Aerothermal Engineering

C204 Turbomachinery

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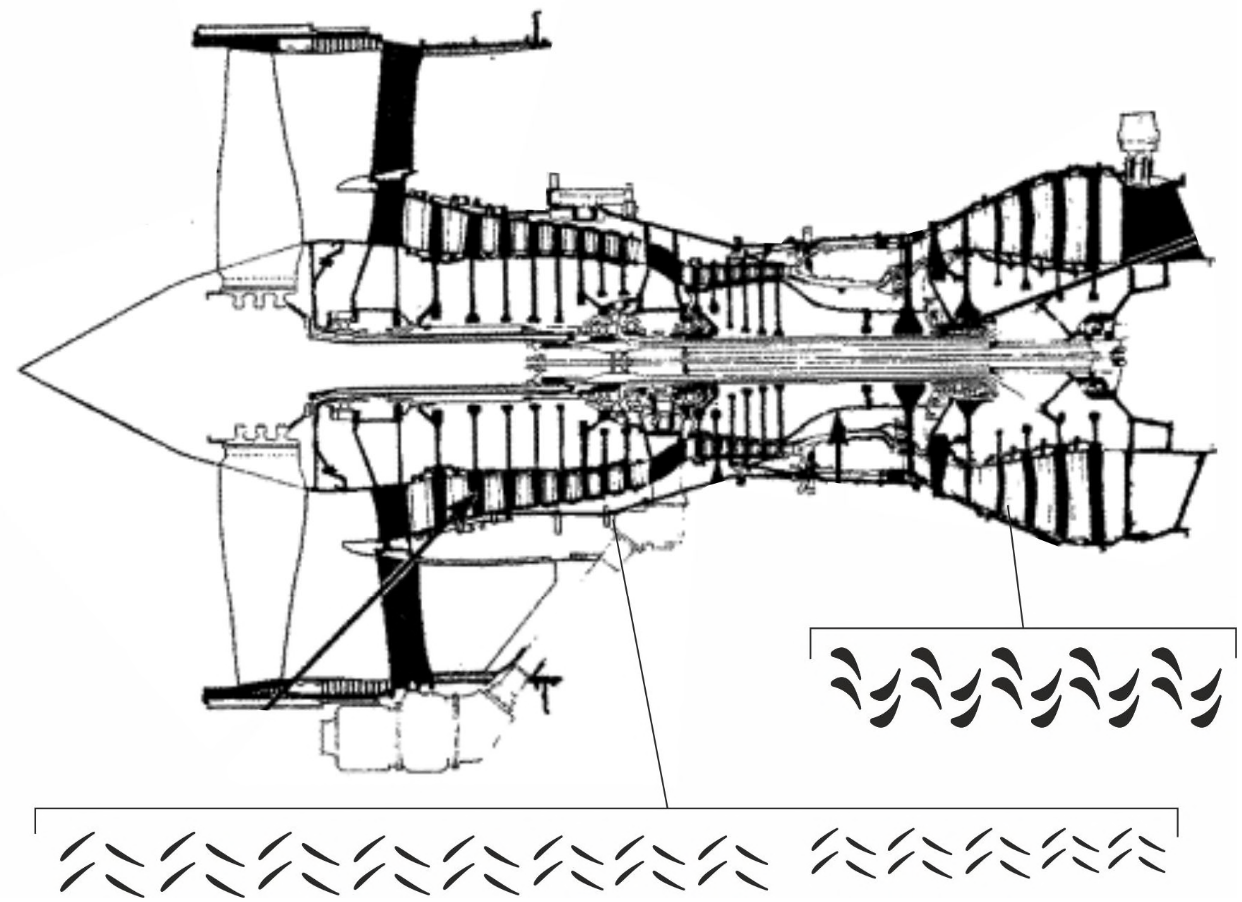
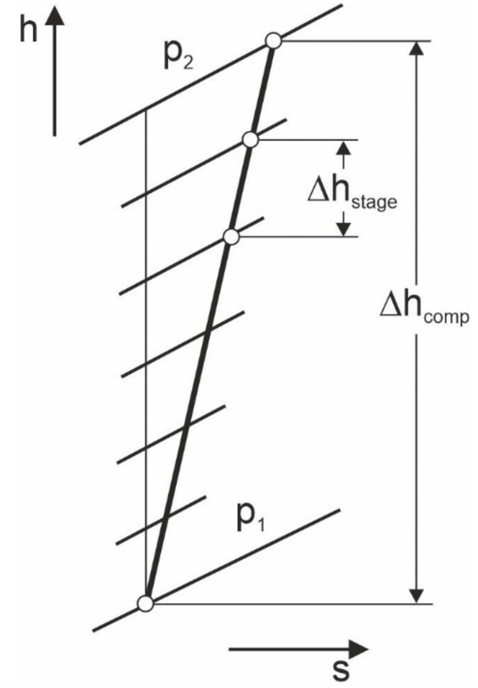
Lecture 2



Power required for the whole machine:

$$P = \dot{m} \times \Delta h_{0,machine}$$

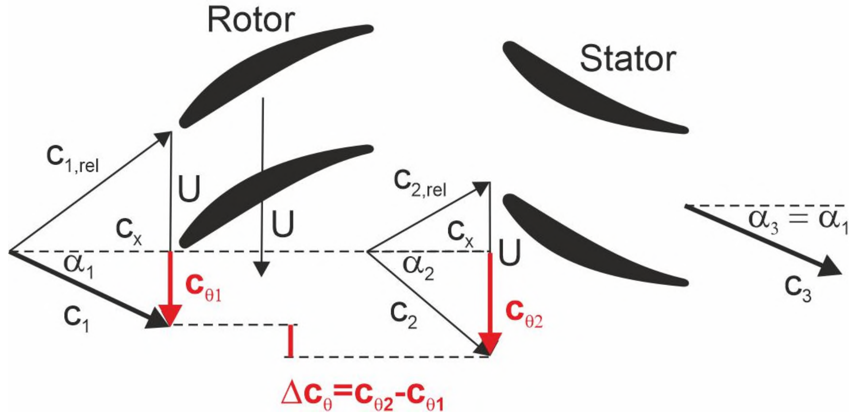
$$N = \frac{\Delta h_{0,overall}}{\Delta h_{0,stage}} = \frac{\Delta h_{0,overall}}{U^2\psi}$$



Fix the velocity triangles for a repeating stage (any three angles must be fixed). Any combination of 3 quantities:

α_1, α_2, β_1, β_2, ϕ, ψ, Λ completely fixes the velocity triangles for a repeating stage.

Common combinations: ϕ, ψ and α_1 or ϕ, ψ and Λ.



$$\boxed{From\ Euler:\ \Delta h_{0,stage} = U_2 c_{\theta 2} - U_1 c_{\theta 1}}$$

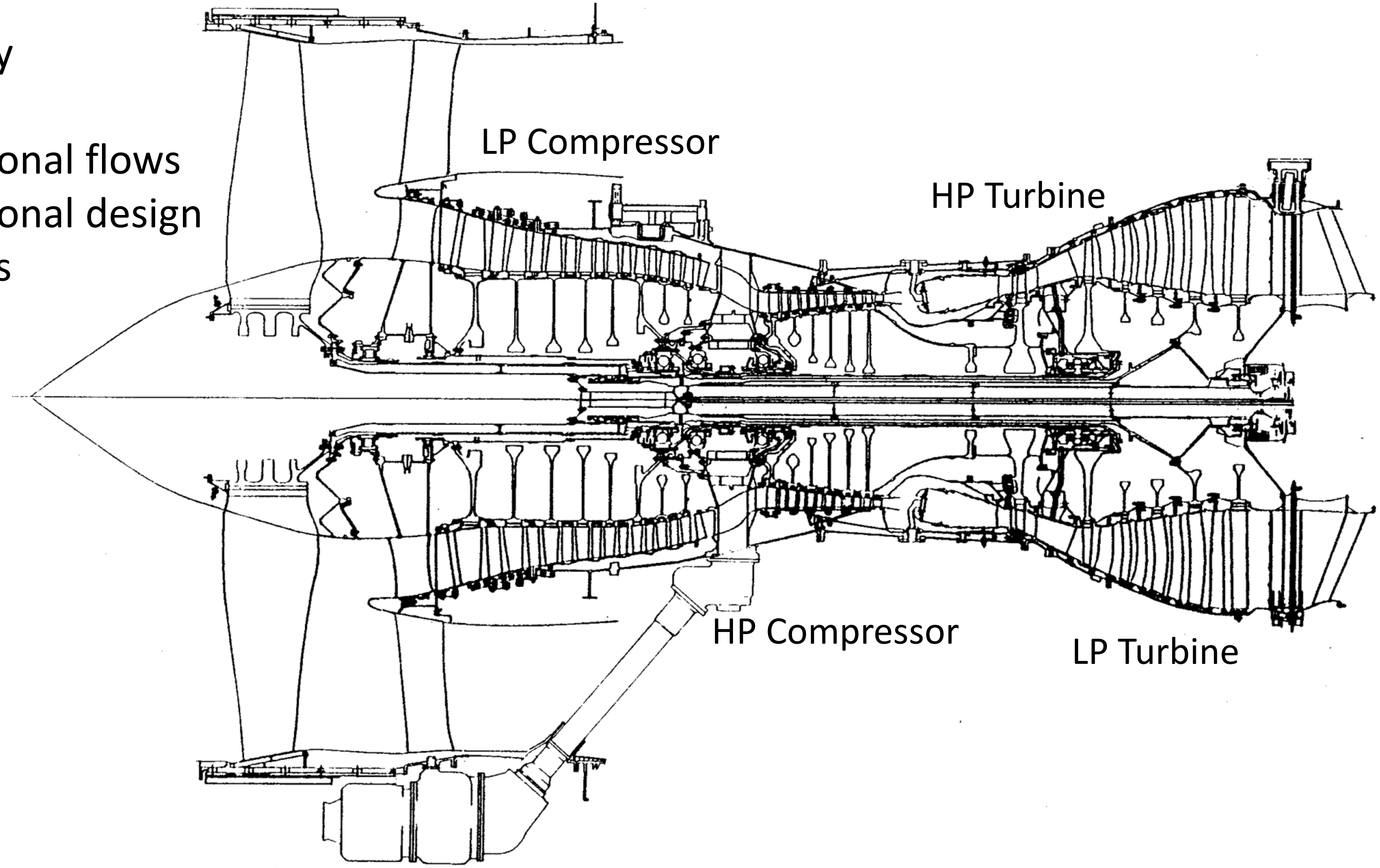
$$\Delta r = \frac{\dot{m}}{\pi D_m \rho c_x} = \frac{\dot{m}}{\pi D_m \rho \phi U}$$

The main goal of a design is to develop a highly efficient engine that will deliver the required propulsive thrust (aero-engine) or power (gas turbine for power gen.)

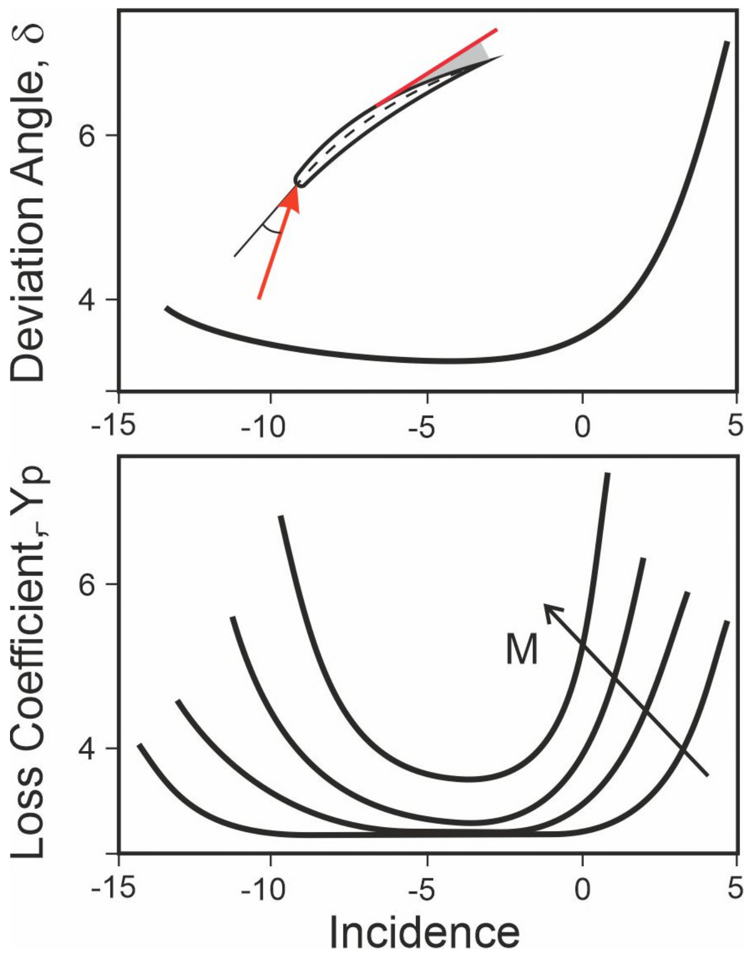
To achieve that all velocity triangles for all compressor and turbine stages have to perform (to be matched) in the real engine as predicted in the design process.

