

# A solution framework based on process mining, optimization, and discrete-event simulation to improve queue performance in an emergency department

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**Abstract.** Long waiting lines are a frequent problem in hospitals' Emergency Departments and can be critical to the patient's health and experience. This study proposes a three-stage solution framework to address this issue: Process Identification, Process Optimization, and Process Simulation. In the first, we use descriptive statistics to understand the data and obtain indicators as well as Process Mining techniques to identify the main process flow; the Optimization phase is composed of a mathematical model to provide an optimal physician schedule that reduces waiting times; and, finally, Simulation is performed to compare the original process flow and scheduling with the optimized solution. We applied the proposed solution framework to a case study in a Brazilian private hospital. Final data comprised of 65,407 emergency cases which corresponded to 399,631 event log registries in a 13-month period. The main metrics observed were the waiting time before the First General Assessment of a physician and the volume of patients within the system per hour and day of the week. When simulated, the optimal physician scheduling resulted in more than 40% reduction in waiting times and queue length, a 29.3% decrease of queue occurrences, and 54.2% less frequency of large queues.

**Keywords:** Emergency Department, Process Discovery, Physician Scheduling.

## 1 Introduction

A hospital's Emergency Department (ED) is the main gateway for a great part of its patients. One of the most frequent problems found in EDs is long waiting times, which result in negative experiences and can worsen the patient's condition. As the use of EDs has a random nature, patient arrival is difficult to predict, which makes tactical and operational hospital planning more complicated. Literature has shown that excessive workload to healthcare staff is prejudicial to patient's safety [1]. Also, in busy hours, patients that are not in severe conditions can end up going through long lines [2], as they do not have priority on getting serviced.

Therefore, it is important to understand and deal with the flow of patients, minimizing waiting times. For that purpose, several studies use discrete-event simulation as an auxiliary tool for operations management in healthcare. One of the most frequent types of work is to simulate the flow of patients, whether concerning clinics [3], surgery divisions [4] or emergency departments [5].

Among the studies regarding simulation, a few also consider Process Mining as an auxiliary tool to better represent the agents' behavior [6]. Liu et al. [7] highlight the difficulties found in the construction of the simulation model that can be overcome through the use of Process Mining software, which automatically creates the process flow; Kovalchuk et al. [8] use process mining, machine learning, discrete-event simulation and queuing theory to simulate the flow of patients; Wang et al. [9] use fuzzy logic to get to the simulation model along with Process Mining; and Rojas et al. [10] implement a performance analysis for emergency rooms through Process Mining.

One way to deal with long lines and improve patient experience in EDs is to raise the number of physicians on duty (increase capacity). However, it can be unsustainable for hospitals due to possible increases in costs, and constraints of workload and law. Therefore, tools to try to find optimal staffing levels can be applied. Savage et al. [11], for example, adopted a mathematical programming model while Green et al. [12] used queuing theory and Kuo [13] proposed a simulation-optimization approach, exploring solutions iteratively, which were evaluated with simulations at each step.

Hence, this paper proposes a solution framework that incorporates Process Mining, optimization, and discrete-event simulation to improve staff schedule and reduce waiting times. Firstly, descriptive statistics and a Process Mining tool provide the identification of the “as is” process and indicators of the system, which are the basis to the simulation model. Then, physician levels are proposed by an optimization model which aims to reduce lines. Lastly, these results are applied in the simulation model to test the optimization results and to represent potential gains with the optimized schedule. We applied this methodology using data from a large sized private hospital in Brazil.

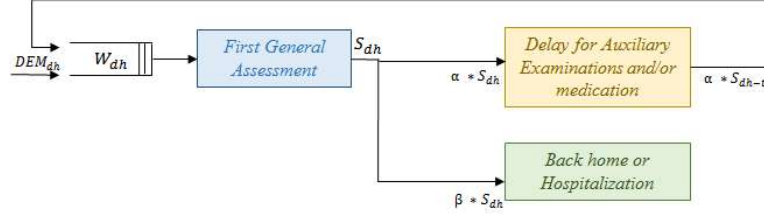
The remainder of this paper is structured as follows: Section 2 presents the problem definition; Section 3 describes the solution framework; Section 4 shows the numerical experiments performed; Section 5 presents the conclusions.

## 2 Problem Definition

This paper is driven by a hospital case study, and it aims to present a framework to analyze the current status of an ED, determine the optimal levels of its staffing at each hour and day of the week and to develop a simulation model that can accurately represent current and future situations.

