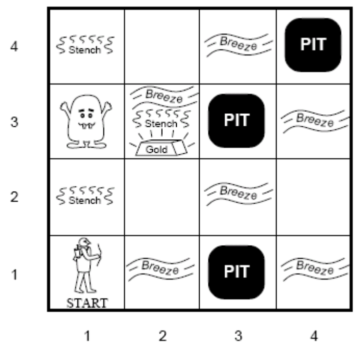
COS30019: Introduction to Artificial Intelligence

Uncertain Knowledge and Reasoning

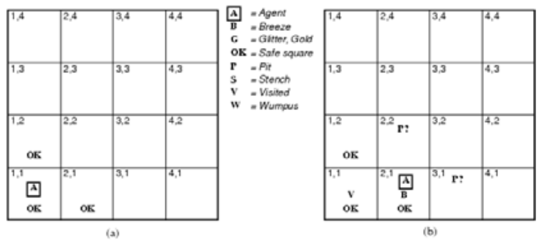
Outline

- Uncertainty
- Probability
- Syntax and Semantics
- Inference
- Independence and Bayes' Rule

Agent in a Wumpus world



Agent in a Wumpus world



The problem of incomplete information:

$P_{2,2} \vee P_{3,1}$

$\Rightarrow$ AI planners are struggling to deal with (disjunctions in state representations)

Agent in a Wumpus world

Uncertainty

- Home is 40-minute drive from the airport
- Let action A_t = *leave for airport t minutes before flight*.
  Will A_t get me there on time?
- Problems
  - partial observability (road state, other drivers' plans, etc.)
  - noisy sensors (traffic reports, etc.)
  - uncertainty in outcomes (flat tire, etc.)
  - immense complexity modeling and predicting traffic

Can we take a purely logical approach?

- Risks falsehood: "A_{45} will get me there on time"
- Leads to conclusions that are too weak for decision making:
  - *A_{45} will get me there on time if there is no accident on the bridge and it doesn't rain and my tires remain intact, etc.*
  - *A_{1440} might reasonably be said to get me there on time but I'd have to stay overnight at the airport!*
- Logic represents uncertainty by disjunction but cannot tell us how likely the different conditions are.

Methods for handling uncertainty

- Default or nonmonotonic logic:
  - Assume my car does not have a flat tire
  - Assume A_{45} works unless contradicted by evidence
  - Issues: What assumptions are reasonable? How to handle contradiction?
- Rules with *ad-hoc* fudge factors:
  - A_{45} $\mapsto_{0.3}$ get there on time
  - *Sprinkler* $\mapsto_{0.99}$ *WetGrass*
  - *WetGrass* $\mapsto_{0.7}$ *Rain*
  - Issues: Problems with combination, e.g., *Sprinkler* causes *Rain*??
- Probability
  - Model agent's degree of belief
  - Given the available evidence, A_{45} will get me there on time with probability 0.04
  - Probabilities have a clear calculus of combination

Our Alternative: Use Probability

- *Given the available evidence, A_{45} will get me there on time with probability 0.04*

  $P(A_{45}) = 0.04$ (prior/unconditional probability)
- Probabilistic assertions summarize the effects of
  - *Laziness*: too much work to list the complete set of antecedents or consequents to ensure no exceptions
  - *Theoretical ignorance*: medical science has no complete theory for the domain
  - *Uncertainty:* Even if we know all the rules, we might be uncertain about a particular patient

Uncertainty (Probabilistic Logic): Foundations

- Probability theory provides a quantitative way of encoding likelihood
- Frequentist
  - Probability is inherent in the process
  - Probability is estimated from measurements
- Subjectivist (Bayesian)
  - Probability is a model of *your* degree of belief

Subjective (Bayesian) Probability

- Probabilities relate propositions to *one's own state of knowledge*
  - Example: *$P(A_{45}$|no reported accidents) = 0.06*

    (Conditional probability)
- These are *not* assertions about the world
- Probabilities of propositions change with new evidence
  - Example: *$P(A_{45}$|no reported accidents, 5am) = 0.15*

Making decisions under uncertainty

Suppose I believe the following:

$P(A_{45}$ gets me there on time | …)	= 0.04
$P(A_{90}$ gets me there on time | …)	= 0.70
$P(A_{180}$ gets me there on time | …)	= 0.95
$P(A_{1440}$ gets me there on time | …)	= 0.9999

- Which action to choose?

