Theoretische Physik III

Quantenmechanik

Vorlesung von Prof. Dr. Andreas Buchleitner im Sommersemester 2019

Markus Österle   Damian Lanzenstiel

29. April 2019

Inhaltsverzeichnis

0 Einleitung **2**
0.1 Wichtige Infos 2
0.2 Programm 2
0.3 Litaratur 2

1 Quantenmechanik - Intro **4**
1.1 Wave-particle duality at the double-slit 5
1.1.1 Consequences and terminology 5
1.2 Measure, filtering and spectral decomposition 6

Kapitel 0

Einleitung

0.1 Wichtige Infos

Professor Andreas Buchleitner Zi. 901
`abu@uni-freiburg.de`
`buchleitner_office@physik.uni-freiburg.de`

Sekretäre Gislinde Bühler & Susanne Trantke Zi. 804

Übungsleiter Eduardo Carnio, Zi. 910
`eduardo.carnio@physik.uni-freiburg.de`

ILLIAS Theorie III: Password: `TPIIIss19`

Klausur 15. Juli 13:00 - 16:00 Uhr im großen Hörsaal

0.2 Programm

- Proseminar (BSc) zus. mit MSc-Seminar
  QM für Liebhaber & Interpretation of QM
- Kolloquium montags 17:15 Uhr 27. Mai Göttinger Erklärung, CF v. W.
- 23.-27. September DPG Fall Meeting, Quantum Sciences and IT

0.3 Litaratur

(auch auf ILLIAS gelistet)

- C. Cohen-Tannodji, B. Diu, F. Lafoë, Mécauique quantique, F,D,E, Vol I + II
- O. Hittmeier Lehrbuch d. Quantenmechanik, Thienig 1972
- B. - G. Engert, Lectures on quantum mechanics, I - IV, World Scientific 2006
- M. Bartelmann et al, Theoretische Physik, Springer 2015
- J.J. Sakurai, Modern Quantumechanics, Addison-Wesley 1995
- A. Peres, Quantum Theory: Concepts and Methods, Kluwer 1995
- M.A. Nielson, I. L. Chang, Quantum Computation & Quantum Information, Cambridge University Press 2000

- Landau & Lifschitz, Lehrbuch der Theoretischen Physik Bd. III

Formelsammlung: Bronstein & Sememdiciev. Taschenbuch d. Mathematik

Kapitel 1

Quantenmechanik - Intro

Quantenmechanik (QM) beschreibt den Mikrokosmos (im Gegensatz zum Makrokosmos).
→ im CD-Player
→ im Handy
→ Kernspin
→ Zeitstandards

QM ist „merkwürdig“ insofern, als anthropomorpher Anschauung unangepasst. ⇒ Sie sorgt noch heute für hitzige und kontroverse Debatten.

→ siehe Podcasts PI - Kolloquium, z.B. Nicoles Gisin 15.04.2009, Reinhard Werner 24.12.2007

→ mathematischer Rahmen relativ einfach, doch Interpretation schwierig
⇒ Feynman: „Shut up and calculate!“

Historische Genese: Wie (fast?) alle physikalischen Theorien aus experimenteller Evidenz, die mit der„klassischen“ Theorie nicht vereinbar war.

Aus **theoretischer „Notlage“ angesichts bestehender Experimente:**
Balmer-Linien (1885), Franck-Hertz-Versuch (1913), Photoeffekt (Hallweds 1888 & Einstein 1905), Schwarzkörperspektrum (Planck 1900), Compton-Effekt (1921), Kernspaltung (Halm, Meitner und Strassmann 1939), Stern-Gerlach-Versuch (1921).

Große Namen: N. Bohr, W. Heisenberg, E. Schrödinger, M. Born, John v. Neumann, A. Sommerfeld, L. de Broglie, P. Dirac, W. Pauli, L. Szilárd, R. Oppenheimer, Gamow, Siegelt, Hellmann, Etore Majorana.

Zu Majorana: Leonardo Sciascia: La Scomparsa di Majorana (Das Verschwinden des Majorana).

Buchempfehlung: Richard Rhodes: Die Atombombe oder die Geschichte des 8. Schöpfungstages

Weitere Quantenmechaniker: E. Teller, A. Sacharov, L. Landau, J. Belt, M. Gutzwiller.

Korrespondenzprinzip: Wie korrespondieren die QM-Theorien mit den klassischen Theorien? Wie sieht der Übergang vom diskreten zu einem kontinuierlichen Spektrum aus?

Beispiel: Atommodell mit quantisierten Elektronen-Orbitalen von Bohr und dem Klassischeren Modell von Rutherford und kontinuierlichen Kepler-Orbitalen.
Die Energieniveaus eines Wasserstoff Atoms sind: $E = \frac{1}{2n^2}$. Daraus folgt, dass höhere Energieniveaus immer näher aneinander liegen. Die Einergiedifferenzen $E_{n+1} - E_n \sim \hbar\omega_{\text{Kepler}}$ werden also immer geringer. Die Umlauffrequenz kann also mit zunehmender Hauptquantenzahl immer genauer bestimmbar.

