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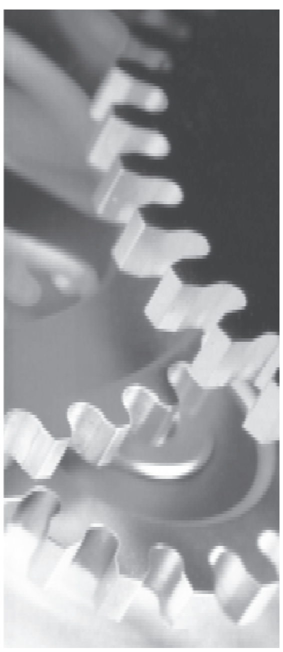

LEHMAN BROTHERS

Bond-Implied CDS Term Structures and Relative Value Measures for Basis Trading

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1. INTRODUCTION

In a recent paper (Berd, Mashal and Wang [2003]) we introduced a new methodology for direct estimation of implied term structures of survival probabilities from credit bond prices. We have shown that this methodology is more robust than the traditional implementations of reduced-form default models (for the latter, see Litterman and Iben [1991], Jarrow and Turnbull [1995], and Duffie and Singleton [1999]). More importantly, it is more consistent with underlying market and legal practices such as debt acceleration and equal priority recovery for same-seniority bonds.

Our methodology is well suited to a direct comparison with credit derivatives, particularly credit default swaps, whose valuation is driven by the modeling of default probabilities. In this paper we introduce new relative value measures, which take advantage of the internal consistency of this pricing methodology. In particular, we define:

- Bond-Implied CDS (BCDS) term structure
- CDS-Cash curve basis
- Systematic and full bond-specific basis to CDS curve
- Risk-free-equivalent coupon (RFC) streams for credit-risky bonds

We also introduce and discuss static replication/hedging strategies of credit risk in cash bonds using forward and spot CDS. In particular, we demonstrate in detail how these strategies can be used to hedge the default risk of a credit bond with an arbitrary coupon, given an arbitrary term structure of risk-free rates. The complete hedge of credit risk in these strategies is reflected in the complementarity between the risk-free-equivalent coupon streams and the CDS hedging costs, which is formally proven in the Appendix.

2. BOND-IMPLIED CDS TERM STRUCTURE

Credit default swaps are by far the largest component of the rapidly growing credit derivatives market. They comprise as much as 70% of the market by notional volume. The outstanding notional of CDS is comparable with that of cash bonds, and their liquidity often exceeds that of the cash market for the top 200 or so names. At the same time, cash credit bonds often cover a greater range of maturities than CDS and have far more extensive historical data associated with them, providing fertile ground for research and back-testing trading strategies. One can therefore hope that an implied CDS measure derived from bond prices will be a valuable tool for consistent comparisons between the two markets across the entire range of maturities.

Deriving the Bond-Implied CDS spread term structure

The survival-based valuation approach is well suited to the CDS market. In fact it has been the market practice since its inception. By deriving the bond-implied CDS spreads within the same framework we are aiming to give investors an apples-to-apples relative value measure across the bond and CDS markets.

The pricing relationship for credit default swaps simply states that the expected present value of the premium leg is equal to the expected present value of the contingent payment (see O'Kane and Turnbull [2003]):

$$[1] \qquad \frac{S}{f} \cdot \sum_{i=1}^{N} Q(t,t_i) \cdot Z(t,t_i) = (1-R) \cdot \sum_{i=1}^{N} \left(Q(t,t_{i-1}) - Q(t,t_i) \right) \cdot Z(t,t_i)$$

where, S is the annual spread, f is the payment frequency ($f = 4$), $Q(t,t_i)$ is the issuer's survival probability between time t to time t_i, $Z(t,t_i)$ stands for the (default risk-free) discount factor from time t to time t_i, and R stands for the recovery rate. This formulation is consistent with the methodology described in Berd *et al.* (2003).

The bond-implied CDS spread term structure, hereafter denoted as BCDS term structure, is defined by substituting the survival probability term structure fitted from bond prices, $Q_{bond}(t,t_i)$, into the following equation for par CDS spreads:

$$[2] \qquad BCDS(t,t_N) = f \cdot (1-R) \cdot \frac{\sum_{i=1}^{N} \left(Q_{bond}(t,t_{i-1}) - Q_{bond}(t,t_i) \right) \cdot Z(t,t_i)}{\sum_{i=1}^{N} Q_{bond}(t,t_i) \cdot Z(t,t_i)}$$

The different payment frequencies for bonds and CDS do not represent a problem because the fitted survival probability term structure is continuous and can be evaluated at any frequency. Furthermore, the CDS contracts stipulate that at default any accrued protection premium must be paid.

The above equations approximate the settlement process of the CDS at default assuming that all defaults during a payment period are settled at the end of the period and, correspondingly, the entire period premium is paid (see O'Kane and Turnbull [2003] for a more accurate approximation). We also ignore the cheapest-to-deliver option.

Comparison with conventional spread measures

The BCDS term structure complements bond-based valuation measures defined in Berd *et al.* (2003), namely the par Libor and par Treasury spreads, and constant coupon price (CCP) term structures. The BCDS term structure is closely related to, but not equivalent to, par Libor spreads. In fact, under certain circumstances, the BCDS term structure may be significantly different from conventional measures such as a bond's z-spread and Libor spread (see "Credit Spreads Explained" by O'Kane and Sen in this issue for definitions of the conventional spread measures).

Market participants often use the z-spread as a proxy for comparing bonds with CDS. Such analysis may be misleading because the derivation of z-spreads is based on the valuation of credit bonds with spread-based discount functions which we have shown to be incorrect in our earlier paper. This is due to the fact that credit bonds do not have fixed cashflows – they only have fixed promised cashflows, while realized cashflows may well turn out to be different from the pro-forma projections. Hence, a survival-based approach is needed to correctly model default-risky bonds.

