SWINBURNE UNIVERSITY OF TECHNOLOGY

COS30019: Introduction to Artificial Intelligence

Agents and

First-Order Logic

Outline

- Why FOL?
- Syntax and semantics of FOL
- Using FOL
- Wumpus world in FOL
- Knowledge engineering in FOL

Pros and cons of propositional logic

☺ Propositional logic is declarative

☺ Propositional logic allows partial/disjunctive/negated information

- (unlike most data structures and databases)

☺ Propositional logic is compositional:

- meaning of $B_{1,1} \wedge P_{1,2}$ is derived from meaning of $B_{1,1}$ and of $P_{1,2}$

☺ Meaning in propositional logic is context-independent

- (unlike natural language, where meaning depends on context)

☹ Propositional logic has very limited expressive power

- (unlike natural language)
- E.g., cannot say "pits cause breezes in adjacent squares"
  - except by writing one sentence for each square

First-order logic

- Whereas propositional logic assumes the world contains facts,
- first-order logic (like natural language) assumes the world contains
  - Objects: people, houses, numbers, colors, baseball games, wars, …
  - Relations: red, round, prime, brother of, bigger than, part of, comes between, …
  - Functions: father of, best friend, one more than, plus, …

Logics in General

- Ontological Commitment: What exists in the world — TRUTH
  - PL : facts hold or do not hold.
  - FL : objects with relations between them that hold or do not hold
- Epistemological Commitment: What an agent believes about facts — BELIEF

Language	Ontological Commitment	Epistemological Commitment
Propositional logic	facts	true/false/unknown
First-order logic	facts, objects, relations	true/false/unknown
Temporal logic	facts, objects, relations, times	true/false/unknown
Probability theory	facts	degree of belief $\in [0,1]$
Fuzzy logic	degree of truth $\in [0,1]$	known interval value

Syntax of FOL: Basic elements

Constants	KingJohn, 2, HIT3002,...
Predicates	Brother, >,...
Functions	Sqrt, LeftLegOf,...
Variables	x, y, a, b,...
Connectives	$\neg, \Rightarrow, \wedge, \vee, \Leftrightarrow$
Equality	=
Quantifiers	$\forall, \exists$

Atomic sentences

Atomic sentence = *predicate* (*term*$_1$,...,*term*$_n$)
or *term*$_1$ = *term*$_2$

Term = *function* (*term*$_1$,...,*term*$_n$)
or *constant* or *variable*

- E.g., *Brother*(*KingJohn*,*RichardTheLionheart*),

  \> (*Length*(*LeftLegOf*(*Richard*)), *Length*(*LeftLegOf*(*KingJohn*)))

Complex sentences

- Complex sentences are made from atomic sentences using connectives

  $\neg S$, $S_1 \wedge S_2$, $S_1 \vee S_2$, $S_1 \Rightarrow S_2$, $S_1 \Leftrightarrow S_2$,

E.g. *Sibling(KingJohn, Richard)* $\Rightarrow$ *Sibling(Richard, KingJohn)*

$>(1,2) \vee \leq (1,2)$

$>(1,2) \wedge \neg >(1,2)$

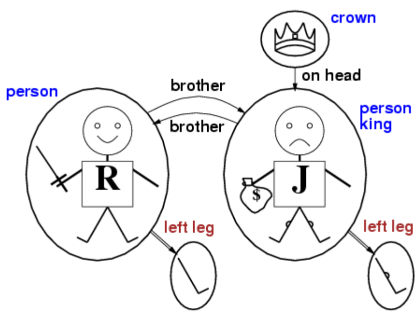
Truth in first-order logic

- Sentences are true with respect to a model and an interpretation
- Model contains objects (domain elements) and relations among them
-
- Interpretation specifies referents for

  constant symbols → objects
  predicate symbols → relations
  function symbols → functional relations

- An atomic sentence *predicate(term*$_1$*,...,term*$_n$*)* is true iff the objects referred to by *term*$_1$*,...,term*$_n$ are in the relation referred to by *predicate*.

