

PROCESS MODEL & KPI'S

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1. PROCESS MODEL

We begin by deciding which process to model. This process will be a thermal tank. We choose this in order to be a surrogate for boiler operation. We will consider this thermal tank as a control volume that is well-mixed, is of fixed fluid mass, and has uniform temperature, a lumped-parameter system. This thermal tank will be heated by a heater and will have imperfect insulation properties (heat will escape through the tank itself).

1.1. Assumptions.

(a) mass is conserved in the control volume.

$$\Delta m_{cv} = \sum m_{in} - \sum m_{out}, \quad \frac{dm_{cv}}{dt} = \sum \dot{m}_{in} - \sum \dot{m}_{out}$$

(a) energy is conserved in the control volume.

$$\Delta E_{cv} = \sum E_{in} - \sum E_{out}, \quad \frac{dE_{cv}}{dt} = \sum \dot{E}_{in} - \sum \dot{E}_{out}$$

(a) ($T(t) = T_{out}$) temperature of the fluid that leaves the CV is the same net temperature of the fluid in the CV (well-mixed property).

1.2. Definitions.

$T(t)$: temperature of the tank ($^{\circ}\text{C}$)

T_{amb} : ambient temperature of the system ($^{\circ}\text{C}$)

T_{in} : temperature of the fluid as it enters the system; inlet temperature ($^{\circ}\text{C}$)

m_{cv} : fluid mass of the control volume (CV) (kg)

\dot{m}_{cv} : mass flow rate of the CV (kg/s)

E_{cv} : internal energy of the fluid in the CV (kJ)

E_{mass} : energy of the mass in CV (kJ)

$Q(t)$: heat (kW)

$W(t)$: work (kJ)

\dot{Q} : heat flow (kW/s)

\dot{W} : work flow (power) (kJ/s, kW)

hA : overall heat loss coefficient (conduction and convection effects) (kW/°C)
 c_p : specific heat capacity coefficient at constant pressure (kJ/kg·°C)

1.3. Identities.

$E_{cv} = m_{cv}c_pT$ since energy storage is proportional to temperature, with proportionality $m_{cv}c_p$. This follows from our approximately constant pressure assumption.

$\dot{Q}_{\text{loss}} = hA(T - T_{\text{amb}})$ by Newton's law of cooling.

1.4. Derivation. Our efforts here will be to produce a governing equation for our system.

A thermal tank has a constant maximum volume, so the volume is controlled. We want to monitor the temperature within the thermal tank and be able to keep it within a boundary. Thus, this system is in essence an energy balance problem. So we turn to the principles and laws of thermodynamics to start this derivation.

The first law of thermodynamics (a version of the law of conservation of energy, adapted for thermodynamic processes) says the change in internal energy of the system (ΔE_{cv}) is equal to the difference between the heat supplied to the system (Q) and the work (W) done by the system on its surroundings.

$$\Delta E_{cv} = Q - W. \quad (1)$$

Since we are dealing with a control volume, an additional mechanism can change the energy of a system: mass flow in and out of the control volume. Therefore, a control volume undergoing a process is expressed as

$$\Delta E_{cv} = Q - W + \sum E_{\text{in, mass}} - \sum E_{\text{out, mass}} = Q - W + E_{\text{mass}}. \quad (2)$$

We take all quantities as variable w.r.t time t and express (2) in flow rate form:

$$\frac{dE_{cv}}{dt} = \dot{Q} - \dot{W} + \frac{dE_{\text{mass}}}{dt}. \quad (3)$$

Component-wise we have

$$\begin{aligned} \frac{dE_{cv}}{dt} &= \frac{d}{dt} [m_{cv}c_pT(t)] = m_{cv}c_p \frac{dT}{dt}, \\ \dot{Q} &= \dot{Q}_{\text{in}} - \dot{Q}_{\text{loss}} = \dot{Q}_{\text{in}} - hA(T - T_{\text{amb}}), \quad \dot{W} = 0, \\ \frac{dE_{\text{mass}}}{dt} &= \dot{m}c_p(T_{\text{out}} - T_{\text{in}}) = \dot{m}c_p(T - T_{\text{in}}), \end{aligned}$$

which from (3) gives our governing equation

$$m_{cv}c_p \frac{dT}{dt} = \dot{Q}_{in} - hA(T - T_{amb}) - \dot{m}c_p(T - T_{in}), \quad (4)$$

a first-order, lumped-parameter differential equation describing our thermal process.

2. KPIs

Key performance indicators (KPIs) use mathematical formulas and statistical analysis on real-time data to measure operational performance, efficiency, and quality. we will introduce three different KPIs for our simulated problem.

2.1. KPI 1: Thermal Efficiency. This KPI is responsible for measuring how much of the input energy E_{in} is actually being transferred to the process.

$$\eta(t) = \frac{\dot{m}c_p(T - T_{in})}{Q_{in}} \quad (5)$$

2.2. KPI 2: Efficiency Deviation. This KPI is used for maintenance insight and detecting degradation.

$$\Delta\eta(t) = \eta(t) - \eta_{nominal} \quad (6)$$

2.3. KPI 3: Rolling Average Efficiency. This KPI is used for trend analysis and management dashboards.

$$\bar{\eta}(t) = \frac{1}{\tau} \int_{t-\tau}^t \eta(s) ds \quad (7)$$