The problem consists of a hospital's ED, where patients go to the screening process with a nurse, who determines whether they need vital medical care. If the patients do need vital care, they go straight to the physician assessment; else, they stay in a waiting room until a physician is available. After that, the patient can be directed to receive medication, stay in observation or be examined, and, afterward, is evaluated again, preferably by the same physician. These steps can be repeated until the patient is admitted

to the hospital or discharged. Patient's arrival might depend on the day of the week (d) and the hour of the day (h); (t) stands for the time between patients' assessments. Figure 1 represents a basic patient flow, which can be adapted to have other activities and, therefore, lines.



**Fig. 1.** Basic ED patient flow

In this study, we evaluated indicators of performance regarding the queue for the First General Assessment activity - the average waiting time, the frequency of queue, the frequency of queues with more than ten patients (large queue) and the average number of patients in the queue (queue length). Those indicators belong to the door-to-doctor stage of the process, and their evaluation corresponds to an important measure of ED performance [14].

The problem defined here consists, basically, in addressing physician staffing to meet patient arrival and volume, considering that there would be no increase in the available number of physicians, to minimize patient's waiting time. Furthermore, even though the proposed solution framework was applied in a specific ED in this paper, it can be adapted to other cases.

### 3 Solution framework

The framework proposed in this article consists of three general phases: Process Identification, that uses Process Mining and descriptive statistics to define and understand the “as is” process and its current indicators, especially waiting time; Process optimization, in which we defined the best physician staffing to the ED using a Mixed-Integer Programming (MIP) model; and Process simulation, that evaluated the MIP proposed solution and its expected benefits, considering the “as is” process and service times. The proposed framework is represented in Figure 2, and each phase is described in the following sections.

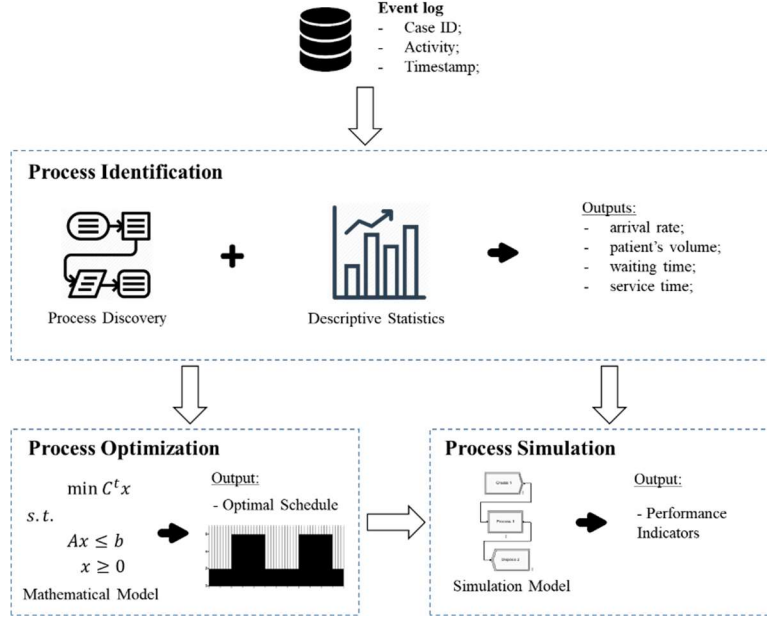


Fig. 2. Diagram of the proposed solution framework.

### 3.1 Process identification

In this first phase, we considered Process Mining (PM) techniques to identify the main flows within activities from data of each patient. Therefore, we obtained the event log from the ED and performed process discovery to identify the “as is” flow of activities. For this purpose, we considered the variants that accounted for 80% of cases as the main process flow to be studied [15]. Furthermore, we obtained and analyzed indicators from the process: the arrival rate of patients, average service time, number of patients in the system, and the current staff scheduling.

### 3.2 Process optimization

The optimization model is based on the process model identified with the Process Mining software. It represents the basic ED flow described in Section 2, but it can be adapted to have more activities and lines, that can be obtained in the Process Identification phase. Its objective is to determine the number of physicians that should be in service during each hour and day of the week, in accordance with service requirements. The objective of the model is to minimize the total number of patients waiting per hour in the ED. Table 1 presents the domains in which our model attributes are defined, while Tables 2 and 3 provide a complete listing of parameter and variable definitions.

