P \leq \sigma_B + \sigma_{eP} \leq \sigma_B + CTEV* \tag{6}$$

By substituting (4) in (5), this can be expressed as:

$$\sigma_P \leq \sigma_B\left(1+\gamma*\right) \tag{7}$$

So a conditional TEV constraint not only controls relative active risk but also controls total portfolio volatility relative to benchmark volatility.

Of course, the correlation between the portfolio and the benchmark is relevant for more precise statements about total portfolio risk. Therefore, we now also consider portfolio systematic risk with respect to the benchmark. The conditional TEV constraint (4) implies:

$$TEV = \sqrt{\sigma_P^2 + \sigma_B^2 - 2\rho_{P,B}\sigma_P\sigma_B} \leq \gamma*\sigma_B \tag{8}$$

which can be rewritten as:

$$\sqrt{\frac{\sigma_P^2}{\sigma_B^2} + 1 - 2\beta_{P,B}} \leq \gamma* \tag{9}$$

where $\beta_{P,B} = \rho_{P,B} \cdot \sigma_P / \sigma_B$ is the beta of the portfolio with respect to the benchmark. This shows that also controlling portfolio beta makes sense when trying to control total portfolio volatility with respect to benchmark volatility.[7]

We can support our intuitive argument to link the TEV to the benchmark volatility with a theoretical argument. We first tie the TEV to the XSD and we next show that that the XSD is related to the benchmark volatility.

[7] Roll [1992] already showed that a beta deviating from unity has a large impact on a portfolio's TEV. This implies that TEV optimizations should include a unit beta restriction.

The link between TEV, XSD, and benchmark volatility

A portfolio manager deviates from the benchmark in the hope of attaining outperformance. In deviating from the benchmark composition, the manager exposes himself to the XSD in stock returns. When XSD is low, we have homogeneous market: all security returns are in a relatively narrow range. In such a market, deviating from the benchmark has little consequences for alpha. After all, in that case the returns on the over-weighed positions are very close to the returns on the under-weighed positions. In contrast, when XSD is high, the security returns are scattered over a wide range. Hence, the future active return from over- and under-weighing can cover a wide range and consequently we expect the TEV to be large. Summarizing: deviating from the benchmark is more risky in a heterogeneous market than in a homogeneous market and consequently the *ex ante* TEV will be higher when XSD is high than when XSD is low. Indeed, De Silva ea [2001] and Ankrim and Ding [2002] document the empirical relation between XSD and the range in active returns, whereas Yu and Sharaiha [2007] explicitly link XSD to the opportunity to generate alpha. More recently, Gorman et al [2010] find that the cross-sectional dispersion of U.S. stock returns (and the VIX) provide forecasts of alpha dispersion across high-performing and low-performing stock portfolios. Agapova et al [2011] report that XSD explains approximately 30% of the variation in the average relative returns of actively managed large-cap strategies, and Petajisto [2013] finds that active stock selection (i.e. reaping alpha) is most successful at times of high XSD.

Having established the positive relation between TEV (and alpha and variation in alpha) and XSD, we now turn to the relation between benchmark volatility and XSD. Under mild assumptions, it can be derived that this relation is positive. As shown in the Appendix, the cross-section variance of securities in the investment opportunity set in period *t*, represented by the squared XSD, is given by:[8]

$$XSV\left(\tilde{r}_{i,t}\right) \approx XSV\left(\mu_i\right) + \left(\tilde{r}_{B,t} - \mu_B\right)^2 \cdot XSV\left(b_i\right) + XSV\left(\tilde{z}_{i,t}\right) \qquad (10)$$

where $XSV(\cdot)$ denotes the cross-sectional variance. Eq.(10) shows that the "heterogeneity" of the opportunity set is a function of

i. the cross-section variance of expected returns μ_i,

ii. the instantaneous variance of the benchmark $(\tilde{r}_{B,t} - \mu_B)^2$ and the range in the securities' benchmark exposures $XSV\left(b_i\right)$, representing the systematic squared XSD, and

iii. the cross-section variance of the residual returns $\tilde{z}_{i,t}$ (i.e. the residual squared XSD).

