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QUANTITATIVE MANAGEMENT OF GLOBAL BOND PORTFOLIOS

- We discuss major secular factors contributing to the gradual transformation over the last decade of global portfolio management from multi-currency Treasury investing to the inclusion of spread products. There has been a corresponding migration of mandates from the Global Treasury to the Global Aggregate Index.
- We discuss plans for a global risk model as we present an overview of our revised U.S. multi-factor risk model and the first release of a risk model for Euro-denominated assets.
- A detailed overview of methodologies for replication of the Global Aggregate Index and its major components shows the Index to be quite replicable with just a limited number of securities or even with derivatives. A simple methodology for MBS index replication allows investors to avoid both the technical details and back office adjustments of this asset class.
- We discuss a possible solution to translation of an incomplete market outlook into an optimal asset allocation in a global portfolio by combining it with a condition of market equilibrium.

Introduction

Over the last few years a major transformation in the composition of global portfolios got underway from multi-currency Treasuries to a broad acceptance of credit, MBS and other collateralized products. In some ways, it is moving down the same path as the U.S. domestic portfolio management did in the last 25 years: from U.S. Treasury Indices to Government/Corporate to Aggregate, which include a large MBS component.

Some of the reasons for this global transformation are the same as they were in the U.S.: diversification of risk and reduction in return volatility, superior performance over long periods of time in line with the extra spread, superior Sharpe ratios or risk adjusted performance. Historical evidence supports a case for a Global Aggregate Index versus both the domestic indices (U.S. or Euro) and the Global Treasury Index by any of the above measures.[1]

Another important advantage of a macro index over its component indices is the opportunity for diversification of style for its outperformance. Our research shows[2] that the information ratio achieved by a portfolio manager at a given level of market timing skills depends on the number of independent decisions made. The Global Aggregate portfolio offers the widest possible variety of choices for outperformance in an investment grade product.

Finally, an interesting property of the Global Aggregate Index is revealed by comparison of the market capitalization weight of its components to the historically optimal weight. It turns out that its allocation to MBS and JGBs, assets with the lowest correlation to the

[1] *A Case for Credit in Global Benchmarks.* In Lehman Brothers GRV, July 2001.
[2] Value of Security Selection versus Asset Allocation in Credit Markets: Part II – An "Imperfect Foresight" Study. *Journal of Portfolio Management,* Vol. 27, No. 1, Fall 2000.

rest of the index, is constructive in terms of risk minimization for the whole index. It is sufficient to make a difference in the overall volatility of the index returns and not excessive enough to cause a destabilizing effect.

From a Global Treasury Portfolio to a Global Aggregate

All of the above reasons for the transformation to the Global Aggregate Index are typical of any migration from a sector index to a macro index. Historical data would provide similar supporting evidence for moving from Treasury indices in the U.S. to the U.S. Aggregate Index. However, the past decade witnessed several major events of particular significance to global portfolio managers, which led them to re-examine their investment strategies. Some are common to all investors, others are of particular importance to a certain group, such as official institutions and Central Banks.

First, a global Treasury portfolio was subject to the secular shrinkage in the supply of U.S. Treasuries. Since the early 1990s the reversal in the trend of supply of U.S. Treasuries has been truly remarkable, both in terms of magnitude and suddenness. As a percentage of the Lehman U.S. Aggregate Index, Treasuries now (December 2001) comprise only 22% of the Index, compared to 46% at the beginning of the 1990s. In contrast, the share of credit assets in the Index has increased from 19% to 27% and MBS from 26% to 35%. The large shift in the relative supply of Treasuries and spread product has contributed to the re-pricing of spread assets. The spread to Treasuries of 10-year AA-rated industrial corporate bonds rose steadily since 1996-1997 after it became apparent that absolute Treasury supply was declining. Recently, the supply of U.S. Treasuries has begun to increase somewhat in response to the U.S. slowdown and tax cuts. However, future supply of U.S. Treasuries is expected to increase modestly, at best, while the supply of other fixed-income assets continues to grow.

The prolonged economic slowdown in Japan resulted in an increased share of low-yielding JGB securities in the global Treasury market (from 25% in December 1991 to 31% in December 2001) at the expense of higher yielding U.S. Treasury bonds (from 41% in December 1991 to 23% in December 2001). Inclusion of spread products in a global aggregate portfolio tilts the market weights back in favor of U.S. dollar denominated securities (as of December 2001 the Lehman Global Aggregate Index had 22% of its market value in Yen-denominated securities versus 46% in denominated in U.S. dollar).

A major event of the past decade was the introduction of the Euro, which led to a reduction in diversification opportunities and choice of out-performance styles available to a manager of a global Treasury portfolio. Maximizing information ratio (out-performance per unit of tracking error) requires diversification of risk and styles of portfolio management, both of which can be achieved by the inclusion of credit and collateralized products in the portfolio.

The emergence and deepening of the Euro credit market has been another event that has caused global portfolio managers to re-examine their investment strategies. In the past, a sizable credit investment was only possible in U.S. dollars. This limitation was a major disincentive for investors to expend the time and resources to develop credit expertise. Today, there exists a thriving Euro credit market that is expected to grow significantly in the future, further increasing the potential rewards of credit analysis.

Another powerful but frequently overlooked factor in the transition from a global Treasury to a global Aggregate portfolio is the transformation in the composition of Central Bank reserves. Over the next decade it could have a significant influence on the behavior of spread products. We would like to give it some extra attention.

New Developments in the Management of Central Bank Reserves

Perhaps the most startling event in Central Bank reserve management has been the shrinkage in the supply of U.S. Treasuries, which historically have comprised a substantial percentage of dollar reserves held by non-U.S. official institutions. Non-U.S. official institutions' holdings of marketable U.S. Government debt comprised about 20% of outstanding marketable supply.[3]

National reserves managers have responded to the reduction in Treasury supply and the increased relative attractiveness of spread product. Many official institutions expanded their investments in other asset classes that had long been acceptable such as Aaa-rated sovereign and U.S. Agency debt. Others began exploring Aaa-rated asset backed securities. These particular assets have several attributes that make them a reasonable substitute for a portion of official U.S. Treasury holdings. First, they are all Aaa-rated, which allows institutions to add these assets without having to staff-up a large credit analysis operation. Second, these assets involve very little "headline" or "C-1" risk.[4]

However, for official institutions heretofore reluctant to hold spread assets, the sharp decline in the supply of U.S. Treasuries raises the real possibility of having to invest a substantial portion of their dollar holdings in U.S. credit assets rated lower than Aaa and involving "headline" risk. This adds to the demand on the part of central banks for diversification and risk management tools.

The second major event of the past decade to influence reserve management practices was the emergence of the Euro. The ECB can meet the liquidity requirements on behalf of all members with a much smaller amount of reserves freeing up a significant amount of NCB reserves from the need to provide immediate liquidity. The ECB has given the NCBs the opportunity to manage their reserves with more of a total return objective.

A related event during the 1990s was a reconsideration of the efficacy of foreign exchange intervention on a massive scale. Many central banks no longer manage their reserves with

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[3] Federal Reserve Board of Governors, *Flow of Funds Accounts of the United States,* Z.1 Release, 8 June 2001 and U.S. Treasury *Monthly Statement of the Public Debt of the United States,* July 2001.
[4] "C-1" risk refers to an event that figures prominently on the front page of the *Money & Investments* section of the *Wall Street Journal.*

the expectation that they would have to liquidate them in a single day. Instead, they can credibly and effectively defend their currencies by demonstrating a commitment to liquidate the reserves as needed over time. This change in intervention thinking also allows central banks to manage their dollar reserves with more of a total return and less of a liquidity objective.

A more recent event is the willingness of central banks to gradually sell portions of their gold holdings. It is reasonable to expect that some proceeds from gold sales will be re-deployed in higher yielding fixed-income assets.

Implications for Asset Managers

The inevitable inclusion of spread products into global portfolios gives managers unsurpassed freedom of choice for outperformance but also raises a number of important concerns.

- With so many dimensions of portfolio risk at play, how can we quantify the risk of deviation from the market and ascribe it to specific portfolio exposures (currency, curve, credit, prepayments, issuer concentration, etc)?
- Are global indices replicable? Does presence of a wide variety of currencies and thousands of securities make it impossible to follow them and outperform?
- Given the significant weight of U.S. MBS securities and their highly technical nature, should a global portfolio manager always look to have them managed externally? Is there a low cost, low barrier to entry alternative for replicating the MBS market?
- The traditional mean-variance framework of optimal asset allocation requires return forecasts for all assets in question. It is nearly impossible to furnish them for a universe as diverse as the Global Aggregate Index. How can views on some markets be combined with equilibrium assumptions on other markets?

This report is structured to give an overview of the solutions to the above questions, with most of the details of the models and empirical studies left outside of this discussion. We will be referring to corresponding subject papers where appropriate.

