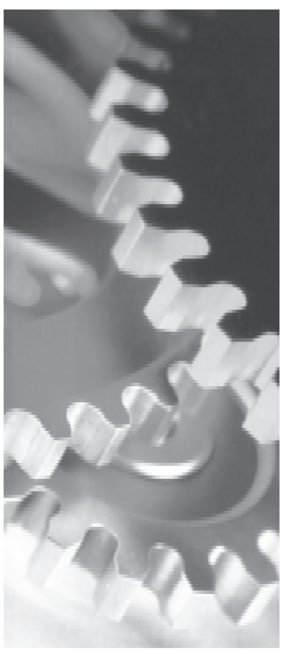
Fixed Income
Quantitative Credit Research

May 2002

The New Lehman Brothers Credit Risk Model

(Extracted from the QCR Quarterly, vol. 2002-Q2)

Marco Naldi, Kenny Chu and Gary Wang



LEHMAN BROTHERS

The New Lehman Brothers Credit Risk Model

Marco Naldi[1]
212-526-1728
mnaldi@lehman.com

Kenny Chu
kchu@lehman.com

Gary Wang
gwang@lehman.com

We describe the new Lehman Brothers Risk Model as it applies to the U.S. High-Grade Credit universe. The discussion focuses on our choices for splitting total return and modeling its stochastic components. We explain how to use this framework to estimate the Tracking Error Volatility of a given portfolio vs. a pre-specified benchmark, and offer some out-of-sample evidence on the performance of our estimator. Finally, we illustrate some of the Risk Model output available on POINT, our new portfolio analytic system.

1. INTRODUCTION

During the past decade, the Lehman Brothers Multi-Factor Risk Model (for the latest update, see Dynkin, Hyman and Wu [1999]) has served as an effective risk management tool for an increasing number of portfolio managers. The common practice of constraining asset allocation choices with the assignment of predetermined benchmarks has created a need for quantitative measurements of relative risk. As a result, the risk model's coverage of a variety of asset classes and its ability to measure the deviation risk between a portfolio and a pre-assigned benchmark have made it a unique tool for managers who need to track the return of a highly diversified index.

These days, credit managers are paying more attention to the importance of an adequate level of portfolio diversification. The current spread volatility and significant tail-risk are making the management of benchmarked positions harder than ever. A unique feature of our model is its ability to decompose systematic and non-systematic deviation risk, thus allowing for a quantitative evaluation of the benefits that a given position may derive from an increased level of diversification. We believe the utilization of these capabilities, combined with the employment of other quantitative tools, will become increasingly important for credit fund managers, since the complexity and volatility of the underlying credit marketplace are likely to remain elevated.

This article describes the framework underlying the new Lehman Brothers Risk Model for the U.S. investment grade credit universe. The model is calibrated using historical observations from the IG Credit Index, and it can be applied to portfolios that include any modeled issue in our database. The new U.S. Risk Model, covering the Treasury, Agency, and Credit indices is currently available on Lehman Brothers' portfolio system POINT. We are extending the model to MBS, ABS, and CMBS in order to complete the coverage of the Lehman Brothers U.S. Aggregate Index.

Relative to the previous version of the model, we have introduced a new way of splitting bond returns, a different way of measuring interest rate and volatility risk, and different specifications for systematic and non-systematic spread risk. Moreover, robust econometric techniques have been introduced for parameter estimation. What we have retained is

[1] Several people contributed to the development and the implementation of the U.S. Risk Model, including Albert Desclee, Lev Dynkin, Hank Haligowski, Jay Hyman, Dick Kazarian, Vasant Naik, and Nancy Roth. We would like to thank Arthur Berd, Ivan Gruhl, Mark Howard, Arthur Tetyevsky, Minh Trinh and Stuart Turnbull for their comments and suggestions.

the flexible and computationally convenient framework of a linear factor model and the general calibration methodology based on cross-sectional regression analysis.

Beyond providing the user with an intuitive measure of statistical distance between a given portfolio and a pre-assigned benchmark (tracking error volatility--from now on, TEV), the model lends itself to a number of applications such as:

- Understanding detailed factor exposures for a given portfolio;
- Comparing risks associated with different views;
- Identifying trades to achieve a reduction in deviation risk;
- Creating index proxies using a relatively low number of assets;
- Decomposing risk across asset classes, allowing for risk budgeting at different levels; and
- Decomposing systematic and non-systematic risk.

In the remainder of this article, we will describe the model (section 2), test its out-of-sample performance (section 3), challenge its robustness with small-portfolio examples (section 4), and illustrate some of the reports currently available to the user (section 5).

2. THE MODEL

Our methodology relies on the itemization of a security's total return into well-defined components. The model quantifies the statistical behavior of each of these sources of return. The new Credit Risk Model improves upon the existing framework in a number of ways:

- We have redefined benchmark (Treasury) risk in an OAS-based setting. Callable securities are handled in the same way as non-callable securities. We have eliminated the use of static measures that ignore embedded options (such as "yield to worst" or "ZV duration"). We allow for non-parallel changes in the benchmark Treasury curve by using 6 key rates loaded by the corresponding key-rate durations and use option-adjusted convexity as an additional risk loading.
- We incorporate volatility risk in an OAS-based framework for all volatility-sensitive securities. This risk has taken on increasing importance as the cash and derivative markets have become increasingly integrated.
- We have significantly improved our approach to handling systematic spread risk. We explain spread return by means of an industry-rating cross effect based on a new partition of the credit universe. The spread factor realizations are estimated using a robust statistical procedure that minimizes the impact of outliers. These estimated factors are easily interpretable and explain a significant fraction of each security's spread return.

2.1. Return Splitting

From a modeling perspective, our final goal is to come up with a linear factor representation for the total return of every modeled corporate bond in our database. As a first step, we employ a proprietary pricing model to perform a historical decomposition of every monthly total return in the IG Credit Index.

We first write total return as the sum of coupon return R_coup and full-price return R_fp:

$$R\_tot = R\_coup + R\_fp$$

A pricing model can be interpreted as a function *P* that relates current time t, benchmark curve YC_t (Treasury par curve), implied volatility surface Vol_t, and option-adjusted spread OAS_t to the full price of the bond fp_t:

$$fp_t = P(t, YC_t, Vol_t, OAS_t).$$

We now define time return *R_time* as the portion of full-price return due solely to the passage of time:

$$R\_time_{t+1} = P(t+1, YC_t, Vol_t, OAS_t) / P(t, YC_t, Vol_t, OAS_t) - 1,$$

Analogously, we define benchmark return *R_bmk* as that portion of full-price return due solely to the movement of the benchmark curve,

$$R\_bmk_{t+1} = P(t, YC_{t+1}, Vol_t, OAS_t) / P(t, YC_t, Vol_t, OAS_t) - 1,$$

and volatility return *R_vol* as

$$R\_vol_{t+1} = P(t, YC_t, Vol_{t+1}, OAS_t) / P(t, YC_t, Vol_t, OAS_t) - 1.$$

Lastly, we approximate spread return as the residual component of this decomposition

$$R\_fp - R\_time - R\_bmk - R\_vol = R\_spread.$$

This is an approximation, because *R_spread* not only contains the portion of return due to the movement in the bond's *OAS*, but also includes all cross-effects.

