

# Parallel Huffman Coding

Francesco Bozzo  
DISI, University of Trento  
Trento, Italy  
francesco.bozzo@studenti.unitn.it  
229312

Michele Yin  
DISI, University of Trento  
Trento, Italy  
michele.yin@studenti.unitn.it  
229359

**Abstract**—This report aims to explain the design of a parallel encoding and decoding Huffman algorithm. The application is developed with the C99 programming language, using MPI for multiprocessing and OpenMP for multithreading to scale horizontally with increasing hardware resources. Our tool is both able to exploit multiprocessing to process concurrently multiple files, and to split the Huffman coding of the same file among multiple threads.

Comparing this tool with other online implementations, we can state that...

**Index Terms**—Huffman, MPI, OpenMP, High Performance Computing

## I. INTRODUCTION

Even if the Huffman algorithm is not directly used nowadays, its prefix mechanism is still part of Deflate (PKZIP's algorithm), JPEG, and MP3 compression algorithms

## II. THE HUFFMAN ALGORITHM

The Huffman algorithm has the objective of finding a more convenient bit representation to store information through lossless compression. Instead of considering groups of eight bits as the way to encode data, the Huffman algorithm uses variable-length sequences of bits with prefixes defined as *alphabet*.

### A. Priority Queue

The Huffman algorithm uses a minimum priority queue to build its alphabet efficiently. This specific data structure ensures logarithmic insertion and deletion time with respect to its size. We implemented the minimum priority queue by using a minimum heap. Practically speaking, to implement the min-heap tree, we used a standard C array ensuring that the min-heap property still holds at every insertion and deletion:

$$A[i] \leq A[l(i)], A[i] \leq A[r(i)] \quad (1)$$

where  $A[i]$ ,  $A[l(i)]$ , and  $A[r(i)]$  are respectively a node, its left child, and its right child in a min-heap tree.

To implement the priority queue also a parallel approach has been considered [1], but since it contains only 256 elements (1 byte), we did not consider it too important in terms of performance.

### B. Encoding

The Huffman encoding procedure makes use of a minimum priority queue to build its alphabet efficiently. The idea is to build a tree similar to Figure 1 that defines all the variable-length prefix sequences of bits: these sequences are represented by the path from the tree root to a leaf. Using a greedy approach, the Huffman encoding ensures that the less frequent a byte is in a file, the more probable is to have a longer Huffman representation, which means that its path from the root to its specific leaf is longer.

Once having computed the frequencies for each one of the 256 different bytes in a file, the Huffman algorithm populates a min priority queue with Huffman tree nodes storing the byte value and its frequency. Successively, it removes the least two frequent bytes from the queue, creates a new node with children the two extracted nodes, assigns to it a dummy character and the sum of the frequencies of its two children, and inserts it into the min priority queue. After  $n-1$  iterations, the last node is the root of the Huffman tree [2]. Algorithm 1 explains in the detail this procedure.

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**Algorithm 1:** Build the Huffman tree

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```
1 // Populate the min priority queue with characters and
  their frequencies
2 for  $i = 1$  to  $n - 1$  do
3   Q.insert(f[i], Tree(f[i], c[i]))
4 // Repeat until the queue has only a single element left
5 for  $i = 1$  to  $n - 1$  do
6   // Get the two least frequent nodes
7   z1 = Q.deleteMin()
8   z2 = Q.deleteMin()
9   // Create an inner tree node and insert it into the
    queue
10  z = Tree(z1.f + z2.f, null)
11  z.left = z1
12  z.right = z2
13  Q.insert(z.f, z)
14 // The last element in the queue is the root of the
    Huffman tree
15 return Q.deleteMin()
```

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Once built the Huffman tree, it is possible to generate the

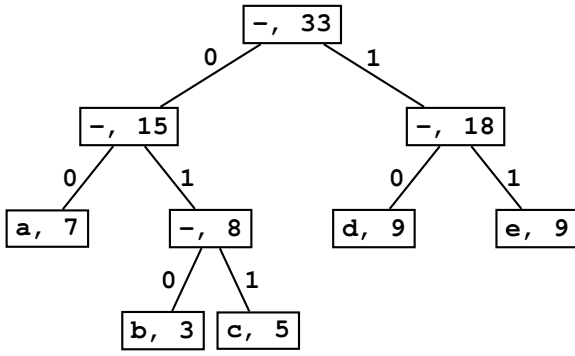


Fig. 1: An example of Huffman tree.

Huffman alphabet visiting the tree using a DFS algorithm, assigning 0 to each left-child traverse and 1 for the right one as presented in Algorithm 2. Finally, it is possible to compress the file by creating a stream of bits corresponding to its content using the Huffman alphabet.

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**Algorithm 2:** Encode using Huffman tree

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```

1 while not eof() do
2   bit = read()
3   if bit == 0 then
4     encode(node.left)
5   else
6     encode(node.right)
7   if node is leaf then
8     return node.value

```

---

### C. Decoding

Once having the encoded file and the Huffman tree, it is straightforward to decompress the file to its original shape. The prefix alphabet ensures to have that it is possible to visit the tree using a DFS approach and get a unique decoded version of the file using an approach similar to Algorithm 2.

### III. SERIAL HUFFMAN IMPLEMENTATION

To be able to implement the Huffman encoding and decoding algorithm, several things have been taken into account.

- The byte frequencies computed during the encoding process are saved in the compressed file. In this way, the decoding procedure can easily rebuild the Huffman tree.
- Both the encoding and decoding procedures make use of buffers to improve I/O performance: the streams of bytes are first written inside the buffer and then saved on the disk.
- The encoding procedure works with chunks of 4096 bytes. This ensures that the tool can handle even large files when dealing with bit buffers. The decoding procedure deals with chunks of maximum size of  $4096 * 32$

bytes, since the maximum length of a Huffman tree traverse from root to leaf is of 256 bits, which is 32 times a byte. As explained later, dealing with chunks makes handy the parallel implementation of the encoding and decoding algorithms.

- In the encoded file, the chunk offsets are saved at the end of the file: this ensures that the compressed stream of prefixed sequences of bits is still meaningful.

### IV. PARALLEL HUFFMAN IMPLEMENTATION

To parallelize the serial Huffman algorithm over threads and processes, we decided to follow this simple yet effective structure:

- Multiple processes should handle groups of file and folders separately.
- Multiple threads of the same process should work on different chunks of the same file in parallel.

