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Quantitative Strategy
Portfolios Under Construction

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A Cookbook for Custom Factor Attribution

Holdings based attribution framework
We introduce a unified holdings based portfolio attribution framework that provides flexibility for devising customized attribution schemes. It also produces more accurate and timely estimates than return based approaches.

Flexibility is key
A meaningful attribution analysis should be one tailored to a manager's objectives. We demonstrate the flexibility in designing custom factor portfolios for attribution purpose in a series of examples: vendor factors, sectors, alpha sources, or more unorthodox cases like stock screens.

Uncovering deficiencies in portfolio construction
By aligning custom factors in the attribution with sources of alpha, we can obtain a consistent image of the implementation efficiency and portfolio construction. Using a real world example, we investigate the effect of constraints such as long-only, risk and turnover on the portfolio and measure their impact on the risk and return profile of the final portfolio.

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Table Of Contents

A letter to our readers 3
A Cookbook for Custom Factor Attribution 3

Model Framework 4
Holding based approach 4
Attribution 5
An illustration 7

Dissecting Mutual Fund Holdings 9
A prolonged view of institutional portfolios 9
Customized attribution by sectors 14

Revisiting Quant GEM 18
Attribution on Robust Multi-Factor (RMF) model 18

Custom Factor Attribution: A Simplified Case 21
An equal-weight two-factor mixture 21
An optimized risk parity basket 23

Custom Factor Attribution: Multiple Alpha Sources 25
Unconstrained MVOs 25
A constrained long/short portfolio 28
Investable long/short portfolios with turnover constraint 30
Vintage alpha sources 33
Attributing on investable factor portfolios 35
Long-only portfolio 37
Long-only portfolio with turnover constraint 39

Unorthodox Applications 40
Analyzing fundamental filters in stock picking portfolios 40
Industry practice 42

Bibliography 44

Technical Appendix 45
From time series regression to holdings regression 45
Holdings regression and cross sectional regression 46
A drilldown on portfolio variance 47

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A letter to our readers

A Cookbook for Custom Factor Attribution

In this paper, we introduce a holdings based attribution methodology and highlight the power of custom factor attribution. It's superior to the returns based approach in accuracy and timeliness, and advantageous over vendor attribution analytics in its capability for customization.

Attribution is, by nature, not a uniform exercise. Ex-ante we are concerned with the sources of risk, alpha, and the implementation efficiency, while ex-post we are concerned with portfolio performance and how it lines up with our forecasts. In the real world, managers have proprietary investment processes, and applying the standard vendor-based attribution does not fully reveal all the salient features or the important details implicit in each manager's strategy. Attribution analysis therefore has to be customized and aligned to the decision making process, in order to generate the most valid and instructive insights. However, the efficacy of custom-based attribution requires the use of a valid methodology that provides the required insight without sacrificing accuracy.

The returns-based time series regression methodology has straightforward applications in attribution and is theoretically flexible to include any factors of interest. However it suffers from severe drawbacks. Most obviously, the curse of dimensionality prevents any practice of deploying too many factors in the attribution. More importantly however, it does not conform well to the dynamic nature of investors and the market. Its efficacy is strongly dependent on the portfolio having stable sensitivities (aka Betas) to each factor over an estimation window; something that is rarely the case. Lastly, altering the factors for customization also changes the covariance structure among assets, which can lead to inaccurate analyses.

Cross sectional methodologies similar to that used by Fama and Macbeth [1973] and advanced by risk model vendors is a step forward in eliminating dimensionality issues and constant beta assumptions and sophisticated risk models have been developed that include an extensive set of factors aimed to capture all systematic risks. However, once we customize our attribution and move away from the risk model factors, we end up losing the risk accuracy provided by the fine-tuned covariance structure used to construct the risk model.

The *holdings-based* approach, developed by Grinold [2006], improves upon the cross sectional approach by providing the flexibility to customize the factors used in the attribution while simultaneously maintaining the covariance structure essential for risk accuracy. In this paper, we showcase various applications of this powerful holdings-based methodology, and illustrate the kinds of insights one can generate from a custom factor attribution grounded by accurate stock level risk forecasts.

For more information on acquiring customized portfolio analyses through the methodology demonstrated in this paper, please contact us by email at DBEQS.Global@db.com.

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Model Framework

We start with the generic st ructure of a factor model, where asset return r_i is segmented to systematic components and a specific component:

$$r_i = \sum_{k=1}^{K} X_{ik} f_k + \varepsilon_i \qquad \text{or in matrix form } \boldsymbol{r} = \boldsymbol{Xf} + \boldsymbol{\varepsilon} \qquad (1)$$

Here, X_{ik} is the exposure (aka Beta) of asset i to factor k, f_k is the return to factor k, and ε_P is the asset's specific (aka idiosyncratic) return. The parameters in the model are typically estimated using a time-series or cross-sectional regression methodology, whereby the cross-sectional approach is prevalent in equity-related analyses.

Risk model vendors specialize in defining the factor structure in (1) in order to capture all the systematic or common drivers of asset returns, leading to quality estimates of the assets' covariance structure. They supply the factor exposures X which forms the input for cross sectional analyses and use cross sectional regressions to estimate the factor returns, f_k .

Working with vendor factors provides a platform for comparing attributes among similar portfolios. However, it is not reflective of manager's decision making process. While the generic form in (1) poses no constraint in the choice of factors, altering the factor set would implicitly disrupt the covariance structure used to construct the risk model, making estimations prone to error.

Flexibility to disentangle risk and return with a custom set of factors while maintaining risk accuracy is paramount.

To customize the attribution factors while preserving the accuracy provided by the vendor risk model, we leverage the framework developed in Grinold [2006] and introduce his holdings based attribution methodology. We show the Technical Appendix, that it is conceptually equivalent to both time series and cross sectional return regressions when the model is depicted by (1). More importantly, it allows us complete flexibility in analyzing the factors of interest, while preserving the covariance structure.

Holding based approach to custom factor attribution

This section proceeds to outline the methodology used to transform the original vendor defined factors to a set of custom factors which will drive the portfolio analytics in the attribution.

Transforming vendor factors to custom factors

At the portfolio level, we transform the factor structure in (1) into a model that uses the custom factors $s_1, \ldots, s_j$, which are designed to yield better insight to the user in the attribution analysis:

$$r_p = \sum_{k=1}^{K} X_{p,k} f_k + \varepsilon_p \quad \Rightarrow \quad r_p = \sum_{j=1}^{J} \beta_{p,j} s_j + \omega_p \qquad (2)$$

We follow Grinold's [2006] holdings-based attribution methodology, which works with the portfolio holdings directly. We can write (2) as:

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$$\boldsymbol{h} = \boldsymbol{S\beta} + \boldsymbol{u} \tag{3}$$

where $\boldsymbol{h}$ is the $N \times 1$ vector corresponding to the portfolio holdings; $\boldsymbol{S}$ is an $N \times J$ matrix where the j th column represents the holdings of the j th custom factor portfolio $\boldsymbol{s}_j$ (we will discuss how to define these in the next section). The exposures to the custom factors (see Technical Appendix) are given by:

$$\boldsymbol{\beta} = (\boldsymbol{S'VS})^{-1}\boldsymbol{S'Vh} \tag{4}$$

where $\boldsymbol{V}$ is the asset-by-asset covariance matrix constructed from the vendor factor structure in (1). In the Technical Appendix, we show the equivalence of this method with the time-series approach in (2) when the Betas are constant over time. However, the advantage of using the holdings based regression in (3) and (4) is that we can deal with a time-varying relationship between the portfolio and the factors to get timely and accurate estimates of $\boldsymbol{\beta}$.

Holding based approach is advantageous over return based approach due to time varying portfolio betas.

Two way street between factor scores and holdings

To perform a holdings regression in (4) we require $\boldsymbol{S}$, i.e. the factor portfolio holdings. In practice, we are often provided with factor scores/exposures, e.g. size exposure is the normalized log market cap, industry exposures are 0/1 flags based on the stock's industry classification. In such cases, we can construct factor portfolios in an ad-hoc way, e.g. an equally-weighted long/short decile portfolio that goes long stocks ranking top 10% by that factor and short bottom 10%. Nonetheless, there is a formal way to translate factors into portfolios, i.e. the factor mimicking portfolio (FMP) approach introduced in Alvarez et al. [2012]:

$$\boldsymbol{H}_{FMP} = (\boldsymbol{X'V}^{-1}\boldsymbol{X})^{-1}\boldsymbol{X'V}^{-1} \tag{5}$$

where $\boldsymbol{X}$ as in (1) is the matrix of stocks' exposures to risk factors. Each (k th) row in $\boldsymbol{H}_{FMP}$ represents the minimum variance portfolio that have unit exposure to the respective (k th) factor and zero exposure on any other factor. The motivation of adopting FMP in our context is the equivalence of holdings regression (3) and the cross-sectional version of returns regression (1), which is detailed in the Technical Appendix.

This grants us another layer of flexibility in terms of the starting point. We can either start from portfolios and compute factor exposures, or start with factor exposures and back out the underlying portfolios. In the former case, factor portfolios intersect each other and exposures are computed in a multivariate regression. In the latter case, exposures are bivariate, and factor portfolios are mutually non-exposed.

Attribution

The regression in (3) and (4) provides a basis for carrying out holdings based attribution analysis. The methodology is widely seen among risk model vendors. Nonetheless, this framework is much more powerful in the application of customized sets of factors. By aligning attribution analysis with investment process, we would be able to draw significant insights into the alpha and portfolio construction processes.

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Risk model vendor analytics

With portfolio holdings and asset-by-asset covariance matrix, a set of ex-ante portfolio-level metrics can be calculated, including risk forecast, predicted beta, expected Information Ratio, expected Value at Risk, expected Transfer Coefficient etc.

Factor level breakdown is derived from (1). Taking the variance on both sides of (1), we can write portfolio risk as (more details in the Technical Appendix)

$$\sigma_p = \Sigma_k X_{pk} \sigma_k \rho_{k,p} + \sigma_\varepsilon \rho_{\varepsilon,p} \qquad (6)$$

(2) decomposes portfolio risk into contributions from each factor and a residual component. The fraction of σ_p that can be explained by factor k is the product of the exposure to factor k in the portfolio (X_{pk}), factor k 's volatility (σ_k), and factor k 's correlation with the portfolio ($\rho_{k,P}$).

Factor exposures and (co)variance items are estimated ex-ante with holdings regression (3), and ex-post return contribution equals the realized return times the ex-ante exposure. The following metrics are commonly seen in vendor analytics.

Factor Exposure - X_{pk}

The portfolio "Beta" on each factor represents the direction and magnitude of the factor loading. Following equation (1), ultimately the return contribution from factor k would be $X_{pk} f_k$, therefore X_{pk} is a straightforward measure of the impact of factor k on portfolio return.

Exposure Volatility - $X_{pk} \sigma_k$

A scaled version of factor exposure that derives from return contribution $X_{pk} f_k$. In most cases, factors are not scaled to have equal volatility, and for two factors with the same exposure X_{pk} , the one with higher volatility σ_k would end up being a bigger driver of portfolio return. Scaling exposure by factor volatility is therefore a more transparent measure of the size of bet on factor k , net of diversification effect. An alternative way to think about volatility-adjusted exposure is as a sort of t-statistic[1].

Risk Contribution - $X_{pk} \sigma_k \rho_{k,p}$

A step further on revealing the risk decomposition by factors in a portfolio context. Positions that are highly correlated to current portfolio add to the total risk, while positions negatively correlated to current portfolio reduce risk through diversification. Scaling factor exposure by factor correlation with portfolio takes into account this effect, making it a better measure of risk composition. Moreover, it follows that

$$\Sigma_k X_{pk} \frac{\sigma_k}{\sigma_p} \rho_{k,p} + \frac{\sigma_\varepsilon}{\sigma_p} \rho_{\varepsilon,p} = 1$$

[1] Consider the case when factors are uncorrelated among each other

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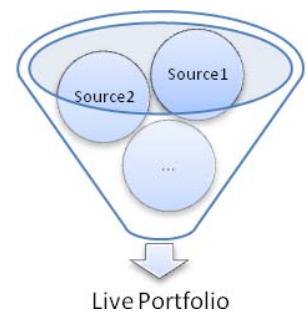
i.e. risk contributions from all factors (including residual) sum to total portfolio risk. Therefore $X_{pk}\sigma_k\rho_{k,p}/\sigma_p$ gives the percentage of risk that is attributable to factor k .

Custom factor attribution

Attribution produced by vendor factors is far from a one-size-fits-all solution, considering that each manager has his/her distinctive investment process. As discussed in Alvarez et al. [2012], the alignment of alpha and risk factors is essential to obtaining a meaningful result in risk/return tradeoff. Suppose we abstract a typical portfolio construction process in Figure 1. For quants the sources would be a set of alpha factors. For fundamental managers, they could be sector/regions, allocation/selection, screening criteria, or even single stocks. For fund of funds they would be the component funds or strategies. Instead of analyzing all portfolios with standard factors irrespective to managers' focuses on particular return sources, we can tailor the factor set to the process in Figure 1 by giving each return source a tangible form, i.e. a portfolio. Holding based attribution would then enable us to break down the risk and return of final portfolio into each source. In the sections to follow, we demonstrate that by applying custom factor attribution in different stage of the portfolio construction process, we can obtain a great deal of insights into the composition and efficiency of our implementation.

Figure 1: Abstraction of portfolio construction process



Source: Deutsche Bank

An illustration

Here we use a simplified example to illustrate the application of holding based attribution. Suppose our investment universe consists of three stocks: X, Y and Z, and we form a 130/30 portfolio with 30% short in X, 50% long in Y and 80% long in Z. We want to see the impact of each stock to this target portfolio. The input would be the following:

$$\boldsymbol{V} = \begin{array}{c} \\ X \\ Y \\ Z \end{array}\begin{array}{c} \begin{array}{ccc} X & Y & Z \end{array} \\ \begin{bmatrix} 0.0232 & 0.0163 & 0.0095 \\ 0.0163 & 0.2601 & 0.0122 \\ 0.0095 & 0.0122 & 0.0174 \end{bmatrix} \end{array}^{2}, \quad \boldsymbol{h} = \begin{array}{c} X \\ Y \\ Z \end{array}\begin{bmatrix} -0.3 \\ 0.5 \\ 0.8 \end{bmatrix}, \quad \boldsymbol{S} = \begin{array}{c} \\ S1 \\ S2 \\ S3 \end{array}\begin{array}{c} \begin{array}{ccc} X & Y & Z \end{array} \\ \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \end{array}, \quad \boldsymbol{R} = \begin{array}{c} X \\ Y \\ Z \end{array}\begin{bmatrix} 6.04\% \\ 17.95\% \\ 4.19\% \end{bmatrix}$$

$\boldsymbol{V}$ is the asset-by-asset matrix, $\boldsymbol{h}$ reflects the target portfolio holdings, and each row in $\boldsymbol{S}$ represents the portfolio holdings of a custom factor, in this case a single stock.

We calculate the exposure to each stock with (4). Portfolio volatilities and covariances are calculated in the usual manner. We can then compute all the attribution metrics in Figure 2.

[2] We round covariance numbers to four decimals for display purpose. Using rounded covariance in computation is subject to errors.

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Figure 2: Attribution output – an illustrative example

	[A] Factor Exposure	[B] Vol. Adj. Exposure	[C] Risk Contribution	[D] Risk Contribution (%)	[E] Return Contribution
X	-0.30	-4.57%	-0.94%	-3.37%	-1.81%
Y	0.50	25.50%	24.07%	85.91%	8.97%
Z	0.80	10.54%	4.89%	17.46%	3.35%
Residual	-	-	-	-	-
Total	-	-	28.02%	100.00%	10.51%

Source: Deutsche Bank

Column A verifies the portfolio weight in each stock. Residual is zero in this case as the target portfolio is a linear combination of three assets. Dividing Column B by Column A gives the standalone volatilities for X, Y and Z, and column C/Column B gives the correlations of X, Y and Z to target portfolio. We can see the short position in X provides diversification to the portfolio as it shows a negative risk contribution. Y, despite having lower weight than Z, has much higher volatility and correlation with the portfolio, and therefore drives target portfolio in a more profound way. Column B gives a more realistic picture of exposures, and column D shows Y is dominating the portfolio risk. Column E shows the ex-post return contribution, which equals the product of Column A and realized stock returns. Again Y is the dominant force in the ex-post performance of target portfolio.

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Dissecting Mutual Fund Holdings

A prolonged view of institutional portfolios

In our previous *Signal Processing* publication (Jussa et al. [2014] "Smart Holding"), we present a snapshot of risk attribution for all mutual fund portfolios in Q2/2013, in order to investigate which style or industry drivers are common across institutional equity portfolio managers. In this section, we extend the study by applying a full scale analysis over a 15-year period from 1997 to 2013. It would reveal whether the commonalities we find across managers are persistent over time.

We construct an aggregate portfolio by adding up the holdings of all mutual fund portfolios available in the Thomson Reuters Ownership Database by their dollar value as reported at each reporting date. It can also be considered as an "average" manager's portfolio, whose positions are representative of the common elements across all institutional managers in aggregate. Our interest lies in the tilts net of the market so we perform the attribution analysis on the aggregate *active* holdings relative to the Russell 3000 index. The goal is to reveal the composition of institutional equity portfolios and draw insight on the prevailing styles and rationale of equity managers.