Models for FOL: Example



Models for FOL

- We can enumerate the models for a given KB vocabulary:

  For each number of domain elements n from 1 to ∞
  For each k-ary predicate P_k in the vocabulary
  For each possible k-ary relation on n objects
  For each constant symbol C in the vocabulary
  For each choice of referent for C from n objects ...

- Computing entailment by enumerating the models will not be easy !!

Quantifiers

- Allows us to express properties of collections of objects instead of enumerating objects by name
- Universal: “for all” $\forall$
- Existential: “there exists” $\exists$

Universal quantification

$\forall$<variables> <sentence>

Everyone at SUT is smart:

$\forall x\ At(x,SUT) \Rightarrow Smart(x)$

$\forall x\ P$ is true in a model *m* iff *P* is true with *x* being each possible object in the model

Roughly speaking, equivalent to the conjunction of instantiations of *P*

At(KingJohn,SUT) $\Rightarrow$ Smart(KingJohn)
$\wedge$ At(Richard,SUT) $\Rightarrow$ Smart(Richard)
$\wedge$ At(SUT,SUT) $\Rightarrow$ Smart(SUT)
$\wedge$ ...

A common mistake to avoid

- Typically, $\Rightarrow$ is the main connective with $\forall$
  - A universally quantifier is also equivalent to a set of implications over all objects
- Common mistake: using $\wedge$ as the main connective with $\forall$:

$\forall x\ At(x, SUT) \wedge Smart(x)$

means "Everyone is at SUT and everyone is smart"

Existential quantification

$\exists$<variables> <sentence>

Someone at SUT is smart:

$\exists x\ At(x, SUT) \wedge Smart(x)$

$\exists x\ P$ is true in a model *m* iff *P* is true with *x* being some possible object in the model

- Roughly speaking, equivalent to the disjunction of instantiations of *P*

At(KingJohn,SUT) $\wedge$ Smart(KingJohn)
$\vee$ At(Richard,SUT) $\wedge$ Smart(Richard)
$\vee$ At(SUT, SUT) $\wedge$ Smart(SUT)
$\vee$ ...

Another common mistake to avoid

- Typically, $\wedge$ is the main connective with $\exists$
- Common mistake: using $\Rightarrow$ as the main connective with $\exists$:

$\exists x\ At(x, SUT) \Rightarrow Smart(x)$

is true even if there is anyone who is not at SUT!

Properties of quantifiers

$\forall x\ \forall y$ is the same as $\forall y\ \forall x$
$\exists x\ \exists y$ is the same as $\exists y\ \exists x$

$\exists x\ \forall y$ is not the same as $\forall y\ \exists x$
$\exists x\ \forall y\ Loves(x,y)$
- "There is a person who loves everyone in the world"

$\forall y\ \exists x\ Loves(x,y)$
- "Everyone in the world is loved by at least one person"

- Quantifier duality: each can be expressed using the other

$\forall x\ Likes(x,IceCream) \quad \neg\exists x\ \neg Likes(x,IceCream)$
$\exists x\ Likes(x,Broccoli) \quad \neg\forall x\ \neg Likes(x,Broccoli)$

Equality

- $term_1 = term_2$ is true under a given interpretation if and only if $term_1$ and $term_2$ refer to the same object
- E.g., definition of *Sibling* in terms of *Parent*:

$\forall x,y\ Sibling(x,y) \Leftrightarrow [\neg(x = y) \wedge \exists m,f\ \neg (m = f) \wedge Parent(m,x) \wedge Parent(f,x) \wedge Parent(m,y) \wedge Parent(f,y)]$

Interacting with FOL KBs

- Suppose a wumpus-world agent is using an FOL KB and perceives a smell and a breeze (but no glitter) at *t=5*:

  `Tell`(KB,Percept([Smell,Breeze,None],5)) (= assertion)
  `Ask`(KB,∃a BestAction(a,5))
  (=queries)

  I.e., does the KB entail some best action at *t=5*?
- Answer: *Yes*, {*a/Shoot*} ← substitution (binding list)
- Given a sentence *S* and a substitution α,
- *S*α denotes the result of plugging α into *S*; e.g.,
  S = Smarter(x,y)
  α = {x/Hillary,y/Bill}
  *S*α = Smarter(Hillary,Bill)
- `Ask`(KB,S) returns some/all α such that KB |= *S*α.