Specifically, there are multiple reasons behind our parallel architectural design for this project. Here follows a list of the main ones:

- In most operating systems a file is a resource that the OS gives to a single process to avoid race conditions. We wanted to follow a similar design philosophy.
- Most operating systems allow multiple processes to open the same file in reading mode, but only few allow to open the same file in writing mode on multiple processes. This is because that leads to potentially concurrency and data integrity issues. By ensuring to have only a single process that open a specific file, we avoid all these issues.
- Because threads of the same process share the address space, we can avoid the expensive data transfer across processes. When data is read or written to file, there is no need to transfer data between threads.

In the next section, we start by explaining the basic multi-threading operations on a single file. Later on, multiprocessing with multiple files is covered.

#### A. Multithreading

The parallel algorithm for a single file follows the exact same procedure as for the serial version, with just a few key differences to allow multithreading.

Suppose we are dealing with  $m$  threads and a single input file. Here follows a common procedure used for both encoding and decoding, with very small differences.

- 1) A single file is divided into chunks of a fixed size. When that is done, the single process forks and creates  $m$  threads.
- 2) A single thread (may be the main one, may not ) then reads  $m$  chunks in a shared memory buffer.
- 3) Each one of the  $m$  threads works in parallel on the processing of its assigned chunk using a buffer. Although all the chunks are in shared memory buffer, each thread is assigned only to a single memory section: in this way there is no need to care about racing conditions when writing in the buffer. This is done by creating a `unsigned char buffer[m][buffer_size]`.

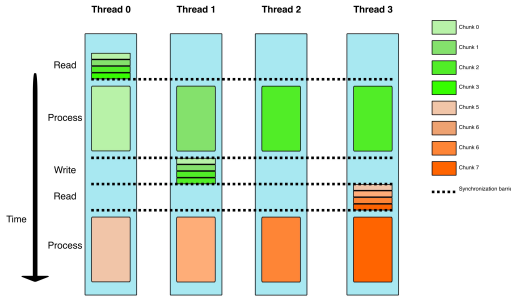


Fig. 2: Simple schema for processing a single file with multiple threads.

- 4) When all threads are done computing, either the encoding or decoding of their chunks, a single thread writes the processed chunks to the output file. Again, since all threads share a memory space there is no need to perform data transfer.

Moreover, our implementation also parallelize the counting of occurrences of a byte in the input file, since this is required in order to create the Huffman tree. This is done only during the encoding phase, as for the decoding the occurrences are no longer required because the Huffman tree is already saved in the encoded data. The procedure is very similar to the one just described, with the key difference that there is no output file to write, but only byte occurrences stored in an in-memory array unique for each thread to avoid concurrency. When the counting is done, all threads join their counts into a single array.

Figure 2 shows a schema of the parallel workflow for processing a single file.

### B. Multiprocessing

Additionally, we wanted our tool to be able to process multiple files and folders: for this use case, we exploit multiprocessing.

To achieve this, we devised a simple algorithm to distribute files across multiple processes, so that each process can work on its own list of files, independently to others. In this way, we can minimize communication and therefore latency and other slowdowns between the different components of the applications. Suppose we are dealing with  $n$  process,  $m$  threads, and an input folder.

- 1) First, the process with rank 0 (the main process) crawls all the files in the input directory recursively in all the subfolders.
- 2) Then the rank 0 process opens all the files and reads their size.
- 3) Once all files and their respective sizes are known, the main process creates a min priority queue where each item represents a process and its priority is the size of files that have been assigned to it.
- 4) Process 0 can iteratively insert files in the priority queue and updates the priority of each process with the

cumulative file size. The main idea is that we can ensure an equal work division among different processes.

- 5) When all this is done, the main process sends the list of files to each process and then each starts to work independently on its own list of files.

### C. Implementation details and other notes

Here follows a list of further comments and design choices for our specific Huffman implementation.

- We decided on an alphabet of 256 for the Huffman coding because this way we can encode each byte of the input data into a different Huffman code. As C language does not allow to address data at lower resolution than a byte, lowering the alphabet would not result in any performance benefit. We also considered encoding multiple bytes into a single Huffman code, but that results in worse compression as the resulting Huffman tree would have more leaves.
- We found that a chunk size of 4096 bytes had the best I/O times. This is probably due to the fact that 4096 B is the size of a page in most Linux based operating systems. The last chunk of a file may be smaller. Reading any other size at the time had resulted in significantly worse I/O times (both increasing and decreasing the chunk size).
- In general, all threads will finish processing their chunks at around the same time, but in some cases there are threads that can take longer. This is due to the very unlucky case where a whole chunk contains exclusively very infrequent bytes of data. Because the bytes are very infrequent, each one is encoded in extremely long sequences, up to 256 bits, resulting in a bigger encoded chunk than the original data. With our Huffman alphabet set to a single byte, the worst case results in having a compressed chunk being 32 times bigger than decompressed one.
- We tried to parallelize the I/O, by having each thread read its own chunk. We found that the standard file descriptor provided by C have a lock to guarantee thread safety. Unfortunately, this lock slows down the reads significantly. We also tried using multiple file descriptors to circumvent this limitation, however we found no improvement over a single-thread sequential read. This is likely because the O.S. schedules I/O requests and serves them one at the time, resulting in an impossibility of having truly parallel I/O.
- We also tested an architecture where we had two dedicated threads to I/O, one for reading and one for writing chunks. The idea is that whenever a chunk is processed, the I/O threads would immediately write the processed chunk to the output file and similarly a new chunk would be read from the input file (very similar to a queue of jobs). Theoretically this would allow to parallelize I/O and computation operations: in this scenario we avoided concurrency problems by synchronizing the I/O with locks. In later analysis we call this architecture "Locks" because of the synchronization method we are using.

- Another detail we noticed on the implementation with two dedicated threads for I/O, is that if there are not enough cores on the CPU or if the operating system decides to allocate all the threads on a single processor, the encoding and decoding times are greatly affected by the scheduler. This is because it might be that the O.S. gives priority to threads that are waiting (for example the writer cannot write any block until the first has finished, even if all others have finished). In the worst case scenario, the multithreading architecture could be even slower than the serial one.

## V. PERFORMANCE AND BENCHMARKING

In this section we are going to discuss the performance evaluation of our algorithm. We start from evaluating some other implementations of Huffman algorithm and then focus on the analysis of our own implementation.

We include in this section some graphs and statistics, but all detailed results are included in the appendix at the end of this report.