MEASURING RISK IN A GLOBAL PORTFOLIO

The global risk modeling project is being executed in two stages. In the first stage, which is now substantially complete, we have focused on developing a multi-factor risk model for Euro-denominated bond portfolios and a new generation of the risk model for dollar-denominated portfolios. In this way, we will have covered the two most important currencies in any global fixed income portfolio. The second and final stage is to build a risk model that will cover portfolios and benchmarks that may include bonds denominated in all major currencies in the Global Aggregate Index. The ultimate objective of this project is to produce a risk model that will account for currency risk, multi-country yield curve risk and spread risk in various sectors of global credit markets. In addition, we aim to account for issuer-level risk, which may depend on such characteristics as the size and the country of origin of an issuer.

The New Risk Model for the U.S. Aggregate Index

During the last decade the Lehman Brothers Risk Model[5] has served as an effective risk management and portfolio structuring tool for a large number of portfolio managers. The common practice of constraining asset allocation choices when managing against a predetermined benchmark has created a need for quantitative measurements of *relative risk*. As a result, the Risk Model's coverage of a variety of asset classes and its ability to measure the rate of return deviation between a portfolio and a benchmark (tracking error) have made it a valuable tool for managers who need to track a highly diversified index.

This note briefly presents the framework underlying the New Lehman Brothers Risk Model for the U.S. Aggregate Index. Relative to the previous version of the model, we have introduced a new way of splitting bond returns, a different way of measuring interest rate and volatility risk, and different models for spread and non-systematic risk. Moreover, robust econometric techniques have been introduced for parameter estimation. What we have retained is the flexible and computationally convenient framework of a linear factor model, as well as the calibration methodology based on cross-sectional regression analysis.

The new Risk Model will be delivered *via* POINT, Lehman Brothers' new portfolio analysis platform. Users will significantly benefit from the integration of a modern portfolio system and a state-of-the-art risk management tool.

Applications

The Risk Model is designed as a complement to fundamental analysis, in that it allows for an objective quantification of the risks associated with active positions taken relative to a benchmark. Beyond providing the user with the Tracking Error ("TE"), a very intuitive measure of statistical distance between the return of a portfolio and a pre-assigned benchmark, the model lends itself to a number of important applications:

- Understanding detailed factor exposures for a given portfolio;
- Comparing risks associated with different views;
- Identifying trades to achieve a reduction in deviation risk;
- Creating index proxies using a relatively low number of assets;
- Decomposing total risk, *e.g.*, across asset classes, thus allowing for risk budgeting at different levels; and
- Decomposing systematic and non-systematic risk.

Modeling Framework

Our methodology relies on an integrated framework for return attribution and risk modeling for assets in the Lehman U.S. Aggregate Index. The return attribution framework decomposes a security's total return into well-defined components. The risk model quantifies the statistical behavior of each of these component sources of return. The new Risk Model improves upon the existing tool in a number of ways:

[5] *The Lehman Brothers Multi-Factor Risk Model, Lehman Brothers*, July 1999.

- Our framework can be applied to any fixed income security for which a pricing model is available. So far, we have completed the models for Treasuries, Agencies, Corporates and MBS.
- A fully integrated approach to return attribution and risk modeling at both the security and portfolio level. The return attribution splits returns into logical components. The risk model then characterizes the statistical behavior of those return components.
- The benchmark risk is redefined in an option adjusted spread (OAS)-based setting. Callable securities (such as callable credits, MBS, etc.) are handled in the same way as non-callable securities. We have eliminated the use of static measures that ignore embedded options (such as "yield to worst" or "ZV duration"). We allow for non-parallel changes in the Treasury yield curve by using 6 key-rate durations, and use option adjusted convexity as an additional risk loading.
- Volatility risk in an OAS-based framework for all volatility-sensitive securities. This risk has taken on increasing importance as the cash and derivative markets become closely integrated. We estimate a set of volatility risk factors that characterize changes in implied volatilities.
- Improved approach to handling spread risk across all asset classes. The spread factors are estimated using a robust statistical procedure that minimizes the impact of outliers. These estimated factors are easily interpretable and explain a significant fraction of each security's spread return.

Yield Curve Risk

We use the Treasury yield curve as our reference curve for interest rates. The yield curve return for a given bond is defined as that portion of its total return that results solely from a change in the yield curve over the month. It is computed by re-pricing the bond using the end of month yield curve, holding all other variables (*e.g.,* time, volatility, and OAS) fixed. The yield curve movement is measured using the Lehman Brothers Treasury Spline curve, which is fitted to the prices of "off-the-run" Treasury securities.

Yield curve return is modeled as a linear combination of 6 key-rate durations (KRDs) multiplied by their respective par yield changes, plus the impact of convexity. Our empirical analysis has shown that the yield curve return is well approximated by KRDs defined at the following maturities: 6-month, 2-year, 5-year, 10-year, 20-year and 30-year.

Volatility Risk

The volatility return for a bond with embedded options is that portion of total return that results solely from a change in the set of implied volatilities over the month.

We model volatility return separately for each asset class. Treasuries, Agencies and Corporates each have one volatility factor, which approximates the parallel movement of the implied volatility surface. The time series of the volatility factors are extracted by running cross-sectional regressions of volatility returns on *vegas* for all bonds with embedded options.

For MBS, the volatility model is slightly more complex, in that we use three unobservable factors, which allows us to capture the non-parallel nature of changes in the implied volatility surface. The yield curve model used for MBS does not assume that rates have a lognormal distribution. As a result, the volatility return can be negative even when Black volatilities drop over the month – as in fact happened over the month ending November 30, 2001. This occurred because model Black volatilities declined as rates increased in the Lehman Brothers MBS Yield Curve Model. Our volatility return includes the impact of changes in volatility on the mortgage rate.

Spread Risk

The spread return is the portion of total return remaining after time, yield curve, and volatility returns have been removed. It is similar to the product of the security's spread duration and its OAS change over the month.

The spread risk models for Treasuries, Agencies and Corporates are conceptually similar. We model systematic spread risk for Treasuries because the reference interest rate curve, as we explained above, is an off-the-run fitted Treasury curve, so that even the prices of Treasury bonds can systematically deviate from the levels implied by that curve.

We partition Agencies into five groups according to issuer, and Corporates into 27 groups formed by intersecting nine industries and three quality buckets (Aa, A, Baa). Each of these groups has its own factor, estimated by cross-sectional regressions of spread returns on spread durations. Given this model specification, the estimated factor series for a given group can be conveniently interpreted as a series of average spread changes within that group. Consistent with investors' intuition, spread factor volatilities are higher as we walk down the credit spectrum. Factors also display correlation patterns that are consistent with previous empirical studies. For example, we observe significant negative correlations between the front-end of the yield curve (the short key rates) and most of the credit spreads.

The models for spread risk also employ a liquidity factor. The sensitivity of a bond to this factor is given by the difference between its OAS and the average OAS of all bonds from the same group. Insofar as the groups we have identified are homogeneous in terms of credit risk, differences in OASs within a group should be related mostly to liquidity. Consistent with this interpretation, we have found that the addition of this liquidity factor captures the return differences between on- and off-the-run Treasury bonds.

For MBS, we model spread returns using 10 unobserved factors that take into account price tier, loan age, guarantee type (i.e., conventional vs. GNMA), and term (30-year, 15-year, and balloon). Adjusting for spread durations, we find that premiums have the highest volatility, followed by discounts and then current coupons, and that seasoned cohorts have higher volatility than unseasoned cohorts.

Non-Systematic Risk

While a factor model captures the statistical properties of the common sources of risk in the market, each asset is also subject to idiosyncratic price changes. Generally speaking, the idiosyncratic portion of risk in a portfolio decreases as the number of assets held increases, something we usually refer to as "the benefits of diversification".

Similarly, the portion of the tracking error due to non-systematic risk becomes negligible when both the investment portfolio and the benchmark are each highly diversified. While this is often true for the benchmark, this is usually not the case for an investor's portfolio. Transaction costs and other constraints generally force asset managers to limit the number of bonds in their portfolios. This implies that tracking error due to idiosyncratic risk may well be a non-negligible number, and a satisfactory measurement tool should capture this dimension of relative risk as well.

To estimate the idiosyncratic portion of the TE between a portfolio and a benchmark, we need to model non-systematic risk. The Risk Model employs the squared residuals from the factor regressions as left-hand-side variables and explains idiosyncratic variances using several bond-level characteristics such as rating, age of the bond and, for securities with embedded options, *vega*.

Putting it All Together

Using the estimated factor series, we compute factor volatilities and correlations. Adding the estimated idiosyncratic variances, we can obtain the TE for any portfolio-benchmark pair.

The Risk Model reports offer the user a number of risk decompositions. The systematic TE is decomposed by subsets of factors (yield curve risk, credit spread risk, etc.), and both isolated and cumulative decompositions are reported. The elements of the isolated decomposition will not add up to the total systematic TE because our factors are not restricted to be orthogonal. By observing how cumulative TE changes as we incrementally add groups of factors, we can spot the effects that factor correlations have on relative risk.

Conclusion

The New Lehman Brothers Risk Model identifies a set of underlying factors in the fixed-income markets and uses their statistical properties to objectively measure portfolio risk. It is an ideal complement to fundamental analysis since it can assign a risk number to any set of market views. By detailing the exposures to the driving risk factors, it substantially simplifies the otherwise complex task of identifying rebalancing strategies to achieve a desired risk exposure.

