Q7: Lotteries in Ghost Kingdom

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Diverse Utilities

Ghost-King (GK) was once great friends with Pacman (P) because he observed that Pacman and he shared the same preference order among all possible event outcomes. Ghost-King, therefore, assumed that he and Pacman shared the same utility function. However, he soon started realizing that he and Pacman had a different preference order when it came to lotteries and, alas, this was the end of their friendship.

Let Ghost-King and Pacman's utility functions be denoted by U_{GK} and U_P respectively. Assume both U_{GK} and U_P are guaranteed to output non-negative values.

Part 1

0.0/2.0 points (graded)

Which of the following relations between U_{GK} and U_P are consistent with Ghost King's observation that U_{GK} and U_P agree, with respect to all event outcomes but not all lotteries?

- [ ] $U_P = aU_{GK} + b \quad (0 < a < 1, b > 0)$
- [ ] $U_P = aU_{GK} + b \quad (a > 1, b > 0)$

- [x] $U_P = U_{GK}^2$ ✔
- [x] $U_P = \sqrt{(U_{GK})}$ ✔

Explanation
For all the above options, U_P and U_{GK} result in the same preference order between two non-lottery events (\)U_{P}(e_1) > U_{P}(e_2) \Leftrightarrow U_{GK}(e_1) > U_{GK}(e_2)\)). However, options 1 and 2 also share the same preference order among all lotteries as well.

Submit You have used 0 of 2 attempts

Answers are displayed within the problem

Part 2

0.0/2.0 points (graded)
In addition to the above, Ghost-King also realized that Pacman was more risk-taking than him . Which of the relations between U_{GK} and U_P are possible?

- [ ] $U_P = aU_{GK} + b \quad (0 < a < 1, b > 0)$
- [ ] $U_P = aU_{GK} + b \quad (a > 1, b > 0)$
- [x] $U_P = U_{GK}^2$ ✔
- [ ] $U_P = \sqrt{(U_{GK})}$

Explanation
As an example, say Ghost-King prefers winning $2 as much as a lottery : winning $0 or $4 with equal probability. For option c), Pacman prefers the lottery much more (more risk-taking) and for option d), Pacman prefers the guaranteed reward.

You have used 0 of 2 attempts

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Answers are displayed within the problem

Guaranteed Return

Pacman often enters lotteries in the Ghost Kingdom. A particular Ghost vendor offers a lottery (for free) with three possible outcomes that are each equally likely: winning $1, $4, or $5.

Let $U_P(m)$ denote Pacman's utility function for $m. Assume that Pacman always acts rationally.

Part 3

0.0/2.0 points (graded)

The vendor offers Pacman a special deal - if Pacman pays $1, the vendor will manipulate the lottery such that Pacman *always gets the highest reward possible*. For which of these utility functions would Pacman choose to pay the $1 to the vendor for the manipulated lottery over the original lottery? (Note that if Pacman pays $1 and wins $m in the lottery, his actual winnings are $m-1.)

- [x] $U_P(m) = m$ ✔
- [x] $U_P(m) = m^2$ ✔

Explanation

a) $U_P(m) = m$:

If pacman does not pay, expected utility = $\frac{1}{3}(5) + \frac{1}{3}(4) + \frac{1}{3}(1) = \frac{10}{3}$

If pacman pays up, expected utility = $1(5-1) + 0(4-1) + 0(1-1) = 4$.

b) $U_P(m) = m^2$:

If pacman does not pay, expected utility = $\frac{1}{3}(5)^2 + \frac{1}{3}(4)^2 + \frac{1}{3}(1)^2 = 14$

If pacman pays up, expected utility = $1(5-1)^2 + 0(4-1)^2 + 0(1-1)^2 = 16$.

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Answers are displayed within the problem

Part 4

0.0/2.0 points (graded)

Now assume that the ghost vendor can only manipulate the lottery such that Pacman *never gets the lowest reward* and the remaining two outcomes become equally likely. For which of these utility functions would Pacman choose to pay the $1 to the vendor for the manipulated lottery over the original lottery?

