Practical-2

Ques.1) Solve z_x + 2xz_y=1+z with Cauchy data z(x,y)=x^2 on y=3x+1

```
In[ ]:= A = D[z[x, y], x] + 2 * x * D[z[x, y], y] == 1 + z[x, y]
```

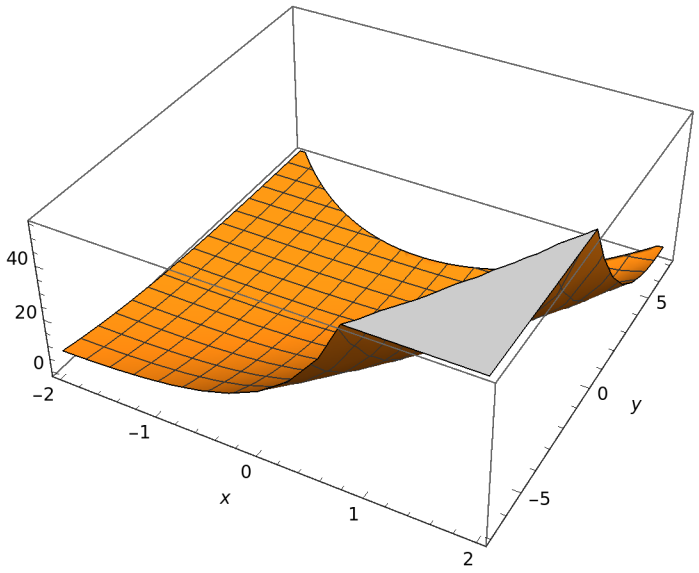
$$\text{Out[ ]= } 2\,x\,z^{(0,1)}[x,\,y] + z^{(1,0)}[x,\,y] == 1 + z[x,\,y]$$

```
In[ ]:= solb = DSolve[{A, z[x, 3 * x + 1] == x^2}, z[x, y], {x, y}]
```

$$\text{Out[ ]= } \left\{\left\{z[x,\,y] \to -\frac{1}{2\,e^{3/2}}\left(2\,e^{3/2} - 13\,e^{x+\frac{1}{2}\sqrt{13+4\,x^2-4\,y}} - 2\,e^{x+\frac{1}{2}\sqrt{13+4\,x^2-4\,y}}\,x^2 + 3\,e^{x+\frac{1}{2}\sqrt{13+4\,x^2-4\,y}}\,\sqrt{13+4\,x^2-4\,y} + 2\,e^{x+\frac{1}{2}\sqrt{13+4\,x^2-4\,y}}\,y\right)\right\}\right\}$$

```
In[ ]:= Plot3D[z[x, y] /. solb, {x, -2, 2}, {y, -7, 7}, AxesLabel → Automatic]
```

Out[ ]=




```
In[ ]:= Clear All
```

```
Out[ ]= All Clear
```

Ques.2) Solve xu_x + yu_y=2xy with Cauchy data u(x,y)=2 on y=x^2

```
In[ ]:= A = x * D[u[x, y], x] + y * D[u[x, y], y] == 2 * x * y
```

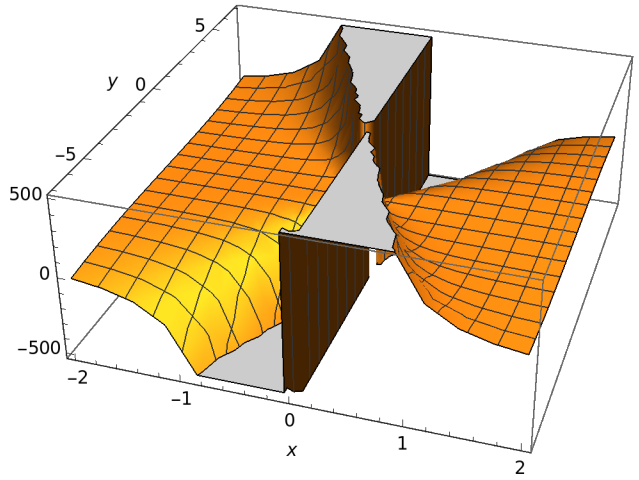
$$\text{Out[ ]= } y\,u^{(0,1)}[x,\,y] + x\,u^{(1,0)}[x,\,y] == 2\,x\,y$$

```
In[ ]:= solb = DSolve[{A, u[x, x^2] == 2}, u[x, y], {x, y}]
```

