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COS10003 (Assignment03):

1)i) Here, there is no repeat of activities and orders matters. Hence the possible activities in a day are:

Permutation (P(n,r)) formulae:

P(n,r)=(n!)/(n-r)!

a) only 1 activity out of the 5: P(5,1)

b) only 2 activity out of the 5: P(5,2)

c)only 3 activity out of the 5: P(5,3)

d)only 4 activity out of the 5: P(5,4)

e) all 5 activity: P(5,5)

Hence total no of different activity patterns: P(5,1)+ P(5,2)+ P(5,3)+ P(5,4)+ P(5,5)

(using P(n,r) formulae) =5+20+60+120+120

=325

ii)only 3 activities out of 5: P(5,3) = 60 (using P(n,r) formulae)

iii)at least 4 activities with shopping at start. Hence it can be:

a) 4 activity with shopping at start: 1* P(4,3) [1st place fixed, then take 3 from 4, but ordered]
b) 5 activity with shopping at start: 1* P(4,4) [1st place fixed, then take 4 from 4, but ordered]

So, total no of patterns: P(4,3)+P(4,4)

(using P(n,r) formulae) = 24+24

=48

b)i) Here total is 12 people. From them we need to choose any 5 people. So order doesn't matter and no repeat. Hence it will be combination.

Combination (C(n,r)) formulae:

C(n,r)=(n!)/((n-r)!(r)!)

C(12,5)

=792

ii)All 12 has been invited. So, possible outcomes are:

a) 0 people out of 12 comes: C(12,0)
b) Only 1 people out of 12 comes: C(12,1)
c) Only 2 people out of 12 comes: C(12,2)
d) Only 3 people out of 12 comes: C(12,3)
e) Only 4 people out of 12 comes: C(12,4)
f) Only 5 people out of 12 comes: C(12,5)
g) Only 6 people out of 12 comes: C(12,6)

h) Only 7 people out of 12 comes: C(12,7)
i) Only 8 people out of 12 comes: C(12,8)
j) Only 9 people out of 12 comes: C(12,9)
k) Only 10 people out of 12 comes: C(12,10)
l) Only 11 people out of 12 comes: C(12,11)
m) All 12 people out of 12 comes: C(12,12)

Hence, total number of outcomes: C(12,0)+ C(12,1)+ C(12,2)+ C(12,3)+ C(12,4)+ C(12,5)+ C(12,6)+ C(12,7)+ C(12,8)+ C(12,9)+ C(12,10)+ C(12,11)+ C(12,12)

=1+12+66+220+495+792+924+792+495+220+66+12+1

=4096

[It can also be done using power set logic(like pizza toping example shown in lecture note solved via: 2^n) (here n will 12). So answer= 2^{12}=4096 which gives the same result, but in fewer steps)]

iii) Here we are going to concert with 11 people. So now total number is 12.

They need to seat in pods of 4. Hence each pod at max will have 4 people. So the arrangement will be 4,4,4. Here the 4 from 1st pod, 2nd pod and 3rd pod can be in each other's place and thus for these 3 groups, we must also consider 3! for ordering. Hence following the logic for identical groups (ie dividing people into group of 4 in tutorial), we get:

12!/(4!4!4!3!) = 5775

iv) No iii can also be done using partition method and combination.

For first 4 people, we choose 4 people from 11 and so: C(12,4) 12-4=8 (remaining people)

For 2nd 4 people, we choose 4 people from 7 and so: C(8,4) 8-4=4(remaining people)

For last batch, we choose 3 people from 3 and so: C(4,4)

Hence no of ways to arrange pods: (C(12,4)* C(8,4)* C(4,4))/3! [as 4 people from 11,7 and 4 can be in each other's position]

=34650/3!=5775

Combination (C(n,r)) formulae:

C(n,r)=(n!)/((n-r)!(r)!)

c)Total number of days= 30. Hence n=30

here it said 20 leaning sessions. So pigeon hole or m =20

We need to show that at least once consecutive. So k+1=2.

