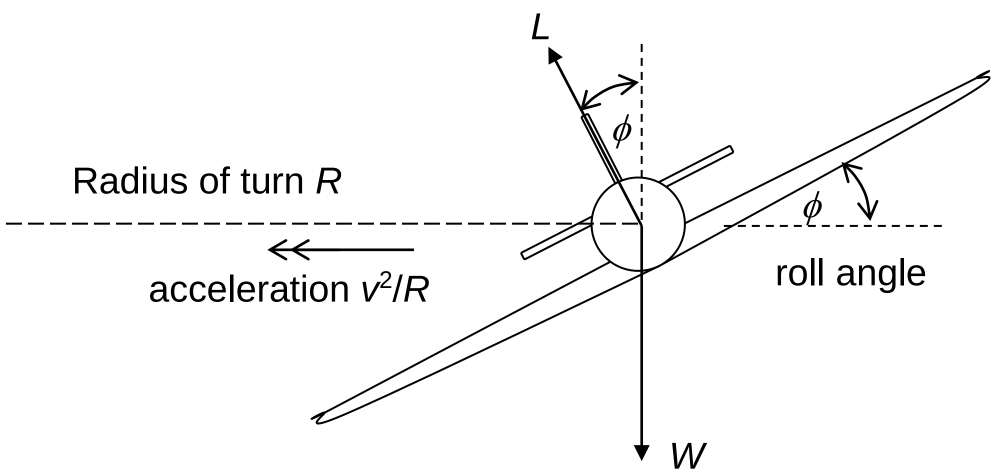
C2 Aircraft Flight and Propulsion – Lecture 4

Prof L. He 4 lectures

Turning Flight

Consider an aircraft in a steady turning flight, banked at an angle ϕ and flying at speed v. The Lift vector is perpendicular to the aircraft and has to balance the weight and provide the centripetal acceleration around the circle.



Resolving vertically: $W = L\cos\phi$

Resolving horizontally: $L\sin\phi = m\dfrac{v^2}{R} = \dfrac{v^2}{Rg}W$

Hence: $$\tan\phi = \frac{v^2}{Rg} \qquad (4\text{-}1)$$

For a given radius, the roll angle increases as velocity increases – more bank is required.

Load Factor

The load factor is defined as:

$$n = \frac{L}{W} = \sec\phi \qquad (4\text{-}2)$$

The forces on the structure will be increased by a factor n compared to level flight, and the pilot will feel n times heavier. The maximum load factor n_{max} is one of the design criteria for the structure of an aircraft.

$$\frac{v^2}{Rg} = \tan\phi = \sqrt{\sec^2\phi - 1}$$

$$\frac{v^2}{Rg} = \sqrt{n^2 - 1} \qquad (4\text{-}3)$$

For a given radius, the velocity is limited by the maximum load factor.

The lift coefficient required for a level turn is given by

$$L = nW = \frac{1}{2}\rho v^2 S C_L$$

The speed in the turn v_t is given by $v_t = \sqrt{\dfrac{2nW}{\rho S C_L}}$, which is $\sqrt{n}$ times the speed required in level flight at the same lift coefficient $v = \sqrt{\dfrac{2W}{\rho S C_L}}$ $\rightarrow$

$$v_t = \sqrt{n}\, v \qquad (4\text{-}4)$$

The aircraft stalls when $C_L = C_{L\,max}$, so the stalling speed increases by the factor $\sqrt{n}$. Example, $\phi = 60^o$, $n = \sec^{-1}(60^o) = 2$ $\rightarrow$ $v_{stt} = 1.41\, v_{st}$ (where v_{stt} is the stalling velocity in a turn and v_{st} is the stalling velocity in level flight).

<u>Minimum Radius of Turn</u>

The minimum radius of turn is limited by the minimum speed, i.e. the speed at which the aircraft will stall, and the maximum load factor n permitted.

$$R = \frac{v^2}{g\sqrt{n^2 - 1}}$$

If n is fixed, then
$$R_{\min} = \frac{v_{stt}^2}{g\sqrt{n_{\max}^2 - 1}} = \frac{v_{st}^2}{g}\frac{n_{\max}}{\sqrt{n_{\max}^2 - 1}} \qquad (4\text{-}5)$$

Example: a sailplane has v_{st} = 70 km/hr (= 19.4 m/s), and the pilot gets uncomfortable for n > 3 (i.e. "3 g's").

$$R_{\min} = \frac{19.4^2}{9.81}\frac{3}{\sqrt{9-1}} = 40.7\,\text{m!}$$

The angle of bank for this is $\phi = \cos^{-1}\left(\frac{1}{3}\right) = 70^{\text{o}}$!

