

Lecture 2: Making Sequences of Good Decisions Given a Model of the World

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CS234 Reinforcement Learning

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Refresh Your Knowledge 1. Participation Poll

In a Markov decision process, a large discount factor γ means that short term rewards are much more influential than long term rewards. [Enter your answer in participation poll]

- True
- False
- Don't know

Refresh Your Knowledge 1. Participation Poll

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- True
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Class Tasks and Updates

- Quiz 0 due Friday at 6pm Stanford time
- Homework 1 out today. Due Friday January 14 at 6pm Stanford time
- Office hours have started. See Ed for days, times of group and 1:1 office hours!

Today's Plan

- Last Time:
 - Introduction
 - Components of an agent: model, value, policy
- This Time:
 - Making good decisions given a Markov decision process
- Next Time:
 - Policy evaluation when don't have a model of how the world works

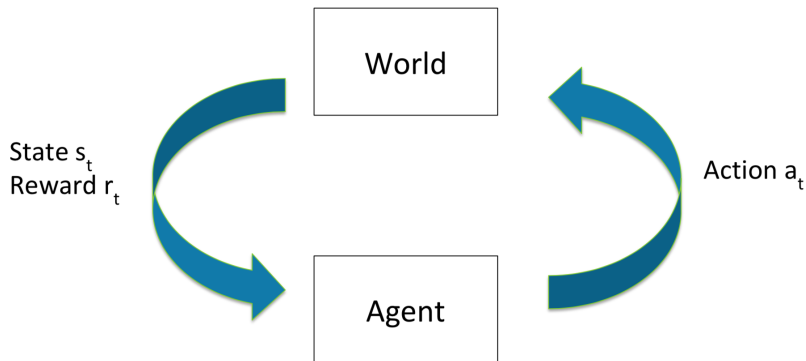
Models, Policies, Values

- **Model:** Mathematical models of dynamics and reward
- **Policy:** Function mapping states to actions
- **Value function:** future rewards from being in a state and/or action when following a particular policy

Today: Given a model of the world

- Markov Processes
- Markov Reward Processes (MRPs)
- Markov Decision Processes (MDPs)
- Evaluation and Control in MDPs

Full Observability: Markov Decision Process (MDP)



- MDPs can model a huge number of interesting problems and settings
 - Bandits: single state MDP
 - Optimal control mostly about continuous-state MDPs
 - Partially observable MDPs = MDP where state is history

Recall: Markov Property

- Information state: sufficient statistic of history
- State s_t is Markov if and only if:

$$p(s_{t+1}|s_t, a_t) = p(s_{t+1}|h_t, a_t)$$

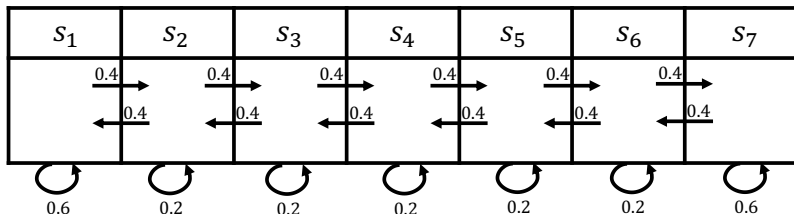
- Future is independent of past given present

Markov Process or Markov Chain

- Memoryless random process
 - Sequence of random states with Markov property
- Definition of Markov Process
 - S is a (finite) set of states ($s \in S$)
 - P is dynamics/transition model that specifies $p(s_{t+1} = s' | s_t = s)$
- Note: no rewards, no actions
- If finite number (N) of states, can express P as a matrix

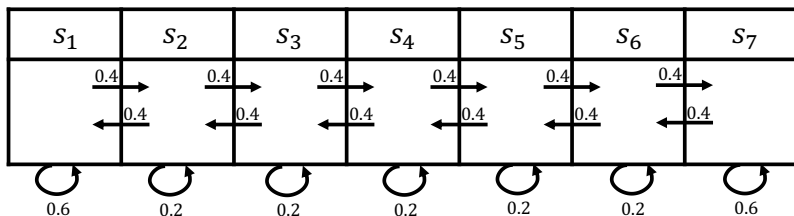
$$P = \begin{pmatrix} P(s_1|s_1) & P(s_2|s_1) & \cdots & P(s_N|s_1) \\ P(s_1|s_2) & P(s_2|s_2) & \cdots & P(s_N|s_2) \\ \vdots & \vdots & \ddots & \vdots \\ P(s_1|s_N) & P(s_2|s_N) & \cdots & P(s_N|s_N) \end{pmatrix}$$