The relative importance of these components is an empirical issue but we will show that the systematic component is substantial. Consequently, we expect XSD to be high (low) during periods in which the benchmark volatility is high (low). Moreover, volatilities are quite persistent over time. Hence, when the *ex ante* benchmark volatility is high, we expect the XSD to be high too, and vice versa. This implies that persistence in benchmark volatility is paired with persistence in XSD. Finally, when there is a surprise in the benchmark volatility, there is also likely to be a surprise in XSD.

We can summarize our argument for a conditional TEV limit as follows. When reaping alpha, a manager deviates from the benchmark composition and as a result exposes himself to the XSD in security returns. Since the XSD is positively related to the benchmark volatility, the TEV will also be positively related to the benchmark volatility. When we do not wish to punish an active manager for the impact of changing overall market dynamics, it makes sense to tie the TEV constraint to the level of the *ex ante* benchmark volatility.

[8] A similar relation has been derived by De Silva ea [2001], Stivers [2003], Yu and Sharaiha [2007], and Grant and Satchell [2014].

An empirical illustration

In this section, we substantiate the contemporaneous relation between TEV and benchmark volatility. We take the Dow Jones EUROSTOXX 50 index as our benchmark portfolio. We collected monthly returns for all stocks comprised in the index during a given month over the period Jan-1999 through Dec-2013 (180 months). We start from XSD. When XSD is low, we have homogeneous market: all security returns are in a relatively narrow range. Alternatively, when XSD is high, the security returns are scattered over a wide range. These two scenarios are illustrated in Exhibit 1 for returns on the Dow Jones EUROSTOXX 50 constituents in the months April and December 2013.

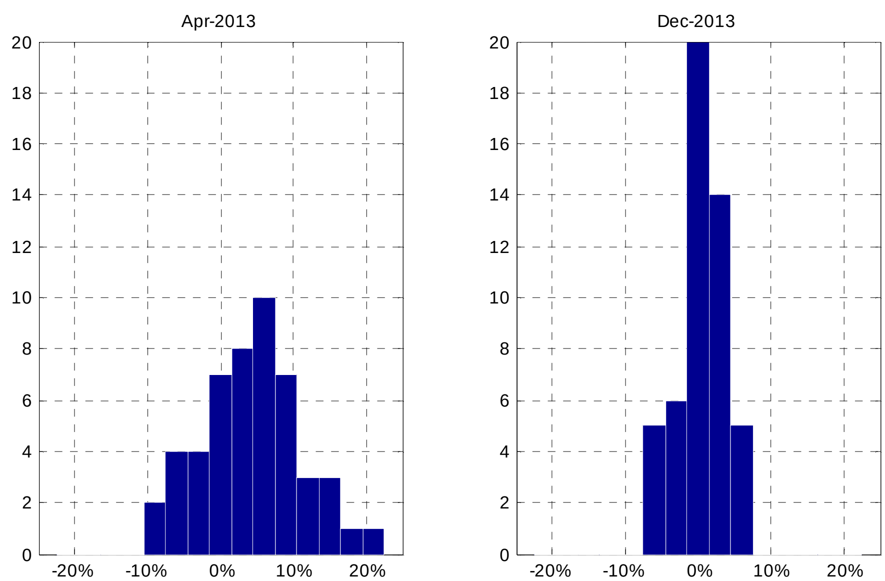

Exhibit 1 The cross-sectional dstribution of returns on the DJ EUROSTOXX 50 index constituents in April and December 2013.

These are two extreme examples of the XSD patterns that can be observed over time. Exhibit 2 plots the monthly cross-sectional return distributions of the stocks in the DJ EUROSTOXX 50 over the full sample period.[9] When XSD is low, deviating from the benchmark has little consequences. After all, the returns on overweighed and under-weighed stocks are quite similar. Hence, the active return is small and consequently the *ex ante* TEV is small. In contrast, when XSD is high, active position returns are dispersed over a wide range, yielding a large active return and a large TEV.

