PRACTICAL 4

- Solution of Vibrating string problem using D,Alembert formula with initial condition .

Ques :- 1 $u_{tt} = u_{xx}$; $-\infty < x < \infty$, $t>0$

$u(x,0)= \text{Sin } x$; $-\infty < x < \infty$

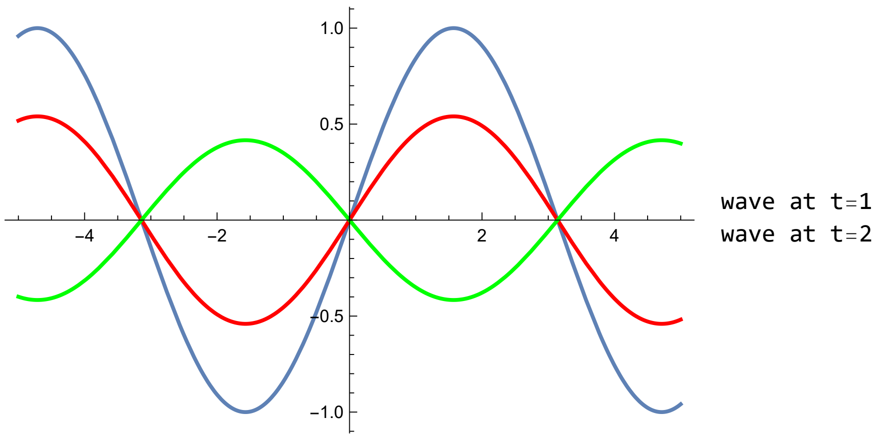
$u_t(x,0)= 0$; $-\infty < x < \infty$

In[101]:=

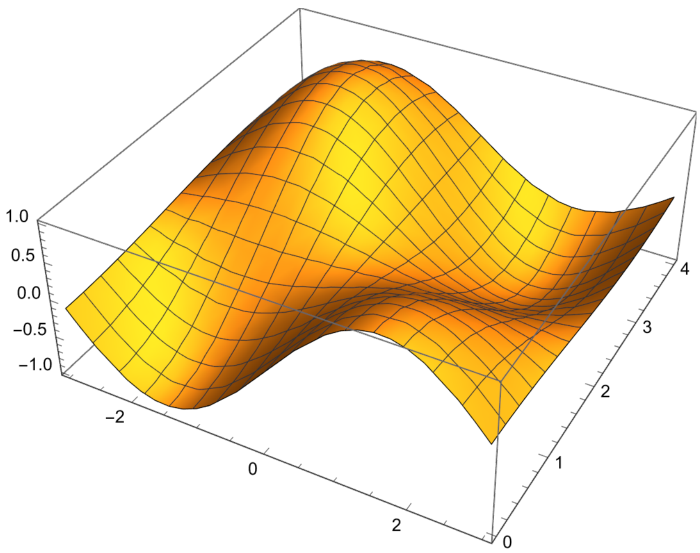
```
c = 1;
f[x_] := Sin[x]
g[x_] := 0
u[x_, t_] := 1/2 * (f[x + c t] + f[x - c t]) + 1/(2 c) * ∫_{x-c t}^{x+c t} g[s] ⅆs
Print[u[x, t]]
h0 = Plot[Evaluate[u[x, 0]], {x, -5, 5}];
h1 =
  Plot[Evaluate[u[x, 1]], {x, -5, 5}, PlotLegends → " wave at t=1", PlotStyle → Red];
h2 =
  Plot[Evaluate[u[x, 2]], {x, -5, 5}, PlotLegends → " wave at t=2", PlotStyle → Green];
Show [h0, h1, h2]
Plot3D[u[x, t], {x, -3, 3}, {t, 0, 4}]
```

