Integrating time series and cross-sectional signals for optimal commodity portfolios *

Regina Hammerschmid†
University of Zurich
Swiss Finance Institute

Harald Lohre‡
Invesco
EMP, Lancaster University Management School

December 15, 2019

ABSTRACT

We study the optimal combination of different commodity signals in a dynamic portfolio theoretic framework. Following Brandt et al. (2006, 2009) we parameterize the portfolio weights of a risk-averse mean-variance investor to integrate information from time series predictors and cross-sectional characteristics of commodity assets. We show that the long end of the futures curve as well as open interest have significant timing power, whereas the short end of the futures curve and past returns are relevant characteristics for tilting commodities. Combining timing and tilting strategies outperforms all factor benchmarks out-of-sample and after transaction costs. Moreover, the optimal commodity allocation is not priced by common risk factors, such as commodity carry or momentum.

Keywords: optimal commodity strategies, parametric portfolio policies, timing and tilting
JEL Classification: G11; G12

*We are grateful to Thorsten Hens, Alexandra Janssen, Carsten Rother, Alexandre Ziegler and seminar participants at the 2018 Quarterly Research Meeting of Invesco Quantitative Strategies in Frankfurt and the Doctoral Seminar at the Department of Banking and Finance at the University of Zurich, for their valuable feedback. Note that this paper expresses the authors' views which do not necessarily coincide with those of Invesco.

†University of Zurich and Swiss Finance Institute, Plattenstrasse 32, 8032 Zurich, Switzerland; regina.hammerschmid@bf.uzh.ch

‡Invesco Quantitative Strategies, An der Welle 5, 60322 Frankfurt/Main, Germany; Centre for Financial Econometrics, Asset Markets and Macroeconomic Policy (EMP), Lancaster University Management School, Bailrigg, Lancaster LA1 4YX, United Kingdom; harald.lohre@invesco.com

I. Introduction

Commodity futures markets experienced an unprecedented inflow of investment capital over the past decade and commodities have established as a viable asset class.[1] The literature on commodity futures pricing has identified a growing number of factors with attractive risk-return profiles capturing either risk premia or market anomalies (see Miffre (2016) and Sakkas and Tessaromatis (2018) for an overview). Most of these long-short strategies are built on single commodity fundamentals, such as term structure variables or past price measures, and only few studies try to integrate several characteristics (Fernandez-Perez, Fuertes, and Miffre (2018)). In this paper, we investigate optimal combinations of different commodity signals in a dynamic portfolio theoretic framework. Instead of implementing simple cross-sectional sorting rules, we maximize expected utility of a mean-variance investor whose weights are parameterized as functions of various commodity fundamentals.

To estimate optimal commodity allocations, we use the parametric portfolio policy approach of Brandt and Santa-Clara (2006) and Brandt, Santa-Clara, and Valkanov (2009) to integrate time series predictors of commodity returns as well as cross-sectional sorting characteristics in a mean-variance framework. In particular, the portfolio weights are parameterized as linear functions of different state variables, and we can directly estimate optimal weights conditional on the information of these fundamentals without estimating the joint distribution of all commodity returns. Furthermore, this approach allows testing for the joint statistical and economic relevance of different state variables in forming optimized commodity portfolio weights out-of-sample. We find that the long end of the commodity futures curve as well as open interest in the futures market have significant timing power, whereas the short end of the futures curve and past returns are highly relevant for cross-sectional tilting of commodity allocations. Overall, a multivariate parametric commodity strategy that combines time series and cross-sectional fundamentals outperforms different factor benchmarks in risk-adjusted terms, out-of-sample and after transaction costs. Regarding the risk exposure of the optimal commodity strategy, we show that it is exposed to—but not priced by—commodity carry and momentum (Bakshi, Gao, and Rossi (2017)) as well

[1] Bhardwaj, Gorton, and Rouwenhorst (2015) document that open interest in the futures market for the average commodity has more than doubled since 2004. This phenomenon is often referred to as financialization of commodity markets (see for example Cheng and Xiong (2014), Henderson, Pearson, and Wang (2014), Singleton (2014), Tang and Xiong (2012) and Basak and Pavlova (2016)).

as basis-momentum (Boons and Prado (2019)). The strategy alpha is highly significant whereas all factor benchmark portfolios' returns are fully spanned by common commodity factors.

Fernandez-Perez, Fuertes, and Miffre (2018) also investigate how to combine different commodity factors into one robust aggregate commodity portfolio. They run a horse race of several integration methods, and their results show that equal weighting of all long-short investment styles delivers the best risk-adjusted performance. While the authors' analysis rests on long-short factors sorted on different commodity characteristics, our parametric portfolio policies directly optimize over individual commodities relating the asset weights to key fundamentals. Also, the approach of Fernandez-Perez, Fuertes, and Miffre (2018) is agnostic to dynamically selecting relevant factors. In contrast, we estimate statistical significance over an in-sample fitting period, which allows us to select relevant state variables in an unbiased fashion for an out-of-sample portfolio study. These parametric policies are anchored in a portfolio theoretic framework maximizing expected utility compared to simple sorting rules. In this vein, the optimized commodity portfolio dominates the equal-weighted factor benchmark proposed in the literature.

Regarding the state variables, we choose key fundamentals arising from different commodity pricing theories and characteristics proposed in the commodity factor investing literature. Trading on the term structure of the futures curve (buying commodities which are in backwardation or selling those in contango) can deliver substantial positive excess returns. The main variables capturing the shape of commodity futures curves are the basis, the hedging pressure or the level of inventories.[2] In addition, there is also a strong momentum effect in commodity futures markets, which stipulates buying past winners and selling past losers.[3] Other characteristics associated with a positive risk premium are value, volatility, skewness or liquidity.[4] Moreover, combining different signals or modifying original signals, usually term structure variables and price measures, tends to

[2]Studies exploiting the roll yield using different variables are Chang (1985), Bessembinder (1992), Erb and Harvey (2006), de Roon, Nijman, and Veld (2000), Gorton, Hayashi, and Rouwenhorst (2012), Basu and Miffre (2013), Dewally, Ederington, and Fernando (2013), Yang (2013), Szymanowska, de Roon, Nijman, and van den Goorbergh (2014), Koijen, Moskowitz, Pedersen, and Vrugt (2018), among others.

[3]The momentum premium in commodity markets has been documented by Miffre and Rallis (2007), Shen, Szakmary, and Sharma (2007), Szakmary, Shen, and Sharma (2010), Asness, Moskowitz, and Pedersen (2013), among others.

[4]References regarding these variables are Asness, Moskowitz, and Pedersen (2013), Szymanowska, de Roon, Nijman, and van den Goorbergh (2014), Fernandez-Perez, Fuertes, and Miffre (2016), Fernandez-Perez, Frijns, Fuertes, and Miffre (2018), among others.

improve the performance of the ensuing long-short commodity strategies, although none of these studies explicitly deal with the optimal combination of all commodity fundamentals.[5]

In addition to these characteristics explaining the cross-section of commodity futures returns, there are more fundamentals capturing time series variation of commodity risk premia. Hong and Yogo (2012) find that the growth rate of open interest has significant predictive power for aggregate commodity returns. Moreover, the slope, curvature and in particular the long end of the futures curve are highly relevant for explaining the time series of commodity futures returns (Hammerschmid (2018), Boons and Prado (2019)).[6] Even if these fundamentals cannot capture cross-sectional return variation, the parametric portfolio policy of Brandt and Santa-Clara (2006) explicitly accounts for state variables explaining the time series variation of returns. Hence, we test for the timing ability of different commodity fundamentals in addition to tapping cross-sectional commodity characteristics.

In terms of methodology, we rely on the seminal work by Brandt and Santa-Clara (2006) to integrate time series predictors and Brandt, Santa-Clara, and Valkanov (2009) to include cross-sectional characteristics. Parametric portfolio policies have also been applied to other asset classes, such as currency returns where they have been shown to improve the carry trade strategy (Barroso and Santa-Clara (2015)), or to navigate a set of equity factors accounting for both time series fundamentals and cross-sectional signals (Dichtl et al. (2018)).[7] To the best of our knowledge, this is the first paper to apply the parametric portfolio theoretic framework to study optimal allocations in the context of commodity futures.

The paper is structured as follows. Section II outlines the concept of parametric portfolio policies for timing and tilting commodity allocations. The commodity data and state variables, as well as some details on the portfolio estimation are explained in Section III. In Section IV we present our main empirical results and in Section V we discuss alternative ways to smooth optimal commodity allocations. Section VI concludes.

[5] Combinations of different commodity signals are discussed in Fuertes, Miffre, and Rallis (2010), Blitz and de Groot (2014), de Groot, Karstanje, and Zhou (2014), Szymanowska, de Roon, Nijman, and van den Goorbergh (2014), Boons and Prado (2019), Fuertes, Miffre, and Fernandez-Perez (2015), Bakshi, Gao, and Rossi (2017), among others.

[6] Other studies also investigate the time series predictability of the basis (Fama and French (1987)), the inventory level (Gorton, Hayashi, and Rouwenhorst (2012)) or different macroeconomic variables (Gargano and Timmermann (2014)).

[7] Additional studies on parametric portfolio policies investigating the role of transaction costs (DeMiguel, Martin-Utrera, Nogales, and Uppal (2019)) or the introduction of leverage constraints (Ammann, Coqueret, and Schade (2016)) are available for equities.

II. Parametric Portfolio Policies

Investors face many decisions when managing portfolios in real time: what signals to use, how to weigh them or how to deal with transaction costs and measurement error. The literature often finds that simple and robust portfolio rules, such as $1/N$ or equally weighted long-short factors sorted on isolated signals, outperform more complex optimized portfolios, in particular out-of-sample (see for example DeMiguel, Garlappi, and Uppal (2009), Jacobs, Müller, and Weber (2014), Fernandez-Perez, Fuertes, and Miffre (2018)).

Parametric portfolio policies allow studying these portfolio decisions in a more realistic setting. First, they are anchored in a portfolio theoretic framework and maximize expected investor utility to determine optimal weights. Second, these policies link the unknown asset weights to some known state variables assuming that the latter convey all relevant information about the assets' conditional distribution of returns. Nevertheless, one avoids estimating the joint distribution of all asset returns, which is inherently difficult to do and hampers portfolio optimization. Instead, one can directly optimize over the coefficients linking the state variables to the asset weights, which represents a tremendous reduction in dimensionality compared to estimating the joint distribution of asset returns. In addition, this approach allows testing for the statistical and economic significance of the state variables, providing an unbiased decision rule for selecting relevant signals. Furthermore, the parametrization transforms the dynamic portfolio selection into a static optimization problem, thus reducing complexity and estimation error. Therefore, parametric portfolio policies are meaningful from an economic and statistical perspective: they translate the information of different state variables to optimal asset weights, while being statistically robust and parsimonious.

Throughout the paper, we model a risk-averse investor with mean-variance preferences who maximizes conditional expected utility

$$\max_{w_t} \ \mathbb{E}_t\left[r_{p,t+1}\right] - \frac{\gamma}{2} Var_t\left(r_{p,t+1}\right) \quad \Leftrightarrow \quad \max_{w_t} \ \mathbb{E}_t\left[w_t' r_{t+1} - \frac{\gamma}{2} w_t' r_{t+1} r_{t+1}' w_t\right], \tag{1}$$

where $r_{p,t+1}$ is the return of the optimized commodity portfolio, r_{t+1} is an $(N \times 1)$ vector of commodity excess returns, w_t are $(N \times 1)$ commodity weights and γ represents the investor's risk aversion. Next, we briefly outline the concepts of parametric timing and tilting portfolio policies that will be used for optimal commodity allocations.

A. Timing policy à la Brandt and Santa-Clara (2006)

To translate the predictive power of K state variables to the N portfolio weights, Brandt and Santa-Clara (2006) assume that the optimal portfolio strategy is linear in K conditioning variables

$$w_t = \theta z_t \,, \tag{2}$$

where θ is an $(N \times K)$ coefficient matrix and z_t is a $(K \times 1)$ vector of observed state variables including a constant. The associated coefficients θ do vary across assets and predictors but are constant over time. We can use this linear timing policy to rewrite the portfolio return as follows

$$r_{p,t+1}(\theta) = w_t' r_{t+1} = (\theta z_t)' r_{t+1} = z_t' \theta' r_{t+1} = \underbrace{vec(\theta)'}_{=:\tilde{w}} \underbrace{(z_t \otimes r_{t+1})}_{=:\tilde{r}_{t+1}} = \tilde{w}' \tilde{r}_{t+1} \,, \tag{3}$$

where $\tilde{w} = vec(\theta)$ is an $(N \cdot K \times 1)$ vector of stacked coefficients and $\tilde{r}_{t+1} = z_t \otimes r_{t+1}$ are $(N \cdot K \times 1)$ excess returns of so-called "managed" portfolios in the augmented asset space. Each managed portfolio invests in a single commodity (basis asset) an amount proportional to the realization of one of the predictor variables. Note that the portfolio return is a function of the time-invariant coefficients θ.[8]

[8]The above definition of the linear timing policy follows Brandt and Santa-Clara (2006) in that z_t is a $(K \times 1)$ vector of state variables. Hence, these conditioning variables are the same across all N assets while the coefficients θ are asset specific. However, the timing policy can easily be extended to include asset specific state variables

$$w_t = \mathrm{diag}(\theta Z_t) \,, \tag{4}$$

where θ is an $(N \times K)$ coefficient matrix and Z_t is a $(K \times N)$ matrix of observed state variables. The portfolio return can then be rewritten as follows

$$r_{p,t+1}(\theta) = w_t' r_{t+1} = \mathrm{diag}(\theta Z_t)' r_{t+1} = \underbrace{vec(\theta)'}_{=:\tilde{w}} \underbrace{\left\langle vec(Z_t'), (\mathbb{1}_{(K \times 1)} \otimes r_{t+1}) \right\rangle}_{=:\tilde{r}_{t+1}} = \tilde{w}' \tilde{r}_{t+1} \,, \tag{5}$$

where $\tilde{w}$ is an $(N \cdot K \times 1)$ vector of stacked coefficients and $\tilde{r}_{t+1}$ are $(N \cdot K \times 1)$ excess returns of managed portfolios in the augmented asset space.