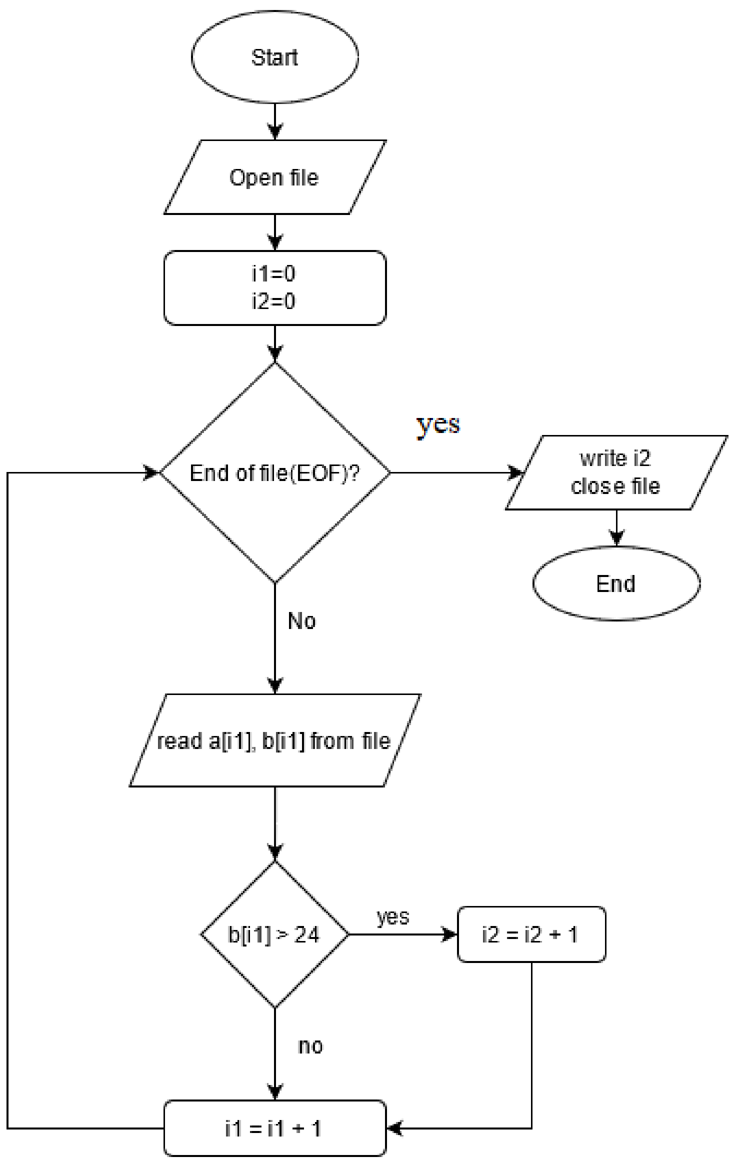
Using the pigeon hole formula, n=mk+1

(placing the values) 30 = 20k +1

K=29/20=1.45

So k+1=2.45 which is close to 2 (when rounded up). Hence we can say that yes, we will need to clean consecutive days at least once.

2) a) See the image attached. (I1 is used as index of array and increment. I2 is used to count no of students who applied to visit campus more than 24 times) (a=name, b=integer)



b) The complexity is O(n). That's because here there is only 1 loop (to read in data a(student name) and b(times the students who applied to visit campus)) and only the parameters inside the loop will be repeated n times. Here, <u>in worst case</u>, we will have: 1 + 2 +(1+2+1+1+1)n +2

=6n+5

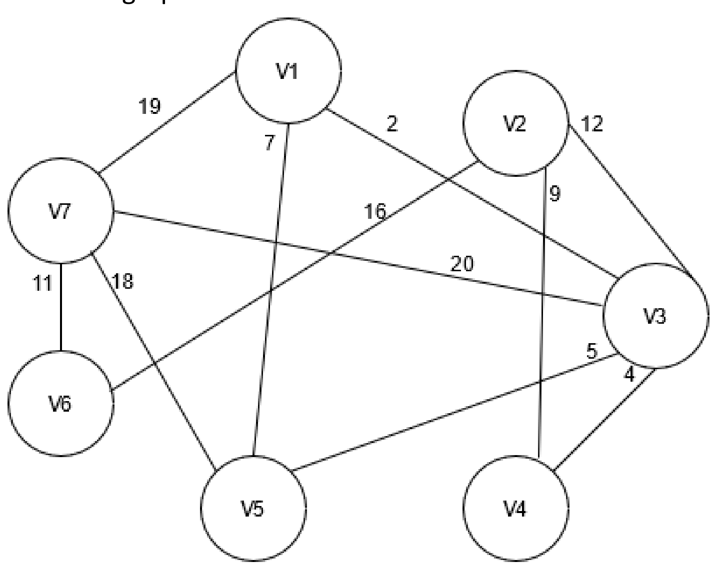
Since we, will only take leading term and ignore the coefficients, the complexity will be = O(n)

3) Here there is double base conditions, one for n =0 and another for n=1. So following the syntax for double base case found in: (there they used two ifs in defining the 2 base cases instead of if and else if)

https://www.tutorialspoint.com/learn_c_by_examples/fibonacci_non_recursive_program_in_c.htm

```
function mystery(n)
        if n==0
            return 1
        if n==1
            return -2
        else
            return   -4*(mystery(n-1))-4*(mystery(n-2))
        end if
end
```

