FORECASTING DEBT PAYDOWN AMONG LEVERAGED EQUITIES

Analysis of U.S. Stocks from 1964 to 2012

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Abstract

Using a random sample of 60% of our cross-sectional data on U.S. stocks from 1964 to 2012, we trained four machine learning algorithms to forecast debt paydown over a one-year horizon. An evaluation of these candidate models on half of the hold-out sample (20% of the original dataset) showed that a boosted trees algorithm can forecast debt paydown with up to 70% precision over the next year. This boosted trees model achieved similar results in a second out-of-sample test on the remaining 20% of original data. While information on one-year-ahead equity returns was not used in training or evaluating any of the models, our results show that stocks with a higher estimated probability of paying down debt in the next year also earn higher average returns in that one-year-ahead period. A back-test of the boosted trees model's forecasts of debt paydown between 1965 and 2012 shows a 10.3 percentage point spread between the average annual returns of portfolios formed from the 10th decile of estimated debt paydown probability versus annual portfolios formed from the 1st decile of estimated debt paydown probability. When the 10th decile is combined with a value investment strategy to focus on cheap leveraged stocks that are most likely to pay down debt, we find a CAPM beta of 1.18 and a statistically significant alpha of 9.3% per year in a four-factor model that controls for the three Fama-French factors and momentum.

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1. *Introduction*

Public companies can return money to shareholders in three ways: dividends, share buybacks, and debt repayment. While much has been written about dividends and share buybacks, this paper contributes to the academic literature through an analysis of how to forecast debt repayment and an investigation of how debt repayment drives equity returns.

We use machine learning to analyze historic financial statements and determine the best predictors of future debt pay-down. Our analysis shows that future debt paydown can be forecasted with precision of up to 70% over a one-year period. The average probability of future debt paydown across all U.S. stocks in our dataset is 37%. Therefore, our algorithm's precision in forecasting debt paydown is up to 1.9x better than the outcome of random guessing.

Forecasting debt paydown is important because leveraged companies that deleverage earn significantly higher equity returns than indebted companies that do not deleverage. Therefore, this paper provides a method for separating attractive leveraged companies (that are likely to pay down debt and earn higher risk-adjusted returns) from unattractive leveraged companies (that are unlikely to pay down debt and usually earn lower risk-adjusted returns). Importantly, the framework in this paper does not rely on predicting earnings growth better than the market, nor on forecasting dividend yields and share buyback yields. Instead, the approach that is presented in this paper is based on a probability assessment of the likelihood of deleveraging and an understanding of the benefits of deleveraging to the financial health and valuation of leveraged companies.

We show that there is a 10.3 percentage point spread in the average annual returns of portfolios formed from the 10^{th} decile of estimated debt paydown probability relative to the returns of portfolios formed from stocks in the 1st decile of estimated debt paydown probability. Moreover, we show that an investment strategy based on the 10th decile of estimated debt paydown probability has higher average annual returns and a higher Sharpe Ratio when applied with a focus on value stocks. Below, we show the back-tested average annual returns of a strategy that focuses on value stocks with high likelihood of future debt paydown according to our algorithm:

Fig 1: Annual Returns of the Decile 10+Value Strategy

Debt Paydown Probability	Decile 10 and Low Price/Book	
	Equal-Weighted	Value-Weighted
Average Annual Return	22.4%	16.4%
Standard Deviation	44.4%	31.4%
Sharpe Ratio	0.39	0.36
Geometric Mean Return (CAGR)	16.7%	13.0%
Avg. Portfolio Size (N stocks)	199	199

While the idea that computers could be effective in forecasting debt paydown may seem surprising, decades of social science research has shown that simple algorithms outperform clinical judgment, and that the most effective methods of predictions rely on the development and application of base rates of probability. Machine learning algorithms incorporate these bases rates when forecasting debt paydown and they can automatically capture non-linear relationships and interactions, resulting in a key set of advantages over linear forecasting models.

In this paper, we use a boosted trees algorithm as our primary classifier and we also compare its efficacy against other forecasting models including logistic regression (logit), random forests and neural networks. The parameters of the boosted trees algorithm were optimized through cross validation in order to minimize the risk of over-fitting to the training data. The boosted trees algorithm provides the best out-of-sample performance among the four models we evaluated, followed by random forests. We believe that this is because tree-based models do particularly well in capturing non-linear relationships and interactions between the variables in our sample.

The rest of our paper proceeds as follows:

- Section 2 describes the data that was used in our analysis.
- Section 3 outlines our process of model selection
- Section 4 presents the results of our final boosted trees model. Based on two out-of-sample tests, we show that debt paydown can be forecasted with precision of up to 70% over a one-year horizon.
- Section 5 connects debt paydown forecasts with expected equity returns and shows that leveraged stocks with higher estimated probabilities of debt paydown have higher expected returns than leveraged stocks with low estimated probabilities of future debt paydown.
- Section 6 presents back-tests of investment strategies that are based on our debt paydown forecasting model from 1965 to 2012.
- Section 7 presents a CAPM and multi factor analysis of an investment strategy that combines debt paydown forecasts and value.
- Section 8 discusses the explanatory variables that were important in the boosted trees model
- Section 9 concludes. Finally, technical details of the boosted trees model are presented in Appendix A.

2. *Data Processing*

Our data was sourced from the University of Chicago's Center for Research on Security Prices (CRSP) database. We compiled fundamental and price information on all NYSE/AMEX/NASDAQ stocks between January 1, 1963 and December 31, 2013. Based on fundamentals that were reported in the prior calendar year, debt paydown predictions are made at the end of March (time T_0) for each stock over the next year. We implemented the following measures in order to pre-process our data for the analysis.

1. We created a binary outcome variable called "future debt paydown" (FDP) to represent debt paydown over the next year. The FDP variable is equal to "1" if a company's long term debt at Year T+1 is less than its long term debt at Year T_0. Companies that increase long term debt or maintain the same amount of long term debt over the next year are tagged with a "0" in our FDP variable. In order to prevent survivorship bias, we also tagged companies that drop out of the data with a "0" as their final outcome in the FDP variable.
2. We excluded one-year-ahead returns from the data during training and evaluation of the algorithms since the analysis is based on information that would have been known at time T_0.
3. To prevent look-ahead bias, we implemented at least a three month lag between the timing of annual accounting information and the market-based variables that are as of the end of March (time T_0) in each year.
4. We removed dates from the data so that historical dates are not used as an explanatory variable in our cross-sectional analysis.
5. We randomly split the data into a training sample (60% of observations, N = 151,151), a validation sample (20% of observations, N = 50,383) and a test sample (20% of observations, N = 50,385).

Figure 2 presents a list of the variables that were used to predict FDP.

