October 31, 2021

[2]:
```python
import scipy.integrate as integr
import numpy as np
import matplotlib.pyplot as plt
from scipy.signal import argrelextrema
import pylab

colors = ["#00FFFF",                # Azure
          "#0000FF",                # Blue
          "#FF0000",                # Red
          "#00FF00",                # Green
          "#D2691E",                #
          "#AAA662",
          "#9A0EEA",
          "#808000",
          "#FFA500",
          "#9ACD32",
          "#ED0DD9",
          "#FC5A50",
          "#929591",
          "#029386",
          "#C79FEF",
          "#FAC205",
          "#6E750E",
          "#06C2AC",
          "#CD5C5C",                # IndianRed
          "#A52A2A",                # Brown
          "#7B68EE",                # MediumState_blue
          "#4682B4",                # SteelBlue
          "#800000"                # Maroon
          ]
colors = 10*colors

mycolors = [
    '#000000',
    '#696969',
    '#A9A9A9',
    '#C0C0C0',
```

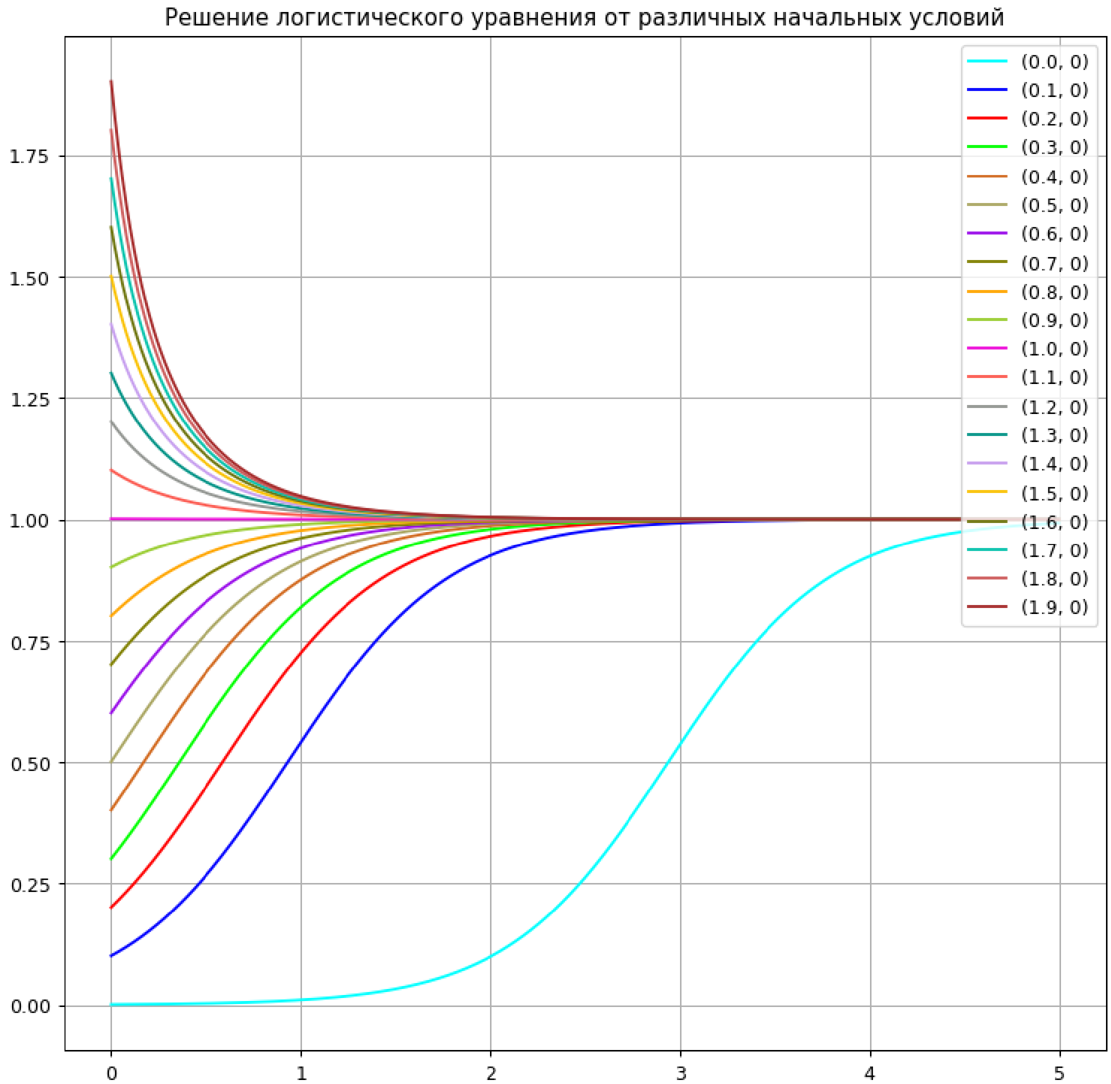
```
    '#D3D3D3',
    '#DCDCDC',
]

def mesh(x, y):
    general = []
    for i in range(len(x)):
        for j in range(len(y)):
            local = []
            local.append(x[i])
            local.append(y[j])
            general.append(local)
    return general

stat_dpi = 90
rect_pics = (7, 7)
big_pics = (10, 10)
dynamics_pics = (14, 7)
```

[3]:
```
def plotFP(y1, y2, centers = None, starts = None, color = "b"):
    fig = plt.figure(facecolor="white", figsize = rect_pics, dpi=stat_dpi)
    plt.plot(y1, y2, c = color)
    if centers is not None:
        for i in centers:
            plt.scatter(i[0], i[1])
    if starts is not None:
        for i in starts:
            plt.scatter(i[0], i[1])
    plt.grid(True)
    plt.show()
```

[4]:
```
def logistic(t, r, x_0, t_0):
    result = 1/(1 + (1/x_0 - 1) * np.exp(-r*(t-t_0)))
    return result
```

[5]:
```
time = np.linspace(0, 5, 500)
#print(y)
#print(time)

fig = plt.figure(facecolor="white", figsize = big_pics, dpi=stat_dpi)
step = 0.1
for i in np.arange(0.001, 2, step):
    plt.plot(time, logistic(time, 2.35, i, 0), c = colors[int(i/step)%30], label = f'({round(i, 2)}, 0)')
plt.title("                                    ")
plt.legend(loc="upper right")
plt.grid(True)
```

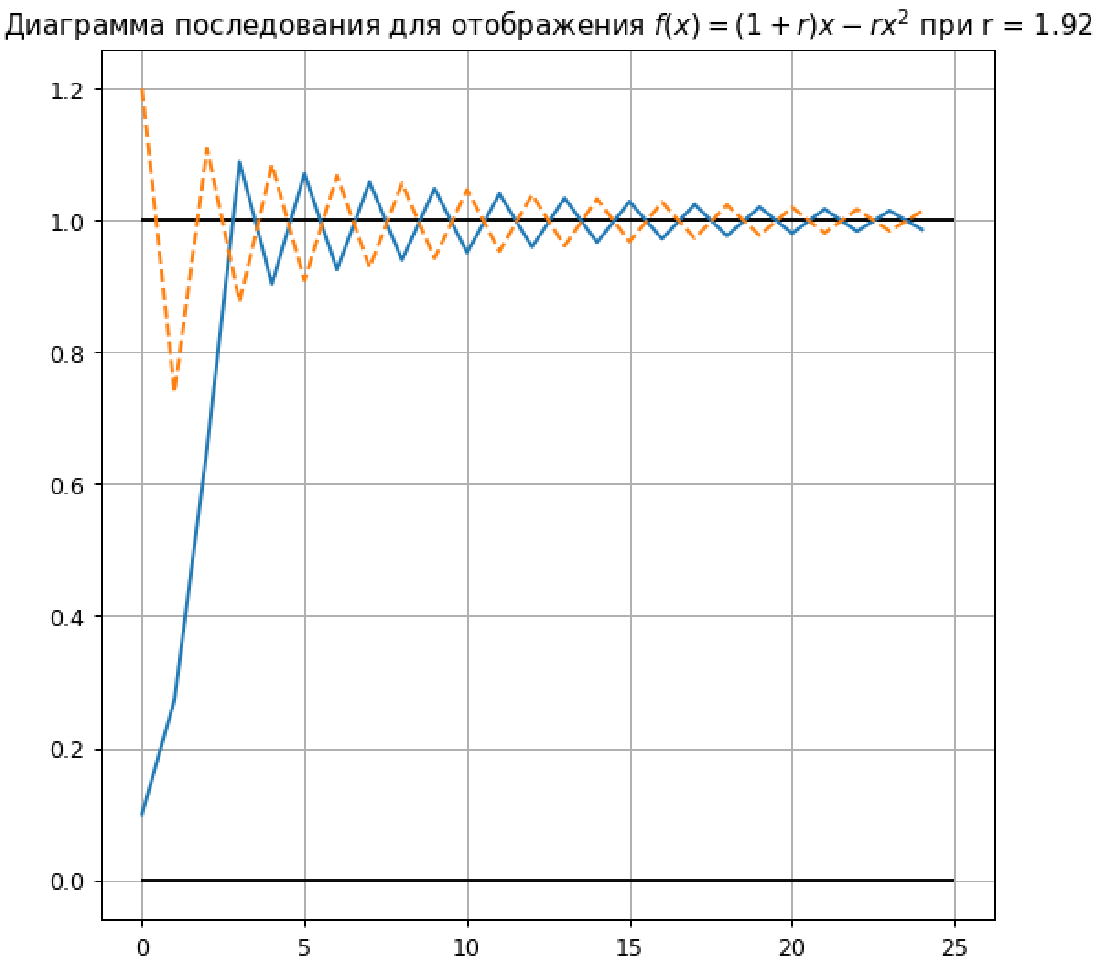
```
plt.show()
```



[6]:
```
def discr_sol(x_0, r, times=0):
    X = [x_0]
    for i in range(times-1):
        X.append((1 + r)*X[-1] - r*X[-1]**2)
    return X
```

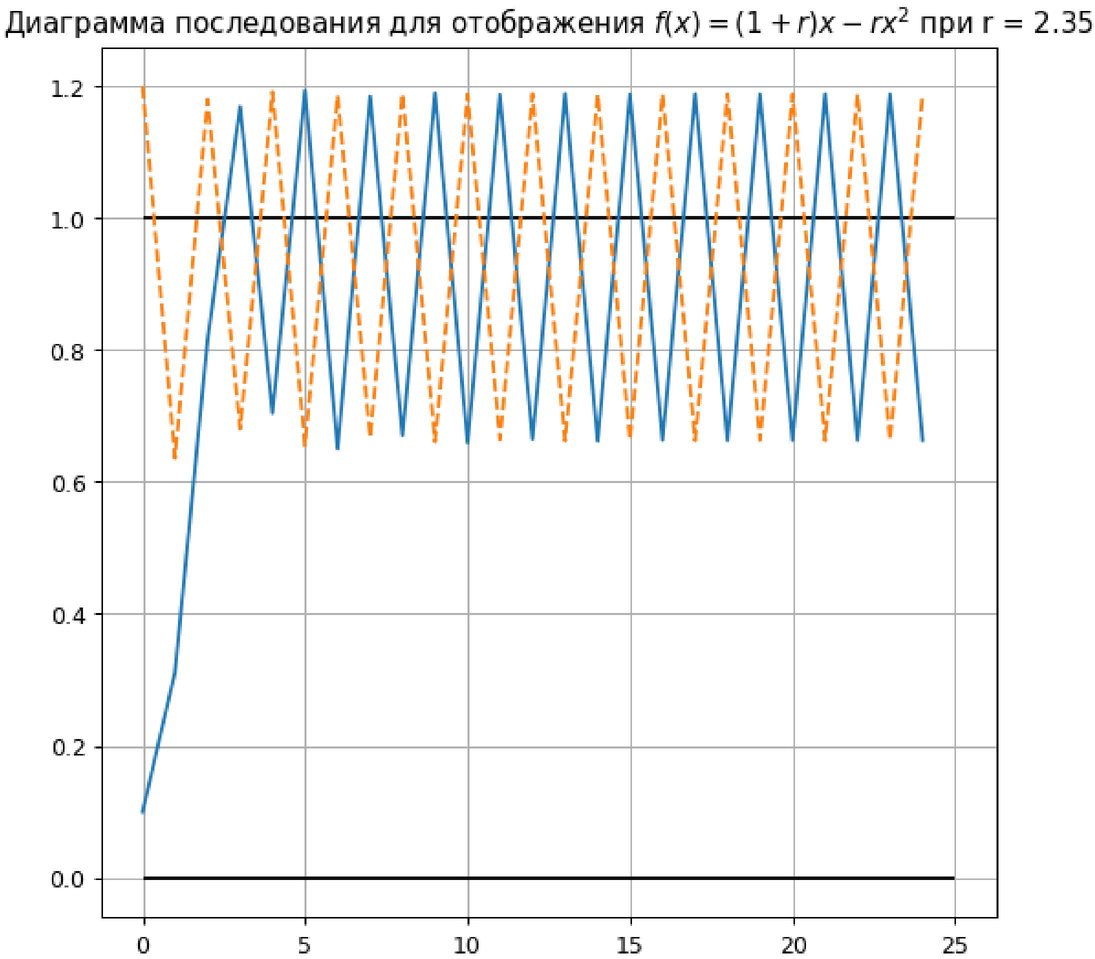
[7]:
```
time = np.arange(0, 25)
fig = plt.figure(facecolor="white", figsize = rect_pics, dpi=stat_dpi)
r=1.92

plt.hlines(1, 0, len(time), color='black')
```

```python
plt.hlines(0, 0, len(time), color='black')
plt.plot(discr_sol(0.1, r, len(time)))
plt.plot(discr_sol(1.2, r, len(time)), linestyle='dashed')
plt.title(r"                              $f(x) = (1+r)x-rx^2$    r = "+str(r))
plt.grid(True)
plt.show()
```



[8]:
```python
time = np.arange(0, 25)
fig = plt.figure(facecolor="white", figsize = rect_pics, dpi=stat_dpi)
r=2.35

plt.hlines(1, 0, len(time), color='black')
plt.hlines(0, 0, len(time), color='black')
plt.plot(discr_sol(0.1, r, len(time)))
plt.plot(discr_sol(1.2, r, len(time)), linestyle='dashed')
plt.title(r"                              $f(x) = (1+r)x-rx^2$    r = "+str(r))
```

