UNIVERSITY OF CAMBRIDGE

MPhil in Scientific Computing

Magnetohydrodynamic Simulation in Complex Geometries

Overview

Controlled nuclear fusion represents one of the most promising and revolutionary advancements in the pursuit of clean and virtually limitless energy. However, the tokamak device, which currently stands as the most feasible approach to achieving controlled nuclear fusion, is not yet fully mature. Research on plasma behavior benefits development of tokamak. Specifically, studies on plasma-wall interactions, particularly the effects on wall resistivity, may benefit the selection of effective wall materials.
This study explore the magnetohydrodynamics (MHD) interaction between plasma and general resistive walls under tokamak scenario. These walls include perfect conducting wall, insulating wall and resistive walls. Their mathematical forms are derived, numerically approximated, and finally visualized in simulations.

Methodology

The core heat plasma within a tokamak vessel is fully ionized, which can be regarded as ideal plasma with no resistivity and viscosity and can be further described with **ideal MHD model**:

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{v}) = 0$$

$$\frac{\partial (\rho \mathbf{v})}{\partial t} + \nabla \cdot \left[\rho \mathbf{v} \otimes \mathbf{v} + \left(p + \frac{1}{2}\mathbf{B}^2 \right) \mathbf{I} - \mathbf{B} \otimes \mathbf{B} \right] = 0$$

$$\frac{\partial U}{\partial t} + \nabla \cdot \left[\left(U + p + \frac{1}{2}\mathbf{B}^2 \right) \mathbf{v} - (\mathbf{v} \cdot \mathbf{B})\mathbf{B} \right] = 0$$

$$\frac{\partial \mathbf{B}}{\partial t} + \nabla \cdot (\mathbf{B} \otimes \mathbf{v} - \mathbf{v} \otimes \mathbf{B}) = 0$$

A first-order **MHD-HLLC** solver [1] is utilized on updating this model while the application of **MUSCL-Hancock** scheme extend the solver into second-order accuracy. **Mixed divergence cleaning** [2] is employed to correct the non-zero magnetic divergence.

Boundary Conditions

In the study of plasma-wall interactions, rigid bodies are introduced. Generally, a reflective ghost fluid method (GFM) [3] is applied on their boundaries. Most of the applied boundary conditions can be described using Dirichlet and Neumann boundary conditions. Dirichlet boundary condition fixes a constant value at the boundary. Neumann boundary condition specifies a fixed change rate across the boundary.

Hydrodynamics effect

For fixed rigid bodies, a zero reflective Dirichlet condition is applied on normal velocity and zero Neumann conditions on tangential velocity and scalar variables, details in [3].

Perfect conducting wall

$$B_n = B_{n0} \qquad \partial B_t / \partial \mathbf{n} = 0$$

implies a constant Dirichlet normal magnetic field and a zero Neumann tangential component.

Insulating wall

$$\partial B_n / \partial \mathbf{n} = 0 \qquad \partial B_t / \partial \mathbf{n} = 0$$

allows penetrations, both zero Neumann conditions

Resistive wall

$$\frac{\partial \mathbf{B}}{\partial t} + \eta_w \nabla \times \nabla \times \mathbf{B} = 0,$$

can not be described as Neumann or Dirichlet. Instead, an equation derived from Maxwell's equations is iterated with GFM.

Validation Tests

Some tests are used for validating the applied methods and some rigid body geometries.

