



Optimal Diversification – A Unified Framework

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Portfolio Modeling | IPRS



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What Portfolios Are Diversified?

- Diversified portfolio
 - US Fixed Income and Equity (Barclays Capital US Aggregate, S&P 500)
 - Some solutions: equal weights, market, weights = 1/vol, minimum volatility

Stats	of Portfolio Retu	rns
%	/mo 1991–2010	

	Volatility	Max Drawdown
Equal Weights	2.30	-27.1
Market Weights	2.79	-34.0
Equal Vol	1.32	-9.4
Minimum Vol	1.07	-5.5

Source: Barclays Capital

Correlation between Portfolios and Assets

	Barclays Capital US Agg	S&P 500
Equal Weights	32%	95%
Market Weights	22%	96%
Equal Vol	73%	71%
Minimum Vol	94%	17%

Source: Barclays Capital

- Portfolios are very different
- Not clear what they have in common and how they differ
- Common feature: No Expected Return Forecast



Objective

- A unified understanding of popular diversification solutions
 - Each solution = a result of assumptions
- Consequences
 - Quality of assumptions → quality of solution
 - Different assumptions → different solution
 - No assumption is universally right → no universal "best" solution



Agenda

- 1. Portfolio Construction General Framework
 - Optimal Risk Return Tradeoff
- 2. From Portfolio Construction to Diversification
 - Diversification: Expected returns assumed a function of risk
- 3. Diversification assumptions about expected returns
 - Construct portfolios
 - Examples



1. Portfolio Construction Framework

- Portfolio construction goal: best risk-return tradeoff
 - Definitions for expected returns, risk, and best tradeoff

Pick one from each column				
Portfolio Risk Measure	Asset Risk Measure	Expected Returns Measure	Portfolio Scaling	
Volatility	Marginal Contribution to portfolio risk	Many possibilities	Target portfolio risk level	
VaR	Total Contribution to portfolio risk		Target portfolio expected return	
Expected Shortfall			Target portfolio leverage	
Downside Deviation				
Other measure				



1. Portfolio Construction Framework (cont)

- Tradeoff: Expected Return/Risk
 - E.g. Sharpe ratio
- Tradeoff can be at portfolio or asset level
 - Portfolio level: best tradeoff → Maximum tradeoff
 - Asset level: best tradeoff → Equal tradeoff across all assets
 - Same result only if Asset Risk Measure = Marginal Contributions



1. Portfolio Construction Framework – Example

Choices Delivering Basic Risk-Parity Setup

Pick one from each column				
Portfolio Risk Measure	Asset Risk Measure	Expected Returns Measure	Portfolio Scaling	Best Tradeoff
Volatility	Marginal Contribution to portfolio risk	The same for all assets	Target portfolio risk level	At portfolio level: Max ER/Risk
VaR	Total Contribution to portfolio risk		Target portfolio expected return	At asset level: ER/Risk the same
Expected Shortfall			Target portfolio leverage	
Downside Deviation				



2. From Portfolio Construction to Diversification

- Portfolio Construction
 - Main goal: best risk-return tradeoff
- Diversification
 - Construct portfolios without explicit expected returns (ER)
 - → A special case of portfolio construction: no ER
- Diversification framework
 - Assume ER depend on risk
 - Each assumption about ER → A different diversified portfolio
 - Simple and intuitive assumptions → Common diversified portfolios



2. Diversification Framework

Special case of general portfolio construction framework

Portfolio Risk Measure	Asset Risk Measure	Expected Returns Measure	Portfolio Scaling	Best Tradeoff
Volatility	Marginal Contribution to portfolio risk	A function of risk - special to diversification	Target portfolio risk level	At portfolio level: Max ER/Risk
VaR	Total Contribution to portfolio risk		Target portfolio expected returns	At asset level: ER/Risk the same
Expected Shortfall			Target portfolio leverage = 0	
Downside Deviation				



3. Diversification Examples

- What we will use for our examples:
 - Mean-Variance Optimal (MVO) portfolios
 - Various assumptions about expected returns

Portfolio Risk Measure	Asset Risk Measure	Expected Returns Measure	Portfolio Scaling	Best Tradeoff
Volatility	Marginal Contribution to portfolio risk	A function of risk - special to diversification	Target portfolio risk level	At portfolio level: Max ER/Risk
VaR	Total Contribution to portfolio risk		Target portfolio expected returns	At asset level: ER/Risk the same
Expected Shortfall			Target portfolio leverage = 0	
Downside Deviation				



Expected Return Assumptions

Typical Expected Returns Assumptions and Resulting Diversified Portfolios

Assumption about ER	Additional Assumptions	Mean-Var Optimum Portfolios
A1.1 Equal across assets	_	Global Minimum Volatility
A1.2 Equal across assets	Volatilities and correlations are constant	Equal Weight
A2.1 Proportional to Volatility	_	Global Minimum Volatility on correlations
A2.2 Proportional to Volatility	Correlations are constant	Weights proportional to 1/Vol ("Equal-vol")
A3.1 Equal to a linear combination factor betas	_	Portfolio of factors
A3.2 Equal to a linear combination factor betas	Factor = market	CAPM (market portfolio)