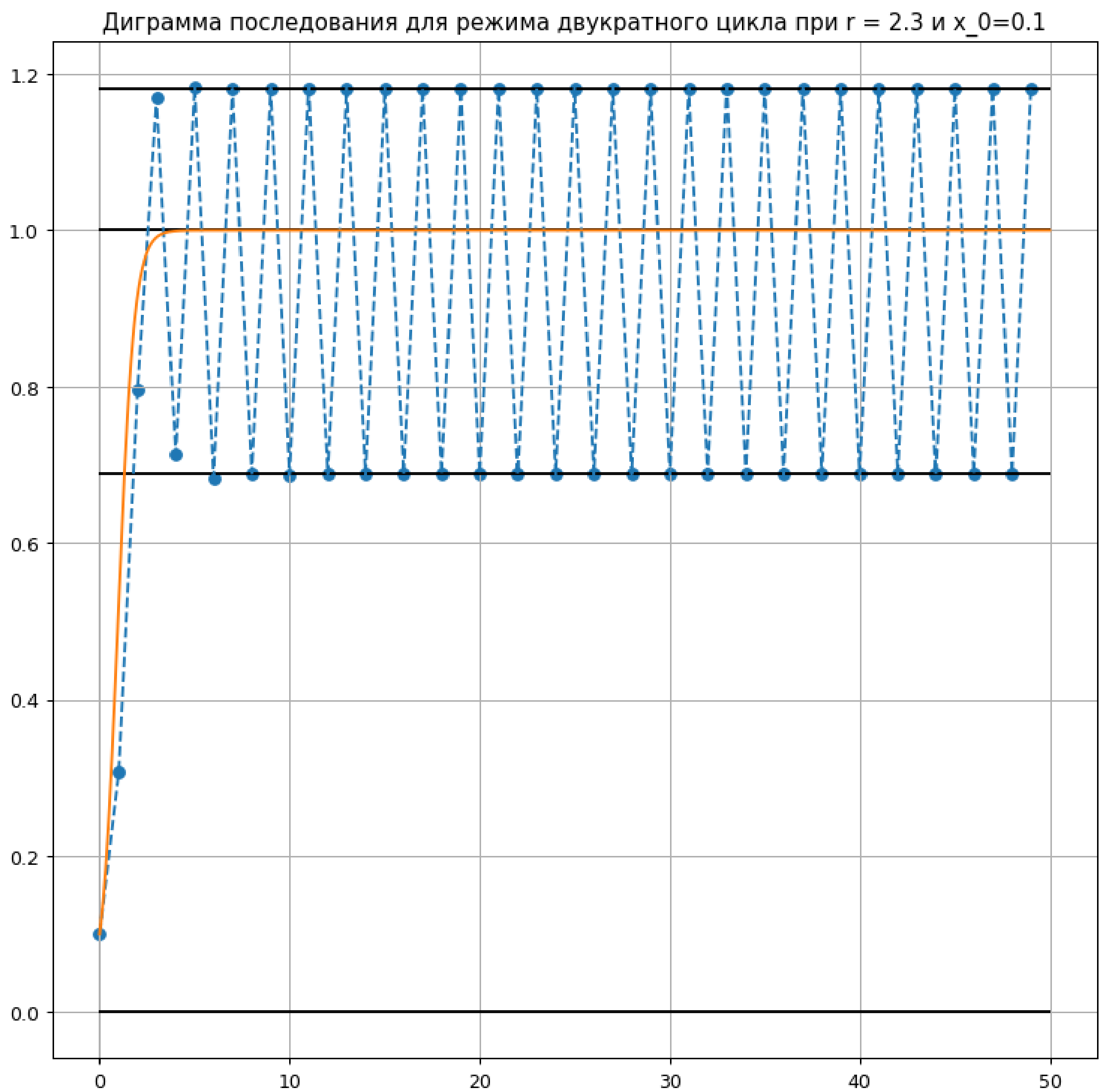
```
plt.grid(True)
plt.show()
```



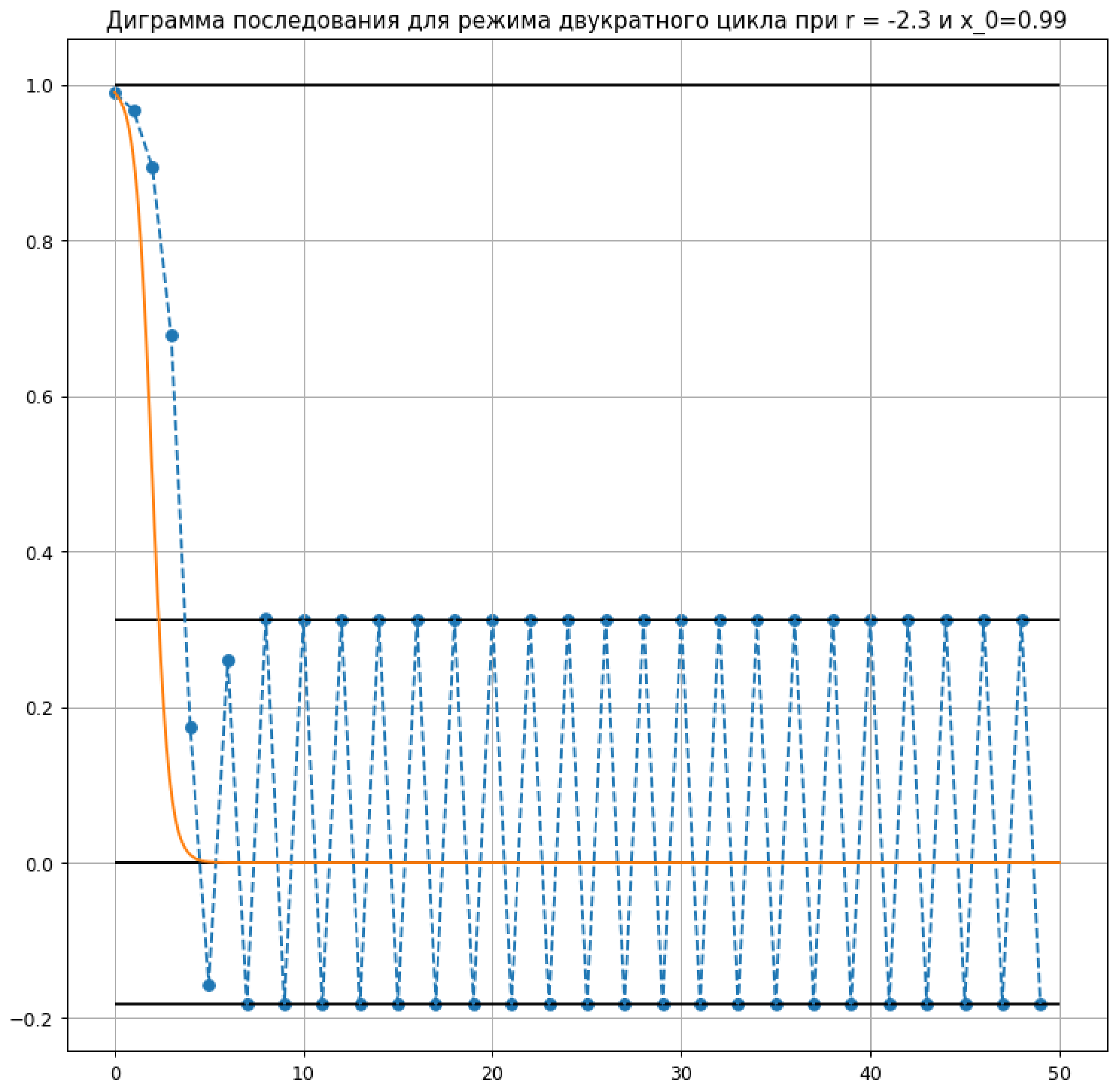

```
[47]: def plotANAL(r, x_0, times=50):
          time = np.arange(0, times)
          fig = plt.figure(facecolor="white", figsize = big_pics, dpi=stat_dpi)
          ff = lambda x : (1 + r) * ((1 + r)*x - r*x**2) - r * ((1+r)*x-r*x**2)**2

          plt.hlines(1, 0, len(time), color='black')
          plt.hlines(0, 0, len(time), color='black')

          xp3 = (2*r + r**2 - r*(-4 + r**2)**0.5)/(2*r**2)
          xp4 = (2*r + r**2 + r*(-4 + r**2)**0.5)/(2*r**2)

          plt.hlines(xp3, 0, len(time), color='black')
          plt.hlines(xp4, 0, len(time), color='black')
```

```
    data = discr_sol(x_0, r, len(time))

    plt.scatter(x=time, y=data)
    plt.plot(time, data, linestyle='dashed')
    plt.plot(np.linspace(0, 50, 1000), logistic(np.linspace(0, 50, 1000), r, x_0, 0))
    plt.title(r"                              r = "+str(r)+f"   {x_0=}")
    plt.grid(True)
    plt.show()
```

[48]:
```
plotANAL(r=2.3, x_0=0.1)
```

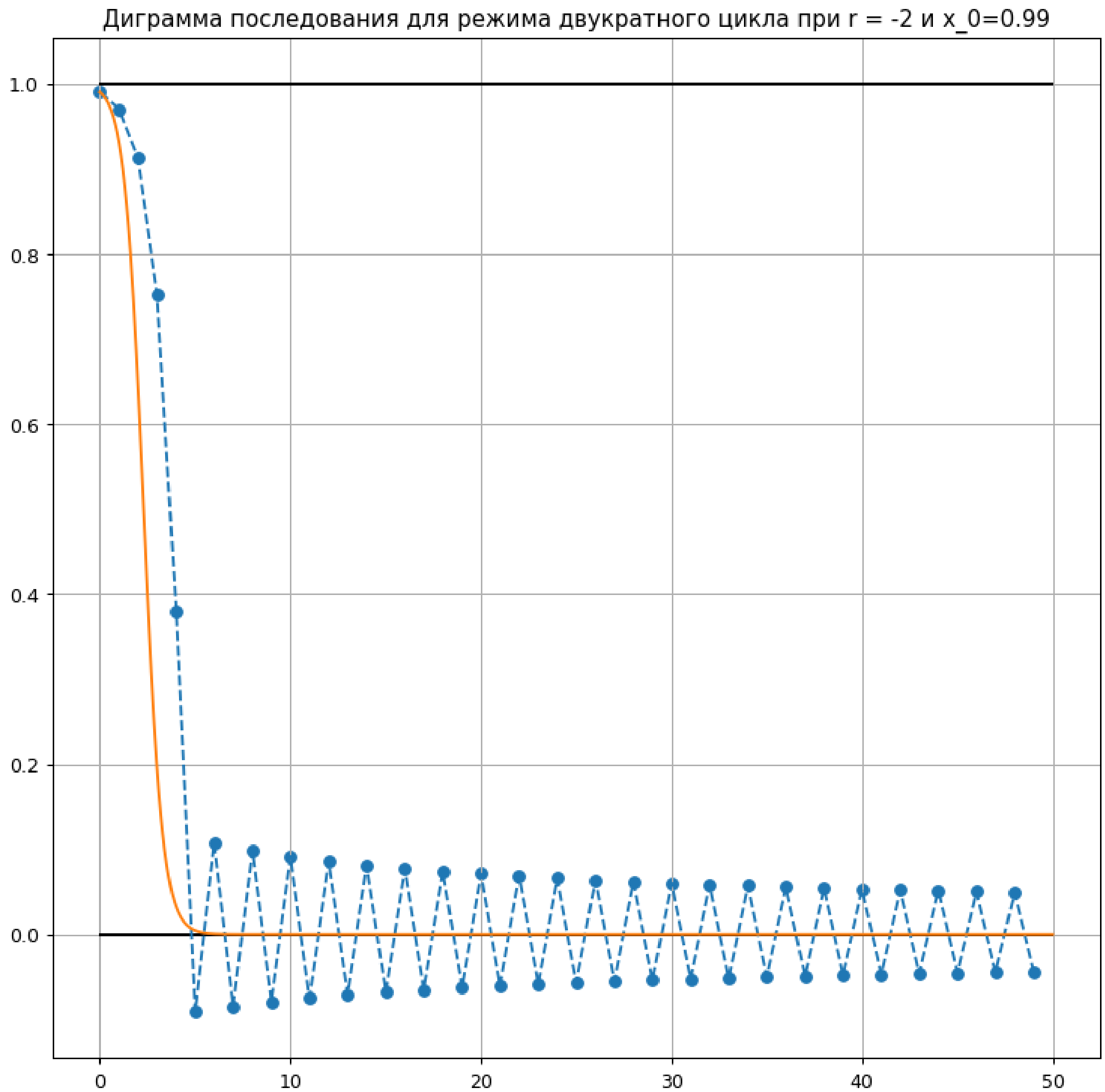


[49]:
```
plotANAL(r=-2.3, x_0=0.99)
```



[50]:
```
# time = np.arange(0, 100)
# fig = plt.figure(facecolor="white", figsize = rect_pics, dpi=stat_dpi)

# r = -2

# plt.hlines(1, 0, len(time), color='black')
# plt.hlines(0, 0, len(time), color='black')

# xp3 = (2*r + r**2 - r*(-4 + r**2)**0.5)/(2*r**2)
# xp4 = (2*r + r**2 + r*(-4 + r**2)**0.5)/(2*r**2)
```

```
# #plt.hlines(xp3, 0, len(time), color='black')
# #plt.hlines(xp4, 0, len(time), color='black')

# #plt.plot(discr_sol(0.1, 2.35, len(time)))
# plt.plot(discr_sol(0.1, r, len(time)))
# plt.title(r"                      $f(x) = (1+r)x-rx^2$    r = "+str(r))
# plt.grid(True)
# plt.show()
plotANAL(r=-2, x_0=0.99)
```




```
[51]: def KenLam(times, x_0, r, with_lines = True, with_arrows = True,
 ↪with_dots=True, with_3cr = False, with_4cr=False, constant_size=False,
 ↪arrow_c=["green"]):
```

