Tutorial Fundamentals – sets: solutions

Solutions

1.

a) $A = \{3, 4, 5, 6, 7, 8, 9, 10\}$
b) $B = \{1, 3, 5, 7, 9\}$
c) C contains no elements, i.e., $C = \varnothing$, the empty set.

2.

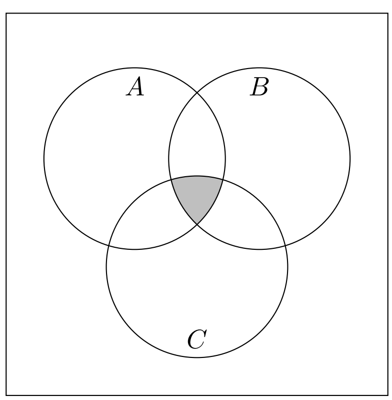
a) $\varnothing \subset A$ because $\varnothing$ is a subset of every set.
b) $A \subset B$ because 1 is the only element of A and it belongs to B.
c) $B \not\subset C$ because 3 is element of B but not of C.
d) $B \subset E$ because the elements of B are also found in E.
e) $C \not\subset D$ because 9 is an element of C but not D.
f) $C \subset E$ because the elements of C are found in E.
g) $D \not\subset E$ because 2 is an element of D but not E.
h) $D \subset U$ because the element of D are found in U.

3.

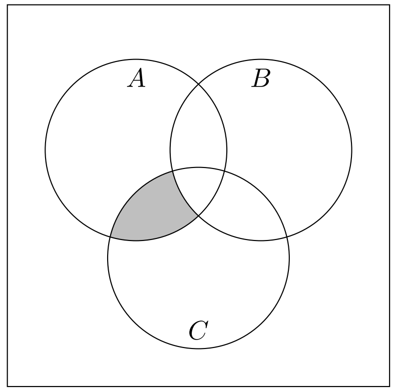
a) True, because all elements of A are in U (so $A \subset U$ also holds true).
b) False, because U has some elements (1 and 2) that are not in D.
c) True, because $B \subseteq C$ and $C \subseteq B$ (N.B. the elements of the set don't have to be listed in the same order).
d) True, because it is an element of every set by definition.

4.

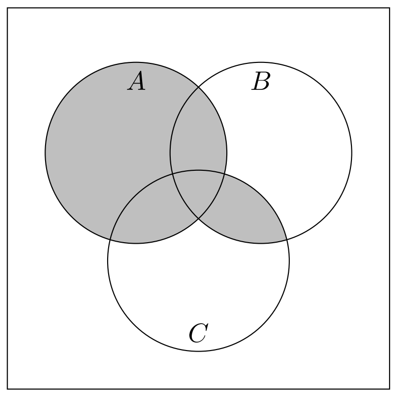
a) This is simply the intersection of A, B and C, so the areas common to all three sets are shaded.



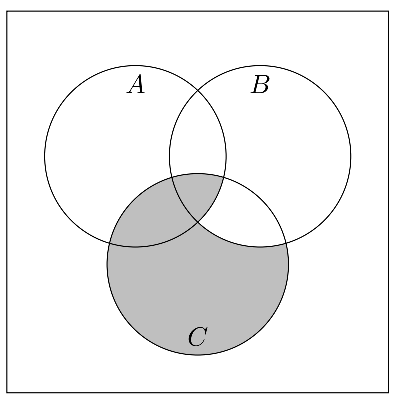
b) B^C can be simply interpreted as not B. Therefore, the intersection is the area common to A and C but not B. That is, shade the area in which A and C overlap, excluding the contribution from B.



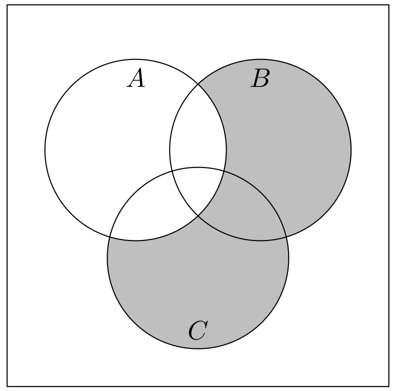
c) This is interpreted as A plus the area common to both B and C (the term in brackets). That is, shade where B and C overlap plus all of A.



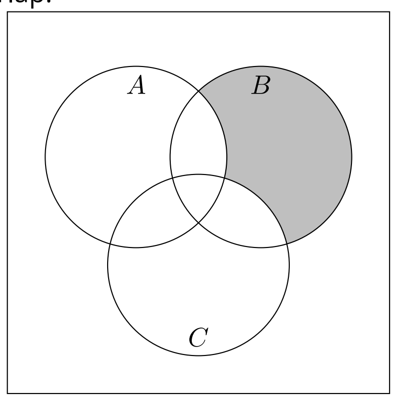
d) This is interpreted as C plus the intersection of A with all sets except B (the term in brackets). Therefore, shade where A overlaps with C plus C except for any overlapping B areas.



e) This is a combination of the areas that are not A with the intersection of B and C (the term in brackets). Therefore, shade the areas in which B and C overlap and all other areas except for A.



f) This is the combination of B and C (the term in brackets) minus C. Therefore shade only the region of B, which does not overlap.



5.

a) $U = \{1,2,3,4,5,6\}$, i.e., all the possible outcomes.
b) $A = \{1,3,5\}$
c) $B = \{3,4,5,6\}$
d) A^C = the set of elements in U not in A, i,e., $A^C = \{2,4,6\}$
e) $B^C = \{1,2\}$
f) $\{3,5\}$, i.e., the set of elements in A and also in B
g) $\{1,3,4,5,6\}$, the set of elements in either A or B
h) $\{1\}$, i.e., the set of elements in A but not in B
i) $\{2\}$, i.e., the set of elements not in A and not in B
j) $\{2\}$, i.e., the set of elements not in $A \cup B$

6.

a) For this, we use De Morgan's to get $(A^C \cap (B^C)^C)$. From here, use the involution law on $(B^C)^C$ to get $(A^C \cap B)$.

b) For both parts of the statement, the absorption law is useful:

- $A \cup (A \cap B^C) = A$

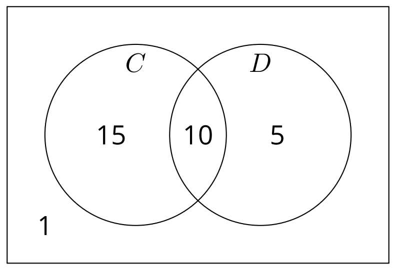
- $A \cap (A \cup B)) = A$

From here, use the idempotent law on $A \cap A$ to get A.

c) Use the distributive law to get $(A \cap B) \cup (A \cap \varnothing)$. Note that the identity law can be used for $A \cap \varnothing$ to obtain $(A \cap B) \cup \varnothing$. The identity law can be used again ($A \cup \varnothing = A$), noting that $(A \cap B)$ can be substituted into the laws for one variable, so we end up with $(A \cap B)$.

   Alternatively, use identity law $(B \cup \varnothing = B)$ to obtain $A \cap (B)$, which is $(A \cap B)$.

7. Where C = houses with cats and D = houses with dogs, the diagram looks like:



a) 1
b) 5

8.

a) 10
b) 25
c) 9
d) 15