$\frac{1}{2}(-\text{Sin}[t-x]+\text{Sin}[t+x])$

Out[109]=



Out[110]=



Ques :- 2 $u_{tt} = 4\, u_{xx}$; $-\infty < x < \infty$, t>0

$u(x,0) = e^{-x^2} \text{Sin } x$; $-\infty < x < \infty$

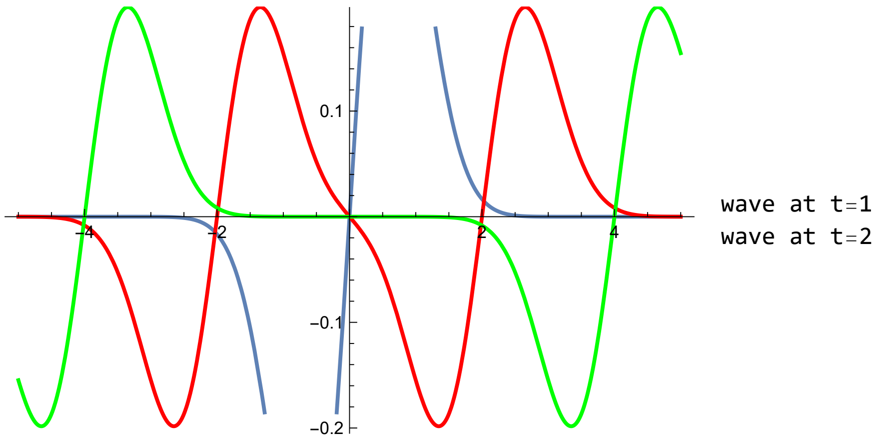
$u_t(x,0)=0$; $-\infty < x < \infty$

In[127]:=

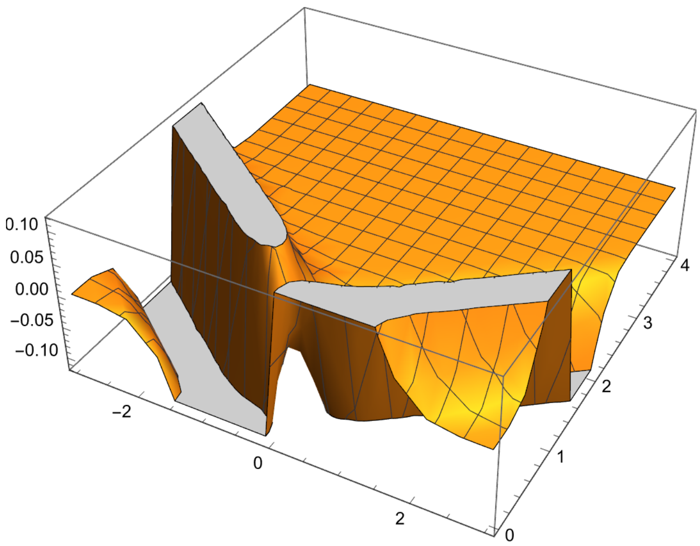
```
c = 2;
f[x_] := e^(-x^2) Sin[x]
g[x_] := 0
u[x_, t_] := 1/2 * (f[x + c t] + f[x - c t]) + 1/(2 c) * ∫_(x-c t)^(x+c t) g[s] ds
Print[u[x, t]]
h0 = Plot[Evaluate[u[x, 0]], {x, -5, 5}];
h1 =
  Plot[Evaluate[u[x, 1]], {x, -5, 5}, PlotLegends → " wave at t=1", PlotStyle → Red];
h2 =
  Plot[Evaluate[u[x, 2]], {x, -5, 5}, PlotLegends → " wave at t=2", PlotStyle → Green];
Show [h0, h1, h2]
Plot3D[u[x, t], {x, -3, 3}, {t, 0, 4}]
```

$$\frac{1}{2}\left(-e^{-(-2t+x)^2}\,\mathrm{Sin}[2t-x]+e^{-(2t+x)^2}\,\mathrm{Sin}[2t+x]\right)$$