Inserting the linear timing policy (2) into the investor's maximization problem (1) and using the transformation to the augmented asset space, we obtain

$$\max_{\tilde{w}} \ \mathbb{E}\left[\tilde{w}'\tilde{r}_{t+1} - \frac{\gamma}{2}\tilde{w}'\tilde{r}_{t+1}\tilde{r}'_{t+1}\tilde{w}\right] . \tag{6}$$

Given that the θ coefficients are constant over time (and therefore the unconditional weights $\tilde{w}$), dynamic portfolio selection is reduced to a static unconditional problem. Hence, the investor maximizes unconditional expected utility over the coefficients θ. To recover the original weights invested in each commodity, we can use the linear timing policy (2) together with the estimated coefficients and realizations of the state variables. Furthermore, the investor's unconditional maximization problem (6) can be formulated as a statistical estimation problem. Specifically, one can express the coefficients θ as a method of moments estimator and use asymptotic theory to derive corresponding standard errors following Hansen (1982). Further details on the statistical inference are given in Appendix A.1.

B. Tilting policy à la Brandt, Santa-Clara, and Valkanov (2009)

To incorporate cross-sectional information of K characteristics, the linear tilting policy of Brandt, Santa-Clara, and Valkanov (2009) assumes that

$$w_t = w_{b,t} + X_t\,\phi\,/N_t\,, \tag{7}$$

where $w_{b,t}$ are $(N \times 1)$ benchmark portfolio weights, X_t is an $(N \times K)$ matrix of observed asset characteristics, ϕ is a $(K \times 1)$ coefficient vector and N_t are the number of assets available at time t. The optimal tilting weights are specified relative to some benchmark, capturing the idea of a benchmark-relative portfolio strategy where active deviations only result from realizations of the characteristics. In the case of zero benchmark weights, the tilting policy will result in long-short factors optimized on the characteristics X_t. Note that the tilting coefficients ϕ are assumed to be constant over time and across assets, whereas the timing coefficients θ are asset specific. However, the characteristics X_t need to be asset specific in order to obtain different active tilts for each asset, while the predictors z_t in the timing policy can be the same across all assets. Furthermore, the characteristics are cross-sectionally standardized, which makes their cross-sectional distribution stationary through time, but more importantly it implies that the tilting portfolio is always fully invested, i.e. the active deviations from the benchmark sum up to 0. Finally, the scaling of the active tilting weights by the number of available assets avoids aggressive allocations due to a change in the asset universe.

We can use the linear tilting policy to rewrite the portfolio return as follows

$$r_{p,t+1}(\phi) = (w_{b,t} + X_t\,\phi\,/N_t)'r_{t+1} = \underbrace{w'_{b,t}r_{t+1}}_{=:r_{b,t+1}} + \phi'\underbrace{X'_t r_{t+1}/N_t}_{=:r_{c,t+1}} = r_{b,t+1} + \phi' r_{c,t+1}\,, \tag{8}$$

where $r_{b,t+1}$ is the return on the benchmark portfolio and $r_{c,t+1}$ is the $(K \times 1)$ characteristic return vector. Inserting the linear tilting policy (7) into the investor's utility maximization problem (1) and expressing the portfolio return as a function of the tilting coefficients we have

$$\max_{\phi} \;\; \mathbb{E}\left[r_{p,t+1}(\phi)\right] - \frac{\gamma}{2} Var\left[r_{p,t+1}(\phi)\right]\,. \tag{9}$$

Due to the time-invariance of the tilting coefficients, the investor now maximizes unconditional expected utility over the coefficients ϕ. Following DeMiguel, Martin-Utrera, Nogales, and Uppal (2019), the unconditional tilting problem (9) can be further split into

$$\max_{\phi} \;\; \phi'\,\mathbb{E}[r_{c,t+1}] - \frac{\gamma}{2}\phi' Var[r_{c,t+1}]\phi - \gamma\phi' Cov[r_{b,t+1}, r_{c,t+1}]\,, \tag{10}$$

which includes the mean and variance of the characteristic return vector as well as the covariance between the characteristic and benchmark returns. To recover the weights invested in each commodity, we can use the linear tilting policy (7) together with estimated coefficients and realizations of the characteristics. Moreover, the unconditional maximization problem with the linear tilting policy can also be expressed as a statistical estimation problem to derive standard errors for the tilting coefficients; see Appendix A.2. for further details.

III. Data, Variables and Estimation

Our investment universe consists of 20 different commodities spread across 5 sectors (Energy, Grains and Oilseeds, Livestock, Metals, Softs) and the sample period is January 1987 to August

2015.[9] For each commodity, we collect daily settlement prices of individual futures contracts available on Bloomberg and end-of-month data on open interest as well as long and short positions of commercial traders (hedgers) published by the Commodity Futures Trading Commission. We select only liquid contracts and discard those contracts that exhibit zero trading volume for at least one year prior to expiration.

A. Commodity returns

The end-of-month return of a fully collateralized long position in a commodity futures contract with maturity T in excess of the risk-free rate is

$$rx_{t+1} = \frac{F_{t+1,T} - F_{t,T}}{F_{t,T}}, \tag{11}$$

where $F_{t,T}$ is the price at the end of month t of the futures contract maturing at the end of month T. We roll over each futures contract to the next nearest contract at the end of the last month before delivery and futures prices are backwards ratio adjusted. Moreover, we assume that $F_{t,T}$ is invested as collateral at the risk-free rate at time t and hence, the futures position is fully collateralized.

Table I provides an overview of the commodity data set used and reports summary statistics for all monthly commodity return time series. Commodities are very heterogeneous and their risk-adjusted returns were not too attractive with negative Sharpe Ratios for at least half of all commodities over the sample period. In addition, annual volatilities are quite high ranging between 13.32 % for Feeder Cattle and 37.80 % for Coffee, paired with large negative drawdowns of 97.88 % in the case of Lumber. Nevertheless, most of these commodity futures returns exhibit positive skewness and excess kurtosis. Despite these challenging risk-return profiles, there is ample room for active long-short strategies to potentially navigate the commodity universe. Regarding the diversification potential, Figure 1 shows the correlation structure among all 20 commodity futures. While commodity returns are positively correlated within each sector, they are largely uncorrelated between different sectors. Note that commodities in the livestock sector are not or even negatively correlated with commodities from other sectors.

[9]The beginning of the sample period as well as the number of commodities used are determined by the data availability of speculators' and hedgers' open positions provided by the Commodity Futures Trading Commission.

[Figure 1 about here.]

B. State variables

We select a comprehensive set of state variables based on commodity pricing theories as well as key characteristics identified in the literature on commodity factor investing and return predictability. A priori, we are agnostic as to whether a state variable captures time series or cross-sectional variation in commodity futures risk premia, and we use all variables both as timing predictors as well as tilting characteristics. Based on the statistical significance of the corresponding coefficients, we then select relevant timing and tilting variables.

The first group of state variables is related to the term structure of commodity futures prices. The n-month *basis* of a commodity future is defined as

$$basis_t^{(n)} = \left(\frac{F_{t,T}}{S_t} \right)^{\frac{1}{(T-t)}} - 1 \,, \tag{12}$$

where $n = T - t$ is the number of months before delivery. To capture the shape of the futures curve, we calculate the basis for $n = 1, 2, 3, 4$ *months* to maturity. For the spot price S_t, we use the nearest-to-maturity futures contract. An upward sloping yield curve (contango) implies a positive basis and a negative roll yield, while an inverted futures curve (backwardation) corresponds to a negative basis and a positive roll yield. The theory of storage (Working (1933), Kaldor (1939), Brennan (1958)) relates the basis to the costs and benefits of owning the physical commodity and postulates that the level of inventories determines the shape of the futures curve. In this vein, numerous studies have documented the predictive power of the basis or inventory levels for future commodity returns (Fama and French (1987), Gorton, Hayashi, and Rouwenhorst (2012), Hammerschmid (2018)), while others identify a positive term structure premium when trading on the commodity basis (Erb and Harvey (2006), Gorton and Rouwenhorst (2006), Szymanowska, de Roon, Nijman, and van den Goorbergh (2014)). Alternatively, the theory of normal backwardation (Keynes (1930), Hicks (1939), Cootner (1960)) argues that commodity futures are primarily used by hedgers to transfer price risk to speculators. Hence, it is the hedging pressure between commodity producers and hedgers which determines the shape of the futures curve as well as the futures risk premium demanded by speculators. Evidence of a positive hedging pressure premium can be found in Chang (1985), Bessembinder (1992), de Roon, Nijman, and Veld (2000) and Basu and Miffre (2013). Accordingly, we compute the *hedging pressure of commercial traders* as the difference between short and long hedge positions divided by the total number of positions held by hedgers.

The second group of state variables captures the idea of trend following. The momentum effect in commodity futures markets has been documented by Miffre and Rallis (2007), Shen, Szakmary and Sharma (2007, 2010) and Fuertes, Miffre, and Rallis (2010), among others. We use *risk-adjusted momentum* which we calculate as cumulative futures excess returns over the past 12 months (excluding the most recent month to avoid any reversal effect) standardized by the realized volatility of past daily returns over the same time period. The individual return volatilities vary a lot across the different commodities, particularly at shorter horizons. To avoid merely picking extremely volatile assets, we use risk-adjusted momentum returns following Moskowitz, Ooi, and Pedersen (2012) and Baltas and Kosowski (2013). Furthermore, we also include *risk-adjusted spot price changes* over the past 3 months to capture some short-term trends in the underlying spot market.

Recently, the literature has identified further state variables which better capture the shape of the futures curve, in particular the slope and curvature. We follow Boons and Prado (2019) and consider basis-momentum which captures the difference between momentum returns of different maturities, i.e. measured at different points on the curve. Specifically, we compute *basis-momentum* as

$$BM_t^{(n)} = \prod_{s=t-11}^{t-1} \left(1 + rx_s^{(1)}\right) - \prod_{s=t-11}^{t-1} \left(1 + rx_s^{(n)}\right) , \tag{13}$$

where $n = 2, 3, 4$ *months* to maturity and $rx_t^{(1)}$ refers to the commodity futures excess return as defined in (11). The futures returns with n month to maturity, $rx_t^{(n)}$, are also calculated according to equation (11) only that individual futures contracts are rolled over to the next nearest maturity contract $n = T - t$ months before delivery. A similar approach to exploit commodity momentum along the futures curve is also taken by de Groot, Karstanje, and Zhou (2014).

Lastly, we include the *growth rate of open interest* following Hong and Yogo (2012), who show that open interest has significant predictive power for aggregate commodity futures returns. We calculate this state variable as the 1-month growth rate in the total number of futures contracts outstanding per commodity.

Overall, we have 11 fundamentals (basis for n1/n2/n3/n4, momentum, spot price change, hedging pressure, open interest and basis-momentum for n1-n2/n1-n3/n1-n4) known to capture variation in commodity futures risk premia. Our choice of state variables is certainly selective and not exhaustive. However, we want to focus on the timing and tilting ability of these key commodity fundamentals within the framework of parametric portfolio policies. Above all, we investigate whether an optimal combination of significant variables outperforms benchmark portfolios of long-short factors sorted on the very same variables.

C. Portfolio estimation

To construct realistic commodity strategies avoiding any look ahead bias, we split the total sample period into an in-sample calibration period from January 1987 to December 2001 and an out-of-sample estimation window starting in January 2002 to August 2015. We calibrate the parametric portfolio policies to estimate the first timing and tilting coefficients for December 2001, such that we can record the first portfolio return in January 2002. Then we re-estimate the models every month using an expanding window of data and apply a monthly rebalancing of commodity allocations. The performance of all strategies is only evaluated over the out-of-sample period.

In order to extract all relevant information of the state variables for the distribution of future commodity returns, we do not impose any constraints on the timing and tilting coefficients, and hence on the asset weights. While these raw coefficients account for the full time series and cross-sectional information, they are expected to induce extreme weights with considerable fluctuations. To tame likely excessive turnover, we rescale the optimal weights every month to a constant total long exposure of 1 and a total short exposure of -1. This rescaling also has the advantage of making the active parametric policies comparable in terms of leverage to the long-short factor benchmarks whose positive and negative weights also sum to 1 and -1, respectively.[10]

Regarding the transaction costs of trading individual commodity futures, we rely on the detailed analysis and estimates provided by Marshall, Nguyen, and Visaltanachoti (2012). In particular, they analyse high-frequency order book data of individual commodity trades, and we use the

[10]For the tiling policy, the rescaling of optimal weights has little effect due to the cross-sectional standardization of characteristics. For the timing policy, we might lose some of the relevant timing information by imposing constant leverage. However, the volatility is reduced by more than half for the univariate timing strategies, and by more than three quarters for the multivariate timing strategies.

commodity-specific transaction cost estimates reported in Table 12 in their paper.[11] Furthermore, we assume a risk aversion of 5 for the mean-variance investor.

