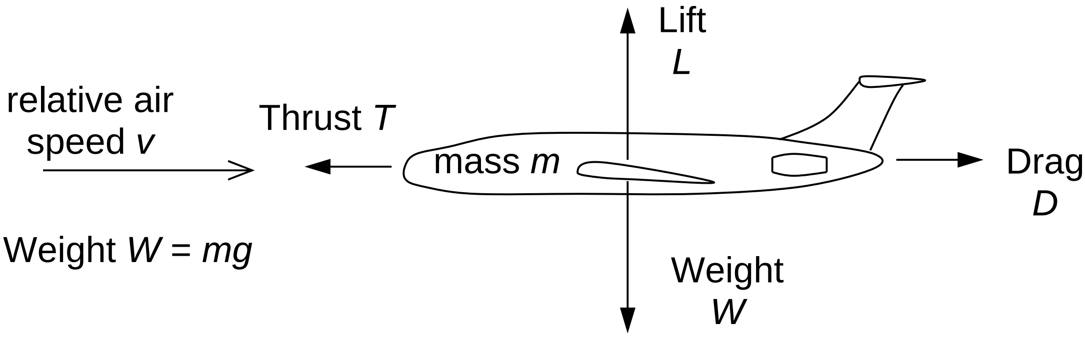
C2 Aircraft Flight and Propulsion – Lecture 2

Prof. L. He 4 lectures

2.1 Force and Power *Required* in Level Flight



Weight $W = mg$

Thrust required T_r

In level steady flight: $T = D$ $= T_r$ the required thrust

$L = W$

$$T_r = \frac{DL}{L} = \frac{DW}{L} = \frac{W}{\left(L/D\right)} \qquad \text{or} \qquad \boxed{T_r = \frac{W}{\left(C_L/C_D\right)}} \qquad (2\text{-}1)$$

Minimum thrust required $(T_r)_{min}$

T_r is a minimum at maximum $\dfrac{C_L}{C_D}$.

Using the simplified model for drag: $$C_D = \underbrace{C_{D0}}_{\text{parasitic + profile drag}} + \underbrace{\frac{kC_L^2}{\pi A}}_{\text{induced drag}}$$

divide by C_L: $$\frac{C_D}{C_L} = \frac{C_{D0}}{C_L} + \frac{kC_L}{\pi A}$$

At max L/D: $$\frac{d(C_D/C_L)}{dC_L} = -\frac{C_{D0}}{C_L^2} + \frac{k}{\pi A} = 0 \qquad \rightarrow \qquad C_{D0} = \frac{kC_L^2}{\pi A}$$

(min D/L)

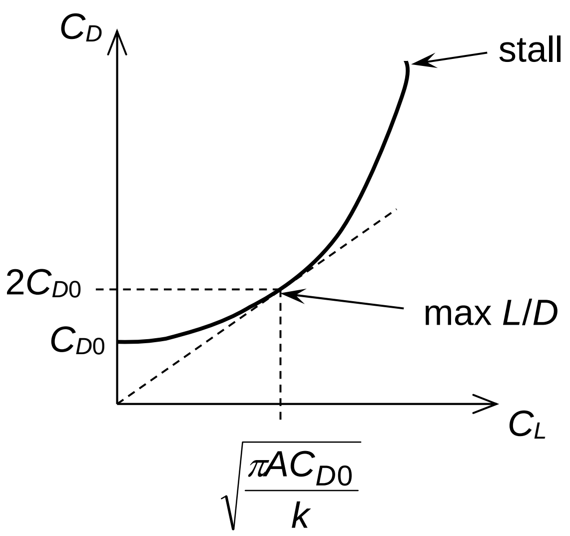
parasitic & profile drag = induced drag

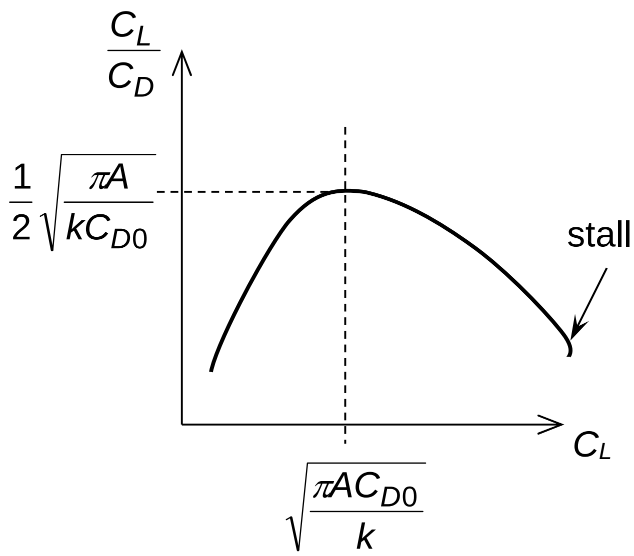
'minimum drag' (i.e. min D/L)

$$(C_L)_{md} = \sqrt{\frac{\pi A C_{D0}}{k}}$$

$$(C_D)_{md} = 2C_{D0} \qquad (2\text{-}2)$$

$$\left(\frac{C_L}{C_D}\right)_{\max} = \frac{1}{2}\sqrt{\frac{\pi A}{k C_{D0}}} = \left(\frac{L}{D}\right)_{\max} \qquad (2\text{-}3)$$





