Preface

This module contains the preface for Fundamentals of Mathematics by Denny Burzynski and Wade Ellis, Jr.

To the next generation of explorers: Kristi, BreAnne, Lindsey, Randi, Piper, Meghan, Wyatt, Lara, Mason, and Sheanna.

Fundamentals of Mathematics is a work text that covers the traditional topics studied in a modern prealgebra course, as well as the topics of estimation, elementary analytic geometry, and introductory algebra. It is intended for students who

1. have had a previous course in prealgebra,
2. wish to meet the prerequisite of a higher level course such as elementary algebra, and
3. need to review fundamental mathematical concepts and techniques.

This text will help the student develop the insight and intuition necessary to master arithmetic techniques and manipulative skills. It was written with the following main objectives:

1. to provide the student with an understandable and usable source of information,
2. to provide the student with the maximum opportunity to see that arithmetic concepts and techniques are logically based,
3. to instill in the student the understanding and intuitive skills necessary to know how and when to use particular arithmetic concepts in subsequent material, courses, and nonclassroom situations, and
4. to give the student the ability to correctly interpret arithmetically obtained results.

We have tried to meet these objectives by presenting material dynamically, much the way an instructor might present the material visually in a classroom. (See the development of the concept of addition and subtraction of fractions in [link], for example.) Intuition and understanding are some of the keys to creative thinking; we believe that the material presented in this text will help the student realize that mathematics is a creative subject.

This text can be used in standard lecture or self-paced classes. To help meet our objectives and to make the study of prealgebra a pleasant and rewarding experience, *Fundamentals of Mathematics* is organized as follows.

Pedagogical Features

The work text format gives the student space to practice mathematical skills with ready reference to sample problems. The chapters are divided into sections, and each section is a complete treatment of a particular topic, which includes the following features:

- **Section Overview**
- **Sample Sets**
- **Practice Sets**
- **Section Exercises**
- **Exercises for Review**
- **Answers to Practice Sets**

The chapters begin with **Objectives** and end with a **Summary of Key Concepts**, an **Exercise Supplement**, and a **Proficiency Exam**.

Objectives
Each chapter begins with a set of objectives identifying the material to be covered. Each section begins with an overview that repeats the objectives for that particular section. Sections are divided into subsections that correspond to the section objectives, which makes for easier reading.

Sample Sets
Fundamentals of Mathematics contains examples that are set off in boxes for easy reference. The examples are referred to as Sample Sets for two reasons:

1. They serve as a representation to be imitated, which we believe will foster understanding of mathematical concepts and provide experience with mathematical techniques.
2. Sample Sets also serve as a preliminary representation of problem-solving techniques that may be used to solve more general and more

complicated problems.

The examples have been carefully chosen to illustrate and develop concepts and techniques in the most instructive, easily remembered way. Concepts and techniques preceding the examples are introduced at a level below that normally used in similar texts and are thoroughly explained, assuming little previous knowledge.

Practice Sets

A parallel Practice Set follows each Sample Set, which reinforces the concepts just learned. There is adequate space for the student to work each problem directly on the page.

Answers to Practice Sets

The Answers to Practice Sets are given at the end of each section and can be easily located by referring to the page number, which appears after the last Practice Set in each section.

Section Exercises

The exercises at the end of each section are graded in terms of difficulty, although they are not grouped into categories. There is an ample number of problems, and after working through the exercises, the student will be capable of solving a variety of challenging problems.

The problems are paired so that the odd-numbered problems are equivalent in kind and difficulty to the even-numbered problems. Answers to the odd-numbered problems are provided at the back of the book.

Exercises for Review

This section consists of five problems that form a cumulative review of the material covered in the preceding sections of the text and is not limited to material in that chapter. The exercises are keyed by section for easy reference. Since these exercises are intended for review only, no work space is provided.

Summary of Key Concepts

A summary of the important ideas and formulas used throughout the chapter is included at the end of each chapter. More than just a list of terms,

the summary is a valuable tool that reinforces concepts in preparation for the Proficiency Exam at the end of the chapter, as well as future exams. The summary keys each item to the section of the text where it is discussed.

Exercise Supplement
In addition to numerous section exercises, each chapter includes approximately 100 supplemental problems, which are referenced by section. Answers to the odd-numbered problems are included in the back of the book.

Proficiency Exam
Each chapter ends with a Proficiency Exam that can serve as a chapter review or evaluation. The Proficiency Exam is keyed to sections, which enables the student to refer back to the text for assistance. Answers to all the problems are included in the Answer Section at the end of the book.

Content

The writing style used in Fundamentals of Mathematics is informal and friendly, offering a straightforward approach to prealgebra mathematics. We have made a deliberate effort not to write another text that minimizes the use of words because we believe that students can best study arithmetic concepts and understand arithmetic techniques by using words and symbols rather than symbols alone. It has been our experience that students at the prealgebra level are not nearly experienced enough with mathematics to understand symbolic explanations alone; they need literal explanations to guide them through the symbols.

We have taken great care to present concepts and techniques so they are understandable and easily remembered. After concepts have been developed, students are warned about common pitfalls. We have tried to make the text an information source accessible to prealgebra students.

Addition and Subtraction of Whole Numbers

This chapter includes the study of whole numbers, including a discussion of the Hindu-Arabic numeration and the base ten number systems. Rounding whole numbers is also presented, as are the commutative and associative properties of addition.

Multiplication and Division of Whole Numbers

The operations of multiplication and division of whole numbers are explained in this chapter. Multiplication is described as repeated addition. Viewing multiplication in this way may provide students with a visualization of the meaning of algebraic terms such as $8x$ when they start learning algebra. The chapter also includes the commutative and associative properties of multiplication.

Exponents, Roots, and Factorizations of Whole Numbers

The concept and meaning of the word root is introduced in this chapter. A method of reading root notation and a method of determining some common roots, both mentally and by calculator, is then presented. We also present grouping symbols and the order of operations, prime factorization of whole numbers, and the greatest common factor and least common multiple of a collection of whole numbers.

Introduction to Fractions and Multiplication and Division of Fractions

We recognize that fractions constitute one of the foundations of problem solving. We have, therefore, given a detailed treatment of the operations of multiplication and division of fractions and the logic behind these operations. We believe that the logical treatment and many practice exercises will help students retain the information presented in this chapter and enable them to use it as a foundation for the study of rational expressions in an algebra course.

Addition and Subtraction of Fractions, Comparing Fractions, and Complex Fractions

A detailed treatment of the operations of addition and subtraction of fractions and the logic behind these operations is given in this chapter. Again, we believe that the logical treatment and many practice exercises will help students retain the information, thus enabling them to use it in the study of rational expressions in an algebra course. We have tried to make explanations dynamic. A method for comparing fractions is introduced, which gives the student another way of understanding the relationship between the words *denominator* and *denomination*. This method serves to show the student that it is sometimes possible to compare two different types of quantities. We also study a method of simplifying complex fractions and of combining operations with fractions.

Decimals

The student is introduced to decimals in terms of the base ten number system, fractions, and digits occurring to the right of the units position. A method of converting a fraction to a decimal is discussed. The logic behind the standard methods of operating on decimals is presented and many examples of how to apply the methods are given. The word of as related to the operation of multiplication is discussed. Nonterminating divisions are examined, as are combinations of operations with decimals and fractions.

Ratios and Rates

We begin by defining and distinguishing the terms *ratio* and *rate*. The meaning of proportion and some applications of proportion problems are described. Proportion problems are solved using the "Five-Step Method." We hope that by using this method the student will discover the value of introducing a variable as a first step in problem solving and the power of organization. The chapter concludes with discussions of percent, fractions of one percent, and some applications of percent.

Techniques of Estimation

One of the most powerful problem-solving tools is a knowledge of estimation techniques. We feel that estimation is so important that we devote an entire chapter to its study. We examine three estimation techniques: estimation by rounding, estimation by clustering, and estimation by rounding fractions. We also include a section on the distributive property, an important algebraic property.

Measurement and Geometry

This chapter presents some of the techniques of measurement in both the United States system and the metric system. Conversion from one unit to another (in a system) is examined in terms of unit fractions. A discussion of the simplification of denominate numbers is also included. This discussion helps the student understand more clearly the association between pure numbers and dimensions. The chapter concludes with a study of perimeter and circumference of geometric figures and area and volume of geometric figures and objects.

Signed Numbers

A look at algebraic concepts and techniques is begun in this chapter. Basic to the study of algebra is a working knowledge of signed numbers. Definitions of variables, constants, and real numbers are introduced. We then distinguish between positive and negative numbers, learn how to read signed numbers, and examine the origin and use of the double-negative property of real numbers. The concept of absolute value is presented both geometrically (using the number line) and algebraically. The algebraic definition is followed by an interpretation of its meaning and several detailed examples of its use. Addition, subtraction, multiplication, and division of signed numbers are presented first using the number line, then with absolute value.

Algebraic Expressions and Equations

The student is introduced to some elementary algebraic concepts and techniques in this final chapter. Algebraic expressions and the process of combining like terms are discussed in [link] and [link]. The method of combining like terms in an algebraic expression is explained by using the interpretation of multiplication as a description of repeated addition (as in [link]).

Acknowledgements

This module contains the authors' acknowledgments and dedication of the book, Fundamentals of Mathematics by Denny Burzynski and Wade Ellis.

Many extraordinarily talented people are responsible for helping to create this text. We wish to acknowledge the efforts and skill of the following mathematicians. Their contributions have been invaluable.

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Denny Burzynski
Wade Ellis, Jr.
San Jose, California
December 1988

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D.B.

Objectives
This module contains the learning objectives for the chapter "Addition and Subtraction of Whole Numbers" from Fundamentals of Mathematics by Denny Burzynski and Wade Ellis, jr.

After completing this chapter, you should

Whole Numbers ([link])

- know the difference between numbers and numerals
- know why our number system is called the Hindu-Arabic numeration system
- understand the base ten positional number system
- be able to identify and graph whole numbers

Reading and Writing Whole Numbers ([link])

- be able to read and write a whole number

Rounding Whole Numbers ([link])

- understand that rounding is a method of approximation
- be able to round a whole number to a specified position

Addition of Whole Numbers ([link])

- understand the addition process
- be able to add whole numbers
- be able to use the calculator to add one whole number to another

Subtraction of Whole Numbers ([link])

- understand the subtraction process
- be able to subtract whole numbers
- be able to use a calculator to subtract one whole number from another whole number

Properties of Addition ([link])

- understand the commutative and associative properties of addition

- understand why 0 is the additive identity

Whole Numbers
This module is from Fundamentals of Mathematics by Denny Burzynski and Wade Ellis, Jr. This module discusses many of aspects of whole numbers, including the Hindu-Arabic numeration system, the base ten positional number system, and the graphing of whole numbers. By the end of this module students should be able to: know the difference between numbers and numerals, know why our number system is called the Hindu-Arabic numeration system, understand the base ten positional number system, and identify and graph whole numbers.

Section Overview

- Numbers and Numerals
- The Hindu-Arabic Numeration System
- The Base Ten Positional Number System
- Whole Numbers
- Graphing Whole Numbers

Numbers and Numerals

We begin our study of introductory mathematics by examining its most basic building block, the **number**.

Number
A **number** is a concept. It exists only in the mind.

The earliest concept of a number was a thought that allowed people to mentally picture the size of some collection of objects. To write down the number being conceptualized, a **numeral** is used.

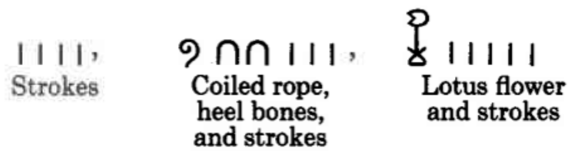
Numeral
A **numeral** is a symbol that represents a number.

In common usage today we do not distinguish between a number and a numeral. In our study of introductory mathematics, we will follow this common usage.

Sample Set A

The following are numerals. In each case, the first represents the number four, the second represents the number one hundred twenty-three, and the third, the number one thousand five. These numbers are represented in different ways.

- Hindu-Arabic numerals
  4, 123, 1005
- Roman numerals
  IV, CXXIII, MV
- Egyptian numerals



Practice Set A

Exercise:

Problem:

Do the phrases "four," "one hundred twenty-three," and "one thousand five" qualify as numerals? Yes or no?

Solution:

Yes. Letters are symbols. Taken as a collection (a written word), they represent a number.

The Hindu-Arabic Numeration System

Hindu-Arabic Numeration System
Our society uses the **Hindu-Arabic numeration system**. This system of numeration began shortly before the third century when the Hindus

invented the numerals

0 1 2 3 4 5 6 7 8 9

Leonardo Fibonacci
About a thousand years later, in the thirteenth century, a mathematician named Leonardo Fibonacci of Pisa introduced the system into Europe. It was then popularized by the Arabs. Thus, the name, Hindu-Arabic numeration system.

The Base Ten Positional Number System

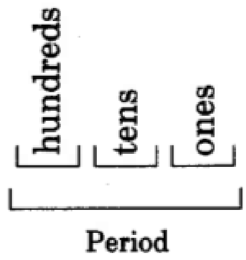
Digits
The Hindu-Arabic numerals 0 1 2 3 4 5 6 7 8 9 are called **digits**. We can form any number in the number system by selecting one or more digits and placing them in certain positions. Each position has a particular value. The Hindu mathematician who devised the system about A.D. 500 stated that "from place to place each is ten times the preceding."

Base Ten Positional Systems
It is for this reason that our number system is called a **positional** number system with **base ten**.

Commas
When numbers are composed of more than three digits, **commas** are sometimes used to separate the digits into groups of three.

Periods
These groups of three are called **periods** and they greatly simplify reading numbers.

In the Hindu-Arabic numeration system, a period has a value assigned to each or its three positions, and the values are the same for each period. The position values are



Thus, each period contains a position for the values of one, ten, and hundred. Notice that, in looking from right to left, the value of each position is ten times the preceding. Each period has a particular name.

As we continue from right to left, there are more periods. The five periods listed above are the most common, and in our study of introductory mathematics, they are sufficient.

The following diagram illustrates our positional number system to trillions. (There are, to be sure, other periods.)

In our positional number system, the **value of a digit** is determined by its *position* in the number.

Sample Set B

Example:
Find the value of 6 in the number 7,261.
Since 6 is in the tens position of the units period, its value is 6 tens.
6 tens = 60

Example:
Find the value of 9 in the number 86,932,106,005.
Since 9 is in the hundreds position of the millions period, its value is 9 hundred millions.
9 hundred millions = 9 hundred million

Example:
Find the value of 2 in the number 102,001.
Since 2 is in the ones position of the thousands period, its value is 2 one thousands.
2 one thousands = 2 thousand

Practice Set B

Exercise:

Problem: Find the value of 5 in the number 65,000.

Solution:

five thousand

Exercise:

Problem: Find the value of 4 in the number 439,997,007,010.

Solution:

four hundred billion

Exercise:

Problem: Find the value of 0 in the number 108.

Solution:

zero tens, or zero

Whole Numbers

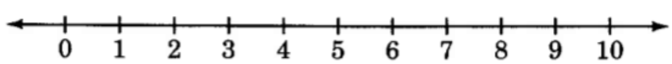
Whole Numbers
Numbers that are formed using only the digits
0 1 2 3 4 5 6 7 8 9
are called **whole numbers**. They are
0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, …

The three dots at the end mean "and so on in this same pattern."

Graphing Whole Numbers

Number Line
Whole numbers may be visualized by constructing a **number line**. To construct a number line, we simply draw a straight line and choose any point on the line and label it 0.

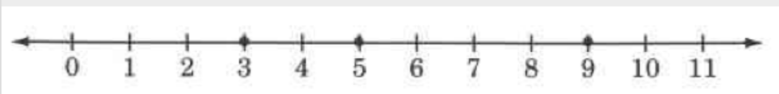
Origin
This point is called the **origin**. We then choose some convenient length, and moving to the right, mark off consecutive intervals (parts) along the line starting at 0. We label each new interval endpoint with the next whole number.



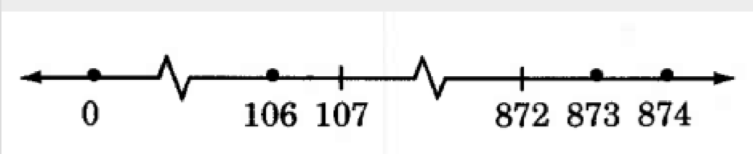
Graphing
We can visually display a whole number by drawing a closed circle at the point labeled with that whole number. Another phrase for visually displaying a whole number is graphing the whole number. The word graph means to "visually display."

Sample Set C

Example:
Graph the following whole numbers: 3, 5, 9.



Example:
Specify the whole numbers that are graphed on the following number line. The break in the number line indicates that we are aware of the whole numbers between 0 and 106, and 107 and 872, but we are not listing them due to space limitations.



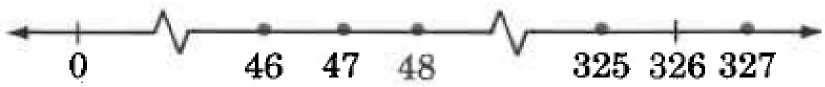
The numbers that have been graphed are
0, 106, 873, 874

Practice Set C

Exercise:

Problem: Graph the following whole numbers: 46, 47, 48, 325, 327.

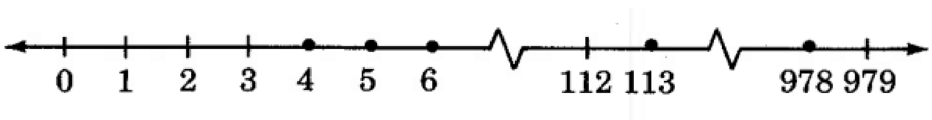
Solution:



Exercise:

Problem:

Specify the whole numbers that are graphed on the following number line.



Solution:

4, 5, 6, 113, 978

A **line** is composed of an endless number of points. Notice that we have labeled only some of them. As we proceed, we will discover new types of numbers and determine their location on the number line.

Exercises

Exercise:

Problem: What is a number?

Solution:

concept

Exercise:

Problem: What is a numeral?

Exercise:

Problem: Does the word "eleven" qualify as a numeral?

Solution:

Yes, since it is a symbol that represents a number.

Exercise:

Problem: How many different digits are there?

Exercise:

Problem:

Our number system, the Hindu-Arabic number system, is a number system with base .

Solution:

positional; 10

Exercise:

Problem:

Numbers composed of more than three digits are sometimes separated into groups of three by commas. These groups of three are called .

Exercise:

Problem:

In our number system, each period has three values assigned to it. These values are the same for each period. From right to left, what are they?

Solution:

ones, tens, hundreds

Exercise:

Problem:

Each period has its own particular name. From right to left, what are the names of the first four?

Exercise:

Problem: In the number 841, how many tens are there?

Solution:

4

Exercise:

Problem: In the number 3,392, how many ones are there?

Exercise:

Problem: In the number 10,046, how many thousands are there?

Solution:

0

Exercise:

Problem:

In the number 779,844,205, how many ten millions are there?

Exercise:

Problem:

In the number 65,021, how many hundred thousands are there?

Solution:

0

For following problems, give the value of the indicated digit in the given number.

Exercise:

Problem: 5 in 599

Exercise:

Problem: 1 in 310,406

Solution:

ten thousand

Exercise:

Problem: 9 in 29,827

Exercise:

Problem: 6 in 52,561,001,100

Solution:

6 ten millions = 60 million

Exercise:

Problem:

Write a two-digit number that has an eight in the tens position.

Exercise:

Problem:

Write a four-digit number that has a one in the thousands position and a zero in the ones position.

Solution:

1,340 (answers may vary)

Exercise:

Problem: How many two-digit whole numbers are there?

Exercise:

Problem: How many three-digit whole numbers are there?

Solution:

900

Exercise:

Problem: How many four-digit whole numbers are there?

Exercise:

Problem: Is there a smallest whole number? If so, what is it?

Solution:

yes; zero

Exercise:

Problem: Is there a largest whole number? If so, what is it?

Exercise:

Problem: Another term for "visually displaying" is .

Solution:

graphing

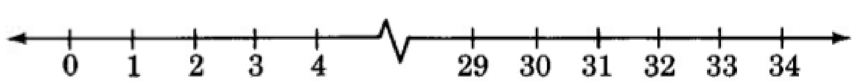
Exercise:

Problem: The whole numbers can be visually displayed on a .

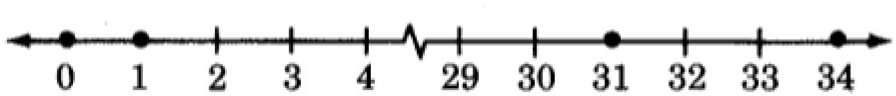
Exercise:

Problem:

Graph (visually display) the following whole numbers on the number line below: 0, 1, 31, 34.



Solution:



Exercise:

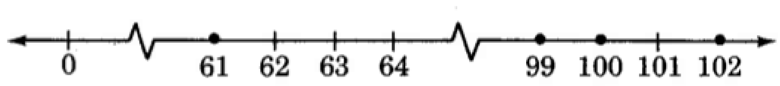
Problem:

Construct a number line in the space provided below and graph (visually display) the following whole numbers: 84, 85, 901, 1006, 1007.

Exercise:

Problem:

Specify, if any, the whole numbers that are graphed on the following number line.



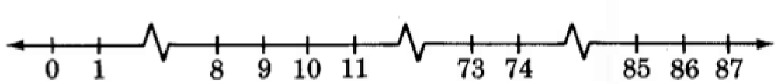
Solution:

61, 99, 100, 102

Exercise:

Problem:

Specify, if any, the whole numbers that are graphed on the following number line.



Reading and Writing Whole Numbers
This module is from Fundamentals of Mathematics by Denny Burzynski and Wade Ellis, Jr. This module discusses how to read and write whole numbers. By the end of this module, students should be able to read and write whole numbers.

Section Overview

- Reading Whole Numbers
- Writing Whole Numbers

Because our number system is a positional number system, reading and writing whole numbers is quite simple.

Reading Whole Numbers

To convert a number that is formed by digits into a verbal phrase, use the following method:

1. Beginning at the right and working right to left, separate the number into distinct periods by inserting commas every three digits.
2. Beginning at the left, read each period individually, saying the period name.

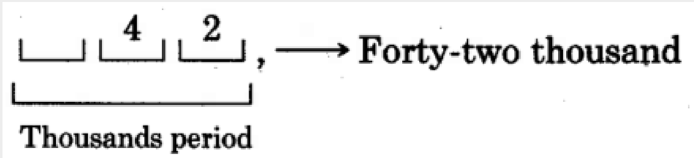
Sample Set A

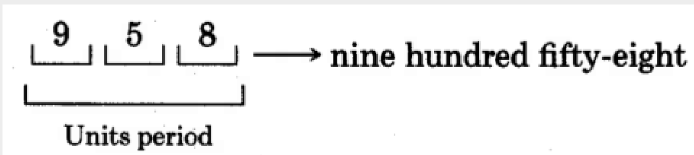
Write the following numbers as words.

Example:
Read 42958.

1. Beginning at the right, we can separate this number into distinct periods by inserting a comma between the 2 and 9.
   42,958

2. Beginning at the left, we read each period individually:

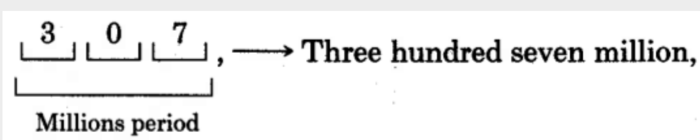


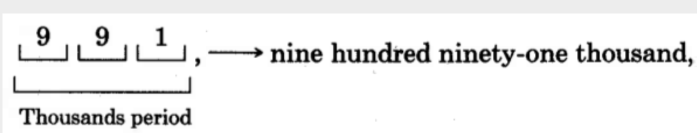


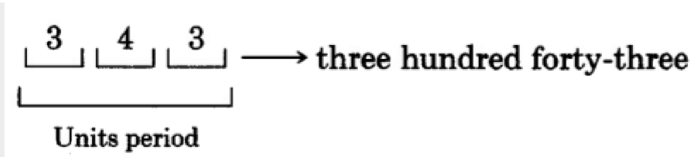
Forty-two thousand, nine hundred fifty-eight.

Example:
Read 307991343.

1. Beginning at the right, we can separate this number into distinct periods by placing commas between the 1 and 3 and the 7 and 9. 307,991,343
2. Beginning at the left, we read each period individually.



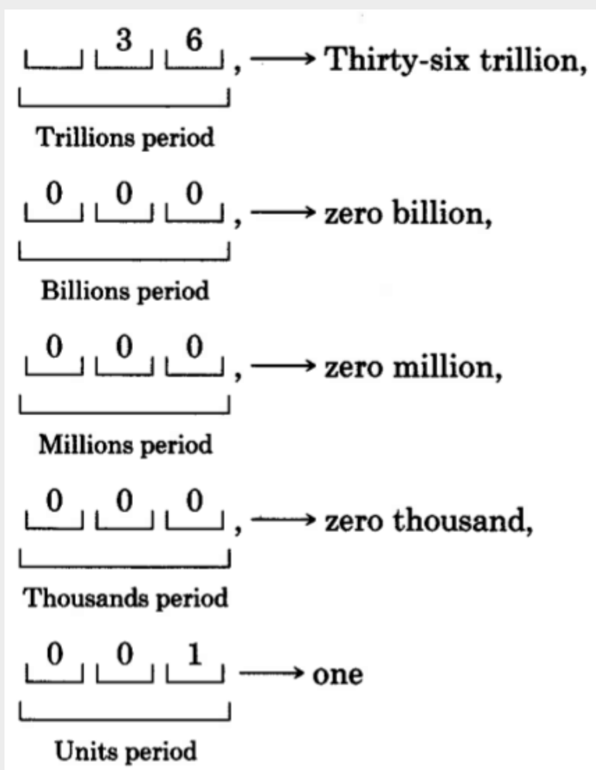




Three hundred seven million, nine hundred ninety-one thousand, three hundred forty-three.

Example:
Read 36000000000001.

1. Beginning at the right, we can separate this number into distinct periods by placing commas. 36,000,000,001
2. Beginning at the left, we read each period individually.



Thirty-six trillion, one.

Practice Set A

Write each number in words.

Exercise:

Problem: 12,542

Solution:

Twelve thousand, five hundred forty-two

Exercise:

Problem: 101,074,003

Solution:

One hundred one million, seventy-four thousand, three

Exercise:

Problem: 1,000,008

Solution:

One million, eight

Writing Whole Numbers

To express a number in digits that is expressed in words, use the following method:

1. Notice first that a number expressed as a verbal phrase will have its periods set off by commas.

2. Starting at the beginning of the phrase, write each period of numbers individually.
3. Using commas to separate periods, combine the periods to form one number.

Sample Set B

Write each number using digits.

Example:
Seven thousand, ninety-two.
Using the comma as a period separator, we have

$$\underbrace{\text{Seven thousand}}, \longrightarrow 7,$$

$$\underbrace{\text{ninety-two}} \longrightarrow 092$$

7,092

Example:
Fifty billion, one million, two hundred thousand, fourteen.
Using the commas as period separators, we have

$$\underbrace{\text{Fifty billion}}, \longrightarrow 50,$$

$$\underbrace{\text{one million}}, \longrightarrow 001,$$

$$\underbrace{\text{two hundred thousand}}, \longrightarrow 200,$$

$$\underbrace{\text{fourteen}} \longrightarrow 014$$

50,001,200,014

Example:
Ten million, five hundred twelve.
The comma sets off the periods. We notice that there is no thousands period. We'll have to insert this ourselves.

$\underbrace{\text{Ten million}}, \longrightarrow 10,$

$\underbrace{\text{zero thousand}}, \longrightarrow 000,$

$\underbrace{\text{five hundred twelve}} \longrightarrow 512$

10,000,512

Practice Set B

Express each number using digits.
Exercise:

Problem: One hundred three thousand, twenty-five.

Solution:

103,025

Exercise:

Problem: Six million, forty thousand, seven.

Solution:

6,040,007

Exercise:

Problem:

Twenty trillion, three billion, eighty million, one hundred nine thousand, four hundred two.

Solution:

20,003,080,109,402

Exercise:

Problem: Eighty billion, thirty-five.

Solution:

80,000,000,035

Exercises

For the following problems, write all numbers in words.

Exercise:

Problem: 912

Solution:

nine hundred twelve

Exercise:

Problem: 84

Exercise:

Problem: 1491

Solution:

one thousand, four hundred ninety-one

Exercise:

Problem: 8601

Exercise:

Problem: 35,223

Solution:

thirty-five thousand, two hundred twenty-three

Exercise:

Problem: 71,006

Exercise:

Problem: 437,105

Solution:

four hundred thirty-seven thousand, one hundred five

Exercise:

Problem: 201,040

Exercise:

Problem: 8,001,001

Solution:

eight million, one thousand, one

Exercise:

Problem: 16,000,053

Exercise:

Problem: 770,311,101

Solution:

seven hundred seventy million, three hundred eleven thousand, one hundred one

Exercise:

Problem: 83,000,000,007

Exercise:

Problem: 106,100,001,010

Solution:

one hundred six billion, one hundred million, one thousand ten

Exercise:

Problem: 3,333,444,777

Exercise:

Problem: 800,000,800,000

Solution:

eight hundred billion, eight hundred thousand

Exercise:

Problem:

A particular community college has 12,471 students enrolled.

Exercise:

Problem:

A person who watches 4 hours of television a day spends 1460 hours a year watching T.V.

Solution:

four; one thousand, four hundred sixty

Exercise:

Problem:

Astronomers believe that the age of the earth is about 4,500,000,000 years.

Exercise:

Problem:

Astronomers believe that the age of the universe is about 20,000,000,000 years.

Solution:

twenty billion

Exercise:

Problem:

There are 9690 ways to choose four objects from a collection of 20.

Exercise:

Problem:

If a 412 page book has about 52 sentences per page, it will contain about 21,424 sentences.

Solution:

four hundred twelve; fifty-two; twenty-one thousand, four hundred twenty-four

Exercise:

Problem:

In 1980, in the United States, there was $1,761,000,000,000 invested in life insurance.

Exercise:

Problem:

In 1979, there were 85,000 telephones in Alaska and 2,905,000 telephones in Indiana.

Solution:

one thousand, nine hundred seventy-nine; eighty-five thousand; two million, nine hundred five thousand

Exercise:

Problem:

In 1975, in the United States, it is estimated that 52,294,000 people drove to work alone.

Exercise:

Problem:

In 1980, there were 217 prisoners under death sentence that were divorced.

Solution:

one thousand, nine hundred eighty; two hundred seventeen

Exercise:

Problem:

In 1979, the amount of money spent in the United States for regular-session college education was $50,721,000,000,000.

Exercise:

Problem:

In 1981, there were 1,956,000 students majoring in business in U.S. colleges.

Solution:

one thousand, nine hundred eighty one; one million, nine hundred fifty-six thousand

Exercise:

Problem:

In 1980, the average fee for initial and follow up visits to a medical doctors office was about $34.

Exercise:

Problem:

In 1980, there were approximately 13,100 smugglers of aliens apprehended by the Immigration border patrol.

Solution:

one thousand, nine hundred eighty; thirteen thousand, one hundred

Exercise:

Problem:

In 1980, the state of West Virginia pumped 2,000,000 barrels of crude oil, whereas Texas pumped 975,000,000 barrels.

Exercise:

Problem: The 1981 population of Uganda was 12,630,000 people.

Solution:

twelve million, six hundred thirty thousand

Exercise:

Problem:

In 1981, the average monthly salary offered to a person with a Master's degree in mathematics was $1,685.

For the following problems, write each number using digits.

Exercise:

Problem: Six hundred eighty-one

Solution:

681

Exercise:

Problem: Four hundred ninety

Exercise:

Problem: Seven thousand, two hundred one

Solution:

7,201

Exercise:

Problem: Nineteen thousand, sixty-five

Exercise:

Problem: Five hundred twelve thousand, three

Solution:

512,003

Exercise:

Problem:

Two million, one hundred thirty-three thousand, eight hundred fifty-nine

Exercise:

Problem: Thirty-five million, seven thousand, one hundred one

---

Solution:

35,007,101

Exercise:

Problem: One hundred million, one thousand

Exercise:

Problem: Sixteen billion, fifty-nine thousand, four

Solution:

16,000,059,004

Exercise:

Problem:

Nine hundred twenty billion, four hundred seventeen million, twenty-one thousand

Exercise:

Problem: Twenty-three billion

Solution:

23,000,000,000

Exercise:

Problem:

Fifteen trillion, four billion, nineteen thousand, three hundred five

Exercise:

Problem: One hundred trillion, one

Solution:

100,000,000,000,001

Exercises for Review

Exercise:

Problem: ([link]) How many digits are there?

Exercise:

Problem: ([link]) In the number 6,641, how many tens are there?

Solution:

4

Exercise:

Problem: ([link]) What is the value of 7 in 44,763?

Exercise:

Problem: ([link]) Is there a smallest whole number? If so, what is it?

Solution:

yes, zero

Exercise:

Problem:

([link]) Write a four-digit number with a 9 in the tens position.

Rounding Whole Numbers
This module is from Fundamentals of Mathematics by Denny Burzynski and Wade Ellis, Jr. This module discusses how to round whole numbers. By the end of the module students should be able to understand that rounding is a method of approximation and round a whole number to a specified position.

Section Overview

- Rounding as an Approximation
- The Method of Rounding Numbers

Rounding as an Approximation

A primary use of whole numbers is to keep count of how many objects there are in a collection. Sometimes we're only interested in the approximate number of objects in the collection rather than the precise number. For example, there are *approximately* 20 symbols in the collection below.

The *precise* number of symbols in the above collection is 18.

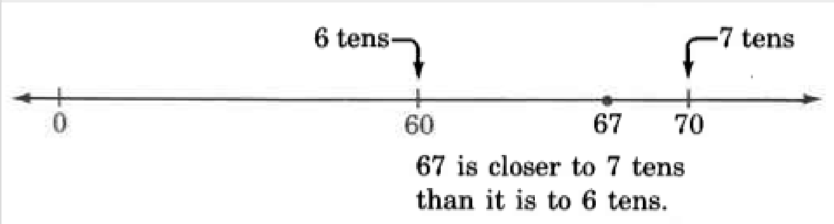
Rounding
We often approximate the number of objects in a collection by mentally seeing the collection as occurring in groups of tens, hundreds, thousands, etc. This process of approximation is called **rounding**. Rounding is very useful in estimation. We will study estimation in Chapter 8.

When we think of a collection as occurring in groups of tens, we say we're *rounding to the nearest ten*. When we think of a collection as occurring in groups of hundreds, we say we're *rounding to the nearest hundred*. This idea of rounding continues through thousands, ten thousands, hundred thousands, millions, etc.

The process of rounding whole numbers is illustrated in the following examples.

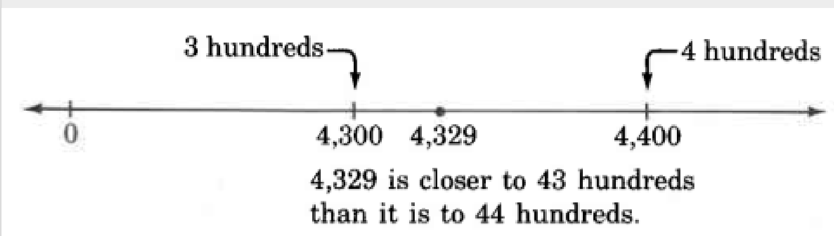
Example:

Round 67 to the nearest ten.
On the number line, 67 is more than halfway from 60 to 70. The digit immediately to the right of the tens digit, the round-off digit, is the indicator for this.



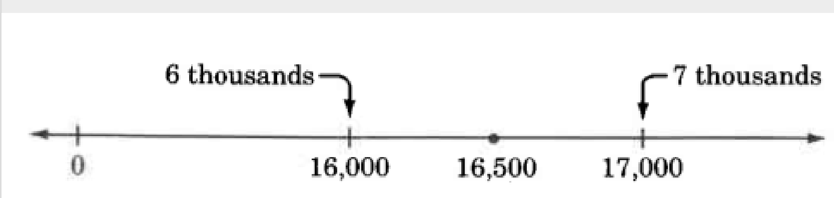
Thus, 67, rounded to the nearest ten, is 70.

Example:
Round 4,329 to the nearest hundred.
On the number line, 4,329 is less than halfway from 4,300 to 4,400. The digit to the immediate right of the hundreds digit, the round-off digit, is the indicator.



Thus, 4,329, rounded to the nearest hundred is 4,300.

Example:
Round 16,500 to the nearest thousand.
On the number line, 16,500 is exactly halfway from 16,000 to 17,000.



By convention, when the number to be rounded is *exactly halfway* between two numbers, it is rounded to the *higher* number.
Thus, 16,500, rounded to the nearest thousand, is 17,000.

Example:
A person whose salary is $41,450 per year might tell a friend that she makes $41,000 per year. She has rounded 41,450 to the nearest thousand. The number 41,450 is closer to 41,000 than it is to 42,000.

The Method of Rounding Whole Numbers

From the observations made in the preceding examples, we can use the following method to **round a whole number** to a particular position.

1. Mark the position of the round-off digit.
2. Note the digit to the immediate right of the round-off digit.

    a. If it is less than 5, replace it and all the digits to its right with zeros. Leave the round-off digit unchanged.
    b. If it is 5 or larger, replace it and all the digits to its right with zeros. Increase the round-off digit by 1.

Sample Set A

Use the method of rounding whole numbers to solve the following problems.

Example:
Round 3,426 to the nearest ten.

1. We are rounding to the tens position. Mark the digit in the tens position

2. Observe the digit immediately to the right of the tens position. It is 6. Since 6 is greater than 5, we *round up* by replacing 6 with 0 and adding 1 to the digit in the tens position (the round-off position): $2 + 1 = 3$.
3,430

Thus, 3,426 rounded to the nearest ten is 3,430.

Example:
Round 9,614,018,007 to the nearest ten million.

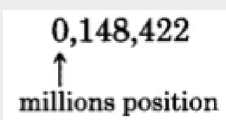
1. We are rounding to the nearest ten million.

2. Observe the digit immediately to the right of the ten millions position. It is 4. Since 4 is less than 5, we *round down* by replacing 4 and all the digits to its right with zeros.
9,610,000,000

Thus, 9,614,018,007 rounded to the nearest ten million is 9,610,000,000.

Example:
Round 148,422 to the nearest million.

1. Since we are rounding to the nearest million, we'll have to *imagine* a digit in the millions position. We'll write 148,422 as 0,148,422.

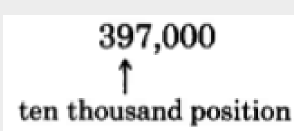


2. The digit immediately to the right is 1. Since 1 is less than 5, we'll *round down* by replacing it and all the digits to its right with zeros.
0,000,000
This number is 0.

Thus, 148,422 rounded to the nearest million is 0.

Example:
Round 397,000 to the nearest ten thousand.

1. We are rounding to the nearest ten thousand.



2. The digit immediately to the right of the ten thousand position is 7. Since 7 is greater than 5, we round up by replacing 7 and all the digits to its right with zeros and adding 1 to the digit in the ten thousands position. But $9 + 1 = 10$ and we must carry the 1 to the next (the hundred thousands) position.
   400,000

Thus, 397,000 rounded to the nearest ten thousand is 400,000.

Practice Set A

Use the method of rounding whole numbers to solve each problem.
Exercise:

Problem: Round 3387 to the nearest hundred.

Solution:

3400

Exercise:

Problem: Round 26,515 to the nearest thousand.

Solution:

27,000

Exercise:

Problem: Round 30,852,900 to the nearest million.

Solution:

31,000,000

Exercise:

Problem: Round 39 to the nearest hundred.

Solution:

0

Exercise:

Problem: Round 59,600 to the nearest thousand.

Solution:

60,000

Exercises

For the following problems, complete the table by rounding each number to the indicated positions.

Exercise:

Problem: 1,642

hundred	thousand	ten thousand	million

Solution:

hundred	thousand	ten thousand	million
1,600	2000	0	0

Exercise:

Problem: 5,221

hundred	thousand	ten thousand	million

Exercise:

Problem: 91,803

Hundred	thousand	ten thousand	million

Solution:

Hundred	thousand	ten thousand	million
91,800	92,000	90,000	0

Exercise:

Problem: 106,007

hundred	thousand	ten thousand	million

Exercise:

Problem: 208

hundred	thousand	ten thousand	million

Solution:

hundred	thousand	ten thousand	million
200	0	0	0

Exercise:

Problem: 199

hundred	thousand	ten thousand	million

Exercise:

Problem: 863

hundred	thousand	ten thousand	million

Solution:

hundred	thousand	ten thousand	million
900	1,000	0	0

Exercise:

Problem: 794

hundred	thousand	ten thousand	million

Exercise:

Problem: 925

hundred	thousand	ten thousand	million

Solution:

hundred	thousand	ten thousand	million
900	1,000	0	0

Exercise:

Problem: 909

hundred	thousand	ten thousand	million

Exercise:

Problem: 981

hundred	thousand	ten thousand	million

Solution:

hundred	thousand	ten thousand	million
1,000	1,000	0	0

Exercise:

Problem: 965

hundred	thousand	ten thousand	million

Exercise:

Problem: 551,061,285

hundred	thousand	ten thousand	million

Solution:

hundred	thousand	ten thousand	million
551,061,300	551,061,000	551,060,000	551,000,000

Exercise:

Problem: 23,047,991,521

hundred	thousand	ten thousand	million

Exercise:

Problem: 106,999,413,206

Hundred	thousand	ten thousand	million

Solution:

hundred	thousand	ten thousand	million
106,999,413,200	106,999,413,000	106,999,410,000	106,999,000,000

Exercise:

Problem: 5,000,000

hundred	thousand	ten thousand	million

Exercise:

Problem: 8,006,001

hundred	thousand	ten thousand	million

Solution:

Hundred	Thousand	ten thousand	Million
8,006,000	8,006,000	8,010,000	8,000,000

Exercise:

Problem: 94,312

hundred	thousand	ten thousand	million

Exercise:

Problem: 33,486

hundred	thousand	ten thousand	million

Solution:

hundred	thousand	ten thousand	million
33,500	33,000	30,000	0

Exercise:

Problem: 560,669

hundred	thousand	ten thousand	million

Exercise:

Problem: 388,551

hundred	thousand	ten thousand	million

Solution:

hundred	thousand	ten thousand	million
388,600	389,000	390,000	0

Exercise:

Problem: 4,752

hundred	thousand	ten thousand	million

Exercise:

Problem: 8,209

hundred	thousand	ten thousand	million

Solution:

hundred	thousand	ten thousand	million
8,200	8,000	10,000	0

Exercise:

Problem:

In 1950, there were 5,796 cases of diphtheria reported in the United States. Round to the nearest hundred.

Exercise:

Problem:

In 1979, 19,309,000 people in the United States received federal food stamps. Round to the nearest ten thousand.

Solution:

19,310,000

Exercise:

Problem:

In 1980, there were 1,105,000 people between 30 and 34 years old enrolled in school. Round to the nearest million.

Exercise:

Problem:

In 1980, there were 29,100,000 reports of aggravated assaults in the United States. Round to the nearest million.

Solution:

29,000,000

For the following problems, round the numbers to the position you think is most reasonable for the situation.

Exercise:

Problem:

In 1980, for a city of one million or more, the average annual salary of police and firefighters was $16,096.

Exercise:

Problem:

The average percentage of possible sunshine in San Francisco, California, in June is 73%.

Solution:

70% or 75%

Exercise:

Problem:

In 1980, in the state of Connecticut, $3,777,000,000 in defense contract payroll was awarded.

Exercise:

Problem:

In 1980, the federal government paid $5,463,000,000 to Viet Nam veterans and dependants.

Solution:

$5,500,000,000

Exercise:

Problem: In 1980, there were 3,377,000 salespeople employed in the United States.

Exercise:

Problem:

In 1948, in New Hampshire, 231,000 popular votes were cast for the president.

Solution:

230,000

Exercise:

Problem: In 1970, the world production of cigarettes was 2,688,000,000,000.

Exercise:

Problem:

In 1979, the total number of motor vehicle registrations in Florida was 5,395,000.

Solution:

5,400,000

Exercise:

Problem: In 1980, there were 1,302,000 registered nurses the United States.

Exercises for Review

Exercise:

Problem:

([link]) There is a term that describes the visual displaying of a number. What is the term?

Solution:

graphing

Exercise:

Problem: ([link]) What is the value of 5 in 26,518,206?

Exercise:

Problem: ([link]) Write 42,109 as you would read it.

Solution:

Forty-two thousand, one hundred nine

Exercise:

Problem: ([link]) Write "six hundred twelve" using digits.

Exercise:

Problem: ([link]) Write "four billion eight" using digits.

Solution:

4,000,000,008

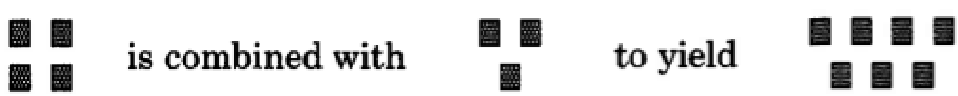
Addition of Whole Numbers
This module is from Fundamentals of Mathematics by Denny Burzynski and Wade Ellis, Jr. This module discusses how to add whole numbers. By the end of this module, students should be able to understand the addition process, add whole numbers, and use the calculator to add one whole number to another.

Section Overview

- Addition
- Addition Visualized on the Number Line
- The Addition Process
- Addition Involving Carrying
- Calculators

Addition

Suppose we have two collections of objects that we combine together to form a third collection. For example,



We are combining a collection of four objects with a collection of three objects to obtain a collection of seven objects.

Addition
The process of combining two or more objects (real or intuitive) to form a third, the total, is called **addition**.

In addition, the numbers being added are called **addends** or **terms**, and the total is called the **sum**. The **plus symbol** (+) is used to indicate addition, and the **equal symbol** (=) is used to represent the word "equal." For example, $4 + 3 = 7$ means "four added to three equals seven."

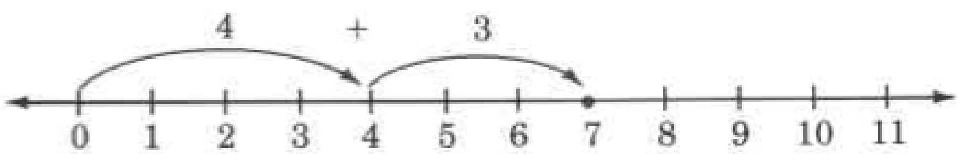
Addition Visualized on the Number Line

Addition is easily visualized on the number line. Let's visualize the addition of 4 and 3 using the number line.

To find $4+3$,

1. Start at 0.
2. Move to the right 4 units. We are now located at 4.
3. From 4, move to the right 3 units. We are now located at 7.

Thus, $4+3=7$.



The Addition Process

We'll study the process of addition by considering the sum of 25 and 43.

$$\begin{array}{r} 25 \\ +43 \end{array} \quad \text{means}$$

$$\begin{array}{r} 2 \text{ tens} + 5 \text{ ones} \\ +4 \text{ tens} + 3 \text{ ones} \\ \hline 6 \text{ tens} + 8 \text{ ones} \end{array}$$

We write this as 68.

We can suggest the following procedure for adding whole numbers using this example.

Example:
The Process of Adding Whole Numbers
To **add whole numbers**,
The process:

1. Write the numbers vertically, placing corresponding positions in the same column.

$$\begin{array}{r} 25 \\ +43 \end{array}$$

2. Add the digits in each column. Start at the right (in the ones position) and move to the left, placing the sum at the bottom.

$$\begin{array}{r} 25 \\ +43 \\ 68 \end{array}$$

Note: Confusion and incorrect sums can occur when the numbers are *not aligned* in columns properly. Avoid writing such additions as

$$\begin{array}{l} 25 \\ +43 \end{array}$$

$$\begin{array}{l} \quad 25 \\ +43 \end{array}$$

Sample Set A

Example:

Add 276 and 103.

$$\begin{array}{rl} 276 & 6+3=9. \\ +103 & 7+0=7. \\ 379 & 2+1=3. \end{array}$$

Example:

Add 1459 and 130

$$\begin{array}{r} 1459 \\ +130 \\ \hline 1589 \end{array} \qquad \begin{array}{l} 9+0=9. \\ 5+3=8. \\ 4+1=5. \\ 1+0=1. \end{array}$$

In each of these examples, each individual sum does not exceed 9. We will examine individual sums that exceed 9 in the next section.

Practice Set A

Perform each addition. Show the expanded form in problems 1 and 2.

Exercise:

Problem: Add 63 and 25.

Solution:

88

$$\begin{array}{r} 6 \text{ tens} + 3 \text{ ones} \\ +2 \text{ tens} + 5 \text{ ones} \\ \hline 8 \text{ tens} + 8 \text{ ones} \end{array}$$

Exercise:

Problem: Add 4,026 and 1,501.

Solution:

5,527

$$
\begin{array}{rcrcrcr}
4 \text{ thousands} & + & 0 \text{ hundreds} & + & 2 \text{ tens} & + & 6 \text{ ones} \\
+1 \text{ thousand} & + & 5 \text{ hundreds} & + & 0 \text{ tens} & + & 1 \text{ one} \\
\hline
5 \text{ thousands} & + & 5 \text{ hundreds} & + & 2 \text{ tens} & + & 7 \text{ ones}
\end{array}
$$

Exercise:

Problem: Add 231,045 and 36,121.

Solution:

267,166

Addition Involving Carrying

It often happens in addition that the sum of the digits in a column will exceed 9. This happens when we add 18 and 34. We show this in expanded form as follows.

$$
\begin{array}{rcl}
18 & = & 1 \text{ ten} + \overset{\text{This sum exceeds 9.}}{8} \text{ ones} \\
\underline{+34} & = & \underline{3 \text{ tens} + 4 \text{ ones}} \\
 & & 4 \text{ tens} + 12 \text{ ones} = 4 \text{ tens} + \overbrace{1 \text{ ten} + 2 \text{ ones}}^{12 \text{ ones}} \\
 & & \qquad\qquad\qquad\quad = 5 \text{ tens} + 2 \text{ ones} \\
 & & \qquad\qquad\qquad\quad = 52
\end{array}
$$

Notice that when we add the 8 ones to the 4 ones we get 12 ones. We then convert the 12 ones to 1 ten and 2 ones. In vertical addition, we show this conversion by **carrying** the ten to the tens column. We write a 1 at the top of the tens column to indicate the carry. This same example is shown in a shorter form as follows:

$$
\begin{array}{r}
{}^{1} \\
18 \\
+34 \\
\hline
52
\end{array}
$$

$8 + 4 = 12$ Write 2, carry 1 ten to the top of the next column to the left.

Sample Set B

Perform the following additions. Use the process of carrying when needed.

Example:
Add 1875 and 358.

$$\begin{array}{r} {\scriptstyle 111} \\ 1875 \\ +\ \ 358 \\ \hline 2233 \end{array}$$

$5 + 8 = 13$	Write 3, carry 1 ten.
$1 + 7 + 5 = 13$	Write 3, carry 1 hundred.
$1 + 8 + 3 = 12$	Write 2, carry 1 thousand.
$1 + 1 = 2$	

The sum is 2233.

Example:
Add 89,208 and 4,946.

$$\begin{array}{r} {\scriptstyle 11\ \ \ 1} \\ 89{,}208 \\ +\ \ 4{,}946 \\ \hline 94{,}154 \end{array}$$

$8 + 6 = 14$	Write 4, carry 1 ten.
$1 + 0 + 4 = 5$	Write the 5 (nothing to carry).
$2 + 9 = 11$	Write 1, carry one thousand.
$1 + 9 + 4 = 14$	Write 4, carry one ten thousand.
$1 + 8 = 9$	

The sum is 94,154.

Example:

Add 38 and 95.

$$\begin{array}{r} {\scriptstyle 11} \\ 38 \\ +\ 95 \\ \hline 133 \end{array}$$

$8 + 5 = 13$	Write 3, carry 1 ten.
$1 + 3 + 9 = 13$	Write 3, carry 1 hundred.
$1 + 0 = 1$	

As you proceed with the addition, it is a good idea to keep in mind what is actually happening.

$$\begin{array}{rlrr} \begin{array}{r} 38 \\ \underline{+95} \end{array} & \text{means} & 3 \text{ tens} & +\ \ 8 \text{ ones} \\ & & \underline{+\ \ 9 \text{ tens}} & \underline{+\ \ 5 \text{ ones}} \\ & & 12 \text{ tens} & +13 \text{ ones} \\ & = & 12 \text{ tens} + 1 \text{ ten} & +\ \ 3 \text{ ones} \\ & = & 13 \text{ tens} & +\ \ 3 \text{ ones} \\ & = 1 \text{ hundred} + & 3 \text{ tens} & +\ \ 3 \text{ ones} \\ & = 133 & & \end{array}$$

The sum is 133.

Example:
Find the sum 2648, 1359, and 861.

$$
\begin{array}{r}
{\scriptstyle 111} \\
2648 \\
1359 \\
+\ \ 861 \\
\hline
4868
\end{array}
$$

$8 + 9 + 1 = 18$	Write 8, carry 1 ten.
$1 + 4 + 5 + 6 = 16$	Write 6, carry 1 hundred.
$1 + 6 + 3 + 8 = 18$	Write 8, carry 1 thousand.
$1 + 2 + 1 = 4$	

The sum is 4,868.

Numbers other than 1 can be carried as illustrated in [link].

Example:
Find the sum of the following numbers.

$$
\begin{array}{r}
{\scriptstyle 132\ 1} \\
878016 \\
9905 \\
38951 \\
+\ \ 56817 \\
\hline
983689
\end{array}
$$

$6+5+1+7=19$	Write 9, carry the 1.
$1+1+0+5+1=8$	Write 8.
$0+9+9+8=26$	Write 6, carry the 2.
$2+8+9+8+6=33$	Write 3, carry the 3.
$3+7+3+5=18$	Write 8, carry the 1.
$1+8=9$	Write 9.

The sum is 983,689.

Example:
The number of students enrolled at Riemann College in the years 1984, 1985, 1986, and 1987 was 10,406, 9,289, 10,108, and 11,412, respectively. What was the total number of students enrolled at Riemann College in the years 1985, 1986, and 1987?
We can determine the total number of students enrolled by adding 9,289, 10,108, and 11,412, the number of students enrolled in the years 1985, 1986, and 1987.

$$\begin{array}{r} {}^{1}\ \ {}^{11}\ \ \\ 9{,}289 \\ 10{,}108 \\ +\,11{,}412 \\ \hline 30{,}809 \end{array}$$

The total number of students enrolled at Riemann College in the years 1985, 1986, and 1987 was 30,809.

Practice Set B

Perform each addition. For the next three problems, show the expanded form.
Exercise:

Problem: Add 58 and 29.

Solution:

87

$$\begin{array}{r} 5\text{ tens} + 8\text{ ones} \\ +2\text{ tens} + 9\text{ ones} \\ \hline 7\text{ tens} + 17\text{ ones} \end{array}$$

$$\begin{aligned} &= 7\,\text{tens} + 1\,\text{ten} + 7\,\text{ones} \\ &= 8\,\text{tens} + 7\,\text{ones} \\ &= 87 \end{aligned}$$

Exercise:

Problem: Add 476 and 85.

Solution:

561

$$\begin{array}{rrr} 4\text{ hundreds} + & 7\text{ tens} + & 6\text{ ones} \\ + & 8\text{ tens} + & 5\text{ ones} \\ \hline 4\text{ hundreds} + & 15\text{ tens} + & 11\text{ ones} \end{array}$$

$$\begin{aligned} &= 4\text{ hundreds} + 15\text{ tens} + 1\text{ ten} + 1\text{ one} \\ &= 4\text{ hundreds} + 16\text{ tens} + 1\text{ one} \\ &= 4\text{ hundreds} + 1\text{ hundred} + 6\text{ tens} + 1\text{ one} \\ &= 5\text{ hundreds} + 6\text{ tens} + 1\text{ one} \\ &= 561 \end{aligned}$$

Exercise:

Problem: Add 27 and 88.

Solution:

115

$$\begin{array}{r} 2 \text{ tens} + \ \ 7 \text{ ones} \\ + \ \ 8 \text{ tens} + \ \ 8 \text{ ones} \\ \hline 10 \text{ tens} + 15 \text{ ones} \end{array}$$

$= 10 \text{ tens} + 1 \text{ ten} + 5 \text{ ones}$
$= 11 \text{ tens} + 5 \text{ ones}$
$= 1 \text{ hundred} + 1 \text{ ten} + 5 \text{ ones}$
$= 115$

Exercise:

Problem: Add 67,898 and 85,627.

Solution:

153,525

For the next three problems, find the sums.

Exercise:

Problem:
$$\begin{array}{r} 57 \\ 26 \\ 84 \end{array}$$

Solution:

167

Exercise:

Problem:

$$\begin{array}{r} 847 \\ 825 \\ 796 \end{array}$$

Solution:

2,468

Exercise:

Problem:

$$\begin{array}{r} 16,945 \\ 8,472 \\ 387,721 \\ 21,059 \\ 629 \end{array}$$

Solution:

434,826

Calculators

Calculators provide a very simple and quick way to find sums of whole numbers. For the two problems in Sample Set C, assume the use of a calculator that does not require the use of an ENTER key (such as many Hewlett-Packard calculators).

Sample Set C

Use a calculator to find each sum.

Example:

$34 + 21$		Display Reads
Type	34	34
Press	+	34
Type	21	21
Press	=	55

The sum is 55.

Example:

$106 + 85 + 322 + 406$		Display Reads	
Type	106	106	The calculator keeps a running subtotal
Press	+	106	
Type	85	85	

Press	=	191	$\leftarrow 106 + 85$
Type	322	322	
Press	+	513	$\leftarrow 191 + 322$
Type	406	406	
Press	=	919	$\leftarrow 513 + 406$

The sum is 919.

Practice Set C

Use a calculator to find the following sums.

Exercise:

Problem: $62 + 81 + 12$

Solution:

155

Exercise:

Problem: $9,261 + 8,543 + 884 + 1,062$

Solution:

19,750

Exercise:

Problem: $10,221 + 9,016 + 11,445$

Solution:

30,682

Exercises

For the following problems, perform the additions. If you can, check each sum with a calculator.

Exercise:

Problem: $14 + 5$

Solution:

19

Exercise:

Problem: $12 + 7$

Exercise:

Problem: $46 + 2$

Solution:

48

Exercise:

Problem: $83 + 16$

Exercise:

Problem: $77 + 21$

Solution:

98

Exercise:

Problem: $$\begin{array}{r} 321 \\ +\ \ 42 \end{array}$$

Exercise:

Problem: $$\begin{array}{r} 916 \\ +\ \ 62 \end{array}$$

Solution:

978

Exercise:

Problem: $$\begin{array}{r} 104 \\ +561 \end{array}$$

Exercise:

Problem: $$\begin{array}{r} 265 \\ +103 \end{array}$$

Solution:

368

Exercise:

Problem: $552 + 237$

Exercise:

Problem: $8{,}521 + 4{,}256$

Solution:

12,777

Exercise:

Problem: $\begin{array}{r} 16{,}408 \\ +\ \ 3{,}101 \end{array}$

Exercise:

Problem: $\begin{array}{r} 16{,}515 \\ +42{,}223 \end{array}$

Solution:

58,738

Exercise:

Problem: $616{,}702 + 101{,}161$

Exercise:

Problem: $43{,}156{,}219 + 2{,}013{,}520$

Solution:

45,169,739

Exercise:

Problem: $17 + 6$

Exercise:

Problem: $25 + 8$

Solution:

33

Exercise:

Problem: $\begin{array}{r} 84 \\ + \;\; 7 \end{array}$

Exercise:

Problem: $\begin{array}{r} 75 \\ + \;\; 6 \end{array}$

Solution:

81

Exercise:

Problem: $36 + 48$

Exercise:

Problem: $74 + 17$

Solution:

91

Exercise:

Problem: $486 + 58$

Exercise:

Problem: $743 + 66$

Solution:

809

Exercise:

Problem: $381 + 88$

Exercise:

Problem: $\begin{array}{r} 687 \\ +175 \end{array}$

Solution:

862

Exercise:

Problem: $\begin{array}{r} 931 \\ +853 \end{array}$

Exercise:

Problem: $1{,}428 + 893$

Solution:

2,321

Exercise:

Problem: $12{,}898 + 11{,}925$

Exercise:

Problem: $\begin{array}{r} 631{,}464 \\ +509{,}740 \end{array}$

Solution:

1,141,204

Exercise:

Problem: $\begin{array}{r} 805{,}996 \\ +\ \ 98{,}516 \end{array}$

Exercise:

Problem: $\begin{array}{r} 38{,}428{,}106 \\ +522{,}936{,}005 \end{array}$

Solution:

561,364,111

Exercise:

Problem: $5{,}288{,}423{,}100 + 16{,}934{,}785{,}995$

Exercise:

Problem: $98{,}876{,}678{,}521{,}402 + 843{,}425{,}685{,}685{,}658$

Solution:

942,302,364,207,060

Exercise:

Problem: $41 + 61 + 85 + 62$

Exercise:

Problem: $21 + 85 + 104 + 9 + 15$

Solution:

234

Exercise:

Problem:
$$\begin{array}{r} 116 \\ 27 \\ 110 \\ 110 \\ + \quad 8 \end{array}$$

Exercise:

Problem:
$$\begin{array}{r} 75{,}206 \\ 4{,}152 \\ +16{,}007 \end{array}$$

Solution:

95,365

Exercise:

Problem:
$$\begin{array}{r} 8{,}226 \\ 143 \\ 92{,}015 \\ 8 \\ 487{,}553 \\ 5{,}218 \end{array}$$

Exercise:

Problem:

50,006
1,005
100,300
20,008
1,000,009
800,800

Solution:

1,972,128

Exercise:

Problem:

616
42,018
1,687
225
8,623,418
12,506,508
19
2,121
195,643

For the following problems, perform the additions and round to the nearest hundred.

Exercise:

Problem:

1,468
2,183

Solution:

3,700

Exercise:

Problem:
928,725
15,685

Exercise:

Problem:
82,006
3,019,528

Solution:

3,101,500

Exercise:

Problem:
18,621
5,059

Exercise:

Problem:
92
48

Solution:

100

Exercise:

Problem:
16
37

Exercise:

Problem:
21
16

Solution:

0

Exercise:

Problem:
11,172
22,749
12,248

Exercise:

Problem:
240
280
210
310

Solution:

1,000

Exercise:

Problem:
9,573
101,279
122,581

For the next five problems, replace the letter m with the whole number that will make the addition true.

Exercise:

Problem: $\begin{array}{r} 62 \\ + \quad m \\ 67 \end{array}$

Solution:

5

Exercise:

Problem: $\begin{array}{r} 106 \\ + \quad m \\ 113 \end{array}$

Exercise:

Problem: $\begin{array}{r} 432 \\ + \quad m \\ 451 \end{array}$

Solution:

19

Exercise:

Problem: $\begin{array}{r} 803 \\ + \quad m \\ 830 \end{array}$

Exercise:

Problem: $$\begin{array}{r} 1{,}893 \\ +\quad m \\ 1{,}981 \end{array}$$

Solution:

88

Exercise:

Problem:

The number of nursing and related care facilities in the United States in 1971 was 22,004. In 1978, the number was 18,722. What was the total number of facilities for both 1971 and 1978?

Exercise:

Problem:

The number of persons on food stamps in 1975, 1979, and 1980 was 19,179,000, 19,309,000, and 22,023,000, respectively. What was the total number of people on food stamps for the years 1975, 1979, and 1980?

Solution:

60,511,000

Exercise:

Problem:

The enrollment in public and nonpublic schools in the years 1965, 1970, 1975, and 1984 was 54,394,000, 59,899,000, 61,063,000, and 55,122,000, respectively. What was the total enrollment for those years?

Exercise:

Problem:

The area of New England is 3,618,770 square miles. The area of the Mountain states is 863,563 square miles. The area of the South Atlantic is 278,926 square miles. The area of the Pacific states is 921,392 square miles. What is the total area of these regions?

Solution:

5,682,651 square miles

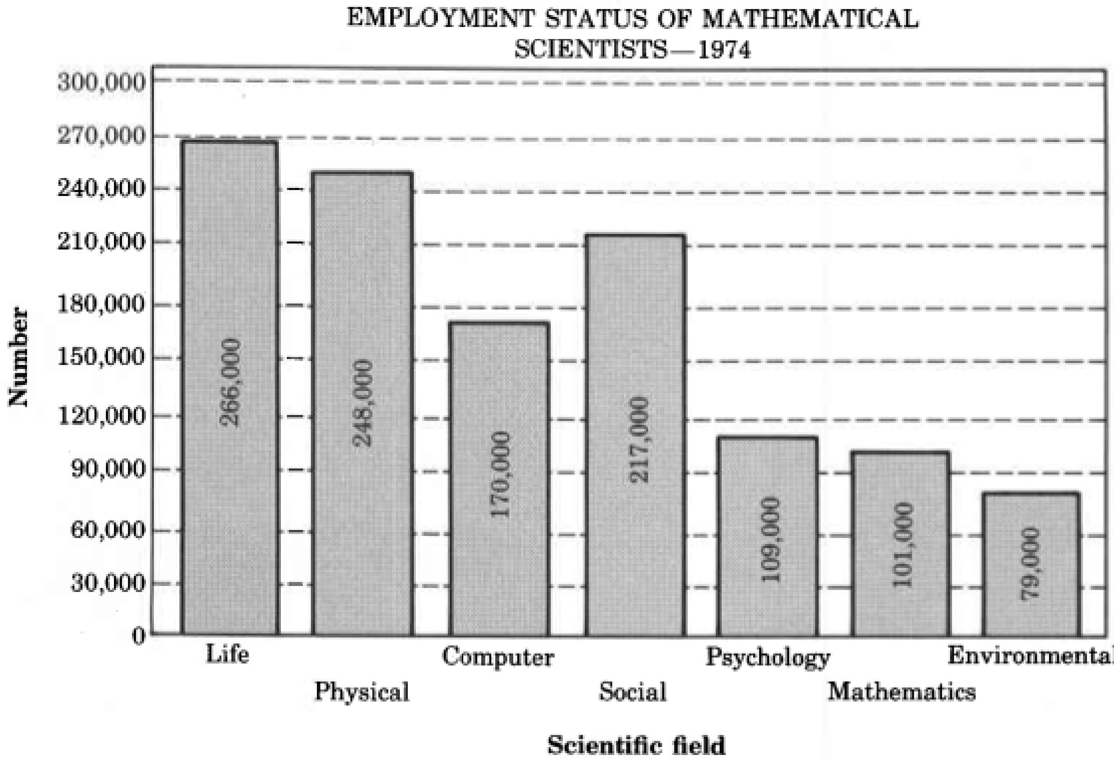
Exercise:

Problem:

In 1960, the IRS received 1,188,000 corporate income tax returns. In 1965, 1,490,000 returns were received. In 1970, 1,747,000 returns were received. In 1972 —1977, 1,890,000; 1,981,000; 2,043,000; 2,100,000; 2,159,000; and 2,329,000 returns were received, respectively. What was the total number of corporate tax returns received by the IRS during the years 1960, 1965, 1970, 1972 —1977?

Exercise:

Problem: Find the total number of scientists employed in 1974.



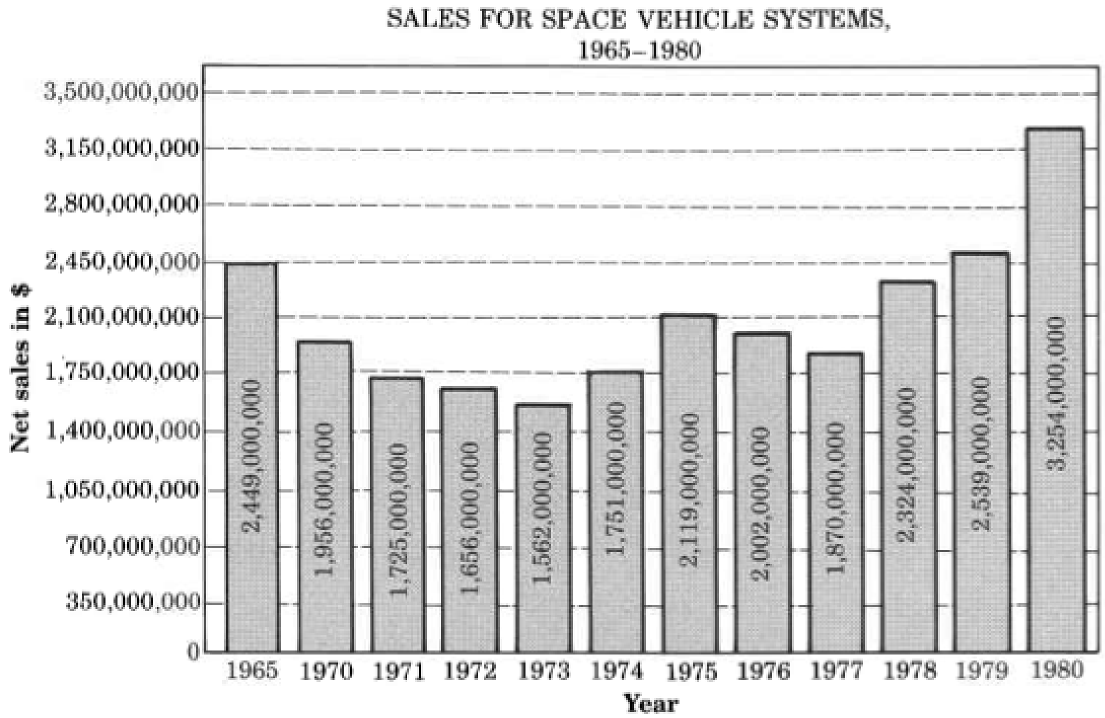
Solution:

1,190,000

Exercise:

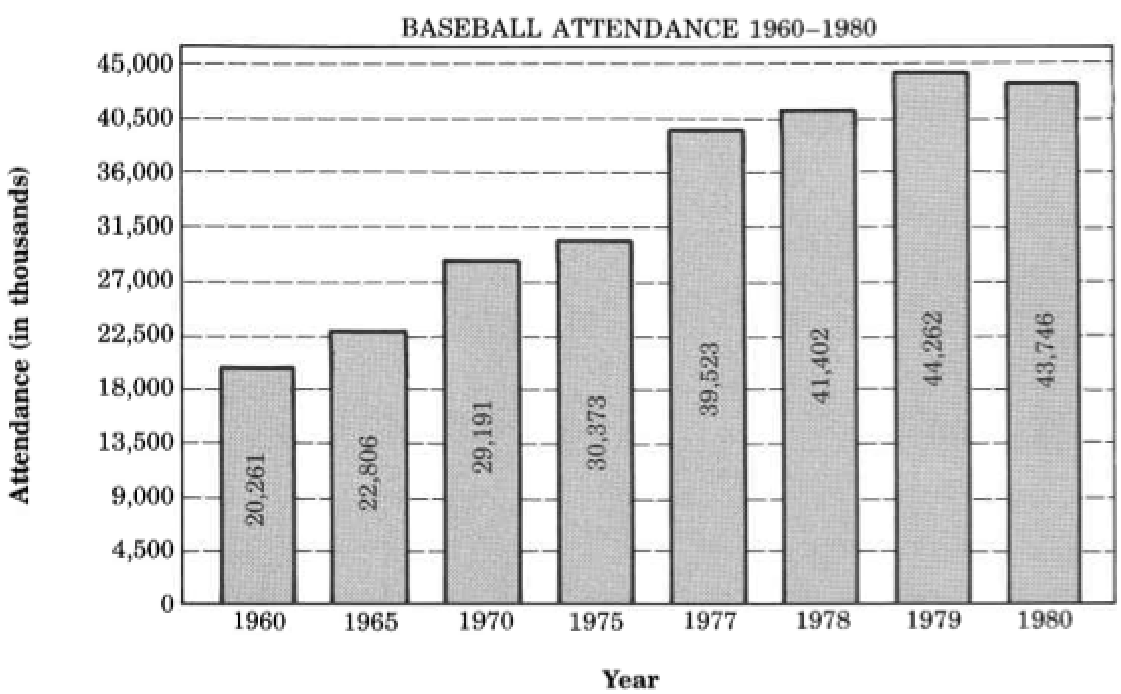
Problem:

Find the total number of sales for space vehicle systems for the years 1965-1980.



Exercise:

Problem: Find the total baseball attendance for the years 1960-1980.



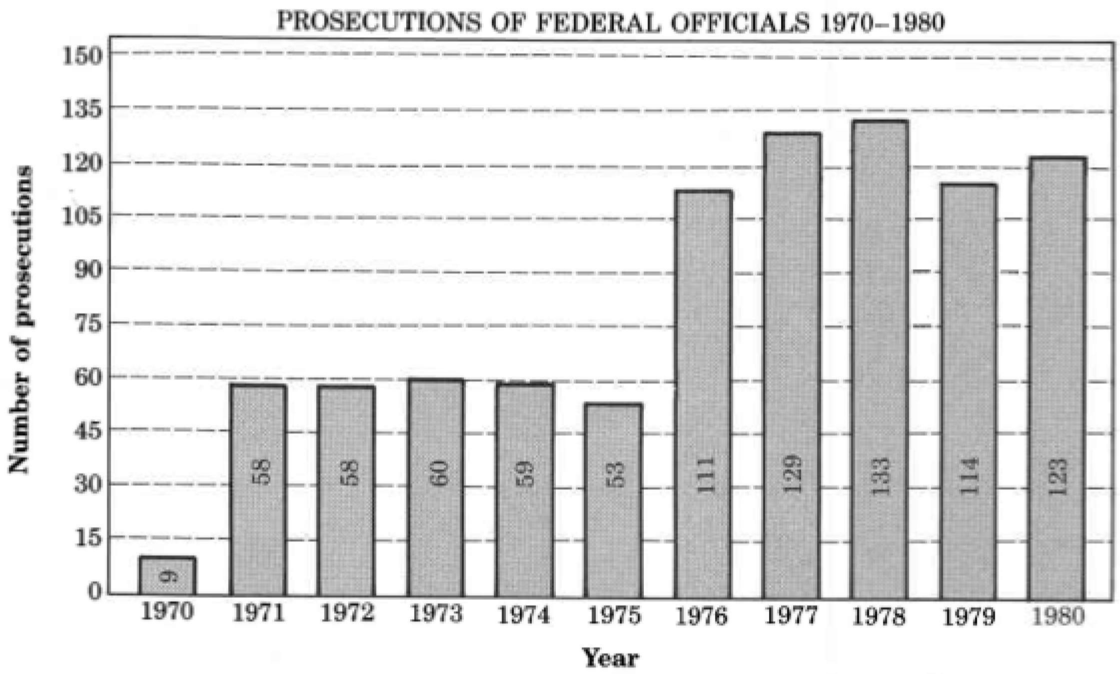
Solution:

271,564,000

Exercise:

Problem:

Find the number of prosecutions of federal officials for 1970-1980.



For the following problems, try to add the numbers mentally.

Exercise:

Problem:

$$\begin{array}{r} 5 \\ 5 \\ 3 \\ 7 \end{array}$$

Solution:

20

Exercise:

Problem: 8
2
6
4

Exercise:

Problem: 9
1
8
5
2

Solution:

25

Exercise:

Problem: 5
2
5
8
3
7

Exercise:

Problem: 6
4
3
1
6
7
9
4

Solution:

40

Exercise:

Problem: 20
30

Exercise:

Problem: 15
35

Solution:

50

Exercise:

Problem: 16
14

Exercise:

Problem: 23
27

Solution:

50

Exercise:

Problem: 82
18

Exercise:

Problem: 36
14

Solution:

50

Exercises for Review

Exercise:

Problem:

([link]) Each period of numbers has its own name. From right to left, what is the name of the fourth period?

Exercise:

Problem:

([link]) In the number 610,467, how many thousands are there?

Solution:

0

Exercise:

Problem: ([link]) Write 8,840 as you would read it.

Exercise:

Problem: ([link]) Round 6,842 to the nearest hundred.

Solution:

6,800

Exercise:

Problem: ([link]) Round 431,046 to the nearest million.

Subtraction of Whole Numbers
This module is from Fundamentals of Mathematics by Denny Burzynski and Wade Ellis, Jr. This module discusses how to subtract whole numbers. By the end of this module, students should be able to understand the subtraction process, subtract whole numbers, and use a calculator to subtract one whole number from another whole number.

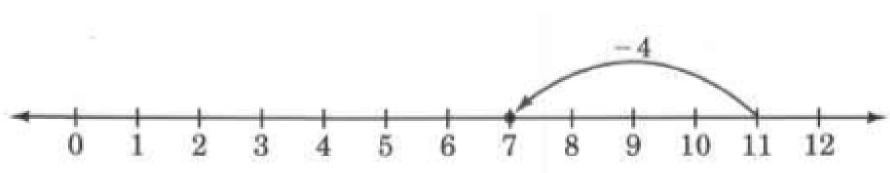
Section Overview

- Subtraction
- Subtraction as the Opposite of Addition
- The Subtraction Process
- Subtraction Involving Borrowing
- Borrowing From Zero
- Calculators

Subtraction

Subtraction
Subtraction is the process of determining the remainder when part of the total is removed.

Suppose the sum of two whole numbers is 11, and from 11 we remove 4. Using the number line to help our visualization, we see that if we are located at 11 and move 4 units to the left, and thus remove 4 units, we will be located at 7. Thus, 7 units remain when we remove 4 units from 11 units.



The Minus Symbol
The **minus symbol** (-) is used to indicate subtraction. For example, $11 - 4$ indicates that 4 is to be subtracted from 11.

Minuend

The number immediately in front of or the minus symbol is called the **minuend**, and it represents the *original* number of units.

Subtrahend
The number immediately following or below the minus symbol is called the **subtrahend**, and it represents the number of units *to be removed*.

Difference
The *result* of the subtraction is called the **difference** of the two numbers. For example, in $11 - 4 = 7$, 11 is the minuend, 4 is the subtrahend, and 7 is the difference.

Subtraction as the Opposite of Addition

Subtraction can be thought of as the opposite of addition. We show this in the problems in Sample Set A.

Sample Set A

Example:
$8 - 5 = 3$ since $3 + 5 = 8$.

Example:
$9 - 3 = 6$ since $6 + 3 = 9$.

Practice Set A

Complete the following statements.
Exercise:

Problem: $7 - 5 =$ since $+5 = 7$.

Solution:

$7 - 5 = 2$ since $2 + 5 = 7$

Exercise:

Problem: $9 - 1 =$ since $+1 = 9$.

Solution:

$9 - 1 = 8$ since $8 + 1 = 9$

Exercise:

Problem: $17 - 8 =$ since $+8 = 17$.

Solution:

$17 - 8 = 9$ since $9 + 8 = 17$

The Subtraction Process

We'll study the process of the subtraction of two whole numbers by considering the difference between 48 and 35.

$$\begin{array}{r} 48 \\ \underline{-35} \end{array} \quad \text{means} \quad \begin{array}{r} 4 \text{ tens} + 8 \text{ ones} \\ \underline{-3 \text{ tens} - 5 \text{ ones}} \\ 1 \text{ ten} \ \ + 3 \text{ ones} \end{array}$$

which we write as 13.

Example:

The Process of Subtracting Whole Numbers

To subtract two whole numbers,

The process

1. Write the numbers vertically, placing corresponding positions in the same column.

   $$\begin{array}{r} 48 \\ -35 \end{array}$$

2. Subtract the digits in each column. Start at the right, in the ones position, and move to the left, placing the difference at the bottom.

   $$\begin{array}{r} 48 \\ -35 \\ 13 \end{array}$$

Sample Set B

Perform the following subtractions.

Example:

$$\begin{array}{r} 275 \\ -142 \\ 133 \end{array}$$

$5 - 2 = 3.$

$7 - 4 = 3.$

$2 - 1 = 1.$

Example:

$$\begin{array}{r} 46{,}042 \\ -\ \ 1{,}031 \\ \hline 45{,}011 \end{array}$$

$2 - 1 = 1.$

$4 - 3 = 1.$

$0 - 0 = 0.$

$6 - 1 = 5.$

$4 - 0 = 4.$

Example:
Find the difference between 977 and 235.
Write the numbers vertically, placing the larger number on top. Line up the columns properly.

$$\begin{array}{r} 977 \\ -235 \\ \hline 742 \end{array}$$

The difference between 977 and 235 is 742.

Example:
In Keys County in 1987, there were 809 cable television installations. In Flags County in 1987, there were 1,159 cable television installations. How many more cable television installations were there in Flags County than in Keys County in 1987?
We need to determine the difference between 1,159 and 809.

$$\begin{array}{r} {\scriptstyle 1\ 1} \\ 1{,}159 \\ -\ \ 809 \\ \hline 350 \end{array}$$

There were 350 more cable television installations in Flags County than in Keys County in 1987.

Practice Set B

Perform the following subtractions.

Exercise:

Problem: $$\begin{array}{r} 534 \\ -203 \end{array}$$

Solution:

331

Exercise:

Problem: $$\begin{array}{r} 857 \\ -\ \ 43 \end{array}$$

Solution:

814

Exercise:

Problem: $$\begin{array}{r} 95{,}628 \\ -34{,}510 \end{array}$$

Solution:

61,118

Exercise:

Problem: $\begin{array}{r} 11{,}005 \\ -\ \ 1{,}005 \end{array}$

Solution:

10,000

Exercise:

Problem: Find the difference between 88,526 and 26,412.

Solution:

62,114

In each of these problems, each bottom digit is less than the corresponding top digit. This may not always be the case. We will examine the case where the bottom digit is greater than the corresponding top digit in the next section.

Subtraction Involving Borrowing

Minuend and Subtrahend
It often happens in the subtraction of two whole numbers that a digit in the **minuend** (top number) will be less than the digit in the same position in the **subtrahend** (bottom number). This happens when we subtract 27 from 84.

$$\begin{array}{r} 84 \\ -27 \end{array}$$

We do not have a name for $4 - 7$. We need to rename 84 in order to continue. We'll do so as follows:

$$\begin{array}{rcl} 84 &=& 8 \text{ tens} + 4 \text{ ones} \\ \underline{-27} &=& \underline{2 \text{ tens} + 7 \text{ ones}} \end{array}$$

$$\begin{array}{l} 7 \text{ tens} + 1 \text{ ten} + 4 \text{ ones} \\ \underline{2 \text{ tens} \qquad\qquad\quad + 7 \text{ ones}} \end{array}$$

$$\begin{array}{l} 7 \text{ tens} + 10 \text{ ones} + 4 \text{ ones} \\ \underline{2 \text{ tens} \qquad\qquad\qquad + 7 \text{ ones}} \end{array}$$

Our new name for 84 is 7 tens + 14 ones.

$$\begin{array}{l} 7 \text{ tens} + 14 \text{ ones} \\ \underline{2 \text{ tens} + 7 \text{ ones}} \\ 5 \text{ tens} + 7 \text{ ones} \end{array}$$

$= 57$

Notice that we converted 8 tens to 7 tens + 1 ten, and then we converted the 1 ten to 10 ones. We then had 14 ones and were able to perform the subtraction.

Borrowing
The process of **borrowing** (converting) is illustrated in the problems of Sample Set C.

Sample Set C

Example:

$$\begin{array}{r} {\scriptstyle 7\,14} \\ \cancel{8}4 \\ -27 \\ \hline 57 \end{array}$$

1. Borrow 1 ten from the 8 tens. This leaves 7 tens.
2. Convert the 1 ten to 10 ones.
3. Add 10 ones to 4 ones to get 14 ones.

Example:

$$\begin{array}{r} {\scriptstyle 5\,17} \\ \cancel{6}\cancel{7}2 \\ -\ \ 91 \\ \hline 581 \end{array}$$

1. Borrow 1 hundred from the 6 hundreds. This leaves 5 hundreds.
2. Convert the 1 hundred to 10 tens.
3. Add 10 tens to 7 tens to get 17 tens.

Practice Set C

Perform the following subtractions. Show the expanded form for the first three problems.

Exercise:

Problem: $\begin{array}{r} 53 \\ -35 \end{array}$

Solution:

$$\begin{array}{ll} 18, & 5 \text{ tens} + 3 \text{ ones} \\ - & \underline{3 \text{ tens} + 5 \text{ ones}} \\ & 4 \text{ tens} + 1 \text{ ten} + 3 \text{ ones} \\ - & \underline{3 \text{ tens} \qquad\quad + 5 \text{ ones}} \\ & 4 \text{ tens} + 13 \text{ ones} \\ - & \underline{3 \text{ tens} + \ \ 5 \text{ ones}} \\ & 1 \text{ ten} \ \ + \ \ 8 \text{ ones} \\ & = 18 \end{array}$$

Exercise:

Problem: $\begin{array}{r} 76 \\ -28 \end{array}$

Solution:

$$\begin{array}{ll} 48, & 7 \text{ tens} + 6 \text{ ones} \\ - & \underline{2 \text{ tens} + 8 \text{ ones}} \\ & 6 \text{ tens} + 1 \text{ ten} + 6 \text{ ones} \\ - & \underline{2 \text{ tens} \qquad\quad + 8 \text{ ones}} \\ & 6 \text{ tens} + 16 \text{ ones} \\ - & \underline{2 \text{ tens} + \ \ 8 \text{ ones}} \\ & 4 \text{ tens} + \ \ 8 \text{ ones} \\ & = 48 \end{array}$$

Exercise:

Problem: $\begin{array}{r} 872 \\ -565 \end{array}$

Solution:

$$
\begin{array}{ll}
307, & 8 \text{ hundreds} + 7 \text{ tens} + 2 \text{ ones} \\
- & \underline{5 \text{ hundreds} + 6 \text{ tens} + 5 \text{ ones}} \\
 & 8 \text{ hundreds} + 6 \text{ tens} + 1 \text{ ten} + 2 \text{ ones} \\
- & \underline{5 \text{ hundreds} + 6 \text{ tens} \qquad\qquad + 5 \text{ ones}} \\
 & 8 \text{ hundreds} + 6 \text{ tens} + 12 \text{ ones} \\
- & \underline{5 \text{ hundreds} + 6 \text{ tens} + \;\; 5 \text{ ones}} \\
 & 3 \text{ hundreds} + 0 \text{ tens} + \;\; 7 \text{ ones} \\
 & = 307
\end{array}
$$

Exercise:

Problem: $\begin{array}{r} 441 \\ -356 \end{array}$

Solution:

85

Exercise:

Problem: $\begin{array}{r} 775 \\ -\;\;66 \end{array}$

Solution:

709

Exercise:

Problem: $\begin{array}{r} 5,663 \\ -2,559 \end{array}$

Solution:

3,104

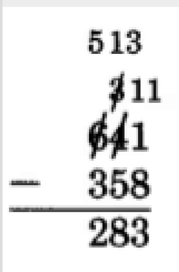
Borrowing More Than Once

Sometimes it is necessary to **borrow more than once**. This is shown in the problems in [link].

Sample Set D

Perform the Subtractions. Borrowing more than once if necessary

Example:



1. Borrow 1 ten from the 4 tens. This leaves 3 tens.
2. Convert the 1 ten to 10 ones.
3. Add 10 ones to 1 one to get 11 ones. We can now perform $11 - 8$.
4. Borrow 1 hundred from the 6 hundreds. This leaves 5 hundreds.
5. Convert the 1 hundred to 10 tens.
6. Add 10 tens to 3 tens to get 13 tens.
7. Now $13 - 5 = 8$.
8. $5 - 3 = 2$.

Example:

$$\begin{array}{r} 12 \\ 4\cancel{2}14 \\ \cancel{5}\cancel{3}4 \\ -\quad 85 \\ \hline 449 \end{array}$$

1. Borrow 1 ten from the 3 tens. This leaves 2 tens.
2. Convert the 1 ten to 10 ones.
3. Add 10 ones to 4 ones to get 14 ones. We can now perform $14 - 5$.
4. Borrow 1 hundred from the 5 hundreds. This leaves 4 hundreds.
5. Convert the 1 hundred to 10 tens.
6. Add 10 tens to 2 tens to get 12 tens. We can now perform $12 - 8 = 4$.
7. Finally, $4 - 0 = 4$.

Example:

71529

- 6952

After borrowing, we have

$$\begin{array}{r} 10 \\ 14 \\ 6\cancel{0}\cancel{4}12 \\ 71529 \\ -\ \ 6952 \\ \hline 64577 \end{array}$$

Practice Set D

Perform the following subtractions.

Exercise:

Problem: $\begin{array}{r} 526 \\ -358 \end{array}$

Solution:

168

Exercise:

Problem: $\begin{array}{r} 63,419 \\ -\ \ 7,779 \end{array}$

Solution:

55,640

Exercise:

Problem: $\begin{array}{r} 4,312 \\ -3,123 \end{array}$

Solution:

1,189

Borrowing from Zero

It often happens in a subtraction problem that we have to borrow from one or more zeros. This occurs in problems such as

1. $\begin{array}{r} 503 \\ -\ \ 37 \end{array}$

   and

2. $\begin{array}{r} 5000 \\ -\quad 37 \end{array}$

We'll examine each case.

Example:
Borrowing from a single zero.

Consider the problem $\begin{array}{r} 503 \\ -\ 37 \end{array}$

Since we do not have a name for $3 - 7$, we must borrow from 0.

$$\begin{array}{rr} 503 = & 5 \text{ hundreds} + 0 \text{ tens} + 3 \text{ ones} \\ -\ \ 37 & 3 \text{ tens} + 7 \text{ ones} \\ \hline \end{array}$$

Since there are no tens to borrow, we must borrow 1 hundred. One hundred = 10 tens.

$$\begin{array}{r} 4 \text{ hundreds} + 10 \text{ tens} + 3 \text{ ones} \\ 3 \text{ tens} + 7 \text{ ones} \\ \hline \end{array}$$

We can now borrow 1 ten from 10 tens (leaving 9 tens). One ten = 10 ones and 10 ones + 3 ones = 13 ones.

$$\begin{array}{r} 4 \text{ hundreds} + 9 \text{ tens} + 13 \text{ ones} \\ 3 \text{ tens} + \ \ 7 \text{ ones} \\ \hline 4 \text{ hundreds} + 6 \text{ tens} + \ \ 6 \text{ ones} = 466 \end{array}$$

Now we can suggest the following method for borrowing from a single zero.

Borrowing from a Single Zero
To borrow from a single zero,

1. Decrease the digit to the immediate left of zero by one.
2. Draw a line through the zero and make it a 10.
3. Proceed to subtract as usual.

Sample Set E

Example:

Perform this subtraction.

$$\begin{array}{r} 503 \\ -\ \ 37 \end{array}$$

The number 503 contains a single zero

1. The number to the immediate left of 0 is 5. Decrease 5 by 1.

   $5 - 1 = 4$

   $$\begin{array}{rccc} & 4 & 10 & \\ & \cancel{5} & \cancel{0} & 3 \\ - & & 3 & 7 \\ \hline \end{array}$$

2. Draw a line through the zero and make it a 10.

3. Borrow from the 10 and proceed.

   $$\begin{array}{rccc} & & 9 & \\ & 4 & \cancel{10} & 13 \\ & \cancel{5} & \cancel{0} & \cancel{3} \\ - & & 3 & 7 \\ \hline & 4 & 6 & 6 \end{array}$$

   $1 \text{ ten} + 10 \text{ ones}$

   $10 \text{ ones} + 3 \text{ ones} = 13 \text{ ones}$

Practice Set E

Perform each subtraction.

Exercise:

Problem: $$\begin{array}{r} 906 \\ -\ \ 18 \end{array}$$

Solution:

888

Exercise:

Problem: $$\begin{array}{r} 5102 \\ -\ \ 559 \end{array}$$

Solution:

4,543

Exercise:

Problem: $$\begin{array}{r} 9055 \\ -\ \ 386 \end{array}$$

Solution:

8,669

Example:

Borrowing from a group of zeros

Consider the problem $\begin{array}{r} 5000 \\ -\quad 37 \end{array}$

In this case, we have a group of zeros.

$$\begin{array}{rrr} & 5000 = & 5 \text{ thousands} + 0 \text{ hundred} + 0 \text{ tens} + 0 \text{ ones} \\ - & 37 = & 3 \text{ tens} + 7 \text{ ones} \\ \hline \end{array}$$

Since we cannot borrow any tens or hundreds, we must borrow 1 thousand. One thousand = 10 hundreds.

$$\begin{array}{r} 4 \text{ thousands} + 10 \text{ hundreds} + 0 \text{ tens} + 0 \text{ ones} \\ 3 \text{ tens} + 7 \text{ ones} \\ \hline \end{array}$$

We can now borrow 1 hundred from 10 hundreds. One hundred = 10 tens.

$$\begin{array}{r} 4 \text{ thousands} + 9 \text{ hundreds} + 10 \text{ tens} + 0 \text{ ones} \\ 3 \text{ tens} + 7 \text{ ones} \\ \hline \end{array}$$

We can now borrow 1 ten from 10 tens. One ten = 10 ones.

$$\begin{array}{r} 4 \text{ thousands} + 9 \text{ hundreds} + 9 \text{ tens} + 10 \text{ ones} \\ 3 \text{ tens} + \ \ 7 \text{ ones} \\ \hline 4 \text{ thousands} + 9 \text{ hundreds} + 6 \text{ tens} + \ \ 3 \text{ ones} = 4{,}963 \end{array}$$

From observations made in this procedure we can suggest the following method for borrowing from a group of zeros.

Borrowing from a Group of zeros

To borrow from a group of zeros,

1. Decrease the digit to the immediate left of the group of zeros by one.
2. Draw a line through each zero in the group and make it a 9, except the rightmost zero, make it 10.
3. Proceed to subtract as usual.

Sample Set F

Perform each subtraction.

Example:

$$\begin{array}{r} 40{,}000 \\ -\quad 125 \end{array}$$

The number 40,000 contains a group of zeros.

1. The number to the immediate left of the group is 4. Decrease 4 by 1.

   $4 - 1 = 3$

2. Make each 0, except the rightmost one, 9. Make the rightmost 0 a 10.

   $$\begin{array}{r} 3\;9\;\;9\;9\;10 \\ \cancel{4}\cancel{0},\cancel{0}\cancel{0}\cancel{0} \\ -\qquad 125 \\ \hline \end{array}$$

3. Subtract as usual.

   $$\begin{array}{r} 3\;9\;\;9\;9\;10 \\ \cancel{4}\cancel{0},\cancel{0}\cancel{0}\cancel{0} \\ -\qquad 125 \\ \hline 39{,}875 \end{array}$$

Example:

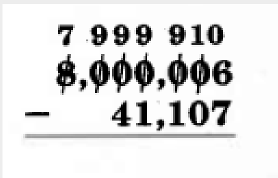
$$\begin{array}{r} 8{,}000{,}006 \\ -\quad 41{,}107 \end{array}$$

The number 8,000,006 contains a group of zeros.

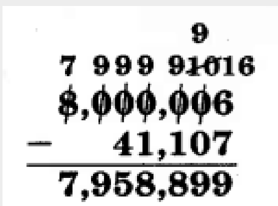
1. The number to the immediate left of the group is 8. Decrease 8 by 1.

$8 - 1 = 7$

2. Make each zero, except the rightmost one, 9. Make the rightmost 0 a 10.



3. To perform the subtraction, we’ll need to borrow from the ten.



$1 \text{ ten} = 10 \text{ ones}$

$10 \text{ ones} + 6 \text{ ones} = 16 \text{ ones}$

Practice Set F

Perform each subtraction.

Exercise:

Problem: $\begin{array}{r} 21,007 \\ - \;\; 4,873 \\ \hline \end{array}$

Solution:

16,134

Exercise:

Problem: $$\begin{array}{r} 10{,}004 \\ -\ \ 5{,}165 \\ \hline \end{array}$$

Solution:

4,839

Exercise:

Problem: $$\begin{array}{r} 16{,}000{,}000 \\ -\ \ \ \ 201{,}060 \\ \hline \end{array}$$

Solution:

15,789,940

Calculators

In practice, calculators are used to find the difference between two whole numbers.

Sample Set G

Find the difference between 1006 and 284.

Display Reads		
Type	1006	1006

Press	–	1006
Type	284	284
Press	=	722

The difference between 1006 and 284 is 722.

(What happens if you type 284 first and then 1006? We'll study such numbers in [link]Chapter 10.)

Practice Set G

Exercise:

Problem:

Use a calculator to find the difference between 7338 and 2809.

Solution:

4,529

Exercise:

Problem:

Use a calculator to find the difference between 31,060,001 and 8,591,774.

Solution:

22,468,227

Exercises

For the following problems, perform the subtractions. You may check each difference with a calculator.

Exercise:

Problem: $\begin{array}{r} 15 \\ -\ \ 8 \end{array}$

Solution:

7

Exercise:

Problem: $\begin{array}{r} 19 \\ -\ \ 8 \end{array}$

Exercise:

Problem: $\begin{array}{r} 11 \\ -\ \ 5 \end{array}$

Solution:

6

Exercise:

Problem: $\begin{array}{r} 14 \\ -\ \ 6 \end{array}$

Exercise:

Problem: $\begin{array}{r} 12 \\ -\ \ 9 \end{array}$

Solution:

3

Exercise:

Problem: $\begin{array}{r} 56 \\ -12 \end{array}$

Exercise:

Problem: $\begin{array}{r} 74 \\ -33 \end{array}$

Solution:

41

Exercise:

Problem: $\begin{array}{r} 80 \\ -61 \end{array}$

Exercise:

Problem: $\begin{array}{r} 350 \\ -141 \end{array}$

Solution:

209

Exercise:

Problem: $\begin{array}{r} 800 \\ -650 \end{array}$

Exercise:

Problem: $$\begin{array}{r} 35,002 \\ -14,001 \end{array}$$

Solution:

21,001

Exercise:

Problem: $$\begin{array}{r} 5,000,566 \\ -2,441,326 \end{array}$$

Exercise:

Problem: $$\begin{array}{r} 400,605 \\ -121,352 \end{array}$$

Solution:

279,253

Exercise:

Problem: $$\begin{array}{r} 46,400 \\ -\quad 2,012 \end{array}$$

Exercise:

Problem: $$\begin{array}{r} 77,893 \\ -\qquad 421 \end{array}$$

Solution:

77,472

Exercise:

Problem: $\begin{array}{r} 42 \\ -18 \end{array}$

Exercise:

Problem: $\begin{array}{r} 51 \\ -27 \end{array}$

Solution:

24

Exercise:

Problem: $\begin{array}{r} 622 \\ -\ \ 88 \end{array}$

Exercise:

Problem: $\begin{array}{r} 261 \\ -\ \ 73 \end{array}$

Solution:

188

Exercise:

Problem: $\begin{array}{r} 242 \\ -158 \end{array}$

Exercise:

Problem: $\begin{array}{r} 3,422 \\ -1,045 \end{array}$

Solution:

2,377

Exercise:

Problem: $\begin{array}{r} 5,565 \\ -3,985 \end{array}$

Exercise:

Problem: $\begin{array}{r} 42,041 \\ -15,355 \end{array}$

Solution:

26,686

Exercise:

Problem: $\begin{array}{r} 304,056 \\ -\quad 20,008 \end{array}$

Exercise:

Problem: $\begin{array}{r} 64,000,002 \\ -\quad 856,743 \end{array}$

Solution:

63,143,259

Exercise:

Problem: $\begin{array}{r} 4,109 \\ -\quad 856 \end{array}$

Exercise:

Problem: $\begin{array}{r} 10{,}113 \\ -\ \ 2{,}079 \end{array}$

Solution:

8,034

Exercise:

Problem: $\begin{array}{r} 605 \\ -\ \ 77 \end{array}$

Exercise:

Problem: $\begin{array}{r} 59 \\ -26 \end{array}$

Solution:

33

Exercise:

Problem: $\begin{array}{r} 36{,}107 \\ -\ \ 8{,}314 \end{array}$

Exercise:

Problem: $\begin{array}{r} 92{,}526{,}441{,}820 \\ -59{,}914{,}805{,}253 \end{array}$

Solution:

32,611,636,567

Exercise:

Problem: $\begin{array}{r} 1,605 \\ -\ \ 881 \end{array}$

Exercise:

Problem: $\begin{array}{r} 30,000 \\ -26,062 \end{array}$

Solution:

3,938

Exercise:

Problem: $\begin{array}{r} 600 \\ -216 \end{array}$

Exercise:

Problem: $\begin{array}{r} 9,000,003 \\ -\ \ 726,048 \end{array}$

Solution:

8,273,955

For the following problems, perform each subtraction.

Exercise:

Problem: Subtract 63 from 92.

Note: The word "from" means "beginning at." Thus, 63 from 92 means beginning at 92, or $92 - 63$.

Exercise:

Problem: Subtract 35 from 86.

Solution:

51

Exercise:

Problem: Subtract 382 from 541.

Exercise:

Problem: Subtract 1,841 from 5,246.

Solution:

3,405

Exercise:

Problem: Subtract 26,082 from 35,040.

Exercise:

Problem: Find the difference between 47 and 21.

Solution:

26

Exercise:

Problem: Find the difference between 1,005 and 314.

Exercise:

Problem: Find the difference between 72,085 and 16.

Solution:

72,069

Exercise:

Problem: Find the difference between 7,214 and 2,049.

Exercise:

Problem: Find the difference between 56,108 and 52,911.

Solution:

3,197

Exercise:

Problem: How much bigger is 92 than 47?

Exercise:

Problem: How much bigger is 114 than 85?

Solution:

29

Exercise:

Problem: How much bigger is 3,006 than 1,918?

Exercise:

Problem: How much bigger is 11,201 than 816?

Solution:

10,385

Exercise:

Problem: How much bigger is 3,080,020 than 1,814,161?

Exercise:

Problem:

In Wichita, Kansas, the sun shines about 74% of the time in July and about 59% of the time in November. How much more of the time (in percent) does the sun shine in July than in November?

Solution:

15%

Exercise:

Problem:

The lowest temperature on record in Concord, New Hampshire in May is 21°F, and in July it is 35°F. What is the difference in these lowest temperatures?

Exercise:

Problem:

In 1980, there were 83,000 people arrested for prostitution and commercialized vice and 11,330,000 people arrested for driving while intoxicated. How many more people were arrested for drunk driving than for prostitution?

Solution:

11,247,000

Exercise:

Problem:

In 1980, a person with a bachelor's degree in accounting received a monthly salary offer of $1,293, and a person with a marketing degree a monthly salary offer of $1,145. How much more was offered to the person with an accounting degree than the person with a marketing degree?

Exercise:

Problem:

In 1970, there were about 793 people per square mile living in Puerto Rico, and 357 people per square mile living in Guam. How many more people per square mile were there in Puerto Rico than Guam?

Solution:

436

Exercise:

Problem:

The 1980 population of Singapore was 2,414,000 and the 1980 population of Sri Lanka was 14,850,000. How many more people lived in Sri Lanka than in Singapore in 1980?

Exercise:

Problem:

In 1977, there were 7,234,000 hospitals in the United States and 64,421,000 in Mainland China. How many more hospitals were there in Mainland China than in the United States in 1977?

Solution:

57,187,000

Exercise:

Problem:

In 1978, there were 3,095,000 telephones in use in Poland and 4,292,000 in Switzerland. How many more telephones were in use in Switzerland than in Poland in 1978?

For the following problems, use the corresponding graphs to solve the problems.

Exercise:

Problem:

How many more life scientists were there in 1974 than mathematicians? ([link])

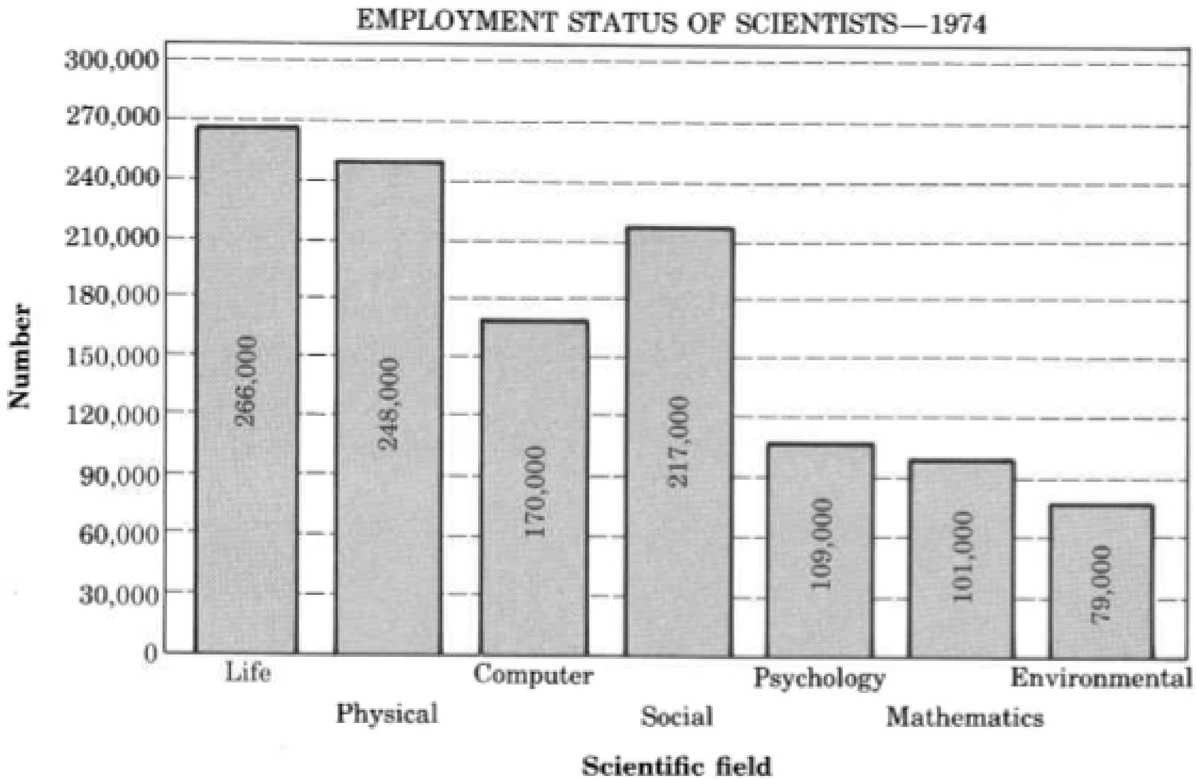
Solution:

165,000

Exercise:

Problem:

How many more social, psychological, mathematical, and environmental scientists were there than life, physical, and computer scientists? ([link])



Exercise:

Problem:

How many more prosecutions were there in 1978 than in 1974? ([link])

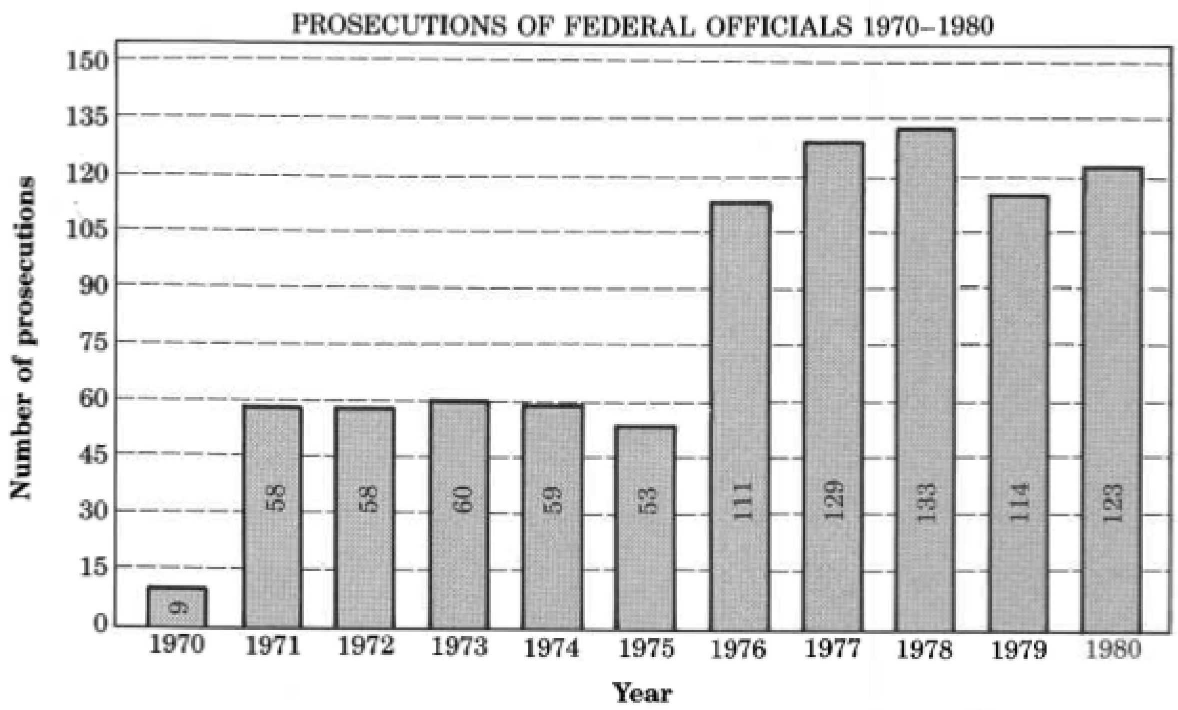
Solution:

74

Exercise:

Problem:

How many more prosecutions were there in 1976-1980 than in 1970-1975? ([link])



Exercise:

Problem:

How many more dry holes were drilled in 1960 than in 1975? ([link])

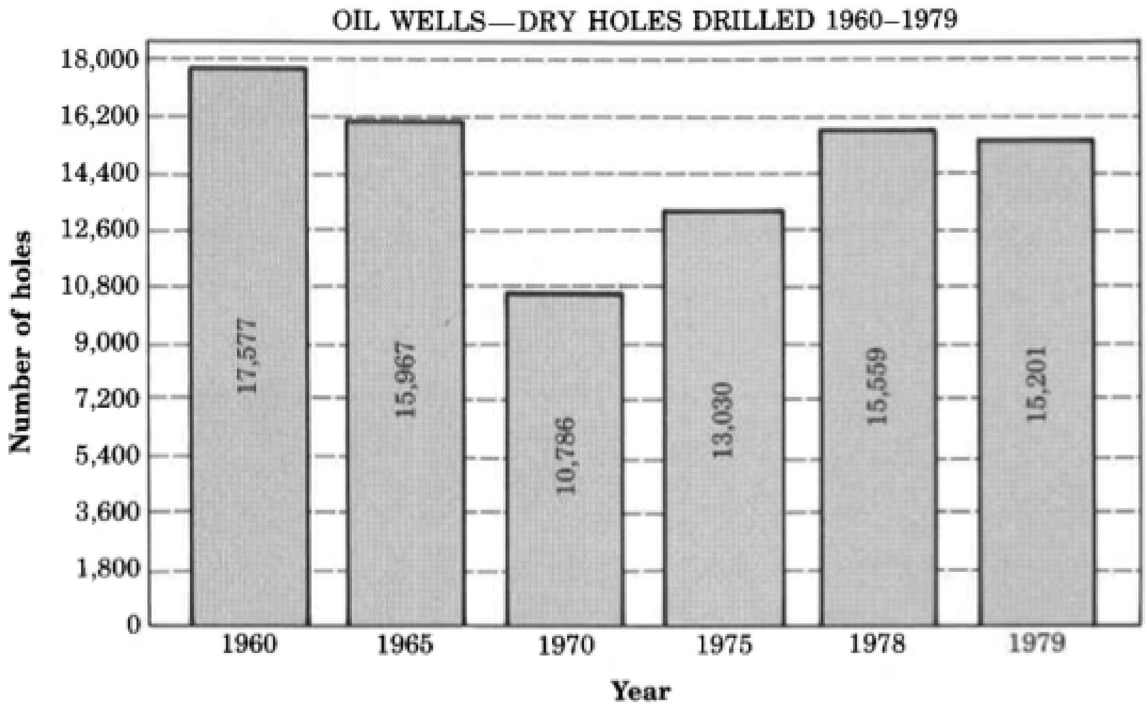
Solution:

4,547

Exercise:

Problem:

How many more dry holes were drilled in 1960, 1965, and 1970 than in 1975, 1978 and 1979? ([link])



For the following problems, replace the ☐ with the whole number that will make the subtraction true.

