PERFORMANCE ATTRIBUTION AS A PORTFOLIO MANAGEMENT TOOL

The Barclays Capital Hybrid Performance Attribution® Model

Performance measurement and attribution is an important function of portfolio management. It helps bring clarity to the sources of portfolio risk and performance and identify the contributions of individual decision-makers. Performance attribution algorithms typically follow either a sector-based allocation or a factor-based allocation methodology, but neither is sufficient to cope with the complexity of modern portfolio management. In this paper, we introduce a model that blends the two and has the flexibility to adapt to the particular management structure of most fixed-income and equity portfolios. The Barclays Capital Hybrid Performance Attribution® (HPA) Model, available through POINT® — Barclays Capital's cross-asset portfolio management system – is a multi-currency attribution model with daily outperformance calculation and compounding, covering the majority of asset classes including fixed-income and equity cash instruments; interest rate, currency, credit and equity derivatives; and ETFs and funds of funds. In this paper, we present a generic hybrid performance attribution framework and a detailed description of its implementation and capabilities in POINT®.[1]

Anthony Lazanas
212-526-3127
Anthony.Lazanas@barcap.com

Chris Sturhahn
+1 212 526 8987
chris.sturhahn@barcap.com

Pamela Zhong
+44 (0)20 313 47577
yanqiu.zhong@barcap.com

www.barcap.com

[1] The authors owe gratitude to a plethora of people that have made invaluable contributions to the conceptualization, formulation, development and application of this model over the years: Lev Dynkin for introducing us to the idea of hybrid attribution; Jonathan Carmel for producing the initial specifications; Jay Hyman for insightful ideas and comments; Bruce Moskowitz and Eric Wilson for turning the model into a useful tool and in the process contributing numerous improvements; Attakrit Asvanunt, whose help putting this paper together was instrumental; everyone in the POINT® organisation, particularly the Development, Marketing and Modeling teams, who spent innumerable hours implementing and testing the model and helped us steer it towards robustness and practicality; and, of course, our clients, who have adopted the model and are continuously giving us valuable feedback and ideas.

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Introduction

Managing a global diversified portfolio is a complex task that comprises adopting medium- and long-term views in different markets and implementing them with daily actions. These functions interact with each other in a way that makes it very difficult to quantify the contribution of each decision-maker to the performance of the portfolio. Performance attribution is a mathematical framework that attempts to split the total outperformance of a portfolio versus a benchmark into the contributions of individual decisions.

Complex portfolios are typically managed by breaking down the investment process into a sequence of decisions made by different managers, each of whom specializes in a specific market or type of security. Traditionally, this process follows product and sector lines. Risk-factor-based portfolio management became popular with the advent of modern portfolio risk management and the realization that certain risks are common for the entire fixed-income asset class. FX and interest rates are examples of risks that are now commonly being managed separately for the entire portfolio while product/sector experts focus on managing excess returns.

The total outperformance of a portfolio depends on the combined outcome of the decisions of all portfolio agents, as well as the market performances. Often, the effects of different agents' decisions depend on each other in a way that makes it difficult to disentangle them. A successful performance attribution algorithm will do so in a way that satisfies three important requirements:

- *Additivity:* The contribution of two or more agents combined is equal to the sum of the contributions from those agents.
- *Completeness:* The sum of all outperformance contributions is equal to the total portfolio outperformance.
- *Fairness:* The allocation of outperformance to different interacting agents is performed in a way that is perceived to be fair by all agents.

These requirements are important because performance attribution is often used to measure the skill level and determine the compensation of portfolio managers. The fair decomposition of portfolio outperformance must take into account the management structure of the portfolio.

The Barclays Capital HPA model provides the flexibility to match the attribution algorithm to the specific management structure of a portfolio. In particular, it allows users to specify which risk factors are managed portfolio-wide and then performs top-down performance allocation on excess returns. Explanation of performance is based on analytically calculated exposures (option-adjusted duration (OAD), partial (key-rate) durations, vega, spread duration, etc.) to risk factors that characterize each component of return. Exposures are obtained using the Barclays Capital security pricing models.

An additional complexity of performance attribution comes from dynamic management and market timing. Allocations to risk factors may change frequently because of trading or market moves, and the attribution algorithm must have a sampling frequency that is high enough to capture this effect. The POINT® implementation employs daily frequency which is sufficient for most portfolios. Daily results are compounded over the attribution period to produce the final outperformance breakdown.

Daily compounding generates the requirement for high-quality daily analytics for all securities in a portfolio and its benchmark. Due to the detailed security-level calculations, it also acts as a magnifying lens for any underlying problems with the analytics. Inconsistent prices, indicatives and analytics lead to large residuals that can be used to uncover and correct the problems. To ease the data requirements, the algorithm is equipped with the ability to cope with situations in which analytics are missing or presumed bad and even handle a reasonable amount of missing security prices.

In Section 1, we go over a practical example of the use of a flexible performance attribution platform for a multi-currency portfolio and illustrate how performance attribution can be a very useful portfolio management tool.

In Section 2, we discuss how an analytics-based performance attribution algorithm can highlight bad data and, therefore, be used as an efficient data quality tool.

Section 3 describes the mathematical concepts of performance attribution and outlines the many different possible ways the algorithm can be applied. This section concludes with a generic algorithm that is the basis of the HPA framework in POINT®.

Sections 4 and 5 describe in details the implementation of the HPA model in POINT®, including the handling of multi-currency attribution, FX allocation and hedging, local currency return splits and the various local currency attribution models currently supported.

Section 6 details the handling of derivatives, including interest rate, currency and credit derivatives.

In Section 7, we argue that a practical attribution system must deal with several special situations that arise in portfolio management and discuss how the HPA implementation in POINT® handles several issues such as transactions and trading P&L, missing or bad prices and analytics, as well as daily compounding.

1. Performance Attribution as a Portfolio Management Tool

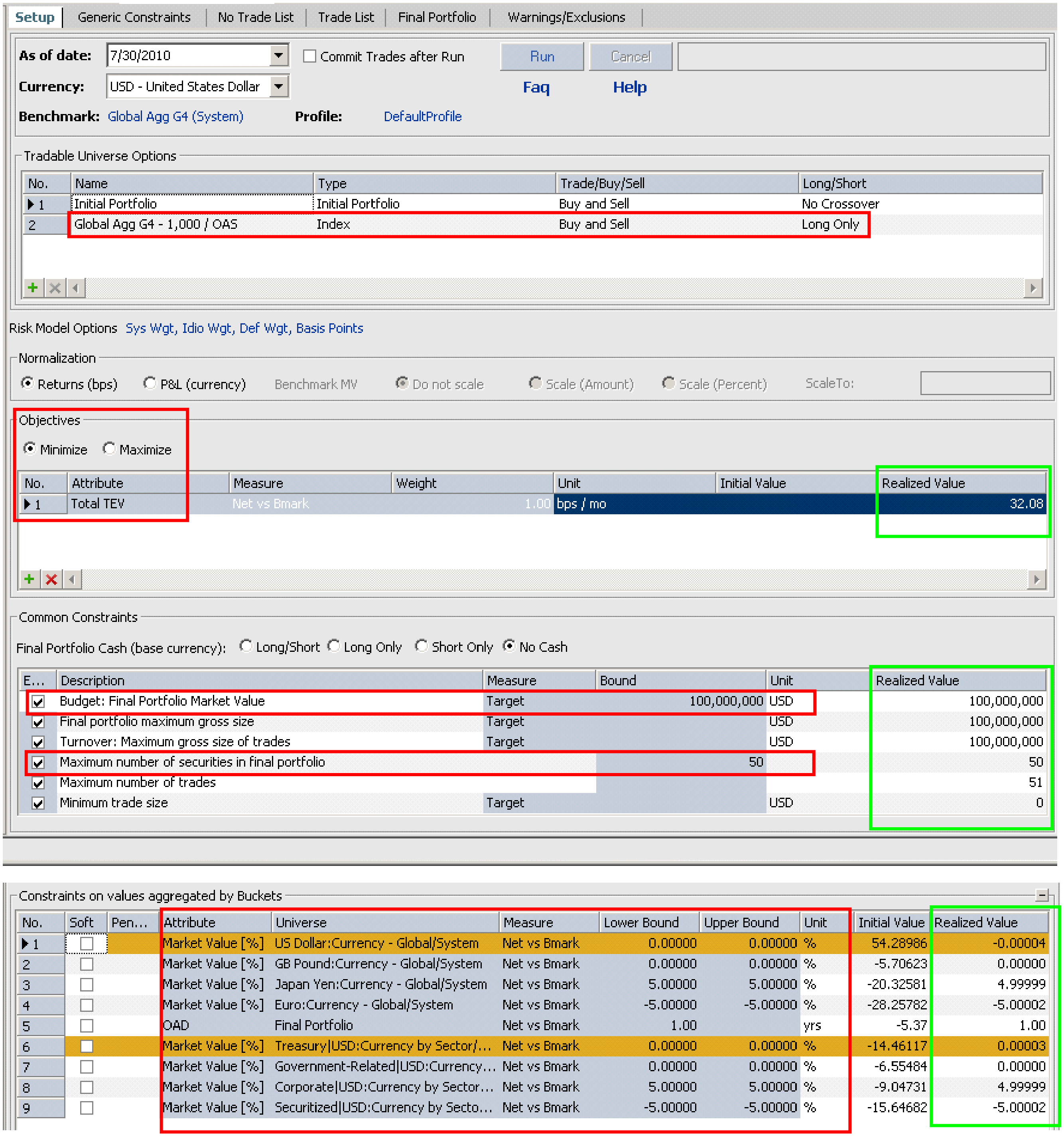
Detailed and flexible performance attribution can bring clarity to the process of portfolio management by systematically exposing sources of risk and performance. By linking ex-post outperformance contributions to ex-ante views and management decisions, a manager can identify unanticipated sources of risk and return and adjust the management process accordingly.

To illustrate this point and provide an overview of the practical use of the HPA model, let us consider the following example. A portfolio manager benchmarked to the Barclays Capital Global Aggregate G4 Index[2] has the latitude to implement certain market views in order to enhance the portfolio returns, as long as the tracking error volatility of the portfolio relative to its benchmark stays below 40bp per month. Let us assume that the size of the portfolio is fairly small, say $100,000,000, so the manager has decided to have no more than 50 bonds in it.

At the end of July 2010, the manager is bullish on the yen, negative on the euro, and neutral on the dollar and the pound. She believes that global rates are falling. She has no sector views except in the US markets, where she likes corporates but dislikes the securitised sector. She wants to implement these views while making sure that the tracking error volatility of the resulting portfolio relative to the benchmark is less than the 40bp per month risk budget.

[2] A subset of the Barclays Capital Global Aggregate Index that invests in bonds denominated only in the USD, EUR, GBP and JPY.

Figure 1: Portfolio Construction Problem Setup



Setup | Generic Constraints | No Trade List | Trade List | Final Portfolio | Warnings/Exclusions

As of date: 7/30/2010 Commit Trades after Run Run Cancel

Currency: USD - United States Dollar Faq Help

Benchmark: Global Agg G4 (System) **Profile:** DefaultProfile

Tradable Universe Options

No.	Name	Type	Trade/Buy/Sell	Long/Short
1	Initial Portfolio	Initial Portfolio	Buy and Sell	No Crossover
2	Global Agg G4 - 1,000 / OAS	Index	Buy and Sell	Long Only

Risk Model Options Sys Wgt, Idio Wgt, Def Wgt, Basis Points

Normalization

Returns (bps) P&L (currency) Benchmark MV Do not scale Scale (Amount) Scale (Percent) ScaleTo:

Objectives

Minimize Maximize

No.	Attribute	Measure	Weight	Unit	Initial Value	Realized Value
1	Total TEV	Net vs Bmark	1.00	bps / mo		32.08

Common Constraints

Final Portfolio Cash (base currency): Long/Short Long Only Short Only No Cash

E...	Description	Measure	Bound	Unit	Realized Value
☑	Budget: Final Portfolio Market Value	Target	100,000,000	USD	100,000,000
☑	Final portfolio maximum gross size	Target		USD	100,000,000
☑	Turnover: Maximum gross size of trades	Target		USD	100,000,000
☑	Maximum number of securities in final portfolio		50		50
☑	Maximum number of trades				51
☑	Minimum trade size	Target		USD	0

Constraints on values aggregated by Buckets

No.	Soft	Pen...	Attribute	Universe	Measure	Lower Bound	Upper Bound	Unit	Initial Value	Realized Value
1	☐		Market Value [%]	US Dollar:Currency - Global/System	Net vs Bmark	0.00000	0.00000	%	54.28986	-0.00004
2	☐		Market Value [%]	GB Pound:Currency - Global/System	Net vs Bmark	0.00000	0.00000	%	-5.70623	0.00000
3	☐		Market Value [%]	Japan Yen:Currency - Global/System	Net vs Bmark	5.00000	5.00000	%	-20.32581	4.99999
4	☐		Market Value [%]	Euro:Currency - Global/System	Net vs Bmark	-5.00000	-5.00000	%	-28.25782	-5.00002
5	☐		OAD	Final Portfolio	Net vs Bmark	1.00		yrs	-5.37	1.00
6	☐		Market Value [%]	Treasury|USD:Currency by Sector/...	Net vs Bmark	0.00000	0.00000	%	-14.46117	0.00003
7	☐		Market Value [%]	Government-Related|USD:Currency...	Net vs Bmark	0.00000	0.00000	%	-6.55484	0.00000
8	☐		Market Value [%]	Corporate|USD:Currency by Sector...	Net vs Bmark	5.00000	5.00000	%	-9.04731	4.99999
9	☐		Market Value [%]	Securitized|USD:Currency by Secto...	Net vs Bmark	-5.00000	-5.00000	%	-15.64682	-5.00002

Source: POINT®

We created such a portfolio using the Portfolio Optimiser[3] in POINT®. To enhance liquidity, we restricted the universe of securities to be used in the portfolio to those with an amount outstanding of at least $1bn and with an option-adjusted spread (OAS) to the risk free curve between -50 and +400. We asked the optimiser to select securities from this universe attempting to minimise the tracking error volatility of the portfolio versus the benchmark while abiding by constraints that express the manager's views. The set-up of the portfolio construction problem is shown in Figure 1. The optimiser is asked to select securities from the custom index "Global Agg G4 – 1,000/OAS," which has been filtered to contain only bonds fulfilling the liquidity constraints described above, and construct a portfolio with a market value of $100,000,000 holding a maximum of 50 bonds.

The objective is to minimise the tracking error volatility of the portfolio relative to the Barclays Capital Global Agg G4 index while having 1) no net exposure to the dollar or the pound, a 5% overweight to the yen, and a 5% underweight to the euro; 2) an overweight to global OAD of at least 1 year; and 3) in the US, no net exposure to the Treasury and government-related sectors, a 5% overweight to corporates, and a 5% underweight to securitised.

The optimiser indeed creates a portfolio of 50 bonds that abides by all constraints (within reasonable tolerance bounds[4]) and has an estimated tracking error volatility of 32.08bp per month. The manager implements the particular portfolio on July 30, 2010, and represents it in POINT® under the name "REPL: GlAgg G4 50." The portfolio is held without further transactions during August, at the end of which the HPA module is run and generates a multi-page report with the breakdown of the portfolio performance relative to its benchmark index.

The HPA module in POINT® gives users the flexibility to customise the attribution algorithm so that it is consistent with the portfolio management structure.[5] For our initial analysis, we will assume that the report is generated with default system settings.

Ultimately, performance attribution is the decomposition of a single number, the total outperformance of the portfolio over the benchmark, into the sum of the contributions of various market movements and managers' decisions. The output of the HPA model is hierarchical in that it first provides a summary decomposition and subsequently provides recursive decomposition and explanatory analysis for each number in the summary.

Figure 2 displays the Global Portfolio Summary report of the HPA model. The Global Outperformance panel (Box A) registers a portfolio outperformance of +38.7bp. In the Global Outperformance Details panel (Box B), we see a first decomposition of the global outperformance into three components: FX Allocation and Hedging (+28.3bp), Local Market Allocation (-8.7bp) and Local Management (+19.1bp). The latter is further decomposed in the Local Management Details" panel (Box C) into contributions from Yield Curve exposures (+20.6bp), Asset Allocation decisions (0.0bp), Security Selection (-1.7bp) and three more components that, while important for practical performance attribution (see Section 7), are not significant drivers of return in this particular example.

[3] For more details on how to use the POINT® Portfolio Optimiser, see Kumar 2009 and Kumar and Lazanas 2009.
[4] The optimiser uses a heuristic search algorithm to deal with integer constraints such us the maximum number of bonds in a portfolio. The algorithm will return a solution even if it does not satisfy all constraints. Constraints that are breached by more than a tolerance bound are highlighted such as constraints #1 and #6 in the example above.
[5] For a detailed description of the performance attribution set of options and the report package available in POINT®, please see Asvanunt and Zhong 2010.

Figure 2: Global Portfolio Summary Report

POINT® Hybrid Performance Attribution®

Global Portfolio Summary

Portfolio: REPL : GlAgg G4 50
Benchmark : Global Agg G4
Partition : Class 1 / System

Period: 7/30/2010 to 8/31/2010
Base Currency: USD
Curve Type: Treasury

A

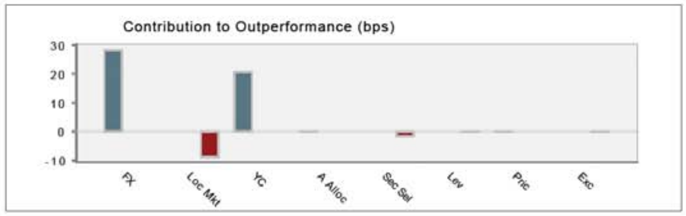
Global Outperformance	
Portfolio Return (bps)	188.7
Benchmark Return (bps)	150.0
Outperformance (bps)	**38.7**

Global Outperformance Details	
FX Allocation & Hedging	28.3
Local Market Allocation	-8.7
Local Management	19.1

B

C

Local Management Details	
Yield Curve	20.6
Asset Allocation	0.0
Security Selection	-1.7
Leverage	-0.0
Pricing Differences	0.1
Exclusions	-0.0



Global Outperformance Contribution	JPY	EUR	USD	GBP
Hedged O/W (%)	5.0	-4.9	-0.1	-0.0
Unhedged O/W (%)	5.0	-4.9	-0.1	-0.0
Total Outperformance	13.5	14.7	14.6	-4.2
FX Allocation & Hedging	16.2	12.1	0.0	0.0
Local Market Allocation	-5.0	-3.7	0.0	-0.0
Local Management	2.4	6.3	14.6	-4.2

Local Management Outperformance Contribution	USD	EUR	GBP	JPY
Total Local Management	14.6	6.3	-4.2	2.4
Asset Allocation	-2.5	1.5	1.1	-0.1
Security Selection	5.9	-13.3	6.2	-0.5
Common Factors/Others	11.2	18.1	-11.5	3.0

Yield Curve Outperformance Contribution	Avg	6mo	2y	5y	10y	20y	30y	Rest	Carry	Total
EUR (Port wt: 23.2%, Outperf. Contrib. 17.9bp)										
Dur o/w	1.1	-0.1	0.0	-0.6	2.0	0.3	-0.6			
Yld Chn	-50.2	0.6	-22.6	-43.1	-57.3	-64.7	-69.6			
Outperf	54.2	3.1	-1.3	4.0	14.1	3.5	-11.5	8.3	2.9	77.4
GBP (Port wt: 5.8%, Outperf. Contrib. -11.5bp)										
Dur o/w	-3.8	-0.0	0.6	-0.0	0.5	-2.1	-2.7			
Yld Chn	-44.9	-5.3	-22.1	-43.4	-46.7	-49.5	-44.9			
Outperf	-172.4	1.2	-14.7	-0.1	0.9	-10.1	0.1	0.9	-5.0	-199.2
USD (Port wt: 45.5%, Outperf. Contrib. 11.2bp)										
Dur o/w	0.5	0.0	0.1	-0.2	-0.5	1.4	-0.3			
Yld Chn	-31.6	1.3	-5.2	-26.1	-45.7	-48.7	-42.1			
Outperf	14.6	-0.5	-3.5	1.4	-6.6	23.8	-3.8	-0.6	-0.3	24.6

Best/Worst Buckets by Asset Allocation Contribution

Bucket name	Avg Weight (%)		Ret ex Common		Outperf	
	Port	Bench	Port	Bench	Alloc	Select
EUR : Corporate	10.2	4.9	18.4	-23.0	2.8	4.2
EUR : Treasury	10.2	15.6	-275.8	-122.4	1.4	-15.5
GBP : Corporate	2.9	1.3	182.8	47.7	0.5	3.9
GBP : Treasury	0.0	3.7	0.0	2.3	0.5	0.0
JPY : Corporate	4.7	1.1	-0.4	10.3	0.2	-0.5
USD : Securitized	10.3	15.2	-23.3	-19.0	-0.1	-0.5
JPY : Government-Related	0.0	2.1	0.0	12.9	-0.2	0.0
EUR : Government-Related	1.7	4.3	-119.4	-6.3	-1.1	-1.9
EUR : Securitized	1.0	3.3	11.3	26.3	-1.6	-0.2
USD : Corporate	14.0	9.0	-33.5	-70.2	-2.3	5.1

Best/Worst Instruments by Security Selection Contribution

Instrument	Bucket	Port wt (%)	Instr Ret	Bucket Ret	Outperf
TITIM 6.375 6/24/2	GBP : Corporate	2.9	182.8	47.7	3.9
ICO 4.5 3/7/2013	GBP : Government	2.9	101.7	21.2	2.3
GE 4.75 7/30/2014	EUR : Corporate	2.4	54.4	-23.0	1.9
MS 5.625 1/9/2012	USD : Corporate	1.4	57.7	-70.2	1.8
RABOBK 4.2 5/13/20	USD : Corporate	1.6	39.0	-70.2	1.7
POLAND 5.25 1/20/2	EUR : Government	1.7	-119.4	-6.3	-1.9
BTPS 5.0 3/1/2025	EUR : Treasury	1.9	-253.7	-122.4	-2.4
CMCSA 6.45 3/15/20	USD : Corporate	1.4	-280.0	-70.2	-2.9
BTPS 3.75 8/1/2021	EUR : Treasury	1.6	-433.4	-122.4	-4.8
BTPS 4.75 8/1/2023	EUR : Treasury	1.7	-420.6	-122.4	-5.0

Source: POINT®

1.1. Outperformance from Currency Exposures

The manager presumes that the positive contribution of FX exposures to outperformance must mean that her currency views were correct and wants to know more details. These can be found in the FX Allocation & Hedging report shown in Figure 3. There, it can be confirmed that the yen rallied and the euro fell versus the dollar, contributing +16.1 and +12.0bp of outperformance, respectively (Box B). The fact that the pound also fell generates no outperformance since the net exposure to the pound was designed to be zero, as confirmed in Box C. In the same box, we confirm that the mean net exposure to the yen was approximately +5% and to the pound approximately -5%, as designed. The mean exposure is calculated over the period of attribution (here, August) and may drift from the original allocations. This report allows users to understand the extent of such drift by also displaying the minimum and maximum net exposures over the period of attribution. In this particular case, the drift is negligible.

Figure 3: FX Outperformance Breakdown

POINT® Hybrid Performance Attribution®

FX Allocation & Hedging

Portfolio: REPL : GlAgg G4 50
Benchmark : Global Agg G4
Partition : Class 1 / System

Period: 7/30/2010 to 8/31/2010
Base Currency: USD
Curve Type: Treasury

Currency	FX	Currency Rates and Returns						Market Weight (%)							Outperformance (bps)		
								Average				Overweight					
		FX Rate Begin	FX Rate End	Avg. Depo. Rate (%)	FX + Depo. Return (bps)	Excess over Base Currency Depo.	Benchmark Local Market Return	Portfolio Core	Portfolio Hedges	Portfolio With Hedges	Benchmark With Hedges	Mean	Min	Max	Unhedged FX Allocation	Hedged FX Allocation	FX / Local Cross Term
USD	(base currency)	1.00000	1.00000	0.2	2.1	0.0	129.8	45.5	0.0	45.5	45.6	-0.1	-0.2	-0.0	0.0	0.0	0.0
EUR	EUR/USD	1.30280	1.27095	0.3	-242.1	-244.1	241.7	23.2	0.0	23.2	28.1	-4.9	-5.0	-4.9	12.0	12.0	0.2
JPY	JPY/USD	0.01154	0.01191	0.1	321.5	319.4	65.3	25.6	0.0	25.6	20.5	5.0	5.0	5.1	16.1	16.1	0.0
GBP	GBP/USD	1.56610	1.53690	0.6	-181.5	-183.5	445.5	5.8	0.0	5.8	5.8	-0.0	-0.1	0.0	-0.0	-0.0	0.1
Total								**100.0**	**0.0**	**100.0**	**100.0**				**28.1**	**28.1**	**0.2**

Source: POINT®

Like all attribution reports, the FX report provides many details to help users understand the sources of the reported outperformance contributions. For example, the +16.1bp of outperformance coming from the yen exposure can be explained by multiplying the excess FX return of the yen versus the dollar (+319.4bp in Box A) by the overweight (5%). Indeed, 5% x 319.4 = +16.0bp. The calculation is not exact since the HPA algorithm employs a complex model to compound the daily outperformance and its drivers over a longer period (see Section 7.1), but is sufficiently accurate when the net exposure does not fluctuate significantly over the attribution period, as in this example. Finally, the completeness of FX outperformance decomposition is ensured by accounting for the interaction effect between FX and local returns. In this case, it is only +0.2bp and is reported per currency in the last column of the report in Figure 3.

1.2. Outperformance from Allocation to Local Markets

The manager now turns her attention to the second global outperformance component, the Local Market Allocation, which had a negative contribution of -8.7bp. The Local Market Allocation report in Figure 4 is very similar to the FX Allocation & Hedging report. In this case, the driver of outperformance is the local return over the deposit rate (Box A).

Naturally, only the yen and pound markets contribute since the net exposure to the other two is (very close to) zero (Boxes B and C). The yen market overweight is not a good decision in this case since the local return over deposit of that market is only +64.2bp, much lower than the average benchmark local return over deposit of +164.1bp (Box A). Therefore, it generates underperformance that can be approximately calculated as (64.2 – 164.1) x 5% = -5.0bp, as reported in Box B. The euro market underweight is not good either, since the local market had a return of +239.2bp, which is higher than the benchmark average of +164.1bp, and it generates underperformance of (239.2 – 164.1) x -5% = -3.8bp. The actual contribution is -3.7bp, as reported in Box B and, together with the -5.0bp of the yen market contribution, fully explains the Local Markets Allocation outperformance component.

The exposures to the local markets might be thought as incidental to the manager's FX views and a result of not properly differentiating the views on FX versus the views on the local market performance. In this case, it might make sense to combine the two contributions and allocate 28.3 – 8.7 = +19.6bp of outperformance to the views on the currency markets.

