C201 Viscous Flow and Turbulence

Lecture 1

Part 1: Statistical tools

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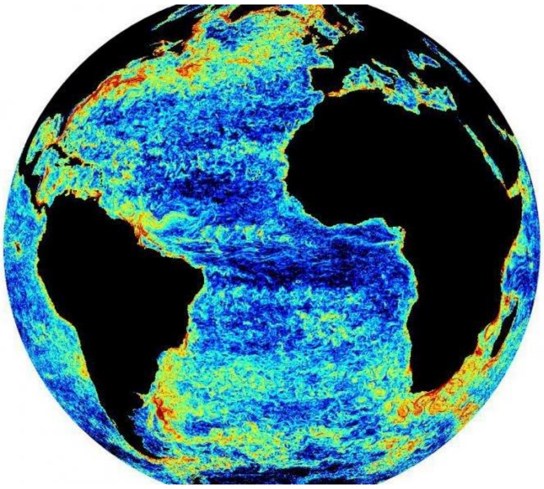


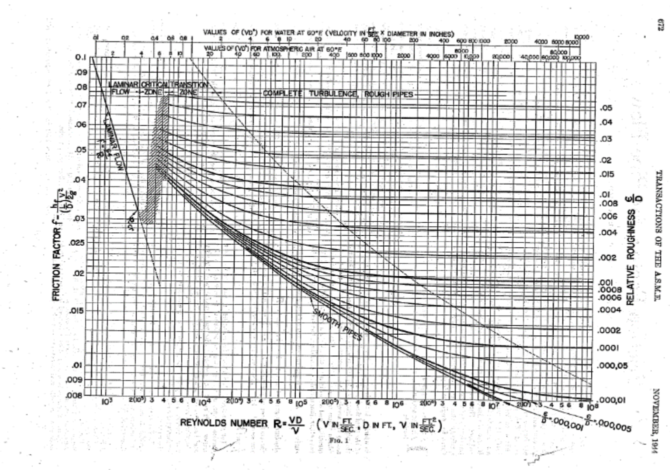


Figure 1: left, Energy dissipation in the Earth’s oceans, Brown University; right: Moody’s chart from his original paper in 1944: Moody, L. F. (1944), Friction factors for pipe flow, Transactions of the ASME, 66 (8): 671–684

1 Introduction

Turbulent flows are ubiquitous in the natural world. The flow in most waterways, in the oceans, in the atmosphere and even the Earth mantel is turbulent. In biological systems, the flow in the aorta of most mammals – including humans – is turbulent.

The flows of interest in most large-scale engineering systems are also turbulent. Since your first and second year studies you have become familiar with the idea that turbulence causes a drastic change in the dependence of quantities such as drag, friction or heat transfer coefficients on the Reynolds numbers.

We need to understand the properties of turbulent flows for to

- Understand physical phenomena around us
- Because of the influence turbulence has on engineering

1.1 The main features of turbulent flows

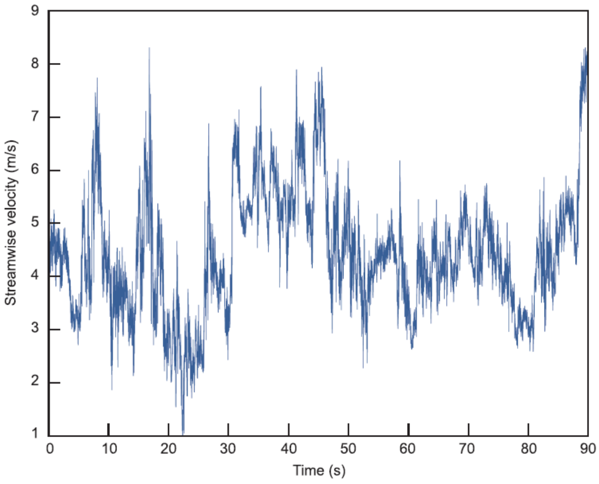


Figure 3: A time series of velocities for a turbulent boundary layer at $Re \sim 2 \times 107$.

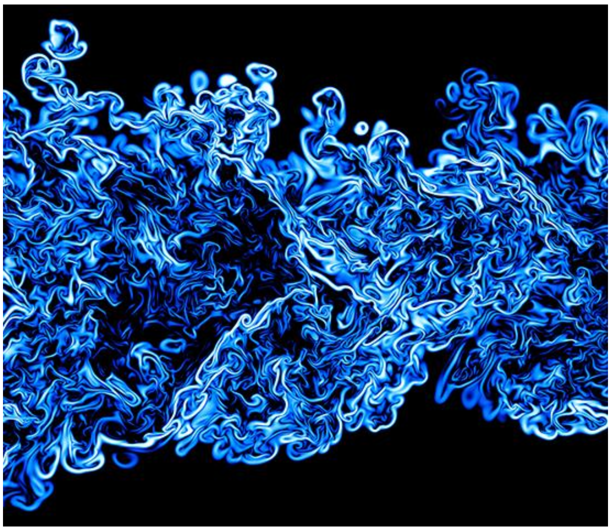


Figure 3: Vorticity field of a turbulent jet. DNS by Michael Gauding, Copyright: *CNRS UMR 6614 CORIA and JSC*

