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Q1. ① equation of the trajectory

$$\begin{cases} x_0 = 30 \cdot \cos(2\pi \cdot 15 \cdot u) \\ y_0 = 30 \cdot \sin(2\pi \cdot 15 \cdot u) \\ z_0 = 15 \cdot 10 u \end{cases} \quad u \in [0,1]$$

② a unique sphere $1-(x-x_0)^2-(y-y_0)^2-(z-z_0)^2=0$

③ to specify the uniform speed $u = \frac{k-1}{160-1}$ $k=1,2,\dots 160$

Q2. The definition of the line is

$$\begin{cases} x_0 = 2 \cdot u \\ y_0 = 2 \cdot u \\ z_0 = 0 \end{cases}$$

The rotation matrix is

$$\begin{bmatrix} \cos(\frac{3}{2}\pi) & 0 & \sin(\frac{3}{2}\pi) & 0 \\ 0 & 1 & 0 & 0 \\ -\sin(\frac{3}{2}\pi) & 0 & \cos(\frac{3}{2}\pi) & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Code

```
x0=2*u;
y0=2*u;
z0=0;
tau=sin(pi/2*t);
z=z0*cos(1.5*pi*tau) - x0*sin(1.5*pi*tau);
y=y0;
x=z0*sin(1.5*pi*tau) + x0*cos(1.5*pi*tau);
```

$$\therefore \begin{cases} z = z_0 \cdot \cos(\frac{3}{2}\pi \cdot \tau) - x_0 \sin(\frac{3}{2}\pi \cdot \tau) \\ y = y_0 \\ x = z_0 \cdot \sin(\frac{3}{2}\pi \cdot \tau) + x_0 \cos(\frac{3}{2}\pi \cdot \tau) \end{cases} \quad \tau = \sin(\frac{\pi}{2} \cdot t) \quad t \in [0,1]$$

Q3. A: $\begin{cases} x = 2 \cdot V \cdot \cos(2\pi u) \\ y = V \cdot \sin(2\pi u) \\ z = 0 \end{cases}$ B: $\begin{cases} x = V \cdot \cos(2\pi u) \\ y = 2 \cdot V \cdot \sin(2\pi u) \\ z = 0 \end{cases}$

$\therefore$ the transformation is

$$\begin{cases} x = (1-s)(2V\cos(2\pi u)) + s \cdot (V\cos(2\pi u)) \\ y = (1-s)(V \cdot \sin(2\pi u)) + s \cdot (2V\sin(2\pi u)) \\ z = 0 \end{cases}$$

Code

```
xA=2*v*cos(2*pi*u);
yA=v*sin(2*pi*u);
xB=v*cos(2*pi*u);
yB=2*v*sin(2*pi*u);
x=(1-t)*xA+t*xB;
y=(1-t)*yA+t*yB;
z=0;
```

$s = t$, $t \in [0,1]$

Q4. sphere

$$\begin{cases} x_1 = w\cos(-\frac{\pi}{2}+\pi u)\cdot\sin(-\pi+2\pi v) \\ y_1 = w\sin(-\frac{\pi}{2}+\pi u) \\ z_1 = w\cos(-\frac{\pi}{2}+\pi u)\cos(-\pi+2\pi v) \end{cases} \qquad u,v,w\in[0,1]$$

cylinder

$$\begin{cases} x_2 = 2u\cos(2\pi v)-2 \\ y_2 = -4+4w \\ z_2 = 2u\sin(2\pi v)+2 \end{cases} \qquad u,v,w\in[0,1]$$

$\therefore$ the morphing process is $\begin{cases} x = (1-\tau)x_1+\tau x_2 \\ y = (1-\tau)y_1+\tau y_2 \\ z = (1-\tau)z_1+\tau z_2 \end{cases}$

$$\tau = \sin\left(\frac{\pi}{2}\cdot\frac{k-1}{160-1}\right) \quad k=1,2,\dots,160$$

Code

```
x1=w*cos(-0.5*pi+pi*u)*sin(-pi+2*pi*v);
y1=w*sin(-0.5*pi+pi*u);
z1=w*cos(-0.5*pi+pi*u)*cos(-pi+2*pi*v);

x2=2*u*cos(2*pi*v)-2;
y2=-4+4*w;
z2=2*u*sin(2*pi*v)+2;


x=(1-tau)*x1+tau*x2;
y=(1-tau)*y1+tau*y2;
z=(1-tau)*z1+tau*z2;
```