The z-spread of a credit bond is consistent with a correct survival-based valuation framework only under the assumption of zero recovery rates. Generally speaking, such an assumption is far from observed statistics. Given that historical average recovery values are about 40% (30% during the recent credit downturn) the z-spread overestimates the losses in case of default by a significant amount. Therefore, it should not be surprising that the BCDS spread term structure can differ from the z-spread by substantial margins.

An example of this is presented in Figures 1a and 1b, which depict the relationship between the survival-based BCDS curve, the Libor spread curve and the z-spreads of individual bonds, based on spread-based discount function methodology. Figure 1a shows the results for Georgia Pacific Co. as of December 31, 2002 – ie, at a time when the bonds of the company were substantially distressed. We observe that the shape of the BCDS curve bears no resemblance to the shape of the traditionally fitted Libor spread curve or to the z-spreads of individual bonds. Figure 1b, on the other hand, shows the same set of bonds as of December 31, 2003 – when the spreads have generally tightened and the bonds no longer trade at large discounts. Here, the BCDS curve is much closer to the conventional Libor spreads.

We would like to emphasize that the methodology developed in Berd *et al.* (2003) is based on fitting the *prices* of bonds rather than their spreads, and that the deviation of the BCDS curve from bond z-spreads does not indicate a poor fit of bond values. To confirm this, we also show the constant coupon price term structures (ie, the projected prices of hypothetical bonds with 6%, 8% and 10% coupons) from the same model compared with the bond prices as of December 31, 2002. As we can see, the CCP term structures neatly envelop the scatter plot of bond prices, as indeed they should.

Figures 1a and 1b also shed light on the interesting issue regarding the "slope of the spread curve". In both cases, the Libor OAS curve which is fitted using the conventional methodology, is inverted, while the BCDS curve is not (or at least not for all maturities). One often hears that distressed bond pricing is always accompanied by an inverted spread curve. However, this is only true in the conventional, spread-discount-factor based models, and it is a consequence of the inability of these models to correctly capture the peculiar aspects of distressed bond pricing – namely the fact that such bonds trade "on price" or "to recovery".

Indeed, in case of default, all bonds of the same seniority, regardless of their maturity, should trade at the same price (equal to recovery). As the credit quality of the issuer deteriorates, the market begins pricing these bonds closer and closer to recovery scenario. As a result, bonds begin trading at similar low dollar prices across all maturities. In the conventional spread-discount-function based methodology, one can explain an $80 price of a 20-year bond with a spread of 500bp. However, to explain an $80 price of a 5-year bond, one would need to raise the spread to very large levels to achieve the required discounting effect. Thus, the inversion of the spread curve is due to the bonds trading on price. In the survival-based methodology, the low prices of bonds are explained by high default rates, which need not have an inverted term structure. Therefore, the BCDS term structure also need not be inverted in order to capture the credit risk of distressed bonds.

Figure 1a. BCDS and Libor OAS term structures and bond z-spreads, GP as of 12/31/02

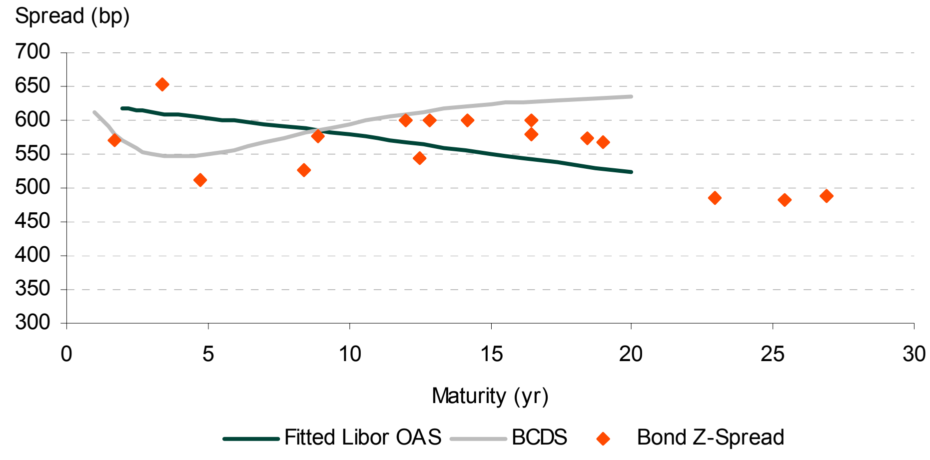


Figure 1b. BCDS and Libor OAS term structures and bond z-spreads, GP as of 12/31/03