$$\text{Out[ ]= } \left\{\left\{u[x,\,y] \to \frac{2\,x^3 + x^4\,y - y^3}{x^3}\right\}\right\}$$

In[ ]:=
```
Plot3D[u[x, y] /. solb, {x, -2, 2}, {y, -7, 7}, AxesLabel → Automatic]
```

Out[ ]=



In[ ]:=
```
Clear All
```

Out[ ]=
```
All Clear
```

Ques.3) Solve xu_x + yu_y=u+1 with Cauchy data u(x,y)=x^2 on y=x^2

In[ ]:=
```
A = x * D[u[x, y], x] + y * D[u[x, y], y] == u[x, y] + 1
```

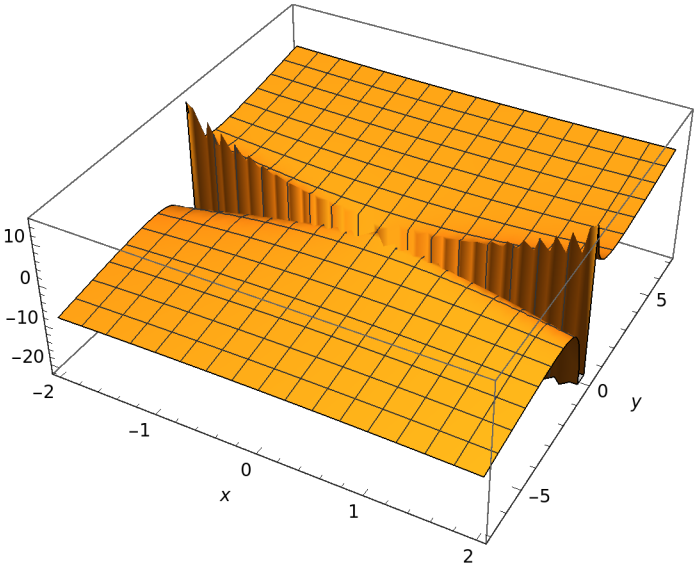
Out[ ]=
$$y\, u^{(0,1)}[x, y] + x\, u^{(1,0)}[x, y] == 1 + u[x, y]$$

In[ ]:=
```
solb = DSolve[{A, u[x, x^2] == x^2}, u[x, y], {x, y}]
```

Out[ ]=
$$\left\{\left\{u[x, y] \to \frac{x^2 - y + y^2}{y}\right\}\right\}$$

In[ ]:=
```
Plot3D[u[x, y] /. solb, {x, -2, 2}, {y, -7, 7}, AxesLabel → Automatic]
```

Out[ ]=



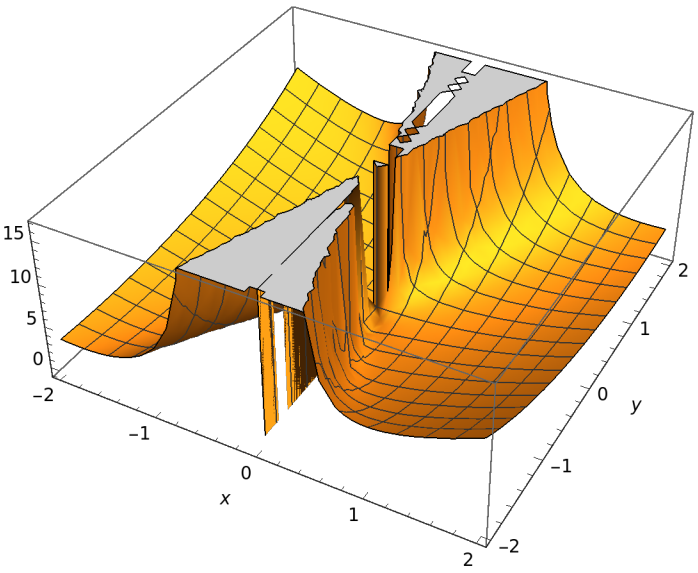
```
In[ ]:= Clear All
Out[ ]= All Clear
```

