

Parallel Huffman Coding

Francesco Bozzo
DISI, University of Trento
Trento, Italy
francesco.bozzo@studenti.unitn.it
229312

Michele Yin
DISI, University of Trento
Trento, Italy
michele.yin@studenti.unitn.it
229359

Abstract—This report aims to explain the design of a parallel encoding and decoding Huffman algorithm. The application is developed with the C99 programming language, using MPI for multiprocessing and OpenMP for multithreading to scale horizontally with increasing hardware resources. Our tool is both able to exploit multiprocessing to process concurrently multiple files, and to split the Huffman coding of the same file among multiple threads.

Comparing this tool with other online implementations, we can state that...

Index Terms—Huffman, MPI, OpenMP, High Performance Computing

I. INTRODUCTION

Even if the Huffman algorithm is not directly used nowadays, its prefix mechanism is still part of Deflate (PKZIP's algorithm), JPEG, and MP3 compression algorithms

II. THE HUFFMAN ALGORITHM

The Huffman algorithm has the objective of finding a more convenient bit representation to store information through lossless compression. Instead of considering groups of eight bits as the way to encode data, the Huffman algorithm uses variable-length sequences of bits with prefixes defined as *alphabet*.

A. Priority Queue

The Huffman algorithm uses a minimum priority queue to build its alphabet efficiently. This specific data structure ensures logarithmic insertion and deletion time with respect to its size. We implemented the minimum priority queue by using a minimum heap. Practically speaking, to implement the min-heap tree, we used a standard C array ensuring that the min-heap property still holds at every insertion and deletion:

$$A[i] \leq A[l(i)], A[i] \leq A[r(i)] \quad (1)$$

where $A[i]$, $A[l(i)]$, and $A[r(i)]$ are respectively a node, its left child, and its right child in a min-heap tree.

To implement the priority queue also a parallel approach has been considered [1], but since it contains only 256 elements (1 byte), we did not consider it too important in terms of performance.

B. Encoding

The Huffman encoding procedure makes use of a minimum priority queue to build its alphabet efficiently. The idea is to build a tree similar to Figure 1 that defines all the variable-length prefix sequences of bits: these sequences are represented by the path from the tree root to a leaf. Using a greedy approach, the Huffman encoding ensures that the less frequent a byte is in a file, the more probable is to have a longer Huffman representation, which means that its path from the root to its specific leaf is longer.

Once having computed the frequencies for each one of the 256 different bytes in a file, the Huffman algorithm populates a min priority queue with Huffman tree nodes storing the byte value and its frequency. Successively, it removes the least two frequent bytes from the queue, creates a new node with children the two extracted nodes, assigns to it a dummy character and the sum of the frequencies of its two children, and inserts it into the min priority queue. After $n-1$ iterations, the last node is the root of the Huffman tree [2]. Algorithm 1 explains in the detail this procedure.

Algorithm 1: Build the Huffman tree

```
1 // Populate the min priority queue with characters and
  their frequencies
2 for  $i = 1$  to  $n - 1$  do
3    $Q.insert(f[i], Tree(f[i], c[i]))$ 
4 // Repeat until the queue has only a single element left
5 for  $i = 1$  to  $n - 1$  do
6   // Get the two least frequent nodes
7    $z1 = Q.deleteMin()$ 
8    $z2 = Q.deleteMin()$ 
9   // Create an inner tree node and insert it into the
    queue
10   $z = Tree(z1.f + z2.f, null)$ 
11   $z.left = z1$ 
12   $z.right = z2$ 
13   $Q.insert(z.f, z)$ 
14 // The last element in the queue is the root of the
    Huffman tree
15 return  $Q.deleteMin()$ 
```

Once built the Huffman tree, it is possible to generate the

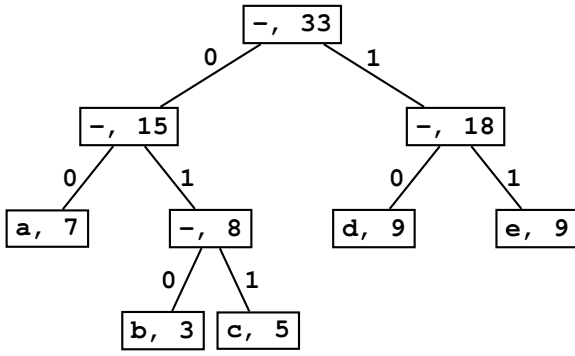


Fig. 1: An example of Huffman tree.

Huffman alphabet visiting the tree using a DFS algorithm, assigning 0 to each left-child traverse and 1 for the right one as presented in Algorithm 2. Finally, it is possible to compress the file by creating a stream of bits corresponding to its content using the Huffman alphabet.

Algorithm 2: Encode using Huffman tree

```

1 while not eof() do
2   bit = read()
3   if bit == 0 then
4     encode(node.left)
5   else
6     encode(node.right)
7   if node is leaf then
8     return node.value

```

C. Decoding

Once having the encoded file and the Huffman tree, it is straightforward to decompress the file to its original shape. The prefix alphabet ensures to have that it is possible to visit the tree using a DFS approach and get a unique decoded version of the file using an approach similar to Algorithm 2.

III. SERIAL HUFFMAN IMPLEMENTATION

To be able to implement the Huffman encoding and decoding algorithm, several things have been taken into account.

- The byte frequencies computed during the encoding process are saved in the compressed file. In this way, the decoding procedure can easily rebuild the Huffman tree.
- Both the encoding and decoding procedures make use of buffers to improve I/O performance: the streams of bytes are first written inside the buffer and then saved on the disk.
- The encoding procedure works with chunks of 4096 bytes. This ensures that the tool can handle even large files when dealing with bit buffers. The decoding procedure deals with chunks of maximum size of $4096 * 32$

bytes, since the maximum length of a Huffman tree traverse from root to leaf is of 256 bits, which is 32 times a byte. As explained later, dealing with chunks makes handy the parallel implementation of the encoding and decoding algorithms.

- In the encoded file, the chunk offsets are saved at the end of the file: this ensures that the compressed stream of prefixed sequences of bits is still meaningful.

IV. PARALLEL HUFFMAN IMPLEMENTATION

To parallelize the serial Huffman algorithm over threads and processes, we decided to follow this simple yet effective structure:

- Multiple processes should handle groups of file and folders separately.
- Multiple threads of the same process should work on different chunks of the same file in parallel.

