Practical-3

Ques.1)Solve P.D.E. u_x - u_y =1, with Cauchy data u(x,0)= x^2

In[ ]:=
```
sol1 = DSolve[
  {x '[t] == 1, y '[t] == -1, u '[t] == 1, x[0] == s, y[0] == 0, u[0] == s ^ 2}, {x[t], y[t], u[t]}, t]
```

Out[ ]= $\{\{x[t] \to s + t,\ y[t] \to -t,\ u[t] \to s^2 + t\}\}$

```
Print["x[t]=", sol1[[1, 1, 2]]]
Print["y[t]=", sol1[[1, 2, 2]]]
Print["u[t]=", sol1[[1, 3, 2]]]
```

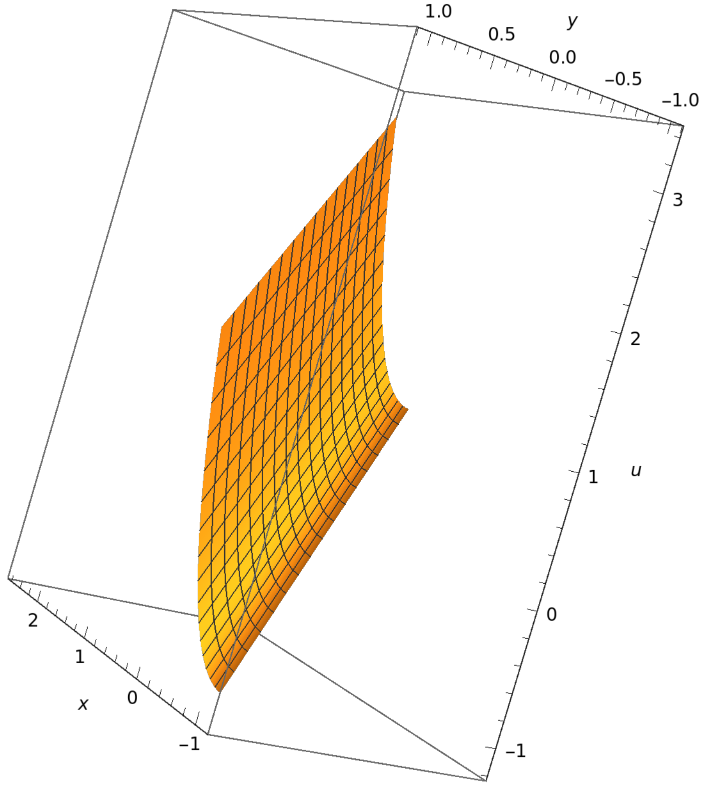
x[t]=s + t

y[t]=-t

u[t]=s^2 + t

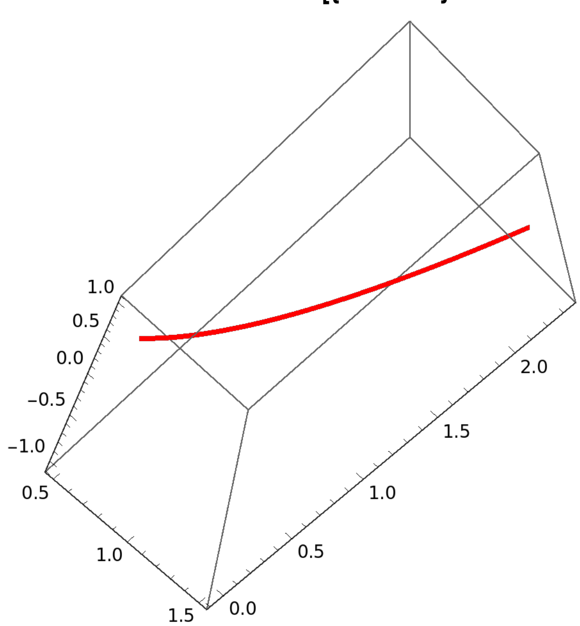
In[∘]:= **nfig1 = ParametricPlot3D[{sol1[[1, 1, 2]], sol1[[1, 2, 2]], sol1[[1, 3, 2]]},
{t, -1, 1}, {s, 0, 1.5}, AxesLabel → {x, y, u}]**

