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Tail Risk Estimation

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POINT Portfolio Modeling
Index, Portfolio and Risk Solutions

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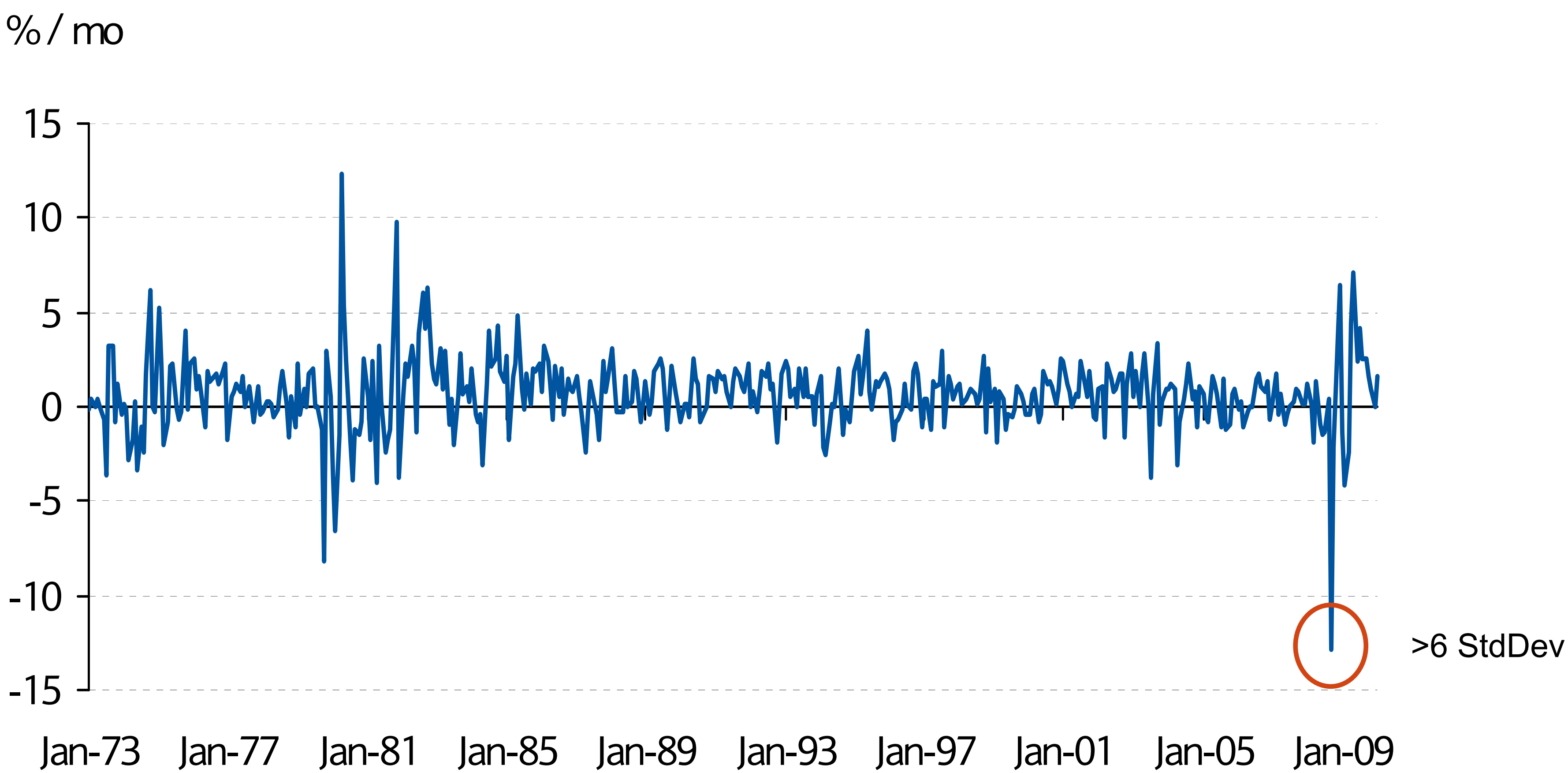
Tail Risk Example

Barclays Capital US Financials IG Index – Total Return



Source: Barclays Capital.

Importance of Tail Risk

- Extreme losses
- Maximum allowed portfolio loss: Exceedance → fund / institution failure
- Magnitude of extreme losses: **Loosely** linked to volatility
- Mean-variance framework
  - Poor predictor under extreme scenarios
- Failures of the framework
  - Likelihood of extreme events is higher than predicted by normal distribution
  - Behavior of “mean-variance hedges” under stressed scenarios
  - Asymmetry of positive vs. negative extreme scenarios
- Maximum allowed loss may change under extreme macro events

Agenda

- Measurement
- Tail Risk Models
  - Historical
  - Marginal / Copula approach
- Marginal / Copula in detail
- Direct modeling of extreme events: EVT, CAViaR
- Testing
- VaR by horizon

Measures of Tail Risk

- VaR: Best of worst case scenarios
  - Use: When looking at a threshold, e.g., default
  - Problematic: No diversification ($VaR_1 + VaR_2 < VaR_{1,2}$), unless normal or T
- CVaR/ES: Average of worst case scenarios
  - Use: Understand the portfolio behavior under stressed scenarios
  - Coherent measure, i.e., monotonic, homogeneous, translational invariant, sub-additive
  - Sub-additive: $CVaR_1 + CVaR_2 > CVaR_{1,2}$
  - Better behaved in optimization problems
- Normal distribution → VaR/CVaR = simple volatility rescaling