This can be done only by understanding all physical processes in an engine and incorporating their effects on engine performance into the design process:

- Compressibility
- Viscous effects
- Three-dimensional flows
- Three-dimensional design
- Unsteady flows



Cascade tests to produce: losses, Y_p deviation, δ



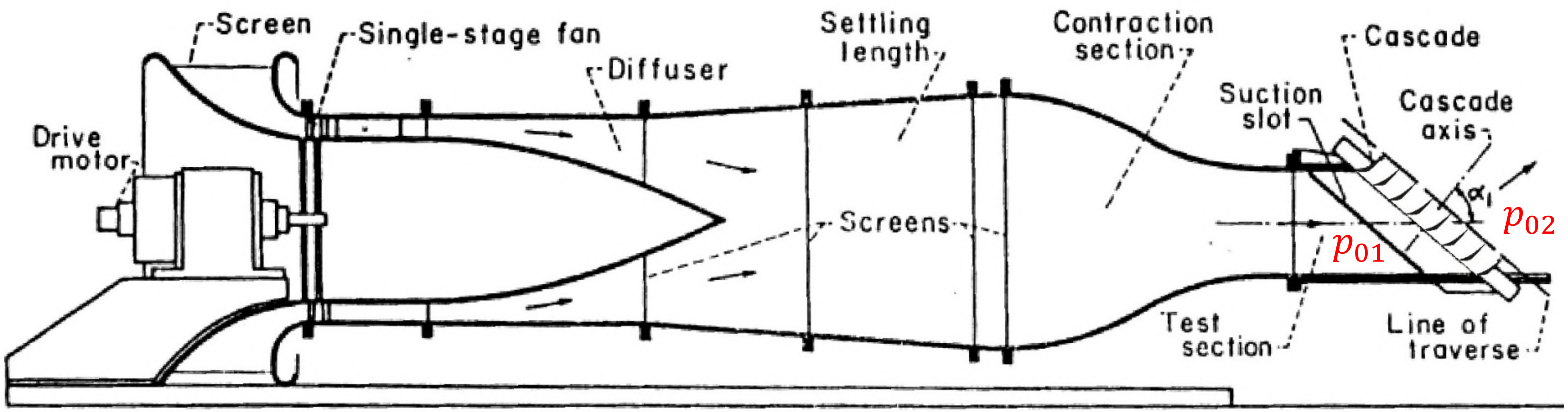
Compressors:

$$Y_p = \frac{p_{02,is} - p_{02}}{p_{01} - p_1} = \frac{p_{01} - p_{02}}{p_{01} - p_1}$$

Turbines:

$$Y_p = \frac{p_{02,is} - p_{02}}{p_{02} - p_2} = \frac{p_{01} - p_{02}}{p_{02} - p_2}$$

(For rotating blade rows use relative flow quantities.)

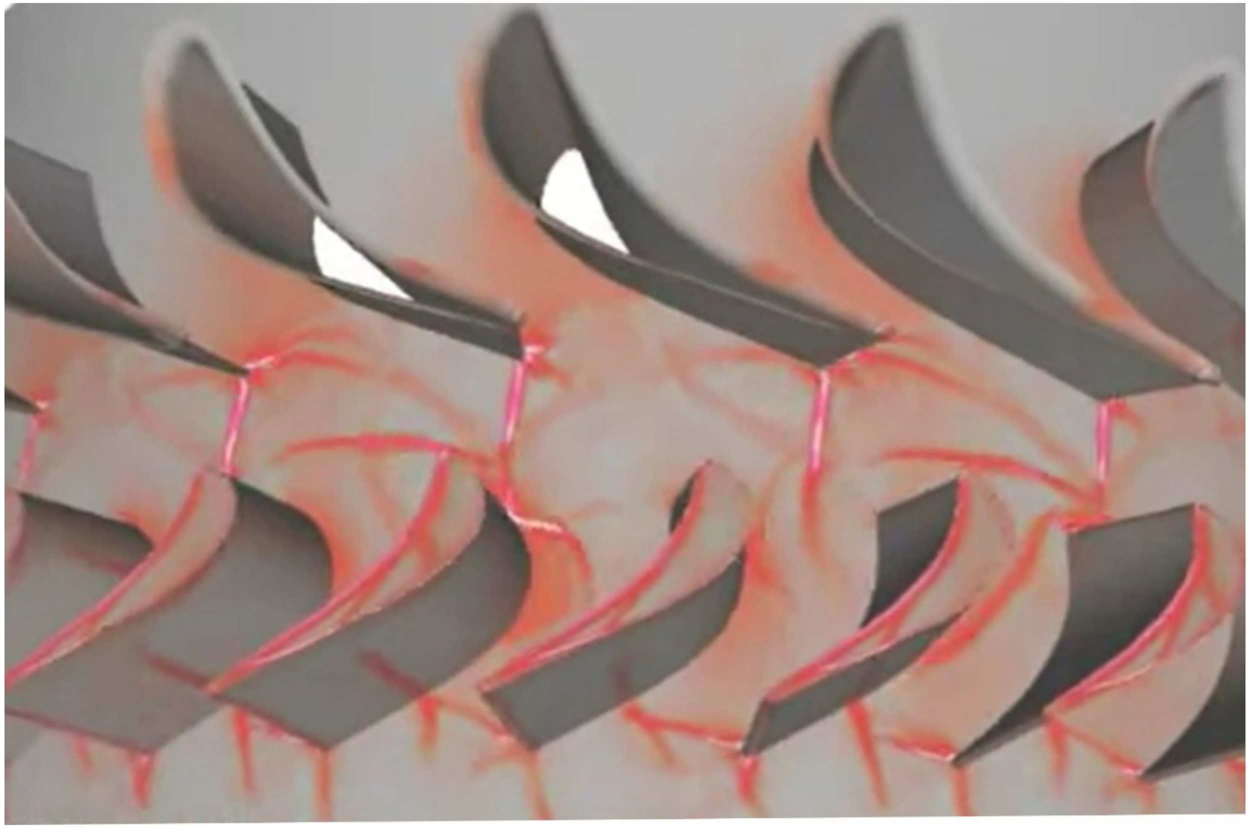


Compressibility and Ma Number Effects

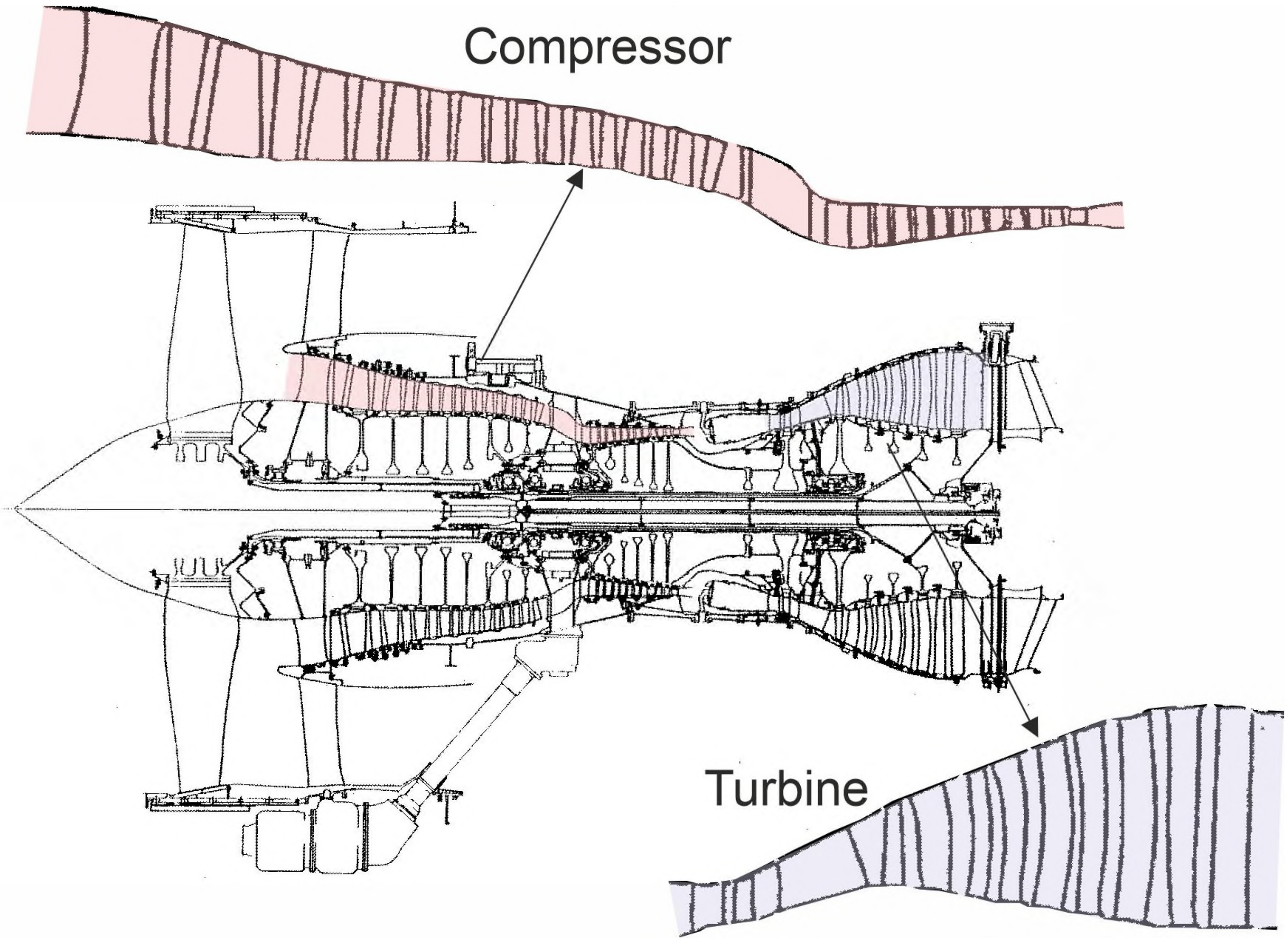
If Ma number is greater than 0.3, and that is actually the case for most jet engine or gas turbine components, the compressibility effects must be incorporated into the design:

- Due to density change across the engine, the volume flow rate varies and velocity triangles have to be constructed with this effect implemented.
- Aerodynamic Losses (profile losses, trailing edge losses, shock-boundary layer interaction)
- Flow angle deviation, etc.

Complex shocks structure within a transonic turbine stage



R Pecnik (ITTM TU Graz) Numerical simulation of transonic turbine stage (density gradient contours)


Compressor
Turbine

Example - Non-dimensional groups for compressible flow machines

$$\frac{p_2}{p_1} = f_1\left(\gamma, \frac{\dot{m}\sqrt{C_p T_1}}{D^2 p_1}, \frac{ND}{\sqrt{C_p T_1}}\right)$$

$$\frac{T_2}{T_1} = f_2\left(\gamma, \frac{\dot{m}\sqrt{C_p T_1}}{D^2 p_1}, \frac{ND}{\sqrt{C_p T_1}}\right)$$

$$\eta = f_3\left(\gamma, \frac{\dot{m}\sqrt{C_p T_1}}{D^2 p_1}, \frac{ND}{\sqrt{C_p T_1}}\right)$$

$$\frac{\dot{W}_x}{\dot{m} C_p T_1} = f_4\left(\gamma, \frac{\dot{m}\sqrt{C_p T_1}}{D^2 p_1}, \frac{ND}{\sqrt{C_p T_1}}\right)$$