Thrust required for a Level Turn

Flying in a turn at constant C_L and hence constant C_D:

$$L = nW$$

$$v_t = \sqrt{n}\,v$$

The lift-to-drag ratio is constant since $\frac{L}{D} = \frac{C_L}{C_D}$ hence the drag flying in a level turn is increased by a factor n higher than the drag in level flight. The thrust is also therefore increased by n $\rightarrow$ $(T_r)_t = nT_r$.

The minimum required thrust still occurs at $\left(\frac{C_L}{C_D}\right)_{\max}$, but the minimum drag speed in the turn will be $\sqrt{n}$ higher than in level flight .

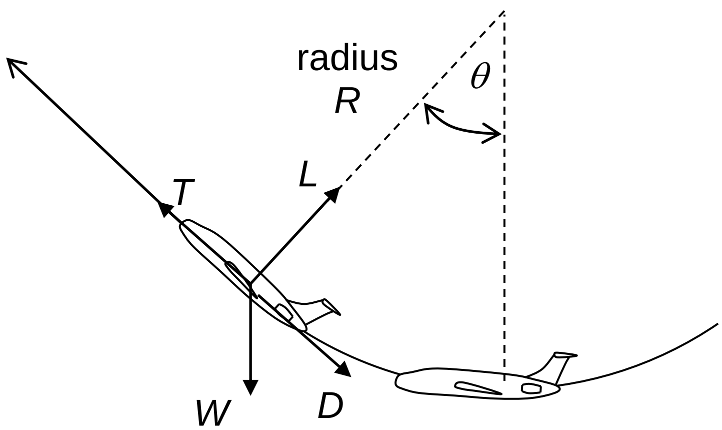
Power Required in a Level Turn

Flying at constant C_L and C_D, $v_t = \sqrt{n}\,v$ and $(T_r)_t = nT_r$, so the power required in turn will be a factor $n^{3/2}$ greater than the power required in level flight:

$$(P_r)_t = (T_r)_t\, v_t = nT_r\,\sqrt{n}\,v = n^{3/2}P_r\,. \qquad (4\text{-}6)$$

The minimum power in a level turn will still occur at $\left(\frac{C_L^{3/2}}{C_D}\right)_{\max}$ as for level flight.

Pull-Ups



$$\nearrow\!\!\swarrow \quad L - W\cos\theta = m\frac{v^2}{R} = \frac{v^2}{Rg}W$$

$$\nwarrow\!\!\searrow \quad T - D = W\sin\theta$$

Load factor is still defined as $n = L/W$,

$$W(n - \cos\theta) = \frac{v^2}{Rg}W$$

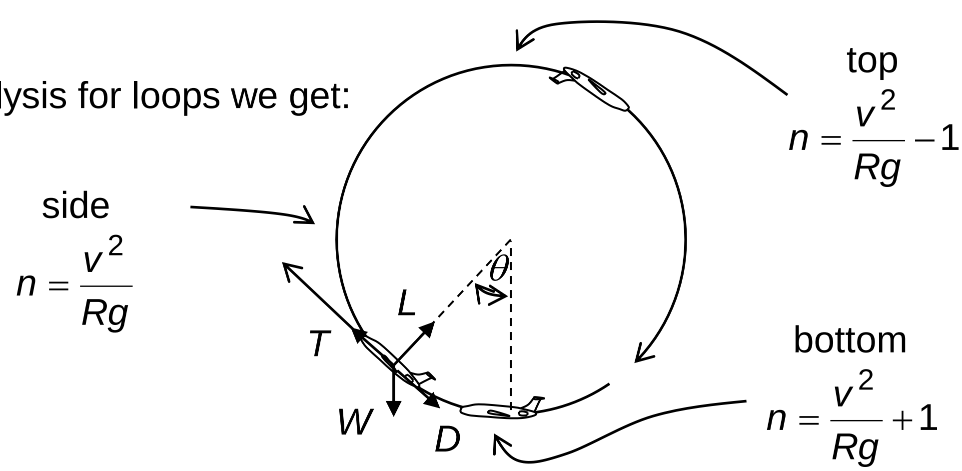
$$n = \frac{v^2}{Rg} + \cos\theta \qquad (4\text{-}7)$$