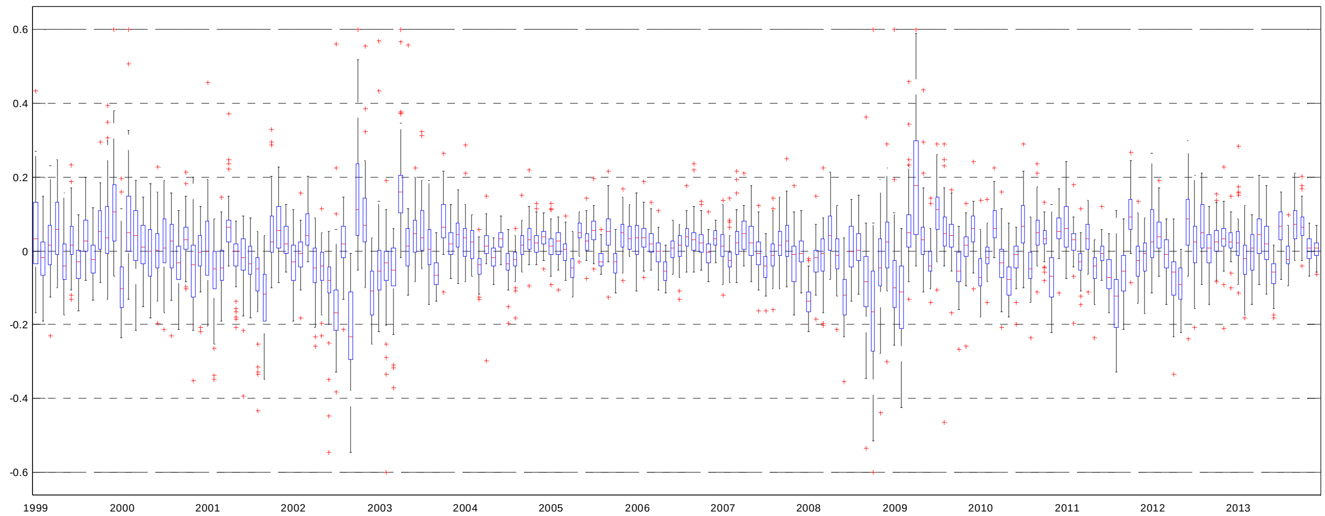

Exhibit 2 The cross-sectional return distributions of the DJ EUROSTOXX 50 index constituents, monthly returns over the period Jan 1999 – Dec 2013. Source: Bloomberg.

We now investigate the empirical relation between XSD and benchmark volatility. In Exhibit 3, we plot the monthly XSD and monthly benchmark (DJ EURO STOXX 50) volatility. We calculate the monthly benchmark volatility using intra-month daily data, thus proxying instantaneous monthly volatility. For comparison, we also plot the ultimo month VSTOXX, the

[9] Hallerbach et al [2005] use these cross-sectional return distributions to describe and analyze market opportunities and dynamics.

implied volatility index of the DJ EURO STOXX 50 index.[10] The graph shows a very strong relation between XSD and the two (instantaneous and *ex ante*) volatility measures. The correlation between the XSD and the realized monthly volatility (VSTOXX) is 64% (68%).

