



# Two-level Implementation

CS207 Chapter 4

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# NAND and NOR implementation



- Digital circuits are frequently constructed with NAND or NOR gates rather than with AND and OR gates. 逻辑电路一般用 而不是
  - NAND and NOR gates are easier to fabricate with electronic components.
  - They are the basic gates used in all IC digital logic families.
- Rules and procedures have been developed for the conversion from Boolean functions given in terms of AND, OR, and NOT into equivalent NAND and NOR logic diagrams.



# NAND circuits

- The NAND gate is said to be a universal gate.
- A convenient way to implement a Boolean function with NAND gates:
  - Obtain the simplified Boolean function in terms of Boolean operators;
  - Convert the function to NAND logic.
- Recall that:

$$A \text{ --- } \boxed{\text{NAND}} \text{ --- } F = A' \quad \quad A \text{ --- } \boxed{\text{NAND}} \text{ --- } \boxed{\text{NAND}} \text{ --- } F = AB$$

$$\begin{array}{c} A \text{ --- } \boxed{\text{NAND}} \\ B \text{ --- } \boxed{\text{NAND}} \end{array} \text{ --- } \boxed{\text{NAND}} \text{ --- } F = (A'B')' = A + B$$

$$\begin{array}{c} A \text{ --- } \bullet \\ B \text{ --- } \bullet \end{array} \begin{array}{c} \text{--- } \boxed{\text{NAND}} \\ \text{--- } \boxed{\text{NAND}} \end{array} \text{ --- } \boxed{\text{NAND}} \text{ --- } F = A \oplus B$$

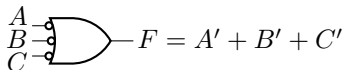
# NAND circuits

- To facilitate the conversion to NAND logic, it is convenient to define an alternative graphic symbol for the gate.

AND-invert:



Invert-OR:



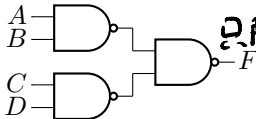
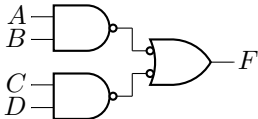
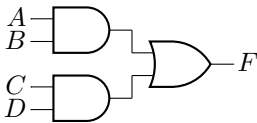


## NAND circuits

- The implementation of Boolean functions with NAND gates requires that the functions be in sum-of-products form.

两次进相对于值没变

- Take  $F = AB + CD$  as an example:



只用一种门大大简化

- $F = AB + CD = ((AB)'(CD'))'$  according to DeMorgan property.

# NAND circuits

- Example: Implement the following Boolean function with NAND gates:

$$F(x, y, z) = \sum(1, 2, 3, 4, 5, 7).$$

$x \backslash yz$	00	01	11	10
0		1	1	1
1	1	1	1	

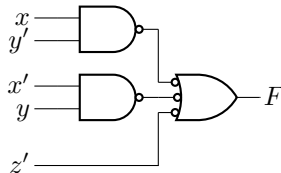
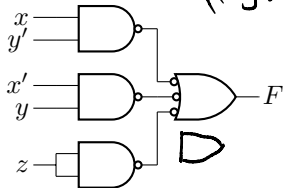
- $F = xy' + x'y + z.$

# NAND circuits

- Example: Implement the following Boolean function with NAND gates:

$$F = xy' + x'y + z.$$

$$((xy')' + (x'y)' + z')'$$



# NAND circuits

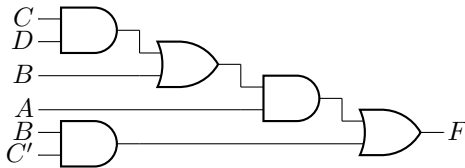


- A Boolean function can be implemented with two levels of NAND gates.
  - ① Simplify the function and express it in **sum-of-products form**. 先简化方程.
  - ② Draw **a NAND gate for each product term** of the expression that has at least two literals. The inputs to each NAND gate are the literals of the term. This procedure produces a group of first-level gates. 再对每个 Product 做门 (1级门)
  - ③ Draw **a single gate** using the AND-invert or the invert-OR graphic symbol **in the second level**, with inputs coming from outputs of first-level gates. 对 Sum 用 And; (2级门)
  - ④ A term with **a single literal requires an inverter** in the first level. However, if the single literal is complemented, it can be connected directly to an input of the second level NAND gate. 如果 Product 只有一个元素, 则可以直操作接入 2 级门