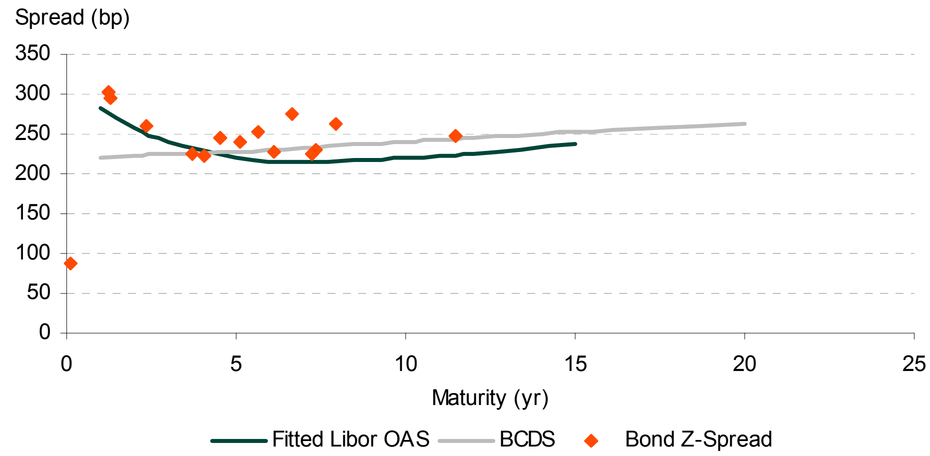
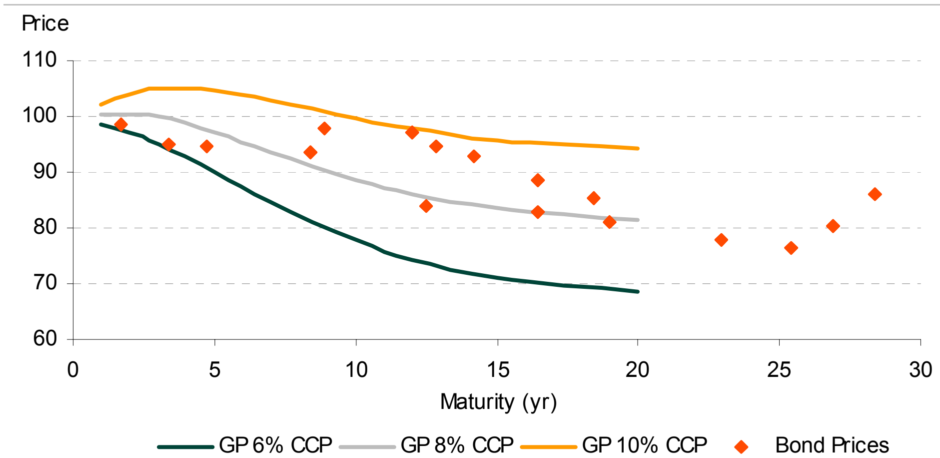


Figure 1c. CCP term structures and bond prices, GP as of 12/31/02



3. STATIC HEDGING OF BONDS WITH CDS

In the previous section we defined the BCDS term structure and explained why it is a better valuation measure than the traditional Libor OAS or z-spread measures. However, in order to be sure that the BCDS term structure corresponds to the fair value of the CDS as seen by the bond market, one should explicitly construct a hedging strategy using CDS that would completely eliminate the credit risk of any given cash bond.

Such a strategy is well known for a par bond when the underlying credit risk-free curve (which is usually assumed to coincide with the Libor swaps curve) and the hazard rates are flat. Under such assumptions, the hedging strategy turns out to be simple – buy the credit bond and buy an equal face value of CDS. The resulting combination exhibits no risk due to default. The interest rate risk of the hedged position coincides with the risk of a credit risk-free bond with a lower coupon equal to the difference of the credit-risky coupon and the CDS spread. This risk can be hedged using interest rate swaps, although residual timing risk may remain if the swap is not terminated upon default.

The situation quickly becomes more complex when the above assumptions are not valid, most notably in case of non-par bonds and/or a Libor curve that is significantly different from flat. Some of the difficulties and possible remedies have been discussed in McAdie and O'Kane (2001). There it was shown that, when hedging a given fixed coupon bond with a CDS of matching maturity, neither the face value hedge ratio nor the market value hedge ratio based on the current price of the bond is satisfactory, because they lead either to a residual mark-to-market risk during the life of the bond or to a substantial dependence of the carry cost of the hedged position on the price of the bond. McAdie and O'Kane conclude that, for a hedge using a single CDS position, an intermediate strategy called a zero-recovery market value hedge provides the best compromise.

In this section we further extend the intuition put forward in McAdie and O'Kane (2001). As it turns out, the complete hedging strategy cannot in general be accomplished by a single CDS position. Indeed, under generic conditions, a bond will have a non-trivial term structure of forward prices computed with today's risk-free discount curve and hazard rate term structure. If a bond is well hedged today, it will likely be either over- or under-hedged in the future, when its price is expected to change. After all, if the bond actually survives to maturity its price will pull to par. So the hedge at maturity should always be based on the expected price at that point, which is close to par. The question is – what should happen at prior times?

Staggered hedging strategy with forward CDS

The precise answer to this question is presented in the Appendix, where we define the notion of the *risk-free-equivalent coupon stream* (RFC) and show its complementarity with the BCDS curve. The complementarity condition defines the unique static hedging strategy based on a sequence of forward CDS contracts with notional that depends on the bond's forward price for each maturity.

A forward CDS contract provides protection during a future time period in exchange for premiums which are paid during that period but whose level is preset today. In case of a credit event prior to the starting date of that period, the forward CDS knocks out without a payment and provides no further protection. However, if a credit event occurs during the contractual future period – the CDS pays the loss amount $N_{fwd} \cdot (1-R)$. Since, based on today's calculations, the bond is expected to have a forward price $P_{fwd} \neq 100\%$ at that future date, then in order to insure the full forward price of the bond we must choose the hedge notional

[3] $$N_{fwd}(t) = \frac{P_{fwd}(t) - R}{1 - R}$$

This intuition is precisely right, as shown in the Appendix. The sequence of forward CDS hedges does, in fact, completely hedge out the credit risk of a fixed coupon bond. If we subtract from the bond's coupon the cost of hedging according to [3], the residual cashflows will coincide with the above mentioned risk-free-equivalent coupon stream. The RFC is such a pre-set sequence of term-dependent coupons which, if discounted with risk-free rate, correspond to the same term structure of forward prices as the one obtained for the credit-risky bond under consideration (see Appendix for more details).

To be more accurate, we must mention that the hedge notional shown above is only valid for the case of continuously compounded coupons and spreads. In a more realistic case of semi-annual coupon payments, one must take into account the present value of those coupons as well as the fact that the CDS market conventions stipulate a payment of the accrued premium upon default. The result is a slightly modified expression for the hedge notional applicable for the payment period ending at t_i:

[4] $$N_{fwd}(t_i) = \frac{0.5 \cdot \left(P_{fwd}(t_{i-1}) + P_{fwd}(t_i) + w \cdot C\right) - R}{1 - R}$$

The hedge depends on the average forward price during the preceding coupon period as well as the potential loss of coupon in case of default. The effective weight w depends on the assumed coupon recovery. We have found empirically that for coupon recovery of 50% the weight is also 50%, while for coupon recovery of 0% the weight is 25%.