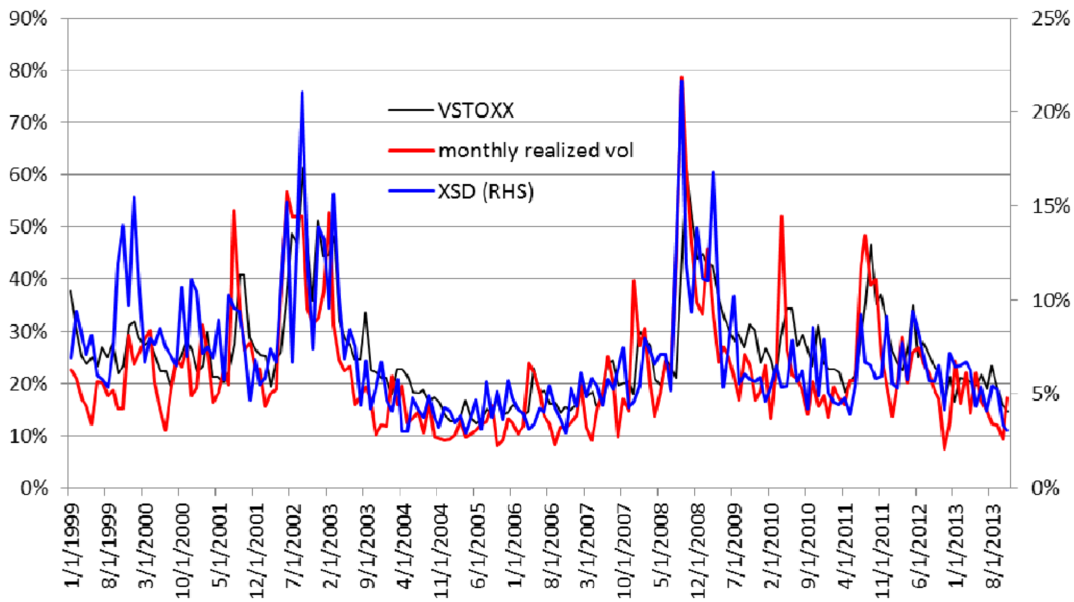

Exhibit 3 The realized volatility (estimated from daily returns in each month) of the DJ EUROSTOXX 50 index, together with the VSTOXX (implied volatility at the beginning of each month) and the XSD. Monthly data over the period Jan 1999 – Dec 2013. Source: Bloomberg.

Exhibit 4 plots the relation between the instantaneous monthly volatility (using intra-month daily data) and the XSD. The relation is positive and quite strong.

[10] See http://www.stoxx.com/indices/index_information.html?symbol=V2TX .

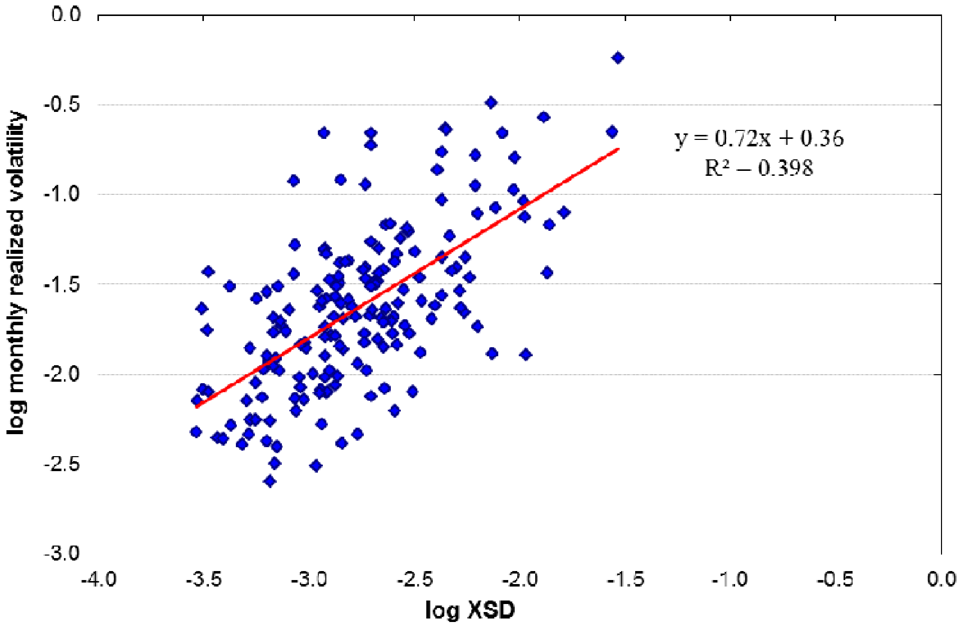

Exhibit 4 The relation between the instantaneous volatility and the XSD of the DJ EUROSTOXX 50 index. Instantaneous volatility is computed using the daily returns within each month. Because of the right-skewness in volatilities, we consider their $\log s_{10}$. Monthly data over the period Jan 1999 – Dec 2013.

The relation between the XSD and the *ex ante* monthly volatility from the VSTOXX is plotted in Exhibit 5. Especially the stronger link between XSD and VSTOXX is relevant since the VSTOXX is forward-looking (hence we take the VSTOXX at the beginning of each month).

