UNIVERSITY OF CAMBRIDGE

MPhil in Scientific Computing

Magnetohydrodynamic Simulation in Complex Geometries

Overview

The study explores the interaction between plasma and the resistive wall, with a focus on implementing the outcomes model through simulations under tokamaks scenario. After a brief introduction, the boundary conditions for perfect conducting walls, insulating walls, and resistive walls are mathematically derived and numerically approximated. These boundary conditions are then applied in validation tests. Ultimately, a comprehensive model is utilized for plasma simulations within a resistive tokamak vessel.

Methodology

A ideal MHD model is applied on modeling the core plasma. A first-order MHD-HLLC solver is utilized and extended to second-order accuracy through the application of the MUSCL-Hancock scheme. Mixed divergence cleaning is applied to correct the non-zero divergence arising from plasma-wall interactions, details in [1].

Boundary Conditions

In the selection of boundary conditions, different approaches are applied to the hydrodynamic effects and the magnetic field. On hydrodynamics effect, reflective Dirichlet condition is applied on velocity field and Neumann condition on scalar variables. For details, in [2]. On magnetic field, there mainly three boundary conditions:

Perfect conducting wall

$$\frac{\partial\left(\mathbf{n}\cdot\mathbf{B}\right)}{\partial t}=0 \quad \Rightarrow \quad \begin{array}{rl} B_n = B_{n0} & \text{Dirichlet} \\ \partial B_t/\partial\mathbf{n} = 0 & \text{Neumann} \end{array}$$

Insulating wall

$$\begin{array}{r} \nabla\times\mathbf{B}=0 \\ \nabla\cdot\mathbf{B}=0 \end{array} \quad \Rightarrow \quad \begin{array}{rl} \partial B_n/\partial\mathbf{n} = 0 & \text{Neumann} \\ \partial B_t/\partial\mathbf{n} = 0 & \text{Neumann} \end{array}$$

Resistive wall

$$\frac{\partial\mathbf{B}}{\partial t}+\eta_w\nabla\times\nabla\times\mathbf{B}=0,$$

updated using ghost fluid method.

Plasma in Resistive Tokamak Vessel

The following graph shows B_z in simulation within a resistive tokamak-shaped vessel. This figure is taken from the moment $t=0.2$ after the core dense plasma hitting the vessel wall with resistivity $\eta_w=0.05$. The interaction on magnetic field between plasma and vessel wall result in significant increase in B_z on wall.

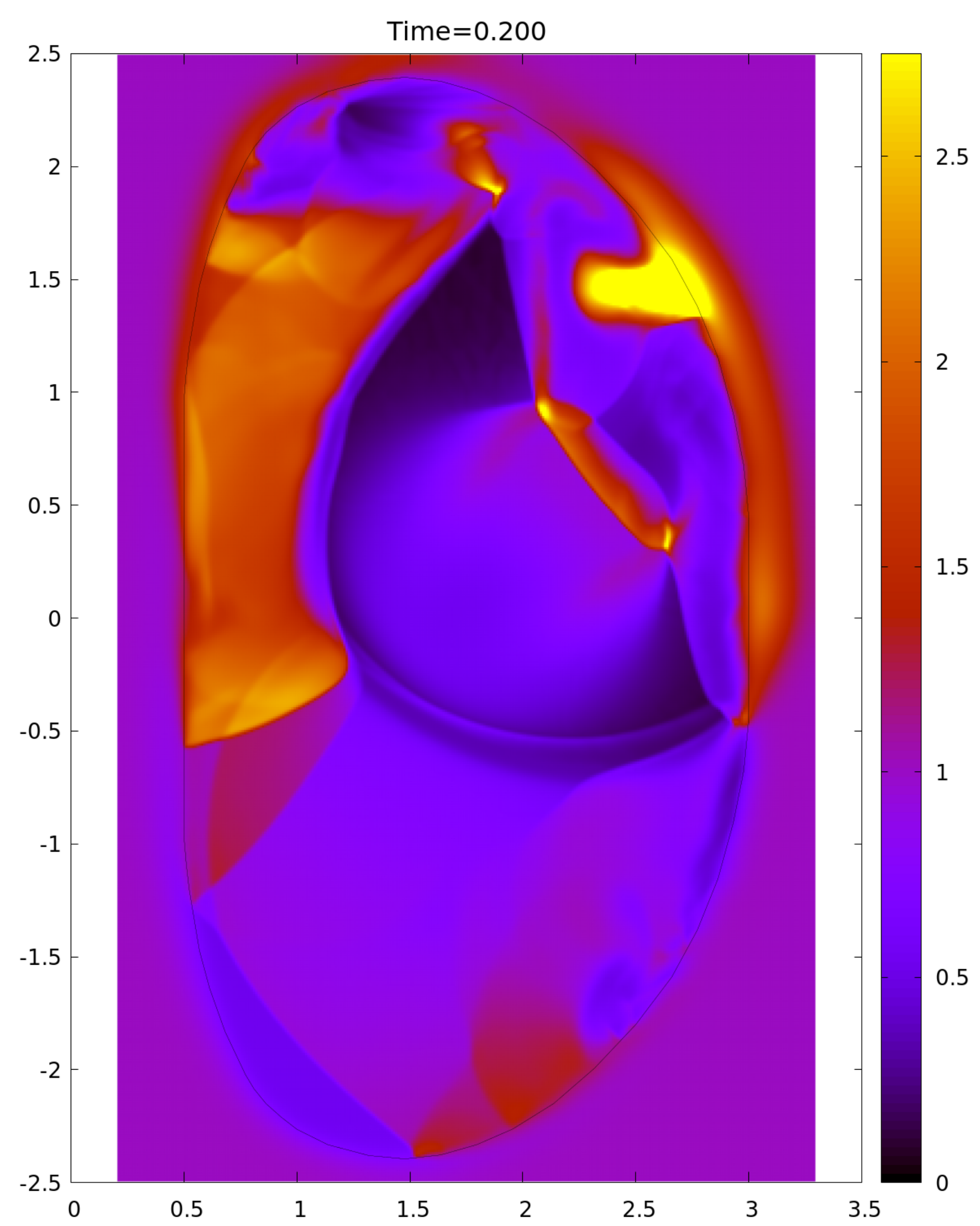

Figure 1: B_z of plasma in a cylindrical equilibrium test within a tokamak-shaped vessel, outlined by a thin black line. It is taken after the core plasma hitting $\eta_w=0.05$ resistive wall. Interaction between plasma and wall result in some penetration on wall.

Consistent Equation

After discussion on boundary condition consistency, a consistent equation across conditions is proposed

$$\nabla^2\mathbf{B}=\frac{nq_e^2}{m\nu}\frac{\partial\mathbf{B}}{\partial t}-\frac{nq_e^2}{m\nu}e^{-\nu t}\frac{\partial\mathbf{B}}{\partial t}$$

References and Acknowledgements

[1] J. Vides et al. *ESAIM: Proceedings*. Vol. 43. EDP Sciences. 2013.

[2] S. K. Sambasivan and H. UdayKumar. *AIAA journal* 47.12 (2009).

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