Exercise:

Problem: $\begin{array}{r} 14 \\ -\square \\ 3 \end{array}$

Solution:

11

Exercise:

Problem: $\begin{array}{r} 21 \\ -\square \\ 14 \end{array}$

Exercise:

Problem: $\begin{array}{r} 35 \\ -\square \\ \hline 25 \end{array}$

Solution:

10

Exercise:

Problem: $\begin{array}{r} 16 \\ -\square \\ \hline 9 \end{array}$

Exercise:

Problem: $\begin{array}{r} 28 \\ -\square \\ \hline 16 \end{array}$

Solution:

12

For the following problems, find the solutions.

Exercise:

Problem: Subtract 42 from the sum of 16 and 56.

Exercise:

Problem: Subtract 105 from the sum of 92 and 89.

Solution:

76

Exercise:

Problem: Subtract 1,127 from the sum of 2,161 and 387.

Exercise:

Problem: Subtract 37 from the difference between 263 and 175.

Solution:

51

Exercise:

Problem: Subtract 1,109 from the difference between 3,046 and 920.

Exercise:

Problem:

Add the difference between 63 and 47 to the difference between 55 and 11.

Solution:

60

Exercise:

Problem:

Add the difference between 815 and 298 to the difference between 2,204 and 1,016.

Exercise:

Problem:

Subtract the difference between 78 and 43 from the sum of 111 and 89.

Solution:

165

Exercise:

Problem:

Subtract the difference between 18 and 7 from the sum of the differences between 42 and 13, and 81 and 16.

Exercise:

Problem:

Find the difference between the differences of 343 and 96, and 521 and 488.

Solution:

214

Exercises for Review

Exercise:

Problem:

([link]) In the number 21,206, how many hundreds are there?

Exercise:

Problem:

([link]) Write a three-digit number that has a zero in the ones position.

Solution:

330 (answers may vary)

Exercise:

Problem: ([link]) How many three-digit whole numbers are there?

Exercise:

Problem: ([link]) Round 26,524,016 to the nearest million.

Solution:

27,000,000

Exercise:

Problem: ([link]) Find the sum of $846 + 221 + 116$.

Properties of Addition
This module is from Fundamentals of Mathematics by Denny Burzynski and Wade Ellis, Jr. This module discusses properties of addition. By the end of the module students should be able to understand the commutative and associative properties of addition and understand why 0 is the additive identity.

Section Overview

- The Commutative Property of Addition
- The Associative Property of Addition
- The Additive Identity

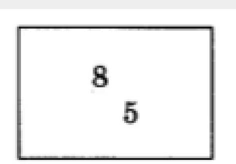
We now consider three simple but very important properties of addition.

The Commutative Property of Addition

Commutative Property of Addition
If two whole numbers are added in any order, the sum will not change.

Sample Set A

Example:
Add the whole numbers



$8 + 5 = 13$
$5 + 8 = 13$

The numbers 8 and 5 can be added in any order. Regardless of the order they are added, the sum is 13.

Practice Set A

Exercise:

Problem:

Use the commutative property of addition to find the sum of 12 and 41 in two different ways.

12
41

Solution:

$12 + 41 = 53$ and $41 + 12 = 53$

Exercise:

Problem: Add the whole numbers

837
1,958

Solution:

$837 + 1{,}958 = 2{,}795$ and $1{,}958 + 837 = 2{,}795$

The Associative Property of Addition

Associative Property of Addition
If three whole numbers are to be added, the sum will be the same if the first two are added first, then that sum is added to the third, or, the second two are added first, and that sum is added to the first.

Using Parentheses
It is a common mathematical practice to **use parentheses** to show which pair of numbers we wish to combine first.

Sample Set B

Example:
Add the whole numbers.

43
16
27

43 and 16 are associated.
$$(43 + 16) + 27 = 59 + 27 = 86.$$
$$43 + (16 + 27) = 43 + 43 = 86.$$
16 and 27 are associated.

Practice Set B

Exercise:

Problem:

Use the associative property of addition to add the following whole numbers two different ways.

17
32
25

Solution:

$(17+32)+25=49+25=74$ and
$17+(32+25)=17+57=74$

Exercise:

Problem:

1,629
806
429

Solution:

$(1{,}629+806)+429=2{,}435+429=2{,}864$

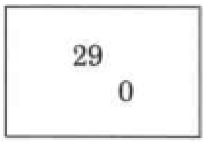
$1{,}629+(806+429)=1{,}629+1{,}235=2{,}864$

The Additive Identity

0 Is the Additive Identity
The whole number 0 is called the **additive identity**, since when it is added to any whole number, the sum is identical to that whole number.

Sample Set C

Example:
Add the whole numbers.



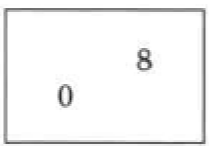
$29 + 0 = 29$
$0 + 29 = 29$
Zero added to 29 does not change the identity of 29.

Practice Set C

Add the following whole numbers.

Exercise:

Problem:

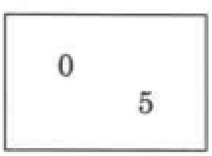


Solution:

8

Exercise:

Problem:



Solution:

5

Suppose we let the letter x represent a choice for some whole number. For the first two problems, find the sums. For the third problem, find the sum provided we now know that x represents the whole number 17.

Exercise:

Problem:

$$\begin{array}{r} x \\ 0 \end{array}$$

Solution:

x

Exercise:

Problem:

$$\begin{array}{r} 0 \\ x \end{array}$$

Solution:

x

Exercise:

Problem:

$$\begin{array}{r} 0 \\ x \end{array}$$

Solution:

17

Exercises

For the following problems, add the numbers in two ways.

Exercise:

Problem:

8
29

Solution:

37

Exercise:

Problem:

36
12

Exercise:

Problem:

36
48

Solution:

45

Exercise:

Problem:

26
117

Exercise:

Problem:

456
112

Solution:

568

Exercise:

Problem:

1,096
4,251

Exercise:

Problem:

73,205
49,118

Solution:

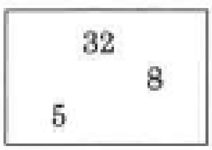
122,323

Exercise:

Problem:

Exercise:

Problem:

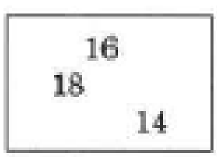


Solution:

45

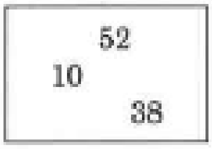
Exercise:

Problem:



Exercise:

Problem:

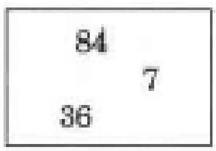


Solution:

100

Exercise:

Problem:



Exercise:

Problem:

Solution:

556

Exercise:

Problem:

Exercise:

Problem:

37,728
1,261
4,472

Solution:

43,461

For the following problems, show that the pairs of quantities yield the same sum.

Exercise:

Problem: $(11+27)+9$ and $11+(27+9)$

Exercise:

Problem: $(80+52)+6$ and $80+(52+6)$

Solution:

$132+6=80+58=138$

Exercise:

Problem: $(114+226)+108$ and $114+(226+108)$

Exercise:

Problem: $(731+256)+171$ and $731+(256+171)$

Solution:

$987+171=731+427=1{,}158$

Exercise:

Problem:

The fact that (a first number + a second number) + third number = a first number + (a second number + a third number) is an example of the property of addition.

Exercise:

Problem:

The fact that 0 + any number = that particular number is an example of the property of addition.

Solution:

Identity

Exercise:

Problem:

The fact that a first number + a second number = a second number + a first number is an example of the property of addition.

Exercise:

Problem:

Use the numbers 15 and 8 to illustrate the commutative property of addition.

Solution:

$15 + 8 = 8 + 15 = 23$

Exercise:

Problem:

Use the numbers 6, 5, and 11 to illustrate the associative property of addition.

Exercise:

Problem:

The number zero is called the additive identity. Why is the term identity so appropriate?

Solution:

…because its partner in addition remains identically the same after that addition

Exercises for Review

Exercise:

Problem: ([link]) How many hundreds in 46,581?

Exercise:

Problem: ([link]) Write 2,218 as you would read it.

Solution:

Two thousand, two hundred eighteen.

Exercise:

Problem: ([link]) Round 506,207 to the nearest thousand.

Exercise:

Problem: ([link]) Find the sum of $\begin{array}{r} 482 \\ +\ \ 68 \end{array}$

Solution:

550

Exercise:

Problem: ([link]) Find the difference: $\begin{array}{r} 3{,}318 \\ -\ \ 429 \end{array}$

Summary of Key Concepts
This module is from Fundamentals of Mathematics by Denny Burzynski and Wade Ellis, Jr. This module reviews the key concepts from the chapter Addition and Subtraction of Whole Numbers.

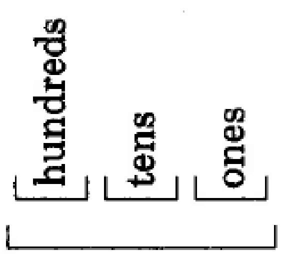
Summary of Key Concepts

Number / Numeral ([link])
A **number** is a concept. It exists only in the mind. A **numeral** is a symbol that represents a number. It is customary not to distinguish between the two (but we should remain aware of the difference).

Hindu-Arabic Numeration System ([link])
In our society, we use the **Hindu-Arabic** numeration system. It was invented by the Hindus shortly before the third century and popularized by the Arabs about a thousand years later.

Digits ([link])
The numbers 0, 1, 2, 3, 4, 5, 6, 7, 8, 9 are called **digits**.

Base Ten Positional System ([link])
The Hindu-Arabic numeration system is a positional number system with **base ten**. Each position has value that is ten times the value of the position to its right.

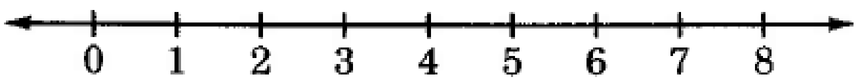


Commas / Periods ([link])
Commas are used to separate digits into groups of three. Each group of three is called a **period**. Each period has a name. From right to left, they are ones, thousands, millions, billions, etc.

Whole Numbers ([link])
A **whole number** is any number that is formed using only the digits (0, 1, 2, 3, 4, 5, 6, 7, 8, 9).

Number Line ([link])
The **number line** allows us to visually display the whole numbers.

Graphing ([link])
Graphing a whole number is a term used for visually displaying the whole number. The graph of 4 appears below.



Reading Whole Numbers ([link])
To express a whole number as a verbal phrase:

1. Begin at the right and, working right to left, separate the number into distinct periods by inserting commas every three digits.
2. Begin at the left, and read each period individually.

Writing Whole Numbers ([link])
To rename a number that is expressed in words to a number expressed in digits:

1. Notice that a number expressed as a verbal phrase will have its periods set off by commas.
2. Start at the beginning of the sentence, and write each period of numbers individually.
3. Use commas to separate periods, and combine the periods to form one number.

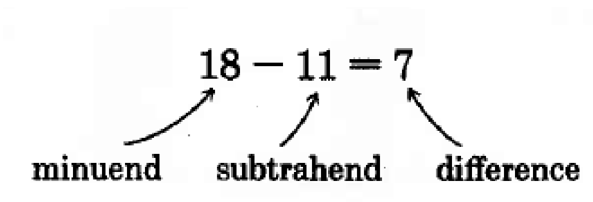
Rounding ([link])
Rounding is the process of approximating the number of a group of objects by mentally "seeing" the collection as occurring in groups of tens, hundreds, thousands, etc.

Addition ([link])
Addition is the process of combining two or more objects (real or intuitive) to form a new, third object, the total, or sum.

Addends / Sum ([link])
In addition, the numbers being added are called **addends** and the result, or total, the **sum**.

Subtraction ([link])
Subtraction is the process of determining the remainder when part of the total is removed.

Minuend / Subtrahend Difference ([link])



Commutative Property of Addition ([link])
If two whole numbers are added in either of two orders, the sum will not change.
$3+5=5+3$

Associative Property of Addition ([link])
If three whole numbers are to be added, the sum will be the same if the first two are added and that sum is then added to the third, or if the second two are added and the first is added to that sum.
$(3+5)+2=3+(5+2)$

Parentheses in Addition ([link])
Parentheses in addition indicate which numbers are to be added first.

Additive Identity ([link])
The whole number 0 is called the **additive identity** since, when it is added to any particular whole number, the sum is identical to that whole number.
$0+7=7$
$7+0=7$

Exercise Supplement
This module is from Fundamentals of Mathematics by Denny Burzynski and Wade Ellis, Jr. This module is an exercise supplement for the chapter Addition and Subtraction of Whole Numbers and contains many exercise problems. Odd problems are accompanied by solutions.

Exercise Supplement

For problems 1-35, find the sums and differences.

Exercise:

Problem: $\begin{array}{r} 908 \\ +\ \ 29 \end{array}$

Solution:

937

Exercise:

Problem: $\begin{array}{r} 529 \\ +161 \end{array}$

Exercise:

Problem: $\begin{array}{r} 549 \\ +\ \ 16 \end{array}$

Solution:

565

Exercise:

Problem: $\begin{array}{r} 726 \\ +892 \end{array}$

Exercise:

Problem: $\begin{array}{r} 390 \\ +169 \end{array}$

Solution:

559

Exercise:

Problem: $\begin{array}{r} 166 \\ +660 \end{array}$

Exercise:

Problem: $\begin{array}{r} 391 \\ +951 \end{array}$

Solution:

1,342

Exercise:

Problem: $\begin{array}{r} 48 \\ +36 \end{array}$

Exercise:

Problem: $\begin{array}{r} 1{,}103 \\ +\ \ 898 \end{array}$

Solution:

2,001

Exercise:

Problem: $$\begin{array}{r} 1{,}642 \\ +\ \ 899 \end{array}$$

Exercise:

Problem: $$\begin{array}{r} 807 \\ +1{,}156 \end{array}$$

Solution:

1,963

Exercise:

Problem: $$\begin{array}{r} 80{,}349 \\ +\ \ 2{,}679 \end{array}$$

Exercise:

Problem: $$\begin{array}{r} 70{,}070 \\ +\ \ 9{,}386 \end{array}$$

Solution:

79,456

Exercise:

Problem: $$\begin{array}{r} 90{,}874 \\ +\ \ 2{,}945 \end{array}$$

Exercise:

Problem: $\begin{array}{r} 45,292 \\ +51,661 \end{array}$

Solution:

96,953

Exercise:

Problem: $\begin{array}{r} 1,617 \\ +54,923 \end{array}$

Exercise:

Problem: $\begin{array}{r} 702,607 \\ +\ \ 89,217 \end{array}$

Solution:

791,824

Exercise:

Problem: $\begin{array}{r} 6,670,006 \\ +\qquad 2,495 \end{array}$

Exercise:

Problem: $\begin{array}{r} 267 \\ +8,034 \end{array}$

Solution:

8,301

Exercise:

Problem: $\begin{array}{r} 7{,}007 \\ +11{,}938 \end{array}$

Exercise:

Problem: $\begin{array}{r} 131{,}294 \\ +\quad 9{,}087 \end{array}$

Solution:

140,381

Exercise:

Problem: $\begin{array}{r} 5{,}292 \\ +\quad 161 \end{array}$

Exercise:

Problem: $\begin{array}{r} 17{,}260 \\ +58{,}964 \end{array}$

Solution:

76,224

Exercise:

Problem: $\begin{array}{r} 7{,}006 \\ -5{,}382 \end{array}$

Exercise:

Problem: $\begin{array}{r} 7{,}973 \\ -3{,}018 \end{array}$

Solution:

4,955

Exercise:

Problem: $\begin{array}{r} 16,608 \\ -\;\;1,660 \end{array}$

Exercise:

Problem: $\begin{array}{r} 209,527 \\ -\;\;23,916 \end{array}$

Solution:

185,611

Exercise:

Problem: $\begin{array}{r} 584 \\ -226 \end{array}$

Exercise:

Problem: $\begin{array}{r} 3,313 \\ -1,075 \end{array}$

Solution:

2,238

Exercise:

Problem: $\begin{array}{r} 458 \\ -122 \end{array}$

Exercise:

Problem: $\begin{array}{r} 1{,}007 \\ +\ \ 331 \end{array}$

Solution:

1,338

Exercise:

Problem: $\begin{array}{r} 16{,}082 \\ +\ \ 2{,}013 \end{array}$

Exercise:

Problem: $\begin{array}{r} 926 \\ -\ \ 48 \end{array}$

Solution:

878

Exercise:

Problem: $\begin{array}{r} 736 \\ +5{,}869 \end{array}$

Exercise:

Problem: $\begin{array}{r} 676{,}504 \\ -\ \ 58{,}277 \end{array}$

Solution:

618,227

For problems 36-39, add the numbers.

Exercise:

Problem:

769
795
298
746

Exercise:

Problem:

554
184
883

Solution:

1,621

Exercise:

Problem:

30,188
79,731
16,600
66,085
39,169
95,170

Exercise:

Problem:

$$\begin{array}{r} 2{,}129 \\ 6{,}190 \\ 17{,}044 \\ 30{,}447 \\ 292 \\ 41 \\ 428{,}458 \end{array}$$

Solution:

484,601

For problems 40-50, combine the numbers as indicated.

Exercise:

Problem: $2{,}957 + 9{,}006$

Exercise:

Problem: $19{,}040 + 813$

Solution:

19,853

Exercise:

Problem: $350{,}212 + 14{,}533$

Exercise:

Problem: $970 + 702 + 22 + 8$

Solution:

1,702

Exercise:

Problem: $3{,}704 + 2{,}344 + 429 + 10{,}374 + 74$

Exercise:

Problem: $874 + 845 + 295 - 900$

Solution:

1,114

Exercise:

Problem: $904 + 910 - 881$

Exercise:

Problem: $521 + 453 - 334 + 600$

Solution:

1,300

Exercise:

Problem: $892 - 820 - 9$

Exercise:

Problem: $159 + 4{,}085 - 918 - 608$

Solution:

2,718

Exercise:

Problem: $2{,}562 + 8{,}754 - 393 - 385 - 910$

For problems 51-63, add and subtract as indicated.

Exercise:

Problem: Subtract 671 from 8,027.

Solution:

7,356

Exercise:

Problem: Subtract 387 from 6,342.

Exercise:

Problem: Subtract 2,926 from 6,341.

Solution:

3,415

Exercise:

Problem: Subtract 4,355 from the sum of 74 and 7,319.

Exercise:

Problem: Subtract 325 from the sum of 7,188 and 4,964.

Solution:

11,827

Exercise:

Problem: Subtract 496 from the difference of 60,321 and 99.

Exercise:

Problem: Subtract 20,663 from the difference of 523,150 and 95,225.

Solution:

407,262

Exercise:

Problem:

Add the difference of 843 and 139 to the difference of 4,450 and 839.

Exercise:

Problem:

Add the difference of 997,468 and 292,513 to the difference of 22,140 and 8,617.

Solution:

718,478

Exercise:

Problem:

Subtract the difference of 8,412 and 576 from the sum of 22,140 and 8,617.

Exercise:

Problem:

Add the sum of 2,273, 3,304, 847, and 16 to the difference of 4,365 and 864.

Solution:

9,941

Exercise:

Problem:

Add the sum of 19,161, 201, 166,127, and 44 to the difference of the sums of 161, 2,455, and 85, and 21, 26, 48, and 187.

Exercise:

Problem:

Is the sum of 626 and 1,242 the same as the sum of 1,242 and 626? Justify your claim.

Solution:

$626 + 1{,}242 = 1{,}242 + 626 = 1{,}868$

Proficiency Exam
This module is from Fundamentals of Mathematics by Denny Burzynski and Wade Ellis, Jr. This module is a proficiency exam to the chapter Addition and Subtraction of Whole Numbers. Each problem is accompanied with a reference link pointing back to the module that discusses the type of problem demonstrated in the question. The problems in this exam are accompanied by solutions.

Proficiency Exam

Exercise:

Problem: ([link]) What is the largest digit?

Solution:

9

Exercise:

Problem:

([link]) In the Hindu-Arabic number system, each period has three values assigned to it. These values are the same for each period. From right to left, what are they?

Solution:

ones, tens, hundreds

Exercise:

Problem:

([link]) In the number 42,826, how many hundreds are there?

Solution:

8

Exercise:

Problem: ([link]) Is there a largest whole number? If so, what is it?

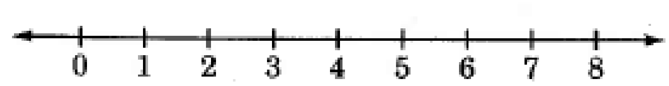
Solution:

no

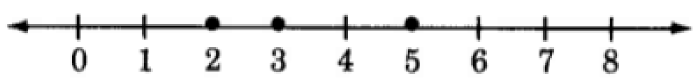
Exercise:

Problem:

([link]) Graph the following whole numbers on the number line: 2, 3, 5.



Solution:



Exercise:

Problem: ([link]) Write the number 63,425 as you would read it aloud.

Solution:

Sixty-three thousand, four hundred twenty-five

Exercise:

Problem:

([link]) Write the number eighteen million, three hundred fifty-nine thousand, seventy-two.

Solution:

18,359,072

Exercise:

Problem: ([link]) Round 427 to the nearest hundred.

Solution:

400

Exercise:

Problem: ([link]) Round 18,995 to the nearest ten.

Solution:

19,000

Exercise:

Problem:

([link]) Round to the most reasonable digit: During a semester, a mathematics instructor uses 487 pieces of chalk.

Solution:

500

For problems 11-17, find the sums and differences.

Exercise:

Problem: ([link]) $\begin{array}{r} 627 \\ +\ \ 48 \end{array}$

Solution:

675

Exercise:

Problem: ([link]) $3106 + 921$

Solution:

4,027

Exercise:

Problem: ([link]) $\begin{array}{r} 152 \\ + \;\; 36 \end{array}$

Solution:

188

Exercise:

Problem: ([link]) $\begin{array}{r} 5{,}189 \\ 6{,}189 \\ 4{,}122 \\ +8{,}001 \end{array}$

Solution:

23,501

Exercise:

Problem: ([link]) $21 + 16 + 42 + 11$

Solution:

90

Exercise:

Problem: ([link]) $520 - 216$

Solution:

304

Exercise:

Problem: ([link]) $\begin{array}{r} 80{,}001 \\ -\ \ 9{,}878 \end{array}$

Solution:

70,123

Exercise:

Problem: ([link]) Subtract 425 from 816.

Solution:

391

Exercise:

Problem: ([link]) Subtract 712 from the sum of 507 and 387.

Solution:

182

Exercise:

Problem:

([link]) Is the sum of 219 and 412 the same as the sum of 412 and 219? If so, what makes it so?

Solution:

Yes, commutative property of addition

Objectives
This module contains Chapter 2 of Fundamentals of Mathematics by Denny Burzynski and Wade Ellis, Jr.

After completing this chapter, you should
Multiplication of Whole Numbers ([link])

- understand the process of multiplication
- be able to multiply whole numbers
- be able to simplify multiplications with numbers ending in zero
- be able to use a calculator to multiply one whole number by another

Concepts of Division of Whole Numbers ([link])

- understand the process of division
- understand division of a nonzero number into zero
- understand why division by zero is undefined
- be able to use a calculator to divide one whole number by another

Division of Whole Numbers ([link])

- be able to divide a whole number by a single or multiple digit divisor
- be able to interpret a calculator statement that a division results in a remainder

Some Interesting Facts about Division ([link])

- be able to recognize a whole number that is divisible by 2, 3, 4, 5, 6, 8, 9, or 10

Properties of Multiplication ([link])

- understand and appreciate the commutative and associative properties of multiplication
- understand why 1 is the multiplicative identity

Multiplication of Whole Numbers
This module is from Fundamentals of Mathematics by Denny Burzynski and Wade Ellis, Jr. This module discusses how to multiply whole numbers. By the end of the module students should be able to understand the process of multiplication, multiply whole numbers, simplify multiplications with numbers ending in zero, and use a calculator to multiply one whole number by another.

Section Overview

- Multiplication
- The Multiplication Process With a Single Digit Multiplier
- The Multiplication Process With a Multiple Digit Multiplier
- Multiplication With Numbers Ending in Zero
- Calculators

Multiplication

Multiplication is a description of repeated addition.

In the addition of

$5 + 5 + 5$

the number 5 is repeated 3 times. Therefore, we say we have three times five and describe it by writing

3×5

Thus,

$3 \times 5 = 5 + 5 + 5$

Multiplicand
In a multiplication, the repeated addend (number being added) is called the **multiplicand**. In 3×5, the 5 is the multiplicand.

Multiplier
Also, in a multiplication, the number that records the number of times the multiplicand is used is called the **multiplier**. In 3×5, the 3 is the multiplier.

Sample Set A

Express each repeated addition as a multiplication. In each case, specify the multiplier and the multiplicand.

Example:
$7 + 7 + 7 + 7 + 7 + 7$
6×7. Multiplier is 6. Multiplicand is 7.

Example:
$18 + 18 + 18$
3×18. Multiplier is 3. Multiplicand is 18.

Practice Set A

Express each repeated addition as a multiplication. In each case, specify the multiplier and the multiplicand.

Exercise:

Problem: $12 + 12 + 12 + 12$

. Multiplier is . Multiplicand is .

Solution:

4×12. Multiplier is 4. Multiplicand is 12.

Exercise:

Problem: $36 + 36 + 36 + 36 + 36 + 36 + 36 + 36$

. Multiplier is . Multiplicand is .

Solution:

8×36. Multiplier is 8. Multiplicand is 36.

Exercise:

Problem: $0 + 0 + 0 + 0 + 0$

. Multiplier is . Multiplicand is .

Solution:

5×0. Multiplier is 5. Multiplicand is 0.

Exercise:

Problem: $\underbrace{1847 + 1847 + ... + 1847}_{12,000 \text{ times}}$

. Multiplier is . Multiplicand is .

Solution:

$12{,}000 \times 1{,}847$. Multiplier is 12,000. Multiplicand is 1,847.

Factors
In a multiplication, the numbers being multiplied are also called **factors**.

Products
The result of a multiplication is called the **product**. In $3 \times 5 = 15$, the 3 and 5 are not only called the multiplier and multiplicand, but they are also called factors. The product is 15.

Indicators of Multiplication $\times, \cdot, (\ )$
The multiplication symbol ($\times$) is not the only symbol used to indicate multiplication. Other symbols include the dot ($\cdot$) and pairs of parentheses ( ). The expressions

$3 \times 5,\ 3 \cdot 5,\ 3(5),\ (3)5,\ (3)(5)$

all represent the same product.

The Multiplication Process With a Single Digit Multiplier

Since multiplication is repeated addition, we should not be surprised to notice that **carrying** can occur. Carrying occurs when we find the product of 38 and 7:

$$\begin{array}{r} {\scriptstyle 5\ } \\ 38 \\ \times\ \ 7 \\ \hline 266 \end{array}$$

First, we compute $7 \times 8 = 56$. Write the 6 in the ones column. Carry the 5. Then take $7 \times 3 = 21$. Add to 21 the 5 that was carried: $21 + 5 = 26$. The product is 266.

Sample Set B

Find the following products.

Example:

$$\begin{array}{r} {}^{1} \\ 64 \\ \times \quad 3 \\ \hline 192 \end{array}$$

$3 \times 4 = 12$ Write the 2, carry the 1.

$3 \times 6 = 18$ Add to 18 the 1 that was carried: $18 + 1 = 19$.

The product is 192.

Example:

$$\begin{array}{r} {}^{1\,3} \\ 526 \\ \times \quad 5 \\ \hline 2{,}630 \end{array}$$

$5 \times 6 = 30$ Write the 0, carry the 3.

$5 \times 2 = 10$ Add to 10 the 3 that was carried: $10 + 3 = 13$. Write the 3, carry the 1.

$5 \times 5 = 25$ Add to 25 the 1 that was carried: $25 + 1 = 6$.

The product is 2,630.

Example:

$$\begin{array}{r} {}^{7\;\;3} \\ 1{,}804 \\ \times \quad 9 \\ \hline 16{,}236 \end{array}$$

$9 \times 4 = 36$ Write the 6, carry the 3.

$9 \times 0 = 0$ Add to the 0 the 3 that was carried: $0 + 3 = 3$. Write the 3.

$9 \times 8 = 72$ Write the 2, carry the 7.

$9 \times 1 = 9$ Add to the 9 the 7 that was carried: $9 + 7 = 16$.

Since there are no more multiplications to perform, write both the 1 and 6.

The product is 16,236.

Practice Set B

Find the following products.

Exercise:

Problem: $\begin{array}{r} 37 \\ \times \quad 5 \\ \hline \end{array}$

Solution:

185

Exercise:

Problem: $\begin{array}{r} 78 \\ \times \quad 8 \\ \hline \end{array}$

Solution:

624

Exercise:

Problem: $\begin{array}{r} 536 \\ \times \quad 7 \\ \hline \end{array}$

Solution:

3,752

Exercise:

Problem: $\begin{array}{r} 40,019 \\ \times \quad 8 \\ \hline \end{array}$

Solution:

320,152

Exercise:

Problem: $\begin{array}{r} 301,599 \\ \times \quad 3 \\ \hline \end{array}$

Solution:

904,797

The Multiplication Process With a Multiple Digit Multiplier

In a multiplication in which the multiplier is composed of two or more digits, the *multiplication must take place in parts*. The process is as follows:

- **Part 1 First Partial Product** Multiply the multiplicand by the ones digit of the multiplier. This product is called the **first partial product**.
- **Part 2 Second Partial Product** Multiply the multiplicand by the tens digit of the multiplier. This product is called the **second partial product**. Since the tens digit is used as a factor, the second partial product is written below the first partial product so that its rightmost digit appears in the tens column.
- **Part 3** If necessary, continue this way finding partial products. Write each one below the previous one so that the rightmost digit appears in the column directly below the digit that was used as a factor.
- **Part 4 Total Product** Add the partial products to obtain the **total product**.

Note: It may be necessary to carry when finding each partial product.

Sample Set C

Example:
Multiply 326 by 48.

- **Part 1**

$$\begin{array}{r} {}^{2\,4} \\ 326 \\ \times\ \ 48 \\ \hline 2608 \end{array} \longleftarrow \textbf{First partial product.}$$

- **Part 2**

$$\begin{array}{r} {}^{1\,2} \\ {}^{2\,4} \\ 326 \\ \times\ \ 48 \\ \hline 2608 \\ 1304 \end{array} \longleftarrow \textbf{Second partial product.}$$

- **Part 3**This step is unnecessary since all of the digits in the multiplier have been used.
- **Part 4**Add the partial products to obtain the total product.

```
    12
    24
   326
 ×  48
  2608
+1304
 15648   ⟵ Total product.
```

- The product is 15,648.

Example:
Multiply 5,369 by 842.

- **Part 1**

```
  11
 5369
× 842
10738   ⟵ First partial product.
```

- **Part 2**

```
   123
    11
  5369
×  842
 10738
21476     ⟵ Second partial product.
```

- **Part 3**

```
   257
   123
    11
  5369
×  842
 10738
21476
42952       ⟵ Third partial product.
4520698   ⟵ Total product (Part 4).
```

- The product is 4,520,698.

Example:
Multiply 1,508 by 206.

- **Part 1**

$$\begin{array}{r} {\scriptstyle 3\ 4\ \ } \\ 1508 \\ \times\ \ 206 \\ \hline 9048 \end{array} \longleftarrow \textbf{First partial product (in first column from the right).}$$

- **Part 2**

$$\begin{array}{r} {\scriptstyle 3\ 4\ \ } \\ 1508 \\ \times\ \ 206 \\ \hline 9048 \end{array}$$

  Since 0 times 1508 is 0, the partial product will not change the identity of the total product (which is obtained by addition). Go to the next partial product.

- **Part 3**

$$\begin{array}{r} {\scriptstyle 1\ 1\ \ } \\ {\scriptstyle 3\ 4\ \ } \\ 1508 \\ \times\ 206 \\ \hline 9048 \\ 3016\ \ \ \\ \hline 310648 \end{array} \begin{array}{l} \\ \\ \\ \\ \\ \longleftarrow \textbf{Third partial product (in third column from the right).} \\ \longleftarrow \textbf{Total product (Part 4).} \end{array}$$

- The product is 310,648

Practice Set C

Exercise:

Problem: Multiply 73 by 14.

Solution:

1,022

Exercise:

Problem: Multiply 86 by 52.

Solution:

4,472

Exercise:

Problem: Multiply 419 by 85.

Solution:

35,615

Exercise:

Problem: Multiply 2,376 by 613.

Solution:

1,456,488

Exercise:

Problem: Multiply 8,107 by 304.

Solution:

2,464,528

Exercise:

Problem: Multiply 66,260 by 1,008.

Solution:

66,790,080

Exercise:

Problem: Multiply 209 by 501.

Solution:

104,709

Exercise:

Problem: Multiply 24 by 10.

Solution:

240

Exercise:

Problem: Multiply 3,809 by 1,000.

Solution:

3,809,000

Exercise:

Problem: Multiply 813 by 10,000.

Solution:

8,130,000

Multiplications With Numbers Ending in Zero

Often, when performing a multiplication, one or both of the factors will end in zeros. Such multiplications can be done quickly by aligning the numbers so that the rightmost nonzero digits are in the same column.

Sample Set D

Perform the multiplication $(49{,}000)(1{,}200)$.

$$(49{,}000)(1{,}200) \quad = \quad \begin{array}{r} 49000 \\ \times \quad 1200 \end{array}$$

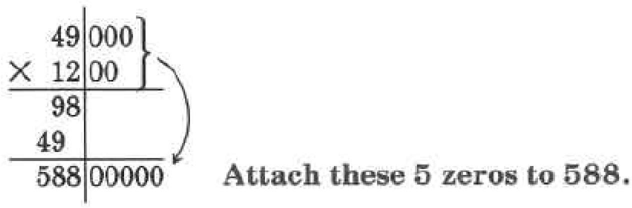
Since 9 and 2 are the rightmost nonzero digits, put them in the same column.

$$\begin{array}{r} 49000 \\ \times 1200 \\ \hline \end{array}$$

Draw (perhaps mentally) a vertical line to separate the zeros from the nonzeros.

$$\begin{array}{r|l} 49 & 000 \\ \times 12 & 00 \\ \hline \end{array}$$

Multiply the numbers to the left of the vertical line as usual, then attach to the right end of this product the total number of zeros.



The product is 58,800,000

Practice Set D

Exercise:

Problem: Multiply 1,800 by 90.

Solution:

162,000

Exercise:

Problem: Multiply 420,000 by 300.

Solution:

126,000,000

Exercise:

Problem: Multiply 20,500,000 by 140,000.

Solution:

2,870,000,000,000

Calculators

Most multiplications are performed using a calculator.

Sample Set E

Example:
Multiply 75,891 by 263.

		Display Reads
Type	75891	75891
Press	×	75891
Type	263	263
Press	=	19959333

The product is 19,959,333.

Example:
Multiply 4,510,000,000,000 by 1,700.

		Display Reads
Type	451	451
Press	×	451
Type	17	17
Press	=	7667

The display now reads 7667. We'll have to add the zeros ourselves. There are a total of 12 zeros. Attaching 12 zeros to 7667, we get 7,667,000,000,000,000.
The product is 7,667,000,000,000,000.

Example:
Multiply 57,847,298 by 38,976.

		Display Reads

Type	57847298	57847298
Press	×	57847298
Type	38976	38976
Press	=	2.2546563 12

The display now reads 2.2546563 12. What kind of number is this? This is an example of a whole number written in **scientific notation**. We'll study this concept when we get to decimal numbers.

Practice Set E

Use a calculator to perform each multiplication.

Exercise:

Problem: 52×27

Solution:

1,404

Exercise:

Problem: $1{,}448 \times 6{,}155$

Solution:

8,912,440

Exercise:

Problem: $8{,}940{,}000 \times 205{,}000$

Solution:

1,832,700,000,000

Exercises

For the following problems, perform the multiplications. You may check each product with a calculator.

Exercise:

Problem: $\begin{array}{r} 8 \\ \times 3 \end{array}$

Solution:

24

Exercise:

Problem: $\begin{array}{r} 3 \\ \times 5 \end{array}$

Exercise:

Problem: $\begin{array}{r} 8 \\ \times 6 \end{array}$

Solution:

48

Exercise:

Problem: $\begin{array}{r} 5 \\ \times 7 \end{array}$

Exercise:

Problem: 6×1

Solution:

6

Exercise:

Problem: 4×5

Exercise:

Problem: 75×3

Solution:

225

Exercise:

Problem: 35×5

Exercise:

Problem: $\begin{array}{r} 45 \\ \times\ \ 6 \end{array}$

Solution:

270

Exercise:

Problem: $\begin{array}{r} 31 \\ \times\ \ 7 \end{array}$

Exercise:

Problem: $\begin{array}{r} 97 \\ \times\ \ 6 \end{array}$

Solution:

582

Exercise:

Problem: $\begin{array}{r} 75 \\ \times 57 \end{array}$

Exercise:

Problem: $\begin{array}{r} 64 \\ \times 15 \end{array}$

Solution:

960

Exercise:

Problem: $\begin{array}{r} 73 \\ \times 15 \end{array}$

Exercise:

Problem: $\begin{array}{r} 81 \\ \times 95 \end{array}$

Solution:

7,695

Exercise:

Problem: $\begin{array}{r} 31 \\ \times 33 \end{array}$

Exercise:

Problem: 57×64

Solution:

3,648

Exercise:

Problem: 76×42

Exercise:

Problem: 894×52

Solution:

46,488

Exercise:

Problem: 684×38

Exercise:

Problem: $\begin{array}{r} 115 \\ \times \quad 22 \end{array}$

Solution:

2,530

Exercise:

Problem: $\begin{array}{r} 706 \\ \times\ \ 81 \end{array}$

Exercise:

Problem: $\begin{array}{r} 328 \\ \times\ \ 21 \end{array}$

Solution:

6,888

Exercise:

Problem: $\begin{array}{r} 550 \\ \times\ \ 94 \end{array}$

Exercise:

Problem: 930×26

Solution:

24,180

Exercise:

Problem: 318×63

Exercise:

Problem: $\begin{array}{r} 582 \\ \times 127 \end{array}$

Solution:

73,914

Exercise:

Problem: $\begin{array}{r} 247 \\ \times 116 \end{array}$

Exercise:

Problem: $\begin{array}{r} 305 \\ \times 225 \end{array}$

Solution:

68,625

Exercise:

Problem: $\begin{array}{r} 782 \\ \times 547 \end{array}$

Exercise:

Problem: $\begin{array}{r} 771 \\ \times 663 \end{array}$

Solution:

511,173

Exercise:

Problem: $\begin{array}{r} 638 \\ \times 516 \end{array}$

Exercise:

Problem: $1,905 \times 710$

Solution:

1,352,550

Exercise:

Problem: $5,757 \times 5,010$

Exercise:

Problem: $\begin{array}{r} 3,106 \\ \times 1,752 \end{array}$

Solution:

5,441,712

Exercise:

Problem: $\begin{array}{r} 9{,}300 \\ \times 1{,}130 \end{array}$

Exercise:

Problem: $\begin{array}{r} 7{,}057 \\ \times 5{,}229 \end{array}$

Solution:

36,901,053

Exercise:

Problem: $\begin{array}{r} 8{,}051 \\ \times 5{,}580 \end{array}$

Exercise:

Problem: $\begin{array}{r} 5{,}804 \\ \times 4{,}300 \end{array}$

Solution:

24,957,200

Exercise:

Problem: $\begin{array}{r} 357 \\ \times 16 \end{array}$

Exercise:

Problem: $\begin{array}{r} 724 \\ \times \quad 0 \end{array}$

Solution:

0

Exercise:

Problem: $\begin{array}{r} 2{,}649 \\ \times \quad 41 \end{array}$

Exercise:

Problem: $\begin{array}{r} 5{,}173 \\ \times \quad 8 \end{array}$

Solution:

41,384

Exercise:

Problem: $\begin{array}{r} 1{,}999 \\ \times \quad 0 \end{array}$

Exercise:

Problem: $\begin{array}{r} 1{,}666 \\ \times \quad 0 \end{array}$

Solution:

0

Exercise:

Problem: $\begin{array}{r} 51{,}730 \\ \times \quad 142 \end{array}$

Exercise:

Problem: $\begin{array}{r} 387 \\ \times 190 \end{array}$

Solution:

73,530

Exercise:

Problem: $\begin{array}{r} 3{,}400 \\ \times \quad 70 \end{array}$

Exercise:

Problem: $\begin{array}{r} 460{,}000 \\ \times \quad 14{,}000 \end{array}$

Solution:

6,440,000,000

Exercise:

Problem: $\begin{array}{r} 558,000,000 \\ \times \quad 81,000 \end{array}$

Exercise:

Problem: $\begin{array}{r} 37,000 \\ \times \quad 120 \end{array}$

Solution:

4,440,000

Exercise:

Problem: $\begin{array}{r} 498,000 \\ \times \quad 0 \end{array}$

Exercise:

Problem: $\begin{array}{r} 4,585,000 \\ \times \quad 140 \end{array}$

Solution:

641,900,000

Exercise:

Problem: $\begin{array}{r} 30,700,000 \\ \times \quad 180 \end{array}$

Exercise:

Problem: $\begin{array}{r} 8,000 \\ \times \quad 10 \end{array}$

Solution:

80,000

Exercise:

Problem:

Suppose a theater holds 426 people. If the theater charges $4 per ticket and sells every seat, how much money would they take in?

Exercise:

Problem:

In an English class, a student is expected to read 12 novels during the semester and prepare a report on each one of them. If there are 32 students in the class, how many reports will be prepared?

Solution:

384 reports

Exercise:

Problem:

In a mathematics class, a final exam consists of 65 problems. If this exam is given to 28 people, how many problems must the instructor grade?

Exercise:

Problem:

A business law instructor gives a 45 problem exam to two of her classes. If each class has 37 people in it, how many problems will the instructor have to grade?

Solution:

3,330 problems

Exercise:

Problem:

An algebra instructor gives an exam that consists of 43 problems to four of his classes. If the classes have 25, 28, 31, and 35 students in them, how many problems will the instructor have to grade?

Exercise:

Problem:

In statistics, the term "standard deviation" refers to a number that is calculated from certain data. If the data indicate that one standard deviation is 38 units, how many units is three standard deviations?

Solution:

114 units

Exercise:

Problem:

Soft drinks come in cases of 24 cans. If a supermarket sells 857 cases during one week, how many individual cans were sold?

Exercise:

Problem:

There are 60 seconds in 1 minute and 60 minutes in 1 hour. How many seconds are there in 1 hour?

Solution:

3,600 seconds

Exercise:

Problem:

There are 60 seconds in 1 minute, 60 minutes in one hour, 24 hours in one day, and 365 days in one year. How many seconds are there in 1 year?

Exercise:

Problem:

Light travels 186,000 miles in one second. How many miles does light travel in one year? (*Hint:* Can you use the result of the previous problem?)

Solution:

5,865,696,000,000 miles per year

Exercise:

Problem:

An elementary school cafeteria sells 328 lunches every day. Each lunch costs $1. How much money does the cafeteria bring in in 2 weeks?

Exercise:

Problem:

A computer company is selling stock for $23 a share. If 87 people each buy 55 shares, how much money would be brought in?

Solution:

$110,055

Exercises for Review

Exercise:

Problem: ([link]) In the number 421,998, how may ten thousands are there?

Exercise:

Problem: ([link]) Round 448,062,187 to the nearest hundred thousand.

Solution:

448,100,000

Exercise:

Problem: ([link]) Find the sum. 22,451 + 18,976.

Exercise:

Problem: ([link]) Subtract 2,289 from 3,001.

Solution:

712

Exercise:

Problem:

([link]) Specify which property of addition justifies the fact that (a first whole number + a second whole number) = (the second whole number + the first whole number)

Concepts of Division of Whole Numbers
This module is from Fundamentals of Mathematics by Denny Burzynski and Wade Ellis, Jr. This module discusses how to divide whole numbers. By the end of the module students should be able to understand the process of division, understand division of a nonzero number into zero, understand why division by zero is undefined, and use a calculator to divide one whole number by another.

Section Overview

- Division
- Division into Zero (Zero As a Dividend: $\frac{0}{a}$, $a \neq 0$)
- Division by Zero (Zero As a Divisor: $\frac{0}{a}$, $a \neq 0$)
- Division by and into Zero (Zero As a Dividend and Divisor: $\frac{0}{0}$)
- Calculators

Division

Division is a description of repeated subtraction.

In the process of division, the concern is how many times one number is contained in another number. For example, we might be interested in how many 5's are contained in 15. The word *times* is significant because it implies a relationship between division and multiplication.

There are several notations used to indicate division. Suppose Q records the number of times 5 is contained in 15. We can indicate this by writing

$$\begin{array}{r} Q \\ 5\overline{)15} \end{array} \qquad \frac{15}{5} = Q$$

5 into 15 15 divided by 5

$$15/5 = Q \quad 15 \div 5 = Q$$

15 divided by 5 15 divided by 5

Each of these division notations describes the *same* number, represented here by the symbol Q. Each notation also converts to the same multiplication form. It is $15 = 5 \times Q$

In division,

Dividend
the number being divided into is called the **dividend**.

Divisor
the number dividing into the dividend is the **divisor**.

Quotient
the result of the division is called the **quotient**.

$$\text{divisor}\overline{\smash{)}\text{dividend}}^{\text{quotient}}$$

$$\frac{\text{dividend}}{\text{divisor}} = \text{quotient}$$

$$\text{dividend}/\text{divisor} = \text{quotient} \quad \text{dividend} \div \text{divisor} = \text{quotient}$$

Sample Set A

Find the following quotients using multiplication facts.

Example:
$18 \div 6$
Since $6 \times 3 = 18$,
$18 \div 6 = 3$
Notice also that

$$\begin{array}{rl} 18 & \\ -6 & \\ 12 & \\ -6 & \text{Repeated subtraction} \\ 6 & \\ -6 & \\ 0 & \end{array}$$

Thus, 6 is contained in 18 three times.

Example:

$\frac{24}{3}$

Since $3 \times 8 = 24$,

$\frac{24}{3} = 8$

Notice also that 3 could be subtracted exactly 8 times from 24. This implies that 3 is contained in 24 eight times.

Example:

$\frac{36}{6}$

Since $6 \times 6 = 36$,

$\frac{36}{6} = 6$

Thus, there are 6 sixes in 36.

Example:

$9\overline{)72}$

Since $9 \times 8 = 72$,

$$\begin{array}{r} 8 \\ 9\overline{)72} \end{array}$$

Thus, there are 8 nines in 72.

Practice Set A

Use multiplication facts to determine the following quotients.

Exercise:

Problem: $32 \div 8$

Solution:

4

Exercise:

Problem: $18 \div 9$

Solution:

2

Exercise:

Problem: $\frac{25}{5}$

Solution:

5

Exercise:

Problem: $\frac{48}{8}$

Solution:

6

Exercise:

Problem: $\frac{28}{7}$

Solution:

4

Exercise:

Problem: $4\overline{)36}$

Solution:

9

Division into Zero (Zero as a Dividend: $\frac{0}{a}$, $a \neq 0$)

Let's look at what happens when the dividend (the number being divided into) is zero, and the divisor (the number doing the dividing) is any whole number except zero. The question is

What number, if any, is $\frac{0}{\text{any nonzero whole number}}$?

Let's represent this unknown quotient by Q. Then,

$$\frac{0}{\text{any nonzero whole number}} = Q$$

Converting this division problem to its corresponding multiplication problem, we get

$$0 = Q \times (\text{any nonzero whole number})$$

From our knowledge of multiplication, we can understand that if the product of two whole numbers is zero, then one or both of the whole numbers must be zero. Since any nonzero whole number is certainly not zero, Q must represent zero. Then,

$$\frac{0}{\text{any nonzero whole number}} = 0$$

Zero Divided By Any Nonzero Whole Number Is Zero
Zero divided any nonzero whole number is zero.

Division by Zero (Zero as a Divisor: $\frac{a}{0}$, $a \neq 0$)

Now we ask,

What number, if any, is $\frac{\text{any nonzero whole number}}{0}$?

Letting Q represent a possible quotient, we get

$$\frac{\text{any nonzero whole number}}{0} = Q$$

Converting to the corresponding multiplication form, we have

$$(\text{any nonzero whole number}) = Q \times 0$$

Since $Q \times 0 = 0$, $(\text{any nonzero whole number}) = 0$. But this is absurd. This would mean that $6 = 0$, or $37 = 0$. A nonzero whole number *cannot* equal 0! Thus,

$\frac{\text{any nonzero whole number}}{0}$ *does not* name a number

Division by Zero is Undefined
Division by zero does not name a number. It is, therefore, undefined.

Division by and Into Zero (Zero as a Dividend and Divisor: $\frac{0}{0}$)

We are now curious about zero divided by zero $\left(\frac{0}{0}\right)$. If we let Q represent a potential quotient, we get

$$\frac{0}{0} = Q$$

Converting to the multiplication form,

$$0 = Q \times 0$$

This results in

$0 = 0$

This is a statement that is true regardless of the number used in place of Q. For example,

$\frac{0}{0} = 5$, since $0 = 5 \times 0$.

$\frac{0}{0} = 31$, since $0 = 31 \times 0$.

$\frac{0}{0} = 286$, since $0 = 286 \times 0$.

A *unique* quotient cannot be determined.

Indeterminant
Since the result of the division is inconclusive, we say that $\frac{0}{0}$ is **indeterminant**.

$\frac{0}{0}$ is Indeterminant
The division $\frac{0}{0}$ is indeterminant.

Sample Set B

Perform, if possible, each division.

Example:
$\frac{19}{0}$. Since division by 0 does not name a whole number, no quotient exists, and we state $\frac{19}{0}$ is undefined

Example:

$0\overline{)14}$. Since division by 0 does not name a defined number, no quotient exists, and we state $0\overline{)14}$ is undefined

Example:
$9\overline{)0}$. Since division into 0 by any nonzero whole number results in 0, we have $\begin{array}{r} 0 \\ 9\overline{)0} \end{array}$

Example:
$\frac{0}{7}$. Since division into 0 by any nonzero whole number results in 0, we have $\frac{0}{7} = 0$

Practice Set B

Perform, if possible, the following divisions.

Exercise:

Problem: $\frac{5}{0}$

Solution:

undefined

Exercise:

Problem: $\frac{0}{4}$

Solution:

0

Exercise:

Problem: $0\overline{)0}$

Solution:

indeterminant

Exercise:

Problem: $0\overline{)8}$

Solution:

undefined

Exercise:

Problem: $\frac{9}{0}$

Solution:

undefined

Exercise:

Problem: $\frac{0}{1}$

Solution:

0

Calculators

Divisions can also be performed using a calculator.

Sample Set C

Example:
Divide 24 by 3.

Display Reads		
Type	24	24
Press	÷	24
Type	3	3
Press	=	8

The display now reads 8, and we conclude that $24 \div 3 = 8$.

Example:
Divide 0 by 7.

Display Reads		
Type	0	0

Press	÷	0
Type	7	7
Press	=	0

The display now reads 0, and we conclude that $0 \div 7 = 0$.

Example:
Divide 7 by 0.
Since division by zero is undefined, the calculator should register some kind of error message.

Display Reads		
Type	7	7
Press	÷	7
Type	0	0
Press	=	Error

The error message indicates an undefined operation was attempted, in this case, division by zero.

Practice Set C

Use a calculator to perform each division.

Exercise:

Problem: $35 \div 7$

Solution:

5

Exercise:

Problem: $56 \div 8$

Solution:

7

Exercise:

Problem: $0 \div 6$

Solution:

0

Exercise:

Problem: $3 \div 0$

Solution:

An error message tells us that this operation is undefined. The particular message depends on the calculator.

Exercise:

Problem: $0 \div 0$

Solution:

An error message tells us that this operation cannot be performed. Some calculators actually set $0 \div 0$ equal to 1. We know better! $0 \div 0$ is indeterminant.

Exercises

For the following problems, determine the quotients (if possible). You may use a calculator to check the result.

Exercise:

Problem: $4\overline{)32}$

Solution:

8

Exercise:

Problem: $7\overline{)42}$

Exercise:

Problem: $6\overline{)18}$

Solution:

3

Exercise:

Problem: $2\overline{)14}$

Exercise:

Problem: $3\overline{)27}$

Solution:

9

Exercise:

Problem: $1\overline{)6}$

Exercise:

Problem: $4\overline{)28}$

Solution:

7

Exercise:

Problem: $\frac{30}{5}$

Exercise:

Problem: $\frac{16}{4}$

Solution:

4

Exercise:

Problem: $24 \div 8$

Exercise:

Problem: $10 \div 2$

Solution:

5

Exercise:

Problem: $21 \div 7$

Exercise:

Problem: $21 \div 3$

Solution:

7

Exercise:

Problem: $0 \div 6$

Exercise:

Problem: $8 \div 0$

Solution:

not defined

Exercise:

Problem: $12 \div 4$

Exercise:

Problem: $3\overline{)9}$

Solution:

3

Exercise:

Problem: $0\overline{)0}$

Exercise:

Problem: $7\overline{)0}$

Solution:

0

Exercise:

Problem: $6\overline{)48}$

Exercise:

Problem: $\frac{15}{3}$

Solution:

5

Exercise:

Problem: $\frac{35}{0}$

Exercise:

Problem: $56 \div 7$

Solution:

8

Exercise:

Problem: $\frac{0}{9}$

Exercise:

Problem: $72 \div 8$

Solution:

9

Exercise:

Problem: Write $\frac{16}{2} = 8$ using three different notations.

Exercise:

Problem: Write $\frac{27}{9} = 3$ using three different notations.

Solution:

$27 \div 9 = 3$; $9\overline{)27} = 3$; $\frac{27}{9} = 3$

Exercise:

Problem: In the statement $\begin{array}{r} 4 \\ 6\overline{)24} \end{array}$

6 is called the .

24 is called the .

4 is called the .

Exercise:

Problem: In the statement $56 \div 8 = 7$,

7 is called the .

8 is called the .

56 is called the .

Solution:

7 is quotient; 8 is divisor; 56 is dividend

Exercises for Review

Exercise:

Problem: ([link]) What is the largest digit?

Exercise:

Problem: ([link]) Find the sum. $\begin{array}{r} 8{,}006 \\ +4{,}118 \end{array}$

Solution:

12,124

Exercise:

Problem: ([link]) Find the difference. $\begin{array}{r} 631 \\ -589 \end{array}$

Exercise:

Problem:

([link]) Use the numbers 2, 3, and 7 to illustrate the associative property of addition.

Solution:

$$\begin{aligned}(2+3)+7 &= 2+(3+7) = 12 \\ 5+7 &= 2+10 = 12\end{aligned}$$

Exercise:

Problem: ([link]) Find the product. $\begin{array}{r} 86 \\ \times 12 \end{array}$

Division of Whole Numbers
This module is from Fundamentals of Mathematics by Denny Burzynski and Wade Ellis, Jr. This module discusses how to divide whole numbers. By the end of the module students should be able to be able to divide a whole number by a single or multiple digit divisor and interpret a calculator statement that a division results in a remainder.

Section Overview

- Division with a Single Digit Divisor
- Division with a Multiple Digit Divisor
- Division with a Remainder
- Calculators

Division with a Single Digit Divisor

Our experience with multiplication of whole numbers allows us to perform such divisions as $75 \div 5$. We perform the division by performing the corresponding multiplication, $5 \times Q = 75$. Each division we considered in [link] had a one-digit quotient. Now we will consider divisions in which the quotient may consist of two or more digits. For example, $75 \div 5$.

Let's examine the division $75 \div 5$. We are asked to determine how many 5's are contained in 75. We'll approach the problem in the following way.

1. Make an educated guess based on experience with multiplication.
2. Find how close the estimate is by multiplying the estimate by 5.
3. If the product obtained in step 2 is less than 75, find out how much less by subtracting it from 75.
4. If the product obtained in step 2 is greater than 75, decrease the estimate until the product is less than 75. Decreasing the estimate makes sense because we do not wish to exceed 75.

We can suggest from this discussion that the process of division consists of
The Four Steps in Division

1. an educated guess

2. a multiplication
3. a subtraction
4. bringing down the next digit (if necessary)

The educated guess can be made by determining how many times the divisor is contained in the dividend by using only one or two digits of the dividend.

Sample Set A

Example:

Find $75 \div 5$.

$5\overline{)75}$ Rewrite the problem using a division bracket.

$$\begin{array}{r} 10 \\ 5\overline{)75} \end{array}$$

Make an educated guess by noting that one 5 is contained in 75 at most 10 times.

Since 7 is the tens digit, we estimate that 5 goes into 75 at most 10 times.

$$\begin{array}{r} 10 \\ 5\overline{)75} \\ \underline{-50} \\ 25 \end{array}$$

Now determine how close the estimate is.

10 fives is $10 \times 5 = 50$. Subtract 50 from 75.

Estimate the number of 5's in 25.

There are exactly 5 fives in 25.

$$\begin{array}{r} \left.\begin{array}{r} 5 \\ 10 \end{array}\right\} \begin{array}{l} \text{10 fives + 5 fives = 15 fives.} \\ \text{There are 15 fives contained in 75.} \end{array} \\ 5\overline{)75} \\ \underline{-50} \\ 25 \\ \underline{-25} \\ 0 \end{array}$$

Check:

$$75 \stackrel{?}{=} 15 \times 5$$
$$75 \stackrel{?}{=} 75$$

Thus, $75 \div 5 = 15$.
The notation in this division can be shortened by writing.

$$\begin{array}{r} 15 \\ 5\overline{)75} \\ \underline{-5\downarrow} \\ 25 \\ \underline{-25} \\ 0 \end{array}$$

$$\begin{cases} \text{Divide:} & \text{5 goes into 7 at most 1 time.} \\ \text{Multiply:} & 1 \times 5 = 5. \text{ Write 5 below 7.} \\ \text{Subtract:} & 7 - 5 = 2. \text{ Bring down the 5.} \end{cases}$$

$$\begin{cases} \text{Divide:} & \text{5 goes into 25 exactly 5 times.} \\ \text{Multiply:} & 5 \times 5 = 25. \text{ Write 25 below 25.} \\ \text{Subtract:} & 25 - 25 = 0. \end{cases}$$

Example:
Find $4{,}944 \div 8$.

$$8\overline{)4944}$$

Rewrite the problem using a division bracket.

$$\begin{array}{r} 600 \\ 8\overline{)4944} \\ \underline{-4800} \\ 144 \end{array}$$

8 goes into 49 at most 6 times, and 9 is in the hundreds column. We'll guess 600.
Then, $8 \times 600 = 4800$.

$$\begin{array}{r} 10\\ 600\\ 8\overline{)4944}\\ \underline{-4800}\\ 144\\ \underline{-\ \ 80}\\ 64 \end{array}$$

8 goes into 14 at most 1 time, and 4 is in the tens column. We'll guess 10.

$$\begin{array}{r} 8\\ 10\\ 600\\ 8\overline{)4944}\\ \underline{-4800}\\ 144\\ \underline{-\ \ 80}\\ 64\\ \underline{-64}\\ 0 \end{array}$$

8 goes into 64 exactly 8 times.
600 eights + 10 eights + 8 eights = 618 eights.
Check:

$$\begin{aligned} 4944 &\stackrel{?}{=} 8 \times 618\\ 4944 &\stackrel{\checkmark}{=} 4944 \end{aligned}$$

Thus, $4{,}944 \div 8 = 618$.
As in the first problem, the notation in this division can be shortened by eliminating the subtraction signs and the zeros in each educated guess.

$$\begin{array}{r} 618\\ 8\overline{)4944}\\ \underline{48\downarrow\ \ }\\ 14\ \ \\ \underline{8\downarrow}\\ 64\\ \underline{64}\\ 0 \end{array}$$

$$\begin{cases} \text{Divide:} & \text{8 goes into 49 at most 6 times.} \\ \text{Multiply:} & 6 \times 8 = 48. \text{ Write 48 below 49.} \\ \text{Subtract:} & 49 - 48 = 1. \text{ Bring down the 4.} \end{cases}$$
$$\begin{cases} \text{Divide:} & \text{8 goes into 14 at most 1 time.} \\ \text{Multiply:} & 1 \times 8 = 8. \text{ Write 8 below 14.} \\ \text{Subtract:} & 14 - 8 = 6. \text{ Bring down the 4.} \end{cases}$$
$$\begin{cases} \text{Divide:} & \text{8 goes into 64 exactly 8 times.} \\ \text{Multiply:} & 8 \times 8 = 64. \text{ Write 64 below 64.} \\ \text{Subtract:} & 64 - 64 = 0. \end{cases}$$

Note: Not all divisions end in zero. We will examine such divisions in a subsequent subsection.

Practice Set A

Perform the following divisions.

Exercise:

Problem: $126 \div 7$

Solution:

18

Exercise:

Problem: $324 \div 4$

Solution:

81

Exercise:

Problem: $2{,}559 \div 3$

Solution:

853

Exercise:

Problem: $5{,}645 \div 5$

Solution:

1,129

Exercise:

Problem: $757{,}125 \div 9$

Solution:

84,125

Division with a Multiple Digit Divisor

The process of division also works when the divisor consists of two or more digits. We now make educated guesses using the first digit of the divisor and one or two digits of the dividend.

Sample Set B

Example:
Find $2{,}232 \div 36$.

$36\overline{)2232}$
Use the first digit of the divisor and the first two digits of the dividend to make the educated guess.
3 goes into 22 at most 7 times.
Try 7: $7 \times 36 = 252$ which is greater than 223. Reduce the estimate.
Try 6: $6 \times 36 = 216$ which is less than 223.

$$\begin{array}{r} 6 \\ 36\overline{)\;2232} \\ -216\downarrow \\ \hline 72 \end{array}$$

Multiply: $6 \times 36 = 216$. Write 216 below 223.

Subtract: 223 - 216 = 7. Bring down the 2.

Divide 3 into 7 to estimate the number of times 36 goes into 72. The 3 goes into 7 at most 2 times.
Try 2: $2 \times 36 = 72$.

$$\begin{array}{r} 62 \\ 36\overline{)2232} \\ \underline{216\downarrow} \\ 72 \\ \underline{-72} \\ 0 \end{array}$$

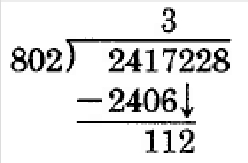
Check:

$$\begin{array}{l} 2232 \stackrel{?}{=} 36 \times 62 \\ 2232 \stackrel{\checkmark}{=} 2232 \end{array}$$

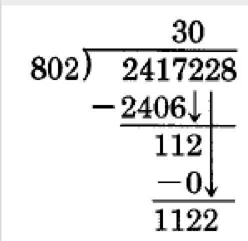
Thus, $2{,}232 \div 36 = 62$.

Example:
Find $2{,}417{,}228 \div 802$.
$802\overline{)2417228}$
First, the educated guess: $24 \div 8 = 3$. Then $3 \times 802 = 2406$, which is less than 2417. Use 3 as the guess. Since $3 \times 802 = 2406$, and 2406 has

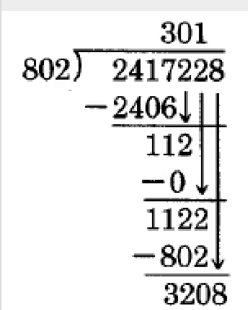
four digits, place the 3 above the fourth digit of the dividend.



Subtract: 2417 - 2406 = 11.
Bring down the 2.
The divisor 802 goes into 112 at most 0 times. Use 0.



Multiply: $0 \times 802 = 0$.
Subtract: 112 - 0 $= 112$.
Bring down the 2.
The 8 goes into 11 at most 1 time, and $1 \times 802 = 802$, which is less than 1122. Try 1.



Subtract $1122 - 802 = 320$
Bring down the 8.
8 goes into 32 at most 4 times.
$4 \times 802 = 3208$.

Use 4.

$$
\begin{array}{r|l}
 & 3014 \\
\hline
802 & 2417228 \\
 & -2406 \\
 & \underline{}112 \\
 & -0 \\
 & 1122 \\
 & -802 \\
 & 3208 \\
 & -3208 \\
 & 0
\end{array}
$$

Check:

$$
\begin{array}{l}
2417228 \stackrel{?}{=} 3014 \times 802 \\
2417228 \stackrel{\checkmark}{=} 2417228
\end{array}
$$

Thus, $2{,}417{,}228 \div 802 = 3{,}014$.

Practice Set B

Perform the following divisions.

Exercise:

Problem: $1{,}376 \div 32$

Solution:

43

Exercise:

Problem: $6{,}160 \div 55$

Solution:

112

Exercise:

Problem: $18,605 \div 61$

Solution:

305

Exercise:

Problem: $144,768 \div 48$

Solution:

3,016

Division with a Remainder

We might wonder how many times 4 is contained in 10. Repeated subtraction yields

$$\begin{array}{r} 10 \\ \underline{-\ \ 4} \\ 6 \\ \underline{-4} \\ 2 \end{array}$$

Since the remainder is less than 4, we stop the subtraction. Thus, 4 goes into 10 two times with 2 remaining. We can write this as a division as follows.

$$\begin{array}{r} 2 \\ 4\overline{)10} \\ -\ 8 \\ \hline 2 \end{array}$$

$$\begin{array}{ll} \text{Divide:} & \text{4 goes into 10 at most 2 times.} \\ \text{Multiply:} & 2 \times 4 = 8. \text{ Write 8 below 0.} \\ \text{Subtract:} & \text{10 - 8} = 2. \end{array}$$

Since 4 does not divide into 2 (the remainder is less than the divisor) and there are no digits to bring down to continue the process, we are done. We write

$$\begin{array}{r} 2\,\mathrm{R}\mathit{2} \\ 4\overline{)\ \ 10} \\ -8 \\ \hline 2 \end{array} \text{ or } 10 \div \mathit{4} = \underbrace{\mathit{2}\mathrm{R}\mathit{2}}_{\text{2 with remainder 2}}$$

Sample Set C

Example:

Find $85 \div 3$.

$$\begin{array}{r} 28 \\ 3\overline{)85} \\ 6\downarrow \\ \hline 25 \\ 24 \\ \hline 1 \end{array}$$

$$\left\{\begin{array}{ll} \text{Divide:} & \text{3 goes into 8 at most 2 times.} \\ \text{Multiply:} & 2 \times 3 = 6. \text{ Write 6 below 8.} \\ \text{Subtract:} & \text{8 - 6} = 2. \text{ Bring down the 5.} \end{array}\right.$$

$$\begin{cases} \text{Divide:} & \text{3 goes into 25 at most 8 times.} \\ \text{Multiply:} & 3 \times 8 = 24. \text{ Write 24 below 25.} \\ \text{Subtract:} & 25 - 24 = 1. \end{cases}$$

There are no more digits to bring down to continue the process. We are done. One is the remainder.

Check: Multiply 28 and 3, then add 1.

$$\begin{array}{r} 28 \\ \underline{\times \;\; 3} \\ 84 \\ \underline{+ \;\; 1} \\ 85 \end{array}$$

Thus, $85 \div 3 = 28\,\mathrm{R}1$.

Example:

Find $726 \div 23$.

$$\begin{array}{r} 31 \\ 23\overline{)726} \\ \underline{69\downarrow} \\ 36 \\ \underline{23} \\ 13 \end{array}$$

Check: Multiply 31 by 23, then add 13.

$$\begin{array}{r} 31 \\ \underline{\times \;\; 23} \\ 93 \\ \underline{62} \\ 713 \\ \underline{+ \;\; 13} \\ 726 \end{array}$$

Thus, $726 \div 23 = 31R13$.

Practice Set C

Perform the following divisions.

Exercise:

Problem: $75 \div 4$

Solution:

18 R3

Exercise:

Problem: $346 \div 8$

Solution:

43 R2

Exercise:

Problem: $489 \div 21$

Solution:

23 R6

Exercise:

Problem: $5{,}016 \div 82$

Solution:

61 R14

Exercise:

Problem: $41{,}196 \div 67$

Solution:

614 R58

Calculators

The calculator can be useful for finding quotients with single and multiple digit divisors. If, however, the division should result in a remainder, the calculator is unable to provide us with the particular value of the remainder. Also, some calculators (most nonscientific) are unable to perform divisions in which one of the numbers has more than eight digits.

Sample Set D

Use a calculator to perform each division.

Example:
$328 \div 8$

Type	328
Press	÷
Type	8
Press	=

The display now reads 41.

Example:
$53,136 \div 82$

Type	53136
Press	÷
Type	82
Press	=

The display now reads 648.

Example:
$730,019,001 \div 326$
We first try to enter 730,019,001 but find that we can only enter 73001900. If our calculator has only an eight-digit display (as most nonscientific calculators do), we will be unable to use the calculator to perform this division.

Example:
$3727 \div 49$

Type	3727
Press	÷
Type	49
Press	=

The display now reads 76.061224.
This number is an example of a decimal number (see [link]). When a decimal number results in a calculator division, we can conclude that the division produces a remainder.

Practice Set D

Use a calculator to perform each division.

Exercise:

Problem: $3,330 \div 74$

Solution:

45

Exercise:

Problem: $63,365 \div 115$

Solution:

551

Exercise:

Problem: $21,996,385,287 \div 53$

Solution:

Since the dividend has more than eight digits, this division cannot be performed on most nonscientific calculators. On others, the answer is 415,026,137.4

Exercise:

Problem: $4{,}558 \div 67$

Solution:

This division results in 68.02985075, a decimal number, and therefore, we cannot, at this time, find the value of the remainder. Later, we will discuss decimal numbers.

Exercises

For the following problems, perform the divisions.

The first 38 problems can be checked with a calculator by multiplying the divisor and quotient then adding the remainder.

Exercise:

Problem: $52 \div 4$

Solution:

13

Exercise:

Problem: $776 \div 8$

Exercise:

Problem: $603 \div 9$

Solution:

67

Exercise:

Problem: $240 \div 8$

Exercise:

Problem: $208 \div 4$

Solution:

52

Exercise:

Problem: $576 \div 6$

Exercise:

Problem: $21 \div 7$

Solution:

3

Exercise:

Problem: $0 \div 0$

Exercise:

Problem: $140 \div 2$

Solution:

70

Exercise:

Problem: $528 \div 8$

Exercise:

Problem: $244 \div 4$

Solution:

61

Exercise:

Problem: $0 \div 7$

Exercise:

Problem: $177 \div 3$

Solution:

59

Exercise:

Problem: $96 \div 8$

Exercise:

Problem: $67 \div 1$

Solution:

67

Exercise:

Problem: $896 \div 56$

Exercise:

Problem: $1{,}044 \div 12$

Solution:

87

Exercise:

Problem: $988 \div 19$

Exercise:

Problem: $5{,}238 \div 97$

Solution:

54

Exercise:

Problem: $2{,}530 \div 55$

Exercise:

Problem: $4{,}264 \div 82$

Solution:

52

Exercise:

Problem: $637 \div 13$

Exercise:

Problem: $3{,}420 \div 90$

Solution:

38

Exercise:

Problem: $5{,}655 \div 87$

Exercise:

Problem: $2{,}115 \div 47$

Solution:

45

Exercise:

Problem: $9{,}328 \div 22$

Exercise:

Problem: $55{,}167 \div 71$

Solution:

777

Exercise:

Problem: $68{,}356 \div 92$

Exercise:

Problem: $27{,}702 \div 81$

Solution:

342

Exercise:

Problem: $6{,}510 \div 31$

Exercise:

Problem: $60{,}536 \div 94$

Solution:

644

Exercise:

Problem: $31{,}844 \div 38$

Exercise:

Problem: $23{,}985 \div 45$

Solution:

533

Exercise:

Problem: $60{,}606 \div 74$

Exercise:

Problem: $2{,}975{,}400 \div 285$

Solution:

10,440

Exercise:

Problem: $1{,}389{,}660 \div 795$

Exercise:

Problem: $7{,}162{,}060 \div 879$

Solution:

8,147 remainder 847

Exercise:

Problem: $7{,}561{,}060 \div 909$

Exercise:

Problem: $38 \div 9$

Solution:

4 remainder 2

Exercise:

Problem: $97 \div 4$

Exercise:

Problem: $199 \div 3$

Solution:

66 remainder 1

Exercise:

Problem: $573 \div 6$

Exercise:

Problem: $10{,}701 \div 13$

Solution:

823 remainder 2

Exercise:

Problem: $13{,}521 \div 53$

Exercise:

Problem: $3{,}628 \div 90$

Solution:

40 remainder 28

Exercise:

Problem: $10{,}592 \div 43$

Exercise:

Problem: $19{,}965 \div 30$

Solution:

665 remainder 15

Exercise:

Problem: $8{,}320 \div 21$

Exercise:

Problem: $61{,}282 \div 64$

Solution:

957 remainder 34

Exercise:

Problem: $1{,}030 \div 28$

Exercise:

Problem: $7{,}319 \div 11$

Solution:

665 remainder 4

Exercise:

Problem: $3{,}628 \div 90$

Exercise:

Problem: $35{,}279 \div 77$

Solution:

458 remainder 13

Exercise:

Problem: $52{,}196 \div 55$

Exercise:

Problem: $67{,}751 \div 68$

Solution:

996 remainder 23

For the following 5 problems, use a calculator to find the quotients.

Exercise:

Problem: $4,346 \div 53$

Exercise:

Problem: $3,234 \div 77$

Solution:

42

Exercise:

Problem: $6,771 \div 37$

Exercise:

Problem: $4,272,320 \div 520$

Solution:

8,216

Exercise:

Problem: $7,558,110 \div 651$

Exercise:

Problem:

A mathematics instructor at a high school is paid $17,775 for 9 months. How much money does this instructor make each month?

Solution:

$1,975 per month

Exercise:

Problem:

A couple pays $4,380 a year for a one-bedroom apartment. How much does this couple pay each month for this apartment?

Exercise:

Problem:

Thirty-six people invest a total of $17,460 in a particular stock. If they each invested the same amount, how much did each person invest?

Solution:

$485 each person invested

Exercise:

Problem:

Each of the 28 students in a mathematics class buys a textbook. If the bookstore sells $644 worth of books, what is the price of each book?

Exercise:

Problem:

A certain brand of refrigerator has an automatic ice cube maker that makes 336 ice cubes in one day. If the ice machine makes ice cubes at a constant rate, how many ice cubes does it make each hour?

Solution:

14 cubes per hour

Exercise:

Problem:

A beer manufacturer bottles 52,380 ounces of beer each hour. If each bottle contains the same number of ounces of beer, and the manufacturer fills 4,365 bottles per hour, how many ounces of beer does each bottle contain?

Exercise:

Problem:

A computer program consists of 68,112 bits. 68,112 bits equals 8,514 bytes. How many bits in one byte?

Solution:

8 bits in each byte

Exercise:

Problem:

A 26-story building in San Francisco has a total of 416 offices. If each floor has the same number of offices, how many floors does this building have?

Exercise:

Problem:

A college has 67 classrooms and a total of 2,546 desks. How many desks are in each classroom if each classroom has the same number of desks?

Solution:

38

Exercises for Review

Exercise:

Problem: ([link]) What is the value of 4 in the number 124,621?

Exercise:

Problem: ([link]) Round 604,092 to the nearest hundred thousand.

Solution:

600,000

Exercise:

Problem: ([link]) What whole number is the additive identity?

Exercise:

Problem: ([link]) Find the product. $6,256 \times 100$.

Solution:

625,600

Exercise:

Problem: ([link]) Find the quotient. $0 \div 11$.

Some Interesting Facts about Division
This module is from Fundamentals of Mathematics by Denny Burzynski and Wade Ellis, Jr. This module discusses interesting facts about diving whole numbers. By the end of the module students should be able to recognize a whole number that is divisible by 2, 3, 4, 5, 6, 8, 9, or 10.

Section Overview

- Division by 2, 3, 4, and 5
- Division by 6, 8, 9, and 10

Quite often, we are able to determine if a whole number is divisible by another whole number just by observing some simple facts about the number. Some of these facts are listed in this section.

Division by 2, 3, 4, and 5

Division by 2
A whole number is **divisible by 2** if its *last digit* is 0, 2, 4, 6, or 8.

The numbers 80, 112, 64, 326, and 1,008 are all divisible by 2 since the last digit of each is 0, 2, 4, 6, or 8, respectively.

The numbers 85 and 731 are *not* divisible by 2.

Division by 3
A whole number is **divisible by 3** if the *sum of its digits* is divisible by 3.

The number 432 is divisible by 3 since $4+3+2=9$ and 9 is divisible by 3.

$432 \div 3 = 144$

The number 25 is *not* divisible by 3 since $2+5=7$, and 7 is not divisible by 3.

Division by 4

A whole number is **divisible by 4** if its *last two digits* form a number that is divisible by 4.

The number 31,048 is divisible by 4 since the last two digits, 4 and 8, form a number, 48, that is divisible by 4.

$31048 \div 4 = 7262$

The number 137 is not divisible by 4 since 37 is not divisible by 4.

Division by 5
A whole number is **divisible by 5** if its *last digit* is 0 or 5.

Sample Set A

Example:
The numbers 65, 110, 8,030, and 16,955 are each divisible by 5 since the last digit of each is 0 or 5.

Practice Set A

State which of the following whole numbers are divisible by 2, 3, 4, or 5. A number may be divisible by more than one number.
Exercise:

Problem: 26

Solution:

2

Exercise:

Problem: 81

Solution:

3

Exercise:

Problem: 51

Solution:

3

Exercise:

Problem: 385

Solution:

5

Exercise:

Problem: 6,112

Solution:

2, 4

Exercise:

Problem: 470

Solution:

2, 5

Exercise:

Problem: 113,154

Solution:

2, 3

Division by 6, 8, 9, 10

Division by 6
A number is **divisible by 6** if it is divisible by *both* 2 and 3.

The number 234 is divisible by 2 since its last digit is 4. It is also divisible by 3 since $2 + 3 + 4 = 9$ and 9 is divisible by 3. Therefore, 234 is divisible by 6.

The number 6,532 is *not* divisible by 6. Although its last digit is 2, making it divisible by 2, the sum of its digits, $6 + 5 + 3 + 2 = 16$, and 16 is not divisible by 3.

Division by 8
A whole number is **divisible by 8** if its *last three digits* form a number that is divisible by 8.
The number 4,000 is divisible by 8 since 000 is divisible by 8.
The number 13,128 is divisible by 8 since 128 is divisible by 8.
The number 1,170 is *not* divisible by 8 since 170 is not divisible by 8.

Division by 9
A whole number is **divisible by 9** if the *sum of its digits* is divisible by 9.
The number 702 is divisible by 9 since $7 + 0 + 2$ is divisible by 9.
The number 6588 is divisible by 9 since $6 + 5 + 8 + 8 = 27$ is divisible by 9.
The number 14,123 is *not* divisible by 9 since $1 + 4 + 1 + 2 + 3 = 11$ is not divisible by 9.

Division by 10

A Whole number is **divisible by 10** if its *last digit* is 0.

Sample Set B

Example:
The numbers 30, 170, 16,240, and 865,000 are all divisible by 10.

Practice Set B

State which of the following whole numbers are divisible 6, 8, 9, or 10. Some numbers may be divisible by more than one number.

Exercise:

Problem: 900

Solution:

6, 9, 10

Exercise:

Problem: 6,402

Solution:

6

Exercise:

Problem: 6,660

Solution:

6, 9, 10

Exercise:

Problem: 55,116

Solution:

6, 9

Exercises

For the following 30 problems, specify if the whole number is divisible by 2, 3, 4, 5, 6, 8, 9, or 10. Write "none" if the number is not divisible by any digit other than 1. Some numbers may be divisible by more than one number.

Exercise:

Problem: 48

Solution:

2, 3, 4, 6, 8

Exercise:

Problem: 85

Exercise:

Problem: 30

Solution:

2, 3, 5, 6, 10

Exercise:

Problem: 83

Exercise:

Problem: 98

Solution:

2

Exercise:

Problem: 972

Exercise:

Problem: 892

Solution:

2, 4

Exercise:

Problem: 676

Exercise:

Problem: 903

Solution:

3

Exercise:

Problem: 800

Exercise:

Problem: 223

Solution:

none

Exercise:

Problem: 836

Exercise:

Problem: 665

Solution:

5

Exercise:

Problem: 4,381

Exercise:

Problem: 2,195

Solution:

5

Exercise:

Problem: 2,544

Exercise:

Problem: 5,172

Solution:

2, 3, 4, 6

Exercise:

Problem: 1,307

Exercise:

Problem: 1,050

Solution:

2, 3, 5, 6, 10

Exercise:

Problem: 3,898

Exercise:

Problem: 1,621

Solution:

none

Exercise:

Problem: 27,808

Exercise:

Problem: 45,764

Solution:

2, 4

Exercise:

Problem: 49,198

Exercise:

Problem: 296,122

Solution:

2

Exercise:

Problem: 178,656

Exercise:

Problem: 5,102,417

Solution:

none

Exercise:

Problem: 16,990,792

Exercise:

Problem: 620,157,659

Solution:

none

Exercise:

Problem: 457,687,705

Exercises for Review

Exercise:

Problem: ([link]) In the number 412, how many tens are there?

Solution:

1

Exercise:

Problem: ([link]) Subtract 613 from 810.

Exercise:

Problem: ([link]) Add 35, 16, and 7 in two different ways.

Solution:

$$(35 + 16) + 7 = 51 + 7 = 58$$
$$35 + (16 + 7) = 35 + 23 = 58$$

Exercise:

Problem: ([link]) Find the quotient $35 \div 0$, if it exists.

Exercise:

Problem: ([link]) Find the quotient. $3654 \div 42$.

Solution:

Properties of Multiplication
This module is from Fundamentals of Mathematics by Denny Burzynski and Wade Ellis, Jr. This module discusses properties of multiplication of whole numbers. By the end of the module students should be able to understand and appreciate the commutative and associative properties of multiplication and understand why 1 is the multiplicative identity.

Section Overview

- The Commutative Property of Multiplication
- The Associative Property of Multiplication
- The Multiplicative Identity

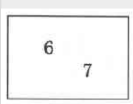
We will now examine three simple but very important properties of multiplication.

The Commutative Property of Multiplication

Commutative Property of Multiplication
The product of two whole numbers is the same regardless of the order of the factors.

Sample Set A

Example:
Multiply the two whole numbers.

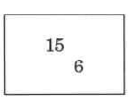


$6 \cdot 7 = 42$
$7 \cdot 6 = 42$
The numbers 6 and 7 can be multiplied in any order. Regardless of the order they are multiplied, the product is 42.

Practice Set A

Use the commutative property of multiplication to find the products in two ways.
Exercise:

Problem:

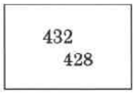


Solution:

$15 \cdot 6 = 90$ and $6 \cdot 15 = 90$

Exercise:

Problem:



Solution:

$432 \cdot 428 = 184{,}896$ and $428 \cdot 432 = 184{,}896$

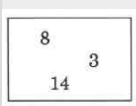
The Associative Property of Multiplication

Associative Property of Multiplication
If three whole numbers are multiplied, the product will be the same if the first two are multiplied first and then that product is multiplied by the third, or if the second two are multiplied first and that product is multiplied by the first. Note that the order of the factors is maintained.

It is a common mathematical practice to *use parentheses* to show which pair of numbers is to be combined first.

Sample Set B

Example:
Multiply the whole numbers.

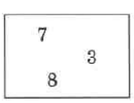


$(8 \cdot 3) \cdot 14 = 24 \cdot 14 = 336$
$8 \cdot (3 \cdot 14) = 8 \cdot 42 = 336$

Practice Set B

Use the associative property of multiplication to find the products in two ways.
Exercise:

Problem:

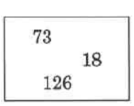


Solution:

168

Exercise:

Problem:



Solution:

165,564

The Multiplicative Identity

The Multiplicative Identity is 1
The whole number 1 is called the **multiplicative identity**, since any whole number multiplied by 1 is not changed.

Sample Set C

Example:
Multiply the whole numbers.

12
1

$12 \cdot 1 = 12$
$1 \cdot 12 = 12$

Practice Set C

Multiply the whole numbers.
Exercise:

Problem:

843
1

Solution:

843

Exercises

For the following problems, multiply the numbers.
Exercise:

Problem:

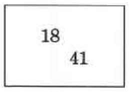
9
26

Solution:

234

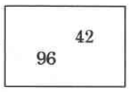
Exercise:

Problem:



Exercise:

Problem:

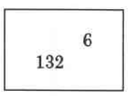


Solution:

4,032

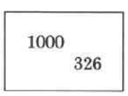
Exercise:

Problem:



Exercise:

Problem:

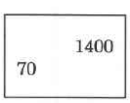


Solution:

326,000

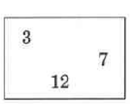
Exercise:

Problem:



Exercise:

Problem:

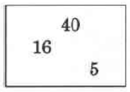


Solution:

252

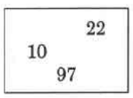
Exercise:

Problem:



Exercise:

Problem:

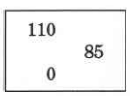


Solution:

21,340

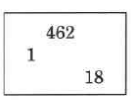
Exercise:

Problem:



Exercise:

Problem:

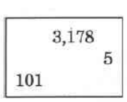


Solution:

8,316

Exercise:

Problem:



For the following 4 problems, show that the quantities yield the same products by performing the multiplications.

Exercise:

Problem: $(4 \cdot 8) \cdot 2$ and $4 \cdot (8 \cdot 2)$

Solution:

$32 \cdot 2 = 64 = 4 \cdot 16$

Exercise:

Problem: $(100 \cdot 62) \cdot 4$ and $100 \cdot (62 \cdot 4)$

Exercise:

Problem: $23 \cdot (11 \cdot 106)$ and $(23 \cdot 11) \cdot 106$

Solution:

$23 \cdot 1{,}166 = 26{,}818 = 253 \cdot 106$

Exercise:

Problem: $1 \cdot (5 \cdot 2)$ and $(1 \cdot 5) \cdot 2$

Exercise:

Problem:

The fact that
$(\text{a first number} \cdot \text{a second number}) \cdot \text{a third number} = \text{a first number} \cdot (\text{a second number} \cdot \text{a third num}$
is an example of the property of multiplication.

Solution:

associative

Exercise:

Problem:

The fact that $1 \cdot \text{any number} = \text{that particular number}$is an example of the property of multiplication.

Exercise:

Problem: Use the numbers 7 and 9 to illustrate the commutative property of multiplication.

Solution:

$7 \cdot 9 = 63 = 9 \cdot 7$

Exercise:

Problem: Use the numbers 6, 4, and 7 to illustrate the associative property of multiplication.

Exercises for Review

Exercise:

Problem: ([link]) In the number 84,526,098,441, how many millions are there?

Solution:

6

Exercise:

Problem: ([link]) Replace the letter m with the whole number that makes the addition true. $\begin{array}{r} 85 \\ + \quad m \\ \hline 97 \end{array}$

Exercise:

Problem: ([link]) Use the numbers 4 and 15 to illustrate the commutative property of addition.

Solution:

$4 + 15 = 19$

$15 + 4 = 19$

Exercise:

Problem: ([link]) Find the product. $8{,}000{,}000 \times 1{,}000$.

Exercise:

Problem: ([link]) Specify which of the digits 2, 3, 4, 5, 6, 8,10 are divisors of the number 2,244.

Solution:

2, 3, 4, 6

Summary of Key Concepts
This module is from Fundamentals of Mathematics by Denny Burzynski and Wade Ellis, Jr. This module summarizes the concepts discussed in the chapter "Multiplication and Division of Whole Numbers."

Summary of Key Concepts

Multiplication ([link])
Multiplication is a description of repeated addition.

$$\underbrace{7+7+7+7}_{\text{7 appears 4 times}}$$

This expression is described by writing 4 X 7.

Multiplicand/Multiplier/Product ([link])
In a multiplication of whole numbers, the repeated addend is called the **multiplicand**, and the number that records the number of times the multiplicand is used is the **multiplier**. The result of the multiplication is the **product**.

Factors ([link])
In a multiplication, the numbers being multiplied are also called **factors**. Thus, the multiplicand and the multiplier can be called factors.

Division ([link])
Division is a description of repeated subtraction.

Dividend/Divisor/Quotient ([link])
In a division, the number divided into is called the **dividend**, and the number dividing into the dividend is called the **divisor**. The result of the division is called the **quotient**.

$$\begin{array}{r} \text{quotient} \\ \text{divisor}\overline{)\text{dividend}} \end{array}$$

Division into Zero ([link])
Zero divided by any nonzero whole number is zero.

Division by Zero ([link])

Division by zero does not name a whole number. It is, therefore, undefined. The quotient $\frac{0}{0}$ is indeterminant.

Division by 2, 3, 4, 5, 6, 8, 9, 10 ([link])
Division by the whole numbers 2, 3, 4, 5, 6, 8, 9, and 10 can be determined by noting some certain properties of the particular whole number.

Commutative Property of Multiplication ([link])
The product of two whole numbers is the same regardless of the order of the factors. $3 \times 5 = 5 \times 3$

Associative Property of Multiplication ([link])
If three whole numbers are to be multiplied, the product will be the same if the first two are multiplied first and then that product is multiplied by the third, or if the second two are multiplied first and then that product is multiplied by the first.
$(3 \times 5) \times 2 = 3 \times (5 \times 2)$
Note that the order of the factors is maintained.

Multiplicative Identity ([link])
The whole number 1 is called the **multiplicative identity** since any whole number multiplied by 1 is not changed.
$4 \times 1 = 4$
$1 \times 4 = 4$

Exercise Supplement
This module is from Fundamentals of Mathematics by Denny Burzynski and Wade Ellis, Jr. This module is an exercise supplement for the chapter "Multiplication and Division of Whole Numbers" and contains many exercise problems. Odd problems are accompanied by solutions.

Exercise Supplement

Multiplication of Whole Numbers ([link])

Exercise:

Problem:

In the multiplication $5 \times 9 = 45$, 5 and 9 are called and 45 is called the .

Solution:

factors; product

Exercise:

Problem:

In the multiplication $4 \times 8 = 32$, 4 and 8 are called and 32 is called the .

Concepts of Division of Whole Numbers ([link])

Exercise:

Problem:

In the division $24 \div 6 = 4$, 6 is called the , and 4 is called the .

Solution:

divisor; quotient

Exercise:

Problem:

In the division $36 \div 2 = 18$, 2 is called the , and 18 is called the .

Some Interesting Facts about Division ([link])

Exercise:

Problem: A number is divisible by 2 only if its last digit is .

Solution:

an even digit (0, 2, 4, 6, or 8)

Exercise:

Problem:

A number is divisible by 3 only if of its digits is divisible by 3.

Exercise:

Problem:

A number is divisible by 4 only if the rightmost two digits form a number that is .

Solution:

divisible by 4

Multiplication and Division of Whole Numbers ([link],[link])

Find each product or quotient.

Exercise:

Problem: $\begin{array}{r} 24 \\ \times\ 3 \end{array}$

Exercise:

Problem: $\begin{array}{r} 14 \\ \times\ 8 \end{array}$

Solution:

112

Exercise:

Problem: $21 \div 7$

Exercise:

Problem: $35 \div 5$

Solution:

7

Exercise:

Problem: $\begin{array}{r} 36 \\ \times 22 \end{array}$

Exercise:

Problem: $\begin{array}{r} 87 \\ \times 35 \end{array}$

Solution:

3,045

Exercise:

Problem: $\begin{array}{r} 117 \\ \times 42 \end{array}$

Exercise:

Problem: $208 \div 52$

Solution:

4

Exercise:

Problem: $\begin{array}{r} 521 \\ \times \;\; 87 \end{array}$

Exercise:

Problem: $\begin{array}{r} 1005 \\ \times \;\;\; 15 \end{array}$

Solution:

15,075

Exercise:

Problem: $1338 \div 446$

Exercise:

Problem: $2814 \div 201$

Solution:

14

Exercise:

Problem: $\begin{array}{r} 5521 \\ \times 8 \end{array}$

Exercise:

Problem: $\begin{array}{r} 6016 \\ \times \quad 7 \end{array}$

Solution:

42,112

Exercise:

Problem: $576 \div 24$

Exercise:

Problem: $3969 \div 63$

Solution:

63

Exercise:

Problem: $\begin{array}{r} 5482 \\ \times \quad 322 \end{array}$

Exercise:

Problem: $\begin{array}{r} 9104 \\ \times \;\; 115 \end{array}$

Solution:

1,046,960

Exercise:

Problem: $\begin{array}{r} 6102 \\ \times 1000 \end{array}$

Exercise:

Problem: $\begin{array}{r} 10101 \\ \times 10000 \end{array}$

Solution:

101,010,000

Exercise:

Problem: $162{,}006 \div 31$

Exercise:

Problem: $0 \div 25$

Solution:

0

Exercise:

Problem: $25 \div 0$

Exercise:

Problem: $4280 \div 10$

Solution:

428

Exercise:

Problem: $2126000 \div 100$

Exercise:

Problem: $84 \div 15$

Solution:

5 remainder 9

Exercise:

Problem: $126 \div 4$

Exercise:

Problem: $424 \div 0$

Solution:

not defined

Exercise:

Problem: $1198 \div 46$

Exercise:

Problem: $995 \div 31$

Solution:

32 remainder 3

Exercise:

Problem: $0 \div 18$

Exercise:

Problem: $\begin{array}{r} 2162 \\ \times 1421 \end{array}$

Solution:

3,072,202

Exercise:

Problem: 0×0

Exercise:

Problem: 5×0

Solution:

0

Exercise:

Problem: 64×1

Exercise:

Problem: 1×0

Solution:

0

Exercise:

Problem: $0 \div 3$

Exercise:

Problem: $14 \div 0$

Solution:

not defined

Exercise:

Problem: $35 \div 1$

Exercise:

Problem: $1 \div 1$

Solution:

1

Properties of Multiplication ([link])

Exercise:

Problem:

Use the commutative property of multiplication to rewrite 36×128.

Exercise:

Problem:

Use the commutative property of multiplication to rewrite 114×226.

Solution:

$226 \cdot 114$

Exercise:

Problem:

Use the associative property of multiplication to rewrite $(5 \cdot 4) \cdot 8$.

Exercise:

Problem:

Use the associative property of multiplication to rewrite $16 \cdot (14 \cdot 0)$.

Solution:

$(16 \cdot 14) \cdot 0$

Multiplication and Division of Whole Numbers ([link],[link])

Exercise:

Problem:

A computer store is selling diskettes for $4 each. At this price, how much would 15 diskettes cost?

Exercise:

Problem:

Light travels 186,000 miles in one second. How far does light travel in 23 seconds?

Solution:

4,278,000

Exercise:

Problem:

A dinner bill for eight people comes to exactly $112. How much should each person pay if they all agree to split the bill equally?

Exercise:

Problem:

Each of the 33 students in a math class buys a textbook. If the bookstore sells $1089 worth of books, what is the price of each book?

Solution:

$33

Proficiency Exam
This module is from Fundamentals of Mathematics by Denny Burzynski and Wade Ellis, Jr. This module is a proficiency exam to the chapter "Multiplication and Division of Whole Numbers." Each problem is accompanied with a reference link pointing back to the module that discusses the type of problem demonstrated in the question. The problems in this exam are accompanied by solutions.

Proficiency Exam

Exercise:

Problem:

([link]) In the multiplication of $8 \times 7 = 56$, what are the names given to the 8 and 7 and the 56?

Solution:

8 and 7 are factors; 56 is the product

Exercise:

Problem:

([link]) Multiplication is a description of what repeated process?

Solution:

Addition

Exercise:

Problem:

([link]) In the division $12 \div 3 = 4$, what are the names given to the 3 and the 4?

Solution:

3 is the divisor; 4 is the quotient

Exercise:

Problem:

([link]) Name the digits that a number must end in to be divisible by 2.

Solution:

0, 2, 4, 6, or 8

Exercise:

Problem:

([link]) Name the property of multiplication that states that the order of the factors in a multiplication can be changed without changing the product.

Solution:

commutative

Exercise:

Problem: ([link]) Which number is called the multiplicative identity?

Solution:

1

For problems 7-17, find the product or quotient.

Exercise:

Problem: ([link]) 14×6

Solution:

84

Exercise:

Problem: ([link]) 37×0

Solution:

0

Exercise:

Problem: ([link]) 352×1000

Solution:

352,000

Exercise:

Problem: ([link]) 5986×70

Solution:

419,020

Exercise:

Problem: ([link]) 12×12

Solution:

252

Exercise:

Problem: ([link]) $856 \div 0$

Solution:

not defined

Exercise:

Problem: ([link]) $0 \div 8$

Solution:

0

Exercise:

Problem: ([link]) $136 \div 8$

Solution:

17

Exercise:

Problem: ([link]) $432 \div 24$

Solution:

18

Exercise:

Problem: ([link]) $5286 \div 37$

Solution:

142 remainder 32

Exercise:

Problem: ([link]) 211×1

Solution:

211

For problems 18-20, use the numbers 216, 1,005, and 640.

Exercise:

Problem: ([link]) Which numbers are divisible by 3?

Solution:

216; 1,005

Exercise:

Problem: ([link]) Which number is divisible by 4?

Solution:

216; 640

Exercise:

Problem: ([link]) Which number(s) is divisible by 5?

Solution:

1,005; 640

Objectives
This module contains the learning objectives for the chapter "Exponents, Roots, and Factorizations of Whole Numbers" from Fundamentals of Mathematics by Denny Burzynski and Wade Ellis, jr.

After completing this chapter, you should

Exponents and Roots ([link])

- understand and be able to read exponential notation
- understand the concept of root and be able to read root notation
- be able to use a calculator having the y^x key to determine a root

Grouping Symbols and the Order of Operations ([link])

- understand the use of grouping symbols
- understand and be able to use the order of operations
- use the calculator to determine the value of a numerical expression

Prime Factorization of Natural Numbers ([link])

- be able to determine the factors of a whole number
- be able to distinguish between prime and composite numbers
- be familiar with the fundamental principle of arithmetic
- be able to find the prime factorization of a whole number

The Greatest Common Factor ([link])

- be able to find the greatest common factor of two or more whole numbers

The Least Common Multiple ([link])

- be able to find the least common multiple of two or more whole numbers

Exponents and Roots
This module is from Fundamentals of Mathematics by Denny Burzynski and Wade Ellis, Jr. This module discusses exponents and roots. By the end of the module students should be able to understand and be able to read exponential notation, understand the concept of root and be able to read root notation, and use a calculator having the y^x key to determine a root.

Section Overview

- Exponential Notation
- Reading Exponential Notation
- Roots
- Reading Root Notation
- Calculators

Exponential Notation

Exponential Notation
We have noted that multiplication is a description of repeated addition. **Exponential notation** is a description of repeated multiplication.

Suppose we have the repeated multiplication

$8 \cdot 8 \cdot 8 \cdot 8 \cdot 8$

Exponent
The factor 8 is repeated 5 times. Exponential notation uses a *superscript* for the number of times the factor is repeated. The superscript is placed on the repeated factor, 8^5, in this case. The superscript is called an **exponent**.

The Function of an Exponent
An **exponent** records the number of identical factors that are repeated in a multiplication.

Sample Set A

Write the following multiplication using exponents.

Example:
$3 \cdot 3$. Since the factor 3 appears 2 times, we record this as
3^2

Example:
$62 \cdot 62 \cdot 62 \cdot 62 \cdot 62 \cdot 62 \cdot 62 \cdot 62 \cdot 62$. Since the factor 62 appears 9 times, we record this as
62^9

Expand (write without exponents) each number.

Example:
12^4. The exponent 4 is recording 4 factors of 12 in a multiplication. Thus,
$12^4 = 12 \cdot 12 \cdot 12 \cdot 12$

Example:
706^3. The exponent 3 is recording 3 factors of 706 in a multiplication. Thus,
$706^3 = 706 \cdot 706 \cdot 706$

Practice Set A

Write the following using exponents.
Exercise:

Problem: $37 \cdot 37$

Solution:

37^2

Exercise:

Problem: $16 \cdot 16 \cdot 16 \cdot 16 \cdot 16$

Solution:

16^5

Exercise:

Problem: $9 \cdot 9 \cdot 9 \cdot 9 \cdot 9 \cdot 9 \cdot 9 \cdot 9 \cdot 9 \cdot 9$

Solution:

9^{10}

Write each number without exponents.

Exercise:

Problem: 85^3

Solution:

$85 \cdot 85 \cdot 85$

Exercise:

Problem: 4^7

Solution:

$4 \cdot 4 \cdot 4 \cdot 4 \cdot 4 \cdot 4 \cdot 4$

Exercise:

Problem: $1{,}739^2$

Solution:

$1{,}739 \cdot 1{,}739$

Reading Exponential Notation

In a number such as 8^5,

Base
8 is called the **base**.

Exponent, Power
5 is called the **exponent**, or **power**. 8^5 is read as "eight to the fifth power," or more simply as "eight to the fifth," or "the fifth power of eight."

Squared
When a whole number is raised to the second power, it is said to be **squared**. The number 5^2 can be read as

5 to the second power, or
5 to the second, or
5 squared.

Cubed
When a whole number is raised to the third power, it is said to be **cubed**. The number 5^3 can be read as

5 to the third power, or
5 to the third, or
5 cubed.

When a whole number is raised to the power of 4 or higher, we simply say that that number is raised to that particular power. The number 5^8 can be read as

5 to the eighth power, or just
5 to the eighth.

Roots

In the English language, the word "root" can mean a source of something. In mathematical terms, the word "root" is used to indicate that one number is the source of another number through repeated multiplication.

Square Root
We know that $49 = 7^2$, that is, $49 = 7 \cdot 7$. Through repeated multiplication, 7 is the source of 49. Thus, 7 is a root of 49. Since two 7's must be multiplied together to produce 49, the 7 is called the second or **square root** of 49.

Cube Root
We know that $8 = 2^3$, that is, $8 = 2 \cdot 2 \cdot 2$. Through repeated multiplication, 2 is the source of 8. Thus, 2 is a root of 8. Since three 2's must be multiplied together to produce 8, 2 is called the third or **cube root** of 8.

We can continue this way to see such roots as fourth roots, fifth roots, sixth roots, and so on.

Reading Root Notation

There is a symbol used to indicate roots of a number. It is called the radical sign $\sqrt[n]{\ }$

The Radical Sign $\sqrt[n]{\ }$
The symbol $\sqrt[n]{\ }$ is called a **radical sign** and indicates the nth root of a number.

We discuss *particular roots* using the radical sign as follows:

Square Root
$\sqrt[2]{\text{number}}$ indicates the **square root** of the number under the radical sign. It is customary to drop the 2 in the radical sign when discussing square roots. The symbol $\sqrt{}$ is understood to be the square root radical sign.

$\sqrt{49}$ = 7 since $7 \cdot 7 = 7^2 = 49$

Cube Root
$\sqrt[3]{\text{number}}$ indicates the **cube root** of the number under the radical sign.

$\sqrt[3]{8} = 2$ since $2 \cdot 2 \cdot 2 = 2^3 = 8$

Fourth Root
$\sqrt[4]{\text{number}}$ indicates the **fourth root** of the number under the radical sign.

$\sqrt[4]{81} = 3$ since $3 \cdot 3 \cdot 3 \cdot 3 = 3^4 = 81$

In an expression such as $\sqrt[5]{32}$

Radical Sign
$\sqrt{}$ is called the **radical sign**.

Index
5 is called the **index**. (The index describes the indicated root.)

Radicand
32 is called the **radicand**.

Radical
$\sqrt[5]{32}$ is called a **radical** (or radical expression).

Sample Set B

Find each root.

Example:
$\sqrt{25}$ To determine the square root of 25, we ask, "What whole number squared equals 25?" From our experience with multiplication, we know this number to be 5. Thus,
$\sqrt{25} = 5$
Check: $5 \cdot 5 = 5^2 = 25$

Example:
$\sqrt[5]{32}$ To determine the fifth root of 32, we ask, "What whole number raised to the fifth power equals 32?" This number is 2.
$\sqrt[5]{32} = 2$
Check: $2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 = 2^5 = 32$

Practice Set B

Find the following roots using only a knowledge of multiplication.
Exercise:

Problem: $\sqrt{64}$

Solution:

8

Exercise:

Problem: $\sqrt{100}$

Solution:

10

Exercise:

Problem: $\sqrt[3]{64}$

Solution:

4

Exercise:

Problem: $\sqrt[6]{64}$

Solution:

2

Calculators

Calculators with the $\sqrt{x}$, y^x, and $1/x$ keys can be used to find or approximate roots.

Sample Set C

Example:
Use the calculator to find $\sqrt{121}$

		Display Reads
Type	121	121
Press	$\sqrt{x}$	11

Example:
Find $\sqrt[7]{2187}$.

		Display Reads
Type	2187	2187
Press	y^x	2187
Type	7	7
Press	$1/x$	.14285714
Press	=	3

$\sqrt[7]{2187} = 3$ (Which means that $3^7 = 2187$.)

Practice Set C

Use a calculator to find the following roots.
Exercise:

Problem: $\sqrt[3]{729}$

Solution:

9

Exercise:

Problem: $\sqrt[4]{8503056}$

Solution:

54

Exercise:

Problem: $\sqrt{53361}$

Solution:

231

Exercise:

Problem: $\sqrt[12]{16777216}$

Solution:

4

Exercises

For the following problems, write the expressions using exponential notation.

Exercise:

Problem: $4 \cdot 4$

Solution:

4^2

Exercise:

Problem: $12 \cdot 12$

Exercise:

Problem: $9 \cdot 9 \cdot 9 \cdot 9$

Solution:

9^4

Exercise:

Problem: $10 \cdot 10 \cdot 10 \cdot 10 \cdot 10 \cdot 10$

Exercise:

Problem: $826 \cdot 826 \cdot 826$

Solution:

826^3

Exercise:

Problem: $3{,}021 \cdot 3{,}021 \cdot 3{,}021 \cdot 3{,}021 \cdot 3{,}021$

Exercise:

Problem: $\underbrace{6 \cdot 6 \cdots\cdots 6}_{\text{85 factors of 6}}$

Solution:

6^{85}

Exercise:

Problem: $\underbrace{2 \cdot 2 \cdots\cdots 2}_{\text{112 factors of 2}}$

Exercise:

Problem: $\underbrace{1 \cdot 1 \cdots\cdots 1}_{\text{3,008 factors of 1}}$

Solution:

1^{3008}

For the following problems, expand the terms. (Do not find the actual value.)

Exercise:

Problem: 5^3

Exercise:

Problem: 7^4

Solution:

$7 \cdot 7 \cdot 7 \cdot 7$

Exercise:

Problem: 15^2

Exercise:

Problem: 117^5

Solution:

$117 \cdot 117 \cdot 117 \cdot 117 \cdot 117$

Exercise:

Problem: 61^6

Exercise:

Problem: 30^2

Solution:

$30 \cdot 30$

For the following problems, determine the value of each of the powers. Use a calculator to check each result.

Exercise:

Problem: 3^2

Exercise:

Problem: 4^2

Solution:

$4 \cdot 4 = 16$

Exercise:

Problem: 1^2

Exercise:

Problem: 10^2

Solution:

$10 \cdot 10 = 100$

Exercise:

Problem: 11^2

Exercise:

Problem: 12^2

Solution:

$12 \cdot 12 = 144$

Exercise:

Problem: 13^2

Exercise:

Problem: 15^2

Solution:

$15 \cdot 15 = 225$

Exercise:

Problem: 1^4

Exercise:

Problem: 3^4

Solution:

$3 \cdot 3 \cdot 3 \cdot 3 = 81$

Exercise:

Problem: 7^3

Exercise:

Problem: 10^3

Solution:

$10 \cdot 10 \cdot 10 = 1{,}000$

Exercise:

Problem: 100^2

Exercise:

Problem: 8^3

Solution:

$8 \cdot 8 \cdot 8 = 512$

Exercise:

Problem: 5^5

Exercise:

Problem: 9^3

Solution:

$9 \cdot 9 \cdot 9 = 729$

Exercise:

Problem: 6^2

Exercise:

Problem: 7^1

Solution:

$7^1 = 7$

Exercise:

Problem: 1^{28}

Exercise:

Problem: 2^7

Solution:

$2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 = 128$

Exercise:

Problem: 0^5

Exercise:

Problem: 8^4

Solution:

$8 \cdot 8 \cdot 8 \cdot 8 = 4{,}096$

Exercise:

Problem: 5^8

Exercise:

Problem: 6^9

Solution:

$6 \cdot 6 \cdot 6 \cdot 6 \cdot 6 \cdot 6 \cdot 6 \cdot 6 \cdot 6 = 10{,}077{,}696$

Exercise:

Problem: 25^3

Exercise:

Problem: 42^2

Solution:

$42 \cdot 42 = 1{,}764$

Exercise:

Problem: 31^3

Exercise:

Problem: 15^5

Solution:

$15 \cdot 15 \cdot 15 \cdot 15 \cdot 15 = 759{,}375$

Exercise:

Problem: 2^{20}

Exercise:

Problem: 816^2

Solution:

$816 \cdot 816 = 665{,}856$

For the following problems, find the roots (using your knowledge of multiplication). Use a calculator to check each result.