Specifically, there are multiple reasons behind our parallel architectural design for this project. Here follows a list of the main ones:

- In most operating systems a file is a resource that the OS gives to a single process to avoid race conditions. We wanted to follow a similar design philosophy.
- Most operating systems allow multiple processes to open the same file in reading mode, but only few allow to open the same file in writing mode on multiple processes. This is because that leads to potentially concurrency and data integrity issues. By ensuring to have only a single process that open a specific file, we avoid all these issues.
- Because threads of the same process share the address space, we can avoid the expensive data transfer across processes. When data is read or written to file, there is no need to transfer data between threads.

In the next section, we start by explaining the basic multi-threading operations on a single file. Later on, multiprocessing with multiple files is covered.

A. Multithreading

The parallel algorithm for a single file follows the exact same procedure as for the serial version, with just a few key differences to allow multithreading.

Suppose we are dealing with m threads and a single input file. Here follows a common procedure used for both encoding and decoding, with very small differences.

- 1) A single file is divided into chunks of a fixed size. When that is done, the single process forks and creates m threads.
- 2) A single thread (may be the main one, may not) then reads m chunks in a shared memory buffer.
- 3) Each one of the m threads works in parallel on the processing of its assigned chunk using a buffer. Although all the chunks are in shared memory buffer, each thread is assigned only to a single memory section: in this way there is no need to care about racing conditions when writing in the buffer. This is done by creating a `unsigned char buffer[m][buffer_size]`.

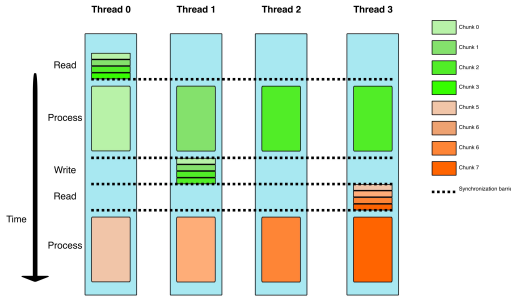


Fig. 2: Simple schema for processing a single file with multiple threads.

- 4) When all threads are done computing, either the encoding or decoding of their chunks, a single thread writes the processed chunks to the output file. Again, since all threads share a memory space there is no need to perform data transfer.

Moreover, our implementation also parallelize the counting of occurrences of a byte in the input file, since this is required in order to create the Huffman tree. This is done only during the encoding phase, as for the decoding the occurrences are no longer required because the huffman tree is already saved in the encoded data. The procedure is very similar to the one just described, with the key difference that there is no output file to write, but only byte occurrences stored in an in-memory array unique for each thread to avoid concurrency. When the counting is done, all threads join their counts into a single array.

Figure 2 shows a schema of the parallel workflow for processing a single file.

B. Multiprocessing

Additionally, we wanted our tool to be able to process multiple files and folders: for this use case, we exploit multiprocessing.

To achieve this, we devised a simple algorithm to distribute files across multiple processes, so that each process can work on its own list of files, independently to others. In this way, we can minimize communication and therefore latency and other slowdowns between the different components of the applications. Suppose we are dealing with n process, m threads, and an input folder.

- 1) First, the process with rank 0 (the main process) crawls all the files in the input directory recursively in all the subfolders.
- 2) Then the rank 0 process opens all the files and reads their size.
- 3) Once all files and their respective sizes are known, the main process creates a min priority queue where each item represents a process and its priority is the size of files that have been assigned to it.
- 4) Process 0 can iteratively insert files in the priority queue and updates the priority of each process with the

cumulative file size. The main idea is that we can ensure an equal work division among different processes.

- 5) When all this is done, the main process sends the list of files to each process and then each starts to work independently on its own list of files.

C. Implementation details and other notes

Here follows a list of further comments and design choices for our specific Huffman implementation.

- We decided on an alphabet of 256 for the Huffman coding because this way we can encode each byte of the input data into a different Huffman code. As C language does not allow to address data at lower resolution than a byte, lowering the alphabet would not result in any performance benefit. We also considered encoding multiple bytes into a single huffman code, but that results in worse compression as the resulting huffman tree would have more leaves.
- We found that a chunk size of 4096 bytes had the best I/O times. This is probably due to the fact that 4096 B is the size of a page in most Linux based operating systems. The last chunk of a file may be smaller. Reading any other size at the time had resulted in significantly worse I/O times (both increasing and decreasing the chunk size).
- In general, all threads will finish processing their chunks at around the same time, but in some cases there are threads that can take longer. This is due to the very unlucky case where a whole chunk contains exclusively very infrequent bytes of data. Because the bytes are very infrequent, each one is encoded in extremely long sequences, up to 256 bits, resulting in a bigger encoded chunk than the original data. With our Huffman alphabet set to a single byte, the worst case results in having a compressed chunk being 32 times bigger than decompressed one.
- We tried to parallelize the I/O, by having each thread read its own chunk. We found that the standard file descriptor provided by C have a lock to guarantee thread safety. Unfortunately, this lock slows down the reads significantly. We also tried using multiple file descriptors to circumvent this limitation, however we found no improvement over a single-thread sequential read. This is likely because the O.S. schedules I/O requests and serves them one at the time, resulting in an impossibility of having truly parallel I/O.
- We also tested an architecture where we had two dedicated threads to I/O, one for reading and one for writing chunks. The idea is that whenever a chunk is processed, the I/O threads would immediately write the processed chunk to the output file and similarly a new chunk would be read from the input file (very similar to a queue of jobs). Theoretically this would allow to parallelize I/O and computation operations: in this scenario we avoided concurrency problems by synchronizing the I/O with locks. In later analysis we call this architecture "Locks" because of the synchronization method we are using.

- Another detail we noticed on the implementation with two dedicated threads for I/O, is that if there are not enough cores on the CPU or if the operating system decides to allocate all the threads on a single processor, the encoding and decoding times are greatly affected by the scheduler. This is because it might be that the O.S. gives priority to threads that are waiting (for example the writer cannot write any block until the first has finished, even if all others have finished). In the worst case scenario, the multithreading architecture could be even slower than the serial one.

V. PERFORMANCE AND BENCHMARKING

Here we discuss the performance evaluation of our algorithm. We start from a evaluation of some other implementations of Huffman algorithm and then focus on the analysis of our own implementation.

We include in this section some graphs and statistics but all detailed results are included in the appendix at the end this report.

