Aerothermal Engineering

C204 Turbomachinery

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Lecture 1

C2 Aerothermal Engineering

Michaelmas Term 2022

** Each block 4 lectures and 1 class sheet*

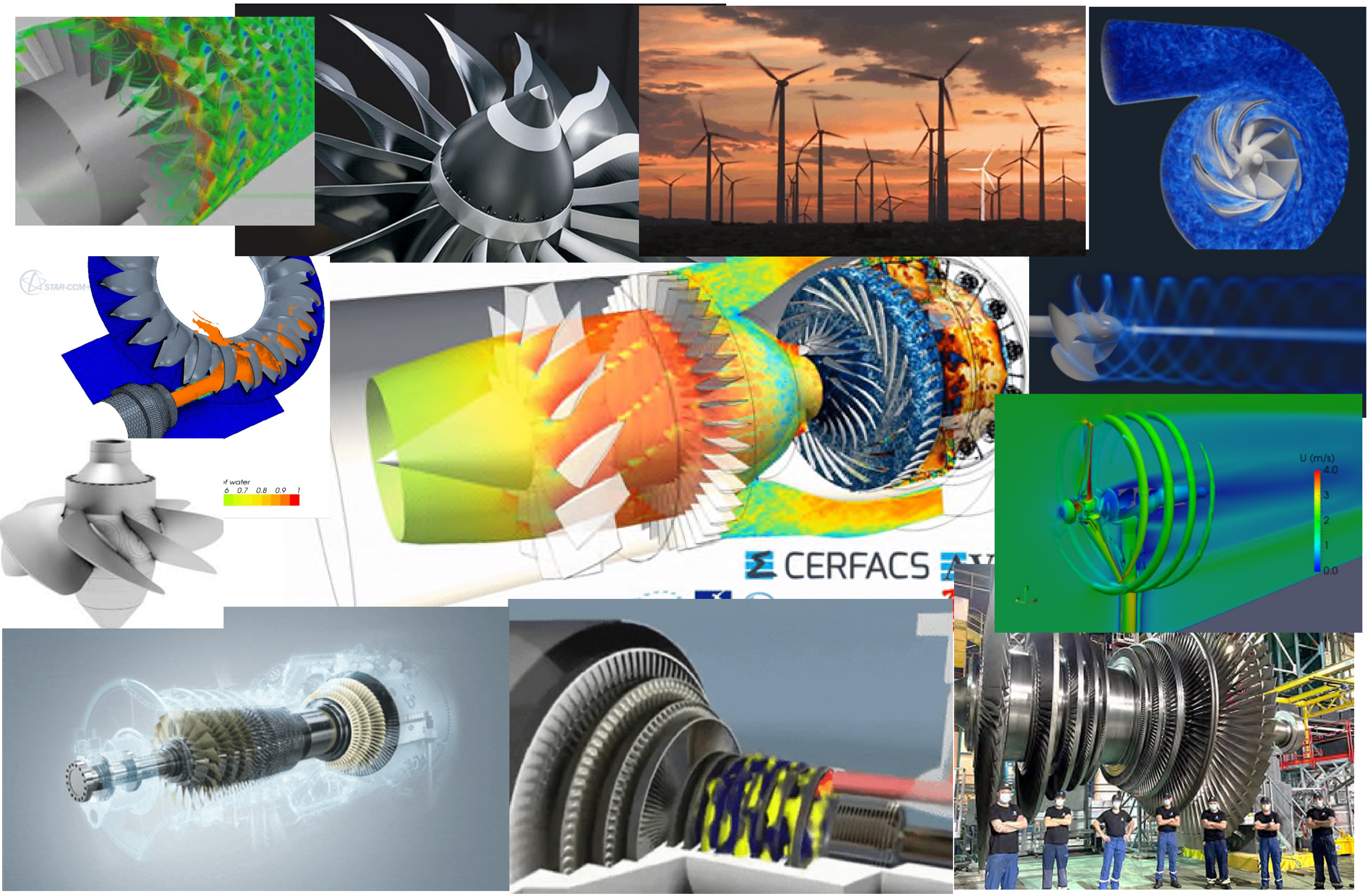
Week 1 & 2 Viscous Flow and Turbulence (Luca Di Mare)

Week 3 & 4 Turbomachinery (Budimir Rosic)

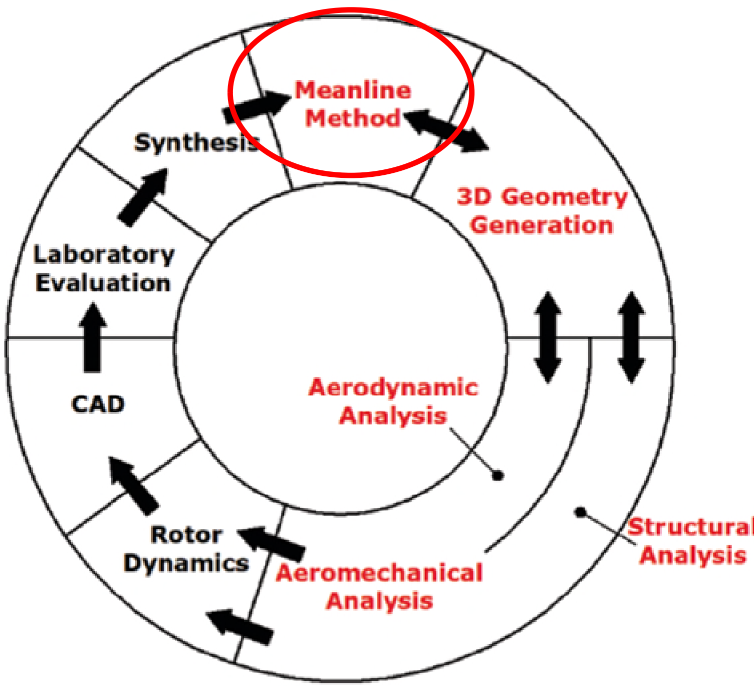
Week 5 & 6 Wing Theory and Compressible Flow (David Gillespie)

Week 7 & 8 Aircraft Flight and Propulsion (Li He)

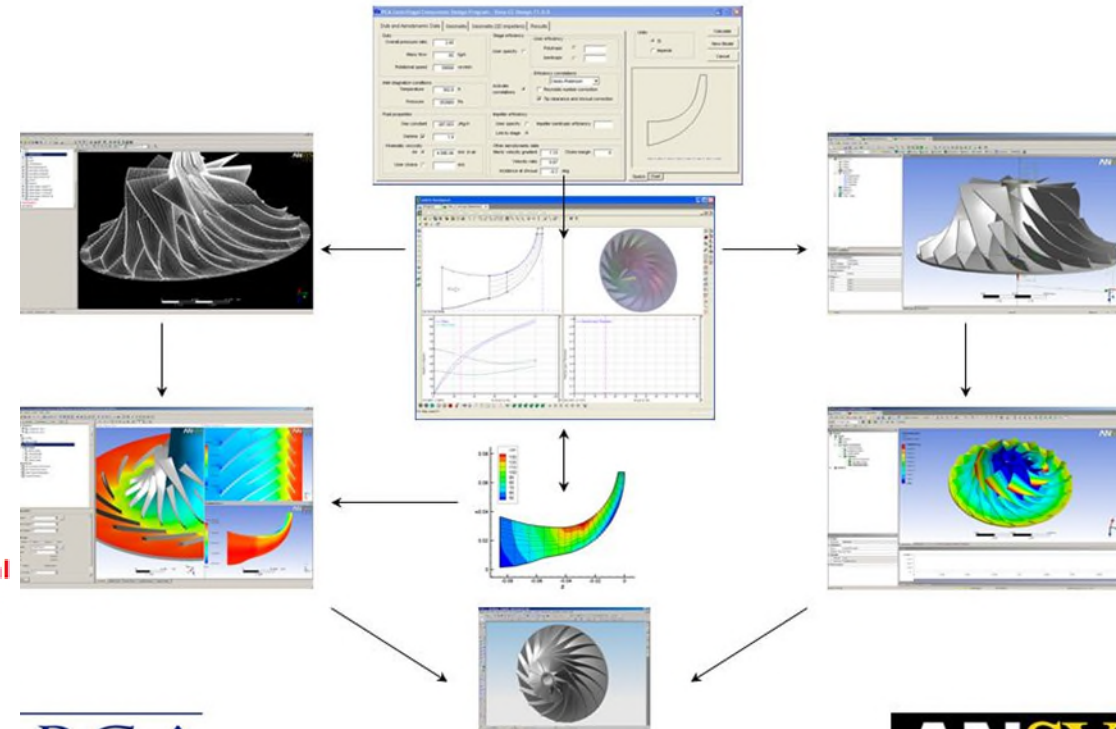
Turbomachines are the best means for transferring the mechanical energy to / from a fluid



What was Covered Last Year in B19? - Meanline Analysis



A typical turbomachinery design system
(*Japikse, D., and Baines, N. C., 1994, Introduction to turbomachinery*)



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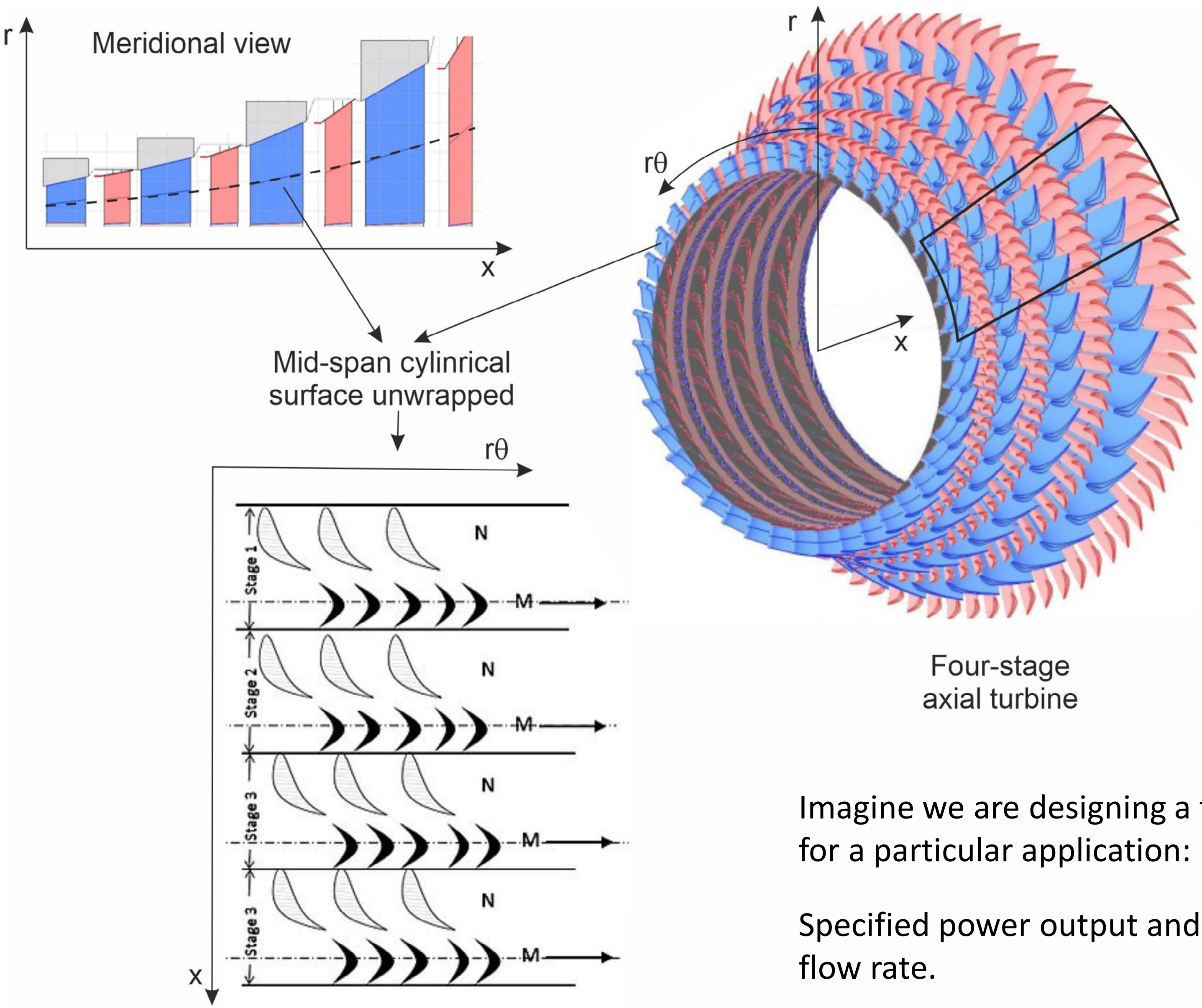
ANSYS

