

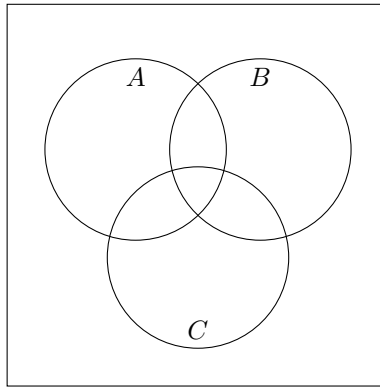
Tutorial: Fundamentals – sets

Aim

The aim of this tutorial is for students to work confidently with sets and start to logically solve problems.

Questions

1. If $\mathbb{N} = \{1, 2, 3, \dots\}$ is the set of the natural numbers, list the elements of the following sets:
 - a) $A = \{z : z \in \mathbb{N}, 2 < z < 11\}$
 - b) $B = \{z : z \in \mathbb{N}, z \text{ is odd}, z < 11\}$
 - c) $C = \{z : z \in \mathbb{N}, 2 + z = 1\}$
2. Consider the following sets: $\emptyset, A = \{1\}, B = \{1, 3\}, C = \{1, 5, 9\}, D = \{1, 2, 3, 4, 5\}, E = \{1, 3, 5, 7, 9\}, U = \{1, 2, \dots, 8, 9\}$. Insert the correct symbol \subset or $\not\subset$ between the following pairs:
 - a) $\emptyset \quad A$
 - b) $A \quad B$
 - c) $B \quad C$
 - d) $B \quad E$
 - e) $C \quad D$
 - f) $C \quad E$
 - g) $D \quad E$
 - h) $D \quad U$
3. If the universal set $U = \{1, 2, 3, 4, 5, 6\}$, $A = \{1, 3, 5\}$, $B = \{2, 4, 6\}$, $C = \{4, 2, 6\}$, and $D = \{3, 4, 5, 6\}$. Which of the following statements are true:
 - a) $A \subseteq U$
 - b) $U \subseteq D$
 - c) $B = C$
 - d) $\emptyset \subset B$
4. Considering the following Venn diagram, shade the following sets:



- a) $A \cap B \cap C$
- b) $A \cap B^C \cap C$
- c) $A \cup (B \cap C)$
- d) $C \cap (A \cup B^C)$
- e) $A^C \cap (B \cup C)$
- f) $(A^C \cap B) \setminus C$

5. Given a six sided dice, find:

- a) the universal set (U) for tossing a dice once.
- b) $A = \{z \in U, z \text{ is odd}\}$
- c) $B = \{z \in U, z > 2\}$
- d) A^C
- e) B^C
- f) $A \cap B$
- g) $A \cup B$
- h) $A \cap B^C$
- i) $A^C \cap B^C$
- j) $(A \cup B)^C$

6. Using the set algebra laws, show the following.

- a) $(A \cup B^C)^C = A^C \cap B$
- b) $(A \cup (A \cap B^C)) \cap (A \cap (A \cup B)) = A$
- c) $A \cap (B \cup \emptyset) = A \cap B$

For extra fun, draw the two Venn diagrams to check your answers.

7. A survey of 31 homes in Burwood Road found the following data on pet ownership: 15 homes had a dog; 25 homes had a cat, 10 homes had both a dog and cat.

- a) How many homes had neither a dog nor a cat?
- b) How many homes had only dogs?

8. A survey of ski areas is summarised below.

Area/Condition	Easy access, expensive lifts	Easy access, cheap ski lifts	Difficult access, expensive lifts	Difficult access, cheap lifts
Hotham	5	2	10	12
Perisher	4	1	8	15
Queenstown	3	6	7	9

Let H = Hotham, P = Perisher and Q = Queenstown. Let A = easy access and E = expensive lifts. Find the number of ski areas in the following sets:

- $H \cap A^C \cap E$
- $(P \cap E^C) \cup (Q \cap A)$
- $(H \cup P)^C \cap A$
- The number of ski areas in Perisher that have neither expensive lifts nor easy access.

Extension questions

9. Use the laws of set algebra to prove the following:

$$(A \setminus B) \setminus C = A \setminus (B \cup C)$$

You will need to work out $A \setminus B$ first in terms of complement and intersection.

Note: If you need more practice on some of the elementary set questions, then there are some questions listed in Canvas from other sources that you might find useful.