A. setup

All data is resulting average of 3 runs to minimize the effect random variance in between runs. We tested everything on HPC2 cluster of University of Trento. We also used a single node, using the `--map-by` command to map the correct number of cores to each MPI process. Each time we submitted as a job to the cluster individually, waiting for the previous to finish. This is to prevent hiccups due to multiple instances of the program trying to access the same data, which we noticed greatly affect I/O times.

The correctness of our algorithm was easily verified by encoding and then decoding a file, then comparing the decoded result to the original with the `diff` command.

B. datasets

The dataset was pragmatically generated using a C program, creating text files with characters that follow the frequency of letters in English language. Although for this benchmark are only using the 26 letters of the English alphabet, our algorithm reads bytes of data and can therefore work on any type of file.

In particular we chose to analyze these file sizes 1 MiB, 5 MiB, 10 MiB, 50 MiB, 100 MiB, 500 MiB, 1 GiB, 5 GiB, 10 GiB. This should be enough range to understand different algorithms scale with increasing file sizes.

Secondly, because our algorithm also works with many files at once, we used Linux kernel as benchmark for that scenario. As the time of writing, it is about 1.3 GiB of total size, with about ~ 84.000 files, which on average are each 16MiB in size.

Considering the dataset sizes and testing we have done, we estimate to have read or wrote about ~ 6 TiB of data for our whole benchmark analysis.

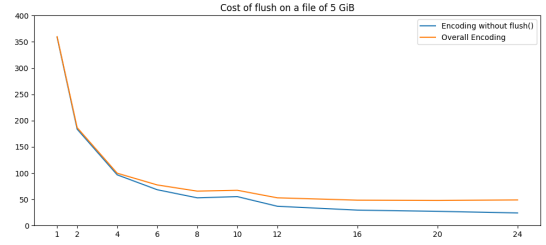


Fig. 3: Encoding times with and without the flush()

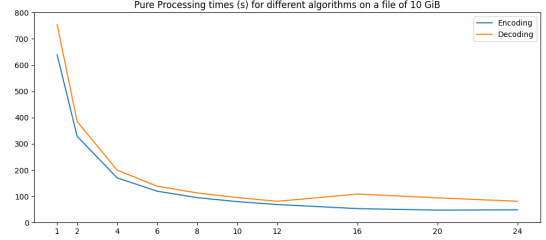


Fig. 4: Encoding vs Decoding times

C. Other works

D. Multithreading evaluation

In this section we evaluate our algorithm on single files, using multithreading and testing our parallelization performances with increasing number of threads available.

We can say that with small files, less than a few MiBs, the algorithm is actually slower with more threads. This is likely because the data is not large enough to offset the cost of creating multiple threads. Considering the cost parallel processing, we highly suggest not doing parallelization for small files.

However when the data is large enough our multithreaded approach scales up.

With the largest file (10 GiB) our algorithm has an efficiency of 93% when tested with 4 threads. This number steadily decreases with the number of threads and we hit 32% with 24 threads.

It has to be noted that we are heavily limited by the I/O. As we are unable to parallelize the I/O, we are heavily limited by that. Especially writing to disk takes a considerable amount of time.

As a proof of this, if we consider only the processing section of our algorithm, and ignore the time required by the `fflush()` operation before a `fclose()`, we see significantly better times. Although we start from similar efficiency at low threading, we achieve a 54% efficiency with 24 threads.

If we consider only CPU time, which doesn't count the time spent waiting in I/O, we get incredible results.

We also noticed worse times in decoding than encoding. This is probably due to the fact that we are reading fixed chunks of 4096 bytes in encoding, but when we decode, we read the same chunks which is now of a different size than 4096 bytes.

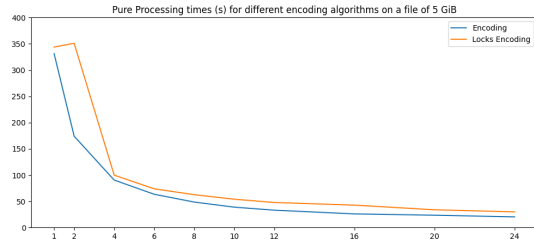


Fig. 5: Barrier vs Lock based synchronization performance

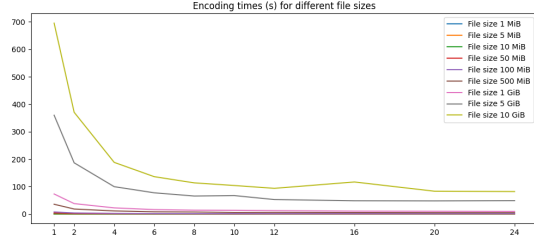


Fig. 6: Encoding times

As cited in the previous section, we also tested a version with lock based synchronization instead of barrier. Although theoretically we should get better results, in practice we almost equal results across the board, with only the largest file having some significant difference. This difference is greater in decoding than in encoding.

E. Multiprocessing

Here we test on Linux

F. Memory

Our memory requirements don't scale upwards with the size of the files we need to encode, because at any given time, the most we only store is the size of two buffers, each is a `unsigned char buffer[num_threads][4096]`. These buffers are emptied at each I/O cycle, therefore leading to a high memory efficiency. Only when processing multiple files our memory requirements scale up with the number of files, as each process needs to store information about the files is assigned to process.

VI. CONCLUSIONS

To conclude.

REFERENCES

- [1] G. S. Brodal, J. L. Träff, and C. D. Zaroliagis, "A parallel priority queue with constant time operations," *Journal of Parallel and Distributed Computing*, vol. 49, no. 1, pp. 4–21, 1998. [Online]. Available: <https://www.sciencedirect.com/science/article/pii/S0743731598914253>
- [2] A. Bertossi and A. Montresor, *Algoritmi e strutture di dati*. CittàStudi, 2010. [Online]. Available: <https://books.google.es/books?id=WKWSQAACAAJ>

