QUANTITATIVE PORTFOLIO MANAGEMENT CONFERENCE

Estimating Volatilities and Correlations over Time and Across Asset Classes

Arne Staal
POINT Portfolio Modeling
Index, Portfolio and Risk Solutions

April 8, 2010

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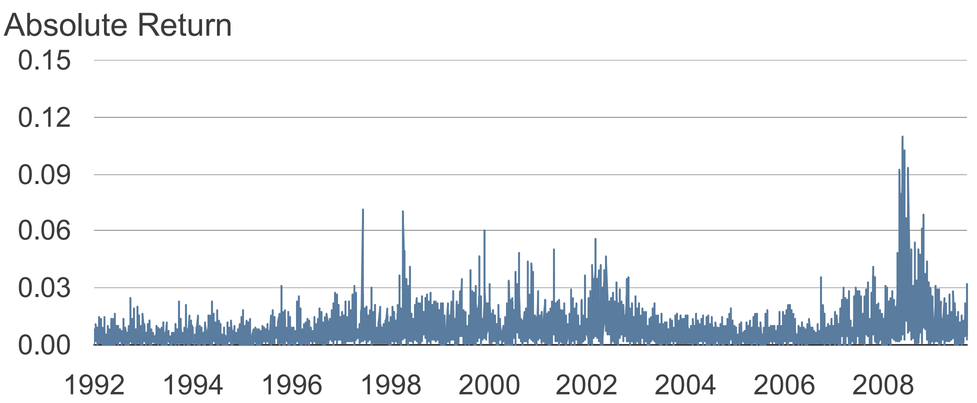
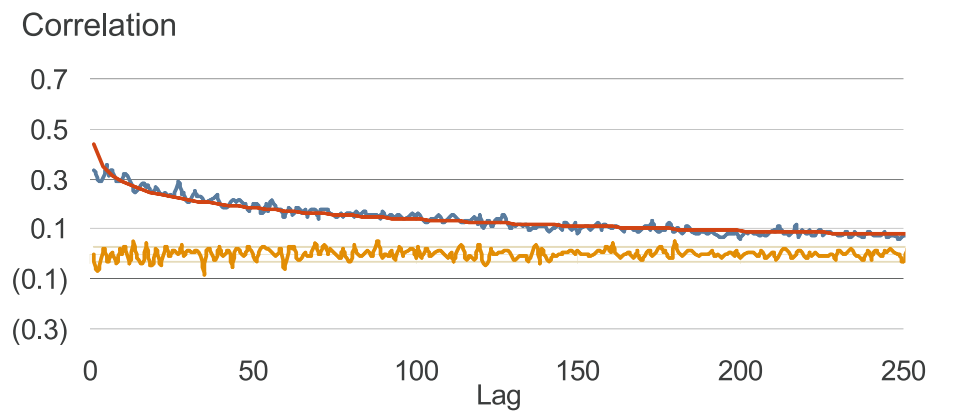
The Covariance Matrix – A Critical Input

- Many practical problems in finance require an understanding of the volatility and correlations of asset returns
  - Risk measurement
  - Portfolio construction and asset allocation
  - Pricing, hedging, trading
- Unfortunately, the covariance matrix is inherently unobservable
  - Volatilities are highly dynamic, with complex behavior over time

S&P 500 Daily Absolute Returns



Autocorrelations S&P 500 Daily (Absolute) Ret



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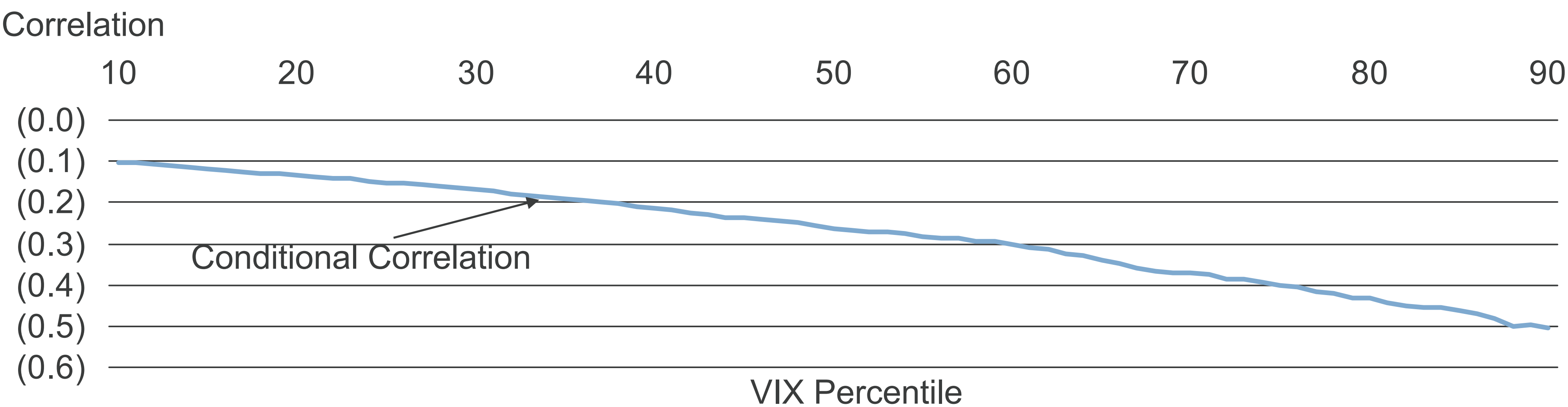
Measuring, Understanding, and Forecasting …

- Correlations change substantially over time in response to market conditions

Equity-Treasury Correlation Conditional on VIX Level



- The search for better understanding and improved models is of major concern in both academia and industry
  - How can we best measure dynamic volatility and correlations ex-ante and ex-post?
  - What drives volatility?

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Outline

Volatilities and correlations across assets and asset classes

- Covariance matrix estimation: past and present
- Does the mean matter?
- Three areas of innovation
  - Decomposing the covariance matrix: volatilities and correlations
  - Mixed frequency estimation
  - Conditioning information
- Testing

Volatilities and correlations over different horizons

- The term structure of risk
  - Is sqrt(time) scaling reasonable?
  - A dynamic term structure model

Conclusion

- We have seen only the tip of the iceberg!