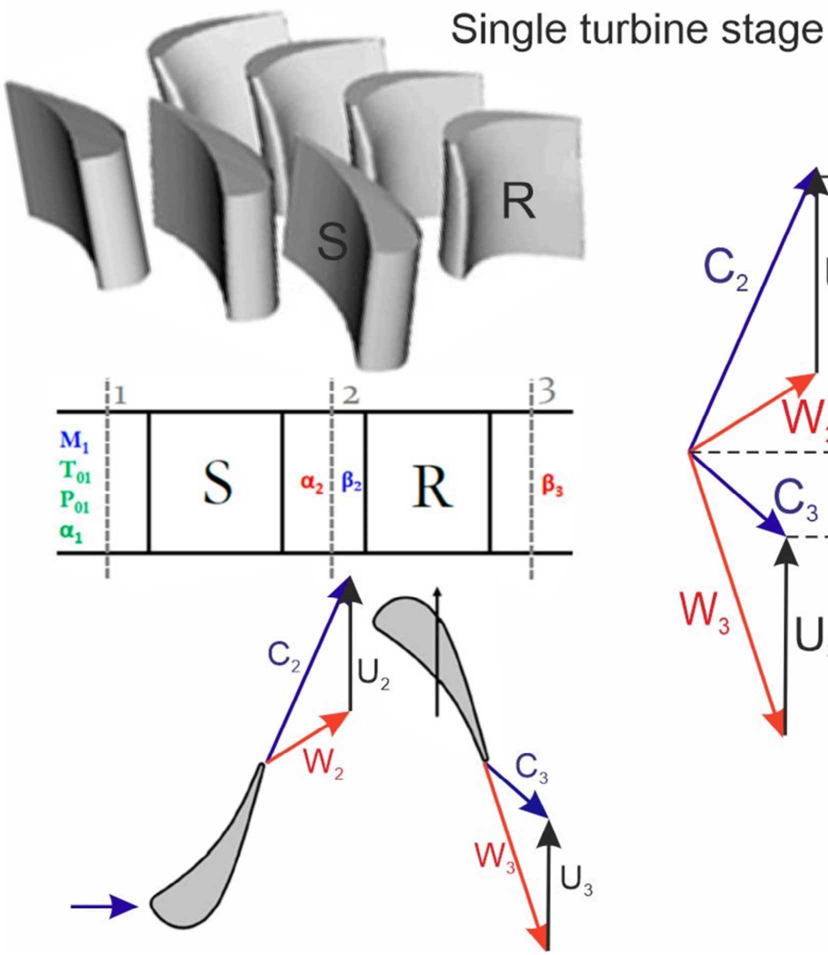
What was Covered Last Year in B19? - Meanline Analysis



Imagine we are designing a turbine for a particular application:

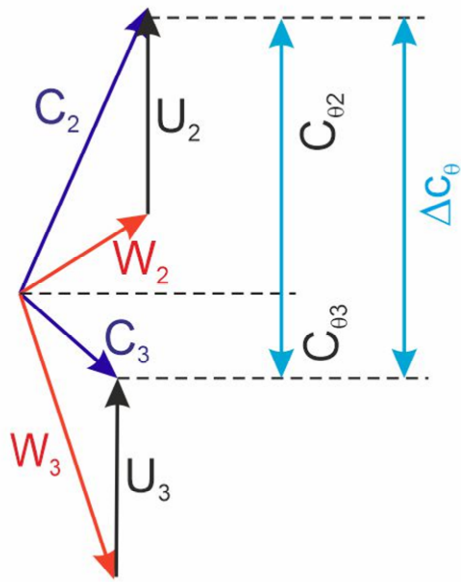
Specified power output and mass flow rate.

Meanline Analysis



Euler Work Equation

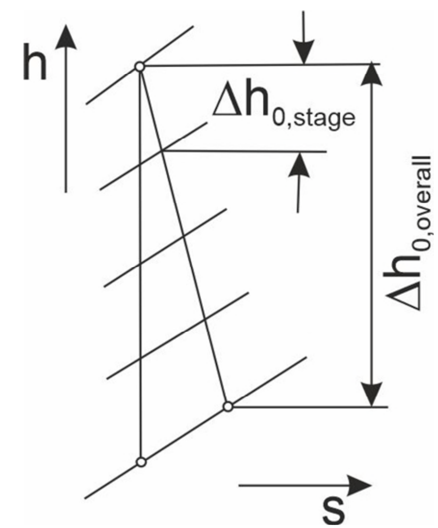
$$w = h_{02} - h_{03} = \Delta h_{0,stage} = U_2 c_{\theta 2} - U_3 c_{\theta 3}$$



The blade speed is usually fixed by stressing requirements and the overall enthalpy change is fixed by the operating requirements. Given these, a choice of the loading coefficient enables the number of stages to be fixed via

$$Power = \dot{m}\Delta h_{0,overall}$$

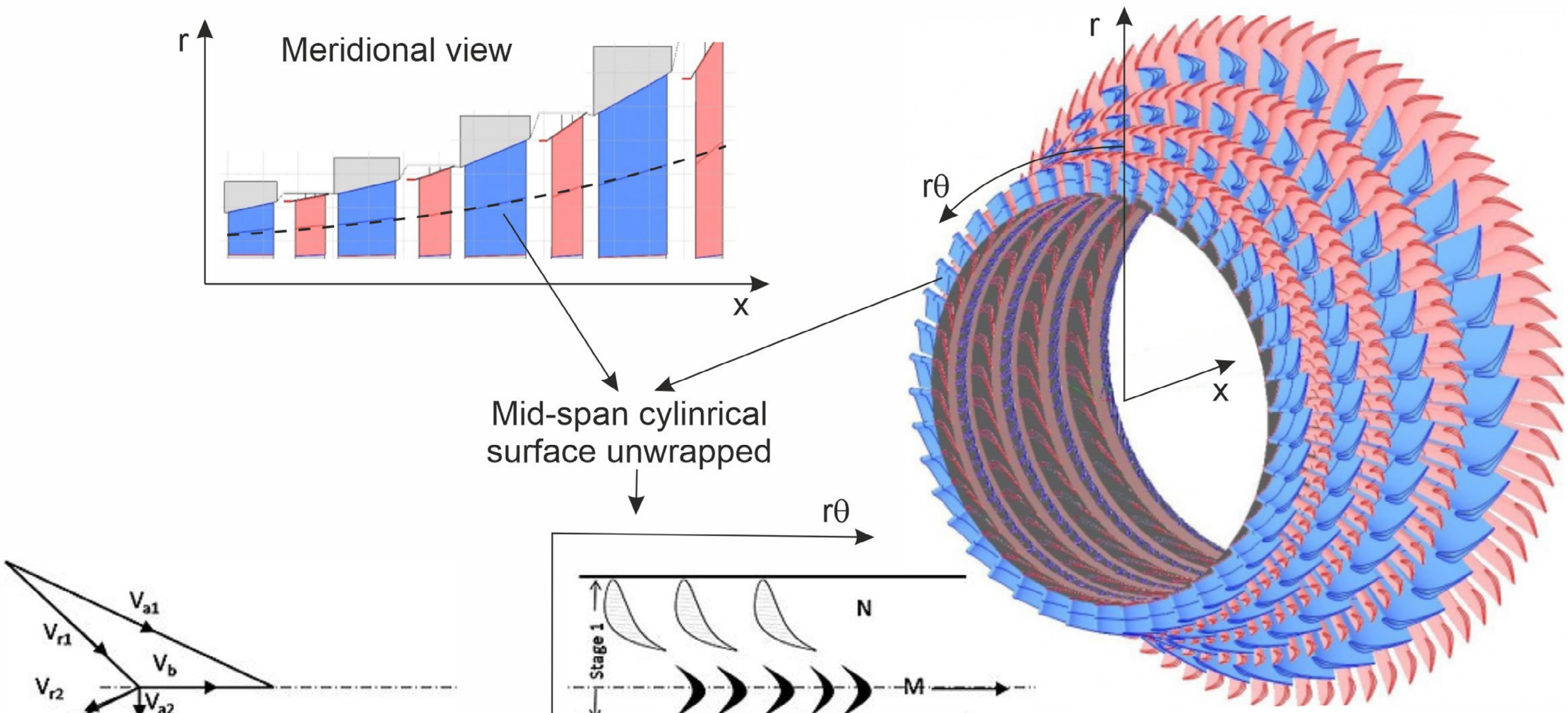
$$N = \frac{\Delta h_{0,overall}}{\Delta h_{0,stage}} = \frac{\Delta h_{0,overall}}{(U^2\psi)}$$

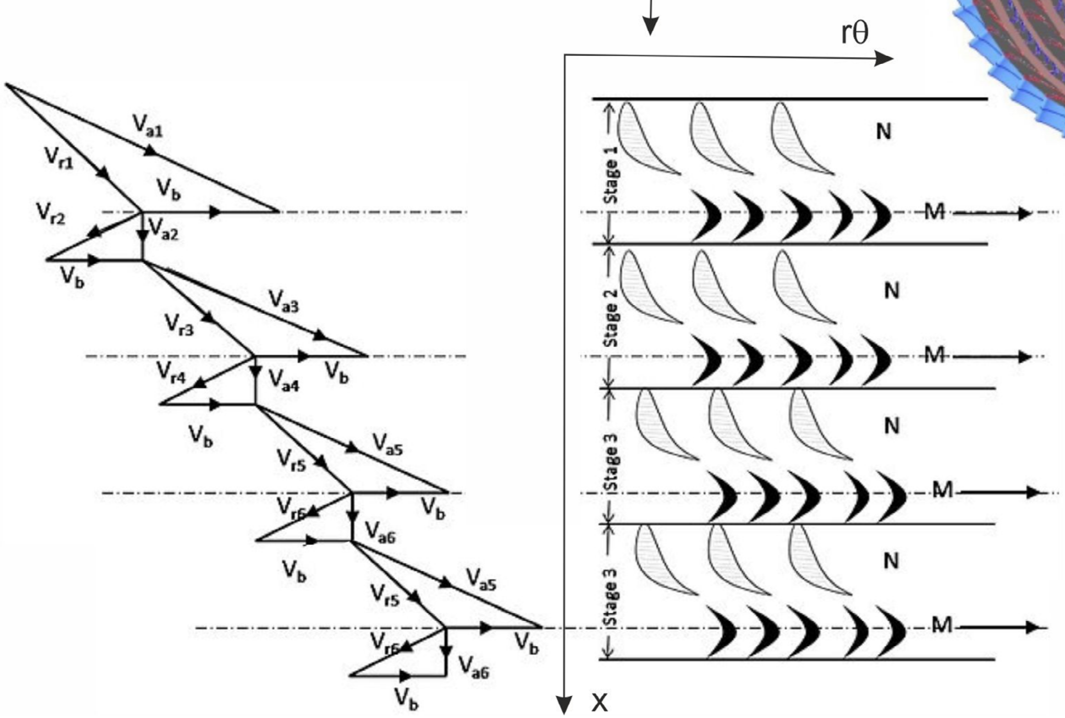


Any combination of 3 quantities chosen from: α_1, α_2, β_1, β_2, ϕ, ψ, Λ completely fixes <u>the velocity triangles for a repeating stage</u>.

(Common combinations are to fix ϕ, ψ and α_1 or ϕ, ψ and Λ.)

Mean Line Analysis



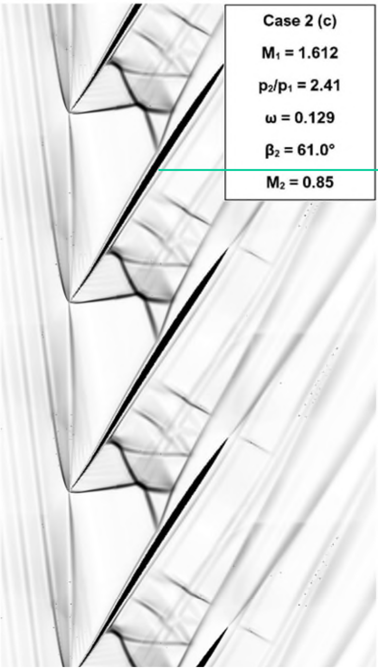


The mass flow rate is usually specified and the mean diameter, D_m, of the machine is also usually fixed, if the flow coefficient, ϕ, is chosen then the height of each stage, Δr, can be found from

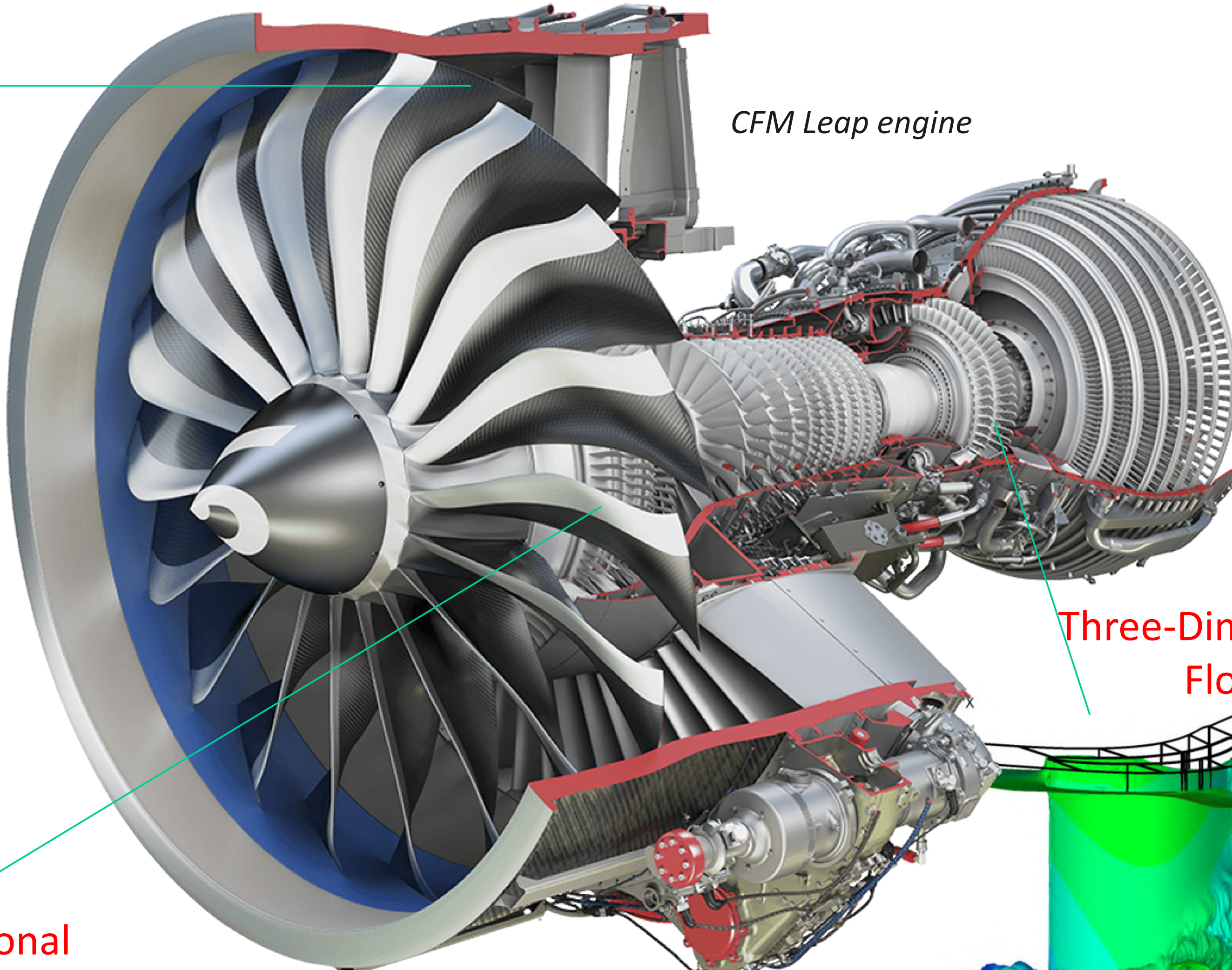
$$\Delta r = \frac{\dot{m}}{\pi D_m \rho C_x} = \frac{\dot{m}}{\pi D_m \rho \phi U}$$

The density must be calculated separately for each stage of a compressible flow machine.