Example: Mars Rover Markov Chain Transition Matrix, P



$$P = \begin{pmatrix} 0.6 & 0.4 & 0 & 0 & 0 & 0 & 0 \\ 0.4 & 0.2 & 0.4 & 0 & 0 & 0 & 0 \\ 0 & 0.4 & 0.2 & 0.4 & 0 & 0 & 0 \\ 0 & 0 & 0.4 & 0.2 & 0.4 & 0 & 0 \\ 0 & 0 & 0 & 0.4 & 0.2 & 0.4 & 0 \\ 0 & 0 & 0 & 0 & 0.4 & 0.2 & 0.4 \\ 0 & 0 & 0 & 0 & 0 & 0.4 & 0.6 \end{pmatrix}$$

Example: Mars Rover Markov Chain Episodes



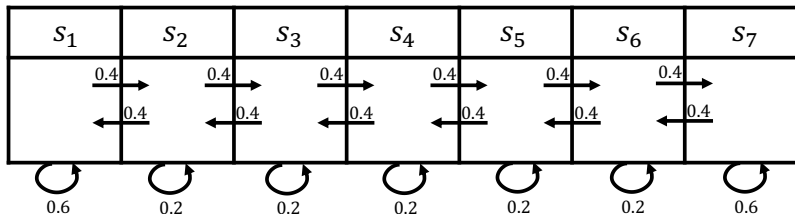
Example: Sample episodes starting from S_4

- $S_4, S_5, S_6, S_7, S_7, \dots$
- $S_4, S_4, S_5, S_4, S_5, S_6, \dots$
- $S_4, S_3, S_2, S_1, \dots$

Markov Reward Process (MRP)

- Markov Reward Process is a Markov Chain + rewards
- Definition of Markov Reward Process (MRP)
 - S is a (finite) set of states ($s \in S$)
 - P is dynamics/transition model that specifies $P(s_{t+1} = s' | s_t = s)$
 - R is a reward function $R(s_t = s) = \mathbb{E}[r_t | s_t = s]$
 - Discount factor $\gamma \in [0, 1]$
- Note: no actions
- If finite number (N) of states, can express R as a vector

Example: Mars Rover MRP



- Reward: +1 in s_1 , +10 in s_7 , 0 in all other states

Return & Value Function

- Definition of Horizon (H)
 - Number of time steps in each episode
 - Can be infinite
 - Otherwise called **finite** Markov reward process

- Definition of Return, G_t (for a MRP)

- Discounted sum of rewards from time step t to horizon H

$$G_t = r_t + \gamma r_{t+1} + \gamma^2 r_{t+2} + \cdots + \gamma^{H-1} r_{t+H-1}$$

- Definition of State Value Function, $V(s)$ (for a MRP)

- Expected return from starting in state s

$$V(s) = \mathbb{E}[G_t | s_t = s] = \mathbb{E}[r_t + \gamma r_{t+1} + \gamma^2 r_{t+2} + \cdots + \gamma^{H-1} r_{t+H-1} | s_t = s]$$

Discount Factor

- Mathematically convenient (avoid infinite returns and values)
- Humans often act as if there's a discount factor < 1
- $\gamma = 0$: Only care about immediate reward
- $\gamma = 1$: Future reward is as beneficial as immediate reward
- If episode lengths are always finite ($H < \infty$), can use $\gamma = 1$

Computing the Value of a Markov Reward Process

- Markov property provides structure
- MRP value function satisfies

$$V(s) = \underbrace{R(s)}_{\text{Immediate reward}} + \underbrace{\gamma \sum_{s' \in S} P(s'|s) V(s')}_{\text{Discounted sum of future rewards}}$$

Matrix Form of Bellman Equation for MRP

- For finite state MRP, we can express $V(s)$ using a matrix equation

$$\begin{pmatrix} V(s_1) \\ \vdots \\ V(s_N) \end{pmatrix} = \begin{pmatrix} R(s_1) \\ \vdots \\ R(s_N) \end{pmatrix} + \gamma \begin{pmatrix} P(s_1|s_1) & \cdots & P(s_N|s_1) \\ P(s_1|s_2) & \cdots & P(s_N|s_2) \\ \vdots & \ddots & \vdots \\ P(s_1|s_N) & \cdots & P(s_N|s_N) \end{pmatrix} \begin{pmatrix} V(s_1) \\ \vdots \\ V(s_N) \end{pmatrix}$$
$$V = R + \gamma PV$$