**Table 1.** Sets, subsets and corresponding domains

Sets	Indexes	Domain	Description
Days ( $D$ )	$d$	$\{1, \dots, \text{PLAN\_HORIZON}\}$	Days of the planning horizon
Hours ( $H$ )	$h, h'$	$\{0, \dots, 23\}$	Hours of the day (24-hour clock format)
Shift Type ( $K$ )	$k$	$\{1, \dots,  K \}$	Type of shifts

**Table 2.** Model parameters

Parameters	Description	Unit
$PLAN\_HORIZON$	Number of days in the planning horizon	-
$DUR_k$	Duration of each shift $k$	Hour
$WL\_TOTAL$	Total workload available for the planning horizon	Hour
$CAP\_MIN_{dh}$	Minimum number of physicians required per hour $h$ and day $d$	-
$I\_QUEUE$	Initial queue at the beginning of the planning horizon	-
$START\_COMB_{h,h',k}$	Indicates the hour $h'$ covered if shift type $k$ start in hour $h$	$\{0, 1\}$
$\alpha$	Rate of patients seen who need ancillary exams and/or medication	-
$DEM_{dh}$	Demand (arrival of patients for each day $d$ and hour $h$ )	-
$CAP_{dh}$	Capacity (service rate - number of patients seen per physician each day $d$ and hour $h$ )	-

**Table 3.** Model variables

Variables	Description	Domain
$wait_{dh}$	Number of patients waiting in each hour $h$ and each day $d$	$\mathbb{R}_+$
$x_{dhk}$	Number of physicians assigned in the shift type $k$ starting in hour $h$ in the day $d$	$\mathbb{Z}_+$
$ic_{dh}$	Idle capacity in each hour $h$ and day $d$	$\mathbb{R}_+$
$served_{dh}$	Number of patients seen in each hour $h$ and day $d$	$\mathbb{R}_+$
$n_{dh}$	Number of physicians required for each hour $h$ and each day $d$	$\mathbb{Z}_+$

The proposed mathematical formulation is as follows:

$$\text{Min } Z = \sum_d \sum_h wait_{dh} \quad (1)$$

$$\sum_d \sum_h \sum_k x_{dhk} * DUR_k \leq WL\_TOTAL \quad (2)$$

$$\sum_h \sum_k x_{dhk} * START\_COMB_{h,h',k} \geq n_{dh} + CAP\_MIN_{dh} \quad \forall h, \forall d \quad (3)$$

$$wait_{dh} = I\_QUEUE + DEM_{dh} - served_{dh} \quad \forall d \mid d=1, \forall h \mid h=0 \quad (4)$$

$$wait_{dh} = wait_{d-1, '23'} + DEM_{dh} + \alpha * served_{d-1, '22'} - served_{dh} \quad \forall d \mid d \neq 1, \forall h \mid h = 0 \quad (5)$$

$$wait_{dh} = wait_{d, h-1} + DEM_{dh} + \alpha * served_{d, h-t} - served_{dh} \quad \forall d, \forall h \mid h > 0 \quad (6)$$

$$served_{dh} = CAP_{dh} * n_{dh} - ic_{dh} \quad \forall d, \forall h \quad (7)$$

$$wait_{dh} \in R_+, \quad \forall h \in H, \forall d \in D \quad (8)$$

$$x_{dhk} \in Z_+, \quad \forall d \in D, \forall h \in H, \forall k \in K \quad (9)$$

$$ic_{dh} \in R_+, \quad \forall h \in H, \forall d \in D \quad (10)$$

$$served_{dh} \in R_+, \quad \forall h \in H, \forall d \in D \quad (11)$$

$$n_{dh} \in Z_+, \quad \forall h \in H, \forall d \in D \quad (12)$$

The objective function in (1) consists of minimizing the total number of patients waiting each day and hour. Constraint (2) guarantees that the sum of the shift durations assigned is less than or equal to the total allowable workload. Constraint (3) enforces that the total of physicians assigned in each shift type ( $x_{dhk}$ ) meets hourly the staffing level required ( $n_{dh}$ ). Queue, number of patients served and idle capacity are computed through (4)–(7) based on the number of physicians required per hour to meet demand. Finally, constraints (8)–(12) define the domain of the decision variables.

### 3.3 Process simulation

In this framework, we apply Discrete Event Simulation (DES) because of its flexibility, versatility, and ability to model processes in a great level of detail [16]. We use it to evaluate the changes that would be caused if the schedule derived from the optimization phase was applied in the ED. The design of the model is formulated based on the process identification phase. To validate the model and evaluate the optimized schedule, we considered queue-related metrics such as the average number of patients in the queue, average waiting time, frequency of queues and frequency of queues with more than ten patients (large queues).