Exercise:

Problem: $\sqrt{9}$

Exercise:

Problem: $\sqrt{16}$

Solution:

4

Exercise:

Problem: $\sqrt{36}$

Exercise:

Problem: $\sqrt{64}$

Solution:

8

Exercise:

Problem: $\sqrt{121}$

Exercise:

Problem: $\sqrt{144}$

Solution:

12

Exercise:

Problem: $\sqrt{169}$

Exercise:

Problem: $\sqrt{225}$

Solution:

15

Exercise:

Problem: $\sqrt[3]{27}$

Exercise:

Problem: $\sqrt[5]{32}$

Solution:

2

Exercise:

Problem: $\sqrt[4]{256}$

Exercise:

Problem: $\sqrt[3]{216}$

Solution:

6

Exercise:

Problem: $\sqrt[7]{1}$

Exercise:

Problem: $\sqrt{400}$

Solution:

20

Exercise:

Problem: $\sqrt{900}$

Exercise:

Problem: $\sqrt{10,000}$

Solution:

100

Exercise:

Problem: $\sqrt{324}$

Exercise:

Problem: $\sqrt{3,600}$

Solution:

60

For the following problems, use a calculator with the keys $\sqrt{x}$, y^x, and $1/x$ to find each of the values.

Exercise:

Problem: $\sqrt{676}$

Exercise:

Problem: $\sqrt{1,156}$

Solution:

34

Exercise:

Problem: $\sqrt{46,225}$

Exercise:

Problem: $\sqrt{17,288,964}$

Solution:

4,158

Exercise:

Problem: $\sqrt[3]{3,375}$

Exercise:

Problem: $\sqrt[4]{331,776}$

Solution:

24

Exercise:

Problem: $\sqrt[8]{5,764,801}$

Exercise:

Problem: $\sqrt[12]{16,777,216}$

Solution:

4

Exercise:

Problem: $\sqrt[8]{16,777,216}$

Exercise:

Problem: $\sqrt[10]{9,765,625}$

Solution:

5

Exercise:

Problem: $\sqrt[4]{160{,}000}$

Exercise:

Problem: $\sqrt[3]{531{,}441}$

Solution:

81

Exercises for Review

Exercise:

Problem:

([link]) Use the numbers 3, 8, and 9 to illustrate the associative property of addition.

Exercise:

Problem:

([link]) In the multiplication $8 \cdot 4 = 32$, specify the name given to the numbers 8 and 4.

Solution:

8 is the multiplier; 4 is the multiplicand

Exercise:

Problem: ([link]) Does the quotient $15 \div 0$ exist? If so, what is it?

Exercise:

Problem: ([link]) Does the quotient $0 \div 15$exist? If so, what is it?

Solution:

Yes; 0

Exercise:

Problem:

([link]) Use the numbers 4 and 7 to illustrate the commutative property of multiplication.

Grouping Symbols and the Order of Operations
This module is from Fundamentals of Mathematics by Denny Burzynski and Wade Ellis, Jr. This module discusses grouping symbols and the order of operations. By the end of the module students should be able to understand the use of grouping symbols, understand and be able to use the order of operations and use the calculator to determine the value of a numerical expression.

Section Overview

- Grouping Symbols
- Multiple Grouping Symbols
- The Order of Operations
- Calculators

Grouping Symbols

Grouping symbols are used to indicate that a particular collection of numbers and meaningful operations are to be grouped together and considered as one number. The grouping symbols commonly used in mathematics are the following:

( ), [ ], { },
Parentheses: ( )
Brackets: [ ]
Braces: { }
Bar:

In a computation in which more than one operation is involved, grouping symbols indicate which operation to perform first. If possible, we perform operations *inside* grouping symbols first.

Sample Set A

If possible, determine the value of each of the following.

Example:
$9+(3\cdot 8)$
Since 3 and 8 are within parentheses, they are to be combined first.
$$\begin{aligned} 9+(3\cdot 8) &= 9+24 \\ &= 33 \end{aligned}$$
Thus,
$9+(3\cdot 8)=33$

Example:
$(10\div 0)\cdot 6$
Since $10\div 0$ is undefined, this operation is meaningless, and we attach no value to it. We write, "undefined."

Practice Set A

If possible, determine the value of each of the following.

Exercise:

Problem: $16 - (3 \cdot 2)$

Solution:

10

Exercise:

Problem: $5 + (7 \cdot 9)$

Solution:

68

Exercise:

Problem: $(4 + 8) \cdot 2$

Solution:

24

Exercise:

Problem: $28 \div (18 - 11)$

Solution:

4

Exercise:

Problem: $(33 \div 3) - 11$

Solution:

0

Exercise:

Problem: $4 + (0 \div 0)$

Solution:

not possible (indeterminant)

Multiple Grouping Symbols

When a set of grouping symbols occurs *inside* another set of grouping symbols, we perform the operations within the innermost set first.

Sample Set B

Determine the value of each of the following.

Example:
$2+(8\cdot 3)-(5+6)$
Combine 8 and 3 first, then combine 5 and 6.
$2+24-11\quad \text{Now combine left to right.}$

$26-11$

15

Example:
$10+[30-(2\cdot 9)]$
Combine 2 and 9 since they occur in the innermost set of parentheses.
$10+[30-18]\quad \text{Now combine 30 and 18.}$

$10+12$

22

Practice Set B

Determine the value of each of the following.
Exercise:

Problem: $(17+8)+(9+20)$

Solution:

54

Exercise:

Problem: $(55-6)-(13\cdot 2)$

Solution:

23

Exercise:

Problem: $23+(12 \div 4)-(11 \cdot 2)$

Solution:

4

Exercise:

Problem: $86+[14 \div (10-8)]$

Solution:

93

Exercise:

Problem: $31+\{9+[1+(35-2)]\}$

Solution:

74

Exercise:

Problem: $\{6-[24 \div (4 \cdot 2)]\}^3$

Solution:

27

The Order of Operations

Sometimes there are no grouping symbols indicating which operations to perform first. For example, suppose we wish to find the value of $3+5 \cdot 2$. We could do *either* of two things:

Add 3 and 5, then multiply this sum by 2.

$$\begin{aligned} 3+5 \cdot 2 &= 8 \cdot 2 \\ &= 16 \end{aligned}$$

Multiply 5 and 2, then add 3 to this product.

$$\begin{aligned} 3+5 \cdot 2 &= 3+10 \\ &= 13 \end{aligned}$$

We now have two values for one number. To determine the correct value, we must use the *accepted order of operations*.

Order of Operations

1. Perform all operations *inside* grouping symbols, beginning with the innermost set, in the order 2, 3, 4 described below,
2. Perform all exponential and root operations.
3. Perform all multiplications and divisions, moving left to right.
4. Perform all additions and subtractions, moving left to right.

Sample Set C

Determine the value of each of the following.

Example:

$21 + 3 \cdot 12$	Multiply first.
$21 + 36$	Add.
57	

Example:

$(15 - 8) + 5 \cdot (6 + 4).$	Simplify inside parentheses first.
$7 + 5 \cdot 10$	Multiply.
$7 + 50$	Add.
57	

Example:

$63 - (4 + 6 \cdot 3) + 76 - 4$	Simplify first within the parenthesis by multiplying, then adding.
$63 - (4 + 18) + 76 - 4$	
$63 - 22 + 76 - 4$	Now perform the additions and subtractions, moving left to right.
$41 + 76 - 4$	Add 41 and 76: $41 + 76 = 117.$
$117 - 4$	Subtract 4 from 117: $117 - 4 = 113.$
113	

Example:

$7 \cdot 6 - 4^2 + 1^5$	Evaluate the exponential forms, moving left to right.
$7 \cdot 6 - 16 + 1$	Multiply 7 and 6: $7 \cdot 6 = 42$
$42 - 16 + 1$	Subtract 16 from 42: $42 - 16 = 26$
$26 + 1$	Add 26 and 1: $26 + 1 = 27$
27	

Example:

$6 \cdot (3^2 + 2^2) + 4^2$	Evaluate the exponential forms in the parentheses: $3^2 = 9$ and $2^2 = 4$
$6 \cdot (9 + 4) + 4^2$	Add the 9 and 4 in the parentheses: $9 + 4 = 13$
$6 \cdot (13) + 4^2$	Evaluate the exponential form: $4^2 = 16$
$6 \cdot (13) + 16$	Multiply 6 and 13: $6 \cdot 13 = 78$
$78 + 16$	Add 78 and 16: $78 + 16 = 94$
94	

Example:

$\frac{6^2+2^2}{4^2+6 \cdot 2^2} + \frac{1^3+8^2}{10^2-19 \cdot 5}$	Recall that the bar is a grouping symbol. The fraction $\frac{6^2+2^2}{4^2+6 \cdot 2^2}$ is equivalent to $(6^2 + 2^2) \div (4^2 + 6 \cdot 2^2)$
$\frac{36+4}{16+6 \cdot 4} + \frac{1+64}{100-19 \cdot 5}$	
$\frac{36+4}{16+24} + \frac{1+64}{100-95}$	
$\frac{40}{40} + \frac{65}{5}$	
$1 + 13$	
14	

Practice Set C

Determine the value of each of the following.

Exercise:

Problem: $8 + (32 - 7)$

Solution:

33

Exercise:

Problem: $(34 + 18 - 2 \cdot 3) + 11$

Solution:

57

Exercise:

Problem: $8(10) + 4(2 + 3) - (20 + 3 \cdot 15 + 40 - 5)$

Solution:

0

Exercise:

Problem: $5 \cdot 8 + 4^2 - 2^2$

Solution:

52

Exercise:

Problem: $4(6^2 - 3^3) \div (4^2 - 4)$

Solution:

3

Exercise:

Problem: $(8 + 9 \cdot 3) \div 7 + 5 \cdot (8 \div 4 + 7 + 3 \cdot 5)$

Solution:

125

Exercise:

Problem: $\frac{3^3+2^3}{6^2-29} + 5\left(\frac{8^2+2^4}{7^2-3^2}\right) \div \frac{8 \cdot 3+1^8}{2^3-3}$

Solution:

7

Calculators

Using a calculator is helpful for simplifying computations that involve large numbers.

Sample Set D

Use a calculator to determine each value.

Example:
$9{,}842 + 56 \cdot 85$

	Key		Display Reads
Perform the multiplication first.	Type	56	56
	Press	×	56
	Type	85	85
Now perform the addition.	Press	+	4760
	Type	9842	9842
	Press	=	14602

The display now reads 14,602.

Example:
$42(27+18)+105(810 \div 18)$

	Key		Display Reads
Operate inside the parentheses	Type	27	27
	Press	+	27
	Type	18	18
	Press	=	45
Multiply by 42.	Press	×	45
	Type	42	42
	Press	=	1890

Place this result into memory by pressing the memory key.

	Key		Display Reads
Now operate in the other parentheses.	Type	810	810
	Press	÷	810
	Type	18	18
	Press	=	45
Now multiply by 105.	Press	×	45
	Type	105	105
	Press	=	4725
We are now ready to add these two quantities together.	Press	+	4725
Press the memory recall key.			1890
	Press	=	6615

Thus, $42(27+18)+105(810 \div 18)=6{,}615$

Example:
16^4+37^3

Nonscientific Calculators		
Key		Display Reads
Type	16	16
Press	×	16
Type	16	16
Press	×	256
Type	16	16
Press	×	4096

Type	16	16
Press	=	65536
Press the memory key		
Type	37	37
Press	×	37
Type	37	37
Press	×	1396
Type	37	37
Press	×	50653
Press	+	50653
Press memory recall key		65536
Press	=	116189

Calculators with y^x Key		
Key		Display Reads
Type	16	16
Press	y^x	16
Type	4	4
Press	=	4096
Press	+	4096
Type	37	37
Press	y^x	37
Type	3	3

Press	=	116189

Thus, $16^4 + 37^3 = 116{,}189$
We can certainly see that the more powerful calculator simplifies computations.

Example:
Nonscientific calculators are unable to handle calculations involving very large numbers.
$85612 \cdot 21065$

Key		Display Reads
Type	85612	85612
Press	×	85612
Type	21065	21065
Press	=	

This number is too big for the display of some calculators and we'll probably get some kind of error message. On some scientific calculators such large numbers are coped with by placing them in a form called "scientific notation." Others can do the multiplication directly. (1803416780)

Practice Set D

Use a calculator to find each value.
Exercise:

Problem: $9{,}285 + 86(49)$

Solution:

13,499
Exercise:

Problem: $55(84 - 26) + 120(512 - 488)$

Solution:

6,070

Exercise:

Problem: $106^3 - 17^4$

Solution:

1,107,495

Exercise:

Problem: $6{,}053^3$

Solution:

This number is too big for a nonscientific calculator. A scientific calculator will probably give you $2.217747109 \times 10^{11}$

Exercises

For the following problems, find each value. Check each result with a calculator.

Exercise:

Problem: $2 + 3 \cdot (8)$

Solution:

26

Exercise:

Problem: $18 + 7 \cdot (4 - 1)$

Exercise:

Problem: $3 + 8 \cdot (6 - 2) + 11$

Solution:

46

Exercise:

Problem: $1 - 5 \cdot (8 - 8)$

Exercise:

Problem: $37 - 1 \cdot 6^2$

Solution:

1

Exercise:

Problem: $98 \div 2 \div 7^2$

Exercise:

Problem: $(4^2 - 2 \cdot 4) - 2^3$

Solution:

0

Exercise:

Problem: $\sqrt{9} + 14$

Exercise:

Problem: $\sqrt{100} + \sqrt{81} - 4^2$

Solution:

3

Exercise:

Problem: $\sqrt[3]{8} + 8 - 2 \cdot 5$

Exercise:

Problem: $\sqrt[4]{16} - 1 + 5^2$

Solution:

26

Exercise:

Problem: $61 - 22 + 4[3 \cdot (10) + 11]$

Exercise:

Problem: $121 - 4 \cdot [(4) \cdot (5) - 12] + \frac{16}{2}$

Solution:

97

Exercise:

Problem: $\frac{(1+16)-3}{7}+5\cdot(12)$

Exercise:

Problem: $\frac{8\cdot(6+20)}{8}+\frac{3\cdot(6+16)}{22}$

Solution:

29

Exercise:

Problem: $10\cdot[8+2\cdot(6+7)]$

Exercise:

Problem: $21\div7\div3$

Solution:

1

Exercise:

Problem: $10^2\cdot3\div5^2\cdot3-2\cdot3$

Exercise:

Problem: $85\div5\cdot5-85$

Solution:

0

Exercise:

Problem: $\frac{51}{17}+7-2\cdot5\cdot\left(\frac{12}{3}\right)$

Exercise:

Problem: $2^2\cdot3+2^3\cdot(6-2)-(3+17)+11(6)$

Solution:

90

Exercise:

Problem: $26-2\cdot\left\{\frac{6+20}{13}\right\}$

Exercise:

Problem: $2 \cdot \{(7+7)+6 \cdot [4 \cdot (8+2)]\}$

Solution:

508

Exercise:

Problem: $0+10(0)+15 \cdot \{4 \cdot 3+1\}$

Exercise:

Problem: $18+\frac{7+2}{9}$

Solution:

19

Exercise:

Problem: $(4+7) \cdot (8-3)$

Exercise:

Problem: $(6+8) \cdot (5+2-4)$

Solution:

144

Exercise:

Problem: $(21-3) \cdot (6-1) \cdot (7)+4(6+3)$

Exercise:

Problem: $(10+5) \cdot (10+5)-4 \cdot (60-4)$

Solution:

1

Exercise:

Problem: $6 \cdot \{2 \cdot 8+3\}-(5) \cdot (2)+\frac{8}{4}+(1+8) \cdot (1+11)$

Exercise:

Problem: $2^5+3 \cdot (8+1)$

Solution:

52

Exercise:

Problem: $3^4 + 2^4 \cdot (1 + 5)$

Exercise:

Problem: $1^6 + 0^8 + 5^2 \cdot (2 + 8)^3$

Solution:

25,001

Exercise:

Problem: $(7) \cdot (16) - 3^4 + 2^2 \cdot (1^7 + 3^2)$

Exercise:

Problem: $\frac{2^3-7}{5^2}$

Solution:

$\frac{1}{25}$

Exercise:

Problem: $\frac{(1+6)^2+2}{3 \cdot 6+1}$

Exercise:

Problem: $\frac{6^2-1}{2^3-3} + \frac{4^3+2 \cdot 3}{2 \cdot 5}$

Solution:

14

Exercise:

Problem: $\frac{5\left(8^2-9 \cdot 6\right)}{2^5-7} + \frac{7^2-4^2}{2^4-5}$

Exercise:

Problem: $\frac{(2+1)^3+2^3+1^{10}}{6^2} - \frac{15^2-[2 \cdot 5]^2}{5 \cdot 5^2}$

Solution:

0

Exercise:

Problem: $\frac{6^3-2\cdot10^2}{2^2}+\frac{18(2^3+7^2)}{2(19)-3^3}$

Exercise:

Problem: $2\cdot\left\{6+\left[10^2-6\sqrt{25}\right]\right\}$

Solution:

152

Exercise:

Problem: $181-3\cdot\left(2\sqrt{36}+3\sqrt[3]{64}\right)$

Exercise:

Problem: $\frac{2\cdot\left(\sqrt{81}-\sqrt[3]{125}\right)}{4^2-10+2^2}$

Solution:

$\frac{4}{5}$

Exercises for Review

Exercise:

Problem:

([link]) The fact that 0 + any whole number = that particular whole number is an example of which property of addition?

Exercise:

Problem: ([link]) Find the product. $4{,}271\times630$.

Solution:

2,690,730

Exercise:

Problem: ([link]) In the statement $27\div3=9$, what name is given to the result 9?

Exercise:

Problem: ([link]) What number is the multiplicative identity?

Solution:

1

Exercise:

Problem: ([link]) Find the value of 2^4.

Prime Factorization of Natural Numbers
This module is from Fundamentals of Mathematics by Denny Burzynski and Wade Ellis, Jr. This module discusses prime factorization of natural numbers. By the end of the module students should be able to determine the factors of a whole number, distinguish between prime and composite numbers, be familiar with the fundamental principle of arithmetic and find the prime factorization of a whole number.

Section Overview

- Factors
- Determining the Factors of a Whole Number
- Prime and Composite Numbers
- The Fundamental Principle of Arithmetic
- The Prime Factorization of a Natural Number

Factors

From observations made in the process of multiplication, we have seen that

$$(\text{factor}) \cdot (\text{factor}) = \text{product}$$

Factors, Product
The two numbers being multiplied are the **factors** and the result of the multiplication is the **product**. Now, using our knowledge of division, we can see that a first number is a factor of a second number if the first number divides into the second number a whole number of times (without a remainder).

One Number as a Factor of Another
A first number is a **factor** of a second number if the first number divides into the second number a whole number of times (without a remainder).

We show this in the following examples:

Example:
3 is a factor of 27, since $27 \div 3 = 9$, or $3 \cdot 9 = 27$.

Example:
7 is a factor of 56, since $56 \div 7 = 8$, or $7 \cdot 8 = 56$.

Example:
4 is *not* a factor of 10, since $10 \div 4 = 2\text{R}2$. (There is a remainder.)

Determining the Factors of a Whole Number

We can use the tests for divisibility from [link] to determine *all* the factors of a whole number.

Sample Set A

Example:
Find all the factors of 24.

Try 1:	$24 \div 1 = 24$	1 and 24 are factors
Try 2:	24 is even, so 24 is divisible by 2.	
	$24 \div 2 = 12$	2 and 12 are factors
Try 3:	$2 + 4 = 6$ and 6 is divisible by 3, so 24 is divisible by 3.	
	$24 \div 3 = 8$	3 and 8 are factors
Try 4:	$24 \div 4 = 6$	4 and 6 are factors
Try 5:	$24 \div 5 = 4R4$	5 is not a factor.

The next number to try is 6, but we already have that 6 is a factor. Once we come upon a factor that we already have discovered, we can stop.
All the whole number factors of 24 are 1, 2, 3, 4, 6, 8, 12, and 24.

Practice Set A

Find all the factors of each of the following numbers.

Exercise:

Problem: 6

Solution:

1, 2, 3, 6

Exercise:

Problem: 12

Solution:

1, 2, 3, 4, 6, 12

Exercise:

Problem: 18

Solution:

1, 2, 3, 6, 9, 18

Exercise:

Problem: 5

Solution:

1, 5

Exercise:

Problem: 10

Solution:

1, 2, 5, 10

Exercise:

Problem: 33

Solution:

1, 3, 11, 33

Exercise:

Problem: 19

Solution:

1, 19

Prime and Composite Numbers

Notice that the only factors of 7 are 1 and 7 itself, and that the only factors of 3 are 1 and 3 itself. However, the number 8 has the factors 1, 2, 4, and 8, and the number 10 has the factors 1, 2, 5, and 10. Thus, we can see that a whole number can have only *two* factors (itself and 1) and another whole number can have *several* factors.

We can use this observation to make a useful classification for whole numbers: prime numbers and composite numbers.

Prime Number
A whole number (greater than one) whose only factors are itself and 1 is called a **prime number**.

The Number 1 is *Not* a Prime Number
The first seven prime numbers are 2, 3, 5, 7, 11, 13, and 17. Notice that the whole number 1 is *not* considered to be a prime number, and the whole number 2 is the *first prime* and the *only even prime* number.

Composite Number
A whole number composed of factors other than itself and 1 is called a **composite number**. Composite numbers are not prime numbers.

Some composite numbers are 4, 6, 8, 9, 10, 12, and 15.

Sample Set B

Determine which whole numbers are prime and which are composite.

Example:
39. Since 3 divides into 39, the number 39 is composite: $39 \div 3 = 13$

Example:
47. A few division trials will assure us that 47 is only divisible by 1 and 47. Therefore, 47 is prime.

Practice Set B

Determine which of the following whole numbers are prime and which are composite.
Exercise:

Problem: 3

Solution:

prime

Exercise:

Problem: 16

Solution:

composite

Exercise:

Problem: 21

Solution:

composite

Exercise:

Problem: 35

Solution:

composite

Exercise:

Problem: 47

Solution:

prime

Exercise:

Problem: 29

Solution:

prime

Exercise:

Problem: 101

Solution:

prime

Exercise:

Problem: 51

Solution:

composite

The Fundamental Principle of Arithmetic

Prime numbers are very useful in the study of mathematics. We will see how they are used in subsequent sections. We now state the Fundamental Principle of Arithmetic.

Fundamental Principle of Arithmetic
Except for the order of the factors, every natural number other than 1 can be factored in one and only one way as a product of prime numbers.

Prime Factorization
When a number is factored so that all its factors are prime numbers. the factorization is called the **prime factorization** of the number.

The technique of prime factorization is illustrated in the following three examples.

1. $10 = 5 \cdot 2$. Both 2 and 5 are primes. Therefore, $2 \cdot 5$ is the prime factorization of 10.
2. 11. The number 11 is a prime number. Prime factorization applies only to composite numbers. Thus, 11 has *no* prime factorization.
3. $60 = 2 \cdot 30$. The number 30 is not prime: $30 = 2 \cdot 15$.

$60 = 2 \cdot 2 \cdot 15$

The number 15 is not prime: $15 = 3 \cdot 5$

$60 = 2 \cdot 2 \cdot 3 \cdot 5$

We'll use exponents.

$60 = 2^2 \cdot 3 \cdot 5$

The numbers 2, 3, and 5 are each prime. Therefore, $2^2 \cdot 3 \cdot 5$ is the prime factorization of 60.

The Prime Factorization of a Natural Number

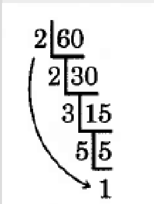
The following method provides a way of finding the prime factorization of a natural number.
The Method of Finding the Prime Factorization of a Natural Number

1. Divide the number repeatedly by the smallest prime number that will divide into it a whole number of times (without a remainder).
2. When the prime number used in step 1 no longer divides into the given number without a remainder, repeat the division process with the next largest prime that divides the given number.
3. Continue this process until the quotient is smaller than the divisor.
4. The prime factorization of the given number is the *product* of all these prime divisors. If the number has no prime divisors, it is a prime number.

We may be able to use some of the tests for divisibility we studied in [link] to help find the primes that divide the given number.

Sample Set C

Example:
Find the prime factorization of 60.
Since the last digit of 60 is 0, which is even, 60 is divisible by 2. We will repeatedly divide by 2 until we no longer can. We shall divide as follows:



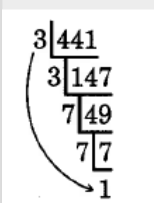
30 is divisible by 2 again.

15 is not divisible by 2, but it is divisible by 3, the next prime.

5 is not divisble by 3, but it is divisible by 5, the next prime.

The quotient 1 is finally smaller than the divisor 5, and the prime factorization of 60 is the product of these prime divisors.
$60 = 2 \cdot 2 \cdot 3 \cdot 5$
We use exponents when possible.
$60 = 2^2 \cdot 3 \cdot 5$

Example:
Find the prime factorization of 441.
441 is not divisible by 2 since its last digit is not divisible by 2.
441 is divisible by 3 since $4 + 4 + 1 = 9$ and 9 is divisible by 3.



147 is divisible by$3 (1 + 4 + 7 = 12)$.

49 is not divisible by 3, nor is it divisible by 5. It is divisible by 7.

The quotient 1 is finally smaller than the divisor 7, and the prime factorization of 441 is the product of these prime divisors.
$441 = 3 \cdot 3 \cdot 7 \cdot 7$
Use exponents.
$441 = 3^2 \cdot 7^2$

Example:
Find the prime factorization of 31.

31 is not divisible by 2	Its last digit is not even
	$31 \div 2 = 15\text{R}1$
	The quotient, 15, is larger than the divisor, 3. Continue.
31 is not divisible by 3	The digits $3 + 1 = 4$, and 4 is not divisible by 3.
	$31 \div 3 = 10\text{R}1$
	The quotient, 10, is larger than the divisor, 3. Continue.
31 is not divisible by 5	The last digit of 31 is not 0 or 5.
	$31 \div 5 = 6\text{R}1$
	The quotient, 6, is larger than the divisor, 5. Continue.
31 is not divisible by 7.	Divide by 7.
	$31 \div 7 = 4\text{R}1$
	The quotient, 4, is smaller than the divisor, 7.
	We can stop the process and conclude that 31 is a prime number.
The number 31 is a prime number	

Practice Set C

Find the prime factorization of each whole number.

Exercise:

Problem: 22

Solution:

$22 = 2 \cdot 11$

Exercise:

Problem: 40

Solution:

$40 = 2^3 \cdot 5$

Exercise:

Problem: 48

Solution:

$48 = 2^4 \cdot 3$

Exercise:

Problem: 63

Solution:

$63 = 3^2 \cdot 7$

Exercise:

Problem: 945

Solution:

$945 = 3^3 \cdot 5 \cdot 7$

Exercise:

Problem: 1,617

Solution:

$1617 = 3 \cdot 7^2 \cdot 11$

Exercise:

Problem: 17

Solution:

17 is prime

Exercise:

Problem: 61

Solution:

61 is prime

Exercises

For the following problems, determine the missing factor(s).

Exercise:

Problem: $14 = 7 \cdot$

Solution:

2

Exercise:

Problem: $20 = 4 \cdot$

Exercise:

Problem: $36 = 9 \cdot$

Solution:

4

Exercise:

Problem: $42 = 21\cdot$

Exercise:

Problem: $44 = 4 \cdot$

Solution:

11

Exercise:

Problem: $38 = 2 \cdot$

Exercise:

Problem: $18 = 3 \cdot$.

Solution:

$3 \cdot 2$

Exercise:

Problem: $28 = 2 \cdot$.

Exercise:

Problem: $300 = 2 \cdot 5 \cdot$.

Solution:

$2 \cdot 3 \cdot 5$

Exercise:

Problem: $840 = 2 \cdot$ . .

For the following problems, find all the factors of each of the numbers.

Exercise:

Problem: 16

Solution:

1, 2, 4, 8, 16

Exercise:

Problem: 22

Exercise:

Problem: 56

Solution:

1, 2, 4, 7, 8, 14, 28, 56

Exercise:

Problem: 105

Exercise:

Problem: 220

Solution:

1, 2, 4, 5, 10, 11, 20, 22, 44, 55, 110, 220

Exercise:

Problem: 15

Exercise:

Problem: 32

Solution:

1, 2, 4, 8, 16, 32

Exercise:

Problem: 80

Exercise:

Problem: 142

Solution:

1, 2, 71, 142

Exercise:

Problem: 218

For the following problems, determine which of the whole numbers are prime and which are composite.

Exercise:

Problem: 23

Solution:

prime

Exercise:

Problem: 25

Exercise:

Problem: 27

Solution:

composite

Exercise:

Problem: 2

Exercise:

Problem: 3

Solution:

prime

Exercise:

Problem: 5

Exercise:

Problem: 7

Solution:

prime

Exercise:

Problem: 9

Exercise:

Problem: 11

Solution:

prime

Exercise:

Problem: 34

Exercise:

Problem: 55

Solution:

composite ($5 \cdot 11$)

Exercise:

Problem: 63

Exercise:

Problem: 1,044

Solution:

composite

Exercise:

Problem: 924

Exercise:

Problem: 339

Solution:

composite

Exercise:

Problem: 103

Exercise:

Problem: 209

Solution:

composite ($11 \cdot 19$)

Exercise:

Problem: 667

Exercise:

Problem: 4,575

Solution:

composite

Exercise:

Problem: 119

For the following problems, find the prime factorization of each of the whole numbers.

Exercise:

Problem: 26

Solution:

$2 \cdot 13$

Exercise:

Problem: 38

Exercise:

Problem: 54

Solution:

$2 \cdot 3^3$

Exercise:

Problem: 62

Exercise:

Problem: 56

Solution:

$2^3 \cdot 7$

Exercise:

Problem: 176

Exercise:

Problem: 480

Solution:

$2^5 \cdot 3 \cdot 5$

Exercise:

Problem: 819

Exercise:

Problem: 2,025

Solution:

$3^4 \cdot 5^2$

Exercise:

Problem: 148,225

Exercises For Review

Exercise:

Problem: ([link]) Round 26,584 to the nearest ten.

Solution:

26,580

Exercise:

Problem: ([link]) How much bigger is 106 than 79?

Exercise:

Problem: ([link]) True or false? Zero divided by any nonzero whole number is zero.

Solution:

true

Exercise:

Problem: ([link]) Find the quotient. $10,584 \div 126$.

Exercise:

Problem: ([link]) Find the value of $\sqrt{121} - \sqrt{81} + 6^2 \div 3$.

Solution:

14

The Greatest Common Factor
This module is from Fundamentals of Mathematics by Denny Burzynski and Wade Ellis, Jr. This module discusses the greatest common factor. By the end of the module students should be able to find the greatest common factor of two or more whole numbers.

Section Overview

- The Greatest Common Factor (GCF)
- A Method for Determining the Greatest Common Factor

The Greatest Common Factor (GCF)

Using the method we studied in [link], we could obtain the prime factorizations of 30 and 42.

$30 = 2 \cdot 3 \cdot 5$

$42 = 2 \cdot 3 \cdot 7$

Common Factor
We notice that 2 appears as a factor in both numbers, that is, 2 is a **common factor** of 30 and 42. We also notice that 3 appears as a factor in both numbers. Three is also a common factor of 30 and 42.

Greatest Common Factor (GCF)
When considering two or more numbers, it is often useful to know if there is a largest common factor of the numbers, and if so, what that number is. The largest common factor of two or more whole numbers is called the **greatest common factor**, and is abbreviated by **GCF**. The greatest common factor of a collection of whole numbers is useful in working with fractions (which we will do in [link]).

A Method for Determining the Greatest Common Factor

A straightforward method for determining the GCF of two or more whole numbers makes use of both the prime factorization of the numbers and

exponents.

Finding the GCF
To find the **greatest common factor (GCF)** of two or more whole numbers:

1. Write the prime factorization of each number, using exponents on repeated factors.
2. Write each base that is common to each of the numbers.
3. To each base listed in step 2, attach the *smallest exponent* that appears on it in either of the prime factorizations.
4. The GCF is the product of the numbers found in step 3.

Sample Set A

Find the GCF of the following numbers.

Example:
12 and 18

1. $12 = 2 \cdot 6 = 2 \cdot 2 \cdot 3 = 2^2 \cdot 3$
   $18 = 2 \cdot 9 = 2 \cdot 3 \cdot 3 = 2 \cdot 3^2$
2. The common bases are 2 and 3.
3. The *smallest exponents* appearing on 2 and 3 in the prime factorizations are, respectively, 1 and 1 ($2^1$and 3^1), or 2 and 3.
4. The GCF is the product of these numbers.

   $2 \cdot 3 = 6$

The GCF of 30 and 42 is 6 because 6 is the largest number that divides both 30 and 42 without a remainder.

Example:
18, 60, and 72

1. $18 = 2 \cdot 9 = 2 \cdot 3 \cdot 3 = 2 \cdot 3^2$
   $60 = 2 \cdot 30 = 2 \cdot 2 \cdot 15 = 2 \cdot 2 \cdot 3 \cdot 5 = 2^2 \cdot 3 \cdot 5$
   $72 = 2 \cdot 36 = 2 \cdot 2 \cdot 18 = 2 \cdot 2 \cdot 2 \cdot 9 = 2 \cdot 2 \cdot 2 \cdot 3 \cdot 3 = 2^3 \cdot 3^2$
2. The common bases are 2 and 3.
3. The smallest exponents appearing on 2 and 3 in the prime factorizations are, respectively, 1 and 1:

   2^1 from 18

   3^1 from 60
4. The GCF is the product of these numbers.

   GCF is $2 \cdot 3 = 6$

Thus, 6 is the largest number that divides 18, 60, and 72 without a remainder.

Example:
700, 1,880, and 6,160

$$\begin{aligned} 700 &= 2 \cdot 350 = 2 \cdot 2 \cdot 175 = 2 \cdot 2 \cdot 5 \cdot 35 \\ &= 2 \cdot 2 \cdot 5 \cdot 5 \cdot 7 \\ &= 2^2 \cdot 5^2 \cdot 7 \\ 1{,}880 &= 2 \cdot 940 = 2 \cdot 2 \cdot 470 = 2 \cdot 2 \cdot 2 \cdot 235 \\ &= 2 \cdot 2 \cdot 2 \cdot 5 \cdot 47 \\ &= 2^3 \cdot 5 \cdot 47 \\ 6{,}160 &= 2 \cdot 3{,}080 = 2 \cdot 2 \cdot 1{,}540 = 2 \cdot 2 \cdot 2 \cdot 770 \\ &= 2 \cdot 2 \cdot 2 \cdot 2 \cdot 385 \\ &= 2 \cdot 2 \cdot 2 \cdot 2 \cdot 5 \cdot 77 \\ &= 2 \cdot 2 \cdot 2 \cdot 2 \cdot 5 \cdot 7 \cdot 11 \\ &= 2^4 \cdot 5 \cdot 7 \cdot 11 \end{aligned}$$

1. (factorizations above)
2. The common bases are 2 and 5
3. The smallest exponents appearing on 2 and 5 in the prime factorizations are, respectively, 2 and 1.

   2^2 from 700.

   5^1 from either 1,880 or 6,160.

4. The GCF is the product of these numbers.

   GCF is $2^2 \cdot 5 = 4 \cdot 5 = 20$

Thus, 20 is the largest number that divides 700, 1,880, and 6,160 without a remainder.

Practice Set A

Find the GCF of the following numbers.
Exercise:

Problem: 24 and 36

Solution:

12

Exercise:

Problem: 48 and 72

Solution:

24

Exercise:

Problem: 50 and 140

Solution:

10

Exercise:

Problem: 21 and 225

Solution:

3

Exercise:

Problem: 450, 600, and 540

Solution:

30

Exercises

For the following problems, find the greatest common factor (GCF) of the numbers.

Exercise:

Problem: 6 and 8

Solution:

2

Exercise:

Problem: 5 and 10

Exercise:

Problem: 8 and 12

Solution:

4

Exercise:

Problem: 9 and 12

Exercise:

Problem: 20 and 24

Solution:

4

Exercise:

Problem: 35 and 175

Exercise:

Problem: 25 and 45

Solution:

5

Exercise:

Problem: 45 and 189

Exercise:

Problem: 66 and 165

Solution:

33

Exercise:

Problem: 264 and 132

Exercise:

Problem: 99 and 135

Solution:

9

Exercise:

Problem: 65 and 15

Exercise:

Problem: 33 and 77

Solution:

11

Exercise:

Problem: 245 and 80

Exercise:

Problem: 351 and 165

Solution:

3

Exercise:

Problem: 60, 140, and 100

Exercise:

Problem: 147, 343, and 231

Solution:

7

Exercise:

Problem: 24, 30, and 45

Exercise:

Problem: 175, 225, and 400

Solution:

25

Exercise:

Problem: 210, 630, and 182

Exercise:

Problem: 14, 44, and 616

Solution:

2

Exercise:

Problem: 1,617, 735, and 429

Exercise:

Problem: 1,573, 4,862, and 3,553

Solution:

11

Exercise:

Problem: 3,672, 68, and 920

Exercise:

Problem: 7, 2,401, 343, 16, and 807

Solution:

1

Exercise:

Problem: 500, 77, and 39

Exercise:

Problem: 441, 275, and 221

Solution:

1

Exercises for Review

Exercise:

Problem: ([link]) Find the product. $2,753 \times 4,006$.

Exercise:

Problem: ([link]) Find the quotient. $954 \div 18$.

Solution:

53

Exercise:

Problem:

([link]) Specify which of the digits 2, 3, or 4 divide into 9,462.

Exercise:

Problem: ([link]) Write $8 \times 8 \times 8 \times 8 \times 8 \times 8$ using exponents.

Solution:

$8^6 = 262{,}144$

Exercise:

Problem: ([link]) Find the prime factorization of 378.

The Least Common Multiple
This module is from Fundamentals of Mathematics by Denny Burzynski and Wade Ellis, Jr. This module discusses the least common multiple. By the end of the module students should be able to find the least common multiple of two or more whole numbers.

Section Overview

- Multiples
- Common Multiples
- The Least Common Multiple (LCM)
- Finding the Least Common Multiple

Multiples

When a whole number is multiplied by other whole numbers, with the exception of zero, the resulting products are called **multiples** of the given whole number. Note that any whole number is a multiple of itself.

Sample Set A

Multiples of 2	Multiples of 3	Multiples of 8	Multiples of 10
$2\times1=2$	$3\times1=3$	$8\times1=8$	$10\times1=10$
$2\times2=4$	$3\times2=6$	$8\times2=16$	$10\times2=20$
$2\times3=6$	$3\times3=9$	$8\times3=24$	$10\times3=30$
$2\times4=8$	$3\times4=12$	$8\times4=32$	$10\times4=40$

$2 \times 5 = 10$	$3 \times 5 = 15$	$8 \times 5 = 40$	$10 \times 5 = 50$
$\vdots$	$\vdots$	$\vdots$	$\vdots$

Practice Set A

Find the first five multiples of the following numbers.

Exercise:

Problem: 4

Solution:

4, 8, 12, 16, 20

Exercise:

Problem: 5

Solution:

5, 10, 15, 20, 25

Exercise:

Problem: 6

Solution:

6, 12, 18, 24, 30

Exercise:

Problem: 7

Solution:

7, 14, 21, 28, 35

Exercise:

Problem: 9

Solution:

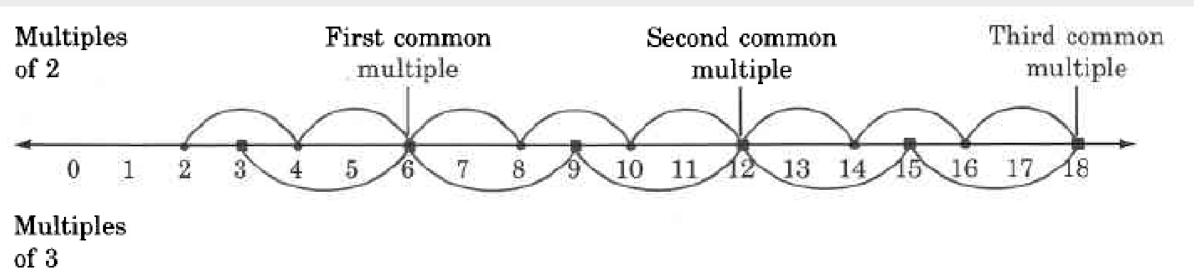
9, 18, 27, 36, 45

Common Multiples

There will be times when we are given two or more whole numbers and we will need to know if there are any multiples that are common to each of them. If there are, we will need to know what they are. For example, some of the multiples that are common to 2 and 3 are 6, 12, and 18.

Sample Set B

Example:
We can visualize common multiples using the number line.



Notice that the common multiples can be divided by *both* whole numbers.

Practice Set B

Find the first five common multiples of the following numbers.

Exercise:

Problem: 2 and 4

Solution:

4, 8, 12, 16, 20

Exercise:

Problem: 3 and 4

Solution:

12, 24, 36, 48, 60

Exercise:

Problem: 2 and 5

Solution:

10, 20, 30, 40, 50

Exercise:

Problem: 3 and 6

Solution:

6, 12, 18, 24, 30

Exercise:

Problem: 4 and 5

Solution:

20, 40, 60, 80, 100

The Least Common Multiple (LCM)

Notice that in our number line visualization of common multiples (above), the first common multiple is also the smallest, or **least common multiple**, abbreviated by **LCM**.

Least Common Multiple
The **least common multiple**, **LCM**, of two or more whole numbers is the smallest whole number that each of the given numbers will divide into without a remainder.

The least common multiple will be extremely useful in working with fractions ([link]).

Finding the Least Common Multiple

Finding the LCM
To find the LCM of two or more numbers:

1. Write the prime factorization of each number, using exponents on repeated factors.
2. Write each base that appears in each of the prime factorizations.
3. To each base, attach the *largest exponent* that appears on it in the prime factorizations.
4. The LCM is the product of the numbers found in step 3.

There are some major differences between using the processes for obtaining the GCF and the LCM that we must note carefully:
The Difference Between the Processes for Obtaining the GCF and the LCM

1. Notice the difference between step 2 for the LCM and step 2 for the GCF. For the GCF, we use only the bases that are *common* in the prime factorizations, whereas for the LCM, we use *each* base that appears in the prime factorizations.
2. Notice the difference between step 3 for the LCM and step 3 for the GCF. For the GCF, we attach the *smallest* exponents to the common bases, whereas for the LCM, we attach the *largest* exponents to the bases.

Sample Set C

Find the LCM of the following numbers.

Example:
9 and 12

1. $9 = 3 \cdot 3 = 3^2$
   $12 = 2 \cdot 6 = 2 \cdot 2 \cdot 3 = 2^2 \cdot 3$
2. The bases that appear in the prime factorizations are 2 and 3.
3. The *largest exponents* appearing on 2 and 3 in the prime factorizations are, respectively, 2 and 2:

   2^2 from 12.

   3^2 from 9.
4. The LCM is the product of these numbers.

   $\text{LCM} = 2^2 \cdot 3^2 = 4 \cdot 9 = 36$

Thus, 36 is the smallest number that both 9 and 12 divide into without remainders.

Example:
90 and 630

1. $$\begin{aligned} 90 &= 2 \cdot 45 = 2 \cdot 3 \cdot 15 = 2 \cdot 3 \cdot 3 \cdot 5 = 2 \cdot 3^2 \cdot 5 \\ 630 &= 2 \cdot 315 = 2 \cdot 3 \cdot 105 = 2 \cdot 3 \cdot 3 \cdot 35 = 2 \cdot 3 \cdot 3 \cdot 5 \cdot 7 \\ &= 2 \cdot 3^2 \cdot 5 \cdot 7 \end{aligned}$$
2. The bases that appear in the prime factorizations are 2, 3, 5, and 7.
3. The largest exponents that appear on 2, 3, 5, and 7 are, respectively, 1, 2, 1, and 1:

   2^1 from either 90 or 630.

3^2 from either 90 or 630.
5^1 from either 90 or 630.
7^1 from 630.

4. The LCM is the product of these numbers.

$$\text{LCM} = 2 \cdot 3^2 \cdot 5 \cdot 7 = 2 \cdot 9 \cdot 5 \cdot 7 = 630$$

Thus, 630 is the smallest number that both 90 and 630 divide into with no remainders.

Example:
33, 110, and 484

1. $$\begin{aligned} 33 &= 3 \cdot 11 \\ 110 &= 2 \cdot 55 = 2 \cdot 5 \cdot 11 \\ 484 &= 2 \cdot 242 = 2 \cdot 2 \cdot 121 = 2 \cdot 2 \cdot 11 \cdot 11 = 2^2 \cdot 11^2. \end{aligned}$$
2. The bases that appear in the prime factorizations are 2, 3, 5, and 11.
3. The largest exponents that appear on 2, 3, 5, and 11 are, respectively, 2, 1, 1, and 2:

   2^2 from 484.
   3^1 from 33.
   5^1 from 110
   11^2 from 484.

4. The LCM is the product of these numbers.

$$\begin{aligned} \text{LCM} &= 2^2 \cdot 3 \cdot 5 \cdot 11^2 \\ &= 4 \cdot 3 \cdot 5 \cdot 121 \\ &= 7260 \end{aligned}$$

Thus, 7260 is the smallest number that 33, 110, and 484 divide into without remainders.

Practice Set C

Find the LCM of the following numbers.

Exercise:

Problem: 20 and 54

Solution:

540

Exercise:

Problem: 14 and 28

Solution:

28

Exercise:

Problem: 6 and 63

Solution:

126

Exercise:

Problem: 28, 40, and 98

Solution:

1,960

Exercise:

Problem: 16, 27, 125, and 363

Solution:

6,534,000

Exercises

For the following problems, find the least common multiple of the numbers.

Exercise:

Problem: 8 and 12

Solution:

24

Exercise:

Problem: 6 and 15

Exercise:

Problem: 8 and 10

Solution:

40

Exercise:

Problem: 10 and 14

Exercise:

Problem: 4 and 6

Solution:

12

Exercise:

Problem: 6 and 12

Exercise:

Problem: 9 and 18

Solution:

18

Exercise:

Problem: 6 and 8

Exercise:

Problem: 5 and 6

Solution:

30

Exercise:

Problem: 7 and 8

Exercise:

Problem: 3 and 4

Solution:

12

Exercise:

Problem: 2 and 9

Exercise:

Problem: 7 and 9

Solution:

63

Exercise:

Problem: 28 and 36

Exercise:

Problem: 24 and 36

Solution:

72

Exercise:

Problem: 28 and 42

Exercise:

Problem: 240 and 360

Solution:

720

Exercise:

Problem: 162 and 270

Exercise:

Problem: 20 and 24

Solution:

120

Exercise:

Problem: 25 and 30

Exercise:

Problem: 24 and 54

Solution:

216

Exercise:

Problem: 16 and 24

Exercise:

Problem: 36 and 48

Solution:

144

Exercise:

Problem: 24 and 40

Exercise:

Problem: 15 and 21

Solution:

105

Exercise:

Problem: 50 and 140

Exercise:

Problem: 7, 11, and 33

Solution:

231

Exercise:

Problem: 8, 10, and 15

Exercise:

Problem: 18, 21, and 42

Solution:

126

Exercise:

Problem: 4, 5, and 21

Exercise:

Problem: 45, 63, and 98

Solution:

4,410

Exercise:

Problem: 15, 25, and 40

Exercise:

Problem: 12, 16, and 20

Solution:

240

Exercise:

Problem: 84 and 96

Exercise:

Problem: 48 and 54

Solution:

432

Exercise:

Problem: 12, 16, and 24

Exercise:

Problem: 12, 16, 24, and 36

Solution:

144

Exercise:

Problem: 6, 9, 12, and 18

Exercise:

Problem: 8, 14, 28, and 32

Solution:

224

Exercise:

Problem: 18, 80, 108, and 490

Exercise:

Problem: 22, 27, 130, and 225

Solution:

193,050

Exercise:

Problem: 38, 92, 115, and 189

Exercise:

Problem: 8 and 8

Solution:

8

Exercise:

Problem: 12, 12, and 12

Exercise:

Problem: 3, 9, 12, and 3

Solution:

36

Exercises for Review

Exercise:

Problem: ([link]) Round 434,892 to the nearest ten thousand.

Exercise:

Problem: ([link]) How much bigger is 14,061 than 7,509?

Solution:

6,552

Exercise:

Problem: ([link]) Find the quotient. $22{,}428 \div 14$.

Exercise:

Problem: ([link]) Expand 84^3. Do not find the value.

Solution:

$84 \cdot 84 \cdot 84$

Exercise:

Problem: ([link]) Find the greatest common factor of 48 and 72.

Summary of Key Concepts
This module is from Fundamentals of Mathematics by Denny Burzynski and Wade Ellis, Jr. This module reviews the key concepts from the chapter "Exponents, Roots, Factorization of Whole Numbers."

Summary of Key Concepts

Exponential Notation ([link])
Exponential notation is a description of repeated multiplication.

Exponent ([link])
An **exponent** records the number of identical factors repeated in a multiplication.

In a number such as 7^3,

Base ([link])
7 is called the **base**.

Exponent ([link])
3 is called the **exponent**, or power.

Power ([link])
7^3 is read "seven to the third power," or "seven cubed."

Squared, Cubed ([link])
A number raised to the second power is often called **squared**. A number raised to the third power is often called **cubed**.

Root ([link])
In mathematics, the word **root** is used to indicate that, through repeated multiplication, one number is the source of another number.

The Radical Sign $\sqrt{}$ ([link])
The symbol $\sqrt{}$ is called a **radical sign** and indicates the square root of a number. The symbol $\sqrt[n]{}$ represents the nth root.

Radical, Index, Radicand ([link])

An expression such as $\sqrt[4]{16}$ is called a **radical** and 4 is called the **index**. The number 16 is called the **radicand**.

Grouping Symbols ([link])

Grouping symbols are used to indicate that a particular collection of numbers and meaningful operations are to be grouped together and considered as one number. The grouping symbols commonly used in mathematics are

Parentheses: ( )
Brackets: [ ]
Braces: { }
Bar:

Order of Operations ([link])

1. Perform all operations inside grouping symbols, beginning with the innermost set, in the order of 2, 3, and 4 below.
2. Perform all exponential and root operations, moving left to right.
3. Perform all multiplications and division, moving left to right.
4. Perform all additions and subtractions, moving left to right.

One Number as the Factor of Another ([link])

A first number is a factor of a second number if the first number divides into the second number a whole number of times.

Prime Number ([link])

A whole number greater than one whose only factors are itself and 1 is called a **prime number**. The whole number 1 is not a prime number. The whole number 2 is the first prime number and the only even prime number.

Composite Number ([link])

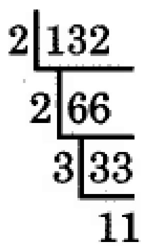
A whole number greater than one that is composed of factors other than itself and 1 is called a **composite number**.

Fundamental Principle of Arithmetic ([link])

Except for the order of factors, every whole number other than 1 can be written in one and only one way as a product of prime numbers.

Prime Factorization ([link])
The prime factorization of 45 is $3 \cdot 3 \cdot 5$. The numbers that occur in this factorization of 45 are each prime.

Determining the Prime Factorization of a Whole Number ([link])
There is a simple method, based on division by prime numbers, that produces the prime factorization of a whole number. For example, we determine the prime factorization of 132 as follows.



The prime factorization of 132 is $2 \cdot 2 \cdot 3 \cdot 11 = 2^2 \cdot 3 \cdot 11$.

Common Factor ([link])
A factor that occurs in each number of a group of numbers is called a **common factor**. 3 is a common factor to the group 18, 6, and 45

Greatest Common Factor (GCF) ([link])
The largest common factor of a group of whole numbers is called the **greatest common factor**. For example, to find the greatest common factor of 12 and 20,

1. Write the prime factorization of each number.

   $$\begin{aligned} 12 &= 2 \cdot 2 \cdot 3 = 2^2 \cdot 3 \\ 60 &= 2 \cdot 2 \cdot 3 \cdot 5 = 2^2 \cdot 3 \cdot 5 \end{aligned}$$
2. Write each base that is common to each of the numbers:

   2 and 3

3. The smallest exponent appearing on 2 is 2.
   The smallest exponent appearing on 3 is 1.
4. The GCF of 12 and 60 is the product of the numbers 2^2 and 3.

   $$2^2 \cdot 3 = 4 \cdot 3 = 12$$

Thus, 12 is the largest number that divides both 12 and 60 without a remainder.

Finding the GCF ([link])
There is a simple method, based on prime factorization, that determines the GCF of a group of whole numbers.

Multiple ([link])
When a whole number is multiplied by all other whole numbers, with the exception of zero, the resulting individual products are called **multiples** of that whole number. Some multiples of 7 are 7, 14, 21, and 28.

Common Multiples ([link])
Multiples that are common to a group of whole numbers are called **common multiples**. Some common multiples of 6 and 9 are 18, 36, and 54.

The LCM ([link])
The **least common multiple** (LCM) of a group of whole numbers is the smallest whole number that each of the given whole numbers divides into without a remainder. The least common multiple of 9 and 6 is 18.

Finding the LCM ([link])
There is a simple method, based on prime factorization, that determines the LCM of a group of whole numbers. For example, the least common multiple of 28 and 72 is found in the following way.

1. Write the prime factorization of each number

   $$\begin{aligned} 28 &= 2 \cdot 2 \cdot 7 = 2^2 \cdot 7 \\ 72 &= 2 \cdot 2 \cdot 2 \cdot 3 \cdot 3 = 2^3 \cdot 3^2 \end{aligned}$$

2. Write each base that appears in each of the prime factorizations, 2, 3, and 7.
3. To each of the bases listed in step 2, attach the *largest* exponent that appears on it in the prime factorization.

   2^3, 3^2, and 7
4. The LCM is the product of the numbers found in step 3.

   $2^3 \cdot 3^2 \cdot 7 = 8 \cdot 9 \cdot 7 = 504$

Thus, 504 is the smallest number that both 28 and 72 will divide into without a remainder.

The Difference Between the GCF and the LCM ([link])
The GCF of two or more whole numbers is the largest number that divides into each of the given whole numbers. The LCM of two or more whole numbers is the smallest whole number that each of the given numbers divides into without a remainder.

Exercise Supplement
This module is from Fundamentals of Mathematics by Denny Burzynski and Wade Ellis, Jr. This module is an exercise supplement for the chapter "Exponents, Roots, Factorization of Whole Numbers" and contains many exercise problems. Odd problems are accompanied by solutions.

Exercise Supplement

Exponents and Roots ([link])

For problems 1 -25, determine the value of each power and root.

Exercise:

Problem: 3^3

Solution:

27

Exercise:

Problem: 4^3

Exercise:

Problem: 0^5

Solution:

0

Exercise:

Problem: 1^4

Exercise:

Problem: 12^2

Solution:

144

Exercise:

Problem: 7^2

Exercise:

Problem: 8^2

Solution:

64

Exercise:

Problem: 11^2

Exercise:

Problem: 2^5

Solution:

32

Exercise:

Problem: 3^4

Exercise:

Problem: 15^2

Solution:

225

Exercise:

Problem: 20^2

Exercise:

Problem: 25^2

Solution:

625

Exercise:

Problem: $\sqrt{36}$

Exercise:

Problem: $\sqrt{225}$

Solution:

15

Exercise:

Problem: $\sqrt[3]{64}$

Exercise:

Problem: $\sqrt[4]{16}$

Solution:

2

Exercise:

Problem: $\sqrt{0}$

Exercise:

Problem: $\sqrt[3]{1}$

Solution:

1

Exercise:

Problem: $\sqrt[3]{216}$

Exercise:

Problem: $\sqrt{144}$

Solution:

12

Exercise:

Problem: $\sqrt{196}$

Exercise:

Problem: $\sqrt{1}$

Solution:

1

Exercise:

Problem: $\sqrt[4]{0}$

Exercise:

Problem: $\sqrt[6]{64}$

Solution:

2

Section 3.2

For problems 26-45, use the order of operations to determine each value.

Exercise:

Problem: $2^3 - 2 \cdot 4$

Exercise:

Problem: $5^2 - 10 \cdot 2 - 5$

Solution:

0

Exercise:

Problem: $\sqrt{81} - 3^2 + 6 \cdot 2$

Exercise:

Problem: $15^2 + 5^2 \cdot 2^2$

Solution:

325

Exercise:

Problem: $3 \cdot \left(2^2 + 3^2\right)$

Exercise:

Problem: $64 \cdot \left(3^2 - 2^3\right)$

Solution:

64

Exercise:

Problem: $\frac{5^2+1}{13} + \frac{3^3+1}{14}$

Exercise:

Problem: $\frac{6^2-1}{5\cdot 7} - \frac{49+7}{2\cdot 7}$

Solution:

-3

Exercise:

Problem: $\frac{2\cdot\left[3+5\left(2^2+1\right)\right]}{5\cdot 2^3-3^2}$

Exercise:

Problem: $\frac{3^2\cdot\left[2^5-1^4\left(2^3+25\right)\right]}{2\cdot 5^2+5+2}$

Solution:

$-\frac{9}{57}$

Exercise:

Problem: $\frac{\left(5^2-2^3\right)-2\cdot 7}{2^2-1}+5\cdot\left[\frac{3^2-3}{2}+1\right]$

Exercise:

Problem: $(8-3)^2+\left(2+3^2\right)^2$

Solution:

146

Exercise:

Problem: $3^2\cdot\left(4^2+\sqrt{25}\right)+2^3\cdot\left(\sqrt{81}-3^2\right)$

Exercise:

Problem: $\sqrt{16+9}$

Solution:

5

Exercise:

Problem: $\sqrt{16}+\sqrt{9}$

Exercise:

Problem:

Compare the results of problems 39 and 40. What might we conclude?

Solution:

The sum of square roots is not necessarily equal to the square root of the sum.

Exercise:

Problem: $\sqrt{18 \cdot 2}$

Exercise:

Problem: $\sqrt{6 \cdot 6}$

Solution:

6

Exercise:

Problem: $\sqrt{7 \cdot 7}$

Exercise:

Problem: $\sqrt{8 \cdot 8}$

Solution:

8

Exercise:

Problem:

An records the number of identical factors that are repeated in a multiplication.

Prime Factorization of Natural Numbers ([link])

For problems 47- 53, find all the factors of each number.

Exercise:

Problem: 18

Solution:

1, 2, 3, 6, 9, 18

Exercise:

Problem: 24

Exercise:

Problem: 11

Solution:

1, 11

Exercise:

Problem: 12

Exercise:

Problem: 51

Solution:

1, 3, 17, 51,

Exercise:

Problem: 25

Exercise:

Problem: 2

Solution:

1, 2

Exercise:

Problem: What number is the smallest prime number?

Grouping Symbol and the Order of Operations ([link])

For problems 55 -64, write each number as a product of prime factors.

Exercise:

Problem: 55

Solution:

$5 \cdot 11$

Exercise:

Problem: 20

Exercise:

Problem: 80

Solution:

$2^4 \cdot 5$

Exercise:

Problem: 284

Exercise:

Problem: 700

Solution:

$2^2 \cdot 5^2 \cdot 7$

Exercise:

Problem: 845

Exercise:

Problem: 1,614

Solution:

$2 \cdot 3 \cdot 269$

Exercise:

Problem: 921

Exercise:

Problem: 29

Solution:

29 is a prime number

Exercise:

Problem: 37

The Greatest Common Factor ([link])

For problems 65 - 75, find the greatest common factor of each collection of numbers.

Exercise:

Problem: 5 and 15

Solution:

5

Exercise:

Problem: 6 and 14

Exercise:

Problem: 10 and 15

Solution:

5

Exercise:

Problem: 6, 8, and 12

Exercise:

Problem: 18 and 24

Solution:

6

Exercise:

Problem: 42 and 54

Exercise:

Problem: 40 and 60

Solution:

20

Exercise:

Problem: 18, 48, and 72

Exercise:

Problem: 147, 189, and 315

Solution:

21

Exercise:

Problem: 64, 72, and 108

Exercise:

Problem: 275, 297, and 539

Solution:

11

The Least Common Multiple ([link])

For problems 76-86, find the least common multiple of each collection of numbers.

Exercise:

Problem: 5 and 15

Exercise:

Problem: 6 and 14

Solution:

42

Exercise:

Problem: 10 and 15

Exercise:

Problem: 36 and 90

Solution:

180

Exercise:

Problem: 42 and 54

Exercise:

Problem: 8, 12, and 20

Solution:

120

Exercise:

Problem: 40, 50, and 180

Exercise:

Problem: 135, 147, and 324

Solution:

79, 380

Exercise:

Problem: 108, 144, and 324

Exercise:

Problem: 5, 18, 25, and 30

Solution:

450

Exercise:

Problem: 12, 15, 18, and 20

Exercise:

Problem: Find all divisors of 24.

Solution:

1, 2, 3, 4, 6, 8, 12, 24

Exercise:

Problem: Find all factors of 24.

Exercise:

Problem: Write all divisors of $2^3 \cdot 5^2 \cdot 7$.

Solution:

1, 2, 4, 5, 7, 8, 10, 14, 20, 25, 35, 40, 50, 56, 70, 100, 140, 175, 200, 280, 700, 1,400

Exercise:

Problem: Write all divisors of $6 \cdot 8^2 \cdot 10^3$.

Exercise:

Problem: Does 7 divide $5^3 \cdot 6^4 \cdot 7^2 \cdot 8^5$?

Solution:

yes

Exercise:

Problem: Does 13 divide $8^3 \cdot 10^2 \cdot 11^4 \cdot 13^2 \cdot 15$?

Proficiency Exam
This module is from Fundamentals of Mathematics by Denny Burzynski and Wade Ellis, Jr. This module is a proficiency exam to the chapter "Exponents, Roots, Factorization of Whole Numbers." Each problem is accompanied with a reference link pointing back to the module that discusses the type of problem demonstrated in the question. The problems in this exam are accompanied by solutions.

Proficiency Exam

Exercise:

Problem:

([link]) In the number 8^5, write the names used for the number 8 and the number 5.

Solution:

base; exponent

Exercise:

Problem:

([link]) Write using exponents. $12 \times 12 \times 12 \times 12 \times 12 \times 12 \times 12$

Solution:

12^7

Exercise:

Problem: ([link]) Expand 9^4.

Solution:

$9^4 = 9 \cdot 9 \cdot 9 \cdot 9 = 6{,}561$

For problems 4-15, determine the value of each expression.

Exercise:

Problem: ([link]) 4^3

Solution:

64

Exercise:

Problem: ([link]) 1^5

Solution:

1

Exercise:

Problem: ([link]) 0^3

Solution:

0

Exercise:

Problem: ([link]) 2^6

Solution:

64

Exercise:

Problem: ([link]) $\sqrt{49}$

Solution:

7

Exercise:

Problem: ([link]) $\sqrt[3]{27}$

Solution:

3

Exercise:

Problem: ([link]) $\sqrt[8]{1}$

Solution:

1

Exercise:

Problem: ([link]) $16+2\cdot(8-6)$

Solution:

20

Exercise:

Problem: ([link]) $5^3-\sqrt{100}+8\cdot 2-20\div 5$

Solution:

127

Exercise:

Problem: ([link]) $3\cdot\frac{8^2-2\cdot 3^2}{5^2-2}\cdot\frac{6^3-4\cdot 5^2}{29}$

Solution:

24

Exercise:

Problem: ([link]) $\frac{20+2^4}{2^3 \cdot 2-5 \cdot 2} \cdot \frac{5 \cdot 7-\sqrt{81}}{7+3 \cdot 2}$

Solution:

8

Exercise:

Problem:

([link]) $\left[(8-3)^2+\left(33-4\sqrt{49}\right)\right]-2\left[\left(10-3^2\right)+9\right]-5$

Solution:

5

For problems 16-20, find the prime factorization of each whole number. If the number is prime, write "prime."

Exercise:

Problem: ([link]) 18

Solution:

$3^2 \cdot 2$

Exercise:

Problem: ([link]) 68

Solution:

$2^2 \cdot 17$

Exercise:

Problem: ([link]) 142

Solution:

$2 \cdot 71$

Exercise:

Problem: ([link]) 151

Solution:

prime

Exercise:

Problem: ([link]) 468

Solution:

$2^2 \cdot 3^2 \cdot 13$

For problems 21 and 22, find the greatest common factor.

Exercise:

Problem: ([link]) 200 and 36

Solution:

4

Exercise:

Problem: ([link]) 900 and 135

Solution:

45

Exercise:

Problem: ([link]) Write all the factors of 36.

Solution:

1, 2, 3, 4, 6, 9, 12, 18, 36

Exercise:

Problem: ([link]) Write all the divisors of 18.

Solution:

1, 2, 3, 6, 9, 18

Exercise:

Problem: ([link]) Does 7 divide into $5^2 \cdot 6^3 \cdot 7^4 \cdot 8$? Explain.

Solution:

Yes, because one of the (prime) factors of the number is 7.

Exercise:

Problem: ([link]) Is 3 a factor of $2^6 \cdot 3^2 \cdot 5^3 \cdot 4^6$? Explain.

Solution:

Yes, because it is one of the factors of the number.

Exercise:

Problem: ([link]) Does 13 divide into $11^3 \cdot 12^4 \cdot 15^2$? Explain.

Solution:

No, because the prime 13 is not a factor any of the listed factors of the number.

For problems 28 and 29, find the least common multiple.
Exercise:

Problem: ([link]) 432 and 180

Solution:

2,160

Exercise:

Problem: ([link]) 28, 40, and 95

Solution:

5,320

Objectives
This module contains the learning objectives for the chapter "Introduction to Fractions and Multiplication and Division of Fractions" from Fundamentals of Mathematics by Denny Burzynski and Wade Ellis, jr.

After completing this chapter, you should
Fractions of Whole Numbers ([link])

- understand the concept of fractions of whole numbers
- be able to recognize the parts of a fraction

Proper Fractions, improper Fractions, and Mixed Numbers ([link])

- be able to distinguish between proper fractions, improper fractions, and mixed numbers
- be able to convert an improper fraction to a mixed number
- be able to convert a mixed number to an improper fraction

Equivalent Fractions, Reducing Fractions to Lowest Terms, and Raising Fractions to Higher Terms ([link])

- be able to recognize equivalent fractions
- be able to reduce a fraction to lowest terms
- be able to raise a fraction to higher terms

Multiplication of Fractions ([link])

- understand the concept of multiplication of fractions
- be able to multiply one fraction by another
- be able to multiply mixed numbers
- be able to find powers and roots of various fractions

Division of Fractions ([link])

- be able to determine the reciprocal of a number
- be able to divide one fraction by another

Applications Involving Fractions ([link])

- be able to solve missing product statements
- be able to solve missing factor statements

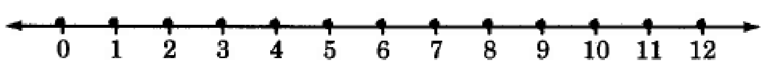
Fractions of Whole Numbers
This module is from Fundamentals of Mathematics by Denny Burzynski and Wade Ellis, Jr. This module discusses fractions of whole numbers. By the end of the module students should be able to understand the concept of fractions of whole numbers and recognize the parts of a fraction.

Section Overview

- More Numbers on the Number Line
- Fractions of Whole Numbers
- The Parts of a Fraction
- Reading and Writing Fractions

More Numbers on the Number Line

In Chapters [link], [link], and [link], we studied the whole numbers and methods of combining them. We noted that we could visually display the whole numbers by drawing a number line and placing closed circles at whole number locations.



By observing this number line, we can see that the whole numbers do not account for every point on the line. What numbers, if any, can be associated with these points? In this section we will see that many of the points on the number line, including the points already associated with whole numbers, can be associated with numbers called *fractions*.

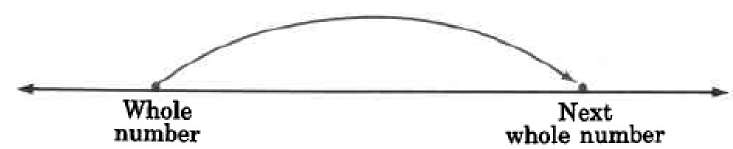
Fractions of Whole Numbers

The Nature of the Positive Fractions

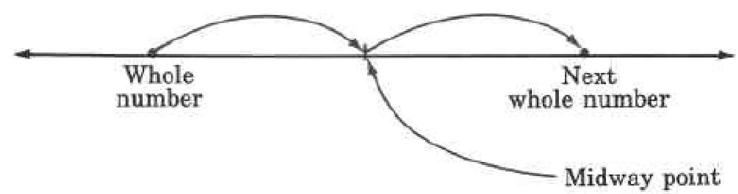
We can extend our collection of numbers, which now contains only the whole numbers, by including fractions of whole numbers. We can determine the nature of these fractions using the number line.

If we place a pencil at some whole number and proceed to travel to the right to the next whole number, we see that our journey can be *broken* into different types of equal parts as shown in the following examples.

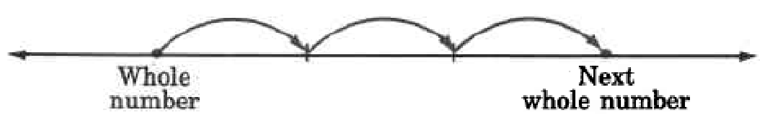
a. 1 part.



b. 2 equal parts.



c. 3 equal parts.

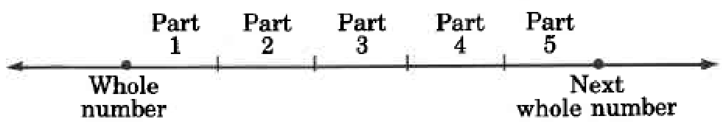


d. 4 equal parts.

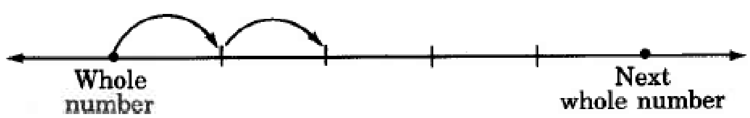
The Latin Word Fractio

Notice that the number of parts, 2, 3, and 4, that we are breaking the original quantity into is always a *nonzero whole number*. The idea of breaking up a whole quantity gives us the word *fraction*. The word fraction comes from the Latin word "fractio" which means a breaking, or fracture.

Suppose we break up the interval from some whole number to the next whole number into five equal parts.



After starting to move from one whole number to the next, we decide to stop after covering only two parts. We have covered 2 parts of 5 equal parts. This situation is described by writing $\frac{2}{5}$.



Positive Fraction
A number such as $\dfrac{2}{5}$ is called a **positive fraction**, or more simply, a **fraction**.

The Parts of a Fraction

A fraction has *three parts*.

1. The fraction bar — .

   Fraction Bar
   The **fraction bar** serves as a grouping symbol. It separates a quantity into individual groups. These groups have names, as noted in 2 and 3 below.
2. The nonzero number below the fraction bar.

   Denominator
   This number is called the **denominator** of the fraction, and it indicates the number of parts the whole quantity has been divided into. Notice

that the denominator must be a nonzero whole number since the least number of parts any quantity can have is one.

3. The number above the fraction bar.

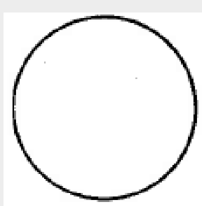
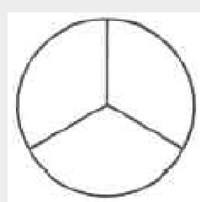
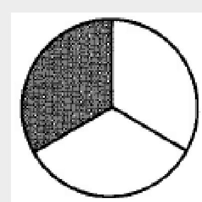
   Numerator
   This number is called the **numerator** of the fraction, and it indicates how many of the specified parts are being considered. Notice that the numerator can be any whole number (including zero) since any number of the specified parts can be considered.

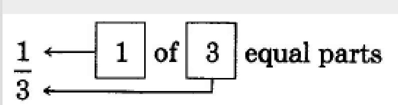
$$\frac{\text{whole number}}{\text{nonzero whole number}} \leftrightarrow \frac{\text{numerator}}{\text{denominator}}$$

Sample Set A

The diagrams in the following problems are illustrations of fractions.

Example:

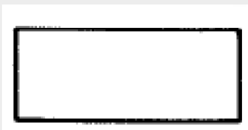
Diagrams	A whole circle	The whole circle divided into 3 equal parts	1 of the 3 equal parts
			



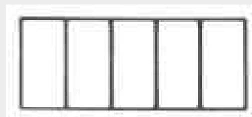
The fraction $\frac{1}{3}$ is read as "one third."

Example:

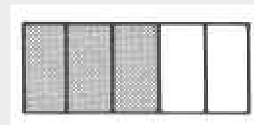
A whole rectangle

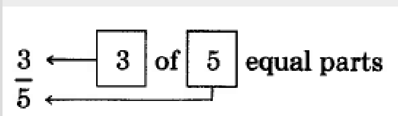


The whole rectangle divided into 5 equal parts



3 of the 5 equal parts

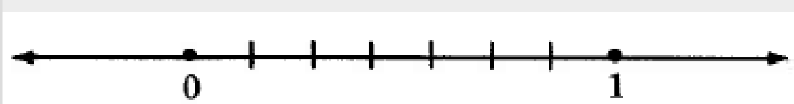




The fraction $\frac{3}{5}$ "is read as "three fifths."

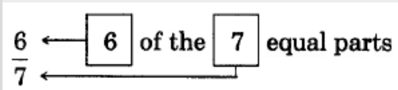
Example:

The number line between 0 and 1



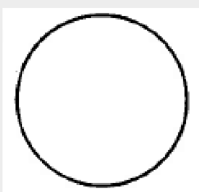
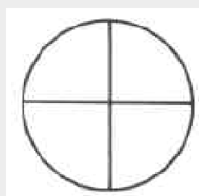
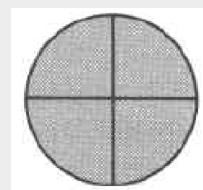
The number line between 0 and 1 divided into 7 equal parts

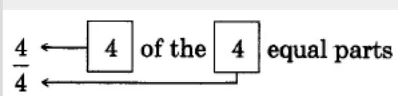
6 of the 7 equal parts



The fraction $\frac{6}{7}$ is read as "six sevenths."

Example:

A whole circle	The whole circle divided into 4 equal parts	4 of the 4 equal parts
		



When the numerator and denominator are equal, the fraction represents the entire quantity, and its value is 1.

$\frac{\text{nonzero whole number}}{\text{same nonzero whole number}} = 1$

Practice Set A

Specify the numerator and denominator of the following fractions.

Exercise:

Problem: $\frac{4}{7}$

Solution:

4, 7

Exercise:

Problem: $\frac{5}{8}$

Solution:

5, 8

Exercise:

Problem: $\frac{10}{15}$

Solution:

10, 15

Exercise:

Problem: $\frac{1}{9}$

Solution:

1, 9

Exercise:

Problem: $\frac{0}{2}$

Solution:

0, 2

Reading and Writing Fractions

In order to properly translate fractions from word form to number form, or from number form to word form, it is necessary to understand the use of the *hyphen*.

Use of the Hyphen
One of the main uses of the **hyphen** is to tell the reader that two words not ordinarily joined are to be taken in combination as a unit. Hyphens are *always* used for numbers between and including 21 and 99 (except those ending in zero).

Sample Set B

Write each fraction using whole numbers.

Example:
Fifty three-hundredths. The hyphen joins the words three and hundredths and tells us to consider them as a single unit. Therefore,
fifty three-hundredths translates as $\frac{50}{300}$

Example:
Fifty-three hundredths. The hyphen joins the numbers fifty and three and tells us to consider them as a single unit. Therefore,
fifty-three hundredths translates as $\frac{53}{100}$

Example:
Four hundred seven-thousandths. The hyphen joins the words seven and thousandths and tells us to consider them as a single unit. Therefore,
four hundred seven-thousandths translates as $\frac{400}{7,000}$

Example:
Four hundred seven thousandths. The absence of hyphens indicates that the words *seven* and *thousandths* are to be considered individually.
four hundred seven thousandths translates as $\frac{407}{1000}$

Write each fraction using words.

Example:
$\frac{21}{85}$ translates as twenty-one eighty-fifths.

Example:
$\frac{200}{3,000}$ translates as two hundred three-thousandths. A hyphen is needed between the words three and thousandths to tell the reader that these words are to be considered as a single unit.

Example:
$\frac{203}{1,000}$ translates as two hundred three thousandths.

Practice Set B

Write the following fractions using whole numbers.
Exercise:

Problem:one tenth

Solution:

$\frac{1}{10}$

Exercise:

Problem: eleven fourteenths

Solution:

$\frac{11}{14}$

Exercise:

Problem: sixteen thirty-fifths

Solution:

$\frac{16}{35}$

Exercise:

Problem: eight hundred seven-thousandths

Solution:

$\frac{800}{7,000}$

Write the following using words.

Exercise:

Problem: $\frac{3}{8}$

Solution:

three eighths

Exercise:

Problem: $\frac{1}{10}$

Solution:

one tenth

Exercise:

Problem: $\frac{3}{250}$

Solution:

three two hundred fiftieths

Exercise:

Problem: $\frac{114}{3,190}$

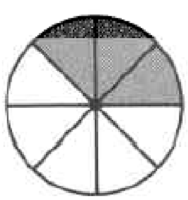
Solution:

one hundred fourteen three thousand one hundred ninetieths

Name the fraction that describes each shaded portion.

Exercise:

Problem:

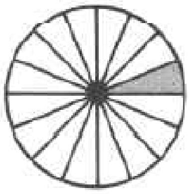


Solution:

$\frac{3}{8}$

Exercise:

Problem:



Solution:

$\frac{1}{16}$

In the following 2 problems, state the numerator and denominator, and write each fraction in words.

Exercise:

Problem:

The number $\frac{5}{9}$ is used in converting from Fahrenheit to Celsius.

Solution:

5, 9, five ninths

Exercise:

Problem: A dime is $\frac{1}{10}$ of a dollar.

Solution:

1, 10, one tenth

Exercises

For the following 10 problems, specify the numerator and denominator in each fraction.

Exercise:

Problem: $\frac{3}{4}$

Solution:

numerator, 3; denominator, 4

Exercise:

Problem: $\frac{9}{10}$

Exercise:

Problem: $\frac{1}{5}$

Solution:

numerator, 1; denominator, 5

Exercise:

Problem: $\frac{5}{6}$

Exercise:

Problem: $\frac{7}{7}$

Solution:

numerator, 7; denominator, 7

Exercise:

Problem: $\frac{4}{6}$

Exercise:

Problem: $\frac{0}{12}$

Solution:

numerator, 0; denominator, 12

Exercise:

Problem: $\frac{25}{25}$

Exercise:

Problem: $\frac{18}{1}$

Solution:

numerator, 18; denominator, 1

Exercise:

Problem: $\frac{0}{16}$

For the following 10 problems, write the fractions using whole numbers.

Exercise:

Problem: four fifths

Solution:

$\frac{4}{5}$

Exercise:

Problem: two ninths

Exercise:

Problem: fifteen twentieths

Solution:

$\frac{15}{20}$

Exercise:

Problem: forty-seven eighty-thirds

Exercise:

Problem: ninety-one one hundred sevenths

Solution:

$\frac{91}{107}$

Exercise:

Problem: twenty-two four hundred elevenths

Exercise:

Problem: six hundred five eight hundred thirty-fourths

Solution:

$\frac{605}{834}$

Exercise:

Problem: three thousand three forty-four ten-thousandths

Exercise:

Problem: ninety-two one-millionths

Solution:

$\frac{92}{1{,}000{,}000}$

Exercise:

Problem: one three-billionths

For the following 10 problems, write the fractions using words.

Exercise:

Problem: $\frac{5}{9}$

Solution:

five ninths

Exercise:

Problem: $\frac{6}{10}$

Exercise:

Problem: $\frac{8}{15}$

Solution:

eight fifteenths

Exercise:

Problem: $\frac{10}{13}$

Exercise:

Problem: $\frac{75}{100}$

Solution:

seventy-five one hundredths

Exercise:

Problem: $\frac{86}{135}$

Exercise:

Problem: $\frac{916}{1,014}$

Solution:

nine hundred sixteen one thousand fourteenths

Exercise:

Problem: $\frac{501}{10,001}$

Exercise:

Problem: $\frac{18}{31,608}$

Solution:

eighteen thirty-one thousand six hundred eighths

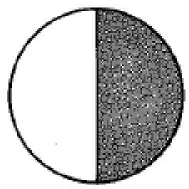
Exercise:

Problem: $\frac{1}{500,000}$

For the following 4 problems, name the fraction corresponding to the shaded portion.

Exercise:

Problem:

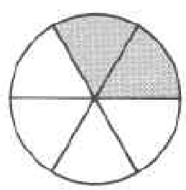


Solution:

$\frac{1}{2}$

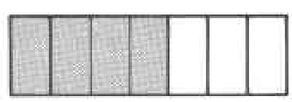
Exercise:

Problem:



Exercise:

Problem:

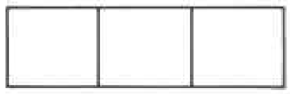


Solution:

$\frac{4}{7}$

Exercise:

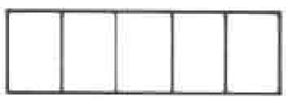
Problem:



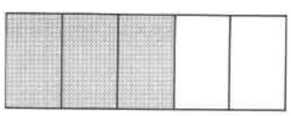
For the following 4 problems, shade the portion corresponding to the given fraction on the given figure.

Exercise:

Problem: $\frac{3}{5}$

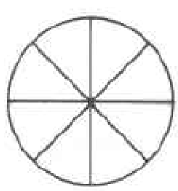


Solution:



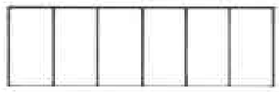
Exercise:

Problem: $\frac{1}{8}$

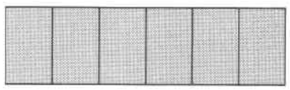


Exercise:

Problem: $\frac{6}{6}$

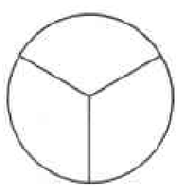


Solution:



Exercise:

Problem: $\frac{0}{3}$



State the numerator and denominator and write in words each of the fractions appearing in the statements for the following 10 problems.

Exercise:

Problem: A contractor is selling houses on $\frac{1}{4}$ acre lots.

Solution:

Numerator, 1; denominator, 4; one fourth

Exercise:

Problem:

The fraction $\frac{22}{7}$ is sometimes used as an approximation to the number π. (The symbol is read “pi.")

Exercise:

Problem: The fraction $\frac{4}{3}$ is used in finding the volume of a sphere.

Solution:

Numerator, 4; denominator, 3; four thirds

Exercise:

Problem: One inch is $\frac{1}{12}$ of a foot.

Exercise:

Problem:

About $\frac{2}{7}$ of the students in a college statistics class received a “B” in the course.

Solution:

Numerator, 2; denominator, 7; two sevenths

Exercise:

Problem:

The probability of randomly selecting a club when drawing one card from a standard deck of 52 cards is $\frac{13}{52}$.

Exercise:

Problem:

In a box that contains eight computer chips, five are known to be good and three are known to be defective. If three chips are selected at random, the probability that all three are defective is $\frac{1}{56}$.

Solution:

Numerator, 1; denominator, 56; one fifty-sixth

Exercise:

Problem:

In a room of 25 people, the probability that at least two people have the same birthdate (date and month, not year) is $\frac{569}{1000}$.

Exercise:

Problem:

The mean (average) of the numbers 21, 25, 43, and 36 is $\frac{125}{4}$.

Solution:

Numerator, 125; denominator, 4; one hundred twenty-five fourths

Exercise:

Problem:

If a rock falls from a height of 20 meters on Jupiter, the rock will be $\frac{32}{25}$ meters high after $\frac{6}{5}$ seconds.

Exercises For Review

Exercise:

Problem:

([link]) Use the numbers 3 and 11 to illustrate the commutative property of addition.

Solution:

$3 + 11 = 11 + 3 = 14$

Exercise:

Problem: ([link]) Find the quotient. $676 \div 26$

Exercise:

Problem: ([link]) Write $7 \cdot 7 \cdot 7 \cdot 7 \cdot 7$ using exponents.

Solution:

7^5

Exercise:

Problem: ([link]) Find the value of $\frac{8 \cdot (6+20)}{8} + \frac{3 \cdot (6+16)}{22}$.

Exercise:

Problem: ([link]) Find the least common multiple of 12, 16, and 18.

Solution:

144

Proper Fractions, Improper Fractions, and Mixed Numbers
This module is from Fundamentals of Mathematics by Denny Burzynski and Wade Ellis, Jr. This module discusses proper fractions, improper fractions, and mixed numbers. By the end of the module students should be able to distinguish between proper fractions, improper fractions, and mixed numbers, convert an improper fraction to a mixed number and convert a mixed number to an improper fraction.

Section Overview

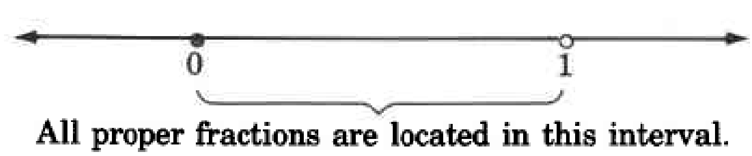
- Positive Proper Fractions
- Positive Improper Fractions
- Positive Mixed Numbers
- Relating Positive Improper Fractions and Positive Mixed Numbers
- Converting an Improper Fraction to a Mixed Number
- Converting a Mixed Number to an Improper Fraction

Now that we know what positive fractions are, we consider three types of positive fractions: proper fractions, improper fractions, and mixed numbers.

Positive Proper Fractions

Positive Proper Fraction

Fractions in which the whole number in the numerator is strictly less than the whole number in the denominator are called **positive proper fractions**. On the number line, proper fractions are located in the interval from 0 to 1. Positive proper fractions are always less than one.



The closed circle at 0 indicates that 0 is included, while the open circle at 1 indicates that 1 is not included.

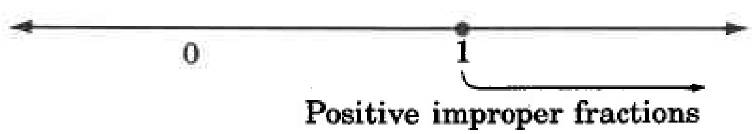
Some examples of positive proper fractions are

$\frac{1}{2}$, $\frac{3}{5}$, $\frac{20}{27}$, and $\frac{106}{255}$

Note that $1 < 2$, $3 < 5$, $20 < 27$, and $106 < 225$.

Positive Improper Fractions

Positive Improper Fractions
Fractions in which the whole number in the numerator is greater than or equal to the whole number in the denominator are called **positive improper fractions**. On the number line, improper fractions lie to the right of (and including) 1. Positive improper fractions are always greater than or equal to 1.



Some examples of positive improper fractions are

$\frac{3}{2}$, $\frac{8}{5}$, $\frac{4}{4}$, and $\frac{105}{16}$

Note that $3 \geq 2$, $8 \geq 5$, $4 \geq 4$, and $105 \geq 16$.

Positive Mixed Numbers

Positive Mixed Numbers
A number of the form

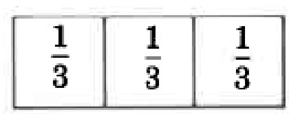
$$\text{nonzero whole number} + \text{proper fraction}$$

is called a **positive mixed number**. For example, $2\frac{3}{5}$ is a mixed number. On the number line, mixed numbers are located in the interval to the right of (and including) 1. Mixed numbers are always greater than or equal to 1.

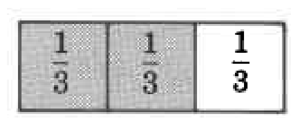
Relating Positive Improper Fractions and Positive Mixed Numbers

A relationship between improper fractions and mixed numbers is suggested by two facts. The first is that improper fractions and mixed numbers are located in the same interval on the number line. The second fact, that mixed numbers are the sum of a natural number and a fraction, can be seen by making the following observations.

Divide a whole quantity into 3 equal parts.

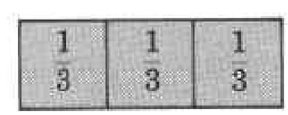


Now, consider the following examples by observing the respective shaded areas.



In the shaded region, there are 2 one thirds, or $\frac{2}{3}$.

$2\left(\frac{1}{3}\right) = \frac{2}{3}$



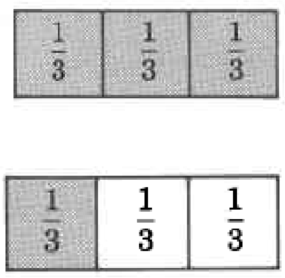
There are 3 one thirds, or $\frac{3}{3}$, or 1.

$3\left(\frac{1}{3}\right) = \frac{3}{3}$ or 1

Thus,

$\frac{3}{3} = 1$

Improper fraction = whole number.



There are 4 one thirds, or $\frac{4}{3}$, or 1 and $\frac{1}{3}$.

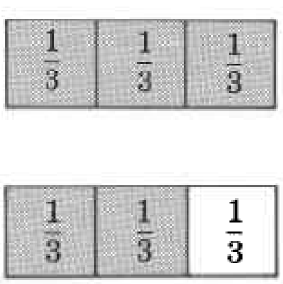
$4\left(\frac{1}{3}\right) = \frac{4}{3}$or $1 \text{ and } \frac{1}{3}$

The terms 1 and $\frac{1}{3}$ can be represented as $1 + \frac{1}{3}$ or $1\frac{1}{3}$

Thus,

$\frac{4}{3} = 1\frac{1}{3}$.

Improper fraction = mixed number.



There are 5 one thirds, or $\frac{5}{3}$, or 1 and $\frac{2}{3}$.

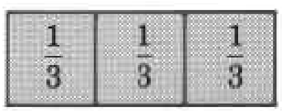
$5\left(\frac{1}{3}\right) = \frac{5}{3} \text{ or } 1 \text{ and } \frac{2}{3}$

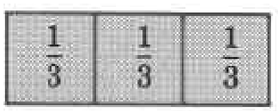
The terms 1 and $\frac{2}{3}$ can be represented as $1 + \frac{2}{3}$ or $1\frac{2}{3}$.

Thus,

$\frac{5}{3} = 1\frac{2}{3}$.

Improper fraction = mixed number.





There are 6 one thirds, or $\frac{6}{3}$, or 2.

$6\left(\frac{1}{3}\right) = \frac{6}{3} = 2$

Thus,

$\frac{6}{3} = 2$

Improper fraction = whole number.

The following important fact is illustrated in the preceding examples.

Mixed Number = Natural Number + Proper Fraction
Mixed numbers are the *sum* of a natural number and a proper fraction.
Mixed number = (natural number) + (proper fraction)

For example $1\frac{1}{3}$ can be expressed as $1 + \frac{1}{3}$ The fraction $5\frac{7}{8}$ can be expressed as $5 + \frac{7}{8}$.

It is important to note that a number such as $5 + \frac{7}{8}$ does *not* indicate multiplication. To indicate multiplication, we would need to use a

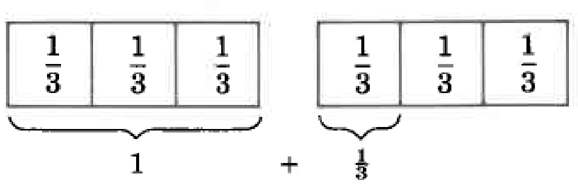
multiplication symbol (such as ·)

Note: $5\frac{7}{8}$ means $5+\frac{7}{8}$ and *not* $5\cdot\frac{7}{8}$, which means 5 times $\frac{7}{8}$ or 5 multiplied by $\frac{7}{8}$.

Thus, mixed numbers may be represented by improper fractions, and improper fractions may be represented by mixed numbers.

Converting Improper Fractions to Mixed Numbers

To understand how we might convert an improper fraction to a mixed number, let's consider the fraction, $\frac{4}{3}$.



$$\begin{aligned} \frac{4}{3} &= \frac{1}{3}+\frac{1}{3}+\frac{1}{3}+\frac{1}{3} \\ & \qquad 1 \\ &= 1+\frac{1}{3} \\ &= 1\frac{1}{3} \end{aligned}$$

Thus, $\frac{4}{3}=1\frac{1}{3}$.

We can illustrate a procedure for converting an improper fraction to a mixed number using this example. However, the conversion is *more easily* accomplished by dividing the numerator by the denominator and using the result to write the mixed number.

Converting an Improper Fraction to a Mixed Number

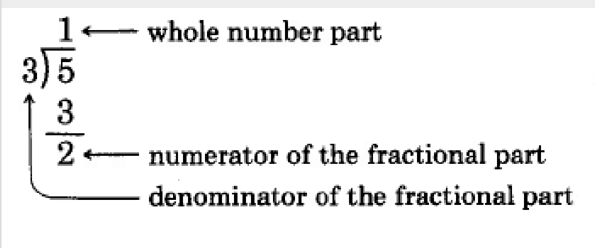
To convert an improper fraction to a mixed number, divide the numerator by the denominator.

1. The whole number part of the mixed number is the quotient.
2. The fractional part of the mixed number is the remainder written over the divisor (the denominator of the improper fraction).

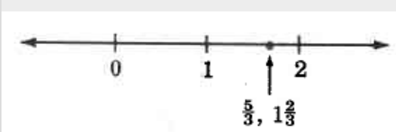
Sample Set A

Convert each improper fraction to its corresponding mixed number.

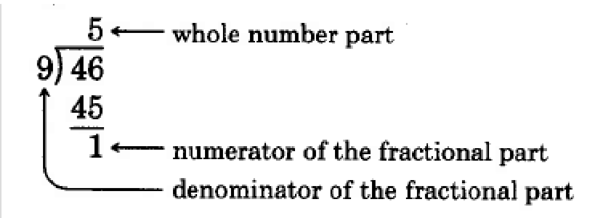
Example:
$\frac{5}{3}$ Divide 5 by 3.



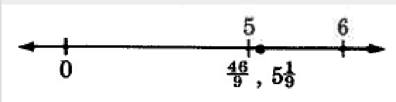
The improper fraction $\frac{5}{3} = 1\frac{2}{3}$.



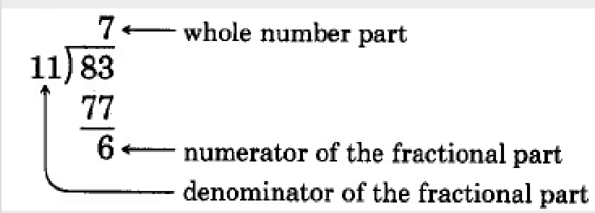
Example:
$\frac{46}{9}$. Divide 46 by 9.



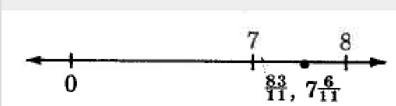
The improper fraction $\frac{46}{9} = 5\frac{1}{9}$.



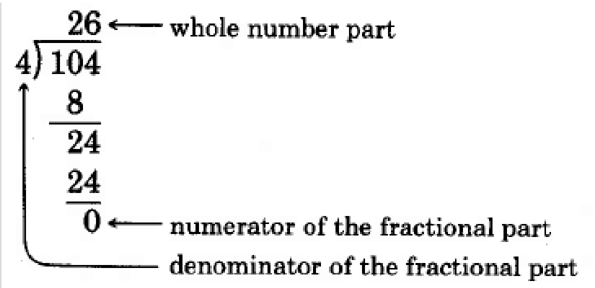
Example:
$\frac{83}{11}$. Divide 83 by 11.



The improper fraction $\frac{83}{11} = 7\frac{6}{11}$.

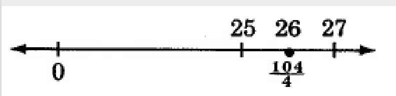


Example:
$\frac{104}{4}$ Divide 104 by 4.



$\frac{104}{4} = 26\frac{0}{4} = 26$

The improper fraction $\frac{104}{4} = 26$.



Practice Set A

Convert each improper fraction to its corresponding mixed number.

Exercise:

Problem: $\frac{9}{2}$

Solution:

$4\frac{1}{2}$

Exercise:

Problem: $\frac{11}{3}$

Solution:

$3\frac{2}{3}$

Exercise:

Problem: $\frac{14}{11}$

Solution:

$1\frac{3}{11}$

Exercise:

Problem: $\frac{31}{13}$

Solution:

$2\frac{5}{13}$

Exercise:

Problem: $\frac{79}{4}$

Solution:

$19\frac{3}{4}$

Exercise:

Problem: $\frac{496}{8}$

Solution:

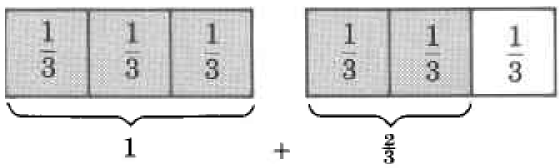
62

Converting Mixed Numbers to Improper Fractions

To understand how to convert a mixed number to an improper fraction, we'll recall

mixed number = (natural number) + (proper fraction)

and consider the following diagram.



$$1\frac{2}{3} = \underbrace{\overbrace{\frac{1}{3}+\frac{1}{3}+\frac{1}{3}}^{1+} + \overbrace{\frac{1}{3}+\frac{1}{3}}^{\frac{2}{3}}}_{5\cdot\frac{1}{3}=\frac{5}{3}}$$

Recall that multiplication describes repeated addition.

Notice that $\frac{5}{3}$ can be obtained from $1\frac{2}{3}$ using multiplication in the following way.

Multiply: $3 \cdot 1 = 3$

$$1\frac{2}{3}$$

Add: $3 + 2 = 5$. Place the 5 over the 3: $\frac{5}{3}$

The procedure for converting a mixed number to an improper fraction is illustrated in this example.

Converting a Mixed Number to an Improper Fraction
To convert a mixed number to an improper fraction,

1. Multiply the denominator of the fractional part of the mixed number by the whole number part.
2. To this product, add the numerator of the fractional part.

3. Place this result over the denominator of the fractional part.

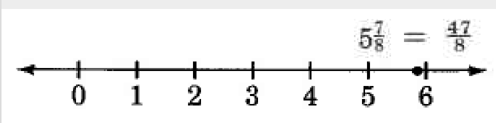
Sample Set B

Convert each mixed number to an improper fraction.

Example:
$5\frac{7}{8}$

1. Multiply: $8 \cdot 5 = 40$.
2. Add: $40 + 7 = 47$.
3. Place 47 over 8: $\frac{47}{8}$.

Thus, $5\frac{7}{8} = \frac{47}{8}$.



Example:
$16\frac{2}{3}$

1. Multiply: $3 \cdot 16 = 48$.
2. Add: $48 + 2 = 50$.
3. Place 50 over 3: $\frac{50}{3}$

Thus, $16\frac{2}{3} = \frac{50}{3}$

Practice Set B

Convert each mixed number to its corresponding improper fraction.

Exercise:

Problem: $8\frac{1}{4}$

Solution:

$\frac{33}{4}$

Exercise:

Problem: $5\frac{3}{5}$

Solution:

$\frac{28}{5}$

Exercise:

Problem: $1\frac{4}{15}$

Solution:

$\frac{19}{15}$

Exercise:

Problem: $12\frac{2}{7}$

Solution:

$\frac{86}{7}$

Exercises

For the following 15 problems, identify each expression as a proper fraction, an improper fraction, or a mixed number.

Exercise:

Problem: $\frac{3}{2}$

Solution:

improper fraction

Exercise:

Problem: $\frac{4}{9}$

Exercise:

Problem: $\frac{5}{7}$

Solution:

proper fraction

Exercise:

Problem: $\frac{1}{8}$

Exercise:

Problem: $6\frac{1}{4}$

Solution:

mixed number

Exercise:

Problem: $\frac{11}{8}$

Exercise:

Problem: $\frac{1,001}{12}$

Solution:

improper fraction

Exercise:

Problem: $191\frac{4}{5}$

Exercise:

Problem: $1\frac{9}{13}$

Solution:

mixed number

Exercise:

Problem: $31\frac{6}{7}$

Exercise:

Problem: $3\frac{1}{40}$

Solution:

mixed number

Exercise:

Problem: $\frac{55}{12}$

Exercise:

Problem: $\frac{0}{9}$

Solution:

proper fraction

Exercise:

Problem: $\frac{8}{9}$

Exercise:

Problem: $101\frac{1}{11}$

Solution:

mixed number

For the following 15 problems, convert each of the improper fractions to its corresponding mixed number.

Exercise:

Problem: $\frac{11}{6}$

Exercise:

Problem: $\frac{14}{3}$

Solution:

$4\frac{2}{3}$

Exercise:

Problem: $\frac{25}{4}$

Exercise:

Problem: $\frac{35}{4}$

Solution:

$8\frac{3}{4}$

Exercise:

Problem: $\frac{71}{8}$

Exercise:

Problem: $\frac{63}{7}$

Solution:

9

Exercise:

Problem: $\frac{121}{11}$

Exercise:

Problem: $\frac{165}{12}$

Solution:

$13\frac{9}{12}$ or $13\frac{3}{4}$

Exercise:

Problem: $\frac{346}{15}$

Exercise:

Problem: $\frac{5{,}000}{9}$

Solution:

$555\frac{5}{9}$

Exercise:

Problem: $\frac{23}{5}$

Exercise:

Problem: $\frac{73}{2}$

Solution:

$36\frac{1}{2}$

Exercise:

Problem: $\frac{19}{2}$

Exercise:

Problem: $\frac{316}{41}$

Solution:

$7\frac{29}{41}$

Exercise:

Problem: $\frac{800}{3}$

For the following 15 problems, convert each of the mixed numbers to its corresponding improper fraction.

Exercise:

Problem: $4\frac{1}{8}$

Solution:

$\frac{33}{8}$

Exercise:

Problem: $1\frac{5}{12}$

Exercise:

Problem: $6\frac{7}{9}$

Solution:

$\frac{61}{9}$

Exercise:

Problem: $15\frac{1}{4}$

Exercise:

Problem: $10\frac{5}{11}$

Solution:

$\frac{115}{11}$

Exercise:

Problem: $15\frac{3}{10}$

Exercise:

Problem: $8\frac{2}{3}$

Solution:

$\frac{26}{3}$

Exercise:

Problem: $4\frac{3}{4}$

Exercise:

Problem: $21\frac{2}{5}$

Solution:

$\frac{107}{5}$

Exercise:

Problem: $17\frac{9}{10}$

Exercise:

Problem: $9\frac{20}{21}$

Solution:

$\frac{209}{21}$

Exercise:

Problem: $5\frac{1}{16}$

Exercise:

Problem: $90\frac{1}{100}$

Solution:

$\frac{9001}{100}$

Exercise:

Problem: $300\frac{43}{1,000}$

Exercise:

Problem: $19\frac{7}{8}$

Solution:

$\frac{159}{8}$

Exercise:

Problem: Why does $0\frac{4}{7}$ not qualify as a mixed number?

Note: See the definition of a mixed number.

Exercise:

Problem: Why does 5 qualify as a mixed number?

Note: See the definition of a mixed number.

Solution:

… because it may be written as $5\frac{0}{\text{n}}$, where n is any positive whole number.

Calculator Problems

For the following 8 problems, use a calculator to convert each mixed number to its corresponding improper fraction.

Exercise:

Problem: $35\frac{11}{12}$

Exercise:

Problem: $27\frac{5}{61}$

Solution:

$\frac{1{,}652}{61}$

Exercise:

Problem: $83\frac{40}{41}$

Exercise:

Problem: $105\frac{21}{23}$

Solution:

$\frac{2{,}436}{23}$

Exercise:

Problem: $72\frac{605}{606}$

Exercise:

Problem: $816\frac{19}{25}$

Solution:

$\frac{20,419}{25}$

Exercise:

Problem: $708\frac{42}{51}$

Exercise:

Problem: $6,012\frac{4,216}{8,117}$

Solution:

$\frac{48,803,620}{8,117}$

Exercises For Review

Exercise:

Problem: ([link]) Round 2,614,000 to the nearest thousand.

Exercise:

Problem: ([link]) Find the product. $1,004 \cdot 1,005$.

Solution:

1,009,020

Exercise:

Problem: ([link]) Determine if 41,826 is divisible by 2 and 3.

Exercise:

Problem: ([link]) Find the least common multiple of 28 and 36.

Solution:

252

Exercise:

Problem:

([link]) Specify the numerator and denominator of the fraction $\frac{12}{19}$.

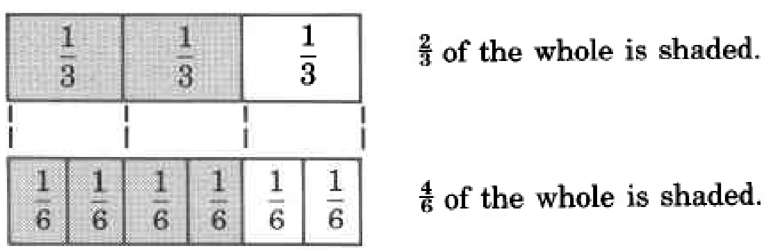
Equivalent Fractions, Reducing Fractions to Lowest Terms, and Raising Fractions to Higher Terms
This module is from Fundamentals of Mathematics by Denny Burzynski and Wade Ellis, Jr. This module discusses equivalent fractions, reducing fractions to lowest terms, and raising fractions to higher terms. By the end of the module students should be able to recognize equivalent fractions, reduce a fraction to lowest terms and be able to raise a fraction to higher terms.

Section Overview

- Equivalent Fractions
- Reducing Fractions to Lowest Terms
- Raising Fractions to Higher Terms

Equivalent Fractions

Let's examine the following two diagrams.



Notice that both $\frac{2}{3}$ and $\frac{4}{6}$ represent the *same part* of the whole, that is, they represent the same number.

Equivalent Fractions
Fractions that have the same value are called **equivalent fractions**. Equivalent fractions may look different, but they are still the same point on the number line.

There is an interesting property that equivalent fractions satisfy.

$$\frac{2}{3} \quad \frac{4}{6}$$

A Test for Equivalent Fractions Using the Cross Product
These pairs of products are called **cross products**.

$$2 \cdot 6 \overset{?}{=} 3 \cdot 4$$
$$12 \overset{\checkmark}{=} 12$$

If the cross products are equal, the fractions are equivalent. If the cross products are not equal, the fractions are not equivalent.

Thus, $\frac{2}{3}$ and $\frac{4}{6}$ are equivalent, that is, $\frac{2}{3} = \frac{4}{6}$.

Sample Set A

Determine if the following pairs of fractions are equivalent.

Example:
$\frac{3}{4}$ and $\frac{6}{8}$. Test for equality of the cross products.

$$\frac{3}{4} \quad \frac{6}{8}$$

$$3 \cdot 8 \overset{?}{=} 6 \cdot 4$$
$$24 \overset{\checkmark}{=} 24$$

The cross products are equals.
The fractions $\frac{3}{4}$ and $\frac{6}{8}$ are equivalent, so $\frac{3}{4} = \frac{6}{8}$.

Example:
$\frac{3}{8}$ and $\frac{9}{16}$. Test for equality of the cross products.

$\frac{3}{8} \times \frac{9}{16}$

$3 \cdot 16 \stackrel{?}{=} 9 \cdot 8$
$48 \neq 72$

The cross products are *not* equal.
The fractions $\frac{3}{8}$ and $\frac{9}{16}$ are not equivalent.

Practice Set A

Determine if the pairs of fractions are equivalent.

Exercise:

Problem: $\frac{1}{2}, \frac{3}{6}$

Solution:

$6 \stackrel{?}{=} 6$

, yes

Exercise:

Problem: $\frac{4}{5}, \frac{12}{15}$

Solution:

$60 \stackrel{?}{=} 60$

, yes

Exercise:

Problem: $\frac{2}{3}, \frac{8}{15}$

Solution:

$30 \neq 24$, no

Exercise:

Problem: $\frac{1}{8}, \frac{5}{40}$

Solution:

$40 \not\leq 40$

, yes

Exercise:

Problem: $\frac{3}{12}, \frac{1}{4}$

Solution:

$12 \not\leq 12$

, yes

Reducing Fractions to Lowest Terms

It is often very useful to *conver*t one fraction to an equivalent fraction that has reduced values in the numerator and denominator. We can suggest a method for doing so by considering the equivalent fractions $\frac{9}{15}$ and $\frac{3}{5}$. First, divide both the numerator and denominator of $\frac{9}{15}$ by 3. The fractions $\frac{9}{15}$ and $\frac{3}{5}$ are equivalent.

(Can you prove this?) So, $\frac{9}{15} = \frac{3}{5}$. We wish to convert $\frac{9}{15}$ to $\frac{3}{5}$. Now divide the numerator and denominator of $\frac{9}{15}$ by 3, and see what happens.

$\frac{9 \div 3}{15 \div 3} = \frac{3}{5}$

The fraction $\frac{9}{15}$ is converted to $\frac{3}{5}$.

A natural question is "Why did we choose to divide by 3?" Notice that

$$\frac{9}{15} = \frac{3 \cdot 3}{5 \cdot 3}$$

We can see that the *factor* 3 is common to both the numerator and denominator.

Reducing a Fraction
From these observations we can suggest the following method for converting one fraction to an equivalent fraction that has reduced values in the numerator and denominator. The method is called **reducing a fraction**.