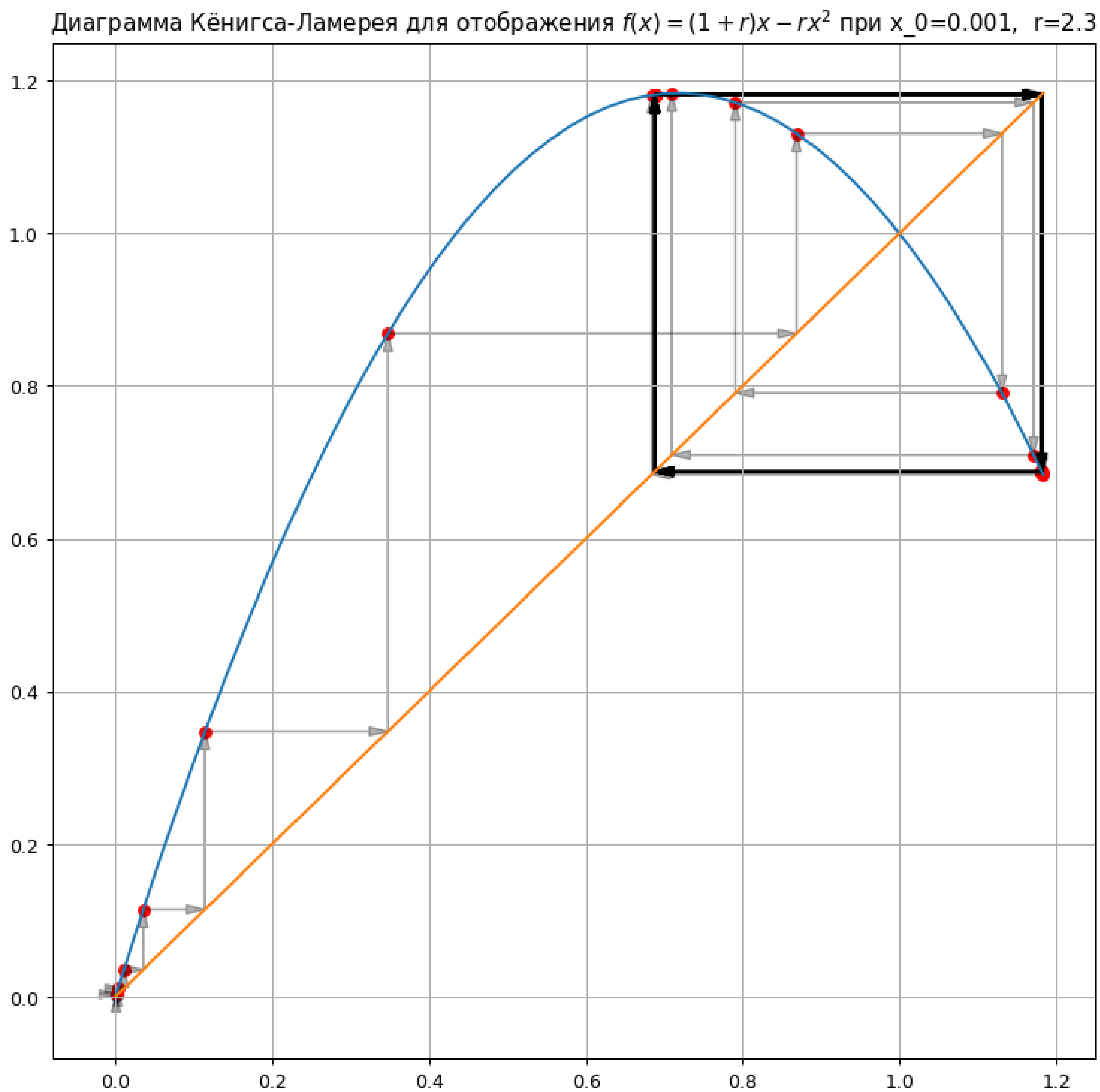
```
    f = lambda x : (1 + r)*x - r*x**2
    fig = plt.figure(facecolor="white", figsize = big_pics, dpi=stat_dpi)
    dtime = np.arange(x_0, times)
    X = [x_0]
    Y = [x_0]
    plt.scatter(X, Y)

    for i in dtime:
        y = f(X[-1])
        X.append(X[-1])
        Y.append(y)
        X.append(y)
        Y.append(Y[-1])

    scale = (max(X)-min(X))
    for i in range(1, len(X)):
        if with_arrows:
            plt.arrow(X[i-1], Y[i-1], X[i]-X[i-1],
                  Y[i]-Y[i-1], head_width=0.01*scale, head_length=0.02*scale,
                      color=arrow_c[i%(len(arrow_c))], width=0.0005*scale,␣
↪alpha=0.3, length_includes_head=True)



    time = np.linspace(min(X), max(X))
    if constant_size:
        time = np.linspace(0, (1+r)/r)
    plt.plot(time, f(time))
    if with_arrows:
        plt.plot(time, time)
    if with_3cr:
        plt.plot(time, f(f(time)))
    if with_4cr:
        plt.plot(time, f(f(f(time))), color='yellow')


    if with_lines:
        plt.plot([X[i] for i in range(len(X)) if i%2==0][:-1], [X[i] for i in␣
↪range(len(X)) if i%2==0][1:], color="red")
    if with_dots:
        plt.scatter([X[i] for i in range(len(X)) if i%2==0][:-1], [X[i] for i␣
↪in range(len(X)) if i%2==0][1:], color="red")
    #print([X[i] for i in range(len(X)) if i%2==0])
    plt.title(r"          -               $f(x) = (1+r)x - rx^2$"+f"   {x_0=}, ␣
↪{r=}")
    plt.grid(True)
    plt.show()
```

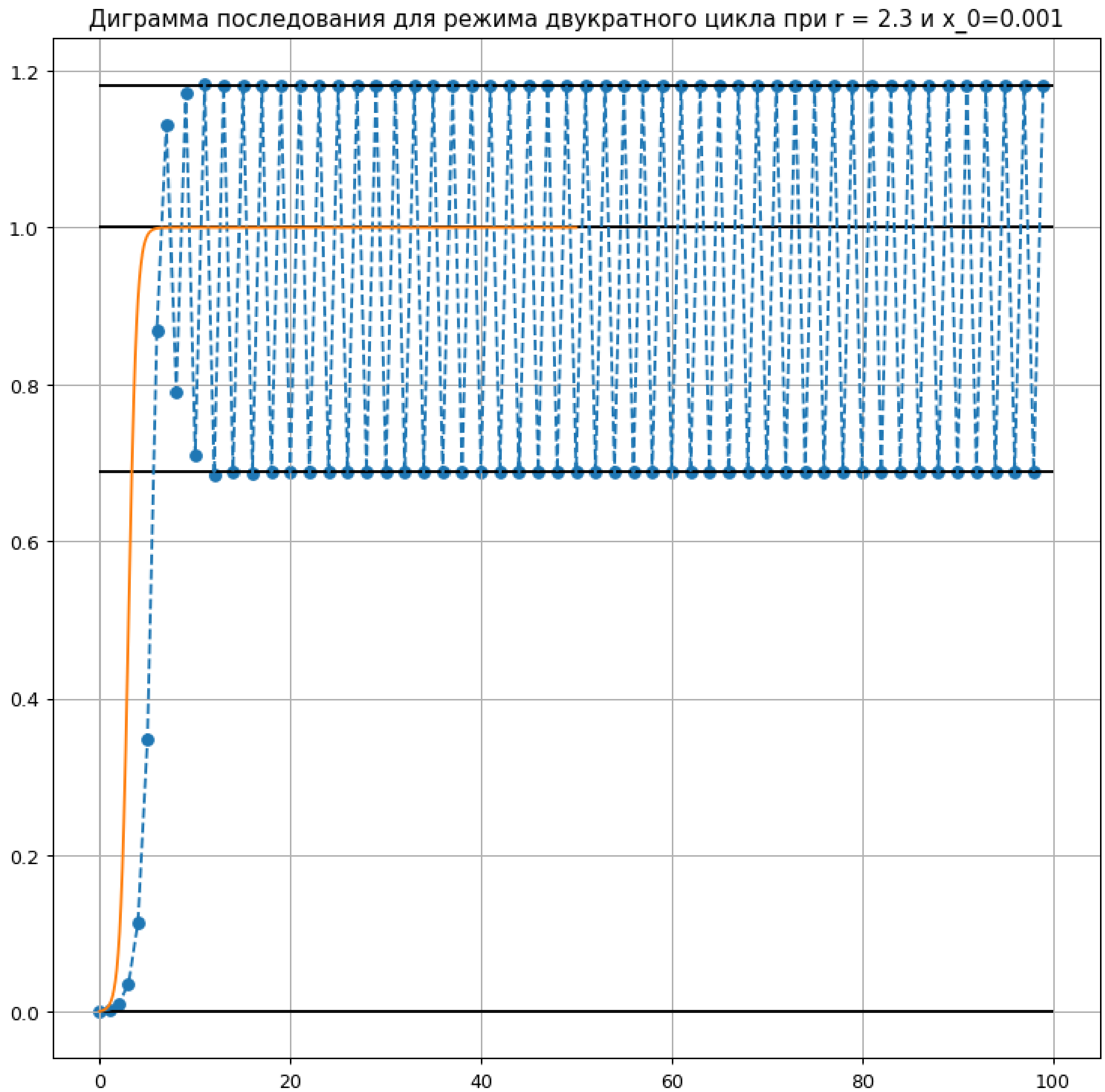
```
[67]: def analyze(configs):
    for i in configs:
        KenLam(i['times'], i['x_0'], i['r'], with_arrows=i['with_arrows'],␣
↪with_dots=i['with_dots'], constant_size=i['constant_size'],
               with_3cr=i['with_3cr'], with_4cr=i['with_4cr'],␣
↪arrow_c=i['arrow_colors'], with_lines=i['with_lines'])
        plotANAL(i['r'], i['x_0'], i['times'])

def calculateStable(r):
    return (2*r + r**2 - r*(-4 + r**2)**0.5)/(2*r**2), (2*r + r**2 + r*(-4 +␣
↪r**2)**0.5)/(2*r**2)

configs=[
    {
        'times': 100,
        'x_0': 0.001,
        'r': 2.3,
        'with_arrows': True,
        'with_dots': True,
        'with_3cr': False,
        'with_4cr': False,
        'arrow_colors': ['black'],
        'with_lines': False,
        'constant_size': False
    },
    {
        'times': 100,
        'x_0': 0.1,
        'r': 1.9,
        'with_arrows': True,
        'with_dots': True,
        'with_3cr': False,
        'with_4cr': False,
        'arrow_colors': ['black'],
        'with_lines': False,
        'constant_size': False
    },
    {
        'times': 100,
        'x_0': 0.6,
        'r': 2,
```

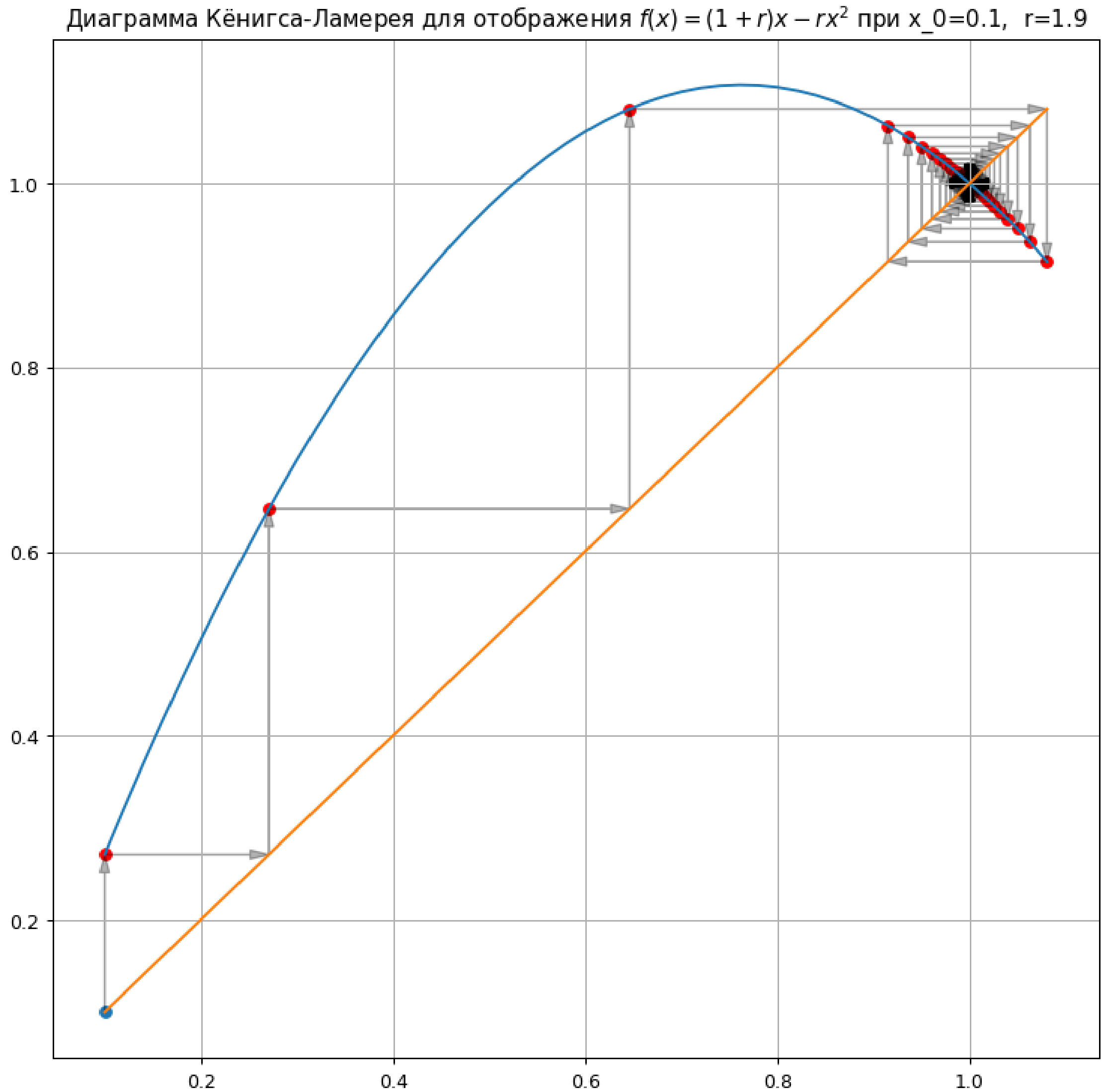
```
        'with_arrows': True,
        'with_dots': True,
        'with_3cr': False,
        'with_4cr': False,
        'arrow_colors': ['black'],
        'with_lines': False,
        'constant_size': False
    },
    {
        'times': 100,
        'x_0': calculateStable(2.05)[0],
        'r': 2.05,
        'with_arrows': True,
        'with_dots': True,
        'with_3cr': False,
        'with_4cr': False,
        'arrow_colors': ['black'],
        'with_lines': False,
        'constant_size': True
    },
    {
        'times': 100,
        'x_0': calculateStable(2.05)[1],
        'r': 2.05,
        'with_arrows': True,
        'with_dots': True,
        'with_3cr': False,
        'with_4cr': False,
        'arrow_colors': ['black'],
        'with_lines': False,
        'constant_size': True
    },
    {
        'times': 100,
        'x_0': 0.1,
        'r': 2.55,
        'with_arrows': True,
        'with_dots': True,
        'with_3cr': False,
        'with_4cr': False,
        'arrow_colors': ['black'],
        'with_lines': False,
        'constant_size': False
    },
    {
        'times': 100,
        'x_0': 0.01,
```

```
        'r': -2.46,
        'with_arrows': True,
        'with_dots': True,
        'with_3cr': False,
        'with_4cr': False,
        'arrow_colors': ['black'],
        'with_lines': False,
        'constant_size': False
    }
]


analyze(configs)

# KenLam(500, 0.001, -2.3, with_arrows=True, with_3cr=False, with_4cr=True,␣
 ↪arrow_c=['black'], with_lines=False)
# KenLam(500, 0.1, 1.9, with_arrows=True, with_3cr=False, with_4cr=True,␣
 ↪arrow_c=['black'], with_lines=False)

# r = 2
# x=(2*r + r**2 - r*(-4 + r**2)**0.5)/(2*r**2)
# KenLam(100, x, r, with_arrows=True, with_3cr=True, constant_size=True,␣
 ↪arrow_c=['black'])
# x = (2*r + r**2 + r*(-4 + r**2)**0.5)/(2*r**2)
# KenLam(100, x, r, with_arrows=True, with_3cr=True, constant_size=True,␣
 ↪arrow_c=['black'])

# r = 2.05
# x=(2*r + r**2 - r*(-4 + r**2)**0.5)/(2*r**2)
# KenLam(100, x, r, with_arrows=True, with_3cr=True, constant_size=True,␣
 ↪arrow_c=['black'])
# x = (2*r + r**2 + r*(-4 + r**2)**0.5)/(2*r**2)
# KenLam(100, x, r, with_arrows=True, with_3cr=True, constant_size=True,␣
 ↪arrow_c=['black'])

# KenLam(100, 0.1, 2.1, with_arrows=True, with_3cr=True, constant_size=True,␣
 ↪arrow_c=['black'], with_lines=False)
# KenLam(100, 0.1, 2.3, with_arrows=True, with_3cr=True, constant_size=True,␣
 ↪arrow_c=['black'], with_lines=False)
# KenLam(100, 0.1, 2.45, with_arrows=True, with_3cr=True, constant_size=True,␣
 ↪arrow_c=['black'], with_lines=False)
# KenLam(500, 0.1, 2.5, with_arrows=True, with_3cr=False, with_4cr=True,␣
 ↪arrow_c=['black'], with_lines=False)
# KenLam(500, 0.1, 2.55, with_arrows=True, with_3cr=False, with_4cr=True,␣
 ↪arrow_c=['black'], with_lines=False)
```

