Tutorial Logic: solutions

Solutions

1.

a) It is not cold.
b) It is cold and raining.
c) It is cold or it is raining.
d) It is raining or it is not cold.
e) It is not cold and it is not raining.
f) It is not true that it is not raining.

2.

a) $p \wedge q$
b) $p \wedge \neg q$
c) $\neg(\neg p \vee q)$
d) $\neg p \wedge \neg q$
e) $p \vee (\neg p \wedge q)$
f) $\neg(\neg p \vee \neg q)$

3.

a)

p	q	$\neg p$	$\neg p \wedge q$
T	T	F	F
T	F	F	F
F	T	T	T
F	F	T	F

b)

p	q	$p \vee q$	$\neg(p \vee q)$
T	T	T	F
T	F	T	F
F	T	T	F
F	F	F	T

c)

p	q	$\neg q$	$p \to \neg q$
T	T	F	F
T	F	T	T
F	T	F	T
F	F	T	T

4.

a)

p	q	r	$p \land q$	$(p \land q) \to r$
T	T	T	T	T
T	T	F	T	F
T	F	T	F	T
T	F	F	F	T
F	T	T	F	T
F	T	F	F	T
F	F	T	F	T
F	F	F	F	T

b)

p	q	$\neg q$	$\neg q \land p$	$q \leftrightarrow (\neg q \land p)$
T	T	F	F	F
T	F	T	T	F
F	T	F	F	F
F	F	T	F	T

c)

p	q	r	$p \to r$	$q \to r$	$(p \to r) \lor (q \to r)$
T	T	T	T	T	T
T	T	F	F	F	F
T	F	T	T	T	T
T	F	F	F	T	T
F	T	T	T	T	T
F	T	F	T	F	T
F	F	T	T	T	T
F	F	F	T	T	T

d) a) and c) are equivalent.

5.

a)

p	q	$p \vee q$	$\neg p \wedge \neg q$	$(p \vee q) \wedge (\neg p \wedge \neg q)$
T	T	T	F	F
T	F	T	F	F
F	T	T	F	F
F	F	F	T	F

$(p \vee q) \wedge (\neg p \wedge \neg q)$ is a contradiction.

b)

p	q	$p \wedge q$	$(p \wedge q) \rightarrow q$
T	T	T	T
T	F	F	T
F	T	F	T
F	F	F	T

$(p \wedge q) \rightarrow q$ is a tautology.

c)

p	q	$p \wedge q$	$(p \wedge q) \leftrightarrow \neg p$
T	T	T	F
T	F	F	T
F	T	F	F
F	F	F	F

$(p \wedge q) \leftrightarrow \neg p$ is neither a contradiction nor a tautology.

6.

a) If he has courage then he will win. Let p denote “He has courage” and q denote “He will win”. The conditional statement above can be symbolically written as $p \rightarrow q$. Its contrapositive is symbolically written as $\neg q \rightarrow \neg p$ which translates to “If he will not win, then he has no courage”. Note that both the original conditional proposition and its contrapositive are logically equivalent.

b) Only if he does not tire will he win. Let p denote “He will win” and q denote “He tires”. The conditional statement above can be symbolically written as $p \to \neg q$. Its contrapositive is symbolically written as $\neg\neg q \to \neg p$ which translates to “If he tires, then he will not win”.

7.

a)

$$p \lor (p \land q) \equiv p \qquad \text{(absorption law)}$$

Another approach:

$$\begin{aligned} p \lor (p \land q) &\equiv (p \land T) \lor (p \land q) && \text{(identity law)} \\ &\equiv p \land (T \lor q) && \text{(distributive law)} \\ &\equiv p \land T && \text{(identity law)} \\ &\equiv p && \text{(identity law)} \end{aligned}$$

b)

$$\begin{aligned} \neg(p \lor q) \lor (\ p \land q) &\equiv (\neg p \land \neg q) \lor (\neg p \land q) && \text{(de Morgan's law)} \\ &\equiv \neg p \land (\neg q \lor q) && \text{(distributive law)} \\ &\equiv \neg p \land T && \text{(complement law)} \\ &\equiv \neg p && \text{(identity law)} \end{aligned}$$

8. The simplest forms are:

a) p
b) $p \land q$
c) $\neg p \land \neg q$

9.

a)

p	q	$\neg p$	$\neg p \to q$	$\neg q$
T	T	F	**T**	**F**
T	F	F	**T**	**T**
F	T	T	T	F

p	q	$\neg p$	$\neg p \rightarrow q$	$\neg q$
F	F	T	F	T

The argument above is not valid (i.e. fallacy) because whenever the truth values of $\neg p \rightarrow q$ and p are true, the truth value of $\neg q$ is not always true (as evident from the first row in the truth table above).

b)

p	q	r	$p \rightarrow q$	$r \rightarrow \neg q$	$r \rightarrow \neg p$
T	T	T	T	F	F
T	T	F	**T**	**T**	**T**
T	F	T	F	T	F
T	F	F	F	T	T
F	T	T	T	F	T
F	T	F	**T**	**T**	**T**
F	F	T	**T**	**T**	**T**
F	F	F	**T**	**T**	**T**

The argument above is valid because whenever $p \rightarrow q$ and $r \rightarrow \neg q$ are true, $r \rightarrow \neg p$ is always true.

10. Using contradiction:

a) Set the conclusion to false, so q must be true. If q is true, then p can be true or false to make P_1 true. If p can be true, then P_2 can be true. There is no contradiction, therefore the argument does not hold.

b) Set the conclusion to be false, $r \rightarrow \neg p$ must be false so r must be true and p must be true ($T \rightarrow F$). If p is true, then q must be true for P_1 to hold. If r is true and p is true, then P_2 is false ($T \rightarrow F$), so a contradiction is found and the argument holds.