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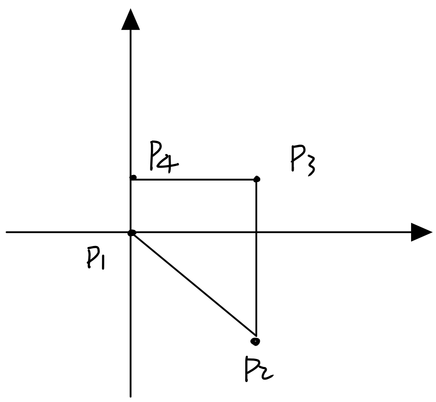
Q1. through the homogeneous coordinate,
we can therefore determine the 2D Cartesian coordinate
$P_1(0,-1)$ $P_2(2,-2)$ $P_3(2,1)$ $P_4(0,1)$



thus, it is a trapzium

Q2 assume the affine transformation is defined by

$$x' = ax + by + m$$
$$y' = cx + dy + n$$

$V_a = (0,0)$ $V_a' = (10,8)$
$V_b = (4,0)$ $V_b' = (6,4)$
$V_c = (4,4)$ $V_c' = (14,0)$
$V_d = (2,6)$ $V_d' = (20,0)$
$V_e = (0,4)$ $V_e' = (18,4)$

$\therefore 10 = m$
$8 = n$
$6 = 4a + m \Rightarrow a = -1$
$4 = 4c + n \Rightarrow c = -1$
$14 = 4a + 4b + m \Rightarrow b = 2$
$0 = 4c + 4d + n \Rightarrow d = -1$

$\therefore x' = -x + 2y + 10$
$y' = -x - y + 8$

Given the point $P = (2,3)$

$P' = (14, 3)$

Q3. first, move it to the original point $M_1 = \begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & -1 \\ 0 & 0 & 1 \end{bmatrix}$

then scale it in X-corrodinate by 5 times and y-corrodinate 3 times $S = \begin{bmatrix} 5 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

then move it back position $M_2 = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}$

then rotate this matrix (clockwise) 90° $R_1 = \begin{bmatrix} \cos(-90) & -\sin(-90) & 0 \\ \sin(-90) & \cos(-90) & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$$= \begin{bmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

then reflect through y-axis $R_2 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$\therefore$ the whole transformation matrix $T = R_1 R_2 M_2 S M_1$

$$T = \begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} 5 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & -1 \\ 0 & 0 & 1 \end{bmatrix}$$

Q4. $R = \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$ $S = \begin{bmatrix} S_x & 0 & 0 \\ 0 & S_y & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$$SR = \begin{bmatrix} S_x\cos\theta & S_x(-\sin\theta) & 0 \\ S_y\sin\theta & S_y\cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$RS = \begin{bmatrix} S_x\cos\theta & S_y(-\sin\theta) & 0 \\ S_x\sin\theta & S_y\cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} S_x\cos\theta & S_x(-\sin\theta) & 1 \\ S_y\sin\theta & S_y\cos\theta & 2 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & -2 \\ 0 & 0 & 1 \end{bmatrix}$$

$$T_1 = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} S_x\cos\theta & S_x(-\sin\theta) & 0 \\ S_y\sin\theta & S_y\cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & -2 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} S_x\cos\theta & S_x(-\sin\theta) & -S_x\cos\theta + 2S_x\sin\theta + 1 \\ S_y\sin\theta & S_y\cos\theta) & -S_y\sin\theta - 2S_y\cos\theta + 2 \\ 0 & 0 & 1 \end{bmatrix}$$

$$T_2 = \begin{bmatrix} S_x\cos\theta & S_y(-\sin\theta) & -S_x\cos\theta + 2S_y\sin\theta + 1 \\ S_x\sin\theta & S_y\cos\theta & -S_x\sin\theta - 2S_y\cos\theta + 2 \\ 0 & 0 & 1 \end{bmatrix}$$

if $T_1 = T_2 \Rightarrow$ $S_x(-\sin\theta) = S_y(-\sin\theta)$
$S_x(\sin\theta) = S_y(\sin\theta)$

the condition is either
1. $S_x = S_y$ or
2. $\sin\theta = -\sin\theta = 0 \Rightarrow \theta = k\pi \ (k \in \mathbb{Z})$