

# Preface

This updated edition is intended for a one- or two-term introductory course in discrete mathematics, based on my experience in teaching this course over many years and requests from users of previous editions. Formal mathematics prerequisites are minimal; calculus is not required. There are no computer science prerequisites. The book includes examples, exercises, figures, tables, sections on problem-solving, sections containing problem-solving tips, section reviews, notes, chapter reviews, self-tests, and computer exercises to help the reader master introductory discrete mathematics. In addition, an Instructor's Solutions Manual and website are available.

In the early 1980s there were few textbooks appropriate for an introductory course in discrete mathematics. However, there was a need for a course that extended students' mathematical maturity and ability to deal with abstraction, which also included useful topics such as combinatorics, algorithms, and graphs. The original edition of this book (1984) addressed this need and significantly influenced the development of discrete mathematics courses. Subsequently, discrete mathematics courses were endorsed by many groups for several different audiences, including mathematics and computer science majors. A panel of the Mathematical Association of America (MAA) endorsed a year-long course in discrete mathematics. The Educational Activities Board of the Institute of Electrical and Electronics Engineers (IEEE) recommended a freshman discrete mathematics course. The Association for Computing Machinery (ACM) and IEEE accreditation guidelines mandated a discrete mathematics course. This edition, like its predecessors, includes topics such as algorithms, combinatorics, sets, functions, and mathematical induction endorsed by these groups. It also addresses understanding and constructing proofs and, generally, expanding mathematical maturity.

## New to This Edition

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The changes in this book, the eighth edition, result from comments and requests from numerous users and reviewers of previous editions of the book. This edition includes the following changes from the seventh edition:

- The web icons in the seventh edition have been replaced by short URLs, making it possible to quickly access the appropriate web page, for example, by using a hand-held device.
- The exercises in the chapter self-tests no longer identify the relevant sections making the self-test more like a real exam. (The hints to these exercises *do* identify the relevant sections.)

- Examples that are worked problems clearly identify where the solution begins and ends.
- The number of exercises in the first three chapters (Sets and Logic; Proofs; and Functions, Sequences, and Relations) has been increased from approximately 1640 worked examples and exercises in the seventh edition to over 1750 in the current edition.
- Many comments have been added to clarify potentially tricky concepts (e.g., “subset” and “element of,” collection of sets, logical equivalence of a sequence of propositions, logarithmic scale on a graph).
- There are more examples illustrating diverse approaches to developing proofs and alternative ways to prove a particular result [see, e.g., Examples 2.2.4 and 2.2.8; Examples 6.1.3(c) and 6.1.12; Examples 6.7.7, 6.7.8, and 6.7.9; Examples 6.8.1 and 6.8.2].
- A number of definitions have been revised to allow them to be more directly applied in proofs [see, e.g., one-to-one function (Definition 3.1.22) and onto function (Definition 3.1.29)].
- Additional real-world examples (see descriptions in the following section) are included.
- The altered definition of sequence (Definition 3.2.1) provides more generality and makes subsequent discussion smoother (e.g., the discussion of subsequences).
- Exercises have been added (Exercises 40–49, Section 5.1) to give an example of an algebraic system in which prime factorization does not hold.
- An application of the binomial theorem is used to prove Fermat’s little theorem (Exercises 40 and 41, Section 6.7).
- There is now a randomized algorithm to search for a Hamiltonian cycle in a graph (Algorithm 8.3.10).
- The Closest-Pair Problem (Section 13.1 in the seventh edition) has been integrated into Chapter 7 (Recurrence Relations) in the current edition. The algorithm to solve the closest-pair problem is based on merge sort, which is discussed and analyzed in Chapter 7. Chapter 13 in the seventh edition, which has now been removed, had only one additional section.
- A number of recent books and articles have been added to the list of references, and several book references have been updated to current editions.
- The number of exercises has been increased to nearly 4500. (There were approximately 4200 in the seventh edition.)

## Contents and Structure

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### Content Overview

#### Chapter 1 Sets and Logic

Coverage includes quantifiers and features practical examples such as using the Google search engine (Example 1.2.13). We cover translating between English and symbolic expressions as well as logic in programming languages. We also include a logic game (Example 1.6.15), which offers an alternative way to determine whether a quantified propositional function is true or false.

## Chapter 2 Proofs

Proof techniques discussed include direct proofs, counterexamples, proof by contradiction, proof by contrapositive, proof by cases, proofs of equivalence, existence proofs (constructive and nonconstructive), and mathematical induction. We present loop invariants as a practical application of mathematical induction. We also include a brief, optional section on resolution proofs (a proof technique that can be automated).

