Tutorial: Logic

Aim

The aim of this tutorial is for students to be able to work confidently with propositional logic, in particular, connectives, truth tables, simplification, and arguments.

Questions

1. Let p be "It is cold" and let q be "It is raining". Give simple sentences that describe the following statements:

   a) $\neg p$
   b) $p \wedge q$
   c) $p \vee q$
   d) $q \vee \neg p$
   e) $\neg p \wedge \neg q$
   f) $\neg\neg q$

2. Let p be "She is tall" and let q be "She is clever". Write each of the following statements in symbolic form:

   a) She is tall and clever.
   b) She is tall but not clever.
   c) It is false that she is short or clever.
   d) She is neither tall nor clever.
   e) She is tall, or she is short and clever.
   f) It is not true that she is short or not clever.

3. Draw the truth tables for:

   a) $\neg p \wedge q$
   b) $\neg(p \vee q)$
   c) $p \to \neg q$

4. Draw the truth tables for the following statements:

   a) $(p \wedge q) \to r$
   b) $q \leftrightarrow (\neg q \wedge p)$
   c) $(p \to r) \vee (q \to r)$
   d) Do you notice anything about these expressions?

5. Use a truth table to determine whether the following are a tautology, contradiction or neither.

   a) $(p \vee q) \wedge (\neg p \wedge \neg q)$
   b) $(p \wedge q) \to q$

c) $(p \wedge q) \leftrightarrow \neg p$

6. Determine the contrapositive of each of following conditional statements:

a) If he has courage then he will win.
b) Only if he does not tire will he win.

Also try converse and inverse if you have time.

7. Use the laws of logic to simply the following propositions:

a) $p \vee (p \wedge q)$
b) $\neg(p \vee q) \vee (\neg p \wedge q)$

8. Use the laws of logic to obtain the simplest possible representation of the following expressions:

a) $(p \vee \neg q) \wedge (p \vee q)$
b) $\neg(p \rightarrow \neg(p \wedge q))$
c) $\neg(p \vee (q \wedge \neg p))$

9. Use a truth table to determine the validity of the following arguments.

a) $\neg p \rightarrow q, p \models \neg q$
b) $p \rightarrow q, r \rightarrow \neg q \models r \rightarrow \neg p$

Extension questions

10. Use the laws of logic/looking for a contradiction to determine the validity of Q9.

Note: for extra practice, there are some suggested questions from other sources in Canvas.