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Risk Modelling and Performance Attribution for Inflation-linked Securities

Anthony Lazanas and Jeremy Rosten

LEHMAN BROTHERS

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Anthony Lazanas
+1 212 526 3127
alazanas@lehman.com

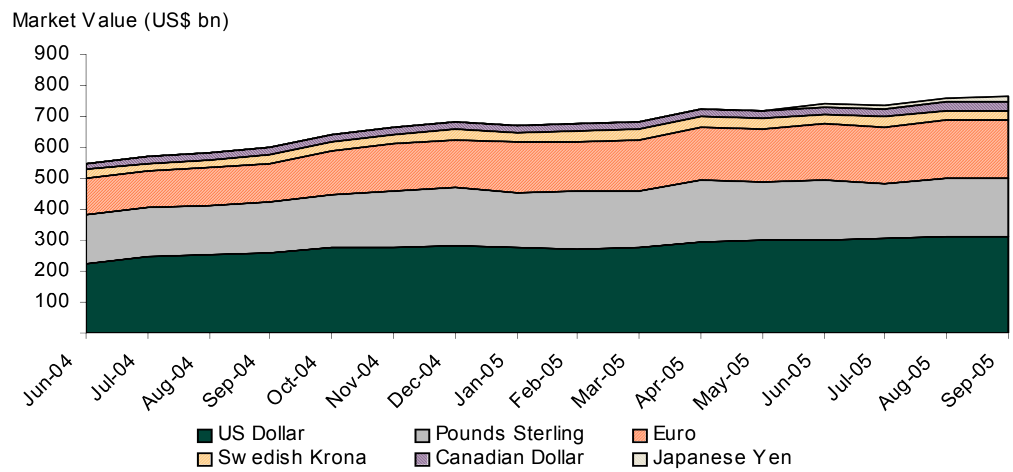
Jeremy Rosten
+44 (0)20 7102 1020
jrosten@lehman.com

Accessed through POINT[2], our Global Risk Model and Hybrid Performance Attribution Model allow a comprehensive analysis of the risk and return characteristics of global fixed income portfolios. As part of the continuing expansion of the universe of securities handled by these two models, we present an overview of our newly developed approach to the analytics of inflation-linked debt. We outline our methodology for the integration of this asset class within the existing frameworks of the models and arrive at what we consider to be a simple yet powerful formulation of the risk factors driving returns, leading to an intuitive treatment in terms of both risk and return.

1. INTRODUCTION

In October 1997, the global market for inflation-linked government bonds accounted for approximately 2% of the total government bond market. By the end of September 2005, the figure had risen to around 7.2%[3] totalling some $766 billion. Moreover, we think there are reasons to expect that the size of the inflation-linked market will continue to increase both in total size and as a proportion of the global government debt market. Figures 1 and 2 show the recent growth in the market as well as the current distribution of inflation-linked debt across the main sovereign issuers.

Figure 1. Recent growth of the inflation-linked market



Source: POINT.

Figure 2. Current distribution of market value across major sovereign inflation issuers[4]

	US Dollar	Euro	Pounds Sterling	Swedish Krona	Canadian Dollar	Japanese Yen	Total
Market Value ($Bn)	311.9	189.6	185.5	32.3	29.4	17.3	766

Source: POINT.

1 *The authors would like to thank Vasant Naik, Prafulla Nabar, Adam Purzitsky and Adam Shergold for their help, advice and comments.*
2 *POINT is the Lehman Brothers Portfolio and Index analysis Tool.*
3 *The figures represent market values in the Lehman Brothers Global Inflation-linked Index plus the Japan Inflation-linked Index as compared with that of the Lehman Brothers Global Treasury Index as of September 30, 2005.*
4 *All figures are as of September 30, 2005 – Source POINT*

Given the size and projected growth of this asset class, the need for analytics, risk measurement and performance-attribution tools has never been greater. As part of the growing asset-class coverage of POINT, the Lehman Brothers Global Risk and Hybrid Performance Attribution models have this year seen the inclusion of inflation-linked bonds. This article outlines the approaches that we have taken in modelling these securities.

In our approach, we consider the exposure of inflation-linked securities to nominal yields and their exposure to inflation-related factors separately. Although there is a strong correlation between nominal rates and inflation (of the order of 50%), we prefer to keep these two risk factors separate so as to remain consistent with the risk framework of other security classes. Inflation risk affects these securities in two ways: (a) the risk that realised inflation could be different from market expectations, thus affecting the size of the paid cash flows; and (b) the risk that the market expected inflation over the life of the bond may change. We introduce three new factors in the Global Risk Model in each market[5]: one for realised inflation and two (a short-term and a long-term factor) for expected inflation.

In the Hybrid Performance Attribution model, we follow a similar approach: Yield curve return is extracted to isolate inflation-driven return. The latter comes once again from two main sources: (a) the difference between monthly realised inflation and expected inflation; and (b) changes in expected inflation over the lifetime of the bond. We introduce a technique to smooth the discrete (monthly) nature of the first component and report this return as a daily "carry" (representing the difference between the short-term realised inflation and the long-term expected inflation implied by the bond price) plus smaller monthly "inflation surprise" return shocks. With respect to the expected inflation component of return, we allow the user to implicitly break it down into smaller components by partitioning a benchmark index, similar to the treatment of credit securities.

The structure of the article is as follows:

In **section 2** we review some essential analytical concepts as they apply to the inflation-linked market, together with inflation-specific analytical terms. We look at yield and sensitivity definitions and conventions and relate them to some widely used measures.

Section 3 describes the hurdles faced in seeking to model inflation risk and attribute returns. We then develop the formal mathematical framework and the pricing equation that provides the basis for the identification and specification of factors driving risk and return.

Section 4 builds on the results of this analysis to describe the risk factors for the inflation-linked market in the Global Risk Model. We comment on some sample output of the Risk Model.

Section 5 presents the handling of these securities in our Performance Attribution model.

In **section 6** we make our concluding remarks.

Appendix A lays out the cash flow calculations and conventions referred to throughout the body of the article.

Appendix B presents the details of how we convert the discrete month-to-month inflation accretion return (occurring on index announcement days) into a smooth daily return.

5 In some markets, only one expected inflation factor is employed. See section 4.2 for more details.

2. BASIC ANALYTICS OF INFLATION-LINKED BONDS: TERMINOLOGY AND CONVENTIONS

Inflation-linked bonds (also called "reals", "inflation linkers" or simply "linkers") have numerous specific concepts and definitions in terms of yields and sensitivities. We take a whistle-stop tour of some of the principal conventions and measures in the market. This gives us not only a better understanding of how the market operates, but also helps us in our modelling the risk and return characteristics of this asset class.

2.1. Cash flows of inflation-linked bonds

An inflation-linked bond is typically a coupon-paying instrument with cash flows linked to a specified price index.[6] A *real coupon*, *c*, is defined along with the mechanics of indexation: the price index to be used and the rules to calculate the delivered cash flow. For a cashflow occurring at time *t*, we first define the Index Ratio, IR_t, as the ratio of the price index level to be used at time *t* to the index level at the issue of the bond (see appendix A for details). The index level to be used at time *t* is typically a lagged value of the price index. Then, the coupon (and similarly the principal[7]) payable at time *t* is given by:

$$c_t = c \cdot IR_t$$

In this way, the cash flows grow with the index and provide protection against inflation.

2.2. Real vs. nominal yield

Once we have such an index and securities with cash flows defined with reference to it, we can proceed to define and derive the standard analytical measures and concepts.

2.2.1. Real Yield

We define the *real yield* as the yield of the inflation-linked bond in terms of index units. That is, let B_t be the nominal price of the inflation-linked bond at time *t*, IR_t be the index ratio at time *t* and *c* the real coupon of bond. Translated into index units $\frac{B_t}{IR_t}$ is the real value of the bond and *c* is that of the real coupon (since the coupon will grow with the index it remains a constant in terms of index units). We can then define the real yield as the value of *r* that solves the pricing equation:

$$\frac{B_t}{IR_t} = \sum_{n=1}^{N} \frac{c \cdot 100}{(1+r)^n} + \frac{100}{(1+r)^N} \tag{1}$$

6 Inflation-linked bonds in the US are linked to the Index of Consumer Prices for Urban Consumers (CPURNSA). The indices in the UK, France, Japan, Canada, and Sweden are respectively, the General Index of Retail Prices all-items non-seasonally adjusted (UKRPI), Consumer Price Index excluding tobacco for all households residing in mainland France (FRCPXTOB), Japan CPI Nationwide General Ex Fresh Food (JCPNGENF), all-items CPI non-seasonally adjusted, published monthly by Statistics Canada (CACPI) and Sweden CPI (SWCPI) respectively. For Eurozone inflation-linked securities the index is Eurostat Eurozone HICP Ex Tobacco Unrevised series NSA(CPTFEMU)

7 In some markets floors exist for principal payments. See Appendix A for details.

2.2.2. Nominal Yield

The *nominal yield* can be defined in a number of ways. The most common method is that produced by making an explicit assumption, π , of future inflation that is constant across the life of the bond. Then, by inflating all future cash flows at this rate and computing an internal rate of return (IRR) for the resulting pricing equation, one arrives at the nominal yield.[8] Assuming a constant inflation rate of π , at time *t*, the n^{th} coupon, c_n, can be written:

$$c_n = IR_{t+n} \cdot c = \frac{IR_{t+n}}{IR_t} \cdot IR_t \cdot c = (1+\pi)^n \cdot IR_t \cdot c$$

To obtain the nominal yield of the inflation-linked bond we solve the following pricing equation for *y*:

$$B_t = IR_t \cdot \sum_{n=1}^{N} \frac{c \cdot 100 \cdot (1+\pi)^n}{(1+y)^n} + IR_t \cdot \frac{100 \cdot (1+\pi)^N}{(1+y)^N} \qquad (2)$$

On its inflation analytics page, Bloomberg uses the most recent realisation of annual CPI growth as the inflation assumption (index ratio for the current settlement date divided by the index ratio at the settlement date for one year in the past[9]). This is also the methodology currently implemented in POINT.[10]

Intuitively, the nominal yield differs from the corresponding real yield by some measure of expected[11] inflation across the life of the bond. This brings us to the concept of *Breakeven Inflation*.

2.2.3 Breakeven Inflation (BEI)

Just as yields are defined to be the single discount rate that, if applied to all cash flows, correctly prices the bond, we can define the inflation *breakeven* yield as the single inflation assumption which, if applied to inflate all cash flows, correctly prices the inflation-linked bond. To define breakeven inflation we first need to identify a benchmark nominal bond with respect to which the breakeven inflation is measured. The term "breakeven" inflation is indeed derived from the fact that if inflation over the life of the bond is constant and equal to the breakeven rate, then the yield to maturity of the real bond exactly equals that of the nominal benchmark bond. Hence the investor "breaks even". We denote by y^{bench} the yield of the nominal government security benchmark and define, π^{bei} , the inflation breakeven by equation (3):

8 *A second methodology is to generate a forward inflation curve, derived from the prices of inflation-linked securities in the market. Then, using the resultant curve, the appropriate breakeven rate,* π_{t,t_n} *, is applied for the period from the pricing date to the date of the nth cash flow. This method gives a set of projected cash flows consistent with current market expectations of inflation. This security, consisting of the projected cash flows, can be correctly priced with one yield. Formally, we define the n-th cash flow as follows:*

$$cf_n = \begin{cases} c(1+\pi_{t,t_n})^n & n < N \\ (1+c)(1+\pi_{t,t_N})^N & n = N \end{cases}$$

This leads to the pricing equation defining the nominal yield, y, thus: $B_t = IR_t \cdot \sum_{n=1}^{N} \frac{100 \cdot cf_n}{(1+y)^n}$

9 *That is, take the index ratio that applies to the settlement date for a trade done exactly one year in the past. In fact this will in general need two historical values of CPI and an interpolation between them. In so far as this differs from the breakeven inflation, it results in a nominal yield that differs from that of the corresponding nominal Treasury bond.*

10 *The inflation assumption can be viewed in POINT in the index/portfolio contents view in the inflation field entitled 'Assumed Inflation'.*

11 *Strictly speaking, this is an expectation under the risk-neutral measure.*

$$B_t = IR_t \cdot \sum_{n=1}^{N} \frac{c \cdot 100 \cdot \left(1+\pi^{\text{bei}}\right)^n}{\left(1+y^{\text{bench}}\right)^n} + IR_t \cdot \frac{100 \cdot \left(1+\pi^{\text{bei}}\right)^N}{\left(1+y^{\text{bench}}\right)^N} \qquad (3)$$

With this definition, π^{bei} represents a (risk-adjusted) expectation of average inflation over the life of the bond, and the nominal yield of the bond is equal to the yield of the equivalent nominal benchmark. It is straightforward to see that the real yield, *r*, the equivalent nominal benchmark yield[12], y^{bench}, and the corresponding breakeven inflation, π^{bei}, are related via the equation:

$$(1+r)(1+\pi^{\text{bei}}) = 1 + y^{\text{bench}}$$

While this is a consistent definition of breakeven inflation, it is not the only convention used when quoting breakevens. For example, the simplest and most common approach is to use the difference between the yield of the benchmark Treasury bond and the real yield of the inflation-linked security which, by the above formula, will in general be a good approximation.