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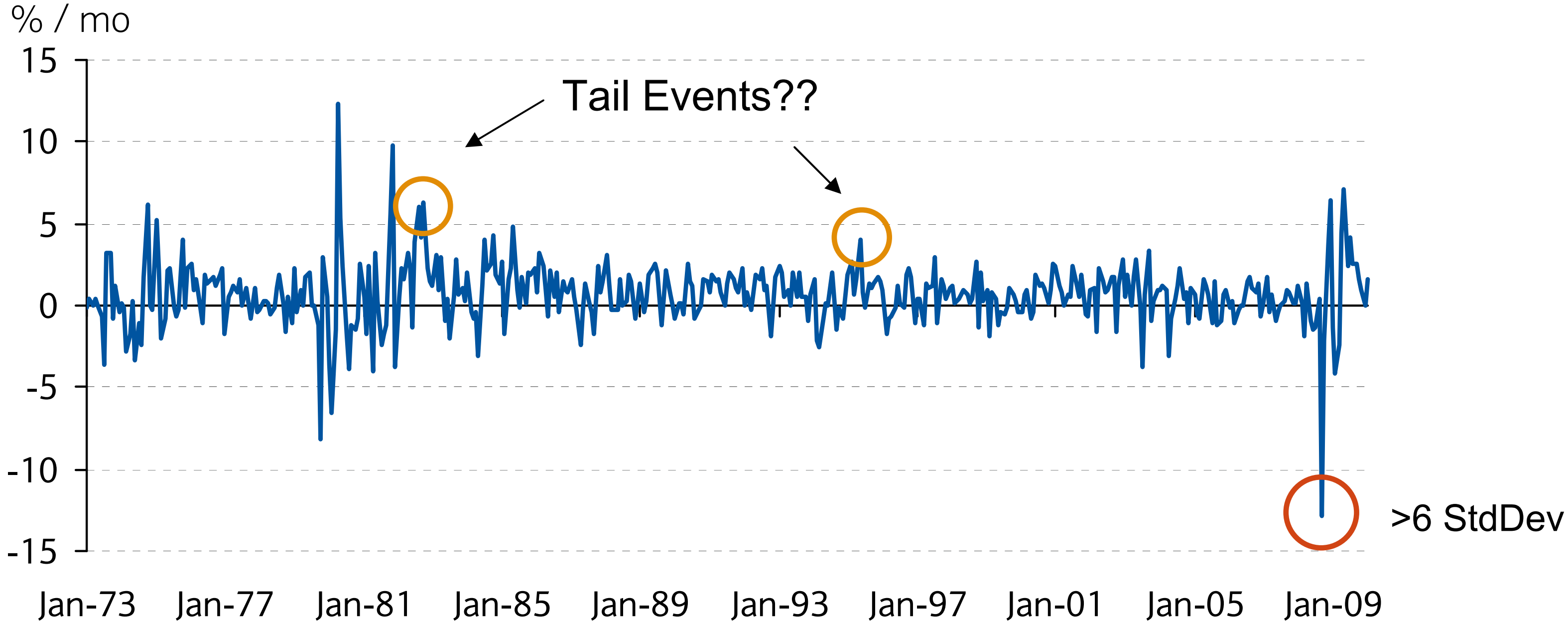
Initial Models: Historical Option

- Historical option: Construct portfolio values using time-series of pricing inputs
  - No distributional assumption → more robust but less precise
  - Backward looking, in-sample
  - Cannot do weighting, otherwise lose older events
  - We can do better

Barclays Capital US Financials IG Index – Total Return



Source: Barclays Capital.

State-of-the-Art: Marginal-Copula

- Forecast the distribution of pricing inputs (factors) instead of portfolio
  - Limited types of distributions and better behaved
  - Fit a distribution: Can look as far out in the tail as a user wants
  - Fit a **multivariate** distribution
- Multivariate distributions
  - MV Normal: Simple, covariance matrix defines everything
  - MV Student-T: Add one parameter
    - All individual factors' (marginal) distributions are the same (link 1)
    - Marginal distributions linked to factors inter-dependency (link 2)
- **Marginal-Copula**: It breaks the links
  - Model each marginal distribution separately,
  - Then join them with a function (=copula), which defines factor dependency
  - POINT approach

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Marginal-Copula Models

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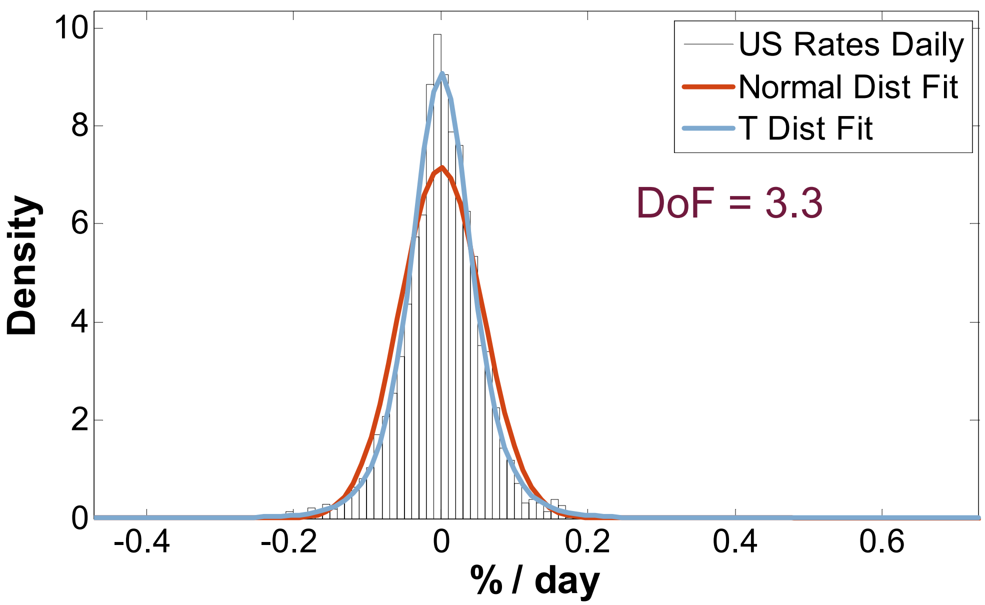
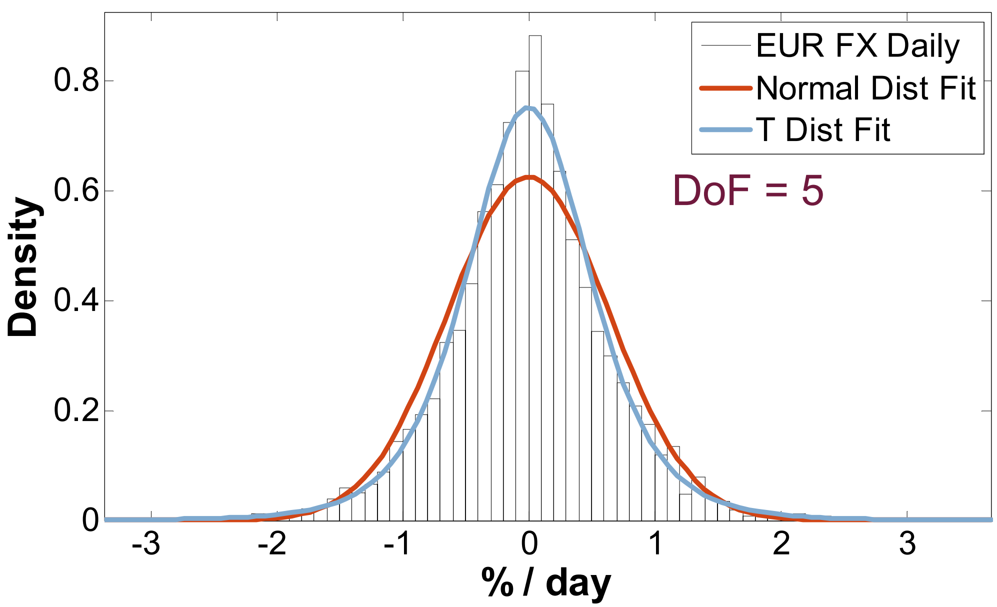
Marginal Distributions