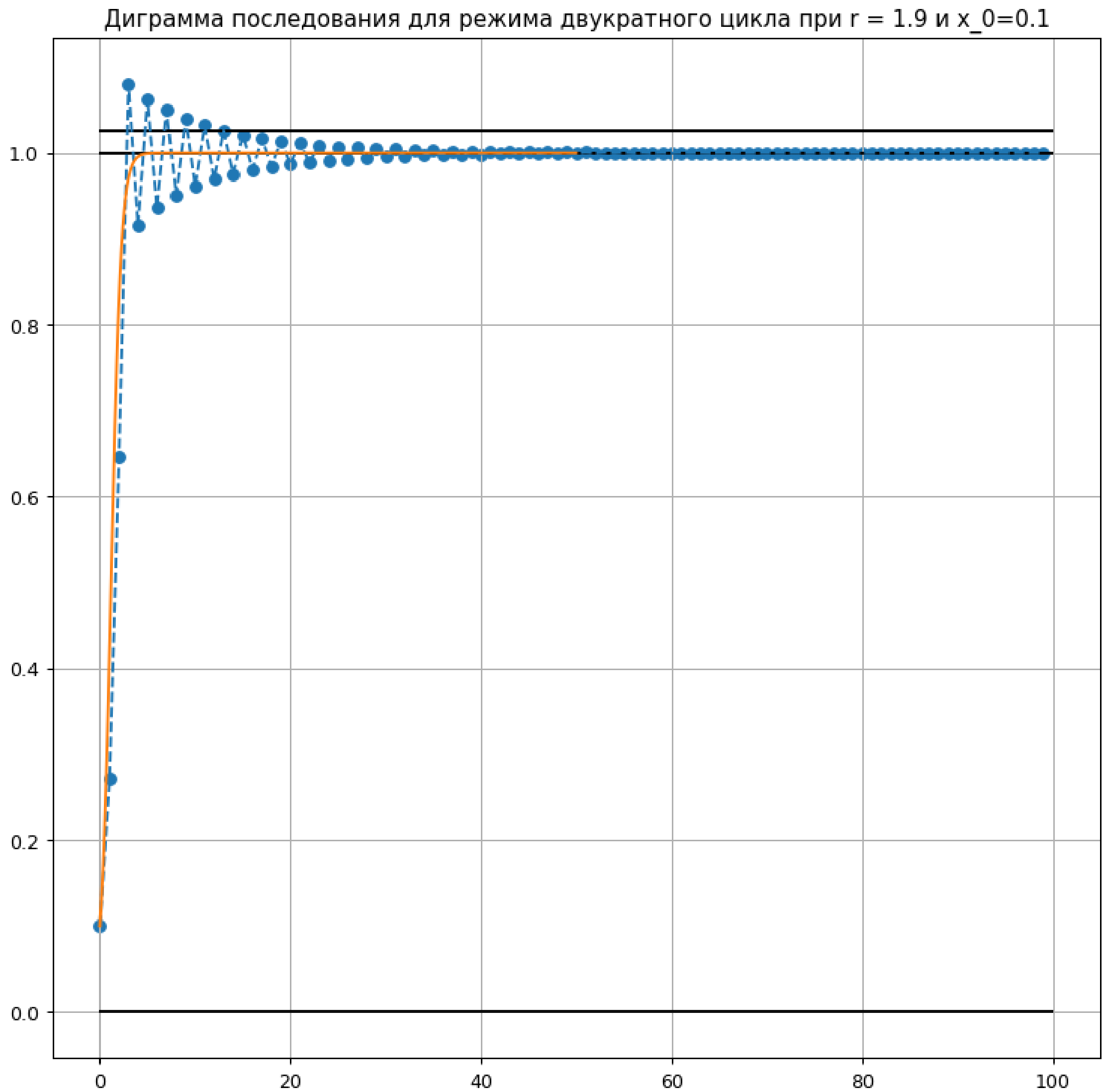
```
# KenLam(500, 0.1, 2.57, with_arrows=True, with_3cr=False, with_4cr=True,␣
↪arrow_c=['black'], with_lines=False)
# KenLam(1000, 0.01, -2.46, with_arrows=True, with_3cr=False, with_4cr=True,␣
↪arrow_c=['black'], with_lines=False)
```




Диграмма последования для режима двукратного цикла при r = 2.3 и x_0=0.001
1.2
1.0
0.8
0.6
0.4
0.2
0.0
0
20
40
60
80
100




```
c:\program files\python39\lib\site-packages\numpy\ma\core.py:3379:
ComplexWarning: Casting complex values to real discards the imaginary part
  _data[indx] = dval
c:\program files\python39\lib\site-packages\matplotlib\cbook\__init__.py:1333:
ComplexWarning: Casting complex values to real discards the imaginary part
  return np.asarray(x, float)
```



Диграмма последования для режима двукратного цикла при r = 1.9 и x_0=0.1
1.0
0.8
0.6
0.4
0.2
0.0
0
20
40
60
80
100

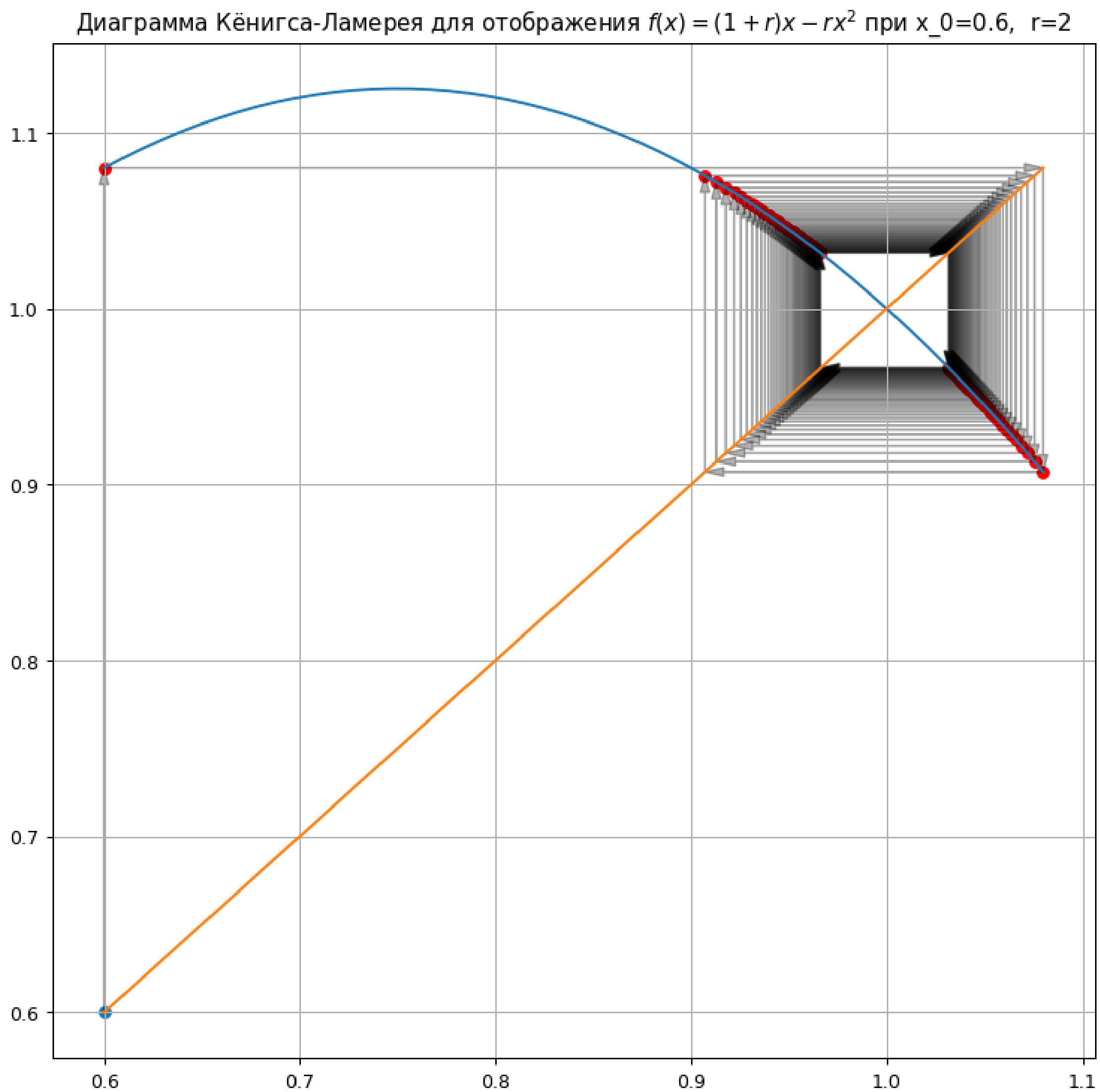


Диаграмма Кёнигса-Ламерея для отображения $f(x) = (1+r)x - rx^2$ при x_0=0.6, r=2
1.1
1.0
0.9
0.8
0.7
0.6
0.6
0.7
0.8
0.9
1.0
1.1