A1: Expected Returns Equal across Assets

Portfolio Risk **Asset Risk Measure Best Tradeoff Expected Returns Portfolio Scaling Marginal Contribution Target portfolio** At portfolio level: **Volatility Equal across assets** to portfolio risk leverage = 0 Max ER/Risk

- Optimal Portfolio
 - Global Minimum Volatility (GMV)
 - Equal weight portfolio (if no views on corrs and vols)
- If assets on different scales
 - Wrong assumption → No diversification

Example 1: Barclays Capital US Treasury S&P 500 **S&P** Commodities

Correlation between Various Portfolios and Assets, 1990-2010

	US Tsy	S&P 500	Comm
GMV	86%	19%	18%
Equal Weights	9%	64%	84%
40/40/20	14%	81%	66%

Source: Barclays Capital



A2: Expected Returns/Vol Are the Same

Portfolio Risk	Asset Risk Measure	Expected Returns	Portfolio Scaling	Best Tradeoff
Volatility	Marginal Contribution to portfolio risk	Proportional to Vol	Target portfolio leverage = 0	At portfolio level: Max ER/Risk

- Optimal Portfolio
 - Global Minimum Volatility portfolio using correlation matrix (MVC)
 - Volatility-weighted portfolio (if no views on correlations); "Equal Vol"

- Do analysis in terms of vol-stabilized assets $\tilde{r}_i = r_i / \sigma_i$
 - They have the same ER → apply results A1
 - Covariance(\tilde{r}) = Correlation(r)
 - Weights \widetilde{w}_i depend only on correlations
 - No correlation views $\rightarrow \widetilde{w}_i = 1/N \leftrightarrow w_i = const/\sigma_i$
 - Beta of \tilde{r}_i on a factor $f = \rho_{i,f} * const$



Example 1 (cont'd): Asset Class Portfolio

- US Tsy, Equity, and Commodities: Barclays Capital US Tsy, S&P 500, S&P GS Comm
- Four diversification methods: Min Vol on Corr (MVC), Equal Vol, Equal Wgt, 40/40/20

Stats of Portfolio Returns %/mo 1983-2010 Difference Difference

due to vols due to corrs Min Vol **Equal Equal** (MVC) Vol Weight 40/40/20 Mean 0.40 0.34 0.38 0.41 **Volatility** 2.29 1.54 1.57 2.57 Drawdown -11.1 -16.2 -40.2 -33.8

Source: Barclays Capital

Correlation Between Optimal Portfolio and Assets

	US Tsy	S&P 500	Comm
Min Vol (MVC)	55%	54%	54%
Equal Vol	52%	57%	62%
Equal Weight	9%	64%	84%
40/40/20	14%	81%	66%

Source: Barclays Capital

Risk optimization lowers portfolio vol and drawdown, staying fully invested



Example 1 (cont'd): Asset Class Portfolio

- How "Min Vol on Corr" (MVC) works
 - Less weight (vol adjusted) on more correlated asset;
 - Still all assets have the same correlation with final portfolio

Optimum "Min Vol on Corr" Portfolio and Underlying Assets

	US Tsy	S&P 500	Comm
Avg correlation w/other assets	3%	9%	0%
Avg weight (vol-adjusted)	34%	30%	36%
Avg weight (original assets)	71%	12%	17%
Correlation w/realized portfolio, monthly 1983–2010	55%	54%	54%

Source: Barclays Capital



Example 2: S&P 500 Sector Portfolio

- 22 industry sectors of SP500
- Homogenous large universe; correlations high; correlations and vols similar
- What to expect from theory

 - Equal Vol ↔ Equal Weight because vols are similar
 - Min Vol ↔ Market because correlations are high

Stats of Portfolio Returns %/mo 1993-2010

			Difference due to vols	
	Min Vol (MVC)	Equa Vol	l Equal Weight	Market Weight
Mean	0.33	0.53	0.56	0.47
Volatility	4.09	4.11	4.39	4.43
Drawdown	-58.1	-50.4	-53.1	-54.8

Source: Barclays Capital

Empirical results match expectations

Stats of Correlations between **Optimal Portfolios and Assets**

	Min Corr	Max Corr	Avg Corr
Min Vol (MVC)	51%	79%	64%
Equal Vol	50%	88%	70%
Equal Weight	47%	88%	70%
Market Weight	44%	86%	67%

Source: Barclays Capital

For homogenous and correlated universe all these assumptions are reasonable



Example 2 (cont'd): S&P 500 Sector Portfolio

- Why correlations may fail: we cannot estimate them well and weights are small
 - Errors in corr forecast create 1:1 errors in weights
- Behavior in two correlation regimes of the utilities sector w/other sectors
 - Jul-00 Jun-02: realized vs. forecasted corrs are different
 - Jan-03 Dec-04: realized vs. forecasted corrs are similar

