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2017
MCM/ICM
Summary Sheet

Sustainable cities are needed

With the development of urbanization， the sustainable city is needed urgently. Thus, we make a metric Q to analyze whether the city is a sustainable city, and then we use the metric Q to analyze other questions.

Firstly, we select a series of factors to reflect the three E's evaluation criteria. And then we make a basic evaluation model. We can evaluate whether its growth fits smart growth by the unary linear regression model.

Secondly, in order to make the evaluation more convenient, we make the multiple linear regression model to define a metric value Q. And then we calculate the standard Q in different years by the Gaussian distribution. We can also choose a sustainable city by the first step, and then we can establish the multiple linear regression model with the help of MATLAB and SPSS. So we can evaluate the city whether is sustainable by the value Q. That method can make the problem easy.

Thirdly, we establish the incidence matrix advantage analysis model to analyze that how a city can become sustainable. If a city wants to fit the smart growth, we need to analyze which factors are more important than others. For this, we should consider its data and realistic situation. So we use ArcGIS to analyze the geography of the city. That makes our plans fit local area better. And we can also use this model to rank the importance of factors.

Finally, we use STRIPAT model to calculate factors that can support the population of each city by 2050. With the help of SPSS, we use the least-squares analysis method to calculate the factors. And then we can make a conclusion.

In this paper, we choose two mid-sized cities on two different continents to meet the requirement of ICM. We choose Hainan Tibetan Autonomous Prefecture in China and Canberra in Australia. The two cities can also test our models.

Contents

1 Introduction

1.1 Background

With the development of the social economy, urbanization process is greatly accelerated. It has a lot of advantages in many aspects of people's lives. For example, the transport of cities is more convenient and people's life expectancy significantly improved. But, some scientists find that the rapid development of cities makes a lot of problems against the sustainable development. Some places even in some mid-sized cities of China have problems with environment, resources, and social equality, etc. Thus sustainable cities are needed necessarily.

1.2 Smart growth

Smart growth is an approach to development that encourages a mix of building types and uses, diverse housing and transportation options, development within existing neighborhoods, and community engagement. Smart growth focuses on building cities that embrace the three E' s(Economically prosperous, socially Equitable, and Environmentally Sustainable)of sustainability. The ten principles for smart growth are

1 Mix land uses
2 Take advantage of compact building design
3 Create a range of housing opportunities and choices
4 Create walkable neighborhoods
5 Foster distinctive, attractive communities with a strong sense of place
6 Preserve open space, farmland, natural beauty, and critical environmental areas
7 Strengthen and direct development towards existing communities
8 Provide a variety of transportation choices
9 Make development decisions predictable, fair, and cost effective
10 Encourage community and stakeholder collaboration in development decisions

2 Basic assumptions and symbol descriptions

2.1 General assumptions

- Cities that we consider won't have major disaster. We assuming that because cities that we choose haven't experienced major disaster. So we can believe that the population growth in line with natural growth.
- When we choose the two mid-sized cities, we only consider resident population and we don't consider floating population.
- There haven't been large-scale population movements in cities that we choose.
- Economic growth is stable, has not suffered greatly from the impact of the financial crisis.

2.2 Symbol descriptions

Symbols	descriptions
$x_i(i=1,2,\ldots,n)$	the independent variables of unary linear regression equation(different factor values)
$y_i(i=1,2,\ldots,n)$	the dependent variables of unary linear regression equation (different factor values)
$k_i(i=1,2,\ldots,n)$	coefficients of the independent variables in unary linear regression equation
b	The constant term of the independent variables in unary linear regression equation
$p_i(i=1,2,\ldots,n)$	the independent variables of multiple linear regression equation
Q	the dependent variables of multiple linear regression equation
$a_i(i=1,2,\ldots,n)$	weights of the independent variables in multiple linear regression equation
c	The constant term of the multiple variables in unary linear regression equation
r_{ij}	The level of reference series associated with compared series
R_{ij}	Incidence matrix

$P_i(i=1,2,\ldots,n)$	The different factors
P_1	Financial revenue （Billion yuan）
P_2	Average path length （10^{-2}km/ people）
P_3	Per capita area of roads （$10^{-2}km^2$/people）
P_4	Population density （10-1km2/people）
P_5	Per capita GDP （10^7 yuan/people）
P_6	The green rate （10^{-1} percent）

P_7	Per capita arable land （10^{-4}km2/ people）
P_8	Pollution treatment capacity （$10^4 m^3$/day）

3 The basic smart growth's evaluation model

In order to meet ten principles for smart growth and the three E's of sustainability, we find some reasonable factors. By studying these factories, we can evaluate the smart city [1].

- Firstly, we choose factors of the city development to meet the smart city's requirement.
- Secondly, we consider the relationship of two factors with the method of the unary linear regression model.
- Thirdly, through the conclusion of the unary linear regression equation, determine them whether meet linear relationship (index relationship).
- Finally we make the conclusion by analyzing the last step.

3.1 Choosing reasonable factors

First, we assume that the growth that meets the three E's of sustainability is smart growth. We start considering from three E's of sustainability, and then we choose some reasonable factors to meet that. In order to consider economically prosperous, we choose financial expenditure, population density and per capita GDP to study; About socially equitable, we find average path length, per capita area of roads and per capita arable land to study; About environmentally sustainable, we choose the green rate, pollution treatment capacity to consider.

