

Big Data and AI Strategies

A Practitioner's Introduction to Neural Networks

- The report aims to demystify neural networks for our readers in a practitioner-friendly way.
- The neural network architecture is explained by comparing it to the familiar linear regression model.
- Using simulated data, we demonstrate how to construct a neural network from scratch in R.
- We then move on to using real world data and examine the correspondence between neural networks and existing, well-known financial models for volatility forecasting:
 - **Feedforward neural network** vs. **ARCH**
 - **Recurrent neural network** vs. **GARCH(1, 1)**
 - **Long short term memory network** vs. **GARCH(p, q)**
- Finally LSTM is used to forecast volatility of S&P 500 and EURUSD, and its performance is compared against GARCH(1, 1).

Global Quantitative and Derivatives Strategy

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Introduction

In this report we aim to demystify neural network for our readers by putting it in a more familiar financial context. When there is already an abundance of literature on neural networks available, why the need for another report?

Compared to other fields in which neural networks have been widely applied, the problems we face in finance are relatively unique, and therefore require separate treatments. We summarize below what are, in our view, the unique challenges investors face when learning about neural networks.

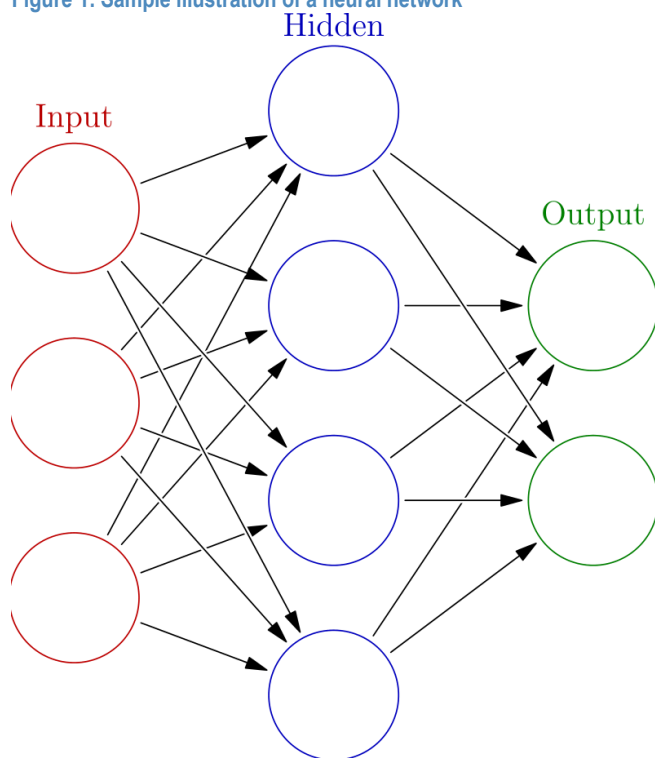
Classification vs. regression

Neural networks are most often discussed in the context of classification, where y variables are either Boolean or categorical. The vast majority of the interesting problems in finance are concerned with continuous variables and are therefore regression problems.

Financial models are not expressed as flow charts

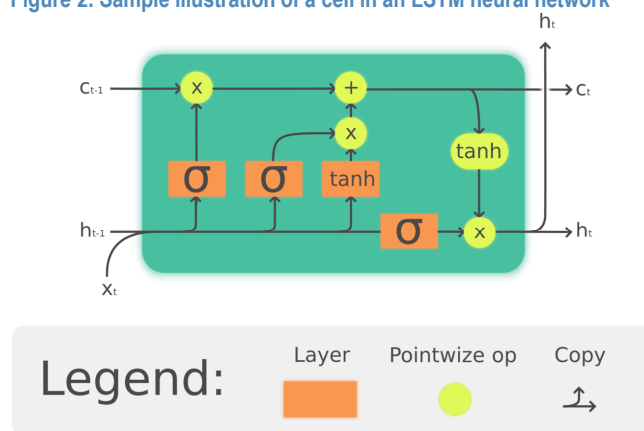
Neural networks are almost always illustrated by graphs seen in Figure 1 and Figure 2, which are unfamiliar to investors since financial models are rarely expressed as such.

Figure 1: Sample illustration of a neural network



Source: Wikipedia/Glosser.ca

Figure 2: Sample illustration of a cell in an LSTM neural network



Source: Wikipedia/Guillaume Chevalier

Unfamiliar terminology

In addition, terms such as **activation function**, **bias**, **forward** and **backpropagation** are not commonly found in our glossary, even though very similar concepts already exist in finance.

Lack of working examples

While our colleagues [demonstrated the many challenges](#) that neural networks pose when applied to forecasting equity return, finance-related examples, especially the ones that demonstrate predictive power, are few and far between.

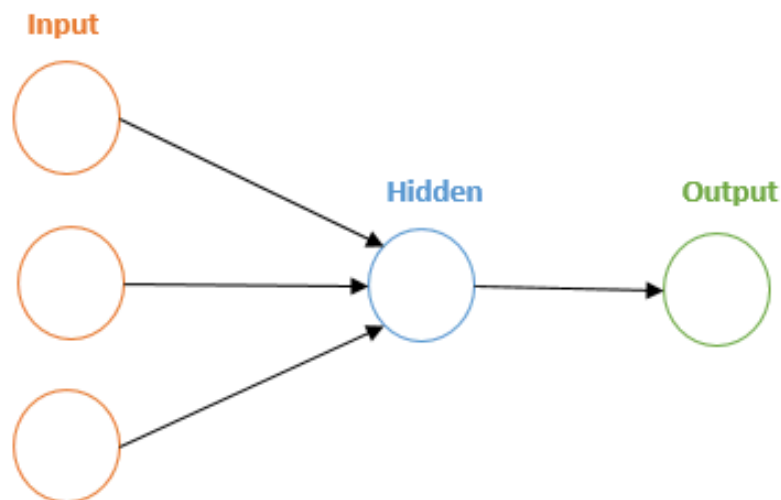
The goal of this report is to introduce neural networks in a practitioner-friendly way, taking into account the points raised above. We first explain the neural network architecture by comparing it to the familiar linear regression model. We then demonstrate how to construct a neural network from scratch in R using simulated data. Finally we move on to real world data and examine the correspondence between neural networks and existing, well-known financial models for volatility forecasting.

Revisiting the Neural Network Architecture

Linear model as a special case of neural network

How would we interpret the graphical representation of a neural network? We start out by showing that a linear regression model can be represented similarly. For instance, the model $y = \alpha + \beta_1 x_1 + \beta_2 x_2 + \beta_3 x_3$ can be seen as a neural network with one hidden layer and one hidden neuron, as shown in Figure 3:

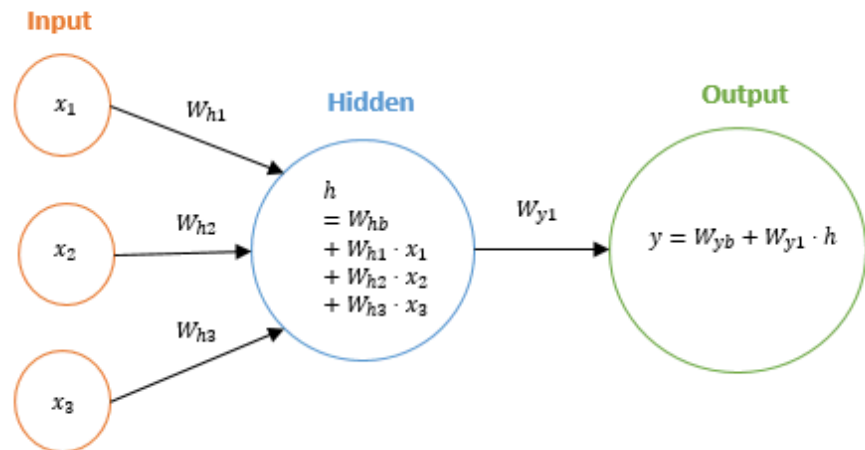
Figure 3: A neural network with one hidden neuron and one hidden layer



Source: J.P. Morgan

The **input layer** nodes (or **neurons**) are explanatory variables. The three nodes indicate there are three explanatory variables, i.e. x_1, x_2, x_3 . We can think of the arrows connecting the nodes as regression coefficients, or **weights**. Each node is a linear combination of the nodes connected into it. We may also add an intercept, a.k.a. **bias**, which may not be explicitly shown, to each node. If we fully annotate the graph, it would look like Figure 4.