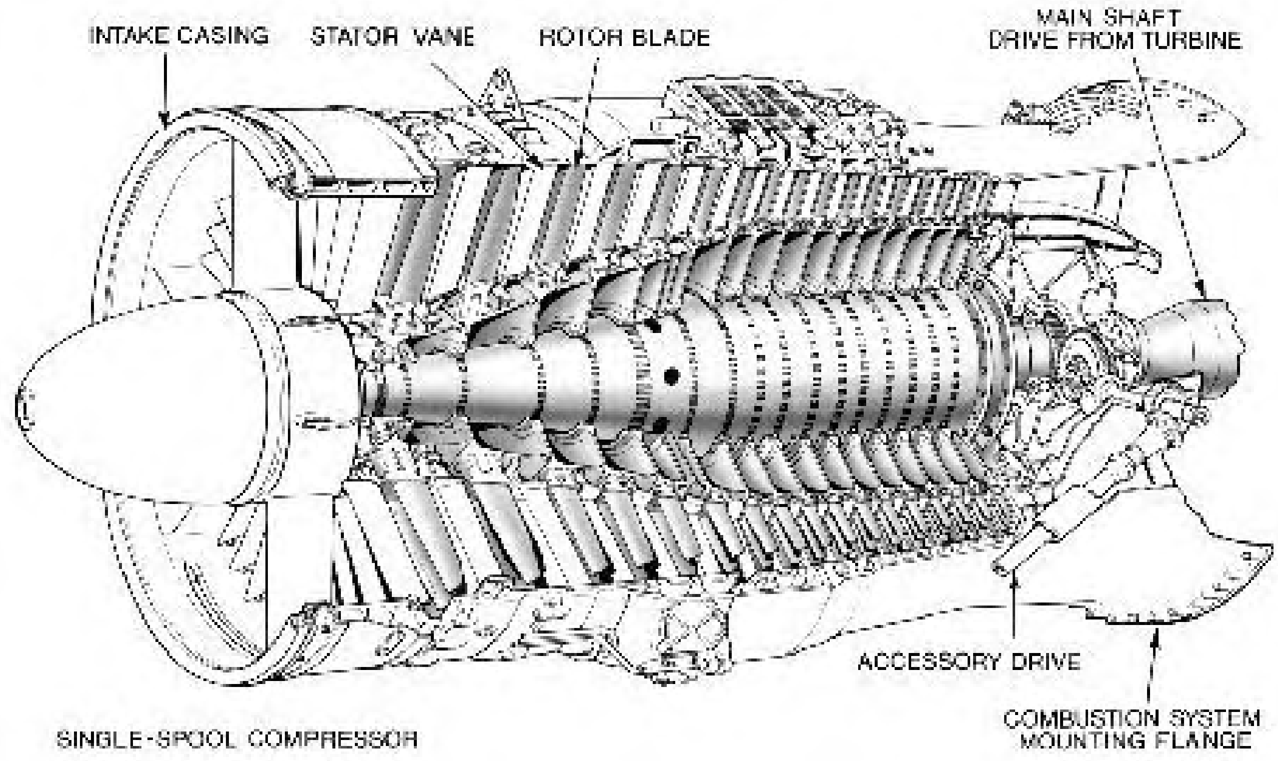
In practice it is more convenient to use the stagnation properties (as they are easier to measure)

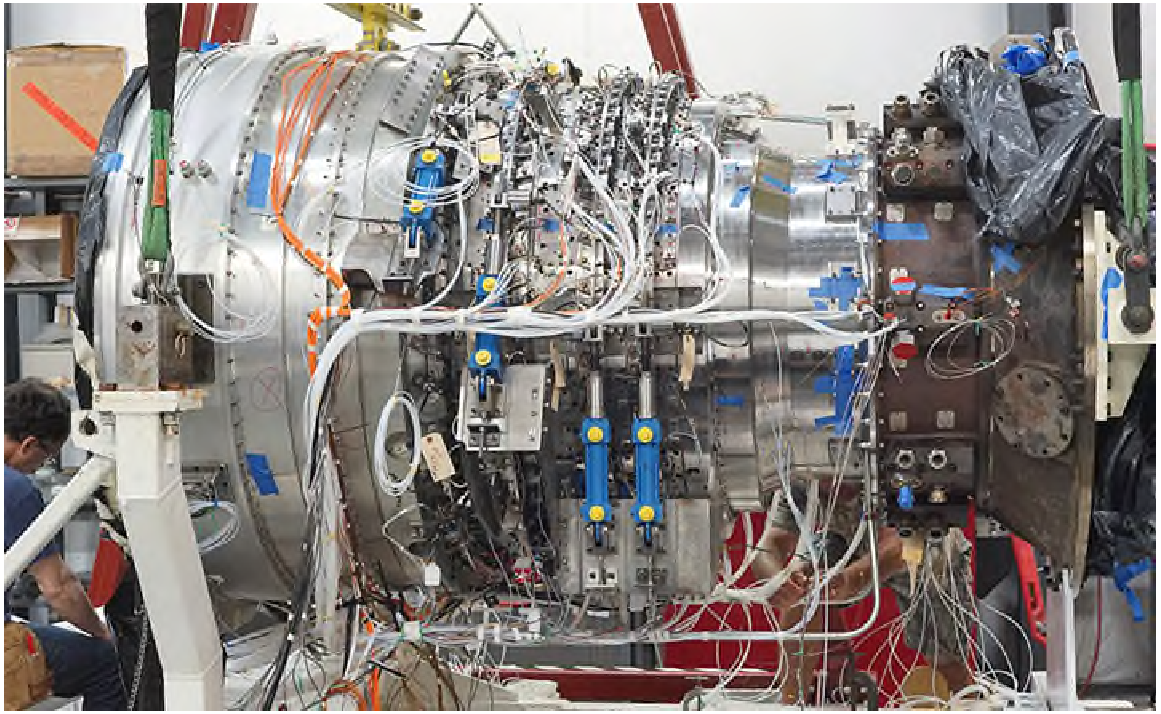
Example – Multistage Compressor Map

The compressor with the performance map shown is tested at sea level on a stationary test bed (the atmospheric temperature and pressure is 298 K and 101 kPa, respectively.) When running at its design operating point, the mass flow rate through the compressor is measured as 15 kg/s and the rotational speed is 6200 rpm.

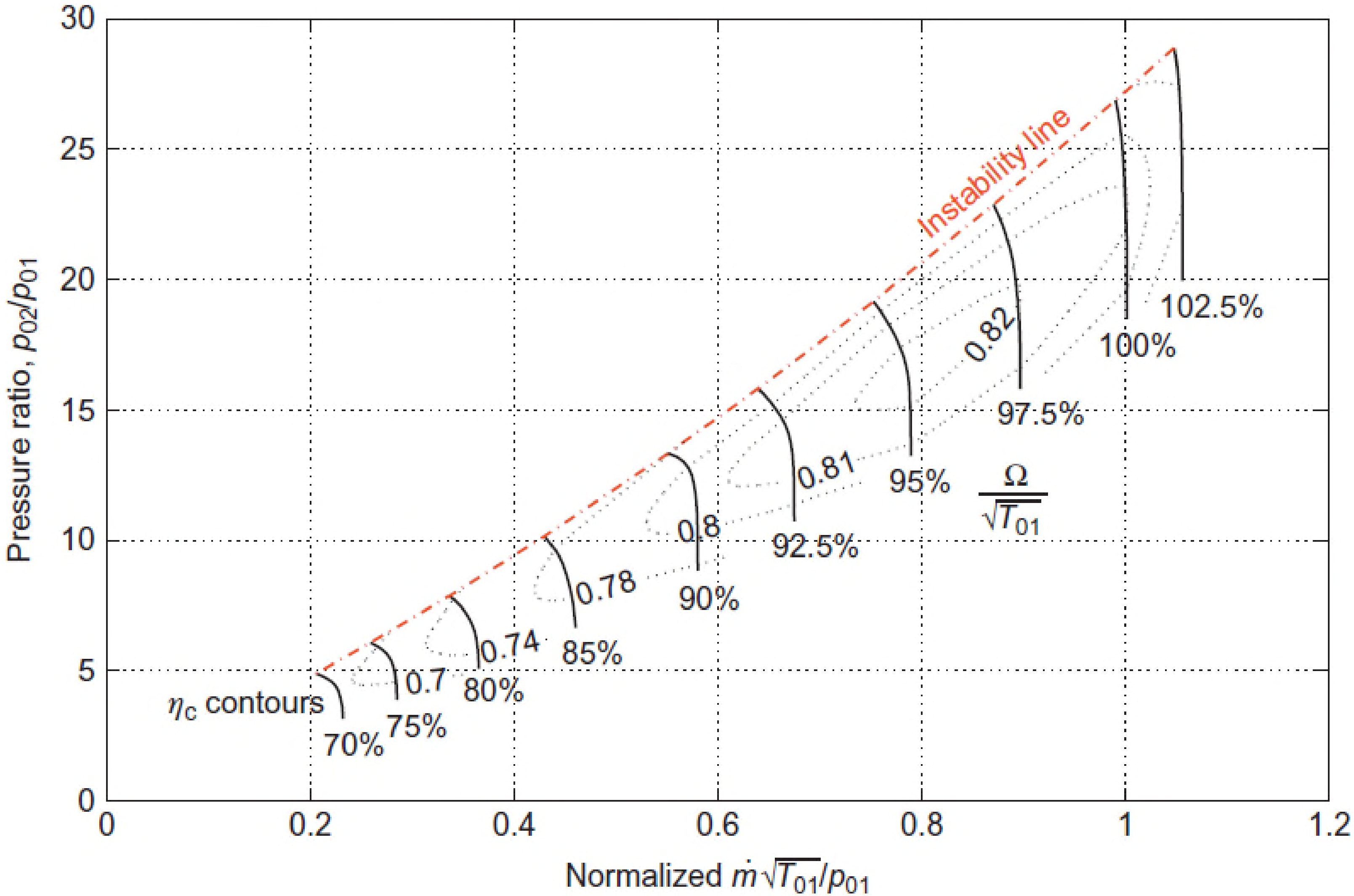
Determine the mass flow rate and rotational speed when the compressor is operating at the design operating point during high altitude cruise with an inlet stagnation temperature of 236 K and an inlet stagnation pressure of 10.2 kPa.

The design pressure ratio of the compressor is 22. Using the compressor characteristic determine the compressor isentropic and polytropic efficiency at the design point. Hence calculate the required power input at the cruise condition. Assume throughout for air that $\gamma = 1.4$ and $c_p = 1005\ J/kgK$.





https://blog.softinway.com/performance
-testing-of-axial-compressors/



Performance map of a multistage high-speed axial compressor

Solution

At cruise and during the sea-level stationary test the compressor is operating at its design operating point – at both conditions all the non-dimensional performance parameters of the compressor will be the same.

The non-dimensional mass flow: $$\left[\frac{\dot{m}\sqrt{c_p T_{01}}}{A p_{01}}\right]_{test} = \left[\frac{\dot{m}\sqrt{c_p T_{01}}}{A p_{01}}\right]_{cruise}$$

No change in the dimensions of the compressor or in the gas properties of the working fluid:

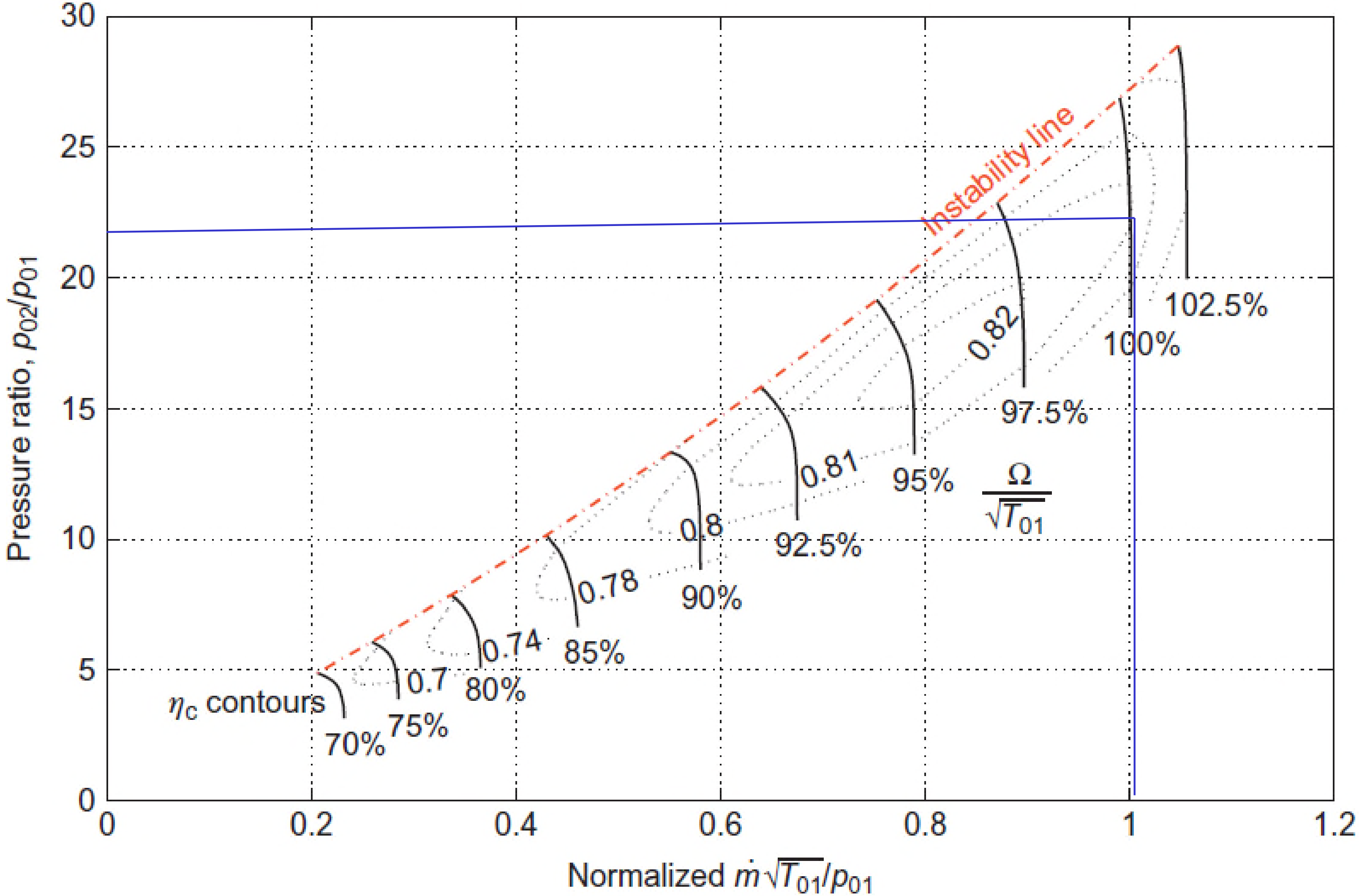
$$\left[\frac{\dot{m}\sqrt{T_{01}}}{p_{01}}\right]_{test} = \left[\frac{\dot{m}\sqrt{T_{01}}}{p_{01}}\right]_{cruise}$$