Maximum load factor will occur at the bottom of the pull-up when $\cos\theta = 1$:

$$n = \frac{v^2}{Rg} + 1$$

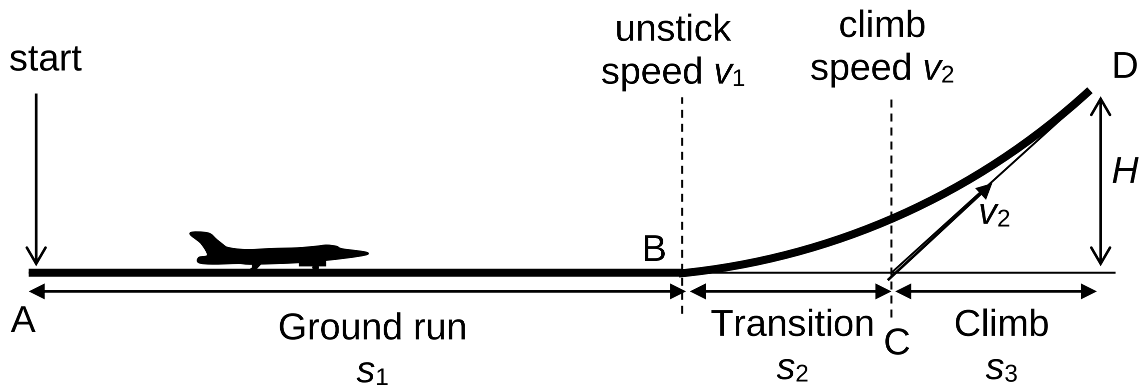
Loops

Following the same analysis for loops we get:



Take-Off Distance

Take off distance s is defined as the horizontal distance required to clear an obstruction of height H from a standing start. H is often set to be 50ft or 15m. There are three stages: ground run, transition, and climb.



1. During the ground run AB, the aircraft accelerates along the runway. As the velocity increases, the lift generated increases, and the unstick point is defined as the point at which the lift is sufficient to raise the aircraft off the ground. The corresponding velocity is the unstick speed v_1, and is usually around $1.1 \times v_{stall}$.

The curved path BD is approximated by:

2. a transition BC where the aircraft accelerates just above the ground from v_1 to a safe climb speed v_2, which is approximately $1.2 \times v_{stall}$; and
3. a constant climb CD at velocity v_2 and climb angle γ.

Note that books vary with their approximations used to calculate $s_1 + s_2 + s_3$.

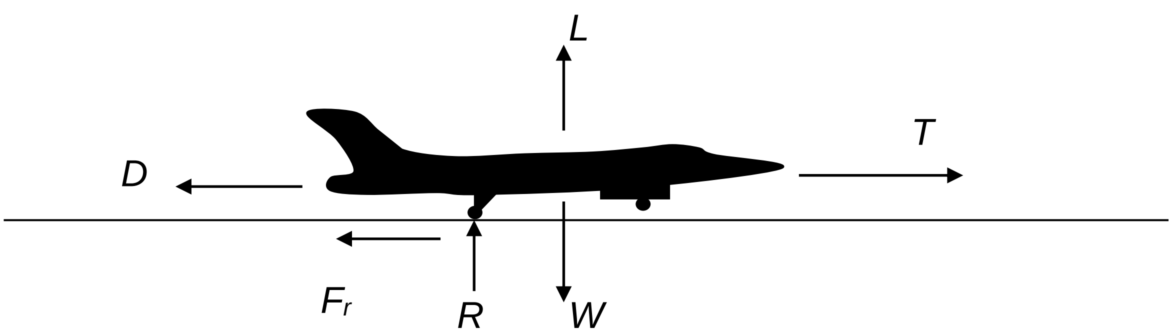
Ground Run

The rolling friction of wheels on the surface is given by:

$$F_r = \mu_r R = \mu_r (W - L)$$

where μ_r is the coefficient of resistance and R is the normal reaction (= $W - L$). Typical values for μ_r are:

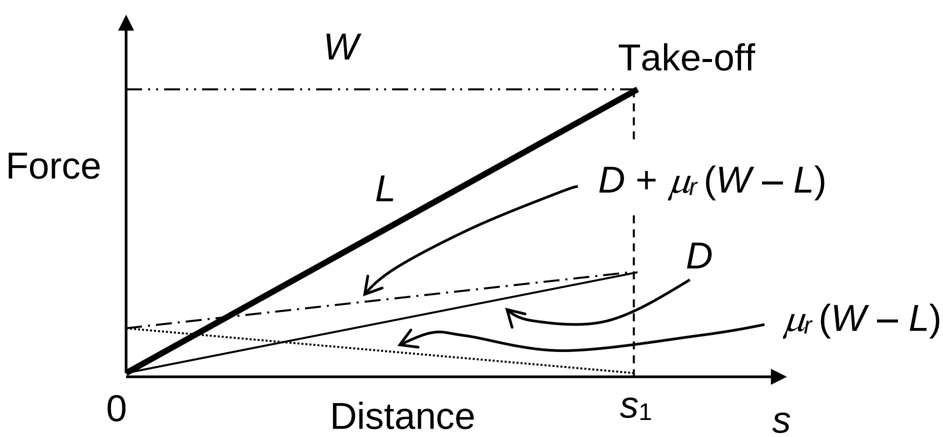
Surface	μ_r
Paved	0.02
Short grass	0.05
Long wet grass	0.13



The acceleration is given by:

$$T - D - \mu_r (W - L) = m\frac{dv}{dt} = \frac{W}{g} v \frac{dv}{ds} \tag{4-8}$$

The above forces can vary during take-off:



The thrust T can be considered constant. Drag is given by:

$$D = \frac{1}{2}\rho v^2 S C_D = \frac{1}{2}\rho v^2 S\left(C_{D0} + \phi\frac{k}{\pi A}C_L^2\right)$$

where ϕ is a ground effect factor due to a reduced induced drag factor k close to the ground. As an approximation:

$$\phi = \frac{\left(\frac{16h}{b}\right)^2}{1+\left(\frac{16h}{b}\right)^2} \tag{4-9}$$

where h = height of wing above ground, and b = wingspan.

The lift is given by $L = \frac{1}{2}\rho v^2 S C_L$.

Assume that the lift coefficient is held at its value for the unstick speed C_{L1}.

Noting that:

$$v\frac{dv}{ds} = \frac{d}{ds}\left(\frac{v^2}{2}\right)$$

the acceleration equation becomes:

$$\frac{W}{g}\frac{d}{ds}\left(\frac{v^2}{2}\right) = T - D - \mu_r(W - L)$$

Substituting for the drag, with $C_L = C_{L1}$:

$$\frac{W}{g}\frac{d}{ds}\left(\frac{v^2}{2}\right) = T - \frac{1}{2}\rho v^2 S\left(C_{D0} + \phi\frac{k}{\pi A}C_{L1}^2\right) - \mu_r(W - L)$$

$$\frac{W}{g}\frac{d}{ds}\left(\frac{v^2}{2}\right) = (T - \mu_r W) - \frac{1}{2}\rho v^2 S\left(C_{D0} + \phi\frac{k}{\pi A}C_{L1}^2 - \mu_r C_{L1}\right)$$

$$\frac{W}{g}\frac{d}{ds}\left(\frac{v^2}{2}\right) = a_1 - a_2\left(\frac{v^2}{2}\right)$$

Solve by integration:

$$s_1 = \int_0^{s1} ds = \int_0^{v1} \frac{\frac{W}{g} d\left(\frac{v^2}{2}\right)}{\left(a_1 - a_2\left(\frac{v^2}{2}\right)\right)}$$

$$s_1 = -\frac{W}{g a_2}\left[\ln\left(a_1 - a_2\left(\frac{v^2}{2}\right)\right)\right]_0^{v1}$$

Therefore the ground run is given by:

$$s_1 = -\frac{W}{g a_2} \ln\left(1 - \frac{a_2}{a_1}\left(\frac{v_1^2}{2}\right)\right) \qquad \text{(4-10)}$$

where $a_1 = (T - \mu_r W)$, and $a_2 = \rho S\left(C_{D0} + \phi \frac{k}{\pi A} C_{L1}^2 - \mu_r C_{L1}\right)$

<u>Example:</u>

Calculate the ground run for an Airbus A300 with T = 500 kN, W = 1.2 MN, b = 45 m, h = 4m, C_{D0} = 0.02, k = 1.3, C_L = 1.0, S = 260 m^2, μ_r = 0.02, and ρ = 1.225 kg/m^3.