We summarize those factors as following figure shows:

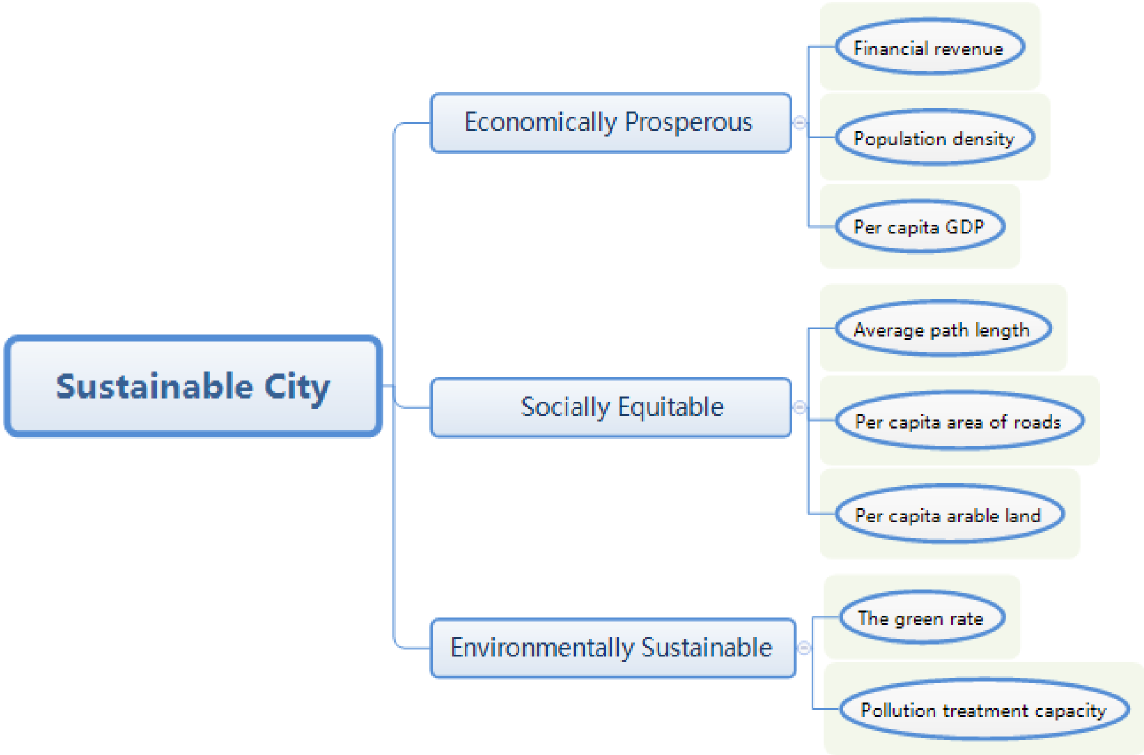

Figure 1 The reasonable factors from three E's of sustainability

3.2 The unary linear regression model

In order to evaluate whether the city with the smart growth, we choose two factors for a group. We can learn whether the two factors have linear (or index) relationship with the unary linear regression method:

If they have linear relationship, they need to meet the following formula:

$$y_i = kx_i + b(i = 1,2,...,n) \tag{1}$$

If they have index relationship, they need to meet the following formula:

$$y_i = a^{x_i} + b(i = 1,2,...,n) \tag{2}$$

We can also convert that index relationship to linear relationship :

$$\ln y_i = x_i \ln a + b' (i = 1,2,...n) \tag{3}$$

If the most factors have linear (or index) relationship with others, we can consider that city into smart growth by our analysis [1]. We make that evaluation model to prepare for the multiple linear regression model.

4 The multiple linear regression model

Considering the requirement of The International City Management Group (ICM), we should judge the level of smart city development. In order to make the judgement more convenient, we decide to quantitate the judgement standard. Because there are many factors influencing urban smart growth, we choose multiple linear regression model to discuss this question. And describe the level of the factors effect with the method of giving weight value. We can get the basic multiple liner regression model:

$$Q = a_1 p_1 + a_2 p_2 + ... + a_n p_n + c \qquad (4)$$

In order to establish the multiple linear regression model, we should determine the weight value a_i(i=1,2,..,n).

- Firstly, we choose a smart growth city generally by the city's previous data. That city isn't restricted by the number of the population.
- Secondly, we check that city whether meet the smart growth's requirement. We check that by the basic smart growth's evaluation model we established at page 3 in our paper.
- Thirdly, we define the metric Q by the Gaussian distribution. The function of the metric Q is to measure the success of smart growth of a city. And then, we use SPSS to determine the weight value a_i(i=1,2,..,n) in the multiple linear regression model.
- Finally, we establish the multiple linear regression model. And then we can determine the both cities that we choose whether with the smart growth.

4.1 Finding a smart growth city

In order to get more reasonable weight values , we choose Beijing's previous data to start our studying. Because according to the scientific research in 2000 year. Beijing has been meeting smart growth principles. Beijing's previous data is in the following table 1:

Table 1 Beijing's previous data[2]