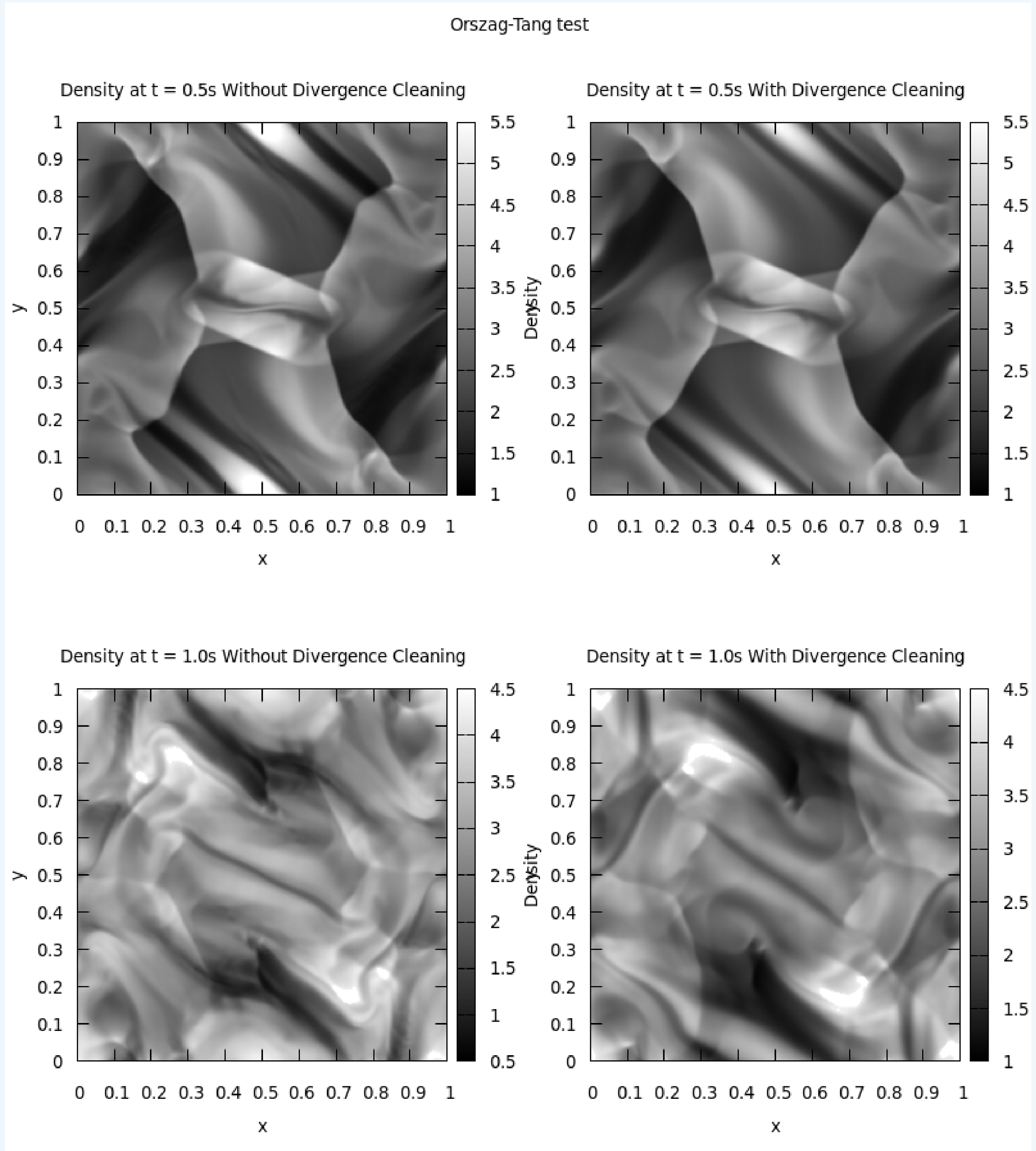

Figure 1: Results of the Orszag-Tang test at $t = 0.5$ (upper) and $t = 1.0$ (lower), comparing the simulations with divergence cleaning (right) to the one without (left).

