Management Practice

14. Assignment in a global context

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MPiE

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Course

Literature for the course:

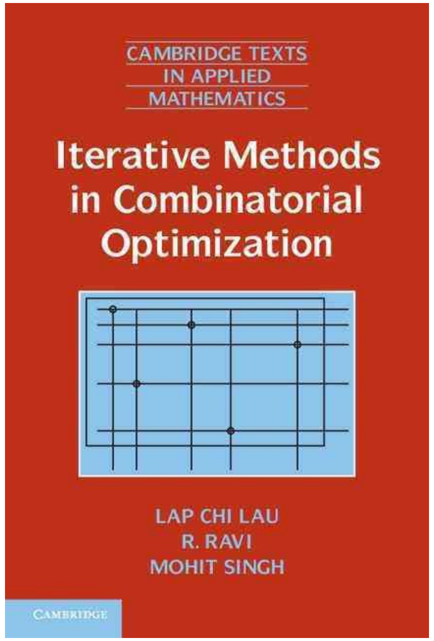
Eisner, Howard. *Essentials of project and systems engineering management*. John Wiley & Sons, 2008.

Learning objective for this session:

- Understand what an assignment problem is
- Able to solve a Integer linear constrained optimization problem
- Able to apply the Hungarian Method
- Able to discuss difference between assignments methods

Literature for this session:

Lap Chi Lau, Ramamoorthi Ravi, and Mohit Singh. *Iterative methods in combinatorial optimization*. Vol. 46. Cambridge University Press, 2011.



Assignment

Assignment is the allocation of a job or task to someone.

The assignment supports the matching of personnel to specific tasks or more generically, assigning jobs to machines.

There is a benefit if we can optimise the assignments against a certain cost parameter.

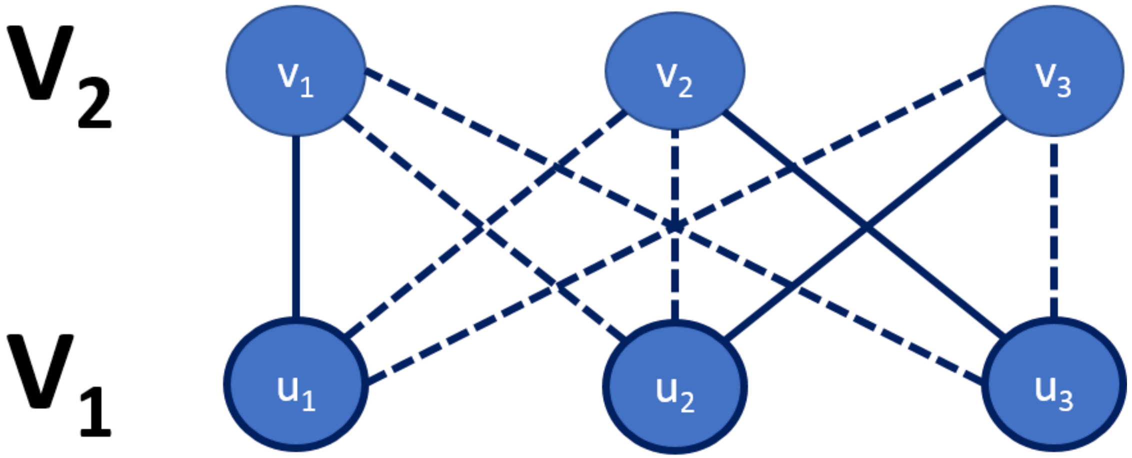
Assignment problem

- The assignment problem is one of the fundamental combinatorial optimization problems (finding in a finite set of objects the optimum) .
- Combinatorial optimization explores a finite (although countably infinite is also possible) set of potential solutions in search for an optimal solution.
- A set is countably infinite if its elements can be put in one-to-one correspondence with the set of natural numbers
- A criterion function that can be minimized or maximized can be used to define optimality.

Source: John and Lei, General assignment problem

Classic assignment problem

- Given a bipartite graph $G=(V_1 \cup V_2, E)$ and weight w, the objective is to match every vertex in V_1 with a distinct vertex in V_2 to minimize the total weight (cost) of the matching. This is also know as the minimum weight bipartite matching problem and is a fundamental problem in combinatorial optimisation.