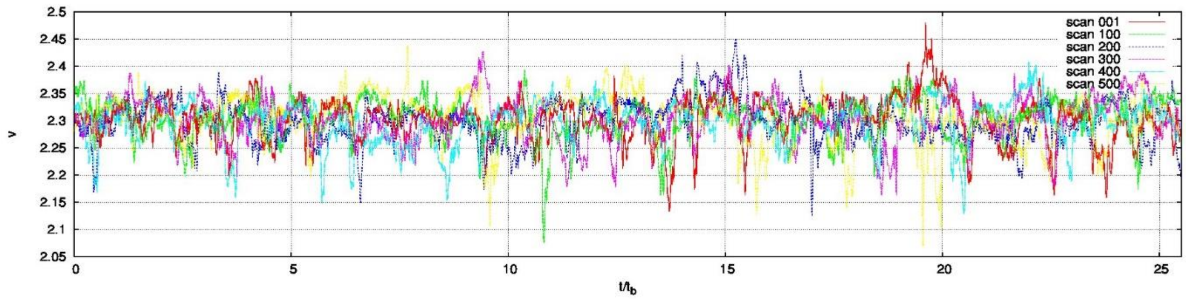
Turbulent flows have some specific properties that set them aside from the laminar flow past the same geometric configuration.

The most notable feature of a turbulent flow is the irregular and chaotic variation of the flow properties. In general, a probe immersed in a turbulent flow will produce a signal not dissimilar from the one shown in Figure 3.

The flow quantities in a turbulent field are chaotic in the sense that it is difficult to predict their values in the future given their recent history. This property becomes apparent in problems such as weather forecast: the accuracy of weather forecast rapidly deteriorates with the length of the forecast.

Turbulence is invariably associated to a three-dimensional vorticity field. The appearance of the vorticity field for a turbulent jet is shown in Figure 3. The three dimensional vorticity field of a turbulent flow plays an important role in the way kinetic energy is transformed into internal energy in a turbulent flow. We will see in more detail during this module that turbulence is sustained at the expense of the kinetic energy of the mean flow. The energy drained from the mean flow by turbulence is invariably dissipated into heat.

The rapid fluctuations of flow quantities in a turbulent flow also cause turbulence to considerably enhance the rate of transport of momentum, mass and energy compared to an analogous laminar flow. It is this enhanced rate of transport that makes the study of turbulence important in engineering applications.



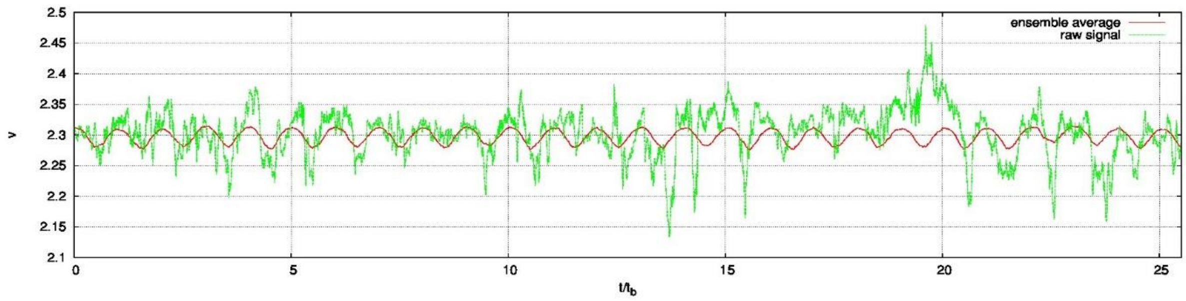


Figure 4: six repetition of a velocity measurement in a research compressor (top) and the ensemble average of a large number of repetitions of the same measurement (bottom).