Notice that *R_time* takes on a negative value over months in which a coupon is paid out, since the full-price jumps down accordingly. To obtain a smooth measure of the deterministic component of return, we define carry return a

$$R\_carry = R\_coup + R\_time.$$

Using the previous definitions, we can now represent total return as

$$R\_tot = R\_carry + R\_bmk + R\_vol + R\_spread.$$

This is the basic return decomposition that we are going to use for modeling total return risk. Since *R_carry* is known in advance, we now need to specify a linear factor representation for the remaining three stochastic components. In the next sections, we index the *ith* bond with *i* while we omit the time subscript.

2.2. Benchmark Risk

The benchmark return for a bond is defined as that portion of total return that results solely from a change in the Treasury curve over the month. It is computed by re-pricing the bond using the end-of-month yield curve, holding all other variables (time, volatility, *OAS*) fixed. The curve movement is measured using the Lehman Brothers Treasury curve, which is fitted to the prices of "off-the-run" Treasury securities.

An extremely precise approximation of *R_bmk* is given by a linear combination of 6 key-rate durations (*KRD*) multiplied by their respective par yield changes (Δy), plus the impact of a convexity factor that mimics the second-order movement of the curve:

$$R\_bmk_i \cong \sum_j KRD_{i,j} * \Delta y_j + OAC_i * \left(\overline{\Delta y}\right)^2.$$

Here, *OAC* represents the option-adjusted convexity of the bond, and $\left(\overline{\Delta y}\right)^2$ is the squared average key-rate change, which proxies for the second-order movement of the curve. Extensive empirical analysis has shown that benchmark return is best approximated by using the following six maturities on the fitted par curve as key rates: 6-month, 2-year, 5-year, 10-year, 20-year, and 30-year.

2.3. Volatility Risk

The volatility return for a bond with embedded options is that portion of total return that results solely from a change in the set of implied volatilities over the month. It is, of course, equal to zero for all bonds without embedded options.

We explain volatility return using one volatility factor, which approximates the parallel movement of the implied volatility surface. The time series of the factor realizations is extracted by running cross-sectional regressions of volatility returns on vegas for all bonds with embedded options, i.e.,

$$R\_vol_i \cong \frac{vega_i}{fp_i} * F^{vol}$$

2.4. Spread Risk

The spread return is the portion of total return remaining after time, benchmark, and volatility returns have been removed. It can be approximated by the product of the security's spread duration and its *OAS* change (ΔOAS) over the period:

$$R\_spread_i \cong -oasd_i * \Delta OAS_i$$

To specify a linear factor model for ΔOAS, we first partition the IG Credit Index into 27 cells formed by intersecting 9 industries and 3 quality groups. Each of these 27 cells has its own generic spread factor. The industry partition is chosen so that a minimum number of bonds is retained in each of the cells for every month in the sample. This ensures that the estimated factor realizations are not at any time dominated by the idiosyncratic behavior of a small number of bonds, a feature which is especially important in the current market environment. To minimize the resulting loss of information, the aggregation of certain industries relies on the merger of highly correlated groups. Figure 1 describes the cells composition and assigns them short acronyms used in some of the reports.

Figure 1. **Industry—Rating Partition**

	AAA/AA	A	BBB
Banking and Brokerage	BAN1	BAN2	BAN3
Basic Industries	BAS1	BAS2	BAS3
Consumer Cyclicals	CCY1	CCY2	CCY3
Communication and Technology	COM1	COM2	COM3
Energy and Transportation	ENE1	ENE2	ENE3
Financial Companies and Insurance	FIN1	FIN2	FIN3
Consumer Non-Cyclicals	NCY1	NCY2	NCY3
Non-Corporate	NON1	NON2	NON3
Utilities	UTI1	UTI2	UTI3

Beyond cell-specific factors, we employ a slope factor for the generic credit spread curve and an *OAS* factor, on which bonds with relatively high and relatively low *OAS* load with opposite signs ("relatively" refers to the distance from the median *OAS* of all cell peers). The *OAS* factor is meant to capture liquidity risk, since liquidity is a significant determinant of the *OAS* distribution within a given cell.

Finally, we include an additive effect for non-U.S. bonds. Specifying a set of factors for different geographical areas assumed to be homogeneous in terms of spread risk would require us to change this definition as the riskiness of different countries in the same area changes over time. Therefore, we use three non-U.S. factors corresponding to the three rating groups defined above (*AAA/AA*, *A*, *BBB*). This way, a systematic change in the creditworthiness of a given country will automatically reassign its issuers to a different non-U.S. factor.

In summary, we model the spread return of bond *i* belonging to cell c as

$$R\_spread_{i,c} = -oasd_i * [\, F_c + (tomat_i - \overline{tomat_c}) * F_{twist} + (OAS_i - \overline{OAS_c}) * F_{OAS} + F_{c,nonUS} \,] + e_i ,$$

where , F_c =1,2,…,27 represent cell-specific factors, F_{twist} is the slope factor, F_{OAS} is the *OAS* factor, $F_{c,\,nonUS}$ denotes three non-U.S. factors (since $F_{p,\,nonUS} = F_{q,\,nonUS}$ whenever cells *p* and *q* have the same rating), *tomat* is time-to-maturity, $\overline{x}_c$ indicates the median value of variable *x* in cell *c*, and e_i represents the idiosyncratic (asset-specific) component of spread return.

We estimate the time series of the 27+1+1+3=32 factor realizations by running cross-sectional regressions, one for each month in the panel. To control for outliers and pricing errors, we employ a robust estimator designed optimally to reduce their influence on the estimated factor series. [2]

Notice that the estimated series for F_c , c =1,2,…,27 can be interpreted as the series of spread changes of generic U.S. bonds with median time to maturity and median *OAS* within their cell.

[2] We employ an *M*-estimator based on iterative generalized least squares. See Wilcox (97) for more details.

2.5. Idiosyncratic Risk

The factors described in the previous section capture the systematic portion of spread returns. The error terms e_i, on the other hand, are a source of asset-specific risk. The covariance structure of the systematic factors is going to be sufficient to describe the risk of highly diversified portfolios, since diversification is going to drive to (almost) zero the variance of the portfolio of the asset-specific components. For portfolios with a moderate number of bonds, however, idiosyncratic risk can represent a significant portion of the total TEV.

Using the panel of residuals from the factor regressions, we estimate a simple model to describe the dependence of asset-specific variance on bond characteristics. To explain the idiosyncratic variance of a bond, we use its rating, its age (the percentage of its original maturity remaining, as a proxy of liquidity), and its vega (which is going to be non-zero if and only if the bond has embedded optionalities).

2.6. Putting It All Together

In matrix notation, we can think of the new Credit Risk Model for the vector of total return *R_tot* as:

$$R\_tot_{t+1} = R\_carry_{t+1} + L_t F_{t+1} + e_{t+1},$$

where L_t denotes the matrix of all loadings (key-rate durations, vegas, spread durations, etc.) at time t and F_{t+1} is the vector of factor realizations over the period $(t,t+1)$.

