

## 2P04 Lab 3

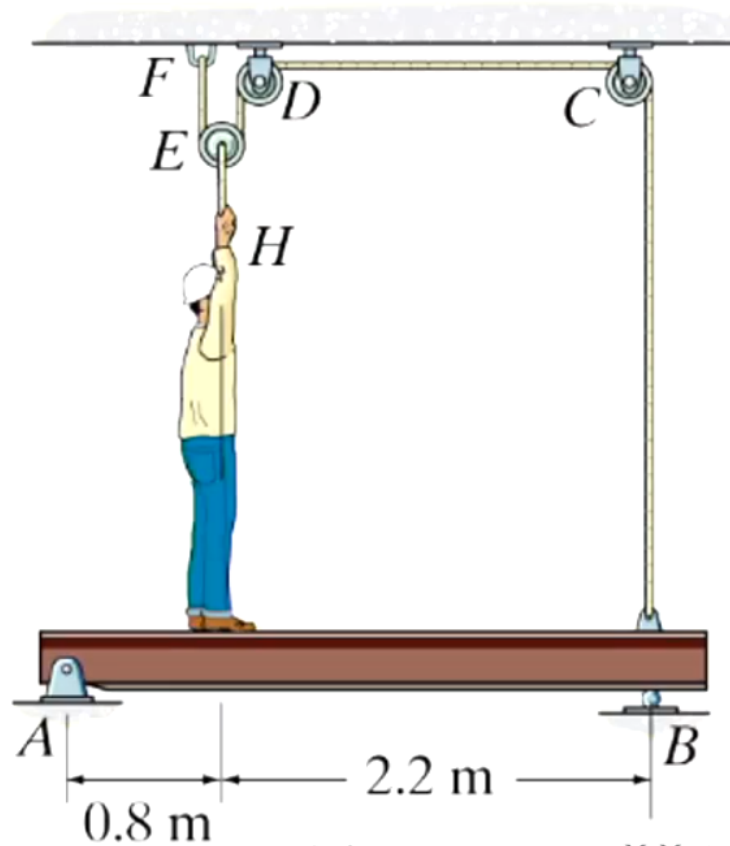
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### 1 Lecture 9

#### 1.1 Lecture Question

A 750 N weight man tries to lift the end of a beam he's standing on by pulling down on the pulley cord below. How much of his weight does he need to shift to the pulley for the beam to start lifting?



We know that the system starts off with in equilibrium due to the reaction force at A from the man's weight and the reaction moment at A due to the torque from the man's weight. We're asked to find when the net torque shifts in such a manner that a clockwise torque is produced instead. We know that  $HE = 2DH$ . Therefore, the tension in the rope pulling up the man is double the weight he's shifting to the pulley.

We can create the following equation for net torque:

$$3T - 0.8(750 - 2T) > 0$$

We know that the tension the man transfers over is half his weight, so in order to see how much weight he loses from where he's standing we need to subtract double the tension. Now we can go ahead and solve the above inequality to find the minimum tension required:

$$3T - 600 + 1.6T > 0$$

$$4.6T > 600$$

$$T > 130.43$$

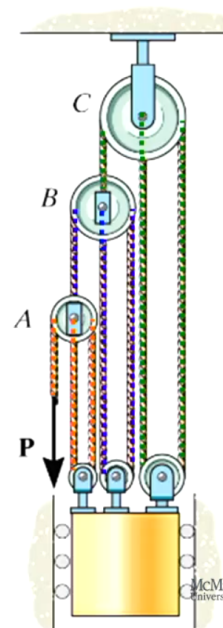
Therefore, the weight that the man needs to transfer over in order to shift the net torque to a clockwise direction is 260.86 N. This makes his current weight 489.14 N.

## 1.2 Lecture Quiz

### Quiz:

If  $P = 100$  N and the structure is in equilibrium, **determine the weight of the cheese block** being lifted at constant speed by this machine.

Note: there are 3 separate ropes here, indicated by colour



We know that the orange rope must be equal to  $P$  since the pulleys are assumed to be ideal without any friction.

We know that  $3A = B$  since pulley A must be in equilibrium.

We know that  $3B = C$  since pulley B must be in equilibrium.