Out[∘]=



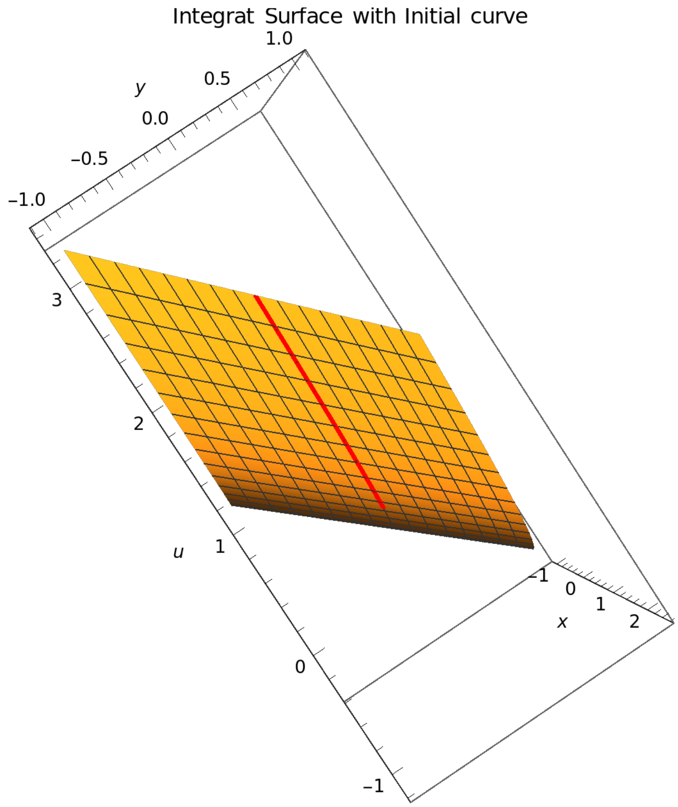
In[∘]:= **nfig2 = ParametricPlot3D[{s, 0, s^2}, {s, 0.5, 1.5}, PlotStyle → {Thick, Red}]**

Out[∘]=



In[ ]:=
```
Show[nfig1, nfig2, PlotLabel → "Integrat Surface with Initial curve"]
```

Out[ ]=



In[ ]:=
```
Clear All
```

Out[ ]= All Clear

Ques.2)Solve P.D.E. u_x +xu_y =0, with Cauchy data u(0,y)= Sinx

In[ ]:=
```
sol1 = DSolve[
  {x'[t] == 1, y'[t] == x[t], u'[t] == 0, x[0] == 0, y[0] == s, u[0] == Sin[s]}, {x[t], y[t], u[t]}, t]
```

Out[ ]= $\left\{\left\{x[t] \to t,\ y[t] \to \frac{1}{2}\left(2\,s + t^2\right),\ u[t] \to \mathrm{Sin}[s]\right\}\right\}$

In[ ]:=
```
Print["x[t]=", sol1[[1, 1, 2]]]
Print["y[t]=", sol1[[1, 2, 2]]]
Print["u[t]=", sol1[[1, 3, 2]]]
```

