

# Packet Reachability of VANET in Bidirectional Road Scenario

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**Abstract**—Bidirectional road is the basic and ordinary scenario of Vehicular Ad hoc Network (VANET) which is an emerging and challenging research applying mobile ad hoc technology to transportation system. According to different forwarding mechanisms in routing protocol of such network, the connections are divided into end-to-end (E2E) type and Store-Carry-Forward (SCF) type. The packet reachability probabilities, the representative of network connectivity, of both connection types are calculated and compared quantitatively using our proposed probability model. The results are validated by simulations in our experiments. Our work provides important and useful reference basis for network and routing protocol designers.

**Keywords**—VANET; network connectivity; packet reachability

## I. INTRODUCTION

Vehicular Ad hoc Networks (VANET) represents a rapid emerging and challenging class of Mobile Ad Hoc Network (MANET). In such networks nodes (or vehicles) communicate with each other using equipped wireless transmission devices in hop by hop manner. This capability is particularly useful for distributing information such as traffic and safety information, weather information, and navigation information without the need of costly infrastructure. In order to facilitate the information dissemination in VANET, network connectivity is needed.

The connectivity of VANET was studied by means of network simulation method in many literatures [1-4], while few of them analyzed connectivity theoretically. Connectivity of the network was discussed by percolation theory in [5]. Authors think the network almost sure cannot be fully connected for one dimensional dynamic network. The paper [6] considers VANET as a nominal system with disturbance. Under constant disturbance conditions, the lower bound of reachable neighbors for each vehicle to maintain a high connectivity is analytically derived only in one way street scenario. For bidirectional road scenarios in VANET, packets from a source vehicle should be able to propagate and reach all the vehicles on the road segment, so packet reachability probability (PRP) is used to evaluate network connectivity in this paper.

There are two different connection types in the existed routing protocols of VANET. The first one is traditional end-to-end connection always adopted by topology-based routing. It is called E2E connection in the following. In such case, packets sent from the source vehicle are forwarded by the intermediate node along the pre-calculated route to the destination. Once the forwarding node can not find the proper

next hop for some reason, i.e., vehicle mobility, it has to drop the packet and the source node cannot communicate with the destination along the formed path any more. The second connection type is store-carry-forward (SCF) always used by opportunistic routing. Packets can be cached by the intermediate vehicle when failed to find next hop node. When the new forwarding opportunity occurs, the packet is relayed thus it can be transmitted to the destination at last. Our goal is to analyze the network connectivity of both connection types theoretically in terms of packet reachability probability. The validity of our theoretical analysis is also verified by simulation. Conclusions are meaningful in designing and developing self-organizing traffic network.

The reminder of this paper is organized as follows. In Section II we describe our probability model and assumptions. Section III analyzes packet reachability probability of E2E and SCF theoretically and Section IV is the experimental validation of our analysis. Finally, Section V concludes this paper.

## II. PROBABILITY MODEL AND ASSUMPTIONS

The basic bidirectional road scenario is illustrated as Fig. 1. We propose a probability model to analyze the network connectivity probability in VANET. The connectivity probability is defined as the probability that packets can be transmitted from the leftmost vehicle to the rightmost vehicle in Fig. 1. In fact, many factors affect the packet transmissions of such inter-vehicle network including radio range, radio interference and time-varying vehicle mobility, etc. For simplicity, some assumptions are used in the model as follows: 1) The bidirectional road considered is long enough and its width can be ignored, so the network can be viewed as one dimensional structure. The vehicle length is also ignored. 2) Vehicle moving along the road can overtake each other like in multiple lanes. 3) All vehicles in the network has the same transmission range  $r$  and also two vehicles can communicate directly if the distance between them is less or equal to  $r$ . 4) Since vehicles are uniformly distributed on the road, for efficient large road length and the number of nodes, it can be shown that the amount of vehicles in any segment is considered as Poisson distribution and the distance between two neighbors is exponential distributed [10].