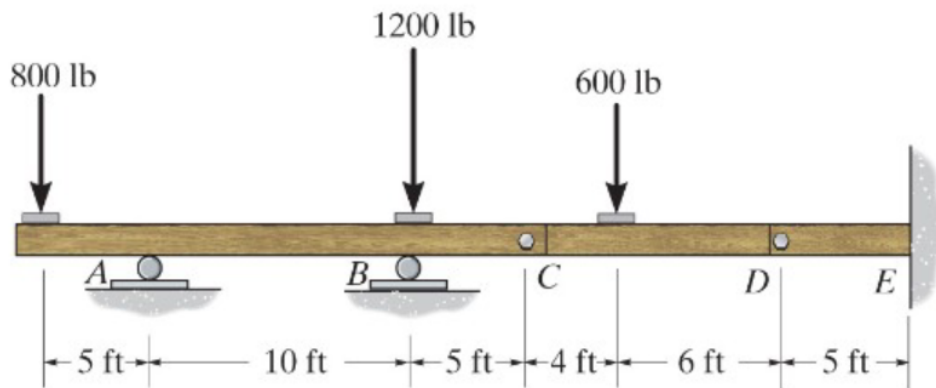
And given how the ropes are connected to the cheese block, we know that  $2A + 2B + 2C = \text{cheese}$ .

If  $P = 100$  N, then  $A = 100$  N,  $B = 300$  N,  $C = 900$  N, and the cheese block weighs 2600 N.

Reflection: In this lecture we were introduced to frames and machines and how we can solve for the forces and moments in them. We learned how to calculate the forces and moments in a system with multiple components. This is useful in real life applications where we have to deal with multiple components in a system. Moreover, we learned how to calculate the forces and moments in a system with multiple components.

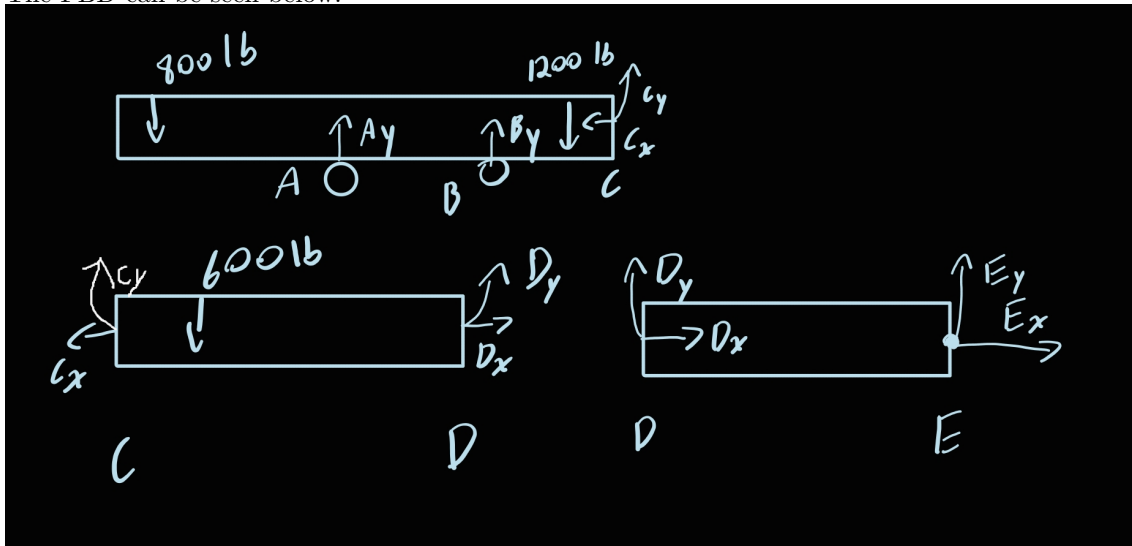
### 1.3 Problem Bank Question

This compound beam is supported by a fixed support at E and rollers at A and B, and has pins at C and D.



- Draw an FBD for each of the three sections of the compound beam.
- Determine the reactions at the supports, and the x and y components of the force in each pin.

The FBD can be seen below:



We can create equations for each of the sections and solve for the unknowns.

Maple code to solve:

```
restart: solve([
-800 + Ay + By - 1200 + Cy,
Cy-600+Dy,
Dy+Ey,
Cx,
Cx-Dx,
Dx+Ex,
800 * 5 - 1200*10 + 10*By + 15*Cy,
800*20 - 15*Ay - 5*By + 5*1200 - 600*4 + Dy*10
]);
```