Portfolio risk is complex and often unintuitive, due to the interplay of many underlying forces. For this reason, benchmarked investors should analyze their positions within a structured framework: the new Lehman Brothers U.S. Risk Model provides a unique analytical tool for the objective measurement and decomposition of deviation risk.

The New Risk Model for the Euro Aggregate Index

During 2001, we introduced the Lehman Brothers Euro Risk Model. For the first time, our portfolio analytics platform can calculate tracking errors for non-dollar portfolios. The model applies the multi-factor approach to the analysis of risk for the Lehman Brothers Euro-Aggregate Index, and essentially covers all Euro-denominated fixed-rate investment-grade debt.

The Euro Risk Model leverages our experience in development and application of the U.S. Risk Model, which has been in use for more than 10 years. It takes a long-term view of volatility calibration. This feature helped make the model stable even in highly volatile market conditions such as those witnessed in the financial crisis of Autumn 1998. It uses key rate durations to measure term structure exposures, as does the new U.S. Risk Model described above. To help compensate for the relatively small amount of historical data available for Euro-denominated spread product, the model has been calibrated using a judicious combination of historical data from the Euro and USD credit markets.

Yet for all the benefits of our U.S. experience, the model has a decidedly European flavor, with several features specific to the Euro market. To reflect the fact that multiple countries issue treasury debt in a single currency, the model includes risk factors for spread changes in different categories of treasury securities. As the swap curve plays a critical role in the pricing of many sectors of European credit markets (notably in the collateralized sector), there are risk factors representing changes in Euro swap spreads of different maturities. In addition, the sector/quality partition used to define risk factors for changes in spreads (over swaps) was specifically based on the structure of the Euro credit markets.

Like its U.S. counterpart, the Euro Risk Model measures both systematic and non-systematic risk. Systematic risk is based on exposures to common risk factors that drive bond returns across the market. Non-systematic risk is the volatility of returns that are specific to a particular issue or issuer. Both types of risk contribute to the total return volatility of a portfolio or benchmark and to the tracking error between them (the volatility of the return difference).

Systematic Risk

The Euro Risk Model contains 34 systematic risk factors, covering changes to interest rates, swap spreads and sector spreads over LIBOR. These factors are summarized in Figure 1. For each category of risk factor, the appropriate security attributes are used to estimate the sensitivities of individual bonds to each risk factor. These factor exposures can then be for a portfolio or an index. A covariance matrix summarizes the historical volatilities and correlations of historical changes in the systematic risk factors. The set of factor exposures, together with the covariance matrix, is used to compute systematic risk.

All bonds are exposed to the first three sets of risk factors, i.e., key rates, convexity and swap spreads. In addition, bonds are exposed to the risk factor capturing sector-level LIBOR spread changes. For this, the universe of Euro-denominated bonds is classified into the 24 sectors shown in Figure 2. Every bond is exposed only to the spread factor of the sector to which it belongs.

For the Aaa-treasury component of the market, Germany, France, Netherlands and Spain are classified as large issuers. Government bonds issued by these countries are bucketed by maturity. Of the three buckets, spread factors are included in the risk model for the latter two. This spread accounts for liquidity or “specialness premia” that are embedded in longer-dated treasury bonds in the Eurozone. (No such factor is included for the shortest

Figure 1. **Summary of Systematic Risk Factors and Exposures**

Category	Risk Factors	Factor Exposures
Yield curve: key rates	Five points along the Aaa Euro Treasury Curve (6mo, 2yr, 5yr, 10yr, and 30yr)	Key rate durations
Yield curve: convexity	Squared average change in the five key rates	Convexity
Spreads: swap spreads	Five points on the swap spread curve (2yr, 5yr, 7yr, 10yr, and 30yr)	Swap spread durations
Spreads: sector spreads (over LIBOR)	Average spreads over LIBOR for each of 23 market sectors (see Figure 2)	Option-adjusted spread duration

Figure 2. **Classification Scheme for Sector Spread Risk Factors**

Treasury	Agency and Local Govt.	Collateralized	Non-Corp. Credit	Corporate Credit
Aaa Large Issuers 8.5-10.5yr	Aaa	All	Aaa	Finance Aaa
Aaa Large Issuers 10.5+yr	Aa/A		Aa	Finance Aa
Aa/A Large Issuers 0-8.5yr			A	Finance A
Aa/A Large Issuers 8.5+yr			Baa	Utility Aaa/Aa
Aaa Small Issuers				Utility A
Aa/A Small Issuers				Industrial Aaa/Aa
				Industrial A
				Telecom Aaa/Aa
				Telecom A
				All Baa

bucket, which is assumed to follow the underlying yield curve with no additional spread.) For the Aa/A treasury component, large issuers included Italy and Spain up until December 2001. In December, the rating accorded to Spanish treasury bonds was upgraded to AAA and the large Aa/A bucket only includes Italy at the current time. All other treasuries fall under small issuers. The Agency and Local Government component of the universe is grouped by rating: Aaa and Aa/A. All collateralized bonds are grouped in one bucket. The non-corporate credit component is represented by four spread risk factors based on quality rating. Finally, the corporate credit component of the market is divided into ten sub-groups by rating and sector.

Non-systematic Risk

While the systematic risk factors capture all market-wide and sector-wide drivers of returns, they are not the only sources of risk fixed-income investors are faced with. A bond's return is also affected by factors specific to the issuer and at times by factors specific to a particular issue. The Euro Risk Model captures this effect by calculating the non-systematic tracking error obtained from the residual returns unexplained by the systematic risk factors. The total exposure of a portfolio or index to a particular issuer is given by the contribution to spread duration (percent of market value times spread duration). For the purpose of assigning non-systematic spread volatilities, the market is divided along a finer partition than that shown in Figure 2, with a total of 35 categories.

Using the Model

The primary application of the risk model is to analyze the risk of a portfolio relative to its benchmark. To accomplish this, the model computes the active portfolio exposure, the net difference between the risk factor exposures of the portfolio and the benchmark. The active portfolio exposure is multiplied by the covariance matrix to obtain the systematic part of the projected tracking error. The active issuer exposures are multiplied by the non-systematic spread volatilities to compute the non-systematic component of tracking error. Active portfolio exposures to non-systematic risk generally stem from large issuer concentrations, and can be decreased by diversification.

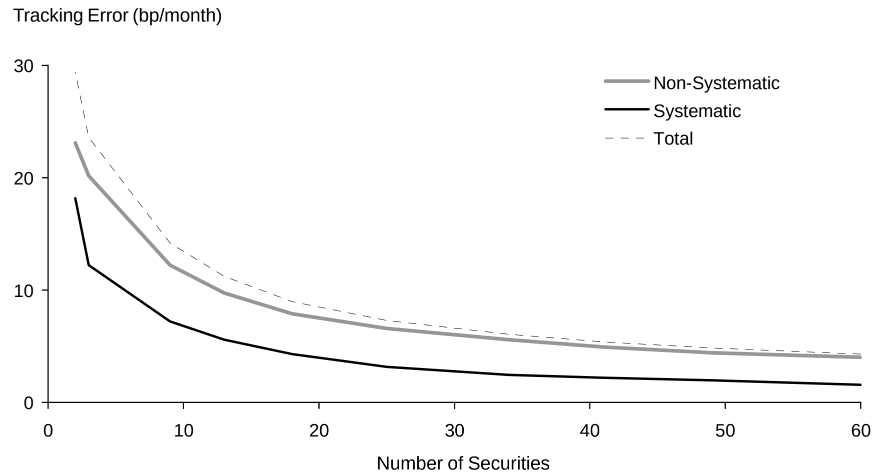
The model is currently implemented on two portfolio analytics platforms, POINT and PointWeb, described in more detail in a later section. The model projects the total return volatilities of a portfolio and its benchmark, as well as the tracking error between them. All of these quantities can be broken down into components due to curve risk, swap spread risk, sector spread risk, and nonsystematic risk. A comprehensive set of supporting reports traces the systematic risk to factor exposures and volatilities, and a display of the largest 20 active issuer exposures shows the source of non-systematic tracking error.

We have also constructed an optimizer that uses the risk model to minimize tracking error subject to constraints. This can help carry out many types of portfolio optimization tasks. For instance, we can minimize tracking error subject to a minimum required portfolio expected return or OAS. By varying the minimum required return, we can build an efficient frontier.

The optimizer can also be used for index replication. Unconstrained minimization of tracking error gives the best tracking portfolio that can be constructed from a given set of securities. Investors often seek a portfolio of relatively few securities that can closely track the performance of an index. Figure 3 shows the optimal tracking error as a function of the number of securities. It demonstrates that a portfolio of just 50 securities can replicate the returns of the Euro-Aggregate Index with a tracking error of just 5 basis points per month.