- Focus on only individual (univariate) distributions
- Use various types: Normal, Student-T, Skew-T, etc.
- Trade-off: Complexity vs. robustness
- Type varies by asset class: e.g., IR are skewed, EUR FX are less

Distribution of Daily US Rates Changes



Distribution of Daily EUR FX Changes

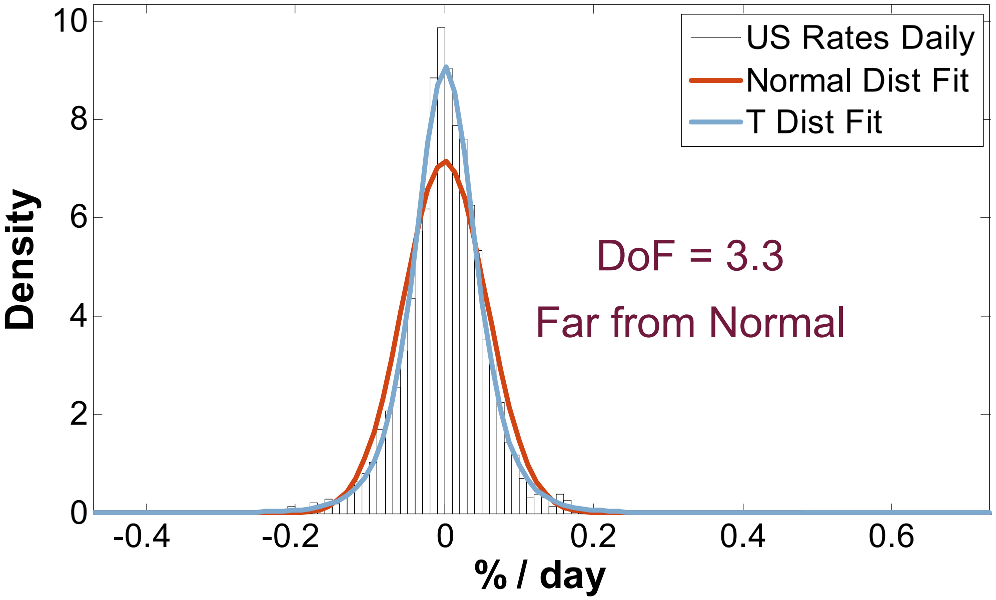


Source: Barclays Capital.

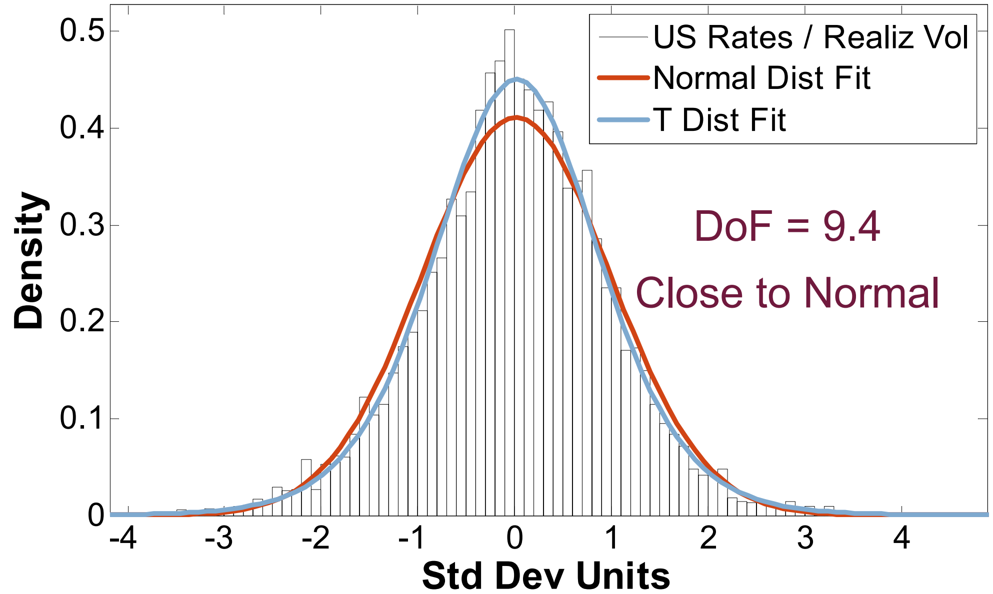
Marginal Distributions: Conditioning

- Mixing simple distributions with varying volatility → complex unconditional distributions
  - Divide by volatility before defining distribution type → constant vol
  - Resulting distributions are closer to normal
  - Focus on a good **volatility forecast model** instead of fancy distribution