Figure 2a illustrates this staggered forward CDS strategy in the case of a hypothetical 8% coupon 5-year bond which trades at a high initial price premium of 116.69%. The top rows of the table show the term to maturity, the spot BCDS curve and the forward BCDS curve with forward horizon equal to 0.5 years (see Berd [2003] for the relationship between spot and forward CDS spreads). The middle of the table shows how the hedge notionals are related to the forward prices of the credit bond and how they gradually decrease as the forward prices exhibit pull to par – compare the notional for a given term (column) with the forward price shown in the shaded area on the row corresponding to the same term (in this example we used 50% principal and coupon recovery in equation [4]). The next four rows demonstrate the complementarity between the hedging costs and the risk-free-equivalent coupon stream (RFC), which holds very accurately. Finally, we show that the forward prices of the resulting residual cashflow streams discounted with risk-free interest rates do indeed coincide with high accuracy with the forward prices of the original bond for all future horizons.

Staggered hedging strategy with spot CDS

One could, in principle, construct a very similar strategy using the spot instead of forward CDS. As discussed in McAdie *et al.* (2003) and Berd (2003), a forward CDS protection contract is closely related to a long-short pair trade in spot CDS, with equal notionals of the long protection position at the longer maturity and of the short protection position at the shorter maturity. Although in terms of credit protection the long-short trade is indeed equivalent to a forward CDS contract, the premium legs of these two strategies will generally differ, resulting in different forward mark-to-markets.

If we execute the hedging strategy with long-short pairs, the result becomes a staggered hedge which is nearly 100% notional for the final maturity, and which includes some additional relatively small long (or short) positions for shorter maturities depending on the forward prices of the credit bond being hedged. Each such position hedges the incremental digital price risk (with no recovery) corresponding to the next maturity interval on the hedging grid:

[5] $$N_{pair}(t_i) = \frac{P_{fwd}(t_i) - P_{fwd}(t_{i+1})}{1-R}$$

Figure 2b shows an implementation of this strategy for the same hypothetical high premium bond. While the current credit risk of this bond is indeed hedged, as evidenced by close replication of spot price, this hedge does drift away from perfection with time (but remains quite close nevertheless). This is because the pair trades are not exactly equivalent to a series of forward CDS.

The coupon and price premium/discount dependence

From the discussion and examples shown above, it is clear that both the coupon level of the credit bond and the term structure of the underlying interest rates and issuer's hazard rates may substantially affect the hedging strategy with forward CDS or long-short CDS pairs. Its dependence on the underlying bond is depicted in Figures 3a, 3b and 3c.

Figure 3a shows a case of a bond with a high coupon equal to 8% and a high current price of 116.69% – the same as Figures 2a and 2b. The forward CDS hedge notional starts as high as 133% of the face value, and gradually decreases toward 100%. Despite the decrease in the hedge notional, the semi-annual cost of hedging grows gradually from 40bp to 44bp as a result of a relatively steep forward CDS curve term structure.

Figure 3b shows a case of a bond with a very low coupon equal to 3% and a current discount price equal to 94.33%. The forward CDS hedge notional starts at 89% of the face value, and gradually increases toward 100%. The semi-annual cost of hedging grows more steeply from 27bp to 43bp as both forward CDS rates and hedge notionals grow.

Figure 3c shows a case of a bond with near-par coupon equal to 4.25% and a current price of 99.93%. Despite the fact that this is a par bond, the forward bond prices and hedge notionals exhibit a non-trivial term structure, starting near 100%, then dropping to lower levels and only pulling back to par near final maturity. The semi-annual cost of hedging grows from 33bp to 43bp, which is somewhere between the high and low coupon cases.

Notes on practical implementation

The hedging strategy using a sequence of forward CDS is difficult to implement in practice. Although less precise, the strategy using spot CDS is generally easier to put to work. However, the precise hedge amounts stipulated by BCDS complementarity generally correspond to odd-lot notionals for intermediate terms, which would likely result in a loss of liquidity. As a compromise between accuracy and liquidity, we suggest a coarse-grained staggered hedge which can be constructed using a maturity grid with longer intervals. The forward price changes in these intervals will yield lumpier intermediate hedge notionals, according to [5]. The optimal hedging grid will depend on the bond coupon level and the underlying interest rates. For bonds with modest premium or discount, just one or two additional hedges can result in sufficient accuracy.

Figure 2a. Complete hedge of a premium credit bond with forward CDS, 8% coupon 5-year maturity bond