  Depends on my preferences for missing flight vs. time spent waiting, etc.

Decision Theory

- *Decision Theory* develops methods for making optimal decisions in the presence of uncertainty.
  - Decision Theory = utility theory + probability theory
- *Utility theory* is used to represent and infer preferences: Every state has a degree of usefulness
- An agent is rational if and only if it chooses an action that yields the highest *expected* utility, averaged over all possible outcomes of the action.

Random variables

- A *discrete random variable* is a function that
  - takes discrete values from a countable domain and
  - maps them to a number between 0 and 1
  - Example: Weather is a discrete (propositional) random variable that has domain <sunny,rain,cloudy,snow>.
    - *sunny* is an abbreviation for *Weather = sunny*
    - *P(Weather=sunny)=0.72, P(Weather=rain)=0.1*, etc.
    - Can be written: *P(sunny)=0.72, P(rain)=0.1,* etc.
    - Domain values must be exhaustive and mutually exclusive
- Other types of random variables:
  - *Boolean random variable* has the domain <true,false>,
    - e.g., *Cavity* (special case of discrete random variable)
  - *Continuous random variable* as the domain of real numbers, e.g., *Temp*

Propositions

- Elementary proposition constructed by assignment of a value to a random variable:
  - e.g. *Weather = sunny*
  - e.g.*Cavity = false* (abbreviated as $\neg$*cavity*)
- Complex propositions formed from elementary propositions & standard logical connectives
  - e.g. *Weather = sunny* $\vee$ *Cavity = false*

Atomic Events

- *Atomic event*:
  - A *complete* specification of the state of the world about which the agent is uncertain
  - E.g., if the world consists of only two Boolean variables *Cavity* and *Toothache*, then there are 4 distinct atomic events:

    Cavity = false $\wedge$ *Toothache = false*

    Cavity = false $\wedge$ *Toothache = true*

    Cavity = true $\wedge$ *Toothache = false*

    Cavity = true $\wedge$ *Toothache = true*
- Atomic events are mutually exclusive and exhaustive

Axioms of Probability

- For any proposition a, $0 \leq P(a) \leq 1$
- $P(true) = 1$ and $P(false) = 0$
- $P(A \vee B) = P(A) + P(B) – P(A \wedge B)$

Example:

- $P(a \vee \neg a) = P(a) + P(\neg a) – P(a \wedge \neg a)$
- $P(true) = P(a) + P(\neg a) – P(false)$
- $1 = P(a) + P(\neg a)$
- $P(\neg a) = 1 – P(a)$

Prior probability

- *Prior (unconditional) probability*
  - corresponds to belief prior to arrival of any (new) evidence
  - *P(sunny)=0.72, P(rain)=0.1*, etc.
- *Probability distribution* gives values for all possible assignments:
  - Vector notation: Weather is one of <0.72, 0.1, 0.08, 0.1>, where weather is one of <sunny,rain,cloudy,snow>.
  - *P(Weather) = <0.72,0.1,0.08,0.1>*
  - Sums to 1 over the domain

Conditional Probability

- E.g., $P(A_{45}|\text{no reported accidents}) = 0.06$

  The probability of plan A_{45} getting us there in time is *0.06*, given that all we know is there are *no reported accidents*

- Definition of Conditional Probability:
  $P(A \mid B) = P(A \wedge B)/P(B)$
- Product rule gives an alternative formulation:
  $P(A \wedge B) = P(A \mid B) * P(B)$
  $= P(B \mid A) * P(A)$
- A general version holds for whole distributions:
  $P(Weather,Cavity) = P(Weather \mid Cavity) * P(Cavity)$

Joint probability distribution

- Probability assignment to all combinations of values of random variables

	Toothache	¬ Toothache
Cavity	0.04	0.06
¬ Cavity	0.01	0.89

- The sum of the entries in this table has to be 1
- *Every question about a domain can be answered by the joint distribution*
- Probability of a proposition is the sum of the probabilities of atomic events in which it holds
  - P(cavity) = 0.1 [add elements of cavity row]
  - P(toothache) = 0.05 [add elements of toothache column]

Probabilistic Inference

- *Probabilistic inference:* the computation
  - from *observed evidence*
  - from *prior* and *conditional probabilities*
  - for *query propositions*.
- We use the *full joint distribution* as the "knowledge base" from which answers to questions may be derived.
- E.g., three Boolean variables *Toothache (T), Cavity (C), ShowsOnXRay (X)*

	T		¬T	
	X	¬X	X	¬X
C	0.108	0.012	0.072	0.008
¬C	0.016	0.064	0.144	0.576

- Probabilities in joint distribution sum to 1

Probabilistic Inference II

	T		¬T	
	X	¬X	X	¬ X
C	0.108	0.012	0.072	0.008
¬C	0.016	0.064	0.144	0.576

- Probability of any proposition computed by finding atomic events where proposition is true and adding their probabilities
  - $P(cavity \vee toothache)$
    $= 0.108 + 0.012 + 0.072 + 0.008 + 0.016 + 0.064$
    $= 0.28$
  - $P(cavity)$
    $= 0.108 + 0.012 + 0.072 + 0.008$
    $= 0.2$

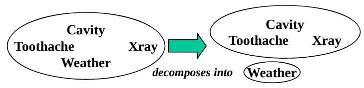
Conditioning

- *Idea:* Use *conditional probabilities* instead of joint probabilities
- $P(A) = P(A \wedge B) + P(A \wedge \neg B)$
  $= P(A \mid B) * P(B) + P(A \mid \neg B) * P(\neg B)$

  <u>*Example*</u>:
  $P(symptom) =$
  $P(symptom|disease) * P(disease) +$
  $P(symptom| \neg disease) * P(\neg disease)$
- More generally: $P(Y) = \sum_z P(Y|z) * P(z)$
- Conditioning is a useful rule for derivations involving probability expressions.

Independence

- A and B are <u>*independent*</u> iff
  - $P(A \wedge B) = P(A) * P(B)$
  - $P(A \mid B) = P(A)$
  - $P(B \mid A) = P(B)$
- Independence is essential for efficient probabilistic reasoning



$P(T, X, C, W) = P(T, X, C) \bullet P(W)$

- 32 entries reduced to 12; for *n* independent unbiased coins, $O(2^n) \rightarrow O(n)$
- Absolute independence powerful but rare
- Dentistry is a large field with hundreds of variables, none of which are independent. What to do?

Conditional Independence

- A and B are *conditionally independent given C* iff
  - $P(A \mid B, C) = P(A \mid C)$
  - $P(B \mid A, C) = P(B \mid C)$
  - $P(A \wedge B \mid C) = P(A \mid C) * P(B \mid C)$
- Toothache (T), Spot in Xray (X), Cavity (C)
  - None of these propositions are independent of one other
  - But *T and X are conditionally independent given C*

Conditional Independence II

- In most cases, the use of conditional independence reduces the size of the representation of the joint distribution from exponential in *n* to linear in *n*.
- *Conditional independence is our most basic and robust form of knowledge about uncertain environments.*

Bayes' Rule

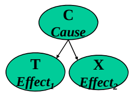
- $P(A \mid B) = (P(B \mid A) * P(A)) / P(B)$
- $P(disease \mid symptom) = \dfrac{P(symptom \mid disease) * P(disease)}{P(symptom)}$
- Useful for assessing diagnostic probability from causal probability:
  - $P(Cause|Effect) = (P(Effect|Cause) * P(Cause)) / P(Effect)$
- Imagine
  - disease = Zika, symptom = fever
  - *P(disease | symptom)* is different in Zika-indicated country vs. Australia
  - *P(symptom | disease)* should be the same
    - It is more useful to learn *P(symptom | disease)*
  - What about P(symptom)?
    - Use *conditioning*

Combining Evidence

- Bayesian updating given two pieces of information

$$P(C|T,X) = \frac{P(T,X|C)P(C)}{P(T,X)}$$

- Assume that T and X are conditionally independent given C (naïve Bayes Model)



$$P(C|T,X) = \frac{P(T|C)P(X|C)P(C)}{P(T,X)}$$

- We can do the evidence combination sequentially

Summary

- Probability is a rigorous formalism for uncertain knowledge
- Joint probability distribution specifies probability of every atomic event
- Queries can be answered by summing over atomic events
- For nontrivial domains, we must find a way to reduce the joint size
- Independence and conditional independence provide the tools
- Conditioning and Bayes' rule provide basic and powerful mechanisms for probabilistic inference