Изначальная функция последования и производная по x от нее

```
In[7]:= F[x_] = (1 + r) * x - r * x^2;
```

```
In[8]:= G[x_] = D[F[x], x];
```

Исследования точек равновесия

```
In[9]:= Reduce[Abs[G[0]] < 1, r][[1]]
```

Out[9]= $-2 < \mathrm{Re}[r] < 0$

```
In[10]:= Reduce[Abs[G[0]] == 0, r]
```

Out[10]= $r == -1$

```
In[11]:= Reduce[Abs[G[1]] < 1, r][[1]]
```

Out[11]= $0 < \mathrm{Re}[r] < 2$

```
In[12]:= Reduce[Abs[G[1]] == 0, r]
```

Out[12]= $r == 1$

Исследования двухкратного цикла

```
In[13]:= U[x_] = F[F[x]];
```

```
In[14]:= Solve[U[x] == x, x]
```

Out[14]= $\left\{\{x \to 0\}, \{x \to 1\}, \left\{x \to \frac{2\,r + r^2 - r\sqrt{-4 + r^2}}{2\,r^2}\right\}, \left\{x \to \frac{2\,r + r^2 + r\sqrt{-4 + r^2}}{2\,r^2}\right\}\right\}$

```
In[15]:= x3 = Solve[U[x] == x, x][[3]][[1]][[2]];
x4 = Solve[U[x] == x, x][[4]][[1]][[2]];
```

Найдены границы двухкратного цикла. Найдем значения r при которых возникает состояние супер-устойчивости. Для этого посчитаем производную от функции F[F[x]]

```
In[17]:= dU[x_] = D[U[x]]
```

Out[17]= $(1 + r) \times \left((1 + r)\, x - r\, x^2\right) - r\left((1 + r)\, x - r\, x^2\right)^2$

```
In[18]:= Reduce[dU[x3] == 0, r]
```

Out[18]= $r == -2$

```
In[19]:= Reduce[dU[x4] == 0, r]
```

Out[19]= $r == -2$

Таким образом, получам двухкратный цикл в состоянии супер-устойчивости при r = -2

```
In[20]:= dU[x]
```

Out[20]= $(1 + r) \times \left((1 + r)\, x - r\, x^2\right) - r\left((1 + r)\, x - r\, x^2\right)^2$

```
In[21]:= U[x]
```

Out[21]= $(1 + r) \times \left((1 + r)\, x - r\, x^2\right) - r\left((1 + r)\, x - r\, x^2\right)^2$

In[22]:= `K[x_] = F[F[F[x]]]`

Out[22]= $(1+r)\times\left((1+r)\times\left((1+r)\,x-r\,x^2\right)-r\left((1+r)\,x-r\,x^2\right)^2\right)-r\left((1+r)\times\left((1+r)\,x-r\,x^2\right)-r\left((1+r)\,x-r\,x^2\right)^2\right)^2$

In[23]:= `Solve[K[x] == x, x]`

Out[23]= $\{\{x\to 0\},\{x\to 1\},$
$\{x\to \text{Root}[3+3r+r^2+(-6r-9r^2-5r^3-r^4)\,\#1+(9r^2+15r^3+9r^4+2r^5)\,\#1^2+(-10r^3-16r^4-8r^5-r^6)\,\#1^3+(8r^4+10r^5+3r^6)\,\#1^4+(-4r^5-3r^6)\,\#1^5+r^6\,\#1^6\,\&,1]\},$
$\{x\to \text{Root}[3+3r+r^2+(-6r-9r^2-5r^3-r^4)\,\#1+(9r^2+15r^3+9r^4+2r^5)\,\#1^2+(-10r^3-16r^4-8r^5-r^6)\,\#1^3+(8r^4+10r^5+3r^6)\,\#1^4+(-4r^5-3r^6)\,\#1^5+r^6\,\#1^6\,\&,2]\},$
$\{x\to \text{Root}[3+3r+r^2+(-6r-9r^2-5r^3-r^4)\,\#1+(9r^2+15r^3+9r^4+2r^5)\,\#1^2+(-10r^3-16r^4-8r^5-r^6)\,\#1^3+(8r^4+10r^5+3r^6)\,\#1^4+(-4r^5-3r^6)\,\#1^5+r^6\,\#1^6\,\&,3]\},$
$\{x\to \text{Root}[3+3r+r^2+(-6r-9r^2-5r^3-r^4)\,\#1+(9r^2+15r^3+9r^4+2r^5)\,\#1^2+(-10r^3-16r^4-8r^5-r^6)\,\#1^3+(8r^4+10r^5+3r^6)\,\#1^4+(-4r^5-3r^6)\,\#1^5+r^6\,\#1^6\,\&,4]\},$
$\{x\to \text{Root}[3+3r+r^2+(-6r-9r^2-5r^3-r^4)\,\#1+(9r^2+15r^3+9r^4+2r^5)\,\#1^2+(-10r^3-16r^4-8r^5-r^6)\,\#1^3+(8r^4+10r^5+3r^6)\,\#1^4+(-4r^5-3r^6)\,\#1^5+r^6\,\#1^6\,\&,5]\},$
$\{x\to \text{Root}[3+3r+r^2+(-6r-9r^2-5r^3-r^4)\,\#1+(9r^2+15r^3+9r^4+2r^5)\,\#1^2+(-10r^3-16r^4-8r^5-r^6)\,\#1^3+(8r^4+10r^5+3r^6)\,\#1^4+(-4r^5-3r^6)\,\#1^5+r^6\,\#1^6\,\&,6]\}\}$

Проверить систему на наличие трехкратных и более кратных циклов не представляет возможности

October 30, 2021

[1]:
```python
import scipy.integrate as integr
import numpy as np
import matplotlib.pyplot as plt
from scipy.signal import argrelextrema
import pylab

colors = ["#00FFFF",                # Azure
          "#0000FF",                # Blue
          "#FF0000",                # Red
          "#00FF00",                # Green
          "#D2691E",                #
          "#AAA662",
          "#9A0EEA",
          "#808000",
          "#FFA500",
          "#9ACD32",
          "#ED0DD9",
          "#FC5A50",
          "#929591",
          "#029386",
          "#C79FEF",
          "#FAC205",
          "#6E750E",
          "#06C2AC",
          "#CD5C5C",                # IndianRed
          "#A52A2A",                # Brown
          "#7B68EE",                # MediumState_blue
          "#4682B4",                # SteelBlue
          "#800000"                # Maroon
          ]
colors = 10*colors

mycolors = [
    '#000000',
    '#696969',
    '#A9A9A9',
    '#C0C0C0',
```

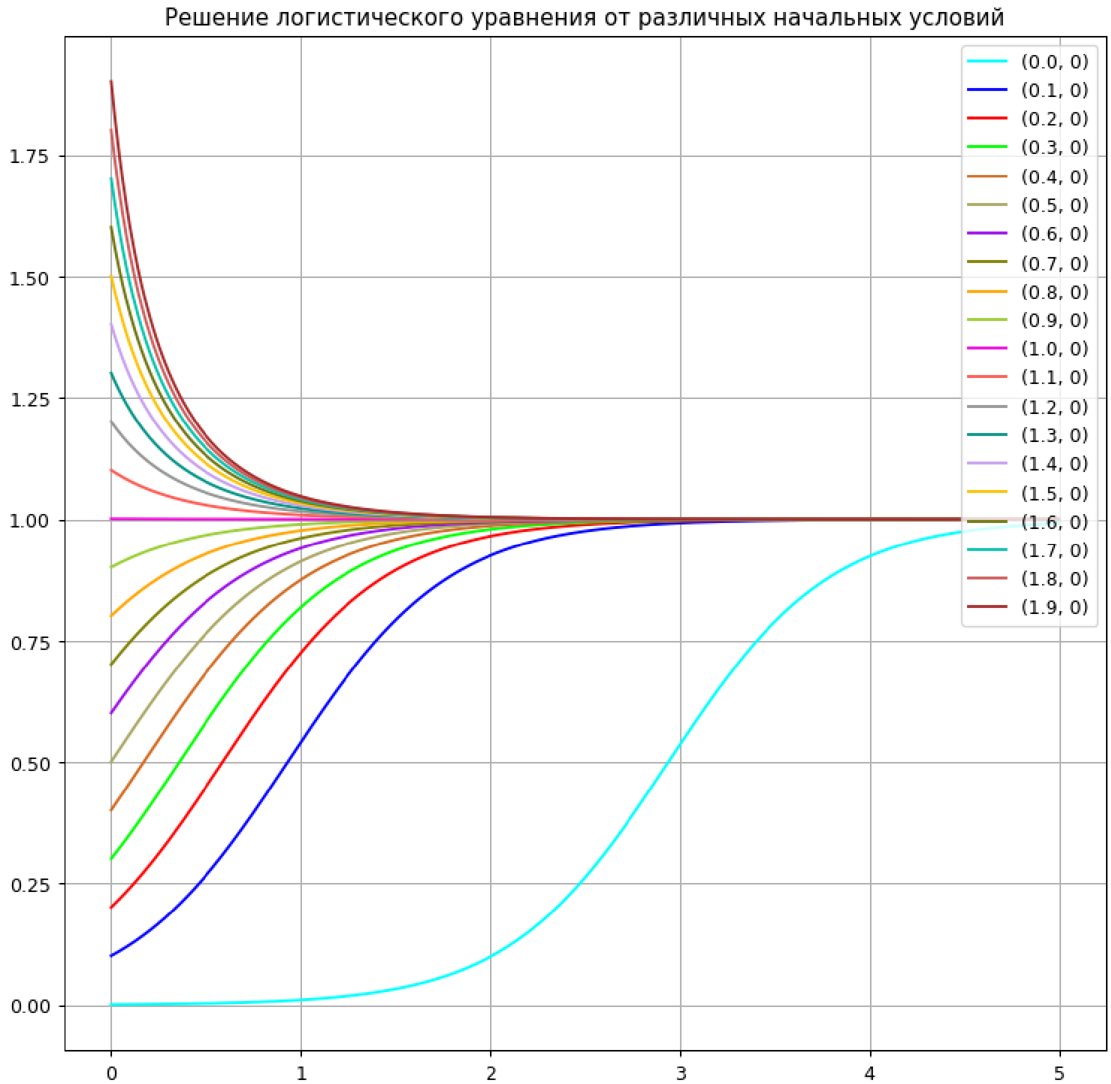
```
    '#D3D3D3',
    '#DCDCDC',
]

def mesh(x, y):
    general = []
    for i in range(len(x)):
        for j in range(len(y)):
            local = []
            local.append(x[i])
            local.append(y[j])
            general.append(local)
    return general

stat_dpi = 90
rect_pics = (7, 7)
big_pics = (10, 10)
dynamics_pics = (14, 7)
```

[2]:
```
def plotFP(y1, y2, centers = None, starts = None, color = "b"):
    fig = plt.figure(facecolor="white", figsize = rect_pics, dpi=stat_dpi)
    plt.plot(y1, y2, c = color)
    if centers is not None:
        for i in centers:
            plt.scatter(i[0], i[1])
    if starts is not None:
        for i in starts:
            plt.scatter(i[0], i[1])
    plt.grid(True)
    plt.show()
```