	Term	*0.00*	*0.50*	*1.00*	*1.50*	*2.00*	*2.50*	*3.00*	*3.50*	*4.00*	*4.50*	*5.00*
	Fwd BCDS	0.60%	0.60%	0.63%	0.66%	0.69%	0.72%	0.74%	0.77%	0.80%	0.83%	0.86%
	Spot BCDS	0.60%	0.60%	0.62%	0.63%	0.65%	0.66%	0.67%	0.69%	0.70%	0.71%	0.72%
Term	**Fwd Price**											
0.00	116.70%											
0.50	113.84%		1.33									
1.00	111.31%			1.27								
1.50	109.09%				1.22							
2.00	107.16%					1.18						
2.50	105.50%						1.15					
3.00	104.08%							1.12				
3.50	102.84%								1.09			
4.00	101.77%									1.07		
4.50	100.84%										1.05	
5.00	100.00%											1.03
	Hedge Notional		**1.33**	**1.27**	**1.22**	**1.18**	**1.15**	**1.12**	**1.09**	**1.07**	**1.05**	**1.03**
	Protection CF		0.40%	0.40%	0.40%	0.41%	0.41%	0.42%	0.42%	0.43%	0.43%	0.44%
	Coupon less Protection CF		3.60%	3.60%	3.60%	3.59%	3.59%	3.58%	3.58%	3.57%	3.57%	3.56%
	RCF		3.60%	3.60%	3.60%	3.60%	3.59%	3.59%	3.58%	3.57%	3.56%	3.56%
	CF Diff		*-0.01%*	*0.00%*	*0.00%*	*0.00%*	*0.00%*	*0.00%*	*0.00%*	*0.00%*	*0.00%*	*0.00%*
	Projected Fwd Price	116.69%	113.83%	111.30%	109.09%	107.16%	105.51%	104.08%	102.85%	101.77%	100.84%	100.00%
	Fwd Price	116.70%	113.84%	111.31%	109.09%	107.16%	105.50%	104.08%	102.84%	101.77%	100.84%	100.00%
	Price Diff	*-0.02%*	*-0.01%*	*-0.01%*	*0.00%*	*0.00%*	*0.00%*	*0.00%*	*0.00%*	*0.00%*	*0.00%*	*0.00%*

Figure 2b. Present value hedge of a premium credit bond with spot CDS, 8% coupon 5-year maturity bond

	Term	*0.00*	*0.50*	*1.00*	*1.50*	*2.00*	*2.50*	*3.00*	*3.50*	*4.00*	*4.50*	*5.00*
	Fwd BCDS	0.60%	0.60%	0.63%	0.66%	0.69%	0.72%	0.74%	0.77%	0.80%	0.83%	0.86%
	Spot BCDS	0.60%	0.60%	0.62%	0.63%	0.65%	0.66%	0.67%	0.69%	0.70%	0.71%	0.72%
Term	**Fwd Price**											
0.00	116.70%											
0.50	113.84%		1.33									
1.00	111.31%		-1.27	1.27								
1.50	109.09%			-1.22	1.22							
2.00	107.16%				-1.18	1.18						
2.50	105.50%					-1.15	1.15					
3.00	104.08%						-1.12	1.12				
3.50	102.84%							-1.09	1.09			
4.00	101.77%								-1.07	1.07		
4.50	100.84%									-1.05	1.05	
5.00	100.00%										-1.03	1.03
	Hedge Notional		0.05	0.05	0.04	0.04	0.03	0.03	0.02	0.02	0.02	1.03
	Protection CF		0.46%	0.45%	0.43%	0.42%	0.41%	0.40%	0.39%	0.38%	0.37%	0.37%
	Coupon less Protection CF		3.54%	3.55%	3.57%	3.58%	3.59%	3.60%	3.61%	3.62%	3.63%	3.63%
	RCF		3.60%	3.60%	3.60%	3.60%	3.59%	3.59%	3.58%	3.57%	3.56%	3.56%
	CF Diff		-0.07%	-0.05%	-0.03%	-0.01%	0.00%	0.02%	0.03%	0.05%	0.06%	0.07%
	Projected Fwd Price	116.76%	113.96%	111.48%	109.30%	107.39%	105.73%	104.29%	103.02%	101.91%	100.91%	100.00%
	Fwd Price	116.70%	113.84%	111.31%	109.09%	107.16%	105.50%	104.08%	102.84%	101.77%	100.84%	100.00%
	Price Diff	0.06%	0.13%	0.17%	0.21%	0.22%	0.22%	0.21%	0.18%	0.14%	0.08%	0.00%

