PRACTICAL - 5

Solution of the heat equation ut = kuxx with the given initial equations.

1.) ut - kuxx = 0 , 0<x<5 , t>0

u(x,0) = 0 , u(0,t) = sint , u(5,t) = 0 , k= 1.

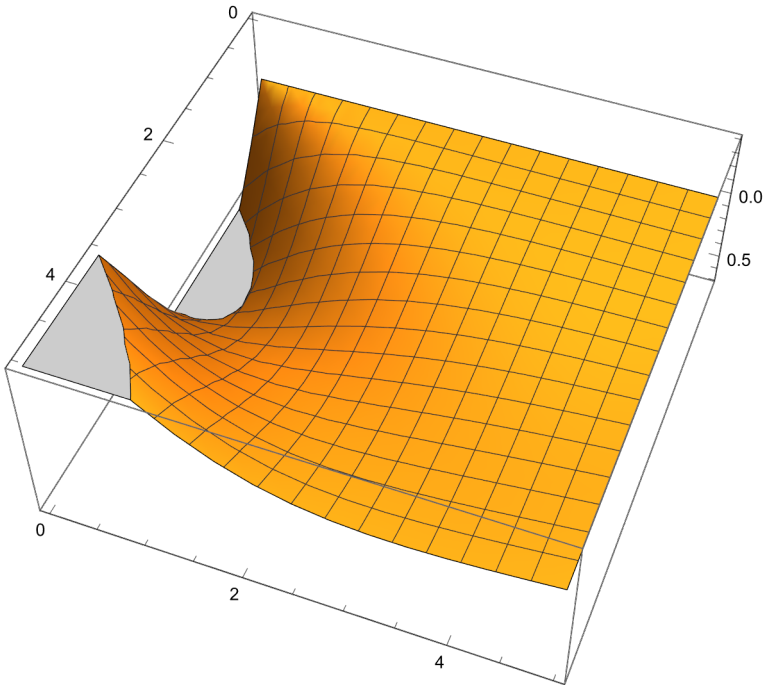
In[81]:=
```
k = 1;
L = 5;
pde = D[u[x, t], t] == k D[u[x, t], {x, 2}];
initialCondition = u[x, 0] == 0;
boundaryCondition1 = u[0, t] == Sin[t];
boundaryCondition2 = u[L, t] == 0;
solution = NDSolve[{pde, initialCondition, boundaryCondition1, boundaryCondition2},
  u[x, t], {x, 0, L}, {t, 0, 5} ]
```

Out[87]=

$$\left\{\left\{u[x, t] \to \text{InterpolatingFunction}\left[\text{Domain: } \{\{0., 5.\}, \{0., 5.\}\} \text{ Output: scalar}\right][x, t]\right\}\right\}$$

In[93]:=
```
Plot3D[u[x, t] /. solution, {x, 0, 5}, {t, 0, 5}]
```

Out[93]=



In[134]:=
```
ClearAll[x, u, t]
```

2.) ut - kuxx = 0 , 0<x<5 , t>0

u(x,0) = Sinx, u(0,t) = 0 , u(π,t) = 0 , k= 1.

In[199]:=

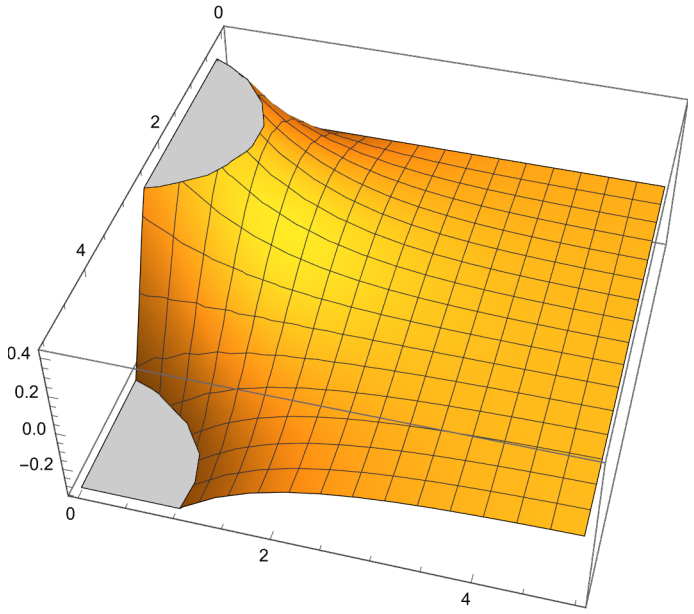
```
k = 1;

pde1 = D[u[x, t], t] == k D[u[x, t], {x, 2}];
in = u[x, 0] == Sin[x];
b1 = u[0, t] == 0;
b2 = u[π, t] == 0;
sol = NDSolve[{pde1, in, b1, b2}, u[x, t], {x, 0, π}, {t, 0, 5} ]
Plot3D[u[x, t] /. sol, {x, 0, 5}, {t, 0, 5}]
```