2.3. Durations and measures of yield curve exposure

Inflation-linked bonds have a more complex relationship with the nominal yield curve than nominal government bonds or, indeed, corporates. This arises from the fact that breakeven and realised inflation are often highly correlated with movements in the underlying nominal curve. Therefore, a full picture of the reaction of the security's price to a shift or reshaping of the nominal yield curve necessarily requires a consideration of where inflation is likely to go given nominal rate movements. Despite this, some simple duration measures remain in use. We survey the most common measure of sensitivities used in the market.

2.3.1. Real modified duration

Just as we defined the real yield of an inflation-linked bond, so we may define the real modified duration as the sensitivity of price to a change in that yield. We simply reprice the bond in real terms according to equation (1), shifting the current yield and calculating the price sensitivity.

Formally, we define the real duration as (minus) the proportional change in the price of the linker per unit change in the real yield.

$$D_t^{real} = -\frac{1}{B_t}\left(\frac{\partial B_t}{\partial r}\right)$$

12 In practice, where there is no government benchmark bond of the same maturity as the inflation-linked bond interpolation is used between yields of adjacent government par bonds.

2.3.2. Nominal duration (OAD)

This measures the sensitivity of the inflation-linked bond with respect to a (small) parallel shift in the level of the nominal par curve. It is calculated by making a single inflation assumption[13], as in the definition of nominal yield, and then computing sensitivity to the shift of the nominal curve on present value of the newly projected cash flows. In that the inflation growth is fixed, it is a measure of the sensitivity of the security's price to movements of the nominal yield curve assuming no accompanying movement in average inflation rates across the life of the bond.[14]

2.3.3. Empirical duration

Empirical duration seeks to estimate the empirical sensitivity of an inflation-linked security to nominal yields with reference to their effect on real yields. We have noted above in 2.3.1 that, given a movement in the real yield of a linker, we can approximate the return implications via the Real Modified Duration. We couple this with an estimate of how that real yield reacts to a movement in nominal yields, for example by regressing changes in the observed real yield of the inflation-linked bond, Δy_t^{real}, against changes in the yield of the nominal benchmark, $\Delta y_t^{\text{nominal}}$. We can then translate a movement in the latter into a return for the inflation-linked security via the real duration.

Formally, we estimate the sensitivity, β, of real yields to nominal by regression, choosing β in the equation below, to minimise the sum of the squares of the error terms over the period considered:[15]

$$\Delta y_t^{\text{real}} = \beta \Delta y_t^{\text{nominal}} + \varepsilon_t \qquad (4)$$

This measure is principally designed to provide a hedge ratio for a position in a linker with respect to the benchmark nominal security.[16]

By way of comparison, Figure 3 shows the real, nominal and empirical durations of the OATi 2029 for the period May-04 to Sep-05.

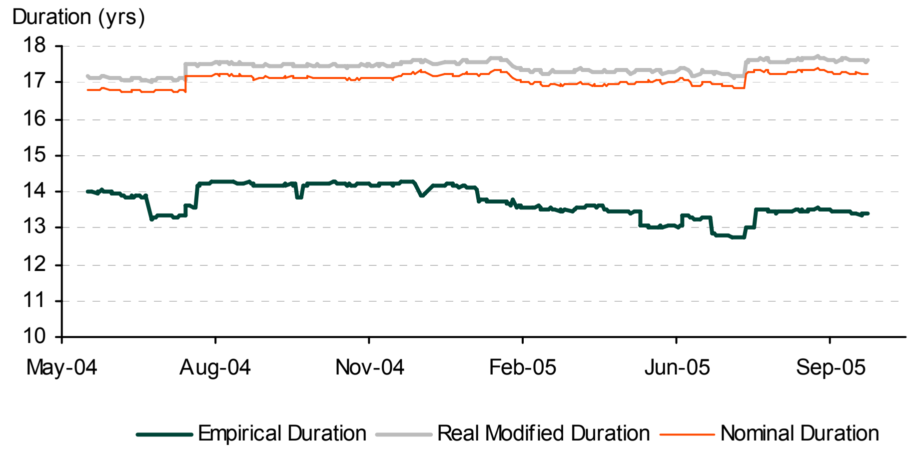
13 See section 4.5.2 for details on the inflation assumption in POINT for this calculation.

14 This assumption of zero correlation of inflation and nominal yields, whilst artificial and at odds with experience, is of value in two particular contexts: First, by way of comparison with spread markets where spreads, whilst known to have non-zero correlation with yields, are kept constant in the calculation of durations. Second, such a measure allows the user to define their own scenarios for nominal and real rates even if those scenarios are not in keeping with the historically observed relationship between the two sources of risk. The user is free, in other words, to choose her own betas for real versus nominal yields in a 'what-if' analysis.

15 Technically, we should guard against the term dependence of the correlation and note that the maturities of the linker change across time. To be precise, we should compute time series of constant maturity real and nominal yields at the current maturity of the bond in question and use the correlation at this point on the curve. Given, though, the short time window over which the correlation is estimated and that the interpolation will be required to construct constant maturity yields at the particular maturity of the bond we are seeking to model, this approximation seems reasonable. Also note, we consciously omit the estimation of the constant term so as to produce an empirical sensitivity that is independent of the magnitude of the move in nominal rates.

16 Denote by D^{real} the Real Modified Duration of the inflation-linked security and by D^{nominal} the nominal modified duration of the treasury benchmark. Then \$1 Market Value of the real bond could be approximately hedged with respect to the nominal yield curve by a short position of $\beta \cdot \left(D^{\text{real}} / D^{\text{nominal}} \right)$ in the nominal security.

Figure 3. Nominal and Empirical Correlations For OATi 2029



Source: LehmanLive.

As would be expected, the empirical duration lies below the other non-correlation based measures.

In section 4 we demonstrate another measure of empirical duration that is based on the historical monthly correlations of inflation and nominal risk factors.

3. MODELLING INFLATION RISK AND RETURN

3.1. Consistency with existing frameworks

The risk and performance attribution models for inflation-linked bonds is a part of our Global Risk and Hybrid Performance Attribution Models. It is therefore necessary that the analytical framework used for inflation-linked bonds is consistent with that used for other asset classes. In our models, we measure risk and return in nominal terms. As a result, inflation-linked securities whose cash flows depend on future inflation realizations are exposed to inflation risk in addition to their exposure to nominal risk sources.[17] We, therefore, seek to isolate inflation factors that reflect the exposure of linkers to inflation-related sources of nominal return. From the point of view of the Global Risk Model, we need to identify those market-wide drivers of risk and return that are associated with a given inflation index. These should also be specified so as to complement the existing systematic factors.

To identify these risk factors we need to perform an attribution of the monthly returns of inflation-linked securities as we do for all other securities in the Risk Model universe. This enables us to isolate a "residual" return; a component of return that is above and beyond that due to (nominal) yield curve movements, volatility changes and time-related effects such as rolldown and accretion. It is this term that represents the return arising from the inflation-linked nature of the security and which we need to model.[18]

17 *One could argue that it is precisely inflation-linked products, protected as they are against the effects of the increase in prices, that are immune to inflation effects and that inflation risk is present only in nominal securities. This implies that the riskless asset is cash growing with inflation. Common convention, however, dictates that we consider unadjusted cash as the riskless asset. Under this convention, it is inflation-linked securities that are exposed to inflation risk.*

18 *In so far as the security has other components of return, such as corporate spreads for corporate inflation-linked securities, or country spreads for European sovereign issuers (e.g. Greece or Italy which trade at a spread to the French-German Euro Treasury curve that is the risk free EUR curve in POINT as a whole), these effects will be stripped out before calibration of the inflation risk factors. In practice, due to lack of liquidity, non-treasury linkers do not take part in the estimation of inflation risk factors.*

To perform this attribution, however, we must first define a pricing equation that forms a basis for measuring the impact of the passage of time, changes in the yield curve and potential volatility effects. Such an equation would also be needed to compute analytical sensitivities to allow us to extract risk factors from the residual return described above.

As a first step towards achieving a consistency between our modelling of inflation-linked bonds and that of other asset classes, we establish below a parallel between the approaches we use for inflation-linked bonds and for corporate bullets.

3.2. The parallel between inflation linked bonds and corporate bullets

The difficulty in modelling an inflation-linked security is the uncertainty regarding its cash flows. This uncertainty is a different one from that encountered in the credit asset class, for example. In this asset class the uncertainty concerns whether the coupon will be paid at all. For inflation-linked securities issued by national treasuries, the uncertainty faced is with regard to the size of the payment, which depends on the announced level of a particular index at some point in the future. Nonetheless, despite these differences, the analytical approach taken in handling credit securities can be adapted to our purposes.

For a security with credit risk, we note that uncertainty regarding the cash flows is reflected in the market price of the bond. This correspondingly defines an option-adjusted spread, oas_t over the relevant discounting curve, described by the zero rates $\{i_{t,t_i}\}_{i\geq 1}$, as in (5):

$$B_t = \sum_{i=1}^{N} \frac{100.c}{(1+i_{t,t_i}+oas_t)^{t_i-t}} + \frac{100}{(1+i_{t,t_N}+oas_t)^{t_N-t}} \qquad (5)$$

or

$$B_t = B(c, YC_t, T-t, oas_t)$$

Effectively, we assume that the cash flows, as given by the cash flow schedule at the time of evaluation, are assured and we solve for the fixed spread, the OAS, over the discount curve that correctly prices those cash flows.

For inflation-linked securities we can adopt a similar approach that allows us to look at these bonds as simple variations on bullet bonds with an appropriate spread. The idea here is that whereas for a credit security the risk of default is *discounting* the expected value of a given cash flow leading to a positive spread, the effect of index linkage is to *increase* the expected value (from that of the real coupon) resulting in a *negative* spread. We develop the relevant mathematics in the following section.

3.3. The pricing equation

To aid intuition, we initially present a simplified analysis, where we assume that the inflation index is observed continuously and without lag. Relaxing this assumption, we develop a full pricing equation in the subsequent section.

3.3.1. A simplified analysis

We recall from section 3.1 that the coupon c_{t_i} at time t_i is given by:

$$c_{t_i} = c \cdot IR_{t_i}$$

Where c is the real coupon given at the issue of the bond and IR_t is the index ratio which applies to a cash flow at time t, as set out in the terms of the bond issue.

Our next step is to distinguish between that part of the Index Ratio, IR_{t_i}, that applies to time t_i at some time in the future, that is known at evaluation time *t* and that part that is unknown and represents the future effects of inflation. So, for a coupon, c_{t_i}, at a time t_i, we can re-write the index ratio for that cash flow, IR_{t_i} as follows:

$$IR_{t_i} = \left(IR_{t_i} \big/ IR_t \right) \cdot IR_t = (1+\pi_{t,t_i})^{t_i - t} \cdot IR_t \qquad (6)$$

The index ratio, IR_t, is known at time *t* and the quantity $\left(IR_{t_i} \big/ IR_t \right)$ represents the effect of inflation between time *t* and time t_i This last term has been rewritten in the form $(1+\pi_{t,t_i})^{t_i - t}$ where we define a variable π_{t,t_i} to account for the growth in the index ratio between time t and t_i.