The mass flow at cruise: $$\dot{m}_{cruise} = \left[\frac{\dot{m}\sqrt{T_0}}{p_{01}}\right]_{test} \times \left[\frac{p_{01}}{\sqrt{T_0}}\right]_{cruise} = \frac{15\sqrt{298}}{101}\frac{10.2}{\sqrt{236}} = 1.70\ kg/s$$

The non-dimensional speed: $$\left[\frac{\Omega}{\sqrt{T_{01}}}\right]_{test} = \left[\frac{\Omega}{\sqrt{T_{01}}}\right]_{cruise}$$

$$\Omega_{cruise} = \sqrt{T_{01cruise}}\left[\frac{\Omega}{\sqrt{T_{01}}}\right]_{test} = \sqrt{236} \cdot \frac{6200}{\sqrt{298}} = 5520\ RPM$$

From the compressor performance map, at 100% speed and pressure ratio 22 $\rightarrow \eta_C = 0.81$



Performance map of a multistage high-speed axial compressor

$$\frac{T_{02}}{T_{01}} = \frac{(p_{02}/p_{01})^{(\gamma-1)/\gamma} - 1}{\eta_C} + 1 = \frac{22^{1/3.5} - 1}{0.81} + 1 = 2.751$$

The polytropic efficiency: $\eta_P = \frac{\gamma - 1}{\gamma} \frac{\ln(p_{02}/p_{01})}{\ln(T_{02}/T_{01})} = 0.286 \frac{ln22}{ln2.751} = 0.873$

(The polytropic efficiency higher than the isentropic efficiency at this pressure ratio.)

The input power to the compressor at the cruise condition can be found using the fact that the non-dimensional power coefficient $\Delta T_0/T_0$ is unchanged between the two conditions:

$$\dot{W}_x = \dot{m} C_p \Delta T_0 \qquad \left[\frac{\dot{W}_x}{\dot{m} C_p T_{01}}\right]_{test} = \left[\frac{\dot{W}_x}{\dot{m} C_p T_{01}}\right]_{cruise} \rightarrow \left[\frac{\Delta T_0}{T_{01}}\right]_{test} = \left[\frac{\Delta T_0}{T_{01}}\right]_{cruise}$$

$$\left[\frac{\Delta T_0}{T_{01}}\right]_{cruise} = \frac{T_{02}}{T_{01}} - 1 = 2.751 - 1 = 1.751$$

$$\dot{W}_{x\,cruise} = \left[\dot{m} C_p \Delta T_0\right]_{cruise} = \left[\dot{m} C_p T_{01}\right]_{cruise} \frac{\Delta T_0}{T_{01}} = 1.70 \cdot 1005 \cdot 236 \cdot 1.751 = 706kW$$

Solving Procedure for Compressible Flow Axial Machines

In the absolute frame of reference:

$$\frac{T}{T_0}, \frac{p}{p_0}, \frac{c}{\sqrt{c_p T_0}}, \frac{\dot{m}\sqrt{c_p T_0}}{A p_0}, = Functions(M)$$

(tabulated for different Mach numbers)

Remembering that all static quantities (e.g. p, T) are the same in both absolute and relative frames, the above relations are also true for relative stagnation quantities:

$$\frac{T}{T_{0,rel}}, \frac{p}{p_{0,rel}}, \frac{c_{rel}}{\sqrt{c_p T_{0,rel}}}, \frac{\dot{m}\sqrt{c_p T_{0,rel}}}{A p_{0,rel}}, = Functions(M)$$

Therefore, we can use the <u>same tables for both absolute and relative flows</u> providing we use appropriate stagnation quantities (e.g. T_0 or $T_{0,rel}$) and appropriate Mach number (M or M_{rel}).
A – is the effective flow area normal to the appropriate flow vector (c or c_r).

Stationary Blade Row

The loss of stagnation pressure through the blade is usually given as a stagnation pressure loss coefficient Y_p

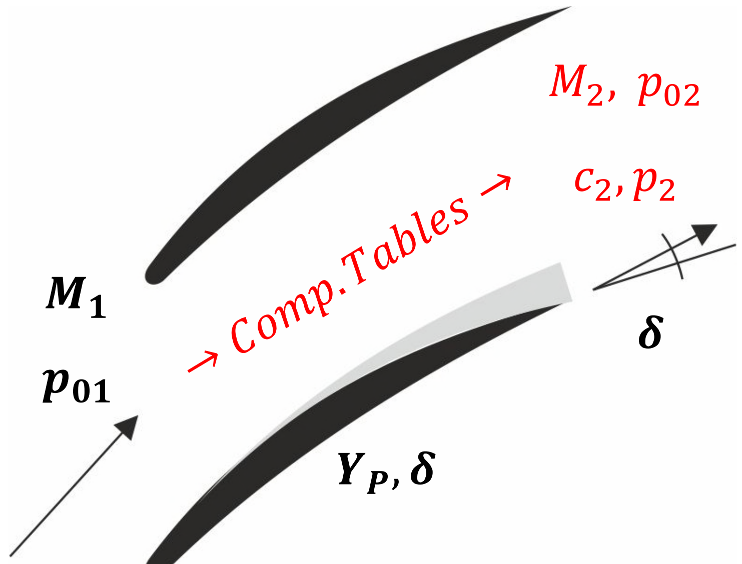
$$Y_P = \frac{p_{01} - p_{02}}{p_{01} - p_1} \quad (compressor\ cascade)$$

Given the inlet Mach number and Y_p it is easy to calculate the outlet Mach number for a stator blade as follows:

$$\frac{p_1}{p_{01}} = F_1(M_1)$$ F_1 from compressible flow tables

$$p_{02} = p_{01} - Y_P(p_{01} - p_1)$$

given $$\frac{p_{02}}{p_{01}} = 1 - Y_p\left(1 - \frac{p_1}{p_{01}}\right) \quad \rightarrow \quad p_{02}$$



$$\frac{\dot{m}\sqrt{C_p T_{01}}}{A_{a1}\cos(\alpha_1) p_{01}} = F_2(M_1)$$ F_2 from compressible flow tables

(From SFEE) For a stationary blade row $T_{02} = T_{01}$

$$\frac{\dot{m}\sqrt{c_p T_{01}}}{A_{a2}\cos(\alpha_2) p_{02}} = F_2(M_2) = \frac{\dot{m}\sqrt{c_p T_{01}}}{A_{a1}\cos(\alpha_1) p_{01}} \frac{p_{01}}{p_{02}} \frac{A_{a1}}{A_{a2}} \frac{\cos(\alpha_1)}{\cos(\alpha_2)} = F_2(M_1) \frac{p_{01}}{p_{02}} \frac{A_{a1}}{A_{a2}} \frac{\cos(\alpha_1)}{\cos(\alpha_2)}$$

$F_2(M_1)$

Given M_1, Y_P, the angles α_1 *and* α_2 the annulus areas of stream tube thickness A_{a1}, A_{a2}, $F_2(M_2)$ is easily found and M_2 can then be obtained from the compressible flow tables.

$$F_2(M_2) = F_2(M_1) \frac{p_{01}}{p_{02}} \frac{A_{a1}}{A_{a2}} \frac{\cos(\alpha_1)}{\cos(\alpha_2)} \quad \rightarrow Comp.Tables \quad \rightarrow \quad \boldsymbol{M_2}$$

From M_2 and P_{02}, the downstream static pressure and velocity can be found, again from compressible flow tables.

$$M_2,\ p_{02}, \rightarrow Comp.Tables \quad \rightarrow \quad c_2, p_2$$

Rotating Blade Row

A similar analysis can be applied to a rotor row where the main difference is that the stagnation pressures and temperatures must be *relative* ones, as seen by an observer on the rotor.

The relative stagnation temperature will change through a rotor if the stream surface radius changes. In this case $T_{02,rel}$ can be found from ***Euler's Work equation*** in the form (derived in B19):

$$\boxed{h_{0,rel} - 0.5\Omega^2 r^2 = constant\ along\ a\ stream\ surface}$$

$$In\ general\ if\ U_1 \neq U_2 \quad \rightarrow \quad T_{01,rel} \neq T_{02,rel}$$

$$\frac{p_{02rel,is}}{p_{01,rel}} = \left(\frac{T_{02rel}}{T_{01rel}}\right)^{\gamma/(\gamma-1)} \quad \rightarrow\ In\ general\ \ p_{02rel,is} \neq p_{01,rel}$$

Hence, strictly speaking, blade row loss coefficients Y_p are not directly applicable in machines where, in a rotating blade row, the relative stagnation pressure and the relative stagnation enthalpy can change as a result of changes in radius without there being any implied loss of efficiency.

$$r = const \ \rightarrow\ U = r\Omega = const \ \rightarrow\ h_{0,rel} = constant \ \rightarrow\ T_{01,rel} = T_{02,rel}$$

$$\frac{p_{02rel,is}}{p_{01,rel}} = \left(\frac{T_{02rel}}{T_{01rel}}\right)^{\gamma/(\gamma-1)} \quad \rightarrow \quad p_{02rel,is} = p_{02,rel} = p_{01,rel}$$

$$h_{0,rel} - 0.5\Omega^2 r^2 = constant$$

This will give $T_{02,rel}$

and the isentropic $P_{02,rel}$ can then be found from

$$\frac{p_{02rel,is}}{p_{01,rel}} = \left(\frac{T_{02rel}}{T_{01rel}}\right)^{\gamma/(\gamma-1)}$$

Subtracting the loss of relative stagnation pressure as obtained from Y_P from $P_{02,rel,is}$ then gives the actual relative stagnation pressure at exit.

$$p_{02rel} = p_{02rel,is} - Y_p(p_{01rel} - p_1)$$

$$\frac{p_{02rel}}{p_{01rel}} = \frac{p_{02rel,is}}{p_{01rel}} - Y_p\left(1 - \frac{p_1}{p_{01rel}}\right)$$

(*Ideal* p_{02} *is not* p_{01} *in the case of a rotor!*)

The compressible flow relations can then be applied exactly as before to find the exit flow conditions.

$$\frac{T}{T_{0,rel}}, \frac{p}{p_{0,rel}}, \frac{c_{rel}}{\sqrt{c_p T_{0,rel}}}, \frac{\dot{m}\sqrt{c_p T_{0,rel}}}{A p_{0,rel}}, = Functions(M)$$

The above analysis applies equally to turbine blades, with the only difference that Yp is defined differently for turbines (we use the exit dynamic head in the denominator).
This analysis will not be presented separately for turbines.

Importance of Understanding of Real Blade Row Aerodynamic Performance for the Stage Design

Example – deviation angle, δ
Measurements or accurate numerical predictions (CFD) of the flow angle for different M numbers and incidence angles are necessary for accurate prediction of flow parameters.

After the initial value for the exit flow angle α_2 is set by the mean-line analysis.

