# PORTFOLIO OPTIMIZATION

## Barclays Capital Portfolio Optimizer in POINT

Anuj Kumar
+1 212 526 8058
anuj.kumar@barcap.com

www.barcap.com

The Barclays Capital Portfolio Optimizer, available through the POINT system, provides advanced portfolio construction, hedging, and re-balancing capabilities. The optimizer creates optimal portfolios by automatically trading off risk, return, transaction costs, and other portfolio analytics using client-specified preferences. Users can enforce a diverse set of constraints on the portfolios, reflecting their investment goals and regulatory requirements.

The optimizer is integrated with the rest of the analytical tools in POINT, including the Global Risk Model (GRM), benchmark indices, and quantitative analytics based on state-of-the-art pricing engines. It supports a diverse universe of tradable securities including cash bonds, equities, options, futures, and other derivatives and allows users to incorporate their market views (e.g., security-level return forecast, analyst ratings, etc.) in the optimization process. Investors can control portfolio leverage, turnover, budget, notional, number of positions, and number of trades. In addition, the optimizer supports generic sector constraints on a number of portfolio analytics.

The optimizer is a large-scale quadratic constrained quadratic programming (QCQP) solver utilizing state-of-the-art optimization technology with support for a number of non-convex and combinatorial constraints. We believe our generic approach is superior to optimization tools based on traditional mean-variance frontiers and gradient descent approaches. It operates at the security level and recommends optimal trades in investable securities. It supports large optimization universes and runs interactively, with a turnaround time of minutes. It can also be programmed through the POINT batch processing engine to deliver trade lists at a given sequence of dates, essentially defining the evolution of optimal portfolios over time and facilitating the back-testing of portfolio construction strategies.

The optimizer allows users to construct optimal multi-currency portfolios, long-short portfolios, and overlay strategies. It incorporates the relationships between diverse asset classes (e.g., fixed-income and equities) to construct optimal portfolios. Users can easily construct optimal index-tracking portfolios (e.g., minimizing tracking error volatility with few positions), portfolios maximizing risk-adjusted expected returns, and portfolios targeting specific risk factors or in specific market sectors. The optimizer can be used to optimally and partially liquidate a benchmarked portfolio to generate cash outflows or invest new cash injections while controlling trading costs and turnover. It can be used to construct optimal hedges using derivatives and tradable portfolio instruments to target a specific component of risk.

Albert Desclee, Anthony Lazanas, Bruce Phelps and Antonio Silva contributed significantly to the writing of this report. We would like to thank Fatih Karakurt, Mark McQueeney, Jesus Ruiz-Mata, Basiru Samba, Ken Yanovsky, and Gary Wang, who contributed to the implementation of the optimizer in POINT.

# CONTENTS

# LIST OF FIGURES

# I. THE OPTIMIZATION APPROACH

## 1. Introduction

Managing and constructing global diversified portfolios is a complex task that comprises adopting long- and medium-term views in different markets and implementing them with regular trading activity. With the globalization and increasing complexity of investment opportunities, portfolio management systems have become indispensable in the investment management process. Barclays Capital POINT is an industry-leading global, multi-asset class portfolio management system. POINT provides advanced quantitative portfolio analytics and functions, including portfolio risk (e.g., tracking error volatility and value at risk), performance attribution, and scenario analysis. In addition, POINT provides an advanced portfolio optimization tool that is integrated with the rest of its functionalities to assist portfolio managers in constructing, rebalancing, and hedging their portfolios.

To help both internal and external clients navigate the daunting landscape of portfolio optimization, we have developed a tool that creates and analyzes optimal portfolios in a fast, flexible, reliable, scalable, and user-friendly fashion. Over the past two decades, optimization tools that use risk models, quantitative analytics, and alpha models to forecast (and optimize) investment performance have become increasingly popular for portfolio construction, rebalancing, and hedging.

- Portfolio construction deals with designing a model portfolio that satisfies certain prescribed criteria. Constructing a portfolio with few securities that has a low tracking error against a specified benchmark is one such example. Synthesizing a factor-mimicking portfolio that has exposure to only certain risk factor(s) in a fundamental risk model is another.

- Portfolio rebalancing entails recommending a small number of trades to optimize an existing portfolio, with characteristics such as controlled risk, low market impact (transaction costs), and high expected return. Frequent rebalancing is invariably required for most investment strategies to re-align the portfolio with the investment strategy as markets move over time.

- Portfolio hedging involves designing a transaction-cost-aware hedge that mitigates specific components of portfolio risk as much as possible. A portfolio manager may want to use non-cash derivative instruments such as futures, exchange-traded funds, interest rate swaps, credit derivatives, or other hedging instruments to achieve significant risk reduction by targeting specific components of portfolio risk in a cost-effective manner.

Regardless of whether these problems arise in an asset allocation context or in a security selection context, they are amenable to our optimization framework. The optimizer takes a unified approach to solve all these categories of problems in a flexible way.

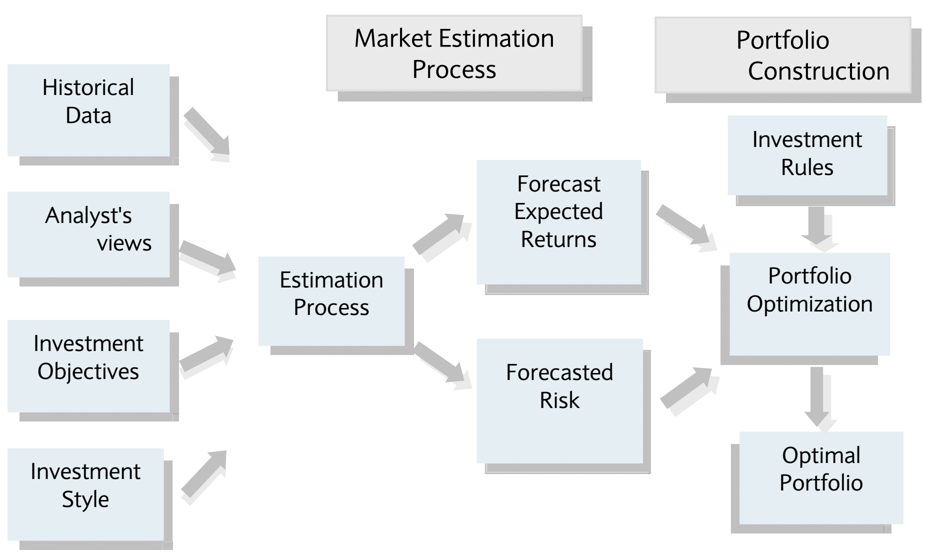
## 2. Motivation for the portfolio optimization approach

Figure 1 depicts a typical quantitative portfolio construction process consisting of three steps. In step 1, the set of investable securities is priced using algorithms to generate quantitative analytics such as sensitivities (duration, industry beta), spreads, yields, etc., and the corresponding technical infrastructure (databases, servers) to manage the analytics. In step 2, a market estimation process is set up to forecast the risk (volatilities, correlations, factor models, etc.) and expected return (excess return forecasts, outperformance rankings, etc.) of the investable securities. In step 3, an optimizer is used to construct a portfolio using the risk and expected return forecasts from step 2 by translating the investment strategy (benchmarking or total return portfolios, etc.) into a portfolio optimization problem while incorporating regulatory and business constraints.

Portfolio management skills and the quality of portfolio management tools are significant determinants of success in the portfolio construction process. Many components of the process are subjective. For example, portfolio risk can be forecasted using the factor model-based parametric approach or a scenario analysis-based approach that represents risk in a large number of market scenarios obtained by re-pricing all securities in likely future market environments. Similarly, a variety of pricing models are available to price the set of investable securities.

A qualitative approach to fixed-income portfolio construction involves manually picking a set of diversified names from a large security pool. The market views are qualitatively incorporated in this subjective process. In a quantitative approach, managers may have preferences over quantitative measures such as duration, spread, liquidity, etc., of the portfolio. The portfolio is still constructed manually using trial and error schemes to achieve targeted quantitative characteristics.

**Figure 1: Portfolio construction process**




Source: Barclays Capital

A portfolio optimizer automates much of step 3, making it objective by constructing "best" portfolios given the manager's preferences over the set of quantitative portfolio characteristics. Having said that, the optimizer has to be flexible in terms of the manager's ability to specify, control, and experiment with preferences to be of much help. The POINT platform facilitates all three steps discussed above. The optimizer, in particular, delivers on all points mentioned above. It is flexible and based on advanced mathematical principles, with a user-friendly interface for uploading expected return forecasts and other security characteristics.

Next, we discuss how various investment themes are amenable to the portfolio optimization process.

The goal in passive portfolio management is to track a benchmark as closely as possible, with low transaction costs and high operational efficiency. This approach can be translated into a portfolio optimization problem that minimize tracking error volatility (TEV) and imposes limits on turnover and the number of positions held in the portfolio. In contrast, the goal in active portfolio management is to outperform the benchmark by optimally trading off expected return against portfolio risk and trading costs. To enable a wider space for generating returns, an active management strategy may relax the no-leverage, strong diversification, and long-only constraints. Relatively speaking, an active portfolio can be concentrated, and its turnover can be high, so its risk and transaction costs must be carefully weighted against potential alpha to minimize performance erosion. In practice, most portfolio management strategies fall somewhere between passive (benchmarking) and active (total return) management.

An index-enhancing strategy is considered semi-active, in that it aims to find the optimal balancing point between outperforming the benchmark and keeping the tracking error low. A portable-alpha management strategy includes both passive and active management using a risk-budgeting process. Most of the investment strategies on this spectrum of passive and active management can be translated into portfolio optimization problems in which the goal is to optimize a set of quantitative performance measures while satisfying a set of portfolio constraints. Thus, we believe that most portfolio managers will find the portfolio optimizer useful.

Portfolio optimization has long been recognized for its importance thanks to the ground-breaking work of Harry Markowitz in the 1950s. The use of optimization techniques in portfolio construction has gained wide acceptance, but has not reached its full potential, in our opinion. Three major forces are making portfolio optimization more practical. First, advances in computer technology and optimization algorithms make it possible to solve a complex portfolio optimization problem with several thousand securities in a matter of seconds. Second, the financial markets have become complex and competitive, making a disciplined optimization-oriented portfolio management approach a mandate in many leading financial organizations. In equities, for example, wide acceptance of active quantitative portfolio management (Fabozzi [1998]) indicates that using sub-optimal approaches such as equal weighting and market capitalization weighting puts portfolio managers at a competitive disadvantage. Third, portfolio optimization requires a lot of potentially noisy estimates such as covariance matrices, transaction cost models, and alpha models, thereby raising concerns that optimizers amplify the estimation noise inherent in the input parameters. In the past decade, great strides have been made in reducing the noise level of these estimates using robust risk factor models, advanced constraints in the optimization process, and transaction cost models. In our risk models, we strive to reduce noise in estimates by ensuring that the risk factors satisfy the invariance principle (Lazanas [2007]) and incorporating a hierarchical approach to reduce the dimensionality of the estimation problem.

## 3. Leveraging POINT's Global Risk Model (GRM) and quantitative analytics

The "quality" of portfolios constructed using the optimizer depends heavily on the quality of quantitative analytics and risk models covering the set of investable securities. The optimizer is fully integrated with various pricing and risk models in POINT, which delivers quantitative content produced by Barclays Capital's Research. In particular, it provides access to:

- Valuation models for credit, mortgages, and interest rates securities.
- The global family of Barclays Capital benchmark indices, a large number of custom indices, and the corresponding analytics.

The platform supports a large set of currencies with extensive global security coverage, including:

- Treasuries/agencies
- Credit (investment grade and high yield)
- MBS, fixed rate and hybrid ARMs, and CMOs
- ABS/CMBS
- Interest rate and credit derivatives
- FX forwards, futures, and cross-currency swaps
- Global cash equities and equity ETFs
- Global equity derivatives and convertibles
- US municipal securities

Clients can also create user-defined instruments (UDIs) in POINT. In particular, they can create custom credit and interest rate derivatives, as well as cash bonds. POINT supports alternative asset classes (e.g., commodities) by allowing users to upload their own price (or return) history. A model to forecast the risk of such instruments is then built automatically using this price (or return) history.

In addition, POINT delivers returns and analytics based on user-defined market scenarios. Users can create joint market scenarios using shifts in multiple interest rate curves, sector credit curves, etc. Security-level scenario returns and analytics are obtained by re-pricing all securities at these market scenarios.

POINT's Global Risk Model (GRM) uses a factor-based approach to support all the asset classes mentioned above to forecast portfolio risk at monthly and daily risk horizons.

The Portfolio Optimizer is designed to leverage all of quantitative analytics and risk models supported in POINT. In particular, it allows users to:

- Implement a fixed-income global macro strategy by constructing portfolios by optimal allocation to 23 global currencies.
- Construct market-neutral long-short equity portfolios based on user-defined views on expected return and other performance measures.

- Construct liquid index replicating portfolios for a large number of Barclays and third-party benchmark indices available in POINT.
- Construct replicating portfolios based on the stratified sampling approach by matching portfolio analytics to the benchmark in multiple diversified cells.
- Construct portfolios that perform well under multiple user-defined market scenarios.
- Construct portfolios with varying degrees of leverage (using either derivatives or by taking short positions) to study the risk-return profile as a function of leverage.
- Construct portfolios with custom trade-off between security selection and asset allocation by making the asset allocation and security selection decisions jointly (e.g., using different weights for idiosyncratic and systematic risk in the optimization process).
- Implement optimal total return overlay strategies by constructing alpha maximizing sub-portfolios while controlling the tracking error of the total portfolio with respect to the core benchmark.
- Construct non-linear hedges based on the risk model, as well as standard sensitivity-based linear hedges.
- Implement sector views to optimize security selection by constructing minimal tracking error portfolios subject to specific asset allocation (sector weights).
- Perform back-testing of portfolio construction strategies using historical returns, analytics, and risk model calibrations for a diverse database of securities.

## 4. Security idiosyncratic risk in portfolio construction

The typical approach to portfolio construction is to, first decide the asset allocation based on the benchmark weights and try to match (or take active positions in) the portfolio against its benchmark along relevant dimensions (duration buckets, credit sectors, MBS pricing tiers, etc.). Since matching the portfolio along all risk dimensions is not typically possible, managers use their judgment to give relative importance to different dimensions. Once the asset allocation has been decided, the manager selects securities in each sector to satisfy the asset allocation using subjective preferences over the universe of investable securities. This process tends to be ad hoc and time consuming, but transparent. In addition, if the manager is required to control the total risk of the portfolio as measured by a specific risk model, then the process may require several tedious iterations.

A portfolio optimizer can be very useful in the above process. First, the optimizer allows users simultaneously to adhere to risk controls and to match the portfolio with the benchmark along the desirable set of analytics dimensions. If the optimizer is interactive, the manager can easily experiment with the constraints to decide on a portfolio that has acceptable risk and analytics (sector weights, betas, and durations, etc.) mismatches with respect to the benchmark. Second, the optimizer makes security selection an objective process in which the manager can make optimal decisions based on preferences (idiosyncratic risk, security attributes such as liquidity, age, etc.) over the set of investable securities. Finally, separating asset allocation and security selection is not necessarily appropriate. In practice, arguably a lot can be gained if the allocation and selection decisions are made jointly. For example, it might be sensible to overweight (or underweight) a specific sector independent of the expected return information if it has securities with low average idiosyncratic risk compared with other sectors that have securities with high idiosyncratic risk. The quantification of this tradeoff is not easy without the aid of a good risk model and an optimizer. Our optimizer explicitly models such situations based on an advanced idiosyncratic and default risk model, allowing users to control the portfolio's total risk while trading off between systematic and idiosyncratic risk if they so desire.

## 5. Practical considerations in portfolio optimization

Markovitz [1952] first introduced mean-variance portfolio optimization by representing portfolio risk as the variance of portfolio returns using historical security-by-security covariance. The Markowitz mean-variance model intends to optimize the tradeoff between expected return and total risk. Mean-variance optimization has been only marginally successful as a portfolio construction tool because of the inherent difficulty in predicting security expected returns due to estimation errors. Furthermore, portfolio characteristics other than return mean and variance are important determinants of portfolio performance and, hence, need to be incorporated in the portfolio construction process. For example, curve exposure for fixed-income portfolios and leverage for long-short equity portfolios need to be considered.

The Markovitz model assumes zero transaction costs. For institutional investors, transaction costs can be as important as return and risk. Transaction costs include adverse market movements and bid-offer costs. Transaction cost analysis should not be an afterthought and needs to be integrated into the optimization framework. In equities, transaction costs consist of two parts – the adverse movement in price (price effect) and broker commission costs. Typically, the price component is modeled by fitting order-book data to calibrate the transaction costs as a function of the order size (Zhang, Manwani, and Vandermark [2008]). In fixed-income markets, where the majority of asset classes are traded over the counter, transaction cost models are relatively simple. Measures such as total amount outstanding, age, and trading volume[1] are used as qualitative proxies for trading costs. The transaction price for buy trades is assumed to be higher than the market price, and the transaction price for sell trades is assumed to be lower than the market price by an amount equal to the security's transaction cost. More sophisticated models use historical data to establish a relationship between transaction costs, historical bid-ask spreads, market environment, and various security characteristics (e.g., Dastidar and Phelps [2009] and Edwards, Harris, and Piwowar [2007]). Users can construct optimal portfolios based[2] on the liquidity cost scores (LCS) available in POINT or their own estimates of transaction costs from any of the above models. Alternatively, quantitative proxies such as amount outstanding can be used directly.

Most money managers are judged based on the tracking error between the performance of their portfolio and a benchmark index rather than the portfolio's total return, so the benchmark universe is of vital importance to both passive and active portfolio managers. Therefore, a unified optimization framework that accommodates both total risk and tracking error risk is desirable. To that end, the optimizer allows users to specify a benchmark universe. If the manager is interested only in total risk, the benchmark universe can be defaulted to base currency cash. The benchmark can be either public (i.e., a well-defined market index) or private (i.e., a user-defined portfolio or index). Any of the benchmark indices available in POINT (e.g., US Aggregate, Global Aggregate) or any custom index/portfolio (e.g., long-short equity) can be used as a benchmark universe. This ensures that the optimizer uses high quality native index analytics for all constituent securities in the Barclays Capital's benchmark indices.

[1] For example, the average daily trading volume in individual US credit bonds, which can be used as a measure of liquidity, is available in the TRACE system.

[2] Users can request access to liquidity cost scores (LCS) as described in Dastidar and Phelps [2009] available in POINT. POINT provides an easy-to-use interface to upload security-specific custom data (e.g. estimates of transaction costs).

It is essential that the optimizer can solve large-scale problems involving thousands of securities within an acceptable amount of time. The state-of-the-art technology in POINT's optimizer allows users to use up to 5,000 tradable securities in a single optimization problem. The benchmark size is not limited, thus supporting most popular indices as benchmarks in the optimizer. The optimizer takes a predictable solution time (a few minutes) to solve such large problems.

In practice, a variety of operational, regulatory, and strategy-specific constraints need to be taken into account in the portfolio construction process. For example, the manager may want to limit the number of positions held in the target portfolio and may specify a threshold for holding or trading a security to achieve desirable operational efficiency. The top holding in the target portfolio cannot be more than five percent of the portfolio gross notional because of investment policy constraints. For an enhanced indexing strategy, managers may specify the extent to which they are willing to overweight or underweight certain risk exposures (e.g., sectors, regions, countries, etc.) without incurring an unnecessarily high tracking error. For a long-short strategy, the manager may want to be dollar neutral, beta-neutral, or duration-neutral at a sector or portfolio level. To be of practical relevance, the portfolio optimization framework needs to accommodate all these commonly used constraints. However, not all constraints are comparable in their importance or the complexity that they pose to the optimizer. The risk constraint requires quadratic formulation; thus, problems involving preferences for portfolio risk take longer to solve than problems with only linear analytics constraints. The constraint on minimum trade size requires combinatorial formulation and is much more challenging to solve than a purely continuous formulation.

In an optimization process, we want to choose a set of decision variables, guided by an objective function that we want to maximize or minimize. The set of possible decision variables is restricted by a given set of constraints. In a portfolio optimization problem, the basic decision variables are which securities to trade and how much. The objective function is the combination of various portfolio characteristics (risk, return, etc.), and the constraints are the restrictions on these portfolio characteristics (long-only, turnover, etc.). The following representation describes key elements of most practical portfolio optimization problems.

**Minimize** risk - $\lambda_r$ expected return - $\lambda_c$ transaction costs - $\lambda_p$ portfolio penalty

Subject to: Return >= a given return threshold

Risk <= a given risk threshold

Transaction costs <= a given constant

Portfolio penalty <= a given constant

Any other constraint required.

Portfolio penalty is a generic term to denote any other objective function term that is a function of the quantitative analytics or custom data; for example, penalties for trading illiquid bonds or bonds issued a long time ago. Constraints include any other portfolio restriction required because of regulatory requirements or investor preferences or imposed by the portfolio manager to improve performance.

Variations in the definitions of the terms mentioned above give rise to different categories of portfolio optimization problems. For example, the risk term can be total risk or active risk with respect to a benchmark, or it can be systematic risk or idiosyncratic risk. The return measure can be total return or excess return, etc. Similarly, values of various weights $\lambda$ s that define the objective function and the variation in the set of constraints create different categories of optimization problems; for example, risk minimization, return maximization, risk-adjusted returns maximization, etc. The optimizer is based on this unified and general formulation; this framework allows users to define and solve diverse problems simply by changing parameters and various attributes used in the problem definition.

## 6. Implementability of optimal portfolios

It is difficult for portfolio managers to express all of their preferences in the context of an optimization problem. In fact, many of the preferences are not quantifiable. As a result, managers may want to adjust the optimal portfolio constructed by the optimizer. One reason is that certain optimization parameters are highly subjective. For example, it is difficult to quantify risk aversion in an unambiguous fashion. Risk tolerance tends to be situation specific and time varying. As such, it can be hard to rank risk tolerance in a consistent fashion, let alone represent it with a single number. What is optimal for one manager can be suboptimal for another. In some cases, there are multiple decision makers, so compromise on "optimality" may be needed to reach consensus. In practice, making business sense is more important than just pursuing a mathematically perfect solution. Thus portfolio optimizer should not be thought of as a black box that makes decisions for the portfolio managers. Instead, it is a quantitative tool that can help managers in making decisions if it is flexible and used with caution.

The portfolios generated by the optimizer are only as good as their data input (sensitivity analytics, expected return estimates, analyst rankings, and the risk models). Even though risk and transaction cost models are calibrated using historical data and past performance is not indicative of future performance, these models have proven to be reasonable forecasting and control tools. Multi-factor risk models, with their low[3] dimensional quadratic structure, permit a continuum of portfolios with similar levels of systematic risk. The subset of these portfolios with a similar level of diversification (i.e., idiosyncratic risk) is also large. Hence, leaving it up to the optimizer to choose a portfolio from this large set of portfolios simply to minimize risk may lead to unstable solutions (which can change with slight perturbations in the estimated parameters). This is especially true for total return portfolios.

The optimizer supports generic portfolio constraints that can be used to selectively eliminate the dependence of the optimal solutions on specific risk model parameters. For example, in multi-currency fixed-income portfolios, currency risk constitutes a large component of portfolio risk. In a default black-box formulation, the optimizer may suggest portfolios that trade off active exposure to the fixed-income risk factor for active exposure to foreign currencies based on the risk model correlation estimates between the same. Most quantitative analysts would agree that these time-varying correlations are difficult to estimate. The optimizer provides a simple remedy for this situation: constrain the active foreign currency exposure to zero in most major currencies. It is easy to deduce that the optimal solution to this constrained problem is independent of the estimated correlations between currency and fixed-income factors. If the manager wants to implement FX positions, it can be done by optimizing a pure FX overlay (e.g., constructed using the currency forwards and futures). An example of managing FX risk can be found in Section III.

[3] The number of risk factors is typically an order of magnitude smaller than the number of securities in the universe. For example, there are 34 risk factors in our US equity risk model covering tens of thousands of stocks. It is sufficient to match the exposure to these 34 factors to eliminate systematic risk completely according to such model specifications. There are a large number of distinct portfolios with the same systematic risk profile.

Expected return forecasts are more difficult to estimate and subject to a lot of noise. The difficulties in expected return estimation are well known and have theoretical underpinnings in the efficient market theory. These problems can be alleviated using various techniques, but cannot be eliminated. Our optimizer allows users to construct portfolios based on multiple expected return profiles to alleviate sensitivity to a single expected return forecast, thus making the solution robust.

All optimization models, however sophisticated, are an approximation of market reality. There are many requirements that cannot be modeled exactly or efficiently. For example, it is normally hard to solve an optimization problem that requires portfolio risk to be exactly equal to some number because of the *non-convexity* of this constraint. For another example, the requirement that at most 'k' securities can be selected in an optimal portfolio is difficult to meet for a large-scale portfolio optimization problem because of the *combinatorial* nature of this constraint. To obtain theoretically optimal solutions in these cases can be computationally and theoretically prohibitive – especially for large-scale problems. Therefore, it should be acceptable to obtain a near-optimal solution for such optimization problems. In most cases, the near-optimal solution is far superior to what can be achieved through a manual approach, so a careful balance is needed between optimality and practicality when it comes to solving complex portfolio optimization problems.