Fig 2: Summary of Variables Used in the Analysis

	Explanatory Variable	Description
1	*asset_growth*	Asset growth (%) between Year T-2 and Year T-1
2	*asset_turnover*	Binary variable = 1 if sales growth (%) > asset growth (%) over the past year, otherwise 0
3	*assets_total*	Total assets
4	*bv_per_share*	Book value per share
5	*debt_assets*	Long term debt / Total assets
6	*debtpy_debt*	Binary variable = 1 if long term debt reported in Year T-1 < long term debt reported in Year T-2, otherwise 0
7	*ebitda*	Earnings before interest, taxes and depreciation (EBITDA)
8	*ebitda_ev*	EBITDA / Enterprise value
9	*ebitda_margin*	EBITDA / Revenue
10	*enterprise_value*	Enterprise value (defined as market capitalization + long term debt)
11	*gprofit_assets*	Gross profit / Total assets
12	*lt_debt_ev*	Long term debt / Enterprise value
13	*lt_debt_total*	Long term debt
14	*marketcap_t*	Market capitalization at the end of March in Year T_0
15	*pb_index*	Deciles of [Price/Book] for each year
16	*price_book*	Price/Book
17	*prior_1yr_return*	Return from the end of March in Year T-1 to the end of March in Year T_0 (including dividends)
18	*profit_index*	Deciles of [Gross profit / Total assets] for each year
19	*pyreturn_below_median*	Binary variable = 1 if the return of a stock in the past year was below the median of all stocks, otherwise 0
20	*pyreturn_index*	Deciles of prior one-year return for each year
21	*revenue*	Revenue (Sales)
22	*sales_growth*	Sales growth (%) between Year T-2 and Year T-1
23	*sharespy_shares*	Binary variable = 1 if there are fewer shares outstanding in Year T-1 vs Year T-2, otherwise 0
24	*size_index*	Deciles of market capitalization for each year
25	*value_index*	Deciles of [EBITDA / Enterprise value] for each year
	Outcome Variable: FDP	**Binary variable = 1 for companies that pay down debt over the next year, otherwise 0**

We then confirmed that each random sample of the data was balanced in terms of the outcome variable, future debt paydown, ("FDP"). As shown in the table below, the average probability of future debt paydown in each sample is 37%.

Fig 3: Samples Used in the Analysis

	Future Debt Paydown		
Sample	**No = 0**	**Yes = 1**	**% of Sample = 1**
Training	94,800	56,351	37.3%
Validation	31,585	18,798	37.3%
Test	31,684	18,701	37.1%

3. Analysis and Model Selection

The training sample was used to "train" our algorithms to identify relationships between the 25 explanatory variables and the outcome variable, FDP. As it turns out, the best models were able to capture non-linear interactions between the explanatory variables in order to provide a more complete picture of how these variables relate to FDP. Throughout the training process, the algorithms did not "see" the validation sample so this information could be used as out-of-sample data to evaluate the performance of each algorithm's predictions. During the evaluation stage, each algorithm assigned a probability of future debt paydown to every stock in the validation sample (without access to the actual result in FDP). Once probabilities were assigned to the validation sample, the model's performance was evaluated on the basis of how closely the probabilities matched the actual outcome in FDP. Without seeing the 1s and 0s in each stock's FDP variable within the validation sample, a good model would assign higher probabilities to stocks that turn out to have a 1 in the FDP variable, and it would assign low probabilities to stocks that turn out to have a 0 in FDP.

In terms of model selection, we started by using a logistic regression model. Probabilities from the logit model are based on a logistic transformation of a linear regression of FDP against the explanatory variables. We used a lasso regularization procedure with cross validation to select a logit model that minimized the out-of-sample deviance of our forecasts.[2] The selected logit model used 23 of the original 25 explanatory variables. Once this logit model had been trained on the training sample, the debt paydown probabilities it calculated for each stock in the validation sample were based on the function below, where X_i represents an explanatory variable.

$$Prob(FDP = 1|\boldsymbol{X}) = \frac{e^{\beta_0+\beta_1 X_1+ \,\ldots.\, +\beta_N X_N}}{1+e^{\beta_0+\beta_1 X_1+ \,\ldots.\, +\beta_N X_N}}$$

While the logit model provides reasonable out-of-sample predictions, it is not the best model for forecasting debt paydown. One reason for this is because logit is based on a linear model so it does not automatically capture non-linear relationships between the explanatory variables and the outcome. Secondly, in cases where there are interactions between the explanatory variables, logit does not automatically capture these interactions.

The other machine learning algorithms we evaluated were boosted trees, random forests, and neural networks. Boosted trees and random forests both work by utilizing decision trees to estimate probabilities. Each branch of the tree represents one of the explanatory variables and the nodes at the bottom of the tree provide the probability estimates. A generalized example of how decision trees work is provided in the chart below.

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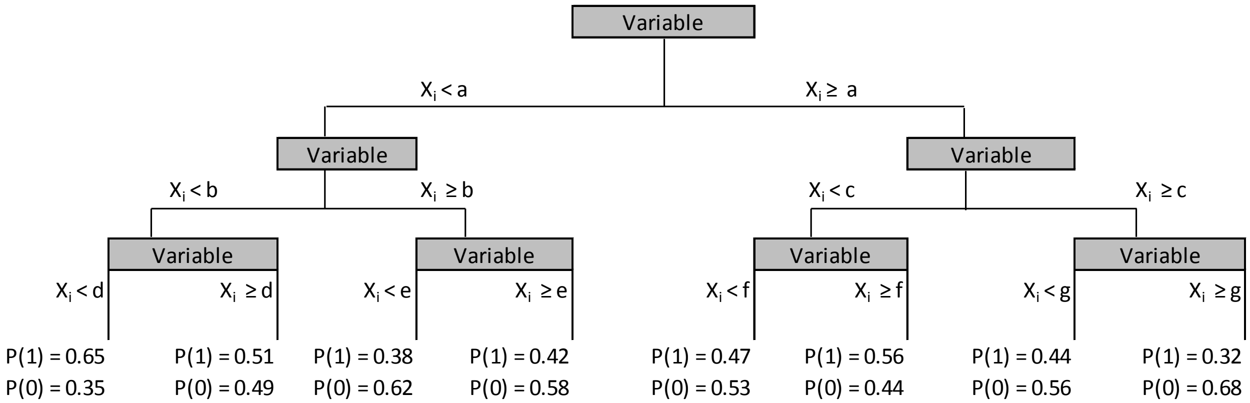
[2] "One-Step Estimator Paths for Concave Regularization" Matt Taddy (2015)

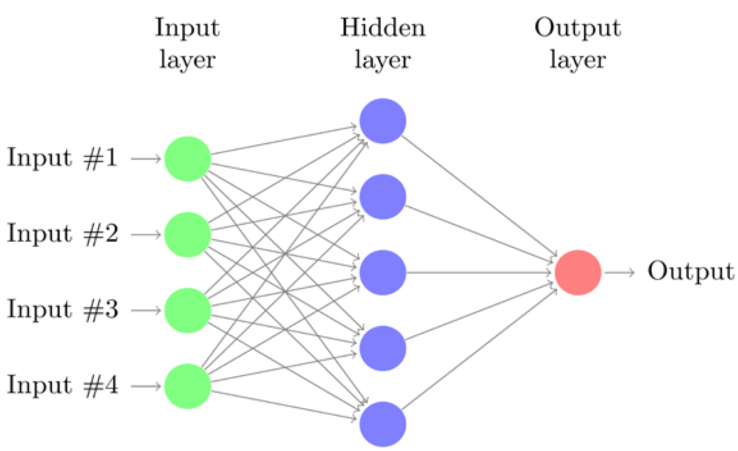
Fig 4: Generalized Decision Tree Example



A major advantage of decision trees is that they capture interactions between explanatory variables particularly well. In the case of boosting and random forests, hundreds or thousands of trees are fit to the training data in order to develop a model of how the explanatory variables relate to the outcome.

Neural networks are based on a model of the neurons in the human brain. Each explanatory variable provides an input to the neural network. One or more hidden layers of "neurons" then develop a model of how the inputs interact with each other and relate to the output variable, as illustrated below.

