A10586W1

TRINITY TERM 2018

SECOND PUBLIC EXAMINATION

CONTROL SYSTEMS (PAPER B15)

Honour School of Engineering Science

Thursday 07 June 2018 09:30 – 11:00

Answers to not more than **THREE** *questions should be submitted, and each question must be answered in a separate booklet.*

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Note that:

- *The approximate allocation of marks is given in the margin.*
- *Permitted calculator series: Casio fx-83, Casio fx-85, Sharp EL-531.*
- *A copy of Engineering Tables & Data (HLT) is provided.*

1. A high capacity magnetic hard disk drive has a control system to move the read head to a specific position on the disk. A colleague has already defined a basic non-dimensional model of the system

$$\ddot{\phi} = u + d\ ,$$

where ϕ is the read head position, u is the motor torque, and d is an exogenous disturbance torque.

(a) Derive a state-space model of the system.

[2 marks]

(b) Design a state feedback controller that places closed-loop poles to ensure a damping ratio of 0.7 and natural frequency of $\omega_0 = 1000\,\mathrm{rad\,s^{-1}}$.

[4 marks]

(c) Owing to mechanical imperfections there is an offset between the centre of the tracks and the rotational centre of the disc. This results in a sinusoidal disturbance torque

$$d = A\sin(\omega_d t)\ ,$$

where A is unknown, and $\omega_d = 5000\,\mathrm{rpm}$ is the rotational speed of the disk.

(i) Derive an appropriate observer model to estimate the disturbance, and hence suggest a control scheme to ensure zero tracking error. You may assume that the read head position can be measured directly and is the only available measurement. Prove that such a system is workable by checking its observability.

(ii) A colleague has used your derivations to set the observer poles at $-2000 \pm \mathrm{j}1500$ $\mathrm{rad\,s^{-1}}$ and $-4000 \pm \mathrm{j}3000\,\mathrm{rad\,s^{-1}}$. Explain the reasoning behind this choice.

[10 marks]

2. (a) The Hamiltonian matrix M and its matrix of eigenvectors W that correspond to stable eigenvalues are

$$M = \begin{bmatrix} A & -BR^{-1}B^T \\ -Q & -A^T \end{bmatrix}, \qquad W = \begin{bmatrix} W_{11} \\ W_{21} \end{bmatrix}.$$

Demonstrate that $P = W_{21}W_{11}^{-1}$ satisfies the steady state Algebraic Riccati equation

$$PA + A^T P - PBR^{-1}B^T P = -Q\ .$$

You may assume that W_{11} is invertible.

[4 marks]

(b) To ensure optimum performance, it is necessary to regulate the temperature of a battery in a hybrid electric aircraft around a steady state reference temperature T_0 with zero error. This is to be achieved despite the presence of the *unknown* but *constant* internal heating power Q. The model of the dynamics is shown in Figure 1.

(i) By introducing an extra state, z, sketch a control scheme that achieves zero steady-state error without the use of an observer, and explain the reason for your choice.

(ii) Write down the corresponding state-space equations of the augmented open-loop system with states T and z. Hence determine a steady-state condition for u that maintains output $y = T_0$.

(iii) Hence formulate a state-space model in terms of deviations about the steady-state conditions.

[8 marks]

(c) Given $Q = I$ and $R = 4$, the Hamiltonian for the system in part (b) has two stable eigenvectors $w_1 = [1\ \ 1\ \ 0\ \ 1]^T$ and $w_2 = [1\ \ 2\ \ -2\ \ 4]^T$, when the state vector is written as $x = [T\ \ z]^T$. Determine the optimal feedback gain.

[4 marks]

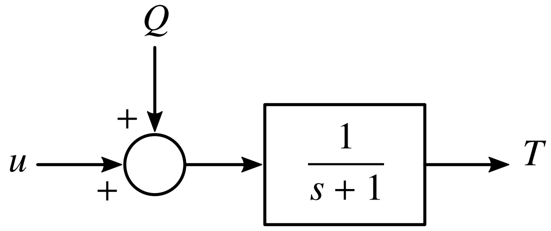

Figure 1

3. The dynamics of a second order process are given by

$$\begin{aligned} \dot{x}_1 &= v \\ \dot{x}_2 &= x_1 - a x_2 \\ y &= b x_2 + w\,, \end{aligned}$$

where w is a white noise process with power spectral density σ_w^2 and $E[v(t)v(t+\tau)] = r\delta(\tau)$.

(a) For $a = 1$, $b = 1$:

(i) compute the Kalman Filter Covariance matrix P (you may assume all elements of P are positive);

(ii) compute the steady-state Kalman filter gain.

[8 marks]

(b) Derive the Kalman filter transfer function from the measurement y to the Kalman filter estimate $\hat{y}$.

[4 marks]

(c) Use frequency domain interpretation or otherwise to explain the filter behaviour for the cases when $(\sigma_w^2/r) \to \infty$ and $(\sigma_w^2/r) \to 0$.

[4 marks]

4. (a) A dynamic system is described by the differential equation

$$\frac{d^2y}{dt^2} + 2\frac{dy}{dt} + 5y(t) = u(t) \ .$$

(i) Determine the impulse response for the system.

(ii) Use the impulse response to determine whether the system is bounded-input, bounded-output (BIBO) stable.

(iii) If $G(s)$ denotes the transfer function of the system, determine $\|G\|_\infty$.

[10 marks]

(b) The system is now incorporated in a *positive* feedback loop as shown in Figure 2, with a proportional controller $C = 5$. Derive the impulse response of the closed-loop system and use this to determine whether the closed-loop system is stable.

[6 marks]

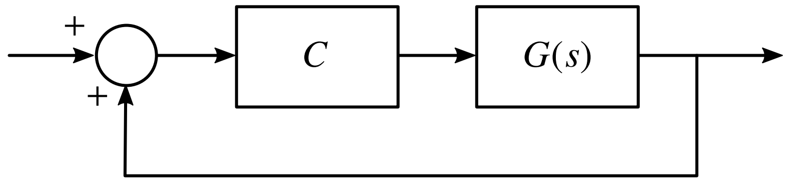

Figure 2