Out[135]=



Out[136]=



Ques :- 3 $u_{tt} = 2\,u_{xx}$; $-\infty < x < \infty$, t>0

u(x,0)= {0, x< -1 }; $-\infty < x < \infty$

$\{1, -1 \leq x \leq 1\}; -\infty < x < \infty$

$\{0, x > 1\}; -\infty < x < \infty$

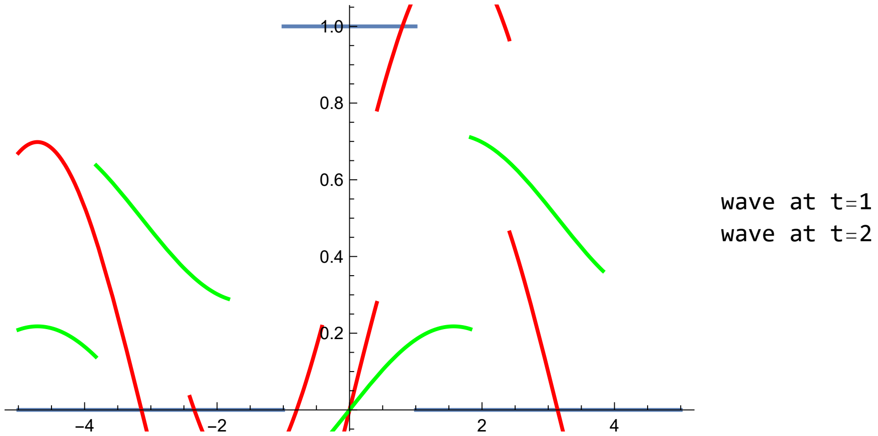
$u_t(x,0) = \text{Sin } x; -\infty < x < \infty$

In[137]:=

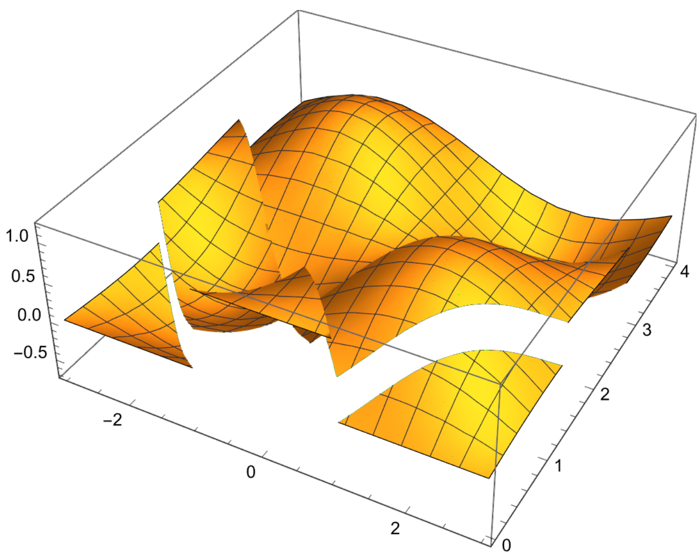
```
c = √2;
f[x_] := Piecewise[{{0, x < -1}, {1, -1 ≤ x ≤ 1}, {0, x > 1}}]
g[x_] := Sin[x]
u[x_, t_] := 1/2 * (f[x + c t] + f[x - c t]) + 1/(2 c) * ∫_{x-c t}^{x+c t} g[s] ⅆs
Print[u[x, t]]
h0 = Plot[Evaluate[u[x, 0]], {x, -5, 5}];
h1 =
  Plot[Evaluate[u[x, 1]], {x, -5, 5}, PlotLegends → " wave at t=1", PlotStyle → Red];
h2 =
  Plot[Evaluate[u[x, 2]], {x, -5, 5}, PlotLegends → " wave at t=2", PlotStyle → Green];
Show [h0, h1, h2]
Plot3D[u[x, t], {x, -3, 3}, {t, 0, 4}]
```

$$\frac{1}{2}\left(\left(\begin{cases} 0 & -\sqrt{2}\,t + x < -1 \\ 1 & -1 \leq -\sqrt{2}\,t + x \leq 1 \\ 0 & \text{True} \end{cases}\right) + \left(\begin{cases} 0 & \sqrt{2}\,t + x < -1 \\ 1 & -1 \leq \sqrt{2}\,t + x \leq 1 \\ 0 & \text{True} \end{cases}\right)\right) + \frac{\text{Sin}[\sqrt{2}\,t]\,\text{Sin}[x]}{\sqrt{2}}$$

