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The American Economic Review, Vol. 80, No. 5 (Dec., 1990), 999-1021.

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Market Liquidity, Hedging, and Crashes

By GERARD GENNOTTE AND HAYNE LELAND*

In the absence of significant news, hedging strategies were blamed for the stock market crash of October 1987; but traditional models cannot explain how a relatively small amount of selling could cause so large a price drop. We develop a rational expectations model in which prices play an important role in shaping expectations; markets are much less liquid in our model than in traditional models. Discontinuities (or "crashes") can occur even with relatively little hedging. The model is consistent with theories as disparate as Keynes' "beauty contest" insight and Thom's "catastrophe" analysis and suggests means to reduce volatility. (JEL 313)

Immediately following the stock market crash of October 19, 1987, both practitioners and academics sought an explanation based on external events. While several trends were clearly "bad news" for the market, these trends had been revealing themselves over the previous months. It was difficult to isolate new events occurring between October 16 and October 19 that were of sufficient importance to explain the magnitude of the price fall.

The Brady Commission's examination of the October break (Nicholas Brady et al., 1988) therefore centered on internal market causes rather than external events. In particular, the Commission focused attention on a number of large institutions following "price insensitive" strategies such as portfolio insurance.¹

In dramatic language, the Brady Report painted a picture of enormous waves of institutional selling driving down prices ex-

cessively. The report claimed that such sellers suffered from an "illusion of liquidity"; and it buttressed this conclusion by pointing out that a few large sellers alone sold about \$6 billion of stock and stock index futures.

Although there has been a unanimous positive response to the Brady Commission's marshalling of facts, there has not been a unanimous acceptance of their interpretations of these facts. Formal portfolio-insurance strategies were followed by less than 3 percent of stock market funds.² While the \$6 billion sold by portfolio insurers seems a large amount, it represented only 15 percent of total stock and stock index futures volume on October 19. In absolute terms, the \$6 billion amounts to less than 0.2 percent of the roughly \$3.5 trillion of equity value at the beginning of the day. Is it reasonable to think that selling 0.2 percent can drive down prices by over 20 percent—that selling \$6 billion can cause losses of \$700 billion? Of course the answer depends upon the elasticity of demand for stocks. But traditional models imply an elasticity much greater than the market exhibited on October 19, 1987.

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¹Portfolio insurance strategies are dynamic hedging strategies which provide protection by replicating a put option (see Mark Rubinstein and Leland, 1981). These strategies have the property that they tend to sell after the market has declined and to buy after market rises.

²Best estimates suggested \$70–\$100 billion in funds were following formal portfolio insurance programs. On a precrash total stocks value of about \$3.5 trillion, this represents 2–3 percent. Of course, informal hedging strategies such as stop-loss selling may have amounted to considerably more than this (see the survey of Robert Shiller [1987]).

A recent study by Michael Brennan and Eduardo Schwartz (1989) suggests that a 5-percent use of portfolio insurance by investors would have a minimal impact on market prices and volatility. Their model and other commonly studied portfolio/consumption models suggest elasticities of demand for stock far greater than 1—more than 100 times the elasticity implied by the Brady Commission's conclusions. Information changes, rather than selling by portfolio insurers, are needed to explain October 19 in these standard models.

Other evidence does not seem to confirm a strong connection between the crash and portfolio insurance. If short-run selling were the cause of the decline, we might expect a quick reversal, but this did not occur. Furthermore, Richard Roll's (1989) cross-market studies showed little correlation between October 1987 performance and various aspects of markets, including whether portfolio insurance was used.

In sum, the crash of 1987 presents the following dilemma to current financial models: the amount of selling seems insufficient to explain the large price drop observed on October 19. Thus, information changes seem necessary to explain the drop, but no such information changes can be documented.

Parallels with the crash of 1929 may be useful in understanding the crash of 1987. Like 1987, no significant economic news was associated with the period immediately surrounding the earlier crash. Several large declines preceded the crash of 1929—as they did in 1987. Volatility increased markedly in the weeks preceding both drops. In both cases, hedging strategies were discussed as a possible contributing factor: stop-loss orders in 1929 and portfolio insurance in 1987. In 1929, stop-loss strategies were used for portfolio protection but were also triggered by margin calls, which led to greater controls over margined stock-buying following the crash.

Because of these similarities, we would hope that an explanation of the 1987 crash would also be relevant to the 1929 crash. The explanation cannot be entirely in terms of futures markets and portfolio insurance, since neither existed in 1929.

In this paper, we develop an explanation of market “crashes” that reconciles the strands of evidence above. This explanation is not based on important changes in information, and therefore it is consistent with the failure to observe any significant events that directly “caused” the 1987 (or 1929) decline. In fact, we define a “crash” to be a discontinuity in the relationship between the underlying environment and stock prices: an infinitesimal shift in information (or other small shock) can lead to a major change in stock market level.³

Our explanation of crashes is based on unobserved plans of investors to hedge against losses. In 1929, stop-loss strategies were used. In 1987, portfolio insurance and stop-loss strategies were followed. We develop a “price pressure” argument akin to that of Sanford Grossman (1988a). However, this argument must meet two criticisms:

- (1) How can relatively small amounts of hedging drive down prices significantly?
- (2) Why didn't stock prices rebound the moment such selling pressure stopped?

Our model answers the first question by examining the determinants of market liquidity. An important aspect of financial markets is that only a small proportion of investors actively gather information on future economic prospects or asset supply. Other investors look to current prices to impute information about future prices. This dual role of prices—affecting demand both through the budget constraint and through expectations—leads to very different price elasticities than traditional models, in which prices play only the first role. Only recently have financial economists begun to explore

³Such discontinuities are commonly observed in physical systems and have been the recent subject of study by mathematicians examining “catastrophe theory.” In a preliminary paper (Gennotte and Leland, 1987), we considered a simple model of stock market discontinuities.

the implications for markets in which prices play both roles.⁴

In such environments, there is an important difference between observed and unobserved supply changes. If there are relatively few informed investors, markets may be much less liquid (and therefore more fragile) than traditional models predict when unobserved supply shocks occur.⁵ We show that relatively small unobserved supply shocks can have pronounced effects—more than 100 times greater than the effects of observed supply shocks—on current market prices.

Unobserved supply shocks have greater price impact as a consequence of investors inferring information from prices. A supply shock leads to lower prices, which in turn (since the shock is unobserved) leads uninformed investors to revise downwards their expectations. This limits these investors' willingness to absorb the extra supply and causes a magnified price response.⁶

Our model answers the second question by showing how a discontinuity in market prices can occur if hedging plans generate very large trades. Hedging plans create additional supply as price falls. A small information change can trigger lower prices, which, because of hedging, lead to greater excess supply and a further fall in prices. Thus, a small change in information can lead to a dramatic fall in prices, with no immediate rebound occurring. This feature of crashes distinguishes our results from

those of Grossman (1988a,b). We demonstrate that such a "meltdown" scenario obtains only for an unrealistically large hedging activity when the hedging trades are perfectly anticipated. If, however, investors (or a fraction of investors) are unaware of hedging plans, crashes can occur for much smaller levels of hedging activity. The discontinuities arise because investors are unable to perfectly distinguish hedging activity from information-based trades and therefore adjust downward their expectations of future prices. Imperfect anticipation of hedging activity relaxes the rational expectation requirement, but in a realistic fashion. Moreover, our estimate of hedging plans in place in 1987 approximates the threshold at which discontinuities occur in the imperfect anticipation case, providing a potential explanation for the 1987 crash.

Finally, our model suggests that some changes in market organization can radically reduce the likelihood of crashes. The most important such change is increasing market knowledge about the size and trading requirements of hedging programs. Preannouncement of trading requirements can lessen the impact of such trades by a factor greater than 100. To the extent that the specialist's book helps reveal the nature of order flow, this information should be made available to all traders. As suggested by Grossman, the use of put options to implement hedging may also serve to smooth markets.

The model builds from the work of Sanford Grossman and Joseph Stiglitz (1980), Martin Hellwig (1980), Douglas Diamond and Robert Verrecchia (1981), Anat Admati (1985), Gennotte (1985), and Albert Kyle (1985). We postulate a subset of informed investors who receive noisy signals about future market values. Random supply keeps these investors from perfectly inferring the aggregate information from price. Some investors, however, whom we characterize as market-makers, receive information about the size of the random supply. This information enables market-makers to distinguish, at least partially, selling that is information-based from selling that is motivated by liquidity considerations. We show

⁴See Grossman (1976), Grossman and Joseph Stiglitz (1980), Martin Hellwig (1980), Douglas Diamond and Robert Verrecchia (1981), Albert Kyle (1985), and Anat Admati (1985). These papers focus on the role of prices in aggregating information.

⁵This point is discussed informally in Leland and Rubinstein (1988) and in D. Cutler, James Poterba, and Lawrence Summers (1989). Fischer Black (1988) describes a model in which shocks to expectations—rather than supply—can cause large price changes.

⁶Such models reflect a rational expectations view of Keynes' famous "beauty contest" metaphor, that successful investors must base their investments on their expectations of others' expectations of value, rather than solely on their own estimates of value. Thus price, reflecting others' expectations, rationally conditions each individual investor's expectations, and bandwagon or "herd" effects can result.

that market-makers provide a significant source of liquidity in meeting random supply shocks.

We also allow for the presence of hedging programs such as stop-loss orders and portfolio insurance. These hedging strategies are usually nonlinear functions of the equilibrium price; hence, the resulting rational expectations equilibrium price is, in general, a nonlinear function of the signals. We examine the effects of these strategies on market equilibrium and stability for alternative specifications about the observability of hedging. The possibility of price discontinuities, with the implication of crashes, is new to our model.