<u>Variation of T_r with v</u>

$$T_r = D = \frac{1}{2}\rho v^2 S C_D$$

$$T_r = \frac{1}{2}\rho v^2 S\left(C_{D0} + \frac{k}{\pi A}C_L^2\right) = \frac{1}{2}\rho v^2 S\left(C_{D0} + \frac{k}{\pi A}\left(\frac{W}{\frac{1}{2}\rho v^2 S}\right)^2\right) \qquad (L = W)$$

$$\boxed{T_r = D = \frac{1}{2}\rho v^2 S C_{D0} + \frac{2kW^2}{\pi A \rho v^2 S}} \qquad (2\text{-}4)$$

parasitic + profile drag $\propto v^2$

induced drag $\propto \frac{1}{v^2}$

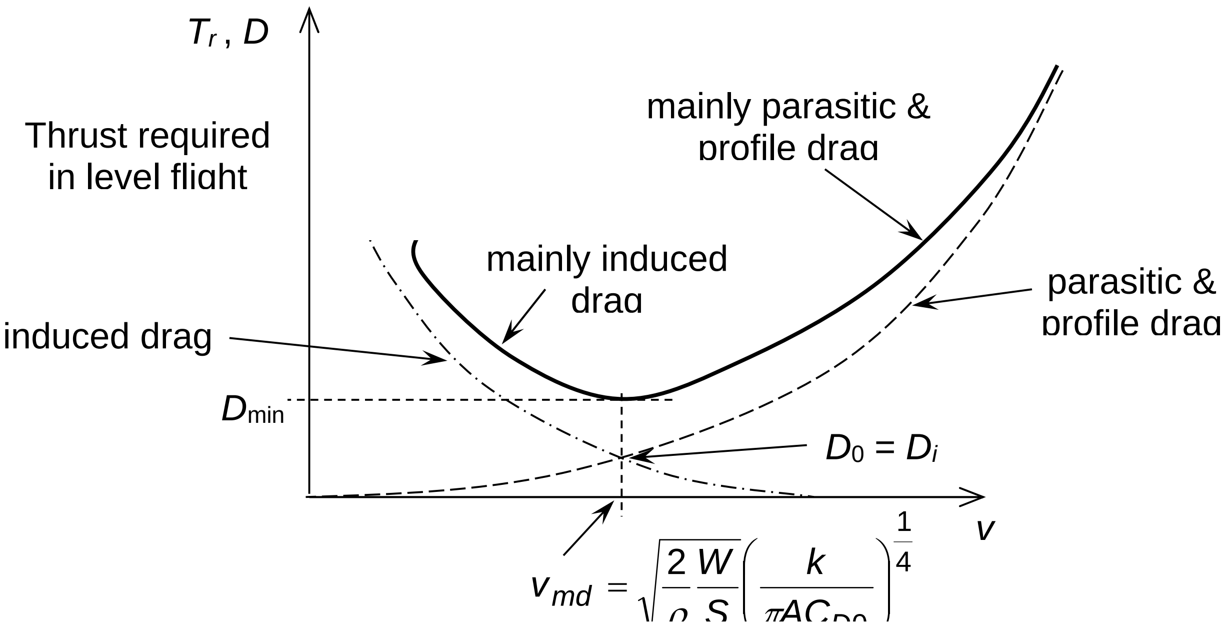
Since $W = L = \frac{1}{2}\rho v^2 S C_L \quad \rightarrow \quad \boxed{v = \sqrt{\frac{2}{\rho C_L}\frac{W}{S}}}$ (2-5)

wing loading, W/S (N/m^2)

Minimum drag speed

$$v_{md} = \sqrt{\frac{2}{\rho (C_L)_{md}}\left(\frac{W}{S}\right)} = \sqrt{\frac{2}{\rho}\left(\frac{W}{S}\right)\sqrt{\frac{k}{\pi A C_{D0}}}}$$

$$v_{md} = \sqrt{\frac{2}{\rho}}\sqrt{\frac{W}{S}}\left(\frac{k}{\pi A C_{D0}}\right)^{\frac{1}{4}}$$



Note: for $v < v_{md}$, more engine thrust is required to fly slower!

Minimum drag $\quad D_{min} = \frac{1}{2}\rho v_{md}^2 S(2C_{D0}) = \frac{1}{2}\rho\frac{2}{\rho}\frac{W}{S}\left(\frac{k}{\pi A C_{D0}}\right)^{\frac{1}{2}} S(2C_{D0})$

$$D_{min} = 2W\sqrt{\frac{k C_{D0}}{\pi A}}$$

Note: D_{min} does not depend on air density ρ or wing area S.