Figure 4: Neural network fully annotated



Source: J.P. Morgan

The biases are denoted by W_{hb} , W_{yb} , and the weights are denoted by W_{h1} , W_{h2} , W_{h3} , W_{y1} . If we substitute in the hidden node value h into output node value y , we can see that it is exactly the same our linear model $y = \alpha + \beta_1 x_1 + \beta_2 x_2 + \beta_3 x_3$:

$$\begin{aligned} y &= W_{yb} + W_{y1} \cdot h \\ &= W_{yb} + W_{y1} \cdot (W_{hb} + W_{h1} \cdot x_1 + W_{h2} \cdot x_2 + W_{h3} \cdot x_3) \\ &= (W_{yb} + W_{y1} \cdot W_{hb}) + (W_{y1} \cdot W_{h1})x_1 + (W_{y1} \cdot W_{h2})x_2 + (W_{y1} \cdot W_{h3})x_3 \\ &= \alpha + \beta_1 x_1 + \beta_2 x_2 + \beta_3 x_3 \end{aligned}$$

Where:

$$\begin{aligned} \alpha &= W_{yb} + W_{y1} \cdot W_{hb} \\ \beta_1 &= W_{y1} \cdot W_{h1} \\ \beta_2 &= W_{y1} \cdot W_{h2} \\ \beta_3 &= W_{y1} \cdot W_{h3} \end{aligned}$$

The procedure for **training** this neural network model is the same as fitting a linear regression, where we choose the biases and weights to minimize a **loss function**, e.g. least squares.

In fact, all neural networks, regardless of the number of neurons or layers, will reduce to a linear regression, if the **activation functions** are linear, which is what we have implicitly chosen for the model above. We now take a closer look at activation functions.

Activation functions

They are functions used to introduce nonlinearity to the neural network. For instance, if we are modeling a probability and would like to restrict our output values y to be between 0 and 1, we can change the output node to:

$$y = \sigma(W_{yb} + W_{y1} \cdot h)$$

Where $\sigma(\cdot)$ is the logistic, a.k.a. **sigmoid** function. It is familiar to us in the context of logistic regression, which is used to model the probability of an event.

$$\sigma(x) = \frac{e^x}{1 + e^x}$$

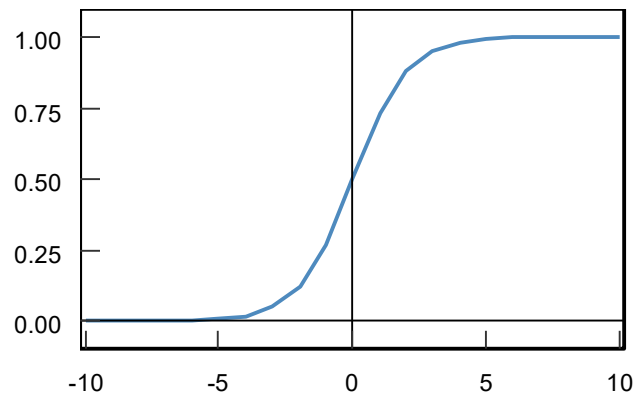
Figure 5 shows how it looks graphically. As we can see, the range of the function is strictly between 0 and 1. Also worth noting is that for x values between -5 and 5, the shape of the function is approximately linear. We will discuss the implications later in the report.

Naturally, activation functions can be applied to any layer besides the output layer. If we apply the sigmoid function to the hidden layer instead, the model will look like:

$$y = W_{yb} + W_{y1} \cdot \sigma(h)$$

In this case, the y variable can still take on all real values.

Figure 5: Sigmoid function

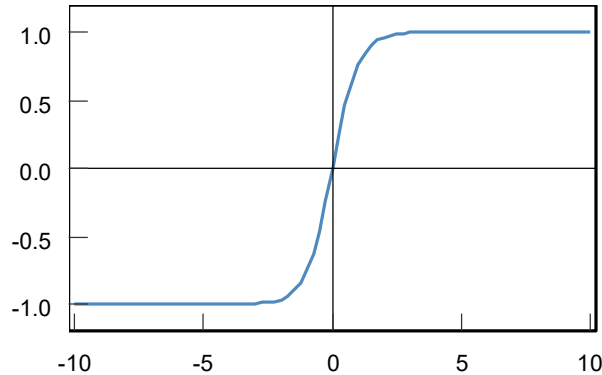


Source: J.P. Morgan

A couple of additional examples that are commonly used can be seen in Figure 6 and Figure 7. They can also be put into economic contexts. **Tanh** (Figure 6) is similar to sigmoid, but the range is between ± 1 . It may be useful for modelling portfolio weights, restricted between +100% to -100%. **ReLU** (Figure 7) may be used for modelling option strategies given its shape is exactly that of a call option payoff.

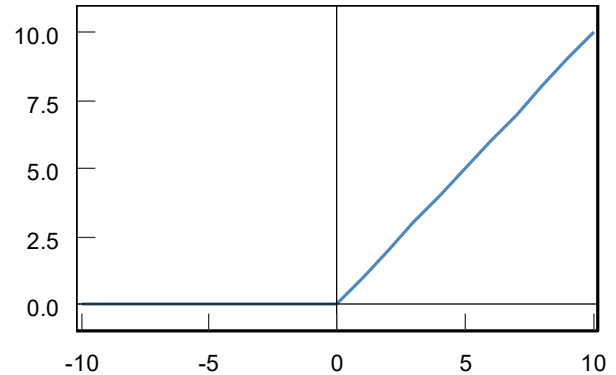
Activation functions are at the heart of neural networks. If we only use linear activation functions, then it would be no different from a linear regression. We suggest a couple of best practices for utilizing activation functions. Firstly, for regression problem, we suggest using linear activation functions in order to map the output values to all real values. Secondly, input variables (x_1, x_2, x_3) should be normalized to within reasonable domains of the activation function. For instance, if sigmoid is used, it's best to transform all x variables so that most of them fall between ± 5 (e.g. converting them into z-scores). Otherwise, very large or very small values are at risk of being truncated and useful information may be lost.

Figure 6: Tanh function



Source: J.P. Morgan

Figure 7: Rectified linear unit (ReLU)



Source: J.P. Morgan

In theory, neural networks are capable of approximating any function regardless of the choice of activation function. However, in practice it is important to be thoughtful about which activation function to use. We will come back to this topic when we demonstrate how to build a neural network from scratch.