Out[145]=



Out[146]=



Ques :- 4 $u_{tt} = 2\, u_{xx}$; $-\infty < x < \infty$, $t>0$

$u(x,0)=$ Sin x ; $-\infty < x < \infty$

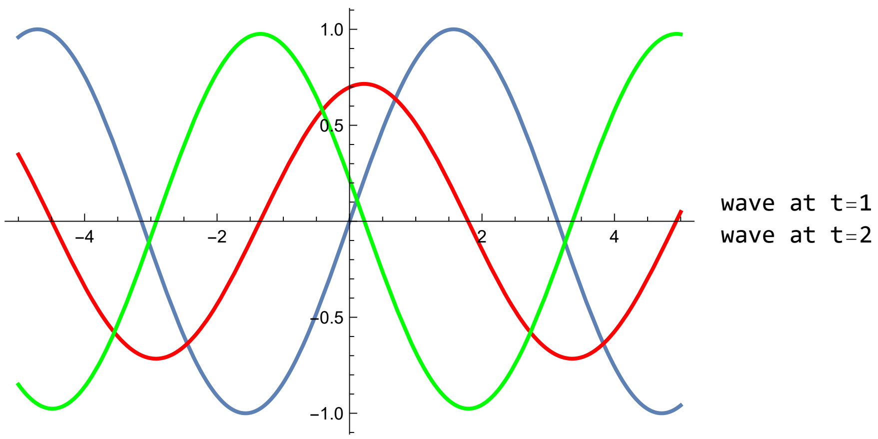
$u_t(x,0)=$ Cos x ; $-\infty < x < \infty$

In[147]:=

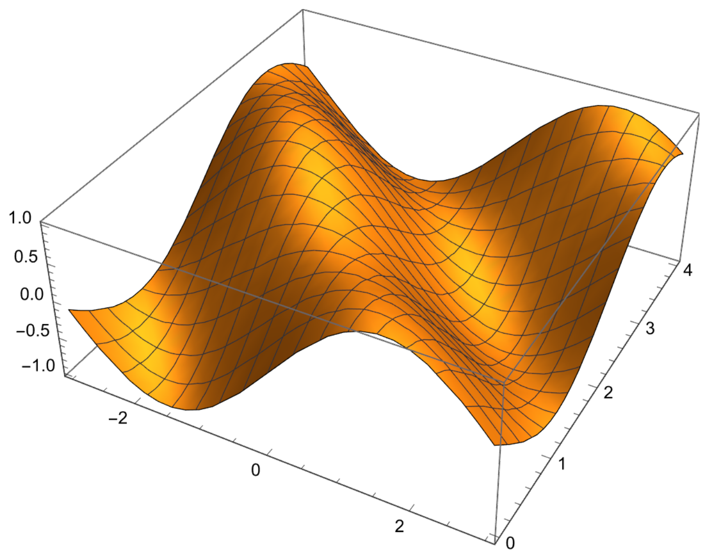
```
c = √2;
f[x_] := Sin[x]
g[x_] := Cos[x]
u[x_, t_] := 1/2 * (f[x + c t] + f[x - c t]) + 1/(2 c) * ∫_{x-c t}^{x+c t} g[s] ds
Print[u[x, t]]
h0 = Plot[Evaluate[u[x, 0]], {x, -5, 5}];
h1 =
  Plot[Evaluate[u[x, 1]], {x, -5, 5}, PlotLegends → " wave at t=1", PlotStyle → Red];
h2 =
  Plot[Evaluate[u[x, 2]], {x, -5, 5}, PlotLegends → " wave at t=2", PlotStyle → Green];
Show [h0, h1, h2]
Plot3D[u[x, t], {x, -3, 3}, {t, 0, 4}]
```

$$\frac{\mathrm{Cos}[x]\,\mathrm{Sin}[\sqrt{2}\,t]}{\sqrt{2}} + \frac{1}{2}\left(-\mathrm{Sin}[\sqrt{2}\,t - x] + \mathrm{Sin}[\sqrt{2}\,t + x]\right)$$