The notations used in our model are defined as follows: 1)  $L$ : road length, unit is m. 2)  $\lambda$ : vehicle density of the road in one direction, unit is #/m. 3)  $N$ : total number of vehicles in the road and  $N = 2\lambda L$ . 4)  $r$ : wireless radio range, unit is m.

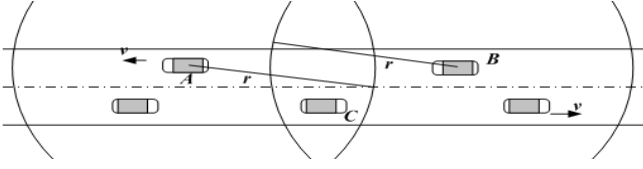


Figure 1 Illustration of bidirectional roads of VANET

### III. PACKET REACHABILITY PROBABILITY ANALYSIS

#### A. Packet Reachability of E2E connection

As our assumption 3), the distance  $X_i$  between two neighbors in one direction are exponential distributed and are independent and identically distributed (i.i.d) random variables. Since  $E(X) = 1/\lambda$ , the cumulative distribution function (cdf) is

$$F(x) = \begin{cases} 1 - e^{-\lambda x}, & x \geq 0 \\ 0, & x < 0 \end{cases} \quad (1)$$

If all vehicles along this direction can communicate with each other, the distances between any of consecutive vehicles are no more than  $r$ . So the packet reachability probability of this connection type can be calculated as

$$\begin{aligned} P_1 &= \Pr\{X_1 \leq r, X_2 \leq r, \dots, X_i \leq r\} \\ &= \prod_{i=1}^{\lambda L - 1} \Pr\{X_i \leq r\} \\ &= (1 - e^{-\lambda r})^{\lambda L - 1} \end{aligned} \quad (2)$$

The vehicle density of our assumed bidirectional road is twice of one-way road, so packet reachability probability of E2E class is

$$P_{E2E} = (1 - e^{-2\lambda r})^{N-1} = (1 - e^{-2\lambda r})^{2\lambda L - 1} \quad (3)$$

From (3) we can see the connectivity of E2E is determined by the vehicle density, wireless range and road length. Fig. 2 shows the connectivity probability as the function of vehicle density when  $L=5000m$ . The values of  $r$  are 150m and 300m which mean the wireless radius of 802.11b devices and 802.11p devices, respectively. It is observed that, for a given transmission range, the connectivity probability increases with the increase of vehicle density. In other words, the more nodes moves on the road, the higher connectivity the network achieves. The results are meaningful for network designers, for example, it shows that there are at least 26 vehicles per kilometers for high performance (PRP more than 90%) for 802.11p network.

#### B. Packet Reachability of SCF connection

When packets are transmitted with SCF procedure, although two consecutive nodes along the same direction are far away more than  $r$  at some point, they may also communicate with the forwarding of other vehicles moving along opposite direction in the following intervals. As an example, node A and B cannot communicate directly now in

Fig1 and after a short period node C appears between them, so packets can be forwarded to B from A with the help of C.

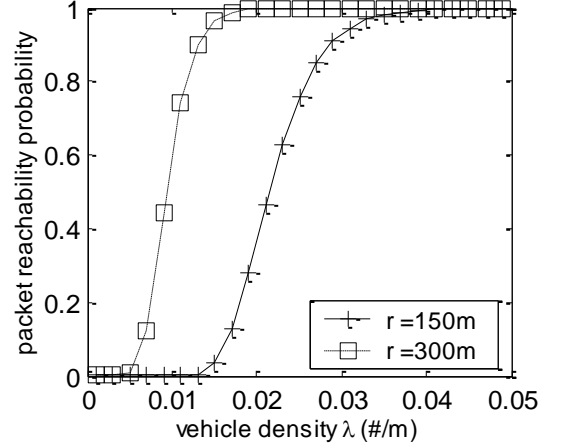


Figure 2 Packet reachability of E2E connection

Let  $N_{\min}$  be the minimum amount of intermediate nodes needed by two communicating nodes far from  $d$ . Since the network are assumed one dimensional,