year	Financial revenue (Billion yuan)	Average path length (10^{-2}km/ peoplle)	Per capita area of roads (10^{-2}km^2/people)	Population density (10^{-1}km^2/people)	Per capita GDP (10^7 yuan/people)	The green rate (10^{-1} percent)	Per capita arable land (10^{-4}km^2/ people)	Pollution treatment capacity (10^4m^3/day)
1978	50.46	238.4394722	184.8537005	636.9124181	124.842226	223	492.5232358	23.2
1979	47.75	237.5431947	180.3589343	655.621492	133.8758221	223	475.8443875	23.2
1980	51.29	241.6233551	184.0097313	660.8834191	153.8206347	201	470.8614398	23.2
1981	49.12	243.0374238	189.5126197	671.7726848	151.4360313	201	461.9429939	25.2
1982	47.25	285.6684492	224.3850267	683.3196913	165.6684492	201	453.3048128	25.2
1983	39.84	296.8421053	238.4210526	694.2820393	192.7368421	201	445.1263158	25.2
1984	45.62	303.4196891	247.9792746	705.2443873	224.4559585	201	437.0373057	25.2
1985	52.44	303.6697248	253.312946	716.9375585	262.0795107	221	428.7084608	25.2
1986	60.34	295.5252918	248.9299611	751.2862488	277.1400778	228.6	407.4990272	26.4
1987	63.62	294.8424069	251.2893983	765.1718896	312.1298949	229	398.9627507	26.4
1988	68.11	296.9839774	254.5711593	775.4034144	386.6163996	250	391.9557022	26.4
1989	71.05	300.9302326	261.8604651	785.6349392	424.1860465	260	385.5683721	26.4
1990	74.01	301.6574586	267.4953959	793.6739944	461.1418048	280	380.0322284	30.4
1991	77.02	302.3765996	286.4716636	799.52058	547.440585	284.3	375.8473492	4.5
1992	80.25	289.3829401	291.4700544	805.3671656	643.4664247	303.3	371.0136116	4.5
1993	84.1	295.4136691	305.5755396	812.6753976	796.942446	313.3	364.7149281	24.5
1994	99.85	294.7555556	308.4444444	822.1760992	1018.044444	323.9	357.5235556	58.5
1995	115.26	255.2953401	279.2742387	914.3329046	1205.099512	326.8	315.23859	58.5
1996	150.9	291.0115928	302.2868032	920.3987371	1420.676513	332.4	273.0840083	58.5
1997	209.91	293.3064516	327.5	906.2207671	1675.080645	342.2	276.0983871	58.5
1998	262.01	298.731535	338.3108542	910.313377	1908.477842	356	273.8094091	58.5
1999	320.44	298.5205218	346.2456252	918.7909261	2130.766783	363	269.1568565	58.5
2000	398.39	302.5814022	360.8829569	996.5505145	2318.64183	365	241.4549721	128.5
2001	507.68	311.3132626	437.6579308	1012.263213	2677.063028	387.8	210.5335355	143.5
2002	600.96	382.5182687	537.1697583	1040.107577	3031.899944	405.7	193.0234682	180.6
2003	665.94	209.7638012	367.0008239	1064.370907	3438.066465	408.7	178.4264625	215
2004	830.03	272.4593019	429.8921418	1090.89979	4041.803443	419.1	158.3956589	255
2005	1007.35	264.8244473	483.550065	1124.00608	4531.53446	420	151.7561118	324
2006	1235.78	276.0149906	453.3416615	1170.047942	5070.455965	425	145.2685821	331
2007	1882.04	266.1097852	455.3699284	1224.859682	5875.178998	430	138.5365752	348
2008	2282.03	349.2941841	504.8560136	1294.287886	6276.115189	435	130.8233766	329.4
2009	2678.77	335.8602151	493.4946237	1359.331151	6533.870968	444	122.1346398	356
2010	3810.9	323.9206891	478.8725215	1433.802035	7193.842704	450	114.0625924	365
2011	4359.09	310.0168434	453.9780046	1475.23971	8051.075002	456	109.9555137	369.4
2012	4512.85	303.0493404	446.3345093	1512.292446	8640.313149	462	106.7299087	388.5
2013	5566.08	297.6640817	454.4637791	1545.544902	9362.965765	468	104.5759883	393
2014	7214.54	298.6614612	464.8633575	1572.439196	9913.924521	474	102.2256925	425

And then, We check that city by the basic smart growth's evaluation model we established at page 2 in our paper. We need to analyze their relationships by that model.

First, we choose "per capita GDP (y_1)" and "population density(x_1)" to check their relationship. We use matlab software to draw their relationship by their data in figure 2.

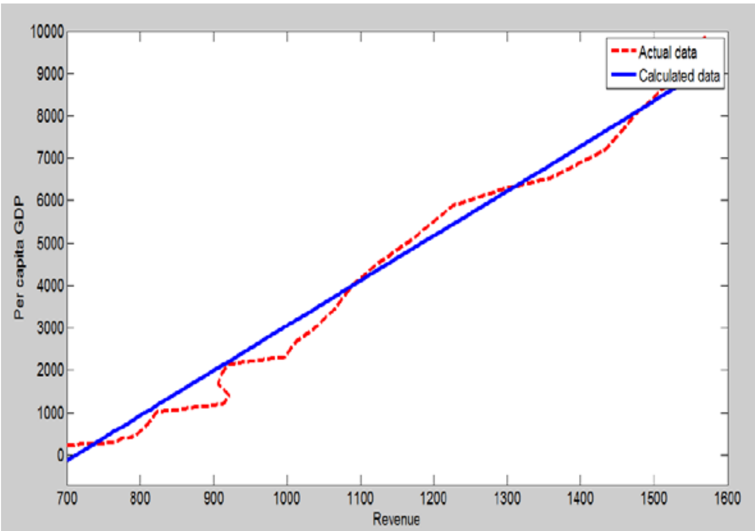

Figure 2 The relationship of population density and per capita GDP

As we can see by the actual data, we can get the general relationship between them. And then, we use calculated data to fit actual data. Thus, we get a straight line with the calculated data. Finally, we can get the formula:

$$y_1 = 10.6x_1 - 7549.2 \qquad (5)$$

We can see that, the formula (5) meets the formula (1). Thus, we can make a conclusion that they are in a linear relationship. And then we analyze that linear relationship with the reality. With the increasing of the population the per capita also increases, we can know that the growth of the population doesn't influence the Economically Prosperous. So by the analysis of those two factors, we can get their relationship fits the smart growth.

Second, we choose "population density (y_2)" and "Financial revenue(x_2)" to check their relationship. We use matlab software to draw their relationship by their data in figure 3.