In this course: Axial-Flow Turbine and Compressor



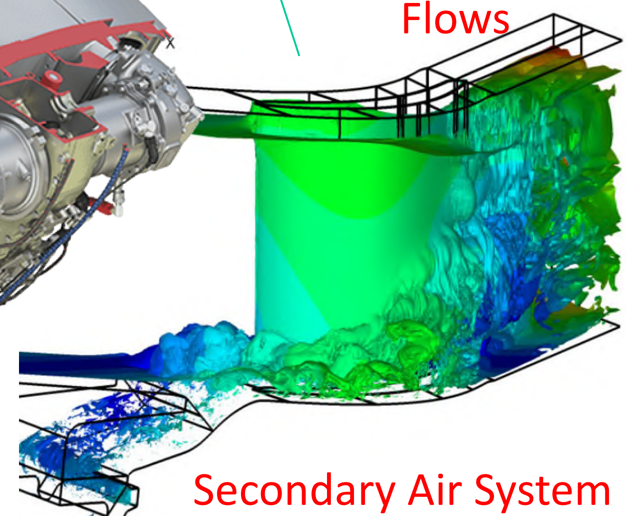
Compressibility Effects
(shock structure at the fan blade tip)



CFM Leap engine

Three-Dimensional Blade Design
(Fan blade – lean, twist, sweep)

Three-Dimensional Flows



Secondary Air System

Illustration of Three-Dimensional Nature of Flow in an Axial Turbine Stage

In this course

Lecture 1
Review of axial compressor and turbine stage characteristics and non-dimensional parameters.
Dimensional analysis of compressible flows in turbomachines.

Lecture 2
Two-Dimensional cascade compressible flow. Axial flow turbine and compressor stage analysis and design.

Lecture 3
Three-dimensional flows in axial turbomachines (radial equilibrium theory). Free-vortex turbine stage design. Three-dimensional design aspects (lean, sweep endwall profiling).

Lecture 4
Three-dimensional flow features in axial turbomachines (secondary flows, leakages).
Unsteady interactions. Secondary air system and cooling flows in gas turbines.

Learning Outcomes

1. To be able to calculate stage performance and different stage non-dimensional parameters for compressible axial flow turbine and compressor stages.
2. To perform mean-line analysis for compressible flow axial turbomachines.
3. To understands and be able to apply the principles of radial equilibrium theory.
4. To be able to apply different 'vortex design' concepts
5. To understand physics of different three-dimensional flow features in axial turbomachines.
6. To understand three-dimensional design aspects in axial turbomachines.

Reading List

Fluid Mechanics and Thermodynamics of Turbomachinery (7th Edition), S. L. Dixon, C. A. Hall; (Publisher: Elsevier)

Gas Turbine Theory, H. Cohen, G. F. C. Rogers, G.F.C., H. I. H. Saravanamuttoo; (Publisher: Pearson)

Compressor Aerodynamics N. A. Cumpsty; (Publisher: Krieger)

The Jet Engine, Rolls-Royce

Notation

Flow Parameters:

α_{rel}, β	Flow angle in rotating coordinate (from axial direction)
$\dot{m}$	Mass flow rate
$\dot{Q}$	Volumetric flow rate
α	Flow angle in absolute coordinate system
δ	Deviation (($\alpha_2 - \chi_2$) for compressor, ($\chi_2 - \alpha_2$) for turbine)
ϕ	Flow coefficient $= V_x/U$ or $Q/\Omega D^3$
ψ	Stage loading $= \Delta h_o/U^2$
h	Enthalpy
i	Incidence
Λ	Reaction
M	Mach number
μ	Dynamic viscosity
p	Static pressure
p_0	Stagnation pressure
T	Static temperature
T_0	Stagnation temperature
U	Blade speed
V, c	Absolute velocity
V_{rel}, w	Relative velocity
W	Work
η	Efficiency

Geometric Parameters

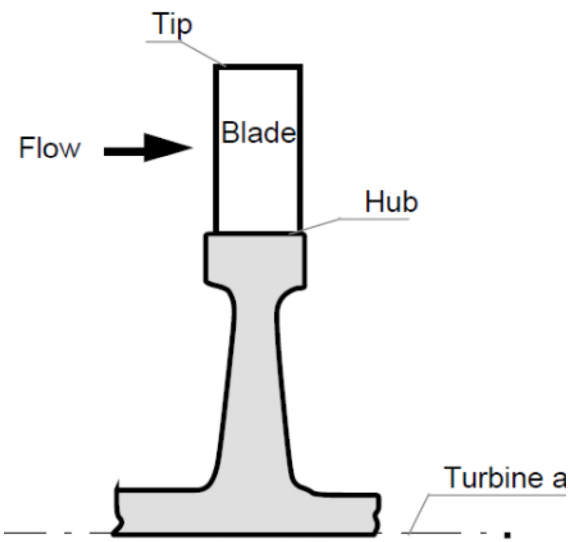
a	Annulus area $= \pi(r_{tip}^2 - r_{hub}^2) \approx 2\pi r_{mean} h$
A	Effective flow area – measured perpendicular to velocity vector $= acos\alpha$
χ	Metal blade angle (from axial direction)
C	Blade chord
C_x	Axial chord
D	Diameter (usually mean or tip)
γ	Stagger angle
h	Annulus height, blade height, span
o	Throat (opening)
θ	Blade camber ($(\chi_1 - \chi_2)$ for compressor, $(\chi_2 - \chi_1)$ for turbine
r	Radius
s	Pitch (spacing)

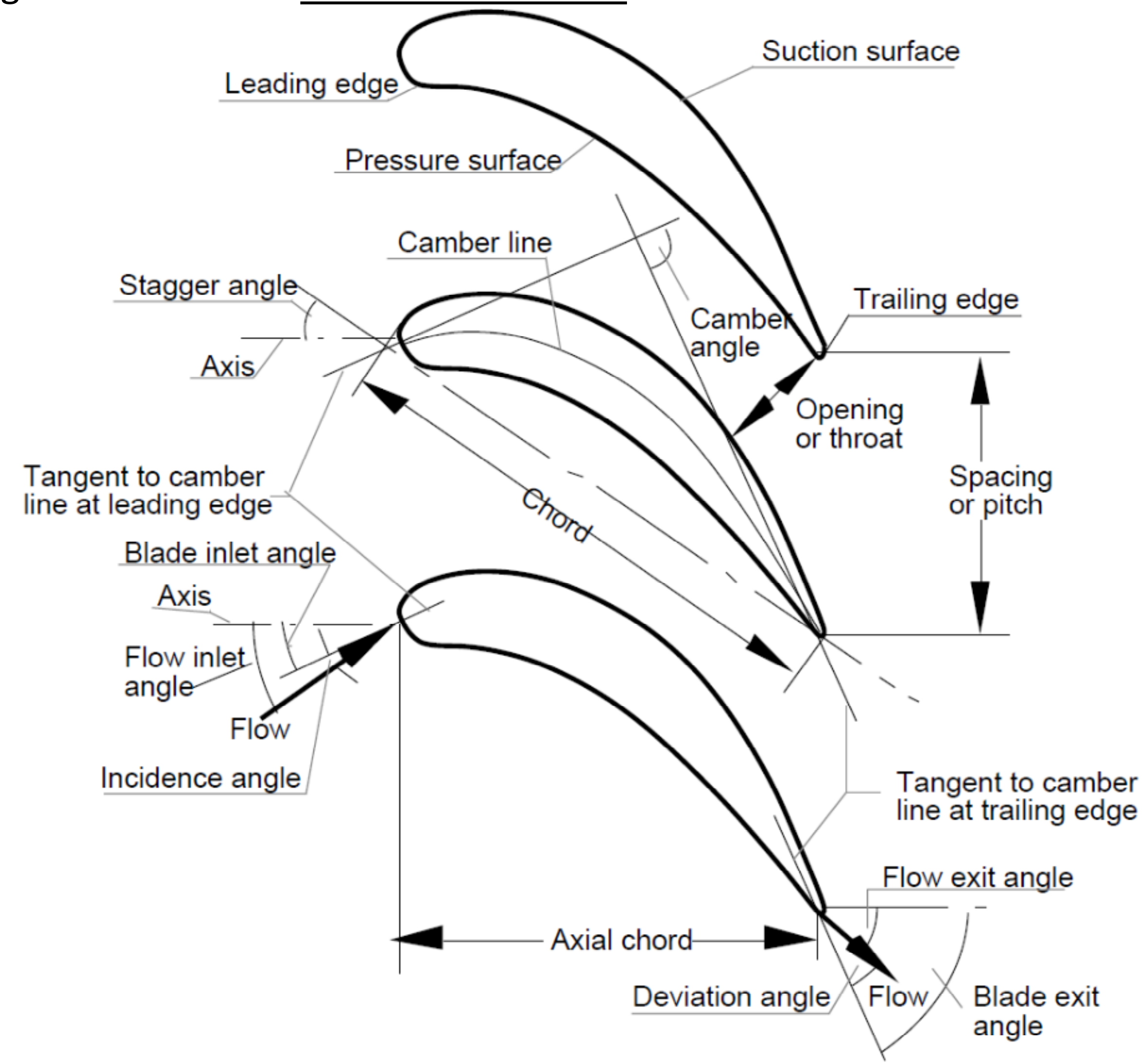
Suffices:

0	Stagnation
1	Inlet
2	Cascade exit or 1st blade row exit / 2nd blade row inlet
3	Stage exit / 2nd blade row exit / 3rd blade row inlet
m	Meridional or mean
$mean$	Value at mean radius ($r_{mean} = \frac{1}{2}(r_{tip} + r_{hub})$)
P	Polytropic (efficiency)
r	Radial
rel	Relative frame of reference (usually rotating frame implied)
x	Axial
y, θ	Tangential

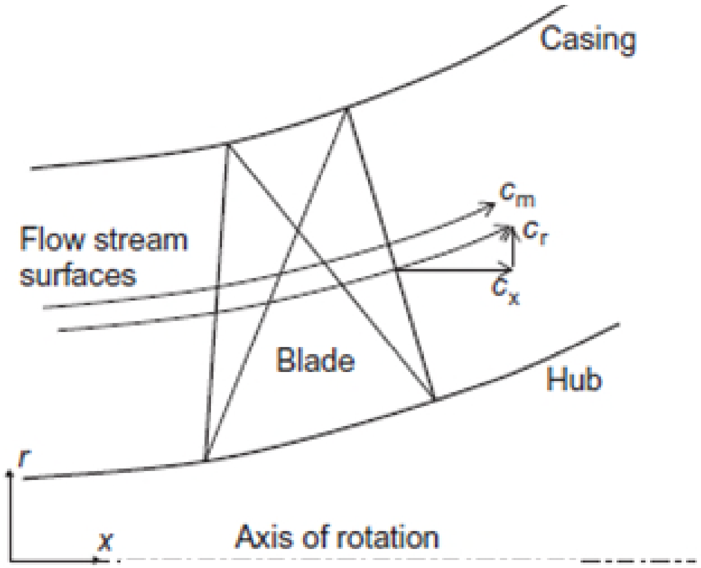
Blade section nomenclature

The blade itself is defined by a camber line and a thickness distribution (locally normal to the camber line), although other definition of shape are in use.
Blade camber angle $\theta = (\chi_1 - \chi_2)$ for a compressor blade and $\theta = (\chi_2 - \chi_1)$ for a turbine blade.
Blade metal angle and flow angle are measured <u>from axial direction</u>.

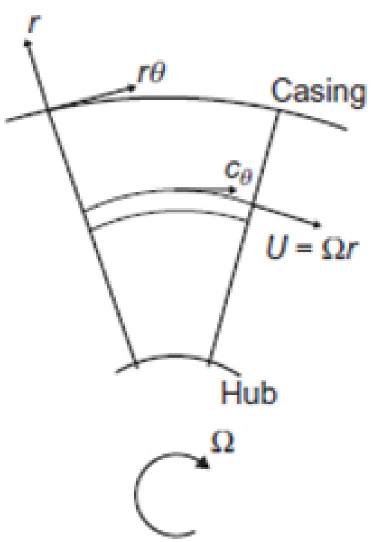




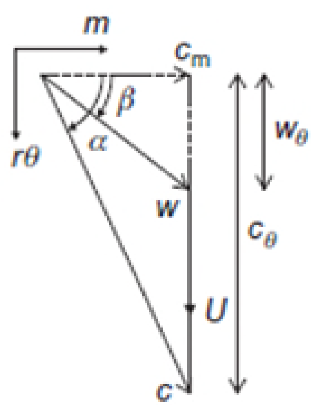
The Coordinate System and Flow Velocities within a Turbomachine



Meridional or side view



View along the axis



View looking down onto a stream surface.

