Assignment1

Question1

Decide whether you think the following statement is true or false. If it is true, give a short explanation. If it is false, give a counterexample.

> *True or false? In **every** instance of the Stable Matching Problem, there is a stable matching containing a pair (m, w) such that **m is ranked first on the preference list of w and w is ranked first on the preference list of m**.*

False

We can put forward an example:

Suppose we have man A and B, woman X and Y

Preference List:

A	**B**	**X**	**Y**
X	Y	B	A
Y	X	A	B

We can run stable-matching algorithm and see what happens:

1. A proposed to X, A-X become a match.
2. B proposed to Y, B-Y become a match.
3. Algorithm finish

Here we can find that there **is not** contain a pair (m, w) such that m is ranked first on the preference list of w and w is ranked first on the preference list of m.

Question2

Decide whether you think the following statement is true or false. If it is true, give a short explanation. If it is false, give a counterexample.

> *True or false? Consider an instance of the Stable Matching Problem in which there* ***exists*** *a man m and a woman w such that* ***m is ranked first on the preference list of w and w is ranked first on the preference list of m****. Then in* ***every*** *stable matching S for this instance, the* ***pair (m, w) belongs to S****.*

True

We can prove this by running the stable-matching algorithm

1. At the beginning, all the man are free.
2. When it is man m's turn, he is free and will adefinitely propose to woman w.
    a. If w has had a match with another man m'
        i. She will break up with m' and accept m.
    b. If w doesn't have a match with other man
        i. She will accept m.
3. After that, if any other man proposed to w, because w has had the best chosen, she will never break up anymore.

Question3

There are many other settings in which we can ask questions related to some type of “stability” principle. Here’s one, involving competition between two enterprises.

Suppose we have two television networks, whom we’ll call A and B. **There are n prime-time programming slots, and each network has n TV shows**. Each network wants to devise a schedule—an assignment of each show to a distinct slot—so as to attract **as much market share as possible**.

Here is the way we determine how well the two networks perform relative to each other, given their schedules. **Each show has a fixed rating**, which is based on the number of people who watched it last year; we’ll **assume that no two shows have exactly the same rating**. A network wins a given time slot if the show that it schedules for the time slot has a larger rating than the show the other network schedules for that time slot. The goal of each network is to win as many time slots as possible.

Suppose in the opening week of the fall season, Network A reveals a schedule S and Network B reveals a schedule T. On the basis of this pair of schedules, each network wins certain time slots, according to the rule above. We'll say that the pair of schedules **(S, T)is stable** if **neither network can unilaterally change its own schedule and win more time slots**. That is, there is no schedule S such that Network A wins more slots with the pair (S' , T) than it did with the pair (S, T); and symmetrically, there is no schedule T such that Network B wins more slots with the pair (S', T) than it did with the pair (S, T).

The analogue of Gale and Shapley's question for this kind of stability is the following: **For every set of TV shows and ratings, is there always a stable pair of schedules?** Resolve this question by doing one of the following two things:

(a) give an algorithm that, for any set of TV shows and associated ratings, produces a stable pair of schedules; or

(b) give an example of a set of TV shows and associated ratings for which there is no stable pair of schedules.

We can find that in marriage match problem, each man and woman give his/her preference order, and everyone has the same weight in rating.

For example, if woman X's preference order is A, B; woman Y's preference order is B, A, this means the status of A to X is the same as B to Y.

But here in TV shows, we observe that not two shows have exactly the same rating, which means the **rating is differ in each show**.

We can put forward one example showes that **there is no stable pair of schedules**.

Suppose Network A has two showes $\{a_1, a_2\}$, with ratings $a_1 = 10, a_2 = 20$; and Network B has two showes $\{b_1, b_2\}$, with rating $b_1 = 5, b_2 = 20$.

- First, Network A wins all the schedule becaule $a_1 > b_1, a_2 > b_2$.
- Then, Network B will want to switch the order of the shows in it's schedule (so that it will win one slot rather than none). It's schedule will be $\{b_2, b_1\}$.
- In this case, Network B will win one slot because $a_1 < b_2$. However, Network A will want to switch the order of shows in it's schedule (so that it will win two slots rather than one). It's schedule will be $\{a_2, a_1\}$.
- The situation repeating...

Question8

For this problem, we will explore the issue of truthfulness in the **Stable Matching Problem** and specifically in the **Gale-Shapley algorithm**. The basic question is: Can a man or a woman **end up better off** by **lying** about his or her preferences? More concretely, we suppose each participant has a true preference order. Now consider a woman w. Suppose w prefers man m to m' , but both m and m' are low on her list of preferences. Can it be the case that by switching the order of m and m' on her list of preferences (**i.e., by falsely claiming that she prefers m' to m) and running the algorithm with this false preference list, w will end up with a man m'' that she truly prefers to both m and m' ?** (We can ask the same question for men, but will focus on the case of women for purposes of this question.)

Resolve this question by doing one of the following two things:

(a) Give a proof that, for any set of preference lists, switching the order of a pair on the list cannot improve a woman’s partner in the GaleShapley algorithm; or

(b) Give an example of a set of preference lists for which there is a switch that would improve the partner of a woman who switched preferences.

Yes, it can happen.

We will put forward an example to show the result.

Suppose we have three man A, B, C; three woman X,Y,Z.

Their preference order are as follow:

A	**B**	**C**	**X**	**Y**	**Z**	**Z'**
Z	X	Z	A	A	B	B
X	Z	X	B	B	A	C
Y	Y	Y	C	C	C	A

Column Z showes the true prefrences of woman Z, while in column Z'she pretend she prefers man C to A.

If Z keeps her true preference order, the reult will become:

- A-Z
- B-X
- C-Y

Hovever, if Z is lying, the resule will become:

- A-X
- B-Z
- C-Y

In this case, Z can married to the man B who she prefer more than A.