Typically, small trades are not desirable in the trade list. Simple post-processing could lead to substantially sub-optimal solutions. For example, when managers deal with futures contracts with large sizes, there are significant differences between not trading and trading a single contract. Hence, it is desirable to be able to specify some threshold that has to be met for holding or trading a position. In addition, the manager normally wants to place a limit on the number of names held or traded to ensure operational efficiency. Our optimizer supports all these constraints. The "maximum number of securities in the final portfolio," "maximum number of trades," and "minimum trade size" constraints all require the so-called combinatorial formulation. Since obtaining a truly optimal solution in a reasonable amount of time for a mid- or large-sized problem with such constraints is unrealistic and often theoretically impossible,[4] we use a custom proprietary heuristic algorithm for these problems. We discuss the quality of solutions obtained using these heuristics in the next section.

The rest of this report is organized as follows. Section II presents the modeling framework of the optimizer and the motivation behind it. We compare our approach and its theoretical underpinnings with alternative approaches. Section III presents a number of useful and practical applications of the optimizer. Readers interested primarily in practical applications may jump to Section III. The last section features our concluding remarks.

[4] For example, consider the problem of picking trades from US Aggregate Index (approximately 10,000 securities). There are $2^{10,000}$ distinct subsets of securities in which the optimizer can recommend trades.

# II. THE PORTFOLIO OPTIMIZATION FRAMEWORK

This section presents a high-level description of POINT's optimization framework. The Barclays Capital Portfolio Optimizer is based on large-scale quadratic constrained quadratic programming (QCQP) technology, with support for a number of non-convex and combinatorial constraints. For non-convex and combinatorial constraints, we use internally developed heuristics that perform well in practice.

Since the optimizer leverages POINT's Global Risk Model (GRM) to define portfolio risk, we first provide an abstract description of this model as a general linear factor model without resorting to actual modeling details (risk factors, definitions, etc.) used in various asset classes. Please refer to the Barclays Capital Global Risk Model guide, available on POINT's documentation page (Dynkin, Joneja et. al. [2005]), for a detailed discussion. This abstract description suffices for the purpose of describing the optimization framework.

## 1. Global Risk Model: A linear factor model

The Global Risk Model (GRM) defines the tracking error volatility of a portfolio with respect to a given benchmark as the standard deviation of the portfolio's active return (i.e., net of benchmark) against the given benchmark. GRM is a linear multi-factor risk model that decomposes the return of each security into components driven by systematic market variables (yield curve movements, sector spread changes, equity sector returns, FX rate changes, etc., depending on asset class) and a security-specific idiosyncratic return.[5]

$r_i = \sum_j L_{ij} f_j + \varepsilon_i$ for each security $i$ in the universe,

where $\sum_j L_{ij} f_j$ represents the return explained by the systematic risk factors and $\varepsilon_i$ represents the security-specific idiosyncratic return. The risk model is designed to forecast the portfolio risk at monthly and daily time horizons. The model is calibrated to market data, macroeconomic data, and firm-specific accounting data etc., to estimate the factor covariance matrix, $S = Cov(f, f)$ the loadings matrix $L$, and the idiosyncratic covariance matrix[6] $I = Cov(\varepsilon, \varepsilon)$ at regular intervals. Thus, the security-by-security covariance matrix is constructed as $L' SL + I$. Note that we are using vector and matrix notation here. $L$, $S$, and $I$ are $n-by-k$, $k-by-k$, and $n-by-n$ matrices, respectively, where $n$ is the number of securities in the universe and $k$ is the number of risk factors in the risk model.

The total tracking error variance of a portfolio is defined as follows.

---

[5] GRM has an additional security-specific default risk term for a subset of fixed-income securities (e.g. high yield bonds, credit derivatives etc) modeled using the estimates of issuer-specific default probabilities and loss given default. The default component of return is assumed to be the negative of loss given default with a forecast probability of default and zero otherwise. This component is assumed to be uncorrelated with the other components of the return. Furthermore, a default event correlation model is used to estimate the cross-sectional correlations. Since GRM assumes zero correlations between the idiosyncratic returns and default returns, the default covariance matrix is simply added to the idiosyncratic matrix in the optimizer. Thus, default risk is embedded in the idiosyncratic risk in the optimizer.

[6] Classical asset pricing theory, the motivating force behind the factor models commonly used for risk modeling, assumes that the idiosyncratic returns are cross-sectionally independent. This assumption is relaxed in POINT's global risk model. There are several examples of asset classes covered in POINT's risk model for which a cross-section correlation structure is modeled among idiosyncratic returns, e.g., the bonds issued by the same issuer, and equity and debt securities of the same company. Hence, in the optimizer, we do not assume that the idiosyncratic covariance matrix is diagonal.

$$TTeV(x, x^B) = (x - x^B)'(L'SL + I)(x - x^B)$$

Similarly, the systematic and idiosyncratic tracking error variance are, respectively, defined as

$$STeV(x, x^B) = (x - x^B)'(L'SL)(x - x^B) \text{, and}$$

$$ITeV(x, x^B) = (x - x^B)'(I)(x - x^B)$$

where

$x$ = vector of portfolio positions, and

$x^B$ = vector of benchmark positions.

Notice that the vector of portfolio positions $x$ is the decision variable in a typical portfolio optimization problem, and the tracking error variance is a quadratic function of the decision vector. Thus, portfolio optimization models that support risk minimization and constraints on risk require support for quadratic objective functions and quadratic constraints, leading to a quadratic constrained quadratic programming (QCQP) formulation. The estimated systematic covariance matrix $S$ and the idiosyncratic covariance matrix $I$ must be positive semi-definite to preclude a long-short portfolio with negative variance forecast. This condition ensures that the quadratic function defining the active portfolio variance is a convex function. Hence, the QCQP formulation is necessarily convex. Convex optimization problems satisfy the elegant properties of duality theory, such as global optimal solutions, and are easier to solve in theory and practice (Appendix A).

The complete risk model (i.e., the factor and idiosyncratic covariance matrix) is calibrated on specific calibration dates. In addition, all the analytics (e.g., key rate durations, credit spreads, etc.) that feed the loadings calculations are computed daily by the pricing engines. Thus risk model is parameterized by:

- The calibration date.
- The risk measurement date.
- The time horizon for which it designed to forecast risk.

Furthermore, GRM allows users to choose several additional parameters; for example:

- The choice of using weighted or unweighted estimation methods.
- The implicitly hedging of risk from certain user-specified risk factors, etc.

The optimizer is flexible to allow users to select any such parametric risk model to define portfolio risk – including a specification of the relative risk aversion between systematic and idiosyncratic risk. We discuss many of these features, as well as details of how to set up and solve interesting and practical problems, in the *Portfolio Optimizer User Guide* (Kumar and Lazanas [2009]) and Section III of this report.

## 2. Mean-variance optimization models

Mean-variance analysis studies the tradeoffs between a portfolio's expected return and its total risk as measured by its variance forecast. It is a major building block in the foundation of modern portfolio theory and is the basis of the various optimization models over the past fifty years. Mean-variance is widely used and is a prerequisite to understand more recent models.

This model assumes that all investors are risk averse and interested in maximizing the quadratic utility function, $\mu' x - \lambda \cdot x'(L'SL + I)x$, where $\lambda \geq 0$ is the risk aversion parameter and $\mu$ represents the vector of expected return. In addition, the model enforces the budget constraint that the market value of the portfolio is equal to the budget[7] available.

$$\sum_{i=1}^{n} x_i = B$$

Thus, the most basic formulation leads to the following optimization problem.

Maximize $\mu' x - \lambda \cdot x'(L'SL + I)x$

Subject to: $e' x = B$

Where $e$ is a vector of all 1s. Note that there are no constraints on the signs of the positions in each security; in particular, the portfolio is allowed to be long-short. This simple model can be solved analytically by applying first-order optimality conditions to the Lagrangian function to obtain the following analytical solution.

$$x^* = \frac{1}{2\lambda}(L'SL + I)^{-1}(\mu - \delta e)$$

Where $\delta$ represents the Lagrange multiplier corresponding to the budget constraint.

$$\delta = \frac{e'(L'SL + I)^{-1}\mu - 2\lambda B}{e'(L'SL + I)^{-1}\mu}$$

The optimal portfolios above are said to be mean-variance efficient, and the set of portfolios obtained by varying the risk aversion parameter $\lambda$ are said to constitute the efficient frontier. Plotting the expected return and volatility (square root of portfolio variance in this model) of the set of optimal portfolios gives rise to the ubiquitous two-dimensional representation of the efficient frontiers. This basic model can alternatively be formulated by minimizing risk subject to achieving a particular expected return $\underline{\mu}$ and maximizing expected return subject to risk being lower than a particular threshold $\overline{\sigma}$. The set of solutions obtained by varying the expected return threshold $\underline{\mu}$ in the first formulation and the set of solutions obtained by varying the risk threshold $\overline{\sigma}$ in the second formulation leads to the same efficient frontier as the original formulations. This elegant result is based on the duality theory of convex optimization. Furthermore, adding a risk-free security to the set of investable securities gives rise to the famous two-fund theorem, in which the set of efficient portfolios is a linear combination of the risk-free asset and an optimal risky portfolio. The risky portfolio is also called the tangency portfolio and is one of the mean-variance efficient portfolios with respect to the set of all risky assets.

Note that the basic model discussed above supports many elegant results, but it is impractical and symbolic at best. Next, we discuss various extensions of the mean-variance

[7] This model can be formulated with security weights or the monetary amounts to be invested in each security as the decision variable. We chose to use monetary amounts because it encompasses situations in which security weights are not defined. This is the case, for example, when the portfolio is infinitely leveraged (i.e., has a market value of zero).

analysis needed in practice and their implications for the optimal portfolios. The POINT optimizer supports all these extensions.

## A. Long-short and concentration constraints

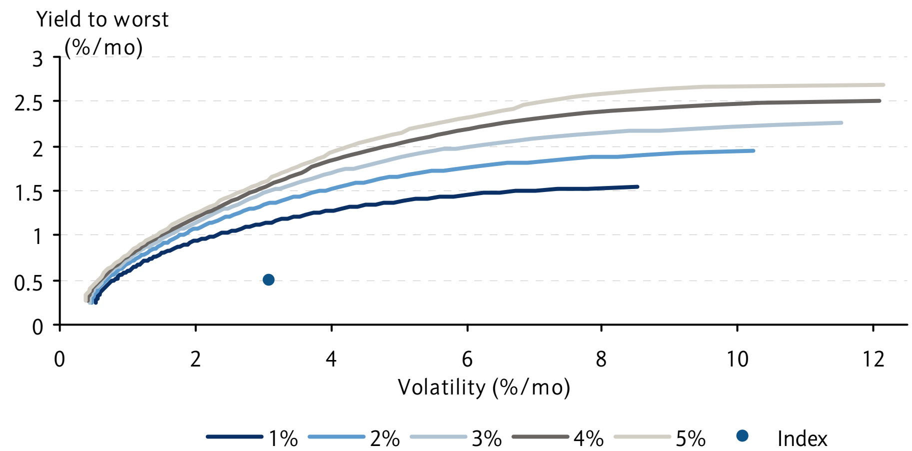
Many investment mandates require managers to use long-only portfolios. Thus, managers need to enforce the long-only constraint in the portfolio construction process. To construct long-only portfolios, we add the constraint $x_i \geq 0$ for all securities. In long-only optimal portfolios, many security weights can be zero at the optimal solution, and there are potentially large allocations to some securities based on the mean-variance optimization in the positive orthant. Naïve mean-variance optimal portfolios tend to concentrate on assets with the best risk-return characteristics. Since mean-variance optimization ignores the portfolio's tail risk, diversification achieved based on risk minimization alone may not be sufficient to control concentration risk. To protect against tail risk events (e.g., corporate defaults), it is common practice to limit the exposure to sources of risk that can produce extreme losses. Such "model-free" constraints are used in addition to the mean-variance preferences. For example, it may be desirable to limit the maximum amount invested in any single security in the optimizer. This is achieved by specifying a vector of upper bounds on the exposure to individual issuers, securities, or risk factors.

These concentration constraints restrict the set of feasible portfolios and have important implications for the risk-return profile of the optimal portfolios. To demonstrate this, we study the optimal long-only total return portfolios constructed using the bonds from the Barclays Capital US Credit Index as of May-end 2009 at various concentration limits. The index constitutes 3,611 bonds (the return universe constituents for June 2009). We use POINT's Global Risk Model's June calibrations to define the risk forecast. The systematic covariance matrix uses 66 risk factors including curve, swap spreads, and percentage changes in spread-based credit factors in 25 sectors. The idiosyncratic matrix is non-diagonal (to incorporate idiosyncratic correlations between bonds issued by the same issuer), but sparse. We use the yield to worst of the index bonds as an expected return measure in this exercise.

Figure 2 plots the optimal risk-return frontiers obtained at 1, 2, 3, 4, and 5% concentration limits. As expected, the frontier with higher concentration limits dominates the frontiers with lower concentration limits. The minimum risk portfolio in all fives cases is well diversified, and the concentration limit is redundant for this portfolio. This explains the convergence between the five frontiers at the bottom left of the figure.

Note that the index itself does not lie on the efficient frontier (Figure 2). There are several reasons for this. First, the yield to worst is not the best measure of the corporate bond's expected returns based on the equilibrium risk pricing arguments. Second (and the point that we would like to emphasize), there is much to be said about the implementability of the portfolios on this efficient frontier. By implementability, we mean whether these risk-return profiles can be achieved in practice, as we are not accounting for many practical considerations such as liquidity and transaction costs. As we enforce additional practical constraints, the efficient frontier would shift right and down – making the index closer to being efficient.

**Figure 2: Mean-variance frontier: US credit bonds**




Source: Barclays Capital

## B. Active risk and returns

The goal in most investment mandates is to outperform a given benchmark index in terms of return while keeping the active risk relative to the index low. Total return optimization can be thought as a specific example of benchmarked portfolio optimization with base currency cash as the benchmark portfolio. Optimization using active risk, return, and other quantitative analytics can be easily formulated by introducing *active* holding variables. Active holding variables are the net holding minus the holding amount of the same security in the appropriately scaled benchmark universe - $x - x^B$ where $x^B$ is the known vector of appropriately scaled benchmark holdings.

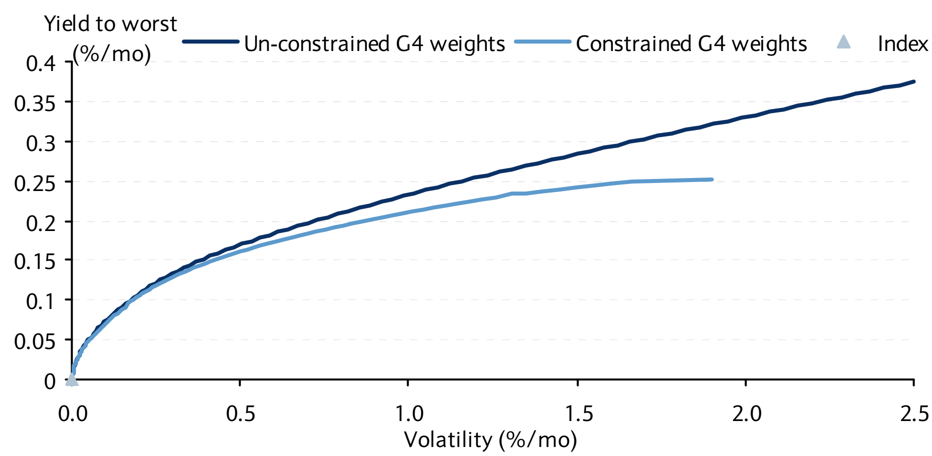
The resulting optimization model remains a convex QCQP problem and, in particular, does not add additional computational complexity to the model. But the qualitative characteristics of the optimal portfolios are significantly different. For example, the presence of a risk-free investable security (i.e., base currency cash) does not make the set of all efficient portfolios a linear combination of the risky tangency portfolio and the cash. Instead, the efficient frontier is always curved in the active risk-return space, and full replication is the only viable zero active risk option.

To discuss portfolio optimization in the active return space, we construct optimal long-only bond portfolios from the Global Treasury Index benchmarked to the same index. The analytics and risk model calibration is taken as of May-end 2009 using POINT. The foreign currency risk is measured against the USD. The bonds constituting this index come from 36 countries across the globe and are denominated in 21 different currencies. We construct the mean-variance efficient frontier of portfolios containing bonds from this universe by maximizing the net yield-to-worst of the portfolio vs. the index subject to a given maximum level of active risk.

Figure 3 plots two such efficient frontiers. The dark line represents problems with no additional constraints, and the light line represents problems where portfolios are required to have zero net exposure to G4 currencies (USD, GBP, EUR, and JPY). The x-axis measures the active portfolio risk constraint while the y-axis shows the maximum achievable net yield-to-worst subject to the particular risk constraint. Clearly, the index itself has zero active risk and zero net yield and lies at the origin of the graph.

In the unconstrained case, the optimizer is free to use active country/currency allocations, in addition to picking bonds with relatively higher yield within each currency. In the constrained case the optimizer must match G4 currency weights and therefore is prevented from increasing the aggregate exposure to non-G4 currencies (which may have higher yields). Therefore, yield maximization can be only achieved by security selection within the non-G4 currencies. Naturally, the unconstrained frontier dominates the constrained one, both in terms of achieving higher excess yield for the same level of risk and in its ability to reach levels of risk that are unattainable in the constrained problems.

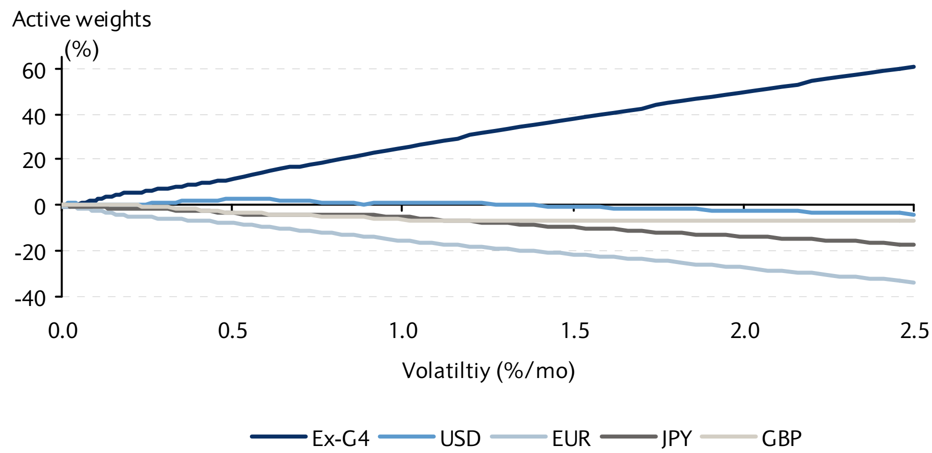
**Figure 3: Global treasury optimal portfolios: Active currency allocation**




Source: Barclays Capital

Figure 4 displays the active weights to the G4 currencies and the aggregate non-G4 weights of optimal portfolios in the unconstrained case. Observe that the optimal active weights are positively related to the average yield of the respective currencies, as the optimizer achieves higher yield at the cost of increasing portfolio risk. Specifically, the optimal portfolios overweight the non-G4 sector, which includes a large number of bonds with relatively higher yield, and underweight the rest of the currencies. Since treasury bonds have a low level of idiosyncratic risk, the optimal portfolio closely matches the exposure to different parts of the curve for all four large (by market weight) underlying currency/countries.

**Figure 4: Active weights along the frontier: Global treasury portfolios**




Source: Barclays Capital

## C. Sector Constraints

The ability to incorporate preferences over the subset of the portfolio along the risk factor and security sectors dimensions is an important requirement for a practical portfolio optimizer. Typically, mean-variance optimization leads to optimal portfolios that can be concentrated in particular asset classes or load heavily on particular risk factors. This is because estimation errors in the risk and return forecasting may lead to corner point solutions with concentrated exposures to a sector or risk factor. Furthermore, volatility models do not model the tail risk; hence, the optimal mean-variance exposure may not be acceptable from a tail risk perspective. Thus, it is desirable to be able to limit these exposures explicitly in a model-free fashion (independent of covariance forecasts). As discussed in the global treasury-based portfolio construction example, these constraints have significant implications for the risk-return profile of the optimal portfolios. The constraints can be translated to generic linear constraints of the following form on the security holdings.

$$\underline{b} \leq \sum_{i=1}^{n} c_i x_i \leq \overline{b}$$

We use the term exposure generically here. It can be represented by any security analytics that are sensible for the asset class or the risk factor. For example, market weights or industry beta exposures are appropriate for equities, but spread duration or DTS (duration times spread) exposure is a more appropriate risk measure for credit bonds. Limits on the active exposures to individual ticker or issuers give rise to similar constraints. Managers may be interested in inputting constraints based on custom data, in addition to the large number of quantitative analytics available in POINT.

If the investable universe includes portfolio instruments such as exchange-traded funds, index options, index futures, total return swaps, etc., then the sector constraints should reflect the partial sector contribution of such instruments. For example, the ETF SPY tracking S&P500 index contributes to all equity sectors. These intricate constraints can also be represented as generic linear constraints. POINT can look through such complex instruments and decompose exposures from their constituents accurately.

## D. Leverage constraints

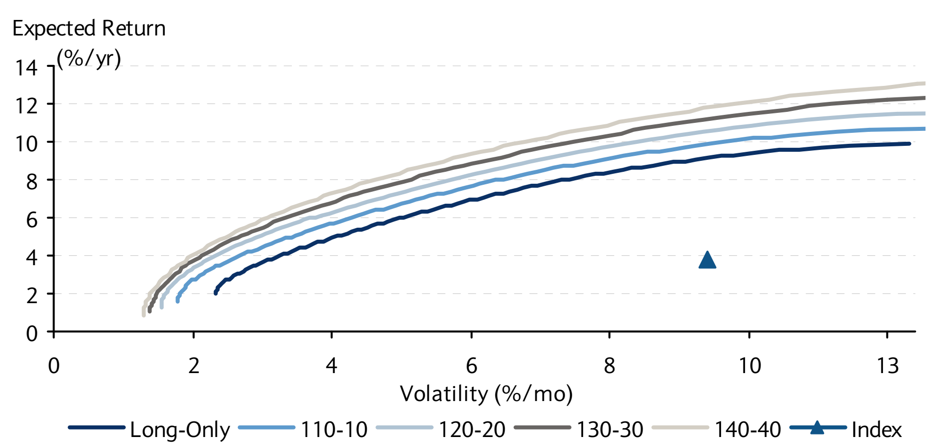
Leverage can be generated in portfolio construction by the use of derivatives, or by taking short positions. Leverage increases the portfolio's risk and return sensitivity to the budget; hence, the higher order terms, often ignored in linear factor models, become significant. For the same reasons, tail risk considerations and portfolio bankruptcy risk become important. Hence, leverage is an additional dimension that manager wants to control in addition to risk and return and is also typically mandated by the investors. This is modelled using the gross notional constraint,

$$\sum_{i=1}^{n} c_i |x_i| \le L$$

where $c_i$ is the notional amount per unit holding in security $i$. For cash (funded) securities the notional is simply the position size (market value), but for derivatives such as futures contracts, the notional is defined based on a security's risk characteristics. Notice that these gross size constraints are not linear, but they can easily be re-formulated as linear constraints by splitting the decision variables into their positive and negative parts as long as the constraint is convex[8] (chapter 1 in Bertsimas and Tsitsiklis, [1997]).

To demonstrate the effect of leverage constraints on the risk-return characteristics of optimal portfolios, we constructed long-short portfolios by optimal allocation to the Russell 3000 constituent stocks based on analytics as of may-end 2009 with varying degree of leverage, and risk aversion. A simple equilibrium-price-of-risk based model is used to compute the expected return numbers for each stock for demonstration purposes. We again use POINT's risk model calibrations to define portfolio risk. The risk model uses 85 systematic risk factors to forecast the risk of this index. Both portfolio risk and expected returns are defined with respect to the (net) market value of the portfolio. In addition, a 1% concentration limit is enforced on both the long and the short side. The concentration limit is enforced to obtain a minimal model-free diversification for all portfolios.

**Figure 5: US equities: long-short mean-variance frontiers with varying leverage**




Source: Barclays Capital

[8] The lower bound constraint on leverage is not convex.

Figure 5 plots the five efficient frontiers obtained at different level of leverage. In contrast to the US credit bond example, the minimum variance portfolios here have different risk levels at different leverage. This is because the long-short portfolios are able to achieve a better idiosyncratic risk diversification using the extra exposure obtained by the short side per unit of market value. Hence, the minimum achievable risk decreases as we increase the portfolio leverage. Since we are changing only leverage across these five frontiers, high leverage frontiers dominate the low leverage frontiers.

### E. Rebalancing and hedging problems

In portfolio hedging and rebalancing, we want to optimize an existing portfolio. In rebalancing, we want to generically trade any security, including the positions in the current portfolio, to improve the portfolio's performance and re-align it with the market. In portfolio hedging, we typically do not want to trade the current portfolio, but use a small set of hedge instruments (derivatives) to reduce portfolio risk by hedging out specific components.