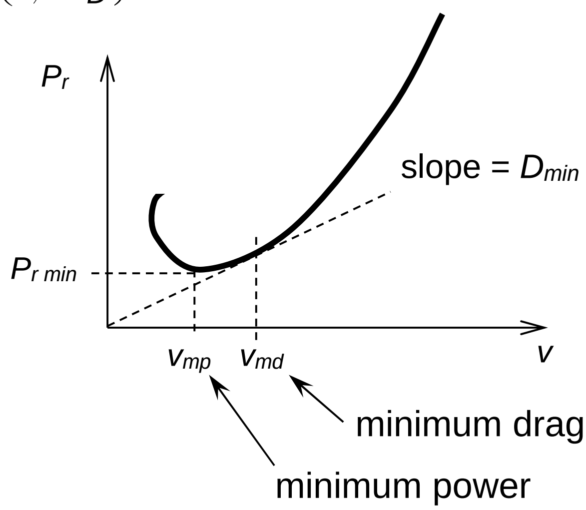
Power required P_r in level flight

$$T_r = \frac{W}{\left(C_L / C_D\right)}, \quad \text{also} \quad v = \sqrt{\frac{2}{\rho C_L}\frac{W}{S}}$$

Power $\quad P_r = T_r\, v$

$$P_r = \frac{W}{\left(C_L / C_D\right)}\sqrt{\frac{2}{\rho C_L}\frac{W}{S}}$$

$$P_r = W\sqrt{\frac{2}{\rho}\frac{W}{S}}\;\frac{1}{\left(\dfrac{C_L^{3/2}}{C_D}\right)} \qquad \text{(2-6)}$$



so $\quad P_r \propto \dfrac{1}{\left(\dfrac{C_L^{3/2}}{C_D}\right)} \qquad$ c.f. $\quad T_r \propto \dfrac{1}{\left(\dfrac{C_L}{C_D}\right)}$

Minimum power at maximum $\left(\dfrac{C_L^{3/2}}{C_D}\right)$

slope of line through origin on $P_r : v$ graph = $\dfrac{P_r}{v} = \dfrac{T_r v}{v} = T_r = D$

Using the approximate model for drag, $C_D = C_{D0} + \dfrac{kC_L^2}{\pi A}$

divide by $C_L^{3/2}$, $\qquad \dfrac{C_D}{C_L^{3/2}} = C_{D0}C_L^{-3/2} + \dfrac{kC_L^{1/2}}{\pi A}$

$$\frac{d}{dC_L}\left(\frac{C_D}{C_L^{3/2}}\right) = -\frac{3}{2}C_{D0}C_L^{-5/2} + \frac{1}{2}\frac{kC_L^{-1/2}}{\pi A} = 0 \qquad \rightarrow \qquad 3C_{D0} = \frac{kC_L^2}{\pi A}$$

i.e. induced drag = 3 × (parasitic and profile drag), thus:

$$\boxed{(C_D)_{mp} = 4C_{D0}} \qquad \text{(2-7)}$$

$$(C_L)_{mp} = \sqrt{\frac{3\pi A C_{D0}}{k}}$$

Note: $\quad \boxed{(C_L)_{mp} = \sqrt{3}\,(C_L)_{md}} \qquad$ (2-8)

Velocity for minimum power (Eq.2-5)

$$v_{mp} = \sqrt{\frac{2}{\rho}\frac{W}{S}}\frac{1}{\sqrt{(C_L)_{mp}}}$$

velocity for minimum power:

$$\boxed{v_{mp} = \sqrt{\frac{2}{\rho}\frac{W}{S}}\frac{1}{3^{1/4}}\left(\frac{k}{\pi A C_{D0}}\right)^{\frac{1}{4}}}$$

$$\left(v_{mp} = \frac{1}{3^{1/4}}v_{md} = 0.76\,v_{md}\right)$$

Minimum power

$$\left(\frac{C_D}{C_L^{3/2}}\right)_{\min} = C_{D0}\left(C_{L\,mp}\right)^{-3/2} + \frac{k\left(C_{L\,mp}\right)^{1/2}}{\pi A}$$

$$= C_{D0}\left(\frac{3\pi A C_{D0}}{k}\right)^{-\frac{3}{4}} + \frac{k}{\pi A}\left(\frac{3\pi A C_{D0}}{k}\right)^{\frac{1}{4}} = 4C_{D0}\left(\frac{k}{3\pi A C_{D0}}\right)^{\frac{3}{4}}$$

$$\left(\frac{C_L^{3/2}}{C_D}\right)_{\max} = \frac{1}{4}\left(\frac{3\pi A}{k}\right)^{\frac{3}{4}}C_{D0}^{-\frac{1}{4}}$$

minimum power (Eq 2-6):