A fraction can be **reduced** by dividing *both* the numerator and denominator by the *same* nonzero whole number.

$$\frac{9}{12} = \frac{9 \div 3}{12 \div 3} = \frac{3}{4} \qquad \frac{16}{30} = \frac{16 \div 2}{30 \div 2} = \frac{8}{15}$$

Notice that $\frac{3}{3} = 1$ and $\frac{2}{2} = 1$

Consider the collection of equivalent fractions

$$\frac{5}{20}, \frac{4}{16}, \frac{3}{12}, \frac{2}{8}, \frac{1}{4}$$

Reduced to Lowest Terms
Notice that each of the first four fractions can be *reduced* to the last fraction, $\frac{1}{4}$, by dividing both the numerator and denominator by, respectively, 5, 4, 3, and 2. When a fraction is converted to the fraction that has the smallest numerator and denominator in its collection of equivalent fractions, it is said to be **reduced to lowest terms**. The fractions $\frac{1}{4}$, $\frac{3}{8}$, $\frac{2}{5}$, and $\frac{7}{10}$ are all reduced to lowest terms.

Observe a very important property of a fraction that has been reduced to lowest terms. The *only* whole number that divides *both* the numerator and denominator without a remainder is the number 1. When 1 is the only whole number that divides two whole numbers, the two whole numbers are said to be **relatively prime**.

Relatively Prime
A fraction is reduced to lowest terms if its numerator and denominator are **relatively prime**.

Methods of Reducing Fractions to Lowest Terms

Method 1: Dividing Out Common Primes

1. Write the numerator and denominator as a product of primes.
2. Divide the numerator and denominator by each of the common prime factors. We often indicate this division by drawing a slanted line through each divided out factor. This process is also called **cancelling common factors**.
3. The product of the remaining factors in the numerator and the product of remaining factors of the denominator are relatively prime, and this fraction is reduced to lowest terms.

Sample Set B

Reduce each fraction to lowest terms.

Example:

$\frac{6}{18} = \frac{\overset{1}{\cancel{2}} \cdot \overset{1}{\cancel{3}}}{\underset{1}{\cancel{2}} \cdot \underset{1}{\cancel{3}} \cdot 3} = \frac{1}{3}$ 1 and 3 are relatively prime.

Example:

$\frac{16}{20} = \frac{\overset{1}{\cancel{2}} \cdot \overset{1}{\cancel{2}} \cdot 2 \cdot 2}{\underset{1}{\cancel{2}} \cdot \underset{1}{\cancel{2}} \cdot 5} = \frac{4}{5}$ 4 and 5 are relatively prime.

Example:

$\frac{56}{104} = \frac{\overset{1}{\cancel{2}} \cdot \overset{1}{\cancel{2}} \cdot \overset{1}{\cancel{2}} \cdot 7}{\underset{1}{\cancel{2}} \cdot \underset{1}{\cancel{2}} \cdot \underset{1}{\cancel{2}} \cdot 13} = \frac{7}{13}$ 7 and 13 are relatively prime (and also truly prime)

Example:

$\frac{315}{336} = \frac{\overset{1}{\cancel{3}} \cdot 3 \cdot 5 \cdot \overset{1}{\cancel{7}}}{2 \cdot 2 \cdot 2 \cdot 2 \cdot \underset{1}{\cancel{3}} \cdot \underset{1}{\cancel{7}} \cdot} = \frac{15}{16}$ 15 and 16 are relatively prime.

Example:

$\frac{8}{15} = \frac{2 \cdot 2 \cdot 2}{3 \cdot 5}$ No common prime factors, so 8 and 15 are relatively prime. The fraction $\frac{8}{15}$ is reduced to lowest terms.

Practice Set B

Reduce each fraction to lowest terms.

Exercise:

Problem: $\frac{4}{8}$

Solution:

$\frac{1}{2}$

Exercise:

Problem: $\frac{6}{15}$

Solution:

$\frac{2}{5}$

Exercise:

Problem: $\frac{6}{48}$

Solution:

$\frac{1}{8}$

Exercise:

Problem: $\frac{21}{48}$

Solution:

$\frac{7}{16}$

Exercise:

Problem: $\frac{72}{42}$

Solution:

$\frac{12}{7}$

Exercise:

Problem: $\frac{135}{243}$

Solution:

$\frac{5}{9}$

Method 2: Dividing Out Common Factors

1. Mentally divide the numerator and the denominator by a factor that is common to each. Write the quotient above the original number.
2. Continue this process until the numerator and denominator are relatively prime.

Sample Set C

Reduce each fraction to lowest terms.

Example:

$\frac{25}{30}$. 5 divides into both 25 and 30.

$\frac{\overset{5}{\cancel{25}}}{\underset{6}{\cancel{30}}} = \frac{5}{6}$ 5 and 6 are relatively prime.

Example:

$\frac{18}{24}$. Both numbers are even so we can divide by 2.

$\frac{\overset{9}{\cancel{18}}}{\underset{12}{\cancel{24}}}$ Now, both 9 and 12 are divisible by 3.

$\frac{\overset{\overset{3}{\cancel{9}}}{\cancel{18}}}{\underset{\underset{4}{\cancel{12}}}{\cancel{24}}} = \frac{3}{4}$ 3 and 4 are relatively prime.

Example:

$\frac{\overset{\overset{7}{\cancel{21}}}{\cancel{210}}}{\underset{\underset{5}{\cancel{15}}}{\cancel{150}}} = \frac{7}{5}$. 7 and 5 are relatively prime.

Example:
$\frac{36}{96} = \frac{18}{48} = \frac{9}{24} = \frac{3}{8}$. 3 and 8 are relatively prime.

Practice Set C

Reduce each fraction to lowest terms.

Exercise:

Problem: $\frac{12}{16}$

Solution:

$\frac{3}{4}$

Exercise:

Problem: $\frac{9}{24}$

Solution:

$\frac{3}{8}$

Exercise:

Problem: $\frac{21}{84}$

Solution:

$\frac{1}{4}$

Exercise:

Problem: $\frac{48}{64}$

Solution:

$\frac{3}{4}$

Exercise:

Problem: $\frac{63}{81}$

Solution:

$\frac{7}{9}$

Exercise:

Problem: $\frac{150}{240}$

Solution:

$\frac{5}{8}$

Raising Fractions to Higher Terms

Equally as important as reducing fractions is raising fractions to higher terms. Raising a fraction to higher terms is the process of constructing an equivalent fraction that has higher values in the numerator and denominator than the original fraction.

The fractions $\frac{3}{5}$ and $\frac{9}{15}$ are equivalent, that is, $\frac{3}{5} = \frac{9}{15}$. Notice also,

$\frac{3 \cdot 3}{5 \cdot 3} = \frac{9}{15}$

Notice that $\frac{3}{3} = 1$ and that $\frac{3}{5} \cdot 1 = \frac{3}{5}$. We are not changing the value of $\frac{3}{5}$.

From these observations we can suggest the following method for converting one fraction to an equivalent fraction that has higher values in the numerator and denominator. This method is called **raising a fraction to higher terms**.

Raising a Fraction to Higher Terms

A fraction can be raised to an equivalent fraction that has higher terms in the numerator and denominator by multiplying both the numerator and denominator by the same nonzero whole number.

The fraction $\frac{3}{4}$ can be raised to $\frac{24}{32}$ by multiplying both the numerator and denominator by 8.

$$\frac{3}{4} = \frac{3 \cdot 8}{4 \cdot 8} = \frac{24}{32}$$

Notice that $\frac{8}{8} = 1$.

Most often, we will want to convert a given fraction to an equivalent fraction with a higher specified denominator. For example, we may wish to convert $\frac{5}{8}$ to an equivalent fraction that has denominator 32, that is,

$$\frac{5}{8} = \frac{?}{32}$$

This is possible to do because we know the process. We must multiply *both* the numerator and denominator of $\frac{5}{8}$ by the *same* nonzero whole number in order to 8 obtain an equivalent fraction.

We have some information. The denominator 8 was raised to 32 by multiplying it by some nonzero whole number. Division will give us the proper factor. Divide the original denominator into the new denominator.

$$32 \div 8 = 4$$

Now, multiply the numerator 5 by 4.

$$5 \cdot 4 = 20$$

Thus,

$$\frac{5}{8} = \frac{5 \cdot 4}{8 \cdot 4} = \frac{20}{32}$$

So,

$\frac{5}{8} = \frac{20}{32}$

Sample Set D

Determine the missing numerator or denominator.

Example:
$\frac{3}{7} = \frac{?}{35}$. Divide the original denominator into the new denominator.
$35 \div 7 = 5$ The quotient is 5. Multiply the original numerator by 5.
$\frac{3}{7} = \frac{3 \cdot 5}{7 \cdot 5} = \frac{15}{35}$ The missing numerator is 15.

Example:
$\frac{5}{6} = \frac{45}{?}$. Divide the original numerator into the new numerator.
$45 \div 5 = 9$ The quotient is 9. Multiply the original denominator by 9.
$\frac{5}{6} = \frac{5 \cdot 9}{6 \cdot 9} = \frac{45}{54}$ The missing denominator is 45.

Practice Set D

Determine the missing numerator or denominator.
Exercise:

Problem: $\frac{4}{5} = \frac{?}{40}$

Solution:

32

Exercise:

Problem: $\frac{3}{7} = \frac{?}{28}$

Solution:

12

Exercise:

Problem: $\frac{1}{6} = \frac{?}{24}$

Solution:

4

Exercise:

Problem: $\frac{3}{10} = \frac{45}{?}$

Solution:

150

Exercise:

Problem: $\frac{8}{15} = \frac{?}{165}$

Solution:

88

Exercises

For the following problems, determine if the pairs of fractions are equivalent.

Exercise:

Problem: $\frac{1}{2}, \frac{5}{10}$

Solution:

equivalent

Exercise:

Problem: $\frac{2}{3}, \frac{8}{12}$

Exercise:

Problem: $\frac{5}{12}, \frac{10}{24}$

Solution:

equivalent

Exercise:

Problem: $\frac{1}{2}, \frac{3}{6}$

Exercise:

Problem: $\frac{3}{5}, \frac{12}{15}$

Solution:

not equivalent

Exercise:

Problem: $\frac{1}{6}, \frac{7}{42}$

Exercise:

Problem: $\frac{16}{25}, \frac{49}{75}$

Solution:

not equivalent

Exercise:

Problem: $\frac{5}{28}, \frac{20}{112}$

Exercise:

Problem: $\frac{3}{10}, \frac{36}{110}$

Solution:

not equivalent

Exercise:

Problem: $\frac{6}{10}, \frac{18}{32}$

Exercise:

Problem: $\frac{5}{8}, \frac{15}{24}$

Solution:

equivalent

Exercise:

Problem: $\frac{10}{16}, \frac{15}{24}$

Exercise:

Problem: $\frac{4}{5}, \frac{3}{4}$

Solution:

not equivalent

Exercise:

Problem: $\frac{5}{7}, \frac{15}{21}$

Exercise:

Problem: $\frac{9}{11}, \frac{11}{9}$

Solution:

not equivalent

For the following problems, determine the missing numerator or denominator.

Exercise:

Problem: $\frac{1}{3} = \frac{?}{12}$

Exercise:

Problem: $\frac{1}{5} = \frac{?}{30}$

Solution:

6

Exercise:

Problem: $\frac{2}{3} = \frac{?}{9}$

Exercise:

Problem: $\frac{1}{5} = \frac{?}{30}$

Solution:

12

Exercise:

Problem: $\frac{2}{3} = \frac{?}{9}$

Exercise:

Problem: $\frac{3}{4} = \frac{?}{16}$

Solution:

12

Exercise:

Problem: $\frac{5}{6} = \frac{?}{18}$

Exercise:

Problem: $\frac{4}{5} = \frac{?}{25}$

Solution:

20

Exercise:

Problem: $\frac{1}{2} = \frac{4}{?}$

Exercise:

Problem: $\frac{9}{25} = \frac{27}{?}$

Solution:

75

Exercise:

Problem: $\frac{3}{2} = \frac{18}{?}$

Exercise:

Problem: $\frac{5}{3} = \frac{80}{?}$

Solution:

48

Exercise:

Problem: $\frac{1}{8} = \frac{3}{?}$

Exercise:

Problem: $\frac{4}{5} = \frac{?}{100}$

Solution:

80

Exercise:

Problem: $\frac{1}{2} = \frac{25}{?}$

Exercise:

Problem: $\frac{3}{16} = \frac{?}{96}$

Solution:

18

Exercise:

Problem: $\frac{15}{16} = \frac{225}{?}$

Exercise:

Problem: $\frac{11}{12} = \frac{?}{168}$

Solution:

154

Exercise:

Problem: $\frac{9}{13} = \frac{?}{286}$

Exercise:

Problem: $\frac{32}{33} = \frac{?}{1518}$

Solution:

1,472

Exercise:

Problem: $\frac{19}{20} = \frac{1045}{?}$

Exercise:

Problem: $\frac{37}{50} = \frac{1369}{?}$

Solution:

1,850

For the following problems, reduce, if possible, each of the fractions to lowest terms.

Exercise:

Problem: $\frac{6}{8}$

Exercise:

Problem: $\frac{8}{10}$

Solution:

$\frac{4}{5}$

Exercise:

Problem: $\frac{5}{10}$

Exercise:

Problem: $\frac{6}{14}$

Solution:

$\frac{3}{7}$

Exercise:

Problem: $\frac{3}{12}$

Exercise:

Problem: $\frac{4}{14}$

Solution:

$\frac{2}{7}$

Exercise:

Problem: $\frac{1}{6}$

Exercise:

Problem: $\frac{4}{6}$

Solution:

$\frac{2}{3}$

Exercise:

Problem: $\frac{18}{14}$

Exercise:

Problem: $\frac{20}{8}$

Solution:

$\frac{5}{2}$

Exercise:

Problem: $\frac{4}{6}$

Exercise:

Problem: $\frac{10}{6}$

Solution:

$\frac{5}{3}$

Exercise:

Problem: $\frac{6}{14}$

Exercise:

Problem: $\frac{14}{6}$

Solution:

$\frac{7}{3}$

Exercise:

Problem: $\frac{10}{12}$

Exercise:

Problem: $\frac{16}{70}$

Solution:

$\frac{8}{35}$

Exercise:

Problem: $\frac{40}{60}$

Exercise:

Problem: $\frac{20}{12}$

Solution:

$\frac{5}{3}$

Exercise:

Problem: $\frac{32}{28}$

Exercise:

Problem: $\frac{36}{10}$

Solution:

$\frac{18}{5}$

Exercise:

Problem: $\frac{36}{60}$

Exercise:

Problem: $\frac{12}{18}$

Solution:

$\frac{2}{3}$

Exercise:

Problem: $\frac{18}{27}$

Exercise:

Problem: $\frac{18}{24}$

Solution:

$\frac{3}{4}$

Exercise:

Problem: $\frac{32}{40}$

Exercise:

Problem: $\frac{11}{22}$

Solution:

$\frac{1}{2}$

Exercise:

Problem: $\frac{27}{81}$

Exercise:

Problem: $\frac{17}{51}$

Solution:

$\frac{1}{3}$

Exercise:

Problem: $\frac{16}{42}$

Exercise:

Problem: $\frac{39}{13}$

Solution:

3

Exercise:

Problem: $\frac{44}{11}$

Exercise:

Problem: $\frac{66}{33}$

Solution:

2

Exercise:

Problem: $\frac{15}{1}$

Exercise:

Problem: $\frac{15}{16}$

Solution:

already reduced

Exercise:

Problem: $\frac{15}{40}$

Exercise:

Problem: $\frac{36}{100}$

Solution:

$\frac{9}{25}$

Exercise:

Problem: $\frac{45}{32}$

Exercise:

Problem: $\frac{30}{75}$

Solution:

$\frac{2}{5}$

Exercise:

Problem: $\frac{121}{132}$

Exercise:

Problem: $\frac{72}{64}$

Solution:

$\frac{9}{8}$

Exercise:

Problem: $\frac{30}{105}$

Exercise:

Problem: $\frac{46}{60}$

Solution:

$\frac{23}{30}$

Exercise:

Problem: $\frac{75}{45}$

Exercise:

Problem: $\frac{40}{18}$

Solution:

$\frac{20}{9}$

Exercise:

Problem: $\frac{108}{76}$

Exercise:

Problem: $\frac{7}{21}$

Solution:

$\frac{1}{3}$

Exercise:

Problem: $\frac{6}{51}$

Exercise:

Problem: $\frac{51}{12}$

Solution:

$\frac{17}{4}$

Exercise:

Problem: $\frac{8}{100}$

Exercise:

Problem: $\frac{51}{54}$

Solution:

$\frac{17}{18}$

Exercise:

Problem:

A ream of paper contains 500 sheets. What fraction of a ream of paper is 200 sheets? Be sure to reduce.

Exercise:

Problem:

There are 24 hours in a day. What fraction of a day is 14 hours?

Solution:

$\frac{7}{12}$

Exercise:

Problem:

A full box contains 80 calculators. How many calculators are in $\frac{1}{4}$ of a box?

Exercise:

Problem:

There are 48 plants per flat. How many plants are there in $\frac{1}{3}$ of a flat?

Solution:

16

Exercise:

Problem:

A person making $18,000 per year must pay $3,960 in income tax. What fraction of this person's yearly salary goes to the IRS?

For the following problems, find the mistake.

Exercise:

Problem: $\frac{3}{24} = \frac{\cancel{3}}{\cancel{3} \cdot 8} = \frac{0}{8} = 0$

Solution:

Should be $\frac{1}{8}$; the cancellation is division, so the numerator should be 1.

Exercise:

Problem: $\frac{8}{10} = \frac{\cancel{2} + 6}{\cancel{2} + 8} = \frac{6}{8} = \frac{3}{4}$

Exercise:

Problem: $\frac{7}{15} = \frac{\cancel{7}}{\cancel{7}+8} = \frac{1}{8}$

Solution:

Cancel factors only, not addends; $\frac{7}{15}$ is already reduced.

Exercise:

Problem: $\frac{6}{7} = \frac{\cancel{5}+1}{\cancel{5}+2} = \frac{1}{2}$

Exercise:

Problem: $\frac{\cancel{9}}{\cancel{9}} = \frac{0}{0} = 0$

Solution:

Same as [link]; answer is $\frac{1}{1}$ or 1.

Exercises for Review

Exercise:

Problem: ([link]) Round 816 to the nearest thousand.

Exercise:

Problem: ([link]) Perform the division: $0 \div 6$.

Solution:

0

Exercise:

Problem: ([link]) Find all the factors of 24.

Exercise:

Problem: ([link]) Find the greatest common factor of 12 and 18.

Solution:

6

Exercise:

Problem: ([link]) Convert $\frac{15}{8}$ to a mixed number.

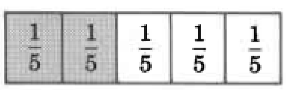
Multiplication of Fractions
This module is from Fundamentals of Mathematics by Denny Burzynski and Wade Ellis, Jr. This module discusses multiplication of fractions. By the end of the module students should be able to understand the concept of multiplication of fractions, multiply one fraction by another, multiply mixed numbers and find powers and roots of various fractions.

Section Overview

- Fractions of Fractions
- Multiplication of Fractions
- Multiplication of Fractions by Dividing Out Common Factors
- Multiplication of Mixed Numbers
- Powers and Roots of Fractions

Fractions of Fractions

We know that a fraction represents a part of a whole quantity. For example, two fifths of one unit can be represented by

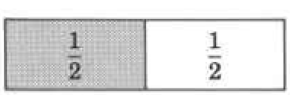


$\frac{2}{5}$ of the whole is shaded.

A natural question is, what is a fractional part of a fractional quantity, or, what is a fraction of a fraction? For example, what $\frac{2}{3}$ of $\frac{1}{2}$?

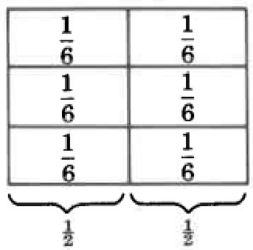
We can suggest an answer to this question by using a picture to examine $\frac{2}{3}$ of $\frac{1}{2}$.

First, let's represent $\frac{1}{2}$.



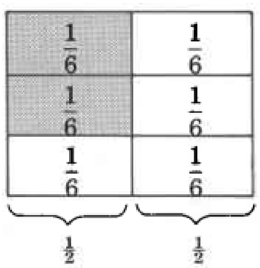
$\frac{1}{2}$ of the whole is shaded.

Then divide each of the $\frac{1}{2}$ parts into 3 equal parts.



Each part is $\frac{1}{6}$ of the whole.

Now we'll take $\frac{2}{3}$ of the $\frac{1}{2}$ unit.



$\frac{2}{3}$ of $\frac{1}{2}$ is $\frac{2}{6}$, which reduces to $\frac{1}{3}$.

Multiplication of Fractions

Now we ask, what arithmetic operation (+, –, ×, ÷) will produce $\frac{2}{6}$ from $\frac{2}{3}$ of $\frac{1}{2}$?

Notice that, if in the fractions $\frac{2}{3}$ and $\frac{1}{2}$, we multiply the numerators together and the denominators together, we get precisely $\frac{2}{6}$.

$\frac{2 \cdot 1}{3 \cdot 2} = \frac{2}{6}$

This reduces to $\frac{1}{3}$ as before.

Using this observation, we can suggest the following:

1. **The Word "OF" Indicates Multiplication** The word "of" translates to the arithmetic operation "times."
2. **The Method of Multiplying Fractions** To multiply two or more fractions, multiply the numerators together and then multiply the denominators together. Reduce if necessary.

$$\frac{\text{numerator 1}}{\text{denominator 1}} \cdot \frac{\text{numerator 2}}{\text{denominator 2}} = \frac{\text{numerator 1}}{\text{denominator 1}} \cdot \frac{\text{numerator 2}}{\text{denominator 2}}$$

Sample Set A

Perform the following multiplications.

Example:

$\frac{3}{4} \cdot \frac{1}{6} = \frac{3 \cdot 1}{4 \cdot 6} \quad = \frac{3}{24}$ Now, reduce.

$= \frac{\overset{1}{\cancel{3}}}{\underset{8}{\cancel{24}}} = \frac{1}{8}$

Thus

$\frac{3}{4} \cdot \frac{1}{6} = \frac{1}{8}$

This means that $\frac{3}{4}$ of $\frac{1}{6}$ is $\frac{1}{8}$, that is, $\frac{3}{4}$ of $\frac{1}{6}$ of a unit is $\frac{1}{8}$ of the original unit.

Example:

$\frac{3}{8} \cdot 4$. Write 4 as a fraction by writing $\frac{4}{1}$

$\frac{3}{8} \cdot \frac{4}{1} = \frac{3 \cdot 4}{8 \cdot 1} = \frac{12}{8} = \frac{\overset{3}{\cancel{12}}}{\underset{2}{\cancel{8}}} = \frac{3}{2}$

$\frac{3}{8} \cdot 4 = \frac{3}{2}$

This means that $\frac{3}{8}$ of 4 whole units is $\frac{3}{2}$ of one whole unit.

Example:

$$\frac{2}{5}\cdot\frac{5}{8}\cdot\frac{1}{4}=\frac{2\cdot5\cdot1}{5\cdot8\cdot4}=\frac{\overset{1}{\cancel{10}}}{\underset{16}{\cancel{160}}}=\frac{1}{16}$$

This means that $\frac{2}{5}$ of $\frac{5}{8}$ of $\frac{1}{4}$ of a whole unit is $\frac{1}{16}$ of the original unit.

Practice Set A

Perform the following multiplications.

Exercise:

Problem: $\frac{2}{5}\cdot\frac{1}{6}$

Solution:

$\frac{1}{15}$

Exercise:

Problem: $\frac{1}{4}\cdot\frac{8}{9}$

Solution:

$\frac{2}{9}$

Exercise:

Problem: $\frac{4}{9}\cdot\frac{15}{16}$

Solution:

$\frac{5}{12}$

Exercise:

Problem: $\left(\frac{2}{3}\right)\left(\frac{2}{3}\right)$

Solution:

$\frac{4}{9}$

Exercise:

Problem: $\left(\frac{7}{4}\right)\left(\frac{8}{5}\right)$

Solution:

$\frac{14}{5}$

Exercise:

Problem: $\frac{5}{6} \cdot \frac{7}{8}$

Solution:

$\frac{35}{48}$

Exercise:

Problem: $\frac{2}{3} \cdot 5$

Solution:

$\frac{10}{3}$

Exercise:

Problem: $\left(\frac{3}{4}\right)(10)$

Solution:

$\frac{15}{2}$

Exercise:

Problem: $\frac{3}{4} \cdot \frac{8}{9} \cdot \frac{5}{12}$

Solution:

$\frac{5}{18}$

Multiplying Fractions by Dividing Out Common Factors

We have seen that to multiply two fractions together, we multiply numerators together, then denominators together, then reduce to lowest terms, if necessary. The reduction can be tedious if the numbers in the fractions are large. For example,

$$\frac{9}{16} \cdot \frac{10}{21} = \frac{9 \cdot 10}{16 \cdot 21} = \frac{90}{336} = \frac{45}{168} = \frac{15}{28}$$

We avoid the process of reducing if we divide out common factors *before* we multiply.

$$\frac{9}{16} \cdot \frac{10}{21} = \frac{\overset{3}{\cancel{9}}}{\underset{8}{\cancel{16}}} \cdot \frac{\overset{5}{\cancel{10}}}{\underset{7}{\cancel{21}}} = \frac{3 \cdot 5}{8 \cdot 7} = \frac{15}{56}$$

Divide 3 into 9 and 21, and divide 2 into 10 and 16. The product is a fraction that is reduced to lowest terms.

The Process of Multiplication by Dividing Out Common Factors
To multiply fractions by dividing out common factors, divide out factors that are common to both a numerator and a denominator. The factor being divided out can appear in any numerator and any denominator.

Sample Set B

Perform the following multiplications.

Example:

$\frac{4}{5} \cdot \frac{5}{6}$

$\frac{\overset{2}{\cancel{4}}}{\underset{1}{\cancel{5}}} \cdot \frac{\overset{1}{\cancel{5}}}{\underset{3}{\cancel{6}}} = \frac{2 \cdot 1}{1 \cdot 3} = \frac{2}{3}$

Divide 4 and 6 by 2

Divide 5 and 5 by 5

Example:

$\frac{8}{12} \cdot \frac{8}{10}$

$\frac{\overset{4}{\cancel{8}}}{\underset{3}{\cancel{12}}} \cdot \frac{\overset{2}{\cancel{8}}}{\underset{5}{\cancel{10}}} = \frac{4 \cdot 2}{3 \cdot 5} = \frac{8}{15}$

Divide 8 and 10 by 2.

Divide 8 and 12 by 4.

Example:

$8 \cdot \frac{5}{12} = \frac{\overset{2}{\cancel{8}}}{1} \cdot \frac{5}{\underset{3}{\cancel{12}}} = \frac{2 \cdot 5}{1 \cdot 3} = \frac{10}{3}$

Example:

$\frac{35}{18} \cdot \frac{63}{105}$

$\frac{\overset{1}{\cancel{\overset{7}{\cancel{35}}}}}{\underset{2}{\cancel{18}}} \cdot \frac{\overset{7}{\cancel{63}}}{\underset{\underset{3}{\cancel{21}}}{\cancel{105}}} = \frac{1 \cdot 7}{2 \cdot 3} = \frac{7}{6}$

Example:

$\frac{13}{9} \cdot \frac{6}{39} \cdot \frac{1}{12}$

$\frac{\overset{1}{\cancel{13}}}{9} \cdot \frac{\overset{\overset{1}{\cancel{2}}}{\cancel{6}}}{\underset{\underset{1}{\cancel{3}}}{\cancel{39}}} \cdot \frac{1}{\underset{6}{\cancel{12}}} = \frac{1 \cdot 1 \cdot 1}{9 \cdot 1 \cdot 6} = \frac{1}{54}$

Practice Set B

Perform the following multiplications.

Exercise:

Problem: $\frac{2}{3} \cdot \frac{7}{8}$

Solution:

$\frac{7}{12}$

Exercise:

Problem: $\frac{25}{12} \cdot \frac{10}{45}$

Solution:

$\frac{25}{54}$

Exercise:

Problem: $\frac{40}{48} \cdot \frac{72}{90}$

Solution:

$\frac{2}{3}$

Exercise:

Problem: $7 \cdot \frac{2}{49}$

Solution:

$\frac{2}{7}$

Exercise:

Problem: $12 \cdot \frac{3}{8}$

Solution:

$\frac{9}{2}$

Exercise:

Problem: $\left(\frac{13}{7}\right)\left(\frac{14}{26}\right)$

Solution:

1

Exercise:

Problem: $\frac{16}{10} \cdot \frac{22}{6} \cdot \frac{21}{44}$

Solution:

$\frac{14}{5}$

Multiplication of Mixed Numbers

Multiplying Mixed Numbers

To perform a multiplication in which there are mixed numbers, it is convenient to first convert each mixed number to an improper fraction, then

multiply.

Sample Set C

Perform the following multiplications. Convert improper fractions to mixed numbers.

Example:

$1\frac{1}{8} \cdot 4\frac{2}{3}$

Convert each mixed number to an improper fraction.

$1\frac{1}{8} = \frac{8 \cdot 1 + 1}{8} = \frac{9}{8}$

$4\frac{2}{3} = \frac{4 \cdot 3 + 2}{3} = \frac{14}{3}$

$\frac{\cancelto{3}{9}}{\cancelto{4}{8}} \cdot \frac{\cancelto{7}{14}}{\cancelto{1}{3}} = \frac{3 \cdot 7}{4 \cdot 1} = \frac{21}{4} = 5\frac{1}{4}$

Example:

$16 \cdot 8\frac{1}{5}$

Convert $8\frac{1}{5}$ to an improper fraction.

$8\frac{1}{5} = \frac{5 \cdot 8 + 1}{5} = \frac{41}{5}$

$\frac{16}{1} \cdot \frac{41}{5}$.

There are no common factors to divide out.

$\frac{16}{1} \cdot \frac{41}{5} = \frac{16 \cdot 41}{1 \cdot 5} = \frac{656}{5} = 131\frac{1}{5}$

Example:

$9\frac{1}{6} \cdot 12\frac{3}{5}$

Convert to improper fractions.

$9\frac{1}{6} = \frac{6 \cdot 9 + 1}{6} = \frac{55}{6}$

$$12\frac{3}{5} = \frac{5 \cdot 12+3}{5} = \frac{63}{5}$$

$$\frac{\overset{11}{\cancel{55}}}{\underset{2}{\cancel{6}}} \cdot \frac{\overset{21}{\cancel{63}}}{\underset{1}{\cancel{5}}} = \frac{11 \cdot 21}{2 \cdot 1} = \frac{231}{2} = 115\frac{1}{2}$$

Example:

$$\begin{aligned} \frac{11}{8} \cdot 4\frac{1}{2} \cdot 3\frac{1}{8} &= \frac{11}{8} \cdot \frac{\overset{3}{\cancel{9}}}{\underset{1}{\cancel{2}}} \cdot \frac{\overset{5}{\cancel{10}}}{\underset{1}{\cancel{3}}} \\ &= \frac{11 \cdot 3 \cdot 5}{8 \cdot 1 \cdot 1} = \frac{165}{8} = 20\frac{5}{8} \end{aligned}$$

Practice Set C

Perform the following multiplications. Convert improper fractions to mixed numbers.

Exercise:

Problem: $2\frac{2}{3} \cdot 2\frac{1}{4}$

Solution:

6

Exercise:

Problem: $6\frac{2}{3} \cdot 3\frac{3}{10}$

Solution:

22

Exercise:

Problem: $7\frac{1}{8} \cdot 12$

Solution:

$85\frac{1}{2}$

Exercise:

Problem: $2\frac{2}{5} \cdot 3\frac{3}{4} \cdot 3\frac{1}{3}$

Solution:

30

Powers and Roots of Fractions

Sample Set D

Find the value of each of the following.

Example:
$\left(\frac{1}{6}\right)^2 = \frac{1}{6} \cdot \frac{1}{6} = \frac{1 \cdot 1}{6 \cdot 6} = \frac{1}{36}$

Example:
$\sqrt{\frac{9}{100}}$. We're looking for a number, call it ?, such that when it is squared, $\frac{9}{100}$ is produced.
$(?)^2 = \frac{9}{100}$
We know that
$3^2 = 9$ and $10^2 = 100$

We'll try $\frac{3}{10}$. Since

$$\left(\frac{3}{10}\right)^2 = \frac{3}{10} \cdot \frac{3}{10} = \frac{3 \cdot 3}{10 \cdot 10} = \frac{9}{100}$$

$$\sqrt{\frac{9}{100}} = \frac{3}{10}$$

Example:

$$4\frac{2}{5} \cdot \sqrt{\frac{100}{121}}$$

$$\frac{\overset{2}{\cancel{22}}}{\underset{1}{\cancel{5}}} \cdot \frac{\overset{2}{\cancel{10}}}{\underset{1}{\cancel{11}}} = \frac{2 \cdot 2}{1 \cdot 1} = \frac{4}{1} = 4$$

$$4\frac{2}{5} \cdot \sqrt{\frac{100}{121}} = 4$$

Practice Set D

Find the value of each of the following.

Exercise:

Problem: $\left(\frac{1}{8}\right)^2$

Solution:

$\frac{1}{64}$

Exercise:

Problem: $\left(\frac{3}{10}\right)^2$

Solution:

$\frac{9}{100}$

Exercise:

Problem: $\sqrt{\frac{4}{9}}$

Solution:

$\frac{2}{3}$

Exercise:

Problem: $\sqrt{\frac{1}{4}}$

Solution:

$\frac{1}{2}$

Exercise:

Problem: $\frac{3}{8} \cdot \sqrt{\frac{1}{9}}$

Solution:

$\frac{1}{8}$

Exercise:

Problem: $9\frac{1}{3} \cdot \sqrt{\frac{81}{100}}$

Solution:

$8\frac{2}{5}$

Exercise:

Problem: $2\frac{8}{13} \cdot \sqrt{\frac{169}{16}}$

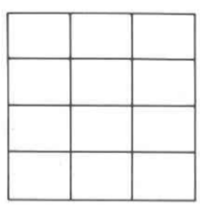
Solution:

$8\frac{1}{2}$

Exercises

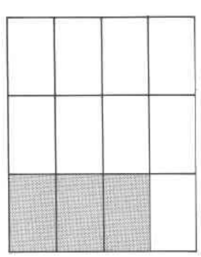
For the following six problems, use the diagrams to find each of the following parts. Use multiplication to verify your result.
Exercise:

Problem: $\frac{3}{4}$ of $\frac{1}{3}$



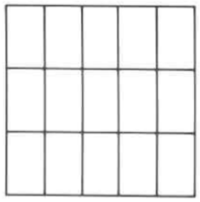
Solution:

$\frac{1}{4}$



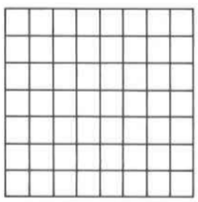
Exercise:

Problem: $\frac{2}{3}$ of $\frac{3}{5}$



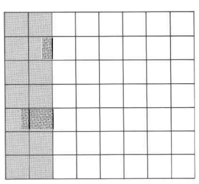
Exercise:

Problem: $\frac{2}{7}$ of $\frac{7}{8}$



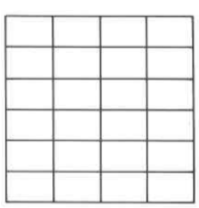
Solution:

$\frac{1}{4}$



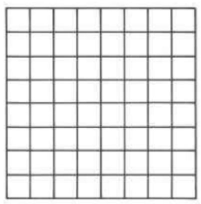
Exercise:

Problem: $\frac{5}{6}$ of $\frac{3}{4}$



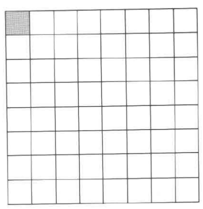
Exercise:

Problem: $\frac{1}{8}$ of $\frac{1}{8}$



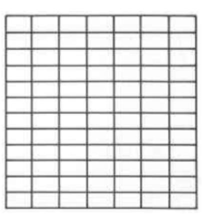
Solution:

$\frac{1}{64}$



Exercise:

Problem: $\frac{7}{12}$ of $\frac{6}{7}$



For the following problems, find each part without using a diagram.

Exercise:

Problem: $\frac{1}{2}$ of $\frac{4}{5}$

Solution:

$\frac{2}{5}$

Exercise:

Problem: $\frac{3}{5}$ of $\frac{5}{12}$

Exercise:

Problem: $\frac{1}{4}$ of $\frac{8}{9}$

Solution:

$\frac{2}{9}$

Exercise:

Problem: $\frac{3}{16}$ of $\frac{12}{15}$

Exercise:

Problem: $\frac{2}{9}$ of $\frac{6}{5}$

Solution:

$\frac{4}{15}$

Exercise:

Problem: $\frac{1}{8}$ of $\frac{3}{8}$

Exercise:

Problem: $\frac{2}{3}$ of $\frac{9}{10}$

Solution:

$\frac{3}{5}$

Exercise:

Problem: $\frac{18}{19}$ of $\frac{38}{54}$

Exercise:

Problem: $\frac{5}{6}$ of $2\frac{2}{5}$

Solution:

2

Exercise:

Problem: $\frac{3}{4}$ of $3\frac{3}{5}$

Exercise:

Problem: $\frac{3}{2}$ of $2\frac{2}{9}$

Solution:

$\frac{10}{3}$ or $3\frac{1}{3}$

Exercise:

Problem: $\frac{15}{4}$ of $4\frac{4}{5}$

Exercise:

Problem: $5\frac{1}{3}$ of $9\frac{3}{4}$

Solution:

52

Exercise:

Problem: $1\frac{13}{15}$ of $8\frac{3}{4}$

Exercise:

Problem: $\frac{8}{9}$ of $\frac{3}{4}$ of $\frac{2}{3}$

Solution:

$\frac{4}{9}$

Exercise:

Problem: $\frac{1}{6}$ of $\frac{12}{13}$ of $\frac{26}{36}$

Exercise:

Problem: $\frac{1}{2}$ of $\frac{1}{3}$ of $\frac{1}{4}$

Solution:

$\frac{1}{24}$

Exercise:

Problem: $1\frac{3}{7}$ of $5\frac{1}{5}$ of $8\frac{1}{3}$

Exercise:

Problem: $2\frac{4}{5}$ of $5\frac{5}{6}$ of $7\frac{5}{7}$

Solution:

126

For the following problems, find the products. Be sure to reduce.

Exercise:

Problem: $\frac{1}{3} \cdot \frac{2}{3}$

Exercise:

Problem: $\frac{1}{2} \cdot \frac{1}{2}$

Solution:

$\frac{1}{4}$

Exercise:

Problem: $\frac{3}{4} \cdot \frac{3}{8}$

Exercise:

Problem: $\frac{2}{5} \cdot \frac{5}{6}$

Solution:

$\frac{1}{3}$

Exercise:

Problem: $\frac{3}{8} \cdot \frac{8}{9}$

Exercise:

Problem: $\frac{5}{6} \cdot \frac{14}{15}$

Solution:

$\frac{7}{9}$

Exercise:

Problem: $\frac{4}{7} \cdot \frac{7}{4}$

Exercise:

Problem: $\frac{3}{11} \cdot \frac{11}{3}$

Solution:

1

Exercise:

Problem: $\frac{9}{16} \cdot \frac{20}{27}$

Exercise:

Problem: $\frac{35}{36} \cdot \frac{48}{55}$

Solution:

$\frac{28}{33}$

Exercise:

Problem: $\frac{21}{25} \cdot \frac{15}{14}$

Exercise:

Problem: $\frac{76}{99} \cdot \frac{66}{38}$

Solution:

$\frac{4}{3}$

Exercise:

Problem: $\frac{3}{7} \cdot \frac{14}{18} \cdot \frac{6}{2}$

Exercise:

Problem: $\frac{4}{15} \cdot \frac{10}{3} \cdot \frac{27}{2}$

Solution:

12

Exercise:

Problem: $\frac{14}{15} \cdot \frac{21}{28} \cdot \frac{45}{7}$

Exercise:

Problem: $\frac{8}{3} \cdot \frac{15}{4} \cdot \frac{16}{21}$

Solution:

$7\frac{13}{21}$ or $\frac{160}{21}$

Exercise:

Problem: $\frac{18}{14} \cdot \frac{21}{35} \cdot \frac{36}{7}$

Exercise:

Problem: $\frac{3}{5} \cdot 20$

Solution:

12

Exercise:

Problem: $\frac{8}{9} \cdot 18$

Exercise:

Problem: $\frac{6}{11} \cdot 33$

Solution:

18

Exercise:

Problem: $\frac{18}{19} \cdot 38$

Exercise:

Problem: $\frac{5}{6} \cdot 10$

Solution:

$\frac{25}{3}$ or $8\frac{1}{3}$

Exercise:

Problem: $\frac{1}{9} \cdot 3$

Exercise:

Problem: $5 \cdot \frac{3}{8}$

Solution:

$\frac{15}{8} = 1\frac{7}{8}$

Exercise:

Problem: $16 \cdot \frac{1}{4}$

Exercise:

Problem: $\frac{2}{3} \cdot 12 \cdot \frac{3}{4}$

Solution:

6

Exercise:

Problem: $\frac{3}{8} \cdot 24 \cdot \frac{2}{3}$

Exercise:

Problem: $\frac{5}{18} \cdot 10 \cdot \frac{2}{5}$

Solution:

$\frac{10}{9} = 1\frac{1}{9}$

Exercise:

Problem: $\frac{16}{15} \cdot 50 \cdot \frac{3}{10}$

Exercise:

Problem: $5\frac{1}{3} \cdot \frac{27}{32}$

Solution:

$\frac{9}{2} = 4\frac{1}{2}$

Exercise:

Problem: $2\frac{6}{7} \cdot 5\frac{3}{5}$

Exercise:

Problem: $6\frac{1}{4} \cdot 2\frac{4}{15}$

Solution:

$\frac{85}{6} = 14\frac{1}{6}$

Exercise:

Problem: $9\frac{1}{3} \cdot \frac{9}{16} \cdot 1\frac{1}{3}$

Exercise:

Problem: $3\frac{5}{9} \cdot 1\frac{13}{14} \cdot 10\frac{1}{2}$

Solution:

72

Exercise:

Problem: $20\frac{1}{4} \cdot 8\frac{2}{3} \cdot 16\frac{4}{5}$

Exercise:

Problem: $\left(\frac{2}{3}\right)^2$

Solution:

$\frac{4}{9}$

Exercise:

Problem: $\left(\frac{3}{8}\right)^2$

Exercise:

Problem: $\left(\frac{2}{11}\right)^2$

Solution:

$\frac{4}{121}$

Exercise:

Problem: $\left(\frac{8}{9}\right)^2$

Exercise:

Problem: $\left(\frac{1}{2}\right)^2$

Solution:

$\frac{1}{4}$

Exercise:

Problem: $\left(\frac{3}{5}\right)^2 \cdot \frac{20}{3}$

Exercise:

Problem: $\left(\frac{1}{4}\right)^2 \cdot \frac{16}{15}$

Solution:

$\frac{1}{15}$

Exercise:

Problem: $\left(\frac{1}{2}\right)^2 \cdot \frac{8}{9}$

Exercise:

Problem: $\left(\frac{1}{2}\right)^2 \cdot \left(\frac{2}{5}\right)^2$

Solution:

$\frac{1}{25}$

Exercise:

Problem: $\left(\frac{3}{7}\right)^2 \cdot \left(\frac{1}{9}\right)^2$

For the following problems, find each value. Reduce answers to lowest terms or convert to mixed numbers.

Exercise:

Problem: $\sqrt{\frac{4}{9}}$

Solution:

$\frac{2}{3}$

Exercise:

Problem: $\sqrt{\frac{16}{25}}$

Exercise:

Problem: $\sqrt{\frac{81}{121}}$

Solution:

$\frac{9}{11}$

Exercise:

Problem: $\sqrt{\frac{36}{49}}$

Exercise:

Problem: $\sqrt{\frac{144}{25}}$

Solution:

$\frac{12}{5} = 2\frac{2}{5}$

Exercise:

Problem: $\frac{2}{3} \cdot \sqrt{\frac{9}{16}}$

Exercise:

Problem: $\frac{3}{5} \cdot \sqrt{\frac{25}{81}}$

Solution:

$\frac{1}{3}$

Exercise:

Problem: $\left(\frac{8}{5}\right)^2 \cdot \sqrt{\frac{25}{64}}$

Exercise:

Problem: $\left(1\frac{3}{4}\right)^2 \cdot \sqrt{\frac{4}{49}}$

Solution:

$\frac{7}{8}$

Exercise:

Problem: $\left(2\frac{2}{3}\right)^2 \cdot \sqrt{\frac{36}{49}} \cdot \sqrt{\frac{64}{81}}$

Exercises for Review

Exercise:

Problem: ([link]) How many thousands in 342,810?

Solution:

2

Exercise:

Problem: ([link]) Find the sum of 22, 42, and 101.

Exercise:

Problem: ([link]) Is 634,281 divisible by 3?

Solution:

yes

Exercise:

Problem: ([link]) Is the whole number 51 prime or composite?

Exercise:

Problem: ([link]) Reduce $\frac{36}{150}$ to lowest terms.

Solution:

$\frac{6}{25}$

Division of Fractions
This module is from Fundamentals of Mathematics by Denny Burzynski and Wade Ellis, Jr. This module discusses division of fractions. By the end of the module students should be able to determine the reciprocal of a number and divide one fraction by another.

Section Overview

- Reciprocals
- Dividing Fractions

Reciprocals

Reciprocals
Two numbers whose product is 1 are called **reciprocals** of each other.

Sample Set A

The following pairs of numbers are reciprocals.

Example:
$\frac{3}{4}$ and $\frac{4}{3}$

$\frac{3}{4} \cdot \frac{4}{3} = 1$

Example:
$\frac{7}{16}$ and $\frac{16}{7}$

$\frac{7}{16} \cdot \frac{16}{7} = 1$

Example:
$\frac{1}{6}\text{and}\frac{6}{1}$

$\frac{1}{6} \cdot \frac{6}{1} = 1$

Notice that we can find the reciprocal of a nonzero number in fractional form by inverting it (exchanging positions of the numerator and denominator).

Practice Set A

Find the reciprocal of each number.

Exercise:

Problem: $\frac{3}{10}$

Solution:

$\frac{10}{3}$

Exercise:

Problem: $\frac{2}{3}$

Solution:

$\frac{3}{2}$

Exercise:

Problem: $\frac{7}{8}$

Solution:

$\frac{8}{7}$

Exercise:

Problem: $\frac{1}{5}$

Solution:

5

Exercise:

Problem: $2\frac{2}{7}$

> **Note:** Write this number as an improper fraction first.

Solution:

$\frac{7}{16}$

Exercise:

Problem: $5\frac{1}{4}$

Solution:

$\frac{4}{21}$

Exercise:

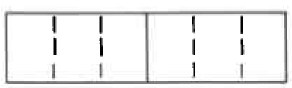
Problem: $10\frac{3}{16}$

Solution:

$\frac{16}{163}$

Dividing Fractions

Our concept of division is that it indicates *how many times* one quantity is contained in another quantity. For example, using the diagram we can see that there are 6 one-thirds in 2.



There are 6 one-thirds in 2.

Since 2 contains six $\frac{1}{3}$'s we express this as

$$2 \div \boxed{\frac{1}{3}} = 6$$

Note also that $2 \cdot \boxed{3} = 6$

$\frac{1}{3}$ and 3 are reciprocals

Using these observations, we can suggest the following method for dividing a number by a fraction.

Dividing One Fraction by Another Fraction
To divide a first fraction by a second, nonzero fraction, multiply the first traction by the reciprocal of the second fraction.

Invert and Multiply
This method is commonly referred to as **"invert the divisor and multiply."**

Sample Set B

Perform the following divisions.

Example:
$\frac{1}{3} \div \frac{3}{4}$. The divisor is $\frac{3}{4}$. Its reciprocal is $\frac{4}{3}$. Multiply $\frac{1}{3}$ by $\frac{4}{3}$.
$\frac{1}{3} \cdot \frac{4}{3} = \frac{1 \cdot 4}{3 \cdot 3} = \frac{4}{9}$
$\frac{1}{3} \div \frac{3}{4} = \frac{4}{9}$

Example:
$\frac{3}{8} \div \frac{5}{4}$ The divisor is $\frac{5}{4}$. Its reciprocal is $\frac{4}{5}$. Multiply $\frac{3}{8}$ by $\frac{4}{5}$.
$\frac{3}{\underset{2}{\cancel{8}}} \cdot \frac{\overset{1}{\cancel{4}}}{5} = \frac{3 \cdot 1}{2 \cdot 5} = \frac{3}{10}$
$\frac{3}{8} \div \frac{5}{4} = \frac{3}{10}$

Example:
$\frac{5}{6} \div \frac{5}{12}$. The divisor is $\frac{5}{12}$. Its reciprocal is $\frac{12}{5}$. Multiply $\frac{5}{6}$ by $\frac{12}{5}$.
$\frac{\overset{1}{\cancel{5}}}{\underset{1}{\cancel{6}}} \cdot \frac{\overset{2}{\cancel{12}}}{\underset{1}{\cancel{5}}} = \frac{1 \cdot 2}{1 \cdot 1} = \frac{2}{1} = 2$
$\frac{5}{6} \div \frac{5}{12} = 2$

Example:
$2\frac{2}{9} \div 3\frac{1}{3}$. Convert each mixed number to an improper fraction.
$2\frac{2}{9} = \frac{9 \cdot 2 + 2}{9} = \frac{20}{9}$.
$3\frac{1}{3} = \frac{3 \cdot 3 + 1}{3} = \frac{10}{3}$.
$\frac{20}{9} \div \frac{10}{3}$ The divisor is $\frac{10}{3}$. Its reciprocal is $\frac{3}{10}$. Multiply $\frac{20}{9}$ by $\frac{3}{10}$.
$\frac{\overset{2}{\cancel{20}}}{\underset{3}{\cancel{9}}} \cdot \frac{\overset{1}{\cancel{3}}}{\underset{1}{\cancel{10}}} = \frac{2 \cdot 1}{3 \cdot 1} = \frac{2}{3}$
$2\frac{2}{9} \div 3\frac{1}{3} = \frac{2}{3}$

Example:

$\frac{12}{11} \div 8$. First conveniently write 8 as $\frac{8}{1}$.

$\frac{12}{11} \div \frac{8}{1}$ The divisor is $\frac{8}{1}$. Its reciprocal is $\frac{1}{8}$. Multiply $\frac{12}{11}$ by $\frac{1}{8}$.

$$\frac{\overset{3}{\cancel{12}}}{11} \cdot \frac{1}{\underset{2}{\cancel{8}}} = \frac{3 \cdot 1}{11 \cdot 2} = \frac{3}{22}$$

$\frac{12}{11} \div 8 = \frac{3}{22}$

Example:

$\frac{7}{8} \div \frac{21}{20} \cdot \frac{3}{35}$. The divisor is $\frac{21}{20}$. Its reciprocal is $\frac{20}{21}$.

$$\frac{\overset{1}{\cancel{7}}}{\underset{2}{\cancel{8}}} \cdot \frac{\overset{\overset{1}{\cancel{5}}}{\cancel{20}}}{\underset{\underset{1}{\cancel{3}}}{\cancel{21}}} \frac{\overset{1}{\cancel{3}}}{\underset{7}{\cancel{35}}} = \frac{1 \cdot 1 \cdot 1}{2 \cdot 1 \cdot 7} = \frac{1}{14}$$

$\frac{7}{8} \div \frac{21}{20} \cdot \frac{3}{25} = \frac{1}{14}$

Example:

How many $2\frac{3}{8}$-inch-wide packages can be placed in a box 19 inches wide?

The problem is to determine how many two and three eighths are contained in 19, that is, what is $19 \div 2\frac{3}{8}$?

$2\frac{3}{8} = \frac{19}{8}$ Convert the divisor $2\frac{3}{8}$ to an improper fraction.

$19 = \frac{19}{1}$ Write the dividend 19 as $\frac{19}{1}$.

$\frac{19}{1} \div \frac{19}{8}$ The divisor is $\frac{19}{8}$. Its reciprocal is $\frac{8}{19}$.

$$\frac{\overset{1}{\cancel{19}}}{1} \cdot \frac{8}{\underset{1}{\cancel{19}}} = \frac{1 \cdot 8}{1 \cdot 1} = \frac{8}{1} = 8$$

Thus, 8 packages will fit into the box.

Practice Set B

Perform the following divisions.

Exercise:

Problem: $\frac{1}{2} \div \frac{9}{8}$

Solution:

$\frac{4}{9}$

Exercise:

Problem: $\frac{3}{8} \div \frac{9}{24}$

Solution:

1

Exercise:

Problem: $\frac{7}{15} \div \frac{14}{15}$

Solution:

$\frac{1}{2}$

Exercise:

Problem: $8 \div \frac{8}{15}$

Solution:

15

Exercise:

Problem: $6\frac{1}{4} \div \frac{5}{12}$

Solution:

15

Exercise:

Problem: $3\frac{1}{3} \div 1\frac{2}{3}$

Solution:

2

Exercise:

Problem: $\frac{5}{6} \div \frac{2}{3} \cdot \frac{8}{25}$

Solution:

$\frac{2}{5}$

Exercise:

Problem:

A container will hold 106 ounces of grape juice. How many $6\frac{5}{8}$-ounce glasses of grape juice can be served from this container?

Solution:

16 glasses

Determine each of the following quotients and then write a rule for this type of division.

Exercise:

Problem: $1 \div \frac{2}{3}$

Solution:

$\frac{3}{2}$

Exercise:

Problem: $1 \div \frac{3}{8}$

Solution:

$\frac{8}{3}$

Exercise:

Problem: $1 \div \frac{3}{4}$

Solution:

$\frac{4}{3}$

Exercise:

Problem: $1 \div \frac{5}{2}$

Solution:

$\frac{2}{5}$

Exercise:

Problem: When dividing 1 by a fraction, the quotient is the .

Solution:

is the reciprocal of the fraction.

Exercises

For the following problems, find the reciprocal of each number.

Exercise:

Problem: $\frac{4}{5}$

Solution:

$\frac{5}{4}$ or $1\frac{1}{4}$

Exercise:

Problem: $\frac{8}{11}$

Exercise:

Problem: $\frac{2}{9}$

Solution:

$\frac{9}{2}$ or $4\frac{1}{2}$

Exercise:

Problem: $\frac{1}{5}$

Exercise:

Problem: $3\frac{1}{4}$

Solution:

$\frac{4}{13}$

Exercise:

Problem: $8\frac{1}{4}$

Exercise:

Problem: $3\frac{2}{7}$

Solution:

$\frac{7}{23}$

Exercise:

Problem: $5\frac{3}{4}$

Exercise:

Problem: 1

Solution:

1

Exercise:

Problem: 4

For the following problems, find each value.

Exercise:

Problem: $\frac{3}{8} \div \frac{3}{5}$

Solution:

$\frac{5}{8}$

Exercise:

Problem: $\frac{5}{9} \div \frac{5}{6}$

Exercise:

Problem: $\frac{9}{16} \div \frac{15}{8}$

Solution:

$\frac{3}{10}$

Exercise:

Problem: $\frac{4}{9} \div \frac{6}{15}$

Exercise:

Problem: $\frac{25}{49} \div \frac{4}{9}$

Solution:

$\frac{225}{196}$ or $1\frac{29}{196}$

Exercise:

Problem: $\frac{15}{4} \div \frac{27}{8}$

Exercise:

Problem: $\frac{24}{75} \div \frac{8}{15}$

Solution:

$\frac{3}{5}$

Exercise:

Problem: $\frac{5}{7} \div 0$

Exercise:

Problem: $\frac{7}{8} \div \frac{7}{8}$

Solution:

1

Exercise:

Problem: $0 \div \frac{3}{5}$

Exercise:

Problem: $\frac{4}{11} \div \frac{4}{11}$

Solution:

1

Exercise:

Problem: $\frac{2}{3} \div \frac{2}{3}$

Exercise:

Problem: $\frac{7}{10} \div \frac{10}{7}$

Solution:

$\frac{49}{100}$

Exercise:

Problem: $\frac{3}{4} \div 6$

Exercise:

Problem: $\frac{9}{5} \div 3$

Solution:

$\frac{3}{5}$

Exercise:

Problem: $4\frac{1}{6} \div 3\frac{1}{3}$

Exercise:

Problem: $7\frac{1}{7} \div 8\frac{1}{3}$

Solution:

$\frac{6}{7}$

Exercise:

Problem: $1\frac{1}{2} \div 1\frac{1}{5}$

Exercise:

Problem: $3\frac{2}{5} \div \frac{6}{25}$

Solution:

$\frac{85}{6}$ or $14\frac{1}{6}$

Exercise:

Problem: $5\frac{1}{6} \div \frac{31}{6}$

Exercise:

Problem: $\frac{35}{6} \div 3\frac{3}{4}$

Solution:

$\frac{28}{18} = \frac{14}{9}$ or $1\frac{5}{9}$

Exercise:

Problem: $5\frac{1}{9} \div \frac{1}{18}$

Exercise:

Problem: $8\frac{3}{4} \div \frac{7}{8}$

Solution:

10

Exercise:

Problem: $\frac{12}{8} \div 1\frac{1}{2}$

Exercise:

Problem: $3\frac{1}{8} \div \frac{15}{16}$

Solution:

$\frac{10}{3}$ or $3\frac{1}{3}$

Exercise:

Problem: $11\frac{11}{12} \div 9\frac{5}{8}$

Exercise:

Problem: $2\frac{2}{9} \div 11\frac{2}{3}$

Solution:

$\frac{4}{21}$

Exercise:

Problem: $\frac{16}{3} \div 6\frac{2}{5}$

Exercise:

Problem: $4\frac{3}{25} \div 2\frac{56}{75}$

Solution:

$\frac{3}{2}$ or $1\frac{1}{2}$

Exercise:

Problem: $\frac{1}{1000} \div \frac{1}{100}$

Exercise:

Problem: $\frac{3}{8} \div \frac{9}{16} \cdot \frac{6}{5}$

Solution:

$\frac{4}{5}$

Exercise:

Problem: $\frac{3}{16} \cdot \frac{9}{8} \cdot \frac{6}{5}$

Exercise:

Problem: $\frac{4}{15} \div \frac{2}{25} \cdot \frac{9}{10}$

Solution:

3

Exercise:

Problem: $\frac{21}{30} \cdot 1\frac{1}{4} \div \frac{9}{10}$

Exercise:

Problem: $8\frac{1}{3} \cdot \frac{36}{75} \div 4$

Solution:

1

Exercises for Review

Exercise:

Problem: ([link]) What is the value of 5 in the number 504,216?

Exercise:

Problem: ([link]) Find the product of 2,010 and 160.

Solution:

321,600

Exercise:

Problem:

([link]) Use the numbers 8 and 5 to illustrate the commutative property of multiplication.

Exercise:

Problem: ([link]) Find the least common multiple of 6, 16, and 72.

Solution:

144

Exercise:

Problem: ([link]) Find $\frac{8}{9}$ of $6\frac{3}{4}$.

Applications Involving Fractions
This module is from Fundamentals of Mathematics by Denny Burzynski and Wade Ellis, Jr. This module discusses applications involving fractions. By the end of the module students should be able to solve missing product statements and solve missing factor statements.

Section Overview

- Multiplication Statements
- Missing Product Statements
- Missing Factor Statements

Multiplication Statements

Statement, Multiplication Statement
A *statement* is a sentence that is either true or false. A mathematical statement of the form

product = (factor 1) · (factor 2)

is a **multiplication statement**. Depending on the numbers that are used, it can be either true or false.

Omitting exactly one of the three numbers in the statement will produce exactly one of the following three problems. For convenience, we'll represent the omitted (or missing) number with the letter *M* (*M* for Missing).

1. *M* = (factor 1) · (factor 2) Missing *product* statement.
2. *M* · (factor 2) = product Missing *factor* statement.
3. (factor 1) · *M* = product Missing *factor* statement.

We are interested in developing and working with methods to determine the missing number that makes the statement true. Fundamental to these methods is the ability to translate two words to mathematical symbols. The word

of translates to *times*
is translates to *equals*

Missing Products Statements

The equation $M = 8 \cdot 4$ is a *missing product* statement. We can find the value of *M* that makes this statement true by *multiplying* the known factors.

Missing product statements can be used to determine the answer to a question such as, "What number is fraction 1 of fraction 2?

Sample Set A

Find $\frac{3}{4}$ of $\frac{8}{9}$. We are being asked the question, "What number is $\frac{3}{4}$ of $\frac{8}{9}$?" We must translate from words to mathematical symbols.

$$\begin{array}{cccccccl}
\underbrace{\text{What number}} & \text{is} & \frac{3}{4} & \text{of} & \frac{8}{9} & & & \textbf{becomes} \\
 & \downarrow & \downarrow & \downarrow & \downarrow & & & \\
M & = & \frac{3}{4} & \cdot & \frac{8}{9} & & & \textbf{Multiply.} \\
\text{missing product} & & \text{known factor} & & \text{known factor} & & &
\end{array}$$

$$M = \frac{\overset{1}{\cancel{3}}}{\underset{1}{\cancel{4}}} \cdot \frac{\overset{2}{\cancel{8}}}{\underset{3}{\cancel{9}}} = \frac{1 \cdot 2}{1 \cdot 3} = \frac{2}{3}$$

Thus, $\frac{3}{4}$ of $\frac{8}{9}$ is $\frac{2}{3}$.

$$\begin{array}{ccccc}
\underbrace{\text{What number}} & \text{is} & \frac{3}{4} & \text{of} & 24 \\
 & \downarrow & \downarrow & \downarrow & \downarrow \\
M & = & \frac{3}{4} & \cdot & 24 \\
\text{missing product} & & \text{known factor} & & \text{known factor}
\end{array}$$

$$M = \frac{3}{\cancel{4}_1} \cdot \frac{\cancel{24}^6}{1} = \frac{3 \cdot 6}{1 \cdot 1} = \frac{18}{1} = 18$$

Thus, 18 is $\frac{3}{4}$ of 24.

Practice Set A

Exercise:

Problem: Find $\frac{3}{8}$ of $\frac{16}{15}$.

Solution:

$\frac{2}{5}$

Exercise:

Problem: What number is $\frac{9}{10}$ of $\frac{5}{6}$?

Solution:

$\frac{3}{4}$

Exercise:

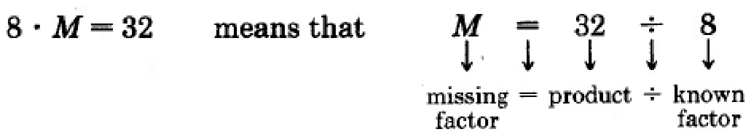
Problem: $\frac{11}{16}$ of $\frac{8}{33}$ is what number?

Solution:

$\frac{1}{6}$

Missing Factor Statements

The equation $8 \cdot M = 32$ is a *missing factor* statement. We can find the value of M that makes this statement true by dividing (since we know that $32 \div 8 = 4$).

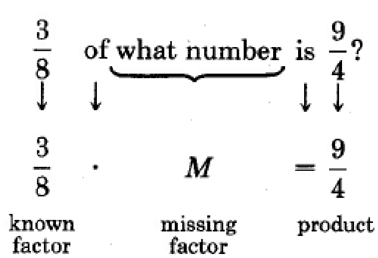


Finding the Missing Factor
To find the missing factor in a missing factor statement, divide the product by the known factor.
missing factor = (product) ÷ (known factor)

Missing factor statements can be used to answer such questions as

1. $\frac{3}{8}$ of what number is $\frac{9}{4}$?
2. What part of $1\frac{2}{7}$ is $1\frac{13}{14}$?

Sample Set B



Now, using

missing factor = (product) ÷ (known factor)

We get

$$M = \frac{9}{4} \div \frac{3}{8} = \frac{9}{4} \cdot \frac{8}{3} = \frac{\overset{3}{\cancel{9}}}{\underset{1}{\cancel{4}}} \cdot \frac{\overset{2}{\cancel{8}}}{\underset{1}{\cancel{3}}}$$

$$= \frac{3 \cdot 2}{1 \cdot 1}$$

$$= 6$$

Check: $\frac{3}{8} \cdot 6 \stackrel{?}{=} \frac{9}{4}$

$$\frac{3}{\underset{4}{\cancel{8}}} \cdot \frac{\overset{3}{\cancel{6}}}{1} \stackrel{?}{=} \frac{9}{4}$$

$$\frac{3 \cdot 3}{4 \cdot 1} \stackrel{?}{=} \frac{9}{4}$$

$$\frac{9}{4} \stackrel{\checkmark}{=} \frac{9}{4}$$

Thus, $\frac{3}{8}$ of 6 is $\frac{9}{4}$.

$$\begin{array}{ccccc} \underbrace{\text{What part}} & \text{of} & 1\frac{2}{7} & \text{is} & 1\frac{13}{14}? \\ & \downarrow & \downarrow & \downarrow & \downarrow \\ M & \cdot & 1\frac{2}{7} & = & 1\frac{13}{14} \\ \text{missing factor} & & \text{known factor} & & \text{product} \end{array}$$

For convenience, let's convert the mixed numbers to improper fractions.

$$M \cdot \frac{9}{7} = \frac{27}{14}$$

Now, using

missing factor = (product)÷(known factor)

we get

$$M = \frac{27}{14} \div \frac{9}{7} = \frac{27}{14} \cdot \frac{7}{9} = \frac{\overset{3}{\cancel{27}}}{\underset{2}{\cancel{14}}} \cdot \frac{\overset{1}{\cancel{7}}}{\underset{1}{\cancel{9}}}$$

$$= \frac{3 \cdot 1}{2 \cdot 1}$$

$$= \frac{3}{2}$$

Check:

$$\frac{3}{2} \cdot \frac{9}{7} \stackrel{?}{=} \frac{27}{14}$$

$$\frac{3 \cdot 9}{2 \cdot 7} \stackrel{?}{=} \frac{27}{14}$$

$$\frac{27}{14} \stackrel{\checkmark}{=} \frac{27}{14}$$

Thus, $\frac{3}{2}$ of $1\frac{2}{7}$ is $1\frac{13}{14}$.

Practice Set B

Exercise:

Problem: $\frac{3}{5}$ of what number is $\frac{9}{20}$?

Solution:

$\frac{3}{4}$

Exercise:

Problem: $3\frac{3}{4}$ of what number is $2\frac{2}{9}$?

Solution:

$\frac{16}{27}$

Exercise:

Problem: What part of $\frac{3}{5}$ is $\frac{9}{10}$?

Solution:

$1\frac{1}{2}$

Exercise:

Problem: What part of $1\frac{1}{4}$ is $1\frac{7}{8}$?

Solution:

$1\frac{1}{2}$

Exercises

Exercise:

Problem: Find $\frac{2}{3}$ of $\frac{3}{4}$.

Solution:

$\frac{1}{2}$

Exercise:

Problem: Find $\frac{5}{8}$ of $\frac{1}{10}$.

Exercise:

Problem: Find $\frac{12}{13}$ of $\frac{13}{36}$.

Solution:

$\frac{1}{3}$

Exercise:

Problem: Find $\frac{1}{4}$ of $\frac{4}{7}$.

Exercise:

Problem: $\frac{3}{10}$ of $\frac{15}{4}$ is what number?

Solution:

$\frac{9}{8}$ or $1\frac{1}{8}$

Exercise:

Problem: $\frac{14}{15}$ of $\frac{20}{21}$ is what number?

Exercise:

Problem: $\frac{3}{44}$ of $\frac{11}{12}$ is what number?

Solution:

$\frac{1}{16}$

Exercise:

Problem: $\frac{1}{3}$ of 2 is what number?

Exercise:

Problem: $\frac{1}{4}$ of 3 is what number?

Solution:

$\frac{3}{4}$

Exercise:

Problem: $\frac{1}{10}$ of $\frac{1}{100}$ is what number?

Exercise:

Problem: $\frac{1}{100}$ of $\frac{1}{10}$ is what number?

Solution:

$\frac{1}{1,000}$

Exercise:

Problem: $1\frac{5}{9}$ of $2\frac{4}{7}$ is what number?

Exercise:

Problem: $1\frac{7}{18}$ of $\frac{4}{15}$ is what number?

Solution:

$\frac{10}{27}$

Exercise:

Problem: $1\frac{1}{8}$ of $1\frac{11}{16}$ is what number?

Exercise:

Problem: Find $\frac{2}{3}$ of $\frac{1}{6}$ of $\frac{9}{2}$.

Solution:

$\frac{1}{2}$

Exercise:

Problem: Find $\frac{5}{8}$ of $\frac{9}{20}$ of $\frac{4}{9}$.

Exercise:

Problem: $\frac{5}{12}$ of what number is $\frac{5}{6}$?

Solution:

2

Exercise:

Problem: $\frac{3}{14}$ of what number is $\frac{6}{7}$?

Exercise:

Problem: $\frac{10}{3}$ of what number is $\frac{5}{9}$?

Solution:

$\frac{1}{6}$

Exercise:

Problem: $\frac{15}{7}$ of what number is $\frac{20}{21}$?

Exercise:

Problem: $\frac{8}{3}$ of what number is $1\frac{7}{9}$?

Solution:

$\frac{2}{3}$

Exercise:

Problem: $\frac{1}{3}$ of what number is $\frac{1}{3}$?

Exercise:

Problem: $\frac{1}{6}$ of what number is $\frac{1}{6}$?

Solution:

1

Exercise:

Problem: $\frac{3}{4}$ of what number is $\frac{3}{4}$?

Exercise:

Problem: $\frac{8}{11}$ of what number is $\frac{8}{11}$?

Solution:

1

Exercise:

Problem: $\frac{3}{8}$ of what number is 0?

Exercise:

Problem: $\frac{2}{3}$ of what number is 1?

Solution:

$\frac{3}{2}$ or $1\frac{2}{3}$

Exercise:

Problem: $3\frac{1}{5}$ of what number is 1?

Exercise:

Problem: $1\frac{9}{12}$ of what number is $5\frac{1}{4}$?

Solution:

3

Exercise:

Problem: $3\frac{1}{25}$ of what number is $2\frac{8}{15}$?

Exercise:

Problem: What part of $\frac{2}{3}$ is $1\frac{1}{9}$?

Solution:

$\frac{5}{3}$ or $1\frac{2}{3}$

Exercise:

Problem: What part of $\frac{9}{10}$ is $3\frac{3}{5}$?

Exercise:

Problem: What part of $\frac{8}{9}$ is $\frac{3}{5}$?

Solution:

$\frac{27}{40}$

Exercise:

Problem: What part of $\frac{14}{15}$ is $\frac{7}{30}$?

Exercise:

Problem: What part of 3 is $\frac{1}{5}$?

Solution:

$\frac{1}{15}$

Exercise:

Problem: What part of 8 is $\frac{2}{3}$?

Exercise:

Problem: What part of 24 is 9?

Solution:

$\frac{3}{8}$

Exercise:

Problem: What part of 42 is 26?

Exercise:

Problem: Find $\frac{12}{13}$ of $\frac{39}{40}$.

Solution:

$\frac{9}{10}$

Exercise:

Problem: $\frac{14}{15}$ of $\frac{12}{21}$ is what number?

Exercise:

Problem: $\frac{8}{15}$ of what number is $2\frac{2}{5}$?

Solution:

$\frac{9}{2} = 4\frac{1}{2}$

Exercise:

Problem: $\frac{11}{15}$ of what number is $\frac{22}{35}$?

Exercise:

Problem: $\frac{11}{16}$ of what number is 1?

Solution:

$\frac{16}{11}$ or $1\frac{5}{11}$

Exercise:

Problem: What part of $\frac{23}{40}$ is $3\frac{9}{20}$?

Exercise:

Problem: $\frac{4}{35}$ of $3\frac{9}{22}$ is what number?

Solution:

$\frac{30}{77}$

Exercises for Review

Exercise:

Problem:

([link]) Use the numbers 2 and 7 to illustrate the commutative property of addition.

Exercise:

Problem: ([link]) Is 4 divisible by 0?

Solution:

no

Exercise:

Problem: ([link]) Expand 3^7. Do not find the actual value.

Exercise:

Problem: ([link]) Convert $3\frac{5}{12}$ to an improper fraction.

Solution:

$\frac{41}{12}$

Exercise:

Problem: ([link]) Find the value of $\frac{3}{8} \div \frac{9}{16} \cdot \frac{6}{5}$.

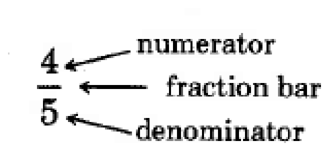
Summary of Key Concepts
This module is from Fundamentals of Mathematics by Denny Burzynski and Wade Ellis, Jr. This module reviews the key concepts from the chapter "Introduction to Fractions and Multiplication and Division of Fractions."

Summary of Key Concepts

Fraction ([link])
The idea of breaking up a whole quantity into equal parts gives us the word **fraction**.

Fraction Bar, Denominator, Numerator ([link])
A fraction has three parts:

1. The fraction bar — -
2. The nonzero whole number below the fraction bar is the **denominator**.
3. The whole number above the fraction bar is the **numerator**.



Proper Fraction ([link])
Proper fractions are fractions in which the numerator is strictly less than the denominator.

$\frac{4}{5}$ is a proper fraction

Improper Fraction ([link])
Improper fractions are fractions in which the numerator is greater than or equal to the denominator. Also, any nonzero number placed over 1 is an improper fraction.

$\frac{5}{4}$, $\frac{5}{5}$, and $\frac{5}{1}$ are improper fractions

Mixed Number ([link])

A **mixed number** is a number that is the sum of a whole number and a proper fraction.

$1\frac{1}{5}$ is a mixed number $\left(1\frac{1}{5} = 1 + \frac{1}{5}\right)$

Correspondence Between Improper Fractions and Mixed Numbers ([link])
Each improper fraction corresponds to a particular mixed number, and each mixed number corresponds to a particular improper fraction.

Converting an Improper Fraction to a Mixed Number ([link])
A method, based on division, converts an improper fraction to an equivalent mixed number.

$\frac{5}{4}$ can be converted to $1\frac{1}{4}$

Converting a Mixed Number to an Improper Fraction ([link])
A method, based on multiplication, converts a mixed number to an equivalent improper fraction.

$5\frac{7}{8}$ can be converted to $\frac{47}{8}$

Equivalent Fractions ([link])
Fractions that represent the same quantity are **equivalent fractions**.

$\frac{3}{4}$ and $\frac{6}{8}$ are equivalent fractions

Test for Equivalent Fractions ([link])
If the **cross products** of two fractions are equal, then the two fractions are equivalent.

$$\frac{3}{4} \quad \frac{6}{8}$$
$$3 \cdot 8 \stackrel{?}{=} 4 \cdot 6$$
$$24 = 24$$

Thus, $\frac{3}{4}$ and $\frac{6}{8}$ are equivalent.

Relatively Prime ([link])
Two whole numbers are **relatively prime** when 1 is the only number that divides both of them.

3 and 4 are relatively prime

Reduced to Lowest Terms ([link])
A fraction is **reduced to lowest terms** if its numerator and denominator are relatively prime.

The number $\frac{3}{4}$ is reduced to lowest terms, since 3 and 4 are relatively prime.

The number $\frac{6}{8}$ is *not* reduced to lowest terms since 6 and 8 are not relatively prime.

Reducing Fractions to Lowest Terms ([link])
Two methods, one based on dividing out common primes and one based on dividing out any common factors, are available for reducing a fraction to lowest terms.

Raising Fractions to Higher Terms ([link])
A fraction can be raised to higher terms by multiplying both the numerator and denominator by the same nonzero number.

$$\frac{3}{4} = \frac{3 \cdot 2}{4 \cdot 2} = \frac{6}{8}$$

The Word "OF" Means Multiplication ([link])
In many mathematical applications, the word "of" means multiplication.

Multiplication of Fractions ([link])
To multiply two or more fractions, multiply the numerators together and multiply the denominators together. Reduce if possible.

$$\frac{5}{8} \cdot \frac{4}{15} = \frac{5 \cdot 4}{8 \cdot 15} = \frac{20}{120} = \frac{1}{6}$$

Multiplying Fractions by Dividing Out Common Factors ([link])

Two or more fractions can be multiplied by first dividing out common factors and then using the rule for multiplying fractions.

$$\frac{\overset{1}{\cancel{5}}}{\underset{2}{\cancel{8}}} \cdot \frac{\overset{1}{\cancel{4}}}{\underset{3}{\cancel{15}}} = \frac{1 \cdot 1}{2 \cdot 3} = \frac{1}{6}$$

Multiplication of Mixed Numbers ([link])
To perform a multiplication in which there are mixed numbers, first convert each mixed number to an improper fraction, then multiply. This idea also applies to division of mixed numbers.

Reciprocals ([link])
Two numbers whose product is 1 are reciprocals.

7 and $\frac{1}{7}$ are reciprocals

Division of Fractions ([link])
To divide one fraction by another fraction, multiply the dividend by the reciprocal of the divisor.

$$\frac{4}{5} \div \frac{2}{15} = \frac{4}{5} \cdot \frac{15}{2}$$

Dividing 1 by a Fraction ([link])
When dividing 1 by a fraction, the quotient is the reciprocal of the fraction.

$$\frac{1}{\frac{3}{7}} = \frac{7}{3}$$

Multiplication Statements ([link])
A mathematical statement of the form

product = (factor 1) (factor 2)

is a multiplication statement.

By omitting one of the three numbers, one of three following problems result:

1. M = (factor 1) · (factor 2) Missing product statement.
2. product = (factor 1) · M Missing factor statement.
3. product = M · (factor 2) Missing factor statement.

Missing products are determined by simply multiplying the known factors. *Missing factors* are determined by

missing factor = (product) ÷ (known factor)

Exercise Supplement
This module is from Fundamentals of Mathematics by Denny Burzynski and Wade Ellis, Jr. This module is an exercise supplement for the chapter "Introduction to Fractions and Multiplication and Division of Fractions" and contains many exercise problems. Odd problems are accompanied by solutions.

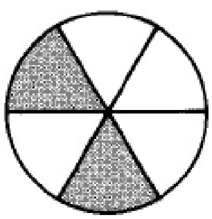
Exercise Supplement

Fractions of Whole Numbers ([link])

For Problems 1 and 2, name the suggested fraction.

Exercise:

Problem:

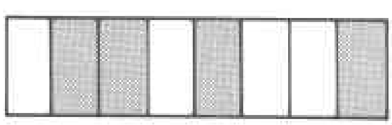


Solution:

$\frac{2}{6}$ or $\frac{1}{3}$

Exercise:

Problem:



For problems 3-5, specify the numerator and denominator.

Exercise:

Problem: $\frac{4}{5}$

Solution:

numerator, 4; denominator, 5

Exercise:

Problem: $\frac{5}{12}$

Exercise:

Problem: $\frac{1}{3}$

Solution:

numerator, 1; denominator, 3

For problems 6-10, write each fraction using digits.

Exercise:

Problem: Three fifths

Exercise:

Problem: Eight elevenths

Solution:

$\frac{8}{11}$

Exercise:

Problem: Sixty-one forty firsts

Exercise:

Problem: Two hundred six-thousandths

Solution:

$\frac{200}{6,000}$

Exercise:

Problem: zero tenths

For problems 11-15, write each fraction using words.

Exercise:

Problem: $\frac{10}{17}$

Solution:

ten seventeenths

Exercise:

Problem: $\frac{21}{38}$

Exercise:

Problem: $\frac{606}{1431}$

Solution:

six hundred six, one thousand four hundred thirty-firsts

Exercise:

Problem: $\frac{0}{8}$

Exercise:

Problem: $\frac{1}{16}$

Solution:

one sixteenth

For problems 16-18, state each numerator and denominator and write each fraction using digits.

Exercise:

Problem: One minute is one sixtieth of an hour.

Exercise:

Problem:

In a box that contains forty-five electronic components, eight are known to be defective. If three components are chosen at random from the box, the probability that all three are defective is fifty-six fourteen thousand one hundred ninetieths.

Solution:

numerator, 56; denominator, 14,190

Exercise:

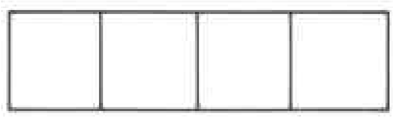
Problem:

About three fifths of the students in a college algebra class received a “B” in the course.

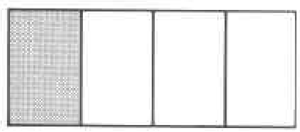
For problems 19 and 20, shade the region corresponding to the given fraction.

Exercise:

Problem: $\frac{1}{4}$

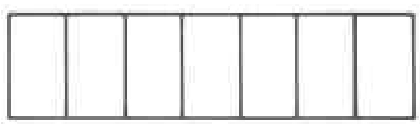


Solution:



Exercise:

Problem: $\frac{3}{7}$



Proper Fraction, Improper Fraction, and Mixed Numbers ([link])

For problems 21-29, convert each improper fraction to a mixed number.

Exercise:

Problem: $\frac{11}{4}$

Solution:

$2\frac{3}{4}$

Exercise:

Problem: $\frac{15}{2}$

Exercise:

Problem: $\frac{51}{8}$

Solution:

$6\,\frac{3}{8}$

Exercise:

Problem: $\frac{121}{15}$

Exercise:

Problem: $\frac{356}{3}$

Solution:

$118\,\frac{2}{3}$

Exercise:

Problem: $\frac{3}{2}$

Exercise:

Problem: $\frac{5}{4}$

Solution:

$1\,\frac{1}{4}$

Exercise:

Problem: $\frac{20}{5}$

Exercise:

Problem: $\frac{9}{3}$

Solution:

3

For problems 30-40, convert each mixed number to an improper fraction.

Exercise:

Problem: $5\frac{2}{3}$

Exercise:

Problem: $16\frac{1}{8}$

Solution:

$\frac{129}{8}$

Exercise:

Problem: $18\frac{1}{3}$

Exercise:

Problem: $3\frac{1}{5}$

Solution:

$\frac{16}{5}$

Exercise:

Problem: $2\frac{9}{16}$

Exercise:

Problem: $17\frac{20}{21}$

Solution:

$\frac{377}{21}$

Exercise:

Problem: $1\frac{7}{8}$

Exercise:

Problem: $1\frac{1}{2}$

Solution:

$\frac{3}{2}$

Exercise:

Problem: $2\frac{1}{2}$

Exercise:

Problem: $8\frac{6}{7}$

Solution:

$\frac{62}{7}$

Exercise:

Problem: $2\frac{9}{2}$

Exercise:

Problem: Why does $0\frac{1}{12}$ not qualify as a mixed number?

Solution:

because the whole number part is zero

Exercise:

Problem: Why does 8 qualify as a mixed number?

Equivalent Fractions, Reducing Fractions to Lowest Terms, and Raising Fractions to Higher Term ([link])

For problems 43-47, determine if the pairs of fractions are equivalent.
Exercise:

Problem: $\frac{1}{2}, \frac{15}{30}$

Solution:

equivalent

Exercise:

Problem: $\frac{8}{9}, \frac{32}{36}$

Exercise:

Problem: $\frac{3}{14}, \frac{24}{110}$

Solution:

not equivalent

Exercise:

Problem: $2\frac{3}{8}$, $\frac{38}{16}$

Exercise:

Problem: $\frac{108}{77}$, $1\frac{5}{13}$

Solution:

not equivalent

For problems 48-60, reduce, if possible, each fraction.

Exercise:

Problem: $\frac{10}{25}$

Exercise:

Problem: $\frac{32}{44}$

Solution:

$\frac{8}{11}$

Exercise:

Problem: $\frac{102}{266}$

Exercise:

Problem: $\frac{15}{33}$

Solution:

$\frac{5}{11}$

Exercise:

Problem: $\frac{18}{25}$

Exercise:

Problem: $\frac{21}{35}$

Solution:

$\frac{3}{5}$

Exercise:

Problem: $\frac{9}{16}$

Exercise:

Problem: $\frac{45}{85}$

Solution:

$\frac{9}{17}$

Exercise:

Problem: $\frac{24}{42}$

Exercise:

Problem: $\frac{70}{136}$

Solution:

$\frac{35}{68}$

Exercise:

Problem: $\frac{182}{580}$

Exercise:

Problem: $\frac{325}{810}$

Solution:

$\frac{65}{162}$

Exercise:

Problem: $\frac{250}{1000}$

For problems 61-72, determine the missing numerator or denominator.

Exercise:

Problem: $\frac{3}{7} = \frac{?}{35}$

Solution:

15

Exercise:

Problem: $\frac{4}{11} = \frac{?}{99}$

Exercise:

Problem: $\frac{1}{12} = \frac{?}{72}$

Solution:

6

Exercise:

Problem: $\frac{5}{8} = \frac{25}{?}$

Exercise:

Problem: $\frac{11}{9} = \frac{33}{?}$

Solution:

27

Exercise:

Problem: $\frac{4}{15} = \frac{24}{?}$

Exercise:

Problem: $\frac{14}{15} = \frac{?}{45}$

Solution:

42

Exercise:

Problem: $\frac{0}{5} = \frac{?}{20}$

Exercise:

Problem: $\frac{12}{21} = \frac{96}{?}$

Solution:

168

Exercise:

Problem: $\frac{14}{23} = \frac{?}{253}$

Exercise:

Problem: $\frac{15}{16} = \frac{180}{?}$

Solution:

192

Exercise:

Problem: $\frac{21}{22} = \frac{336}{?}$

Multiplication and Division of Fractions ([link], [link])

For problems 73-95, perform each multiplication and division.

Exercise:

Problem: $\frac{4}{5} \cdot \frac{15}{16}$

Solution:

$\frac{3}{4}$

Exercise:

Problem: $\frac{8}{9} \cdot \frac{3}{24}$

Exercise:

Problem: $\frac{1}{10} \cdot \frac{5}{12}$

Solution:

$\frac{1}{24}$

Exercise:

Problem: $\frac{14}{15} \cdot \frac{7}{5}$

Exercise:

Problem: $\frac{5}{6} \cdot \frac{13}{22} \cdot \frac{11}{39}$

Solution:

$\frac{5}{36}$

Exercise:

Problem: $\frac{2}{3} \div \frac{15}{7} \cdot \frac{5}{6}$

Exercise:

Problem: $3\frac{1}{2} \div \frac{7}{2}$

Solution:

1

Exercise:

Problem: $2\frac{4}{9} \div \frac{11}{45}$

Exercise:

Problem: $\frac{8}{15} \cdot \frac{3}{16} \cdot \frac{5}{24}$

Solution:

$\frac{1}{48}$

Exercise:

Problem: $\frac{8}{15} \div 3\frac{3}{5} \cdot \frac{9}{16}$

Exercise:

Problem: $\frac{14}{15} \div 3\frac{8}{9} \cdot \frac{10}{21}$

Solution:

$\frac{4}{35}$

Exercise:

Problem: $18 \cdot 5\frac{3}{4}$

Exercise:

Problem: $3\frac{3}{7} \cdot 2\frac{1}{12}$

Solution:

$\frac{50}{7} = 7\frac{1}{7}$

Exercise:

Problem: $4\frac{1}{2} \div 2\frac{4}{7}$

Exercise:

Problem: $6\frac{1}{2} \div 3\frac{1}{4}$

Solution:

2

Exercise:

Problem: $3\frac{5}{16} \div 2\frac{7}{18}$

Exercise:

Problem: $7 \div 2\frac{1}{3}$

Solution:

3

Exercise:

Problem: $17 \div 4\frac{1}{4}$

Exercise:

Problem: $\frac{5}{8} \div 1\frac{1}{4}$

Solution:

$\frac{1}{2}$

Exercise:

Problem: $2\frac{2}{3} \cdot 3\frac{3}{4}$

Exercise:

Problem: $20 \cdot \frac{18}{4}$

Solution:

90

Exercise:

Problem: $0 \div 4\frac{1}{8}$

Exercise:

Problem: $1 \div 6\frac{1}{4} \cdot \frac{25}{4}$

Solution:

1

Applications Involving Fractions ([link])

Exercise:

Problem: Find $\frac{8}{9}$ of $\frac{27}{2}$.

Exercise:

Problem: What part of $\frac{3}{8}$ is $\frac{21}{16}$?

Solution:

$\frac{7}{2}$ or $3\frac{1}{2}$

Exercise:

Problem: What part of $3\frac{1}{5}$ is $1\frac{7}{25}$?

Exercise:

Problem: Find $6\frac{2}{3}$ of $\frac{9}{15}$.

Solution:

4

Exercise:

Problem: $\frac{7}{20}$ of what number is $\frac{14}{35}$?

Exercise:

Problem: What part of $4\frac{1}{16}$ is $3\frac{3}{4}$?

Solution:

$\frac{12}{13}$

Exercise:

Problem: Find $8\frac{3}{10}$ of $16\frac{2}{3}$.

Exercise:

Problem: $\frac{3}{20}$ of what number is $\frac{18}{30}$?

Solution:

4

Exercise:

Problem: Find $\frac{1}{3}$ of 0.

Exercise:

Problem: Find $\frac{11}{12}$ of 1.

Solution:

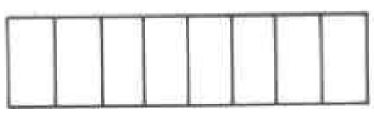
$\frac{11}{12}$

Proficiency Exam
This module is from Fundamentals of Mathematics by Denny Burzynski and Wade Ellis, Jr. This module is a proficiency exam to the chapter "Introduction to Fractions and Multiplication and Division of Fractions." Each problem is accompanied with a reference link pointing back to the module that discusses the type of problem demonstrated in the question. The problems in this exam are accompanied by solutions.

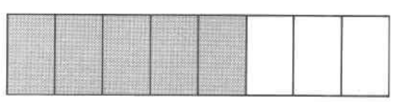
Proficiency Exam

Exercise:

Problem: ([link]) Shade a portion that corresponds to the fraction $\frac{5}{8}$.



Solution:



Exercise:

Problem:

([link]) Specify the numerator and denominator of the fraction $\frac{5}{9}$.

Solution:

Numerator, 5; denominator, 9

Exercise:

Problem: ([link]) Write the fraction five elevenths.

Solution:

$\frac{5}{11}$

Exercise:

Problem: ([link]) Write, in words, $\frac{4}{5}$.

Solution:

Four fifths

Exercise:

Problem:

([link]) Which of the fractions is a proper fraction? $4\frac{1}{12}$, $\frac{5}{12}$, $\frac{12}{5}$

Solution:

$\frac{5}{12}$

Exercise:

Problem: ([link]) Convert $3\frac{4}{7}$ to an improper fraction.

Solution:

$\frac{25}{7}$

Exercise:

Problem: ([link]) Convert $\frac{16}{5}$ to a mixed number.

Solution:

$3\frac{1}{5}$

Exercise:

Problem: ([link]) Determine if $\frac{5}{12}$ and $\frac{20}{48}$ are equivalent fractions.

Solution:

yes

For problems 9-11, reduce, if possible, each fraction to lowest terms.

Exercise:

Problem: ([link]) $\frac{21}{35}$

Solution:

$\frac{3}{5}$

Exercise:

Problem: ([link]) $\frac{15}{51}$

Solution:

$\frac{5}{17}$

Exercise:

Problem: ([link]) $\frac{104}{480}$

Solution:

$\frac{13}{60}$

For problems 12 and 13, determine the missing numerator or denominator.

Exercise:

Problem: ([link]) $\frac{5}{9} = \frac{?}{36}$

Solution:

20

Exercise:

Problem: ([link]) $\frac{4}{3} = \frac{32}{?}$

Solution:

24

For problems 14-25, find each value.

Exercise:

Problem: ([link]) $\frac{15}{16} \cdot \frac{4}{25}$

Solution:

$\frac{3}{20}$

Exercise:

Problem: ([link]) $3\frac{3}{4} \cdot 2\frac{2}{9} \cdot 6\frac{3}{5}$

Solution:

55

Exercise:

Problem: ([link]) $\sqrt{\frac{25}{36}}$

Solution:

$\frac{5}{6}$

Exercise:

Problem: ([link]) $\sqrt{\frac{4}{9}} \cdot \sqrt{\frac{81}{64}}$

Solution:

$\frac{3}{4}$

Exercise:

Problem: ([link]) $\frac{11}{30} \cdot \sqrt{\frac{225}{121}}$

Solution:

$\frac{1}{2}$

Exercise:

Problem: ([link]) $\frac{4}{15} \div 8$

Solution:

$\frac{1}{30}$

Exercise:

Problem: ([link]) $\frac{8}{15} \cdot \frac{5}{12} \div 2\frac{4}{9}$

Solution:

$\frac{1}{11}$

Exercise:

Problem: ([link]) $\left(\frac{6}{5}\right)^3 \div \sqrt{1\frac{11}{25}}$

Solution:

$\frac{36}{25} = 1\frac{11}{25}$

Exercise:

Problem: ([link]) Find $\frac{5}{12}$ of $\frac{24}{25}$.

Solution:

$\frac{2}{5}$

Exercise:

Problem: ([link]) $\frac{2}{9}$ of what number is $\frac{1}{18}$?

Solution:

$\frac{1}{4}$

Exercise:

Problem: ([link]) $1\frac{5}{7}$ of $\frac{21}{20}$ is what number?

Solution:

$\frac{9}{5} = 1\frac{4}{5}$

Exercise:

Problem: ([link]) What part of $\frac{9}{14}$ is $\frac{6}{7}$?

Solution:

$\frac{4}{3}$ or $1\frac{1}{3}$

Objectives

This module contains the learning objectives for the chapter "Addition and Subtraction of Fractions" from Fundamentals of Mathematics by Denny Burzynski and Wade Ellis, jr.

After completing this chapter, you should

Addition and Subtraction of Fractions with Like Denominators ([link])

- be able to add and subtract fractions with like denominators

Addition and Subtraction of Fractions with Unlike Denominators ([link])

- be able to add and subtract fractions with unlike denominators

Addition and Subtraction of Mixed Numbers ([link])

- be able to add and subtract mixed numbers

Comparing Fractions ([link])

- understand ordering of numbers and be familiar with grouping symbols
- be able to compare two or more fractions

Complex Fractions ([link])

- be able to distinguish between simple and complex fractions
- be able to convert a complex fraction to a simple fraction

Combinations of Operations with Fractions ([link])

- gain a further understanding of the order of operations

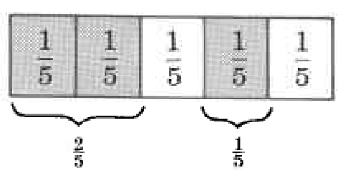
Addition and Subtraction of Fractions with Like Denominators
This module is from Fundamentals of Mathematics by Denny Burzynski and Wade Ellis, Jr. This module discusses how to add and subtract fractions with like denominators. By the end of the module students should be able to add and subtract fractions with like denominators.

Section Overview

- Addition of Fraction With Like Denominators
- Subtraction of Fractions With Like Denominators

Addition of Fraction With Like Denominators

Let's examine the following diagram.



2 one-fifths and 1 one fifth is shaded.

It is shown in the shaded regions of the diagram that

(2 one-fifths) + (1 one-fifth) = (3 one-fifths)

That is,

$\frac{2}{5} + \frac{1}{5} = \frac{3}{5}$

From this observation, we can suggest the following rule.

Method of Adding Fractions Having Like Denominators
To add two or more fractions that have the same denominators, add the numerators and place the resulting sum over the common denominator. Reduce, if necessary.

Sample Set A

Find the following sums.

Example:
$\frac{3}{7}+\frac{2}{7}$. The denominators are the same. Add the numerators and place that sum over 7.
$\frac{3}{7}+\frac{2}{7}=\frac{3+2}{7}=\frac{5}{7}$

Example:
$\frac{1}{8}+\frac{3}{8}$. The denominators are the same. Add the numerators and place the sum over 8. Reduce.
$\frac{1}{8}+\frac{3}{8}=\frac{1+3}{8}=\frac{4}{8}=\frac{1}{2}$

Example:
$\frac{4}{9}+\frac{5}{9}$. The denominators are the same. Add the numerators and place the sum over 9.
$\frac{4}{9}+\frac{5}{9}=\frac{4+5}{9}=\frac{9}{9}=1$

Example:
$\frac{7}{8}+\frac{5}{8}$. The denominators are the same. Add the numerators and place the sum over 8.
$\frac{7}{8}+\frac{5}{8}=\frac{7+5}{8}=\frac{12}{8}=\frac{3}{2}$

Example:

To see what happens if we *mistakenly add the denominators* as well as the numerators, let's add

$$\frac{1}{2}+\frac{1}{2}$$

Adding the numerators and *mistakenly* adding the denominators produces

$$\frac{1}{2}+\frac{1}{2}=\frac{1+1}{2+2}=\frac{2}{4}=\frac{1}{2}$$

This means that two $\frac{1}{2}$'s is the same as one $\frac{1}{2}$. Preposterous! **We do not add denominators**.

Practice Set A

Find the following sums.

Exercise:

Problem: $\frac{1}{10}+\frac{3}{10}$

Solution:

$\frac{2}{5}$

Exercise:

Problem: $\frac{1}{4}+\frac{1}{4}$

Solution:

$\frac{1}{2}$

Exercise:

Problem: $\frac{7}{11}+\frac{4}{11}$

Solution:

1

Exercise:

Problem: $\frac{3}{5}+\frac{1}{5}$

Solution:

$\frac{4}{5}$

Exercise:

Problem:

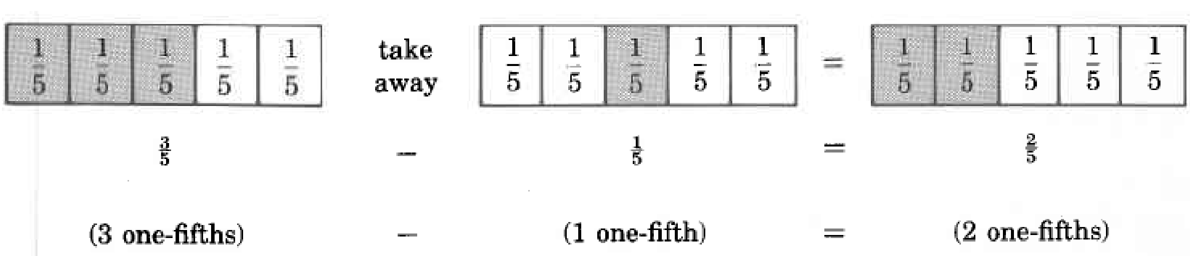
Show why adding both the numerators and denominators is preposterous by adding $\frac{3}{4}$ and $\frac{3}{4}$ and examining the result.

Solution:

$\frac{3}{4}+\frac{3}{4}=\frac{3+3}{4+4}=\frac{6}{8}=\frac{3}{4}$, so two $\frac{3}{4}$'s= one $\frac{3}{4}$ which is preposterous.

Subtraction of Fractions With Like Denominators

We can picture the concept of subtraction of fractions in much the same way we pictured addition.



From this observation, we can suggest the following rule for subtracting fractions having like denominators:

Subtraction of Fractions with Like Denominators
To subtract two fractions that have like denominators, subtract the numerators and place the resulting difference over the common denominator. Reduce, if possible.

Sample Set B

Find the following differences.

Example:
$\frac{3}{5} - \frac{1}{5}$. The denominators are the same. Subtract the numerators. Place the difference over 5.
$\frac{3}{5} - \frac{1}{5} = \frac{3-1}{5} = \frac{2}{5}$

Example:
$\frac{8}{6} - \frac{2}{6}$. The denominators are the same. Subtract the numerators. Place the difference over 6.
$\frac{8}{6} - \frac{2}{6} = \frac{8-2}{6} = \frac{6}{6} = 1$

Example:
$\frac{16}{9} - \frac{2}{9}$. The denominators are the same. Subtract numerators and place the difference over 9.
$\frac{16}{9} - \frac{2}{9} = \frac{16-2}{9} = \frac{14}{9}$

Example:
To see what happens if we *mistakenly* subtract the denominators, let's consider
$\frac{7}{15} - \frac{4}{15} = \frac{7-4}{15-15} = \frac{3}{0}$
We get division by zero, which is undefined. **We do not subtract denominators.**

Practice Set B

Find the following differences.

Exercise:

Problem: $\frac{10}{13} - \frac{8}{13}$

Solution:

$\frac{2}{13}$

Exercise:

Problem: $\frac{5}{12} - \frac{1}{12}$

Solution:

$\frac{1}{3}$

Exercise:

Problem: $\frac{1}{2} - \frac{1}{2}$

Solution:

0

Exercise:

Problem: $\frac{26}{10} - \frac{14}{10}$

Solution:

$\frac{6}{5}$

Exercise:

Problem:

Show why subtracting both the numerators and the denominators is in error by performing the subtraction $\frac{5}{9} - \frac{2}{9}$.

Solution:

$\frac{5}{9} - \frac{2}{9} = \frac{5-2}{9-9} = \frac{3}{0}$, which is undefined

Exercises

For the following problems, find the sums and differences. Be sure to reduce.

Exercise:

Problem: $\frac{3}{8} + \frac{2}{8}$

Solution:

$\frac{5}{8}$

Exercise:

Problem: $\frac{1}{6} + \frac{2}{6}$

Exercise:

Problem: $\frac{9}{10} + \frac{1}{10}$

Solution:

1

Exercise:

Problem: $\frac{3}{11} + \frac{4}{11}$

Exercise:

Problem: $\frac{9}{15} + \frac{4}{15}$

Solution:

$\frac{13}{15}$

Exercise:

Problem: $\frac{3}{10} + \frac{2}{10}$

Exercise:

Problem: $\frac{5}{12} + \frac{7}{12}$

Solution:

1

Exercise:

Problem: $\frac{11}{16} - \frac{2}{16}$

Exercise:

Problem: $\frac{3}{16} - \frac{3}{16}$

Solution:

0

Exercise:

Problem: $\frac{15}{23} - \frac{2}{23}$

Exercise:

Problem: $\frac{1}{6} - \frac{1}{6}$

Solution:

0

Exercise:

Problem: $\frac{1}{4} + \frac{1}{4} + \frac{1}{4}$

Exercise:

Problem: $\frac{3}{11} + \frac{1}{11} + \frac{5}{11}$

Solution:

$\frac{9}{11}$

Exercise:

Problem: $\frac{16}{20} + \frac{1}{20} + \frac{2}{20}$

Exercise:

Problem: $\frac{12}{8} + \frac{2}{8} + \frac{1}{8}$

Solution:

$\frac{15}{8}$

Exercise:

Problem: $\frac{1}{15} + \frac{8}{15} + \frac{6}{15}$

Exercise:

Problem: $\frac{3}{8} + \frac{2}{8} - \frac{1}{8}$

Solution:

$\frac{1}{2}$

Exercise:

Problem: $\frac{11}{16} + \frac{9}{16} - \frac{5}{16}$

Exercise:

Problem: $\frac{4}{20} - \frac{1}{20} + \frac{9}{20}$

Solution:

$\frac{3}{5}$

Exercise:

Problem: $\frac{7}{10} - \frac{3}{10} + \frac{11}{10}$

Exercise:

Problem: $\frac{16}{5} - \frac{1}{5} - \frac{2}{5}$

Solution:

$\frac{13}{5}$

Exercise:

Problem: $\frac{21}{35} - \frac{17}{35} + \frac{31}{35}$

Exercise:

Problem: $\frac{5}{2} + \frac{16}{2} - \frac{1}{2}$

Solution:

10

Exercise:

Problem: $\frac{1}{18} + \frac{3}{18} + \frac{1}{18} + \frac{4}{18} - \frac{5}{18}$

Exercise:

Problem: $\frac{6}{22} - \frac{2}{22} + \frac{4}{22} - \frac{1}{22} + \frac{11}{22}$

Solution:

$\frac{9}{11}$

The following rule for addition and subtraction of two fractions is preposterous. Show why by performing the operations using the rule for the following two problems.

Preposterous Rule
To add or subtract two fractions, simply add or subtract the numerators and place this result over the sum or difference of the denominators.
Exercise:

Problem: $\frac{3}{10} - \frac{3}{10}$

Exercise:

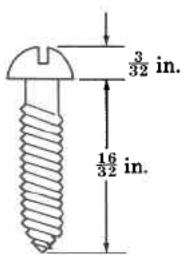
Problem: $\frac{8}{15} + \frac{8}{15}$

Solution:

$\frac{16}{30} = \frac{8}{5}$(using the preposterous rule)

Exercise:

Problem: Find the total length of the screw.



Exercise:

Problem:

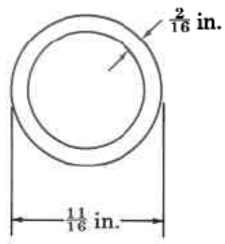
Two months ago, a woman paid off $\frac{3}{24}$ of a loan. One month ago, she paid off $\frac{5}{24}$ of the total loan. This month she will again pay off $\frac{5}{24}$ of the total loan. At the end of the month, how much of her total loan will she have paid off?

Solution:

$\frac{13}{24}$

Exercise:

Problem: Find the inside diameter of the pipe.



Exercises for Review

Exercise:

Problem: ([link]) Round 2,650 to the nearest hundred.

Solution:

2700

Exercise:

Problem:

([link]) Use the numbers 2, 4, and 8 to illustrate the associative property of addition.

Exercise:

Problem: ([link]) Find the prime factors of 495.

Solution:

$3^2 \cdot 5 \cdot 11$

Exercise:

Problem: ([link]) Find the value of $\frac{3}{4} \cdot \frac{16}{25} \cdot \frac{5}{9}$.

Exercise:

Problem: ([link]) $\frac{8}{3}$ of what number is $1\frac{7}{9}$?

Solution:

$\frac{2}{3}$

Addition and Subtraction of Fractions with Unlike Denominators
This module is from Fundamentals of Mathematics by Denny Burzynski and Wade Ellis, Jr. This module discusses how to add and subtract fractions with unlike denominators. By the end of the module students should be able to add and subtract fractions with unlike denominators.

Section Overview

- A Basic Rule
- Addition and Subtraction of Fractions

A Basic Rule

There is a basic rule that must be followed when adding or subtracting fractions.

A Basic Rule
Fractions can only be added or subtracted conveniently if they have like denominators.

To see why this rule makes sense, let's consider the problem of adding a quarter and a dime.

$1 \text{ quarter} + 1 \text{ dime} = 35 \text{ cents}$

Now,

$$\left.\begin{array}{r} 1 \ \text{ quarter} = \frac{25}{100} \\ 1 \ \text{ dime} = \frac{10}{100} \end{array}\right\} \text{ same denominations}$$
$$35, ¢ = \frac{35}{100}$$

$\frac{25}{100} + \frac{10}{100} = \frac{25+10}{100} = \frac{35}{100}$

In order to combine a quarter and a dime to produce 35¢, we convert them to quantities of the same denomination.

Same denomination → same denominator

Addition and Subtraction of Fractions

Least Common Multiple (LCM) and Least Common Denominator (LCD)
In [link], we examined the least common multiple (LCM) of a collection of numbers. If these numbers are used as denominators of fractions, we call the least common multiple, the least common denominator (LCD).

Method of Adding or Subtracting Fractions with Unlike Denominators
To add or subtract fractions having unlike denominators, convert each fraction to an equivalent fraction having as a denominator the least common denominator (LCD) of the original denominators.

Sample Set A

Find the following sums and differences.

Example:

$\frac{1}{6}+\frac{3}{4}$. The denominators are not the same. Find the LCD of 6 and 4.

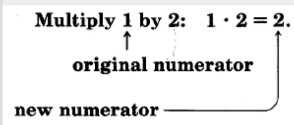
$$\left.\begin{array}{r} 6=2\cdot 3 \\ 4=2^2 \end{array}\right\} \text{The LCD} = 2^2\cdot 3 = 4\cdot 3 = 12$$

Write each of the original fractions as a new, equivalent fraction having the common denominator 12.

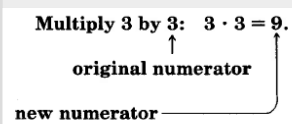
$\frac{1}{6}+\frac{3}{4}=\frac{}{12}+\frac{}{12}$

To find a new numerator, we divide the original denominator into the LCD. Since the original denominator is being multiplied by this quotient, we must multiply the original numerator by this quotient.

$12\div 6=2$



$12\div 4=3$



$$\begin{aligned} \frac{1}{6}+\frac{3}{4} &= \frac{1\cdot 2}{12}+\frac{3\cdot 3}{12} \\ &= \frac{2}{12}+\frac{9}{12} && \text{Now the denominators are the same.} \\ &= \frac{2+9}{12} && \text{Add the numerators and place the sum over the common denominator.} \\ &= \frac{11}{12} \end{aligned}$$

Example:

$\frac{1}{2}+\frac{2}{3}$. The denominators are not the same. Find the LCD of 2 and 3.

$\text{LCD}=2\cdot 3=6$

Write each of the original fractions as a new, equivalent fraction having the common denominator 6.

$\frac{1}{2}+\frac{2}{3}=\frac{}{6}+\frac{}{6}$

To find a new numerator, we divide the original denominator into the LCD. Since the original denominator is being multiplied by this quotient, we must multiply the original numerator by this quotient.

$6\div 2=3$ Multiply the numerator 1 by 3.

$6\div 2=3$ Multiply the numerator 2 by 2.

$$\begin{aligned}\frac{1}{2}+\frac{2}{3} &= \frac{1\cdot 3}{6}+\frac{2\cdot 3}{6}\\ &= \frac{3}{6}+\frac{4}{6}\\ &= \frac{3+4}{6}\\ &= \frac{7}{6} \text{ or } 1\frac{1}{6}\end{aligned}$$

Example:

$\frac{5}{9}-\frac{5}{12}$. The denominators are not the same. Find the LCD of 9 and 12.

$$\left.\begin{aligned}&9 = 3\cdot 3 = 3^2\\ &12 = 2\cdot 6 = 2\cdot 2\cdot 3 = 2^2\cdot 3\end{aligned}\right\} \quad \text{LCD } = 2^2\cdot 3^2 = 4\cdot 9 = 36$$

$\frac{5}{9}-\frac{5}{12} = \frac{}{36}-\frac{}{36}$

$36 \div 9 = 4$ Multiply the numerator 5 by 4.

