Homework
Eigenfaces

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Computational linear algebra for large scale problems

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1 Introduction

The main goal of this homework is to solve and analyze the face recognition problem. The proposed solution is based on the Principal component analysis. This strategy applied on the proposed problem is also known as Eigenfaces method.
Two possible solutions are proposed based on the PCA and the 2DPCA, in order to solve gray-scale faces recognition and color faces recognition tasks. This methods are combined with a simple distance based classifier to achieve the face classification problem.
The last part is dedicated to the analysis and the comparison of the results obtained on a specific dataset constructed from the Pain dataset.

2 Principal Component Analysis

In this section a brief description of the two main methods used for this tasks is presented. This simple application on the face recognition problem provides good results that in some cases are competitive with other more complex methods.
In general, the PCA is a method for dimensionality reduction that is largely used for data analysis. Very briefly, PCA allows to reduce the features of a dataset preserving most of the information.
In this way, the analysis is simplified in term of computational costs, without significant loss of accuracy. For this reason, this method is useful in face recognition where data under analysis are images which contains a large number variables (pixels).

2.1 Principal Component Analysis

The PCA is an orthogonal linear transformation that maps the data to a new coordinate system, in which the most of the variance is explained on the first new components. The change-of-base matrix used to make this projection is composed by the eigenvectors of the covariance matrix of the input data. In the case of face recognition task this eigevectors are also known as eigenfaces. The PCA can be performed in two ways, through eigendecomposition or singular value decomposition. Here is analyzed the second alternative because is the one used in this homework to perform the PCA.

The SVD is a matrix factorization of $A \in R^{mxn}$ where m is the number of samples and n the number of features, such that

$$A = U\Sigma V^T$$

Where $U \in R^{mxm}$ and $V \in R^{nxn}$ are orthogonal matrices and $\Sigma \in R^{mxn}$ is a diagonal matrix where the diagonal correspond to the singular values σ_i of the matrix A.

The main purpose of the PCA is to find the eigenvectors and eigenvalues of the covariance matrix of A that is a centered matrix. This is achieved using SVD because it can be proved that the columns of V^T correspond to the eigenvectors of the covariance matrix, while its eigenvalues are given by $\lambda_i = \frac{\sigma_i^2}{n-1}$.

Considering the singular values in matrix Σ in a decreasing order (i.e. $\sigma_1 \geq \sigma_2 \geq \sigma_2 \geq ... \geq \sigma_n \geq 0$) then the corresponding eigenvectors in V are ordered according to their explained variance.

In order to find the Principal components the data are projected on the new space, generated by an arbitrary portion of the principal axes contained in V.

2.2 2 Dimensional PCA

The 2DPCA has the same aim of the standard PCA. However, the PCA needs a matrix as input data, while the 2DPCA can be applied directly to a tensor. This approach results to be useful in specific cases, such as the recognition of color faces [1].

Instead of using SVD in this case is used the eigendecomposition strategy. This process consists in finding the covariance matrix of the tensor and calculate its eigenvectors ordered by decreasing order of the the correspondent eigenvalues.

The covariance matrix of a tensor A is calculated in the following way :

$$G = \mathbf{E}[(\mathbf{A} - \mathbf{E}[\mathbf{A}])^{\mathbf{T}})(\mathbf{A} - \mathbf{E}[\mathbf{A}])]$$

Then, the matrix of eigenvectors is obtained and the principal components are calculated multiplying each matrix of the tensor with the principal directions matrix.

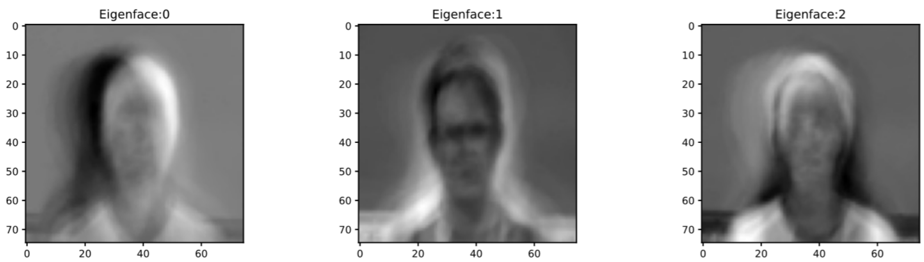


Figure 1: Eigenfaces obtained in experiments of Section 5

3 Faces recognition using PCA Eigenfaces

In this section is analyzed a simple task of face recognition taking advantage from the PCA explained in 2.1.
This approach takes inspiration from the paper [2].