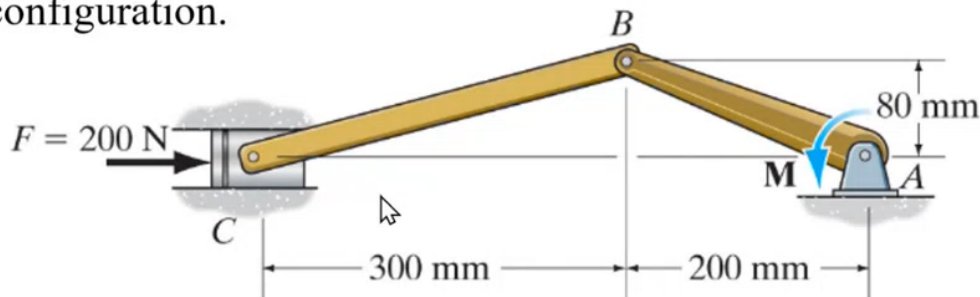
The output is as follows:

$\{A_y = 1380, B_y = 260, C_x = 0, C_y = 360, D_x = 0, D_y = 240, E_x = 0, E_y = -240\}$

## 2 Lecture 10

### 2.1 Lecture Question

If a 200 N force is applied to the piston in the current position of the mechanism shown below by a spring that is unstretched when AB is vertical, determine the spring constant and the moment **M** required to hold the system in static equilibrium in this configuration.



We can start by finding the interior angles at A, B, and C. Then, we can find the force of BC as well as the torque produced around A by BC. From that, we have found the resulting moment. Then, we can figure out how much the spring is stretched by from its natural resting position. Using that we can find the spring constant.

Some Maple code to help with the calculations:

```
restart;
LAB:=sqrt(0.08^2 + 0.2^2); LBC:=sqrt(0.08^2 + 0.3^2);
theta:=arctan(0.08/0.2); phi:=arctan(0.08/0.3);
BC:=solve(BC*cos(phi)+200);
M:=-LAB*BC*sin(phi+theta);
sVertical:=sqrt(LBC^2-LAB^2);
stretch:=0.5 - sVertical;
k:=200/stretch;
```

The output from this code is:

$$BC := -206.9889960$$

$$M := 26.66666667$$

$$k := 723.6067976$$

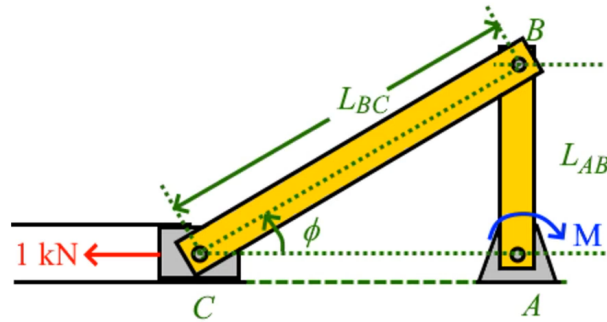
Therefore, the moment at A is 26.6 N\*m and the spring constant is 723.6 N/m.

## 2.2 Lecture Quiz

### 2P L10 Quiz:

Suppose two pin-connected bars with lengths  $L_{AB} = 215 \text{ mm}$  and  $L_{BC} = 310 \text{ mm}$  are connected to a frictionless piston.

Determine the moment  $M$  (in the direction specified) required to hold the system in equilibrium for the specified force applied to the piston.



We know that there must be a reaction force and a reaction moment at A to counteract the forces and moments from other parts of the system. We can start by find  $\phi$  which will give us the force in the BC component. Given that, we can take the cross product of the force and the distance to find the moment at A.

Some Maple code to help with the calculations:

```
restart;
LAB:=0.215:LBC:=0.310;
phi:=arcsin(LAB/LBC);
BC:=solve(BC*cos(phi)-1000);
M:=-LAB*BC*sin(Pi/2+phi);
```

The output from this code is:

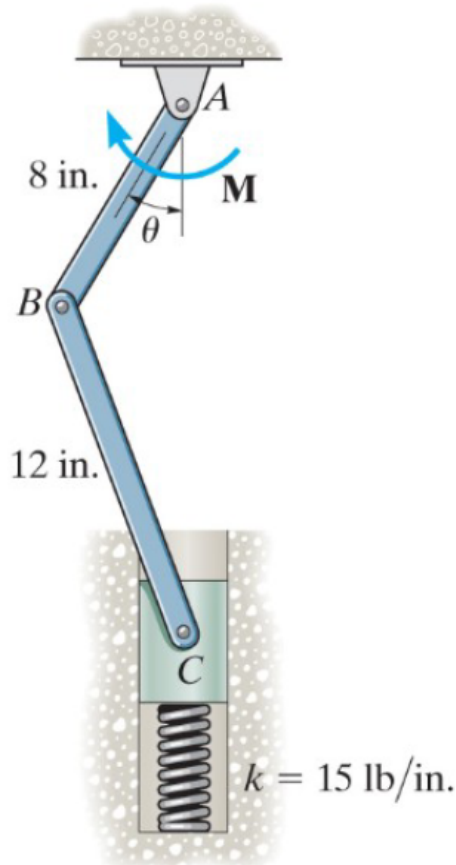
$BC := 1388.098355$   
 $M := -214.9999999$

Therefore, the resulting moment at A is 215 N\*m in the clockwise direction.