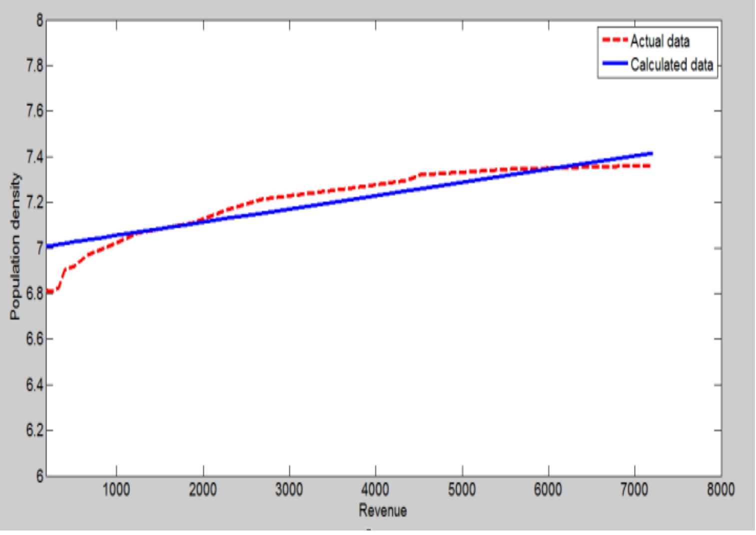

Figure 3 The relationship of population density and per capita GDP

Repeat the first process of the analysis. We can see the actual data, their relationship seems like index relationship. So we convert y_2 into $\ln y_2$ to get the calculated data. And then we get the formula:

$$\ln y_2 = 5.84 \times 10^{-5} x_2 + 6.9952 \qquad (6)$$

And then we analyze that index relationship with the reality. The increasing of the population doesn't make the great influence on the revenue of the local government. So we can get that the bigger number of the population doesn't pose a burden to the government. Thus, we can get their relationship fits the smart growth.

Third, we choose "per capita area of roads (y_3)" and "the green rate(x_3)" to check their relationship. Repeat the first process of the analysis.

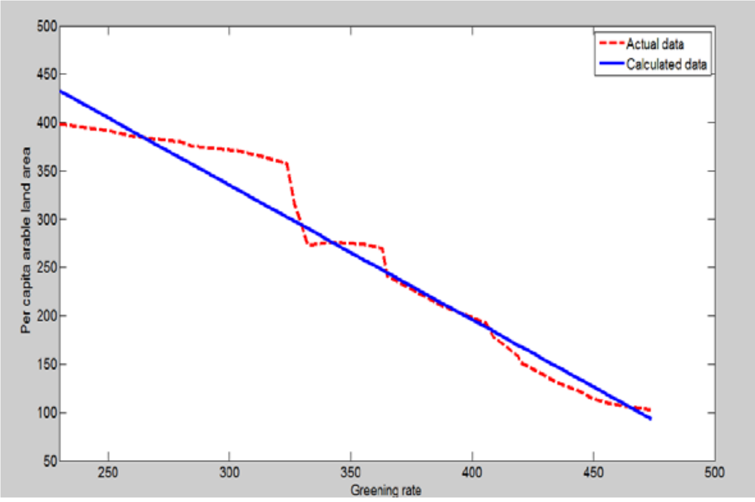

Figure 4　The relationship of per capita area of roads and the green rate

Repeat the first process of the analysis. We can get the formula:

$$y_3 = -1.392x_3 + 752.8817 \qquad (7)$$

And then we analyze that linear relationship with the reality. We can get that the quantity of the decreasing of the per capita area can exactly fit the quantity of the increasing of the green rate. Thus, Thus, we can get their relationship fits the smart growth.

Last, we choose "the green rate (y_4)" and "population density (x_4)" to check their relationship. Repeat the first process of the analysis.

According to our analyze, we find that Beijing's data meet the requirement of the smart growth city. Thus, we can proceed the next step.

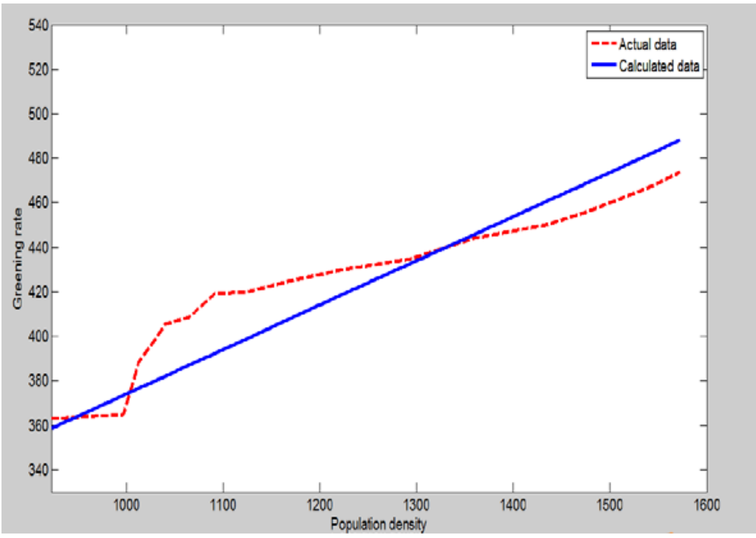

Figure 5　The relationship of the green rate and population density

Repeat the first process of the analysis. We can get the formula:

$$y_4 = 0.199x_4 + 175.261 \qquad (8)$$

And then we analyze that linear relationship with the reality. With the increasing of the population, the green rate also grows. Thus we can make a conclusion that that relationship can fit the requirement of the Environmentally Sustainable.

4.2 Define the metric

We choose the Gaussian distribution [3] to define the metric, the Probability Density Function is:

$$f(x) = \frac{1}{\sqrt{2\pi\sigma}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}, -\infty < x < \infty \quad (9)$$

We can easily get $\int_{-\infty}^{+\infty} f(x)dx = 1$. And then, we define the Probability Distribution Function:

$$F(x) = \int_{-\infty}^{+\infty} f(x)dx \quad (10)$$

We know that Probability Distribution curves meet a lot of growths of natural things. So we assume that the smart city growth also meets the Probability Distribution curves. And then, we use MATLAB to make the Probability Distribution. If we define the growth limit is 1000, we can get the metric value that fit into different years.