Using the estimated factor series, we can compute the factor covariance Σ, and with the fitted values from the asset-specific risk model, we can construct the idiosyncratic covariance matrix Ω. The latter has mostly zeroes off the main diagonal, except for cells corresponding to pairs of bonds issued by the same ticker, in which we use the conservative assumption of unit correlation. We now have:

$$Cov_t\left(R\_tot_{t+1}\right) = L_t \Sigma L_t' + \Omega.$$

If we define the vectors

$$\begin{aligned} &\theta_P = Portfolio \quad , \theta_B = Benchmark \\ &\theta = \theta_P - \theta_B, \end{aligned}$$

then we can write the corresponding tracking error volatility as

$$\begin{aligned} TEV_t(\theta) &= \sqrt{\theta' Cov_t\left(R\_tot_{t+1}\right)\theta} \\ &= \sqrt{\theta' L_t \Sigma L_t' \theta + \theta' \Omega \theta} \\ &= \sqrt{STEV_t^2(\theta) + ITEV_t^2(\theta)} \end{aligned},$$

where the last equality highlights the decomposition into a systematic (STEV) and an idiosyncratic (ITEV) component.

The new Credit Risk Model also reports an expected outperformance. The expected total return of each bond is computed as the carry return plus a convexity term. Using vector notation, we can write the expected outperformance for the difference portfolio θ as

$$E_t(\theta' R\_tot_{t+1}) = \theta' R\_carry_{t+1} + \theta' OAC_t * E_t\left(\overline{\Delta y}\right)^2.$$

All other factors have a time-series average that is statistically insignificant and is, therefore, constrained to zero. This estimate of expected return relies on the observation that the risk premium of a given bond will be reflected in its price and, therefore, in its carry return.

In the next section, we offer two tests for the out-of-sample performance of the model. For a given portfolio-benchmark pair (i.e., for a given θ), they are both based on the standardized realized outperformance (STR) defined as

$$STR_{t+1}(\theta) = \frac{\theta' R\_tot_{t+1} - E_t(\theta' R\_tot_{t+1})}{TEV_t(\theta)}.$$

3. TESTING THE MODEL PERFORMANCE

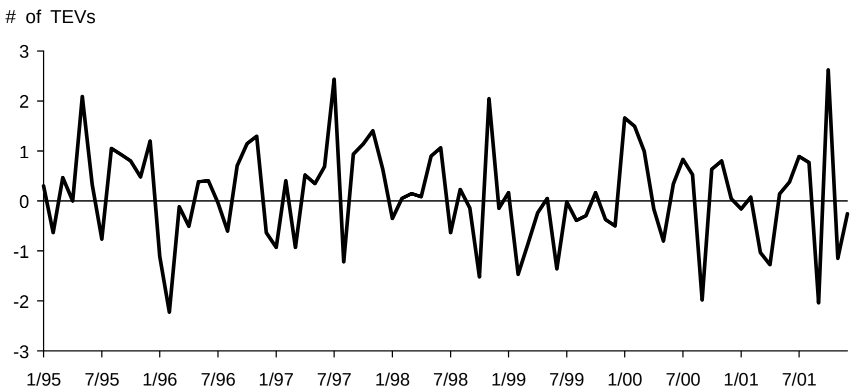
Evaluating the out-of-sample performance of a volatility estimate (such as TEV) is not an easy task because of an inconvenient property of volatility—its ex-post unobservability. Volatility is a parameter of a distribution, and, as such, it will never be "realized." Expressions such as "realized volatility" refer to a particular estimator (usually sample volatility) and not to volatility itself, which will never be known. When the predicted variable is not ex-post observable, straightforward performance measures such as "mean square prediction error" are ineffective: it is impossible to compute the distance between a predicted value and a realized value when the latter is unknown. It is, therefore, necessary to adopt different evaluation methods. In this section, we propose two simple and intuitive tests for the evaluation of the TEV statistic produced by the new Credit Risk Model.

3.1. Unit Variance Test

Our first test builds on the following intuition: dividing each realization of a process by its estimated standard deviation will produce a series of unit-variance realizations, if the estimated volatilities are correct. The chi-square test that we discuss in this section is simply a formalization of this idea.

Consider holding a market-value-weighted portfolio of all issues in the IG Credit Index with option-adjusted duration between 3 and 5 years when benchmarked to the whole IG Credit Index. Now imagine doing the same thing each month from January 1995 to December 2001 (84 periods). At the beginning of each month, we estimate TEV using the Credit Risk Model calibrated to the history available up to that point. At the end of each month, we observe the realized outperformance of

Figure 2. **Standardized Outperformance**
Portfolio = 3- to 5-yr OAD Credit Index, Benchmark = Credit Index



the portfolio. Figure 2 depicts the series of standardized realized outperformances (STR). Its estimated standard deviation is 0.97, with a 95% confidence interval (0.84,1.15), showing that we cannot reject the null of unit variance at any reasonable significance level. The graph also shows that the absolute value of the realized outperformance has exceeded one TEV 29% of the time and two TEVs 7% of the time, a behavior compatible with a distribution with slightly fatter tails than the normal. Experimenting with a variety of portfolios and benchmarks, we have found that the frequency of hits through the one-TEV band ranges between 25% and 35%. Hits in the tails become more frequent as portfolio diversification decreases, as one would expect.

3.2. Runs Test

A risk model allows the user to make probabilistic statements about uncertain events, and, in particular, it provides a framework for computing interval forecasts. Value at risk, the most widely used risk measure in the finance industry, is simply the bound of a one-sided interval forecast.

As we saw above, the risk model's conditional estimate of TEV is based on the time-varying loadings L_t and the factor covariance matrix Σ. Since the latter is estimated as the full sample covariance of the factor series, we should check whether the estimated TEV fails to capture a possible persistency in volatility.

Christoffersen (1998) has proposed a non-parametric methodology for evaluating conditional interval forecasts. Suppose that at time *t*, we form a forecasting interval (D_t, U_t) with probability coverage *p* for the realization of variable *x* over the next period. If we denote with H_t the information known at time *t*, we have

$$\Pr\left(x_{t+1} \in \left(D_t, U_t\right) \mid H_t\right) = p.$$

This simply means that, conditional on what is known at time t, the probability that the realization in period $(t, t+1)$ falls in the interval (D_t, U_t) is equal to *p*. Thus, *p* is usually called the "conditional coverage" of the interval. Next, define a sequence of random indicators $\{I_t\}_t$ as

$$I_{t+1} = \begin{cases} 1, x_{t+1} \in (D_t, U_t) \\ 0, x_{t+1} \notin (D_t, U_t) \end{cases},$$

which says that the indicator I_{t+1} takes a unit value if the realization x_{t+1} falls in the interval (a "hit") and zero if it falls outside.

The main result that we are going to use is that the sequence of interval forecasts has correct conditional coverage if and only if the hit sequence $\{I_t\}_t$ is an i.i.d. Bernoulli (p) sequence. To see this, set $\{I_t, I_{t-1}, I_{t-2}, \ldots\} = H_t$ and observe that correct conditional coverage implies that

$$p = \Pr(I_{t+1} = 1 \mid I_t, I_{t-1}, I_{t-2}, \ldots) \text{ and}$$

$$p = E[E[I_{t+1} \mid I_t, I_{t-1}, I_{t-2}, \ldots]] = E[I_{t+1}] = \Pr(I_{t+1} = 1),$$

i.e., the hit sequence is serially independent. That it is also identically distributed follows from observing that each I_t is simply a Bernoulli(p) random variable. The reverse implication is trivial.