I. Informed and Uninformed Investors

We assume that investors may be informed or uninformed. The informed investors can be subdivided into two types, who differ in terms of the signals they are able to observe. Thus, in all there are three classes of investors:

- (1) *uninformed* investors (denoted U), who observe only the equilibrium price p_0 ;
- (2) *price-informed* investors (I), who observe a personal, unbiased signal p_i' on future price (or liquidation value) p and also observe p_0 ;
- (3) *supply-informed* investors (SI), who observe a common supply signal S and the equilibrium price p_0 .⁷

The price-informed investors can be thought of as having (personal) information about economic fundamentals which are noisy predictors of future price. Supply-informed investors can be thought of as market-makers who have information about the sources of order flows: the size of new issues, portfolio restructurings, and other elements of liquidity trading.⁸

Our model allows arbitrary proportions of investors in each class. The relative proportion of investors who are informed versus uninformed is a key determinant of market liquidity. Informed investors, particularly supply-informed investors, will absorb a substantial proportion of liquidity-trading demands. Even when they are relatively few, informed investors constitute an important fraction of the supply of liquidity.

Thus, an important empirical question is the relative number of investors of each type. While data on this question are difficult to gather, we do have some evidence that informed traders, particularly supply-informed traders, are relatively small as a fraction of total market capital.

Among supply-informed investors are specialists and other market-makers (including "upstairs" desks) who adjust their positions in response to changing demand for liquidity. Because of their role as market-makers, these investors have special information on the nature of demand. Through the order book or simply on the basis of their knowledge of institutional trading, market-makers can learn (perhaps imperfectly) about the volume of noninformation (or "liquidity") trading versus trading based on information.

The funds committed to supply-information gathering (and providing liquidity) depend upon the return to this activity. In some cases, competitive forces will determine the amount provided. In other cases, institutional factors such as the specialist system may limit the number of potential entrants. We shall see below that such limitations can importantly affect the stability of markets.

There is no way to provide an exact quantification of market-making capital. However, it clearly is small relative to the \$3.5 trillion of equity investment. For example, the total capital of New York Stock Ex-

⁷Investors who are both price-informed and supply-informed can easily be incorporated in this framework. Adding an I/SI investor has the same impact as adding an I and an SI investor separately.

⁸Our "market-makers" behave competitively, taking prices as given. In contrast with Lawrence Glosten and

Paul Milgrom (1985), we do not require that all trades be completed through the market-makers, or that market-makers are risk-neutral. Our market-makers absorb the aggregate excess supply generated by other traders at the equilibrium price. We discuss their contribution to market liquidity in Section IV.

change specialists, including lines of credit, is approximately \$3 billion (Brady et al., 1988 p. VI-40). Total capital committed by “upstairs” trading houses and other forms of market-making may be four or five times this number;⁹ but at \$15–\$20 billion, these supply-informed funds would represent about 0.5 percent of total market capital.

Capital devoted to “price-informed” market timing is even more difficult to estimate. It would include funds that explicitly gather information about future economic prospects (“fundamentals”) and engage in market timing strategies reflecting this information. While many funds actively alter their exposures to individual stocks, most do not actively alter their total stock exposure based on information about future economic trends, perhaps because long-term success stories have been so rare (see Roy Henriksen, 1984); but a few do. The single most prominent market timing strategy is “tactical asset allocation,” utilized by perhaps \$20 billion of assets. A total of \$70 billion, or about 2 percent, might be a guess for price-informed funds which actively gather information about future prices and trade on it.¹⁰

This leaves most investors in the class we term “uninformed.” “Passive” might be a somewhat less pejorative description of these investors, who participate in the market “for the long haul” and do not move in and out based on information about fundamentals or current liquidity trading. The relative lack of popularity of information-based market timing strategies suggests that most investors belong to this class.

II. Market Equilibrium

A single risky security is traded. Its future price (or liquidation value) p is a normally

distributed random variable with unconditional variance Σ and unconditional expectation \bar{p} . All investors share this prior distribution of future price. Current price is determined by supply and demand. Riskless bonds are also traded, and the riskless rate is zero.¹¹

A. Supply

The net supply of stocks is a fixed amount \bar{m} , modified by two additional factors:

- (1) A random and exogenously determined net supply created by “liquidity traders.” This shock is composed of two pieces: an unobserved liquidity shock, L , distributed $N(0, \Sigma_L)$; and a liquidity shock, S , distributed $N(0, \Sigma_S)$, which is observed by all supply-informed investors.¹² S and L are assumed to be independently distributed.
- (2) A deterministic supply by hedgers, rebalancers, and others who utilize dynamic strategies akin to portfolio insurance. The supply of stock from these strategies is a known function of the current market price p_0 . We denote this supply by π , a decreasing differentiable function of p_0 . This hedging demand will be observed by different market participants in the alternative environments that are considered below.

Thus, the total supply is

$$\bar{m} + L + S + \pi$$

or

$$(1) \quad m + \pi$$

where $m = \bar{m} + L + S$.

⁹Conversations with officers at major investment banking firms.

¹⁰We assume that the informed investors have information of value. It is difficult to assess the fraction of the market timing based on “quality” fundamental information, but it is most probably smaller than the fraction engaged in timing in general. Investors trading on spurious information would add to the amount of random “liquidity” trading.

¹¹All the results extend in a straightforward way to the case of nonzero interest rates.

¹²An equivalent formulation would allow the supply-informed investors to receive a noisy signal about total liquidity trading and not distinguish S from L . A simple transformation of variables allows the alternative interpretation.

B. Demand

As discussed above, there are three classes j of investors characterized by the information signals (if any) they receive. All investors maximize expected utility of terminal wealth over a single period. Preferences are assumed to exhibit constant absolute risk aversion. The utility function of each investor in class j is a function of terminal wealth W and is given by

$$U_j(W) = -\exp(-W/a_j).$$

Expectations depend upon the signals that investors observe. Each price-informed investor i observes $p_i^t = p + \varepsilon_i$, where p is the true future price and ε_i denotes a noise term uncorrelated with other random variables and uncorrelated across price-informed investors. Both p and ε_i are assumed to be normally distributed, as $N(\bar{p}, \Sigma)$ and $N(0, \Sigma_\varepsilon)$, respectively.¹³ Supply-informed (SI) investors receive a common signal, S , where S is the liquidity supply, distributed $N(0, \Sigma_S)$ and observed only by SI investors. All investors can observe the current market price and use this (and their other information) to determine their conditional distributions of future price.

All investors in class j have the same conditional variance Z_j for future price. The expectation of the future price conditional on the information available to an agent i belonging to class j is denoted \bar{p}_j^i . It is well known that, given exponential utility and normality, the portfolio optimization problem leads to a demand for shares by investor i in class j equal to

$$n_j^i = a_j Z_j^{-1} (\bar{p}_j^i - p_0).$$

There are w_j investors of type j . Demand per investor in class j , $n_j \equiv \sum_i n_j^i / w_j$, is equal to

$$n_j = a_j Z_j^{-1} (\bar{p}_j - p_0)$$

where \bar{p}_j is the mean expected future price for investors in class j . All supply-informed investors observe the same signals and hence have the same conditional expected price. This also is the case for uninformed investors. Price-informed investors receive different signals, ε_i . However, as their number increases, the mean expected future price converges to the actual future price p by the law of large numbers.¹⁴

The relative market power of investor class j , k_j , is defined as the ratio of the weighted risk tolerance of the class to the sum of the weighted tolerances:

$$k_j \equiv a_j w_j / \sum_j a_j w_j.$$

Define the normalized total demand D as the sum of the individual classes' demands divided by the weighted sum of the three classes' risk tolerance, $\sum_j w_j a_j$:

$$(2) \quad D \equiv \frac{\sum_j w_j n_j}{\sum_j w_j a_j} = \sum_j k_j Z_j^{-1} (\bar{p}_j - p_0)$$

Similarly, the supply parameters: π , \bar{m} , S , and L are normalized by dividing the original parameters by $\sum_j w_j a_j$; we do, however, keep the same notation. Our analysis focuses on relative proportions and is thus unaffected by this normalization.

C. Equilibrium

Equilibrium of supply (1) and demand (2) yields the equation for equilibrium price:

$$Z^{-1} p_0 + \pi(p_0) = \sum_j k_j Z_j^{-1} \bar{p}_j - m$$

¹³For simplicity and notational convenience, we assume that the ε_i are i.i.d. across agents. Differences in information precision would not affect our results.

¹⁴Hellwig (1980) shows the error terms ε_i do cancel in the limit of a sequence of finite economies where the relative proportion of investors in each class remains fixed and where the total number of investors and the supply parameters grow without bound at the same rate. Individual agents are thus price-takers and, importantly, the individual error terms ε_i do not affect prices.

where Z^{-1} is $\sum_j k_j Z_j^{-1}$. We may now characterize the equilibrium price function relating current price to future price (as revealed by the average of individual signals p_i), unobserved liquidity supply (L), and observed supply (S).

In our postulated environment, with all investors cognizant of hedging supply $\pi(p_0)$, we can show the following.

THEOREM 1: *There exists a rational expectations equilibrium (REE) of the form*

$$(3) \quad p_0 = f(p - \bar{p} - HL - IS)$$

where $f(\cdot)$ is a correspondence and where H and I are real constants that depend only on the agents' relative market power and on the means and variances of the random variables.

The proof and all derivations, as well as expressions for $f^{-1}(\cdot)$, H , and I , are provided in the Appendix.

The price correspondence $f(\cdot)$ in Theorem 1 can be discontinuous and multivalued. We can, however, characterize precisely the situations in which crashes can be ruled out as follows.