↓

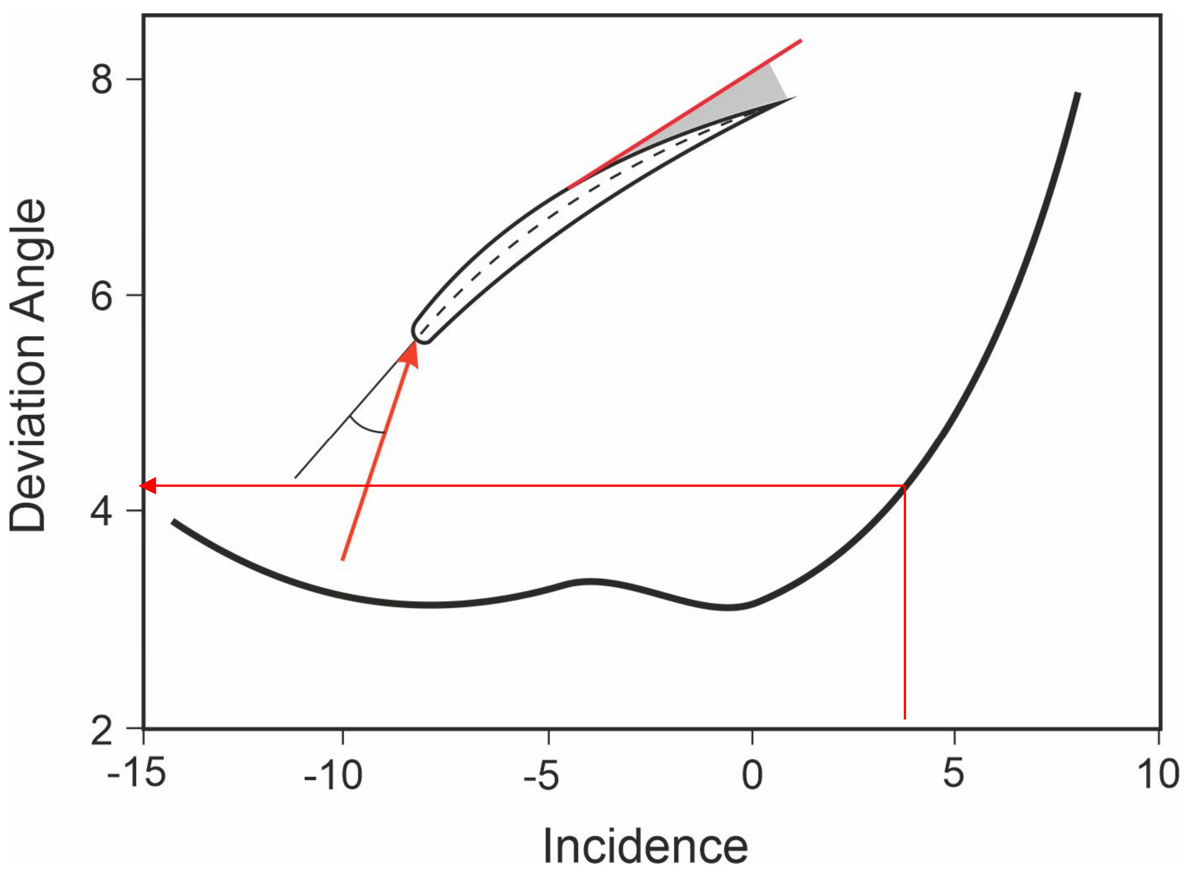
Empirical correlations (Measurements)
$\delta = f(M, Incidence)$

↓

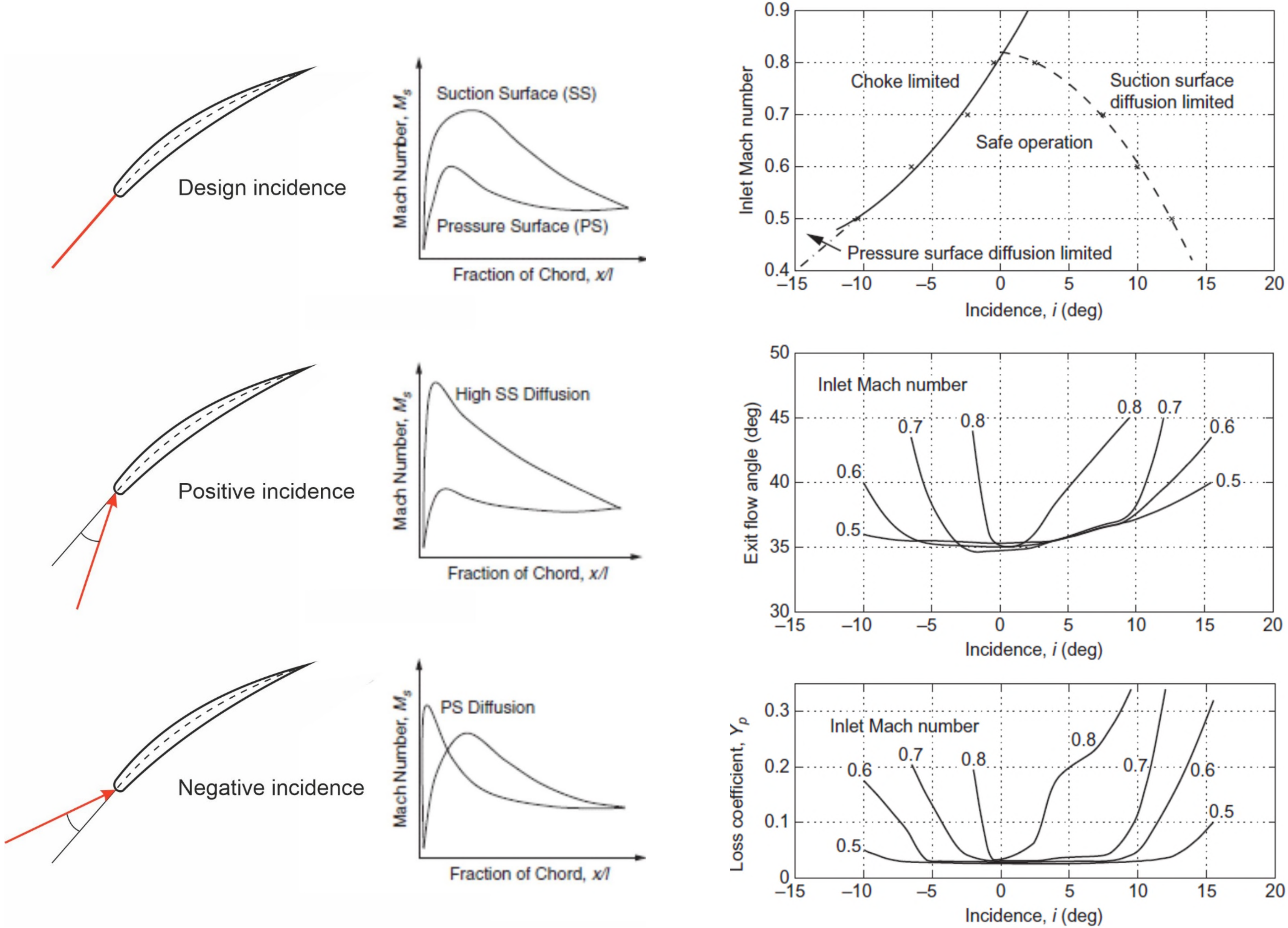
Updated α_2

↓

$$\frac{\dot{m}\sqrt{c_p T_{02}}}{A_{a2}\cos(\alpha_2)p_{02}} = F_2(M_2)$$



Influence of inlet Mach number on compressor cascade operation



Example

A two-dimensional compressor cascade is tested with an inlet Mach number of 0.78, an inlet flow angle of 45° and an exit flow angle of 5°, at an inlet stagnation pressure of 1 bar and an inlet stagnation temperature of 300 K. (Take $\gamma = 1.4 \; and \; c_p = 1005 \; J/kgK$)

a) Determine the mass flow rate per unit frontal area.

b) Calculate the exit Mach number and the static pressure ratio:
   i) when the flow is isentropic
   ii) when the stagnation pressure loss coefficient is 0.033.

a) $\boldsymbol{Given: M_1, \alpha_1, \alpha_2, p_{01}, T_{01}}$

$$\frac{\dot{m}\sqrt{c_p T_{01}}}{\mathrm{A}p_{01}} = F(M_1)$$

$$M_1 = 0.78 \;\; \rightarrow F(M_1) = 1.223$$

$$\frac{\dot{m}\sqrt{c_p T_{01}}}{A_a \cos(\alpha_1) p_{01}} = F(M_1)$$

$$\frac{\dot{m}}{A_a} = \frac{\cos(\alpha_1) p_{01}}{\sqrt{c_p T_{01}}} F(M_1) = \frac{\cos 45^o \cdot 1 \cdot 10^5}{\sqrt{1005 \cdot 300}} \cdot 1.223 = 157.5 \; kg/m^2$$

Given: $\boldsymbol{M_1, \alpha_1, \alpha_2, p_{01}, T_{01}}$

i) The flow is isentropic $\quad p_{01} = p_{02}$

$$\frac{\dot{m}\sqrt{c_p T_{02}}}{A_a \cos(\alpha_2) p_{02}} = \frac{\dot{m}}{A_a} \frac{\sqrt{c_p T_{01}}}{\cos(\alpha_2) p_{01}} = 157.5 \frac{\sqrt{1005 \cdot 300}}{10^5 cos5^O} = 0.8681 = F(M_2) \rightarrow M_2 = 0.439$$

$$\frac{p_2}{p_1} = \frac{p_{01}}{p_1} \cdot \frac{p_{02}}{p_{01}} \cdot \frac{p_2}{p_{02}} = \left(1 + \frac{\gamma - 1}{2} M_1^2\right)^{\frac{\gamma}{\gamma-1}} \cdot 1 \cdot \left(1 + \frac{\gamma - 1}{2} M_2^2\right)^{-\frac{\gamma}{\gamma-1}}$$

Isentropic flow

$$\frac{p_2}{p_1} = \left(\frac{1 + 0.2 \cdot 0.78^2}{1 + 0.2 \cdot 0.439^2}\right)^{3.5} = 1.309$$

ii) Loss coefficient Y_P = 0.033

$$Y_P = \frac{p_{01} - p_{02}}{p_{01} - p_1} = 0.033 \quad \rightarrow \frac{p_{02}}{p_{01}} = 1 - Y_P\left(1 - \frac{p_1}{p_{01}}\right)$$

$$\rightarrow \frac{p_{02}}{p_{01}} = 1 - Y_P\left(1 - \left(1 + \frac{\gamma - 1}{2} M_1^2\right)^{-\frac{\gamma}{\gamma-1}}\right) = 1 - 0.033\left(1 - \left(1 + \frac{1.4 - 1}{2} 0.78^2\right)^{-3.5}\right) = 0.98908$$

$$p_{01} = 1bar \quad \rightarrow \quad p_{02} = 0.98908\ bar$$

$$\frac{\dot{m}\sqrt{c_p T_{02}}}{A_a \cos(\alpha_2) p_{02}} = \frac{\dot{m}}{A_a} \frac{\sqrt{c_p T_{02}}}{\cos(\alpha_2) p_{02}} = 157.5 \frac{\sqrt{1005 \cdot 300}}{0.98908 \cdot 10^5 cos5^O} = 0.8777(M_2) \rightarrow M_2 = 0.446$$

$$\frac{p_2}{p_1} = \frac{p_{01}}{p_1} \cdot \frac{p_{02}}{p_{01}} \cdot \frac{p_2}{p_{02}} = \left(1 + \frac{\gamma - 1}{2} M_1^2\right)^{\frac{\gamma}{\gamma-1}} \cdot 0.98908 \cdot \left(1 + \frac{\gamma - 1}{2} M_2^2\right)^{-\frac{\gamma}{\gamma-1}}$$

$$\frac{p_2}{p_1} = 0.98908\left(\frac{1 + 0.2 \cdot 0.78^2}{1 + 0.2 \cdot 0.446^2}\right)^{3.5} = 1.290$$

Example

For the high Mach number compressor rotor blade row find the static pressure ratio (p_2/p_1) across the blade row and the absolute exit flow angle, a_2 and the exit Mach number M_2.