This gives the following expression for the cash flow, c_{t_i}, at time t_i

$$c_{t_i} = c \cdot IR_t \cdot \left(1+\pi_{t,t_i}\right)^{t_i - t}$$

Standing at time t, and employing the above notation, we can write that the price of the inflation-linked bond at time *t* is the present value of this set of cash flows:

$$B_t = IR_t \cdot PV_t\left(\left\{c.100.(1+\pi_{t,t_i})^{t_i - t}\right\}_{i=1\ldots N-1}, (1+c).100.(1+\pi_{t,t_N})^{t_N - t}\right)$$

Our next step is to solve for a *single* inflation rate that prices the bond. In other words, we seek π_t that satisfies:

$$B_t = IR_t \cdot \sum_{i=1}^{N} \frac{c.100.(1+\pi_t)^{t_i - t}}{(1+i_{t,t_i})^{t_i - t}} + IR_t \cdot \frac{100.(1+\pi_t)^{t_N - t}}{(1+i_{t,t_N})^{t_N - t}} \qquad (7)$$

This should be compared with equation (2) for the nominal yield above. There we derive a single yield to discount cash flows, calculated with reference to a single inflation assumption; here we work with the observed nominal discount curve, deriving instead an inflation rate across the life of the bond. Thus we can write:

$$B_t = IR_t M(YC_t, \pi_t, T-t)$$

where M is the price, at time *t*, of a "model bond", whose cash flows are coupons of size *c* (the real coupon) and principal repayment at maturity $T = t_N$, and where YC_t is the nominal yield curve at *t*. In particular π_t represents a measure of break-even inflation expectations across the life of the bond.[19]

19 Strictly speaking π_t will be a risk-neutral spread and will include risk premium, liquidity risk and other effects above and beyond inflation expectations.

Now we are ready to establish the parallel with a credit security.[20] The two formulations are equivalent when we write $oas_t \approx -\pi_t$. In the limit, as we tend toward continuous compounding, the equivalence is exact.[21] Thus, our information is equivalent to a credit bond with spread *under* the nominal curve of magnitude π_t, the "breakeven inflation" derived in (7).[22]

Under this formulation, the sources of risk of such a security would therefore be movements in nominal rates, changes in the realised rate of inflation, IR_t and movements in this π_t or breakeven inflation.

This is the essence of the approach we adopt. We associate with the inflation-linked security a spread and proceed to treat it in much the same way as any other spread product. Nonetheless, this is not the final word. The idiosyncrasies of the inflation-linked market necessitate a more complex variation of the above formulation. We outline these in the next section.

3.4. The full pricing equation

There are two further issues that we must address and that result in a pricing equation that is a refinement of equation (7). We outline each in turn along with the approach that we have taken in resolving them. We then present the final pricing equation. Details of the method are shown in Appendix B

3.4.1. Lag effects

In section 3.3, we have assumed that the price index is observed continuously and without lag. Under this assumption, the index ratio applicable to cash flows arriving at t becomes known at t. In fact, this is not the case. Price indices are announced with a lag and at monthly intervals. As a result, the index ratio applicable to cash flows arriving at t is known before t.

Consider an example of a French Local Inflation-linked bond.[23] Suppose we wish to know the cash flow of an inflation-linked bond that is scheduled to occur on July 25, 2005. By the terms of the bond, this is determined by the level of the French Local Price index (FRCPXTOB) for April and May 2004. The latter is published at some time mid June 2005. This means that already in mid-June 2005 we know the coupon that we will receive some weeks later on July 25, 2005.

This "foreknowledge" of the index ratio has three implications for our pricing equation:

(a) At any given moment in time, the future cash flows of an inflation-linked security can be broken down into those that are known at the time of evaluation, CF_t^{known}, and those,

20 *To express equation 7 in the form of equation 5, i.e. in classical credit spread terms, we need merely solve equations of the form:*

$$\left(\frac{1+\pi_t}{1+i_t}\right)^n \equiv \frac{1}{\left(1+i_t+oas_t\right)^n} \qquad \Leftrightarrow \qquad \frac{1+\pi_t}{1+i_t} = \frac{1}{1+i_t+oas_t}$$

$$\Leftrightarrow \qquad oas_t = -\pi_t\left(1+\frac{i_t-\pi_t}{1+\pi_t}\right)$$

21 *That is, we write* $\exp\left\{-\left(t_n-t\right)\cdot\left(i_{t,t_n}-\pi_t\right)\right\}$ *for the inflation-linked securities 'discount factor' and* $\exp\left\{-\left(t_n-t\right)\cdot\left(i_{t,t_n}+oas_t\right)\right\}$ *for that of the credit bond. These are the same for all n iff* $oas_t = -\pi_t$

22 *In section 2.2.3 we defined breakeven inflation with respect to the yield of the nominal benchmark security. Here we are defining it with respect to the entire nominal yield curve. This is more precise. Henceforth, we use the term break-even inflation in this sense.*

23 *The same will be true for a US TIPS or an HICP linked bond since they have the same conventions – see Appendix A.*

CF_t^{unknown}, that are not. This means that we can write more comprehensively that the (inflated) price of an inflation-linked security is given by:

$$\begin{aligned} B_t &= PV\,(CF_t^{\text{known}}) + PV\,(CF_t^{\text{unknown}}) \\ &= PV\,(CF_t^{\text{known}}) + IR_t M_t (YC_t, \pi_t, T - t) \end{aligned} \qquad (8)$$

The first term represents the present value of a vanilla bullet security and the second is our familiar spread formulation of the security with inflation linkage. Note that this decomposition into known and unknown cash flows means that we allow inflation risk to apply only to those cash flows that are truly exposed to it and incorporate the latest information in the market regarding realised inflation.

(b) In equation (6), we should use the index ratio computed using the most recently announced price index level This ensures that we are modelling as stochastic only that part of inflation thus far unknown to the market.

(c) A consequence of (b) is that we must change the exponent in the spread terms $(1+\pi_t)^{t_i - t}$ to something akin to $(1+\pi_t)^{t_i - t - 1}$ (or $(1+\pi_t)^{t_i - t - 6}$ in the UK market where the lag is 8 months not 3 as for HICP above).

3.4.2. Smoothing the return

Because inflation-linked securities are linked to the level of a discretely published index – CPI or RPI – traditional methods of performance attribution lead to discontinuous and unintuitive results. This is because in our pricing equation, inflation effects take two different forms. One representation of inflation is through π_t, the spread. This term results in a spread carry as time passes as would be the case for a conventional credit security. The second way in which inflation effects are reflected in the pricing equation is via the index ratio term. As we have described in the previous section, this uses the *latest* index ratio corresponding to the most recent CPI announcement. This term is, therefore, static throughout the month until the day of publication of the next CPI. On that day, it jumps with a daily return equal to the entire growth of the index for that month.

For example, consider an inflation-linked security with a spread of -365bp as described by equation (7) with π_t=365bp.[24] All other things remaining equal, spread accretion takes place at the rate of approximately -1bp per day.[25] This large negative spread carry is typically countered by an equally large positive jump in the index ratio on the next CPI announcement day.[26] The result is a series of daily attributions with a large negative excess return, with the exception of a large discontinuity on one particular day which is misleading and undesirable.

To deal with this problem, we use a projection for the next index level, replacing the latest index ratio by a moving update. This allows us to smooth the monthly growth of the index ratio. We define a date, t_{proj}, up to which all cash flows are assumed to be known and form the corresponding index ratio $IR_{t,t_{proj}}$. This moves forward each day until the new CPI is published, at which point the process of projection and updating begins again. We leave the details of this procedure for Appendix B, but the underlying idea is straightforward: we predict a month into the future and advance the latest index ratio daily.

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24 So that 365bp represents a measure of inflation expectations across the life of the bond.

25 This is calculated by dividing the spread by the number of days used for accretion which we have assumed for illustration to be 365,

26 This discontinuity on the day of CPI announcement, subject to seasonal effects and shocks to underlying inflation, will largely offset the spread accretion term, in that it is the monthly return on the inflation index.

3.4.3. The full equation

With the modifications described in sections 3.4.1 and 3.4.2 in mind, we state the new pricing equation. If we now consider the index ratio as known up to the point t_{proj}, we can rewrite the pricing equation (7) as follows (we now switch to continuous notation for the discounting and spread effects):

$$B_t = \sum_i^{t_i \le t_{proj}} IR_{t,t_i} cf_i \cdot df_{t,t_i} + \sum_i^{t_i > t_{proj}} IR_{t,t_{proj}} cf_i \cdot df_{t,t_i} e^{-s_t \cdot (t_i - t_{proj})} \qquad (9)$$

where $cf_i = \begin{cases} 100 \cdot c & if \quad t_i < T \\ 100 \cdot (1+c) & if \quad t_i = T \end{cases}$

Here we explicitly include the known (and projected) cash flows under the first summation sign on the right hand side and write df_{t,t_i} for the discount factor between times t and t_i.

Further, we use s_t to denote the inflation spread which is equal to the negative of the breakeven inflation for the bond.

4. INFLATION RISK FACTORS IN THE GLOBAL RISK MODEL

Having arrived at a modelling framework, we now describe the application of our analysis in the risk modelling arena and thereafter to performance attribution.

4.1. Return splitting

Having cast the inflation-linked security into a familiar form, namely that of a bullet credit bond with the associated spread, we can apply the standard risk model machinery to identify risk factors specific to the inflation market. In this way we seek to model the sources of return of this asset class over and above those that they have in common with nominal securities.

4.1.1. Risk modelling for traditional securities

The calibration of the risk model factors begins, in all cases, with the process of *return splitting*. Here we decompose the total return of each security over the preceding month into various components:

$$\mathrm{Ret}_t^{\mathrm{Total}} = \mathrm{Ret}_t^{\mathrm{FX}} + \mathrm{Ret}_t^{\mathrm{Time}} + \mathrm{Ret}_t^{\mathrm{YC}} + \mathrm{Ret}_t^{\mathrm{Vol}} + \mathrm{Ret}_t^{\mathrm{Res}}$$

Having stripped out the return due to currency movements, the passage of time, the movements and reshaping of the yield curve, and volatility effects[27], we are left with the residual return, $\mathrm{Ret}_t^{\mathrm{Res}}$.

27 This term comes in to play for securities with embedded optionality such as callable bonds.

In the context of a credit security, this component of return would be associated with the movements in spread of an issue.[28] This spread return can be modelled in terms of systematic risk factors and an idiosyncratic residual. Once we have decided the structure of these market-wide sources of risk, we can use our monthly data to put numbers to these risk factors month by month, thereby arriving at time series of factors. We regress the spread return of all the bonds in the relevant calibration universe against both indicative and continuous variables to obtain monthly time series of systematic factors.[29]

4.1.2. Adaptation to Inflation-linked Securities

In the case of inflation-linked securities, residual return relates to inflation factors, both long-term expectations and short-term effects. We see from (9) that, in addition to those sources of return for a nominal security, we have to consider the effect of the realised index ratio. In addition, the interpretation of spread return is now in terms of movements in breakeven inflation rates. Our return splitting equation takes on the following modified form:

$$\mathrm{Ret}_t^{\mathrm{Total}} = \mathrm{Ret}_t^{\mathrm{FX}} + \mathrm{Ret}_t^{\mathrm{Time}} + \mathrm{Ret}_t^{\mathrm{YC}} + \mathrm{Ret}_t^{\mathrm{Vol}} + \mathrm{Ret}_t^{\mathrm{IR}} + \mathrm{Ret}_t^{\mathrm{BEI}}$$

where now $\mathrm{Ret}_t^{\mathrm{IR}}$ is the monthly return due to the change in the $IR_{t,t_{proj}}$ term in the second summation in equation (9) and $\mathrm{Ret}_t^{\mathrm{BEI}}$ is that return due to changes in breakeven inflation, the “spread” of the inflation-linked security over the reference curve.[30]

We see, therefore, that there are two new dimensions of risk for an inflation-linked security over and above those that apply to nominal bonds: changes in realised index ratios and changes in long-term inflation expectations. We consider each category of factor in turn.