- [x] $U_P(m) = m$ ✔
- [ ] $U_P(m) = m^2$

Explanation

a) $U_P(m) = m$:

If pacman does not pay, expected utility = $\frac{1}{3}(5) + \frac{1}{3}(4) + \frac{1}{3}(1) = \frac{10}{3}$

If pacman pays up, expected utility = $\frac{1}{2}(5-1) + \frac{1}{2}(4-1) + 0\,(1-1) = 3.5$.

b) $U_P(m) = m^2$:

If pacman does not pay, expected utility = $\frac{1}{3}(5)^2 + \frac{1}{3}(4)^2 + \frac{1}{3}(1)^2 = 14$

If pacman pays up, expected utility = $\frac{1}{2}(5-1)^2 + \frac{1}{2}(4-1)^2 + 0(1-1)^2 = 12.5$.

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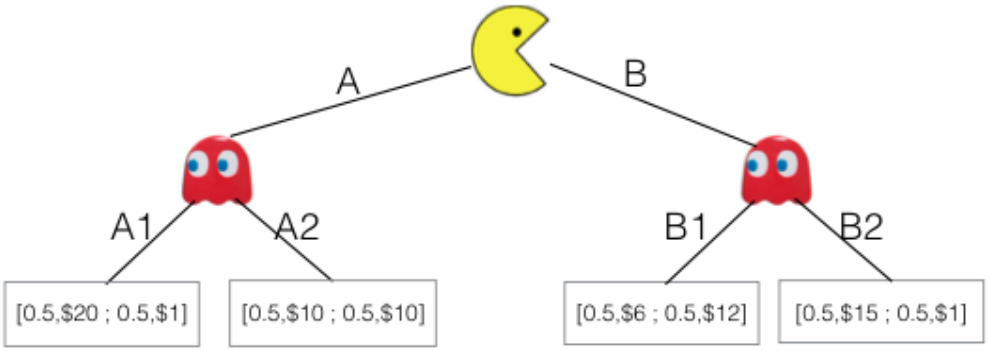
Minimizing Other Utility



The Ghost-King, angered by Pacman's continued winnings, decided to revolutionize the lotteries in his Kingdom. There are now 4 lotteries (A1, A2, B1, B2), each with two equally likely outcomes. Pacman, who wants to maximize his expected utility, can pick one of two lottery types (A, B). The ghost vendor thinks that Pacman's utility function is $U'_P(m) = m$ and minimizes accordingly. However, Pacman's real utility function $U_P(m)$ may be different.

For each of the following utility functions for Pacman, select the lottery corresponding to the outcome of the game. Note that Pacman knows how the ghost vendor is going to behave.

Pacman's expected utility for the 4 lotteries, under various utility functions, are as follows :
$U_P(m) = m$ *: [A1 : 10.5; A2 : 10; B1 : 9; B2 : 8]*
$U_P(m) = m^2$ *: [A1 : 200.5; A2 : 100; B1 : 90; B2 : 113]*
$U_P(m) = \sqrt{m}$ *: [A1 : 2.74; A2 : 3.16; B1 : 2.96; B2 : 2.44]*

Part 5

0.0/1.0 point (graded)
$U_P(m) = m$

- ○ A1
- ◉ A2 ✔
- ○ B1
- ○ B2

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Answers are displayed within the problem

Part 6

0.0/1.0 point (graded)

$U_P(m) = m^2$

- A1
- A2
- B1
- B2 ✔

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Answers are displayed within the problem

Part 7

0.0/1.0 point (graded)

$U_P(m) = \sqrt{m}$

- A1
- A2 ✔

- B1
- B2

Explanation

Since vendor minimizes $U_P'(m) = m$, If Pacman chooses A, vendor would pick A2 and if Pacman chooses B, vendor would pick B2. For $U_P(m) = m$ and $U_P(m) = \sqrt{m}$, Pacman prefers A2 over B2 and for $U_P(m) = m^2$, Pacman prefers B2 over A2 and acts accordingly.

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Answers are displayed within the problem