### A. Setup

All the tests have been performed on the HPC2 cluster of the University of Trento. To reduce as much as possible the I/O timings, which are crucial for our application, jobs were run in a very specific setting:

- Threads are allocated in the same node and, if possible, on the same socket to guarantee fast communication between them. This has been achieved by using the MPI option `--map-by`.
- Processes are allocated in different nodes, by using the PBS directive `place=scatter:excl`. This is because in our application different processes do not communicate very often between each other, but they just split their work and use the file-system.

The provided timings are obtained by averaging the result of three runs with the exact same configurations: this helps to minimize the effect of random variance between runs due to potential hardware congestion. Each job has been submitted to the cluster individually, waiting for the previous to finish preventing hiccups due to multiple instances of the program trying to access the same file-system resource, which we noticed to greatly affect I/O times.

We extensively verified the correctness of our algorithm by encoding and then decoding a file, then comparing the decoded result to the original file by using the `diff` command. Moreover, Valgrind has been used to spot any memory leaks that could cause runtime errors.

### B. Datasets

The evaluation dataset has been randomly generated by using a simple C program: we created text files with character frequencies that follow the occurrences of the letters in English language. Although for benchmarking we are only using ASCII characters, please note that our algorithm reads bytes of data and can therefore work with any file. The dataset is

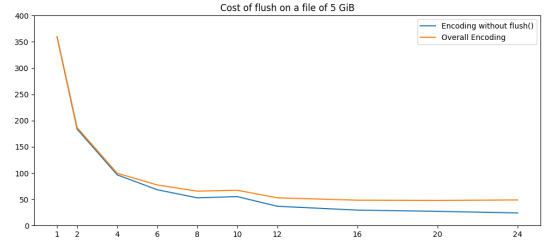


Fig. 3: Encoding times with and without the flush()

composed by files of increasing size: 1 MiB, 5 MiB, 10 MiB, 50 MiB, 100 MiB, 500 MiB, 1 GiB, 5 GiB, and 10 GiB. This should be enough range to understand how different algorithms scale with increasing file sizes.

Secondly, because our algorithm also works with many files at once, we chose some GitHub repository as benchmark: Numpy, PyTorch and Linux kernel as benchmarks. They are respectively 30MiB, 200MiB and 1.3 GiB of total size, with about  $\sim 2,000$ ,  $\sim 12,000$ , and  $\sim 80,000$  files, which on average are each 16MiB in size.

Considering the dataset sizes and amount testing we have done, we estimate to have accessed the disk (both read and writes) for about  $\sim 6$  TiB of data for our whole benchmark analysis.

### C. Other works

#### D. Multithreading evaluation

In this section we evaluate our algorithm on single files, by using multithreading and testing our parallelization performances with an increasing number of threads.

With small files (i.e. less than a few MiBs) the algorithm is actually slower with more threads. This is likely to be the case because the data is not large enough to offset the cost of creating multiple threads. Considering the cost of parallel processing, we highly suggest not doing parallelization for small files.

However, when the data is large enough our multithreaded approach scales up. With the largest file (10 GiB) our algorithm has an efficiency of 93% when tested with 4 threads. This number steadily decreases with the number of threads, and we hit 32% with 24 threads.

Please note that our tool is heavily limited by the cluster I/O, since we can not make it parallel: especially writings to disk require a considerable amount of time. As a proof of this, if we consider only the processing section of our algorithm, and ignore the time required by the `fflush()` operation before a `fclose()`, we see significantly decreased times. Although we start from similar efficiency at low threading, we achieve a 54% efficiency with 24 threads in this case: if we consider only CPU time, which does not count the time spent waiting in I/O, we get very promising results.

We also noticed worse times in the decoding procedure with respect to encoding. This is probably due to the fact that the tool is reading fixed chunks of 4096 bytes in encoding, but during decoding the chunk can have a variable length.

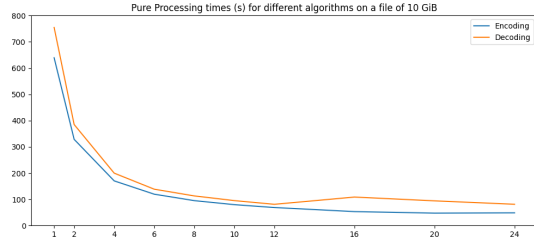


Fig. 4: Encoding vs Decoding times

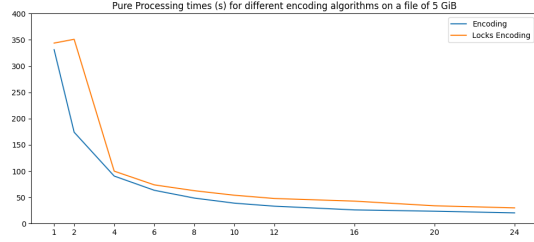


Fig. 5: Barrier vs Lock based synchronization performance

As already discussed previously, we also tested a version on a lock-based synchronization instead of barriers. Although theoretically we should get better results, in practice we obtain almost equal performances across the board, with only the largest files having some significant difference. This difference is greater in decoding than in encoding.

#### E. Multiprocessing

We tested on different folders as benchmark the parallelization performances of our algorithm. Theoretically, our algorithm should scale very well with increasing number of processes and files, because once the main process sends the list of files to encode to each process there is no further communication between different processes. However, we find that real world results are different. While we don't expect much increasing in the performance by the number of threads, because the files are on average very small, we should get better results by using many processes. In reality our testing shows that this is not the case. Our efficiencies are extremely low. We believe the main limitation of our algorithm is the I/O of the cluster. When tested on a local machine (MacBook A2442) with one thread and 1,2,4 processes, we find that our algorithm does indeed scale with the number of processes.

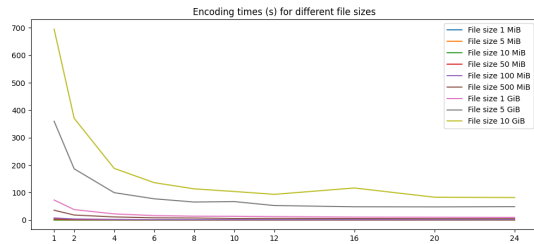


Fig. 6: Encoding times

Moreover, we were not able to test Linux on the cluster due to extreme I/O times.

#### F. Memory, storage and other considerations

Our memory requirements don't scale upwards with the size of the files we need to encode, because at any given time, the most we only store is the size of two buffers, each is a `unsigned char buffer[num_threads][4096]`. These buffers are emptied at each I/O cycle, therefore leading to a high memory efficiency. Only when processing multiple files our memory requirements scale up with the number of files, as each process needs to store information about the files is assigned to process.