The component of velocity along an axi-symmetric stream surface is called the meridional velocity:

$$c_m = \sqrt{c_x^2 + c_r^2}$$

In purely axial flow machines the radius of the flow path is constant and, $c_m = c_x$

The swirl, or tangential, angle is the angle between the flow direction and the meridional direction: $\alpha = tan^{-1}(c_\theta / c_m)$

Relative velocities (In a frame of reference that is stationary relative to the blades)

The relative velocity, *w* is the vector subtraction of the local velocity of the blade U from the absolute velocity of the flow c,

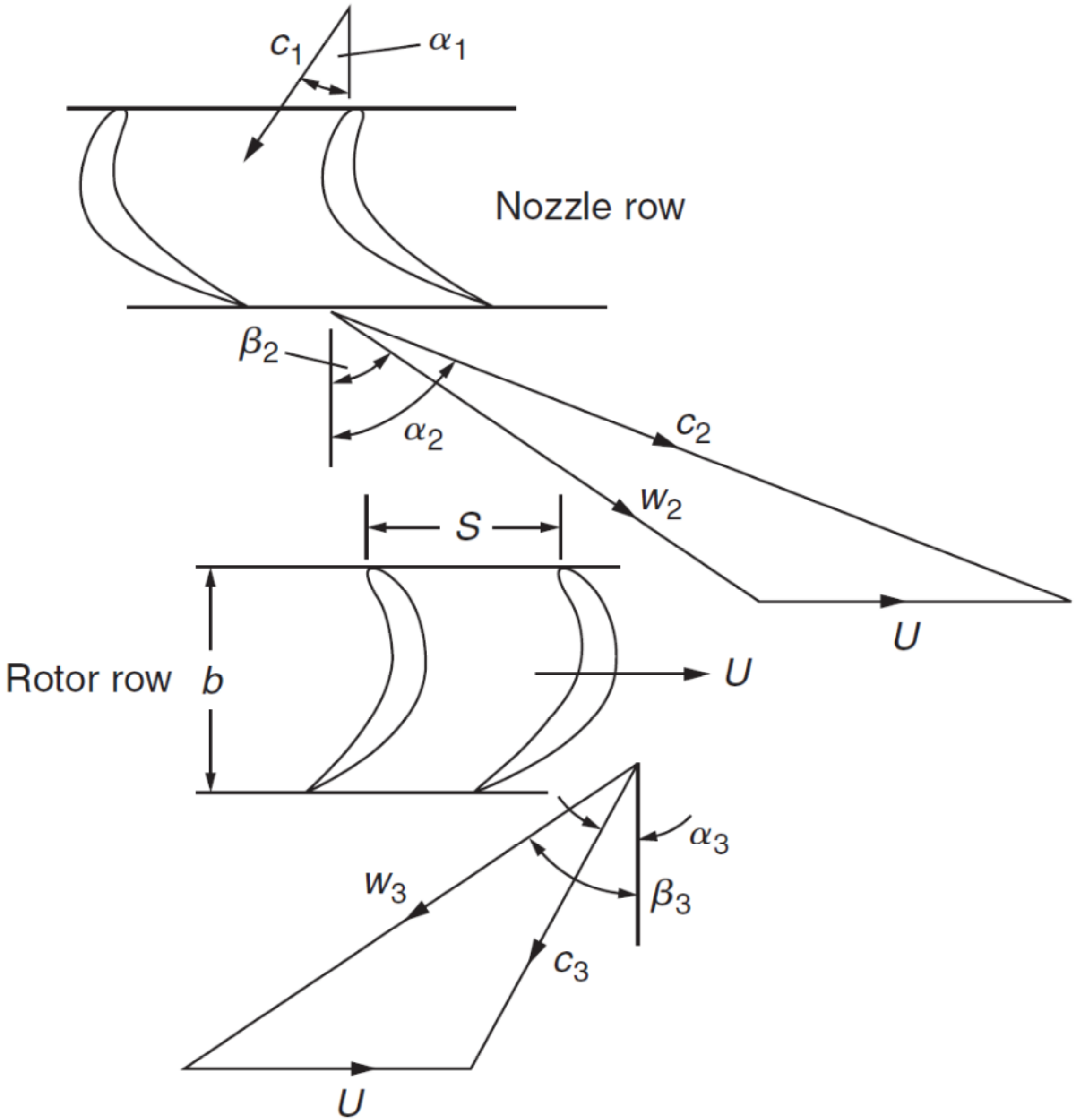
$$w_\theta = c_\theta - U$$

The relative flow angle is the angle between the relative flow direction and the meridional direction: $\beta = tan^{-1}(w_\theta / c_m)$

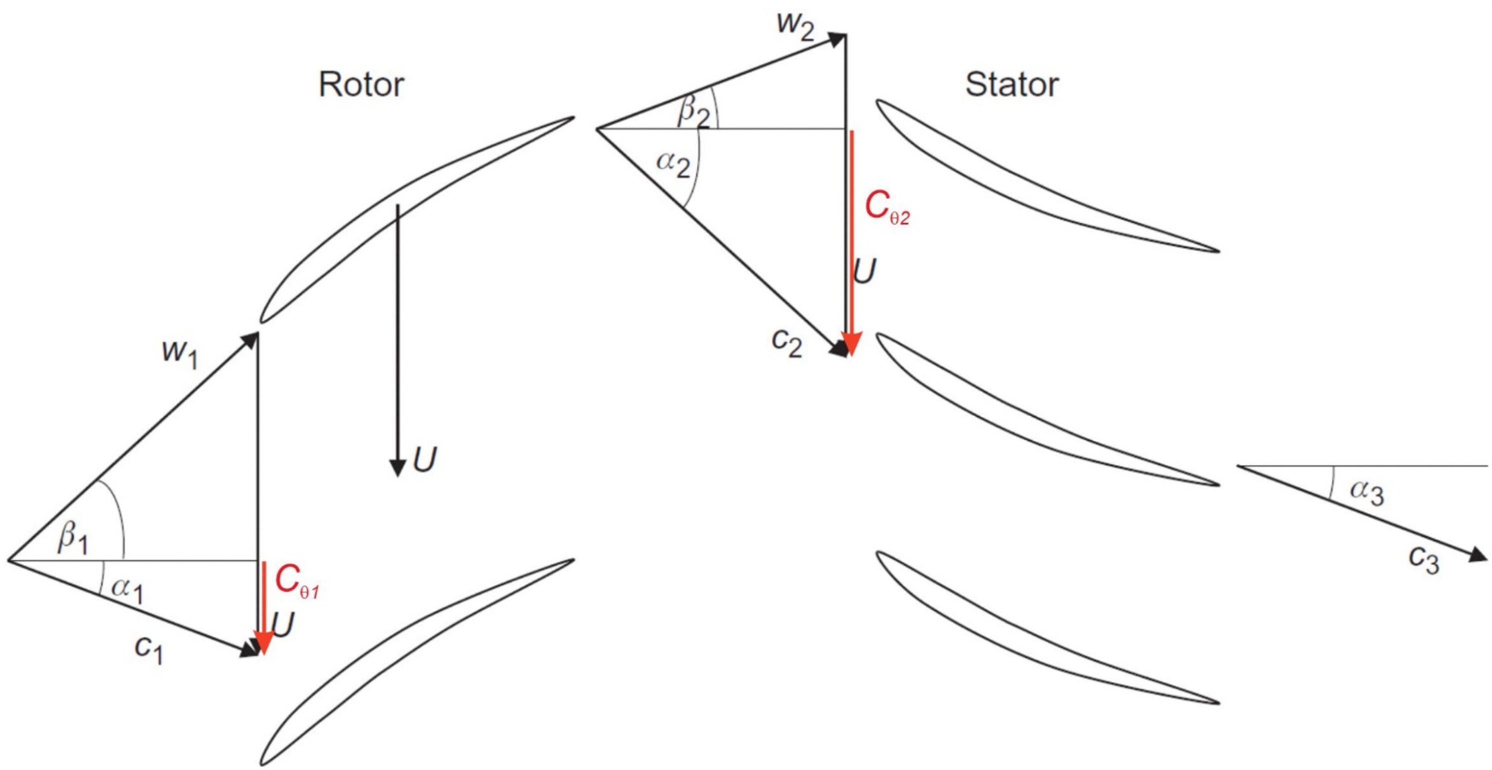
$$tan\beta = tan\alpha - U/c_m$$

Sign Convention: is to use positive values for tangential velocities and angles that are in the direction of rotation

Axial Turbine Stage – Velocity Triangles

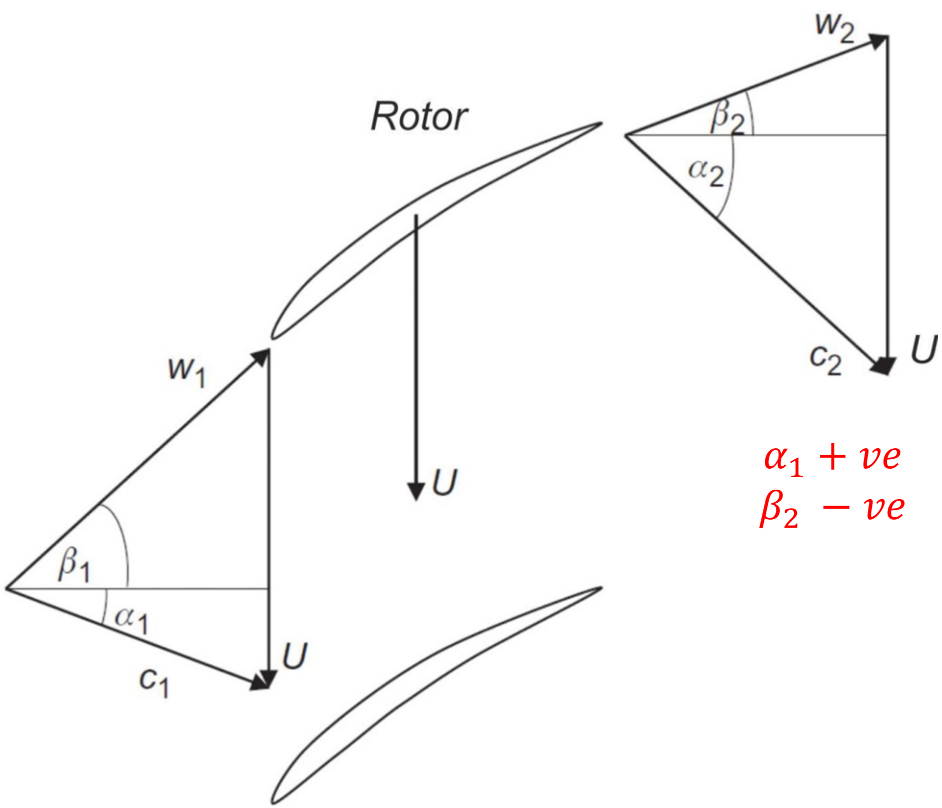


Axial Compressor Stage – Velocity Triangles



Example - Axial Compressor Stage – Velocity Triangles

The axial velocity through an axial flow fan is constant and equal to 30 m/s. The flow angles for the stage are $\alpha_1 = 23^o$ and $\beta_1 = 60^o$
Determine the blade speed U and, if the mean radius of the fan is 0.15 m, find the rotational speed of the rotor.



$\alpha_1 + ve$
$\beta_2 \; - ve$

$$c_{\theta 1} = w_{\theta 1} + U$$

$$c_{\theta 1} = c_x tan\alpha_1$$
$$w_{\theta 1} = c_x tan\beta_1$$

$$U = c_{\theta 1} - w_{\theta 1}$$

$$U = c_x tan\alpha_1 - c_x tan\beta_1 =$$
$$= c_x(tan\alpha_1 - tan\beta_1)$$

$$U = 30(tan23^o - tan(-60^o)) = 64.8\, m/s$$

$$\Omega = \frac{U}{r_m} = \frac{64.8}{0.15} = 432\frac{rad}{s}$$
$$or \quad 432 \cdot \frac{30}{\pi} = 4125.3RPM$$

α_1 *is* $+ ve$ *(in the direction of blade rotation)*

β_1 *is* $- ve$

Fundamental Laws

Continuity: $\dot{m} = \rho_1 c_1 A_{n1} = \rho_2 c_2 A_{n2}$,
A_{n1} and A_{n2} are the areas normal to the flow direction at station 1 and 2 along a passage.