The Euro Risk Model is a single-currency model, and can be used to analyze the risk of Euro-denominated portfolios and benchmarks. However, the use of our U.S. and Euro risk models in tandem on the respective portions of a global portfolio provides risk coverage for more than two thirds of the Global Aggregate Index by market value. Since these models are based on similar yield curve models in the two currencies, we are well on the way towards our next goal of providing a global risk model.

Figure 3. **Tracking Errors of Optimal Index Replication Portfolios vs. the Lehman Brothers Euro-Aggregate Index**



REPLICATING THE GLOBAL AGGREGATE INDEX AND ITS COMPONENTS

With an index as broad as the Global Aggregate, one might expect the replication of index returns to be a difficult, or nearly impossible, task. Ironically, however, it is the diversity of the index that makes it surprisingly easy to replicate. Simple replication schemes for each of the major component markets, even if tracking errors are relatively large within each market, can be combined to form a global aggregate portfolio with quite low tracking errors. This is because the replication errors in the different markets are not correlated with each other, which allows their contribution to overall tracking error to be reduced quite nicely by diversification.

This diversification effect can be seen in our studies of two methods for replicating the returns of the Global Aggregate Index, either with cash instruments or derivatives. For example, by merely matching index exposures to the four largest currencies and their yield curves using the Treasury futures markets, one can track the index quite well (average tracking error of 20.8 basis points per month).[6] By using non-Treasury instruments (e.g., swaps) to track the spread components of the Global Aggregate Index, one can obtain increasingly better index replication.

One of the most effective tools to construct replicating proxy portfolios is to use an optimizer based on a risk model. As illustrated in the previous section, tracking error minimization can find the portfolio that best matches all of the systematic risk exposures

[6] For details please refer to *The Lehman Global Aggregate Index: Replication with Derivatives and Cash Instruments,* forthcoming.

of an index while taking the least amount of security-specific risk as well. Eventually, with the implementation of a global risk model, tracking error minimization may provide a comprehensive solution for index replication with cash instruments. At present, we have risk model implementations only for the U.S. and Euro-denominated segments of the Global Aggregate Index. The respective risk models can be used to help design replicating portfolios for these markets, which constitute a majority of the index. Other methods (such as the JGB index replication described below) will need to be used to complete the coverage of the index.

Fortunately, formal tracking error minimization using a risk model is not the only road to index replication. Another widely used technique is stratified sampling, in which an index is divided into cells, and a small number of securities is used to replicate the returns of each cell. We have successfully used variations on this approach to replicate many different indices. The Global Aggregate and its components are no exception.

Many investors may be interested in passive replication of just a part of the Global Aggregate Index. For instance, investors not familiar with the U.S. mortgage market may be concerned about the unique challenges posed by this market. A typical strategy for outperformance is to take active positions in market segments where an investor has the greatest expertise, while using passive replication techniques to reduce risk in other market segments.

In this section, we explore many aspects of index replication, for both the Global Aggregate Index as a whole and for its various component markets, using both cash instruments and derivatives.

Replication of the MBS Component with TBAs

Mortgage securities comprise a significant portion of the Lehman U.S. Aggregate Index (35%) and the Lehman Global Aggregate Index (17%). Investors wishing to track these indices must take exposure to the U.S. mortgage market. However, some investors are reluctant to do so due to its technical nature. We designed two MBS Index replication strategies that invest in highly tradable and liquid MBS securities. Both are relatively simple in design and easy to implement and maintain. Most important, neither strategy requires the investor to have detailed knowledge of the U.S. mortgage market.

Unlike most bond indices, the MBS index contains only non-traded “generic” securities. As a result, the replication process has to start with forming the tracking proxy portfolio out of these generics and then proceed to select actual mortgage pools for each generic. Yet there is no assurance that a particular pool will perform identically to its generic. This added layer of decision-making has a potential for significant additional tracking error.

The replication of the MBS Index by stratified sampling would direct investors to buy certain amounts of seasoned product at the outset of the portfolio. The obvious benefit is the close replication of the benchmark from the very beginning. However, seasoned pools may be difficult to obtain. More importantly, investors buying seasoned pools might be delivered small pools that create a real possibility for the added tracking error. Ensuring that all seasoned holdings are implemented with large pools is difficult, if not impossible, in practice. Finally, MBS replication by sampling requires the investor to have full back-office capabilities for MBS transaction processing from day one.

To address this problem we conducted a study[7] that resulted in two practical strategies for replicating the MBS index with liquid instruments. These strategies deal with the issue of pool-specific risk and let the investor build MBS transaction processing capabilities gradually while being fully invested in the market. For both strategies, the mechanics of constructing proxy portfolios are the same. The two replication strategies differ in terms of their definition of the "available set." The first strategy ("TBAs-only") uses only TBA contracts on actively traded, recently originated mortgage coupons. As the composition of the new-issue mortgage market changes over time, this strategy adjusts its holdings of TBA contracts so as to always reflect the most recent and active portion of the mortgage market. The second strategy ("large pools-only") buys large MBS pools of current production but allows some holdings to remain in the portfolio and season over time. In both cases, the resulting proxy portfolio is rebalanced quarterly.

Figure 4 presents performance results for both replication strategies. One potential drawback of the large pools-only strategy is the steadily increasing number of pools in the proxy portfolio. However, by limiting the number of issues to roughly match that of the TBAs-only strategy we show that the improved tracking of the large pools strategy is due primarily to the presence of seasoned issues in the proxy and not to a larger number of securities.

Both strategies provide investors with an effective way to replicate the MBS index without detailed pool-level knowledge of the mortgage market. The relative simplicity of the two strategies may encourage some investors to attempt the MBS index replication on their own.

[7] *Tradable Proxy Portfolios for the Lehman MBS Index*, Lehman Brothers, May 2001.

Figure 4. **Performance Summary for the MBS Index Replication Strategies**, January 1994-May 2001, bp

	TBAs-Only	Large Pools-Only	Large Pools Only (12 Issues Max)
Monthly return difference (portfolio vs. the index)			
Average	-1.2	0.2	0.6
Standard deviation (realized tracking error)	6.0	4.4	4.4
Minimum	-18.7	-9.2	-9.2
Maximum	17.6	23.6	24.4
Realized tracking error (annualized):	20.9	15.2	15.2

Replication of the JGB Index with Cash Securities

Japanese Government Bonds (JGBs) currently constitute about 77% of the market value of the Lehman Brothers' Japan Aggregate Index. This means that the JGB Index is a close substitute for the Japan Aggregate Index. Deteriorating fiscal conditions in Japan are likely to force the Japanese government to continue its massive debt financing program, leading to an even higher proportion of JGBs in the Japan Aggregate Index. The rest of the Japan Aggregate is evenly divided between municipal and corporate bonds. However, recent downgrades of the Japanese sovereign rating by three major international rating agencies also dilute the credit spread advantage that conventional credit indices command over their sovereign counterpart. These facts suggest strong interest from investors in understanding how to replicate the JGB Index effectively. We show how a carefully designed bucketing/cell strategy can be employed to achieve the JGB Index replication with minimal tracking error.

JGB Index Composition and Market Segmentation

One of the challenges in replicating the JGB Index can be attributed to market segmentation. Specifically, market players trade JGBs according to their maturity at issuance. For example, bonds issued with an original maturity of 20 years ("old 20s"), known as super-longs, are usually traded as a group distinctly different from their 10-year counterpart ("old 10s"). Currently, JGBs are being issued with maturities of 2, 5, 10, 20, and 30 years. Previously, 4-year and 6-year JGBs were issued as well, but they were discontinued in 2001. Figure 5 shows the distribution of outstanding JGBs by maturity at issuance.

Clearly, 10-year JGBs represent the largest of all maturity categories, and the old 10s, though issued many years ago, are still the most liquid group. Since the Bank of Japan adopted the zero interest rate policy in March 2001, short-dated bonds are increasingly becoming popular for domestic investors because of their attractive carry income and limited downside risk. This has led to improved buying interest and market liquidity for the 5-year JGBs (old 10s as well as new 5s). On the other hand, given its super low yields, 2-year JGBs are considered to be more or less an investment tool for cash management.

Figure 5. **Shares of JGBs by Maturity at Issuance, as of 10/31/01**

Maturity at Issuance	Share of all JGBs
2	11.1%
4	4.0%
5	9.9%
6	5.9%
10	60.3%
20	8.3%
30	0.5%
Total	100%

Replication Methodology

Using stratified sampling, we break the JGB Index into six *remaining maturity* buckets or cells, while making an effort to accommodate the market segmentation issue described above. The six maturity buckets and the market value of the bonds in each bucket are reported in Figure 6.

In Figure 6, the first bucket represents JGBs that have 1-3.5 years remaining to maturity. Given the BOJ's zero interest rate policy, yields on these bonds are approximately 10 basis points. The second bucket (3.5-6 years) predominantly includes JGBs issued with original maturity of 4, 5, 6 and 10 years. The next bucket (6-8 years) includes JGBs most highly correlated with the 10-year JGB futures or the "cheapest-to-deliver" (CTD) sector. The liquidity for this sector is good. The fourth bucket (8-12 years) tracks bonds near the 10-year sector, which is also highly liquid. Since we have included the super-longs in this bucket, the return for the bucket is affected not only by the performance of the 10-year JGBs, but by the 20-year super-longs as well. The fifth bucket contains super-longs with 12-16 years to maturity, while the last bucket consists of bonds with maturities from 16 to 30 years. As 30-year JGBs have a different investor clientele compared to the 20-years, the two markets trade differently.

