PHIL 5201 Homework 1

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Problem Set

1. (a) Let $A = \{a, b, c\}$. Name all of the subsets of A

$$\emptyset, \{a\}, \{a, b\}, \{a, c\}, \{b, c\}, \{a, b, c\}$$

(b) Apply the definition of a finite sequence in order to explicitly name the set-theoretic object that $\langle a, b, c, d\rangle$ is identical to.
Solution:

$$\{\{\{\{\{\{a\}, \{a, b\}\}\}, \{\{\{a\}, \{a, b\}\}, c\}\}\}, \{\{\{\{\{a\}, \{a, b\}\}\}, \{\{\{a\}, \{a, b\}\}, c\}\}, d\}\}$$

2. For each of the following, prove the statement or show that it is false by a providing counterexample:

(a) For all sets of sets X,

$$X = \mathcal{P}\left(\bigcup X\right)$$

Solution: False. The empty set is a member of the power set but not a set X

(b) For all sets of sets X,

$$X = \bigcup(\mathcal{P}X)$$

Solution:

Let b denote a member of $\mathcal{P}(X)$. By definition of the power set, $b \subseteq X$.
moreover, the $\bigcup$ operation returns a set of all the elements of the operand set.
Thus the $\bigcup$ of all b's, or members, gives a set of the elements of $\mathcal{P}(X)$ which are in X.
Conversely, any element-or combination of elements-of X belong some b to so
consequently the $\bigcup$ of all such combinations is equivalent to X

(c) let X be a set of sets that is pairwise disjoint. Then,

$$\bigcap X = \emptyset$$

Solution:

True. Since X is pairwise disjoint, meaning no two sets in the collection share a element, which implies that there does not exist a shared element between all the sets, and thus the $\bigcap$ of all such sets is the empty set.

3. Recall the defining feature of ordered pairs:

$$\langle a, b\rangle = \langle c, d\rangle \text{ iff } a = c \text{ and } b = d.$$

Prove that the definition of ordered pairs satifies this feature That is, prove that

$$\{\{a\}, \{a, b\}\} = \{c\}, \{c, d\}\} \text{ iff } a = c \text{ and } b = d$$

Solution:

$$S_1 = \{\{a\}, \{a, b\}\}, \quad S_2 = \{\{c\}, \{c, d\}\}$$

$$p \Leftrightarrow q = (p \rightarrow q) \wedge (q \rightarrow p)$$

$$P = (S_1 = S_2) \quad Q = (a = c \text{ and } b = d)$$

Case 1:

$$P \rightarrow Q$$

The principle of extensionality, which states that two sets, A, B are only identical if every member of A is in B and conversely. The set $\{a\}$ is in S_1 and therefore must be equal to an element in S_2. Since a set of 1 element can't be equivalent to a set of 2 elements, $\{a\}$ must be equal $\{c\}$. Now we can substitute $a = c$ into the sets with 2 elements we have $\{a, b\} = \{a, d\}$, so $b = d$

Case 2:

$$Q \rightarrow P$$

By substitution, $(\{\{a\}, \{a, b\}\} = \{c\}, \{c, d\}\}) = (\{\{c\}, \{c, d\}\}\} = \{a\}, \{a, b\}\})$

4. Let $O = \{1, 3, 5, \ldots\}$ be the set off odd natural numbers. prove that $O \sim \mathbb{N}$

Solution:

Two sets are equinumerous iff there is a bijection between the sets.
by definition, an odd number x can be expressed by the bijective function:

$$f(k) = 2k + 1$$

where k is natural number. To prove O $\sim \mathbb{N}$, we can rearrange the formula to solve for k

$$k = \frac{x - 1}{2}$$

since $f(k) = 2k + 1$ is bijective, it's inverse $k = \frac{x-1}{2}$ will also be bijective.