PRACTICAL 7

Solve the following questions theoretically and numerically using Mathematica. Plot the initial function, characteristic base curves and the traffic density function.

Q1. Find the traffic density $u(x, t)$ satisfying

$$\frac{\partial u}{\partial t} + 2u\frac{\partial u}{\partial x} = 0.$$

with the initial condition $u(x, 0) = \begin{cases} 1 & x < 0 \\ 1 - x & 0 < x < 1 \\ 0 & 1 \le x \end{cases}$

In[1]:=
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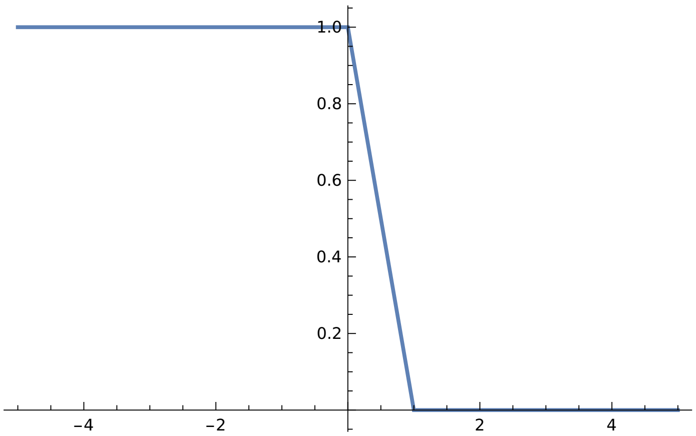
Plot[Piecewise[{{1, x < 0}, {1 - x, 0 < x < 1}, {0, x > 1}}], {x, -5, 5}]
f[x_] := Piecewise[{{1, x < 0}, {1 - x, 0 < x < 1}, {0, x > 1}}];
eq = D[u[x, t], t] + 2 D[u[x, t], x] == 0;
sol = DSolve[{eq, u[x, 0] == f[x]}, u[x, t], {x, t}]
Plot3D[u[x, t] /. sol, {x, -1, 3}, {t, 0, 1}]
```

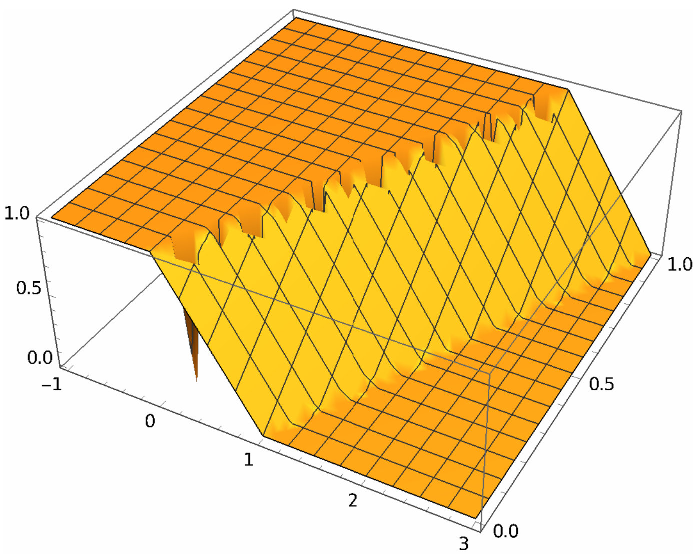
Out[1]=



Out[4]=

$$\left\{\left\{u[x, t] \to \begin{cases} 1 & -2\left(t - \frac{x}{2}\right) < 0 \\ 1 + 2t - x & 0 < -2\left(t - \frac{x}{2}\right) < 1 \\ 0 & \text{True} \end{cases}\right\}\right\}$$

Out[5]=



Q2. Find the traffic density u (x, t) satisfying