4.1.3. The index ratio risk factor

With respect to the first of these risk factors, we note from (9) that the exposure of a security to this factor is simply the fraction of its market value that is represented by the unknown cash flows. The risk factor is, then, the returns of the index ratio. We note that we use the latest projected index ratio available at the time of calibration and not the official index ratio used for quotation of the security price in the market place.

The interpretation of this risk factor should be as a measure of the short-term movements of the inflation index, reflecting the seasonality that plays such a significant part in CPI behaviour.[31] It seeks to capture the difference between the expectations of inflation in the near term and those being released on a monthly basis.[32] Insofar as the CPI growth from one month to the next is not generally in line with that implied by the breakeven inflation rate for the security, a distinct risk factor is necessary for a full description of the risk profile of an inflation-linked security.

28 *For credit securities we further strip out the return due to movements of Swap spreads.*
29 *This process is described fully in out publication ‘publication ‘The Lehman Brothers Global Risk Model: A Portfolio Manager’s Guide’ (D.Joneja/L.Dynkin April 2005).*
30 *We ignore the second order cross terms between* $\mathrm{Ret}_t^{\mathrm{IR}}$ *and other sources of return for tractability in the same way we already do for FX return in general.*
31 *For an assessment of the significance of seasonality in inflation indices see ‘Understanding Seasonality in Price Indices’, Brondolo and Giani July 2005.*
32 *To be sure, such a difference may be due to a number of factors besides seasonality such as the slope of the breakeven curve, liquidity and inflation risk premia.*

4.2. Breakeven factors

Having performed the return split detailed above and isolated the breakeven return term, $\mathrm{Ret}_t^{\mathrm{BEI}}$, we proceed to model the implied change in breakeven inflation for the bond in the following way:

Using a linear approximation for spread return (in excess of carry) we write:

$$\mathrm{Ret}_t^{\mathrm{BEI}} \approx -OASD_t * \Delta s_t$$
$$\Rightarrow \qquad \mathrm{Ret}_t^{\mathrm{BEI}} \big/ OASD_t \approx -\Delta s_t = \Delta \pi_t$$

This means that our risk factors take the form of changes in breakeven inflation.

So, for each security, we can isolate a measure of the change in its breakeven inflation across the month. Having done this, we need to identify a parsimonious set of risk factors that capture, as far as possible, the movements of these expected inflation rates for all inflation-linked securities in the various markets.

The description of yield curve and swap spread risk factors currently employed in the model is one of modelling certain key tenors along the respective curves. Intuitively, it makes sense to consider risk factors capturing the movements at different points along the breakeven curve in the same way. In this way, we would be approximating the changes in level and shape of the breakeven inflation curve in a piecewise linear fashion. A one-factor model would correspond to a model that captured only changes in the overall level of the curve, the shift; two factors to one that reflected the shift and some measure of the change in slope; three to shift, slope and curvature effects, and so on. In this way, we avoid the difficult task of fitting an entire inflation curve across all maturities. This choice of factors must not only accurately describe the behaviour of inflation, but must also have an eye to the future. The factors must remain relevant going forward in that sufficient issuance should be expected at the chosen tenors. In addition, they must be chosen in such a way as to be mutually compatible, allowing comparison and integration of inflation risk across markets.

From a practical point of view, the decision of which tenors to use must also address several additional considerations. Firstly, we need to have sufficient issuance across the maturity spectrum to justify the factors. For example, in the Japanese government inflation-linked market, at the time of writing, all four of the distinct outstanding bonds have maturities in the 8-10 year range, making any more than one breakeven factor both unnecessary and unsuitable. Secondly, we seek a parsimonious model, keeping the number of risk factors to a minimum given a certain lower bound on explanatory power of the model. Finally, we want to make the model consistent across markets, choosing the same points on the curve where data permit.

In the search for the optimal factor specification, analysis was carried out to gauge the relative effectiveness of using different number of factors to capture more and more of the inflation curve behaviour. Figure 4 shows the effect on explanatory power, in the sense of monthly cross-sectional (weighted) R^2 for the Sterling index-linked Gilts market[33] using one, two and three terms to model the behaviour of breakeven inflation.[34] Each chart shows the increase in R^2 together with the level of the R^2 for the two-factor model.[35]

33 This experiment focused on the Sterling market as it was felt most likely to add explanatory power here given the downward pressures on the end of the Sterling curve peculiar to this market.

34 NOTE: The 3-factor model uses tenors of 5, 15 and 30 years as compared with 5 and 20 years for the 2-factor model. As such, since the 3 factors are not a super-set of those in the 2- factor model, risk factors will not necessarily give a higher R-squared, as is the case in a few months.

35 Note that because the R2is both adjusted and weighted negative values are possible.

Figure 4 demonstrates that there is little evidence that the introduction of a third factor is justified. With the exception of a handful of months, it adds relatively little in terms of explanatory power. Changes in the slope and level of the inflation curve appear to be capturing most of the behaviour of the curve as a whole.

With this is mind, and for markets with a good number and spread of issues, we specify a two-factor model with a short and long factor as described in the equation below:

$$\Delta\pi_{i,t} = \alpha_{i,t} f_t^{short} + (1-\alpha_{i,t}) f_t^{long} + \varepsilon_{i,t}$$

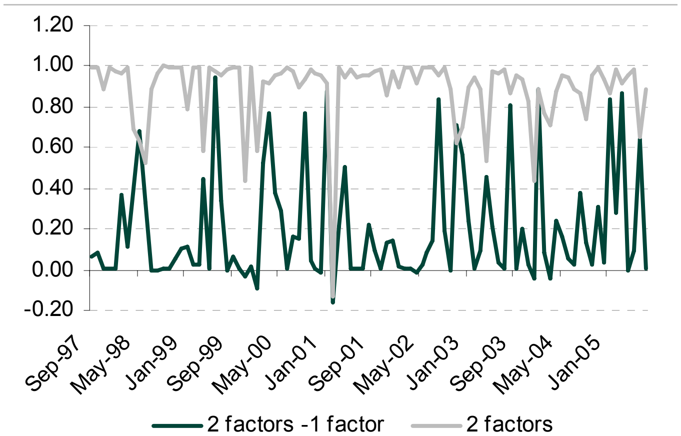
Here, $\pi_{i,t}$ denotes the break-even inflation of bond i at time t and $\alpha_{i,t}$ is a bond-specific interpolation factor determining the degree to which bond *i* at time *t* loads on the short and long factors.

Here, the change in the breakeven inflation of security *i* from time *t-1* to time *t* is modelled as an interpolated value between a short and a long breakeven factor. Once we have decided on the tenors of these two factors, we can then apply standard regression techniques (as we do across the entire Risk Model universe of asset classes). We use our cross-sectional data each month to estimate the time series of factor values that best describe the observed movements in breakeven inflation across all securities in that market during the course of the previous month.

The resultant time series allow us to measure the historical relationships between these new sources of risk in our new asset class and those already present in our model and so incorporate them into the Risk Model covariance matrix.

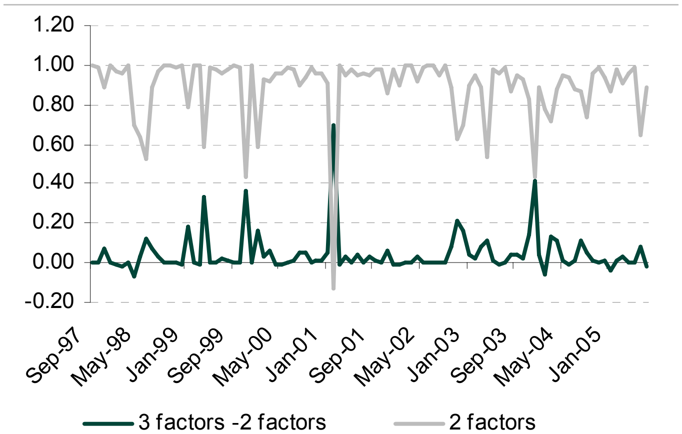
Figure 4. Increase in explanatory power by adding breakeven risk factors (UK market)

Figure 4a. Comparing one- and two-factor specifications for the breakeven spreads



Source: Lehman Brothers.

Figure 4b. Comparing Two- and Three-Factor Specifications for the Break-even Spreads

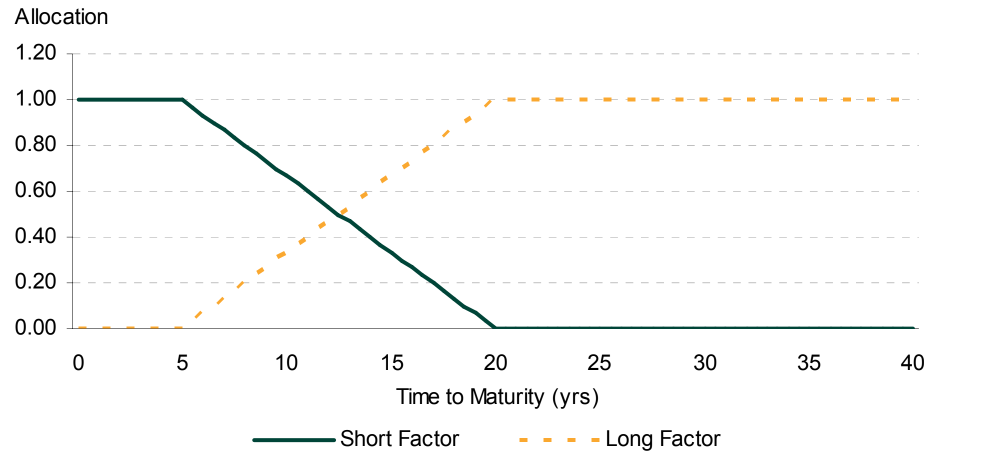


Source: Lehman Brothers.

Having considered the current structure of the markets and indicators as to the likely future points of issuance of the major inflation issuers across the globe, we have opted for 5-year and 20-year factors to capture the short and long end of the inflation curve respectively. In markets where the use of two factors is felt to be excessive, one parallel shift factor is employed.[36] Thus, the loading of bonds on these two factors is determined by their time to maturity in accordance with Figure 5 below:

36 This is the case for Canadian, Japanese and French local inflation markets.

Figure 5. Loadings on breakeven inflation factors as a function of time to maturity



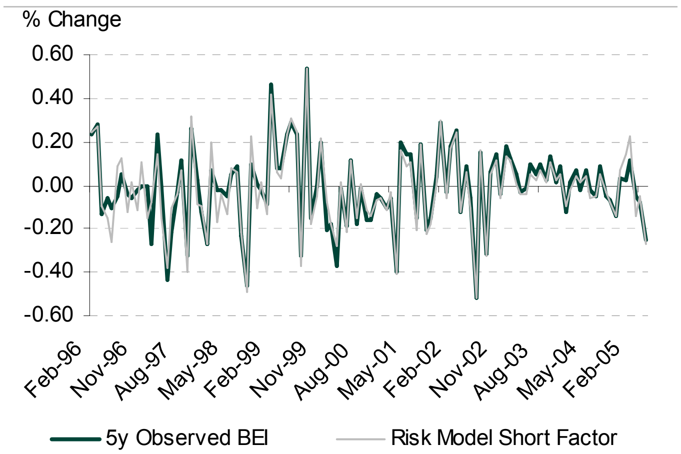
Source: Lehman Brothers.

So, an inflation-linked security of maturity 5-years or less loads only on the short inflation factor, and similarly, one of maturity 20-years or above only on the long inflation factor. Any bond of intermediate tenor loads in a natural fashion on both of the long and short factors.

Figures 6a and 6b show a comparison of the resultant long and short risk model breakeven factors with the observed changes in corresponding fitted constant maturity breakevens in the sterling market (these are interpolated from observed breakeven data).