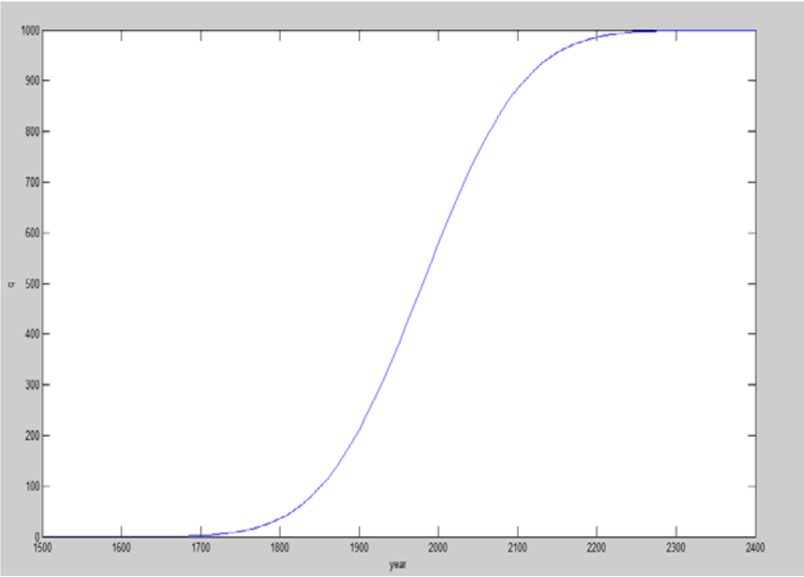

Figure 6 The metric value (Q) that fit into different years

According to the figure 6, we can get the metric value (Q) that meet the years we know the data, in table 2:

Table 2 The standard metric value (Q) that meet the different years

year	1978	1979	1980	1982	1983	1984	1985	1986
Q	492.0217	496.0106	500	507.9783	511.9665	515.9534	519.9388	523.92218

year	1987	1988	1989	1990	1991	1992	1993	1994
Q	527.9032	531.8814	535.8564	539.8278	543.7953	547.7584	551.7168	555.67

year	1995	1996	1997	1998	1999	2000	2001	2002
Q	559.6177	563.5595	567.4949	571.4237	575.3454	579.2597	583.1662	587.06442

year	2003	2004	2005	2006	2007	2008	2009	2010
Q	590.9541	594.8349	598.7063	602.5681	606.4199	610.2612	614.0919	617.9114

year	2011	2012	2013	2014	2015	2016	2017	2018
Q	621.7195	625.5158	629.3	633.0717	636.8306	640.5764	644.3087	648.0273

4.3 Define the weight value

According to the multiple linear regression model, we should know the weight value $a_i(i=1,2,..,n)$. So we use the data in table 1. And then we can define the weight value by the SPSS software.

Table 3 Model summary in the SPSS

Model Summary

Model	R	R Square	Adjusted R Square	Std. Error of the Estimate
1	.996[a]	.993	.991	4.05627

a. Predictors: (Constant), p8, p2, p1, p6, p3, p7, p4, p5

Table 4 the conclusion of the SPSS

Coefficients^{a}

Model		Unstandardized Coefficients B	Unstandardized Coefficients Std. Error	Standardized Coefficients Beta	t	Sig.
1	(Constant)	521.589	46.147		11.303	.000
	p1	.002	.003	.072	.613	.545
	p2	.023	.068	.017	.338	.738
	p3	.003	.061	.008	.054	.957
	p4	.056	.038	.370	1.448	.159
	p5	-.002	.004	-.152	-.582	.565
	p6	.140	.062	.310	2.243	.033
	p7	-.189	.056	-.592	-3.346	.002
	p8	-.006	.005	-.202	-1.243	.224

a. Dependent Variable: q

According to table 3 we can get that, R^2=0.993>0.4. Thus, we can use this multiple linear regression model to study our questions.[4] And then, in the table 4 we can get all the weight values in the multiple linear regression equation. Thus, we can complete the multiple linear regression equation:

$$Q = 0.002p_1 + 0.023p_2 + 0.003p_3 + 0.056p_4 - 0.002p_5 + 0.14p_6 - 0.189p_7 - 0.006p_8 + 521.589 \tag{10}$$

4.4 Conclusion: evaluate the growth of the city

In order to meet the requirement of The International City Management Group (ICM). We choose two reasonable cities: Canberra (with a population of 368000) in Australia and Hainan Tibetan Autonomous Prefecture (with a population of 349240) in China.

We can get the data of the two cities in 2013. Thus, we can evaluate whether the city's current plan meets the smart growth. We can calculate Q by the equation (10). And comparing Q of the mid-sized city to the smart growth city in the same year, we can evaluate whether the city meets the smart growth city.

Table 5 Data of the two mid-sized cities[5]

City	Year	Financial revenue (Billion yuan)	Average path length (10^{-2}km/ people)	Per capita area of roads (10^{-2}km^2/people)	Population density (10^{-1}km^2/people)	Per capita GDP (10^7 yuan/people)	The green rate (10^{-1} percent)	Per capita arable land (10^{-4}km^2/ people)	Pollution treatment capacity (10^4m^3/day)
Hainan Tibetan Autonomous Prefecture	2013	332.6	4350.6	5767.9	10.259	2562.8	649	828.53	107.18
Canberra	2013	368.64	8549.22	11917.1	16.36	41678.9	594	14.99	803.269

Table 6 Q of the two mid-sized cities

City	Year	Q
Hainan Tibetan Autonomous Prefecture	2013	568.78
Canberra	2013	744.69

By comparing, we find the Q of the smart city is 629.3 (Table 3). But the Q of Hainan Tibetan Autonomous Prefecture is only 568.78<629.3. At the same time, the Q of Canberra is 744.69>629.3.Finally,we make a conclusion the growth of Hainan Tibetan Autonomous Prefecture doesn't meet the smart growth. And the growth of Canberra meets the smart growth.

5. Incidence matrix advantage analysis model

In order to make the plan for the two cities we need to know how to distribute the weight to the factors (P_i) we choose. Thus we use the advantage analysis method [6].