Figure 3a. **Hedging strategy for high coupon, premium bond**

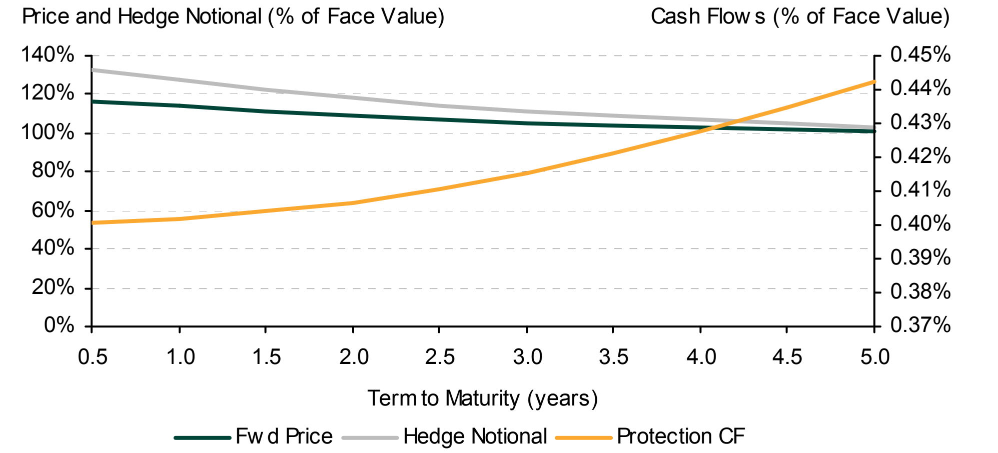


Figure 3b. **Hedging strategy for low coupon, discount bond**

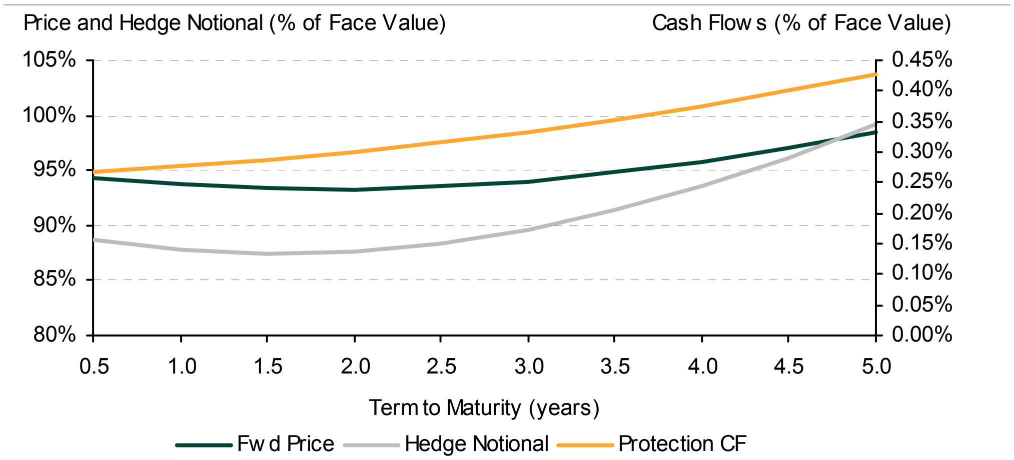
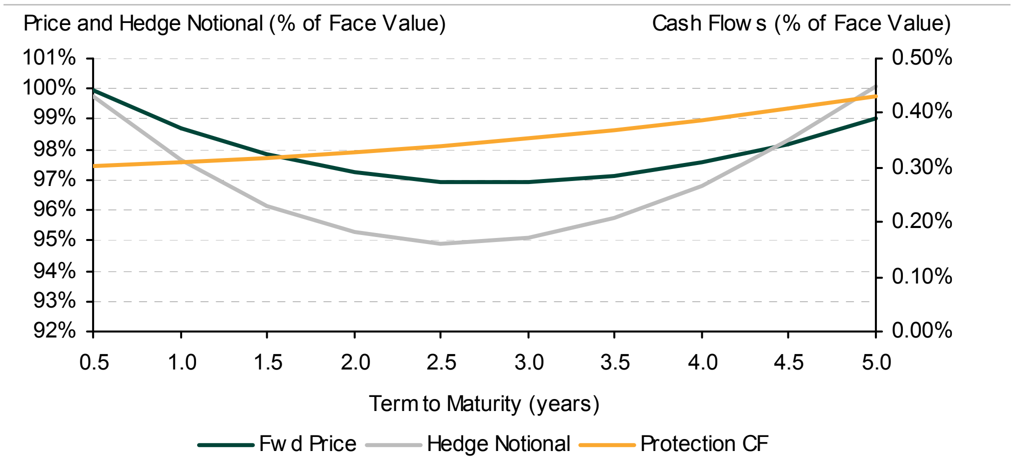


Figure 3c. **Hedging strategy for medium coupon, near-par bond**



4. RELATIVE VALUE MEASURES FOR BASIS TRADING

Although CDS and cash bonds reflect the same underlying issuer credit risk, there are important fundamental and technical reasons why the CDS and bond markets can sometimes diverge from the economic parity (see McAdie and O'Kane [2001]). Such divergences, commonly referred to as the CDS-Cash basis, are closely monitored by many credit investors. Trading the CDS-Cash basis is one of the widely used strategies for generation of excess returns using CDS. To facilitate basis trading, one must have a reliable measure of relative value between cash bonds and CDS. Here we suggest such a measure based on our approach.

The CDS-Cash curve basis

If the bonds of a given issuer were perfectly priced according to our framework, their prices would satisfy the survival-based fair value, and the BCDS term structure derived from the issuer hazard rates would correspond to a complete hedge of the credit risk as discussed in the previous section. Therefore, if the market-observed CDS term structure coincided with the BCDS term structure, then one could claim that there is no basis between the two markets since the hedged cash bond would have zero expected excess return over risk-free rates. If the market-observed CDS spreads were tighter than BCDS, one could hedge the credit bond at a cheaper price than that implied by the model, thus locking in a positive expected excess return. Vice versa, if the market CDS spreads were wider than BCDS, the hedge would have a higher cost, and the expected excess return of the hedged position would be negative.

We therefore introduce the **CDS-Cash curve basis** as a consistent relative value measure:

[6] $$\text{CurveBasis}(0,T) = CDS_{market}(0,T) - BCDS(0,T)$$

This measure corrects for the biases associated with commonly used asset swap spreads and z-spreads. Since this basis measure is well defined across the entire range of maturities, one can use it not only to screen for issuers that correspond to attractive CDS-Cash trading opportunities, but also to pinpoint maturities for which such trades would be most beneficial.

Bond-specific basis and incremental relative value

The curve basis does not fully reflect the differences in the costs of hedging implied by the fair value BCDS term structure and by the market observed CDS term structure. These differences depend, as explained in the previous section, on the projected forward price term structure of cash bonds, and therefore on the coupon level of the bond under consideration.

According to the staggered hedging strategy presented in the previous section, the notional of the final maturity hedge using the spot CDS is approximately equal to the face value and therefore its contribution to the CDS-Cash basis is simply the curve basis. However, the incremental hedges for each maturity horizon are proportional to the change in the projected forward price of the bond during the short maturity span around that horizon [5]. The present value of every basis point of difference between the BCDS and the market observed CDS curve for each of these incremental hedges is given by the risky PV01 calculated in accordance with the BCDS curve. Therefore, the incremental hedging cost differential $HCD_{curve}(T)$ due to intermediate maturity curve basis is equal to:

[7]
$$HCD_{curve}(T) = -\int_0^T \text{CurveBasis}(0,t) \cdot \frac{1}{1-R} \cdot \frac{\partial P_{fwd}(t,T)}{\partial t} \cdot \text{RiskyPV01}(0,t) \cdot dt$$

To convert the hedging cost differential into a spread-equivalent measure, we divide it by the risky PV01 of the final maturity BCDS. This has a meaning of a weighted average curve basis in units corresponding to matching maturity CDS. We call it a **systematic bond basis**:

$$\text{BondBasis}_{\text{systematic}}(T) = \text{CurveBasis}(0,T) + \frac{HCD_{curve}(T)}{\text{RiskyPV01}(0,T)} \quad [8]$$

Generally speaking, if the bond price premium or discount is not large, then the corrections to the systematic bond basis due to intermediate maturities will be very small, and one can use the curve basis for the final maturity as a good proxy of the systematic bond basis. However, one must note that if the matching maturity curve basis is exactly zero, then the correction term will become the main component of the systematic bond basis. Its dependence on the bond price and curve basis is non-trivial.

