CZ2003 Computer Graphic and Visual

Tutorial Answer

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CZ2003 Tutorial 1 (2022/23, Semester 1)

Coordinate Systems and Vectors

1. A straight line is defined by equation $y = 3x + 4$ in Cartesian coordinate system *XY*.
   (i) Define this straight line in polar coordinates *r, a* as an explicit function $r = f(\alpha)$.
   (ii) Specify the domain for the polar coordinate α in both radians and degrees for this straight line.

2. (i) Define in polar coordinates $r = f(\alpha)$ the origin-centred circle with radius *R*. Specify the domain for the polar coordinate α.

   (ii) Define in polar coordinates $r = f(\alpha)$ a circle with radius *R* and the centre at the Cartesian coordinates (*R*, 0). Specify the domain for the polar coordinate α.

3. With reference to Figure Q3, write formulas deriving Cartesian coordinates *x*, *y*, *z*, from the cylindrical *r*, α, *h* and spherical coordinates *r*, α, β. Notice that the axes layout is different in the two cases.

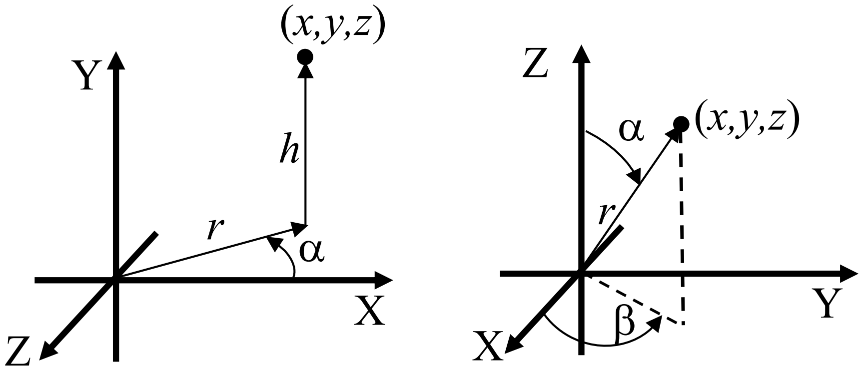


Figure Q3

4. (i) With reference to Figure Q4, calculate coordinates (numbers) of the unit (magnitude is equal to 1) normal vector **N**.
   (ii) What are the coordinates of the unit normal vector to the opposite side of the triangle?

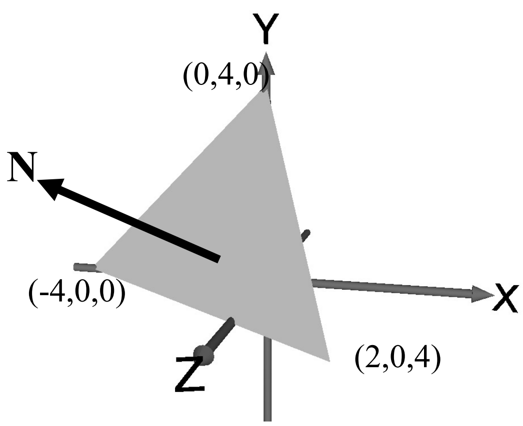


Figure Q4

Q1.

(i) $y = 3x + 4$

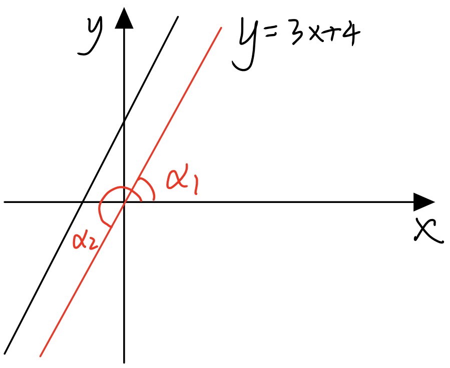
in polar coordinate system $y = r \cdot \sin\alpha$ $x = r \cdot \cos\alpha$

$r \cdot \sin\alpha = 3r \cdot \cos\alpha + 4$

$\Rightarrow r(\sin\alpha - 3\cos\alpha) = 4$

$\Rightarrow r = \dfrac{4}{\sin\alpha - 3\cos\alpha}$

(ii) in polar system, as shown in the below graph the range of the α will be in (α_1, α_2)



$\alpha_1 = \arctan(3) = 71.57°$

$\alpha_2 = \arctan(3) + \pi = 251.57°$

Thus, the domain of α is

$$\begin{cases} \text{radian: } (\arctan(3), \arctan(3)+\pi) \\ \text{degree: } (71.57°, 251.57°) \end{cases}$$

Q2

(i) in cartesian system, we define a circle

$x^2 + y^2 = R^2$

in polar system, that is

$r^2\cos^2\alpha + r^2\sin^2\alpha = R^2$

$\Rightarrow r^2 = R^2$, since $r > 0$ in usual

$\Rightarrow r = R \quad \alpha \in [0, 2\pi)$ in usual

(ii) in cartesian system, we define a circle

$(x-R)^2 + y^2 = R^2$

in polar system, that is

$(r\cos\alpha - R)^2 + r^2\sin\alpha^2 = R^2$

$\Rightarrow r^2\cos^2\alpha - 2Rr\cos\alpha + R^2 + r^2\sin\alpha^2 = R^2$

$\Rightarrow r^2 - 2Rr\cos\alpha = 0$

$\Rightarrow r = 2R\cos\alpha$

in usual $r \geq 0$

$\Rightarrow 2R\cos\alpha \geq 0$, since $R > 0$ in usual

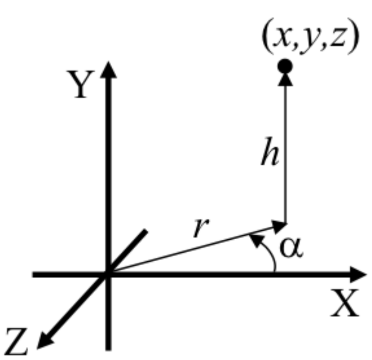
$\Rightarrow \cos\alpha > 0$

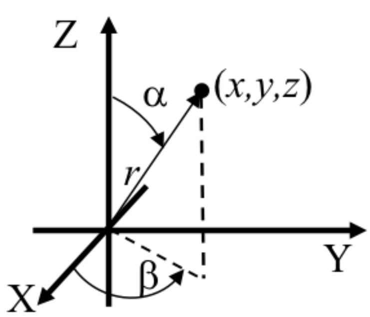
$\Rightarrow \alpha \in [0, \frac{\pi}{2}]$

Q3

(i) $\begin{cases} x = r \cdot \cos\alpha \\ y = h \\ z = -r \cdot \sin\alpha \end{cases}$

(ii) $\begin{cases} x = r \cdot \cos\alpha \cdot \cos\beta \\ y = r \cdot \cos\alpha \cdot \sin\beta \\ z = r \cdot \cos\alpha \end{cases}$





Q4

(i) suppose three points are

$A(-4,0,0)$ $B(0,4,0)$ $C(2,0,4)$

then $\vec{AB}=(4,4,0)$

$\vec{BC}=(2,-4,4)$

$\vec{CA}=(-6,0,-4)$

since $\vec{N}$ is the normal vector

suppose $\vec{N}=(x,y,z)$ then

$$\begin{cases} 4x+4y=0 \\ 2x-4y+4z=0 \\ -6x-4z=0 \\ x^2+y^2+z^2=1 \end{cases}$$

solve this formula, we can obtain

$$\vec{N}=\left(\frac{-2\sqrt{17}}{17}, \frac{2\sqrt{17}}{17}, \frac{3\sqrt{17}}{17}\right)$$

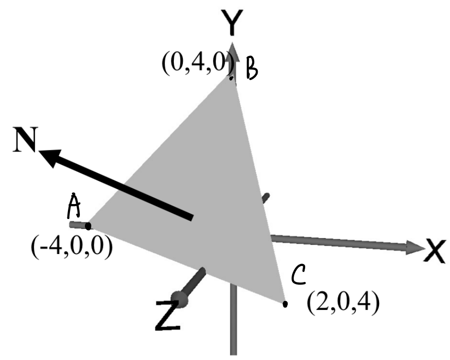


Figure Q4

(ii) the unit vector to the opposite side of the triangle is

$$\vec{N}_0=\left(\frac{2\sqrt{17}}{17}, -\frac{2\sqrt{17}}{17}, -\frac{3\sqrt{17}}{17}\right)$$