D. Benchmark portfolios

The tilting policy given in equation (7) is defined relative to some benchmark weights. For the development of optimal commodity allocations, we use cash as our benchmark which avoids anchoring the information of state variables to any benchmark allocation. Instead, our goal is to translate all relevant information of key commodity fundamentals to optimal portfolio weights. With cash as benchmark, the tilting policy will mimic the pure long-short factors sorted on the same characteristics with zero total weight. Regarding the timing policy, there is no explicit benchmark formulation given in equation (2). However, we impose the weights of the basis assets to be equal to the benchmark weights; that is they are set to zero in the case of cash. Consequently, we only consider managed portfolios that capture the interaction between commodity returns and predictor variables.[12] Again, we focus on the maximal timing power of different predictors for individual commodity returns without imposing any constraint or benchmark anchors.

To evaluate the out- or underperformance of the parametric portfolio policies, we resort to external benchmarks. First, we compare our parametric strategies to the performance of the aggregate commodity market, in particular we look at different indices such as the S&P-GSCI, the BCOM or the CRB commodity index. However, these indices may not represent a fair performance benchmark as they are long-only and lack any active element. Following the literature exploiting commodity risk premia, we construct long-short factors sorted on the same state variables used for the parametric policies. Every month, all 20 commodities are ranked according to one of the state variables and one buys the best 25 % and sells the worst 25 % of commodities, applying equal weights within each basket.[13] As an external performance benchmark for the multivariate timing and tilting portfolios, we further integrate different long-short factors into one aggregate factor

[11] As a robustness test, we also use other transaction cost estimates of Marshall, Nguyen, and Visaltanachoti (2012), either based on commodity-specific tick data (Table 7) or a proportional cost of 8.6 bps following Fernandez-Perez, Fuertes, and Miffre (2018). Still, we prefer to go with the most conservative transaction cost estimate in the presentation of this paper's results.

[12] Mechanically, this means that the vector z_t does not contain a constant in the case of a cash benchmark. Alternatively, if z_t includes a constant, then the first column of the θ matrix corresponds to the benchmark weights.

[13] As a robustness test, we also use rank weighted long-short factors. However, these usually underperform the equal-weighted factors in risk-adjusted terms despite diversifying across all 20 commodities.

benchmark. Following Fernandez-Perez, Fuertes, and Miffre (2018), we use equal weighting of all long-short factors with rescaling of weights to +1 for long positions and -1 for short positions.

IV. Empirical Results

Our empirical results are structured as follows. First, we use each state variable separately, either as a timing predictor or as a tilting characteristic, and compare their univariate performance to long-short factors sorted on the same signal. Based on in-sample parameter significance, we select relevant timing and tilting variables, which we then use jointly in a multivariate setting. Furthermore, we discuss several routes to combining the multivariate timing and tilting policies into one aggregate parametric portfolio to capture both time series and cross-sectional information at the same time. Lastly, we analyse the risk exposures of our parametric commodity portfolios using common factor models proposed in the literature.

A. Univariate portfolio policies

Table II summarizes the out-of-sample net performance (i.e. after transaction costs) of univariate timing (Panel A) and tilting policies (Panel B), as well as the corresponding long-short factors sorted on the same variables (Panel C). In addition, Panel D reports summary statistics for aggregate commodity indices during the same time period, which was marked by large price decreases in the commodity sector as evidenced by the negative performance of the S&P-GSCI and an annual average return of 20 bps for the BCOM index. Thus, one may wonder whether an active commodity strategy can earn positive risk-adjusted returns even during such market conditions.

First, the net returns of the univariate timing policies are not overly attractive, with only three positive Sharpe Ratios and large negative drawdowns. Across all state variables, only the long end of the futures curve (in particular the Basis with 3 months to maturity) outperforms the corresponding benchmark factor in risk-adjusted terms. It has a Sharpe Ratio of 0.27 compared to 0.15 for the long-short factor. On the other hand, the performance of the univariate tilting policies looks more promising with only two negative Sharpe Ratios. Moreover, in terms of risk-adjusted returns the tilting portfolios outperform the factor benchmark strategies in 6 out of 11 cases.

Hence, it seems that the state variables work better in tilting the cross-section of commodities than timing individual commodity returns, at least when considering univariate policies.

In general, when we only use one state variable to classify future commodity returns, the value added of these optimized parametric portfolio policies seems limited compared to simple sorting rules without any estimation risk. Furthermore, the parametric portfolio policies invest in all 20 commodities (unless the realization of a certain state variable is zero), while the long-short factors are only invested in half of all commodities (by buying the top 25 % and selling the bottom 25 %). This might be one reason for the weaker performance of the parametric strategies.[14] In a similar vein, Barroso and Santa-Clara (2015) document that univariate tilting policies for currency returns do not consistently outperform the benchmark portfolios, although they argue it is mainly due to transaction costs.

B. Variable selection

One of the main advantages of parametric portfolio policies is that they can readily accommodate several state variables to determine asset weights accounting for possible interdependencies and correlations among fundamentals. In addition, the parametrization allows testing for the statistical significance of individual variables, representing an objective framework to select relevant determinants of future asset returns. We next turn to optimally combining different state variables in a multivariate setting without any look ahead bias but in a way which can be implemented in real time.

In particular, we estimate univariate timing and tilting portfolios for each state variable separately using only data over the in-sample calibration period from January 1987 to December 2001.[15] Due to high multicollinearity among the term structure and trend following variables, we refrain from including all state variables in a given timing or tilting policy. Otherwise, the resulting coefficients might not be deemed statistically significant, even if they actually are relevant determinants of future commodity returns. Moreover, the timing policy could be rendered an underspecified estimation problem, as the parametrization requires a separate coefficient for

[14] The effect of investing in all commodities versus only the top and bottom 25 % is particularly strong for the case of hedging pressure, where the tilting strategy achieves a Sharpe Ratio of -0.24 while the long-short factor has a Sharpe Ratio of 0.20.

[15] Barroso and Santa-Clara (2015) pursue a similar variable selection strategy by first testing each fundamental separately over an in-sample period and then only selecting significant ones for out-of-sample forecasts.

each asset and predictor, e.g. for 5 predictors and 20 assets this would imply 100 coefficients. Hence, one would need to enlarge the in-sample calibration period at the expense of a shorter out-of-sample evaluation window.

The estimated timing (Panel A) and tilting (Panel B) coefficients together with standard errors are reported in Table III, where each column represents a separate parametric portfolio policy. Given the asset specific nature of the timing policy, we count the number of θ coefficients significant at the 10 % level and report the corresponding percentage numbers. We select those state variables for which at least half of all commodity assets have significant coefficients: the selected variables are the long end of the futures curve (Basis n3 and n4) and the growth rate of open interest. In fact, these fundamentals are also found in the literature to be most predictive for future commodity returns (Hong and Yogo (2012), Szymanowska, de Roon, Nijman, and van den Goorbergh (2014), Hammerschmid (2018)). Conversely, the parametrization of the tilting portfolios is more parsimonious by design as one estimates only one coefficient per state variable applied across all commodities. Hence, we directly select those characteristics with a ϕ coefficient significant at the 10 % level: the selected variables are the short end of the futures curve (Basis n1 and n2), both trend following variables (Momentum and Spot price change) as well as a combination of both, namely Basis-Momentum with 3 and 4 months to maturity.[16] Moreover, the sign of the estimated coefficients is in line with theory predictions. For example, in a backwardated commodity futures market (characterized by a negative basis) one expects to earn positive returns on a long position, and this conjecture is confirmed by the negative ϕ coefficients for all term structure variables. Similarly, past winners are expected to outperform past losers in the near future, and this momentum effect is reflected in a positive ϕ coefficient for the trend following variables. Only the sign of the hedging pressure coefficient is not consistent with the theory of normal backwardation; however it is not significant.

In total, we thus choose 3 variables for the multivariate timing policy and 6 fundamentals for the multivariate tilting portfolio. Note that the two multivariate sets are not overlapping in terms of selected state variables. Despite being agnostic as to whether a specific fundamental captures time series or cross-sectional variation of the commodity futures risk premia, our variable

[16] In fact, the selected tilting characteristics all have p-values far below 5 %, thus, the significance level of 10 % is not binding here. For the timing selection, there is a trade-off between the significance level and the number of assets with significant coefficients. We find that the three selected timing fundamentals are always among the most significant variables independent of the calibration of trade-off parameters.

selection procedure indicates a distinct difference between timing predictors and cross-sectional characteristics.

C. Multivariate portfolio policies

Table IV summarizes the net performance of different multivariate commodity portfolios evaluated over the out-of-sample period from January 2002 to August 2015. Panel A reports summary statistics for the timing and tilting portfolios as well as two combined strategies including both time series and cross-sectional information. Panel B summarizes the equally weighted factor benchmark portfolios and Panel C gives the performance of several aggregate commodity market indices. Regarding the factor benchmarks, we either consider all 11 long-short factors (Fernandez-Perez, Fuertes, and Miffre (2018)) or only those factors which use significant timing or tilting state variables (reported in Table III) as a sorting signal. This subset is denoted as PPP variables in Table IV. The latter choice (built on the same information set as the combined parametric portfolio) outperforms the benchmark based on all possible factors with a Sharpe Ratio of 0.61 compared to 0.56. Furthermore, the average annual return after transaction costs of these benchmark portfolios is about 10 % with an annualized volatility of 16 % and a positive skewness around 0.4. Clearly, these active factor benchmarks dominate long-only market indices in terms of risk-adjusted performance.

Considering Panel A, the multivariate timing policy using the basis n3, n4 and open interest, achieves an out-of-sample performance after transaction costs of 8.67 % annualized mean return with 15.51 % annualized volatility, implying a Sharpe Ratio of 0.56. The performance of the multivariate tilting portfolio using the basis n1, n2, momentum, spot change and basis-momentum n3, n4 as state variables, is very similar with an annualized mean return of 9.57 % and an annualized volatility of 17.45 %, which results in a Sharpe Ratio of 0.55. In terms of crash risk, the tilting strategy is fairly resilient given a positive skewness and excess kurtosis, while the returns of the timing portfolio are negatively skewed with a maximum drawdown of 32.12 %. Furthermore, the risk-adjusted performance of each multivariate timing and tilting policy is very similar to the benchmark portfolio using all long-short factors but slightly lower than the factor benchmark based only on significant timing and tilting variables. Hence, it seems that integrating time series and cross-sectional information to determine future asset returns is the key challenge offering large diversification potential. Next, we discuss two possible routes to optimally combining the multivariate timing and tilting policies.

First, we employ a simple and robust aggregation method by investing 50 % in each parametric portfolio implying equal weighting of time series and cross-sectional information.[17] This 50-50 combination achieves a remarkable out-of-sample performance with an average annualized return of 15.77 % and an annualized volatility of 14.45 %. While the mean return nearly doubles compared to the individual timing or tilting portfolios, the volatility of the aggregate portfolio decreases slightly. As a result, risk-adjusted performance is improved and the Sharpe Ratio after transaction costs of the 50-50 combined commodity portfolio is 1.09, which clearly outperforms the best factor benchmark with a Sharpe Ratio of 0.61. Moreover, the returns of the aggregate parametric portfolio are nearly normally distributed and the maximum drawdown is reduced to 19.59 %. Hence, diversifying across time series and cross-sectional information certainly adds value. These results also show that optimizing over different fundamentals to determine asset weights and mixing the resulting optimal allocations is much better than integrating the same set of information via simple long-short sorting rules and equal weighting.

Second, we investigate a more sophisticated approach and combine the two multivariate policies using the Black-Litterman (1991, 1992) framework. They propose a Bayesian model to combine investor specific views regarding the performance of various assets with market equilibrium expected returns where the relative importance of the investor's views depends on the degree of the respective view confidence. In a mean-variance setting the resulting aggregate portfolio also overcomes the problem of highly concentrated and input-sensitive allocations given the shrinkage effect on the optimized inputs. Further details on the general Black-Litterman model are given in Appendix B.1. For the combination of the multivariate timing and tilting policies, this implies that we obtain a data-driven smoothed aggregate commodity portfolio. Importantly, the relative importance of time series versus cross-sectional information depends on the prevailing uncertainty of the corresponding timing or tilting view. In particular, we use reverse mean-variance optimization to extract the expected commodity returns which would result in the optimized timing and

[17] We do not take the average of the multivariate timing and tilting portfolio returns which would imply that the average return of the aggregate portfolio must lie between 8.67 % and 9.57 %. Instead, we average the individual timing and tilting weights and thus get a new "combo" weight for each individual commodity. If the timing and tilting view on an individual commodity disagree, i.e. one is short and the other long, the resulting combo weight will be very small. On the other hand, the 50-50 combination has a large exposure to commodities where the timing and tilting models agree.

tilting weights. These implied timing and tilting returns are then our investor specific timing and tilting views. The confidence level we put in these views is proportional to the variance of past asset returns, following He and Litterman (2002). For the prior or market equilibrium, we use the view implied by our internal benchmark portfolio, i.e. cash. Given these inputs, we can use the Black-Litterman formula to mix the prior with the parametric timing and tilting views to obtain the posterior distribution of expected commodity returns. Lastly, we revert to standard mean-variance optimization using the posterior mean and variance to obtain smoothed commodity weights integrating both time series and cross-sectional information. Further details on the Black-Litterman combination in the context of timing and tilting policies are given in Appendix B.2.

