# Tutorial Relations and functions: solutions

## Solutions

1. See figure with Solution 1, Relation diagrams. Note we are no longer doing diagram a) as part of this unit.


Figure 1: Solution 1, Relation diagrams

2.

a) We first determine those numbers in $A$ that are divisible by $1, 2, 3, 4,$ and $6$:

$$1|1, 1|2, 1|3, 1|4, 1|6, 2|2, 2|4, 2|6, 3|3, 3|6, 4|4, 6|6$$

Therefore: $R = \{(1,1), (1,2), (1,3), (1,4), (1,6), (2,2), (2,4), (2,6), (3,3), (3,6), (4,4), (6,6)\}$

b) Draw a digraph of the relation: Write the elements of the set and use $R$ to draw the arrows. See figure with Solution 2b, Digraph.

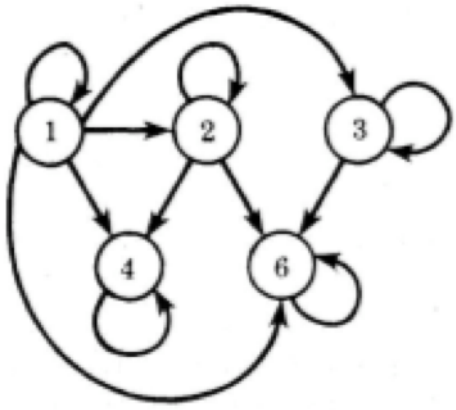


Figure 2: Solution 2b, Digraph

3.

a) Yes.

- Reflexivity: for all $a$, $a + a$ is even
- Symmetry: if $a + b$ is even, then $b + a$ is even
- Transitivity: if $a + b$ is even, and $b + c$ is even, then $a + c$ is even

b) No.
- Reflexivity: for all $a$, $a + a$ is not odd. Case: $a + a = 2a$ is always even if $a$ is an integer.

c) Yes.
- Reflexivity: for all $a$, $a \times a \geq 0$
- Symmetry: if $a \times b \geq 0$, then $b \times a \geq 0$
- Transitivity: if $a \times b \geq 0$, and $b \times c \geq 0$, then $a \times c \geq 0$

d) No.
- Reflexivity: for all $a$, $\mid a - a \mid \leq 2$
- Symmetry: if $\mid a - b \mid \leq 2$ , then $\mid b - a \mid \leq 2$
- Transitivity: if $\mid a - b \mid \leq 2$ , and $\mid b - c \mid \leq 2$, then if is not necessarily so that $\mid a - c \mid \leq 2$. Case: $a = 5, b = 3, c = 1, a - c = 4 \nleq 2$.

4. Only c) is a valid partition. a) has $c$ twice, b) has the empty set, and d) does not have all the elements.

5.

a) No, because there is nothing assigned to the element $b \in A$.
b) No, because two elements $x$ and $z$ are assigned $c \in A$.
c) Yes, because all the elements of $A$ are assigned and the assignments are unique (unlike case b).

6.

a) Partial, as odd inputs do not have outputs.
b) Total.
c) Partial; it is only valid when $x \geq y/2$.

7.

a) $f^{-1} : \mathbb{R} \to \mathbb{R}, f^{-1}(x) = (x - 2)/3$
b) No inverse function, not a bijection
c) No inverse function, not a bijection

8.

a) See figure with Soultion 8a, Bijection
b) Surjection and injection.

9.

a) $(f \circ f)(x) = 16x - 15$
b) $(f \circ g)(x) = 4x^2 + 1$

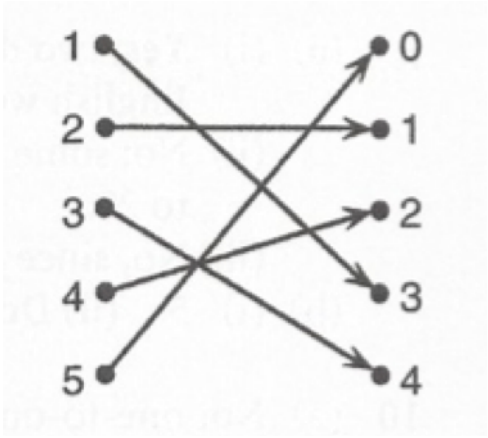


Figure 3: Soultion 8a, Bijection

c)

$$(h \circ f)(x) = \begin{cases} 1 & x \geq \frac{3}{4} \\ 0 & x < \frac{3}{4} \end{cases}$$

10.

a) `"COMPUTER LOGIC ESSENTIALS"`
b) `"COMPUTER LOGIC ESSENTIALS"`