A widely used test for the null hypothesis that a sequence of indicators is i.i.d. is the so-called runs test. A "run" is a subsequence of adjacent zeroes or ones in the sequence $\{I_t\}_t$. For example, the sequence {1110000} contains two runs, while the sequence {1100100} contains four runs. We can test the null hypothesis that $\{I_t\}_t$ is an i.i.d. sequence using the statistic

$$Z = \frac{R - 2np(1-p)}{2(np(1-p)(1-3p(1-p)))^{1/2}}.$$

Here, R denotes the number of runs in the sequence, n is the number of elements in the sequence, and p is the probability of a hit (the interval coverage). Under the null hypothesis that $\{I_t\}_t$ is an i.i.d. sequence, Z is asymptotically standard normal.[3] The test is usually made operational by substituting p with its maximum likelihood estimate $\hat{p}$.

We conduct the test by producing time series of interval forecasts based on the estimated TEVs. At each time t, we set the interval forecast equal to ±1 TEV and compute the out-of-sample outperformance over $(t,t+1)$ to derive the value of the indicator I_{t+1}.

For the same portfolio and benchmark used in the previous section, the p-value for the one-sided test of the hypothesis that the hit sequence is serially independent is 15%. This shows that we cannot reject the null hypothesis at the usual 5% significance level and that the significance can be raised substantially before rejecting the null.

3 See Campbell, Lo, and MacKinlay (1997).

Figures 3. **5-Year TRAINS Portfolio,** as of 1/15/02

Issuer	**Coupon**	**Maturity**	**Amt** (000s)	**Moody's**	**S&P**
AMERICAN ELECTRIC POWER	6.125	5/15/06	23,000	Baa1	BBB+
AOL TIME WARNER	6.125	4/15/06	50,600	Baa1	BBB+
BANK ONE CORP	6.5	2/1/06	50,600	Aa3	A
BEAR STEARNS COMPANIES I	5.7	1/15/07	50,600	A2	A
BRISTOL-MYERS SQUIBB	4.75	10/1/06	50,600	Aa2	AAA
BRITISH TELECOMMUNICATIO	7.875	12/15/05	46,000	Baa1	A-
CANADA (GOVERNMENT OF)	6.75	8/28/06	46,000	Aa1	AA+
CITIGROUP INC	5.75	5/10/06	59,800	Aa1	AA-
CONOCO FUNDING CO	5.45	10/15/06	50,600	Baa1	BBB+
FIRSTENERGY CORP	5.5	11/15/06	27,600	Baa2	BBB-
FLEETBOSTON FINL CORP	4.875	12/1/06	50,600	A1	A
FORD MOTOR CREDIT COMPANY	6.5	1/25/07	50,600	A3	BBB+
FRANCE TELECOM SA	7.7	3/1/06	32,200	Baa1	BBB+
GENERAL ELECTRIC CAPITAL	6.8	11/1/05	36,800	Aaa	AAA
GENERAL MOTORS ACCEPT CO	6.125	9/15/06	50,600	A2	BBB+
ITALY (REPUBLIC OF)	4.375	10/25/06	50,600	Aa3	AA
KRAFT FOODS INC	4.625	11/1/06	50,600	A2	A-
NATIONAL RURAL UTILITIES	6	5/15/06	36,800	A1	AA-
SPRINT CAPITAL CORP	6	1/15/07	50,600	Baa1	BBB+
TYCO INTERNATIONAL GROUP	6.375	2/15/06	50,600	Baa1	BBB
UNILEVER NV	6.875	11/1/05	50,600	A1	A+
VERIZON WIRELESS INC	5.375	12/15/06	46,000	A2	A+
WAL-MART STORES	5.45	8/1/06	46,000	Aa2	AA
WASHINGTON MUTUAL INC	5.625	1/15/07	46,000	A3	BBB+
WORLDCOM INC	8	5/15/06	46,000	Baa2	BBB

Figures 4. **10-Year TRAINS Portfolio,** as of 1/15/02

Issuer	**Coupon**	**Maturity**	**Amt** (000s)	**Moody's**	**S&P**
ALCOA ALUMINIO S A	6.5	6/1/11	48,840	A1	A+
AMERICAN TELEPHONE & TEL	6	3/15/09	39,960	A3	BBB+
AOL TIME WARNER	6.75	4/15/11	48,840	Baa1	BBB+
BANK ONE CORP	7.875	8/1/10	48,840	A1	A-
BANKAMERICA CORPN	7.4	1/15/11	35,520	Aa3	A
BELLSOUTH CORPORATION	6	10/15/11	48,840	Aa3	A+
CITIGROUP INC	7.25	10/1/10	39,960	Aa2	A+
CONAGRA INC	6.75	9/15/11	35,520	Baa1	BBB+
CREDIT SUISSE FB USA INC	6.5	1/15/12	48,840	Aa3	AA-
DAIMLERCHRYSLER NORTH AM	7.3	1/15/12	48,840	A3	BBB+
FIRSTENERGY CORP	6.45	11/15/11	26,640	Baa2	BBB-
FORD MOTOR CREDIT COMPANY	7.25	10/25/11	48,840	A3	BBB+
FRANCE TELECOM SA	8.25	3/1/11	48,840	Baa1	BBB+
GENERAL MOTORS ACCEPT CO	6.875	9/15/11	48,840	A2	BBB+
HONEYWELL INT'L	7.5	3/1/10	48,840	A2	A
INTER-AMERICAN DEVELOPME	7.375	1/15/10	35,520	Aaa	AAA
ITALY (REPUBLIC OF)	6	2/22/11	39,960	Aa3	AA
KRAFT FOODS INC	5.625	11/1/11	48,840	A2	A-
NISOURCE FINANCE CORP	7.875	11/15/10	26,640	Baa3	BBB
PROGRESS ENERGY INC	7.1	3/1/11	48,840	Baa1	BBB
QUEBEC (PROVINCE OF)	6.125	1/22/11	48,840	A1	A+
SEARS ROEBUCK ACCEPTANCE	6.75	8/15/11	48,840	A3	A-
TYCO INTERNATIONAL GROUP	6.375	10/15/11	48,840	Baa1	BBB
WAL-MART STORES	6.875	8/10/09	48,840	Aa2	AA
WORLDCOM INC	7.5	5/15/11	48,840	Baa2	BBB