$36 \div 12 = 3$ Multiply the numerator 5 by 3.

$$\begin{aligned}\frac{5}{9}-\frac{5}{12} &= \frac{5\cdot 4}{36}-\frac{5\cdot 3}{36}\\ &= \frac{20}{36}-\frac{15}{36}\\ &= \frac{20-15}{36}\\ &= \frac{5}{36}\end{aligned}$$

Example:

$\frac{5}{6}-\frac{1}{8}+\frac{7}{16}$ The denominators are not the same. Find the LCD of 6, 8, and 16

$$\left.\begin{aligned}6 &= 2\cdot 3\\ 8 &= 2\cdot 4 = 2\cdot 2\cdot 2 = 2^3\\ 16 &= 2\cdot 8 = 2\cdot 2\cdot 4 = 2\cdot 2\cdot 2\cdot 2 = 2^4\end{aligned}\right\} \text{The LCD is } 2^4\cdot 3 = 48$$

$\frac{5}{6}-\frac{1}{8}+\frac{7}{16} = \frac{}{48}-\frac{}{48}+\frac{}{48}$

$48 \div 6 = 8$ Multiply the numerator 5 by 8

$48 \div 8 = 6$ Multiply the numerator 1 by 6

$48 \div 16 = 3$ Multiply the numerator 7 by 3

$$\begin{aligned}\frac{5}{6}-\frac{1}{8}+\frac{7}{16} &= \frac{5\cdot 8}{48}-\frac{1\cdot 6}{48}+\frac{7\cdot 3}{48}\\ &= \frac{40}{48}-\frac{6}{48}+\frac{21}{48}\\ &= \frac{40-6+21}{48}\\ &= \frac{55}{48} \text{ or } 1\frac{7}{48}\end{aligned}$$

Practice Set A

Find the following sums and differences.

Exercise:

Problem: $\frac{3}{4}+\frac{1}{12}$

Solution:

$\frac{5}{6}$

Exercise:

Problem: $\frac{1}{2} - \frac{3}{7}$

Solution:

$\frac{1}{14}$

Exercise:

Problem: $\frac{7}{10} - \frac{5}{8}$

Solution:

$\frac{3}{40}$

Exercise:

Problem: $\frac{15}{16} + \frac{1}{2} - \frac{3}{4}$

Solution:

$\frac{11}{16}$

Exercise:

Problem: $\frac{1}{32} - \frac{1}{48}$

Solution:

$\frac{1}{96}$

Exercises

Exercise:

Problem:

A most basic rule of arithmetic states that two fractions may be added or subtracted conveniently only if they have .

Solution:

The same denominator

For the following problems, find the sums and differences.

Exercise:

Problem: $\frac{1}{2} + \frac{1}{6}$

Exercise:

Problem: $\frac{1}{8} + \frac{1}{2}$

Solution:

$\frac{5}{8}$

Exercise:

Problem: $\frac{3}{4} + \frac{1}{3}$

Exercise:

Problem: $\frac{5}{8} + \frac{2}{3}$

Solution:

$\frac{31}{24}$

Exercise:

Problem: $\frac{1}{12} + \frac{1}{3}$

Exercise:

Problem: $\frac{6}{7} - \frac{1}{4}$

Solution:

$\frac{17}{28}$

Exercise:

Problem: $\frac{9}{10} - \frac{2}{5}$

Exercise:

Problem: $\frac{7}{9} - \frac{1}{4}$

Solution:

$\frac{19}{36}$

Exercise:

Problem: $\frac{8}{15} - \frac{3}{10}$

Exercise:

Problem: $\frac{8}{13} - \frac{5}{39}$

Solution:

$\frac{19}{39}$

Exercise:

Problem: $\frac{11}{12} - \frac{2}{5}$

Exercise:

Problem: $\frac{1}{15} + \frac{5}{12}$

Solution:

$\frac{29}{60}$

Exercise:

Problem: $\frac{13}{88} - \frac{1}{4}$

Exercise:

Problem: $\frac{1}{9} - \frac{1}{81}$

Solution:

$\frac{8}{81}$

Exercise:

Problem: $\frac{19}{40} + \frac{5}{12}$

Exercise:

Problem: $\frac{25}{26} - \frac{7}{10}$

Solution:

$\frac{17}{65}$

Exercise:

Problem: $\frac{9}{28} - \frac{4}{45}$

Exercise:

Problem: $\frac{22}{45} - \frac{16}{35}$

Solution:

$\frac{2}{63}$

Exercise:

Problem: $\frac{56}{63} + \frac{22}{33}$

Exercise:

Problem: $\frac{1}{16} + \frac{3}{4} - \frac{3}{8}$

Solution:

$\frac{7}{16}$

Exercise:

Problem: $\frac{5}{12} - \frac{1}{120} + \frac{19}{20}$

Exercise:

Problem: $\frac{8}{3} - \frac{1}{4} + \frac{7}{36}$

Solution:

$\frac{47}{18}$

Exercise:

Problem: $\frac{11}{9} - \frac{1}{7} + \frac{16}{63}$

Exercise:

Problem: $\frac{12}{5} - \frac{2}{3} + \frac{17}{10}$

Solution:

$\frac{103}{30}$

Exercise:

Problem: $\frac{4}{9} + \frac{13}{21} - \frac{9}{14}$

Exercise:

Problem: $\frac{3}{4} - \frac{3}{22} + \frac{5}{24}$

Solution:

$\frac{217}{264}$

Exercise:

Problem: $\frac{25}{48} - \frac{7}{88} + \frac{5}{24}$

Exercise:

Problem: $\frac{27}{40} + \frac{47}{48} - \frac{119}{126}$

Solution:

$\frac{511}{720}$

Exercise:

Problem: $\frac{41}{44} - \frac{5}{99} - \frac{11}{175}$

Exercise:

Problem: $\frac{5}{12} + \frac{1}{18} + \frac{1}{24}$

Solution:

$\frac{37}{72}$

Exercise:

Problem: $\frac{5}{9} + \frac{1}{6} + \frac{7}{15}$

Exercise:

Problem: $\frac{21}{25} + \frac{1}{6} + \frac{7}{15}$

Solution:

$\frac{221}{150}$

Exercise:

Problem: $\frac{5}{18} - \frac{1}{36} + \frac{7}{9}$

Exercise:

Problem: $\frac{11}{14} - \frac{1}{36} - \frac{1}{32}$

Solution:

$\frac{1,465}{2,016}$

Exercise:

Problem: $\frac{21}{33} + \frac{12}{22} + \frac{15}{55}$

Exercise:

Problem: $\frac{5}{51} + \frac{2}{34} + \frac{11}{68}$

Solution:

$\frac{65}{204}$

Exercise:

Problem: $\frac{8}{7} - \frac{16}{14} + \frac{19}{21}$

Exercise:

Problem: $\frac{7}{15} + \frac{3}{10} - \frac{34}{60}$

Solution:

$\frac{1}{5}$

Exercise:

Problem: $\frac{14}{15} - \frac{3}{10} - \frac{6}{25} + \frac{7}{20}$

Exercise:

Problem: $\frac{11}{6} - \frac{5}{12} + \frac{17}{30} + \frac{25}{18}$

Solution:

$\frac{607}{180}$

Exercise:

Problem: $\frac{1}{9} + \frac{22}{21} - \frac{5}{18} - \frac{1}{45}$

Exercise:

Problem: $\frac{7}{26} + \frac{28}{65} - \frac{51}{104} + 0$

Solution:

$\frac{109}{520}$

Exercise:

Problem:

A morning trip from San Francisco to Los Angeles took $\frac{13}{12}$ hours. The return trip took $\frac{57}{60}$ hours. How much longer did the morning trip take?

Exercise:

Problem:

At the beginning of the week, Starlight Publishing Company's stock was selling for $\frac{115}{8}$ dollars per share. At the end of the week, analysts had noted that the stock had gone up $\frac{11}{4}$ dollars per share. What was the price of the stock, per share, at the end of the week?

Solution:

$\$\frac{137}{8}\text{or}\$17\frac{1}{8}$

Exercise:

Problem:

A recipe for fruit punch calls for $\frac{23}{3}$ cups of pineapple juice, $\frac{1}{4}$ cup of lemon juice, $\frac{15}{2}$ cups of orange juice, 2 cups of sugar, 6 cups of water, and 8 cups of carbonated non-cola soft drink. How many cups of ingredients will be in the final mixture?

Exercise:

Problem:

The side of a particular type of box measures $8\frac{3}{4}$ inches in length. Is it possible to place three such boxes next to each other on a shelf that is $26\frac{1}{5}$ inches in length? Why or why not?

Solution:

No; 3 boxes add up to $26\frac{1''}{4}$, which is larger than $25\frac{1''}{5}$.

Exercise:

Problem:

Four resistors, $\frac{3}{8}$ ohm, $\frac{1}{4}$ ohm, $\frac{3}{5}$ ohm, and $\frac{7}{8}$ ohm, are connected in series in an electrical circuit. What is the total resistance in the circuit due to these resistors? ("In series" implies addition.)

Exercise:

Problem:

A copper pipe has an inside diameter of $2\frac{3}{16}$ inches and an outside diameter of $2\frac{5}{34}$ inches. How thick is the pipe?

Solution:

No pipe at all; inside diameter is greater than outside diameter

Exercise:

Problem:

The probability of an event was originally thought to be $\frac{15}{32}$. Additional information decreased the probability by $\frac{3}{14}$. What is the updated probability?

Exercises for Review

Exercise:

Problem: ([link]) Find the difference between 867 and 418.

Solution:

449

Exercise:

Problem: ([link]) Is 81,147 divisible by 3?

Exercise:

Problem: ([link]) Find the LCM of 11, 15, and 20.

Solution:

660

Exercise:

Problem: ([link]) Find $\frac{3}{4}$ of $4\frac{2}{9}$.

Exercise:

Problem: ([link]) Find the value of $\frac{8}{15} - \frac{3}{15} + \frac{2}{15}$.

Solution:

$\frac{7}{15}$

Addition and Subtraction of Mixed Numbers
This module is from Fundamentals of Mathematics by Denny Burzynski and Wade Ellis, Jr. This module discusses how to add and subtract mixed numbers. By the end of the module students should be able to add and subtract mixed numbers.

Section Overview

- The Method of Converting to Improper Fractions

To add or subtract mixed numbers, convert each mixed number to an improper fraction, then add or subtract the resulting improper fractions.

Sample Set A

Find the following sums and differences.

Example:
$8\frac{3}{5} + 5\frac{1}{4}$. Convert each mixed number to an improper fraction.
$8\frac{3}{5} = \frac{5 \cdot 8 + 3}{5} = \frac{40+3}{5} = \frac{43}{5}$
$5\frac{1}{4} = \frac{4 \cdot 5 + 1}{4} = \frac{20+1}{4} = \frac{21}{4}$ Now add the improper fractions $\frac{43}{5}$ and $\frac{21}{4}$.
$\frac{43}{5} + \frac{21}{4}$ The LCD = 20.

$$\begin{aligned} \frac{43}{5} + \frac{21}{4} &= \frac{43 \cdot 4}{20} + \frac{21 \cdot 5}{20} \\ &= \frac{172}{20} + \frac{105}{20} \\ &= \frac{172+105}{20} \\ &= \frac{277}{20} && \text{Convert this improper fraction to a mixed number.} \\ &= 13\frac{17}{20} \end{aligned}$$

Thus, $8\frac{3}{5} + 5\frac{1}{4} = 13\frac{17}{20}$.

Example:
$3\frac{1}{8} - \frac{5}{6}$. Convert the mixed number to an improper fraction.
$3\frac{1}{8} = \frac{3 \cdot 8 + 1}{8} = \frac{24+1}{8} = \frac{25}{8}$
$\frac{25}{8} - \frac{5}{6}$ The LCD = 24.

$$\begin{aligned} \frac{25}{8} - \frac{5}{6} &= \frac{25 \cdot 3}{24} - \frac{5 \cdot 4}{24} \\ &= \frac{75}{24} - \frac{20}{24} \\ &= \frac{75-20}{24} \\ &= \frac{55}{24} \qquad \text{Convert his improper fraction to a mixed number.} \\ &= 2\frac{7}{24} \end{aligned}$$

Thus, $3\frac{1}{8} - \frac{5}{6} = 2\frac{7}{24}$.

Practice Set A

Find the following sums and differences.

Exercise:

Problem: $1\frac{5}{9} + 3\frac{2}{9}$

Solution:

$4\frac{7}{9}$

Exercise:

Problem: $10\frac{3}{4} - 2\frac{1}{2}$

Solution:

$8\frac{1}{4}$

Exercise:

Problem: $2\frac{7}{8} + 5\frac{1}{4}$

Solution:

$8\frac{1}{8}$

Exercise:

Problem: $8\frac{3}{5} - \frac{3}{10}$

Solution:

$8\frac{3}{10}$

Exercise:

Problem: $16+2\frac{9}{16}$

Solution:

$18\frac{9}{16}$

Exercises

For the following problems, perform each indicated operation.

Exercise:

Problem: $3\frac{1}{8}+4\frac{3}{8}$

Solution:

$7\frac{1}{2}$

Exercise:

Problem: $5\frac{1}{3}+6\frac{1}{3}$

Exercise:

Problem: $10\frac{5}{12}+2\frac{1}{12}$

Solution:

$12\frac{1}{2}$

Exercise:

Problem: $15\frac{1}{5}-11\frac{3}{5}$

Exercise:

Problem: $9\frac{3}{11}+12\frac{3}{11}$

Solution:

$21\frac{6}{11}$

Exercise:

Problem: $1\frac{1}{6}+3\frac{2}{6}+8\frac{1}{6}$

Exercise:

Problem: $5\frac{3}{8}+1\frac{1}{8}-2\frac{5}{8}$

Solution:

$3\frac{7}{8}$

Exercise:

Problem: $\frac{3}{5}+5\frac{1}{5}$

Exercise:

Problem: $2\frac{2}{9}-\frac{5}{9}$

Solution:

$1\frac{2}{3}$

Exercise:

Problem: $6+11\frac{2}{3}$

Exercise:

Problem: $17-8\frac{3}{14}$

Solution:

$8\frac{11}{14}$

Exercise:

Problem: $5\frac{1}{3} + 2\frac{1}{4}$

Exercise:

Problem: $6\frac{2}{7} - 1\frac{1}{3}$

Solution:

$4\frac{20}{21}$

Exercise:

Problem: $8\frac{2}{5} + 4\frac{1}{10}$

Exercise:

Problem: $1\frac{1}{3} + 12\frac{3}{8}$

Solution:

$13\frac{17}{24}$

Exercise:

Problem: $3\frac{1}{4} + 1\frac{1}{3} - 2\frac{1}{2}$

Exercise:

Problem: $4\frac{3}{4} - 3\frac{5}{6} + 1\frac{2}{3}$

Solution:

$2\frac{7}{12}$

Exercise:

Problem: $3\frac{1}{12} + 4\frac{1}{3} + 1\frac{1}{4}$

Exercise:

Problem: $5\frac{1}{15}+8\frac{3}{10}-5\frac{4}{5}$

Solution:

$7\frac{17}{30}$

Exercise:

Problem: $7\frac{1}{3}+8\frac{5}{6}-2\frac{1}{4}$

Exercise:

Problem: $19\frac{20}{21}+42\frac{6}{7}-\frac{5}{14}+12\frac{1}{7}$

Solution:

$74\frac{25}{42}$

Exercise:

Problem: $\frac{1}{16}+4\frac{3}{4}+10\frac{3}{8}-9$

Exercise:

Problem: $11-\frac{2}{9}+10\frac{1}{3}-\frac{2}{3}-5\frac{1}{6}+6\frac{1}{18}$

Solution:

$21\frac{1}{3}$

Exercise:

Problem: $\frac{5}{2}+2\frac{1}{6}+11\frac{1}{3}-\frac{11}{6}$

Exercise:

Problem: $1\frac{1}{8}+\frac{9}{4}-\frac{1}{16}-\frac{1}{32}+\frac{19}{8}$

Solution:

$5\,\frac{21}{32}$

Exercise:

Problem: $22\,\frac{3}{8} - 16\,\frac{1}{7}$

Exercise:

Problem: $15\,\frac{4}{9} + 4\,\frac{9}{16}$

Solution:

$20\,\frac{1}{144}$

Exercise:

Problem: $4\,\frac{17}{88} + 5\,\frac{9}{110}$

Exercise:

Problem: $6\,\frac{11}{12} + \frac{2}{3}$

Solution:

$7\,\frac{7}{12}$

Exercise:

Problem: $8\,\frac{9}{16} - \frac{7}{9}$

Exercise:

Problem: $5\,\frac{2}{11} - \frac{1}{12}$

Solution:

$5\,\frac{13}{132}$

Exercise:

Problem: $18\,\frac{15}{16} - \frac{33}{34}$

Exercise:

Problem: $1\frac{89}{112} - \frac{21}{56}$

Solution:

$1\frac{47}{212}$

Exercise:

Problem: $11\frac{11}{24} - 7\frac{13}{18}$

Exercise:

Problem: $5\frac{27}{84} - 3\frac{5}{42} + 1\frac{1}{21}$

Solution:

$3\frac{1}{4}$

Exercise:

Problem: $16\frac{1}{48} - 16\frac{1}{96} + \frac{1}{144}$

Exercise:

Problem:

A man pours $2\frac{5}{8}$ gallons of paint from a bucket into a tray. After he finishes pouring, there are $1\frac{1}{4}$ gallons of paint left in his bucket. How much paint did the man pour into the tray?

Note: Think about the wording.

Solution:

$2\frac{5}{8}$ gallons

Exercise:

Problem:

A particular computer stock opened at $37\frac{3}{8}$ and closed at $38\frac{1}{4}$. What was the net gain for this stock?

Exercise:

Problem:

A particular diet program claims that $4\frac{3}{16}$ pounds can be lost the first month, $3\frac{1}{4}$ pounds can be lost the second month, and $1\frac{1}{2}$ pounds can be lost the third month. How many pounds does this diet program claim a person can lose over a 3-month period?

Solution:

$8\frac{15}{16}$ pounds

Exercise:

Problem:

If a person who weighs $145\frac{3}{4}$ pounds goes on the diet program described in the problem above, how much would he weigh at the end of 3 months?

Exercise:

Problem:

If the diet program described in the problem above makes the additional claim that from the fourth month on, a person will lose $1\frac{1}{8}$ pounds a month, how much will a person who begins the program weighing $208\frac{3}{4}$ pounds weight after 8 months?

Solution:

$194\frac{3}{16}$ pounds

Exercises for Review

Exercise:

Problem: ([link]) Use exponents to write $4 \cdot 4 \cdot 4$.

Exercise:

Problem: ([link]) Find the greatest common factor of 14 and 20.

Solution:

2

Exercise:

Problem: ([link]) Convert $\frac{16}{5}$ to a mixed number.

Exercise:

Problem: ([link]) Find the sum. $\frac{4}{9} + \frac{1}{9} + \frac{2}{9}$.

Solution:

$\frac{7}{9}$

Exercise:

Problem: ([link]) Find the difference. $\frac{15}{26} - \frac{3}{10}$.

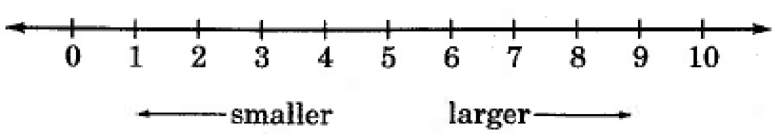
Comparing Fractions
This module is from Fundamentals of Mathematics by Denny Burzynski and Wade Ellis, Jr. This module discusses how to compare fractions. By the end of the module students should be able to understand ordering of numbers and be familiar with grouping symbols and compare two or more fractions.

Section Overview

- Order and the Inequality Symbols
- Comparing Fractions

Order and the Inequality Symbols

Our number system is called an **ordered number system** because the numbers in the system can be placed in order from smaller to larger. This is easily seen on the number line.



On the number line, a number that appears to the right of another number is larger than that other number. For example, 5 is greater than 2 because 5 is located to the right of 2 on the number line. We may also say that 2 is less than 5.

To make the inequality phrases "greater than" and "less than" more brief, mathematicians represent them with the symbols > and <, respectively.

Symbols for Greater Than > and Less Than <
> represents the phrase "greater than."
< represents the phrase "less than."

5 > 2 represents "5 is greater than 2."

2 < 5 represents "2 is less than 5."

Comparing Fractions

Recall that the fraction $\frac{4}{5}$ indicates that we have 4 of 5 parts of some whole quantity, and the fraction $\frac{3}{5}$ indicates that we have 3 of 5 parts. Since 4 of 5 parts is more than 3 of 5 parts, $\frac{4}{5}$ is greater than $\frac{3}{5}$; that is,

$$\frac{4}{5} > \frac{3}{5}$$

We have just observed that when two fractions have the same denominator, we can determine which is larger by comparing the numerators.

Comparing Fractions
If two fractions have the same denominators, the fraction with the larger numerator is the larger fraction.

Thus, to compare the sizes of two or more fractions, we need only convert each of them to equivalent fractions that have a common denominator. We then compare the numerators. It is convenient if the common denominator is the LCD. The fraction with the larger numerator is the larger fraction.

Sample Set A

Example:
Compare $\frac{8}{9}$ and $\frac{14}{15}$.
Convert each fraction to an equivalent fraction with the LCD as the denominator. Find the LCD.

$$\left.\begin{array}{rcl} 9 & = & 3^2 \\ 15 & = & 3 \cdot 5 \end{array}\right\} \text{The LCD} = 3^2 \cdot 5 = 9 \cdot 5 = 45$$

$$\frac{8}{9} = \frac{8 \cdot 5}{45} = \frac{40}{45}$$

$$\frac{14}{15} = \frac{14 \cdot 3}{45} = \frac{42}{45}$$

Since $40 < 42$,

$\frac{40}{45} < \frac{42}{45}$

Thus $\frac{8}{9} < \frac{14}{15}$.

Example:

Write $\frac{5}{6}$, $\frac{7}{10}$, and $\frac{13}{15}$ in order from smallest to largest.

Convert each fraction to an equivalent fraction with the LCD as the denominator.

Find the LCD.

$$\begin{aligned} 6 &= 2 \cdot 3 \\ 10 &= 2 \cdot 5 \quad \text{The LCD} = 2 \cdot 3 \cdot 5 = 30 \\ 15 &= 3 \cdot 5 \end{aligned}$$

$\frac{5}{6} = \frac{5 \cdot 5}{30} = \frac{25}{30}$

$\frac{7}{10} = \frac{7 \cdot 3}{30} = \frac{21}{30}$

$\frac{13}{15} = \frac{13 \cdot 2}{30} = \frac{26}{30}$

Since $21 < 25 < 26$,

$\frac{21}{30} < \frac{25}{30} < \frac{26}{30}$

$\frac{7}{10} < \frac{5}{6} < \frac{13}{15}$

Writing these numbers in order from smallest to largest, we get $\frac{7}{10}$, $\frac{5}{6}$, $\frac{13}{15}$.

Example:

Compare $8\frac{6}{7}$ and $6\frac{3}{4}$.

To compare mixed numbers that have different whole number parts, we need only compare whole number parts. Since 6 < 8,

$6\frac{3}{4} < 8\frac{6}{7}$

Example:

Compare $4\frac{5}{8}$ and $4\frac{7}{12}$

To compare mixed numbers that have the same whole number parts, we need only compare fractional parts.

$$\left.\begin{array}{rcl} 8 & = & 2^3 \\ 12 & = & 2^2 \cdot 3 \end{array}\right\} \text{The LCD} = 2^3 \cdot 3 = 8 \cdot 3 = 24$$

$\frac{5}{8} = \frac{5 \cdot 3}{24} = \frac{15}{24}$

$\frac{7}{12} = \frac{7 \cdot 2}{24} = \frac{14}{24}$

Since 14 < 15,

$\frac{14}{24} < \frac{15}{24}$

$\frac{7}{12} < \frac{5}{8}$

Hence, $4\frac{7}{12} < 4\frac{5}{8}$

Practice Set A

Exercise:

Problem: Compare $\frac{3}{4}$ and $\frac{4}{5}$.

Solution:

$\frac{3}{4} < \frac{4}{5}$

Exercise:

Problem: Compare $\frac{9}{10}$ and $\frac{13}{15}$.

Solution:

$\frac{13}{15} < \frac{9}{10}$

Exercise:

Problem: Write $\frac{13}{16}$, $\frac{17}{20}$, and $\frac{33}{40}$ in order from smallest to largest.

Solution:

$\frac{13}{16}, \frac{33}{40}, \frac{17}{20}$

Exercise:

Problem: Compare $11\frac{1}{6}$ and $9\frac{2}{5}$.

Solution:

$9\frac{2}{5} < 11\frac{1}{6}$

Exercise:

Problem: Compare $1\frac{9}{14}$ and $1\frac{11}{16}$.

Solution:

$1\frac{9}{14} < 1\frac{11}{16}$

Exercises

Arrange each collection of numbers in order from smallest to largest.

Exercise:

Problem: $\frac{3}{5}$, $\frac{5}{8}$

Solution:

$\frac{3}{5} < \frac{5}{8}$

Exercise:

Problem: $\frac{1}{6}$, $\frac{2}{7}$

Exercise:

Problem: $\frac{3}{4}$, $\frac{5}{6}$

Solution:

$\frac{3}{4} < \frac{5}{6}$

Exercise:

Problem: $\frac{7}{9}$, $\frac{11}{12}$

Exercise:

Problem: $\frac{3}{8}$, $\frac{2}{5}$

Solution:

$\frac{3}{8} < \frac{2}{5}$

Exercise:

Problem: $\frac{1}{2}$, $\frac{5}{8}$, $\frac{7}{16}$

Exercise:

Problem: $\frac{1}{2}$, $\frac{3}{5}$, $\frac{4}{7}$

Solution:

$\frac{1}{2} < \frac{4}{7} < \frac{3}{5}$

Exercise:

Problem: $\frac{3}{4}$, $\frac{2}{3}$, $\frac{5}{6}$

Exercise:

Problem: $\frac{3}{4}$, $\frac{7}{9}$, $\frac{5}{4}$

Solution:

$\frac{3}{4} < \frac{7}{9} < \frac{5}{4}$

Exercise:

Problem: $\frac{7}{8}$, $\frac{15}{16}$, $\frac{11}{12}$

Exercise:

Problem: $\frac{3}{14}$, $\frac{2}{7}$, $\frac{3}{4}$

Solution:

$\frac{3}{14} < \frac{2}{7} < \frac{3}{4}$

Exercise:

Problem: $\frac{17}{32}$, $\frac{25}{48}$, $\frac{13}{16}$

Exercise:

Problem: $5\frac{3}{5}$, $5\frac{4}{7}$

Solution:

$5\frac{4}{7} < 5\frac{3}{5}$

Exercise:

Problem: $11\frac{3}{16}$, $11\frac{1}{12}$

Exercise:

Problem: $9\frac{2}{3}$, $9\frac{4}{5}$

Solution:

$9\frac{2}{3} < 9\frac{4}{5}$

Exercise:

Problem: $7\frac{2}{3}$, $8\frac{5}{6}$

Exercise:

Problem: $1\frac{9}{16}$, $2\frac{1}{20}$

Solution:

$1\frac{9}{16} < 2\frac{1}{20}$

Exercise:

Problem: $20\frac{15}{16}$, $20\frac{23}{24}$

Exercise:

Problem: $2\frac{2}{9}$, $2\frac{3}{7}$

Solution:

$2\frac{2}{9} < 2\frac{3}{7}$

Exercise:

Problem: $5\frac{8}{13}$, $5\frac{9}{20}$

Exercises for Review

Exercise:

Problem: ([link]) Round 267,006,428 to the nearest ten million.

Solution:

270,000,000

Exercise:

Problem: ([link]) Is the number 82,644 divisible by 2? by 3? by 4?

Exercise:

Problem: ([link]) Convert $3\frac{2}{7}$ to an improper fraction.

Solution:

$\frac{23}{7}$

Exercise:

Problem: ([link]) Find the value of $\frac{5}{6} + \frac{3}{10} - \frac{2}{5}$

Exercise:

Problem: ([link]) Find the value of $8\frac{3}{8} + 5\frac{1}{4}$.

Solution:

$13\frac{5}{8}\text{or}\frac{109}{8}$

Complex Fractions
This module is from Fundamentals of Mathematics by Denny Burzynski and Wade Ellis, Jr. This module discusses complex fractions. By the end of the module students should be able to distinguish between simple and complex fractions and convert a complex fraction to a simple fraction.

Section Overview

- Simple Fractions and Complex Fractions
- Converting Complex Fractions to Simple Fractions

Simple Fractions and Complex Fractions

Simple Fraction

A **simple fraction** is any fraction in which the numerator is any whole number and the denominator is any nonzero whole number. Some examples are the following:

$\frac{1}{2}$, $\frac{4}{3}$, $\frac{763}{1,000}$

Complex Fraction

A **complex fraction** is any fraction in which the numerator and/or the denominator is a fraction; it is a fraction of fractions. Some examples of complex fractions are the following:

$\frac{\frac{3}{4}}{\frac{5}{6}}$, $\frac{\frac{1}{3}}{2}$, $\frac{6}{\frac{9}{10}}$, $\frac{4+\frac{3}{8}}{7-\frac{5}{6}}$

Converting Complex Fractions to Simple Fractions

The goal here is to convert a complex fraction to a simple fraction. We can do so by employing the methods of adding, subtracting, multiplying, and dividing fractions. Recall from [link] that a fraction bar serves as a grouping symbol separating the fractional quantity into two individual groups. We proceed in simplifying a complex fraction to a simple fraction by simplifying the numerator and the denominator of the complex fraction separately. We will simplify the numerator and denominator *completely* before removing the

fraction bar by dividing. This technique is illustrated in problems 3, 4, 5, and 6 of [link].

Sample Set A

Convert each of the following complex fractions to a simple fraction.

Example:

$$\frac{\frac{3}{8}}{\frac{15}{16}}$$

Convert this complex fraction to a simple fraction by performing the indicated division.

$$\frac{\frac{3}{8}}{\frac{15}{16}} = \frac{3}{8} \div \frac{15}{16} \qquad \text{The divisor is } \frac{15}{16}\text{. Invert } \frac{15}{16} \text{ and multiply.}$$

$$= \frac{\overset{1}{\cancel{3}}}{\underset{1}{\cancel{8}}} \cdot \frac{\overset{2}{\cancel{16}}}{\underset{5}{\cancel{15}}} = \frac{1 \cdot 2}{1 \cdot 5} = \frac{2}{5}$$

Example:

$$\frac{\frac{4}{9}}{6} \qquad \text{Write 6 as } \frac{6}{1} \text{ and divide.}$$

$$\frac{\frac{4}{9}}{\frac{6}{1}} = \frac{4}{9} \div \frac{6}{1}$$

$$= \frac{\overset{2}{\cancel{4}}}{9} \cdot \frac{1}{\underset{3}{\cancel{6}}} = \frac{2 \cdot 1}{9 \cdot 3} = \frac{2}{27}$$

Example:

$$\frac{5+\frac{3}{4}}{46} \qquad \text{Simplify the numerator.}$$

$$\frac{\frac{4 \cdot 5+3}{4}}{46} = \frac{\frac{20+3}{4}}{46} = \frac{\frac{23}{4}}{46} \qquad \text{Write 46 as } \frac{46}{1}.$$

$$\frac{\frac{23}{4}}{\frac{46}{1}} = \frac{23}{4} \div \frac{46}{1}$$

$$= \frac{\overset{1}{\cancel{23}}}{4} \cdot \frac{1}{\underset{2}{\cancel{46}}} = \frac{1 \cdot 1}{4 \cdot 2} = \frac{1}{8}$$

Example:

$$\frac{\frac{1}{4}+\frac{3}{8}}{\frac{1}{2}+\frac{13}{24}} = \frac{\frac{2}{8}+\frac{3}{8}}{\frac{12}{24}+\frac{13}{24}} = \frac{\frac{2+3}{8}}{\frac{12+13}{24}} = \frac{\frac{5}{8}}{\frac{25}{24}} = \frac{5}{8} \div \frac{25}{24}$$

$$\frac{5}{8} \div \frac{25}{24} = \frac{\overset{1}{\cancel{5}}}{\underset{1}{\cancel{8}}} \cdot \frac{\overset{3}{\cancel{24}}}{\underset{5}{\cancel{25}}} = \frac{1 \cdot 3}{1 \cdot 5} = \frac{3}{5}$$

Example:

$$\frac{4+\frac{5}{6}}{7-\frac{1}{3}} = \frac{\frac{4 \cdot 6+5}{6}}{\frac{7 \cdot 3-1}{3}} = \frac{\frac{29}{6}}{\frac{20}{3}} = \frac{29}{6} \div \frac{20}{3}$$

$$= \frac{29}{\underset{2}{\cancel{6}}} \cdot \frac{\overset{1}{\cancel{3}}}{20} = \frac{29}{40}$$

Example:

$$\frac{11+\frac{3}{10}}{4\frac{4}{5}} = \frac{\frac{11 \cdot 10+3}{10}}{\frac{4 \cdot 5+4}{5}} = \frac{\frac{110+3}{10}}{\frac{20+4}{5}} = \frac{\frac{113}{10}}{\frac{24}{5}} = \frac{113}{10} \div \frac{24}{5}$$

$$\frac{113}{10} \div \frac{24}{5} = \frac{113}{\underset{2}{\cancel{10}}} \cdot \frac{\overset{1}{\cancel{5}}}{24} = \frac{113 \cdot 1}{2 \cdot 24} = \frac{113}{48} = 2\frac{17}{48}$$

Practice Set A

Convert each of the following complex fractions to a simple fraction.

Exercise:

Problem: $\frac{\frac{4}{9}}{\frac{8}{15}}$

Solution:

$\frac{5}{6}$

Exercise:

Problem: $\frac{\frac{7}{10}}{28}$

Solution:

$\frac{1}{40}$

Exercise:

Problem: $\frac{5+\frac{2}{5}}{3+\frac{3}{5}}$

Solution:

$\frac{3}{2}$

Exercise:

Problem: $\frac{\frac{1}{8}+\frac{7}{8}}{6-\frac{3}{10}}$

Solution:

$\frac{10}{57}$

Exercise:

Problem: $\frac{\frac{1}{6}+\frac{5}{8}}{\frac{5}{9}-\frac{1}{4}}$

Solution:

$2\frac{13}{22}$

Exercise:

Problem: $\frac{16-10\frac{2}{3}}{11\frac{5}{6}-7\frac{7}{6}}$

Solution:

$1\frac{5}{11}$

Exercises

Simplify each fraction.

Exercise:

Problem: $\frac{\frac{3}{5}}{\frac{9}{15}}$

Solution:

1

Exercise:

Problem: $\frac{\frac{1}{3}}{\frac{1}{9}}$

Exercise:

Problem: $\frac{\frac{1}{4}}{\frac{5}{12}}$

Solution:

$\frac{3}{5}$

Exercise:

Problem: $\frac{\frac{8}{9}}{\frac{4}{15}}$

Exercise:

Problem: $\frac{6+\frac{1}{4}}{11+\frac{1}{4}}$

Solution:

$\frac{5}{9}$

Exercise:

Problem: $\frac{2+\frac{1}{2}}{7+\frac{1}{2}}$

Exercise:

Problem: $\frac{5+\frac{1}{3}}{2+\frac{2}{15}}$

Solution:

$\frac{5}{2}$

Exercise:

Problem: $\frac{9+\frac{1}{2}}{1+\frac{8}{11}}$

Exercise:

Problem: $\frac{4+\frac{10}{13}}{\frac{12}{39}}$

Solution:

$\frac{31}{2}$

Exercise:

Problem: $\frac{\frac{1}{3}+\frac{2}{7}}{\frac{26}{21}}$

Exercise:

Problem: $\frac{\frac{5}{6}-\frac{1}{4}}{\frac{1}{12}}$

Solution:

7

Exercise:

Problem: $\frac{\frac{3}{10}+\frac{4}{12}}{\frac{19}{90}}$

Exercise:

Problem: $\frac{\frac{9}{16}+\frac{7}{3}}{\frac{139}{48}}$

Solution:

1

Exercise:

Problem: $\frac{\frac{1}{288}}{\frac{8}{9}-\frac{3}{16}}$

Exercise:

Problem: $\dfrac{\frac{27}{429}}{\frac{5}{11}-\frac{1}{13}}$

Solution:

$\frac{1}{6}$

Exercise:

Problem: $\dfrac{\frac{1}{3}+\frac{2}{5}}{\frac{3}{5}+\frac{17}{45}}$

Exercise:

Problem: $\dfrac{\frac{9}{70}+\frac{5}{42}}{\frac{13}{30}-\frac{1}{21}}$

Solution:

$\frac{52}{81}$

Exercise:

Problem: $\dfrac{\frac{1}{16}+\frac{1}{14}}{\frac{2}{3}-\frac{13}{60}}$

Exercise:

Problem: $\dfrac{\frac{3}{20}+\frac{11}{12}}{\frac{19}{7}-1\frac{11}{35}}$

Solution:

$\frac{16}{21}$

Exercise:

Problem: $\frac{2\frac{2}{3}-1\frac{1}{2}}{\frac{1}{4}+1\frac{1}{16}}$

Exercise:

Problem: $\frac{3\frac{1}{5}+3\frac{1}{3}}{\frac{6}{5}-\frac{15}{63}}$

Solution:

$\frac{686}{101}$

Exercise:

Problem: $\frac{\frac{1\frac{1}{2}+15}{5\frac{1}{4}-3\frac{5}{12}}}{\frac{8\frac{1}{3}-4\frac{1}{2}}{11\frac{2}{3}-5\frac{11}{12}}}$

Exercise:

Problem: $\frac{\frac{5\frac{3}{4}+3\frac{1}{5}}{2\frac{1}{5}+15\frac{7}{10}}}{\frac{9\frac{1}{2}-4\frac{1}{6}}{\frac{1}{8}+2\frac{1}{120}}}$

Solution:

$\frac{1}{3}$

Exercises for Review

Exercise:

Problem: ([link]) Find the prime factorization of 882.

Exercise:

Problem: ([link]) Convert $\frac{62}{7}$ to a mixed number.

Solution:

$8\frac{6}{7}$

Exercise:

Problem: ([link]) Reduce $\frac{114}{342}$ to lowest terms.

Exercise:

Problem: ([link]) Find the value of $6\frac{3}{8} - 4\frac{5}{6}$.

Solution:

$1\frac{13}{24}$ or $\frac{37}{24}$

Exercise:

Problem: ([link]) Arrange from smallest to largest: $\frac{1}{2}, \frac{3}{5}, \frac{4}{7}$.

Combinations of Operations with Fractions
This module is from Fundamentals of Mathematics by Denny Burzynski and Wade Ellis, Jr. This module discusses combinations of operations with fractions. By the end of the module students should gain a further understanding of the order of operations.

Section Overview

- The Order of Operations

The Order of Operations

To determine the value of a quantity such as

$$\frac{1}{2}+\frac{5}{8}\cdot\frac{2}{15}$$

where we have a combination of operations (more than one operation occurs), we must use the accepted order of operations.
The Order of Operations:

1. In the order (2), (3), (4) described below, perform all operations inside grouping symbols: ( ), [ ], ( ), —. Work from the innermost set to the outermost set.
2. Perform exponential and root operations.
3. Perform all multiplications and divisions moving left to right.
4. Perform all additions and subtractions moving left to right.

Sample Set A

Determine the value of each of the following quantities.

Example:
$$\frac{1}{4}+\frac{5}{8}\cdot\frac{2}{15}$$

a. Multiply first.

$$\frac{1}{4} + \frac{\overset{1}{\cancel{5}}}{\underset{4}{\cancel{8}}} \cdot \frac{\overset{1}{\cancel{2}}}{\underset{3}{\cancel{15}}} = \frac{1}{4} + \frac{1 \cdot 1}{4 \cdot 3} = \frac{1}{4} + \frac{1}{12}$$

b. Now perform this addition. Find the LCD.

$$\left.\begin{array}{r} 4 = 2^2 \\ 12 = 2^2 \cdot 3 \end{array}\right\} \text{The LCD} = 2^2 \cdot 3 = 12.$$

$$\begin{aligned} \frac{1}{4} + \frac{1}{12} &= \frac{1 \cdot 3}{12} + \frac{1}{12} = \frac{3}{12} + \frac{1}{12} \\ &= \frac{3+1}{12} = \frac{4}{12} = \frac{1}{3} \end{aligned}$$

Thus, $\frac{1}{4} + \frac{5}{8} \cdot \frac{2}{15} = \frac{1}{3}$

Example:

$\frac{3}{5} + \frac{9}{44}\left(\frac{5}{9} - \frac{1}{4}\right)$

a. Operate within the parentheses first, $\left(\frac{5}{9} - \frac{1}{4}\right)$.

$$\left.\begin{array}{r} 9 = 3^2 \\ 4 = 2^2 \end{array}\right\} \text{The LCD} = 2^2 \cdot 3^2 = 4 \cdot 9 = 36.$$

$$\frac{5 \cdot 4}{36} - \frac{1 \cdot 9}{36} = \frac{20}{36} - \frac{9}{36} = \frac{20-9}{36} = \frac{11}{36}$$

Now we have

$\frac{3}{5} + \frac{9}{44}\left(\frac{11}{36}\right)$

b. Perform the multiplication.

$$\frac{3}{5} + \frac{\overset{1}{\cancel{9}}}{\underset{4}{\cancel{44}}} \cdot \frac{\overset{1}{\cancel{11}}}{\underset{4}{\cancel{36}}} = \frac{3}{5} + \frac{1 \cdot 1}{4 \cdot 4} = \frac{3}{5} + \frac{1}{16}$$

c. Now perform the addition. The LCD=80.

$$\frac{3}{5} + \frac{1}{16} = \frac{3 \cdot 16}{80} + \frac{1 \cdot 5}{80} = \frac{48}{80} + \frac{5}{80} = \frac{48+5}{80} = \frac{53}{80}$$

Thus, $\frac{3}{5} + \frac{9}{44}\left(\frac{5}{9} - \frac{1}{4}\right) = \frac{53}{80}$

Example:

$$8 - \frac{15}{426}\left(2 - 1\frac{4}{15}\right)\left(3\frac{1}{5} + 2\frac{1}{8}\right)$$

a. Work within each set of parentheses individually.

$$\begin{aligned}
2 - 1\frac{4}{15} &= 2\frac{1 \cdot 15 + 4}{15} = 2 - \frac{19}{15} \\
&= \frac{30}{15} - \frac{19}{15} = \frac{30-19}{15} = \frac{11}{15} \\
3\frac{1}{5} + 2\frac{1}{8} &= \frac{3 \cdot 5 + 1}{5} + \frac{2 \cdot 8 + 1}{8} \\
&= \frac{16}{5} + \frac{17}{8} \text{LCD} = 40 \\
&= \frac{16 \cdot 8}{40} + \frac{17 \cdot 5}{40} \\
&= \frac{128}{40} + \frac{85}{40} \\
&= \frac{128+85}{40} \\
&= \frac{213}{40}
\end{aligned}$$

Now we have

$$8 - \frac{15}{426}\left(\frac{11}{15}\right)\left(\frac{213}{40}\right)$$

b. Now multiply.

$$8-\frac{\cancelto{1}{15}}{\cancelto{2}{426}}\cdot\frac{11}{\cancelto{1}{15}}\cdot\frac{\cancelto{1}{213}}{40}=8-\frac{1\cdot 11\cdot 1}{2\cdot 1\cdot 40}=8-\frac{11}{80}$$

c. Now subtract.

$$8-\frac{11}{80}=\frac{80\cdot 8}{80}-\frac{11}{80}=\frac{640}{80}-\frac{11}{80}=\frac{640-11}{80}=\frac{629}{80}\text{ or }7\frac{69}{80}$$

Thus, $8-\frac{15}{426}\left(2-1\frac{4}{15}\right)\left(3\frac{1}{5}+2\frac{1}{8}\right)=7\frac{69}{80}$

Example:

$\left(\frac{3}{4}\right)^2\cdot\frac{8}{9}-\frac{5}{12}$

a. Square $\frac{3}{4}$.

$$\left(\frac{3}{4}\right)^2=\frac{3}{4}\cdot\frac{3}{4}=\frac{3\cdot 3}{4\cdot 4}=\frac{9}{16}$$

Now we have

$$\frac{9}{16}\cdot\frac{8}{9}-\frac{5}{12}$$

b. Perform the multiplication.

$$\frac{\cancelto{1}{9}}{\cancelto{2}{16}}\cdot\frac{\cancelto{1}{8}}{\cancelto{1}{9}}-\frac{5}{12}=\frac{1\cdot 1}{2\cdot 1}-\frac{5}{12}=\frac{1}{2}-\frac{5}{12}$$

c. Now perform the subtraction.

$$\frac{1}{2}-\frac{5}{12}=\frac{6}{12}-\frac{5}{12}=\frac{6-5}{12}=\frac{1}{12}$$

Thus, $\left(\frac{4}{3}\right)^2\cdot\frac{8}{9}-\frac{5}{12}=\frac{1}{12}$

Example:

$$2\frac{7}{8}+\sqrt{\frac{25}{36}}\div\left(2\frac{1}{2}-1\frac{1}{3}\right)$$

a. Begin by operating inside the parentheses.

$$\begin{aligned}2\frac{1}{2}-1\frac{1}{3} &= \frac{2\cdot2+1}{2}-\frac{1\cdot3+1}{3}=\frac{5}{2}-\frac{4}{3}\\ &= \frac{15}{6}-\frac{8}{6}=\frac{15-8}{6}=\frac{7}{6}\end{aligned}$$

b. Now simplify the square root.

$$\sqrt{\frac{25}{36}}=\frac{5}{6}\left(\text{since}\left(\frac{5}{6}\right)^2=\frac{25}{36}\right)$$

Now we have

$$2\frac{7}{8}+\frac{5}{6}\div\frac{7}{6}$$

c. Perform the division.

$$2\frac{7}{8}+\frac{5}{\cancel{6}_1}\cdot\frac{\cancel{6}^1}{7}=2\frac{7}{8}+\frac{5\cdot1}{1\cdot7}=2\frac{7}{8}+\frac{5}{7}$$

d. Now perform the addition.

$$\begin{aligned}2\frac{7}{8}+\frac{5}{7} &= \frac{2\cdot8+7}{8}+\frac{5}{7}=\frac{23}{8}+\frac{5}{7}\quad \text{LCD}=56.\\ &= \frac{23\cdot7}{56}+\frac{5\cdot8}{56}=\frac{161}{56}+\frac{40}{56}\\ &= \frac{161+40}{56}=\frac{201}{56}\text{ or }3\frac{33}{56}\end{aligned}$$

Thus, $2\frac{7}{8}+\sqrt{\frac{25}{36}}\div\left(2\frac{1}{2}-1\frac{1}{3}\right)=3\frac{33}{56}$

Practice Set A

Find the value of each of the following quantities.

Exercise:

Problem: $\frac{5}{16} \cdot \frac{1}{10} - \frac{1}{32}$

Solution:

0

Exercise:

Problem: $\frac{6}{7} \cdot \frac{21}{40} \div \frac{9}{10} + 5\frac{1}{3}$

Solution:

$\frac{35}{6}$ or $5\frac{5}{6}$

Exercise:

Problem: $8\frac{7}{10} - 2\left(4\frac{1}{2} - 3\frac{2}{3}\right)$

Solution:

$\frac{211}{30}$ or $7\frac{1}{30}$

Exercise:

Problem: $\frac{17}{18} - \frac{58}{30}\left(\frac{1}{4} - \frac{3}{32}\right)\left(1 - \frac{13}{29}\right)$

Solution:

$\frac{7}{9}$

Exercise:

Problem: $\left(\frac{1}{10} + 1\frac{1}{2}\right) \div \left(1\frac{4}{5} - 1\frac{6}{25}\right)$

Solution:

$2\,\frac{6}{7}$

Exercise:

Problem: $\frac{\frac{2}{3}-\frac{3}{8}\cdot\frac{4}{9}}{\frac{7}{16}\cdot 1\frac{1}{3}+1\frac{1}{4}}$

Solution:

$\frac{3}{11}$

Exercise:

Problem: $\left(\frac{3}{8}\right)^2+\frac{3}{4}\cdot\frac{1}{8}$

Solution:

$\frac{15}{64}$

Exercise:

Problem: $\frac{2}{3}\cdot 2\frac{1}{4}-\sqrt{\frac{4}{25}}$

Solution:

$\frac{11}{10}$

Exercises

Find each value.

Exercise:

Problem: $\frac{4}{3}-\frac{1}{6}\cdot\frac{1}{2}$

Solution:

$\frac{5}{4}$

Exercise:

Problem: $\frac{7}{9} - \frac{4}{5} \cdot \frac{5}{36}$

Exercise:

Problem: $2\frac{2}{7} + \frac{5}{8} \div \frac{5}{16}$

Solution:

$4\frac{2}{7}$

Exercise:

Problem: $\frac{3}{16} \div \frac{9}{14} \cdot \frac{12}{21} + \frac{5}{6}$

Exercise:

Problem: $\frac{4}{25} \div \frac{8}{15} - \frac{7}{20} \div 2\frac{1}{10}$

Solution:

$\frac{2}{15}$

Exercise:

Problem: $\frac{2}{5} \cdot \left(\frac{1}{19} + \frac{3}{38}\right)$

Exercise:

Problem: $\frac{3}{7} \cdot \left(\frac{3}{10} - \frac{1}{15}\right)$

Solution:

$\frac{1}{10}$

Exercise:

Problem: $\frac{10}{11} \cdot \left(\frac{8}{9} - \frac{2}{5}\right) + \frac{3}{25} \cdot \left(\frac{5}{3} + \frac{1}{4}\right)$

Exercise:

Problem: $\frac{2}{7} \cdot \left(\frac{6}{7} - \frac{3}{28}\right) + 5\frac{1}{3} \cdot \left(1\frac{1}{4} - \frac{1}{8}\right)$

Solution:

$6\frac{3}{14}$

Exercise:

Problem: $\frac{\left(\frac{6}{11} - \frac{1}{3}\right) \cdot \left(\frac{1}{21} + 2\frac{13}{42}\right)}{1\frac{1}{5} + \frac{7}{40}}$

Exercise:

Problem: $\left(\frac{1}{2}\right)^2 + \frac{1}{8}$

Solution:

$\frac{3}{8}$

Exercise:

Problem: $\left(\frac{3}{5}\right)^2 - \frac{3}{10}$

Exercise:

Problem: $\sqrt{\frac{36}{81}} + \frac{1}{3} \cdot \frac{2}{9}$

Solution:

$\frac{20}{27}$

Exercise:

Problem: $\sqrt{\frac{49}{64}} - \sqrt{\frac{9}{4}}$

Exercise:

Problem: $\frac{2}{3} \cdot \sqrt{\frac{9}{4}} - \frac{15}{4} \cdot \sqrt{\frac{16}{225}}$

Solution:

0

Exercise:

Problem: $\left(\frac{3}{4}\right)^2 + \sqrt{\frac{25}{16}}$

Exercise:

Problem: $\left(\frac{1}{3}\right)^2 \cdot \sqrt{\frac{81}{25}} + \frac{1}{40} \div \frac{1}{8}$

Solution:

$\frac{2}{5}$

Exercise:

Problem: $\left(\sqrt{\frac{4}{49}}\right)^2 + \frac{3}{7} \div 1\frac{3}{4}$

Exercise:

Problem: $\left(\sqrt{\frac{100}{121}}\right)^2 + \frac{21}{(11)^2}$

Solution:

1

Exercise:

Problem: $\sqrt{\frac{3}{8}+\frac{1}{64}}-\frac{1}{2}\div 1\frac{1}{3}$

Exercise:

Problem: $\sqrt{\frac{1}{4}}\cdot\left(\frac{5}{6}\right)^2+\frac{9}{14}\cdot 2\frac{1}{3}-\sqrt{\frac{1}{81}}$

Solution:

$\frac{125}{72}$

Exercise:

Problem: $\sqrt{\frac{1}{9}}\cdot\sqrt{\frac{6\frac{3}{8}+2\frac{5}{8}}{16}}+7\frac{7}{10}$

Exercise:

Problem: $\frac{3\frac{3}{4}+\frac{4}{5}\cdot\left(\frac{1}{2}\right)^3}{\frac{67}{240}+\left(\frac{1}{3}\right)^4\cdot\left(\frac{9}{10}\right)}$

Solution:

$\frac{252}{19}$

Exercise:

Problem: $\sqrt{\sqrt{\frac{16}{81}}+\frac{1}{4}\cdot 6}$

Exercise:

Problem: $\sqrt{\sqrt{\frac{81}{256}}} - \frac{3}{32} \cdot 1\frac{1}{8}$

Solution:

$\frac{165}{256}$

Exercises for Review

Exercise:

Problem:

([link]) True or false: Our number system, the Hindu-Arabic number system, is a positional number system with base ten.

Exercise:

Problem:

([link]) The fact that 1 times any whole number = that particular whole number illustrates which property of multiplication?

Solution:

multiplicative identity

Exercise:

Problem: ([link]) Convert $8\frac{6}{7}$ to an improper fraction.

Exercise:

Problem: ([link]) Find the sum. $\frac{3}{8} + \frac{4}{5} + \frac{5}{6}$.

Solution:

$\frac{241}{120}$ or $2\frac{1}{120}$

Exercise:

Problem: ([link]) Simplify $\frac{6+\frac{1}{8}}{6-\frac{1}{8}}$.

Summary of Key Concepts
This module is from Fundamentals of Mathematics by Denny Burzynski and Wade Ellis, Jr. This module reviews the key concepts from the chapter "Addition and Subtraction of Fractions, Comparing Fractions, and Complex Fractions."

Summary of Key Concepts

Addition and Subtraction of Fractions with Like Denominators ([link])
To *add or subtract two fractions that have the same denominators*, add or subtract the numerators and place the resulting sum or difference over the common denominator. Reduce, if necessary. Do not add or subtract the denominators.

$\frac{1}{8}+\frac{5}{8}=\frac{1+5}{8}=\frac{6}{8}=\frac{3}{4}$

Basic Rule for Adding and Subtracting Fractions ([link])
Fractions can be added or subtracted conveniently only if they have like denominators.

Addition and Subtraction of Fractions with Unlike Denominators ([link])
To *add or subtract fractions having unlike denominators*, convert each fraction to an equivalent fraction having as denominator the LCD of the original denominators.

Addition and Subtraction of Mixed Numbers ([link])
1. To *add or subtract mixed numbers*, convert each mixed number to an improper fraction, then add or subtract the fractions.

Ordered Number System ([link])
Our number system is *ordered* because the numbers in the system can be placed in order from smaller to larger.

Inequality Symbols ([link])
> represents the phrase "greater than."
< represents the phrase "less than."

Comparing Fractions ([link])
If two fractions have the same denominators, the fraction with the larger numerator is the larger fraction.

$\frac{5}{8} > \frac{3}{8}$

Simple Fractions ([link])
A *simple fraction* is any fraction in which the numerator is any whole number and the denominator is any nonzero whole number.

Complex Fractions ([link])
A *complex fraction* is any fraction in which the numerator and/or the denominator is a fraction.

Complex fractions can be converted to simple fractions by employing the methods of adding, subtracting, multiplying, and dividing fractions.

Exercise Supplement
This module is from Fundamentals of Mathematics by Denny Burzynski and Wade Ellis, Jr. This module is an exercise supplement for the chapter "Addition and Subtraction of Fractions, Comparing Fractions, and Complex Fractions" and contains many exercise problems. Odd problems are accompanied by solutions.

Exercise Supplement

Addition and Subtractions of Fractions with Like and Unlike Denominators, and Addition and Subtraction of Mixed Numbers ([link], [link], [link])

For problems 1-53, perform each indicated operation and write the result in simplest form.

Exercise:

Problem: $\frac{3}{4} + \frac{5}{8}$

Solution:

$\frac{11}{8}$ or $1\frac{3}{8}$

Exercise:

Problem: $\frac{9}{16} + \frac{1}{4}$

Exercise:

Problem: $\frac{1}{8} + \frac{3}{8}$

Solution:

$\frac{1}{2}$

Exercise:

Problem: $\frac{5}{7} + \frac{1}{14} + \frac{5}{21}$

Exercise:

Problem: $\frac{5}{6} + \frac{1}{3} + \frac{5}{21}$

Solution:

$\frac{59}{42} = 1\frac{17}{42}$

Exercise:

Problem: $\frac{2}{5} + \frac{1}{8}$

Exercise:

Problem: $\frac{1}{4} + \frac{1}{8} + \frac{1}{4}$

Solution:

$\frac{5}{8}$

Exercise:

Problem: $\frac{1}{16} + \frac{1}{10}$

Exercise:

Problem: $\frac{2}{7} + \frac{1}{3}$

Solution:

$\frac{13}{21}$

Exercise:

Problem: $2\frac{1}{3}+\frac{1}{6}$

Exercise:

Problem: $3\frac{11}{16}+\frac{3}{4}$

Solution:

$4\frac{7}{16}$

Exercise:

Problem: $5\frac{1}{12}+3\frac{1}{8}$

Exercise:

Problem: $16\frac{2}{5}+8\frac{1}{4}$

Solution:

$8\frac{3}{20}$

Exercise:

Problem: $1\frac{1}{7}+2\frac{4}{7}$

Exercise:

Problem: $1\frac{3}{8}+0$

Solution:

$1\frac{3}{8}$

Exercise:

Problem: $3\frac{1}{10}+4$

Exercise:

Problem: $18\frac{2}{3}+6$

Solution:

$24\frac{2}{3}$

Exercise:

Problem: $1\frac{4}{3}+5\frac{5}{4}$

Exercise:

Problem: $\frac{21}{4}+\frac{2}{3}$

Solution:

$\frac{71}{12}=5\frac{11}{12}$

Exercise:

Problem: $\frac{15}{16}-\frac{1}{8}$

Exercise:

Problem: $\frac{9}{11}-\frac{5}{22}$

Solution:

$\frac{13}{22}$

Exercise:

Problem: $6\frac{2}{15} - 1\frac{3}{10}$

Exercise:

Problem: $5\frac{2}{3} + 8\frac{1}{5} - 2\frac{1}{4}$

Solution:

$11\frac{37}{60}$

Exercise:

Problem: $8\frac{3}{10} - 4\frac{5}{6} - 3\frac{1}{15}$

Exercise:

Problem: $\frac{11}{12} + \frac{1}{9} - \frac{1}{16}$

Solution:

$\frac{139}{144}$

Exercise:

Problem: $7\frac{2}{9} - 5\frac{5}{6} - 1\frac{1}{3}$

Exercise:

Problem: $16\frac{2}{5} - 8\frac{1}{6} - 3\frac{2}{15}$

Solution:

$5\frac{1}{10}$

Exercise:

Problem: $4\frac{1}{8}+0-\frac{32}{8}$

Exercise:

Problem: $4\frac{1}{8}+0-\frac{32}{8}$

Solution:

$\frac{1}{8}$

Exercise:

Problem: $8-2\frac{1}{3}$

Exercise:

Problem: $4-3\frac{5}{16}$

Solution:

$\frac{11}{16}$

Exercise:

Problem: $6\frac{3}{7}+4$

Exercise:

Problem: $11\frac{2}{11}-3$

Solution:

$8\frac{2}{11}$

Exercise:

Problem: $21\frac{5}{8} - \frac{5}{8}$

Exercise:

Problem: $\frac{3}{4} + \frac{5}{16} \cdot \frac{4}{5}$

Solution:

1

Exercise:

Problem: $\frac{11}{12} + \frac{15}{16} \div 2\frac{1}{2}$

Exercise:

Problem: $1\frac{3}{10} + 2\frac{2}{3} \div \frac{4}{9}$

Solution:

$7\frac{3}{10}$

Exercise:

Problem: $8\frac{3}{5} - 1\frac{1}{14} \cdot \frac{3}{7}$

Exercise:

Problem: $2\frac{3}{8} \div 3\frac{9}{16} - \frac{1}{9}$

Solution:

$\frac{5}{9}$

Exercise:

Problem: $15\frac{2}{5} \div 50 - \frac{1}{10}$

Complex Fractions and Combinations of Operations with Fractions ([link],[link])

Exercise:

Problem: $\frac{\frac{9}{16}}{\frac{21}{32}}$

Solution:

$\frac{6}{7}$

Exercise:

Problem: $\frac{\frac{10}{21}}{\frac{11}{14}}$

Exercise:

Problem: $\frac{1\frac{7}{9}}{1\frac{5}{27}}$

Solution:

$\frac{3}{2}$ or $1\frac{1}{2}$

Exercise:

Problem: $\frac{\frac{15}{17}}{\frac{50}{51}}$

Exercise:

Problem: $\frac{1\frac{9}{16}}{2\frac{11}{12}}$

Solution:

$\frac{15}{28}$

Exercise:

Problem: $\frac{8\frac{4}{15}}{3}$

Exercise:

Problem: $\frac{9\frac{1}{18}}{6}$

Solution:

$\frac{163}{108}$ or $1\frac{55}{108}$

Exercise:

Problem: $\frac{3\frac{1}{4}+2\frac{1}{8}}{5\frac{1}{6}}$

Exercise:

Problem: $\frac{3+2\frac{1}{2}}{\frac{1}{4}+\frac{5}{6}}$

Solution:

$\frac{66}{13}$ or $5\frac{1}{13}$

Exercise:

Problem: $\frac{4+1\frac{7}{10}}{9-2\frac{1}{5}}$

Exercise:

Problem: $\frac{1\frac{2}{5}}{9-\frac{2}{2}}$

Solution:

$\frac{7}{40}$

Exercise:

Problem: $\frac{1\frac{2}{3}\cdot\left(\frac{1}{4}+\frac{1}{5}\right)}{1\frac{1}{2}}$

Exercise:

Problem: $\frac{\frac{10}{23}\cdot\left(\frac{5}{6}+2\right)}{\frac{8}{9}}$

Solution:

$\frac{255}{184}$ or $1\frac{71}{184}$

Comparing Fractions ([link])

For problems 54-65, place each collection in order from smallest to largest.

Exercise:

Problem: $\frac{1}{8}, \frac{3}{16}$

Exercise:

Problem: $\frac{3}{32}, \frac{1}{8}$

Solution:

$\frac{3}{32}, \frac{1}{8}$

Exercise:

Problem: $\frac{5}{16}, \frac{3}{24}$

Exercise:

Problem: $\frac{3}{10}, \frac{5}{6}$

Solution:

$\frac{3}{10}, \frac{5}{6}$

Exercise:

Problem: $\frac{2}{9}, \frac{1}{3}, \frac{1}{6}$

Exercise:

Problem: $\frac{3}{8}, \frac{8}{3}, \frac{19}{6}$

Solution:

$\frac{3}{8}, \frac{8}{3}, \frac{19}{6}$

Exercise:

Problem: $\frac{3}{5}, \frac{2}{10}, \frac{7}{20}$

Exercise:

Problem: $\frac{4}{7}$, $\frac{5}{9}$

Solution:

$\frac{5}{9}$, $\frac{4}{7}$

Exercise:

Problem: $\frac{4}{5}$, $\frac{5}{7}$

Exercise:

Problem: $\frac{5}{12}$, $\frac{4}{9}$, $\frac{7}{15}$

Solution:

$\frac{5}{12}$, $\frac{4}{9}$, $\frac{7}{15}$

Exercise:

Problem: $\frac{7}{36}$, $\frac{1}{24}$, $\frac{5}{12}$

Exercise:

Problem: $\frac{5}{8}$, $\frac{13}{16}$, $\frac{3}{4}$

Solution:

$\frac{5}{8}$, $\frac{3}{4}$, $\frac{13}{16}$

Proficiency Exam
This module is from Fundamentals of Mathematics by Denny Burzynski and Wade Ellis, Jr. This module is a proficiency exam to the chapter "Addition and Subtraction of Fractions, Comparing Fractions, and Complex Fractions." Each problem is accompanied with a reference link pointing back to the module that discusses the type of problem demonstrated in the question. The problems in this exam are accompanied by solutions.

Proficiency Exam

For problems 1-12, perform each indicated operation and write the result in simplest form.

Exercise:

Problem: ([link]) $\frac{3}{16}+\frac{1}{8}$

Solution:

$\frac{5}{16}$

Exercise:

Problem: ([link]) $2\frac{2}{3}+5\frac{1}{6}$

Solution:

$7\frac{5}{6}$

Exercise:

Problem: ([link]) $\frac{7}{15}\cdot\frac{20}{21}+\frac{5}{9}$

Solution:

1

Exercise:

Problem: ([link]) $\frac{3}{11} + \frac{5}{11}$

Solution:

$\frac{8}{11}$

Exercise:

Problem: ([link]) $6\,\frac{2}{9} \cdot 1\,\frac{17}{28} - \left(3\,\frac{4}{17} - \frac{21}{17}\right)$

Solution:

8

Exercise:

Problem: ([link]) $5\,\frac{1}{8} - 2\,\frac{4}{5}$

Solution:

$2\,\frac{13}{40}$

Exercise:

Problem: ([link]) $\frac{\frac{7}{12}}{\frac{8}{21}}$

Solution:

$\frac{49}{32}$ or $\frac{17}{32}$

Exercise:

Problem: ([link]) $\frac{\frac{1}{8}+\frac{3}{4}}{1\frac{7}{8}}$

Solution:

$\frac{7}{15}$

Exercise:

Problem: ([link]) $4\frac{5}{16}+1\frac{1}{3}-2\frac{5}{24}$

Solution:

$3\frac{7}{16}$

Exercise:

Problem: ([link]) $\frac{5}{18}\cdot\left(\frac{15}{16}-\frac{3}{8}\right)$

Solution:

$\frac{5}{32}$

Exercise:

Problem: ([link]) $4+2\frac{1}{3}$

Solution:

$6\frac{1}{3}$ or $\frac{19}{3}$

Exercise:

Problem: ([link]) $8\frac{3}{7}-5$

Solution:

$3\frac{3}{7}$

For problems 13-15, specify the fractions that are equivalent.

Exercise:

Problem: ([link]) $\frac{4}{5}$, $\frac{12}{15}$

Solution:

equivalent

Exercise:

Problem: ([link]) $\frac{5}{8}$, $\frac{24}{40}$

Solution:

not equivalent

Exercise:

Problem: ([link]) $\frac{5}{12}$, $\frac{80}{192}$

Solution:

equivalent

For problems 16-20, place each collection of fractions in order from smallest to largest.

Exercise:

Problem: ([link]) $\frac{8}{9}$, $\frac{6}{7}$

Solution:

$\frac{6}{7}$, $\frac{8}{9}$

Exercise:

Problem: ([link]) $\frac{5}{8}, \frac{7}{9}$

Solution:

$\frac{5}{8}, \frac{7}{9}$

Exercise:

Problem: ([link]) $11\frac{5}{16}, 11\frac{5}{12}$

Solution:

$11\frac{5}{16}, 11\frac{5}{12}$

Exercise:

Problem: ([link]) $\frac{2}{15}, \frac{3}{10}, \frac{1}{6}$

Solution:

$\frac{2}{15}, \frac{1}{6}, \frac{3}{10}$

Exercise:

Problem: ([link]) $\frac{19}{32}, \frac{9}{16}, \frac{5}{8}$

Solution:

$\frac{9}{16}, \frac{19}{32}, \frac{5}{8}$

Objectives
This module contains the learning objectives for the chapter "Decimals" from Fundamentals of Mathematics by Denny Burzynski and Wade Ellis, jr.

After completing this chapter, you should

Reading and Writing Decimals ([link])

- understand the meaning of digits occurring to the right of the ones position
- be familiar with the meaning of decimal fractions
- be able to read and write a decimal fraction

Converting a Decimal to a Fraction ([link])

- be able to convert an ordinary decimal and a complex decimal to a fraction

Rounding Decimals ([link])

- be able to round a decimal number to a specified position

Addition and Subtraction of Decimals ([link])

- understand the method used for adding and subtracting decimals
- be able to add and subtract decimals
- be able to use the calculator to add and subtract decimals

Multiplication of Decimals ([link])

- understand the method used for multiplying decimals
- be able to multiply decimals
- be able to simplify a multiplication of a decimal by a power of 10
- understand how to use the word "of" in multiplication

Division of Decimals ([link])

- understand the method used for dividing decimals
- be able to divide a decimal number by a nonzero whole number and by another, nonzero, decimal number

- be able to simplify a division of a decimal by a power of 10

Nonterminating Divisions ([link])

- understand the meaning of a nonterminating division
- be able to recognize a nonterminating number by its notation

Converting a Fraction to a Decimal ([link])

- be able to convert a fraction to a decimal

Combinations of Operations with Decimals and Fractions ([link])

- be able to combine operations with decimals

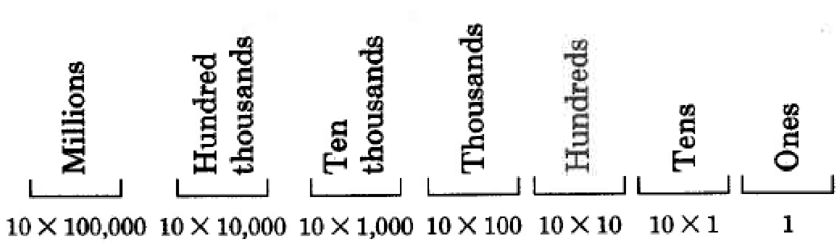
Reading and Writing Decimals
This module is from Fundamentals of Mathematics by Denny Burzynski and Wade Ellis, Jr. This module discusses how to read and write decimals. By the end of the module students should understand the meaning of digits occurring to the right of the ones position, be familiar with the meaning of decimal fractions and be able to read and write a decimal fraction.

Section Overview

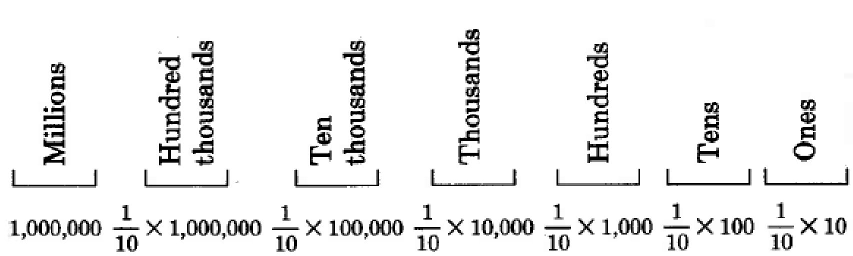
- Digits to the Right of the Ones Position
- Decimal Fractions
- Reading Decimal Fractions
- Writing Decimal Fractions

Digits to the Right of the Ones Position

We began our study of arithmetic ([link]) by noting that our number system is called a positional number system with base ten. We also noted that each position has a particular value. We observed that each position has ten times the value of the position to its right.



This means that each position has $\frac{1}{10}$ the value of the position to its left.



Thus, a digit written to the right of the units position must have a value of $\frac{1}{10}$ of 1. Recalling that the word "of" translates to multiplication $(\cdot)$, we can see that the value of the *first position* to the right of the units digit is $\frac{1}{10}$ of 1, or

$$\frac{1}{10} \cdot 1 = \frac{1}{10}$$

The value of the *second position* to the right of the units digit is $\frac{1}{10}$ of $\frac{1}{10}$, or

$$\frac{1}{10} \cdot \frac{1}{10} = \frac{1}{10^2} = \frac{1}{100}$$

The value of the third position to the right of the units digit is $\frac{1}{10}$ of $\frac{1}{100}$, or

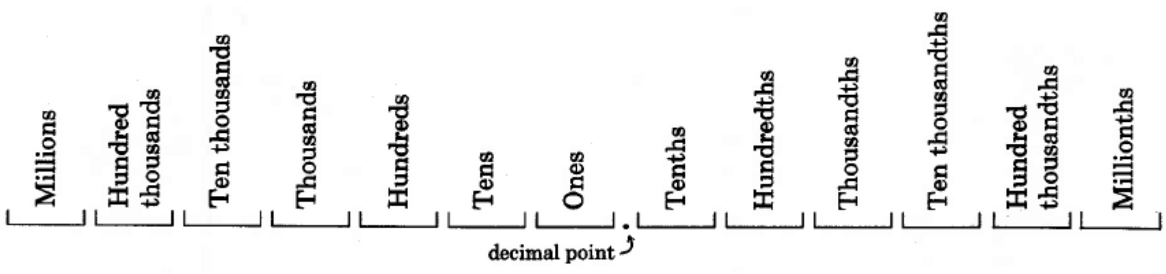
$$\frac{1}{10} \cdot \frac{1}{100} = \frac{1}{10^3} = \frac{1}{1000}$$

This pattern continues.

We can now see that if we were to write digits in positions to the right of the units positions, those positions have values that are fractions. Not only do the positions have fractional values, but the fractional values are all powers of 10 $\left(10,10^2,10^3,\ldots\right)$.

Decimal Fractions

Decimal Point, Decimal
If we are to write numbers with digits appearing to the right of the units digit, we must have a way of denoting where the whole number part ends and the fractional part begins. Mathematicians denote the separation point of the units digit and the tenths digit by writing a **decimal point**. The word *decimal* comes from the Latin prefix "deci" which means ten, and we use it because we use a base ten number system. Numbers written in this form are called **decimal fractions**, or more simply, **decimals**.



Notice that decimal numbers have the suffix "th."

Decimal Fraction
A **decimal fraction** is a fraction in which the denominator is a power of 10.

The following numbers are examples of decimals.

1. 42.6

   The 6 is in the tenths position.

   $42.6 = 42\frac{6}{10}$
2. 9.8014

   The 8 is in the tenths position.
   The 0 is in the hundredths position.
   The 1 is in the thousandths position.
   The 4 is in the ten thousandths position.

   $9.8014 = 9\frac{8014}{10{,}000}$
3. 0.93

   The 9 is in the tenths position.
   The 3 is in the hundredths position.

   $0.93 = \frac{93}{100}$

> **Note:**Quite often a zero is inserted in front of a decimal point (in the units position) of a decimal fraction that has a value less than one.

This zero helps keep us from overlooking the decimal point.

4. 0.7

   The 7 is in the tenths position.

   $0.7 = \frac{7}{10}$

   Note: We can insert zeros to the right of the right-most digit in a decimal fraction without changing the value of the number.
   $\frac{7}{10} = 0.7 = 0.70 = \frac{70}{100} = \frac{7}{10}$

Reading Decimal Fractions

Reading a Decimal Fraction
To read a decimal fraction,

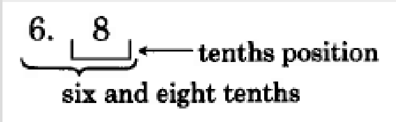
1. Read the whole number part as usual. (If the whole number is less than 1, omit steps 1 and 2.)
2. Read the decimal point as the word "and."
3. Read the number to the right of the decimal point as if it were a whole number.
4. Say the name of the position of the last digit.

Sample Set A

Read the following numbers.

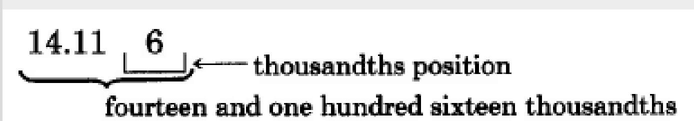
Example:

6.8

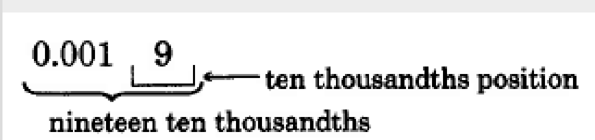


Note: Some people read this as "six point eight." This phrasing gets the message across, but technically, "six *and* eight tenths" is the correct phrasing.

Example:
14.116



Example:
0.0019



Example:
81

Eighty-one
In this problem, the indication is that any whole number is a decimal fraction. Whole numbers are often called *decimal numbers*.
$81 = 81.0$

Practice Set A

Read the following decimal fractions.
Exercise:

Problem: 12.9

Solution:

twelve and nine tenths

Exercise:

Problem: 4.86

Solution:

four and eighty-six hundredths

Exercise:

Problem: 7.00002

Solution:

seven and two hundred thousandths

Exercise:

Problem: 0.030405

Solution:

thirty thousand four hundred five millionths

Writing Decimal Fractions

Writing a Decimal Fraction
To write a decimal fraction,

1. Write the whole number part.
2. Write a decimal point for the word "and."
3. Write the decimal part of the number so that the right-most digit appears in the position indicated in the word name. If necessary, insert zeros to the right of the decimal point in order that the right-most digit appears in the correct position.

Sample Set B

Write each number.

Example:
Thirty-one and twelve hundredths.
The decimal position indicated is the hundredths position.
31.12

Example:
Two and three hundred-thousandths.
The decimal position indicated is the hundred thousandths. We'll need to insert enough zeros to the immediate right of the decimal point in order to locate the 3 in the correct position.
2.00003

Example:
Six thousand twenty-seven and one hundred four millionths.
The decimal position indicated is the millionths position. We'll need to insert enough zeros to the immediate right of the decimal point in order to locate the 4 in the correct position.
6,027.000104

Example:
Seventeen hundredths.
The decimal position indicated is the hundredths position.
0.17

Practice Set B

Write each decimal fraction.

Exercise:

Problem: Three hundred six and forty-nine hundredths.

Solution:

306.49

Exercise:

Problem: Nine and four thousandths.

Solution:

9.004

Exercise:

Problem: Sixty-one millionths.

Solution:

0.000061

Exercises

For the following three problems, give the decimal name of the position of the given number in each decimal fraction.

Exercise:

Problem:
1. 3.941
9 is in the position.
4 is in the position.
1 is in the position.

Solution:

Tenths; hundredths, thousandths

Exercise:

Problem:
17.1085
1 is in the position.
0 is in the position.
8 is in the position.
5 is in the position.

Exercise:

Problem:
652.3561927
9 is in the position.
7 is in the position.

Solution:

Hundred thousandths; ten millionths

For the following 7 problems, read each decimal fraction by writing it.

Exercise:

Problem: 9.2

Exercise:

Problem: 8.1

Solution:

eight and one tenth

Exercise:

Problem: 10.15

Exercise:

Problem: 55.06

Solution:

fifty-five and six hundredths

Exercise:

Problem: 0.78

Exercise:

Problem: 1.904

Solution:

one and nine hundred four thousandths

Exercise:

Problem: 10.00011

For the following 10 problems, write each decimal fraction.

Exercise:

Problem: Three and twenty one-hundredths.

Solution:

3.20

Exercise:

Problem: Fourteen and sixty seven-hundredths.

Exercise:

Problem: One and eight tenths.

Solution:

1.8

Exercise:

Problem: Sixty-one and five tenths.

Exercise:

Problem: Five hundred eleven and four thousandths.

Solution:

511.004

Exercise:

Problem: Thirty-three and twelve ten-thousandths.

Exercise:

Problem: Nine hundred forty-seven thousandths.

Solution:

0.947

Exercise:

Problem: Two millionths.

Exercise:

Problem: Seventy-one hundred-thousandths.

Solution:

0.00071

Exercise:

Problem: One and ten ten-millionths.

Calculator Problems

For the following 10 problems, perform each division using a calculator. Then write the resulting decimal using words.

Exercise:

Problem: $3 \div 4$

Solution:

seventy-five hundredths

Exercise:

Problem: $1 \div 8$

Exercise:

Problem: $4 \div 10$

Solution:

four tenths

Exercise:

Problem: $2 \div 5$

Exercise:

Problem: $4 \div 25$

Solution:

sixteen hundredths

Exercise:

Problem: $1 \div 50$

Exercise:

Problem: $3 \div 16$

Solution:

one thousand eight hundred seventy-five ten thousandths

Exercise:

Problem: $15 \div 8$

Exercise:

Problem: $11 \div 20$

Solution:

fifty-five hundredths

Exercise:

Problem: $9 \div 40$

Exercises for Review

Exercise:

Problem: ([link]) Round 2,614 to the nearest ten.

Solution:

2610

Exercise:

Problem: ([link]) Is 691,428,471 divisible by 3?

Exercise:

([link]) Determine the missing numerator.

Problem: $\frac{3}{14} = \frac{?}{56}$

Solution:

12

Exercise:

Problem: ([link]) Find $\frac{3}{16}$ of $\frac{32}{39}$

Exercise:

Problem: ([link]) Find the value of $\sqrt{\frac{25}{81}} + \left(\frac{2}{3}\right)^2 + \frac{1}{9}$

Solution:

$\frac{10}{9}$ or $1\frac{1}{9}$

Converting a Decimal to a Fraction
This module is from Fundamentals of Mathematics by Denny Burzynski and Wade Ellis, Jr. This module discusses how to covert a decimal to a fraction. By the end of the module students should be able to convert an ordinary decimal and a complex decimal to a fraction.

Section Overview

- Converting an Ordinary Decimal to a Fraction
- Converting a Complex Decimal to a Fraction

Converting an Ordinary Decimal to a Fraction

We can convert a decimal fraction to a fraction, essentially, by saying it in words, then writing what we say. We may have to reduce that fraction.

Sample Set A

Convert each decimal fraction to a proper fraction or a mixed number.

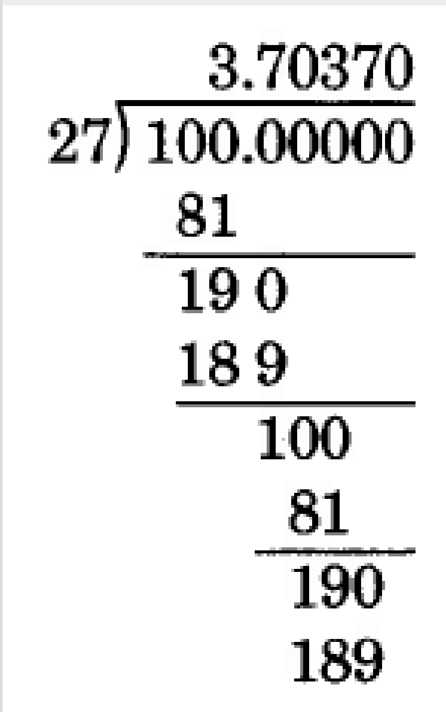
Example:

Reading: six tenths→ $\frac{6}{10}$.
Reduce: $\frac{3}{5}$.

Example:

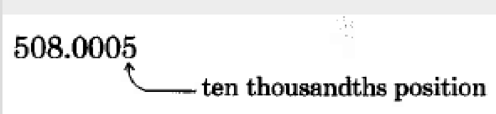
Reading: nine hundred three thousands → $\frac{903}{1000}$.

Example:



Reading: eighteen and sixty-one hundredths → $18\frac{61}{100}$.

Example:



Reading: five hundred eight and five ten thousandths → $508\frac{5}{10,000}$.
Reduce: $508\frac{1}{2,000}$.

Practice Set A

Convert the following decimals to fractions or mixed numbers. Be sure to reduce.

Exercise:

Problem: 16.84

Solution:

$16\frac{21}{25}$

Exercise:

Problem: 0.513

Solution:

$\frac{513}{1,000}$

Exercise:

Problem: 6,646.0107

Solution:

$6,646\frac{107}{10,000}$

Exercise:

Problem: 1.1

Solution:

$1\frac{1}{10}$

Converting A Complex Decimal to a Fraction

Complex Decimals
Numbers such as $0.11\frac{2}{3}$ are called **complex decimals**. We can also convert complex decimals to fractions.

Sample Set B

Convert the following complex decimals to fractions.

Example:
$0.11\frac{2}{3}$
The $\frac{2}{3}$ appears to occur in the thousands position, but it is referring to $\frac{2}{3}$ of a hundredth. So, we read $0.11\frac{2}{3}$ as "eleven and two-thirds hundredths."

$$\begin{aligned} 0.11\frac{2}{3} = \frac{11\frac{2}{3}}{100} &= \frac{\frac{11 \cdot 3+2}{3}}{100} \\ &= \frac{\frac{35}{3}}{\frac{100}{1}} \\ &= \frac{35}{3} \div \frac{100}{1} \\ &= \frac{\overset{7}{\cancel{35}}}{3} \cdot \frac{1}{\underset{20}{\cancel{100}}} \\ &= \frac{7}{60} \end{aligned}$$

Example:
$4.006\frac{1}{4}$
Note that $4.006\frac{1}{4} = 4 + .006\frac{1}{4}$

$$
\begin{aligned}
4+.006\frac{1}{4} &= 4+\frac{6\frac{1}{4}}{1000} \\
&= 4+\frac{\frac{25}{4}}{\frac{1000}{1}} \\
&= 4+\frac{\overset{1}{\cancel{25}}}{4}\cdot\frac{1}{\underset{40}{\cancel{1000}}} \\
&= 4+\frac{1\cdot 1}{4\cdot 40} \\
&= 4+\frac{1}{160} \\
&= 4\frac{1}{160}
\end{aligned}
$$

Practice Set B

Convert each complex decimal to a fraction or mixed number. Be sure to reduce.

Exercise:

Problem: $0.8\frac{3}{4}$

Solution:

$\frac{7}{8}$

Exercise:

Problem: $0.12\frac{2}{5}$

Solution:

$\frac{31}{250}$

Exercise:

Problem: $6.005\frac{5}{6}$

Solution:

$6\frac{7}{1,200}$

Exercise:

Problem: $18.1\frac{3}{17}$

Solution:

$18\frac{2}{17}$

Exercises

For the following 20 problems, convert each decimal fraction to a proper fraction or a mixed number. Be sure to reduce.

Exercise:

Problem: 0.7

Solution:

$\frac{7}{10}$

Exercise:

Problem: 0.1

Exercise:

Problem: 0.53

Solution:

$\frac{53}{100}$

Exercise:

Problem: 0.71

Exercise:

Problem: 0.219

Solution:

$\frac{219}{1,000}$

Exercise:

Problem: 0.811

Exercise:

Problem: 4.8

Solution:

$4\frac{4}{5}$

Exercise:

Problem: 2.6

Exercise:

Problem: 16.12

Solution:

$16\frac{3}{25}$

Exercise:

Problem: 25.88

Exercise:

Problem: 6.0005

Solution:

$6\,\frac{1}{2,000}$

Exercise:

Problem: 1.355

Exercise:

Problem: 16.125

Solution:

$16\,\frac{1}{8}$

Exercise:

Problem: 0.375

Exercise:

Problem: 3.04

Solution:

$3\,\frac{1}{25}$

Exercise:

Problem: 21.1875

Exercise:

Problem: 8.225

Solution:

$8\frac{9}{40}$

Exercise:

Problem: 1.0055

Exercise:

Problem: 9.99995

Solution:

$9\frac{19{,}999}{20{,}000}$

Exercise:

Problem: 22.110

For the following 10 problems, convert each complex decimal to a fraction.

Exercise:

Problem: $0.7\frac{1}{2}$

Solution:

$\frac{3}{4}$

Exercise:

Problem: $0.012\frac{1}{2}$

Exercise:

Problem: $2.16\frac{1}{4}$

Solution:

$2\frac{13}{80}$

Exercise:

Problem: $5.18\frac{2}{3}$

Exercise:

Problem: $14.112\frac{1}{3}$

Solution:

$14\frac{337}{3,000}$

Exercise:

Problem: $80.0011\frac{3}{7}$

Exercise:

Problem: $1.40\frac{5}{16}$

Solution:

$1\frac{129}{320}$

Exercise:

Problem: $0.8\frac{5}{3}$

Exercise:

Problem: $1.9\frac{7}{5}$

Solution:

$2\frac{1}{25}$

Exercise:

Problem: $1.7\frac{37}{9}$

Exercises for Review

Exercise:

Problem:

([link]) Find the greatest common factor of 70, 182, and 154.

Solution:

14

Exercise:

Problem:

([link]) Find the greatest common multiple of 14, 26, and 60.

Exercise:

Problem: ([link]) Find the value of $\frac{3}{5} \cdot \frac{15}{18} \div \frac{5}{9}$.

Solution:

$\frac{9}{10}$

Exercise:

Problem: ([link]) Find the value of $5\frac{2}{3}+8\frac{1}{12}$.

Exercise:

Problem:

([link]) In the decimal number 26.10742, the digit 7 is in what position?

Solution:

thousandths

Rounding Decimals
This module is from Fundamentals of Mathematics by Denny Burzynski and Wade Ellis, Jr. This module discusses how to round decimals. By the end of the module students should be able to round a decimal number to a specified position.

Section Overview

- Rounding Decimal Numbers

Rounding Decimal Numbers

We first considered the concept of rounding numbers in [link] where our concern with rounding was related to whole numbers only. With a few minor changes, we can apply the same rules of rounding to decimals.

To round a decimal to a particular position:

1. Mark the position of the round-off digit (with an arrow or check).
2. Note whether the digit to the immediate right of the marked digit is

    a. *less than 5*. If so, leave the round-off digit unchanged.
    b. *5 or greater*. If so, add 1 to the round-off digit.

3. If the round-off digit is

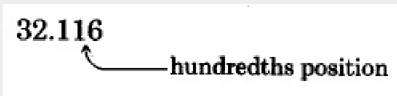
    a. to the right of the decimal point, eliminate all the digits to its right.
    b. to the left of the decimal point, replace all the digits between it and the decimal point with zeros and eliminate the decimal point and all the decimal digits.

Sample Set A

Round each decimal to the specified position. (The numbers in parentheses indicate which step is being used.)

Example:
Round 32.116 to the nearest hundredth.

- **1**

- **2b** The digit immediately to the right is 6, and $6 > 5$, so we add 1 to the round-off digit:

  $1 + 1 = 2$
- **3a** The round-off digit is to the right of the decimal point, so we eliminate all digits to its right.

  32.12

The number 32.116 rounded to the nearest hundredth is 32.12.

Example:
Round 633.14216 to the nearest hundred.

- **1**

- **2a** The digit immediately to the right is 3, and $3 < 5$ so we leave the round-off digit unchanged.
- **3b** The round-off digit is to the left of 0, so we replace all the digits between it and the decimal point with zeros and eliminate the decimal point and all the decimal digits.

  600

The number 633.14216 rounded to the nearest hundred is 600.

Example:
1,729.63 rounded to the nearest ten is 1,730.

Example:
1.0144 rounded to the nearest tenth is 1.0.

Example:
60.98 rounded to the nearest one is 61.

Sometimes we hear a phrase such as "round to three decimal places." This phrase means that the round-off digit is the third decimal digit (the digit in the thousandths position).

Example:
67.129 rounded to the second decimal place is 67.13.

Example:
67.129558 rounded to 3 decimal places is 67.130.

Practice Set A

Round each decimal to the specified position.
Exercise:

Problem: 4.816 to the nearest hundredth.

Solution:

4.82

Exercise:

Problem: 0.35928 to the nearest ten thousandths.

Solution:

0.3593

Exercise:

Problem: 82.1 to the nearest one.

Solution:

82

Exercise:

Problem: 753.98 to the nearest hundred.

Solution:

800

Exercise:

Problem: Round 43.99446 to three decimal places.

Solution:

43.994

Exercise:

Problem: Round 105.019997 to four decimal places.

Solution:

105.0200

Exercise:

Problem: Round 99.9999 to two decimal places.

Solution:

100.00

Exercises

For the first 10 problems, complete the chart by rounding each decimal to the indicated positions.

Exercise:

Problem: 20.01071

Tenth	Hundredth	Thousandth	Ten Thousandth

Solution:

Tenth	Hundredth	Thousandth	Ten Thousandth
20.0	20.01	20.011	20.0107

Exercise:

Problem: 3.52612

Tenth	Hundredth	Thousandth	Ten Thousandth
	3.53		

Exercise:

Problem: 531.21878

Tenth	Hundredth	Thousandth	Ten Thousandth

Solution:

Tenth	Hundredth	Thousandth	Ten Thousandth
531.2	531.22	531.219	531.2188

Exercise:

Problem: 36.109053

Tenth	Hundredth	Thousandth	Ten Thousandth
36.1			

Exercise:

Problem: 1.999994

Tenth	Hundredth	Thousandth	Ten Thousandth

Solution:

Tenth	Hundredth	Thousandth	Ten Thousandth
2.0	2.00	2.000	2.0000

Exercise:

Problem: 7.4141998

Tenth	Hundredth	Thousandth	Ten Thousandth
		7.414	

Exercise:

Problem: 0.000007

Tenth	Hundredth	Thousandth	Ten Thousandth

Solution:

Tenth	Hundredth	Thousandth	Ten Thousandth
0.0	0.00	0.000	0.0000

Exercise:

Problem: 0.00008

Tenth	Hundredth	Thousandth	Ten Thousandth
			0.0001

Exercise:

Problem: 9.19191919

Tenth	Hundredth	Thousandth	Ten Thousandth

Solution:

Tenth	Hundredth	Thousandth	Ten Thousandth
9.2	9.19	9.192	9.1919

Exercise:

Problem: 0.0876543

Tenth	Hundredth	Thousandth	Ten Thousandth

Calculator Problems

For the following 5 problems, round 18.4168095 to the indicated place.

Exercise:

Problem: 3 decimal places.

Solution:

18.417

Exercise:

Problem: 1 decimal place.

Exercise:

Problem: 5 decimal places.

Solution:

18.41681

Exercise:

Problem: 6 decimal places.

Exercise:

Problem: 2 decimal places.

Solution:

18.42

Calculator Problems

For the following problems, perform each division using a calculator.

Exercise:

Problem: $4 \div 3$ and round to 2 decimal places.

Exercise:

Problem: $1 \div 8$ and round to 1 decimal place.

Solution:

0.1

Exercise:

Problem: $1 \div 27$ and round to 6 decimal places.

Exercise:

Problem: $51 \div 61$ and round to 5 decimal places.

Solution:

0.83607

Exercise:

Problem: $3 \div 16$ and round to 3 decimal places.

Exercise:

Problem: $16 \div 3$ and round to 3 decimal places.

Solution:

5.333

Exercise:

Problem: $26 \div 7$ and round to 5 decimal places.

Exercises for Review

Exercise:

Problem: ([link]) What is the value of 2 in the number 421,916,017?

Solution:

Ten million

Exercise:

Problem: ([link]) Perform the division: $378 \div 29$.

Exercise:

Problem: ([link]) Find the value of 4^4.

Solution:

256

Exercise:

Problem: ([link]) Convert $\frac{11}{3}$ to a mixed number.

Exercise:

Problem: ([link]) Convert 3.16 to a mixed number fraction.

Solution:

$3\frac{4}{25}$

Addition and Subtraction of Decimals
This module is from Fundamentals of Mathematics by Denny Burzynski and Wade Ellis, Jr. This module discusses how to add and subtract decimals. By the end of the module students should understand the method used for adding and subtracting decimals, be able to add and subtract decimals and be able to use the calculator to add and subtract decimals.

Section Overview

- The Logic Behind the Method
- The Method of Adding and Subtracting Decimals
- Calculators

The Logic Behind the Method

Consider the sum of 4.37 and 3.22. Changing each decimal to a fraction, we have

$4\frac{37}{100} + 3\frac{22}{100}$ Performing the addition, we get

$$\begin{aligned} 4.37 + 3.22 = 4\frac{37}{100} + 3\frac{22}{100} &= \frac{4 \cdot 100 + 37}{100} + \frac{3 \cdot 100 + 22}{100} \\ &= \frac{437}{100} + \frac{322}{100} \\ &= \frac{437 + 322}{100} \\ &= \frac{759}{100} \\ &= 7\frac{59}{100} \\ &= \text{seven and fifty-nine hundredths} \\ &= 7.59 \end{aligned}$$

Thus, $4.37 + 3.22 = 7.59$.

The Method of Adding and Subtracting Decimals

When writing the previous addition, we could have written the numbers in columns.

$$\begin{array}{r} 4.37 \\ +3.22 \\ \hline 7.59 \end{array}$$

This agrees with our previous result. From this observation, we can suggest a method for adding and subtracting decimal numbers.

Method of Adding and Subtracting Decimals
To add or subtract decimals:

1. Align the numbers vertically so that the decimal points line up under each other and the corresponding decimal positions are in the same column.
2. Add or subtract the numbers as if they were whole numbers.
3. Place a decimal point in the resulting sum or difference directly under the other decimal points.

Sample Set A

Find the following sums and differences.

Example:
$9.813 + 2.140$

$$\begin{array}{rl} 9.813 & \text{The decimal points are aligned in the same column.} \\ +2.140 & \\ \hline 11.953 & \end{array}$$

Example:
$841.0056 + 47.016 + 19.058$

841.0056

47.016

+19.058

To insure that the columns align properly, we can write a 0 in the position at the end of the numbers 47.016 and 19.058 without changing their values.

$$\begin{array}{r} {\scriptstyle 11\quad\ \ 1\quad} \\ 841.0056 \\ 47.0160 \\ +19.0580 \\ \hline 907.0796 \end{array}$$

Example:

1.314 – 0.58

1.314

−0.58 Write a 0 in the thousandths position.

$$\begin{array}{r} {\scriptstyle 12\ \ } \\ {\scriptstyle 0\ \cancel{2}\,11\ } \\ \cancel{1}.\cancel{3}\cancel{1}4 \\ -0.580 \\ \hline 0.734 \end{array}$$

Example:

16.01 – 7.053

16.01

−7.053 Write a 0 in the thousandths position.

$$\begin{array}{r} {\scriptstyle 15\ \ 9\,10\,10} \\ 1\cancel{6}.\cancel{0}\cancel{1}\cancel{0} \\ -\ \ 7.053 \\ \hline 8.957 \end{array}$$

Example:
Find the sum of 6.88106 and 3.5219 and round it to three decimal places.

$$\begin{array}{l} \quad 6.88106 \\ +3.5219 \qquad \text{Write a 0 in the ten thousandths position.} \end{array}$$

$$\begin{array}{r} {\scriptstyle 1\ 1} \\ 6.88106 \\ +\ 3.52190 \\ \hline 10.40296 \end{array}$$

We need to round the sum to the thousandths position. Since the digit in the position immediately to the right is 9, and 9>5, we get
10.403

Example:
Wendy has $643.12 in her checking account. She writes a check for $16.92. How much is her new account balance?
To find the new account balance, we need to find the difference between 643.12 and 16.92. We will subtract 16.92 from 643.12.

$$\begin{array}{r} {\scriptstyle 3\ 12\ 11} \\ 643.12 \\ -\ 16.92 \\ \hline 626.20 \end{array}$$

After writing a check for $16.92, Wendy now has a balance of $626.20 in her checking account.

Practice Set A

Find the following sums and differences.
Exercise:

Problem: $3.187 + 2.992$

Solution:

6.179

Exercise:

Problem: $14.987 - 5.341$

Solution:

9.646

Exercise:

Problem: $0.5261 + 1.0783$

Solution:

1.6044

Exercise:

Problem: $1.06 - 1.0535$

Solution:

0.0065

Exercise:

Problem: $16{,}521.07 + 9{,}256.15$

Solution:

25,777.22

Exercise:

Problem:

Find the sum of 11.6128 and 14.07353, and round it to two decimal places.

Solution:

25.69

Calculators

The calculator can be useful for finding sums and differences of decimal numbers. However, calculators with an eight-digit display cannot be used when working with decimal numbers that contain more than eight digits, or when the sum results in more than eight digits. In practice, an eight-place decimal will seldom be encountered. There are some inexpensive calculators that can handle 13 decimal places.

Sample Set B

Use a calculator to find each sum or difference.

Example:
42.0638 + 126.551

		Display Reads

Type	42.0638	42.0638
Press	+	42.0638
Type	126.551	126.551
Press	=	168.6148

The sum is 168.6148.

Example:
Find the difference between 305.0627 and 14.29667.

		Display Reads
Type	305.0627	305.0627
Press	—	305.0627
Type	14.29667	14.29667
Press	=	290.76603

The difference is 290.76603

Example:
51.07 + 3,891.001786

Since 3,891.001786 contains more than eight digits, we will be unable to use an eight-digit display calculator to perform this addition. We can, however, find the sum by hand.

$$\begin{array}{r} 51.070000 \\ 3891.001786 \\ \hline 3942.071786 \end{array}$$

The sum is 3,942.071786.

Practice Set B

Use a calculator to perform each operation.

Exercise:

Problem: $4.286 + 8.97$

Solution:

13.256

Exercise:

Problem: $452.0092 - 392.558$

Solution:

59.4512

Exercise:

Problem: Find the sum of 0.095 and 0.001862

Solution:

0.096862

Exercise:

Problem: Find the difference between 0.5 and 0.025

Solution:

0.475

Exercise:

Problem: Find the sum of 2,776.00019 and 2,009.00012.

Solution:

Since each number contains more than eight digits, using some calculators may not be helpful. Adding these by "hand technology," we get 4,785.00031

Exercises

For the following 15 problems, perform each addition or subtraction. Use a calculator to check each result.

Exercise:

Problem: $1.84 + 7.11$

Solution:

8.95

Exercise:

Problem: $15.015 - 6.527$

Exercise:

Problem: $11.842 + 28.004$

Solution:

39.846

Exercise:

Problem: $3.16-2.52$

Exercise:

Problem: $3.55267+8.19664$

Solution:

11.74931

Exercise:

Problem: $0.9162-0.0872$

Exercise:

Problem: $65.512-8.3005$

Solution:

57.2115

Exercise:

Problem: $761.0808-53.198$

Exercise:

Problem: $4.305+2.119-3.817$

Solution:

2.607

Exercise:

Problem: $19.1161 + 27.8014 + 39.3161$

Exercise:

Problem: $0.41276 - 0.0018 - 0.00011$

Solution:

0.41085

Exercise:

Problem: $2.181 + 6.05 + 1.167 + 8.101$

Exercise:

Problem: 1.0031+6.013106+0.00018+0.0092+2.11

Solution:

9.135586

Exercise:

Problem: $27 + 42 + 9.16 - 0.1761 + 81.6$

Exercise:

Problem: $10.28 + 11.111 + 0.86 + 5.1$

Solution:

27.351

For the following 10 problems, solve as directed. A calculator may be useful.

Exercise:

Problem: Add 6.1121 and 4.916 and round to 2 decimal places.

Exercise:

Problem: Add 21.66418 and 18.00184 and round to 4 decimal places.

Solution:

39.6660

Exercise:

Problem: Subtract 5.2121 from 9.6341 and round to 1 decimal place.

Exercise:

Problem: Subtract 0.918 from 12.006 and round to 2 decimal places.

Solution:

11.09

Exercise:

Problem:

Subtract 7.01884 from the sum of 13.11848 and 2.108 and round to 4 decimal places.

Exercise:

Problem:

A checking account has a balance of $42.51. A check is written for $19.28. What is the new balance?

Solution:

$23.23

Exercise:

Problem:

A checking account has a balance of $82.97. One check is written for $6.49 and another for $39.95. What is the new balance?

Exercise:

Problem:

A person buys $4.29 worth of hamburger and pays for it with a $10 bill. How much change does this person get?

Solution:

$5.71

Exercise:

Problem:

A man buys $6.43 worth of stationary and pays for it with a $20 bill. After receiving his change, he realizes he forgot to buy a pen. If the total price of the pen is $2.12, and he buys it, how much of the $20 bill is left?

Exercise:

Problem:

A woman starts recording a movie on her video cassette recorder with the tape counter set at 21.93. The movie runs 847.44 tape counter units. What is the final tape counter reading?

Solution:

869.37

Exercises for Review

Exercise:

Problem: ([link]) Find the difference between 11,206 and 10,884.

Exercise:

Problem: ([link]) Find the product, $820 \cdot 10{,}000$.

Solution:

8,200,000

Exercise:

Problem: ([link]) Find the value of $\sqrt{121} - \sqrt{25} + 8^2 + 16 \div 2^2$.

Exercise:

Problem: ([link]) Find the value of $8\frac{1}{3} \cdot \frac{36}{75} \div 2\frac{2}{5}$.

Solution:

$\frac{20}{9} = \frac{5}{3}$ or $2\frac{2}{9}$

Exercise:

Problem: ([link]) Round 1.08196 to the nearest hundredth.

Multiplication of Decimals
This module is from Fundamentals of Mathematics by Denny Burzynski and Wade Ellis, Jr. This module discusses how to multiply decimals. By the end of the module students should understand the method used for multiplying decimals, be able to multiply decimals, be able to simplify a multiplication of a decimal by a power of 10 and understand how to use the word "of" in multiplication.

Section Overview

- The Logic Behind the Method
- The Method of Multiplying Decimals
- Calculators
- Multiplying Decimals By Powers of 10
- Multiplication in Terms of “Of”

The Logic Behind the Method

Consider the product of 3.2 and 1.46. Changing each decimal to a fraction, we have

$$\begin{aligned}(3.2)(1.46) &= 3\tfrac{2}{10} \cdot 1\tfrac{46}{100} \\ &= \tfrac{32}{10} \cdot \tfrac{146}{100} \\ &= \tfrac{32 \cdot 146}{10 \cdot 100} \\ &= \tfrac{4672}{1000} \\ &= 4\tfrac{672}{1000} \\ &= \text{four and six hundred seventy-two thousandths} \\ &= 4.672\end{aligned}$$

Thus, $(3.2)(1.46) = 4.672$.

Notice that the factor

$$\begin{array}{r} 3.2 \text{ has 1 decimal place,} \\ 1.46 \text{ has 2 decimal places,} \\ \text{and the product} \\ 4.672 \text{ has 3 decimal places.} \end{array} \quad 1 + 2 = 3$$

Using this observation, we can suggest that the sum of the number of decimal places in the factors equals the number of decimal places in the product.

$$\begin{array}{rlr} & \overset{1}{1.46} \longleftarrow & 2 \text{ decimal places} \\ \times & 3.2 \longleftarrow & \underline{+1 \text{ decimal place}} \\ \hline & 292 & \\ & 438 & \\ \hline & 4.672 \longleftarrow & 3 \text{ decimal places} \end{array}$$

The Method of Multiplying Decimals

Method of Multiplying Decimals
To multiply decimals,

1. Multiply the numbers as if they were whole numbers.
2. Find the sum of the number of decimal places in the factors.
3. The number of decimal places in the product is the sum found in step 2.

Sample Set A

Find the following products.

Example:
$6.5 \cdot 4.3$

$$\left.\begin{array}{r l} 6.5 & \longleftarrow \text{1 decimal plate} \\ 4.3 & \longleftarrow \text{1 decimal place} \end{array}\right\} 1 + 1 = 2 \text{ decimal places in the product.}$$

$$\begin{array}{r l} 6.5 & \\ \times\ 4.3 & \\ \hline 195 & \\ 260 & \\ \hline 27.95 & \longleftarrow \text{2 decimal places} \end{array}$$

Thus, $6.5 \cdot 4.3 = 27.95$.

Example:

$23.4 \cdot 1.96$

$$\left.\begin{array}{r l} 23.4 & \longleftarrow \text{1 decimal place} \\ 1.96 & \longleftarrow \text{2 decimal places} \end{array}\right\} 1 + 2 = 3 \text{ decimal places in the product.}$$

$$\begin{array}{r l} 23.4 & \\ \times\ 1.96 & \\ \hline 1404 & \\ 2106 & \\ 234 & \\ \hline 45.864 & \longleftarrow \text{3 decimal places} \end{array}$$

Thus, $23.4 \cdot 1.96 = 45.864$.

Thus, $23.4 \cdot 1.96 = 45.864$.

Example:

Find the product of 0.251 and 0.00113 and round to three decimal places.

$$\left.\begin{array}{r l} 0.251 & \longleftarrow \text{3 decimal places} \\ 0.00113 & \longleftarrow \text{5 decimal places} \end{array}\right\} 3 + 5 = 8 \text{ decimal places in the product.}$$

$$\begin{array}{r l} 0.251 & \\ \times\ 0.00113 & \\ \hline 753 & \\ 251 & \\ 251 & \\ \hline 0.00028363 & \longleftarrow \text{We need to add three zeros to get 8 decimal places.} \end{array}$$

Now, rounding to three decimal places, we get

$$0.251 \cdot 0.00113 = 0.000$$ to three decimal places.

Practice Set A

Find the following products.

Exercise:

Problem: $5.3 \cdot 8.6$

Solution:

45.58

Exercise:

Problem: $2.12 \cdot 4.9$

Solution:

10.388

Exercise:

Problem: $1.054 \cdot 0.16$

Solution:

0.16864

Exercise:

Problem: $0.00031 \cdot 0.002$

Solution:

0.00000062

Exercise:

Problem:

Find the product of 2.33 and 4.01 and round to one decimal place.

Solution:

9.3

Exercise:

Problem: $10 \cdot 5.394$

Solution:

53.94

Exercise:

Problem: $100 \cdot 5.394$

Solution:

539.4

Exercise:

Problem: $1000 \cdot 5.394$

Solution:

5,394

Exercise:

Problem: $10,000 \cdot 5.394$

Solution:

59,340

Calculators

Calculators can be used to find products of decimal numbers. However, a calculator that has only an eight-digit display may not be able to handle numbers or products that result in more than eight digits. But there are plenty of inexpensive ($50 - $75) calculators with more than eight-digit displays.

Sample Set B

Find the following products, if possible, using a calculator.

Example:
$2.58 \cdot 8.61$

		Display Reads
Type	2.58	2.58
Press	×	2.58
Type	8.61	8.61
Press	=	22.2138

The product is 22.2138.

Example:
$0.006 \cdot 0.0042$

		Display Reads
Type	.006	.006
Press	×	.006
Type	.0042	0.0042
Press	=	0.0000252

We know that there will be seven decimal places in the product (since $3 + 4 = 7$). Since the display shows 7 decimal places, we can assume the product is correct. Thus, the product is 0.0000252.

Example:
$0.0026 \cdot 0.11976$
Since we expect $4 + 5 = 9$ decimal places in the product, we know that an eight-digit display calculator will not be able to provide us with the exact value. To obtain the exact value, we must use "hand technology." Suppose, however, that we agree to round off this product to three decimal places. We then need only four decimal places on the display.

		Display Reads
Type	.0026	.0026
Press	×	.0026
Type	.11976	0.11976
Press	=	0.0003114

Rounding 0.0003114 to three decimal places we get 0.000. Thus, $0.0026 \cdot 0.11976 = 0.000$ to three decimal places.

Practice Set B

Use a calculator to find each product. If the calculator will not provide the exact product, round the result to four decimal places.

Exercise:

Problem: $5.126 \cdot 4.08$

Solution:

20.91408

Exercise:

Problem: $0.00165 \cdot 0.04$

Solution:

0.000066

Exercise:

Problem: $0.5598 \cdot 0.4281$

Solution:

0.2397

Exercise:

Problem: $0.000002 \cdot 0.06$

Solution:

0.0000

Multiplying Decimals by Powers of 10

There is an interesting feature of multiplying decimals by powers of 10. Consider the following multiplications.