Figure 4: Local Market Allocation Report

POINT® Hybrid Performance Attribution®

Local Market Allocation

Portfolio: REPL : GlAgg G4 50
Benchmark : Global Agg G4
Partition : Class 1 / System

Period: 7/30/2010 to 8/31/2010
Base Currency: USD
Curve Type: Treasury

A C

Currency Market	Local Market Returns (bps)				Market Weight (%)					Outperformance (bps)	
	Benchmark Local Market Returns (bps)			Portfolio Local	Average		Overweight				
	Total Return	Depo. Rate Return	Return over Deposit		Port	Bench	Mean	Min	Max	Local Market Allocation	Local Management
USD	129.8	2.1	127.8	161.6	45.5	45.6	-0.1	-0.2	-0.0	0.0	14.6
EUR	241.7	2.5	239.2	269.0	23.2	28.1	-4.9	-5.0	-4.9	-3.7	6.3
JPY	65.3	1.1	64.2	74.8	25.6	20.5	5.0	5.0	5.1	-5.0	2.4
GBP	445.5	5.1	440.5	371.5	5.8	5.8	-0.0	-0.1	0.0	-0.0	-4.2
Total	**166.0**	**2.2**	**164.1**	**176.3**	**100.0**	**100.0**				**-8.7**	**19.1**

B

Source: POINT®

1.3. Outperformance from Management of Local Markets

Let us now look deeper into the +19.1bp of Local Management outperformance. We have already seen a breakdown of that number in Box C of Figure 2 and know that the dominant contributor is yield curve exposures with +20.6bp. The Global Outperformance by Currency report shown in Figure 5 lists all components of outperformance by currency.

We can see that positive yield curve return came primarily from the dollar and euro markets, whereas the pound curve exposure resulted in a loss of -11.5bp. We also see that asset allocation within currencies was not a significant driver of outperformance, but that security selection decisions within each currency were significant, although the net result across all currencies was small at -1.7bp. In particular, the managers responsible for the dollar and pound portfolios must be rewarded because they chose securities that contributed +5.9bp and +6.2bp respectively, but the euro manager must be penalised, as her contribution was significantly negative at -13.3bp.

Figure 5: Breakdown of Local Outperformance per Currency

POINT® Hybrid Performance Attribution®

Global Outperformance by Currency

Portfolio: REPL : GlAgg G4 50
Benchmark : Global Agg G4
Partition : Class 1 / System

Period: 7/30/2010 to 8/31/2010
Base Currency: USD
Curve Type: Treasury

A

	USD	EUR	JPY	GBP								Total
Unhedged Portfolio Weight (%)	45.5	23.2	25.6	5.8								100.0
Outperformance	14.6	14.7	13.5	-4.2								38.7
FX Allocation & Hedging	**0.0**	**12.1**	**16.2**	**0.0**								**28.3**
Hedged FX Allocation	0.0	12.0	16.1	-0.0								28.1
FX / Local Cross Term	0.0	0.2	0.0	0.1								0.2
Hedging Effects												0.0
Local Market Allocation	**0.0**	**-3.7**	**-5.0**	**-0.0**								**-8.7**
Local Management	**14.6**	**6.3**	**2.4**	**-4.2**								**19.1**
Yield Curve	11.2	17.9	3.0	-11.5								20.6
Asset Allocation	-2.5	1.5	-0.1	1.1								0.0
Security Selection	5.9	-13.3	-0.5	6.2								-1.7
Leverage	-0.0	0.0	0.0	0.0								-0.0
Pricing Differences	0.0	0.1	-0.0	-0.0								0.1
Exclusions	-0.0	0.0	0.0	0.0								-0.0

B

Source: POINT®

1.3.1. Outperformance due to yield curve exposure

Focusing on the large yield curve outperformance contribution, the manager wants to understand how her directive to go long duration by at least 1 year was implemented in the four currencies. Figure 6 shows the yield curve outperformance breakdown for the US dollar. This is part of a set of local reports produced for each currency market. All numbers in the local reports are reported on an un-weighted basis, i.e. without accounting for the market value weight allocated to each currency market. Therefore, to calculate their contributions to total outperformance, one has to multiply them by the corresponding currency weight in the portfolio. In this example, the currency weights can be found in the Global Outperformance by Currency report in Figure 5 (Box A); they are 45.5% for the dollar, 23.2% for the euro, 25.6% for the yen and 5.8% for the pound market.

Figure 6: Yield Curve Outperformance Breakdown for USD

POINT® Hybrid Performance Attribution®

USD: Yield Curve (Treasury)

Portfolio: REPL : GlAgg G4 50
Benchmark : Global Agg G4
Partition : Class 1 / System

Period: 7/30/2010 to 8/31/2010
Base Currency: USD
Curve Type: Treasury

	Yield				Curve-Matched Market Weight (%)					Duration (yrs)					Outperformance (bps)		
	Level (%)		Change		Average		Overweight			Average		Overweight			Explained by Yield Curve		
	Port	Bench	Port	Bench	Port	Bench	Average	Min	Max	Port	Bench	Average	Min	Max	Carry	Change	Total
Parallel Shift																	
Average	1.311	1.245	-31.6	-31.6	99.7	100.0	-0.3	-0.4	0.0	4.66	4.18	0.48	0.45	0.50	-0.3	14.6	14.3
Key Rates & Cash																	
Cash	0.241	0.241	-1.3	-1.3	-18.2	-11.2	-7.0	-7.3	-6.5	0.00	0.00	0.00	0.00	0.00	1.0	0.0	1.0
6m	0.179	0.179	1.3	1.3	35.3	32.4	2.8	2.2	3.3	0.18	0.16	0.01	0.01	0.02	-0.3	-0.5	-0.8
2y	0.534	0.534	-5.2	-5.2	42.3	35.6	6.7	6.5	7.0	0.84	0.71	0.13	0.13	0.14	-0.5	-3.5	-4.0
5y	1.628	1.628	-26.1	-26.1	19.8	24.9	-5.1	-5.4	-4.9	0.96	1.21	-0.25	-0.26	-0.24	-0.4	1.4	1.0
10y	3.055	3.055	-45.7	-45.7	7.1	12.1	-5.0	-5.3	-4.7	0.64	1.09	-0.45	-0.48	-0.43	-1.0	-6.6	-7.6
20y	3.795	3.795	-48.7	-48.7	12.3	3.2	9.1	9.1	9.2	1.83	0.47	1.36	1.33	1.38	1.6	23.8	25.3
30y	4.000	4.000	-42.1	-42.1	1.1	2.9	-1.8	-1.8	-1.7	0.21	0.54	-0.33	-0.35	-0.31	-0.3	-3.8	-4.1
Rest of Curve & Convexity																	
															-0.0	-0.6	-0.6
Total Yield Curve Levels & Shifts																	
															-0.3	**24.9**	**24.6**

Source: POINT®

The yield curve report is very detailed, but the manager can quickly see that the US Treasury curve experienced bull-flattening with long rates falling in excess of 40bp (Box A). She can also see that the portfolio is generally long duration by about 0.5y (4.66y vs. 4.18y in Box B). This is consistent with the general desire to be long duration, and with rates generally falling, it creates outperformance. More precisely, the HPA report estimates the yield change at the average maturity point of the benchmark (which must be close to 6 years, as one can imply from the 4.18y duration of the benchmark) to be -31.6bp (Box A). Multiplied by the (negative of) the average duration overweight of 0.48y results in approximately +15.2bp of outperformance contribution. The HPA algorithm calculates that this number is actually +14.6bp (Box C top) by performing daily compounding and taking into account that duration exposure fluctuated between 0.45 and 0.50y during the month. The total US curve outperformance, accounting for the re-shaping of the yield curve and the contributions from carry, is calculated to be +24.6bp (Box C bottom). Most of the additional outperformance comes from a butterfly position on the curve, where an overweight of the 20y point of about 1.36y is partially offset with underweighting the 5, 10 and 30y points (Box D). Since the 20y point is the one with the largest drop (48.7bp), this butterfly position results in additional outperformance. HPA calculates and reports the excess contribution of each curve point separately.

The manager should get concerned that such a large component of total outperformance comes from an inadvertent position in the re-shaping of the US curve (about +4.7bp, which is estimated as the non-parallel US curve outperformance 24.6 – 14.3 = +10.3bp, times the dollar market weight of 45.5%) and should seek to control exposure to the various parts of yield curves more carefully. This can be achieved in the POINT® optimiser by adding constraints on the partial (key-rate) duration exposures of the various yield curves. Figure 7 displays the yield curve reports for the other three currencies.

The euro curve outperformance analysis is very similar to the dollar one. There is a duration overweight of about 1 year, as well as a butterfly position. Like the dollar curve, the euro curve also bull-flattens, resulting in (un-weighted) outperformance of +77.4bp and a contribution to total outperformance of +17.9bp (remember that the average portfolio weight of the euro market is 23.2%).

The yen has an even bigger duration overweight, about 2.7y, but the curve movement is not very dramatic, with average yields falling about 7bp. Significant curve re-shaping positions exist as well, with the 20y point being overweighted by more than 3y, while the 20y point is underweighted by about 1.7y. Nevertheless, the yen curve change is not sufficiently pronounced and generates only +11.8bp of (un-weighted) outperformance, contributing +3.0bp to the total outperformance (the average portfolio weight of the yen market is 25.6%).

The picture is completely different for the pound. Here, the portfolio has a significant duration underweight of about 3.8y, combined with a strong bull-flattening move of the pound yield curve (a yield change of -44.9bp at the long end). This underweight results in a dramatic (un-weighted) underperformance of -199.2bp. The small average portfolio weight of the Pound market (5.8%) prevents this from being a major drag in the portfolio performance but still its contribution to total outperformance is a significant -11.5bp.

At this point, it should be clear to the portfolio manager that understanding and managing the detailed exposure to the yield curve of each currency is of paramount importance for proper control of the portfolio performance. She may also decide that global curve management is so important that it has to be managed before any local allocation and management decisions are made. In this case, a slightly different attribution algorithm must be used, one that excludes yield curve outperformance from the Local Market Allocation and Local Management Components. HPA supports this style of portfolio management, where global curve exposure is managed centrally. A detailed discussion follows in Section 5.5.1.

Figure 7: Yield Curve Outperformance Breakdown for EUR, JPY and GBP

POINT® Hybrid Performance Attribution®

EUR: Yield Curve (Treasury)

Portfolio: REPL : GlAgg G4 50
Benchmark : Global Agg G4
Partition : Class 1 / System

Period: 7/30/2010 to 8/31/2010
Base Currency: USD
Curve Type: Treasury

	Yield				Curve-Matched Market Weight (%)					Duration (yrs)					Outperformance (bps)		
	Level (%)		Change		Average		Overweight			Average		Overweight			Explained by Yield Curve		
	Port	Bench	Port	Bench	Port	Bench	Average	Min	Max	Port	Bench	Average	Min	Max	Carry	Change	Total
Parallel Shift																	
Average	2.030	1.698	-50.2	-50.2	100.0	100.0	-0.0	-0.0	0.0	6.68	5.62	1.05	1.00	1.11	2.9	54.2	57.1
Key Rates & Cash																	
Cash	0.279	0.279	0.0	0.0	-13.1	-14.5	1.3	0.8	1.5	0.00	0.00	0.00	0.00	0.00	0.2	0.0	0.2
6m	0.381	0.381	0.6	0.6	4.0	16.2	-12.2	-12.8	-11.6	0.02	0.08	-0.06	-0.06	-0.06	1.8	3.1	5.0
2y	0.771	0.771	-22.6	-22.6	39.5	37.4	2.2	1.4	3.1	0.79	0.74	0.04	0.03	0.06	-1.4	-1.3	-2.6
5y	1.726	1.726	-43.1	-43.1	22.0	33.7	-11.7	-11.8	-11.6	1.07	1.64	-0.57	-0.57	-0.56	-1.2	4.0	2.8
10y	2.708	2.708	-57.3	-57.3	40.1	18.5	21.6	21.4	21.9	3.65	1.68	1.97	1.96	1.98	0.8	14.1	14.9
20y	3.377	3.377	-64.7	-64.7	7.5	5.8	1.7	1.3	2.1	1.16	0.89	0.26	0.21	0.31	-0.1	3.5	3.4
30y	3.324	3.324	-69.6	-69.6	0.0	3.0	-3.0	-3.0	-2.9	0.00	0.59	-0.59	-0.60	-0.58	-0.1	-11.5	-11.6
Rest of Curve & Convexity																	
															0.0	8.3	8.3
Total Yield Curve Levels & Shifts																	
															2.9	**74.4**	**77.4**

JPY: Yield Curve (Treasury)

Portfolio: REPL : GlAgg G4 50
Benchmark : Global Agg G4
Partition : Class 1 / System

Period: 7/30/2010 to 8/31/2010
Base Currency: USD
Curve Type: Treasury

	Yield				Curve-Matched Market Weight (%)					Duration (yrs)					Outperformance (bps)		
	Level (%)		Change		Average		Overweight			Average		Overweight			Explained by Yield Curve		
	Port	Bench	Port	Bench	Port	Bench	Average	Min	Max	Port	Bench	Average	Min	Max	Carry	Change	Total
Parallel Shift																	
Average	0.822	0.617	-7.0	-7.0	100.0	100.0	-0.0	-0.0	0.0	9.54	6.85	2.69	2.66	2.71	0.7	18.5	19.2
Key Rates & Cash																	
Cash	0.126	0.126	-1.2	-1.2	-6.6	-13.2	6.6	6.3	6.9	0.00	0.00	0.00	0.00	0.00	-0.5	0.0	-0.5
6m	0.124	0.124	-1.3	-1.3	-0.3	15.3	-15.6	-16.5	-14.8	-0.00	0.08	-0.08	-0.08	-0.07	1.2	0.4	1.6
2y	0.145	0.145	-3.3	-3.3	19.2	33.9	-14.7	-16.8	-12.8	0.38	0.68	-0.29	-0.34	-0.25	1.0	1.3	2.3
5y	0.344	0.344	-5.3	-5.3	51.8	19.7	32.1	31.1	33.3	2.58	0.98	1.59	1.55	1.65	-1.0	-3.0	-4.0
7y	0.616	0.616	-12.2	-12.2	2.6	14.9	-12.3	-12.5	-12.2	0.18	1.03	-0.85	-0.86	-0.85	-0.6	-4.6	-5.1
10y	1.034	1.034	-5.1	-5.1	-1.1	16.2	-17.3	-17.6	-17.0	-0.11	1.57	-1.67	-1.70	-1.65	-1.5	3.2	1.7
20y	1.776	1.776	-8.1	-8.1	27.5	10.0	17.5	17.3	17.7	4.85	1.77	3.09	3.03	3.13	1.2	2.7	3.9
30y	1.876	1.876	-7.6	-7.6	6.9	3.1	3.8	3.6	4.0	1.65	0.75	0.90	0.88	0.94	0.2	0.7	0.9
Rest of Curve & Convexity																	
															-0.0	-8.1	-8.1
Total Yield Curve Levels & Shifts																	
															0.7	**11.1**	**11.8**

GBP: Yield Curve (Treasury)

Portfolio: REPL : GlAgg G4 50
Benchmark : Global Agg G4
Partition : Class 1 / System

Period: 7/30/2010 to 8/31/2010
Base Currency: USD
Curve Type: Treasury

	Yield				Curve-Matched Market Weight (%)					Duration (yrs)					Outperformance (bps)		
	Level (%)		Change		Average		Overweight			Average		Overweight			Explained by Yield Curve		
	Port	Bench	Port	Bench	Port	Bench	Average	Min	Max	Port	Bench	Average	Min	Max	Carry	Change	Total
Parallel Shift																	
Average	1.990	2.833	-44.9	-44.9	100.0	100.0	0.0	-0.0	0.0	4.78	8.58	-3.81	-3.98	-3.69	-5.0	-172.4	-177.4
Key Rates & Cash																	
Cash	0.572	0.573	-0.4	-0.4	-11.7	-10.5	-1.2	-1.6	-0.3	0.00	0.00	0.00	0.00	0.00	-0.3	0.0	-0.3
6m	0.579	0.579	-5.3	-5.3	5.2	11.2	-5.9	-6.1	-5.8	0.03	0.06	-0.03	-0.03	-0.03	1.8	1.2	3.0
2y	0.936	0.936	-22.1	-22.1	53.3	21.1	32.2	32.0	32.5	1.06	0.42	0.64	0.63	0.64	-2.9	-14.7	-17.7
5y	2.205	2.205	-43.4	-43.4	24.7	24.8	-0.1	-0.4	0.2	1.19	1.19	-0.00	-0.02	0.01	1.2	-0.1	1.2
10y	3.417	3.417	-46.7	-46.7	28.4	23.0	5.4	4.8	5.8	2.51	2.03	0.48	0.42	0.51	1.9	0.9	2.7
20y	4.210	4.210	-49.5	-49.5	0.0	14.9	-14.9	-15.0	-14.9	0.00	2.14	-2.14	-2.20	-2.10	-1.1	-10.1	-11.2
30y	4.277	4.277	-44.9	-44.9	0.0	15.5	-15.5	-15.6	-15.4	0.00	2.74	-2.74	-2.82	-2.69	-0.6	0.1	-0.5
Rest of Curve & Convexity																	
															0.0	0.9	0.9
Total Yield Curve Levels & Shifts																	
															-5.0	**-194.2**	**-199.2**

Source: POINT®

1.3.2. Asset Allocation and Security Selection

The manager now shifts her attention to the remaining sources of outperformance. First, she wants to understand how her views on the performance of US assets panned out. From the "Global Outperformance by Currency" report of Figure 5, she already knows that USD asset allocation contributed -2.5bp to total outperformance. Figure 8 displays the USD Asset Allocation report, where we can see that most of the underperformance is caused from the overweight to corporates. Remember that all numbers in this page are un-weighted; i.e., they have to be scaled by the 45.5% portfolio weight of the US market allocation in order to convert them into total contribution numbers. In particular, the 5% overweight sought to US corporates during the portfolio construction is represented here as an overweight of 10.9% (30.9% - 19.8% = 11.1%, approximately equal to 5%/45.5%), and the -2.5bp of underperformance is reported as -5.5bp (approximately equal to -2.5bp / 45.5%). The corporate sector contributes -5.2bp, something that can be explained by the significant underperformance of the corporate sector in the benchmark (-70.2bp) relative to the benchmark itself (-22.0bp). The approximate calculation results in 11.1% x (-70.2 + 22.0) = -5.35bp, very close to the -5.2bp reported. The Treasury and government-related sectors have no contribution to asset allocation, as their weights are matched exactly between the portfolio and the benchmark, as designed. Finally, the securitized sector has minimal contribution to asset allocation, despite the significant underweight, because its performance (-19.0bp) is close to that of the benchmark (-22.0bp).

Figure 8: US Asset Allocation Breakdown

POINT® Hybrid Performance Attribution®

USD: Asset Allocation

Portfolio: REPL : GlAgg G4 50
Benchmark : Global Agg G4
Partition : Class 1 / System

Period: 7/30/2010 to 8/31/2010
Base Currency: USD
Curve Type: Treasury

Level 1: Class 1
Portfolio Market Weight: 100.0

Partition Bucket	Market Weight (%) Average		Ret ex Common Factors		Outperformance (bps)	
	Port	Bench	Port	Bench	Asset Allocation	Security Selection
Total	100.0	100.0	-14.6	-22.0	-5.5	13.0
Treasury	31.7	31.7	-4.7	-2.2	-0.0	-0.8
Government-Related	14.3	14.3	17.6	-7.7	-0.0	3.7
Corporate	30.7	19.8	-33.5	-70.2	-5.2	11.1
Securitized	22.8	33.4	-23.3	-19.0	-0.3	-1.0
Cash	0.5	0.8	0.0	0.0	-0.1	0.0

Source: POINT®

The same report also lists the contribution of each sector to security selection. We see that once again, the corporate sector is the dominant contributor with +11.1bp out of the total +13.0bp of (un-weighted) security selection in the dollar market. To further understand the sources of security selection outperformance, the manager studies the detailed USD Security Selection report displayed in Figure 9. This lists the chosen securities in each sector and their excess to curve return. We can see that the manager responsible for security picks in the US corporate sector has indeed picked mostly securities whose excess return beat the benchmark. One notable exception is the 6.45% 2037 Comcast bond (Box B), which represents 10.1% of the portfolio holdings in this sector (an overweight of 10% versus the benchmark) and which experienced -280.0bp of excess return (much worse than the benchmark, which lost only 70.2bp).

Figure 9: US Security Selection Breakdown

POINT® Hybrid Performance Attribution®

USD: Security Selection

Portfolio: REPL : GlAgg G4 50
Benchmark : Global Agg G4
Partition : Class 1 / System

Period: 7/30/2010 to 8/31/2010
Base Currency: USD
Curve Type: Treasury

Bucket/Issue	Issuer	Coupon	Maturity	Ticker	MV (%)		Ret ex Common Factors		Outperformance			
					Port	Bench	Port	Bench	Security Selection	Pricing	Total	
USD					100.0	100.0	-14.6	-22.0	13.0	0.0	13.0	
Treasury					31.7	31.7	-4.7	-2.2	-0.8	-0.0	-0.8	
912828MQ	US TREASURY NOTES	0.88	2/29/2012	US/T	54.9	0.9	-1.0	-1.0	0.2	0.0	0.2	
912810FF	US TREASURY BONDS	5.25	11/15/2028	US/T	19.5	0.2	-1.1	-1.1	0.1	-0.0	0.1	
912810FP	US TREASURY BONDS	5.38	2/15/2031	US/T	25.6	0.4	-15.5	-15.5	-1.1	-0.0	-1.1	
Bmark Securities	Not in Portfolio					98.5		-2.1	-0.0	0.0	-0.0	
Government-Related					14.3	14.3	17.6	-7.7	3.7	0.0	3.7	
46513EFF	ISRAEL STATE OF	5.50	9/18/2023	AID	14.3	0.1	68.0	68.0	1.5	0.0	1.5	
31359MRK	FEDERAL NATL MTG ASSN-GLOBAL	4.62	5/1/2013	FNMA	62.0	0.1	9.0	9.0	1.5	0.0	1.5	
31359MTP	FEDERAL NATL MTG ASSN-GLOBAL	5.12	1/2/2014	FNMA	22.0	0.1	7.5	7.5	0.5	-0.0	0.5	
XS0114288789	RUSSIA GLOBAL	7.50	3/31/2030	RUSSIA	1.7	0.9	43.3	43.3	0.1	0.0	0.1	A
Bmark Securities	Not in Portfolio					98.9		-8.2	0.1	0.0	0.1	
Corporate					30.7	19.8	-33.5	-70.2	11.1	0.0	11.1	
61746BCW	MORGAN STANLEY DEAN WITTER	5.62	1/9/2012	MS	10.0	0.1	57.7	57.7	3.9	0.0	3.9	
74977RBQ	RABOBANK	4.20	5/13/2014	RABOBK	11.5	0.1	39.0	39.0	3.8	0.0	3.8	
46625HHN	JP MORGAN CHASE & CO	4.65	6/1/2014	JPM	13.9	0.1	3.7	3.7	3.1	0.0	3.1	
90261XFY	UNION BANK OF SWITZERLAND	3.88	1/15/2015	UBS	10.0	0.1	22.2	22.2	2.8	-0.0	2.8	
025816BA	AMERICAN EXPRESS CO	7.25	5/20/2014	AXP	10.4	0.0	9.7	9.7	2.5	0.0	2.5	
96008YAB	WESTFIELD CAPITAL CORP	5.12	11/15/2014	WDCAU	5.8	0.0	59.9	59.9	2.3	0.0	2.3	
38141GEE	GOLDMAN SACHS GROUP-GLOBAL	5.35	1/15/2016	GS	9.7	0.1	-33.3	-33.3	1.1	-0.0	1.1	
25459HAG	DIRECTV HOLDINGS/FING	7.62	5/15/2016	DTV	9.5	0.1	-76.9	-76.9	-0.3	-0.0	-0.3	B
94973VAK	WELLPOINT INC-GLOBAL	5.25	1/15/2016	WLP	9.2	0.0	-133.8	-133.8	-1.8	-0.0	-1.8	
20030NAM	COMCAST CORPORATION	6.45	3/15/2037	CMCSA	10.1	0.1	-280.0	-280.0	-6.4	0.0	-6.4	
Bmark Securities	Not in Portfolio					99.4		-70.4	0.1	0.0	0.1	
Securitized					22.8	33.4	-23.3	-19.0	-1.0	0.0	-1.0	
FNA05003	FNMA Conventional Long T. 30yr	5.00	7/1/2032	FNMA	21.3	2.0	14.2	14.2	1.5	0.0	1.5	
FNA04403	FNMA Conventional Long T. 30yr	4.50	9/1/2032	FNMA	11.2	0.4	6.0	6.0	0.6	0.0	0.6	
FGB04403	FHLM Gold Guar Single F. 30yr	4.50	8/1/2032	FHLMC	10.3	0.2	-21.0	-21.0	-0.1	0.0	-0.1	
FNA04409	FNMA Conventional Long T. 30yr	4.50	2/1/2039	FNMA	19.7	5.8	-26.5	-26.5	-0.2	0.0	-0.2	
GNA04403	GNMA I Single Family 30yr	4.50	7/1/2032	GNMA	8.6	0.0	-50.7	-50.7	-0.6	0.0	-0.6	
GNF04403	GNMA I Single Family 15yr	4.50	2/1/2018	GNMA	29.0	0.0	-52.7	-52.7	-2.2	0.0	-2.2	
Bmark Securities	Not in Portfolio					91.5		-19.4	0.1	0.0	0.1	
Cash					0.5	0.8	0.0	0.0	0.0	0.0	0.0	
USD - Unsettled	CASH - U.S. Dollar - Unsettled			USD	52.3		0.0		0.0	0.0	0.0	
USD - Settled	CASH - U.S. Dollar - Settled			USD	47.7	62.0	0.0	0.0	0.0	0.0	0.0	
Bmark Securities	Not in Portfolio					38.0		0.0	0.0	0.0	0.0	

Source: POINT®

To estimate the contribution of the bond to the security selection outperformance coming from US corporates, one must take into account the weight of the corporate sector in the US portfolio, which is 30.7% (Box A), to get 30.7% x 10% x (-280.0 + 70.2) = +6.4bp. After scaling by the 45.5% weight of the US portfolio, we calculate the contribution of the particular Comcast bond to global outperformance as 45.5% x -6.4bp = -2.9bp. This is quite significant and highlights the importance of name selection in a portfolio with a relatively small number of positions.

The manager also takes a look at the details of security selection in the euro market, which, as reported in Figure 5, reduces the total outperformance by a significant 13.3bp. The report in Figure 10 lists the security selection outperformance as -57.4bp (which is equal to the -13.3bp de-scaled by the market weight of the euro market of 23.2%) and identifies Italian government bonds as the main culprits. The fact that, absent any specific direction otherwise, the euro government sector was constructed by mostly Italian government bonds, leading to significant underperformance because of sovereign credit deterioration in Italy, indicates that the manager should attempt to take control of country exposure within the euro sector.