Out[204]=

$$\left\{\left\{u[x, t] \to \text{InterpolatingFunction}\left[\text{Domain: } \{\{0., 3.14\}, \{0., 5.\}\} \text{ Output: scalar}\right][x, t]\right\}\right\}$$

Out[205]=



In[206]:=

```
ClearAll[x, t, u]
```

3.) ut - kuxx = 0 , 0<x<5 , t>0

u(x,0) = Tanhx, u(0,t) = t , u(10,t) = 0 , k= 1.

In[222]:=

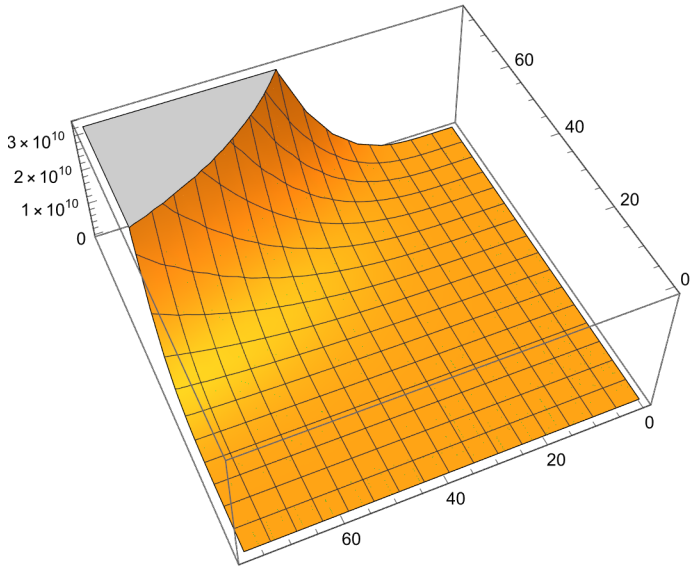
```
k = 1;
L = 10;
pde = D[u[x, t], t] == k D[u[x, t], {x, 2}];
initialCondition = u[x, 0] == Tanh[x];
boundaryCondition1 = u[0, t] == t;
boundaryCondition2 = u[L, t] == 0;
solution = NDSolve[{pde, initialCondition, boundaryCondition1, boundaryCondition2},
  u[x, t], {x, 0, L}, {t, 0, 5} ]
Plot3D[u[x, t] /. sol, {x, 0, 78}, {t, 0, 78}]
```

NDSolve: Warning: boundary and initial conditions are inconsistent.

Out[228]=

$$\left\{\left\{u[x, t] \to \text{InterpolatingFunction}\left[\text{Domain: } \{\{0., 10.\}, \{0., 5.\}\} \text{ Output: scalar}\right][x, t]\right\}\right\}$$

Out[229]=



4.) ut - kuxx = 0 , 0<x<5 , t>0

u(x,0) = Sinπx, u(0,t) = t , u(1,t) = t^2 , k= 1.

In[246]:=

```
k = 1;
L = 1;
pde = D[u[x, t], t] == k D[u[x, t], {x, 2}];
initialCondition = u[x, 0] == Sinπ[x];
boundaryCondition1 = u[0, t] == t;
boundaryCondition2 = u[L, t] == t^2;
solution = DSolve[{pde, initialCondition, boundaryCondition1, boundaryCondition2},
  u[x, t], {x, 0, L}, {t, 0, 5} ]
Plot3D[u[x, t] /. sol, {x, 0, 5}, {t, 0, 5}]
```

Out[252]=

$$\left\{\left\{u[x, t] \to t + (-1 + t)\, t\, x + \sum_{K[1]=1}^{\infty} \sqrt{2} \left(e^{-\pi^2 t K[1]^2} \text{Integrate}\left[\sqrt{2} \text{Sin}[\pi x K[1]] \text{Sin}\pi[x], \right.\right.\right.$$

