Theoretische Physik II
Elektrodynamik

Vorlesung von Prof. Dr. Michael Thoss im Wintersemester 2018

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Kapitel 1

Mathe

1.1 Grundbegriffe

Ladung

$$\text{diskrete Ladungsverteilung} \qquad Q = \sum_{i=1}^{n} q_i$$

$$\text{kontinuierliche Ladungsverteilung} \qquad Q = \int_V \rho(\boldsymbol{r})\ \mathrm{d}^3 r$$

$$\text{Punktladung} \qquad \rho(\boldsymbol{r}) = q\delta(\boldsymbol{r} - \boldsymbol{r}_0)$$

$$\text{Poisson-Gleichung} \qquad \Delta\Phi(\boldsymbol{r}) = -\frac{1}{\varepsilon_0}\rho(\boldsymbol{r})$$

Coulomb'sches Gesetz

$$\text{zwei Punktladungen} \qquad \boldsymbol{F}_{12} = \frac{1}{4\pi\varepsilon_0}\, q_1 q_2\, \frac{\boldsymbol{r}_1 - \boldsymbol{r}_2}{|\boldsymbol{r}_1 - \boldsymbol{r}_2|^3} = -\boldsymbol{F}_{21}$$

$$n \text{ Punktladungen} \qquad \boldsymbol{F}_1 = \frac{1}{4\pi\varepsilon_0}\, q_1 \sum_{j=2}^{n} q_j\, \frac{\boldsymbol{r} - \boldsymbol{r}_j}{|\boldsymbol{r} - \boldsymbol{r}_j|^3}$$

Elektrisches Feld

$$\text{im bel. Raumpunkt} \qquad \boldsymbol{E}(\boldsymbol{r}) = \frac{q}{4\pi\varepsilon_0}\, \frac{\boldsymbol{r} - \boldsymbol{r}_0}{|\boldsymbol{r} - \boldsymbol{r}_0|^3}$$

$$\text{diskrete Ladungsverteilung} \qquad \boldsymbol{E}(\boldsymbol{r}) = \frac{1}{4\pi\varepsilon_0} \sum_{j=2}^{n} q_j\, \frac{\boldsymbol{r} - \boldsymbol{r}_j}{|\boldsymbol{r} - \boldsymbol{r}_j|^3}$$

$$\text{kontinuierliche Ladungsverteilung} \qquad \boldsymbol{E}(\boldsymbol{r}) = \frac{1}{4\pi\varepsilon_0} \int \rho(\boldsymbol{r}')\, \frac{\boldsymbol{r} - \boldsymbol{r}'}{|\boldsymbol{r} - \boldsymbol{r}'|^3}\ \mathrm{d}^3 r'$$

$$\text{Gradientenfeld} \qquad \boldsymbol{E}(\boldsymbol{r}) = -\nabla\, \varphi(\boldsymbol{r})$$

1.2 Einführung

Dirac'sche Delta-Funktion

Definition

$$\int_V \delta(\boldsymbol{r}-\boldsymbol{r}_0)\,\mathrm{d}^3 r := \begin{cases} 1\,, & r_0 \in V \\ 0\,, & \text{sonst} \end{cases}$$
$$\delta(\boldsymbol{r}-\boldsymbol{r}_0) = 0 \qquad \forall\, \boldsymbol{r} \neq \boldsymbol{r_0}$$

Bemerkung: Die δ-Funktion ist keine Funktion im üblichen mathematischen Sinne. Man bezeichnet sie deshalb als **uneigentliche Funktion** oder als **Distribution**. Heuristisch:

$$\delta(x) = \begin{cases} +\infty\,, & x = 0 \\ 0\,, & x \neq 0 \end{cases}$$
$$\int_{-\infty}^{\infty} \delta(x)\,dx = 1$$

Formeln

$$\int_a^b f(x)\,\delta(x-x_0)\,dx = \begin{cases} f(x_0)\,, & a < x_0 < b \\ \frac{1}{2} f(x_0)\,, & x_0 = a \vee b \\ 0 & \text{sonst} \end{cases}$$
$$\text{“}\ f(x)\,\delta'(x-x_0) = -f'(x)\,\delta(x-x_0)\ \text{”} \qquad \text{(heuristisch)}$$
$$\delta(x-x_0) = \frac{d}{dx}\Theta(x-x_0) \qquad (\Theta \quad \text{sei die Stufenfunktion})$$

Mehrdimensionale Delta-Funktion

Kartesisch (x,y,z) $\quad \delta(\boldsymbol{r}-\boldsymbol{r}_0) = \delta(x-x_0)\,\delta(y-y_0)\,\delta(z-z_0)$