If the curve basis is positive at intermediate maturities, and the bond is trading at premium and is expected to gradually accrete to par, the systematic bond basis will be positive – ie, it will cost more to hedge the bond than the BCDS curve would suggest. If, on the contrary, the curve basis is positive but the bond is trading at a discount and is expected to accrete up to par, then the systematic bond basis will be negative – ie, it will cost less to hedge the bond with CDS than the bond market itself implies. This is because the staggered hedge strategy in this case actually requires selling protection at intermediate maturities, and therefore the positive curve basis will net extra benefits for the hedger. In the other cases when either the curve basis changes sign at intermediate maturities or the bond trades near par and its forward price may both increase and decrease over the future time horizon, the hedging cost differential can easily turn out to be either positive or negative – one would have to perform the full calculation to find out.

In addition to the fair value hedging cost differential embodied in the curve basis, there will also be an issue-specific pricing differential corresponding to the OAS-to-Fit (OASF) measure introduced in Berd *et. al.* (2003). This measure captures the pricing differential between the given bond and the issuer's fitted survival curve. As such, it reflects liquidity and other technical aspects of bond pricing unrelated to credit risk.

We define the **full bond-specific CDS-Cash** basis as the systematic bond basis minus an incremental amount equal to the bond's OASF:

$$\text{BondBasis}_{\text{full}}(T) = \text{BondBasis}_{\text{systematic}}(T) - OASF \quad [9]$$

Depending on the sign and magnitude of OAS-to-Fit, a given bond may have a negative basis while the issuer curve basis is positive, and vice versa. Having said this, since the average OASF across all bonds is zero by construction (see the discussion in our previous paper) then our definition of the bond-specific basis does not add any new bias to the systematic (or curve) basis. It only serves to assist investors in picking the best candidates for execution of either positive or negative basis strategies. For example, if the investor believes that a negative curve basis will converge, then picking the cheapest bond (most positive OASF) will add an incremental expected return to the trade.

Examples of curve and bond-specific basis

Let us consider an example of curve basis in one of the most actively traded issuers, Altria Group (ticker MO). Using the data as of February 6, 2004 we can see the difference between the shapes of the BCDS and market CDS curves. A clear hump in the relative curve basis in intermediate maturities is related to significant hedging activity in the CDS market as volatility picked up in late January and early February.

If an investor's view is that the curve basis is a transient phenomenon and is expected to converge, then picking a two- to three-year maturity range would maximize the convergence potential. In this maturity range the CDS market looked cheap to cash using our relative value measures (we use the bid-side CDS marks since the bond valuation measures including BCDS term structure are also based on bid-side quotes).

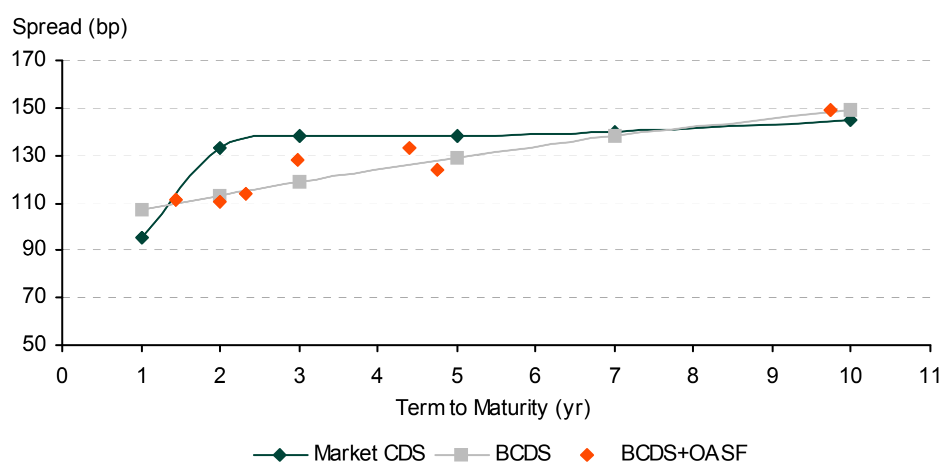
On a bond-by-bond comparison the 5-year MO 5.625 of 2008 (P=104.12) has the largest negative OAS-to-Fit and is therefore the richest cash bond among those included in this analysis. However, when considering the combined BCDS+OASF measure, the largest total basis is exhibited by the 2-year bonds MO 6.375 of 2006 (P=106.07) and MO 6.95 of 2006 (P=107.77). Note that all these bonds trade at a relatively modest premium, and therefore the systematic bond basis approximately coincides with the matching maturity curve basis.

Finally, we would like to note that comparing the BCDS and market CDS term structures may be a valuable tool not only for basis trading but for curve trading as well. For example, by looking at Figure 4, an investor may conclude that the CDS curve is too flat compared with the cash market-based BCDS curve. Depending on the investor's directional views regarding Altria, this opinion could be implemented in either bullish or bearish fashion.

A bearish curve steepener would be to sell 2-year protection and buy 7-year protection in equal notional amounts. Such a trade has an overall negative Credit01, and therefore will produce positive returns if spreads widen and/or steepen. A bullish curve steepener would be to sell 2-year protection and buy 7-year protection in amounts which make the trade Credit01-neutral. Then the overall spread moves will not affect the trade much, while the excess tightening in the front end as the spreads rally will produce positive returns.