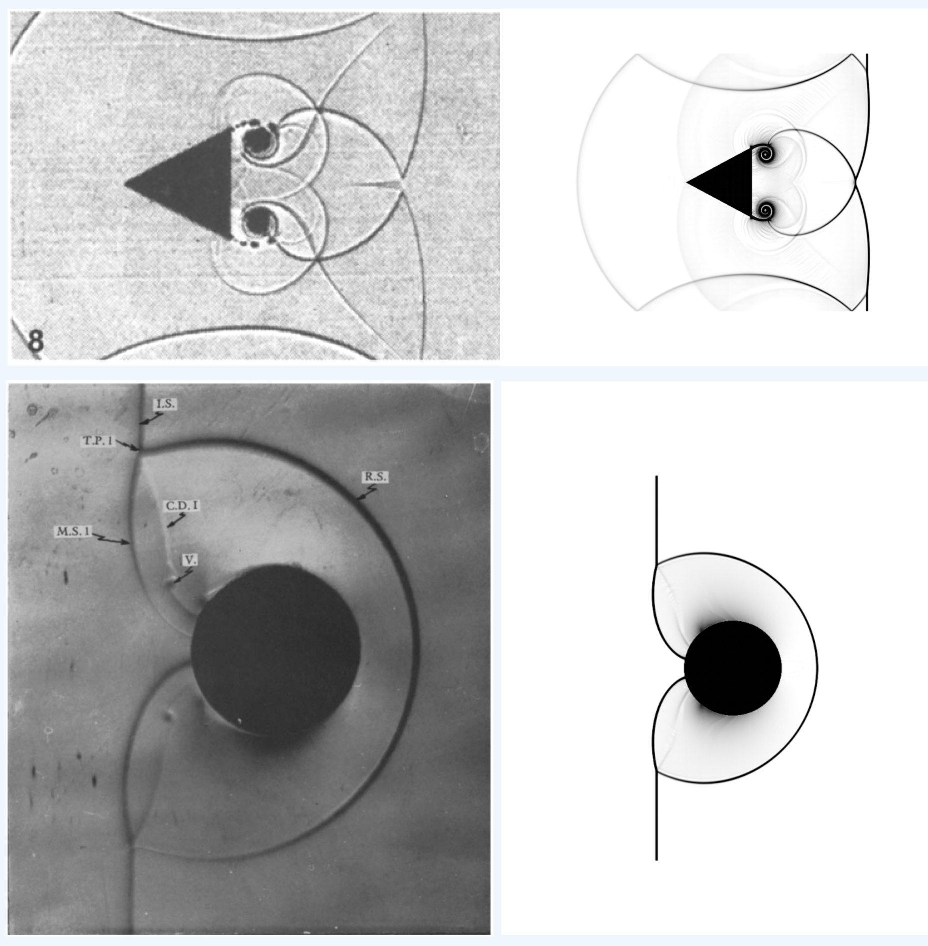

Figure 2: Schlieren plots for Shock diffraction over wedge or cylinder, comparing the results from simulations (right) to the experiment results (left).

Validation for Resistive Wall

The validation for resistive wall is adopted from a plasma cylindrical equilibrium test of Ferraro *et al.* [4]. It is designed to model the resistive wall mode within a simplified straight cylindrical geometry. The Figure 3 demonstrates the initial state of cylindrical equilibrium test:

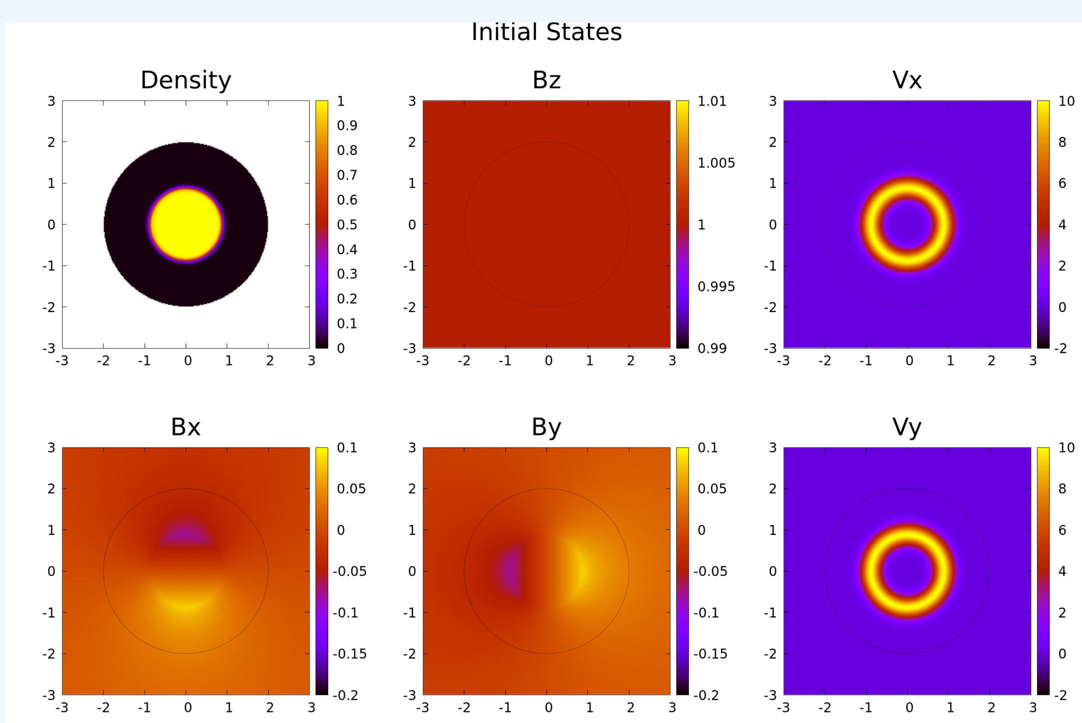

Figure 3: Initial state of cylindrical equilibrium.