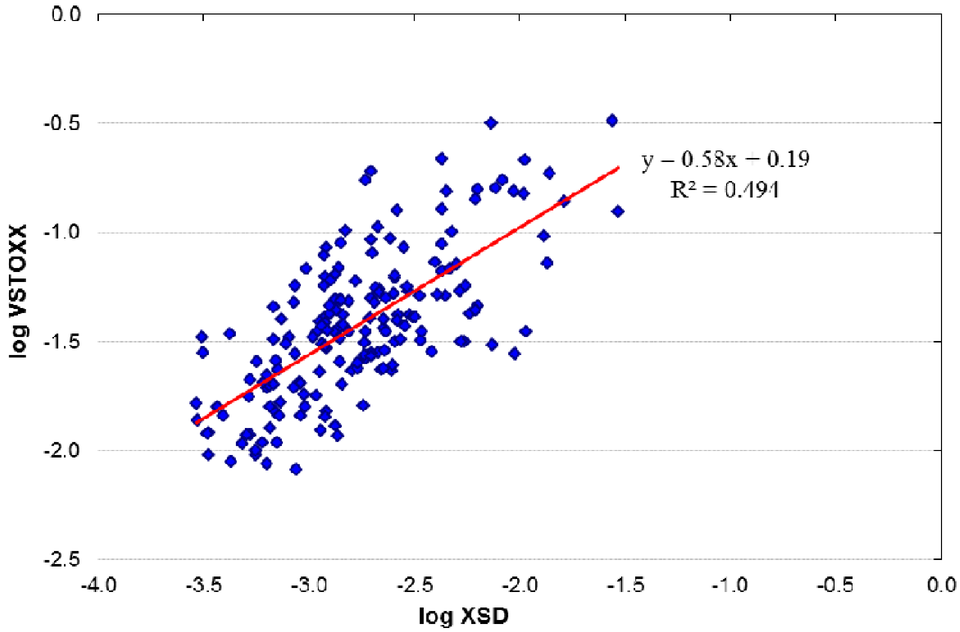

Exhibit 5 The relation between the implied volatility VSTOXX (at the beginning of each month) and the XSD of the DJ EUROSTOXX 50 index. Because of the right-skewness in volatilities, we consider their logs_{10}. Monthly data over the period Jan 1999 – Dec 2013.

Exhibit 6 displays the monthly auto-correlations up to lag 6 of three volatility measures and the XSD, together with their standard errors. The instantaneous benchmark volatility (proxied by the absolute values of the monthly realized returns) shows no significant persistence (at the usual 95% level) over any of the 6 monthly lags. Note, however, that although the link with the benchmark variance in eq.(10) is most obvious, this is a quite noisy volatility estimator. Using the daily returns within each month improves estimation efficiency and Exhibit 6 shows that this realized monthly volatility is very persistent (with significant auto-correlations for the full six lags). The implied volatility VSTOXX displays the greatest persistence (again with significant auto-correlations for the full six lags). As expected, the XSD also shows a very persistent auto-correlation pattern and can be positioned in-between the monthly realized volatility and the VSTOXX.

lag:		-1M	-2M	-3M	-4M	-5M	-6M
instantaneous bmk volatility		0.076	0.069	0.146	0.029	0.100	0.092
	s.e.	*0.076*	*0.076*	*0.076*	*0.076*	*0.076*	*0.076*
realized monthly bmk volatility		0.728	0.555	0.438	0.366	0.345	0.361
	s.e.	*0.052*	*0.064*	*0.069*	*0.071*	*0.072*	*0.071*
VSTOXX		0.856	0.742	0.658	0.579	0.531	0.515
	s.e.	*0.040*	*0.051*	*0.058*	*0.062*	*0.065*	*0.066*
XSD		0.658	0.591	0.614	0.520	0.455	0.501
	s.e.	*0.058*	*0.062*	*0.060*	*0.065*	*0.068*	*0.066*

Exhibit 6 The auto-correlations of the instantaneous benchmark volatility (proxied by the absolute values of the monthly realized returns), the realized monthly volatility (estimated from daily returns within each month), the implied volatility VSTOXX, and the XSD. Because of the right-skewness in volatilities, we consider their logs_{10}. Standard errors are shown in italics. Monthly data over the period Jan-1999 through Dec-2013.

Incorporating a conditional TEV limit in mandates – further thoughts

In conventional mandates, an absolute TEV limit TEV* is formulated. For risk monitoring, the *ex ante* TEV is compared against the TEV*. When TEV > TEV*, the manager has breached the *ex ante* TEV limit. This breach could be caused by excessive active positions, but it is also possible that the *ex ante* TEV is high because the *ex ante* benchmark volatility is high. When it is not deemed appropriate to punish an active manager for increased market dynamics (and hence she is allowed to maintain the prevailing active share corresponding to her alpha convictions), then it makes sense to formulate a conditional TEV limit.