Aspect ratio: $A = \frac{b^2}{S} = \frac{45^2}{260} = 7.8$

Ground effect factor: $\phi = \left(\frac{16h}{b}\right)^2 \Big/ \left(1 + \left(\frac{16h}{b}\right)^2\right) = 0.67$

$$a_1 = (T - \mu_r W) = 500 \times 10^3 - 0.02 \times 1.2 \times 10^6 = 476\,\text{kN}$$

$$a_2 = \rho S\left(C_{D0} + \phi \frac{k}{\pi A} C_{L1}^2 - \mu_r C_{L1}\right)$$

$$a_2 = 1.225 \times 260\left(0.02 + 0.67 \frac{1.3}{\pi 7.8} 1^2 - 0.02 \times 1\right) = 11.3$$

Unstick speed $v_1 = \sqrt{\dfrac{2W}{\rho S C_{L1}}}$ =86.8 m/s

$$s_1 = -\frac{W}{g a_2}\ln\left(1-\frac{a_2}{a_1}\left(\frac{v_1^2}{2}\right)\right) = -\frac{1.2\times10^6}{9.81\times11.3}\ln\left(1-\frac{11.3}{476\times10^3}\left(\frac{86.8^2}{2}\right)\right)$$

$$-\frac{1.2\times10^6}{9.81\times11.3}\ln(1-0.089)$$

$$s_1 = 1014 \text{ m}$$

(Heathrow runways are ~ 3km).

<u>Approximation for Ground Run:</u>

Since $\dfrac{a_2}{a_1}\left(\dfrac{v_1^2}{2}\right) << 1$ use $\ln(1-x) \approx -x$

$$s_1 \approx \frac{W}{g a_2}\frac{a_2}{a_1}\left(\frac{v_1^2}{2}\right) = \frac{W}{g a_1}\left(\frac{v_1^2}{2}\right) = 986\,\text{m} \qquad (\text{~}4.5\%\text{ low} \ \rightarrow\ \text{ok})$$

$$s_1 \approx \frac{W\left(v_1^2/2\right)}{g(T_r - \mu_r W)} \tag{4-11}$$

Use this approximation to assess what factors affect the ground run:

Noting that $v_1^2/2 = \dfrac{W}{\rho S C_{L1}}$ $\rightarrow$ $s_1 \approx \dfrac{W^2}{g\rho S C_{L1}(T_r - \mu_r W)}$

Effect of altitude: take-off distance increases with altitude.

To minimize s_1:

- maximize S $\rightarrow$ large wing area
- maximise C_{L1} $\rightarrow$ flaps and leading-edge slats are used to increase C_L, but without increasing the drag too much
- maximise T $\rightarrow$ powerful engines
- μ_r $\rightarrow$ wet grass can cause problems for low-powered aircraft.

Alternative Approximation for the Ground Run:

Acceleration is given by:

$$T - D - \mu_r (W - L) = m\frac{dv}{dt} = \frac{W}{g} v \frac{dv}{ds}$$

Consider the overall force $T - D - \mu_r (W - L)$ to be constant at the value corresponding to $v_{av} = \dfrac{v_1}{\sqrt{2}}$ $\rightarrow$ so $D_{av} = \dfrac{D_1}{2}$ and $L_{av} = \dfrac{W}{2}$.

$$s_1 = \int_0^{s1} ds = \frac{W}{g} \int_0^{v1} \frac{v dv}{\left[T - D - \mu_r (W - L)\right]_{av}}$$

$$s_1 = \frac{W}{g} \frac{v_1^2}{2} \frac{1}{\left[T - \frac{D_1}{2} - \mu_r \left(W - \frac{W}{2}\right)\right]} \qquad (4\text{-}12)$$

Example: For the Airbus A300:

Using velocity, $v_{av} = \dfrac{v_1}{\sqrt{2}} = \dfrac{86.8}{\sqrt{2}} = 61.4$ m/s

$$D_{av} = \frac{1}{2} 1.225 \times 61.4^2 \times 260 \left(0.02 + 0.67 \frac{1.3}{\pi 7.8} 1^2\right) = 33.3\,\text{kN}$$

$$s_1 \approx \frac{1.2 \times 10^6}{9.81} \times \frac{86.8^2}{2} \times \frac{1}{\left[500 \times 10^3 - 33.3 \times 10^3 - 0.02\left(\frac{1.2 \times 10^6}{2}\right)\right]}$$

$s_1 \approx 1013$ m

$\rightarrow$ approximation is very accurate!