PROPOSITION 1: *$f(\cdot)$ is a continuous function if and only if $Z^{-1}p_0 + \pi(p_0)$ is strictly monotonic in p_0 .*

PROPOSITION 2: *In the absence of hedgers [$\pi(p_0) = 0$], $f(\cdot)$ is a continuous function, and no "crashes" can occur.*

In the absence of hedgers or if the hedging supply is a linear function of equilibrium prices, f is a linear function [equation (A4) in the Appendix]. The REE function f is nonlinear if the hedging supply is a nonlinear function of equilibrium prices p_0 . Even though current price p_0 is not normally distributed due to the nonlinearity of $f(\cdot)$, investors recognize that a simple transformation of p_0 , namely $f^{-1}(p_0)$, is normally distributed and can be used to condition expectations of future price p . In the following two sections, we examine aspects of equilibrium price behavior in the absence of hedgers.

III. The Nature of Equilibrium Pricing: An Example

Consistent with our earlier discussion, we consider an example in which there are relatively few price-informed and supply-informed investors. We assume that 0.5 percent of investors (market-makers) are supply-informed and that 2 percent of investors are price-informed. There is no hedging supply π ; this will be introduced in Section IV.

Several other parameters must be specified before the model is complete. A key parameter is the quality of the information signal received by the price-informed investors. The better the signal, as expressed by the signal-to-noise ratio, the lower the conditional variance Z_1 for the price-informed investors.

We assume that the quality of the signal received by each price-informed investor is not very high. Specifically, we assume that the price-informed investors' signal-to-noise ratio is 0.2. Thus, if Σ is the *ex ante* variance of future price and Σ_e is the variance of each price-informed investors' future price signal, then Σ_e is five times Σ . This assumption implies that (in equilibrium) the price-informed investors' conditional standard deviation for future price is 19.1 percent, rather than the 20 percent of uninformed investors, who observe price only. This slight improvement seems consistent with the perceived difficulty in predicting future market prices.¹⁵

Also important is the fraction of total liquidity-supply shock that, on average, can be observed by supply-informed investors. Since S is observed and L is not, this fraction can be parameterized by the ratio Σ_S / Σ_L . If the ratio is high, then supply-informed investors on average will observe most of the total liquidity shock. We assume that the ratio is 1: On average, supply-

¹⁵While each individual signal about future price is quite noisy, the average signal perfectly reflects future price, as in Hellwig (1980). But individual investors cannot "back out" the true future price from current price, because supply is noisy.

informed investors receive a signal that reveals information about half the total liquidity-supply shock. Conditional on the supply signal, a supply-informed investor estimates a 19.2-percent standard deviation for future price.

Our example is consistent with a linear rational expectations price function as in Theorem 1. We choose Σ , the *ex ante* variance of future price, and \bar{m} , the fixed supply, to equate the expected return on the risky security to 6 percent and the standard deviation (conditional on current price) to 20 percent.¹⁶ Finally, we find a variance of supply Σ_m such that the variance of p_0 , the current price, is equal to the variance of the future price p , conditional on p_0 . This provides the example with intertemporal consistency: expected future price volatility (given current price) equals current price volatility. Parameters for the example are summarized in the Appendix.

The rational expectations equilibrium price function (3) for our example is

$$(4) \quad p_0 = 0.5(p - \bar{p} - 19.95L - 8.14S) + 1.$$

Given this price function and the volatilities of future price and liquidity surprises, the standard deviation of p_0 is 20 percent, as is the standard deviation of p conditional on p_0 . The example is chosen to reflect "reasonable" parameters when the model's single risky security is interpreted as the stock market portfolio and will be used in subsequent sections to illustrate aspects of market behavior.

IV. Stock Market Liquidity

Because of the Brady Commission's focus on limited stock market liquidity, we examine the impact of changes in supply on market price. We postulate a small percent-

age change in supply and determine the resulting percentage change in the equilibrium price from the pricing relation (3). This will then determine the (inverse of the) price elasticity of the market. We interpret greater price elasticity as a more liquid market. In this section, we continue to assume that there is zero net hedging: $\pi = 0$.

We study three possibilities: supply increases, and

- (1) the increase in supply is known to all investors;
- (2) the supply-informed investors (only) receive an accurate signal about the increase in supply;
- (3) no signal is received by the supply-informed investors (or anyone else).

The first possibility is modeled by letting the expected supply \bar{m} change. A change in expected supply will be common knowledge and will not affect investors' expected future price. From equations (3) and (A4) in the Appendix,

$$\text{Elasticity} = - \frac{\frac{\delta \bar{m}}{\bar{m}}}{\frac{\delta p_0}{p_0}} = \frac{p_0}{\bar{m}} Z^{-1}.$$

Given the example (4), we find an elasticity of 17: a 1-percent observed supply increase will lead to a 0.06-percent fall in price. Such a high elasticity is very much in line with the predictions of traditional models, which do not postulate that investors learn from market prices. Investor classes participate proportionately to their market power k_j in absorbing the increase in supply.

The second possibility is modeled by a small increase in the random supply S which is observed only by the supply-informed investors. In this case,

$$\text{Elasticity} = - \frac{\frac{\delta s}{\bar{m}}}{\frac{\delta p_0}{p_0}} = \frac{p_0}{\bar{m}} \frac{1}{FI}$$

¹⁶Recall that the interest rate is normalized to 0. Thus, the assumed return of 6 percent represents a 6-percent premium over the riskless interest rate. The risk and excess return of the risky security in our example are consistent with the long-term risk and excess return of the stock market as estimated by Ibbotson Associates (1985).

where F is the slope of the price function f , which is linear in our example.

In the example (4), we find an elasticity of 0.16: a 1-percent partially observed increase in supply will lower price by 6 percent. Remarkably, this is only 1 percent of the elasticity above. This is because investors, with the exception of the supply-informed investors, revise downward their expectations (which are conditional on p_0) as price falls. Thus, they are less willing to absorb the increased supply. Indeed, in our example, the supply-informed investors absorb about 54 percent of any increase in liquidity supply, which they observe, even though they constitute only 0.5 percent of investors.¹⁷

Price-informed traders, who represent 2 percent of investors, absorb another 18 percent of the increase in liquidity supply. They are more willing to buy as prices fall because (on average) they receive signals about future price that moderate the fall of expected future price. Uninformed investors have no such signals and can only infer from current price. Because they impute a lower future price as current price falls, they absorb but 28 percent of the increased supply, despite the fact that they constitute 97.5 percent of investors.

The final possibility is modeled by an increase in L . In this case, elasticity falls even further, since the supply-informed traders will not observe the increase in supply and will not increase their demand. From (3), we can determine that

$$\text{Elasticity} = - \frac{\frac{\delta L}{\bar{m}}}{\frac{\delta p_0}{p_0}} = \frac{p_0}{\bar{m}} \frac{1}{FH}.$$

¹⁷The exponential utility model does not limit purchases by investors to their initial wealth. If we imposed a "no leverage" condition, elasticity in this case would be even lower. This is because in our equilibrium, supply-informed investors will buy tremendous amounts of stock (on a per capita basis) when prices fall. This would only be possible if they can undertake levered stock positions.

In the example (4), elasticity will be 0.07, or about 1/250 of the elasticity predicted by traditional portfolio/consumption models. A 1-percent unobserved increase in supply will lower prices by 14 percent. In this case, price-informed investors will absorb about 40 percent of the supply increase, and uninformed investors absorb the remaining 60 percent.¹⁸ But price must fall precipitously to induce them to absorb the extra supply.

The model therefore resolves the paradox of low versus high demand elasticity. If supply changes are unobserved, all investors will revise downward their expected future price and will absorb the increased supplies only after price has fallen substantially. Price-informed investors will have somewhat greater elasticity of demand than uninformed investors, since they receive independent information about future prices. However, their contribution will be minimal if they are few, or if their price information is very noisy.

How supply-informed investors, or market-makers, contribute to market liquidity (and therefore to price volatility) depends on the quality of the supply signal they observe. When their signal has low precision, they side with the uninformed investors, who always sell when prices rise and always buy when prices fall. Their average selling price therefore is higher than their average buying price, providing a positive spread much like the uninformed market-maker in Lawrence Glosten and Paul Milgrom (1985). Because their actions are not aggressive and their numbers are relatively small, market-makers with poor supply information will reduce volatility only marginally.

When market-makers receive more precise information about the extent of liquidity trading, their behavior changes. They aggressively take the other side of observed liquidity trades, thereby reducing the price volatility associated with liquidity shocks;

¹⁸Supply-informed investors play little role in this scenario, since they observe $S = 0$. In fact, they actually sell (a small amount) rather than buy, since observing $S = 0$ implies that price information is more likely to be negative, given the fall in prices.

but supply-informed investors will trade in the same direction as informed traders when their information indicates that there is little liquidity trading. This behavior tends to accentuate price moves related to information and suggests that a further examination of the market-makers' role is warranted.¹⁹

Only the supply-informed market-makers will have a high elasticity of demand and the consequent ability to absorb liquidity trades. But these participants are relatively few in number and risk aversion ultimately limits their ability to absorb liquidity selling without a substantial price drop. In sum, traditional models will grossly overestimate the liquidity of financial markets, unless all investors observe the increase in supply.

V. Hedging Strategies and Market Stability

We now consider the situation when there is hedging activity in the market. Hedgers sell as stock prices fall. They do so to protect themselves against further potential losses. Whether hedge programs are carried out by portfolio insurance programs or by less formal means such as stop-loss orders, the result is the same: supply increases as prices fall. This selling must be absorbed by other investors.