1.2 Statistical description of turbulent flows

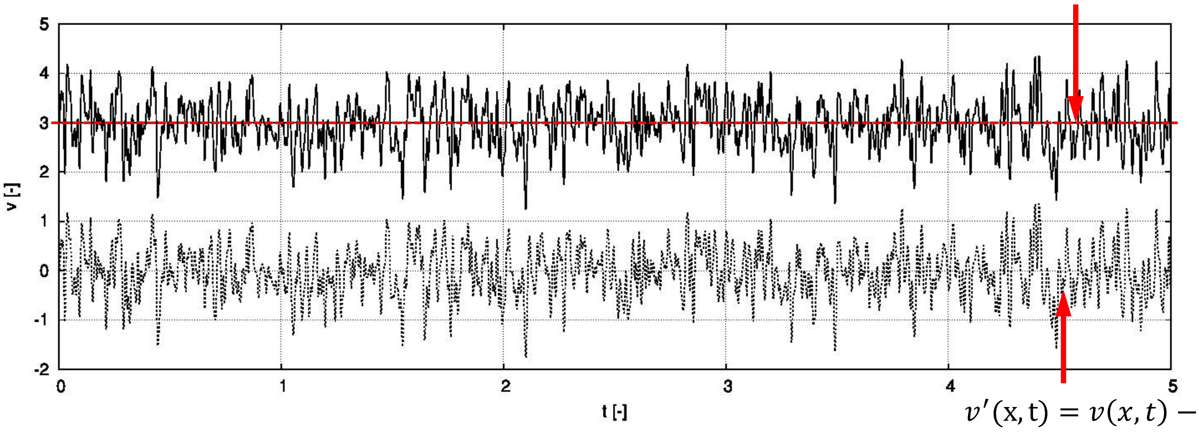
One of the consequences of the chaotic nature of turbulent flows is the fact that they are not strictly reproducible. As an example, we could repeat a velocity measurement a large number of times (see Figure 4 top). Each repetitions of the experiment will give us different signals. If we take an ensemble average of a large number of such measurements, however, we will find that the ensemble average is a well-behaved function of the boundary conditions and of the other parameters of the experiment (see Figure 4 bottom graph).

A detailed description of a turbulent flow is therefore very challenging, but it is also not strictly necessary: engineering calculations are mainly concerned with average flow quantities over long times (e.g. in plants that operate for long times like power stations) or over large fleets of geometrically similar devices (e.g. mass-produced pumps).

In the rest of this module we will mainly concerned with a statistical description of flow quantities in a turbulent flow because 1) they are accessible experimentally 2) they are of actual interest in engineering calculations.

1.3 Reynolds decomposition

$$\bar{v}(\mathrm{x}) = \lim_{T \to \infty} \frac{1}{T} \int_{\tau}^{T+\tau} v(x,t)dt$$



$$v'(\mathrm{x},\mathrm{t}) = v(x,t) - \bar{v}(x)$$

Figure 5: the mean value and the fluctionation in Reynolds' decomposition of turbulent flow quantities.

The main tool in the statistical description of turbulent flows is the Reynolds decomposition. The Reynolds decomposition splits every quantity into a mean value (which we will denote with an overbar, e.g. $\bar{v}$) and a fluctuation (which we will denote with a a prime, e.g. v'):

$$v(x,t) = V(x) + v'(x,t)$$

An example illustrating the relationship between the turbulent quantity $v(x,t)$, its mean $V(x)$ and its fluctuating component $v'(x,t)$ is given in Figure 5. The turbulent variable in Figure 5 is denoted by the solid black line. Its mean (approximately 3) is denoted by the red dashed line and its fluctuating component is denoted by the dashed black line. In general, turbulent variables are functions of both time and position. The mean value is in general a function of position only and the fluctuating part is still a function of both time and position.

The mean value can be defined as a time average, over a suitably long period of time:

$$V\,(x) = \lim_{T \to \infty} \frac{1}{T} \int_{\tau}^{T+\tau} v(x,t)dt$$

We will mainly be interested in stationary processes, where the ensemble average is identical to the time average. The ensemble average is the average over a large number of repetitions of the same experiment:

$$V(x) = \lim_{N \to \infty} \frac{1}{N} \sum v(x,t_i)$$

We will use time and ensemble averages interchangeably in the rest of this module.

1.4 Some consequences of the Reynolds decomposition

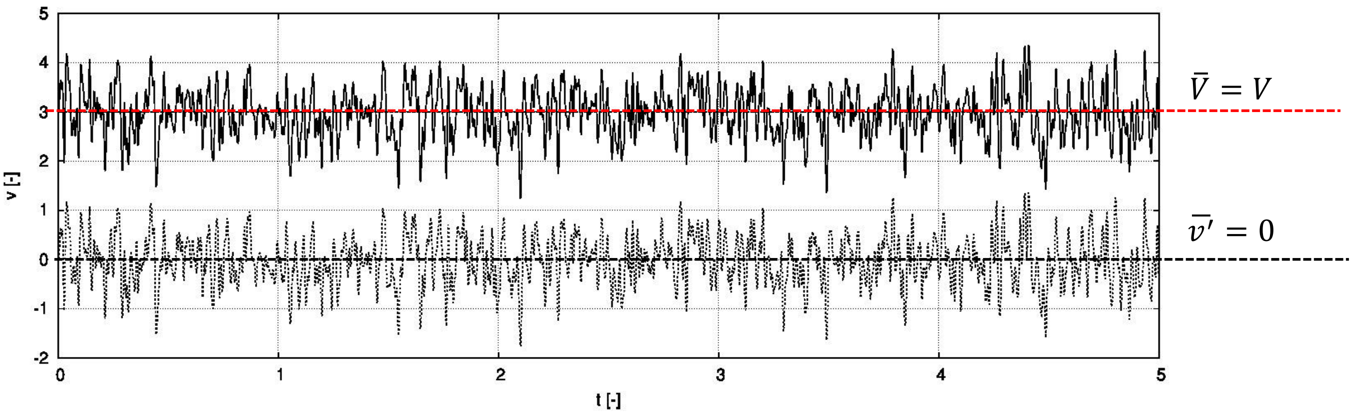


Figure 6: mean value of a mean value and of a fluctuating component.