Figures 5. **Long TRAINS Portfolio,** as of 2/1/02

Issuer	**Coupon**	**Maturity**	**Amt** (000s)	**Moody's**	**S&P**
ABBEY NATIONAL PLC	7.95	10/26/29	30,000	Aa3	AA-
AMERICAN TELEPHONE & TEL	6.5	3/15/29	30,000	A3	BBB+
BELLSOUTH CAP FDG CORP	7.875	2/15/30	30,000	Aa3	A+
BRITISH TELECOMMUNICATIO	8.875	12/15/30	30,000	Baa1	A-
CONOCO INC	6.95	4/15/29	30,000	Baa1	BBB+
DEVON ENERGY CORP	7.875	9/30/31	30,000	Baa2	BBB
DOW CHEM CO	7.375	11/1/29	30,000	A3	A
EL PASO ENERGY CORP MTN	7.75	1/15/32	30,000	Baa2	BBB
FIRSTENERGY CORP	7.375	11/15/31	30,000	Baa2	BBB-
FORD MTR CO DEL	7.45	7/16/31	30,000	Baa1	BBB+
FRANCE TELECOM SA	9	3/1/31	30,000	Baa1	BBB+
GENERAL MOTORS ACCEPT CO	8	11/1/31	30,000	A2	BBB+
I.B.R.D. (WORLD BANK)	7.625	1/19/23	30,000	Aaa	AAA
ITALY (REPUBLIC OF)	6.875	9/27/23	30,000	Aa3	AA
KELLOGG CO	7.45	4/1/31	30,000	Baa2	BBB
LOCKHEED MARTIN	8.5	12/1/29	30,000	Baa2	BBB
PEPSI BOTTLING GRP	7	3/1/29	30,000	A3	A-
QUEBEC PROV CDA	7.125	2/9/24	30,000	A1	A+
ROHM & HAAS CO	7.85	7/15/29	30,000	A3	A-
SPRINT CAPITAL CORP	6.9	5/1/19	30,000	Baa2	BBB+
TIME WARNER ENTMT CO L P	8.375	3/15/23	30,000	Baa1	BBB+
VERIZON GLOBAL FDG CORP	7.75	12/1/30	30,000	A1	A+
VIACOM INTERNATIONAL INC	7.875	7/30/30	30,000	A3	A-
WAL MART STORES INC	7.55	2/15/30	30,000	Aa2	AA
WORLDCOM INC	8.25	5/15/31	30,000	Baa2	BBB

In summary, our test excludes the presence of serial correlation in the hit sequence by detecting the presence of a sufficiently high number of runs in the sample. This suggests that our TEV estimator adequately captures the volatility dynamics. We obtained analogous results for a large number of test portfolios and benchmarks.

4. A ROBUSTNESS CHECK: TRACKING WITH TRAINS

On January 15, 2002, Lehman Brothers launched the 5- and 10-year TRAINS (targeted return index securities), while a longer portfolio was issued short afterward, on February 1. Each TRAINS replicates the return on a basket of 25 of the most actively traded High-Grade bonds and is constructed with the goal of tracking a specific curve segment of the overall IG Credit Index. These baskets offer investors a diversified exposure to the credit market through a single transaction. Figures 3-5 describe the portfolios on the respective closing dates.

Since inception, a couple of credits included in the TRAINS have experienced extreme idiosyncratic movements. With the benefit of hindsight, we can now perform a fairly challenging robustness check for the new Credit Risk Model.

Standing on 1/15/02 (i.e., using only information available to this date), we estimate the TEV for the 5-year TRAINS versus the 3.5- to 7-year-to-maturity portion of the Credit Index. The realized underperformance of the 5-year TRAINS over the following three-month period (ending 4/15/02) represents a 3.88 standard-deviation event.

Repeating the same exercise for the 10-year TRAINS versus the 7- to 15-year-to-maturity Credit Index , we observe a 2.76 standard-deviation underperformance. Finally, the long TRAIN realized a 1.9 standard-deviation underperformance versus the 20+year-to-maturity portion of the Credit Index.

These small-portfolio examples with extreme realizations add to the evidence presented in the previous section that the new Credit Risk Model produces reliable volatility estimates. In particular, these results witness the importance of specifying appropriate models for both the systematic and the idiosyncratic components of the return process, the latter being a crucial determinant of volatility when portfolio diversification is somewhat limited.

5. POINT REPORTS

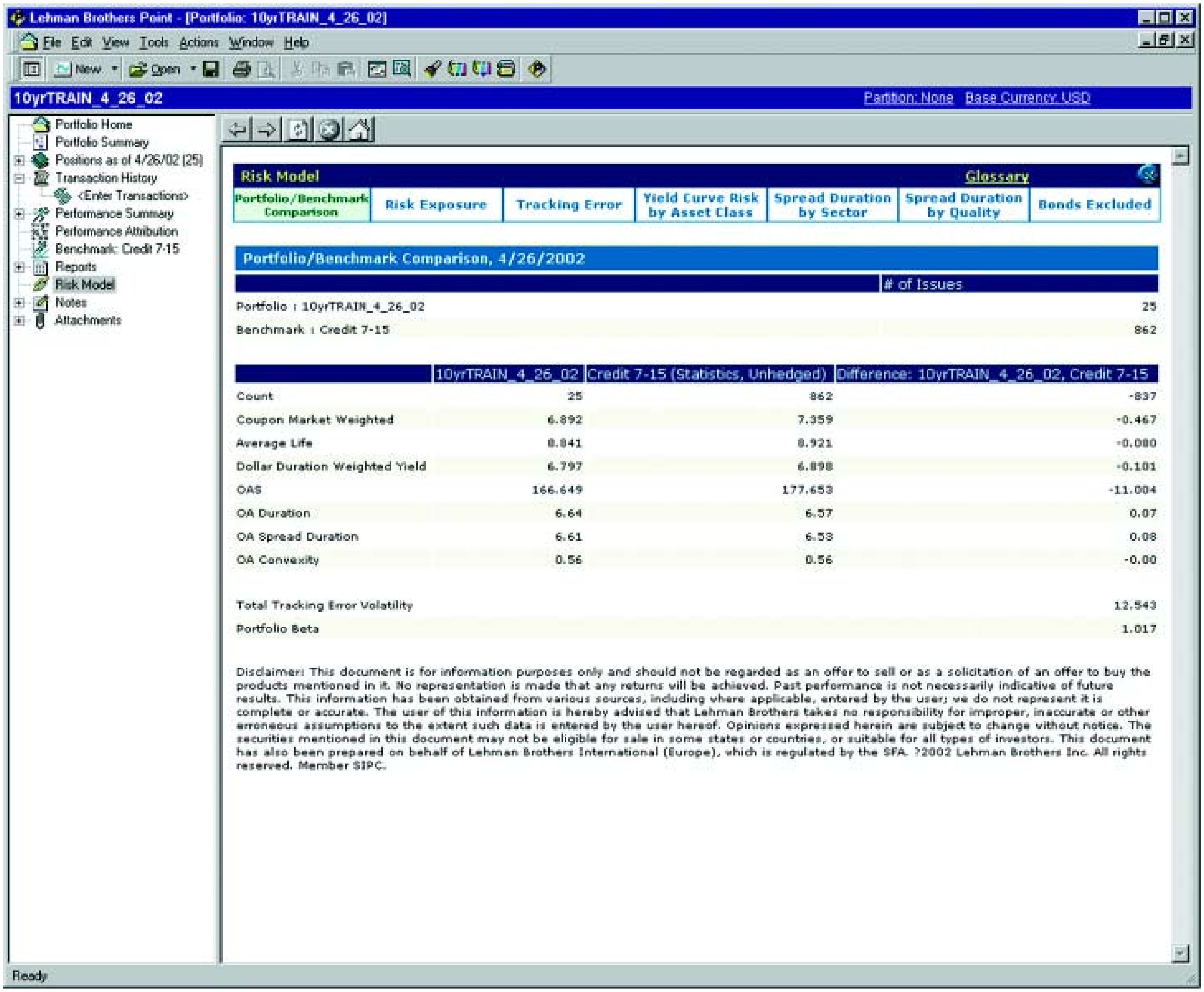
In this section, we offer a view of some of the new Credit Risk Model reports available on Lehman Brothers' portfolio system POINT.

