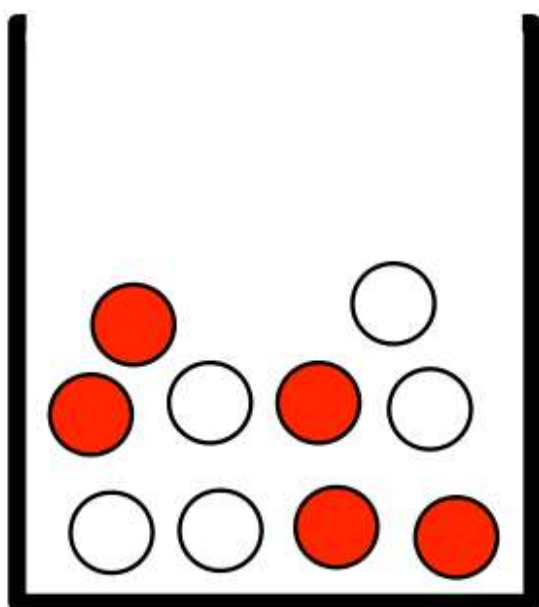


## Q8: Utilities: Low/High

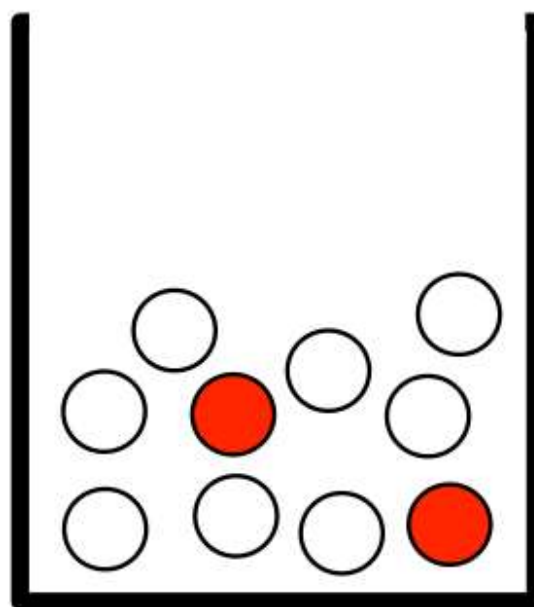
### Problem 8: Utilities: Low/High

After a tiring day of eating food and escaping from ghosts, Pacman heads to the casino for some well-deserved rest and relaxation! This particular casino has two games, Low and High, which are both free to play.

The two games are set up very similarly. In each game, there is a bin of marbles. The Low bin contains 5 white and 5 red marbles, and the High bin contains 8 white and 2 red marbles:



Low  
\$100



High  
\$1000

Play for each game proceeds as follows: the dealer draws a single marble at random from the bin. If a red marble is drawn, the game pays out. The Low payout is \$100, and the High payout is \$1000. The payout is divided evenly among everyone playing that game. For

example, if two people are playing Low and a red marble is drawn, they each receive \$50. If a white marble is drawn, they receive nothing. The draws for both games are done simultaneously, and only once per night (there is no repeated play).

---

## Part 1: Expectations

0.0/2.0 points (ungraded)

Suppose Pacman is at the casino by himself (there are no other players). What are his expected winnings for the two cases below? Do not include the dollar symbol (\$) in your answer.

i. From playing a single round of Low:

Answer: 50

ii. From playing a single round of High:

Answer: 200

Submit

You have used 0 of 2 attempts

---

 Answers are displayed within the problem

---

## Part 2: Preferences

Pacman is still at the casino by himself. Let  $p$  denote the amount of money Pacman wins, and let his utility be given by some function  $U(p)$ . Assume that Pacman is a rational agent who acts to maximize expected utility.

---

### Part 2.1

0.0/3.0 points (ungraded)

If you observe that Pacman chooses to play Low, which of the following must be true about  $U(p)$ ? Assume  $U(0) = 0$ . Check all that apply.