Out[155]=



Out[156]=



Ques :- 5 $u_{tt} = \pi u_{xx}$; $-\infty < x < \infty$, t>0

$u(x,0)= 0$; $-\infty < x < \infty$

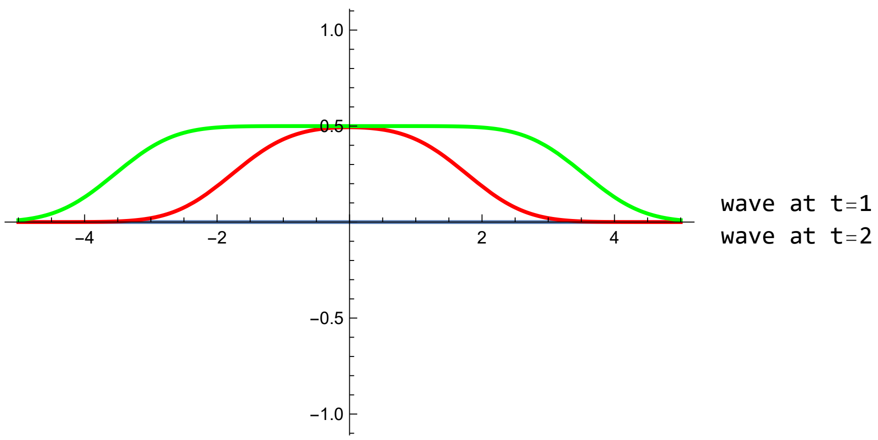
$u_t(x,0)= e^{-x^2}$; $-\infty < x < \infty$

In[157]:=

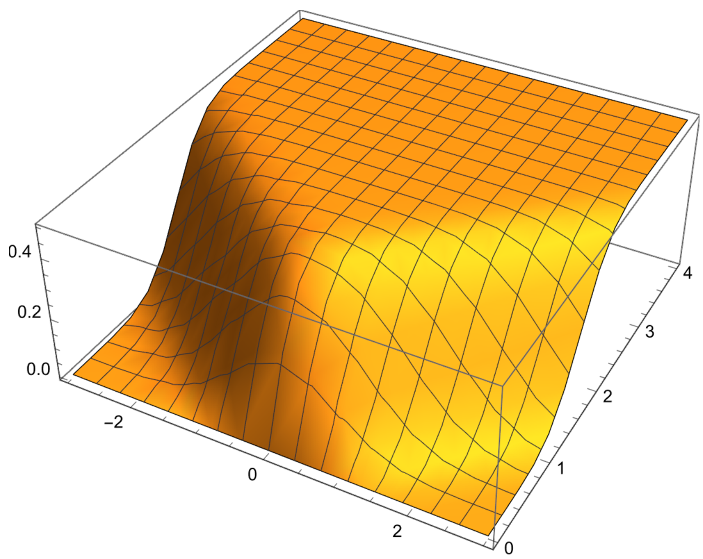
```
c = √π;
f[x_] := 0
g[x_] := e^(-x^2)
u[x_, t_] := 1/2 * (f[x + c t] + f[x - c t]) + 1/(2 c) * ∫_(x-c t)^(x+c t) g[s] ds
Print[u[x, t]]
h0 = Plot[Evaluate[u[x, 0]], {x, -5, 5}];
h1 =
  Plot[Evaluate[u[x, 1]], {x, -5, 5}, PlotLegends → " wave at t=1", PlotStyle → Red];
h2 =
  Plot[Evaluate[u[x, 2]], {x, -5, 5}, PlotLegends → " wave at t=2", PlotStyle → Green];
Show [h0, h1, h2]
Plot3D[u[x, t], {x, -3, 3}, {t, 0, 4}]
```

$$\frac{1}{4}\left(\text{Erf}\left[\sqrt{\pi}\, t - x\right] + \text{Erf}\left[\sqrt{\pi}\, t + x\right]\right)$$

Out[165]=



Out[166]=



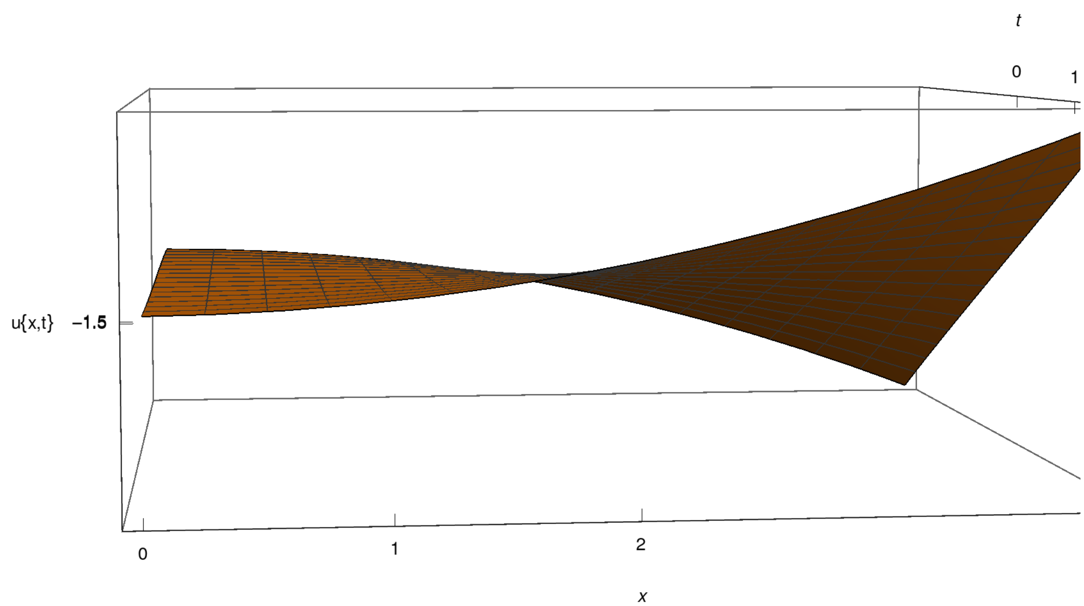
Ques .1 Solve wave equation $u_{tt} - 4 u_{xx} = 0$ with cauchy data $u(x, 0) = x^3$, $u_t(x, 0) = x$; $-\infty < x < \infty$, $t > 0$.