Distribution of Daily US Rates Changes



Distribution of Daily US Rates Chg / Realized Vol



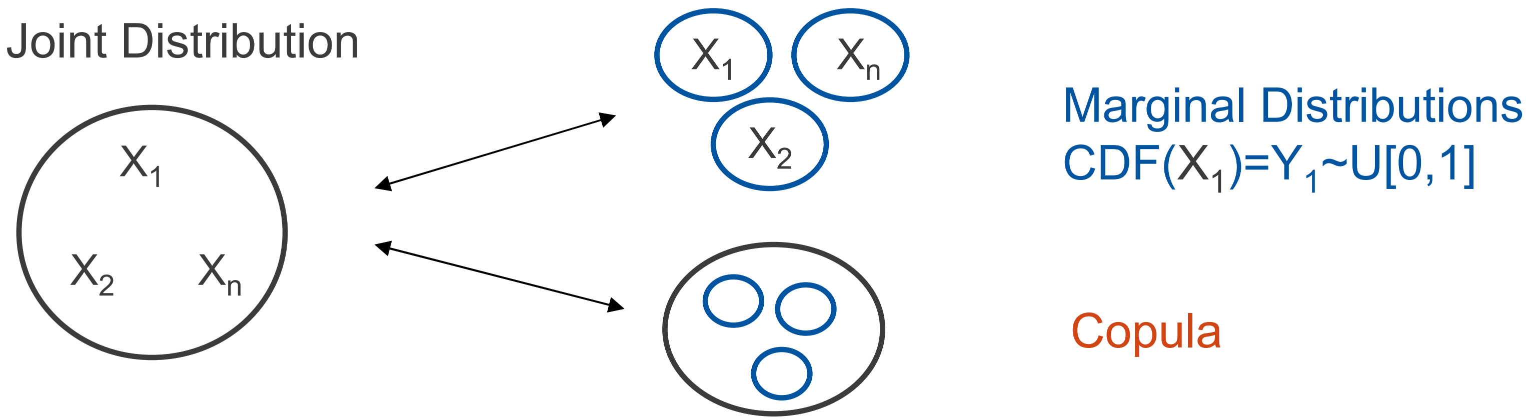
- Volatility forecast can depend on assumed distribution – Interdependence

Source: Barclays Capital.

Copulae

- Links marginal distributions $M_1(X_1),\ldots M_n(X_n)$ into the joint distribution *F*

$$F(X_1, X_2 \ldots X_n) = C(M_1(X_1), M_2(X_2)\ldots M_n(X_n), parms)$$



- Types: Normal Copula, Student-T Copula, Clayton, Gumbell
- Focus on only capturing factors' dependence
  - Incorporate correlations
  - Can go beyond correlations: Tail dependence, asymmetry
  - Measures: Pearson, Spearman, Kendall correlations; coeff of tail dependence
    - No perfect measure
- Harder to calibrate than marginals

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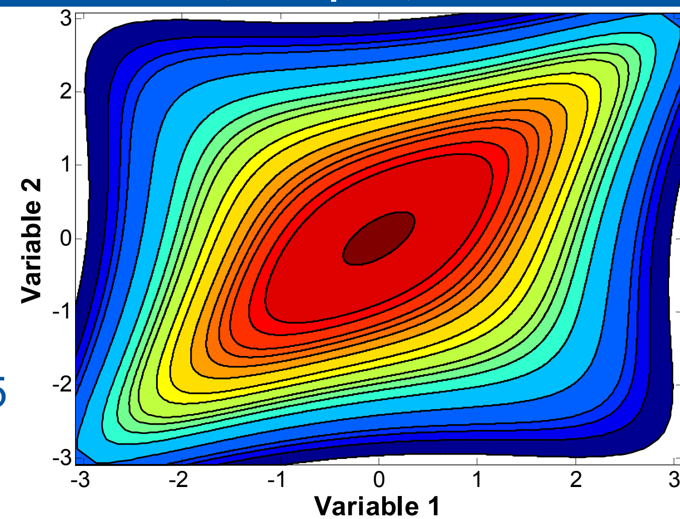
Copulae – Examples

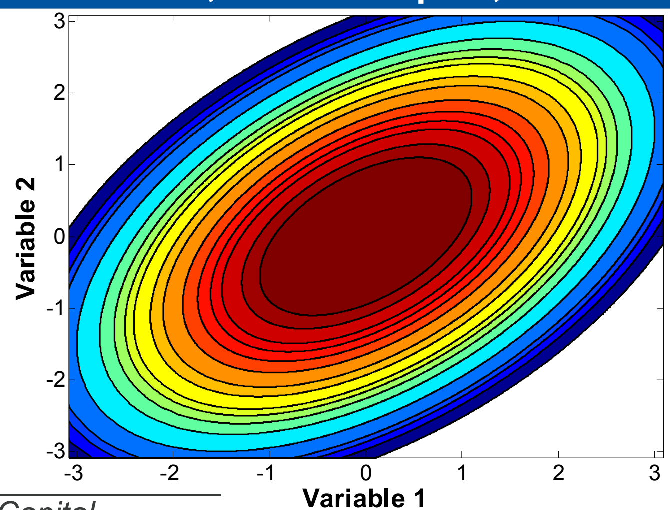
- A pair of variables, both have the same marginal distribution, N[0,1], same corr=.5
- Compute 1%VaR Long-Short and Long-Long Portfolios for various copulae

Constant Prob Curves, T Copula, corr=0.5 DoF=3



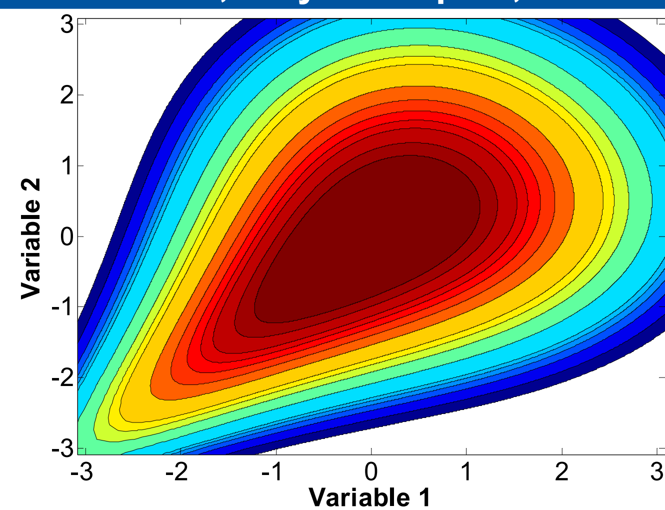
1% VaR
L/S = -1.35
L/L = -2.11

Constant Prob Curves, Normal Copula, corr=0.5



1% VaR
L/S = -1.17

Constant Prob Curves, Clayton Copula, theta=1



1% VaR
L/L = -2.23

Source: Barclays Capital.