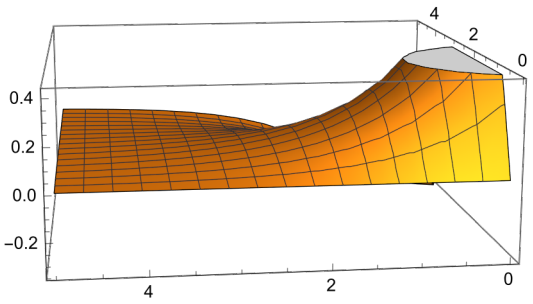
$$\left.\left.\left. \{x, 0, 1\}, \text{Assumptions} \to \{x, t\} \in \text{Rectangle}[\{0, 0\}, \{1, 5 \right.\right.\right.$$

$$\left.\left.\left. \frac{\sqrt{2} \left(-2 (-1)^{K[1]} + \pi^2 \left(-1 + 2 (-1)^{K[1]} t\right) K[1]^2 + e^{-\pi^2 t K[1]^2} \left(2 (- \right.\right.\right.\right.$$

$$\left.\left.\left.\left. \pi^5 K[1]^5 \right.\right.\right.\right.$$

$$\left.\left.\left.\left. x K[1]] \right.\right.\right.\right.$$

Out[253]=



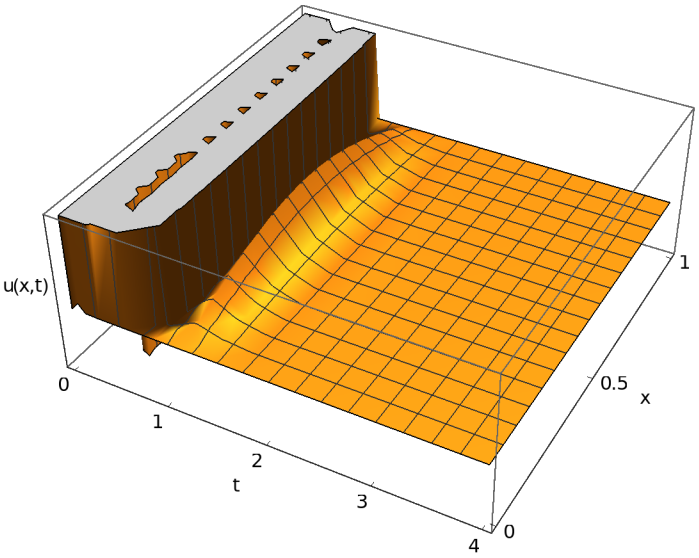
Question : $u_t = 4\,u_{xx}$, $u[x, 0] = x^2\,(1 - x)$, $u[0, t] = 0$, $u[1, t] = 0$, $0 < x < 1$, $t > 0$;

In[19]:=
```
Solution :
  eqn = {D[u[x, t], t] == 4 * D[u[x, t], {x, 2}], u[x, 0] == x^2 * (1 - x^2), u[0, t] == 0, u[1, t] == 0}
sol1 = u[x, t] /. NDSolve[eqn, u[x, t], {x, 0, 1}, {t, 0, 4}, PrecisionGoal → 3][[1]]
Plot3D[sol1, {t, 0, 4}, {x, 0, 1}, AxesLabel → {"t", "x", "u(x,t)"},
 Ticks → {{0, 1, 2, 3, 4}, {0, 0.5, 1}, {-1, 1}}]
```

Out[19]= $\left\{u^{(0,1)}[x, t] == 4\,u^{(2,0)}[x, t],\ u[x, 0] == x^2\,(1 - x^2),\ u[0, t] == 0,\ u[1, t] == 0\right\}$

Out[20]= InterpolatingFunction[ Domain: {{0., 1.}, {0., 4.}}
Output: scalar][x, t]