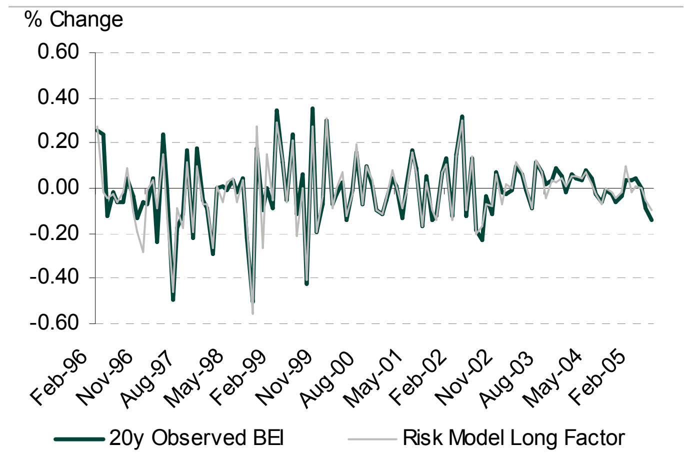
Figure 6. Breakeven inflation risk factors vs. observed (constant maturity) breakevens (UK market)

Figure 6a. Short (5y) factor



Source: Lehman Brothers.

Figure 6b. Long (20y) factor



Source: Lehman Brothers.

The above charts demonstrate that the factors that are produced by our model are very closely related to observed breakevens, with correlations with the constant maturity breakevens in excess of 90% and with similar volatilities.

Figure 7 presents some key correlations and volatilities of the inflation factors in various markets, together with correlations with some nominal factors of particular interest.

Figure 7 Key correlations and volatilities of inflation risk factors in the Global Risk Model[37]

	Volatility (Bp/M)	Correlation				
		Realised Inflation	Expected Inflation		Nominal Par Rate[38]	
			5y	20y	5y	30y
EUR (HICP)						
Realised Inflation	33.2	1.00	-0.26	-0.25	-0.14	-0.25
Expected Inflation						
Short (5y)	11.8	-0.26	1.00	0.45	0.50	0.50
Long (20y)	10.7	-0.25	0.45	1.00	0.19	0.26
UK						
Realised Inflation	30.6	1.00	-0.30	-0.26	-0.08	-0.05
Expected Inflation						
Short (5y)	14.6	-0.30	1.00	0.61	0.47	0.38
Long (20y)	8.7	-0.26	0.61	1.00	0.37	0.46
USD						
Realised Inflation	32.4	1.00	-0.32	-0.30	-0.06	-0.10
Expected Inflation						
Short (5y)	18.0	-0.32	1.00	0.69	0.52	0.53
Long (20y)	14.3	-0.30	0.69	1.00	0.43	0.54

Source: Lehman Brothers.

As we would have expected, correlations of inflation expectations and nominal rates are generally high, at about 50%.

4.3. Risk factor loadings: a worked example

We consider the 2015 $1\frac{5}{8}$ % US TIPS, trading with a closing (real) price on 12 September of \$99.547. If we price this bond using our methodology above, we find a breakeven inflation of 239bp. (The traditional breakeven inflation, defined as the difference between the maturity-matched par yield from the local-market fitted curve and the real yield[39], is 253bp[40]). Using this as the spread under the treasury curve and applying the standard procedure for spread duration calculation (namely moving the spread a small amount while keeping all else constant and repricing), we obtain an OASD of 8.67 years. Similarly, holding the spread constant and shifting the treasury par curve we obtain an OAD of 8.56 years, with exposures to the movement of key rate points along the curve of -0.02, -0.14, 0.70, 8.03, 0.00 and 0.00 years at the 6m, 2y, 5y, 10y, 20y and 30y points, respectively.[41]

Figure 8 shows the loadings of the bond on the relevant risk factors in the Risk Model. Note, the loading on the long-term (breakeven) inflation expectation factors is determined by the time to maturity of the security. In this case, the bond has a time to maturity of 9.34 years.

37 All volatilities and correlations are as of the end of May 2005 and are calculated using weighting with a half life of 1 year.

38 Note: Cross asset-class correlations are modelled via smaller sets of factors known as core factors.

39 Note: the fitted par yield used is a linear interpolated value between the nearest available fitted par yields (these are available at half-year intervals as explained in footnote 12).

40 All figures are taken from POINT.

41 Due to the heavy weighting of the later cash flows for a linker, in particular the principal repayment, analytically inflation-linked bonds bear some resemblance to nominal zero coupon bonds. This accounts for some negative key rate durations which are also observed in zeroes.

This implies a loading of $\frac{20-9.34}{20-5}=0.71$ on the 5-year factor and $1-0.71=0.29$ on the 20-year factor. This must then be multiplied by the spread duration, 8.67, to obtain the exposure contributions to each factor.

Figure 8. Loadings of 2015 $1\frac{5}{8}$ US TIPS on risk factors in the Global Risk Model

Risk Factor Name	Unit	Loading
Nominal Yield Curve Factors		
USD 6M key rate	Years of Duration	-0.02
USD 2Y key rate	Years of Duration	-0.14
USD 5Y key rate	Years of Duration	0.70
USD 10Y key rate	Years of Duration	8.03
USD 20Y key rate	Years of Duration	0.00
USD 30Y key rate	Years of Duration	0.00
USD Convexity	Years^2 of Convexity	0.81
Inflation Factors		
US Realised Inflation	% Market Value	100%
US Expected Inflation Short (5y)	Years of Spread Duration	0.71*8.67=6.2
US Expected Inflation Long (20y)	Years of Spread Duration	0.29*8.67=2.5

Source: Lehman Brothers.

4.4. Extending the model to other asset classes

This approach can be applied to the modelling of other asset classes and we briefly describe two.

(a) **Corporate inflation-linked securities**

The above framework naturally lends itself to the integration of credit securities whose cash flows are linked to an inflation index. Our approach is simply to consider the spread of such a security as having two distinct components: an inflation breakeven, as given by that of the nearest government benchmark, and a credit-related spread whose risk characteristics are governed, as they would be for any nominal credit security, by reference to its sector and quality together with the other spread risk factors. In terms of its exposure to risk factors, it simply has exposure to both inflation factors and nominal credit risk factors (in addition to yield curve, swap spread and volatility as per any bond).

(b) **Inflation swaps**

The inflation swap is, in theory, little more than a position in a nominal zero coupon bond with an equal and opposite position in a zero coupon inflation-linked bond.[42] In addition, there will be an exposure to the "inflation swap spread", the spread of the inflation swap curve over the inflation bond curve as for interest rate swaps. An approach that models this inflation swap spread, whose characteristics may differ from their nominal counterpart[43], and combines the risk factors already calibrated for the cash bond inflation market, provides an immediate method for the inclusion of inflation swaps into the risk and performance attribution models.

42 Here we have in mind the zero coupon inflation swap being the prevalent form of the instrument at present.
43 A thorough empirical study to assess the nature of this risk factor is warranted.

4.5. Risk model implied analytics

We are now in a position to outline two new analytical measures made possible by the framework developed in the preceding sections.

4.5.1. Risk model empirical OAD

Equipped with our new inflation risk factors, we are in a position to calculate additional measures of exposure to nominal yield curve movements. We now have use of the historical relationships between changes in key rates on the nominal curve and those of the CPI and breakeven factors, as summarised in the risk model covariance matrix. This allows us to produce empirical sensitivities to parallel shifts in the nominal curve; an OAD that incorporates the correlations of inflation and nominal yields.

One particular difference exists between the approaches we have already outlined in section 2.3.3 and this new risk model based approach. The previous methodologies are based, in practice at least, on daily regressions, whereas risk model covariances use data at monthly intervals and so monthly correlations are the result. They differ, often substantially. Noise that is prominent in daily data should be smoothed by using monthly frequency data. All other things being equal, this should, lead to higher correlations. Patterns of high or low correlations that persist for short time periods are not captured by methodologies based on monthly data, whereas if data at daily intervals are used these will be reflected in correlation estimates.

In short, it is a case of "horses for courses". Daily data-driven betas of real yields against one nominal benchmark yield suit those who seek to establish hedge ratios for the real bond using the closest nominal and whose intuition is that betas change significantly even on a very short-term basis.[44] Monthly-based correlations, across multiple points on the nominal curve with a wider set of inflation risk factors, are possibly more relevant to managers of real and hybrid real-nominal portfolios rebalancing at monthly intervals who believe in longer term underlying betas between real and nominal yields. They may wish to compare sensitivities of their real bonds with the classical OAD of nominal securities, defined as the sensitivity to a parallel shift of the entire yield curve.[45]

4.5.2. Nominal OAD

Finally, we note that our pricing equation (9) allows us to compute a nominal OAD where we choose the single inflation assumption to be that breakeven spread that is the result of pricing the security. In POINT, it is this "Model Bond OAD" that is reported as simply "OAD" for real securities.

4.6. Risk reports for portfolios with inflation-linked bonds

This section briefly samples the implementation of the model that we have described above as seen in POINT. We begin with an analysis of a specific portfolio against the Lehman Brothers Global Treasury Index (nominal). We choose a portfolio that is a composite of inflation-linked and nominal treasury indices in US dollar, sterling and euro.[46]

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44 Or more accurately, who believe that daily historical relationships are more relevant in predicting likely future behaviour than a longer term outlook.

45 It should be noted that the sensitivity of a real yield to the benchmark yield will not in general give the same results since the parallel shift corresponds to a more restrictive subset of yield curve scenarios than the more general movements of one particular point on that curve. To do this in a similar framework, changes in real yields could be simultaneously regressed against several (preferably constant maturity) points on the nominal curve such as key rates. The sum of the resulting estimated coefficients would yield the required sensitivity to changes in the level of the nominal curve.

46 We use a composite of 70% nominal Treasuries (40% US dollar, 20% euro and 10% sterling) and 30% inflation-linked treasuries (10% US TIPS, 10% euro inflation-linked and 10% inflation-linked Gilts).

We begin by looking at Figure 9 which shows the Tracking Error Report. This report summarises the estimated risk of the portfolio relative to the benchmark by high level asset class components. The first column, labelled "Isolated TEV" is a measure of risk adjusted exposure that combines exposures and covariances that relate to yield curve factors exclusively, ignoring cross terms with other components of risk. In our example, we note that the exposure to yield curve factors is approximately 58bp of return volatility per month. This compares with 23bp of risk arising from inflation-related sources. These two are the principal sources of risk in the portfolio and combine, taking cross correlations into account, to 49bp per month, as seen in the second row of column 2 ('Cumulative TEV'). It is immediately apparent therefore, that the inflation exposure serves to reduce the overall risk implied by the yield curve exposure. This effect is also observable from the fourth column, "Percentage of tracking error variance". In this column, on the line corresponding to inflation risk, we see -7.68% as the estimate for the contribution to risk arising from the inflation-linked sources of risk as seen in a portfolio context. This implies that the positive correlation between inflation expectations and nominal yields in effect reduces exposure to the yield curve. Indeed, the difference between the analytical nominal OAD of the portfolio and benchmark is calculated to be 0.99 years, whereas when the correlation between inflation factors and nominal yields is taken into account this drops to zero.[47]

Figure 9. Sample tracking error volatility report

LEHMAN BROTHERS | POINT Global Risk Model

Tracking Error 10/7/2005

Portfolio : composite_LMRQ
Benchmark : Global Treasury

Global Risk Factor	Isolated TEV (bps)	Cumulative TEV (bps)	Contribution to TEV (bps)	Percentage of tracking error variance (%)	Systematic beta
Global					
Yield Curve	57.89	57.89	53.25	107.75	1.54
Inflation	22.72	49.56	-3.8	-7.68	
Swap Spreads	0.64	49.58	0.03	0.06	0.81
Volatility	0.01	49.58	-0.0	-0.01	2.04
Investment-Grade Spreads	1.34	49.33	-0.23	-0.46	0.72
Treasury Spreads	1.34	49.33	-0.23	-0.46	0.72
Systematic risk	49.33	49.33	49.25	99.66	1.38
Idiosyncratic risk	2.88	49.42	0.17	0.34	
Credit default risk	0.0	49.42	0.0	0.0	
Total risk		**49.42**	**49.42**	**100.0**	
Portfolio volatility (bps/month)					114.7
Benchmark volatility (bps/month)					78.24

Source: POINT.

Figure 10 shows the details of the active exposures to inflation related risk factors.

The portfolio holds only inflation-linked bonds, and so it is only in the first of the two columns that we see non-zero exposures.