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Alternative Modeling Frameworks

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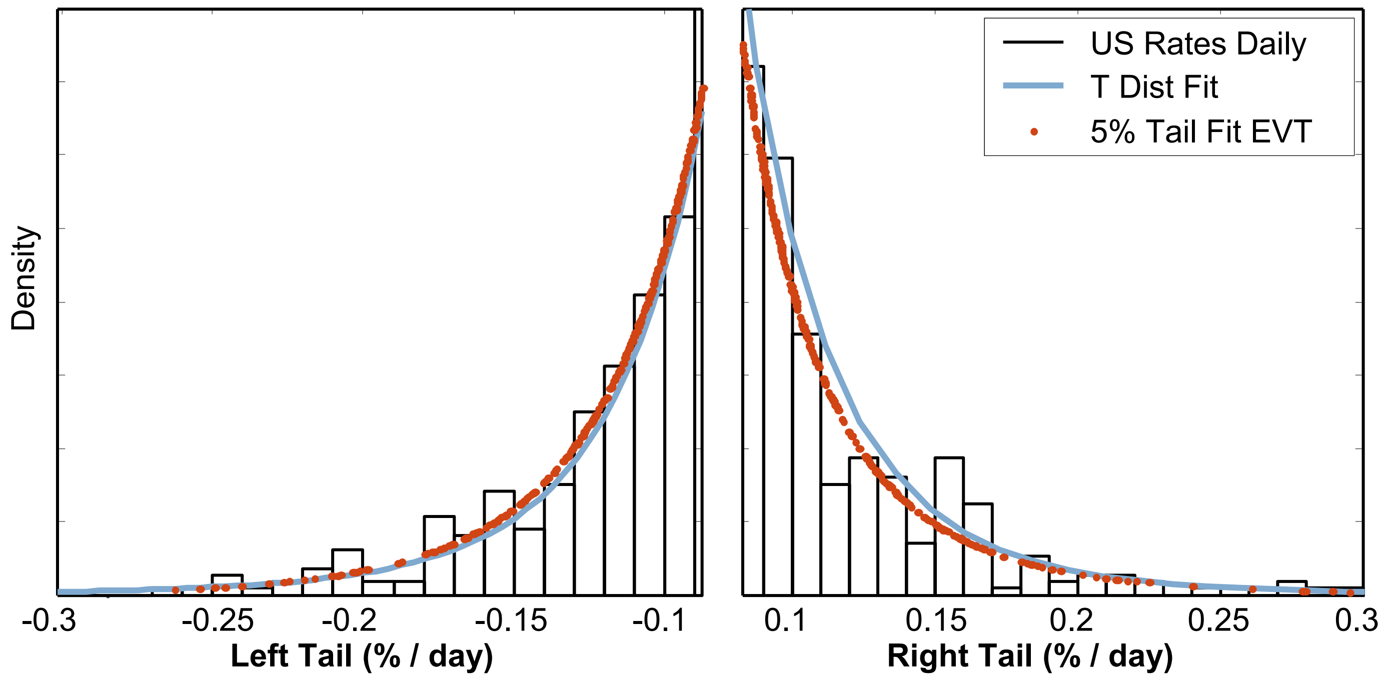
Direct Modeling of Tails: EVT

- Focus directly on tails, ignore the rest of the distribution
- Extreme Value Theory: The asymptotic distribution of min / max of a sample (=VaR)
  - Assumptions on tail distribution: What is “tail”?
  - Difficult to do multivariate distribution
  - Use: When hard to calibrate entire distribution
- E.g., fit the 5% tails using a power-law (EVT)

US Rates Daily Chg: 5% Tail Fit vs. Entire Distribution Fit

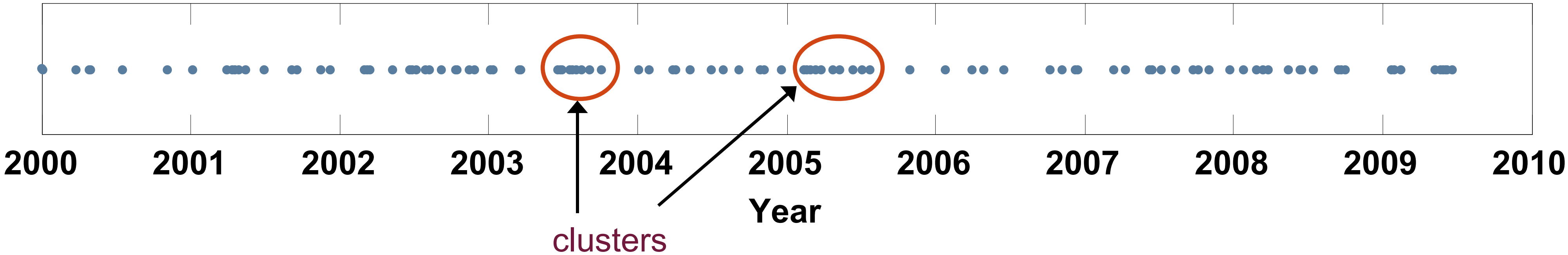


Source: Barclays Capital.

Direct Modeling of Tails: CAViaR

- Use time-series dependence: Current tails are a function of past realizations
  - Plot only realizations (scaled by vol) above forecasted VaR

Days when (US Rate Chg / GARCH Volatility) < -1.65



- Same intuition as GARCH: Extreme observations clustering

$$VaR_t^{95\%} = \omega + \beta \cdot VaR_{t-1}^{95\%} + \alpha \cdot f\left(r_{t-1}\right)$$

  - Estimate with quantile regressions
- CAViaR (Engle Manganelli 2002)

Source: Barclays Capital.