VII. APPENDIX

A. Encoding

TABLE I: Overall encoding times

| File sizes \ Threads | 1 | 2 | 4 | 6 | 8 | 10 | 12 | 16 | 20 | 24 |
|----------------------|----------|----------|----------|----------|----------|----------|---------|----------|---------|---------|
| 1 MiB | 0.1084 | 0.1029 | 0.0826 | 0.0905 | 0.0718 | 0.1100 | 0.1338 | 0.2110 | 0.1705 | 0.2531 |
| 5 MiB | 0.4073 | 0.2008 | 0.1255 | 0.1037 | 0.0970 | 0.0786 | 0.0789 | 0.0767 | 0.0906 | 0.0883 |
| 10 MiB | 0.7611 | 0.4737 | 0.2658 | 0.2084 | 0.1804 | 0.1861 | 0.2204 | 0.3353 | 0.2432 | 0.3930 |
| 50 MiB | 3.9241 | 2.1378 | 1.2788 | 1.0203 | 0.8575 | 0.7589 | 0.7773 | 0.6583 | 0.7523 | 0.9954 |
| 100 MiB | 7.7270 | 4.0680 | 2.4028 | 1.8596 | 1.5545 | 1.4324 | 1.2994 | 1.2468 | 1.2308 | 1.5375 |
| 500 MiB | 35.7753 | 18.3831 | 11.4959 | 8.3511 | 7.5589 | 5.7548 | 6.0857 | 5.5698 | 5.1431 | 5.0972 |
| 1 GiB | 73.1120 | 38.1679 | 22.5066 | 16.3364 | 14.2131 | 13.7565 | 12.6196 | 10.8261 | 10.3345 | 9.7426 |
| 5 GiB | 359.8495 | 186.6294 | 99.8038 | 77.4620 | 65.6555 | 67.3056 | 52.9090 | 48.4594 | 48.0126 | 48.8973 |
| 10 GiB | 694.9750 | 370.5490 | 188.2278 | 136.1719 | 113.5878 | 104.0254 | 93.7227 | 116.8249 | 83.2084 | 82.0529 |

TABLE II: Pure encoding times

| File sizes \ Threads | 1 | 2 | 4 | 6 | 8 | 10 | 12 | 16 | 20 | 24 |
|----------------------|----------|----------|----------|----------|---------|---------|---------|---------|---------|---------|
| 1 MiB | 0.0637 | 0.0338 | 0.0184 | 0.0128 | 0.0100 | 0.0085 | 0.0388 | 0.0938 | 0.0351 | 0.1032 |
| 5 MiB | 0.2980 | 0.1522 | 0.0782 | 0.0537 | 0.0413 | 0.0341 | 0.0296 | 0.0243 | 0.0207 | 0.0264 |
| 10 MiB | 0.6547 | 0.3523 | 0.1845 | 0.1254 | 0.0977 | 0.0812 | 0.0730 | 0.1533 | 0.0570 | 0.1505 |
| 50 MiB | 3.3027 | 1.7545 | 0.9147 | 0.6244 | 0.4884 | 0.4326 | 0.3432 | 0.2658 | 0.2953 | 0.3141 |
| 100 MiB | 6.7211 | 3.4461 | 1.8267 | 1.2448 | 0.9903 | 0.8153 | 0.6873 | 0.5415 | 0.4856 | 0.6148 |
| 500 MiB | 32.3825 | 16.6046 | 8.6817 | 6.2457 | 4.8624 | 4.0588 | 3.4132 | 2.6616 | 2.3853 | 2.0639 |
| 1 GiB | 67.1756 | 34.0970 | 17.7353 | 12.6548 | 9.8790 | 8.2365 | 7.0042 | 5.4307 | 4.8915 | 4.2098 |
| 5 GiB | 331.2526 | 174.0889 | 90.5618 | 63.4905 | 48.6453 | 38.9458 | 33.1852 | 26.1005 | 23.6226 | 20.4274 |
| 10 GiB | 639.3369 | 328.5801 | 170.4086 | 119.6824 | 95.1232 | 79.8054 | 68.8753 | 53.3415 | 47.5727 | 48.6436 |

TABLE III: Overall encoding efficiency

| File sizes \ Threads | 1 | 2 | 4 | 6 | 8 | 10 | 12 | 16 | 20 | 24 |
|----------------------|-----|--------|--------|--------|--------|--------|--------|--------|--------|--------|
| 1 MiB | 1.0 | 0.5272 | 0.3283 | 0.1997 | 0.1889 | 0.0986 | 0.0676 | 0.0321 | 0.0318 | 0.0179 |
| 5 MiB | 1.0 | 1.0144 | 0.8111 | 0.6544 | 0.5250 | 0.5182 | 0.4302 | 0.3317 | 0.2247 | 0.1922 |
| 10 MiB | 1.0 | 0.8034 | 0.7158 | 0.6086 | 0.5273 | 0.4090 | 0.2878 | 0.1419 | 0.1565 | 0.0807 |
| 50 MiB | 1.0 | 0.9178 | 0.7671 | 0.6410 | 0.5721 | 0.5171 | 0.4207 | 0.3726 | 0.2608 | 0.1643 |
| 100 MiB | 1.0 | 0.9497 | 0.8040 | 0.6925 | 0.6213 | 0.5394 | 0.4956 | 0.3873 | 0.3139 | 0.2094 |
| 500 MiB | 1.0 | 0.9731 | 0.7780 | 0.7140 | 0.5916 | 0.6217 | 0.4899 | 0.4014 | 0.3478 | 0.2924 |
| 1 GiB | 1.0 | 0.9578 | 0.8121 | 0.7459 | 0.6430 | 0.5315 | 0.4828 | 0.4221 | 0.3537 | 0.3127 |
| 5 GiB | 1.0 | 0.9641 | 0.9014 | 0.7742 | 0.6851 | 0.5346 | 0.5668 | 0.4641 | 0.3747 | 0.3066 |
| 10 GiB | 1.0 | 0.9378 | 0.9231 | 0.8506 | 0.7648 | 0.6681 | 0.6179 | 0.3718 | 0.4176 | 0.3529 |