Reflection: In this lecture, we took the concepts of frames in relation to machines and applied more complex geometry to them. We learned how to calculate the forces and moments in a system with more complex geometry. This is useful in real life applications where we have to deal with more complex systems. Namely, we used triangles to calculate the forces and moments in the system.

## 2.3 Problem Bank Question

The piston moves vertically between two smooth walls. If the spring is unstretched when  $\theta = 0$ , determine how much moment  $\mathbf{M}$  (in lb-in) is needed to hold this mechanism in equilibrium when  $\theta = 20^\circ$ . Present your answer in both lb-in and in N-m.



Hints:

- Because of the moment, AB is not a 2-force member.
- You'll need to do some trig (possibly even using things like the sine law and/or cos law), to find the angle BC makes with the vertical (i.e., with the line between C and A)
- Try to work out the conversion factor by starting with the following things you should memorize: 2.54 cm/in, 2.2 lbs of weight per kg of mass (on Earth with  $g = 9.81 \text{ N/kg}$ ). Then look it up online to check if you got it right. Messing up or forgetting unit conversions has been responsible for some major engineering disasters like the loss of NASA's Mars Climate Orbiter.

We can start off by getting the length of the spring when it's unstretched so we can figure out it's current extension and thus force. Then we can find the force of the spring and convert it to the force on BC. Finally take the cross product to find the moment.

Some Maple code to help with the calculations:

```
restart:k:=15:
starting:=8+12:
# Cosine law
evalf(solve([12^2=8^2+ending^2-2*8*ending*cos(20*Pi/180), ending>0], ending)):
assign(%):
stretch:=starting-ending:
Fspring:=15*stretch:
# Sine law
solve([sin(20*Pi/180)/12 = sin(phi)/8]): assign(evalf(%)):
# Force of CB
solve([FCB*cos(phi)=Fspring]): assign(%):
tau:=-8*FCB*sin(phi+20*Pi/180):
tauMetric:=0.11298*tau:
```

The output from this code is:

$$F_{spring} := 11.97844770$$

$$\tau := -53.86271779$$

$$\tau_{Metric} := -6.085409856$$

Therefore, the moment is -53.9 lb\*in or -6.1 N\*m.

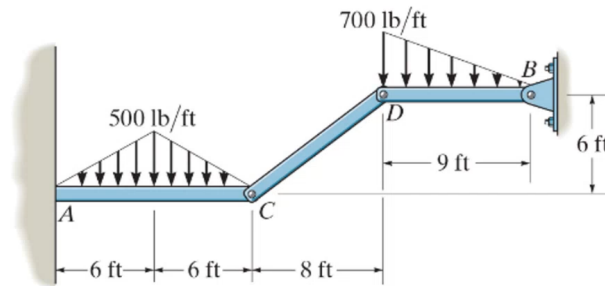
### 3 Lecture 11

#### 3.1 Lecture Question

##### \*Distributed Loads

Find the reactions in the supports and the tension in CD.

Find equivalent point loads for the DLs.



We can start off by finding the force of CD from the right triangle. We know that the x reaction force at B counteracts the force from CD, and the y reaction force at B counteracts the y component of CD plus the sum of the loaded force. The loaded force can be found by getting the integral of the distributed load. Since we have 3 unknowns, we can use the x and y equations as described in addition to the torque about point D.

After getting these values, we can move on and repeat this process on the left triangle to get the moment as well.

Some Maple code to help with the calculations:

```
restart:
p1:=piecewise(x<6,500*x/6, x>=6, 500*(1-(x-6)/6)):
p2:=700*(1-x/9):
tauP1A:=int(-x*pi,x=0..12):
solve([
-CD*8/sqrt(8^2+6^2) + BX,
-CD*6/sqrt(8^2+6^2) - int(p2, x=0..9) + BY,
-int(x*p2,x=0..9) + 9*BY]): assign(%):
solve([
-AX+CD*8/sqrt(8^2 + 6^2),
AY-int(p1,x=0..12)+CD*6/sqrt(8^2+6^2),
M-int(x*p1,x=0..12)+12*CD*6/sqrt(8^2+6^2)]):
```

