SWINBURNE UNIVERSITY OF TECHNOLOGY

# COS30019: Introduction to Artificial Intelligence

Games/Adversarial Search

## Outline

- What are games?
- Optimal decisions in games
  - Which strategy leads to success?
- $\alpha$-$\beta$ pruning
- Games of imperfect information
- Games that include an element of chance

## What are and why study games?

- Games are a form of *multi-agent environment*
  - What do other agents do and how do they affect our success?
  - Cooperative vs. competitive multi-agent environments.
  - Competitive multi-agent environments give rise to adversarial search a.k.a. *games*
- Why study games?
  - Fun; historically entertaining
  - Interesting subject of study because they are hard
  - Easy to represent and agents restricted to small number of actions

## Relation of Games to Search

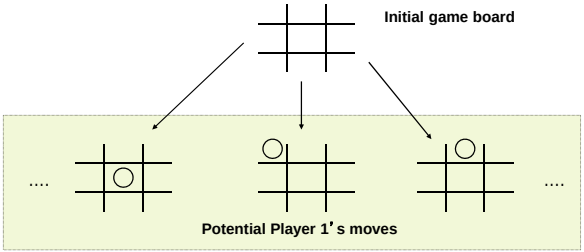
- Search – no adversary
  - Solution is (heuristic) method for finding goal
  - Heuristics can find *optimal* solution
    - Also Constraint Satisfaction Problem (CSP)
  - Evaluation function: estimate of cost from start to goal through given node
  - Examples: path planning, scheduling activities
- Games – adversary
  - Solution is strategy (strategy specifies move for every possible opponent reply).
  - Time limits force an *approximate* solution
  - Evaluation function: evaluate "goodness" of game position
  - Examples: chess, checkers, Othello, backgammon

## Types of Games

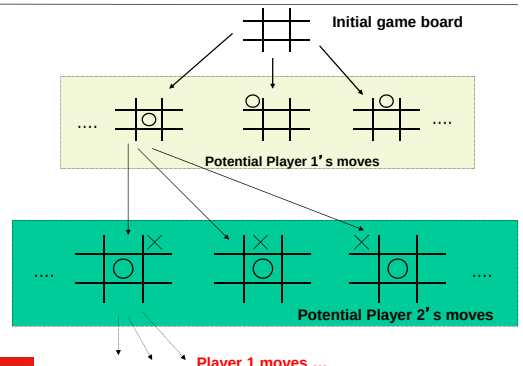
| | deterministic | chance |
|---|---|---|
| perfect information | chess, checkers, go, othello | backgammon monopoly |
| imperfect information | | bridge, poker, scrabble nuclear war |

## Game Tree…

- Games playing problem can be categorized into a search tree – this is known as a "*game tree*"



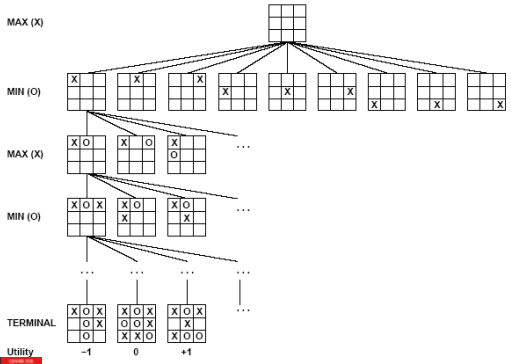
## Game Tree




## Game setup

- Two players: MAX and MIN
- MAX moves first and they take turns until the game is over. Winner gets award, loser gets penalty.
- Games as search:
  - Initial state: e.g. board configuration of chess
  - Successor function: list of (move,state) pairs specifying legal moves.
  - Terminal test: Is the game finished?
  - Utility function: Gives numerical value of terminal states. E.g. win (+1), loose (-1) and draw (0) in tic-tac-toe (next)
- MAX uses search tree to determine next move.

## Partial Game Tree for Tic-Tac-Toe



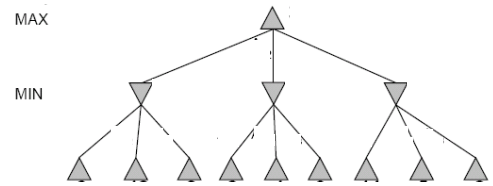

## Optimal strategies

- Find the contingent *strategy* for MAX assuming an infallible MIN opponent.
- Assumption: Both players play optimally !!
- Given a game tree, the optimal strategy can be determined by using the minimax value of each node:

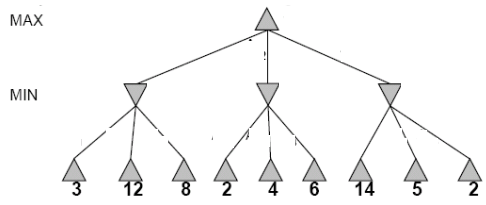
MINIMAX-VALUE($n$)=

| | |
|---|---|
| UTILITY($n$) | If $n$ is a terminal |
| $\max_{s \in successors(n)}$ MINIMAX-VALUE($s$) | If $n$ is a max node |
| $\min_{s \in successors(n)}$ MINIMAX-VALUE($s$) | If $n$ is a min node |

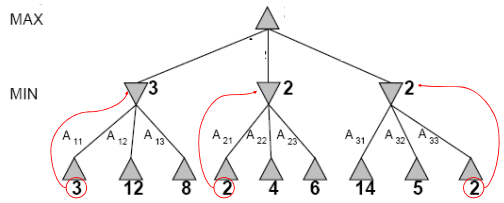
## Two-Ply Game Tree



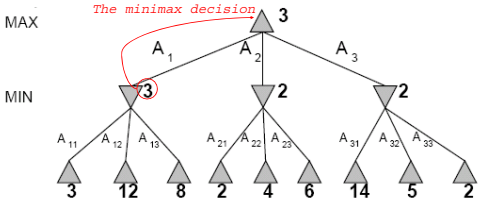

## Two-Ply Game Tree



## Two-Ply Game Tree




## Two-Ply Game Tree




***Minimax maximizes the worst-case outcome for max.***

## What if MIN does not play optimally?

- Definition of optimal play for MAX assumes MIN plays optimally: maximizes worst-case outcome for MAX.
- But if MIN does not play optimally, MAX will do even better. [proven.]

## Minimax Algorithm

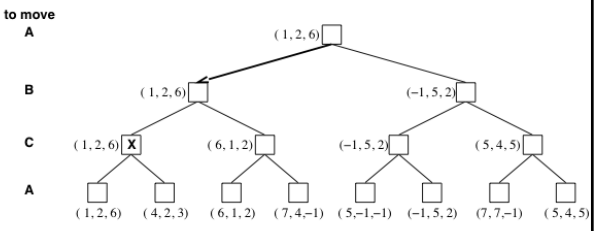
```
function MINIMAX-DECISION(state) returns an action
  inputs: state, current state in game
  v←MAX-VALUE(state)
  return the action in SUCCESSORS(state) with value v

function MAX-VALUE(state) returns a utility value
  if TERMINAL-TEST(state) then return UTILITY(state)
  v ← -∞
  for a,s in SUCCESSORS(state) do
    v ← MAX(v,MIN-VALUE(s))
  return v

function MIN-VALUE(state) returns a utility value
  if TERMINAL-TEST(state) then return UTILITY(state)
  v ← ∞
  for a,s in SUCCESSORS(state) do
    v ← MIN(v,MAX-VALUE(s))
  return v
```

## Properties of Minimax

| Criterion | Minimax |
|---|---|
| Time | $O(b^m)$ ☹ |
| Space | $O(bm)$ ☺ |

## Multiplayer games

- Games allow more than two players
- Single minimax values become vectors



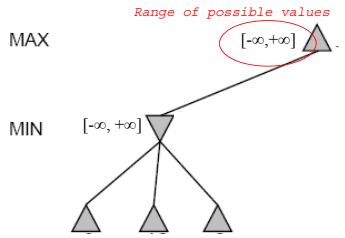
## Problem of minimax search

- Number of games states is exponential to the number of moves.
  - Solution: Do not examine every node

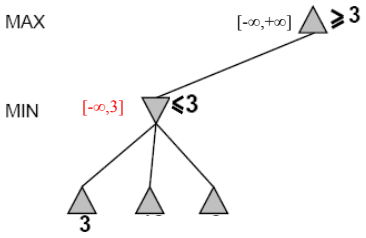
    Basic idea: "If you have an idea that is surely bad, don't take the time to see how truly awful it is."
    -- Pat Winston
- Revisit example …