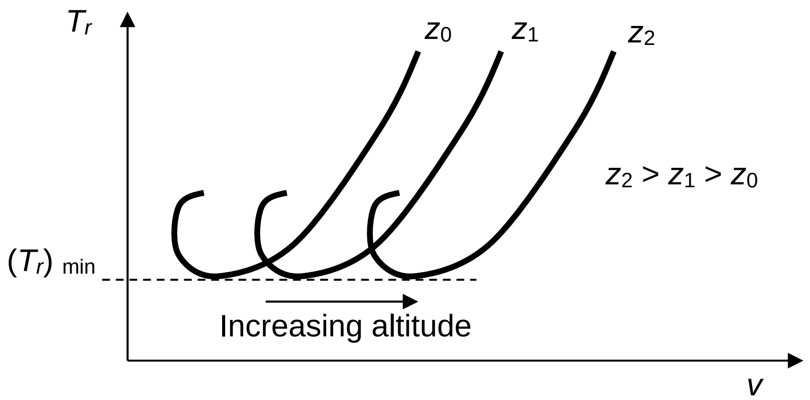
$$\boxed{\left(P_r\right)_{\min} = 4W\sqrt{\frac{2}{\rho}\frac{W}{S}}\left(\frac{k}{3\pi A}\right)^{\frac{3}{4}}C_{D0}^{\frac{1}{4}}} \qquad (2\text{-}9)$$

Effects of altitude

Both thrust T_r and power P_r are functions of density $\left(\rho = \dfrac{p}{RT}\right)$, which decreases with altitude. The International Standard Atmosphere (HLT p.68) tabulates atmospheric conditions over a range of altitudes, and can be used for aircraft design; density is tabulated as ρ/ρ_0 where ρ_0 = 1.225 kg/m^3 is taken to be the density at sea level.

For steady level flight, velocity (Eq.2-7), $v = \sqrt{\dfrac{2}{\rho C_L}\dfrac{W}{S}} = \sqrt{\dfrac{\rho_0}{\rho}} v_{SL}$, (2-10)

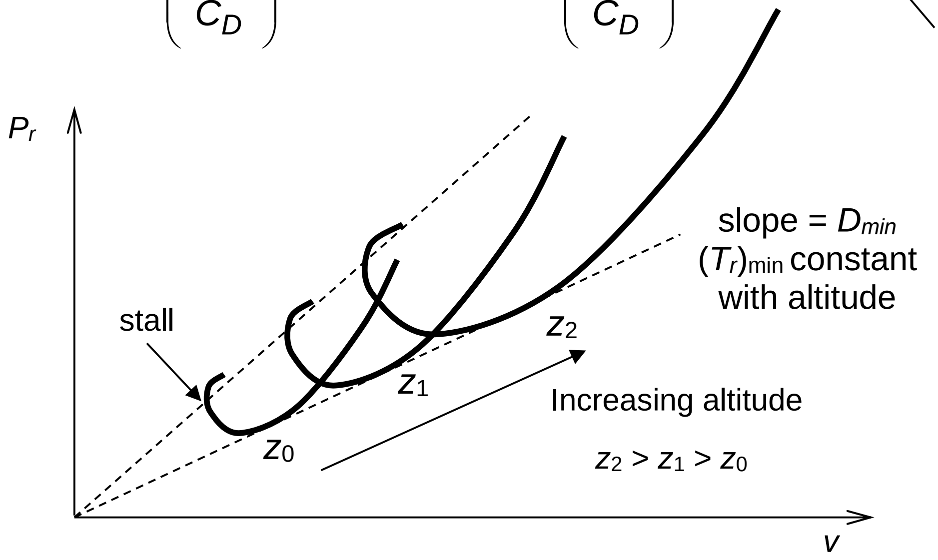
where v_{SL} is the velocity at sea level. For constant incidence (constant lift coefficient), and constant weight, aircraft have to fly faster at higher altitudes to compensate for the lower densities; therefore the T_r : v plot is shifted to the right with increasing altitude. Thrust, $T_r = \dfrac{W}{\left(C_L / C_D\right)}$ is unchanged for constant W and constant incidence (constant C_L / C_D); so there is no vertical shift of the T_r : v plot with altitude.



Power, $P_r = T_r\, v$; and for constant W and incidence, v increases with altitude; so there is a vertical shift of the $P_r : v$ plot with altitude. The $P_r : v$ plot also shifts to the right due to v increasing with altitude.

$$P_r = W\sqrt{\frac{2}{\rho}\frac{W}{S}}\,\frac{1}{\left(\frac{C_L^{3/2}}{C_D}\right)} = \sqrt{\frac{\rho_0}{\rho}}W\sqrt{\frac{2}{\rho_0}\frac{W}{S}}\,\frac{1}{\left(\frac{C_L^{3/2}}{C_D}\right)} = \sqrt{\frac{\rho_0}{\rho}}\left(P_r\right)_{SL} \qquad (2\text{-}11)$$