Generalization to multiple neurons and multiple layers

Multiple neurons and layers are extensions of the aforementioned framework. In case of multiple neurons, the neural network can be expressed as follows:

$$\begin{aligned} y &= W_{yb} + W_{y1} \cdot \sigma_1(h_1) + W_{y2} \cdot \sigma_2(h_2) + \dots \\ &= W_{yb} + \sum_i W_{yi} \cdot \sigma_i(h_i) \end{aligned}$$

Where:

$$h_i = W_{ib} + W_{i1} \cdot x_1 + W_{i2} \cdot x_2 + \dots$$

If one more layer is added on, it can be expressed as the following:

$$\begin{aligned} y &= W_{yb} + W_{y1} \cdot \sigma_{11} \left(W_{1b} + \sum_i W_{1i} \cdot \sigma_{1i}(h_{1i}) \right) + W_{y2} \\ &\quad \cdot \sigma_{12} \left(W_{2b} + \sum_i W_{2i} \cdot \sigma_{2i}(h_{2i}) \right) + \dots \end{aligned}$$

By increasing the neurons/layers, the number of weight parameters increases accordingly, and the neural network becomes a massive nonlinear regression. Our goal remains choosing the weights to minimize the sum of least squares between the model and actual y values.

In Table 1, we show the number of weight parameters assuming we still have three input features, but vary the number of neurons and layers. In building neural networks we would be cautious on deploying too many neurons/layers, especially if the data set is not particularly large. While in-sample fit will inevitably increase, too many parameters increases the risk of overfitting. Additionally, the training

(optimization) process becomes much more computationally intensive and the risk of non-convergence increases.

Table 1: Number of parameters (weights) for a neural network with three input features

| | | Layers | | | | | | | | | |
|---------|----|--------|-----|-----|-----|-----|-----|-----|-----|-----|------|
| | | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
| Neurons | 1 | 6 | 8 | 10 | 12 | 14 | 16 | 18 | 20 | 22 | 24 |
| | 2 | 11 | 17 | 23 | 29 | 35 | 41 | 47 | 53 | 59 | 65 |
| | 3 | 16 | 28 | 40 | 52 | 64 | 76 | 88 | 100 | 112 | 124 |
| | 4 | 21 | 41 | 61 | 81 | 101 | 121 | 141 | 161 | 181 | 201 |
| | 5 | 26 | 56 | 86 | 116 | 146 | 176 | 206 | 236 | 266 | 296 |
| | 6 | 31 | 73 | 115 | 157 | 199 | 241 | 283 | 325 | 367 | 409 |
| | 7 | 36 | 92 | 148 | 204 | 260 | 316 | 372 | 428 | 484 | 540 |
| | 8 | 41 | 113 | 185 | 257 | 329 | 401 | 473 | 545 | 617 | 689 |
| | 9 | 46 | 136 | 226 | 316 | 406 | 496 | 586 | 676 | 766 | 856 |
| | 10 | 51 | 161 | 271 | 381 | 491 | 601 | 711 | 821 | 931 | 1041 |

Source: J.P. Morgan

Building a neural network from scratch

In this section, we show how to build a simple neural network in R from scratch, using only base R functions.¹

A neural network is made up of two parts. The choice of the number of neurons, hidden layers, and activation functions determines how x variables are transformed into y , given some fixed weights. The transformation process is known as the **forward propagation** step.

Defining the forward propagation is only half of the work, since we want to find out the weights that get us closest to the known y values. The optimization process is known as **backpropagation** for a neural network. Training a neural network involves using an initial guess of the weights, going through the forward propagation and comparing the output with the actual y values, then going back and choosing a new set of weights based on an optimization algorithm (e.g. gradient descent) and repeating the process until the results converge to actual y values within reasonable tolerances.

Backpropagation is a strictly numerical procedure which, although important, carries a lot of technical complexity but little economic intuition. Therefore, we suggest delegating this process to canned optimization routine. In practice, this may require substantial computational power depending on the complexity of the model. For the purpose of our demonstration, the *optim* function built into base R is used.

Below we show how to implement in R the forward propagation step, and use *optim* for the backpropagation.

¹ Parts of the code are modified from <https://selbydavid.com/2018/01/09/neural-network> by David Selby

We use sigmoid as the activation function for our hidden layer, and linear function for our output layer. The sigmoid is defined in Figure 8.

Figure 8: Define sigmoid function

```
1 sigmoid <- function(x) {  
2   exp(x) / (1+exp(x))  
3 }
```

Source: J.P. Morgan

We now move on to the forward propagation step. The `cbind(1, ...)` in lines 6 and 8 add the intercepts (biases). The sigmoid function is applied to the hidden node in line 7.

Figure 9: Define forward propagation function

```
5 fwdprop <- function(x, wh, wy) {  
6   h <- cbind(1, x) %*% wh  
7   h <- sigmoid(h) ###hidden layer  
8   output <- cbind(1, h) %*% wy ####output layer  
9   return(list(output = output, h = h))  
10 }
```

Source: J.P. Morgan

Similar to OLS, our loss function is defined to be least squares. The `init.w` variable is a vector which contains all the parameters including the intercepts (biases). For now we hard code the first four values to the W_h terms and the last two variables to the W_y terms, for the sake of simplicity.

Figure 10: Define loss function

```
12 loss.fun <- function(init.w, x, y) {  
13   wh = init.w[1:4]  
14   wy = init.w[5:6]  
15   y.hat <- fwdprop(x, wh, wy)$output  
16   return(sum((y - y.hat)^2))  
17 }
```

Source: J.P. Morgan

The code above constitutes a one layer, one neuron neural network model. It can be expanded relatively easily to accommodate multiple layers and neurons.

Before moving onto the backpropagation step, we first simulate some sample data in order to train the neural network. As opposed to using actual data, we are able to specify the exact data generating process in our simulation. Lines 20 – 25 generate 500 normal random variables with mean zero and standard deviation 0.1. In line 26 we define y as a linear function of x_1, x_2, x_3 , and some added noise. The neural network will attempt to use the sigmoid to fit to the linear relationship. This exercise will demonstrate the implication of the choice of activation functions.

Figure 11: Simulate in-sample data

```
19 set.seed(1) ##seed for random number generator
20 nobs <- 500
21 mymean <- 0
22 mysd <- 0.1
23 x1 <- rnorm(nobs, mean = mymean, sd = mysd)
24 x2 <- rnorm(nobs, mean = mymean, sd = mysd)
25 x3 <- rnorm(nobs, mean = mymean, sd = mysd)
26 y1 <- 1 + 1*x1 + 0.5*x2 + -0.75*x3 + rnorm(nobs, sd = mysd)
```

Source: J.P. Morgan

Given the data, we use R's built-in optimizer *optim()* to train the neural network. The first argument is the initial guess of the weights. We assign them six normally distributed random variables. The second argument is the loss function, and the subsequent arguments pass the data into the loss function.

Figure 12: Train the model using *optim()*

```
28 mysolution <- optim(par = rnorm(6), fn = loss.fun,
29   x = cbind(x1, x2, x3), y = y1)
```

Source: J.P. Morgan

We can retrieve the weights from the variable *mysolution* by examining its *par* object. However, we will not be able to recover the true coefficients since we used a sigmoid activation function in our neural network.

To perform an out-of-sample test, we simulate another 500 variables with the same data generating process, but different random variables (lines 31 – 34). In lines 35 – 37, we feed the trained parameters into the forward propagation function and obtain the predictions.