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Volatilities and Correlations Across Assets

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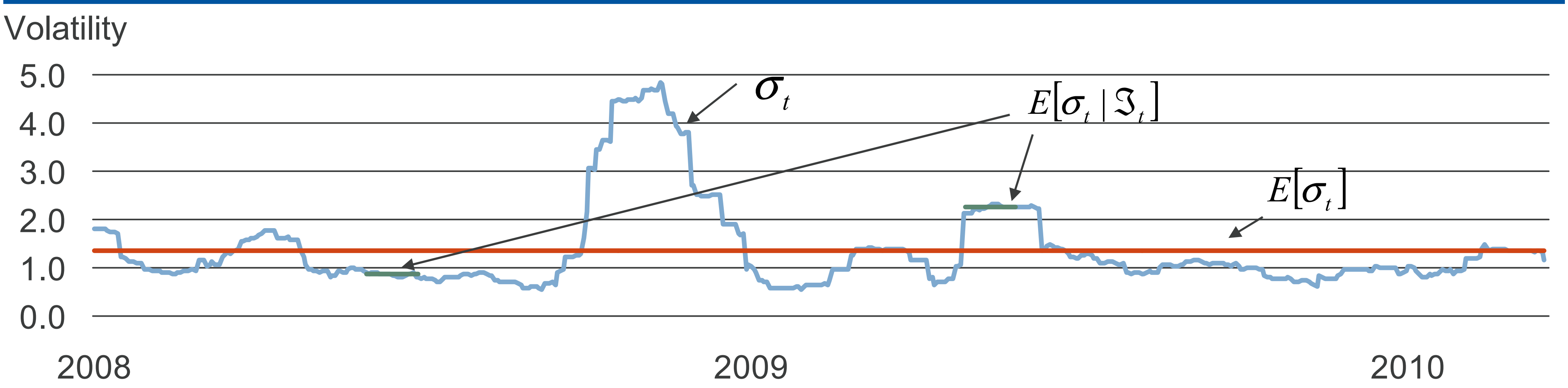
Volatility – What Do We Mean?

- Implied, realized, stochastic, (un)conditional … volatility
- The return process can be decomposed into a conditional mean and a conditionally orthogonal innovation process

$$r_t = \underbrace{E_{t-1}[r_t]}_{\text{expected}} + \underbrace{\sigma_{t-1} \cdot \eta_t}_{\text{surprise}} \qquad \eta_t \sim iid(0,1)$$

- Conditional vs. unconditional estimates: point-in-time vs. average over time

Barclays Capital US Corp BBB Index Spread Volatility



Does the Mean Matter?

- For a given time window, volatility estimation is more precise with more data; mean estimation is not (Merton 1980)

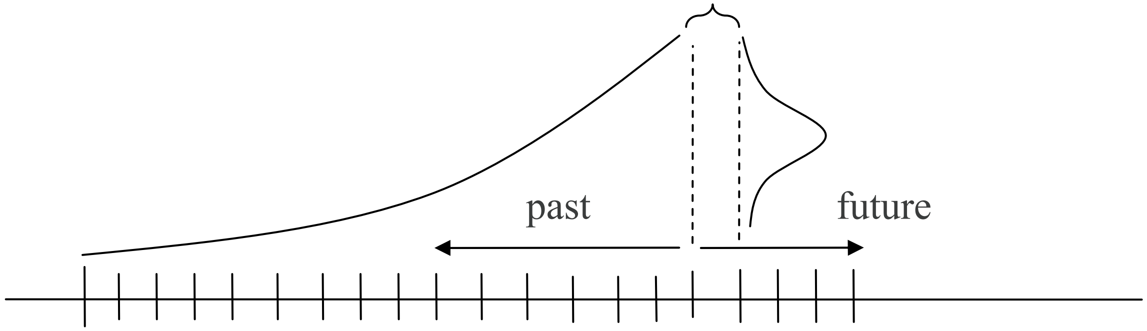
- Mean estimation effect on (co) variance estimates
  - Theoretically important; imposing the wrong mean introduces biases
  - Practically, not so important; means are very small relative to volatility for most asset classes over short periods

- Specifying a mean process is a highly subjective task because of the low signal-to-noise ratio
  - Statistical: ARMA, etc.
  - Economic: Alpha models based on accounting, price, and macro data
  - Market implied: CAPM, etc.

Covariance Matrix Estimation – The Basics

- (G)ARCH (Engle [1982], Bollerslev [1986])
  - The benchmark: Does anything beat a GARCH(1,1)? (Hansen & Lunde 2005)
  - A zoology of GARCH models: EGARCH, TGARCH, ZGARCH …
  - Special case: Exponential weighted moving average (EWMA)
- Multivariate GARCH
  - Problem is dimensionality: Impose structure on N(N-1)/2 parameters
  - EWMA: Extends to high dimensions
  - DCC (Engle [2002, 2005]): Particularly convenient for big systems
- Intuition: All of these models define a weighting function on past squared returns to predict volatility over the next period

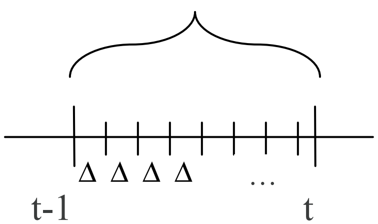
$$\sigma_{t+1}^{2} = \overline{\sigma} + \sum_{l=1}^{L} w_{t-l} r_{t-l,t-l+1}^{2}$$