Using FOL

The kinship domain:

- Brothers are siblings
  ∀x,y *Brother(x,y)* ⇔ *Sibling(x,y)*
- One's mother is one's female parent
  ∀m,c *Mother(c)* = m ⇔ *(Female(m)* ∧ *Parent(m,c))*
- "Sibling" is symmetric
  ∀x,y *Sibling(x,y)* ⇔ *Sibling(y,x)*
- A first cousin is a child of a parent's sibling
  ∀x,y *FirstCousin(x,y)* ⇔ ∃p,ps Parent(p,x) ∧ Sibling(ps,p) ∧ Parent(ps,y)

Using FOL

The set domain:

- $\forall s\ Set(s) \Leftrightarrow (s = \{\}) \vee (\exists x,s_2\ Set(s_2) \wedge s = \{x|s_2\})$
- $\neg\exists x,s\ \{x|s\} = \{\}$
- $\forall x,s\ x \in s \Leftrightarrow s = \{x|s\}$
- $\forall x,s\ x \in s \Leftrightarrow [\exists y,s_2\} (s = \{y|s_2\} \wedge (x = y \vee x \in s_2))]$
- $\forall s_1,s_2\ s_1 \subseteq s_2 \Leftrightarrow (\forall x\ x \in s_1 \Rightarrow x \in s_2)$
- $\forall s_1,s_2\ (s_1 = s_2) \Leftrightarrow (s_1 \subseteq s_2 \wedge s_2 \subseteq s_1)$
- $\forall x,s_1,s_2\ x \in (s_1 \cap s_2) \Leftrightarrow (x \in s_1 \wedge x \in s_2)$
- $\forall x,s_1,s_2\ x \in (s_1 \cup s_2) \Leftrightarrow (x \in s_1 \vee x \in s_2)$

FOL Version of Wumpus World

- Typical percept sentence:
  Percept([Stench,Breeze,Glitter,None,None],5)
- Actions:
  Turn(Right), Turn(Left), Forward, Shoot, Grab, Release, Climb
- To determine best action, construct query:
  ∀ a BestAction(a,5)
- ASK solves this and returns {a/Grab}
  - And TELL about the action.

Knowledge base for the wumpus world

- Perception
  - ∀b,g,t Percept([Smell,b,g],t) ⇒ Smelt(t)
  - ∀s,b,t Percept([s,b,Glitter],t) ⇒ Glitter(t)
- Reflex
  - ∀t Glitter(t) ⇒ BestAction(Grab,t)
- Reflex with internal state
  - ∀t Glitter(t) ∧¬Holding(Gold,t) ⇒ BestAction(Grab,t)

Holding(Gold,t) can not be observed: keep track of change.

All *synchronic* sentences!

Deducing hidden properties

Environment definition:

∀x,y,a,b *Adjacent*([x,y],[a,b]) ⇔
[a,b] ∈ {[x+1,y], [x-1,y],[x,y+1],[x,y-1]}

Properties of locations:

∀s,t *At*(Agent,s,t) ∧ Smelt(t) ⇒ Smelly(s)
∀s,t *At*(Agent,s,t) ∧ Breeze(t) ⇒ Breezy(s)

Squares are breezy near a pit:

- Diagnostic rule---infer cause from effect
  ∀s Breezy(s) ⇔ ∃ r Adjacent(r,s) ∧ Pit(r)
- Causal rule---infer effect from cause (model based reasoning)
  ∀r Pit(r) ⇒ [∀s Adjacent(r,s) ⇒ Breezy(s)]

Knowledge engineering in FOL

1. Identify the task (what will the KB be used for)
2. Assemble the relevant knowledge
   Knowledge acquisition.
3. Decide on a vocabulary of predicates, functions, and constants
   Translate domain-level knowledge into logic-level names.
4. Encode general knowledge about the domain
   define axioms
5. Encode a description of the specific problem instance
6. Pose queries to the inference procedure and get answers
7. Debug the knowledge base

Summary

- First-order logic:
  - objects and relations are semantic primitives
  - syntax: constants, functions, predicates, equality, quantifiers.
- Increased expressive power: sufficient to define wumpus world