We also find that we achieve on average 48% compression rate of our synthetic data. Instead, on Linux kernel, because contains more varied data which consists of mostly code, we achieve a lower 62% compression rate. Testing on other already compressed data as .zip or AVI resulted in a compression rate of 100%.

## VI. CONCLUSIONS

To conclude.

## REFERENCES

- [1] G. S. Brodal, J. L. Träff, and C. D. Zaroliagis, "A parallel priority queue with constant time operations," *Journal of Parallel and Distributed Computing*, vol. 49, no. 1, pp. 4–21, 1998. [Online]. Available: <https://www.sciencedirect.com/science/article/pii/S0743731598914253>
- [2] A. Bertossi and A. Montresor, *Algoritmi e strutture di dati*. CittàStudi, 2010. [Online]. Available: <https://books.google.es/books?id=WKWLSQAACAAJ>

## VII. APPENDIX

### A. Encoding

TABLE I: Overall encoding times

Threads Size	1	2	4	6	8	10	12	16	20	24	32	48	64
1 MiB	0.108	0.103	0.083	0.091	0.072	0.110	0.134	0.211	0.170	0.030	<b>0.025</b>	0.027	0.026
5 MiB	0.407	0.201	0.126	0.104	0.097	0.079	0.079	0.077	0.091	0.181	0.056	0.058	<b>0.053</b>
10 MiB	0.761	0.474	0.266	0.208	0.180	0.186	0.220	0.335	0.243	0.102	<b>0.089</b>	0.096	0.170
50 MiB	3.924	2.138	1.279	1.020	0.857	0.759	0.777	0.658	0.752	0.424	0.537	0.417	<b>0.394</b>
100 MiB	7.727	4.068	2.403	1.860	1.554	1.432	1.299	1.247	1.231	0.856	0.945	<b>0.720</b>	0.888
500 MiB	35.775	18.383	11.496	8.351	7.559	5.755	6.086	5.570	5.143	3.800	3.669	5.378	<b>3.538</b>
1 GiB	73.112	38.168	22.507	16.336	14.213	13.756	12.620	10.826	10.335	7.810	<b>7.468</b>	10.505	8.655
5 GiB	359.849	186.629	99.804	77.462	65.655	67.306	52.909	48.459	48.013	<b>36.637</b>	38.346	59.600	47.238
10 GiB	694.975	370.549	188.228	136.172	113.588	104.025	93.723	116.825	83.208	76.289	73.011	<b>65.807</b>	85.032

TABLE II: Pure encoding times

Threads Size	1	2	4	6	8	10	12	16	20	24	32	48	64
1 MiB	0.064	0.034	0.018	0.013	0.010	0.009	0.039	0.094	0.035	<b>0.004</b>	<b>0.004</b>	<b>0.004</b>	<b>0.004</b>
5 MiB	0.298	0.152	0.078	0.054	0.041	0.034	0.030	0.024	0.021	0.018	0.017	0.015	<b>0.013</b>
10 MiB	0.655	0.352	0.184	0.125	0.098	0.081	0.073	0.153	0.057	0.034	<b>0.029</b>	0.031	0.040
50 MiB	3.303	1.755	0.915	0.624	0.488	0.433	0.343	0.266	0.295	0.177	0.139	0.178	<b>0.132</b>
100 MiB	6.721	3.446	1.827	1.245	0.990	0.815	0.687	0.542	0.486	0.380	0.482	<b>0.242</b>	0.462
500 MiB	32.383	16.605	8.682	6.246	4.862	4.059	3.413	2.662	2.385	1.929	1.386	3.187	<b>1.233</b>
1 GiB	67.176	34.097	17.735	12.655	9.879	8.237	7.004	5.431	4.892	3.982	<b>2.936</b>	5.939	4.277
5 GiB	331.253	174.089	90.562	63.491	48.645	38.946	33.185	26.100	23.623	19.312	<b>14.208</b>	44.206	32.709
10 GiB	639.337	328.580	170.409	119.682	95.123	79.805	68.875	53.342	47.573	39.444	40.491	<b>29.375</b>	56.942

TABLE III: Overall encoding efficiency

Threads Size	1	2	4	6	8	10	12	16	20	24	32	48	64
1 MiB	1.000	0.527	0.328	0.200	0.189	0.099	0.068	0.032	0.032	0.151	0.135	0.083	0.064
5 MiB	1.000	1.014	0.811	0.654	0.525	0.518	0.430	0.332	0.225	0.094	0.229	0.146	0.119
10 MiB	1.000	0.803	0.716	0.609	0.527	0.409	0.288	0.142	0.156	0.311	0.267	0.165	0.070
50 MiB	1.000	0.918	0.767	0.641	0.572	0.517	0.421	0.373	0.261	0.386	0.229	0.196	0.156
100 MiB	1.000	0.950	0.804	0.693	0.621	0.539	0.496	0.387	0.314	0.376	0.255	0.224	0.136
500 MiB	1.000	0.973	0.778	0.714	0.592	0.622	0.490	0.401	0.348	0.392	0.305	0.139	0.158
1 GiB	1.000	0.958	0.812	0.746	0.643	0.531	0.483	0.422	0.354	0.390	0.306	0.145	0.132
5 GiB	1.000	0.964	0.901	0.774	0.685	0.535	0.567	0.464	0.375	0.409	0.293	0.126	0.119
10 GiB	1.000	0.938	0.923	0.851	0.765	0.668	0.618	0.372	0.418	0.380	0.297	0.220	0.128

TABLE IV: Pure encoding efficiency

Threads Size	1	2	4	6	8	10	12	16	20	24	32	48	64
1 MiB	1.000	0.943	0.868	0.830	0.796	0.747	0.137	0.042	0.091	0.626	0.495	0.326	0.233
5 MiB	1.000	0.979	0.952	0.925	0.902	0.873	0.840	0.765	0.719	0.676	0.540	0.422	0.346
10 MiB	1.000	0.929	0.887	0.870	0.837	0.806	0.748	0.267	0.574	0.810	0.713	0.441	0.254
50 MiB	1.000	0.941	0.903	0.882	0.845	0.764	0.802	0.777	0.559	0.779	0.741	0.386	0.392
100 MiB	1.000	0.975	0.920	0.900	0.848	0.824	0.815	0.776	0.692	0.736	0.436	0.578	0.227
500 MiB	1.000	0.975	0.932	0.864	0.832	0.798	0.791	0.760	0.679	0.699	0.730	0.212	0.410
1 GiB	1.000	0.985	0.947	0.885	0.850	0.816	0.799	0.773	0.687	0.703	0.715	0.236	0.245
5 GiB	1.000	0.951	0.914	0.870	0.851	0.851	0.832	0.793	0.701	0.715	0.729	0.156	0.158
10 GiB	1.000	0.973	0.938	0.890	0.840	0.801	0.774	0.749	0.672	0.675	0.493	0.453	0.175