To model the portfolio hedging and rebalancing problems, a vector of initial holding in each security $x^0$ is defined. Given $x^0$, the vector of trading amount in each security is given by $x - x^0$. We need to model both these variables to support preferences over a set of analytics that are a function of both the final holding amounts and the trading amounts. In addition, buys need to be separated from sells, and longs need to be separated from shorts. Having access to the long, short, buy, sell, and gross holding and trading amounts in each tradable security in our solver allows us to implement our preferences in the large set of analytics at the front end. In particular, the concentration, leverage, and sector constraints, discussed previously, are also supported for the trading portfolio (i.e., portfolio defined by the collection of trades).

### F. Trading costs

Bid-ask spreads and broker-dealers' commissions are direct measures of transaction costs. In addition, the market typically moves adversely for large trades, creating market impact costs. Both of these costs have a direct effect on portfolio performance and, hence, should be incorporated in the optimization process. In practice, the market impact and the transaction costs are combined in a trading cost model that gives the per-unit cost of trading a security. We represent these trading costs as:

$$\sum_{i=1}^{n} \int_{x_i^0}^{x_i} TC_i (x_i - x_i^0) dx \approx \sum_{i=1}^{n} TC_i \left| x_i - x_i^0 \right|,$$

where $x_i^0$ represents the initial holding and the $TC_i$ is the model trading cost for the i[th] security. Portfolio managers may want to use this term in the objective function to trade off the transaction costs with the risk and expected return or to input a constraint. This absolute value functional form is modelled using linear models by decomposing the trading variables, $x_i - x_i^0$ into positive and negative parts similar to the leverage constraints.

Trading costs are generally modelled as convex piecewise linear functions of the trading amount in more detailed models. This is especially true for equities, for which the exchange trading volume and trade-by-trade price movement data are available, making the calibration of such models easy. The piecewise linear and convex functional form does not impose any additional complexity on the model and can easily be translated to a linear model if the fixed trading costs are assumed to be zero. In case of fixed costs, a theoretically correct models leads to a mixed integer formulation[9] that is hard to solve. In practice, a convex piecewise linear approximation of the function is used instead by modifying the coefficients to incorporate the fixed cost effects in the proportional costs (Boyd, S. and Vandenberghe, L. [2004]).

## G. Constraints on the number of holdings

Mean-variance optimal portfolios typically hold a large number of positions based on the quadratic optimization to diversify the idiosyncratic risk. In practice, managers may not want to hold a large number of securities in their portfolios, as this is a robust and direct proxy for management and transaction costs.

Typical passive management mandates are benchmarked to market indices that contain a large number of securities. Full replication (by holding all benchmark securities in proportion) is possible only at very large portfolio size and rarely practiced.[10] Typically, to track the performance of the benchmark index, these mandates requires managers to maintain a low tracking error volatility with respect to the underlying benchmark. This leads to a clear trade-off between the achievable portfolio tracking error volatility and the number of securities in the portfolio. Using a large enough number of securities, exposures can be matched to minimize the systematic risk, but to achieve an acceptable level of idiosyncratic risk, a larger number of securities are needed.

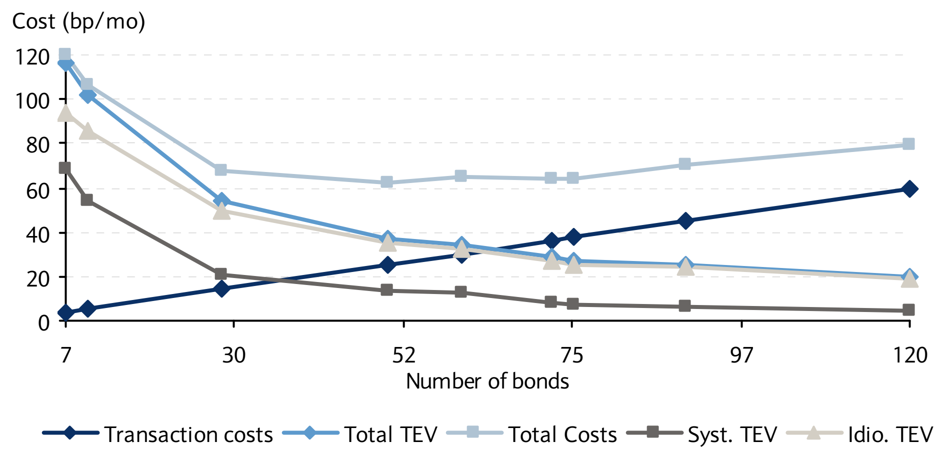
The number of security constraints is modelled by defining binary (0,1) holding indicator variables. Each variable assumes a value of 1 if and only if the corresponding security is present in the optimal portfolio. The sum of these binary variables is equal to the number of securities in the portfolio. Constraints on this sum (i.e., the number of securities in the portfolio) give rise to the integer programming formulation. We discuss the implications in a later section.

We now discuss the replication of Barclays Capital US Credit Index using a limited number of positions. We construct an optimal portfolio by minimizing total tracking error volatility (TEV) with respect to the index using a given number of bonds in the portfolio as of March 31, 2009. Figure 6 plots the value of tracking error volatility (TEV) of the optimal portfolios as a function of the number of bonds in the portfolio. It should not be surprising that this function is decreasing. The minimum tracking error volatility (TEV) decreases from 120bp/month using seven bonds to 20bp/month using 120 bonds. Although systematic risk (as calculated by a factor risk model) can be reduced by matching the factor exposures of the index and even become "zero" when the number of bonds in the replicating portfolio is equal to the number of systematic risk factors, idiosyncratic risk is reduced only by diversification and decreases at a rate roughly proportional to the square root of the number of bonds in the replicating portfolio. The replication process initially seeks to reduce systematic risk but, as the number of bonds increases, idiosyncratic risk dominates and the rate of risk reduction decreases. If we now assume that portfolio management cost is some increasing function of the number of bonds in the portfolio, there exists an optimal trade-off between risk-reduction achieved by increasing the number of replicating bonds versus management cost increase. Figure 6 illustrates this point. The darker straight increasing line represents transaction and management cost as a function of the number of securities. The three decreasing lines represent the minimum achievable total, systematic and idiosyncratic TEV as a function of the number of securities used. It can be seen in the graph that systematic TEV drops quickly and remains very low above 30 bonds or so. On the other

[9] Additional binary (0,1) trading indicator variables are needed to model the fixed costs.
[10] This is especially true for fixed-income portfolios. In equities, full replication can be practical at large portfolio size because of relatively low costs for exchange traded stocks and high tracking error risk.

hand idiosyncratic risk decreases more gradually. The top line represents the sum of transaction cost and optimal total TEV. Assuming that the user is willing to substitute 1bp of TEV with 1bp of management cost, the minimum of this function represents the optimal number of securities for this replication problem. Different preferences between cost and risk will lead to different solutions.

**Figure 6: Replicating the US Credit Index: Tracking error volatility versus number of bonds in the portfolio**




Source: Barclays Capital

The Portfolio Optimizer in POINT allows portfolio managers to construct portfolios by targeting transaction cost and tracking error jointly by expressing the price of substitution as the relative weights of the two terms in the objective function. In practice, portfolio managers have a good idea about the optimal number of bonds in their portfolio and want to construct portfolios subject to a maximum number of positions constraint. Such constraint is supported in the optimizer as well.

## H. Constraint on number of trades

The number of recommended trades is a model independent proxy for turnover and transaction costs in portfolio rebalancing and hedging problems. Managers typically want to rebalance their portfolios using a few buys and sells. In a hedging problem, we want to choose as few trades as possible from appropriately chosen hedge universe. Thus, the ability to control the number of trades is important in constructing optimal rebalances and hedges.

The number of trades constraint is modelled by introducing a trading indicator variable for each security that assumes a value of 1 if and only if the corresponding security is being traded. The sum of all these indicator variables is equal to the number of trades. Constraints on this sum also give rise to a combinatorial optimization model.

In portfolio construction problems, the trading variables are the same as the holding variables because the initial portfolio is blank. Thus, the number of trades and the minimum trade size are same as the number of holdings and the minimum holding size, respectively.

## I. Small trades/positions

In portfolio construction, the mean-variance optimal portfolio may (and typically does) contain small positions. Similarly, in rebalancing and hedging problems, the optimizer may recommend small trades based on the quadratic optimization. In practice, managers often have a strict preference not to hold small positions or trade small amounts. Small positions and trades incur the fixed transaction and management cost without appreciably helping with portfolio objectives.

The constraint on the minimum trading and holding size is an *if-then-else* constraint, i.e., if trading in a particular security, trade at least a given amount. These constraints are implemented using the trading and holding indicator variables mentioned previously and also lead to a combinatorial formulation.

### J. Stratified Sampling

Stratified sampling refers to the risk model independent (aka correlation free) portfolio construction approach in which the tradable securities universe is partitioned into a number of security cells along relevant risk dimensions and a few securities are selected from each cell to match the benchmark or a target exposure in that cell. Stratified sampling is an alternative approach to risk minimization often used in passive index replication.

In addition to matching the cell exposures independent of each other, it is desirable to implement portfolio-level targets. For example, to construct a credit portfolio, we may want to match portfolio duration to the benchmark while matching the spread duration exposure in each credit sector—at the same time maximizing the liquidity[11] of the portfolio. This approach leads to a class of linear programming based optimization models.

Note that adding any one of the above features (sections II.2.A-II.2.J) to an unconstrained mean-variance model renders the analytical solution impossible. In most practical problems involving a large number of investable securities, a professional solver is required to solve the optimization model.

In practice, managers can express a preference for a multitude of portfolio characteristics. So the "real" efficient frontier is a high dimensional surface consisting of portfolios obtained by varying the relative importance given to the various characteristics/dimensions while enforcing all relevant constraints. This set is different across managers and investment themes; hence, a portfolio constructed based on the basic mean-variance optimization is of little use in practice. Our optimizer is not based on mean-variance frontiers and extends the mean-variance model along the dimensions discussed in this section.

## 3. Generic POINT portfolio optimization problem

Our optimization model maximizes (or minimizes) a combination of portfolio performance measures (e.g., risk, return, transaction costs, penalty, yields, and spreads) using client provided tradeoffs among these measures. This combination is termed the objective function of the optimization problem, and the tradeoff parameters are termed the objective function weights. Individual quantitative measures in the objective functions are termed the objective function terms. Thus, the objective function is a linear weighted sum of objective function terms.

The model optimizes the objective function subject to a number of user-provided constraints. Mathematically, these constraints can be categorized as linear, quadratic, convex piece-wise linear, and combinatorial constraints. In summary, it is designed to solve the following generic[12] optimization problem.

[11] Assuming that a quantitative measure of liquidity (e.g., liquidity cost scores in Phelps and Dastidar [2009]) is available.
[12] In Appendix C, we introduce various categories of optimization problems as they relate to the mathematical optimization model underlying POINT's optimizer.

**Maximize objective function**

Subject to:

1. Budget constraint.
2. Turnover constraints.
3. Upper bound on number of securities in the final portfolio.
4. Upper bound on number of trades recommended.
5. Lower bound on the size of individual trades suggested.
6. Generic linear constraints on the final and trading portfolio analytics computed at
   - Generic security buckets defined using generic security partition, and
   - with respect to a user specified benchmark.
7. Upper bound on the total, systematic or idiosyncratic portfolio risk with respect to cash or a specified benchmark.

Where the **objective function** is specified as the weighted sum of the following terms with user-specified weights:

1. The systematic, idiosyncratic, or total tracking error variance of the portfolio with respect to a user-specified benchmark.
2. The user-provided security-specific expected return forecasts.
3. A large number of POINT supported quantitative analytics across all asset classes.
4. Transaction costs.

Most of the functionalities in the generic problems can be mathematically modeled as a **convex programming problem**, but the following functionality requires a combinatorial optimization model.

1. Upper bound constraint on the number of securities held in the final portfolio.
2. Upper bound constraint on the number of trades recommended.
3. Lower bound constraint on the each trading positions.

We solve the optimization model using state-of-the-art large-scale QCQP technology if it does not involve combinatorial constraints. The combinatorial constraints are handled using internally developed proprietary heuristic algorithms in conjunction with the QCQP technology. These heuristics are designed to solve the practical models in a predictable running time in a close-to-optimal way. Because of the heuristic nature, the solutions are not guaranteed to be theoretically optimal, but in our experience, these heuristics successfully solve most practical models in the above category. Also, the heuristics allow us to solve *large-scale problems* with combinatorial constraints.

## 4. Alternative approaches to portfolio optimization

We discuss competing optimization models sometimes used in the context of our generic model. The following three optimization schemes are common.

## A. Gradient descent algorithms

Optimizers based on this approach take a one-trade-at-a-time iterative approach. At each step, the optimizer guides the users to select a buy-sell pair swap trade. The set of possible swap trades is ranked based on the achieved objective function if an optimal trade is performed in the given pair of securities. The *gradient* of the objective function with respect to the trading amount in the swap is used to compute the rankings and the optimal trading amount at each iteration. The objective function is typically fixed (for example, tracking error volatility). To construct new portfolios, the user manually starts the procedure at a cash-only portfolio and successively picks the swap trades until an acceptable portfolio is achieved.

The most important limitations of such models are that they do not support any portfolio constraints and are significantly sub-optimal. Even though users have control over the pair trade in the intermediate step, the process tends to be highly manual and time consuming. For example, to construct a 50-bond minimum tracking portfolio, the user has to make 50 decisions at the intermediate iterations.

## B. Hierarchical optimization: Constructing efficient frontiers

The hierarchical optimization approach separates the portfolio construction problem into an asset allocation problem and a security selection problem. In the asset allocation step, the security-level constraints and preferences are ignored, and portfolio weights to broader marker sectors are determined to optimize the risk-return tradeoff based on risk and return forecasts at the broader market level. Once the asset allocation decision is finalized, individual security weights are determined within each sector to match this asset allocation. Managers typically focus on the idiosyncratic risk and outperformance forecasts of individual securities in this step.

Because asset allocation decisions do not incorporate security- or trading-level constraints, the resulting mean-variance optimization problem is relatively simple. In particular, the combinatorial constraints that are needed at the security level are not present. Given the asset class-level regulatory constraints, the complete efficient frontier is generated, and asset allocation is chosen on this frontier. The security selection problems in each market sectors are then solved independent of each other.

A major drawback of this approach is that it is difficult to flexibly incorporate generic portfolio-level constraints. For example, to construct a portfolio to track the Barclays US Aggregate Index, suppose that the asset allocation is done using various sectors in the credit, treasury, government-related, and securitized markets. Individual bonds are then selected in each sector. It is difficult to guarantee that the portfolio duration will match the overall index or a given target in an automated fashion if the security selection is done independently in each sector. The security selection part of this problem can be implemented in the optimizer using the stratified sampling approach discussed previously. In this scheme, the security selection decisions across different market sectors are *jointly* made. For example, the market weights in each sector can be constrained to match the target asset allocation while including a portfolio-level constraint that the active duration of the portfolios is zero.

It is difficult to incorporate the portfolio rebalancing and hedging problems in this framework. The optimal asset allocations and actual trades depend heavily on the initial portfolio. Trading costs and benchmark turnover are more important in these problems, in which asset allocation can be changed only marginally.

Furthermore, this approach ignores the interaction between security selection and asset allocation. First, conditional on the security selection decisions, the risk forecasts (i.e., the covariance between various market sectors) can be significantly different from the risk forecasts that were used in the asset allocation optimization. Second, it might be beneficial to take security-level preference such as liquidity, idiosyncratic risk, etc., into account in the asset allocation decisions. These preferences clearly influence the manager's ability to implement a security selection outcome that is consistent with the risk-return profile of the various asset classes used in the asset allocation step.

In the US Aggregate Index-based portfolio construction example mentioned above, the number of systematic factors and the idiosyncratic level in the treasury sector is much lower than the credit sector. Hence, views in the treasury sector (e.g., curve flatteners) can be implemented more easily than views in the credit sector. The number of bonds to be selected in the credit sector also affects this tradeoff. Hence, asset allocation based purely on the sector-level risk-return forecast is a simple, but sub-optimal, mechanism. Our framework allows a more sophisticated asset allocation mechanism using the allocation to portfolio instruments that deliver the total returns of the corresponding market sectors. The asset class-level risk model in this case is implied from the underlying security-level risk model using the benchmark weights. This model includes idiosyncratic risk and constraints on exposure to finer subsectors. We illustrate this using a practical portfolio construction example (Section III.8) in the next section, which present various portfolio management applications.

In conclusion, this approach to portfolio construction can be thought of as an application of the portfolio optimizer. Managers can use the optimizer for asset allocation and risk-budgeting applications, as well as automating security selection procedures to implement a given asset allocation.

## C. Combinatorial optimization using integer programming

As discussed previously, there are several reasons to introduce integer variables to portfolio optimization models. In particular, the constraints on the number of securities in the portfolio and the number of recommended trades lead to this formulation. Similarly, the conditional structure of the constraint on smallest holding/trading size gives rise to a combinatorial formulation. We need to find a feasible portfolio with the best performance measure (risk, return) among those satisfying the combinatorial constraints.

It is well known that all these problems can be properly formulated by introducing a number of 0-1 integer variables. However, solving the resulting optimization problems containing integer variables is considered to be an intractably difficult problem. The reason is that the model includes a non-linear (quadratic) risk term. Unfortunately, there is virtually no **efficient** algorithm for solving nonlinear integer programming problems. The various branch-and-bound schemes proposed in the literature and available in commercial solvers take exponential running time and can handle only small-scale problems (e.g., Mitra, Ellison and Scowcroft [2007], Bienstock [1996], Bertsimas, B, and Shioda [2009] and the reference therein). Also, the solutions obtained by these methods are not reliable, and the solution time tends to be highly unpredictable. A sample risk minimization problem with 100 tradable securities with these constraints can sometimes be solved in few minutes, but takes hours if the risk model is slightly changed (e.g., optimization as of a different date).

For these reasons, we use Lagrangian function-based iterative heuristics to solve these problems. We use the QCQP technology at the intermediate iterations of the heuristics. These heuristics are custom-tailored for our optimization model and improved based on performance regressions on a large category of problems. In our experience, even though the theoretical optimality is not guaranteed, these heuristics deliver a close-to-optimal solution to most practical problems.

To benchmark the performance of the heuristic algorithms, we performed a number index replication exercises using the optimizer for various benchmark indices. The optimization problem is to minimize the TEV relative to the benchmark index subject to picking at most 50 positions in the portfolio. Figure 7 display a set of sample results. The size of the benchmark indices range from about 12,000 securities for the largest, the Barclays Capital Global Aggregate Index, to 500 securities for the smallest, the S&P 500 index. The low tracking error volatility of the optimal portfolios intuitively indicates the close-to-optimality of the solutions. The last row reports the total running time (including data loading, etc.) of the optimizer to solve these problems. The optimizer consistently solves similar-sized problems in a comparable running time. For large-scale problems, the data loading time, which grows roughly linearly with problem size, constitutes a major component (90% on an average for these problems) of the total running time.

**Figure 7: Sample optimizer performance for index replication problems**

| Index | Global Agg | US Aggregate | US Credit | Global Tsy | Russell 3000 | MBS | Russell 2000 | S&P 500 |
|---|---|---|---|---|---|---|---|---|
| Universe Size | ~12,000 | ~9,000 | ~3,500 | ~1,100 | ~3,000 | ~1,800 | 2,000 | 500 |
| TeV in bp/mo (Feb '09 end) | 14.2 | 18.0 | 40.0 | 7.1 | 75.9 | 1.4 | 127.8 | 72.6 |
| TeV in bp/mo (Sept '08 end) | 10.0 | 11.7 | 19.2 | 5.8 | 60.2 | 1.2 | 114.7 | 54.5 |
| Total Solution Time (minutes) | ~20 | ~10 | ~5 | ~1 | ~3 | ~2 | ~2 | <1 |

Replicating portfolios hold a maximum of 50 securities, TEV in bp/mo
Source: Barclays Capital

## D. Alternative risk measures

In addition to the mean-variance framework, a number of alternative optimization models have been proposed by academician and practitioners. These models primarily differ from our framework in that they propose alternative risk metrics.

Optimization based on tail risk measures such as VaR and alternative expected return forecasting frameworks (Bayesian models, etc.) are two popular themes. Tail risk optimization includes models for optimizing VaR and CVaR. Note that under the normality assumption, the tail risk optimization boils down to the mean-variance optimization, because mean and volatility completely parameterizes the normal distribution. In POINT's tail risk model, the non-normality of the security returns is modelled using a large number of Monte Carlo scenarios to compute the tail risk measures[13]. CVaR optimization based on this framework can be modelled using a linear programming model (Rockafellar and Uryasev [2000]). To obtain accurate risk measures, Monte Carlo models use a large number of scenarios. This leads to a very large-scale linear programming problem corresponding to the CVaR optimization. For problems involving a large number of investable securities, the size becomes a prohibitive factor even for state-of-the-art solvers.

In general, the complete distribution of portfolio return is important to investors. Volatility, VaR, and CVar are three statistics of this distribution, and might not completely define the distribution. In addition, skew, kurtosis, and so forth can be targeted in an optimization model. Such models remain impractical because of well-recognized difficulties in estimating the underlying parameters.

[13] See the Barclays Capital portfolio modeling publication on the tail risk model in POINT.

Bayesian models include the Black-Litterman framework, which updates the risk model and expected returns forecasts conditional on the user's view on market risk factors and individual securities. The important feature in these models is the ability to combine and update the user's views on markets and forecasts based on historical data. In the end, the portfolio optimization problem in this framework is a mean-variance optimization problem with conditionally updated risk model and expected return measures.

Recently, optimization models based on the robust optimization approach (Iyengar and Goldfarb [2003]) have been widely discussed. These models explicitly model the quality (uncertainty) of the risk model and expected return estimates using confidence intervals around the estimate. The onus of providing a quantitative measure of the uncertainty in the estimates is on the user before these models can be used. Because of this, only simple uncertainty structures involving a few parameters are implementable. An argument can be made about the user's ability to estimate the uncertainly parameter. Furthermore, a closer look at these model indicates that they can be broken down to mean-variance optimization with additional constraints automatically constructed using the problem data including the uncertainty parameters. We would argue that instead of supplying the unobservable uncertainty parameters, users can directly enforce relevant constraints on the observable portfolio weights. For example, Fan, Zhang, et al. (2008), advocate the use of gross value constraints on portfolio holdings in long-short portfolio construction as a robustness measure that improves performance. Our optimizer allows gross value constraints on holdings, as well as trading variables, in addition to flexible sector constraints based on net value and gross value analytics. For more details on the constraint functionality available, see the optimizer user guide (Kumar and Lazanas [2009]).

# III. OPTIMIZER APPLICATIONS IN PORTFOLIO MANAGEMENT

We present a number of portfolio management applications to demonstrate how many practical problems are amenable to the optimization process. We believe that the Portfolio Optimizer can help construct benchmark solutions to such problems. The examples in this section illustrate the optimizer's functionality and are not intended to be exhaustive. We want to emphasize the generality of the formulation that allows users to solve a wide variety of problems related to diverse portfolio management themes. Managers should be able to perform more practical optimization case studies using the features illustrated here.

The applications can be broadly classified into three categories – portfolio construction, portfolio rebalancing, and portfolio hedging. We assume that our readers are familiar with the optimizer based on the *Optimizer User Guide* (Kumar and Lazanas [2009]).

## 1. Hedging portfolio risk

Hedging a portfolio is akin to buying insurance: the portfolio manager is willing to incur some cost in order to insulate a portfolio against adverse movements in particular market risk factors. For example, fixed income portfolio managers may want to hedge against adverse movements in risk-free rates, equity managers may want to hedge against a big drop in the equity markets, and so forth. Liquid derivative instruments (e.g., bond futures, equity index options, and currency forwards) are commonly used to create hedges because of their funding advantage. The hedges can be constructed to protect against one-sided movements (e.g., a big drop in the equity markets) using non-linear derivative instruments (e.g., equity index options) or to protect against two-sided movements (e.g., active interest rate risk) using linear hedging instruments (e.g., interest rate futures). Hedges eventually become less effective as the market moves and the portfolio's sensitivity to risk factors changes. Hence, hedges need to be adjusted periodically. It is essential that hedge instruments are very liquid; otherwise, the transaction costs incurred in frequent trading would make the hedging exercise too costly to be practical.

In a typical linear factor model, the active return of a global mixed portfolio is decomposed into the various components: carry (i.e., time return), FX, curve, spread changes, equities or other allocation factors, and idiosyncratic return.

$$r^P - r^B = \underbrace{r_C^P - r_C^B}_{time} + \underbrace{\sum_c \left(MV[\%]_c^P - MV[\%]_c^B\right)\cdot \Delta FX_c^P}_{FX} +$$

$$\sum_i \underbrace{\sum_k \left(KRD_k^P - KRD_k^B\right)\cdot \Delta KR_k^P}_{Curve_i} + \underbrace{\sum_k \left(L_k^P - L_k^B\right)\cdot \Delta f_k^P}_{Spread\ \&equity\ ..etc.} + \cdots + \underbrace{\left(\varepsilon_k^P - \varepsilon_k^B\right)}_{Idio}$$

Specific components of a portfolio's systematic risk are targeted in a hedging exercise using instruments that provide a relatively clean exposure to these components. In its most common form, the hedging exercise attempts to minimize the portfolio net loadings to targeted risk factors. We characterize this form of hedging as linear. Linear hedging requires at least as many hedge instruments as the number of risk factors being hedged. For example, linear FX hedging entails matching the FX factor exposures to the benchmark using FX derivatives. If a portfolio has net exposure to ten currencies (with one of them being the base currency, i.e., nine risk factors), nine FX forwards are required to fully hedge FX risk.