Reynolds' decomposition has some properties we will use in following derivations.

The mean of a mean value is the mean value itself:

$$\bar{V} = V$$

The mean of a fluctuating component is 0:

$$\overline{v'} = 0$$

The mean of the product of a fluctuation times a mean value is also 0:

$$\overline{u'\,V} = \overline{U\,v'} = 0$$

However, in general, the mean of the product of two fluctuations is not zero:

$$\overline{u'v'} \neq 0$$

This property is indeed a fundamental ingredient of how turbulence can affect the transport of mass, momentum and energy in flow fields.

The time and ensemble average commute with spatial operators (derivatives and integrals)

$$\overline{\frac{\partial v}{\partial x}} = \frac{\partial\, V}{\partial x}$$

2 The dynamic behaviour of turbulence – the RANS equations

In the previous chapter we have introduced Reynolds' decomposition to districate the mean behaviour of turbulent flows from their fluctuating instantaneous values.

We now would like to make statements about turbulent flows related to the conservation laws of mass, momentum and energy of classical mechanics and we would like to learn what form these statements take for turbulent flows.

We start from the Navier-Stokes equations written for an incompressible fluid of constant kinematic viscosity ν and density ρ (See foreword for index notation)

$$\frac{\partial u_i}{\partial x_i} = 0$$

$$\frac{\partial u_i}{\partial t} + u_j \frac{\partial u_i}{\partial x_j} = -\frac{1}{\rho}\frac{\partial p}{\partial x_i} + \nu \frac{\partial^2 u_i}{\partial x_j \partial x_j}$$

The equations written above apply at any instant in time whether the flow is turbulent or laminar. As we have mentioned, their predictive usefulness for turbulent flows is limited by the sensitivity of the flow history on the precise initial conditions.

In order to extract useful information we can apply Reynolds' decomposition:

$$u_i(\boldsymbol{x}, t) = U_i(\boldsymbol{x}) + u_i'(\boldsymbol{x}, t)$$

2.1 The continuity equation

The continuity equation becomes:

$$\frac{\partial}{\partial x_i}(U_i + u_i') = 0$$

Applying time averaging to the continuity equation we find

$$\overline{\frac{\partial}{\partial x_\iota}(U_\iota + u_\iota')} = \overline{\frac{\partial U_\iota}{\partial x_\iota}} = \frac{\partial U_i}{\partial x_i} = 0$$

The steps above are justified by the commutative properties of time/ensemble averages and spatial operators (see previous chapter).

Subtracting the mean continuity equation from the original equation we have

$$\frac{\partial u_i}{\partial x_i} - \frac{\partial U_i}{\partial x_i} = 0 \rightarrow \frac{\partial u_i'}{\partial x_i} = 0$$

So we have shown that if the flow is divergence-free, then both the mean velocity and the its fluctuating component must be divergence-free.

2.2 The momentum equation

Substituting the Reynolds decomposition into the the momentum equations we find

$$\frac{\partial(U_i+u_i')}{\partial t}+\left(U_j+u_j'\right)\frac{\partial}{\partial x_j}(U_i+u_i')=-\frac{1}{\rho}\frac{\partial}{\partial x_i}(P+p')+\nu\frac{\partial^2}{\partial x_j\partial x_j}(U_i+u_i')$$

Applying time averaging we notice that the time derivative must disappear because the mean value is constant in time and because if the process is statistically stationary no average property can be time-dependent. The surviving terms are:

$$\overline{\left(\bar{u}_j+u_j'\right)\frac{\partial}{\partial x_j}(\bar{u}_i+u_i')}=-\frac{1}{\rho}\frac{\partial}{\partial x_i}\overline{\bar{p}+p'}+\nu\frac{\partial^2}{\partial x_j\partial x_j}\overline{\bar{u}_i+u_i'}$$

2.2.1 Convective derivative

In the convective derivative we can expand the products to find:

$$\overline{\left(U_j+u_j'\right)\frac{\partial}{\partial x_j}(U_i+u_i')}=\overline{U_j\frac{\partial U_i}{\partial x_j}+u_j'\frac{\partial U_i}{\partial x_j}+U_j\frac{\partial u_i'}{\partial x_j}+u_j'\frac{\partial u_i'}{\partial x_j}}$$

The terms $u_j'\frac{\partial U_i}{\partial x_j}+U_j\frac{\partial u_i'}{\partial x_j}$ average to 0. We can also write

$$u_j'\frac{\partial u_i'}{\partial x_j}=\frac{\partial}{\partial x_j}u_i'u_j'-u_i\frac{\partial u_j}{\partial x_j}=\frac{\partial}{\partial x_j}u_i'u_j'$$

because

$$\frac{\partial u_j'}{\partial x_j}=0$$

as shown earlier.