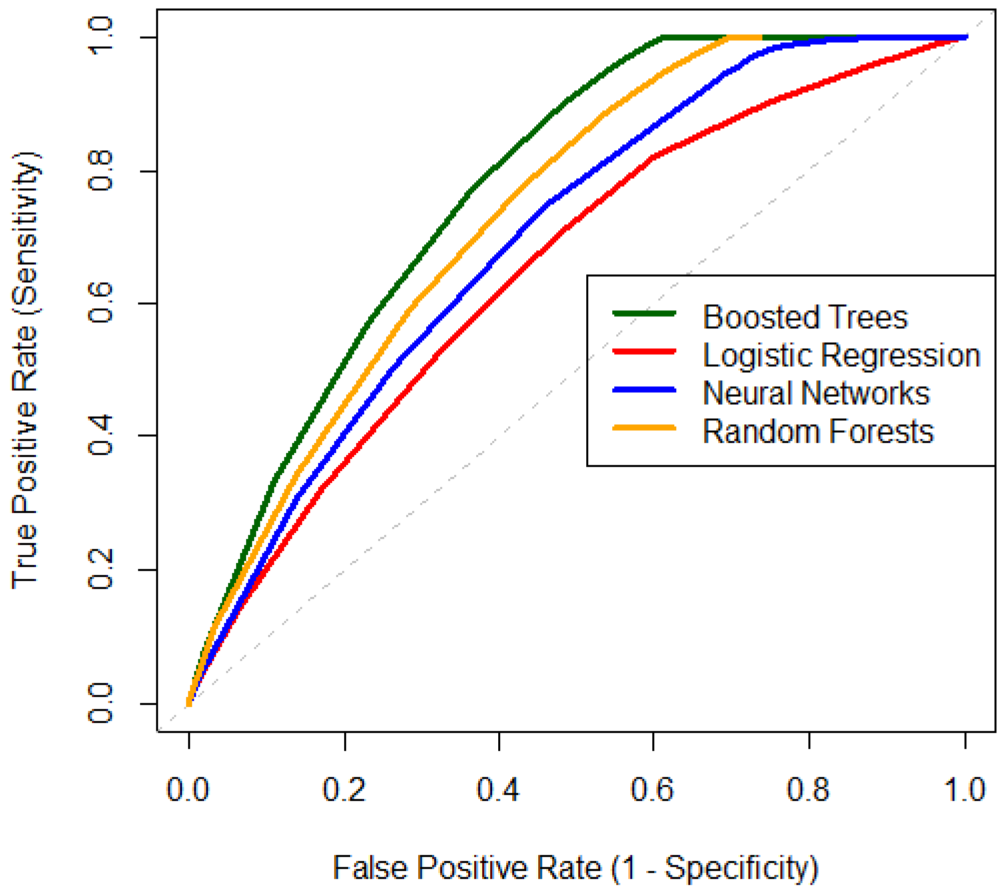
Fig 5: Neural Network Example



Source: Texample.net

After we trained algorithms under each method, our next step was to compare the performance of each model. A convenient tool for comparing the out-of-sample performance of each model is the ROC Curve analysis.[3] The best prediction model has the largest area under its ROC Curve by providing the highest proportion of true positives relative to false positives. As shown in the table below, the ROC Curve for predicting FDP shows that boosted trees provide the best out-of-sample performance when applied to the validation data.

Fig 6: ROC Curves (Out-of-Sample Performance on the Validation Data)



[3] The analysis presented in this paper was performed in R. R Core Team (2015). "R: A language and environment for statistical computing." *R Foundation for Statistical Computing, Vienna, Austria*

4. *Results of the Boosted Trees Model*

As shown in the ROC Curve above, there is a trade-off between the number of true positives (TP) captured by a model and the number of false positives (FP). In the case of predicting future debt paydown, we can measure this trade-off through the *precision* of the model. Precision is defined as the proportion of true positives relative to the total number of positives predicted by the model. $Precision = \frac{TP}{(TP+FP)}$

The ROC Curve helps us to maximize precision by determining the optimal probability threshold for considering a prediction as a "1" or a "0". For example, if we want to target the top 5% of predicted 1s, the ROC Curve for boosted trees suggests setting the probability threshold at 0.658. All stocks in the validation data with debt paydown probabilities greater than 0.658 are estimated as 1s, and stocks with debt paydown probabilities less than or equal to 0.658 are estimated as 0s. At this threshold, the boosted trees model provides 70.2% precision in its predictions of which stocks will pay down debt in the next year. The *accuracy* of the model is 64.4% at this threshold. Accuracy is defined as the total number of predictions that the model got right after accounting for true negatives (TN) and false negatives (FN). $Accuracy = \frac{TP+TN}{(TP+FP+TN+FN)}$

These results are summarized in the table below.

Fig 7: Performance of Boosted Trees at Probability Threshold $\hat{P}$ > 0.658 (Validation Data)

	Actual			
Boosting ($\hat{P}$ > 0.658)	**0**	**1**	**Performance**	
Predicted 0	30,946	17,295	Accuracy	64.4%
Predicted 1	639	1,503	Precision	70.2%

If we wanted to anticipate a greater number of actual 1s we could lower the probability threshold. However, this increase in the number of predicted 1s would come at the cost of lower precision, as shown in the table below. It's important to note that the average probability of future debt paydown across the dataset is 0.37 so even if we lower the probability threshold in our model to near that level, we still get precision that is better than random chance at 52.6%.

Fig 8: Precision of the Boosted Trees Model at Various Probability Thresholds

Target % of Predicted 1s	Probability Threshold	Accuracy	Precision	# of Predicted 1s
Top 5%	0.658	64.4%	70.2%	2,142
Top 20%	0.572	68.1%	63.9%	9,828
Top 40%	0.498	69.5%	59.4%	18,102
Top 60%	0.433	68.6%	55.7%	26,017
Top 80%	0.376	66.0%	52.6%	32,022

As a final out-of-sample test of our model, we combined the training and validation sample and trained the boosted trees algorithm on this combined sample. We then tested this model on the test sample; a second dataset that the model had not "seen" up to that point. If the results of this second out-of-sample test are similar to the model's performance at the validation stage, then we can be more confident in the model's stability when predicting debt paydown in new datasets.

Fig 9: Performance of Boosted Trees Model at $\hat{P}$ > 0.658 and $\hat{P}$ > 0.376 (Test Data)

	Actual			
$\hat{P}$ > 0.658	0	1	Performance	
Predicted 0	31,052	17,218	Accuracy	64.6%
Predicted 1	632	1,483	Precision	70.1%

	Actual			
$\hat{P}$ > 0.376	0	1	Performance	
Predicted 0	16,509	2,036	Accuracy	65.8%
Predicted 1	15,175	16,665	Precision	52.3%

As shown in the above results, the boosted trees model achieves a precision of 70.1% on the test data at a threshold of $\hat{P}$ > 0.658. If we lower the threshold to $\hat{P}$ > 0.376, the model's precision is 52.3%. These out-of-sample results are very similar to the model's performance at the validation stage, which is what we would expect from a stable prediction model.

