

CZ2003 Computer Graphic and Visual

Tutorial Answer

CZ2003 Tutorial 2 (2022/23, Semester 1)

Mathematical functions in computer graphics

1. Give a definition of mathematical function.
2. What ways of defining mathematical functions do you know?
3. Given an explicit function $y = \sin(x) + \cos(x)$, propose how to convert it to the respective parametric functions $x = f_1(t)$ $y = f_2(t)$?
4. (i) Given parametric functions $x = \sin^2(t)$ and $y = \cos(t)$, obtain the respective implicit function $f(x,y) = 0$.

(ii) Given parametric functions $x = 2 + 3t$ and $y = 3 + t$, obtain the respective implicit function $f(x,y) = 0$.

Q1. Mathematics function associate one quantity, the argument of the function (input) with another quantity, the value of the function (output)

A function assigns exactly one output to each input.

Values from the input domain map to the values of the output domain

Q2. {

- ① Explicit way
eg. $y = f(x)$ $z = f(x, y)$
- ② Implicit way
eg. $f(x) = 0$, $f(x, y) = 0$
- ③ Parametric way
eg. $x = f_x(t)$ $y = f_y(t)$
 $x = f_x(u, v)$, $y = f_y(u, v)$

Q3. $y = \sin(x) + \cos(x)$ can be converted into

$$\begin{cases} x = t \\ y = \sin(t) + \cos(t) \end{cases}$$

Q4.

$$\begin{aligned} \text{(i)} \quad x &= \sin^2(t) \\ y &= \cos(t) \quad \Rightarrow \quad x + y^2 = \sin^2(t) + \cos^2(t) = 1 \end{aligned}$$

Then we can find implicit function
 $f(x, y) = x + y^2 - 1 = 0$

$$\begin{aligned} \text{(ii)} \quad x &= 2 + 3t \\ \Rightarrow t &= \frac{x-2}{3} \end{aligned}$$

$$y = 3 + \frac{x-2}{3}$$

$$\Rightarrow 3y = 9 + x - 2$$

$$\Rightarrow 3y = x + 7$$

Then we can find implicit function
 $f(x, y) = 3y - x - 7 = 0$