PRACTICAL - 5

Solution of the heat equation ut = kuxx with the given initial equations.

1.) ut - kuxx = 0 , 0<x<5 , t>0

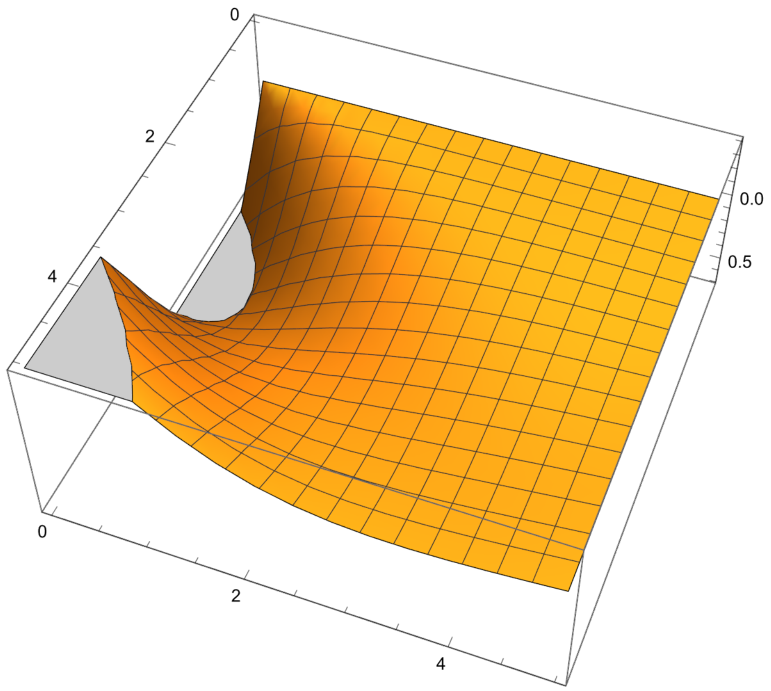
u(x,0) = 0 , u(0,t) = sint , u(5,t) = 0 , k= 1.

In[81]:=
```
k = 1;
L = 5;
pde = D[u[x, t], t] == k D[u[x, t], {x, 2}];
initialCondition = u[x, 0] == 0;
boundaryCondition1 = u[0, t] == Sin[t];
boundaryCondition2 = u[L, t] == 0;
solution = NDSolve[{pde, initialCondition, boundaryCondition1, boundaryCondition2},
  u[x, t], {x, 0, L}, {t, 0, 5} ]
```

Out[87]=
```
{{u[x, t] → InterpolatingFunction[ Domain: {{0., 5.}, {0., 5.}} Output: scalar ][x, t]}}
```

In[93]:=
```
Plot3D[u[x, t] /. solution, {x, 0, 5}, {t, 0, 5}]
```

Out[93]=



In[134]:=
```
ClearAll[x, u, t]
```

2.) ut - kuxx = 0 , 0<x<5 , t>0

u(x,0) = Sinx, u(0,t) = 0 , u(π,t) = 0 , k= 1.

In[199]:=

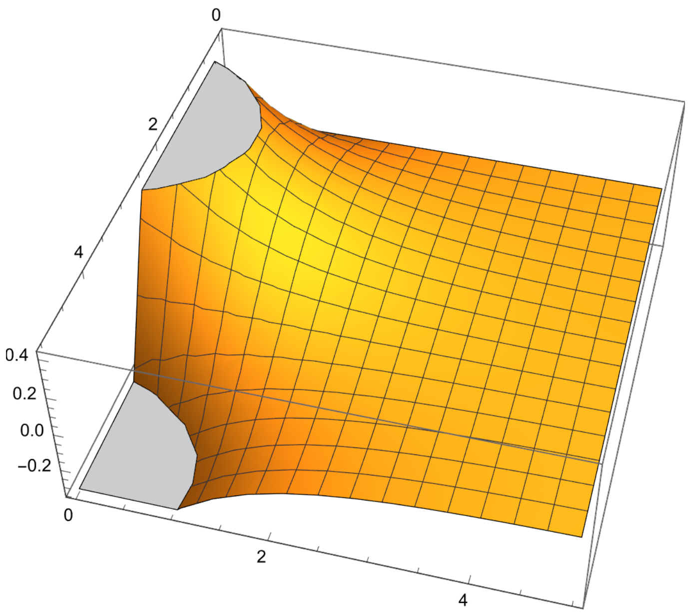
```
k = 1;

pde1 = D[u[x, t], t] == k D[u[x, t], {x, 2}];
in = u[x, 0] == Sin[x];
b1 = u[0, t] == 0;
b2 = u[π, t] == 0;
sol = NDSolve[{pde1, in, b1, b2}, u[x, t], {x, 0, π}, {t, 0, 5}]
Plot3D[u[x, t] /. sol, {x, 0, 5}, {t, 0, 5}]
```