Figure 13: Simulate out-of-sample data

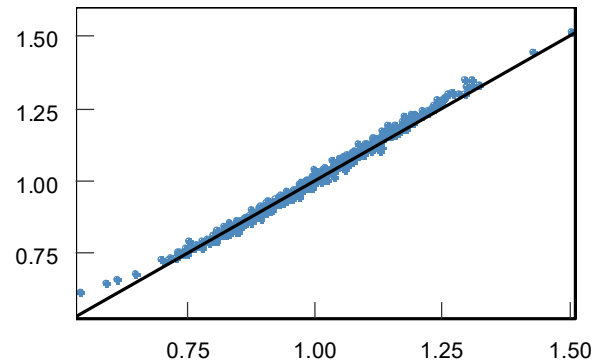
```
31 xx1 <- rnorm(nobs, mean = mymean, sd = mysd)
32 xx2 <- rnorm(nobs, mean = mymean, sd = mysd)
33 xx3 <- rnorm(nobs, mean = mymean, sd = mysd)
34 yy1 <- 1 + 1*xx1 + 0.5*xx2 + -0.75*xx3
35 mypredictions <- fwdprop(x = cbind(xx1, xx2, xx3),
36   wh = mysolution$par[1:4],
37   wy = mysolution$par[5:6])$output
```

Source: J.P. Morgan

The out-of-sample predictions appear reasonable (Figure 14), which is reassuring given the mismatch between the data generating process and our chosen activation function. We observe some nonlinearity for the more extreme values as a result of the sigmoid function.

Figure 14: Predicted vs. actual y values

Neural network predictions



Source: J.P. Morgan

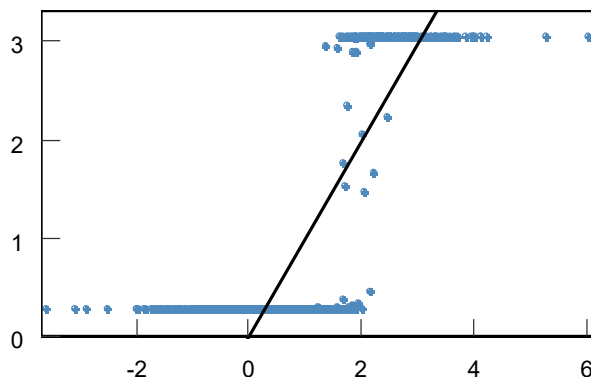
Actual values

The above exercise is of course an idealized example. In practice, to ensure a reasonable solution, there are many important considerations in the backpropagation process such as the choice of weight initialization, learning rate, etc.

Although the specifics are beyond the scope of this report, we provide a simple illustration by slightly modifying the data. We set the *mysd* value to 1 instead of 0.1 in line 22. By having variables with a wider dispersion and larger magnitude, we end up having a number of observations falling outside of the linear region of the sigmoid function (i.e. between +5 and -5 as shown in Figure 5). As a result, the fit becomes much worse (Figure 15). There are a couple of ways to remedy the problem. One option, as discussed previously, is to normalize our data to make sure that it works well with the activation function. Another option is to use more sophisticated optimization function. Here we choose simulated annealing, which is a stochastic gradient descent method, and can see that the fit is vastly improved (Figure 16).

Figure 15: Predicted vs. actual y values using Nelder Mead algorithm

Neural network predictions

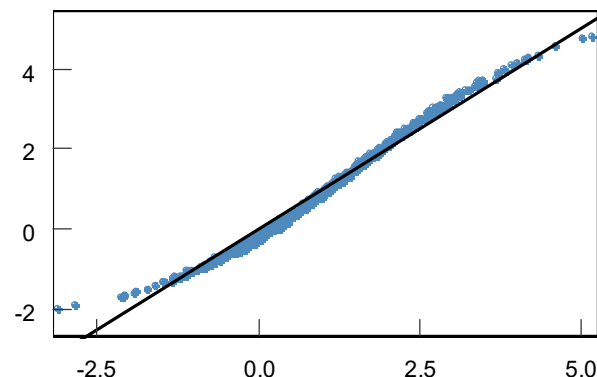


Source: J.P. Morgan

Actual values

Figure 16: Predicted vs. actual y values using simulated annealing

Neural network predictions



Source: J.P. Morgan

Actual values

Fortunately, in our code, simulated annealing only involves one additional parameter (*method* = 'SANN' in Figure 17).

Figure 17: Extra argument required for simulated annealing

```
28 mysolution <- optim(par = rnorm(6), fn = loss.fun,
29 x = cbind(x1, x2, x3), y = y1, method = 'SANN')
```

Source: J.P. Morgan

Case study: volatility forecasting

Moving on to real world examples, some of the more sophisticated models in finance are also special cases of neural networks. Specifically, we show that there is a one to one correspondence between **ARCH/GARCH** used in volatility forecasting and **feedforward/recurrent neural networks**, respectively. For simplicity, all neural networks shown in this section consist of one hidden layer with one neuron. Adding more neurons and layers does not alter the relationships discussed.

In addition, for demonstration with real data, we use the EURUSD daily data over the last 10 years. The first six rows of the data are seen in Figure 18. *Ret1D* is the log returns of *ClosePrice*, and *Var* is *Ret1D* squared. The *VarLn* variables are *Var* lagged by *n* days. Here we lag the data for up to five days for our ARCH(5) model below.

Figure 18: Define sigmoid function

| | CloseDate | ClosePrice | Ret1D | Var | VarL1 | VarL2 | VarL3 | VarL4 | VarL5 |
|---|------------|------------|---------------|--------------|--------------|--------------|--------------|--------------|--------------|
| 1 | 2009-08-10 | 1.4140 | -0.0061339072 | 3.762482e-05 | 8.529179e-05 | 6.270020e-06 | 8.148381e-07 | 1.926603e-08 | 1.458829e-04 |
| 2 | 2009-08-11 | 1.4131 | -0.0006366949 | 4.053804e-07 | 3.762482e-05 | 8.529179e-05 | 6.270020e-06 | 8.148381e-07 | 1.926603e-08 |
| 3 | 2009-08-12 | 1.4211 | 0.0056453470 | 3.186994e-05 | 4.053804e-07 | 3.762482e-05 | 8.529179e-05 | 6.270020e-06 | 8.148381e-07 |
| 4 | 2009-08-13 | 1.4277 | 0.0046335382 | 2.146968e-05 | 3.186994e-05 | 4.053804e-07 | 3.762482e-05 | 8.529179e-05 | 6.270020e-06 |
| 5 | 2009-08-14 | 1.4215 | -0.0043521057 | 1.894082e-05 | 2.146968e-05 | 3.186994e-05 | 4.053804e-07 | 3.762482e-05 | 8.529179e-05 |
| 6 | 2009-08-17 | 1.4077 | -0.0097554853 | 9.516949e-05 | 1.894082e-05 | 2.146968e-05 | 3.186994e-05 | 4.053804e-07 | 3.762482e-05 |

Source: J.P. Morgan

Feedforward Neural Network \Leftrightarrow ARCH

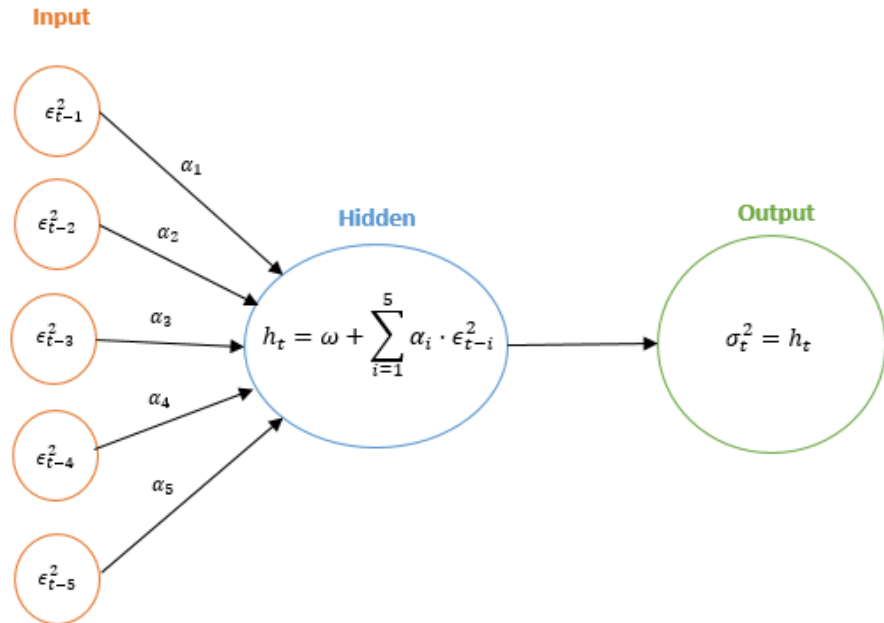
A simple ARCH(p) model is specified as follows, where $\mathcal{N}(\mu, \sigma)$ is the normal distribution. Although one can use ARCH to model asset returns (r_t), here we are mainly interested in volatility forecasting, i.e. σ_t^2 .