$$N_{\min} = \lceil d/r \rceil - 1 \quad (4)$$

For two consecutive vehicles A and B moving along the same direction, for example the left, packets cannot be transmitted between them when there are less than  $N_{\min}$  nodes moving along the opposite direction between A and B.  $p_0(N_{\min})$  denotes the probability of this occurrence. Since the amount  $k$  of vehicles on the  $l$ -length road is considered as Poisson distribution, its probability mass function (PMF) is

$$p_p(k) = \frac{(\lambda l)^k}{k!} e^{-\lambda l} \quad k = 0, 1, 2, \dots \quad (5)$$

So,

$$\begin{aligned} p_0(N_{\min}) &= \Pr\{k = 0\} + \Pr\{k = 1\} + \dots \\ &\quad + \Pr\{k = N_{\min} - 1\} \\ &= \sum_{i=0}^{N_{\min}-1} p_p\{i\} \\ &= \sum_{i=0}^{N_{\min}-1} \frac{(\lambda d)^i}{i!} e^{-\lambda d} \quad (d > r). \end{aligned} \quad (6)$$

At the same time, the probability  $P_{d-r}(i)$  of the event  $d \in [ir, (i+1)r)$  is

$$\begin{aligned} P_{d-r}(i) &= F((i+1)r) - F(ir) \\ &= e^{-i\lambda r} - e^{-(i+1)\lambda r}. \end{aligned} \quad (7)$$

Two consecutive vehicles A and B cannot communicate even with the help of forwarding nodes moving along the opposite direction between them in the current moment or the later time and the probability  $P_0$  of this happens is

$$\begin{aligned}
P_0 &= p_{d-r}(1)p_0(1) + p_{d-r}(2)p_0(2) + \dots \\
&\quad + p_{d-r}(\lceil L/r \rceil - 1)p_0(\lceil L/r \rceil - 1) \\
&= \sum_{k=1}^{\lceil L/r \rceil - 1} p_{d-r}(k)p_0(k) \\
&= \sum_{k=1}^{\lceil L/r \rceil - 1} \left( \sum_{i=0}^{k-1} \frac{(i\lambda r)^i}{i!} e^{-i\lambda r} \right) (e^{-k\lambda r} - e^{-(k+1)\lambda r}).
\end{aligned} \tag{8}$$

Furthermore, when there are  $j$  cases that distance between neighbors along the same direction is more than  $r$ , the probability that network is connected yet with the forwarding of reverse vehicles is

$$P_{connect \setminus j \text{ far}} = (1 - P_0)^j. \tag{9}$$

Specifically, for a pair of two consecutive nodes with same direction, the probability that they cannot communicate with each other directly is

$$P_{far} = 1 - F(r) = e^{-\lambda r}. \tag{10}$$

There are  $\lambda L - 1$  links between neighbors along one direction if the vehicle density and road length are given. Since the distances between pairs of neighbors are independent, the probability of the event that the distance is larger than or equal to  $r$  allows binomial distribution. Thus, the probability that such event occurs  $j$  times is

$$P_{j \text{ far}} = \binom{\lambda L - 1}{j} P_{far}^j (1 - P_{far})^{\lambda L - 1 - j}. \tag{11}$$

As a result, the probability  $P_{SCF}$  that packets can be transmitted from vehicle locating at one end of road to the destination locating at the other end with SCF procedure can be calculated as following.