As explained in section 4.3 the loading on the realised inflation factor in each market will be the total market value represented by the inflation-linked securities in that market.[48] Since our portfolio is in fact 30% by market value in inflation linkers, 10% in each of euro, US dollar and sterling the four inflation expectation exposures (US, HICP, French Local and

47 However, it should be noted that the bulk of the yield curve risk remains as it is spread across several curves of different sovereigns and is only neutral to a simultaneous parallel shift of equal amount in all currencies.

48 To be precise, it will be the market value of the unknown cash flows considering the index projection as known for this purpose.

UK) sum to 30%. If we move along the first row of this section of the report we see that the volatility of this factor in the US is approximately 32bp. The fifth column labelled "TE impact of an isolated 1 std. dev. up change" brings together the exposure of 10% and the volatility of 32bp to imply an outperformance of 3.2bp for a 1 standard deviation upward move in this factor, holding all else constant. The next column presents the result of the same movement but now on a correlated basis, moving all other risk factors by their conditional expectation given the 32bp move in US realised inflation. On this basis the outperformance is expected to be rather less, at approximately 1.6bp. This is due to the positive correlation between CPI and nominal rates with respect to which we are long duration.

If we now consider the expected inflation, or breakeven, factors we note that the overall exposure of an inflation-linked bond to this factor will be its option-adjusted spread duration (OASD) multiplied by the interpolation factor that determined how much it loads on the short factor and how much on the long. For US short expected inflation for example, on row 2 of the report, we see that the total exposure to this factor is 0.337 years of spread duration. The volatility of the factor is estimated at 18.5bp/m implying an outperformance of 6.2bp for an upward move of 1 standard deviation. On a correlated basis, however, reflecting the likely upward move in nominal yields that would accompany an increase in inflation expectations, this changes to an underperformance of about 12.6bp. For further explanation of the report see our publication *The Lehman Brothers Global Risk Model: A Portfolio Manager's Guide* (D.Joneja/L.Dynkin April 2005).

Figure 10. Inflation risk factors in a sample factor exposure report

Factor name	Sensitivity/Exposure	Portfolio exposure	Benchmark exposure	Net exposure	Factor volatility	TE impact of an isolated 1 std. dev. up change (bps)	TE impact of a correlated 1 std. dev. up change (bps)	Marginal contribution to TEV (bps)	Percentage of tracking error variance (%)	Contribution to TEV (bps)
INFLATION										
US Realized Inflation	MW%	10.0	0.0	10.0	0.32	3.2	1.63	0.011	0.21	0.11
US Expected Inflation Short	OASD (Yr)	0.337	0.0	0.337	18.46	6.22	-12.62	-4.715	-3.21	-1.59
US Expected Inflation Long	OASD (Yr)	0.497	0.0	0.497	14.54	7.22	-10.61	-3.12	-3.13	-1.55
EuroZone Realized Inflation	MW%	6.49	0.0	6.49	0.34	2.21	8.21	0.057	0.74	0.37
EuroZone Expected Inflation Short	OASD (Yr)	0.242	0.0	0.242	11.96	2.89	-7.05	-1.707	-0.83	-0.41
EuroZone Expected Inflation Long	OASD (Yr)	0.389	0.0	0.389	10.6	4.12	5.85	1.256	0.99	0.49
French Realized Inflation	MW%	3.51	0.0	3.51	0.29	1.03	7.4	0.044	0.31	0.15
French Expected Inflation	OASD (Yr)	0.275	0.0	0.275	9.44	2.6	0.17	0.033	0.02	0.01
UK Realized Inflation	MW%	10.0	0.0	10.0	0.31	3.14	2.94	0.019	0.38	0.19
UK Expected Inflation Short	OASD (Yr)	0.338	0.0	0.338	14.95	5.05	-4.52	-1.368	-0.93	-0.46
UK Expected Inflation Long	OASD (Yr)	0.788	0.0	0.788	8.87	6.99	-7.76	-1.393	-2.22	-1.1

Source: POINT.

The next section deals with the return attribution for inflation-linked securities.

5. Performance attribution

Having adopted a risk view for inflation linkers that accounts for the exposure to the nominal government curve and to inflation growth expectations separately, we follow the same methodology for the attribution of daily returns, making the analysis of inflation-linked bonds similar to the analysis of credit securities. We calculate the following return components:

(a) *Return due to the change of the government curve*. The sensitivities that drive this return are the key-rate durations of the security (computed by holding the inflation spread constant, i.e. ignoring any correlation between nominal rates and inflation).

(b) *Carry return earned because of the exposure to the government curve*. Here, the sensitivities are the weights of a portfolio of hypothetical par government securities that have the same key-rate durations as the linker (curve-matched portfolio). This carry return is partially offset by the carry return lost because of the negative inflation spread.

(c) *Return due to changes of implied volatility.* The exposure of inflation bonds to implied volatility is negligible as their only optionality stems from the principal guarantee of most bonds, which is unlikely to have any value as inflation is generally positive.

(d) *Return due to changes of inflation spread.* Return due to changes in long-term expected inflation growth, liquidity and risk premia etc as expressed by the change of the spread of the issue. The sensitivity for this component is the spread duration of the security.

(e) *A negative spread carry*. This arises because of the negative spread that represents the expected growth of inflation. Here, the sensitivity is the percentage of the market value weight of the unknown cash flows.

5.1. The inflation accretion return

All the above return components are similar to those of any other credit issue. However, inflation-linked securities have an additional component of return, namely that due to inflation accretion. This return, which is driven by the monthly realisation of the inflation index, counterbalances the return lost because of the negative spread carry. In a risk neutral and prescient world, the return lost because of spread carry would be exactly equal to the return gained from inflation accretion, such that the total return of the linker would be equal to the return of a standard nominal security of the same term. In practice, of course, these returns do not cancel each other, although they do have opposite signs most of the time. In the short term, seasonality effects and differences between short-term and long-term inflation growth expectations may generate significant discrepancies between these two returns. In the long term, their difference should reflect the risk and liquidity premium (or penalty) of inflation linkers.

The exact amount of inflation accretion becomes known discretely once a month, on the announcement day. As discussed before, if we updated our pricing equation monthly, any return analysis performed between two announcement days would show an accretion return of zero, something less than desirable. For this reason, we have decided to project the inflation for the first unknown month on a daily basis – one day at a time – following the inflation announcement day. We use this estimate (which may change from day to day) to compute accretion for each passing day, thus countering the daily carry lost to spread until the next announcement day. On that day, when the true inflation becomes known, we treat it as a final (accurate this time) estimate and we correct our monthly accretion. Thus on each day we have two accretion components of return:

(f) *Inflation accretion.* One-day accretion using the inflation projection.

(g) *Inflation surprise.* A correction term for all days elapsed since the last announcement day reflecting the difference between yesterday's and today's inflation projection (or realisation).

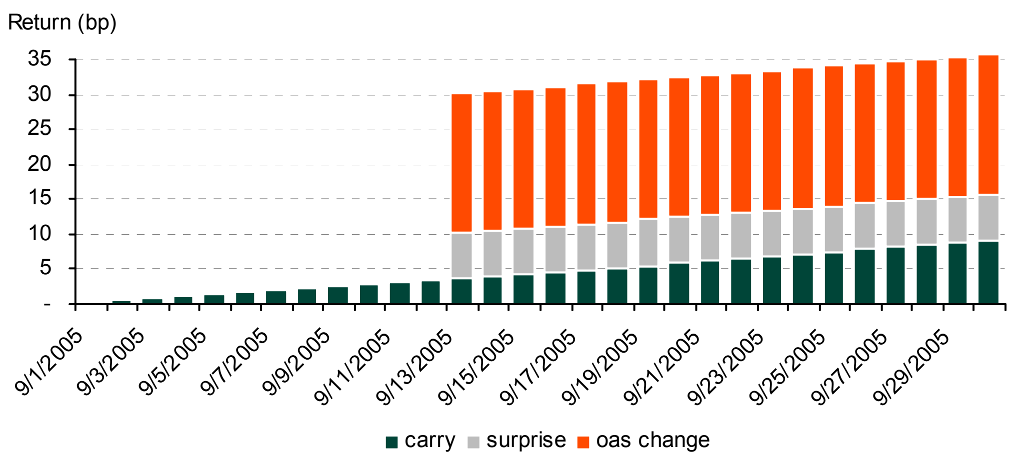
5.2. An example

Consider an inflation-linked bond which trades at a spread of -200bp relative to the nominal curve. This means that the average inflation expectation over the life of the security is approximately 2% per annum. Let us also assume that because of seasonality the short-term inflation for the next month is projected to come in higher at 3%. The security begins earning a daily carry return of (300 – 200)/365 = 0.27bp per day.

Let us assume that nothing changes until the index announcement day, at which point the announced inflation comes in higher than the expectation at a 3.5% annual rate. This means that the annual carry rate used since the previous announcement day (a period of approximately one month) was lower by 50bp. We correct for that by adding a one-time "inflation surprise" return component of approximately 50/12 = 4.1bp.

After the realised inflation is announced, longer term inflation expectations are affected and the spread of the bond grows in magnitude by falling to -210bp. At the same time we form a projection for the inflation rate of the subsequent month, let us assume 2.5%. The rate at which carry is earned now is (250 – 210)/265 = 0.11bp per day. Finally, the change of long-term inflation expectations generates a return proportional to the spread duration of the bond. Even if we assume that the spread duration is small, say 2 years, this return will be quite significant at – (–210 + 200)×2 = +20bp. Figure 11 below illustrates the inflation return growth of a linker over the course of a month.

Figure 11. Return growth of an inflation-linked security



Source: POINT.

5.3. Return components – detail

Let us now examine how the pricing equation (9) is being used to calculate the various return components of inflation-linked securities. Recall that equation (9) sets the inflated price of the security equal to a function of time, nominal interest rates, the inflation spread and the currently projected index ratio, as follows:

$$B_t = \sum_{i}^{t_i \le t_{proj}} IR_{t,t_i} cf_i \cdot df_{t,t_i} + IR_{t,t_{proj}} \sum_{i}^{t_i > t_{proj}} cf_i \cdot df_{t,t_i} e^{-s_t \cdot (t_i - t_{proj})}$$

(a) Curve change return

The exact effect of the change of the discounting curve is computed by repricing the security using the last business day's nominal closing curve but keeping today's inflation spread unchanged, following the methodology established for standard credit securities.

$$\hat{B}_t = \sum_i^{t_i \le t_{proj}} IR_{t,t_i} cf_i \cdot df_{t,t_i}^{prev} + IR_{t,t_{proj}} \sum_i^{t_i > t_{proj}} cf_i \cdot df_{t,t_i}^{prev} e^{-s_t \cdot (t_i - t_{proj})}$$

Assuming no cash flows[49] at time t, the return due to curve change is: $\dfrac{B_t - \hat{B}_t}{\hat{B}_t}$.

In addition, we compute the key-rate durations in a similar manner to any other security, by perturbing the nominal par curve but keeping the inflation spread over the resultant discount curve unchanged. We can then split the above total return due to curve change into the contribution of each key-rate separately, as well as a residual term that includes the incremental effect of all other curve points plus convexity.

(b) Curve carry

Curve carry is also computed in the same way as for standard securities. A key-rate duration-matching portfolio of hypothetical par zero-spread bonds and cash is constructed, and its carry return is theoretically calculated. The curve carry return of the security is set equal to the carry return of the curve-matching portfolio.

(c) Volatility return

Volatility return is computed in a similar fashion to the curve change calculation, by repricing the security using today's curve and spread but yesterday's volatility parameters.

(d) Spread change return

As mentioned before, for all practical purposes we treat the inflation spread as equivalent to the option-adjusted spread of the security. We thus define the return that is induced by changes in inflation spread to be the spread change return of linkers. We also define the price sensitivity to changes of the inflation spread as the option-adjusted spread duration of the security. Note however, that unlike regular securities where the spread discounting is applied to all cash flows after the settlement day, the inflation spread discounting (actually accretion since the spread is negative) term is only applied to cash flows after the last projected ratio date.