## 4 Numerical experiments

### 4.1 Case study and data

This study was conducted in a large sized private hospital located in the city of Rio de Janeiro, Brazil. Data comprised of 81,736 emergency cases over 13 months for patients aged over 14 years old and with a length-of-stay up to 24 hours in the general emergency, resulting in an event log of 544,226 registries. For each record, there are timestamps for their different stages in the emergency flow.

In the original schedule, physicians and nurses work in 12-hour shifts. The number of physicians varies during the hour and day of the week: 5 physicians in the first shift (8 am to 8 pm) and 3 in the following shift from Monday to Wednesday, and 4 physicians in the first shift and 2 in the following shift, from Thursday to Sunday. The doctors' entry time into the ED could be modified to meet the demand better. However,

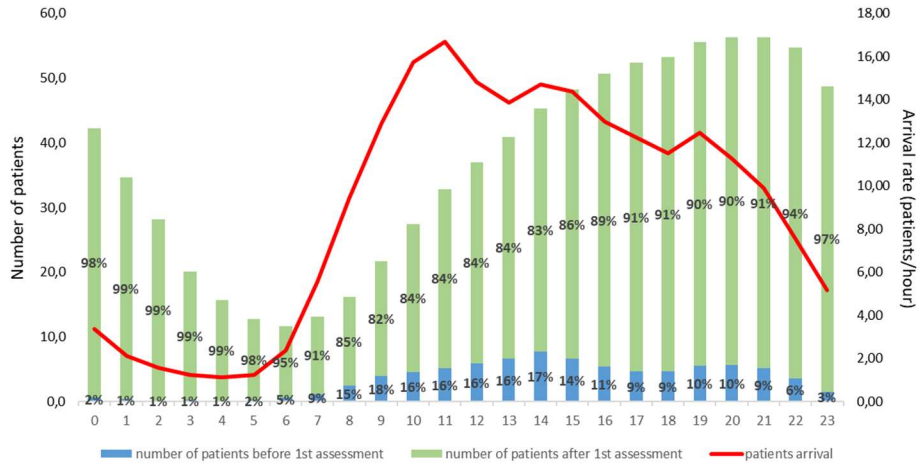
this is a difficult task without the support of a decision-making tool. The number of nurses and receptionists were considered as invariable during the whole day. We emphasize that no information that could identify the patients from each emergency case was provided.

In the Process Identification stage, we used *Disco* for process discovery and *R 3.5.2* (*tidy verse* package) for data analysis. In Process Optimization, the mathematical programming model was implemented and solved using AIMMS 4.64 and GUROBI 8.1 with default settings in Intel i5-7200U 2.5 GHz 20GB RAM computer. The optimal solution was obtained within 19 seconds (548 constraints and 883 variables). In Process Simulation, the model was implemented using Arena Simulation software version 14.7.

## 4.2 Results

To analyze the current status of the process, we estimated the hourly demand and capacity of each weekday, in terms of patient arrival and volume, as well as service time and waiting time for the First General Assessment. In Figure 3, we show the proportions regarding the average volume of patients before the first assessment (low blue bars) and during or after the first assessment (top green bars), and the average arrival rate (red line), for each hour of the day.

One can observe that the arrival rate of patients has an uprising trend until its peak at hour 11 (17 patients/hour), which also increases the proportion of patients waiting for the first assessment at this time. The total number of patients in the ED is the highest at around hour 20 (56 patients), which is also the time of shift change, and some of these patients stay in the ED until the next morning (hour 6).



**Figure 3.** Distribution of the number of patients and patient arrival rate in the ED

For the process discovery, we obtained the process variants that accounted for 80% of the emergency cases from the original event log. The comparison of the data from the original and the selected event log is shown in Table 4.

**Table 4.** Comparison between original and selected event log data

Data Information	Complete	Selected Variants	%
Emergency Cases	81,736	65,407	80
Event Log size	544,226	399,631	73
No. of Variants	1,731	22	1.3

We observed that the selection resulted in 22 out of 1,731 of all the identified process variants, which corresponded to 65,407 emergency cases and composed 73% of the events. Thus, there is a large variability of flows in the ED, which may be associated to different situations and high dynamics in this department due to the severity of patients, complexity of treatments and organizational procedures.