☐  $U(50) \geq U(1000)$

☐  $U(100) \geq U(1000)$

☒  $\frac{1}{2}U(100) \geq \frac{2}{10}U(1000)$  ✓

☐  $U(50) \geq U(100)$

☐  $U(50) \geq U(200)$

You have used 0 of 1 attempt

**i** Answers are displayed within the problem

## Part 2.2

0.0/3.0 points (ungraded)

Given that Pacman plays Low, which of the following are possibilities for  $U(p)$ ? Check all that apply.

☐  $p$

☐  $2^p - 1$

☐  $p^2$

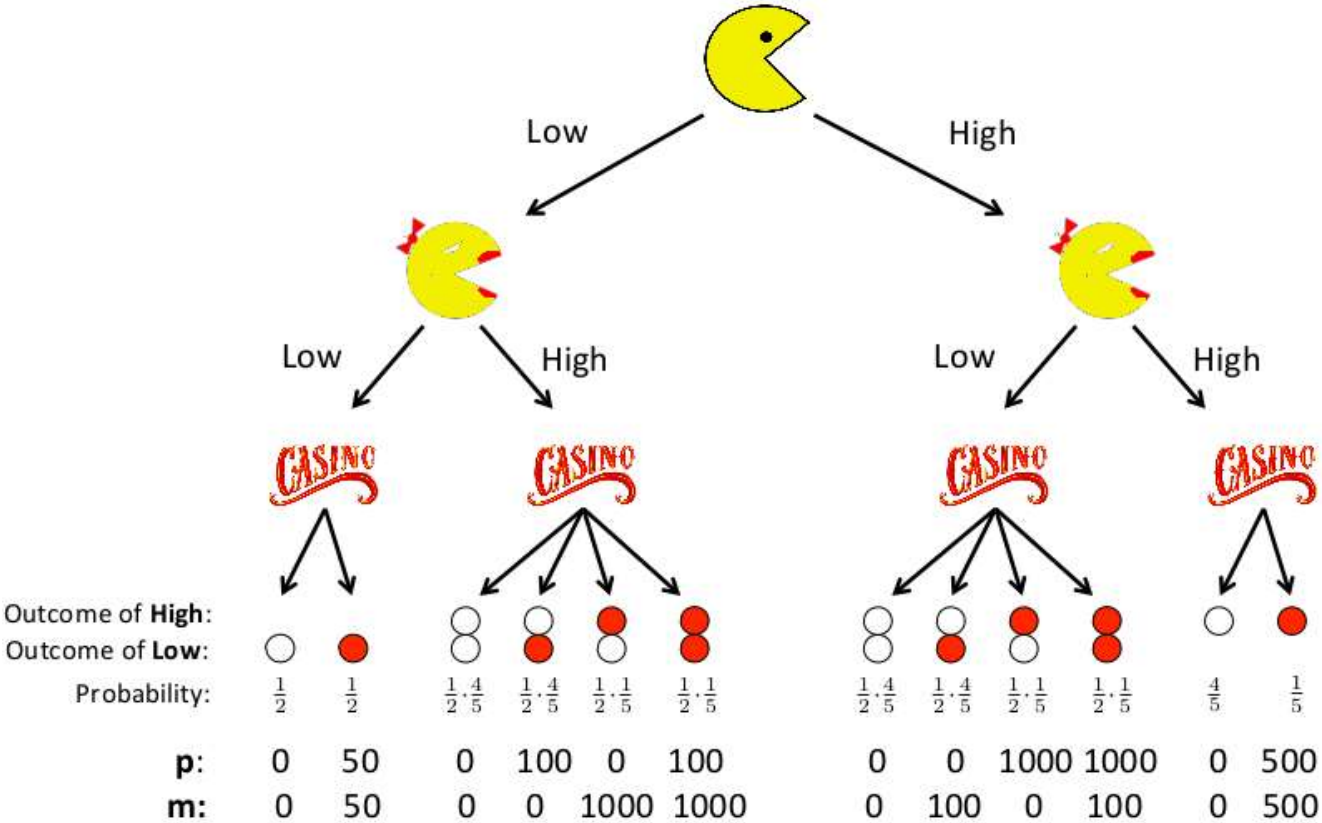
☒  $\sqrt[3]{p}$  ✓

You have used 0 of 1 attempt

**i** Answers are displayed within the problem

### Part 3: Multiple Players

Ms. Pacman is joining Pacman at the casino! Assume that Pacman arrives first and chooses which game he will play, and then Ms. Pacman arrives and chooses which game she will play. Let  $p$  denote Pacman's winnings and  $m$  denote Ms. Pacman's winnings. Since both Pacman and Ms. Pacman are rational agents, we can describe Pacman's utility with a function  $U_1(p, m)$  and Ms. Pacman's utility with a function  $U_2(p, m)$ . You might find it helpful to refer to the game tree shown in the figure below.



#### Part 3.1

0.0/4.0 points (ungraded)

Suppose  $U_1(p, m) = p$  and  $U_2(p, m) = m$ ; that is, both players are attempting to maximize their own expected winnings. Compute the expected utilities of both players, for each combination of games they could play:

Pacman	Ms. Pacman	$\mathbb{E}[U_1(p, m)]$	$\mathbb{E}[U_2(p, m)]$
--------	------------	-------------------------	-------------------------

Low	Low	<div>25</div> <div>Answer: 25</div>	<div>25</div> <div>Answer: 25</div>
Low	High	<div>50</div> <div>Answer: 50</div>	<div>200</div> <div>Answer: 200</div>
High	Low	<div>200</div> <div>Answer: 200</div>	<div>50</div> <div>Answer: 50</div>
High	High	<div>100</div> <div>Answer: 100</div>	<div>100</div> <div>Answer: 100</div>

Submit

You have used 0 of 2 attempts

## Part 3.2

0.0/2.0 points (ungraded)

Given that Pacman chooses first, which of the following are possibilities for the games Pacman and Ms. Pacman respectively choose to play? Check all that apply.