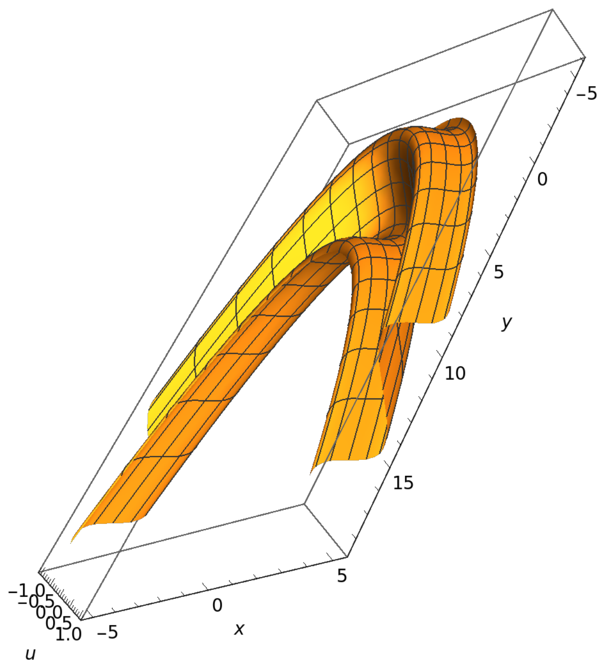
x[t]=t

y[t]= $\frac{1}{2}\left(2\,s + t^2\right)$

u[t]=Sin[s]

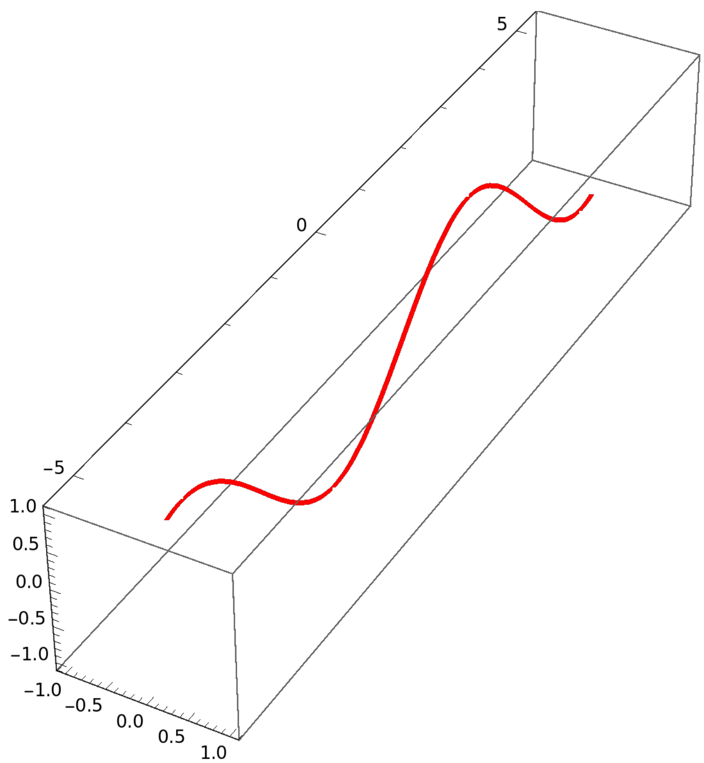
```
In[ ]:= nfig1 = ParametricPlot3D[{sol1[[1, 1, 2]], sol1[[1, 2, 2]], sol1[[1, 3, 2]]},
          {t, -5, 5}, {s, -5, 5}, AxesLabel → {x, y, u}]
```

Out[ ]=



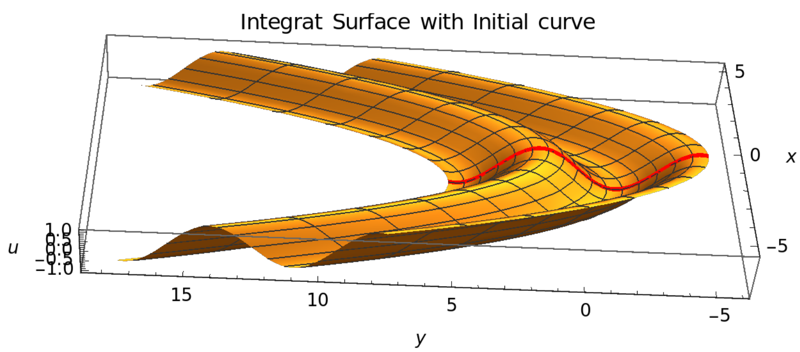

```
In[ ]:= nfig2 = ParametricPlot3D[{0, s, Sin[s]}, {s, -5, 5}, PlotStyle → {Thick, Red}]
```

