Theoretische Physik II
Elektrodynamik

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Kapitel 1

Mathe

1.1 Einführung

Dirac'sche Delta-Funktion

Definition

$$\int_V \mathrm{d}^3 r \; \delta(\boldsymbol{r} - \boldsymbol{r_0}) := \begin{cases} 1\,, & r_0 \in V \\ 0\,, & \text{sonst} \end{cases}$$

$$\delta(\boldsymbol{r} - \boldsymbol{r_0}) = 0 \qquad \forall\, \boldsymbol{r} \neq \boldsymbol{r_0}$$

Bemerkung: Die δ-Funktion ist keine Funktion im üblichen mathematischen Sinne. Man bezeichnet sie deshalb als **uneigentliche Funktion** oder als **Distribution**. Heuristisch:

$$\delta(x) = \begin{cases} +\infty\,, & x = 0 \\ 0\,, & x \neq 0 \end{cases}$$

$$\int_{-\infty}^{\infty} \delta(x)\, dx = 1$$

Formeln

$$\int_a^b f(x)\,\delta(x - x_0)\,dx = \begin{cases} f(x_0)\,, & a < x_0 < b \\ \frac{1}{2} f(x_0)\,, & x_0 = a \vee b \\ 0 & \text{sonst} \end{cases}$$

$$\text{“}\; f(x)\,\delta'(x - x_0) = -f'(x)\,\delta(x - x_0)\;\text{”} \qquad \text{(heuristisch)}$$

$$\delta(x - x_0) = \frac{d}{dx}\Theta(x - x_0) \qquad (\Theta \;\text{ sei die Stufenfunktion})$$

Mehrdimensionale Delta-Funktion

Kartesisch (x,y,z) $\quad \delta(\boldsymbol{r} - \boldsymbol{r_0}) = \delta(x - x_0)\,\delta(y - y_0)\,\delta(z - z_0)$

Kugel (r, θ, φ) $\quad \delta(\boldsymbol{r} - \boldsymbol{r_0}) = \dfrac{1}{r_0^2 \sin\theta_0}\,\delta(r - r_0)\,\delta(\theta - \theta_0)\,\delta(\varphi - \varphi_0)$

Zylinder (ρ, ϕ, z) $\quad \delta(\boldsymbol{r} - \boldsymbol{r_0}) = \dfrac{1}{\rho_0}\,\delta(\rho - \rho_0)\,\delta(\varphi - \varphi_0)\,\delta(z - z_0)$

Produktformeln

f, g sind skalare Felder, $\boldsymbol{F}, \boldsymbol{G}$ sind vektor Felder:

$$
\begin{aligned}
\boldsymbol{\nabla}(fg) &= f\boldsymbol{\nabla}(g) + g\boldsymbol{\nabla}(f) \\
\boldsymbol{\nabla} \cdot (f\boldsymbol{G}) &= f\boldsymbol{\nabla} \cdot (\boldsymbol{G}) + \boldsymbol{G} \cdot \boldsymbol{\nabla}(f) \\
\boldsymbol{\nabla}(\boldsymbol{F} \times \boldsymbol{G}) &= \boldsymbol{G} \cdot \boldsymbol{\nabla} \times (\boldsymbol{F}) - \boldsymbol{F} \cdot \boldsymbol{\nabla} \times (\boldsymbol{G}) \\
\boldsymbol{\nabla} \times (f\boldsymbol{G}) &= f\boldsymbol{\nabla} \times (\boldsymbol{G}) - \boldsymbol{G} \times \boldsymbol{\nabla}
\end{aligned}
$$

Identitäten

$$
\begin{aligned}
\boldsymbol{\nabla} \times (\boldsymbol{\nabla}\boldsymbol{F}) &= \boldsymbol{\nabla}(\boldsymbol{\nabla} \cdot \boldsymbol{F}) - \Delta\boldsymbol{F} \\
\boldsymbol{\nabla} \times (\boldsymbol{\nabla} f) &= \boldsymbol{0} \\
\boldsymbol{\nabla} \cdot (\boldsymbol{\nabla} \times \boldsymbol{F}) &= 0 \\
\boldsymbol{\nabla} \times (\boldsymbol{a} \times \boldsymbol{\nabla} f) &= \boldsymbol{a}\Delta f - \boldsymbol{\nabla}(\boldsymbol{a} \cdot \boldsymbol{\nabla} f)
\end{aligned}
$$

Kugelkoordinaten

$$
\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} \rho \sin\phi \cos\theta \\ \rho \sin\phi \sin\theta \\ \rho \cos\phi \end{pmatrix}, \qquad \int\limits_{-\infty}^{+\infty} f(\boldsymbol{r})\ \mathrm{d}r = \int\limits_0^{2\pi} \int\limits_0^{\pi} \int\limits_0^{\infty} f(r, \phi, \theta) r^2 \sin\phi\ \mathrm{d}r\mathrm{d}\phi\mathrm{d}\theta
$$

Gradienten eines skalaren Feldes

$$
\boldsymbol{\nabla} g(\boldsymbol{r}) = \sum_i \boldsymbol{e}_i \frac{1}{\left|\frac{\partial \boldsymbol{r}}{\partial u_i}\right|} \frac{\partial}{\partial u_i} g(\boldsymbol{r})
$$

1.2 Integralsätze

Gaußscher Satz

$$
\int_V \boldsymbol{\nabla} \cdot \boldsymbol{E}(\boldsymbol{r})\ \mathrm{d}^3 r = \oint_F \boldsymbol{E}\ \mathrm{d}\boldsymbol{f}
$$

Stokes Satz

$$
\int_F \boldsymbol{\nabla} \times \boldsymbol{a}\ \mathrm{d}\boldsymbol{f} = \int_{\partial F} \boldsymbol{a}\ \mathrm{d}\boldsymbol{r}
$$

Greensche Identitäten

$$
\int_V \boldsymbol{\nabla} g(\boldsymbol{r}) \cdot \boldsymbol{\nabla} h(\boldsymbol{r}) + g(\boldsymbol{r})\Delta h(\boldsymbol{r})\ \mathrm{d}^3 r = \oint_{\partial V} g(\boldsymbol{r})\boldsymbol{\nabla} h(\boldsymbol{r})\ \mathrm{d}\boldsymbol{f}
$$

$$
\int_V g(\boldsymbol{r})\Delta h(\boldsymbol{r}) g(\boldsymbol{r}) - h(\boldsymbol{r})\Delta g(\boldsymbol{r})\ \mathrm{d}^3 r = \oint_{\partial V} g(\boldsymbol{r})\boldsymbol{\nabla} h(\boldsymbol{r}) - h(\boldsymbol{r})\boldsymbol{\nabla} g(\boldsymbol{r})\ \mathrm{d}\boldsymbol{f}
$$