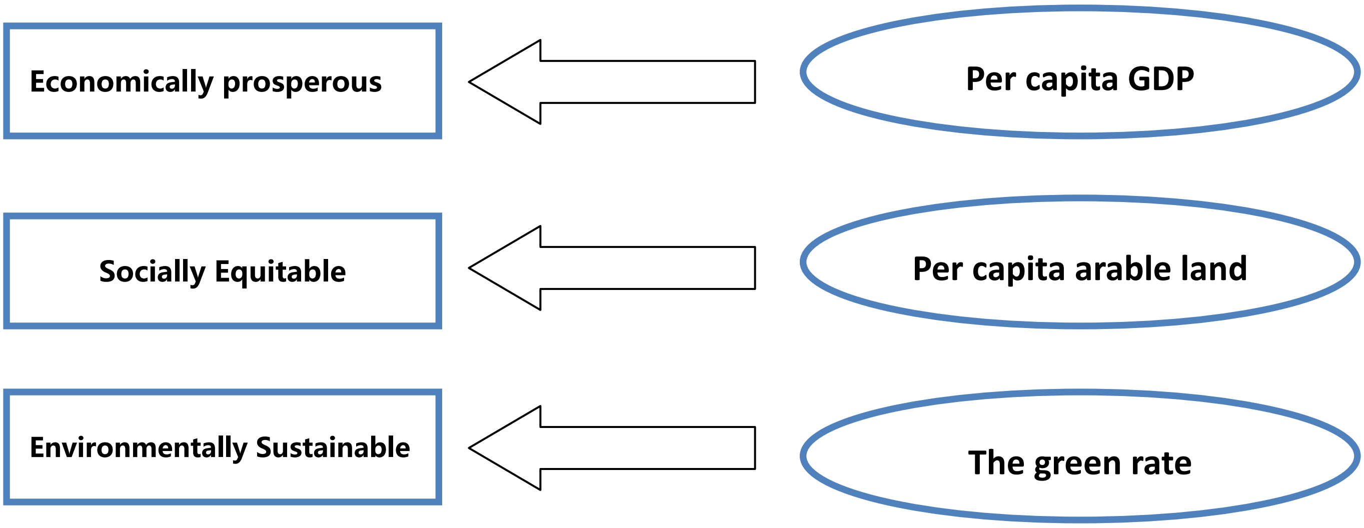

Figure 7 three factors that can reflect three E

- Firstly, we need to select the reference series. We choose three E as our references, but we can't have the direct data. Thus, we select three elements(P_5, P_6, P_7) to reflect three E as the reference series.(as the figure 7 shows)
- Secondly, we consider the other factors (P_1,P_2,P_3,P_4,P_8) into compared series.
- Thirdly, we define r_{ij} as the level of reference series associated with compared series. And then we can make the incidence matrix $R = (r_{ij})_{m\times n}$ by MATLAB.
- Finally, we can get which factors have the great influence on the three E. We can increase

the intensity of development on those factors.

5.1 City 1 Hainan Tibetan Autonomous Prefecture

Table 7 Data of Hainan Tibetan Autonomous Prefecture[5]

Year	P_1	P_2	P_3	P_4	P_5	P_6	P_7	P_8
2007	1132	1468	2.89	23495.25	83.94	42.79	40.8665	24.5
2008	1846	1664	3.88	25810.11	81.67	43.17	51.6	58.5
2009	1937	1799	4.32	26340.23	76.34	43.56	58.4	58.5
2010	2243	1926	5.41	27030.813	75.54	44.17	69.89	128.5
2011	2436	2167	6.44	30135.74	76.28	44.6	82.65	163.6
2012	2559	2366	8.13	33445.13	75.66	45.1	104.35	215
2013	2635	1983	11.67	37863.86	74.9	45.72	117.12	228.5
2014	3002	2429	12.08	43288.54	72.13	45.9	130.72	248.3
2015	3164	2589	14.175	48113.66	70.31	46.43	140.2	260.4

By using MATLAB we can get a incidence matrix R_1.

$$R_1 = \begin{array}{c} \\ P_5 \\ P_6 \\ P_7 \end{array} \begin{array}{c} \begin{array}{ccccc} P_1 & P_2 & P_3 & P_4 & P_8 \end{array} \\ \begin{bmatrix} 0.8344 & 0.7510 & 0.8773 & 0.5278 & 0.7104 \\ 0.8694 & 0.9650 & 0.8572 & 0.5440 & 0.9395 \\ 0.8311 & 0.9334 & 0.8250 & 0.5521 & 0.9875 \end{bmatrix} \end{array}$$

By analyzing the incidence matrix R_1 we can know that, the data in the fourth row is obviously smaller than others. Thus, we can consider P_6 into the least important factor. Compared other rows, we can find that the data in the first row and the third row are uniformly distributed. So we can consider P_1 and P_3 into important factors.

5.1.1 The growth plans

According to the analysis of the value in incidence matrix R_1, we can make a growth plan for city 1. They should put the road in the first place. We combine the geography, expected growth rates, and economic opportunities into the road constructing. We analyze the geography element by using ArcGIS. As we can see in the figure 8, Hainan Tibetan Autonomous Prefecture is a landlocked city. And it is a hilly area, so it should establish tunnels to make the transportation more convenient. We analyze it’s geology by ArcGIS, we find there are much Custer terrain and the soil is loose in someplace. So when they establish tunnels they must focus more on local geology in case of landslide disaster. Because of the influence of the Custer terrain, the reasonable areas to establish tunnels are restricted greatly. So we advise that they should broad the road established to make the transportation convenient and reduce the risk of the landslide disaster. Besides tunnels they also can develop air transport. And they also should establish the walkable road in the village. The road establishing also can provide more employment opportunities to promote the development of the economy.

