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Testing the Lehman Brothers Agency Risk Model

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LEHMAN BROTHERS

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In this paper, we discuss some specific issues related to the Lehman Brothers Agency Risk Model. In particular, we focus on the intra-issuer correlation of idiosyncratic risk. We find that our systematic factors explain most of the return co-movements between bonds of the same issuer, and that intra-issuer correlation of asset-specific risk is therefore negligible. We also propose a simple out-of-sample test to study the accuracy of the model's predictions.[1]

1. INTRODUCTION

For several years, the Lehman Brothers Risk Model has been used by portfolio managers as an effective risk management tool.[2] The latest release of our US Risk Model – delivered via the Lehman Brothers proprietary POrtfolio and INdex Tool (POINT) – covers the Lehman Brothers US Aggregate Index, which includes six major asset classes: Treasury, Agency, Credit, Mortgage, CMBS and ABS. In this paper, we briefly review the Agency Risk Model, and discuss the issue of intra-issuer correlation of idiosyncratic risk. We then perform a simple out-of-sample test to provide evidence for the historical performance of the model.

2. THE LEHMAN BROTHERS AGENCY RISK MODEL

The Lehman Brothers multifactor risk model is based on an intuitive decomposition of the total return of a bond into deterministic and stochastic components:

$$Tot\ R_t = Carry\ R_t + YC\ R_t + Vol\ R_t + Sprd\ R_t \tag{1}$$

The first term on the right-hand side – the "carry return" – represents the deterministic portion of the return, and it is solely due to the passage of time. The stochastic part of the total return is further decomposed into a yield curve return (YCR), a volatility return (Vol R) and a spread return (Sprd R). Each component is then modeled as a linear combination of a set of risk factors.

The yield curve return is modelled as:

$$YCR_t \approx -\sum_{j=1}^{6} KRD_{j,t-1} * \Delta KR_{j,t} + OAC_{t-1} * \left(\overline{\Delta KR_t}\right)^2 \tag{2}$$

where six key-rate changes and $\Delta KR_{i,t}$ the squared average key-rate change $\left(\overline{\Delta KR_t}\right)^2$ represent observable systematic factors which are loaded by the bond's key-rate durations (KRD) and option-adjusted convexity (OAC), respectively.

The volatility return is non-zero only for bonds with embedded options. It is modelled as:

$$Vol\ R_t \approx \frac{100}{\text{Price}_{t-1}} * Vega_{t-1} * F_t^{Vol} = -VolDur_{t-1} * F_t^{Vol} \tag{3}$$

where F_t^{Vol} is a latent, non-observable volatility factor, and the second equality defines the concept of "volatility duration."

[1] *Reprinted from the Liquid Market Quarterly 2003-Q2.*

[2] *See Dykin, Hyman and Wu (1999), Naldi, Chu and Wang (2002), and Berd and Naldi (2002).*

The model for spread return is primarily based on the issuer of the agency bond. There are four major issuers in the Agency Index: Fannie Mae (FNMA), Freddie Mac (FHMC), Federal Home Loan Bank (FHLB) and Farmer Mac (FARM). The remaining bonds – issued by smaller issuers – are lumped together into a residual group. We employ five issuer-related systematic factors, as well as a slope factor and an OAS factor. Formally, spread return for bond *i* is modelled as:

$$SprdR_t^i = -OASD_{t-1}^i \cdot \left(F_t^{Issuer} + \left(TTM_{t-1}^i - \overline{TTM}_{t-1}^{Issuer}\right)F_t^{Twist} + \left(OAS_{t-1}^i - \overline{OAS}_{t-1}^{Issuer}\right)F_t^{Liq}\right) + \varepsilon_t^i, \quad (4)$$

where TTM denotes time to maturity, and $\overline{x}^{Issuer}$ represents the median value of variable x in the corresponding issuer group. Notice that, for a given period *(t-1, t)*, the realisations of the five issuer factors F^{Issuer} can be interpreted as the OAS changes of five hypothetical bonds with median maturity and median OAS within their issuer group. The realisations of the unobserved latent factors F_t^{Vol}, F_t^{Issuer}, F_t^{Twist} and F_t^{Liq} are all estimated by robust cross-sectional regressions.