Figure 10: EUR Security Selection Breakdown

POINT® Hybrid Performance Attribution®

EUR: Security Selection

Portfolio: REPL : GlAgg G4 50
Benchmark : Global Agg G4
Partition : Class 1 / System

Period: 7/30/2010 to 8/31/2010
Base Currency: USD
Curve Type: Treasury

Bucket/Issue	Issuer	Coupon	Maturity	Ticker	MV (%)		Ret ex Common Factors		Outperformance		
					Port	Bench	Port	Bench	Security Selection	Pricing	Total
EUR					100.0	100.0	-121.0	-69.7	-57.4	0.6	-56.8
Treasury					44.0	55.4	-275.8	-122.4	-66.6	0.6	-66.1
IT0003644769	ITALY (REPUBLIC OF)	4.50	2/1/2020	BTPS	4.7	0.5	-270.5	-270.5	-2.7	0.0	-2.7
BE0000308172	BELGIUM (KINGDOM OF)	4.00	3/28/2022	BGB	38.5	0.3	-149.7	-149.7	-3.9	-0.0	-3.9
IT0004594930	ITALY (REPUBLIC OF)	4.00	9/1/2020	BTPS	5.6	0.5	-321.9	-321.9	-4.5	0.0	-4.5
IT0004513641	ITALY (REPUBLIC OF)	5.00	3/1/2025	BTPS	18.3	0.5	-253.7	-253.7	-10.5	0.6	-9.9
IT0004009673	ITALY (REPUBLIC OF)	3.75	8/1/2021	BTPS	15.8	0.6	-433.4	-433.4	-20.7	-0.0	-20.7
IT0004356843	ITALY (REPUBLIC OF)	4.75	8/1/2023	BTPS	17.1	0.5	-420.6	-420.6	-21.6	0.0	-21.6
Bmark Securities	Not in Portfolio					97.0		-116.2	-2.7	0.0	-2.7
Government-Related					7.5	15.3	-119.4	-6.3	-8.2	0.0	-8.2
XS0479333311	POLAND (REPUBLIC OF)	5.25	1/20/2025	POLAND	100.0	0.3	-119.4	-119.4	-8.2	0.0	-8.2
Bmark Securities	Not in Portfolio					99.7		-6.0	-0.0	0.0	-0.0
Corporate					43.8	17.6	18.4	-23.0	18.2	-0.0	18.2
XS0441800579	GE CAPITAL CORP	4.75	7/30/2014	GE	24.0	0.2	54.4	54.4	8.1	0.0	8.1
XS0430052869	ROYAL BANK OF SCOTLAND	5.75	5/21/2014	RBS	6.8	0.1	108.4	108.4	3.9	-0.0	3.9
XS0284891297	TELEFONICA EMISONES SAU	4.67	2/7/2014	TELEFO	11.8	0.1	49.1	49.1	3.8	-0.0	3.8
XS0415108892	IBERDROLA	4.88	3/4/2014	IBESM	22.7	0.1	1.1	1.1	2.3	0.0	2.3
XS0275431111	IMPERIAL TOBACCO FIN PLC	4.38	11/22/2013	IMTLN	16.7	0.1	0.9	0.9	1.7	-0.0	1.7
XS0302633168	MERRILL LYNCH & CO INC	4.88	5/30/2014	BAC	13.2	0.1	5.5	5.5	1.6	0.0	1.6
XS0185490934	CITIGROUP INC	4.75	2/10/2014	C	4.8	0.1	-181.4	-181.4	-3.4	-0.0	-3.4
Bmark Securities	Not in Portfolio					99.1		-23.3	0.1	0.0	0.1
Securitized					4.4	11.7	11.3	26.3	-0.7	-0.0	-0.7
ES0312298237	AYT CEDULAS CAJAS GLOBAL	4.25	7/29/2014	AYTCED	100.0	0.2	11.3	11.3	-0.7	-0.0	-0.7
Bmark Securities	Not in Portfolio					99.8		26.3	-0.0	0.0	-0.0
Cash					0.3	0.0	-0.0	-0.0	0.0	0.0	0.0
EUR - Unsettled	CASH - European Monetary Unit - Unsettled			EUR	0.0		0.0		0.0	0.0	0.0
EUR - Settled	CASH - European Monetary Unit - Settled			EUR	100.0	100.0	-0.0	-0.0	0.0	0.0	0.0

Source: POINT®

By now, the manager has a good understanding of most major contributors to the outperformance of the portfolio in August and has also reached some useful conclusions regarding the management of a small global portfolio:

- Yield curve exposure is the dominant risk factor and must be managed explicitly in each currency.
- It is not sufficient to manage yield curve exposure using just the duration. Exposure to curve re-shaping must also be controlled explicitly.
- It may make sense to manage yield curve exposure globally and not allow it to be determined by local allocation and management decisions.
- Name risk is very significant in a small portfolio; exposure to corporate issuers must be scrutinised and questionable names excluded.
- Country risk in the euro area must be managed separately.

Further insight can be gained by using a mode of performance attribution in which returns and outperformance are fully decomposed using all available analytics from pricing models. In this model, any part of outperformance that cannot be explained by analytics is reported as residual. Running the HPA algorithm using the Fully Analytical mode, we get the Global Portfolio Summary report shown in Figure 11.

Figure 11: Global Portfolio Summary using the Fully Analytical Model

POINT®

Global Portfolio Summary

Portfolio: REPL : GlAgg G4 50
Benchmark : Global Agg G4
Partition : Class 1 / System

Global Outperformance	
Portfolio Return (bps)	188.7
Benchmark Return (bps)	150.0
Outperformance (bps)	**38.7**

Global Outperformance Details	
FX Allocation & Hedging	28.3
Local Market Allocation	-8.7
Local Management	19.1

Local Management Details	
Yield Curve	20.6
Implied Volatility	0.9
Asset Allocation	-0.9
Security Selection	-2.2
Leverage	-0.0
Mortgage	-3.7
Pricing Differences	0.1
Residual	4.3
Others	-0.0

Source: POINT®

Comparing it with the report of Figure 2 we see that the Global Outperformance Details breakdown has not changed, but that the Local Management Details panel has. While the yield curve contribution remains at +20.6bp, asset allocation is now -0.9bp instead of 0.0bp, and security selection is now -2.2bp instead of -1.7bp. New terms have appeared, in particular Implied Volatility with +0.9bp of contribution, Mortgage with -3.7bp of contribution and Residual with a prominent +4.3bp. The Fully Analytical model also offers multiple reports, allowing users to understand the sources of the various types of outperformance; it will be discussed in detail in Section 4. In this example we will review only the EUR Security Selection report shown in Figure 12. We can see that not only is the portfolio concentrated on Italian government bonds, but that it also contains bonds of very long maturity (Box A), further over-weighting the entire European government sector in terms of spread duration exposure. This may lead the manager to draw yet another conclusion:

- The spread duration exposure of sectors with credit risk must be carefully controlled.

2. Performance Attribution as a Data Quality Tool

The report of Figure 12 has another interesting piece of information: one particular Italian government bond (the 5.00% of 2025 highlighted in Box B) is almost single-handedly responsible for the entire reported residual. Indeed, it contributes +17.4bp of residual to the euro portfolio, which, when scaled by the 23.2% that is the euro portfolio market weight, results in +4.0bp of residual (out of the total of +4.3bp reported in Figure 11).

A diligent manager will promptly investigate residuals coming from a Fully Analytical attribution model. These may mean one of four things: the attribution algorithm or implementation has a problem, the analytics used are not capturing all return components of a security, bad analytics, or finally bad returns. All four are extremely important for the portfolio manager to know.

Over time, model deficiencies will presumably be corrected, leaving data quality problems in the form of bad analytics or bad returns to be the primary cause of attribution residuals. A comprehensive and detailed performance attribution platform such as the HPA algorithm implemented in POINT® serves as a magnifying lens for data quality problems and allows users to pinpoint and correct such issues quickly.

Figure 12: EUR Security Selection Report Using the Fully Analytical Model

POINT® — Hybrid Performance Attribution®

EUR: Security Selection

Portfolio: REPL : GlAgg G4 50
Benchmark : Global Agg G4
Partition : Class 1 / System

Period: 7/30/2010 to 8/31/2010
Base Currency: USD
Curve Type: Treasury

Bucket/Issue	Issuer/Coupon/Maturity	OAS	OAS Change	OASD	MV (%) Port	MV (%) Bench	OASD Contrib (%) Port	OASD Contrib (%) Bench	Outperformance: Security Selection: Spread Carry	Spread Change	Other: Pricing	Residual	Total
EUR		P 162.8	P 23.4	P 6.5	100.0	100.0	100.0	100.0	2.9	-53.7	0.6	17.2	-33.0
		B 104.5	B 14.4	B 5.5									
Treasury		P 108.1	P 34.0	P 9.4	44.0	55.4	63.3	64.8	1.5	-59.3	0.6	16.9	-40.3
		B 67.9	B 19.8	B 6.5									
BE0000308172	BELGIUM (KINGDOM OF) 4.00 '22	57.1	15.9	9.4	38.5	0.3	38.4	0.5	-0.2	6.3	-0.0	0.0	6.1
IT0003644769	ITALY (REPUBLIC OF) 4.50 '20	137.2	35.7	7.8	4.7	0.5	3.9	0.7	0.1	-2.1	0.0	-0.0	-2.0
IT0004594930	ITALY (REPUBLIC OF) 4.00 '20	136.7	40.2	8.2	5.6	0.5	4.9	0.7	0.1	-3.6	0.0	-0.0	-3.5
IT0004513641	ITALY (REPUBLIC OF) 5.00 '25	153.1	47.5	10.3	18.3	0.5	20.0	0.7	0.6	-22.2	0.6	17.4	-3.7
IT0004356843	ITALY (REPUBLIC OF) 4.75 '23	143.0	43.4	9.8	17.1	0.5	17.8	0.7	0.5	-16.8	0.0	-0.1	-16.4
IT0004009673	ITALY (REPUBLIC OF) 3.75 '21	123.7	48.7	9.0	15.8	0.6	15.1	0.9	0.3	-17.2	-0.0	-0.1	-16.9
Bmark Securities	Not in Portfolio	66.0	18.9	6.4		97.0		95.8	0.1	-3.6	0.0	-0.3	-3.8
Government-Related		P 204.0	P 12.8	P 10.1	7.5	15.3	11.6	12.4	0.7	-7.2	0.0	-0.5	-6.9
		B 93.8	B 3.6	B 4.5									
POLAND	POLAND (REPUBLIC OF)	204.0	12.8	10.1	100.0	2.1	100.0	2.9	0.7	-6.9	0.0	-0.3	-6.5
Bmark Securities	Not in Portfolio	92.7	3.3	4.4		97.9		97.1	0.0	-0.3	0.0	-0.2	-0.4
Corporate		P 194.1	P 0.3	P 3.4	43.8	17.6	22.6	13.6	-0.1	14.1	-0.0	0.8	14.9
		B 196.0	B 9.7	B 4.3									
GE	GE CAPITAL CORP	145.6	-10.4	3.7	24.0	2.4	26.3	2.6	-0.4	7.8	0.0	0.3	7.6
RBS	ROYAL BANK OF SCOTLAND	245.0	-24.0	3.5	6.8	1.6	7.0	1.8	0.1	3.3	-0.0	0.1	3.4
TELEFO	TELEFONICA EMISONES SAU	186.4	-9.3	3.2	11.8	1.1	11.3	1.2	-0.0	3.1	-0.0	0.1	3.2
BAC	MERRILL LYNCH & CO INC	231.5	4.8	3.5	13.2	1.7	13.7	1.5	0.1	1.3	0.0	0.2	1.6
IBESM	IBERDROLA	166.6	4.7	3.3	22.7	0.7	22.1	0.6	-0.2	1.5	0.0	0.2	1.5
IMTLN	IMPERIAL TOBACCO FIN PLC	159.3	4.9	3.0	16.7	0.5	15.0	0.4	-0.2	0.9	-0.0	0.2	0.9
C	CITIGROUP INC	533.7	71.4	3.2	4.8	1.4	4.6	1.6	0.6	-3.7	-0.0	0.1	-3.1
Bmark Securities	Not in Portfolio	192.1	9.6	4.3		90.5		90.4	0.1	-0.1	0.0	-0.3	-0.3
Securitized		P 341.6	P 5.7	P 3.7	4.4	11.7	2.5	9.2	0.7	-1.4	-0.0	-0.1	-0.7
		B 154.6	B -2.5	B 4.3									
ES0312298237	AYT CEDULAS CAJAS GLOB 4.25 '14	341.6	5.7	3.7	100.0	0.2	100.0	0.1	0.7	-1.4	-0.0	0.1	-0.5
Bmark Securities	Not in Portfolio	154.3	-2.5	4.3		99.8		99.9	0.0	-0.0	0.0	-0.2	-0.2
Cash		P 0.0	P 0.0	P 0.0	0.3	0.0	0.0	0.0	0.0	0.0	0.0	-0.0	-0.0
		B 0.0	B 0.0	B 0.0									
EUR - Unsettled	CASH - European Monetary Unit	0.0	0.0	0.0	0.0		0.0		0.0	0.0	0.0	0.0	0.0
EUR - Settled	CASH - European Monetary Unit	0.0	0.0	0.0	100.0	100.0	0.0	0.0	0.0	0.0	0.0	-0.0	-0.0

Source: POINT®

In the case of this portfolio, a quick investigation reveals that the offending bond had a coupon payment of 2.50% on August 31, 2010, that has been recorded twice. The price of the bond on August 30, 2010, was approximately 110; therefore, one would expect a residual of +227.3bp at the bond level. Indeed, if the net weight of the bond in the portfolio is taken into account, the residual contribution of the bond to the euro portfolio is 44.0% * (18.3% - 0.5%) * 227.3 = +17.8bp,[6] very close to the +17.4bp of reported residual.

Re-running the Fully Analytical model after correcting the double entry produces a mostly identical report (Figure 13) to the one before (Figure 11), except that the return of the portfolio is lower by the correction amount (about +4.2bp), the outperformance is +34.5bp instead of +38.7bp, and the residual term has been reduced correspondingly from +4.3bp to +0.2bp.

[6] 44.0% is the weight of the government sector in the euro portfolio; 18.3% is the market weight of the bond in the euro government sector in the portfolio, and 0.5% is the market weight of the same bond in the corresponding benchmark sector.

Figure 13: Results from the Fully Analytical Model after the Data Correction

POINT®

Global Portfolio Summary

Portfolio: REPL : GlAgg G4 50
Benchmark : Global Agg G4
Partition : Class 1 / System

Global Outperformance	
Portfolio Return (bps)	184.5
Benchmark Return (bps)	150.0
Outperformance (bps)	**34.5**

Global Outperformance Details	
FX Allocation & Hedging	28.3
Local Market Allocation	-8.7
Local Management	14.9

Local Management Details	
Yield Curve	20.6
Implied Volatility	0.9
Asset Allocation	-0.9
Security Selection	-2.2
Leverage	-0.0
Mortgage	-3.7
Exclusions	-0.0
Residual	0.2
Others	0.0

Source: POINT®

However, if we re-run the original standard model, the results are quite different (Figure 14). Now, the correction is absorbed almost entirely by the security selection term, which drops from -1.7 to -5.8bp, in particular by the allocation to long duration Italian government bonds in the euro market portfolio, which, after the removal of the erroneous coupon, looks even worse than before the correction.

Figure 14: Results from the Standard Model after the Data Correction

POINT®

Global Portfolio Summary

Portfolio: REPL : GlAgg G4 50
Benchmark : Global Agg G4
Partition : Class 1 / System

Global Outperformance	
Portfolio Return (bps)	184.5
Benchmark Return (bps)	150.0
Outperformance (bps)	**34.5**

Global Outperformance Details	
FX Allocation & Hedging	28.3
Local Market Allocation	-8.7
Local Management	14.9

Local Management Details	
Yield Curve	20.6
Asset Allocation	0.1
Security Selection	-5.8
Leverage	-0.0
Pricing Differences	0.0
Exclusions	-0.0

Source: POINT®

In conclusion, data problems have the potential to affect the quality of outperformance measurement of a portfolio significantly, as well as its breakdown to the various sources. Proper use of an analytics-based attribution platform can help identify and correct potential issues promptly.

3. The Mathematical Framework of Performance Attribution

3.1. The Classic Asset Allocation / Security Selection Equations

In its simplest form,[7] portfolio management versus a benchmark is modelled as a two-step process that consists of market weight allocation to various sectors and security selection within each sector. Portfolio return is the market value-weighted sum of the individual sector returns:

$$R^P = \sum_s w_s^P R_s^P$$

Similarly, benchmark return is the weighted sum of benchmark sector returns:

$$R^B = \sum_s w_s^B R_s^B$$

Total outperformance $R^P - R^B$ is split into three components: the outperformance that would be achieved only by allocation (Asset Allocation), the outperformance achieved only by security selection using the benchmark allocation weights (Security Selection), and an interaction term that can be attributed to the joint effect of asset allocation and security selection.

Asset Allocation	$\sum_s \left(w_s^P - w_s^B\right) \cdot R_s^B$	(1)
Security Selection	$\sum_s w_s^B \cdot \left(R_s^P - R_s^B\right)$	(2)
Interaction	$\sum_s \left(w_s^P - w_s^B\right) \cdot \left(R_s^P - R_s^B\right)$	(3)

Interaction terms arise every time two factors contribute to outperformance through a non-additive function such as the product. Interaction terms are philosophically undesirable, since the stated goal of performance attribution is to credit agents separately for their contribution. Thus, we attempt to merge interaction terms into other outperformance components that can be uniquely attributed to a single agent. The criteria for consolidation are usually dictated by the order in which decisions are made, or the relative size of the interaction term.

The standard assumption is that asset allocation precedes security selection (i.e., the portfolio is managed in a top-down fashion), so it is common to fold the interaction term into the latter:

Asset Allocation	$\sum_s \left(w_s^P - w_s^B\right) \cdot R_s^B$	(4)
Security Selection	$\sum_s w_s^P \cdot \left(R_s^P - R_s^B\right)$	(5)

If, on the other hand, the expected returns of sectors in the portfolio are different than those in the benchmark (for example, because of manager skill or leverage), then it might be more appropriate to fold the interaction term into asset allocation instead. As a result, asset

[7] This methodology is usually called the "Brinson model," and it was first laid out in Brinson and Fachler 1985.

allocation would be given by $\sum_s \left(w_s^P - w_s^B\right)\cdot R_s^P$ and security selection would be given by $\sum_s w_s^B \cdot \left(R_s^P - R_s^B\right)$.

It is easy to see that in both cases, the outperformance breakdown is complete; i.e., the sum of the two components is equal to the total outperformance.

The above formulas would assign positive contribution to asset allocation outperformance to any overweighted sector with A positive benchmark return. Thus, if all sectors have positive benchmark returns, overweighting the sector with the worst return would still be considered a good decision. This is intuitively wrong, as the weight allocated to such sector would be better used in sectors with higher returns. Therefore, instead of using the absolute benchmark sector return in the above asset allocation formula, one can use the sector return relative to the benchmark return (which represents the weighted average return of all sectors):

Asset Allocation $$\sum_s \left(w_s^P - w_s^B\right)\cdot \left(R_s^B - R^B\right) \tag{6}$$

Security Selection $$\sum_s w_s^P \cdot \left(R_s^P - R_s^B\right) \tag{7}$$

Notice that while the contribution of each sector to asset allocation changes, the total asset allocation outperformance remains the same as long as $\sum_s w_s^P = \sum_s w_s^B$.

The constraint that the sum of the portfolio and the benchmark sector weights are equal to one necessitates the comparison of the benchmark sector returns with the overall benchmark return. If infinite leveraging is allowed, such that there is no constraint on each sector's weight, then each overweighted sector with a positive return should indeed have a positive contribution to outperformance. In such a case, we do not need to compare the return of each sector to the return of the benchmark. In practice, however, leverage constraints do exist (usually dictated by risk constraints), and neither the benchmark return nor zero is an appropriate choice for the *hurdle rate* for each sector. In the following discussion, we will use the term R^H generically for the hurdle rate. In the HPA implementation, we have made specific choices for the hurdle rate at various points of the application, which will be discussed throughout the paper.

3.2. Recursive Allocation

Most large portfolios have many management layers by which allocation decisions are made successively. For example, the global strategist can specify the allocation into global markets, the US strategist can determine the allocation to major asset classes in the US (equities, bonds, commodities, etc.), the fixed-income strategist can determine the allocations into the various sectors, and so forth.

In such a case, what we called security selection above contains not only security selection, but also further asset allocation, as well as security selection for each sub-sector. In fact, each sector can be considered as a smaller portfolio for which the above performance breakdown equations can be applied recursively. For this reason we will use the term "Sector Management" instead of "Security Selection", and we will reserve the term "Security Selection" for the last layer of management when no further sector decomposition occurs.

For each sector *s,* its contribution to the previous layer sector management $w_s^P \cdot \left(R_s^P - R_s^B\right)$, is recursively split into the following components:

Asset Allocation	$w_s^P \sum_r \left(\frac{w_r^P}{w_s^P} - \frac{w_r^B}{w_s^B}\right) \cdot \left(R_r^B - R_s^H\right)$	(8)
Sector Management	$\sum_r w_r^P \cdot \left(R_r^P - R_r^B\right)$	(9)

Typically, the benchmark sector return R_s^B is used as the hurdle rate as for the calculation of asset allocation to sub-sectors. In this fashion, we can keep decomposing sector management outperformance until we reach the last layer of portfolio management. Since at each level the decomposition is complete, the total outperformance decomposition is also complete. This algorithm has two of the three required properties of performance attribution: *additivity* and *completeness*. As we will see in the following discussion, it also has sufficient flexibility to be applied under different management structures such that it is considered to be *fair*, thereby satisfying all three requirements of performance attribution.

3.2.1. Empty Sectors

The above formulas can be used for sectors that are empty in the portfolio by simply setting $w_r^P = 0$. In this case, the sector management component is zero and the contribution of the empty sector to asset allocation is $-w_s^P \cdot \frac{w_r^B}{w_s^B} \cdot \left(R_r^B - R_s^H\right)$.

However, it is not clear how to handle out-of-benchmark sectors, as their benchmark return is not defined. Instead, we need to use a suitable replacement for that sector, which we will refer to as the reference return. By setting $w_r^B = 0$, the asset allocation contribution of an out-of-benchmark sector is $w_r^P \cdot \left(R_r^{ref} - R_s^H\right)$, and the security selection is $w_r^P \cdot \left(R_r^P - R_r^{ref}\right)$. Choosing an appropriate reference return algorithmically is not straightforward and is best determined by the user. The POINT® implementation sets the reference return equal to the portfolio return of the sector. Thus, the security selection term becomes zero, and the asset allocation term from that sector is equal to $w_r^P \cdot \left(R_r^P - R_s^H\right)$. This implies that the outperformance from an out-of-benchmark sector is entirely attributed to asset allocation, and nothing is attributed to security selection.

Another interesting note is that the contribution of out-of-benchmark sectors to sector management (which is zero when the portfolio return is chosen as the reference return) can still be recursively split into non-zero asset-allocation contributions of its sub-sectors. Of course, the algorithm guarantees that the sum of all such contributions is zero.

3.3. Factor-Based Attribution

The attribution framework described so far is often referred to as "sector-based" attribution, since the investment universe is decomposed into the sectors where a percentage of the portfolio market value is allocated. This framework is gradually losing ground to a "factor-based" decomposition of the investment universe, where the return of the security in the portfolio/benchmark is decomposed into the contributions of common risk factors plus a residual. The residual represents the return not captured by the risk factors. The coefficients of the factors, α_k, are called *loadings* or *sensitivities* of the portfolio to each factor, f_k. Typically, the decomposition is linear and is represented as:

$$R^P = \sum_k \alpha_k^P f_k^P + \varepsilon^P$$

Such risk factors include interest rates, implied volatility, spreads, etc., and the loadings are the corresponding sensitivities to the exposure of each factor (option-adjusted duration, partial (key-rate) durations, vega, spread duration, etc.). Factor-based attribution essentially entails the decomposition of outperformance to the contribution of each risk factor.

$$R^P - R^B = \sum_k \left(\alpha_k^P - \alpha_k^B\right) \cdot f_k^P + \left(\varepsilon^P - \varepsilon^B\right) \tag{10}$$

Pure factor-based attribution systems attempt to use a set of factors that can explain all of the systematic return of a portfolio. In this case the excess return is idiosyncratic and therefore quite small for a well diversified portfolio. The disadvantages of this approach are that it does not attribute idiosyncratic returns of small portfolios to security selection decisions, and there is usually little flexibility in defining the risk factors. The latter shortcoming may be overcome with factor re-mapping (see Silva (2009)), which allows each portfolio manager to attain a personalized view of risk and return decomposition.

3.4. Hybrid Attribution: A General Framework of Outperformance Decomposition

We will now combine the ideas we developed in the preceding sections into a unified framework underlying the HPA model that bridges the sector-based and the factor-based attribution. It is a conceptually simple recursive algorithm that can be described in three steps:

STEP 1: RETURN SPLITTING

The total return of the portfolio and benchmark is split into the linear contributions of factors:

$$R^P = \sum_k \alpha_k^P f_k^P \qquad R^B = \sum_k \alpha_k^B f_k^B$$

Note that residual is included here as one of the factors with a loading of one.

STEP 2: FACTOR RETURN ATTRIBUTION

The factors from step 1 are categorised as either common or allocated. We denote the set of common factors by C and the set of allocated factors by A and write the total outperformance as:

$$R^P - R^B = \left(\sum_{k \in C} \alpha_k^P f_k^P - \sum_{k \in C} \alpha_k^B f_k^B\right) + \left(\sum_{k \in A} \alpha_k^P f_k^P - \sum_{k \in A} \alpha_k^B f_k^B\right) \tag{11}$$

The outperformance from each factor is then handled separately. For each factor, we partition the investment universe into sectors[8] and decompose the outperformance from the factor.

$$\alpha_k^P f_k^P - \alpha_k^B f_k^B = \sum_s \alpha_{k,s}^P f_{k,s}^P - \sum_s \alpha_{k,s}^B f_{k,s}^B \tag{12}$$

Bottom-Up Aggregation of Common Factors

Outperformance from common factors is explained using bottom-up aggregation, namely via net exposures and factor movements at each level of the aggregation. Furthermore, if a

[8] A different partition may be used for each factor.

factor is truly common, where the factor movement for the portfolio is equal to the factor movement for the benchmark, then the bottom-up aggregation becomes:

$$\alpha_k^P f_k^P - \alpha_k^B f_k^B = \sum_s \left(\alpha_{k,s}^P - \alpha_{k,s}^B\right) \cdot f_{k,s} \tag{13}$$

Top-Down Decomposition Of Allocated Factors

Outperformance from allocated factors is explained using one of the two allocation methods:

Absolute Allocation

In this case, the portfolio exposure to this factor is determined by the decisions of the asset allocator and sector managers. Each sector may take a different view as to the expected direction of each factor.

Asset Allocation $$\sum_s \left(\alpha_{k,s}^P - \alpha_{k,s}^B\right) \cdot f_{k,s}^B \tag{14}$$

Sector Management $$\sum_s \alpha_{k,s}^P \cdot \left(f_{k,s}^P - f_{k,s}^B\right) \tag{15}$$

Relative Allocation

The total exposure to a factor is determined at the portfolio level (or the higher hierarchical level). The asset allocator subsequently determines the sector exposures to the factor subject to the constraint imposed by the portfolio exposure. Each sector may take a different view as to the relative movement of each factor against the overall benchmark factor return.

Asset Allocation $$\alpha_k^P \cdot \sum_s \left(\frac{\alpha_{k,s}^P}{\alpha_k^P} - \frac{\alpha_{k,s}^B}{\alpha_k^B}\right) \cdot \left(f_{k,s}^B - f_k^H\right) \tag{16}$$

Sector Management $$\sum_s \alpha_{k,s}^P \cdot \left(f_{k,s}^P - f_{k,s}^B\right) \tag{17}$$

Top-level Exposure $$\left(\alpha_k^P - \alpha_k^B\right) \cdot f_k^H \tag{18}$$

Notice that in the relative allocation framework, the factor hurdle rate f_k^H essentially becomes a bottom-up common portfolio return factor.