## Chapter 3 Functions, Sequences, and Relations

The chapter includes strings, sum and product notations, and motivating examples such as the Luhn algorithm for computing credit card check digits, which opens the chapter. Other examples include an introduction to hash functions (Example 3.1.15), pseudo-random number generators (Example 3.1.16), a real-world example of function composition showing its use in making a price comparison (Example 3.1.45), an application of partial orders to task scheduling (Section 3.3), and relational databases (Section 3.6).

## Chapter 4 Algorithms

The chapter features a thorough discussion of algorithms, recursive algorithms, and the analysis of algorithms. We present a number of examples of algorithms before getting into big-oh and related notations (Sections 4.1 and 4.2), thus providing a gentle introduction and motivating the formalism that follows. We then continue with a full discussion of the “big oh,” omega, and theta notations for the growth of functions (Section 4.3). Having all of these notations available makes it possible to make precise statements about the growth of functions and the time and space required by algorithms.

We use the algorithmic approach throughout the remainder of the book. We mention that many modern algorithms do not have all the properties of classical algorithms (e.g., many modern algorithms are not general, deterministic, or even finite). To illustrate the point, we give an example of a randomized algorithm (Example 4.2.4). Algorithms are written in a flexible form of pseudocode, which resembles currently popular languages such as C, C++, and Java. (The book does not assume any computer science prerequisites; the description of the pseudocode used is given in Appendix C.) Among the algorithms presented are:

- Tiling (Section 4.4)
- Euclidean algorithm for finding the greatest common divisor (Section 5.3)
- RSA public-key encryption algorithm (Section 5.4)
- Generating combinations and permutations (Section 6.4)
- Merge sort (Section 7.3)
- Finding a closest pair of points (Section 7.4)
- Dijkstra’s shortest-path algorithm (Section 8.4)
- Backtracking algorithms (Section 9.3)
- Breadth-first and depth-first search (Section 9.3)
- Tree traversals (Section 9.6)
- Evaluating a game tree (Section 9.9)
- Finding a maximal flow in a network (Section 10.2)

## Chapter 5 Introduction to Number Theory

The chapter includes classical results (e.g., divisibility, the infinitude of primes, fundamental theorem of arithmetic), as well as algorithmic number theory (e.g., the Euclidean algorithm to find the greatest common divisor, exponentiation using repeated squaring, computing  $s$  and  $t$  such that  $\gcd(a, b) = sa + tb$ , computing an inverse modulo an inte-

ger). The major application is the RSA public-key cryptosystem (Section 5.4). The calculations required by the RSA public-key cryptosystem are performed using the algorithms previously developed in the chapter.

### **Chapter 6 Counting Methods and the Pigeonhole Principle**

Coverage includes combinations, permutations, discrete probability (optional Sections 6.5 and 6.6), and the Pigeonhole Principle. Applications include internet addressing (Section 6.1) and real-world pattern recognition problems in telemarketing (Example 6.6.21) and virus detection (Example 6.6.22) using Bayes' Theorem.

### **Chapter 7 Recurrence Relations**

The chapter includes recurrence relations and their use in the analysis of algorithms.

### **Chapter 8 Graph Theory**

Coverage includes graph models of parallel computers, the knight's tour, Hamiltonian cycles, graph isomorphisms, and planar graphs. Theorem 8.4.3 gives a simple, short, elegant proof of the correctness of Dijkstra's algorithm.

### **Chapter 9 Trees**

Coverage includes binary trees, tree traversals, minimal spanning trees, decision trees, the minimum time for sorting, and tree isomorphisms.

### **Chapter 10 Network Models**

Coverage includes the maximal flow algorithm and matching.

### **Chapter 11 Boolean Algebras and Combinatorial Circuits**

Coverage emphasizes the relation of Boolean algebras to combinatorial circuits.

### **Chapter 12 Automata, Grammars, and Languages**

Our approach emphasizes modeling and applications. We discuss the *SR* flip-flop circuit in Example 12.1.11, and we describe fractals, including the von Koch snowflake, which can be described by special kinds of grammars (Example 12.3.19).

### **Book frontmatter and endmatter**

Appendixes include coverage of matrices, basic algebra, and pseudocode. A reference section provides more than 160 references to additional sources of information. Front and back endpapers summarize the mathematical and algorithm notation used in the book.