## Alpha-Beta Example

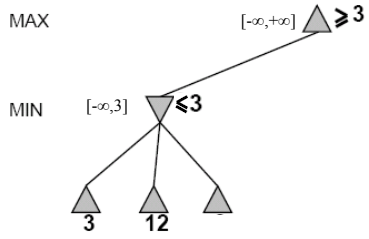
**Do DF-search until first leaf**



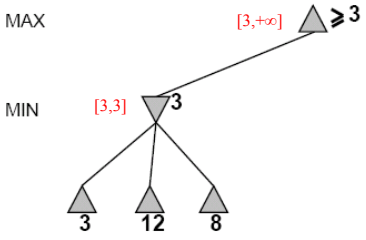

## Alpha-Beta Example (continued)



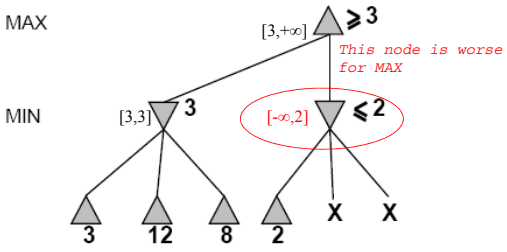

## Alpha-Beta Example (continued)



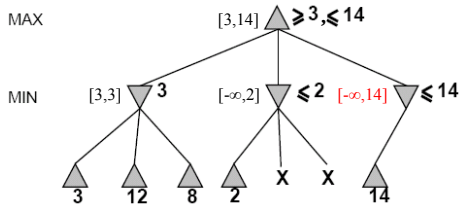

## Alpha-Beta Example (continued)




## Alpha-Beta Example (continued)



## Alpha-Beta Example (continued)

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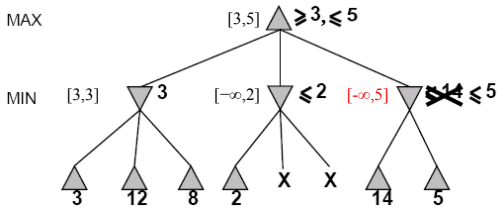
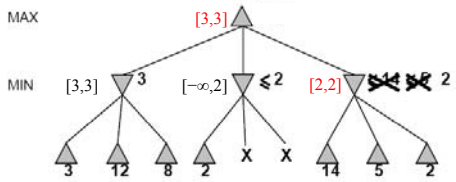
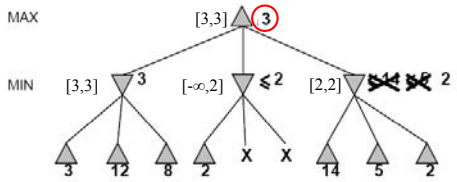



## Alpha-Beta Example (continued)




## Alpha-Beta Example (continued)




## Alpha-Beta Algorithm

**function** ALPHA-BETA-SEARCH(*state*) **returns** *an action*
  **inputs:** *state*, current state in game
  *v*←MAX-VALUE(*state*, $-\infty$ , $+\infty$)
  **return** the *action* in SUCCESSORS(*state*) with value *v*

---

**function** MAX-VALUE(*state*,$\alpha$ , $\beta$) **returns** *a utility value*
  **if** TERMINAL-TEST(*state*) **then return** UTILITY(*state*)
  $v \leftarrow -\infty$
  **for** *a*,*s* in SUCCESSORS(*state*) **do**
    $v \leftarrow$ MAX($v$,MIN-VALUE(*s*, $\alpha$ , $\beta$))
    **if** $v \geq \beta$ **then return** $v$
    $\alpha \leftarrow$ MAX($\alpha$ ,$v$)
  **return** $v$

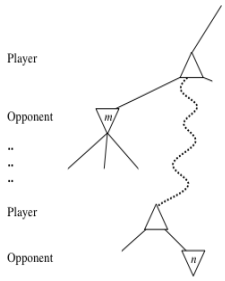
## Alpha-Beta Algorithm

---

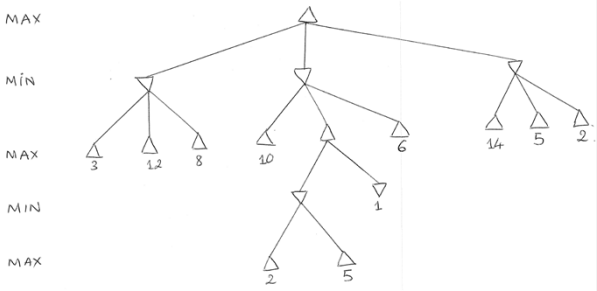
**function** MIN-VALUE(*state*, $\alpha$ , $\beta$) **returns** *a utility value*
  **if** TERMINAL-TEST(*state*) **then return** UTILITY(*state*)
  $v \leftarrow +\infty$
  **for** *a*,*s* in SUCCESSORS(*state*) **do**
    $v \leftarrow$ MIN($v$,MAX-VALUE(*s*, $\alpha$ , $\beta$))
    **if** $v \leq \alpha$ **then return** $v$
    $\beta \leftarrow$ MIN($\beta$ ,$v$)
  **return** $v$

## General alpha-beta pruning

- Consider a node *n* somewhere in the tree
- If player has a better choice at
  - Parent node of n
  - Or any choice point further up
- *n* will never be reached in actual play.
- Hence when enough is known about *n*, it can be pruned.




## General alpha-beta pruning




Can you determine which nodes will be pruned in the above game tree? Let's answer it in the tutorial (next week).

## Final Comments about Alpha-Beta Pruning

- Pruning does not affect final results
- Entire subtrees can be pruned.
- Good move *ordering* improves effectiveness of pruning
- With "perfect ordering," time complexity is $O(b^{m/2})$
  - Branching factor of sqrt(b) !!
  - Alpha-beta pruning can look twice as far as minimax in the same amount of time
- Repeated states are again possible.
  - Store them in memory = transposition table

## Games of imperfect information

- Minimax and alpha-beta pruning require too much leaf-node evaluations.
- May be impractical within a reasonable amount of time.
- SHANNON (1950):
  - Cut off search earlier (replace TERMINAL-TEST by CUTOFF-TEST)
  - Apply heuristic evaluation function EVAL (replacing utility function of alpha-beta)

## Cutting off search

- Change:
  - **if** TERMINAL-TEST(*state*) **then return** UTILITY(*state*)

  into
  - **if** CUTOFF-TEST(*state*,*depth*) **then return** EVAL(*state*)
- Introduces a fixed-depth limit *depth*
  - Is selected so that the amount of time will not exceed what the rules of the game allow.
- When cuttoff occurs, the evaluation is performed.

## Heuristic EVAL

- Idea: produce an estimate of the expected utility of the game from a given position.
- Performance depends on quality of EVAL.
- Requirements:
  - EVAL should order terminal-nodes in the same way as UTILITY.
  - Computation may not take too long.
  - For non-terminal states the EVAL should be strongly correlated with the actual chance of winning.
- Only useful for quiescent (no wild swings in value in near future) states

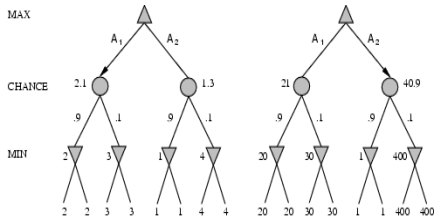
## Expected minimax value

EXPECTED-MINIMAX-VALUE($n$)=

| | |
|---|---|
| UTILITY($n$) | If $n$ is a terminal |
| $\max_{s \in successors(n)}$ MINIMAX-VALUE(s) | If $n$ is a max node |
| $\min_{s \in successors(n)}$ MINIMAX-VALUE(s) | If $n$ is a max node |
| $\sum_{s \in successors(n)} P(s)$ . EXPECTEDMINIMAX(s) | If $n$ is a chance node |

These equations can be backed-up recursively all the way to the root of the game tree.

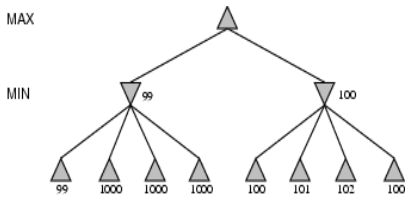
## Position evaluation with chance nodes




- Left, A1 wins
- Right A2 wins
- Outcome of evaluation function may not change when values are scaled differently.
- Behavior is preserved only by a *positive linear* transformation of EVAL.

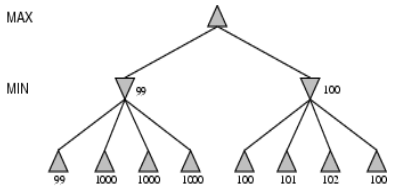
## Discussion

- Examine section on state-of-the-art games yourself
- Minimax assumes right tree is better than left, yet …
  - ☐ Return probability distribution over possible values
  - ☐ Yet expensive calculation




## Discussion

- Utility of node expansion
  - ☐ Only expand those nodes which lead to significantly better moves
- Both suggestions require meta-reasoning




## State-of-the-Art Game Programs

- Chess: Human world champion Garry Kasparov defeated by Deep Blue in six-game match in 1997. Deep Blue searched 200 million positions per second, used very sophisticated evaluation function, and undisclosed methods to extend some line of search up to 40 plies. Currently, a good program on PC can match with human world champion.
- Othello: Human world champions refuse to play against computers, who are too good.
- Checkers: Chinook ended 40-year reign of human world champion Marion Tinsley in 1994. It ran on regular PCs and used alpha-beta search.
- Go: In March 2016, Google's AlphaGo beat Lee Sedol in a five-game match, the first time a computer Go program has beaten a 9-dan professional without handicaps. Although it lost to Lee Sedol in the fourth game, Lee resigned the final game, giving a final score of 4 games to 1 in favour of AlphaGo. In recognition of beating Lee Sedol, AlphaGo was awarded an honorary 9-dan by the Korea Baduk Association.

## Summary

- Games are fun (and dangerous)
- They illustrate several important points about AI
  - ☐ Perfection is unattainable -> approximation
  - ☐ Good idea what to think about
  - ☐ Uncertainty constrains the assignment of values to states
- Games are to AI as grand prix racing is to automobile design.