Geometrical Data:		
Mean radius (constant)	r	0.300 m
Blade height (span, constant)	h	0.050 m
Annulus cross-sectional area	A_a	0.0942
Operating Conditions:		
Blade speed	U	250 m/s
Mass flow rate	$\dot{m}$	16.0 kg/s
Inlet stagnation pressure (abs.)	p_{01}	1.4 bar
Inlet stagnation temperature	T_{01}	340 K
Absolute inlet swirl	α_1	10°
Blade performance		
Rotor pressure loss coefficient	Y_P	0.034
Rotor relative exit angle	$\alpha_{2,rel}$	-35°

(Working fluid: Air, $\gamma = 1.4, R = 287\, J/(kgK), c_p = 1005\, J/(kgK)$)

Upstream of Rotor
(absolute frame, find inlet flow conditions)
Effective flow area $A = A_a cos\alpha_1$

$$\frac{\dot{m}\sqrt{c_p T_{01}}}{A_{a1}\cos(\alpha_1)p_{01}} = \frac{16\sqrt{1005 \cdot 340}}{0.0942 \cdot cos10° \cdot 1.4 \cdot 10^5} = 0.72$$

$$Tables\ (\gamma = 1.4)\ \rightarrow\ \ M_1 = 0.350$$

$$Tables\ (\gamma = 1.4)$$

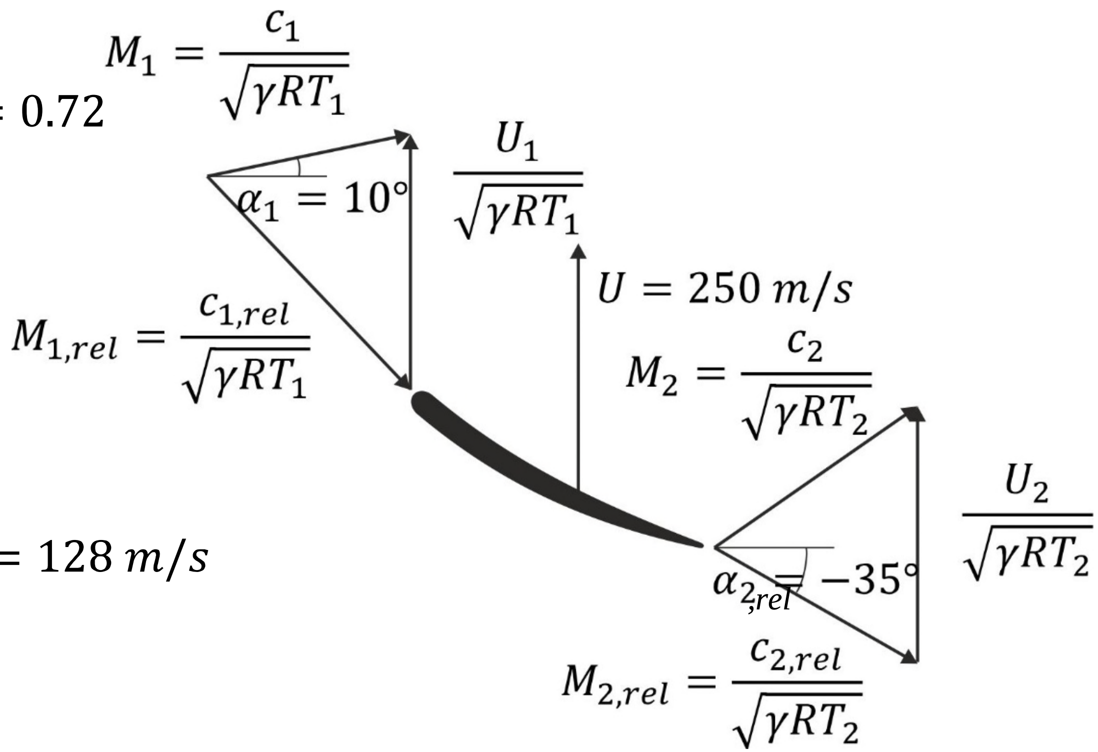
$$\frac{c_1}{\sqrt{c_p T_{01}}} = 0.219\ \rightarrow\ \ c_1 = 0.219\sqrt{1005 \cdot 340} = 128\ m/s$$

$$\frac{T_1}{T_{01}} = 0.976\ \ \rightarrow\ \ T_1 = 331.8\ K$$

$$\frac{p_1}{p_{01}} = 0.919\ \ \rightarrow\ \ p_1 = 1.287\ \text{bar}$$

Velocity / Mach triangle

(Velocity triangles may be converted by dividing all velocities by $\sqrt{\gamma RT}$ (local speed of sound) to produce geometrically similar Mach number triangles.)



Upstream of Rotor (relative frame, convert absolute conditions to relative frame)

$$c_{x1} = c_1 cos\alpha_1 = 128 \cdot cos10° = 126.1 m/s$$

$$c_{\theta 1} = c_1 sin\alpha_1 = 128 \cdot sin10° = 22.2 m/s$$

$$c_{\theta 1,rel} = c_{\theta 1} - U = -227.8 \text{ m/s}$$

$$c_{1rel} = \sqrt{c_{x1}^2 + c_{\theta 1rel}^2} = \sqrt{126.1^2 + 227.8^2} = 260.4\, m/s$$

$$tan\alpha_{1rel} = \frac{c_{\theta 1rel}}{c_{x1}} \quad \rightarrow \alpha_{1rel} = tan^{-1}\left(\frac{c_{\theta 1rel}}{c_{x1}}\right) = tan^{-1}\left(\frac{-227.8}{126.1}\right) = 61°$$

$$M_{1,rel} = \frac{c_{1,rel}}{\sqrt{\gamma R T_1}} = \frac{260.4}{\sqrt{1.4 \cdot 287 \cdot 331.8}} = 0.713$$

Tables:

$$\frac{T_1}{T_{01rel}} = 0.908 \qquad \rightarrow \qquad T_{01rel} = 365.4\, K$$

(Alternative: $T_{01rel} = T_1 + c_{1rel}^2/2c_p = 365.5\, K$)

$$\frac{p_1}{p_{01rel}} = 0.713 \qquad \rightarrow \qquad p_{01rel} = 1.805 \text{ bar}$$

Rotor Performance (relative frame, apply loss and flow turning)

$T_{01rel} = T_{02rel} = 365.4\ K$ $\qquad h_{0,rel} - 0.5\Omega^2 r^2 = constant\ along\ a\ stream\ surface$

$U = const \quad \rightarrow h_{0,rel} = const$

$P_{02rel,is} = p_{01rel} = 1.805\ bar$

Loss coefficient: $Y_P = \dfrac{p_{01rel} - p_{02rel}}{p_{01rel} - p_1} = 0.034 \quad \rightarrow p_{02rel} = p_{01rel} - Y_P(p_{01rel} - p_1) = 1.787$ bar

Exit angle $\alpha_{2rel} = -35°$ is given.

Rotor Exit (relative frame, find exit conditions in relative frame)

$$\frac{\dot{m}\sqrt{c_p T_{02rel}}}{A_{a1}\cos(\alpha_{1rel}) p_{01rel}} = \frac{16\sqrt{1005 \cdot 365.4}}{0.0942 \cdot \cos(-35°) \cdot 1.787 \cdot 10^5} = 0.703$$

Tables ($\gamma = 1.4$) $\quad \rightarrow \quad M_{2rel} = 0.340$

$\dfrac{c_{2rel}}{\sqrt{c_p T_{02rel}}} = 0.213 \rightarrow \quad c_{2rel} = 129.1\ m/s$

(Alternative: $\frac{c_{2rel}}{\sqrt{\gamma R T_2}} = 0.340 \rightarrow \quad c_1 = 0.340\sqrt{1.4 \cdot 287 \cdot 357} = 128.8\ m/s$)

$\dfrac{T_2}{T_{02rel}} = 0.977 \quad \rightarrow \quad T_2 = 357.0\ K$

$\dfrac{p_2}{p_{02rel}} = 0.923 \quad \rightarrow \quad p_2 = 1.649$ bar

Pressure ratio: $\dfrac{p_2}{p_1} = \dfrac{1.649}{1.287} = 1.28$

Downstream of Rotor (absolute frame, convert back to absolute flow conditions)

$$c_{x2} = c_{2rel} cos\alpha_{2rel} = 129.1 \cdot \cos(-35°) = 105.8\, m/s$$

$$c_{\theta 2rel} = c_{2rel} sin\alpha_{2rel} = 129.1 \cdot \sin(-35°) = -74.0\, m/s$$

$$c_{\theta 2} = c_{\theta 2rel} + U = 176.0 \text{ m/s}$$

$$c_2 = \sqrt{c_{x2}^2 + c_{\theta 2}^2} = \sqrt{105.8^2 + 176.0^2} = 205.4\, m/s$$

$$tan\alpha_2 = \frac{c_{\theta 2}}{c_{x2}} \quad \rightarrow \alpha_2 = tan^{-1}\left(\frac{c_{\theta 2}}{c_{x2}}\right) = tan^{-1}\left(\frac{176.0}{105.8}\right) = 59°$$

$$M_2 = \frac{c_2}{\sqrt{\gamma R T_2}} = \frac{205.4}{\sqrt{1.4 \cdot 287 \cdot 357}} = 0.542$$

Tables ($\gamma = 1.4$) $M_2 = 0.542$

$$\frac{T_2}{T_{02}} = 0.945 \quad \rightarrow \quad T_{02} = 377.8\ K \qquad \Delta T_0 = 377.8 - 340 = 37.8\ K$$

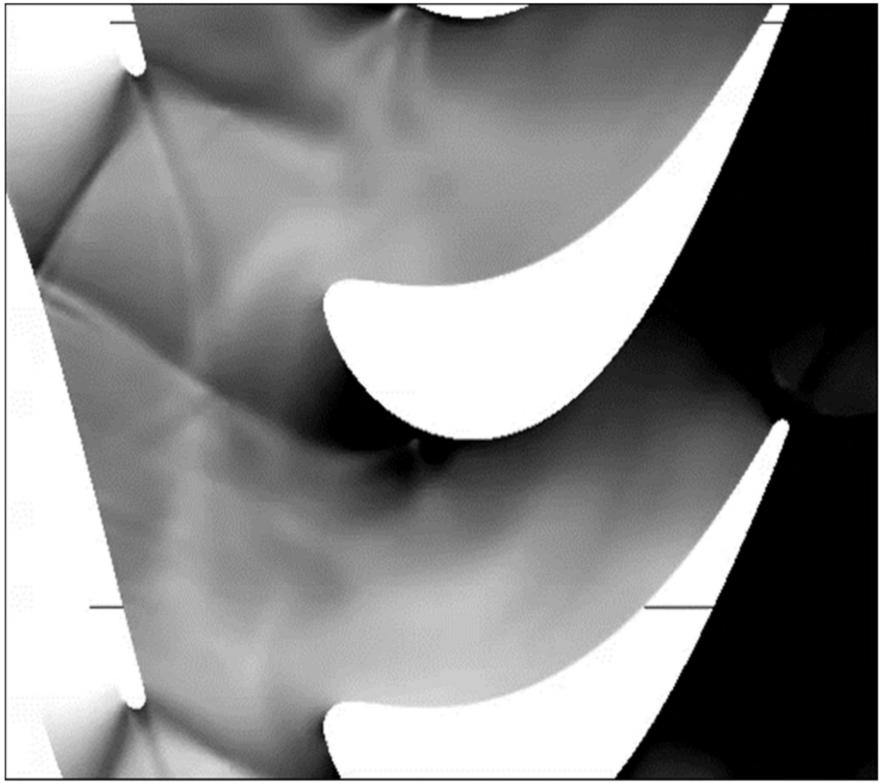
Check (using Euler's Work Equation) $h_{02} - h_{01} = U(c_{\theta 2} - c_{\theta 1}) = 250(176 - 22.2) = 38.1\ kJ/kg$

$$\rightarrow \Delta T_0 = 38.1\ K$$

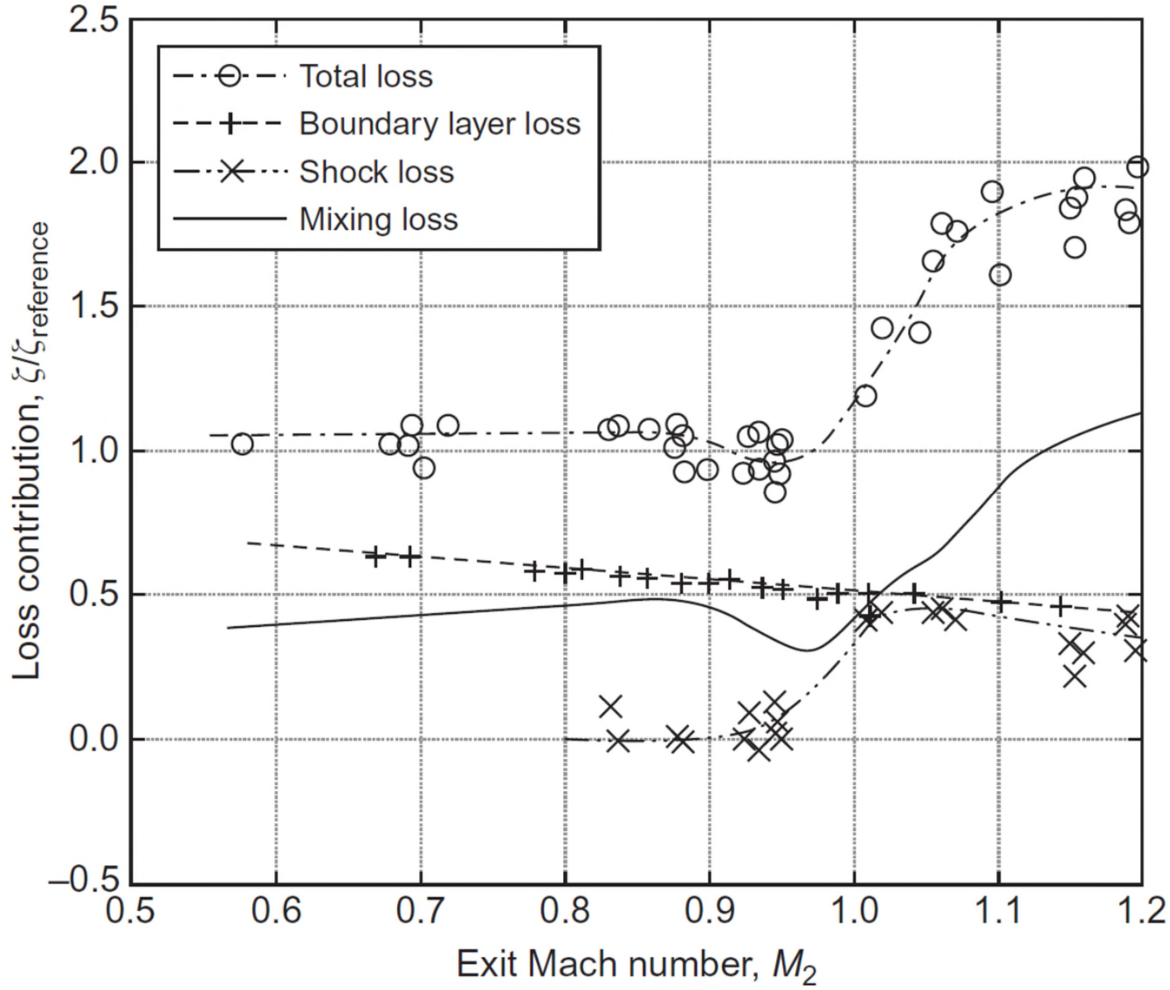
$V_{x2} < V_{x1}$ this is due to compressibility