The Black-Litterman combination of multivariate timing and tilting views achieves an out-of-sample net performance of 15.63 % annualized average return with 16.16 % volatility, implying a Sharpe Ratio of 0.97. Hence, the risk-adjusted performance of the Black-Litterman mixture is slightly lower than the 50-50 combination, however both aggregate parametric commodity portfolios strongly outperform the equally weighted factor benchmarks. Moreover, the Black-Litterman approach represents a purely data-driven integration of time series and cross-sectional information rather than an ad hoc constant mixture rule such as the 50-50 weights. The Black-Litterman combination also allows for time variation in weighting the timing and tilting views depending on the prediction uncertainty of each view. In particular, the timing view is more volatile than the tilting view resulting in more extreme bets for individual assets whereas the tilting policy is based on a long-short cross-sectional ranking of all commodities. Consequently, the Black-Litterman combination assigns less weight to the timing portfolio than to the tilting portfolio because of the inherently higher prediction uncertainty. The approach is thus more flexible in adapting to market circumstances and less risky than a constant 50-50 combination, which might suffer more strongly from an unfavourable commodity allocation.

The sizeable outperformance of the combined parametric portfolios over the factor benchmarks is testament of the substantial benefit of estimating optimal relations between significant asset fundamentals and future asset returns as well as the separate modelling of time series and cross-sectional relations. Even if the univariate performance of the long-short factors is deemed more attractive than univariate parametric portfolios, an equal weighting integration does not fully account for the interactions and cross-correlations among different state variables.

Figure 2 plots the cumulative performance of the different multivariate commodity portfolios. First, comparing the performance of the timing (blue) and tilting (green) portfolios, one can clearly see how they would complement each other over time, highlighting the different information content and relative strengths of time series predictors vis-à-vis cross-sectional characteristics. While the financial crisis marked a sharp decline in aggregate commodity prices (see S&P-GSCI performance shown in grey), the tilting policy experienced strong positive performance. Conversely, the timing policy documents substantial positive returns from 2014 onwards. Combining the two views to realize the diversification benefits leads to the remarkable outperformance of the aggregate parametric commodity portfolios (the 50-50 combination is shown in red and the Black-Litterman combination in dark red). The combined strategies outperform not only the individual timing and tilting views but also the equal-weighted factor benchmark based on the same set of state variables across the whole sample period. Overall, the parametric commodity portfolios produce stable positive returns, with a remarkable increase during 2014 driven by the strong timing view and only a small decline during 2011. In contrast, the performance of the equal-weighted factor benchmark strategy is very close to that of the multivariate tilting portfolio but clearly less profitable than the combined parametric portfolio, particularly from 2013 onwards.

[Figure 2 about here.]

In Figure 3 we plot the out-of-sample portfolio weights of the 50-50 combined strategy over time. Each commodity weight is split into the timing (blue) and tilting (red) component.[18] Obviously, the parametric policy is very active with high turnover: still it is profitable to thus capture the time variation in the underlying commodity fundamentals. These cyclicalities and seasonalities are well captured in particular by the timing view, e.g. see the case of livestock or agricultural commodities. The weights of crude oil or gold exhibit longer swings probably corresponding to the business cycle. For soybeans or soyoil, one can readily observe the interplay of the timing and tilting view, resulting in a smooth aggregate weight not plagued by extreme bets from either information set.

[Figure 3 about here.]

[18]The individual asset weights of the Black-Litterman combined strategy cannot be split up into a timing and tilting component as they derive from a mean-variance optimization that is based on the posterior mean and variance which integrate already the timing and tilting views.

In Figure 4 we further split up the original unscaled timing (Panel A) and tilting (Panel B) weights into the individual state variables constituting the multivariate views. Again, we note the significant amplitude of the timing views, which ultimately leads to a lower relative weight in the Black-Litterman combination versus the much smoother tilting weights. It is noteworthy that the basis n3 and basis n4 often have opposite signs in the multivariate timing policy. In line with the literature, particularly the long end of the futures curve proves highly relevant for predicting commodity returns. Moreover, we find that the growth rate of open interest has strong timing power for agriculturals while it is less relevant for metals or energy commodities. Conversely, there are fewer opposing effects within the multivariate tilting view and the sign of the individual characteristics is mostly aligned.

[Figure 4 about here.]

D. Risk factor exposure

To analyse the risk exposure of the parametric commodity portfolios and test whether the optimal strategies earn positive and significant excess returns, we rely on the three-factor model proposed by Bakshi, Gao, and Rossi (2017) including the average commodity market return, a carry factor capturing the term structure premium and a momentum factor. In addition, we consider the basis-momentum factor of Boons and Prado (2019). For every multivariate commodity portfolio we estimate

$$R_{p,t} = \alpha + \beta_{AVG} AVG_t + \beta_{CARRY} CARRY_t + \beta_{MOM} MOM_t + \beta_{BM} BM_t + \varepsilon_t \, , \tag{14}$$

where AVG_t is the average return across all commodities, $CARRY_t$ is the return on the long-short factor sorted on the basis n1, MOM_t is the momentum factor based on past 12-month cumulative returns and BM_t is the return on the basis-momentum n3 factor.[19] Table V summarizes the pricing results for the parametric portfolio policies in Panel A and the factor benchmark portfolios in Panel B. For each commodity strategy, we estimate the three- and four-factor model to price out-of-sample returns after transaction costs.

[19]For the basis and basis-momentum factor we have also tested other maturities, however they have lower explanatory power.

First, we find that the multivariate timing policy has a negative exposure to carry and momentum, which together explain about 7 % of total timing returns. The remaining alpha is positive and highly significant. On the other hand, the multivariate tilting returns are positively exposed to and fully explained by commodity carry, momentum and basis-momentum with an insignificant alpha and an adjusted R^2 of over 46 %. On the aggregate level, we find that both the 50-50 and Black-Litterman combined commodity portfolios positively load on momentum and basis-momentum. However, only 15 % of return variation is explained by these risk factors, and the alpha of the combined strategies is always positive and highly significant. While the tilting returns are fully spanned, the timing view adds substantial value to the aggregate portfolio. Thus enjoying diversification benefits, the combined strategy earns significant excess returns.

In line with the above findings on the pricing of the parametric commodity strategies, we see that the returns of the benchmark factor portfolios are almost fully explained by the four commodity risk factors, with adjusted R^2 values up to 70 % as well as very low and insignificant alpha estimates. Thus, this risk factor analysis proves once more that the parametric portfolio policies successfully integrate time series and cross-sectional information of asset fundamentals in an optimal way and offer significant positive excess returns above common risk factors which cannot be harvested by simply investing in long-short factors.

V. Smoothing the Optimal Allocations

Although parametric commodity portfolios offer significant excess returns, they include high turnover. So far, we have tamed extreme allocations, excessive leverage and volatility by rescaling the weights to sum to +1 for the long leg and -1 for the short leg. We want to pursue an alternative route to smooth optimal timing and tilting allocations without rescaling of weights or imposing constraints which could limit the timing and tilting potential of the state variables. To do so, we use the Black-Litterman (1991, 1992) framework and smooth the timing and tilting views by accounting for their inherent prediction uncertainty. This can be seen as a shrinkage to optimization inputs away from very uncertain commodity state variables towards less volatile fundamentals, and ultimately resulting in more stable commodity allocations. The prior view assumes that there are no state variables affecting classic mean-variance optimization over the commodity space. Under this prior, we set the timing and tilting coefficients, θ and ϕ, to zero.

Our subjective view is based on the multivariate sets of timing and tilting state variables which determine the distribution of future commodity returns. Following the parametric policies to transform the mean-variance optimization problem, we specify absolute views on each managed portfolio (timing) or characteristics portfolio (tilting). Using the Black-Litterman formula, we can combine these two sets of views to get the posterior mean and variance of the augmented asset returns (timing) or characteristics return vector (tilting). These Black-Litterman estimates then serve as inputs for mean-variance optimization over the transformed asset space to obtain smoothed θ or ϕ coefficients and, hence, optimal timing and tilting weights. We pursue this smoothing procedure separately for the multivariate timing and tilting policy and then combine both views using either 50-50 weighting or the Black-Litterman integration. Further details on the smoothing of timing and tilting views are given in Appendix B.3.

Table VI summarizes the out-of-sample net performance of different smoothed multivariate commodity portfolios, which include the timing (Panel A) and tilting (Panel B) portfolios, the combined 50-50 (Panel C) and combined Black-Litterman (Panel D) strategy as well as the factor benchmarks (Panel E); the latter are as before. In addition to the smoothed strategy, we include the performance of the original portfolio with and without rescaling of weights. Across all portfolios, the largest benefit of this smoothing procedure compared to the original unscaled version is the substantial reduction in volatility, leading to similar volatility levels as for the rescaled portfolios. Consequently, the risk-adjusted performance improves and the Sharpe Ratio increases to 0.57 for the smoothed multivariate timing policy and to 0.62 for the tilting policy. Interestingly, the timing returns are now positively skewed while all other moments across all portfolios are more or less the same as in the rescaled version. Furthermore, combining the smoothed timing and tilting views with a 50-50 mix achieves a Sharpe Ratio of 0.94; for the Black-Litterman integration the Sharpe Ratio is 0.79. Although the risk-adjusted performance of the smoothed combined strategies is slightly lower compared to the combined portfolios using rescaled views, the combined strategies still outperform the equally weighted factor benchmarks (maximum Sharpe Ratio of 0.61). Overall, we conclude that the proposed Black-Litterman procedure to smooth optimal timing and tilting allocations offers a data-driven and intuitive way to tame excessive leverage and volatility. Moreover, it avoids unintuitive rescaling of weights but achieves a comparably attractive risk-return profile.

VI. Conclusion

The optimal combination of different commodity signals in a portfolio theoretic framework achieves attractive risk-adjusted returns, out-of-sample and after transaction costs. Despite the estimation risk inherent in mean-variance optimization, the optimal parametric strategy consistently outperforms the naive equally weighted factor benchmark, which is often deemed the method of choice to combine long-short factors. Based on the same set of commodity fundamentals, the optimized commodity portfolio achieves a net Sharpe Ratio of 1.09 compared to 0.61 for the equally weighted benchmark portfolio. Moreover, the optimal commodity portfolio is exposed to but not priced by commodity carry, momentum, and basis-momentum, which in total explain only about 15 % of return variation. Its alpha is highly significant and the optimal strategy offers positive excess returns above common risk factors. Conversely, the factor benchmark portfolio is fully priced by these risk factors, with an insignificant alpha and an adjusted R^2 of nearly 70 %. Hence, our results indicate that portfolio optimization over different fundamentals substantially enhances overall performance.

The parametric commodity strategy optimally integrates information from time series predictors and cross-sectional characteristics of commodity assets. In addition to the optimal combination, the parametric policies also offer a framework to select state variables which have a significant effect on the future distribution of asset returns. For the commodity universe, we find two distinct sets of variables which capture either time series or cross-sectional variation in commodity futures risk premia. Specifically, the long end of the futures curve together with the growth rate of open interest have significant timing power, while the short end of the futures curve and past returns are highly relevant for cross-sectional tilting of commodities. These fundamentals effectively capture the seasonalities and cyclicalities of individual commodity returns and the parametric policies ultimately translate these notions to time-varying asset weights. Furthermore, we investigate an alternative way to combine timing and tilting views by accounting for their inherent prediction uncertainty. While this Black-Litterman approach does not improve overall risk-adjusted performance, it offers an intuitive way of mixing different views that can be very valuable in turbulent times.

References

Ammann, M., G. Coqueret, and J.P. Schade, 2016, Characteristics-based portfolio choice with leverage constraints, *Journal of Banking & Finance* 70, 23–37.

Asness, C.S., T.J. Moskowitz, and L.H. Pedersen, 2013, Value and momentum everywhere, *Journal of Finance* 68, 929–985.

Bakshi, G., X. Gao, and A.G. Rossi, 2017, Understanding the sources of risk underlying the cross section of commodity returns, *Management Science*.

Baltas, N., and R. Kosowski, 2013, Momentum strategies in futures markets and trend-following funds, Working paper, Imperial College Business School London.

Barroso, P., and P. Santa-Clara, 2015, Beyond the carry trade: Optimal currency portfolios, *Journal of Financial and Quantitative Analysis* 50, 1037–1056.

Basak, S., and A. Pavlova, 2016, A model of financialization of commodities, *Journal of Finance* 71, 1511–1556.

Basu, D., and J. Miffre, 2013, Capturing the risk premium of commodity futures: The role of hedging pressure, *Journal of Banking & Finance* 37, 2652–2664.

Bessembinder, H., 1992, Systematic risk, hedging pressure, and risk premiums in futures markets, *Review of Financial Studies* 5, 637–667.

Bhardwaj, G., G. Gorton, and G. Rouwenhorst, 2015, Facts and fantasies about commodity futures ten years later, Working paper, National Bureau of Economic Research.

Black, F., and R. Litterman, 1991, Asset allocation: Combining investor views with market equilibrium, *Journal of Fixed Income* 1, 7–18.