Analytic Solution for Value of MRP

- For finite state MRP, we can express $V(s)$ using a matrix equation

$$\begin{pmatrix} V(s_1) \\ \vdots \\ V(s_N) \end{pmatrix} = \begin{pmatrix} R(s_1) \\ \vdots \\ R(s_N) \end{pmatrix} + \gamma \begin{pmatrix} P(s_1|s_1) & \cdots & P(s_N|s_1) \\ P(s_1|s_2) & \cdots & P(s_N|s_2) \\ \vdots & \ddots & \vdots \\ P(s_1|s_N) & \cdots & P(s_N|s_N) \end{pmatrix} \begin{pmatrix} V(s_1) \\ \vdots \\ V(s_N) \end{pmatrix}$$

$$V = R + \gamma PV$$

$$V - \gamma PV = R$$

$$(I - \gamma P)V = R$$

$$V = (I - \gamma P)^{-1}R$$

- Solving directly requires taking a matrix inverse $\sim O(N^3)$
- Note that $(I - \gamma P)$ is invertible

Iterative Algorithm for Computing Value of a MRP

- Dynamic programming
- Initialize $V_0(s) = 0$ for all s
- For $k = 1$ until convergence
 - For all s in S

$$V_k(s) = R(s) + \gamma \sum_{s' \in S} P(s'|s) V_{k-1}(s')$$

- Computational complexity: $O(|S|^2)$ for each iteration ($|S| = N$)

Markov Decision Process (MDP)


- Markov Decision Process is Markov Reward Process + actions
- Definition of MDP
 - S is a (finite) set of Markov states $s \in S$
 - A is a (finite) set of actions $a \in A$
 - P is dynamics/transition model for **each action**, that specifies $P(s_{t+1} = s' | s_t = s, a_t = a)$
 - R is a reward function¹

$$R(s_t = s, a_t = a) = \mathbb{E}[r_t | s_t = s, a_t = a]$$

- Discount factor $\gamma \in [0, 1]$
- MDP is a tuple: (S, A, P, R, γ)

¹Reward is sometimes defined as a function of the current state, or as a function of the (state, action, next state) tuple. Most frequently in this class, we will assume reward is a function of state and action

Example: Mars Rover MDP

s_1	s_2	s_3	s_4	s_5	s_6	s_7
						

$$P(s'|s, a_1) = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{pmatrix} \quad P(s'|s, a_2) = \begin{pmatrix} 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

- 2 deterministic actions

- Policy specifies what action to take in each state
 - Can be deterministic or stochastic
- For generality, consider as a conditional distribution
 - Given a state, specifies a distribution over actions
- Policy: $\pi(a|s) = P(a_t = a | s_t = s)$

- MDP + $\pi(a|s)$ = Markov Reward Process
- Precisely, it is the MRP $(S, R^\pi, P^\pi, \gamma)$, where

$$R^\pi(s) = \sum_{a \in A} \pi(a|s) R(s, a)$$

$$P^\pi(s'|s) = \sum_{a \in A} \pi(a|s) P(s'|s, a)$$

- Implies we can use same techniques to evaluate the value of a policy for a MDP as we could to compute the value of a MRP, by defining a MRP with R^π and P^π

MDP Policy Evaluation, Iterative Algorithm

- Initialize $V_0(s) = 0$ for all s
- For $k = 1$ until convergence
 - For all s in S

$$V_k^\pi(s) = \sum_a \pi(a|s) \left[r(s, a) + \gamma \sum_{s' \in S} p(s'|s, a) V_{k-1}^\pi(s') \right]$$

- This is a **Bellman backup** for a particular policy
- Note that if the policy is deterministic then the above update simplifies to

$$V_k^\pi(s) = r(s, \pi(s)) + \gamma \sum_{s' \in S} p(s'|s, \pi(s)) V_{k-1}^\pi(s')$$


Check Your Understanding L2N1: MDP 1 Iteration of Policy Evaluation, Mars Rover Example