TABLE IV: Pure encoding efficiency

| File sizes \ Threads | 1 | 2 | 4 | 6 | 8 | 10 | 12 | 16 | 20 | 24 |
|----------------------|-----|--------|--------|--------|--------|--------|--------|--------|--------|--------|
| 1 MiB | 1.0 | 0.9432 | 0.8677 | 0.8304 | 0.7960 | 0.7472 | 0.1370 | 0.0424 | 0.0908 | 0.0257 |
| 5 MiB | 1.0 | 0.9788 | 0.9521 | 0.9253 | 0.9025 | 0.8729 | 0.8402 | 0.7654 | 0.7194 | 0.4697 |
| 10 MiB | 1.0 | 0.9290 | 0.8872 | 0.8703 | 0.8373 | 0.8061 | 0.7476 | 0.2668 | 0.5743 | 0.1813 |
| 50 MiB | 1.0 | 0.9412 | 0.9027 | 0.8816 | 0.8454 | 0.7635 | 0.8021 | 0.7767 | 0.5593 | 0.4381 |
| 100 MiB | 1.0 | 0.9752 | 0.9198 | 0.8999 | 0.8484 | 0.8244 | 0.8149 | 0.7757 | 0.6920 | 0.4555 |
| 500 MiB | 1.0 | 0.9751 | 0.9325 | 0.8641 | 0.8325 | 0.7978 | 0.7906 | 0.7604 | 0.6788 | 0.6538 |
| 1 GiB | 1.0 | 0.9851 | 0.9469 | 0.8847 | 0.8500 | 0.8156 | 0.7992 | 0.7731 | 0.6867 | 0.6649 |
| 5 GiB | 1.0 | 0.9514 | 0.9144 | 0.8696 | 0.8512 | 0.8505 | 0.8318 | 0.7932 | 0.7011 | 0.6757 |
| 10 GiB | 1.0 | 0.9729 | 0.9379 | 0.8903 | 0.8401 | 0.8011 | 0.7735 | 0.7491 | 0.6720 | 0.5476 |

TABLE V: Overall encoding times

| File sizes \ Threads | 1 | 2 | 4 | 6 | 8 | 10 | 12 | 16 | 20 | 24 |
|----------------------|----------|----------|----------|----------|----------|----------|---------|---------|---------|---------|
| 1 MiB | 0.0880 | 0.0837 | 0.0732 | 0.0654 | 0.0724 | 0.0724 | 0.0672 | 0.0667 | 0.0752 | 0.0712 |
| 5 MiB | 0.4246 | 0.4062 | 0.2753 | 0.1725 | 0.1674 | 0.1642 | 0.1479 | 0.1389 | 0.1387 | 0.1362 |
| 10 MiB | 0.7128 | 0.6782 | 0.2417 | 0.1939 | 0.1762 | 0.1471 | 0.1676 | 0.1286 | 0.1403 | 0.1570 |
| 50 MiB | 3.5854 | 3.3710 | 1.1411 | 0.8380 | 0.7478 | 0.6985 | 0.9610 | 0.9681 | 0.7327 | 0.8949 |
| 100 MiB | 6.8047 | 6.6007 | 2.1585 | 1.6222 | 1.3589 | 1.2004 | 1.0981 | 0.9736 | 0.8877 | 0.8418 |
| 500 MiB | 33.4643 | 31.9953 | 10.5641 | 7.8537 | 6.6834 | 6.0406 | 5.5178 | 5.9307 | 4.5767 | 4.2484 |
| 1 GiB | 68.4891 | 65.9418 | 21.1728 | 16.4254 | 13.6481 | 12.0767 | 10.9664 | 9.7954 | 9.0356 | 8.3424 |
| 5 GiB | 367.2282 | 370.8502 | 110.2986 | 84.1276 | 75.3192 | 66.4084 | 59.6663 | 59.4788 | 51.1808 | 48.9043 |
| 10 GiB | 676.5289 | 642.9220 | 187.6202 | 135.9907 | 123.8217 | 109.7696 | 96.6426 | 93.2259 | 90.8253 | 88.1542 |

TABLE VI: Pure encoding times

| File sizes \ Threads | 1 | 2 | 4 | 6 | 8 | 10 | 12 | 16 | 20 | 24 |
|----------------------|----------|----------|----------|----------|---------|---------|---------|---------|---------|---------|
| 1 MiB | 0.0612 | 0.0621 | 0.0360 | 0.0278 | 0.0303 | 0.0216 | 0.0244 | 0.0239 | 0.0301 | 0.0158 |
| 5 MiB | 0.3611 | 0.3633 | 0.2093 | 0.1092 | 0.1031 | 0.0923 | 0.0792 | 0.0657 | 0.0530 | 0.0501 |
| 10 MiB | 0.6167 | 0.6074 | 0.1689 | 0.1167 | 0.0905 | 0.0756 | 0.0784 | 0.0540 | 0.0529 | 0.0590 |
| 50 MiB | 3.1314 | 3.0569 | 0.8470 | 0.5946 | 0.4641 | 0.4157 | 0.6712 | 0.6516 | 0.4349 | 0.4997 |
| 100 MiB | 6.0792 | 6.0371 | 1.6635 | 1.1509 | 0.8906 | 0.7352 | 0.6421 | 0.5210 | 0.4383 | 0.3862 |
| 500 MiB | 31.0011 | 30.4712 | 8.2873 | 5.8097 | 4.5192 | 3.7783 | 3.2265 | 2.6096 | 2.2273 | 1.9458 |
| 1 GiB | 62.7481 | 62.3551 | 17.2300 | 11.9387 | 9.1530 | 7.5818 | 6.5052 | 5.3627 | 4.6125 | 4.1011 |
| 5 GiB | 343.8756 | 351.0666 | 99.9091 | 73.8247 | 62.6944 | 54.0495 | 47.8488 | 42.7941 | 33.9437 | 29.9516 |
| 10 GiB | 629.4486 | 623.7099 | 172.8286 | 120.5519 | 94.6229 | 78.7281 | 68.0250 | 55.2154 | 46.7232 | 41.4739 |

TABLE VII: Overall encoding efficiency

| File sizes \ Threads | 1 | 2 | 4 | 6 | 8 | 10 | 12 | 16 | 20 | 24 |
|----------------------|-----|--------|--------|--------|--------|--------|--------|--------|--------|--------|
| 1 MiB | 1.0 | 0.5258 | 0.3004 | 0.2242 | 0.1519 | 0.1216 | 0.1091 | 0.0825 | 0.0585 | 0.0515 |
| 5 MiB | 1.0 | 0.5227 | 0.3856 | 0.4104 | 0.3171 | 0.2585 | 0.2392 | 0.1911 | 0.1531 | 0.1299 |
| 10 MiB | 1.0 | 0.5255 | 0.7371 | 0.6127 | 0.5057 | 0.4845 | 0.3543 | 0.3465 | 0.2541 | 0.1892 |
| 50 MiB | 1.0 | 0.5318 | 0.7855 | 0.7130 | 0.5994 | 0.5133 | 0.3109 | 0.2315 | 0.2447 | 0.1669 |
| 100 MiB | 1.0 | 0.5155 | 0.7881 | 0.6991 | 0.6260 | 0.5669 | 0.5164 | 0.4368 | 0.3833 | 0.3368 |
| 500 MiB | 1.0 | 0.5230 | 0.7919 | 0.7102 | 0.6259 | 0.5540 | 0.5054 | 0.3527 | 0.3656 | 0.3282 |
| 1 GiB | 1.0 | 0.5193 | 0.8087 | 0.6949 | 0.6273 | 0.5671 | 0.5204 | 0.4370 | 0.3790 | 0.3421 |
| 5 GiB | 1.0 | 0.4951 | 0.8323 | 0.7275 | 0.6095 | 0.5530 | 0.5129 | 0.3859 | 0.3588 | 0.3129 |
| 10 GiB | 1.0 | 0.5261 | 0.9015 | 0.8291 | 0.6830 | 0.6163 | 0.5834 | 0.4536 | 0.3724 | 0.3198 |

