PRACTICAL 4

- Solution of Vibrating string problem using D,Alembert formula with initial condition .

Ques :- 1 $u_{tt} = u_{xx}$; $-\infty < x < \infty$, $t>0$

$u(x,0)= \text{Sin } x$; $-\infty < x < \infty$

$u_t(x,0)= 0$; $-\infty < x < \infty$

In[101]:=

```
c = 1;
f[x_] := Sin[x]
g[x_] := 0
u[x_, t_] := 1/2 * (f[x + c t] + f[x - c t]) + 1/(2 c) * ∫_{x-c t}^{x+c t} g[s] ⅆs
Print[u[x, t]]
h0 = Plot[Evaluate[u[x, 0]], {x, -5, 5}];
h1 =
  Plot[Evaluate[u[x, 1]], {x, -5, 5}, PlotLegends → " wave at t=1", PlotStyle → Red];
h2 =
  Plot[Evaluate[u[x, 2]], {x, -5, 5}, PlotLegends → " wave at t=2", PlotStyle → Green];
Show [h0, h1, h2]
Plot3D[u[x, t], {x, -3, 3}, {t, 0, 4}]
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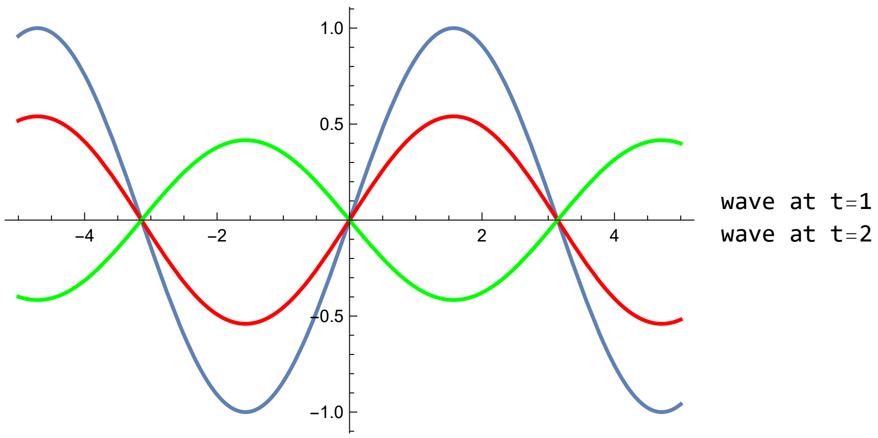
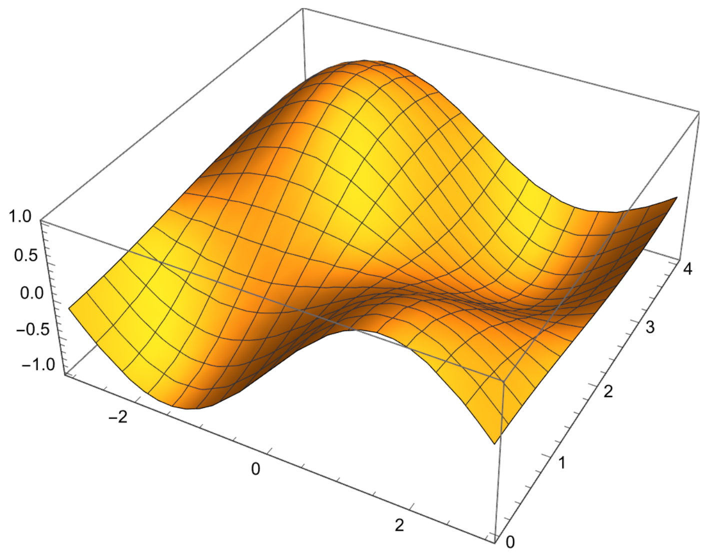
$\frac{1}{2}(-\mathrm{Sin}[t-x]+\mathrm{Sin}[t+x])$

Out[109]=



Out[110]=



Ques :- 2 $u_{tt} = 4\, u_{xx}$; $-\infty < x < \infty$, $t>0$

$u(x,0) = e^{-x^2}\,\mathrm{Sin}\, x$; $-\infty < x < \infty$

$u_t(x,0) = 0$; $-\infty < x < \infty$

In[127]:=