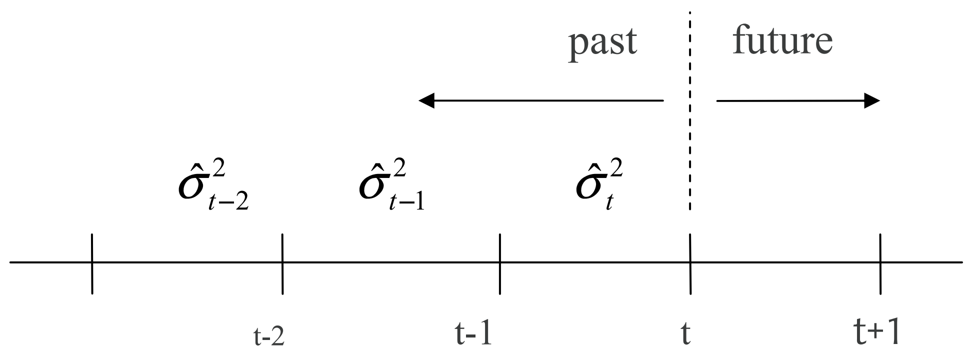


Covariance Matrix Estimation – Beyond the Basics

- Stochastic volatility
  - Not a filter; volatility is a random variable with respect to the information set
  - More relevant to pricing than risk estimation and portfolio construction
- Realized quadratic variation (Andersen et al. [2000, 2001, 2003])
  - If the true price process is a semi-martingale, we can measure ex-post realized volatility infinitely precisely given high-frequency data
  - Volatility and correlations become observable (up to micro-structure noise)
  - Can model volatility and correlations directly; volatility appears to have long memory

$$\hat{\sigma}_t^2 = \sum_{1}^{1/\Delta} r_{t-1+i\Delta}^2 \rightarrow \sigma_t^2$$



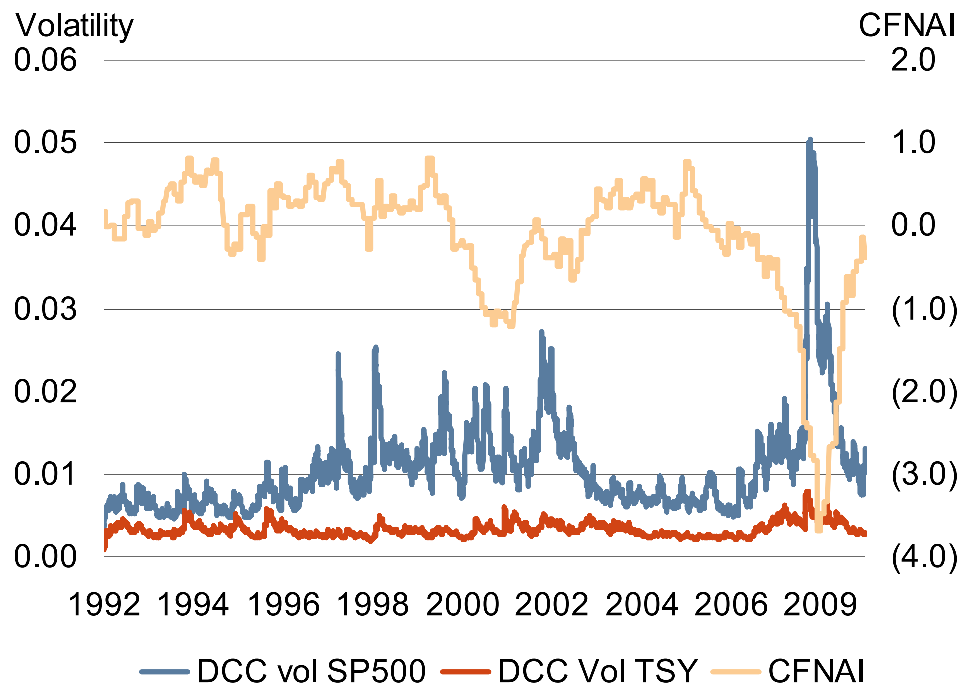


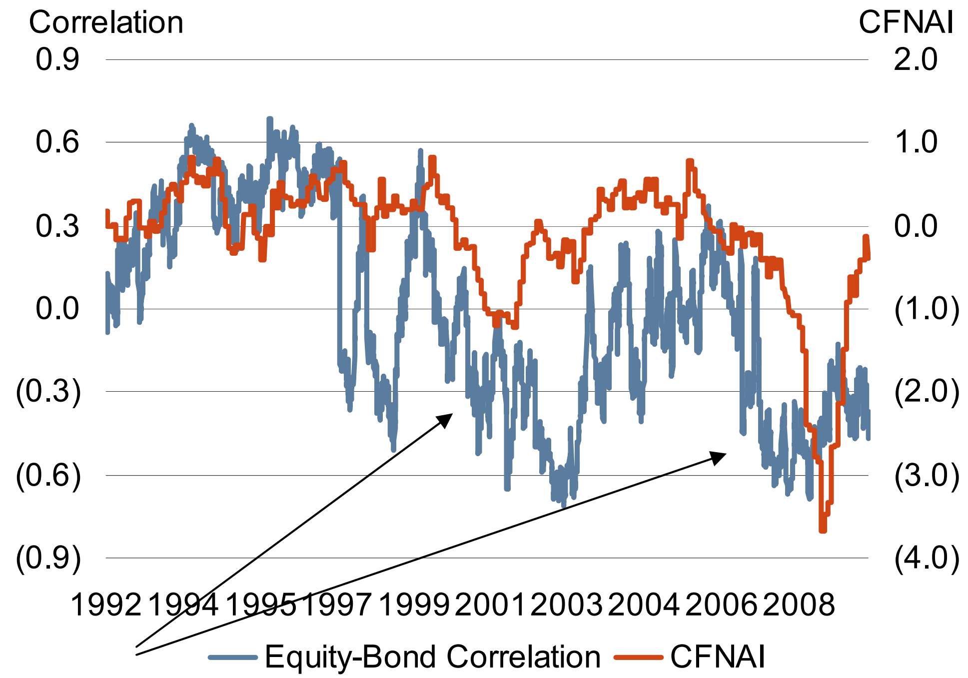
High frequency estimation of volatility → ***Volatility becomes (almost) observable!***

A Classic Example – Equities and Treasury Bonds

- Volatilities and correlations of treasury bonds and equities are highly dependent on the state of the economy: significant risk, hedging, and portfolio implications

Equity and Treasury Co-(Variances) Based on a Daily DCC Model





Flight-to-quality

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Innovations in Covariance Modeling

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Decomposing the Covariance Matrix

- We can always separate the properties of the second moments of the marginal distribution and the multivariate distribution