$$\frac{\partial u}{\partial t} + c\,(1 - 2u)\,\frac{\partial u}{\partial x} = 0.$$

with the initial condition $u(x, 0) = \begin{cases} \frac{1}{3} & x < 0 \\ \frac{1}{3} + \frac{5}{12}x & 0 < x < 1 \\ \frac{3}{4} & 1 \le x \end{cases}$

In[6]:=
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c = 1;
pde = D[u[x, t], t] + c (1 - 2 u[x, t]) D[u[x, t], x] == 0;

int = {u[x, 0] == Piecewise[{{1/3, x < 0}, {1/3 + 5/12 x, 0 ≤ x < 1}, {3/4, x ≥ 1}}]}
Plot[ Piecewise[{{1/3, x < 0}, {1/3 + 5/12 x, 0 ≤ x < 1}, {3/4, x ≥ 1}}], {x, -5, 5}]

sol = NDSolve[{pde, int}, u[x, t], {x, -2, 2}, {t, 0, 2}];

Plot3D[u[x, t] /. sol, {x, -2, 2}, {t, 0, 2}, AxesLabel → {"x", "t", "u(x,t)"},
 PlotLabel → "Traffic Density u(x,t)"]
```
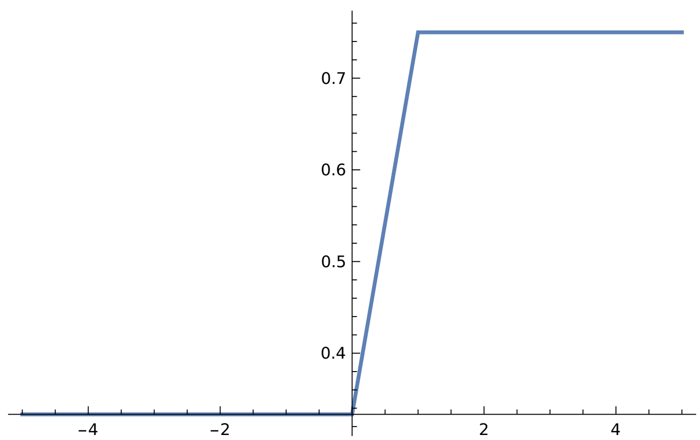
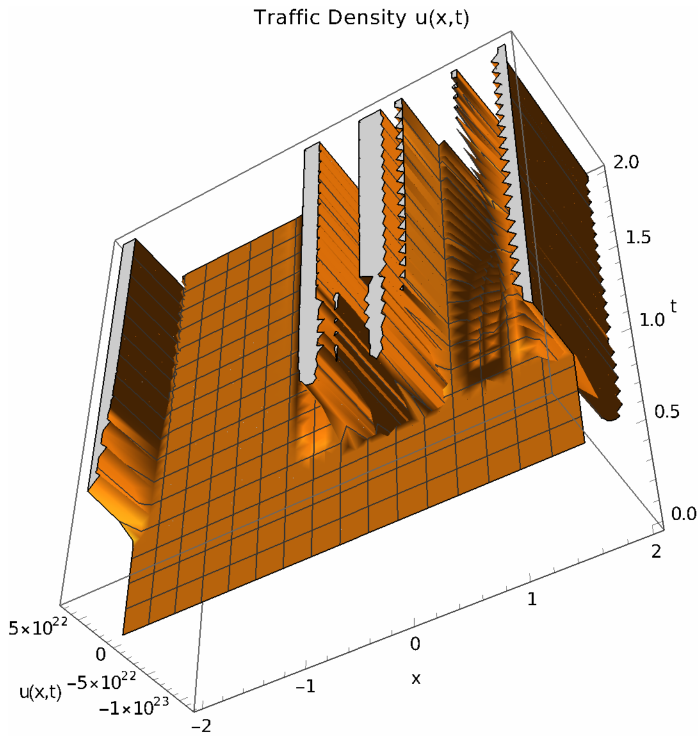

Out[8]= $\left\{u[x, 0] == \left(\begin{cases} \frac{1}{3} & x < 0 \\ \frac{1}{3} + \frac{5x}{12} & 0 \le x < 1 \\ \frac{3}{4} & x \ge 1 \\ 0 & \text{True} \end{cases}\right)\right\}$

Out[9]=



Out[11]=


Traffic Density u(x,t)
2.0
1.5
1.0
t
0.5
0.0
2
1
0
x
-1
-2
5×10^22
0
u(x,t)
-5×10^22
-1×10^23