## **Features of Content Coverage**

- **A strong emphasis on the interplay among the various topics.** Examples of this include:
  - We closely tie mathematical induction to recursive algorithms (Section 4.4).
  - We use the Fibonacci sequence in the analysis of the Euclidean algorithm (Section 5.3).
  - Many exercises throughout the book require mathematical induction.
  - We show how to characterize the components of a graph by defining an equivalence relation on the set of vertices (see the discussion following Example 8.2.13).
  - We count the number of nonisomorphic  $n$ -vertex binary trees (Theorem 9.8.12).
- **A strong emphasis on reading and doing proofs.** We illustrate most proofs of theorems with annotated figures and/or motivate them by special Discussion sec-

tions. Separate sections (Problem-Solving Corners) show students how to attack and solve problems and how to do proofs. Special end-of-section Problem-Solving Tips highlight the main problem-solving techniques of the section.

- **A large number of applications, especially applications to computer science.**
- **Figures and tables** illustrate concepts, show how algorithms work, elucidate proofs, and motivate the material. Several figures illustrate proofs of theorems. The captions of these figures provide additional explanation and insight into the proofs.

## Textbook Structure

Each chapter is organized as follows:

Chapter X Overview  
 Section X.1  
 Section X.1 Review Exercises  
 Section X.1 Exercises  
 Section X.2  
 Section X.2 Review Exercises  
 Section X.2 Exercises  
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 Chapter X Notes  
 Chapter X Review  
 Chapter X Self-Test  
 Chapter X Computer Exercises

In addition, most chapters have **Problem-Solving Corners** (see “Hallmark Features” for more information about this feature).

**Section review exercises** review the key concepts, definitions, theorems, techniques, and so on of the section. All section review exercises have answers in the back of the book. Although intended for reviews of the sections, section review exercises can also be used for placement and pretesting.

**Chapter notes** contain suggestions for further reading. **Chapter reviews** provide reference lists of the key concepts of the chapters. **Chapter self-tests** contain exercises based on material from throughout the chapter, with answers in the back of the book.

**Computer exercises** include projects, implementation of some of the algorithms, and other programming related activities. Although there is no programming prerequisite for this book and no programming is introduced in the book, these exercises are provided for those readers who want to explore discrete mathematics concepts with a computer.

## Hallmark Features

### Exercises

The book contains nearly 4500 exercises, approximately 150 of which are computer exercises. We use a star to label exercises felt to be more challenging than average. Exercise numbers in color (approximately one-third of the exercises) indicate that the exercise has a hint or solution in the back of the book. The solutions to most of the remaining exercises may be found in the Instructor’s Guide. A handful of exercises are clearly identified as requiring calculus. No calculus concepts are used in the main body of the book and, except for these marked exercises, no calculus is needed to solve the exercises.

## Examples

The book contains almost 650 worked examples. These examples show students how to tackle problems in discrete mathematics, demonstrate applications of the theory, clarify proofs, and help motivate the material.

## Problem-Solving Corners

The Problem-Solving Corner sections help students attack and solve problems and show them how to do proofs. Written in an informal style, each is a self-contained section centered around a problem. The intent of these sections is to go beyond simply presenting a proof or a solution to the problem: we show alternative ways of attacking a problem, discuss what to look for in trying to obtain a solution to a problem, and present problem-solving and proof techniques.

Each Problem-Solving Corner begins with a statement of a problem. We then discuss ways to attack the problem, followed by techniques for finding a solution. After we present a solution, we show how to correctly write it up in a formal manner. Finally, we summarize the problem-solving techniques used in the section. Some sections include a Comments subsection, which discusses connections with other topics in mathematics and computer science, provides motivation for the problem, and lists references for further reading about the problem. Some Problem-Solving Corners conclude with a few exercises.

## Supplements and Technology

### Instructor's Solution Manual (downloadable)

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The Instructor's Solutions Manual, written by the author, provides worked-out solutions for most exercises in the text. It is available for download to qualified instructors from the Pearson Instructor Resource Center [www.pearsonglobaleditions.com](http://www.pearsonglobaleditions.com).

### Web Support

The short URLs in the margin of the text provide students with direct access to relevant content at point-of-use, including:

- Expanded explanations of difficult material and links to other sites for additional information about discrete mathematics topics.
- Computer programs (in C or C++).

The URL `goo.gl/STo1E7` provides access to all of the above resources plus an errata list for the text.

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### NOTE:

When you enter URLs that appear in the text, take care to distinguish the following characters:

- l = lowercase l
- I = uppercase I
- 1 = one
- O = uppercase O
- 0 = zero

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Richard Johnsonbaugh

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