Course: COS10003

ID: 103172423

Name: S M Ragib Rezwan

Assignment 2:

1) Total number of students (U) = 465 (U) or |U| is the universal set
Let students who joined Student Club be => S
Let students who ate at café be => A
Let students who went to a gym be => G

- Student joined a student club (S) = 220
- Students who ate at a café (A) = 159
- Students who went to a gym (G) = 208
- Students who joined a student club and ate at a café (S ∩ A) = 68
- Students who joined a student club and went to a gym (S ∩ G) =126
- Students who joined a student club, ate at a cafe and went to a gym (S ∩ A ∩ G) =32
- Students who did not join a student club, eat at a cafe, or go to a gym (S ∪ A ∪ G)'= 101

a) Missing value: The students who ate at café and went to a gym (A ∩ G)
We can find it using the formulae for finding (S ∪ A ∪ G) and placing respective values in it:

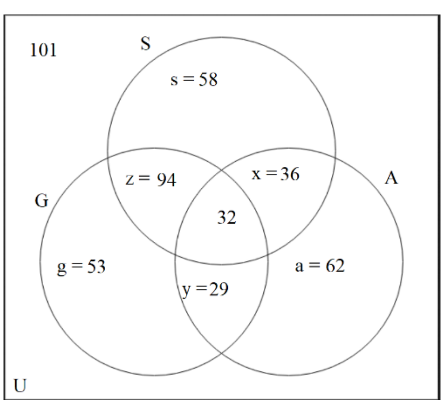
$|S \cup A \cup G| = |S| + |A| + |G| - |S \cap A| - |A \cap G| - |S \cap G| + |S \cap A \cap G|$

We know, $|S \cup A \cup G| = |U| - |S \cup A \cup G|'$

Therefore,
$|U| - |S \cup A \cup G|' = |S| + |A| + |G| - |S \cap A| - |A \cap G| - |S \cap G| + |S \cap A \cap G|$
⇨ $465 - 101 = 220 + 159 + 208 - 68 - |A \cap G| - 126 + 32$
⇨ $364 = 425 - |A \cap G|$

Hence, $|A \cap G| = 425 – 364 = 61$

b) Venn diagram (and related calculation):



From (S ∩ A) => 68 = 32 + x => x = 36

From (A ∩ G) => 61 = 32 + y => y = 29

From (S ∩ G) => 126 = 32 + z => z = 94

From (S) => 220 = 36 + 32 + 94 + s => s = 58

From (A) => 159 = 36 + 32 + 29 + a => a = 62

From (G) => 208 = 94 + 32 + 29 + g => g = 53 c)i) Not go to Gym:

$G' = (U) - G = 465 - 208 = 257$

ii) Joined Club, but not eat at café:

$S \cap A' = S - (S \cap A) = (58 + 94 + 32 + 36) - (32 + 36) = 152$

iii)Did only one of Club, ate at café and gym:

only "s" + only "a" + only "g" = 58 + 62 + 53 = 173

[Note: no. iii, can also be done using set notation and using values from Venn diagram via:

"s" + "a" + "g" = $(S \cup A \cup G) - (S \cap A) - (S \cap G) + (S \cap A \cap G) - (A \cap G) + (S \cap A \cap G)$

$= 364 - 68 - 126 + 32 - 61 + 32$

$= 173$]

d)i) not go to gym, nor eat at club:

$(G \cup A)'$

ii)Joined club, ate in café, but not go to Gym:

$(S \cap A) \cap G'$

2)a)i) Huyen plays cricket and either David plays esports or Adita plays esports.

ii) If David plays esports, then either Adita doesn't play esports or Huyen plays cricket.

iii) Huyen is not playing cricket, nor is David playing esports.

b)i) $d \rightarrow a$

ii) $\neg(a \vee d)$

iii) $a \leftrightarrow (h \wedge d)$

3) (height <= 100 or width > 10) and (height > 100 or width > 10) and height <= 100

Let height <= 100 be => h Hence, height > 100 be => ¬h

Let width > 10 be => w

Therefore, the statement can be rewritten as: $(h \vee w) \wedge (\neg h \vee w) \wedge (h)$

Using Associative Law: $(h) \wedge (h \vee w) \wedge (\neg h \vee w)$

Using Absorption Law: $(h) \wedge (\neg h \vee w)$

Using Distributive Law: $(h \wedge \neg h) \vee (h \wedge w)$

Using Complement Law: $\mathbf{F} \vee (h \wedge w)$

Using Identity Law: $(h \wedge w)$

Using Associative Law: $(w \wedge h)$

Hence, simplified version in words: width > 10 and height <= 100

4)Given, S ➔ relations of Z x Z Z={ a, b, c, d, e}

And, S = {(a, a),(b, b),(a, b),(b, a),(c, c),(d, d),(e, e),(c, e),(d, e),(e, c),(e, d)}

In order to be an Equivalence relation, it needs to be

A) Reflexive
B) Symmetric
C) Transitive

A) Testing for Reflexive:
Condition: for all "a" in Z, (a, a) ∈ R
Here, Z has elements a, b, c, d, and e
And relation S has (a,a), (b,b) (c,c) (d,d) and (e,e)
Hence, yes, it is reflexive.