When exploring the 22 selected variants, we observed two main types of flows: either a patient gets discharged right after the first contact with a physician or the patient needs some extra assistance. The former comprehends the most frequent variant, while in the latter type the patient may need nursing assistance for medication, undergo in laboratory or image tests (e.g., x-ray, MRI, etc.), or stay in a dedicated bed for observation. In Table 5, we summarized the activities, events, and time from Variants 1 to 4, which corresponded to 70% of the emergency cases within the previously selected variants.

**Table 5.** Activities and proportions of Variants 1 - 4

Activity	Variant 1	Variant 2	Variant 3	Variant 4
1	Door	Door	Door	Door
2	Screening	Screening	Screening	Screening
3	Registration	Registration	Registration	Registration
4	FGA	FGA	FGA	FGA
5	ED discharge	In observation	Lab Tests	Lab Tests
6		ED discharge	In observation	Image Tests
7			ED discharge	In observation
8				ED discharge
No of Events	143,280	62,676	24,710	24,976
No of Cases	28,656 (43.8% )	10,446 (16% )	3,530 (5.4% )	3,122 (4.8% )

FGA - First General Assessment

For all variants, the Door-To-Doctor flow was similar. Variant 1 represents the patients that were discharged right after the First General Assessment (about 44%). “In observation” was common among Variants 2 to 4. Therefore, we considered the “as is” process as the set of activities from the 22 selected variants, considering the two main flows observed in Variant 1 and Variants 2 - 22.

In Table 6, we display the average number of cases, service time, and capacity per day of the week, regarding the First General Assessment activity. Monday has been the day with the largest demand, with an average of 197.4 cases, and it also presented the lowest average service time, 13.5 minutes, being the highest on Sunday, 16.3 minutes. Saturday presented the smallest demand (average of 141.1 patients). In overall, the ser-

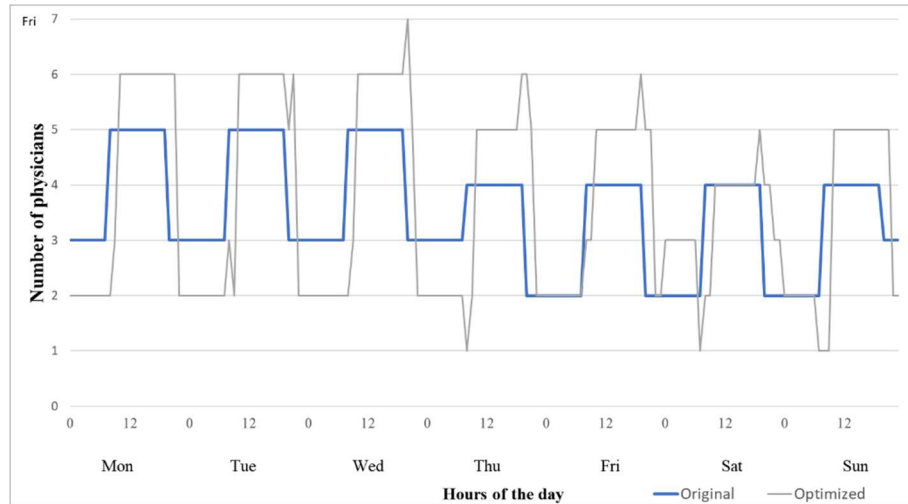


vice times presented a high variability, with standard deviations greater than the average times. We suggest that large patient volume could influence in physician performance by speeding up the First General Assessment. Estimated capacity also shows a slight decrease throughout the week, as the demand is also reduced, although it fluctuates around the average of 4.1 patients per hour.

**Table 6.** Demand, performance, and capacity of the emergency department

Days of the week	Number of cases		Service Time (min)		Estimated capacity per hour	
	Average	SD	Average	SD	Average	SD
Monday	197.4	31.4	13.5	22.8	4.5	2.6
Tuesday	180.3	25.5	14.5	23.2	4.1	2.6
Wednesday	179.7	19.8	15.0	21.5	4.0	2.8
Thursday	162.0	20.2	14.6	35.6	4.1	1.7
Friday	156.4	20.2	15.4	59.6	3.9	1.0
Saturday	141.1	19.3	13.8	28.8	4.4	2.1
Sunday	141.6	19.0	16.3	52.2	3.7	1.2
Total	165.2	29.9	14.7	36.4	4.1	1.6

These statistics were used as input to the Process Optimization and the Simulation. The optimization model provided a schedule considered optimal for this problem to improve the waiting time. The optimal and original schedules are shown in Figure 4.