As an alternative to linear hedging, managers can take advantage of the relationships between risk factors and attempt to use a small number of hedge instruments to minimize a specific component of risk rather than completely eliminate it. A risk model is a useful tool for this exercise. In this case, it is not necessary to neutralize the factor exposures completely. Instead, the estimated covariance between various risk factors is used to construct "optimal" hedges. Any number of hedge instruments can be used in this method.

As an example, consider the exercise of hedging interest rate risk in a USD fixed income portfolio. Most managers measure interest rate exposure using the key-rate durations (KRDs) of the portfolio to a discrete number of curve points (POINT uses six[14] such KRDs for USD rates). If managers want to eliminate all interest rate exposure as measured by KRDs, they need to use as many hedge instruments as the number of KRDs. Alternatively, because the bulk of curve risk can be explained by three risk factors (level, slope, and convexity), managers can employ a risk model-based hedge using a smaller number of highly liquid instruments. The four front-month US Treasury bond futures are commonly used in this context.

The Portfolio Optimizer in POINT can be used to perform both linear and non-linear hedging. In addition, it allows users to quantify the effect of hedging on the portfolio yield, expected returns, and transaction costs. There is a clear tradeoff between the amount of risk reduction achieved and the reduction in portfolio carry and the transaction costs incurred. An optimal tradeoff is chosen to determine the desired hedging strategy.

The optimizer supports a variety of hedging applications, including:

- Hedging interest rate curve risk using interest rate swaps, treasury bonds futures, and other interest rate derivatives.
- Hedging foreign exchange risk in multi-currency portfolios using short-maturity FX futures and FX forwards.
- Hedging mortgage portfolios using liquid TBA (to-be-announced) contracts.
- Hedging excess credit risk using single-name CDS or liquid portfolio default swaps (CDX in North America and ITraxx in Europe)
- Hedging global equity portfolios or trading books using exchange-traded funds and equity futures.
- Hedging volatility exposure using volatility products – both in fixed-income using caps, floors, and swaptions and in equities using index options.

All of the above problems can be set up in the optimizer and solved in the linear hedging (matching analytics) and non-linear hedging (minimizing risk) framework while incorporating trading costs and liquidity preferences. Below, we present examples of curve, FX, and excess (credit and equity) risk hedging.

### A. Hedging curve risk

Consider a total return US credit bond portfolio manager who implements credit positions by constructing a diversified portfolio of individual issuers. Suppose that we want to minimize the portfolio's exposure to the Treasury curve using Treasury bond futures to minimize curve risk while maintaining the credit exposure. As an example, we construct curve hedges for a diversified portfolio of 119 US credit bonds as of March 31, 2009. We use the four front Treasury bond futures as the hedge instruments. To construct the linear six KRDs hedge, the hedge universe is expanded with a treasury bill and a long-maturity treasury strip (Figure 8).

[14] The USD treasury curve risk in POINT is modeled using six (6m, 2y, 5y, 10y, 20y, and 30y) key rate points.

**Figure 8 : Curve risk hedge instruments**

| Identifier | Description | OAD | 6mo KRD | 2yr KRD | 5yr KRD | 10yr KRD | 20yr KRD | 30yr KRD |
|---|---|---|---|---|---|---|---|---|
| 912795P5 | US TREASURY BILLS | 0.50 | 0.49 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| TUM9:CBT | 2y Treasury Notes | 1.95 | 0.03 | 1.92 | 0.00 | 0.00 | 0.00 | 0.00 |
| FVM9:CBT | 5y Treasury Notes | 4.17 | 0.01 | 0.86 | 3.34 | 0.00 | 0.00 | 0.00 |
| TYM9:CBT | 10y Treasury Notes | 6.20 | 0.01 | 0.11 | 3.70 | 2.50 | 0.00 | 0.00 |
| USM9:CBT | 30y US Treasury Bonds | 10.81 | 0.01 | 0.09 | 0.39 | 4.91 | 5.82 | 0.00 |
| 912834AU | US TREASURY STRIPS | 30.40 | -0.04 | -0.26 | -1.05 | -4.32 | -10.62 | 46.68 |
| Initial Portfolio | Aggregate analytics | 5.99 | 0.06 | 0.77 | 1.44 | 1.75 | 0.94 | 1.03 |

As of March 31, 2009. OAD and KRDs in years. Source: Barclays Capital

To set up the linear hedging problem in the optimizer, users should do the following.

- Create a portfolio containing all hedge instruments as of the hedging date and use this portfolio as the tradable universe in the optimizer.
- Constrain the optimizer from trading securities in the initial portfolio, but allow it to take both long and short positions in the hedge instruments.
- Add six generic constraints specifying that all key rate durations of the resulting portfolio should be zero.
- To keep the hedging process cash neutral, specify zero market value change in the budget constraint. The optimizer includes a counter trade in base currency cash to fund the trades in the hedge instruments. Since the hedges are expected to be short positions, it would generate a cash inflow (due to trades in the T-strip and T-bill, if any).
- Leave the objective function blank.

Below, we compare the perfect linear hedge (Version 1) – in which six instruments are used to hedge out the six key rate durations[15] – to alternative hedging options. In Version 2, we use all six instruments to hedge the curve risk of the portfolio by minimizing isolated curve risk based on the risk model. The non-curve components of portfolio risk are implicitly hedged (i.e., zeroed out in the risk definition) to define isolated curve risk. In Versions 3 and 4, we use only the four futures contracts. Similar to Version 2, Version 3 minimizes isolated curve risk. Version 4 zeros out the sum of 6m and 2y KRDs, the 5y KRD, the 10y KRD, and the sum of the 20y and 30y KRDs (four constraints).

**Figure 9: Optimal curve hedges**

| Identifier | Description | V1: Perfect Hedge | V2: RM Hedge | V3: RM Hedge (Futures only) | V4: Linear Hedge (Futures only) |
|---|---|---|---|---|---|
| 912795P5 | US TREASURY BILLS | -8,630 | -9,077 | NA | NA |
| TUM9:CBT | 2y Treasury Notes | -32,802 | -33,115 | -3,305 | -22,611 |
| FVM9:CBT | 5y Treasury Notes | -3,330 | -2,503 | -91,557 | -30,052 |
| TYM9:CBT | 10y Treasury Notes | -27,251 | -28,084 | 53,229 | -2,496 |
| USM9:CBT | 30y US Treasury | -15,621 | -15,431 | -39,199 | -26,193 |
| 912834AU | US TREASURY STRIPS | -6,739 | -6,854 | NA | NA |
| USD | CASH - U.S. Dollar | 10,824 | 11,308 | NA | NA |

Numbers in 1000s (position amount). Source: Barclays Capital POINT

[15] Hedging six KRDs using six hedge instruments leads to a system of six linear equations in six variables, resulting in a unique solution.

Figures 9, 10, and 11 display the resulting optimal hedges, KRD profiles, and portfolio curve risk and return, respectively. The hedges in V1 and V2 are very similar in terms of hedging positions (Figure 9), KRD exposure (Figure 10), and curve risk achieved (Figure 11). This is because the six key rate risk factors explain the majority of the treasury curve risk, and risk minimization leads to a solution similar to linear KRD hedging. Indeed, if the six key rates were the only factors capturing curve risk in the POINT risk model, the hedges would be identical. The small difference is due to the fact that the POINT risk model is also using a second-order rate risk factor – the square of the average change of key rates. Exposure to this factor is measured by the portfolio's convexity. The strip bond has a significant convexity of 9.33. The optimizer uses a larger position in the strip bond to hedge out the convexity factor in V2 even if this causes KRDs not to be perfectly matched. As expected, V2 achieves lower curve risk relative to V1 (Figure 11) as measured by the POINT risk model, but the difference is so small that it does not have statistical significance given the estimation uncertainty in the parameters of the risk model. However, in many situations risk-model based hedging may reveal risk factors whose effect may not be captured by standard linear hedging.[16]

It is interesting to compare the V3 and V4 solutions. Both use only the four futures and cannot match all six key-rate durations. In V4, by combining the 20y and 30y exposures we force the optimizer to use the 20y futures contract to match the sum of the two exposures. As a result the hedged portfolio is long 30y rates and short 20y rates, effectively having a flattener position in the long end of the curve. The opposite occurs at the short end of the curve where we bucket the 6mo with the 2y key-rate exposure, but the overall exposure and risk contribution is much smaller. In V3 the optimizer balances the 20y negative net KRD with a combination of long exposures to the 10y and the 30y points effectively creating a butterfly position. Although the magnitude of the key-rate exposures in V3 is significantly larger than those in V4, the overall risk is smaller (9.0bp vs. 12.1bp, see Figure 11) because a butterfly position has much lower risk than an outright flattener. This risk reduction is not insignificant but it comes at the cost of increasing the notional of the hedges from about 81mn to 187mn (Figure 11), more than doubling transaction costs. As always, managers must trade-off transaction cost with hedge effectiveness.

**Figure 10: Initial and hedged KRD profile**

| | Initial Value | V1: Perfect Hedge | V2: RM Hedge | V3: RM Hedge (Futures only) | V4: Linear Hedge (Futures only) |
|---|---|---|---|---|---|
| 6mo KRD | 0.06 | 0.00 | 0.00 | 0.05 | 0.04 |
| 2yr KRD | 0.77 | 0.00 | 0.00 | -0.21 | -0.04 |
| 5yr KRD | 1.44 | 0.00 | 0.00 | 0.05 | 0.00 |
| 10yr KRD | 1.75 | 0.00 | -0.01 | 0.90 | 0.00 |
| 20yr KRD | 0.94 | 0.00 | 0.02 | -2.01 | -1.03 |
| 30yr KRD | 1.03 | 0.00 | -0.02 | 1.03 | 1.03 |

KRD units in years. Source: Barclays Capital POINT

In addition to portfolio curve risk, Figure 11 also displays the ex-post return of the hedged portfolios attributable to the changes in the treasury curve during April 2009. The unhedged portfolio lost more than 2% of its value because of the changes in the Treasury curve over the month. The four versions of hedged portfolios experienced a curve return ranging from

[16] For example, hedging treasury curve exposure with swaps introduces exposure to swap spreads. A KRD linear hedge completely ignores this factor, yet during the crisis of 2008 swap spreads' volatility significantly reduced the effectiveness of swap hedges. A risk model based hedge would explicitly account for this risk.

-2.3bp to 11.8bp, indicating that the hedges were effective to a large extent. The ex-post curve return is not zero, as one might expect, because of hedge slippage and convexity effects over the course of the month.

We note that the realized curve return under V1 and V2 is significantly larger than the predicted curve return volatility. We measure the "curve return" over the month using the exact scenario-based return splitting methodology employed in POINT's Hybrid Performance Attribution model, rather than using the seven factors (i.e., six key-rate changes and average key-rate square change) employed by the risk model. The risk model incorporates any curve risk in excess of the seven factors into the excess return risk and idiosyncratic models. In general, as with any factor model, we should exercise caution when isolating specific components of risk, especially as such factor-based risk estimates can be driven close to zero in the context of portfolio hedging.

**Figure 11: Initial and hedged portfolio's isolated curve risk and ex-post curve returns**

| | Initial Value | V1: Perfect Hedge | V2: RM Hedge | V3: RM Hedge | V4: Linear Hedge |
|---|---|---|---|---|---|
| Curve Risk (bp/mo) | 210.2 | 1.4 | 1.2 | 9.0 | 12.1 |
| April '09 Curve Return (bp) | -214.8 | 11.8 | 11.6 | -2.3 | 6.5 |
| Notional of hedges ('000) | | 94,373 | 95,064 | 187,290 | 81,352 |

Source: Barclays Capital POINT

The optimizer is also effective when the number of hedge instruments is larger than the number of risk factors. In this case, there are infinite solutions to the linear hedging problem; hence, it is recommended that users specify a preference criterion in addition to neutralizing the risk factor exposure. In the absence of such criteria, the optimizer chooses a random corner point solution. Users can choose to optimize transaction costs, idiosyncratic risk, turnover, expected returns, or a combination of these to choose an optimal solution from the set of many feasible solutions.

## B. Hedging FX risk

The POINT system has unified and extensive support for portfolios containing securities denominated in multiple currencies. All return and analytics calculations are performed in the user-specified base currency. Similarly, users specify a base currency in each optimization problem, and the optimizer defines the portfolio attributes, including risk, relative to the base currency. Exposures to all currencies except the base currency incur FX risk in the risk model based on FX risk factors. Below, we demonstrate how the optimizer can be used to construct FX hedges using FX forwards by neutralizing exposure to the currency risk factors and minimizing FX risk based on the estimated factor covariance matrix.

### Using central rolling FX forwards to hedge FX risk

POINT provides users with access to a collection of *central* portfolios containing at-the-money rolling hedge instruments to facilitate easy creation of appropriate hedge universes. This collection includes several portfolios containing the FX forwards in various currency pairs and various maturities. The instruments contained in these portfolios are rolling in the sense that they are created on the fly when these portfolios are used as tradable universes in the optimizer. The definition of these instrument changes with the optimization date, since they are always constant-maturity at-the money instruments by construction. Readers can consult the optimizer user guide and other POINT documentation for a precise description of the behavior of the rolling instruments.

In practice, global portfolios are typically managed on an FX-hedged basis, and FX hedging is performed separately. We use the central FX forward portfolios in constructing currency hedges for a global credit portfolio benchmarked to the Barclays Global Credit Corp index as of July-end 2009. The base currency is assumed to be USD. We use eleven 1m FX forwards. A long position in each of these contracts has USD as the receive currency and the rest of the currencies spanned by the benchmark index as the pay currency.

We construct three different hedging schemes. H1 minimizes the isolated FX risk of the total portfolio. H2 matches the portfolio weights to the benchmark in EUR, JPY, GBP, and CAD only. H3 minimizes a combination of isolated FX risk and transaction costs. The optimal hedge in H2 uses exactly four forwards to satisfy the four constraints on the respective currency buckets (Figure 12). Since the portfolio's major active currency exposure comes from these four currencies, this hedge is able to reduce the isolated FX TEV from 48.1 to 1.2bp/month. H1, as expected, completely neutralizes the TEV, but uses ten hedge positions, five of which are very small. In comparison, H3 reduces transaction costs[17] by approximately 20% at the cost of increasing the ex-ente risk to 4.6bp/month (Figure 12).

**Figure 12: Comparing various FX hedges**

| | Initial | H1 | H1 | H3 |
|---|---|---|---|---|
| Isolated FX TEV (bp/mo) | 48.1 | 0.0 | 1.2 | 4.6 |
| # of hedges | | 10 | 4 | 5 |
| Transaction Costs (USD) | | 10,750 | 10,380 | 8,657 |

Source: Barclays Capital

Figure 13 displays the initial active currency exposure (market value %) profile of the initial portfolio, as well as the three hedged portfolios, to the 12 currencies spanned by the benchmark index. The active exposures[18] in H1 are almost zero. The active exposures of EUR, JPY, GBP, and CAD are zero, as constrained, in H2. This hedge does not change exposures in the rest of the currencies, and base currency is used to fund all four hedges, thus increasing the active exposure to 0.30% from -6.29%. H3 chooses an optimal tradeoff between reducing transaction costs and reducing active exposures to minimize the TEV, resulting in significant active exposure after the hedges.

**Figure 13: Currency exposure: Comparing various FX hedges**

| Market Value [%] | Portfolio | Global Credit Corp | Active (portfolio) | H1 | H2 | H3 |
|---|---|---|---|---|---|---|
| United States Dollar | 47.09 | 53.38 | -6.29 | 0.00 | 0.30 | 1.74 |
| European Euro | 44.88 | 31.22 | 13.66 | 0.00 | 0.00 | 2.77 |
| Pounds Sterling | 3.52 | 7.06 | -3.54 | 0.00 | 0.00 | -2.13 |
| Japanese Yen | 3.04 | 5.58 | -2.54 | -0.01 | 0.00 | -1.08 |
| Canadian Dollar | 1.30 | 2.30 | -1.00 | 0.06 | 0.00 | -1.00 |
| Australian Dollar | 0.17 | 0.26 | -0.09 | -0.09 | -0.09 | -0.09 |
| Korean Won | | 0.09 | -0.09 | 0.01 | -0.09 | -0.09 |
| Singapore Dollar | | 0.04 | -0.04 | 0.04 | -0.04 | -0.04 |
| New Zealand Dollar | | 0.03 | -0.03 | -0.03 | -0.03 | -0.03 |
| S. African Rand | | 0.02 | -0.02 | 0.03 | -0.02 | -0.02 |
| Danish Krone | | 0.01 | -0.01 | 0.01 | -0.01 | -0.01 |
| Norwegian Krone | | 0.01 | -0.01 | -0.01 | -0.01 | -0.01 |

Source: Barclays Capital POINT

[17] A transaction cost of 5bp is assumed for all FX forwards. A 2:1 tradeoff is used between the TEV and the transaction costs.

[18] Even though an at-the-money FX forward contract is market value neutral, it changes the allocation (market value weights) across the two underlying currencies (the hedging effect).

## C. Hedging excess of curve risk

Treasury futures, swaps, and other interest rate derivatives are effective in hedging against the common risk in fixed-income portfolios – curve risk. Liquid derivative products to hedge portfolio risk in excess of the curve are now available. Relatively low trading costs make these derivative products effective means of hedging excess risk. For example, liquid credit derivatives can be used to hedge credit risk in cash credit portfolios. Similarly, in global portfolios, local market hedges are placed to mitigate risk in excess of FX and curve risk. In practice, various specialist groups are assigned the task of hedging various components of portfolio risk, and separate hedges are designed for currency, curve, and credit risk.

The optimizer allows users to hedge excess risk using the implicit hedging mechanism in the risk model. Implicit hedging allows users to create a custom risk model by zeroing out the risk model loadings for specific categories of risk factors for each security in the universe. For example, if currency and curve risk are implicitly hedged in a global credit portfolio, the optimizer exclusively focuses on the credit risk in excess of FX and curve.

We present an example of hedging the credit risk of a US credit portfolio benchmarked to the Barclays US Credit index using the CDX contracts at June-end 2009. Assume that the portfolio is rebalanced quarterly and derivatives are used to hedge its risk relative to the index between the rebalancing dates.

To define this hedging problem in the optimizer, we create a tradable universe containing the 5y and 10y CDX-NA-IG Series 12 instruments. These are the two most liquid CDX contracts. The initial portfolio has a market value of $100mn and consists[19] of 50 US corporate credit bonds. The portfolio happens to be short spread duration relative to the index at June-end 2009; hence, any potential hedge should take additional credit exposure using these derivatives. A simple approach is to choose 10y CDX as the hedge instrument because its spread duration is closer to the index, and choose a hedge ratio to neutralize active spread duration exposure. The portfolio's active spread duration is equal to 5.38 - 5.88 = -0.50 years. Thus, a spread duration-neutral hedge is to go long (i.e., buy protection) a 0.50 * 100/7.75 = 6.45mn notional of the 10y CDX contract.

The risk minimization-based optimal hedge suggested by the optimizer is to take a 7.58mn notional long position in the 10y CDX contract. The optimizer did not choose any position in the 5y contract. Although such a hedge reduces the portfolio exposure (as measured by spread duration) to credit spreads measurably, the overall risk of the portfolio is reduced only marginally, to 36.6bp/month from 40.8bp/month. Figure 14 presents the portfolio's active spread duration contributions and isolated TEV decomposition into various credit sectors before and after applying the hedge.

[19] This demo portfolio is constructed to replicate the benchmark credit index as of December 31, 2008. By the hedging date (June 30, 2009), the portfolio has become significantly underweight US corporate credit relative to the index.

Figure 14: Effect of CDX hedge on portfolio's risk decomposition

| | Portfolio | | | Portfolio + Hedge | | |
|---|---|---|---|---|---|---|
| Security Partition Bucket | Active OASD Contribution (yrs) | Total CTEV | Isolated TEV | Active OASD Contribution (yrs) | Total CTEV | Isolated TEV |
| Total | -0.50 | 40.8 | 40.8 | 0.06 | 36.6 | 36.6 |
| Banking | -0.22 | 10.0 | 17.4 | -0.21 | 9.5 | 17.3 |
| Finance | 0.19 | 5.5 | 15.6 | 0.26 | 6.4 | 16.0 |
| Basic | -0.08 | 4.8 | 9.5 | 0.00 | 4.2 | 9.3 |
| Consumer Cyclical | 0.06 | 1.9 | 10.0 | 0.16 | 1.8 | 10.4 |
| Consumer Non-Cyclical | -0.23 | 7.1 | 12.5 | -0.14 | 4.4 | 11.2 |
| Energy & Transport | -0.12 | 5.7 | 9.3 | -0.05 | 3.8 | 8.4 |
| Technology & Communications | 0.11 | -0.6 | 10.1 | 0.20 | 1.0 | 11.3 |
| Utility | -0.18 | 8.9 | 14.0 | -0.15 | 5.7 | 13.5 |
| Non-Corporate | 0.00 | -2.5 | 14.6 | 0.00 | 0.0 | 14.5 |
| Others | 0.00 | 0.0 | 0.0 | 0.00 | 0.0 | 0.0 |

TEV in bp/mo. Source: Barclays Capital POINT

The optimal CDX hedge is less effective in reducing tracking error volatility than one might expect because of the basis risk between the CDX contract and the underlying cash market. It is well recognized that in the recent liquidity crisis, CDX instruments turned out to be an ineffective hedge for cash credit portfolios. The optimal hedge in the optimizer is reflecting this recent history. The CDX basis is modelled using additional basis factors. The basis factors explain almost half of the mark-to-market risk (i.e., risk attributable to daily spread changes) in the CDX contract based on the June 2009 calibrations in the risk model.

## D. Hedging sector/country exposures

The optimizer can also be used to construct optimal hedges to mitigate portfolio risk emanating from particular market sectors. Users can choose to hedge specific sector exposure using sector-level constraints or use the risk model to automatically allow the optimizer to focus on sectors with large contribution to the TEV. Examples of this type of hedging problem include using liquid sector ETFs to hedge equity portfolios and using on-the-run treasury bonds to hedge portfolio duration exposure in various maturity buckets.

Suppose an equity desk would like to hedge its stock inventory on the close of July 31, 2009, using liquid sector ETFs. The inventory can be loaded in POINT as an equity portfolio. We use a diverse collection of 91 randomly weighted US stocks as a sample portfolio. The hedge universe consists of eight SPDR ETFs covering the eight sectors (spanned by the portfolio being hedged) of the S&P 500 index. We construct two different hedges. SH1 minimizes the total risk of the portfolio. SH2 additionally enforces eight constraints to neutralize the portfolio's exposure to the corresponding eight sectors. We use GICS level 1 sector classification to define the sector constraints.

The optimal hedges short the sector ETFs to hedge the corresponding equity exposure in the portfolio (Figure 15). The first row, corresponding to cash, is simply the cash inflow generated by the short trades.

Figure 15: Optimal sector ETF hedge for an equity portfolio

| | | | SH1 | | SH2 | |
|---|---|---|---|---|---|---|
| Identifier | Description | Price | Shares traded | Trade Market Value [%] | Shares traded | Trade Market Value [%] |
| USD | CASH - U.S. Dollar | 1.00 | 1,316,279 | 111.8% | 1,209,565 | 102.7% |
| XLB UP | MATERIALS SELECT SECTOR SPDR | 29.12 | -6,503 | -16.1% | -6,289 | -15.6% |
| XLE UP | ENERGY SELECT SECTOR SPDR | 50.51 | -2,288 | -9.8% | -1,874 | -8.0% |
| XLI UP | INDUSTRIAL SELECT SECT SPDR | 23.88 | -15,704 | -31.9% | -14,056 | -28.5% |
| XLK UP | TECHNOLOGY SELECT SECT SPDR | 19.70 | -8,062 | -13.5% | -10,508 | -17.6% |
| XLP UP | CONSUMER STAPLES SPDR | 24.52 | -1,997 | -4.2% | -1,539 | -3.2% |
| XLU UP | UTILITIES SELECT SECTOR SPDR | 28.94 | -4,366 | -10.7% | -3,509 | -8.6% |
| XLV UP | HEALTH CARE SELECT SECTOR | 27.87 | -4,168 | -9.9% | -3,468 | -8.2% |
| XLY UP | CONSUMER DISCRETIONARY SELT | 25.27 | -7,362 | -15.8% | -6,061 | -13.0% |

Source: Barclays Capital POINT

Figure 16 displays the effect of two hedges on the portfolio's exposure and contribution to the total TEV corresponding to the eight sectors. We see that SH1 and SH2 are able to reduce the total TEV of the portfolio from 9.15%/month to 1.25%/month and 1.46%/month, respectively. Since SH2 is a constrained version of SH1, the risk reduction is lower in SH2. Note that the optimal hedges reduce the "contribution to TEV," as well as the weights, uniformly across the sectors (Figure 16).