Following the holding-based methodology from the last section, we attribute portfolio risk to the set of systematic factors provided by a risk model vendor (Axioma US2 medium horizon model). The seventy-eight systematic factors include 10 style factors and 68 industry factors. Figure 3 offers a snapshot of the top-level risk decomposition in the institutional equity space averaged over Q2/1997 – Q2/2013.

Figure 3: Active risk summary

Active Risk	1.30%
Benchmark Risk	18.55%
Factor Risk	1.08%
Specific Risk	0.68%

Source: Thomson Reuters, Compustat, IBES, Russell, S&P, Axioma, Deutsche Bank Quantitative Strategy

In aggregate, institutions have an average tracking error of 1.3% over the 15-year horizon, confirming that mutual funds as a whole track the market pretty closely. Within active risk, 1.1% is attributable to style factors, while 0.7% is due to idiosyncratic stock selection. The low level of active risk relative to benchmark risk indicates that alphas get diversified across managers, leaving mostly factor beta exposures. These style factor exposures are increasingly accessible through cheaper alternatives, e.g. smart beta products.

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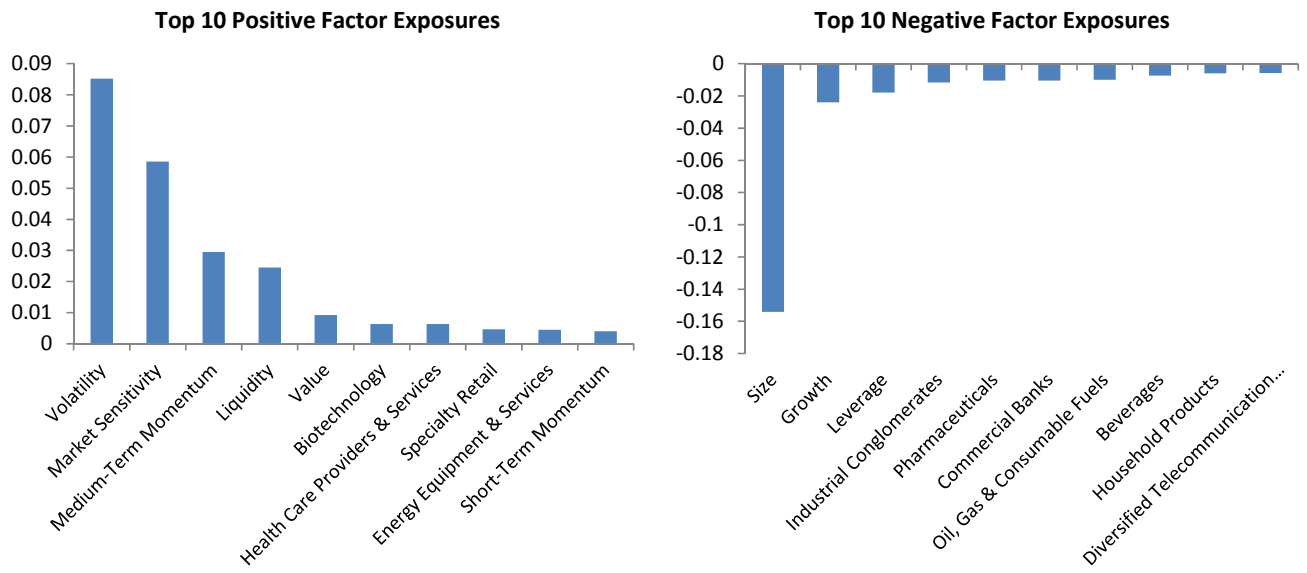
Figure 4 illustrates the factors that managers, on average, had greater exposure to Volatility, Beta (Market Sensitivity), Momentum and Liquidity over the past 15 years, which coincides with our findings for Q2 2013. Similarly, the strong negative exposure on size, which we've seen in Q2 2013, appears again in the 15-year averages, implying that equity managers averagely had a small cap size bias over the period. Instead of questioning whether its association with the bullish market in mid-2013, our doubt this time is whether the average value is driven by a few extraordinary periods when managers actively seeking small-cap opportunities.

Figure 4:Top positive and negative factor exposures



Source: Thomson Reuters, Compustat, IBES, Russell, S&P, Axioma, Deutsche Bank Quantitative Strategy

Alternatively, as we posit last time, the small cap tilt could be an unintended exposure that results from naïve portfolio construction. This bias will arise anytime portfolio holdings are not calibrated to avoid a market cap bias against the benchmark. For example, equally weighted stock portfolios have a natural small cap size bias relative to a capitalization weighted benchmark. Furthermore, this bias is amplified as the number of stocks in the portfolio decreases and can be quite significant for the vast number of manager portfolios which hold between 30 to 40 stocks. Interestingly enough, this effect cannot be mitigated by simply avoiding small or mid cap stocks. It will be present in the portfolio unless the weights are deliberately modified to adjust for the size exposure.

For this second explanation to hold water, we would expect a consistent and systematic negative size exposure in the aggregate portfolio over time. Indeed, Figure 5 shows that the negative size bias perpetuates over the entire 15-year of the study.

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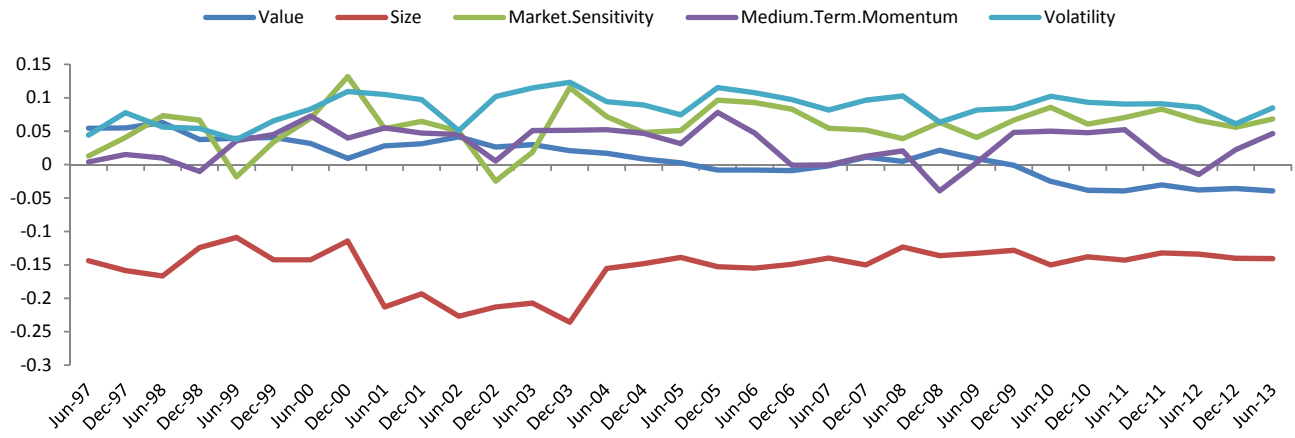
Figure 5:Consistency of negative size exposure



Source: Thomson Reuters, Compustat, IBES, Russell, S&P, Axioma, Deutsche Bank Quantitative Strategy

Similar to the small cap bias, the positive loadings on the Volatility and Market Sensitivity factors show that the tendency of institutional managers to hold relatively riskier stocks isn't a one-time phenomenon. There are various explanations for this persistent volatility bias. One argument presented in the literature centers on the "call option value" present in the compensation structure (base salary + outperformance bonus) of active managers. In this scenario, managers are incentivized to hold riskier stocks in order to maximize the payoff of this embedded option (see Baker and Haugen [2012]).

A second explanation takes us back to the side effects of naïve portfolio construction. Similar to the negative size bias, equally weighted stock portfolios will have a natural positive volatility bias. Interestingly, it can be shown that the induced volatility tilt is not just a product of the size tilt; a positive volatility tilt would develop even in the case when the equally weighted stocks were of the same size. Lastly, we note that these results are also in line with many other arguments underlying the "low volatility anomaly".

A better way to gauge factor exposure

Looking at factor exposure alone can be deceiving because factor volatilities can differ substantially. Given equal exposures, the risk arising from the more volatile factor will generally have more impact on the return and risk of the portfolio. Therefore we look to the volatility-adjusted exposure as an improved measure of the impact of the factor bet.

Figure 6 shows the largest volatility-adjusted exposures of the aggregate active portfolio. Size and volatility remain the most prominent factors. However, industry factors have more presence in these rankings because of their higher factor volatilities relative to the other style factors. In aggregate, managers have been more exposed to Biotechnology, Energy Equipment & Services, Healthcare Providers & Services, and Semiconductors, and underweighted Industrial Conglomerates, Oil, Gas & Consumable Fuels, Pharmaceuticals and Commercial Banks.

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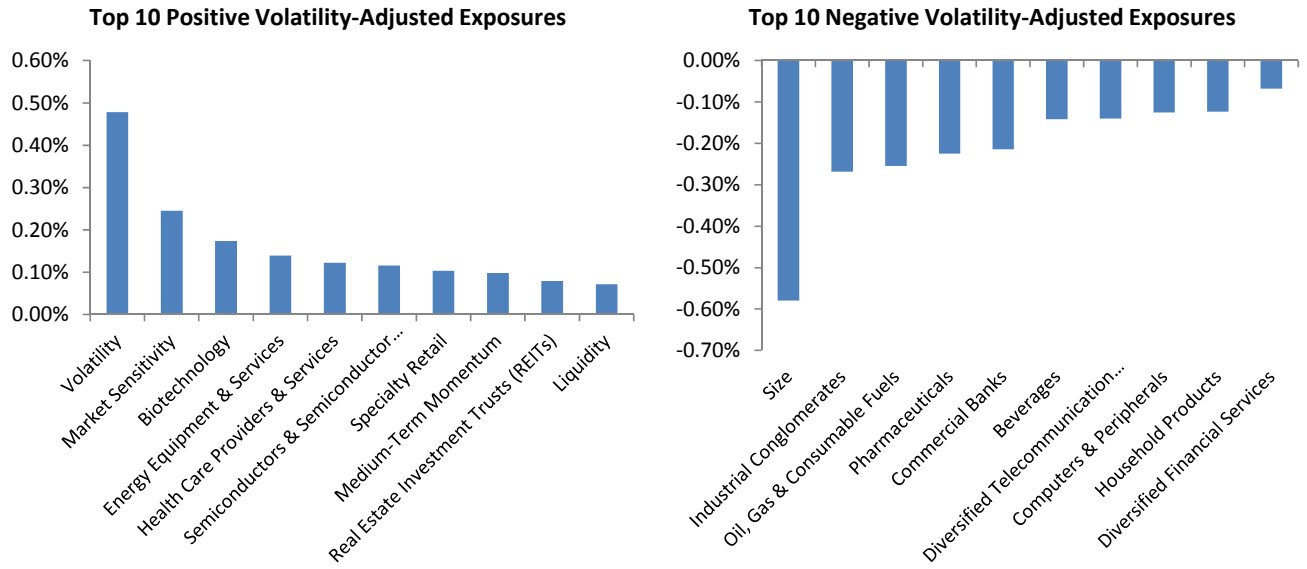
Figure 6:Top positive and negative volatility adjusted factor exposures



Source: Thomson Reuters, Compustat, IBES, Russell, S&P, Axioma, Deutsche Bank Quantitative Strategy

Taking it one step further, we compute the *risk contribution* attributable to each factor, which incorporates the diversification effect of a factor by accounting for its correlation with portfolio. The appealing aspect of risk contribution is that it is an additive metric – all the components' risk contributions sum up to the portfolio risk. Therefore we can obtain an additive percentage breakdown of portfolio risk by factor.

Figure 7 and Figure 8 illustrate the risk decomposition of an average institutional portfolio. 60% of active risk is attributable to styles, of which 23% is due to the small size bias. Volatility, Market Sensitivity (Beta) and Momentum are also major risk contributors among style factors. Industry in total contributes 14% of active risk. The top industry risk is Biotechnology, contributing 5% of active risk. Finally, 26% of total active risk is produced by specific component, which corresponds to managers' skills in active stock selection.

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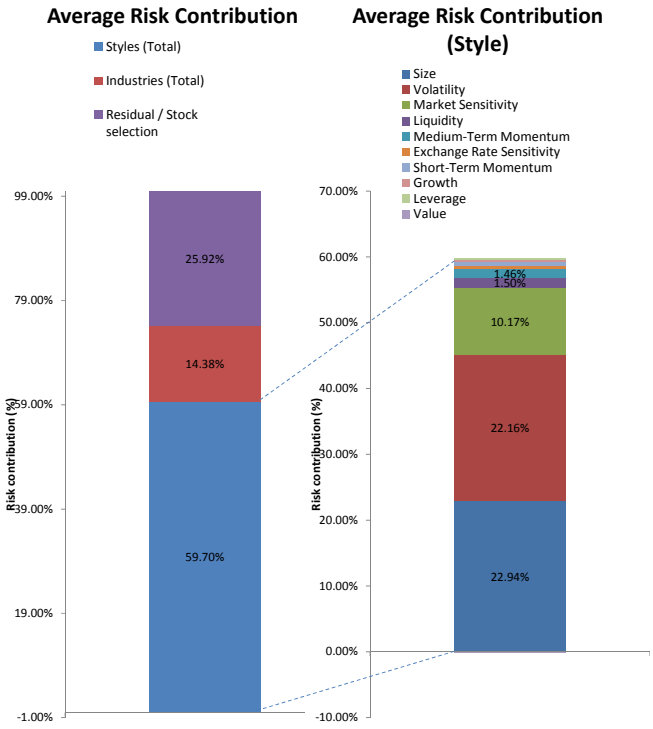
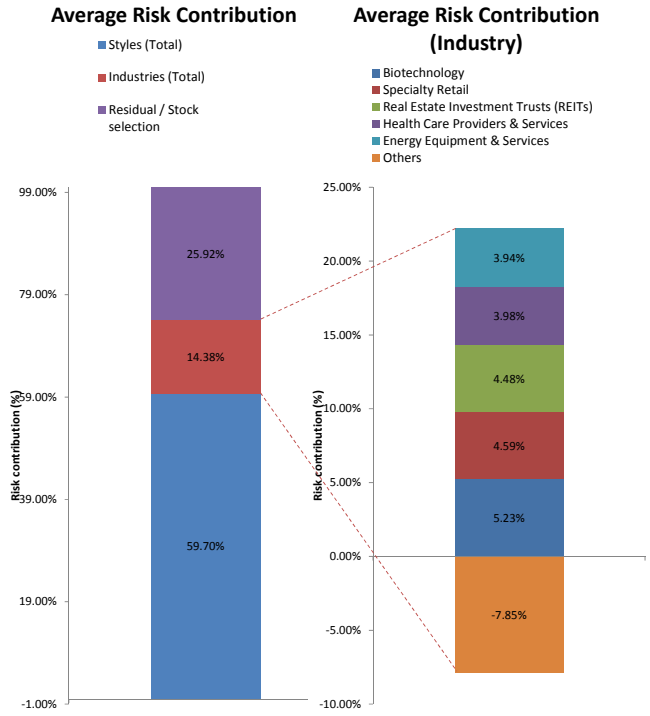
Figure 7: Average risk contribution by Axioma style factors



Source: Thomson Reuters, Compustat, Russell, S&P, Axioma Deutsche Bank Quantitative Strategy

Figure 8: Average risk contribution by Axioma industry factors



Source: Thomson Reuters, Compustat, Russell, S&P, Axioma, Deutsche Bank Quantitative Strategy

Overall, the combination of size, volatility, beta and momentum contribute up to 57% of active risk. Figure 9 below presents the full details of the attribution result.

