Tutorial Probability and statistics: solutions

Solutions

1.

a) Firstly, we observe that there are $C(52,2) = 1326$ ways of drawing 2 cards from 52 cards. There are $C(13,2) = 78$ ways to draw 2 clubs from 13 clubs. Therefore,

$$p = \frac{\text{number of ways 2 clubs can be drawn}}{\text{number of ways 2 cards can be drawn}} = \frac{78}{1326} = \frac{1}{17}$$

b) There are 13 clubs and 13 spades, resulting in 13 x 13 = 169 ways to draw a club and a spade, therefore,

$$p = \frac{169}{1326} = \frac{13}{102}$$

2. Let M = {Australians who have not visited Melbourne} and S = {Australians who have not visited Sydney}; which means $P(M) = 0.25$, $P(S) = 0.15$ and $P(M \cap S) = 0.1$.

a) The probability that a person has visited Melbourne given that she has not visited Sydney is:

$$P(M \mid S) = \frac{P(M \cap S)}{P(S)} = \frac{0.1}{0.15} = \frac{2}{3}$$

b) The probability that a person has not visited Sydney given that she has not visited Melbourne is:

$$P(S \mid M) = \frac{P(M \cap S)}{P(M)} = \frac{0.1}{0.25} = \frac{2}{5}$$

c) The probability that a person has not visited either Melbourne or Sydney is:

$$P(M \cup S) = P(M) + P(S) - P(M \cap S) = 0.25 + 0.15 - 0.10 = 0.30$$

3.

a) The sample space S of the experiment is as follows:

$$S = \{(1,H),(2,H),(3,H),(4,H),(5,H),(6,H),(1,T),(2,T),(3,T),(4,T),(5,T),(6,T)\}$$

b) Let E be the event “the die shows an odd number and the coin shows a head’’. Event E may be described as follows:

$$E = \{(1,H),(3,H),(5,H)\}$$

The probability P(E) is given by:

$$P(E) = \frac{n(E)}{n(S)} = \frac{3}{12} = \frac{1}{4}$$

4.

a) Independent as $P(A)P(B) = P(A \cap B)$.
b) Dependent as $P(A)P(B) \neq P(A \cap B)$.
c) Independent as $P(A)P(B) = 0.4 \times 0.4 = 0.16 = P(A \cap B)$.

5.

a) Kerry's expectation (E) is:

$$E = \frac{\$98,000}{1000} + \frac{\$18,000}{1000} + \frac{\$8,000}{1000} + \frac{-\$2,000 \times 997}{1000} = -\$1870$$

b) To be fair, E must be zero, so solving for x we find:

$$\frac{\$100,000 - x}{1000} + \frac{\$20,000 - x}{1000} + \frac{\$10,000 - x}{1000} + \frac{\$0 - 997x}{1000} = \$0$$

$$\$100,000 - x + \$20,000 - x + \$10,000 - x - 997x = \$0$$

$$\$1,000x = \$130,000$$

$$x = \$130$$

6. Use the binomial distribution function with $n = 4$, $p = 0.2$ and $q = 1 - p = 0.8$.

a) $P(X = 2; 4, 0.2) = C(4, 2) \times 0.2^2 \times 0.8^2 = 0.1536$
b) $P(X = 3; 4, 0.2) = C(4, 3) \times 0.2^3 \times 0.8^1 = 0.0256$
c) $P(X \geq 1) = 1 - P(X = 0; 4, 0.2) = 1 - 0.8^4 = 0.5904$

7. This is another example of the use of the binomial distribution with $n = 4$, $p = 2/3$ and $q = 1 - p = 1/3$. Carlton wins more than half the games if it wins 3 or 4 games, which means:

$$P\left(X = 3; 4, \frac{2}{3}\right) + P\left(X = 4; 4, \frac{2}{3}\right) = \binom{4}{3} \times \left(\frac{2}{3}\right)^3 \times \left(\frac{1}{3}\right) + \binom{4}{4} \times \left(\frac{2}{3}\right)^4 = \frac{32}{81} + \frac{16}{81} = \frac{16}{27}$$

8. We are taking 2 from 2 Turkish Delight chocolates and 4 from 10 normal chocolates. In total we are selecting 6 chocolates from 12. This is an example of the hypergeometric distribution with $n = 6$, $r = 2$, $W = 10$ and $R = 2$.

$$\frac{\binom{2}{2} \times \binom{10}{4}}{\binom{12}{6}} = \frac{1 \times 210}{924} = 0.2273$$

9. This is an example of the Poisson distribution because changes in weather qualify as rare events. We use $\lambda = 4$ and the formula $P(X = k) = e^{-\lambda}\lambda^k/k!$.

a) We need to find $P(X = 8)$.

$P(X = 8) = e^{-4}\frac{4^8}{8!} = 0.0298$

b) For this part, we need to find $P(X \geq 8)$ or $P(X > 7)$.

$P(X > 7) = 1 - [P(X = 0) + P(X = 1) + \ldots P(X = 7)]$
$P(X = 0) = e^{-4}\frac{4^0}{0!} = 0.0183$
$P(X = 1) = e^{-4}\frac{4^1}{1!} = 0.0733$
$\ldots$
$P(X = 7) = e^{-4}\frac{4^7}{7!} = 0.0595$

$P(X > 7) = 1 - [0.0183 + 0.0733 + 0.1465 + 0.1954 + 0.1954 + 0.1563 + 0.1042 + 0.0595] = 0.0511$

10. This is an example of the geometric distribution, with $p = 0.3$ and using the formula $P(X = k) = (1 - p)^{k-1}p$ for the k^{th} trial being a success.

a) $P(X = 3) = 0.7^2 \times 0.3 = 0.147$
b) $P(X > 4) = 1 - P(X \leq 4) = 1 - (0.3 + 0.7^1 \times 0.3 + 0.7^2 \times 0.3 + 0.7^3 \times 0.3) = 1 - [0.3 + 0.21 + 0.147 + 0.1029] = 0.2401$

Note this is the same as $1 - (1 - 0.7^4)$.

11. Since 20% of goods have 1 or more defect, that means $P(X \geq 1) = 0.2$, i.e., $1 - P(X = 0) = 0.2$ from which we obtain $e^{-\lambda} = 0.8$, $\lambda = 0.2232$ which is the average number of defects per specimen. Therefore $P(X = 1) = 0.1786$ or 17.86% have only 1 defect.