We can obviously see that the population density factor has the least influence on the smart growth. Thus, we advise that they don't need to focus more on controlling the increasing of the population. Because Hainan Tibetan Autonomous Prefecture is located in the Southwest China, the level of the economic development is relatively low. The transportation of that city isn't convenient. Thus, there is a small population living in that city.

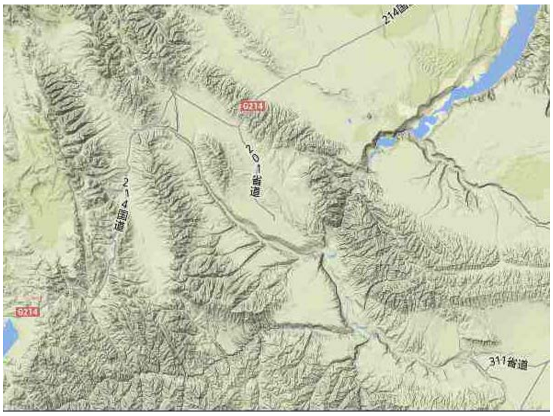

Figure 8 Location map and topographic map of Hainan Tibetan Autonomous Prefecture

5.1.2 Evaluation of the growth plans

According to our plans for city 1, we predict a series of values to evaluate the success of the growth plans. We predict values based on the Gaussian distribution, and we increase the value of P_3 and the value of P_4 manually to meet our plans.

Table 8 the predictive factor values in 2030

Year	P1	P2	P3	P4	P5	P6	P7	P8
2030	4253	3204	40.25	6023.52	58.28	48.32	250.3	453.2

We can calculate metric Q of that city in 2030 by the multiple linear regression model. The Q is 703.254. Compared with the standard Q 691.426, we find it can meet the requirement of the sustainable city. So we can make a conclusion that our plans are successful.

5.1.3 Rank the factors as the most potential to the least potential

In the incidence matrix R_1, Based on comparisons between lines can be seen that: P_6 is better than P_7 and P_7 is better than P_5. Based on comparisons between lines can be seen that: the ranking of the compared series can be calculated.

Table 9 Ranking of the factors

P_i	P_1	P_2	P_3	P_4	P_5	P_6	P_7	P_8
rank	4	7	5	8	3	1	2	6

5.2 City 2 Canberra

Table 10 Factor values of Canberra [5]

year	P1	P2	P3	P4	P5	P6	P7	P8
2008	2860	2130	272.3	692.439	55.6	35.2	521.6	621.86
2009	3109	2205	295.2	650.8137	56.2	36.4	782.5	640.26
2010	3408	2316	310.1	637.2462	58.4	36.8	923.6	689.94
2011	3750	24770	350.2	635.109	57.1	37.2	1007.2	716.8
2012	3960	2840	331.1	611.2743	58.4	37.8	1220.8	746.22
2013	4020	2860	340.2	590.505	58.8	38.1	1360.5	760.83
2014	4363	3129	350.5	576.6407	59.1	38.5	1510.2	791.11
2015	4637	3304	368.64	516.4472	59.4	38.6	1608.81	803.27

By using MATLAB we can get a incidence matrix R_2.

$$R_2 = \begin{array}{c} \\ P_5 \\ P_6 \\ P_7 \end{array} \begin{array}{c} \begin{array}{ccccc} P_1 & P_2 & P_3 & P_4 & P_8 \end{array} \\ \begin{bmatrix} 0.8534 & 0.5340 & 0.8773 & 0.9032 & 0.8125 \\ 0.8395 & 0.5286 & 0.8572 & 0.9328 & 0.7330 \\ 0.8572 & 0.5723 & 0.8250 & 0.9501 & 0.5032 \end{bmatrix} \end{array}$$

By analyzing the incidence matrix R_2 we can know that, the data in the second row is obviously smaller than others. Thus, we can consider P_2 into the least important factor. Compared other rows, we can find that the data in the fourth row is bigger than others. So we can consider P_4 into important factors.

5.2.1 The growth plans

According to the analysis of the value in incidence matrix R_2, we can make a growth plan for city 1. They should put the population in the first place. We combine the geography, expected growth rates, and economic opportunities into the population analysis. We use ArcGIS to analyze the geography of Canberra. As we can see in the figure 9, Canberra is a coastal city at southeast of Australia. Canberra is in the mid-latitude area. Thus, the weather is suitable for people to live. So they should control the population increasing in that city. Controlling the population can also benefit the socially equitable development. As we can see in the right picture the traffic network is very developed. So they can focus less on the road construction and enjoy the convenient transportation.

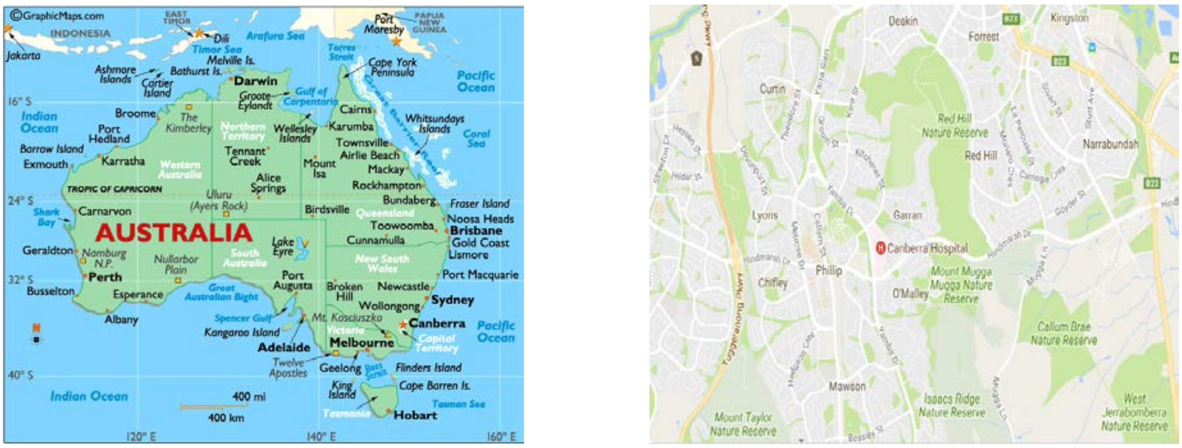

Figure 9 Location map and topographic map of Canberra

5.2.2 Evaluation of the growth plans

According to our plans for city 1, we predict a series of values to evaluate the success of the growth plans. We predict values based on the Gaussian distribution, and we increase the value of P_4 and decrease the value of P_2 manually to meet our plans.