1.1 Wave-particle duality at the double-slit

⋆hier fehlt eine Grafik⋆

caption: Double-slit Experiment by Young (1803)

For Waves the complex amplitudes $E_1(x)$ and $E_2(x)$ coming from slits 1 and 2 and arriving at point x on the screen add up

$$E(x) = E_1(x) + E_2(x) \tag{1.1}$$

The corresponding intensity reads:

$$I(x) \propto |E(x)|^2 = \underbrace{|E_1(x)|^2 + |E_2(x)|^2}_{\substack{\text{“classical” intensities} \\ \text{e.g. with Tennis balls} \\ \textbf{ladder}\text{ contribution}}} + \underbrace{2\Re\left(E_1^*(x)E_2(x)\right)}_{\substack{\text{intereference term} \\ \sim\cos(\varphi_2(x)-\varphi_1(x)) \\ \text{contains phase information} \\ \textbf{cross}\text{ term}}} \tag{1.2}$$

⋆hier fehlt eine Grafik⋆

$$\begin{matrix} E_1 - E_1^* \\ E_2 - E_2^* \end{matrix} \qquad \begin{matrix} E_1 \\ E_2 \end{matrix} \times \begin{matrix} E_1^* \\ E_2^* \end{matrix} \qquad \Rightarrow \qquad |E_1 + E_2|^2 = (E_1 + E_2)(E_1^* + E_2^*) = |E_1|^2 + |E_2|^2 + 2\Re|E_1^* E_2|$$

→ **Light is a wave phenomenon**

⋆hier fehlt eine Grafik⋆

If we make the source weaker and weaker and have a sufficiently sensitive screen/detector, we observe the arrival of **single point-like** photons on the screen (photo-electric effect Einstein (1905) (→ corpusculan hypothesis))

By making the source sufficiently weak, we can ensure that at most 1 photon is present in the interferometer at a given time. → no possible interaction between photons!

If we **integrate** over many single detection events, we recover the interference pattern.

We can make statistical predictions about the position of individual detection events (the integrated signal forms a probabilistic distribution) but the individual photons clearly don't have a deterministic trajectory (otherwise no interference).

Summary

- Upon detection, light behaves like an assembly of particles.
- The density of detection events reproduces the predictions of the wave picture (classical electromagnetism).
- We cannot explain the appearance of an interference pattern if we treat the photons as classical particles. Each photon goes through both slits 1 and 2: two classically exclusive alternatives.

1.1.1 Consequences and terminology

The agreement of the probability distribution for individual detections events with the predictions of optics (classical field theory) justifies referring to $E(x, t)$ as a **probability amplitude** of the photons. $|E(x, t)|^2$ is the corresponding **probability density** (normalized, real, other attributes . . .) for detection at point x and time t. Later on, we use $\Psi(x, t)$ instead of $E(x, t)$ for the **wavefunction**.

The appearance of both wave and particle properties in the behavior of microscopic objects is known as **wave-particle duality**.

This raises the question of the "**critical scale**" below which these phenomenon take place and above which our classical representations hold.

Remarks:

I) In optics, interference follows from the superposition principle, which is a consequence of the linearity of the field equations.

Correspondingly the equations of QM are also linear and the superpositions principle applies.

II) The probabilistic predictions of the wave picture can only be accessed by accumulating many individual detection events, i.e. by repeating the same experiment.

Individual events are not predictable, QM only makes statistical predictions.

III) These is a strong effect to push this critical scale function into the macroscopic world (e.g. experiments M. Arndt, Vienna interference of C_{60} molecules).

1.2 Measure, filtering and spectral decomposition

⋆hier fehlt eine Grafik⋆ caption: Linearly polarized beam of monochromatic light

Classical field:

$$\boldsymbol{E}(\boldsymbol{r},t) = E_0 \hat{e}_\varphi e^{i(kz-\omega t)} \tag{1.3}$$

$$\begin{aligned}\boldsymbol{E}(\boldsymbol{r},t) = & \; E_0 \cos\theta \boldsymbol{e}_x e^{i(kz-\omega t)} \\ & + E_0 \sin\theta \boldsymbol{e}_y e^{i(kz-\omega t)}\end{aligned}$$

After the filter $\boldsymbol{E}(\boldsymbol{r},t) = E_0 \cos\theta \boldsymbol{e}_x e^{i(kz-\omega t)}$
Intensity:

$$I' \propto |E'|^2 \qquad I \propto |E|^2$$

Malus' law

$$I' = I \cos^2\theta \tag{1.4}$$

What happens if we weaken the source to obtain single photons?
→ 2 possible options:

- a single photons passes completely (click on detector): event "1"
- or it does not pass at all (no click): event "0"

For each photon, the outcome cannot be predicted with certainty.

Upon averaging over a large number of photons, the fraction which makes it through is:

$$\frac{N_1}{n_1 + n_0} \to \cos^2\theta$$

in accordance with Malus' law (1.4).

E.g.: `10010001110101`

Special casses:

- $\theta = 0°$ $\rightarrow$ all photons go through: `1111...1`
- $\theta = 90°$ $\rightarrow$ no photon goes through: `0000...0`

$\Rightarrow$ in this case the output is certain but experiment must be repeated many times to prove that this is the case.

In these cases, we say that the photon finds itself in an **eigenstate**. One state: $|1\rangle$ for $\hat{e}_x$ polarized and one state $|0\rangle$ for $\hat{e}_y$ polarized which is associated with the particular outcomes for **eigenvalues** 1 and 0.