5. *Future Debt Paydown and Expected Returns*

Having shown that the boosted trees model is effective at forecasting future debt paydown through two out-of-sample tests, we now turn our attention to the link between debt paydown and expected returns. Up until this point, we have not used any information on one-year-ahead returns in our analysis. The boosted tress model has simply been trained and evaluated in order to optimize its ability to forecast debt paydown over the next year. In Figure 10 below, we incorporate new information on one-year-ahead returns for each stock to show that forecasting future debt paydown is related to expected returns. Specifically, there is a higher expected return associated with stocks that are identified by the boosted trees model as having a higher probability of paying down debt in the next year. In the table below, we applied the boosted trees model to all stocks in our dataset that have a positive long term debt balance at time T_0. We then grouped these 195,884 stocks into deciles according to their estimated probability of future debt paydown. These deciles are shown in the first column of the table below. The second column shows the average outcome per decile in terms stocks that paid down debt in the next year (1s) and stocks that did not pay down debt in the next year (0s). Since this table mostly reflects in-sample forecasts (approximately 80% of this data was used to train the algorithm) we can see a monotonic relationship between estimated debt paydown probabilities and actual outcomes. This is what we would expect from the model. The new information, however, is in the third column of the table where we see that stocks in the higher deciles of estimated debt paydown probability also earn higher average returns in the subsequent year. This relationship is monotonic across the deciles, apart from the first decile. There is a greater proportion of missing returns in that first decile, which may be contributing to this discrepancy in the trend.

Fig 10: Mean One-Year-Ahead Return by Estimated Debt Paydown Probability Decile

Probability Decile	Mean FDP (Actual Outcome)	Next 1-Year Return	Standard Error	Number of Obs
10	0.7129	30.22%	1.21%	19,204
9	0.6349	21.44%	1.43%	18,693
8	0.5863	20.25%	1.37%	18,348
7	0.5414	18.21%	1.41%	18,275
6	0.5135	17.27%	1.58%	18,104
5	0.4729	11.09%	1.29%	17,828
4	0.4310	9.52%	1.18%	17,242
3	0.3840	5.37%	0.66%	17,593
2	0.3318	4.37%	0.56%	18,338
1	0.1824	8.40%	1.82%	14,749

It is important to note that the returns shown in Figure 10 are equal-weighted. Moreover, the sample is drawn from all leveraged NYSE/NASDAQ/AMEX stocks without any cutoff for market capitalization. Therefore, the returns of microcap stocks carry the same weight as the returns of large caps. Finally, while Fig 10 shows that there is a higher expected return associated with stocks that are more likely to pay down debt, Fig 10 does not reflect the volatility that is associated with investing in these leveraged stocks. In order to address these issues, we conducted a back-test between 1965 and 2012 where we formed annual portfolios according to the estimated debt paydown probability deciles in each year. In order to capture stocks that are accessible to institutional capital, the annual portfolios only included stocks from market cap deciles 3 to 10 in each respective year. All stocks in annual size deciles 1 and 2 were excluded from the portfolios in our back-test.

6. Back-test Results

Figure 11 presents the results of this back-test over 48 years. Portfolios of stocks that have a higher estimated probability of paying down debt have a higher average annual return than portfolios of stocks with a low estimated probability of paying down debt in the next year. As expected for leveraged equities, the standard deviations of these annual returns are also relatively high. On a value-weighted basis, the Decile 10 portfolios (with the highest estimated probability of future debt paydown) earn a 12.3% average annual return with a standard deviation of 29.3% and a 0.24 Sharpe Ratio. In contrast, the Decile 1 portfolios (with the lowest estimated probability of future debt paydown) earn a 2.0% average annual return with a standard deviation of 17.1% and a -0.19 Sharpe Ratio. Therefore, there is a 10.3 percentage point spread in the average annual returns of Decile 10 portfolios and Decile 1 portfolios. At $t = 2.10$, this difference in average annual returns is reliably different from zero.

Fig 11: Summary of Annual Portfolio Returns

In the tables below, we summarize the average annual returns, standard deviation, and Sharpe Ratio of each set of annual portfolios. The top table presents equal-weighted portfolio returns and the bottom table presents value-weighted portfolio returns. 48 portfolios were formed per decile of estimated debt paydown probability in every year from 1965 to 2012.

Equal-Weighted Returns

Debt Paydown Probability	Decile 10	Decile 9	Decile 8	Decile 7	Decile 6	Decile 5	Decile 4	Decile 3	Decile 2	Decile 1
Average Annual Return	19.2%	13.1%	10.8%	9.4%	9.5%	6.1%	5.1%	4.7%	2.9%	3.5%
Standard Deviation	41.9%	32.0%	28.1%	28.1%	25.9%	23.1%	21.8%	20.3%	19.0%	17.6%
Sharpe Ratio	0.33	0.25	0.20	0.15	0.16	0.04	-0.01	-0.03	-0.12	-0.10
Geometric Mean Return (CAGR)	13.8%	9.4%	7.7%	6.2%	6.6%	3.7%	3.0%	2.7%	1.1%	2.0%
Avg. Portfolio Size (N stocks)	325	318	314	311	307	306	310	312	323	243

Value-Weighted Returns

Debt Paydown Probability	Decile 10	Decile 9	Decile 8	Decile 7	Decile 6	Decile 5	Decile 4	Decile 3	Decile 2	Decile 1
Average Annual Return	12.3%	8.4%	7.1%	5.3%	5.1%	5.7%	2.4%	2.6%	1.5%	2.0%
Standard Deviation	29.3%	22.5%	23.0%	20.1%	18.9%	17.7%	17.5%	17.7%	16.6%	17.1%
Sharpe Ratio	0.24	0.14	0.08	0.01	-0.01	0.03	-0.16	-0.15	-0.23	-0.19
Geometric Mean Return (CAGR)	9.1%	6.2%	4.7%	3.4%	3.5%	4.2%	0.93%	0.96%	0.09%	0.56%
Avg. Portfolio Size (N stocks)	325	318	314	311	307	306	310	312	323	243

Financial theory and empirical evidence both show that leverage and value are related. Holding enterprise value constant, a company that has a high debt balance will have a lower market value of equity than an otherwise identical company with low leverage. Therefore, a natural extension of this back-test is to combine it with a value investment strategy. To this end, we formed combined portfolios of Decile 10 stocks (by estimated debt paydown probability) with value stocks that independently rank in deciles 1 to 5 by Price/Book Value. In other words, the annual portfolios in this new "Decile 10+Value" strategy comprise stocks that independently rank in the top 10% of estimated debt paydown probability and the bottom 50% of Price/Book Value in a given year. As shown in the table below, combining debt paydown forecasts and a value strategy leads to an improvement in average annual returns, both on an absolute and a risk-adjusted basis. On a value-weighted basis, the Decile 10+Value strategy has a 16.4% average annual return with a 31.4% standard deviation, resulting in a 0.36 Sharpe Ratio.

Fig 12: Annual Returns of the Decile 10+Value Strategy

Debt Paydown Probability	**Decile 10 and Low Price/Book**	
	Equal-Weighted	**Value-Weighted**
Average Annual Return	22.4%	16.4%
Standard Deviation	44.4%	31.4%
Sharpe Ratio	0.39	0.36
Geometric Mean Return (CAGR)	16.7%	13.0%
Avg. Portfolio Size (N stocks)	199	199

7. CAPM and Factor Analysis of the Decile 10+Value Strategy

Evidently, it is better to apply the debt paydown forecasting model in combination with a value investment strategy. Given the fact that value tends to outperform growth over long horizons, it makes intuitive sense that cheap leveraged stocks should outperform expensive leveraged stocks in the long run; particularly when those cheap leveraged stocks are paying down debt.

Of course, risk and return are related. In order to evaluate the risk factor exposure of the Decile 10+Value strategy, we regressed the excess value-weighted returns of this strategy against the market factor in order to assess the CAPM beta of the strategy. This CAPM beta is 1.18 with a t-statistic of 6.02, as shown below in the first panel of Figure 13. The second panel of Figure 13 presents a four factor model that controls for the three Fama-French[4] factors—excess return of the market (MKT), size (SMB) and value (HML)—as well as the momentum factor (UMD).[5] This four factor model shows that the Decile 10+Value strategy has positive betas to the market, size and value factors. The strategy has a negative beta to momentum. After accounting for these risk factors, the annual risk-adjusted return (α) of the strategy is 9.30%, which is statistically significant at t = 2.64.