Figure 6 refers to the 10-year TRAINS portfolio versus the 7- to 15-year-to-maturity Credit Index as of 4/26/2002. The table on the top gives summary statistics such as number of bonds included, option-adjusted spread, option-adjusted duration, etc., for the two portfolios, as well as a preliminary glance at TEV (in bp per month) and the portfolio's beta with respect to the chosen benchmark.

The bottom table reports the estimated TEV and its decomposition into different sources corresponding to different subsets of factors. This decomposition highlights both the isolated effect of a particular set of factors and its cumulative effect. Since risk factors are correlated, the cumulative decomposition depends on the order in which new sets of factors are added. It is therefore offered using two different sequences, the first focusing on adding one asset class at a time (Treasury, Agency, Credit), the second adding one type of risk at a time (benchmark risk, volatility risk, spread risk). Of course, risk measures related to asset classes other than credit are all equal to zero in the tables, since our portfolios and benchmarks both belong to the credit universe. It is interesting to notice that the idiosyncratic portion of the TEV is actually larger than the systematic part, showing again the importance of modeling idiosyncratic variance in order to capture the deviation risk of small portfolios correctly.

Figure 7 refers to the example used in section 3 (i.e., 3- to 5-year option-adjusted duration versus the whole Credit Index). This report is also produced as of 4/26/2002. This time, both the portfolio and the benchmark are highly diversified, so it is natural to expect that the idiosyncratic risk be almost eliminated, as shown by the TEV decomposition. Also, since there is a huge curve mismatch, it is not surprising that a large portion of the TEV comes from interest rate risk. It is also instructive to observe that credit spread risk, although large when isolated, does not increase the overall TEV by much when added to benchmark risk, because of the well-documented negative correlation between interest rates and credit spreads, which is captured in the factor covariance matrix.

Figure 6a. **Portfolio = 10-year TRAINS; Benchmark = 7- to 15-Year Maturity Credit Index**
Portfolio/Benchmark Summary



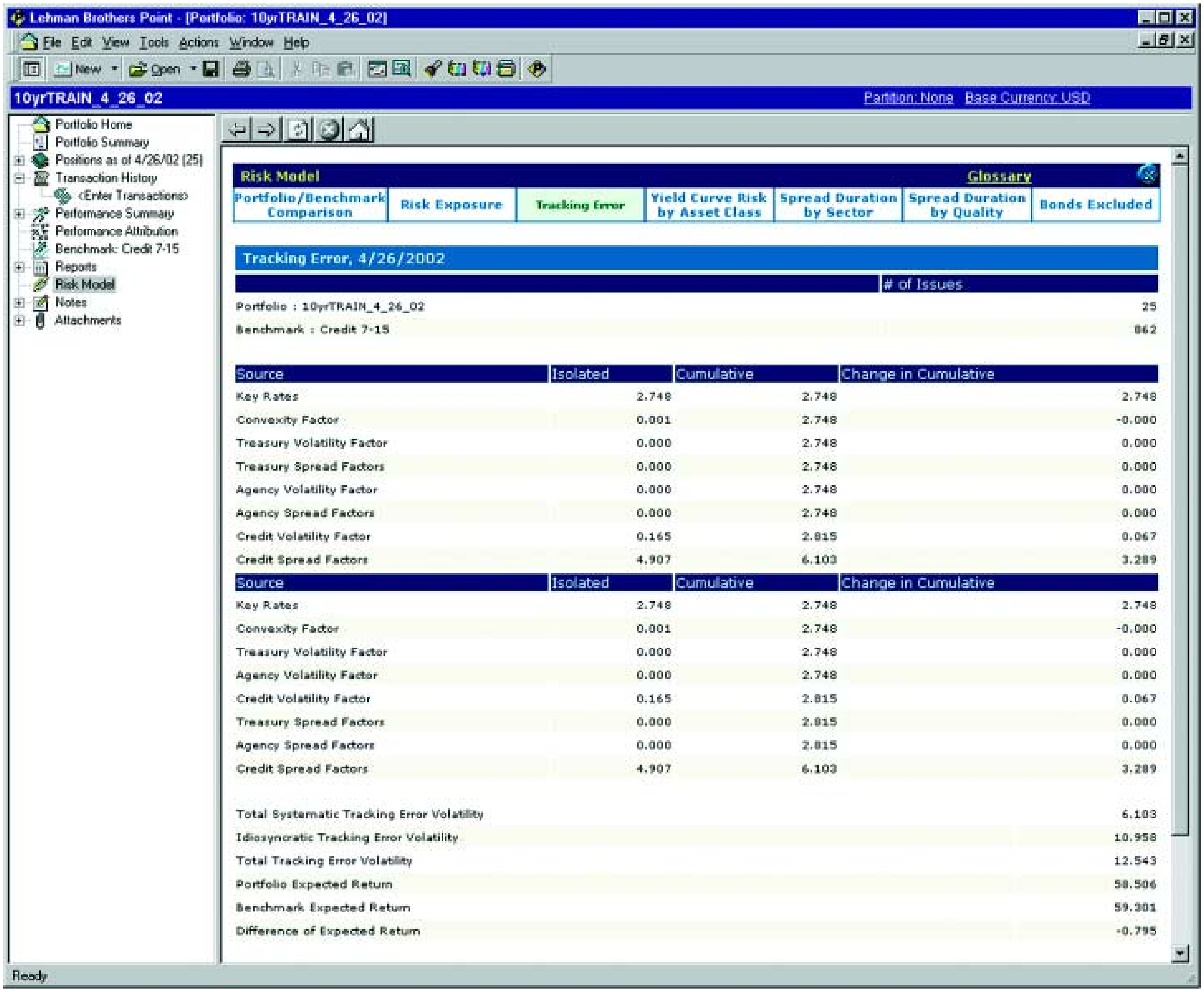
	# of Issues
Portfolio : 10yrTRAIN_4_26_02	25
Benchmark : Credit 7-15	862

	10yrTRAIN_4_26_02	Credit 7-15 (Statistics, Unhedged)	Difference: 10yrTRAIN_4_26_02, Credit 7-15
Count	25	862	-837
Coupon Market Weighted	6.892	7.359	-0.467
Average Life	8.841	8.921	-0.080
Dollar Duration Weighted Yield	6.797	6.898	-0.101
OAS	166.649	177.653	-11.004
OA Duration	6.64	6.57	0.07
OA Spread Duration	6.61	6.53	0.08
OA Convexity	0.56	0.56	-0.00

Total Tracking Error Volatility	12.543
Portfolio Beta	1.017

Disclaimer: This document is for information purposes only and should not be regarded as an offer to sell or as a solicitation of an offer to buy the products mentioned in it. No representation is made that any returns will be achieved. Past performance is not necessarily indicative of future results. This information has been obtained from various sources, including where applicable, entered by the user; we do not represent it is complete or accurate. The user of this information is hereby advised that Lehman Brothers takes no responsibility for improper, inaccurate or other erroneous assumptions to the extent such data is entered by the user hereof. Opinions expressed herein are subject to change without notice. The securities mentioned in this document may not be eligible for sale in some states or countries, or suitable for all types of investors. This document has also been prepared on behalf of Lehman Brothers International (Europe), which is regulated by the SFA. ?2002 Lehman Brothers Inc. All rights reserved. Member SIPC.