STEP 3: RECURSIVE APPLICATION

Each of the sector management terms from top-down allocations, $f_{k,s}^P - f_{k,s}^B$, can be now decomposed separately using the above algorithm recursively.

At the last stage of the decomposition, in which a sector is not decomposed further into sub-sectors, the sector management term becomes security selection. The top-down decomposition can be applied one last time to break down the security selection outperformance of a sector into the contributions of individual securities. In this case, the management term is typically zero since the factor driving the return of the same security in the portfolio and the benchmark must be the same.[9]

[9] In certain situations, this is not true. For example, this term is non-zero if the security price used to calculate portfolio returns is different from the price used for the same security in the benchmark, or if the market variables or analytics used to decompose the return of a security are not the same between the portfolio and the benchmark. HPA recognizes such cases and captures these contributions to outperformance in a special term called *Pricing Differences* as explained in Section 7.3.1.

Example: Hybrid Performance Attribution

To illustrate how this algorithm can be applied in practice, consider a very simple model in which the return of a security is split into two components: the time return that is measured by the yield of the security and a market return that is measured as the product of the duration of the security times the negative of its yield change. This decomposition may not be complete, so it is reasonable to assume the presence of a residual.

$$R_i = Y_i \, \Delta t - D_i \, \Delta Y_i + \varepsilon_i$$

The portfolio (or benchmark) return is the weighted average return of the securities in the portfolio (or benchmark). The summation extends over a universe defined by the union of securities in the portfolio and benchmark. Securities not present in either get correspondingly zero weight.

$$R^P = \sum_i w_i^P R_i = \sum_i w_i^P Y_i \, \Delta t - w_i^P D_i \, \Delta Y_i + w_i^P \varepsilon_i$$
$$R^B = \sum_i w_i^B R_i = \sum_i w_i^B Y_i \, \Delta t - w_i^B D_i \, \Delta Y_i + w_i^B \varepsilon_i$$

Aggregation of yield and duration is the usual weighted average; however, yield changes must be aggregated using duration contribution weights to retain their natural interpretation.

$$Y^P = \sum_i w_i^P Y_i \qquad Y^B = \sum_i w_i^B Y_i$$
$$D^P = \sum_i w_i^P D_i \qquad D^B = \sum_i w_i^B D_i$$

$$\Delta \mathrm{Y}^P = \sum_i \frac{w_i^P D_i}{D^P} \Delta \mathrm{Y}_i \qquad \Delta \mathrm{Y}^B = \sum_i \frac{w_i^B D_i}{D^B} \Delta \mathrm{Y}_i$$

We can now re-write the portfolio and benchmark returns using top-level analytics:

$$R^P = Y^P \Delta t - D^P \Delta Y^P + \sum_i w_i^P \varepsilon_i$$
$$R^B = Y^B \Delta t - D^B \Delta Y^B + \sum_i w_i^B \varepsilon_i$$

Let us assume that time return and residual are allocated as common factors.

Carry $\quad (Y^P - Y^B) \cdot \Delta t$

Residual $\quad \sum_i (w_i^P - w_i^B) \cdot \varepsilon_i$

On the other hand, market return is managed by allocating duration exposure $w_s^P D_s^P$ to a set of sectors and allowing managers to pick securities within each sector.

If the top-level portfolio duration is not constrained and is completely determined by the choices of the sectors' net spread duration exposures, we should use the absolute allocation algorithm by setting $\alpha_k \leftarrow D$, $\alpha_{k,s} \leftarrow w_s D_s$, $f_k \leftarrow -\Delta Y$, $f_{k,s} \leftarrow \Delta Y_s$ and $f_k^H = 0$. The duration and yield change of each sector in the portfolio and the benchmark are calculated in the same weighted average fashion we discussed above. Since the asset allocation agent is free to determine the duration exposure of each sector, the hurdle rate is set to zero and the total outperformance solely distributed between asset allocation and security selection.

Asset Allocation $$-\sum_{s}\left(w_s^P D_s^P - w_s^B D_s^B\right)\cdot \Delta Y_s^B$$

Security Selection $$-\sum_{s} w_s^P D_s^P \cdot \left(\Delta Y_s^P - \Delta Y_s^B\right)$$

On the other hand, if the net duration of the portfolio is actively managed, we need to use the relative allocation method. In this case, we set the hurdle rate equal to the benchmark yield change.

Asset Allocation $$-D^P \cdot \sum_{s}\left(\frac{w_s^P D_s^P}{D^P} - \frac{w_s^B D_s^B}{D^B}\right)\cdot \left(\Delta Y_s^B - \Delta Y^B\right)$$

Security Selection $$-\sum_{s} w_s^P D_s^P \cdot \left(\Delta Y_s^P - \Delta Y_s^B\right)$$

Top-level Exposure $$-\left(D^P - D^B\right)\cdot \Delta Y^B$$

This framework has enough richness to describe a large family of attribution methodologies. The Barclays Capital HPA model, which we will describe in detail in the following sections, is a particular implementation of the above framework.

4. The Barclays Capital Hybrid Performance Attribution® Model

We begin the discussion of our attribution model by explaining in detail the return splitting algorithm, followed by the outperformance breakdown for a single currency portfolio. Multi-currency performance attribution, including the details of our approach to handle FX allocation and hedging, is discussed in Section 5.

4.1. Local Currency Return Splitting

Securities with deterministic cash flows are priced by discounting the cash flows off a reference yield curve.

When future cash flows are not known but can be reasonably predicted by modelling them as functions of economic variables (most commonly interest rates) and security characteristics, more complex statistical diffusion models are used. Such models produce a set of projected interest rate paths that are consistent with the current yield curve and the market implied volatility of liquid interest-rate options. Projected cash flows on each path are discounted at the corresponding discount factor, and the present value of the security is calculated as the average present value across all interest-rate paths. In this case, the present value of a security is a function of interest rates, their implied volatility and possibly other variables in the model.

Finally, in certain situations, such as stocks, expected cash flows are unpredictable and the value of the security is not driven by such expectations. In such cases, usually no pricing model is being used.

A pricing model produces the "model value" of a security. In the case of OTC securities such as derivatives, the model value is usually used to mark to market positions in the security. When a security is publicly traded, its market price generally does not agree with the model price. To make them, equal model parameters are tweaked ("calibrated") to match the market price. For example, in the case of bonds, this usually entails additional discounting of cash flows at a flat rate across all maturities. In the majority of models, in which cash flows are discounted at a risk-free reference curve (the government or swap curve in a particular currency), this additional discounting rate is the well-known OAS, which captures the extra discounting required to account for credit, liquidity and other types of risks. Analytics produced by such a model are therefore known as option-adjusted analytics.

Generically, a pricing model can be expressed with the following formula:

Full Price = *f* (Time, Curve, Volatility, Other Inputs, OAS)

For a single currency portfolio, the HPA methodology begins with splitting the local currency return of each security into the contributions of various factors. Our return-splitting algorithm accomplishes that in two stages:

STAGE I:	Scenario-based return decomposition into broad categories
STAGE II:	Analytics-based return decomposition into fine categories

In Stage I, our scenario decomposition of daily return begins with the market value of the position today and then moves one parameter at a time back to yesterday's value until all of the parameters have changed and yesterday's market value is obtained. At each step, we obtain the market value of the security by fully recalculating its present value by changing one input parameter at a time. At the end of this stage, we have the following broad return categories:

Surprise Return	Difference of actual cash flows and notional changes from model predictions
Time Return	The effect of elapsing time
Curve Change Return	The effect of changes in the yield curve
Volatility Change Return	The effect of changes in the implied volatility surface
Other Market Return	The effect of changes in any other market parameters
Spread Return	The effect of changes in the option-adjusted spread

In Stage II, we use the sensitivity of the security to various risk factors (analytics) to decompose the return further.

Surprise Return

Surprise return is relevant to securities whose cash flows and amount outstanding are not deterministic. For mortgage securities, it is prepayments that generate the random behavior of cash flows and amount outstanding. For inflation-linked securities, it is the dependence of cash flows on the reference inflation index.

Our security valuation models make assumptions about inputs to pricing models and use such projections to value securities until realised values are known. The return explained by running scenarios or using model analytics is the return that would be realised if the uncertain parameters were consistent with model predictions. If the realised values of the parameters are different from the projected ones, the difference between the actual return and the return explained by the model is captured as surprise return.

Example: Mortgage Prepayments Surprise Return

Consider an MBS security that is trading at a price of 105 and zero accrued. The prepayment model projects a 2.4% prepayment over the next month (25% CPR), thus predicting a prepayment return of -11.4bp[10]. If the actual prepayment is 1.3% (15% CPR), resulting in a prepayment return of -6.2bp, the difference of +5.2bp will be registered as prepayment surprise return. The model-expected -11.4bp of prepayment return will be part of the time return component.

Example: Inflation Surprise Return

Consider an inflation-linked security that is trading at a price consistent with projected inflation for next month of 3.0%. If the actual inflation announcement is 3.5%, there is (3.5% - 3.0%) x (1/12) = +4.1bp of inflation surprise return to adjust the inflation accretion accumulated over the month at the projected inflation rate.

Time Return

Time return is the deterministic component of the return, i.e., the return predicted by the pricing model if market parameters remain unchanged.[11] For most fixed-income securities, we decompose time return into curve carry, spread carry and volatility decay. Inflation-linked securities also have inflation accretion and inflation spread carry. These time return components are measured using analytics, as we will describe in detail in following sections. The difference between the sum of these components and the scenario-based time return is referred to as time residual, which we report as part of the total residual.

Curve Change Return

Changes of interest rates affect securities returns primarily through the change of discount factors. In addition, some securities have cash flows that are modelled as functions of interest rates (MBS, floating-rate securities). The component of return that is due to the fluctuation of the yield curve is captured by the curve change return component.

The exposure of the security value to the yield curve is captured by its sensitivity to parallel shifts of the curve (OAD), movements of a specified set of points (key-rate points) representing the curve (key-rate durations, or KRDs), and its second-order sensitivity (convexity or OAC).

Since we measure returns daily, the potential change of yields is limited (moves above 15bp are very uncommon); therefore, the convexity contribution is typically much smaller than the duration contribution. For this reason, we do not explicitly measure the effect of curve convexity.[12]

[10] -11.4bp = (par-105) * 2.4% / 105.
[11] We discuss the definition of "unchanged" for the yield curve in Section 4.4.1.
[12] For currencies with high volatility of yields, convexity can become a significant determinant even for daily returns. In such a case, it is straightforward to include convexity as an explanatory variable to our algorithm.

Example: Curve Change Return Decomposition

How total curve change return is broken down between duration and convexity depends to a large extent on the frequency at which the duration exposure of a portfolio is managed. In the HPA algorithm, we have assumed that this happens at a daily frequency, consistent with the practice of the majority of portfolio managers. To understand the frequency implications, consider a portfolio with interest-rate duration (*D*) of 5 and an interest-rate convexity (*C*) of -2. Let us assume that over the two weeks, interest rates keep falling at 10bp/business day for a total of 100bp. We will use the duration-convexity formula to explain portfolio returns caused by changes in interest rates:

$$R^{\Delta YC} = -D \cdot \Delta r + 0.5 \cdot (C/100) \cdot \Delta r^2 ,$$

where both the returns and the rates change is expressed in bp.

If we apply this formula for the entire two weeks, we get:

Duration Return:	-5 x (-100) =	+500bp
Convexity Return:	0.5 x (-2/100) x $(-100)^2$ =	-100bp
Total Return:		+400bp

On the other hand, if we apply it daily, we get quite a different breakdown. The daily algorithm must take into account additional complexities such as the duration drift of the portfolio, as well as compounding. Indeed, we cannot simply calculate the daily duration and convexity return as -5 x (-10) = +50bp and 0.5 x (-2/100) x $(-10)^2$ = -1bp, respectively, and just aggregate over 10 days to get +500bp duration return and -10bp convexity return, for a total return of +490bp. Instead, we first need to estimate the daily duration drift of the portfolio due to falling interest rates. Using the definitions of duration and convexity, we estimate the duration change due to a change in interest rates as:

$$\Delta D = (D^2 - 100 \cdot C) \cdot \Delta r / 10{,}000 ,$$

where duration and convexity have the usual units and change of rates is measured in basis points.

Due to the high convexity of the portfolio, over the course of the ten days the duration of the portfolio shrinks by about two years.

Next, we need to apply daily compounding both to the total return of the portfolio and the contributions of duration and convexity (see Section 7.1.1 for details). After both adjustments, the total return calculation and breakdown are as follows:

Duration Return:	+408bp
Convexity Return:	-10bp
Total Return:	+398bp

The total return is very close with the one estimated by analytics at the beginning of the period, but the breakdown between duration and convexity is dramatically different. In the daily model, the convexity contribution is much smaller than the duration contribution.

Any curve change return (as estimated by the scenario-based total curve change return) in excess of what can be explained by the key-rates is called the "Rest of Curve" return, as it captures the contribution of the curve movements between the key-rate points, as well as the convexity and other second-order terms.

In POINT®, we produce two sets of analytics corresponding to using the government curve and the swap curve to discount cash flows. Our algorithm allows the user to select which curve to use for performance attribution. For tax-exempt municipal securities, the model can use a decomposition based on the municipal AAA curve instead of the Treasury curve.

Volatility Change Return

Securities with optionality require reference implied volatilities to calibrate their interest rate diffusion model. In the fixed income world, most models use the Black swaption-implied volatility surface as input and calculate the sensitivity of the security value to the changes in implied volatility with a single analytic, vega, which is the price change of the security for a 1% parallel shift in the Black implied volatility surface.

Vega does not capture the effects of non-parallel movements of the implied volatility surface or the effect of yield curve moves on the diffusion parameters of the interest rate diffusion model.[13] Clearly, it does not sufficiently represent the sensitivity of the security value to changes in implied volatility surface. For this reason, capturing the effects of implied volatility changes through scenarios is imperative.

We decompose the scenario-based implied volatility change return into a parallel shift component that is equal to the product of vega and the average volatility shift, and a remainder that contains the effects of non-parallel volatility movements and interest rate changes.

Other Market Return

Although yield curve and implied volatility are the most important factors in the pricing models of most fixed-income instruments, they do not capture every aspect of the cash flows, so more factors may be required. Securities or derivatives with cash flows that depend on external factors, such as prepayments or inflation levels, may use additional inputs to the pricing model. Changes in such inputs will generate return that is not attributable to yield curve or implied volatility surface changes. The effects of the changes of all others factors are captured together as Other Market Return.

For example, the model-projected cash flows of MBS securities depend on market mortgage rates, as well as additional model-specific parameters such as home price appreciation expectations or swap spreads. All of these parameters contribute to Other Market Return. The breakdown to the various factors follows an instrument specific algorithm. In particular for mortgages, we estimate the mortgage rate change effect by using the mortgage-rate duration and assign the remainder Other Market Return to other mortgage factors.

Spread Return

The effect of OAS changes to return is captured in the spread return. Sensitivity to spread movements is measured directly using both spread duration (OASD) and spread convexity (OASC). In general, spread can realize very large daily moves (100s of bp) making use of spread convexity necessary. The difference between the sum of spread return captured by OASD and OASC and the scenario-based spread return contributes to the residual.

Residual

Portions of the return that cannot be explained by our analytics are reported together as "residual." Our definition of this captures any spread return beyond what can be explained

[13] In the simple lognormal model, the stochastic component of the relative interest rate changes is proportional to a scalar, the lognormal (Black) volatility; therefore no dependence on interest rates exists. Other models that better capture the true behavior of interest rates make the stochastic term a function of black volatilities, as well as the level of interest rates.

by OASD and OASC, time residual, and any other components of return that were not considered in our scenario splitting exercise. It will also capture the effect of pricing parameters that were not included in the Other Market Return calculation.

The left panel of Figure 15 shows broad return categories from our return splitting scenarios and its components. The right panel shows how we regroup the return components into major return factors: curve, implied volatility, spread, mortgage, inflation and residual.

Figure 15: Return Splitting Components

Split by return categories	
Surprise	Mtg Prepay Surprise
	Inflation Surprise
Time Return	Curve Carry
	Spread Carry
	Volatility Decay
	Inflation Accretion
	Inflation Spread Carry
	Time Residual
Curve Change	Key-Rate Changes
	Rest of Curve Change
Volatility Change	Parallel Black
	Rest of Volatility Change
Other Change	Mtg Spread Change
	Other Mtg Factors
	Inflation Spread Change
Spread Change	Spread Duration
	Spread Convexity
	Spread Residual

Split by return factors	
Curve	Curve Carry
	Key-Rate Changes
	Rest of Curve Change
Volatility	Volatility Decay
	Parallel Black
	Rest of Volatility Change
Spread	Spread Carry
	Spread Duration
	Spread Convexity
Mortgage	Mtg Prepay Surprise
	Mtg Spread Change
	Other Mtg Factors
Inflation	Inflation Surprise
	Inflation Accretion
	Inflation Spread Carry
	Inflation Spread Change
Residual	Spread Residual
	Time Residual

Source: Barclays Capital

The components of return splits along with the residual can be found in the Security Returns Split report (Figure 16). Return splits are reported for each security separately, as well as aggregated by hierarchical partition buckets.

So far, the discussion of return splitting has been fixed income-centric. Our current implementation does not attempt to split the return of equity securities. As a result, we simply include equity price and dividend return as additional return factors.

The six return factors on the right panel of Figure 15 and the equity factors are comprehensive in capturing the daily return of a security. However, to provide a complete and practical outperformance analysis between a portfolio and a benchmark, we introduce three other factors in our return-splitting algorithm: intra-day, pricing difference and exclusion. Intra-day, or "trading," return captures the difference between the executed price and the end-of-day price. It appears only on the days when a trade is executed. Pricing difference captures the difference between the price of the same security in the portfolio and the benchmark. Finally, exclusion includes the returns of the securities for which we do not have complete analytics to perform the regular return splits. These three additional factors are discussed in more details in Section 7.2 and 7.3.

Figure 16: Security Return Splits Report

POINT® Hybrid Performance Attribution®

USD: Security Return Splits

Portfolio: REPL : GlAgg G4 50
Benchmark : Global Agg G4
Partition : Class 1 / System

Period: 7/30/2010 to 8/31/2010
Base Currency: USD
Curve Type: Treasury

Bucket/Issue	Issuer/Coupon/Maturity	MV%	Curve		Volatility		OAS		PPM	Pre-	Mtg		Resi	Total
			Carry	Chng	Decay	Chng	Carry	Chng	Sw Sprd	pay	Sprd	Exclusions	dual	Return
USD		100.0	16.9	159.3	1.0	-6.1	6.4	-25.7	-0.2	1.3	8.5	0.0	0.2	161.6
Treasury		31.7	19.0	301.1	-0.0	0.0	0.1	-3.5	0.0	0.0	0.0	0.0	-1.2	315.4
912828MQ	US TREASURY NOTES 0.88 '12	54.9	6.6	7.3	-0.0	0.0	0.1	-0.5	0.0	0.0	0.0	0.0	-0.6	13.0
912810FP	US TREASURY BONDS 5.38 '31	25.6	33.6	684.5	0.0	0.0	0.2	-12.1	0.0	0.0	0.0	0.0	-3.6	702.6
912810FF	US TREASURY BONDS 5.25 '28	19.5	35.5	646.2	0.0	0.0	0.0	-1.2	0.0	0.0	0.0	0.0	0.1	680.5
Government-Related		14.3	15.4	91.0	0.0	0.0	6.9	9.5	0.0	0.0	0.0	0.0	1.2	123.9
31359MRK	FEDERAL NATL MTG ASSN-GLOBAL 4.62 '13	62.0	10.8	18.4	0.0	0.0	7.0	0.5	0.0	0.0	0.0	0.0	1.5	38.2
31359MTP	FEDERAL NATL MTG ASSN-GLOBAL 5.12 '14	22.0	14.3	37.0	0.0	0.0	8.0	-3.3	0.0	0.0	0.0	0.0	2.8	58.8
46513EFF	ISRAEL STATE OF 5.50 '23	14.3	36.6	485.3	0.0	0.0	3.3	66.2	0.0	0.0	0.0	0.0	-1.4	589.9
XS0114288789	RUSSIA GLOBAL 7.50 '30	1.7	23.3	186.8	0.0	0.0	16.8	32.3	0.0	0.0	0.0	0.0	-5.7	253.4
Corporate		30.7	19.0	122.4	0.3	0.8	15.6	-52.6	0.0	0.0	0.0	0.0	2.4	107.9
46625HHN	JP MORGAN CHASE & CO 4.65 '14	13.9	16.8	53.5	0.0	0.0	11.6	-11.5	0.0	0.0	0.0	0.0	3.5	74.0
74977RBQ	RABOBANK 4.20 '14	11.5	16.6	51.8	0.0	0.0	10.8	24.5	0.0	0.0	0.0	0.0	3.7	107.4
025816BA	AMERICAN EXPRESS CO 7.25 '14	10.4	15.9	49.1	0.0	0.0	15.0	-8.8	0.0	0.0	0.0	0.0	3.5	74.7
20030NAM	COMCAST CORPORATION 6.45 '37	10.1	32.5	614.2	0.0	-0.0	17.0	-297.3	0.0	0.0	0.0	0.0	0.3	366.8
90261XFY	UNION BANK OF SWITZERLAND 3.88 '15	10.0	21.2	85.9	0.0	0.0	15.3	3.7	0.0	0.0	0.0	0.0	3.3	129.4
61746BCW	MORGAN STANLEY DEAN WITTER 5.62 '12	10.0	6.2	6.5	0.0	-0.0	17.2	40.7	0.0	0.0	0.0	0.0	-0.2	70.4
38141GEE	GOLDMAN SACHS GROUP-GLOBAL 5.35 '16	9.7	25.9	135.7	-0.0	-0.0	18.1	-53.8	0.0	0.0	0.0	0.0	2.4	128.3
25459HAG	DIRECTV HOLDINGS/FING 7.62 '16	9.5	11.0	33.6	2.8	8.1	25.2	-114.5	0.0	0.0	0.0	0.0	1.5	-32.4
94973VAK	WELLPOINT INC-GLOBAL 5.25 '16	9.2	25.9	136.3	0.0	0.0	12.7	-148.6	0.0	0.0	0.0	0.0	2.2	28.4
96008YAB	WESTFIELD CAPITAL CORP 5.12 '14	5.8	19.6	74.3	0.0	0.0	15.7	40.7	0.0	0.0	0.0	0.0	3.5	153.8
Securitized		22.8	12.5	59.9	4.2	-27.5	2.4	-42.6	-1.0	6.1	36.9	0.0	-1.8	49.1
GNF04403	GNMA I Single Family 15 4.50 '18	29.0	14.1	51.9	0.2	-1.4	4.8	-71.1	-0.3	1.5	4.4	0.0	9.4	13.3
FNA05003	FNMA Conventional Long T. 30 5.00 '32	21.3	8.2	12.7	5.8	-28.4	0.8	-26.3	-3.1	12.1	61.5	0.0	-8.3	35.0
FNA04409	FNMA Conventional Long T. 30 4.50 '39	19.7	13.2	88.3	6.7	-46.7	1.8	-33.0	0.4	6.2	45.8	0.0	-7.7	75.0
FNA04403	FNMA Conventional Long T. 30 4.50 '32	11.2	11.6	57.2	5.8	-38.3	0.8	-16.0	-1.1	9.3	52.4	0.0	-6.9	74.9
FGB04403	FHLM Gold Guar Single F. 30 4.50 '32	10.3	12.9	74.7	5.6	-38.8	1.8	-35.5	-1.1	6.5	46.5	0.0	-6.1	66.7
GNA04403	GNMA I Single Family 30 4.50 '32	8.6	16.9	125.1	4.3	-42.4	2.8	-51.7	-0.8	1.5	33.8	0.0	1.7	91.2
Cash		0.5	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
USD - Unsettled	CASH - U.S. Dollar - Unsettled	52.3	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
USD - Settled	CASH - U.S. Dollar - Settled	47.7	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0

Source: POINT®

4.2. Local Outperformance Breakdown

Having split the returns in such detail based on analytical exposures to market factors, one might be tempted to proceed with bottom-up aggregation of outperformance per factor. However, not every portfolio manager uses the factor approach to management, nor do all agree on which factors should represent portfolio risk. Many portfolios are managed by sectors or asset classes where top-down decomposition of outperformance is more appropriate, as the returns of the portfolio and the benchmark in each asset class or sector may differ. Seeing applications of both methods, we chose a flexible hybrid approach for performance attribution.

Hybrid Performance Attribution® allows users to split total return into two parts: that driven by common factors for the entire portfolio and that in excess of common factors, which is further explained by asset allocation/security selection based on a user-defined hierarchical partition.

Common components are explained by bottom-up aggregation as the difference of the exposure of the portfolio and the benchmark multiplied by the factor return. For example, if we describe curve exposure with duration, the outperformance explained by curve movement would be the duration overweight of the portfolio (over the benchmark) multiplied by the negative of the curve change.

Excess of common or allocated components is explained by top-down decomposition using a user-defined hierarchical partition and the recursive algorithm of Section 3.4, as will be detailed below.

Our approach has the ability to accommodate different portfolio management styles. It also allows users to compare different ways of analyzing returns and outperformance and leads to a better understanding of what drives portfolio outperformance. For example, many high yield managers do not like to split total return into curve, implied volatility and spread components, citing the high negative correlation between curve and implied volatility return versus spread return. Instead, they prefer to manage total return as a whole. And even when they do wish to separate out curve and implied volatility return and manage excess return, they usually do not rely on spread duration and spread convexity to measure their exposure. However, when spreads were very tight in 2005, many high yield managers began experimenting with duration-based management. Further, other risk exposure measures, such as the "duration-times-spread" (DTS),[14] can be readily used in the hybrid approach.

While any return split component could be considered as a bottom-up common return component or as an excess return component that participates in the top-down allocation algorithm, POINT® offers three common configurations that support most portfolio management decision-making structures.

Model I: The Total Return model

This is the simplest model we offer in HPA, where the (local currency) total return is considered as a single allocated factor. In other words, there is no common factor,[15] and the total return is explained by top-down decomposition using market value weights and user-defined security partition.

Model II: The Excess Return model

This model takes advantage of the return splitting algorithm to separate the factors contributing to return into common and allocated factors. Returns from common factors are explained by bottom-up aggregation using appropriate analytic weights (e.g., OAD, vega, market beta, etc.). For example, yield curve and/or implied volatility are typically considered as common factors for fixed-income securities, and market or industry return (e.g., S&P500) for equities. Returns from allocated factors (excess-over-common factor returns) are explained by top-down decomposition as in the previous model.

Model III: The Fully Analytical model

This is the most detailed model in HPA. It takes the full advantage of the return splitting algorithm and the hybrid allocation algorithm, where each factor can be explained either by the top-down or the bottom-up aggregation using appropriate weights.

The following sections discuss the usage and implementation details of these different types of models in HPA.