TABLE V: Overall encoding times

Threads Size	1	2	4	6	8	10	12	16	20	24	32	48	64
1 MiB	0.088	0.084	0.073	0.065	0.072	0.072	0.067	0.067	0.075	0.071	0.031	<b>0.027</b>	0.033
5 MiB	0.425	0.406	0.275	0.172	0.167	0.164	0.148	0.139	0.139	0.136	<b>0.058</b>	0.064	0.059
10 MiB	0.713	0.678	0.242	0.194	0.176	0.147	0.168	0.129	0.140	0.157	0.102	<b>0.094</b>	0.102
50 MiB	3.585	3.371	1.141	0.838	0.748	0.698	0.961	0.968	0.733	0.895	0.430	<b>0.388</b>	0.573
100 MiB	6.805	6.601	2.159	1.622	1.359	1.200	1.098	0.974	0.888	0.842	0.771	<b>0.722</b>	0.735
500 MiB	33.464	31.995	10.564	7.854	6.683	6.041	5.518	5.931	4.577	4.248	4.140	<b>3.551</b>	3.811
1 GiB	68.489	65.942	21.173	16.425	13.648	12.077	10.966	9.795	9.036	8.342	8.007	8.001	<b>7.690</b>
5 GiB	367.228	370.850	110.299	84.128	75.319	66.408	59.666	59.479	51.181	48.904	38.824	<b>38.677</b>	40.676
10 GiB	676.529	642.922	187.620	135.991	123.822	109.770	96.643	93.226	90.825	88.154	63.886	<b>61.847</b>	64.598

TABLE VI: Pure encoding times

Threads Size	1	2	4	6	8	10	12	16	20	24	32	48	64
1 MiB	0.061	0.062	0.036	0.028	0.030	0.022	0.024	0.024	0.030	0.016	0.011	<b>0.009</b>	0.011
5 MiB	0.361	0.363	0.209	0.109	0.103	0.092	0.079	0.066	0.053	0.050	0.023	0.023	<b>0.017</b>
10 MiB	0.617	0.607	0.169	0.117	0.090	0.076	0.078	0.054	0.053	0.059	<b>0.039</b>	<b>0.030</b>	<b>0.035</b>
50 MiB	3.131	3.057	0.847	0.595	0.464	0.416	0.671	0.652	0.435	0.500	0.186	<b>0.145</b>	0.160
100 MiB	6.079	6.037	1.664	1.151	0.891	0.735	0.642	0.521	0.438	0.386	0.327	0.285	<b>0.249</b>
500 MiB	31.001	30.471	8.287	5.810	4.519	3.778	3.226	2.610	2.227	1.946	1.814	1.430	<b>1.404</b>
1 GiB	62.748	62.355	17.230	11.939	9.153	7.582	6.505	5.363	4.612	4.101	3.698	2.966	<b>2.833</b>
5 GiB	343.876	351.067	99.909	73.825	62.694	54.050	47.849	42.794	33.944	29.952	18.749	14.847	<b>14.376</b>
10 GiB	629.449	623.710	172.829	120.552	94.623	78.728	68.025	55.215	46.723	41.474	32.493	27.176	<b>23.935</b>

TABLE VII: Overall encoding efficiency

Threads Size	1	2	4	6	8	10	12	16	20	24	32	48	64
1 MiB	1.000	0.526	0.300	0.224	0.152	0.122	0.109	0.082	0.059	0.051	0.089	0.067	0.042
5 MiB	1.000	0.523	0.386	0.410	0.317	0.259	0.239	0.191	0.153	0.130	0.229	0.138	0.113
10 MiB	1.000	0.525	0.737	0.613	0.506	0.485	0.354	0.346	0.254	0.189	0.219	0.158	0.109
50 MiB	1.000	0.532	0.786	0.713	0.599	0.513	0.311	0.231	0.245	0.167	0.261	0.192	0.098
100 MiB	1.000	0.515	0.788	0.699	0.626	0.567	0.516	0.437	0.383	0.337	0.276	0.196	0.145
500 MiB	1.000	0.523	0.792	0.710	0.626	0.554	0.505	0.353	0.366	0.328	0.253	0.196	0.137
1 GiB	1.000	0.519	0.809	0.695	0.627	0.567	0.520	0.437	0.379	0.342	0.267	0.178	0.139
5 GiB	1.000	0.495	0.832	0.728	0.609	0.553	0.513	0.386	0.359	0.313	0.296	0.198	0.141
10 GiB	1.000	0.526	0.901	0.829	0.683	0.616	0.583	0.454	0.372	0.320	0.331	0.228	0.164

TABLE VIII: Pure encoding efficiency

Threads Size	1	2	4	6	8	10	12	16	20	24	32	48	64
1 MiB	1.000	0.493	0.425	0.367	0.252	0.283	0.209	0.160	0.101	0.161	0.172	0.138	0.085
5 MiB	1.000	0.497	0.431	0.551	0.438	0.391	0.380	0.344	0.341	0.300	0.491	0.331	0.334
10 MiB	1.000	0.508	0.913	0.881	0.852	0.816	0.655	0.714	0.583	0.436	0.495	0.426	0.275
50 MiB	1.000	0.512	0.924	0.878	0.843	0.753	0.389	0.300	0.360	0.261	0.525	0.449	0.306
100 MiB	1.000	0.503	0.914	0.880	0.853	0.827	0.789	0.729	0.693	0.656	0.581	0.444	0.381
500 MiB	1.000	0.509	0.935	0.889	0.857	0.821	0.801	0.742	0.696	0.664	0.534	0.452	0.345
1 GiB	1.000	0.503	0.910	0.876	0.857	0.828	0.804	0.731	0.680	0.638	0.530	0.441	0.346
5 GiB	1.000	0.490	0.860	0.776	0.686	0.636	0.599	0.502	0.507	0.478	0.573	0.483	0.374
10 GiB	1.000	0.505	0.911	0.870	0.832	0.800	0.771	0.712	0.674	0.632	0.605	0.483	0.411