As a comparison, we also present a five factor model in the third panel of Figure 13. The five factor model includes the three Fama-French factors, momentum, and Pástor-Stambaugh's traded liquidity risk factor (LIQ).[6] This five factor model shows that the Decile 10+Value strategy has a negative beta to LIQ and therefore, a low sensitivity to aggregate liquidity risk.

As shown by the model evaluation criteria at the bottom of each panel in Figure 13, the four factor and five factor models provide better explanations for the variation in returns of the Decile 10+Value strategy. The four factor model has a 74.9% Adjusted-R^2 and the five factor model has a 77.9% Adjusted-R^2, whereas the CAPM model's Adjusted-R^2 is only 42.8%. Moreover, the four factor model minimizes the Akaike Information Criterion (AIC) and the Bayes Information Criterion (BIC). Since smaller values of AIC and BIC indicate that a model is closer to the truth, the four factor model seems to be the best candidate model for explaining the variation in returns of the Decile 10+Value strategy.

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[4] "The Cross-Section of Expected Stock Returns" Eugene Fama and Kenneth French (1992)
[5] "Value and Momentum Everywhere" Clifford Asness, Tobias Moskowitz, and Lasse Heje Pedersen (2013)
[6] "Liquidity Risk and Expected Stock Returns" Luboš Pástor and Robert F. Stambaugh (2002)

Fig 13: CAPM and Multi Factor Models of Annual Decile 10+Value Returns

Panel I

CAPM Model	
	Decile 10+Value
Factor (β)	
MKT (R_M - R_F)	1.18
	(6.02)
Intercept (CAPM α)	4.22%
	(1.14)
R^2	44.04%
Adjusted-R^2	42.83%
AIC	2.796
BIC	6.538
Number of obs	48

Panel II

Four Factor Model	
	Decile 10+Value
Factor (β)	
MKT (R_M - R_F)	0.96
	(6.68)
SMB	0.91
	(4.50)
HML	0.30
	(1.61)
UMD	-0.94
	-(5.07)
Intercept (α)	9.30%
	(2.64)
R^2	77.06%
Adjusted-R^2	74.93%
AIC	-34.009
BIC	-24.653
Number of obs	48

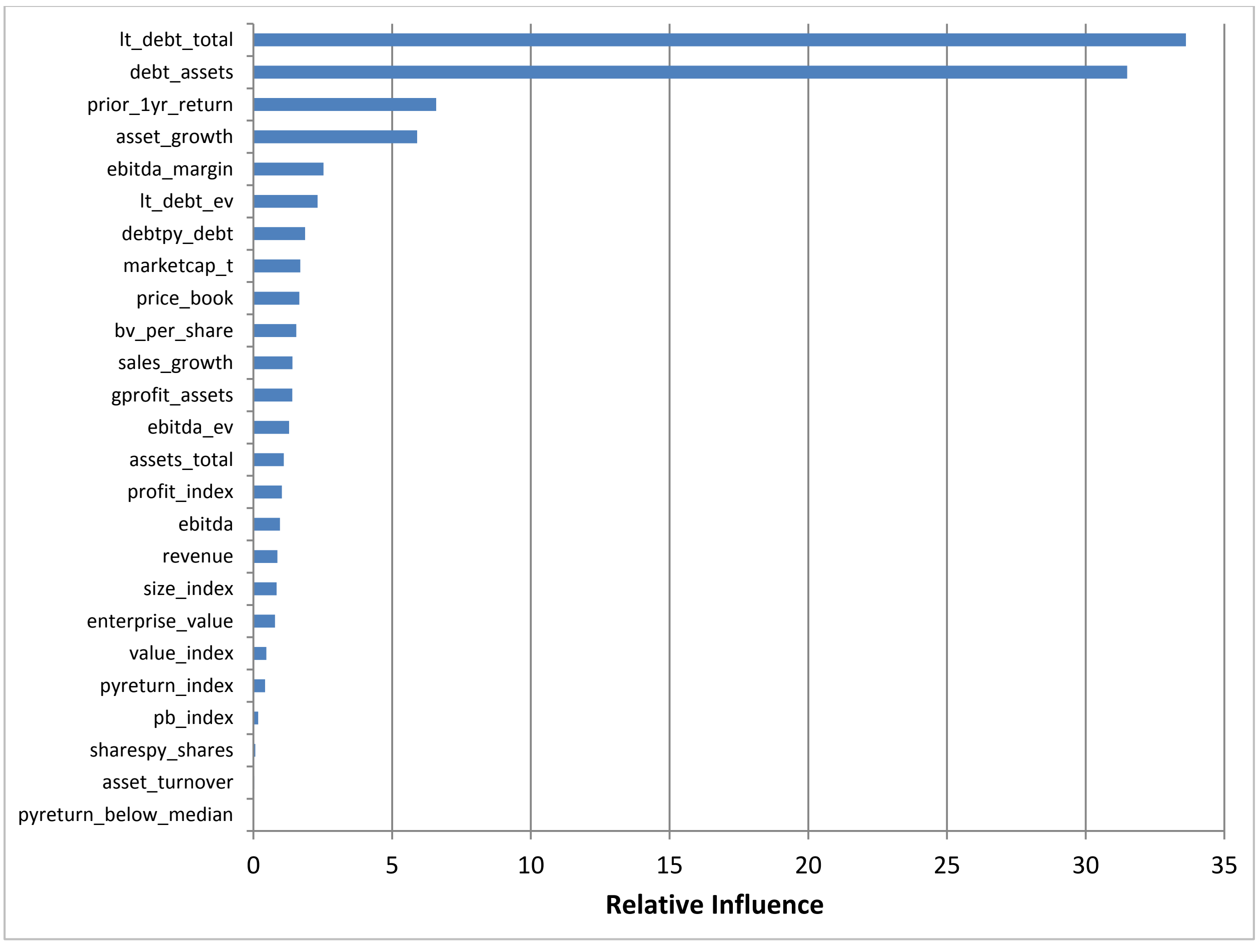
Panel III

Five Factor Model	
	Decile 10+Value
Factor (β)	
MKT (RM - RF)	0.91
	(6.39)
SMB	0.97
	(4.54)
HML	0.29
	(1.53)
UMD	-1.01
	-(5.29)
LIQ	-0.43
	-(2.07)
Intercept (α)	12.34%
	(3.37)
R^2	80.41%
Adjusted-R^2	77.90%
AIC	-33.629
BIC	-22.789
Number of obs	45

8. *Analysis of Variables in the Boosted Trees Model*

Having explained the variation in returns associated with forecasting debt paydown among leveraged value stocks, we now turn to explaining the variables that were important in developing the original debt paydown forecasts. The table below shows the relative importance of explanatory variables in the boosted trees model. It is important to keep in mind that this measure of variable importance is not ordinal (e.g. we cannot say that *Debt/Assets* is 5 times more important than *Prior 1 Year Return)*. However, the chart below provides a useful summary of variables that were important for forecasting future debt paydown in the boosted trees model.