For each maturity bucket we pick two bonds in such a way that the duration of the two-bond portfolio matches the bucket's duration. Typically, we pick one bond with a duration less than the bucket duration and the other bond with a duration larger than the bucket duration. Moreover, we always try to pick bonds that have better liquidity relative to other bonds in the bucket. To take care of the market segmentation issue, we sometimes pick bonds across different maturity at issuance groups. Specifically, for the first bucket we pick old 10-year bonds, while for the second bucket we pick one bond from the old 10s and the other bond from the old 6s. For the third and fourth buckets we choose bonds from old 10s, realizing that potential tracking errors can exist due to the lack of super-longs in the replicating portfolio. The last two buckets are replicated only by 20-year super-longs. 30-year super-longs are never used because of small issue size and lower liquidity in this sector.

Figure 6. **Shares of JGBs by Remaining Maturity Bucket as of 10/31/01**

Remaining Maturity Bucket	Share of Total
1-3.5	26%
3.5-6	28%
6-8	21%
8-12	18%
12-16	2%
16-30	4%
Total	100%

Performance and Tracking Errors

Figure 7 summarizes the performance of this replication strategy for each bucket as well as for the entire replicating portfolio. Our replication experiment covers a period of 15 months, starting from July 2000 when Lehman started publishing the Lehman Brothers Japan Aggregate Index.

Overall, the performance results are very encouraging. For the replicating portfolio, the average monthly return difference between the replicating portfolio and the JGB Index is close to zero, while the monthly tracking error is 2.1 basis points. Our results show significant improvement in tracking performance relative to those reported in a recent study in which a similar replication was carried out *via* JGB futures and swaps.[8]

The worst performing bucket in Figure 7 is the sixth bucket, with a monthly tracking error of 19.6 basis points. Note that this bucket comprises bonds with maturities from 16 to 30 years. Since we are replicating this bucket with 20-year bonds only, and the returns on the 30-year bonds have lagged those of 20-years in our sample period, it is not too surprising to have a relatively large tracking error. However, the average monthly return difference for this bucket remains close to zero. Indeed, the average monthly return differences are satisfactory for all six buckets. Tracking errors for the second, fourth, and fifth buckets are around 5-6 basis points. These buckets are also somewhat affected by the market segmentation problem as in the case of the sixth bucket. Clearly, the replicating portfolio benefited significantly from diversification, registering a much smaller tracking error.

Replicating the Lehman Brothers JGB Index using a bucketing approach is promising. However, several practical issues remain. For example, transaction costs such as bid-offer spreads should be included in the tracking analysis. To the extent possible, we have set the holding period of each bond in the replicating portfolio to be at least three months so as to reduce transaction costs due to rebalancing. Market liquidity or market impact is also an important consideration for those pursuing an index replication strategy, as JGBs in certain sectors are almost impossible to trade. This is why it may be necessary to analyze tracking errors for a replicating strategy that is restricted to selecting only liquid bonds. Future research will address these practical issues.

[8] *Replication with Derivatives: The Global Aggregate Index and the Japanese Aggregate Index*, March 2001.

Figure 7. **Monthly Return Differences between the Replicating Portfolio and the JGB Index,** bp

	Bucket 1	2	3	4	5	6	Replication Strategy
Average	0.2	-0.9	-1.0	1.7	-0.2	0.9	-0.1
Standard Deviation	1.8	4.9	2.9	6.4	5.0	19.6	2.1

Replication of the Global Aggregate Index with Liquid Bonds ("Cash" Replication Strategies)

We now turn our attention to designing proxy portfolios for the Global Aggregate Index as a whole. We present three cash replication strategies that differ in the breadth of the investable universe.[9] The simplest strategy ("Treasury-only") is limited to treasury securities of the four major index currencies. This reduced set only allows replicating index allocations across currencies and term structure. Exposures to spread movements are ignored. The second strategy ("Treasury-plus") expands the available set to include agency and collateralized (except asset-backed) securities. Finally, the third strategy ("All-sectors") fully exploits advantages of bond replication by adding all major credit sectors in the index.

Replication Methodology

All three replication strategies rely on the stratified sampling technique, which is implemented as a linear optimization process that maximizes portfolio liquidity subject to a number of constraints. We consider minimizing illiquidity (represented by the illiquidity score explained below) a reasonable objective function, but in the case of constructing proxy portfolios, the constraints are the most important element. Besides the investable set, the choice of optimization constraints is the primary determinant of the strategy performance.

For each calendar month the optimizer selected bonds and their weights in the replicating portfolio, based on contemporaneous index characteristics. We then measured the portfolio's return for the month. The accumulated time series of monthly return differences between our replicating portfolio and the index were used to calculate the tracking errors.

As noted above, we limited the replicating portfolio to the four main currencies in the index: U.S. Dollar, Euro, Japanese Yen, and Pound Sterling. Together these four currencies account for over 95% of index market value. All remaining currencies are mapped to one of the four, which essentially assumes their perfect correlation with the corresponding large currency. Another way to distribute the remaining 5 percent was to allocate them proportionately to the four big pieces. The outcomes of the two methods were found to be practically equivalent.

All strategies match two index cell attributes: market value and contribution to duration. The sampling is done along five dimensions: term structure (26 buckets), quality rating (4 buckets), sector (7 buckets), currency bloc (4 buckets), and country bloc (4 buckets).[10] Constraints are also applied to combinations of these dimensions, such as sector by country, and quality rating by country (Figure 8).

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[9] *Replication of the Lehman Global Aggregate Index with Derivatives and Cash Instruments,* Lehman Brothers, December 2001.

[10] Some countries in the Global Aggregate Index have large credit issues in several currencies. If only currency, but not country, allocations are constrained, a replicating portfolio may end up heavily overweighted in one country.

Figure 8. **Cash Replication: Optimization Constraints**

Term structure constraints (matching duration contribution)

6 KRD points (the overall portfolio)	1, 3, 5, 7, 10, and 30 years
5 KRD points (4 main currencies)	1, 3, 5, 10, and 30 years

One-dimension cell constraints (matching market value and duration contribution)

4 currency blocs	USD, EUR, JPY, GBP
7 sectors	Treasury, Agency, Finance, Utility, Industrial, MBS, ABS
4 country blocs	US, Europe, Japan, Other
4 rating categories	Aaa, Aa, A, Baa

Combination constraints (matching duration contribution)

	Examples
Currencies x curve points (max 20)	Duration contribution to the 5-yr USD
Country blocs x sectors (max 14)	European-based Finance, irrespective of currency
Country blocs x rating (max 9)	Japanese Baa and A (together), irrespective of currency

The sampling scheme is intended to capture the main sources of return volatility while reflecting a standard and intuitive index partitioning. Allocation by currency represents the risk of divergence among yield movements in the major markets of the index. Sectors and quality rating partitioning describes the sources of spread movements. The term structure matching is done via a simplified key rate duration (KRD) mechanism.

We made the three strategies differ in the level of detail in the stratification scheme to test the performance of different portfolio concentrations. The number of bonds in the replicating portfolio is usually close to the number of imposed constraints (lower when constraints overlap).
While only one or two bonds end up representing most buckets, the local tracking errors tend to be uncorrelated and therefore offset each other. The bias in favor of the most liquid securities in the index, introduced by all strategies, is a systematic risk that does not get diversified away. However, because the mean return of the replicating portfolios was observed to be very close to the index, we can reasonably assume that this liquidity bias is minimal.

To force issue-level diversification, we constrained the amount that can be invested in any single security. The magnitude of the single bond constraint is a function of the bond's sector and rating quality. This flexibility ensures higher diversification in less homogeneous segments of the index, where individual issues are more likely to display higher idiosyncratic volatility.

For the first two strategies, we constructed the monthly proxy portfolio "from scratch", discarding the previous month proxy. This may not be a realistic approach for real-life tracking portfolios that are subject to transaction costs. Nevertheless, for the purposes of comparing the relative merits of different replication strategies this approach makes results less dependent on the portfolio inception date or particular study period. In a practical implementation, we would certainly seek to actively limit turnover.

No bond in the index is explicitly excluded from the investable set. Rather, the optimizer's preferences are influenced through a system of incentives, implemented as an illiquidity score[11] computed for every bond in the index. This score is an increasing function of the bond's age, and also reflects the bond's relative liquidity within its peer group. In fact, at any point in time, no more than a third of the index holdings are typically considered investable. Given the sufficient availability of investable bonds across all major index sectors, the tracking portfolio never invested outside of this narrow universe.

In the all-sectors strategy, we introduced one exception to this illiquidity score mechanism to reduce turnover: a special treatment of bonds already in the portfolio. For such bonds the score is set to a very small number irrespective of the bond's outstanding amount or age. If a bond in the portfolio drops out of the investable universe, we set the maximum allowed holding to the current position. As a result, we can continue to hold this bond (or sell it) but cannot buy more of it. A sell is forced only if the bond leaves the index.