Transition:

The aircraft has left the ground so the frictional force is zero:

$$\frac{W}{g}\frac{d}{ds}\left(\frac{v^2}{2}\right) = T - D$$

During this phase, the aircraft is flying close to the minimum drag speed, and it is assumed that the thrust and the drag are both approximately constant at their unstick speed v_1 values.

$$s_2 = \int_0^{s2} ds = \frac{W}{g}\int_{v1}^{v2}\frac{d\left(v^2/2\right)}{\left(T - D_1\right)}$$

$$s_2 = \frac{W}{2g}\frac{\left(v_2^2 - v_1^2\right)}{\left(T - D_1\right)} = \frac{Wv_1^2\left(\left(\frac{v_2}{v_1}\right)^2 - 1\right)}{2g\left(T - D_1\right)} \qquad (4\text{-}13)$$

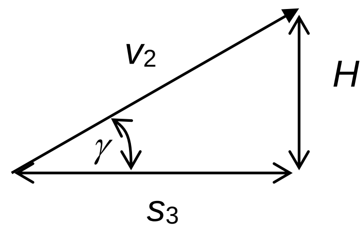
where $D_1 = \frac{1}{2}\rho v_1^2 S\left(C_{D0} + \phi\frac{k}{\pi A}C_{L1}^2\right)$. Aircraft flies just above the ground so the ground effect factor is still used to reduce the induced drag term.

Example: For the Airbus A300 with $v_2 = 1.2v_{stall}$:

$$D_1 = \frac{1}{2}1.225 \times 86.8^2 \times 260\left(0.02 + 0.67\frac{1.3}{\pi 7.8}1^2\right) = 66.6 \text{ kN}$$

$$s_2 = \frac{1.2 \times 10^6 \times 86.8^2\left(\left(\frac{1.2}{1.1}\right)^2 - 1\right)}{2 \cdot 9.81(500 - 66.6) \times 10^3} = 202 \text{ m}$$

Climb



Steady climb at velocity v_2 out of the ground effect.

From Lecture 3: $\sin\gamma = \dfrac{(T - D_2)}{W}$

$$s_3 = H\cot\gamma = H\sqrt{\frac{1}{\sin^2\gamma} - 1} = H\sqrt{\left(\frac{W}{T - D_2}\right)^2 - 1} \qquad (4\text{-}14)$$

Example: For the Airbus A300:

$$C_{L2} = C_{L1}\left(\frac{V_1}{v_2}\right)^2 = 0.84\,,\; v_2 = \sqrt{\frac{2W}{\rho S C_{L2}}} = 94.7 \text{ m/s}$$

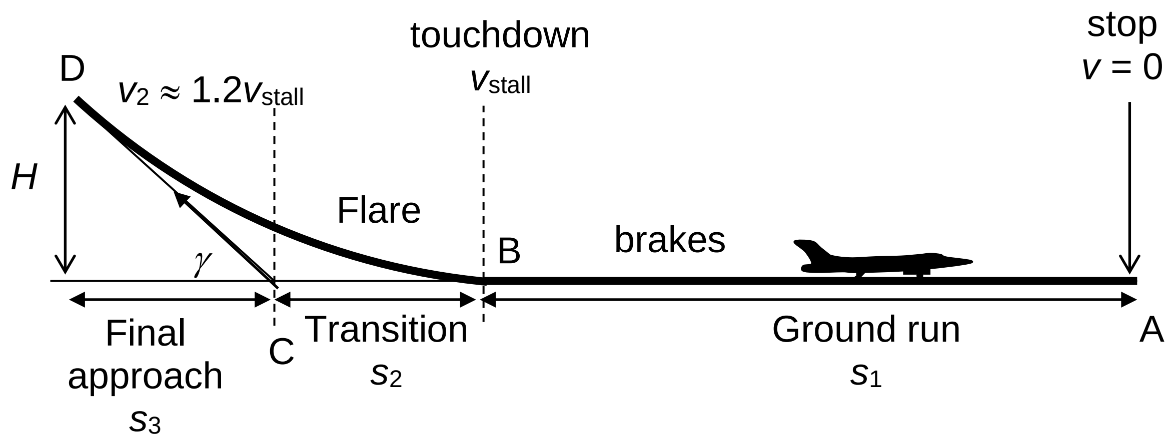
$$D_2 = \frac{1}{2}1.225\times 94.7\times 260\left(0.02 + \frac{1.3}{\pi 7.8}0.84^2\right) = 82 \text{ kN}$$