Method 1

In[19]:=
```
pde1 = {D[u[x, t], {t, 2}] - 4 D[u[x, t], {x, 2}] == 0, u[x, 0] == x^3,
    Derivative[0, 1][u][x, 0] == x};
Sol1 = DSolve[pde1, u[x, t], {x, t}]
Plot3D[u[x, t] /. Sol1, {x, -2, 2}, {t, 0, 4}, AxesLabel → {t, x, "u{x,t}"},
 Ticks → {{0, 1, 2, 3, 4, 5}, {-2, -1, 0, 1, 2}, {-1.5, 1.5}}, PlotStyle → {Dashed, Thick}]
```

Out[20]= $\left\{\left\{u[x,\ t] \rightarrow \frac{1}{4}\left(-\frac{1}{2}(-2\,t+x)^2+\frac{1}{2}(2\,t+x)^2\right)+\frac{1}{2}\left((-2\,t+x)^3+(2\,t+x)^3\right)\right\}\right\}$

Out[21]=



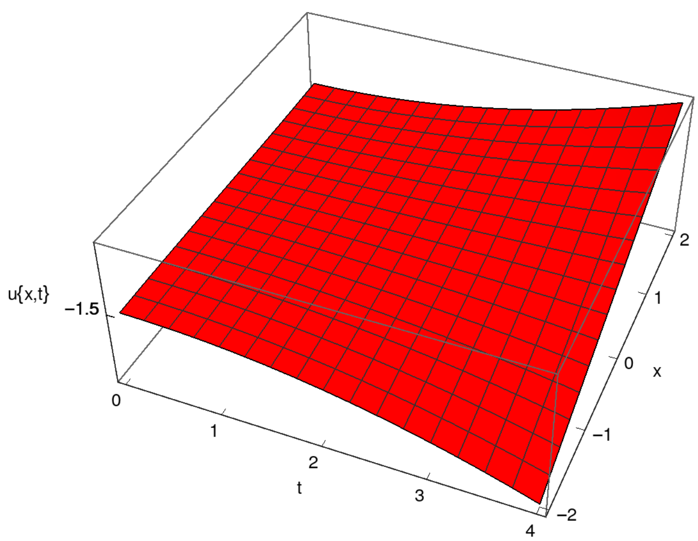
Method 2

In[25]:=

```
npde1 = {D[u[x, t], {t, 2}] - 4 D[u[x, t], {x, 2}] == 0, u[x, 0] == x^3,
    Derivative[0, 1][u][x, 0] == x};
nsol1 = u[x, t] /. NDSolve[npde1, u[x, t], {x, -2, 2}, {t, 0, 4}, PrecisionGoal → 4]
Plot3D[nsol1, {t, 0, 4}, {x, -2, 2}, AxesLabel → {"t", "x", "u{x,t}"},
 Ticks → {{0, 1, 2, 3, 4, 5}, {-2, -1, 0, 1, 2}, {-1.5, 1.5}}, PlotStyle → Red]
```

NDSolve: Warning: an insufficient number of boundary conditions have been specified for the direction of independent variable x. Artificial boundary effects may be present in the solution.