It is generally believed that hedge programs can make markets more volatile. But can they lead to a crash or "meltdown," in which selling begets selling and prices plunge without stop? Phrased more formally, can the function relating price to underlying information become discontinuous?

We examine these questions by adding a hedging supply (π) to a market that previously did not have such a supply. At the initial equilibrium price ($p_0 = 1$), we normalize hedging supply to be zero. For prices below $p_0 = 1$, the hedging supply will be positive; for prices above, negative. We assume that the hedging supply is a continuous and differentiable function of p_0 .

The key to the stability of markets is the extent to which hedging strategies are observed by investors. Parallel to our discussion of market liquidity, we consider three cases: all investors observe the hedging supply function π ; only supply-informed investors observe hedging; and no investors observe hedging. In the first case, agents have perfect knowledge of the market structure; in the others, some agents underestimate hedging activity.²⁰

Theorem 1 characterized equilibrium when all investors can observe the hedging supply function $\pi(\cdot)$. We can further show the following.

PROPOSITION 3: *When all investors observe hedging supply π , (i) the current equilibrium price will be more volatile than when there is no hedging supply, and (ii) the equilibrium price function can be discontinuous when hedging supply is sufficiently large.*

Although discontinuities and, therefore, crashes can occur with full observability of π , we shall show that crashes are unlikely in this environment.

We now characterize equilibrium when hedging is partially observed or unobserved.

THEOREM 2: *If hedging supply $\pi(\cdot)$ can be observed only by supply-informed investors or is totally unobserved, there exists a price equilibrium*

$$p_0 = f(p - \bar{p} - HL - IS)$$

²⁰Through time, investors might learn of the existence of hedgers. Modeling this would require a multi-period framework. However, the amount of hedge trading typically is small if the price level is not "too" close to the critical points, making it difficult to infer the extent of hedging. Further, when hedgers first enter the market, there is no mechanism whereby uninformed traders could immediately recognize their presence. An extension of our model might allow investors to have some prior probability distribution of hedging supply given current price and use observed prices to infer the likelihood of hedging activity as well as liquidity shocks and future price information. We hope to explore this more complex problem in subsequent work.

¹⁹Perhaps recognizing the incentives of supply-informed market-makers to accentuate price movements by their own trading activities, exchange rules require specialists to maintain "orderly markets."

where $f(\cdot)$ is a correspondence that depends upon the extent of observability, but H and I are real constants which are identical to those when hedging supply is fully observed.

Partial observability or nonobservability therefore does not affect the argument of $f(\cdot)$ but does affect its functional form. In the context of Theorem 2, some investors do not observe hedging supply and thus make an erroneous assumption on the functional form for equilibrium prices. This incorrect assumption is reflected in the functional form of actual equilibrium prices. The agents who are aware of hedging plans, however, know the actual functional form. For the cases in which some agents are aware of hedging plans, the same equilibrium would obtain if the agents were unaware of hedging plans but were able to identify the hedging trades as liquidity (i.e., information-free) trades. This property explains why the difference in the excess demand equations (5) below amounts to adding hedging activity π to the expected supply \bar{m} [in (5i)], to the observed liquidity trades S [in (5ii)], and to the unobserved liquidity trades L [in (5iii)].

We further show the following.

PROPOSITION 4: *When only supply-informed investors observe hedging activity π , (i) the current price will be more volatile than when all investors observe a similar amount of hedging activity, and (ii) the equilibrium price can become discontinuous at a lower level of hedging activity than when all investors observe hedging.*

PROPOSITION 5: *When all investors are ignorant of hedging activity π , (i) current price is even more volatile than when only supply-informed investors observe hedging activity, and (ii) discontinuities can occur at even lower levels of hedging activity.*

The maximal hedging level before price discontinuities or “crashes” can occur thus depends critically on whether hedging is observed. It also depends upon the nature of hedging strategies. We assume that a fraction ω of assets are protected by a put-

option replicating strategy.²¹ The supply created by this portfolio-insurance hedging strategy will depend on the current stock price p_0 . The incremental hedging supply when future price is p_0 , relative to the supply at the initial equilibrium price ($p_0 = 1$), is given by

$$\pi = \omega \{ N[d_1(1)] - N[d_1(p_0)] \}$$

where ω is the fraction of assets subject to the hedging strategy, $N(\cdot)$ is the cumulative normal distribution function, and d_1 is given by the Black-Scholes formula

$$d_1(p_0) = \frac{\ln\left(\frac{p_0}{K}\right) + \frac{1}{2}\sigma^2}{\sigma}$$

where K is the striking price of the option and σ is the standard deviation of p conditional on p_0 .²² Note that $\pi'(p_0)$, the derivative of the hedging supply with respect to p_0 , is negative for large p_0 and becomes more negative as p_0 falls, before eventually approaching zero as p_0 falls to zero.

With the three alternative specifications above for $f(\cdot)$ depending on observability, we can derive the excess-demand functions as p_0 varies. From the Appendix, the equations for excess demand are given by

$$(5i) \quad XD_A = \frac{1}{H} \left[p - \bar{p} - HL - IS + \frac{Z^{-1}(\bar{p} - p_0) - (\bar{m} + \pi)}{Z^{-1} - \Sigma^{-1}} \right]$$

²¹Put-replicating strategies are just one possible type of hedging. Others might include stop-loss, “constant proportion of surplus” policies, or do-it-yourself strategies. We examine put-option replication because it was the most prevalent of formal protection strategies on October 19.

²²See Black and Myron Scholes (1973). Our formula assumes that the interest rate has been normalized to 0, and assumes a one-year time horizon. Note that the Black-Scholes hedge replicates a put option when future price p follows a lognormal process; our model presumes that p is normally distributed.

when all investors observe π ,

$$(5ii) \quad XD_P = \frac{1}{H} \left[p - \bar{p} - HL - IS + \frac{Z^{-1}(\bar{p} - p_0) - \bar{m}}{Z^{-1} - \Sigma^{-1}} \right] - \frac{I}{H} \pi$$

when only supply-informed investors observe π , and

$$(5iii) \quad XD_U = \frac{1}{H} \left[p - \bar{p} - HL - IS + \frac{Z^{-1}(\bar{p} - p_0) - \bar{m}}{Z^{-1} - \Sigma^{-1}} \right] - \pi$$

when no investors observe π .

These three functions are graphed in Figure 1 for the parameters in our earlier example, with 5 percent of investors following a put replicating hedge strategy with the protected level being 90 percent of initial price ($\omega = 0.05$, $K = 0.9$). Note that the fully anticipated excess-demand function is the flattest; neither it nor the partially anticipated excess-demand function is "backward bending." However, the unanticipated excess demand function is backward-bending. The three curves in Figure 1 intersect at $p_0 = 1$. Thus, in the absence of future price or liquidity shocks, the price $p_0 = 1$ is an equilibrium in all three cases.

Now, suppose that information signals about future price become slightly more pessimistic: $p - \bar{p} = -0.01$, a 1-percent downward shock. This will cause demand to fall slightly, thereby shifting all three curves to the left by the same small amount. Figure 2 depicts this shift.

To restore equilibrium, price will fall in all three cases, until excess demand again is zero. The excess-demand curve when hedging is unobserved has the steepest slope: the resulting price drop (2.7 percent) to restore equilibrium will be greater than the price drop (0.7 percent) to restore equilibrium in the partially observable case, which in turn will be greater than the price drop (0.5 percent) to restore equilibrium in the fully observable case.

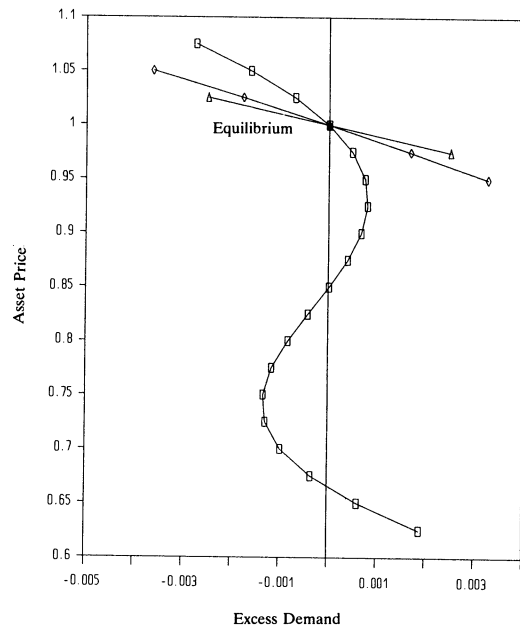


FIGURE 1. AGGREGATE EXCESS DEMAND IN THE ABSENCE OF FUTURE PRICE OR LIQUIDITY SHOCK. SYMBOLS: \square = UNOBSERVED; \diamond = PARTIALLY OBSERVED; \triangle = ANTICIPATED

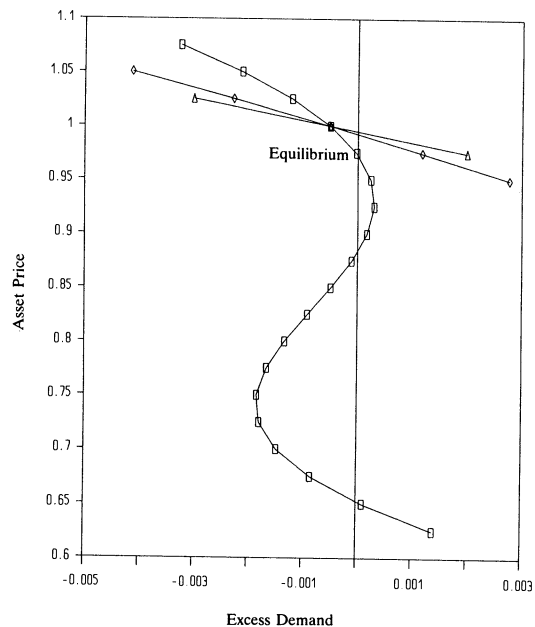


FIGURE 2. AGGREGATE EXCESS DEMAND WITH 1-PERCENT DOWNWARD SHOCK. SYMBOLS: \square = UNOBSERVED; \diamond = PARTIALLY OBSERVED; \triangle = ANTICIPATED

In short, the market price is more volatile in response to future price shocks when hedging supply is unobservable. It will also be greater in all cases if ω , the proportion of hedgers, becomes larger. When hedging activity is unobserved, volatility increases because investors believe a change in fundamentals is more probable, creating a magnified price response.