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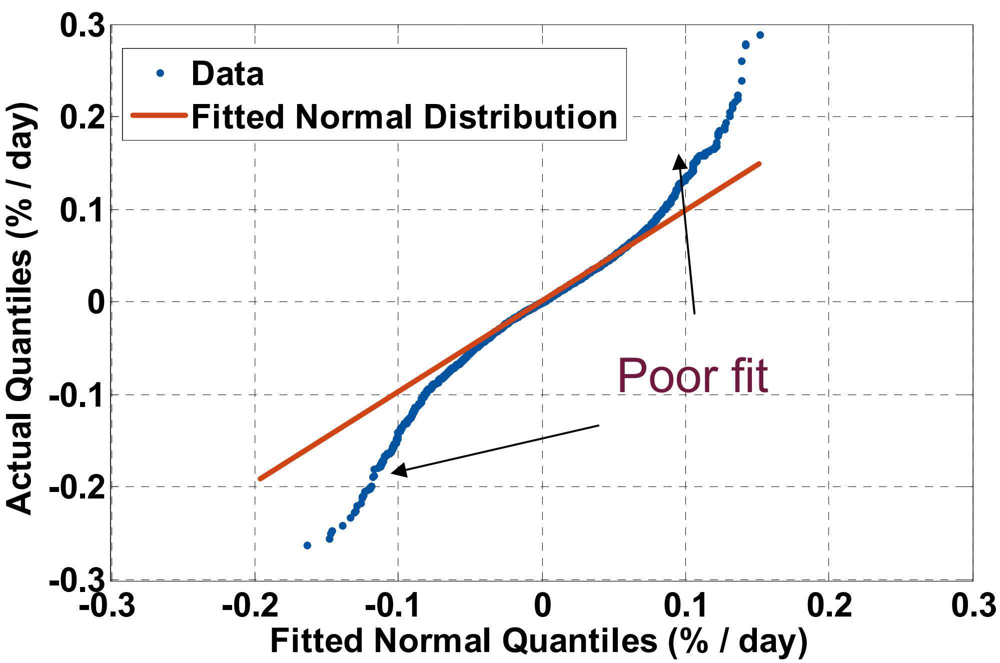
Testing Models

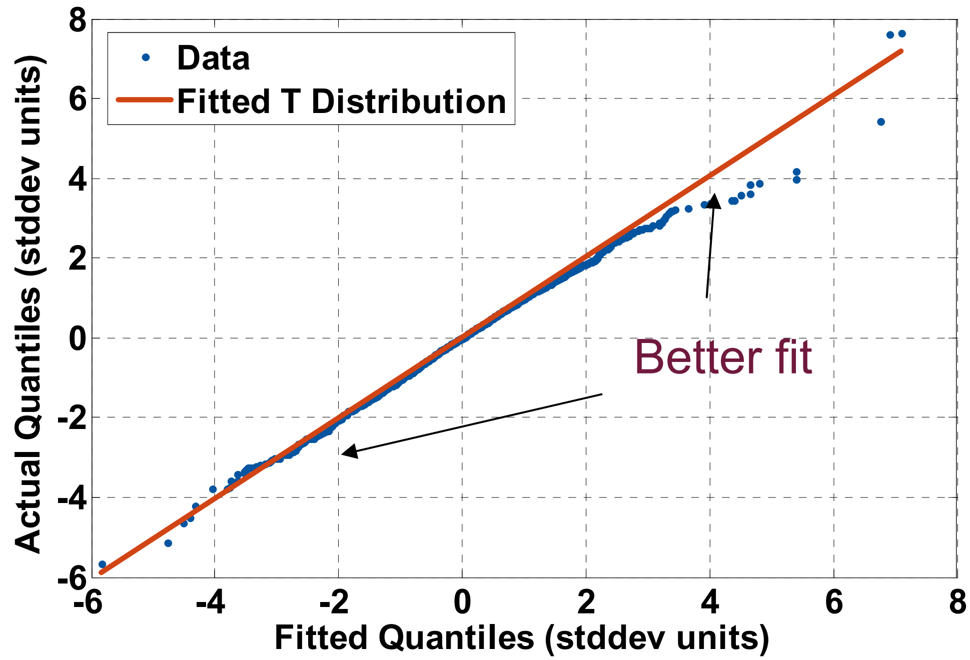
Testing: QQ Plots

- In-sample test
- Plot quantiles of fitted distribution vs. real data (QQ Plot), see if they match
- Cannot do out-of-sample because fitted distribution varies over time
- QQ plots for two tail-risk models

Model 1: Fit Daily US Rates Using Normal Distribution



Model 2: Fit Daily (US Rates / GARCH Vol) Using Student-T Distribution



Source: Barclays Capital.

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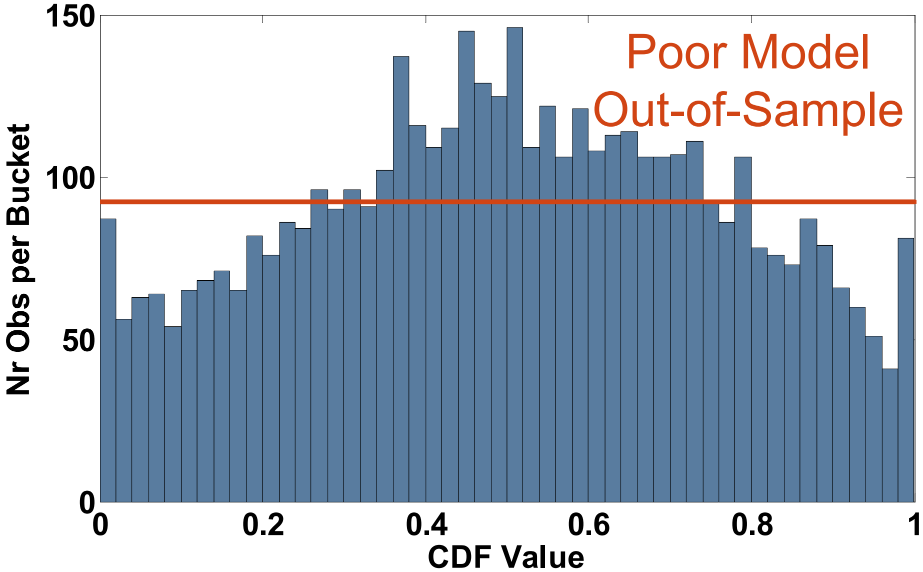
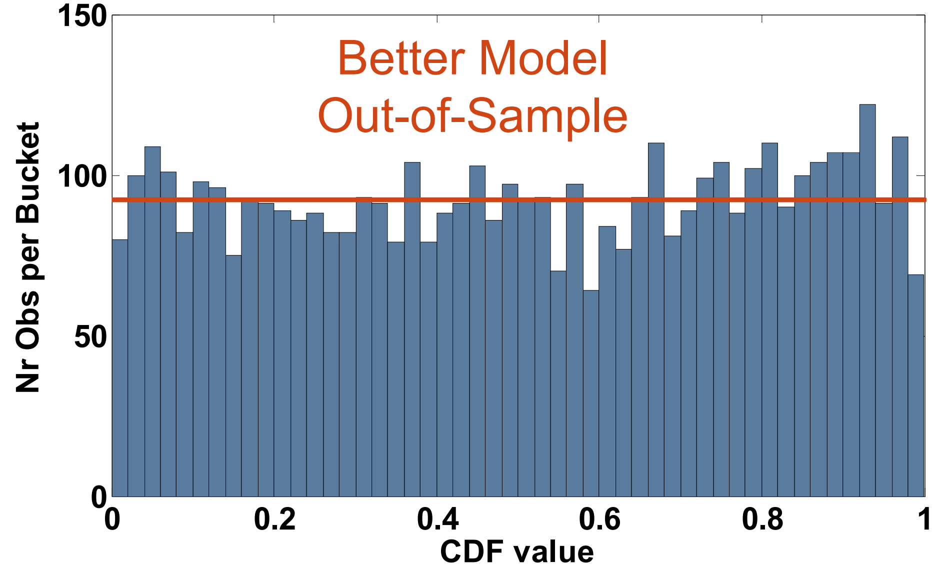
Testing: CDF of Data