Putting together the models for the different stochastic components, the total return for agency bond *i* can be represented as:

$$R_t^i = Carry_{t-1}^i + L_{t-1}^i F_t' + \varepsilon_t^i, \quad (5)$$

where F_t is the (1 x K) vector of the systematic factors, L_{t-1}^i is the vector of risk exposures for bond i and ε_t^i is the idiosyncratic return.

Once a Risk Model user has specified a portfolio and a benchmark, the Tracking Error (TE) is defined as the return difference between the two, ie,

$$TE_t = R_t^{Portfolio} - R_t^{Benchmark} = \sum_i (\theta_i^P - \theta_i^B) R_t^i, \quad (6)$$

where θ_i^P is the weight of security i in the portfolio and θ_i^B is the weight in the benchmark. At a given point in time, the Tracking Error over the following period is a random variable, and the main goal of the risk model is to provide an estimate of its volatility. Using the linear factor specification in (5), the Tracking Error Volatility (i.e., the square root of the variance of the TE) can be computed as:

$$\begin{aligned} TEV_{t-1} &= \sqrt{VAR_{t-1}[TE_t]} \\ &= \sqrt{\theta' L'_{t-1} \Sigma_{t-1} L_{t-1} \theta + \theta' \Omega_{t-1} \theta} \end{aligned} \quad (7)$$

where $\theta = \theta^P - \theta^B$ is a (N x 1) vector of the weight difference between the benchmark and the portfolio, L_{t-1} is a (N x K) matrix of risk exposures at the beginning of the period, Σ_{t-1} is the (K x K) covariance matrix of the systematic risk factors and Ω_{t-1} is the N x N covariance matrix of the idiosyncratic components.

The systematic covariance matrix Σ_{t-1} and the diagonal of the idiosyncratic variance matrix Ω_{t-1} are estimated using historical information. The off-diagonal terms of Ω_{t-1}, on the other hand, are set to zero by the assumption that the residual terms ε_t^i in the factor model (5) are uncorrelated with each other. This assumption is strictly related to the specification of the linear factor model in (5). If the model in (5) was mis-specified and the residual terms ε_t^i were correlated with each other, then the calculation of the TEV would be biased. The purpose of the next sections is to look at the intra-issuer correlations of the idiosyncratic returns produced by the Agency Risk Model, and investigate the out-of-sample properties of the TEV estimates.

3. INTRA-ISSUER CORRELATION

In this section, we investigate the intra-issuer correlation of the idiosyncratic components of return. As we discussed in the previous section, there are four systematic factors that capture the average OAS change for the four major issuers: FNMA, FHMC, FHLB and FARM. One would therefore expect that the correlations among the idiosyncratic return of bonds issued by these four issuers will be small, because the systematic factors should capture the common movements for the same issuer.

However, the bonds issued by all other issuers are lumped into a residual group and assigned a common systematic factor. We need to investigate whether this modelling choice produces correlated residuals from the systematic regressions and, therefore, whether we should allow for correlated idiosyncratic terms for the bonds included in this residual group. Assuming that idiosyncratic terms are orthogonal when in fact they are not would bias our estimate of the idiosyncratic tracking error volatility (ITEV). To see this, let us look at the expression for ITEV:

$$ITEV^2 = \theta'\Omega_{t-1}\theta = \sum_i \theta_i^2 \sigma_i^2 + 2\sum_{i \neq j} \theta_i\, \theta_j\, \sigma_i\, \sigma_j\, \rho_{ij} \tag{8}$$