Historical Simulation Results

All historical simulations were performed over a 28-month period from February 1999 through May 2001. To obtain more pertinent conclusions from simulations results, we defined the index over the entire study period using the current minimum outstanding amount of U.S. $300 million.

Figure 9 presents the summary performance results for the three replication strategies. The treasury-only strategy resulted in the smallest portfolios, averaging 22 holdings over the study period. As more sectors are made available for replication, the average portfolio size increases to 28 bonds for the treasury-plus strategy, and to 50 for the all-sectors strategy. Predictably, tracking errors decline as the index composition is matched more closely. Currency risk contributes significantly to tracking error, increasing it by 14% for the treasury-only strategy and by 40% for the all-sectors strategy.

Replication of the Global Aggregate Index with Futures and Swaps

Instead of using index securities to gain exposure to an index return, investors can gain exposure synthetically, by replicating an index with futures and swaps. This way, a

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[11] (((1 + Age x Age Multiplier) x AvePeerSize) / Outstanding) + 100 x IneligibleFlag,

where:

Age Multiplier is 0 for treasuries, 0.5 for agencies, and 1 for all other bonds;
AvePeerSize = average amount outstanding across the four major sectors (treasuries, agencies and supranational, collateralized, credit); and
IneligibleFlag is 1 for illiquid, callable (putable) bonds, very old issues, and currencies other than the main four; 0 for all other bonds.

Figure 9. **Cash Replication: Performance Summary**
February 1999 – May 2001

	Treasury		All	All Sectors
	Only	**Plus**	**Sectors**	**Feb 1999-Nov 2001**
Average Number of Bonds in the Replicating Portfolio	22	28	51	50
USD Tracking Error (bp/month):				
Hedged	11.7	6.3	4.2	4.1
Unhedged	13.3	7.7	5.9	5.9
USD Average Monthly Return Difference (basis points):				
Hedged	-2.7	-0.8	0.0	0.3
Unhedged	-2.4	-0.4	0.5	0.6

portfolio manager can easily put new cash to work. This can be a stopgap measure until such time as the funds can be fully invested in index securities, or a part of a long-term strategy. Alternatively, utilizing positions in liquid derivatives allows for rapid implementation of asset allocation shifts from one market to another. Replication portfolios can also be used in a "portable alpha" strategy, in which a manager uses derivatives to replicate the yield curve returns of the benchmark, but actually invests funds in a different market, where he anticipates greater profit opportunities.

We have investigated the tracking errors that can be achieved when using derivatives to replicate the U.S. Aggregate, Euro Aggregate, and Global Aggregate indices.[12] We have found that even within a single market, it is advantageous to use a hybrid strategy, such as Treasury futures contracts for the Treasury component and swaps for the spread product component of an index. This is because using different instruments for different index components leads to a reduced tracking error for each component, and equally important, produces uncorrelated replication errors for the different subcomponents. For example, as shown in Figure 10, from February 1999 through May 2001, the U.S. Aggregate component of the Global Aggregate Index was tracked to within 16.0 basis points per month by a hybrid Treasury futures and swaps strategy. This compares to 33.2 basis points per month using only Treasury futures, or 26.3 basis points per month using only swaps for the full index. Over the same time period, a strategy using Euro swaps tracked the Euro-Aggregate Index to within 16.7 basis points per month, and Japanese interest rate swaps tracked the Japanese Treasury Index to within 20.8 basis points per month. Combining these three strategies with a position in four Gilt swaps to represent the sterling market, we were able to replicate the unhedged Global Aggregate Index with a tracking error of 10.9 basis points per month (see Figure 11).

[12] *Replication of the Lehman Global Aggregate Index with Derivatives and Cash Instruments,* Lehman Brothers, December 2001.

Figure 10. **Means and Tracking Errors of Outperformance of Replication Portfolios Versus Selected Local Markets' Indices,** February 1999–May 2001

Index Replication Portfolio	U.S.Aggregate Index			Euro-AggregateIndex			Yen AggregateIndex		Sterling Aggregate Index	
	Treasury Futures	Swaps	Tsy Compt: Tsy Futures; Credit: Swaps	Treasury Futures	Swaps	Tsy Compt: Tsy Futures; Credit: Swaps	10-yr. Treasury Futures	Swaps	Long Gilt Futures	Swaps
Tracking Error (bp/m)	33.2	26.3	16.0	17.1	16.7	13.1	38.8	20.8	49.4	40.8
Mean (bp/m)	0.4	-0.3	-0.3	1.5	0.5	0.5	12.9	2.1	-8.3	12.4

Figure 11. **Tracking Errors for Unhedged Global Aggregate Index Dollar Based Returns,** February 1999–May 2001

Aggregate Index				Tracking Error	Mean Outperformance
US	**Euro**	**Yen**	**Sterling**		
Treasury Futures	Treasury Futures	10-year Treasury Futures	None	**23.7**	**2.4**
Tsy Comp: Tsy Futures; Credit: Swaps	Swaps	Swaps	None	**14.9**	**-0.8**
Treasury Futures	Treasury Futures	10-year Tsy Futures	10-year Tsy Futures	**20.8**	**4.2**
Tsy Comp: Tsy Futures; Credit: Swaps	Swaps	Swaps	Swaps	**10.9**	**2.0**

We also used empirical hedge ratios in some other replication simulations. Although they proved very promising for some individual markets, for the Global Aggregate Index they did not lead to tracking errors below the 10.9 basis points per month indicated above. We found the cost of the derivatives replication portfolios quite manageable, on the order of about 0.6 basis points per month for a U.S. $1 billion portfolio.

Replication of Credit Indices with Baskets of Credit Default Swaps: Example of Japanese Corporates

The credit derivatives market in Japan has been growing rapidly over the last few years. This growth, along with the much-improved liquidity, has increased the investment opportunity set for credit investors. Credit derivatives are being used as a tool for creating synthetic credit exposures. This is important since the supply of credit products is much more limited in Japan compared to the U.S. Currently, the credit derivatives market is driven by two classes of investors. Insurance companies sell protection to generate income or to enhance portfolio returns, while banks and other financial institutions buy protection to reduce their credit risk exposure. Investors who were earlier restricted to the cash bond market can now branch into credit derivatives to create risk and return equivalent to their cash counterpart. Increasingly, credit derivatives are becoming a source of price discovery for cash bonds, since their liquidity may be better than their cash equivalent. Thus, there exists a symbiotic relationship between the cash bond market and the credit derivatives market. We can empirically demonstrate this relationship and show how to replicate Japanese corporate bond returns using default swaps.

Our objective here is to give a sense of how much higher the tracking error relative to the Credit Index will be for a basket of credit default swaps compared to a portfolio of underlying corporate bonds.

In a default swap transaction, the protection buyer pays a premium, known as the default swap spread (DSS), to compensate the protection seller for taking on the risk of default by a given reference name. In the event of default, the protection seller steps in to indemnify the protection buyer by compensating the buyer with the notional loss beyond the recovery value.

The theoretical relationship between corporate bonds and default swaps can be best established via floating rate notes (FRNs), as default swap spreads are most effective in mimicking the spreads of an FRN trading at par. Selling protection and investing the notional amount in a LIBOR deposit would generate a coupon of LIBOR plus a spread, while buying a FRN would also lead to the same coupon. In the event of default, both investments would deliver the recovery value of the notional amount. In an arbitrage-free environment, the FRN spread should be identical to the default swap spread.[13]

The relationship between the fixed rate bond and default swap spreads is slightly more complicated, however, as the fixed rate bond may be priced significantly away from par. A default swap is a *par* asset that compensates the protection buyer against the loss on the par value of the asset. In contrast, the fixed rate bond may expose investors to a greater (or lower) credit risk than the par value if the bond is priced significantly above (or below) par. In order to demonstrate the same relationship, we use an asset swap contract in which the fixed coupons of a corporate bond are swapped into a series of floating rate coupons, thus making a fixed rate bond equivalent to a floating rate note. This relationship implies a high correlation between the par floater spreads and fixed rate bond spreads, and thus a high correlation between the default swap spreads and fixed rate bond spreads.

Methodology

We created the following two excess returns series to demonstrate how to generate corporate bond returns using default swaps.

[13] For an explanation, please see “Trading the Default Swap Basis” in Lehman Brothers’ *Quantitative Credit Research Quarterly Oct. 18, 2001,* by Robert McAdie and Dominic O’Kane.

To calculate excess returns on default swaps we assume that an investor enters a default swap as the protection seller. This position receives a fixed spread for the life of the contract or until default, whichever is earlier. In case of default, the seller compensates the protection buyer by the par value of the notional minus the recovery value that debtors would receive from the defaulted bond. The return calculated as such shall be viewed as returns in excess of LIBOR, as we are not making an associated LIBOR investment. Since we only have adequate data for 5-year default swaps, we roll the default swap over to a new 5-year term contract at the end of each month. In other words, we are computing excess returns for investing in 5-year constant maturity default swaps.[14]

We create a time series of corporate bond excess returns using total returns on 5-year constant maturity (*i.e.*, rolling over to a new 5-year bond at the end of each month) corporate bonds. We construct excess returns by subtracting from the bond returns a return equivalent to investing in a constant maturity 5-year LIBOR "bond," *i.e.*, a hypothetical 5-year bond with its coupon rate set to be the 5-year swap rate. This allows us to compare fairly the excess returns to LIBOR of a fixed rate bond with that of a similar term default swap.