Source: Lau et al, 2011

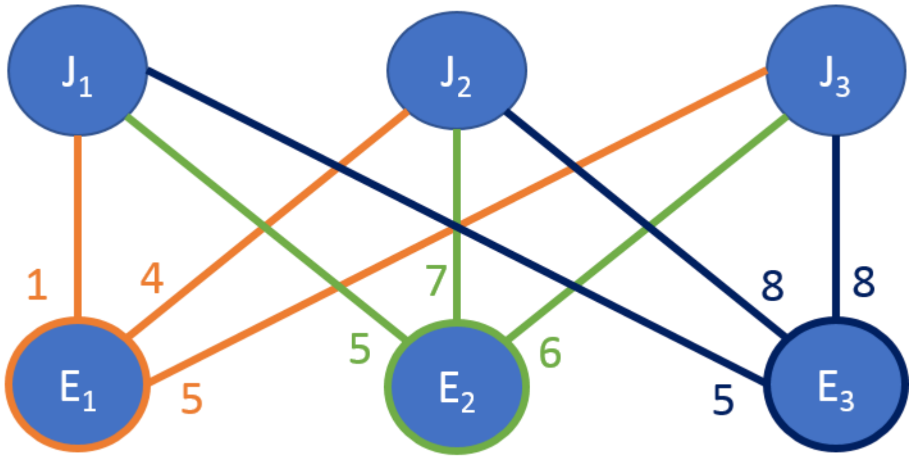
Example assignment problem

- 3 employees can be put on 3 jobs
- Each employee can only work on one job.
- The suitability of each employee for each of the jobs can be captured by a cost value. The cost will be **lower** if the employee is more suitable for that job.

Source: Shun Y. Cheung, 2012

Example assignment problem

- Find a maximum matching (assign jobs to as many employees as possible) for which the sum of the cost of the edges is minimized