Ques.4) Solve xu_x + (x+y)u_y=u+1 with Cauchy data u(x,y)=x^2 on y=0

```
In[ ]:= A = x * D[u[x, y], x] + (x + y) * D[u[x, y], y] == u[x, y] + 1
Out[ ]= (x + y) u^(0,1)[x, y] + x u^(1,0)[x, y] == 1 + u[x, y]

In[ ]:= solb = DSolve[{A, u[x, 0] == x^2}, u[x, y], {x, y}]
Out[ ]= {{u[x, y] → e^(-y/x) (-e^(y/x) + e^(2 y/x) + x^2)}}

In[ ]:= Plot3D[u[x, y] /. solb, {x, -2, 2}, {y, -2, 2}, AxesLabel → Automatic]
   General: Exp[-768.] is too small to represent as a normalized machine number; precision may be lost.
   General: Exp[-1536.] is too small to represent as a normalized machine number; precision may be lost.
   General: Exp[-896.] is too small to represent as a normalized machine number; precision may be lost.
   General: Further output of General::munfl will be suppressed during this calculation.
Out[ ]=
```




```
In[ ]:= Clear All
Out[ ]= All Clear
```

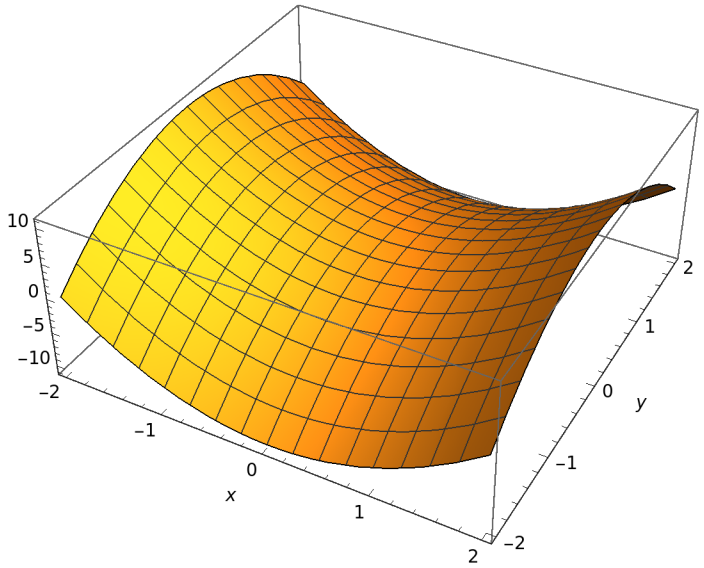
Ques.5) Solve yu_x + xu_y=0 with Cauchy data u(0,y)=exp(-y^2) on x=0

```
In[ ]:= A = y * D[u[x, y], x] + x * D[u[x, y], y] == 0
Out[ ]= x u^(0,1)[x, y] + y u^(1,0)[x, y] == 0

In[ ]:= solb = DSolve[{A, u[0, y] == e^(-y^2)}, u[x, y], {x, y}]
Out[ ]= {{u[x, y] → e^(x^2 - y^2)}}
```

In[ ]:=
```
Plot3D[u[x, y] /. solb, {x, -2, 2}, {y, -2, 2}, AxesLabel → Automatic]
```

Out[ ]=



In[ ]:=
```
Clear All
```

Out[ ]=
```
All Clear
```