Out[ ]=



```
In[ ]:= Show[nfig1, nfig2, PlotLabel → "Integrat Surface with Initial curve"]
```

Out[ ]=




```
In[ ]:= Clear All
Out[ ]= All Clear
```

Ques.3)Solve P.D.E. $3u_x + 2u_y$ =0, with Cauchy data u(x,0)= sin x

```
In[ ]:= sol1 = DSolve[
          {x'[t] == 3, y'[t] == 2, u'[t] == 0, x[0] == s, y[0] == 0, u[0] == Sin[s]}, {x[t], y[t], u[t]}, t]
Out[ ]= {{x[t] → s + 3 t, y[t] → 2 t, u[t] → Sin[s]}}
```

```
In[ ]:= Print["x[t]=", sol1[[1, 1, 2]]]
        Print["y[t]=", sol1[[1, 2, 2]]]
        Print["u[t]=", sol1[[1, 3, 2]]]
```

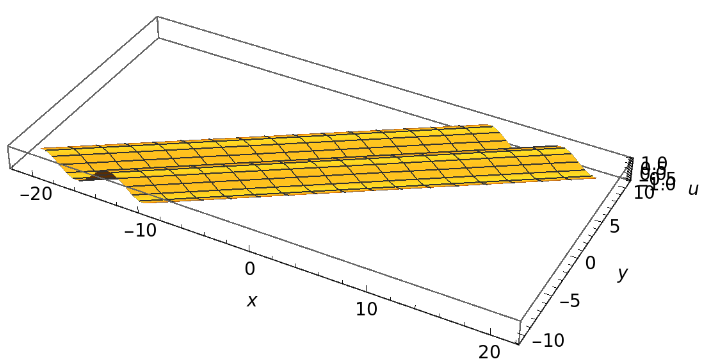
x[t]=s + 3 t

y[t]=2 t

u[t]=Sin[s]

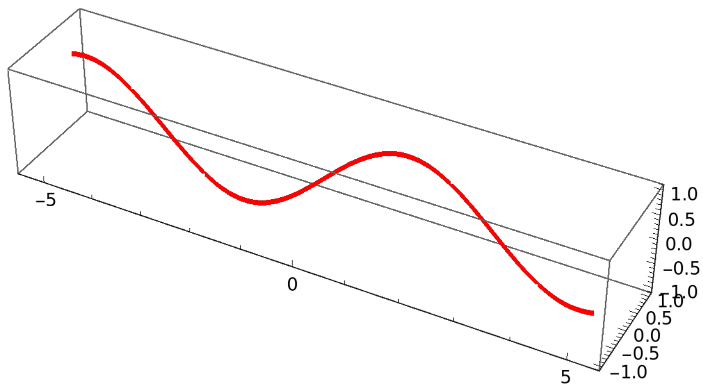
```
In[ ]:= nfig1 = ParametricPlot3D[{sol1[[1, 1, 2]], sol1[[1, 2, 2]], sol1[[1, 3, 2]]},
          {t, −5, 5}, {s, −5, 5}, AxesLabel → {x, y, u}]
```