Source: Shun Y. Cheung, 2012

Potential solution for assignment problem

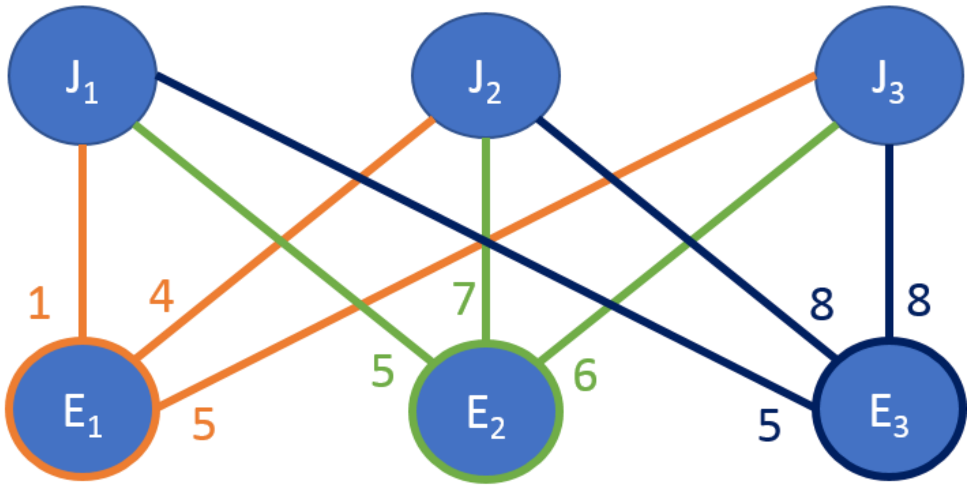
- Find all maximum matchings

{E_1→J_1, E_2→J_2, E_3→J_3}; {E_1→J_2, E_2→J_1, E_3→J_3}, …

- Sum the cost of the edges of each maximum matching

{16}; {17}, …

- Select the maximum matching with the lowest possible cost



Source: Shun Y. Cheung, 2012

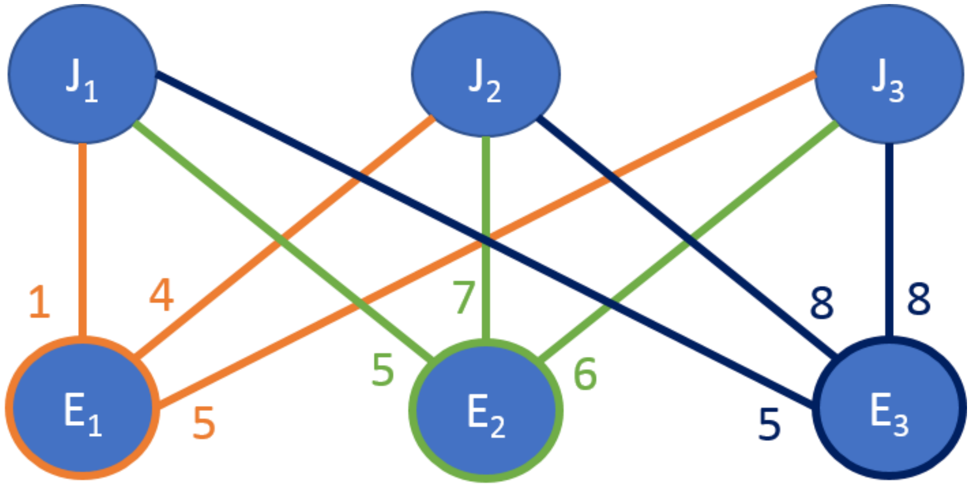
Optimal assignment

- An assignment is a set of *n* entry positions in the cost matrix, no two of which lie in the same row or column.
- The sum of the *n* entries of an assignment is its cost.
- An assignment with the smallest possible cost is called an optimal assignment.

Source: Shun Y. Cheung, 2012

Integer linear constrained optimization problem (IP)

- Set up cost matrix



C	J_1	J_2	J_3
E_1	1	4	5
E_2	5	7	6
E_3	5	8	8

Source: Shun Y. Cheung, 2012

Integer linear constrained optimization problem (IP)

C	J_1	J_2	J_3
E_1	1	4	5
E_2	5	7	6
E_3	5	8	8

- Cost matrix $C = [c_{ij}]$ where c_{ij} is the cost of Employee *i* working on Job *j*
- A variable x_{ij} is generated that has a binary set: [0] OR [1]
- The value [1] indicates that for x_{ij} the Employee *i* is assigned Job *j*.
- The value [0] is used otherwise (no assignment took place)

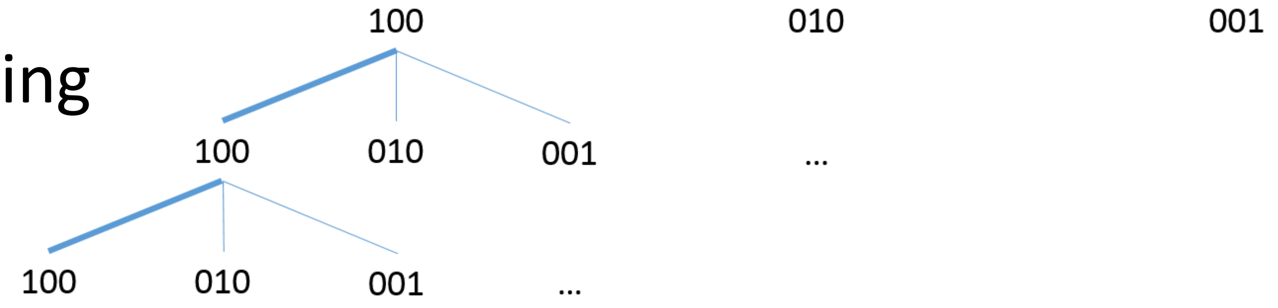
Source: Shun Y. Cheung, 2012

IP

- Minimize: $1x_{11}+4x_{12}+5x_{13}+5x_{21}+7x_{22}+6x_{23}+5x_{31}+8x_{32}+8x_{33}$