Check the mass conservation: $\rho_1 c_{x1} A_{a1} = \rho_2 c_{x2} A_{a2}$

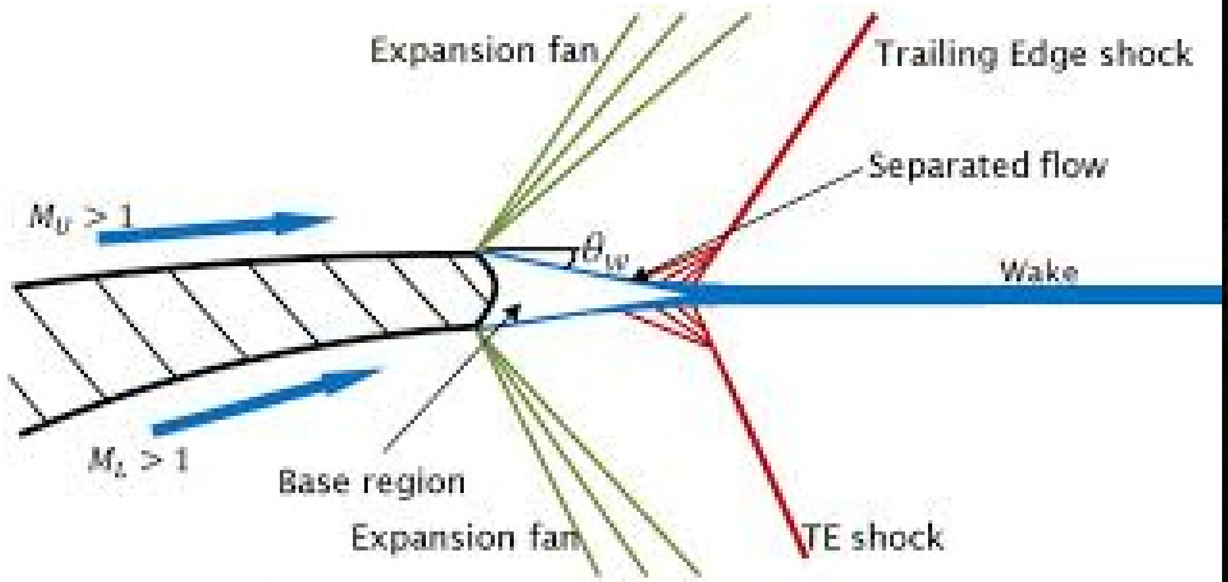
Effect of Mach number on 2D Turbine Cascade Flow

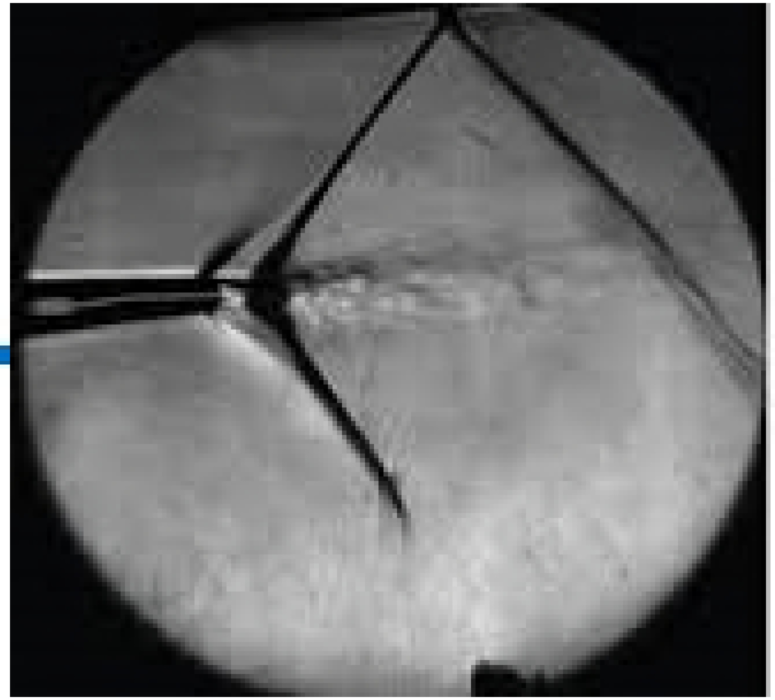


Variation of loss coefficient with Mach number for a turbine cascade at a Reynolds number of $1x10^6$.



Losses In Shock Waves at Turbine Cascade Trailing Edge

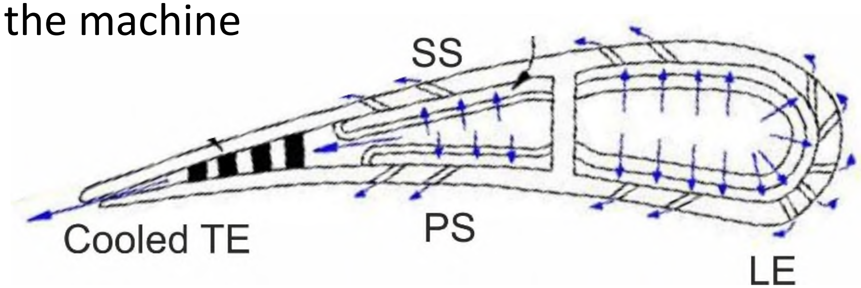




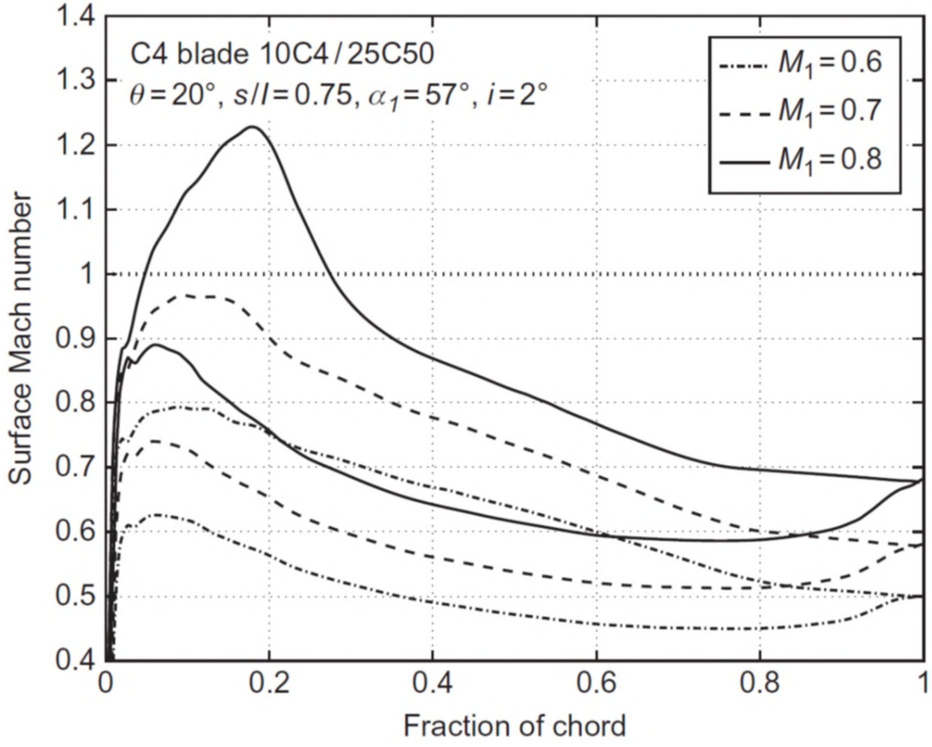
The shock system at the trailing edge.
The low base pressure formed behind the TE can generate a very large TE loss. The flow expands around the TE to this low pressure and is then recompressed by a strong shock wave at the point where the SS and PS flows meet. The loss generation comes from the intense viscous dissipation at the edges of the separated region immediately behind the TE and from the strong shock formed at the close of this region.

Shock loss: The irreversibility occurs due to heat conduction and high viscous normal stresses within the shock wave (only a few molecular free paths in thickness).

The TE of cooled turbine blades is quite thick and this may be the largest single source of loss in the machine

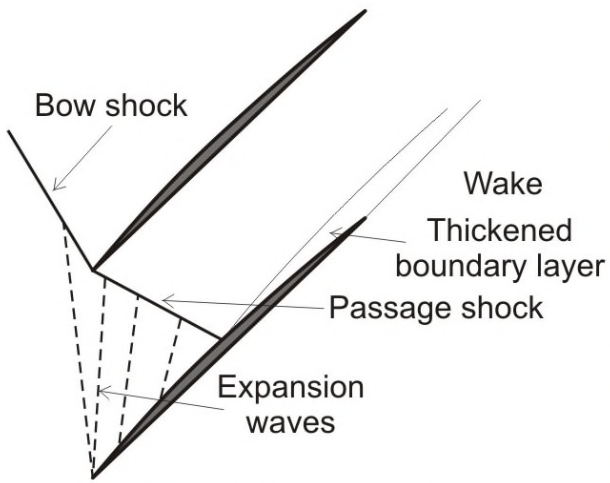


Effect of Mach number on 2D Compressor Cascade Flow

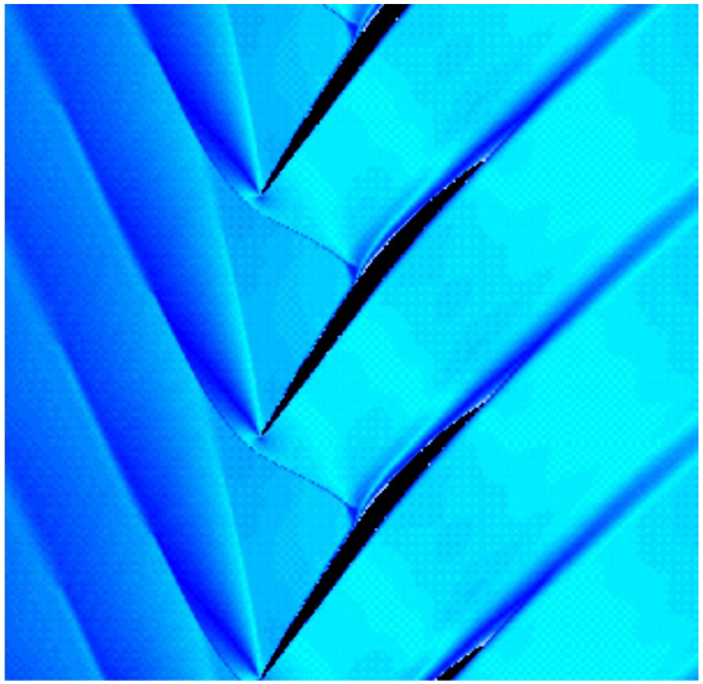


Surface Mach number variation for different inlet Mach number values

The effects of high Mach inlet numbers can be reduced by using very thin blades with very low turning. Both these features help to make the peak suction surface Mach number not much higher than the inlet Mach number. Such blades are used for transonic compressors with inlet Mach numbers up to about 1.5. The figure shows a typical blade shape and shock pattern in such blades.