3.1 Data preparation and Eigenfaces construction

Images are complex structured data, in particular they are tensors with depth equal to three in case of color images, or simple matrices for greyscale images. This approach, since works with a one dimensional PCA, needs data in a matrix form as well. So, the first step is a good data representation of the input. A good possibility is to reshape each image in a row vector and then concatenate them in a matrix M. So considering a set of data containing k greyscale images of dimension nxm the data are reshaped to a matrix form k x n·m. In case of color images the process is the same but ends with a matrix of dimension k x $n \cdot m \cdot 3$ (considering an RGB color image).
Then a z-score normalization step is applied to the data to reduce some noise related to the background or different lights in the images.
After that the PCA is applied obtaining the Eigenfaces matrix ($n \cdot m$ x d) where d is the number of PCs chosen. Finally the matrix M is transformed using the eigenfaces matrix, obtaining a reduced matrix $\tilde{M}$.

3.2 Classification

In the classification task a new set of p images is analyzed in order to assign to each of them a label, that in our case is the identity of the represented person.
To do that the steps presented in the previous paragraph are applied also on the test set. So, the images are represented in a matrix format, obtaining a new matrix T. Then it is normalized and using the previous Eigenfaces matrix is mapped in a new space obtaining $\tilde{T}$.
The classification step is performed with a simple Nearest-neighbor classifier. Briefly, the euclidean distance between a image of $\tilde{T}$ and each row of $\tilde{M}$ is computed. The label associated with the row with minimum distance corresponds to the label associated to the new instance.

$$d(p, q_i) = \sqrt{\sum_{j=1}^{d} (p_j - q_{ij})^2}$$

where p_j represents the j-th element of a row of $\tilde{T}$ and q_{ij} represents the j-th element of the i-th row of $\tilde{M}$.
This process is iterated over each row of T and at the end each image is classified.

4 Faces recognition using 2D-PCA Eigenfaces

In this section is presented the last method for face recognition. In this case is not necessary to transform the data related to the images in vectors because 2D-PCA allows to work with tensor data. However also in this case a data preparation step is required for color images as suggested by [3].

4.1 Data preparation and Eigenfaces construction

With this approach the dimension of the set of k color images m x n x 3 is reshaped in a tensor. Each image tensor is converted in a matrix where the rows represent the colors encoded (in case of RGB color images there are 3 rows) and the columns the pixels of the image. After this process the input data are represented by a tensor G of dimension k x 3 x mn. Instead, considering a set of greyscale images the data transformation is not necessary

and the input set can be directly used because it is just a tensor.
Then the 2DPCA is applied on the tensor G obtaining the Eigenfaces matrix with dimension nmx d for color images and n x d for greyscale. At the end the Eigenfaces matrix is used to map the data in the new dimensional space.

4.2 Classification

The classification task is very similar to the classification of the PCA presented in the previous section. In the same way, A new set of unlabelled images R is processed with the steps presented in the last paragraph.
The classificator is as before a NN but the formula changes taking into account the different shape of the data.

$$d(p, q_j) = \sum_{l=1}^{d} \|X_l - X_l^{(j)}\|$$

where $\|X_l - X_l^{(j)}\|$ is the norm between the vectors $X_l, X_l^{(j)}$ of the matrix associated with an image p of test and the j-th image of the train set.

5 Experiments

In this section some experiments with the two proposed methods and their results are presented, considering as evaluation parameter the accuracy of the classification.
The experiments of the PCA Eigenfaces are performed on greyscale images while the 2DPCA method is applied both on color and greyscale images. At the end a study about the impact on the accuracy of different percentages of explained variance is showed.

5.1 Dataset

Face recognition problem is very common nowadays, however is not easy to find good datasets for this task because the faces are sensible private data. For this reason is used a subset of the Pain dataset constructed by us. The original dataset contains color images of same people with different facial expressions. For the purpose of this homework 250 images of 10 different persons are selected, half male and half female. The dataset is balanced because contains 25 images for person in which the expression and the perspective changes. As the aim of this homework is the face recognition the images are labelled in according to the person represented.

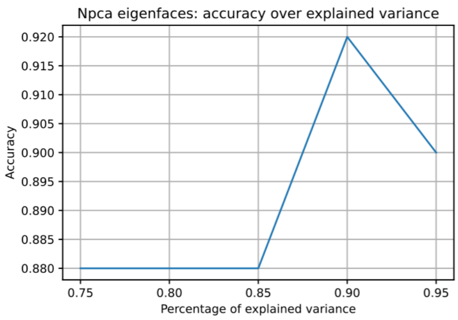
In order to have a good comparison between greyscale and color faces recognition results, the same dataset is also converted in greyscale obtaining a new one.

For the classification the two datasets are splitted in train and test set, where the tests account 20% of the total data. To make the results totally comparable the same random seed is used in the two splits, in order to have the same images in train and test both in the greyscale and color cases.