So the convective derivative is reduced to:

$$\overline{\left(U_j+u_j'\right)\frac{\partial}{\partial x_j}(U_i+u_i')}=\bar{u}_j\frac{\partial\bar{u}_i}{\partial x_j}+\frac{\partial}{\partial x_j}\overline{u_i'u_j'}$$

2.2.2 The RANS momentum equation

Finally, we obtain the the Reynolds Averaged Navier-Stokes (RANS) equations

$$\bar{u}_j\frac{\partial\bar{u}_i}{\partial x_j}=-\frac{1}{\rho}\frac{\partial\,\bar{p}}{\partial x_i}+\nu\frac{\partial^2\bar{u}_i}{\partial x_j\partial x_j}-\frac{\partial}{\partial x_j}\overline{u_i'u_j'}$$

We notice that the RANS equation is formally quite similar to the initial Navier-Stokes equation for a steady flow – it misses the time derivative – but it also contains an additional term, which is the divergence operator applied to the tensor: $\overline{u_i'u_j'}$. This tensor is called **Reynolds stress** tensor and it represents the average effect of turbulent fluctuations on the transport of momentum.

The Reynolds stress represents the effect of turbulence on transport of momentum

- In full:

$$U\frac{\partial U}{\partial x}+V\frac{\partial U}{\partial y}+W\frac{\partial U}{\partial z}=-\frac{1}{\rho}\frac{\partial \bar{p}}{\partial x}+\nu\left(\frac{\partial^2 U}{\partial x^2}+\frac{\partial^2 U}{\partial y^2}+\frac{\partial^2 U}{\partial z^2}\right)-\frac{\partial \overline{u'^2}}{\partial x}-\frac{\partial \overline{u'v'}}{\partial y}-\frac{\partial \overline{u'w'}}{\partial z}$$

$$U\frac{\partial V}{\partial x}+V\frac{\partial V}{\partial y}+W\frac{\partial V}{\partial z}=-\frac{1}{\rho}\frac{\partial \bar{p}}{\partial y}+\nu\left(\frac{\partial^2 V}{\partial x^2}+\frac{\partial^2 V}{\partial y^2}+\frac{\partial^2 V}{\partial z^2}\right)-\frac{\partial \overline{u'v'}}{\partial x}-\frac{\partial \overline{v'^2}}{\partial y}-\frac{\partial \overline{v'w'}}{\partial z}$$

$$U\frac{\partial W}{\partial x}+V\frac{\partial W}{\partial y}+W\frac{\partial W}{\partial z}=-\frac{1}{\rho}\frac{\partial \bar{p}}{\partial x}+\nu\left(\frac{\partial^2 W}{\partial x^2}+\frac{\partial^2 W}{\partial y^2}+\frac{\partial^2 W}{\partial z^2}\right)-\frac{\partial \overline{u'w'}}{\partial x}-\frac{\partial \overline{v'w'}}{\partial y}-\frac{\partial\, \overline{w'^2}}{\partial z}$$

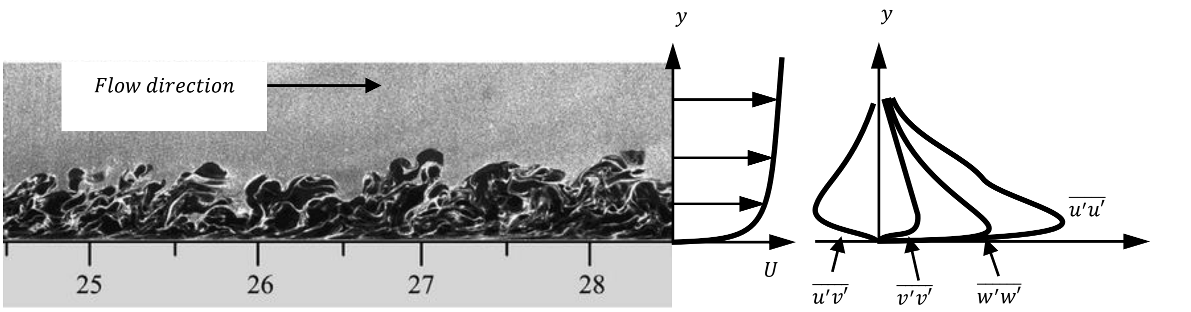


Figure 7: a boundary layer and the distribution of the non-zero components of the Reynolds stress.