A. Prelude to a Crash

We continue to examine market behavior as information about future price becomes (continuously) more pessimistic. Figure 3 indicates the situation for $p - \bar{p} = -0.016$. Relative to our initial equilibrium (at $p_0 = 1$), the average of future price signals is now 1.6 percent more pessimistic than the case in Figure 2. Of course, price must drop further to restore equilibrium. This in turn creates further portfolio insurance selling.

How far does price drop? This depends on how completely portfolio-insurance selling is observed. If every investor observes π , the price falls from 1 to 0.992, or 0.8 percent; if it is observed only by the supply-informed, price falls by 1.2 percent; but if no one can observe π , price will fall 7.25 percent in response to the signal (-0.016), almost t n times as far as when π is fully observed.

Note that, in the case of unobserved π , the market also becomes more sensitive to future price signals. This can be seen in Figures 2 and 3 by noting the fact that excess demand is becoming a steeper function of p_0 . Thus, volatility of the market is increasing as p_0 falls.

The move from the situation in Figure 1 to the situation in Figure 3 seems to correspond to the steady erosion of confidence that occurred during the month leading up to October 19, 1987. As the Brady Commission Report documented, a number of negative economic trends came to light during this period: interest rates were rising; the dollar was weakening; tensions in the Middle East were increasing; and so on. In our model, this is reflected by a sequence of negative signals about future price.

As the market fell, portfolio-insurance programs became more active. At higher

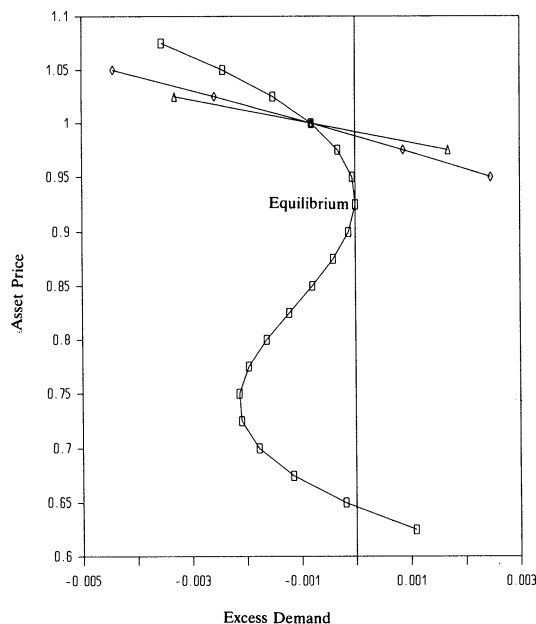


FIGURE 3. AGGREGATE EXCESS DEMAND WITH 1.6-PERCENT DOWNWARD SHOCK. SYMBOLS: \square = UNOBSERVED; \diamond = PARTIALLY OBSERVED; \triangle = ANTICIPATED

market levels, not much hedging was necessary, given the relatively low levels of protection chosen by many pension funds. As the market fell closer to the desired protection level, greater hedging was needed, and the market became more volatile. Yet although portfolio insurance was beginning to attract some public attention, it was largely unknown to the majority of investors and not fully understood even by market professionals. It was Friday, October 16, 1987.

B. The Crash

Figure 3 shows the market at a critical point when hedging strategies are not observed (such as on October 16). Prior to October 16, 1987, prices had fallen strongly over the previous several trading sessions, with great volatility. Over the weekend, a bit more negative news came into the market—nothing earthshaking, but enough to shift the backward-bending excess-demand curve a fractional amount further to the left.

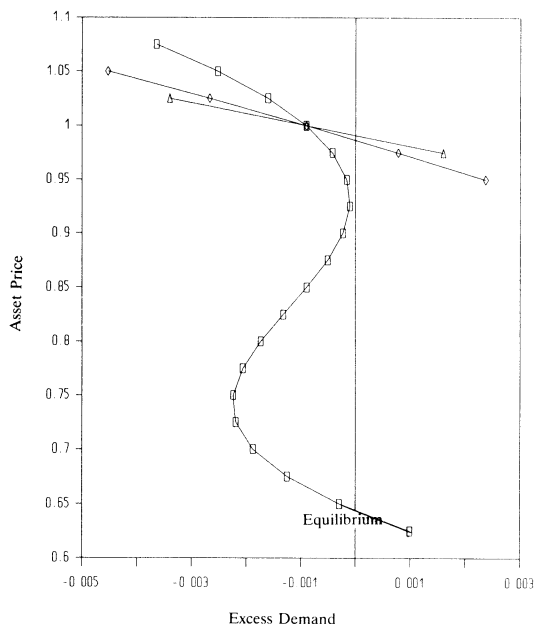


FIGURE 4. AGGREGATE EXCESS DEMAND WITH 1.8-PERCENT DOWNWARD SHOCK. SYMBOLS: \square = UNOBSERVED; \diamond = PARTIALLY OBSERVED; \triangle = ANTICIPATED

Figure 4 illustrates the situation as it may have been on Monday morning, October 19, 1987. The value of $p - \bar{p}$ has fallen to -0.018 , slightly below its previous level. The marginally negative news over the weekend, coupled with further portfolio-insurance selling (including some resulting from Friday's decline) led to rapidly falling prices.

Observing these falling prices, uninformed investors (rationally) concluded that highly negative information must have been received by the price-informed investors. (Indeed, the following day's newspapers vainly sought the information event which "must" have triggered the crash.) As reported by Robert Shiller (1987), the majority of investors stood on the sidelines or bought only limited amounts, consistent with a conviction that something unknown but terrible must have happened. Investors surveyed by Shiller reacted more to the crash itself than to outside news. Meanwhile, hedgers were selling ever larger amounts.

As Figure 4 shows, excess supply actually increased as prices began to fall, leading

them to fall even further. The feared meltdown was actually happening. Only when hedgers had largely completed their selling did the market stabilize, but at a much lower level. In Figure 4, our example shows a postcrash equilibrium price of $p_0 = 0.64$: a 30 percent drop from its previous closing price in Figure 3. While the market on October 19 did not fall quite this far, it also is the case that many hedgers scaled back the size of their hedging programs in the face of extraordinarily high transactions costs.²³

A similar story could be told about the 1929 crash. The only difference is that portfolio-insurance hedging would be replaced by stop-loss hedging. While less exact in delivering desired results, stop-loss orders have the effect of increasing liquidity supply as prices fall. It is no accident that investigators focused on the role of stop-loss orders and margined stock buying, since the latter forced additional stop-loss selling as the market descended.

It should be emphasized that a crash in our model is not due to a discontinuous change in the underlying information. Rather, the market reaches a critical point, and a "catastrophe" occurs, both in practice and in theory.²⁴ While hedging strategies are an important part of our explanation of the crash, equally important is the market structure which precludes observing these hedging strategies. Figures 1–4 also plot the excess-demand functions associated with partial or complete observability. These "regular" (i.e., not backward-bending) excess-demand functions eliminate the possibility of crashes in our example. Indeed, if π programs had been fully observable, prices would have fallen a modest 1 percent; and the fall would have been about 1.5 percent if supply-informed investors (only) had observed the extent of π sales.

²³For a description of how hedging programs were modified in the presence of high trading costs, see Leland (1988).

²⁴Hal Varian (1979) discusses catastrophe theory and its relation to economic models. Our discontinuity represents a "cusp catastrophe," as discussed in Section VI.

This is not to say that crashes are impossible with partial observability. If we had assumed a 15-percent use of portfolio insurance ($\omega = 0.15$), the excess-demand curve with partial observation would be backward-bending. A 15-percent use would represent over \$500 billion, or more than five times the total amount estimated for formal programs. Shiller's survey suggested that formal portfolio-insurance programs were "the tip of the iceberg" relative to total hedging, so it is possible that the crash could have occurred in our example even with supply-informed traders aware of hedging supply.

C. After the Fall

A new low-price equilibrium is established after the crash. If information about future prices now reverses itself, returning it to precrash levels of optimism, will the market rebound to its former level? The answer is no. In Figure 4, a small rightward shift of the excess-demand function will lead to a small increase in equilibrium price p_0 from the 0.64 level. Even if the upper branch of the excess-demand curve intersects the zero-excess-demand line, implying the possibility of multiple equilibria, the lower equilibrium price is locally stable and can be expected to prevail. Since the slope of the excess-demand curve is less steep at $p_0 = 0.64$ than just before the crash (when $p_0 = 0.9275$), price volatility will return to lower levels.

Eventually, if information becomes still more favorable (to $p - \bar{p} = 0.026$, well above precrash levels) and if the hedging function π remains the same as before the crash, the excess-demand curve will shift sufficiently to the right such that its lower branch is just tangent to the vertical zero-excess-demand line (see Fig. 5). This will be accompanied by higher volatility. Any further increase in future price expectations could lead to an upward jump in prices: a "meltdown" rather than a meltdown. In our example, the discontinuous jump would commence at $p_0 = 0.74$ (15 percent above the market low) and jump to $p_0 = 1.043$.