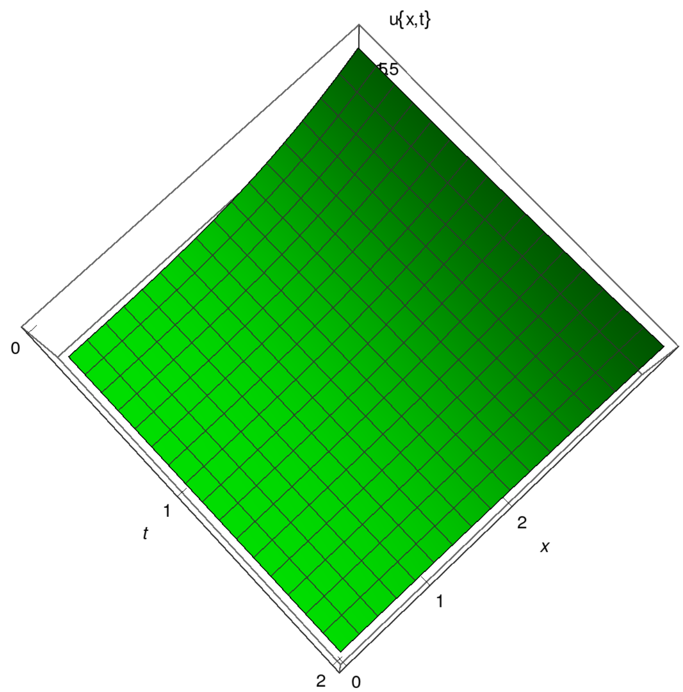
Out[26]= {InterpolatingFunction[Domain: {{-2., 2.}, {0., 4.}} Output: scalar][x, t]}



Ques .2 Solve wave equation u_tt - 4 u_xx = 0 with cauchy data u (x, 0) = x ^ 3, u_t(x, 0) = x; 0 < x < inf, t > 0 .

In[22]:=
```
pde2 = {D[u[x, t], {t, 2}] - 4 D[u[x, t], {x, 2}] == 0, u[x, 0] == x^3,
    Derivative[0, 1][u][x, 0] == x, Derivative[1, 0][u][0, t] == 0};
sol2 = DSolve[pde2, u[x, t], {x, t}]
Plot3D[u[x, t] /. sol2, {x, 0, 2}, {t, 0, 4}, AxesLabel → {t, x, "u{x,t}"},
 Ticks → {{0, 1, 2, 3, 4, 5}, {0, 1, 2}, {-1.5, 1.5}}, PlotStyle → Green]
```

Out[23]= $\left\{\left\{u[x, t] \to t\,x + 12\,t^2\,x + x^3 + 2\,\mathbb{c}_1\,\text{DiracDelta}[2\,t - x] + 2\,\mathbb{c}_1\,\text{DiracDelta}[2\,t + x] + (2\,t - x)^3\,\text{HeavisideTheta}\left[t - \frac{x}{2}\right] + \left(t - \frac{x}{2}\right)^2\,\text{HeavisideTheta}\left[t - \frac{x}{2}\right]\right\}\right\}$



Ques. 3 $\partial_{tt}u - 9\,\partial_{xx}u = 0$, u[x, 0] = Sin[x], $\frac{\partial u}{\partial t}$[x, 0] = x^3, $\frac{\partial u}{\partial x}$[0, t] = 0, 0 < x < 1, t > 0 .

```
In[28]:= aA = {D[u[x, t], {t, 2}] - D[u[x, t], {x, 2}] == 0, u[x, 0] == Sin[x],
            Derivative[0, 1][u][x, 0] == x^3, Derivative[1, 0][u][0, t] == 0};
         sol = u[x, t] /. NDSolve[aA, u[x, t], {x, 0, 1}, {t, 0, 4}, PrecisionGoal → 3]
         Plot3D[sol, {x, 0, 1}, {t, 0, 4}, AxesLabel → {t, x, "u{x,t}"},
          Ticks → {{0, 1, 2, 3, 4, 5}, {0, 1}, {-3, 0}}, PlotStyle → Gray]
```

NDSolve: Warning: boundary and initial conditions are inconsistent.

```
Out[29]= {InterpolatingFunction[ Domain: {{0., 1.}, {0., 4.}}
                                 Output: scalar ][x, t]}
```

Out[30]=