Figure 7 shows a side view of a typical turbulent boundary layer and, with it, a typical distribution of the non-zero components of the Reynolds stress tensor. The part of the flow where the Reynolds stresses are non-zero is the part of the flow where we see turbulent eddies in the visualization. The presence of the eddies induces a correlation between the wall-normal and streamwise velocity fluctuations that results in a negative Reynolds shear stress $\overline{u'v'}$. The Reynolds shear stress acts “generally” - but not universally – in the direction of the negative velocity gradient. We will discuss this mechanism more in detail later on in the module. The presence of the turbulent eddies also causes the normal stresses $\overline{u'u'}, \overline{v'v'}$ and $\overline{w'w'}$ to be non-zero, as these are just the variance of the velocity components. Note that if the flow is statistically two-dimensional – i.e. mean flow is not a function of z and $W = 0$ (z axis goes into the page in Figure 7) then the stresses $\overline{u'w'}$ and $\overline{u'w'}$ must also be zero.

2.3 The kinetic energy of the mean flow

When studying correlations and turbulence spectra we saw that the spectrum represents how the energy of turbulence is distributed across a range of time- and length-scales.

We now turn our attention to deriving transport equations for the kinetic energy of the mean flow and the kinetic energy of turbulence. We derive these equations to identify the mechanism by which turbulence drains energy from the mean flow.

In order to derive an equation for the kinetic energy of the mean flow we start from the RANS equation

$$U_j \frac{\partial U_i}{\partial x_j} = -\frac{1}{\rho} \frac{\partial P}{\partial x_i} + \nu \frac{\partial^2 U_i}{\partial x_j \partial x_j} - \frac{\partial}{\partial x_j} \overline{u_i' u_j'}$$

Taking the product by the mean velocity component $\bar{u}_i$ and summing over the subscript i yields:

$$U_i U_j \frac{\partial U_i}{\partial x_j} = -\frac{1}{\rho} U_i \frac{\partial P}{\partial x_i} + U_i \nu \frac{\partial^2 U_i}{\partial x_j \partial x_j} - U_i \frac{\partial}{\partial x_j} \overline{u_i' u_j'}$$

In the following we will apply the following identity – due to the product rule:

$$U_i \frac{\partial b_{ij}}{\partial x_j} = \frac{\partial}{\partial x_j} U_i b_{ij} - b_{ij} \frac{\partial U_i}{\partial x_j}$$

where b_{ij} is a tensor.

Applying the identity above to the terms containing the viscous stress gives:

$$U_i \frac{\partial^2 U_i}{\partial x_j \partial x_j} = \frac{\partial}{\partial x_j} U_i \frac{\partial U_i}{\partial x_j} - \frac{\partial U_i}{\partial x_j} \frac{\partial U_i}{\partial x_j} = \frac{\partial^2 K}{\partial x_j \partial x_j} - \frac{\partial U_i}{\partial x_j} \frac{\partial U_i}{\partial x_j}$$

Applying the same identity to the terms containing the Reynolds stress gives:

$$U_i \frac{\partial \overline{u_i' u_j'}}{\partial x_j} = \frac{\partial}{\partial x_j} U_i \overline{u_i' u_j'} - \overline{u_i' u_j'} \frac{\partial U_i}{\partial x_j}$$

By denoting the kinetic energy of the mean flow as k

$$K = \frac{1}{2} U_i U_i$$

We can write

$$U_j \frac{\partial K}{\partial x_j} = -\frac{1}{\rho} U_i \frac{\partial P}{\partial x_i} + \nu \frac{\partial^2 K}{\partial x_j \partial x_j} - \nu \frac{\partial U_i}{\partial x_j} \frac{\partial U_i}{\partial x_j} - \frac{\partial}{\partial x_j} U_i \overline{u_i' u_j'} + \overline{u_i' u_j'} \frac{\partial U_i}{\partial x_j}$$

We make the following observation about the role of terms containing divergence-like expressions and terms containing the scalar product of a tensor by the velocity gradient.

Divergence-like terms represent transport mechanisms: application of Gauss' theorem shows that these terms cannot change the total amount of kinetic energy in the flow domain, unless there are fluxes at the boundary.

Product-like terms must represent creaction/destruction mechanisms as these terms can have a finite integral over the flow domain whether or not they are active at the boundaries.

We can therefore put forward the following interpretation of the terms in the mean-flow kinetic energy budget:

$$\underbrace{U_j \frac{\partial K}{\partial x_j}}_{\text{Convection}} = \underbrace{-\frac{1}{\rho} U_i \frac{\partial P}{\partial x_i}}_{\text{Bernoulli}} \underbrace{+ \nu \frac{\partial^2 K}{\partial x_j \partial x_j}}_{\substack{\text{Diffusion}\\ \text{(viscous)}}} \underbrace{- \nu \frac{\partial U_i}{\partial x_j} \frac{\partial U_i}{\partial x_j}}_{\substack{\text{Dissipation}\\ \text{(viscous)}}} \underbrace{- \frac{\partial}{\partial x_j} U_i \overline{u_i' u_j'}}_{\substack{\text{Diffusion}\\ \text{(turbulent)}}} \underbrace{+ \overline{u_i' u_j'} \frac{\partial U_i}{\partial x_j}}_{?}$$

We leave the interpretation of the term

$$\overline{u_i' u_j'} \frac{\partial U_i}{\partial x_j}$$

open for now and defer it to the discussion of the TKE budget equation.