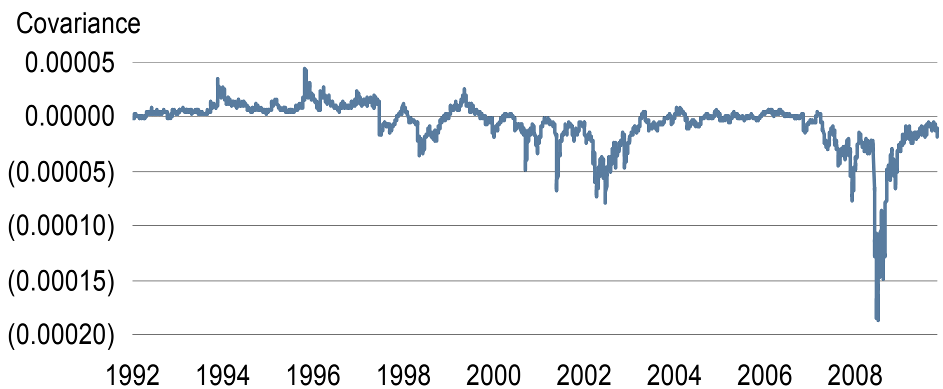
$$\Sigma_t = D_t R_t D_t$$

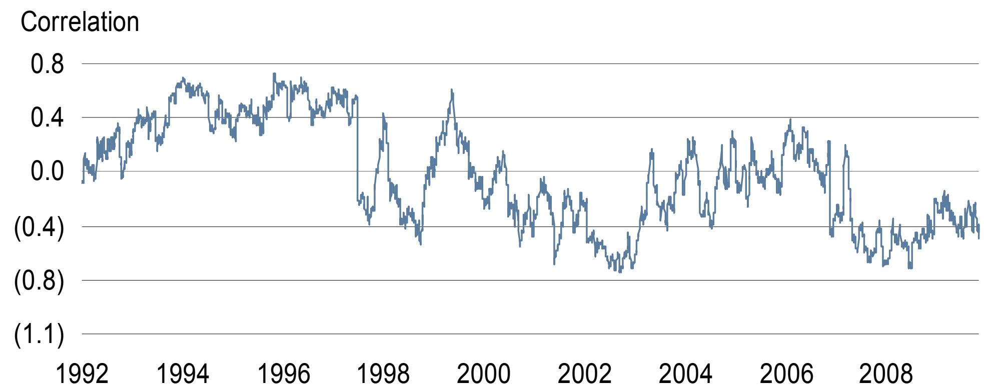
$$D_t \sim diagonal\ volatility\ matrix$$

$$R_t \sim correlation\ matrix$$

- Why make life complicated?
  - Easier interpretation: We have a more intuitive understanding of volatilities and correlations
  - Better statistical models (we use this approach in our equity risk models!)

Equity and Treasury Covariance vs. Correlation





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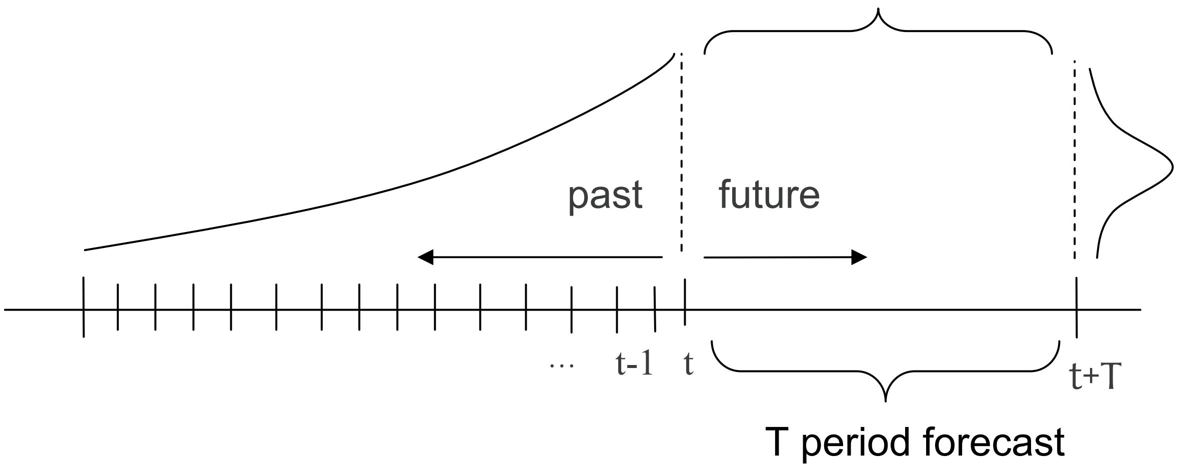
Mixed Frequency Estimation

- Higher-frequency observations are used to model and forecast lower-frequency time series properties (e.g., monthly volatility predicted from daily returns)

$$\tilde{\sigma}^2_{t+T} = \sum_{l=1}^{L} w_{t-l} r^2_{t-l,t-l+1}$$



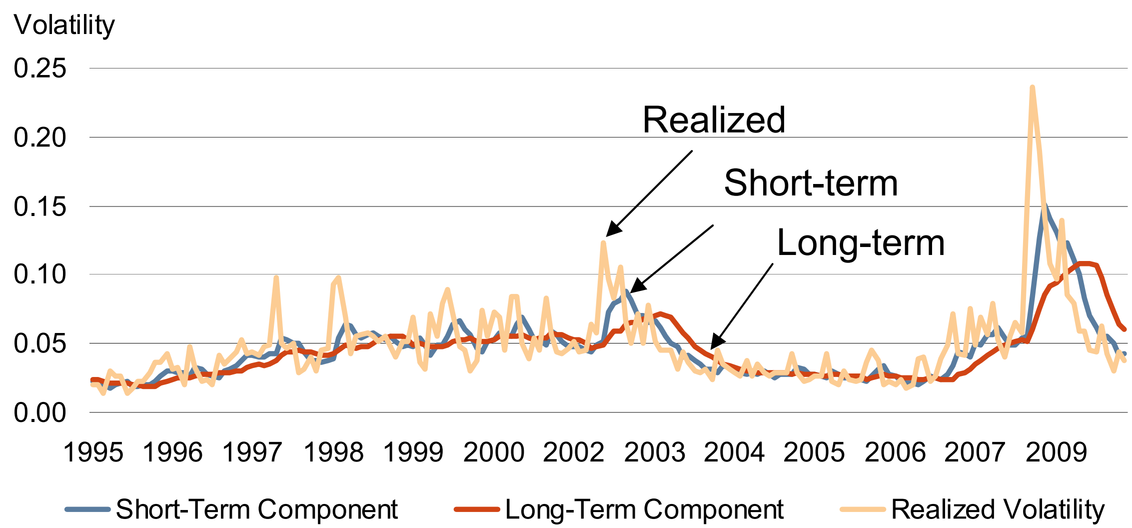
- Mixed frequency estimation benefits and concerns
  - Allows for more precise estimates and improved dynamics in liquid markets
  - Noise vs. information
  - The Epps effect