Perhaps such an upward jump is possible only in the mind of the theorist. However,

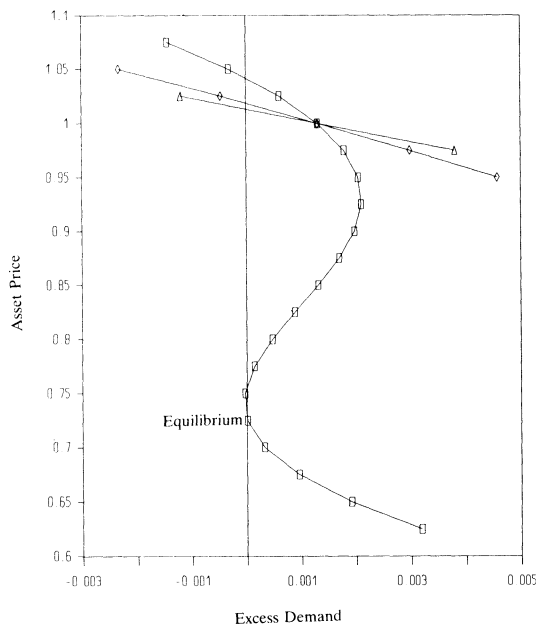


FIGURE 5. AGGREGATE EXCESS DEMAND WITH 2.6-PERCENT UPWARD SHOCK. SYMBOLS: \square = UNOBSERVED; \diamond = PARTIALLY OBSERVED; \triangle = ANTICIPATED

over the period 1928–88, 22 of the 38 one-day stock market moves that exceeded 7 percent were upward jumps, and the financial press occasionally remarks on such a possibility (see Anise Wallace, 1989).

Figure 6 graphs the equilibrium price function relating p_0 to the future price surprise, $p - \bar{p}$, for the three different observability cases. For the case with unobserved π , we see that the point of discontinuity on the upper branch of the function is at $p_0 = 0.927$, and the discontinuity on the lower branch is at $p_0 = 0.740$. For the other two cases, there are no discontinuities given our example's parameters.

Future price surprises are not the only possible sources of discontinuous price behavior. A random liquidity-supply shock could also lead to discontinuous behavior. But whatever the cause, the critical price (i.e., where the discontinuity occurs) will remain the same. This leads us to examine the general nature of critical points: when do they occur, and what determines their level?

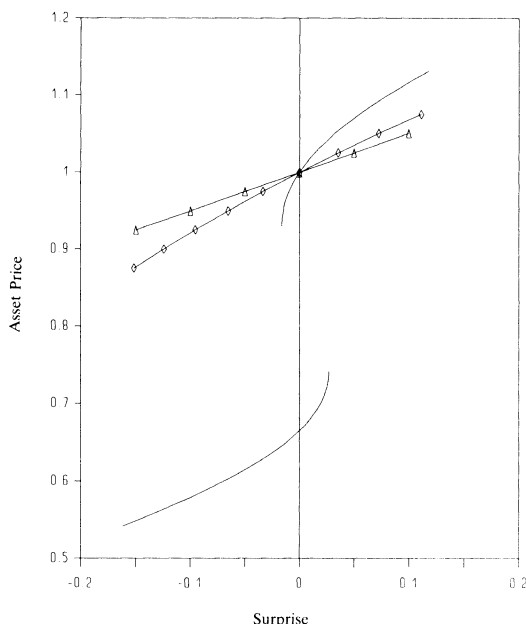


FIGURE 6. EQUILIBRIUM PRICE AS A FUNCTION OF $p - E(p)$. SYMBOLS: — = UNOBSERVED; \diamond = PARTIALLY OBSERVED; Δ = ANTICIPATED

VI. Discontinuities: Some General Results

We now characterize price levels at which the price function becomes discontinuous and the minimum amount of hedging with which a “crash” can occur. These critical points depend upon the extent to which hedging can be observed.

First consider the case in which hedging strategies are unobservable: investors are unaware of hedging strategies and thus do not distinguish them from unobservable liquidity trades. The equilibrium price p_0 is the price level for which excess demand is equal to zero [eq. (5iii)].

Discontinuities will occur if the root (or roots) of (5iii) are discontinuous functions of the variables p , L , and S . Since excess demand is continuous and differentiable in p_0 , discontinuities will take place at points where the function reaches an extremum. Differentiating (5iii) with respect to p_0 yields

$$\frac{\delta XD_U}{\delta p_0} = - \left(\frac{1}{FH} + \pi' \right)$$

where $F = 1 - Z/\Sigma > 0$ is the coefficient that obtains in the case of no hedging (agents are unaware of hedging in this case) and where

$$\pi' = -\omega \frac{N'(d_1)}{p_0 \sigma} (< 0).$$

The derivative of the demand for hedging (π') tends to zero as prices become large and as prices become small; hence, the derivative of excess demand is negative at both very high and very low prices.

The equilibrium price function will be discontinuous if and only if there exists a p_0 such that $(1 + FH\pi') < 0$. This implies prices at which the excess-demand curve is backward-bending and also implies that

$$(6) \quad 1 + FH\pi' = 0$$

admits a solution. Then, since π has a unique inflection point in this case, equation (6) has two solutions, the critical prices c_1 and c_2 ($c_2 > c_1$). Excess demand is an increasing function of equilibrium price p_0 in the interval (c_1, c_2) and decreasing elsewhere. It can also be shown that the first critical point, c_1 , decreases as FH increases and the second, c_2 , increases as FH increases. Equation (6) has two roots if and only if

$$\omega FH > Ke^{-(3/2)\sigma^2} (2\pi\sigma^2)^{1/2} \equiv \phi_{\min}$$

implying

$$\omega_{\min} = (FH)^{-1} \phi_{\min}.$$

The root is unique (and there is no discontinuity) when equality obtains. ω_{\min} represents the largest proportion of hedgers for which a crash does not occur. Note that ω_{\min} also is the upper bound of ω for which the inverse price function $f^{-1}(p_0)$ is monotonically increasing.

The critical prices, c_1 and c_2 , are given by

$$(7) \quad c_1 = Ke^{-2\sigma^2} e^{-[2\sigma^2 \ln(\omega FH / \phi_{\min})]^{1/2}}$$

$$c_2 = Ke^{-2\sigma^2} e^{[2\sigma^2 \ln(\omega FH / \phi_{\min})]^{1/2}}$$

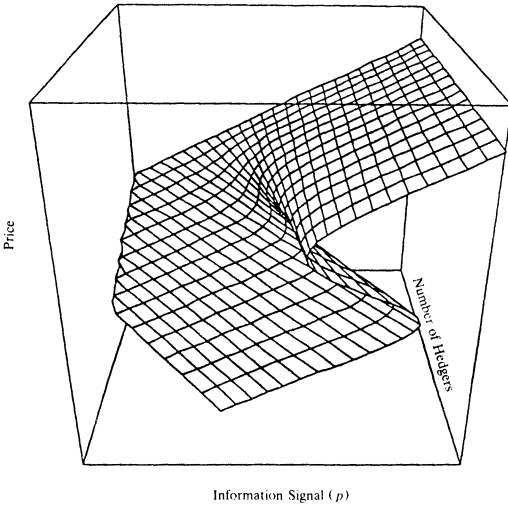


FIGURE 7. EQUILIBRIUM PRICE AS A FUNCTION OF FUNDAMENTALS AND THE NUMBER OF HEDGERS

The difference $c_2 - c_1$ is the range of prices p_0 for which no stable equilibrium exists; however, the amount of the price drop when c_2 is reached from above is larger than this difference. In our base case, the discontinuity occurs for a value of ϕ_{\min} of 0.425, which is reached for $\omega_{\min} = 4.26$ percent. This percentage of hedgers can create a market crash in our example. Conversely, the market meltup takes place when the equilibrium price reaches c_1 from below and the price jumps to a level higher than c_2 (see Fig. 5). Figure 7 graphs equilibrium price as a function of information signals and the fraction of hedgers ω . The graph indicates a “cusp catastrophe”: for low values of ω , there is a unique price equilibrium for each signal realization; for higher values, three distinct equilibria exist in the “fold” area. The fold points correspond to the critical points of Figure 6. The cusp point corresponds to ω_{\min} . Note that the interval (c_1, c_2) corresponds to the interval over which the function $p_0 + FH\pi(p_0)$ is decreasing in p_0 and, therefore, is the range of prices over which the inverse price function $f^{-1}(\cdot)$ is decreasing and multiple price equilibria exist.

Now consider the case in which there is partial observation of hedging strategies: supply-informed investors can observe the

sum of S and π . The same reasoning leads to an equation analogous to (6):

$$1 + FI\pi' = 0$$

implying

$$\omega_{\min} = (FI)^{-1} \phi_{\min}.$$

The critical points are obtained by substituting I for H in (7). Since $H > I$ in all cases, the minimum fraction ω_{\min} (10.4 percent in our example) is higher than in the previous case, c_1 is larger, c_2 smaller, and ceteris paribus the price drop is smaller.

Finally, in the fully observed case, identical results obtain provided that FH is replaced with Z : discontinuities require $1 + Z\pi' < 0$. Note that this does not appear to be a major restriction: ω would have to be enormous (over 100 percent).²⁵

In summary, crashes are most likely to occur in the unobserved case, since the inequality is satisfied for the lowest values of ω_{\min} . Because the critical-point difference $c_2 - c_1$ is greatest in this environment, the “crashes” associated with this environment will also be the largest.