$$\begin{aligned}
P_{SCF} &= \sum_{j=0}^{N-1} P_{j \text{ far}} \cdot P_{connect \setminus j \text{ far}} \\
&= \sum_{j=0}^{N-1} \binom{\lambda L - 1}{j} (1 - P_0)^j P_{far}^j (1 - P_{far})^{\lambda L - 1 - j} \\
&= \sum_{j=0}^{N-1} \binom{\lambda L - 1}{j} e^{-j\lambda r} (1 - e^{-\lambda r})^{\lambda L - 1 - j} (1 - P_0)^j
\end{aligned} \tag{12}$$

The values of  $P_{SCF}$  as a function of vehicle densities are depicted in Fig. 4, where  $L=10\text{km}$  and  $r=150\text{m}$  and  $300\text{m}$ . Packet reachability probability increases with the increasing vehicle density when  $r$  has both different values. The main reason for this is that more forwarding chances can be obtained by vehicles when the amount of nodes increases for a given road length. Besides, the packet reachability with larger transmission range ( $r=300\text{m}$ ) is better than that with the lower ( $r=150\text{m}$ ) which is mainly because that longer wireless transmission distance makes more chance to find neighbors and decrease forwarding hops.

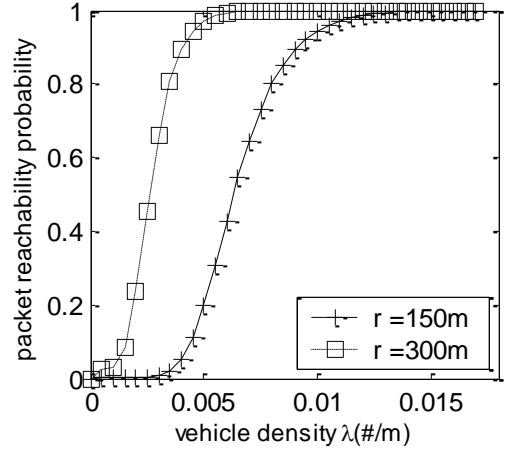


Figure 4 Packet reachability of SCF connection

### C. Results Comparison of E2E and SCF

The current forwarding vehicle in SCF mechanism selects next hop node also from nodes which may become its neighbors in a short moment besides from its current neighbors which was the case in E2E. Thus, the network connectivity of SCF connection should be better than E2E because the different forwarding process mechanisms. Seen from (3) and (12), parameters  $L$ ,  $\lambda$  and  $r$  are common important factors affecting the packet reachability probability of E2E and SCF. Here the PRP value of both connection types are compared varying vehicle densities  $\lambda$  given  $L$  and  $r$ , for example,  $L=5\text{km}$  and  $r=150\text{m}$ . The comparison results are shown in Fig. 5.

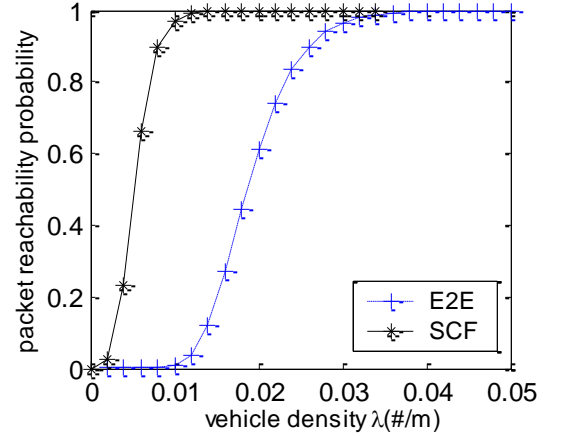


Figure 5 PRP comparison of E2E and SCF

From the comparison figure, it is seen that for a given vehicle density  $\lambda$  the PRP of SCF has distinct advantages over E2E. As an example, the PRP values of SCF and E2E are 0.8 and almost 0 when  $\lambda=0.004$ , respectively. That's mainly because of the different mechanisms when forwarding packets. The results prove that SCF procedure has much more advantages than E2E in bidirectional road scenarios in VANET.