Kugel (r,θ,φ) $\quad \delta(\boldsymbol{r}-\boldsymbol{r}_0) = \dfrac{1}{r_0^2 \sin\theta_0}\,\delta(r-r_0)\,\delta(\theta-\theta_0)\,\delta(\varphi-\varphi_0)$

Zylinder (ρ,ϕ,z) $\quad \delta(\boldsymbol{r}-\boldsymbol{r}_0) = \dfrac{1}{\rho_0}\,\delta(\rho-\rho_0)\,\delta(\varphi-\varphi_0)\,\delta(z-z_0)$

Produktformeln

f, g sind skalare Felder, $\boldsymbol{F}, \boldsymbol{G}$ sind vektor Felder:

$$\begin{aligned} \boldsymbol{\nabla}(fg) &= f\boldsymbol{\nabla}(g) + g\boldsymbol{\nabla}(f) \\ \boldsymbol{\nabla}\cdot(f\boldsymbol{G}) &= f\boldsymbol{\nabla}\cdot(\boldsymbol{G}) + \boldsymbol{G}\cdot\boldsymbol{\nabla}(f) \\ \boldsymbol{\nabla}(\boldsymbol{F}\times\boldsymbol{G}) &= \boldsymbol{G}\cdot\boldsymbol{\nabla}\times(\boldsymbol{F}) - \boldsymbol{F}\cdot\boldsymbol{\nabla}\times(\boldsymbol{G}) \\ \boldsymbol{\nabla}\times(f\boldsymbol{G}) &= f\boldsymbol{\nabla}\times(\boldsymbol{G}) - \boldsymbol{G}\times\boldsymbol{\nabla} \end{aligned}$$

Identitäten

$$\begin{aligned} \boldsymbol{\nabla}\times(\boldsymbol{\nabla}\boldsymbol{F}) &= \boldsymbol{\nabla}(\boldsymbol{\nabla}\cdot\boldsymbol{F}) - \Delta\boldsymbol{F} \\ \boldsymbol{\nabla}\times(\boldsymbol{\nabla}f) &= \boldsymbol{0} \\ \boldsymbol{\nabla}\cdot(\boldsymbol{\nabla}\times\boldsymbol{F}) &= 0 \\ \boldsymbol{\nabla}\times(\boldsymbol{a}\times\boldsymbol{\nabla}f) &= \boldsymbol{a}\Delta f - \boldsymbol{\nabla}(\boldsymbol{a}\cdot\boldsymbol{\nabla}f) \end{aligned}$$

Kugelkoordinaten

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} \rho \sin\phi \cos\theta \\ \rho \sin\phi \sin\theta \\ \rho \cos\phi \end{pmatrix}, \qquad \int\limits_{-\infty}^{+\infty} f(\boldsymbol{r}) \ \mathrm{d}r = \int\limits_0^{2\pi} \int\limits_0^{\pi} \int\limits_0^{\infty} f(r,\phi,\theta) r^2 \sin\phi \ \mathrm{d}r \mathrm{d}\phi \mathrm{d}\theta$$

Gradienten eines skalaren Feldes

$$\boldsymbol{\nabla} g(\boldsymbol{r}) = \sum_i \boldsymbol{e}_i \frac{1}{\left|\frac{\partial \boldsymbol{r}}{\partial u_i}\right|} \frac{\partial}{\partial u_i} g(\boldsymbol{r})$$

1.3 Integralsätze

Gaußscher Satz

$$\int_V \boldsymbol{\nabla} \cdot \boldsymbol{E}(\boldsymbol{r}) \ \mathrm{d}^3 r = \oint_F \boldsymbol{E} \ \mathrm{d}\boldsymbol{f}$$

Stokes Satz

$$\int_F \boldsymbol{\nabla} \times \boldsymbol{a} \ \mathrm{d}\boldsymbol{f} = \int_{\partial F} \boldsymbol{a} \ \mathrm{d}\boldsymbol{r}$$

Greensche Identitäten

$$\int_V \boldsymbol{\nabla} g(\boldsymbol{r}) \cdot \boldsymbol{\nabla} h(\boldsymbol{r}) + g(\boldsymbol{r}) \Delta h(\boldsymbol{r}) \ \mathrm{d}^3 r = \oint_{\partial V} g(\boldsymbol{r}) \boldsymbol{\nabla} h(\boldsymbol{r}) \ \mathrm{d}\boldsymbol{f}$$

$$\int_V g(\boldsymbol{r}) \Delta h(\boldsymbol{r}) g(\boldsymbol{r}) - h(\boldsymbol{r}) \Delta g(\boldsymbol{r}) \ \mathrm{d}^3 r = \oint_{\partial V} g(\boldsymbol{r}) \boldsymbol{\nabla} h(\boldsymbol{r}) - h(\boldsymbol{r}) \boldsymbol{\nabla} g(\boldsymbol{r}) \ \mathrm{d}\boldsymbol{f}$$