VII. Making Markets More Stable

Our analysis suggests that unobserved hedging strategies can destabilize a market, leading to greater volatility and ultimately to a crash. Are there private or governmental policies that would lessen the chances of such an event in the future?

Outlawing hedging strategies is one such possibility, but it is neither practical nor desirable. It is not practical because it is not enforceable. An investor following a stop-loss or portfolio-insurance hedging strategy can always claim he is doing so for other reasons: an anticipated expense, a forecast of weak markets, etc. Short of prohibiting selling for any reason, it is impractical to

²⁵This means that selling by hedgers following a Black-Scholes put-option-replicating strategy would be met by the buying of investors as prices fell continuously, even if hedgers' selling (as prices fell to zero) were 100 percent of initial supply. Indeed, hedge selling would have to be 10 times more intensive than this before price could fall discontinuously.

prohibit selling for hedging purposes. Nor would it be desirable. Investors are willing to participate in a market because they can sell whenever they wish to, including for risk-avoidance purposes.

We note that the market is partially self-correcting. Stop-loss and dynamic hedging strategies are fully effective only when prices move continuously. The possibility of a crash will limit the use of dynamic protection strategies. Of course, if relatively few investors follow such strategies, crashes are unlikely to occur.

Portfolio protection is a legitimate aim of private investors. Is there a way in which investors can achieve protection without contributing to—or suffering from—discontinuous markets? Our analysis provides some clues. The most important result is that widespread knowledge of dynamic hedging usage can minimize its impact on markets. The preceding section showed that the unobserved hedging, which created a 30-percent crash in market prices, would have less than a 1-percent impact on prices if it were observed by all investors. Does this seem preposterous? Some postcrash evidence suggests that it is not. On October 19, 1988, exactly one year after the crash, the Japanese government sold over \$24 billion of a single stock, Nippon Telephone and Telegraph (NT&T). This was four times the amount of all stocks that portfolio insurers had sold the year before. Yet NT&T stock did not decline by a significant amount (either at sale or at the time of initial announcement), because investors had prior knowledge that the sale did not reflect an informational change. Interestingly, portfolio insurers were anxious to disseminate information about their trading requirements prior to the crash, but events happened more quickly than regulatory approval.²⁶

²⁶With the assistance of a major portfolio-insurance firm (LOR), the New York Futures Exchange (NYFE) had requested the right to publicize large futures sales in advance. The theory behind the request was that preannouncement would allow time for the market to organize a competitive response. Prior to the crash, the NYFE proposal had been withdrawn, reportedly because there were insufficient means of electronically disseminating the information.

An alternative is for hedgers to use static instruments that provide the same results as dynamic hedging strategies. For example, put options provide protection without requiring further trading. They would seem to be the ideal instrument to avoid the problems of trading in uninformed (and therefore illiquid) markets. A criticism of this argument is that it simply pushes the problem back one level: the sellers of the put option will need to protect themselves through a dynamic hedging strategy. Even if this is true, however, at least there will be publicly available information about the number of outstanding put options. Astute observers can “reverse engineer” the dynamic strategies that the open interest in such options imply. If this information is widely disseminated, we will have nearly universal observation of π strategies.

Short of all investors being aware of hedging plans, our analysis also shows that the stability of markets is strongly affected by the number of supply-informed traders who can observe these plans. These market-makers play a role far beyond their numbers in increasing market liquidity. The crash that occurred in our example with no investors observing hedging could have been prevented if there had been as few as 0.03 percent supply-informed investors (given $\omega = 0.05$) observing hedging supply π .

To the extent that stock-exchange specialists have privileged access to information on the nature of order flows, they play a key role in providing stability. Rules that limit free entry to this activity will leave markets considerably more vulnerable than otherwise. Electronic “open books” should be a seriously considered reform, and other forms of market organization (such as single-price auctions) should be examined.

Low margin requirements in stock or derivatives markets can lead to an increased level of forced margin sales as prices fall. In effect, low margins increase the likely amount of stop-loss sales. If the extent of forced margin sales is difficult to observe, low margin requirements could increase the market’s vulnerability to crashes.

Would price limits help? The answer is no—unless such limits (and the trading halts caused by their being reached) permitted

better dissemination of information on hedgers' selling. Absent this, price limits would only delay the ultimate crash by a bit, without modifying its magnitude. Certainly the market did not seem to benefit from the "trading halt" created by the weekend of October 17–18.

VIII. Conclusion

We have shown that information differences among market participants can cause financial markets to be relatively illiquid. A small unobserved supply shock can create a large fall in prices. This is because the fall in prices affects investors' expectations as well as their budgets. Traditional models which do not recognize that many investors are poorly informed will grossly overestimate the liquidity of stock markets.

A consequence of diminished liquidity is that even relatively small unobserved trades by hedging programs can have a destabilizing effect. We developed an example in which a market crash occurred when only 5 percent of investors were following a hedging program replicating a put option.

Our model suggests how a crash caused by hedging in this country could be propagated to foreign markets, even when these markets do not have hedging programs such as portfolio insurance. Foreign investors, observing the large price drop in the U.S. market but ignorant of the extent of hedging in that market, rationally infer that significant negative information must have been received by U.S. investors. To the extent that this information is also significant for their own markets, foreign investors revise downward their expectations, causing prices to fall globally.

Our model also indicates policies to minimize the chance of future crashes. These include the wide dissemination of knowledge about hedgers' actions, marginal positions, and the use of put options or related securities that provide hedging without requiring dynamic trading. This recommendation supports a similar contention by Grossman (1988a). Allowing wider access to the information in specialists' books might also help to stabilize the market. In contrast, price limits are unlikely to have useful ef-

fects unless they are combined with greater dissemination of trading information at the time limits are reached.

APPENDIX

Notation (with Example Parameters in Parentheses)

Prices

p_0 :	current equilibrium price
p :	realized end-of-period price
\bar{p} :	unconditional expected end-of-period price (1.06)
\bar{p}_i :	investor i 's conditional expectation of end-of-period price
Σ :	unconditional variance of end-of-period price (0.08)
Z_j :	class j investor-conditional variance of p
Z :	market power-weighted average conditional variance of p

Information

m :	supply of shares divided by the sum of risk-tolerance coefficients; expectation \bar{m} (1.503), variance Σ_m (0.00034)
p'_i :	$p + \varepsilon_i$ price signal observed by investor i in class I
ε_i :	price signal noise, uncorrelated across investors, uncorrelated with other random variables; <i>ex ante</i> variance Σ_ε (0.4)
S :	liquidity supply observed by investors SI; mean 0 and variance Σ_S (0.00017)
L :	unobserved liquidity supply; mean 0 and variance Σ_L (0.00017); L and S are independent

Investors

SI:	supply-informed investor class; observe p_0 and S
I:	price-informed investor class; observe p_0 and p'_i
U:	uninformed investor class; observe p_0
j :	investor class SI, I, or U
a_j :	investor-class j risk tolerance
w_j :	number of investors in class j

- k_j : relative market power of class j ;
ratio of the products of w_j and
 a_j to the sum across classes:
 $k_j \equiv a_j w_j / \sum a_j w_j$ ($k_I = 0.02$,
 $k_{SI} = 0.005$, $k_U = 0.975$)
- $\pi(p_0)$: hedging share supply
- ω : fraction of share total hedged
(5 percent)

PROOF OF THEOREM 1:

We will assume that investors believe the function f^{-1} to be well-defined (i.e., a given equilibrium price level p_0 obtains for only one possible realization of the argument of the function f). Subsequently we will show that this belief is confirmed in equilibrium. The variance-covariance matrix \mathbf{V} of the three-signal vector

$$\begin{bmatrix} p' \\ S \\ f^{-1}(p_0) \end{bmatrix}$$

and the covariance vector \mathbf{W} of the signal vector with the future price are given by

$$\mathbf{V} = \begin{bmatrix} \Sigma + \Sigma_\epsilon & 0 & \Sigma \\ 0 & \Sigma_S & -I\Sigma_S \\ \Sigma & -I\Sigma_S & \Sigma + H^2\Sigma_L + I^2\Sigma_S \end{bmatrix}$$

$$\mathbf{W} = \begin{bmatrix} \Sigma \\ 0 \\ \Sigma \end{bmatrix}$$

For simplicity, we have omitted the subscript i (for investor i) of p' . The distribution of end-of-period prices conditional on all three signals is normal with expectation \bar{p}_{II} and variance Z_{II} . Defining $[A_{II}, B_{II}, C_{II}] \equiv \mathbf{W}^\top \mathbf{V}^{-1}$, where \mathbf{W}^\top denotes the transpose of \mathbf{W} , leads to

$$\bar{p}_{II} = \bar{p} + \mathbf{W}^\top \mathbf{V}^{-1} \begin{bmatrix} p' - \bar{p} \\ S \\ f^{-1}(p_0) \end{bmatrix}$$

$$= \bar{p} + [A_{II}, B_{II}, C_{II}] \begin{bmatrix} p' - \bar{p} \\ S \\ f^{-1}(p_0) \end{bmatrix}$$

$$Z_{II} = \Sigma - \text{Cov}\{p, [p', S, f^{-1}(p_0)]\}^\top$$

$$\mathbf{V}^{-1} \text{Cov}\{p, [p', S, f^{-1}(p_0)]\}$$

$$Z_{II} = \Sigma - (A_{II}\Sigma + C_{II}\Sigma)$$

(see, e.g., Morris DeGroot, 1975). Straightforward and lengthy manipulation of the equations leads to

$$Z_{II} = \left[\frac{1}{\Sigma} + \frac{1}{\Sigma_\epsilon} + \frac{1}{H^2\Sigma_L} \right]^{-1}$$