Figure 9:Portfolio attribution with Axioma factors

		Factor Exposure	Vol-Adj Exposures	Risk Contribution	Risk Contribution (%)
Style	Size	-0.15	-0.58%	0.27%	22.94%
	Volatility	0.09	0.48%	0.27%	22.16%
	Market Sensitivity	0.06	0.25%	0.13%	10.17%
	Liquidity	0.02	0.07%	0.02%	1.50%
	Medium-Term Momentum	0.03	0.10%	0.02%	1.46%
	Exchange Rate Sensitivity	0.00	0.00%	0.01%	0.50%
	Short-Term Momentum	0.00	0.02%	0.01%	0.49%
	Growth	-0.02	-0.04%	0.01%	0.31%
	Leverage	-0.02	-0.03%	0.00%	0.24%
	Value	0.01	0.02%	0.00%	-0.06%
Styles (Total)		**0.02**	**0.29%**	**0.73%**	**59.70%**
Industry	Biotechnology	0.01	0.17%	0.07%	5.23%
	Specialty Retail	0.00	0.10%	0.05%	4.59%
	Real Estate Investment Trusts (REITs)	0.00	0.08%	0.04%	4.48%
	Health Care Providers & Services	0.01	0.12%	0.04%	3.98%
	Energy Equipment & Services	0.00	0.14%	0.04%	3.94%
	Others	-0.03	-0.48%	-0.07%	-7.85%
Industries (Total)		**0.00**	**0.14%**	**0.18%**	**14.38%**
Residual / Stock selection				**0.39%**	**25.92%**
Total Active Risk (forecast)				**1.30%**	**100.00%**

Source: Thomson Reuters, Compustat, IBES, Russell, S&P, Axioma, Deutsche Bank Quantitative Strategy

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Customized attribution by sectors

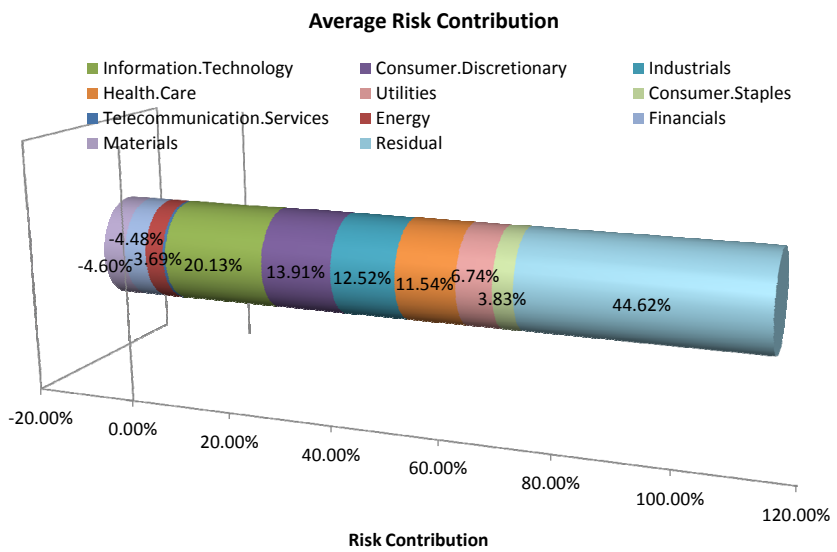
An attribution analysis with Axioma systematic factors reveals interesting insights. Yet allocating risk to 78 factors may not be a standard practice for many mangers. With 68 industry factors plus 10 style factors, it has a very granular view by industries for which the results are controlled for many other factors, making it less informative. However, the flexibility of our framework allows us to customize the attribution as needed. Here we demonstrate how to apply our attribution tool to a traditional sector attribution.

Attribution on sector mimicking portfolio

We start with the binary exposure of the 10 GICS sectors rather than 68 Axioma industries. Following the FMP approach, we back out the sector mimicking portfolios from the factor exposure matrix. Attribution is performed on the sector mimicking portfolios.

Figure 10 shows the risk contribution averaged over the sample period. The top sectors are information technology, consumer discretionary, industrials, and healthcare. Sectors like telecommunication and energy contribute little or negative risk to the aggregate portfolio. There is 45% of active risk not captured by sectors.

Figure 10: Average risk contribution by sectors



Source: Thomson Reuters, Compustat, IBES, Russell, S&P, Axioma, Deutsche Bank Quantitative Strategy

Looking at the sector exposure over time in Figure 11, information technology and consumer discretionary have consistently positive exposures over time. Information technology rose to the top at the height of dot-com bubble. Healthcare has been positively exposed since 2002. Consumer staples and utilities have consistently negative exposures.

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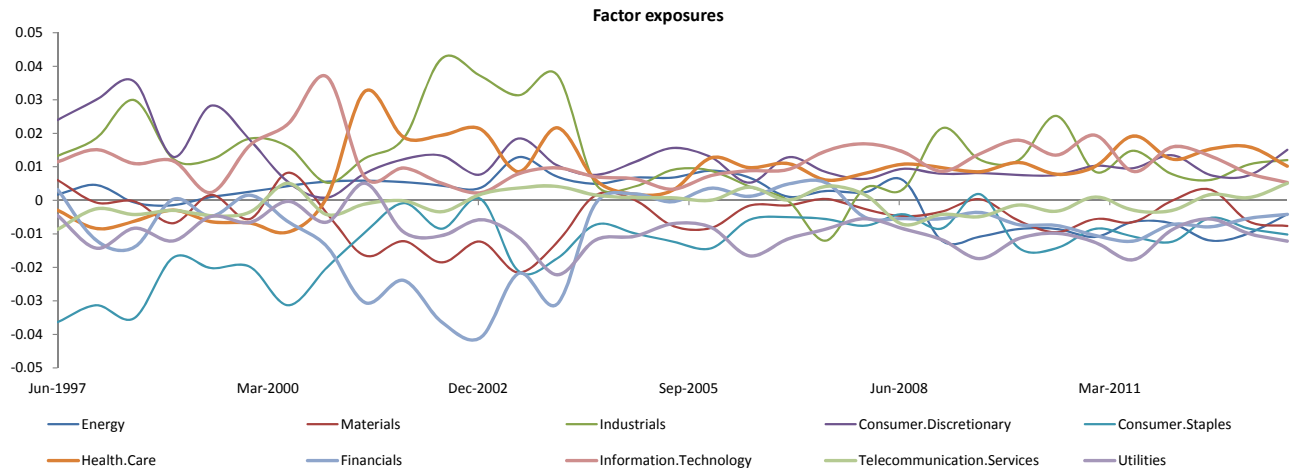
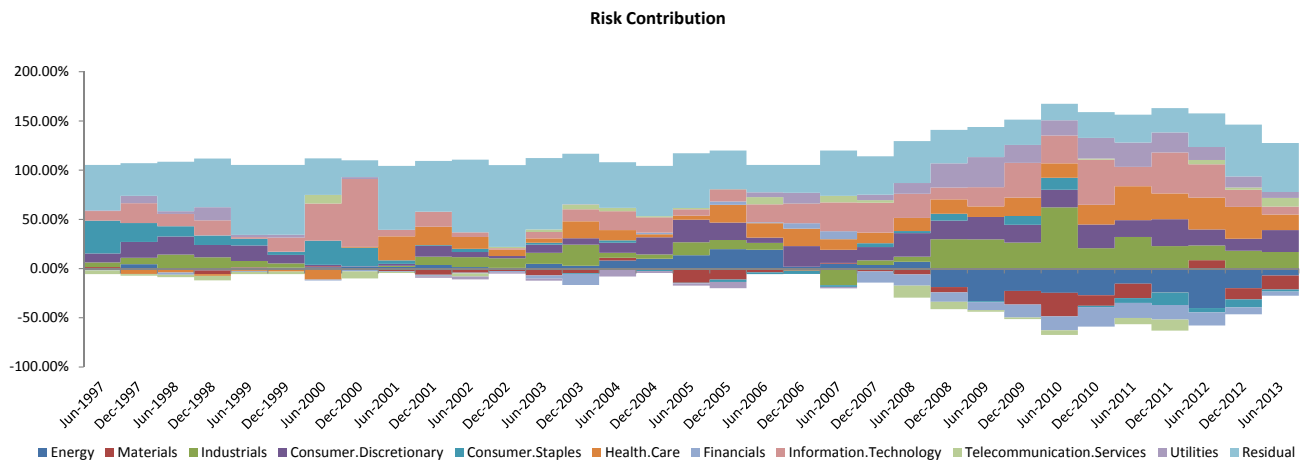
Figure 11: Exposures to sectors over time



Source: Thomson Reuters, Compustat, IBES, Russell, S&P, Axioma, Deutsche Bank Quantitative Strategy

Figure 11 presents a refined way to look at transition among sectors in terms of risk decomposition over time. Consumer staples was the most influential sector pre-2001, while information technology and healthcare rose in importance thereafter. Starting from 2008, a divergence of appetite across sectors becomes more noticeable. Cyclical sectors like: Information technology, healthcare and consumer discretionary are positively exposed and positively correlated with aggregate institutional holdings, while energy, financials, and telecommunication are underweighted and diversifies the aggregate portfolio.

Figure 12: Risk contribution by sectors over time



Source: Thomson Reuters, Compustat, IBES, Russell, S&P, Axioma, Deutsche Bank Quantitative Strategy

Attribution on cap-weighted sector portfolio

Another, probably more straightforward way to construct a factor portfolio for a certain sector is to take its weight in the benchmark. Indexed on Russell 3000, it is basically a long-only cap-weighted portfolio that includes all stocks that belong to that sector. We construct such a portfolio for each of the ten sectors by GICS classification.

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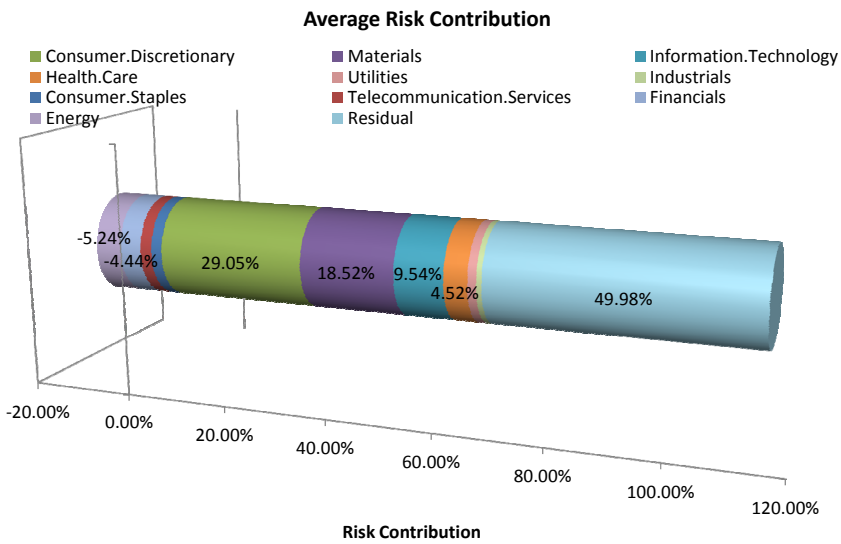
Figure 13: Average risk decomposition by cap-weighted sector portfolios



Source: Thomson Reuters, Compustat, IBES, Russell, S&P, Axioma, Deutsche Bank Quantitative Strategy

Figure 13 summarizes the risk decomposition. We see largely similar results in terms of the direction and magnitude of contributions from sectors. Yet it's notable that materials and industrials seem to have switched places. This result is understandable, given that such sector decomposition is susceptible to collinearity issues. It's a known fact that regression gives erratic results in allocating among factors that are highly correlated. Almost by nature, materials and industrials are highly correlated, and using long-only cap-weighted benchmarks is arguably more vulnerable due to the presence of common factors. Similarly, cyclical sectors such as consumer discretionary, financials, information technology and health care are typically highly correlated among each other, and in certain cases this might result in discrepancies in attribution results.

This example highlights the challenges in performing attribution with custom factors. The best way to tackle the potential collinearity issue is to account for it when selecting the set of custom factors. For example, we can create cyclical and defensive factors by grouping sectors that fall into either category. Or we can go into industry groups or industry levels, select the relevant ones and avoid including highly correlated industries. A good prior is essential to having a sensible attribution scheme that leads to valid outputs.

Allocation vs. Selection

Lastly, we illustrate how our attribution tool can be applied to Brinson style attribution that decomposes active performance into allocation and selection effects based on sectors. According to Brinson and Fachler [1985],

$$r^A = \Sigma_i w_i^A (r_i^B - r^{BM}) + \Sigma_i w_i^P (r_i^P - r_i^B) .$$

The first item represents allocation effect. Each factor $(r_i^B - r^{BM})$ is return of sector i relative to benchmark, which corresponding to a "relative sector portfolio" that goes long sector i and short the benchmark. The exposure on each sector is the active weight for sector i, i.e. w_i^A.

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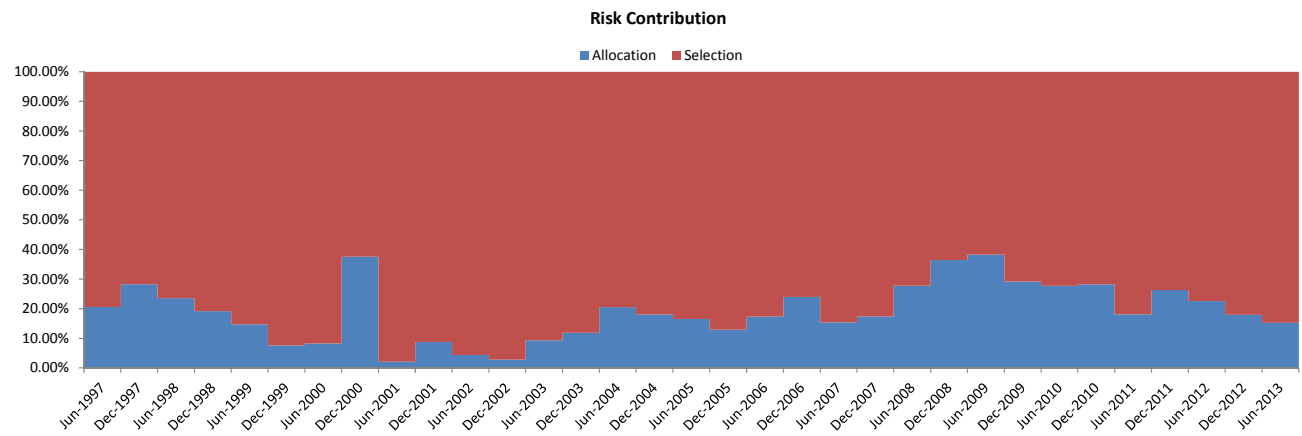
The second item is selection effect. The factor $(r_i^P - r_i^B)$ is the active return on sector i. Its underlying portfolio is an "active sector portfolio" that would take the stocks from sector i in our portfolio by their total weight, rebase to 100%, and subtract sector i benchmark. The exposure on this factor is the total weight on sector i, i.e. w_i^P.

Once we construct the factor portfolios, the attribution is readily in place. Figure 14 shows the risk decomposition by two effects over time. Allocation effect accounts for 19% of active risk on average. It peaks in end-2000 and mid-2009 at around 38%. Selection effect on average accounts for 81% of average risk. One reason for the uneven decomposition is in the exposures - w_i^P is always bigger than w_i^A. Yet correlation also plays a part. "Active sector portfolio" in this case has significantly higher correlation to active holdings than "relative sector portfolio" (see Figure 15).

Figure 14: Risk decomposition: Allocation vs. Selection



Source: Thomson Reuters, Compustat, IBES, Russell, S&P, Axioma, Deutsche Bank Quantitative Strategy

Figure 15: Average correlation with active portfolio

	Energy	Materials	Industrials	Consumer Discretionary	Consumer Staples	Health Care	Financials	Information Technology	Telecom Services	Utilities
Relative sector portfolio	-9%	26%	14%	34%	-52%	-26%	0%	29%	-23%	-28%
Active sector portfolio	56%	18%	44%	48%	39%	65%	25%	60%	38%	15%

Source: Thomson Reuters, Compustat, IBES, Russell, S&P, Axioma, Deutsche Bank Quantitative Strategy

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Revisiting Quant GEM

Attribution on Robust Multi-Factor (RMF) model

In our previous research of Emerging Market strategy (Wang et al. [2013]), we developed Robust Multi-Factor (RMF) model by dynamically weighting a basket of quant factors in Alpha Risk Parity scheme and apply region and sector neutralization. We also implemented with practical constraints a long-only active portfolio benchmarked to the MSCI EM index. In this section we run an attribution analysis to examine the risk exposure of this strategy in the period December 2006 to September 2013.

Figure 17 presents a summary of the attribution on active holdings of long-only RMF portfolio by Axioma factors, while Figure 18 graphs the risk decomposition. Styles in total contribute 15%. A small size tilt is noticeable relative to benchmark. Top overweighted industries are Transport, Automobiles, Utilities and Telecom, while Banks is heavily underweighted. Top country exposures come from overweights in Taiwan, Korea, Turkey and Thailand, and underweights in Russia and Mexico. There is 62% residual risk, which is typical for an optimized portfolio. Compared to the average mutual fund portfolio, our RMF takes significantly higher percentage of stock-selection risk that delivers a much higher information ratio.

