

## Q4: Value Functions

### Problem 4: Value Functions

Consider a general search problem defined by:

- A set of states,  $\mathcal{S}$ .
- A start state  $s_0$ .
- A set of goal states  $G$ , with  $G \subset \mathcal{S}$ .
- A successor function  $\text{Succ}(s)$  that gives the set of states  $s'$  that you can go to from the current state  $s$ .
- For each successor  $s'$  of  $s$ , the cost (weight)  $W(s, s')$  of that action.

As usual, the search problem is to find a lowest-cost path from the start state  $s_0$  to a goal  $g \in G$ . You may assume that each non-goal state has at least one successor, that the weights are all positive, and that all states can reach a goal state in a finite number of steps.

Define  $C(s)$  to be the *optimal cost* of the state  $s$ ; that is, the cost of the lowest-cost path from  $s$  to any goal state. For  $g \in G$ , clearly  $C(g) = 0$ .

### Part 1

0.0/4.0 points (ungraded)

Select the Bellman-style (one-step lookahead) equation that expresses  $C(s)$  for a non-goal state  $s$  in terms of the optimal costs of other states.

☐ 
$$C(s) = \sum_{s' \in \text{Succ}(s)} W(s, s')$$

☐  $C(s) = \min_{s' \in \text{Succ}(s)} W(s, s')$

☐  $C(s) = \max_{s' \in \text{Succ}(s)} W(s, s')$

☐  $C(s) = \sum_{s' \in \text{Succ}(s)} [W(s, s') + C(s')]$

☒  $C(s) = \min_{s' \in \text{Succ}(s)} [W(s, s') + C(s')] \checkmark$

☐  $C(s) = \max_{s' \in \text{Succ}(s)} [W(s, s') + C(s')]$

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You have used 0 of 1 attempt

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**i** Answers are displayed within the problem

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## Part 2

0.0/2.0 points (ungraded)

Consider a heuristic function  $h(s)$  with  $h(s) \geq 0$ . What relation must hold between  $h(s)$  and  $C(s)$  for  $h(s)$  to be an admissible heuristic?

☐  $h(s) = C(s), \forall s \in S$


☒  $h(s) \leq C(s), \forall s \in S \checkmark$

☐  $h(s) \geq C(s), \forall s \in S$

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### Part 3

By analogy to value iteration, define  $C_k(s)$  to be the minimum cost of any plan starting from  $s$  that is either of length  $k$  or reaches a goal state in at most  $k$  actions. Define  $C_0(s) = 0$  for all  $s$ .  $C(s)$  is defined as before to be the cost of the lowest-cost path from  $s$  to any goal state, without any restriction on the number of actions starting from  $s$ .

Suppose we use  $C$  and  $C_k$  as heuristic functions.

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#### Part 3.1

0.0/2.0 points (ungraded)

Which of the following statements regarding the admissibility of  $C$  and  $C_k$  are true? Check all that apply.

☐  $C_k$  might be inadmissible for any given value of  $k$ .

☒  $C_k$  is admissible for all  $k$ . ✓

☐ Let  $L$  be the length of the longest path from a state to a goal.  $C_k$  is only guaranteed to be admissible if  $k \geq L$ .

☒  $C$  is admissible. ✓

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#### Part 3.2

0.0/2.0 points (ungraded)

Which of the following statements regarding the consistency of  $C$  and  $C_k$  are true? Check all that apply.

☐  $C_k$  might be inconsistent for any given value of  $k$ .

☒  $C_k$  is consistent for all  $k$ . ✓

☐ Let  $L$  be the length of the longest path from a state to a goal.  $C_k$  is only guaranteed to be consistent if  $k \geq L$ .

☒  $C$  is consistent. ✓

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You have used 0 of 1 attempt

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