$$r_t = \epsilon_t, \epsilon_t \sim \mathcal{N}(0, \sigma_t)$$

$$\sigma_t^2 = \omega + \alpha_1 \epsilon_{t-1}^2 + \alpha_2 \epsilon_{t-2}^2 + \dots + \alpha_p \epsilon_{t-p}^2$$

As we can see, the ARCH model is essentially a linear model and can therefore be approximated by a simple neural network (Figure 19). It is also known as a feedforward neural network, in contrast to a recurrent neural network, which we will get to in the next section.

Figure 19: ARCH expressed as a feedforward neural network



Source: J.P. Morgan

To make the neural network consistent with ARCH, we choose the linear activation function for the hidden layer and the identity function for the output layer. Moreover, since ARCH is commonly estimated using maximum likelihood estimation (MLE), we do the same for our loss function. Specifically, we set the loss function to minimize negative sum of log likelihood, as seen below, where $f(\mu, \sigma)$ is the likelihood function of the normal distribution:

$$\sum_i -\log(f(y_t, \sigma_t))$$

Compare the code for the classical ARCH and neural network expression:

Figure 20: Classical ARCH definition in R

```
1 myARCHfit <- function(params) {
2   sigma2t <- with(eurusd, params[1] +
3     params[2] * VarL1 +
4     params[3] * VarL2 +
5     params[4] * VarL3 +
6     params[5] * VarL4 +
7     params[6] * VarL5)
8   sigmat <- sqrt(sigma2t)
9   log.likelihood <- log(dnorm(eurusd$Ret1D, 0, sigmat))
10  return(-1 * sum(log.likelihood))
11 }
12
13 init.params <- c(0, rep(0.2, 5))
14 arch <- optim(par = init.params, fn = myARCHfit)
```

Source: J.P. Morgan

Figure 21: ARCH expressed as a feedforward neural network in R

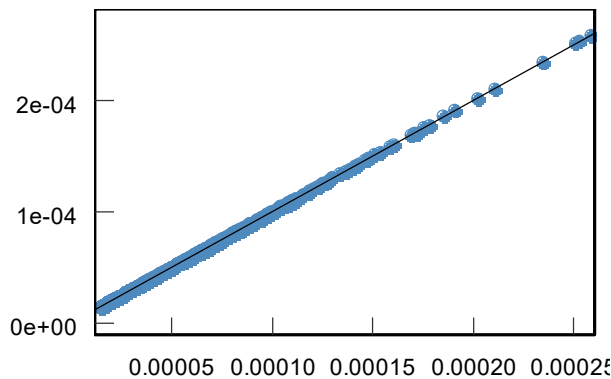
```
1 actfun <- function(x) {x} ###linear activation function
2 fwdprop <- function(x, w){
3   h <- actfun(cbind(1, x) %*% (w))
4   y <- h ###output layer: identity function
5   list(output = y)
6 }
7
8 loss.fun <- function(init.w, x, y){
9   y.hat <- fwdprop(x, init.w)$output
10  return(-sum(log(dnorm(y, 0, sqrt(y.hat)))))
11 }
12 init.guess <- c(0, rep(0.2, 5))
13
14 arch.nn <- optim(par = init.guess, fn = loss.fun,
15 x = as.matrix(eurusd[, c('VarL1', 'VarL2', 'VarL3',
16 'VarL4', 'VarL5')]), y = eurusd[, 'Ret1D'])
```

Source: J.P. Morgan

In Figure 22 and Table 2 we compare the output and parameter values, and find the two models to be identical.

Figure 22: ARCH vs. neural network output values

σ_t^2 (Neural network)



Source: J.P. Morgan

σ_t^2 (ARCH)

Table 2: ARCH vs. neural network parameter values

| | ARCH | NN |
|--------|----------|----------|
| omega | 0.000014 | 0.000014 |
| alpha1 | 0.065149 | 0.065149 |
| alpha2 | 0.277980 | 0.277980 |
| alpha3 | 0.186251 | 0.186251 |
| alpha4 | 0.195344 | 0.195344 |
| alpha5 | 0.087819 | 0.087819 |

Source: J.P. Morgan

Recurrent Neural Network \Leftrightarrow GARCH(1, 1)

It is generally accepted that ARCH with longer lags tends to produce better results. GARCH(1, 1) is a parsimonious way of parametrizing ARCH(∞).

$$r_t = \epsilon_t, \epsilon_t \sim \mathcal{N}(0, \sigma_t)$$

$$\sigma_t^2 = \omega + \alpha \epsilon_{t-1}^2 + \beta \sigma_{t-1}^2$$

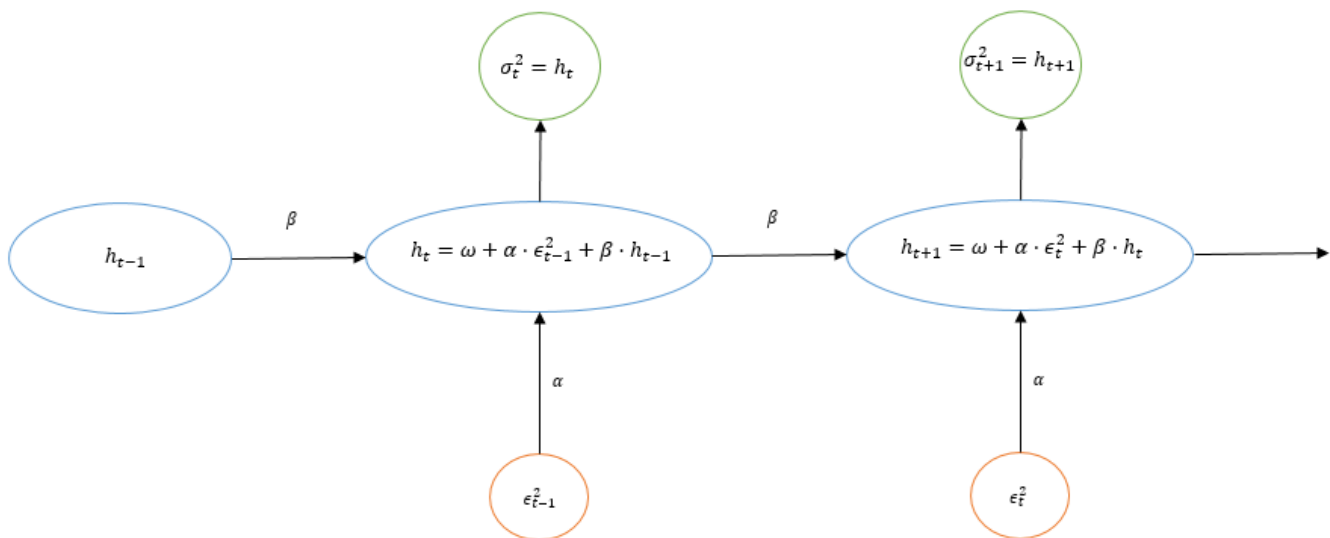
By expanding the σ_{t-1}^2 term we can see it contains all the previous ϵ_{t-i} terms, and the weights of ϵ_{t-i} decay exponentially with i .