```
c = 2;
f[x_] := e^(-x^2) Sin[x]
g[x_] := 0
u[x_, t_] := 1/2 * (f[x + c t] + f[x - c t]) + 1/(2 c) * Integrate[g[s], {s, x - c t, x + c t}]
Print[u[x, t]]
h0 = Plot[Evaluate[u[x, 0]], {x, -5, 5}];
h1 =
  Plot[Evaluate[u[x, 1]], {x, -5, 5}, PlotLegends → " wave at t=1", PlotStyle → Red];
h2 =
  Plot[Evaluate[u[x, 2]], {x, -5, 5}, PlotLegends → " wave at t=2", PlotStyle → Green];
Show [h0, h1, h2]
Plot3D[u[x, t], {x, -3, 3}, {t, 0, 4}]
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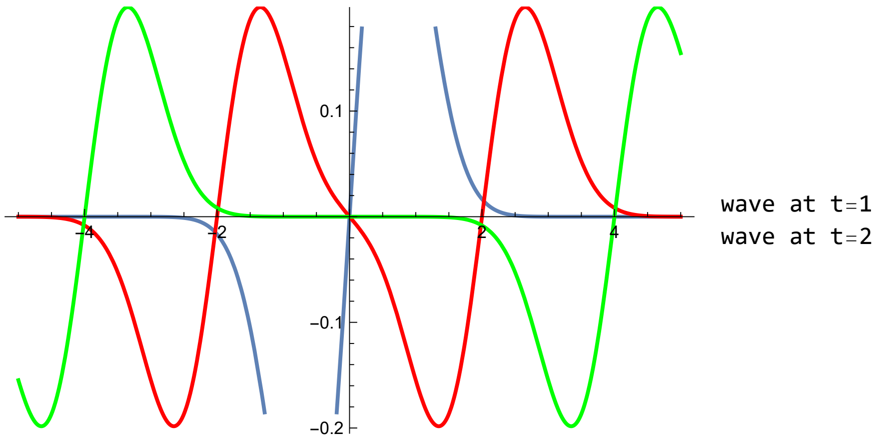
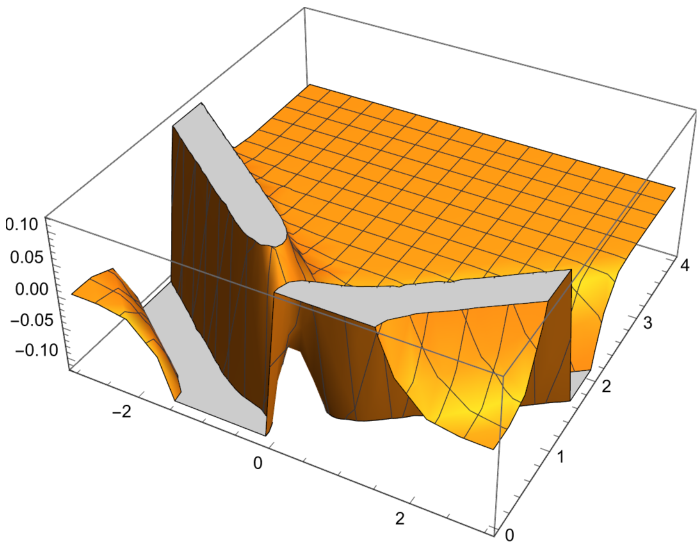
$$\frac{1}{2}\left(-e^{-(-2t+x)^2}\,\mathrm{Sin}[2t-x] + e^{-(2t+x)^2}\,\mathrm{Sin}[2t+x]\right)$$