sea level

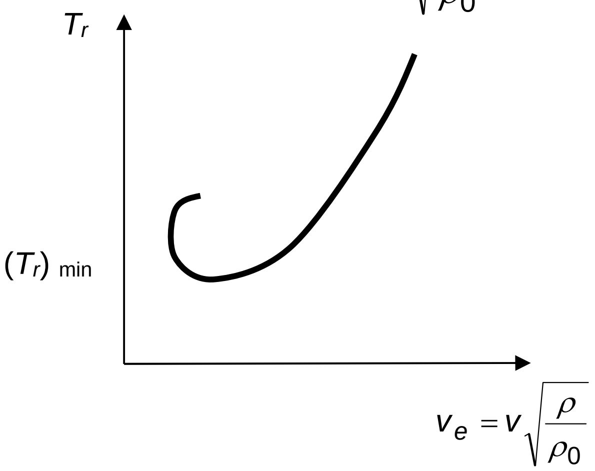


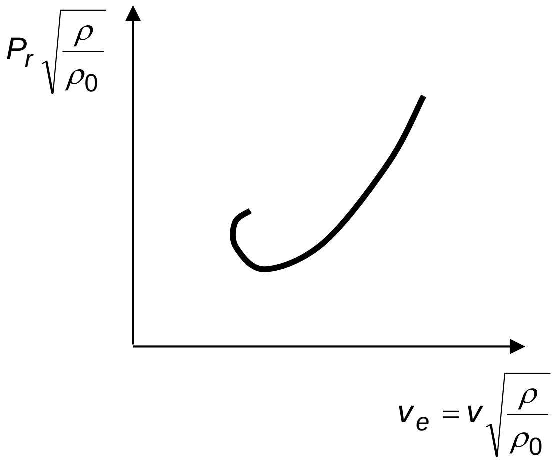
Note that stalling speed increases with altitude.

Equivalent airspeed

The $T_r : v$ and $P_r : v$ curves collapse onto single curves using the concept of

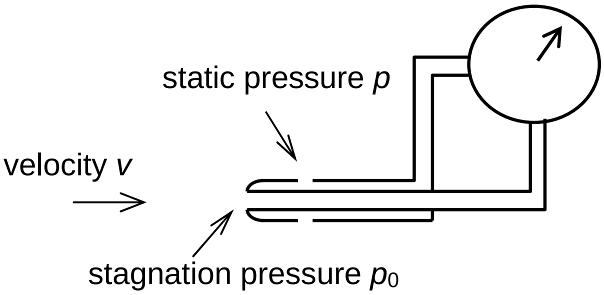
equivalent airspeed, $v_e = v\sqrt{\dfrac{\rho}{\rho_0}}$. (2-12)





At low Mach numbers where Bernoulli's equation applies (M_∞ < ~0.3), equivalent airspeed is the same as the <u>indicated airspeed</u> (IAS) measured by a pitot-static probe:

Bernoulli: $p_0 - p = \frac{1}{2}\rho v^2 \quad \rightarrow v = \sqrt{\frac{2(p_0 - p)}{\rho}}$



differential pressure gauge calibrated at ρ_0, standard sea level.

$$\text{IAS} = \sqrt{\frac{2(p_0 - p)}{\rho_0}} = \sqrt{\frac{\rho}{\rho_0}}v = v_e$$

Note: aircraft always stall at about the same v_e or IAS.

2.2 Engine Characteristics *(Thrust and Power Available)*

An aircraft's engine provides the necessary thrust for flight. There are four main types of aircraft engines available: propeller engines, turbojet engines, turbofan engines, and turbo-prop engines. Turbofan engines comprise a turbojet core, where the turbine also powers a large fan ahead of the compressor, and are a successful compromise between the high thrust achieved by turbojets and the high efficiency associated with propellers. Turboprop engines have propellers driven by a turbojet engine, where most of the work extracted by the turbine is used to drive the propeller; these engines produce more thrust than piston engine-driven propellers, and with efficiencies between the piston engine-driven propeller and the turbofan. In this course we will consider propulsion using piston engine-driven propellers, and jet engines.

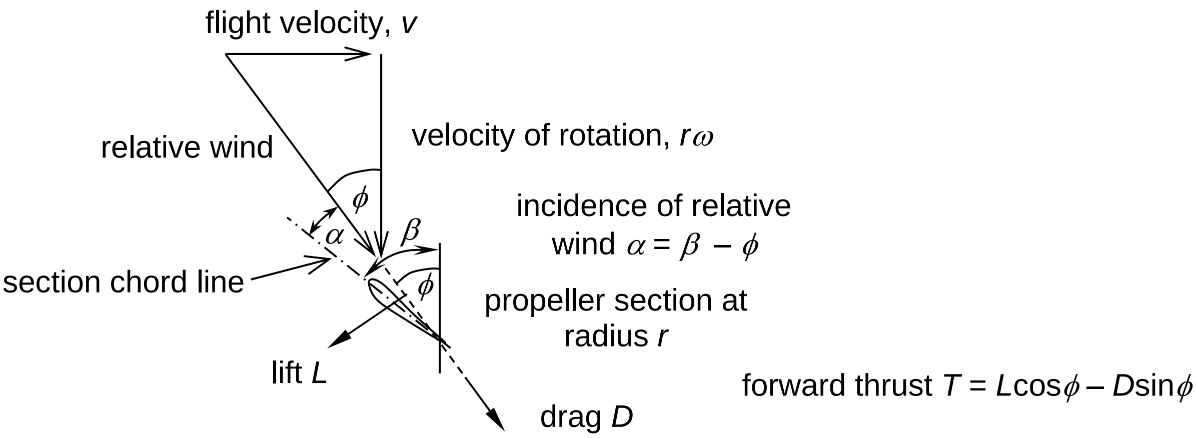
Propeller driven by a piston engine:

A propeller is made up of aerofoil sections that generate lift in much the same way as a wing. The lift generated produces the thrust required to propel the aircraft forward. A piston engine provides the brake power P_B that drives the propeller shaft. The available power for a propeller engine is given by

$$P_{av} = \eta_{pr} P_B$$

where η_{pr} is the propeller efficiency. η_{pr} is a function of the advance ratio $\frac{v}{ND}$, a dimensionless quantity where N is the rotational speed of the propeller (rev s^{-1}) and D is the propeller diameter.

The relative wind seen by the propeller at a particular radial section (radius r) is the vector sum of the flight velocity and the velocity of rotation $r\omega$ at that section, where ω is the angular velocity of the propeller.

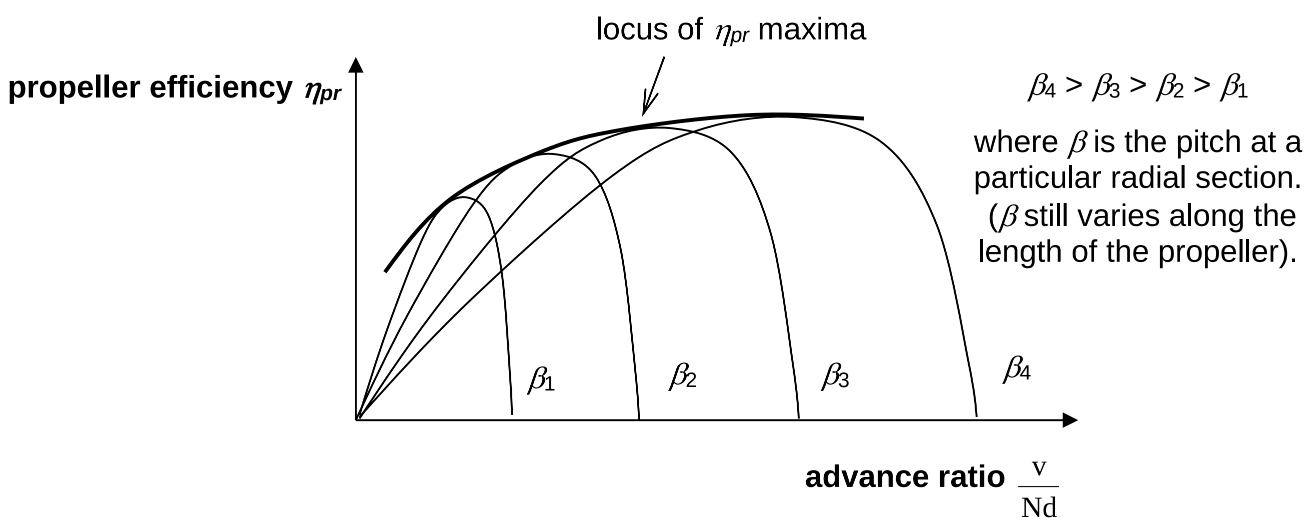


β is the pitch angle of the propeller; propellers are generally twisted with β almost 90° at the root, to β almost 0° at the tip, to account for the increase of $r\omega$ with radius.

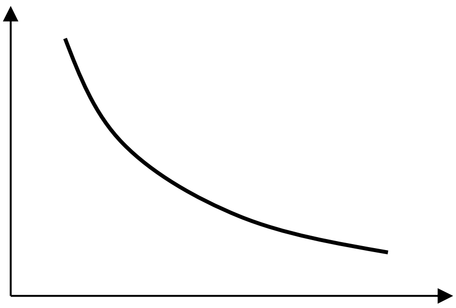
The lift (and drag) produced by the propeller depend strongly on the incidence of the relative wind, which is given by $\alpha = \beta - \phi$; and the flow angle ϕ

is set by the ratio $\frac{v}{r\omega}$. For a fixed-speed, fixed-pitch propeller, ϕ increases with flight velocity, reducing the incidence of the relative wind, which reduces both the lift and drag produced by the propeller. However the L/D ratio of the propeller initially increases with flight velocity to $(L/D)_{max}$, then decreases rapidly as the lift reduces to zero (and eventually reverses if the flight velocity is increased further). Note that $\frac{v}{(r\omega)_{tip}} = \frac{v}{\left(\frac{d}{2}\right)(2\pi N)} = \frac{v}{\pi N d} = \frac{1}{\pi} \times \text{advance ratio}$.