C	J_1	J_2	J_3
E_1	1	4	5
E_2	5	7	6
E_3	5	8	8

- Solution tree (3^3)
- Select only maximum matching



Source: Shun Y. Cheung, 2012

IP

- Algorithm output provides two possible options

C=15	J_1	J_2	J_3
E_1	0	4	0
E_2	0	0	6
E_3	5	0	0
C=15	**J_1**	**J_2**	**J_3**
E_1	1	0	0
E_2	0	0	6
E_3	0	8	0

- This was selected from 6 possible outcomes {16, **15**, 17, **15**, 18, 17}

Source: Shun Y. Cheung, 2012

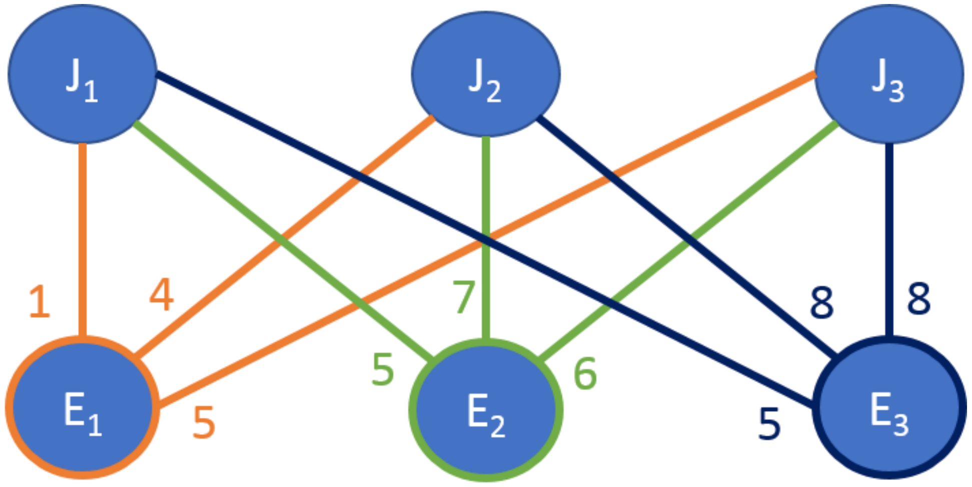
Simplex Algorithm for network problems

- A general purpose algorithm to find the optimum of a linear cost function with linear constraints is the Simplex Algorithm.
- A specially adapted Simplex algorithm is the Hungarian Algorithm.
- Although it was developed by Harold Kuhn, much of the work relied on the Hungarians Jenő Egerváry and Dénes Kőnig.
- If a number is added to or subtracted from all of the entries of any one row or column of a cost matrix, then on optimal assignment for the resulting cost matrix is also an optimal assignment for the original cost matrix.

Source: Shun Y. Cheung, 2012; Derek Bruff 2005

Hungarian Method

- Original cost matrix



C	J_1	J_2	J_3
E_1	1	4	5
E_2	5	7	6
E_3	5	8	8

Source: Shun Y. Cheung, 2012

Hungarian Method

(1) Subtract the smallest entry in each row from all the entries of its row.

C	J_1	J_2	J_3
E_1	**1**	4	5
E_2	**5**	7	6
E_3	**5**	8	8

C	J_1	J_2	J_3
E_1	0	3	4
E_2	0	2	1
E_3	0	3	3

Source: Shun Y. Cheung, 2012

Hungarian Method

(2) Subtract the smallest entry in each column from all the entries of its column.

C	J_1	J_2	J_3
E_1	0	3	4
E_2	0	**2**	**1**
E_3	0	3	3

C	J_1	J_2	J_3
E_1	0	1	3
E_2	0	0	0
E_3	0	1	2

Source: Shun Y. Cheung, 2012

Hungarian Method

- (3) Draw lines through appropriate rows and columns so that all the zero entries of the cost matrix are covered and the minimum number of such lines is used.

C	J_1	J_2	J_3
E_1	0	1	3
E_2	0	0	0
E_3	0	1	2

Source: Shun Y. Cheung, 2012

Hungarian Method

- (4) Test for Optimality:

(i) If the minimum number of covering lines is *n* (number of rows or columns), an optimal assignment of zeros is possible and we are finished.