- Challenge: Aggregation mechanisms in forecasting

Mixed Frequency Estimation – Direct Approach

- Mixed frequency data sampling: MIDAS (Gheysels et al. [2004])
  - Use low-dimensional weighting functions in a regression framework with variables at different frequencies (e.g., regress monthly realized variance on past daily absolute returns)
- The MIDAS approach can be extended to many different settings
  - Example: Mixed frequency vol model – A two-factor model in which the short-term component moves around a long-term component
  - Much better predictive power than GARCH (R^2 is 35% higher for our model!)

S&P 500 Mixed Frequency Vol Model



$$r_t \sim N\left(\mu_t, \sigma_t^2\right)$$

$$\sigma_t^2 = \underbrace{\mu_{\sigma,t}}_{\text{Long-Term component}} + \underbrace{\phi_H \sum_{k=0}^{k^{\max}} b_H(k,\theta) r_{t-k,t-k-1}^2}_{\text{Short-Term component}}$$

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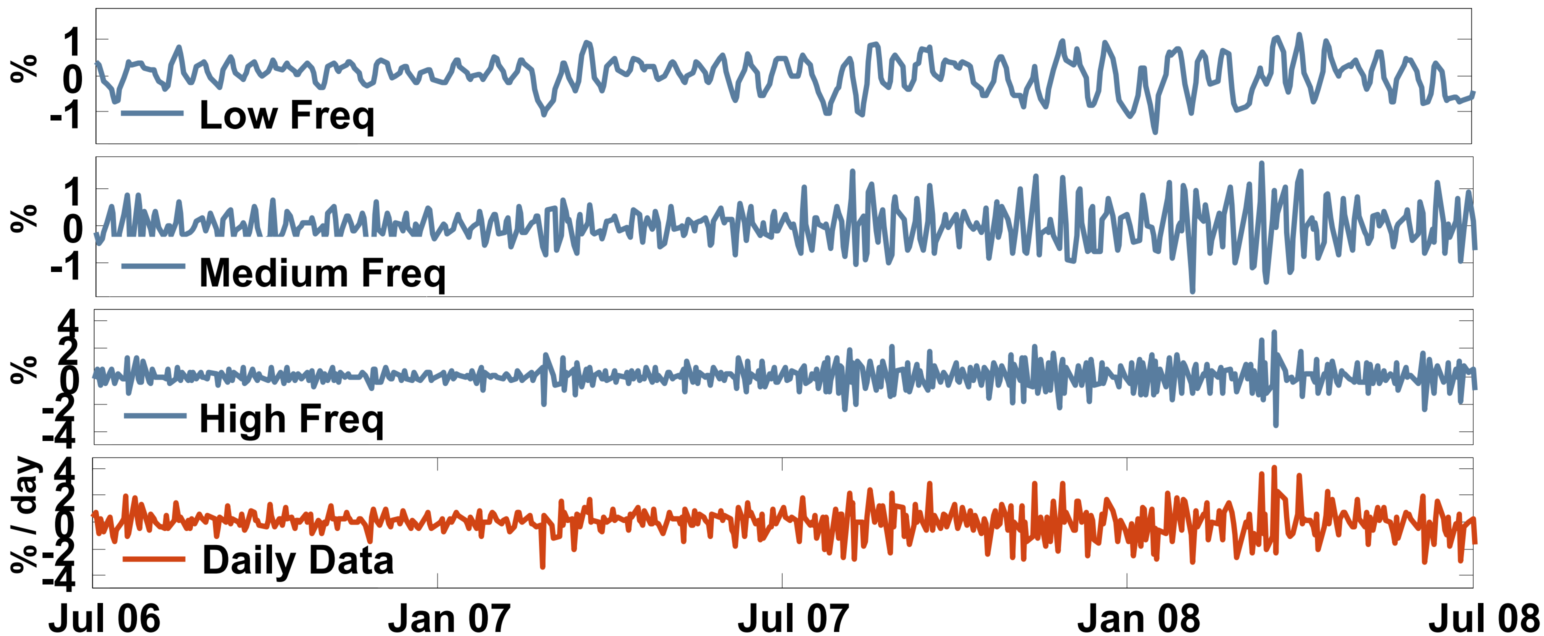
Mixed Frequency Estimation – Wavelets