The output from this code is:

$$\{BX = -2800, BY = 1050, CD = -3500\}$$

$$\{AX = -2800, AY = 5100, M = 43200\}$$

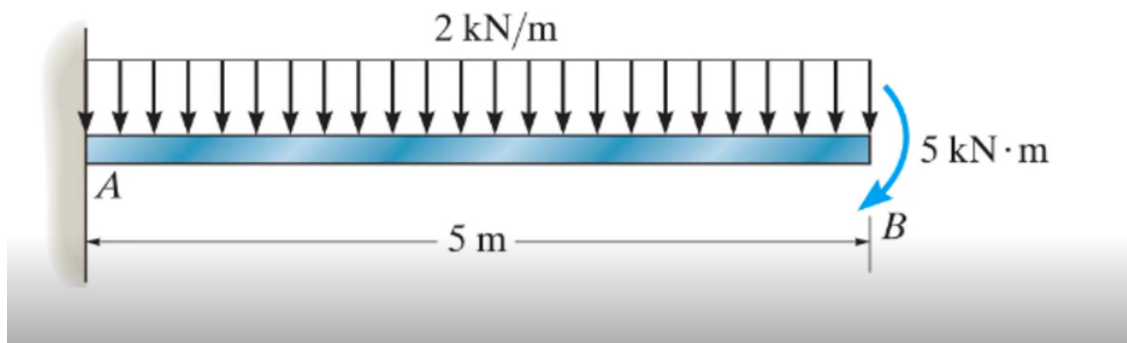
Therefore, the force of tension in CD is 3500 lb and the moment is 43200 lb\*in counterclockwise.

### 3.2 Lecture Quiz

2P L11 Quiz:

Determine the torque the distributed load produces at point A.

Note: you don't need to determine the reaction moment at A, or the total torque produced by the applied load and moment at A, only the torque about point A produced by the distributed load.

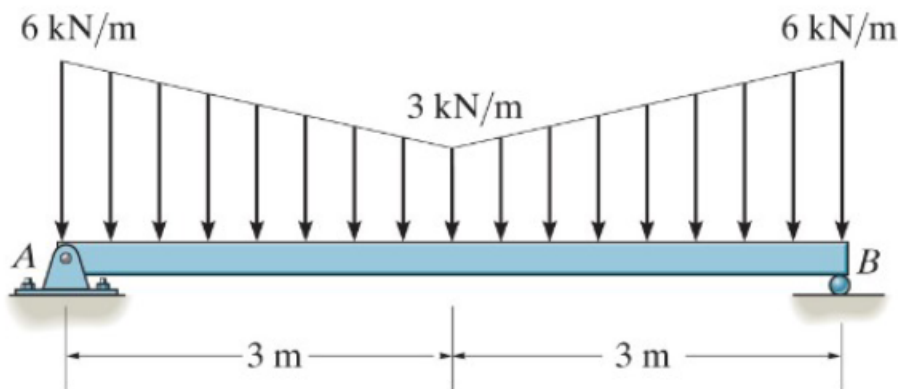


This can be solved by finding the torque from the distributed load, which we know is  $\tau = \int x \cdot F dx$ . And then we can add the 5 kN moment to that to get the total torque at point A. We can analytically calculate the integral  $\int x \cdot F dx + 5000$  to be  $-25000 \text{ N}\cdot\text{m}$ .

Reflection: In this lecture we learned about distributed loads and how to calculate the torque from them. This is useful in real life applications where we have to deal with distributed loads. We also learned how to calculate the torque from a moment and a distributed load.

### 3.3 Problem Bank Question

Solve for the support reactions in the following structure:



Note: remember to always draw an FBD to help you solve problems and assign your reaction directions. If the TA can't tell which directions are positive for your reactions because you didn't include an FBD (or at least explain it clearly enough without one) they won't be able to award you the marks.

$A_x$  is positive towards the right and upwards.  $B_y$  is positive upwards and no x component exists.

We can model the distributed load as a function of x and then integrate it to get the total force. Then we can use the equations of equilibrium to solve for the unknowns.

We can use the following Maple code to help with the calculations:



```

restart :
p:=piecewise (x<3,6-x,x>=3,x);
force:=int (p,x=0..6);
tau:=int (x*p,x=0..6);
solve ([
Ay+By-force ,
Ax,
-tau+6*By]): evalf(%);

```

The output from this code is:

```

p := { 6 - x    x < 3
      x        3 ≤ x
force := 27
      τ := 81
{Ax = 0., Ay = 13.50000000, By = 13.50000000}

```

Therefore, the x component of A is 0, and the y components of both A and B are 13.5 kN upwards.