Table 11 the predictive factor values in 2030

Year	P1	P2	P3	P4	P5	P6	P7	P8
2030	6320	3852	4471.24	443.1245	65.1	41.3	2935.47	976.57

We can calculate metric Q of that city in 2030 by the multiple linear regression model. The Q is 712.631. Compared with the standard Q 691.426, we find it can meet the requirement of the sustainable city. So we can make a conclusion that our plans are successful.

5.2.3 Rank the factors as the most potential to the least potential

In the incidence matrix R_1, Based on comparisons between lines can be seen that: P_5 is better than P_6 and P_6 is better than P_7. Based on comparisons between lines can be seen that: the ranking of the compared series can be calculated.

Table 12 Ranking of the factors

P_i	P_1	P_2	P_3	P_4	P_5	P_6	P_7	P_8
rank	6	8	5	4	1	2	3	7

6. Supporting population by 2050

(We analyze the city 1 at first ,and the city two have the same process)

In order to calculate which factors' value can support the increasing population by 2050, we use STRIPAT model for reference[8].

$$S = kP^{\alpha}A^{\beta}U^{\lambda}T^{\gamma}\varepsilon \qquad (11)$$

Where:

S: The value of the factor that is needed to calculate.

U: The value that we need to support in the future.

P, A, T: The value of the other factors by selecting.

α, β, λ, γ: When P, A, U,T changing 1% the causing percent of S.

ε: Random perturbation item

In this question, we can regard the population (P_4) as U in STRIPAT model. And then, we need to calculate the value of city's GDP(P_5) to support the increasing population. Firstly, we should select three factors that associated with P_5 greatly. We use the unary linear regression model (we establish at page 4) to analyze other factors' connection level with P_5. By using MATLAB ,we can rank other factors. (ranking 1 represents associate with P_5 well). This model needs the data in the table 7.

Table 13 the ranking of factors by the connection level with P_5

P_i	P_1	P_2	P_3	P_6	P_7	P_8
rank	2	1	4	3	5	6

Secondly, we select P_2, P_1, P_6,in the STRIPAT model to calculate S(p_5).

$$p_5 = p_1^{\alpha} p_2^{\beta} p_4^{\lambda} p_6^{\gamma} \quad (12)$$

$$\ln p_5 = \alpha \ln p_1 + \beta \ln p_2 + \lambda \ln p_4 + \gamma \ln p_6 \quad (13)$$

Thirdly, we calculate α, β, λ, γ by SPSS with the method of principal components analysis.

And then we choose two principle elements, they can explain the 99.934% of the original variable. We use symbols FAC_1 and FAC_2 to represent the two principle elements. We get that:

$$FAC_1 = 0.255 \ln p_1 + 0.251 \ln p_2 + 0.255 \ln p_4 + 0.254 \ln p_6 \quad (14)$$

$$FAC_2 = -1.418 \ln p_1 + 2.677 \ln p_2 + 0.537 \ln p_4 - 1.759 \ln p_6 \quad (15)$$

Table 13 Component score coefficient Matrix

Component Score Coefficient Matrix

	Component	
	1	2
p1	.255	-1.418
p2	.251	2.677
p4	.255	.537
p6	.254	-1.759

Extraction Method: Principal Component Analysis.

Finally, we consider $\ln p_5$ as the variable that is interpreted. FAC_1 and FAC_2 are variables that can interpret $\ln p_5$. We put FAC_1 and FAC_2 into SPSS to go on least-squares analysis.

Table 14 model regression summary

Model Summary

Equation 1	Multiple R	.302
	R Square	.091
	Adjusted R Square	-1.726
	Std. Error of the Estimate	.372

As we can see in the table 14, R^2=0.91. It can show that they fit very well. Thus, we can get the equation:

$$\ln p_5 = 8.460 - 0.34FAC_1 - 0.15FAC_2 \quad (16)$$

According to equation (14), (15), (16). We can get that:

$$p_5 = 8642.321 p_1^{0.5144} p_2^{0.324} p_4^{0.843} p_6^{0.251} \quad (17)$$

According to equation (17), we can get that p_4 increases 1% will cause p_5 increase 0.843%. Thus, we can get the value of GDP (p5) that can support the population by 2050.

We can make a conclusion (the city 2 has the same process at the city 1) that:

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Table 15 The value of the other factors that can support population in city 1

p1	p2	p3	p5	p6	p7	p8
6313	5329	143.28	530.17	58.23	40.26	673.94

Table 16 The value of the other factors that can support population in city 2

p1	p2	p3	p5	p6	p7	p8
14634.24	11834.67	265.34	930.72	59.28	14.38	1289.61

7. Strengths and Weaknesses

7.1 Strengths

(1) We use Gaussian distribution to calculate the standard metric Q. And we establish the multiple linear regression model with the help of MATLAB and SPSS. Thus we can evaluate the growth of the cities by comparing the value of Q. That makes problem more convenient.

(2) When we make plans of the developments of cities. We consider the local area with the help of ArcGIS.

(3) In this paper, we establish models from basic to complex. And we analyze the questions from theory to realistic situation.

7.2 Weaknesses

(1) We can’t analyze all the factors that fit the ten principles.
(2) When we make the plan for the city, we don’t give the planning value. And we only give the general value in one year.

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