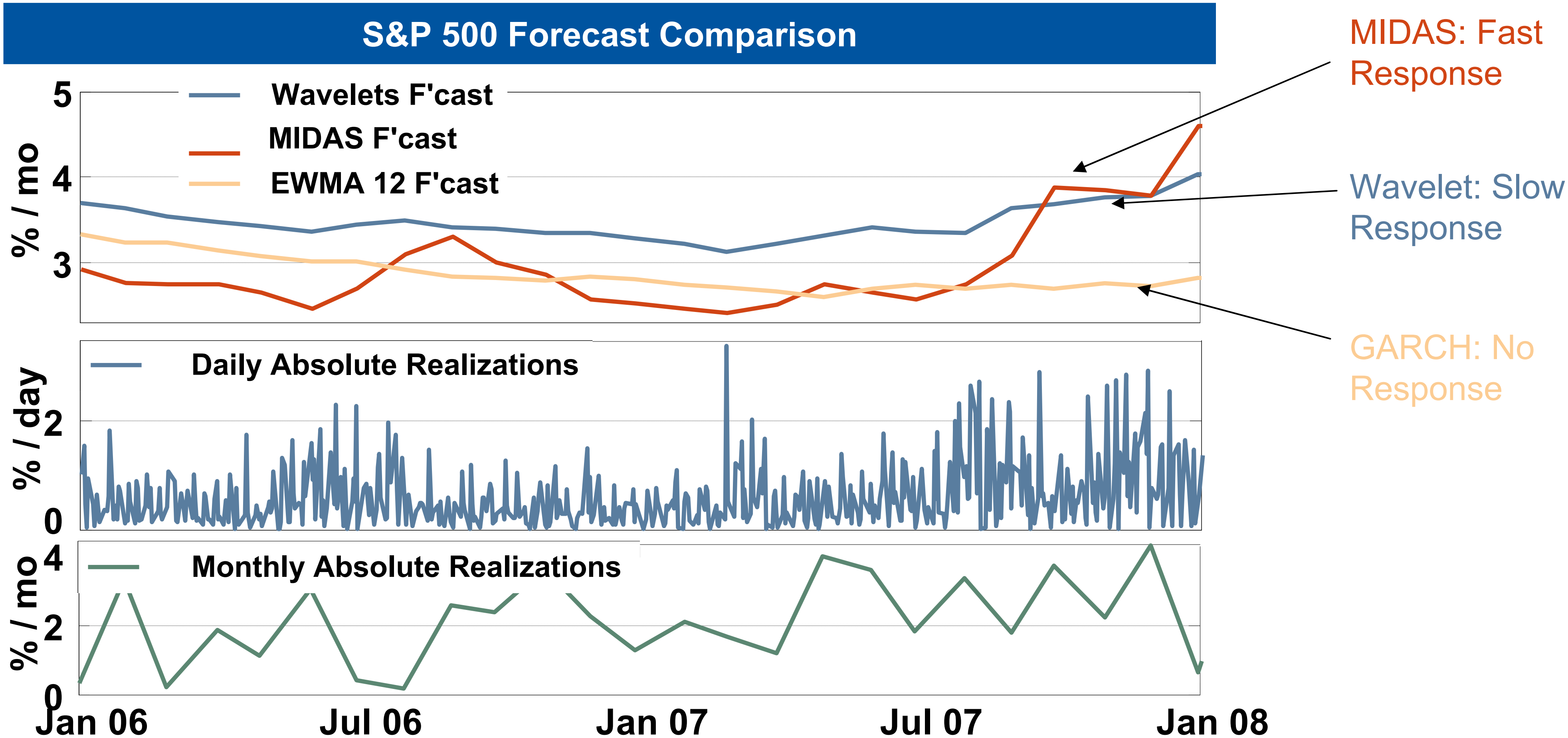
- Wavelets are a promising class of models that we are studying for use in risk modeling and trading strategies
- Wavelets allow to capture the time-frequency analysis parsimoniously
  - Decompose the time series into several time series for each frequency range
  - Examine and forecast each frequency and aggregate as desired

S&P 500 Return Wavelet Decomposition



Mixed Frequency – Volatility Forecast Behavior

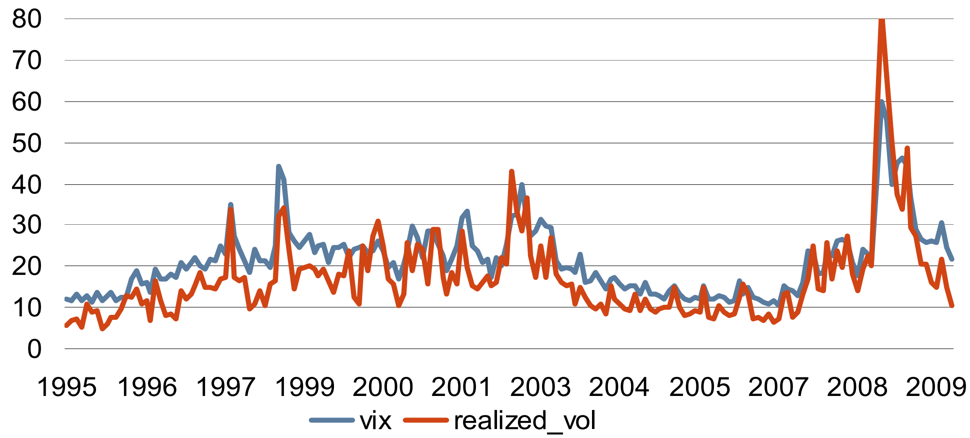
- Mixed frequency modeling allows for much more dynamic behavior, in turbulent markets



Conditioning Information

- The forecasting problem: Construct an "optimal" forecast based on the available information set
  - Information set consists of more than past returns!
- Conditioning information
  - State variables capture non-time-series-specific information that will determine future behavior: State of the economy, market sentiment, etc.
  - Example: The VIX is an excellent, but not sufficient, predictor of future S&P 500 volatility

S&P 500 – Realized Volatility vs. VIX



Model Comparison

	α	β	R^2
VIX	(6.8)	1.1	79%
MFVM	0.8	0.95	49%
MFVM+VIX	(1.4)	1.05	60%

MFVM= Mixed Frequency Vol Model

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How to Judge a Model?

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What Makes for a Good Model?

- The literature on (co)variance estimation is enormous and expanding at a rapid pace
  - How to choose your model?
  - What are the tradeoffs of the different models?