Out[204]=

```
{{u[x, t] → InterpolatingFunction[ Domain: {{0., 3.14}, {0., 5.}} Output: scalar ][x, t]}}
```

Out[205]=



In[206]:=

```
ClearAll[x, t, u]
```

3.) ut - kuxx = 0 , 0<x<5 , t>0

u(x,0) = Tanhx, u(0,t) = t , u(10,t) = 0 , k= 1.

In[222]:=

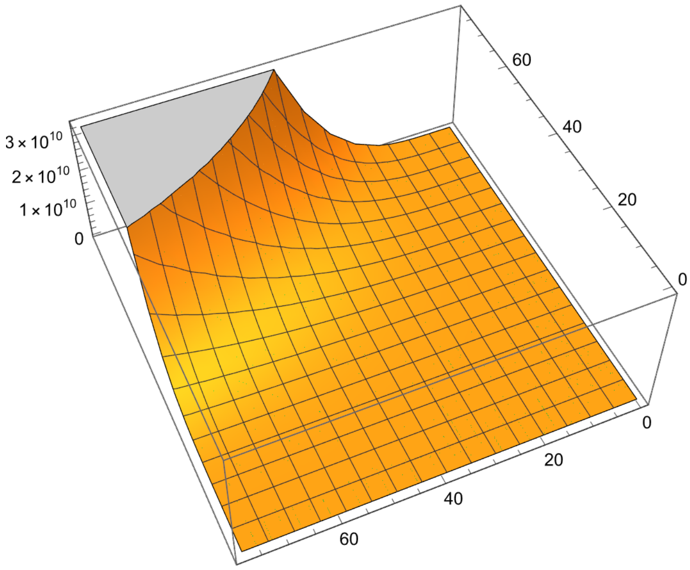
```
k = 1;
L = 10;
pde = D[u[x, t], t] == k D[u[x, t], {x, 2}];
initialCondition = u[x, 0] == Tanh[x];
boundaryCondition1 = u[0, t] == t;
boundaryCondition2 = u[L, t] == 0;
solution = NDSolve[{pde, initialCondition, boundaryCondition1, boundaryCondition2},
  u[x, t], {x, 0, L}, {t, 0, 5} ]
Plot3D[u[x, t] /. sol, {x, 0, 78}, {t, 0, 78}]
```

NDSolve: Warning: boundary and initial conditions are inconsistent.

Out[228]=

{{u[x, t] → InterpolatingFunction[Domain: {{0., 10.}, {0., 5.}} Output: scalar][x, t]}}

Out[229]=



4.) ut - kuxx = 0 , 0<x<5 , t>0

u(x,0) = Sinπx, u(0,t) = t , u(1,t) = t^2 , k= 1.

In[246]:=

```
k = 1;
L = 1;
pde = D[u[x, t], t] == k D[u[x, t], {x, 2}];
initialCondition = u[x, 0] == Sinπ[x];
boundaryCondition1 = u[0, t] == t;
boundaryCondition2 = u[L, t] == t^2;
solution = DSolve[{pde, initialCondition, boundaryCondition1, boundaryCondition2},
  u[x, t], {x, 0, L}, {t, 0, 5}]
Plot3D[u[x, t] /. sol, {x, 0, 5}, {t, 0, 5}]
```

Out[252]=

$$\left\{\left\{u[x,t] \to t + (-1+t)\,t\,x + \sum_{K[1]=1}^{\infty} \sqrt{2}\left(e^{-\pi^2 t K[1]^2}\,\text{Integrate}\left[\sqrt{2}\,\text{Sin}[\pi\,x\,K[1]]\,\text{Sin}\pi[x],\right.\right.\right.\right.$$

$$\{x, 0, 1\}, \text{Assumptions} \to \{x, t\} \in \text{Rectangle}[\{0, 0\}, \{1, 5$$

$$\frac{\sqrt{2}\left(-2(-1)^{K[1]} + \pi^2\left(-1+2(-1)^{K[1]}\,t\right)K[1]^2 + e^{-\pi^2 t K[1]^2}\left(2\,(\cdot\right.\right.}{\pi^5\,K[1]^5}$$

$$x\,K[1]]$$

Out[253]=