$$\begin{aligned} \sigma_t^2 &= \omega + \alpha \epsilon_{t-1}^2 + \beta \sigma_{t-1}^2 \\ &= \omega + \alpha \epsilon_{t-1}^2 + \beta(\omega + \alpha \epsilon_{t-2}^2 + \beta(\omega + \alpha \epsilon_{t-3}^2 + \beta(\omega + \alpha \epsilon_{t-4}^2 + \dots))) \end{aligned}$$

$$= \sum_{i=0}^{\infty} \beta^i (\omega + \alpha \epsilon_{t-1-i}^2)$$

While a feedforward neural network is not able to capture the recursive structure, a recurrent neural network (RNN) does exactly that. In Figure 23, we show what happens in an RNN. The horizontal arrows no longer indicate connections between neurons, but between one time period and the next within a single neuron. The input and output layers remain the same as a feedforward neural network, but the h_t in the hidden layer will take h_{t-1} as an additional input. In other words, h_t plays the role of σ_t^2 in the GARCH model.

Figure 23: GARCH expressed as a recurrent neural network



Source: J.P. Morgan

Again compare the classical GARCH vs. the RNN expression:

Figure 24: Classical GARCH definition in R

```
1 myGARCHfit <- function(params) {
2   omega <- params[1]
3   alphaL1 <- params[2]
4   betaL1 <- params[3]
5   sigma2t <- rep(mean(eurusd$VarL1), nrow(eurusd))
6   for (i in 2:nrow(eurusd)) {
7     sigma2t[i] <- omega + alphaL1 * eurusd$VarL1[i] +
8       betaL1 * sigma2t[i-1]
9   }
10  sigmat <- sqrt(sigma2t)
11  log.likelihood <- log(dnorm(eurusd$Ret1D, 0, sigmat))
12  return(-1 * sum(log.likelihood))
13 }
14
15 init.params <- c(0, rep(0.5, 2))
16 garch <- optim(par = init.params, fn = myGARCHfit)
```

Source: J.P. Morgan

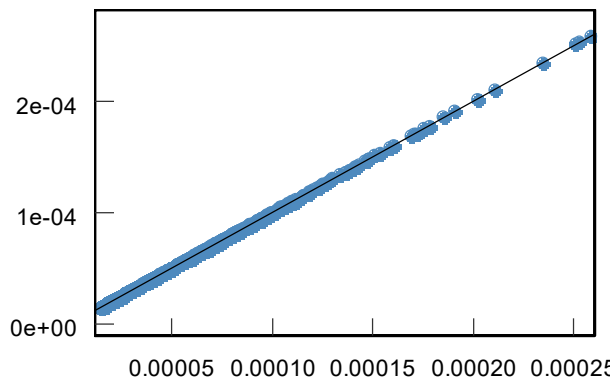
Figure 25: GARCH expressed as a recurrent neural network in R

```
1 actfun <- function(x) {x} ##linear activation function
2 rnn.fwdprop <- function(x, w){
3   h <- matrix(nrow = nrow(x))
4   h[1, 1] <- mean(x)
5   for (k in 2:nrow(h)){
6     h[k, 1] <- actfun(cbind(1, x[k, 1], h[k-1, 1]) %*% w)
7   }
8   y <- h ###identity function output
9   list(output = y)
10 }
11
12 loss.fun <- function(init.w, x, y){
13   y.hat <- rnn.fwdprop(x, init.w)$output
14   return(-sum(log(dnorm(y, 0, sqrt(y.hat)))))
15 }
16
17 init.guess <- c(0, rep(0.5, 2))
18 garch.nn <- optim(par = init.guess, fn = loss.fun,
19   x = as.matrix(eurusd[, c('VarL1')]),
20   y = eurusd[, 'Ret1D'])
```

Source: J.P. Morgan

The results are shown below and they produce the same output as expected.

Figure 26: GARCH vs. RNN output values
 σ_t^2 (Neural network)



Source: J.P. Morgan

Table 3: GARCH vs. RNN parameter values

| | GARCH | RNN |
|-------|----------|----------|
| omega | 4.84E-08 | 4.84E-08 |
| alpha | 0.026377 | 0.026377 |
| beta | 0.972016 | 0.972016 |

Source: J.P. Morgan

σ_t^2 (GARCH)

LSTM \Leftrightarrow GARCH(p, q)

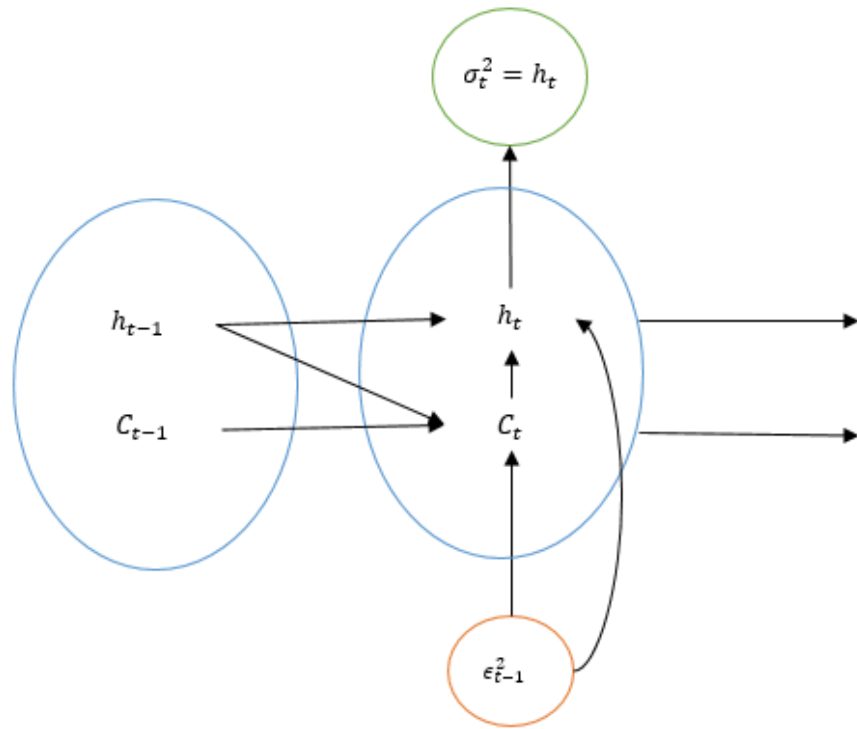
While GARCH(1, 1) is a parsimonious way of parameterizing long lags, long term memory still tends to get lost. Recall in GARCH(1, 1), the weights for ϵ_t^2 decay at a rate of β every period. The importance of the more distant historical observations will decay to close to 0 relatively quickly. For instance, in our example above, with $\beta = 0.972016$, the weights for observations more than nine months old will be close to 0.

$$\begin{aligned}\sigma_t^2 &= \omega + \alpha\epsilon_{t-1}^2 + \beta\sigma_{t-1}^2 \\ &= \omega + \alpha\epsilon_{t-1}^2 + \beta(\omega + \alpha\epsilon_{t-2}^2 + \beta(\omega + \alpha\epsilon_{t-3}^2 + \beta(\omega + \alpha\epsilon_{t-4}^2 + \dots)))\end{aligned}$$

$$= \sum_{i=0}^{\infty} \beta^i (\omega + \alpha \epsilon_{t-1-i}^2)$$

One way to remedy the problem is to introduce more lagged terms, such as GARCH(2, 2). When researchers encountered the same problem with RNN, the solution proposed was Long Short Term Memory (LSTM), which is roughly illustrated in Figure 27. In addition to the h_t time series, LSTM maintains an alternative time series C_t which allows for historical information to be retained at an alternative, perhaps slower, decay rate. In addition to the usual RNN inputs, h_t is also a function of C_t .