[3]:
```
def logistic(t, r, x_0, t_0):
    result = 1/(1 + (1/x_0 - 1) * np.exp(-r*(t-t_0)))
    return result
```

[4]:
```
time = np.linspace(0, 5, 500)
#print(y)
#print(time)

fig = plt.figure(facecolor="white", figsize = big_pics, dpi=stat_dpi)
step = 0.1
for i in np.arange(0.001, 2, step):
    plt.plot(time, logistic(time, 2.35, i, 0), c = colors[int(i/step)%30], label = f'({round(i, 2)}, 0)')
plt.title("                                    ")
plt.legend(loc="upper right")
plt.grid(True)
```

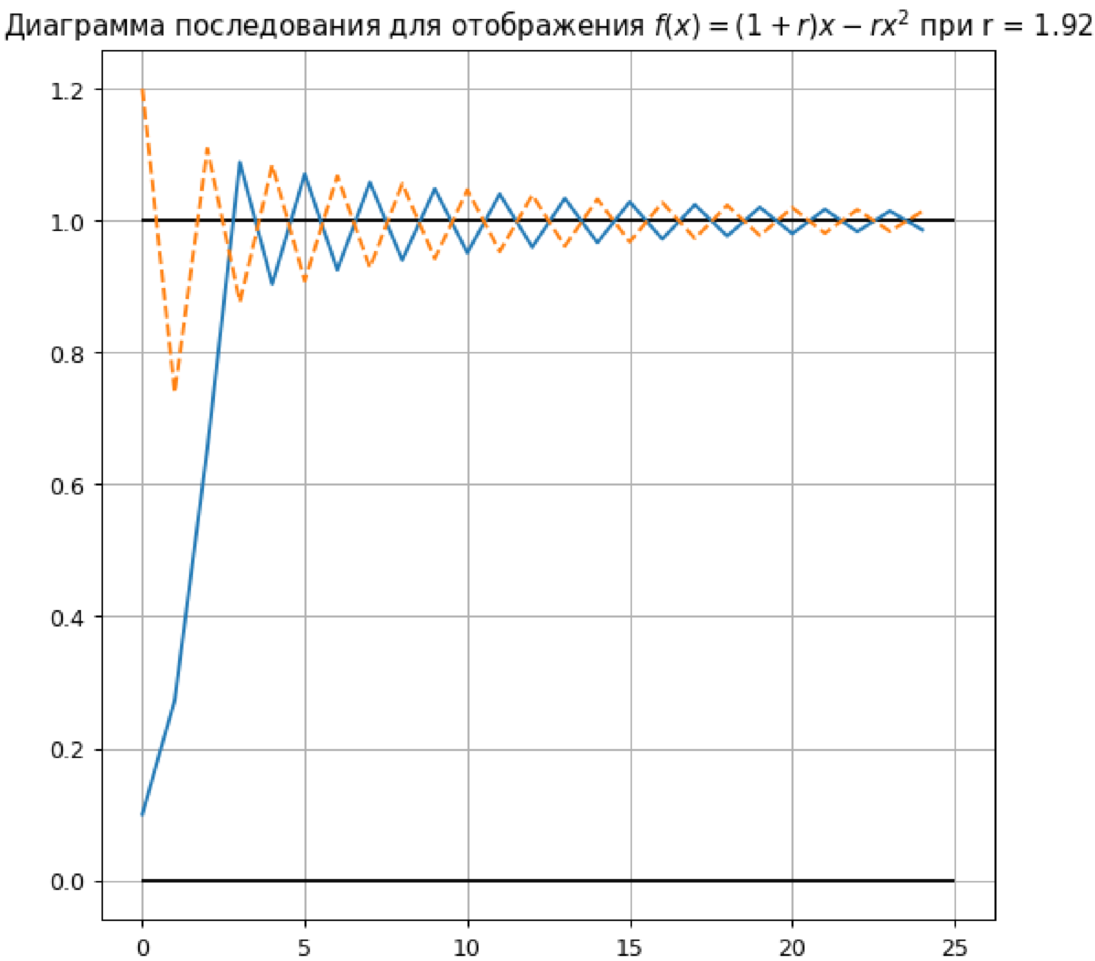
```
plt.show()
```



[5]:
```
def discr_sol(x_0, r, times=0):
    X = [x_0]
    for i in range(times-1):
        X.append((1 + r)*X[-1] - r*X[-1]**2)
    return X
```

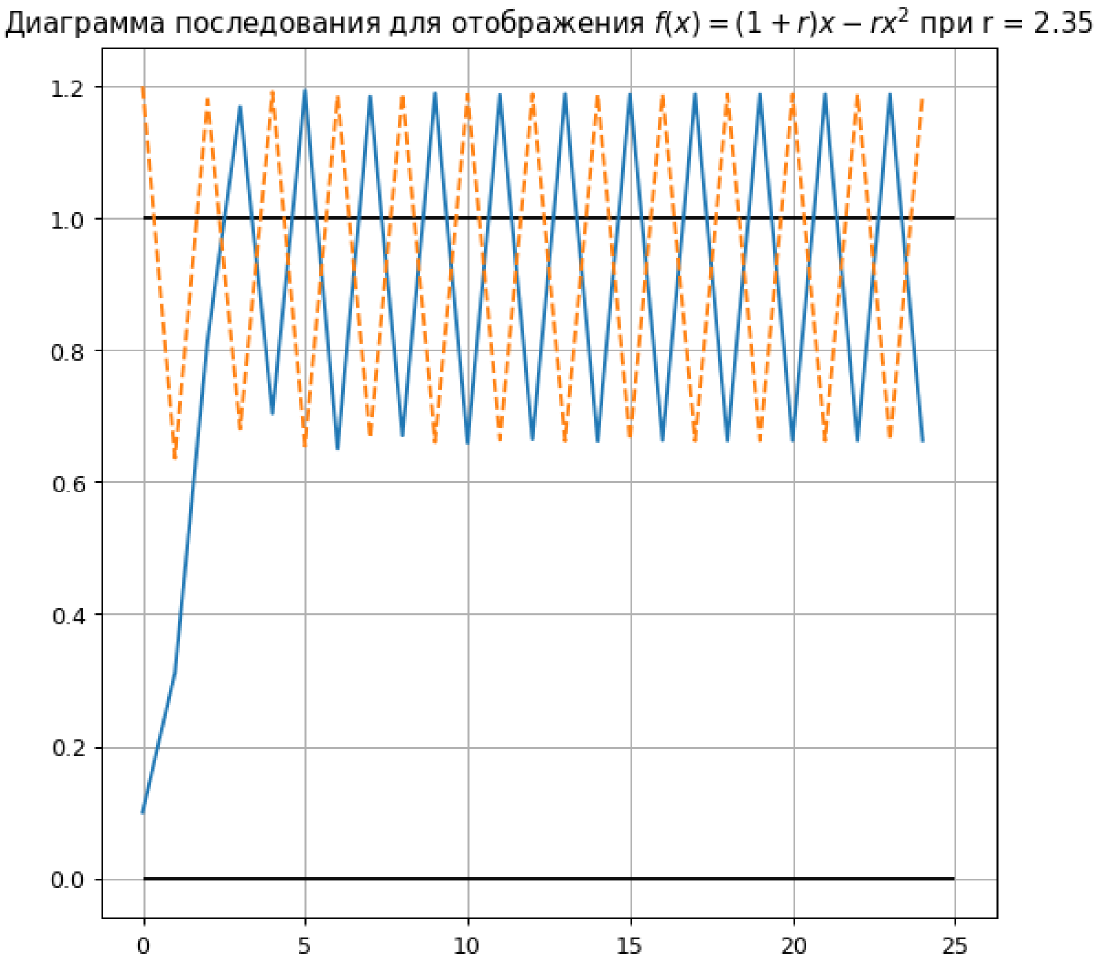
[6]:
```
time = np.arange(0, 25)
fig = plt.figure(facecolor="white", figsize = rect_pics, dpi=stat_dpi)
r=1.92

plt.hlines(1, 0, len(time), color='black')
```

```
plt.hlines(0, 0, len(time), color='black')
plt.plot(discr_sol(0.1, r, len(time)))
plt.plot(discr_sol(1.2, r, len(time)), linestyle='dashed')
plt.title(r"                              $f(x) = (1+r)x-rx^2$    r = "+str(r))
plt.grid(True)
plt.show()
```



[7]:
```
time = np.arange(0, 25)
fig = plt.figure(facecolor="white", figsize = rect_pics, dpi=stat_dpi)
r=2.35

plt.hlines(1, 0, len(time), color='black')
plt.hlines(0, 0, len(time), color='black')
plt.plot(discr_sol(0.1, r, len(time)))
plt.plot(discr_sol(1.2, r, len(time)), linestyle='dashed')
plt.title(r"                              $f(x) = (1+r)x-rx^2$    r = "+str(r))
```