Out[ ]=



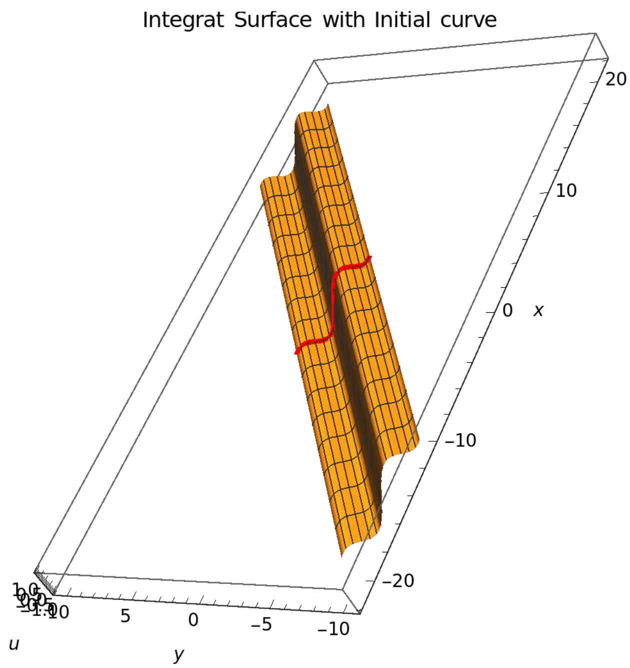
In[∘]:= `nfig2 = ParametricPlot3D[{s, 0, Sin[s]}, {s, -5, 5}, PlotStyle → {Thick, Red}]`

Out[∘]=



In[∘]:= `Show[nfig1, nfig2, PlotLabel → "Integrat Surface with Initial curve"]`

Out[∘]=



In[∘]:= `Clear All`

Out[∘]= `All Clear`

Ques.4) Solve P.D.E. $y u_x + x u_y = 0$, with Cauchy data u(0,y)= y^2

In[1]:= `sol1 = DSolve[{x'[t] == y[t], y'[t] == x[t], u'[t] == 0, x[0] == 0, y[0] == s, u[0] == s^2}, {x[t], y[t], u[t]}, t]`

Out[1]= $\left\{\left\{x[t] \to \frac{1}{2} e^{-t}\left(-1+e^{2t}\right) s,\ y[t] \to \frac{1}{2} e^{-t}\left(1+e^{2t}\right) s,\ u[t] \to s^2\right\}\right\}$

In[2]:=

```
Print["x[t]=", sol1[[1, 1, 2]]]
Print["y[t]=", sol1[[1, 2, 2]]]
Print["u[t]=", sol1[[1, 3, 2]]]
```

$x[t]=\frac{1}{2}e^{-t}\left(-1+e^{2t}\right)s$

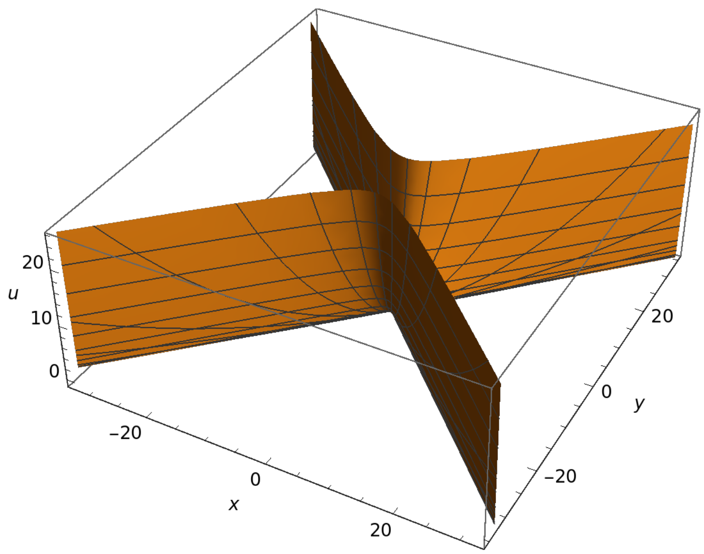
$y[t]=\frac{1}{2}e^{-t}\left(1+e^{2t}\right)s$