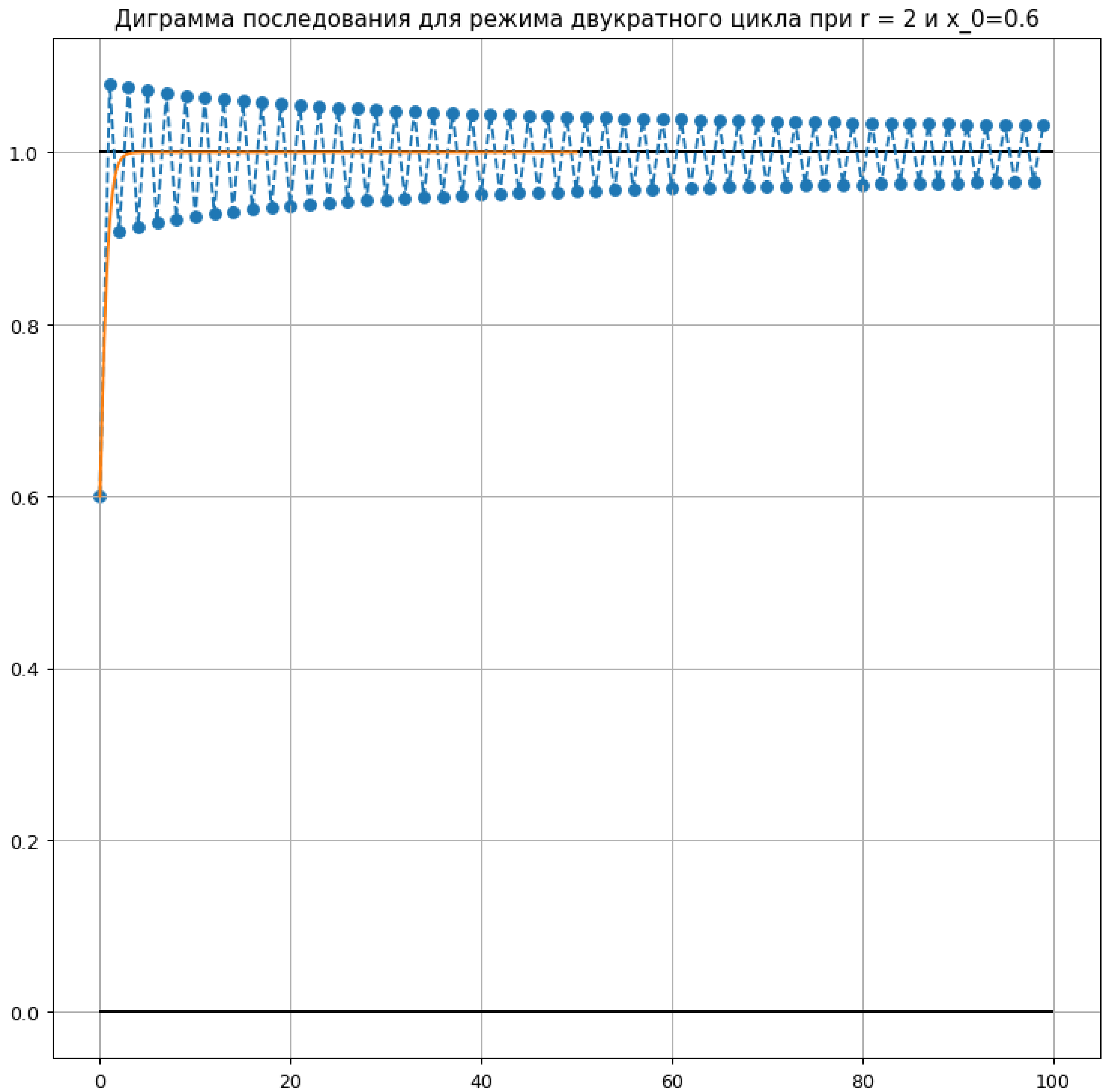
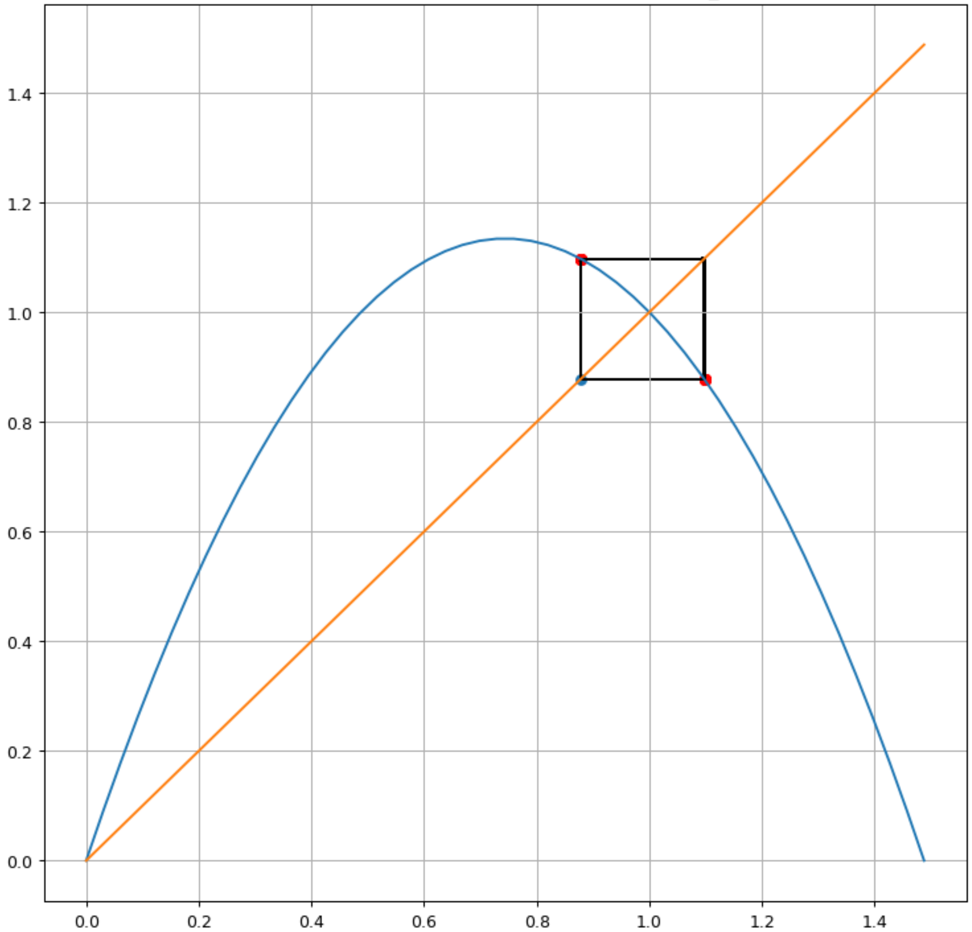
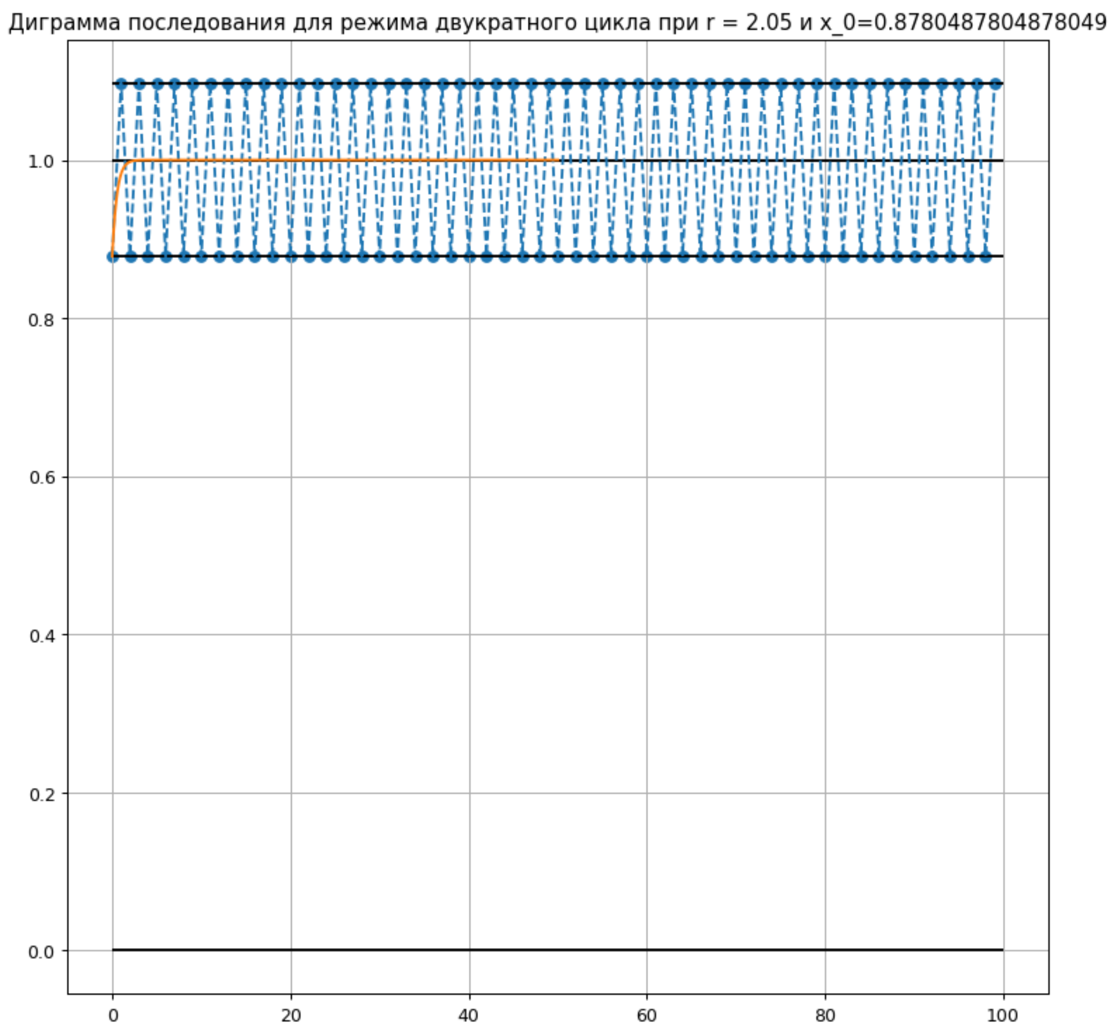

Диграмма последования для режима двукратного цикла при r = 2 и x_0=0.6
1.0
0.8
0.6
0.4
0.2
0.0
0
20
40
60
80
100

Диаграмма Кёнигса-Ламерея для отображения $f(x) = (1+r)x - rx^2$ при x_0=0.8780487804878049, r=2.05




Диграмма последования для режима двукратного цикла при r = 2.05 и x_0=0.8780487804878049
1.0
0.8
0.6
0.4
0.2
0.0
0
20
40
60
80
100

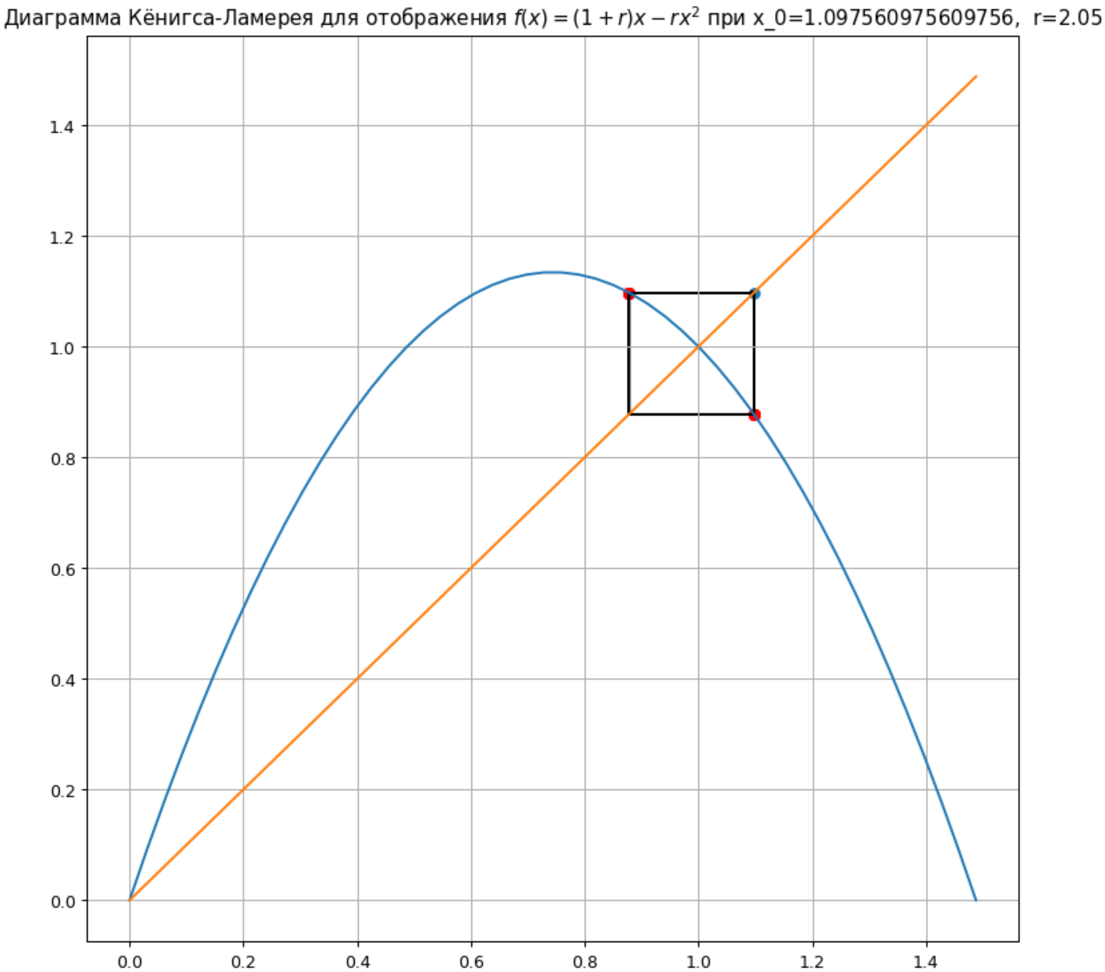
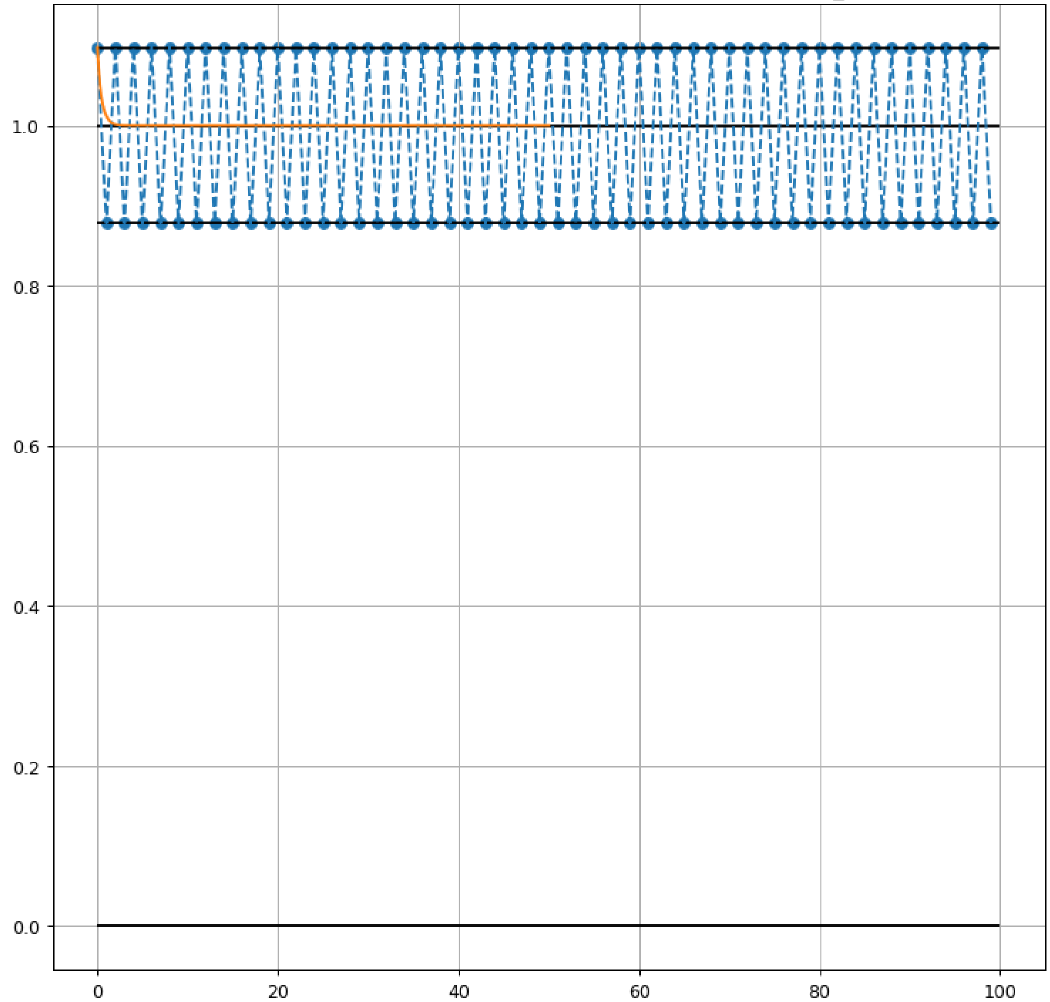


Диаграмма Кёнигса-Ламерея для отображения $f(x) = (1+r)x - rx^2$ при x_0=1.097560975609756, r=2.05
0.0
0.2
0.4
0.6
0.8
1.0
1.2
1.4

Диграмма последования для режима двукратного цикла при r = 2.05 и x_0=1.097560975609756



