COS30019 - Introduction to Artificial Intelligence
Tutorial Problems Week 7

Task 1: Use truth tables to show that the following are valid (i.e. that the equivalences hold).

$P \wedge (Q \vee R) \Leftrightarrow (P \wedge Q) \vee (P \wedge R)$	(Distribution of $\wedge$)
$\neg(P \wedge Q) \Leftrightarrow \neg P \vee \neg Q$	(de Morgan's Law)
$\neg(P \vee Q) \Leftrightarrow \neg P \wedge \neg Q$	(de Morgan's Law)
$(P \Rightarrow Q) \Leftrightarrow (\neg Q \Rightarrow \neg P)$	(Contraposition)
$(P \Rightarrow Q) \Leftrightarrow \neg P \vee Q$	

P	**Q**	**R**	Q∨R	P∧(Q∨R)	P∧Q	P∧R	(P∧Q)∨(P∧R)	P∧(Q∨R)⇔(P∧Q)∨(P∧R)
F	F	F	F	F	F	F	F	T
F	F	T	T	F	F	F	F	T
F	T	F	T	F	F	F	F	T
F	T	T	T	F	F	F	F	T
T	F	F	F	F	F	F	F	T
T	F	T	T	T	F	T	T	T
T	T	F	T	T	T	F	T	T
T	T	T	T	T	T	T	T	T

P	**Q**	P∧Q	¬(P∧Q)	¬P	¬Q	¬P ∨ ¬Q	¬(P ∧ Q) ⇔ ¬P ∨ ¬Q
F	F	F	T	T	T	T	T
F	T	F	T	T	F	T	T
T	F	F	T	F	T	T	T
T	T	T	F	F	F	F	T

P	**Q**	P∨Q	¬(P∨Q)	¬P	¬Q	¬P ∧ ¬Q	¬(P ∨ Q) ⇔ ¬P ∧ ¬Q
F	F	F	T	T	T	T	T
F	T	T	F	T	F	F	T
T	F	T	F	F	T	F	T
T	T	T	F	F	F	F	T

P	**Q**	P ⇒ Q	¬Q	¬P	¬Q ⇒ ¬P	(P ⇒ Q) ⇔ (¬Q ⇒ ¬P)
F	F	T	T	T	T	T
F	T	T	F	T	T	T
T	F	F	T	F	F	T
T	T	T	F	F	T	T

P	**Q**	P ⇒ Q	¬P	¬P ∨ Q	(P ⇒ Q) ⇔ ¬P ∨ Q
F	F	T	T	T	T
F	T	T	T	T	T
T	F	F	F	F	T
T	T	T	F	T	T

Task 2: Decide whether each of the following sentences is valid, unsatisfiable, or neither. Verify your decisions using truth tables or the equivalence rules from the lecture.

a. Smoke ⇒ Smoke
b. Smoke ⇒ Fire
c. (Smoke ⇒ Fire) ⇒ (¬Smoke ⇒ ¬Fire)
d. Smoke ∨ Fire ∨ ¬Fire
e. ((Smoke ∧ Heat) ⇒ Fire) ⇔ ((Smoke ⇒ Fire) ∨ (Heat ⇒ Fire))
f. (Smoke ⇒ Fire) ⇒ ((Smoke ∧ Heat) ⇒ Fire)
g. Big ∨ Dumb ∨ (Big ⇒ Dumb)
h. (Big ∧ Dumb) ∨ ¬Dumb

a.

Smoke	Smoke ⇒ Smoke
F	T
T	T

VALID (all rows are True)

b.

Smoke	**Fire**	Smoke ⇒ Fire
F	F	T
F	T	T
T	F	F
T	T	T

NEITHER: Row 1 is T (not unsatisfiable) and Row 3 is F (not valid)

Sm: Smoke & Fr: Fire & Ht: Heat

c.

