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Q1. The rotation axis is Y axis
it rotates counterclockwise, and rotates 60°

Q2. the line passes through (1,0,0) and (0,0,2)
one of the direction of the line is (−1,0,2)

① move point (1,0,0) to the origin

$$T_1 = \begin{bmatrix} 1 & 0 & 0 & -1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$a=-1 \quad b=0 \quad c=2$

$d=2 \quad L=\sqrt{5}$

② aligning a vector to z-axis

$$R_x(\alpha) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad R_y(-\beta) = \begin{bmatrix} \frac{2}{\sqrt{5}} & 0 & \frac{1}{\sqrt{5}} & 0 \\ 0 & 1 & 0 & 0 \\ \frac{-1}{\sqrt{5}} & 0 & \frac{2}{\sqrt{5}} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

③ reflection

$$R_f = \begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

④ Reverse Steps

$$R_y(\beta) = \begin{bmatrix} \frac{2}{\sqrt{5}} & 0 & \frac{-1}{\sqrt{5}} & 0 \\ 0 & 1 & 0 & 0 \\ \frac{1}{\sqrt{5}} & 0 & \frac{2}{\sqrt{5}} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad R_x(-\alpha) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

⑤ Move Back

$$T_2 = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Thus, the whole matrix is

$$T = T_2 \cdot R_x(-\alpha) \cdot R_y(\beta) \cdot R_f \cdot R_y(-\beta) \cdot R_x(-\alpha) \cdot T_1$$

Q3.

$$\begin{bmatrix} 2 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix} = \begin{bmatrix} 2x+y \\ y+1 \\ z \\ 1 \end{bmatrix} \begin{matrix} = x \\ = y' \\ = z' \\ \end{matrix}$$

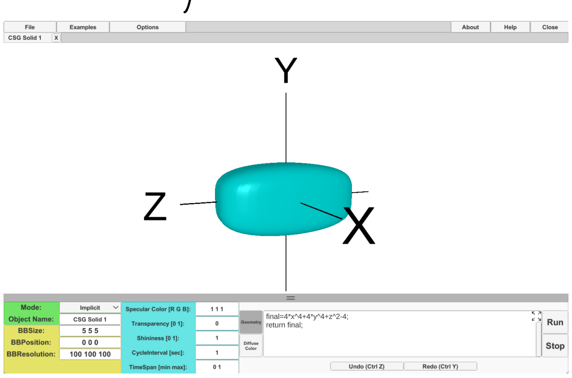
$$\Rightarrow \quad x = \tfrac{1}{2}(x' - y' + 1)$$
$$y = y' - 1$$
$$z = z$$

satisfy: $4 \times \left(\tfrac{1}{2}(x' - y' + 1)\right)^4 + 4(y' - 1)^4 + z'^2 - 4 = 0$

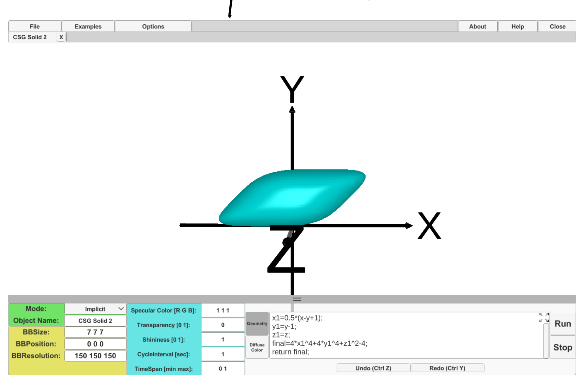
the implicit function of the new object is

$$4 \cdot \left(\tfrac{1}{2}(x - y + 1)\right)^4 + 4(y-1)^4 + z^2 - 4 = 0$$

origin



transformed



Q4. original curve = $\begin{cases} x = \sin(\pi u) \\ y = 0 \\ z = \cos(\pi u) + 1 \end{cases}$ $u \in [0,1]$

$$R_y(\theta) \begin{bmatrix} \cos(\theta) & 0 & \sin(\theta) & 0 \\ 0 & 1 & 0 & 0 \\ -\sin(\theta) & 0 & \cos(\theta) & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad \begin{matrix} \theta \in [0, 2\pi] \\ \theta = 2\pi v \\ v \in [0,1] \end{matrix}$$

$$T = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 2v \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad v \in [0,1]$$

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 2v \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos(\theta) & 0 & \sin(\theta) & 0 \\ 0 & 1 & 0 & 0 \\ -\sin(\theta) & 0 & \cos(\theta) & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ 0 \\ z \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} \cos\theta & 0 & \sin\theta & 0 \\ 0 & 1 & 0 & 2v \\ -\sin\theta & 0 & \cos\theta & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ 0 \\ z \\ 1 \end{bmatrix} = \begin{bmatrix} \cos\theta\, x + \sin\theta\, z \\ 2v \\ -\sin\theta\, x + \cos\theta\, z \\ 1 \end{bmatrix}$$

thus, the parametric representation of the surface is

$$\begin{cases} x = \sin(\pi u) \cdot \cos(2\pi v) + (\cos(\pi u) + 1) \cdot \sin(2\pi v) \\ y = 2v \\ z = \sin(\pi u) \cdot (-\sin(2\pi v)) + (\cos(\pi u) + 1) \cdot \cos(2\pi v) \end{cases} \quad \begin{matrix} u \in [0,1] \\ v \in [0,1] \end{matrix}$$