The Steady Flow Energy Equation (SFEE)
$\dot{Q} - \dot{W} = \dot{m}(h_{02} - h_{01})$
The stagnation enthalpy is constant in any process that does not involve a work or heat transfer.
Most turbomachinery flow processes are adiabatic.
For work producing machines (turbines), $\dot{W} > 0$: $\dot{W}_t = \dot{m}(h_{01} - h_{02})$
For work absorbing machines (compressors), $\dot{W} < 0$: $\dot{W}_c = \dot{m}(h_{02} - h_{01})$

The Moment of Momentum – The Euler Work Equation

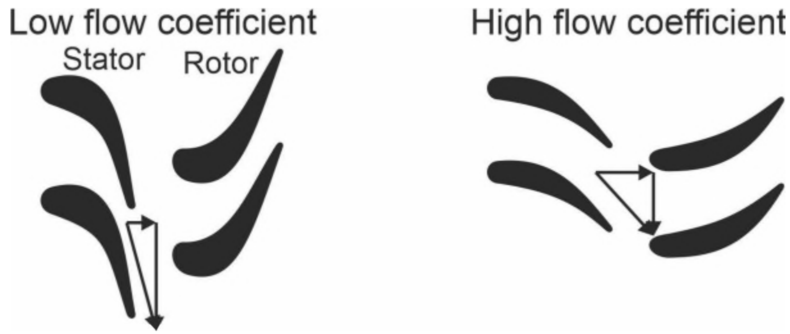
$$\dot{W} = \dot{m}(U_2 c_{\theta 2} - U_1 c_{\theta 1})$$

For any adiabatic turbomachine (turbine or compressor) applying the SFEE:

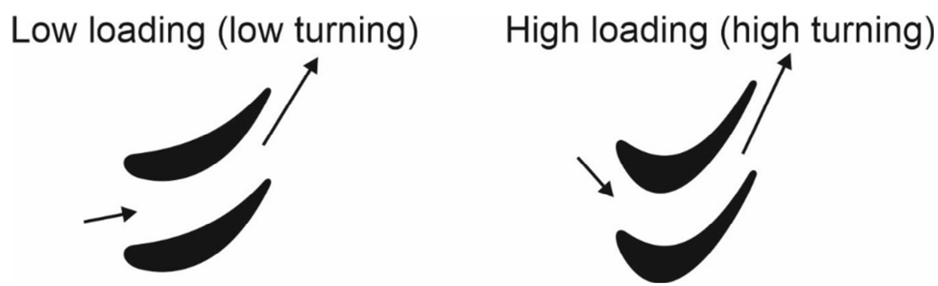
$$w = h_{01} - h_{02} = U_1 c_{\theta 1} - U_2 c_{\theta 2}$$

Stage Characteristics and Non-dimensional Parameters

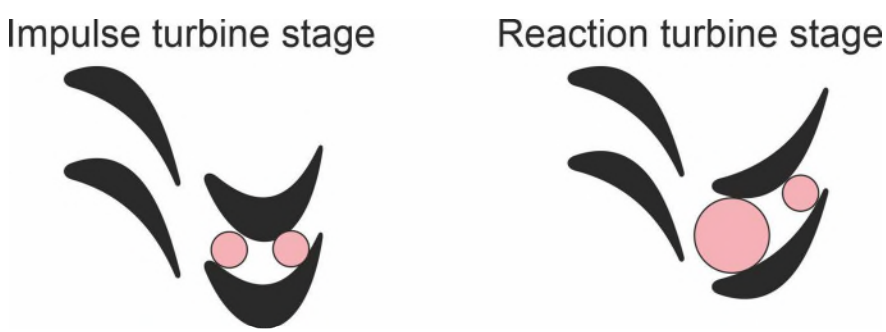
$\boldsymbol{\phi = c_x/U}$ - Flow coefficient - the ratio of axial flow velocity to blade speed



$\boldsymbol{\psi = \Delta h_0/U^2}$ - Stage loading - the ratio of the stagnation enthalpy change through a stage to the square of the blade speed.



$\boldsymbol{\Lambda = \Delta h_{rotor}/\Delta h_{stage}}$ - Stage reaction – ratio of the change in static enthalpy through the rotor to that through the stage.



Definitions of Efficiency

Efficiency of Turbines:

The isentropic efficiency:

$$\eta_{is} = \frac{Power\ output\ \ to\ rotor}{Maximum\ energy\ difference\ possible\ for\ the\ fluid\ in\ unit\ time}$$

Mechanical efficiency :

$$\eta_m = \frac{Power\ output\ to\ coupling\ of\ shaft}{Power\ output\ to\ rotor}$$

Represents the mechanical energy losses that occur between the turbine rotor and the output shaft coupling as a result of the work done against friction at the bearings, glands, etc. (For small machines (several kilowatts) it may amount to 5% or more, but for medium and large machines this loss ratio may become as little as 1%)

The overall efficiency:

$$\eta_o = \frac{Power\ output\ to\ coupling\ of\ shaft}{Maximum\ energy\ difference\ possible\ for\ the\ fluid\ in\ unity\ time}$$

$$\eta_o = \eta_m \eta_{is}$$

Definitions of Efficiency

Efficiency of Compressors:

The isentropic efficiency:

$$\eta_{is} = \frac{Useful\ mechanical\ energy\ input\ to\ fluid\ in\ unit\ time}{Power\ input\ to\ rotor}$$

Mechanical efficiency :

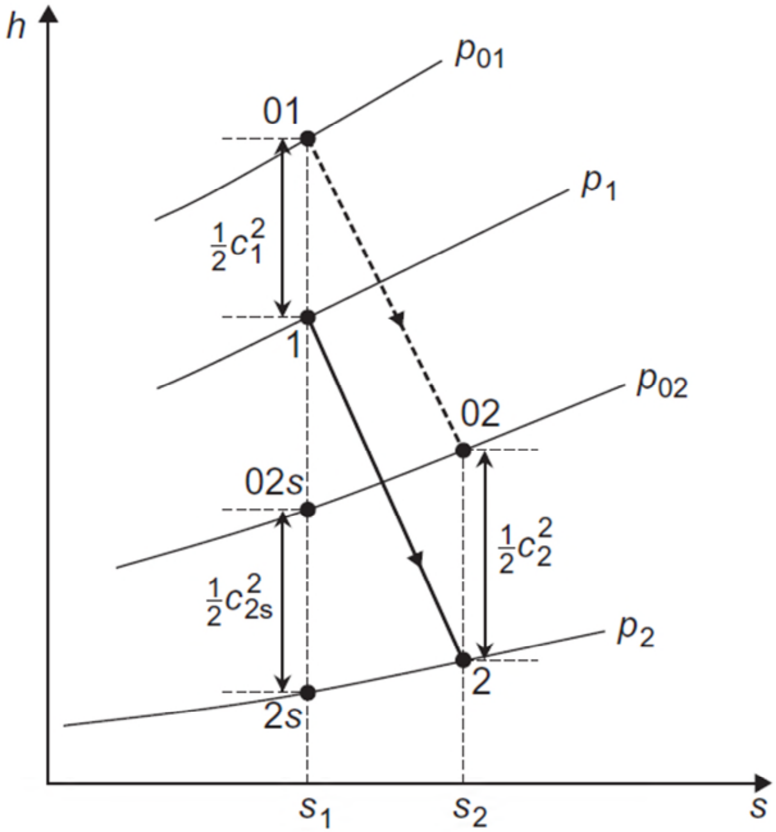
$$\eta_m = \frac{Power\ output\ to\ rotor}{Power\ output\ to\ coupling\ of\ shaft}$$

The overall efficiency:

$$\eta_o = \frac{Useful\ mechanical\ energy\ input\ to\ fluid\ in\ unit\ time}{Power\ input\ to\ coupling\ of\ shaft}$$

$$\eta_o = \eta_m \eta_{is}$$

Efficiency of Turbines



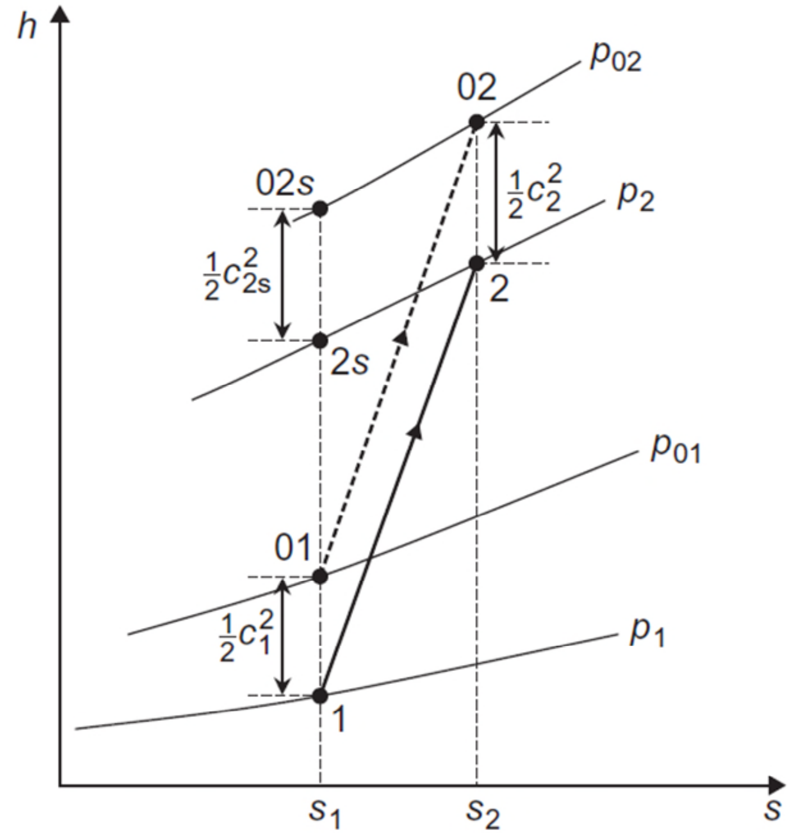
Total-to-total isentropic efficiency:

$$\eta_{tt} = \frac{h_{01} - h_{02}}{h_{01} - h_{02s}}$$

Total-to-static isentropic efficiency:

$$\eta_{ts} = \frac{h_{01} - h_{02}}{h_{01} - h_{2s}}$$

Efficiency of Compressors



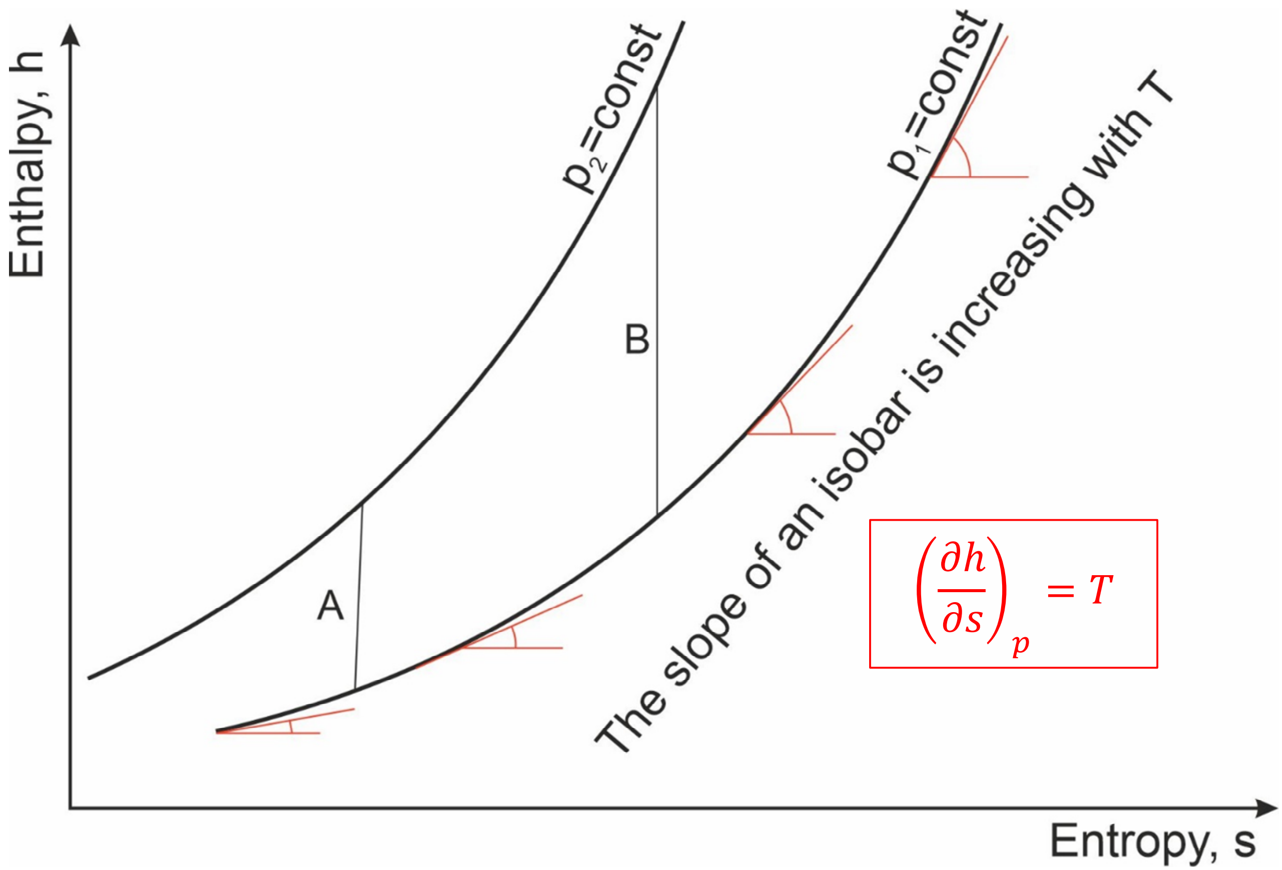
Total-to-total isentropic efficiency:

$$\eta_{tt} = \frac{h_{02s} - h_{01}}{h_{02} - h_{01}}$$

Total-to-static isentropic efficiency:

$$\eta_{ts} = \frac{h_{2s} - h_{01}}{h_{02} - h_{01}}$$

Small Stage or Polytropic Efficiency

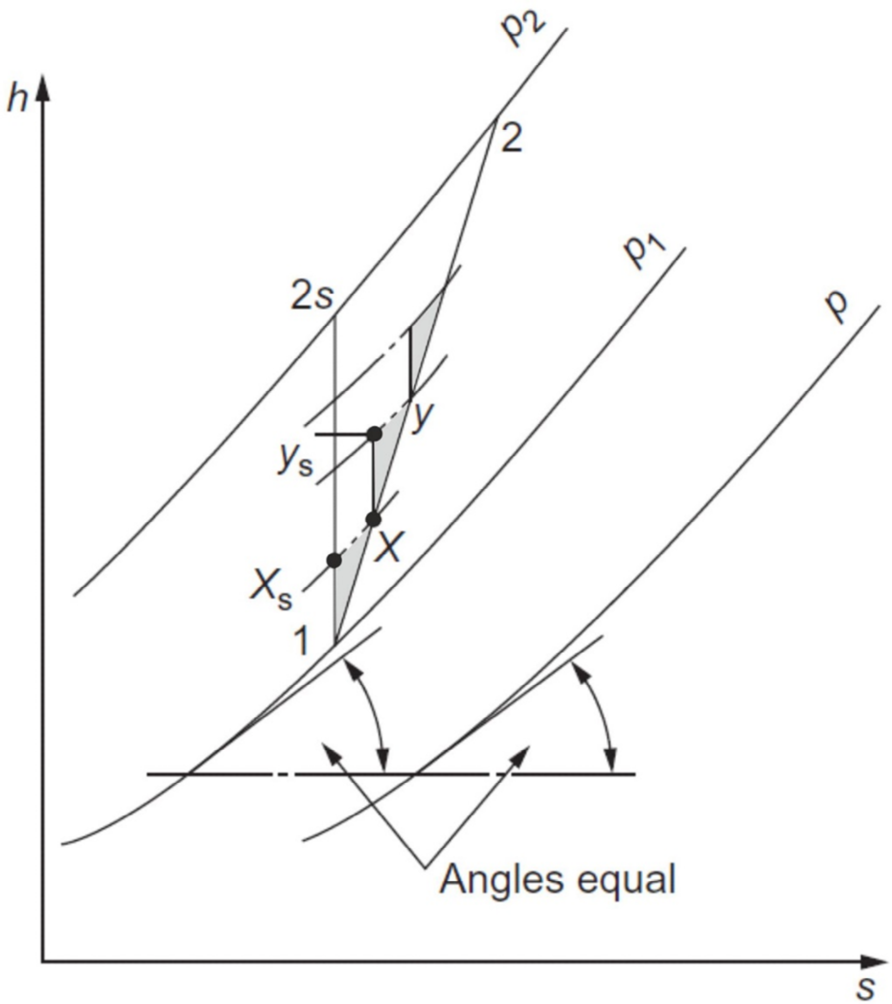


From the relation $Tds = dh - vdp$, for a constant pressure process, $(\partial h/\partial s)_p = T$

$T \uparrow \quad \rightarrow \quad greater\ the\ slope\ of\ p = const\ line$

Small Stage or Polytropic Efficiency

The isentropic efficiency can be misleading if used for comparing the efficiencies of turbomachines of differing pressure ratios.
Regard a turbomachine as being composed of a large number of very small stages (irrespective of the actual number of stages). If each small stage has the same efficiency, η_P, then the isentropic efficiency of the whole machine will be different from the small stage efficiency.



Compression Process

Adiabatic compression between pressures p_1 and p_2: between states 1 and 2; isentropic line 1 to 2s.

Divide the compression process into a large number of small stages of equal efficiency η_P. For each small stage (grey areas) the actual work input is δW and the corresponding ideal work in the isentropic process is δW_{min}.

$$\eta_P = \frac{\delta W_{min}}{\delta W} = \frac{h_{xs} - h_1}{h_x - h_1} = \frac{h_{ys} - h_x}{h_y - h_x} = \cdots$$

Each small stage has the same efficiency, hence:

$$\eta_P = \sum \delta W_{min} / \sum \delta W$$

$$\sum dW = (h_x - h_1) + (h_y - h_x) + \cdots = (h_2 - h_1)$$

$$\eta_P = [(\boldsymbol{h_{xs}} - \boldsymbol{h_1}) + (\boldsymbol{h_{ys}} - \boldsymbol{h_x}) + \cdots]/(h_2 - h_1)$$

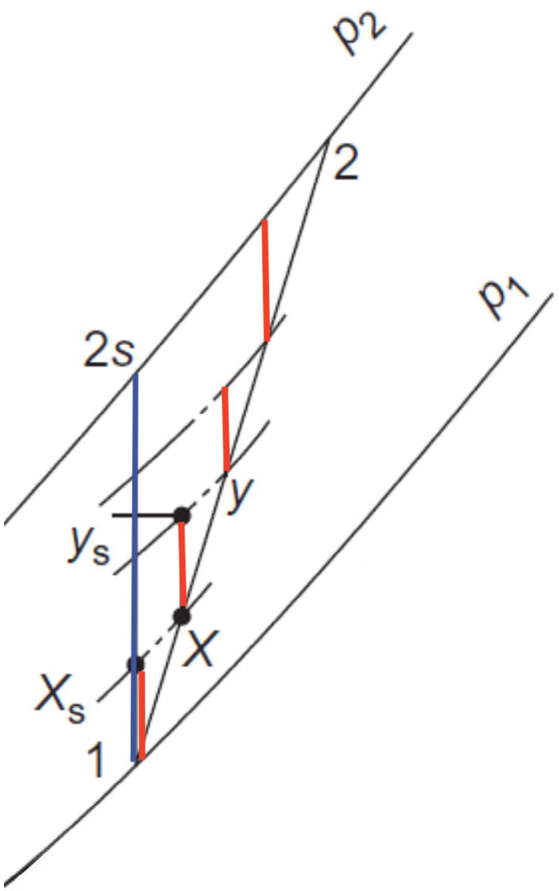
Isentropic efficiency for the whole compression process:

$$\eta_{is,c} = (\boldsymbol{h_{2s}} - \boldsymbol{h_1})/(h_2 - h_1)$$

Due to divergence of the constant pressure lines:

$$[(\boldsymbol{h_{xs}} - \boldsymbol{h_1}) + (\boldsymbol{h_{ys}} - \boldsymbol{h_x}) + \cdots] > (\boldsymbol{h_{2s}} - \boldsymbol{h_1})$$

$$\sum \delta W_{min} > W_{min}$$



$$\boxed{\begin{array}{c}\eta_P > \eta_{is,c} \\ (Difference\ being\ dependent\ upon\ the\ divergence\ of\ the\ constant\ pressure\ lines)\end{array}}$$

Small stage efficiency for a perfect gas

Assume an infinitesimal compressor stage (an incremental change in pressure is dp)
The polytropic efficiency for the small stage is:

$$\eta_P = \frac{dh_{is}}{dh}$$

For an isentropic process $Tds = 0 = dh_{is} - vdp$,

$$\eta_P = \frac{vdp}{C_p dT}$$

The perfect gas $v = RT/p,\ C_p = \gamma R/(\gamma - 1)$

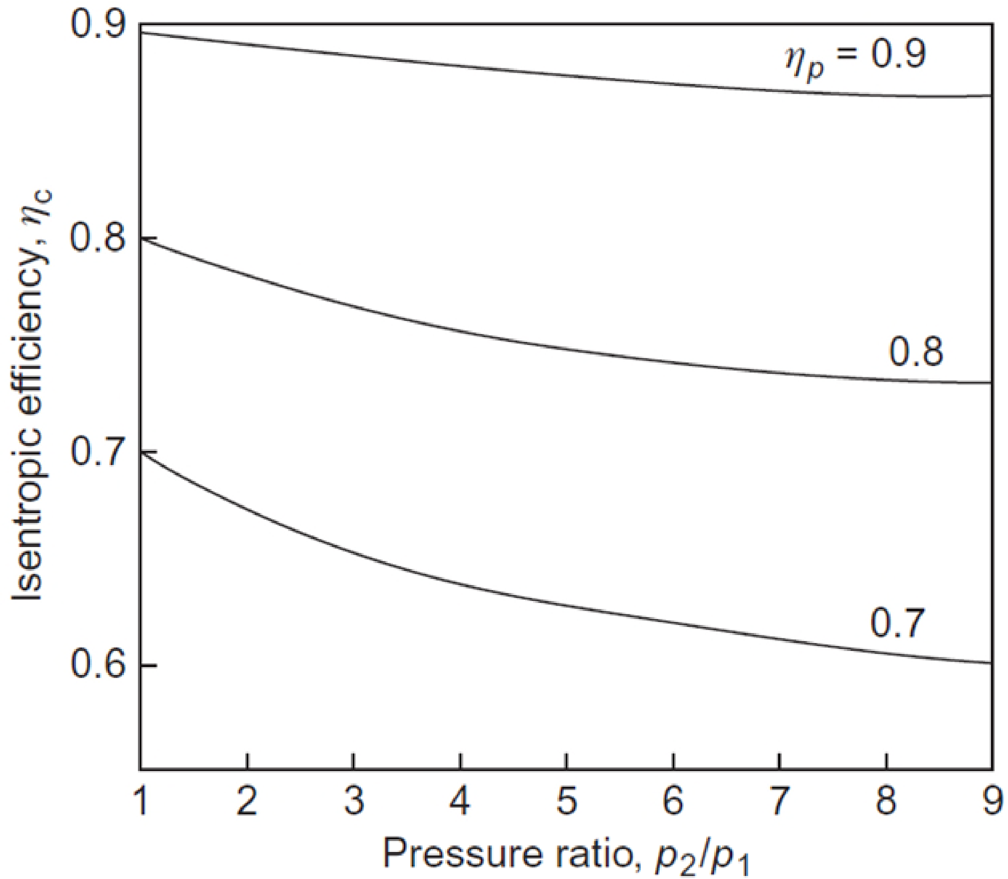
$$\frac{dT}{T} = \frac{(\gamma-1)}{\gamma\eta_P}\frac{dp}{p} \quad \rightarrow$$

$$\frac{T_2}{T_1} = \left(\frac{p_2}{p_1}\right)^{(\gamma-1)/\eta_P\gamma} \quad \rightarrow$$

$$\eta_P = \frac{\gamma-1}{\gamma}\frac{\ln(p_2/p_1)}{\ln(T_2/T_1)}$$

The isentropic efficiency for the whole compression process is:

$$\eta_{is,C} = (T_{2s} - T_1)/(T_2 - T_1)$$



The ideal compression process: $\eta_P = 1 \quad \rightarrow \qquad {}^{T_{2s}}/_{T_1} = ({}^{p_2}/_{p_1})^{(\gamma-1)/\gamma}$

$$\eta_{is,C} = \frac{T_{2s} - T_1}{T_2 - T_1} = \frac{\dfrac{T_{2s}}{T_1} - 1}{\dfrac{T_2}{T_1} - 1} = \frac{\left(\dfrac{p_2}{p_1}\right)^{(\gamma-1)/\gamma} - 1}{\left(\dfrac{p_2}{p_1}\right)^{(\gamma-1)/\eta_P\gamma} - 1}$$

Example

An axial flow air compressor is designed to provide an overall total-to-total pressure ratio of 8 to 1. At inlet and outlet the stagnation temperatures are 300 K and 600 K, respectively. Determine the overall total-to-total efficiency and the polytropic efficiency for the compressor. Assume air as the working fluid $\gamma = 1.4$.

Solution

$$\eta_{is,C} = \frac{T_{02s} - T_{01}}{T_{02} - T_{01}} = \frac{\left(\dfrac{p_{02}}{p_{01}}\right)^{(\gamma-1)/\gamma} - 1}{\dfrac{T_{02}}{T_{01}} - 1} = \frac{8^{0.4/1.4} - 1}{2 - 1} = 0.81$$

$$\eta_P = \frac{\gamma - 1}{\gamma} \frac{\ln(p_{02}/p_{01})}{\ln(T_{02}/T_{01})} = 0.286 \frac{ln8}{ln2} = 0.286 \frac{2.08}{0.693} = 0.858$$