Sm	**Fr**	Sm⇒ Fr	¬Sm	¬Fr	¬Sm⇒¬Fr	(Sm⇒ Fr)⇒(¬Sm ⇒ ¬Fr)
F	F	T	T	T	T	T
F	T	T	T	F	F	F
T	F	F	F	T	T	T
T	T	T	F	F	T	T

NEITHER: Row 1 is T (not unsatisfiable) and Row 2 is F (not valid)

d.

Sm	**Fr**	Sm ∨ Fr	¬Fr	Sm ∨ Fr ∨ ¬Fr
F	F	F	T	T
F	T	T	F	T
T	F	T	T	T
T	T	T	F	T

VALID (all rows are True)

e.

Sm	**Fr**	**Ht**	$Sm \wedge Ht$	$(Sm \wedge Ht) \Rightarrow Fr$	$Sm \Rightarrow Fr$	$Ht \Rightarrow Fr$	$(Sm \Rightarrow Fr) \vee (Ht \Rightarrow Fr)$	$((Sm \wedge Ht) \Rightarrow Fr) \Leftrightarrow ((Sm \Rightarrow Fr) \vee (Ht \Rightarrow Fr))$
F	F	F	F	T	T	T	T	T
F	F	T	F	T	T	F	T	T
F	T	F	F	T	T	T	T	T
F	T	T	F	T	T	T	T	T
T	F	F	F	T	F	T	T	T
T	F	T	T	F	F	F	F	T
T	T	F	F	T	T	T	T	T
T	T	T	T	T	T	T	T	T

VALID (all rows are True)

You can try logical equivalence too:

$(Sm \wedge Ht) \Rightarrow Fr \equiv \neg (Sm \wedge Ht) \vee Fr \equiv (\neg Sm \vee \neg Ht) \vee Fr \equiv (\neg Sm \vee \neg Ht) \vee Fr \vee Fr \equiv (\neg Sm \vee Fr) \vee (\neg Ht \vee Fr) \equiv ((Sm \Rightarrow Fr) \vee (Ht \Rightarrow Fr))$

Thus, $(Sm \wedge Ht) \Rightarrow Fr \equiv ((Sm \Rightarrow Fr) \vee (Ht \Rightarrow Fr))$;
which means the sentence $((Sm \wedge Ht) \Rightarrow Fr) \Leftrightarrow ((Sm \Rightarrow Fr) \vee (Ht \Rightarrow Fr))$ is VALID.

f.

Sm	**Fr**	**Ht**	$Sm \Rightarrow Fr$	$Sm \wedge Ht$	$((Sm \wedge Ht) \Rightarrow Fr)$	$(Sm \Rightarrow Fr) \Rightarrow ((Sm \wedge Ht) \Rightarrow Fr)$
F	F	F	T	F	T	T
F	F	T	T	F	T	T
F	T	F	T	F	T	T
F	T	T	T	F	T	T
T	F	F	F	F	T	T
T	F	T	F	T	F	T
T	T	F	T	F	T	T
T	T	T	T	T	T	T

VALID (all rows are True)

Questions g. and h. will be the exercises for you to get some practice.

Task 3: Represent the following sentences in propositional logic. Can you prove that the unicorn is mythical? What about magical? Horned?

KB	If the unicorn is mythical, then it is immortal, but if it is not mythical, then it is a mortal mammal. If the unicorn is either immortal or a mammal, then it is horned. The unicorn is magical if it is horned.

Let (propositional variables):

- Mythical – My
- Immortal / Mortal – I / ¬I
- Mammal - Mam
- Horned – H
- Magical – Ma

Sentences will be translated to:

- If the unicorn is mythical, then it is immortal, but if it is not mythical, then it is a mortal mammal.
  (My ⇒ I)
  (¬My ⇒ ¬I ∧ Mam)
- If the unicorn is either immortal or a mammal, then it is horned.
  I ∨ Mam ⇒ H
- The unicorn is magical if it is horned.
  H ⇒ Ma

  KB = (My ⇒ I) ∧ (¬My ⇒ ¬I ∧ Mam) ∧ (I ∨ Mam ⇒ H) ∧ (H ⇒ Ma)

Can you prove that the unicorn is mythical?
We consider the first column in green in the cases where our KB is true. We can see that in the first instance My=0(False), which means that **KB|/=My**. That's why we cannot prove that unicorn is mythical.
What about magical? Horned?
We consider the column in red in the cases where KB is true. We can see that **KB|=Ma**, which means that we can prove that unicorn is magical. Similarly with horned (column in orange), we get **KB|=H**, which means that we can prove that unicorn is horned.