Fig 14: Variable Importance from Boosted Trees



As discussed earlier, a major advantage of the boosted trees model is its ability to capture interactions between explanatory variables. Since there are 25 explanatory variables in our data, these interactions have many dimensions. However, we can visualize two-way interactions in three dimensions for separate pairs of explanatory variables. Examples of these visualizations are presented below in Figures 15 to 18. In these perspective plots, we combined the *Prior Debt Paydown* binary variable with *Asset Growth* over the past year, *Prior 1 Year Return*, *Debt/Assets,* and *Debt/EV* respectively. In each chart, the vertical axis ("fitted value") represents the average probability of future debt paydown at various levels of the two explanatory variables. It is interesting to note that—in all cases—the estimated probability of future debt paydown is higher for companies that have paid down debt in the previous year. This relationship is evident in the observation of a jump in the plotted surface at 0.5 on the *Prior Debt Paydown* axis. Since *Prior Debt Paydown* is a binary variable, values of 0.5 and above represent a "1" in the *Prior Debt Paydown* variable and values below 0.5 represent a "0" in this variable. With respect to the second explanatory variable, which is continuous in all charts, each graph plots the 90% of observations that are between the 5^{th} and 95^{th} percentile of that variable's range.

The perspective plot with historical *Asset Growth* (Figure 15) is particularly interesting because it shows that companies that *reduced* assets in the past year are more likely to pay down debt in the next year—especially if they also paid down debt in the past year. This observation is intuitive. If a company sold assets in the past year and used some of the proceeds to pay down debt, it is probably in a divestiture process and will likely pay down debt in the next year as well.

Figure 16 suggests that the likelihood of future debt paydown is higher within the tails of the *Prior 1-Year Return* distribution. Moreover, for any given value of *Prior 1-Year Return,* the average probability of future debt paydown is higher for companies that paid down debt in the past year. This relationship also makes intuitive sense because companies that experience poor returns in the previous year may have deteriorating fundamentals and they may need to deleverage in order to avoid financial distress. Conversely, Figure 16 suggests that companies that delivered very high returns in the past year are also more likely to pay down debt in the next year. This may be due to improving fundamentals that increase these companies' cashflows, thereby enabling them to pay down debt in the next year.

Figure 17 and 18 plot the probability of future debt paydown with respect to two measures of leverage: *Debt/Assets* and *Debt/EV* for companies that have a positive long term debt balance. In both cases, the probability of future debt paydown is higher among companies that paid down debt in the past year.

Overall, the perspective plots in Figures 15 to 18 capture non-linear relationships between the explanatory variables and the probability of future debt paydown. This ability to automatically capture non-linear relationships and interactions is one of the fundamental advantages of machine learning over linear forecasting models.

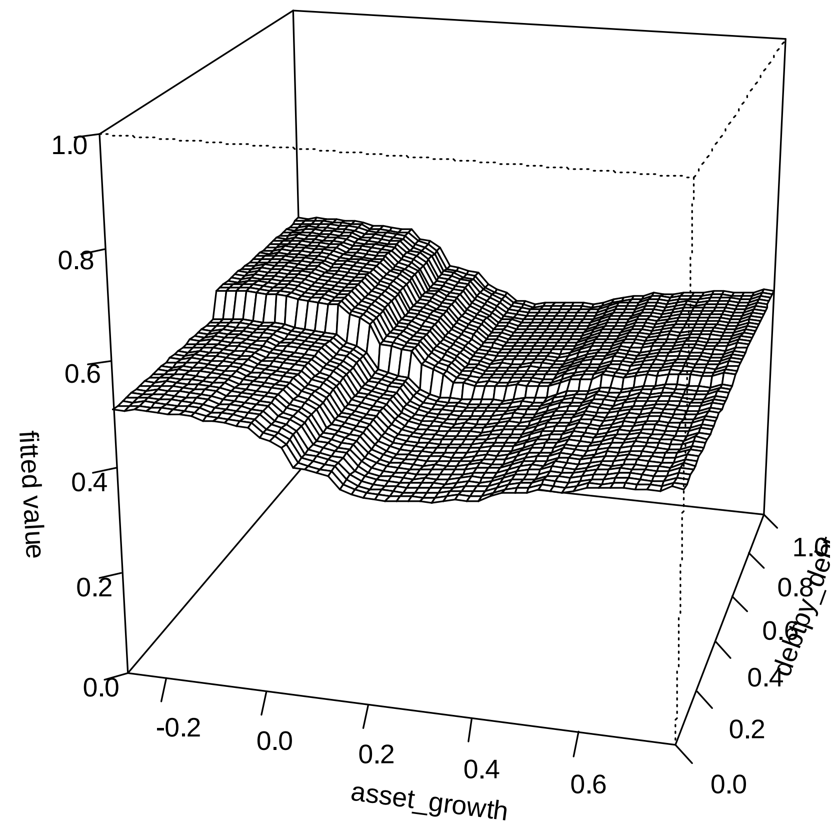
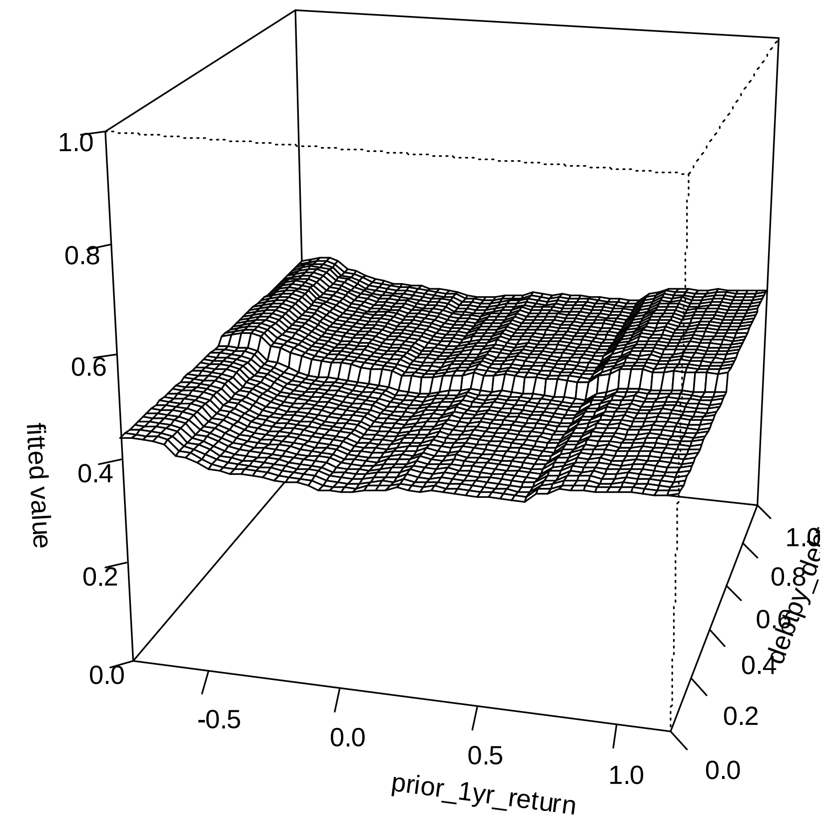
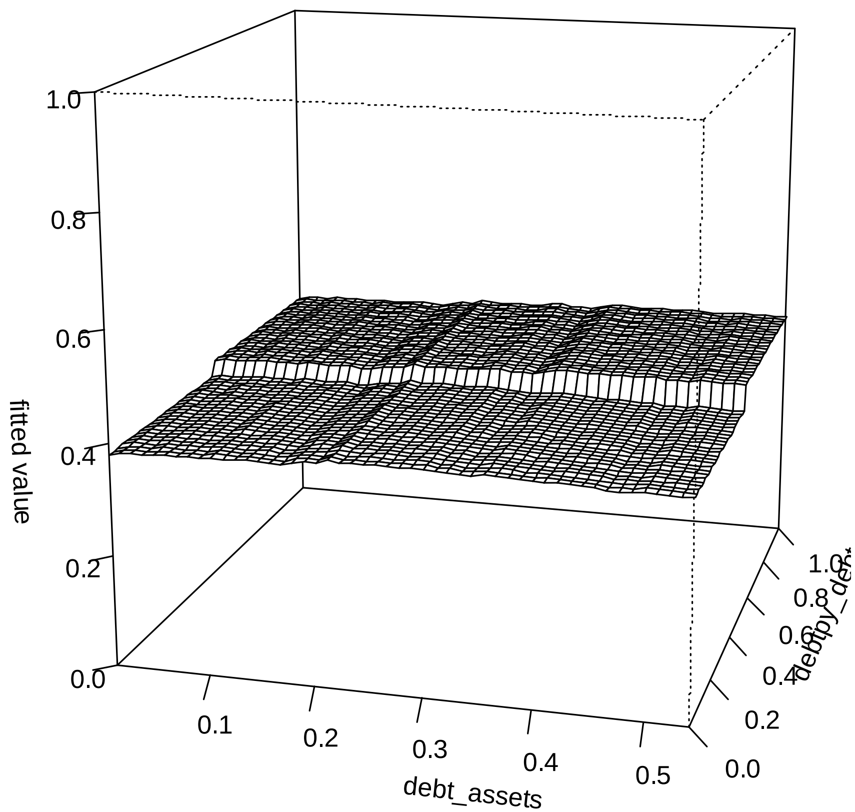
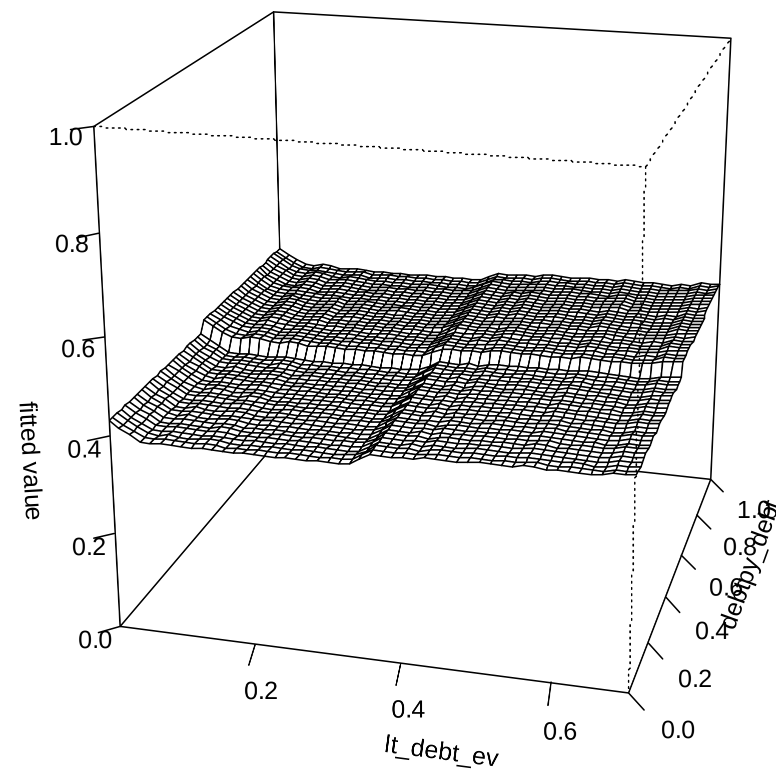
Fig 15: Asset Growth (%) and Prior Debt Paydown (0 or 1)	Fig 16: Prior 1 Year Return (%) and Prior Debt Paydown (0 or 1)
	