$$s_3 = 15\sqrt{\left(\frac{1.2\times 10^6}{(500 - 82)\times 10^3}\right)^2 - 1} = 40\text{m}, \quad \gamma = 20^{\circ} \rightarrow \text{steep}$$

Take-off distance for the Airbus A300 = $s_1 + s_2 + s_3$ = 1014 + 202 + 40 = 1256 m

Landing Distance

Landing is the reverse process to take-off (v_1 replaced by v_{stall}) :



Final approach: $$s_3 = H\sqrt{\left(\frac{W}{T - D_2}\right)^2 - 1} \qquad (4\text{-}15)$$

Transition: $$s_2 = \frac{W v_{stall}^2 \left(\left(\frac{v_2}{v_{stall}}\right)^2 - 1\right)}{2g\left(D_{stall} - T\right)} = \frac{W^2 \left(\left(\frac{v_2}{v_{stall}}\right)^2 - 1\right)}{g\rho S C_{Lstall}\left(D_{stall} - T\right)} \qquad (4\text{-}16)$$

Ground run: Use brakes with braking coefficient $\mu_b \approx 0.4$ for paved surface.

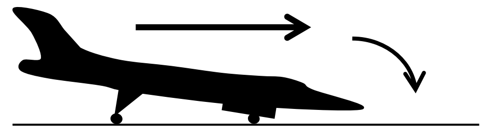
use approximation: $$s_1 = \frac{W v_{stall}^2}{2g\left[\left(D - T\right) + \mu_b\left(W - L\right)\right]_{av}}, \qquad (4\text{-}17)$$

where $v_{av} = \frac{v_{st}}{\sqrt{2}}$

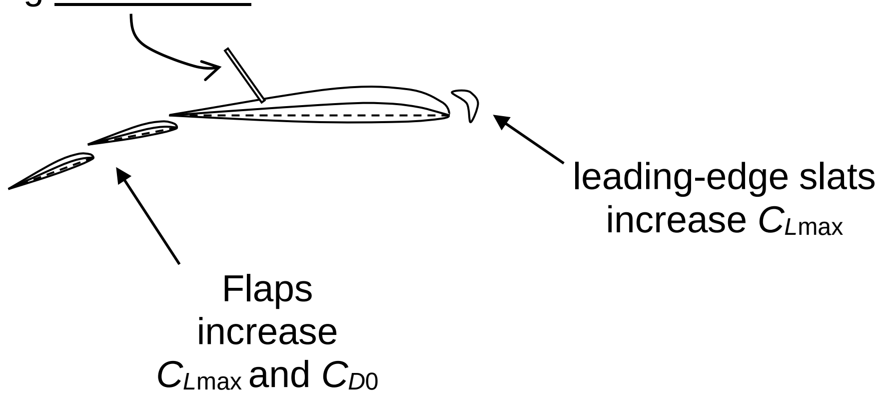
Minimisation of Landing Distance

1. Throttle back so that $T \approx 0$
2. Use large flaps and leading-edge slats to increase $C_{L\mathrm{stall}}$ and C_{D0}

   → this reduces v_{stall} and v_2 and increases γ.
3. Reduce lift after touchdown by lowering the nose:



   or by using aerobrakes:



4. Use reverse thrust $T = -T_R$ after touchdown. $T_R \approx 0.4T_{max}$.

Example: Landing distance for the Airbus A300

T = 500 kN, $W_{\text{take-off}}$ = 1.2 MN, b = 45 m, h = 4m, C_{D0} = 0.02, k = 1.3,

C_L = 1.0, S = 260 m^2, μ_r = 0.02, and ρ = 1.225 kg/m^3.

extra information: $C_{L\ \max}$ = 1.2 (clean), W = 900 kN (landing weight)