Our data sample includes thirteen firms for which sufficient historical data was available. These firms cover a broad range of industry sectors including financials, utilities, electronics, industrials, telecommunication, trading, automobiles, and transportation. These thirteen names represent 46.4% of the total outstanding amount in the Japan Corporate Index. To generate the return series, we apply our in-house valuation tool to obtain the mark-to-market valuation of default swaps. These mark-to-market gains and losses at the end of the month are divided by the notional amount to produce our constant maturity default swap returns.

Results

We report our empirical results by first examining the relationship between the default swap spreads and corporate spreads. We then compare the excess returns to LIBOR between default swaps and corporate bonds.

(a) Default Swap Spreads vs. Corporate Spreads

A comparison of month-end default swap spreads and the corresponding corporate bond spreads shows that the default swap spread is significantly higher than the corresponding corporate spread. The average spreads for the corporates in our sample vary from around -3.5 basis points to 70 basis points, while the average default swap spreads vary from 15 basis points to 97 basis points. For the thirteen firms in our sample, the average corporate spread is around 15 basis points with a standard deviation of 11 basis points, while the average default swap spread is approximately 43 basis points with a standard deviation of 19 basis points. On average, the default swap spread is roughly 28 basis points

[14] Some of the default swaps are denominated in U.S. dollars, in such cases we compute returns by assuming the currency exposure to be perfectly hedged.

higher than its corporate counterpart. Figure 12 depicts this stylized relationship for one of the thirteen firms: Bank of Tokyo Mitsubishi.

The data also show that default swap spreads are highly correlated with corporate spreads. For the thirteen firms the average correlation was 64% with the correlation ranging from 16% to 86% (Figure 13). The correlations were calculated for rolling windows of 12 months as well as for the whole sample. Figure 13 demonstrates that the correlations vary considerably over the period. Finally, the data reveal that default swap spreads are more volatile than corporate spreads. We used an F-test to determine if the DSS and the corporate spreads differed in variances and discovered at standard significance levels that the variances were significantly higher for default swaps.

Figure 12. **BOTM Default Swap Spread Compared to the Corporate Spread**

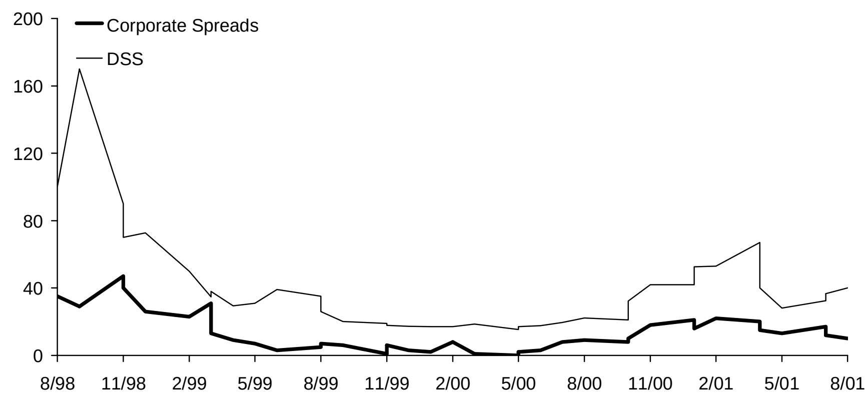

Figure 13. **Correlation between DSS and Corporate Spreads,** (1999-2001) (%)

	Max	**Min**	**Overall**
BOTM	92	20	75
IBJ	97	12	66
Sanwa	81	-1	74
Sumitomo	74	36	72
TEPCO	96	-22	82
NTT Corp	95	-74	67
Toyota Motor	90	0	54
NEC	91	35	61
NIPPON Steel	97	7	86
Mitsubishi Corp.	89	-13	16
East Japan Railway	84	2	71
ANA	97	-39	85
FUJITSU	91	-92	20
Average	90	-10	64

(b) Default Swaps Excess Returns vs. Corporate Bond Excess Returns

Our hypothesis is that default swap spreads, which trade at higher spreads than corporates, generate returns comparable to corporates (if not outperform). The data in Figure 14 support this hypothesis. For ten out of the thirteen issuers, default swaps produced higher excess returns than the corresponding corporates. On average, default swaps produced 1.7 bp/month more in excess returns, although the average standard deviation for default swaps is higher. Default swaps outperformed corporates most (6.6 bp/month) in the case of ANA, while the minimum was in the case of Toyota Motors (-1.1 bp/month).

There are two main forces driving these relative excess returns. First, default swap spreads have been falling for the past three years, after peaking during the Russian crisis in 1998. This trend induces positive relative excess returns for default swaps for our sample period. Second, what makes default swap excess returns relatively more attractive, as stated earlier, is the fact that the default swap spreads have been considerably higher than their corporate counterparts. In fact, the average default swap spreads were 70% higher. As the market becomes more liquid and credit derivatives become more acceptable by investors, such a large difference may not be sustainable.

Figure 14 presents the average excess return difference between default swaps and corporates for each of the thirteen firms. Figure 14 also reports the standard deviation of these monthly return differences (i.e. tracking error). For the thirteen firms in our sample, the average monthly return difference is a positive 1.7 basis points, while the monthly tracking error averages to 9.1 basis points. The average annualized information ratio is about 0.7, indicating that cash bond returns can indeed be replicated effectively by default swaps.

Figure 14. **Monthly Tracking Performance**

	Monthly Return Difference (bp)	Monthly Tracking Error (bp)	Annualized Information Ratio
BOTM	-0.4	14.2	-0.1
IBJ	2.4	17.9	0.5
Sanwa	0.9	13.0	0.3
Sumitomo	-0.5	11.9	-0.1
TEPCO	2.0	4.7	1.5
NTT Corp	1.3	4.5	1.0
Toyota Motor	-1.1	3.9	-0.9
NEC	0.5	10.6	0.2
Nippon Steel	1.8	8.1	0.8
Mitsubishi	-0.3	6.2	-0.2
JR East	2.1	6.7	1.1
ANA	6.6	7.3	3.1
Fujitsu	6.1	9.4	2.3
Average	**1.7**	**9.1**	**0.7**

Default swaps are a viable alternative for investing in Japanese credit. This conclusion is particularly relevant for Japanese investors since the supply of corporate securities is rather limited and credit derivatives often have better liquidity. Our empirical analysis shows that the strategy of selling default protection yields higher average returns, with the same type of credit exposure, than a portfolio of corporate bonds. As such, investors can effectively use default swaps as a tool to outperform their Japanese credit benchmarks.

OUTPERFORMING THE GLOBAL AGGREGATE INDEX

As we mentioned in the introduction, one of the main benefits of managing a portfolio relative to the Global Aggregate Index is the potential for diversification of management style. Investors could engage in currency timing, duration or yield curve timing, sector rotations or security selection. The choice depends on the level of attainable skills in market timing or credit research as well as on the expected information ratio (risk adjusted outperformance) for a given level of skills. We can simulate results expected of a skillful manager historically by employing the "imperfect foresight" methodology. In it we isolate a particular pure strategy, placing our view relative to the index along one dimension, while remaining neutral in all the others. In doing so we tilt the outcome in favor of the "correct" decision proportionally to the level of skill.

As an example, we recently measured the historical relative value of three core-plus strategies relative to the U.S. Aggregate Index[15] : currency overlay and exposures to high-yield and emerging markets. In addition, we studied the performance of a simple duration timing strategy.

Some investors try to maximize expected returns for a given level of risk using the mean-variance analysis framework. However, other investors find this framework difficult to use due, in part, to the sensitivity of portfolio allocations to small changes in expected returns. This is a particular problem when using historical data to estimate expected returns because of the volatility of these estimates depending on the time period studied. Below we discuss Lehman's new asset allocation model, which uses expected returns based partly on historical data and partly on prior information (either a reasonable guess or a market neutral view). The advantage of this model is the construction of portfolios with better risk-adjusted expected performance, which is key to outperforming a benchmark.

Global Asset Allocation Model: Combining Views with Market Data

The Markowitz analytical framework—the first asset pricing contribution to the field of portfolio management—remains one of the dominant models in investment science. The theory introduces the concept of efficient or optimal portfolios and reduces portfolio selection to a simple optimization problem. Under this framework, risk-adverse investors select efficient portfolios that provide maximum expected return for a given level of risk or minimum risk for a given level of expected return. Risk is defined as the variance of

[15] *Valuing Macro Allocation Decisions: A "Foresight" Study,* Lehman Brothers, July 2001.

asset returns. Stated otherwise, investors have a mean-variance (MV) decision criterion defined over a single-period rate of return on the portfolio. Graphically, the set of efficient portfolios is represented in MV space by a curve known as the efficient frontier that exhibits a familiar parabolic shape.

