Article 3 : Review of functional data analysis

Functional data analysis (FDA) is a field of study concerned with the analysis and theory of data represented as functions, images, shapes, or other generalized objects. The foundational concept in FDA is the function, where each subject in a dataset is associated with one or more functions. The term "functional data analysis" was coined by Ramsay in 1982, but the roots of this area trace back to the work of Grenander and Rao in the 1950s and 1960s.

In FDA, the data are typically considered to be infinite-dimensional, posing challenges for both theory and computation. The high dimensionality of functional data varies depending on how the data were sampled. The prevailing methods in FDA involve non- and semi-parametric approaches due to the need for flexibility and the richness of information inherent in high-dimensional data.

First-generation functional data typically consist of independent real-valued functions recorded over a compact interval. These functions are often viewed as realizations of a stochastic process, typically assumed to be in a Hilbert space, such as 2()*L*2(*I*). Nonparametric smoothing techniques are commonly used in FDA, with the smoothness of individual random functions being important for regularization.

Next-generation functional data refer to more complex data objects, such as multivariate or correlated functional data, or data involving images or shapes. Examples include brain and neuroimaging data. While scientific interest often lies in understanding the underlying stochastic process, this process is often latent and cannot be directly observed. Data are usually collected discretely over time, leading to challenges in analysis.

Sparse and irregularly sampled functional data, often referred to as longitudinal data, present additional challenges in theory and methodology compared to densely sampled data. Various statistical tools and methods are available for handling functional data, including smoothing methods, functional analysis, and stochastic processes. Several software packages are also available for analyzing functional data.

Overall, FDA is a rapidly evolving field with a wide range of applications and research opportunities, spanning from traditional linear methods to more advanced nonlinear approaches. Despite its challenges, FDA offers powerful tools for analyzing and interpreting complex functional datasets.