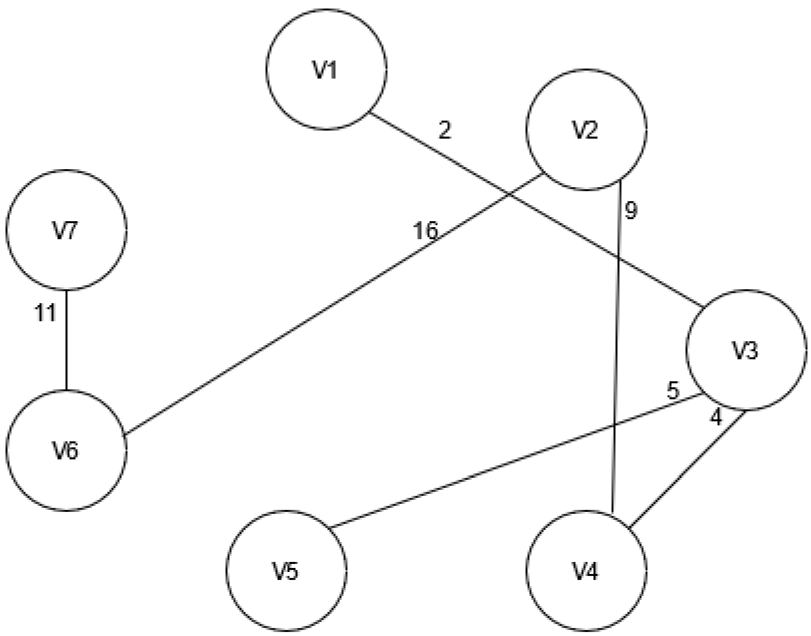
4)a) See attached picture for graph G



b)i)order of edges (kept or discarded list) as they are removed from S:

1) W(1,3) = 2 Kept
2) W(3,4) = 4 Kept
3) W(3,5) = 5 Kept
4) W(1,5) = 7 Discarded as (V1,V3,V5,V1) will make cycle
5) W(2,4) = 9 Kept
6) W(6,7) = 11 Kept
7) W(2,3) = 12 Discarded as (V2,V3,V4,V2) will make cycle
8) W(2,6) = 16 Kept
9) W(5,7) = 18 Discarded as (V5,V7,V6,V2,V4,V3,V5) will make cycle
10) W(1,7) = 19 Discarded as (V1, V7,V6,V2,V4,V3,V1) will make cycle
11) W(3,7) = 20 Discarded as (V3, V7,V6,V2,V4,V3) will make cycle

ii)See attached picture for spanning tree F



5)a)i) (fair deck, expected value)

E=\$(50-10)*(any Red face card) + \$(20-10)*(any Black card)+ \$(-10)*(remaining cards)

=\$40*(6/52) + \$10*(26/52) - \$10*((52-26-6)/52)

=\$40*(6/52) + \$10*(26/52) - \$10*(20/52)

=\$60/13 + \$5 - \$50/13

=\$75/13 = \$5.77 (2d.p.)

ii) 0 = \$(50-x)*(any Red face card) + \$(20-x)*(any Black card)+ \$(-x)*(remaining cards)

0 = \$75/13 + \$10 - \$x

0 = \$205/13 - \$x

\$x = \$205/13

= \$15.77 (decimal places rounded to nearest cent) [ie 15 dollars and 77 cents or 1577 cents]

b) (1) Probability To win \$(50-10) = need 3 red face cards

total no of red face cards=6

total no of cards = 52

so probability to pick 3 red face cards back to back is:

= P(1st red face card) *P(2nd red face card from remaining cards) *P(3rd red face card from remaining cards)

=(total red face card/total cards) * (total red face card-1/total cards-1) *=(total red face card-2/total cards-2)

=(6/52) *(5/51)*(4/50)

=1/1105

(2) Probability To win \$(20-10) = need 2 red card 1 black card

=P(1st card red)*P(2nd card red)*P(3rd card black)

=(total red card/total cards) * (total red card-1/total cards-1) *=(total black card/total cards-2)

=(26/52) *(25/51)*(26/50)

=13/102

(3) Probability to get -\$10 = 1-1/1105-13/102

=5779/6630

Hence, E= \$(50-10)* 1/1105 + \$(20-10) * 13/102 - \$10* 5779/6630

=\$8/221 + \$65/51 -\$5779/663

= -\$4910/663

= -\$7.41(2d.p.)