Black, F., and R. Litterman, 1992, Global portfolio optimization, *Financial Analysts Journal* 48, 28–43.

Blitz, D., and W. de Groot, 2014, Strategic allocation to commodity factor premiums, *Journal of Alternative Investments* 17, 103.

Boons, M., and M.P. Prado, 2019, Basis-momentum, *Journal of Finance* 74, 239–279.

Brandt, M.W., and P. Santa-Clara, 2006, Dynamic portfolio selection by augmenting the asset space, *Journal of Finance* 61, 2187–2217.

Brandt, M.W., P. Santa-Clara, and R. Valkanov, 2009, Parametric portfolio policies: Exploiting characteristics in the cross-section of equity returns, *Review of Financial Studies* 22, 3411–3447.

Brennan, M.J., 1958, The supply of storage, *American Economic Review* 48, 50–72.

Chang, E.C., 1985, Returns to speculators and the theory of normal backwardation, *Journal of Finance* 40, 193–208.

Cheng, I.H., and W. Xiong, 2014, Financialization of commodity markets, *Annual Review of Financial Economics* 6, 419–441.

Cootner, P.H., 1960, Returns to speculators: Telser versus Keynes, *Journal of Political Economy* 68, 396–404.

de Groot, W., D. Karstanje, and W. Zhou, 2014, Exploiting commodity momentum along the futures curves, *Journal of Banking & Finance* 48, 79–93.

de Roon, F.A., T.E. Nijman, and C. Veld, 2000, Hedging pressure effects in futures markets, *Journal of Finance* 55, 1437–1456.

DeMiguel, V., L. Garlappi, and R. Uppal, 2009, Optimal versus naive diversification: How inefficient is the 1/n portfolio strategy?, *Review of Financial Studies* 22, 1915–1953.

DeMiguel, V., A. Martin-Utrera, F.J. Nogales, and R. Uppal, 2019, A transaction-cost perspective on the multitude of firm characteristics, Working paper, London Business School.

Dewally, M., L.H. Ederington, and C.S. Fernando, 2013, Determinants of trader profits in commodity futures markets, *Review of Financial Studies* 26, 2648–2683.

Dichtl, H., W. Drobetz, H. Lohre, C. Rother, and P. Vosskamp, 2018, Optimal timing and tilting of equity factors, Working paper, University of Hamburg.

Erb, C.B., and C.R. Harvey, 2006, The strategic and tactical value of commodity futures, *Financial Analysts Journal* 62, 69–97.

Fama, E.F., and K.R. French, 1987, Commodity futures prices: Some evidence on forecast power, premiums, and the theory of storage, *Journal of Business* 60, 55–73.

Fernandez-Perez, A., B. Frijns, A.M. Fuertes, and J. Miffre, 2018, The skewness of commodity futures returns, *Journal of Banking & Finance* 86, 143–158.

Fernandez-Perez, A., A.M. Fuertes, and J. Miffre, 2016, Is idiosyncratic volatility priced in commodity futures markets?, *International Review of Financial Analysis* 46, 219–226.

Fernandez-Perez, A., A.M. Fuertes, and J. Miffre, 2018, A flexible style-integration framework, Working paper, Auckland University of Technology.

Fuertes, A.M., J. Miffre, and A. Fernandez-Perez, 2015, Commodity strategies based on momentum, term structure, and idiosyncratic volatility, *Journal of Futures Markets* 35, 274–297.

Fuertes, A.M., J. Miffre, and G. Rallis, 2010, Tactical allocation in commodity futures markets: Combining momentum and term structure signals, *Journal of Banking & Finance* 34, 2530–2548.

Gargano, A., and A. Timmermann, 2014, Forecasting commodity price indexes using macroeconomic and financial predictors, *International Journal of Forecasting* 30, 825–843.

Gorton, G.B., F. Hayashi, and K.G. Rouwenhorst, 2012, The fundamentals of commodity futures returns, *Review of Finance* 17, 35–105.

Gorton, G.B., and K.G. Rouwenhorst, 2006, Facts and fantasies about commodity futures, *Financial Analysts Journal* 62, 47–68.

Hammerschmid, R., 2018, Commodity return predictability, Working paper, University of Zurich.

Hansen, L.P., 1982, Large sample properties of generalized method of moments estimators, *Econometrica* 50, 1029–1054.

He, G., and R. Litterman, 2002, The intuition behind Black-Litterman model portfolios, Working paper, Goldman Sachs Asset Management.

Henderson, B.J., N.D. Pearson, and L. Wang, 2014, New evidence on the financialization of commodity markets, *Review of Financial Studies* 28, 1285–1311.

Hicks, J.R., 1939, *Value and capital*. (Oxford: Clarendon Press).

Hong, H., and M. Yogo, 2012, What does futures market interest tell us about the macroeconomy and asset prices?, *Journal of Financial Economics* 105, 473–490.

Jacobs, H., S. Müller, and M. Weber, 2014, How should individual investors diversify? An empirical evaluation of alternative asset allocation policies, *Journal of Financial Markets* 19, 62–85.

Kaldor, N., 1939, Speculation and economic stability, *Review of Economic Studies* 7, 1–27.

Keynes, J.M., 1930, *A treatise on money*. (London Macmillan).

Koijen, R.S.J., T.J. Moskowitz, L.H. Pedersen, and E.B. Vrugt, 2018, Carry, *Journal of Financial Economics* 127, 197–225.

Marshall, B.R., N.H. Nguyen, and N. Visaltanachoti, 2012, Commodity liquidity measurement and transaction costs, *Review of Financial Studies* 25, 599–638.

Miffre, J., 2016, Long-short commodity investing: A review of the literature, *Journal of Commodity Markets* 1, 3–13.

Miffre, J., and G. Rallis, 2007, Momentum strategies in commodity futures markets, *Journal of Banking & Finance* 31, 1863–1886.

Moskowitz, T.J., Y.H. Ooi, and L.H. Pedersen, 2012, Time series momentum, *Journal of Financial Economics* 104, 228–250.

Sakkas, A., and N. Tessaromatis, 2018, Factor based commodity investing, Working paper, EDHEC.

Shen, Q., A.C. Szakmary, and S.C. Sharma, 2007, An examination of momentum strategies in commodity futures markets, *Journal of Futures Markets* 27, 227–256.

Singleton, K.J., 2014, Investor flows and the 2008 boom/bust in oil prices, *Management Science* 60, 300–318.

Szakmary, A.C., Q. Shen, and S.C. Sharma, 2010, Trend-following trading strategies in commodity futures: A re-examination, *Journal of Banking & Finance* 34, 409–426.

Szymanowska, M., F. de Roon, T. Nijman, and R. van den Goorbergh, 2014, An anatomy of commodity futures risk premia, *Journal of Finance* 69, 453–482.

Tang, K., and W. Xiong, 2012, Index investment and the financialization of commodities, *Financial Analysts Journal* 68, 54–74.

Working, H., 1933, Price relations between July and September wheat futures at Chicago since 1885, *Wheat Studies of the Food Research Institute* 9, 187–238.

Yang, F., 2013, Investment shocks and the commodity basis spread, *Journal of Financial Economics* 110, 164–184.

A. Statistical Inference

1. Timing policy

The sample moment of the unconditional timing portfolio problem defined in equation (6) is

$$\max_{\tilde{w}} \; \frac{1}{T}\sum_{t=0}^{T-1}\left[\tilde{w}'\tilde{r}_{t+1} - \frac{\gamma}{2}\tilde{w}'\tilde{r}_{t+1}\tilde{r}'_{t+1}\tilde{w}\right]. \tag{15}$$

The maximum expected utility estimate $\hat{\tilde{w}}$ satisfies the following first-order conditions

$$\frac{1}{T}\sum_{t=0}^{T-1} h(\tilde{r}_{t+1};\tilde{w}) = \frac{1}{T}\sum_{t=0}^{T-1}\left(\tilde{r}_{t+1} - \gamma\tilde{r}_{t+1}\tilde{r}'_{t+1}\tilde{w}\right) = 0\,, \tag{16}$$

which can be seen as a method of moments estimator. Following Hansen (1982), the corresponding asymptotic covariance matrix of a general method of moments (GMM) estimator is

$$AVar(\tilde{w}) = \frac{1}{T}\left[G'V^{-1}G\right]^{-1}, \tag{17}$$

$$\text{where} \quad G = \frac{1}{T}\sum_{t=0}^{T-1}\frac{\partial h(\tilde{r}_{t+1};\tilde{w})}{\partial\tilde{w}} = \frac{1}{T}\sum_{t=0}^{T-1}\left(-\gamma\tilde{r}_{t+1}\tilde{r}'_{t+1}\right) \tag{18}$$

$$\text{and} \quad V = \frac{1}{T}\sum_{t=0}^{T-1}\left[\tilde{r}_{t+1} - \gamma\tilde{r}_{t+1}\tilde{r}'_{t+1}\tilde{w}\right]\left[\tilde{r}_{t+1} - \gamma\tilde{r}_{t+1}\tilde{r}'_{t+1}\tilde{w}\right]'. \tag{19}$$

Hence, the standard errors of the timing coefficients θ are

$$se(\tilde{w}) = \sqrt{\text{diag}(AVar(\tilde{w}))}\,, \quad \text{where} \quad \tilde{w} = vec(\theta)\,. \tag{20}$$

2. Tilting policy

The sample moment of the unconditional tilting portfolio problem in terms of the characteristic and benchmark return defined in equation (10) is

$$\max_{\phi} \; \frac{1}{T}\sum_{t=0}^{T-1}\left[\phi' r_{c,t+1} - \gamma\phi' r_{c,t+1} r_{b,t+1} - \frac{\gamma}{2}\phi' r_{c,t+1} r'_{c,t+1}\phi\right]. \tag{21}$$

The maximum expected utility estimate $\hat{\phi}$ satisfies the following first-order conditions

$$\frac{1}{T}\sum_{t=0}^{T-1} h(r_{c,t+1};\phi) = \frac{1}{T}\sum_{t=0}^{T-1}\left(r_{c,t+1} - \gamma r_{c,t+1} r_{b,t+1} - \gamma r_{c,t+1} r'_{c,t+1}\phi\right) = 0\,, \tag{22}$$

which can be seen as a method of moments estimator. Following Hansen (1982), the corresponding asymptotic covariance matrix of a GMM estimator is

$$AVar(\phi) = \frac{1}{T}\left[G' V^{-1} G\right]^{-1}\,, \tag{23}$$

$$\text{where} \quad G = \frac{1}{T}\sum_{t=0}^{T-1}\frac{\partial h(r_{c,t+1};\phi)}{\partial \phi} = \frac{1}{T}\sum_{t=0}^{T-1}\left(-\gamma r_{c,t+1} r'_{c,t+1}\right) \tag{24}$$

$$\text{and} \quad V = \frac{1}{T}\sum_{t=0}^{T-1}\left[r_{c,t+1} - \gamma r_{c,t+1} r_{b,t+1} - \gamma r_{c,t+1} r'_{c,t+1}\phi\right]\left[r_{c,t+1} - \gamma r_{c,t+1} r_{b,t+1} - \gamma r_{c,t+1} r'_{c,t+1}\phi\right]'\,. \tag{25}$$

Hence, the standard errors of the tilting coefficients ϕ are

$$se(\phi) = \sqrt{\text{diag}(AVar(\phi))}\,. \tag{26}$$

B. Black-Litterman for Parametric Portfolio Policies

1. Black-Litterman framework

The Black-Litterman (1991, 1992) model uses a Bayesian approach to combine investor specific views with market equilibrium expected returns. The prior assumes that the investor holds the market portfolio and, given the corresponding market weights w_m, one can use reverse mean-variance optimization to determine the implied equilibrium returns Π. The *prior distribution of expected returns* is

$$\mathbb{E}[r] \sim \mathcal{N}(\Pi, \tau\Sigma)\,, \tag{27}$$

$$\text{where} \quad \Pi = \gamma \Sigma w_m\,, \quad \Sigma = Var(r_t)\,.$$

Therein, γ is the risk aversion coefficient and τ is a proportionality constant measuring the investor's confidence in the prior estimates. Throughout our analysis, we assume a risk aversion of 5 and we set τ equal to 0.05. The investor can specify K views about future expected returns with a certain level of confidence. The parametrization of the subjective views is

$$P\,\mathbb{E}[r] = Q + \varepsilon\,, \quad \varepsilon \sim \mathcal{N}(0, \Sigma)\,, \tag{28}$$

where P is a $(K \times N)$ projection matrix of the K views on the N asset weights, Q is a $(K \times 1)$ vector of return expectations for each view and Σ is a $(K \times K)$ covariance matrix of the investor's views. We follow He and Litterman (2002) and assume that the variance of the subjective views is a diagonal matrix proportional to the variance of the prior

$$\Omega = \mathrm{diag}(P(\tau\Sigma)P')\,. \tag{29}$$

Hence, the *distribution of the investor's views* is

$$P\,\mathbb{E}[r] \,|\, \mathbb{E}[r] \sim \mathcal{N}(Q, \Sigma)\,. \tag{30}$$

The *posterior distribution of expected returns* is then

$$\begin{aligned} &\mathbb{E}[r] \,|\, P\,\mathbb{E}[r] \sim \mathcal{N}(\mu^*, M)\,, \\ \text{where} \quad &\mu^* = \Big((\tau\Sigma)^{-1} + P'\Omega^{-1}P\Big)^{-1} \Big((\tau\Sigma)^{-1}\Pi + P'\Omega^{-1}Q\Big) \\ \text{and} \quad &M = \Big((\tau\Sigma)^{-1} + P'\Omega^{-1}P\Big)^{-1}\,. \end{aligned} \tag{31}$$

Black and Litterman use the Bayes formula to combine the equilibrium prior (27) with the investor's views (30) to obtain the posterior mean and variance of expected returns. Finally, the *posterior distribution of returns* is

$$\begin{aligned} &r \sim \mathcal{N}(\mu^*, \Sigma^*)\,, \\ \text{where} \quad &\mu^* = \Big((\tau\Sigma)^{-1} + P'\Omega^{-1}P\Big)^{-1} \Big((\tau\Sigma)^{-1}\Pi + P'\Omega^{-1}Q\Big) \\ &\quad = \Pi + (\tau\Sigma)P'\left(P(\tau\Sigma)P' + \Omega\right)^{-1}(Q - P\Pi) \\ \text{and} \quad &\Sigma^* = \Sigma + M = \Sigma + \Big((\tau\Sigma)^{-1} + P'\Omega^{-1}P\Big)^{-1} \\ &\quad = (1+\tau)\Sigma - \tau^2\Sigma P'\left(P(\tau\Sigma)P' + \Omega\right)^{-1} P\Sigma\,. \end{aligned} \tag{32}$$

Then, one can use the posterior mean μ^* and variance Σ^* as inputs to mean-variance optimization in order to get smoothed asset weights accounting for the investor specific views.