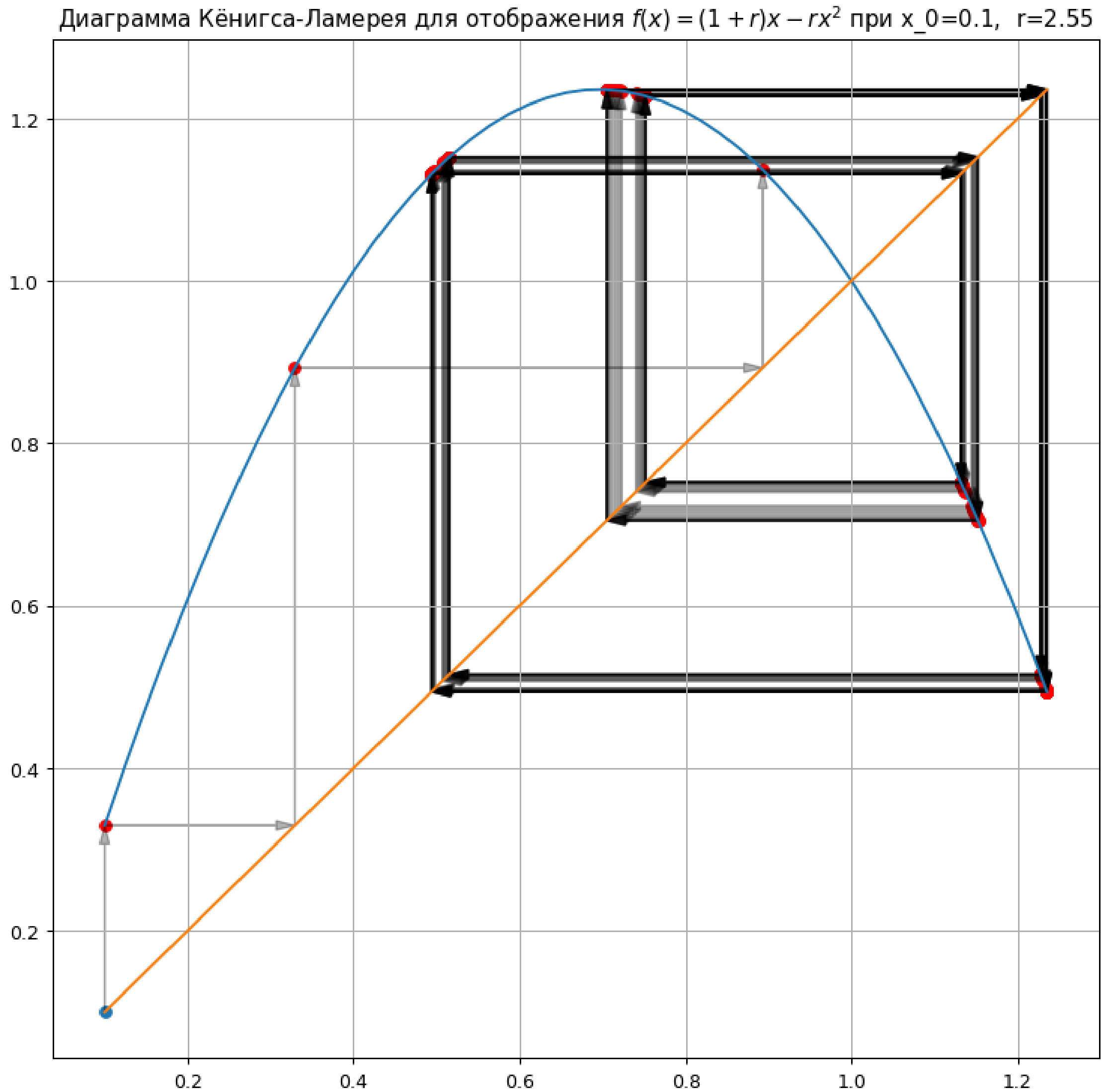
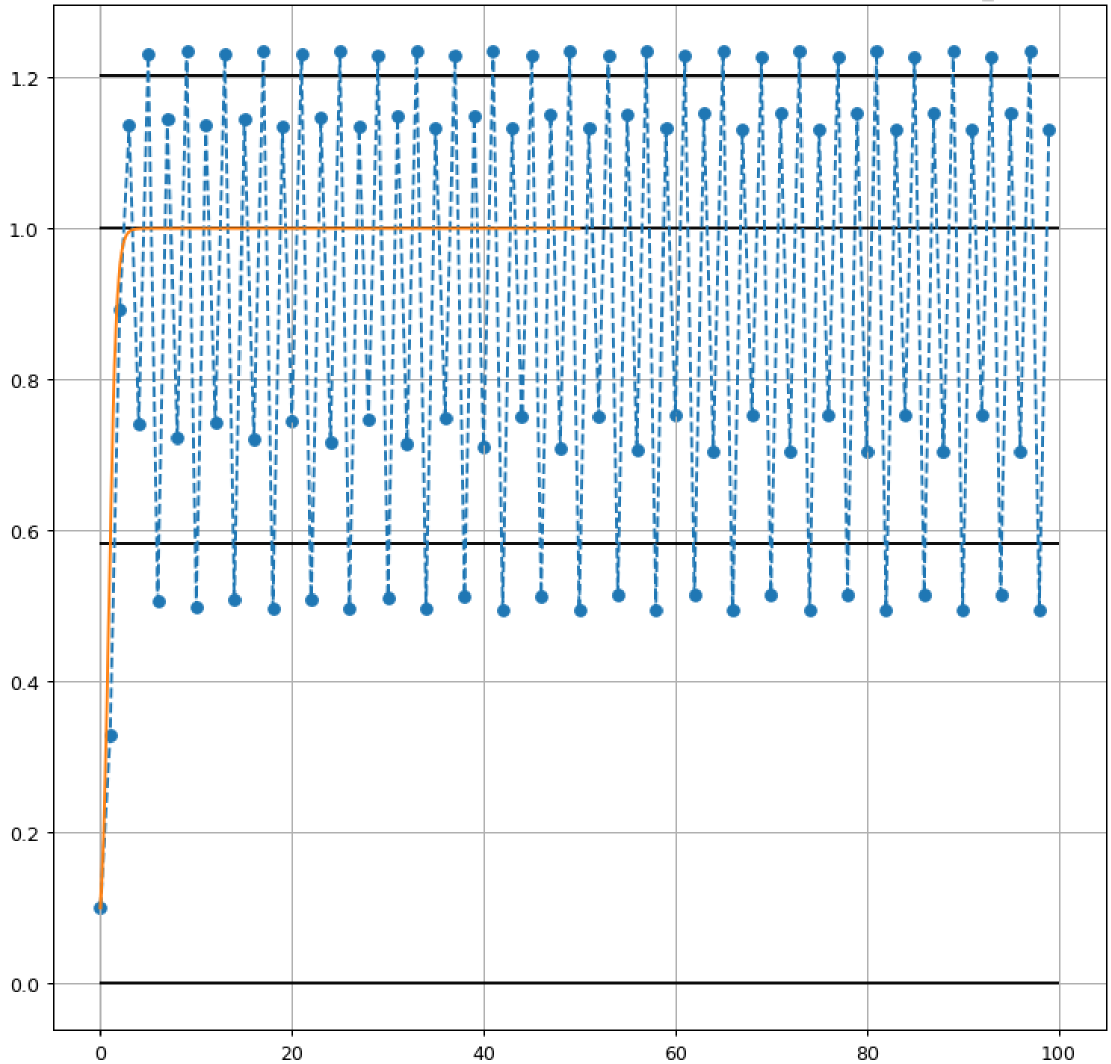


Диаграмма Кёнигса-Ламерея для отображения $f(x) = (1+r)x - rx^2$ при x_0=0.1, r=2.55
1.2
1.0
0.8
0.6
0.4
0.2
0.2
0.4
0.6
0.8
1.0
1.2

Диграмма последования для режима двукратного цикла при r = 2.55 и x_0=0.1



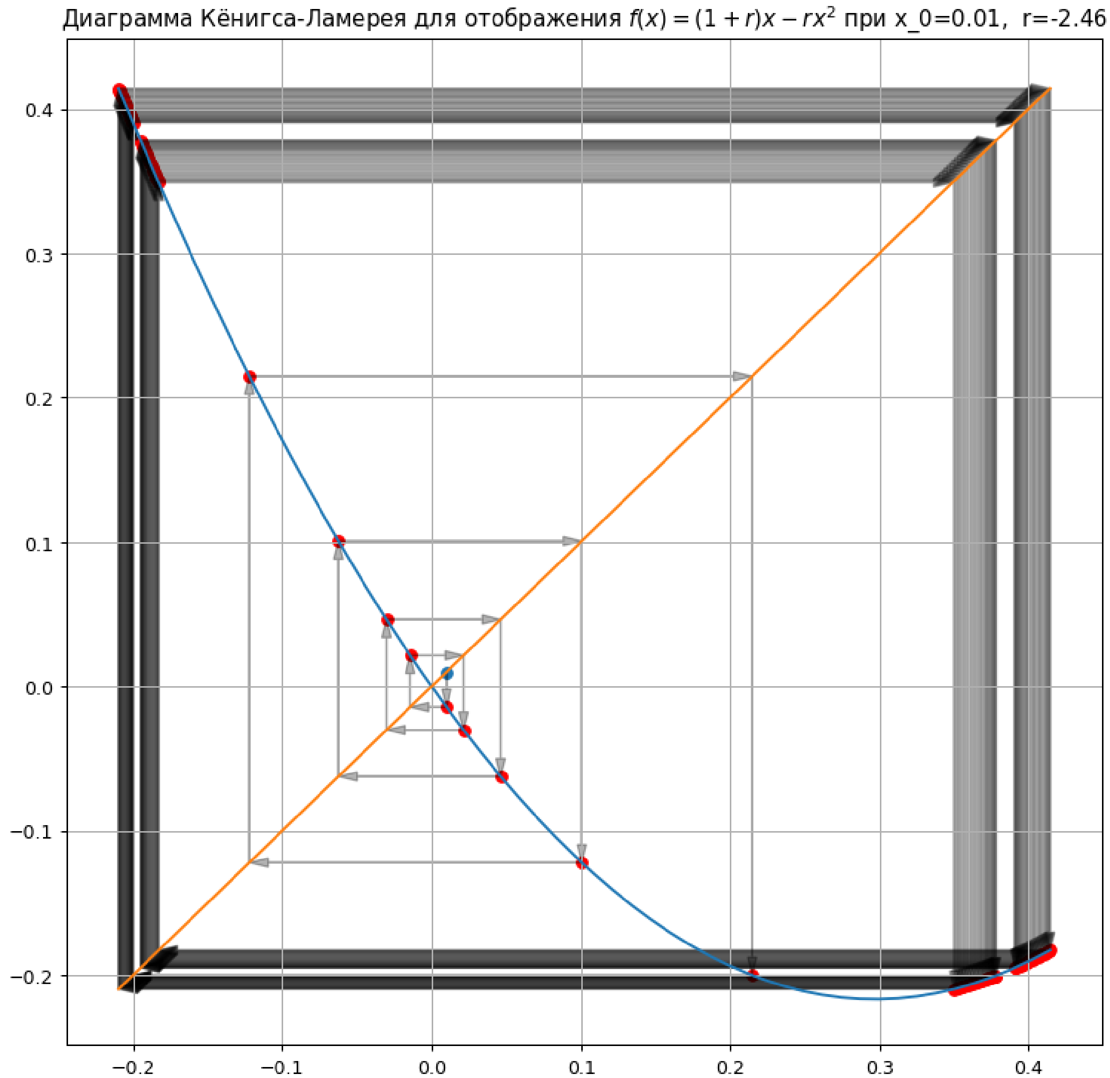
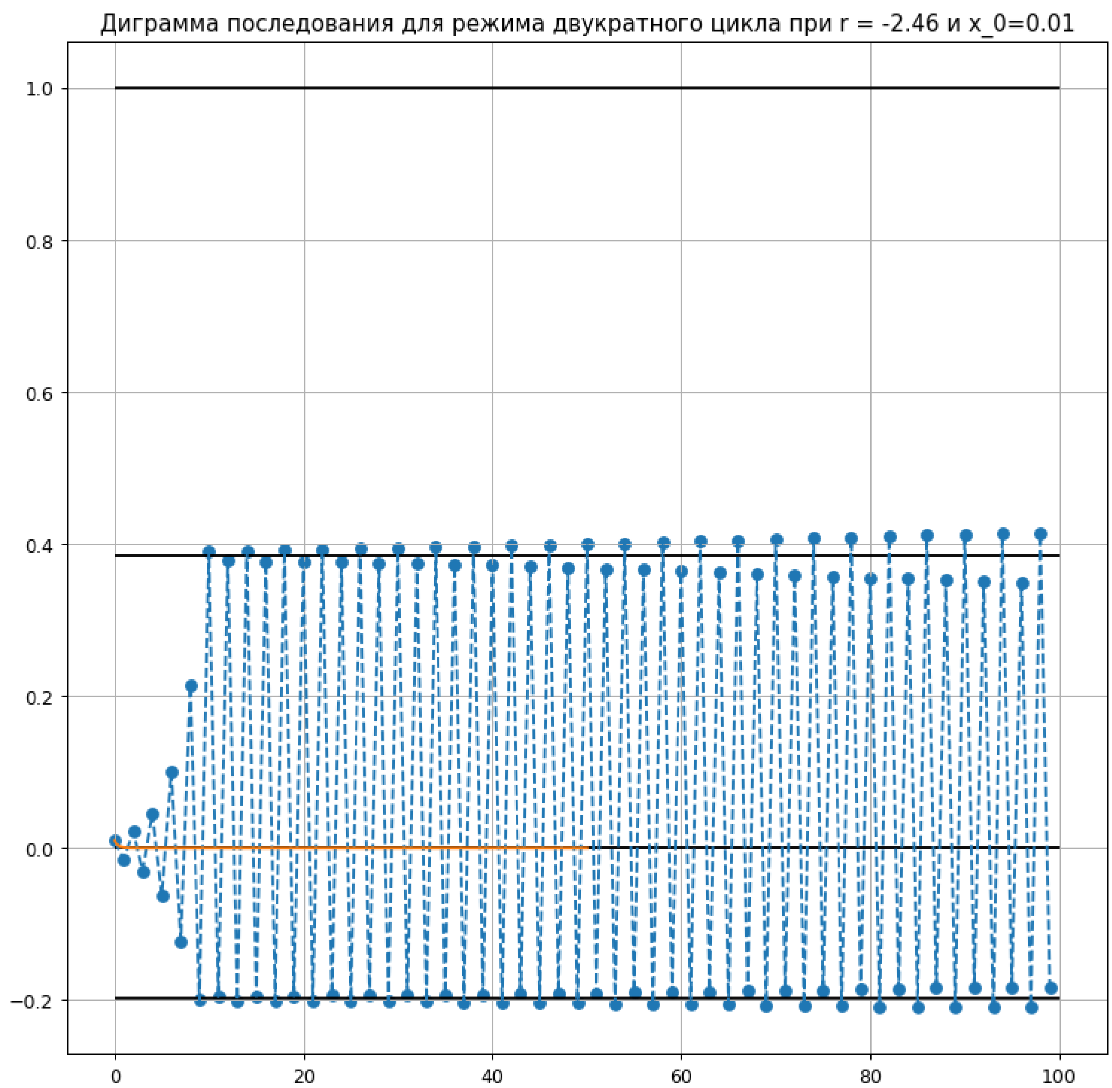


Диаграмма Кёнигса-Ламерея для отображения $f(x) = (1+r)x - rx^2$ при x_0=0.01, r=-2.46
0.4
0.3
0.2
0.1
0.0
−0.1
−0.2
−0.2
−0.1
0.0
0.1
0.2
0.3
0.4



0.1

[110]:
```
config2=[
    {
        'times': 250,
        'x_0': calculateStable(2.6)[0],
        'r': 2.6,
        'with_arrows': True,
        'with_dots': True,
        'with_3cr': False,
        'with_4cr': False,
        'arrow_colors': ['black'],
        'with_lines': False,
```

```python
            'constant_size': False
    },
    {
        'times': 250,
        'x_0': calculateStable(2.6)[0]+0.01,
        'r': 2.6,
        'with_arrows': True,
        'with_dots': True,
        'with_3cr': False,
        'with_4cr': False,
        'arrow_colors': ['black'],
        'with_lines': False,
        'constant_size': False
    },
    {
        'times': 150,
        'x_0': 0.05,
        'r': -2.91,
        'with_arrows': True,
        'with_dots': True,
        'with_3cr': False,
        'with_4cr': False,
        'arrow_colors': ['black'],
        'with_lines': False,
        'constant_size': False
    },
    {
        'times': 150,
        'x_0': 0.1,
        'r': -2.91,
        'with_arrows': True,
        'with_dots': True,
        'with_3cr': False,
        'with_4cr': False,
        'arrow_colors': ['black'],
        'with_lines': False,
        'constant_size': False
    }
]

analyze(config2)
```