- Compute Y = forecasted CDF at realized values
  - Y should be IID Uniform [0,1]
  - Testing: Plot the histogram of Y; formally, use Kolmogorov-Smirnov and Chi-sq tests
- Out-of-Sample
- Allows for changing distribution
- Histogram of forecasted CDF using previous two tail-risk models

Model 1: Daily US Rates



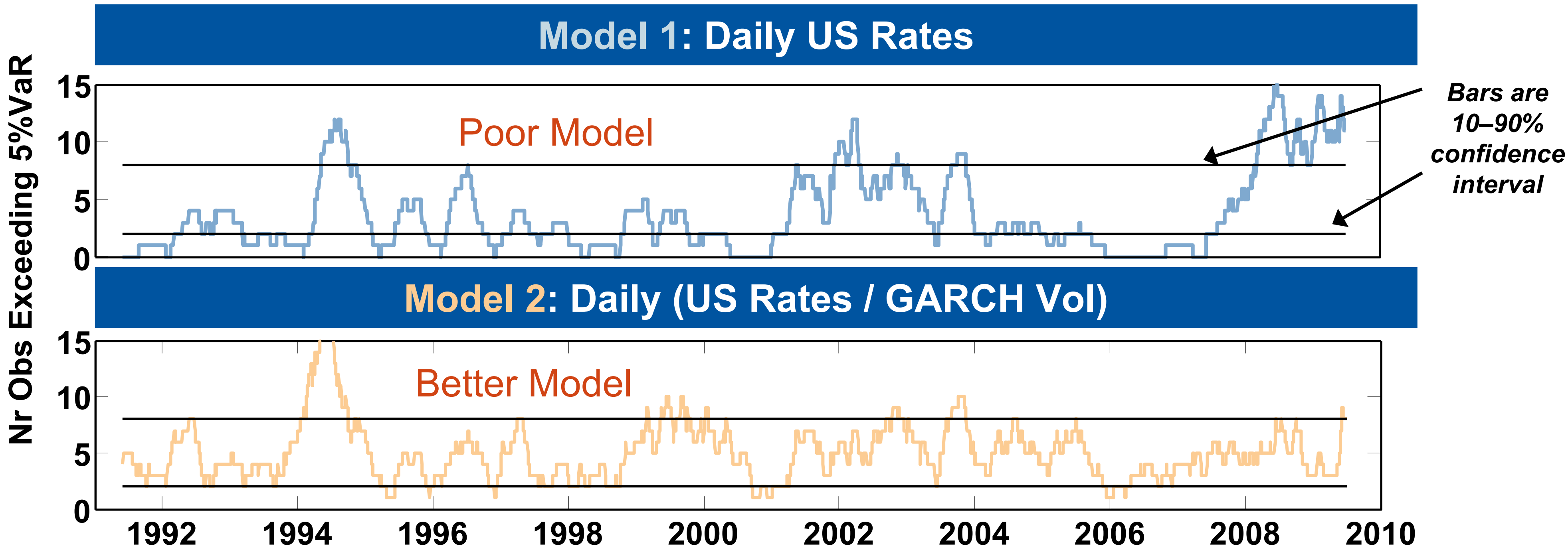
Model 2: Daily (US Rates / GARCH Vol)



Source: Barclays Capital.

Testing: VaR Exceedances

- Analyze N = % of realizations above Q%VaR
  - Used also when forecast quantile Q directly (EVT, CAViaR)
  - N should = Q on average, not vary too much, and be unpredictable
- Plot N over a 100-day period, rolling, using previous two tail-risk models



- Conditional tests: Regress indicators on predictors; test $N{\cdot}R\text{-}sq \sim X^2_N$

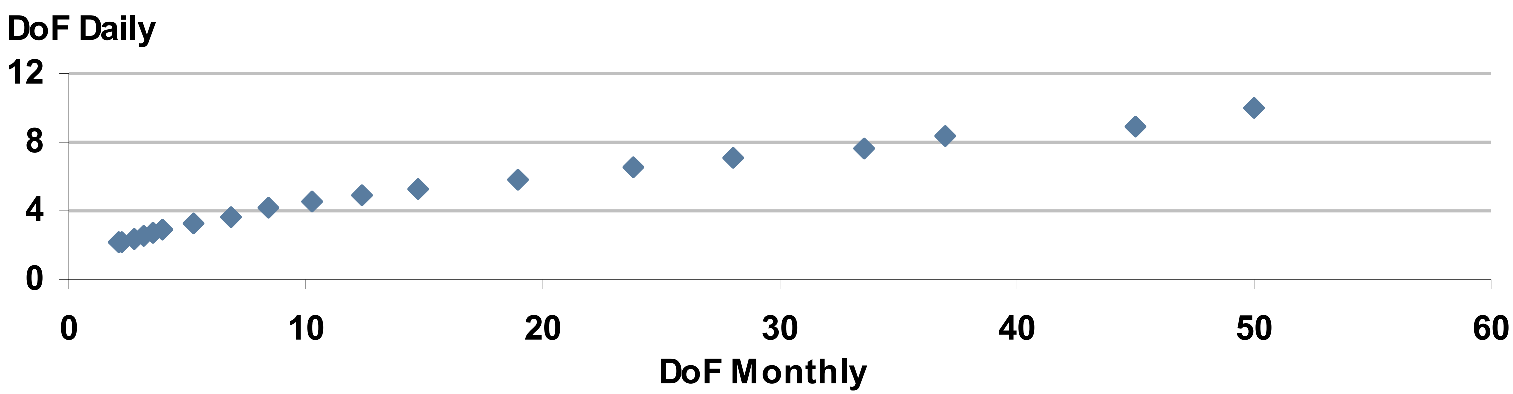
Source: Barclays Capital.