TABLE VIII: Pure encoding efficiency

| File sizes \ Threads | 1 | 2 | 4 | 6 | 8 | 10 | 12 | 16 | 20 | 24 |
|----------------------|-----|--------|--------|--------|--------|--------|--------|--------|--------|--------|
| 1 MiB | 1.0 | 0.4928 | 0.4246 | 0.3672 | 0.2522 | 0.2831 | 0.2086 | 0.1598 | 0.1015 | 0.1611 |
| 5 MiB | 1.0 | 0.4969 | 0.4312 | 0.5511 | 0.4378 | 0.3912 | 0.3801 | 0.3437 | 0.3409 | 0.3001 |
| 10 MiB | 1.0 | 0.5077 | 0.9129 | 0.8805 | 0.8520 | 0.8159 | 0.6554 | 0.7144 | 0.5827 | 0.4356 |
| 50 MiB | 1.0 | 0.5122 | 0.9243 | 0.8778 | 0.8433 | 0.7532 | 0.3888 | 0.3003 | 0.3601 | 0.2611 |
| 100 MiB | 1.0 | 0.5035 | 0.9136 | 0.8804 | 0.8532 | 0.8268 | 0.7890 | 0.7292 | 0.6934 | 0.6560 |
| 500 MiB | 1.0 | 0.5087 | 0.9352 | 0.8893 | 0.8575 | 0.8205 | 0.8007 | 0.7425 | 0.6959 | 0.6638 |
| 1 GiB | 1.0 | 0.5032 | 0.9104 | 0.8760 | 0.8569 | 0.8276 | 0.8038 | 0.7313 | 0.6802 | 0.6375 |
| 5 GiB | 1.0 | 0.4898 | 0.8605 | 0.7763 | 0.6856 | 0.6362 | 0.5989 | 0.5022 | 0.5065 | 0.4784 |
| 10 GiB | 1.0 | 0.5046 | 0.9105 | 0.8702 | 0.8315 | 0.7995 | 0.7711 | 0.7125 | 0.6736 | 0.6324 |

TABLE IX: Overall decoding times

| File sizes \ Threads | 1 | 2 | 4 | 6 | 8 | 10 | 12 | 16 | 20 | 24 |
|----------------------|----------|----------|----------|----------|----------|----------|----------|----------|----------|----------|
| 1 MiB | 0.1304 | 0.0907 | 0.0793 | 0.0986 | 0.0912 | 0.0975 | 0.1179 | 0.1152 | 0.1185 | 0.1039 |
| 5 MiB | 0.4219 | 0.2527 | 0.1772 | 0.1663 | 0.1411 | 0.1563 | 0.1507 | 0.1395 | 0.1296 | 0.1214 |
| 10 MiB | 0.8712 | 0.5173 | 0.3839 | 0.2955 | 0.2931 | 0.4331 | 0.3716 | 0.3557 | 0.3683 | 0.3546 |
| 50 MiB | 4.1082 | 2.2768 | 1.5108 | 1.2518 | 1.2762 | 1.3824 | 1.4124 | 1.4088 | 1.4343 | 1.3277 |
| 100 MiB | 7.9605 | 4.5073 | 2.9151 | 2.2752 | 1.8842 | 2.4183 | 2.4003 | 2.4222 | 2.3432 | 2.2511 |
| 500 MiB | 37.6189 | 22.5113 | 13.5012 | 11.3339 | 10.0352 | 9.3255 | 9.6405 | 11.1822 | 11.0692 | 10.0214 |
| 1 GiB | 76.6794 | 43.2517 | 27.5849 | 23.2715 | 18.5904 | 17.8240 | 18.5319 | 22.0271 | 19.8127 | 17.1276 |
| 5 GiB | 361.8713 | 193.1122 | 108.8124 | 96.7901 | 81.0661 | 84.8969 | 79.1422 | 73.7254 | 79.3745 | 75.9761 |
| 10 GiB | 759.0719 | 391.2016 | 212.3814 | 156.3964 | 135.8435 | 135.4010 | 134.0983 | 137.9998 | 134.0065 | 132.2553 |

TABLE X: Pure decoding times

| File sizes \ Threads | 1 | 2 | 4 | 6 | 8 | 10 | 12 | 16 | 20 | 24 |
|----------------------|----------|----------|----------|----------|----------|---------|---------|----------|---------|---------|
| 1 MiB | 0.0833 | 0.0579 | 0.0519 | 0.0737 | 0.0693 | 0.0712 | 0.0920 | 0.0903 | 0.0924 | 0.0761 |
| 5 MiB | 0.3397 | 0.1883 | 0.1231 | 0.1014 | 0.0843 | 0.0942 | 0.0893 | 0.0789 | 0.0729 | 0.0623 |
| 10 MiB | 0.7516 | 0.4136 | 0.2667 | 0.1912 | 0.1905 | 0.2648 | 0.2672 | 0.2459 | 0.2821 | 0.2490 |
| 50 MiB | 3.6338 | 1.8884 | 1.0578 | 0.7449 | 0.6699 | 0.9762 | 0.9542 | 0.9429 | 0.9658 | 0.8575 |
| 100 MiB | 7.1318 | 3.6134 | 2.0121 | 1.3884 | 1.1225 | 1.5401 | 1.5201 | 1.5413 | 1.4775 | 1.4684 |
| 500 MiB | 36.3947 | 18.5941 | 9.8736 | 6.9800 | 5.4581 | 4.7071 | 5.7395 | 6.9697 | 6.4991 | 5.4733 |
| 1 GiB | 74.7637 | 38.4079 | 19.7644 | 13.8396 | 10.9082 | 10.2419 | 9.2313 | 12.5040 | 12.0604 | 9.8245 |
| 5 GiB | 359.8834 | 188.8855 | 97.7879 | 82.3554 | 56.9994 | 59.9356 | 50.8868 | 39.2907 | 48.5617 | 48.8765 |
| 10 GiB | 754.8416 | 385.0213 | 199.8499 | 138.7388 | 113.1048 | 95.2621 | 81.1119 | 108.7269 | 94.3171 | 81.2615 |