- Main question: What is the objective?
  - Horizon and rebalancing frequency
  - Risk measurement, asset allocation, hedging, pricing

- Desirable properties in a covariance model
  - Intuitive relationships across and within asset classes and regions
  - Adaptive but not noisy, robust to outliers (jumps)
  - Unbiased risk estimates over any reasonable period of time, invariant conditional distributions
  - Model building and evaluation is both art and science

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Testing Your Estimates

- Direct: Testing based on realized volatility and correlations
  - Regressions of predictions on forecasts and other variables, MSFE

$$\sigma_t^{realized} = \alpha + \beta \cdot \sigma_t^{forecast} + \varepsilon_t, \quad H_0 : \alpha = 0, \beta = 1$$

$$MSFE = \sum \left(\sigma_t^{realized} - \sigma_t^{forecast} \right)^2$$

- Indirect: Testing of standardized residuals
  - Unbiased?

$$H_0 : E\left[\left(\frac{\varepsilon_t}{\sigma_t^{forecast}} \right)^2 \right] = 1$$

- Invariant distributions?

$$H_0 : \frac{\varepsilon_t}{\sigma_t^{forecast}} \sim iid \quad F$$

- Multivariate testing: GMV, long-short, focus portfolios

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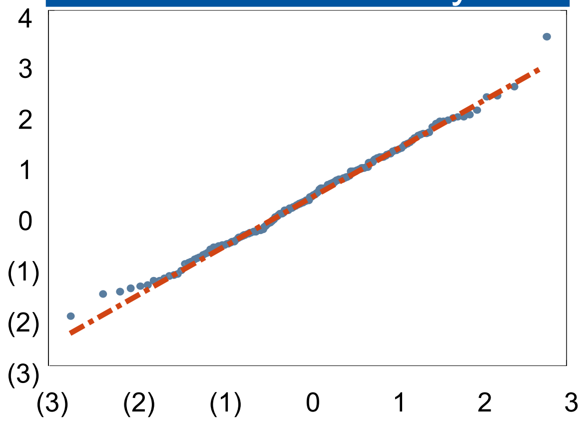
Testing Your Estimates – S&P 500 Volatility

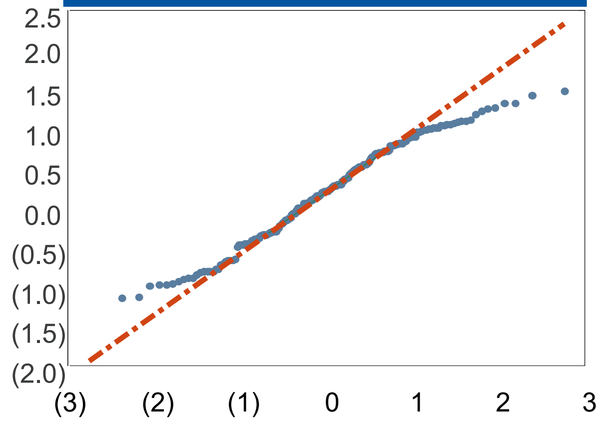
QQ-Plots and Bias Tests of Standardized S&P 500 Returns 1995–2009

Realized volatility



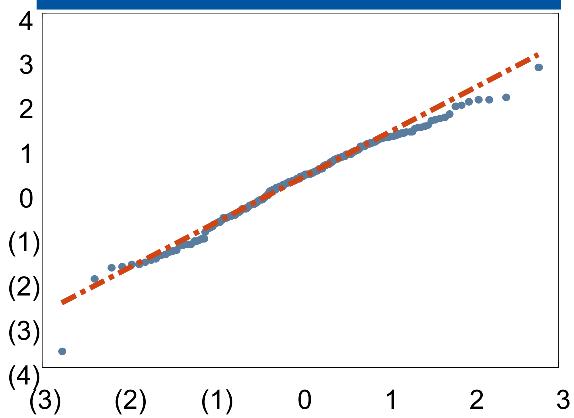
$$stdev\left(\frac{r_t}{\sigma_t}\right) = 0.97$$

VIX



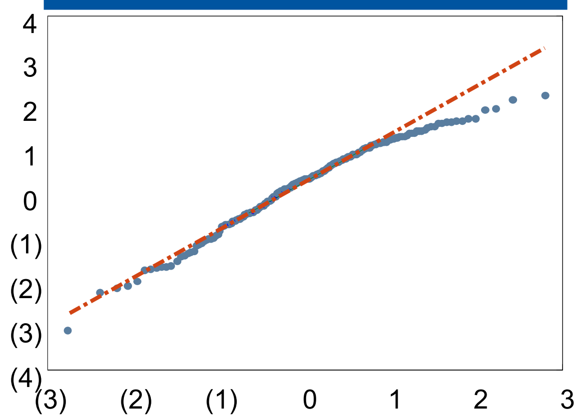
$$stdev\left(\frac{r_t}{\sigma_t}\right) = 0.64$$

MFVM



$$stdev\left(\frac{r_t}{\sigma_t}\right) = 0.99$$

MFVM+VIX



$$stdev\left(\frac{r_t}{\sigma_t}\right) = 0.99$$

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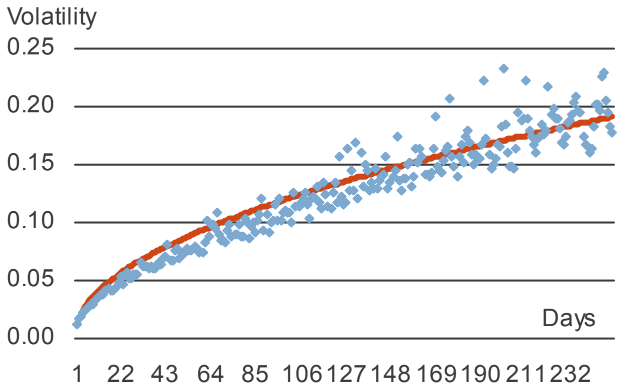
The Term Structure of Volatility

The Term Structure of Risk – Time Scaling

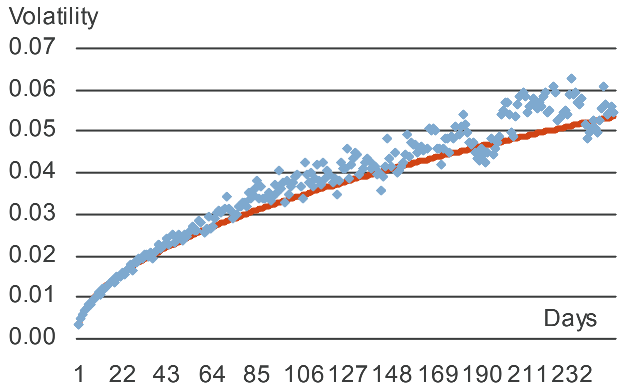
- Different portfolio management problems require risk forecasts at different horizons
- The term structure of volatility: What exactly do we mean?
  - The *unconditional* term structure of risk: Sqrt(t) scaling?
  - The *conditional* term structure of risk: A complex problem
    - Mean-reversion in volatility?
    - Possible shapes?
- Square-root-time rule
  - For financial returns, square-root-t scaling works fairly well *unconditionally*