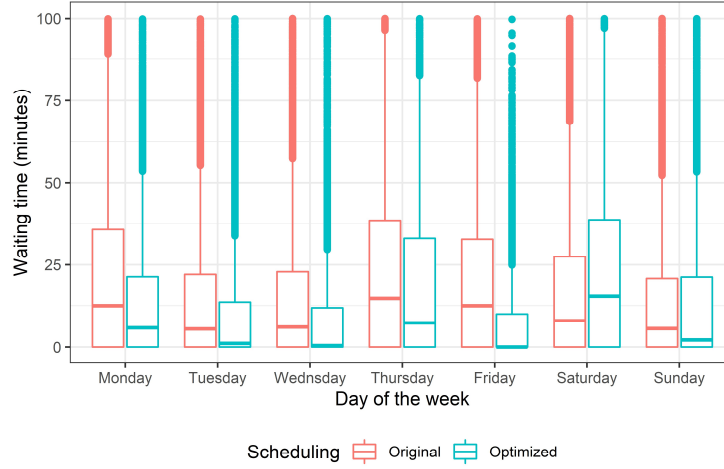
**Figure 4.** Original (blue) and optimal (gray) schedules

In the optimized schedule, there were usually more physicians during the day, between 10 am and 7 pm, which is the period of the day that generally has the highest volume of patients in the system. In the original schedule, there is a maximum of 5 physicians at a given time, while in the optimized schedule, the maximum is 7.

Therefore, using the results from the previous two phases, the distribution of times and arrival rates, the process flow, and the optimal scheduling, we obtained a simulation model that represents the studied ED. We simulated one week of the ED process, starting on Monday, and considered one week for a warm-up period. Patients' inter-arrivals times followed an exponential distribution and service times for each activity in the model were fitted to the most adequate distribution.

To validate the simulation model, we compared its results to data contained in the hospital's information system, such as the average number of patients that arrive in the ED and the service time for the First General Assessment in each day of the week, considering the original physician scheduling. The simulation presented an overall average of 160.7 people arriving in the ED per day ( $SD = 27$ ), which is a difference of 2.7% to the overall average from the ED's data in Table 6. Service time also had similar results (average of 14.4 min,  $SD = 8.3$ , a difference of 2%), being the largest difference on Monday, 11%. These results have a satisfactory accuracy level for our research interest, which also remarks on the credibility of the process knowledge derived from the results provided by the process discovery phase.

Then, we simulated the ED flow with the schedule provided by the optimization model. In Figure 5, we compare the distribution of waiting times at the First General Assessment obtained from simulating the two schedules for each day of the week.



**Figure 5.** Waiting times for the original and optimal schedules

The simulation model showed that the optimized schedule could provide reductions in the waiting time for every day of the week, except for Saturday, in which there was an increase of 3% in the average waiting time (less than a minute), which could be influenced by the assignment of fewer physicians in this day as shown in Fig. 4. Results regarding the other indicators of queue performance are in Table 7.

**Table 7** – Queue indicators for original and optimal schedules (simulation)

Indicators	Original Schedule	Optimized Schedule	$\Delta\%$
Average number of patients in queue	3.0	1.6	-46.7
Average waiting time (min)	26.5	14.6	-44.9
Frequency of queues (%)	48.8	34.5	-29.3
Frequency of queues > 10 patients (%)	8.3	3.8	-54.2

The proposed schedule could lead to considerable reductions in queue performance indicators, according to the Simulation model. Both the overall average waiting time and the number of patients in the queue for the FGA activity were decreased by more than 40%, considering all the simulation runs. Also, the frequency of queues was 29.3% lower, and the presence of longer lines decreased by more than 50%. Hence, those results indicate improvements in the process that can provide better patient's experience and care.

## 5 Conclusion

In this study, we proposed a solution framework to improve waiting times in an emergency department. It is comprised of descriptive statistics to analyze data, process mining to discover the process flow, optimization to find the optimal schedule, and discrete-event simulation to test the proposed schedule. The application of the methodology was performed using data from a large sized private hospital. According to the Simulation, our solution framework could provide an average reduction of 45% in the waiting times and 47% in the number of patients in queue. Moreover, the simulation showed that the frequency of lines could decrease by 29%, with 54% less large queues (more than 10 patients). We emphasize that the proposed schedule has not yet been applied to the ED, however, the simulation results show promising improvements. Our solution framework could easily be adapted to different contexts of ED's process evaluation and improvement. Suggestions for future works include analyzing ED indicators other than Door-to-Doctor time and developing a more thorough model that includes other lines, and the severity of cases (priorities).

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