Fourier Series Expansion of a Square Wave

$$f(x) = \left\{ \begin{array}{ll} 1 & \text{if } 0 \leq x \geq L \\ -1 & \text{if } L \leq x \geq 2L \end{array} \right\} \tag{1}$$

Fourier series expansion is given by

$$f(x) = \sum_{n=-\infty}^{\infty} A_n e^{\frac{\iota 2\pi n x}{2L}} \tag{2}$$

$$where\ A_n = \frac{1}{2L} \int_{-L}^{L} f(x) e^{\frac{\iota 2\pi n x}{2L}} dx \tag{3}$$

Using 3 for $n = 0$,

$$A_0 = \frac{1}{2L} \int_{-L}^{L} f(x) dx = 0 \tag{4}$$

For $n \neq 0$,

$$A_n = \frac{1}{2L} \left[\int_{0}^{L} e^{\frac{\iota \pi n x}{L}} dx - \frac{1}{2L} \int_{L}^{2L} e^{\frac{\iota \pi n x}{L}} dx \right] \tag{5}$$

$$= \frac{1}{\iota \pi n} (1 - (1)^n) \tag{6}$$

Equation 2 can be simplified using the formula

$$\frac{e^{\iota x} - e^{-\iota x}}{2\iota} = sin(x) \tag{7}$$

This gives us

$$f(x) = \sum_{n=1}^{\infty} \frac{2(1 - (-1)^n)}{\pi n} sin \left(\frac{\pi n x}{L} \right) \tag{8}$$