Lecture6 Motions and Morphing

1. Introduction

Animation

Animation can be viewed as visual changes in a scene with time.

An active research field in computer graphics

- Motion caption
- Physics-based simulation
- Natural phenomena simulation

Edwin E. Catmull
- co-founder of Pixar
- president of Walt Disney Animation Studios

Patrick M. Hanrahan
- the Canon USA Professor of Stanford University

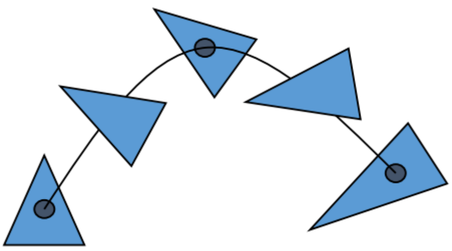
- winners of 2019 A.M.Turing Award
- For fundamental contributions to 3D computer graphics, and the impact of **computer-generated imagery** (CGI) in **filmmaking** and other applications.

Basic concepts

Basic idea is to introduce **time** into the definitions of shape, position, orientation, etc.

Two fundamental aspects

- Change of shape, position, orientation, etc.

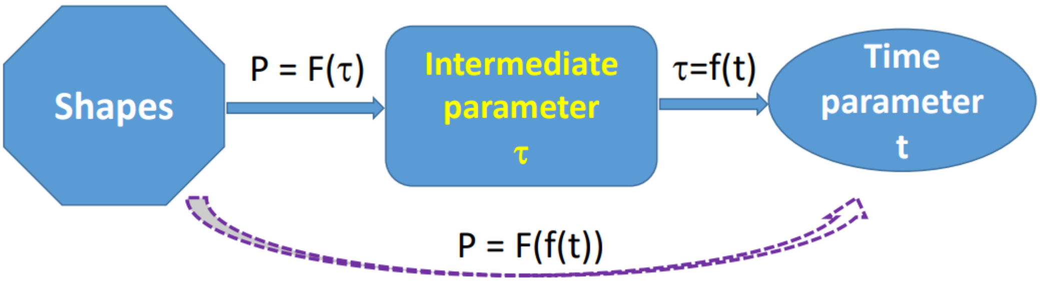


- Speed control

2. Speed Specification

Strategy

- Define the shape, position, or orientation, etc., by functions of some parameter (say, τ)
- Define function $\tau = f(t)$ to relate the change to time



Speed $\tau = f(t)$

How to define an appropriate function $\tau = f(t)$ to simulate

Uniform

$$\tau = f(t) = \frac{t - t_1}{t_2 - t_1}, t \in [t_1, t_2]$$

Acceleration

$$\tau = f(t) = 1 - cos(\frac{\pi}{2}\frac{t - t_1}{t_2 - t_1}), t \in [t_1, t_2]$$

Deceleration

$$\tau = f(t) = sin(\frac{\pi}{2}\frac{t - t_1}{t_2 - t_1}), t \in [t_1, t_2]$$

Two descriptions of animation models

Use time parameter

$$\tau = f(t), t \in [t_1, t_2]$$

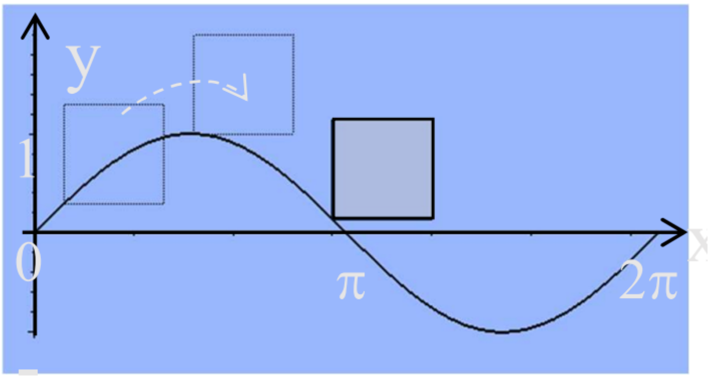
Use frame index

$$\tau = f(k), k = 1, 2, 3...$$

- k: frame index, $1 \le k \le m$
- m: total number of frames

3. Motion-by-path

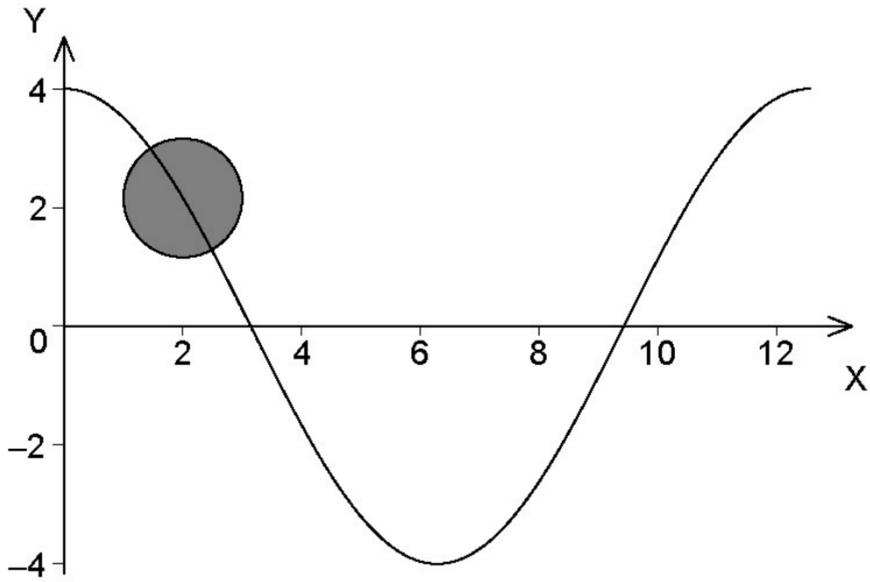
"Motion by path" approach: define a motion or animation via explicitly specifying the motion path



1. Represent the path by parametric equations: $(x, y, z) = (x(\tau), y(\tau), z(\tau)), \tau \in [0, 1]$
2. By linking the path to the target animation, derive the representation of the motion object
3. Appropriately define $\tau = f(t), \tau \in [t_1, t_2]$ or $\tau = f(k), k = 1, \ldots, m$ to control the speed

Example

The following figure shows a unit disk moving on the XY plane. During the movement, the center of the disk moves along a trajectory defined by $y = 4cos(\frac{x}{2})$ from point $(0, 4)$ to point $(4\pi, 4)$. Propose a mathematical model in implicit representation for this motion. The motion consists of 100 frames and involves deceleration



Step1: The equation of the trajectory is

$$\begin{cases} x_0 = 4\pi u \\ y_0 = 4\cos(4\pi u/2) = 4\cos(2\pi u) \end{cases} \quad u \in [0,1]$$

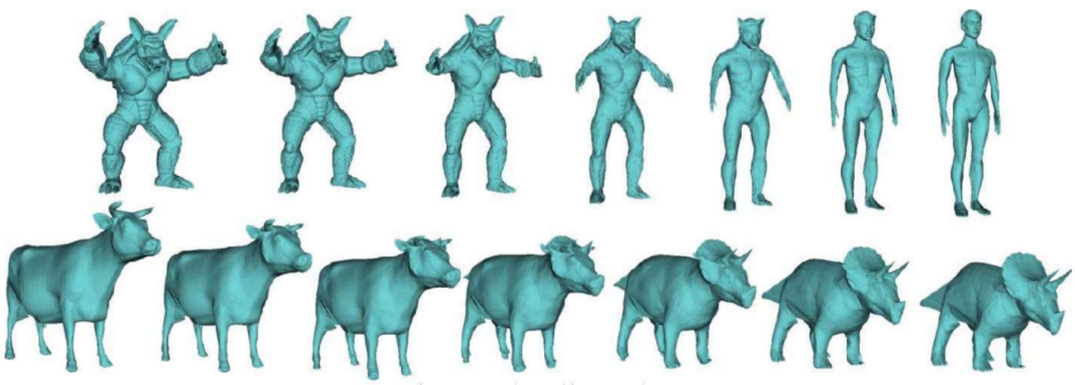
Step2: The moving disk can thus be represented implicitly by

$$1 - (x - 4\pi u)^2 - (y - 4cos(2\pi u))^2 \geq 0$$

Step3: To specify the speed, let

$$u = sin(\frac{\pi}{2}\frac{k-1}{100-1}), k = 1, 2, \ldots, 100$$

4. Morphing



Given two items A and B of the same type, we want to compute an intermediate item $v(\tau)$ which gradually changes from A to B at a constant rate

Linear interpolation model

$$v(\tau) = (1 - \tau)A + \tau B, 0 \leq \tau \leq 1$$

- When $\tau = 0, v(\tau) = A$
- When $\tau = 1, v(\tau) = B$
- For intermediate values of parameter τ, $v(\tau)$ is a linear combination of A and B