4.3. Total Return Model

The Total Return attribution model is the most basic performance attribution model and follows the classical, sector-based, asset allocation/security selection framework. It is appropriate for portfolios whose managers make decisions solely based on the total returns

[14] For more details, see Ben Dor et al. 2010.
[15] Except for intra-day return on the days when securities are traded.

of the securities. A typical usage is an equity portfolio whose managers are allocating the capital based on their views of the sector and security returns, rather than risk factors.

However, POINT® offers the flexibility to define the allocation buckets via a user-defined partition using any of the hundreds of security attributes available. Thus, users can create alternative allocations such as geography, size, momentum, etc., essentially mimicking factor-based allocation.

HPA excludes intra-day from the total allocated outperformance, and the rest is explained by top-down decomposition into asset allocation and security selection per user-defined hierarchical partition. The partition should be defined to mimic the management structure of the portfolio. For instance, if the investment decision is determined first by allocating capital to industry sectors, followed by selecting the securities within those sectors, then industry sector is an appropriate partition to use.

The Total Return allocation model uses the relative allocation method of the top-down decomposition algorithm, where sector market value weight w_s is the allocation weight $\alpha_{k,s}$, local currency total return of sector *s* is the only factor return $f_{k,s}$, and local currency total return of the benchmark is the hurdle rate.

Sector Allocation
$$w^P \cdot \sum_s \left(\frac{w_s^P}{w^P} - \frac{w_s^B}{w^B} \right) \cdot \left(TR_s^B - TR^B \right) \tag{19}$$

Sector Management
$$\sum_s w_s^P \cdot \left(TR_s^P - TR_s^B \right) \tag{20}$$

Top-Level Exposure
$$\left(w^P - w^B \right) \cdot TR^B \tag{21}$$

Absent leverage, the top-level weights of the portfolio and the benchmark are both equal to one, so the top-level exposure term is always zero.

Figure 17: USD Portfolio Summary using the Total Return Model

POINT®

USD: Summary

Portfolio: REPL : GlAgg G4 50
Benchmark : Global Agg G4
Partition : Class 1 / System

Outperformance (USD)	
Portfolio Return (bps)	161.6
Benchmark Return (bps)	129.8
Outperformance (bps)	**31.8**

Outperformance Details	
Asset Allocation	18.9
Security Selection	13.2
Leverage	-0.1

Source: POINT®

Let us now return to the example of Section 1 and review the outperformance breakdown using different attribution models. We will focus on the Dollar portion of the portfolio, which had an (un-weighted) return of +31.8bp,[16] mostly from interest rate exposure. Using the

[16] This number can be approximated in Figure 5 as the ratio of the return contribution of the dollar portfolio (+14.6bp) divided by the dollar portfolio weight (45.5%). The calculation is not exact because the weight of the dollar portfolio fluctuates over the monthly attribution period.

Total Return model will fold the interest rate contribution into asset allocation (+18.9bp) and security selection (+13.2bp). The contribution of the allocation to each asset class is calculated based on the total return of each sector (including yield curve return), as detailed in the Asset Allocation report (Figure 18).

Figure 18: USD Asset Allocation using the Total Return Model

POINT® Hybrid Performance Attribution®

USD: Asset Allocation

Portfolio: REPL : GlAgg G4 50
Benchmark : Global Agg G4
Partition : Class 1 / System

Period: 7/30/2010 to 8/31/2010
Base Currency: USD
Curve Type: Treasury

Level 1: Class 1
Portfolio Market Weight: 100.0

Partition Bucket	Market Weight (%)		Ret ex Common Factors		Outperformance (bps)	
	Average				Asset Allocation	Security Selection
	Port	Bench	Port	Bench		
Total	100.0	100.0	161.6	129.8	18.9	13.2
Treasury	31.7	31.7	315.4	201.1	0.0	36.3
Government-Related	14.3	14.3	123.9	129.5	-0.0	-0.8
Corporate	30.7	19.8	107.9	196.1	7.4	-27.3
Securitized	22.8	33.4	49.1	27.1	11.0	5.1
Cash	0.5	0.8	0.0	0.0	0.5	0.0

Source: POINT®

For example, the corporate sector contributes +7.4bp to the outperformance, since it has been overweighted by about 10.9% (portfolio average weight: 30.7%, benchmark average weight: 19.8%) and its performance (+196.1bp) is much higher than the performance of the benchmark (+129.8bp). The approximate calculation yields 10.9% x (196.1 – 129.8) = +7.2bp, close to the +7.4bp reported.

The contribution to security selection is also listed and captures the performance advantage of the portfolio versus the benchmark, weighted by the portfolio weight of each sector. For the corporate sector, this calculation yields 30.7% x (107.9 – 196.1) = -27.1bp close to the -27.3bp reported.

In this analysis, we assumed that asset allocation occurs only at the major asset class level (Treasuries, government-related, corporates, securitized and cash). As we discussed above, both the HPA algorithm and its POINT® implementation support multiple decision layers. For example, let us assume that after the broad asset class allocation, there is a second allocation layer to the sectors of each asset class. Such a decision structure can be represented in POINT with a hierarchical nested partition. Running the same portfolio with a nested partition (but still using the Total Return model) yields dramatically different results, as seen in Figure 19. Asset allocation has shrunk from +18.9bp to +2.9bp, while security selection has grown from +13.2bp to +29.2bp.

Figure 19: USD Portfolio Summary using the Total Return Model (Nested Partition)

POINT®

USD: Summary

Portfolio: REPL : GlAgg G4 50
Benchmark : Global Agg G4
Partition : Class 2 Nested / System

Outperformance (USD)	
Portfolio Return (bps)	161.6
Benchmark Return (bps)	129.8
Outperformance (bps)	**31.8**

Outperformance Details	
Asset Allocation	2.9
Security Selection	29.2
Leverage	-0.1

Source: POINT®

To understand this change, let us look into the details of the Asset Allocation report, as shown in Figure 20. In the Level 1 report, we see that the there are two columns for asset allocation (Box A), a top-level one labelled simply Asset Allocation and identical to the asset allocation from the simple partition (+18.9bp) and a Further Allocation column that captures subsequent allocation decisions (-16.0bp). The sum of the two gives the +2.9bp that was reported as asset allocation in the summary report. Three Level 2 reports illustrate the details of the sector allocation decisions within each of the three asset classes that are split into sectors (the partition does not have sub-sectors for Treasuries or cash).

For example, the corporate sector shows -8.6bp of underperformance contribution from Further Allocation (Box B). Its Level 2 decomposition table explains why: the portfolio had a large overweight to financial institutions, an underweight to industrials and no allocation to utilities (Box C). While both industrials and utilities outperformed the corporate benchmark, financials had a relatively lacklustre performance of +157.0bp relative to +196.1bp of the corporates (Box D). Therefore, all three sectors have negative contribution to outperformance, with the largest (-5.1bp) coming from financial institutions. Once again, this can be approximated with the recursive application of Equation (19) as follows: 30.7% x (80.4% - 38.5%) x (157.0 – 196.1) = -5.0bp close to the -5.1bp reported. The security selection numbers reported in Box E represent security selection within each sub-sector.

4.4. Excess Return Model

The Excess Return attribution model uses the return splitting algorithm to extract common factor returns from the total return and considers the remaining as excess return. In the fixed income world, portfolio managers like to manage their exposures to common factors such as the movement of the yield curve and implied volatility surface separately from their investment decision process. For example, consider a manager who chooses to hedge the overall exposure to the yield curve and makes investment decisions purely based on expectations of excess returns. In this case, she would like to see the total outperformance breakdown into curve and excess return so that she can see the effectiveness of the hedge, as well as the asset allocation/security selection choices separately. A similar approach can be used for factor-based returns in equity portfolios, using a return component to represent market return and allocating specific return using top-down attribution.

Figure 20: USD Asset Allocation using the Total Return Model (Nested Partition)

POINT® Hybrid Performance Attribution®

USD: Asset Allocation

Portfolio: REPL : GlAgg G4 50
Benchmark : Global Agg G4
Partition : Class 2 Nested / System

Period: 7/30/2010 to 8/31/2010
Base Currency: USD
Curve Type: Treasury

Level 1: Class 2 Nested
Portfolio Market Weight: 100.0

Partition Bucket	Market Weight (%) Average		Ret ex Common Factors		Outperformance (bps)		
	Port	Bench	Port	Bench	Asset Allocation	Further Allocation	Security Selection
Total	100.0	100.0	161.6	129.8	18.9	-16.0	29.2
Treasury	31.7	31.7	315.4	201.1	0.0		36.3
Government-Related	14.3	14.3	123.9	129.5	-0.0	-4.7	3.9
Corporate	30.7	19.8	107.9	196.1	7.4	-8.6	-18.7
Securitized	22.8	33.4	49.1	27.1	11.0	-2.6	7.6
Cash	0.5	0.8	0.0	0.0	0.5		0.0

Level 2: Government-Related: Class 2
Portfolio Market Weight: 14.3

Partition Bucket	Market Weight (%) Average		Ret ex Common Factors		Outperformance (bps)	
	Port	Bench	Port	Bench	Asset Allocation	Security Selection
Government-Related	100.0	100.0	123.9	129.5	-4.7	3.9
Agency	98.3	70.5	121.7	94.3	-1.4	3.9
Local Authority		6.8		330.7	-2.0	0.0
Sovereign	1.7	12.6	253.4	232.7	-1.6	0.0
Supranational		10.0		113.5	0.2	0.0

Level 2: Corporate: Class 2
Portfolio Market Weight: 30.7

Partition Bucket	Market Weight (%) Average		Ret ex Common Factors		Outperformance (bps)	
	Port	Bench	Port	Bench	Asset Allocation	Security Selection
Corporate	100.0	100.0	107.9	196.1	-8.6	-18.7
Industrial	19.6	51.1	171.3	214.6	-1.8	-2.6
Utility		10.4		251.2	-1.8	0.0
Financial Institutions	80.4	38.5	92.4	157.0	-5.1	-16.1

Level 2: Securitized: Class 2
Portfolio Market Weight: 22.8

Partition Bucket	Market Weight (%) Average		Ret ex Common Factors		Outperformance (bps)	
	Port	Bench	Port	Bench	Asset Allocation	Security Selection
Securitized	100.0	100.0	49.1	27.1	-2.6	7.6
MBS Passthrough	100.0	92.2	49.1	15.9	-0.2	7.6
ABS		0.6		101.2	-0.1	0.0
CMBS		6.4		181.3	-2.3	0.0
Covered		0.8		35.5	-0.0	0.0

Source: POINT®

Currently, POINT® offers two versions of the Excess Return model: The Excess of Curve Return model, in which interest rates are considered to be the only common factor of return, and the Excess of Curve & Volatility Return model, in which both interest rates and implied volatility are considered to be common factors for fixed income securities. Other models such as alternative weights or common factors for equities will be introduced over time. In both models, we also exclude intra-day, pricing difference and exclusions from the excess return.

The outperformance due to exposure to common factors is explained by bottom-up aggregation, which we will discuss in detail in the following sections. The excess return outperformance is explained by top-down decomposition using the relative allocation model of Section 3.4, with sector excess returns and sector market value weights as in the Total Return attribution model.

Sector Allocation	$w^P \cdot \sum_s \left(\frac{w_s^P}{w^P} - \frac{w_s^B}{w^B} \right) \cdot \left(ER_s^B - ER^B \right)$	(22)
Sector Management	$\sum_s w_s^P \cdot \left(ER_s^P - ER_s^B \right)$	(23)
Top-Level Exposure	$\left(w^P - w^B \right) \cdot ER^B$	(24)

Once again, if neither the portfolio nor the benchmark is leveraged, the top-level weights are equal to one and the top-level exposure term is equal to zero.

Figure 21 shows the outperformance breakdown when using the Excess Return model with the nested partition. The left-hand side table shows the results when interest rates are treated as common return factors. Most of the outperformance (+24.6bp) comes from the yield curve exposure. Asset allocation is responsible for -2.2bp and security selection for +9.7bp. Since this model has dependence on pricing analytics (interest rates and implied volatility sensitivities), other terms also appear (*Pricing Differences* and *Exclusions*) but are insignificant. For more details on these terms, refer to Section 7.3.

Figure 21: USD Portfolio Summary using the Excess Return Model (Nested Partition)

Excess of Curve

Outperformance Details	
Yield Curve	24.6
Asset Allocation	-2.2
Security Selection	9.7
Leverage	-0.0
Pricing Differences	0.0
Exclusions	-0.0

Excess of Curve and Implied Volatility

Outperformance Details	
Yield Curve	24.6
Implied Volatility	1.9
Asset Allocation	-4.0
Security Selection	9.6
Leverage	-0.0
Pricing Differences	0.0
Exclusions	-0.0

Source: POINT®

The right-hand table shows the results when, in addition to interest rates, implied volatility is also treated as a common factor. The Implied Volatility term contributes +1.9bp to outperformance, altering the asset allocation contribution to -4.0bp and the security selection contribution to +9.6bp.

4.4.1. Outperformance due to curve exposure

As discussed in Section 4.1, the pricing framework for most fixed-income securities essentially amounts to projecting future cash flows and discounting them at projected paths of interest rates. Different interest rate curves can be used as the reference curve, depending on the type of the security. In POINT®, we support option-adjusted analysis with respect to both the government curve and the swap curve in most currencies.[17] In addition, for tax-exempt municipal securities, we offer option-adjusted analysis with respect to the municipal AAA curve. In HPA, the user can choose which curve to use as the basis of the analysis.

The curve contribution to return comprises curve change (the effect of instantaneous rate moves) and curve carry (the effect of the change of curve discount factors or interest rate-related projected cash flows with the passage of time, assuming that rates stay unchanged).

[17] Both government-curve and swap-curve based analytics are available for the USD, EUR, GBP, JPY, AUD, NZD, CAD, CHF, SEK, NOK, DKK, CZK, HUF, PLN, RUB and ZAR. Only swap-curve based analytics are available for the ARS, BRL, CLP, CNY, COP, HKD, IDR, ILS, INR, ISK, KRW, MXN, MYR, PEN, PHP, RON, SGD, THB, TRY and TWD.

There exist two different philosophies with respect to the definition of what constitutes a rate move, and they result in different breakdowns between curve carry and curve change, but the total curve effect (their sum) is identical under both methodologies.

Methodology I: Rolling on forwards

A curve is deemed unchanged if the rates realize the values implied by the forward rates in the previous period. For example, if we look at the curve one month into the future, the 1y rate is equal to the one month forward 1y rate calculated from today's curve.

In this methodology, a curve change is calculated with respect to the projected forward rates, and carry always accrues at the short rate.

This method is not very popular (particularly among cash investors) because it introduces significant complexity in the estimation of curve change return, which is the most important component of fixed income returns. In addition, forward rates are not particularly good predictors of the realised path of interest rates, further reducing the appeal of this method.

Methodology II: Parallel translation

A curve is deemed unchanged if the level and shape of the curve remains unchanged. For example, if we look at the curve one month into the future, the 1y rate is equal to the 1y rate in today's curve.

In this methodology, a curve change is calculated with respect to today's curve, and carry accrues at a different rate for each cash flow.

However, the representation of the yield curve (e.g., par rates, zero rates, forward rates) is important and affects the breakdown of total curve return between carry and change. If instantaneous forward rates are used to represent the curve, then each cash flow accrues carry at a rate equal to the forward rate corresponding to the cash flow date. Securities with deterministic cash flows accrue carry at a rate equal to the average of the forward rates corresponding to the date of each cash flow, weighted by the present value of each cash flow. Unfortunately, the forward rate representation is not very popular since forward rates are not directly observable in the market. Investors prefer the better understood zero or par rates to represent a curve. Under both representations, the calculation of the carry of a set of cash flows is complicated.

POINT® uses par rates to represent yield curves and the parallel translation methodology to estimate changes to such curves.

Since interest rate curves are infinite-dimension objects, we need to represent them with a finite collection of factors. The simplest method is to choose a small discrete set of curve points (key-rate points) dispersed along the curve. The premise is that the dynamics of the curve can be represented with the dynamics of the key-rates with a high degree of accuracy. Intermediate curve points are assumed to move as a linear combination of the moves of adjacent key-rates. We define the key-rates to be the 6m, 2y, 5y, 10y, 20y and 30y points for most currencies (one exception is the JPY, for which we use one additional key-rate at the 7y point).

Although our key-rates representation of the curve is highly accurate from a risk perspective (captures more than 99.9% of its variance), there may be deviations from a return perspective. Indeed, there are periods where, for example, the 3-yr US Treasury rate moves very differently than what is implied from the 2-yr and 5-yr rates. For this reason, we use scenario-based return splits to capture the total contribution of yield curve changes to the return of each security. We use analytics (key-rate durations) to explain the bulk of curve outperformance, but we also have the scenario-based outperformance calculation that captures any residual outperformance from intermediate curve points and convexity.

Curve Carry Calculations

As discussed above, curve carry calculation is complex even for bullet securities under par rate representations of yield curves. For this reason, we employ an approximation method for the calculation of curve carry called the Curve-Matching Portfolio (CMP) method.

The basic idea is to define a set of "par bonds" whose curve carry can be easily computed analytically, and then construct a carry-matching portfolio of such bonds for each portfolio or benchmark security. The matching algorithm attempts to create a portfolio that earns the same curve carry as the security.

1. We construct one par bond corresponding to each key-rate point in the currency
2. Each par bond has a coupon equal to the par key-rate of the corresponding maturity and a price of par (100). The payment frequency and day count convention match those used for curve construction. As a result, all par bonds on the day of construction have an option-adjusted spread of zero by construction.
3. For each security in the portfolio and the benchmark, we construct a CMP consisting of these par bonds and cash, such that the KRDs of this portfolio are equal to the KRDs of the corresponding security. Cash is needed to make sure that the CMP and the security are market value matched.

The amount of cash needed to replicate any particular bond can vary according to the duration profile and price of the bond. Matching a bond with a CMP is akin to replicating the cash flows of the bond with the set of par bonds. Intuitively, if a bond has a higher coupon than the current par rates, we must use leverage in the matching portfolio (borrowing at the short rate to fund the future cash flows) to match the cash flow profile of the bond. This leads to a negative amount of cash in the CMP.

HPA reports an average curve carry outperformance, as well as curve carry outperformance at each key-rate point. If we approximate the carry return of each par bond or cash as its yield (by definition the corresponding key-rate level) multiplied by the elapsed time $y_j \Delta t$ and we use ω_j to denote the CMP weight of the portfolio or benchmark allocated to each par bond or cash, then the curve carry outperformance can be broken down per key-rate point contribution as $\sum_j \left(\omega_j^P - \omega_j^B\right) \cdot y_j \cdot \Delta t$. If we define the average portfolio or benchmark yield as the average of all key-rates using the CMP portfolio weights, the above formula can be re-written as $\left(y_{avg}^P - y_{avg}^B\right) \cdot \Delta t$. The excess curve carry contribution of each key-rate over the average carry can be defined, but the sum over all key rates will be zero. Thus, curve carry can be broken down generically using the following equations:

Outperformance from avg. carry	$\left(y_{avg}^P - y_{avg}^B\right) \cdot \Delta t$	(25)
Key-rate contributions	$\sum_j \left(\omega_j^P \left(y_j - y_{avg}^P\right) - \omega_j^B \left(y_j - y_{avg}^B\right)\right) \cdot \Delta t$	(26)

If the average yield is set to zero for both portfolio and benchmark, then the average carry term is zero and the curve carry outperformance is attributed to each key-rate. If the average yield is calculated using the CMP weights for portfolio and benchmark, then the average carry is equal to the total carry contribution and the sum of the excess key-rate contributions is equal to zero.

Curve Change

Curve change outperformance is a result of duration profile differences between the portfolio and the benchmark and the change in the curve. We use the scenario-based return decomposition described in Section 4.1 to compute the total curve change return and hence the outperformance. Then, by using the analytics (OAD and KRDs), we further decompose the total curve change outperformance into that due to average parallel shift, reshaping (at the key-rate points), rest of curve, and convexity.

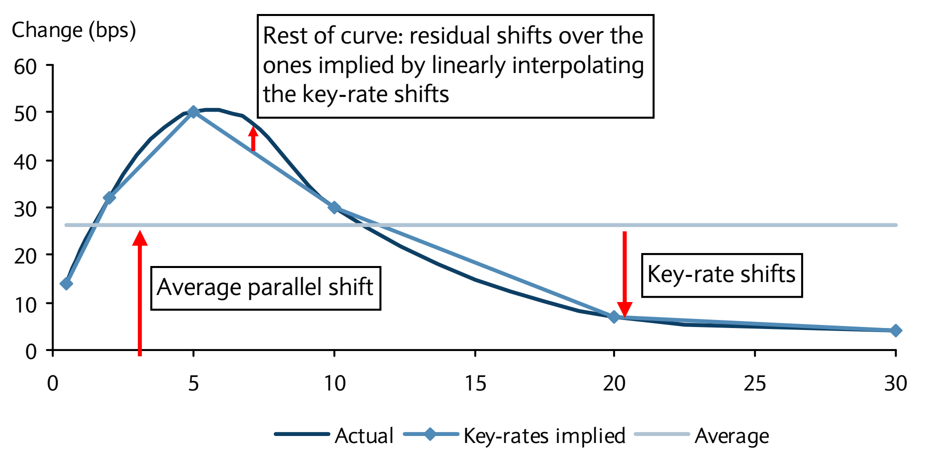
The outperformance from average parallel curve shift is computed by applying the bottom-up aggregation, using key-rate durations as loadings and key-rate changes as factors. Similar to curve carry, we also introduce the notion of an average yield change (parallel shift) that can be used to control how the curve change outperformance is distributed between the key-rate contributions and the parallel shift term.

Outperformance from avg. parallel shifts	$-\left(OAD^P - OAD^B\right)\cdot \Delta y_{avg}$	(27)
Outperformance from reshaping	$-\sum_j \left(KRD_j^P - KRD_j^B\right)\cdot\left(\Delta y_j - \Delta y_{avg}\right)$	(28)

If the average yield change is set to zero, the curve change outperformance will be attributed to each key-rate separately. If the average yield change is set equal to an appropriately defined parallel shift, the key-rate terms capture the outperformance in excess of the parallel shift; i.e., the curve re-shaping effect on outperformance.

Since we represent the shape of the curve by key-rate points, there will always be a small fraction of curve change outperformance not captured by the above methodology, as explained at the beginning of this section. Figure 22 illustrates the change not captured by the key-rates.

Figure 22: Approximation of Curve Change by Key-rates



Source: Barclays Capital

HPA reports the curve outperformance in excess of key-rates, as well as return from convexity and other higher-order terms as Rest of Curve and Convexity. It is calculated as the difference between total curve change return from the scenario-based decomposition and the sum of the average parallel shift and reshaping return.

The detailed Yield Curve report, which helps users understand the breakdown of the yield curve-related outperformance, has been discussed in Section 1.3.1. In particular, we analyzed the calculation of the outperformance due to yield changes, but did not discuss the contributions from curve carry which were much smaller. To complete the analysis, we now illustrate how the contributions of curve carry are calculated by focusing on the euro yield curve of the example in Section 1 which has the highest carry contribution of all currencies[18]. In Figure 23, we repeat the euro Yield Curve report first seen in Figure 7. In Box A, we highlight the average yield of the portfolio and the benchmark calculated as the yield of the corresponding curve-matching portfolios. Since the portfolio is longer than the benchmark in an upward sloping curve, its average yield is higher than the benchmark (2.030% versus 1.698%). The corresponding average curve carry contribution can be approximated using Equation (25) as the excess yield of the portfolio times the elapsed time of one month, (2.030% - 1.698%) x (1/12) = +2.8bp, close to the reported +2.9bp in Box B. The additional contributions of key-rate points can be estimated using Equation (26). For example, the 2y point contributes approximately 39.5% x (0.771% - 2.030%) – 37.4% x (0.771% - 1.698%) x (1/12) = -1.3bp, close to the -1.4bp reported in Box C.

Figure 23: Yield Curve Report

POINT® — Hybrid Performance Attribution®

EUR: Yield Curve (Treasury)

Portfolio: REPL : GlAgg G4 50
Benchmark : Global Agg G4
Partition : Class 2 Nested / System

Period: 7/30/2010 to 8/31/2010
Base Currency: USD
Curve Type: Treasury

A B C

	Yield				Curve-Matched Market Weight (%)					Duration (yrs)					Outperformance (bps)		
	Level (%)		Change		Average		Overweight			Average		Overweight			Explained by Yield Curve		
	Port	Bench	Port	Bench	Port	Bench	Average	Min	Max	Port	Bench	Average	Min	Max	Carry	Change	Total
Parallel Shift																	
Average	2.030	1.698	-50.2	-50.2	100.0	100.0	-0.0	-0.0	0.0	6.68	5.62	1.05	1.00	1.11	2.9	54.2	57.1
Key Rates & Cash																	
Cash	0.279	0.279	0.0	0.0	-13.1	-14.5	1.3	0.8	1.5	0.00	0.00	0.00	0.00	0.00	0.2	0.0	0.2
6m	0.381	0.381	0.6	0.6	4.0	16.2	-12.2	-12.8	-11.6	0.02	0.08	-0.06	-0.06	-0.06	1.8	3.1	5.0
2y	0.771	0.771	-22.6	-22.6	39.5	37.4	2.2	1.4	3.1	0.79	0.74	0.04	0.03	0.06	-1.4	-1.3	-2.6
5y	1.726	1.726	-43.1	-43.1	22.0	33.7	-11.7	-11.8	-11.6	1.07	1.64	-0.57	-0.57	-0.56	-1.2	4.0	2.8
10y	2.708	2.708	-57.3	-57.3	40.1	18.5	21.6	21.4	21.9	3.65	1.68	1.97	1.96	1.98	0.8	14.1	14.9
20y	3.377	3.377	-64.7	-64.7	7.5	5.8	1.7	1.3	2.1	1.16	0.89	0.26	0.21	0.31	-0.1	3.5	3.4
30y	3.324	3.324	-69.6	-69.6	0.0	3.0	-3.0	-3.0	-2.9	0.00	0.59	-0.59	-0.60	-0.58	-0.1	-11.5	-11.6
Rest of Curve & Convexity																	
															0.0	8.3	8.3
Total Yield Curve Levels & Shifts																	
															2.9	**74.4**	**77.4**

Source: POINT®

4.4.2. *Outperformance due to implied volatility exposure*

For fixed-income securities with optionality, our pricing models are calibrated to reflect the Black swaption-implied volatility surface prevailing in the market. Therefore, changes in implied volatility create returns on the embedded option that must be attributed correctly in performance attribution. Similar to how we measure the contribution of curve changes, we use both scenario- and analytics-based calculations to break down the contribution of

[18] In Figure 6 and
Figure 7, we see the un-weighted curve carry outperformance of all four currencies. The highest one comes from the pound, but because of the small market weight of the pound portfolio, its contribution is very small. The highest contribution comes from the euro portfolio, which has an un-weighted carry outperformance of +2.9bp and a weight of 23.2%, for a total contribution of +0.7bp.

implied volatility changes to outperformance. HPA reports outperformance due to changes in implied volatility in terms of volatility decay, parallel shift, and surface reshaping.

Volatility Decay

Volatility decay captures the change in option value due to the lapse of time. The return from volatility decay is difficult to capture exactly, as it requires re-simulating interest rate paths with the shifted time parameter of volatility while keeping other time parameters unchanged. However, we can approximate volatility decay by subtracting all other components of the total time return, such as curve and spread carry.