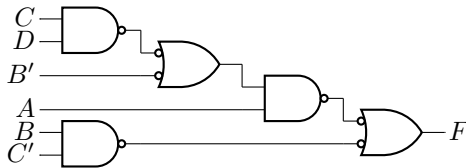
# Multilevel NAND circuits

- The standard form of expressing Boolean functions results in a two-level implementation.
  - There are occasions when the design of digital systems results in gating structures with three or more levels.
  - Example:  $F = A(CD + B) + BC'$ .



# Multilevel NAND circuits

- The standard form of expressing Boolean functions results in a two-level implementation.
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# Multilevel NAND circuits



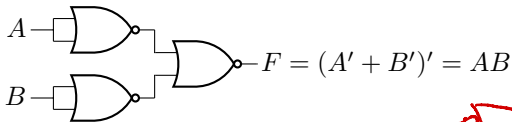
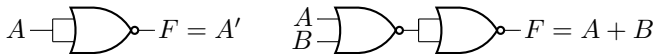
- The general procedure for converting a multilevel AND-OR diagram into an all-NAND diagram using mixed notation is as follows:
  - ① Convert all AND gates to NAND gates with AND-invert graphic symbols.
  - ② Convert all OR gates to NAND gates with invert-OR graphic symbols.
  - ③ Check all the bubbles in the diagram. For every bubble that is not compensated by another small circle along the same line, insert an inverter (a one-input NAND gate) or complement the input literal.



# NOR circuits

NOR 操作是 NAND 操作的对偶

- The NOR operation is the dual of the NAND operation.
- All procedures and rules for NOR logic are the duals of the corresponding procedures and rules developed for NAND logic. NOR 就是为了 NAND 而出现

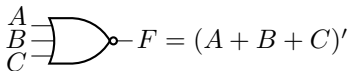


# NOR circuits

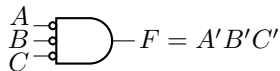


- To facilitate the conversion to NOR logic, it is convenient to define an alternative graphic symbol for the gate.

OR-invert:



Invert-AND:





# NOR circuits

两级门

- A two-level implementation with NOR gates requires that the function be simplified into product-of-sums form.
- Change the OR gates to NOR gates with OR-invert graphic symbols and the AND gate to a NOR gate with an invert-AND graphic symbol.

圆圈在前面

# Non-degenerate forms



- It will be instructive from a theoretical point of view to find out how many two-level combinations of gates are possible. 找到两级门的个数.
- We consider four types of gates: AND, OR, NAND, and NOR.
  - There are 16 possible combinations of two-level forms.  $2^4$
- Eight of these combinations are said to be *degenerate* forms. 有八种组合称为简并形式
  - They degenerate to a single operation.
  - Example: AND in the first level and second level degenerates to an AND of all inputs.
- The remaining eight nondegenerate forms produce an implementation in sum-of-products form or product-of-sums form.
  - 1) AND-OR 2) OR-AND 3) NAND-NAND 4) NOR-NOR
  - 5) NOR-OR 6) NAND-AND 7) OR-NAND 8) AND-NOR

一共有16种组合, 其中8种  
可以直接用一个门解决或化简

# AND-OR-INVERT implementation

- The two forms, NAND-AND and AND-NOR, are equivalent.
  - Both perform the AND-OR-INVERT function.
  - Example:  $F = (AB + CD + E)'$ .
- An AND-OR implementation requires an expression in sum-of-products form.
- The AND-OR-INVERT implementation is similar, except for the inversion. 取反
  - If the complement of the function is simplified into sum-of-products form (by combining the 1's in the map), it will be possible to implement  $F$  with the AND-OR part of the function.



