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### *Dedication*

To Pat, my wife, for her continuous support through my many book projects, for formally and informally copy-editing my books, for maintaining good cheer throughout, and for preventing all *egregious* mistakes that would have otherwise found their way into print. Her contributions are deeply appreciated.

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