Plotting propeller efficiency η_{pr} against advance ratio $\frac{v}{Nd}$ for various fixed-pitch propellers, the maximum η_{pr} values initially increase with flight velocity then remain approximately constant over a wide range of velocities. Variable-pitch propellers are designed such that the pitch of the propeller is able to be varied mechanically in order to always operate at maximum η_{pr}. For this course it will be assumed that the propeller-driven aircraft have variable-pitch propellers and that the available power $P_{av} = \eta_{pr} P_B$ is approximately constant with flight velocity.

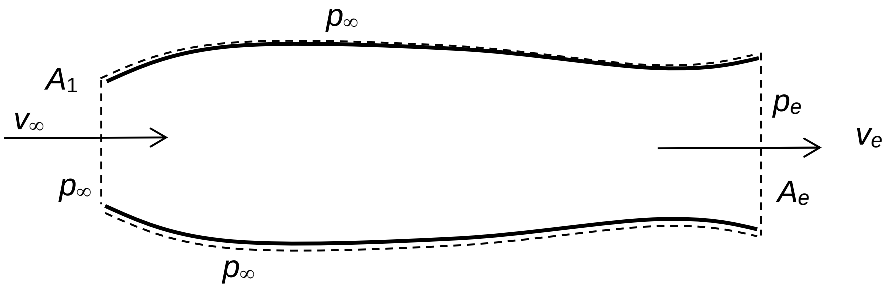


The available thrust for a propeller-driven aircraft is given by $T_{av} = \dfrac{P_{av}}{v}$, and therefore decreases with flight velocity since P_{av} is approximately constant.



At constant altitude, P_B is to a good approximation constant and independent of flight velocity. The brake power provided by a piston engine decreases with altitude due to the lower density. This reduction in brake power can be overcome in supercharged engines by compressing the captured air to higher pressures before it enters the pistons.

Jet engines

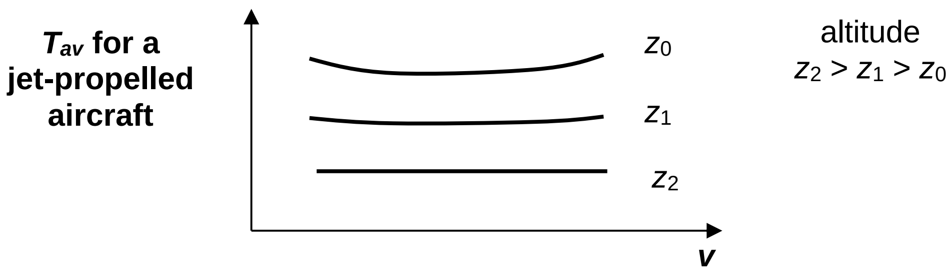


The thrust equation is obtained by applying the momentum equation to the control volume:

$$T_{av} = (\dot{m}_a + \dot{m}_f)v_e - \dot{m}_a v_\infty + (p_e - p_\infty)A_e \qquad (2\text{-}13)$$

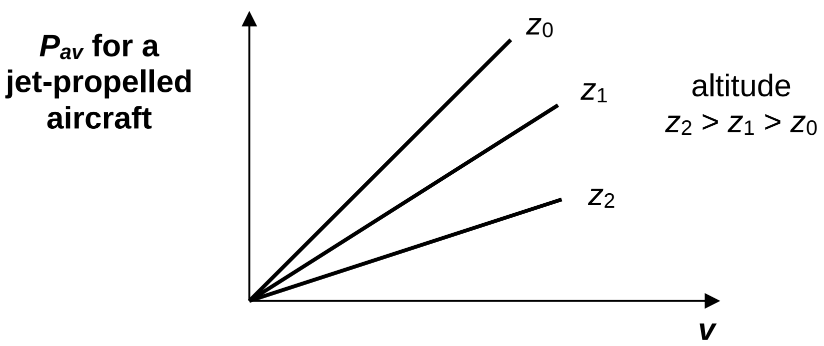
The $(p_e - p_\infty)A_e$ term is generally much smaller than the rate of change of momentum term, and the mass flow rate of fuel $\dot{m}_f$ is small compared to the mass flow rate of air $\dot{m}_a$.

As the flight velocity v_∞ increases, the mass flow rate of air $\dot{m}_a = \rho_\infty A_\infty v_\infty$ increases; but also since the exit jet velocity v_e is a function of the compression, combustion and expansion process within the engine and does not vary strongly with v_∞, $(v_e - v_\infty)$ decreases with flight velocity. The net effect is that the thrust available T_{av} for a jet engine is approximately constant with flight velocity. T_{av} increases slightly at high velocities due to the 'ram effect': the increased pressure rise obtained from the initial diffusion of the captured air to low velocities before it enters the compressor, and T_{av} also increases slightly at low velocities where v_e produced by the engine tends to be slightly higher. These effects are less pronounced at altitude:



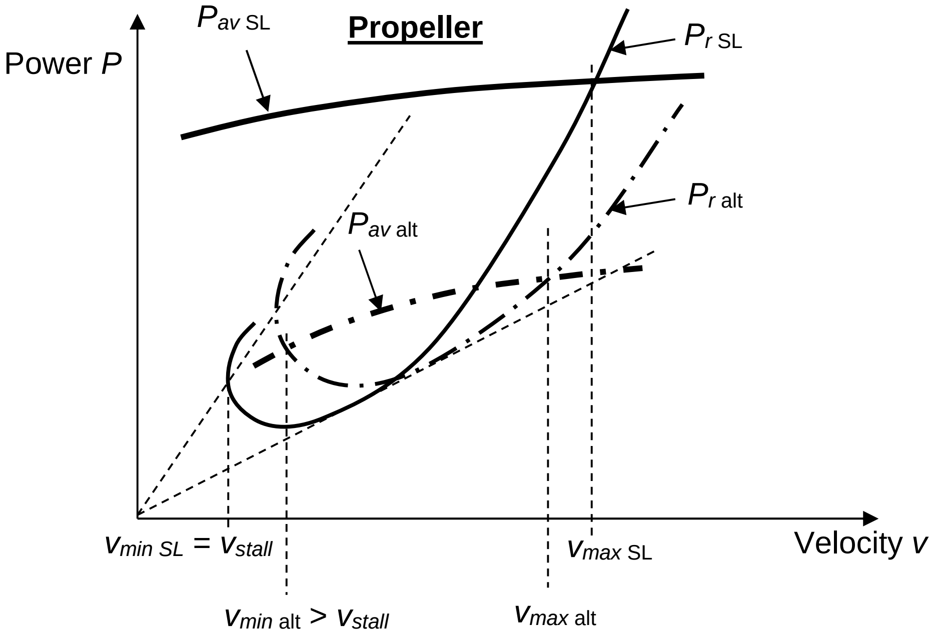
Thrust decreases at altitude because of the lower mass flow rates of air resulting from lower density.

The available power for a jet engine is given by $P_{av} = T_{av}\, v$ and therefore increases linearly with velocity since T_{av} is approximately constant.

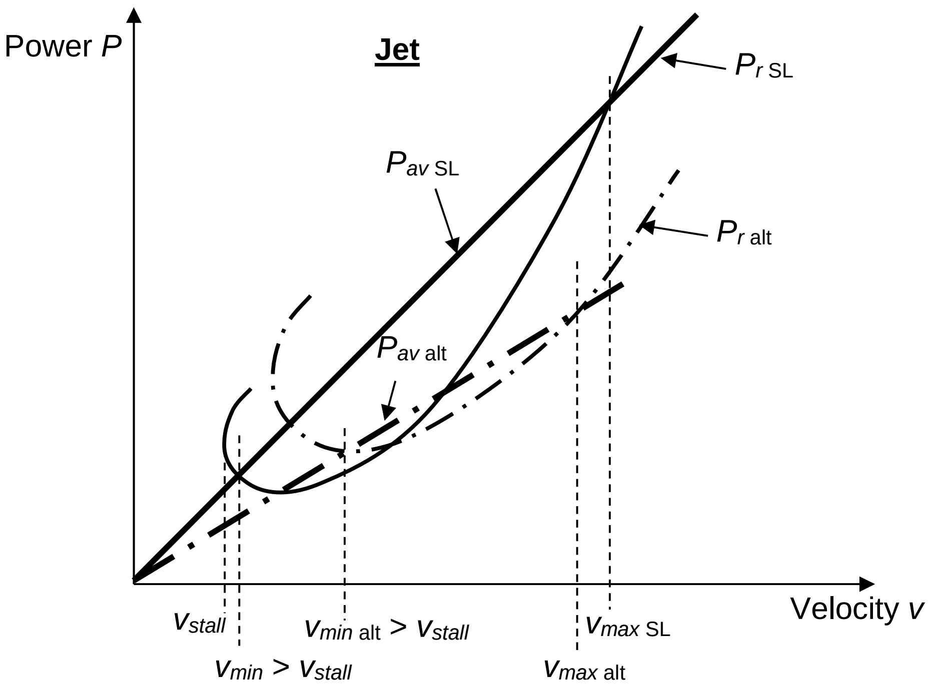


Flight envelopes

Propeller



Jet



The maximum velocity of an aircraft v_{max} is determined by the high-speed intersection of the power required and the power available curves (or the thrust required and thrust available curves). The minimum velocity v_{min} is the low-speed intersection of these curves, or the stall velocity, depending on which is greater. For jet-powered aircraft it is more convenient to perform these calculations on a thrust-velocity diagram since thrust available is approximately constant with velocity for a jet.

Propeller-driven aircraft have excess power at take-off from sea level, which is safe. Jet powered aircraft have $v_{min} > v_{stall}$ at sea level, and excess power is small at take-off.

Flying at higher altitudes reduces v_{max} and increases v_{min} above v_{stall}. Maximum altitude is attained when the curves just touch $\rightarrow$ when this occurs, $v_{max} = v_{min}$.

In practice, the maximum velocity may be determined by structural limitations, or by Mach number limits when shockwaves acting on lifting surfaces lead to a sudden increase of the power required and thrust required curves, or when shockwaves form on the propeller creating a sudden reduction of the power available and thrust available curves.