Multiplication	Number of Zeros in the Power of 10	Number of Positions the Decimal Point Has Been Moved to the Right
$10 \cdot 8.315274 = 83.15274$	1	1
$100 \cdot 8.315274 = 831.5274$	2	2
$1{,}000 \cdot 8.315274 = 8{,}315.274$	3	3

$10{,}000 \cdot 8.315274 = 83{,}152.74$	4	4

Multiplying a Decimal by a Power of 10
To multiply a decimal by a power of 10, move the decimal place to the *right* of its current position as many places as there are zeros in the power of 10. Add zeros if necessary.

Sample Set C

Find the following products.

Example:
100 · 34.876. Since there are 2 zeros in 100, Move the decimal point in 34.876 two places to the right.

$$\begin{aligned} 100 \cdot 34.876 &= 3487.6 \\ &= 3{,}487.6 \end{aligned}$$

Example:
1,000 · 4.8058. Since there are 3 zeros in 1,000, move the decimal point in 4.8058 three places to the right.

$$\begin{aligned} 1{,}000 \cdot 4.8058 &= 4805.8 \\ &= 4{,}805.8 \end{aligned}$$

Example:

$10,000 \cdot 56.82$. Since there are 4 zeros in 10,000, move the decimal point in 56.82 four places to the right. We will have to add two zeros in order to obtain the four places.

$$\begin{aligned} 10,000 \cdot 56.82 &= 568200. \\ &= 568,200 \end{aligned}$$

Since there is no fractional part, we can drop the decimal point.

Example:

$$\begin{aligned} (1,000,000)(2.57) &= 2570000. \\ &= 2,570,000 \end{aligned}$$

Example:

$$\begin{aligned} (1,000)(0.0000029) &= 0\,000.0029 \\ &= 0.0029 \end{aligned}$$

Practice Set C

Find the following products.

Exercise:

Problem: $100 \cdot 4.27$

Solution:

427

Exercise:

Problem: $10{,}000 \cdot 16.52187$

Solution:

165,218.7

Exercise:

Problem: $(10)(0.0188)$

Solution:

0.188

Exercise:

Problem: $(10{,}000{,}000{,}000)(52.7)$

Solution:

527,000,000,000

Multiplication in Terms of “Of”

Recalling that the word "of" translates to the arithmetic operation of multiplication, let's observe the following multiplications.

Sample Set D

Example:
Find 4.1 of 3.8.
Translating "of" to "×", we get

$$\begin{array}{r} 4.1 \\ \times 3.8 \\ 328 \\ 123 \\ 15.58 \end{array}$$

Thus, 4.1 of 3.8 is 15.58.

Example:
Find 0.95 of the sum of 2.6 and 0.8.
We first find the sum of 2.6 and 0.8.

$$\begin{array}{r} 2.6 \\ +0.8 \\ 3.4 \end{array}$$

Now find 0.95 of 3.4

$$\begin{array}{r} 3.4 \\ \times 0.95 \\ 170 \\ 306 \\ 3.230 \end{array}$$

Thus, 0.95 of $(2.6 + 0.8)$ is 3.230.

Practice Set D

Exercise:

Problem: Find 2.8 of 6.4.

Solution:

17.92

Exercise:

Problem: Find 0.1 of 1.3.

Solution:

0.13

Exercise:

Problem: Find 1.01 of 3.6.

Solution:

3.636

Exercise:

Problem: Find 0.004 of 0.0009.

Solution:

0.0000036

Exercise:

Problem: Find 0.83 of 12.

Solution:

9.96

Exercise:

Problem: Find 1.1 of the sum of 8.6 and 4.2.

Solution:

14.08

Exercises

For the following 30 problems, find each product and check each result with a calculator.

Exercise:

Problem: $3.4 \cdot 9.2$

Solution:

31.28

Exercise:

Problem: $4.5 \cdot 6.1$

Exercise:

Problem: $8.0 \cdot 5.9$

Solution:

47.20

Exercise:

Problem: $6.1 \cdot 7$

Exercise:

Problem: $(0.1)(1.52)$

Solution:

0.152

Exercise:

Problem: $(1.99)(0.05)$

Exercise:

Problem: $(12.52)(0.37)$

Solution:

4.6324

Exercise:

Problem: $(5.116)(1.21)$

Exercise:

Problem: $(31.82)(0.1)$

Solution:

3.182

Exercise:

Problem: $(16.527)(9.16)$

Exercise:

Problem: $0.0021 \cdot 0.013$

Solution:

0.0000273

Exercise:

Problem: $1.0037 \cdot 1.00037$

Exercise:

Problem: $(1.6)(1.6)$

Solution:

2.56

Exercise:

Problem: $(4.2)(4.2)$

Exercise:

Problem: $0.9 \cdot 0.9$

Solution:

0.81

Exercise:

Problem: $1.11 \cdot 1.11$

Exercise:

Problem: $6.815 \cdot 4.3$

Solution:

29.3045

Exercise:

Problem: $9.0168 \cdot 1.2$

Exercise:

Problem: $(3.5162)(0.0000003)$

Solution:

0.00000105486

Exercise:

Problem: $(0.000001)(0.01)$

Exercise:

Problem: $(10)(4.96)$

Solution:

49.6

Exercise:

Problem: $(10)(36.17)$

Exercise:

Problem: $10 \cdot 421.8842$

Solution:

4,218.842

Exercise:

Problem: $10 \cdot 8.0107$

Exercise:

Problem: $100 \cdot 0.19621$

Solution:

19.621

Exercise:

Problem: $100 \cdot 0.779$

Exercise:

Problem: $1000 \cdot 3.596168$

Solution:

3,596.168

Exercise:

Problem: $1000 \cdot 42.7125571$

Exercise:

Problem: $1000 \cdot 25.01$

Solution:

25,010

Exercise:

Problem: $100,000 \cdot 9.923$

Exercise:

Problem: $(4.6)(6.17)$

Actual product	Tenths	Hundreds	Thousandths

Solution:

Actual product	Tenths	Hundreds	Thousandths
28.382	28.4	28.38	28.382

Exercise:

Problem: $(8.09)(7.1)$

Actual product	Tenths	Hundreds	Thousandths

Exercise:

Problem: $(11.1106)(12.08)$

Actual product	Tenths	Hundreds	Thousandths

Solution:

Actual product	Tenths	Hundreds	Thousandths
134.216048	134.2	134.22	134.216

Exercise:

Problem: $0.0083 \cdot 1.090901$

Actual product	Tenths	Hundreds	Thousandths

Exercise:

Problem: $7 \cdot 26.518$

Actual product	Tenths	Hundreds	Thousandths

Solution:

Actual product	Tenths	Hundreds	Thousandths
185.626	185.6	185.63	185.626

For the following 15 problems, perform the indicated operations

Exercise:

Problem: Find 5.2 of 3.7.

Exercise:

Problem: Find 12.03 of 10.1

Solution:

121.503

Exercise:

Problem: Find 16 of 1.04

Exercise:

Problem: Find 12 of 0.1

Solution:

1.2

Exercise:

Problem: Find 0.09 of 0.003

Exercise:

Problem: Find 1.02 of 0.9801

Solution:

0.999702

Exercise:

Problem: Find 0.01 of the sum of 3.6 and 12.18

Exercise:

Problem: Find 0.2 of the sum of 0.194 and 1.07

Solution:

0.2528

Exercise:

Problem: Find the difference of 6.1 of 2.7 and 2.7 of 4.03

Exercise:

Problem: Find the difference of 0.071 of 42 and 0.003 of 9.2

Solution:

2.9544

Exercise:

Problem:

If a person earns $8.55 an hour, how much does he earn in twenty-five hundredths of an hour?

Exercise:

Problem: A man buys 14 items at $1.16 each. What is the total cost?

Solution:

$16.24

Exercise:

Problem:

In the problem above, how much is the total cost if 0.065 sales tax is added?

Exercise:

Problem:

A river rafting trip is supposed to last for 10 days and each day 6 miles is to be rafted. On the third day a person falls out of the raft after only $\frac{2}{5}$ of that day’s mileage. If this person gets discouraged and quits, what fraction of the entire trip did he complete?

Solution:

0.24

Exercise:

Problem:

A woman starts the day with \$42.28. She buys one item for \$8.95 and another for \$6.68. She then buys another item for sixty two-hundredths of the remaining amount. How much money does she have left?

Calculator Problems

For the following 10 problems, use a calculator to determine each product. If the calculator will not provide the exact product, round the results to five decimal places.

Exercise:

Problem: $0.019 \cdot 0.321$

Solution:

0.006099

Exercise:

Problem: $0.261 \cdot 1.96$

Exercise:

Problem: $4.826 \cdot 4.827$

Solution:

23.295102

Exercise:

Problem: $(9.46)^2$

Exercise:

Problem: $(0.012)^2$

Solution:

0.000144

Exercise:

Problem: $0.00037 \cdot 0.0065$

Exercise:

Problem: $0.002 \cdot 0.0009$

Solution:

0.0000018

Exercise:

Problem: $0.1286 \cdot 0.7699$

Exercise:

Problem: $0.01 \cdot 0.00000471$

Solution:

0.0000000471

Exercise:

Problem: $0.00198709 \cdot 0.03$

Exercises for Review

Exercise:

Problem: ([link]) Find the value, if it exists, of $0 \div 15$.

Solution:

0

Exercise:

Problem:

([link]) Find the greatest common factor of 210, 231, and 357.

Exercise:

Problem:([link]) Reduce $\frac{280}{2,156}$ to lowest terms.

Solution:

$\frac{10}{77}$

Exercise:

Problem:

([link]) Write "fourteen and one hundred twenty-one ten-thousandths, using digits."

Exercise:

Problem:

([link]) Subtract 6.882 from 8.661 and round the result to two decimal places.

Solution:

1.78

Division of Decimals
This module is from Fundamentals of Mathematics by Denny Burzynski and Wade Ellis, Jr. This module discusses how to divide decimals. By the end of the module students should understand the method used for dividing decimals, be able to divide a decimal number by a nonzero whole number and by another, nonzero, decimal number and be able to simplify a division of a decimal by a power of 10.

Section Overview

- The Logic Behind the Method
- A Method of Dividing a Decimal By a Nonzero Whole Number
- A Method of Dividing a Decimal by a Nonzero Decimal
- Dividing Decimals by Powers of 10

The Logic Behind the Method

As we have done with addition, subtraction, and multiplication of decimals, we will study a method of division of decimals by converting them to fractions, then we will make a general rule.

We will proceed by using this example: Divide 196.8 by 6.

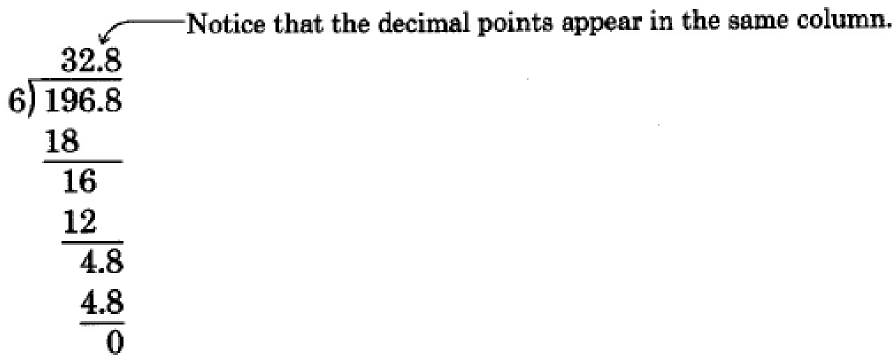
$$\begin{array}{r} 32 \\ 6\overline{)196.8} \\ 18 \\ 16 \\ 12 \\ 4 \end{array}$$

We have, up to this point, divided 196.8 by 6 and have gotten a quotient of 32 with a remainder of 4. If we follow our intuition and bring down the .8, we have the division $4.8 \div 6$.

$$
\begin{aligned}
4.8 \div 6 &= 4\frac{8}{10} \div 6 \\
&= \frac{48}{10} \div \frac{6}{1} \\
&= \frac{\cancelto{8}{48}}{10} \cdot \frac{1}{\cancelto{1}{6}} \\
&= \frac{8}{10}
\end{aligned}
$$

Thus, $4.8 \div 6 = .8$.

Now, our intuition and experience with division direct us to place the .8 immediately to the right of 32.



From these observations, we suggest the following method of division.

A Method of Dividing a Decimal by a Nonzero Whole Number

Method of Dividing a Decimal by a Nonzero Whole Number
To divide a decimal by a nonzero whole number:

1. Write a decimal point above the division line and directly over the decimal point of the dividend.
2. Proceed to divide as if both numbers were whole numbers.
3. If, in the quotient, the first nonzero digit occurs to the right of the decimal point, but not in the tenths position, place a zero in each position between the decimal point and the first nonzero digit of the quotient.

Sample Set A

Find the decimal representations of the following quotients.

Example:

$114.1 \div 7 = 7$

$$\begin{array}{r} 16.3 \\ 7\overline{)114.1} \\ 7 \\ 44 \\ 42 \\ 2.1 \\ 2.1 \\ 0 \end{array}$$

Thus, $114.1 \div 7 = 16.3$.

Check: If $114.1 \div 7 = 16.3$, then $7 \cdot 16.3$ should equal 114.1.

$$\begin{array}{r} {}^{4\,2} \\ 16.3 \\ 7 \\ 114.1 \end{array}$$ True.

Example:

$0.02068 \div 4$

$$\begin{array}{r} 0.00517 \\ 4\overline{)0.02068} \\ \underline{20} \\ 6 \\ \underline{4} \\ 28 \\ \underline{28} \\ 0 \end{array}$$

Place zeros in the tenths and hundredths positions. (See Step 3.)

Thus, $0.02068 \div 4 = 0.00517$.

Practice Set A

Find the following quotients.

Exercise:

Problem: $184.5 \div 3$

Solution:

61.5

Exercise:

Problem: $16.956 \div 9$

Solution:

1.884

Exercise:

Problem: $0.2964 \div 4$

Solution:

0.0741

Exercise:

Problem: $0.000496 \div 8$

Solution:

0.000062

A Method of Dividing a Decimal By a Nonzero Decimal

Now that we can divide decimals by nonzero whole numbers, we are in a position to divide decimals by a nonzero decimal. We will do so by converting a division by a decimal into a division by a whole number, a process with which we are already familiar. We'll illustrate the method using this example: Divide 4.32 by 1.8.

Let's look at this problem as $4\frac{32}{100} \div 1\frac{8}{10}$.

$$\begin{aligned} 4\frac{32}{100} \div 1\frac{8}{10} &= \frac{4\frac{32}{100}}{1\frac{8}{10}} \\ &= \frac{\frac{432}{100}}{\frac{18}{10}} \end{aligned}$$

The divisor is $\frac{18}{10}$. We can convert $\frac{18}{10}$ into a whole number if we multiply it by 10.

$$\frac{18}{10} \cdot 10 = \frac{18}{\cancel{10}_{1}} \cdot \frac{\cancel{10}^{1}}{1} = 18$$

But, we know from our experience with fractions, that if we multiply the denominator of a fraction by a nonzero whole number, we must multiply the numerator by that same nonzero whole number. Thus, when converting $\frac{18}{10}$ to a whole number by multiplying it by 10, we must also multiply the numerator $\frac{432}{100}$ by 10.

$$\begin{aligned} \frac{432}{100} \cdot 10 = \frac{432}{\cancel{100}_{10}} \cdot \frac{\cancel{10}^{1}}{1} &= \frac{432 \cdot 1}{10 \cdot 1} = \frac{432}{10} \\ &= 43\frac{2}{10} \\ &= 43.2 \end{aligned}$$

We have converted the division $4.32 \div 1.8$ into the division $43.2 \div 18$, that is,

$1.8\overline{)4.32} \to 18\overline{)43.2}$

Notice what has occurred.

$1.8\overline{)4.32} \longrightarrow 18.\overline{)43.2}$

If we "move" the decimal point of the divisor one digit to the right, we must also "move" the decimal point of the dividend one place to the right. The word "move" actually indicates the process of multiplication by a power of 10.

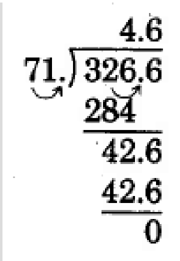
Method of Dividing a Decimal by a Decimal Number
To divide a decimal by a nonzero decimal,

1. Convert the divisor to a whole number by moving the decimal point to the position immediately to the right of the divisor's last digit.
2. Move the decimal point of the dividend to the right the same number of digits it was moved in the divisor.
3. Set the decimal point in the quotient by placing a decimal point directly above the newly located decimal point in the dividend.
4. Divide as usual.

Sample Set B

Find the following quotients.

Example:
$32.66 \div 7.1$
$7.1\overline{)32.66}$



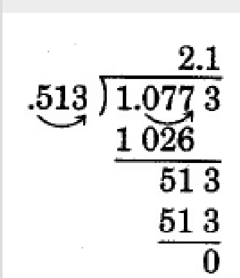
- The divisor has one decimal place.
- Move the decimal point of both the divisor and the dividend 1 place to the right.
- Set the decimal point.
- Divide as usual.

Thus, $32.66 \div 7.1 = 4.6$.
Check: $32.66 \div 7.1 = 4.6$ if $4.6 \times 7.1 = 32.66$

$$\begin{array}{r} 4.6 \\ \times 7.1 \\ 46 \\ 322 \\ 32.66 \end{array}$$

True.

Example:
$1.0773 \div 0.513$

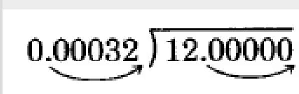


- The divisor has 3 decimal places.
- Move the decimal point of both the divisor and the dividend 3 places to the right.
- Set the decimal place and divide.

Thus, $1.0773 \div 0.513 = 2.1$.
Checking by multiplying 2.1 and 0.513 will convince us that we have obtained the correct result. (Try it.)

Example:
$12 \div 0.00032$
$0.00032\overline{)12.00000}$

- The divisor has 5 decimal places.
- Move the decimal point of both the divisor and the dividend 5 places to the right. We will need to add 5 zeros to 12.
- Set the decimal place and divide.



This is now the same as the division of whole numbers.

$$\begin{array}{r} 37500. \\ 32\overline{)1200000.} \\ 96 \\ 240 \\ 224 \\ 160 \\ 160 \\ 000 \end{array}$$

Checking assures us that $12 \div 0.00032 = 37{,}500$.

Practice Set B

Find the decimal representation of each quotient.
Exercise:

Problem: $9.176 \div 3.1$

Solution:

2.96

Exercise:

Problem: $5.0838 \div 1.11$

Solution:

4.58

Exercise:

Problem: $16 \div 0.0004$

Solution:

40,000

Exercise:

Problem: $8,162.41 \div 10$

Solution:

816.241

Exercise:

Problem: $8,162.41 \div 100$

Solution:

81.6241

Exercise:

Problem: $8{,}162.41 \div 1{,}000$

Solution:

8.16241

Exercise:

Problem: $8{,}162.41 \div 10{,}000$

Solution:

0.816241

Calculators

Calculators can be useful for finding quotients of decimal numbers. As we have seen with the other calculator operations, we can sometimes expect only approximate results. We are alerted to approximate results when the calculator display is filled with digits. We know it is possible that the operation may produce more digits than the calculator has the ability to show. For example, the multiplication

$$\underbrace{0.12345}_{\substack{\text{5 decimal}\\ \text{places}}} \times \underbrace{0.4567}_{\substack{\text{4 decimal}\\ \text{places}}}$$

produces $5 + 4 = 9$ decimal places. An eight-digit display calculator only has the ability to show eight digits, and an approximation results. The way to recognize a possible approximation is illustrated in problem 3 of the next sample set.

Sample Set C

Find each quotient using a calculator. If the result is an approximation, round to five decimal places.

Example:
$12.596 \div 4.7$

		Display Reads
Type	12.596	12.596
Press	÷	12.596
Type	4.7	4.7
Press	=	2.68

Since the display is not filled, we expect this to be an accurate result.

Example:
$0.5696376 \div 0.00123$

		Display Reads

Type	.5696376	0.5696376
Press	÷	0.5696376
Type	.00123	0.00123
Press	=	463.12

Since the display is not filled, we expect this result to be accurate.

Example:
$0.8215199 \div 4.113$

		Display Reads
Type	.8215199	0.8215199
Press	÷	0.8215199
Type	4.113	4.113
Press	=	0.1997373

There are EIGHT DIGITS — DISPLAY FILLED! BE AWARE OF POSSIBLE APPROXIMATIONS.
We can check for a possible approximation in the following way. Since the division $4\overline{)12}^{\,3}$ can be checked by multiplying 4 and 3, we can check our division by performing the multiplication

$$\underset{\text{3 decimal places}}{4.113} \times \underset{\text{7 decimal places}}{0.1997373}$$

This multiplication produces $3 + 7 = 10$ decimal digits. But our suspected quotient contains only 8 decimal digits. We conclude that the answer is an approximation. Then, rounding to five decimal places, we get 0.19974.

Practice Set C

Find each quotient using a calculator. If the result is an approximation, round to four decimal places.

Exercise:

Problem: $42.49778 \div 14.261$

Solution:

2.98

Exercise:

Problem: $0.001455 \div 0.291$

Solution:

0.005

Exercise:

Problem: $7.459085 \div 2.1192$

Solution:

3.5197645 is an approximate result. Rounding to four decimal places, we get 3.5198

Dividing Decimals By Powers of 10

In problems 4 and 5 of [link], we found the decimal representations of $8,162.41 \div 10$ and $8,162.41 \div 100$. Let's look at each of these again and then, from these observations, make a general statement regarding division of a decimal number by a power of 10.

$$
\begin{array}{r}
816.241 \\
10\overline{)8162.410} \\
80 \\
16 \\
10 \\
62 \\
60 \\
24 \\
20 \\
41 \\
40 \\
10 \\
10 \\
0
\end{array}
$$

Thus, $8,162.41 \div 10 = 816.241$.

Notice that the divisor 10 is composed of one 0 and that the quotient 816.241 can be obtained from the dividend 8,162.41 by moving the decimal point one place to the left.

$$
\begin{array}{r}
81.6241 \\
100\overline{)8162.4100} \\
\underline{800} \\
162 \\
\underline{100} \\
62\;4 \\
\underline{60\;0} \\
2\;41 \\
\underline{2\;00} \\
410 \\
\underline{400} \\
100 \\
\underline{100} \\
0
\end{array}
$$

Thus, $8{,}162.41 \div 100 = 81.6241$.

Notice that the divisor 100 is composed of two 0's and that the quotient 81.6241 can be obtained from the dividend by moving the decimal point two places to the left.

Using these observations, we can suggest the following method for dividing decimal numbers by powers of 10.

Dividing a Decimal Fraction by a Power of 10
To divide a decimal fraction by a power of 10, move the decimal point of the decimal fraction to the *left* as many places as there are zeros in the power of 10. Add zeros if necessary.

Sample Set D

Find each quotient.

Example:

$9{,}248.6 \div 100$
Since there are 2 zeros in this power of 10, we move the decimal point 2 places to the left.

$$92\ 48.6 \div 100 = 92.486$$

Example:
$3.28 \div 10{,}000$
Since there are 4 zeros in this power of 10, we move the decimal point 4 places to the left. To do so, we need to add three zeros.

$$0003.28 \div 10{,}000 = 0.000328$$

Practice Set D

Find the decimal representation of each quotient.
Exercise:

Problem: $182.5 \div 10$

Solution:

18.25

Exercise:

Problem: $182.5 \div 100$

Solution:

1.825

Exercise:

Problem: $182.5 \div 1,000$

Solution:

0.1825

Exercise:

Problem: $182.5 \div 10,000$

Solution:

0.01825

Exercise:

Problem: $646.18 \div 100$

Solution:

6.4618

Exercise:

Problem: $21.926 \div 1,000$

Solution:

0.021926

Exercises

For the following 30 problems, find the decimal representation of each quotient. Use a calculator to check each result.

Exercise:

Problem: $4.8 \div 3$

Solution:

1.6

Exercise:

Problem: $16.8 \div 8$

Exercise:

Problem: $18.5 \div 5$

Solution:

3.7

Exercise:

Problem: $12.33 \div 3$

Exercise:

Problem: $54.36 \div 9$

Solution:

6.04

Exercise:

Problem: $73.56 \div 12$

Exercise:

Problem: $159.46 \div 17$

Solution:

9.38

Exercise:

Problem: $12.16 \div 64$

Exercise:

Problem: $37.26 \div 81$

Solution:

0.46

Exercise:

Problem: $439.35 \div 435$

Exercise:

Problem: $36.98 \div 4.3$

Solution:

8.6

Exercise:

Problem: $46.41 \div 9.1$

Exercise:

Problem: $3.6 \div 1.5$

Solution:

2.4

Exercise:

Problem: $0.68 \div 1.7$

Exercise:

Problem: $50.301 \div 8.1$

Solution:

6.21

Exercise:

Problem: $2.832 \div 0.4$

Exercise:

Problem: $4.7524 \div 2.18$

Solution:

2.18

Exercise:

Problem: $16.2409 \div 4.03$

Exercise:

Problem: $1.002001 \div 1.001$

Solution:

1.001

Exercise:

Problem: $25.050025 \div 5.005$

Exercise:

Problem: $12.4 \div 3.1$

Solution:

4

Exercise:

Problem: $0.48 \div 0.08$

Exercise:

Problem: $30.24 \div 2.16$

Solution:

14

Exercise:

Problem: $48.87 \div 0.87$

Exercise:

Problem: $12.321 \div 0.111$

Solution:

111

Exercise:

Problem: $64,351.006 \div 10$

Exercise:

Problem: $64,351.006 \div 100$

Solution:

643.51006

Exercise:

Problem: $64,351.006 \div 1,000$

Exercise:

Problem: $64,351.006 \div 1,000,000$

Solution:

0.064351006

Exercise:

Problem: $0.43 \div 100$

For the following 5 problems, find each quotient. Round to the specified position. A calculator may be used.

Exercise:

Problem: $11.2944 \div 6.24$

Actual Quotient	Tenths	Hundredths	Thousandths

Solution:

Actual Quotient	Tenths	Hundredths	Thousandths
1.81	1.8	1.81	1.810

Exercise:

Problem: $45.32931 \div 9.01$

Actual Quotient	Tenths	Hundredths	Thousandths

Exercise:

Problem: $3.18186 \div 0.66$

Actual Quotient	Tenths	Hundredths	Thousandths

Solution:

Actual Quotient	Tenths	Hundredths	Thousandths
4.821	4.8	4.82	4.821

Exercise:

Problem: $4.3636 \div 4$

Actual Quotient	Tenths	Hundredths	Thousandths

Exercise:

Problem: $0.00006318 \div 0.018$

Actual Quotient	Tenths	Hundredths	Thousandths

Solution:

Actual Quotient	Tenths	Hundredths	Thousandths
0.00351	0.0	0.00	0.004

For the following 9 problems, find each solution.

Exercise:

Problem: Divide the product of 7.4 and 4.1 by 2.6.

Exercise:

Problem:

Divide the product of 11.01 and 0.003 by 2.56 and round to two decimal places.

Solution:

0.01

Exercise:

Problem:

Divide the difference of the products of 2.1 and 9.3, and 4.6 and 0.8 by 0.07 and round to one decimal place.

Exercise:

Problem:

A ring costing \$567.08 is to be paid off in equal monthly payments of \$46.84. In how many months will the ring be paid off?

Solution:

12.11 months

Exercise:

Problem: Six cans of cola cost \$2.58. What is the price of one can?

Exercise:

Problem:

A family traveled 538.56 miles in their car in one day on their vacation. If their car used 19.8 gallons of gas, how many miles per gallon did it get?

Solution:

27.2 miles per gallon

Exercise:

Problem:

Three college students decide to rent an apartment together. The rent is \$812.50 per month. How much must each person contribute toward the rent?

Exercise:

Problem:

A woman notices that on slow speed her video cassette recorder runs through 296.80 tape units in 10 minutes and at fast speed through 1098.16 tape units. How many times faster is fast speed than slow speed?

Solution:

3.7

Exercise:

Problem:

A class of 34 first semester business law students pay a total of $1,354.90, disregarding sales tax, for their law textbooks. What is the cost of each book?

Calculator Problems

For the following problems, use calculator to find the quotients. If the result is approximate (see Sample Set C [link]) round the result to three decimal places.

Exercise:

Problem: $3.8994 \div 2.01$

Solution:

1.94

Exercise:

Problem: $0.067444 \div 0.052$

Exercise:

Problem: $14{,}115.628 \div 484.74$

Solution:

29.120

Exercise:

Problem: $219,709.36 \div 9941.6$

Exercise:

Problem: $0.0852092 \div 0.49271$

Solution:

0.173

Exercise:

Problem: $2.4858225 \div 1.11611$

Exercise:

Problem: $0.123432 \div 0.1111$

Solution:

1.111

Exercise:

Problem: $2.102838 \div 1.0305$

Exercises for Review

Exercise:

Problem: ([link]) Convert $4\frac{7}{8}$ to an improper fraction.

Solution:

$\frac{39}{8}$

Exercise:

Problem: ([link]) $\frac{2}{7}$ of what number is $\frac{4}{5}$?

Exercise:

Problem: ([link]) Find the sum. $\frac{4}{15} + \frac{7}{10} + \frac{3}{5}$.

Solution:

$\frac{47}{30}$ or $1\frac{17}{30}$

Exercise:

Problem: ([link]) Round 0.01628 to the nearest ten-thousandths.

Exercise:

Problem: ([link]) Find the product (2.06)(1.39)

Solution:

2.8634

Nonterminating Divisions
This module is from Fundamentals of Mathematics by Denny Burzynski and Wade Ellis, Jr. This module discusses nonterminating divisions. By the end of the module students should understand the meaning of a nonterminating division and be able to recognize a nonterminating number by its notation.

Section Overview

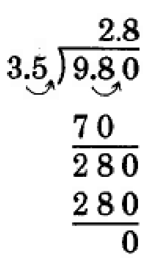
- Nonterminating Divisions
- Denoting Nonterminating Quotients

Nonterminating Divisions

Let's consider two divisions:

1. $9.8 \div 3.5$
2. $4 \div 3$

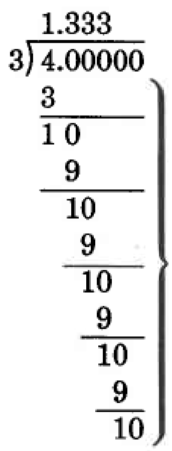
Terminating Divisions
Previously, we have considered divisions like example 1, which is an example of a terminating division. A **terminating division** is a division in which the quotient terminates after several divisions (the *remainder is zero*).



Exact Divisions
The quotient in this problem terminates in the tenths position. Terminating divisions are also called **exact divisions**.

Nonterminating Division

The division in [example 2]() is an example of a nonterminating division. A **non-terminating division** is a division that, regardless of how far we carry it out, *always has a remainder*.



Repeating Decimal
We can see that the pattern in the brace is repeated endlessly. Such a decimal quotient is called a **repeating decimal**.

Denoting Nonterminating Quotients

We use three dots at the end of a number to indicate that a pattern repeats itself endlessly.

$$4 \div 3 = 1.333\ldots$$

Another way, aside from using three dots, of denoting an endlessly repeating pattern is to write a bar (¯) above the repeating sequence of digits.

$$4 \div 3 = 1.3$$

The bar indicates the repeated pattern of 3.

Repeating patterns in a division can be discovered in two ways:

1. As the division process progresses, should the remainder ever be the same as the dividend, it can be concluded that the division is nonterminating and that the pattern in the quotient repeats. This fact is illustrated in [link] of [link].
2. As the division process progresses, should the "product, difference" pattern ever repeat two consecutive times, it can be concluded that the division is nonterminating and that the pattern in the quotient repeats. This fact is illustrated in [link] and 4 of [link].

Sample Set A

Carry out each division until the repeating pattern can be determined.

Example:

$100 \div 27$

$$\begin{array}{r} 3.70370 \\ 27\overline{)100.00000} \\ 81 \\ 19\ 0 \\ 18\ 9 \\ 100 \\ 81 \\ 190 \\ 189 \end{array}$$

When the remainder is identical to the dividend, the division is nonterminating. This implies that the pattern in the quotient repeats.

$100 \div 27 = 3.70370370\ldots$ The repeating block is 703.

$100 \div 27 = 3.703$

Example:

$1 \div 9$

$$\begin{array}{r} .111 \\ 9\overline{)1.000} \\ \underline{9} \\ 10 \\ \underline{9} \\ 10 \\ \underline{9} \\ 1 \end{array}$$

We see that this “product, difference”pattern repeats. We can conclude that the division is nonterminating and that the quotient repeats.

$1 \div 9 = 0.111\ldots$ The repeating block is 1.

$1 \div 9 = 0.1$

Example:

Divide 2 by 11 and round to 3 decimal places.

Since we wish to round the quotient to three decimal places, we'll carry out the division so that the quotient has four decimal places.

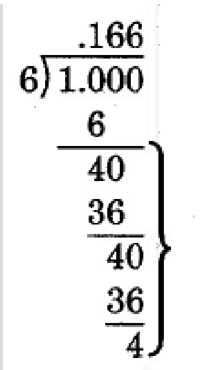
$$\begin{array}{r} .1818 \\ 11\overline{)2.0000} \\ 1\ 1 \\ 90 \\ 88 \\ 20 \\ 11 \\ 90 \end{array}$$

The number .1818 rounded to three decimal places is .182. Thus, correct to three decimal places,

$2 \div 11 = 0.182$

Example:

Divide 1 by 6.



We see that this “product, difference” pattern repeats. We can conclude that the division is nonterminating and that the quotient repeats at the 6.

$1 \div 6 = 0.16$

Practice Set A

Carry out the following divisions until the repeating pattern can be determined.

Exercise:

Problem: $1 \div 3$

Solution:

0.3

Exercise:

Problem: $5 \div 6$

Solution:

0.83

Exercise:

Problem: $11 \div 9$

Solution:

1.2

Exercise:

Problem: $17 \div 9$

Solution:

1.8

Exercise:

Problem: Divide 7 by 6 and round to 2 decimal places.

Solution:

1.17

Exercise:

Problem: Divide 400 by 11 and round to 4 decimal places.

Solution:

36.3636

Exercises

For the following 20 problems, carry out each division until the repeating pattern is determined. If a repeating pattern is not apparent, round the quotient to three decimal places.

Exercise:

Problem: $4 \div 9$

Solution:

0.4

Exercise:

Problem: $8 \div 11$

Exercise:

Problem: $4 \div 25$

Solution:

0.16

Exercise:

Problem: $5 \div 6$

Exercise:

Problem: $1 \div 7$

Solution:

0.142857

Exercise:

Problem: $3 \div 1.1$

Exercise:

Problem: $20 \div 1.9$

Solution:

10.526

Exercise:

Problem: $10 \div 2.7$

Exercise:

Problem: $1.11 \div 9.9$

Solution:

0.112

Exercise:

Problem: $8.08 \div 3.1$

Exercise:

Problem: $51 \div 8.2$

Solution:

6.21951

Exercise:

Problem: $0.213 \div 0.31$

Exercise:

Problem: $0.009 \div 1.1$

Solution:

0.0081

Exercise:

Problem: $6.03 \div 1.9$

Exercise:

Problem: $0.518 \div 0.62$

Solution:

0.835

Exercise:

Problem: $1.55 \div 0.27$

Exercise:

Problem: $0.333 \div 0.999$

Solution:

0.3

Exercise:

Problem: $0.444 \div 0.999$

Exercise:

Problem: $0.555 \div 0.27$

Solution:

2.05

Exercise:

Problem: $3.8 \div 0.99$

Calculator Problems

For the following 10 problems, use a calculator to perform each division.

Exercise:

Problem: $7 \div 9$

Solution:

0.7

Exercise:

Problem: $8 \div 11$

Exercise:

Problem: $14 \div 27$

Solution:

0.518

Exercise:

Problem: $1 \div 44$

Exercise:

Problem: $2 \div 44$

Solution:

0.045

Exercise:

Problem: $0.7 \div 0.9$ (Compare this with [link].)

Exercise:

Problem: $80 \div 110$ (Compare this with [link].)

Solution:

0.72

Exercise:

Problem: $0.0707 \div 0.7070$

Exercise:

Problem: $0.1414 \div 0.2020$

Solution:

0.7

Exercise:

Problem: $1 \div 0.9999999$

Exercise for Review

Exercise:

Problem:

([link]) In the number 411,105, how many ten thousands are there?

Solution:

1

Exercise:

Problem: ([link]) Find the quotient, if it exists. $17 \div 0$.

Exercise:

Problem: ([link]) Find the least common multiple of 45, 63, and 98.

Solution:

4410

Exercise:

Problem:

([link]) Subtract 8.01629 from 9.00187 and round the result to three decimal places.

Exercise:

Problem: ([link]) Find the quotient. $104.06 \div 12.1$.

Solution:

8.6

Converting a Fraction to a Decimal
This module is from Fundamentals of Mathematics by Denny Burzynski and Wade Ellis, Jr. This module discusses how to convert a fraction to a decimal. By the end of the module students should be able to convert a fraction to a decimal.

Now that we have studied and practiced dividing with decimals, we are also able to convert a fraction to a decimal. To do so we need only recall that a fraction bar can also be a division symbol. Thus, $\frac{3}{4}$ not only means "3 objects out of 4," but can also mean "3 divided by 4."

Sample Set A

Convert the following fractions to decimals. If the division is nonterminating, round to two decimal places.

Example:
$\frac{3}{4}$. Divide 3 by 4.

$$\begin{array}{r} .75 \\ 4\overline{)3.00} \\ 2\ 8 \\ 20 \\ 20 \\ 0 \end{array}$$

Thus, $\frac{3}{4} = 0.75$.

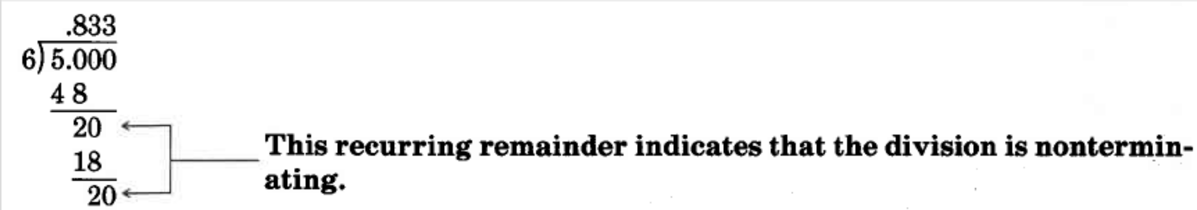
Example:
$\frac{1}{5}$ Divide 1 by 5.

$$\begin{array}{r} .2 \\ 5\overline{)1.0} \\ 1.0 \\ 0 \end{array}$$

Thus, $\frac{1}{5} = 0.2$

Example:

$\frac{5}{6}$. Divide 5 by 6.



$\frac{5}{6} = 0.833\cdots$ We are to round to two decimal places.

Thus, $\frac{5}{6} = 0.83$ to two decimal places.

Example:

$5\frac{1}{8}$. Note that $5\frac{1}{8} = 5 + \frac{1}{8}$.

Convert $\frac{1}{8}$ to a decimal.

$$\begin{array}{r} .125 \\ 8\overline{)1.000} \\ 8 \\ 20 \\ 16 \\ 40 \\ 40 \\ 0 \end{array}$$

$\frac{1}{8} = .125$

Thus, $5\frac{1}{8} = 5 + \frac{1}{8} = 5 + .125 = 5.125$.

Example:

$0.16\frac{1}{4}$. This is a complex decimal.

Note that the 6 is in the hundredths position.

The number $0.16\frac{1}{4}$ is read as "sixteen and one-fourth hundredths."

$$0.16\frac{1}{4} = \frac{16\frac{1}{4}}{100} = \frac{\frac{16\cdot 4+1}{4}}{100} = \frac{\frac{65}{4}}{\frac{100}{1}} = \frac{\overset{13}{\cancel{65}}}{4}\cdot\frac{1}{\underset{20}{\cancel{100}}} = \frac{13\cdot 1}{4\cdot 20} = \frac{13}{80}$$

Now, convert $\frac{13}{80}$ to a decimal.

$$\begin{array}{r}
.1625 \\
80\overline{)13.0000} \\
8\ 0 \\
5\ 00 \\
4\ 80 \\
200 \\
160 \\
400 \\
400 \\
0
\end{array}$$

Thus, $0.16\frac{1}{4} = 0.1625$.

Practice Set A

Convert the following fractions and complex decimals to decimals (in which no proper fractions appear). If the divison is nonterminating, round to two decimal places.

Exercise:

Problem: $\frac{1}{4}$

Solution:

0.25

Exercise:

Problem: $\frac{1}{25}$

Solution:

0.04

Exercise:

Problem: $\frac{1}{6}$

Solution:

0.17

Exercise:

Problem: $\frac{15}{16}$

Solution:

0.9375

Exercise:

Problem: $0.9\frac{1}{2}$

Solution:

0.95

Exercise:

Problem: $8.0126\frac{3}{8}$

Solution:

8.0126375

Exercises

For the following 30 problems, convert each fraction or complex decimal number to a decimal (in which no proper fractions appear).

Exercise:

Problem: $\frac{1}{2}$

Solution:

0.5

Exercise:

Problem: $\frac{4}{5}$

Exercise:

Problem: $\frac{7}{8}$

Solution:

0.875

Exercise:

Problem: $\frac{5}{8}$

Exercise:

Problem: $\frac{3}{5}$

Solution:

0.6

Exercise:

Problem: $\frac{2}{5}$

Exercise:

Problem: $\frac{1}{25}$

Solution:

0.04

Exercise:

Problem: $\frac{3}{25}$

Exercise:

Problem: $\frac{1}{20}$

Solution:

0.05

Exercise:

Problem: $\frac{1}{15}$

Exercise:

Problem: $\frac{1}{50}$

Solution:

0.02

Exercise:

Problem: $\frac{1}{75}$

Exercise:

Problem: $\frac{1}{3}$

Solution:

0.3

Exercise:

Problem: $\frac{5}{6}$

Exercise:

Problem: $\frac{3}{16}$

Solution:

0.1875

Exercise:

Problem: $\frac{9}{16}$

Exercise:

Problem: $\frac{1}{27}$

Solution:

0.037

Exercise:

Problem: $\frac{5}{27}$

Exercise:

Problem: $\frac{7}{13}$

Solution:

0.538461

Exercise:

Problem: $\frac{9}{14}$

Exercise:

Problem: $7\frac{2}{3}$

Solution:

7.6

Exercise:

Problem: $8\frac{5}{16}$

Exercise:

Problem: $1\frac{2}{15}$

Solution:

1.13

Exercise:

Problem: $65\frac{5}{22}$

Exercise:

Problem: $101\frac{6}{25}$

Solution:

101.24

Exercise:

Problem: $0.1\frac{1}{2}$

Exercise:

Problem: $0.24\frac{1}{8}$

Solution:

0.24125

Exercise:

Problem: $5.66\frac{2}{3}$

Exercise:

Problem: $810.3106\frac{5}{16}$

Solution:

810.31063125

Exercise:

Problem: $4.1\frac{1}{9}$

For the following 18 problems, convert each fraction to a decimal. Round to five decimal places.

Exercise:

Problem: $\frac{1}{9}$

Solution:

0.11111

Exercise:

Problem: $\frac{2}{9}$

Exercise:

Problem: $\frac{3}{9}$

Solution:

0.33333

Exercise:

Problem: $\frac{4}{9}$

Exercise:

Problem: $\frac{5}{9}$

Solution:

0.55556

Exercise:

Problem: $\frac{6}{9}$

Exercise:

Problem: $\frac{7}{9}$

Solution:

0.77778

Exercise:

Problem: $\frac{8}{9}$

Exercise:

Problem: $\frac{1}{11}$

Solution:

0.09091

Exercise:

Problem: $\frac{2}{11}$

Exercise:

Problem: $\frac{3}{11}$

Solution:

0.27273

Exercise:

Problem: $\frac{4}{11}$

Exercise:

Problem: $\frac{5}{11}$

Solution:

0.45455

Exercise:

Problem: $\frac{6}{11}$

Exercise:

Problem: $\frac{7}{11}$

Solution:

0.63636

Exercise:

Problem: $\frac{8}{11}$

Exercise:

Problem: $\frac{9}{11}$

Solution:

0.81818

Exercise:

Problem: $\frac{10}{11}$

Calculator Problems

For the following problems, use a calculator to convert each fraction to a decimal. If no repeating pattern seems to exist, round to four decimal places.

Exercise:

Problem: $\frac{16}{125}$

Solution:

0.128

Exercise:

Problem: $\frac{85}{311}$

Exercise:

Problem: $\frac{192}{197}$

Solution:

0.9746

Exercise:

Problem: $\frac{1}{1469}$

Exercise:

Problem: $\frac{4}{21{,}015}$

Solution:

0.0002

Exercise:

Problem: $\frac{81,426}{106,001}$

Exercise:

Problem: $\frac{16,501}{426}$

Solution:

38.7347

Exercises for Review

Exercise:

Problem: ([link]) Round 2,105,106 to the nearest hundred thousand.

Exercise:

Problem: ([link]) $\frac{8}{5}$ of what number is $\frac{3}{2}$?

Solution:

$\frac{15}{16}$

Exercise:

Problem: ([link]) Arrange $1\frac{9}{16}$, $1\frac{5}{8}$, and $1\frac{7}{12}$ in increasing order.

Exercise:

Problem: ([link]) Convert the complex decimal $3.6\frac{5}{4}$ to a fraction.

Solution:

$3\frac{29}{40}$ or 3.725

Exercise:

Problem: ([link]) Find the quotient. $30 \div 1.1$.

Combinations of Operations with Decimals and Fractions
This module is from Fundamentals of Mathematics by Denny Burzynski and Wade Ellis, Jr. This module discusses combinations of operations with decimals and fractions. By the end of the module students should be able to combine operations with decimals.

Having considered operations with decimals and fractions, we now consider operations that involve both decimals and fractions.

Sample Set A

Perform the following operations.

Example:
$0.38 \cdot \frac{1}{4}$. Convert both numbers to decimals or both numbers to fractions. We'll convert to decimals.

$$\begin{array}{r} .25 \\ 4\overline{)1.00} \\ \underline{8} \\ 20 \\ \underline{20} \\ 0 \end{array}$$

To convert $\frac{1}{4}$ to a decimal, divide 1 by 4.
Now multiply 0.38 and .25.

$\frac{1}{4}$

$$\begin{array}{r} .38 \\ \underline{\times .25} \\ 190 \\ \underline{76} \\ .0950 \end{array}$$

Thus, $0.38 \cdot \frac{1}{4} = 0.095$.
In the problems that follow, the conversions from fraction to decimal, or decimal to fraction, and some of the additions, subtraction, multiplications, and divisions will be left to you.

Example:
$1.85 + \frac{3}{8} \cdot 4.1$ Convert $\frac{3}{8}$ to a decimal.
$1.85 + 0.375 \cdot 4.1$ Multiply before adding.
$1.85 + 1.5375$ Now add.
3.3875

Example:

$$\frac{5}{13}\left(\frac{4}{5} - 0.28\right) \text{ Convert 0.28 to a fraction.}$$

$$\begin{aligned}
\frac{5}{13}\left(\frac{4}{5} - \frac{28}{100}\right) &= \frac{5}{13}\left(\frac{4}{5} - \frac{7}{25}\right) \\
&= \frac{5}{13}\left(\frac{20}{25} - \frac{7}{25}\right) \\
&= \frac{\overset{1}{\cancel{5}}}{\underset{1}{\cancel{13}}} \cdot \frac{\overset{1}{\cancel{13}}}{\underset{5}{\cancel{25}}} \\
&= \frac{1}{5}
\end{aligned}$$

Example:

$$\begin{aligned}
\frac{0.125}{1\frac{1}{3}} + \frac{1}{16} - 0.1211 &= \frac{\frac{125}{1000}}{\frac{4}{3}} + \frac{1}{16} - 0.1211 \\
&= \frac{\frac{1}{8}}{\frac{4}{3}} + \frac{1}{16} - 0.1211 \\
&= \frac{1}{8} \cdot \frac{3}{4} + \frac{1}{16} - 0.1211 \\
&= \frac{3}{32} + \frac{1}{16} - 0.1211 \\
&= \frac{3}{32} + \frac{2}{32} - 0.1211 = \frac{5}{32} - 0.1211 \\
&= 0.15625 - 0.1211 \\
&= 0.03515 && \text{Convert this to fraction form} \\
&= \frac{3515}{100{,}000} \\
&= \frac{703}{20{,}000}
\end{aligned}$$

Practice Set A

Perform the following operations.

Exercise:

Problem: $\frac{3}{5} + 1.6$

Solution:

2.2 or $2\frac{1}{5}$

Exercise:

Problem: $8.91 + \frac{1}{5} \cdot 1.6$

Solution:

9.23

Exercise:

Problem: $1\frac{9}{16}\left(6.12+\frac{7}{25}\right)$

Solution:

10

Exercise:

Problem: $\frac{0.156}{1\frac{11}{15}}-0.05$

Solution:

$\frac{1}{25}$ or 0.04

Exercises

Exercise:

Problem: $\frac{3}{10}+0.7$

Solution:

1

Exercise:

Problem: $\frac{1}{5}+0.1$

Exercise:

Problem: $\frac{5}{8}-0.513$

Solution:

0.112

Exercise:

Problem: $0.418-\frac{67}{200}$

Exercise:

Problem: $0.22\cdot\frac{1}{4}$

Solution:

0.055

Exercise:

Problem: $\frac{3}{5} \cdot 8.4$

Exercise:

Problem: $\frac{1}{25} \cdot 3.19$

Solution:

0.1276

Exercise:

Problem: $\frac{3}{20} \div 0.05$

Exercise:

Problem: $\frac{7}{40} \div 0.25$

Solution:

0.7

Exercise:

Problem: $1\frac{1}{15} \div 0.9 \cdot 0.12$

Exercise:

Problem: $9.26 + \frac{1}{4} \cdot 0.81$

Solution:

9.4625

Exercise:

Problem: $0.588 + \frac{1}{40} \cdot 0.24$

Exercise:

Problem: $\frac{1}{20} + 3.62 \cdot \frac{3}{8}$

Solution:

1.4075

Exercise:

Problem: $7 + 0.15 \div \frac{3}{30}$

Exercise:

Problem: $\frac{15}{16} \cdot \left(\frac{7}{10} - 0.5\right)$

Solution:

0.1875

Exercise:

Problem: $0.2 \cdot \left(\frac{7}{20} + 1.1143\right)$

Exercise:

Problem: $\frac{3}{4} \cdot \left(0.875 + \frac{1}{8}\right)$

Solution:

0.75

Exercise:

Problem: $5.198 - 0.26 \cdot \left(\frac{14}{250} + 0.119\right)$

Exercise:

Problem: $0.5\frac{1}{4} + (0.3)^2$

Solution:

0.615

Exercise:

Problem: $(1.4)^2 - 1.6\frac{1}{2}$

Exercise:

Problem: $\left(\frac{3}{8}\right)^2 - 0.000625 + (1.1)^2$

Solution:

1.35

Exercise:

Problem: $(0.6)^2 \cdot \left(\frac{1}{20} - \frac{1}{25}\right)$

Exercise:

Problem: $\left(\frac{1}{2}\right)^2 - 0.125$

Solution:

0.125

Exercise:

Problem: $\frac{0.75}{4\frac{1}{2}} + \frac{5}{12}$

Exercise:

Problem: $\left(\frac{0.375}{2\frac{1}{16}} - \frac{1}{33}\right)$

Solution:

$0.\overline{15}$

Exercise:

Problem: $8\frac{1}{3} \cdot \left(\frac{1\frac{1}{4}}{2.25} + \frac{9}{25}\right)$

Exercise:

Problem: $\frac{\frac{0.32}{\frac{12}{35}}}{0.35}$

Solution:

$2.\overline{6}$

Exercise:

Problem: $\frac{\left(\sqrt{\frac{49}{64}}-5\right)0.125}{1.375}$

Exercises for Review

Exercise:

Problem: ([link]) Is 21,480 divisible by 3?

Solution:

yes

Exercise:

Problem: ([link]) Expand 14^4. Do not find the actual value.

Exercise:

Problem: ([link]) Find the prime factorization of 15,400.

Solution:

$2^3 \cdot 5^2 \cdot 7 \cdot 11$

Exercise:

Problem: ([link]) Convert 8.016 to a fraction.

Exercise:

Problem: ([link]) Find the quotient. $16 \div 27$.

Solution:

$0.\overline{592}$

Summary of Key Concepts
This module is from Fundamentals of Mathematics by Denny Burzynski and Wade Ellis, Jr. This module reviews the key concepts from the chapter "Decimals."

Summary of Key Concepts

Decimal Point ([link])
A *decimal point* is a point that separates the units digit from the tenths digit.

Decimal or Decimal Fraction ([link])
A *decimal fraction* is a fraction whose denominator is a power of ten.

Converting a Decimal to a Fraction ([link])
Decimals can be converted to fractions by saying the decimal number in words, then writing what was said.

Rounding Decimals ([link])
Decimals are rounded in much the same way whole numbers are rounded.

Addition and Subtraction of Decimals ([link])
To add or subtract decimals,

1. Align the numbers vertically so that the decimal points line up under each other and the corresponding decimal positions are in the same column.
2. Add or subtract the numbers as if they were whole numbers.
3. Place a decimal point in the resulting sum directly under the other decimal points.

Multiplication of Decimals ([link])
To multiply two decimals,

1. Multiply the numbers as if they were whole numbers.
2. Find the sum of the number of decimal places in the factors.
3. The number of decimal places in the product is the number found in step 2.

Multiplying Decimals by Powers of 10 ([link])
To multiply a decimal by a power of 10, move the decimal point to the right as many places as there are zeros in the power of ten. Add zeros if necessary.

Division of a Decimal by a Decimal ([link])
To divide a decimal by a nonzero decimal,

1. Convert the divisor to a whole number by moving the decimal point until it appears to the right of the divisor's last digit.
2. Move the decimal point of the dividend to the right the same number of digits it was moved in the divisor.
3. Proceed to divide.
4. Locate the decimal in the answer by bringing it straight up from the dividend.

Dividing Decimals by Powers of 10 ([link])
To divide a decimal by a power of 10, move the decimal point to the left as many places as there are zeros in the power of ten. Add zeros if necessary.

Terminating Divisions ([link])
A *terminating division* is a division in which the quotient terminates after several divisions. Terminating divisions are also called exact divisions.

Nonterminating Divisions ([link])
A *nonterminating division* is a division that, regardless of how far it is carried out, always has a remainder. Nonterminating divisions are also called nonexact divisions.

Converting Fractions to Decimals ([link])
A fraction can be converted to a decimal by dividing the numerator by the denominator.

Exercise Supplement
This module is from Fundamentals of Mathematics by Denny Burzynski and Wade Ellis, Jr. This module is an exercise supplement for the chapter "Decimals" and contains many exercise problems. Odd problems are accompanied by solutions.

Exercise Supplement

Reading and Writing Decimals ([link])

Exercise:

Problem:

The decimal digit that appears two places to the right of the decimal point is in the position.

Solution:

hundredths

Exercise:

Problem:

The decimal digit that appears four places to the right of the decimal point is in the position.

For problems 3-8, read each decimal by writing it in words.

Exercise:

Problem: 7.2

Solution:

seven and two tenths

Exercise:

Problem: 8.105

Exercise:

Problem: 16.52

Solution:

sixteen and fifty-two hundredths

Exercise:

Problem: 5.9271

Exercise:

Problem: 0.005

Solution:

five thousandths

Exercise:

Problem: 4.01701

For problems 9-13, write each decimal using digits.

Exercise:

Problem: Nine and twelve-hundredths.

Solution:

9.12

Exercise:

Problem: Two and one hundred seventy-seven thousandths.

Exercise:

Problem: Fifty-six and thirty-five ten-thousandths.

Solution:

56.0035

Exercise:

Problem: Four tenths.

Exercise:

Problem: Four thousand eighty-one millionths.

Solution:

0.004081

Converting a Decimal to a Fraction ([link])

For problem 14-20, convert each decimal to a proper fraction or a mixed number.

Exercise:

Problem: 1.07

Exercise:

Problem: 58.63

Solution:

$85\frac{63}{100}$

Exercise:

Problem: 0.05

Exercise:

Problem: $0.14\frac{2}{3}$

Solution:

$\frac{11}{75}$

Exercise:

Problem: $1.09\frac{1}{8}$

Exercise:

Problem: $4.01\frac{1}{27}$

Solution:

$4\frac{7}{675}$

Exercise:

Problem: $9.11\frac{1}{9}$

Rounding Decimals ([link])

For problems 21-25, round each decimal to the specified position.

Exercise:

Problem: 4.087 to the nearest hundredth.

Solution:

4.09

Exercise:

Problem: 4.087 to the nearest tenth.

Exercise:

Problem: 16.5218 to the nearest one.

Solution:

17

Exercise:

Problem: 817.42 to the nearest ten.

Exercise:

Problem: 0.9811602 to the nearest one.

Solution:

1

Addition, Subtraction, Multiplication and Division of Decimals, and Nonterminating Divisions ([link],[link],[link],[link])

For problem 26-45, perform each operation and simplify.

Exercise:

Problem: $7.10 + 2.98$

Exercise:

Problem: $14.007 - 5.061$

Solution:

8.946

Exercise:

Problem: $1.2 \cdot 8.6$

Exercise:

Problem: $41.8 \cdot 0.19$

Solution:

7.942

Exercise:

Problem: $57.51 \div 2.7$

Exercise:

Problem: $0.54003 \div 18.001$

Solution:

0.03

Exercise:

Problem: $32{,}051.3585 \div 23{,}006.9999$

Exercise:

Problem: $100 \cdot 1{,}816.001$

Solution:

$181{,}600.1$

Exercise:

Problem: $1{,}000 \cdot 1{,}816.001$

Exercise:

Problem: $10.000 \cdot 0.14$

Solution:

1.4

Exercise:

Problem: $0.135888 \div 16.986$

Exercise:

Problem: $150.79 \div 100$

Solution:

1.5079

Exercise:

Problem: $4.119 \div 10{,}000$

Exercise:

Problem: $42.7 \div 18$

Solution:

2.372

Exercise:

Problem: $6.9 \div 12$

Exercise:

Problem: $0.014 \div 47.6$. Round to three decimal places.

Solution:

0.000

Exercise:

Problem: $8.8 \div 19$. Round to one decimal place.

Exercise:

Problem: $1.1 \div 9$

Solution:

0.12

Exercise:

Problem: $1.1 \div 9.9$

Exercise:

Problem: $30 \div 11.1$

Solution:

2.702

Converting a Fraction to a Decimal ([link])

For problems 46-55, convert each fraction to a decimal.
Exercise:

Problem: $\frac{3}{8}$

Exercise:

Problem: $\frac{43}{100}$

Solution:

0.43

Exercise:

Problem: $\frac{82}{1000}$

Exercise:

Problem: $9\frac{4}{7}$

Solution:

9.571428

Exercise:

Problem: $8\frac{5}{16}$

Exercise:

Problem: $1.3\frac{1}{3}$

Solution:

1.3

Exercise:

Problem: $25.6\frac{2}{3}$

Exercise:

Problem: $125.125\frac{1}{8}$

Solution:

125.125125 (not repeating)

Exercise:

Problem: $9.11\frac{1}{9}$

Exercise:

Problem: $0.0\frac{5}{6}$

Solution:

0.083

Combinations of Operations with Decimals and Fractions ([link])

For problems 56-62, perform each operation.
Exercise:

Problem: $\frac{5}{8} \cdot 0.25$

Exercise:

Problem: $\frac{3}{16} \cdot 1.36$

Solution:

0.255

Exercise:

Problem: $\frac{3}{5} \cdot \left(\frac{1}{2} + 1.75\right)$

Exercise:

Problem: $\frac{7}{2} \cdot \left(\frac{5}{4} + 0.30\right)$

Solution:

5.425

Exercise:

Problem: $19.375 \div \left(4.375 - 1\frac{1}{16}\right)$

Exercise:

Problem: $\frac{15}{602} \cdot \quad 2.6 + 3\frac{1}{4}$

Solution:

0.09343

Exercise:

Problem: $4\frac{13}{18} \div \left(5\frac{3}{14} + 3\frac{5}{21}\right)$

Proficiency Exam
This module is from Fundamentals of Mathematics by Denny Burzynski and Wade Ellis, Jr. This module is a proficiency exam to the chapter "Decimals." Each problem is accompanied with a reference link pointing back to the module that discusses the type of problem demonstrated in the question. The problems in this exam are accompanied by solutions.

Proficiency Exam

Exercise:

Problem:

([link]) The decimal digit that appears three places to the right of the decimal point is in the position.

Solution:

thousandth

Exercise:

Problem: ([link]) Write, using words, 15.036.

Solution:

fifteen and thirty-six thousandths

Exercise:

Problem:

([link]) Write eighty-one and twelve hundredths using digits. 81.12

Solution:

81.12

Exercise:

Problem:

([link]) Write three thousand seventeen millionths using digits.

Solution:

0.003017

Exercise:

Problem: ([link]) Convert 0.78 to a fraction. Reduce.

Solution:

$\frac{39}{50}$

Exercise:

Problem: ([link]) Convert 0.875 to a fraction. Reduce.

Solution:

$\frac{7}{8}$

Exercise:

Problem: ([link]) Round 4.8063 to the nearest tenth.

Solution:

4.8

Exercise:

Problem: ([link]) Round 187.51 to the nearest hundred.

Solution:

200

Exercise:

Problem: ([link]) Round 0.0652 to the nearest hundredth.

Solution:

0.07

For problems 10-20, perform each operation.

Exercise:

Problem: ([link]) $15.026 + 5.971$

Solution:

20.997

Exercise:

Problem: ([link]) $72.15 - 26.585$

Solution:

45.565

Exercise:

Problem: ([link]) $16.2 \cdot 4.8$

Solution:

77.76

Exercise:

Problem: ([link]) $10{,}000 \cdot 0.016$

Solution:

16

Exercise:

Problem: ([link]) $44.64 \div 18.6$

Solution:

2.4

Exercise:

Problem: ([link]) $0.21387 \div 0.19$

Solution:

1.1256

Exercise:

Problem: ([link]) $0.\overline{27} - \frac{3}{11}$

Solution:

0

Exercise:

Problem: ([link]) Convert $6\frac{2}{11}$ to a decimal.

Solution:

$6.\overline{18}$

Exercise:

Problem: ([link]) Convert $0.5\frac{9}{16}$ to a decimal.

Solution:

0.055625

Exercise:

Problem: ([link]) $3\frac{1}{8} + 2.325$

Solution:

5.45

Exercise:

Problem: ([link]) $\frac{3}{8} \times 0.5625$

Solution:

$\frac{27}{128}$ or 0.2109375

Objectives
This module contains the learning objectives for the chapter "Ratios and Rates" from Fundamentals of Mathematics by Denny Burzynski and Wade Ellis, jr.

After completing this chapter, you should

Ratios and Rates ([link])

- be able to distinguish between denominate and pure numbers and between ratios and rates

Proportions ([link])

- be able to describe proportions and find the missing factor in a proportion
- be able to work with proportions involving rates

Applications of Proportions ([link])

- solve proportion problems using the five-step method

Percent ([link])

- understand the relationship between ratios and percents
- be able to make conversions between fractions, decimals, and percents

Fractions of One Percent ([link])

- understand the meaning of a fraction of one percent
- be able to make conversions involving fractions of one percent

Applications of Percents ([link])

- be able to distinguish between base, percent, and percentage
- be able to find the percentage, the percent, and the base

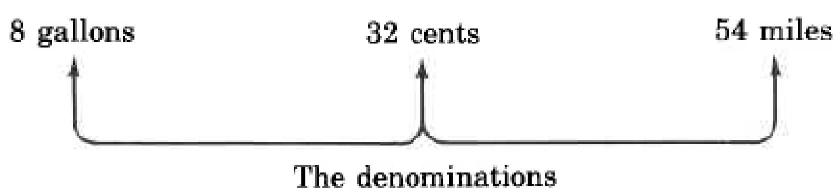
Ratios and Rates
This module is from Fundamentals of Mathematics by Denny Burzynski and Wade Ellis, Jr. This module discusses ratios and rates. By the end of the module students should be able to distinguish between denominate and pure numbers and between ratios and rates.

Section Overview

- Denominate Numbers and Pure Numbers
- Ratios and Rates

Denominate Numbers and Pure Numbers

Denominate Numbers, Like and Unlike Denominate Numbers
It is often necessary or convenient to compare two quantities**. Denominate numbers** are numbers together with some specified unit. If the units being compared are alike, the denominate numbers are called **like denominate numbers**. If units are not alike, the numbers are called **unlike denominate numbers**. Examples of denominate numbers are shown in the diagram:



Pure Numbers
Numbers that exist purely as numbers and do *not* represent amounts of quantities are called **pure numbers**. Examples of pure numbers are 8, 254, 0, $21\frac{5}{8}$, $\frac{2}{5}$, and 0.07.

Numbers can be *compared* in two ways: subtraction and division.

Comparing Numbers by Subtraction and Division
Comparison of two numbers by subtraction indicates how *much more* one number is than another.

Comparison by division indicates how *many times* larger or smaller one number is than another.

Comparing Pure or Like Denominate Numbers by Subtraction
Numbers can be compared by subtraction if and only if they both are like denominate numbers or both pure numbers.

Sample Set A

Example:
Compare 8 miles and 3 miles by subtraction.
$8 \text{ mile} - 3 \text{ miles} = 5 \text{ miles}$
This means that 8 miles is 5 miles more than 3 miles.
Examples of use: I can now jog 8 miles whereas I used to jog only 3 miles. So, I can now jog 5 miles more than I used to.

Example:
Compare 12 and 5 by subtraction.
$12 - 5 = 7$
This means that 12 is 7 more than 5.

Example:
Comparing 8 miles and 5 gallons by subtraction makes no sense.
$8 \text{ miles} - 5 \text{ gallons} = ?$

Example:
Compare 36 and 4 by division.
$36 \div 4 = 9$

This means that 36 is 9 times as large as 4. Recall that $36 \div 4 = 9$ can be expressed as $\frac{36}{4} = 9$.

Example:
Compare 8 miles and 2 miles by division.
$\frac{8 \text{ miles}}{2 \text{ miles}} = 4$
This means that 8 miles is 4 times as large as 2 miles.
Example of use: I can jog 8 miles to your 2 miles. Or, for every 2 miles that you jog, I jog 8. So, I jog 4 times as many miles as you jog.
Notice that when like quantities are being compared by division, we drop the units. Another way of looking at this is that the units divide out (cancel).

Example:
Compare 30 miles and 2 gallons by division.
$\frac{30 \text{ miles}}{2 \text{ gallons}} = \frac{15 \text{ miles}}{1 \text{ gallon}}$
Example of use: A particular car goes 30 miles on 2 gallons of gasoline. This is the same as getting 15 miles to 1 gallon of gasoline.
Notice that when the quantities being compared by division are unlike quantities, we do not drop the units.

Practice Set A

Make the following comparisons and interpret each one.
Exercise:

Problem: Compare 10 diskettes to 2 diskettes by

a. subtraction:
b. division:

Solution:

a. 8 diskettes; 10 diskettes is 8 diskettes more than 2 diskettes.
b. 5; 10 diskettes is 5 times as many diskettes as 2 diskettes.

Exercise:

Problem: Compare, if possible, 16 bananas and 2 bags by

a. subtraction:
b. division:

Solution:

a. Comparison by subtraction makes no sense.
b. $\frac{16 \text{ bananas}}{2 \text{ bags}} = \frac{8 \text{ bananas}}{\text{bag}}$, 8 bananas per bag.

Ratios and Rates

Ratio

A comparison, by division, of two pure numbers or two like denominate numbers is a **ratio**.

The comparison by division of the pure numbers $\frac{36}{4}$ and the like denominate numbers $\frac{8 \text{ miles}}{2 \text{ miles}}$ are examples of ratios.

Rate

A comparison, by division, of two unlike denominate numbers is a **rate**.

The comparison by division of two unlike denominate numbers, such as

$\frac{55 \text{ miles}}{1 \text{ gallon}}$ and $\frac{40 \text{ dollars}}{5 \text{ tickets}}$

are examples of rates.

Let's agree to represent two numbers (pure or denominate) with the letters a and b. This means that we're letting a represent some number and b represent some, perhaps different, number. With this agreement, we can write the ratio of the two numbers a and b as

$\frac{a}{b}$ or $\frac{b}{a}$

The ratio $\frac{a}{b}$ is read as " a to b."

The ratio $\frac{b}{a}$ is read as " b to a."

Since a ratio or a rate can be expressed as a fraction, it may be reducible.

Sample Set B

Example:
The ratio 30 to 2 can be expressed as $\frac{30}{2}$. Reducing, we get $\frac{15}{1}$.
The ratio 30 to 2 is *equivalent* to the ratio 15 to 1.

Example:
The rate "4 televisions to 12 people" can be expressed as $\frac{\text{4 televisions}}{\text{12 people}}$. The meaning of this rate is that "for every 4 televisions, there are 12 people."
Reducing, we get $\frac{\text{1 television}}{\text{3 people}}$. The meaning of this rate is that "for every 1 television, there are 3 people.”
Thus, the rate of "4 televisions to 12 people" is the *same* as the rate of "1 television to 3 people."

Practice Set B

Write the following ratios and rates as fractions.

Exercise:

Problem: 3 to 2

Solution:

$\frac{3}{2}$

Exercise:

Problem: 1 to 9

Solution:

$\frac{1}{9}$

Exercise:

Problem: 5 books to 4 people

Solution:

$\frac{5 \text{ books}}{4 \text{ people}}$

Exercise:

Problem: 120 miles to 2 hours

Solution:

$\frac{60 \text{ miles}}{1 \text{ hour}}$

Exercise:

Problem: 8 liters to 3 liters

Solution:

$\frac{8}{3}$

Write the following ratios and rates in the form "a to b." Reduce when necessary.

Exercise:

Problem: $\frac{9}{5}$

Solution:

9 to 5

Exercise:

Problem: $\frac{1}{3}$

Solution:

1 to 3

Exercise:

Problem: $\frac{25 \text{ miles}}{2 \text{ gallons}}$

Solution:

25 miles to 2 gallons

Exercise:

Problem: $\frac{2 \text{ mechanics}}{4 \text{ wrenches}}$

Solution:

1 mechanic to 2 wrenches

Exercise:

Problem: $\frac{15 \text{ video tapes}}{18 \text{ video tapes}}$

Solution:

5 to 6

Exercises

For the following 9 problems, complete the statements.

Exercise:

Problem:

Two numbers can be compared by subtraction if and only if .

Solution:

They are pure numbers or like denominate numbers.

Exercise:

Problem:

A comparison, by division, of two pure numbers or two like denominate numbers is called a .

Exercise:

Problem:

A comparison, by division, of two unlike denominate numbers is called a .

Solution:

rate

Exercise:

Problem: $\frac{6}{11}$ is an example of a . (ratio/rate)

Exercise:

Problem: $\frac{5}{12}$ is an example of a . (ratio/rate)

Solution:

ratio

Exercise:

Problem: $\frac{7\text{ erasers}}{12\text{ pencils}}$ is an example of a . (ratio/rate)

Exercise:

Problem: $\frac{20\text{ silver coins}}{35\text{ gold coins}}$ is an example of a .(ratio/rate)

Solution:

rate

Exercise:

Problem: $\frac{3\text{ sprinklers}}{5\text{ sprinklers}}$ is an example of a . (ratio/rate)

Exercise:

Problem: $\frac{18\text{ exhaust valves}}{11\text{ exhaust valves}}$ is an example of a .(ratio/rate)

Solution:

ratio

For the following 7 problems, write each ratio or rate as a verbal phrase.

Exercise:

Problem: $\frac{8}{3}$

Exercise:

Problem: $\frac{2}{5}$

Solution:

two to five

Exercise:

Problem: $\frac{8\text{ feet}}{3\text{ seconds}}$

Exercise:

Problem: $\frac{29\text{ miles}}{2\text{ gallons}}$

Solution:

29 mile per 2 gallons or $14\frac{1}{2}$ miles per 1 gallon

Exercise:

Problem: $\frac{30{,}000\text{ stars}}{300\text{ stars}}$

Exercise:

Problem: $\frac{5\text{ yards}}{2\text{ yards}}$

Solution:

5 to 2

Exercise:

Problem: $\frac{164\text{ trees}}{28\text{ trees}}$

For the following problems, write the simplified fractional form of each ratio or rate.

Exercise:

Problem: 12 to 5

Solution:

$\frac{12}{5}$

Exercise:

Problem: 81 to 19

Exercise:

Problem: 42 plants to 5 homes

Solution:

$\frac{42\text{ plants}}{5\text{ homes}}$

Exercise:

Problem: 8 books to 7 desks

Exercise:

Problem: 16 pints to 1 quart

Solution:

$\frac{16 \text{ pints}}{1 \text{ quart}}$

Exercise:

Problem: 4 quarts to 1 gallon

Exercise:

Problem: 2.54 cm to 1 in

Solution:

$\frac{2.54 \text{ cm}}{1 \text{ inch}}$

Exercise:

Problem: 80 tables to 18 tables

Exercise:

Problem: 25 cars to 10 cars

Solution:

$\frac{5}{2}$

Exercise:

Problem: 37 wins to 16 losses

Exercise:

Problem: 105 hits to 315 at bats

Solution:

$\frac{1 \text{ hit}}{3 \text{ at bats}}$

Exercise:

Problem: 510 miles to 22 gallons

Exercise:

Problem: 1,042 characters to 1 page

Solution:

$\frac{1,042 \text{ characters}}{1 \text{ page}}$

Exercise:

Problem: 1,245 pages to 2 books

Exercises for Review

Exercise:

Problem: ([link]) Convert $\frac{16}{3}$ to a mixed number.

Solution:

$5\frac{1}{3}$

Exercise:

Problem: ([link]) $1\frac{5}{9}$ of $2\frac{4}{7}$ is what number?

Exercise:

Problem: ([link]) Find the difference. $\frac{11}{28} - \frac{7}{45}$.

Solution:

$\frac{299}{1260}$

Exercise:

Problem:

([link]) Perform the division. If no repeating patterns seems to exist, round the quotient to three decimal places: $22.35 \div 17$

Exercise:

Problem: ([link]) Find the value of $1.85 + \frac{3}{8} \cdot 4.1$

Solution:

3.3875

Proportions

This module is from Fundamentals of Mathematics by Denny Burzynski and Wade Ellis, Jr. This module discusses proportions. By the end of the module students should be able to describe proportions and find the missing factor in a proportion and be able to work with proportions involving rates.

Section Overview

- Ratios, Rates, and Proportions
- Finding the Missing Factor in a Proportion
- Proportions Involving Rates

Ratios, Rates, and Proportions

Ratio, Rate

We have defined a **ratio** as a comparison, by division, of two pure numbers or two *like* denominate numbers. We have defined a **rate** as a comparison, by division, of two *unlike* denominate numbers.

Proportion

A **proportion** is a statement that two ratios or rates are equal. The following two examples show how to read proportions.

$$\frac{3}{4} = \frac{6}{8}$$

3 is to 4 as 6 is to 8

$$\frac{25 \text{ miles}}{1 \text{ gallon}} = \frac{50 \text{ miles}}{2 \text{ gallons}}$$

25 miles is to 1 gallon as 50 miles is to 2 gallons

Sample Set A

Write or read each proportion.

Example:
$\frac{3}{5} = \frac{12}{20}$
3 is to 5 as 12 is to 20

Example:
$\frac{10 \text{ items}}{5 \text{ dollars}} = \frac{2 \text{ items}}{1 \text{ dollar}}$
10 items is to 5 dollars as 2 items is to 1 dollar

Example:
8 is to 12 as 16 is to 24.
$\frac{8}{12} = \frac{16}{24}$

Example:
50 milligrams of vitamin C is to 1 tablet as 300 milligrams of vitamin C is to 6 tablets.
$\frac{50}{1} = \frac{300}{6}$

Practice Set A

Write or read each proportion.

Exercise:

Problem: $\frac{3}{8} = \frac{6}{16}$

Solution:

3 is to 8 as 6 is to 16

Exercise:

Problem: $\frac{\text{2 people}}{\text{1 window}} = \frac{\text{10 people}}{\text{5 windows}}$

Solution:

2 people are to 1 window as 10 people are to 5 windows

Exercise:

Problem: 15 is to 4 as 75 is to 20.

Solution:

$\frac{15}{4} = \frac{75}{20}$

Exercise:

Problem: 2 plates are to 1 tray as 20 plates are to 10 trays.

Solution:

$\frac{\text{2 plates}}{\text{1 tray}} = \frac{\text{20 plates}}{\text{10 trays}}$

Finding the Missing Factor in a Proportion

Many practical problems can be solved by writing the given information as proportions. Such proportions will be composed of three specified numbers and one unknown number. It is customary to let a letter, such as x, represent the unknown number. An example of such a proportion is

$\frac{x}{4} = \frac{20}{16}$

This proportion is read as " x is to 4 as 20 is to 16."

There is a method of solving these proportions that is based on the equality of fractions. Recall that two fractions are equivalent if and only if their

cross products are equal. For example,

$$\frac{3}{4} = \frac{6}{8} \quad \text{since} \quad \begin{array}{c} \frac{3}{4} \quad \frac{6}{8} \\ 3 \cdot 8 = 6 \cdot 4 \\ 24 = 24 \end{array}$$

Notice that in a proportion that contains three specified numbers and a letter representing an unknown quantity, that regardless of where the letter appears, the following situation always occurs.

$$\underbrace{(\text{number}) \cdot (\text{letter}) = (\text{number}) \cdot (\text{number})}$$

We recognize this as a multiplication statement. Specifically, it is a missing factor statement. (See [link] for a discussion of multiplication statements.) For example,

$$\begin{array}{lll} \frac{x}{4} = \frac{20}{16} & \text{means that} & 16 \cdot x = 4 \cdot 20 \\ \frac{4}{x} = \frac{16}{20} & \text{means that} & 4 \cdot 20 = 16 \cdot x \\ \frac{5}{4} = \frac{x}{16} & \text{means that} & 5 \cdot 16 = 4 \cdot x \\ \frac{5}{4} = \frac{20}{x} & \text{means that} & 5 \cdot x = 4 \cdot 20 \end{array}$$

Each of these statements is a multiplication statement. Specifically, each is a missing factor statement. (The letter used here is x, whereas M was used in [link].)

Finding the Missing Factor in a Proportion
The missing factor in a missing factor statement can be determined by dividing the product by the known factor, that is, if x represents the missing factor, then
$x = (\text{product}) \div (\text{known factor})$

Sample Set B

Find the unknown number in each proportion.

Example:

$\frac{x}{4} = \frac{20}{16}$. Find the cross product.

$$\begin{aligned} 16 \cdot x &= 20 \cdot 4 \\ 16 \cdot x &= 80 && \text{Divide the product 80 by the known factor 16.} \\ x &= \frac{80}{16} \\ x &= 5 && \text{The unknown number is 5.} \end{aligned}$$

This mean that $\frac{5}{4} = \frac{20}{16}$, or 5 is to 4 as 20 is to 16.

Example:

$\frac{5}{x} = \frac{20}{16}$. Find the cross product.

$$\begin{aligned} 5 \cdot 16 &= 20 \cdot \text{x} \\ 80 &= 20 \cdot x && \text{Divide the product 80 by the known factor 20.} \\ \frac{80}{20} &= x \\ 4 &= x && \text{The unknown number is 4.} \end{aligned}$$

This means that $\frac{5}{4} = \frac{20}{16}$, or, 5 is to 4 as 20 is to 6.

Example:

$\frac{16}{3} = \frac{64}{x}$ Find the cross product.

$$\begin{aligned} 16 \cdot x &= 64 \cdot 3 \\ 16 \cdot x &= 192 && \text{Divide 192 by 16.} \\ x &= \frac{192}{16} \\ x &= 12 && \text{The unknown number is 12.} \end{aligned}$$

The means that $\frac{16}{3} = \frac{64}{12}$, or, 16 is to 3 as 64 is to 12.

Example:

$\frac{9}{8} = \frac{x}{40}$ Find the cross products.

$$\begin{array}{rcll} 9 \cdot 40 & = & 8 \cdot \text{x} & \\ 360 & = & 8 \cdot x & \text{Divide 360 by 8.} \\ \frac{360}{8} & = & x & \\ 45 & = & x & \text{The unknown number is 45.} \end{array}$$

Practice Set B

Find the unknown number in each proportion.

Exercise:

Problem: $\frac{x}{8} = \frac{12}{32}$

Solution:

$x = 3$

Exercise:

Problem: $\frac{7}{x} = \frac{14}{10}$

Solution:

$x = 5$

Exercise:

Problem: $\frac{9}{11} = \frac{x}{55}$

Solution:

$x = 45$

Exercise:

Problem: $\frac{1}{6} = \frac{8}{x}$

Solution:

$x = 48$

Proportions Involving Rates

Recall that a rate is a comparison, by division, of unlike denominate numbers. We must be careful when setting up proportions that involve rates. The *form* is important. For example, if a rate involves two types of units, say unit type 1 and unit type 2, we can write

$$\frac{\text{unit type 1}}{\text{unit type 2}} = \frac{\text{unit type 1}}{\text{unit type 2}}$$

Same units appear on same side.

Same units appear on same side.

or

$$\frac{\text{unit type 1}}{\text{unit type 1}} = \frac{\text{unit type 2}}{\text{unit type 2}}$$

Same units appear on same side.

Same units appear on same side.

Both cross products produce a statement of the type

$$(\text{unit type 1}) \cdot (\text{unit type 2}) = (\text{unit type 1}) \cdot (\text{unit type 2})$$

which we take to mean the comparison

$$\underbrace{\underbrace{\text{(unit type 1) is to (unit type 2)}}_{\substack{\text{Comparison of type 1}\\ \text{with type 2}}} \quad \text{as} \quad \underbrace{\text{(unit type 1) is to (unit type 2)}}_{\substack{\text{Comparison of type 1}\\ \text{with type 2}}}}_{\text{Same overall type}}$$

Examples of correctly expressed proportions are the following:

1. $\frac{\text{mi}}{\text{hr}} = \frac{\text{mi}}{\text{hr}}$ Same units appear on the same side
2. $\frac{\text{mi}}{\text{mi}} = \frac{\text{hr}}{\text{hr}}$

Same units appear on the same side.

However, if we write the same type of units on different sides, such as,

$$\frac{\text{unit type 1}}{\text{unit type 2}} = \frac{\text{unit type 2}}{\text{unit type 1}}$$

the cross product produces a statement of the form

$$\underbrace{\underbrace{(\text{unit type 1}) \cdot (\text{unit type 1})}_{\substack{\text{Comparison of type 1} \\ \text{with type 1}}} = \underbrace{(\text{unit type 2}) \cdot (\text{unit type 2})}_{\substack{\text{Comparison of type 2} \\ \text{with type 2}}}}_{\text{Different overall types}}$$

We can see that this is an incorrect comparison by observing the following example: It is *incorrect* to write

$$\frac{\text{2 hooks}}{\text{3 poles}} = \frac{\text{6 poles}}{\text{4 hooks}}$$

for two reason.

1. The cross product is numerically wrong: $(2 \cdot 4 \neq 3 \cdot 6)$.
2. The cross product produces the statement “hooks are to hooks as poles are to poles,” which makes no sense.

Exercises

Exercise:

Problem: A statement that two ratios or are equal is called a .

Solution:

rates, proportion

For the following 9 problems, write each proportion in fractional form.

Exercise:

Problem: 3 is to 7 as 18 is to 42.

Exercise:

Problem: 1 is to 11 as 3 is to 33.

Solution:

$\frac{1}{11} = \frac{3}{33}$

Exercise:

Problem: 9 is to 14 as 27 is to 42.

Exercise:

Problem: 6 is to 90 as 3 is to 45.

Solution:

$\frac{6}{90} = \frac{3}{45}$

Exercise:

Problem: 5 liters is to 1 bottle as 20 liters is to 4 bottles.

Exercise:

Problem:

18 grams of cobalt is to 10 grams of silver as 36 grams of cobalt is to 20 grams of silver.

Solution:

$$\frac{18 \text{ gr cobalt}}{10 \text{ gr silver}} = \frac{36 \text{ gr cobalt}}{20 \text{ gr silver}}$$

Exercise:

Problem:

4 cups of water is to 1 cup of sugar as 32 cups of water is to 8 cups of sugar.

Exercise:

Problem:

3 people absent is to 31 people present as 15 people absent is to 155 people present.

Solution:

$$\frac{3 \text{ people absent}}{31 \text{ people present}} = \frac{15 \text{ people absent}}{155 \text{ people present}}$$

Exercise:

Problem: 6 dollars is to 1 hour as 90 dollars is to 15 hours.

For the following 10 problems, write each proportion as a sentence.

Exercise:

Problem: $\frac{3}{4} = \frac{15}{20}$

Solution:

3 is to 4 as 15 is to 20

Exercise:

Problem: $\frac{1}{8} = \frac{5}{40}$

Exercise:

Problem: $\frac{3 \text{ joggers}}{100 \text{ feet}} = \frac{6 \text{ joggers}}{200 \text{ feet}}$

Solution:

3 joggers are to 100 feet as 6 joggers are to 200 feet

Exercise:

Problem: $\frac{12 \text{ marshmallows}}{3 \text{ sticks}} = \frac{36 \text{ marshmallows}}{9 \text{ sticks}}$

Exercise:

Problem: $\frac{40 \text{ miles}}{80 \text{ miles}} = \frac{2 \text{ gallons}}{4 \text{ gallons}}$

Solution:

40 miles are to 80 miles as 2 gallons are to 4 gallons

Exercise:

Problem: $\frac{4 \text{ couches}}{10 \text{ couches}} = \frac{2 \text{ houses}}{5 \text{ houses}}$

Exercise:

Problem: $\frac{1 \text{ person}}{1 \text{ job}} = \frac{8 \text{ people}}{8 \text{ jobs}}$

Solution:

1 person is to 1 job as 8 people are to 8 jobs

Exercise:

Problem: $\frac{1 \text{ popsicle}}{2 \text{ children}} = \frac{\frac{1}{2}\text{popsicle}}{1 \text{ child}}$

Exercise:

Problem: $\frac{2{,}000 \text{ pounds}}{1 \text{ ton}} = \frac{60{,}000 \text{ pounds}}{30 \text{ tons}}$

Solution:

2,000 pounds are to 1 ton as 60,000 pounds are to 30 tons

Exercise:

Problem: $\frac{1 \text{ table}}{5 \text{ tables}} = \frac{2 \text{ people}}{10 \text{ people}}$

For the following 10 problems, solve each proportion.

Exercise:

Problem: $\frac{x}{5} = \frac{6}{15}$

Solution:

$x = 2$

Exercise:

Problem: $\frac{x}{10} = \frac{28}{40}$

Exercise:

Problem: $\frac{5}{x} = \frac{10}{16}$

Solution:

$x = 8$

Exercise:

Problem: $\frac{13}{x} = \frac{39}{60}$

Exercise:

Problem: $\frac{1}{3} = \frac{x}{24}$

Solution:

$x = 8$

Exercise:

Problem: $\frac{7}{12} = \frac{x}{60}$

Exercise:

Problem: $\frac{8}{3} = \frac{72}{x}$

Solution:

$x = 27$

Exercise:

Problem: $\frac{16}{1} = \frac{48}{x}$

Exercise:

Problem: $\frac{x}{25} = \frac{200}{125}$

Solution:

$x = 40$

Exercise:

Problem: $\frac{65}{30} = \frac{x}{60}$

For the following 5 problems, express each sentence as a proportion then solve the proportion.

Exercise:

Problem: 5 hats are to 4 coats as x hats are to 24 coats.

Solution:

$x = 30$

Exercise:

Problem: x cushions are to 2 sofas as 24 cushions are to 16 sofas.

Exercise:

Problem:

1 spacecraft is to 7 astronauts as 5 spacecraft are to x astronauts.

Solution:

$x = 35$

Exercise:

Problem:

56 microchips are to x circuit boards as 168 microchips are to 3 circuit boards.

Exercise:

Problem:

18 calculators are to 90 calculators as x students are to 150 students.

Solution:

$x = 30$

Exercise:

Problem: x dollars are to $40,000 as 2 sacks are to 1 sack.

Indicate whether the proportion is true or false.

Exercise:

Problem: $\frac{3}{16} = \frac{12}{64}$

Solution:

true

Exercise:

Problem: $\frac{2}{15} = \frac{10}{75}$

Exercise:

Problem: $\frac{1}{9} = \frac{3}{30}$

Solution:

false

Exercise:

Problem: $\frac{6 \text{ knives}}{7 \text{ forks}} = \frac{12 \text{ knives}}{15 \text{ forks}}$

Exercise:

Problem: $\frac{33 \text{ miles}}{1 \text{ gallon}} = \frac{99 \text{ miles}}{3 \text{ gallons}}$

Solution:

true

Exercise:

Problem: $\frac{320\text{ feet}}{5\text{ seconds}} = \frac{65\text{ feet}}{1\text{ second}}$

Exercise:

Problem: $\frac{35\text{ students}}{70\text{ students}} = \frac{1\text{ class}}{2\text{ classes}}$

Solution:

true

Exercise:

Problem: $\frac{9\text{ ml chloride}}{45\text{ ml chloride}} = \frac{1\text{ test tube}}{7\text{ test tubes}}$

Exercises for Review

Exercise:

Problem:

([link]) Use the number 5 and 7 to illustrate the commutative property of addition.

Solution:

$5 + 7 = 12$
$7 + 5 = 12$

Exercise:

Problem:

([link]) Use the numbers 5 and 7 to illustrate the commutative property of multiplication.

Exercise:

Problem: ([link]) Find the difference. $\frac{5}{14}-\frac{3}{22}$.

Solution:

$\frac{17}{77}$

Exercise:

Problem: ([link]) Find the product. $8.06129\cdot 1,000$.

Exercise:

Problem:

([link]) Write the simplified fractional form of the rate "sixteen sentences to two paragraphs."

Solution:

$\frac{8\ \text{sentences}}{1\ \text{paragraph}}$

Applications of Proportions
This module is from Fundamentals of Mathematics by Denny Burzynski and Wade Ellis, Jr. This module discusses applications of proportions. By the end of the module students should be able to solve proportion problems using the five-step method.

Section Overview

- The Five-Step Method
- Problem Solving

The Five-Step Method

In [link] we noted that many practical problems can be solved by writing the given information as proportions. Such proportions will be composed of three specified numbers and one unknown number represented by a letter.

The first and most important part of solving a proportion problem is to determine, by careful reading, what the unknown quantity is and to represent it with some letter.

The Five-Step Method
The five-step method for solving proportion problems:

1. By careful reading, determine what the unknown quantity is and represent it with some letter. There will be only one unknown in a problem.
2. Identify the three specified numbers.
3. Determine which comparisons are to be made and set up the proportion.
4. Solve the proportion (using the methods of [link]).
5. Interpret and write a conclusion in a sentence with the appropriate units of measure.

Step 1 is extremely important. Many problems go unsolved because time is not taken to establish what quantity is to be found.

When solving an applied problem, **always** begin by determining the unknown quantity and representing it with a letter.

Problem Solving

Sample Set A

Example:
On a map, 2 inches represents 25 miles. How many miles are represented by 8 inches?

- **Step 1** The unknown quantity is miles.
  Let $x =$ number of miles represented by 8 inches
- **Step 2** The three specified numbers are
  2 inches
  25 miles
  8 inches
- **Step 3** The comparisons are
  2 inches to 25 miles → $\frac{\text{2 inches}}{\text{25 miles}}$
  8 inches to x miles → $\frac{\text{8 inches}}{\text{x miles}}$
  Proportions involving ratios and rates are more readily solved by suspending the units while doing the computations.
  $\frac{2}{25} = \frac{8}{x}$
- **Step 4** $\frac{2}{25} \quad = \quad \frac{8}{x} \quad \text{Perform the cross multiplication.}$
  $$\begin{aligned} 2 \cdot x &= 8 \cdot 25 \\ 2 \cdot x &= 200 \qquad \text{Divide 200 by 2.} \\ x &= \frac{200}{2} \\ x &= 100 \end{aligned}$$
  In step 1, we let x represent the number of miles. So, x represents 100 miles.
- **Step 5** If 2 inches represents 25 miles, then 8 inches represents 100 miles.
  Try [link] in [link].

Example:
An acid solution is composed of 7 parts water to 2 parts acid. How many parts of water are there in a solution composed of 20 parts acid?

- **Step 1** The unknown quantity is the number of parts of water.
  Let $n =$ number of parts of water.
- **Step 2** The three specified numbers are
  7 parts water
  2 parts acid
  20 parts acid
- **Step 3** The comparisons are
  7 parts water to 2 parts acid → $\frac{7}{2}$
  n parts water to 20 parts acid → $\frac{n}{20}$
  $\frac{7}{2} = \frac{n}{20}$
- **Step 4** $\frac{7}{2} \quad = \quad \frac{n}{20} \quad \text{Perform the cross multiplication.}$
  $$\begin{aligned} 7 \cdot 20 &= 2 \cdot n \\ 140 &= 2 \cdot n \quad \text{Divide 140 by 2.} \\ \frac{140}{2} &= n \\ 70 &= n \end{aligned}$$
  In step 1 we let n represent the number of parts of water. So, n represents 70 parts of water.
- **Step 5** 7 parts water to 2 parts acid indicates 70 parts water to 20 parts acid.
  Try [link] in [link].

Example:
A 5-foot girl casts a $3\frac{1}{3}$-foot shadow at a particular time of the day. How tall is a person who casts a 3-foot shadow at the same time of the day?

- **Step 1** The unknown quantity is the height of the person.
  Let $h =$ height of the person.
- **Step 2** The three specified numbers are
  5 feet (height of girl)
  $3\frac{1}{3}$ feet (length of shadow)
  3 feet (length of shadow)
- **Step 3** The comparisons are
  5-foot girl is to $3\frac{1}{3}$ foot shadow $\rightarrow \frac{5}{3\frac{1}{3}}$
  h-foot person is to 3-foot shadow $\rightarrow \frac{h}{3}$
  $\frac{5}{3\frac{1}{3}} = \frac{h}{3}$
- **Step 4** $$\begin{aligned} \frac{5}{3\frac{1}{3}} &= \frac{h}{3} \\ 5 \cdot 3 &= 3\frac{1}{3} \cdot h \\ 15 &= \frac{10}{3} \cdot h \quad \text{Divide}\, 15\, \text{by}\, \frac{10}{3} \\ \frac{15}{\frac{10}{3}} &= h \\ \frac{\overset{3}{\cancel{15}}}{1} \cdot \frac{3}{\underset{2}{\cancel{10}}} &= h \\ \frac{9}{2} &= h \\ h &= 4\frac{1}{2} \end{aligned}$$
- **Step 5** A person who casts a 3-foot shadow at this particular time of the day is $4\frac{1}{2}$ feet tall.
  Try [link] in [link].

Example:
The ratio of men to women in a particular town is 3 to 5. How many women are there in the town if there are 19,200 men in town?

- **Step 1** The unknown quantity is the number of women in town.
  Let $x =$ number of women in town.

- **Step 2** The three specified numbers are
  3
  5
  19,200
- **Step 3** The comparisons are 3 men to 5 women → $\frac{3}{5}$
  19,200 men to x women → $\frac{19{,}200}{x}$
  $\frac{3}{5} = \frac{19{,}200}{x}$
- **Step 4** $\frac{3}{5} \quad = \quad \frac{19{,}200}{x}$

$$\begin{aligned} 3 \cdot x &= 19{,}200 \cdot 5 \\ 3 \cdot x &= 96{,}000 \\ x &= \frac{96{,}000}{3} \\ x &= 32{,}000 \end{aligned}$$

- **Step 5** There are 32,000 women in town.

Example:
The rate of wins to losses of a particular baseball team is $\frac{9}{2}$. How many games did this team lose if they won 63 games?

- **Step 1** The unknown quantity is the number of games lost.
  Let n = number of games lost.
- **Step 2** Since $\frac{9}{2}$ → means 9 wins to 2 losses, the three specified numbers are
  9 (wins)
  2 (losses)
  63 (wins)
- **Step 3** The comparisons are
  9 wins to 2 losses → $\frac{9}{2}$
  63 wins to n losses → $\frac{63}{n}$
  $\frac{9}{2} = \frac{63}{n}$

- **Step 4** $\begin{aligned} \frac{9}{2} &= \frac{63}{n} \\ 9 \cdot n &= 2 \cdot 63 \\ 9 \cdot n &= 126 \\ n &= \frac{126}{9} \\ n &= 14 \end{aligned}$
- Step 5: This team had 14 losses.
  Try [link] in [link].

Practice Set A

Solve each problem.

Exercise:

Problem:

On a map, 3 inches represents 100 miles. How many miles are represented by 15 inches?

- **Step 1**
- **Step 2**
- **Step 3**
- **Step 4**

- **Step 5**

Solution:

500 miles

Exercise:

Problem:

An alcohol solution is composed of 14 parts water to 3 parts alcohol. How many parts of alcohol are in a solution that is composed of 112 parts water?

- **Step 1**
- **Step 2**
- **Step 3**
- **Step 4**
- **Step 5**

Solution:

24 parts of alcohol

Exercise:

Problem:

A $5\frac{1}{2}$ -foot woman casts a 7-foot shadow at a particular time of the day. How long of a shadow does a 3-foot boy cast at that same time of day?

- **Step 1**
- **Step 2**
- **Step 3**
- **Step 4**
- **Step 5**

Solution:

$3\frac{9}{11}$ feet

Exercise:

Problem:

The rate of houseplants to outside plants at a nursery is 4 to 9. If there are 384 houseplants in the nursery, how many outside plants are there?

- **Step 1**
- **Step 2**
- **Step 3**
- **Step 4**
- **Step 5**

Solution:

864 outside plants

Exercise:

Problem:

The odds for a particular event occurring are 11 to 2. (For every 11 times the event does occur, it will not occur 2 times.) How many times does the event occur if it does not occur 18 times?

- **Step 1**
- **Step 2**
- **Step 3**
- **Step 4**
- **Step 5**

Solution:

The event occurs 99 times.

Exercise:

Problem:

The rate of passing grades to failing grades in a particular chemistry class is $\frac{7}{2}$. If there are 21 passing grades, how many failing grades are there?

- **Step 1**
- **Step 2**
- **Step 3**
- **Step 4**
- **Step 5**

Solution:

6 failing grades

Exercises

For the following 20 problems, use the five-step method to solve each problem.

Exercise:

Problem:

On a map, 4 inches represents 50 miles. How many inches represent 300 miles?

Solution:

24

Exercise:

Problem:

On a blueprint for a house, 2 inches represents 3 feet. How many inches represent 10 feet?

Exercise:

Problem:

A model is built to $\frac{2}{15}$ scale. If a particular part of the model measures 6 inches, how long is the actual structure?

Solution:

45 inches

Exercise:

Problem:

An acid solution is composed of 5 parts acid to 9 parts of water. How many parts of acid are there in a solution that contains 108 parts of water?

Exercise:

Problem:

An alloy contains 3 parts of nickel to 4 parts of silver. How much nickel is in an alloy that contains 44 parts of silver?

Solution:

33 parts

Exercise:

Problem:

The ratio of water to salt in a test tube is 5 to 2. How much salt is in a test tube that contains 35 ml of water?

Exercise:

Problem:

The ratio of sulfur to air in a container is $\frac{4}{45}$. How many ml of air are there in a container that contains 207 ml of sulfur?

Solution:

2328.75

Exercise:

Problem:

A 6-foot man casts a 4-foot shadow at a particular time of the day. How tall is a person that casts a 3-foot shadow at that same time of the day?

Exercise:

Problem:

A $5\frac{1}{2}$-foot woman casts a $1\frac{1}{2}$-foot shadow at a particular time of the day. How long a shadow does her $3\frac{1}{2}$-foot niece cast at the same time of the day?

Solution:

$\frac{21}{22}$ feet

Exercise:

Problem:

A man, who is 6 feet tall, casts a 7-foot shadow at a particular time of the day. How tall is a tree that casts an 84-foot shadow at that same time of the day?

Exercise:

Problem:

The ratio of books to shelves in a bookstore is 350 to 3. How many books are there in a store that has 105 shelves?

Solution:

12,250

Exercise:

Problem:

The ratio of algebra classes to geometry classes at a particular community college is 13 to 2. How many geometry classes does this college offer if it offers 13 algebra classes?

Exercise:

Problem:

The odds for a particular event to occur are 16 to 3. If this event occurs 64 times, how many times would you predict it does not occur?

Solution:

12

Exercise:

Problem:

The odds against a particular event occurring are 8 to 3. If this event does occur 64 times, how many times would you predict it does not occur?

Exercise:

Problem:

The owner of a stationery store knows that a 1-inch stack of paper contains 300 sheets. The owner wishes to stack the paper in units of 550 sheets. How many inches tall should each stack be?

Solution:

$1\frac{5}{6}$

Exercise:

Problem:

A recipe that requires 6 cups of sugar for 15 servings is to be used to make 45 servings. How much sugar will be needed?

Exercise:

Problem:

A pond loses $7\frac{1}{2}$ gallons of water every 2 days due to evaporation. How many gallons of water are lost, due to evaporation, in $\frac{1}{2}$ day?

Solution:

$1\frac{7}{8}$

Exercise:

Problem:

A photograph that measures 3 inches wide and $4\frac{1}{2}$ inches high is to be enlarged so that it is 5 inches wide. How high will it be?

Exercise:

Problem:

If 25 pounds of fertilizer covers 400 square feet of grass, how many pounds will it take to cover 500 square feet of grass?

Solution:

$31\frac{1}{4}$

Exercise:

Problem:

Every $1\frac{1}{2}$ teaspoons of a particular multiple vitamin, in granular form, contains 0.65 the minimum daily requirement of vitamin C. How many teaspoons of this vitamin are required to supply 1.25 the minimum daily requirement?

Exercises for Review

Exercise:

Problem: ([link]) Find the product, $818 \cdot 0$.

Solution:

0

Exercise:

Problem: ([link]) Determine the missing numerator: $\frac{8}{15}=\frac{N}{90}$.

Exercise:

Problem: ([link]) Find the value of $\frac{\frac{3}{10}+\frac{4}{12}}{\frac{19}{20}}$.

Solution:

$\frac{2}{3}$

Exercise:

Problem: ([link]) Subtract 0.249 from the sum of 0.344 and 0.612.

Exercise:

Problem: ([link]) Solve the proportion: $\frac{6}{x}=\frac{36}{30}$.

Solution:

5

Percent
This module is from Fundamentals of Mathematics by Denny Burzynski and Wade Ellis, Jr. This module discusses percents. By the end of the module students should understand the relationship between ratios and percents and be able to make conversions between fractions, decimals, and percents.

Section Overview

- Ratios and Percents
- The Relationship Between Fractions, Decimals, and Percents – Making Conversions

Ratios and Percents

Ratio, Percent
We defined a **ratio** as a comparison, by division, of two pure numbers or two like denominate numbers. A most convenient number to compare numbers to is 100. Ratios in which one number is compared to 100 are called **percents**. The word *percent* comes from the Latin word "per centum." The word "per" means "for each" or "for every," and the word "centum" means "hundred." Thus, we have the following definition.

Percent means “for each hundred," or "for every hundred."

The symbol % is used to represent the word percent.

Sample Set A

Example:
The ratio 26 to 100 can be written as 26%. We read 26% as "twenty-six percent."

Example:
The ratio $\frac{165}{100}$ can be written as 165%.
We read 165% as "one hundred sixty-five percent."

Example:
The percent 38% can be written as the fraction $\frac{38}{100}$.

Example:
The percent 210% can be written as the fraction $\frac{210}{100}$ or the mixed number $2\frac{10}{100}$ or 2.1.

Example:
Since one dollar is 100 cents, 25 cents is $\frac{25}{100}$ of a dollar. This implies that 25 cents is 25% of one dollar.

Practice Set A

Exercise:

Problem: Write the ratio 16 to 100 as a percent.

Solution:

16%

Exercise:

Problem: Write the ratio 195 to 100 as a percent.

Solution:

195%

Exercise:

Problem: Write the percent 83% as a ratio in fractional form.

Solution:

$\frac{83}{100}$

Exercise:

Problem: Write the percent 362% as a ratio in fractional form.

Solution:

$\frac{362}{100}$ or $\frac{181}{50}$

The Relationship Between Fractions, Decimals, and Percents – Making Conversions

Since a percent is a ratio, and a ratio can be written as a fraction, and a fraction can be written as a decimal, any of these forms can be converted to any other.

Before we proceed to the problems in [link] and [link], let's summarize the conversion techniques.

To Convert a Fraction	To Convert a	To Convert a

	Decimal	Percent
To a decimal: Divide the numerator by the denominator	To a fraction: Read the decimal and reduce the resulting fraction	To a decimal: Move the decimal point 2 places to the left and drop the % symbol
To a percent: Convert the fraction first to a decimal, then move the decimal point 2 places to the right and affix the % symbol.	To a percent: Move the decimal point 2 places to the right and affix the % symbol	To a fraction: Drop the % sign and write the number "over" 100. Reduce, if possible.

Conversion Techniques – Fractions, Decimals, Percents

Sample Set B

Example:
Convert 12% to a decimal.
$12\% = \frac{12}{100} = 0.12$
Note that

$$12\% = 12.\% = 0.12$$

The % symbol is dropped, and the decimal point moves 2 places to the left.

Example:
Convert 0.75 to a percent.

$0.75 = \frac{75}{100} = 75\%$

Note that

$0.75 = 75\% = 75.\%$

The % symbol is affixed, and the decimal point moves 2 units to the right.

Example:

Convert $\frac{3}{5}$ to a percent.

We see in [link] that we can convert a decimal to a percent. We also know that we can convert a fraction to a decimal. Thus, we can see that if we first convert the fraction to a decimal, we can then convert the decimal to a percent.

$$\frac{3}{5} \rightarrow \begin{array}{r} .6 \\ 5\overline{)3.0} \\ 3\ 0 \\ 0 \end{array} \text{ or } \frac{3}{5} = 0.6 = \frac{6}{10} = \frac{60}{100} = 60\%$$

Example:

Convert 42% to a fraction.

$42\% = \frac{42}{100} = \frac{21}{50}$

or

$42\% = 0.42 = \frac{42}{100} = \frac{21}{50}$

Practice Set B

Exercise:

Problem: Convert 21% to a decimal.

Solution:

0.21

Exercise:

Problem: Convert 461% to a decimal.

Solution:

4.61

Exercise:

Problem: Convert 0.55 to a percent.

Solution:

55%

Exercise:

Problem: Convert 5.64 to a percent.

Solution:

564%

Exercise:

Problem: Convert $\frac{3}{20}$ to a percent.

Solution:

15%

Exercise:

Problem: Convert $\frac{11}{8}$ to a percent

Solution:

137.5%

Exercise:

Problem: Convert $\frac{3}{11}$ to a percent.

Solution:

$27.\overline{27}\%$

Exercises

For the following 12 problems, convert each decimal to a percent.

Exercise:

Problem: 0.25

Solution:

25%

Exercise:

Problem: 0.36

Exercise:

Problem: 0.48

Solution:

48%

Exercise:

Problem: 0.343

Exercise:

Problem: 0.771

Solution:

77.1%

Exercise:

Problem: 1.42

Exercise:

Problem: 2.58

Solution:

258%

Exercise:

Problem: 4.976

Exercise:

Problem: 16.1814

Solution:

1,618.14%

Exercise:

Problem: 533.01

Exercise:

Problem: 2

Solution:

200%

Exercise:

Problem: 14

For the following 10 problems, convert each percent to a decimal.

Exercise:

Problem: 15%

Solution:

0.15

Exercise:

Problem: 43%

Exercise:

Problem: 16.2%

Solution:

0.162

Exercise:

Problem: 53.8%

Exercise:

Problem: 5.05%

Solution:

0.0505

Exercise:

Problem: 6.11%

Exercise:

Problem: 0.78%

Solution:

0.0078

Exercise:

Problem: 0.88%

Exercise:

Problem: 0.09%

Solution:

0.0009

Exercise:

Problem: 0.001%

For the following 14 problems, convert each fraction to a percent.

Exercise:

Problem: $\frac{1}{5}$

Solution:

20%

Exercise:

Problem: $\frac{3}{5}$

Exercise:

Problem: $\frac{5}{8}$

Solution:

62.5%

Exercise:

Problem: $\frac{1}{16}$

Exercise:

Problem: $\frac{7}{25}$

Solution:

28%

Exercise:

Problem: $\frac{16}{45}$

Exercise:

Problem: $\frac{27}{55}$

Solution:

49.09%

Exercise:

Problem: $\frac{15}{8}$

Exercise:

Problem: $\frac{41}{25}$

Solution:

164%

Exercise:

Problem: $6\frac{4}{5}$

Exercise:

Problem: $9\frac{9}{20}$

Solution:

945%

Exercise:

Problem: $\frac{1}{200}$

Exercise:

Problem: $\frac{6}{11}$

Solution:

54.54%

Exercise:

Problem: $\frac{35}{27}$

For the following 14 problems, convert each percent to a fraction.

Exercise:

Problem: 80%

Solution:

$\frac{4}{5}$

Exercise:

Problem: 60%

Exercise:

Problem: 25%

Solution:

$\frac{1}{4}$

Exercise:

Problem: 75%

Exercise:

Problem: 65%

Solution:

$\frac{13}{20}$

Exercise:

Problem: 18%

Exercise:

Problem: 12.5%

Solution:

$\frac{1}{8}$

Exercise:

Problem: 37.5%

Exercise:

Problem: 512.5%

Solution:

$\frac{41}{8}$ or $5\frac{1}{8}$

Exercise:

Problem: 937.5%

Exercise:

Problem: $9.\overline{9}\%$

Solution:

$\frac{1}{10}$

Exercise:

Problem: $55.\overline{5}\%$

Exercise:

Problem: $22.\overline{2}\%$

Solution:

$\frac{2}{9}$

Exercise:

Problem: $63.\overline{6}\%$

Exercises for Review

Exercise:

Problem: ([link]) Find the quotient. $\frac{40}{54} \div 8\frac{7}{21}$.