Figure 16: Sector weights and contribution to TEV of initial and hedged portfolio

| | Market Weight (%) | | | Contribution to TEV (%/mo) | | |
|---|---|---|---|---|---|---|
| | Initial | SH1 | SH2 | Initial | SH1 | SH2 |
| Total | 0.0 | 0.0 | 0.0 | **9.15** | **1.25** | **1.46** |
| Energy | 8.0 | -1.8 | 0.0 | 0.87 | 0.09 | 0.16 |
| Materials | 15.6 | -0.5 | 0.0 | 1.84 | 0.19 | 0.16 |
| Industrials | 28.5 | -3.3 | 0.0 | 2.83 | 0.38 | 0.49 |
| Consumer Discretionary | 13.0 | -2.8 | 0.0 | 1.62 | 0.14 | 0.34 |
| Consumer Staples | 3.2 | -1.0 | 0.0 | 0.16 | 0.02 | 0.03 |
| Health Care | 8.2 | -1.7 | 0.0 | 0.38 | 0.08 | 0.06 |
| Information Technology | 14.8 | 1.4 | 0.0 | 0.89 | 0.27 | 0.12 |
| Utilities | 8.6 | -2.1 | 0.0 | 0.57 | 0.07 | 0.12 |
| Cash | -100.0 | 11.8 | 0.0 | 0.00 | 0.00 | 0.00 |

Source: Barclays Capital POINT

Similar to this example, equity index futures, index ETFs, and equity index options can be used to hedge equity portfolios in a constrained risk minimization framework.

## 2. Index replication

The goal of passive management is to achieve returns identical to a benchmark market index. This is achieved by constructing index-replicating portfolios seeking to minimize the tracking error relative to the benchmark index. The expectation is that low ex-ante tracking error risk should generate good ex-post tracking performance. The obvious, but impractical, method to achieve this is full replication – buy all benchmark securities in proportion to their market weights in the benchmark. Full replication is feasible only at very large portfolio size, if at all, because of large transaction costs due to many illiquid securities in a typical benchmark index. In practice, managers construct portfolios containing a limited number of securities to replicate the index exposure to most risk factors (i.e., replicate the index beta). This can be achieved using cash securities (bonds, stocks, etc.) or using derivatives (e.g., using interest rate swaps, treasury futures, etc[20]). There are many popular methods for constructing index-replicating cash portfolios. The portfolio can minimize the tracking error volatility (TEV) forecast using a factor risk model subject to using a limited number of positions. Alternatively, a stratified sampling approach, as described in Section II.2.J, can be used. The following index replication examples illustrate the use of the optimizer using various techniques.

## A. Replicating the US Aggregate index using cash bonds

We construct replicating portfolios for the Barclays US Aggregate Index as of May 29, 2009, and discuss the tracking performance of these portfolios in June 2009. The US Aggregate is the most popular benchmark index in the US bond market. The market capitalization-weighted index covers the US investment grade fixed-rate bond market, with index components for government and corporate securities, mortgage pass-through securities, and asset-backed securities.

The returns universe of the index for June 2009 consists of 8,866 bonds. Our objective is to construct a 100-bond portfolio picked from the index constituents so that its total return over the month is as close as possible to the index. Since a large number of the bonds in the index are not liquid, we construct a custom index by filtering the aggregate Index using the amount outstanding criteria as a liquidity measure. All bonds with amounts outstanding smaller than 300mn are excluded. We use this custom index as the tradable universe and the original index as the benchmark in the optimizer.

We construct five different portfolios, depicted in Figure 17. The first four portfolios minimize the total TEV subject to holding at most 100 bonds. Portfolio 1 simply minimizes TEV. Portfolio 2 limits the market weight of any single bond to a maximum of 4%. Portfolio 3 is constrained to match the benchmark index in terms of OAD and OAS. Portfolio 4 is constrained to match the OAD, OAS, and market value weights in class-1[21] sectors to the benchmark index.

**Figure 17: Replicating the US Aggregate index using cash bonds: various formulations**

| Portfolio Index | Description | Total TEV | Systematic TEV | Idiosyncratic TEV |
|---|---|---|---|---|
| 1 | Minimize TEV | 9.7 | 4.6 | 8.5 |
| 2 | Minimize TEV and limit each bond's exposure to 4% | 14.1 | 12.1 | 7.4 |
| 3 | Minimize TEV and match portfolio OAD and OAS to benchmark | 10.0 | 4.6 | 8.8 |
| 4 | Minimize TEV and match portfolio OAD, OAS, and class-1 sector weights to benchmark | 9.7[22] | 4.3 | 8.7 |
| 5 | Stratified sampling | 34.2 | 31.4 | 13.5 |

TEV in bp/month. Source: Barclays Capital

[20] *Replicating Bond Index Baskets: A Synthetic Beta for Fixed Income,* August 2006, Barclays Capital Research Publication.

[21] Classes 1, 2, 3, and 4 are Barclays Capital's issuer-based security classifications.

[22] The TEV achieved in Portfolio-4 is lower than in Portfolio-3, even though Portfolio-4 is a constrained version of Portfolio-3. In theory, this should not happen. As we discussed in the Section III.C, we use custom heuristic algorithms to solve optimization problem involving combinatorial constraints. The heuristic solutions allow for this possibility rarely if the additional constraints are "consistent" with the objective function. Portfolio-4 additionally enforces that class-1 sector weights to be matched to the benchmark. These constraints are consistent with the TEV minimization objective.

The optimization for Portfolio 5 is based on the following stratified sampling approach. The objective function is left blank, and the following constraints are enforced on various portfolio analytics relative to the benchmark.

- Match OAD and the six KRDs to the benchmark index to replicate curve risk at the portfolio level,
- Match portfolio weight and spread duration (OASD) exposure in all class 2 sectors (gov-related agencies, gov-related local authority, gov-related sovereign, gov-related supranational, corp industrials, corp utilities, corp fin inst, MBS agency fixed rate, MBS agency hybrid ARM, ABS, CMBS).
- Match portfolio weight, spread (OAS), and spread duration (OASD) Exposure in the 25 credit sectors based on the Barclays Capital Class 3 credit classification.
- Limit the portfolio weight of individual issues to 5%

Figure 18 displays a partial screen of these constraints in POINT. The above setup leads to 99 binding linear constraints. The optimizer constructs[23] a portfolio with 99 bonds.

**Figure 18: Stratified sampling replication: Sample constraints**

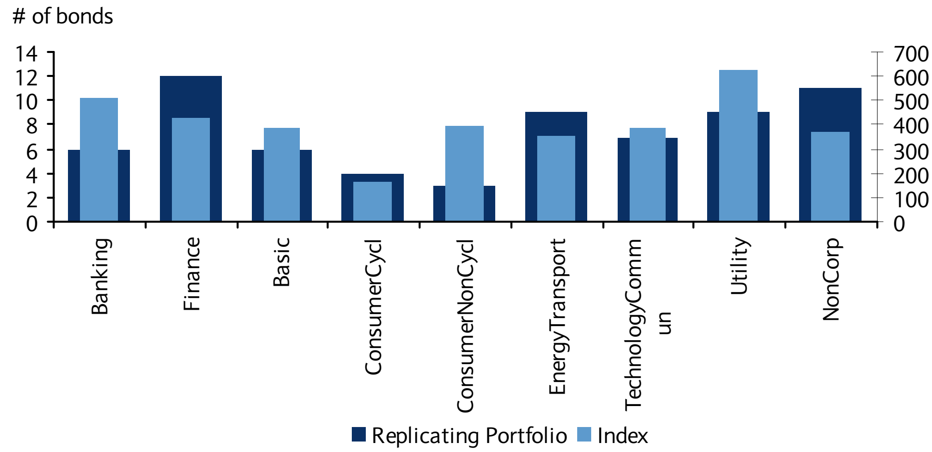
| No. | Attribute | Universe | Measure | Lower Bound | Upper Bound | Unit | Initial Value | Realized Value |
|---|---|---|---|---|---|---|---|---|
| 1 | OAD | Final Portfolio | Net vs Bmark | 0.00 | 0.00 | yrs | -4.14 | 0.00 |
| 2 | KRD 6mo | Final Portfolio | Net vs Bmark | 0.00 | 0.00 | yrs | -0.17 | 0.00 |
| 3 | KRD 2yr | Final Portfolio | Net vs Bmark | 0.00 | 0.00 | yrs | -0.58 | 0.00 |
| 4 | KRD 5yr | Final Portfolio | Net vs Bmark | 0.00 | 0.00 | yrs | -1.14 | 0.00 |
| 5 | KRD 10yr | Final Portfolio | Net vs Bmark | 0.00 | 0.00 | yrs | -1.17 | 0.00 |
| 6 | KRD 20yr | Final Portfolio | Net vs Bmark | 0.00 | 0.00 | yrs | -0.68 | 0.00 |
| 7 | KRD 30yr | Final Portfolio | Net vs Bmark | 0.00 | 0.00 | yrs | -0.41 | 0.00 |
| 8 | Market Value [%] | Treasury:Class 2 Unnested/System | Net vs Bmark | 0.00000 | 0.00000 | % | -25.58695 | -0.00005 |
| 9 | Market Value [%] | Gov-Related Agencies:Class 2 Unnested/System | Net vs Bmark | 0.00000 | 0.00000 | % | -11.03170 | 0.00000 |
| 10 | Market Value [%] | Gov-Related Local Auth:Class 2 Unnested/System | Net vs Bmark | 0.00000 | 0.00000 | % | -0.66793 | 0.00000 |
| 11 | Market Value [%] | Gov-Related Sovereign:Class 2 Unnested/System | Net vs Bmark | 0.00000 | 0.00000 | % | -1.05827 | 0.00000 |
| 12 | Market Value [%] | Gov-Related Supranational:Class 2 Unnested/System | Net vs Bmark | 0.00000 | 0.00000 | % | -1.14302 | 0.00000 |
| 13 | Market Value [%] | Corp Industrials:Class 2 Unnested/System | Net vs Bmark | 0.00000 | 0.00000 | % | -9.76858 | 0.00000 |
| 14 | Market Value [%] | Corp Utilities:Class 2 Unnested/System | Net vs Bmark | 0.00000 | 0.00000 | % | -2.14310 | 0.00000 |
| 15 | Market Value [%] | Corp Fin Inst:Class 2 Unnested/System | Net vs Bmark | 0.00000 | 0.00000 | % | -6.51355 | 0.00000 |
| 16 | Market Value [%] | MBS Agency Fixed Rate:Class 2 Unnested/System | Net vs Bmark | 0.00000 | 0.00000 | % | -35.71734 | -0.00004 |
| 17 | Market Value [%] | MBS Agency Hybrid ARM:Class 2 Unnested/System | Net vs Bmark | 0.00000 | 0.00000 | % | -2.45239 | 0.00000 |
| 18 | Market Value [%] | ABS:Class 2 Unnested/System | Net vs Bmark | 0.00000 | 0.00000 | % | -0.50281 | 0.00000 |
| 19 | Market Value [%] | CMBS:Class 2 Unnested/System | Net vs Bmark | 0.00000 | 0.00000 | % | -3.41436 | 0.00000 |
| 20 | OASD Exposure | Treasury:Class 2 Unnested/System | Net vs Bmark | 0 | 0 | USD | 13,056 | 0 |
| 21 | OASD Exposure | Gov-Related Agencies:Class 2 Unnested/System | Net vs Bmark | 0 | 0 | USD | 3,744 | 0 |
| 22 | OASD Exposure | Gov-Related Local Auth:Class 2 Unnested/System | Net vs Bmark | 0 | 0 | USD | 479 | 0 |
| 23 | OASD Exposure | Gov-Related Sovereign:Class 2 Unnested/System | Net vs Bmark | 0 | 0 | USD | 668 | 0 |
| 24 | OASD Exposure | Gov-Related Supranational:Class 2 Unnested/System | Net vs Bmark | 0 | 0 | USD | 426 | 0 |
| 25 | OASD Exposure | Corp Industrials:Class 2 Unnested/System | Net vs Bmark | 0 | 0 | USD | 6,143 | 0 |
| 26 | OASD Exposure | Corp Utilities:Class 2 Unnested/System | Net vs Bmark | 0 | 0 | USD | 1,499 | 0 |

Source: Barclays Capital POINT

The distribution of the number of bonds in the replicating portfolio and the index in the credit sector (owing to the credit sector constraints mentioned above) is shown in Figure 19. The distribution indicates that the portfolio is well diversified across credit sectors, similar to the index.

[23] This is a linear feasibility problem. The optimizer selects a so-called corner point solution that has same number of securities as the number of binding constraints at the solution.

**Figure 19: Credit sector distribution relative to the index (Portfolio 5)**




Source: Barclays Capital POINT

We used the performance attribution model in POINT to decompose the outperformance of these five replicating portfolios in June 2009. Even though a one-month sample cannot be regarded as indicative of tracking ability, the June 2009 outperformance is a 1.0, 1.4, 0.9, 1.5, and -0.6 sigma event for the five portfolios (Figure 20).

**Figure 20: US Aggregate Index replication: Ex-post return decomposition**

| | Portfolio 1 | Portfolio 2 | Portfolio 3 | Portfolio 4 | Portfolio 5 |
|---|---|---|---|---|---|
| Yield Curve | 0.8 | 5.4 | -3.0 | -2.9 | -1.5 |
| Asset Allocation | 23.9 | 49.0 | 16.3 | 0.1 | 0.5 |
| Security Selection | -15.2 | -35.1 | -4.3 | 17.1 | -19.0 |
| Total | 9.6 | 19.3 | 9.0 | 14.3 | -20.0 |
| TEV Forecast | 9.7 | 14.1 | 10 | 9.7 | 34.2 |

Returns in bp
Source: Barclays Capital

The returns of each portfolio are further decomposed into an asset allocation component and a security selection component (Figure 20). The asset allocation assumes[24] portfolio allocations to the treasury, government-related, securitized, and corporate sectors. We see that the asset allocation is improved as more and more relevant constraints are used in the replication process (asset allocation row in Figure 20).

**Figure 21: US Aggregate Index replication: Ex-post excess return decomposition**

| | Portfolio 1 | | Portfolio 2 | | Portfolio 3 | | Portfolio 4 | | Portfolio 5 | |
|---|---|---|---|---|---|---|---|---|---|---|
| | Alloc | Select | Alloc | Select | Alloc | Select | Alloc | Select | Alloc | Select |
| Total | 23.9 | -15.2 | 49.0 | -35.1 | 16.3 | -4.3 | 0.1 | 17.1 | 0.5 | -19.0 |
| Government-Related | 0.3 | 4.0 | -3.5 | -3.7 | 0.1 | -0.3 | 0.0 | 11.0 | 0.0 | -5.5 |
| Treasury | 4.2 | 0.1 | 16.0 | -1.5 | 2.4 | -0.1 | 0.0 | 1.2 | -0.1 | -0.3 |
| Securitized | 0.8 | 1.8 | 8.6 | -0.1 | 1.3 | 1.8 | 0.0 | 4.5 | -0.2 | 3.1 |
| Corporate | 18.6 | -21.1 | 28.0 | -29.8 | 12.6 | -6.3 | 0.1 | 0.3 | 0.2 | -16.3 |

Return in bp. Source: Barclays Capital

[24] See the documentation on the hybrid performance attribution model available on the POINT documentation page.

The corporate sector has relatively high idiosyncratic risk, which explains its major contribution to the outperformance (Figure 21). Since we match the portfolio weights to the index in the sectors used to define asset allocation returns in the construction process for portfolios 4 and 5, the asset allocation outperformance is close to zero.

The above example indicates that the constraints used in portfolio construction are a significant determinant of the tracking performance. Users can empirically compare the performance of various alternative constraint sets by using POINT's back-testing capabilities (described toward the end of this section).

## B. Replicating the FTSE All World index using global equity index futures

The FTSE All World index, available[25] in POINT, is a large/mid cap aggregate of 2,700 stocks from the FTSE global equity index series. It covers more than 90% of the investable global equity market capitalization, including the US, Europe, Japan, and emerging markets. It has exposure to 36 global currencies spread across 52 countries. We use a collection of 16 liquid equity index futures traded on the major global exchanges to replicate the equity exposure in this index. We perform this exercise as of April 30, 2009, using June 2009 futures.

Figure 22 displays the collection of global equity index futures used in this example. Assume that we want to replicate a net exposure of 100mn euros to the FTSE All World index. We use EUR as the base currency. Since futures are perfectly leveraged instruments, we can approach this problem in two ways. First, we can allow cash in all underlying currencies to be tradable and perform the optimization in return space by specifying a portfolio budget of 100mn euros. This allows the optimizer to create funded equity exposure using a position in the futures contract and the underlying cash to hedge the FX exposure in the benchmark index. Alternatively, we can perform the optimization in the P&L space and constrain the optimizer to construct a zero market value hedge using the 16 futures only, with portfolio size unconstrained. In this case, we need to implicitly hedge out the benchmark FX exposure.

**Figure 22: Global liquid equity index futures**

| Identifier | Description | Primary Exchange | Price | Currency | Country |
|---|---|---|---|---|---|
| TPM9 | TOPIX INDX FUTR Jun09 | Tokyo Stock Exchange | 841.00 | JPY | Japan |
| XPM9 | SPI 200 FUTURES Jun09 | Sydney Futures Exchange | 3,766.00 | AUD | Australia |
| QCM9 | OMXS30 IND FUTURE Jun09 | Stockholm Stock Exchange | 760.07 | SEK | Sweden |
| TWM9 | MSCI TAIWAN INDEX Jun09 | Singapore Exchange (was SIMEX) | 233.00 | USD | Taiwan |
| QZM9 | MSCI SING IX ETS Jun09 | Singapore Exchange (was SIMEX) | 230.30 | SGD | Singapore |
| PTM9 | S&P/TSX 60 IX FUT Jun09 | Montreal Exchange | 563.90 | CAD | Canada |
| STM9 | FTSE/MIB IDX FUT Jun09 | Milan Stock Exchange | 18,715.00 | EUR | Italy |
| IBM9 | IBEX 35 INDX FUTR Jun09 | Meff Renta Variable (Madrid) | 8,893.00 | EUR | Spain |
| Z_M9 | FTSE 100 IDX FUT Jun09 | LIFFE | 4,207.00 | GBP | United Kingdom |
| HIM9 | HANG SENG IDX FUT Jun09 | Hong Kong Futures Exchange | 15,311.00 | HKD | Hong Kong |
| EOM9 | AMSTERDAM IDX FUT Jun09 | EURONEXT Amsterdam | 239.10 | EUR | Netherlands |
| VGM9 | DJ EURO STOXX 50 Jun09 | Eurex Deutschland (was DTB) | 2,328.00 | EUR | Germany |
| GXM9 | DAX INDEX FUTURE Jun09 | Eurex Deutschland (was DTB) | 4,798.50 | EUR | Germany |
| SMM9 | SWISS MKT IX FUTR Jun09 | Eurex Deutschland (was DTB) | 5,216.00 | CHF | Switzerland |
| ESM9 | S&P500 EMINI FUT Jun09 | Chicago Mercantile Exchange | 870.00 | USD | United States |
| CFM9 | CAC40 10 EURO FUT Jun09 | | 3,070.00 | EUR | France |

As-of-date: April 30, 2009. Source: Barclays Capital POINT

[25] The constituent level data for third-party indices are available to POINT clients only if they are permissioned/licensed.

Figure 23 presents the optimal portfolios in the two cases. The minimum TEV achieved using the funded and unfunded approaches are 39.4bp/month and 3,660mn EUR/month, respectively. The risk in P&L optimization translates to 36.6bp/month using the 100mn benchmark notional. This is lower than the achieved TEV with return optimization. There are two reasons for this. First, we assume perfect (implicit) replication of the FX exposure in the P&L optimization, in contrast to the return optimization, in which FX exposure is explicitly included as part of the optimization. The explicit replication is not complete, since the optimizer is allowed to trade the cash only in the underlying currencies of the futures contract, which is a subset of the currencies to which the benchmark index is exposed. Second, the absence of the budget constraint means that the P&L optimization is less constrained.

**Figure 23: Optimal replication of the FTSE All World index using index futures**

| Identifier | Description | Number of Contracts | Notional [%] | Number of Contracts | Notional [%] |
|---|---|---|---|---|---|
| | | Return Optimization | | P&L Optimization | |
| ESM9 | S&P500 EMINI FUT Jun09 | 1,348 | 45.03 | 1,345 | 44.32 |
| TPM9 | TOPIX INDX FUTR Jun09 | 159 | 10.72 | 163 | 10.26 |
| Z_M9 | FTSE 100 IDX FUT Jun09 | 204 | 10.21 | 213 | 9.61 |
| HIM9 | HANG SENG IDX FUT Jun09 | 128 | 7.12 | 94 | 9.53 |
| PTM9 | S&P/TSX 60 IX FUT Jun09 | 67 | 4.61 | 63 | 4.80 |
| GXM9 | DAX INDEX FUTURE Jun09 | 32 | 4.19 | 34 | 3.88 |
| SMM9 | SWISS MKT IX FUTR Jun09 | 103 | 3.70 | 105 | 3.57 |
| CFM9 | CAC40 10 EURO FUT Jun09 | 112 | 3.61 | 115 | 3.43 |
| TWM9 | MSCI TAIWAN INDEX Jun09 | 181 | 2.61 | 49 | 3.18 |
| XPM9 | SPI 200 FUTURES Jun09 | 43 | 2.59 | 145 | 2.25 |
| EOM9 | AMSTERDAM IDX FUT Jun09 | 33 | 1.54 | 32 | 1.58 |
| IBM9 | IBEX 35 INDX FUTR Jun09 | 15 | 1.38 | 15 | 1.32 |
| QCM9 | OMXS30 IND FUTURE Jun09 | 163 | 0.98 | 135 | 1.16 |
| STM9 | FTSE/MIB IDX FUT Jun09 | 6 | 0.87 | 9 | 0.60 |
| QZM9 | MSCI SING IX ETS Jun09 | 21 | 0.84 | 35 | 0.50 |

Source: Barclays Capital POINT

In the case of return optimization, the optimal portfolio weights across currency buckets are displayed in Figure 24. Notice that the optimizer does not fund the euro futures positions, and the portfolio's cash exposure in various currencies does not match the benchmark perfectly. The optimizer is incorporating the risk model correlations that exist between various currency factors and local equity market risk factors, to choose an optimal replication. For example, the optimal portfolio is overweight AUD and TWD to capture the rest of the benchmark exposure to Asia-ex Japan.

The return optimization optimal solution suffers from two common pitfalls. First, the portfolio positions are sensitive to various risk model correlation estimates between the FX factors and the local equity factors and, hence, potentially unstable. Second, it is trading off the equity risk with FX risk. It is typically less costly to hedge the FX risk exposure than the equity risk exposure. FX exposure can be hedged separately using currency forwards and futures, as we demonstrated in the FX hedging example.

Figure 24: Explicit FX hedging: Portfolio and benchmark index weights

| Currency | Portfolio Weight | Index Weight |
|---|---|---|
| United States Dollar | 48.19 | 43.49 |
| European Euro | 0.00 | 14.29 |
| Japanese Yen | 4.77 | 9.19 |
| Pounds Sterling | 6.76 | 8.26 |
| Canadian Dollar | 5.40 | 3.34 |
| Hong Kong Dollar | 0.00 | 3.21 |
| Swiss Franc | 10.93 | 3.06 |
| Australian Dollar | 8.46 | 2.84 |
| Taiwan Dollar | 2.54 | 1.46 |
| Swedish Krona | 7.98 | 1.01 |
| Singapore Dollar | 4.96 | 0.54 |
| Others | 0 | 9.31 |

Source: Barclays Capital POINT

## C. Replicating the Global Aggregate index using cash bonds

The Global Aggregate index is the largest benchmark bond index in the Barclays Capital's family of indices. It provides a broad-based measure of the global investment-grade fixed-income markets. Its three major components are the US Aggregate, the Pan-European Aggregate, and the Asian-Pacific Aggregate Indices. The index also includes Eurodollar and Euro-yen corporate bonds; Canadian government, agency, and corporate securities; and USD investment grade 144A securities. It has exposure to 62 countries and 21 currencies.

Suppose we are interested in constructing a 100-bond replicating portfolio as of June 30, 2009, to track this index denominated in USD. Since the index consist of 12,317 bonds issued globally, full replication is not practical. As discussed in the US Aggregate index replication example above, several formulations can be used to replicate this index with a portfolio containing a small number of bonds. To begin, we can simply ask the optimizer to construct a 100-bond portfolio with minimum tracking error volatility relative to the index. We construct five different versions,[26] MC1 to MC5, of the replicating portfolio by successively adding constraints on portfolio analytics. All the optimization problems minimize portfolio TEV subject to the specified set of constraints.