- Dynamics: $p(s_6|s_6, a_1) = 0.5$, $p(s_7|s_6, a_1) = 0.5$, \dots
- Reward: for all actions, +1 in state s_1 , +10 in state s_7 , 0 otherwise
- Let $\pi(s) = a_1 \forall s$, assume $V_k = [1 \ 0 \ 0 \ 0 \ 0 \ 0 \ 10]$ and $k = 1$, $\gamma = 0.5$
- Compute $V_{k+1}(s_6)$

Select answer on participation poll:

- 0
- 2.5
- 10
- I don't know

Check Your Understanding Poll L2N2

s_1	s_2	s_3	s_4	s_5	s_6	s_7
						

- We will shortly be interested in not just evaluating the value of a single policy, but finding an optimal policy. Given this it is informative to think about properties of the potential policy space.
- First for the Mars rover example [7 discrete states (location of rover); 2 actions: Left or Right]
- How many deterministic policies are there?
- Select answer on the participation poll: 2 / 14 / 7^2 / 2^7 / Not sure
- Is the optimal policy (one with highest value) for a MDP unique?
- Select answer on the participation poll: Yes / No / Not sure

Lecture Break


- At start of next module section will go through check your understanding results and discuss MDP control

Check Your Understanding L2N1: MDP 1 Iteration of Policy Evaluation, Mars Rover Example

- Dynamics: $p(s_6|s_6, a_1) = 0.5$, $p(s_7|s_6, a_1) = 0.5$, ...
- Reward: for all actions, +1 in state s_1 , +10 in state s_7 , 0 otherwise
- Let $\pi(s) = a_1 \forall s$, assume $V_k = [1 \ 0 \ 0 \ 0 \ 0 \ 0 \ 10]$ and $k = 1$, $\gamma = 0.5$
-

$$V_k^\pi(s) = r(s, \pi(s)) + \gamma \sum_{s' \in S} p(s'|s, \pi(s)) V_{k-1}^\pi(s')$$

Check Your Understanding L2N2

s_1	s_2	s_3	s_4	s_5	s_6	s_7
						

- 7 discrete states (location of rover)
 - 2 actions: Left or Right
 - How many deterministic policies are there?
-
- Is the highest reward policy for a MDP always unique?

- Compute the optimal policy

$$\pi^*(s) = \arg \max_{\pi} V^{\pi}(s)$$

- There **exists a unique optimal value function**
- Optimal policy for a MDP in an infinite horizon problem is deterministic

- Compute the optimal policy

$$\pi^*(s) = \arg \max_{\pi} V^{\pi}(s)$$

- There exists a unique optimal value function
- Optimal policy for a MDP in an infinite horizon problem (agent acts forever) is
 - Deterministic
 - Stationary (does not depend on time step)
 - Unique? Not necessarily, may have two policies with identical (optimal) values

Policy Search

- One option is searching to compute best policy
- Number of deterministic policies is $|A|^{|S|}$
- Policy iteration is generally more efficient than enumeration

MDP Policy Iteration (PI)

- Set $i = 0$
- Initialize $\pi_0(s)$ randomly for all states s
- While $i == 0$ or $\|\pi_i - \pi_{i-1}\|_1 > 0$ (L1-norm, measures if the policy changed for any state):
 - $V^{\pi_i} \leftarrow$ MDP V function policy **evaluation** of π_i
 - $\pi_{i+1} \leftarrow$ Policy **improvement**
 - $i = i + 1$

New Definition: State-Action Value Q

- State-action value of a policy

$$Q^{\pi}(s, a) = R(s, a) + \gamma \sum_{s' \in S} P(s'|s, a) V^{\pi}(s')$$

- Take action a , then follow the policy π

Policy Improvement

- Compute state-action value of a policy π_i
 - For s in S and a in A :

$$Q^{\pi_i}(s, a) = R(s, a) + \gamma \sum_{s' \in S} P(s'|s, a) V^{\pi_i}(s')$$

- Compute new policy π_{i+1} , for all $s \in S$

$$\pi_{i+1}(s) = \arg \max_a Q^{\pi_i}(s, a) \quad \forall s \in S$$

MDP Policy Iteration (PI)

- Set $i = 0$
- Initialize $\pi_0(s)$ randomly for all states s
- While $i == 0$ or $\|\pi_i - \pi_{i-1}\|_1 > 0$ (L1-norm, measures if the policy changed for any state):
 - $V^{\pi_i} \leftarrow$ MDP V function policy **evaluation** of π_i
 - $\pi_{i+1} \leftarrow$ Policy **improvement**
 - $i = i + 1$