We conclude that the newly introduced BCDS curve basis and bond-specific basis relative measures should be useful to credit investors in a variety of relative value strategies.

Figure 4. Curve and bond basis in Altria Group (as of 2/06/2004)



APPENDIX: THE COMPLEMENTARITY BETWEEN THE FORWARD CDS AND RISK-FREE-EQUIVALENT COUPON TERM STRUCTURES

Assume that the underlying risk-free discount curve $r(t)$ (usually Libor) and the issuer's hazard rate term structure $h(t)$ are given. The forward discount function for the riskless rate is given by a well known formula, where $r(s)$ is the instantaneous forward rate:

$$[10] \qquad Z(t,T) = \exp\left(- \int_t^T r(s) \cdot ds \right)$$

The forward survival probability $Q(t,T)$ stands for the probability that the issuer will survive during the time period (t,T) provided that it survived until the beginning of that period. It is related to the hazard rate by a similar relationship[1]:

$$[11] \qquad Q(t,T) = \exp\left(- \int_t^T h(s) \cdot ds \right)$$

Consider a credit-risky bond with a given coupon C and final maturity T. The projected forward price of a fixed coupon bond depends on both of these term structures as well as the level of the coupon in the following manner (for simplicity of exposition we assume continuous compounding, but very similar results hold for quarterly, or semi-annual compounding conventions as well):

$$[12] \quad P(t,T) = C \cdot \int_t^T du \cdot e^{-\int_t^u (r(s)+h(s)) \cdot ds} + e^{-\int_t^T (r(s)+h(s)) \cdot ds} + R \cdot \int_t^T du \cdot h(u) \cdot e^{-\int_t^u (r(s)+h(s)) \cdot ds}$$

The first term reflects the present value of the coupon stream under the condition that the bond survived until some intermediate time t, the second term reflects the present value of the final principal payment under the condition that the bond survived until the final maturity T, the third term reflects the recovery of the fraction R of the face value if the issuer defaults at any time between the valuation time and the final maturity.

Let us define the "risk-free-equivalent coupon" stream $RFC(t,T)$ which would reproduce the same forward price term structure but only when discounted with the risk-free discount function, without any default probability. Such a coupon stream will not be constant in general and will have a non-trivial term structure, depending on both the underlying risk-free rates and, through the price of the risky bond, on the issuer hazard rates as well. The defining condition is:

$$[13] \qquad P(t,T) = \int_t^T du \cdot RFC(u,T) \cdot e^{-\int_t^u r(s) \cdot ds} + e^{-\int_t^T r(s) \cdot ds}$$

The concept of a risk-free equivalent coupon stream is necessary for consistent definition of the difference between the default-risky and risk-free bonds when the underlying interest and hazard rates have non-trivial term structures and the bonds are expected to deviate from par pricing either currently or at any time in the future.

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1 *The hazard rate and interest rate can, in general, be stochastic. In this discussion, we restrict our attention to deterministic default intensities and deterministic interest rates.*

To find the relationship between the risk-free-equivalent coupon stream $RFC(t,T)$ and the forward price $P(t,T)$ of the credit-risky bond, let us take a derivative with respect to the valuation time t of both sides of equations [12] and [13]. On one hand we get:

[14] $$\frac{\partial P(t,T)}{\partial t} = \left(r(t) + h(t)\right) \cdot P(t,T) - C - R \cdot h(t)$$

On the other hand, we get:

[15] $$\frac{\partial P(t,T)}{\partial t} = r(t) \cdot P(t,T) - RFC(t,T)$$

Since the left-hand sides are equal by construction, we can equate the right-hand sides and obtain the relationship between the risk-free-equivalent coupon stream and the forward price:

[16] $$C - RFC(t,T) = h(t) \cdot \left(P(t,T) - R\right)$$

Consider now a forward CDS contract for a short time period $(t, t+\Delta t)$. This is a contract which provides protection during the future time period in exchange for premiums which are paid during that period but whose level is preset today. In case of a credit event prior to starting date t, the forward CDS knocks out and provides no further protection during the future period. As explained in Berd (2003), the forward CDS spread is determined by:

[17] $$CDS_{fwd}(t,T) \cdot \int_t^T du \cdot e^{-\int_t^u (r(s)+h(s)) \cdot ds} = (1-R) \cdot \int_t^T du \cdot h(u) \cdot e^{-\int_t^u (r(s)+h(s)) \cdot ds}$$

It is easy to see that for a small future time interval, the forward spread is simply proportional to the hazard rate of the matching horizon:

[18] $$CDS_{fwd}(t, t+\Delta t) = (1-R) \cdot h(t)$$

Substituting this definition into equation [16], we get a complementarity condition between the risk-free-equivalent coupon streams and the forward CDS spreads:

[19] $$C - CDS_{fwd}(t) \cdot N(t,T) = RFC(t,T), \text{ where } N(t,T) = \frac{P(t,T) - R}{1-R}$$

This relationship confirms our intuition about the consistent hedging strategy for non-par credit-risky bonds which consists of a stream of forward CDS with notionals $N(t,T)$ depending on the forward price of the bond. The residual cashflows of the credit-risky bond after paying the required premiums coincide with the projected risk-free-equivalent coupon stream. Although there is still a timing risk associated with this hedging strategy, the notionals of the hedges are such that the recovered value will be equal to the correct forward price of the bond, and therefore the timing risk is unimportant when evaluating the present value of the hedged cashflows to the present time or to any future time before maturity. This is reflected in the fact that discounting these residual cashflows with riskless rates gives the correct forward prices of the bond (compare [13] and [19]).

Also note that the hedge notionals depend on the recovery rate both explicitly and implicitly, via the dependence of the implied hazard rates on the recovery rate. Therefore a credit bond hedged according to this strategy still contains recovery risk (see Berd and Kapoor [2002], and O'Kane and Turnbull [2003] for estimates of recovery dependence).

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