Out[21]=

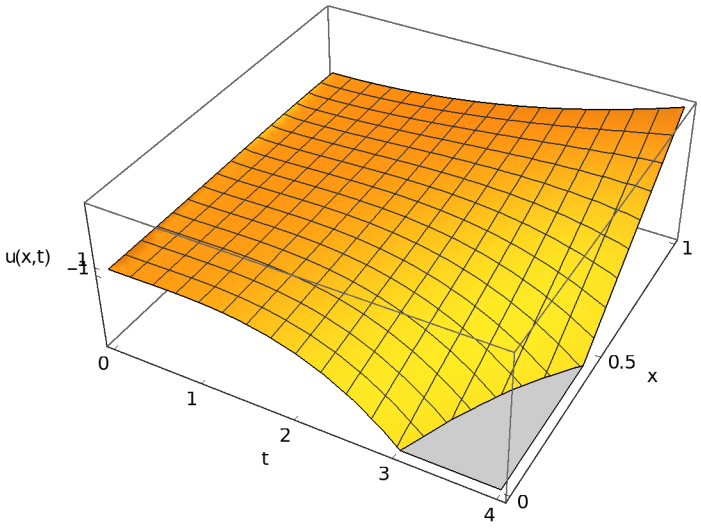


In[25]:=
```
question : eqn = {2 D[u[x, t], t] == 5 * D[u[x, t], {x, 2}],
   u[x, 0] == Sin[x^2] * (1 - x^4), u[0, t] == t^2 * (1 - t), u[1, t] == t^2}
sol1 = u[x, t] /. NDSolve[eqn, u[x, t], {x, 0, 1}, {t, 0, 4}, PrecisionGoal → 3][[1]]
Plot3D[sol1, {t, 0, 4}, {x, 0, 1}, AxesLabel → {"t", "x", "u(x,t)"},
 Ticks → {{0, 1, 2, 3, 4}, {0, 0.5, 1}, {-1, 1}}]
```

Out[25]=
$$\left\{2\, u^{(0,1)}[x, t] == 5\, u^{(2,0)}[x, t],\ u[x, 0] == \left(1 - x^4\right) \text{Sin}\left[x^2\right],\ u[0, t] == (1 - t)\, t^2,\ u[1, t] == t^2\right\}$$

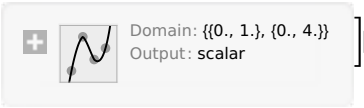
Out[26]=
$$\text{InterpolatingFunction}\left[\ \text{Domain: } \{\{0., 1.\}, \{0., 4.\}\}\ \text{Output: scalar}\ \right][x, t]$$

Out[27]=

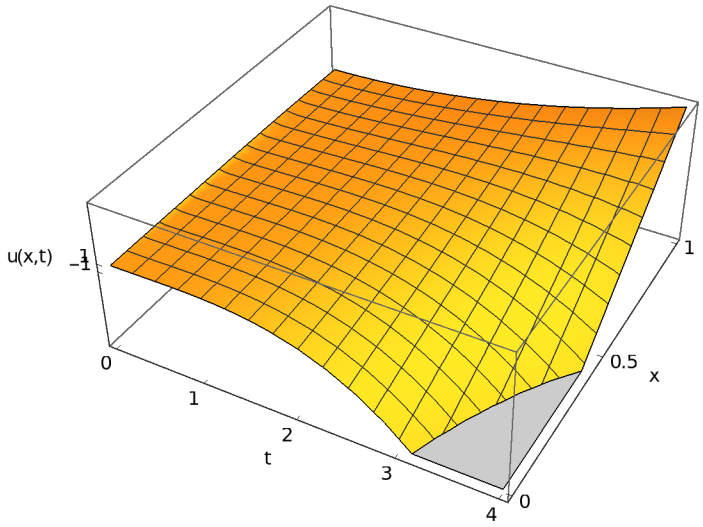


In[7]:=
```
question :
  eqn = {D[u[x, t], t] == 9 * D[u[x, t], {x, 2}],
  u[x, 0] == Sin[x] * (1 - x^3), u[0, t] == t^2 * (1 - t), u[1, t] == t^2}
sol1 = u[x, t] /. NDSolve[eqn, u[x, t], {x, 0, 1}, {t, 0, 4}, PrecisionGoal → 3][[1]]
Plot3D[sol1, {t, 0, 4}, {x, 0, 1}, AxesLabel → {"t", "x", "u(x,t)"},
 Ticks → {{0, 1, 2, 3, 4}, {0, 0.5, 1}, {-1, 1}}]
```