Figure 16: Risk summary of active EM RMF portfolio

Average Active Risk	3.97%
Benchmark Risk	24.67%
Average Factor Risk	1.52%
Average Specific Risk	2.45%
Annualized Active Return	6.71%
Benchmark Return	9.11%
Information Ratio	1.25
Benchmark Sharpe	0.38

Source: Compustat, Worldscope ,MSCI, Axioma, Deutsche Bank Quantitative Strategy

Figure 17: Summary of Axioma factor attribution of active EM RMF portfolio

		Factor Exposure	Exposure Volatility	Risk Contribution	Risk Contribution (%)	Return Contribution
Style	Size	-0.18	-1.22	0.30%	7.65%	0.48%
	Medium-Term Momentum	0.15	0.43	0.07%	1.70%	0.78%
	Value	0.28	0.47	0.06%	1.56%	0.63%
	Short-Term Momentum	-0.02	-0.10	0.06%	1.40%	0.58%
	Liquidity	0.06	0.16	0.04%	0.90%	0.11%
	Volatility	0.02	0.04	0.02%	0.46%	-0.19%
	Growth	0.06	0.07	0.01%	0.36%	0.03%
	Leverage	0.12	0.11	0.01%	0.32%	-0.02%
	Domestic China	0.01	0.15	0.01%	0.17%	0.03%
	Exchange Rate Sensitivity	0.02	0.02	0.01%	0.16%	-0.02%
Styles (Total)		**0.52**	**0.13**	**0.57%**	**14.68%**	**2.42%**
Industry	Semiconductors & Semiconductor Equipment	0.01	0.09	0.02%	0.50%	-0.10%
	Transportation	0.03	0.16	0.02%	0.49%	-0.08%
	Materials	0.02	0.09	0.02%	0.41%	0.06%
	Automobiles & Components	0.02	0.15	0.01%	0.36%	0.13%
	Software & Services	-0.02	-0.15	0.01%	0.34%	-0.13%
	Others	-0.03	-0.11	0.08%	2.07%	0.80%
Industries (Total)		**0.03**	**0.23**	**0.16%**	**4.19%**	**-0.68%**
Country	Russian Federation	-0.03	-0.46	0.07%	1.64%	0.16%
	Brazil	0.00	0.03	0.04%	0.96%	0.21%
	Korea, Republic of	0.02	0.19	0.03%	0.83%	-0.04%
	Taiwan	0.03	0.35	0.03%	0.71%	-0.02%
	Thailand	0.02	0.26	0.03%	0.66%	0.07%
	Others	0.00	0.38	0.14%	3.55%	-3.91%
Countries (Total)		**0.03**	**0.75**	**0.34%**	**8.35%**	**0.68%**
Currency	RUB	-0.03	-0.15	0.07%	1.64%	0.16%
	BRL	0.00	0.00	0.04%	0.96%	0.21%
	KRW	0.02	0.20	0.03%	0.83%	-0.04%
	TWD	0.03	0.11	0.03%	0.71%	-0.02%
	THB	0.02	0.10	0.03%	0.66%	0.07%
	Others	0.00	0.38	0.14%	3.55%	-3.91%
Currency (Total)		**0.03**	**0.29**	**0.34%**	**8.35%**	**0.68%**
Global Market		**0.03**	**0.58**	**0.11%**	**2.46%**	**0.53%**
Residual / Stock selection				**2.45%**	**61.97%**	**3.07%**
Total Active Risk (forecast)				**3.97%**	**100.00%**	**6.71%**

Source: Compustat, Worldscope ,MSCI, Axioma, Deutsche Bank Quantitative Strategy

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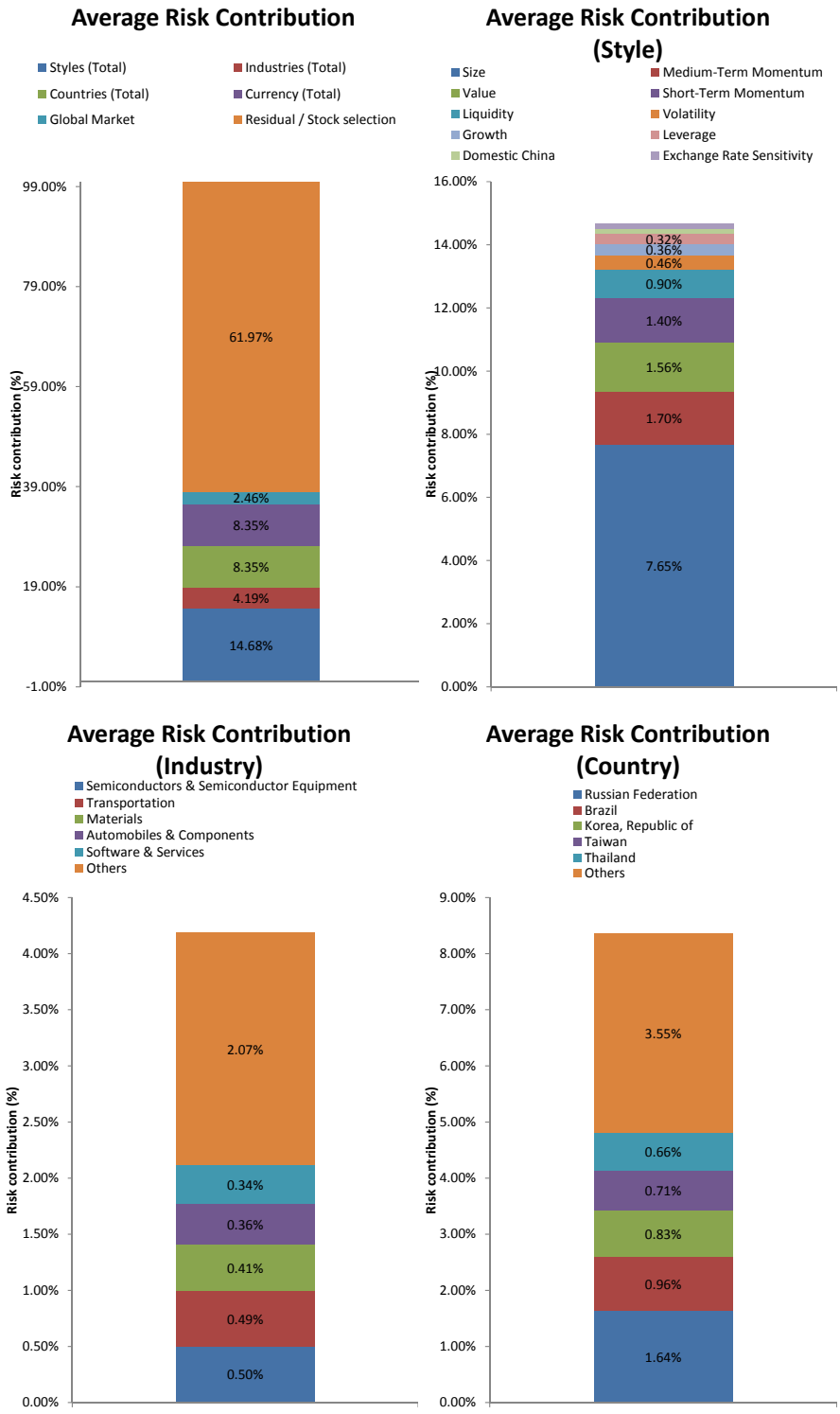
Figure 18: Axioma factor attribution on active EM RMF portfolio



Source: Compustat, Worldscope ,MSCI, Axioma, Deutsche Bank Quantitative Strategy

The above attribution on standard set of factors provides a basis to compare and assess similar strategies.

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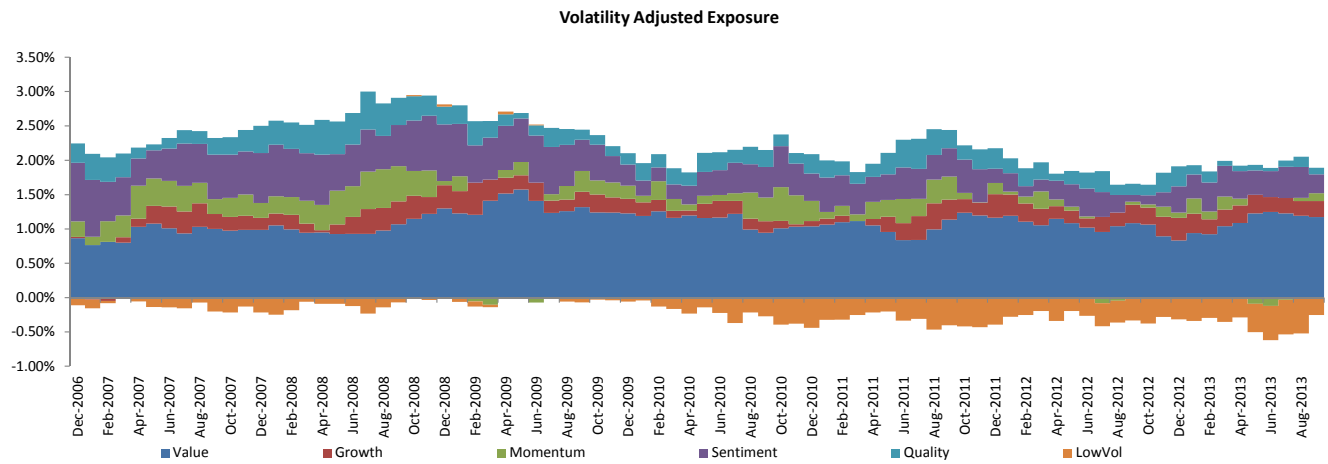
Taking it one step further, we can examine the exposures of each component factors that forms our RMF alpha, to see if the dynamic weighting scheme works well in the implemented portfolio. A custom factor attribution would accomplish the task.

As shown in Figure 19, the active holdings have consistently positive exposures to Value, Growth, Sentiment and Quality, while negative exposure to LowVol. Momentum exposure is sharply tuned down during market turmoil in 2009. Overall, Value is the most prominent factor in the strategy.

Figure 19: Vol. Adj. exposures of component factors in active RMF portfolios.



Source: Compustat, Worldscope ,MSCI, Axioma, Deutsche Bank Quantitative Strategy

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Custom Factor Attribution: A Simplified Case

At the end of the last section, we've seen how custom factor attribution can be applied to the traditional sector-related decomposition. Yet, the virtue of using custom factors is more than a mere substitute for old routines. In this section, we apply custom factor attribution to a simplified example with two factors, to offer some preliminary idea on the sort of insights it generates in signal composition and implementation.

An equal-weight two-factor mixture

Suppose we want to combine two equally-favored alpha sources: Quality (Return on Equity) and Reversal (1-month return), and we create two factor portfolios that long the top decile and short the bottom decile for each factor respectively.

Let's start with a naive approach of averaging the holdings of those two long/short decile portfolios. By construction, this average portfolio has equal exposure to either factor, therefore we call it equal beta portfolio. For illustration purpose, we attribute this portfolio to two factor long/short decile portfolios over January – December 2013 and present the average results in Figure 20.

Figure 20: Attribution of equal beta portfolio on component factors

	Factor Exposure	Exposure Volatility	Variance Contribution	Risk Contribution	Risk Contribution (%)
Quality	0.50	0.04	0.1938%	3.34%	59.24%
Reversal	0.50	0.03	0.1457%	2.40%	40.76%
Residual	-	-	0.0000%	0.00%	0.00%
Total	-	-	0.3394%	5.74%	100.00%

Source: Compustat, Axioma, Thomson Reuters, Deutsche Bank

As expected, the portfolio beta is 0.5 for each of the two factor portfolios. Risk contribution from Quality is higher because of its higher factor volatility during this period. There is no residual risk, as the combined portfolio is a perfect linear combination of two components.

Impact of turnover constraints on factor exposures

Now let's bring turnover constraint into the picture. Suppose we still want to keep the risk level of the equal beta portfolio, but turnover must be capped at 10%. i.e. our portfolio is the solution of the following optimization:

$$\max_h \alpha' h \quad \text{subject to } h'Vh = \sigma(EqualBetaPortfolio) \text{ and } Turnover \leq 10\%$$

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The α here is the alpha implied from the holdings of equal beta portfolio[3]. Without turnover constraint, the solution would be exactly the equal beta portfolio.

Let's call this portfolio equal beta proxy with turnover constraint. Rerunning the attribution of this portfolio on the two original factor portfolios, we get the following result (averaged throughout the year 2013):

Figure 21: Attribution of equal beta proxy with 10% turnover constraint

	Factor Exposure	Exposure Volatility	Variance Contribution	Risk Contribution	Risk Contribution (%)
Quality	0.54	0.04	0.2219%	3.67%	62.59%
Reversal	0.11	0.01	0.0250%	0.37%	5.37%
Residual	-	-	0.0978%	1.74%	32.04%
Total	-	-	0.3448%	5.78%	100.00%

Source: Compustat, Axioma, Thomson Reuters, Deutsche Bank

Figure 21 shows clearly the distortion caused by the turnover constraint. Reversal by nature is a fast moving signal, so the presence of a turnover constraint makes it difficult to load up on reversal. Its exposure dropped sharply from 0.5 to 0.1. In addition, residual risk ballooned to 32% of total risk. By construction, the residual component is orthogonal to all the factors included, meaning nearly one third of the risk in this tracker portfolio is irrelevant to either of the alpha sources we targeted originally.

The slowdown effect of turnover constraint

Furthermore, we can shed some light on how much the Reversal alpha is slowed down by turnover constraint, by dividing Reversal into two components: one is the holdings from previous month which reflects the information in last period; the other is the new positions taken this month, which results from the new signal in current period[4]. In Figure 22, we see that the exposure on lagged reversal even exceeds that on current reversal. For a factor that tends to flip side over month (hence dubbed 'reversal'), loading on month-old information is of no value and often contradictory to current strategy. If we launch this portfolio live, Quality would be the major driver of performance while Reversal would not.

Figure 22: Attribution of equal beta tracker with 10% turnover constraint, adding lagged Reversal factor

	Factor Exposure	Exposure Volatility	Variance Contribution	Risk Contribution	Risk Contribution (%)
Quality	0.54	0.04	0.2206%	3.64%	61.78%
ΔReversal(T)	0.11	0.01	0.0214%	0.31%	4.43%
Reversal(T-1)	0.13	0.01	0.0095%	0.17%	3.14%
Residual	-	-	0.0933%	1.66%	30.65%
Total	-	-	0.3448%	5.78%	100.00%

Source: Compustat, Axioma, Thomson Reuters, Deutsche Bank

[3] Recall that for unconstraint MVO we have $\alpha = 2\lambda V h^*$, thus the alpha can be implied from holdings assuming they result from an unconstraint MVO.

[4] To be precise, we regress current period holdings on lagged holdings and take the innovation as the new positions for current period . This ensures current and lagged components are uncorrelated.

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An optimized risk parity basket

In the previous example, we've seen that when equally weighting two factors portfolios, we are not getting the same risk contribution from each component factors because of their differing volatilities. We now illustrate an example where we have a mean-variance optimized (MVO) portfolio with equal risk contributions from two factors. This can be done by weighting two alphas by the reciprocal of their factor volatilities[5].

We can repeat the attribution sequence from the last example. However, since the final portfolio is an optimized one, using long/short deciles as factor portfolios would not be sensible. The difference in portfolio construction would become the noise that blow up residual risk, while in fact there is no residual component in our alpha construction. In this case, the factor portfolios should also be the mean-variance optimized version based on individual factors. Again we run the attribution over the year 2013.

Figure 23: Attribution of MVO equal RC portfolio on component MVO factor portfolios

	Factor Exposure	Exposure Volatility	Variance Contribution	Risk Contribution	Risk Contribution (%)
Quality	0.38	0.33%	0.0011%	0.24%	50.00%
Reversal	0.62	0.33%	0.0011%	0.24%	50.00%
Residual	-	-	0.0000%	0.00%	0.00%
Total	-	-	0.0022%	0.47%	100.00%

Source: Compustat, Axioma, Thomson Reuters, Deutsche Bank

The average results are illustrated in Figure 23 which indeed show equal risk contribution in two factors. Notice that the volatilities of MVO factor portfolios are significantly lower than long/short decile factor portfolios.

Impact of turnover constraint on factor risk contribution

Now we keep the same risk level and impose the 10% turnover constraint. The setup is straightforward:

$$\max_h \alpha_P' h \quad \text{subject to } h'Vh = \sigma(EqualRCPortfolio) \text{ and } Turnover \le 10\% .$$

Results are shown in Figure 24. Risk contribution from Reversal is taken down to 10%, while residual risk jumps to 41%.

[5] This is easily shown algebraically. For unconstraint MVO, $\alpha = 2\lambda Vh$ so $h = (1/2\lambda)V^{-1}\alpha$. We construct composite alpha as a linear combination of two individual alphas, $\alpha_p = \beta_1\alpha_{qual} + \beta_2\alpha_{rev}$, therefore the portfolio is also a linear combination of two factor portfolios, $V^{-1}\alpha_p / 2\lambda = \beta_1 V^{-1}\alpha_{qual} / 2\lambda + \beta_2 V^{-1}\alpha_{rev} / 2\lambda$. For risk contribution, $RC_i = \beta_i\sigma_i\rho_{i,p} = \beta_i\sigma^2{}_{i,p} / \sigma_p = \beta_i(\beta_i\sigma_i^2 + \beta_j\sigma_{i,j}) / \sigma_p = \beta_i^2\sigma_i^2 / \sigma_p + \beta_i\beta_j\sigma_{i,j} / \sigma_p$. So for two factors to have equal risk contribution, we have $\beta_i\sigma_i = \beta_j\sigma_j$.

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Figure 24: Attribution of MVO equal RC proxy with turnover constraint

	Factor Exposure	Exposure Volatility	Variance Contribution	Risk Contribution	Risk Contribution (%)
Quality	0.37	0.33%	0.0011%	0.23%	48.82%
Reversal	0.27	0.13%	0.0002%	0.05%	10.29%
Residual	-	0.00%	0.0009%	0.20%	40.89%
Total	-	0.00%	0.0023%	0.48%	100.00%

Source: Compustat, Axioma, Thomson Reuters, Deutsche Bank

The slowdown effect of turnover constraint

Again we want to add in lagged reversal into the attribution to see the impact of slowdown in reversal signal. We must be cautious, however, as new positions in MVO portfolios are partly due to the change in covariance matrix over month. We eliminate this effect by bringing the signal forward, i.e. construct an MVO reversal portfolio using the scores from last month. This is the factor portfolio as if we have no updated information this month. Let's call this portfolio Reversal(T-1)@T. Then we compute the new positions for this month relative to Reversal(T-1)@T[6]. Figure 25 shows that 2.5% of portfolio risk is attributable to the stale reversal holdings.