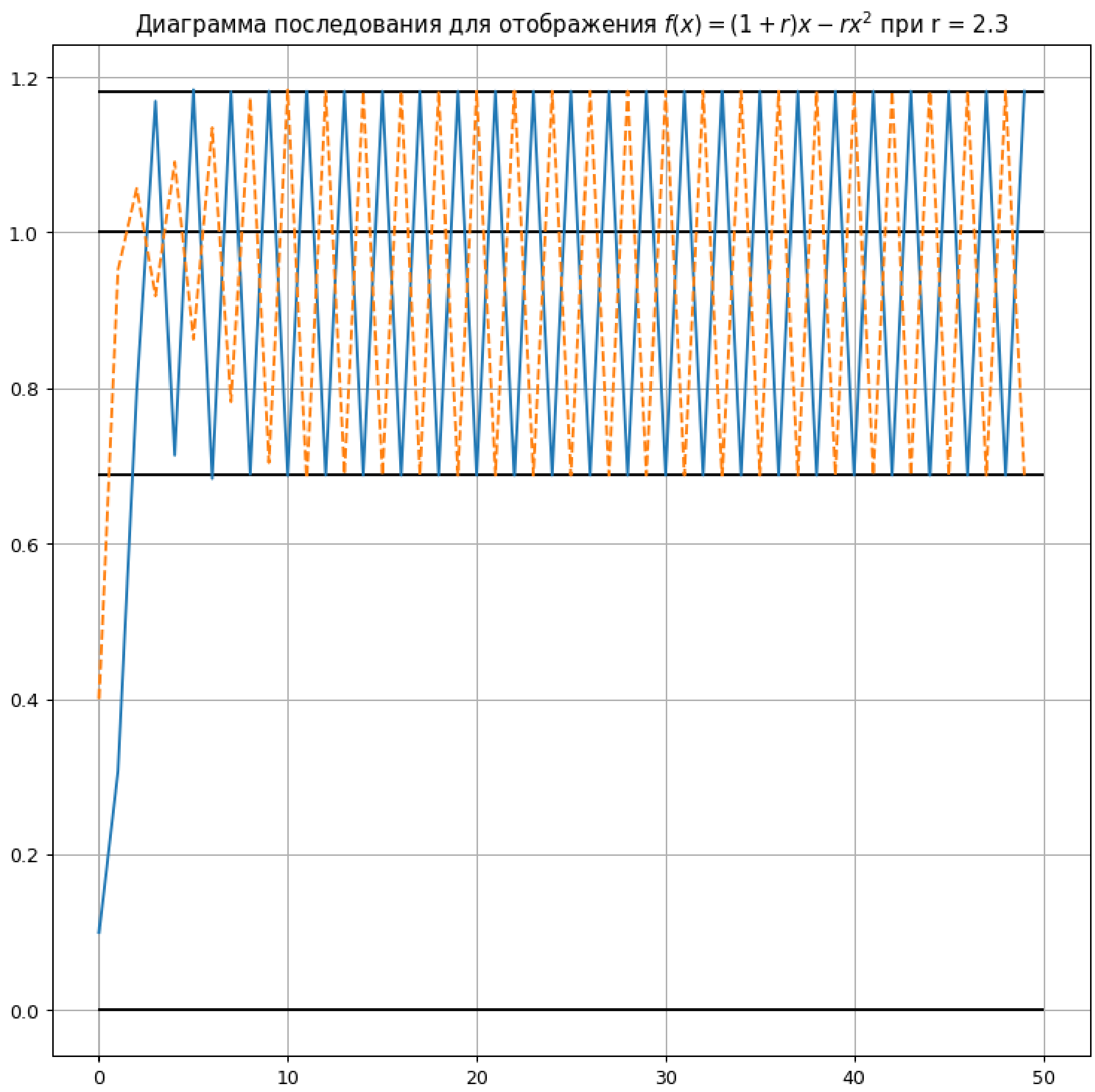
```
plt.grid(True)
plt.show()
```



[8]:

```
time = np.arange(0, 50)
fig = plt.figure(facecolor="white", figsize = big_pics, dpi=stat_dpi)

r = 2.3

plt.hlines(1, 0, len(time), color='black')
plt.hlines(0, 0, len(time), color='black')

xp3 = (2*r + r**2 - r*(-4 + r**2)**0.5)/(2*r**2)
xp4 = (2*r + r**2 + r*(-4 + r**2)**0.5)/(2*r**2)

plt.hlines(xp3, 0, len(time), color='black')
plt.hlines(xp4, 0, len(time), color='black')
```

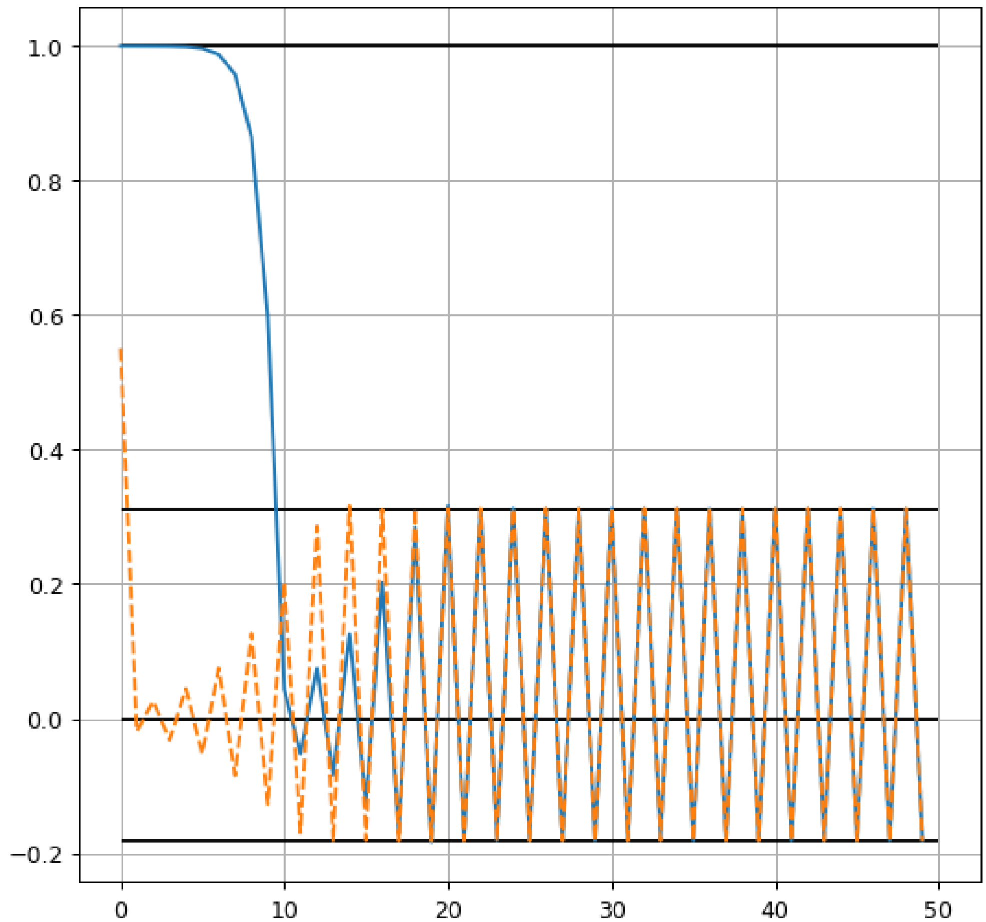
```
#plt.plot(discr_sol(0.1, 2.35, len(time)))
plt.plot(discr_sol(0.1, r, len(time)))
plt.plot(discr_sol(0.4, r, len(time)), linestyle='dashed')
plt.title(r"                       $f(x) = (1+r)x-rx^2$    r = "+str(r))
plt.grid(True)
plt.show()
```



[9]:
```
time = np.arange(0, 50)
fig = plt.figure(facecolor="white", figsize = rect_pics, dpi=stat_dpi)

r = -2.3
```

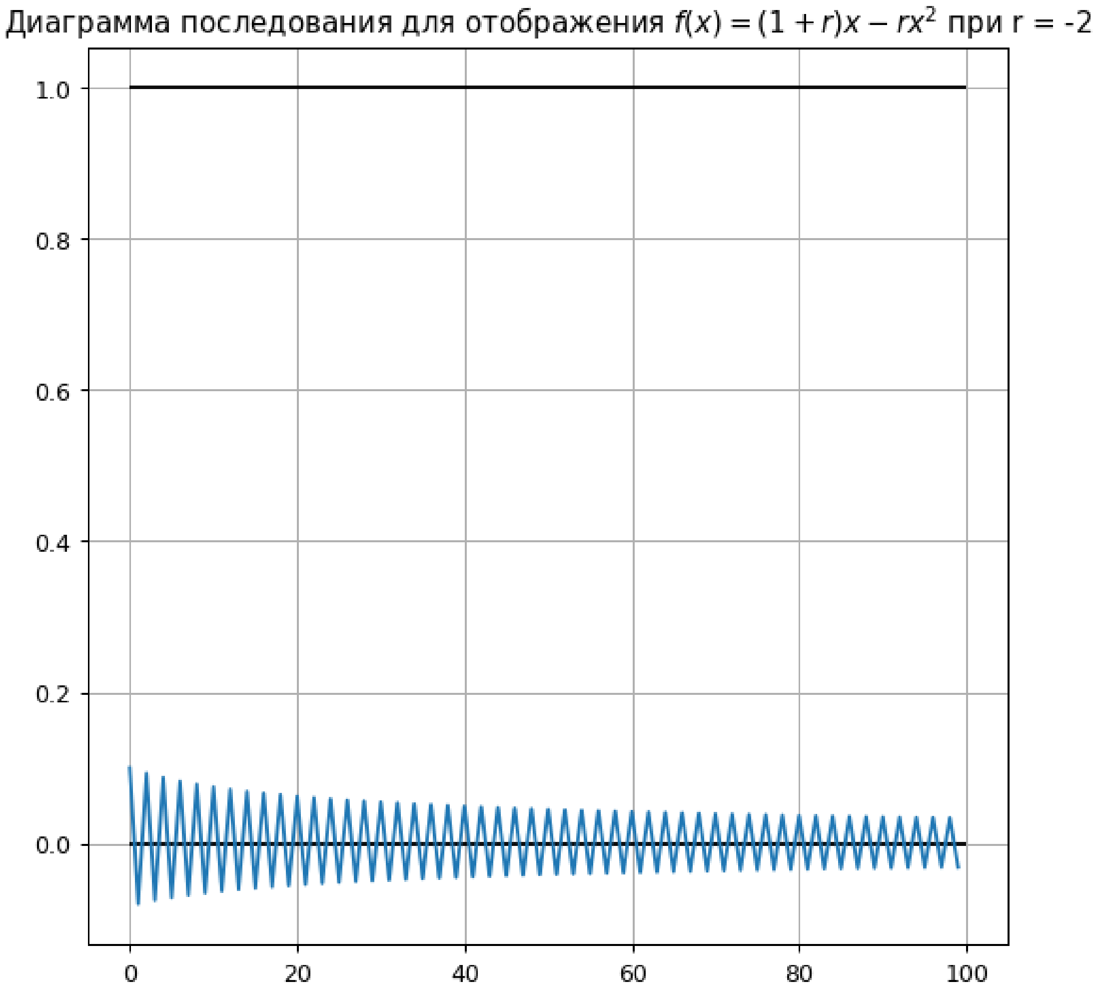
```
plt.hlines(1, 0, len(time), color='black')
plt.hlines(0, 0, len(time), color='black')

xp3 = (2*r + r**2 - r*(-4 + r**2)**0.5)/(2*r**2)
xp4 = (2*r + r**2 + r*(-4 + r**2)**0.5)/(2*r**2)

plt.hlines(xp3, 0, len(time), color='black')
plt.hlines(xp4, 0, len(time), color='black')

plt.plot(discr_sol(0.99999, r, len(time)))
plt.plot(discr_sol(0.55, r, len(time)), linestyle='dashed')
plt.title(r"                      $f(x) = (1+r)x-rx^2$    r = "+str(r))
plt.grid(True)
plt.show()
```

Диаграмма последования для отображения $f(x) = (1+r)x - rx^2$ при r = -2.3



[10]:

```
time = np.arange(0, 100)
fig = plt.figure(facecolor="white", figsize = rect_pics, dpi=stat_dpi)

r = -2

plt.hlines(1, 0, len(time), color='black')
plt.hlines(0, 0, len(time), color='black')

xp3 = (2*r + r**2 - r*(-4 + r**2)**0.5)/(2*r**2)
xp4 = (2*r + r**2 + r*(-4 + r**2)**0.5)/(2*r**2)

#plt.hlines(xp3, 0, len(time), color='black')
#plt.hlines(xp4, 0, len(time), color='black')

#plt.plot(discr_sol(0.1, 2.35, len(time)))
plt.plot(discr_sol(0.1, r, len(time)))
plt.title(r"                              $f(x) = (1+r)x-rx^2$    r = "+str(r))
plt.grid(True)
plt.show()
```




```
[11]: def KenLam(times, x_0, r, with_lines = True, with_arrows = True, with_3cr =␣
      ↪False, with_4cr=False, constant_size=False, arrow_c=["green"]):
          f = lambda x : (1 + r)*x - r*x**2
          fig = plt.figure(facecolor="white", figsize = big_pics, dpi=stat_dpi)
          dtime = np.arange(x_0, times)
          X = [x_0]
          Y = [x_0]
          plt.scatter(X, Y)

          for i in dtime:
              y = f(X[-1])
              X.append(X[-1])
              Y.append(y)
              X.append(y)
              Y.append(Y[-1])
```

```python
    scale = (max(X)-min(X))
    for i in range(1, len(X)):
        if with_arrows:
            plt.arrow(X[i-1], Y[i-1], X[i]-X[i-1],
                  Y[i]-Y[i-1], head_width=0.01*scale, head_length=0.02*scale,
                      color=arrow_c[i%(len(arrow_c))], width=0.0005*scale, alpha=0.3, length_includes_head=True)



    time = np.linspace(min(X), max(X))
    if constant_size:
        time = np.linspace(0, (1+r)/r)
    plt.plot(time, f(time))
    if with_arrows:
        plt.plot(time, time)
    if with_3cr:
        plt.plot(time, f(f(time)))
    if with_4cr:
        plt.plot(time, f(f(f(time))), color='yellow')


    if with_lines:
        plt.plot([X[i] for i in range(len(X)) if i%2==0][:-1], [X[i] for i in range(len(X)) if i%2==0][1:], color="red")
    #print([X[i] for i in range(len(X)) if i%2==0])
    plt.title(r"          -              $f(x) = (1+r)x - rx^2$"+f"    {x_0=}, {r=}")
    plt.grid(True)
    plt.show()
```