Delving Deeper Into Policy Improvement Step

$$Q^{\pi_i}(s, a) = R(s, a) + \gamma \sum_{s' \in S} P(s'|s, a) V^{\pi_i}(s')$$

Delving Deeper Into Policy Improvement Step

$$Q^{\pi_i}(s, a) = R(s, a) + \gamma \sum_{s' \in S} P(s'|s, a) V^{\pi_i}(s')$$

$$\max_a Q^{\pi_i}(s, a) \geq R(s, \pi_i(s)) + \gamma \sum_{s' \in S} P(s'|s, \pi_i(s)) V^{\pi_i}(s') = V^{\pi_i}(s)$$

$$\pi_{i+1}(s) = \arg \max_a Q^{\pi_i}(s, a)$$

- Suppose we take $\pi_{i+1}(s)$ for one action, then follow π_i forever
 - Our expected sum of rewards is at least as good as if we had always followed π_i
- But new proposed policy is to always follow π_{i+1} ...

Monotonic Improvement in Policy

- Definition

$$V^{\pi_1} \geq V^{\pi_2} : V^{\pi_1}(s) \geq V^{\pi_2}(s), \forall s \in S$$

- Proposition: $V^{\pi_{i+1}} \geq V^{\pi_i}$ with strict inequality if π_i is suboptimal, where π_{i+1} is the new policy we get from policy improvement on π_i

Proof: Monotonic Improvement in Policy

$$\begin{aligned} V^{\pi_i}(s) &\leq \max_a Q^{\pi_i}(s, a) \\ &= \max_a R(s, a) + \gamma \sum_{s' \in S} P(s'|s, a) V^{\pi_i}(s') \end{aligned}$$

Proof: Monotonic Improvement in Policy

Check Your Understanding L2N3: Policy Iteration (PI)

- Note: all the below is for finite state-action spaces
- Set $i = 0$
- Initialize $\pi_0(s)$ randomly for all states s
- While $i == 0$ or $\|\pi_i - \pi_{i-1}\|_1 > 0$ (L1-norm, measures if the policy changed for any state):
 - $V^{\pi_i} \leftarrow$ MDP V function policy **evaluation** of π_i
 - $\pi_{i+1} \leftarrow$ Policy **improvement**
 - $i = i + 1$
- **If policy doesn't change, can it ever change again?**
- Select on participation poll: Yes / No / Not sure
- **Is there a maximum number of iterations of policy iteration?**
- Select on participation poll: Yes / No / Not sure

Lecture Break after Policy Iteration

Results for Check Your Understanding L2N3 Policy Iteration

- Note: all the below is for finite state-action spaces
- Set $i = 0$
- Initialize $\pi_0(s)$ randomly for all states s
- While $i \neq 0$ or $\|\pi_i - \pi_{i-1}\|_1 > 0$ (L1-norm, measures if the policy changed for any state):
 - $V^{\pi_i} \leftarrow$ MDP V function policy **evaluation** of π_i
 - $\pi_{i+1} \leftarrow$ Policy **improvement**
 - $i = i + 1$
- **If policy doesn't change, can it ever change again?**

- **Is there a maximum number of iterations of policy iteration?**

Check Your Understanding Explanation of Policy Not Changing

- Suppose for all $s \in S$, $\pi_{i+1}(s) = \pi_i(s)$
- Then for all $s \in S$, $Q^{\pi_{i+1}}(s, a) = Q^{\pi_i}(s, a)$
- Recall policy improvement step

$$Q^{\pi_i}(s, a) = R(s, a) + \gamma \sum_{s' \in S} P(s'|s, a) V^{\pi_i}(s')$$

$$\pi_{i+1}(s) = \arg \max_a Q^{\pi_i}(s, a)$$

$$\pi_{i+2}(s) = \arg \max_a Q^{\pi_{i+1}}(s, a) = \arg \max_a Q^{\pi_i}(s, a)$$

MDP: Computing Optimal Policy and Optimal Value

- Policy iteration computes infinite horizon value of a policy and then improves that policy
- Value iteration is another technique
 - Idea: Maintain optimal value of starting in a state s if have a finite number of steps k left in the episode
 - Iterate to consider longer and longer episodes

Bellman Equation and Bellman Backup Operators

- Value function of a policy must satisfy the Bellman equation

$$V^\pi(s) = R^\pi(s) + \gamma \sum_{s' \in S} P^\pi(s'|s) V^\pi(s')$$

- Bellman backup operator
 - Applied to a value function
 - Returns a new value function
 - Improves the value if possible

$$BV(s) = \max_a \left[R(s, a) + \gamma \sum_{s' \in S} p(s'|s, a) V(s') \right]$$