### IV. VALIDATION OF THEORETICAL ANALYSIS

Since some ideal assumptions are given when deriving the theoretical results, the Monte Carlo simulation method under

MATLAB was used for validation instead of network traffic simulation in simulators.

#### A. Packet Reachability of E2E connection

In our experiments,  $\lambda L$  vehicles are randomly distributed along each direction of the road with length  $L=10\text{km}$ . Vehicle wireless transmission range  $r$  equals to  $150\text{m}$ . All packets can be transmitted from one end of the road to the other or the network is fully connected if and only if all distances between neighbors are no larger than  $r$ . The PRP is thought the ratio of fully connected times to total experiments times ( $N=500$ ) if the experiments are repeated large enough times. The comparison results of theoretical value and simulation value of E2E packet reachability are shown in TABLE I. At all 10 samples, the theoretical and simulative value are almost same with the average error  $0.0081$ , which proves the correctness of the (3).

TABLE I PRP validation of E2E

$\lambda$	theoretical value	simulative value	absolute error
0.001	7.22e-012	0	0.0000
0.006	4.59e-010	0	0.0000
0.011	0.000266	0	0.0003
0.016	0.071636	0.056	0.0156
0.021	0.46296	0.436	0.0270
0.026	0.8084	0.82	0.0116
0.031	0.94498	0.93	0.0150
0.036	0.98544	0.994	0.0086
0.041	0.99628	0.998	0.0017
0.046	0.99907	0.998	0.0011

#### B. Packet Reachability of SCF connection

Since current forwarding node will cache the data packets if it cannot find next hop node in SCF procedure, the cache operation is related to time. While in our analysis, this interval is implicitly assumed large enough. To simulate the cache process, a fix operation is defined to try to fix the link by finding a node from opposite direction lanes between the current forwarding node and its last hop node. When the fix is operated, nodes will be redistributed randomly in opposite direction lanes to simulate vehicle distribution in the later time. So the more times the fix operation is done, the longer time the packets are cached.

Fig. 6 shows the packet reachability as a function of vehicle densities given  $L=5\text{km}$ ,  $r=150\text{m}$ . In our experiment fix times are set 0, 2, 8 and 64, respectively, and each case was repeated 500 times. When fix time is zero, the PRP fits perfectly with the theoretical value of E2E which coincides with that SCF procedure has better performance than E2E. It is observed that the PRP is improved greatly and tends asymptotically towards the theoretical value with the increasing fix times. So when the fix time is large enough, the PRP values can be believed as the same as the theoretical values and (12) is proved in result.

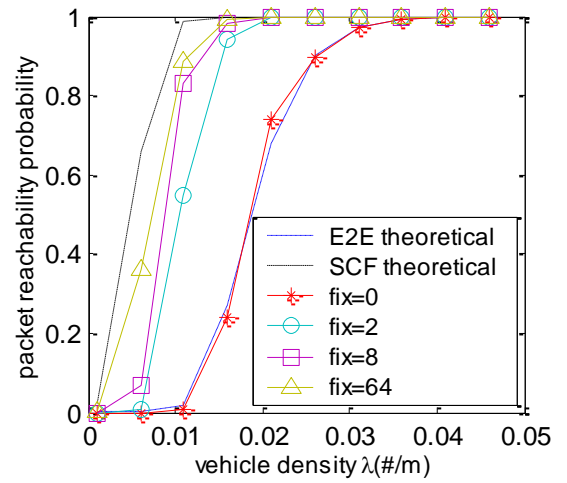


Figure6 PRP validation of SCF

#### V. CONCLUSION

According to different forwarding mechanisms in routing protocol, the connections are divided into end-to-end type and store-carry-forward type. The packet reachability probabilities of both connection types are calculated and compared quantitatively in bidirectional road which is the basic and ordinary scenario of vehicular ad hoc networks. The results are validated by simulations in our experiments. Our work provides important and useful reference basis for network and routing protocol designers.

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