Solution:

$\frac{4}{45}$

Exercise:

Problem: ([link]) $\frac{3}{8}$ of what number is $2\frac{2}{3}$?

Exercise:

Problem: ([link]) Find the value of $\frac{28}{15}+\frac{7}{10}-\frac{5}{12}$.

Solution:

$\frac{129}{60}$ or $2\frac{9}{60}=2\frac{3}{20}$

Exercise:

Problem: ([link]) Round 6.99997 to the nearest ten thousandths.

Exercise:

Problem:

([link]) On a map, 3 inches represent 40 miles. How many inches represent 480 miles?

Solution:

36 inches

Fractions of One Percent
This module is from Fundamentals of Mathematics by Denny Burzynski and Wade Ellis, Jr. This module discusses fractions of one percent. By the end of the module students should understand the meaning of a fraction of one percent and be able to make conversions involving fractions of one percent.

Section Overview

- Conversions Involving Fractions of One Percent
- Conversions Involving Nonterminating Fractions

Conversions Involving Fractions of One Percent

Percents such as $\frac{1}{2}\%$, $\frac{3}{5}\%$, $\frac{5}{8}\%$, and $\frac{7}{11}\%$, where 1% has not been attained, are fractions of 1%. This implies that

$$\frac{1}{2}\% = \frac{1}{2} \text{ of } 1\%$$

$$\frac{3}{5}\% = \frac{3}{5} \text{ of } 1\%$$

$$\frac{5}{8}\% = \frac{5}{8} \text{ of } 1\%$$

$$\frac{7}{11}\% = \frac{7}{11} \text{ of } 1\%$$

Since "percent" means "for each hundred," and "of" means "times," we have

$$\frac{1}{2}\% = \frac{1}{2}\text{of } 1\% = \frac{1}{2} \cdot \frac{1}{100} = \frac{1}{200}$$

$$\frac{3}{5}\% = \frac{3}{5}\text{of } 1\% = \frac{3}{5} \cdot \frac{1}{100} = \frac{3}{500}$$

$$\frac{5}{8}\% = \frac{5}{8}\text{of } 1\% = \frac{5}{8} \cdot \frac{1}{100} = \frac{5}{800}$$

$$\frac{7}{11}\% = \frac{7}{11}\text{of } 1\% = \frac{7}{11} \cdot \frac{1}{100} = \frac{7}{1100}$$

Sample Set A

Example:
Convert $\frac{2}{3}$% to a fraction.

$$\begin{aligned}\frac{2}{3}\% = \frac{2}{3}\text{ of } 1\% &= \frac{\overset{1}{\cancel{2}}}{3} \cdot \frac{1}{\underset{50}{\cancel{100}}} \\ &= \frac{1 \cdot 1}{3 \cdot 50} \\ &= \frac{1}{150}\end{aligned}$$

Example:
Convert $\frac{5}{8}$ % to a decimal.

$$\begin{aligned}\frac{5}{8}\% = \frac{5}{8}\text{ of } 1\% &= \frac{5}{8} \cdot \frac{1}{100} \\ &= 0.625 \cdot 0.01 \\ &= 0.00625\end{aligned}$$

Practice Set A

Exercise:

Problem: Convert $\frac{1}{4}$ % to a fraction.

Solution:

$\frac{1}{400}$

Exercise:

Problem: Convert $\frac{3}{8}$ % to a fraction.

Solution:

$\frac{3}{800}$

Exercise:

Problem: Convert $3\frac{1}{3}$ % to a fraction.

Solution:

$\frac{1}{30}$

Conversions Involving Nonterminating Fractions

We must be careful when changing a fraction of 1% to a decimal. The number $\frac{2}{3}$, as we know, has a nonterminating decimal representation. Therefore, it cannot be expressed exactly as a decimal.

When converting nonterminating fractions of 1% to decimals, it is customary to express the fraction as a rounded decimal with at least three decimal places.

Converting a Nonterminating Fraction to a Decimal
To convert a nonterminating fraction of 1% to a decimal:

1. Convert the fraction as a rounded decimal.
2. Move the decimal point two digits to the left and remove the percent sign.

Sample Set B

Example:
Convert $\frac{2}{3}$ % to a three-place decimal.

1. Convert $\frac{2}{3}$ to a decimal.

   Since we wish the resulting decimal to have three decimal digits, and removing the percent sign will account for two of them, we need to round $\frac{2}{3}$ to one place $(2+1=3)$.

   $\frac{2}{3}\% = 0.7\%$ to one decimal place. $\left(\frac{2}{3} = 0.6666\ldots\right)$
2. Move the decimal point two digits to the left and remove the % sign. We'll need to add zeros to locate the decimal point in the correct location.

   $\frac{2}{3}\% = 0.007$ to 3 decimal places

Example:
Convert $5\frac{4}{11}\%$ to a four-place decimal.

1. Since we wish the resulting decimal to have four decimal places, and removing the percent sign will account for two, we to round $\frac{4}{11}$ to two places.

   $5\frac{4}{11}\% = 5.36\%$ to two decimal places. $\left(\frac{4}{11} = 0.3636\ldots\right)$
2. Move the decimal point two places to the left and drop the percent sign.

   $5\frac{4}{11}\% = 0.0536$ to four decimal places.

Example:
Convert $28\frac{5}{9}\%$ to a decimal rounded to ten thousandths.

1. Since we wish the resulting decimal to be rounded to ten thousandths (four decimal places), and removing the percent sign will account for

two, we need to round $\frac{5}{9}$ to two places.

$28\frac{5}{9}\% = 28.56\%$ to two decimal places. $\left(\frac{5}{9} = 0.5555\ldots\right)$

2. Move the decimal point to the left two places and drop the percent sign.

$28\frac{5}{9}\% = 0.2856$ correct to ten thousandths.

Practice Set B

Exercise:

Problem: Convert $\frac{7}{9}$% to a three-place decimal.

Solution:

0.008

Exercise:

Problem: Convert $51\frac{5}{11}\%$ to a decimal rounded to ten thousandths.

Solution:

0.5145

Exercises

Make the conversions as indicated.

Exercise:

Problem: Convert $\frac{3}{4}$ % to a fraction.

Solution:

$\frac{3}{400}$

Exercise:

Problem: Convert $\frac{5}{6}$% to a fraction.

Exercise:

Problem: Convert $\frac{1}{9}$% to a fraction.

Solution:

$\frac{1}{900}$

Exercise:

Problem: Convert $\frac{15}{19}$ % to a fraction.

Exercise:

Problem: Convert $\frac{5}{4}$ % to a fraction.

Solution:

$\frac{5}{400}$ or $\frac{1}{80}$

Exercise:

Problem: Convert $\frac{7}{3}$% to a fraction.

Exercise:

Problem: Convert $1\frac{6}{7}$% to a fraction.

Solution:

$\frac{13}{700}$

Exercise:

Problem: Convert $2\frac{5}{16}$% to a fraction.

Exercise:

Problem: Convert $25\frac{1}{4}$% to a fraction.

Solution:

$\frac{101}{400}$

Exercise:

Problem: Convert $50\frac{1}{2}$% to a fraction.

Exercise:

Problem: Convert $72\frac{3}{5}$% to a fraction.

Solution:

$\frac{363}{500}$

Exercise:

Problem: Convert $99\frac{1}{8}$% to a fraction.

Exercise:

Problem: Convert $136\frac{2}{3}$% to a fraction.

Solution:

$\frac{41}{30}$

Exercise:

Problem: Convert $521\frac{3}{4}$% to a fraction.

Exercise:

Problem: Convert $10\frac{1}{5}$% to a decimal.

Solution:

$\frac{51}{500} = 0.102$

Exercise:

Problem: Convert $12\frac{3}{4}$% to a decimal.

Exercise:

Problem: Convert $3\frac{7}{8}$% to a decimal.

Solution:

$\frac{31}{800} = 0.03875$

Exercise:

Problem: Convert $7\frac{1}{16}$% to a decimal.

Exercise:

Problem: Convert $\frac{3}{7}$% to a three-place decimal.

Solution:

0.004

Exercise:

Problem: Convert $\frac{1}{9}\%$ to a three-place decimal.

Exercise:

Problem: Convert $6\frac{3}{11}\%$ to a four-place decimal.

Solution:

0.0627

Exercise:

Problem: Convert $9\frac{2}{7}\%$ to a four-place decimal.

Exercise:

Problem: Convert $24\frac{5}{21}\%$ to a three-place decimal.

Solution:

0.242

Exercise:

Problem: Convert $45\frac{8}{27}\%$ to a three-place decimal.

Exercise:

Problem: Convert $11\frac{16}{17}\%$ to a four-place decimal.

Solution:

0.1194

Exercise:

Problem: Convert $5\frac{1}{7}\%$ to a three-place decimal.

Exercises for Review

Exercise:

Problem: ([link]) Write $8 \cdot 8 \cdot 8 \cdot 8 \cdot 8$ using exponents.

Solution:

8^5

Exercise:

Problem: ([link]) Convert $4\frac{7}{8}$ to an improper fraction.

Exercise:

Problem: ([link]) Find the sum. $\frac{7}{10} + \frac{2}{21} + \frac{1}{7}$.

Solution:

$\frac{197}{210}$

Exercise:

Problem: ([link]) Find the product. $(4.21)(0.006)$.

Exercise:

Problem: ([link]) Convert 8.062 to a percent.

Solution:

806.2%

Applications of Percents
This module is from Fundamentals of Mathematics by Denny Burzynski and Wade Ellis, Jr. This module discusses applications of percents. By the end of the module students should be able to distinguish between base, percent, and percentage and be able to find the percentage, the percent, and the base.

Section Overview

- Base, Percent, and Percentage
- Finding the Percentage
- Finding the Percent
- Finding the Base

Base, Percent, and Percentage

There are three basic types of percent problems. Each type involves a base, a percent, and a percentage, and when they are translated from words to mathematical symbols *each becomes a multiplication statement*. Examples of these types of problems are the following:

1. What number is 30% of 50? (Missing product statement.)
2. 15 is what percent of 50? (Missing factor statement.)
3. 15 is 30% of what number? (Missing factor statement.)

In problem 1, the product is missing. To solve the problem, we represent the missing product with P.

$P = 30\% \cdot 50$

Percentage
The missing product P is called the **percentage**. Percentage means *part*, or *portion*. In $P = 30\% \cdot 50$, P represents a particular *part* of 50.

In problem 2, one of the factors is missing. Here we represent the missing factor with Q.

$15 = Q \cdot 50$

Percent
The missing factor is the **percent**. Percent, we know, means *per* 100, or *part* of 100. In $15 = Q \cdot 50$, Q indicates what part of 50 is being taken or considered. Specifically, $15 = Q \cdot 50$ means that if 50 was to be divided into 100 equal parts, then Q indicates 15 are being considered.

In problem 3, one of the factors is missing. Represent the missing factor with B.

$15 = 30\% \cdot B$

Base
The missing factor is the **base**. Some meanings of base are a *source of supply*, or a *starting place*. In $15 = 30\% \cdot B$, B indicates the amount of supply. Specifically, $15 = 30\% \cdot B$ indicates that 15 represents 30% of the total supply.

Each of these three types of problems is of the form

$(\text{percentage}) = (\text{percent}) \cdot (\text{base})$

We can determine any one of the three values given the other two using the methods discussed in [link].

Finding the Percentage

Sample Set A

Example:

$$\begin{array}{ccccccl} \underbrace{\text{What number}} & \text{is} & 30\% & \text{of} & 50? & & \text{Missing product statement.} \\ \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & & \\ \text{(percentage)} & = & \text{(percent)} & \cdot & \text{(base)} & & \\ \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & & \\ P & = & 30\% & \cdot & 50 & & \text{Convert } 30\% \text{ to a decimal.} \\ P & = & .30 & \cdot & 50 & & \text{Multiply.} \\ P & = & 15 & & & & \end{array}$$

Thus, 15 is 30% of 50.

Do [link], [link].

Example:

$$\begin{array}{ccccccl} \underbrace{\text{What number}} & \text{is} & 36\% & \text{of} & 95? & & \text{Missing product statement.} \\ \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & & \\ \text{(percentage)} & = & \text{(percent)} & \cdot & \text{(base)} & & \\ \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & & \\ P & = & 36\% & \cdot & 95 & & \text{Convert } 36\% \text{ to a decimal.} \\ P & = & .36 & \cdot & 95 & & \text{Multiply} \\ P & = & 34.2 & & & & \end{array}$$

Thus, 34.2 is 36% of 95.

Do [link], [link].

Example:

A salesperson, who gets a commission of 12% of each sale she makes, makes a sale of $8,400.00. How much is her commission?

We need to determine what part of $8,400.00 is to be taken. What *part* indicates *percentage*.

$$\begin{array}{ccccccl} \underbrace{\text{What number}} & \text{is} & 12\% & \text{of} & 8{,}400.00? & & \text{Missing product statement.} \\ \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & & \\ \text{(percentage)} & = & \text{(percent)} & \cdot & \text{(base)} & & \\ \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & & \\ P & = & 12\% & \cdot & 8{,}400.00 & & \text{Convert to decimals.} \\ P & = & .12 & \cdot & 8{,}400.00 & & \text{Multiply.} \\ P & = & 1008.00 & & & & \end{array}$$

Thus, the salesperson's commission is $1,008.00.

Do [link], [link].

Example:

A girl, by practicing typing on her home computer, has been able to increase her typing speed by 110%. If she originally typed 16 words per minute, by how many words per minute was she able to increase her speed?

We need to determine what part of 16 has been taken. What *part* indicates *percentage*.

$$\begin{array}{ccccccl} \underbrace{\text{What number}} & \text{is} & 110\% & \text{of} & 16? & & \text{Missing product statement.} \\ & \downarrow & \downarrow & \downarrow & \downarrow & & \\ (\text{percentage}) & = & (\text{percent}) & \cdot & (\text{base}) & & \\ \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & & \\ P & = & 110\% & \cdot & 16 & & \text{Convert to decimals.} \\ P & = & 1.10 & \cdot & 16 & & \text{Multiply.} \\ P & = & 17.6 & & & & \end{array}$$

Thus, the girl has increased her typing speed by 17.6 words per minute. Her new speed is $16 + 17.6 = 33.6$ words per minute.

Do [link], [link].

Example:

A student who makes $125 a month working part-time receives a 4% salary raise. What is the student's new monthly salary?

With a 4% raise, this student will make 100% of the original salary + 4% of the original salary. This means the new salary will be 104% of the original salary. We need to determine what part of $125 is to be taken. What *part* indicates *percentage*.

$$\begin{array}{ccccccl} \underbrace{\text{What number}} & \text{is} & 104\% & \text{of} & 125 & & \text{Missing product statement.} \\ & \downarrow & \downarrow & \downarrow & \downarrow & & \\ (\text{percentage}) & = & (\text{percent}) & \cdot & (\text{base}) & & \\ \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & & \\ P & = & 104\% & \cdot & 125 & & \text{Convert to decimals.} \\ P & = & 1.04 & \cdot & 125 & & \text{Multiply.} \\ P & = & 130 & & & & \end{array}$$

Thus, this student's new monthly salary is $130.

Do [link], [link].

Example:

An article of clothing is on sale at 15% off the marked price. If the marked price is $24.95, what is the sale price?

Since the item is discounted 15%, the new price will be $100\% - 15\% = 85\%$ of the marked price. We need to determine what part of 24.95 is to be taken. What *part* indicates *percentage*.

$$\begin{array}{ccccccl} \underbrace{\text{What number}} & \text{is} & 85\% & \text{of} & \$24.95. & & \text{Missing product statement.} \\ & \downarrow & \downarrow & \downarrow & \downarrow & & \\ (\text{percentage}) & = & (\text{percent}) & \cdot & (\text{base}) & & \\ \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & & \\ P & = & 85\% & \cdot & 24.95 & & \text{Convert to decimals.} \\ P & = & .85 & \cdot & 24.95 & & \text{Multiply.} \\ P & = & 21.2075 & & & & \text{Since this number represents money,} \\ & & & & & & \text{we'll round to 2 decimal places} \\ P & = & 21.21 & & & & \end{array}$$

Thus, the sale price of the item is $21.21.

Practice Set A

Exercise:

Problem: What number is 42% of 85?

Solution:

35.7

Exercise:

Problem:

A sales person makes a commission of 16% on each sale he makes. How much is his commission if he makes a sale of $8,500?

Solution:

$1,360

Exercise:

Problem:

An assembly line worker can assemble 14 parts of a product in one hour. If he can increase his assembly speed by 35%, by how many parts per hour would he increase his assembly of products?

Solution:

4.9

Exercise:

Problem:

A computer scientist in the Silicon Valley makes $42,000 annually. What would this scientist's new annual salary be if she were to receive an 8% raise?

Solution:

$45,360

Finding the Percent

Sample Set B

Example:

$$\begin{array}{cccccccl} 15 & \text{is} & \underbrace{\text{what percent}} & \text{of} & 50? & & & \text{Missing factor statement.} \\ \downarrow & \downarrow & & \downarrow & \downarrow & & & \\ (\text{percentage}) & = & (\text{percent}) & \cdot & (\text{base}) & & & [(\text{product}) = (\text{factor}) \cdot (\text{factor})] \\ \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & & & \\ 15 & = & Q & \cdot & 50 & & & \end{array}$$

Recall that (missing factor) = (product) ÷ (known factor).

$$\begin{aligned} Q &= 15 \div 50 && \text{Divide.} \\ Q &= 0.3 && \text{Convert to a percent.} \\ Q &= 30\% \end{aligned}$$

Thus, 15 is 30% of 50.

Do [link], [link].

Example:

$$\begin{array}{cccccccl} 4.32 & \text{is} & \underbrace{\text{what percent}} & \text{of} & 72? & & & \text{Missing factor statement.} \\ \downarrow & \downarrow & & \downarrow & \downarrow & & & \\ (\text{percentage}) & = & (\text{percent}) & \cdot & (\text{base}) & & & [(\text{product}) = (\text{factor}) \cdot (\text{factor})] \\ \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & & & \\ 4.32 & = & Q & \cdot & 72 & & & \end{array}$$

$$\begin{aligned} Q &= 4.32 \div 72 && \text{Divide.} \\ Q &= 0.06 && \text{Convert to a percent.} \\ Q &= 6\% \end{aligned}$$

Thus, 4.32 is 6% of 72.

Do [link], [link].

Example:

On a 160 question exam, a student got 125 correct answers. What percent is this? Round the result to two decimal places.

We need to determine the percent.

$$\begin{array}{cccccccl} 125 & \text{is} & \underbrace{\text{what percent}} & \text{of} & 160? & & & \text{Missing factor statement.} \\ \downarrow & \downarrow & & \downarrow & \downarrow & & & \\ (\text{percentage}) & = & (\text{percent}) & \cdot & (\text{base}) & & & [(\text{product}) = (\text{factor}) \cdot (\text{factor})] \\ \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & & & \\ 125 & = & Q & \cdot & 160 & & & \end{array}$$

$$\begin{aligned} Q &= 125 \div 160 && \text{Divide.} \\ Q &= 0.78125 && \text{Round to two decimal places.} \\ Q &= .78 \end{aligned}$$

Thus, this student received a 78% on the exam.

Do [link], [link].

Example:

A bottle contains 80 milliliters of hydrochloric acid (HCl) and 30 milliliters of water. What percent of HCl does the bottle contain? Round the result to two decimal places.

We need to determine the percent. The total amount of liquid in the bottle is

$80 \text{ milliliters} + 30 \text{ milliliters} = 110 \text{ milliliters.}$

$$\begin{array}{cccccccl} 80 & \text{is} & \underbrace{\text{what percent}} & \text{of} & 110? & & & \text{Missing factor statement.} \\ \downarrow & \downarrow & & \downarrow & \downarrow & & & \\ (\text{percentage}) & = & (\text{percent}) & \cdot & (\text{base}) & & & [(\text{product}) = (\text{factor}) \cdot (\text{factor})] \\ \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & & & \\ 80 & = & Q & \cdot & 110 & & & \end{array}$$

$$\begin{aligned} Q &= 80 \div 110 && \text{Divide.} \\ Q &= 0.727272\ldots && \text{Round to two decimal places.} \\ Q &\approx 73\% && \text{The symbol "}\approx\text{" is read as "approximately."} \end{aligned}$$

Thus, this bottle contains approximately 73% HCl.
Do [link], [link].

Example:
Five years ago a woman had an annual income of $19,200. She presently earns $42,000 annually. By what percent has her salary increased? Round the result to two decimal places.
We need to determine the percent.

$$\begin{array}{ccccccl} 42{,}000 & \text{is} & \underbrace{\text{what percent}} & \text{of} & 19{,}200? & \text{Missing factor statement.} \\ \downarrow & \downarrow & & \downarrow & \downarrow & \\ (\text{percentage}) & = & (\text{percent}) & \cdot & (\text{base}) & \\ \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \\ 42{,}000 & = & Q & \cdot & 19{,}200 & \end{array}$$

$$\begin{array}{lcll} Q & = & 42{,}000 \div 19{,}200 & \text{Divide.} \\ Q & = & 2.1875 & \text{Round to two decimal places.} \\ Q & = & 2.19 & \text{Convert to a percent.} \\ Q & = & 219\% & \text{Convert to a percent.} \end{array}$$

Thus, this woman's annual salary has increased 219%.

Practice Set B

Exercise:

Problem: 99.13 is what percent of 431?

Solution:

23%

Exercise:

Problem:

On an 80 question exam, a student got 72 correct answers. What percent did the student get on the exam?

Solution:

90%

Exercise:

Problem:

A bottle contains 45 milliliters of sugar and 67 milliliters of water. What fraction of sugar does the bottle contain? Round the result to two decimal places (then express as a percent).

Solution:

40%

Finding the Base

Sample Set C

Example:

$$\begin{array}{ccccc l} 15 & \text{is} & 30\% & \text{of} & \underbrace{\text{what number?}} & \text{Missing factor statement.} \\ \downarrow & \downarrow & \downarrow & \downarrow & & \\ (\text{percentage}) & = & (\text{percent}) & \cdot & (\text{base}) & [(\text{percentage}) = (\text{factor}) \cdot (\text{factor})] \\ \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \\ 15 & = & 30\% & \cdot & B & \text{Convert to decimals.} \\ 15 & = & .30 & \cdot & B & [(\text{missing factor}) = (\text{product}) \div (\text{known factor})] \end{array}$$

$$\begin{array}{lcl} B & = & 15 \div .30 \\ B & = & 50 \end{array}$$

Thus, 15 is 30% of 50.

Try [link] in [link].

Example:

$$\begin{array}{ccccc l} 56.43 & \text{is} & 33\% & \text{of} & \underbrace{\text{what number?}} & \text{Missing factor statement.} \\ \downarrow & \downarrow & \downarrow & \downarrow & & \\ (\text{percentage}) & = & (\text{percent}) & \cdot & (\text{base}) & \\ \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \\ 56.43 & = & 33\% & \cdot & B & \text{Convert to decimals.} \\ 56.43 & = & .33 & \cdot & B & \text{Divide.} \end{array}$$

$$\begin{array}{lcl} B & = & 56.43 \div .33 \\ B & = & 171 \end{array}$$

Thus, 56.43 is 33% of 171.

Try [link] in [link].

Example:

Fifteen milliliters of water represents 2% of a hydrochloric acid (HCl) solution. How many milliliters of solution are there?

We need to determine the total supply. The word *supply* indicates *base*.

$$\begin{array}{ccccc l} 15 & \text{is} & 2\% & \text{of} & \underbrace{\text{what number?}} & \text{Missing factor statement.} \\ \downarrow & \downarrow & \downarrow & \downarrow & & \\ (\text{percentage}) & = & (\text{percent}) & \cdot & (\text{base}) & \\ \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \\ 15 & = & 2\% & \cdot & B & \text{Convert to decimals.} \\ 15 & = & .02 & \cdot & B & \text{Divide.} \end{array}$$

$$\begin{array}{lcl} B & = & 15 \div .02 \\ B & = & 750 \end{array}$$

Thus, there are 750 milliliters of solution in the bottle.

Try [link] in [link].

Example:

In a particular city, a sales tax of $6\frac{1}{2}\%$ is charged on items purchased in local stores. If the tax on an item is \$2.99, what is the price of the item?
We need to determine the price of the item. We can think of *price* as the *starting place*. *Starting place* indicates *base*. We need to determine the base.

$$\begin{array}{ccccccl} 2.99 & \text{is} & 6\frac{1}{2}\% & \text{of} & \underbrace{\text{what number?}} & & \text{Missing factor statement.} \\ \downarrow & \downarrow & \downarrow & \downarrow & & & \\ \text{(percentage)} & = & \text{(percent)} & \cdot & \text{(base)} & & \\ \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & & \\ 2.99 & = & 6\frac{1}{2}\% & \cdot & B & & \text{Convert to decimals.} \\ 2.99 & = & 6.5\% & \cdot & B & & \\ 2.99 & = & .065 & \cdot & B & & [\text{(missing factor)} = \text{(product)} \div \text{(known factor)}] \end{array}$$

$$\begin{array}{lll} B & = & 2.99 \div .065 \quad \text{Divide.} \\ B & = & 46 \end{array}$$

Thus, the price of the item is \$46.00.
Try [link] in [link].

Example:
A clothing item is priced at \$20.40. This marked price includes a 15% discount. What is the original price?
We need to determine the original price. We can think of the original price as the *starting place*. *Starting place* indicates *base*. We need to determine the base. The new price, \$20.40, represents $100\% - 15\% = 85\%$ of the original price.

$$\begin{array}{ccccccl} 20.40 & \text{is} & 85\% & \text{of} & \underbrace{\text{what number?}} & & \text{Missing factor statement.} \\ \downarrow & \downarrow & \downarrow & \downarrow & & & \\ \text{(percentage)} & = & \text{(percent)} & \cdot & \text{(base)} & & \\ \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & & \\ 20.40 & = & 85\% & \cdot & B & & \text{Convert to decimals.} \\ 20.40 & = & .85 & \cdot & B & & [\text{(missing factor)} = \text{(product)} \div \text{(known factor)}] \end{array}$$

$$\begin{array}{lll} B & = & 20.40 \div .85 \quad \text{Divide.} \\ B & = & 24 \end{array}$$

Thus, the original price of the item is \$24.00.
Try [link] in [link].

Practice Set C

Exercise:

Problem: 1.98 is 2% of what number?

Solution:

99

Exercise:

Problem:

3.3 milliliters of HCl represents 25% of an HCl solution. How many milliliters of solution are there?

Solution:

13.2ml

Exercise:

Problem:

A salesman, who makes a commission of $18\frac{1}{4}$% on each sale, makes a commission of $152.39 on a particular sale. Rounded to the nearest dollar, what is the amount of the sale?

Solution:

$835

Exercise:

Problem:

At "super-long play," $2\frac{1}{2}$ hours of play of a video cassette recorder represents 31.25% of the total playing time. What is the total playing time?

Solution:

8 hours

Exercises

For the following 25 problems, find each indicated quantity.

Exercise:

Problem: What is 21% of 104?

Solution:

21.84

Exercise:

Problem: What is 8% of 36?

Exercise:

Problem: What is 98% of 545?

Solution:

534.1

Exercise:

Problem: What is 143% of 33?

Exercise:

Problem: What is $10\frac{1}{2}$% of 20?

Solution:

2.1

Exercise:

Problem: 3.25 is what percent of 88?

Exercise:

Problem: 22.44 is what percent of 44?

Solution:

51

Exercise:

Problem: 0.0036 is what percent of 0.03?

Exercise:

Problem: 31.2 is what percent of 26?

Solution:

120

Exercise:

Problem: 266.4 is what percent of 74?

Exercise:

Problem: 0.0101 is what percent of 0.0505?

Solution:

20

Exercise:

Problem: 2.4 is 24% of what number?

Exercise:

Problem: 24.19 is 41% of what number?

Solution:

59

Exercise:

Problem: 61.12 is 16% of what number?

Exercise:

Problem: 82.81 is 91% of what number?

Solution:

91

Exercise:

Problem: 115.5 is 20% of what number?

Exercise:

Problem: 43.92 is 480% of what number?

Solution:

9.15

Exercise:

Problem: What is 85% of 62?

Exercise:

Problem: 29.14 is what percent of 5.13?

Solution:

568

Exercise:

Problem: 0.6156 is what percent of 5.13?

Exercise:

Problem: What is 0.41% of 291.1?

Solution:

1.19351

Exercise:

Problem: 26.136 is 121% of what number?

Exercise:

Problem: 1,937.5 is what percent of 775?

Solution:

250

Exercise:

Problem: 1 is what percent of 2,000?

Exercise:

Problem: 0 is what percent of 59?

Solution:

0

Exercise:

Problem:

An item of clothing is on sale for 10% off the marked price. If the marked price is $14.95, what is the sale price? (Round to two decimal places.)

Exercise:

Problem:

A grocery clerk, who makes $365 per month, receives a 7% raise. How much is her new monthly salary?

Solution:

390.55

Exercise:

Problem:

An item of clothing which originally sells for $55.00 is marked down to $46.75. What percent has it been marked down?

Exercise:

Problem: On a 25 question exam, a student gets 21 correct. What percent is this?

Solution:

84

Exercise:

Problem: On a 45 question exam, a student gets 40%. How many questions did this student get correct?

Exercise:

Problem:

A vitamin tablet, which weighs 250 milligrams, contains 35 milligrams of vitamin C. What percent of the weight of this tablet is vitamin C?

Solution:

14

Exercise:

Problem:

Five years ago a secretary made $11,200 annually. The secretary now makes $17,920 annually. By what percent has this secretary's salary been increased?

Exercise:

Problem:

A baseball team wins $48\frac{3}{4}$% of all their games. If they won 78 games, how many games did they play?

Solution:

160

Exercise:

Problem:

A typist was able to increase his speed by 120% to 42 words per minute. What was his original typing speed?

Exercise:

Problem:

A salesperson makes a commission of 12% on the total amount of each sale. If, in one month, she makes a total of $8,520 in sales, how much has she made in commission?

Solution:

$1,022.40

Exercise:

Problem:

A salesperson receives a salary of $850 per month plus a commission of $8\frac{1}{2}$% of her sales. If, in a particular month, she sells $22,800 worth of merchandise, what will be her monthly earnings?

Exercise:

Problem:

A man borrows $1150.00 from a loan company. If he makes 12 equal monthly payments of $130.60, what percent of the loan is he paying in interest?

Solution:

36.28%

Exercise:

Problem:

The distance from the sun to the earth is approximately 93,000,000 miles. The distance from the sun to Pluto is approximately 860.2% of the distance from the sun to the Earth. Approximately, how many miles is Pluto from the sun?

Exercise:

Problem:

The number of people on food stamps in Maine in 1975 was 151,000. By 1980, the number had decreased to 59,200. By what percent did the number of people on food stamps decrease? (Round the result to the nearest percent.)

Solution:

61

Exercise:

Problem:

In Nebraska, in 1960, there were 734,000 motor-vehicle registrations. By 1979, the total had increased by about 165.6%. About how many motor-vehicle registrations were there in Nebraska in 1979?

Exercise:

Problem:

From 1973 to 1979, in the United States, there was an increase of 166.6% of Ph.D. social scientists to 52,000. How many were there in 1973?

Solution:

19,500

Exercise:

Problem:

In 1950, in the United States, there were 1,894 daily newspapers. That number decreased to 1,747 by 1981. What percent did the number of daily newspapers decrease?

Exercise:

Problem:

A particular alloy is 27% copper. How many pounds of copper are there in 55 pounds of the alloy?

Solution:

14.85

Exercise:

Problem:

A bottle containing a solution of hydrochloric acid (HCl) is marked 15% (meaning that 15% of the HCl solution is acid). If a bottle contains 65 milliliters of solution, how many milliliters of water does it contain?

Exercise:

Problem:

A bottle containing a solution of HCl is marked 45%. A test shows that 36 of the 80 milliliters contained in the bottle are hydrochloric acid. Is the bottle marked correctly? If not, how should it be remarked?

Solution:

Marked correctly

Exercises For Review

Exercise:

Problem:([link]) Use the numbers 4 and 7 to illustrate the commutative property of multiplication.

Exercise:

Problem:([link]) Convert $\frac{14}{5}$ to a mixed number.

Solution:

$2\frac{4}{5}$

Exercise:

Problem:([link]) Arrange the numbers $\frac{7}{12}$, $\frac{5}{9}$ and $\frac{4}{7}$ in increasing order.

Exercise:

Problem:([link]) Convert 4.006 to a mixed number.

Solution:

$4\frac{3}{500}$

Exercise:

Problem: ([link]) Convert $\frac{7}{8}$ % to a fraction.

Summary of Key Concepts
This module is from Fundamentals of Mathematics by Denny Burzynski and Wade Ellis, Jr. This module reviews the key concepts from the chapter "Ratios and Rates."

Summary of Key Concepts

Denominate Numbers ([link])
Numbers that appear along with units are **denominate numbers**. The amounts 6 dollars and 4 pints are examples of denominate numbers.

Like and Unlike Denominate Numbers ([link])
Like denominate numbers are denominate numbers with like units. If the units are not the same, the numbers are **unlike denominate numbers**.

Pure Numbers ([link])
Numbers appearing without a unit are **pure numbers**.

Comparing Numbers by Subtraction and Division ([link])
Comparison of two numbers by subtraction indicates how much more one number is than another. Comparison by division indicates how many times larger or smaller one number is than another.

Comparing Pure or Like Denominate Numbers by Subtraction ([link])
Numbers can be compared by subtraction if and only if they are pure numbers or like denominate numbers.

Ratio Rate ([link])
A comparison, by division, of two like denominate numbers is a **ratio**. A comparison, by division, of two unlike denominate numbers is a **rate**.

Proportion ([link])
A **proportion** is a statement that two ratios or rates are equal.

$\frac{3 \text{ people}}{2 \text{ jobs}} = \frac{6 \text{ people}}{4 \text{ jobs}}$ is a proportion.

Solving a Proportion ([link])

To **solve a proportion** that contains three known numbers and a letter that represents an unknown quantity, perform the cross multiplication, then divide the product of the two numbers by the number that multiplies the letter.

Proportions Involving Rates ([link])
When writing a proportion involving rates it is very important to write it so that the same type of units appears on the same side of either the equal sign or the fraction bar.

$$\frac{\text{unit type 1}}{\text{unit type 2}} = \frac{\text{unit type 1}}{\text{unit type 2}} \text{ or } \frac{\text{unit type 1}}{\text{unit type 1}} = \frac{\text{unit type 2}}{\text{unit type 2}}$$

Five-Step Method for Solving Proportions ([link])

1. By careful reading, determine what the unknown quantity is and represent it with some letter. There will be only one unknown in a problem.
2. Identify the three specified numbers.
3. Determine which comparisons are to be made and set up the proportion.
4. Solve the proportion.
5. Interpret and write a conclusion.

When solving applied problems, ALWAYS begin by determining the unknown quantity and representing it with a letter.

Percents ([link])
A ratio in which one number is compared to 100 is a **percent**. Percent means "for each hundred."

Conversion of Fractions, Decimals, and Percents ([link])
It is possible to convert decimals to percents, fractions to percents, percents to decimals, and percents to fractions.

Applications of Percents:
The three basic types of percent problems involve a **base**, a **percentage**, and a **percent**.

Base ([link])
The **base** is the number used for comparison.

Percentage ([link])
The **percentage** is the number being compared to the base.

Percent ([link])
By its definition, **percent** means *part of*.

Solving Problems ([link])
$\text{Percentage} = (\text{percent}) \times (\text{base})$
$\text{Percent} = \frac{\text{percentage}}{\text{base}}$
$\text{Base} = \frac{\text{percentage}}{\text{percent}}$

Exercise Supplement
This module is from Fundamentals of Mathematics by Denny Burzynski and Wade Ellis, Jr. This module is an exercise supplement for the chapter "Ratios and Rates" and contains many exercise problems. Odd problems are accompanied by solutions.

Exercise Supplement

Ratios and Rates ([link])

Exercise:

Problem: Compare 250 watts to 100 watts by subtraction.

Solution:

250 watts are 150 watts more than 100 watts

Exercise:

Problem: Compare 126 and 48 by subtraction.

Exercise:

Problem: Compare 98 radishes to 41 radishes by division.

Solution:

98 radishes are 2.39 times as many radishes as 41 radishes

Exercise:

Problem: Compare 144 to 9 by division.

Exercise:

Problem: Compare 100 tents to 5 tents by division.

Solution:

100 tents are 20 times as many tents as 5 tents

Exercise:

Problem: Compare 28 feet to 7 feet by division.

Exercise:

Problem:

Comparison, by division, of two pure numbers or two like denominate numbers is called a .

Solution:

ratio

Exercise:

Problem:

A comparison, by division, of two unlike denominate numbers is called a .

For problems 9-12, express each ratio or rate as a fraction.

Exercise:

Problem: 15 to 5

Solution:

$\frac{3}{1}$

Exercise:

Problem: 72 to 12

Exercise:

Problem:8 millimeters to 5 milliliters

Solution:

$\frac{8\text{ml}}{5\text{ml}}$

Exercise:

Problem:106 tablets to 52 tablets

For problems 13-16, write each ratio in the form "a to b".

Exercise:

Problem: $\frac{9}{16}$

Solution:

9 to 16

Exercise:

Problem: $\frac{5}{11}$

Exercise:

Problem: $\frac{1\ \text{diskette}}{8\ \text{diskettes}}$

Solution:

1 diskette to 8 diskettes

Exercise:

Problem: $\frac{5\ \text{papers}}{3\ \text{pens}}$

For problems 17-21, write each ratio or rate using words.

Exercise:

Problem: $\frac{9}{16} = \frac{18}{32}$

Solution:

9 is to 16 as 18 is to 32

Exercise:

Problem: $\frac{1}{4} = \frac{12}{48}$

Exercise:

Problem: $\frac{8 \text{ items}}{4 \text{ dollars}} = \frac{2 \text{ items}}{1 \text{ dollar}}$

Solution:

8 items are to 4 dollars as 2 items are to 1 dollar

Exercise:

Problem:

150 milligrams of niacin is to 2 tablets as 300 milligrams of niacin is to 4 tablets.

Exercise:

Problem: 20 people is to 4 seats as 5 people is to 1 seat.

Solution:

$\frac{20}{4} = \frac{5}{1}$

20 people are to 4 seats as 5 people are to 1 seat

Proportions ([link])

For problems 22-27, determine the missing number in each proportion.

Exercise:

Problem: $\frac{x}{3} = \frac{24}{9}$

Exercise:

Problem: $\frac{15}{7} = \frac{60}{x}$

Solution:

28

Exercise:

Problem: $\frac{1}{1} = \frac{x}{44}$

Exercise:

Problem: $\frac{3}{x} = \frac{15}{50}$

Solution:

10

Exercise:

Problem: $\frac{\text{15 bats}}{\text{16 balls}} = \frac{\text{x bats}}{\text{128 balls}}$

Exercise:

Problem: $\frac{36 \text{ rooms}}{29 \text{ fans}} = \frac{504 \text{ rooms}}{x \text{ fans}}$

Solution:

406

Applications of Proportions ([link])

Exercise:

Problem:

On a map, 3 inches represents 20 miles. How many miles does 27 inches represent?

Exercise:

Problem:

A salt solution is composed of 8 parts of salt to 5 parts of water. How many parts of salt are there in a solution that contains 50 parts of water?

Solution:

80

Exercise:

Problem:

A model is built to $\frac{4}{15}$ scale. If a particular part of the model measures 8 inches in length, how long is the actual structure?

Exercise:

Problem:

The ratio of ammonia to air in a container is $\frac{3}{40}$ How many milliliters of air should be in a container that contains 8 milliliters of ammonia?

Solution:

$\frac{320}{3}$ or $106\frac{2}{3}$

Exercise:

Problem:

A 4-foot girl casts a 9-foot shadow at a particular time of the day. How tall is a pole that casts a 144-foot shadow at the same time of the day?

Exercise:

Problem:

The odds that a particular event will occur are 11 to 2. If this event occurs 55 times, how many times would you predict it does not occur?

Solution:

10

Exercise:

Problem:

Every $1\frac{3}{4}$ teaspoon of a multiple vitamin, in granular form, contains 0.85 the minimum daily requirement of vitamin A. How many teaspoons of this vitamin are required to supply 2.25 the minimum daily requirement?

Percent and Fractions of One Percent ([link],[link])

For problems 35-39, convert each decimal to a percent.
Exercise:

Problem:0.16

Solution:

16%

Exercise:

Problem:0.818

Exercise:

Problem:5.3536

Solution:

535.36%

Exercise:

Problem:0.50

Exercise:

Problem:3

Solution:

300%

For problems 40-48, convert each percent to a decimal.
Exercise:

Problem:62%

Exercise:

Problem: 1.58%

Solution:

0.0158

Exercise:

Problem: 9.15%

Exercise:

Problem: 0.06%

Solution:

0.0006

Exercise:

Problem: 0.003%

Exercise:

Problem: $5\frac{3}{11}$ % to a three-place decimal

Solution:

0.053

Exercise:

Problem: $\frac{9}{13}$ % to a three-place decimal

Exercise:

Problem: $82\frac{25}{29}$ % to a four-place decimal

Solution:

0.8286

Exercise:

Problem: $18\frac{1}{7}$ % to a four-place decimal

For problems 49-55, convert each fraction or mixed number to a percent.

Exercise:

Problem: $\frac{3}{5}$

Solution:

60%

Exercise:

Problem: $\frac{2}{10}$

Exercise:

Problem: $\frac{5}{16}$

Solution:

31.25%

Exercise:

Problem: $\frac{35}{8}$

Exercise:

Problem: $\frac{105}{16}$

Solution:

656.25%

Exercise:

Problem: $45\frac{1}{11}$

Exercise:

Problem: $6\frac{278}{9}$

Solution:

$3688.\overline{8}\%$

For problems 56-64, convert each percent to a fraction or mixed number.

Exercise:

Problem: 95%

Exercise:

Problem: 12%

Solution:

$\frac{3}{25}$

Exercise:

Problem:83%

Exercise:

Problem:38.125%

Solution:

$\frac{61}{160}$

Exercise:

Problem:$61.\overline{2}\%$

Exercise:

Problem:$\frac{5}{8}\%$

Solution:

$\frac{1}{160}$

Exercise:

Problem: $6\frac{9}{20}\%$

Exercise:

Problem:$15\frac{3}{22}\ \%$

Solution:

$\frac{2977}{19800}$

Exercise:

Problem: $106\frac{19}{45}\%$

Applications of Percents ([link])

For problems 65-72, find each solution.

Exercise:

Problem: What is 16% of 40?

Solution:

6.4

Exercise:

Problem: 29.4 is what percent of 105?

Exercise:

Problem: $3\frac{21}{50}$ is 547.2% of what number?

Solution:

0.625 or $\frac{5}{8}$

Exercise:

Problem: 0.09378 is what percent of 52.1?

Exercise:

Problem: What is 680% of 1.41?

Solution:

9.588

Exercise:

Problem:

A kitchen knife is on sale for 15% off the marked price. If the marked price is $ 39.50, what is the sale price?

Exercise:

Problem:

On an 80 question geology exam, a student gets 68 correct. What percent is correct?

Solution:

85

Exercise:

Problem:

A salesperson makes a commission of 18% of her monthly sales total. She also receives a monthly salary of $1,600.00. If, in a particular month, she sells $4,000.00 worth of merchandise, how much will she make that month?

Proficiency Exam
This module is from Fundamentals of Mathematics by Denny Burzynski and Wade Ellis, Jr. This module is a proficiency exam to the chapter "Ratios and Rates." Each problem is accompanied with a reference link pointing back to the module that discusses the type of problem demonstrated in the question. The problems in this exam are accompanied by solutions.

Proficiency Exam

Exercise:

Problem: ([link]) Compare 4 cassette tapes to 7 dollars.

Solution:

$\frac{4 \text{ cassette tapes}}{7 \text{ dollars}}$

Exercise:

Problem:

([link]) What do we call a comparison, by division, of two unlike denominate numbers?

Solution:

Rate

For problems 3 and 4, express each ratio or rate as a fraction.

Exercise:

Problem: ([link]) 11 to 9

Solution:

$\frac{11}{9}$

Exercise:

Problem: ([link]) 5 televisions to 2 radios

Solution:

$\frac{5 \text{ televisions}}{2 \text{ radios}}$

For problems 5 and 6, write each ratio or rate in the form "a to b."
Exercise:

Problem: ([link]) $\frac{8 \text{ maps}}{3 \text{ people}}$

Solution:

8 maps to 3 people

Exercise:

Problem: ([link]) $\frac{2 \text{ psychologists}}{75 \text{ people}}$

Solution:

two psychologists to seventy-five people

For problems 7-9, solve each proportion.
Exercise:

Problem: ([link]) $\frac{8}{x} = \frac{48}{90}$

Solution:

15

Exercise:

Problem: ([link]) $\frac{x}{7} = \frac{4}{28}$

Solution:

1

Exercise:

Problem: ([link]) $\frac{3 \text{ computers}}{8 \text{ students}} = \frac{24 \text{ computers}}{x \text{ students}}$

Solution:

64

Exercise:

Problem:

([link]) On a map, 4 inches represents 50 miles. How many miles does 3 inches represent?

Solution:

$37\frac{1}{2}$

Exercise:

Problem:

([link]) An acid solution is composed of 6 milliliters of acid to 10 milliliters of water. How many milliliters of acid are there in an acid solution that is composed of 3 milliliters of water?

Solution:

1.8

Exercise:

Problem:

([link]) The odds that a particular event will occur are 9 to 7. If the event occurs 27 times, how many times would you predict it will it not occur?

Solution:

21

For problems 13 and 14, convert each decimal to a percent.

Exercise:

Problem: ([link]) 0.82

Solution:

82%

Exercise:

Problem: ([link]) $3.\overline{7}$

Solution:

$377\frac{7}{9}\%$

For problems 15 and 16, convert each percent to a decimal.

Exercise:

Problem: ([link]) 2.813%

Solution:

0.02813

Exercise:

Problem: ([link]) 0.006%

Solution:

0.00006

For problems 17-19, convert each fraction to a percent.
Exercise:

Problem: ([link]) $\frac{42}{5}$

Solution:

840%

Exercise:

Problem: ([link]) $\frac{1}{8}$

Solution:

12.5%

Exercise:

Problem: ([link]) $\frac{800}{80}$

Solution:

1,000%

For problems 20 and 21, convert each percent to a fraction.
Exercise:

Problem: ([link]) 15%

Solution:

$\frac{3}{20}$

Exercise:

Problem: ([link]) $\frac{4}{27}$ %

Solution:

$\frac{4}{2,700}$ or $\frac{1}{675}$

For problems 22-25, find each indicated quantity.

Exercise:

Problem: ([link]) What is 18% of 26?

Solution:

4.68

Exercise:

Problem: ([link]) 0.618 is what percent of 0.3?

Solution:

206

Exercise:

Problem: ([link]) 0.1 is 1.1% of what number?

Solution:

$9.\overline{09}$

Exercise:

Problem:

([link]) A salesperson makes a monthly salary of $1,000.00. He also gets a commission of 12% of his total monthly sales. If, in a particular month, he sells $5,500.00 worth of merchandise, what is his income that month?

Solution:

$1,660

Objectives
This module contains the learning objectives for the chapter "Techniques of Estimation" from Fundamentals of Mathematics by Denny Burzynski and Wade Ellis, jr.

After completing this chapter, you should
Estimation by Rounding ([link])

- understand the reason for estimation
- be able to estimate the result of an addition, multiplication, subtraction, or division using the rounding technique

Estimation by Clustering ([link])

- understand the concept of clustering
- be able to estimate the result of adding more than two numbers when clustering occurs using the clustering technique

Mental Arithmetic—Using the Distributive Property ([link])

- understand the distributive property
- be able to obtain the exact result of a multiplication using the distributive property

Estimation by Rounding Fractions ([link])

- be able to estimate the sum of two or more fractions using the technique of rounding fractions

Estimation by Rounding
This module is from Fundamentals of Mathematics by Denny Burzynski and Wade Ellis, Jr. This module discusses how to estimate by rounding. By the end of the module students should understand the reason for estimation and be able to estimate the result of an addition, multiplication, subtraction, or division using the rounding technique.

Section Overview

- Estimation By Rounding

When beginning a computation, it is valuable to have an idea of what value to expect for the result. When a computation is completed, it is valuable to know if the result is reasonable.

In the rounding process, it is important to note two facts:

1. The rounding that is done in estimation does not always follow the rules of rounding discussed in [link] (Rounding Whole Numbers). Since estimation is concerned with the expected value of a computation, rounding is done using *convenience* as the guide rather than using hard-and-fast rounding rules. For example, if we wish to estimate the result of the division $80 \div 26$, we might round 26 to 20 rather than to 30 since 80 is more *conveniently* divided by 20 than by 30.
2. Since rounding may occur out of convenience, and different people have different ideas of what may be convenient, results of an estimation done by rounding may vary. For a particular computation, different people may get different estimated results. *Results may vary*.

Estimation
Estimation is the process of determining an expected value of a computation.

Common words used in estimation are *about*, *near*, and *between*.

Estimation by Rounding

The rounding technique estimates the result of a computation by rounding the numbers involved in the computation to one or two nonzero digits.

Sample Set A

Example:
Estimate the sum: $2{,}357 + 6{,}106$.
Notice that 2,357 is near $\underbrace{2{,}400}_{\text{two nonzero digits}}$, and that 6,106 is near $\underbrace{6{,}100}_{\text{two nonzero digits}}$.
The sum can be estimated by $2{,}400 + 6{,}100 = 8{,}500$. (It is quick and easy to add 24 and 61.)
Thus, $2{,}357 + 6{,}106$ is *about* 8,400. In *fact*, $2{,}357 + 6{,}106 = 8{,}463$.

Practice Set A

Exercise:

Problem: Estimate the sum: $4{,}216 + 3{,}942$.

Solution:

$4{,}216 + 3{,}942$: $4{,}200 + 3{,}900$. About 8,100. In fact, 8,158.

Exercise:

Problem: Estimate the sum: $812 + 514$.

Solution:

$812 + 514$: $800 + 500$. About 1,300. In fact, 1,326.

Exercise:

Problem: Estimate the sum: $43{,}892 + 92{,}106$.

Solution:

$43{,}892 + 92{,}106$: $44{,}000 + 92{,}000$. About 136,000. In fact, 135,998.

Sample Set B

Example:
Estimate the difference: $5{,}203 - 3{,}015$.
Notice that 5,203 is near $\underbrace{5{,}200}_{\text{two nonzero digits}}$, and that 3,015 is near $\underbrace{3{,}000}_{\text{one nonzero digit}}$.
The difference can be estimated by $5{,}200 - 3{,}000 = 2{,}200$.
Thus, $5{,}203 - 3{,}015$ is *about* 2,200. In *fact*, $5{,}203 - 3{,}015 = 2{,}188$.
We could make a less accurate estimation by observing that 5,203 is near 5,000. The number 5,000 has only one nonzero digit rather than two (as does 5,200). This fact makes the estimation quicker (but a little less accurate). We then estimate the difference by $5{,}000 - 3{,}000 = 2{,}000$, and conclude that $5{,}203 - 3{,}015$ is about 2,000. This is why we say "answers may vary."

Practice Set B

Exercise:

Problem: Estimate the difference: $628 - 413$.

Solution:

$628 - 413 : 600 - 400$. About 200. In fact, 215.

Exercise:

Problem: Estimate the difference: $7{,}842 - 5{,}209$.

Solution:

$7{,}842 - 5{,}209 : 7{,}800 - 5{,}200$. About 2,600. In fact, 2,633.

Exercise:

Problem: Estimate the difference: $73{,}812 - 28{,}492$.

Solution:

$73{,}812 - 28{,}492 : 74{,}000 - 28{,}000$. About 46,000. In fact, 45,320.

Sample Set C

Example:
Estimate the product: $73 \cdot 46$.
Notice that 73 is near $\underbrace{70}_{\text{one nonzero digit}}$, and that 46 is near $\underbrace{50}_{\text{one nonzero digit}}$.
The product can be estimated by $70 \cdot 50 = 3{,}500$. (Recall that to multiply numbers ending in zeros, we multiply the nonzero digits and affix to this product the total number of ending zeros in the factors. See [link] for a review of this technique.)
Thus, $73 \cdot 46$ is about 3,500. In fact, $73 \cdot 46 = 3{,}358$.

Example:

Estimate the product: $87 \cdot 4{,}316$.
Notice that 87 is close to $\underbrace{90}_{\text{one nonzero digit}}$, and that 4,316 is close to $\underbrace{4{,}000}_{\text{one nonzero digit}}$.
The product can be estimated by $90 \cdot 4{,}000 = 360{,}000$.
Thus, $87 \cdot 4{,}316$ is about 360,000. In fact, $87 \cdot 4{,}316 = 375{,}492$.

Practice Set C

Exercise:

Problem: Estimate the product: $31 \cdot 87$.

Solution:

$31 \cdot 87 : 30 \cdot 90$. About 2,700. In fact, 2,697.

Exercise:

Problem: Estimate the product: $18 \cdot 42$.

Solution:

$18 \cdot 42 : 20 \cdot 40$. About 800. In fact, 756.

Exercise:

Problem: Estimate the product: $16 \cdot 94$.

Solution:

$16 \cdot 94 : 15 \cdot 100$. About 1,500. In fact, 1,504.

Sample Set D

Example:
Estimate the quotient: $153 \div 17$.
Notice that 153 is close to $\underbrace{150}_{\text{two nonzero digits}}$, and that 17 is close to $\underbrace{15}_{\text{two nonzero digits}}$.
The quotient can be estimated by $150 \div 15 = 10$.
Thus, $153 \div 17$ is about 10. In fact, $153 \div 17 = 9$.

Example:
Estimate the quotient: $742{,}000 \div 2{,}400$.
Notice that 742,000 is close to $\underbrace{700{,}000}_{\text{one nonzero digit}}$, and that 2,400 is close to $\underbrace{2{,}000}_{\text{one nonzero digit}}$.
The quotient can be estimated by $700{,}000 \div 2{,}000 = 350$.
Thus, $742{,}000 \div 2{,}400$ is about 350. In fact, $742{,}000 \div 2{,}400 = 309.16$.

Practice Set D

Exercise:

Problem: Estimate the quotient: $221 \div 18$.

Solution:

$221 \div 18$: $200 \div 20$. About 10. In fact, 12.27.

Exercise:

Problem: Estimate the quotient: $4{,}079 \div 381$.

Solution:

$4{,}079 \div 381 : 4{,}000 \div 400$. About 10. In fact, 10.70603675...

Exercise:

Problem: Estimate the quotient: $609{,}000 \div 16{,}000$.

Solution:

$609{,}000 \div 16{,}000 : 600{,}000 \div 15{,}000$. About 40. In fact, 38.0625.

Sample Set E

Example:
Estimate the sum: $53.82 + 41.6$.
Notice that 53.82 is close to $\underbrace{54}_{\text{two nonzero digits}}$, and that 41.6 is close to $\underbrace{42}_{\text{two nonzero digits}}$.
The sum can be estimated by $54 + 42 = 96$.
Thus, $53.82 + 41.6$ is about 96. In fact, $53.82 + 41.6 = 95.42$.

Practice Set E

Exercise:

Problem: Estimate the sum: $61.02 + 26.8$.

Solution:

$61.02 + 26.8 : 61 + 27$. About 88. In fact, 87.82.

Exercise:

Problem: Estimate the sum: $109.12 + 137.88$.

Solution:

$109.12 + 137.88 : 110 + 138$. About 248. In fact, 247. We could have estimated 137.88 with 140. Then $110 + 140$ is an easy mental addition. We would conclude then that $109.12 + 137.88$ is about 250.

Sample Set F

Example:
Estimate the product: $(31.28)(14.2)$.
Notice that 31.28 is close to $\underbrace{30}_{\text{one nonzero digit}}$, and that 14.2 is close to $\underbrace{15}_{\text{two nonzero digits}}$.
The product can be estimated by $30 \cdot 15 = 450$. ($3 \cdot 15 = 45$, then affix one zero.)
Thus, $(31.28)(14.2)$ is about 450. In fact, $(31.28)(14.2) = 444.176$.

Example:
Estimate 21% of 5.42.
Notice that $21\% = .21$ as a decimal, and that .21 is close to $\underbrace{.2}_{\text{one nonzero digit}}$.
Notice also that 5.42 is close to $\underbrace{5}_{\text{one nonzero digit}}$.
Then, 21% of 5.42 can be estimated by $(.2)(5) = 1$.
Thus, 21% of 5.42 is about 1. In fact, 21% of 5.42 is 1.1382.

Practice Set F

Exercise:

Problem: Estimate the product: $(47.8)(21.1)$.

Solution:

$(47.8)(21.1) : (50)(20)$. About 1,000. In fact, 1,008.58.

Exercise:

Problem: Estimate 32% of 14.88.

Solution:

32% of $14.88 : (.3)(15)$. About 4.5. In fact, 4.7616.

Exercises

Estimate each calculation using the method of rounding. After you have made an estimate, find the exact value and compare this to the estimated result to see if your estimated value is reasonable. Results may vary.

Exercise:

Problem: $1{,}402 + 2{,}198$

Solution:

about 3,600; in fact 3,600

Exercise:

Problem: $3{,}481 + 4{,}216$

Exercise:

Problem: $921 + 796$

Solution:

about 1,700; in fact 1,717

Exercise:

Problem: $611 + 806$

Exercise:

Problem: $4{,}681 + 9{,}325$

Solution:

about 14,000; in fact 14,006

Exercise:

Problem: $6{,}476 + 7{,}814$

Exercise:

Problem: $7{,}805 - 4{,}266$

Solution:

about 3,500; in fact 3,539

Exercise:

Problem: $8{,}427 - 5{,}342$

Exercise:

Problem: $14{,}106 - 8{,}412$

Solution:

about 5,700; in fact 5,694

Exercise:

Problem: $26{,}486 - 18{,}931$

Exercise:

Problem: $32 \cdot 53$

Solution:

about 1,500; in fact 1,696

Exercise:

Problem: $67 \cdot 42$

Exercise:

Problem: $628 \cdot 891$

Solution:

about 540,000; in fact 559,548

Exercise:

Problem: $426 \cdot 741$

Exercise:

Problem: $18{,}012 \cdot 32{,}416$

Solution:

about 583,200,000; in fact 583,876,992

Exercise:

Problem: $22,481 \cdot 51,076$

Exercise:

Problem: $287 \div 19$

Solution:

about 15; in fact 15.11

Exercise:

Problem: $884 \div 33$

Exercise:

Problem: $1,254 \div 57$

Solution:

about 20; in fact 22

Exercise:

Problem: $2,189 \div 42$

Exercise:

Problem: $8,092 \div 239$

Solution:

about 33; in fact 33.86

Exercise:

Problem: $2,688 \div 48$

Exercise:

Problem: $72.14 + 21.08$

Solution:

about 93.2; in fact 93.22

Exercise:

Problem: $43.016 + 47.58$

Exercise:

Problem: $96.53 - 26.91$

Solution:

about 70; in fact 69.62

Exercise:

Problem: $115.0012 - 25.018$

Exercise:

Problem: $206.19 + 142.38$

Solution:

about 348.6; in fact 348.57

Exercise:

Problem: $592.131 + 211.6$

Exercise:

Problem: $(32.12)(48.7)$

Solution:

about 1,568.0; in fact 1,564.244

Exercise:

Problem: $(87.013)(21.07)$

Exercise:

Problem: $(3.003)(16.52)$

Solution:

about 49.5; in fact 49.60956

Exercise:

Problem: $(6.032)(14.091)$

Exercise:

Problem: $(114.06)(384.3)$

Solution:

about 43,776; in fact 43,833.258

Exercise:

Problem: $(5{,}137.118)(263.56)$

Exercise:

Problem: $(6.92)(0.88)$

Solution:

about 6.21; in fact 6.0896

Exercise:

Problem: $(83.04)(1.03)$

Exercise:

Problem: $(17.31)(.003)$

Solution:

about 0.0519; in fact 0.05193

Exercise:

Problem: $(14.016)(.016)$

Exercise:

Problem: 93% of 7.01

Solution:

about 6.3; in fact 6.5193

Exercise:

Problem: 107% of 12.6

Exercise:

Problem: 32% of 15.3

Solution:

about 4.5; in fact 4.896

Exercise:

Problem: 74% of 21.93

Exercise:

Problem: 18% of 4.118

Solution:

about 0.8; in fact 0.74124

Exercise:

Problem: 4% of .863

Exercise:

Problem: 2% of .0039

Solution:

about 0.00008; in fact 0.000078

Exercises for Review

Exercise:

Problem: ([link]) Find the difference: $\frac{7}{10} - \frac{5}{16}$.

Exercise:

Problem: ([link]) Find the value $\frac{6-\frac{1}{4}}{6+\frac{1}{4}}$.

Solution:

$\frac{23}{25}$

Exercise:

Problem: ([link]) Convert the complex decimal $1.11\frac{1}{4}$ to a decimal.

Exercise:

Problem:

([link]) A woman 5 foot tall casts an 8-foot shadow at a particular time of the day. How tall is a tree that casts a 96-foot shadow at the same time of the day?

Solution:

60 feet tall

Exercise:

Problem: ([link]) 11.62 is 83% of what number?

Estimation by Clustering
This module is from Fundamentals of Mathematics by Denny Burzynski and Wade Ellis, Jr. This module discusses how to estimate by clustering. By the end of the module students should understand the concept of clustering and be able to estimate the result of adding more than two numbers when clustering occurs using the clustering technique.

Section Overview

- Estimation by Clustering

Cluster
When more than two numbers are to be added, the sum may be estimated using the clustering technique. The rounding technique could also be used, but if several of the numbers are seen to **cluster** (are seen to be close to) one particular number, the clustering technique provides a quicker estimate. Consider a sum such as

$$32 + 68 + 29 + 73$$

Notice two things:

1. There are more than two numbers to be added.
2. Clustering occurs.

    a. Both 68 and 73 cluster around 70, so $68 + 73$ is close to $80 + 70 = 2(70) = 140$.

    $$32 + 68 + 29 + 71$$

    b. Both 32 and 29 cluster around 30, so $32 + 29$ is close to $30 + 30 = 2(30) = 60$.

The sum may be estimated by

$$\begin{aligned}(2 \cdot 30) + (2 \cdot 70) &= 6 + 140 \\ &= 200\end{aligned}$$

In fact, $32 + 68 + 29 + 73 = 202$.

Sample Set A

Estimate each sum. Results may vary.

Example:
$27 + 48 + 31 + 52$.
27 and 31 cluster near 30. Their sum is about $2 \cdot 30 = 60$.
48 and 52 cluster near 50. Their sum is about $2 \cdot 50 = 100$.
Thus, $27 + 48 + 31 + 52$ is about $\begin{aligned}(2 \cdot 30) + (2 \cdot 50) &= 60 + 100 \\ &= 160\end{aligned}$
In fact, $27 + 48 + 31 + 52 = 158$.

Example:
$88 + 21 + 19 + 91$.
88 and 91 cluster near 90. Their sum is about $2 \cdot 90 = 180$.
21 and 19 cluster near 20. Their sum is about $2 \cdot 20 = 40$.
Thus, $88 + 21 + 19 + 91$ is about $\begin{aligned}(2 \cdot 90) + (2 \cdot 20) &= 180 + 40 \\ &= 220\end{aligned}$
In fact, $88 + 21 + 19 + 91 = 219$.

Example:
$17 + 21 + 48 + 18$.
17, 21, and 18 cluster near 20. Their sum is about $3 \cdot 20 = 60$.
48 is about 50.

Thus, $17 + 21 + 48 + 18$ is about $\begin{aligned}(3 \cdot 20) + 50 &= 60 + 50 \\ &= 110\end{aligned}$
In fact, $17 + 21 + 48 + 18 = 104$.

Example:
$61 + 48 + 49 + 57 + 52$.
61 and 57 cluster near 60. Their sum is about $2 \cdot 60 = 120$.
48, 49, and 52 cluster near 50. Their sum is about $3 \cdot 50 = 150$.
Thus, $61 + 48 + 49 + 57 + 52$ is about
$$\begin{aligned}(2 \cdot 60) + (3 \cdot 50) &= 120 + 150 \\ &= 270\end{aligned}$$
In fact, $61 + 48 + 49 + 57 + 52 = 267$.

Example:
$706 + 321 + 293 + 684$.
706 and 684 cluster near 700. Their sum is about $2 \cdot 700 = 1{,}400$.
321 and 293 cluster near 300. Their sum is about $2 \cdot 300 = 600$.
Thus, $706 + 321 + 293 + 684$ is about
$$\begin{aligned}(2 \cdot 700) + (2 \cdot 300) &= 1{,}400 + 600 \\ &= 2{,}000\end{aligned}$$
In fact, $706 + 321 + 293 + 684 = 2{,}004$.

Practice Set A

Use the clustering method to estimate each sum.
Exercise:

Problem: $28 + 51 + 31 + 47$

Solution:

$(2 \cdot 30) + (2 \cdot 50) = 60 + 100 = 160$

Exercise:

Problem: $42 + 39 + 68 + 41$

Solution:

$(3 \cdot 40) + 70 = 120 + 70 = 190$

Exercise:

Problem: $37 + 39 + 83 + 42 + 79$

Solution:

$(3 \cdot 40) + (2 \cdot 80) = 120 + 160 = 280$

Exercise:

Problem: $612 + 585 + 830 + 794$

Solution:

$(2 \cdot 600) + (2 \cdot 800) = 1{,}200 + 1{,}600 = 2{,}800$

Exercises

Use the clustering method to estimate each sum. Results may vary.

Exercise:

Problem: $28 + 51 + 31 + 47$

Solution:

$2(30) + 2(50) = 160 \ \ (157)$

Exercise:

Problem: $42 + 19 + 39 + 23$

Exercise:

Problem: $88 + 62 + 59 + 90$

Solution:

$2(90) + 2(60) = 300 \ \ (299)$

Exercise:

Problem: $76 + 29 + 33 + 82$

Exercise:

Problem: $19 + 23 + 87 + 21$

Solution:

$3(20) + 90 = 150 \ \ (150)$

Exercise:

Problem: $41 + 28 + 42 + 37$

Exercise:

Problem: $89 + 32 + 89 + 93$

Solution:

$3(90) + 30 = 300 \ \ (303)$

Exercise:

Problem: $73 + 72 + 27 + 71$

Exercise:

Problem: $43 + 62 + 61 + 55$

Solution:

$40 + 3(60) = 220 \ \ (221)$

Exercise:

Problem: $31 + 77 + 31 + 27$

Exercise:

Problem: $57 + 34 + 28 + 61 + 62$

Solution:

$3(60) + 2(30) = 240 \ \ (242)$

Exercise:

Problem: $94 + 18 + 23 + 91 + 19$

Exercise:

Problem: $103 + 72 + 66 + 97 + 99$

Solution:

$3(100) + 2(70) = 440 \ \ (437)$

Exercise:

Problem: $42 + 121 + 119 + 124 + 41$

Exercise:

Problem: $19 + 24 + 87 + 23 + 91 + 93$

Solution:

$3(20)+3(90)=330\ \ (337)$

Exercise:

Problem: $108+61+63+96+57+99$

Exercise:

Problem: $518+721+493+689$

Solution:

$2(500)+2(700)=2{,}400\ \ (2{,}421)$

Exercise:

Problem: $981+1208+1214+1006$

Exercise:

Problem: $23+81+77+79+19+81$

Solution:

$2(20)+4(80)=360\ \ (360)$

Exercise:

Problem: $94+68+66+101+106+71+110$

Exercises for Review

Exercise:

Problem: ([link]) Specify all the digits greater than 6.

Solution:

7, 8, 9

Exercise:

Problem: ([link]) Find the product: $\frac{2}{3} \cdot \frac{9}{14} \cdot \frac{7}{12}$.

Exercise:

Problem: ([link]) Convert 0.06 to a fraction.

Solution:

$\frac{3}{50}$

Exercise:

Problem:

([link]) Write the proportion in fractional form: "5 is to 8 as 25 is to 40."

Exercise:

Problem:

([link]) Estimate the sum using the method of rounding: $4{,}882 + 2{,}704$.

Solution:

$4{,}900 + 2{,}700 = 7{,}600 \ \ (7{,}586)$

Mental Arithmetic-Using the Distributive Property
This module is from Fundamentals of Mathematics by Denny Burzynski and Wade Ellis, Jr. This module discusses using the distributive property. By the end of the module students should understand the distributive property and be able to obtain the exact result of a multiplication using the distributive property.

Section Overview

- The Distributive Property
- Estimation Using the Distributive Property

The Distributive Property

Distributive Property
The **distributive property** is a characteristic of numbers that involves both addition and multiplication. It is used often in algebra, and we can use it now to obtain exact results for a multiplication.

Suppose we wish to compute $3(2+5)$. We can proceed in either of two ways, one way which is known to us already (the order of operations), and a new way (the distributive property).

1. Compute $3(2+5)$ using the order of operations.

   $3(2+5)$

   Operate inside the parentheses first: $2+5=7$.

   $3(2+5)=3\cdot 7$

   Now multiply 3 and 7.

   $3(2+5)=3\cdot 7=21$

   Thus, $3(2+5)=21$.
2. Compute $3(2+5)$ using the distributive property.

   We know that multiplication describes repeated addition. Thus,

   $$\begin{aligned} 3(2+5) &= \underbrace{2+5+2+5+2+5}_{2+5 \text{ appears } 3 \text{ times}} \\ &= 2+2+2+5+5+5 && \text{(by the commutative property of addition)} \\ &= 3\cdot 2+3\cdot 5 && \text{(since multiplication describes repeated addition)} \\ &= 6+15 \\ &= 21 \end{aligned}$$

   Thus, $3(2+5)=21$.

   Let's look again at this use of the distributive property.

$$3(2+5) = \underset{2+5 \text{ appears 3 times}}{2+5+2+5+2+5}$$

$$3(2+5) = \underset{2 \text{ appears 3 times}}{2+2+2} + \underset{5 \text{ appears 3 times}}{5+5+5}$$

$$3(2+5) = \underset{\textbf{3 times 2}}{3 \cdot 2} + \underset{\textbf{3 times 5}}{3 \cdot 5}$$

The 3 has been *distributed* to the 2 and 5.

This is the distributive property. We distribute the *factor* to each *addend* in the parentheses. The distributive property works for both sums and differences.

Sample Set A

Example:

$$\begin{aligned} 4(6+2) &= 4 \cdot 6 + 4 \cdot 2 \\ &= 24 + 8 \\ &= 32 \end{aligned}$$

Using the order of operations, we get

$$\begin{aligned} 4(6+2) &= 4 \cdot 8 \\ &= 32 \end{aligned}$$

Example:

$$\begin{aligned} 8(9+6) &= 8 \cdot 9 + 8 \cdot 6 \\ &= 72 + 48 \\ &= 120 \end{aligned}$$

Using the order of operations, we get

$$\begin{aligned} 8(9+6) &= 8 \cdot 15 \\ &= 120 \end{aligned}$$

Example:

$$\begin{aligned} 4(9-5) &= 4 \cdot 9 - 4 \cdot 5 \\ &= 36 - 20 \\ &= 16 \end{aligned}$$

Example:

$$\begin{aligned} 25(20-3) &= 25 \cdot 20 - 25 \cdot 3 \\ &= 500 - 75 \\ &= 425 \end{aligned}$$

Practice Set A

Use the distributive property to compute each value.

Exercise:

Problem: $6(8+4)$

Solution:

$6 \cdot 8 + 6 \cdot 4 = 48 + 24 = 72$

Exercise:

Problem: $4(4+7)$

Solution:

$4 \cdot 4 + 4 \cdot 7 = 16 + 28 = 44$

Exercise:

Problem: $8(2+9)$

Solution:

$8 \cdot 2 + 8 \cdot 9 = 16 + 72 = 88$

Exercise:

Problem: $12(10+3)$

Solution:

$12 \cdot 10 + 12 \cdot 3 = 120 + 36 = 156$

Exercise:

Problem: $6(11-3)$

Solution:

$6 \cdot 11 - 6 \cdot 3 = 66 - 18 = 48$

Exercise:

Problem: $8(9 - 7)$

Solution:

$8 \cdot 9–8 \cdot 7 = 72–56 = 16$

Exercise:

Problem: $15(30 - 8)$

Solution:

$15 \cdot 30 - 15 \cdot 8 = 450 - 120 = 330$

Estimation Using the Distributive Property

We can use the distributive property to obtain exact results for products such as $25 \cdot 23$. The distributive property works best for products when one of the factors ends in 0 or 5. We shall restrict our attention to only such products.

Sample Set B

Use the distributive property to compute each value.

Example:

$25 \cdot 23$

Notice that $23 = 20 + 3$. We now write

$$\begin{aligned} 25 \cdot 23 &= 25(20 + 3) \\ &= 25 \cdot 20 + 25 \cdot 3 \\ &= 500 + 75 \\ &= 575 \end{aligned}$$

Thus, $25 \cdot 23 = 575$

We could have proceeded by writing 23 as $30 - 7$.

$$\begin{aligned} 25 \cdot 23 &= 25(30 - 7) \\ &= 25 \cdot 30 - 25 \cdot 7 \\ &= 750 - 175 \\ &= 575 \end{aligned}$$

Example:
$15 \cdot 37$
Notice that $37 = 30 + 7$. We now write

$$\begin{aligned} 15 \cdot 37 &= 15(30 + 7) \\ &= 15 \cdot 30 + 15 \cdot 7 \\ &= 450 + 105 \\ &= 555 \end{aligned}$$

Thus, $15 \cdot 37 = 555$
We could have proceeded by writing 37 as $40 - 3$.

$$\begin{aligned} 15 \cdot 37 &= 15(40 - 3) \\ &= 15 \cdot 40 - 15 \cdot 3 \\ &= 600 - 45 \\ &= 555 \end{aligned}$$

Example:
$15 \cdot 86$
Notice that $86 = 80 + 6$. We now write

$$\begin{aligned} 15 \cdot 86 &= 15(80 + 6) \\ &= 15 \cdot 80 + 15 \cdot 6 \\ &= 1{,}200 + 90 \\ &= 1{,}290 \end{aligned}$$

We could have proceeded by writing 86 as $90 - 4$.

$$\begin{aligned} 15 \cdot 86 &= 15(90 - 4) \\ &= 15 \cdot 90 - 15 \cdot 4 \\ &= 1{,}350 - 60 \\ &= 1{,}290 \end{aligned}$$

Practice Set B

Use the distributive property to compute each value.
Exercise:

Problem: $25 \cdot 12$

Solution:

$25(10+2)=25 \cdot 10+25 \cdot 2=250+50=300$

Exercise:

Problem: $35 \cdot 14$

Solution:

$35(10+4)=35 \cdot 10+35 \cdot 4=350+140=490$

Exercise:

Problem: $80 \cdot 58$

Solution:

$80(50+8)=80 \cdot 50+80 \cdot 8=4{,}000+640=4{,}640$

Exercise:

Problem: $65 \cdot 62$

Solution:

$65(60+2)=65 \cdot 60+65 \cdot 2=3{,}900+130=4{,}030$

Exercises

Use the distributive property to compute each product.
Exercise:

Problem: $15 \cdot 13$

Solution:

$15(10+3)=150+45=195$

Exercise:

Problem: $15 \cdot 14$

Exercise:

Problem: $25 \cdot 11$

Solution:

$25(10+1) = 250 + 25 = 275$

Exercise:

Problem: $25 \cdot 16$

Exercise:

Problem: $15 \cdot 16$

Solution:

$15(20-4) = 300 - 60 = 240$

Exercise:

Problem: $35 \cdot 12$

Exercise:

Problem: $45 \cdot 83$

Solution:

$45(80+3) = 3600 + 135 = 3735$

Exercise:

Problem: $45 \cdot 38$

Exercise:

Problem: $25 \cdot 38$

Solution:

$25(40-2) = 1{,}000 - 50 = 950$

Exercise:

Problem: $25 \cdot 96$

Exercise:

Problem: $75 \cdot 14$

Solution:

$75(10+4)=750+300=1{,}050$

Exercise:

Problem: $85\cdot 34$

Exercise:

Problem: $65\cdot 26$

Solution:

$65(20+6)=1{,}300+390=1{,}690$ or $65(30-4)=1{,}950-260=1{,}690$

Exercise:

Problem: $55\cdot 51$

Exercise:

Problem: $15\cdot 107$

Solution:

$15(100+7)=1{,}500+105=1{,}605$

Exercise:

Problem: $25\cdot 208$

Exercise:

Problem: $35\cdot 402$

Solution:

$35(400+2)=14{,}000+70=14{,}070$

Exercise:

Problem: $85\cdot 110$

Exercise:

Problem: $95\cdot 12$

Solution:

$95(10+2)=950+190=1{,}140$

Exercise:

Problem: $65 \cdot 40$

Exercise:

Problem: $80 \cdot 32$

Solution:

$80(30 + 2) = 2{,}400 + 160 = 2{,}560$

Exercise:

Problem: $30 \cdot 47$

Exercise:

Problem: $50 \cdot 63$

Solution:

$50(60 + 3) = 3{,}000 + 150 = 3{,}150$

Exercise:

Problem: $90 \cdot 78$

Exercise:

Problem: $40 \cdot 89$

Solution:

$40(90 - 1) = 3{,}600 - 40 = 3{,}560$

Exercises for Review

Exercise:

Problem: ([link]) Find the greatest common factor of 360 and 3,780.

Exercise:

Problem: ([link]) Reduce $\frac{594}{5{,}148}$ to lowest terms.

Solution:

$\frac{3}{26}$

Exercise:

Problem: ([link]) $1\frac{5}{9}$ of $2\frac{4}{7}$ is what number?

Exercise:

Problem: ([link]) Solve the proportion: $\frac{7}{15} = \frac{}{90}$.

Solution:

$= 42$

Exercise:

Problem: ([link]) Use the clustering method to estimate the sum: $88 + 106 + 91 + 114$.

Estimation by Rounding Fractions
This module is from Fundamentals of Mathematics by Denny Burzynski and Wade Ellis, Jr. This module discusses how to estimate by rounding fractions. By the end of the module students should be able to estimate the sum of two or more fractions using the technique of rounding fractions.

Section Overview

- Estimation by Rounding Fractions

Estimation by rounding fractions is a useful technique for estimating the result of a computation involving fractions. Fractions are commonly rounded to $\frac{1}{4}$, $\frac{1}{2}$, $\frac{3}{4}$, 0, and 1. Remember that rounding may cause estimates to vary.

Sample Set A

Make each estimate remembering that results may vary.

Example:
Estimate $\frac{3}{5} + \frac{5}{12}$.
Notice that $\frac{3}{5}$ is about $\frac{1}{2}$, and that $\frac{5}{12}$ is about $\frac{1}{2}$.
Thus, $\frac{3}{5} + \frac{5}{12}$ is about $\frac{1}{2} + \frac{1}{2} = 1$. In fact, $\frac{3}{5} + \frac{5}{12} = \frac{61}{60}$, a little more than 1.

Example:
Estimate $5\frac{3}{8} + 4\frac{9}{10} + 11\frac{1}{5}$.
Adding the whole number parts, we get 20. Notice that $\frac{3}{8}$ is close to $\frac{1}{4}$, $\frac{9}{10}$ is close to 1, and $\frac{1}{5}$ is close to $\frac{1}{4}$.
Then $\frac{3}{8} + \frac{9}{10} + \frac{1}{5}$ is close to $\frac{1}{4} + 1 + \frac{1}{4} = 1\frac{1}{2}$.
Thus, $5\frac{3}{8} + 4\frac{9}{10} + 11\frac{1}{5}$ is close to $20 + 1\frac{1}{2} = 21\frac{1}{2}$.
In fact, $5\frac{3}{8} + 4\frac{9}{10} + 11\frac{1}{5} = 21\frac{19}{40}$, a little less than $21\frac{1}{2}$.

Practice Set A

Use the method of rounding fractions to estimate the result of each computation. Results may vary.
Exercise:

Problem: $\frac{5}{8} + \frac{5}{12}$

Solution:

Results may vary. $\frac{1}{2} + \frac{1}{2} = 1$. In fact, $\frac{5}{8} + \frac{5}{12} = \frac{25}{24} = 1\frac{1}{24}$

Exercise:

Problem: $\frac{7}{9} + \frac{3}{5}$

Solution:

Results may vary. $1+\frac{1}{2}=1\frac{1}{2}$. In fact, $\frac{7}{9}+\frac{3}{5}=1\frac{17}{45}$

Exercise:

Problem: $8\frac{4}{15}+3\frac{7}{10}$

Solution:

Results may vary. $8\frac{1}{4}+3\frac{3}{4}=11+1=12$. In fact, $8\frac{4}{15}+3\frac{7}{10}=11\frac{29}{30}$

Exercise:

Problem: $16\frac{1}{20}+4\frac{7}{8}$

Solution:

Results may vary. $(16+0)+(4+1)=16+5=21$. In fact, $16\frac{1}{20}+4\frac{7}{8}=20\frac{37}{40}$

Exercises

Estimate each sum or difference using the method of rounding. After you have made an estimate, find the exact value of the sum or difference and compare this result to the estimated value. Result may vary.

Exercise:

Problem: $\frac{5}{6}+\frac{7}{8}$

Solution:

$1+1=2\ \left(1\frac{17}{24}\right)$

Exercise:

Problem: $\frac{3}{8}+\frac{11}{12}$

Exercise:

Problem: $\frac{9}{10}+\frac{3}{5}$

Solution:

$1+\frac{1}{2}=1\frac{1}{2}\left(1\frac{1}{2}\right)$

Exercise:

Problem: $\frac{13}{15}+\frac{1}{20}$

Exercise:

Problem: $\frac{3}{20}+\frac{6}{25}$

Solution:

$\frac{1}{4}+\frac{1}{4}=\frac{1}{2}\left(\frac{39}{100}\right)$

Exercise:

Problem: $\frac{1}{12} + \frac{4}{5}$

Exercise:

Problem: $\frac{15}{16} + \frac{1}{12}$

Solution:

$1 + 0 = 1 \left(1 \frac{1}{48}\right)$

Exercise:

Problem: $\frac{29}{30} + \frac{11}{20}$

Exercise:

Problem: $\frac{5}{12} + 6\frac{4}{11}$

Solution:

$\frac{1}{2} + 6\frac{1}{2} = 7 \ \left(6\frac{103}{132}\right)$

Exercise:

Problem: $\frac{3}{7} + 8\frac{4}{15}$

Exercise:

Problem: $\frac{9}{10} + 2\frac{3}{8}$

Solution:

$1 + 2\frac{1}{2} = 3\frac{1}{2} \left(3\frac{11}{40}\right)$

Exercise:

Problem: $\frac{19}{20} + 15\frac{5}{9}$

Exercise:

Problem: $8\frac{3}{5} + 4\frac{1}{20}$

Solution:

$8\frac{1}{2} + 4 = 12\frac{1}{2} \left(12\frac{13}{20}\right)$

Exercise:

Problem: $5\frac{3}{20} + 2\frac{8}{15}$

Exercise:

Problem: $9\frac{1}{15} + 6\frac{4}{5}$

Solution:

$9+7=16\ \left(15\frac{13}{15}\right)$

Exercise:

Problem: $7\frac{5}{12}+10\frac{1}{16}$

Exercise:

Problem: $3\frac{11}{20}+2\frac{13}{25}+1\frac{7}{8}$

Solution:

$3\frac{1}{2}+2\frac{1}{2}+2=8\ \left(7\frac{189}{200}\right)$

Exercise:

Problem: $6\frac{1}{12}+1\frac{1}{10}+5\frac{5}{6}$

Exercise:

Problem: $\frac{15}{16}-\frac{7}{8}$

Solution:

$1-1=0\ \left(\frac{1}{16}\right)$

Exercise:

Problem: $\frac{12}{25}-\frac{9}{20}$

Exercises for Review

Exercise:

Problem:

([link]) The fact that
$(\text{a first number}\cdot\text{a second number})\cdot\text{a third number}=\text{a first number}\cdot(\text{a second number}\cdot\text{a third num}$
is an example of which property of multiplication?

Solution:

associative

Exercise:

Problem: ([link]) Find the quotient: $\frac{14}{15}\div\frac{4}{45}$.

Exercise:

Problem: ([link]) Find the difference: $3\frac{5}{9}-2\frac{2}{3}$.

Solution:

$\frac{8}{9}$

Exercise:

Problem: ([link]) Find the quotient: $4.6 \div 0.11$.

Exercise:

Problem: ([link]) Use the distributive property to compute the product: $25 \cdot 37$.

Solution:

$25(40 - 3) = 1000 - 75 = 925$

Summary of Key Concepts
This module is from Fundamentals of Mathematics by Denny Burzynski and Wade Ellis, Jr. This module reviews the key concepts from the chapter "Techniques of Estimation."

Summary of Key Concepts

Estimation ([link])
Estimation is the process of determining an expected value of a computation.

Estimation By Rounding ([link])
The **rounding technique** estimates the result of a computation by rounding the numbers involved in the computation to one or two nonzero digits. For example, $512 + 896$ can be estimated by $500 + 900 = 1{,}400$.

Cluster ([link])
When several numbers are close to one particular number, they are said to **cluster** near that particular number.

Estimation By Clustering ([link])
The **clustering technique of estimation** can be used when

1. there are more than two numbers to be added, and
2. clustering occurs.

For example, $31 + 62 + 28 + 59$ can be estimated by
$(2 \cdot 30) + (2 \cdot 60) = 60 + 120 = 180$

Distributive Property ([link])
The **distributive property** is a characteristic of numbers that involves both addition and multiplication. For example,
$3(4 + 6) = 3 \cdot 4 + 3 \cdot 6 = 12 + 18 = 30$

Estimation Using the Distributive Property ([link])
The **distributive property** can be used to obtain exact results for a multiplication.

For example,
$15 \cdot 23 = 15 \cdot (20 + 3) = 15 \cdot 20 + 15 \cdot 3 = 300 + 45 = 345$

Estimation by Rounding Fractions ([link])
Estimation by rounding fractions commonly rounds fractions to $\frac{1}{4}$, $\frac{1}{2}$, $\frac{3}{4}$, 0, and 1.
For example,
$\frac{5}{12} + \frac{5}{16}$ can be estimated by $\frac{1}{2} + \frac{1}{4} = \frac{3}{4}$

Exercise Supplement
This module is from Fundamentals of Mathematics by Denny Burzynski and Wade Ellis, Jr. This module is an exercise supplement for the chapter "Techniques of Estimation" and contains many exercise problems. Odd problems are accompanied by solutions.

Exercise Supplement

Estimation by Rounding ([link])

For problems 1-70, estimate each value using the method of rounding. After you have made an estimate, find the exact value. Compare the exact and estimated values. Results may vary.

Exercise:

Problem: $286 + 312$

Solution:

600 (598)

Exercise:

Problem: $419 + 582$

Exercise:

Problem: $689 + 511$

Solution:

(1,200)

Exercise:

Problem: $926 + 1{,}105$

Exercise:

Problem: $1{,}927 + 3{,}017$

Solution:

4,900 (4,944)

Exercise:

Problem: $5{,}026 + 2{,}814$

Exercise:

Problem: $1{,}408 + 2{,}352$

Solution:

3,800 (3,760)

Exercise:

Problem: $1{,}186 + 4{,}228$

Exercise:

Problem: $5{,}771 + 246$

Solution:

6,050 (6,017)

Exercise:

Problem: $8{,}305 + 484$

Exercise:

Problem: $3{,}812 + 2{,}906$

Solution:

6,700 (6,718)

Exercise:

Problem: $5{,}293 + 8{,}007$

Exercise:

Problem: $28{,}481 + 32{,}856$

Solution:

61,400 (61,337)

Exercise:

Problem: $92{,}512 + 26{,}071$

Exercise:

Problem: $87{,}612 + 2{,}106$

Solution:

89,700 (89,718)

Exercise:

Problem: $42{,}612 + 4{,}861$

Exercise:

Problem: $212{,}413 + 609$

Solution:

213,000 (213,022)

Exercise:

Problem: $487{,}235 + 494$

Exercise:

Problem: $2{,}409 + 1{,}526$

Solution:

3,900 (3,935)

Exercise:

Problem: $3{,}704 + 4{,}704$

Exercise:

Problem: $41 \cdot 63$

Solution:

2,400 (2,583)

Exercise:

Problem: $38 \cdot 81$

Exercise:

Problem: $18 \cdot 28$

Solution:

600 (504)

Exercise:

Problem: $52 \cdot 21$

Exercise:

Problem: $307 \cdot 489$

Solution:

150,123 147,000 (150,123)

Exercise:

Problem: $412 \cdot 807$

Exercise:

Problem: $77 \cdot 614$

Solution:

47,278 48,000 (47,278)

Exercise:

Problem: $62 \cdot 596$

Exercise:

Problem: $27 \cdot 473$

Solution:

12,771 14,100 (12,711)

Exercise:

Problem: $92 \cdot 336$

Exercise:

Problem: $12 \cdot 814$

Solution:

8,100 (9,768)

Exercise:

Problem: $8 \cdot 2,106$

Exercise:

Problem: $192 \cdot 452$

Solution:

90,000 (86,784)

Exercise:

Problem: $374 \cdot 816$

Exercise:

Problem: $88 \cdot 4,392$

Solution:

396,000 (386,496)

Exercise:

Problem: $126 \cdot 2,834$

Exercise:

Problem: $3,896 \cdot 413$

Solution:

1,609,048 1,560,000 (1,609,048)

Exercise:

Problem: $5{,}794 \cdot 837$

Exercise:

Problem: $6{,}311 \cdot 3{,}512$

Solution:

22,050,000 (22,164,232)

Exercise:

Problem: $7{,}471 \cdot 5{,}782$

Exercise:

Problem: $180 \div 12$

Solution:

18 (15)

Exercise:

Problem: $309 \div 16$

Exercise:

Problem: $286 \div 22$

Solution:

$14\frac{1}{2}$ (13)

Exercise:

Problem: $527 \div 17$

Exercise:

Problem: $1,007 \div 19$

Solution:

50 (53)

Exercise:

Problem: $1,728 \div 36$

Exercise:

Problem: $2,703 \div 53$

Solution:

54 (51)

Exercise:

Problem: $2,562 \div 61$

Exercise:

Problem: $1,260 \div 12$

Solution:

130 (105)

Exercise:

Problem: $3,618 \div 18$

Exercise:

Problem: $3,344 \div 76$

Solution:

41.25 (44)

Exercise:

Problem: $7,476 \div 356$

Exercise:

Problem: $20,984 \div 488$

Solution:

42 (43)

Exercise:

Problem: $43,776 \div 608$

Exercise:

Problem: $7,196 \div 514$

Solution:

14.4 (14)

Exercise:

Problem: $51,492 \div 514$

Exercise:

Problem: $26{,}962 \div 442$

Solution:

60 (61)

Exercise:

Problem: $33{,}712 \div 112$

Exercise:

Problem: $105{,}152 \div 106$

Solution:

1,000 (992)

Exercise:

Problem: $176{,}978 \div 214$

Exercise:

Problem: $48.06 + 23.11$

Solution:

71.1 (71.17)

Exercise:

Problem: $73.73 + 72.9$

Exercise:

Problem: $62.91 + 56.4$

Solution:

119.4 (119.31)

Exercise:

Problem: $87.865 + 46.772$

Exercise:

Problem: $174.6 + 97.2$

Solution:

272 (271.8)

Exercise:

Problem: $(48.3)(29.6)$

Exercise:

Problem: $(87.11)(23.2)$

Solution:

2,001 (2,020.952)

Exercise:

Problem: $(107.02)(48.7)$

Exercise:

Problem: $(0.76)(5.21)$

Solution:

4.16 (3.9596)

Exercise:

Problem: $(1.07)(13.89)$

Estimation by Clustering ([link])

For problems 71-90, estimate each value using the method of clustering. After you have made an estimate, find the exact value. Compare the exact and estimated values. Results may vary.

Exercise:

Problem: $38 + 51 + 41 + 48$

Solution:

$2(40) + 2(50) = 180$ (178)

Exercise:

Problem: $19 + 73 + 23 + 71$

Exercise:

Problem: $27 + 62 + 59 + 31$

Solution:

$2(30) + 2(60) = 180$ (179)

Exercise:

Problem: $18 + 73 + 69 + 19$

Exercise:

Problem: $83 + 49 + 79 + 52$

Solution:

$2(80) + 2(50) = 260$ (263)

Exercise:

Problem: $67 + 71 + 84 + 81$

Exercise:

Problem: $16 + 13 + 24 + 26$

Solution:

$3(20) + 1(10) = 70$ (79)

Exercise:

Problem: $34 + 56 + 36 + 55$

Exercise:

Problem: $14 + 17 + 83 + 87$

Solution:

$2(15) + 2(80) = 190$ (201)

Exercise:

Problem: $93 + 108 + 96 + 111$

Exercise:

Problem: $18 + 20 + 31 + 29 + 24 + 38$

Solution:

$3(20) + 2(30) + 40 = 160$ (160)

Exercise:

Problem: $32 + 27 + 48 + 51 + 72 + 69$

Exercise:

Problem: $64 + 17 + 27 + 59 + 31 + 21$

Solution:

$2(60) + 2(20) + 2(30) = 220$ (219)

Exercise:

Problem: $81 + 41 + 92 + 38 + 88 + 80$

Exercise:

Problem: $87 + 22 + 91$

Solution:

$2(90) + 20 = 200$ (200)

Exercise:

Problem: $44 + 38 + 87$

Exercise:

Problem: $19 + 18 + 39 + 22 + 42$

Solution:

$3(20) + 2(40) = 140$ (140)

Exercise:

Problem: $31 + 28 + 49 + 29$

Exercise:

Problem: $88 + 86 + 27 + 91 + 29$

Solution:

$3(90) + 2(30) = 330$ (321)

Exercise:

Problem: $57 + 62 + 18 + 23 + 61 + 21$

Mental Arithmetic- Using the Distributive Property ([link])

For problems 91-110, compute each product using the distributive property.

Exercise:

Problem: $15 \cdot 33$

Solution:

$15(30 + 3) = 450 + 45 = 495$

Exercise:

Problem: $15 \cdot 42$

Exercise:

Problem: $35 \cdot 36$

Solution:

$35(40 - 4) = 1400 - 140 = 1{,}260$

Exercise:

Problem: $35 \cdot 28$

Exercise:

Problem: $85 \cdot 23$

Solution:

$85(20 + 3) = 1{,}700 + 225 = 1{,}955$

Exercise:

Problem: $95 \cdot 11$

Exercise:

Problem: $30 \cdot 14$

Solution:

$30(10 + 4) = 300 + 120 = 420$

Exercise:

Problem: $60 \cdot 18$

Exercise:

Problem: $75 \cdot 23$

Solution:

$75(20 + 3) = 1{,}500 + 225 = 1{,}725$

Exercise:

Problem: $65 \cdot 31$

Exercise:

Problem: $17 \cdot 15$

Solution:

$15(20 - 3) = 300 - 45 = 255$

Exercise:

Problem: $38 \cdot 25$

Exercise:

Problem: $14 \cdot 65$

Solution:

$65(10 + 4) = 650 + 260 = 910$

Exercise:

Problem: $19 \cdot 85$

Exercise:

Problem: $42 \cdot 60$

Solution:

$60(40 + 2) = 2{,}400 + 120 = 2{,}520$

Exercise:

Problem: $81 \cdot 40$

Exercise:

Problem: $15 \cdot 105$

Solution:

$15(100+5) = 1{,}500 + 75 = 1{,}575$

Exercise:

Problem: $35 \cdot 202$

Exercise:

Problem: $45 \cdot 306$

Solution:

$45(300+6) = 13{,}500 + 270 = 13{,}770$

Exercise:

Problem: $85 \cdot 97$

Estimation by Rounding Fractions ([link])

For problems 111-125, estimate each sum using the method of rounding fractions. After you have made an estimate, find the exact value. Compare the exact and estimated values. Results may vary.

Exercise:

Problem: $\frac{3}{8} + \frac{5}{6}$

Solution:

$\frac{1}{2} + 1 = 1\frac{1}{2} \left(1\frac{5}{24}\right)$

Exercise:

Problem: $\frac{7}{16} + \frac{1}{24}$

Exercise:

Problem: $\frac{7}{15} + \frac{13}{30}$

Solution:

$\frac{1}{2} + \frac{1}{2} = 1 \ \left(\frac{27}{30} \text{or} \frac{9}{10}\right)$

Exercise:

Problem: $\frac{14}{15} + \frac{19}{20}$

Exercise:

Problem: $\frac{13}{25} + \frac{7}{30}$

Solution:

$\frac{1}{2} + \frac{1}{4} = \frac{3}{4} \left(\frac{113}{150}\right)$

Exercise:

Problem: $\frac{11}{12} + \frac{7}{8}$

Exercise:

Problem: $\frac{9}{32} + \frac{15}{16}$

Solution:

$\frac{1}{4} + 1 = 1\frac{1}{4}\left(\frac{39}{32}\text{or } \ 1\frac{7}{32}\right)$

Exercise:

Problem: $\frac{5}{8} + \frac{1}{32}$

Exercise:

Problem: $2\frac{3}{4} + 6\frac{3}{5}$

Solution:

$2\frac{3}{4} + 6\frac{1}{2} = 9\frac{1}{4}\left(9\frac{7}{20}\right)$

Exercise:

Problem: $4\frac{5}{9} + 8\frac{1}{27}$

Exercise:

Problem: $11\frac{5}{18} + 7\frac{22}{45}$

Solution:

$11\frac{1}{4} + 7\frac{1}{2} = 18\frac{3}{4}\left(18\frac{23}{30}\right)$

Exercise:

Problem: $14\frac{19}{36} + 2\frac{7}{18}$

Exercise:

Problem: $6\frac{1}{20} + 2\frac{1}{10} + 8\frac{13}{60}$

Solution:

$6 + 2 + 8\frac{1}{4} = 16\frac{1}{4}\left(16\frac{11}{30}\right)$

Exercise:

Problem: $5\frac{7}{8} + 1\frac{1}{4} + 12\frac{5}{12}$

Exercise:

Problem: $10\frac{1}{2} + 6\frac{15}{16} + 8\frac{19}{80}$

Solution:

$10\frac{1}{2} + 7 + 8\frac{1}{4} = 25\frac{3}{4}\left(25\frac{27}{40}\right)$

Proficiency Exam
This module is from Fundamentals of Mathematics by Denny Burzynski and Wade Ellis, Jr. This module is a proficiency exam to the chapter "Techniques of Estimation." Each problem is accompanied with a reference link pointing back to the module that discusses the type of problem demonstrated in the question. The problems in this exam are accompanied by solutions.

Proficiency Exam

For problems 1 - 16, estimate each value. After you have made an estimate, find the exact value. Results may vary.

Exercise:

Problem: ([link]) $3,716 + 6,789$

Solution:

10,500 (10,505)

Exercise:

Problem: ([link]) $8,821 + 9,217$

Solution:

18,000 (18,038)

Exercise:

Problem: ([link]) $7,316 - 2,305$

Solution:

5,000 (5,011)

Exercise:

Problem: ([link]) $110{,}812 - 83{,}406$

Solution:

28,000 (27,406)

Exercise:

Problem: ([link]) $82 \cdot 38$

Solution:

3,200 (3,116)

Exercise:

Problem: ([link]) $51 \cdot 92$

Solution:

4,500 (4,692)

Exercise:

Problem: ([link]) $48 \cdot 6{,}012$

Solution:

300,000 (288,576)

Exercise:

Problem: ([link]) $238 \div 17$

Solution:

12 (14)

Exercise:

Problem: ([link]) $2{,}660 \div 28$

Solution:

90 (95)

Exercise:

Problem: ([link]) $43.06 + 37.94$

Solution:

81 (81.00)

Exercise:

Problem: ([link]) $307.006 + 198.0005$

Solution:

505 (505.0065)

Exercise:

Problem: ([link]) $(47.2)(92.8)$

Solution:

4,371 (4,380.16)

Exercise:

Problem: ([link]) $58 + 91 + 61 + 88$

Solution:

$2(60) + 2(90) = 300$ (298)

Exercise:

Problem: ([link]) $43 + 39 + 89 + 92$

Solution:

$2(40) + 2(90) = 260$ (263)

Exercise:

Problem: ([link]) $81 + 78 + 27 + 79$

Solution:

$30 + 3(80) = 270$ (265)

Exercise:

Problem: ([link]) $804 + 612 + 801 + 795 + 606$

Solution:

$3(800) + 2(600) = 3{,}600$ (3,618)

For problems 17-21, use the distributive property to obtain the exact result.

Exercise:

Problem: ([link]) $25 \cdot 14$

Solution:

$25(10 + 4) = 250 + 100 = 350$

Exercise:

Problem: ([link]) $15 \cdot 83$

Solution:

$15(80+3) = 1{,}200 + 45 = 1{,}245$

Exercise:

Problem: ([link]) $65 \cdot 98$

Solution:

$65(100-2) = 6{,}500 - 130 = 6{,}370$

Exercise:

Problem: ([link]) $80 \cdot 107$

Solution:

$80(100+7) = 8{,}000 + 560 = 8{,}560$

Exercise:

Problem: ([link]) $400 \cdot 215$

Solution:

$400(200+15) = 80{,}000 + 6{,}000 = 86{,}000$

For problems 22-25, estimate each value. After you have made an estimate, find the exact value. Results may vary.

Exercise:

Problem: ([link]) $\frac{15}{16} + \frac{5}{8}$

Solution:

$1 + \frac{1}{2} = 1\frac{1}{2}\left(1\frac{9}{16}\right)$

Exercise:

Problem: ([link]) $\frac{1}{25} + \frac{11}{20} + \frac{17}{30}$

Solution:

$0 + \frac{1}{2} + \frac{1}{2} = 1 \left(1 \frac{47}{300}\right)$

Exercise:

Problem: ([link]) $8 \frac{9}{16} + 14 \frac{1}{12}$

Solution:

$8 \frac{1}{2} + 14 = 22 \frac{1}{2} \left(22 \frac{31}{48}\right)$

Exercise:

Problem: ([link]) $5 \frac{4}{9} + 1 \frac{17}{36} + 6 \frac{5}{12}$

Solution:

$5 \frac{1}{2} + 1 \frac{1}{2} + 6 \frac{1}{2} = 13 \frac{1}{2} \left(13 \frac{1}{3}\right)$

Objectives
This module contains the learning objectives for the chapter "Measurement and Geometry" from Fundamentals of Mathematics by Denny Burzynski and Wade Ellis, jr.

After completing this chapter, you should
Measurement and the United States System ([link])

- know what the word measurement means
- be familiar with United States system of measurement
- be able to convert from one unit of measure in the United States system to another unit of measure

The Metric System of Measurement ([link])

- be more familiar with some of the advantages of the base ten number system
- know the prefixes of the metric measures
- be familiar with the metric system of measurement
- be able to convert from one unit of measure in the metric system to another unit of measure

Simplification of Denominate Numbers ([link])

- be able to convert an unsimplified unit of measure to a simplified unit of measure
- be able to add and subtract denominate numbers
- be able to multiply and divide a denominate number by a whole number

Perimeter and Circumference of Geometric Figures ([link])

- know what a polygon is
- know what perimeter is and how to find it
- know what the circumference, diameter, and radius of a circle is and how to find each one
- know the meaning of the symbol and its approximating value

- know what a formula is and four versions of the circumference formula of a circle

Area and Volume of Geometric Figures and Objects ([link])

- know the meaning and notation for area
- know the area formulas for some common geometric figures
- be able to find the areas of some common geometric figures
- know the meaning and notation for volume
- know the volume formulas for some common geometric objects
- be able to find the volume of some common geometric objects

Measurement and the United States System
This module is from Fundamentals of Mathematics by Denny Burzynski and Wade Ellis, Jr. This module discusses the United States System of measurement. By the end of the module students should know what the word measurement means, be familiar with United States system of measurement and be able to convert from one unit of measure in the United States system to another unit of measure.

Section Overview

- Measurement
- The United States System of Measurement
- Conversions in the United States System

Measurement

There are two major systems of measurement in use today. They are the *United States system* and the *metric system*. Before we describe these systems, let's gain a clear understanding of the concept of measurement.

Measurement
Measurement is comparison to some standard.

Standard Unit of Measure
The concept of measurement is based on the idea of direct comparison. This means that measurement is the result of the comparison of two quantities. The quantity that is used for comparison is called the **standard unit of measure**.

Over the years, standards have changed. Quite some time in the past, the standard unit of measure was determined by a king. For example,

1 inch was the distance between the tip of the thumb and the knuckle of the king.
1 inch was also the length of 16 barley grains placed end to end.

Today, standard units of measure rarely change. Standard units of measure are the responsibility of the Bureau of Standards in Washington D.C.

Some desirable properties of a standard are the following:

1. *Accessibility*. We should have access to the standard so we can make comparisons.
2. *Invariance*. We should be confident that the standard is not subject to change.
3. *Reproducibility*. We should be able to reproduce the standard so that measurements are convenient and accessible to many people.

The United States System of Measurement

Some of the common units (along with their abbreviations) for the United States system of measurement are listed in the following table.

Unit Conversion Table	
Length	1 foot (ft) = 12 inches (in.) 1 yard (yd) = 3 feet (ft) 1 mile (mi) = 5,280 feet

Weight	1 pound (lb) =16 ounces (oz) 1 ton (T) = 2,000 pounds
Liquid Volume	1 tablespoon (tbsp) = 3 teaspoons (tsp) 1 fluid ounce (fl oz) = 2 tablespoons 1 cup (c) = 8 fluid ounces 1 pint (pt) = 2 cups 1 quart (qt) = 2 pints 1 gallon (gal) = 4 quarts
Time	1 minute (min) = 60 seconds (sec) 1 hour (hr) = 60 minutes 1 day (da) = 24 hours 1 week (wk) = 7 days

Conversions in the United States System

It is often convenient or necessary to convert from one unit of measure to another. For example, it may be convenient to convert a measurement of length that is given in feet to one that is given in inches. Such conversions can be made using *unit fractions*.

Unit Fraction
A **unit fraction** is a fraction with a value of 1.

Unit fractions are formed by using two equal measurements. One measurement is placed in the numerator of the fraction, and the other in the denominator. **Placement depends on the desired conversion**.

Placement of Units
Place the unit being converted *to* in the **numerator**.
Place the unit being converted *from* in the **denominator**.

For example,

Equal Measurements	Unit Fraction
$1\text{ft} = 12\text{in.}$	$\frac{1\text{ft}}{12\text{in.}}$ or $\frac{12\text{in.}}{1\text{ft}}$
$1\text{pt} = 16\text{ fl oz}$	$\frac{1\text{pt}}{16\text{ fl oz}}$ or $\frac{16\text{ fl oz}}{1\text{pt}}$
$1\text{wk} = 7\text{da}$	$\frac{7\text{da}}{1\text{wk}}$ or $\frac{1\text{wk}}{7\text{da}}$

Sample Set A

Make the following conversions. If a fraction occurs, convert it to a decimal rounded to two decimal places.

Example:
Convert 11 yards to feet.
Looking in the unit conversion table under *length*, we see that $1\text{yd} = 3\text{ ft}$. There are two corresponding unit fractions, $\frac{1\text{ yd}}{3\text{ ft}}$ and $\frac{3\text{ ft}}{1\text{ yd}}$. Which one should we use? Look to see which unit we wish to convert to. Choose the unit fraction with this unit in the *numerator*. We will choose $\frac{3\text{ ft}}{1\text{ yd}}$ since this unit fraction has feet in the numerator.
Now, multiply 11 yd by the unit fraction. Notice that since the unit fraction has the value of 1, multiplying by it does not change the value of 11 yd.

$$\begin{aligned} 11\text{yd} &= \frac{11\text{yd}}{1} \cdot \frac{3\text{ft}}{1\text{yd}} && \text{Divide out common units.} \\ &= \frac{11\cancel{\text{ yd}}}{1} \cdot \frac{3\text{ft}}{1\cancel{\text{ yd}}} && \text{(Units can be added, subtracted, multiplied, and divided, just as numbers can.)} \\ &= \frac{11 \cdot 3\text{ft}}{1} \\ &= 33\text{ft} \end{aligned}$$

Thus, $11\text{yd} = 33\text{ft}$.

Example:
Convert 36 fl oz to pints.
Looking in the unit conversion table under *liquid volume*, we see that $1\text{ pt} = 16\text{ fl oz}$. Since we are to convert to pints, we will construct a unit fraction with pints in the numerator.

$$\begin{aligned} 36\text{fl oz} &= \frac{36\text{fl oz}}{1} \cdot \frac{1\text{pt}}{16\text{fl oz}} && \text{Divide out common units.} \\ &= \frac{36\cancel{\text{ fl oz}}}{1} \cdot \frac{1\text{pt}}{16\cancel{\text{ fl oz}}} \\ &= \frac{36 \cdot 1\text{ pt}}{16} \\ &= \frac{36\text{ pt}}{16} && \text{Reduce.} \\ &= \frac{9}{4}\text{pt} && \text{Convert to decimals:} \frac{9}{4} = 2.25. \end{aligned}$$

Thus, $36\text{ fl oz} = 2.25\text{ pt}$.

Example:
Convert 2,016 hr to weeks.
Looking in the unit conversion table under *time*, we see that $1\text{wk} = 7\text{da}$ and that $1\text{da} = 24\text{ hr}$. To convert from hours to weeks, we must first convert from hours to days and then from days to weeks. We need two unit fractions.
The unit fraction needed for converting from hours to days is $\frac{1\text{ da}}{24\text{ hr}}$. The unit fraction needed for converting from days to weeks is $\frac{1\text{ wk}}{7\text{ da}}$.

$$\begin{aligned} 2{,}016\text{hr} &= \frac{2{,}016\text{hr}}{1} \cdot \frac{1\text{da}}{24\text{hr}} \cdot \frac{1\text{wk}}{7\text{da}} && \text{Divide out common units.} \\ &= \frac{2{,}016\cancel{\text{ hr}}}{1} \cdot \frac{1\cancel{\text{ da}}}{24\cancel{\text{ hr}}} \cdot \frac{1\text{wk}}{7\cancel{\text{ da}}} \\ &= \frac{2{,}016 \cdot 1\text{wk}}{24 \cdot 7} && \text{Reduce.} \\ &= 12\text{wk} \end{aligned}$$

Thus, $2{,}016\text{ hr} = 12\text{ wk}$.