Portfolio Volatilities in Two Correlation Regimes						
	Unstable Correlations	Stable Correlations				
Period	Jul-00–Jun-02	Jan-03-Dec-04				
Vol of MVC Portfolio	4.3	2.8				
Vol of EqVol Portfolio	3.7	3.1				
Vol MVC – Vol EqVol	0.6	-0.3				

Source: Barclays Capital

In large universes, imprecise/unstable correlations create issues



Example 3: US Treasuries Duration Buckets

- 6 duration buckets of BarCap US Treasuries
- Semi-Homogenous small universe; corrs high; corrs and vols differ

Stats of Portfolio Returns %/mo 1994–2010						
	Difference due to corrs Difference due to vols					
	Min Vol (MVC)	Equal Vol	Equal Weight	Market Weight		
Mean	0.14	0.19	0.24	0.20		
Volatility	0.78	1.17	1.70	1.36		
Drawdown	-4.6	-7.2	-9.8	-8.4		
Source: Barclays Capital						

optimal portfolios and assets					
	Min Corr	Max Corr	Avg Corr		
Min Vol	87.7%	96.0%	93.1%		
Equal Vol	85.4%	97.7%	93.5%		
Equal Wgt	79.0%	96.6%	91.7%		
Mkt Wgt	82.6%	97.2%	93.0%		

Stats of correlations between

Source: Barclays Capital

Vols and corrs contribute separately to lower portfolio risk, staying fully invested

- Min Vol on Corr (MVC) portfolio ≈ 50% 1–3y, 50% 20–30y
 - Takes mostly level and slope curve risk
 - If one wants to also take convexity risk → move to a portfolio of risk factors

A better way may be to optimize directly over risk factors



Portfolio Risk	Asset Risk Measure	Expected Returns	Portfolio Scaling	Best Tradeoff
Volatility	Marginal Contribution to portfolio risk	Linear Combination of factor betas	Target portfolio leverage = 0	At portfolio level: Max ER/Risk

- Investors demand compensation to carry certain sources of risk (factors)
 - Higher beta to these risk factors → higher Expected Returns
- Optimum portfolio should contain only risks that carry Expected Returns
 - Simple case: one risk factor, factor = market → CAPM

Optimum portfolio contains only factors

- Multi-factor case: ER are a linear combination of factor betas
- If linear combination of betas = Optimum portfolio of factors then:

Optimum portfolio of assets ↔ Optimum portfolio of factors

- To get optimum factor portfolio → use previous assumptions
 - We need *factor* covariance and assumptions about *factor* ER



Example 4: Re-Do Cross-Assets Using Sector Details

- US Treasuries, S&P 500, S&P GS Commodities
- Use sector details: 6 Tsy duration sectors, 22 equity industry sectors, and 5 commodity sectors
- Assume: one risk factor per asset class
 - Final portfolio contains only three asset class factors
- Factor definitions
 - Market weights (asset class indices)
 - Optimum portfolios within the asset class (see Examples 2–3)

Stats of Factors Constructed with Two Definitions %/mo 1994-2010

	Market-Weight Factors			Optimur	n-Weight	Factors
Asset	US Tsy	SP500	Comm	US Tsy	SP500	Comm
Mean	0.20	0.47	0.35	0.14	0.31	80.0
Volatility	1.36	4.53	6.52	0.78	4.16	3.07
Drawdown	-8.4	-55.8	-67.9	-4.6	-58.1	-44.9

Source: Barclays Capital



Example 4 (cont'd): Cross-Assets Using Sector Details

- Diversification methods we will use
 - Use all 33 assets directly, no factors
 - Use market-weight factors (indices); optimize over factors (same as Example 1)
 - Use optimum asset-class factors; optimize over factors
 - Use a mixture of market-weight and optimum factors; optimize over factors
- "Optimize" = Min Vol on Corr

Stats of Optimum Portfolio Returns for Various Factor Definitions (%/mo 1996-2010)

	No Factors	Market Factors	Optimum Factors	Mixed
Mean	0.02	0.24	0.09	0.11
Volatility	1.25	1.45	0.88	0.85
Drawdown	-15.8	-11.1	-9.1	-7.1

Source: Barclavs Capital

- Using entire universe (no factors) gives an unstable matrix → large drawdown
- Optimum factors portfolio has lower risk than market factors portfolio; their correlation is 85%
- Can use factors with different definitions → mix and match
 - For large homogeneous universes (like equity), market factor makes more sense



Conclusions

- Unified framework on diversification
 - Solutions ← assumptions
- Diversification = Portfolio construction
 - ER = F(Risk)
- Quality of solutions = quality of assumptions
 - Validated for various settings
- Framework customizable based on
 - Assumptions you feel comfortable with
 - Information you have about risk
 - Your preferences



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