2.4 The kinetic energy of turbulence (TKE)

The TKE (Turbulent Kinetic Energy) is the quantity

$$k = \frac{1}{2} \overline{u_i' u_i'}$$

The first step in deriving the TKE budget is to subtract the RANS momentum equation from the instantaneous momentum equation

$$\frac{\partial u_i'}{\partial t} + \left(U_j + u_j'\right) \frac{\partial}{\partial x_j} (U_i + u_i') = -\frac{1}{\rho} \frac{\partial}{\partial x_i} (P + p') + \nu \frac{\partial^2}{\partial x_j \partial x_j} (U_i + u_i')$$

$$U_j \frac{\partial U_i}{\partial x_j} + \frac{\partial}{\partial x_j} \overline{u_i' u_j'} = -\frac{1}{\rho} \frac{\partial P}{\partial x_i} + \nu \frac{\partial^2 U_i}{\partial x_j \partial x_j}$$

to obtain a momentum equation for the fluctuating velocity components only:

$$\frac{\partial u_i'}{\partial t} + u_j' \frac{\partial U_i}{\partial x_j} + U_j \frac{\partial u_i'}{\partial x_j} + \frac{\partial}{\partial x_j} u_j' u_i' - \frac{\partial}{\partial x_j} \overline{u_j' u_i'} = -\frac{1}{\rho} \frac{\partial p'}{\partial x_i} + \nu \frac{\partial^2 u_i'}{\partial x_j \partial x_j}$$

The next step is to take the scalar product by u_i'

$$u_i' \frac{\partial u_i'}{\partial t} + u_i' u_j' \frac{\partial U_i}{\partial x_j} + u_i' U_j \frac{\partial u_i'}{\partial x_j} + u_i' \frac{\partial}{\partial x_j} u_j' u_i' - u_i' \frac{\partial}{\partial x_j} \overline{u_j' u_i'} = -\frac{1}{\rho} u_i' \frac{\partial p'}{\partial x_i} + \nu u_i' \frac{\partial^2 u_i'}{\partial x_j \partial x_j}$$

and to finally take an average in time:

$$\overline{u_i u_j'}\frac{\partial U_i}{\partial x_j} + U_j\frac{\partial k}{\partial x_j} + \frac{\partial}{\partial x_j}\,\overline{u_j' u_i' u_i'} = -\frac{1}{\rho}\,\overline{u_i'\frac{\partial p'}{\partial x_i}} + \nu\frac{\partial^2 k}{\partial x_j \partial x_j} - \nu\,\overline{\frac{\partial u_i'}{\partial x_j}\frac{\partial u_i'}{\partial x_j}}$$

Note that we have performed similar manipulations to those performed deriving the mean flow kinetic energy budget.

2.4.1 Production and dissipation of TKE

We can write the two equations side by side and we notice that they have very similar structures.

$$U_j\frac{\partial K}{\partial x_j} = -\frac{1}{\rho}U_i\frac{\partial P}{\partial x_i} + \nu\frac{\partial^2 K}{\partial x_j \partial x_j} - \nu\frac{\partial U_i}{\partial x_j}\frac{\partial U_i}{\partial x_j} - \frac{\partial}{\partial x_j}U_i\overline{u_i' u_j'} + \overline{u_i' u_j'}\frac{\partial U_i}{\partial x_j}$$

Convection | Bernoulli | Diffusion (viscous) | Dissipation (viscous) | Diffusion (turbulent)

$$U_j\frac{\partial k}{\partial x_j} = -\frac{1}{\rho}\,\overline{u_i'\frac{\partial p'}{\partial x_i}} + \nu\frac{\partial^2 k}{\partial x_j \partial x_j} - \nu\,\overline{\frac{\partial u_i'}{\partial x_j}\frac{\partial u_i'}{\partial x_j}} - \frac{\partial}{\partial x_j}\,\overline{u_j' u_i' u_i'} - \overline{u_i u_j'}\frac{\partial U_i}{\partial x_j}$$

and that very similar mechanisms are at play: convection, turbulent and viscous diffusion, viscous dissipation. We also notice that the term

$$\Pi = \overline{u_i' u_j'}\frac{\partial U_i}{\partial x_j}$$

appears in both equations but with opposite sign. In the brief discussion at the end of the RANS section we have mentioned in passing that the Reynolds stress is "generally" but not universally acting in the direction opposite the velocity gradient. If this is the case then the term Π acts as a sink for the kinetic energy of the mean flow and as a source for the TKE.