Out[7]= $\left\{u^{(0,1)}[x, t] == 9\, u^{(2,0)}[x, t],\ u[x, 0] == \left(1 - x^3\right) \text{Sin}[x],\ u[0, t] == (1 - t)\, t^2,\ u[1, t] == t^2\right\}$

Out[8]= InterpolatingFunction[ [x, t]

Out[9]=



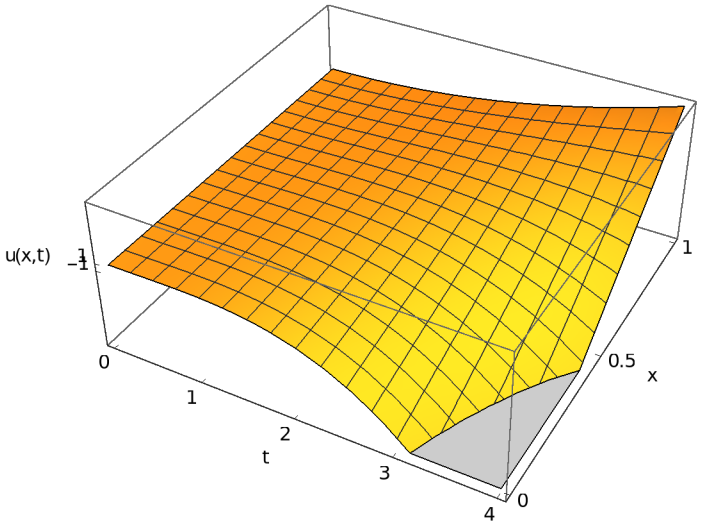

```
question : eqn =
 {3 D[u[x, t], t] == 5 * D[u[x, t], {x, 2}], u[x, 0] == x^3 * (1 - x^2), u[0, t] == 0, u[1, t] == 0}
```

In[10]:=
```
sol1 = u[x, t] /. NDSolve[eqn, u[x, t], {x, 0, 1}, {t, 1, 4}, PrecisionGoal → 3][[1]]
Plot3D[sol1, {t, 0, 4}, {x, 0, 1}, AxesLabel → {"t", "x", "u(x,t)"},
 Ticks → {{0, 1, 2, 3, 4}, {0, 0.5, 1}, {-1, 1}}]
```

Out[10]=
```
InterpolatingFunction[  Domain: {{0., 1.}, {1., 4.}}
Output: scalar ][x, t]
```

InterpolatingFunction: Input value {0.0000715, 0.000286} lies outside the range of data in the interpolating function. Extrapolation will be used.

Out[11]=



NDSolve: Equation or list of equations expected instead of eqn1 in the first argument eqn1.

ReplaceAll: {eqn1} is neither a list of replacement rules nor a valid dispatch table, and so cannot be used for replacing.

Out[14]=
```
u[x, t] /. eqn1
```

ReplaceAll: {eqn1} is neither a list of replacement rules nor a valid dispatch table, and so cannot be used for replacing.

ReplaceAll: {eqn1} is neither a list of replacement rules nor a valid dispatch table, and so cannot be used for replacing.

ReplaceAll: {eqn1} is neither a list of replacement rules nor a valid dispatch table, and so cannot be used for replacing.

General: Further output of ReplaceAll::reps will be suppressed during this calculation.

In[16]:=
```
question : eqn = {2 D[u[x, t], t] == 5 * D[u[x, t], {x, 2}],
   u[x, 0] == Sin[x^2] * (1 - x^4), u[0, t] == t^2 * (1 - t), u[1, t] == t^2}
sol1 = u[x, t] /. NDSolve[eqn, u[x, t], {x, 0, 1}, {t, 0, 4}, PrecisionGoal → 3][[1]]
Plot3D[sol1, {t, 0, 4}, {x, 0, 1}, AxesLabel → {"t", "x", "u(x,t)"},
 Ticks → {{0, 1, 2, 3, 4}, {0, 0.5, 1}, {-1, 1}}]
```

Out[16]=
$$\left\{2\, u^{(0,1)}[x, t] == 5\, u^{(2,0)}[x, t],\ u[x, 0] == \left(1 - x^4\right) \text{Sin}\left[x^2\right],\ u[0, t] == (1 - t)\, t^2,\ u[1, t] == t^2\right\}$$

Out[17]=
InterpolatingFunction[Domain: {{0., 1.}, {0., 4.}} Output: scalar][x, t]

Out[18]=