Validation for Resistive Wall

The Figure 4 below demonstrate the results density and B_z in the cylindrical equilibrium test. The resistive wall leads to a higher peak in B_z showing a qualitative agreement with Chrysanthou's result [5] in Figure 5.

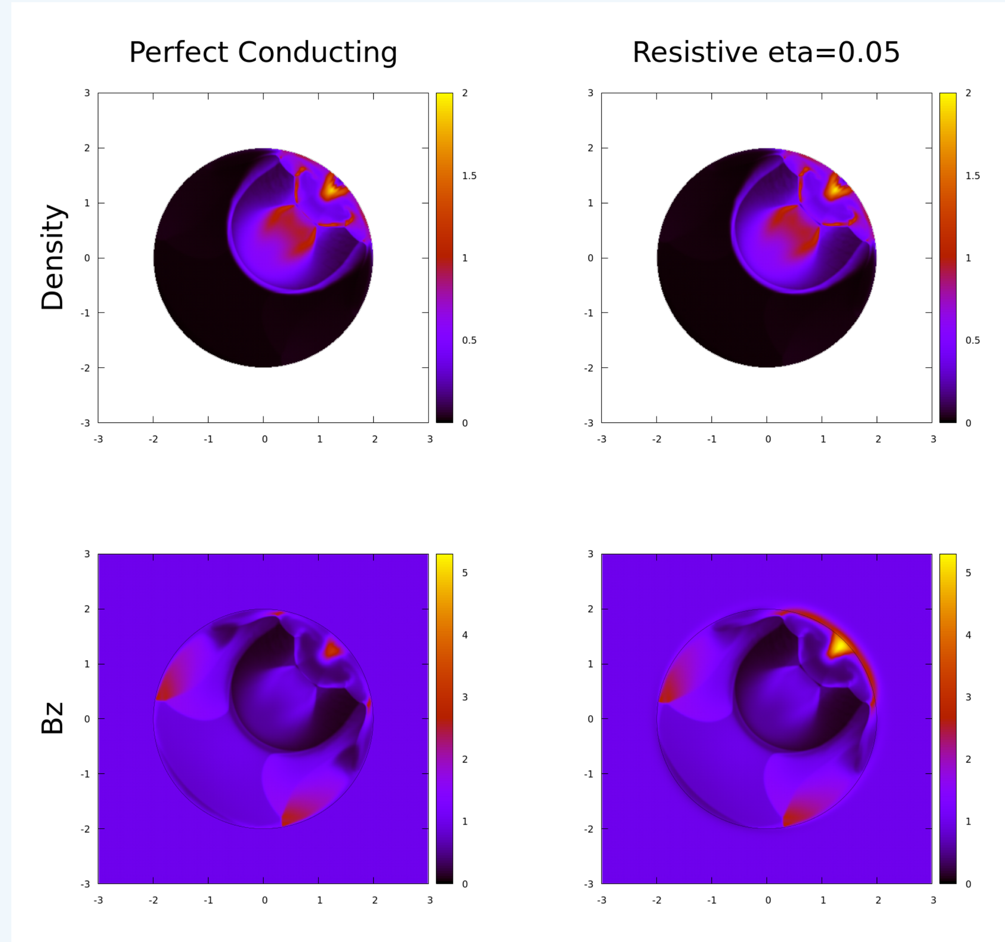

Figure 4: Results of cylindrical equilibrium test comparing densities (upper) and B_z (lower) with resistivities $\eta_w = 0.0$ (left) and $\eta_w = 0.05$ (right).