Traditionally, the MV framework was of particular value to global portfolio managers dealing with a wide variety of loosely correlated global assets. It is the tool of choice for strategic asset allocation. One of its limitations as a tactical portfolio optimization tool within a single currency is the high correlation between returns of all investment grade assets in that currency. The Global Aggregate Index consists of assets with low correlations, making the MV framework even more relevant.

Selection of the best possible combination of assets in a portfolio benefited from the systematic use of the Markowitz analysis. This was analogous to the impact the Black-Merton-Scholes formula had on the pricing of derivatives products. Indeed, the concept of MV-efficiency has the potential to address many issues encountered in portfolio management such as the identification of sources of portfolio risk, the quantitative implementation of investment constraints and objectives, the integration of new information into portfolios, and the determination of an optimal investment policy.

Surprisingly, asset managers have used this appealing yet simple theoretical framework sparingly. Unfortunately, when the framework has been used, it has led to poor out-of-sample performance (measured in terms of ex-post returns and volatility of returns). Investors generally mention five major difficulties when using MV-analysis for portfolio selection:

- The lack of diversification offered by so-called efficient portfolios - the small number of assets in the portfolio compared to the number of assets available in the investment universe even for low levels of risk;
- The extreme sensitivity of optimal portfolio weights to relatively small changes in the value of the inputs to the MV-optimization problem (and particularly expected asset return numbers);
- The difficulty in deriving expected returns that are consistent with prior investment views and in performing neutral or “no-view” optimizations;[16]
- The relevance of the variance as a risk measure, particularly when dealing with asset allocation problems within an asset and liability framework where downside risk measures are supposedly more appropriate; and
- The appropriateness of the Markowitz framework when dealing with long-term, multi-period investment horizons

Market practitioners repeatedly cite these criticisms irrespective of whether the investment policy is set in absolute or in relative terms (i.e., versus an asset or liability benchmark).

[16] For information on Lehman Brothers’ No-View Optimization Model, please see *Total Return Management of Central Bank Reserves,* January 2002.

The Lehman Brothers Asset Allocation Model amends the Markowitz framework by reducing inputs' dependence on data estimated with ordinary statistical methods. This involves forming expectations based partly on historical data and partly on some prior information that is supposedly a reasonable guess of the "real" distribution, particularly as far as expected return is concerned.

To illustrate how the model works, we assume that three fund managers benchmarked against seven asset classes of the Lehman Brothers U.S. Aggregate Index share a common view of wider credit spreads on the back of wider swap spreads over a one-month horizon. In addition, the third fund manager (i.e., a "passive" investor) has no view on the market.

We want to derive a set of efficient portfolios in the tracking-error sense for all four fund managers. As the passive investor has no views on Treasury yield curve dynamics and on spreads dynamics, we will use "no-view" expected returns derived from the Lehman Brothers Asset Allocation Model.[17] The first investor argues as follows before computing expected excess returns: "I consider 5 basis points to be the most plausible value of swap spread widening and I am willing to bet 20 to one that the spread widening is not outside the interval [2.5 basis points, 7.5 basis points]". The second investor differs only in the confidence interval. To reproduce these views in our portfolio, we assume a one-to-one correspondence between swap spreads and the spread of finance bonds to Treasuries.

More precisely we expect the option-adjusted spread of finance bonds to widen by an average of 5 bp over the next month from a starting level of 125 basis points. Given the spread duration, this translates into an annualized monthly excess return differential target of -1.45% between the two asset classes. This view can be considered extreme given the annualized monthly volatility of the return differential that stands at 1.31% and an annualized monthly sample average differential of 0.94%.

In addition, we set the two different confidence intervals for the spread widening intensity at the 95.0% level at [2.5 basis points, 7.5 basis points] and [-10 basis points, 20 basis points] for the first and second investor, respectively. Figure 15 presents the major data necessary for the derivation of tracking-error efficient portfolios and exhibits the impact of the two different levels of confidence on expected excess returns.

Expected excess returns obtained from the Lehman Brothers Asset Allocation Model exhibit less dispersion and differing rank performance to expected excess returns derived from straight sample average calculations. In particular, the expected excess return of finance bonds is dramatically reduced compared with other asset classes, while the differential with treasury bonds is reduced from 0.94% to 0.68%. For the two active investors, the first observation is that although investors had a generic swap spread widening view expressed only through two asset classes, all asset returns are affected.

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[17] For more information regarding the determination of "no-view" expected returns please see *Overview of Single-Period Asset Allocation Models, Understanding Portfolio Selection,* David Prieul and Robin Thompson, Lehman Brothers Analytical Research Series, November 2000.

Figure 15. **Expected Returns and Optimization Results**

Asset class	Sample Return per mo. (%)	No views return (%)	With view [2.5bp, 7.5bp]	With view [-10.0bp, 20.0bp]	Benchmark weight	Sample return weight	No views weight	With view [2.5bp, 7.5bp] weight	With view [-10.0bp, 20.0bp] weight
Treasury	1.91	1.08	-0.03	0.91	26.20	0.00	4.10	76.70	8.60
Agency	1.81	1.04	-0.36	0.82	10.20	0.00	0.00	0.00	0.00
Finance	2.85	1.77	-1.19	1.29	7.00	44.60	7.30	0.00	0.00
Industrial	2.79	1.84	-0.81	1.41	6.80	17.30	0.00	0.00	0.00
Utility	2.56	1.82	-0.79	1.40	1.30	0.00	2.00	0.00	0.00
Yankee	2.79	2.29	-0.70	1.81	5.00	0.00	35.90	0.00	41.00
MBS	2.35	1.44	-0.49	1.13	43.50	38.10	50.70	23.30	50.40

The varying degree of confidence in one's views is illustrated with the size of the confidence intervals. As the confidence interval gets larger, the uncertainty around the prior view increases and the Lehman Brothers updating methodology shrinks data back to the no-view estimates.

For our optimizations, we assume a short selling constraint and a tracking error volatility target of 50 basis points per annum. Given the strength of our average swap spread widening view, it is not surprising to see that the posterior estimate with the tightest confidence bands generates a portfolio that is not well diversified. However, as the confidence in the view decreases, the tracking error efficient portfolio converges to the no-view portfolio. The no-view portfolio itself is still diversified in five out of the seven asset classes that constitute our benchmark compared to only three for the sample return portfolio.

The techniques embedded in the Lehman Brothers Asset Allocation software allow for a better balance of risk across the multiple dimensions of the portfolio, a key feature for systematic out-of-sample performance.

CONCLUSION

We are in the midst of a very exciting transformation in global portfolio management from multi-currency Treasury asset mix to the Global Aggregate Index. The long-term forces driving this transformation, which we tried to summarize in this paper, may well be permanent. This process will inevitably lead to a significant broadening of the investment opportunity set and a resulting improvement in long-term risk adjusted returns.

Contributing to this process is the shift in the asset allocation of Central Bank reserve portfolios that we have started to witness in the late 1990s. Their move from short-dated Treasury securities to a more diversified mix of assets may in fact be one of the most interesting developments in the world of fixed income securities over the next decade.

Despite the large number of securities and the broad asset mix, the Global Aggregate Index is highly replicable, often more so than its components. It can be matched very closely with a limited number of bonds or even a basket of derivatives. Protection from impact of credit events can also be achieved as a practical matter with proper issuer diversification. The most “technical” of its asset classes—U.S. MBS—is one of the easiest to replicate.

In fact, with a broader set of asset classes to consider, the Global Aggregate offers portfolio managers greater opportunity for diversification of style. Instead of only timing currency, duration or curve movements, they will be able to engage in the top-down style of sector rotation, or bottom-up style of security selection, especially in credit products. This style diversity is the most certain way of achieving high information ratios. Empirical studies show which style has been able to deliver the highest historical information ratio at comparable levels of market timing skills.

The increase in dimensions of risk assumed by global portfolio managers will be accompanied by broader endorsement of derivative instruments used to hedge it or to affect asset allocation shifts. The primary candidates could be Treasury futures and interest rate swaps. After the 1998 liquidity crisis, swaps emerged as the hedging instrument of choice for most spread products. They are beginning to be viewed as total return instruments offering participation in spread market movement. Credit default swaps are also finding their way to an increasing number of global portfolios with an objective of participating in credit spread movement while maintaining high liquidity.

Risk diversification is one of the key advantages of investing in a Global Aggregate portfolio. With many dimensions of risk and complex correlations between them, a multi-factor risk model may be the only way of *ex ante* estimation of portfolio risk. The revised U.S. Aggregate and the Euro Aggregate risk models currently available in our portfolio analytics represent a major step towards a global risk model. It is critical for investors to be able to structure replicating portfolios or engage in risk budgeting along different dimensions of risk taking. Along with performance attribution, it forms the cornerstone of our portfolio analysis tools. The ultimate goal is to be able to optimize expected return in a global portfolio subject to a risk constraint and attribute the *ex post* performance to the risks taken.