# Tutorial: Graphs and trees

## Aim

The aim of this tutorial is to be able to define graphs and trees, work with their properties and apply shortest path algorithms.

## Questions

1. Draw the diagram of graph G with:

a) vertices A, B, C, D and edges {{A, B}, {A, D}, {B, C}, {B, D}, {C, D}};
b) vertices a, b, c, d, e, and edges {{a, b}, {a, c}, {b, c}, {d, e}}.
c) vertices a, b, c, d, e, and edges {{a, b}, {a, c}, {b, c}, {c, e}, {d, e}, {e, a}}.
d) vertices a, b, c, d, e, and edges {{a, b}, {a, d}, {b, c}, {b, d}, {e, a}, {d, e}}.

2. Consider the graphs in Q1. Determine the degree of each vertex.

3. Draw or define all the subgraphs of the graph in Figure 1 with at least two vertices.

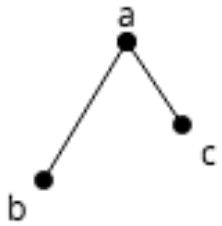


Figure 1: Question 3

4. Draw the following graphs. Find an Eulerian cycle in each graph, or state that one does not exist.

a) $C_5$
b) $K_5$
c) $K_6$
d) $K_{2,4}$
e) $K_{2,3}$

5. Given an undirected, weighted graph as shown in Figure 2, find the cost of the shortest path between $a$ and $z$ using Dijkstra's Algorithm, and enumerate the shortest path.

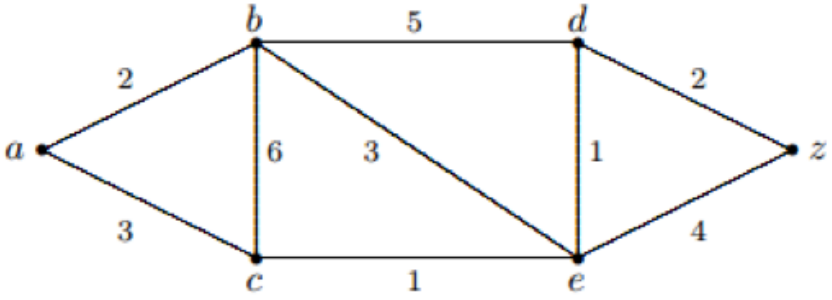


Figure 2: Question 5 and 6

6

. Given the following algorithm for finding a spanning tree (a graph that looks like a tree that connects all the nodes), show the process using the graph in Figure 2: Q5 and Q6.

i. Initialize a tree with a single vertex, chosen arbitrarily from the graph (let us choose $a$).
ii. Grow the tree by one edge: of the edges that connect the tree to vertices not yet in the tree, find the edge with the lowest weight, and transfer it to the tree.
iii. Repeat step 2 (until all vertices are in the tree). (Adapted from https://en.wikipedia.org/wiki/Prim%27s_algorithm)

7. For the tree in Figure 3, write out the nodes by traversing:

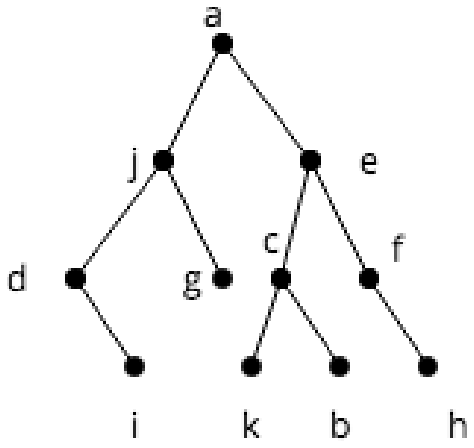


Figure 3: Question 7

a) pre-order
b) post-order

c) in-order

8. For the directed graph in Figure 4, note the order of nodes visited starting from A when using:

a) breadth-first search
b) depth-first search.

In case of a decision between two or more nodes, use ascending alphabetical order (e.g., A before B).

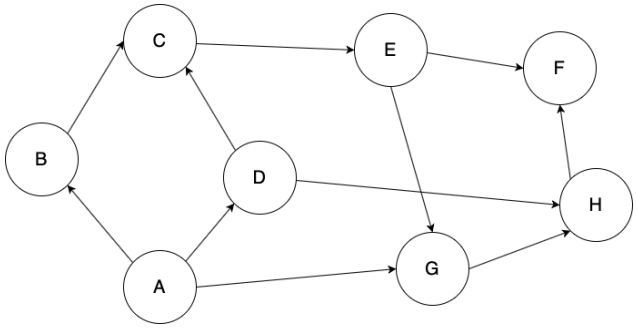


Figure 4: Question 8

## Extra questions

9. Look at the graphs in 1b-d again. Are any of them isomorphic? Which conditions do you need to check for?