TABLE IX: Overall decoding times

Threads Size	1	2	4	6	8	10	12	16	20	24	32	48	64
1 MiB	0.130	0.091	0.079	0.099	0.091	0.097	0.118	0.115	0.119	0.104	0.045	<b>0.026</b>	0.030
5 MiB	0.422	0.253	0.177	0.166	0.141	0.156	0.151	0.139	0.130	0.121	0.087	0.087	<b>0.082</b>
10 MiB	0.871	0.517	0.384	0.296	0.293	0.433	0.372	0.356	0.368	0.355	0.171	<b>0.129</b>	0.161
50 MiB	4.108	2.277	1.511	1.252	1.276	1.382	1.412	1.409	1.434	1.328	0.693	0.594	<b>0.544</b>
100 MiB	7.961	4.507	2.915	2.275	1.884	2.418	2.400	2.422	2.343	2.251	4.392	1.858	<b>1.722</b>
500 MiB	37.619	22.511	13.501	11.334	10.035	9.326	9.640	11.182	11.069	10.021	19.667	<b>5.680</b>	7.006
1 GiB	76.679	43.252	27.585	23.271	18.590	17.824	18.532	22.027	19.813	17.128	37.634	<b>12.183</b>	14.964
5 GiB	361.871	193.112	108.812	96.790	81.066	84.897	79.142	<b>73.725</b>	79.374	75.976	155.945	111.347	81.739
10 GiB	759.072	391.202	212.381	156.396	135.844	135.401	134.098	138.000	134.006	132.255	253.313	<b>106.227</b>	119.210

TABLE X: Pure decoding times

Threads Size	1	2	4	6	8	10	12	16	20	24	32	48	64
1 MiB	0.083	0.058	0.052	0.074	0.069	0.071	0.092	0.090	0.092	0.076	0.015	<b>0.005</b>	<b>0.005</b>
5 MiB	0.340	0.188	0.123	0.101	0.084	0.094	0.089	0.079	0.073	0.062	0.020	0.018	<b>0.016</b>
10 MiB	0.752	0.414	0.267	0.191	0.191	0.265	0.267	0.246	0.282	0.249	0.059	0.032	<b>0.029</b>
50 MiB	3.634	1.888	1.058	0.745	0.670	0.976	0.954	0.943	0.966	0.857	0.271	0.148	<b>0.132</b>
100 MiB	7.132	3.613	2.012	1.388	1.123	1.540	1.520	1.541	1.478	1.468	2.433	<b>0.299</b>	0.511
500 MiB	36.395	18.594	9.874	6.980	5.458	4.707	5.739	6.970	6.499	5.473	16.056	<b>1.514</b>	2.716
1 GiB	74.764	38.408	19.764	13.840	10.908	10.242	9.231	12.504	12.060	9.825	29.059	<b>3.072</b>	5.671
5 GiB	359.883	188.886	97.788	82.355	56.999	59.936	50.887	39.291	48.562	48.877	141.437	<b>15.227</b>	55.262
10 GiB	754.842	385.021	199.850	138.739	113.105	95.262	81.112	108.727	94.317	81.262	245.681	<b>32.584</b>	75.318

TABLE XI: Overall decoding efficiency

Threads Size	1	2	4	6	8	10	12	16	20	24	32	48	64
1 MiB	1.000	0.719	0.411	0.220	0.179	0.134	0.092	0.071	0.055	0.052	0.090	0.103	0.067
5 MiB	1.000	0.835	0.595	0.423	0.374	0.270	0.233	0.189	0.163	0.145	0.152	0.101	0.080
10 MiB	1.000	0.842	0.567	0.491	0.371	0.201	0.195	0.153	0.118	0.102	0.159	0.140	0.085
50 MiB	1.000	0.902	0.680	0.547	0.402	0.297	0.242	0.182	0.143	0.129	0.185	0.144	0.118
100 MiB	1.000	0.883	0.683	0.583	0.528	0.329	0.276	0.205	0.170	0.147	0.057	0.089	0.072
500 MiB	1.000	0.836	0.697	0.553	0.469	0.403	0.325	0.210	0.170	0.156	0.060	0.138	0.084
1 GiB	1.000	0.886	0.695	0.549	0.516	0.430	0.345	0.218	0.194	0.187	0.064	0.131	0.080
5 GiB	1.000	0.937	0.831	0.623	0.558	0.426	0.381	0.307	0.228	0.198	0.073	0.068	0.069
10 GiB	1.000	0.970	0.894	0.809	0.698	0.561	0.472	0.344	0.283	0.239	0.094	0.149	0.099

TABLE XII: Pure decoding efficiency

Threads Size	1	2	4	6	8	10	12	16	20	24	32	48	64
1 MiB	1.000	0.720	0.401	0.188	0.150	0.117	0.075	0.058	0.045	0.046	0.170	0.338	0.245
5 MiB	1.000	0.902	0.690	0.558	0.504	0.361	0.317	0.269	0.233	0.227	0.519	0.404	0.324
10 MiB	1.000	0.909	0.705	0.655	0.493	0.284	0.234	0.191	0.133	0.126	0.397	0.490	0.402
50 MiB	1.000	0.962	0.859	0.813	0.678	0.372	0.317	0.241	0.188	0.177	0.419	0.511	0.429
100 MiB	1.000	0.987	0.886	0.856	0.794	0.463	0.391	0.289	0.241	0.202	0.092	0.497	0.218
500 MiB	1.000	0.979	0.922	0.869	0.834	0.773	0.528	0.326	0.280	0.277	0.071	0.501	0.209
1 GiB	1.000	0.973	0.946	0.900	0.857	0.730	0.675	0.374	0.310	0.317	0.080	0.507	0.206
5 GiB	1.000	0.953	0.920	0.728	0.789	0.600	0.589	0.572	0.371	0.307	0.080	0.492	0.102
10 GiB	1.000	0.980	0.944	0.907	0.834	0.792	0.776	0.434	0.400	0.387	0.096	0.483	0.157