2. Combining optimal timing and tilting views

Given an optimal timing $(w_t(\hat{\theta}))$ and tilting $(w_t(\hat{\phi}))$ policy vector, we can use reverse mean-variance optimization to extract the expected asset returns associated with these optimal parametric weights as

$$Q_t^{time} = \gamma \Sigma w_t(\hat{\theta}) \,, \tag{33}$$

$$Q_t^{tilt} = \gamma \Sigma w_t(\hat{\phi}) \,, \tag{34}$$

where $\Sigma = Var(r_t)$ is an $(N \times N)$ covariance matrix of past commodity returns. Our subjective views combining time series and cross-sectional information is a stacked vector of implied returns

$$Q = \begin{bmatrix} Q_t^{time} \\ Q_t^{tilt} \end{bmatrix} . \tag{35}$$

We consider two absolute views on each commodity, one coming from the timing policy and one from the tilting policy. Therefore, the projection matrix is

$$P = \begin{bmatrix} I_{(N \times N)} \\ I_{(N \times N)} \end{bmatrix} . \tag{36}$$

For the prior view, we assume no timing or tilting information to be relevant, i.e. parametric portfolio policies where $\theta = 0$ and $\phi = 0$. Hence, the prior is equal to the expected returns implied by the internal benchmark weight

$$\Pi = \gamma \Sigma w_{b,t} \,, \tag{37}$$

where $w_{b,t}$ is an $(N \times 1)$ vector of benchmark asset weights. Given all these inputs, we can use the posterior distribution of returns to calculate mean and variance according to (32), which we can further use for mean-variance optimization to obtain the weights of the combined commodity portfolio. Using cash as benchmark, all entries of $w_{b,t}$ are zero and the subjective views are thus shrunk towards zero, ultimately smoothing the ensuing commodity allocation.

3. Smoothing optimal timing and tilting views

Instead of using the Black-Litterman framework to combine given timing and tilting policies into one aggregate commodity portfolio, this section pursues a different route and applies the Black-Litterman model at the level of estimating optimal parametric weights. This allows to account for the inherent prediction uncertainty of state variables already in the estimation of timing or tilting coefficients and to adjust optimal allocations accordingly.

To *smooth optimal timing views*, the Black-Litterman model is employed over the augmented asset space and mean-variance optimization is estimated over θ or $\tilde{w}$, respectively. The prior view assumes that state variables are not relevant for commodities. The corresponding coefficient matrix for the timing policy, θ_{prior}, is equal to the benchmark weight $w_{b,t}$ for the basis assets and it is zero for all "managed" portfolios. Over the augmented asset space, the prior expected returns are

$$\Pi = \gamma \Sigma \tilde{w}_t(\theta_{prior}) \,, \tag{38}$$

where $\Sigma = Var(\tilde{r}_{t+1})$ is an $(N \cdot K \times N \cdot K)$ covariance matrix of augmented asset returns. On the other hand, the investor specific view assumes that there are K state variables relevant for determining the future distribution of asset returns, and we specify the absolute view for each managed portfolio as the historical mean of its return. Thus, the subjective expected returns and the projection matrix are

$$Q = \mathbb{E}[\tilde{r}_{t+1}] \,, \tag{39}$$

$$P = I_{(N \cdot K \times N \cdot K)} \,. \tag{40}$$

Based on these belief specifications, we can calculate the posterior mean and variance of the augmented asset returns according to (32). The Black-Litterman estimates then serve as inputs to mean-variance optimization over the augmented asset space to obtain smoothed θ coefficients and hence optimal timing weights that are less extreme. Also, given the confidence weighting of Black-Litterman, one is attributing lower importance to commodity predictors with higher uncertainty.

To *smooth optimal tilting views*, the Black-Litterman model is employed over the characteristics portfolio and mean-variance optimization is estimated over ϕ. The prior view again assumes that state variables are not relevant for pricing commodities, which implies that the coefficient vector, ϕ_{prior}, is zero and the tilting policy is equal to the benchmark weight, i.e. $w_t(\phi_{prior}) = w_{b,t}$. Hence, the prior expected returns of the characteristics portfolio, Π, are zero since the characteristics return vector does not exist if there are no state variables. For the investor view, we assume that there are K relevant tilting characteristics to determine the future distribution of asset returns, and we specify an absolute view for each characteristics portfolio. The subjective expected returns and the projection matrix are

$$Q = \mathbb{E}[r_{c,t+1}] - \gamma Cov(r_{b,t+1}, r_{c,t+1}) \,, \tag{41}$$

$$P = I_{(K \times K)} \,, \tag{42}$$

where we use the historical mean and covariance of the characteristics and benchmark returns to estimate Q. Further, the covariance matrix of the characteristics return vector is $\Sigma = Var(r_{c,t+1})$. According to (32), we can calculate the posterior mean and variance of the characteristics return vector. Furthermore, we use the Black-Litterman estimates as inputs for mean-variance optimization to get smoothed ϕ coefficients and hence optimal tilting weights.

Table I
Summary statistics

The table gives summary statistics for individual commodity futures excess returns, grouped by commodity sector. It reports annualized mean, standard deviation and Sharpe Ratio, as well as skewness, kurtosis, minimum, maximum, and maximum drawdown of monthly returns. All values are expressed in percentage points and the time period is January 1987 to August 2015.

Sector	Commodity	Mean	Std.Dev	SR	Skew	Kurt	Min	Max	maxDD
Energy	WTI Crude Oil	3.92	32.18	0.12	0.31	4.95	-32.37	46.01	-86.51
	Heating Oil	4.79	30.81	0.16	0.41	4.44	-28.65	36.95	-73.09
Grains & Oilseeds	Corn	-6.08	27.32	-0.22	0.64	6.53	-22.80	45.82	-90.99
	Soybeans	3.69	24.09	0.15	-0.10	4.14	-23.39	23.56	-71.85
	Soymeal	12.73	27.59	0.46	0.37	4.29	-27.15	30.14	-52.71
	Soyoil	-3.62	24.71	-0.15	0.13	4.64	-25.20	26.86	-80.83
	Wheat	-7.89	27.77	-0.28	0.51	5.03	-25.25	37.74	-95.26
Livestock	Feeder Cattle	4.48	13.32	0.34	-0.32	5.51	-20.58	13.77	-34.20
	Lean Hogs	-1.22	25.29	-0.05	0.06	4.00	-25.96	30.20	-85.19
	Live Cattle	3.80	13.41	0.28	-0.33	5.69	-21.09	14.50	-41.40
Metals	Gold	0.04	15.54	0.00	0.19	4.13	-18.46	16.05	-72.77
	Palladium	3.90	31.68	0.12	0.45	6.61	-33.93	46.89	-86.62
	Platinum	1.91	20.60	0.09	-0.43	6.01	-31.86	25.52	-62.76
	Silver	-0.70	28.18	-0.02	0.25	4.04	-27.98	28.23	-78.70
Softs	Lumber	-8.73	30.80	-0.28	0.52	4.87	-26.21	45.24	-97.88
	Cocoa	-3.08	29.47	-0.10	0.54	4.18	-25.01	34.56	-91.56
	Coffee	-6.66	37.80	-0.18	1.07	5.70	-30.39	50.60	-91.14
	Cotton	-3.23	27.36	-0.12	0.19	3.58	-22.64	24.75	-92.62
	Orange Juice	-3.34	30.19	-0.11	0.52	4.10	-22.13	34.15	-92.02
	Sugar	1.73	31.87	0.05	0.41	3.95	-29.70	33.65	-70.77

Table II
Univariate models

The table gives summary statistics for out-of-sample net returns, where each strategy is based on a single state variable. Panel A summarizes results of univariate timing policies, Panel B refers to univariate tilting policies, Panel C to long-short factors and Panel D to aggregate commodity market indices. It reports annualized mean, standard deviation and Sharpe Ratio, as well as skewness, kurtosis, minimum, maximum, and maximum drawdown of monthly returns. All values are expressed in percentage points and the respective out-of-sample evaluation period is January 2002 to August 2015.

Variable	Mean	Std.Dev	SR	Skew	Kurt	Min	Max	maxDD
Panel A: Timing								
Basis n1	-3.89	15.77	-0.25	-0.33	4.83	-19.50	13.83	-64.19
Basis n2	-4.80	15.76	-0.30	-0.20	3.69	-16.54	11.41	-62.97
Basis n3	4.85	17.83	0.27	-0.39	5.45	-23.85	14.76	-44.51
Basis n4	-0.33	17.11	-0.02	0.37	3.24	-11.35	15.66	-55.83
Momentum	0.01	17.07	0.00	0.24	3.48	-13.48	16.87	-45.32
Spot change	1.17	13.37	0.09	-0.08	3.08	-10.34	9.97	-36.48
Hedging pressure	-0.23	16.26	-0.01	-0.10	2.66	-12.93	10.80	-43.58
Open interest	-4.83	14.83	-0.33	0.05	3.78	-14.81	14.81	-57.52
Basis-Mom n2	-2.99	16.38	-0.18	0.51	5.19	-13.69	20.53	-53.53
Basis-Mom n3	-2.95	17.95	-0.16	0.17	3.19	-14.49	14.79	-51.85
Basis-Mom n4	-6.01	17.70	-0.34	0.35	2.82	-11.61	14.20	-73.12
Panel B: Tilting								
Basis n1	2.68	15.93	0.17	-0.06	3.01	-12.19	12.44	-34.66
Basis n2	4.54	16.28	0.28	0.15	3.71	-12.90	15.90	-20.91
Basis n3	4.54	18.43	0.25	0.17	3.34	-12.89	18.22	-30.83
Basis n4	1.38	19.38	0.07	-0.04	3.35	-14.61	18.13	-34.40
Momentum	6.87	18.38	0.37	0.10	2.73	-11.31	15.99	-31.17
Spot change	5.74	18.13	0.32	0.16	3.01	-12.30	14.99	-30.51
Hedging pressure	-4.40	18.70	-0.24	0.24	3.68	-13.44	20.28	-61.95
Open interest	-6.99	15.35	-0.46	-0.14	3.33	-14.35	10.52	-64.08
Basis-Mom n2	0.73	18.12	0.04	0.29	4.54	-14.51	22.36	-57.12
Basis-Mom n3	6.66	20.93	0.32	0.41	4.80	-15.28	26.66	-46.10
Basis-Mom n4	9.37	21.13	0.44	0.32	3.90	-14.67	23.22	-46.72
Panel C: L/S Factor								
Basis n1	8.05	19.61	0.41	0.00	2.84	-12.73	15.37	-27.47
Basis n2	7.82	19.08	0.41	0.00	3.35	-14.31	17.22	-28.57
Basis n3	3.32	22.37	0.15	-0.06	3.44	-15.34	21.76	-45.31
Basis n4	-0.66	22.50	-0.03	-0.09	2.95	-16.01	19.60	-46.32
Momentum	5.84	20.64	0.28	0.14	2.72	-13.21	17.45	-29.41
Spot change	4.47	20.21	0.22	0.20	3.08	-16.00	18.19	-35.97
Hedging pressure	4.06	20.82	0.20	0.03	3.37	-16.99	20.93	-40.13
Open interest	-7.21	16.95	-0.43	0.14	3.75	-15.60	14.38	-76.34
Basis-Mom n2	-2.02	16.36	-0.12	0.07	2.90	-11.31	12.58	-58.82
Basis-Mom n3	5.55	19.68	0.28	-0.38	2.97	-14.94	14.01	-40.75
Basis-Mom n4	9.50	20.49	0.46	-0.13	2.80	-13.64	16.04	-42.59
Panel D: Commodity indices								
CRB	3.50	11.21	0.31	-1.42	12.49	-20.45	9.19	-37.41
S&P-GSCI	-1.43	23.74	-0.06	-0.54	4.17	-28.25	19.65	-74.03
BCOM	0.20	17.22	0.01	-0.61	4.75	-21.34	12.99	-60.96

Table IV
Multivariate models

The table gives summary statistics for out-of-sample net returns of multivariate commodity strategies. Panel A summarizes results of parametric policies, Panel B refers to equally weighted factor benchmarks and Panel C includes aggregate commodity market indices. PPP variables refers to significant timing and tilting state variables selected in Table III. The table reports annualized mean, standard deviation and Sharpe Ratio, as well as skewness, kurtosis, minimum, maximum, and maximum drawdown of monthly returns. All values are expressed in percentage points and the out-of-sample evaluation period is January 2002 to August 2015.