(e) Spread carry return

Since spread discounting is only applied to cash flows after the last projected ratio date, we cannot use the simple $s_t \times \Delta t$ formula to compute spread carry. Instead, we have to carefully compute the sensitivity of the inflated price to the spread discounting term.

$$R_{\text{spread\_carry}} = \frac{1}{B_t} \cdot IR_{t,t_{\text{proj}}} \sum_i^{t_i > t_{\text{proj}}} cf_i \cdot df_{t,t_i} e^{-s_t \cdot (t_i - t_{\text{proj}})} \cdot s_t \cdot \Delta t_{\text{proj}}$$

If we now define the "modified price" as the present value of the cash flows beyond the projected date t_{proj}:

$$B_t^{MOD} = IR_{t,t_{\text{proj}}} \sum_i^{t_i > t_{\text{proj}}} cf_i \cdot df_{t,t_i} e^{-s_t \cdot (t_i - t_{\text{proj}})}$$

we can re-write:

49 On cash flow days the formula is slightly more complicated.

$$R_{\text{spread\_carry}} = \frac{B_t^{MOD}}{B_t} \cdot s_t \cdot \Delta t_{\text{proj}}$$

We have to remember here that while generally the last projection date follows the movement of the pricing date, on inflation announcement days the projection date may move in a discontinuous fashion, as explained in footnote 57 to Appendix B.

(f) Inflation accretion

Each day, the projected index ratio grows by a rate implied by our next month index projection. The index ratio may also change if the CPI projection changes, or on the announcement day, when the actual CPI becomes known. The total change of the index ratio from day t to day t^{next} is derived from equation (6) in Appendix B as:

$$IR_{t^{\text{next}},t_{\text{proj}}^{\text{next}}} - IR_{t,t_{\text{proj}}} = \Delta^{\text{time}}\left(IR_{t,t_{\text{proj}}}\right) + \Delta^{CPI}\left(IR_{t,t_{\text{proj}}}\right)$$

$$\Delta^{\text{time}} IR_{t,t_{\text{proj}}} = \frac{CPI_{t^{\text{next}}}^{\text{proj}} - CPI^{\text{known}}}{DIR_0} \cdot \frac{\text{Days}\left(t_{\text{proj}}^{\text{next}} - t_{\text{proj}}\right)}{\text{DaysOfMonth}\left(t_{\text{known}}\right)}$$

$$\Delta^{CPI} IR_{t,t_{\text{proj}}} = \frac{CPI_{t^{\text{next}}}^{\text{proj}} - CPI_t^{\text{proj}}}{DIR_0} \cdot \frac{\text{Day}\left(t_{\text{proj}}\right) - \text{Day}\left(t_{\text{known}}\right)}{\text{DaysOfMonth}\left(t_{\text{known}}\right)}$$

While the part of the index ratio change that is driven by changes in the projected CPI is addressed separately, the time component of the index ratio change generates an inflation accretion return which is given by the following formula:

$$R_{\text{accretion}} = \frac{B_t^{MOD}}{B_t} \cdot \left(\frac{\Delta^{time} IR_{t,t_{proj}}}{IR_{t,t_{proj}}} \right)$$

On the announcement day we use the announced actual inflation index level instead of the projected level in the above formula.

(g) Inflation surprise

On the announcement day, the announced rate of inflation is in general different from our projection as of the previous day. This causes a "surprise" return driven by the change of the projected index ratio $IR_{t,t_{proj}}$. In addition, any cashflows that fall between the last known ratio date and the last projected ratio date, which have been estimated using the projected inflation rate, must be adjusted to use the correct index ratio computed using the announced value of the index. To simplify the computation of the surprise return, we compute, on a daily basis, an "inflation index duration", $IndexDur$, that measures the sensitivity of the return of the linker to changes in the projected CPI, such that:

$$B_t \cdot IndexDur \equiv \frac{\partial B_t}{\partial CPI_t^{\text{proj}}}$$

Then, to calculate the impact of a revision in the CPI projection (or the effect of the announced CPI differing from the projected level we have been using to date), we simply compute the inflation surprise return as:

$$R_{\text{surprise}} = IndexDur \cdot \Delta CPI_t^{\text{proj}}$$

On the announcement day, the projected index change is set equal to the difference between the realised inflation index and the last projection. On any other day, to the extent that our projection does not change, this term is zero. If the projection is revised, then we classify the return due to this change as a "surprise" return since it is presumably the result of new information.[50]

5.4. Incorporation in the HPA framework

The curve and volatility components of return, as well as the spread change return fall seamlessly into the attribution framework, like the corresponding return of any other security. The spread carry and the inflation accretion components are added together since they jointly represent the liquidity and risk premium return of the security.[51] This combined number is attributed as spread carry return. Finally, the inflation surprise return is reported as a separate component of return in the fashion of a prepayment surprise return in the realm of securitised products such as MBS. Ultimately, it should be accounted together with the inflation accretion component since it is essentially the residual between projected and realised inflation accretion.

Figure 12 below, displays the "Security Return Splits" report that the Lehman Hybrid Attribution Model generates when run on a portfolio of US TIPS for the month of September. We can see that all securities had a large negative (nominal) curve change return which was partly compensated by (inflation) spread change gains.

Figure 12. Return Splits Report

LEHMAN BROTHERS | POINT

USD : Security Return Splits

Portfolio: INFL : TIPS Index **8/31/2005 to 9/30/2005**

Benchmark: Inflation Linked US TIPS **Base Currency: USD**

	Coupon %	Maturity	Ticker	Curve Carry bps	Curve Change bps	Volatility bps	Spread Carry bps	Spread Change bps	Inflation Surprise	Residual bps	Total Return bps
0 - 2 years											
9128272M	3.375	1/15/2007	US/TII	32.7	-43.3	0.0	40.3	16.0	10.5	-4.5	51.7
2 - 4 years											
9128273T	3.625	1/15/2008	US/TII	33.5	-77.6	0.0	41.6	55.2	10.6	-4.6	58.7
9128275W	4.25	1/15/2010	US/TII	33.9	-139.3	0.0	42.7	104.3	10.6	-4.3	47.9
4 - 6 years											
912828CZ	0.875	4/15/2010	US/TII	33.9	-156.4	0.0	38.0	109.0	10.5	1.0	36.0
9128276R	3.5	1/15/2011	US/TII	34.0	-170.5	0.0	41.8	116.5	10.6	-1.9	30.5
9128277J	3.375	1/15/2012	US/TII	35.1	-197.0	0.0	41.2	131.0	10.6	-0.5	20.4
6 - 8 years											
912828AF	3	7/15/2012	US/TII	35.7	-210.3	0.0	40.9	135.2	10.5	0.2	12.2
912828BD	1.875	7/15/2013	US/TII	37.3	-239.2	0.0	39.7	129.9	10.5	1.1	-20.7
8 - 10 years											
912828DH	1.625	1/15/2015	US/TII	40.0	-272.0	0.0	37.7	140.1	10.5	1.6	-42.1
912828CP	2	7/15/2014	US/TII	25.7	-183.3	0.0	20.4	97.6	10.5	2.6	-26.5
912828EA	1.875	7/15/2015	US/TII	40.8	-280.6	0.0	36.8	140.2	10.5	3.2	-49.1
Over 10 years											
912810FD	3.625	4/15/2028	US/TII	40.3	-474.9	0.0	43.0	315.5	10.4	-11.2	-76.9
912810FQ	3.375	4/15/2032	US/TII	39.4	-513.1	0.0	38.9	326.8	10.3	-9.1	-106.8

50 Note though that on the day after the announcement day this term is by definition zero, since no projection is being used to price the security on the announcement day itself.

51 Plus the effects of seasonality and the existence of non-zero inflation slope.

The magnitude of the negative returns due to rising interest rates is about double the magnitude of the positive returns due to increasing inflation expectations. We also see that all securities exhibit a positive inflation (surprise) return, which means that the August index (which was announced on September 15) was higher than the expectation we used in our pricing model. The spread carry return that combines inflation accretion with return loss due to the negative spread is positive, indicating that the short-term expected inflation was much higher than the long-term expected inflation implied from the spread of these securities. Indeed, CPI grew in August by about 50bp, while inflation spreads imply a long-term inflation expectation of about 2.4% per annum generating a loss of about 20bp per month.

In Figure 13 we can see the “Asset Allocation” report from the same run. The portfolio is being compared with the Lehman “Inflation Linked US TIPS” index. A duration partition has been used to identify positions on the term structure of inflation. As we see in the report, the portfolio has overweighted short-term inflation and underweighted long-term inflation. In the column “Benchmark OAS Change” we see how inflation spreads have moved in September for different duration buckets. Short-term inflation expectations increased by about 13.3bp (as indicated by the negative spread change), whereas medium- and long-term inflation expectations rose much more, 27bp and 20bp respectively. The large overweight of short-term inflation was a poor position since short-term inflation increased less than the average for the benchmark. The large underweight in long-term inflation was less important as the move of long-term inflation was similar to the one of the benchmark as a whole.

Figure 13. Asset Allocation Report

LEHMAN BROTHERS | POINT

USD : Asset Allocation - Duration: OA (2 year)

Portfolio: INFL : TIPS Index **8/31/2005 to 9/30/2005**

Benchmark: Inflation Linked US TIPS **Base Currency: USD**

Level 1: Duration: OA (2 year)

Portfolio Market Weight: 100.00

Contribution to Portfolio Spread Duration: 6.41

	Benchmark OAS (bps)		Market Weight (%)					Outperformance (bps)		
			Average		Port Minus Bench			Explained by Allocation		
	Init Level	Change	Port	Bench	Mean	Min	Max	Market Weight	Spread Duration	Total
0 - 2 years	-288.9	-13.3	18.0	5.8	12.2	12.2	12.3	-1.1	-0.9	-2.0
2 - 4 years	-252.6	-27.1	13.4	16.1	-2.7	-2.7	-2.7	0.1	-0.2	-0.1
4 - 6 years	-232.6	-24.4	17.1	13.2	3.8	3.8	3.8	0.1	1.5	1.6
6 - 8 years	-231.2	-18.8	22.2	24.3	-2.2	-2.9	-0.9	-0.1	0.1	0.0
8 - 10 years	-236.6	-16.2	17.1	13.3	3.8	2.6	4.6	0.1	-0.5	-0.4
Over 10 yea	-246.1	-19.5	12.3	27.4	-15.0	-15.1	-15.0	0.1	0.1	0.2
Total	-243.1	-19.7	100	100	0	0	0	-0.8	0.1	-0.7

Extending this algorithm to corporate inflation-linked securities is straightforward once we split the spread of the security into a credit and an inflation component. Having done that, the inflation component will be allocated and reported separately from the credit spread components.

6. Conclusion

We have presented in this paper the approach that we have developed in modelling the risk and return attributes of securities whose cash flows are linked to inflation indices in various markets. We have shown that a simple framework that treats such securities as a variation on the more familiar credit asset class serves us well in enabling analytical properties to be calculated and thus to define and estimate risk factors and performance components. We define a spread-based pricing equation and find that the spread associated with these securities is negative, broadly representing a measure of breakeven inflation, and an expectation of future inflation across the life of the bond.

We defined long and short inflation expectation (or breakeven) factors in the major markets (US TIPS, UK inflation-linked Gilts, HICP linked euro-denominated government debt and Swedish government inflation-linked securities.). In smaller markets or those concentrated about one particular maturity, namely those of Canada, Japan and French local inflation, we use a single-factor to capture the shift in the inflation curve. For each market we use the returns on the (projected) index level as an additional risk factor.

In seeking to attribute the return of such securities we are able to identify and quantify several inflation-related components of return such as inflation surprise due to shocks to underlying inflation as well as a combined term that captures the inflation risk premium, liquidity and seasonal effects.

Our methodology further allows analytical quantities to be defined and calculated such as empirical OAD as a measure of sensitivity to a parallel shift across the nominal yield curve as a whole, as compared with the prevalent beta-based approach which measures sensitivities to a single benchmark nominal yield.

Finally, the application of this work to the asset classes of corporate inflation-linked securities and inflation swaps is straightforward, with the former already being processed by the Risk Model and the latter envisaged as following in the near future.

