Tutorial Counting: solutions

Solutions

There is some Python code at https://github.com/CompLogicEss/Combinatorics which shows how these answers can be calculated as well as explains some of the problem solving process.

1.

a) There are 4 ways to go from A and B and 3 ways to go from B to C. Therefore there are $4 \times 3 = 12$ ways to go from A to C via C.
b) There are 12 ways to go from A to C via B and 12 ways to return. This means there are $12 \times 12 = 144$ ways for the roundtrip.

2. Using the formula:

$$C(n, r) = \binom{n}{r} = \frac{n!}{r!(n-r)!}$$

a)

$$C(16, 3) = \binom{16}{3} = \frac{16 \times 15 \times 14}{3 \times 2 \times 1} = 560$$

b)

$$C(12, 4) = \binom{12}{4} = \frac{12 \times 11 \times 10 \times 9}{4 \times 3 \times 2 \times 1} = 495$$

c)

$$C(15, 5) = \binom{15}{5} = \frac{15 \times 14 \times 13 \times 12 \times 11}{5 \times 4 \times 3 \times 2 \times 1} = 3003$$

3.

a) $8!$ because no letter is repeated.
b) $5!$ because no letter is repeated.
c) Letter 'e' occurs twice, and the letter 's' occurs three times, therefore:

$$\frac{10!}{(2! \times 3!)}$$

4.

$$\frac{12!}{4! \times 4! \times 4! \times 3!} = 5775$$

5.

a) The 4 opals can be chosen from the 11 opals, hence

$$C(11, 4) = \binom{11}{4} = \frac{11!}{4! \times 7!} = \frac{11 \times 10 \times 9 \times 8}{4 \times 3 \times 2 \times 1} = 330$$

b) The 2 white opals can be chosen in $C(6, 2)$ ways and the 2 black opals can be chosen in

$C(5,2)$ ways, therefore:

$$C(6,2) \times C(5,2) = \binom{6}{2} \times \binom{5}{2} = \frac{6!}{2! \times 4!} \times \frac{5!}{2! \times 3!} = \frac{6 \times 5}{2 \times 1} \times \frac{5 \times 4}{2 \times 1} = 150$$

c) There are $C(6,4)$ ways of drawing 4 white opals and $C(5,4)$ ways of drawing 4 black opals. This means there are $C(6,4)+C(5,4)$ ways of drawing 4 opals of the same colour.

$$C(6,4)+C(5,4) = \binom{6}{4}+\binom{5}{4} = \frac{6!}{4! \times 2!}+\frac{5!}{4! \times 1!} = \frac{6 \times 5 \times 4 \times 3}{4 \times 3 \times 2 \times 1}+\frac{5 \times 4 \times 3 \times 1}{4 \times 3 \times 2 \times 1} = 15+5 = 20$$

6. There are $C(20,3) = 1,140$ ways to form a 3-person committee. The only 3-person committees are not allowed are those containing Superman and Batman and a third person. The are $C(18,1)$ such committees, that is, from the 18 others we must choose 1. Therefore there are $1,140 - C(18,1) = 1,122$ committees that don't contain Batman and Superman.

7.

a) There are 26 letters in the alphabet. For there to be five students with the same letter, we need $4 \times 26 + 1 = 105$ students.
b) There are seven days of the week, so $9 \times 7 + 1 = 64$ students.
c) Note there are 366 possible birthdays, so $366 + 1 = 367$ students.