Portfolio	Mean	Std.Dev	SR	Skew	Kurt	Min	Max	maxDD
Panel A: Parametric portfolio policies								
Timing	8.67	15.51	0.56	-0.22	3.18	-14.18	11.34	-32.12
Tilting	9.57	17.45	0.55	0.61	4.06	-11.02	20.10	-29.99
Combo 50-50	15.77	14.45	1.09	0.00	3.53	-9.51	15.18	-19.59
Combo BL	15.63	16.16	0.97	-0.22	3.89	-13.74	15.78	-26.78
Panel B: L/S Factor benchmark								
ALL Variables	9.25	16.65	0.56	0.47	3.66	-9.58	17.25	-21.42
PPP Variables	10.07	16.63	0.61	0.41	3.38	-9.53	16.61	-21.12
Panel C: Commodity indices								
CRB	3.50	11.21	0.31	-1.42	12.49	-20.45	9.19	-37.41
S&P-GSCI	-1.43	23.74	-0.06	-0.54	4.17	-28.25	19.65	-74.03
BCOM	0.20	17.22	0.01	-0.61	4.75	-21.34	12.99	-60.96

Table III
Variable selection

The table reports coefficient estimates together with standard errors in parentheses of univariate timing (Panel A) and tilting (Panel B) policies. * and ** indicate significance at the 10 % or 5 % levels, respectively. The in-sample estimation period is January 1987 to December 2001.

	Basis				Momentum	Spot	Hedging	Open	Basis-Momentum		
	n1	n2	n3	n4		Change	Pressure	Interest	n2	n3	n4
Panel A: Timing coefficients (θ)											
WTI Crude Oil	-0.58**	-0.49**	-0.44**	-0.41**	0.25	0.29**	0.10	0.16	0.11	0.20	0.30**
	(0.15)	(0.14)	(0.14)	(0.15)	(0.18)	(0.19)	(0.19)	(0.12)	(0.13)	(0.13)	(0.12)
Heating Oil	0.35**	0.40**	0.40**	0.39*	0.04	-0.37**	0.17	-0.08	0.16	0.06	0.03
	(0.15)	(0.17)	(0.18)	(0.21)	(0.17)	(0.20)	(0.20)	(0.17)	(0.16)	(0.11)	(0.10)
Corn	0.11	-0.24	-0.14	-0.17*	0.09	0.73**	0.06	-0.06	-0.11	0.19	0.16
	(0.07)	(0.15)	(0.15)	(0.08)	(0.25)	(0.23)	(0.17)	(0.19)	(0.18)	(0.16)	(0.12)
Soybeans	-0.32	-0.01	0.03	0.24	-0.23	-0.55	-0.13	-0.27	0.00	0.15	0.23
	(0.25)	(0.20)	(0.16)	(0.15)	(0.31)	(0.32)	(0.26)	(0.26)	0.15	0.17	0.18
Soymeal	0.06	-0.16	-0.02	-0.23	0.11	0.38	0.43*	0.53*	0.16	0.10	0.22
	(0.20)	(0.21)	(0.18)	(0.21)	(0.23)	(0.23)	(0.22)	(0.30)	0.15	0.12	0.14
Soyoil	-0.30	-0.81**	-0.96**	-0.60**	-0.07	-0.30**	0.07	0.65**	-0.01	-0.09	-0.14
	(0.20)	(0.25)	(0.23)	(0.19)	(0.20)	(0.26)	(0.21)	(0.25)	0.15	0.18	0.17
Wheat	0.33*	-0.40*	0.06	-0.13	0.16	-0.32	0.06	-0.31	0.11	-0.02	-0.05
	(0.18)	(0.23)	(0.19)	(0.22)	(0.20)	(0.23)	(0.23)	(0.24)	0.15	0.14	0.10
Feeder Cattle	-0.06	0.48			0.70	-0.53	-1.14**	0.24	-0.19		
	(0.29)	(0.34)			(0.49)	(0.48)	(0.43)	(0.43)	0.35		
Lean Hogs	0.30	-0.12	-0.05	0.18	0.47**	-0.26	-0.06	-0.67**	0.11	0.26	0.27
	(0.21)	(0.23)	(0.24)	(0.19)	(0.19)	(0.21)	(0.15)	(0.27)	0.16	0.20	0.19
Live Cattle	0.15	0.93**	0.99**	0.64**	0.45	0.47	0.02	1.61**	0.50	0.60	0.66*
	(0.45)	(0.32)	(0.37)	(0.26)	(0.38)	(0.35)	(0.43)	(0.46)	0.39	0.38	0.34
Gold	0.06	-0.29	-0.75**	-0.65**	0.32	-0.79	0.58	0.75*	0.37	0.34	0.28
	(0.65)	(0.27)	(0.36)	(0.23)	(0.33)	(0.45)	(0.38)	(0.45)	0.27	0.22	0.21
Palladium	-0.33**	0.15			0.43**	0.40**	0.06	-0.41**	-0.18*		
	(0.14)	(0.10)			(0.14)	(0.12)	(0.23)	(0.15)	0.10		
Platinum	-0.56**	-0.64*	-0.41**		-0.03	-0.48	-0.04	-0.07	-0.17	0.09	
	(0.27)	(0.34)	(0.21)		(0.21)	(0.23)	(0.27)	(0.33)	0.21	0.22	
Silver	0.93**	0.49**	0.38*	0.27	0.01	0.12	-0.86**	-0.35	-0.27	-0.06	-0.01
	(0.32)	(0.22)	(0.22)	(0.24)	(0.18)	(0.26)	(0.24)	(0.22)	0.18	0.16	0.17
Lumber	0.09	0.36**			0.07	0.12	0.14	-0.27*	0.15		
	(0.19)	(0.16)			(0.12)	(0.11)	(0.17)	(0.14)	0.10		
Cocoa	0.30*	0.09	0.23**	-0.08	0.02	0.06	0.12	-0.26	-0.11	-0.13	-0.10
	(0.17)	(0.17)	(0.11)	(0.15)	(0.12)	(0.15)	(0.15)	(0.18)	0.14	0.17	0.16
Coffee	-0.07	-0.13	-0.17**	-0.09	0.21**	0.17	-0.23*	0.40**	0.02	0.08	0.10
	(0.12)	(0.09)	(0.07)	(0.06)	(0.09)	(0.12)	(0.12)	(0.14)	0.07	0.09	0.09
Cotton	0.11	0.23	0.20	-0.19	-0.09	0.12	0.27	-0.15	-0.46**	-0.14**	-0.24**
	(0.11)	(0.19)	(0.16)	(0.18)	(0.14)	(0.22)	(0.20)	(0.24)	0.16	0.06	0.11
Orange Juice	0.41*	0.07	0.11	0.21*	0.03	0.35**	0.06	0.64**	-0.22**	-0.12	-0.23**
	(0.23)	(0.12)	(0.15)	(0.11)	(0.11)	(0.10)	(0.15)	(0.19)	0.10	0.13	0.12
Sugar	0.14**	0.40**	0.56**	0.30**	0.00	0.24	0.13	0.43**	0.02	0.04	0.12
	(0.03)	(0.14)	(0.13)	(0.15)	(0.13)	(0.13)	(0.17)	(0.20)	0.11	0.09	0.10
θs sign at 10%	45.00	45.00	58.82	50.00	15.00	40.00	20.00	50.00	15.00	5.88	25.00
Selection	-	-	Basis n3	Basis n4	-	-	-	Open Interest	-	-	-
Panel B: Tilting coefficients (ϕ)											
	-2.27**	-2.41**	-1.52	-1.13	2.18**	2.30**	-0.16	0.12	1.11	1.83**	2.12**
	(1.00)	(0.85)	(1.00)	(0.99)	(0.68)	(0.64)	(0.91)	(0.90)	(0.72)	(0.82)	(0.85)
Selection	Basis n1	Basis n2	-	-	Momentum	Spot Change	-	-	-	Basis-Mom n3	Basis-Mom n4

Table V
Risk factor exposure

The table presents results from pricing out-of-sample net returns of multivariate commodity strategies and the long-short factor benchmarks using the three-factor model of Bakshi, Gao, and Rossi (2017) and additionally including the basis-momentum factor of Boons and Prado (2019). Panel A prices multivariate parametric policies and Panel B refers to equally weighted factor benchmarks. For each model, we report coefficient estimates together with standard errors in parentheses and the adjusted R^2. * and ** indicate significance at the 10 % or 5 % levels, respectively. The out-of-sample evaluation period is January 2002 to August 2015.

Portfolio	α	β_{AVG}	β_{CARRY}	β_{MOM}	β_{BM}	R^2_{adj}
Panel A: Parametric portfolio policies						
Timing	0.0107**	0.0360	-0.1352*	-0.1780**		7.51
	(0.0035)	(0.1084)	(0.0725)	(0.0563)		
	0.0106**	0.0341	-0.1365*	-0.1776**	0.0138	6.95
	(0.0033)	(0.1079)	(0.0731)	(0.0553)	(0.0638)	
Tilting	0.0022	0.0568	0.3071**	0.3830**		35.70
	(0.0030)	(0.0920)	(0.0599)	(0.0578)		
	0.0005	0.0164	0.2786**	0.3915**	0.2989**	46.73
	(0.0026)	(0.0989)	(0.0486)	(0.0533)	(0.0527)	
Combo 50-50	0.0105**	0.0526	0.1072*	0.1531**		6.60
	(0.0033)	(0.0791)	(0.0597)	(0.0531)		
	0.0093**	0.0227	0.0861	0.1594**	0.2220**	15.21
	(0.0028)	(0.0767)	(0.0529)	(0.0468)	(0.0551)	
Combo BL	0.0106**	0.0869	0.1008	0.1491**		5.07
	(0.0038)	(0.0954)	(0.0776)	(0.0591)		
	0.0092**	0.0527	0.0767	0.1562**	0.2530**	14.02
	(0.0033)	(0.0891)	(0.0705)	(0.0543)	(0.0647)	
Panel B: L/S Factor benchmark						
ALL Variables	0.0000	0.1610**	0.4411**	0.3677**		56.86
	(0.0027)	(0.0570)	(0.0379)	(0.0538)		
	-0.0017	0.1202**	0.4122**	0.3762**	0.3027**	69.48
	(0.0021)	(0.0437)	(0.0358)	(0.0366)	(0.0463)	
PPP Variables	0.0003	0.1096**	0.4993**	0.3590**		62.30
	(0.0026)	(0.0501)	(0.0343)	(0.0443)		
	-0.0017	0.1202**	0.4122**	0.3762**	0.3027**	69.48
	(0.0021)	(0.0437)	(0.0358)	(0.0366)	(0.0463)	

Table VI
Multivariate models under Black-Litterman

The table gives summary statistics for out-of-sample net returns of multivariate commodity strategies, where the performance of each strategy is evaluated with and without rescaling of weights as well as the Black-Litterman smoothed views. Panel A summarizes results of multivariate timing policies, Panel B refers to multivariate tilting policies, Panels C and D to the 50-50 and Black-Litterman combinations and Panel E includes the equally weighted factor benchmark portfolios. The table reports annualized mean, standard deviation and Sharpe Ratio, as well as skewness, kurtosis, minimum, maximum, and maximum drawdown of monthly returns. All values are expressed in percentage points and the out-of-sample evaluation period is January 2002 to August 2015.

Portfolio	Mean	Std.Dev	SR	Skew	Kurt	Min	Max	maxDD
Panel A: Timing								
w/o rescaling	19.65	78.39	0.25	-0.15	3.45	-62.50	72.46	-95.68
with rescaling	8.67	15.51	0.56	-0.22	3.18	-14.18	11.34	-32.12
with Black-Litterman	13.33	23.41	0.57	0.43	3.85	-17.65	23.11	-29.57
Panel B: Tilting								
w/o rescaling	9.99	20.36	0.49	0.62	4.58	-13.27	24.54	-36.97
with rescaling	9.57	17.45	0.55	0.61	4.06	-11.02	20.10	-29.99
with Black-Litterman	7.99	12.90	0.62	0.56	4.39	-9.45	15.45	-17.36
Panel C: Combo 50-50								
w/o rescaling	27.32	39.23	0.70	-0.05	3.31	-26.87	36.12	-54.52
with rescaling	15.77	14.45	1.09	0.00	3.53	-9.51	15.18	-19.59
with Black-Litterman	12.21	12.94	0.94	0.64	4.09	-8.91	14.23	-18.35
Panel D: Combo BL								
w/o rescaling	18.31	27.76	0.66	-0.04	3.43	-20.09	24.44	-45.66
with rescaling	15.63	16.16	0.97	-0.22	3.89	-13.74	15.78	-26.78
with Black-Litterman	7.20	9.14	0.79	0.72	4.95	-6.93	10.83	-13.06
Panel E: L/S Factor benchmark								
ALL Variables	9.25	16.65	0.56	0.47	3.66	-9.58	17.25	-21.42
PPP Variables	10.07	16.63	0.61	0.41	3.38	-9.53	16.61	-21.12

Figure 1. Contemporaneous commodity return correlations

The figure shows the correlation matrix of individual commodity futures excess returns. The time period is January 1987 to August 2015.