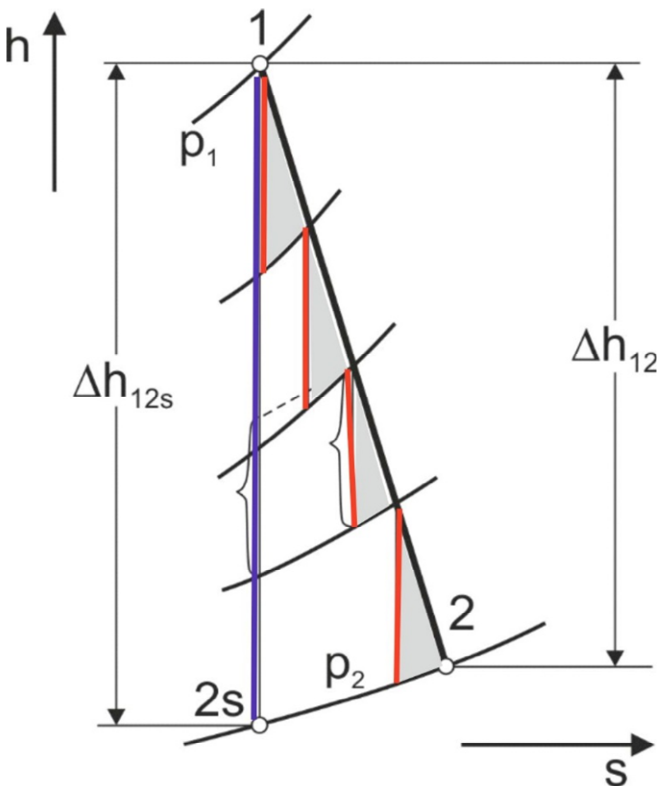
Turbine Polytropic Efficiency

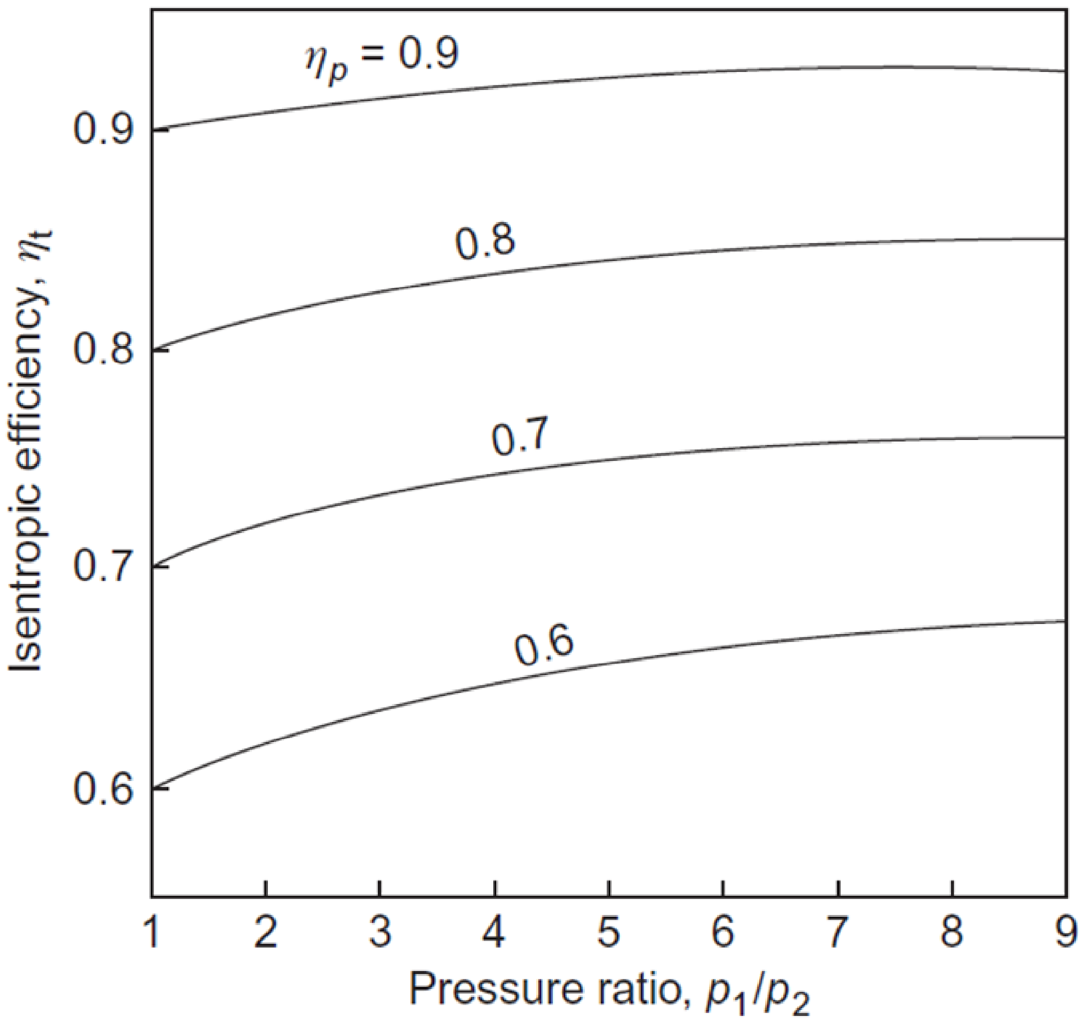
A similar analysis to the compression process.

$$\frac{T_2}{T_1} = \left(\frac{p_2}{p_1}\right)^{\eta_P(\gamma-1)/\gamma}$$

$$\eta_{is,t} = \frac{T_1 - T_2}{T_1 - T_{2s}} = \frac{1 - \left(\frac{p_2}{p_1}\right)^{\eta_P(\gamma-1)/\gamma}}{1 - \left(\frac{p_2}{p_1}\right)^{(\gamma-1)/\gamma}}$$

$$\boxed{\eta_P < \eta_{is,T}}$$





Reheat Factor - R_H

Measure of inefficiency of expansion in steam turbines

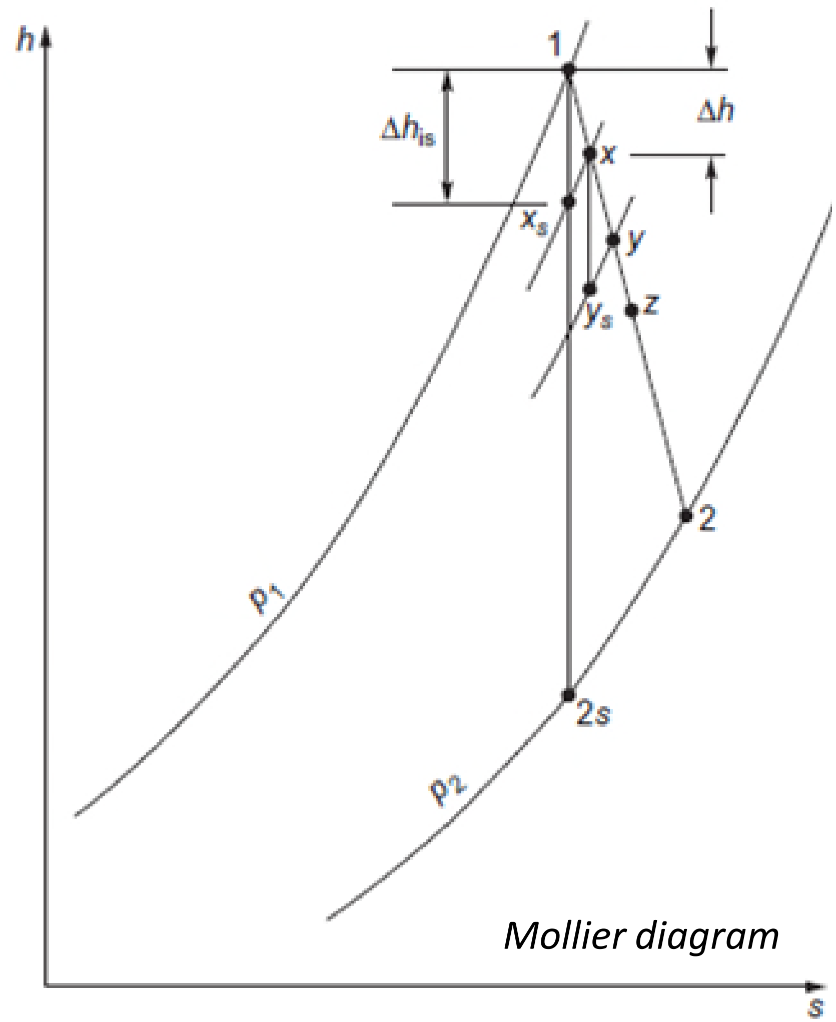
$$R_H = \frac{[(h_1 - h_{xs}) + (h_x - h_{ys}) + \cdots]}{h_1 - h_{2s}} = \left(\sum \Delta h_{is}\right) / (h_1 - h_{2s})$$

Due to divergence of the constant pressure lines, $R_H > 1$

For most steam turbines R_H between 1.03 and 1.08

$$\eta_{is,t} = \frac{h_1 - h_2}{h_1 - h_{2s}} = \frac{h_1 - h_2}{\sum \Delta h_{is}} \cdot \frac{\sum \Delta h_{is}}{h_1 - h_{2s}}$$

$$\eta_{is,t} = \eta_P R_H$$



Mollier diagram

Dimensional Analysis for Compressible Flow Machines

General Principle
The flow in two *geometrically similar* turbomachines is dynamically similar if the streamline patterns of the flows are geometrically similar. i.e. the streamline patterns scale in the same way as the geometry.

This implies that the velocity triangles are similar everywhere in the two machines.
If the flows are dynamically similar, then all dimensionless groups are the same for the two machines.

Method
Perform a "mental experiment" to decide what variables (e.g. $\Omega, D, \dot{m}, \mu, \rho, T, \ldots, etc$) will affect the streamline pattern.
If in doubt include the variable and then you will have to decide by experiment whether it is really significant.

Form these variables into dimensionless groups, these are the independent groups. Then any other dimensionless groups involving other variables that we are interested in, e.g., $\Delta h_0, W_x, \eta$, (dependent variables) are functions of the groups of independent variables.

Dimensional analysis can tell us nothing about exact form of these functional relationships - for this we must perform experiments.

Dimensional Analysis for Compressible Flow Machines

If the density changes then the volume flow rate changes through the machine and the streamline pattern depends on the local volume flow rate. The change in volume flow rate through the machine depends on the pressure change and on the compressibility of the fluid, $d\rho/dp$. The pressure change is chosen to be a dependent variable, determined by the independent variables, but we must include a measure of compressibility as one of independent variables. This is conventionally chosen to be the speed of sound, c since $(dp/d\rho)_s = c^2$.

For simplicity we restrict the analysis to a perfect gas: $c = \sqrt{\gamma RT} = \sqrt{(\gamma - 1)C_P T}$

and so we choose γ and $c_p T_1$ (measure of compressibility) as two independent variables. Since the volume flow rate is no longer constant it is better to use mass flow rate and inlet density $\rho_1$as two other independent variables since these fix the volume flow at inlet. In practice it is customary to choose $p_1$rather than ρ_1 on the grounds that it is more easily measured. This is justifiable since ρ_1 is fixed by $\gamma, c_p, T_1, \ and\ p_1$and we have already chosen the first three of these as independent variables.

If we now fix the above variables to define the inlet flow and the rotational speed of the machine, (which is by convention given the symbol N rather than Ω), then the flow pattern is fixed everywhere within it. The independent variables are:

$$\dot{m}, \gamma, c_p, T_1, D, N, and\ \ p_1$$

(we assumed at the outset that the effects of viscosity can be neglected.)

These independent variables we can form three dimensionless groups which by convention are chosen to be:

$$\gamma \qquad \frac{\dot{m}\sqrt{C_p T_1}}{D^2 p_1} \qquad \frac{ND}{\sqrt{C_p T_1}}$$

γ - is a measure of the gas properties only. It is unlikely to change for any one machine.

$\frac{\dot{m}\sqrt{C_p T_1}}{D^2 p_1}$ is a **dimensionless mass flow rate.** In many (most) cases it is defined using p_{01} and T_{01} rather than p_1 and T_1 because it is easier to measure the stagnation values at machine inlet.

$\frac{ND}{\sqrt{C_p T_1}}$ is a **type of Mach number** based on blade speed. It is a measure of the importance of compressibility in the machine.

Any other dimensionless group involving a dependent variable that we are interested in is a function of the above three groups. e.g.

$$\frac{p_2}{p_1} = f_1\left(\gamma, \frac{\dot{m}\sqrt{C_p T_1}}{D^2 p_1}, \frac{ND}{\sqrt{C_p T_1}}\right)$$

$$\frac{T_2}{T_1} = f_2\left(\gamma, \frac{\dot{m}\sqrt{C_p T_1}}{D^2 p_1}, \frac{ND}{\sqrt{C_p T_1}}\right)$$

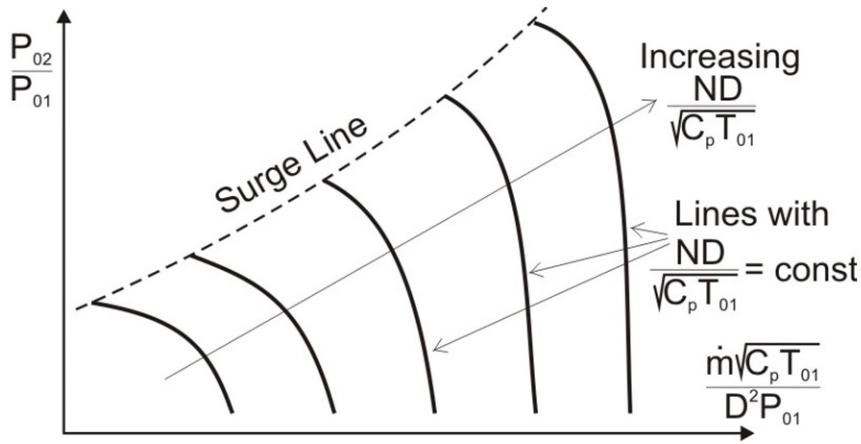
$$\eta = f_3\left(\gamma, \frac{\dot{m}\sqrt{C_p T_1}}{D^2 p_1}, \frac{ND}{\sqrt{C_p T_1}}\right)$$

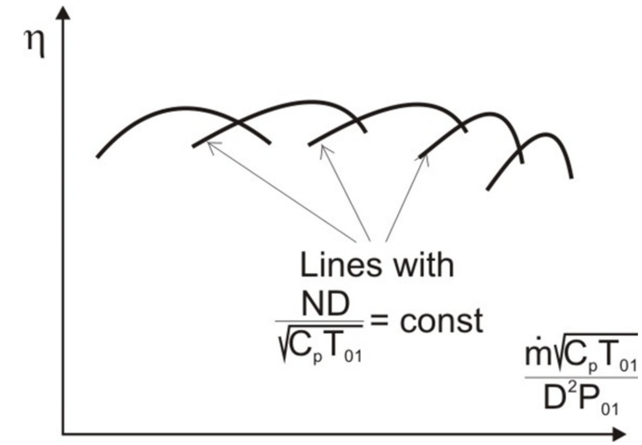
$$\frac{\dot{W}_x}{\dot{m} C_p T_1} = f_4\left(\gamma, \frac{\dot{m}\sqrt{C_p T_1}}{D^2 p_1}, \frac{ND}{\sqrt{C_p T_1}}\right)$$

In practice it is probably most common to use the stagnation pressure and temperature rather then static ones in the above groups (Stagnation quantities easier to measure).