My	I	Mam	H	Ma	My ⇒ I	¬I ∧ Mam	¬My ⇒ ¬I ∧ Mam	I ∨ Mam	I ∨ Mam ⇒ H	H ⇒ Ma	(My ⇒ I) ∧ (¬My ⇒ ¬I ∧ Mam) ∧ I ∨ Mam ⇒ H ∧ H ⇒ Ma
F	F	F	F	F	T	F	F	F	T	T	F
F	F	F	F	T	T	F	F	F	T	T	F
F	F	F	T	F	T	F	F	F	T	F	F
F	F	F	T	T	T	F	F	F	T	T	F
F	F	T	F	F	T	T	T	T	F	T	F
F	F	T	F	T	T	T	T	T	F	T	F
F	F	T	T	F	T	T	T	T	T	F	F
F	**F**	**T**	**T**	**T**	**T**	**T**	**T**	**T**	**T**	**T**	**T**
F	T	F	F	F	T	F	F	T	F	T	F
F	T	F	F	T	T	F	F	T	F	T	F
F	T	F	T	F	T	F	F	T	T	F	F
F	T	F	T	T	T	F	F	T	T	T	F
F	T	T	F	F	T	F	F	T	F	T	F
F	T	T	F	T	T	F	F	T	F	T	F
F	T	T	T	F	T	F	F	T	T	F	F
F	T	T	T	T	T	F	F	T	T	T	F
T	F	F	F	F	F	F	T	F	T	T	F
T	F	F	F	T	F	F	T	F	T	T	F
T	F	F	T	F	F	F	T	F	T	F	F
T	F	F	T	T	F	F	T	F	T	T	F
T	F	T	F	F	F	T	T	T	F	T	F
T	F	T	F	T	F	T	T	T	F	T	F
T	F	T	T	F	F	T	T	T	T	F	F
T	F	T	T	T	F	T	T	T	T	T	F
T	T	F	F	F	T	F	T	T	F	T	F
T	T	F	F	T	T	F	T	T	F	T	F
T	T	F	T	F	T	F	T	T	T	F	F
T	**T**	**F**	**T**	**T**	**T**	**F**	**T**	**T**	**T**	**T**	**T**
T	T	T	F	F	T	F	T	T	F	T	F
T	T	T	F	T	T	F	T	T	F	T	F
T	T	T	T	F	T	F	T	T	T	F	F
T	**T**	**T**	**T**	**T**	**T**	F	**T**	T	**T**	**T**	**T**

-

Is the unicorn Mythical?

-
- If **KB|= My, yes**
- If **KB|=¬My, no**
- Otherwise, **don't know**

Task 4:

a. For each of the following, find a satisfying truth assignment, (values of the propositions which make the formula true), if any exists.
   1. $((a \Rightarrow \neg b) \wedge a)$
   2. $(((a \Rightarrow c) \Rightarrow \neg b) \wedge (a \vee b))$

b. For each of the following, find a falsifying truth assignment, (values of the propositions which make the formula false), if any exists.
   1. $((a \Rightarrow \neg b) \vee a)$
   2. $((\neg b \Rightarrow (a \Rightarrow c)) \vee (a \wedge b))$

Answer:

a. For each of the following, find a **satisfying truth assignment**, (values of the propositions which make the formula true), if any exists.
   1. $((a \Rightarrow \neg b) \wedge a)$

a	**b**	**¬b**	**(a⇒ ¬b)**	**((a⇒ ¬b)∧a)**
F	F	T	T	F
F	T	F	T	F
T	**F**	**T**	**T**	**T**
T	T	F	F	F