Out[135]=



Out[136]=



Ques :- 3 $u_{tt} = 2\,u_{xx}$; $-\infty < x < \infty$, t>0

u(x,0)= {0, x< -1 }; $-\infty < x < \infty$

{1,-1≤x ≤1 }; -∞ < x< ∞

{0, x>1 }; -∞ < x< ∞

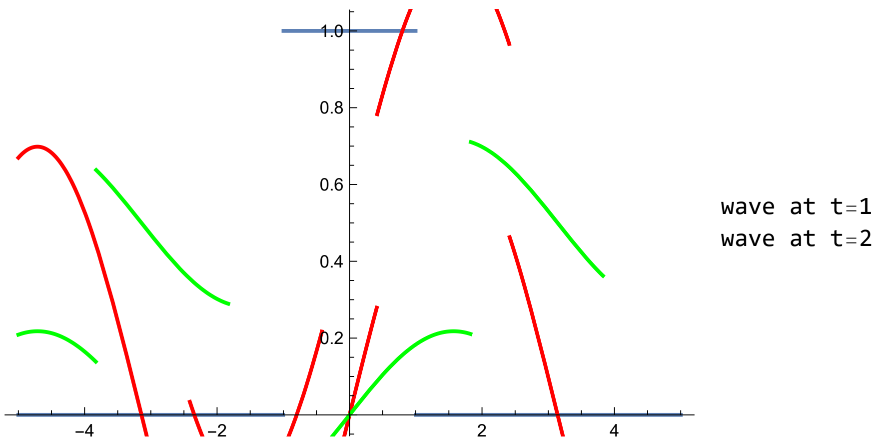
u_t(x,0)= Sin x ; -∞ < x< ∞

In[137]:=

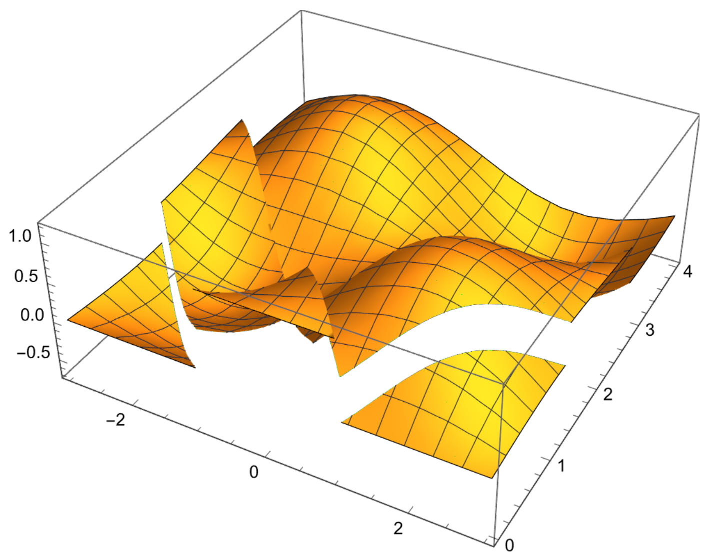
```
c = √2;
f[x_] := Piecewise[{{0, x < -1}, {1, -1 ≤ x ≤ 1}, {0, x > 1}}]
g[x_] := Sin[x]
u[x_, t_] := 1/2 * (f[x + c t] + f[x - c t]) + 1/(2 c) * ∫_{x-c t}^{x+c t} g[s] ds
Print[u[x, t]]
h0 = Plot[Evaluate[u[x, 0]], {x, -5, 5}];
h1 =
  Plot[Evaluate[u[x, 1]], {x, -5, 5}, PlotLegends → " wave at t=1", PlotStyle → Red];
h2 =
  Plot[Evaluate[u[x, 2]], {x, -5, 5}, PlotLegends → " wave at t=2", PlotStyle → Green];
Show [h0, h1, h2]
Plot3D[u[x, t], {x, -3, 3}, {t, 0, 4}]
```

$$\frac{1}{2}\left(\left(\begin{cases} 0 & -\sqrt{2}\,t+x<-1 \\ 1 & -1\le-\sqrt{2}\,t+x\le 1 \\ 0 & \text{True}\end{cases}\right)+\left(\begin{cases} 0 & \sqrt{2}\,t+x<-1 \\ 1 & -1\le\sqrt{2}\,t+x\le 1 \\ 0 & \text{True}\end{cases}\right)\right)+\frac{\text{Sin}[\sqrt{2}\,t]\,\text{Sin}[x]}{\sqrt{2}}$$

Out[145]=



Out[146]=



Ques :- 4 $u_{tt} = 2\, u_{xx}$; $-\infty < x < \infty$, $t>0$

$u(x,0) = \text{Sin } x$; $-\infty < x < \infty$

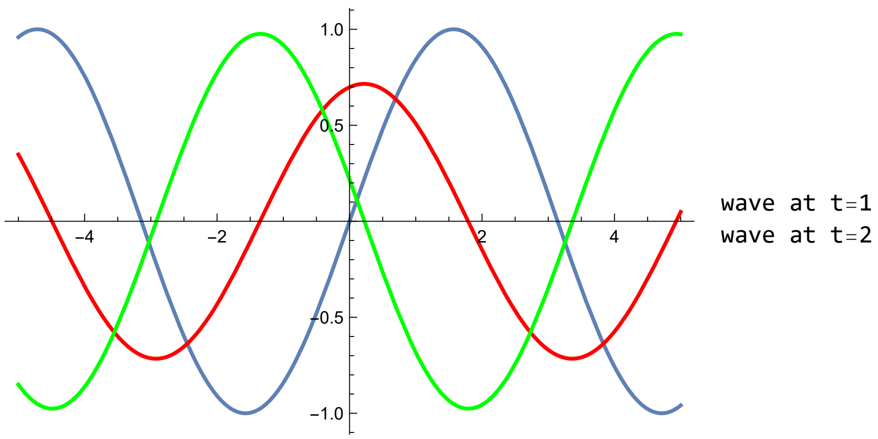
$u_t(x,0) = \text{Cos } x$; $-\infty < x < \infty$