# OR-AND-INVERT implementation



- The two forms, OR-NAND and NOR-OR, are equivalent.
  - Both perform the OR-AND-INVERT function.
  - Example:  $F = [(A + B)(C + D)E]'$ .
- The AND-OR-INVERT implementation requires an expression in product-of-sums form.



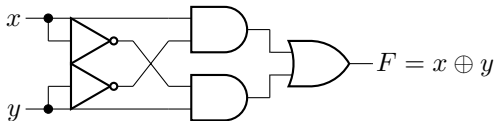
# Exclusive-OR function

- Exclusive-OR, **XOR**:  $x \oplus y = xy' + x'y$ .  $A \Rightarrow B \Rightarrow \text{D-}\bar{F}$
- Exclusive-NOR, **XNOR** or **equivalency**:  $(x \oplus y)' = xy + x'y'$ . 可以用逻辑定义
- The following identities apply to the XOR operation:
  - $x \oplus 0 = x$ .
  - $x \oplus 1 = x'$ .
  - $x \oplus x = 0$ .
  - $x \oplus x' = 1$ .
  - $x \oplus y' = x' \oplus y = (x \oplus y)'$ .

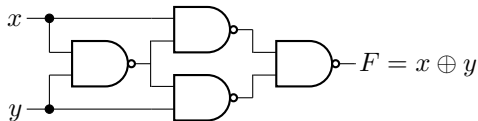
# XOR



- XOR is hard to fabricate, so it is typically constructed by other gates.
  - $(x' + y')x + (x' + y')y = xy' + xy' = x \oplus y$ .



- Or use NAND gates:



- The first NAND gate performs the operation  $(xy)' = x' + y'$ .
- The other two-level NAND circuit produces the sum of products.



# XOR

只有有限数量的布尔函数可以用异或运算表示。

- Only a limited number of Boolean functions can be expressed in terms of XOR operations.
- Particularly useful in arithmetic operations and error detection/correction circuits. 适用算术运算和错误检测



## Odd function

- The XOR operation with three or more variables can be converted into an ordinary Boolean function by replacing the  $\oplus$  with its equivalent Boolean expression.

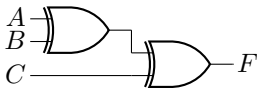
$$\begin{aligned} A \oplus B \oplus C &= (AB' + A'B)C' + (AB + A'B')C \\ &= AB'C' + A'BC' + ABC + A'B'C \\ &= \sum_{0 \leq i \leq 7} (1, 2, 4, 7) \end{aligned}$$

- The three-variable XOR function is equal to 1 if only one variable is equal to 1 or if all three variables are equal to 1.
  - An **odd** number of variables are equal to 1.
  - Odd function.*

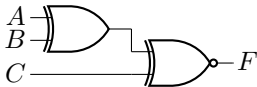
$$A \oplus B \oplus C \oplus D$$

# Odd function

- The three-input odd function is implemented by means of two-input XOR gates.



- Even function* can also be implemented:



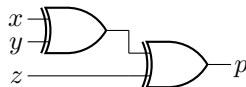
# Parity generation and checking

- XOR functions are very useful in systems requiring error detection and correction codes.
  - A parity bit is an extra bit included with a binary message to make the number of 1's either odd or even.
- The circuit that generates the parity bit in the transmitter is called a *parity generator*. 奇偶校验发射器
- The circuit that checks the parity in the receiver is called a *parity checker*. 奇偶校验器

# Parity generation and checking

- Consider a three-bit message to be transmitted together with an even-parity bit.

$x$	$y$	$z$	Parity bit $p$
0	0	0	0
0	0	1	1
0	1	0	1
0	1	1	0
1	0	0	1
1	0	1	0
1	1	0	0
1	1	1	1

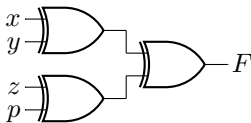


- $p$  constitutes an odd function.



# Parity generation and checking

- The three bits in the message, together with the parity bit, are transmitted to their destination.
- The four bits received must have an even number of 1's with even parity.



偶数个1

$$x \oplus y \oplus z \oplus p$$