- BV yields a value function over all states s

Value Iteration (VI)

- Set $k = 1$
- Initialize $V_0(s) = 0$ for all states s
- Loop until convergence: (for ex. $\|V_{k+1} - V_k\|_\infty \leq \epsilon$)
 - For each state s

$$V_{k+1}(s) = \max_a \left[R(s, a) + \gamma \sum_{s' \in S} P(s'|s, a) V_k(s') \right]$$

- View as Bellman backup on value function

$$V_{k+1} = BV_k$$

$$\pi_{k+1}(s) = \arg \max_a \left[R(s, a) + \gamma \sum_{s' \in S} P(s'|s, a) V_k(s') \right]$$

Policy Iteration as Bellman Operations

- Bellman backup operator B^π for a particular policy is defined as

$$B^\pi V(s) = R^\pi(s) + \gamma \sum_{s' \in S} P^\pi(s'|s) V(s')$$

- Policy evaluation amounts to computing the fixed point of B^π
- To do policy evaluation, repeatedly apply operator until V stops changing

$$V^\pi = B^\pi B^\pi \dots B^\pi V$$

Policy Iteration as Bellman Operations

- Bellman backup operator B^π for a particular policy is defined as

$$B^\pi V(s) = R^\pi(s) + \gamma \sum_{s' \in S} P^\pi(s'|s) V(s')$$

- To do policy improvement

$$\pi_{k+1}(s) = \arg \max_a \left[R(s, a) + \gamma \sum_{s' \in S} P(s'|s, a) V^{\pi_k}(s') \right]$$

Going Back to Value Iteration (VI)

- Set $k = 1$
- Initialize $V_0(s) = 0$ for all states s
- Loop until convergence: (for ex. $\|V_{k+1} - V_k\|_\infty \leq \epsilon$)
 - For each state s

$$V_{k+1}(s) = \max_a \left[R(s, a) + \gamma \sum_{s' \in S} P(s'|s, a) V_k(s') \right]$$

- Equivalently, in Bellman backup notation

$$V_{k+1} = BV_k$$

- To extract optimal policy if can act for $k + 1$ more steps,

$$\pi(s) = \arg \max_a \left[R(s, a) + \gamma \sum_{s' \in S} P(s'|s, a) V_{k+1}(s') \right]$$

Contraction Operator

- Let O be an operator, and $|x|$ denote (any) norm of x
- If $|OV - OV'| \leq |V - V'|$, then O is a contraction operator

Will Value Iteration Converge?

- Yes, if discount factor $\gamma < 1$, or end up in a terminal state with probability 1
- Bellman backup is a contraction if discount factor, $\gamma < 1$
- If apply it to two different value functions, distance between value functions shrinks after applying Bellman equation to each

Proof: Bellman Backup is a Contraction on V for $\gamma < 1$

- Let $\|V - V'\| = \max_s |V(s) - V'(s)|$ be the infinity norm

$$\|BV_k - BV_j\| = \left\| \max_a \left(R(s, a) + \gamma \sum_{s' \in S} P(s' | s, a) V_k(s') \right) - \max_{a'} \left(R(s, a') + \gamma \sum_{s' \in S} P(s' | s, a') V_j(s') \right) \right\|$$

Proof: Bellman Backup is a Contraction on V for $\gamma < 1$

Opportunities for Out-of-Class Practice

- Prove value iteration converges to a unique solution for discrete state and action spaces with $\gamma < 1$
- Does the initialization of values in value iteration impact anything?

Value Iteration for Finite Horizon H

V_k = optimal value if making k more decisions

π_k = optimal policy if making k more decisions

- Initialize $V_0(s) = 0$ for all states s
- For $k = 1 : H$
 - For each state s

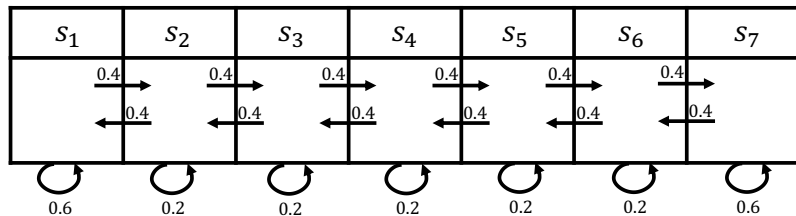
$$V_{k+1}(s) = \max_a \left[R(s, a) + \gamma \sum_{s' \in S} P(s'|s, a) V_k(s') \right]$$

$$\pi_{k+1}(s) = \arg \max_a \left[R(s, a) + \gamma \sum_{s' \in S} P(s'|s, a) V_k(s') \right]$$

