Iterative Methods in Combinatorial Optimization

With the advent of approximation algorithms for NP-hard combinatorial optimization problems, several techniques from exact optimization such as the primal-dual method have proven their staying power and versatility. This book describes a simple and powerful method that is iterative in essence and similarly useful in a variety of settings for exact and approximate optimization. The authors highlight the commonality and uses of this method to prove a variety of classical polyhedral results on matchings, trees, matroids, and flows.

The presentation style is elementary enough to be accessible to anyone with exposure to basic linear algebra and graph theory, making the book suitable for introductory courses in combinatorial optimization at the upper undergraduate and beginning graduate levels. Discussions of advanced applications illustrate their potential for future application in research in approximation algorithms.

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Contents

Preface

Audience

As teachers and students of combinatorial optimization, we have often looked for material that illustrates the elegance of classical results on matchings, trees, matroids, and flows, but also highlights methods that have continued application. With the advent of approximation algorithms, some techniques from exact optimization such as the primal-dual method have indeed proven their staying power and versatility. In this book, we describe what we believe is a simple and powerful method that is iterative in essence and useful in a variety of settings.

The core of the iterative methods we describe relies on a fundamental result in linear algebra that the row rank and column rank of a real matrix are equal. This seemingly elementary fact allows us via a counting argument to provide an alternate proof of the previously mentioned classical results; the method is constructive and the resulting algorithms are iterative with the correctness proven by induction. Furthermore, these methods generalize to accommodate a variety of additional constraints on these classical problems that render them NP-hard – a careful adaptation of the iterative method leads to very effective approximation algorithms for these cases.

Our goal in this book has been to highlight the commonality and uses of this method and convince the readers of the generality and potential for future applications. We have used an elementary presentation style that should be accessible to anyone with introductory college mathematics exposure in linear algebra and basic graph theory. Whatever advanced material in these areas we require, we develop from scratch along the way. Some basic background on approximation algorithms such as is provided in the various books and surveys available on this subject will be useful in appreciating the power of the results we prove in this area. Other than the basic definition of an approximation algorithm and the understanding of polynomial-time complexity, no further technical background is required from this typically more advanced subject.

An important secondary goal of the book is to provide a framework and material for introductory courses in combinatorial optimization at the upper-class undergraduate and beginning graduate levels. We hope the common approach across the chapters gives a comprehensive way to introduce these topics for the first time. The more advanced applications are useful illustrations for graduate students of their potential for future application in their research.

History

This book is inspired by the application of the iterative method in the field of approximation algorithms and its recent adaptations to prove performance guarantees for problems with two objectives. This adaptation showed us how the proof technique can be used to reprove several classical results in combinatorial optimization and also in approximation algorithms in a unified way. The book owes its origin to the paper by Jain [75] describing a 2-approximation algorithm for a large class of minimum cost network-design problems in undirected networks. There are other earlier illustrations of the method in the literature, but it is Jain's work that inspired the adaptation that led to the results in this monograph.

Jain's result itself was a breakthrough when it appeared, and demonstrated the power of his *iterative rounding* method to prove this result that was conjectured based on a long line of earlier papers that applied a different primal-dual method to these problems. In this sense, his method was a purely primal attack on the problem. His method was extended by Lau et al. [88] to degree-bounded network design problems. The adaptation of this method by Singh and Lau [125] to the degree-bounded minimum cost spanning tree problem surprisingly involves no rounding at all! Instead, variables whose value are set to one in the linear programming relaxation are selected, and the program is modified carefully to continue to yield this property. This explains the title of this monograph and also hints at how this adaptation now allows one to prove *exact* results since we no longer have to round any variables and lose optimality.

Acknowledgments

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Dedications

Lau dedicates this work to his parents, his wife Pui Ming, and their children Ching Lam, Sing Chit, and Ching Yiu. Ravi dedicates this work to the memory of his late brother, R. Balasubramaniam, who encouraged him to write a book. Singh dedicates this work to his parents.