$u[t]=s^2$

In[5]:=

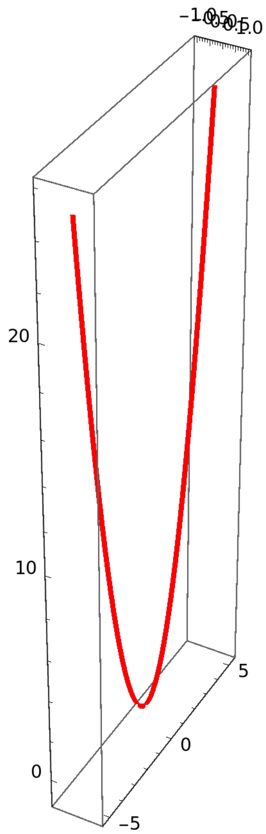
```
nfig1 = ParametricPlot3D[{sol1[[1, 1, 2]], sol1[[1, 2, 2]], sol1[[1, 3, 2]]},
  {t, -5, 5}, {s, -5, 5}, AxesLabel → {x, y, u}]
```

Out[5]=



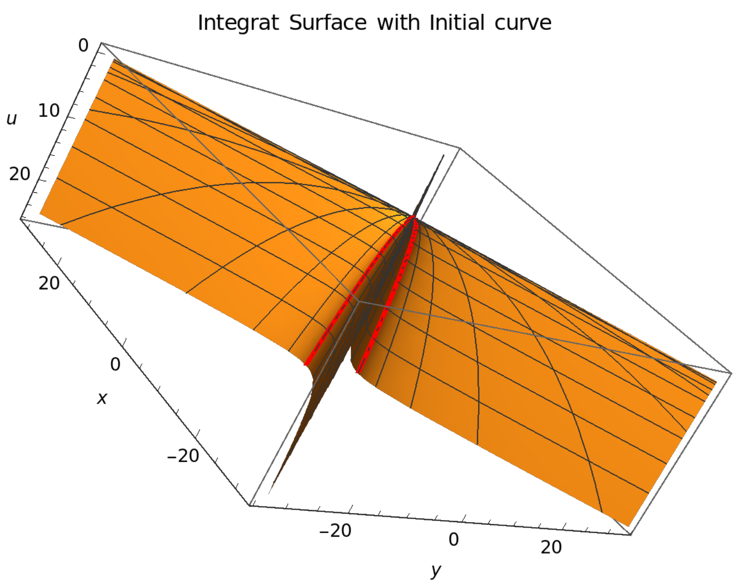
In[8]:= `nfig2 = ParametricPlot3D[{0, s, s^2}, {s, -5, 5}, PlotStyle → {Thick, Red}]`

Out[8]=



In[9]:= `Show[nfig1, nfig2, PlotLabel → "Integrat Surface with Initial curve"]`

Out[9]=



In[10]:= `Clear All`

Out[10]= `All Clear`

Ques.4) Solve P.D.E. $u u_x + u_y = 1/2$, with Cauchy data u(x,y)= 2x on y=x

In[11]:=
```
sol1 = DSolve[
  {x'[t] == u[t], y'[t] == 1, u'[t] == 1/2, x[0] == s, y[0] == s, u[0] == 2 s}, {x[t], y[t], u[t]}, t]
```

Out[11]= $\left\{\left\{u[t] \to \frac{1}{2}(4\,s+t),\ x[t] \to \frac{1}{4}\left(4\,s+8\,s\,t+t^2\right),\ y[t] \to s+t\right\}\right\}$

In[12]:=
```
Print["x[t]=", sol1[[1, 1, 2]]]
Print["y[t]=", sol1[[1, 2, 2]]]
Print["u[t]=", sol1[[1, 3, 2]]]
```