[12]:
```python
KenLam(500, 0.001, -2.3, with_arrows=True, with_3cr=False, with_4cr=True, arrow_c=['black'], with_lines=False)
KenLam(500, 0.1, 1.9, with_arrows=True, with_3cr=False, with_4cr=True, arrow_c=['black'], with_lines=False)

r = 2
x=(2*r + r**2 - r*(-4 + r**2)**0.5)/(2*r**2)
KenLam(100, x, r, with_arrows=True, with_3cr=True, constant_size=True, arrow_c=['black'])
x = (2*r + r**2 + r*(-4 + r**2)**0.5)/(2*r**2)
```

```
KenLam(100, x, r, with_arrows=True, with_3cr=True, constant_size=True,␣
↪arrow_c=['black'])

r = 2.05
x=(2*r + r**2 - r*(-4 + r**2)**0.5)/(2*r**2)
KenLam(100, x, r, with_arrows=True, with_3cr=True, constant_size=True,␣
↪arrow_c=['black'])
x = (2*r + r**2 + r*(-4 + r**2)**0.5)/(2*r**2)
KenLam(100, x, r, with_arrows=True, with_3cr=True, constant_size=True,␣
↪arrow_c=['black'])

KenLam(100, 0.1, 2.1, with_arrows=True, with_3cr=True, constant_size=True,␣
↪arrow_c=['black'], with_lines=False)
KenLam(100, 0.1, 2.3, with_arrows=True, with_3cr=True, constant_size=True,␣
↪arrow_c=['black'], with_lines=False)
KenLam(100, 0.1, 2.45, with_arrows=True, with_3cr=True, constant_size=True,␣
↪arrow_c=['black'], with_lines=False)
KenLam(500, 0.1, 2.5, with_arrows=True, with_3cr=False, with_4cr=True,␣
↪arrow_c=['black'], with_lines=False)
KenLam(500, 0.1, 2.55, with_arrows=True, with_3cr=False, with_4cr=True,␣
↪arrow_c=['black'], with_lines=False)
KenLam(500, 0.1, 2.57, with_arrows=True, with_3cr=False, with_4cr=True,␣
↪arrow_c=['black'], with_lines=False)
```

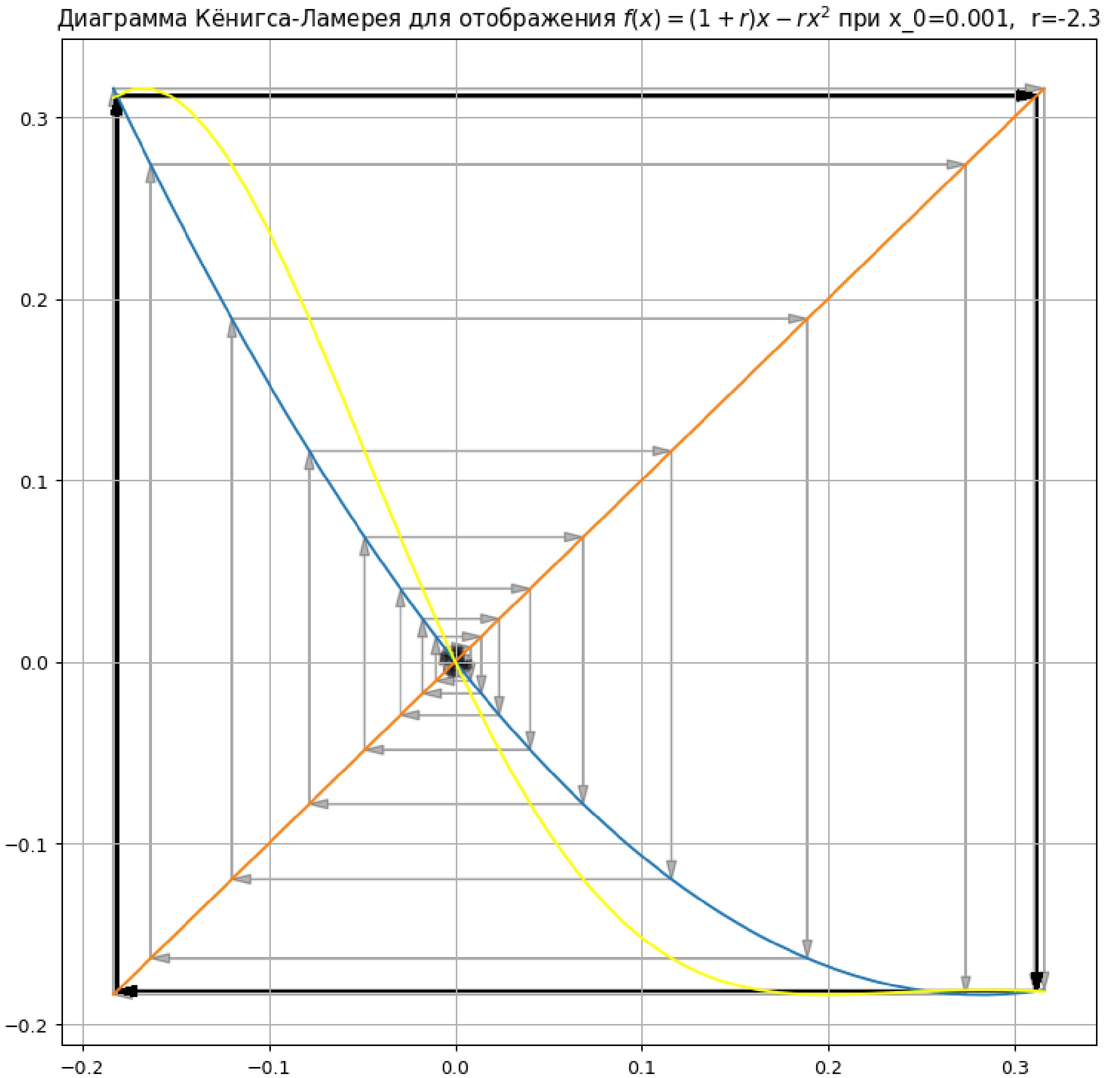


Диаграмма Кёнигса-Ламерея для отображения $f(x) = (1+r)x - rx^2$ при x_0=0.001, r=-2.3
0.3
0.2
0.1
0.0
−0.1
−0.2
−0.2
−0.1
0.0
0.1
0.2
0.3

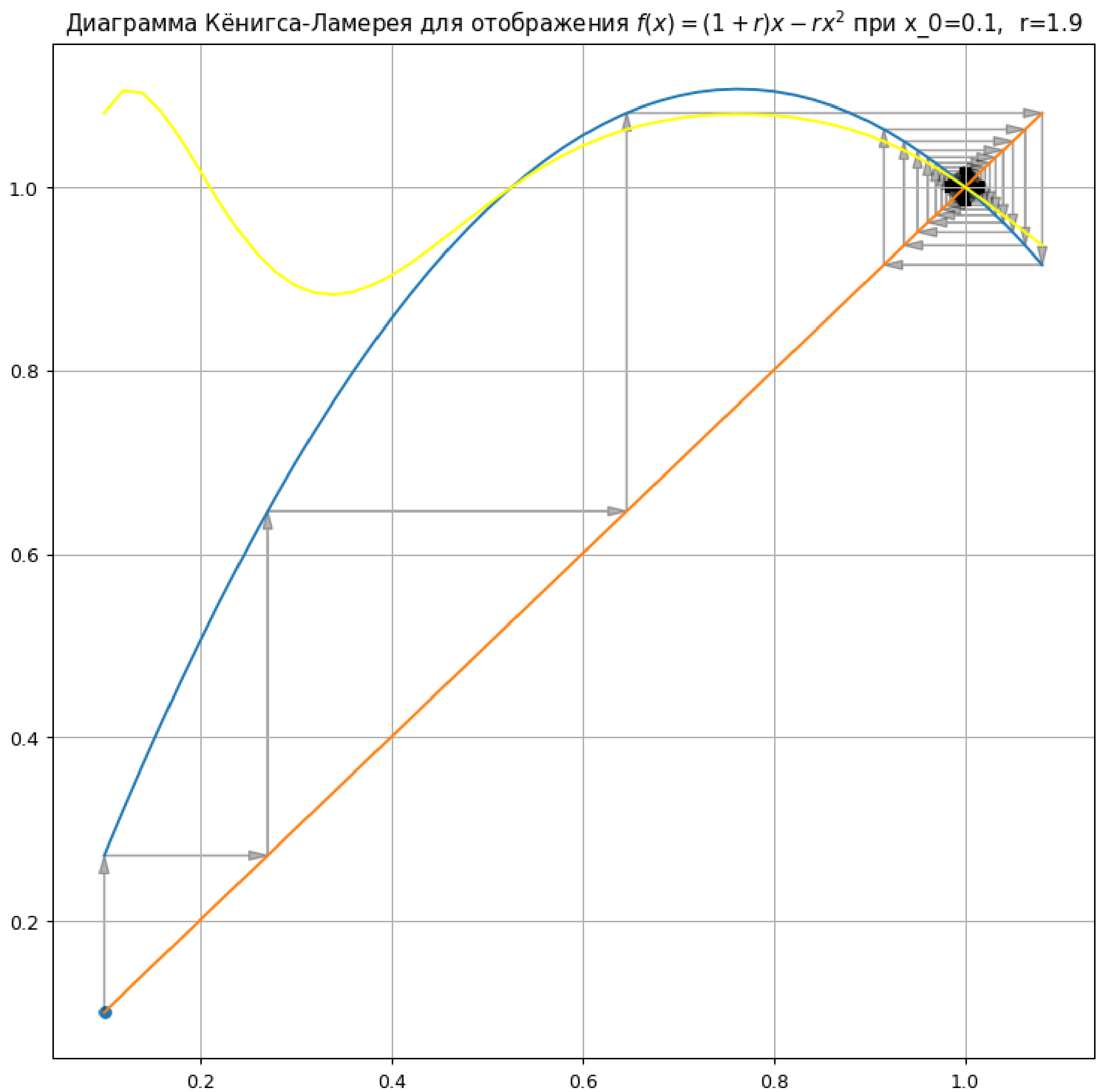


Диаграмма Кёнигса-Ламерея для отображения $f(x) = (1 + r)x - rx^2$ при x_0=0.1, r=1.9
1.0
0.8
0.6
0.4
0.2
0.2
0.4
0.6
0.8
1.0