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APPENDIX A: CASH FLOW CALCULATION

In this appendix we present the mechanics of cash flow calculation in the various inflation-linked markets.[52]

The cash flows for inflation-linked securities are computed as follows. For the coupons, we adjust the stated fixed coupon of the bond (the "real coupon") for inflation, to obtain the actual coupon paid. The formula is:

$$c_{\text{Paid}} = c \cdot IR_t \cdot N \tag{1}$$

Where c and c_{Paid} represent the real and actual coupons paid, respectively, N denotes the par amount, and IR_t the price index ratio. The index ratio is computed by dividing a daily inflation reference index, DIR_t, by the value of the daily index at the "base date"[53] that does not change over the life of the bond. That is:

$$IR_t = \frac{DIR_t}{DIR_0} \tag{2}$$

The daily inflation reference index is necessary in order to compute accrued interest on a daily basis, and is computed by interpolating between two observations of the officially published inflation index. The fact that there is a delay in publishing the inflation figures means that the indexation is with respect to lagged inflation and is, hence, imperfect. Because the price index of a given month m-2 is always published before the end of the following month m-1, it is always available to calculate the daily inflation reference for each day of month m. Let CPI_m denote the index level as of the end of month m. For any day t of a given month m, the inflation reference is obtained by interpolating between CPI_{m-3} and CPI_{m-2} according to the formula:

$$DIR_t = CPI_{m-3} + \frac{nbd(t)-1}{ND_m} \cdot \left(CPI_{m-2} - CPI_{m-3}\right) \tag{3}$$

where $nbd(t)$ denotes the number of business days between date t and the start of the month m, ND_m the number of days in month m, and CPI_{m-i} the price index of month *m-i*. The result is rounded to the fifth decimal place. Note that all indexation is done with respect to the first estimates for inflation and even if these are later updated, the updated numbers are never used for the purpose of recalculating the cash flows..

The bond is redeemed at maturity at the nominal, adjusted for the inflation ratio. If this is lower than the base date, the bond is redeemed at par:

$$PRINCIPAL_{\text{maturity}} = \max\left(100, \frac{IR_T}{IR_0} 100\right) \tag{4}$$

In the US, Canada and the UK, the coupons are paid semi-annually. The US inflation reference is the non-seasonally adjusted US city average all-items CPI for all urban consumers, published by the Bureau of Labor Statistics. The daily reference index is

52 *This appendix is drawn from Desclée, Klaeffling and Mendes-Vives "A Guide to Inflation-linked Government Bonds in the Euro Area" (LMRQ October 2003).*
53 *Typically the issue date of the bond.*

computed as in the French case, with a three-month lag. The US also guarantees redemption at par in case of deflation over the life of the bond.

The Canadian "real return bond" cash flows are computed as in the US, with the inflation index being the all-items CPI non-seasonally adjusted, published monthly by Statistics Canada (Bloomberg ticker CACPI <Index >). Contrary to the US, Canadian inflation-linked bonds do not have a guaranteed redemption at par.

In the UK, the coupons are semi-annual and the index is the General Index of Retail Prices or RPI all-items non-seasonally adjusted (Bloomberg ticker UKRPI <Index>). An important feature of the UK market is that the inflation used is lagged eight months[54] (two months to allow for the compilation and publication of the RPI, and six months to ensure that the nominal size of the next coupon payment is known at the start of each coupon payment for accrued interest calculations). In order to compute the daily inflation reference index, we use the formula:

$$DIR_t = RPI_{m-8} + \frac{nbd(t)-1}{ND_m} \cdot \left(RPI_{m-7} - RPI_{m-8}\right) \tag{5}$$

Note that there is no redemption floor at par in the case of UK index-linked bonds.

In Sweden, apart from the usual inflation-linked coupon bonds (maturing in 2008, 2015, 2020 and 2028), the Riksgalds Kontoret (Swedish National Debt office) has also issued two inflation-linked zero coupons (one already matured in 2004 and one maturing in 2014). Cash flows are computed similarly to the French case (three-month lag). The coupon is annual and paid on December 1. One peculiarity is that only two of the coupon bonds have redemption at par guaranteed in case of deflation (2015 and 2028); all other coupon bonds including the zero coupons, do not.

54 As of June 30, 2005 the DMO announced that it would be issuing inflation-linked gilts with a three-month lag going forward. These will be processed in the Risk Model in the appropriate way.

APPENDIX B: THE PROJECTED INDEX RATIO $IR_{t,t_{proj}}$

As explained in section 3.4.2, we face the problem that observation of the index ratio occurs as a jump process with an announcement at some time during the month. This consists of a statement of the CPI level for a month in the past with reference to which we determine the baseline for all future growth of inflation.

On the index announcement day, t_{ann} , all cash flows occurring on or before a particular future day, t_{known} , are fully specified. The size of cash flows occurring after t_{known} is still uncertain since the appropriate index ratio is unknown. As we move forward in time no new information becomes available until the next announcement day (after about one month), on which date the index ratio for the period of approximately 30 days following t_{known} will become specified.

To remove this discontinuity we introduce an "inflation estimate" for the first unknown month. On each announcement day we project the index value, CPI^{proj}, in each market for the next month. On the next day, $t_{\text{ann}}+1$, we use this projection to calculate a projected value for the index ratio corresponding to the day following t_{known}. We do this by interpolating between the last known index CPI^{known} and the projected index CPI^{proj}. The following day, $t_{\text{ann}}+2$, we interpolate the index level for another day, advancing the *projected index ratio* one further day and so on. This continues throughout the month, using CPI^{proj} to extend our daily index ratio calculations day by day. When we reach the day of the next CPI announcement (that we have been projecting throughout the course of the month) we begin again, now setting the new CPI to the latest known index level and projecting next month's announcement.

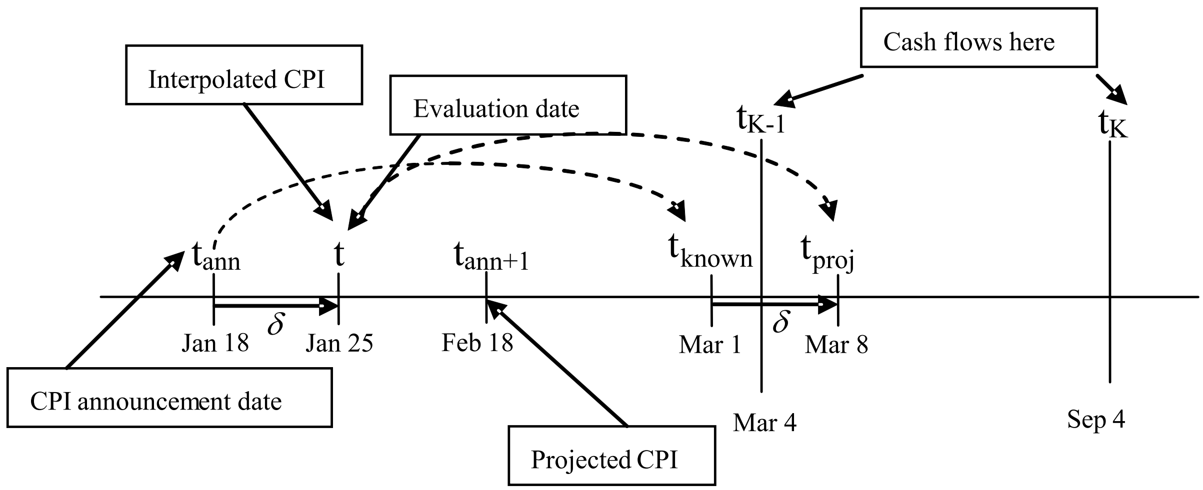
The projected index ratio that advances daily is denoted $IR_{t,t_{\text{proj}}}$, where t_{proj} is the (also daily advancing) date to which it applies. $IR_{t,t_{proj}}$ moves forward each day, and pertains to cash flows and accruals up to a time t_{proj}, thereby smoothing the return under certain projection assumptions. This allows us to approximate a smooth growth in the latest index ratio subject to the accuracy of our CPI prediction.[55] We treat all cash flows up to and including t_{proj} as known and we now use $IR_{t,t_{proj}}$ as the latest "known" index ratio.

Suppose, for example, that on January 18, 2005 the December 2004 HICP index level is announced. The latest index ratio, given by the ratio of the announced CPI level to a base level, determines the potential coupon and accrued that applies to the date of March 1, 2005. This is now both the *latest index ratio date*, t_{known} , and the *projected index ratio date,* t_{proj}. On January 19, the *projected index ratio,* namely that applying to March 2, 2005, is calculated using a projection for January's CPI figure. In this way, the date to which the *projected index ratio* pertains, t_{proj}, moves forward every day and the ratio itself grows daily.

Figure 14 presents is a pictorial version of this procedure.

55 *The effects of the discrepancy between the projected next index value and the later announced value are described in section 5.2 (g) and in Appendix B.*

Figure 14. The mechanics of the CPI projection methodology



Source: Lehman Brothers.

At time t we are in possession of the CPI level as announced a few days previously at time t_{ann}. This CPI, due to the lag effect mentioned above, pertains to the calculation of a cash flow at time t_{known}. (The next actual cash flow for our security in this example is a few days after t_{known} at time t_{k-1}.) As explained above, at time t, we have a projected level for the next, as yet unpublished, CPI (CPI_{proj}). The *official* CPI for this month is published at time t_{ann}+1m, one month later. This means that we are projecting the CPI in such a way as to be able to calculate (as an estimate) the index ratio that applies to a cash flow at time t_{known}+1m and, by the definition of the index ratio, for any date in between t_{known} and t_{known}+1m. At time t, δ days after t_{ann}, we are thus in a position to project the index ratio for time t_{proj} (which is in turn δ days after t_{known}). Thus, as t advances one day at a time, we use the latest[56] value for CPI_{proj} to project the index ratio that applies to the time t_{proj} which, in turn is advancing with t, the evaluation date. Eventually, the new CPI is published and we begin the cycle again, projecting the next CPI and so on.[57]

This approach enables us to smooth the behaviour of the index ratio factor. It further allows us to quantify an inflation shock factor that reflects the genuine jump effect of the variation of the announced value for CPI from the expected one. This estimate should ideally reflect both seasonality and prevailing economic conditions. Once we have made this projection of

56 *The CPI projection could, in theory, be updated midmonth.*

57 *This methodology can result in jumps in the projection date as the interval between CPI announcements is not totally predictable. For example, let us assume that the January '05 CPI was announced on February 23. On this date, the last date for which the index ratio is known for US securities (which have a three-month lag) is April 1. On this day, we also set the last projected ratio date to April 1.The next day, February 24, the last day for which the index is known is still April 1. However, the last projected ratio date is now April 2, and the projected ratio for April 2 is derived by our projection for the February CPI.*

On March 22 (after 27 days have elapsed since the announcement day) the last projected ratio date has moved to April 28. Let us now assume that the February '05 CPI is announced on the next day, March 23. On this day the last projected ratio date is set to the new last day with a known ratio, May 1. So, between March 22 and March 23, although the pricing date moves by only one day, the projection date moves by three.

Indeed, the projection date can move backwards. Consider another case where the November CPI is announced on December 17, and the December CPI is announced on January 19. On December 17 the projection date is set to February 1. On January 18 it has moved 32 days forward to March 5. The next day it moves back four days to March 1. In such eventualities, spread carry and accretion on the jump day will be computed using the jump size.

the CPI, CPI_t^{proj}, the definition of the corresponding projected index ratio $IR_{t,t_{proj}}$ is as for all other index ratio calculations.

Formally, given the CPI projection CPI_t^{proj}, the index ratio is given by:

$$IR_{t,t_{\mathrm{proj}}} = \frac{CPI^{\mathrm{known}}}{DIR_0} + \frac{CPI_t^{\mathrm{proj}} - CPI^{\mathrm{known}}}{DIR_0} \cdot \frac{\mathrm{Day}(t_{\mathrm{proj}}) - \mathrm{Day}(t_{\mathrm{known}})}{\mathrm{DaysOfMonth}(t_{\mathrm{known}})} \tag{6}$$

Where t_{known} denotes the last known ratio date and t_{proj} is the last projected ratio date as described above. This equation is derived from equation (3) in Appendix A by dividing by the base date daily index DIR_0. In addition, here we make no assumption about what day of the month t_{known} is, while equation (3) assumes that it is the first of the month as is typically the case.

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