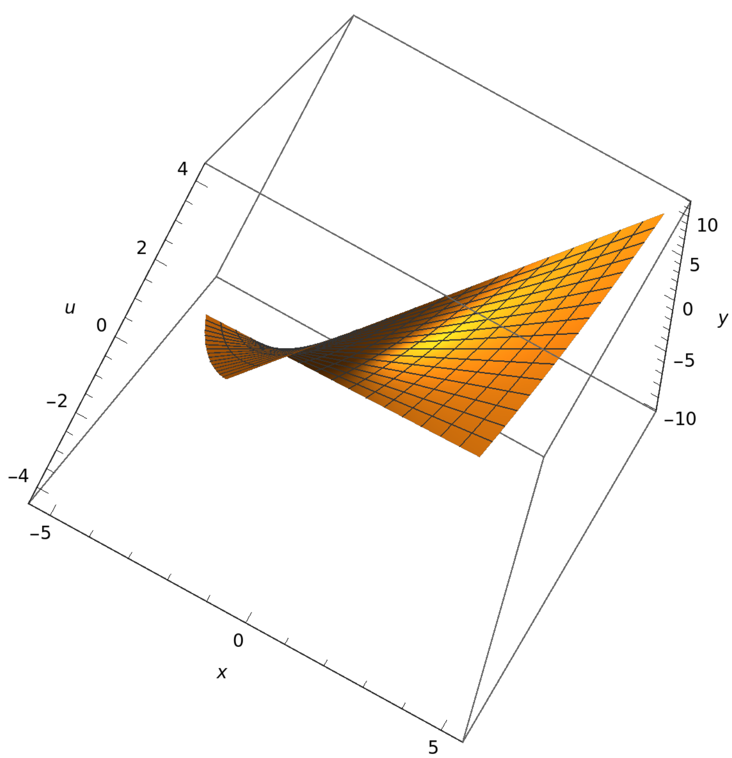
$x[t]=\frac{1}{2}(4\,s+t)$

$y[t]=\frac{1}{4}\left(4\,s+8\,s\,t+t^2\right)$

$u[t]=s+t$

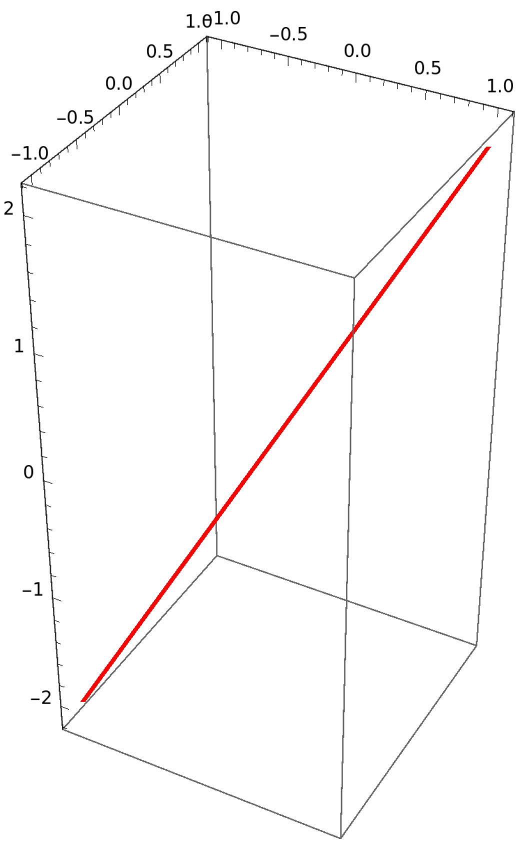
In[18]:=
```
nfig1 = ParametricPlot3D[{sol1[[1, 1, 2]], sol1[[1, 2, 2]], sol1[[1, 3, 2]]},
  {t, -2, 2}, {s, -2, 2}, AxesLabel → {x, y, u}]
```

Out[18]=



In[21]:= `nfig2 = ParametricPlot3D[{s, s, 2 s}, {s, -1, 1}, PlotStyle → {Thick, Red}]`

Out[21]=



In[22]:= `Show[nfig1, nfig2, PlotLabel → "Integrat Surface with Initial curve"]`

Out[22]=