Computing the Value of a Policy in a Finite Horizon

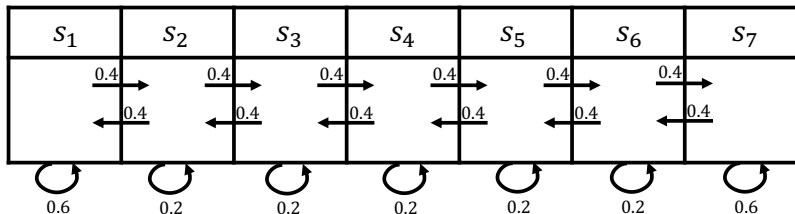
- Alternatively can estimate by simulation
 - Generate a large number of episodes
 - Average returns
 - Concentration inequalities bound how quickly average concentrates to expected value
 - Requires **no assumption** of Markov structure

Example: Mars Rover



- Reward: $+1$ in s_1 , $+10$ in s_7 , 0 in all other states
- Sample returns for sample 4-step ($H=4$) episodes, $\gamma = 1/2$
 - s_4, s_5, s_6, s_7 : $0 + \frac{1}{2} \times 0 + \frac{1}{4} \times 0 + \frac{1}{8} \times 10 = 1.25$

Example: Mars Rover



- Reward: +1 in s_1 , +10 in s_7 , 0 in all other states
- Sample returns for sample 4-step ($H=4$) episodes, start state s_4 , $\gamma = 1/2$
 - s_4, s_5, s_6, s_7 : $0 + \frac{1}{2} \times 0 + \frac{1}{4} \times 0 + \frac{1}{8} \times 10 = 1.25$
 - s_4, s_4, s_5, s_4 : $0 + \frac{1}{2} \times 0 + \frac{1}{4} \times 0 + \frac{1}{8} \times 0 = 0$
 - s_4, s_3, s_2, s_1 : $0 + \frac{1}{2} \times 0 + \frac{1}{4} \times 0 + \frac{1}{8} \times 1 = 0.125$

Question: Finite Horizon Policies

- Set $k = 1$
- Initialize $V_0(s) = 0$ for all states s
- Loop until $k == H$:
 - For each state s

$$V_{k+1}(s) = \max_a R(s, a) + \gamma \sum_{s' \in S} P(s'|s, a) V_k(s')$$

$$\pi_{k+1}(s) = \arg \max_a R(s, a) + \gamma \sum_{s' \in S} P(s'|s, a) V_k(s')$$

Is optimal policy stationary (independent of time step) in finite horizon tasks?

Question: Finite Horizon Policies

- Set $k = 1$
- Initialize $V_0(s) = 0$ for all states s
- Loop until $k == H$:
 - For each state s

$$V_{k+1}(s) = \max_a R(s, a) + \gamma \sum_{s' \in S} P(s'|s, a) V_k(s')$$

$$\pi_{k+1}(s) = \arg \max_a R(s, a) + \gamma \sum_{s' \in S} P(s'|s, a) V_k(s')$$

Is optimal policy stationary (independent of time step) in finite horizon tasks?

Value vs Policy Iteration

- Value iteration:
 - Compute optimal value for horizon $= k$
 - Note this can be used to compute optimal policy if horizon $= k$
 - Increment k
- Policy iteration
 - Compute infinite horizon value of a policy
 - Use to select another (better) policy
 - Closely related to a very popular method in RL: policy gradient

What You Should Know

- Define MP, MRP, MDP, Bellman operator, contraction, model, Q-value, policy
- Be able to implement
 - Value Iteration
 - Policy Iteration
- Give pros and cons of different policy evaluation approaches
- Be able to prove contraction properties
- Limitations of presented approaches and Markov assumptions
 - Which policy evaluation methods require the Markov assumption?

Where We Are

- Last Time:
 - Introduction
 - Components of an agent: model, value, policy
- This Time:
 - Making good decisions given a Markov decision process
- Next Time:
 - Policy evaluation when don't have a model of how the world works