$$Z_{II}^{-1}A_{II} = \frac{1}{\Sigma_\epsilon}$$

$$Z_{II}^{-1}B_{II} = \frac{I}{H^2\Sigma_L}$$

$$Z_{II}^{-1}C_{II} = \frac{1}{H^2\Sigma_L}.$$

These parameters would obtain for an investor who could observe all the signals. To derive the corresponding parameters for the supply-informed investors (SI) it suffices to take the limit of Σ_ϵ at infinity. For investors I and U, who do not observe the signal S , the parameters are obtained by replacing $H^2\Sigma_L$, the contribution to the variance of $f^{-1}(\cdot)$ due to unobserved liquidity trading, with $H^2\Sigma_L + I^2\Sigma_S$ in the expression for the corresponding parameters for II and SI, respectively. This yields

$$Z_{SI} = \left(\frac{1}{\Sigma} + \frac{1}{H^2\Sigma_L} \right)^{-1}$$

$$Z_{SI}^{-1}A_{SI} = 0$$

$$Z_{SI}^{-1}B_{SI} = \frac{I}{H^2\Sigma_L}$$

$$Z_{SI}^{-1}C_{SI} = \frac{1}{H^2\Sigma_L}$$

$$Z_I = \left(\frac{1}{\Sigma} + \frac{1}{\Sigma_\epsilon} + \frac{1}{H^2\Sigma_L + I^2\Sigma_S} \right)^{-1}$$

$$Z_I^{-1}A_I = \frac{1}{\Sigma_\epsilon}$$

$$Z_I^{-1}B_I = 0$$

$$Z_I^{-1}C_I = \frac{1}{H^2\Sigma_L + I^2\Sigma_S}$$

$$Z_U = \left(\frac{1}{\Sigma} + \frac{1}{H^2\Sigma_L + I^2\Sigma_S} \right)^{-1}$$

$$Z_U^{-1}A_U = 0$$

$$Z_U^{-1}B_U = 0$$

$$Z_U^{-1}C_U = \frac{1}{H^2\Sigma_L + I^2\Sigma_S}.$$

The corresponding market power weighted averages are given by

$$Z^{-1} \equiv \sum_j k_j Z_j^{-1} = \frac{1}{\Sigma} + \frac{k_I}{\Sigma_\epsilon} + \frac{H^2\Sigma_L + k_{SI}I^2\Sigma_S}{H^2\Sigma_L(H^2\Sigma_L + I^2\Sigma_S)}$$

$$A \equiv \sum_j k_j Z_j^{-1}A_j = \frac{k_I}{\Sigma_\epsilon}$$

$$B \equiv \sum_j k_j Z_j^{-1}B_j = k_{SI} \frac{I}{H^2\Sigma_L}$$

$$C \equiv \sum_j k_j Z_j^{-1}C_j \\ = k_{SI} \frac{1}{H^2\Sigma_L} + \frac{k_I + k_U}{H^2\Sigma_L + I^2\Sigma_S}.$$

The total demand for shares of the three classes of investors is equal to the total supply plus hedging supply:

$$(A1) \quad \sum_j k_j Z_j^{-1}(\bar{p}_j - p_0) = m + \pi.$$

Reorganizing terms yields, at the limit of economies with an infinite number of agents,

$$(A2) \quad \frac{Z^{-1}p_0 + \pi - Cf^{-1}(p_0) - Z^{-1}\bar{p} + \bar{m}}{A} \\ = p - \bar{p} - \frac{1}{A}L - \frac{1-B}{A}S.$$

This equation is consistent with equation (3) if and only if the following set of equations holds:

$$H = \frac{1}{A} \quad I = \frac{1-B}{A}$$

$$f^{-1}(p_0) = \frac{Z^{-1}p_0 + \pi - Z^{-1}\bar{p} + \bar{m}}{A + C}$$

Substituting A , B , and C yields the unique

solution for H , I , and f^{-1} :

$$(A3) \quad H = \frac{\Sigma_\epsilon}{k_I} \quad I = H - \frac{Hk_{SI}}{H\Sigma_L + k_{SI}}$$

$$f^{-1}(p_0) = \frac{Z^{-1}p_0 + \pi - Z^{-1}\bar{p} + \bar{m}}{Z^{-1} - \Sigma^{-1}}.$$

The solution f^{-1} is a well-defined function, as asserted above.

PROOF OF PROPOSITIONS 1 AND 2:

The function f^{-1} is continuous. Consequently, the function f is well-defined and continuous if and only if f^{-1} , or equivalently $Z^{-1}p_0 + \pi$, is strictly monotonic. If $\pi(p_0) = 0$, f^{-1} is strictly monotonic, since $Z^{-1} > 0$; hence f is well-defined and continuous.

Excess Demand. Substitution of the solutions in equation (A2) yields the excess demand (demand minus supply):

$$XD_A = \frac{1}{H} \left[p - \bar{p} - HL - IS \right. \\ \left. + \frac{Z^{-1}(\bar{p} - p_0) - (\bar{m} + \pi)}{Z^{-1} - \Sigma^{-1}} \right].$$

The Linear Case. When the demand stemming from dynamic strategies is linear in p_0 (i.e., π' is constant), $f(\cdot)$ is a linear function. In the case of no hedging supply ($\pi = 0$), the equilibrium price p_0 is given by

$$(A4) \quad p_0 = \frac{Z^{-1} - \Sigma^{-1}}{Z^{-1}}(p - \bar{p} - HL - IS) \\ + \bar{p} - Z\bar{m}.$$

We will denote by F the slope of the function f ; in this case, $F = 1 - Z/\Sigma$. In the context of our example, we have $Z^{-1} = 25.06$ and

$$p_0 = 0.5(p - 1.06 - 19.95L - 8.14S) + 1.$$

PROOF OF PROPOSITION 3:

We first restrict our attention to the domain of p_0 where $f^{-1}(p_0)$ is strictly increasing. From (A3), $f^{-1}(p_0)$ is also differentiable, with derivative $Z^{-1} + \pi'(p_0) > 0$ over this domain. As hedging activity $\pi(p_0)$ increases, the derivative of $f^{-1}(p_0)$ decreases, since $\pi' < 0$. Therefore, the derivative of $f(\cdot)$ becomes larger, and the current price becomes more sensitive to changes in the signals. Since the signal volatility is exogenous, this in turn implies that the current price is more volatile. For sufficiently large hedging activity, f^{-1} actually decreases over the range of prices p_0 for which

$$(A5) \quad -\pi'(p_0) > Z^{-1}.$$

Therefore, $f(\cdot)$ is multivalued, and discontinuities within the set of stable equilibria can occur, as demonstrated in the example of Section VI.

PROOF OF THEOREM 2:

The proof closely follows that of Theorem 1. The difference consists in the agents' different beliefs about the structure of equilibrium prices. In the first case, supply-informed agents (SI) are aware of hedging strategies and of their impact on prices. SI agents know f^{-1} , the actual inverse price function which obtains in equilibrium. Other agents, ignorant of the presence of hedgers, think that the linear functional form holds. We assume that SI agents believe the coefficients H and I to be unchanged and show that it indeed holds in equilibrium. Equation (A1) still holds, and a similar manipulation leads to the analog of (A2):

$$(A6) \quad \frac{\frac{Z^{-1}(p_0 - (\bar{p} - Z\bar{m}))}{Z^{-1} - \Sigma^{-1}} \left(A + \frac{B}{I} \right) + \pi - C f^{-1}(p_0)}{A + \frac{B}{I}} \\ = p - \bar{p} - \frac{1}{A} L - \frac{1-B}{A} S.$$

Hence, the parameters H and I are unchanged, and $f^{-1}(p_0)$ is given by the left-hand side of equation (A6), because SI agents know the true inverse equilibrium

price function $f^{-1}(\cdot)$. The solution $f^{-1}(p_0)$ is given by

$$(A7) \quad f^{-1}(p_0)$$

$$= \frac{Z^{-1}(p_0 - (\bar{p} - Z\bar{m}))}{Z^{-1} - \Sigma^{-1}} + I\pi.$$

If hedging activity is totally unobserved, similar derivations yield the same parameters H and I as before, and the new inverse equilibrium price function

$$(A8) \quad f^{-1}(p_0)$$

$$= \frac{Z^{-1}(p_0 - (\bar{p} - Z\bar{m}))}{Z^{-1} - \Sigma^{-1}} + H\pi.$$

PROOF OF PROPOSITIONS 4 AND 5:

The derivative of the inverse equilibrium price function $d(f^{-1})/dp_0$ is equal to $F^{-1} + (Z^{-1} - \Sigma^{-1})^{-1}\pi'$ in the fully observed case, to $F^{-1} + I\pi'$ in the partially observed case, and to $F^{-1} + H\pi'$ in the unobserved case [eqs. (A3), (A7), and (A8)]. It is smallest for the unobserved hedging activity case, and it is smaller under partial observation than in the fully observed case, because π' is negative and $H > I > (Z^{-1} - \Sigma^{-1})^{-1}$ [by combining the definition of Z^{-1} and eq. (A3)]. This implies that the derivative of f (and therefore the volatility of p_0) is largest in the case of unobserved hedging activity and least in the case when hedging is fully observed. The derivatives are negative if $-\pi' > Z^{-1} = (Z^{-1} - \Sigma^{-1})F^{-1}$ in the observed case, if $-\pi' > (FI)^{-1}$ in the partially observed case, and if $-\pi' > (FH)^{-1}$ in the unobserved case. Hence, as hedging activity increases, discontinuities appear first in the unobserved case, then in the partially observed case, and finally in the perfectly observed case.

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