Outperformance due to volatility decay is the difference between the weighted average volatility decay return of the portfolio and the benchmark.

Implied Volatility Change

The outperformance due to implied volatility change is a result of the difference in volatility exposures of the portfolio and the benchmark and the change in the implied volatility surface. The exposure to parallel shift is measured by vega, and the parallel shift return is equal to vega multiplied by the average implied volatility change over the entire surface. The implied volatility surface reshaping return is the scenario-based total implied volatility return minus the return from the parallel shift.

The outperformance for both parallel shift and reshaping is the difference between the weighted average return of the portfolio and the benchmark. Figure 24 shows the Implied Volatility report for the dollar portfolio from the example of Section 1, displaying outperformance due to implied volatility exposure broken down by partition buckets.

Figure 24: Implied Volatility Report

POINT® Hybrid Performance Attribution®

USD: Implied Volatility

Portfolio: REPL : GIAgg G4 50
Benchmark : Global Agg G4
Partition : Class 2 Nested / System

Period: 7/30/2010 to 8/31/2010
Base Currency: USD
Curve Type: Treasury

	Volatility				Market Weight (%)		Parallel Vol Exposure					Outperformance (bps)			
	Initial Level (%)		Total Change (%)		Average		Average		Overweight				Explained by Volatility		
	Port	Bench	Port	Bench	Port	Bench	Port	Bench	Mean	Min	Max	Decay	Parallel	Reshaping	Total
Treasury	30.6	30.6	5.712	5.712	31.72	31.66	0.00	0.00	0.00	0.00	0.00	-0.0	0.0	0.0	-0.0
Government-Related	30.6	30.6	5.712	5.712	14.32	14.29	0.00	-0.03	0.03	0.03	0.04	-0.0	0.2	-0.2	-0.0
Corporate	30.6	30.6	5.712	5.712	30.71	19.85	-0.12	-0.01	-0.11	-0.13	-0.08	0.1	-0.6	0.9	0.3
Securitized	30.6	30.6	5.712	5.712	22.76	33.42	-2.49	-3.60	1.10	0.87	1.38	-0.6	6.6	-4.5	1.5
Cash	0.0	0.0	0.000	0.000	0.49	0.78	0.00	0.00	0.00	0.00	0.00	0.0	0.0	0.0	0.0
Total Implied Volatility												-0.5	6.2	-3.8	**1.9**

Source: POINT®

Volatility exposure occurs only in the government-related, corporate and securitized sectors. The ability of an issuer to call a bond and the prepayment option of mortgage-backed securities give issuers of such bonds optionality. Conversely, holders of such bonds are short volatility, as the negative exposure numbers in Box A indicate. By being long the corporate and short the securitized sector, the portfolio is implicitly underweighted in volatility in the corporate sector and overweighted volatility in the securitized. Since on average implied volatilities increased 5.712% during this month (Box B), the corporate sector contributes negatively to parallel volatility outperformance (-0.6bp in Box C), and the securitized sector contributes positively (+6.6bp in Box C). Overall, the portfolio is long volatility since the volatility exposure of the securitized sector is much bigger than that of the corporate and government-related sectors. The total parallel volatility change outperformance contribution is +6.2bp. The total contribution of changes in the implied volatility surface is calculated from the return splits scenarios, as described above. From that, we imply the surface reshaping (net of the parallel shift) contribution to be -3.8bp. Finally, a net long volatility portfolio will suffer loss of return because of volatility decay. In this case, it has been calculated to be -0.5bp. Putting it all together in Box D, we calculate the total contribution of implied volatility to the outperformance of the dollar portfolio to be +1.9bp.

4.5. Fully Analytical Model

The Fully Analytical attribution model offers the flexibility to match the increasing diversity in portfolio management structures. It takes full advantage of the return splits by decomposing returns from each factor individually. The model can accommodate either bottom-up aggregation or top-down decomposition, depending on how the factor exposures are managed in the portfolio. It is suitable for portfolios with multiple asset classes, as they often have exposures to different market risk factors. For example, consider a portfolio consisting of corporate bonds, mortgage-backed securities and inflation-linked bonds. While all of these securities are exposed to the change in the yield curve and implied volatility (for those with optionality), mortgage securities have additional exposure to prepayment risk and mortgage spread, and inflation-linked securities have additional exposure to their reference inflation indices. Similar to how the curve risk is managed, managers of mortgage and inflation-linked securities tend to manage these additional risk factors as well. The ability to break down outperformance due to these specific factors can be very valuable to the managers for explaining the performance of their decisions.

Furthermore, the Fully Analytical model allows portfolio managers to use measures other than market value as allocation weight. This is particularly useful for managers who allocate their capital to various sectors based on risk exposures rather than market values. For instance, a manager taking views on sector spread movement may think of over/underweight in terms of OASD. In the credit space, the concept of DTS has been gaining traction in recent years as an alternative measure of spread risk.[19] As a result, credit portfolio managers may prefer to use OASD or DTS as an allocation weight.

In our current implementation of the full model, we treat the outperformance from curve and implied volatility as the return from common-factors and explain them using bottom-up aggregation, as described in Sections 4.4.1 and 4.4.2. The outperformance from mortgage, inflation and residual is also explained using bottom-up aggregation. We will describe the details of their decomposition in the following sections. These factors are calculated per security and included in the Security Selection report to give full details on the outperformance of securities and sectors.

4.5.1. Outperformance due to option-adjusted spread

Outperformance from OAS is explained using top-down decomposition into asset allocation and security selection per user-defined hierarchical partition or sector. We decompose all three components of spread outperformance (carry, spread duration, and spread convexity) separately using the top-down decomposition algorithm. Currently, there are two versions of the Fully Analytical model: the top-level spread return model and the sector-level spread return model. In both, spread carry outperformance is decomposed by applying the absolute allocation algorithm using market value weights and carry returns. The difference in how we decompose spread change outperformance is explained below.

[19] See Ben Dor et al. 2010.

Top-level spread model

The top-level spread model assumes that there is a decision to over/underweight the OASD against the benchmark at the portfolio level (top level). Subsequent decisions to allocate exposure to various sectors are made relative to the portfolio OASD. This implies that allocation weight is measured by the ratio of the OASD contribution of each sector ($w_s OASD_s$) over the portfolio OASD. In other words, we apply the relative allocation algorithm of the top-down algorithm of Section 3.4 using $\alpha_k \leftarrow OASD$, $\alpha_{k,s} \leftarrow w_s OASD_s$, $f_{k,s} \leftarrow -\Delta OAS_s$ and $-\Delta OAS^B$ as a hurdle rate.

Asset Allocation $$-OASD^P \cdot \sum_s \left(\frac{w_s^P OASD_s^P}{OASD^P} - \frac{w_s^B OASD_s^B}{OASD^B} \right) \cdot \left(\Delta OAS_s^B - \Delta OAS^B \right) \quad (29)$$

Security Selection $$-\sum_s w_s^P OASD_s^P \cdot \left(\Delta OAS_s^P - \Delta OAS_s^B \right) \quad (30)$$

Spread Duration Mismatch $$-\left(OASD^P - OASD^B \right) \cdot \Delta OAS^B \quad (31)$$

The Top-Level Exposure term here has been re-named Spread Duration Mismatch.

Sector-level spread model

In contrast to the top-level model, the sector-level model assumes no such spread duration decision at the portfolio level. Instead, each sector allocation decision is made without any top-level restriction. In this case, the appropriate allocation weight is the absolute, rather than relative, OASD contribution. Furthermore, we set the hurdle rate to zero, as the asset allocator is expressing her view on the absolute (rather than relative) changes in sector OAS. In other words, we apply the absolute top-down decomposition using $\alpha_{k,s} \leftarrow w_s OASD_s$ and $f_{k,s} \leftarrow -\Delta OAS_s$.

Asset Allocation $$-\sum_s \left(w_s^P OASD_s^P - w_s^B OASD_s^B \right) \cdot \Delta OAS_s^B \quad (32)$$

Security Selection $$-\sum_s w_s^P OASD_s^P \cdot \left(\Delta OAS_s^P - \Delta OAS_s^B \right) \quad \mathbf{(33)}$$

In Figure 25 we show the USD Local Management outperformance decomposition of the portfolio from Section 1 when using the Fully Analytical model with either top-level or sector-level spread duration management. In both models, a new category, Mortgage, has appeared and is responsible for -8.2bp of outperformance.[20] In both models, the security selection contribution is +9.3bp. The difference comes from the treatment of spread duration exposure. The top-level model treats it as a portfolio-level decision (essentially a common return factor) contributing -1.8bp of outperformance under the label Spread Duration Mismatch, while asset allocation is listed as +4.9bp. The sector-level model makes spread duration exposure part of the allocation decision and essentially combines the two terms, reporting asset allocation of +3.2bp.

[20] We will discuss this term in more detail below.

Figure 25: USD Portfolio Summary using the Fully Analytical Model (Nested Partition)

Top-Level

Outperformance Details	
Yield Curve	24.6
Implied Volatility	1.9
Asset Allocation	4.9
Security Selection	9.3
Leverage	-0.0
Spread Duration Mismatch	-1.8
Mortgage	-8.2
Residual	1.3
Others	-0.0

Sector-Level

Outperformance Details	
Yield Curve	24.6
Implied Volatility	1.9
Asset Allocation	3.2
Security Selection	9.3
Leverage	-0.0
Mortgage	-8.2
Exclusions	-0.0
Residual	1.3
Others	0.0

Source: POINT®

Figure 26 contains the Asset Allocation report to major asset classes (first level of the nested partition) using the sector-level Fully Analytical attribution model. Outperformance from asset allocation and security selection from each partition bucket is displayed in the report, along with other useful information such as OAS and the changes in OAS, OASD, OASC and market value weights.

Figure 26: Asset Allocation Report for the Sector-Level Fully Analytical Model (Nested Partition)

POINT® Hybrid Performance Attribution®

USD: Asset Allocation

Portfolio: REPL : GlAgg G4 50
Benchmark : Global Agg G4
Partition : Class 2 Nested / System

Period: 7/30/2010 to 8/31/2010
Base Currency: USD
Curve Type: Treasury

Level 1: Class 2 Nested
Portfolio Market Weight: 100.0

Partition Bucket	OAS (bps)				OASD (yrs)		OASC		Market Weight (%)		OASD Contr. (yrs)		Outperformance (bps)								
	Average		Change		Average		Average		Average		Average		Asset Allocation			Further Allocation			Security Selection		
	Port	Bench	Port	Bench	Port	Bench	Port	Bench	Port	Bench	Port	Bench	Carry	Change	Total	Carry	Change	Total	Carry	Change	Total
Total	74.2	57.1	5.5	8.3	4.7	4.5	0.5	0.4	100.0	100.0	4.7	4.5	1.5	2.6	4.1	0.3	-1.2	-0.9	-0.2	9.6	9.3
Treasury	0.8	-1.3	0.5	0.5	6.5	5.2	0.9	0.5	31.7	31.7	2.1	1.6	-0.0	-0.3	-0.3		0.0		0.1	0.0	0.1
Government-Related	80.4	65.1	-2.4	3.7	3.7	4.0	0.2	0.3	14.3	14.3	0.5	0.6	0.0	0.1	0.1	-0.2	0.2	-0.0	0.4	3.1	3.5
Corporate	182.5	186.5	11.8	14.1	4.4	6.2	0.3	0.7	30.7	19.8	1.4	1.2	1.2	-1.7	-0.4	0.9	0.2	1.0	-1.0	2.7	1.7
Securitized	28.1	33.4	13.1	15.9	3.4	3.2	0.3	0.2	22.8	33.4	0.8	1.1	0.2	4.4	4.7	-0.4	-1.5	-1.9	0.3	3.8	4.1
Cash	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.5	0.8	0.0	0.0	0.0	0.0	0.0		0.0		0.0	0.0	0.0

Source: POINT®

The asset classes with the biggest contribution to spread change allocation outperformance are securitized with +4.4bp and corporate with -1.7bp (Box A). These numbers can be explained with information from Boxes B and C. For example, the securitized sector has an underweight of -0.3y in terms of spread duration (0.8 versus 1.1), while spreads in this sector widened 15.9bp. Note that since we use the sector-level model, the hurdle rate is zero; therefore, we do not need to compare the spread widening of each sector with that of the benchmark. The approximate calculation estimates the contribution as -0.3 x -15.9 = +4.8bp, relatively close to the reported +4.4bp in Box A.

Spread carry contributes +1.5bp, coming mostly from the corporate sector, which has high spread (+186.5bp versus +57.1bp for the benchmark) and is market value overweighted (30.7% versus 19.8%). The approximate calculation is (30.7% - 19.8%) x (186.5 – 57.1) x (1/12) = +1.2bp, exactly as reported in Box D.

This report also includes columns for further allocation[21] and security selection, consistent with the nested partition used. Further reports (not shown here) go into the details of each.

From this report, we can also confirm the USD spread duration mismatch term from the top-level model. Indeed, the portfolio is slightly longer than the benchmark in terms of spread duration (4.7 versus 4.5 years in Box B), and benchmark spreads widened 8.3bp on average (Box C). Multiplying the spread overweight by the negative of the benchmark spread change yields +0.2 x (-8.3) = -1.7bp close to the -1.8bp reported in Figure 25.

4.5.2. Mortgage factors

The present value of mortgage-related securities (MBS, CMBS, etc.) depends on additional factors such as prepayments and mortgage rates. HPA uses the Barclays Capital mortgage security pricing model to compute daily mark-to-market returns. To help managers better understand the source of returns from these securities, we report outperformance due to exposure to mortgage factors separately using bottom-up aggregation.

Prepayment Surprise Return

Securities involving prepayments are priced using model predictions of future cash flows. Over a month, the realised prepayment rate is generally different from the one predicted by the model. The difference gives rise to a return component that is not captured by analytics; hence we compute and report separately as Prepayment Surprise return. Following index convention, prepayments are recognised on the first of the month based on a predicted rate (which, if different from the model-predicted one, will generate surprise return). Over the course of the month, the actual prepayment rate can be more accurately predicted and used to adjust the returns, leading to more days where the surprise return is non-zero. Once the actual rate is known and returns are adjusted to reflect it, no more surprise return for that particular month will occur.

Mortgage Spread Change Return

A common factor that affects the cash flows of all mortgage securities is the difference between the model-implied mortgage rate and the actual mortgage rate observed in the market. We call this difference the mortgage spread and the price sensitivity of a security to this spread the mortgage spread duration. The return due to mortgage spread change is simply the product of the mortgage spread duration and the negative of the change in the mortgage spread, i.e., $-MtgRtDur \cdot \Delta MtgSpread$. The outperformance is the difference between the weighted average mortgage spread return of the portfolio and the benchmark.

Other Mortgage Factors

A number of other parameters such as home price appreciation projections and swap spreads are also used in mortgage pricing models. In our scenario-based return splits, we shift all such parameters and record the return as other market return. For mortgage securities, we report the difference between the other market return and the mortgage spread return as return from other mortgage factors.

Figure 27 displays part of the Security Selection report for the USD portfolio of our example running the Fully Analytical model with the nested partition. It is organised across the major asset classes and their sub-sectors. The contribution of each security to security selection coming from spread carry and spread change is detailed and explained using security

[21] It is interesting to note that further allocation uses the relative spread duration allocation model, since the spread duration of each major asset class has been determined at the first level of allocation. The spread duration allocation to sub-sectors must be consistent with the pre-determined spread duration of the asset class.

analytics. The contribution of each security to other sources of outperformance is also shown. In this particular example, the additional columns include mortgage-related factors (prepayments: actually prepayment surprise, mortgage spread, PPM and swap spread: capturing other inputs to the mortgage prepayment and valuation model), pricing, and residual. Other bottom-up terms will also be shown here as appropriate, such as inflation-related return (see next section) as well as trading return (see Section 7.2).

This section of the Security Selection report covers the securitized sector, which is of course solely responsible for the -8.2bp of underperformance attributable to mortgage factors. Box A shows that most of it comes from mortgage spread (-7.0bp), some from prepayments surprise (-1.7bp), while the contribution of other factors is positive (+0.5bp). Contributions of individual securities are also listed.

Figure 27: Security Selection Report for the Fully Analytical Model (Nested Partition)

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USD: Security Selection

Portfolio: REPL : GlAgg G4 50
Benchmark : Global Agg G4
Partition : Class 2 Nested / System

Period: 7/30/2010 to 8/31/2010
Base Currency: USD
Curve Type: Treasury

Bucket/Issue	Issuer/Coupon/Maturity	OAS	OAS Change	OASD	MV (%)		OASD Contrib (%)		Outperformance							
									Security Selection		Other					Total
					Port	Bench	Port	Bench	Spread Carry	Spread Change	Prepay-ments	Mortgage Spread	PPM Swap Spread	Pricing	Resi dual	
74977RBQ	RABOBANK 4.20 '14	126.5	-7.0	3.5	14.3	0.2	13.8	0.1	-0.3	2.6	0.0	0.0	0.0	0.0	0.1	2.4
90261XFY	UNION BANK OF SWI 3.88 '15	178.8	-0.8	4.1	12.4	0.1	14.1	0.1	-0.2	1.9	0.0	0.0	0.0	-0.0	0.1	1.8
61746BCW	MORGAN STANLEY DE 5.62 '12	201.3	-29.6	1.4	12.4	0.2	4.7	0.0	-0.1	1.8	0.0	0.0	0.0	0.0	-0.0	1.7
96008YAB	WESTFIELD CAPITAL 5.12 '14	182.8	-10.4	3.8	7.3	0.1	7.7	0.1	-0.1	1.7	0.0	0.0	0.0	0.0	0.1	1.7
46625HHN	JP MORGAN CHASE & 4.65 '14	136.3	3.3	3.5	17.3	0.2	16.7	0.2	-0.4	1.6	0.0	0.0	0.0	0.0	0.2	1.4
025816BA	AMERICAN EXPRESS 7.25 '14	175.1	2.7	3.3	12.9	0.1	11.9	0.1	-0.2	1.2	0.0	0.0	0.0	0.0	0.1	1.1
38141GEE	GOLDMAN SACHS GRO 5.35 '16	211.5	11.3	4.8	12.0	0.2	15.9	0.2	-0.1	0.4	0.0	0.0	0.0	-0.0	0.1	0.4
94973VAK	WELLPOINT INC-GLO 5.25 '16	148.2	31.0	4.8	11.4	0.1	15.2	0.1	-0.2	-2.3	0.0	0.0	0.0	-0.0	0.1	-2.5
Bmark Securities	Not in Portfolio	237.5	12.9	5.3		98.7		99.1	-0.0	0.1	0.0	0.0	0.0	0.0	-0.1	-0.1
Securitized		P 28.1 B 33.4	P 13.1 B 15.9	P 3.4 B 3.2	22.8	33.4	16.3	23.4	0.3	3.8	-1.7	-7.0	0.5	0.0	1.4	-2.7
MBS Passthrough		P 28.1 B 14.1	P 13.1 B 18.0	P 3.4 B 3.1	22.8	30.8	16.3	21.1	0.3	3.8	-1.7	-7.0	0.5	0.0	1.6	-2.5
FNA05003	FNMA Conventional 5.00 '32	10.0	9.7	3.1	21.3	2.2	19.7	2.2	-0.0	1.2	0.5	2.6	-0.1	0.0	-0.3	3.8
FNA04403	FNMA Conventional 4.50 '32	9.2	5.7	3.6	11.2	0.5	11.9	0.6	-0.0	1.1	0.2	1.3	-0.0	0.0	-0.2	2.4
FNA04409	FNMA Conventional 4.50 '39	21.1	9.7	3.8	19.7	6.3	22.3	7.8	0.0	0.9	0.2	1.2	0.0	0.0	-0.2	2.1
FGB04403	FHLM Gold Guar Si 4.50 '32	20.4	10.4	3.7	10.3	0.2	11.3	0.3	0.0	0.7	0.1	1.1	-0.0	0.0	-0.1	1.7
GNA04403	GNMA I Single Fa 4.50 '32	32.6	12.0	4.3	8.6	0.1	11.0	0.1	0.0	0.5	0.0	0.7	-0.0	0.0	0.0	1.2
GNF04403	GNMA I Single Fa 4.50 '18	54.7	25.4	2.8	29.0	0.0	23.9	0.0	0.2	-1.3	0.1	0.3	-0.0	0.0	0.7	-0.1
Bmark Securities	Not in Portfolio	13.7	19.0	3.0		90.7		89.0	0.0	0.7	-2.8	-14.2	0.8	0.0	1.7	-13.7
ABS		P B 66.9	P B 6.8	P B 3.8		0.2		0.2	0.0	0.0	0.0	0.0	0.0	0.0	-0.0	-0.0
Bmark Securities	Not in Portfolio	66.9	6.8	3.8		100.0		100.0	0.0	0.0	0.0	0.0	0.0	0.0	-0.0	-0.0
CMBS		P B 297.0	P B -5.9	P B 4.0		2.1		1.9	0.0	0.0	-0.0	0.0	0.0	0.0	-0.2	-0.2
Bmark Securities	Not in Portfolio	297.0	-5.9	4.0		100.0		100.0	0.0	0.0	-0.0	0.0	0.0	0.0	-0.2	-0.2
Covered		P B 125.9	P B 24.6	P B 3.8		0.3		0.2	0.0	0.0	0.0	0.0	0.0	0.0	-0.0	-0.0
Bmark Securities	Not in Portfolio	125.9	24.6	3.8		100.0		100.0	0.0	0.0	0.0	0.0	0.0	0.0	-0.0	-0.0

Source: POINT®

4.5.3. *Inflation factors*

Inflation-linked securities have cash flows linked to a specified price index. For example, the principal and interest payment of US TIPS are linked to the Consumer Price Index for All Urban Consumers (CPI-U), non-seasonally adjusted. Therefore, prices of inflation-linked securities also depend on future expectations of inflation.

The analysis of the risk and performance of inflation-linked securities is subject to debate. One school of thought prefers to treat them as independent of nominal interest rates and express risk and performance as a function of real rates (nominal rates minus inflation expectations). In this type of analysis, the return of inflation-linked securities comes from changes of real rates, as well as time return, which is a combination of real rate carry and realised inflation accretion.

Another school of thought prefers to decompose real rates into the difference of nominal interest rates and inflation expectations such that nominal interest rate risk and performance is accounted consistently across all types of bonds. Our current implementation follows the latter approach, as detailed in Lazanas and Rosten (2005). Inflation expectations are captured by a negative discounting spread (similar to OAS), which is called inflation spread. The return of inflation-linked securities is explained by changes in nominal interest rates and inflation spreads. The time return is captured by the nominal interest rate carry reduced by the negative inflation spread carry and increased by realised inflation accretion. Since realised inflation is announced only once a month, we need to make an assumption about the projected inflation of the current month to ensure smooth daily inflation accretion. When realised inflation is announced, it is generally different from projected inflation, producing a return that cannot be otherwise accounted for. This kind of return of inflation-linked securities is called inflation surprise and is accounted separately. Inflation surprise return is also present when current month inflation projections change.

In Figure 28, we show the Security Return Splits attribution report for the Barclays Capital Inflation Linked US TIPS index run using the Fully Analytical model for the period from July 16, 2010 to August 13, 2010. These dates correspond to the CPI-U announcement dates for the index level as of the end of June and July, respectively; therefore, the index should show a realised inflation return equal to the rate of inflation realised during July (but becoming known on August 13).

The total return of the index was +170.8bp, with the majority coming from nominal interest rates exposure (+27.4bp carry and +184.6bp of curve change). Inflation related return was negative at -40.6bp, most of it coming from falling inflation expectations (captured under inflation spread change as -29.7bp). Inflation accretion during July was -3.0bp, indicating that at the beginning of the month, the market actually expected a small amount of deflation at an annualised rate of -0.03% x 12 = -0.36%. The positive inflation surprise of +5.3bp indicates that the realised inflation was actually positive, although quite low. The total realised inflation return is +2.3bp and is the sum of inflation accretion (at the market expected rate) and inflation surprise. From that, we can calculate that July realised annualised inflation was approximately +0.023% x 12 = +0.28%. Indeed, the CPI reading for June was 217.965 and the one for July 218.011, implying an inflation rate for July of 218.011/217.965 -1 = 2.11bp or +0.25%. The inflation spread carry term of -13.1bp reduces the curve carry of +27.7bp that was credited to the index due to exposure to nominal rates. The difference of the two can be thought of as the carry earned by the index due to exposure to real rates, i.e., nominal rates reduced by inflation expectations. Another interesting thing to note is that inflation expectations did not fall uniformly across all maturities. Indeed, short bonds exhibited positive inflation spread return, indicating increasing inflation expectations over the next three years or so. This is also the case for very long bonds, with the 30y bond showing increasing inflation expectations. Inflation expectations fell only for bonds with maturities between 3.5 and 20 years. This indicates that inflation portfolio managers need to pay more attention to the term structure exposures to inflation using more detailed analytics that provide them with partial sensitivities to the inflation term structure.[22]

[22] Such analytics, analogous to key-rate durations for nominal rates, will be introduced to POINT® in an upcoming release.