TABLE XIII: Overall decoding times

Threads Sizes	1	2	4	6	8	10	12	16	20	24	32	48	64
1 MiB	0.272	0.212	0.116	0.101	0.070	0.068	0.066	0.069	0.075	0.093	0.095	<b>0.036</b>	<b>0.036</b>
5 MiB	0.635	0.915	1.265	0.655	0.684	0.289	0.482	0.298	0.330	0.180	0.562	<b>0.124</b>	0.254
10 MiB	0.861	0.501	0.333	0.271	0.269	0.258	0.236	0.217	0.190	<b>0.183</b>	0.604	0.440	0.404
50 MiB	3.991	2.248	1.462	1.176	1.160	1.088	1.017	0.913	0.837	0.775	1.409	0.761	<b>0.726</b>
100 MiB	7.615	33.645	15.250	11.628	13.018	10.761	9.192	3.661	5.691	4.475	2.175	<b>1.312</b>	1.464
500 MiB	37.082	20.422	13.236	11.344	10.774	10.441	9.767	8.661	7.387	7.051	10.581	9.984	<b>6.591</b>
1 GiB	73.855	41.185	26.929	23.051	17.977	18.781	19.731	17.837	16.171	15.437	19.766	14.432	<b>12.636</b>
5 GiB	352.386	475.087	265.894	204.661	156.227	139.894	127.649	109.245	100.918	89.890	104.173	65.082	<b>63.837</b>
10 GiB	714.907	376.029	202.712	190.071	435.224	145.673	133.319	122.774	117.529	<b>115.262</b>	158.281	150.057	148.834

TABLE XIV: Pure decoding times

Threads Sizes	1	2	4	6	8	10	12	16	20	24	32	48	64
1 MiB	0.116	0.064	0.056	0.056	0.041	0.041	0.038	0.041	0.038	0.035	0.061	<b>0.006</b>	0.007
5 MiB	0.565	0.859	1.202	0.598	0.627	0.234	0.425	0.244	0.269	0.113	0.019	0.017	<b>0.016</b>
10 MiB	0.715	0.399	0.234	0.174	0.169	0.159	0.136	0.119	0.095	0.085	0.132	0.031	<b>0.028</b>
50 MiB	3.531	1.848	1.034	0.742	0.730	0.656	0.620	0.486	0.417	0.352	0.703	0.143	<b>0.117</b>
100 MiB	6.874	33.366	14.884	10.786	12.377	9.907	8.348	2.879	5.046	3.695	0.968	<b>0.285</b>	0.510
500 MiB	35.450	18.238	10.009	7.179	6.792	6.317	5.622	4.667	3.831	3.280	6.218	1.394	<b>1.212</b>
1 GiB	72.114	36.926	20.113	14.503	11.600	12.692	11.302	9.281	7.654	6.478	9.440	2.841	<b>2.705</b>
5 GiB	349.890	471.340	257.714	197.061	140.154	124.853	110.634	91.401	74.582	62.539	73.525	14.510	<b>12.708</b>
10 GiB	711.985	370.520	191.792	182.3175	421.918	126.678	113.482	92.082	76.309	64.483	100.849	<b>29.389</b>	54.947

TABLE XV: Overall decoding efficiency

Threads Sizes	1	2	4	6	8	10	12	16	20	24	32	48	64
1 MiB	1.000	0.642	0.585	0.450	0.486	0.402	0.346	0.245	0.182	0.122	0.089	0.157	0.119
5 MiB	1.000	0.347	0.126	0.162	0.116	0.220	0.110	0.133	0.096	0.147	0.035	0.107	0.039
10 MiB	1.000	0.860	0.647	0.529	0.400	0.334	0.303	0.248	0.227	0.196	0.045	0.041	0.033
50 MiB	1.000	0.888	0.683	0.566	0.430	0.367	0.327	0.273	0.238	0.215	0.089	0.109	0.086
100 MiB	1.000	0.113	0.125	0.109	0.073	0.071	0.069	0.130	0.067	0.071	0.109	0.121	0.081
500 MiB	1.000	0.908	0.700	0.545	0.430	0.355	0.316	0.268	0.251	0.219	0.110	0.077	0.088
1 GiB	1.000	0.897	0.686	0.534	0.514	0.393	0.312	0.259	0.228	0.199	0.117	0.107	0.091
5 GiB	1.000	0.371	0.331	0.287	0.282	0.252	0.230	0.202	0.175	0.163	0.106	0.113	0.086
10 GiB	1.000	0.951	0.881	0.625	0.205	0.491	0.447	0.364	0.304	0.258	0.141	0.099	0.075

TABLE XVI: Pure decoding efficiency

Threads Sizes	1	2	4	6	8	10	12	16	20	24	32	48	64
1 MiB	1.000	0.901	0.515	0.349	0.357	0.285	0.255	0.176	0.152	0.138	0.060	0.428	0.260
5 MiB	1.000	0.329	0.118	0.157	0.113	0.242	0.111	0.145	0.105	0.208	0.918	0.703	0.551
10 MiB	1.000	0.897	0.764	0.684	0.529	0.449	0.437	0.376	0.377	0.351	0.169	0.486	0.402
50 MiB	1.000	0.955	0.854	0.793	0.605	0.538	0.474	0.454	0.423	0.418	0.157	0.516	0.471
100 MiB	1.000	0.103	0.115	0.106	0.069	0.069	0.069	0.149	0.068	0.078	0.222	0.502	0.210
500 MiB	1.000	0.972	0.885	0.823	0.652	0.561	0.525	0.475	0.463	0.450	0.178	0.530	0.457
1 GiB	1.000	0.976	0.896	0.829	0.777	0.568	0.532	0.486	0.471	0.464	0.239	0.529	0.417
5 GiB	1.000	0.371	0.339	0.296	0.312	0.280	0.264	0.239	0.235	0.233	0.149	0.502	0.430
10 GiB	1.000	0.9606	0.928	0.651	0.211	0.562	0.523	0.483	0.467	0.460	0.221	0.505	0.202

### E. numpy encoding pack

TABLE XVII: Overall encoding times

Threads Processes	1	2	4
1	39.668	39.316	<b>38.853</b>
2	<b>28.366</b>	29.230	29.015
4	25.048	25.980	<b>24.888</b>
8	26.048	<b>25.678</b>	25.688
10	<b>25.551</b>	25.874	25.738
12	26.608	26.560	<b>24.980</b>
16	26.316	<b>24.455</b>	46.843

TABLE XVIII: Overall encoding efficiency

Threads Processes	1	2	4
1	1.000	0.504	0.255
2	0.699	0.339	0.171
4	0.396	0.191	0.100
8	0.190	0.097	0.048
10	0.155	0.077	0.039
12	0.124	0.062	0.033
16	0.094	0.051	0.013

### F. numpy encoding scatter

TABLE XIX: Overall encoding times

Threads Processes	1	2	4
1	<b>39.346</b>	42.755	39.929
2	28.056	27.240	<b>26.568</b>
4	20.548	<b>19.755</b>	33.501
8	16.588	15.468	<b>15.198</b>
10	27.227	21.108	<b>20.999</b>
12	26.551	25.964	<b>25.659</b>
16	20.122	<b>19.575</b>	19.687