The importance of this observation cannot overstated. We have identified the mechanism by which turbulence drains energy from the mean flow: work done by the Reynolds stress against the mean velocity gradients. The first important consequence we can draw from this observation is that it is not possible to sustain turbulence in a uniform mean flow. Turbulence needs velocity gradients to extract energy from the mean flow and sustain itself.

The energy drawn by turbulence from the mean flow is ultimately destroyed by viscous dissipation through the term:

$$\varepsilon = \nu\,\overline{\frac{\partial u_i'}{\partial x_j}\frac{\partial u_i'}{\partial x_j}}$$

In many flows of interest the TKE production and dissipation rates are similar. Such flows are called equilibrium flows.

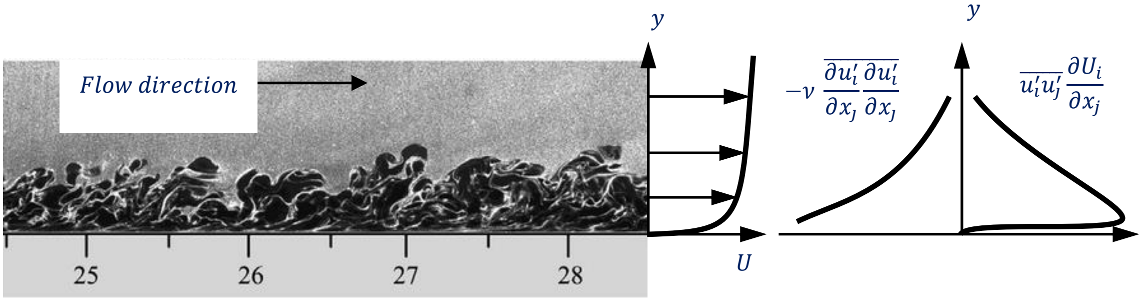


Figure 8: typical production and dissipation rate profiles in a turbulent boundary layer.

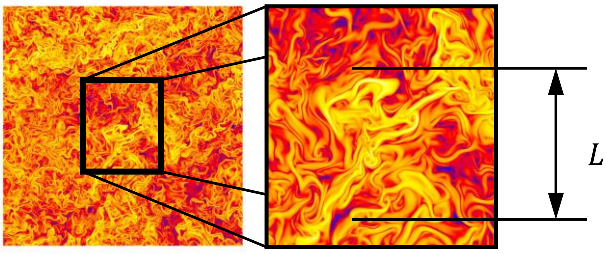


Figure 9: size of turbulent eddies.

2.5 Orders of magnitude of energy budget terms

We examine the mean flow kinetic energy budget:

$$U_j \frac{\partial K}{\partial x_j} = -\frac{1}{\rho} U_i \frac{\partial P}{\partial x_i} + \nu \frac{\partial^2 K}{\partial x_j \partial x_j} - \nu \frac{\partial U_i}{\partial x_j}\frac{\partial U_i}{\partial x_j} - \frac{\partial}{\partial x_j} U_i \overline{u_i' u_j'} + \overline{u_i' u_j'} \frac{\partial U_i}{\partial x_j}$$

and we postulate that the turbulent eddies in the flow have size L (see Figure 9) and RMS velocity fluctuations u'. Then the following order-of-magnitude estimates must hold:

$$\Pi = \overline{u_i' u_j'} \frac{\partial U_i}{\partial x_j} \approx \frac{{u'}^3}{L}$$

$$\varepsilon = \nu \frac{\partial U_i}{\partial x_j}\frac{\partial U_i}{\partial x_j} \approx \frac{\nu {u'}^2}{L^2}$$

The ratio Π/ε is a Reynolds number:

$$\frac{\Pi}{\varepsilon} \approx \frac{u' L}{\nu} = Re_t$$

Re_t is called the turbulence Reynolds number. We notice that at sufficiently high Re_t viscous dissipation is not the main sink for the mean flow kinetic energy. This role is fulfilled by the TKE production.

3 The RANS and the TKE equation: checklist

- Reynolds decomposition applied to NS equations yields the RANS equations
- RANS equations describe behaviour of the mean flow
- Momentum flux due to turbulence appears as the Reynolds stress tensor: $\overline{u_i' u_j'}$
- Scalar product of the RANS momentum equation by the mean velocity gives an equation for the kinetic energy of the mean flow
- Scalar product of the fluctuating component of momentum by the fluctuating velocity gives an equation for the kinetic energy of turbulence
- The term $\overline{u_j' u_i'} \frac{\partial U_j}{\partial x_i}$ appears both in TKE and mean flow KE equations with opposite sign: this term measures the production of TKE at the expense of mean flow KE
- At high enough Reynolds numbers, turbulence production is the main sink of mean flow KE
- Make sure you are familiar with the derivations!