The unconstrained TEV minimization constructs a portfolio with a TEV of 8.0bp/month (MC1). There are active currency exposures in this portfolio. To eliminate these active currency weights, we add four generic constraints to match portfolio active weights in G4 currencies to the benchmark. The optimal portfolio in this formulation has a marginally higher TEV of 8.9bp/month (MC2). Figure 25 compares the market weight, spread, and duration of the optimal portfolio and the benchmark index in the four major currencies. As required in the formulation, there is no active G4 currency exposure in MC2 portfolio.

[26] MC = multi-currency.

Figure 25: Global Agg replication: Duration and spread comparison (MC2)

| | Optimal replicating portfolio | | | | | Global Aggregate index | | | | | Difference | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | OAD (yrs) | Market Value [%] | # of bonds | OAS (bp) | Yield to Worst (%/year) | OAD | Market Value [%] | # of bonds | OAS | Yield to Worst | OAD | Market Value [%] | OAS | Yield to Worst |
| Total | 5.06 | 100.0 | 100 | 122.2 | 3.71 | 5.30 | 100.0 | 12317 | 85.0 | 3.41 | -0.24 | 0.00 | 37.2 | 0.30 |
| USD | 4.25 | 39.8 | 38 | 154.1 | 4.52 | 4.37 | 39.8 | 5773 | 107.3 | 4.10 | -0.12 | 0.00 | 46.8 | 0.48 |
| EUR | 5.27 | 30.7 | 34 | 149.4 | 3.99 | 5.32 | 30.7 | 3109 | 106.3 | 3.59 | -0.05 | 0.00 | 43.1 | 0.40 |
| GBP | 7.38 | 5.3 | 6 | 164.8 | 4.64 | 8.28 | 5.3 | 909 | 117.8 | 4.53 | -0.90 | 0.00 | 47.0 | 0.11 |
| JPY | 6.20 | 17.6 | 16 | 17.2 | 1.05 | 6.44 | 17.6 | 1517 | 7.7 | 0.96 | -0.24 | 0.00 | 9.5 | 0.09 |
| Others | 4.02 | 6.6 | 6 | 49.2 | 3.87 | 5.33 | 6.7 | 1009 | 31.1 | 3.98 | -1.31 | 0.00 | 18.1 | -0.11 |

Source: Barclays Capital POINT.

POINT's Global Risk Model (GRM) models credit risk using percentage changes in sector credit spreads as the risk factor and duration times spread ("DTS") as the risk factor loading for each bond. Hence, it is sufficient to achieve a level of DTS close to the index along diversified credit sectors to achieve a low systematic tracking error volatility. The risk model uses 25 credit sectors that are a combination of class 3 and class 4 sectors. Because there are no specific constrains to match the benchmark OAD or OAS, the optimizer creates a replicating portfolio whose duration and spread do not match the duration and spread of the benchmark. In particular, the portfolio is slightly shorter than the benchmark, and has slightly higher OAS in all G4 currency sectors (Figure 25).

Suppose that we want to avoid this substitution effect between duration and spreads. We duplicate the optimizer report again and add five additional constraints to match portfolio duration to the benchmark in the G4 currency sectors (MC3). The resulting optimal portfolio's market structure is depicted in Figure 26. The MC3 achieves a total TEV of 9.0bp/month.

Figure 26: Global Agg replication: Duration and spread comparison (MC3)

| | Optimal replicating portfolio | | | | | Global Aggregate Index | | | | | Difference | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | OAD (years) | Market Value [%] | # of bonds | OAS (bp) | Yield to Worst (%/year) | OAD | Market Value [%] | Count | OAS | Yield to Worst | OAD | Market Value [%] | OAS | Yield to Worst |
| Total | 5.30 | 100.00 | 100 | 137.2 | 3.900 | 5.30 | 100.00 | 12317 | 85.0 | 3.409 | 0.00 | 0.00 | 52.1 | 0.491 |
| USD | 4.37 | 39.83 | 38 | 183.2 | 4.751 | 4.37 | 39.83 | 5773 | 107.3 | 4.104 | 0.00 | 0.00 | 75.9 | 0.647 |
| EUR | 5.32 | 30.66 | 33 | 151.1 | 4.015 | 5.32 | 30.66 | 3109 | 106.3 | 3.591 | 0.00 | 0.00 | 44.7 | 0.424 |
| GBP | 8.28 | 5.35 | 6 | 154.2 | 4.831 | 8.28 | 5.35 | 909 | 117.8 | 4.531 | 0.00 | 0.00 | 36.4 | 0.300 |
| JPY | 6.44 | 17.60 | 16 | 27.9 | 1.196 | 6.44 | 17.60 | 1517 | 7.7 | 0.965 | 0.00 | 0.00 | 20.2 | 0.231 |
| Others | 5.33 | 6.56 | 7 | 71.9 | 4.690 | 5.33 | 6.56 | 1009 | 31.1 | 3.983 | 0.00 | 0.00 | 40.8 | 0.707 |

Source: Barclays Capital POINT

The optimal portfolio in MC3 still has significantly higher spreads than the benchmark (Figure 26). Next, we add five additional constraints to match the portfolio's OAS to the benchmark in the G4 currency sectors (MC4). The resulting optimal portfolio's market structure is shown in Figure 27. As constrained, the portfolio's OAD, OAS, and market weight matches the benchmark index. The optimal total TEV achieved in MC4 is 9.1bp/month.

**Figure 27: Global Agg replication: Duration and spread comparison (MC4)**

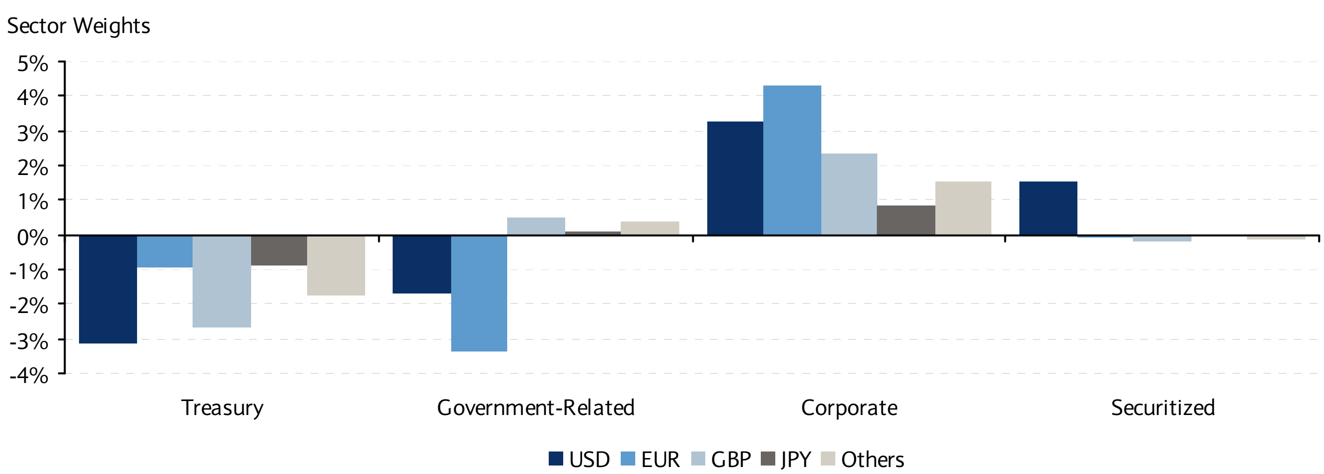
| | Optimal replicating portfolio | | | | | Global Aggregate Index | | | | | Difference | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | OAD (years) | Market Value [%] | # of bonds | OAS (bp) | Yield to Worst (%/year) | OAD | Market Value [%] | Count | OAS | Yield to Worst | OAD | Market Value [%] | OAS | Yield to Worst |
| Total | 5.30 | 100.00 | 100 | 85.0 | 3.479 | 5.30 | 100.00 | 12317 | 85.0 | 3.409 | 0.00 | 0.00 | 0.0 | 0.070 |
| USD | 4.37 | 39.83 | 44 | 107.3 | 4.170 | 4.37 | 39.83 | 5773 | 107.3 | 4.104 | 0.00 | 0.00 | 0.0 | 0.065 |
| EUR | 5.32 | 30.66 | 24 | 106.3 | 3.540 | 5.32 | 30.66 | 3109 | 106.3 | 3.591 | 0.00 | 0.00 | 0.0 | -0.050 |
| GBP | 8.28 | 5.35 | 3 | 117.8 | 4.796 | 8.28 | 5.35 | 909 | 117.8 | 4.531 | 0.00 | 0.00 | 0.0 | 0.265 |
| JPY | 6.44 | 17.60 | 22 | 7.7 | 0.959 | 6.44 | 17.60 | 1517 | 7.7 | 0.965 | 0.00 | 0.00 | 0.0 | -0.006 |
| Others | 5.33 | 6.56 | 7 | 31.1 | 4.690 | 5.33 | 6.56 | 1009 | 31.1 | 3.983 | 0.00 | 0.00 | 0.0 | 0.707 |

Source: Barclays Capital POINT

The distribution of active market weights along the nested buckets defined by class 1 sectors in each of the G4 currency buckets in MC4 is displayed in Figure 28. The portfolio is underweighting treasuries and overweighting the corporate sectors across the major currencies. The negative correlation forecasts between the risk factors explaining the returns of these two asset classes reduce the TEV contributions from the non-zero active weights. Suppose that we want to make the optimal solution insensitive to this correlation forecast. In MC5, we add the constraints that the portfolio should match market weights to the index in the class 1 corporate buckets of the G4 currency sectors. We keep the constraints on the portfolio's active weight and OAD in G4 currency buckets, but replace the spread constraints with a single portfolio-level OAS constraint[27].

Figure 29 displays the active weight to class 1 buckets of the G4 currency sectors in MC5 on the same scale used in Figure 28. We see that corporate bucket active weights are zero as constrained and active weights across the remaining buckets have declined significantly. MC5 achieves an optimal total TEV of 12.2bp/month (9.9 systematic and 7.2bp/month idiosyncratic), only a few basis point higher than the unconstrained optimization.

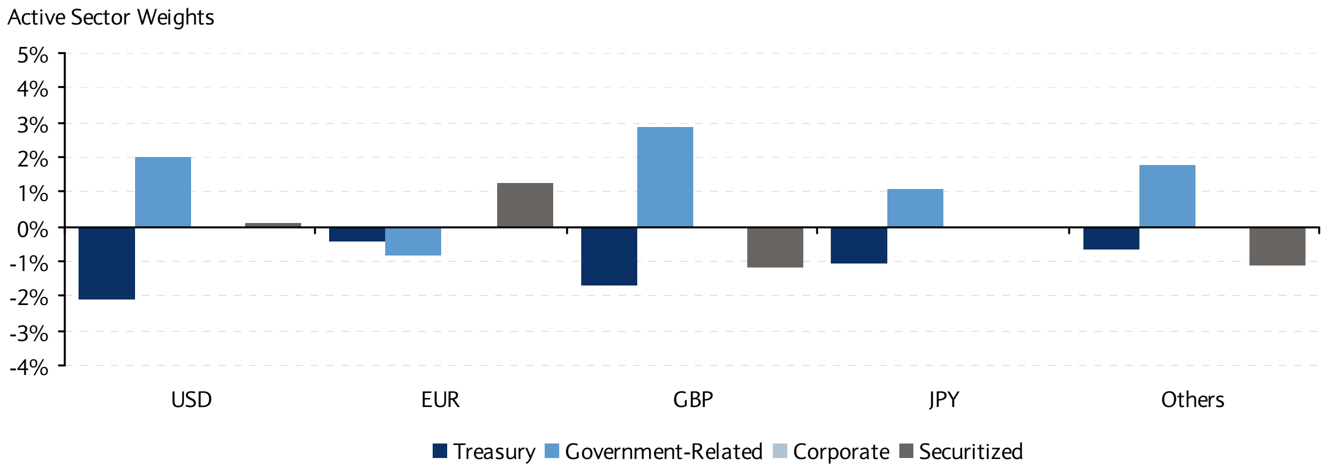
**Figure 28: Global Aggregate Index replication: Sector comparison (MC4)**




Source: Barclays Capital POINT

[27] Keeping currency specific OAS constraints makes the optimization problem infeasible.

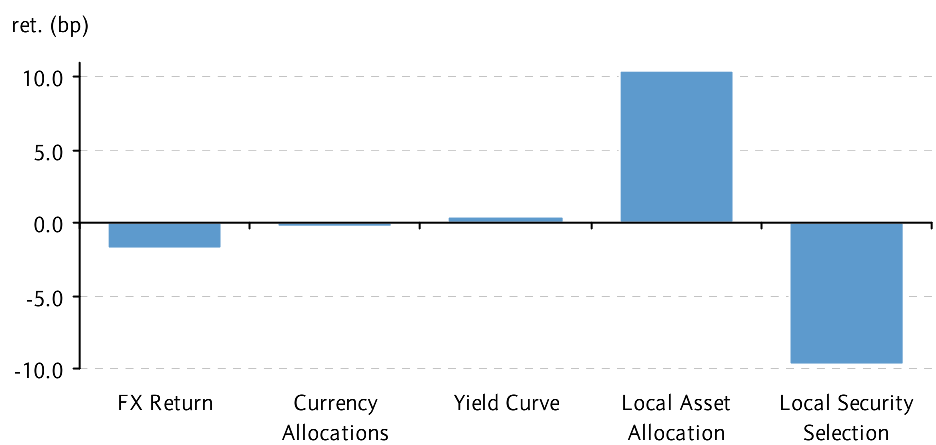
**Figure 29: Global Aggregate Index replication: Sector comparison (MC5)**




Source: Barclays Capital POINT

For illustrative purposes, we study the July 2009 tracking performance of the optimal portfolio obtained in MC5 using the performance attribution model in POINT. The portfolio's total return is 220.7bp, against a total return of 221.4bp from the Global Aggregate Index (a tracking error of less than one basis point). The performance attribution model[28] in POINT further decomposes this performance into FX returns; allocation to various currency buckets, and performance within each currency buckets (called local management). The outperformance within each currency bucket is further decomposed into treasury curve returns and excess returns. The excess return is then decomposed into asset allocation and security selection. Figure 30 displays this return decomposition.

**Figure 30: July 2009 outperformance (active return) decomposition (MC5)**




Source: Barclays Capital POINT

The local outperformance decomposition in the major currency buckets (Figure 31) indicates good tracking performance. Note that the outperformance numbers are small across buckets, and in particular a large positive return component in one bucket is not cancelled by a large negative component in another. Even though we are working with a single sample, this present some evident to indicates that the good tracking performance is not merely a chance event.

[28] See the documentation of POINT's hybrid performance attribution (HPA) model on the documentation page.

**Figure 31: July 2009 outperformance contributions from the G4 markets (MC5)**

| Return (bp) | EUR | USD | CAD | JPY | Others |
|---|---|---|---|---|---|
| Total Local Management | -6.7 | 5.9 | 1.6 | 1.2 | -0.8 |
| Asset Allocation | 3.4 | 3.1 | 3.3 | -0.3 | 1.1 |
| Security Selection | -10.4 | 3.8 | -2.3 | 0.9 | -1.7 |
| Treasury Curve and Other Common Factors | 0.3 | -0.9 | 0.6 | 0.6 | -0.2 |

Source: Barclays Capital POINT

The example above demonstrates how the flexible constraint functionality can be used to add robustness in the portfolio construction process. We successively vetted the optimal solution and added relevant bucket constraints. Note that the TEV of the optimal portfolio increased marginally from 8.9bp/mo in the MC-1 to 12.2bp/mo in MC-5. As we discussed in Section II.6, linear factor models permit a continuum of portfolios with similar levels of systematic risk. For example, to replicate the systematic credit risk in the above example, it is sufficient to closely match various DTS exposures. This can be done by choosing high OAS bonds with shorter duration, low OAS bonds with longer duration, or average OAS bonds with average duration to achieve a similar level of systematic. The subset of these portfolios with a similar level of diversification (i.e., idiosyncratic risk) is also large because of the large and diversified benchmark index. Hence, leaving it up to the optimizer to choose a portfolio from a large set of portfolios with similar risk may lead to unstable solutions (optimal portfolios can be drastically different for small changes in the problem inputs). Typically users will control for this problem by indicating preference for specific securities (e.g., using a rating system) or introducing transaction costs to indicate preference for securities already in the portfolio.

## 3. Rebalancing a portfolio

Portfolio rebalancing refers to the trading activity required to align portfolio characteristics with set targets. Rebalancing is an ongoing function whose frequency is driven by:

- Market volatility that makes a portfolio drift away from its target characteristics.
- Transaction costs.
- User preferences determining the trade-off between rebalancing cost and deviation from the set targets.

Clearly, trading costs – broadly measured by portfolio turnover, number of trades, notional traded, and transaction costs – are an important consideration in the rebalancing process.

Portfolio rebalancing can be generalized to include the trading required to invest new cash injections or to facilitate cash withdrawals from the portfolio. Cash injections and withdrawals are typically a small fraction of the portfolio size. Because of trading costs, securities are seldom bought or sold in proportion to current holdings. Instead, a small number of trades are done to incorporate the cash inflows and outflows as they occur, while the entire portfolio is rebalanced on a regular schedule, e.g., monthly (aka calander rebalancing). Investing the cash flows generated by the portfolio holdings (coupons, dividends, etc.) is also part of the portfolio rebalancing decisions. Once a rebalancing scheme is in place, the optimizer can be used to construct optimal rebalancing trades, as we demonstrate in the following example.

We rebalance a US credit portfolio using ten trades. The initial portfolio consists of 50 US credit bonds, constructed on March 31, 2009, to replicate the benchmark US Credit Index. We solve four different optimization problems (RB-1, RB-2, RB-3, and RB-4) to rebalance this portfolio as of June 30, 2009. The optimizer is allowed to trade the initial portfolio positions, and the final portfolio is constrained to be long only. The objective function is a linear combination of total TEV, yield to worst, and transaction costs.[29]

RB-1 minimizes the total TEV. RB-2 additionally constrains the active weights of the portfolio in the nine credit sectors to be within +/-4%. RB-3 minimizes an equal weighted combination of total TEV and transaction costs while keeping the sector weight constraints. RB-4 minimizes an equal-weighted combination of the total TEV and yield to worst of the portfolio while keeping the sector weight constraints but ignoring transaction costs.

Figures 32 displays the realized values of various performance measures (objective function terms) at the optimal solution to the four problems. As expected, the unconstrained problem (RB-1) achieves the largest reduction in the total TEV. Similarly, RB-3 and RB-4 achieve the lowest transaction costs and the highest yield to worst, respectively, among the four solutions.

**Figure 32: Comparing various rebalancing strategies**

| | Initial Value | RB-1 | RB-2 | RB-3 | RB-4 |
|---|---|---|---|---|---|
| Total TEV (bp/mo) | 40.9 | 30.5 | 31.8 | 33.6 | 35.2 |
| Systematic TEV (bp/mo) | 23.9 | 8.7 | 12.3 | 19.1 | 13.9 |
| Idiosyncratic TEV (bp/mo) | 33.2 | 29.3 | 29.3 | 27.6 | 32.3 |
| Yield to Worst (%/yr) | 6.29 | 6.00 | 5.95 | 5.70 | 6.54 |
| Transaction Cost - P&L ($) | N.A. | 1362 | 1217 | 913 | 1764 |

Source: Barclays Capital POINT

Figure 33 displays the active sector weights of the optimal portfolio. Since all problems except RB-1 impose these sector weights to be within +/- 4%, the rebalanced portfolio is able to neutralize the large underweight in the non-corporate sector and the large overweight in the finance sector in RB-2, RB-3, and RB-4. Notice that certain sectors are hitting the constraint limits.

**Figure 33: Sector active weights (%) before and after rebalancing (RB-3)**

| Universe | Initial Value | RB-1 | RB-2 | RB-3 | RB-4 |
|---|---|---|---|---|---|
| Banking | **4.54** | 0.21 | 2.89 | **4.00** | -0.80 |
| Finance | **9.43** | 8.30 | 3.92 | 2.88 | **4.00** |
| Basic | 0.67 | 0.67 | 0.67 | -0.12 | 0.67 |
| Consumer Cyclical | 1.83 | 0.93 | 1.83 | 1.83 | -0.87 |
| Consumer Non-cyclical | -3.54 | 2.81 | -3.54 | -3.54 | -3.54 |
| Energy and Transport | 0.51 | 0.51 | 0.51 | 0.51 | 0.51 |
| Technology and Communications | -3.50 | -3.51 | -3.51 | -2.79 | 2.80 |
| Utility | 1.23 | 1.23 | 1.23 | 1.23 | 1.23 |
| Non-Corp | **-11.16** | -11.59 | **-4.00** | **-4.00** | **-3.99** |

Source: Barclays Capital POINT

Figure 34 displays the set of trades in the solution to RB-4. Using the ten trades, the optimizer is liquidating five current positions, buying four new positions, and reducing the position in one of the initial holding in the finance sector. Since the objective in RB-4 includes maximizing the yield to worst, the new buys have a relatively high yield to worst. The optimizer is using the sells to reduce the finance sector overweight to satisfy the sector constraints.

[29] The optimizer uses a default transaction cost of 5bp in price units for all non-cash securities. Users can easily upload and use their own estimates of transaction costs.

**Figure 34: Optimal rebalancing trades (RB-4)**

| Identifier | Description | Ticker | Initial Position Amount | Trade Position Amount | Final Position Amount | Initial Market Value [%] | Trade Market Value [%] | Final Market Value [%] | Yield to Worst[%] |
|---|---|---|---|---|---|---|---|---|---|
| 06738CAD | BARCLAYS BANK PLC | BACR | 335,416 | -335,416 | | 3.41% | -3.41% | | 5.85% |
| 59156RAC | METLIFE INC | MET | 327,702 | -327,702 | | 3.47% | -3.47% | | 4.12% |
| 416515AQ | HARTFORD FINANCIAL SERV | HIG | 305,221 | -305,221 | | 3.00% | -3.00% | | 6.92% |
| 803032AF | SANWA BANK LTD | MTFG | 270,887 | -270,887 | | 2.89% | -2.89% | | 4.43% |
| 895953AD | TRICON GLOBAL RESTAURANTS INC | YUM | 243,520 | -243,520 | | 2.70% | -2.70% | | 4.12% |
| 06051GDX | BANK OF AMERICA CORP | BAC | 256,744 | -118,683 | 138,061 | 2.30% | -1.06% | 1.24% | 7.52% |
| 026874AV | AMERICAN INTL GROUP-GLOBAL | AIG | | 125,728 | 125,728 | | 1.04% | 1.04% | 23.01% |
| 060505BS | BANK OF AMERICA CORP-GLOBAL | BAC | | 219,625 | 219,625 | | 2.03% | 2.03% | 6.82% |
| 845335BW | SOUTHWESTERN BELL TEL | T | | 616,450 | 616,450 | | 6.31% | 6.31% | 7.20% |
| 298785EG | EUROPEAN INVESTMENT BANK-GLOBA | EIB | | 661,884 | 661,884 | | 7.16% | 7.16% | 4.10% |

Source: Barclays Capital POINT

Users can experiment with various rebalancing tradeoffs to obtain an implementable solution. For example, the formulation above can be modified to obtain custom rebalancing problems.

- Users can create custom buy and sell universes (e.g., based on their credit analyst forecasts) and use them with "buy only" and "sell only" trading options, respectively.
- Cash injection or withdrawal requirements can be specified using the budget constraint. In this case, the optimizer would use the cash injection (withdrawal) to fund the new buys net of the sells.
- The initial portfolio's trading option can be set to "sell only" with a specified budget reduction to partially and optimally (in terms of risk, returns, and trading costs) liquidate a portfolios using a few sells.

## 4. Long-short optimization

Traditional portfolio management focuses on long-only portfolios. Removing the long-only constraint allows managers extra flexibility to express views but also introduces complexity in the risk-return characteristics of portfolios. Long-short portfolios make it easier to express views on specific risk factors while neutralizing exposure to other risk factors. They also allow the distribution of idiosyncratic risk across all securities, those that are expected to outperform as well as those that are expected to underperform. An optimization tool is very useful in constructing such diversified long-short portfolios because long-short leverage affects portfolio risk and returns in non-trivial ways.

We highlight the use of the optimizer in constructing and analyzing long-short leveraged portfolios in the US equity market. We use POINT’s equity risk model factor loadings (fundamental and quantitative characteristics such as PE ratios, momentum, etc., see Silva, Staal and Ural [2009]) as the performance measures. Furthermore, constraints on the portfolio's ticker exposures are used to achieve model-free diversification.

Leverage in long-short cash equity portfolios is commonly measured using the short holdings as a percentage of the portfolio's net market value. For example, a 130/30 portfolio consists of a short basket that is 30% of its net market value and a long basket that is 130% of its net market value. The gross market exposure in such a portfolio is 160% of its net market value. The goal of the leveraged portfolio is typically to maximize the risk-adjusted return. We presented a long-short total return mean-variance frontier example in Section II.2.D.

Suppose we have a list of 100 buy-recommended stocks and a list of 100 sell-recommended stocks. In addition, assume that the expected return forecasts for all 200 stocks are also available. We want to construct a 130-30 long-short portfolio with short positions in sell-recommended stocks and long positions in buy-recommended stocks. This long-short portfolio construction problem can be solved using the optimizer as follows.