Unconditional Volatility Scaling by Time

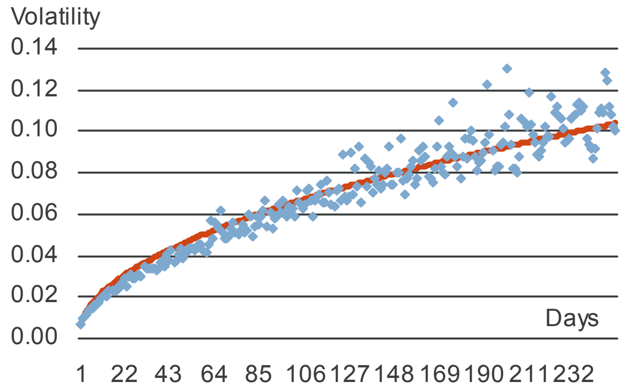
S&P 500



5Yr Treasuries



Equal-Weighted Portfolio



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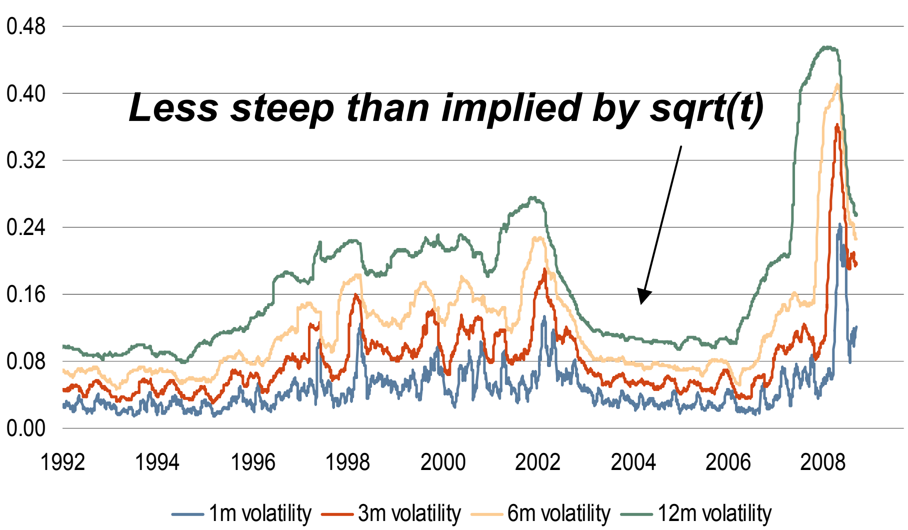
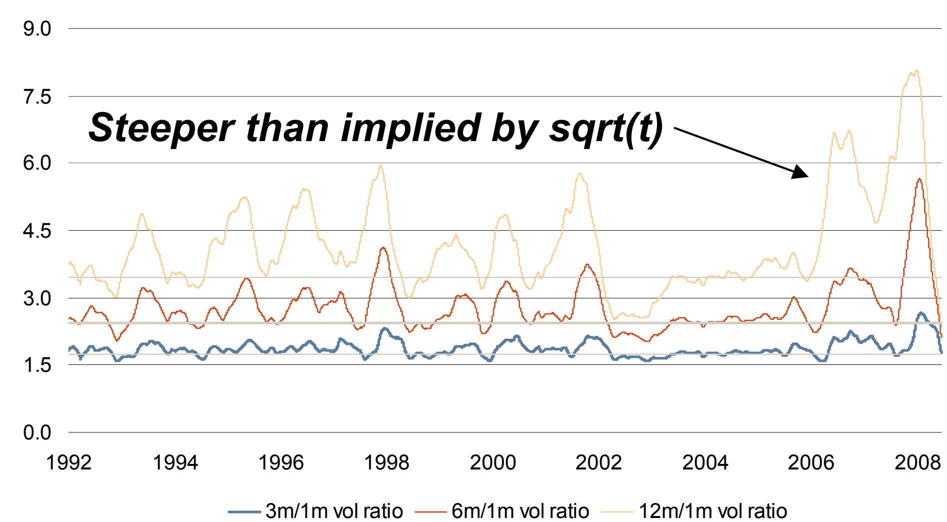
The *Conditional* Term Structure of Risk

- *Conditionally*, we should expect the term structure of risk to be much more dynamic than implied by square-root-t scaling
- We expect a steep curve going into a recession and a flatter curve in expansions
- Volatility might be subject to regime-shifts

S&P 500 Volatility at Different Horizons



Time Scaling One-Month S&P 500 Volatility



- The term structure is dynamic, but is it predictable?

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The Term Structure of Risk – Forecasting

- Three ways to construct conditional longer-term risk forecasts
- Estimate a horizon-specific model
- Iterate over higher-frequency forecasts in e.g., a GARCH
- Mixed-frequency modeling

- A simple multi-scale term structure model of volatility

$$RV_{t+H,t} = \mu_H + \phi_{1m} RV_{t-m,t} + \phi_{2m} RV_{t-3m,t} + \phi_{6m} RV_{t-6m,t} + \phi_{9m} RV_{t-9m,t} + \phi_{12m} RV_{t-12m,t} + \varepsilon_{H,t}$$

Model Comparison

Horizon	TSM MSFE	Sqrt(time) MSFE	TSM R^2	Sqrt(time) R^2
3 months	7.9	10.3	41%	34%
6 months	15.1	19.2	34%	25%
9 months	29.9	33.1	26%	21%
12 months	42.1	44.2	17%	15%

- We can do better than scaling by time, but advantage and predictability decrease with the horizon!

Conclusion

We Have Seen Only the Tip of the Iceberg

- Many important issues have not been discussed
  - Asymmetry in the return-volatility / correlation relationship ("the leverage effect")
  - High-frequency data and estimation
  - Density prediction
  - Relationship to trading volume and economic fundamentals

- Economic evaluation of models
  - Can we quantify in economic terms the gains from a better model?

- On our agenda
  - Mixed frequency estimation across asset classes
  - Term structure of risk
  - Integrating risk management and asset allocation
  - Volatility as a trading signal

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