When switching to the conditional TEV limit, the proportionality parameter γ^* is specified which links the TEV* to the *ex ante* benchmark volatility according to eq.(4). For risk monitoring, the *ex ante* TEV is compared against the *ex ante* benchmark volatility, yielding a value of γ. When $\gamma \leq \gamma^*$, the manager has acted within the conditional TEV limit. In contrast, when $\gamma > \gamma^*$, the manager has breached the conditional TEV limit. Even when adjusting for changed market dynamics, the level of active risk is too high.

Linking TEV* to benchmark volatility also has implications for performance benchmarking. Note that when the TEV equals the TEV* (so the full *ex ante* TE budget is used), then eq.(7) applies as an equality: a position of $(1+\gamma^*)$ in the benchmark has the same volatility as the portfolio. This suggests defining a "neutral portfolio" P^* consisting of $(1+\gamma^*)$ times the benchmark. In order to satisfy the budget restriction, we complement this with a short position in the risk free asset. This portfolio P^* is neutral because it levers the benchmark volatility to the overall portfolio volatility by taking additional benchmark positions instead of adding the active portfolio. The return on P^* can be written as:

$$\tilde{r}_{P^*} = \left(1+\gamma^*\right)\tilde{r}_B - \gamma^* \cdot r_f \qquad (11)$$

where r_f is the risk free rate. The active return that is implied by the neutral portfolio P^* equals:

$$\tilde{r}_{P^*} - \tilde{r}_B = \gamma^* \cdot \left(\tilde{r}_B - r_f\right) \qquad (12)$$

Hence, the information ratio (IR) of the implied active portfolio equals:

$$IR_{P^*} = \frac{\gamma^* \cdot E\left(\tilde{r}_B - r_f\right)}{\gamma^* \cdot \sigma_B} = SR_B \qquad (13)$$

which (as expected) equals the Sharpe Ratio (SR) of the benchmark. So when the active portfolio is a clone of the benchmark, and consequently there is no value-added from active management, then the IR of the portfolio equals the SR of the benchmark. By the same token, whenever the IR of the active portfolio exceeds the SR of the benchmark, the active manager has added value in the active portfolio.

Summary and conclusions

We propose a TEV constraint that is conditional on the underlying benchmark volatility. This conditional TEV constraint CTEV* is motivated by (1) the strong relation between the benchmark volatility and XSD, which can be termed a "stylized fact", and (2) the relation between XSD on the one hand, and alpha and TEV on the other. By budgeting *ex ante* active risk conditional on the benchmark volatility, the portfolio manager is not held responsible for changes in market volatility. This implies that the active manager can focus on identifying over- and under-valued securities without worrying about overall market dynamics. In addition, CTEV* will prevent a manager to deviate too much from his benchmark when market volatility is low, thus mitigating the risk of blow-ups. We also provided a suggestion to benchmark active performance against a "neutral" matching portfolio. Having established the concept of CTEV* and its potential advantages over a conventional TEV constraint, its incorporation in the monitoring of active mandates is left for future research.

Appendix

We start by relating individual security returns to the benchmark return by a linear time series market model:

$$\tilde{r}_{i,t} = \mu_i + b_i\left(\tilde{r}_{B,t} - \mu_B\right) + \tilde{z}_{i,t} \tag{14}$$

where μ_i and μ_B are the expected returns on the security *i* and the benchmark, b_i is the slope coefficient (or beta) and where the zero-mean residual return $\tilde{z}_{i,t}$ is uncorrelated with the benchmark return.

Now consider an opportunity set of *N* stocks, for large *N* (so we can invoke a diversification argument). We define the cross-sectional covariance as:

$$XSC(\tilde{x},\tilde{y}) = \frac{1}{N}\sum_i \left(\tilde{x}_i - \tfrac{1}{N}\sum\nolimits_j \tilde{x}_j\right)\left(\tilde{y}_i - \tfrac{1}{N}\sum\nolimits_k \tilde{y}_k\right) \tag{15}$$

and the cross-sectional variance as $XSV(\tilde{x}) = XSC(\tilde{x},\tilde{x})$. Taking the cross-sectional variance of (14) yields:

$$\begin{aligned} XSV\left(\tilde{r}_{i,t}\right) = {} & XSV\left(\mu_i\right) + \left(\tilde{r}_{B,t} - \mu_B\right)^2 \cdot XSV\left(b_i\right) + XSV\left(\tilde{z}_{i,t}\right) \\ & + 2\left(\tilde{r}_{B,t} - \mu_B\right)\cdot XSC\left(\mu_i, b_i\right) \qquad \text{(A)} \\ & + 2\cdot\left(\tilde{r}_{B,t} - \mu_B\right)\cdot XSC\left(b_i, \tilde{z}_{i,t}\right) + 2\cdot XSC\left(\mu_i, \tilde{z}_{i,t}\right) \qquad \text{(B)} \end{aligned} \tag{16}$$