TABLE XX: Overall encoding efficiency

Threads Processes	1	2	4
1	1.000	0.460	0.246
2	0.701	0.361	0.185
4	0.479	0.249	0.073
8	0.296	0.159	0.081
10	0.145	0.093	0.047
12	0.123	0.063	0.032
16	0.122	0.063	0.031

### G. numpy decoding pack

TABLE XXI: Overall decoding times

Threads Processes	1	2	4
1	43.866	<b>42.586</b>	43.507
2	<b>31.064</b>	33.149	36.228
4	34.853	33.162	<b>30.350</b>
8	30.040	27.924	<b>27.418</b>
10	28.016	28.269	<b>27.062</b>
12	<b>28.995</b>	29.148	29.472
16	30.624	<b>30.456</b>	32.091

TABLE XXII: Overall decoding efficiency

Threads Processes	1	2	4
1	1.000	0.515	0.252
2	0.706	0.331	0.151
4	0.315	0.165	0.090
8	0.183	0.098	0.050
10	0.157	0.078	0.041
12	0.126	0.063	0.031
16	0.090	0.045	0.021

### H. numpy decoding scatter

TABLE XXIII: Overall decoding times

Threads Processes	1	2	4
1	46.248	44.085	<b>43.422</b>
2	30.331	<b>28.251</b>	28.600
4	22.234	21.866	<b>21.471</b>
8	18.648	<b>16.757</b>	16.952
10	16.364	<b>15.773</b>	15.839
12	15.574	15.720	<b>15.148</b>
16	22.756	<b>22.066</b>	22.733

TABLE XXIV: Overall decoding efficiency

Threads Processes	1	2	4
1	1.000	0.525	0.266
2	0.762	0.409	0.202
4	0.520	0.264	0.135
8	0.310	0.172	0.085
10	0.283	0.147	0.073
12	0.247	0.123	0.064
16	0.127	0.065	0.032

*I. pytorch encoding pack*

TABLE XXV: Overall encoding times

Threads Processes	1	2	4
1	247.430	229.251	<b>225.370</b>
2	174.185	<b>168.884</b>	168.979
4	157.589	<b>155.282</b>	157.478
8	161.628	158.789	<b>156.443</b>
10	188.020	167.726	<b>158.378</b>
12	161.222	158.014	<b>154.675</b>
16	159.562	153.257	<b>148.363</b>

TABLE XXVI: Overall encoding efficiency

Threads Processes	1	2	4
1	1.000	0.540	0.274
2	0.710	0.366	0.183
4	0.393	0.199	0.098
8	0.191	0.097	0.049
10	0.132	0.074	0.039
12	0.128	0.065	0.033
16	0.097	0.050	0.026

*J. pytorch encoding scatter*

TABLE XXVII: Overall encoding times

Threads Processes	1	2	4
1	317.663	268.323	<b>265.387</b>
2	184.717	179.515	<b>173.750</b>
4	139.132	<b>123.383</b>	126.569
8	105.950	102.119	<b>100.966</b>
10	98.206	94.839	<b>91.242</b>
12	95.794	96.229	<b>90.712</b>
16	87.427	84.979	<b>84.262</b>

TABLE XXVIII: Overall encoding efficiency

Threads Processes	1	2	4
1	1.000	0.592	0.299
2	0.860	0.442	0.229
4	0.571	0.322	0.157
8	0.375	0.194	0.098
10	0.323	0.167	0.087
12	0.276	0.138	0.073
16	0.227	0.117	0.059

*K. pytorch encoding pack*

TABLE XXIX: Overall encoding times

Threads Processes	1	2	4
1	247.430	229.251	<b>225.370</b>
2	174.185	<b>168.884</b>	168.979
4	157.589	<b>155.282</b>	157.478
8	161.628	158.789	<b>156.443</b>
10	188.020	167.726	<b>158.378</b>
12	161.222	158.014	<b>154.675</b>
16	159.562	153.257	<b>148.363</b>

TABLE XXX: Overall encoding efficiency

Threads Processes	1	2	4
1	1.000	0.540	0.274
2	0.710	0.366	0.183
4	0.393	0.199	0.098
8	0.191	0.097	0.049
10	0.132	0.074	0.039
12	0.128	0.065	0.033
16	0.097	0.050	0.026

*L. pytorch encoding scatter*

TABLE XXXI: Overall encoding times

Threads Processes	1	2	4
1	317.663	268.323	<b>265.387</b>
2	184.717	179.515	<b>173.750</b>
4	139.132	<b>123.383</b>	126.569
8	105.950	102.119	<b>100.966</b>
10	98.206	94.839	<b>91.242</b>
12	95.794	96.229	<b>90.712</b>
16	87.427	84.979	<b>84.262</b>

TABLE XXXII: Overall encoding efficiency

Threads Processes	1	2	4
1	1.000	0.592	0.299
2	0.860	0.442	0.229
4	0.571	0.322	0.157
8	0.375	0.194	0.098
10	0.323	0.167	0.087
12	0.276	0.138	0.073
16	0.227	0.117	0.059

*M. linux encoding pack*

TABLE XXXIII: Overall encoding times

Threads Processes	1	2	4
1	169.893	162.189	<b>146.131</b>
2	120.387	116.712	<b>115.819</b>
4	116.030	<b>111.948</b>	112.458
8	113.034	<b>111.988</b>	113.310
10	<b>113.167</b>	115.369	121.193
12	<b>120.443</b>	121.303	145.063
16	141.595	<b>123.439</b>	126.436

TABLE XXXIV: Overall encoding efficiency

Threads Processes	1	2	4
1	1.000	0.524	0.291
2	0.706	0.364	0.183
4	0.366	0.190	0.094
8	0.188	0.095	0.047
10	0.150	0.074	0.035
12	0.118	0.058	0.024
16	0.075	0.043	0.021

*N. linux encoding scatter*

TABLE XXXV: Overall encoding times

Threads Processes	1	2	4
1	189.447	150.256	<b>147.334</b>
2	122.175	118.490	<b>117.440</b>
4	84.408	81.996	<b>81.776</b>
8	68.110	67.038	<b>43.153</b>
10	63.032	61.393	<b>61.122</b>
12	62.554	<b>61.992</b>	62.346
16	57.539	<b>56.922</b>	56.938

TABLE XXXVI: Overall encoding efficiency

Threads Processes	1	2	4
1	1.000	0.630	0.321
2	0.775	0.400	0.202
4	0.561	0.289	0.145
8	0.348	0.177	0.137
10	0.301	0.154	0.077
12	0.252	0.127	0.063
16	0.206	0.104	0.052