Figure 6b. **Portfolio = 10-year TRAINS; Benchmark = 7- to 15-Year Maturity Credit Index**
Tracking Error



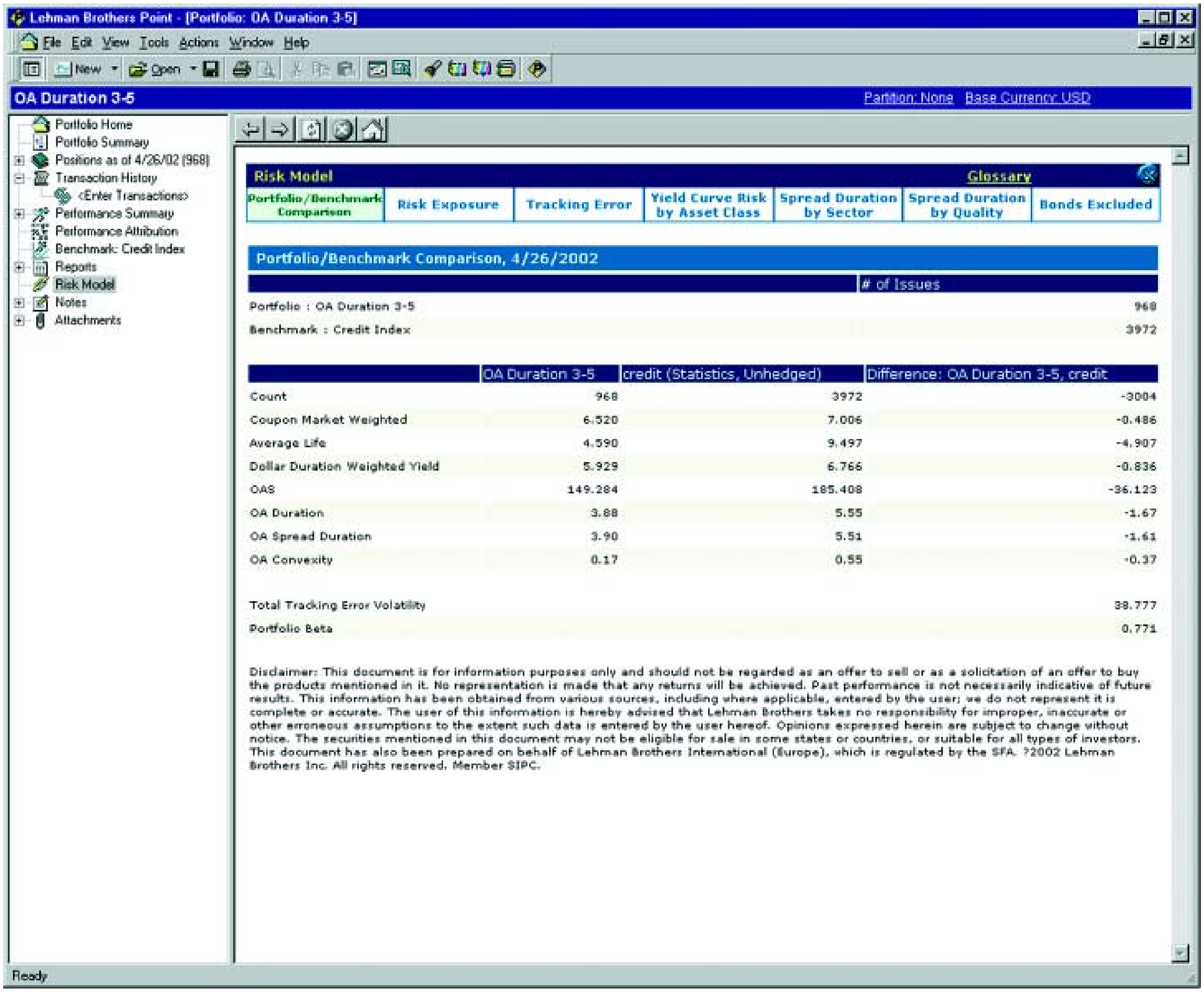
	# of Issues
Portfolio : 10yrTRAIN_4_26_02	25
Benchmark : Credit 7-15	862

Source	Isolated	Cumulative	Change in Cumulative
Key Rates	2.748	2.748	2.748
Convexity Factor	0.001	2.748	-0.000
Treasury Volatility Factor	0.000	2.748	0.000
Treasury Spread Factors	0.000	2.748	0.000
Agency Volatility Factor	0.000	2.748	0.000
Agency Spread Factors	0.000	2.748	0.000
Credit Volatility Factor	0.165	2.815	0.067
Credit Spread Factors	4.907	6.103	3.289

Source	Isolated	Cumulative	Change in Cumulative
Key Rates	2.748	2.748	2.748
Convexity Factor	0.001	2.748	-0.000
Treasury Volatility Factor	0.000	2.748	0.000
Agency Volatility Factor	0.000	2.748	0.000
Credit Volatility Factor	0.165	2.815	0.067
Treasury Spread Factors	0.000	2.815	0.000
Agency Spread Factors	0.000	2.815	0.000
Credit Spread Factors	4.907	6.103	3.289

Total Systematic Tracking Error Volatility	6.103
Idiosyncratic Tracking Error Volatility	10.958
Total Tracking Error Volatility	12.543
Portfolio Expected Return	58.506
Benchmark Expected Return	59.301
Difference of Expected Return	-0.795

Figure 7a. **Portfolio = 3- to 5-yr OAD Credit Index; Benchmark = Credit Index**
Portfolio/Benchmark Summary