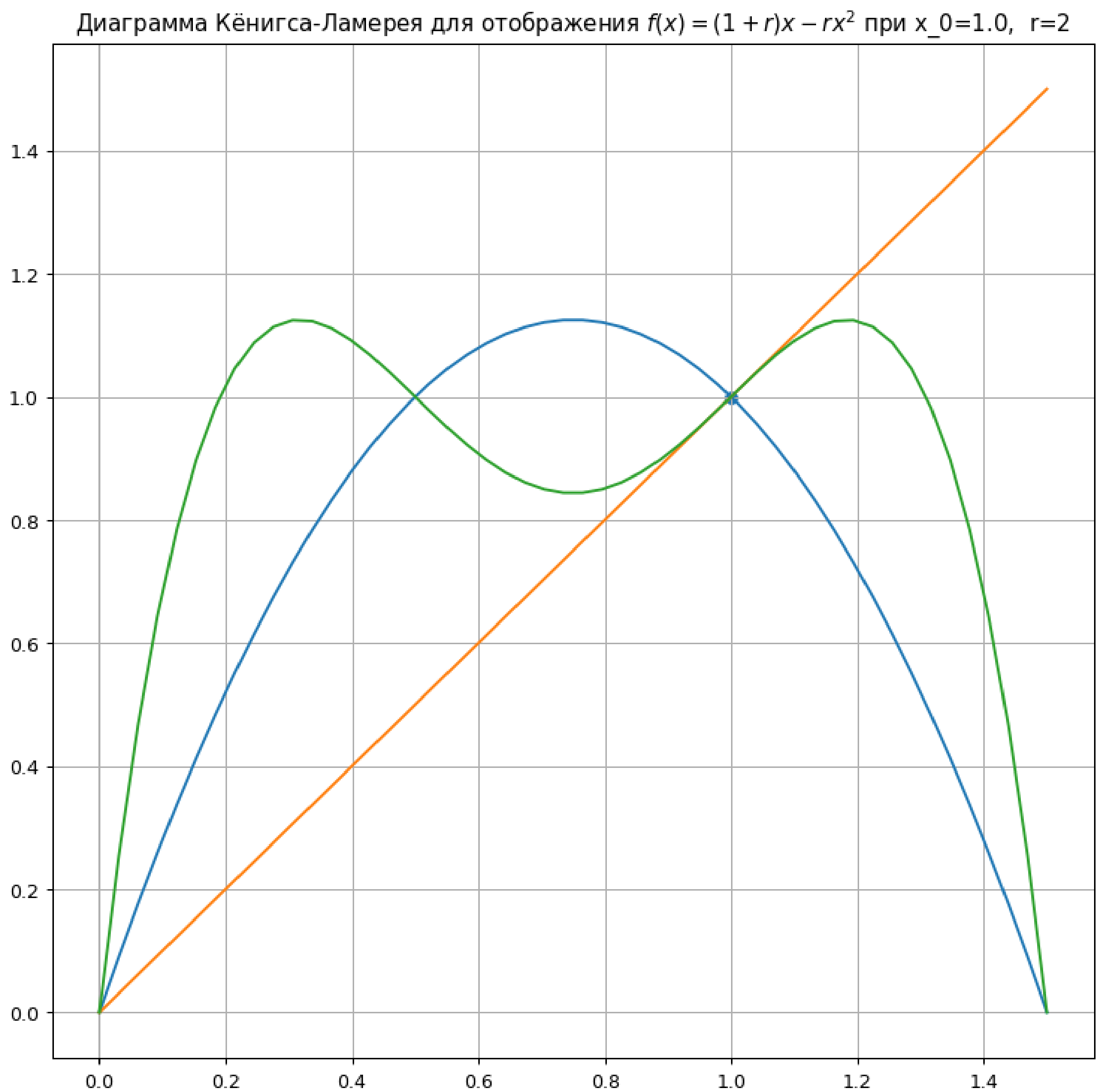


Диаграмма Кёнигса-Ламерея для отображения $f(x) = (1+r)x - rx^2$ при x_0=1.0, r=2
1.4
1.2
1.0
0.8
0.6
0.4
0.2
0.0
0.0
0.2
0.4
0.6
0.8
1.0
1.2
1.4

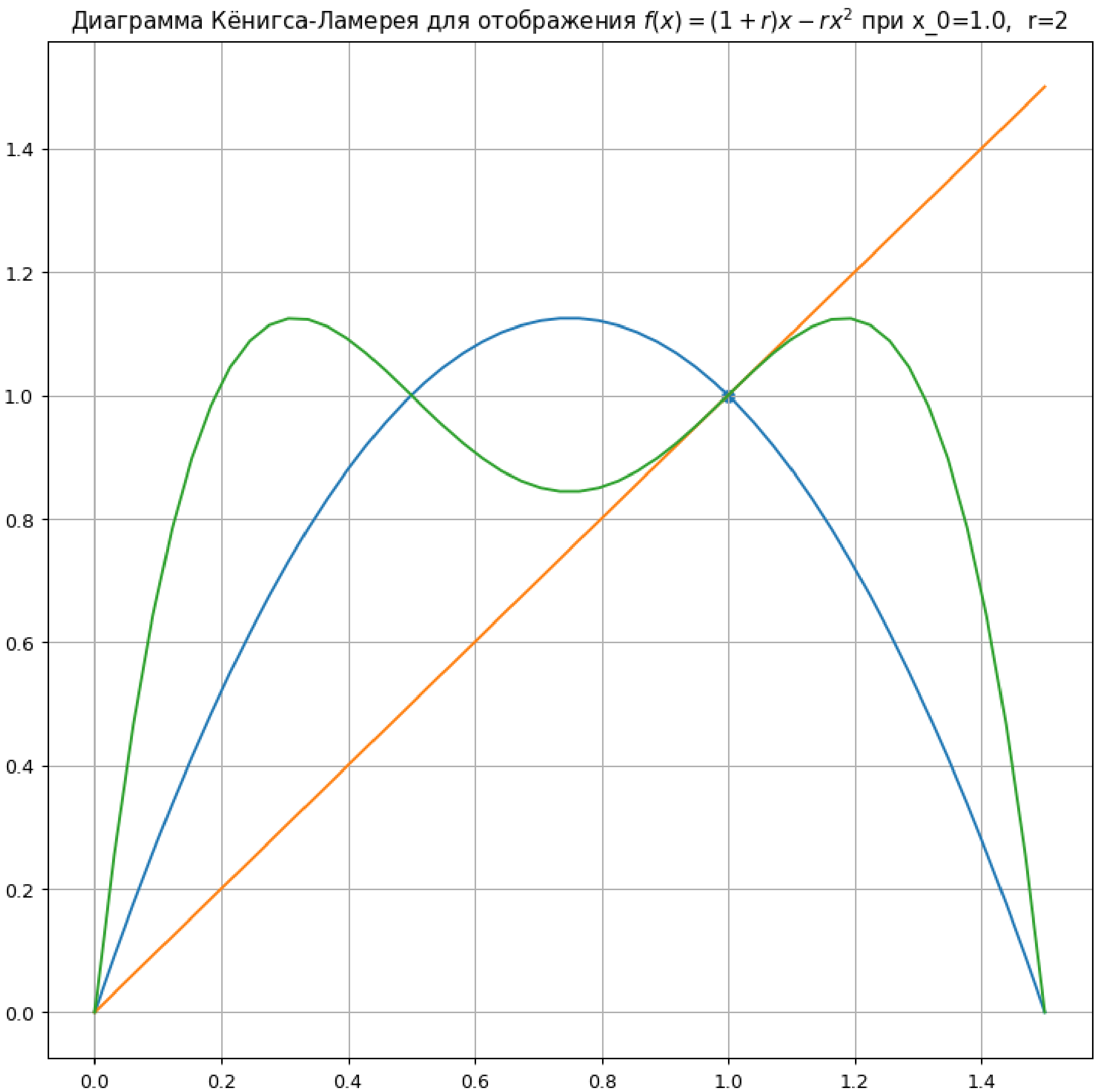
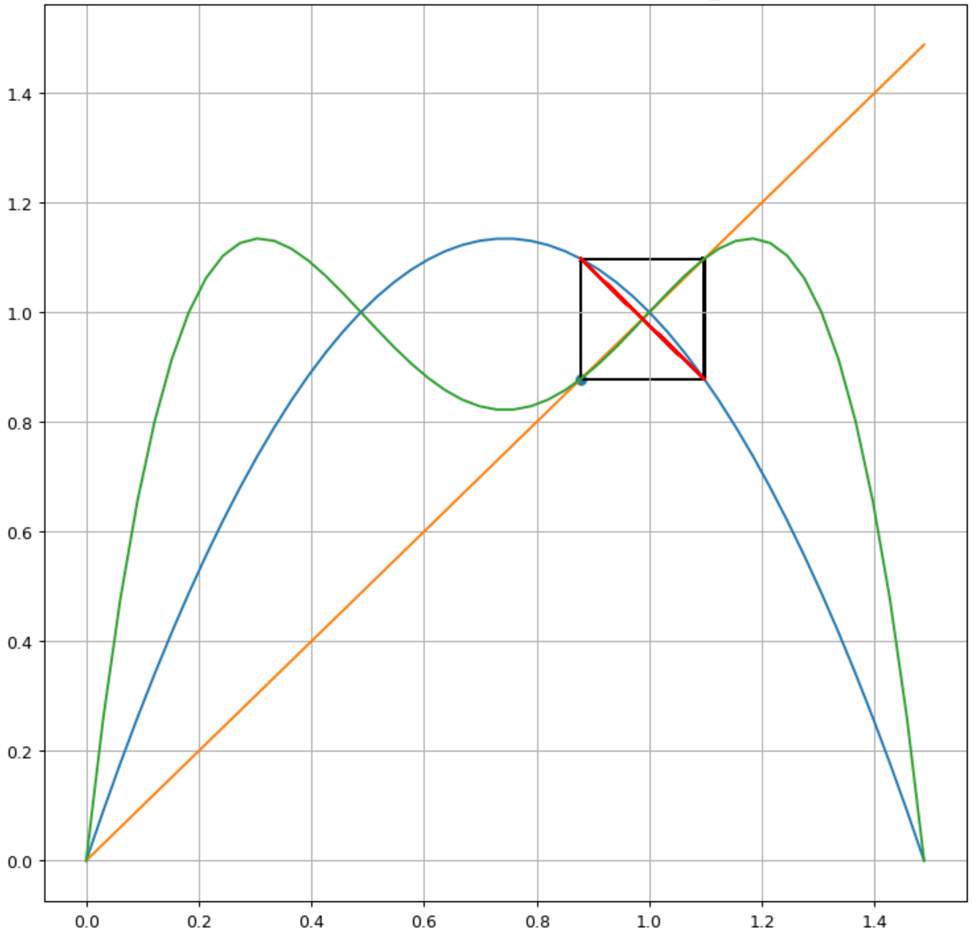