Figure 27: ARCH expressed as a feedforward neural network



Source: J.P. Morgan

Below we implement a basic LSTM network in the volatility forecasting framework²:

$$r_t = \epsilon_t, \epsilon_t \sim \mathcal{N}(0, \sigma_t)$$

$$\begin{aligned} \sigma_t^2 &= o_t C_t \\ C_t &= f_t C_{t-1} + i_t (\omega + \alpha \epsilon_{t-1}^2 + \beta \sigma_{t-1}^2) \\ f_t &= \sigma(w_f + a_f \epsilon_{t-1}^2 + b_f \sigma_{t-1}^2) \\ i_t &= \sigma(w_i + a_i \epsilon_{t-1}^2 + b_i \sigma_{t-1}^2) \\ o_t &= \sigma(w_o + a_o \epsilon_{t-1}^2 + b_o \sigma_{t-1}^2) \end{aligned}$$

² A more thorough explanation of LSTM can be found here: <https://colah.github.io/posts/2015-08-Understanding-LSTMs/> by Christopher Olah

While the model may appear intimidating at first, the intuition is relatively straightforward. First note that f_t and i_t are both sigmoid functions and therefore their values are restricted to between 0 and 1. C_t therefore is a weighted sum between its previous period value (C_{t-1}), i.e. long term memory, and a GARCH model ($\omega + \alpha \epsilon_{t-1}^2 + \beta \sigma_{t-1}^2$), i.e. short term memory. If $i_t = 1, f_t = 0, o_t = 1$, then the LSTM simplifies into an RNN. By making f_t, i_t, o_t (known as **gates**) dependent on ϵ^2 and σ^2 at each point in time, the weights attached to historical observations are much more varied, rather than simply decaying exponentially. If we expand the C_t term, we can see that it contains infinite lags of ϵ_t^2 and σ_t^2 . Therefore, LSTM appears to be a more generalized version of GARCH with infinite lags.

As before, we present two ways of implementing the forward propagation step and loss function for LSTM-GARCH model in Figure 28 and Figure 29.

Figure 28: LSTM-GARCH implementation in R type I

```
1 actfun <- function(x) {x} ##linear activation function
2 sigmoid <- function(x) {exp(x)/(1+exp(x))}
3 lstm.fwdprop <- function(x, w){
4   omega <- w[1]; alpha <- w[2]; beta <- w[3]
5   wf <- w[4]; af <- w[5]; bf <- w[6]
6   wi <- w[7]; ai <- w[8]; bi <- w[9]
7   wo <- w[10]; ao <- w[11]; bo <- w[12]
8
9   h <- matrix(nrow = nrow(x))
10  h[1, 1] <- mean(x)
11
12  ot <- rep(0, nrow(x))
13  ft <- rep(0, nrow(x))
14  it <- rep(0, nrow(x))
15  Ct <- matrix(nrow = nrow(x))
16
17  for (k in 2:nrow(h)){
18    ot[k] <- sigmoid(wo + ao * xdf$VarL1[k] +
19      bo * sigma2t[k-1])
20    ft[k] <- sigmoid(wf + af * xdf$VarL1[k] +
21      bf * sigma2t[k-1])
22    it[k] <- sigmoid(wi + ai * xdf$VarL1[k] +
23      bi * sigma2t[k-1])
24    Ct[k] <- ft[k] * Ct[k-1] +
25      it[k] * actfun(omega + alpha * xdf$VarL1[k] +
26        beta * sigma2t[k-1])
27    h[k, 1] <- ot[k] * actfun(Ct[k])
28  }
29  y <- h ###identity function output
30  list(output = y)
31 }
32
33 loss.fun <- function(init.w, x, y){
34   y.hat <- lstm.fwdprop(x, init.w)$output
35   return(-sum(log(dnorm(y, 0, sqrt(y.hat)))))
36 }
```

Source: J.P. Morgan

Figure 29: LSTM-GARCH implementation in R type II

```
1 sigmoid <- function(x) {exp(x)/(1+exp(x))}
2 myLSTM <- function(params, xdf) {
3   omega <- params[1]; alpha <- params[2]; beta <- params[3]
4   wf <- params[4]; af <- params[5]; bf <- params[6]
5   wi <- params[7]; ai <- params[8]; bi <- params[9]
6   wo <- params[10]; ao <- params[11]; bo <- params[12]
7
8   sigma2t <- rep(mean(xdf$VarL1), nrow(xdf))
9   ot <- rep(0, nrow(xdf))
10  ft <- rep(0, nrow(xdf))
11  it <- rep(0, nrow(xdf))
12  Ct <- rep(0, nrow(xdf))
13
14  for (i in 2:nrow(xdf)) {
15    ot[i] <- sigmoid(wo + ao * xdf$VarL1[i] +
16      bo * sigma2t[i-1])
17    ft[i] <- sigmoid(wf + af * xdf$VarL1[i] +
18      bf * sigma2t[i-1])
19    it[i] <- sigmoid(wi + ai * xdf$VarL1[i] +
20      bi * sigma2t[i-1])
21    Ct[i] <- ft[i] * Ct[i-1] +
22      it[i] * (omega + alpha * xdf$VarL1[i] +
23        beta * sigma2t[i-1])
24    sigma2t[i] <- ot[i] * Ct[i]
25  }
26  sigmat <- sqrt(sigma2t)
27  log.likelihood <- log(dnorm(xdf$Ret1D, 0, sigmat))
28  return(-1 * sum(log.likelihood))
29 }
```

Source: J.P. Morgan

Training an LSTM network becomes considerably more computationally intensive, and therefore we use a more robust optimization library *Rsolnp* which supports parallel processing. Without going into too much detail, the code in Figure 29 trains the LSTM network on historical daily EURUSD data.

Figure 30: Optimization function for training the LSTM

```
30 clusterExport(workers, c('sigmoid', 'myLSTM'))
31 system.time(lstm <- gosolnp(fun = myLSTM, xdf = mydf,
32   LB = c(0, 0.0, 0.0, rep(c(-5, -1.5e5, -1.5e5), 3)),
33   UB = c(0.05, 1, 1, rep(c(5, 1.5e5, 1.5e5), 3)),
34   cluster = workers))
```

Source: J.P. Morgan

LSTM vs. GARCH(1, 1)

Although it has been shown that GARCH specification with longer lags does not significantly outperform GARCH(1, 1)³, the weights in LSTM are much more flexible. To our knowledge, there is no GARCH type model with such a rich parametrization of weights⁴. Therefore, we are curious to see how LSTM performs relative to GARCH.