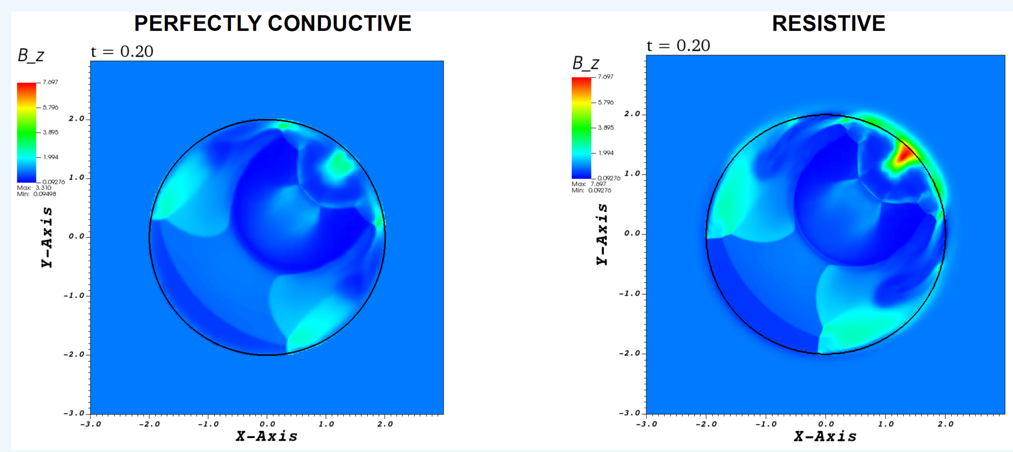

Figure 5: The B_z result in Chrysanthou's test [5].

Plasma in Resistive Tokamak Vessel

The following graph shows density and B_z in simulation within a resistive tokamak-shaped vessel. This figure is taken from the moment $t = 0.2$ after the core dense plasma hitting the vessel wall with resistivity $\eta_w = 0.05$. The interaction on magnetic field between plasma and vessel wall result in significant increase in B_z on wall.

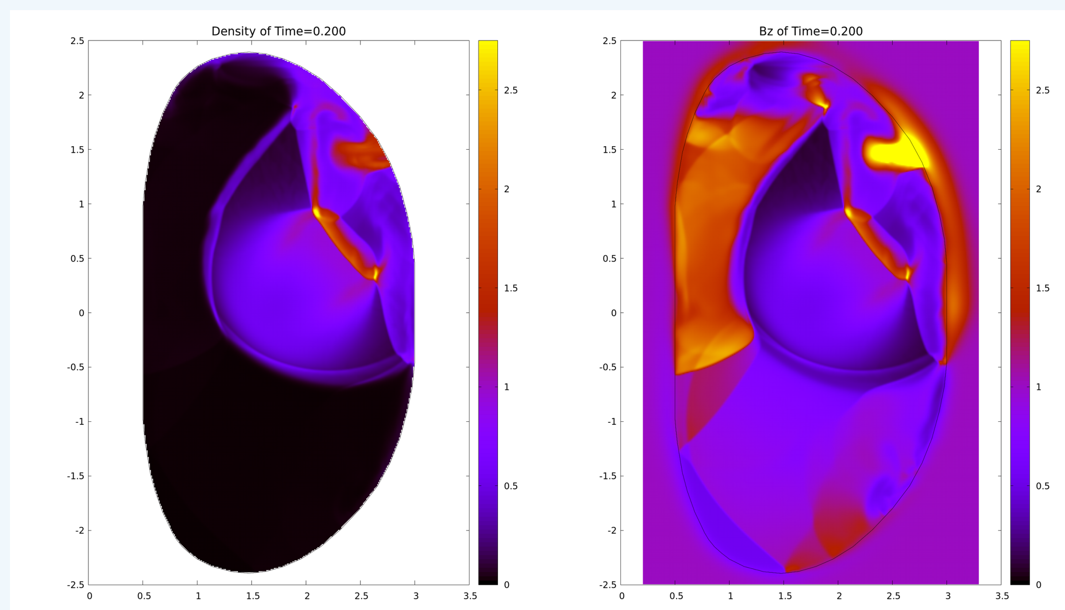

Figure 6: Density (left) and B_z (right) of plasma in a cylindrical equilibrium test within a tokamak-shaped vessel, outlined by a thin black line.

References

[1] S. Li. *Journal of computational physics* 203.1 (2005).

[2] J. Vides et al. *ESAIM: Proceedings.* Vol. 43. EDP Sciences. 2013.

[3] S. K. Sambasivan and H. UdayKumar. *AIAA journal* 47.12 (2009).

[4] N. Ferraro et al. *Physics of Plasmas* 23.5 (2016).

[5] M. Chrysanthou. Master's thesis. University of Cambridge, 2020.

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