- Import the stock level expected return forecast in POINT in the "User Expected Return" field.
- Create a portfolio containing buy-recommended stocks and a portfolio containing sell-recommended stocks and add these two portfolios as the tradable universes in the optimizer report.
- Set up the "buy/sell/trade" options of the two portfolios to "buy only" and "sell only," respectively.
- Apply an upper bound of 160% on the portfolio leverage and maximize the risk-adjusted return objective using a custom tradeoff between risk and return.
- In addition, constrain the portfolio's market exposure from each ticker to 2% on the short side and 3% on the long side.

We constructed one such portfolio as-of April 15, 2009, using a simple market price of risk methodology to imply the expected return of S&P 500 stocks.[30] The portfolio contained 87 positions (41 shorts and 47 longs) with a total risk of 8.37%/month. The portfolio achieved a leverage of exactly 130-30 and an expected return of 10.19%/year. We refrain from discussing the portfolio's constituents and its ex-post performance relative to the forecast performance measures because our objective is simply to illustrate the functionality.

The example above can be extended in several directions. Users can incorporate a separate "neutral" list and allow the optimizer to recommend both buys and sells in this list. Users can upload custom discrete ranking of the stocks (e.g., strong buys, strong sells, etc.) and enforce specific constraints (e.g., market beta exposure) on the buckets of stocks with a particular ranking. As an alternative to the risk model, users can use sector and risk factor exposure constraints to optimize risk.

Users can also construct fully leveraged equity portfolios by specifying a target portfolio budget of zero. The return analytics of such portfolios are not defined; hence, it is necessary to perform the optimization in the P&L space. Applications of fully leveraged portfolios include constructing market value-neutral overlay strategies.

[30] We use the S&P 500 index constituents as the underlying universe. The expected returns are computed as the solution to the following inverse optimization problem. What expected returns would make the S&P 500 index the optimal tangency portfolio based on the US equity risk models and the 1mo Libor rate as the risk-free rate? The top 100 stocks (based on expected returns) are chosen as the buy universe and the bottom 100 stocks are chosen as the sell universe.

## 5. Factor-mimicking portfolios

In its simplest form, factor mimicking entails constructing portfolios of positions that have positive exposure to a specific factor driving risk and return. For example, "momentum" portfolios may be constructed by picking stocks with the "highest momentum" as measured by some algorithm. The number of positions in a factor mimicking portfolio is determined by balancing the portfolio factor sensitivity to portfolio risk. Picking the one stock with the highest momentum provides the highest possible exposure per unit of investment but also exposes the investor to significant name risk. Such risk can be diversified away by including more stocks in the portfolio at the expense of lowering the level of factor exposure per unit of investment.

The simple method described above focuses on a single factor driving risk and returns and ignores the potential exposure of the portfolio to other risk factors. For example, portfolios constructed with this method will necessarily be long the market and may have unintended sector tilts. To avoid such pitfalls and provide isolated exposure to a specific factor, factor-mimicking portfolios can be constructed in the framework of a factor-based risk model and allow short positions to neutralize exposures to other factors such as market, sector, etc. In this framework, factor-mimicking portfolios can be designed to have non-zero exposure to a target risk factor and at the same time zero exposure to other risk factors, thus capturing the risk premium of the given factor. If exposures to all risk factors except the target factor are set to zero, factor-mimicking portfolios can be mathematically defined as the projection of risk factors onto the investable security universe. However, in practice users may choose to neutralize exposure only to the most significant risk factors.

In linear factor models, factor-mimicking portfolios can be constructed by solving a linear feasibility problem. Once again, users must trade-off between the exposure to the target factor per unit of risk versus the overall risk of the portfolio. Typically, users will demand a minimum acceptable exposure to the target factor[31], and seek to minimize total portfolio risk. Because exposure to most other risk factors is constrained to be zero this becomes primarily an idiosyncratic risk minimization exercise.

The concept of factor-mimicking portfolios has widespread application in equities where risk factors are typically unobservable. For example, momentum[32] is widely accepted as a significant risk factor (hence, a determinant of equity returns), but it is difficult to construct an investable portfolio that delivers the equity market momentum risk premium. This is because the risk of momentum-driven returns is mixed with other components of equity risk (industry, volatility, size etc.).

In the following example we use the Point Optimizer to construct "momentum" factor-mimicking portfolios by minimizing the idiosyncratic risk of the portfolio subject to factor exposure constraints. We use the set of US-based stocks in the S&P 500 index as of June 30, 2009 as the tradable universe, a budget of 1mn USD, and a minimum trade size of 10,000 USD. All US equity risk factor loadings except for momentum are constrained to be zero. The exposure to the momentum factor is constrained to be at least one unit. Figure 35 displays the screen of the constraints as they are depicted in the optimizer interface.

[31] Such minimum exposure should be statistically different than zero at an appropriate significance level. The higher the measurement uncertainty the higher the minimum exposure should be set.
[32] The precise methodology used in factor construction varies and is critical in its ability to predict risk and returns.

**Figure 35: Momentum factor-mimicking constraints**

| No. | Attribute | Universe | Measure | Lower Bound | Upper Bound | Unit | Initial Value | Realized Value |
|---|---|---|---|---|---|---|---|---|
| 1 | Book to Price Loading | Final Portfolio | Absolute | 0.00 | 0.00 | | 0.00 | 0.00 |
| 2 | Hybrid Default Probability Loading | Final Portfolio | Absolute | 0.00 | 0.00 | | 0.00 | 0.00 |
| 3 | Discretionary Accruals Loading | Final Portfolio | Absolute | 0.00 | 0.00 | | 0.00 | 0.00 |
| 4 | Forecasted Earnings Price Loading | Final Portfolio | Absolute | 0.00 | 0.00 | | 0.00 | 0.00 |
| 5 | Earnings Price Loading | Final Portfolio | Absolute | 0.00 | 0.00 | | 0.00 | 0.00 |
| 6 | Total Yield Loading | Final Portfolio | Absolute | 0.00 | 0.00 | | 0.00 | 0.00 |
| 7 | Size Loading | Final Portfolio | Absolute | 0.00 | 0.00 | | 0.00 | 0.00 |
| 8 | Momentum 9m Loading | Final Portfolio | Absolute | 1.00 | | | 0.00 | 1.00 |
| 9 | Realized Volatility Loading | Final Portfolio | Absolute | 0.00 | 0.00 | | 0.00 | 0.00 |
| 10 | Share Turnover Loading | Final Portfolio | Absolute | 0.00 | 0.00 | | 0.00 | 0.00 |
| 11 | Beta Auto Components | Final Portfolio | Absolute | 0.0000 | 0.0000 | | 0.0000 | 0.0000 |
| 12 | Beta Banks | Final Portfolio | Absolute | 0.0000 | 0.0000 | | 0.0000 | 0.0000 |
| 13 | Beta Capital Goods | Final Portfolio | Absolute | 0.0000 | 0.0000 | | 0.0000 | 0.0000 |
| 14 | Beta Commercial Svcs | Final Portfolio | Absolute | 0.0000 | 0.0000 | | 0.0000 | 0.0000 |
| 15 | Beta Consumer Durables | Final Portfolio | Absolute | 0.0000 | 0.0000 | | 0.0000 | 0.0000 |
| 16 | Beta Consumer Svcs Supplies | Final Portfolio | Absolute | 0.0000 | 0.0000 | | 0.0000 | 0.0000 |
| 17 | Beta Diversified Financials | Final Portfolio | Absolute | 0.0000 | 0.0000 | | 0.0000 | 0.0000 |
| 18 | Beta Energy | Final Portfolio | Absolute | 0.0000 | 0.0000 | | 0.0000 | 0.0000 |
| 19 | Beta Food Beverage Tobacco | Final Portfolio | Absolute | 0.0000 | 0.0000 | | 0.0000 | 0.0000 |
| 20 | Beta Health Care Equip Svcs | Final Portfolio | Absolute | 0.0000 | 0.0000 | | 0.0000 | 0.0000 |
| 21 | Beta Household Personal Prod | Final Portfolio | Absolute | 0.0000 | 0.0000 | | 0.0000 | 0.0000 |
| 22 | Beta Insurance | Final Portfolio | Absolute | 0.0000 | 0.0000 | | 0.0000 | 0.0000 |
| 23 | Beta Materials | Final Portfolio | Absolute | 0.0000 | 0.0000 | | 0.0000 | 0.0000 |
| 24 | Beta Media | Final Portfolio | Absolute | 0.0000 | 0.0000 | | 0.0000 | 0.0000 |
| 25 | Beta Pharma Bio Life Sci | Final Portfolio | Absolute | 0.0000 | 0.0000 | | 0.0000 | 0.0000 |
| 26 | Beta Real Estate | Final Portfolio | Absolute | 0.0000 | 0.0000 | | 0.0000 | 0.0000 |
| 27 | Beta Retailing | Final Portfolio | Absolute | 0.0000 | 0.0000 | | 0.0000 | 0.0000 |
| 28 | Beta Semiconductor Equip | Final Portfolio | Absolute | 0.0000 | 0.0000 | | 0.0000 | 0.0000 |
| 29 | Beta Software Services | Final Portfolio | Absolute | 0.0000 | 0.0000 | | 0.0000 | 0.0000 |
| 30 | Beta Tech Hardware Equip | Final Portfolio | Absolute | 0.0000 | 0.0000 | | 0.0000 | 0.0000 |
| 31 | Beta Transportation | Final Portfolio | Absolute | 0.0000 | 0.0000 | | 0.0000 | 0.0000 |
| 32 | Beta Telecommunication Svcs | Final Portfolio | Absolute | 0.0000 | 0.0000 | | 0.0000 | 0.0000 |
| 33 | Beta Utilities | Final Portfolio | Absolute | 0.0000 | 0.0000 | | 0.0000 | 0.0000 |
| 34 | Beta Food Staples Retailing | Final Portfolio | Absolute | 0.0000 | 0.0000 | | 0.0000 | 0.0000 |

Source: Barclays Capital POINT

We construct two versions. In the first version, we leave the objective function empty and solve just the feasibility problem. The optimizer presents a solution to this linear feasibility problem consisting of 18 stocks, which is the minimum number of stocks needed. The optimal portfolio holdings are displayed in Figure 36. Generally speaking, the portfolio is short on stocks with high negative momentum and long on low momentum stocks to neutralize the exposure to the rest of the equity factors and keep the momentum exposure, but otherwise stocks are selected randomly without any consideration to risk, liquidity or any other attribute.

Figure 36: Equity momentum mimicking portfolio with minimum number of stocks

| Description | Ticker | Market Value [%] | Momentum factor Loading | Industry Beta |
|---|---|---|---|---|
| Bucket: Materials (2 positions) | | | | |
| BALL CORP | BLL | 52.31 | 0.29 | 0.56 |
| FREEPORT-MCMORAN COPPER | FCX | -18.17 | -0.39 | 1.62 |
| Bucket: Capital Goods (2 positions) | | | | |
| MANITOWOC COMPANY INC | MTW | -8.98 | -2.09 | 1.90 |
| ILLINOIS TOOL WORKS | ITW | 19.18 | -0.24 | 0.89 |
| Bucket: Media (2 positions) | | | | |
| GANNETT CO | GCI | -30.76 | -1.87 | 1.41 |
| TIME WARNER CABLE | TWC | 46.85 | 0.18 | 0.92 |
| Bucket: Health Care Equipment & Services (2 positions) | | | | |
| LABORATORY CRP OF AMER HLDGS | LH | 50.57 | 0.19 | 0.67 |
| TENET HEALTHCARE CORP | THC | -35.53 | -0.45 | 0.96 |
| Bucket: Diversified Financials (4 positions) | | | | |
| MORGAN STANLEY | MS | -40.75 | 0.03 | 1.19 |
| AMERIPRISE FINANCIAL INC | AMP | -107.57 | -0.18 | 0.89 |
| LEGG MASON INC | LM | 29.57 | -1.04 | 0.91 |
| SLM CORP | SLMA | 113.43 | -1.26 | 1.03 |
| Bucket: Insurance (6 positions) | | | | |
| PRINCIPAL FINANCIAL GROUP | PFG | -34.85 | -0.81 | 1.87 |
| AMERICAN INTERNATIONAL GROUP | AIG | -16.31 | -4.35 | 2.01 |
| AON CORP | AOC | 38.00 | 0.03 | 0.54 |
| CINCINNATI FINANCIAL CORP | CINF | 23.04 | 0.11 | 1.02 |
| GENWORTH FINANCIAL INC-CL A | GNW | 76.33 | -1.38 | 2.31 |
| HARTFORD FINANCIAL SVCS GRP | HIG | -56.53 | -2.28 | 2.17 |

Source: Barclays Capital POINT

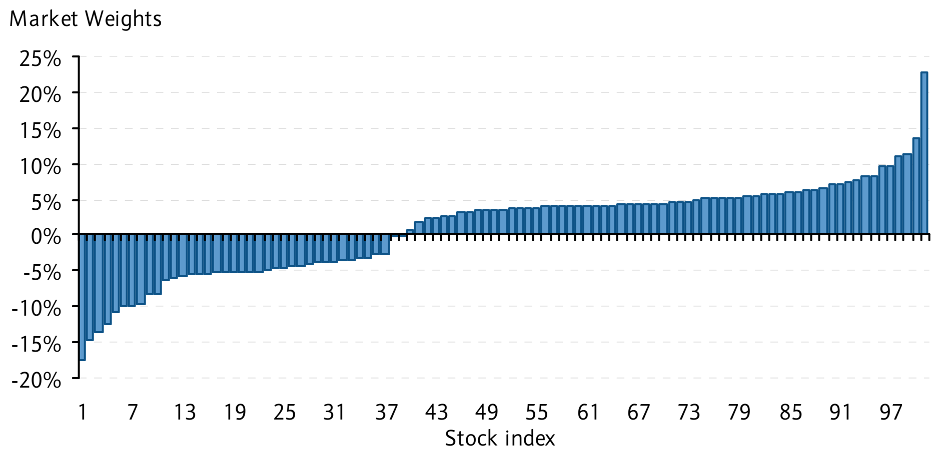
In the second version, we minimize the idiosyncratic risk of the portfolio. We also limit the number of stocks in the portfolio to 100 to restrict the optimizer from choosing a large number of stocks. The systematic risk is equal to the momentum factor volatility in both versions (Figure 37) because both portfolios have unit loading to the momentum factor and zero loadings to all other systematic factors.

Figure 37: Risk forecast of factor-mimicking portfolios

| | Number of stocks | Systematic risk | Idiosyncratic risk | Total risk | Leverage |
|---|---|---|---|---|---|
| Version 1 | 18 | 2.8 | 42.2 | 42.3 | 8:1 |
| Version 2 | 100 | 2.8 | 4.2 | 5.0 | 5.76:1 |

Source: Barclays Capital POINT

**Figure 38: Stock weights in momentum-mimicking portfolio (version 2)**




Source: Barclays Capital

In this example, the factor-mimicking portfolios are necessarily long-short and significantly leveraged. In addition, turnover can be high. Small changes in individual stocks' momentum metric can lead to a drastically different solution. To address leverage and turnover considerations, users can relax the pure mimicking requirement and solve a relaxed and more practical optimization problem. For example:

- Neutralizing the portfolio's exposure to the industry factors only while maximizing exposure to the target factor.
- Neutralizing the portfolio's exposure to the overall market (market beta) while maximizing exposure to the target factor.
- Adding leverage and turnover constraints to the objective function.

The optimizer supports multiple methods of controlling leverage and turnover, and the back-testing functionality can be used to construct a time series of these portfolios. Ultimately, investors can test the ex-post performance of any factor-mimicking portfolio strategy over time, and measure the effect of turnover and leverage constraints on the ability to capture factor returns.

## 6. Constructing overlay strategies

Overlay strategies refer to an add-on investment to a core portfolio, typically managed by a separate manager. The overlay strategies are meant to account for a small fraction of the total portfolio risk, and are meant to be a tactical investment with minimal effect on the strategic allocation of the core portfolio. Examples of overlay strategies include taking short-term views in specific markets (e.g., treasury curve steepener, swap spreads position, etc.), allocation to low correlation beta using futures and other derivatives, or an active alpha strategy managed by a separate manager (e.g., alpha index strategies, hedge fund allocation etc.).

The portfolio optimizer allows users to construct optimal overlay strategies that take into account the risk-return profile of the overlay portfolio in conjunction with the core portfolio. For example, suppose a portable-alpha manager runs a core portfolio with the flexibility to allocate a portion of the total risk budget to relatively high risk-return overlay strategies. This risk budgeting problem can be modelled and solved using the optimizer to determine the optimal allocation to the set of securities representing the trade ideas.

To demonstrate this, we use a core portfolio tracking the Barclays US Aggregate index. The portfolio has positions in 100 bonds and its market value as of 2/28/2009 is 100mn USD. Suppose the manager wants to take a short-term view on the expected US inflation, anticipating inflationary pressure, and plans to implement the inflation views using duration-neutral long-short positions in unfunded US Treasury and US Treasury Tips index total return swaps. Users can model and price unfunded index swaps in POINT delivering the total return of any index (or portfolio) net of financing costs (1mo Libor deposit rate plus user-specified spreads. These swaps can be used as tradable instruments in the optimizer.

The TEV of the initial portfolio is 20bp/mo. Suppose the manager has a total risk budget (including overlays) of 30bp/mo. Since the Treasury TIPS index is expected to outperform the Treasury index with rising inflation expectations, we maximize the allocation to the TIPS TRS subject to the total risk constraints. The overlay portfolio is additionally constrained to be duration-neutral to mitigate the interest rate risk in the overlay portfolio. The optimal duration-neutral overlay positions consist of a short 8.01mn notional in the Treasury index and a long 5.51mn notional position in the Tips index. The risk constraint of 30bp/mo is binding at this solution.

Figure 39 depicts the risk contribution from various buckets of risk factors to the initial, final, and the overlay portfolio. Because the overlay portfolio is unfunded, the bp risk displayed is computed as its P&L risk scaled by the core portfolio market value. We see that most of the risk in the overlay portfolio is contributed by the inflation-linked risk factors (15.5 out of 15.8bp/month). As expected, the low contribution from the curve risk factors (0.3bp/mo) indicated that the duration-neutral requirement is able to reduce the curve risk in the overlay portfolio significantly.

**Figure 39: Overlay strategies: Risk decomposition before and after**

| Risk Factor Partition Bucket | Core Portfolio | Overlay | Core Portfolio + Overlay |
|---|---|---|---|
| **Total** | **20.0** | **15.8** | **30.0** |
| **Systematic** | **9.8** | **15.8** | **22.2** |
| Curve | 0.0 | 0.3 | 0.4 |
| Inflation Linked | 0.0 | 15.5 | 11.0 |
| Swap Spreads | 0.9 | 0.0 | 0.9 |
| Volatility | 0.0 | 0.0 | 0.0 |
| Spread Gov-Related | -0.1 | 0.0 | -0.4 |
| Spread Credit and EMG | 0.3 | 0.0 | -1.0 |
| Spread Securitized | 8.8 | 0.0 | 11.7 |
| **Idiosyncratic** | **9.4** | **0.0** | **6.7** |
| **Credit default** | **0.8** | **0.0** | **0.6** |

Source: Barclays Capital POINT, All numbers in bp/mo.

Since the Tips index outperformed the Treasury index in March 2009 (the total return of 5.84% and 2.18% respectively), the inflation views did pay off. Figure 40 displays the decomposition of the outperformance (against the US Aggregate index) of the core portfolio with and without the overlay strategy in March 2009. The overlay positions enhanced the total return of the portfolio by about 20bp. Note that the majority (15.7bp) of this 20bp is attributable to the derivatives[33] buckets in the asset allocation returns (Figure 40).

[33] The attribution model decomposes fixed income total return in curve return and excess return. The excess return is further decomposed into asset allocation and security selection. We use the Barclays Class-1 security classification scheme to define allocation sectors. The derivative bucket in Figure 40 represents the unfunded overlay swap positions in the Core+Overlay portfolio..

**Figure 40: Overlay Strategies: March 2009 return decomposition before and after**

| | Core Portfolio | Core+Overlay |
|---|---|---|
| Total | 5.8 | 25.4 |
| Yield Curve | 0.8 | 4.7 |
| Asset Allocation | 0.2 | 15.7 |
| Derivatives | | **15.5** |
| Securitized | 0.1 | 1.3 |
| Cash | 0.1 | 0.1 |
| Treasury | 0.0 | 0.0 |
| Government-Related | -0.1 | -0.2 |
| Corporate | 0.0 | -1.1 |
| Security Selection | 4.9 | 5.0 |
| Derivatives | | 0.0 |
| Securitized | 8.5 | 8.8 |
| Cash | 0.0 | 0.0 |
| Treasury | -0.3 | -0.3 |
| Government-Related | -3.3 | -3.3 |
| Corporate | -0.1 | -0.1 |

Source: Barclays Capital POINT, All numbers in bp.

## 7. Cross-asset portfolio optimization

POINT supports optimization across asset classes (rates, inflation, credit, securitized, munis, equities, FX, etc.). The Global Risk Model aggregates risk emanating from different asset classes in a consistent fashion. For example, it accounts for the correlation between the idiosyncratic returns of the equity and debt securities issued by the same issuer. On the other hand, the estimation process based on historical data captures the correlation structure between risk factors used to explain the systematic returns of diverse asset classes. For example, the long-term correlation between equity returns and credit excess returns[34] is estimated to be at about 30%.

We construct a mixed 70/30% fixed-income/equity USD long-only portfolio as of July-end 2009 to demonstrate[35], how the portfolio optimizer can leverage the cross-asset risk models and analytics available in POINT. We maximize risk-adjusted active returns against a 70/30% composite benchmark index. Similar to previous examples, we use yield to worst as the expected return measure for fixed-income securities and risk-model implied expected returns for stocks. The composite benchmark consists of 70% Barclays Capital US Long Gov/Credit index and 30% S&P 500 index. The portfolio maximizes risk-adjusted expected returns as measured by the function[36]

100 * expected returns - 3.46 * portfolio TEV,

subject to a limit of USD100,000 on the minimum holding size. The asset allocation is constrained to be 70% debt and 30% equities and is further constrained to be at least 10% treasuries and 5% government-related. In the equity component, the portfolio's exposure to

[34] As measured by the US Equity Core Factor and the US Credit Core Factor
[35] This problem can easily be set up in the Portfolio Optimizer. See the optimizer user guide (Kumar and Lazanas [2009]) and the sample optimization reports (Kumar [2009]) available in POINT for more details.
[36] The risk aversion parameter (sqrt(12)/100=3.46/100) is chosen translate both risk and expected return terms to P&L units (USD/yr).

individual GICS level-1 sectors is limited to 10%. In addition, a concentration limit of 3% on individual bonds and 1% on individual stocks is enforced.

In this example, the risk model captures the risk aggregation across the capital structure of the same company (i.e., the stocks and bonds with same issuer). Alternatively, users can also user tickers constraints across the capital structure by constraining a measure of exposure (e.g., loss-given-default) for each tradable ticker.

The optimizer constructs a 110-security portfolio (61 bonds and 49 stocks) with a TEV of 108.5bp/month and an expected return of 8.73%/year. The isolated systematic and idiosyncratic TEV are 90.2bp/month and 60.2bp/month, respectively. Furthermore, the equity and fixed income components of the portfolio contribute 52.0 and 56.5bp/month, respectively, to the total TEV.

**Figure 41: Optimal cross-asset portfolio's market structure relative to the benchmark**

| | Number of securities | | Market Value [%] | |
|---|---|---|---|---|
| | Portfolio | Index | Portfolio | Index |
| Total | 110 | 1221 | 100.00 | 100.00 |
| **Debt** | **61** | **1121** | **70.00** | **70.00** |
| Treasury | 4 | 34 | 10.00 | 27.87 |
| Government-Related | 4 | 201 | 5.00 | 9.87 |
| Corporate | 53 | 886 | 55.00 | 32.26 |
| **Equity** | **49** | **100** | **30.00** | **30.00** |
| Energy | 3 | 10 | 1.11 | 4.06 |
| Materials | | 5 | | 0.62 |
| Industrials | 8 | 13 | 5.50 | 2.71 |
| Consumer Discretionary | 6 | 10 | 3.13 | 1.98 |
| Consumer Staples | 1 | 15 | 0.17 | 4.36 |
| Health Care | 8 | 12 | 5.03 | 4.16 |
| Financials | 8 | 14 | 3.61 | 3.80 |
| Information Technology | 13 | 14 | 10.00 | 6.43 |
| Telecommunication Services | 2 | 3 | 1.44 | 1.39 |
| Utilities | 0 | 4 | 0.00 | 0.48 |

Source: Barclays Capital POINT

Figure 41 compares the market weights and the number of securities in the optimal portfolio with the composite benchmark index. By construction, the portfolio does not have equity versus debt active exposure, and is underweight treasuries and overweight corporate bonds to increase the expected returns. Similarly, it takes active sector exposures based on the expected return forecasts across the different equity sectors to improve expected return while controlling the overall active risk of the total portfolio.

