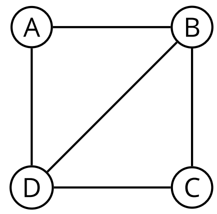
Tutorial Graphs and trees: solutions

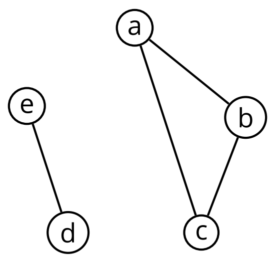
Solutions

1.

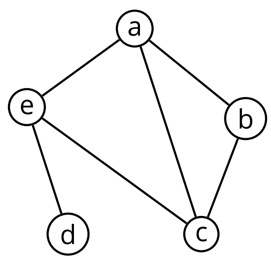
a)



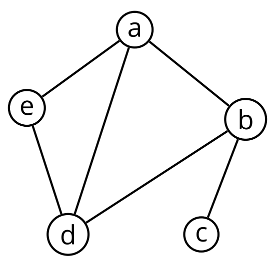
b) Notice that (b) is a disconnected graph.



c)



d)



2.

a) deg(A) = 2, deg(B) = 3, deg(C) = 2, deg(D) = 3

b) deg(a) = 2, deg(b) = 2, deg(c) = 2, deg(d) = 1, deg(e) = 1
c) deg(a) = 3, deg(b) = 2, deg(c) = 3, deg(d) = 1, deg(e) = 3
d) deg(a) = 3, deg(b) = 3, deg(c) = 1, deg(d) = 3, deg(e) = 2

3. For the subgraphs with at least two vertices, we get the following:

- vertices {a,b}, no edges
- vertices {a,c}, no edges
- vertices {b,c}, no edges
- vertices {a,b}, edges {{a,b}}
- vertices {a,c}, edges {{a,c}}
- vertices {a,b,c}, no edges
- vertices {a,b,c}, edges {{a,b}}
- vertices {a,b,c}, edges {{a,c}}
- vertices {a,b,c}, edges ({a,b},{a,c}}

4.

a) This is a neat cycle on its own; the Eulerian cycle is traversing each edge to return to the starting point.
b) All vertices have even degree, so an Eulerian cycle can be found.
c) This graph is slightly problematic, in that the vertices have odd degrees. No Eulerian cycle is possible.
d) This graph has all even degrees, so an Eulerian cycle can be found.
e) This graph has two vertices with odd degrees; this means no Eulerian cycle can be found, but a Eulerian path (that uses all the edges but starts and ends are different vertices) can be found.

5.

	Distance and Paths										
Visited Vertices	a	b	path(b)	c	path(c)	d	path(d)	e	path(e)	z	path(z)
a	0	**2**	a, b	3	a, c	∞	?	∞	?	∞	?
a, b	0	N/A	a, b	**3**	a, c	7	a, b, d	5	a, b, e	∞	?
a, b, c	0	N/A	a, b	N/A	a, c	7	a, b, d	**4**	a, c, e	∞	?
a, b, c, e	0	N/A	a, b	N/A	a, c	**5**	a, c, e, d	N/A	a, c, e	8	a, c, e, z
a, b, c, e, d	0	N/A	a, b	N/A	a, c	N/A	a, c, e, d	N/A	a, c, e	**7**	a, c, e, d, z
a, b, c, e, d, z	STOP here because we have reached the destination z.										

The shortest path from a to z is {a, c, e, d, z} with length 7.

6. The process is:

a) Let's start with a and add that to the tree.
b) The shortest edge out of a that connects elsewhere is 2 connecting to b, so add that edge and b to the tree.

c) The next step is a tie: we could take two edges of weight 3 (a,c) or (b,e). Assume we take the former and add c to the tree.
d) The next vertex to add is e, which is weight 1 from c.
e) Another edge of weight 1 takes us from e to d, so add d to the tree.
f) Finally, z is weight 2 from d, so add z to the tree with an edge from z. Note there are two possible minimum spanning trees, depending on which edge is used in step c.

7.

a) pre-order: a, j, d, i, g, e, c, k, b, f, h
b) post-order: i, d, g, j, k, b, c, h, f, e, a
c) in-order: d, i, j, g, a, k, c, b, e, f, h

8.

a) BFS: A, B, D, G, C, H, E, F
b) DFS: A, B, C, E, F, G, H, D

Extra questions

9. Look for the same number of edges, and then the same number of degrees. c) and d) are isomorphic based on those criteria. The number of simple cycles also matches.