In the cross-section of security returns, we can safely assume that the terms in part (B) of eq.(16) are zero. After all, in the cross-section of securities the residual return $\tilde{z}_{i,t}$ is uncorrelated with the betas b_i and the mean returns μ_i (and the expected realization of $(\tilde{r}_{B,t} - \mu_B)$ is zero). The term in part (A) is likely to be positive when the benchmark is the market portfolio (or the single return generating factor) and the securities are priced under the CAPM or Ross' [1976] Arbitrage Pricing Theory. However, since the expected realization of $(\tilde{r}_{B,t} - \mu_B)$ is zero, we can also ignore part (B) here. Using these results in eq.(16), we obtain:

$$XSV\left(\tilde{r}_{i,t}\right) \approx XSV\left(\mu_i\right) + \left(\tilde{r}_{B,t} - \mu_B\right)^2 \cdot XSV\left(b_i\right) + XSV\left(\tilde{z}_{i,t}\right) \qquad (17)$$

The term $XSV\left(\tilde{r}_{i,t}\right)$ is the squared of the XSD, XSD^2, and the term $(\tilde{r}_{B,t} - \mu_B)^2$ denotes the instantaneous variance of the benchmark return. This establishes the relation between the XSD on the one hand and the benchmark volatility and the residual XSD on the other.

References

Agapova, A., R. Ferguson and J. Greene. (2011) "Market Diversity and the Performance of Actively Managed Portfolios." *The Journal of Portfolio Management* vol. 38 no.1 (Fall): 48-59.

Ankrim, E.M. and Z. Ding. (2002) "Cross-Sectional Volatility and Return Dispersion." *Financial Analysts Journal* vol. 58 no. 5 (Sept-Oct):67-73.

Burmeister, C,, H. Mausser and R. Mendoza. (2005) "Actively Managing Tracking Error." *Journal of Asset Management* vol. 5 no.6: 410-422.

De Silva, H., S. Sapra and S. Thorley. (2001) "Return Dispersion and Active Management." *Financial Analysts Journal* vol. 57 no. 5 (Sept-Oct): 29-42.

Gorman, L.R., S.G. Sapra and R.A. Weigand. (2010) "The Cross-Sectional Dispersion of Stock Returns, Alpha, and the Information Ratio." *The Journal of Investing* vol. 19 no. 3 (Fall): 113-127

Grant, A. and S. Satchell. (2014) "Theoretical Decompositions of the Cross-Sectional Dispersion of Stock Returns." Discussion paper 2014-003, University of Sydney.

Hallerbach, W.G., Ch. Hundack, I. Pouchkarev and J. Spronk. (2005) "Market Dynamics From The Portfolio Opportunity Perspective: The Dax Case." *Zeitschrift für Betriebswirtschaft*, vol. 75 no.7-8: 1-26

Lillo, F., R.N. Mantegna, J.-P. Bouchaud and M. Potters. (2002) "Introducing Variety in Risk Management." *Wilmott Magazine* (Dec): 98-102.

Petajisto, A. (2013) "Active Share and Mutual Fund Performance." *Financial Analysts Journal* vol. 69 no. 4 (Jul-Aug): 1-21.

Roll, R. (1992) "A Mean/Variance Analysis of Tracking Error." *The Journal of Portfolio Management* (Summer): 13-22.

Ross, S.A. (1976) "The Arbitrage Theory of Capital Asset Pricing." *Journal of Economic Theory* vol. 13 no. 3 (dec.): 341-360

Stivers, C. (2003) "Firm-level return dispersion and the future volatility of aggregate stock market returns." Journal of Financial Markets vol. 6: 389-411.

Yu, W. and Y.M. Sharaiha. (2007) "Alpha Budgeting – Cross-sectional Dispersion Decomposed." *Journal of Asset Management* vol. 8 no. 1: 58-72.