It is very seldom that we test a machine or a model with a different gas, so γ is usually dropped from the list of independent groups. An exception is for steam turbines where model turbines sometimes tested in air which has a different γ to steam.

High Speed Compressor Characteristics





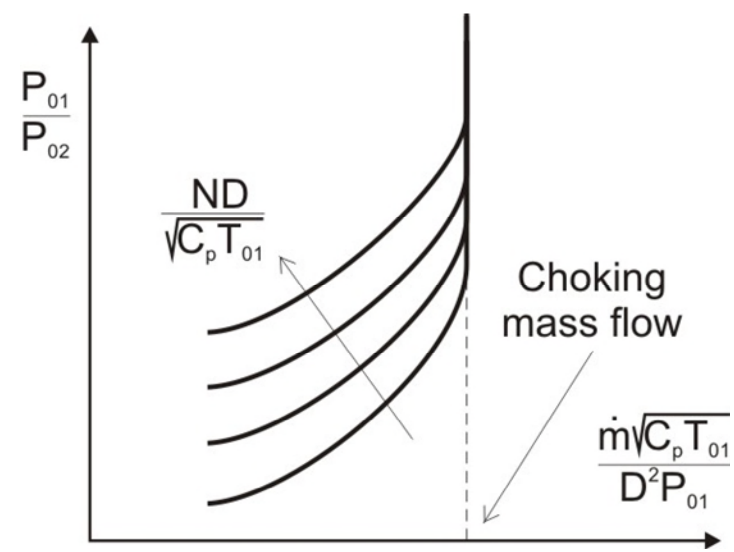
The constant speed lines terminate at the surge (or stall) line where the compressor becomes unstable (Not safe to operate beyond this point).

The maximum pressure ratio of an axial compressor stage is about 2, limited by shock waves.

High Speed Turbine Characteristics

Different behaviour. Because the boundary layers are much more stable, turbines can operate with a high pressure ratio (p_{in}/p_{out}) per stage which soon leads to choking in the blade rows and so to a constant dimensionless mass flow.

Also the lift on a turbine blade is much less dependent on the angle of attack than that on a compressor blade and so the performance is not so much affected by speed.



Effects of Compressibility in Turbomachines

If the flow in a turbomachine is above Mach > 0.3 the effects of compressibility must be included into design calculations. To calculate performance of turbomachines with compressible flows very similar methods are used as those for cases where the flow is considered incompressible.

For example, ordinary velocity triangles can be converted to so called Mach number triangles by dividing all velocities by $\sqrt{\gamma RT}$ (local speed of sound).

Compressible flow relations

$$Mach\ M = \frac{c}{c_{sonic}} = \frac{c}{\sqrt{\gamma RT}}$$

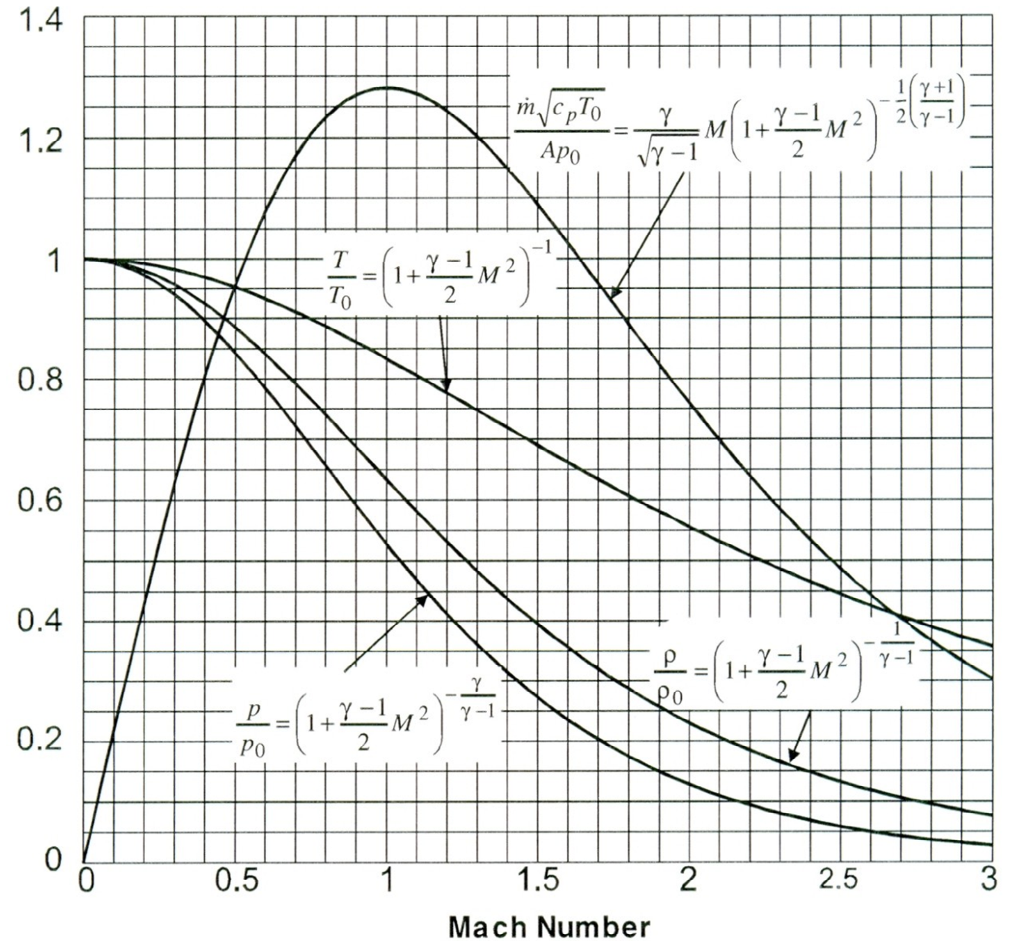
$$\frac{T}{T_0} = \left(1 + \frac{\gamma - 1}{2} M^2\right)^{-1}$$

$$\frac{p}{p_0} = \left(1 + \frac{\gamma - 1}{2} M^2\right)^{-\frac{\gamma}{\gamma-1}}$$

$$\frac{\rho}{\rho_0} = \left(1 + \frac{\gamma - 1}{2} M^2\right)^{-\frac{1}{\gamma-1}}$$

$$\frac{c}{\sqrt{c_p T_0}} = M\sqrt{\gamma - 1}\left(1 + \frac{\gamma - 1}{2} M^2\right)^{-\frac{1}{2}}$$

$$\frac{\dot{m}\sqrt{c_p T_0}}{A p_0} = \frac{\gamma}{\gamma - 1} M \left(1 + \frac{\gamma - 1}{2} M^2\right)^{-\frac{1}{2}\left(\frac{\gamma+1}{\gamma-1}\right)}$$



Tables for the Compressible Flow of a Perfect Gas (Dixon and Hall, Appendix C)
$(\gamma = 1.40, dry\ air\ and\ diatomic\ gases)$

Static and Stagnation Quantities	Flow Relations
$\frac{T}{T_0} = \left(1+\frac{\gamma-1}{2}M^2\right)^{-1}$	$c = M\sqrt{\gamma RT},\ \frac{c}{\sqrt{C_pT_0}} = M\sqrt{\gamma-1}\left(1+\frac{\gamma-1}{2}M^2\right)^{-(1/2)}$
$\frac{p}{p_0} = \left(1+\frac{\gamma-1}{2}M^2\right)^{-\gamma/(\gamma-1)}$	$\dot{m} = \rho c A_n,\ \frac{\dot{m}\sqrt{C_pT_0}}{A_n p_0} = \frac{\gamma}{\sqrt{\gamma-1}}M\left(1+\frac{\gamma-1}{2}M^2\right)^{-(1/2)(\gamma+1)/(\gamma-1)}$
$\frac{\rho}{\rho_0} = \left(1+\frac{\gamma-1}{2}M^2\right)^{-1/(\gamma-1)}$	

Table C.1 (Continued)

M	T/T_0	p/p_0	ρ/ρ_0	$\dot{m}\sqrt{C_pT_0}/A_n p_0$	$c/\sqrt{C_pT_0}$
0.50	0.9524	0.8430	0.8852	0.9561	0.3086
0.51	0.9506	0.8374	0.8809	0.9696	0.3145
0.52	0.9487	0.8317	0.8766	0.9828	0.3203
0.53	0.9468	0.8259	0.8723	0.9958	0.3262
0.54	0.9449	0.8201	0.8679	1.0084	0.3320
0.55	0.9430	0.8142	0.8634	1.0208	0.3378
0.56	0.9410	0.8082	0.8589	1.0328	0.3436
0.57	0.9390	0.8022	0.8544	1.0446	0.3493
0.58	0.9370	0.7962	0.8498	1.0561	0.3551
0.59	0.9349	0.7901	0.8451	1.0672	0.3608
0.60	0.9328	0.7840	0.8405	1.0781	0.3665
0.61	0.9307	0.7778	0.8357	1.0887	0.3722
0.62	0.9286	0.7716	0.8310	1.0990	0.3779
0.63	0.9265	0.7654	0.8262	1.1090	0.3835
0.64	0.9243	0.7591	0.8213	1.1186	0.3891
0.65	0.9221	0.7528	0.8164	1.1280	0.3948
0.66	0.9199	0.7465	0.8115	1.1371	0.4003
0.67	0.9176	0.7401	0.8066	1.1459	0.4059
0.68	0.9153	0.7338	0.8016	1.1544	0.4115
0.69	0.9131	0.7274	0.7966	1.1626	0.4170
0.70	0.9107	0.7209	0.7916	1.1705	0.4225
0.71	0.9084	0.7145	0.7865	1.1782	0.4280
0.72	0.9061	0.7080	0.7814	1.1855	0.4335
0.73	0.9037	0.7016	0.7763	1.1925	0.4389
0.74	0.9013	0.6951	0.7712	1.1993	0.4443
0.75	0.8989	0.6886	0.7660	1.2058	0.4497
0.76	0.8964	0.6821	0.7609	1.2119	0.4551
0.77	0.8940	0.6756	0.7557	1.2178	0.4605
0.78	0.8915	0.6691	0.7505	1.2234	0.4658
0.79	0.8890	0.6625	0.7452	1.2288	0.4711
0.80	0.8865	0.6560	0.7400	1.2338	0.4764
0.81	0.8840	0.6495	0.7347	1.2386	0.4817
0.82	0.8815	0.6430	0.7295	1.2431	0.4869
0.83	0.8789	0.6365	0.7242	1.2474	0.4921
0.84	0.8763	0.6300	0.7189	1.2514	0.4973
0.85	0.8737	0.6235	0.7136	1.2551	0.5025
0.86	0.8711	0.6170	0.7083	1.2585	0.5077
0.87	0.8685	0.6106	0.7030	1.2617	0.5128
0.88	0.8659	0.6041	0.6977	1.2646	0.5179
0.89	0.8632	0.5977	0.6924	1.2673	0.5230

Example

Air flows adiabatically and at high subsonic speed through a duct. At a station A, measured velocity c_A is 250 m/s, the static temperature T_A is 315 K and the static pressure p_A is 180 kPa. Determine the values of the stagnation temperature T_{0A}, the Mach number M_A the stagnation pressure p_{0A} and the stagnation density ρ_{0A}. If the duct cross-sectional area is $0.1\ m^2$, calculate the air mass flow rate. For air take $R = 287\ J/kgK$ and $\gamma = 1.4$.

$$T_{0A} = T_A + \frac{c_A^2}{2c_p} = 346\ K \qquad M_A = \frac{c_A}{\sqrt{\gamma RT}} = 0.703 \qquad p_{0A} = p_A\left(1 + \frac{\gamma - 1}{2}M^2\right)^{\frac{\gamma}{\gamma-1}} = 250\ kPa$$

$$\rho_{0A} = \rho_A\left(1 + \frac{\gamma - 1}{2}{M_A}^2\right)^{\frac{1}{\gamma-1}} = 2.52\ kg/m^3 \qquad where\ \rho_A = \frac{p_A}{RT_A} = 1.991\ kg/m^3$$

$$or\ more\ directly\text{:}\ \rho_{0A} = \frac{p_{0A}}{RT_{0A}} = 2.52\ kg/m^3$$

$$\dot{m} = \rho_A A_A c_A = 1.99 \cdot 0.1 \cdot 250 = 49.8\ kg/s$$

Alternatively, using Table C1 (Dixon & Hall)

$$\frac{\dot{m}\sqrt{c_p T_0}}{A p_0} = f(M_A) = f(0.703) = 1.1728 \qquad \dot{m} = 1.1728\frac{p_{0A}A_A}{\sqrt{c_p T_{0A}}} = 49.7\ kg/s$$

Stagnation pressure loss coefficient, Y_P

$$Y_p = \frac{Loss\ of\ (Relative) Stagnation\ Pressure\ due\ to\ Irreversibilities}{Reference\ Dynamic\ Head}$$

For compressors:

Interested in converting the inlet dynamic head into pressure rise.

Reference dynamic head = inlet dynamic head $= p_{01} - p_1$

$$Y_p = \frac{p_{02,is} - p_{02}}{p_{01} - p_1} = \frac{p_{01} - p_{02}}{p_{01} - p_1}$$

For turbines:

Interested in generating a high exit dynamic head.

Reference dynamic head = exit dynamic head $= p_{02} - p_2$

$$Y_p = \frac{p_{02,is} - p_{02}}{p_{02} - p_2} = \frac{p_{01} - p_{02}}{p_{02} - p_2}$$

(For rotating blade rows use relative flow quantities.)