Диаграмма Кёнигса-Ламерея для отображения $f(x) = (1 + r)x - rx^2$ при x_0=1.0, r=2
0.0
0.2
0.4
0.6
0.8
1.0
1.2
1.4
0.0
0.2
0.4
0.6
0.8
1.0
1.2
1.4

Диаграмма Кёнигса-Ламерея для отображения $f(x) = (1+r)x - rx^2$ при x_0=0.8780487804878049, r=2.05



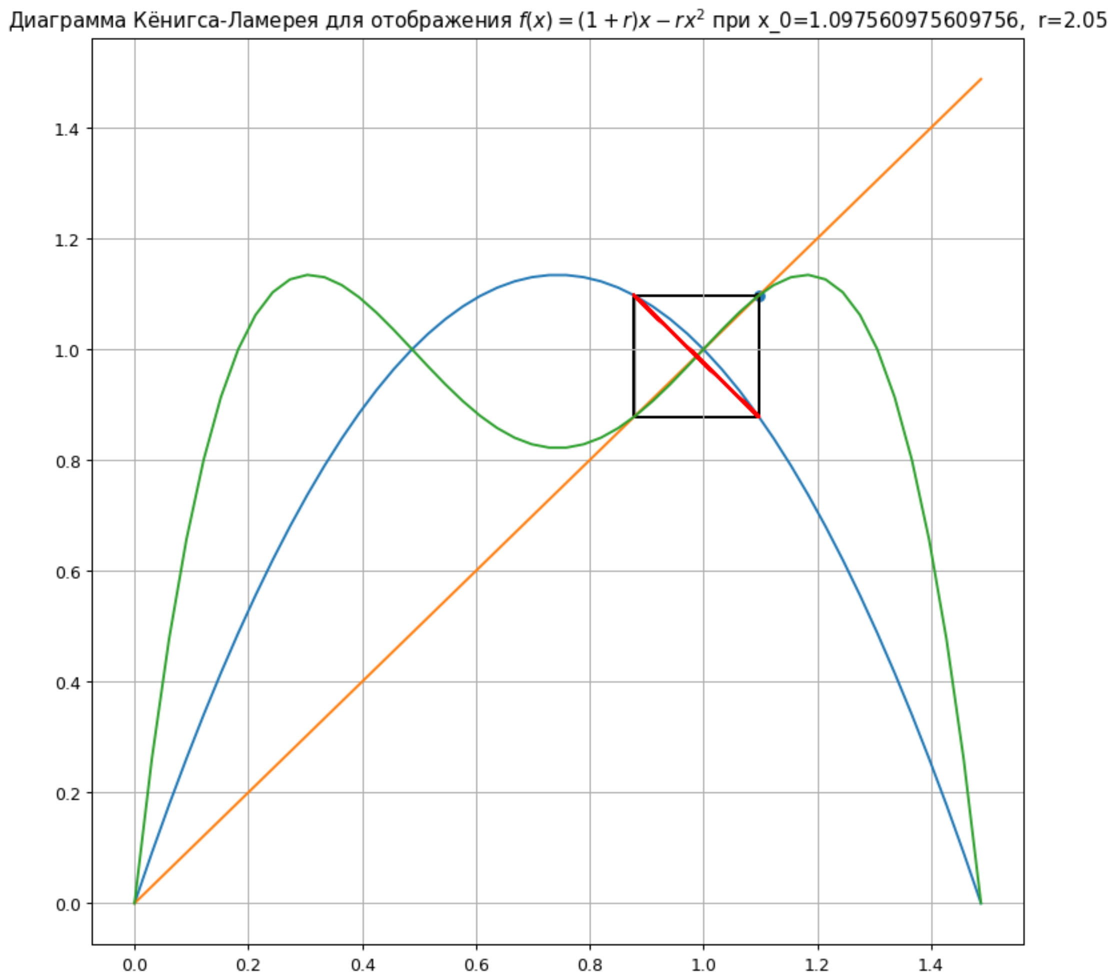
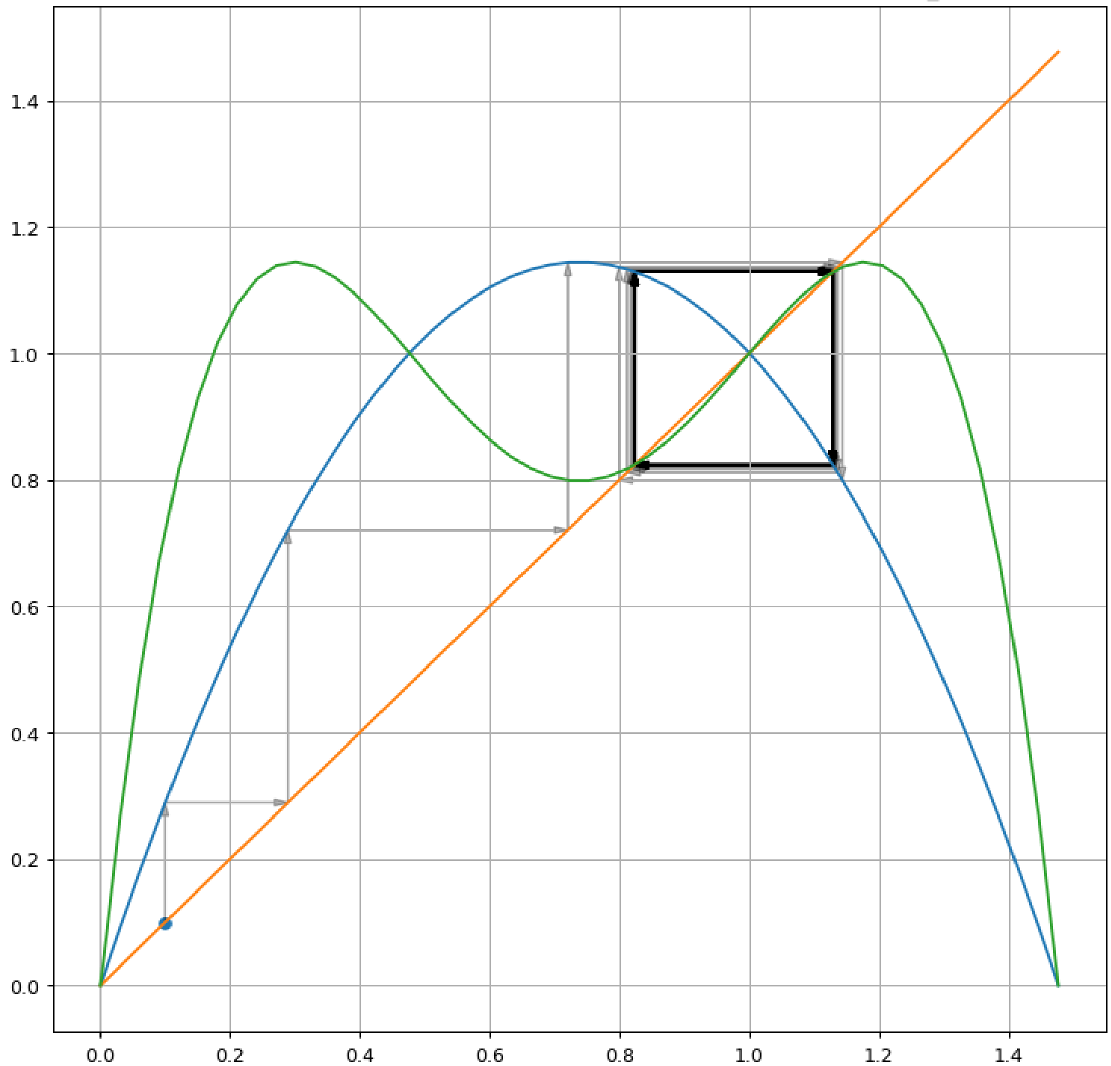


Диаграмма Кёнигса-Ламерея для отображения $f(x) = (1+r)x - rx^2$ при x_0=1.097560975609756, r=2.05
0.0
0.2
0.4
0.6
0.8
1.0
1.2
1.4

Диаграмма Кёнигса-Ламерея для отображения $f(x) = (1+r)x - rx^2$ при x_0=0.1, r=2.1



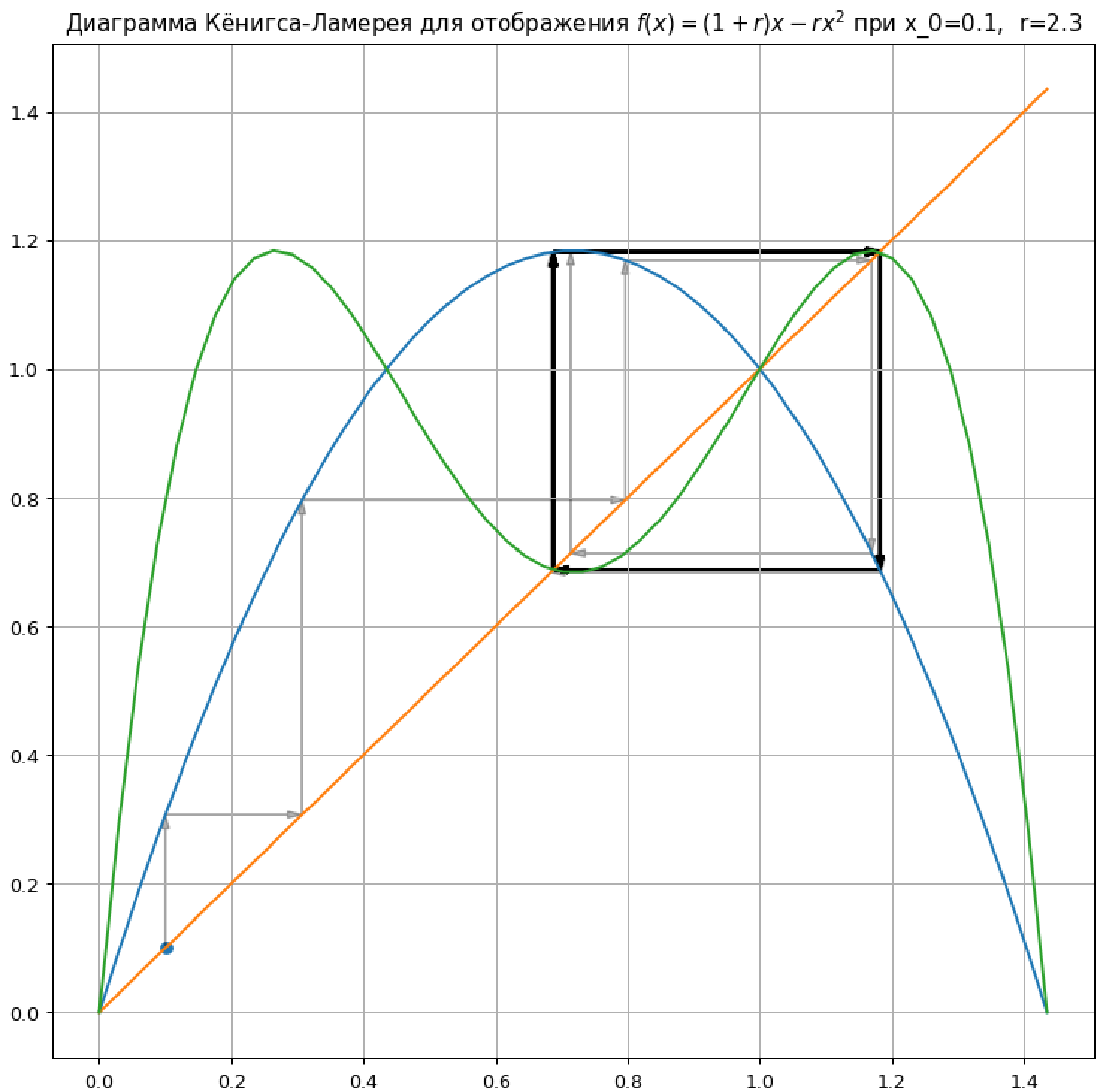


Диаграмма Кёнигса-Ламерея для отображения $f(x) = (1+r)x - rx^2$ при x_0=0.1, r=2.3
1.4
1.2
1.0
0.8
0.6
0.4
0.2
0.0
0.0
0.2
0.4
0.6
0.8
1.0
1.2
1.4