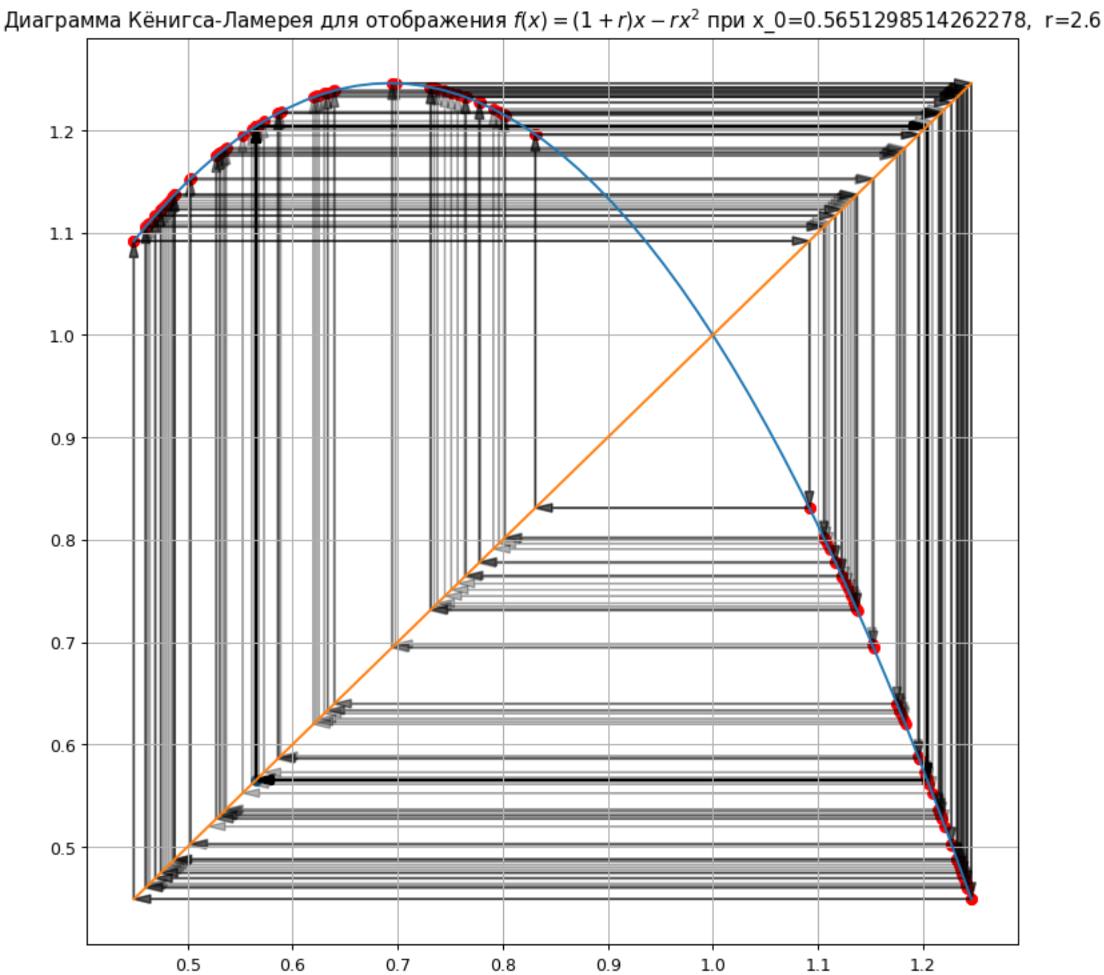


Диаграмма Кёнигса-Ламерея для отображения $f(x) = (1 + r)x - rx^2$ при x_0=0.5651298514262278, r=2.6
1.2
1.1
1.0
0.9
0.8
0.7
0.6
0.5
0.5
0.6
0.7
0.8
0.9
1.0
1.1
1.2

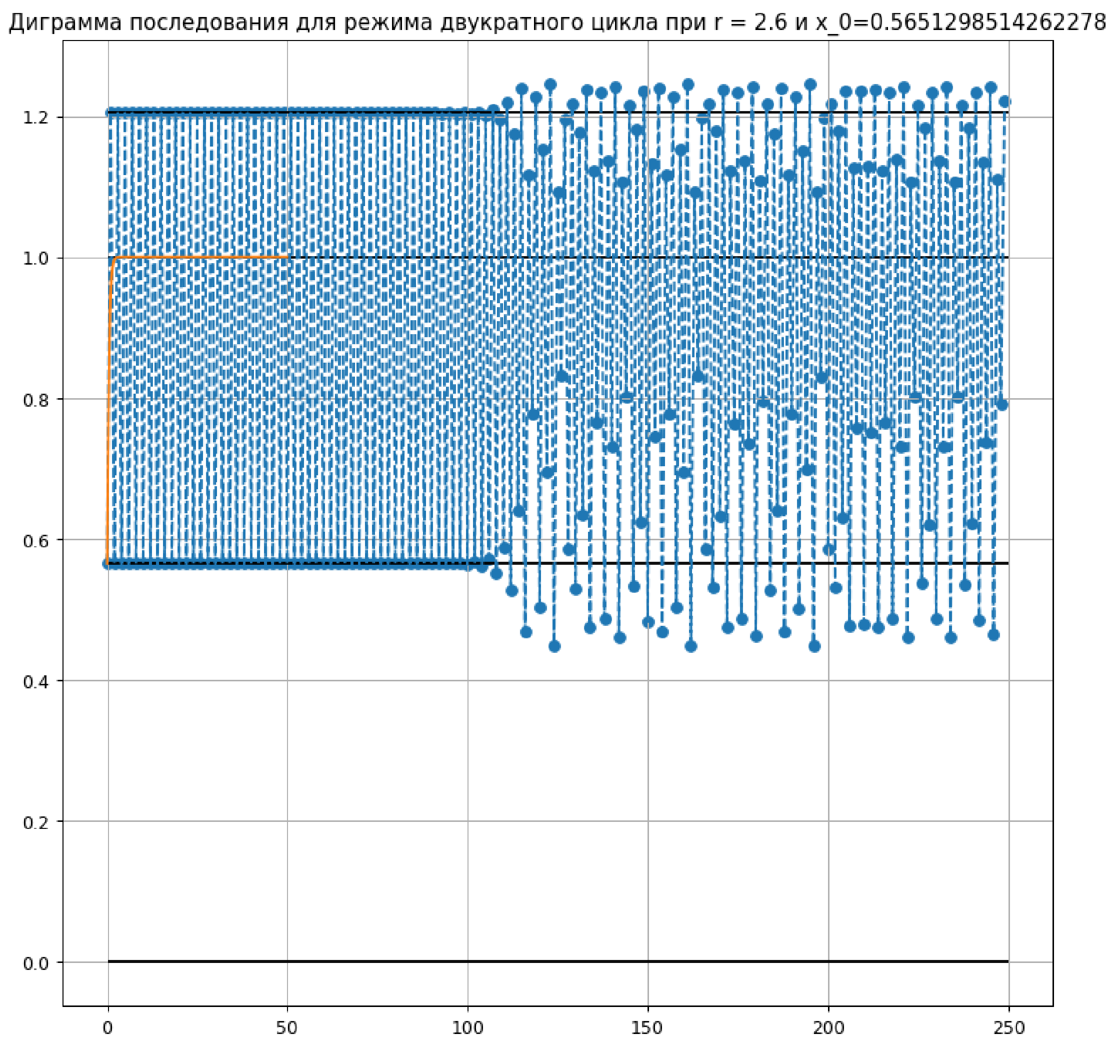
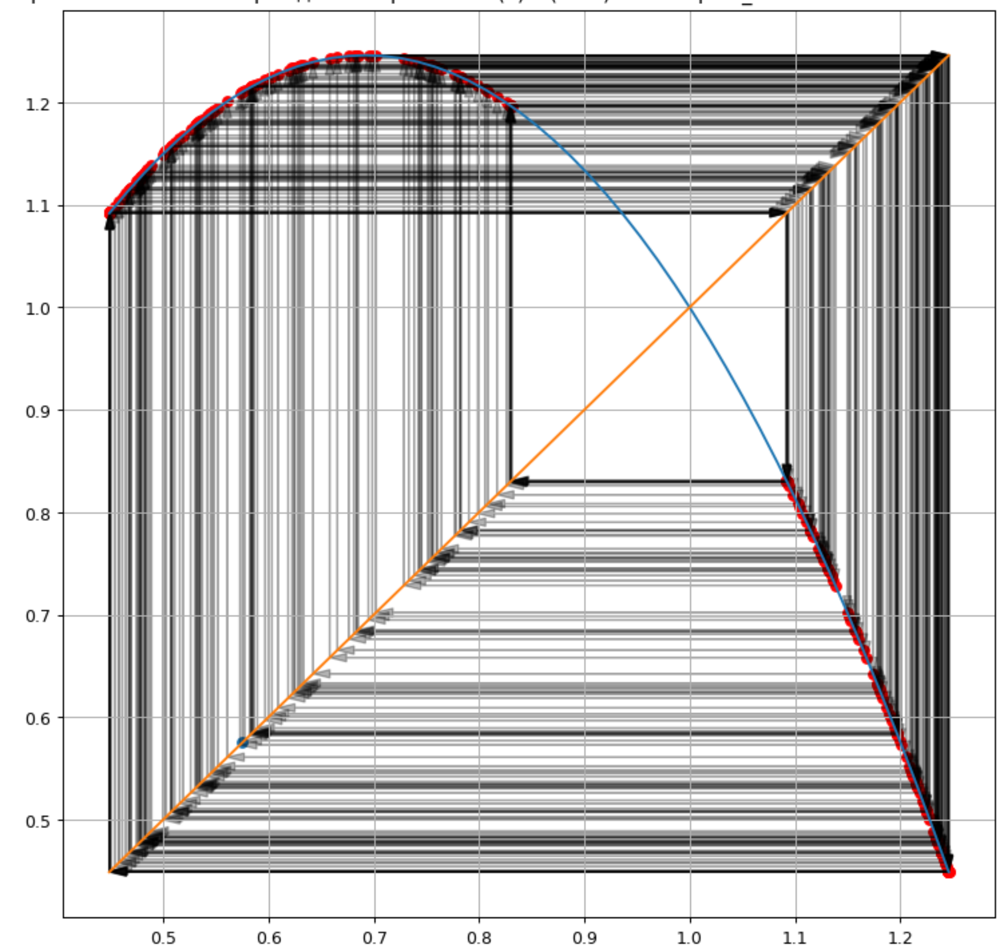
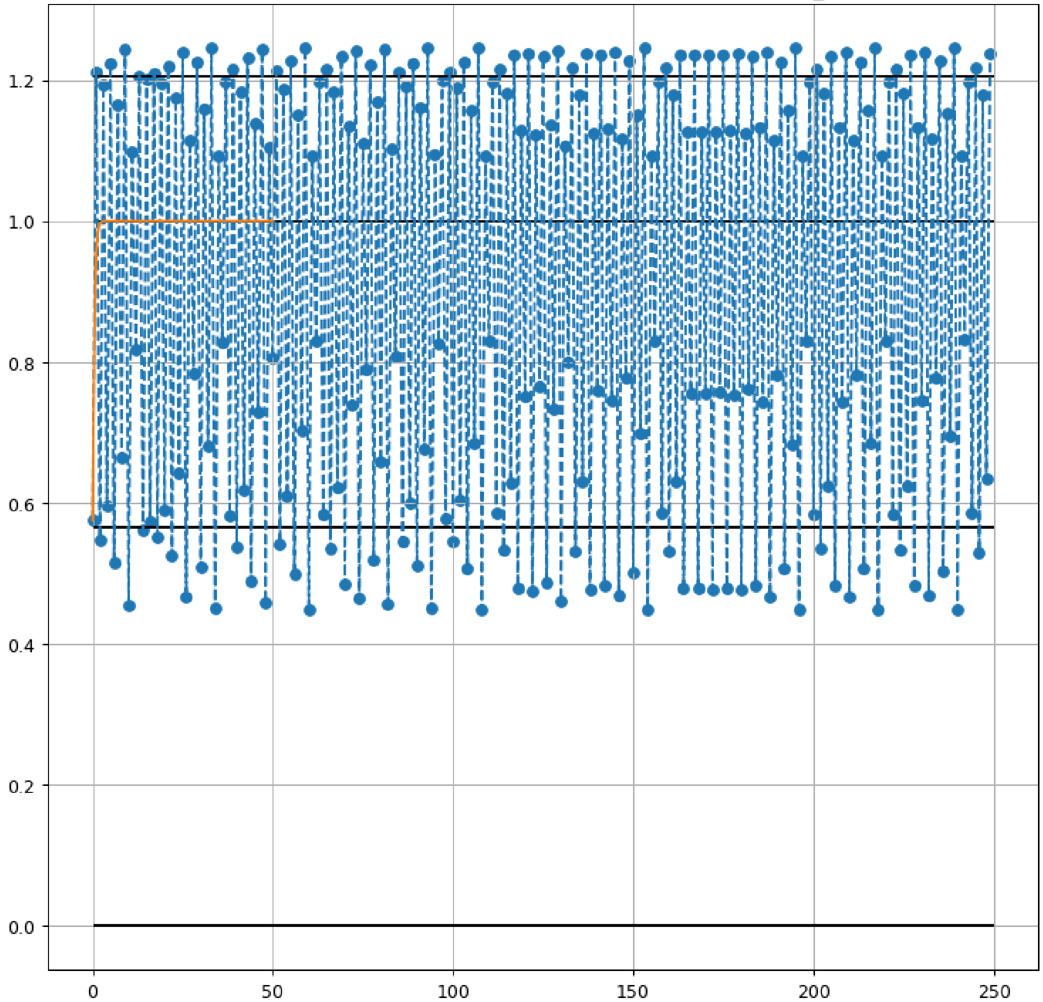

Диграмма последования для режима двукратного цикла при r = 2.6 и x_0=0.5651298514262278
1.2
1.0
0.8
0.6
0.4
0.2
0.0
0
50
100
150
200
250

Диаграмма Кёнигса-Ламерея для отображения $f(x) = (1 + r)x - rx^2$ при x_0=0.5751298514262279, r=2.6



Диграмма последования для режима двукратного цикла при r = 2.6 и x_0=0.5751298514262279