## 8. Optimizing asset allocation

As we discussed in Section II.4.B, for large portfolios, the construction problem is typically decomposed into an asset allocation problem and a security selection problem. In the asset allocation step, security-level constraints and preferences are ignored, and portfolio weights to broader marker sectors are determined to optimize the risk-return tradeoff based on risk and return forecasts at the broader market level. Once the asset allocation decision is finalized, individual security weights are determined within each market sector to match this asset allocation.

Asset allocation decisions are typically made within a mean-variance framework to maximize expected returns for a given level of risk. Each of the decision variables represents an allocation to a market beta (as represented by a broader market sector). These allocations are optimized according to desired targets for risk and return. We demonstrate the use of the Portfolio Optimizer in asset allocation by representing the market sectors using popular benchmark indices (Figure 42).

**Figure 42: Asset allocation: Index volatilities, correlations, and expected return**

| Index | Exp. Ret. (%/yr) | Vol (%/mo benchmarked to USD) | Correlations (%) | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Euro Credit | 5.00 | 4.54 | 100 | 41 | 55 | 25 | 95 | 62 | 32 | 28 | 45 | 63 |
| US HY | 9.00 | 4.92 | 41 | 100 | 52 | 22 | 23 | 64 | 26 | -18 | 46 | 47 |
| US Credit | 6.00 | 2.89 | 55 | 52 | 100 | 45 | 53 | 69 | 8 | 58 | 25 | 32 |
| US MBS | 4.19 | 0.74 | 25 | 22 | 45 | 100 | 28 | 42 | 12 | 53 | 10 | 11 |
| Euro Treasury | 2.90 | 4.31 | 95 | 23 | 53 | 28 | 100 | 53 | 27 | 44 | 36 | 54 |
| US EM | 7.72 | 5.05 | 62 | 64 | 69 | 42 | 53 | 100 | 19 | 29 | 43 | 47 |
| US CMBS | 8.83 | 6.26 | 32 | 26 | 8 | 12 | 27 | 19 | 100 | -11 | 37 | 41 |
| US Government | 2.35 | 1.55 | 28 | -18 | 58 | 53 | 44 | 29 | -11 | 100 | -10 | -3 |
| Russell 3000 | 14.00 | 8.00 | 45 | 46 | 25 | 10 | 36 | 43 | 37 | -10 | 100 | 86 |
| FTSE All World | 13.00 | 7.37 | 63 | 47 | 32 | 11 | 54 | 47 | 41 | -3 | 86 | 100 |

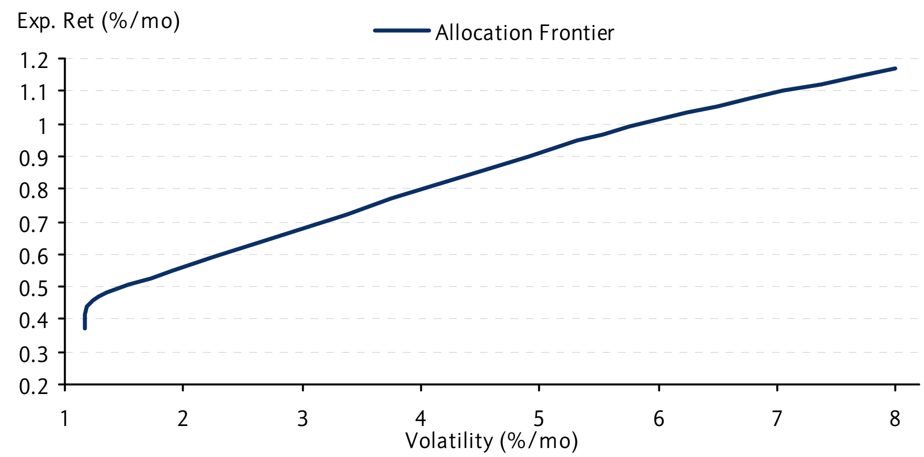
Source: Barclays Capital POINT

We create a set of funded total return swaps (TRSs) in POINT[37] that delivers the monthly total return of the corresponding index. A portfolio containing these ten swaps is created as of June-end 2009. The optimizer uses an aggregation algorithm to imply a variance-covariance structure between these instruments from the constituent information of the underlying indices. The volatilities and correlations showed in Figure 42 are implied from the weighted calibration as of July-end 2009. In general portfolio managers will use their own predictions of asset returns. In this example we estimated expected returns using a bootstrapping algorithm using as inputs the covariance structure and the aggregate yield to worst of bond indices. The high correlations (e.g., 0.86 between Russell 3000 and FTSE All World index) across asset classes in this example are driven by the flight to quality in the liquidity crisis of 2008 as well as a significant overlap of the risk factor exposures (e.g., the rates factor for euro treasury and euro credit).

We construct ten different portfolios that minimize the total risk of the portfolio (consisting of allocations to the above ten total return swaps) subject to achieving a given return. The ten expected return bounds used for the portfolios constitute a set of equidistant points between the expected return of the minimum variance portfolio and the highest achievable return (1.17%/month, corresponding to Russell 3000 index). The risk-return profile of these ten portfolios is plotted in Figure 43.

[37] POINT has flexible pricing support for total return swaps. The underlying could be any POINT index or a portfolio. The TRS could be funded or unfunded at a user-specified spread.

**Figure 43: Asset allocation: Efficient allocation frontier**




Source: Barclays Capital

Figure 44 displays the minimum risk allocation. It is heavily concentrated (almost 80%) in the low risk indices: US Government and US MBS. Furthermore, high correlations inhibit the potential diversification benefit across high volatility indices. The remaining 20% of the allocation is equally distributed among the remaining indices to obtain risk reduction using diversification.

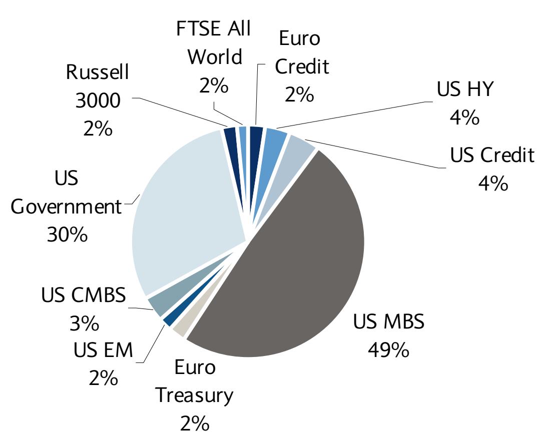
Figure 45 displays an allocation corresponding to the sixth of these increasing risk allocations. It is a 36/64 allocation between equities and fixed income. Within each category, the optimizer chooses the indices with high expected return per unit of volatility and low correlations with the remaining asset classes (Figures 42 and 45).

## 9. Historical optimization: Back-testing portfolio construction strategies

Historical optimization refers to the ability to create optimal portfolios as of a date in the past. The optimization is performed using the risk and performance forecasts available as of the optimization date in order to obtain out-of-sample results. POINT is a transaction-based

**Figure 44: Asset allocation: Minimum risk allocation**

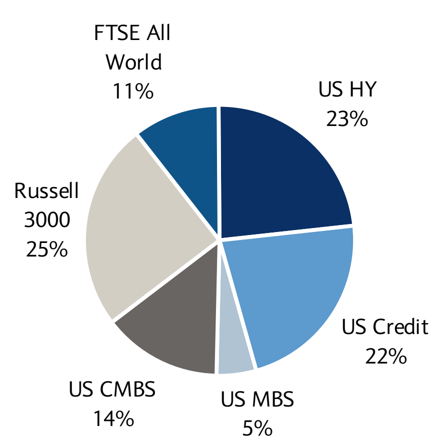
risk 1.18 %/mo - expected return 4.43 %/yr




Source: Barclays Capital POINT

**Figure 45: Asset allocation: Risky optimal allocation**

risk 4.25 %/mo - expected return 7.92 %/yr




Source: Barclays Capital POINT

portfolio system that maintains complete history of portfolios. Users also have access to the complete history of analytics of all central instruments, which includes the constituent bonds of all Barclays Capital benchmark indices. The analytics go as far back as 1990, and the risk model calibrations are available since December 2002. User-defined instruments (UDIs) can be priced to generate the required analytics using our historical curve and other pricing calibrations databases. The optimizer can leverage this historical data to study the performance of complex investment strategies. For example, it can be used to study the quality of risk model TEV forecasts for index-tracking portfolios by generating a sequence of tracking portfolios historically. This test gauges the quality of risk model TEV forecasts using the ex-post realized tracking error of the portfolio.

The optimizer allows users to set up a report to be run on a given portfolio historically in sequence using the **batch processing**[38] functionality – that is, to request the optimizer to commit the recommended trades to the initial portfolio obtained at the current date before running the optimization problem for the next rebalancing date. This makes the optimizer a powerful back-testing tool. We present a simple Global Treasury index replication example.

## A. Backtesting Global Treasury index replication portfolios

We generate a sequence of 82 minimum TEV portfolios, containing 20 bonds each, to track the Barclays Capital Global Treasury index on each month-end starting in December 2002 and ending on October 2009. Since the objective of this exercise is to backtest the global treasury risk model, we give full discretion to the optimizer to choose any bond from the index to construct the minimum TEV portfolios. Each of these 82 portfolios is maintained in the same portfolio by optimizing (i.e., rebalancing) the previous month-end portfolio. Because our objective is to obtain independent samples of the tracking error portfolios, we do not take the rebalancing turnover into consideration in the optimization. For a more practical replication test, the managers would want rebalance the portfolios subject to a controlled turnover.

We use the risk model and return report in POINT to obtain the time series of forecast total TEV and the ex-post tracking error of the replicating portfolio at each month-end. The carry component[39] is excluded from the total TE of the portfolio.

### Testing the TEV forecasts

Figure 46 presents the standard sample statistics of the time series of the total TEV forecasts, as well as the total tracking error net of carry. The worst negative tracking error is a 2.14 standard deviation event, and the worst positive tracking error is a 2.97 standard deviation event. Furthermore, the realized standard deviation of the tracking error (9.4bp/month) is close to the median TEV forecasts (9. 0bp/month).

[38] For a detailed description of the batch processing functionality available in POINT, see POINT's documentation webpage.
[39] The risk model predicts the volatility in excess of carry. Yield to worst is good proxy for the carry. In this example, the replicating portfolio roughly matched the index; hence, carry is not a significant component of the tracking error.

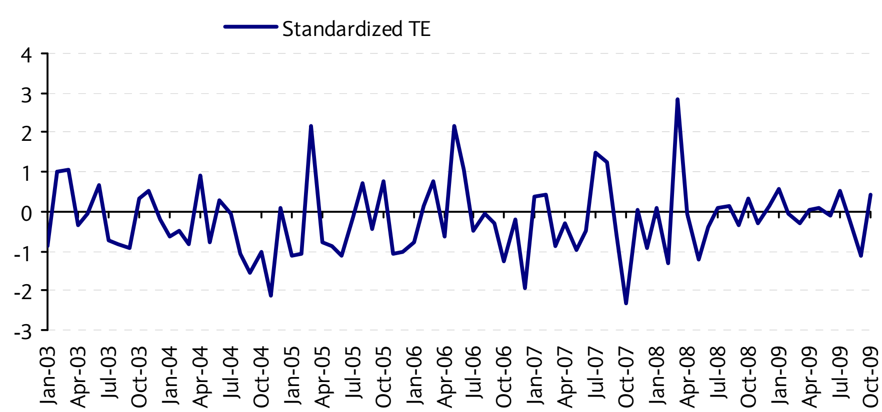
**Figure 46: Global Treasury Index replication using 20 bonds: Total outperformance versus TEV forecasts**

| Statistics | TE (bp) | TEV (bp/mo) | Standardized TE (σ) |
|---|---|---|---|
| Max | 37.3 | 20.7 | 2.97 |
| Min | -22.0 | 4.9 | -2.14 |
| Mean | -1.6 | 10.0 | -0.18 |
| Median | -2.3 | 9.0 | -0.24 |
| Std Dev | 9.4 | 3.7 | 0.92 |

Source: Barclays Capital

Formally evaluating the out-of-sample performance of a time-varying TEV forecast is not easy, because volatility itself is not ex-post observable. Volatility is a parameter of the tracking error distribution and, as such, is not "realized." Realized volatility refers to a particular estimator (usually sample volatility) and not to volatility itself, which will never be known. When the predicted variable is not ex-post observable, straightforward performance measures such as "mean square prediction error" are ineffective: it is not to possible to compute the distance between a predicted value and a realized value when the latter is unknown. This is especially true for time-varying volatility estimates. It is, therefore, necessary to adopt different evaluation methods. We use a simple and intuitive test for the evaluation of the TE statistic generated by the backtest.

**Figure 47: Global Treasury Index replication using 20 bonds: Standardized tracking error time series**




Source: Barclays Capital

If the predicted volatilities are a good estimate of the volatility of the underlying process, dividing each realization of the tracking error by its predicted volatility will produce a series of unit-variance realizations. Figure 47 depicts the series of standardized realized tracking errors. Its estimated standard deviation is 0.92, with a 95% confidence interval[40] [0.80, 1.08], indicating that the null hypothesis, the standardized tracking error time series is generated by a unit variance process, cannot be rejected at a 95% significance level. The graph also shows that the absolute value of the realized tracking error is a one-sigma event 26% of the time and two-sigma event 6% of the time. This is compatible with a distribution

[40] The confidence interval is based on the standard chi-square test used to estimate the population variance from a series of sample.

with slightly fatter tails than normal. Furthermore, 43 of the 82 standardized observations are below the mean, indicating a symmetrical distribution.

Because the optimizer is given full discretion to minimize the TEV forecast to construct the replicating portfolios, it takes full advantage of the estimated parameters in the risk model. Thus, optimizer-based portfolios can expose the weaknesses of a risk model and present a much more stringent test of the TEV forecasts compared with the standard approach of testing the absolute risk of randomized portfolios. This observation also complements our theme of using portfolio constraints, in addition to minimizing TEV, in the portfolio optimization to construct more robust portfolios.

# IV. CONCLUSION

We discussed the modelling framework underlying the Barclays Capital Portfolio Optimizer and used a number of practical portfolio management applications to illustrate its flexibility and generic nature. We conclude that the flexibility to incorporate diverse portfolio preferences – in terms of both constraints and the objective function – is the most important requirement in a practical portfolio optimizer. We believe that the optimizer can be a powerful tool in most portfolio management approaches if it is flexible and is used with caution. We illustrated how various portfolio decision-making problems (portfolio construction, hedging, and rebalancing) can be modelled as optimization problems. Furthermore, the easy-to-use, one-click front end of the optimizer ensures that users can solve problems with ease without knowing the intricate details of the solvers. We believe that the optimizer can significantly enhance the value added by the POINT system to the portfolio management process.

Optimization tends to be an iterative process. Experimenting with risk-return tradeoff to generate the efficient frontier is one such example, since it requires many optimization runs. However, what we mean by “iterative” here is related to which constraints to include or exclude and how to set the objective function parameters and the constraint bounds. If too many constraints are specified, then the problem can become infeasible. It is typically difficult to pinpoint the constraint(s) causing infeasibility because they are often highly intertwined. An over-constrained optimal solution may degrade portfolio performance and can be highly dependent on the constraint bounds. For example, enforcing a tight concentration constraint leads to portfolios with most of securities hitting the concentration bound. Furthermore, it is rare that all the constraints are equally important. Some constraints, such as budget, are normally hard, in that they are not meant to be violated. Other constraints, such as turnover and sector exposure, tend to be soft, in that they are allow a certain amount of leeway. As a guide, we recommend that managers start with the most critical constraints then vet the solution and see whether there is a need to add constraints or fine-tune the constraint parameters. This part of the process is something that an optimizer cannot fully replicate, and human judgment is an integral part of the process to formulate the problem at hand in the optimizer.

To mitigate the model risk arising from the underlying risk and return forecasting models, the optimization formulations need to be stressed by changing underlying risk and pricing parameters, modifying the set of constraints and the objective function to see resulting recommendations; all of the resulting information can and should be used to reach a satisfactory and practically implementable solution. For these reasons, it is important that the optimizer is fast, scalable, and able to present solutions in an interactive fashion, making it helpful tool compared with the manual and ad hoc processes.

In conclusion, the portfolio optimizer completes the POINT platform in delivering a complete and powerful set of portfolio management tools (security modelling and pricing, portfolio analytics, risk modeling, scenario analysis, and portfolio optimization) to our clients.

# REFERENCES


Bertsimas, B, and Shioda, R. (2009), "Algorithm for cardinality-constrained quadratic optimization", *Computational Optimization and Applications*, 43(1), pp 1-22.

Bertsimas, B, and Tsitsiklis, J. N. (1997), "Introduction to linear optimization", *Athena Scientific*, Belmont, Massachusetts, USA.

Bienstock, D. (1996): "Computations Study of a family of mixed-integer programming Problems", *Mathematical Programming*, 74(2), pp 121-140.

Boyd, S. and Vandenberghe, L. (2004) "Convex Optimization", *Cambridge University Press*, New York, NY.

Dastidar, S. and Phelps, B. (2009) "Introducing LCS: Liquidity Cost Scores for US Credit Bonds", *Quantitative Portfolio Strategies*, Barclays Capital Research Publications, New York, NY.

Dynkin, L., A. Desclee, A. Gould, J. Hyman, D. Joneja, R. Kazarian, V. Naik, M. Naldi, B. Phelps, J. Rosten, A. Silva, and G. Wang, (2005), "The Lehman Brothers Global Risk Model: A Portfolio Manager's Guide", *Lehman Brothers Research Publications,* New York, NY.

Fabozzi F. J. (1998) "Active Equity Portfolio Management", *Frank J Fabozzi Associates*, New Hope, PA.

Fan, J. and Zhang, J. and Yu, K. (2008) "Asset Allocation and Risk Assessment with Gross Exposure Constraints for Vast Portfolios", *Working Paper, Princeton University, Princeton, NJ.*

Iyengar, G. and Goldfarb, D. (2003) "Robust portfolio selection problems", *Mathematics of Operations Research*, 28(1), pp.1-37.

Kumar, A. (2009) "The Barclays Capital Portfolio Optimizer – Sample reports", *Barclays Capital Research Publications,* New York, NY.

Kumar, A. and Lazanas, A. (2009) "The Barclays Capital Portfolio Optimizer – Users Guide", *Barclays Capital Research publications*, New York, NY.

Lazanas, A. (2007) "Developments in Modeling Portfolio Risk: The Quest for Invariance", *Lehman Brothers Research publications*, New York, NY.

Markowitz, H. (1952) "Portfolio selection", *The Journal of Finance*, VII(1):77–91.

Mitra, G. and Ellison, F. and Scowcroft A. (2007) "Quadratic Programming for Portfolio Planning: Insights in to algorithms and computational issues Part I – Solving a family of QP models", *Journal of Asset Management*, 8(4), pp 249-258.

Mitra, G. and Ellison, F. and Scowcroft A. (2007), "Quadratic Programming for Portfolio Planning: Insights in to algorithms and computational issues Part II – Processing of portfolio planning models with discrete constraints", *Journal of Asset Management*, 8(3), pp 200-214.

Rockafellar, R.T. and Uryasev, S.P. (2000) "Optimization of Conditional Value-at-Risk", *Journal of Risk (*2), pp 21-42.

Satchell, S. and Scowcroft, A. (2003), "Advances in Portfolio Construction and Implementation", *Butterworth-Heinemann*, Burlington, MA.

Silva, A. and Staal, A. and Ural, C. (2009): “The US Equity risk model”, *Barclays Capital,* New York, NY.

Zhang, L. and Manwani, A. and Vandermark, S. (2008) “PRISE II - The Lehman Brothers Pre-Trade Implementation Shortfall Estimation Model”, *Lehman Brother Research publications,* New York, NY.

# A. MATHEMATICAL MODEL FOR PORTFOLIO OPTIMIZATION PROBLEMS

Mathematical optimization is a well-developed academic field with applications for a wide spectrum of decision-making problems arising in diverse industries including investment management. Academicians in this area are typically interested in the theoretical characteristics of optimal solutions, which depend on the structure of the problems, as well as discovering new numerical algorithms to compute the optimal solutions using these theoretical insights. Two insights are well accepted – first, the solution time grows non-linearly with the size of the problem, and second, the solution time depends heavily on the structure of the problem.

To describe the mathematical model underlying our portfolio optimization problems, we first describe the model for a general optimization problem and then discuss the specific class of problems relevant to our model.

## Mathematical model for optimization problems

In a typical optimization problem, our goal is to maximize a value function (for example, risk-adjusted expected return of a portfolio) or minimize a cost function (e.g., the tracking error variance of a portfolio) subject to a set of restrictions on the choices available. A general[41] mathematical model for an optimization problem can be described as follows.

$$\underset{x \in S}{Minimize} \quad f(x)$$

$$\text{Such that} \quad g_i(x) \le 0 \quad \text{For } i = 1,2,\ldots,K.$$

where $f(x)$ is called the objective function. $x$ is called the decision variable. $S$ is called the domain of the decision variable, and $g_i(x) \le 0$ represents the set of $K$ constraints.

The theoretical framework used to analyze an optimization problem depends on the properties of the objective function, the functions defining the constraints, and the domain of the decision variables. Different types of algorithms are used for different types of optimization problems.

## Convex programming problem

If the domain of the decision variable $S$ is a convex set in the Euclidean space and the functions $f(.)$ and $g_i(.)$ are convex functions, then the optimization problems is said to be a convex programming problem. Convex programming problems have appealing properties – for example, all optimal solutions are globally optimal. Fast numerical algorithms take advantage of these properties to find the optimal solutions. Boyd, S. and Vandenberghe, L. (2004) provides an excellent presentation on this subject.

## Convex quadratic programming problems

Convex quadratic programming problems are a subset of the convex programming problems in which the functions defining the objective function and the constraints are polynomials of at most degree two.

[41] The maximization of a mathematical function is equivalent to minimization of the negative of this function.

### Linear programming problem

Linear programming problems are a special class of convex programming problems in which the objective function and all the constraint functions are linear.

### Combinatorial optimization

If the domain $S$ is not continuous and some of decision variables take discrete values – the optimization problem is said to be a combinatorial optimization problem.

### Integer programming problem

A subset of combinatorial optimization problems is composed of integer programming problems, in which a subset of decision variables is constrained to be integers.

### POINT's optimization model

It is helpful to see the mathematical model underlying the POINT's portfolio optimizer in light of the different category of problems described above. Our model has linear (analytics), quadratic (risk term), and sum of absolute terms (transaction costs) in the objective function, as well as in the constraints. Such problems are classified as quadratic constrained quadratic programming (QCQP). In addition, we have a set of binary variables (the indicator variables for non-zero holding and trading in each security). This leads to an integer programming problem. Such models are called mixed integer quadratic programs (MIQP).

**Analyst Certification(s)**

To the extent that any of the views expressed in this research report are based on the firm's quantitative research model, Barclays Capital Inc. hereby certifies that the views expressed in this research report accurately reflect the firm's quantitative research model and that no part of its analysts compensation was, is or will be directly or indirectly related to the specific recommendations or views expressed herein.

I, Anuj Kumar, hereby certify (1) that the views expressed in this research report accurately reflect my personal views about any or all of the subject securities or issuers referred to in this research report and (2) no part of my compensation was, is or will be directly or indirectly related to the specific recommendations or views expressed in this research report.

**Important Disclosures**

For current important disclosures regarding companies that are the subject of this research report, please send a written request to: Barclays Capital Research Compliance, 745 Seventh Avenue, 17th Floor, New York, NY 10019 or refer to https://ecommerce.barcap.com/research/cgi-bin/all/disclosuresSearch.pl or call 212-526-1072.

Barclays Capital does and seeks to do business with companies covered in its research reports. As a result, investors should be aware that Barclays Capital may have a conflict of interest that could affect the objectivity of this report. Any reference to Barclays Capital includes its affiliates. Barclays Capital and/or an affiliate thereof (the "firm") regularly trades, generally deals as principal and generally provides liquidity (as market maker or otherwise) in the debt securities that are the subject of this research report (and related derivatives thereof). The firm's proprietary trading accounts may have either a long and / or short position in such securities and / or derivative instruments, which may pose a conflict with the interests of investing customers. Where permitted and subject to appropriate information barrier restrictions, the firm's fixed income research analysts regularly interact with its trading desk personnel to determine current prices of fixed income securities. The firm's fixed income research analyst(s) receive compensation based on various factors including, but not limited to, the quality of their work, the overall performance of the firm (including the profitability of the investment banking department), the profitability and revenues of the Fixed Income Division and the outstanding principal amount and trading value of, the profitability of, and the potential interest of the firms investing clients in research with respect to, the asset class covered by the analyst. To the extent that any historical pricing information was obtained from Barclays Capital trading desks, the firm makes no representation that it is accurate or complete. All levels, prices and spreads are historical and do not represent current market levels, prices or spreads, some or all of which may have changed since the publication of this document. Barclays Capital produces a variety of research products including, but not limited to, fundamental analysis, equity-linked analysis, quantitative analysis, and trade ideas. Recommendations contained in one type of research product may differ from recommendations contained in other types of research products, whether as a result of differing time horizons, methodologies, or otherwise.