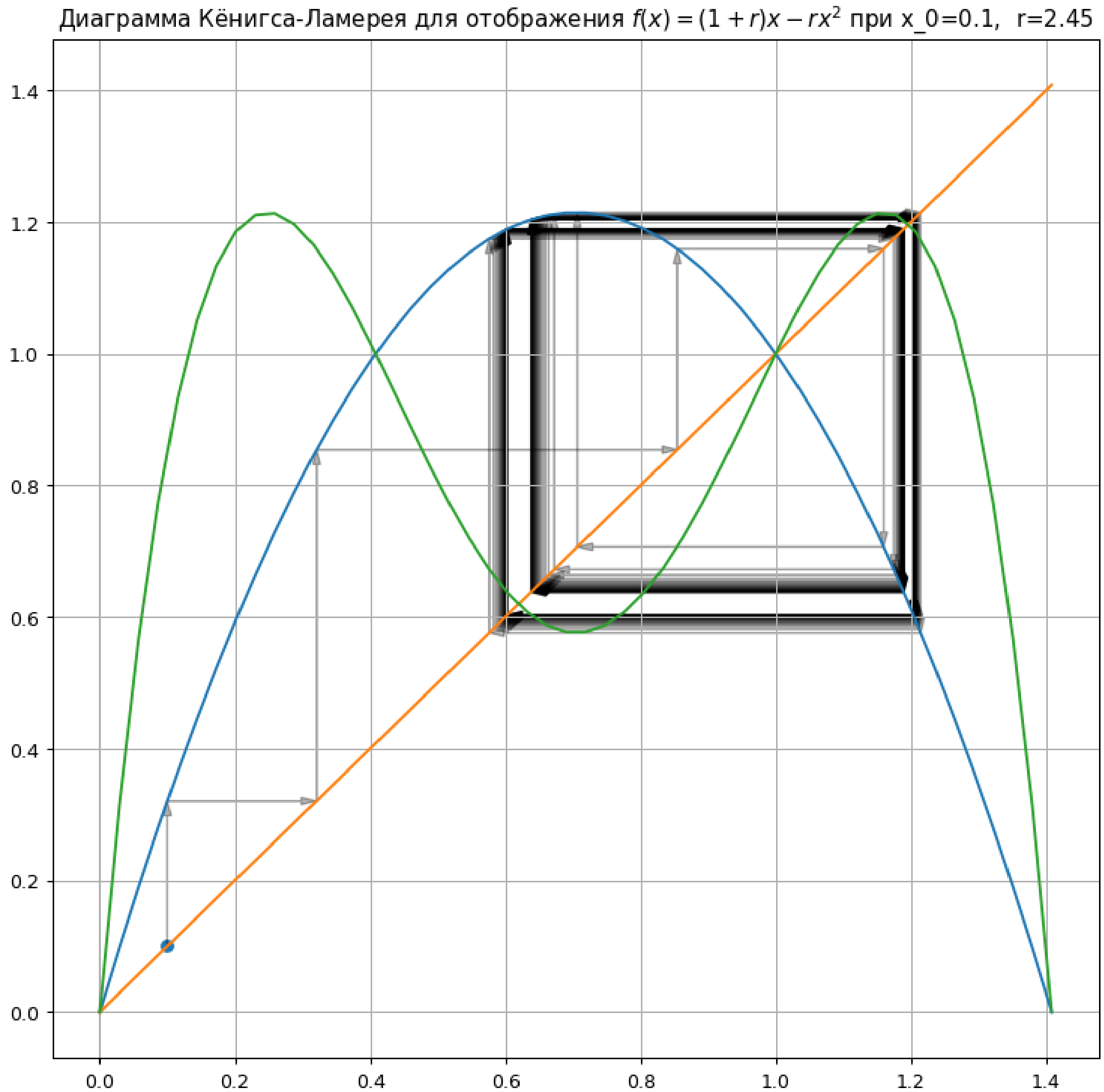


Диаграмма Кёнигса-Ламерея для отображения $f(x) = (1+r)x - rx^2$ при x_0=0.1, r=2.45
0.0
0.2
0.4
0.6
0.8
1.0
1.2
1.4

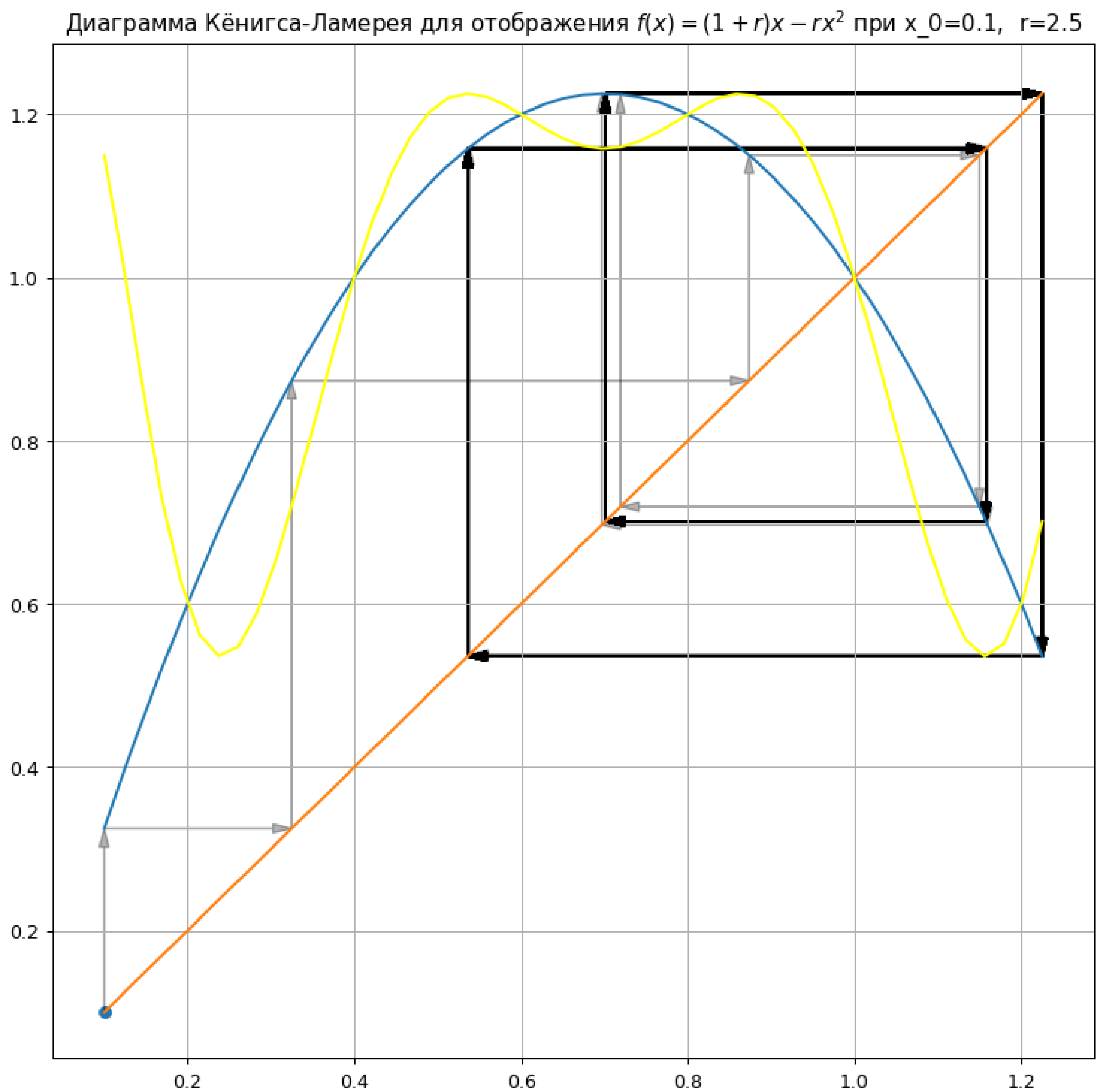


Диаграмма Кёнигса-Ламерея для отображения $f(x) = (1+r)x - rx^2$ при x_0=0.1, r=2.5
1.2
1.0
0.8
0.6
0.4
0.2
0.2
0.4
0.6
0.8
1.0
1.2