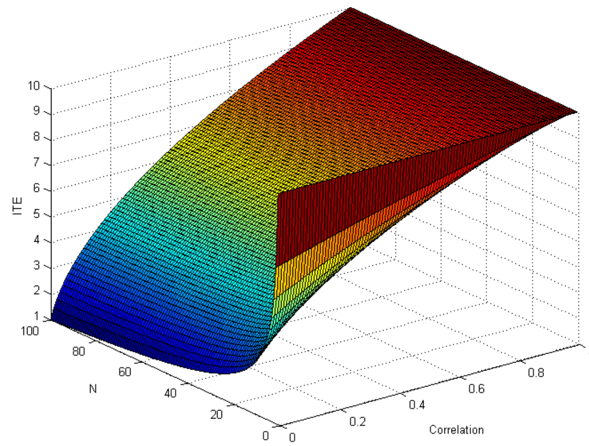
where θ_i is the active weight on bond i, σ_i is the idiosyncratic volatility of bond i and ρ_{ij} is the correlation between the idiosyncratic components of bonds i and j. The Agency Risk Model assumes that all ρ_{ij} are equal to zero, and computes the ITEV using only the first summation term in (8). However, if intra-issuer idiosyncratic correlations were non-zero, the model would be mistakenly neglecting some of the elements of the second summation in (8).

To understand the effect of idiosyncratic correlations on portfolio risk, let us consider an equally weighted portfolio with N securities, and assume that each security has an idiosyncratic volatility of 10bp per month. Figure 1 shows the portfolio idiosyncratic volatility for different values of the idiosyncratic correlation ρ.

Figure 1a. Diversification, idiosyncratic correlation and portfolio idiosyncratic risk

# of bonds	1	2	5	10	100	∞
ρ = 0	10	7.1	4.5	3.2	1	0
ρ = 10%	10	7.4	5.3	4.3	3.3	0
ρ = 25%	10	7.9	6.3	5.7	5.1	5
ρ = 100%	10	10	10	10	10	10

Figure 1b. Diversification, idiosyncratic correlation and portfolio idiosyncratic risk



Figures 1a and 1b show that idiosyncratic correlations have a small effect on the idiosyncratic volatility of a concentrated portfolio. On the other hand, for a portfolio of 100 bonds, the idiosyncratic volatility increases from 1bp to 10bp per month as the idiosyncratic correlations increase from 0% to 100%. Another way to look at this result is that the benefits of portfolio diversification crucially depend on the assumed orthogonality of the bond-specific components.

In the remainder of this section we look at the historical correlations among the idiosyncratic components of returns in the Lehman Brothers Agency Index, which is the same data set employed for the estimation of the Agency Risk Model. The idiosyncratic risk realisation for bond *i* from period *t–1* to period *t* is defined as:

$$\hat{\varepsilon}_t^i = \hat{R}_t^i - Carry_{t-1}^i - L_{t-1}^i \hat{F}_t \qquad (9)$$

where $\hat{R}_t^i$ is the ex-post total return of bond *i*, , L_{t-1}^i is the vector of risk exposures for bond *i* and $\hat{F}_t$ is the estimated factor realisation.

To limit the effects of spurious correlation, we only consider pairs of bonds with at least ten simultaneous observations. Figure 2 shows the average intra-issuer correlation for the five different issuer groups. We report two sets of results using both a time-weighted and an equally-weighted methodology for estimation purposes.

Figure 2. Average intra-issuer correlation of idiosyncratic returns

	FHMC	FNMA	FARM	FHLB	Other
Unweighted	10%	9%	11%	10%	23%
Weighted	8%	12%	10%	7%	21%

Figure 2 shows that the systematic factors employed in the Agency Risk Model capture the co-variation among bonds of the same issuer quite well. The average intra-issuer correlation between idiosyncratic terms across the four major issuers (FNMA, FHMC, FHLB, FARM) is less than 10%, and most of the pairwise correlations are statistically indistinguishable from zero.

The intra-issuer correlations for bonds in the residual group are higher, as we expected, although many of them are hardly significant from a statistical viewpoint. To understand whether this affects the model's performance, we need to analyse more carefully the out-of-sample properties of the model's risk projections.

4. OUT-OF-SAMPLE PERFORMANCE

In this section, we look at the out-of-sample properties of the TEV estimates produced by the Agency Risk Model. Our purpose is to compare alternative assumptions with respect to the idiosyncratic correlations of bonds belonging to the same issuer group.

In general, assessing the performance of a volatility estimator (like TEV) is a challenging task because volatility is not ex-post observable. Hence, we cannot directly compare "predicted" and "realised" volatility. However, we can observe a series of realised tracking errors and ask ourselves how they relate to the "predicted" TEVs.