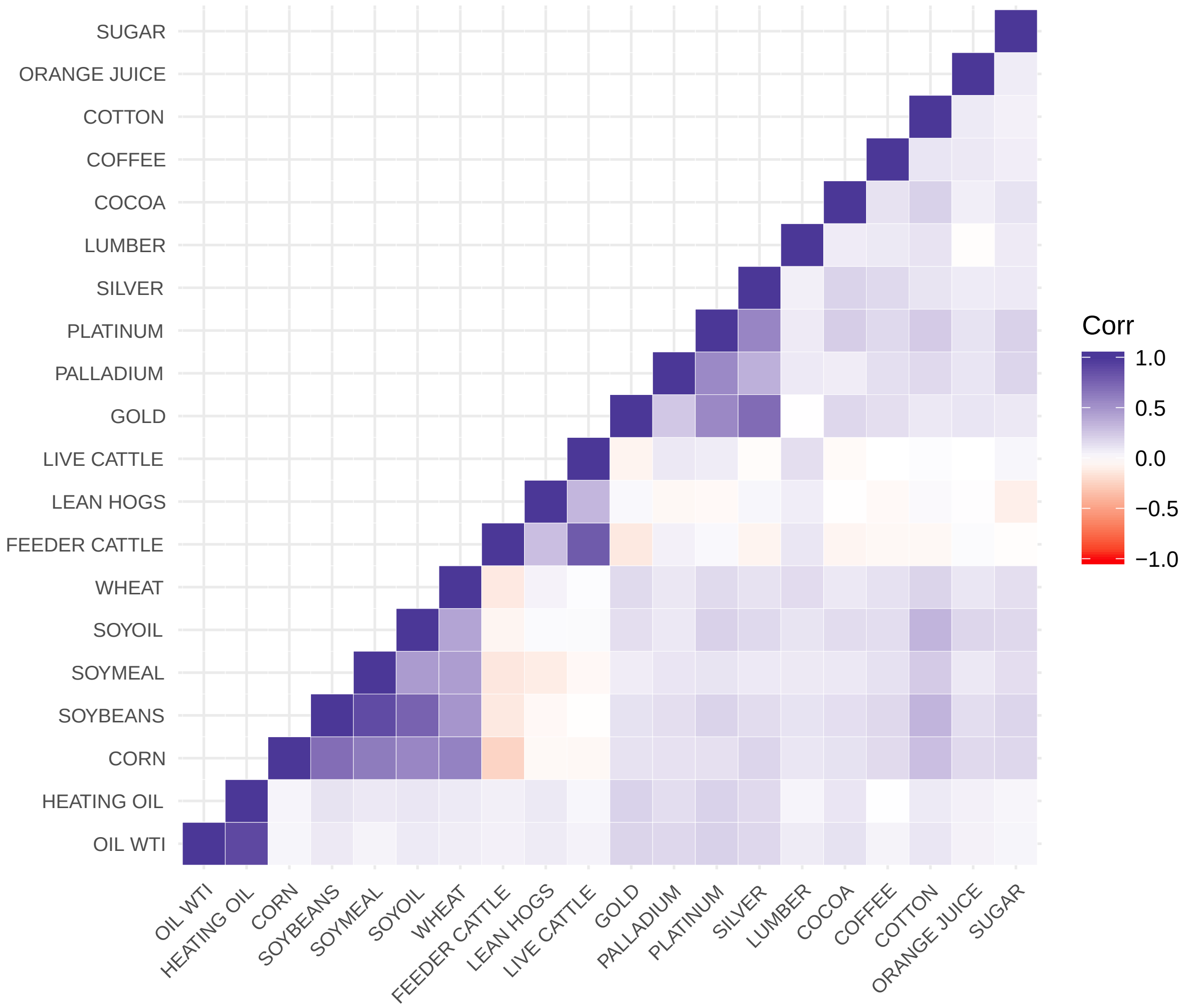


Figure 2. Performance of multivariate commodity strategies

The figure shows the cumulative out-of-sample net performance of different multivariate commodity strategies over the time period January 2002 to August 2015. The green line refers to the multivariate tilting policy, the blue line to the timing policy, the (dark) red to the (Black-Litterman) 50-50 combined strategy, the orange line to the equal-weighted factor benchmark and the grey line documents the performance of the S&P-GSCI.

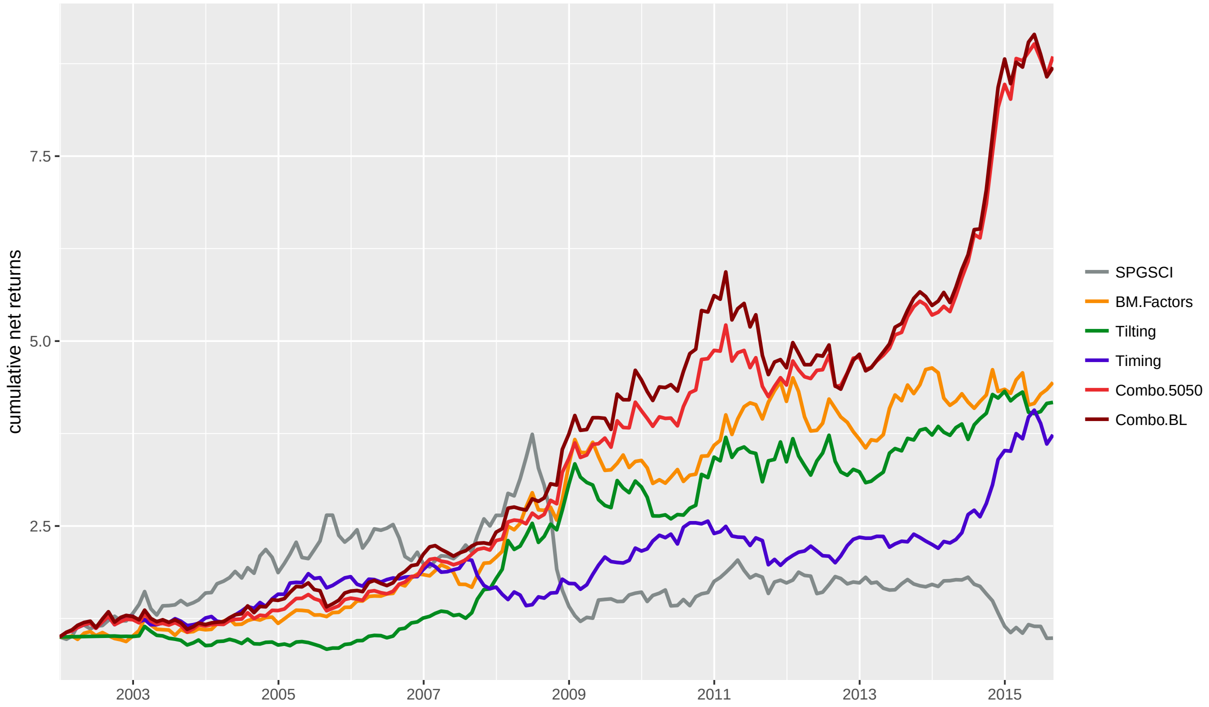


Figure 3. Parametric portfolio weights

The figure shows the portfolio weights of the 50-50 combined strategy, split into the timing (blue) and tilting (red) view, for each commodity. The time period is January 2002 to August 2015.

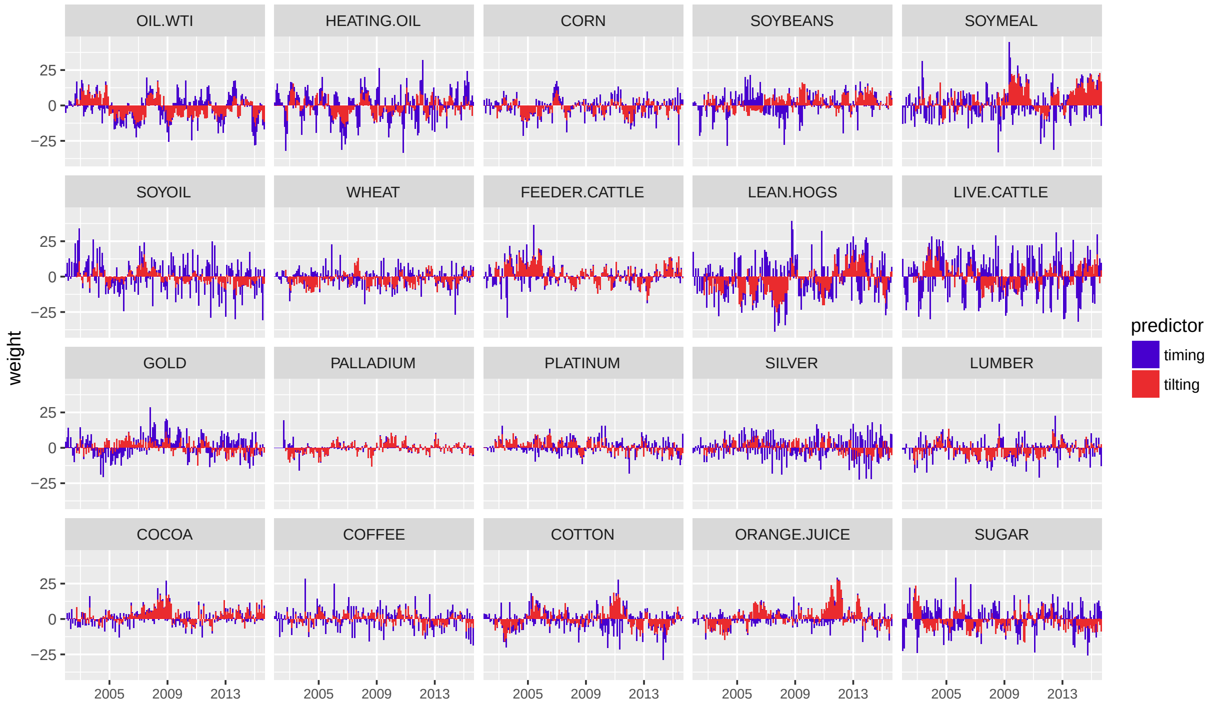
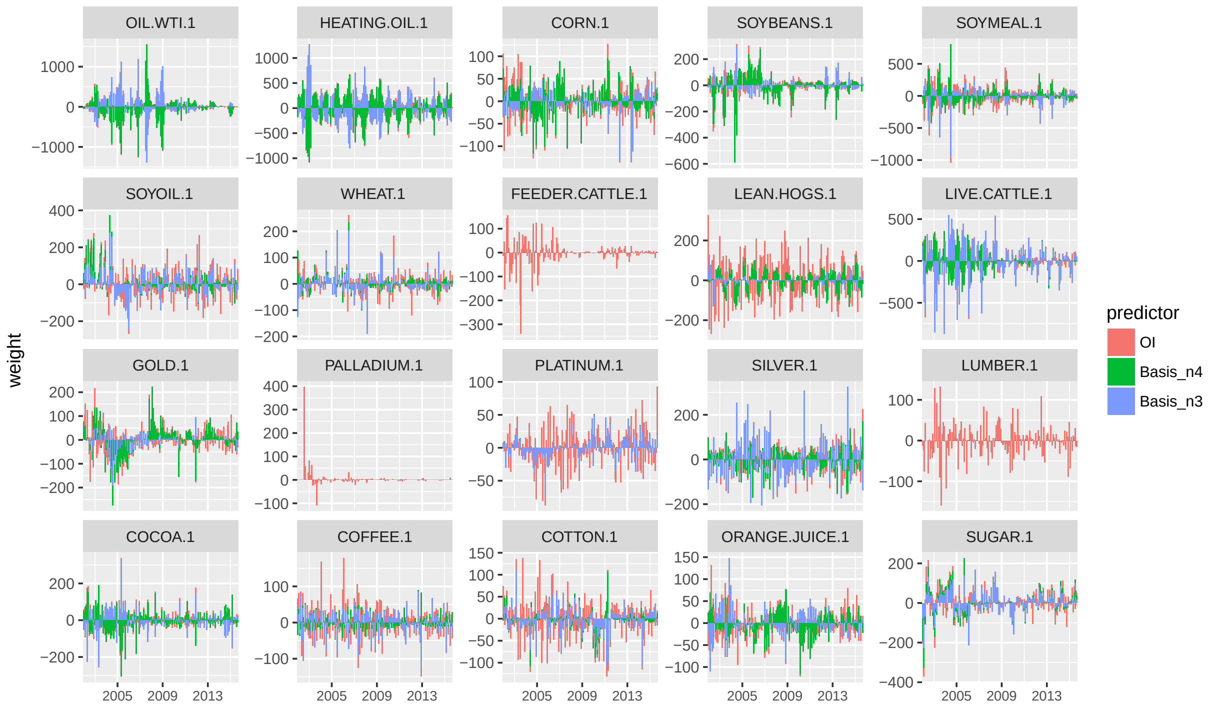


Figure 4. Parametric portfolio weights per state variable

The figure shows the multivariate timing (Panel A) and tilting (Panel B) weights, split up by individual state variables, for each commodity. The time period is January 2002 to August 2015.

Timing



Tilting