☐ (Low, Low)

☐ (Low, High)

☐ (High, Low)

☒ (High, High) ✓

Submit

You have used 0 of 1 attempt

**i** Answers are displayed within the problem

## Part 4: Scenarios

0.0/4.0 points (ungraded)

Now rather than simply maximizing their own winnings, Pacman and Ms. Pacman have different objectives. Here are five utility functions  $U_1(p, m)$  for Pacman:

$$p \quad p + m \quad m \quad (p + m)^2 \quad -m$$

and five utility functions  $U_2(p, m)$  for Ms. Pacman:

$$m \quad p + m \quad -p \quad 2m - p \quad \log_{10}(m)$$

For each of the following scenarios, select the the utility functions listed above which best encode the motivations of each player. A particular function may appear more than once. The first scenario is done for you.

Pacman	Ms. Pacman	Scenario
$p$	$m$	Pacman and Ms. Pacman each want to maximize their own expected winnings.
<div>-m ▼</div> <div>Answer: -m</div>	<div>-p ▼</div> <div>Answer: -p</div>	Pacman and Ms. Pacman have had a terrible fight and are very angry at each other. Each wants the other to lose as much money as possible.
<div>p+m ▼</div> <div>Answer: p+m</div>	<div>m ▼</div> <div>Answer: m</div>	Pacman has gotten over the fight, and now wants to maximize their expected combined winnings (since Pacman and Ms. Pacman share a bank account). However, Ms. Pacman does not trust Pacman to deposit his share, so she just wants to maximize her own expected winnings.
<div>m ▼</div> <div>Answer: m</div>	<div>m ▼</div> <div>Answer: m</div>	Pacman is being extorted by the ghosts, who will immediately confiscate any money that he wins (that is, if Pacman wins 100, he will still have $p = 100$ but does not actually get to keep the money). The ghosts are not monitoring Ms. Pacman and do not know about her winnings, so they will not be confiscated. Ms. Pacman is aware of the extortion, and both Pacman and Ms. Pacman want to maximize the expected total amount the couple gets to keep.

Submit

You have used 0 of 2 attempts