Figure 2: Example of images related to the gray and color dataset

5.2 PCA Eigenface results

Using the method described in the section 3 on the greyscale dataset the results obtained are the following:



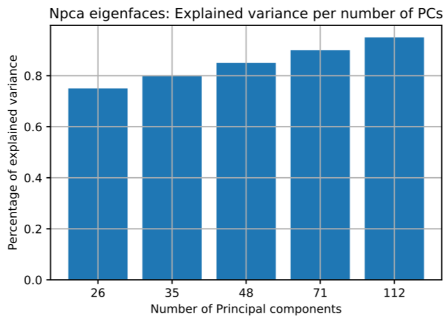
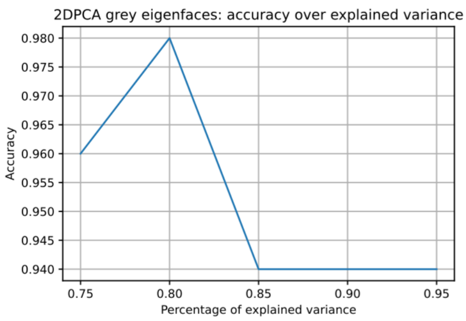


Figure 3: Acc. over explained var. Figure 4: Explained variance by PCs'

In the figure on the right can be seen the number of components required to exploit some percentage of variance. On the left is visible the trend of accuracy changing the percentage of variance explained. It is clear that the best result is obtained with 71 Principal Components that explain the 90% of the variance and reaches an accuracy of 92%.

5.3 2DPCA Grey Eigenface results

Using the same grey dataset, this time processed with the 2DPCA method, the results are shown below :



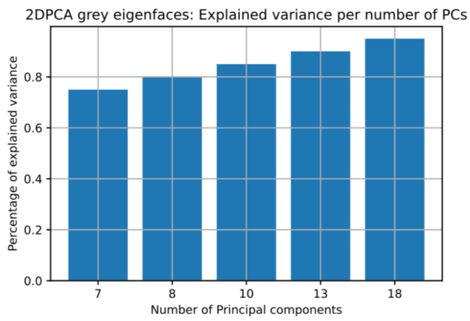


Figure 5: Acc. over explained var. Figure 6: Explained variance by PCs'

Overall, the results are slightly higher. In the graph can be seen that the best result is obtained with only 8 PCs' that exploit the 80% of the variance reaching an accuracy of 98%.

5.4 2DPCA Color Eigenface results

This last experiment is performed with the color dataset using the second proposed method. This experiment is more computationally expensive as the information encoded by color images is higher.

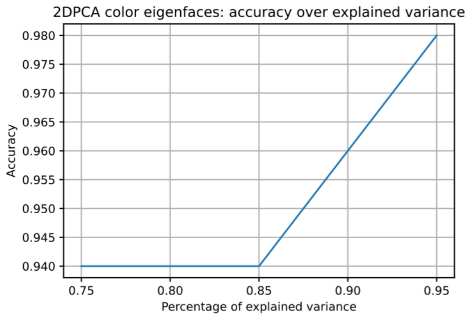


Figure 7: Acc. over explained var.

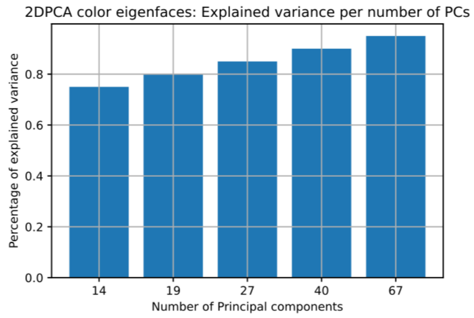


Figure 8: Explained variance by PCs'

From the graph can be seen that the results are comparable with the ones obtained with the greyscale experiments. The best result is an accuracy 98% with the 95% of variance explained by 18 PCs.

6 Conclusions

From the experiments can be seen that the results are enough satisfactory with all the procedure adopted. However, it must be underlined that the chosen dataset is quite simple and adequate to this analysis because images are not affected by significant noise or background effects.

Considering the method applied to the greyscale images, it can be said that the 2DPCA performs better in terms of accuracy with an increasing over the 5%. An interesting thing is that with 2DPCA are obtained better results with less explained variance and the number of PCs' necessary to explain the same percentage variance are less than the ones needed by the PCA method. From a computational point of view is observed that the PCA and the 2DPCA are comparable, even if the PCA uses the sklearn optimized implementation via SVD.

Comparing the experiments using 2DPCA with greyscale and color images the results obtained in terms of accuracy are the same, even if with less variance explained with gray images. Moreover the computational cost of this method applied to color images increases significantly due to the higher number of information encoded. For this reason can be concluded that for very simple tasks of recognition the color does not add significant information.
However, this conclusion should be investigated with more complex and bigger datasets.

References

[1] Jian Yang, D. Zhang, A. F. Frangi, and Jing-yu Yang, "Two-dimensional pca: a new approach to appearance-based face representation and recognition," *IEEE Transactions on Pattern Analysis and Machine Intelligence*, vol. 26, no. 1, pp. 131–137, 2004.

[2] F. Jalled, "Face recognition machine vision system using eigenfaces," 2017.

[3] Chengzhang Wang, Baocai Yin, Xiaoming Bai, and Yanfeng Sun, "Color face recognition based on 2dpca," in *2008 19th International Conference on Pattern Recognition*, pp. 1–4, 2008.