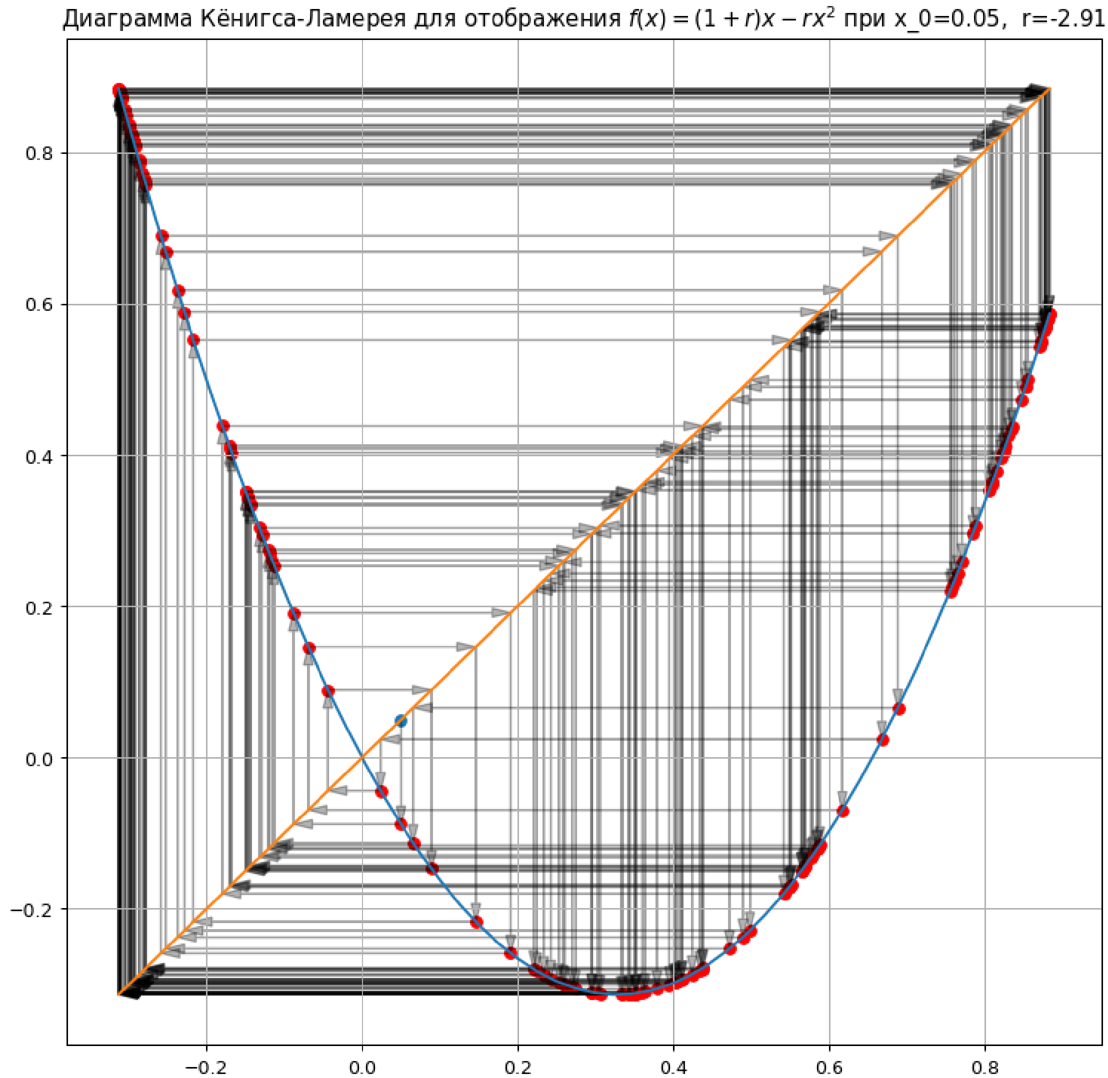


Диаграмма Кёнигса-Ламерея для отображения $f(x) = (1 + r)x - rx^2$ при x_0=0.05, r=-2.91
0.8
0.6
0.4
0.2
0.0
−0.2
−0.2
0.0
0.2
0.4
0.6
0.8

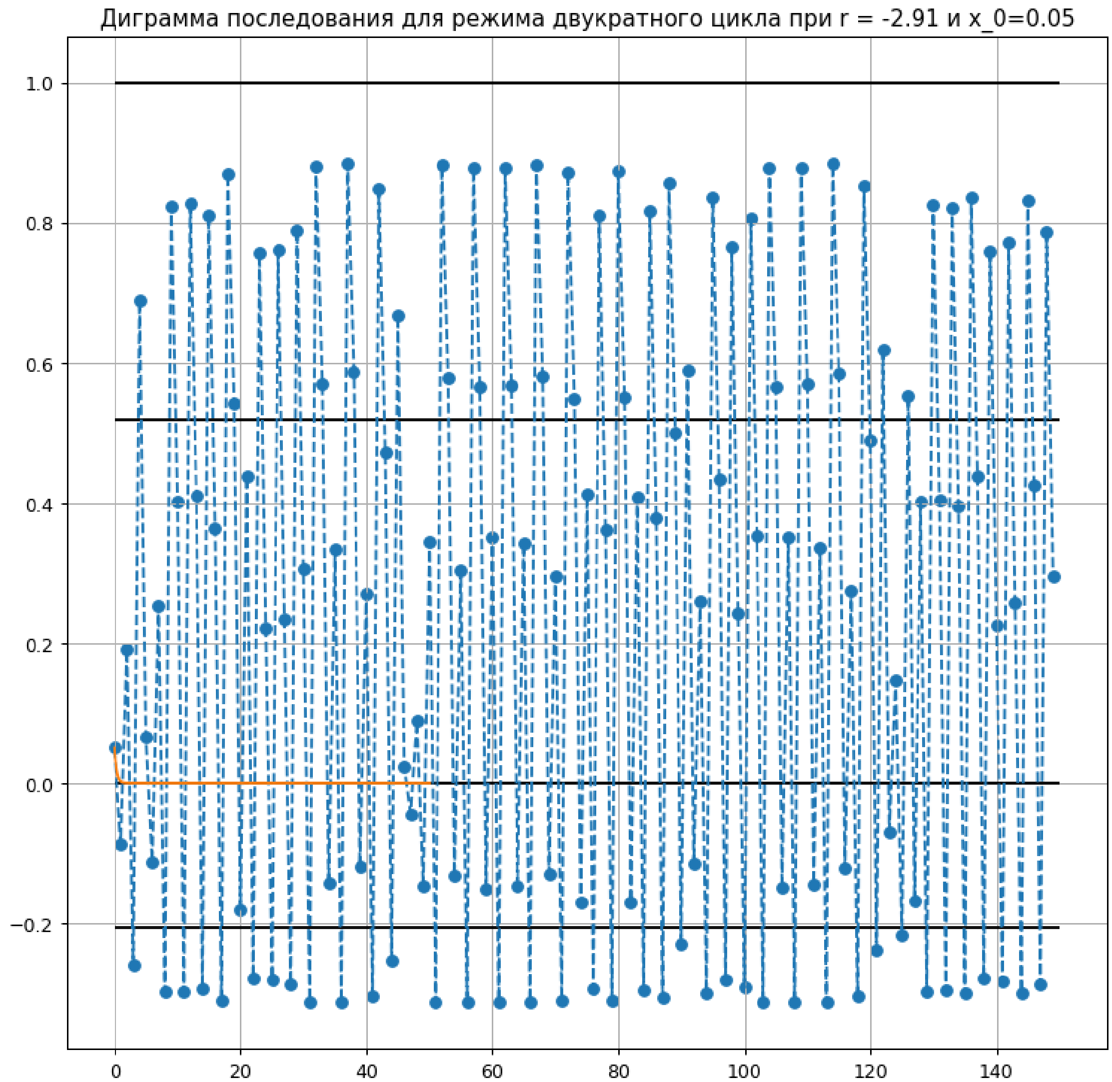
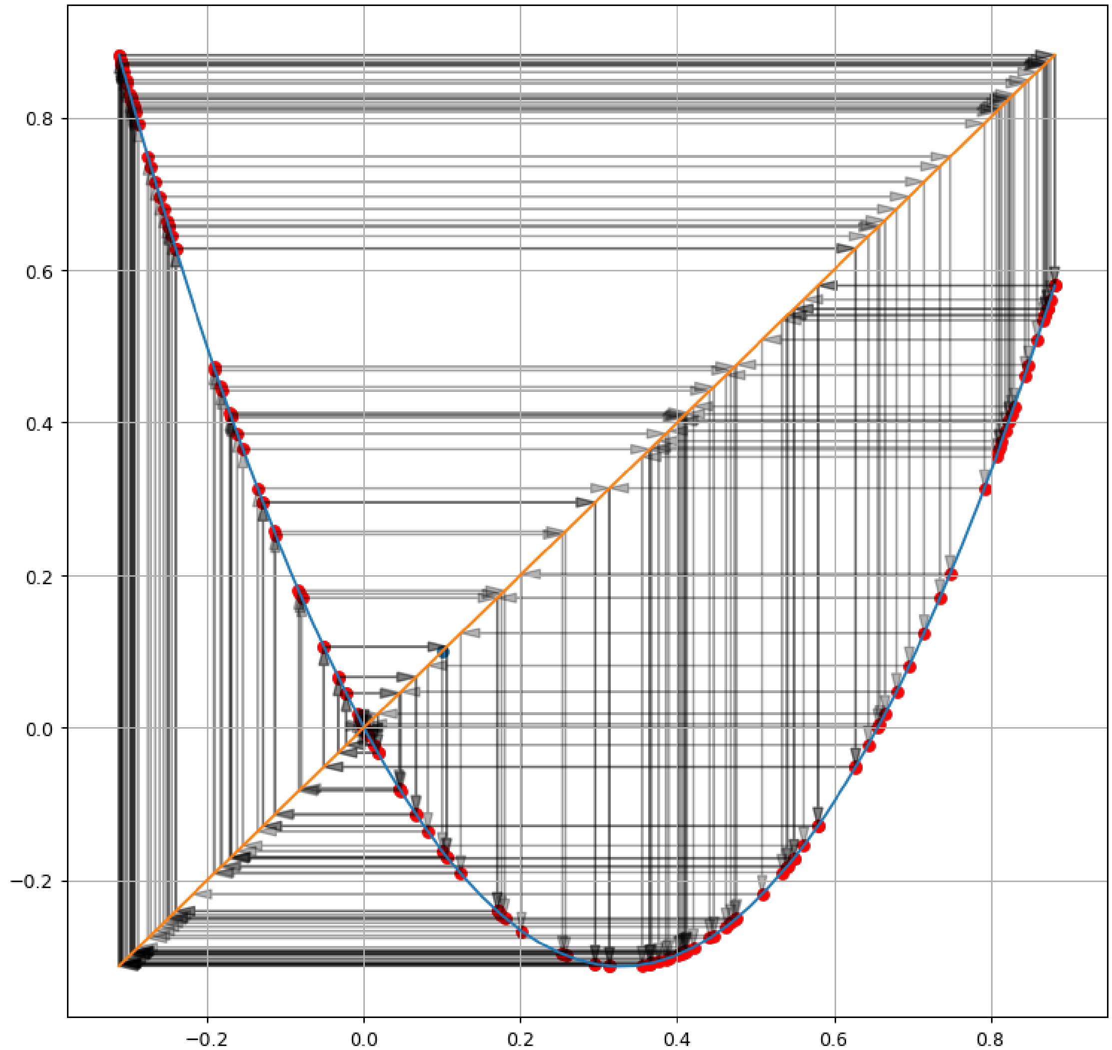
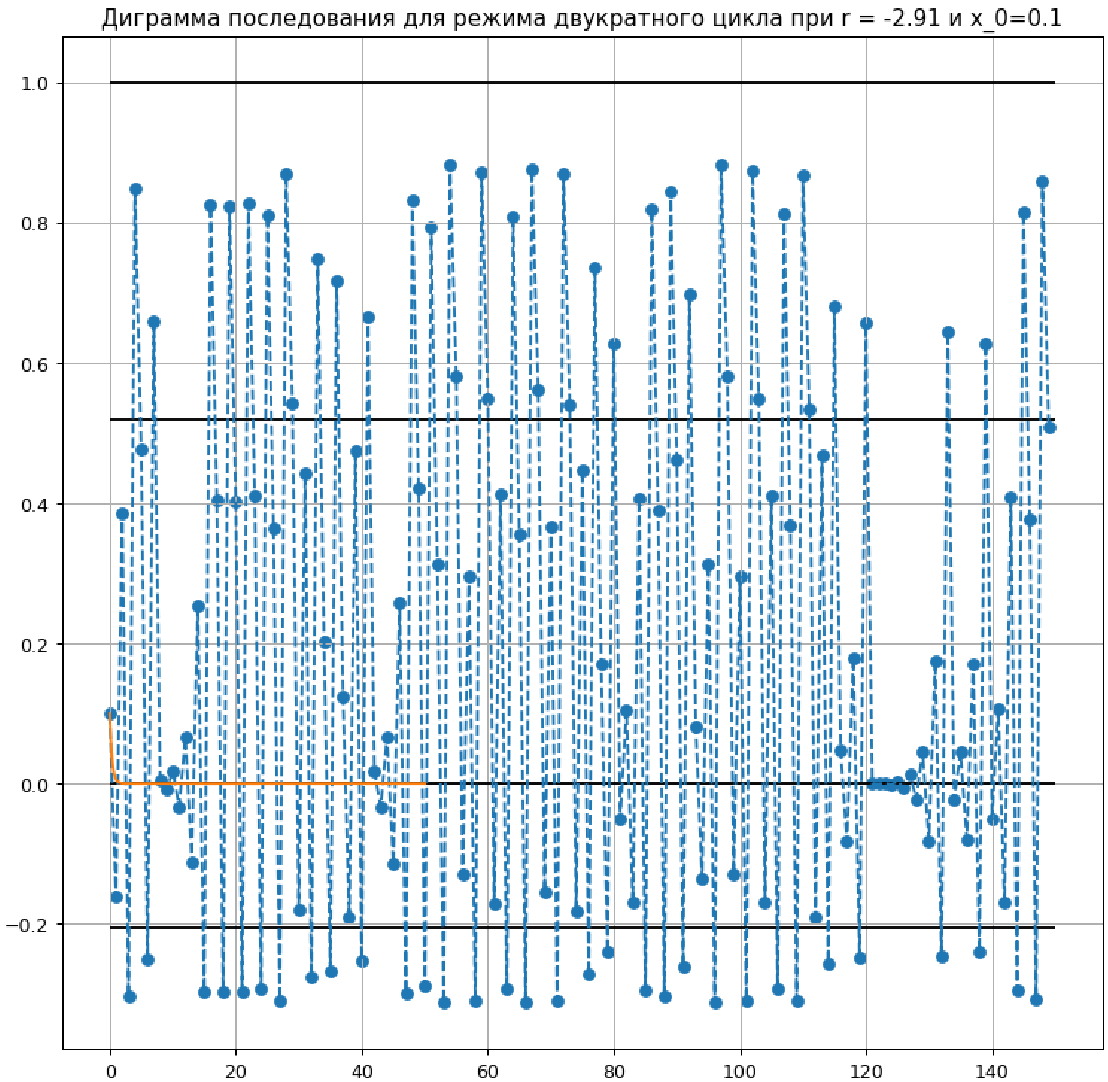

Диграмма последования для режима двукратного цикла при r = -2.91 и x_0=0.05
1.0
0.8
0.6
0.4
0.2
0.0
−0.2
0
20
40
60
80
100
120
140

Диаграмма Кёнигса-Ламерея для отображения $f(x) = (1+r)x - rx^2$ при x_0=0.1, r=-2.91




Диграмма последования для режима двукратного цикла при r = -2.91 и x_0=0.1
1.0
0.8
0.6
0.4
0.2
0.0
−0.2
0
20
40
60
80
100
120
140

[ ]: