

# Lecture5 3D Transformation

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## 1. Basic 3D transformations

### Translation Matrix

$$\begin{bmatrix} 1 & 0 & 0 & l \\ 0 & 1 & 0 & m \\ 0 & 0 & 1 & n \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

### Scaling Matrix

$$\begin{bmatrix} s_x & 0 & 0 & 0 \\ 0 & s_y & 0 & 0 \\ 0 & 0 & s_z & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

### Rotation Matrix

#### Rotation about x-axis

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos \theta & -\sin \theta & 0 \\ 0 & \sin \theta & \cos \theta & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

#### Rotation about y-axis

$$\begin{bmatrix} \cos \theta & 0 & \sin \theta & 0 \\ 0 & 1 & 0 & 0 \\ -\sin \theta & 0 & \cos \theta & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

## Rotation about z-axis

$$\begin{bmatrix} \cos \theta & -\sin \theta & 0 & 0 \\ \sin \theta & \cos \theta & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

## Reflection Matrix

### Over xy plane

$$\text{Ref}_{xy} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

### Over yz plane

$$\text{Ref}_{yz} = \begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

### Over zx plane

$$\text{Ref}_{zx} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

### About axis-x

$$\text{Ref}_x = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

### About axis-y

$$Ref_y = \begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

### About axis-z

$$Ref_z = \begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

## 2. 3D affine transformations

3D Affine transformations can always be represented by

$$\begin{bmatrix} x' \\ y' \\ z' \\ 1 \end{bmatrix} = \begin{bmatrix} a & b & c & m \\ d & e & f & n \\ g & h & p & l \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

### Scaled with relative to a point (l, m, n)

$$\begin{bmatrix} 1 & 0 & 0 & l \\ 0 & 1 & 0 & m \\ 0 & 0 & 1 & n \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} s_x & 0 & 0 & 0 \\ 0 & s_y & 0 & 0 \\ 0 & 0 & s_z & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & -l \\ 0 & 1 & 0 & -m \\ 0 & 0 & 1 & -n \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

### Scaled/Rotate/Reflect with relative to a line

1. Find the direction of the line (vector)
2. Translate the direction of the line to let it pass through the origin
3. Align the line to the z axis
4. Perform scaling/rotation/reflection over the line
5. Reverse the previous steps 3, 2

## Reflect with relative to a plane

1. Find the normal vector of the plane
2. Find any point on the plane
3. Translate the point to the origin
4. Align the normal vector to the z axis
5. Perform reflection over the plane
6. Reverse the steps 4, 3