(ii) If the minimum number of covering lines is less than *n*, an optimal assignment of zeros is not yet possible. In that case, proceed to Step 5.

n>2

C	J_1	J_2	J_3
E_1	0	1	3
E_2	0	0	0
E_3	0	1	2

Source: Shun Y. Cheung, 2012

Hungarian Method

- (5) Determine the smallest entry not covered by any line. **Subtract this entry from each uncovered row**, and then add it to each covered column. Return to Step 3.

C	J_1	J_2	J_3
E_1	0	**1**	3
E_2	0	0	0
E_3	0	1	2

C	J_1	J_2	J_3
E_1	-1	0	2
E_2	0	0	0
E_3	-1	0	1

Source: Shun Y. Cheung, 2012

Hungarian Method

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- (5) Take the smallest entry that was not covered by any line. Subtract this entry from each uncovered row, and **then add it to each covered column**. Return to Step 3.

C	J_1	J_2	J_3
E_1	-1	0	2
E_2	0	0	0
E_3	-1	0	1

C	J_1	J_2	J_3
E_1	0	0	2
E_2	1	0	0
E_3	0	0	1

Source: Shun Y. Cheung, 2012

Hungarian Method

- (3) Draw lines through appropriate rows and columns so that all the zero entries of the cost matrix are covered and the minimum number of such lines is used.

n=3

C	J_1	J_2	J_3
E_1	0	0	2
E_2	1	0	0
E_3	0	0	1

4 (i) If the minimum number of covering lines is *n*, an optimal assignment of zeros is possible and we are finished.

Source: Shun Y. Cheung, 2012

Hungarian Method

- Algorithm output provides two possible options

C=15	J_1	J_2	J_3
E_1	0	**0**	2
E_2	1	0	**0**
E_3	**0**	0	1

C=15	J_1	J_2	J_3
E_1	**0**	0	2
E_2	1	0	**0**
E_3	0	**0**	1

Source: Shun Y. Cheung, 2012

IP

- Algorithm output provides the same two possible options

C=15	J_1	J_2	J_3
E_1	0	**4**	0
E_2	0	0	**6**
E_3	**5**	0	0
C=15	J_1	J_2	J_3
E_1	**1**	0	0
E_2	0	0	**6**
E_3	0	**8**	0

- They yield similar results

Source: Shun Y. Cheung, 2012

Hungarian algorithm

- The original algorithm has a computational complexity of $O(n^4)$.
- The algorithm can be improved by scanning rows and columns in parallell.
- The computational times have been reduced by a range of improvements, but the basic idea still provides the framework for many publsihed variations of the Hungarian algorithm.

Source: Jonker and Volgenant, 1986

Example: global assignment of personnel

- As more companies expand globally, they are also increasing international assignments and relying on expatriates to manage their global operations.
- Around 83 % of employers offer short-term assignments (T< 1 year), 97 % offer long-term assignments (1<T<5 years) and 61% offer permanent transfer.
- The international assignment requires additional specifications that reflect barriers in globalisation
- Determining who is suitable for an international assignment is an important decision point within international people management

Source: Society of HR management, 2017; KPMG's Global Assignment Policies and Practices Survey

International assignment

- Traditionally, organizations have relied on technical, job-related skills as the main criteria for selecting candidates for overseas assignments, but assessing global mindset is equally, if not more, important for successful assignments.

- Research points to three major attributes of successful expatriates:
  - **Intellectual capital.** Knowledge, skills, understanding and cognitive complexity.
  - **Psychological capital.** The ability to function successfully in the host country through internal acceptance of different cultures and a strong desire to learn from new experiences.
  - **Social capital.** The ability to build trusting relationships with local stakeholders, whether they are employees, supply chain partners or customers.

- This can be captured under a suitability value for each employee. This makes the international assignment problem a combinatorial optimization problem.

Source: Society of HR management, 2017; 2018

International assignment additional pointers

- An effective global communication plan will help expatriates feel connected to the home office and will alert them to changes that occur while they are away.
- The Internet, e-mail and intranets are inexpensive and easy ways to bring expatriates into the loop. In addition to formal e-mail communications, organizations should encourage home-office employees to keep in touch with peers on overseas assignments. Employee newsletters that feature global news and expatriate assignments are also encouraged.

Source: Society of HR management, 2017; 2018

Questions?

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