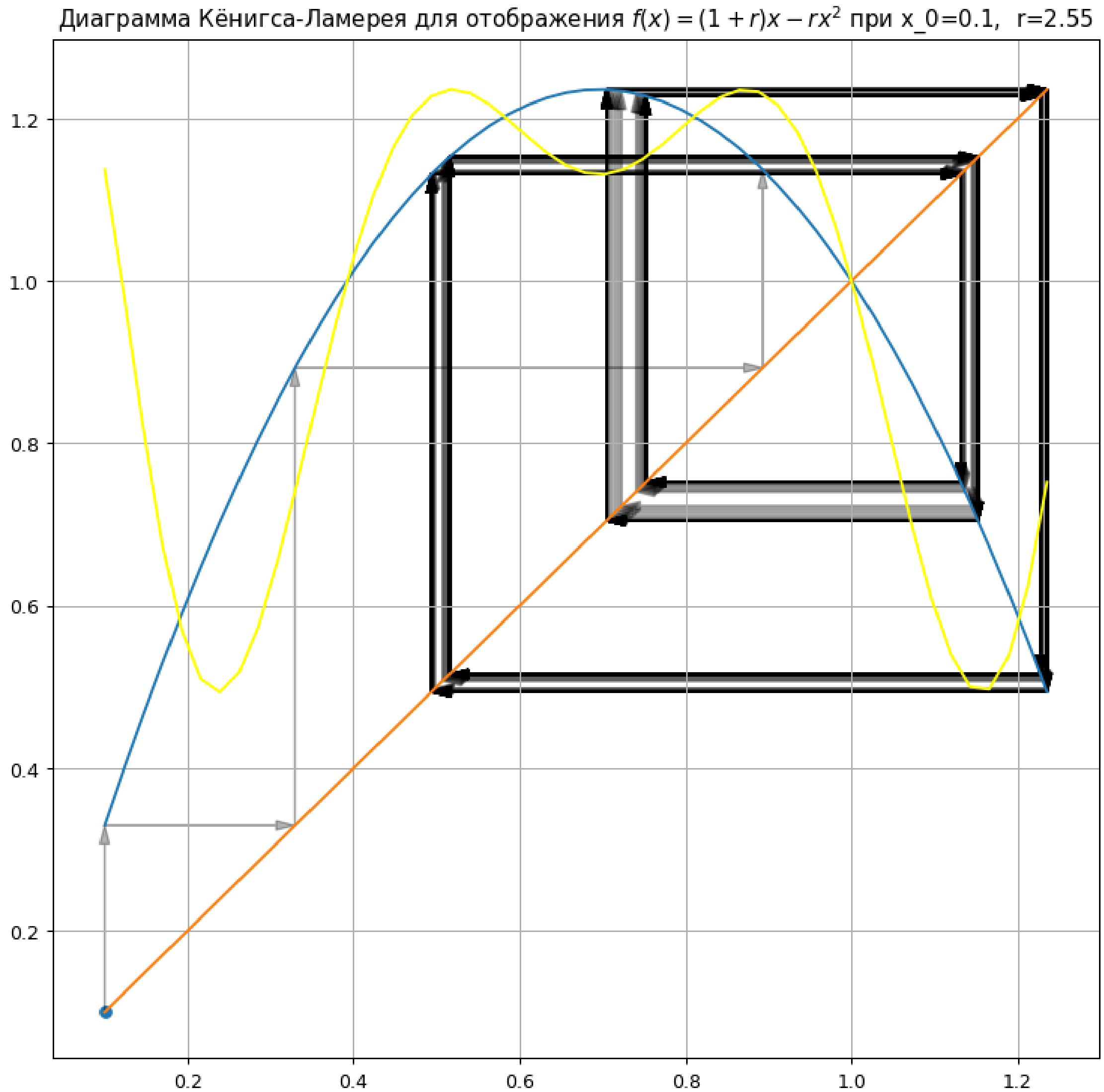
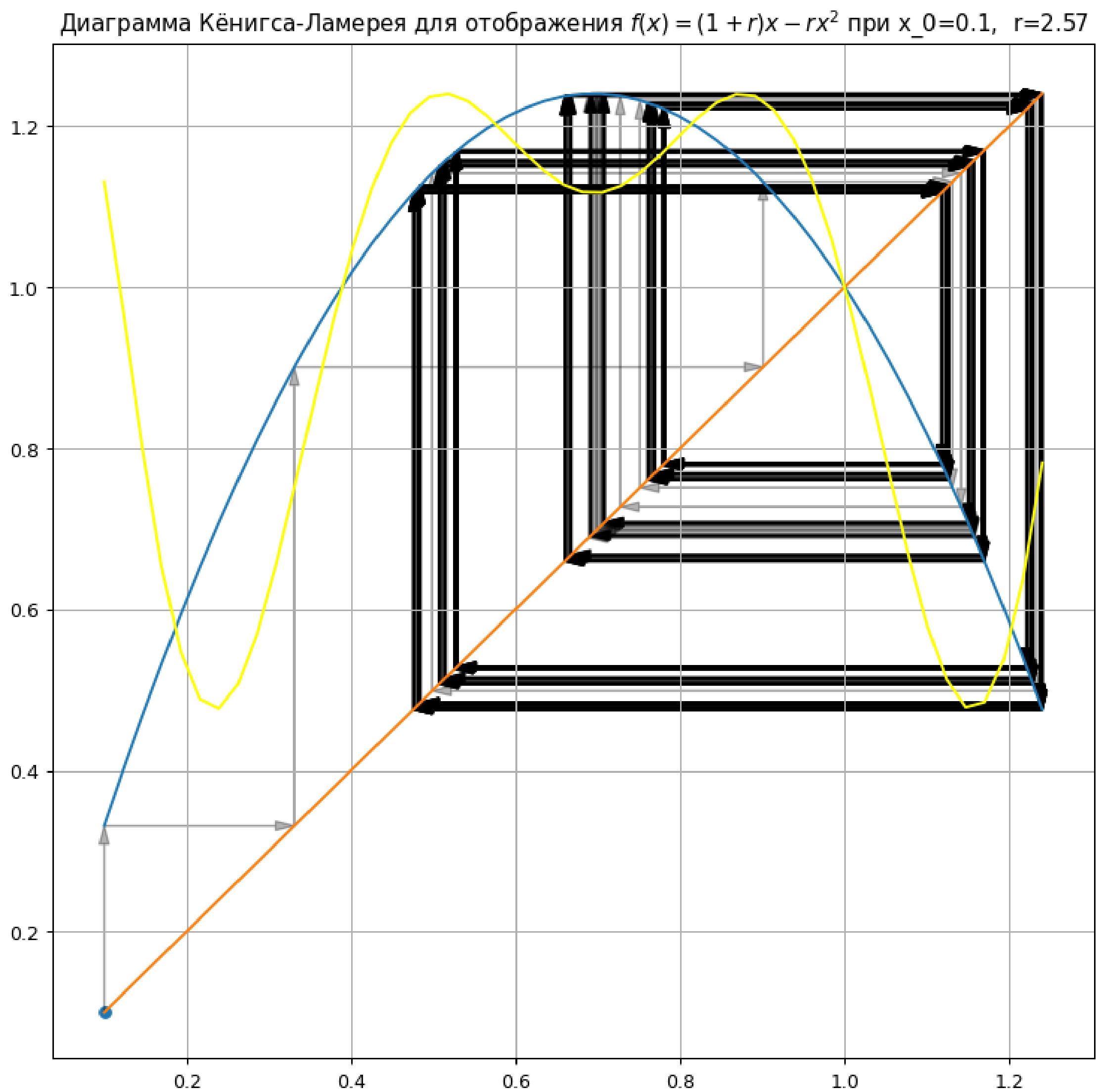
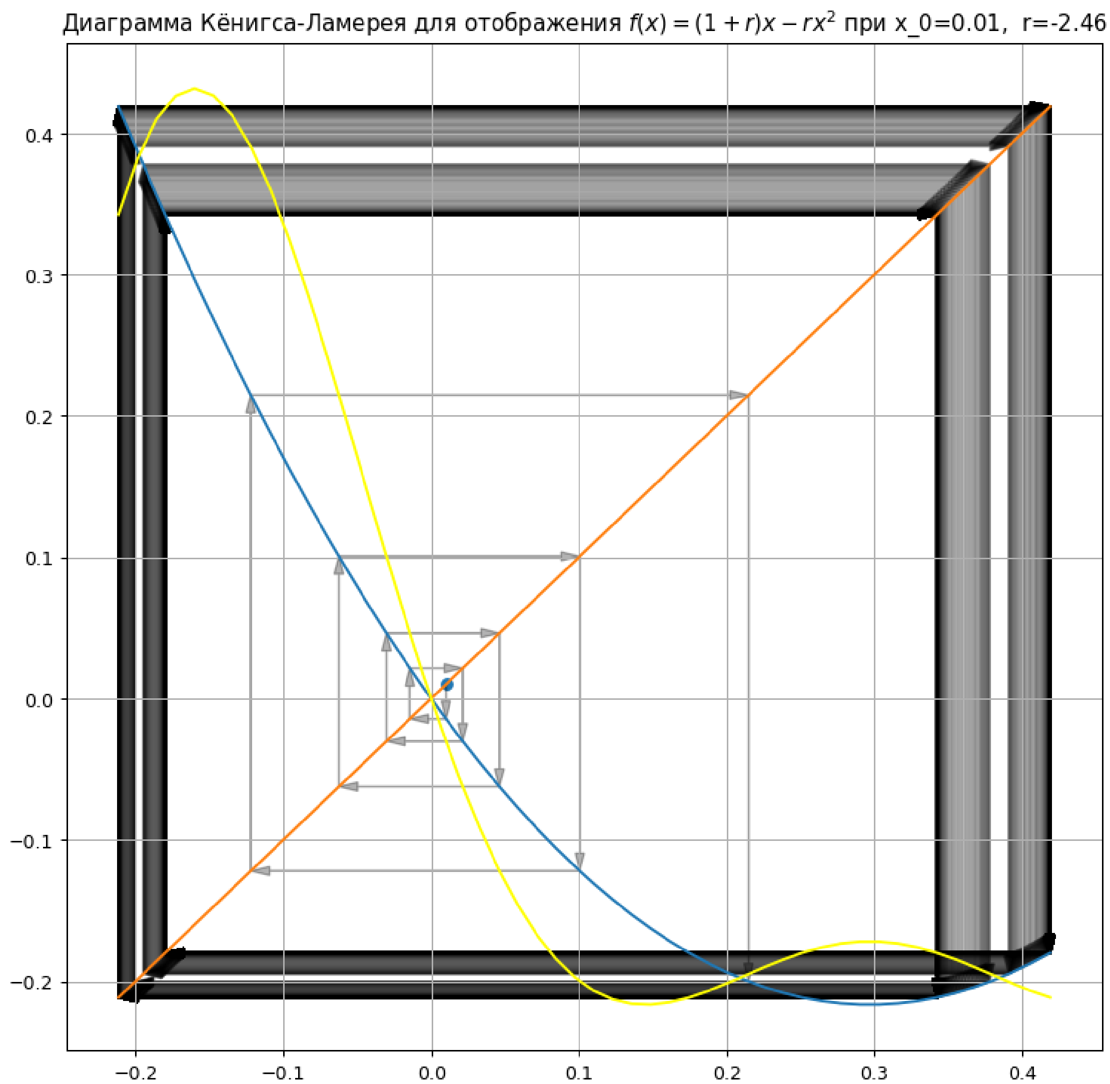


Диаграмма Кёнигса-Ламерея для отображения $f(x) = (1 + r)x - rx^2$ при x_0=0.1, r=2.55
1.2
1.0
0.8
0.6
0.4
0.2
0.2
0.4
0.6
0.8
1.0
1.2



[13]:
```
KenLam(1000, 0.01, -2.46, with_arrows=True, with_3cr=False, with_4cr=True,␣
↪arrow_c=['black'], with_lines=False)
```



[14]:

```
time = np.arange(0, 50)
fig = plt.figure(facecolor="white", figsize = big_pics, dpi=stat_dpi)

r = 2.3
ff = lambda x : (1 + r) * ((1 + r)*x - r*x**2) - r * ((1+r)*x-r*x**2)**2

plt.hlines(1, 0, len(time), color='black')
plt.hlines(0, 0, len(time), color='black')

xp3 = (2*r + r**2 - r*(-4 + r**2)**0.5)/(2*r**2)
xp4 = (2*r + r**2 + r*(-4 + r**2)**0.5)/(2*r**2)

plt.hlines(xp3, 0, len(time), color='black')
```

```
plt.hlines(xp4, 0, len(time), color='black')

x_0 = 1.4

plt.plot(discr_sol(x_0, r, len(time)), linestyle='dashed')
plt.plot(np.linspace(0, 50, 1000), logistic(np.linspace(0, 50, 1000), r, x_0,␣
 ↪0))
plt.title(r"                                    r = "+str(r)+f"   {x_0=}")
plt.grid(True)
plt.show()
```