Figure 25: Attribution of MVO equal RC proxy with turnover constraint, adding lagged Reversal

	Factor Exposure	Exposure Volatility	Variance Contribution	Risk Contribution	Risk Contribution (%)
Quality	0.37	0.32%	0.0010%	0.22%	46.98%
ΔReversal(T)	0.25	0.13%	0.0002%	0.04%	8.54%
Reversal(T-1)@T	0.07	0.07%	0.0001%	0.01%	2.51%
Residual	-	-	0.0009%	0.20%	41.97%
Total	-	-	0.0022%	0.47%	100.00%

Source: Compustat, Axioma, Thomson Reuters, Deutsche Bank

[6] To be precise, we regress current period holdings on lagged holdings and take the innovation as the new positions for current period . This ensures current and lagged components are uncorrelated.

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Custom Factor Attribution: Multiple Alpha Sources

We now look at more realistic cases where there are multiple alpha sources, and they are incorporated in an investable portfolio constructed with realistic constraints. Continuing with Russell 3000 universe, we use five generic style factors:

- Value: trailing 12M earnings yield
- Quality: Return on Equity
- Growth: year-over-year earnings growth
- Momentum: 12M return – 1M return
- Revision three-month

Each factor is normalized and we take the average of the five as the composite alpha score. We can apply custom factor attribution at different stages of the portfolio construction process, to showcase the versatility and depth of the toolkit. We demonstrate in this section that a straightforward custom factor attribution is extremely useful in assessing:

- Alpha composition, by performing attribution on alpha sources;
- Implementation efficiency, by attributing constrained portfolios and analyzing alpha and residual components;
- Timeliness, by incorporating lagged signals in attribution.

We start with the ideal unconstrained implementation, then progressively impose constraints and assess the impacts. For illustration, we run our analysis over the period Jan – Dec 2013.

Unconstrained MVOs

Following the previous section, we know that even before portfolio construction takes place, we can obtain better knowledge on the risk inherited in the alpha by doing attribution on the alpha scores. To facilitate that, we translate factor scores into portfolios by computing the holdings of an unconstrained MVO portfolio based on that factor: $h = (1/2\lambda)V^{-1}\alpha$. Then we can analyze the optimal MVO portfolios using holdings based attribution toolkit.

What can risk vendor factors tell?

As with the mutual fund portfolio, we can run a standard attribution of the optimal MVO portfolios from the final alpha score on Axioma factors to get a sense of the risk decomposition. The result, however, is handicapped by a 90% residual component (see Figure 26). The largest exposures are in growth, momentum and value, which are three of the original components in the composite alpha. Yet their risk contributions are merely 5.7%, 2.3% and 0.2% respectively. There are insignificant loadings on other style and industry factors.

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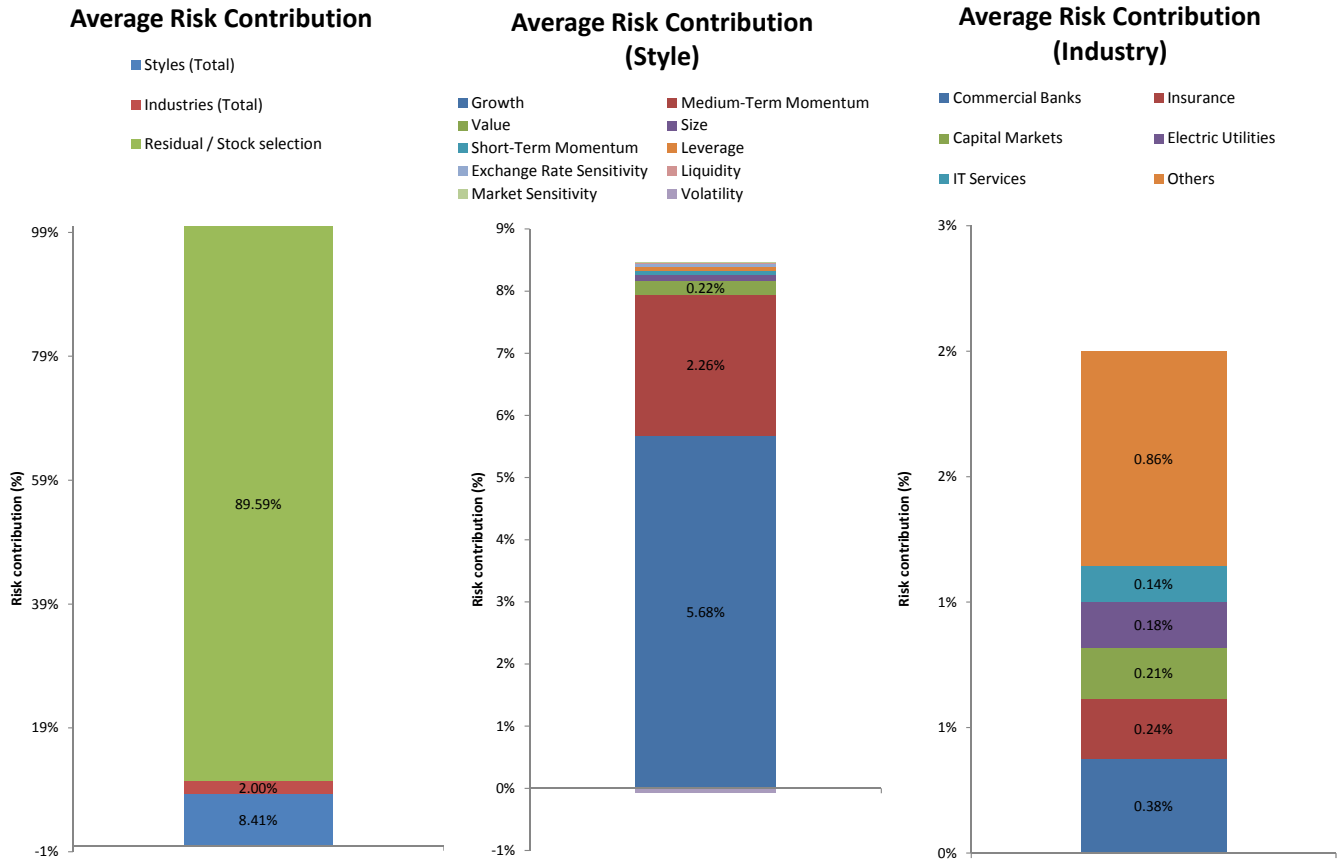
Figure 26: Average risk decomposition by Axioma factors on unconstrained composite alpha MVO portfolio



Source: Compustat, Axioma, Thomson Reuters ,IBES, Russell, S&P, Deutsche Bank Quantitative Strategy

This result exemplifies the pitfall of misalignment between alpha and risk factors in attribution for optimized portfolios as discussed by Alvarez et al.[2012]. Our growth, momentum and value signals are constructed differently than their Axioma counterparts either in the underlying variants or normalization approach. As shown in Alvarez et al. [2012], the slight deviation from risk model factor would be amplified when we constructed an optimized portfolio as risk model would misinterpret the difference as independent diversifiable component, leading to a lower than expected loading on the factor itself and noisy loadings on residual and irrelevant factors. In addition, quality and revision factors are not included in Axioma risk model. They are therefore projected onto industry factors, other irrelevant styles, and possibly large chunk of residuals.

An attribution performed on Axioma factors provides a standard view but does not necessarily reflect the alpha sources in our portfolio. To do that we need to use custom factor attribution.

Aligning attribution factors with alpha source

We can attribute the final alpha MVO portfolio on the five component factor MVO portfolios we constructed. With no constraint present, this attribution should be a straightforward reflection of the composition of our final alpha score. As expected, Figure 27 shows the composite alpha has equal exposure to all five style factors.

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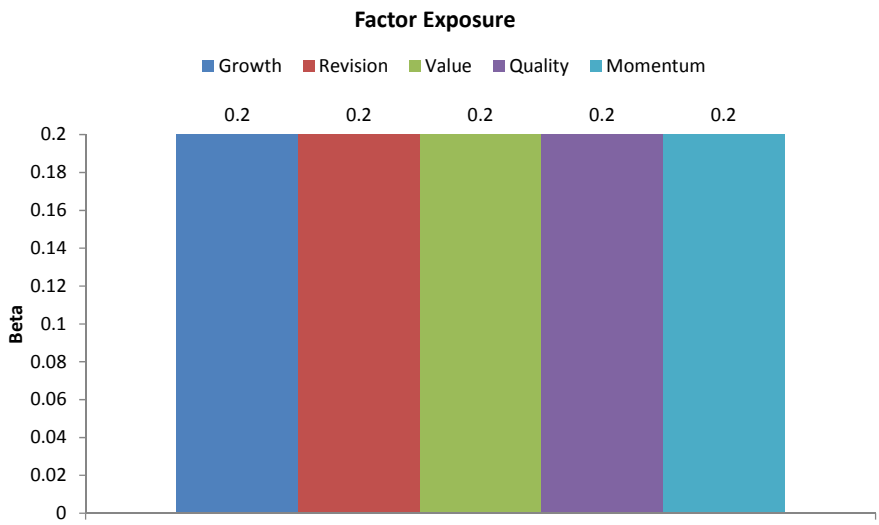
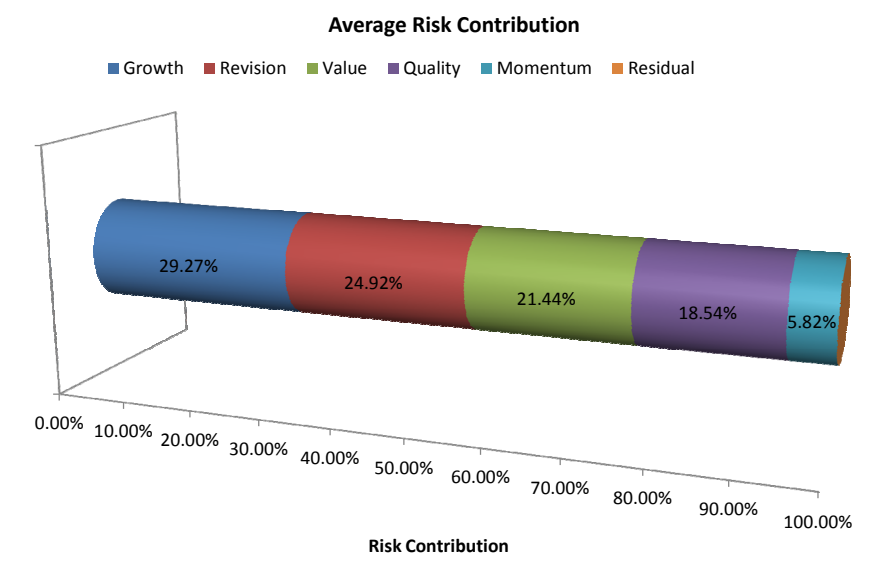
Figure 27: Exposure of composite alpha on five style factor



Source: Compustat, Axioma, Thomson Reuters ,IBES, Russell, S&P, Deutsche Bank Quantitative Strategy

Figure 28: Risk decomposition of composite alpha to five style factors



Source: Compustat, Axioma, Thomson Reuters ,IBES, Russell, S&P, Deutsche Bank Quantitative Strategy

However, the risk contribution in Figure 28 is rather uneven. Despite having the same beta, momentum contributes only 6% of the risk, while growth accounts for 30%. This is due to the different volatilities of the factors and the correlations among them. In Figure 29 we plotted the volatilities of the five MVO factor portfolios. Indeed momentum has much lower volatility than its peers, while growth is the most volatile component. In the absence of any constraint, the volatility of composite alpha would be more driven by growth and less impacted by momentum.

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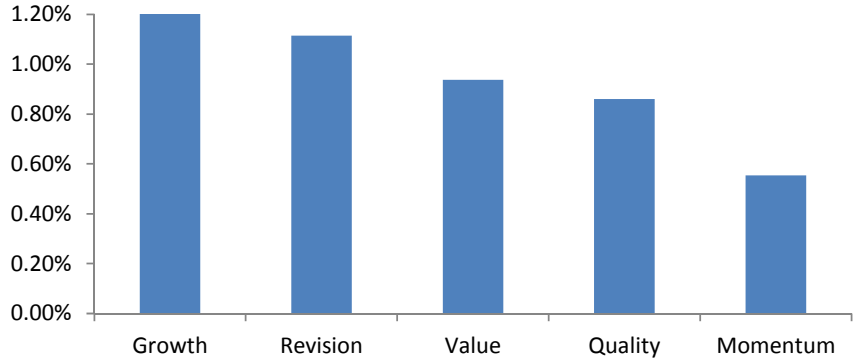
Figure 29: Factor volatilities, computed with MVO factor portfolios



Source: Compustat, Axioma, Thomson Reuters ,IBES, Russell, S&P, Deutsche Bank Quantitative Strategy

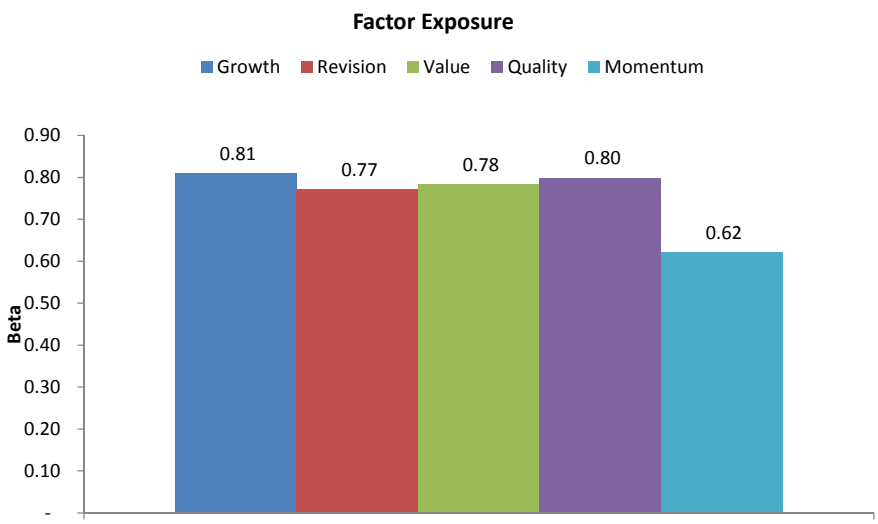
A constrained long/short portfolio

Now let's add a few constraints to make the MVO portfolio more practical.

- Risk target = 3%
- Minimum holding = 1%. It's not allowed to hold any stock with weight less than 1%
- Maximum leverage = 3x. We cap total long position not to exceed 150%, and total short position not to surpass -150%.

These constraints, particularly risk targeting and minimum holding, reduce the number of stocks in the final solution and lead to a portfolio that has averagely around 260 holdings, making it more viable for a hypothetical quant equity manager. Now let's see if our alpha is kept intact after going through such implementation. This can be achieved by rerunning attribution of this constrained long/short portfolio with the five MVO factor portfolios.

Figure 30: Factor exposure of long/short portfolio



Source: Compustat, Axioma, Thomson Reuters ,IBES, Russell, S&P, Deutsche Bank Quantitative Strategy

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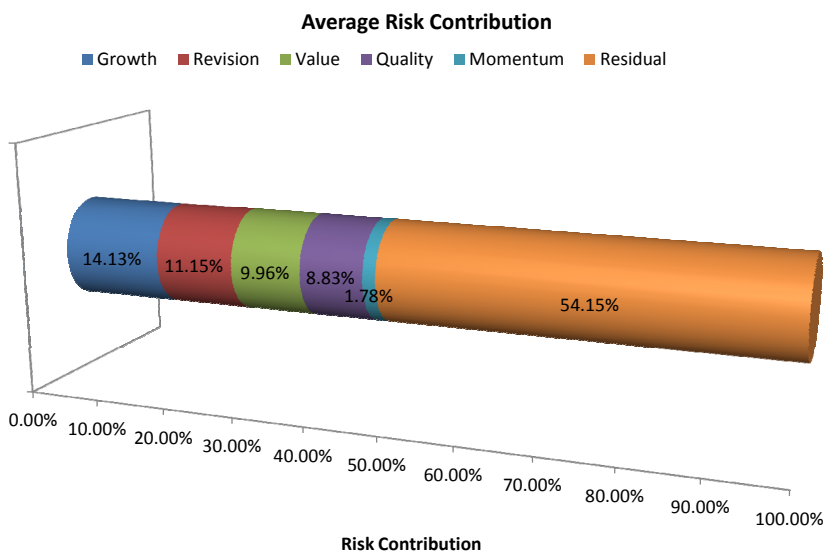
The exposures among factors are still similar except for a dip in momentum. The magnitude of exposures rises significantly, possibly due to the scaling of implied alpha from the imposed constraints. Figure 31 shows the risk contribution drops across all factors, giving away 54% residual that is independent of the component factors we used. The characteristic of the composite alpha is still preserved in a sense that the relative proportion of factor risk attributable to each style is similar to the unconstrained case. But we lose in efficiency by taking 54% of unintended risk due to constraints in implementation.

Figure 31: Risk decomposition of long/short portfolio



Source: Compustat, Axioma, Thomson Reuters ,IBES, Russell, S&P, Deutsche Bank Quantitative Strategy

Implementation efficiency

Let's compare this result with the ubiquitous measure of implementation efficiency, the Transfer Coefficient (TC). TC is the correlation of portfolio returns with the unconstrained optimal case. We can compute ex-ante TC in a holding based fashion:

$$TC = \frac{h_{ls} V h_{mvo}}{\sqrt{h_{ls} V h_{ls}} \sqrt{h_{mvo} V h_{mvo}}}$$

It is equivalent to attributing the target portfolio onto unconstrained MVO one, i.e. the ideal implementation. We plot the time series of TC in Figure 32; it averages 0.68, which translates to 54% of risk contribution from the residual. This measurement matches with our attribution result, yet the attribution analysis reveals more insights when we have multiple alpha sources, as it tracks the change in risk composition for each source in addition to just calculating the total loss in efficiency. It would be advantageous when we have constraints that impact each alpha source differently, which we'll see shortly.

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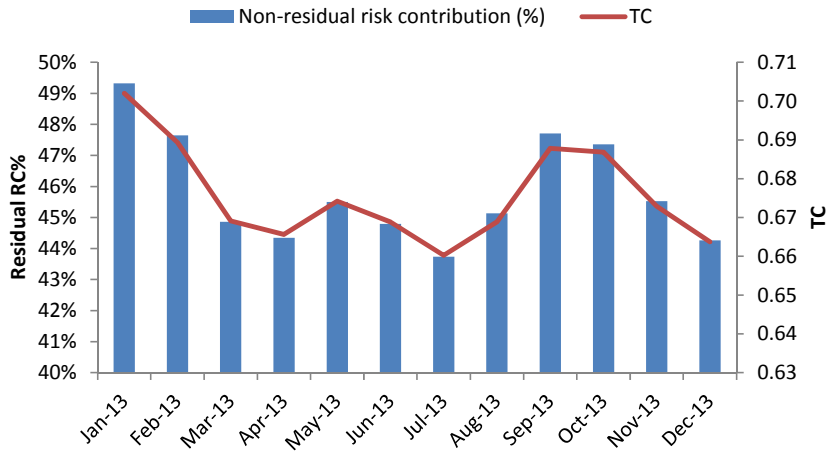
Figure 32: Transfer coefficient (TC) of long/short portfolio vs. Residual RC

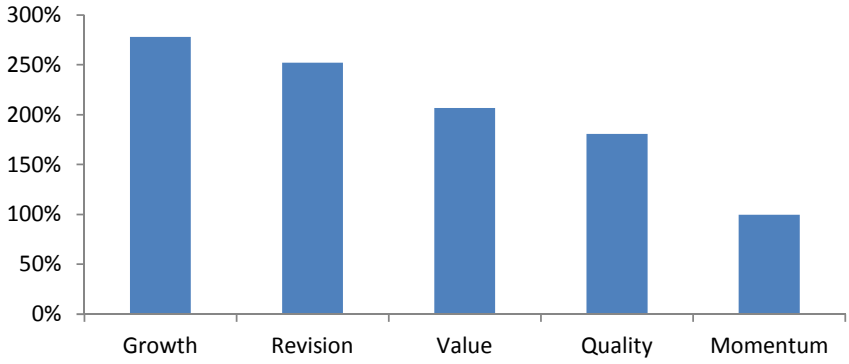


Source: Compustat, Axioma, Thomson Reuters ,IBES, Russell, S&P, Deutsche Bank Quantitative Strategy

Practical long/short portfolios with turnover constraint

The constrained long/short portfolio is not yet a tradable strategy because of its ultra high turnover. We have to constrain portfolio turnover, which would likely have uneven impact on the five style components. As shown in Figure 33, growth and revision are inherently fast signals with high turnover, while momentum in its MVO form has relatively low turnover. We therefore should expect growth and revision to be more severely penalized by a turnover constraint.

Figure 33: Average turnover of MVO factor portfolios



Source: Compustat, Axioma, Thomson Reuters ,IBES, Russell, S&P, Deutsche Bank Quantitative Strategy

Let's do a quick test by trying out a somewhat extreme example and target turnover at 30% two way per month.

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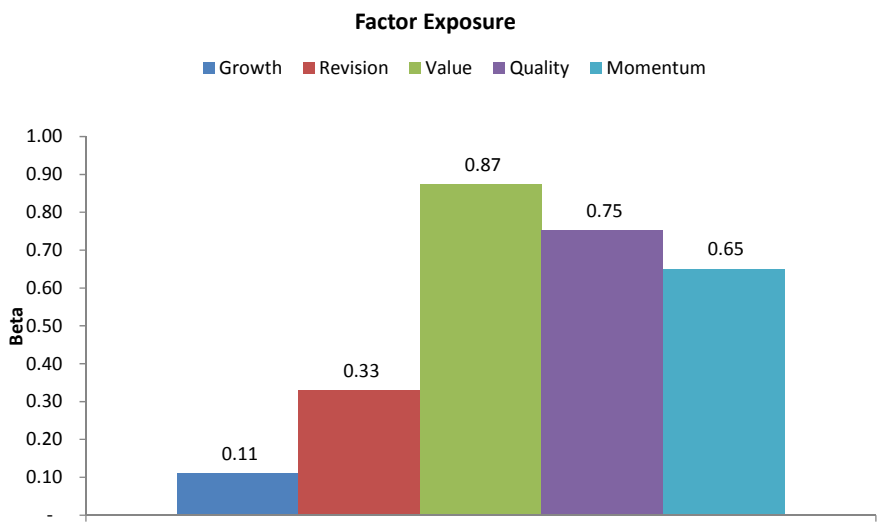
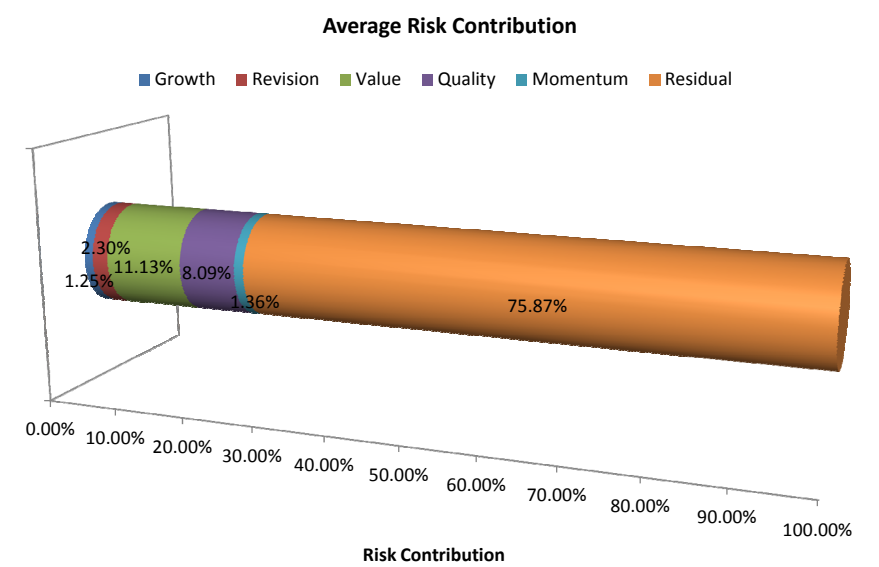
Figure 34: Factor exposure for L/S portfolio with 30% turnover constraint



Source: Compustat, Axioma, Thomson Reuters ,IBES, Russell, S&P, Deutsche Bank Quantitative Strategy

Figure 35: Risk decomposition of L/S portfolio with 30% turnover constraint



Source: Compustat, Axioma, Thomson Reuters ,IBES, Russell, S&P, Deutsche Bank Quantitative Strategy

Figure 34 and Figure 35 illustrate in this extreme case how punitive turnover constraint is to fast signals. Growth slumps from being the most exposed and top contributing risk factor to having the least exposure and accounting for less risk than momentum does. Similar is the case for revision. Value and quality, relatively volatile yet persistent, become the driving factors in alpha. Momentum sees little change in the attribution being the slowest signal among the five. The penalty taken from fast signals boosts residual risk contribution to 76%.

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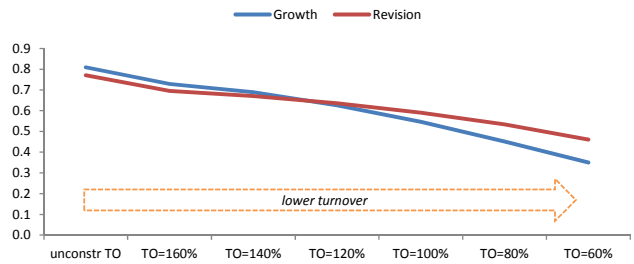
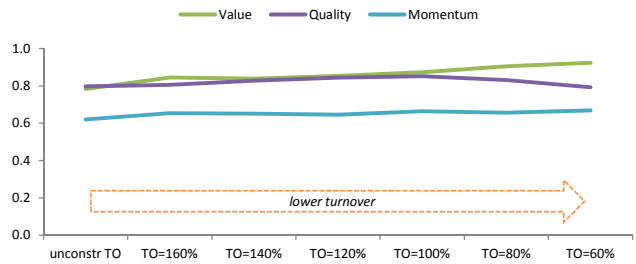
In Figure 36 and Figure 37, we can see how factor exposures and risk contributions would change when turnover is set between the range 60% to 160%. We see clearly the decline in growth and revision, and the stability of value, quality and momentum when we tighten the turnover constraint.

Figure 36: Exposures to high turnover factors



Source: Deutsche Bank

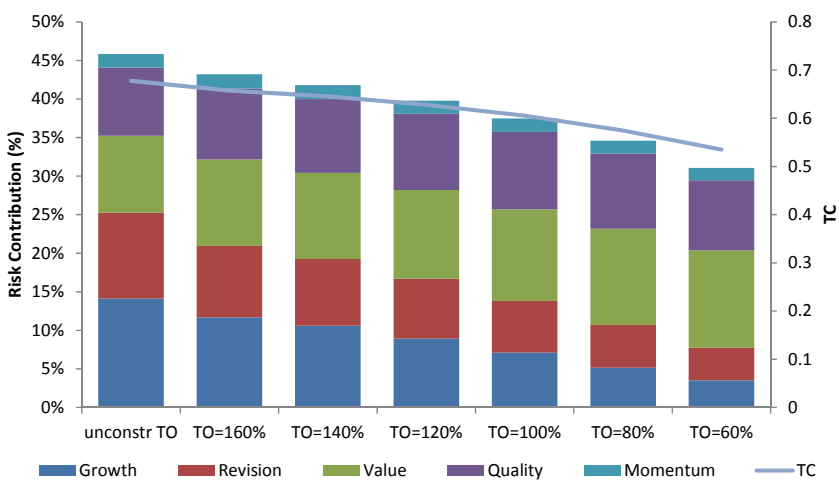
Figure 37: Exposures to low turnover factors



Source: Deutsche Bank

In addition, we can see in Figure 38 that TC declines in line with the decline in total factor risk when turnover constraint tightens. Average TC drops from 0.68 to 0.54, which is in fact lower than what the residual from attribution would imply. It's because we break the alpha down into individual components when running attribution, which leads to a better fit to target portfolio. While TC measures the lump sum loss in efficiency from optimal portfolio, the attribution analysis reveals that the loss comes exclusively from fast signals in growth and revision, and that more balanced signals in value and quality rise in importance in a turnover constrained portfolio.

Figure 38: Evolution of risk decomposition and TC with turnover constraint



Source: Compustat, Axioma, Thomson Reuters ,IBES, Russell, S&P, Deutsche Bank Quantitative Strategy

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Vintage alpha sources

We have seen how our five alpha sources fare at different stages of implementation. It's also possible to add another dimension to it and analyze alpha at different times of arrival. Suppose we want to look at how much our portfolio is loading on new information in the current month versus the stale information up to k months ago. To do that, we use the following factor portfolios:

- Initially, an ideal portfolio constructed today based on the alphas at time T-k. Call it Q(T-k)@T
- At time T-k+1, an incremental portfolio q(T-k+1) that is based solely on information in the month T-k+1. It can be derived from regression:

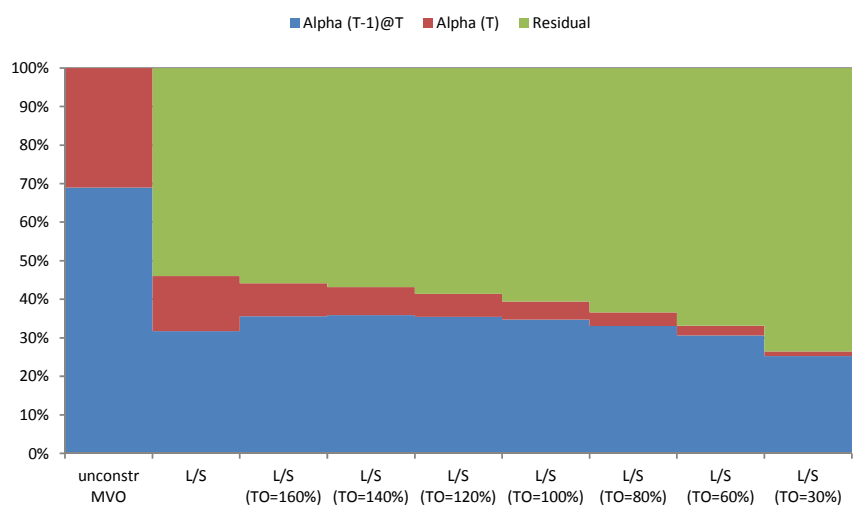
  $$Q_t(T-k+1) = \gamma(T-k+1)Q_t(T-k) + q(T-k+1),$$

  where $\gamma(T-k+1) = \dfrac{Q_t'(T-k+1)VQ_t(T-k)}{Q_t'(T-k)VQ_t(T-k)}$.

  It ensures the new positions q(T-k+1) is uncorrelated with the portfolio from the preceding month Q(T-k)@T

By construction, the unconstrained ideal portfolio would be 100% explained by this set of initial and incremental portfolios. Let's see how our portfolio is exposed to current information versus information dated back one month ago (set k=1).

Figure 39: Risk decomposition by current and lagged alpha



Source: Compustat, Axioma, Thomson Reuters ,IBES, Russell, S&P, Deutsche Bank Quantitative Strategy

We see from Figure 39 that in the ideal case with no constraint, our portfolio should have 31% loading on current information, and 69% loading on information that is over one month old. Once we put in the risk and leverage constraints, 54% is lost to residual risk, and loadings on current and lagged alphas are scaled down proportionally. The turnover constraint, however, has uneven impact on new and dated information. Loading on current alpha shrink monotonically as we tighten the turnover constraint, whereas loading on lagged alpha only change slightly.

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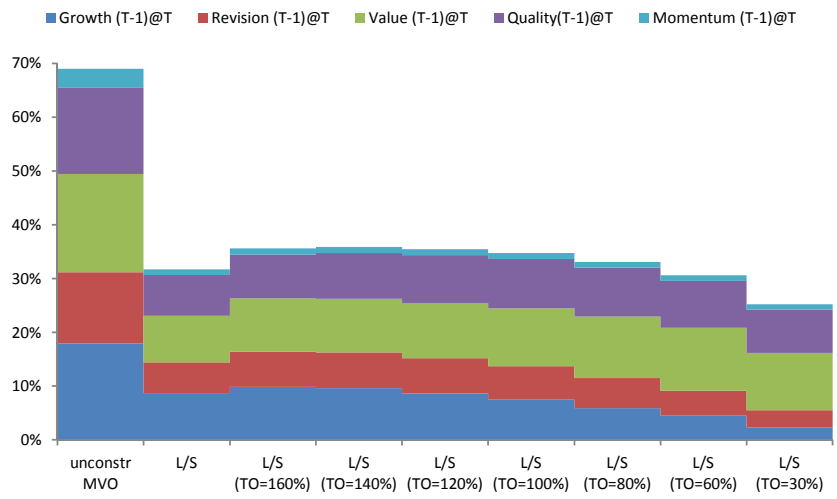
A closer look at the blue portion representing lagged alpha reveals that the small declines come exclusively from fast signals in growth and revision (see Figure 40). Risk contributions from lagged value, quality and momentum are almost unchanged over strengthening turnover constraints. It's a pattern that highly resembles what we observed in the last section. Note that a loose turnover constraint actually increases risk contribution from lagged alphas.

Figure 40: Risk contribution from lagged alphas



Source: Compustat, Axioma, Thomson Reuters ,IBES, Russell, S&P, Deutsche Bank Quantitative Strategy

Figure 41 looks at the current portion of alphas. We can do a quick comparison between Figure 40 and Figure 41. Looking at the numbers for each factor, we see that for fast signals, about half of the risk contribution comes from new positions, while slow signals mostly load on lagged positions. The current portion of our alpha therefore mainly consists of the innovations in fast signals, i.e. growth and revision. Note that momentum actually updates fast, but it is underrepresented in the portfolio to start with.

As the turnover constraint becomes more stringent, we see a similar but more drastic pattern for current alphas. Current growth and revision signals are both heavily suppressed with more stringent turnover constraints. Value, quality and momentum constitute a small portion to start with and are hardly impacted by turnovers.

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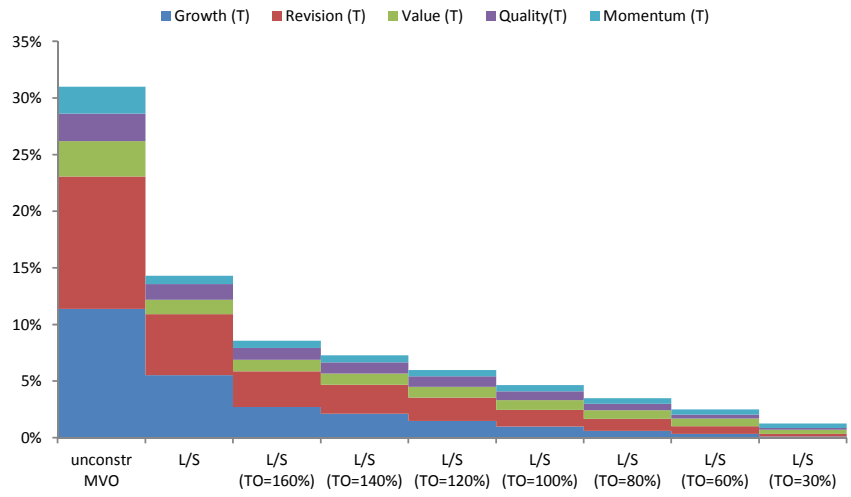
Figure 41: Risk contribution from current alphas



Source: Compustat, Axioma, Thomson Reuters ,IBES, Russell, S&P, Deutsche Bank Quantitative Strategy

In a whole, the optimal long/short portfolio without a turnover constraint has significant loadings on fast signals in the current month: 6% of risk comes from current growth and 5% from current revision. It also loads 9% on lagged growth and 6% on lagged revision, and a total 21% from mainly lagged value, quality and momentum. As we impose more stringent turnover constraints, current and lagged growth and revision are both severely undermined, resulting in the final portfolio having less loading to current information and more reliant on dated information and uninformed residual risk.

Attributing on investable factor portfolios

So far in this section we use the unconstrained MVO portfolio for each style as the factor portfolios to be attributed upon. Such a portfolio is a precise representation in terms of factor risk, but it's often unrealistic with holdings spread out across too many stocks, peculiar leverage and extraordinary turnover. We can, however, substitute it with a tradable portfolio that is constructed on that factor, thus bridging the gap between hypothetical factors and investment practice.

As an example, we now create an equal-weight long/short quintile portfolio for each factor based on its unconstrained MVO holdings. Essentially it is a long/short quintile portfolio built on risk-adjusted factor scores, as the MVO weights are the original scores adjusted by the inverse of asset-by-asset covariance matrix. We can rerun all the previous analyses using this newly constructed set of long/short quintile factor portfolios.

If we attribute the long/short portfolio with 30% turnover constraint in last sub-section onto the new set of factor portfolios, we get the factor exposures in Figure 42 and risk decomposition in Figure 43.

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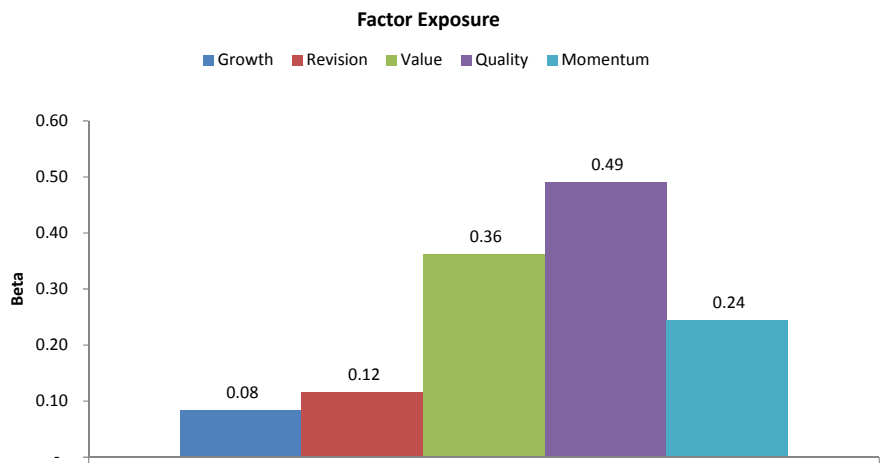
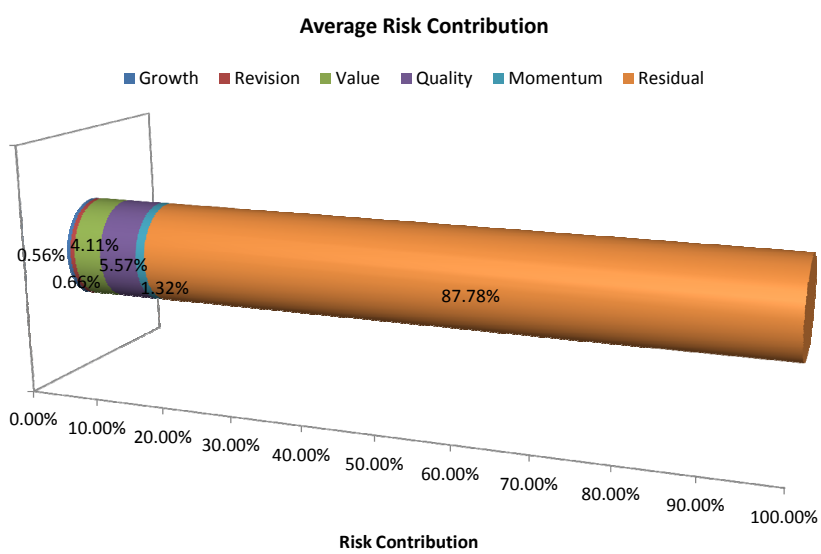
Figure 42: Factor exposures of L/S portfolio with 30% turnover constraint, with L/S quintile factor portfolios



Source: Compustat, Axioma, Thomson Reuters ,IBES, Russell, S&P, Deutsche Bank Quantitative Strategy

Exposures on five factors follow similar order as we've seen in last sub-section. As always, the magnitudes of factor exposures are not only driven by correlations but also factor volatility. Except for growth, other factors have higher volatility in this case due to our use of long/short quintile portfolios as factor representation, therefore making the exposure numbers incomparable to those in the previous case.

Figure 43: Risk decomposition of L/S portfolio with 30% turnover constraint, with L/S quintile factor portfolios



Source: Compustat, Axioma, Thomson Reuters ,IBES, Russell, S&P, Deutsche Bank Quantitative Strategy

Again in Figure 43 we see the same pattern: growth and revision shrinking, value and quality taking top spot, momentum relatively intact. Residual risk is larger and factor risk smaller, which is understandable as the target portfolio is

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constructed in MVO fashion, and it is always more effective (in terms of lowering residuals) to align the construction technique of target and factor portfolios. The approach of using tradable factor portfolios is most relevant when the target portfolio is a mix of distinctive strategies, indices, or other beta instruments.

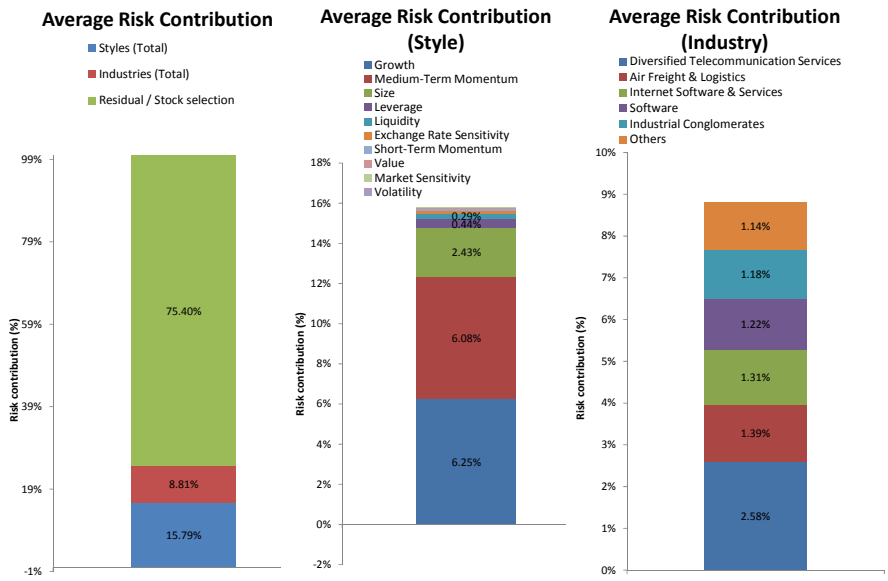
Long-only portfolio

We have seen a number of demonstrations of custom factor attribution with long/short portfolios. In this subsection we apply the same functionalities on long-only portfolios, which could be more relevant to mutual fund managers. The long-only version is constructed with our composite alpha by maximizing final alpha and constraining active risk, with the Russell 3000 index as the benchmark. We want to look into the composition of active holdings, which mimics the long/short portfolio with holdings constraints.

Standard attribution with risk vendor factors

First we run an attribution with Axioma factors (see Figure 44). 75% risk goes to residual. Growth and momentum are the top factors, but contributions from value, quality and revision are not reflected in the results.

Figure 44: Attribution with Axioma factors for long-only portfolio



Source: Compustat, Axioma, Thomson Reuters ,IBES, Russell, S&P, Deutsche Bank Quantitative Strategy

Attribution with custom factors

Now we attribute the active holdings to five component factors. As shown in Figure 45, we see that active exposures to growth, value and revision in the long-only portfolio are cut by half, while quality and momentum exposures drop slightly, compared to the unconstrained case. Again we see that long-only constraint has varying impact on different signals.

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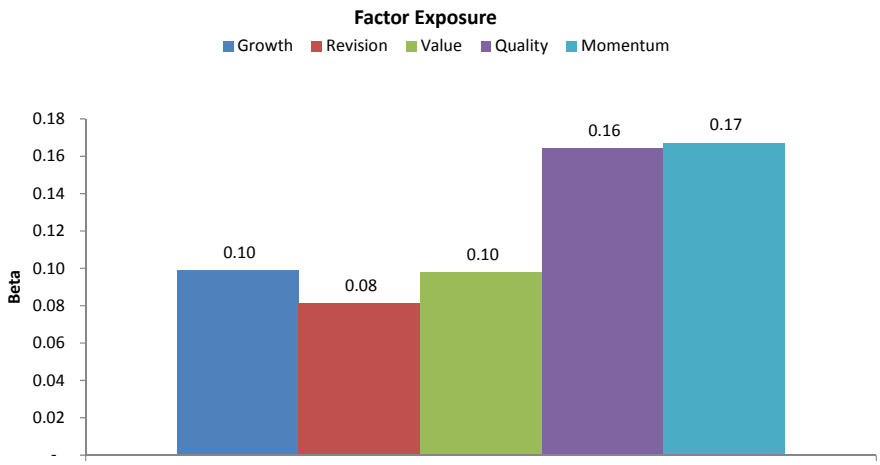
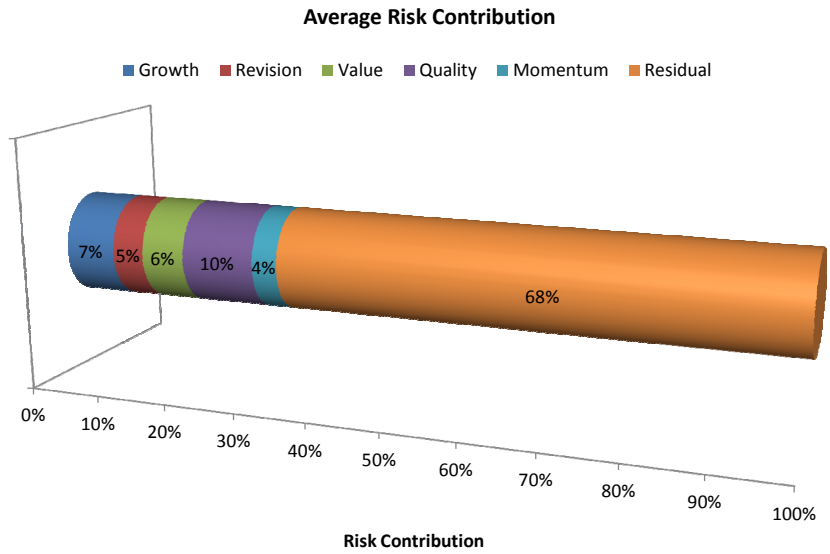
Figure 45: Factor exposure in long-only portfolio



Source: Compustat, Axioma, Thomson Reuters ,IBES, Russell, S&P, Deutsche Bank Quantitative Strategy

Risk decomposition in Figure 46 gives the makeup of active portfolio by component factors. Quality is now the most influential factor contributing 10% of active risk. Compared with the long/short version, contributions from growth, revision and value are cut by half, while contribution from quality is preserved. Moreover, momentum contribution is boosted and close to the level of ideal MVO implementation. It would lead us to hypothesize that factors like growth and revision rely very much on the short side, while momentum during this specific period is more shaped by the long side.

Figure 46: Average risk decomposition of long-only portfolio



Source: Compustat, Axioma, Thomson Reuters ,IBES, Russell, S&P, Deutsche Bank Quantitative Strategy

The residual of 68% is an indicator of our implementation efficiency. By implementing with Long-only constraint we have taken 68% of unrelated risk,

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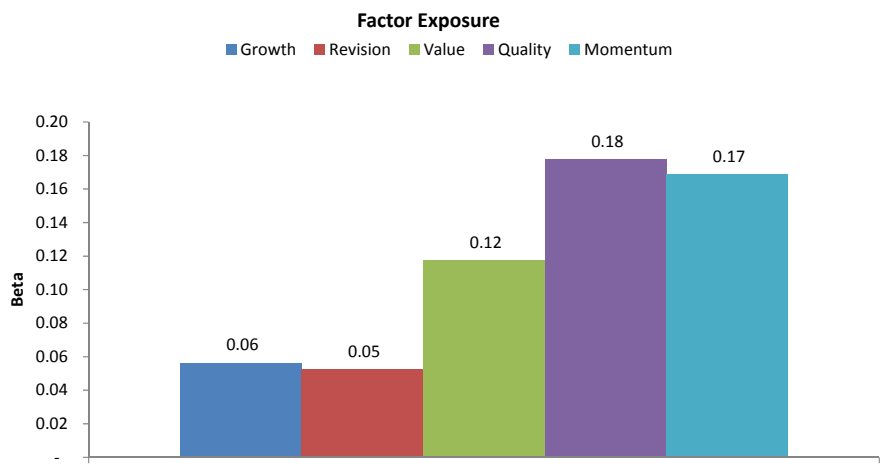
as well as overweight quality and momentum relative to the ideal implementation.

Long-only portfolio with turnover constraint

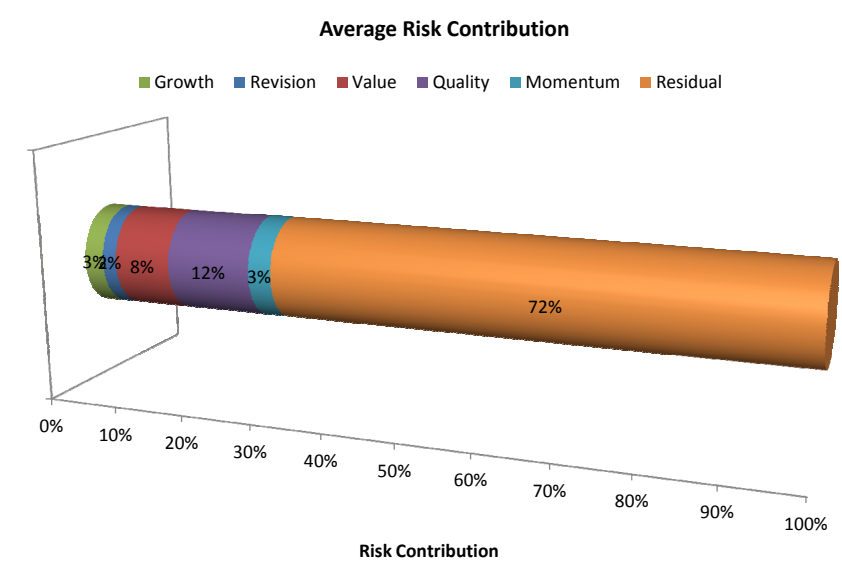
Continue with the setup, we now examine how much efficiency would be lost by imposing turnover constraint on long-only portfolio. We constrain turnover at 10% and reconstruct the portfolio. In Figure 47 Figure 48, we again see that turnover constraint further shrink the impact of growth and revision, while leaving quality and value intact.

Figure 47: Average factor exposures of long-only portfolio with 10% turnover constraint



Source: Deutsche Bank

Figure 48: Average risk decomposition of long-only portfolio with 10% turnover constraint



Source: Compustat, Axioma, Thomson Reuters ,IBES, Russell, S&P, Deutsche Bank Quantitative Strategy

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Unorthodox Applications

In this section we outline a couple of unorthodox applications of the custom factor attribution toolkit, where it aids more traditional types of practices in equity investing.

Analyzing fundamental filters in stock picking portfolios

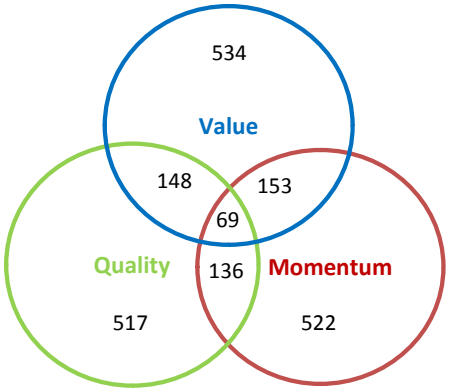
For fundamental mangers, filtering stocks based on a set of fundamental and valuation metrics is a common practice. Yet to scientifically evaluating the impact of each individual filter to the final portfolio is not a straightforward task. Here we attempt to use the attribution tool to gain some insights into the practice of stock filters.

We filter stocks in Russell 3000 universe from December 2012 to December 2013 based on three factors:

- Value: current EV over latest quarter EBITDA
- Quality: Return on Invested Capital
- Momentum: 12M return – 1M return

Suppose we equally weight all stocks in the portfolio. For each factor we keep the stocks that rank in the top quartile by that factor. By using quartiles we ensure all filters have an equal number of stocks included. The intersection of three top quartile baskets is our final portfolio, which has coverage of 69 stocks on average. Figure 49 visualizes the filter mechanism. By looking at pairwise intersections, there seem to be no distinctively strong filters in terms of number of stocks being axed.

Figure 49: Number of stocks by filter sets and intersections (average over Dec 2012 – Dec 2013)



Source: Deutsche Bank

But which one is the key filter in shaping the risk and return of the final portfolio? We can leverage on custom factor attribution to gain a bit more insights on that. Before that, let's first examine whether a standard Axioma attribution would be of any help to the problem (see Figure 50).

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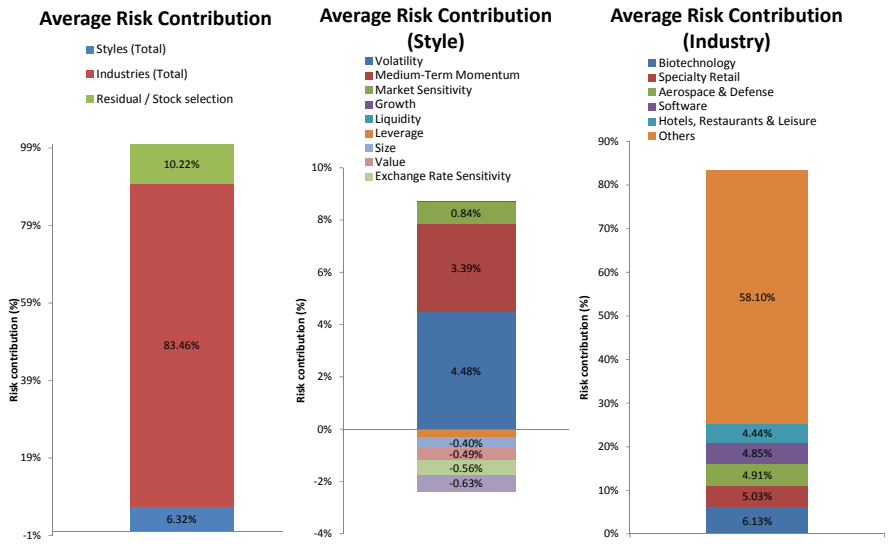
As expected, the output looks generic for a long-only portfolio, with 83% risk in industry factors and 10% in residuals. It's not easy to see the impact of three filters that are behind this portfolio.

Figure 50: Risk decomposition by Axioma factors



Source: Compustat, Axioma, Thomson Reuters ,IBES, Russell, S&P, Deutsche Bank Quantitative Strategy

Now let's look at custom factor attribution by creating factor portfolios that are representative of each stock screens. Three filter portfolios are constructed via equally weighting stocks in the three factor quartile baskets. Each filter portfolio therefore represents the stocks included by the corresponding filter.

We attribute the target portfolio to the three filter portfolios. Figure 51 shows the average risk decomposition, followed by the time series in Figure 52. These results show that if we decompose the risk of final portfolio by three filters, Momentum and Quality filters contribute nearly 100% combined, while Value filter contributes little or negatively to total risk. It suggests the final portfolio is more correlated with Momentum and Quality baskets than to Value basket. If we include more stocks from Value basket in periods like Nov 2013, when Value shows a negative risk contribution, we would likely be diversifying away the risk in the final portfolio.

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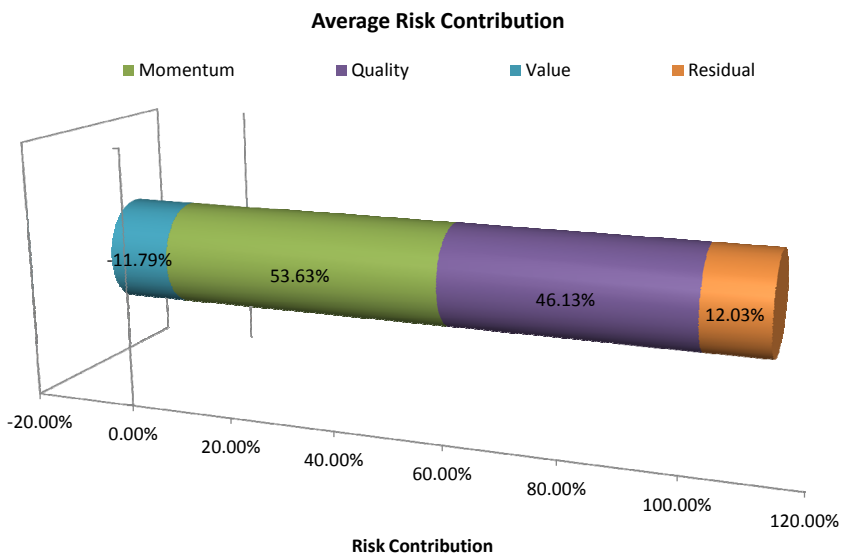
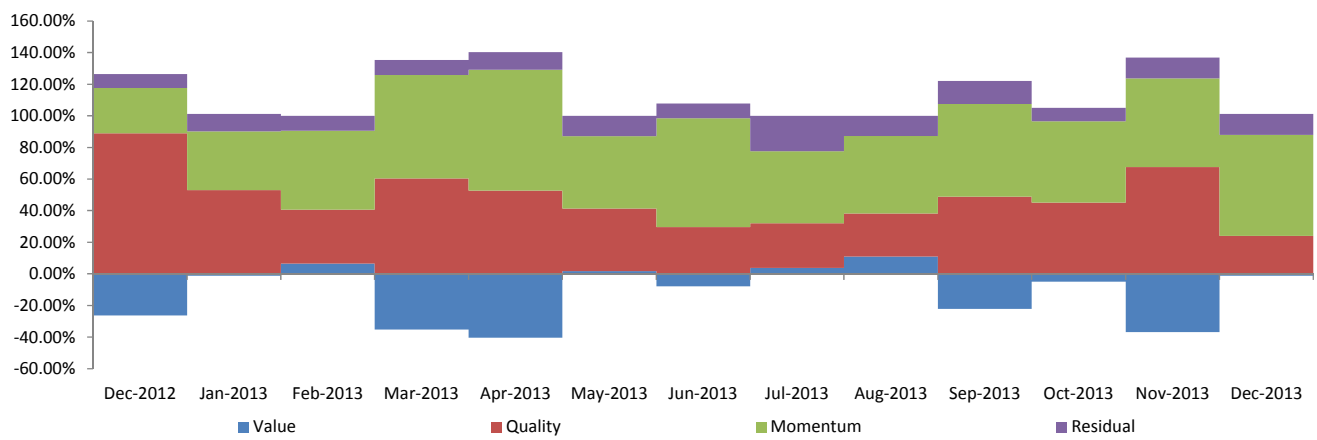
Figure 51: Average risk decomposition by filter portfolios



Source: Compustat, Axioma, Thomson Reuters ,IBES, Russell, S&P, Deutsche Bank Quantitative Strategy

Figure 52: Risk decomposition by filter portfolios over time



Source: Compustat, Axioma, Thomson Reuters ,IBES, Russell, S&P, Deutsche Bank Quantitative Strategy

Industry practice

For a long time the prevalent practice for attribution in the investment world is built on return series. The core of return based approach is the cross sectional regression:

$$r_i^t = \Sigma_k \beta_{i,k}^t f_k^t + \varepsilon_i^t,$$

where f_k^t is the factor return to be estimated. The factors included are usually a common set of systematic factors that one wishes to control, like size and value. In most cases though, it suffers from the misalignment problem as the common factors often fail to capture the alpha sources embedded in cross-

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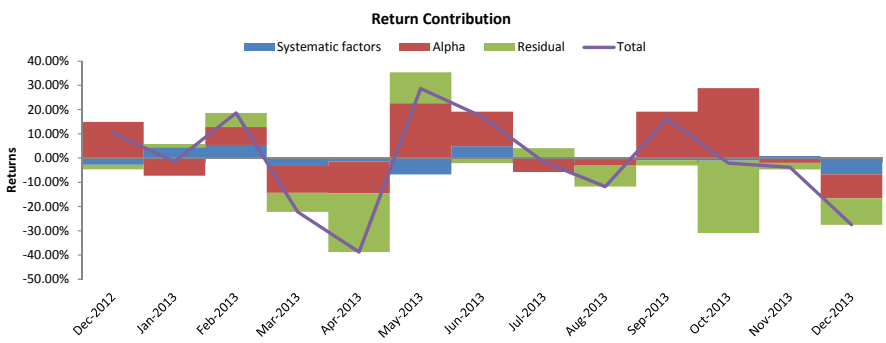
sectional returns. To investigate the contribution from alpha source, another common practice is adopted that adds the alpha factor into the mix:

$$r_i^t = \Sigma_k \beta_{i,k}^t f_k^t + \gamma^t \alpha_i^t + \varepsilon_i^t ,$$

The alpha factor would be more aligned to the returns on LHS. But it's not a radical cure and has unintended consequences. To see that, we follow this cross sectional regression approach and perform an attribution on the long/short portfolio in the five-factor example from previous section, using Axioma systematic factors and composite alpha factor. The result is shown in Figure 53. Note that in 9 out of 13 cases there is negative residual return, and on average residual has -5% return per annum. It turns out the negative residual is not a coincidence – it is seen in many similar attribution cases. The culprit is that we are including lots of redundant factors in the set of systematic factors, and they distort the regression and produce negative residuals that are uninterpretable. In such situations where we have complete knowledge of our alpha sources, it's always proper to use the custom factor approach we demonstrated in the previous sections, to gauge the risk and return decomposition of the portfolio.

Figure 53: Return (annualized) contribution of 5-Factor long/short portfolio



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Technical Appendix

From time series regression to holdings regression

Suppose we have k factor portfolios with holdings

$$S = \begin{pmatrix} fp_{11} & fp_{12} & \cdots & fp_{1k} \\ fp_{21} & fp_{22} & \cdots & fp_{2k} \\ \vdots & \vdots & \cdots & \vdots \\ fp_{n1} & fp_{n2} & \cdots & fp_{nk} \end{pmatrix}.$$

Each column represents a factor portfolio, i.e. column k is the holdings in each of the n stocks in the k th factor portfolio. Let $A = [\alpha_1, \alpha_2, ..., \alpha_n]'$ where α_i is the demeaned total return series for stock i. And let V be the $n \times n$ asset-by-asset covariance matrix, so we have $V = AA'$.

Now consider the items in the factor model (1). We can readily get the return series for portfolio and factor portfolios:

Portfolio return series $R_p = A'h$

Factor return series $F = A'S$ (the matrix form of s in regression (2))

We can compute the factor exposures via time series regression (2) (in matrix form):

$$R_p = F\beta + \varepsilon$$

where

$$R_p = \begin{bmatrix} r_P{}^1 \\ \vdots \\ r_P{}^T \end{bmatrix}, \; F = \begin{bmatrix} f_1{}^1 & \cdots & f_k{}^1 \\ \vdots & \ddots & \vdots \\ f_1{}^T & \cdots & f_k{}^T \end{bmatrix}, \; \beta = \begin{bmatrix} \beta_1 \\ \vdots \\ \beta_k \end{bmatrix}, \text{ and } \varepsilon = \begin{bmatrix} \varepsilon^1 \\ \vdots \\ \varepsilon^T \end{bmatrix}$$

The solution is

$$\beta = (F'F)^{-1} F' R_P = (S'AA'S)^{-1} S'AA'h = (S'VS)^{-1} S'Vh$$

We can get factor volatility readily from factor returns

Factor Covariance Matrix ($k \times k$) = $S'VS$

Factor volatility is the square root of the diagonal elements of $S'VS$

Factor Pairwise Covariance with Portfolio P ($k \times 1$) = $S'VP$

Portfolio Variance = $P'VP$

So risk contribution from factor is

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$RC_F = X(S'VP)/\sqrt{P'VP} = [(S'VS)^{-1}S'VP]S'VP/\sqrt{P'VP}$. RC is a $k \times 1$ vector corresponding to k factor portfolios. Since all risk contributions sum up to portfolio risk, we can get the risk contribution from residual component as $RC_e = \sqrt{P'VP} - sum(RC_F)$

In summary, given factor portfolio holdings, target portfolio holdings and asset-by-asset covariance matrix, we can perform the risk decomposition scheme outline above.

Holdings regression and cross sectional regression

Here we prove a series of results associated with factor mimicking portfolios (FMPs) by showing the equivalence between holdings regression (7) and cross sectional regression (8).

$$h = S\beta + \varepsilon \qquad (7)$$

$$\alpha = Xf + u \qquad (8)$$

Start from cross sectional return regression. Note that α is implied from portfolio holdings h . Therefore we have $\alpha = \lambda Vh$, and similarly $X = \lambda VS$ [7]. Following Grinold [2006],

$X'V^{-1}X = \lambda^2 S'VS$, and

$X'V^{-1}\alpha = \lambda^2 S'Vh$, so that

$$f = (X'V^{-1}X)^{-1}X'V^{-1}\alpha = (S'VS)^{-1}S'Vh = \beta$$

Hence the equivalence.

We strip α from factor return f to get factor mimicking portfolio,

$$FMP = (X'V^{-1}X)^{-1}X'V^{-1}$$

We can verify it with the holding based attribution by letting $S' = FMP$ and plug in (2),

$$\beta = (S'VS)^{-1}S'Vh = X'h$$

So we get back to where we started. The two way street between factor exposures and FMPs is complete.

Lastly, it may seem inconsistent that the equivalence is established in assuming MVO factor portfolios $X = \lambda VS$, while in fact we take $FMP = (X'V^{-1}X)^{-1}X'V^{-1}$ as the factor portfolios in attribution. We can reconcile that by showing both set of portfolios lead to same result in risk contribution,

Case 1: $S = (1/\lambda)V^{-1}X$,

$$VarContribution = \beta * (S'Vh) = \beta * ((1/\lambda^2)X'V^{-1}\alpha = \beta * (1/\lambda^2)X'V^{-1}\alpha$$

[7] This follows the alignment in construction approach between factor portfolios and target portfolio.

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Case 2: $S' = (X'VX)^{-1} X'V^{-1}$,

$$VarContribution = \beta * (S'Vh) = (X'h) * ((X'V^{-1}X)^{-1} X'h) = (1/\lambda) X'V^{-1}\alpha * (1/\lambda)(X'V^{-1}X)^{-1} X'V^{-1}\alpha = (1/\lambda^2) X'V^{-1}\alpha * f$$

We have shown in the equivalence of (7) and (8) that $f = (X'V^{-1}X)^{-1} X'V^{-1}\alpha = (S'VS)^{-1} S'Vh = \beta$. So we see that using MVO and FMP as factor portfolios lead to same decomposition of risk.

A drilldown on portfolio variance

We can write out the decomposition under the factor model structure (1):

$$r_P = \Sigma_k X_{Pk} f_k + \varepsilon_P$$

$$\sigma_P{}^2 = \sigma([\Sigma_k X_{Pk} f_k + \varepsilon_P],\ r_P) = \Sigma_k X_{Pk} \sigma_{k,P} + \sigma_{\varepsilon,P} = \Sigma_k X_{Pk} \sigma_k \rho_{k,P} \sigma_P + \sigma_\varepsilon \rho_{\varepsilon,P} \sigma_P$$

$$\sigma_P = \Sigma_k X_{Pk} \sigma_k \rho_{k,P} + \sigma_\varepsilon \rho_{\varepsilon,P}$$

This is a general result that requires minimum structure in portfolio return. i.e. if we segment portfolio return into an arbitrary set of s components with weights $\{w_1, w_2, ..., w_s\}$ and returns $\{r_1, r_2, ..., r_s\}$:

$$r_P = \Sigma_i w_i r_i$$

Then following exactly the same steps, we can allocate portfolio risk onto each component:

$$\sigma_P = \Sigma_i w_i \sigma_i \rho_{i,P}$$

The set of components are completely flexible. E.g. single stocks, sectors/industries, regions, strategies, allocation/selection etc., they can all be analyzed for risk contribution in this unified framework.

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Appendix 1

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