   2. $(((a \Rightarrow c) \Rightarrow \neg b) \wedge (a \vee b))$

a	**b**	**c**	**(a⇒c)**	**¬b**	**((a⇒c) ⇒ ¬b)**	**(a∨b)**	**(((a⇒c) ⇒ ¬b)∧(a∨b))**
F	F	F	T	T	T	F	F
F	F	T	T	T	T	F	F
F	T	F	T	F	F	T	F
F	T	T	T	F	F	T	F
T	**F**	**F**	**F**	**T**	**T**	**T**	**T**
T	**F**	**T**	**T**	**T**	**T**	**T**	**T**
T	**T**	**F**	**F**	**F**	**T**	**T**	**T**
T	T	T	T	F	F	T	F

b. For each of the following, find a falsifying truth assignment, (values of the propositions which make the formula false), if any exists.
   1. $((a \Rightarrow \neg b) \vee a)$

a	**b**	**¬b**	**(a⇒ ¬b)**	**((a⇒ ¬b)∨a)**
F	F	T	T	T
F	T	F	T	T
T	F	T	T	T
T	T	F	F	T

Does not exist

   2. $((\neg b \Rightarrow (a \Rightarrow c)) \vee (a \wedge b))$

a	b	c	¬b	(a⇒c)	(¬b⇒ (a⇒c))	(a∧b)	((¬b⇒ (a⇒c))∨(a∧b))
F	F	F	T	T	T	F	T
F	F	T	T	T	T	F	T
F	T	F	F	T	T	F	T
F	T	T	F	T	T	F	T
T	**F**	**F**	**T**	**F**	**F**	**F**	**F**
T	F	T	T	T	T	F	T
T	T	F	F	F	T	T	T
T	T	T	F	T	T	T	T

For some of these questions, there is another faster way to do it:
For example, for Question a.1 above to find the satisfying truth assignment for ((a⇒ ¬b)∧a); that is to make sure that it is evaluated to TRUE then both sides of the conjunction ∧ has to be TRUE: (a⇒ ¬b) has to be TRUE and a also has to be TRUE. Because a is TRUE, the implication (a⇒ ¬b) can only be true if ¬b is TRUE. Thus, b must be FALSE. Therefore, **the satisfying truth assignment for** ((a⇒ ¬b)∧a) **is** a= TRUE and b = FALSE.

Task 5: Your friend Tracy argues: “It is bad to be depressed. Watching the news makes me feel depressed. Thus, it's good to avoid watching the news.” Regardless of whether the premises and conclusion are true, show that the **argument is not**, by converting it to propositional logic.

We assume that:
Bad/Good = **B/¬B**
Depressed = **D**
Watching news = **WN**

A **premise** is a statement that the argument will induce or justify the conclusion.
Each letter that can be either true or false is a **propositional variable**.

Tracy’s argument involves three (3) sentences:

It is bad to be depressed. **D ⇒ B**
Watching the news makes me feel depressed. **WN ⇒ D**
It's good to avoid watching the news. **¬WN ⇒ ¬B**

Tracy’s argument starts with 2 premises: **D ⇒ B** and **WN ⇒ D**, and conclude (using THUS) with the conclusion: **¬WN ⇒ ¬B**. Formally, Tracy’s argument is:
((D ⇒ B) ∧ (WN ⇒ D)) ⇒ (¬WN ⇒ ¬B)

D	B	WN	D ⇒ B	WN ⇒ D	(D ⇒ B) ∧(WN ⇒ D)	¬WN ⇒ ¬B	((D ⇒ B) ∧ (WN ⇒ D)) ⇒ (¬WN ⇒ ¬B)
F	F	F	T	T	T	T	T
F	F	T	T	F	F	T	T
F	T	F	T	T	T	F	F
F	T	T	T	F	F	T	T

T	F	F	F	T	F	T	T
T	F	T	F	T	F	T	T
T	T	F	T	T	T	F	F
T	T	T	T	T	T	T	T

The shaded rows indicates the models in which Tracy's argument is clearly flawed. I.e. the sentence representing Tracy's argument is NOT valid.