VaR by Horizon

VaR by Horizon

- VaR has similar horizon issues as volatility / correlation
  - Autocorrelation affects sqrt rule of volatility scaling, affecting VaR
  - May use higher-freq data to better forecast lower-freq moments
- Harder to test as horizon increases – less observations
- Tails become more normal as horizon increases (CLT; need close-to-IID)
  - Distributions may not aggregate to the same type, e.g., Sum of Student-T is not a Student-T
  - But we may assume so and derive aggregation rules for DoF parameter
- Use
  - For intermediate horizons, map parameters from higher frequencies using MC results

DoF Map Between Daily and Monthly Freq.



  - For very long horizons, assume that *conditional on volatility*, distribution is normal

Source: Barclays Capital.

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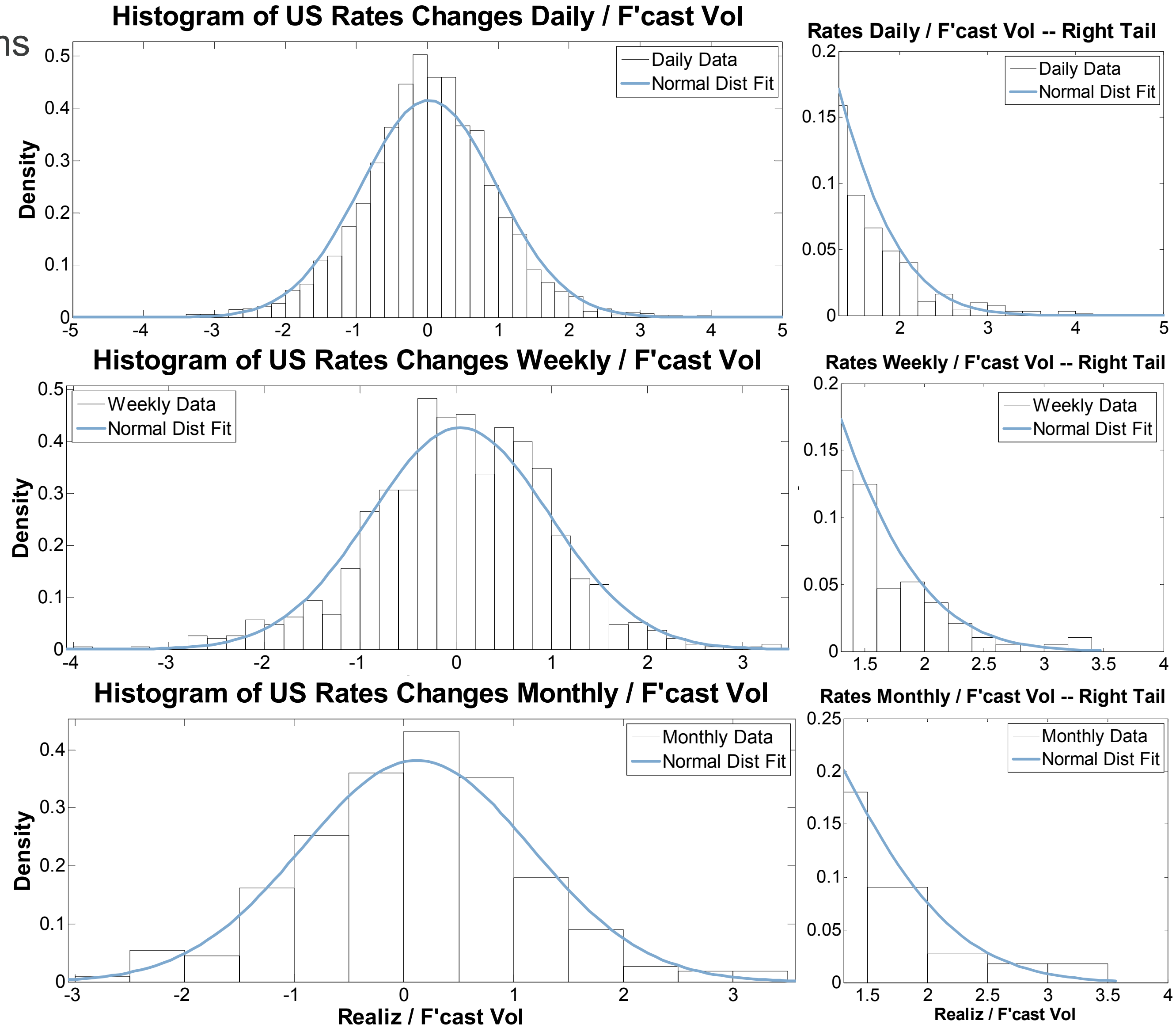
VaR by Horizon: Getting Close to N

- Daily / weekly /monthly histograms fitted by Normal distribution
  - As frequency decreases they get close to N

Jarque-Berra Test for Normality

Frequency	Nr. Obs	Stat	p-val
Daily	4,620	1,485.0	1.00E-20
Weekly	963	32.1	1.00E-04
Monthly	222	3.0	0.18



Source: Barclays Capital.

Conclusion

- Marginal-copula is better than historical: More flexibility, invariance
- Condition on volatility before modeling
  - Need a good volatility model
- Copula defines factor dependence
- Testing is complicated, but not impossible
  - Focus on exceedances and on CDF of realized values
  - Test conditionally if enough data
- As frequency decreases, get close to normal

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