TABLE XI: Overall decoding efficiency

| File sizes \ Threads | 1 | 2 | 4 | 6 | 8 | 10 | 12 | 16 | 20 | 24 |
|----------------------|-----|--------|--------|--------|--------|--------|--------|--------|--------|--------|
| 1 MiB | 1.0 | 0.7186 | 0.4108 | 0.2204 | 0.1787 | 0.1338 | 0.0921 | 0.0707 | 0.0550 | 0.0523 |
| 5 MiB | 1.0 | 0.8349 | 0.5953 | 0.4228 | 0.3739 | 0.2700 | 0.2334 | 0.1890 | 0.1628 | 0.1448 |
| 10 MiB | 1.0 | 0.8421 | 0.5673 | 0.4913 | 0.3715 | 0.2012 | 0.1954 | 0.1531 | 0.1183 | 0.1024 |
| 50 MiB | 1.0 | 0.9022 | 0.6798 | 0.5469 | 0.4024 | 0.2972 | 0.2424 | 0.1823 | 0.1432 | 0.1289 |
| 100 MiB | 1.0 | 0.8831 | 0.6827 | 0.5831 | 0.5281 | 0.3292 | 0.2764 | 0.2054 | 0.1699 | 0.1473 |
| 500 MiB | 1.0 | 0.8356 | 0.6966 | 0.5532 | 0.4686 | 0.4034 | 0.3252 | 0.2103 | 0.1699 | 0.1564 |
| 1 GiB | 1.0 | 0.8864 | 0.6949 | 0.5492 | 0.5156 | 0.4302 | 0.3448 | 0.2176 | 0.1935 | 0.1865 |
| 5 GiB | 1.0 | 0.9369 | 0.8314 | 0.6231 | 0.5580 | 0.4262 | 0.3810 | 0.3068 | 0.2280 | 0.1985 |
| 10 GiB | 1.0 | 0.9702 | 0.8935 | 0.8089 | 0.6985 | 0.5606 | 0.4717 | 0.3438 | 0.2832 | 0.2391 |

TABLE XII: Pure decoding efficiency

| File sizes \ Threads | 1 | 2 | 4 | 6 | 8 | 10 | 12 | 16 | 20 | 24 |
|----------------------|-----|--------|--------|--------|--------|--------|--------|--------|--------|--------|
| 1 MiB | 1.0 | 0.7196 | 0.4013 | 0.1884 | 0.1504 | 0.1170 | 0.0755 | 0.0577 | 0.0451 | 0.0456 |
| 5 MiB | 1.0 | 0.9021 | 0.6897 | 0.5580 | 0.5038 | 0.3606 | 0.3168 | 0.2689 | 0.2330 | 0.2271 |
| 10 MiB | 1.0 | 0.9087 | 0.7046 | 0.6550 | 0.4931 | 0.2839 | 0.2344 | 0.1911 | 0.1332 | 0.1258 |
| 50 MiB | 1.0 | 0.9621 | 0.8588 | 0.8131 | 0.6781 | 0.3722 | 0.3174 | 0.2409 | 0.1881 | 0.1766 |
| 100 MiB | 1.0 | 0.9869 | 0.8861 | 0.8562 | 0.7942 | 0.4631 | 0.3910 | 0.2892 | 0.2413 | 0.2024 |
| 500 MiB | 1.0 | 0.9787 | 0.9215 | 0.8690 | 0.8335 | 0.7732 | 0.5284 | 0.3264 | 0.2800 | 0.2771 |
| 1 GiB | 1.0 | 0.9733 | 0.9457 | 0.9004 | 0.8567 | 0.7300 | 0.6749 | 0.3737 | 0.3100 | 0.3171 |
| 5 GiB | 1.0 | 0.9526 | 0.9201 | 0.7283 | 0.7892 | 0.6004 | 0.5894 | 0.5725 | 0.3705 | 0.3068 |
| 10 GiB | 1.0 | 0.9803 | 0.9443 | 0.9068 | 0.8342 | 0.7924 | 0.7755 | 0.4339 | 0.4002 | 0.3870 |

TABLE XIII: Overall decoding times

| File sizes \ Threads | 1 | 2 | 4 | 6 | 8 | 10 | 12 | 16 | 20 | 24 |
|----------------------|----------|----------|----------|----------|----------|----------|----------|----------|----------|----------|
| 1 MiB | 0.2719 | 0.2116 | 0.1162 | 0.1006 | 0.0699 | 0.0677 | 0.0656 | 0.0695 | 0.0748 | 0.0928 |
| 5 MiB | 0.6351 | 0.9155 | 1.2650 | 0.6553 | 0.6837 | 0.2889 | 0.4825 | 0.2982 | 0.3296 | 0.1795 |
| 10 MiB | 0.8607 | 0.5007 | 0.3328 | 0.2710 | 0.2687 | 0.2580 | 0.2364 | 0.2169 | 0.1899 | 0.1830 |
| 50 MiB | 3.9914 | 2.2478 | 1.4618 | 1.1756 | 1.1596 | 1.0884 | 1.0173 | 0.9127 | 0.8369 | 0.7747 |
| 100 MiB | 7.6155 | 33.6453 | 15.2498 | 11.6282 | 13.0181 | 10.7612 | 9.1921 | 3.6613 | 5.6912 | 4.4747 |
| 500 MiB | 37.0819 | 20.4215 | 13.2360 | 11.3439 | 10.7738 | 10.4405 | 9.7670 | 8.6608 | 7.3871 | 7.0508 |
| 1 GiB | 73.8554 | 41.1849 | 26.9294 | 23.0511 | 17.9771 | 18.7808 | 19.7312 | 17.8374 | 16.1711 | 15.4367 |
| 5 GiB | 352.3856 | 475.0869 | 265.8940 | 204.6614 | 156.2271 | 139.8945 | 127.6489 | 109.2454 | 100.9185 | 89.8899 |
| 10 GiB | 714.9067 | 375.6862 | 202.7149 | 190.6061 | 435.2239 | 145.6734 | 133.3188 | 122.7745 | 117.5286 | 115.2620 |