Final approach:
$$s_3 = H\sqrt{\left(\frac{W}{T - D_2}\right)^2 - 1}$$

Set thrust T = 0 kN

(i) Landing without Flaps etc but $T=0$

$$V_{stall} = \sqrt{\frac{2W}{\rho C_{Lst} S}} = \sqrt{\frac{2 \times 900 \times 10^3}{1{\cdot}227 \times 1{\cdot}2 \times 260}} = 69 \text{ m/s}$$

$$V_2 = 1{\cdot}2 \times 69 = 82 \text{ m/s}, \quad C_{L2} = 1{\cdot}2 \frac{1}{(1{\cdot}2)^2} = \underline{= 0{\cdot}83}$$

$$D_2 = \tfrac{1}{2}\rho V_2^2 S\left(C_{D0} + \frac{k C_{L2}^2}{\pi A}\right)$$
$$= \frac{1{\cdot}227 \times (82)^2}{2} 260\left(0{\cdot}02 + \frac{1{\cdot}3(0{\cdot}83)^2}{\pi \times 7{\cdot}8}\right)$$
$$= 51{\cdot}6 \text{ kN}$$

For $H = 15$ m, $$S_3 = 15\sqrt{\left(\frac{900 \times 10^3}{51{\cdot}6 \times 10^3}\right)^2 - 1}$$

$$S_3 = \underline{261 \text{ m}}\ !$$

Note $$\gamma = \sin^{-1}\frac{D_2}{W} = 3{\cdot}3^\circ$$ $\boxed{\text{Very Shallow approach!}}$

Transition ($\phi = 0.67$) due to ground effect)

at V_{st}, $D_{st} = \frac{1}{2}\rho V_{st}^2 S\left(C_{D0} + \phi \frac{K C_{Lst}^2}{\pi A}\right)$

$= 54\,kN \quad (\approx D_2 !)$ $\leftarrow 0.051$

Which gives $S_2 = \dfrac{900\times10^3(69)^2\left((1.2)^2-1\right)}{2\times9.81\times54\times10^3}$

$= \underline{1.78\ km}$ $\boxed{\text{Long Float!}}$

Ground Run

at $V_{av} = \frac{V_{st}}{\sqrt{2}} = 49\,m/s$,

$D = \frac{D_{st}}{2} = 27\,kN$, $W =$

$L = \frac{W}{2} = 450\,kN$

$s_1 = \dfrac{900\times10^3\times(69)^2}{2\times9.81\left(27\times10^3 + 0.4\times450\times10^3\right)}$

$= \underline{1055\ m}$

Thus, clean Landing Distance for A300

$S = 261 + 1780 + 1055 = \underline{3096\ m}$

This is much too long!

Reduce Landing Distance of A300

1 Flaps fully down give $C_{L_{st}} = 2.2$,
$C_{D0} = 0.04$, $V_{stall} = 69\sqrt{\frac{1.2}{2.2}} = 51\,m/s$
$V_2 = 61\,m/s$, $C_{L_2} = 1.53$

Giving $D_2 = 97.5\,kN$,
$S_3 = 137\,m$, $\gamma = 6.2^\circ$ (Steeper)

Transition $D_{st} = 88\,kN$
$S_2 = 597\,m$

Ground Run $s_1 = \dfrac{900\times10^3 \times (51)^2}{2\times9.81\left(\frac{88}{2}\times10^3 + 0.4\times450\times10^3\right)}$
$= 532\,m.$

So, with flaps $s = 1266\,m$ (Same as T.O. of 1250 m!)

2 Reduce lift: on touchdown $C_L = 0$, $D_{av} = \frac{1}{2}\rho \frac{V_{st}^2}{2} S\, C_{D0}$
$= 8.3\,kN$

The reduced drag is compensated by braking of $\mu_b W$ instead of $\mu_b \frac{W}{2}$ giving
$S_1 = \dfrac{900\times10^3 (51)^2}{2\times9.81\,(8.3\times10^3 + 0.4\times900\times10^3)}$
$= 324\,m$

Add Reverse Thrust $T_R = 0.4 \times 500 = 200\,kN$
$S_1 = 210\,m \longrightarrow s = 944\,m$ $\boxed{\text{Good}}$

For Propeller driven aircraft calcs. are similar. Assume $T =$ constant at take-off thrust.