Fig 17: Debt/Assets (%) and Prior Debt Paydown (0 or 1)	Fig 18: Debt/EV (%) and Prior Debt Paydown (0 or 1)
	

9. *Conclusion*

This paper has shown that debt paydown can be forecasted over a one-year period with precision of up to 70% using a boosted trees algorithm. We performed two out-of-sample tests of the boosted trees model and its performance was similar in both cases. This suggests that the model provides stable forecasts of debt paydown among leveraged companies. We also showed that the ability to forecast debt paydown is positively related to expected returns. Between 1965 and 2012, annual portfolios of stocks in the 10^{th} decile of estimated debt paydown probability outperform the 1^{st} decile by a statistically significant 10.3 percentage points per year on average. We also evaluated a value investment strategy that selects stocks in both the 10^{th} decile of estimated debt paydown probability and the bottom half of Price/Book Value in every year from 1965 to 2012. This Decile 10+Value strategy has a 16.4% average annual return with a 31.4% standard deviation and a 0.36 Sharpe Ratio. After controlling for the 3 Fama-French factors and momentum, the Decile 10+Value strategy has an average risk adjusted return of 9.3% per year.

In terms of limitations, it is important to note that our boosted trees algorithm was trained on U.S. data. Therefore, we do not have any evidence at this point of its precision in forecasting debt paydown among international equities. However, economic theory would suggest that the market forces that influence a company's management to pay down debt should be broadly similar across global markets. In addition, as with any model, our boosted trees algorithm could benefit from the inclusion of additional explanatory variables that are related to the outcome. For example, other debt-related accounting measures such as the Interest Coverage Ratio would likely be beneficial as additional explanatory variables. Therefore, the model presented in this paper is not a *true* model of debt paydown (in the sense that it does not account for all variables that influence debt paydown) but the evidence suggests that is a *useful* model due to its stable performance in two out-of-sample tests. We believe that the most useful application of this model is with respect to separating the attractive leveraged companies (that are paying down debt and earn higher average returns) from the unattractive ones (that are not paying down debt and earn lower average returns). The evidence that debt paydown is generally predictable has implications for public market investors in terms of identifying attractive leveraged stocks for their portfolio. In particular, this framework may be of interest to value investors who hold leveraged stocks, given the positive risk adjusted returns of the Decile 10+Value strategy that selects cheap leveraged stocks with the highest estimated probability of paying down debt.