TABLE XIV: Pure decoding times

| File sizes \ Threads | 1 | 2 | 4 | 6 | 8 | 10 | 12 | 16 | 20 | 24 |
|----------------------|----------|----------|----------|----------|----------|----------|----------|---------|---------|---------|
| 1 MiB | 0.1162 | 0.0645 | 0.0563 | 0.0555 | 0.0407 | 0.0408 | 0.0379 | 0.0413 | 0.0382 | 0.0351 |
| 5 MiB | 0.5650 | 0.8586 | 1.2021 | 0.5983 | 0.6266 | 0.2339 | 0.4245 | 0.2443 | 0.2693 | 0.1130 |
| 10 MiB | 0.7149 | 0.3986 | 0.2338 | 0.1743 | 0.1690 | 0.1591 | 0.1363 | 0.1187 | 0.0949 | 0.0849 |
| 50 MiB | 3.5310 | 1.8481 | 1.0339 | 0.7423 | 0.7299 | 0.6557 | 0.6202 | 0.4858 | 0.4169 | 0.3519 |
| 100 MiB | 6.8744 | 33.3659 | 14.8835 | 10.7860 | 12.3773 | 9.9066 | 8.3482 | 2.8794 | 5.0465 | 3.6955 |
| 500 MiB | 35.4497 | 18.2381 | 10.0089 | 7.1791 | 6.7919 | 6.3173 | 5.6216 | 4.6675 | 3.8306 | 3.2797 |
| 1 GiB | 72.1138 | 36.9260 | 20.1134 | 14.5033 | 11.5997 | 12.6916 | 11.3021 | 9.2808 | 7.6540 | 6.4776 |
| 5 GiB | 349.8904 | 471.3404 | 257.7142 | 197.0613 | 140.1541 | 124.8532 | 110.6343 | 91.4010 | 74.5820 | 62.5393 |
| 10 GiB | 711.9846 | 370.5192 | 191.8006 | 182.3177 | 421.9179 | 126.6782 | 113.4824 | 92.0815 | 76.3088 | 64.4831 |

TABLE XV: Overall decoding efficiency

| File sizes \ Threads | 1 | 2 | 4 | 6 | 8 | 10 | 12 | 16 | 20 | 24 |
|----------------------|-----|--------|--------|--------|--------|--------|--------|--------|--------|--------|
| 1 MiB | 1.0 | 0.6424 | 0.5848 | 0.4504 | 0.4859 | 0.4018 | 0.3456 | 0.2446 | 0.1818 | 0.1220 |
| 5 MiB | 1.0 | 0.3469 | 0.1255 | 0.1615 | 0.1161 | 0.2199 | 0.1097 | 0.1331 | 0.0963 | 0.1474 |
| 10 MiB | 1.0 | 0.8595 | 0.6466 | 0.5293 | 0.4005 | 0.3336 | 0.3033 | 0.2480 | 0.2266 | 0.1959 |
| 50 MiB | 1.0 | 0.8878 | 0.6826 | 0.5659 | 0.4303 | 0.3667 | 0.3269 | 0.2733 | 0.2385 | 0.2147 |
| 100 MiB | 1.0 | 0.1132 | 0.1248 | 0.1092 | 0.0731 | 0.0708 | 0.0690 | 0.1300 | 0.0669 | 0.0709 |
| 500 MiB | 1.0 | 0.9079 | 0.7004 | 0.5448 | 0.4302 | 0.3552 | 0.3164 | 0.2676 | 0.2510 | 0.2191 |
| 1 GiB | 1.0 | 0.8966 | 0.6856 | 0.5340 | 0.5135 | 0.3932 | 0.3119 | 0.2588 | 0.2284 | 0.1993 |
| 5 GiB | 1.0 | 0.3709 | 0.3313 | 0.2870 | 0.2819 | 0.2519 | 0.2300 | 0.2016 | 0.1746 | 0.1633 |
| 10 GiB | 1.0 | 0.9506 | 0.8817 | 0.6251 | 0.2053 | 0.4908 | 0.4469 | 0.3639 | 0.3041 | 0.2584 |

TABLE XVI: Pure decoding efficiency

| File sizes \ Threads | 1 | 2 | 4 | 6 | 8 | 10 | 12 | 16 | 20 | 24 |
|----------------------|-----|--------|--------|--------|--------|--------|--------|--------|--------|--------|
| 1 MiB | 1.0 | 0.9006 | 0.5154 | 0.3487 | 0.3568 | 0.2845 | 0.2554 | 0.1758 | 0.1518 | 0.1379 |
| 5 MiB | 1.0 | 0.3290 | 0.1175 | 0.1574 | 0.1127 | 0.2416 | 0.1109 | 0.1446 | 0.1049 | 0.2083 |
| 10 MiB | 1.0 | 0.8967 | 0.7645 | 0.6835 | 0.5286 | 0.4492 | 0.4369 | 0.3764 | 0.3766 | 0.3507 |
| 50 MiB | 1.0 | 0.9553 | 0.8538 | 0.7928 | 0.6047 | 0.5385 | 0.4745 | 0.4543 | 0.4234 | 0.4181 |
| 100 MiB | 1.0 | 0.1030 | 0.1155 | 0.1062 | 0.0694 | 0.0694 | 0.0686 | 0.1492 | 0.0681 | 0.0775 |
| 500 MiB | 1.0 | 0.9719 | 0.8855 | 0.8230 | 0.6524 | 0.5612 | 0.5255 | 0.4747 | 0.4627 | 0.4504 |
| 1 GiB | 1.0 | 0.9765 | 0.8963 | 0.8287 | 0.7771 | 0.5682 | 0.5317 | 0.4856 | 0.4711 | 0.4639 |
| 5 GiB | 1.0 | 0.3712 | 0.3394 | 0.2959 | 0.3121 | 0.2802 | 0.2635 | 0.2393 | 0.2346 | 0.2331 |
| 10 GiB | 1.0 | 0.9608 | 0.9281 | 0.6509 | 0.2109 | 0.5620 | 0.5228 | 0.4833 | 0.4665 | 0.4601 |