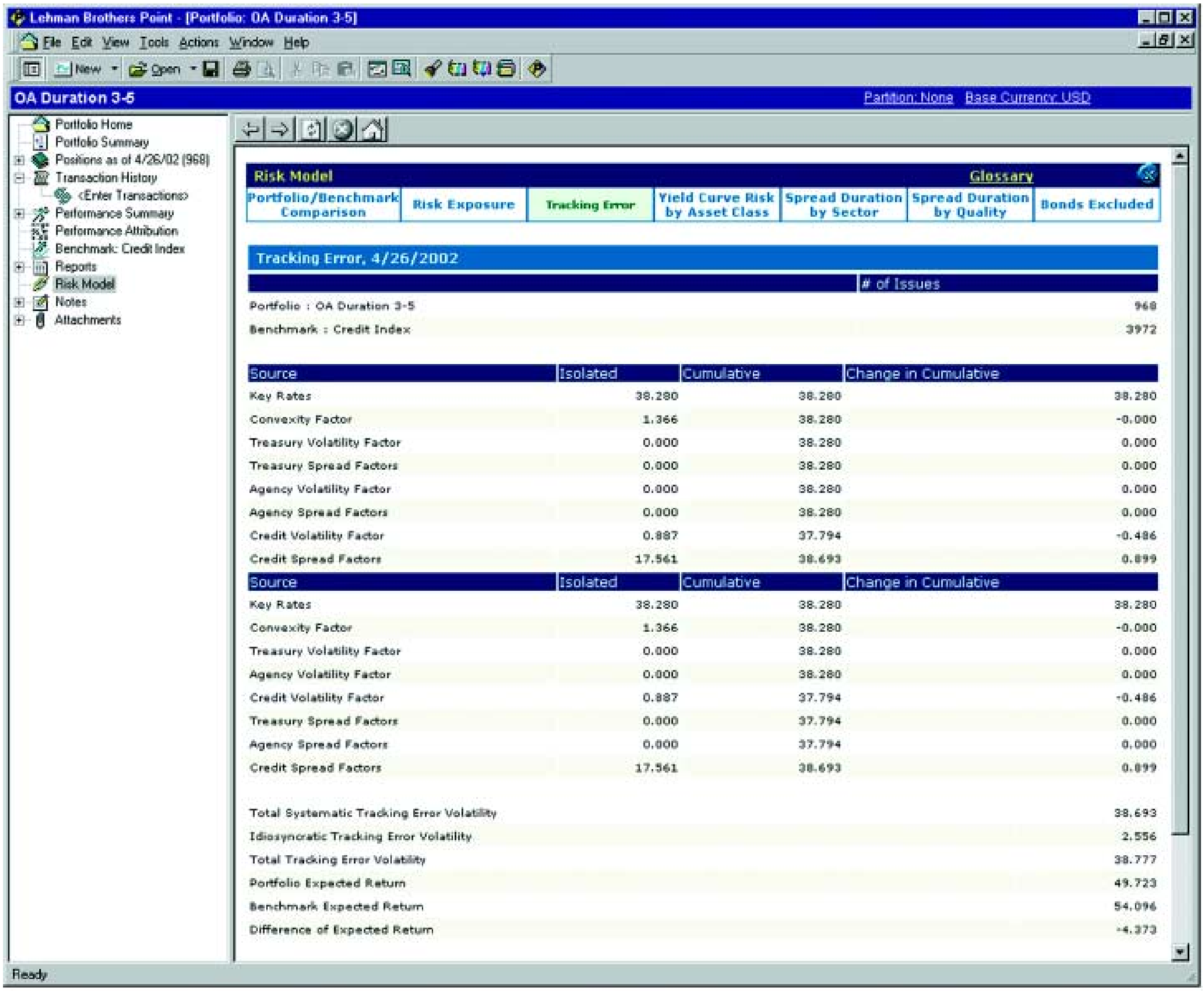
	# of Issues
Portfolio : OA Duration 3-5	968
Benchmark : Credit Index	3972

	OA Duration 3-5	credit (Statistics, Unhedged)	Difference: OA Duration 3-5, credit
Count	968	3972	-3004
Coupon Market Weighted	6.520	7.006	-0.486
Average Life	4.590	9.497	-4.907
Dollar Duration Weighted Yield	5.929	6.766	-0.836
OAS	149.284	185.408	-36.123
OA Duration	3.88	5.55	-1.67
OA Spread Duration	3.90	5.51	-1.61
OA Convexity	0.17	0.55	-0.37

Total Tracking Error Volatility	38.777
Portfolio Beta	0.771

Disclaimer: This document is for information purposes only and should not be regarded as an offer to sell or as a solicitation of an offer to buy the products mentioned in it. No representation is made that any returns will be achieved. Past performance is not necessarily indicative of future results. This information has been obtained from various sources, including where applicable, entered by the user; we do not represent it is complete or accurate. The user of this information is hereby advised that Lehman Brothers takes no responsibility for improper, inaccurate or other erroneous assumptions to the extent such data is entered by the user hereof. Opinions expressed herein are subject to change without notice. The securities mentioned in this document may not be eligible for sale in some states or countries, or suitable for all types of investors. This document has also been prepared on behalf of Lehman Brothers International (Europe), which is regulated by the SFA. ?2002 Lehman Brothers Inc. All rights reserved. Member SIPC.

Figure 7b. **Portfolio = 3- to 5-yr OAD Credit Index; Benchmark = Credit Index**
Tracking Error



	# of Issues
Portfolio : OA Duration 3-5	968
Benchmark : Credit Index	3972

Source	Isolated	Cumulative	Change in Cumulative
Key Rates	38.280	38.280	38.280
Convexity Factor	1.366	38.280	-0.000
Treasury Volatility Factor	0.000	38.280	0.000
Treasury Spread Factors	0.000	38.280	0.000
Agency Volatility Factor	0.000	38.280	0.000
Agency Spread Factors	0.000	38.280	0.000
Credit Volatility Factor	0.887	37.794	-0.486
Credit Spread Factors	17.561	38.693	0.899
Source	**Isolated**	**Cumulative**	**Change in Cumulative**
Key Rates	38.280	38.280	38.280
Convexity Factor	1.366	38.280	-0.000
Treasury Volatility Factor	0.000	38.280	0.000
Agency Volatility Factor	0.000	38.280	0.000
Credit Volatility Factor	0.887	37.794	-0.486
Treasury Spread Factors	0.000	37.794	0.000
Agency Spread Factors	0.000	37.794	0.000
Credit Spread Factors	17.561	38.693	0.899

Total Systematic Tracking Error Volatility	38.693
Idiosyncratic Tracking Error Volatility	2.556
Total Tracking Error Volatility	38.777
Portfolio Expected Return	49.723
Benchmark Expected Return	54.096
Difference of Expected Return	-4.373

A complete Risk Model report contains a few other tables detailing factor mismatches, decomposing interest-rate (benchmark) risk by asset class, and providing standard market structure reports by sector and quality.

6. SUMMARY

Lehman Brothers specializes in advising and working with investors who manage their holdings relative to pre-specified benchmarks. The increasingly complex nature of our fixed-income markets calls for the development of appropriate quantitative tools to evaluate and decompose the associated deviation risk.

More than a decade of experience in risk modeling and a continuous interaction with our clients have proven to be a solid basis for the development of the new Lehman Brothers U.S. Risk Model. In this article we have described its credit-related portion, emphasizing the decomposition of total return and the specification of empirical models for its stochastic components. We have also offered some hints regarding the out-of-sample testing that we use to evaluate the model's output, and a glance at some of the reports available to the user on our new portfolio system POINT.

7. REFERENCES

Campbell, J.Y., A.W. Lo, and A.C. MacKinlay, (1997), *The Econometrics of Financial Markets*, Princeton University Press.

Christoffersen, P.F. (1998), "Evaluating Interval Forecasts," *International Economic Review*, 39, 841-862.

Dynkin, L., J. Hyman and W. Wu (1999), *The Lehman Brothers Multi-Factor Risk Model*, Lehman Brothers publication.

Wilcox, R. (1997), *Introduction to Robust Estimation and Hypothesis Testing*, Academic Press.

Publications—L. Pindyck, A. DiTizio, B. Davenport, W. Lee, D. Kramer, J. Threadgill, R. Madison, A. Acevedo, K. Kim, C. Rial, J. Batstone