Figure 28: Performance Attribution of Inflation-Linked Securities

POINT®

USD Security Return Splits
Portfolio: Portfolio153 — **Period: 7/16/2010 to 8/13/2010**
Benchmark : USD cash — **Base Currency: USD**
Partition : HPA Default Partition / System — **Curve Type: Treasury**

Bucket/Issue	Issuer	Coupon	Maturity	Ticker	MV%	Curve		Inflation					Total
						Carry	Change	Accretion	Carry	Surprise	Spread Change	Residual	Return
USD					100.0	27.7	184.6	-3.0	-13.1	5.3	-29.7	-1.9	169.9
US TIPS					100.0	27.7	184.6	-3.0	-13.1	5.3	-29.7	-1.9	169.9
9128277J	US TREASURY NOTES I/L	3.4	1/15/2012	US/T	1.4	6.4	5.9	-3.0	-6.5	5.2	30.4	-1.0	37.5
912828GN	US TREASURY NOTES I/L	2.0	4/15/2012	US/T	2.9	7.1	8.9	-3.0	-6.1	5.2	31.5	-0.6	43.0
912828AF	US TREASURY NOTES I/L	3.0	7/15/2012	US/T	5.0	7.8	12.5	-3.0	-7.5	5.2	34.4	-0.2	49.2
912828HW	US TREASURY NOTES I/L	0.6	4/15/2013	US/T	2.8	11.5	29.6	-3.0	-8.7	5.2	19.4	1.1	55.0
912828BD	US TREASURY NOTES I/L	1.9	7/15/2013	US/T	4.7	12.7	36.6	-3.0	-8.5	5.2	26.4	1.7	71.1
912828BW	US TREASURY NOTES I/L	2.0	1/15/2014	US/T	4.8	15.7	55.1	-3.0	-8.5	5.2	14.5	2.7	81.8
912828KM	US TREASURY NOTES I/L	1.3	4/15/2014	US/T	3.0	17.4	66.7	-3.0	-10.0	5.2	-7.7	2.9	71.6
912828CP	US TREASURY NOTES I/L	2.0	7/15/2014	US/T	4.4	19.0	77.3	-3.0	-9.6	5.2	-0.2	2.8	91.5
912828DH	US TREASURY NOTES I/L	1.6	1/15/2015	US/T	4.3	22.9	103.4	-3.0	-9.9	5.3	-6.6	2.0	114.0
912828MY	US TREASURY NOTES I/L	0.5	4/15/2015	US/T	2.1	25.5	120.0	-3.0	-12.2	5.2	-31.5	1.4	105.3
912828EA	US TREASURY NOTES I/L	1.9	7/15/2015	US/T	3.8	26.8	128.6	-3.0	-11.0	5.3	-21.2	0.4	125.8
912828ET	US TREASURY NOTES I/L	2.0	1/15/2016	US/T	3.7	28.6	154.3	-3.0	-11.5	5.3	-29.4	0.6	144.9
912828FL	US TREASURY NOTES I/L	2.5	7/15/2016	US/T	3.8	29.8	176.0	-3.0	-12.5	5.3	-49.4	0.7	146.8
912828GD	US TREASURY NOTES I/L	2.4	1/15/2017	US/T	3.4	31.4	199.3	-3.0	-12.6	5.3	-63.7	0.3	156.9
912828GX	US TREASURY NOTES I/L	2.6	7/15/2017	US/T	3.1	32.4	217.5	-3.0	-13.7	5.3	-66.4	0.1	172.1
912828HN	US TREASURY NOTES I/L	1.6	1/15/2018	US/T	2.9	34.5	245.0	-3.0	-14.2	5.3	-80.5	0.1	187.1
912828JE	US TREASURY NOTES I/L	1.4	7/15/2018	US/T	2.8	35.7	265.4	-3.0	-15.2	5.3	-88.9	-0.1	199.3
912828JX	US TREASURY NOTES I/L	2.1	1/15/2019	US/T	2.9	35.8	272.4	-3.0	-15.2	5.3	-88.9	-0.4	205.9
912828LA	US TREASURY NOTES I/L	1.9	7/15/2019	US/T	3.1	36.7	289.4	-3.0	-16.0	5.3	-93.2	-0.7	218.5
912828MF	US TREASURY NOTES I/L	1.4	1/15/2020	US/T	3.5	37.8	309.0	-3.0	-16.4	5.3	-105.6	-0.9	226.3
912810FR	US TREASURY BONDS I/L	2.4	1/15/2025	US/T	5.8	39.9	337.0	-3.0	-17.4	5.4	-66.1	-8.1	287.7
912810FS	US TREASURY BONDS I/L	2.0	1/15/2026	US/T	3.7	39.9	335.1	-3.0	-17.6	5.3	-89.8	-8.7	261.3
912810PS	US TREASURY BONDS I/L	2.4	1/15/2027	US/T	3.1	38.7	320.3	-3.0	-17.6	5.4	-62.0	-8.4	273.3
912810PV	US TREASURY BONDS I/L	1.8	1/15/2028	US/T	2.8	38.2	315.7	-3.0	-17.9	5.4	-33.9	-8.3	296.2
912810FD	US TREASURY BONDS I/L	3.6	4/15/2028	US/T	4.6	36.0	292.1	-3.1	-16.8	5.3	-34.6	-7.5	271.5
912810PZ	US TREASURY BONDS I/L	2.5	1/15/2029	US/T	3.0	36.0	292.8	-3.0	-17.7	5.3	-30.9	-7.3	275.3
912810FH	US TREASURY BONDS I/L	3.9	4/15/2029	US/T	5.5	34.7	279.0	-3.1	-16.9	5.4	-14.2	-6.6	278.3
912810FQ	US TREASURY BONDS I/L	3.4	4/15/2032	US/T	1.4	33.6	237.5	-3.0	-17.7	5.3	11.5	-7.0	260.1
912810QF	US TREASURY BONDS I/L	2.1	2/15/2040	US/T	1.6	35.5	107.3	-3.0	-19.1	5.2	59.6	-9.0	176.4

Source: POINT®

4.5.4. *Selecting an appropriate attribution model*

From the above discussion, it should be clear that many variants of attribution models exist and that a portfolio manager must choose one that best corresponds to the decision structure during portfolio management. We summarize the results from all models applied to the US portion of the portfolio from Section 1 in Figure 29.

Essentially, models differ with respect to what factors of return are managed at the portfolio level via allocations of appropriate exposures (factor-based management) and which component of return is managed via allocation to sectors and security selection within each sector (sector-based management).

Each successive model brings one additional common factor. The Excess of Curve model introduces curve exposure as a common factor; the Excess of Curve and Vol model introduces implied volatilities; the Fully Analytical model with sector-level allocation of spread duration introduces all other factors for which we have explanatory analytics (mortgage factors only, in this particular portfolio) as well as residual; and the top-level Fully Analytical model introduces top-level spread duration as a common factor.

Figure 29: Outperformance Contributions Using Various Models

	Total Return	Excess of Curve	Excess of Curve and Vol	Fully Analytical Sector-level	Fully Analytical Top-level
Yield Curve		24.6	24.6	24.6	24.6
Implied Volatility			1.9	1.9	1.9
Mortgage				-8.2	-9.2
Spread Duration					-1.8
Asset Allocation	2.9	-2.2	-4.0	3.2	4.9
Security Selection	29.2	9.7	9.6	9.3	9.3
Residual				1.3	1.3
Total	32.1	32.1	32.1	32.1	32.1

Source: POINT®

Flexibility is a very important feature of attribution algorithms. Portfolio management styles differ, and an attribution platform must have sufficient richness so that a model representing each management style can be chosen. In addition, running multiple versions of the performance attribution algorithm may help portfolio managers reveal exposures that they have been implicitly taking and help them improve their management process.

In the next section, we discuss how we analyse the performance of multi-currency portfolios.

5. Multi-Currency Attribution

A multi-currency portfolio introduces another layer of complexity to the portfolio management structure. For instance, before we can get to the management of local markets using the framework described in the previous section, there may be decisions made at the global level to determine capital allocation across different local markets, as well as the FX exposures to those markets as illustrated in the example of Section 1. In this section, we describe in more detail how we decompose global total outperformance into FX and local market allocation, as well as local market management.

5.1. FX Return Splitting

Since most portfolio managers prefer to manage FX exposure separately, the first step in return splitting of non-base-currency denominated securities is to decompose the FX component of return from the local component of return. The base currency total return of security *i* (${}^{b}R_i$) can be expressed in terms of its local currency return (R_i) and the spot FX return of the currency in which the security is denominated, F_i. [23]

$$ {}^{b}R_i = F_i + R_i F_i + R_i \tag{34} $$

Here, we assume that all positions with cash flows or dependencies on interest rates in multiple currencies (such as FX swaps, FX forwards, etc.) have been decomposed into legs that have exposure only to the market parameters of a single currency[24]. Note the presence of a cross-term that is the product of the FX return and the local currency return. This can

[23] This formula is applicable to fully funded securities only. When leverage is present, such as in derivatives, it needs to be adjusted appropriately.
[24] This separation is more complicated for FX options. Details on how FX options can be handled in this framework will be discussed in a forthcoming publication when the HPA implementation in POINT® is expanded to support such instruments.

be accounted separately, but in most cases it is combined with the FX return under the assumption that the FX return is typically much larger than local currency returns. While this is generally true for fixed income securities, it may not be true for equity portfolios[25].

To separate the performance due to FX exposures from that due to local management, we need to include in the FX return the cash deposit return in each currency. Indeed, a cash investment in any foreign currency will earn the FX return plus the local deposit return over the attribution period, R_i^{depo}, without incurring any local risk.

$$ {}^{b}R_i = \left(F_i + R_i^{depo}\right) + R_i F_i + \left(R_i - R_i^{depo}\right) \tag{35} $$

The total return of the portfolio can then be written as the sum of returns from "markets" that correspond to the first allocation layer of a multi-currency portfolio manager. A natural such grouping is currency, and this is the standard choice in the HPA algorithm[26]:

$$ {}^{b}R^P = \sum_c {}^{b}w_c^P \cdot \left(F_c^P + R_c^{depo,P}\right) + \sum_c {}^{b}w_c^P R_c^P F_c^P + \sum_c {}^{b}w_c^P \cdot \left(R_c^P - R_c^{depo,P}\right) \tag{36} $$

Here, ${}^{b}w_c^P$ represents the "base-currency portfolio market value weight," i.e., the market value of positions in currency *c* over the market value of the portfolio, both expressed in base currency units. The three terms in Equation (36) correspond to FX return, FX/local cross-term and local returns, respectively.

5.2. FX Hedging

FX hedging – the overlay of FX derivatives with the intent to change the FX exposure profile of a portfolio without affecting its local markets exposures – is a common practice in multi-currency portfolios. It enables managers to disentangle the portfolio construction process between the FX experts and the local market managers. The most common instruments used for hedging are FX forwards, typically with short tenors (e.g., one month) and are rolled regularly. Longer tenors are sometimes used to reduce transaction costs.

Despite the intent to leave the local exposures unchanged, in practice FX hedging instruments have side effects that must be accounted for in a complete attribution framework.

(a) *Exposure to local rates is non-zero[27]:* This effect has been highlighted recently, as short rates exhibited significant volatility in the midst of the global crisis. Even 1-month forwards experienced significant return due to differences in the fluctuation of the short rates across currencies. The HPA model captures the curve return of FX hedges and reports it in the Hedging Effects line under the FX Allocation & Hedging category in the Global Outperformance by Currency report. The presumption is that this return component comes from an unintended exposure to local rates and is not part of the local rates strategy decisions.

(b) *Cash balance effect*: As FX rates fluctuate, the mark-to-market of FX forwards becomes non-zero, essentially representing a cash balance (positive or negative). Unless such cash balance is re-invested regularly, it represents an unintended allocation to cash (which is

[25] This problem is not unlike the problem of attributing the asset allocation/security selection interaction term that was discussed in Section 3 and can be dealt with similar methods.
[26] Other top-level allocation choices are also used in practice. For example, a manager may decide first to allocate to asset classes globally (equities/governments/credit/commodities, etc.) and then consider currency markets. The HPA algorithm supports this case by allowing users to skip currency allocation and allocate instead to user-defined security buckets. However, the FX component of return is always accounted by currency.
[27] For consistency, HPA explains the return of all FX instruments by the spot FX rate and interest rates changes. The basis between the market value FX forwards and the interest rate parity implied fair value is accounted for separately.

equivalent to leveraging if the balance is negative). This effect can be significant and needs to be highlighted during performance attribution. For this reason, the HPA model captures this effect explicitly as the difference of the actual portfolio return minus the hypothetical return that would be achieved if the FX hedges cash balance were reinvested daily in the portfolio. This effect is also reported in the Hedging Effects line under the FX Allocation & Hedging category in the Global Outperformance by Currency report.

Once these side effects have been accounted for, the effect of FX hedges is restricted to the FX component of total return and the cross-term; the local component of return remains unchanged.

5.3. Outperformance from FX Allocation and Hedging

FX outperformance is defined as the part of total outperformance that is due to fluctuations of the exchange rate between the base currency and all non-base currencies present in the portfolio or the benchmark. The first two terms of Equation (36), pure FX and the FX/local cross-term, contribute to FX outperformance.

If we denote the effective portfolio exposure to each FX rate after hedging with ${}^{b}h_c^P$ and the benchmark one with ${}^{b}h_c^B$, we can represent the contribution of FX exposure to the portfolio outperformance as follows:

(a) Pure FX outperformance:

$$ {}^{FX}R^P - {}^{FX}R^B = \sum_c {}^{b}h_c^P \cdot \left(F_c + R_c^{depo}\right) - \sum_c {}^{b}h_c^B \cdot \left(F_c + R_c^{depo}\right) $$

We can now apply the relative allocation model of the top-down decomposition algorithm of Section 3.4 by setting $\alpha_{k,s} = {}^{b}h_c$, $\alpha_k = \sum_c {}^{b}h_c = 1$, $f_{k,c} = F_c + R_c^{depo}$ and $f_k^H = R_b^{depo}$.

Hedged FX Allocation	$\sum_c \left({}^{b}h_c^P - {}^{b}h_c^B\right) \cdot \left(F + R_c^{depo} - R_b^{depo}\right)$	(37)

A similar decomposition can be made to the FX outperformance without the hedges

Un-hedged FX Allocation	$\sum_c \left({}^{b}w_c^P - {}^{b}w_c^B\right) \cdot \left(F + R_c^{depo} - R_b^{depo}\right)$	(38)

On the FX Allocation & Hedging report, both un-hedged and hedged FX allocations are reported. However, only the hedged FX allocation is used in the Global Portfolio Summary and Global Outperformance by Currency reports.

(b) FX-Local Return Cross-term:

Hedges do contribute to the interaction term between FX and local returns. The combined effect can be written as:

FX Cross-term	$\sum_c \left({}^{b}w_c^P R_c^P - {}^{b}w_c^B R_c^B\right) \cdot F_c^B + \left({}^{b}w_c^P - {}^{b}h_c^P - {}^{b}w_c^B + {}^{b}h_c^B\right) \cdot R_c^{depo} \cdot F_c^B$	(39)

Since this term is the product of two returns, its size is usually smaller than either of the two. However over time, the contributions of the cross-term can accumulate (similar to a convexity term) and become significant relative to local and FX returns. The FX cross-term can be a significant component of total return when FX and local returns have opposite signs and similar magnitudes. For example, if FX return is +5% and local return is -4%, the total return is 5% - 4% + 5%x4% = 1.20%. Of that, 0.20%, or about 17%, of the total return is due to the interaction term. Further, if the two returns are opposite but equal in magnitude, the entire total return is due to the interaction term.

While in the majority of the cases the FX/local cross-term is insignificant, it is preferable to account for it explicitly. In our algorithm, we do so and report the contribution of each currency in the last column of the FX Allocation & Hedging report.

Finally, as we mentioned in the previous section, any returns from the hedging instruments are separated from local returns and outperformance. Therefore, we include the local return and market value effects of the hedging instruments, as well as their trading returns, in the Hedging Effects line and report the total hedging effects outperformance under FX Allocation & Hedging, details of which were discussed in Section 1.1.

5.4. Outperformance from Allocation to Local Markets

After the contributions of FX exposure and hedging are taken out, the outperformance due to the local returns in various currencies is:

$$ {}^{L}R^{P} - {}^{L}R^{B} = \sum_{c} {}^{b}w_{c}^{P} \cdot \left(R_{c}^{P} - R_{c}^{depo}\right) - \sum_{c} {}^{b}w_{c}^{B} \cdot \left(R_{c}^{B} - R_{c}^{depo}\right) \tag{40} $$

There are many different ways to decompose this outperformance into the contribution of the various decision-makers. The most straightforward one is to assume that the first decision layer is the allocation of market value exposure to the various currencies. After this, each currency is managed separately versus the corresponding local currency benchmark. Applying the top-down decomposition algorithm of Section 3.4 using $\alpha_{k,c} \leftarrow {}^{b}w_{c}$, $\alpha_{k} \leftarrow \sum_{c} {}^{b}w_{c} = 1$, $f_{k,c} \leftarrow R_{c} - R_{c}^{depo}$ and $f_{k}^{H} \leftarrow R^{B} - R^{depo}$, we get the following terms:

Local Markets Allocation $$ \sum_{c} \left({}^{b}w_{c}^{P} - {}^{b}w_{c}^{B}\right) \cdot \left(R_{c}^{B} - R_{c}^{depo} - \left(R^{B} - R^{depo}\right)\right) \tag{41} $$

Local Management $$ \sum_{c} {}^{b}w_{c}^{P} \cdot \left(R_{c}^{P} - R_{c}^{B}\right) \tag{42} $$

The local management outperformance of each currency *c*, ${}^{b}w_{c}^{P} \cdot \left(R_{c}^{P} - R_{c}^{B}\right)$ can now be decomposed using the local outperformance breakdown described in Section 4. Details of the Local Market Allocation report have been discussed in Section 1.2.

Using currency buckets for local markets allocation is not always consistent with how global portfolios are managed. Sometimes, local buckets need to be smaller, e.g., allocations to specific countries instead of currency blocks such as the euro. In other cases, they need to span currencies, such as regional allocations (e.g., Europe, Americas, Asia Pacific, etc.). Quite often, allocation slices across currencies such as allocation to global asset classes (equities, credit, governments, etc.). The HPA algorithm gives users the flexibility to use an arbitrary security partition to define the top-level allocation to user-defined buckets, rather than the default currency breakdown. In this case, the deposit rate is not necessarily the same between a portfolio and a benchmark bucket, and Equations (41) and (42) must be adjusted appropriately.

5.5. Factor-Based Local Markets Allocation

Market value-weighted local markets allocation hides many of the complexities of the decision process of investing across markets. Indeed, allocation of exposure to local markets consists of the aggregation of exposures to each risk factor that drives local market returns, such as yield curve, implied volatility, spreads, and so forth. To better quantify how much risk is taken in a particular market, we can describe the allocation decision using the appropriate exposure to each risk factor, e.g., interest rate duration for yield curve, vega for implied volatility and spread duration for spreads, etc.

Following the methodology described in Section 3.4, we can re-write Equation (40) by decomposing local returns into the contributions of risk factors as follows:

$$ {}^{L}R^{P} - {}^{L}R^{B} = \sum_{c} {}^{b}w_{c}^{P} \cdot \left(\sum_{k} \alpha_{k,c}^{P} f_{k,c}^{P} - R_{c}^{depo,P} \right) - \sum_{c} {}^{b}w_{c}^{B} \cdot \left(\sum_{k} \alpha_{k,c}^{B} f_{k,c}^{B} - R_{c}^{depo,B} \right) \quad (43)$$

We allow the deposit rate to be different between the portfolio and the benchmark to allow for a generic decomposition and not force currency decomposition only. We can now decompose local outperformance into allocation, management and leverage using different weight for each factor.

Allocation $$\sum_{c} \left(\sum_{k} \left({}^{b}w_{c}^{P} \alpha_{k,c}^{P} - {}^{b}w_{c}^{B} \alpha_{k,c}^{B} \right) \cdot \left(f_{k,c}^{B} - f_{k}^{H} \right) \right) - \sum_{c} \left({}^{b}w_{c}^{P} - {}^{b}w_{c}^{B} \right) \cdot R_{c}^{depo,B} \quad (44)$$

Management $$\sum_{c} \left(\sum_{k} {}^{b}w_{c}^{P} \alpha_{k,c}^{P} \cdot \left(f_{k,c}^{P} - f_{k,c}^{B} \right) \right) - \sum_{c} {}^{b}w_{c}^{P} \cdot \left(R_{c}^{depo,P} - R_{c}^{depo,B} \right) \quad (45)$$

Top-Level $$\sum_{c} \left(\sum_{k} {}^{b}w_{c}^{P} \alpha_{k,c}^{P} - \sum_{k} {}^{b}w_{c}^{B} \alpha_{k,c}^{B} \right) \cdot f_{k}^{H} \quad (46)$$

Below, we discuss an HPA option that uses the above method to decompose the yield curve component of local returns.

5.5.1. Curve Allocation to Local Markets

As the portfolio manager from the example in Section 1 surmised, quite often global rates exposure needs to be managed centrally using key-rate durations to control the exposure to various curves. In the global allocation model of the HPA implementation in POINT®, we use the above methodology to separate curve return from excess return and global key-rate durations as decision variables for global curve outperformance. Any return in excess of the portion that can be explained by key-rate duration exposures is allocated using market value weight, as described in the previous section.

Curve Carry

We combine the outperformance from curve carry and deposit return together and decompose it using the curve-matching portfolio weights $\omega_{c,j}$ to measure carry exposure to individual key-rate points in each currency. Let us also denote the curve return from each individual key-rate point with $R_{c,j}^{carry} = y_{c,j}\Delta t$. From Equations 44-46, using the transformations $\alpha_{k,c} \leftarrow \omega_{c,j}$, $f_{k,c} \leftarrow R_{c,j}^{carry} - R_{c}^{depo}$ and setting the hurdle rate equal to the excess carry return of a reference currency (typically the base currency of the analysis) $f_{k}^{H} \leftarrow R_{ref,j}^{carry} - R_{ref}^{depo}$, we get the following decomposition:

Local Yield Curve Allocation

$$\sum_c \sum_j \left({}^b w_c^P \omega_{c,j}^P - {}^b w_c^B \omega_{c,j}^B\right) \cdot \left(R_{c,j}^{carry,B} - R_c^{depo,B} - \left(R_{ref,j}^{carry,B} - R_{ref}^{depo,B}\right)\right) \quad (47)$$

Local Yield Curve Management

$$\sum_c \sum_j {}^b w_c^P \omega_{c,j}^P \cdot \left(R_{c,j}^{carry,P} - R_c^{depo,P} - \left(R_{c,j}^{carry,B} - R_c^{depo,B}\right)\right) \quad (48)$$

Global Carry over Deposit

$$\sum_c \sum_j \left({}^b w_c^P \omega_{c,j}^P - {}^b w_c^B \omega_{c,j}^B\right) \cdot \left(R_{ref,j}^{carry,B} - R_{ref}^{depo,B}\right) \quad (49)$$

If the top-level buckets represent currency buckets where the interest rates are the same for portfolio and benchmark buckets, then the management term becomes zero. In addition, if no reference currency is specified, then the global term becomes zero as well and global curve carry outperformance is simply broken down per currency and key-rate point:

$$\sum_c \sum_j \left({}^b w_c^P \omega_{c,j}^P - {}^b w_c^B \omega_{c,j}^B\right) \cdot \left(R_{c,j}^{carry} - R_c^{depo}\right)$$

Curve Change

The outperformance due to curve change is decomposed similarly using key-rate durations as allocation weights and the negative of the key-rate yield change as return, i.e., $\alpha_{k,c} \leftarrow KRD_{c,j}$, $f_{k,c} \leftarrow -\Delta y_{c,j}$ and $f_k^H \leftarrow -\Delta y_{ref,j}$.

Local Yield Curve Allocation

$$-\sum_c \sum_j \left({}^b w_c^P KRD_{c,j}^P - {}^b w_c^B KRD_{c,j}^B\right) \cdot \left(\Delta y_{c,j}^B - \Delta y_{ref,j}^B\right) \quad (50)$$

Local Yield Curve Management

$$-\sum_c \sum_j {}^b w_c^P KRD_{c,j}^P \cdot \left(\Delta y_{c,j}^P - \Delta y_{c,j}^B\right) \quad (51)$$

Global Duration Mismatch

$$-\sum_c \sum_j \left({}^b w_c^P KRD_{c,j}^P - {}^b w_c^B KRD_{c,j}^B\right) \cdot \Delta y_{ref,j}^B \quad (52)$$

If the top-level buckets represent currency buckets where interest rate changes are the same for portfolio and benchmark buckets, then the management term becomes zero. In addition, if no reference currency is specified, then the global term becomes zero as well and global curve change outperformance is simply broken down per currency and key-rate point:

$$-\sum_c \sum_j \left({}^b w_c^P KRD_{c,j}^P - {}^b w_c^B KRD_{c,j}^B\right) \cdot \Delta y_{c,j}$$

5.6. Global Allocation in Practice

Let us now revisit the example from Section 1. Let us assume that the portfolio manager has decided to make two changes in the portfolio construction process, namely 1) express currency views using FX forwards and stay neutral in currency markets allocation, and 2) manage global duration using portfolio key-rate durations, more precisely, distribute the recommended one year overweight in global duration evenly between the 2y, 5y, 10y, 20y and 30y points and proportionately across currencies while staying neutral on the short end of the curve.

The constraints in the portfolio construction problem are reformulated, and a new portfolio is constructed, one that has zero net weights in all four currencies and an overall duration of 6.37y versus the 5.37y of the benchmark. Subsequently, two at-the-money FX forward positions are added to the portfolio, one that goes short the euro versus the dollar and one that goes long the yen versus the dollar, both with a notional amount equal to 5% of the portfolio market value. The risk of the resulting portfolio is now calculated to be 39.1bp, still below the 40bp of the portfolio manager's risk budget, as specified in Section 1. Once again, she holds the portfolio for August 2010 and then runs performance attribution, choosing to report the contribution of interest rate curve exposure globally. For simplicity, we choose the Excess of Curve model for local return attribution. The total outperformance and its breakdown per currency and exposure are shown in Figure 30. The total outperformance of the portfolio is now +88.3bp broken down into +27.5bp from FX Allocation & Hedging, +35.6bp from Local Markets Allocation (including global interest rates exposure) and +25.2bp of Local Management (Box A).

The contribution from FX exposure, implemented via currency derivatives, is very similar to the one of the original portfolio (+28.3bp from Figure 5) as designed. The local rates and cash balance effects of the FX forwards are combined to contribute -0.3bp.

Figure 30: Global Outperformance by Currency for the Global Curve Allocation Portfolio

POINT® Hybrid Performance Attribution®

Global Outperformance by Currency

Portfolio: REPL : GlAgg G4 50 - Hedged
Benchmark : Global Agg G4
Partition : HPA Default Partition / System

Period: 7/30/2010 to 8/31/2010
Base Currency: USD
Curve Type: Treasury

	USD	EUR	JPY	GBP	Hedges	Total
Unhedged Portfolio Weight (%)	45.6	28.2	20.5	5.8		100.0
Outperformance	26.4	34.3	26.5	1.4	-0.3	88.3
FX Allocation & Hedging	-0.0	11.8	15.9	0.2	-0.3	27.5
Hedged FX Allocation	0.0	11.8	15.8	0.1		27.7
FX / Local Cross Term	-0.0	-0.0	0.1	0.1		0.1
Hedging Effects					-0.3	-0.3
Local Market Allocation	14.5	18.4	1.3	1.3		35.6
Global Duration	14.5	12.2	11.4	0.2		38.4
Local Yield Curve Advantage	0.0	6.2	-10.1	1.1		-2.8
Local Market Allocation Over Curve	0.0	-0.0	-0.0	-0.0		-0.1
Global Allocation Exclusions Effect	-0.0	0.0	0.0	0.0		-0.0
Local Management	11.8	4.1	9.4	-0.0		25.2
Yield Curve	-0.7	3.0	2.5	0.0		4.8
Asset Allocation	-2.7	-2.2	0.4	-0.8		-5.3
Security Selection	15.2	3.1	6.5	0.8		25.6
Pricing Differences	-0.0	0.2	0.0	-0.0		0.2

A

Source: POINT®

Global interest rates exposure, calculated to be +35.6bp, is displayed under the Local Market Allocation category and is broken into the global duration term, which uses the USD curve as a reference to calculate curve carry and change outperformance,[28] and the local yield curve advantage term, that which captures differential contributions of the other three curves.[29] Allocation of exposure to local markets should have negligible contribution, since, by construction, the portfolio has no net exposure to any local market. Indeed, the local market allocation over curve term is reported to be -0.1bp. Finally, the local management term is broken down into contributions from yield curve (the part that cannot be explained by key-rate durations), asset allocation, security selection and pricing differences. Since the portfolio was constructed from scratch, a new set of securities has been chosen leading to the asset allocation and security selection terms to be different than the ones in the previous portfolio as reported in Figure 5.

Let us now look more carefully into the global curve outperformance. Figure 31 shows a summary report that seeks to explain the +35.6bp of outperformance from interest rates exposure. The first line, Global Duration, shows that the portfolio is 0.98y longer than the benchmark (6.37y versus 5.39y in Box A). Using the US yield curve as reference, we estimate that this duration overweight results in +35.4bp of outperformance (Box B). In addition, since the portfolio is longer in a steep yield curve environment, it enjoys positive curve carry advantage of +2.9bp. As we described in Section 5.5.1, these calculations are done using key-rate durations; however, this report displays aggregated results for brevity. The differential changes of other curves are captured in the second line, Local Yield Curve Advantage, and reported to be -2.8bp (Box B). Once again, calculations use key-rate duration exposures and key-rate changes for each currency. Aggregated results per currency are displayed in this report. For example, the duration overweight of the yen curve exposure leads to relative underperformance of -10.1bp, as the yen curve rallied much less than the dollar curve.