In[147]:=

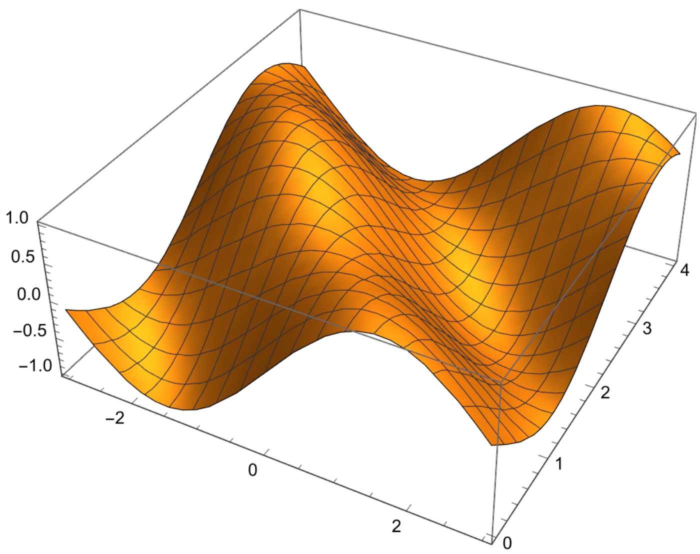
```
c = √2;
f[x_] := Sin[x]
g[x_] := Cos[x]
u[x_, t_] := 1/2 * (f[x + c t] + f[x - c t]) + 1/(2 c) * ∫_{x-c t}^{x+c t} g[s] ⅆs
Print[u[x, t]]
h0 = Plot[Evaluate[u[x, 0]], {x, -5, 5}];
h1 =
  Plot[Evaluate[u[x, 1]], {x, -5, 5}, PlotLegends → " wave at t=1", PlotStyle → Red];
h2 =
  Plot[Evaluate[u[x, 2]], {x, -5, 5}, PlotLegends → " wave at t=2", PlotStyle → Green];
Show [h0, h1, h2]
Plot3D[u[x, t], {x, -3, 3}, {t, 0, 4}]
```

$$\frac{\text{Cos}[x]\,\text{Sin}\left[\sqrt{2}\,t\right]}{\sqrt{2}} + \frac{1}{2}\left(-\text{Sin}\left[\sqrt{2}\,t - x\right] + \text{Sin}\left[\sqrt{2}\,t + x\right]\right)$$

Out[155]=



Out[156]=



Ques :- 5 $u_{tt} = \pi u_{xx}$; $-\infty < x < \infty$, $t>0$

$u(x,0) = 0$; $-\infty < x < \infty$

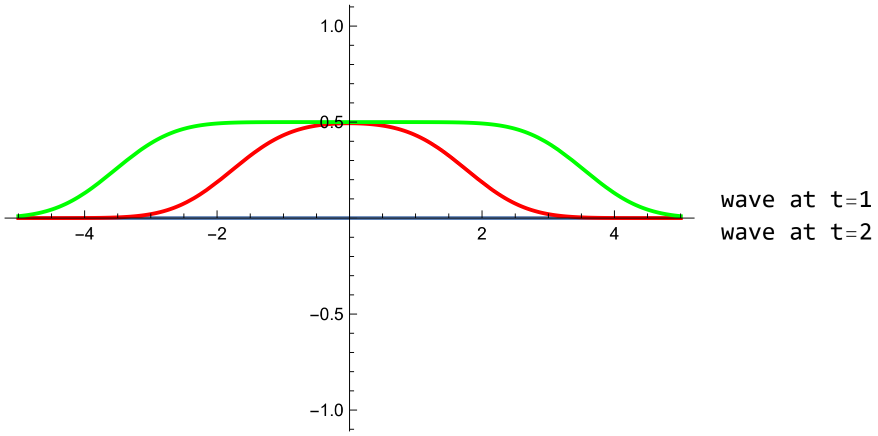
$u_t(x,0) = e^{-x^2}$; $-\infty < x < \infty$

In[157]:=

```
c = √π;
f[x_] := 0
g[x_] := e^(-x^2)
u[x_, t_] := 1/2 * (f[x + c t] + f[x - c t]) + 1/(2 c) * ∫_(x-c t)^(x+c t) g[s] ds
Print[u[x, t]]
h0 = Plot[Evaluate[u[x, 0]], {x, -5, 5}];
h1 =
  Plot[Evaluate[u[x, 1]], {x, -5, 5}, PlotLegends → " wave at t=1", PlotStyle → Red];
h2 =
  Plot[Evaluate[u[x, 2]], {x, -5, 5}, PlotLegends → " wave at t=2", PlotStyle → Green];
Show [h0, h1, h2]
Plot3D[u[x, t], {x, -3, 3}, {t, 0, 4}]
```

$$\frac{1}{4}\left(\text{Erf}\left[\sqrt{\pi}\ t - x\right] + \text{Erf}\left[\sqrt{\pi}\ t + x\right]\right)$$

Out[165]=



Out[166]=