Appendix A: Details of the Boosted Trees Model

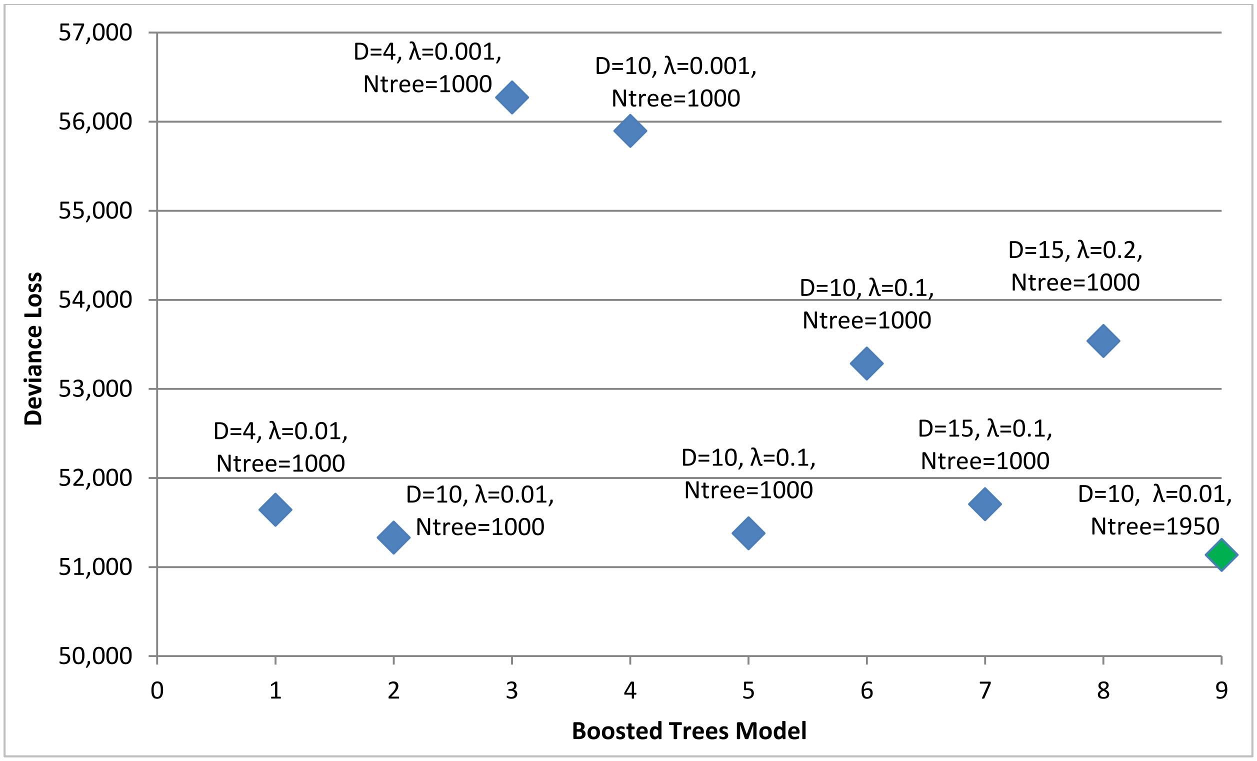
The boosted trees model has three key tuning parameters: (i) the **tree depth (D)**, which determines the complexity of each decision tree, (ii) the **learning rate (λ)**, which sets the pace at which the algorithm "learns" from the data during the training stage, and (iii) the **number of trees** that are fit to the data. The boosted trees algorithm starts by fitting a tree of maximum size D to the training data. This initial fit is then "crushed" or multiplied by the learning rate, λ. A new tree is then fit to what is left unexplained by the crushed fit. This process is then repeated iteratively over hundreds or thousands of trees.

We trained several boosting algorithms with different values of the three parameters. We then evaluated each model according to its deviance loss in the validation sample. Our deviance loss function was as follows:

$$\sum L(x,\hat{y}) = \sum -2\log[P(Y=\hat{y}|x)]$$

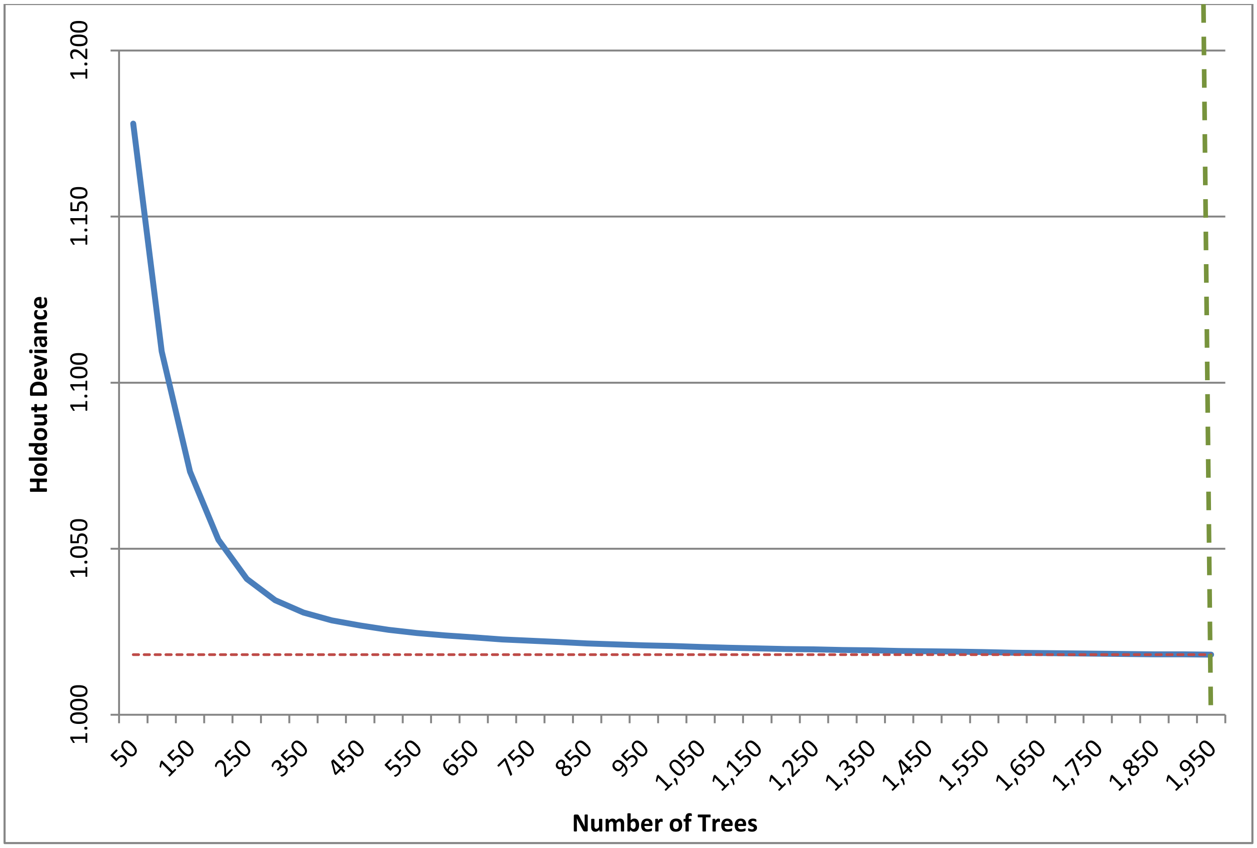
For every observation in the data, $\hat{y}$ is the prediction and Y is the actual outcome in terms of future debt paydown. Each explanatory variable in the data is represented by x. A good model has a high value of $\sum P(Y=\hat{y}|x)$ because most of its predictions are correct. Therefore, a good model will have a *small* value of $\sum -2\log[P(Y=\hat{y}|x)]$ which is the conventional loss function in likelihood analysis. Our final boosting model—with the lowest deviance loss—had parameters of D=10, λ=0.01, and 1,950 trees. A comparison of the deviance loss across the boosted trees models we evaluated is provided below.

Fig 19: Deviance Loss of Boosted Tree Models in the Validation Data



We arrived at the final model (D=10, λ=0.01, 1950 trees) by optimizing the number of trees in model 2 (D=10, λ=0.01, 1000 trees) through cross validation. As shown in Figure 20 below, the optimal number of trees in the final model is 1,950. The optimization procedure held out a portion of the training data and evaluated the deviance loss of the fitted values against this hold-out sample as more trees were added during training. This holdout deviance was minimized at 1,950 trees. Therefore, our final boosting model was based on an algorithm that learned *slowly* (with a small learning rate of 0.01) and had a limited tree complexity (at a maximum depth of 10). In general, it is good to have a model that learns slowly and is not too complex in order to prevent overfitting. The fact that our final model provided similar precision and accuracy in two out-of-sample tests (first on the validation data, and second on the test data) provides further evidence that our final model is stable and generalizable to new data.

Fig 20: Holdout Deviance During Training of the Final Model (D=10, λ=0.01, Ntree=1,950)



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