Figure 31: Global Interest Rates Exposure Outperformance

POINT® Hybrid Performance Attribution®

Global Duration & Local Yield Curve Advantage

Portfolio: REPL : GlAgg G4 50 - Hedged
Benchmark : Global Agg G4
Partition : HPA Default Partition / System

Period: 7/30/2010 to 8/31/2010
Base Currency: USD
Curve Type: Treasury

	Benchmark Yield		Benchmark Carry Over Deposit	Curve-Matched Market Weight (%)					Duration (yrs)					Outperformance (bps)		
				Avg		Overweight			Avg		Overweight			Carry	Change (bps)	Total
	Level (%)	Change (bps)	Total	Port	Bench	Mean	Min	Max	Port	Bench	Mean	Min	Max			
Global Duration				100.0	100.0	0.0	-2.9	3.2	6.37	5.39	0.98	0.87	1.11	2.9	35.4	38.3
Local Yield Curve Advantage														-0.7	-2.0	-2.8
USD (reference)	1.2	0.0	15.1	45.6	45.6	0.0	-1.8	2.1	2.33	1.91	0.43	0.37	0.48	0.0	0.0	0.0
EUR	1.7	0.0	17.0	28.2	28.1	0.1	-0.6	0.7	1.87	1.58	0.29	0.25	0.33	-0.2	6.4	6.2
JPY	0.6	0.0	7.6	20.5	20.5	-0.1	-0.7	0.6	1.60	1.41	0.19	0.17	0.22	-0.5	-9.5	-10.1
GBP	2.8	0.0	25.4	5.7	5.8	-0.0	-0.3	0.3	0.56	0.50	0.07	0.04	0.10	0.0	1.1	1.1
Total														**2.1**	**33.4**	**35.6**

Source: POINT®

To understand all the details of global curve outperformance, the manager needs to look into the detailed curve exposure and curve movement in each currency. Figure 32 provides such detail. We see that according to the manager's instructions, total duration is overweighted by almost a year and distributed proportionately to the market value weight in each currency and fairly evenly to the all curve points except the 6m point. That point is not neutral as directed, but has a small underweight of -0.07y; however, it does not contribute significantly to outperformance since it moved little during the attribution period.

[28] This term contains both the Global Carry over Deposit term from Equation (49) and the Global Duration Mismatch term from Equation (52)

[29] This term contains the curve carry and change Local Yield Curve Allocation terms from Equations (47) and (50). The Local Yield Curve Management term is zero since we used a currency partition.

Figure 32: Interest Rate Duration Exposure in All Currencies

	USD		EUR		GBP		JPY		Total
	Dur O/W	Yld Chng	Dur O/W	Yld Chng	Dur O/W	Yld Chng	Dur O/W	Yld Chng	Dur O/W
Average	0.43		0.29		0.07		0.19		0.98
6-mo	-0.03	*1.3*	-0.02	*0.6*	0.00	*-5.3*	-0.02	*-1.3*	-0.07
2-yr	0.16	*-5.2*	0.02	*-22.6*	0.04	*-22.1*	0.00	*-3.3*	0.22
5-yr	0.09	*-26.1*	0.08	*-43.1*	0.00	*-43.4*	0.00	*-5.3*	0.17
10-yr	-0.11	*-45.7*	0.08	*-57.3*	-0.12	*-46.7*	0.36	*-5.1*	0.21
20-yr	0.32	*-48.7*	-0.01	*-64.7*	-0.12	*-49.5*	0.00	*-8.1*	0.19
30-yr	0.01	*-42.1*	0.14	*-69.6*	0.28	*-44.9*	-0.15	*-7.6*	0.28

Source: POINT®

What is more interesting is that the duration overweight has not been distributed evenly across the curve points in each currency. For example, there is a large steepening exposure to the long end of the pound curve. The reason for the existence of such exposures is that the portfolio optimiser is trying to solve a very constrained problem using only 50 securities while trying to minimize tracking error. Therefore, given the latitude, it will make trade-offs between risks, accepting exposures with relatively smaller estimated risk such us curve re-shaping exposures in order to reduce country, credit, mortgage, and idiosyncratic risk. Should the portfolio manager seek full control over curve exposure, the best method is to use interest-rate derivatives as an overlay, similar to how currency derivatives were used to manipulate currency exposure without affecting local market exposures.

6. Derivatives and Leverage

Derivative instruments are widely used across asset classes as both hedges and means to express market views. Their payoffs are typically tied to changes in the values of underlying securities, and their market values do not generally represent the exposures of the positions. In this section, we discuss how HPA handles derivative instruments to give intuitive and meaningful attribution results.

6.1. Returns and Basis

Returns of derivative instruments can be difficult to measure. The market value of a contract is typically small (or zero) compared with its notional exposure, and the standard definition of return (P&L divided by market value) can result in extremely large (or infinite) values. For example, the market value of instruments that are marked to market daily, such as Treasury futures, is guaranteed to be zero at the end of the day, resulting in a theoretically infinite return.

To maintain the notion of return as a P&L per unit of investment, we seek to use a denominator that represents the size of the position in terms of its exposure. We refer to this quantity as the "return basis" or simply "basis".

To complicate the issue of derivatives further, the standard methodology of multi-period arithmetic return compounding cannot be applied. Consider the following definition of single- and multi-period returns, where PL_t and R_t are the P&L and return over the period (t-1, t).

Portfolio Return	$R_t = \dfrac{PL_t}{MV_{t-1}}$
Multi-Period Return	$R = \left(\prod_{t=1}^{T} (1 + R_t) \right) - 1$

By using market value as a basis for the return and compounding it with the above formula, we get an intuitive relationship between the market values at t=0 and t=T, and the return over the period (0, T):

$$MV_T = MV_0 (1 + R)$$

Changing the basis to a quantity not proportional to the market value will annul this property. This is more evident when returns are large. As an exaggerated example, consider a futures contract that is marked to market daily and therefore always has zero market value. A reasonable basis for returns for a position in the contract is the notional amount invested. Suppose the notional amount is $1,000 and that over two days, the contract produces a mark-to-market P&L of -$500 and $1,000, respectively. Using the notional amount as the basis of returns, the returns for the two days are -50% and +100%, respectively. Using the chain rule to compound returns yields zero return over the two days:

$$(1 - 50\%) \cdot (1 + 100\%) - 1 = 0\%$$

Obviously, this is not consistent with the positive P&L of $500 produced over the two days. Such effects are always present when compounding arithmetic returns using a basis that is different from the market value of the position; however, their effect is much smaller when the magnitude of returns is small. To avoid such non-intuitive results, HPA makes appropriate adjustments to the compounding of returns of derivative securities.

HPA uses an appropriate return basis for all derivative types supported. Most of the time, the basis is set equal to the notional amount, although in many cases special rules are used to define the basis in a bond-equivalent way.

6.2. Returns for Portfolios and Sectors Containing Derivatives

The return of each individual derivative contract is defined using its basis as described above. However, at the portfolio and sector levels, POINT® uses market value as the denominator for all returns and weights.[30] To account for this difference, we define the leverage ratio for each security, β_i , to be the basis divided by the market value of the security. Then the leveraged weight of the security is the product of the market value weight and the leverage ratio, $w_i \beta_i$. The return of the portfolio or sector can then be written as the leveraged weighted sum of the de-leveraged return of each security:

$$R^P = \sum_i \frac{PL_i^P}{MV^P} = \sum_i \frac{MV_i^P}{MV^P} \frac{basis_i^P}{MV_i^P} \frac{PL_i^P}{basis_i^P} = \sum_i w_i^P \beta_i^P R_i^P$$

Although this transformation does not change the return, notice that the weights are no longer constrained to sum to 1 when derivatives are present in the portfolio.

[30] Net basis and gross market value can also be chosen as the denominator for the entire portfolio to allow analysis of highly leveraged and long-short strategy funds.

$$\sum_i w_i \beta_i \neq 1$$

Therefore, in the last step of the top-down decomposition, where we seek to explain the contribution of each security to the security selection outperformance of each bucket, derivatives give rise to an additional term of outperformance that comes from the implicit leverage they introduce.

Example: Leverage from Derivatives

To clarify how HPA deals with derivatives in security selection, consider a bucket with a single credit security (A) experiencing excess of curve return of +10%, while the corresponding benchmark bucket has two equally weighted securities, the one in the portfolio (A) and another one (B) returning 0%. The total return for that bucket in the benchmark is therefore 5%. Let us also assume that the market value weight of this bucket in the portfolio is 20%; therefore, its contribution to security selection according to the Excess Return model is 20% x (10% - 5%) = 1.0%. The contribution of security A to security selection is calculated as 20% x (100% - 50%) x (10% - 5%) = 0.5%, and the one of security B is 20% x (0% - 50%) x (0% - 5%) = 0.5%.

Now assume that the portfolio also contained a short protection credit default swap on the issuer of bond A with a return basis equal to the market value of A, essentially doubling the exposure to issuer A. Let us assume that the default swap is at the money, i.e., its market value is zero, and that it also experiences a return of +10% (using notional as the return basis). Since the market value of the portfolio bucket remains the same but its P&L doubles, the return of the sector is now reported as 20%, and the security selection term becomes 20% x (20% - 5%) = 3.0%. Notice that the market value weight of the bond remains 100% and that the default swap also has a weight of 100% (return basis over market value of the bucket). Bonds A and B contribute to security selection 0.5% exactly as before. The default swap contributes 20% x (100% - 0%) x (10% - 5%) = 1.0%. The contributions of the three securities to security selection sum to 2.0%, leaving 1.0% of security selection unexplained.

This comes from the leverage introduced to the bucket and can be accounted for using the formula for top-level exposure from Equation (24). Indeed, the sum of securities weights in the portfolio is 200%, whereas in the benchmark it is 100%. According to Equation (24), scaled by the 20% sector weight,[31] this causes outperformance of 20% x (200% - 100%) x 5% = 1.0%, completing the decomposition of security selection.

In the context of the use of derivatives, this top-level term is called bucket leverage and reported as a separate line item for each bucket in the Security Selection report. It is folded into the security selection term in summary reports.

6.3. A General Framework for Leverage

Leverage complicates portfolio returns calculations and comparison to the benchmark. Furthermore, there are different means to achieve leverage, and the decision to employ it can occur at any level of the portfolio management decision structure. For instance, cash borrowing can be used to leverage the entire portfolio, while derivatives can be used to leverage specific sectors. The latter case can be further distinguished into a decision made by an asset allocator or sector managers. Below, we outline an analytical way to account for leverage in a consistent framework with the one discussed in Section 3.4.

[31] Scaling is necessary since this represents the second level of the recursive application of the decomposition where all terms must be scaled by the sector weight.

We expand the concept of leverage ratio as introduced in Section 6.2 and apply it to the entire portfolio. We assume that a "fully funded" market value – generally different from the actual market value – can be defined for the portfolio and serve as the basis for the calculation of unleveraged returns. The ratio of funded market value to actual market value is the leverage ratio of the portfolio.

$$R^P_{lever} = \frac{MV^P_{funded}}{MV^P} \cdot \frac{PL^P}{MV^P_{funded}} = \frac{basis^P_S}{MV^P} \cdot \frac{PL^P}{basis^P_S} = \beta^P R^P$$

We can also recursively break down the portfolio return into the contributions of its various sectors by appropriately defining the leverage ratio of each sector:

$$R^P_{lever} = \sum_s \frac{PL^P_s}{MV^P} = \sum_s \frac{MV^P_s}{MV^P} \cdot \frac{basis^P_s}{MV^P_s} \cdot \frac{PL^P_s}{basis^P_s} = \sum_s w^P_s \beta^P_s R^P_s$$

Using similar terminology for the benchmark, the total leveraged outperformance can be expressed as a function of leverage ratios and unleveraged returns as:

$$\beta^P R^P - \beta^B R^B = \sum_s w^P_s \beta^P_s R^P_s - \sum_s w^B_s \beta^B_s R^B_s$$

We will now consider the different ways to break down the total outperformance into contributions of asset allocators and sector managers. It is worthwhile to note that when leverage is allowed, there is no limit to how much exposure managers can allocate to sectors or securities. Consequently, it no longer makes sense to compare sector returns against a hurdle rate to determine contribution from asset allocation decisions.

Separate allocation of market weight and leverage of each sector

In this case, each sector is given a market value budget, but sector managers are free to take leverage within their sectors. Separate agents determine the market value allocation and the leverage in each sector. The simple asset allocation/sector management equations can be readily applied in this case. The comparison of sector returns of the portfolio and the benchmark occurs on a leveraged basis. In this case, the outperformance from leverage decisions is embedded in the sector management term.

Asset Allocation	$\sum_s \left(w^P_s - w^B_s\right) \cdot \beta^B_s R^B_s$	(53)
Sector Management	$\sum_s w^P_s \cdot \left(\beta^P_s R^P_s - \beta^B_s R^B_s\right)$	(54)

Allocation of total exposure to sectors

Alternatively, leverage can be determined by the asset allocator instead of the sector managers. In this case, a single agent decides the total exposure[32] to the unleveraged return of each sector. Once again, the asset allocation/sector management equations can by applied, but we now need to use total exposure instead of market value as the allocation weight. In this case, the outperformance from leverage decisions is embedded in the asset allocation term.

Asset Allocation	$\sum_s \left(w^P_s \beta^P_s - w^B_s \beta^B_s\right) \cdot R^B_s$	(55)
Sector Management	$\sum_s w^P_s \beta^P_s \cdot \left(R^P_s - R^B_s\right)$	(56)

[32] The total exposure is defined here as market weight times leverage.

Relative allocation of total exposure

In general, we can assume that leverage decisions are made at both the portfolio and the sector levels. A top-level manager may determine the total portfolio leverage (relative to the benchmark), and an asset allocator may subsequently determine the total exposure to each sector. Therefore, if total leverage is determined at the portfolio level, then the allocation of total exposure to each sector is constrained by the portfolio leverage decision and must be measured relative to that. Instead of using the absolute total exposure as the allocation weight, we now need to use the relative total exposure (total exposure divided by the portfolio/benchmark leverage). We also need to use a hurdle return, as exposure at the sector level is partially constrained. The resulting outperformance decomposition is summarised as follows:

Asset Allocation	$\beta^P \sum_s \left(\frac{w_s^P \beta_s^P}{\beta^P} - \frac{w_s^B \beta_s^B}{\beta^B} \right) \cdot \left(R_s^B - R^H \right)$	(57)
Sector Management	$\sum_s w_s^P \beta_s^P \cdot \left(R_s^P - R_s^B \right)$	(58)
Top-Level Leverage	$\left(\beta^P - \beta^B \right) \cdot R^H$	(59)
Diversification	$\left(\sum_s w_s^P \beta_s^P - \beta^P \frac{\sum_s w_s^B \beta_s^B}{\beta^B} \right) \cdot R^H$	(60)

To maintain the completeness of the decomposition, we need to introduce an additional term, which we named diversification, to capture the difference of leverage measured at the portfolio level versus the sum of the sectors' leverage contributions. Indeed, if we assume no leverage in the benchmark, the diversification term becomes $\left(\sum_s w_s^P \beta_s^P - \beta^P \right) \cdot R^H$.

Since the leverage ratio is proportional to the return basis, which in turn is typically trying to capture the investment risk associated with a portfolio or a sector, the difference between the sum of the leverage contributions of the sectors and the leverage of the portfolio can be thought of as a diversification effect, hence the name for this term. The diversification term is zero if the leverage is aggregated linearly, i.e. $\beta^P = \sum_s w_s^P \beta_s^P, \quad \beta^B = \sum_s w_s^B \beta_s^B$.

Notice the similarity of the above analysis with the decomposition of the OAS-related outperformance as described in Section 4.5.1. In fact, handling leverage is a generalization of the top-down decomposition algorithm of Section 3.4, where unleveraged returns are the factor returns and leveraged weights are the exposures. Indeed, we can recover Equations (14-18) from Equations (55-60) by setting $w\beta \to \alpha$ and $R \to f$ and assuming linear aggregation of leverage.

6.4. Special Handling of Certain Derivative Types

Although we have derived a general framework for handling derivatives, many times derivatives are used to manage a specific type of risk exposure and are not meant to participate in the general asset allocation/security selection portfolio management framework. We already encountered this issue in Section 5.2, where we discussed how we handle the role of currency derivatives as FX hedging instruments. A similar situation arises with interest rate derivatives as instruments of managing interest rate risk only. In both cases we have to account for return that is not tied to their primary hedging function and for the implicit cash allocation that occurs when derivative instruments acquire market value between rebalancing dates. On the other hand, credit derivatives are typically used along with cash instruments to manage exposures to sectors and issuers; therefore, their treatment is quite different. Below, we discuss how HPA handles each derivative type in detail.

Currency Derivatives

The current HPA implementation in POINT® allows users to treat FX forwards as FX hedging instruments.[33] In this mode, they participate in FX outperformance only, not in the analysis of local outperformance. As discussed in Section 5.2, removing them from local analysis will change the basis of analysis of local returns if such FX forwards have non-zero market value. This difference, which we called the cash-balance effect, is explicitly captured and reported under the Hedging Effects line under FX Allocation and Hedging. This item also captures any return of FX forwards that cannot be explained by spot FX rate changes, the FX hedges local return effect.

Interest Rate Derivatives

Derivatives designed to hedge or gain exposure to interest rate factors are typically overlays onto a portfolio after the core positions are chosen. These instruments, such as futures and interest rate swaps, typically affect the curve return of the portfolio but have little effect on the excess return over yield curve. Typically, they are used in portfolios that lend themselves to the Excess Return and Fully Analytical models. In those two models, IR derivatives contribute only to the curve outperformance, which is a common factor, and they are removed entirely from the decomposition of the allocated factors.

In a similar fashion to currency hedges, the removal of interest rate derivatives from the asset allocation/security selection outperformance analysis introduces a cash-balance effect if the market value of such derivatives is non-zero. Indeed, a positive market value indicates an implicit allocation to cash, whereas a negative market value indicates implicit borrowing and, therefore, leverage. This effect is captured and reported under the top-level outperformance category for each local currency report called Leverage, whether it is true leverage (borrowing) or deleverage (implicit allocation to cash).

Further, exchange-traded derivative instruments may incur return that cannot be explained by their curve exposures. For example, Treasury futures frequently trade at a spread to their theoretical fair value. Changes in this spread result in return that cannot be explained by interest rate movements. Nevertheless, the HPA algorithm does not allow such return to pass through to the asset allocation/security selection algorithm but instead reports it in the Yield Curve report under the Rest of Curve & Convexity category.

This approach has the intuitive property that a portfolio with interest rate derivatives benchmarked against a version of itself without the derivatives will have no asset allocation or security selection in the excess and spread return models. All the outperformance will come from the yield curve and be reported correctly in the Yield Curve report.

Credit Derivatives

In contrast to currency and interest rate derivatives, credit default swaps (CDS), credit index default swaps (CDX or ITRAXX), and total return swaps (TRS) are typically used to manipulate exposure to allocated as well as common factors. HPA considers these

[33] Only FX forwards are treated as hedging instruments currently. FX futures are not yet supported but will also be treated as hedging instruments when available. Currency swaps are not treated as hedges. FX options are not currently supported.

instruments as part of the regular asset allocation/security selection decomposition. CDS and CDX instruments have small interest rate exposures which contribute to the yield curve outperformance, but their primary exposure is to the spread factor of their underlying credits. CDX indices feature look-through functionality – their performance is explained by exploding them into appropriate positions in their constituents. The CDX constituents are then included in the asset allocation/security selection decomposition, ensuring that the correct weights and returns are used to reflect the actual exposure to each partition bucket.

The basis of returns for credit derivatives is not their market value. As a result, they generate bucket leverage, as discussed in Section 5.2.

7. From Theory to Practice

In order to be of any practical use, a performance attribution system must deal with a significant number of special situations that arise in day-to-day portfolio management. Such issues include missing security prices or analytics, bad prices or analytics, intra-period transactions, settlement conventions, pricing discrepancies between the portfolio and its benchmark, etc. In the implementation of the HPA algorithm in POINT®, we strive to account for as many of such situations as possible to create a sophisticated, flexible, yet practical performance attribution system. Below, we discuss some of these issues.

7.1. Return & Outperformance Compounding

7.1.1. Total Return & Return Splits Compounding

It is well known that multi-period arithmetic total return is compounded by taking the product of the single-period total returns:

$$R = \left(\prod_{t=1}^{T} (1 + R_t) \right) - 1$$

The HPA framework begins with the splitting of a single-period total return into a list of factor returns, $R_t = \sum_k \alpha_{k,t} f_{k,t}$. This leads to the question of how the return splits should be compounded so that the sum of the compounded return splits remains equal to the compounded total return. This is necessary to satisfy the additivity and completeness requirements of attribution.

HPA accomplishes this by defining $\hat{R}_k$ as the multi-period compounded factor return. Each period factor return, $f_{k,t}$, is scaled by the compounded total return up to the previous period t-1, i.e.,

$$\hat{R}_k = \sum_{t=1}^{T} \alpha_{k,t} f_{k,t} \prod_{s=1}^{t-1} (1 + R_s), \text{ where } \prod_{s=1}^{0} (1 + R_s) \equiv 1 \tag{61}$$

This guarantees that $R = \sum_k \hat{R}_k$.

7.1.2. Outperformance Compounding

The HPA framework adopted the arithmetic definition of outperformance, $R^P - R^B$. It is well known that arithmetic outperformance does not simply add up over time:

$$R^P - R^B \neq \sum_t (R_t^P - R_t^B)\text{, where } R = \left(\prod_{t=1}^{T} (1 + R_t)\right) - 1$$

The difference, or residual, needs to be distributed. While there are several well-documented methods to handle this problem, we adopted the optimised linking coefficient approach of Menchero (2000). The basic idea behind this approach is to scale each single-period outperformance to incorporate the effects of geometric compounding. In other words, it seeks for a set of coefficients β_t^{Opt}, such that:

$$R^P - R^B = \sum_t \beta_t^{Opt} \left(R_t^P - R_t^B\right) \tag{62}$$

Once β_t^{Opt} is determined, it can be used to scale every outperformance component (e.g., yield curve, asset allocation, security selection, etc.) because of the additivity and completeness properties.

The value of β_t^{Opt} is determined as follows:

$$\begin{aligned} A &= \frac{\left(R^P - R^B\right)/T}{\left(1 + R^P\right)^{1/T} - \left(1 + R^B\right)^{1/T}} \\ C &= \frac{R^P - R^B - A\sum_{t=1}^{T}\left(R_t^P - R_t^B\right)}{\sum_{t=1}^{T}\left(R_t^P - R_t^B\right)^2} \\ \beta_t^{Opt} &= A + C\left(R_t^P - R_t^B\right) \end{aligned} \tag{63}$$

Please refer to Menchero (2000) for a detailed discussion.

7.2. Transactions

Return and outperformance calculations require special treatment of transactions. Security transactions create “intra-period” returns, as well as “unsettled” positions. It is important to recognize intra-period return separately because it does not participate in the top-down decomposition in any of the models described in this paper. It is also important to make the distinction between settled and unsettled positions because only the settled positions earn carry.

(a) Intra-day Return

The HPA algorithm uses daily compounding of returns and outperformance. However, POINT allows users to enter transactions that occur during the day at prices that are the actual transaction prices, rather than the closing prices of the traded security. Thus, the returns of the portfolio, including trading activity, are captured accurately. The difference between the trade price and the closing price used in the HPA algorithm is captured and reported in the Intra-Day return category. In addition, POINT® allows managers to enter offsetting transactions on the same day to capture intra-day trading activity. Such positions are absent from the end-of-day portfolio positions and do not participate in the outperformance analysis algorithm. Nevertheless, their P&L is captured and reported under the intra-day category as well. The contributions of individual positions to intra-day outperformance are reported in the Security Selection report.

(b) Unsettled positions

Most transactions occur on a forward basis; i.e. both cash and the security are exchanged some days in the future. For example, corporate securities transactions settle after three business days (T+3), while most government securities settles after one (T+1). Certain mortgage securities (TBAs) have special settlement rules as they settle on a specific day each month. Generally, portfolio managers are allowed to settle their transactions in POINT® at any forward date.

While buyers of a security are exposed to its clean price fluctuations (market risk) immediately after the transaction, they do not begin earning the coupon return of the security (more accurately, the yield or time return of the security r^{carry}) until after the settlement date. Instead, they earn the yield on the cash (r^{cash}) that they have promised to pay on the settlement date. Therefore, a portfolio holding an unsettled position in this security is earning a return equal to $r^{cash} + r^{price}$. An index holding this position (settled), is earning $r^{yield} + r^{price}$. The return $(r^{cash} - r^{yield}) \cdot \Delta t$, where Δt is the time to settlement constitutes real outperformance (usually negative) that must be accounted for. Correspondingly, when the portfolio sells the position, it is earning the security yield (instead of cash) until the sell settlement date, making back any yield lost when the security was bought. These effects are captured by the outperformance algorithm and reported as appropriate for each model as yield curve and spread carry outperformance.

7.3. Pricing Differences and Exclusions

7.3.1. Pricing Differences

Security prices used to compute returns of benchmark indices (Barclays Capital or third party) may be different from what portfolio managers use to mark their positions.[34] For any outperformance decomposition to be meaningful, it is important for the price and, hence, the return of the same security to be consistent in both universes. In most cases, managers would want to use their own prices to compute their portfolio returns. Therefore, HPA uses security returns from the portfolio by default whenever there are discrepancies.[35] However, since the total return of the benchmark index also needs to be consistent with the published number, we account for the pricing difference effect and report it as a separate source of outperformance.

7.3.2. Missing Prices

When there are missing prices during the attribution period, we either exclude the securities from the analysis entirely or imply prices if that is deemed reasonable. During the period with missing prices, we use previous-day analytics and market changes to imply prices so that we can compute the daily outperformance. This approach is particularly useful if prices for the beginning and end of the period are available so that the correct return over the attribution period can be computed.

[34] This is common for less liquid securities.
[35] HPA also offers the option to use security return from the benchmark.

7.3.3. *Exclusions and Substitutions*

Missing or unreasonable analytics either are substituted by previous values or cause securities to be excluded from the attribution algorithm. If analytics are missing and it is deemed reasonable to use the previous values, we copy them forward and proceed with the usual return splitting and outperformance decomposition process.

However, if the analytics are not within reasonable bounds, the return splitting algorithm will ignore them and the entire total return of the security will be attributed to Exclusions. Since the total return is known, this security still contributes to the total return of the portfolio. In the Excess Return and Fully Analytical models, only the weight of the excluded security is used in asset allocation and security selection, not the return, because we were not able to perform proper return splitting. Exclusion returns are aggregated bottom-up and included in a detailed security-by-security report. This way, the sector outperformance will not be penalised when analytics are missing from the analysis.

All portfolio and benchmark securities with missing prices, implied prices, substitutions and exclusions are displayed on the Warnings and Exclusions report.

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