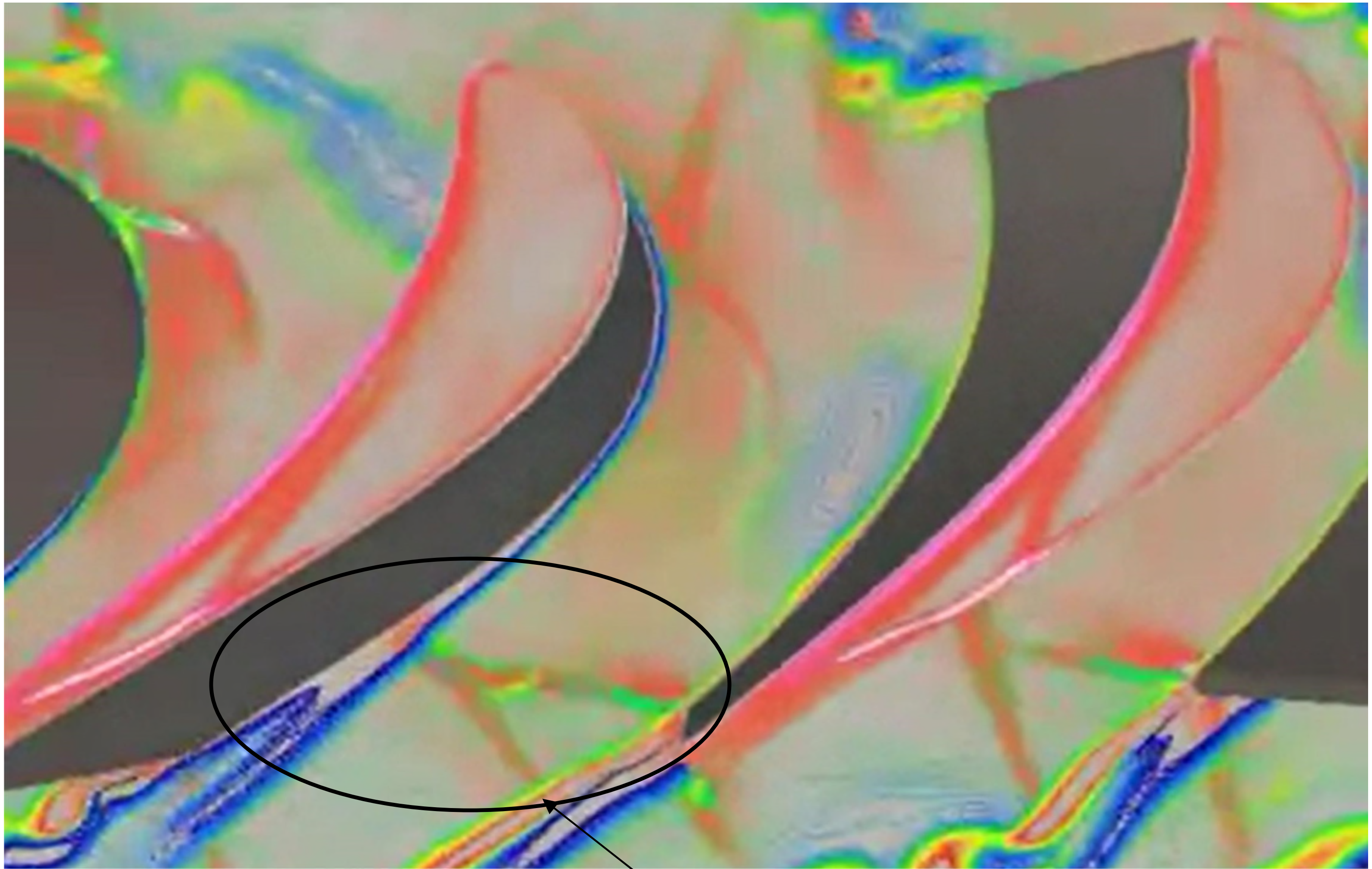


Transonic Compressor Blade



At high Mach numbers the losses associated with shock waves are acceptable as long as they do not cause the boundary layers to separate. Actually, the shock is quite an efficient means of compression for normal Mach numbers less than about 1.5. Stage stagnation pressure ratios of up to 2:1 can be achieved by transonic compressors.

Shock Boundary Layer Interaction



Shock causes the boundary layer separation on the suction side of the rotor blade

This has significant effect on losses and on the flow angle at the rotor exit (flow deviation)

Effect on Blade Exit Angle – Supersonic Deviation

The exit angle of turbine blades is much closer to the "metal" angle at the trailing edge than that of compressor blades. However, its accurate prediction is extremely important because at higher exit angles used for turbine blades the downstream flow area ($scos\alpha_2$) varies rapidly with the angle α_2. e.g. at $\alpha_2 = 70°$ a 1 degree error in angle causes a 5 % error in flow area and hence probably in mass flow and power. The exit angle is best estimated by using the throat area of the blades as follows:

When the blade throat is choked the average Mach number across the throat is 1 and so we have:

$$\frac{\dot{m}\sqrt{c_p T_0}}{o p_{0t}} = F(1)$$

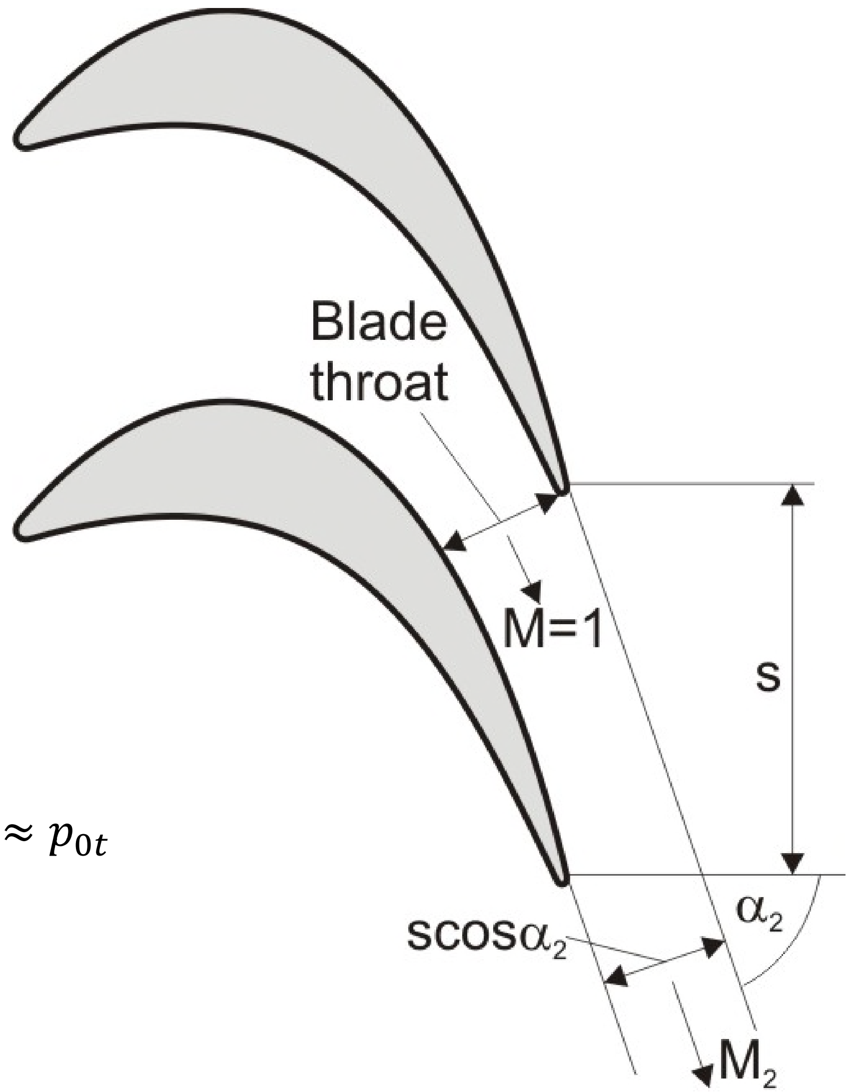
where o is the throat width. Downstream of the blade the flow area is $scos\alpha_2$ and the Mach number is M_2,

$$\frac{\dot{m}\sqrt{c_p T_0}}{scos\alpha_2 p_{02}} = F(M_2)$$

hence $$\cos\alpha_2 = \frac{F(1)}{F(M_2)}\frac{o}{s}\frac{p_{0t}}{p_{02}}$$

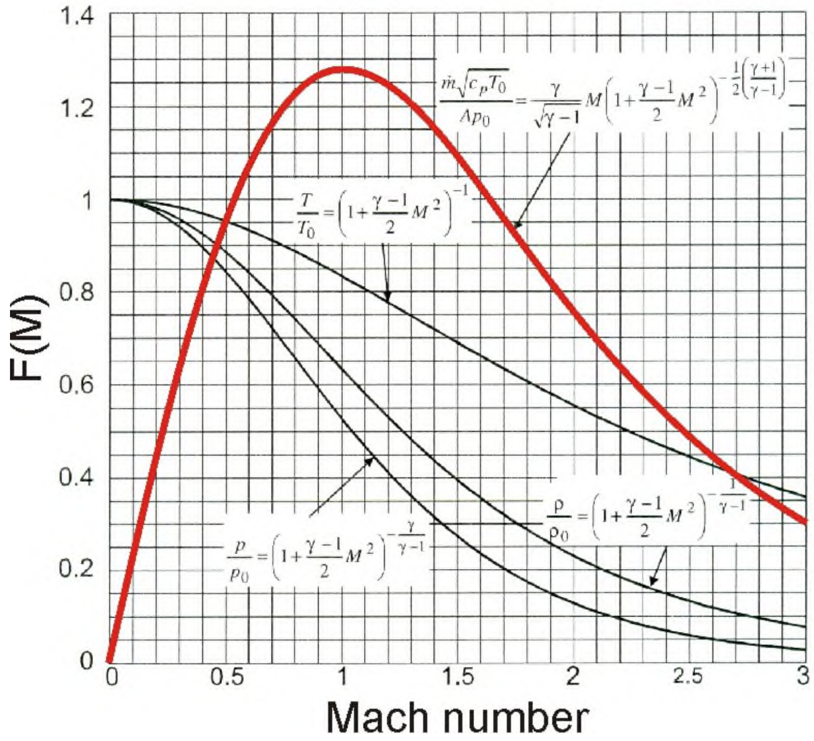
if the losses downstream of the throat are small then $p_{02} \approx p_{0t}$

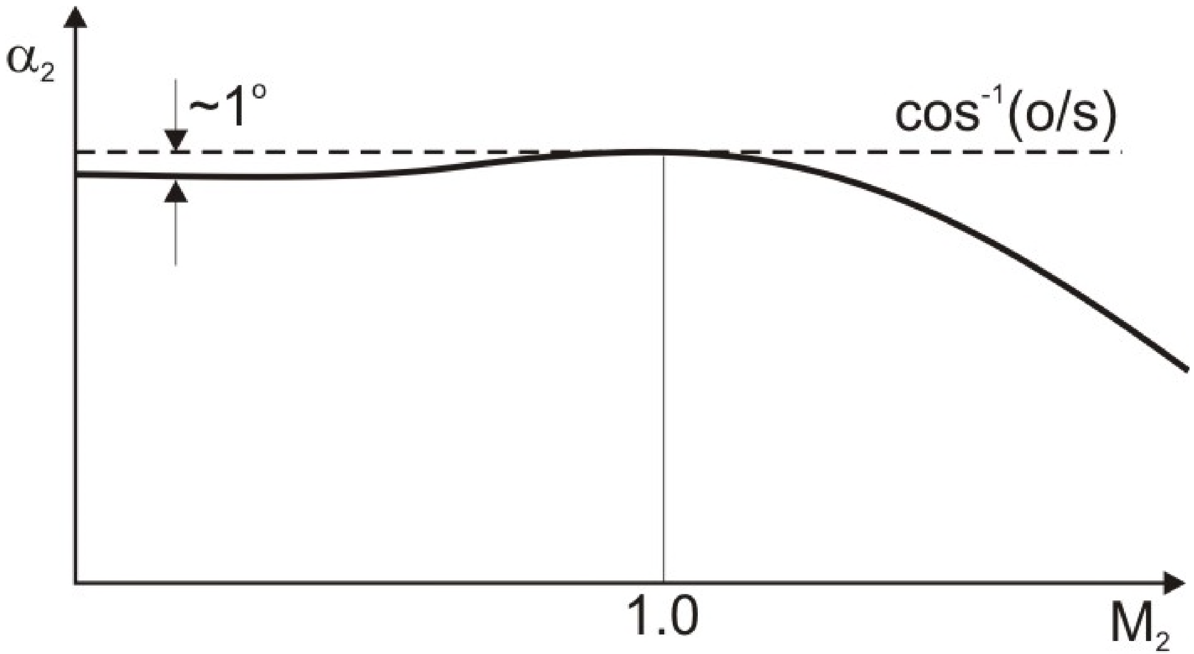
$$cos\alpha_2 = \frac{F(1)}{F(M_2)}\frac{o}{s}$$



and in particular when $M_2 = 1$ then $\alpha_2 = cos^{-1}(o/s)$. It is found that the exit angle varies little with Mach Number for subsonic exit flows and so that above gives a good approximation up to $M_2 = 1$. At supersonic exit flows then $F(M_2)$ is less than F(1) and it follows from the last equation that α_2 decreases.

This reduction in exit angle for supersonic exit flows is known as *supersonic deviation*. The overall variation of exit angle with Mach number is something like





$$\boxed{M_2 > 1 \rightarrow F(M_2) < F(1) \rightarrow \alpha_2 \ decreases}$$

$$\boxed{F(M) = \frac{\dot{m}\sqrt{c_p T_0}}{A p_0} = \frac{\gamma}{\gamma - 1} M \left(1 + \frac{\gamma - 1}{2} M^2\right)^{-\frac{1}{2}\left(\frac{\gamma+1}{\gamma-1}\right)}}$$

Next time:
3D blade design…