³ A forecast comparison of volatility models: Does anything beat a GARCH(1, 1)? – P. H. Hansen, A. Lunde, Journal of Applied Econometrics, Mar 2005

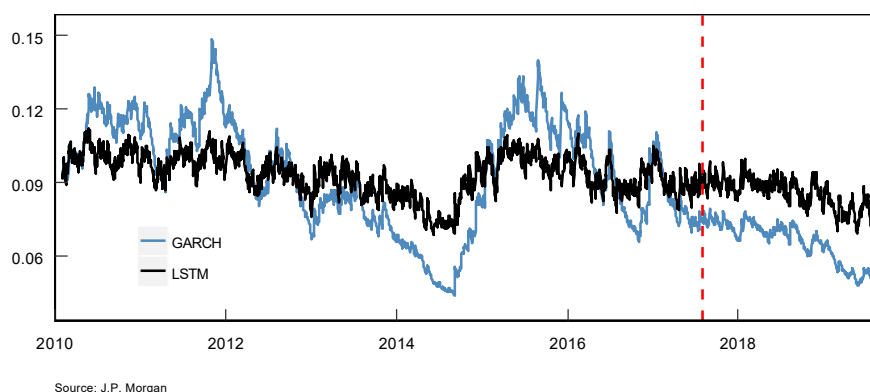
⁴ A list of GARCH family of models: <https://vlab.stern.nyu.edu/docs/volatility> by Robert Engle

Using the last ten years' daily data on EURUSD exchange rate and S&P 500, we benchmark the performance of LSTM against GARCH(1, 1) for forecasting day-ahead volatility.

Specifically, we divide the historical data into **in-sample (first eight years)** and **out-of-sample (most recent two years)**. Both models are fitted to the in-sample data, then the parameters are used to make predictions on out-of-sample data. When analyzing the results below, we leave out the first six months of in-sample data as the 'burn-in' period.

In Figure 31 and Figure 32 we show the LSTM and GARCH(1, 1) outputs for each asset, both in- and out-of-sample (delineated by the red dotted line). Interestingly, we see that for EURUSD the LSTM outputs have highly persistent long term memory. This is also confirmed by inspecting the gate values (Figure 33): f_t which is the importance attached to long term memory is very close to 1, and i_t , the importance of short term memory, is close to 0.

Figure 31: EURUSD volatility forecasts



For S&P 500, the picture is quite different. LSTM and GARCH output very similar values. Again by inspecting the gate values, our observations are confirmed (Figure 34): long term memory weights are close to 0 and short term memory weights are close to 1.

Figure 32: S&P 500 volatility forecasts

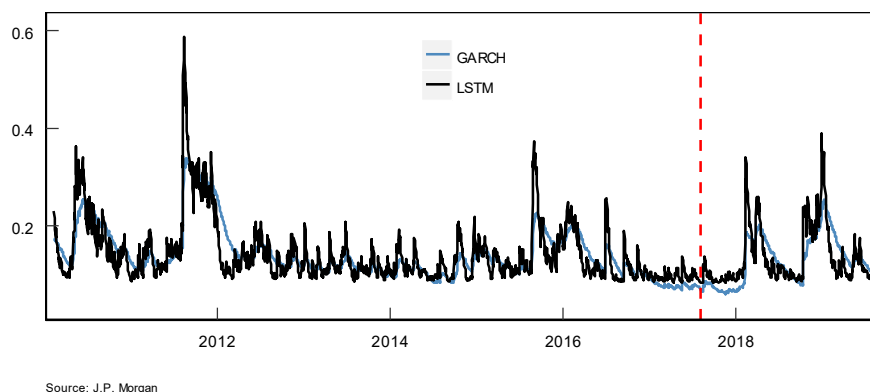
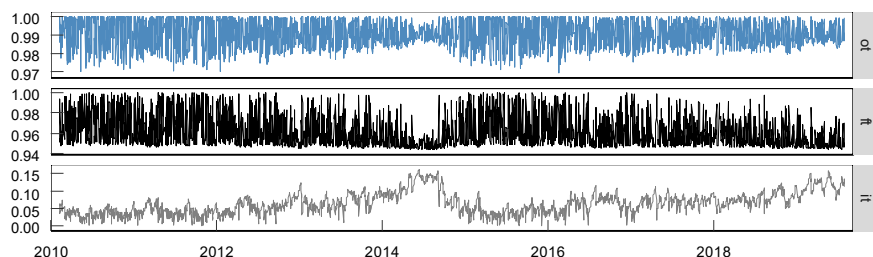
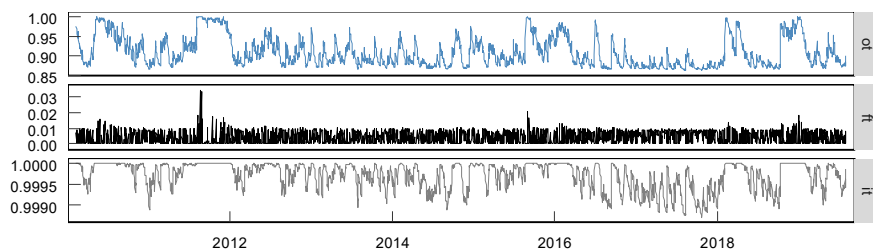


Figure 33: LSTM gate values for EURUSD



Source: J.P. Morgan

Figure 34: LSTM gate values for S&P 500



Source: J.P. Morgan

We consider several metrics to quantify the performance: sum of absolute deviation (SAD), sum of squared errors (SSE), R^2 of regressions between actual and output values, and finally, Akaike information criterion (AIC). SAD takes the absolute value of errors and therefore is robust to outliers. On the other hand, SSE and R^2 take the square of errors and are overweight large errors. AIC measures the sum of likelihood (which is our loss function), after adjusting for the number of parameters. Since GARCH has only 3 parameters and LSTM has 12, such an adjustment is meaningful, in our view.

The results are shown in Table 4. The preferred model is one with smaller out-of-sample values in all three metrics. In general, we find LSTM performs in-line to slightly better than GARCH(1, 1) in forecasting out-of-sample S&P 500 volatility. On the other hand, GARCH(1, 1) does better in forecasting out-of-sample EURUSD volatility.

Table 4: Performance metrics comparison

| | S&P 500 | | EURUSD | |
|-------------|-----------|---------------|-----------|---------------|
| SAD | In-sample | Out-of-sample | In-sample | Out-of-sample |
| GARCH(1, 1) | 0.183 | 0.0441 | 0.0728 | 0.00778 |
| LSTM | 0.182 | 0.0447 | 0.0726 | 0.01120 |
| SSE | In-sample | Out-of-sample | In-sample | Out-of-sample |
| GARCH(1, 1) | 8.90E-05 | 1.94E-05 | 7.78E-06 | 3.13E-02 |
| LSTM | 8.31E-05 | 1.86E-05 | 7.90E-06 | 6.48E-02 |
| R2 | In-sample | Out-of-sample | In-sample | Out-of-sample |
| GARCH(1, 1) | 0.105 | 0.051 | 0.0420 | 0.0210 |
| LSTM | 0.170 | 0.100 | 0.0347 | 0.0090 |
| AIC | In-sample | Out-of-sample | In-sample | Out-of-sample |
| GARCH(1, 1) | -13518 | -3490 | -15558 | -4296 |
| LSTM | -13523 | -3489 | -15450 | -4193 |

Source: J.P. Morgan

Conclusion

In this report we aim to bridge the gap between neural networks and commonly used models in finance, progressively from linear regression to GARCH(p, q). These parallels not only help us understand the inner workings of neural networks, but also allow us to better judge the potential effectiveness of neural networks. For instance, RNN and LSTM appear useful for volatility forecasting, since they share key characteristics with GARCH, which have been proven to work well in volatility forecasting.

At the same time, this exercise also offers potential ways to improve the existing finance model. The rich parametrization of LSTM weights, for example, may be incorporated into GARCH to capture the long memory property of certain volatility time series. As we have shown above, the LSTM enhancements show promising signs in out-of-sample testing and may be of further research interest.

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