B) Testing for Symmetric:
Condition: for every (a,b) is present in S, then (b,a) must be present in S
Here, S has (a,b) and it also has (b,a)
it has (c,e) and it also has (e,c)
it has (d,e) and it also has (e,d)
Hence, yes, it is symmetric.

C) Testing for Transitive:
Condition: for every a,b,c present in Z, if (a,b) and (b,c) are in relation S, so must be (a,c)
Here, (c,e) and (e,d) is present, but (c,d) is not present!
Hence, no, it is not transitive.

Since, it is reflexive and symmetric, but not transitive, it is not an equivalence relation!

5)a) For a function to be valid, each domain value must be connected to a codomain value. Some codomain value can have no domain value connected to them and some codomain value can have more than one domain value connected to them.

Hence, following that restriction and applying to domain X = {x, y} and co-domain Y = {x, y, z},

We can see, that there are 3 + 3 + 3 = 9 possible functions.

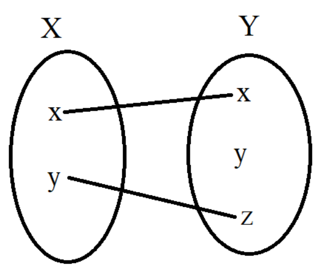
That's because, for (x,x), there can be either (y,x) or (y,y) or (y,z)

for (x,y), there can still be either (y,x) or (y,y) or (y,z)

for (x,z), there can still be either (y,x) or (y,y) or (y,z)

Hence total no of possible functions: 3 + 3 + 3 = 9

Example of a function:



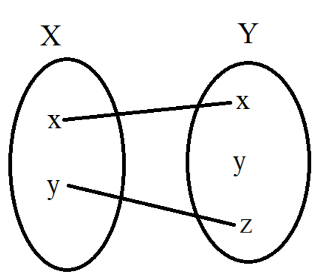
b) To be injective, ∀ a,b ∈ X, $f(a) = f(b)$ → a = b
(and each value in codomain is used at most once!)

Here, there are 6 functions that are injective.
That's because in order to be injective, the possibilities are:

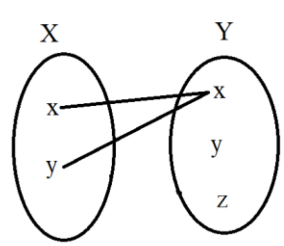
a) If (x,x), it will be accompanied by either (y,y) or (y,z)
b) If (x,y), it will be accompanied by either (y,x) or (y,z)
c) If (x,z), it will be accompanied by either (y,x) or (y,y)

Hence total possibilities for injection function: 2 + 2 + 2 = 6

Example of injection function:



Example of not injection function:



c) No Bijection exists here. That's because for bijection to exist, there must be a function with both surjection and injection. But here no surjection function can exist here. That's because for surjection, each codomain must have either 1 or more domain attached to it. But, here, if we attach 2 domain values with 1 codomain value, the other codomain values will have no domain attached to them. Hence, it won't be surjection. And since no surjection exists, no bijection can exist here.

6)a)i) E = (A + *B*') * (B)

[Note: Using B' to say "not B" as it is not letting me put the "bar" symbol over B in MS word here]

ii) E = (1+0)*1	(A=1, B=1)	1 + 1 = 1	1 * 1 = 1
= 1 * 1		1 + 0 = 1	1 * 0 = 0
= 1		0 + 1 = 1	0 * 1 = 0
		0 + 0 = 0	0 * 0 = 0

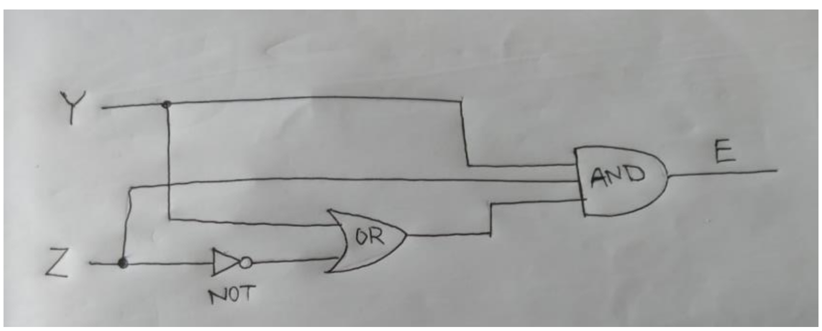
b)i) E = (A + *B*) * (A + C) * (B + C)

ii) E = (1+1)*(1+0)*(1+0) (A=1, B=1, C=0)

= 1 * 1 * 1

= 1

7) a) (Answer given as pic here)



b) E = (z' + y)(y)(z)

Using Distributive Law:	= z'yz + yyz
Using Complement Law:	= 0*y + yyz
Using Idempotent Law:	= 0*y + yz
Using Boundedness Law:	= 0 + yz
Using Identity Law:	= yz

c) Previous Circuit:
Size =3
Depth =3

New Circuit:
size =1
Depth =1

Conclusion: Both depth and size has been decreased by 2 in the new circuit.