Our exercise can be described as follows: at the beginning of each month, we use the available historical data to calibrate the Agency Risk Model and calculate the "predicted" Tracking Error Volatility TEV_{t-1} for a given active portfolio. We then record the "realised" Tracking Error TE_t over the following month, and construct the Standardised Tracking Error (Std_TE) as:

$$Std\_TE_t = \frac{TE_t}{TEV_{t-1}} . \tag{10}$$

We construct a whole time series by repeating this exercise for each month *t*. We can also construct two more time series: the Standardised Systematic TE (Std_STE) and the Standardised Idiosyncratic TE (Std_ITE), defined as the "realised" systematic (idiosyncratic) Tracking Error divided by the "predicted" systematic (idiosyncratic) Tracking Error Volatility.

The portfolio we use for this exercise is a market-value weighted portfolio of all bonds in the residual group (ie, bonds in the Agency Index which are not issued by the four major issuers) with OAD between three and five years. On average, this sample portfolio holds 25 issues during the sample period. As we discussed in the previous section, bonds belonging to the residual issuer group appear to have a higher intra-issuer correlation of idiosyncratic terms than bonds belonging to the other four issuer groups (FNM, FHMC, FHLB, FARM). As our benchmark, we choose the whole Agency Index.

The sample period starts in November 1995 and ends in July 2002, for a total of 80 observations. Figure 3 shows the frequencies with which the Std_TE, Std_STE and Std_ITE time series exceed 1, 2 and 3, under our base assumption of 0% intra-issuer correlation of idiosyncratic returns.

Figure 3. Time series distribution of Std_TE, Std_STE and Std_ITE

Exceed	Std_TE	Std_STE	Std_ITE
1	28.8%	28.8%	33.8%
2	5.0%	3.8%	13.8%
3	1.3%	1.3%	7.5%

0% intra-issuer correlation.

Portfolio = MVW portfolio of Residual Issuer Group with 3- to 5-yr OAD.

Benchmark = US Agency Index.

Figure 3 shows that there is a 28.8% chance of both the Total and Systematic Tracking Error falling outside the one-standard-deviation band, and a 33.8% chance of the Idiosyncratic Tracking Error doing so. Looking at the tails of the distributions, Figure 3 also shows that there is a 13.8% chance of the Idiosyncratic Tracking Error falling outside the two-standard-deviations band and a 7.5% chance of it falling outside the three-standard-deviations threshold.

To see whether this "fat tail" of the Std_ITE series can be ascribed to the assumption of zero intra-issuer idiosyncratic correlations (and the resulting underestimation of the ITEV), we repeat the same exercise assuming 20% intra-issuer correlation of idiosyncratic returns. The results of this exercise are shown in Figure 4.

Figure 4. Time series distribution of Std_TE, Std_STE and Std_ITE

Exceed	Std_TE	Std_STE	Std_ITE
1	27.5%	28.8%	32.5%
2	5.0%	3.8%	13.8%
3	1.3%	1.3%	6.3%

20% intra-issuer correlation

Portfolio = MVW portfolio of Residual Issuer Group with 3- to 5-yr OAD

Benchmark = US Agency Index

Comparing Figures 3 and 4, it is clear that the distribution of Std_ITE changes only slightly when we allow for non-zero intra-issuer idiosyncratic correlations. We have carried out analogous tests for a variety of different test portfolios, such as those consisting of only liquid bonds issued by FNMA or FHMC. The results were analogous to the ones reported above, ie,, the impact of intra-issuer correlations of idiosyncratic returns on the out-of-sample performance of the model was very small.

5. SUMMARY

For more than a decade, the Lehman Brothers Risk Model has helped portfolio managers measure and budget the risk of deviation from their pre-assigned benchmarks. The new Risk Model has improved the fit and the predictive power of the risk projections achieved by the previous framework.

One important assumption underlying the Agency portion of the new Risk Model is the orthogonality of the idiosyncratic returns. In this article we have shown that the bond-specific components of returns are only mildly correlated in sample, and that imposing the assumption of zero correlation does not significantly alter the out-of-sample performance of the Tracking Error Volatility estimates. In particular, the orthogonality assumption does not seem to underestimate significantly the model's estimates of deviation risk.

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