Practice Set A

Make the following conversions. If a fraction occurs, convert it to a decimal rounded to two decimal places.
Exercise:

Problem: Convert 18 ft to yards.

Solution:

6 yd

Exercise:

Problem: Convert 2 mi to feet.

Solution:

10,560 ft

Exercise:

Problem: Convert 26 ft to yards.

Solution:

8.67 yd

Exercise:

Problem: Convert 9 qt to pints.

Solution:

18 pt

Exercise:

Problem: Convert 52 min to hours.

Solution:

0.87 hr

Exercise:

Problem: Convert 412 hr to weeks.

Solution:

2.45 wk

Exercises

Make each conversion using unit fractions. If fractions occur, convert them to decimals rounded to two decimal places.

Exercise:

Problem: 14 yd to feet

Solution:

42 feet

Exercise:

Problem: 3 mi to yards

Exercise:

Problem: 8 mi to inches

Solution:

506,880 inches

Exercise:

Problem: 2 mi to inches

Exercise:

Problem: 18 in. to feet

Solution:

1.5 feet

Exercise:

Problem: 84 in. to yards

Exercise:

Problem: 5 in. to yards

Solution:

0.14 yard

Exercise:

Problem: 106 ft to miles

Exercise:

Problem: 62 in. to miles

Solution:

0.00 miles (to two decimal places)

Exercise:

Problem: 0.4 in. to yards

Exercise:

Problem: 3 qt to pints

Solution:

6 pints

Exercise:

Problem: 5 lb to ounces

Exercise:

Problem: 6 T to ounces

Solution:

192,000 ounces

Exercise:

Problem: 4 oz to pounds

Exercise:

Problem: 15,000 oz to pounds

Solution:

937.5 pounds

Exercise:

Problem: 15,000 oz to tons

Exercise:

Problem: 9 tbsp to teaspoons

Solution:

27 teaspoons

Exercise:

Problem: 3 c to tablespoons

Exercise:

Problem: 5 pt to fluid ounces

Solution:

80 fluid ounces

Exercise:

Problem: 16 tsp to cups

Exercise:

Problem: 5 fl oz to quarts

Solution:

0.16 quart

Exercise:

Problem: 3 qt to gallons

Exercise:

Problem: 5 pt to teaspoons

Solution:

480 teaspoons

Exercise:

Problem: 3 qt to tablespoons

Exercise:

Problem: 18 min to seconds

Solution:

1,080 seconds

Exercise:

Problem: 4 days to hours

Exercise:

Problem: 3 hr to days

Solution:

$\frac{1}{8} = 0.125$ day

Exercise:

Problem: $\frac{1}{2}$ hr to days

Exercise:

Problem: $\frac{1}{2}$ da to weeks

Solution:

$\frac{1}{14} = 0.0714$ week

Exercise:

Problem: $3\frac{1}{7}$ wk to seconds

Exercises for Review

Exercise:

Problem: ([link]) Specify the digits by which 23,840 is divisible.

Solution:

1,2,4,5,8

Exercise:

Problem: ([link]) Find $2\frac{4}{5}$ of $5\frac{5}{6}$ of $7\frac{5}{7}$.

Exercise:

Problem: ([link]) Convert $0.3\frac{2}{3}$ to a fraction.

Solution:

$\frac{11}{30}$

Exercise:

Problem: ([link]) Use the clustering method to estimate the sum: $53 + 82 + 79 + 49$.

Exercise:

Problem: ([link]) Use the distributive property to compute the product: $60 \cdot 46$.

Solution:

$60(50 - 4) = 3{,}000 - 240 = 2{,}760$

The Metric System of Measurement
This module is from Fundamentals of Mathematics by Denny Burzynski and Wade Ellis, Jr. This module discusses the Metric System of measurement. By the end of the module students should be more familiar with some of the advantages of the base ten number system, know the prefixes of the metric measures, be familiar with the metric system of measurement and be able to convert from one unit of measure in the metric system to another unit of measure

Section Overview

- The Advantages of the Base Ten Number System
- Prefixes
- Conversion from One Unit to Another Unit
- Conversion Table

The Advantages of the Base Ten Number System

The metric system of measurement takes advantage of our base ten number system. The advantage of the metric system over the United States system is that in the metric system it is possible to convert from one unit of measure to another simply by multiplying or dividing the given number by a power of 10. This means we can make a conversion simply by moving the decimal point to the right or the left.

Prefixes

Common units of measure in the metric system are the meter (for length), the liter (for volume), and the gram (for mass). To each of the units can be attached a prefix. The **metric prefixes** along with their meaning are listed below.
Metric Prefixes

- **kilo**thousand
- **deci**tenth
- **hecto**hundred
- **centi**hundredth
- **deka**ten
- **milli**thousandth

For example, if length is being measured,

1 kilometer is equivalent to 1000 meters.
1 centimeter is equivalent to one hundredth of a meter.
1 millimeter is equivalent to one thousandth of a meter.

Conversion from One Unit to Another Unit

Let's note three characteristics of the metric system that occur in the metric table of measurements.

1. In each category, the prefixes are the same.
2. We can move from a *larger to a smaller* unit of measure by moving the decimal point to the *right*.
3. We can move from a *smaller to a larger unit* of measure by moving the decimal point to the *left*.

The following table provides a summary of the relationship between the basic unit of measure (meter, gram, liter) and each prefix, and how many places the decimal point is moved and in what direction.

kilo hecto deka unit deci centi milli

Basic Unit to Prefix		Move the Decimal Point
unit to deka	1 to 10	1 place to the left
unit to hector	1 to 100	2 places to the left
unit to kilo	1 to 1,000	3 places to the left
unit to deci	1 to 0.1	1 place to the right
unit to centi	1 to 0.01	2 places to the right

unit to milli	1 to 0.001	3 places to the right

Conversion Table

Listed below, in the unit conversion table, are some of the common metric units of measure.

Unit Conversion Table		
Length	$1\ \text{kilometer}\ \ (\text{km}) = 1{,}000\ \text{meters}\ \ (m)$	$1{,}000 \times 1\text{m}$
	$1\ \text{hectometer}\ \ (\text{hm}) = 100\ \text{meters}$	$100 \times 1\text{m}$
	$1\ \text{dekameter}\ \ (\text{dam}) = 10\ \text{meters}$	$10 \times 1\text{m}$
	1 meter (m)	$1 \times 1\text{m}$
	$1\ \text{decimeter}\ \ (\text{dm}) = \frac{1}{10}\ \text{meter}$	$.1 \times 1\text{m}$
	$1\ \text{centimeter}\ \ (\text{cm}) = \frac{1}{100}\ \text{meter}$	$.01 \times 1\text{m}$
	$1\ \text{millimeter}\ \ (\text{mm}) = \frac{1}{1{,}000}\ \text{meter}$	$.001 \times 1\text{m}$
Mass	$1\ \text{kilogram}\ \ (\text{kg}) = 1{,}000\ \text{grams}\ \ (g)$	$1{,}000 \times 1\text{g}$
	$1\ \text{hectogram}\ \ (\text{hg}) = 100\ \text{grams}$	$100 \times 1\text{g}$
	$1\ \text{dekagram}\ \ (\text{dag}) = 10\ \text{grams}$	$10 \times 1\text{g}$
	$1\ \text{gram}\ \ (\text{g})$	$1 \times 1\text{g}$
	$1\ \text{decigram}\ \ (\text{dg}) = \frac{1}{10}\ \text{gram}$	$.1 \times 1\text{g}$

	$1 \quad \text{centigram} \quad (\text{ cg}) = \frac{1}{100} \quad \text{gram}$	$.01 \times 1\text{g}$
	$1 \quad \text{milligram} \quad (\text{ mg}) = \frac{1}{1{,}000} \quad \text{gram}$	$.001 \times 1\text{g}$
Volume	$1 \quad \text{kiloliter} \quad (\text{ kL}) = 1{,}000 \quad \text{liters} \quad (\ L)$	$1{,}000 \times 1\text{L}$
	$1 \quad \text{hectoliter} \quad (\text{ hL}) = 100 \quad \text{liters}$	$100 \times 1\text{L}$
	$1 \quad \text{dekaliter} \quad (\text{ daL}) = 10 \quad \text{liters}$	$10 \times 1\text{L}$
	1 liter (L)	$1 \times 1\text{L}$
	$1 \quad \text{deciliter} \quad (\text{ dL}) = \frac{1}{10} \quad \text{liter}$	$.1 \times 1\text{L}$
	$1 \quad \text{centiliter} \quad (\text{ cL}) = \frac{1}{100} \quad \text{liter}$	$.01 \times 1\text{L}$
	$1 \quad \text{milliliter} \quad (\text{ mL}) = \frac{1}{1{,}000} \quad \text{liter}$	$.001 \times 1\text{L}$
Time	Same as the United States system	

Distinction Between Mass and Weight
There is a distinction between mass and weight. The **weight** of a body is related to gravity whereas the mass of a body is not. For example, your weight on the earth is different than it is on the moon, but your mass is the same in both places. **Mass** is a measure of a body's resistance to motion. The more massive a body, the more resistant it is to motion. Also, more massive bodies weigh more than less massive bodies.

Converting Metric Units
To convert from one metric unit to another metric unit:

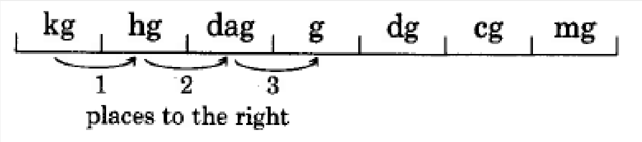
1. Determine the location of the original number on the metric scale (pictured in each of the following examples).
2. Move the decimal point of the original number in the same direction and same number of places as is necessary to move to the metric unit you wish to go to.

We can also convert from one metric unit to another using unit fractions. Both methods are shown in [link] of [link].

Sample Set A

Example:
Convert 3 kilograms to grams.

a. 3 kg can be written as 3.0 kg. Then,



$$3.0 \textbf{ kg} = 3\underbrace{000}_{1\,2\,3}. \textbf{ g}$$

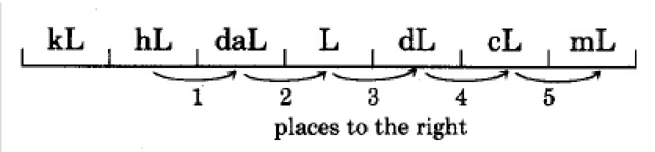
Thus, $3\text{kg} = 3{,}000 \text{ g}$.

b. We can also use unit fractions to make this conversion.

Since we are converting to grams, and $1{,}000 \quad \text{g} = 1 \quad \text{kg}$, we choose the unit fraction $\frac{1{,}000 \quad \text{g}}{1 \quad \text{kg}}$ since grams is in the numerator.

$$\begin{aligned} 3 \quad \text{kg} &= 3 \text{ kg} \cdot \frac{1{,}000 \quad \text{g}}{1 \text{ kg}} \\ &= 3 \quad \cancel{\text{kg}} \cdot \frac{1{,}000 \quad \text{g}}{1 \quad \cancel{\text{kg}}} \\ &= 3 \cdot 1{,}000 \quad \text{g} \\ &= 3{,}000 \quad \text{g} \end{aligned}$$

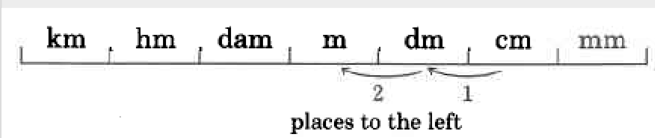
Example:
Convert 67.2 hectoliters to milliliters.



$67.2\text{ hL} = 67\,\underbrace{20000}_{1\,2\,3\,4\,5}.\text{ mL}$

Thus, 67.2 hL = 6,720,000 mL.

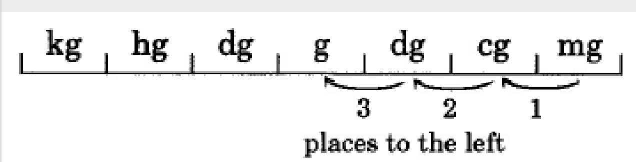
Example:
Convert 100.07 centimeters to meters.



$100.07\text{ cm} = 1.\underbrace{00}_{2\,1}07\text{ m}$

Thus, 100.07 cm = 1.0007 m.

Example:
Convert 0.16 milligrams to grams.



$0.16\text{ mg} = 0.\underbrace{000}_{3\,2\,1}16\text{ g}$

Thus, 0.16 mg = 0.00016 g.

Practice Set A

Exercise:

Problem: Convert 411 kilograms to grams.

Solution:

411,000 g

Exercise:

Problem: Convert 5.626 liters to centiliters.

Solution:

562.6 cL

Exercise:

Problem: Convert 80 milliliters to kiloliters.

Solution:

0.00008 kL

Exercise:

Problem: Convert 150 milligrams to centigrams.

Solution:

15 cg

Exercise:

Problem: Convert 2.5 centimeters to meters.

Solution:

0.025 m

Exercises

Make each conversion.

Exercise:

Problem: 87 m to cm

Solution:

8,700 cm

Exercise:

Problem: 905 L to mL

Exercise:

Problem: 16,005 mg to g

Solution:

16.005 g

Exercise:

Problem: 48.66 L to dL

Exercise:

Problem: 11.161 kL to L

Solution:

11,161 L

Exercise:

Problem: 521.85 cm to mm

Exercise:

Problem: 1.26 dag to dg

Solution:

126 dg

Exercise:

Problem: 99.04 dam to cm

Exercise:

Problem: 0.51 kL to daL

Solution:

5.1 daL

Exercise:

Problem: 0.17 kL to daL

Exercise:

Problem: 0.05 m to dm

Solution:

0.5 dm

Exercise:

Problem: 0.001 km to mm

Exercise:

Problem: 8.106 hg to cg

Solution:

81,060 cg

Exercise:

Problem: 17.0186 kL to mL

Exercise:

Problem: 3 cm to m

Solution:

0.03 m

Exercise:

Problem: 9 mm to m

Exercise:

Problem: 4 g to mg

Solution:

4,000 mg

Exercise:

Problem: 2 L to kL

Exercise:

Problem: 6 kg to mg

Solution:

6,000,000 mg

Exercise:

Problem: 7 daL to mL

Exercises for Review

Exercise:

Problem: ([link]) Find the value of $\frac{5}{8} - \frac{1}{3} + \frac{3}{4}$.

Solution:

$\frac{25}{24} = 1\frac{1}{24}$

Exercise:

Problem: ([link]) Solve the proportion: $\frac{9}{x} = \frac{27}{60}$.

Exercise:

Problem:

([link]) Use the method of rounding to estimate the sum: $8{,}226 + 4{,}118$.

Solution:

12,300 (12,344)

Exercise:

Problem:

([link]) Use the clustering method to estimate the sum:
$87 + 121 + 118 + 91 + 92$.

Exercise:

Problem: ([link]) Convert 3 in. to yd.

Solution:

0.083 yard

Simplification of Denominate Numbers
This module is from Fundamentals of Mathematics by Denny Burzynski and Wade Ellis, Jr. This module discusses how to simplify denominate numbers. By the end of the module students should be able to convert an unsimplified unit of measure to a simplified unit of measure, be able to add and subtract denominate numbers and be able to multiply and divide a denominate number by a whole number.

Section Overview

- Converting to Multiple Units
- Adding and Subtracting Denominate Numbers
- Multiplying a Denominate Number by a Whole Number
- Dividing a Denominate Number by a Whole Number

Converting to Multiple Units

Denominate Numbers
Numbers that have units of measure associated with them are called **denominate numbers**. It is often convenient, or even necessary, to simplify a denominate number.

Simplified Denominate Number
A denominate number is **simplified** when the number of standard units of measure associated With it does not exceed the next higher type of unit.

The denominate number 55 min is simplified since it is smaller than the next higher type of unit, 1 hr. The denominate number 65 min is *not* simplified since it is not smaller than the next higher type of unit, 1 hr. The denominate number 65 min can be simplified to 1 hr 5 min. The denominate number 1 hr 5 min is simplified since the next higher type of unit is day, and 1 hr does not exceed 1 day.

Sample Set A

Example:
Simplify 19 in.
Since 12 in. $=$ 1 ft, and $19 = 12 + 7$,

$$\begin{aligned} 19\text{ in.} &= 12\text{ in.} + 7\text{ in.} \\ &= 1\text{ ft} + 7\text{ in.} \\ &= 1\text{ ft } 7\text{ in.} \end{aligned}$$

Example:
Simplify 4 gal 5 qt.
Since 4 qt $=$ 1 gal, and $5 = 4 + 1$,

$$\begin{aligned} 4\text{ gal } 5\text{ qt} &= 4\text{ gal} + 4\text{ qt} + 1\text{qt} \\ &= 4\text{ gal} + 1\text{ gal} + 1\text{ qt} \\ &= 5\text{ gal} + 1\text{ qt} \\ &= 5\text{ gal } 1\text{ qt} \end{aligned}$$

Example:
Simplify 2 hr 75 min.
Since 60 min $=$ 1 hr, and $75 = 60 + 15$,

$$\begin{aligned} 2\text{ hr } 75\text{ min} &= 2\text{ hr} + 60\text{ min} + 15\text{ min} \\ &= 2\text{ hr} + 1\text{ hr} + 15\text{ min} \\ &= 3\text{ hr} + 15\text{ min} \\ &= 3\text{ hr } 15\text{ min} \end{aligned}$$

Example:
Simplify 43 fl oz.
Since 8 fl oz $=$ 1 c (1 cup), and $43 \div 8 = 5R3$,

$$\begin{aligned} 43 \text{ fl oz} &= 40 \text{ fl oz} + 3 \text{ fl oz} \\ &= 5 \cdot 8 \text{ fl oz} + 3 \text{ fl oz} \\ &= 5 \cdot 1 \text{ c} + 3 \text{ fl oz} \\ &= 5 \text{ c} + 3 \text{ fl oz} \end{aligned}$$

But, 2 c = 1 pt and $5 \div 2 = 2R1$. So,

$$\begin{aligned} 5 \text{ c} + 3 \text{ fl oz} &= 2 \cdot 2 \text{ c} + 1 \text{ c} + 3 \text{ fl oz} \\ &= 2 \cdot 1 \text{ pt} + 1 \text{ c} + 3 \text{ fl oz} \\ &= 2 \text{ pt} + 1 \text{ c} + 3 \text{ fl oz} \end{aligned}$$

But, 2 pt = 1 qt, so

$2 \text{ pt} + 1 \text{ c} + 3 \text{ fl oz} = 1 \text{ qt } 1 \text{ c } 3 \text{ fl oz}$

Practice Set A

Simplify each denominate number. Refer to the conversion tables given in [link], if necessary.

Exercise:

Problem: 18 in.

Solution:

1 ft 6 in.

Exercise:

Problem: 8 gal 9 qt

Solution:

10 gal 1 qt

Exercise:

Problem: 5 hr 80 min

Solution:

6 hr 20 min

Exercise:

Problem: 8 wk 11 da

Solution:

9 wk 4 da

Exercise:

Problem: 86 da

Solution:

12 wk 2 da

Adding and Subtracting Denominate Numbers

Adding and Subtracting Denominate Numbers
Denominate numbers can be added or subtracted by:

1. writing the numbers vertically so that the like units appear in the same column.
2. adding or subtracting the number parts, carrying along the unit.
3. simplifying the sum or difference.

Sample Set B

Example:

Add 6 ft 8 in. to 2 ft 9 in.

$$\begin{array}{rl} & 6 \text{ ft} \;\; 8 \text{ in.} \\ + & 2 \text{ ft} \;\; 9 \text{ in.} \\ \hline & 8 \text{ ft } 17 \text{ in.} \end{array} \quad \text{Simplify this denominate number.}$$

Since 12 in. = 1 ft,

$$\begin{aligned} 8 \text{ ft} + 12 \text{ in.} + 5 \text{ in.} &= 8 \text{ ft} + 1 \text{ ft} + 5 \text{ in.} \\ &= 9 \text{ ft} + 5 \text{ in.} \\ &= 9 \text{ ft } 5 \text{ in.} \end{aligned}$$

Example:

Subtract 5 da 3 hr from 8 da 11 hr.

$$\begin{array}{rl} & 8 \text{ da } 11 \text{ hr} \\ - & 5 \text{ da } \;\; 3 \text{ hr} \\ \hline & 3 \text{ da } \;\; 8 \text{ hr} \end{array}$$

Example:

Subtract 3 lb 14 oz from 5 lb 3 oz.

$$\begin{array}{rl} & 5 \text{ lb } \;\; 3 \text{ oz} \\ - & 3 \text{ lb } 14 \text{ oz} \end{array}$$

We cannot directly subtract 14 oz from 3 oz, so we must borrow 16 oz from the pounds.

$$\begin{aligned} 5 \text{ lb } 3 \text{ oz} &= 5 \text{ lb} + 3 \text{ oz} \\ &= 4 \text{ lb} + 1 \text{ lb} + 3 \text{ oz} \\ &= 4 \text{ lb} + 16 \text{ oz} + 3 \text{ oz} \quad (\text{Since } 1 \text{ lb} = 16 \text{ oz.}) \\ &= 4 \text{ lb} + 19 \text{ oz} \\ &= 4 \text{ lb } 19 \text{ oz} \end{aligned}$$

$$\begin{array}{rl} & 4 \text{ lb } 19 \text{ oz} \\ - & 3 \text{ lb } 14 \text{ oz} \\ \hline & 1 \text{ lb } \;\; 5 \text{ oz} \end{array}$$

Example:
Subtract 4 da 9 hr 21 min from 7 da 10 min.

$$\begin{array}{r} 7\text{ da }0\text{ hr }10\text{ min} \\ -4\text{ da }9\text{ hr }21\text{ min} \end{array}$$ Borrow 1 da from the 7 da.

$$\begin{array}{r} 6\text{ da }24\text{ hr }10\text{ min} \\ -4\text{ da }\ \ 9\text{ hr }21\text{ min} \end{array}$$ Borrow 1 hr from the 24 hr.

$$\begin{array}{r} 6\text{ da }23\text{ hr }70\text{ min} \\ -4\text{ da }\ \ 9\text{ hr }21\text{ min} \\ \hline 2\text{ da }14\text{ hr }49\text{ min} \end{array}$$

Practice Set B

Perform each operation. Simplify when possible.

Exercise:

Problem: Add 4 gal 3 qt to 1 gal 2 qt.

Solution:

6 gal 1 qt

Exercise:

Problem: Add 9 hr 48 min to 4 hr 26 min.

Solution:

14 hr 14 min

Exercise:

Problem: Subtract 2 ft 5 in. from 8 ft 7 in.

Solution:

6 ft 2in.

Exercise:

Problem: Subtract 15 km 460 m from 27 km 800 m.

Solution:

12 km 340 m

Exercise:

Problem: Subtract 8 min 35 sec from 12 min 10 sec.

Solution:

3 min 35 sec

Exercise:

Problem: Add 4 yd 2 ft 7 in. to 9 yd 2 ft 8 in.

Solution:

14 yd 2 ft 3 in

Exercise:

Problem: Subtract 11 min 55 sec from 25 min 8 sec.

Solution:

13 min 13 sec

Multiplying a Denominate Number by a Whole Number

Let's examine the repeated sum

$$\underbrace{4 \text{ ft } 9 \text{ in.} + 4 \text{ ft } 9 \text{ in.} + 4 \text{ ft } 9 \text{ in.}}_{3 \text{ times}} = 12 \text{ ft } 27 \text{ in.}$$

Recalling that multiplication is a description of repeated addition, by the distributive property we have

$$\begin{aligned} 3(4 \text{ ft } 9 \text{ in.}) &= 3(4 \text{ ft} + 9 \text{ in.}) \\ &= 3 \cdot 4 \text{ ft} + 3 \cdot 9 \text{ in.} \\ &= 12 \text{ ft} + 27 \text{ in.} \qquad \text{Now, } 27 \text{ in.} = 2 \text{ ft } 3 \text{ in.} \\ &= 12 \text{ ft} + 2 \text{ ft} + 3 \text{ in.} \\ &= 14 \text{ ft} + 3 \text{ in.} \\ &= 14 \text{ ft } 3 \text{ in.} \end{aligned}$$

From these observations, we can suggest the following rule.

Multiplying a Denominate Number by a Whole Number
To multiply a denominate number by a whole number, multiply the number part of each unit by the whole number and affix the unit to this product.

Sample Set C

Perform the following multiplications. Simplify if necessary.

Example:

$$\begin{aligned} 6 \cdot (2 \text{ ft } 4 \text{ in.}) &= 6 \cdot 2 \text{ ft} + 6 \cdot 4 \text{ in.} \\ &= 12 \text{ ft} + 24 \text{ in.} \end{aligned}$$

Since $3 \text{ ft} = 1 \text{ yd}$ and $12 \text{ in.} = 1 \text{ ft}$,

$$\begin{aligned} 12 \text{ ft} + 24 \text{ in.} &= 4 \text{ yd} + 2 \text{ ft} \\ &= 4 \text{ yd } 2 \text{ ft} \end{aligned}$$

Example:

$$\begin{aligned}
8 \cdot (5 \text{ hr } 21 \text{ min } 55 \text{ sec}) &= 8 \cdot 5\text{hr} + 8 \cdot 21 \text{ min} + 8 \cdot 55 \text{ sec} \\
&= 40 \text{ hr} + 168 \text{ min} + 440\text{sec} \\
&= 40 \text{ hr} + 168 \text{ min} + 7 \text{ min} + 20 \text{ sec} \\
&= 40 \text{ hr} + 175 \text{ min} + 20 \text{ sec} \\
&= 40 \text{ hr} + 2 \text{ hr} + 55 \text{ min} + 20 \text{ sec} \\
&= 42 \text{ hr} + 55 \text{ min} + 20 \text{ sec} \\
&= 24\text{hr} + 18\text{hr} + 55 \text{ min} + 20 \text{ sec} \\
&= 1 \text{ da} + 18 \text{ hr} + 55 \text{ min} + 20 \text{ sec} \\
&= 1 \text{ da } 18 \text{ hr } 55 \text{ min } 20 \text{ sec}
\end{aligned}$$

Practice Set C

Perform the following multiplications. Simplify.

Exercise:

Problem: $2 \cdot (10 \text{ min})$

Solution:

20 min

Exercise:

Problem: $5 \cdot (3 \text{ qt})$

Solution:

$15 \text{ qt} = 3 \text{ gal } 3 \text{ qt}$

Exercise:

Problem: $4 \cdot (5\text{ft } 8 \text{ in.})$

Solution:

20 ft 32 in. $=$ 7 yd 1 ft 8 in.

Exercise:

Problem: $10 \cdot (2\text{hr } 15 \text{ min } 40 \text{ sec})$

Solution:

20 hr 150 min 400 sec $=$ 22 hr 36 min 40 sec

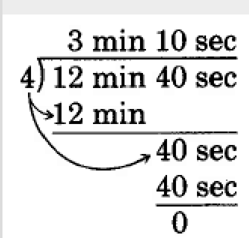
Dividing a Denominate Number by a Whole Number

Dividing a Denominate Number by a Whole Number
To divide a denominate number by a whole number, divide the number part of each unit by the whole number beginning with the largest unit. Affix the unit to this quotient. Carry any remainder to the next unit.

Sample Set D

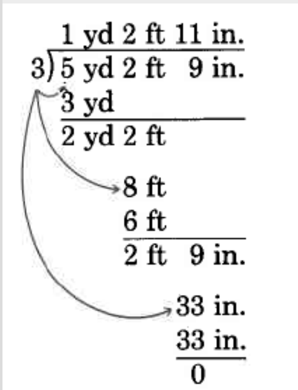
Perform the following divisions. Simplify if necessary.

Example:
$(12 \text{ min } 40 \text{ sec}) \div 4$



Thus $(12 \text{ min } 40 \text{ sec}) \div 4 = 3\text{min } 10 \text{ sec}$

Example:
(5 yd 2 ft 9 in.) $\div$ 3



Convert to feet: 2 yd 2 ft $=$ 8 ft .
Convert to inches: 2 ft 9 in . $=$ 33 in .
Thus (5 yd 2 ft 9 in.) $\div 3 =$ 1yd 2 ft 11 in.

Practice Set D

Perform the following divisions. Simplify if necessary.

Exercise:

Problem: (18 hr 36 min) $\div$ 9

Solution:

2 hr 4 min

Exercise:

Problem: (34 hr 8 min.) $\div$ 8

Solution:

4 hr 16 min

Exercise:

Problem: $(13 \text{ yd } 7 \text{ in.}) \div 5$

Solution:

2 yd 1 ft 11 in

Exercise:

Problem: $(47 \text{ gal } 2 \text{ qt } 1 \text{ pt}) \div 3$

Solution:

15 gal 3 qt 1 pt

Exercises

For the following 15 problems, simplify the denominate numbers.
Exercise:

Problem: 16 in.

Solution:

1 foot 4 inches

Exercise:

Problem: 19 ft

Exercise:

Problem: 85 min

Solution:

1 hour 25 minutes

Exercise:

Problem: 90 min

Exercise:

Problem: 17 da

Solution:

2 weeks 3 days

Exercise:

Problem: 25 oz

Exercise:

Problem: 240 oz

Solution:

15 pounds

Exercise:

Problem: 3,500 lb

Exercise:

Problem: 26 qt

Solution:

6 gallons 2 quarts

Exercise:

Problem: 300 sec

Exercise:

Problem: 135 oz

Solution:

8 pounds 7 ounces

Exercise:

Problem: 14 tsp

Exercise:

Problem: 18 pt

Solution:

2 gallons 1 quart

Exercise:

Problem: 3,500 m

Exercise:

Problem: 16,300 mL

Solution:

16 liters 300 milliliters (or 1daL 6 L 3dL)

For the following 15 problems, perform the indicated operations and simplify the answers if possible.

Exercise:

Problem: Add 6 min 12 sec to 5 min 15 sec.

Exercise:

Problem: Add 14 da 6 hr to 1 da 5 hr.

Solution:

15 days 11 hours

Exercise:

Problem: Add 9 gal 3 qt to 2 gal 3 qt.

Exercise:

Problem: Add 16 lb 10 oz to 42 lb 15 oz.

Solution:

59 pounds 9 ounces

Exercise:

Problem: Subtract 3 gal 1 qt from 8 gal 3 qt.

Exercise:

Problem: Subtract 3 ft 10 in. from 5 ft 8 in.

Solution:

1 foot 10 inches

Exercise:

Problem: Subtract 5 lb 9 oz from 12 lb 5 oz.

Exercise:

Problem: Subtract 10 hr 10 min from 11 hr 28 min.

Solution:

1 hour 18 minutes

Exercise:

Problem: Add 3 fl oz 1 tbsp 2 tsp to 5 fl oz 1 tbsp 2 tsp.

Exercise:

Problem: Add 4 da 7 hr 12 min to 1 da 8 hr 53 min.

Solution:

5 days 16 hours 5 minutes

Exercise:

Problem: Subtract 5 hr 21 sec from 11 hr 2 min 14 sec.

Exercise:

Problem: Subtract 6 T 1,300 lb 10 oz from 8 T 400 lb 10 oz.

Solution:

1 ton 1,100 pounds (or 1T 1,100 lb)

Exercise:

Problem: Subtract 15 mi 10 in. from 27 mi 800 ft 7 in.

Exercise:

Problem:

Subtract 3 wk 5 da 50 min 12 sec from 5 wk 6 da 20 min 5 sec.

Solution:

2 weeks 23 hours 29 minutes 53 seconds

Exercise:

Problem: Subtract 3 gal 3 qt 1 pt 1 oz from 10 gal 2 qt 2 oz.

Exercises for Review

Exercise:

Problem: ([link]) Find the value: $\left(\frac{5}{8}\right)^2 + \frac{39}{64}$.

Solution:

1

Exercise:

Problem: ([link]) Find the sum: $8 + 6\frac{3}{5}$.

Exercise:

Problem: ([link]) Convert $2.05\frac{1}{11}$ to a fraction.

Solution:

$2\frac{14}{275}$

Exercise:

Problem:

([link]) An acid solution is composed of 3 parts acid to 7 parts water. How many parts of acid are there in a solution that contains 126 parts water?

Exercise:

Problem: ([link]) Convert 126 kg to grams.

Solution:

126,000 g

Perimeter and Circumference of Geometric Figures
This module is from Fundamentals of Mathematics by Denny Burzynski and Wade Ellis, Jr. This module discusses perimeter and circumference of geometric figures. By the end of the module students should know what a polygon is, know what perimeter is and how to find it, know what the circumference, diameter, and radius of a circle is and how to find each one, know the meaning of the symbol π and its approximating value and know what a formula is and four versions of the circumference formula of a circle.

Section Overview

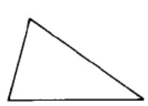
- Polygons
- Perimeter
- Circumference/Diameter/Radius
- The Number π
- Formulas

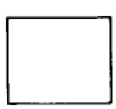
Polygons

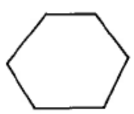
We can make use of conversion skills with denominate numbers to make measurements of geometric figures such as rectangles, triangles, and circles. To make these measurements we need to be familiar with several definitions.

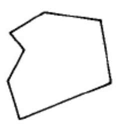
Polygon
A **polygon** is a closed plane (flat) figure whose sides are line segments (portions of straight lines).

Polygons

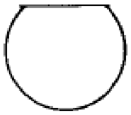








Not polygons



Perimeter

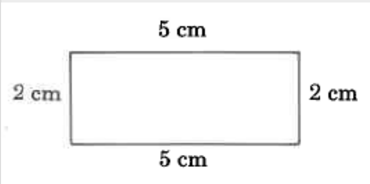
Perimeter
The **perimeter** of a polygon is the distance around the polygon.

To find the perimeter of a polygon, we simply add up the lengths of all the sides.

Sample Set A

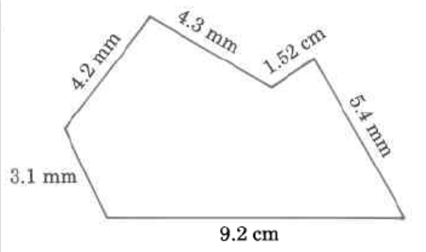
Find the perimeter of each polygon.

Example:



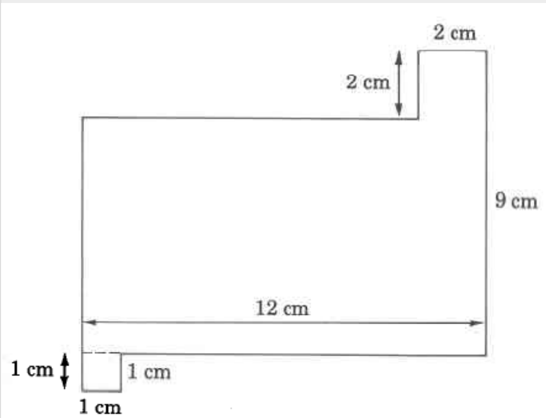
$$\begin{aligned} \text{Perimeter} &= 2 \text{ cm} + 5 \text{ cm} + 2 \text{ cm} + 5 \text{ cm} \\ &= 14 \text{ cm} \end{aligned}$$

Example:

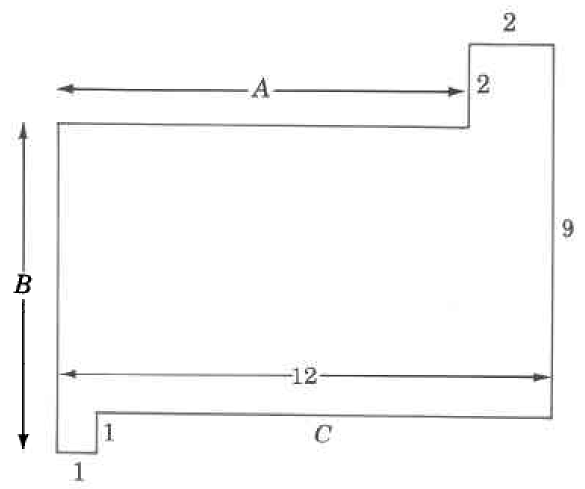


$$\begin{array}{rrl} \text{Perimeter} & = & 3.1 \text{ mm} \\ & & 4.2 \text{ mm} \\ & & 4.3 \text{ mm} \\ & & 1.52 \text{ mm} \\ & & 5.4 \text{ mm} \\ & & +9.2 \text{ mm} \\ & & 27.72 \text{ mm} \end{array}$$

Example:



Our first observation is that three of the dimensions are missing. However, we can determine the missing measurements using the following process. Let A, B, and C represent the missing measurements. Visualize



$A = 12\text{m} - 2\text{m} = 10\text{m}$
$B = 9\text{m} + 1\text{m} - 2\text{m} = 8\text{m}$
$C = 12\text{m} - 1\text{m} = 11\text{m}$

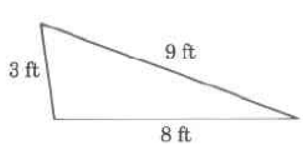
$$\begin{array}{rr} \text{Perimeter} = & 8 \text{ m} \\ & 10 \text{ m} \\ & 2 \text{ m} \\ & 2 \text{ m} \\ & 9 \text{ m} \\ & 11 \text{ m} \\ & 1 \text{ m} \\ + & 1 \text{ m} \\ \hline & 44 \text{ m} \end{array}$$

Practice Set A

Find the perimeter of each polygon.

Exercise:

Problem:

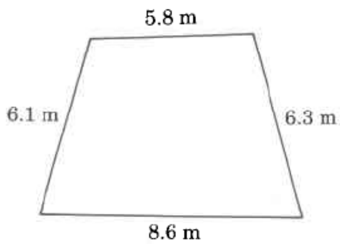


Solution:

20 ft

Exercise:

Problem:

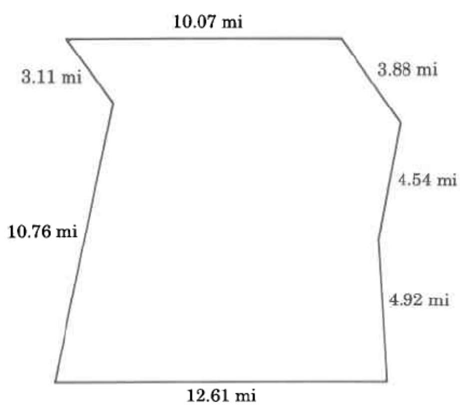


Solution:

26.8 m

Exercise:

Problem:



Solution:

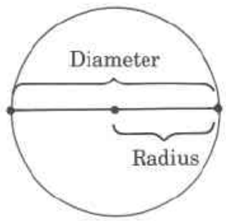
49.89 mi

Circumference/Diameter/Radius

Circumference
The **circumference** of a circle is the distance around the circle.

Diameter
A **diameter** of a circle is any line segment that passes through the center of the circle and has its endpoints on the circle.

Radius
A **radius** of a circle is any line segment having as its endpoints the center of the circle and a point on the circle.
The radius is one half the diameter.



The Number π

The symbol π, read "pi," represents the nonterminating, nonrepeating decimal number 3.14159 $\ldots$. This number has been computed to millions of decimal places without the appearance of a repeating block of digits.

For computational purposes, π is often approximated as 3.14. We will write $\pi \approx 3.14$ to denote that π is approximately equal to 3.14. The symbol "≈" means "approximately equal to."

Formulas

To find the circumference of a circle, we need only know its diameter or radius. We then use a formula for computing the circumference of the circle.

Formula
A **formula** is a rule or method for performing a task. In mathematics, a formula is a rule that directs us in computations.

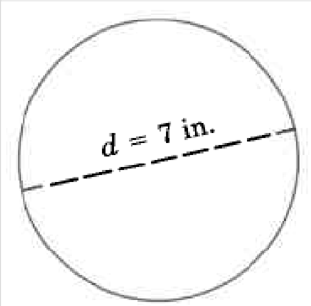
Formulas are usually composed of letters that represent important, but possibly unknown, quantities.

If C, d, and r represent, respectively, the circumference, diameter, and radius of a circle, then the following two formulas give us directions for computing the circumference of the circle.
Circumference Formulas

1. $C = \pi d$ or $C \approx (3.14)d$
2. $C = 2\pi r$ or $C \approx 2(3.14)r$

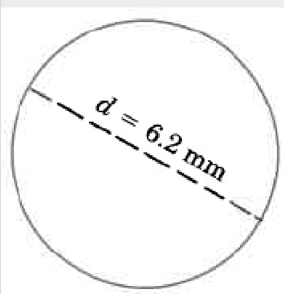
Sample Set B

Example:
Find the exact circumference of the circle.



Use the formula $C = \pi d$.
$C = \pi \cdot 7\text{in.}$
By commutativity of multiplication,
$C = 7\text{in.} \cdot \pi$
$C = 7\pi\text{in.}$, exactly
This result is exact since π has not been approximated.

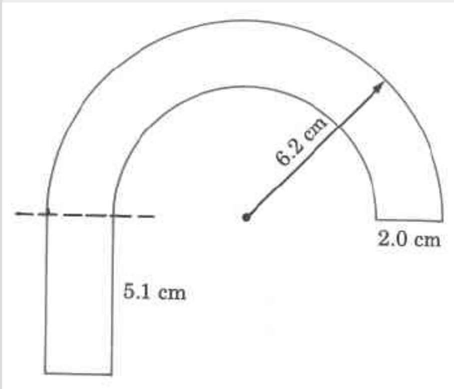
Example:
Find the approximate circumference of the circle.



Use the formula $C = \pi d$.
$C \approx (3.14)(6.2)$
$C \approx 19.648 \text{ mm}$
This result is approximate since π has been approximated by 3.14.

Example:
Find the approximate circumference of a circle with radius 18 inches.
Since we're given that the radius, r, is 18 in., we'll use the formula $C = 2\pi r$.
$C \approx (2)(3.14)(18 \text{ in.})$
$C \approx 113.04 \text{ in.}$

Example:
Find the approximate perimeter of the figure.



We notice that we have two semicircles (half circles).
The larger radius is 6.2 cm.
The smaller radius is 6.2 cm - 2.0 cm = 4.2 cm.
The width of the bottom part of the rectangle is 2.0 cm.

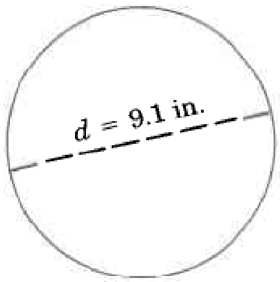
$$\begin{array}{lll} \text{Perimeter} = & 2.0 \text{ cm} & \\ & 5.1 \text{ cm} & \\ & 2.0 \text{ cm} & \\ & 5.1 \text{ cm} & \\ & (0.5)\cdot(2)\cdot(3.14)\cdot(6.2 \text{ cm}) & \text{Circumference of outer semicircle.} \\ + & (0.5)\cdot(2)\cdot(3.14)\cdot(4.2 \text{ cm}) & \text{Circumference of inner semicircle} \\ & & 6.2 \text{ cm} - 2.0 \text{ cm} = 4.2 \text{ cm} \\ & & \text{The 0.5 appears because we want the} \\ & & \text{perimeter of only } half \text{ a circle.} \end{array}$$

$$\begin{array}{lrl} \text{Perimeter} \approx & 2.0 & \text{cm} \\ & 5.1 & \text{cm} \\ & 2.0 & \text{cm} \\ & 5.1 & \text{cm} \\ & 19.468 & \text{cm} \\ & +13.188 & \text{cm} \\ & 48.856 & \text{cm} \end{array}$$

Practice Set B

Exercise:

Problem: Find the exact circumference of the circle.

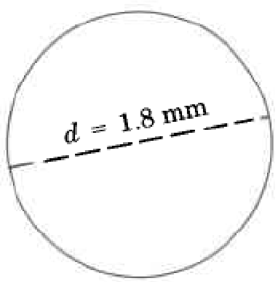


Solution:

9.1π in.

Exercise:

Problem: Find the approximate circumference of the circle.



Solution:

5.652 mm

Exercise:

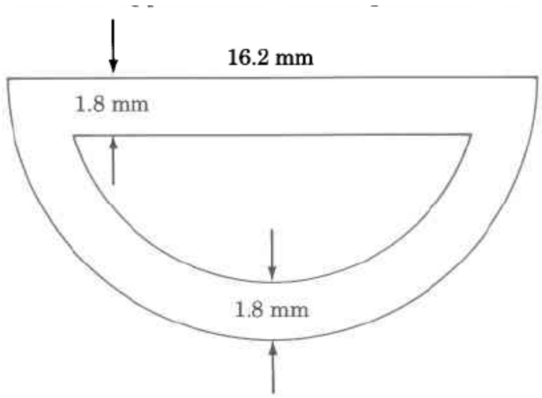
Problem: Find the approximate circumference of the circle with radius 20.1 m.

Solution:

126.228 m

Exercise:

Problem: Find the approximate outside perimeter of



Solution:

41.634 mm

Exercises

Find each perimeter or approximate circumference. Use $\pi = 3.14$.
Exercise:

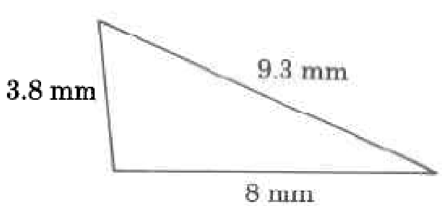
Problem:

Solution:

21.8 cm

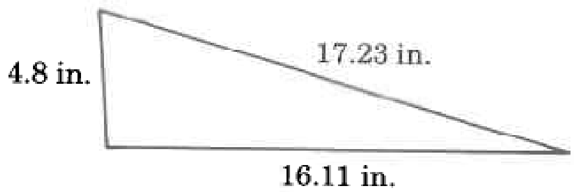
Exercise:

Problem:



Exercise:

Problem:

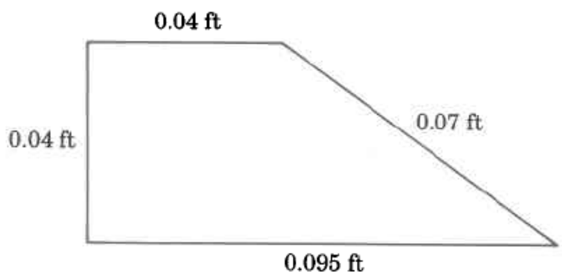


Solution:

38.14 inches

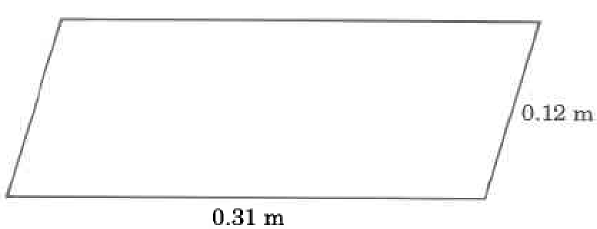
Exercise:

Problem:



Exercise:

Problem:

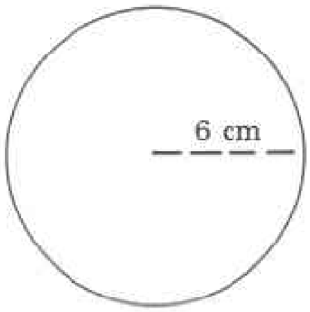


Solution:

0.86 m

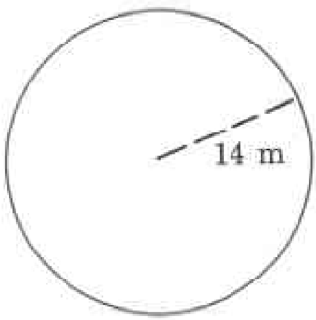
Exercise:

Problem:



Exercise:

Problem:

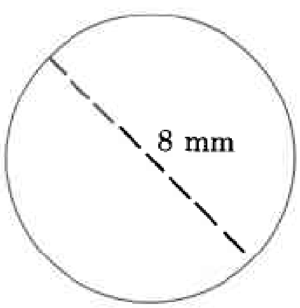


Solution:

87.92 m

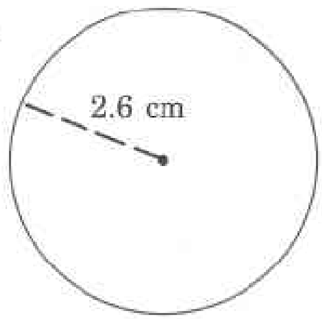
Exercise:

Problem:



Exercise:

Problem:

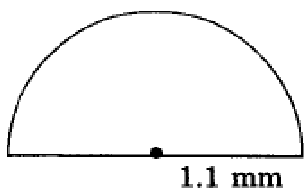


Solution:

16.328 cm

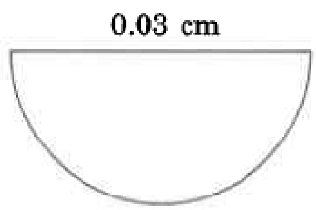
Exercise:

Problem:



Exercise:

Problem:

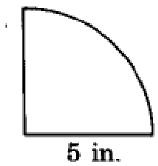


Solution:

0.0771 cm

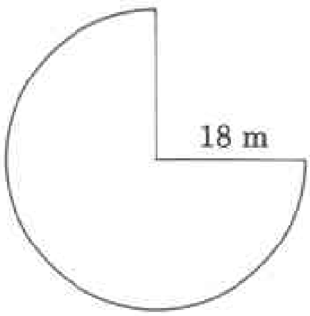
Exercise:

Problem:



Exercise:

Problem:

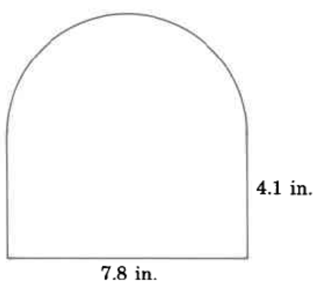


Solution:

120.78 m

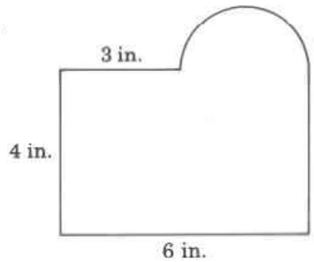
Exercise:

Problem:



Exercise:

Problem:

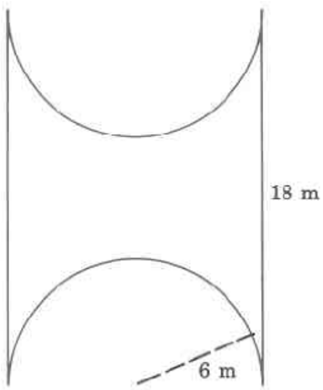


Solution:

21.71 inches

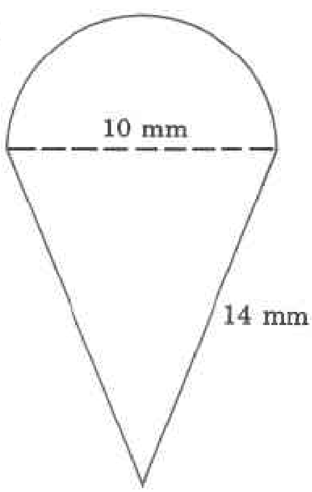
Exercise:

Problem:



Exercise:

Problem:

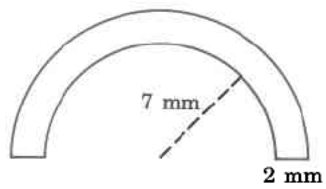


Solution:

43.7 mm

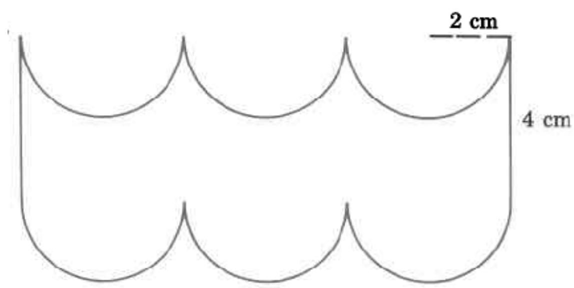
Exercise:

Problem:



Exercise:

Problem:

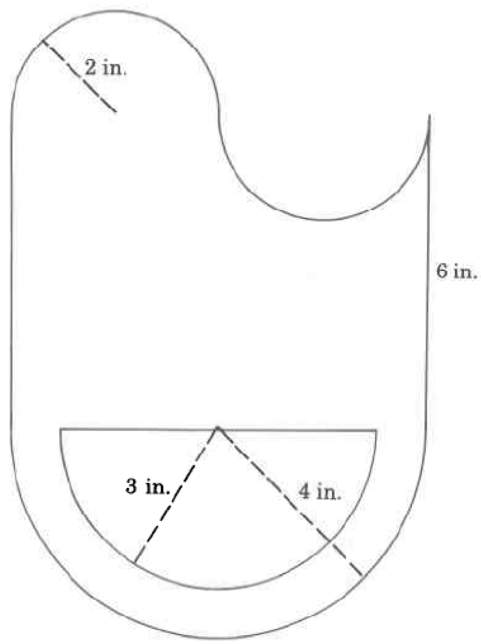


Solution:

45.68 cm

Exercise:

Problem:



Exercises for Review

Exercise:

Problem: ([link]) Find the value of $2\frac{8}{13} \cdot \sqrt{10\frac{9}{16}}$.

Solution:

8.5 or $\frac{17}{2}$ or $8\frac{1}{2}$

Exercise:

Problem: ([link]) Find the value of $\frac{8}{15} + \frac{7}{10} + \frac{21}{60}$.

Exercise:

Problem: ([link]) Convert $\frac{7}{8}$ to a decimal.

Solution:

0.875

Exercise:

Problem:

([link]) What is the name given to a quantity that is used as a comparison to determine the measure of another quantity?

Exercise:

Problem: ([link]) Add 42 min 26 sec to 53 min 40 sec and simplify the result.

Solution:

1 hour 36 minutes 6 seconds

Area and Volume of Geometric Figures and Objects
This module is from Fundamentals of Mathematics by Denny Burzynski and Wade Ellis, Jr. This module discusses area and volume of geometric figures and objects. By the end of the module students should know the meaning and notation for area, know the area formulas for some common geometric figures, be able to find the areas of some common geometric figures, know the meaning and notation for volume, know the volume formulas for some common geometric objects and be able to find the volume of some common geometric objects.

Section Overview

- The Meaning and Notation for Area
- Area Formulas
- Finding Areas of Some Common Geometric Figures
- The Meaning and Notation for Volume
- Volume Formulas
- Finding Volumes of Some Common Geometric Objects

Quite often it is necessary to multiply one denominate number by another. To do so, we multiply the number parts together and the unit parts together. For example,

$$\begin{aligned} 8 \text{ in.} \cdot 8 \text{ in.} &= 8 \cdot 8 \cdot \text{in.} \cdot \text{in.} \\ &= 64 \text{ in.}^2 \end{aligned}$$

$$\begin{aligned} 4\text{mm} \cdot 4 \text{ mm} \cdot 4 \text{ mm} &= 4 \cdot 4 \cdot 4 \cdot \text{mm} \cdot \text{mm} \cdot \text{mm} \\ &= 64 \text{ mm}^3 \end{aligned}$$

Sometimes the product of units has a physical meaning. In this section, we will examine the meaning of the products $(\text{length unit})^2$ and $(\text{length unit})^3$.

The Meaning and Notation for Area

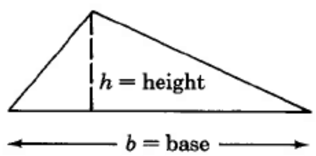
The product $(\text{length unit}) \cdot (\text{length unit}) = (\text{length unit})^2$, or, square length unit (sq length unit), can be interpreted physically as the *area* of a surface.

Area

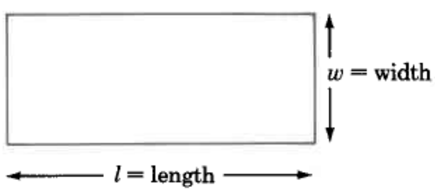
The **area** of a surface is the amount of square length units contained in the surface.

For example, 3 sq in. means that 3 squares, 1 inch on each side, can be placed precisely on some surface. (The squares may have to be cut and rearranged so they match the shape of the surface.)

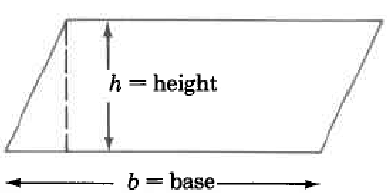
We will examine the area of the following geometric figures.



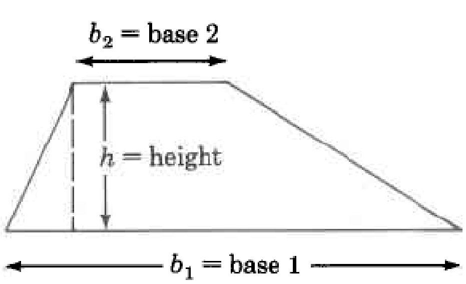
Triangles



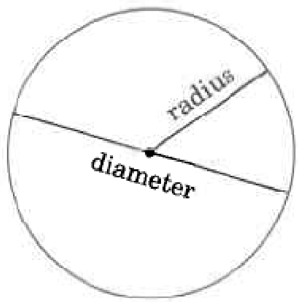
Rectangles



Parallelograms



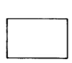
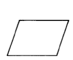
Trapezoids

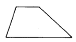
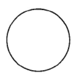


Circles

Area Formulas

We can determine the areas of these geometric figures using the following formulas.

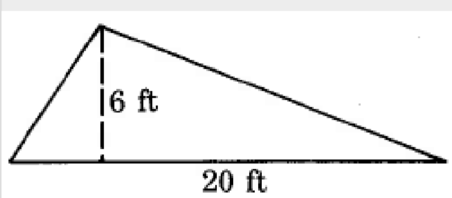
	Figure	Area Formula	Statement
	Triangle	$A_T = \frac{1}{2} \cdot b \cdot h$	Area of a triangle is one half the base times the height.
	Rectangle	$A_R = l \cdot w$	Area of a rectangle is the length times the width.
	Parallelogram	$A_P = b \cdot h$	Area of a parallelogram is base times the height.

	Trapezoid	$A_{\text{Trap}} = \frac{1}{2} \cdot (b_1 + b_2) \cdot h$	Area of a trapezoid is one half the sum of the two bases times the height.
	Circle	$A_C = \pi r^2$	Area of a circle is π times the square of the radius.

Finding Areas of Some Common Geometric Figures

Sample Set A

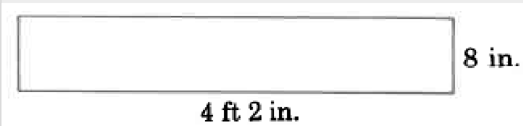
Example:
Find the area of the triangle.



$$\begin{aligned} A_T &= \frac{1}{2} \cdot b \cdot h \\ &= \frac{1}{2} \cdot 20 \cdot 6 \ \text{ sq ft} \\ &= 10 \cdot 6 \ \text{ sq ft} \\ &= 60 \text{ sq ft} \\ &= 60 \text{ ft}^2 \end{aligned}$$

The area of this triangle is 60 sq ft, which is often written as 60 ft^2.

Example:
Find the area of the rectangle.



Let's first convert 4 ft 2 in. to inches. Since we wish to convert to inches, we'll use the unit fraction $\frac{12 \text{ in.}}{1 \text{ ft}}$ since it has inches in the numerator. Then,

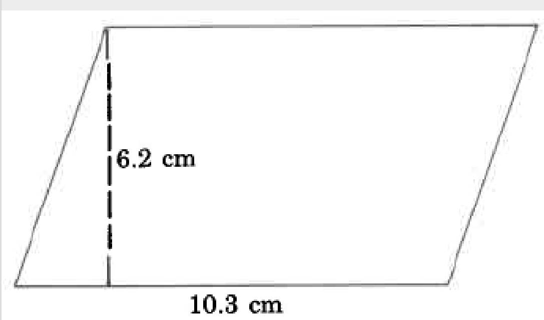
$$\begin{aligned} 4\text{ft} &= \frac{4 \text{ ft}}{1} \cdot \frac{12\text{in.}}{1 \text{ ft}} \\ &= \frac{4 \cancel{\text{ft}}}{1} \cdot \frac{12\text{in.}}{1 \cancel{\text{ft}}} \\ &= 48\text{in.} \end{aligned}$$

Thus, 4 ft 2 in. $=$ 48 in. $+$ 2 in. $=$ 50 in.

$$\begin{aligned} A_R &= l \cdot w \\ &= 50 \text{ in.} \cdot 8 \text{ in.} \\ &= 400 \text{ sq in.} \end{aligned}$$

The area of this rectangle is 400 sq in.

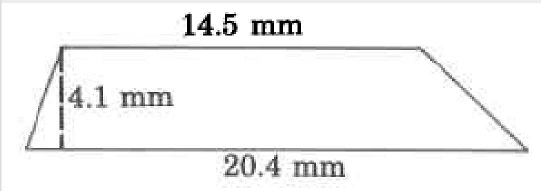
Example:
Find the area of the parallelogram.



$$\begin{aligned} A_P &= b \cdot h \\ &= 10.3 \text{ cm} \cdot 6.2 \text{ cm} \\ &= 63.86 \text{ sq cm} \end{aligned}$$

The area of this parallelogram is 63.86 sq cm.

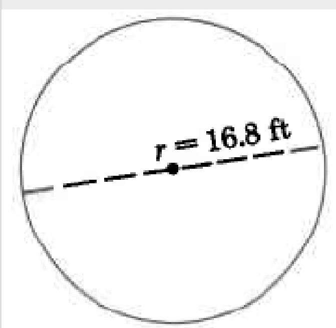
Example:
Find the area of the trapezoid.



$$\begin{aligned} A_{Trap} &= \frac{1}{2} \cdot (b_1 + b_2) \cdot h \\ &= \frac{1}{2} \cdot (14.5 \text{ mm}, +, 20.4 \text{ mm}) \cdot (4.1 \text{ mm}) \\ &= \frac{1}{2} \cdot (34.9 \text{ mm}) \cdot (4.1 \text{ mm}) \\ &= \frac{1}{2} \cdot (143.09 \text{ sq mm}) \\ &= 71.545 \text{ sq mm} \end{aligned}$$

The area of this trapezoid is 71.545 sq mm.

Example:
Find the approximate area of the circle.



$$\begin{aligned} A_c &= \pi \cdot r^2 \\ &\approx (3.14) \cdot (16.8 \text{ ft})^2 \\ &\approx (3.14) \cdot (282.24 \text{ sq ft}) \\ &\approx 888.23 \text{ sq ft} \end{aligned}$$

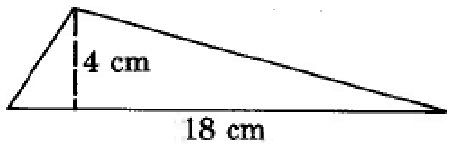
The area of this circle is approximately 886.23 sq ft.

Practice Set A

Find the area of each of the following geometric figures.

Exercise:

Problem:

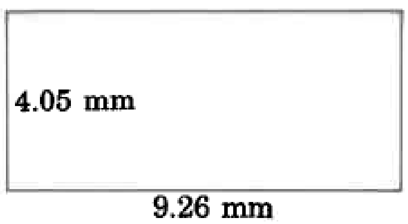


Solution:

36 sq cm

Exercise:

Problem:

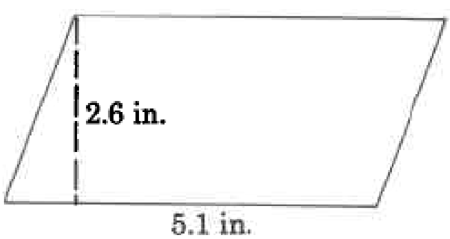


Solution:

37.503 sq mm

Exercise:

Problem:

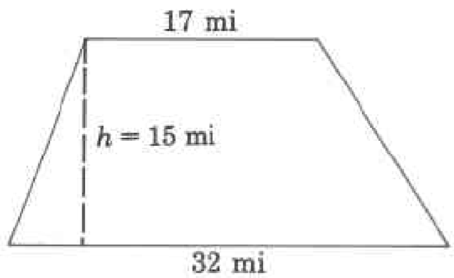


Solution:

13.26 sq in.

Exercise:

Problem:

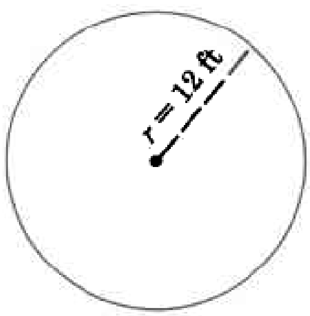


Solution:

367.5 sq mi

Exercise:

Problem:



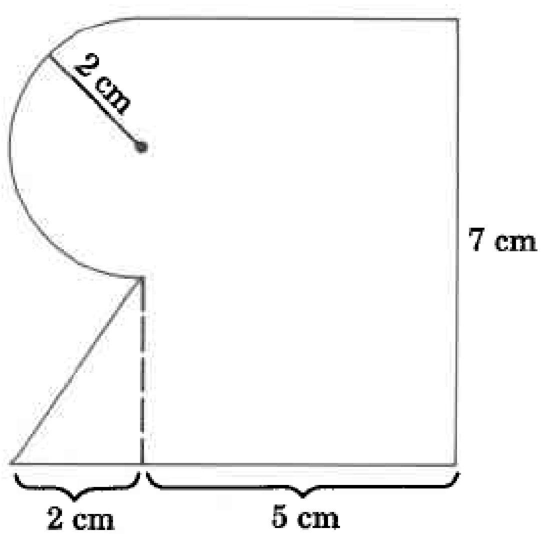
(approximate)

Solution:

452.16 sq ft

Exercise:

Problem:



Solution:

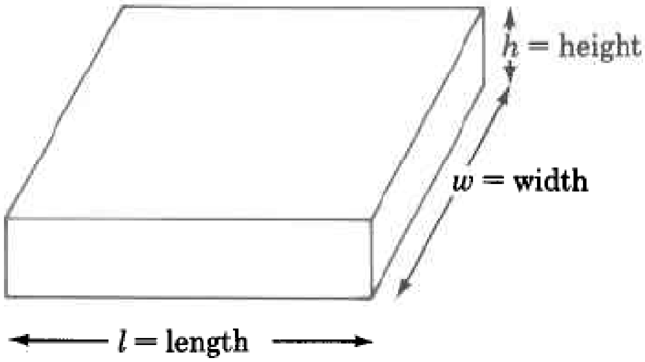
44.28 sq cm

The Meaning and Notation for Volume

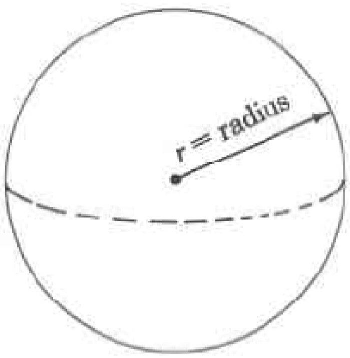
The product $(\text{length unit})(\text{length unit})(\text{length unit}) = (\text{length unit})^3$, or cubic length unit (cu length unit), can be interpreted physically as the *volume* of a three-dimensional object.

Volume
The **volume** of an object is the amount of cubic length units contained in the object.

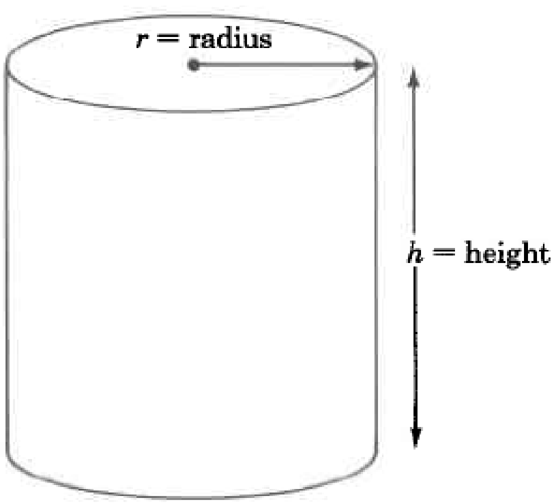
For example, 4 cu mm means that 4 cubes, 1 mm on each side, would precisely fill some three-dimensional object. (The cubes may have to be cut and rearranged so they match the shape of the object.)



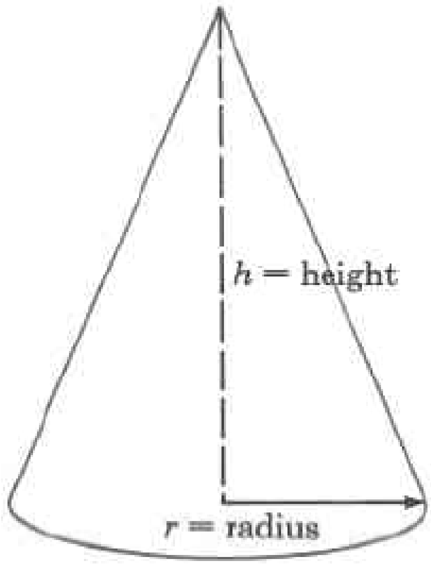
Rectangular solid



Sphere



Cylinder



Cone

Volume Formulas

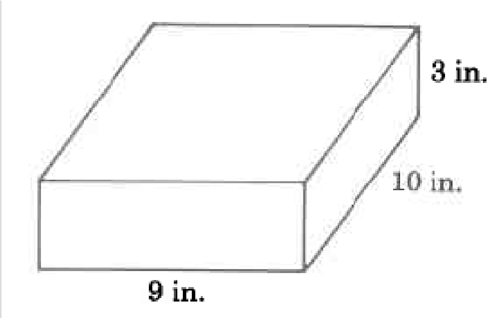
	Figure	Volume Formula	Statement
	Rectangular solid	$$\begin{aligned} V_R &= l \cdot w \cdot h \\ &= (\text{area of base}) \cdot (\text{height}) \end{aligned}$$	The volume of a rectangular solid is the length times the width times the height.
	Sphere	$V_S = \frac{4}{3} \cdot \pi \cdot r^3$	The volume of

			a sphere is $\frac{4}{3}$ times π times the cube of the radius.
	Cylinder	$$\begin{aligned} V_{\text{Cyl}} &= \pi \cdot r^2 \cdot h \\ &= (\text{area of base}) \cdot (\text{height}) \end{aligned}$$	The volume of a cylinder is π times the square of the radius times the height.
	Cone	$$\begin{aligned} V_c &= \frac{1}{3} \cdot \pi \cdot r^2 \cdot h \\ &= (\text{area of base}) \cdot (\text{height}) \end{aligned}$$	The volume of a cone is $\frac{1}{3}$ times π times the square of the radius times the height.

Finding Volumes of Some Common Geometric Objects

Sample Set B

Example:
Find the volume of the rectangular solid.

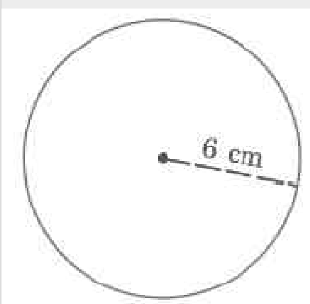


$$
\begin{aligned}
V_R &= l \cdot w \cdot h \\
&= 9 \text{ in.} \cdot 10 \text{ in.} \cdot 3 \text{ in.} \\
&= 270 \text{ cu in.} \\
&= 270 \text{ in.}^3
\end{aligned}
$$

The volume of this rectangular solid is 270 cu in.

Example:

Find the approximate volume of the sphere.

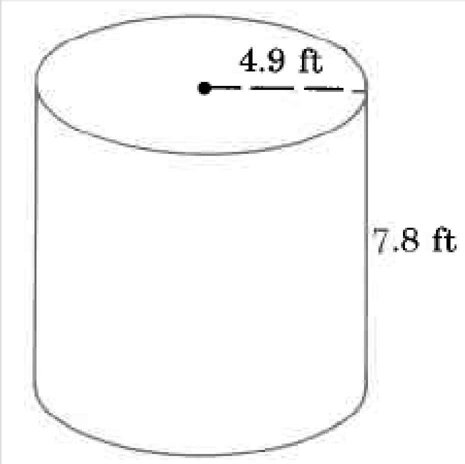


$$
\begin{aligned}
V_S &= \frac{4}{3} \cdot \pi \cdot r^3 \\
&\approx \left(\frac{4}{3}\right) \cdot (3.14) \cdot (6 \text{ cm})^3 \\
&\approx \left(\frac{4}{3}\right) \cdot (3.14) \cdot (216 \text{ cu cm}) \\
&\approx 904.32 \text{ cu cm}
\end{aligned}
$$

The approximate volume of this sphere is 904.32 cu cm, which is often written as 904.32 cm^3.

Example:

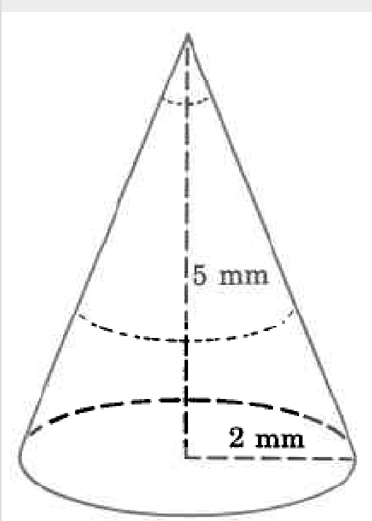
Find the approximate volume of the cylinder.



$$\begin{aligned} V_{\text{Cyl}} &= \pi \cdot r^2 \cdot h \\ &\approx (3.14) \cdot (4.9 \text{ ft})^2 \cdot (7.8 \text{ ft}) \\ &\approx (3.14) \cdot (24.01 \text{ sq ft}) \cdot (7.8 \text{ ft}) \\ &\approx (3.14) \cdot (187.278 \text{ cu ft}) \\ &\approx 588.05292 \text{ cu ft} \end{aligned}$$

The volume of this cylinder is approximately 588.05292 cu ft. The volume is approximate because we approximated π with 3.14.

Example:
Find the approximate volume of the cone. Round to two decimal places.



$$\begin{aligned} V_c &= \frac{1}{3} \cdot \pi \cdot r^2 \cdot h \\ &\approx \left(\frac{1}{3}\right) \cdot (3.14) \cdot (2 \text{ mm})^2 \cdot (5 \text{ mm}) \\ &\approx \left(\frac{1}{3}\right) \cdot (3.14) \cdot (4 \text{ sq mm}) \cdot (5 \text{ mm}) \\ &\approx \left(\frac{1}{3}\right) \cdot (3.14) \cdot (20 \text{ cu mm}) \\ &\approx 20.93 \ \text{ cu mm} \\ &\approx 20.93 \text{ cu mm} \end{aligned}$$

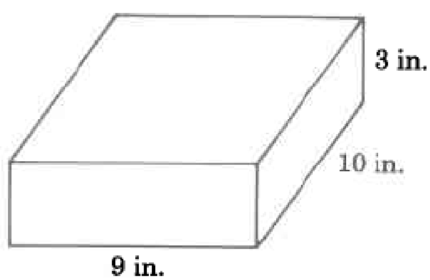
The volume of this cone is approximately 20.93 cu mm. The volume is approximate because we approximated π with 3.14.

Practice Set B

Find the volume of each geometric object. If π is required, approximate it with 3.14 and find the approximate volume.

Exercise:

Problem:

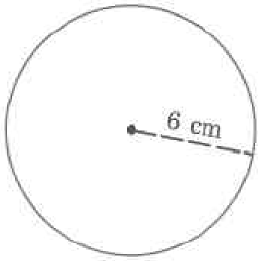


Solution:

21 cu in.

Exercise:

Problem: Sphere

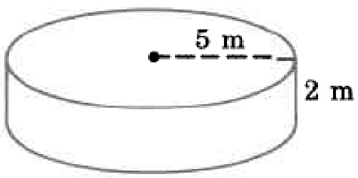


Solution:

904.32 cu ft

Exercise:

Problem:

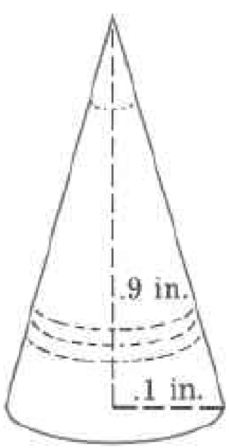


Solution:

157 cu m

Exercise:

Problem:



Solution:

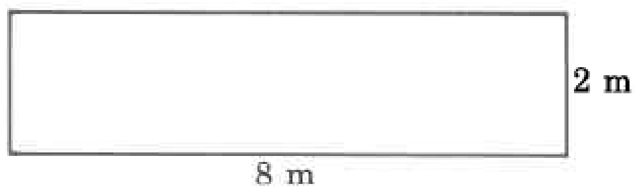
0.00942 cu in.

Exercises

Find each indicated measurement.

Exercise:

Problem: Area

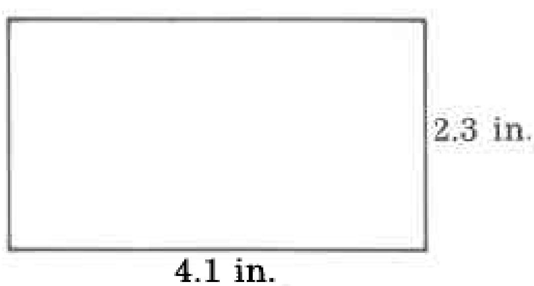


Solution:

16 sq m

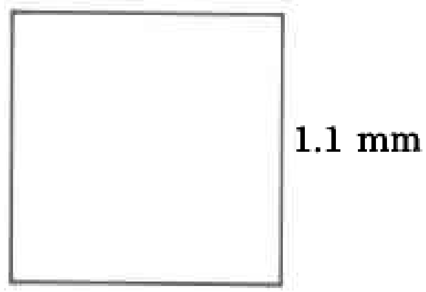
Exercise:

Problem: Area



Exercise:

Problem: Area

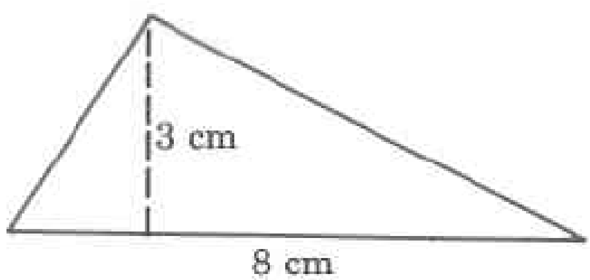


Solution:

1.21 sq mm

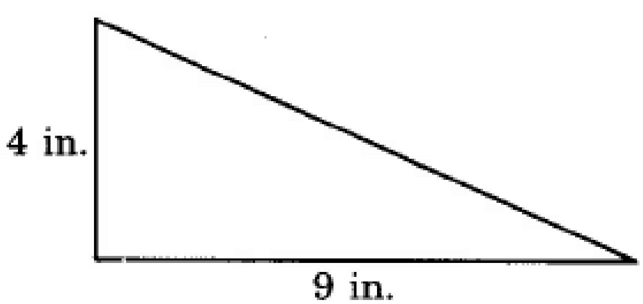
Exercise:

Problem: Area



Exercise:

Problem: Area

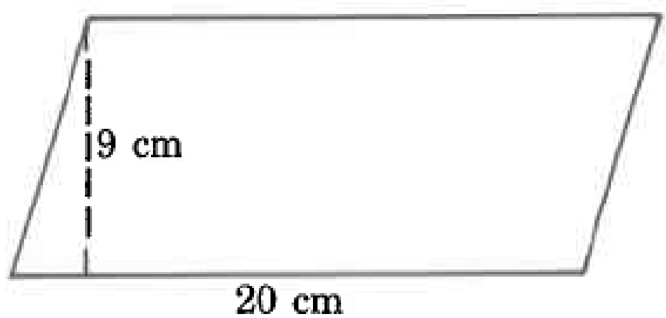


Solution:

18 sq in.

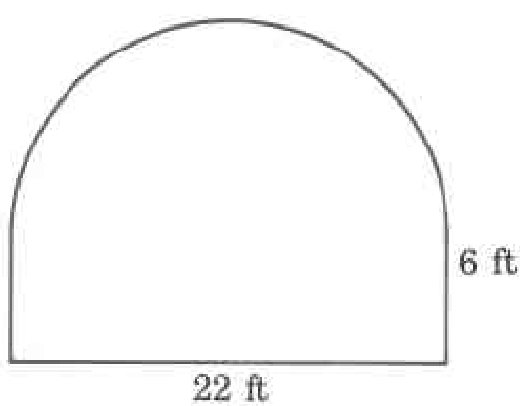
Exercise:

Problem: Area



Exercise:

Problem: Exact area

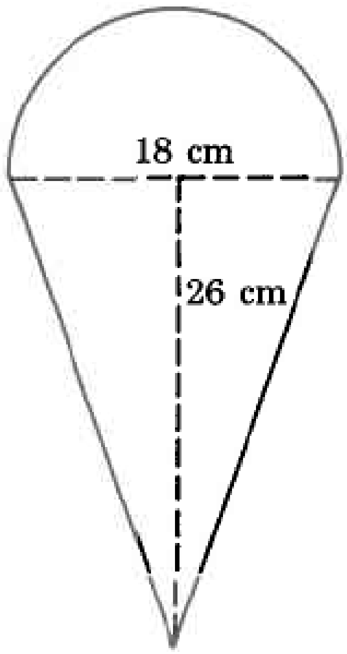


Solution:

$(60.5\pi + 132)$ sq ft

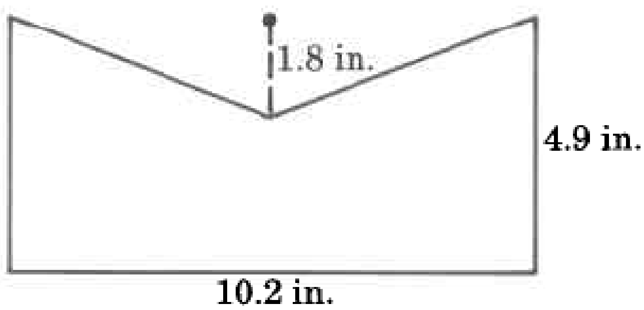
Exercise:

Problem: Approximate area



Exercise:

Problem: Area

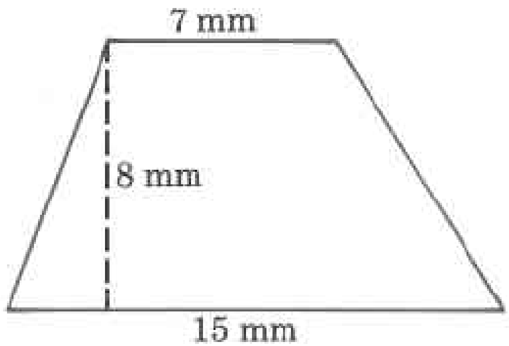


Solution:

40.8 sq in.

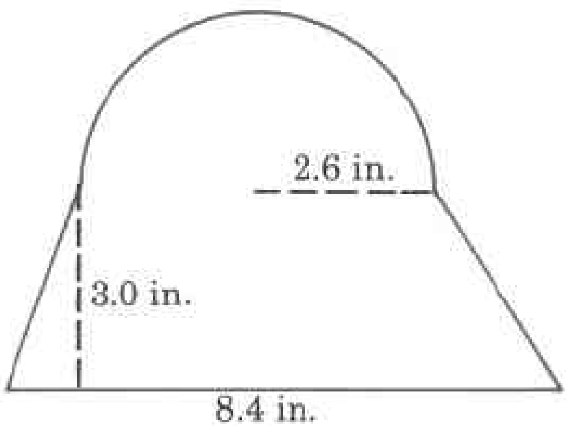
Exercise:

Problem: Area



Exercise:

Problem: Approximate area

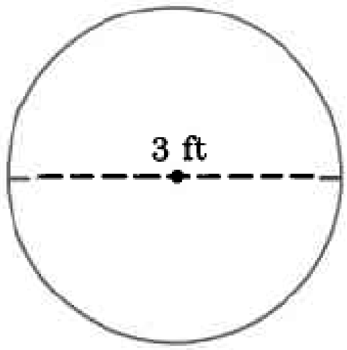


Solution:

31.0132 sq in.

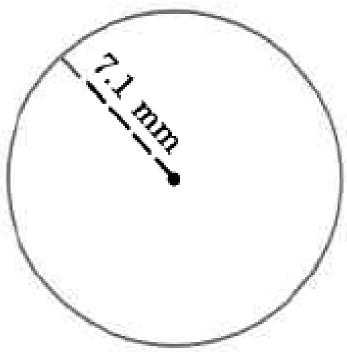
Exercise:

Problem: Exact area



Exercise:

Problem: Approximate area

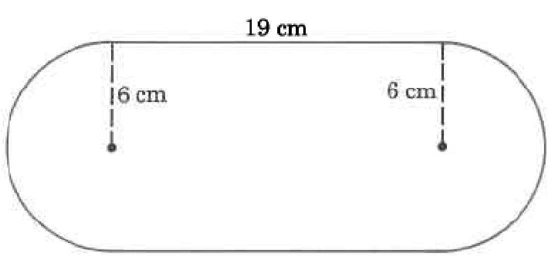


Solution:

158.2874 sq mm

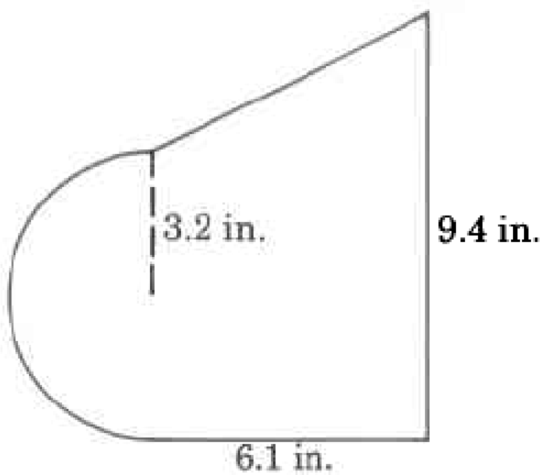
Exercise:

Problem: Exact area



Exercise:

Problem: Approximate area

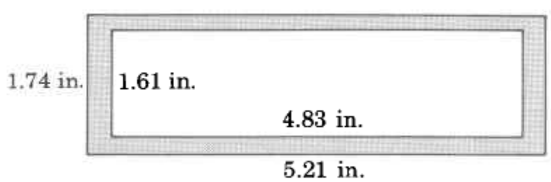


Solution:

64.2668 sq in.

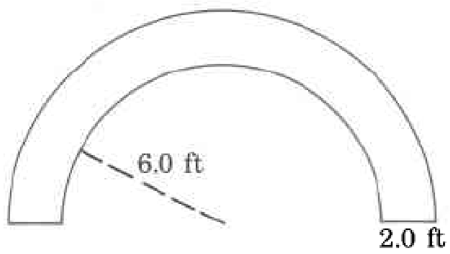
Exercise:

Problem: Area



Exercise:

Problem: Approximate area

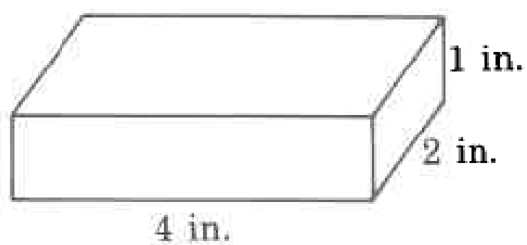


Solution:

43.96 sq ft

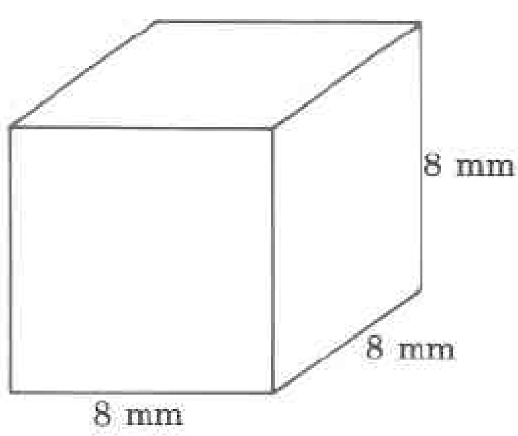
Exercise:

Problem: Volume



Exercise:

Problem: Volume

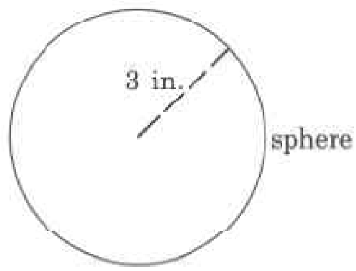


Solution:

512 cu cm

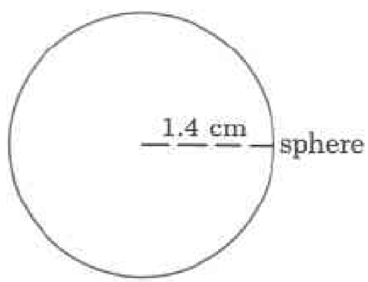
Exercise:

Problem: Exact volume



Exercise:

Problem: Approximate volume

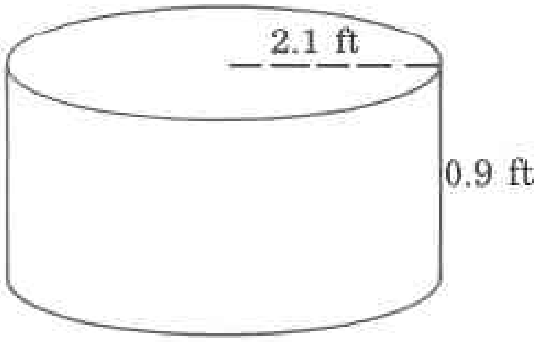


Solution:

11.49 cu cm

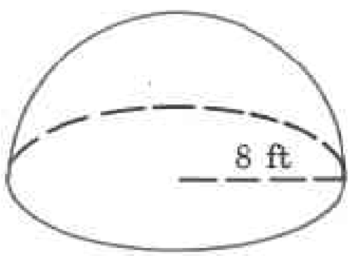
Exercise:

Problem: Approximate volume



Exercise:

Problem: Exact volume

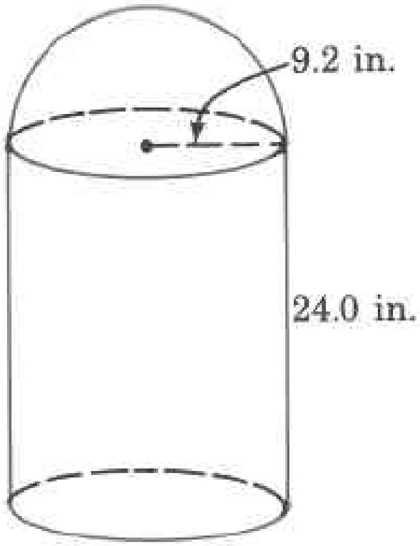


Solution:

$\frac{1024}{3}\pi$ cu ft

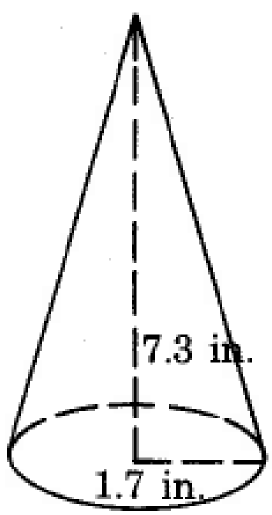
Exercise:

Problem: Approximate volume



Exercise:

Problem: Approximate volume

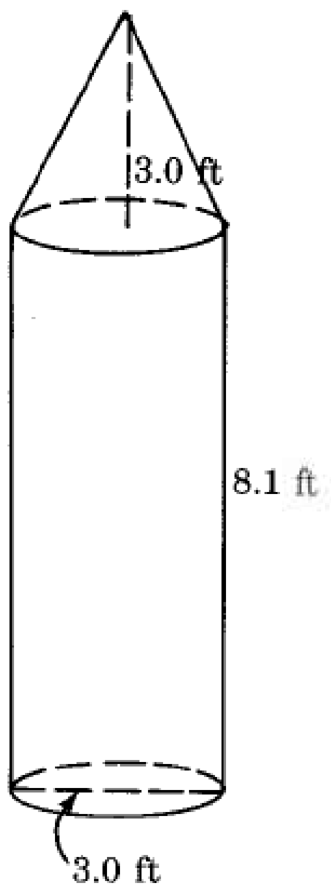


Solution:

22.08 cu in.

Exercise:

Problem: Approximate volume



Exercises for Review

Exercise:

Problem: ([link]) In the number 23,426, how many hundreds are there?

Solution:

4

Exercise:

Problem: ([link]) List all the factors of 32.

Exercise:

Problem: ([link]) Find the value of $4\frac{3}{4} - 3\frac{5}{6} + 1\frac{2}{3}$.

Solution:

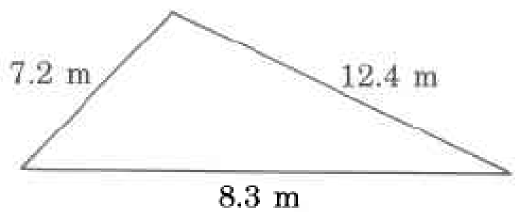
$\frac{31}{12} = 2\frac{7}{12} = 2.58$

Exercise:

Problem: ([link]) Find the value of $\frac{5+\frac{1}{3}}{2+\frac{2}{15}}$.

Exercise:

Problem: ([link]) Find the perimeter.



Solution:

27.9m

Summary of Key Concepts
This module is from Fundamentals of Mathematics by Denny Burzynski and Wade Ellis, Jr. This module reviews the key concepts from the chapter "Measurement and Geometry."

Summary of Key Concepts

Measurement ([link])
Measurement is comparison to some standard.

Standard Unit of Measure ([link])
A quantity that is used for comparison is called a **standard unit of measure**.

Two Types of Measurement Systems ([link])
There are two major types of measurement systems in use today. They are the **United States system** and the **metric system**.

Unit Fraction ([link])
A **unit fraction** is a fraction that has a value of 1. Unit fractions can be used to convert from one unit of measure to another.

Meter, Liter, Gram, and associated prefixes ([link])
Common units of measure in the metric system are the **meter** (m), for length, the **liter** (L), for volume, and the **gram** (g), for mass. To each of these units, a prefix can be attached.

- **kilo**thousand
- **deci**tenth
- **hecto**hundred
- **centi**hundredth
- **deka**ten
- **milli**thousandth

Metric Conversions ([link])
To **convert** from one metric unit to another:

1. Determine the location of the original number on the metric scale.

2. Move the decimal point of the original number in the same direction and the same number of places as is necessary to move to the metric unit you wish to convert to.

Denominate Numbers ([link])
Numbers that have units of measure associated with them are **denominate numbers**. The number 25 mg is a denominate number since the mg unit is associated with the pure number 25. The number 82 is not a denominate number since it has no unit of measure associated with it.

Simplified Denominate Number ([link])
A denominate number is **simplified** when the number of standard units of measure associated with it does not exceed the next higher type of unit. 55 min is simplified, whereas 65 min is not simplified

Addition and Subtraction of Denominate Numbers ([link])
Denominate numbers can be **added** or **subtracted** by

1. writing the numbers vertically so that the like units appear in the same column.
2. adding or subtracting the number parts, carrying along the unit.
3. simplifying the sum or difference.

Multiplying a Denominate Number by a Whole Number ([link])
To **multiply** a denominate number by a whole number, multiply the number part of each unit by the whole number and affix the unit to the product.

Dividing a Denominate Number by a Whole Number ([link])
To **divide** a denominate number by a whole number, divide the number part of each unit by the whole number beginning with the largest unit. Affix the unit to this quotient. Carry the remainder to the next unit.

Polygon ([link])
A **polygon** is a closed plane (flat) figure whose sides are line segments (portions of straight lines).

Perimeter ([link])
The **perimeter** of a polygon is the distance around the polygon.

Circumference, Diameter, Radius ([link])
The **circumference** of a circle is the distance around the circle. The **diameter** of a circle is any line segment that passes through the center of the circle and has its endpoints on the circle. The **radius** of a circle is one half the diameter of the circle.

The number π ([link])
The symbol π, read "pi," represents the nonterminating, nonrepeating decimal number 3.14159... . For computational purposes, π is often approximated by the number 3.14.

Formula ([link])
A **formula** is a rule for performing a task. In mathematics, a formula is a rule that directs us in computations.

Circumference Formulas ([link])
$C = \pi \cdot d \; C \approx (3.14)d$
$C = 2 \cdot \pi \cdot r \; C \approx 2(3.14)r$

Area ([link])
The **area** of a surface is the amount of square length units contained in the surface.

Volume ([link])
The **volume** of an object is a measure of the amount of cubic length units contained in the object.

Area Formulas ([link])
Triangle: $A = \frac{1}{2} \cdot b \cdot h$
Rectangle: $A = l \cdot w$
Parallelogram: $A = b \cdot h$
Trapezoid: $A = \frac{1}{2} \cdot (b_1 + b_2) \cdot h$
Circle: $A = \pi \cdot r^2$

Volume Formulas ([link])
Rectangle solid: $V = l \cdot w \cdot h$
Sphere: $V = \frac{4}{3} \cdot \pi \cdot r^3$

Cylinder: $V = \pi \cdot r^2 \cdot h$
Cone: $V = \frac{1}{3} \cdot \pi \cdot r^2 \cdot h$

Exercise Supplement
This module is from Fundamentals of Mathematics by Denny Burzynski and Wade Ellis, Jr. This module is an exercise supplement for the chapter "Measurement and Geometry" and contains many exercise problems. Odd problems are accompanied by solutions.

Exercise Supplement

Measurement and the United States System ([link])

Exercise:

Problem: What is measurement?

Solution:

Measurement is comparison to a standard (unit of measure).

For problems 2-6, make each conversion. Use the conversion table given in Section 9.1.

Exercise:

Problem: 9 ft= yd

Exercise:

Problem: 32 oz= lb

Solution:

2 pounds

Exercise:

Problem: 1,500 mg = g

Exercise:

Problem: 12,000 lb = T

Solution:

6 tons

Exercise:

Problem: 5,280 ft= mi

For problems 7-23, make each conversion.

Exercise:

Problem: 23 yd to ft

Solution:

69 feet

Exercise:

Problem: $2\frac{1}{2}$ mi to yd

Exercise:

Problem: 8 in. to ft

Solution:

$\frac{2}{3} = 0.666$ feet

Exercise:

Problem: 51 in. to mi

Exercise:

Problem: 3 qt to pt

Solution:

6 pints

Exercise:

Problem: 8 lb to oz

Exercise:

Problem: 5 cups to tbsp

Solution:

80 tablespoons

Exercise:

Problem: 9 da to hr

Exercise:

Problem: $3\frac{1}{2}$ min to sec

Solution:

210 seconds

Exercise:

Problem: $\frac{3}{4}$ wk to min

The Metric System of Measurement ([link])

Exercise:

Problem: 250 mL to L

Solution:

$\frac{1}{4} = 0.25\text{L}$

Exercise:

Problem: 18.57 cm to m

Exercise:

Problem: 0.01961 kg to mg

Solution:

19,610 mg

Exercise:

Problem: 52,211 mg to kg

Exercise:

Problem: 54.006 dag to g

Solution:

540.06 g

Exercise:

Problem: 1.181 hg to mg

Exercise:

Problem: 3.5 kL to mL

Solution:

3,500,000 mL

Simplification of Denominate Numbers ([link])

For problems 24-31, perform the indicated operations. Simplify, if possible.

Exercise:

Problem: Add 8 min 50 sec to 5 min 25 sec.

Exercise:

Problem: Add 3 wk 3 da to 2 wk 5 da

Solution:

6 weeks 1 day

Exercise:

Problem: Subtract 4 gal 3 qt from 5 gal 2 qt.

Exercise:

Problem: Subtract 2 gal 3 qt 1pt from 8 gal 2 qt.

Solution:

5 gallons 2 quarts 1 pint

Exercise:

Problem: Subtract 5 wk 4 da 21 hr from 12 wk 3 da 14 hr.

Exercise:

Problem: Subtract 2 T 1,850 lb from 10 T 1,700 lb.

Solution:

7 T 1,850 pounds

Exercise:

Problem:

Subtract the sum of 2 wk 3 da 15 hr and 5 wk 2 da 9 hr from 10 wk.

Exercise:

Problem:

Subtract the sum of 20 hr 15 min and 18 hr 18 min from the sum of 8 da 1 hr 16 min 5 sec.

Solution:

7 days, 11 hours, 56 minutes, 7 seconds

For problems 32-43, simplify, if necessary.

Exercise:

Problem: 18 in.

Exercise:

Problem: 4 ft

Solution:

1 yard 1 foot

Exercise:

Problem: 23 da

Exercise:

Problem: 3,100 lb

Solution:

1 ton 1,100 pounds

Exercise:

Problem: 135 min

Exercise:

Problem: 4 tsp

Solution:

1 tablespoon 1 teaspoon

Exercise:

Problem: 10 fl oz

Exercise:

Problem: 7 pt

Solution:

3 quarts 1 pint

Exercise:

Problem: 9 qt

Exercise:

Problem: 2,300 mm

Solution:

2.3 meters

Exercise:

Problem: 14,780 mL

Exercise:

Problem: 1,050 m

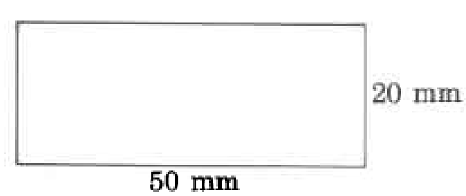
Solution:

1.05 km

Perimeter, Circumference, Area and Volume of Geometric Figures and Objects ([link],[link])

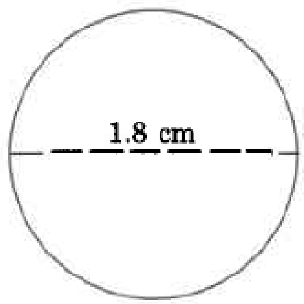
For problems 44-58, find the perimeter, circumference, area or volume.
Exercise:

Problem: Perimeter, area



Exercise:

Problem: Approximate circumference

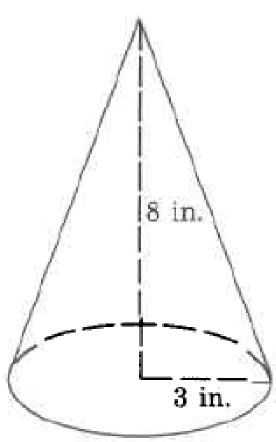


Solution:

5.652 sq cm

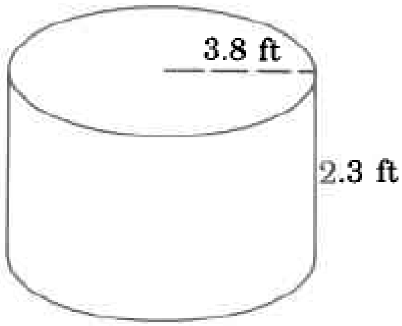
Exercise:

Problem: Approximate volume



Exercise:

Problem: Approximate volume

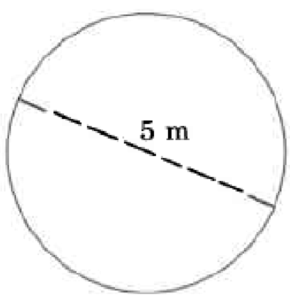


Solution:

104.28568 cu ft

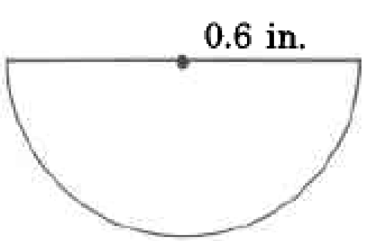
Exercise:

Problem: Exact area



Exercise:

Problem: Exact area

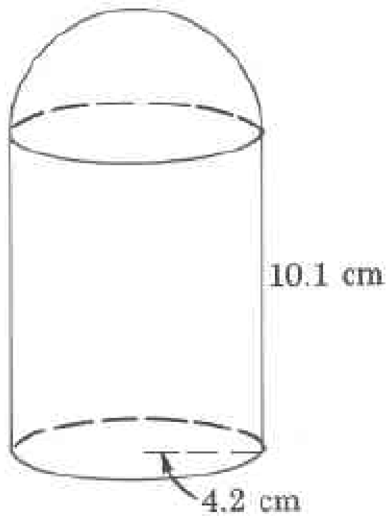


Solution:

0.18π sq in.

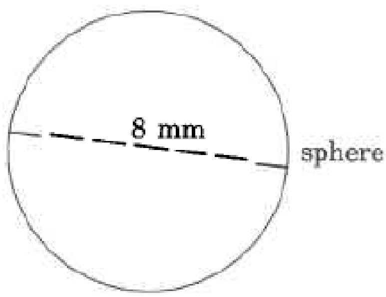
Exercise:

Problem: Exact volume



Exercise:

Problem: Approximate volume

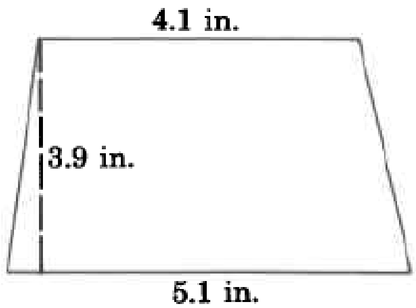


Solution:

267.94667 cu mm

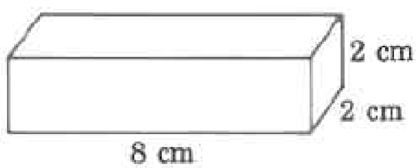
Exercise:

Problem: Area



Exercise:

Problem: Volume

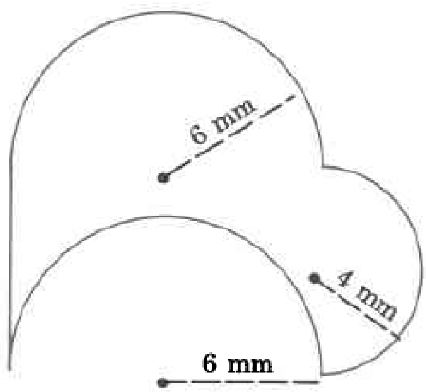


Solution:

32 cu cm

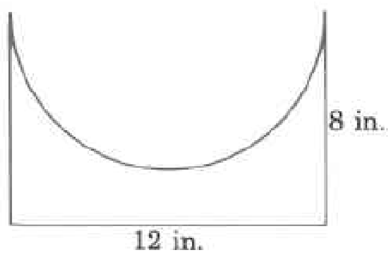
Exercise:

Problem: Exact area



Exercise:

Problem: Approximate area

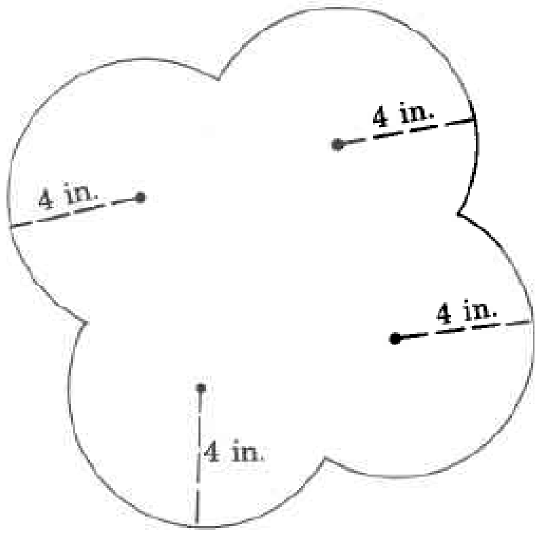


Solution:

39.48 sq in.

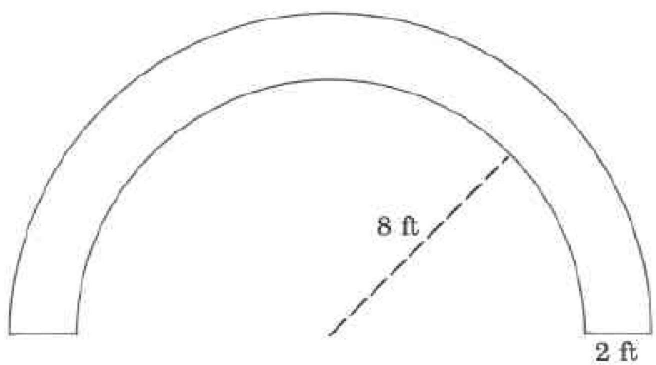
Exercise:

Problem: Exact area



Exercise:

Problem: Approximate area

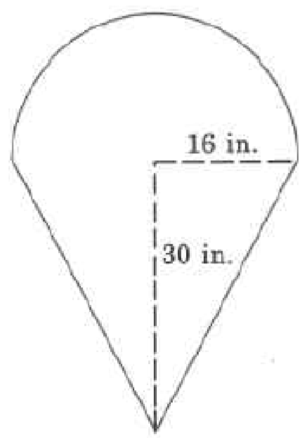


Solution:

56.52 sq ft

Exercise:

Problem: Approximate area


16 in.
30 in.

Proficiency Exam
This module is from Fundamentals of Mathematics by Denny Burzynski and Wade Ellis, Jr. This module is a proficiency exam to the chapter "Measurement and Geometry." Each problem is accompanied with a reference link pointing back to the module that discusses the type of problem demonstrated in the question. The problems in this exam are accompanied by solutions.

Proficiency Exam

Exercise:

Problem:

([link]) The process of determining, by comparison to some standard, the size of something is called .

Solution:

measurement

For problems 2-9, make each conversion.

Exercise:

Problem: ([link]) 14 yards to feet

Solution:

42 feet

Exercise:

Problem: ([link]) 51 feet to inches

Solution:

612 inches

Exercise:

Problem: ([link]) $\frac{1}{3}$ yard to feet

Solution:

1 foot

Exercise:

Problem: ([link]) $2\frac{1}{4}$ minutes to seconds

Solution:

135 seconds

Exercise:

Problem: ([link]) 8,500 mg to cg

Solution:

850 cg

Exercise:

Problem: ([link]) 5.8623 L to kL

Solution:

0.0058623 kL

Exercise:

Problem: ([link]) 213.1062 mm to m

Solution:

0.2132062 m

Exercise:

Problem: ([link]) 100,001 kL to mL

Solution:

100,001,000,000 mL

For problems 10-13, simplify each number.

Exercise:

Problem: ([link]) 23 da

Solution:

3 weeks 2 days

Exercise:

Problem: ([link]) 88 ft

Solution:

29 yards 1 foot

Exercise:

Problem: ([link]) 4216 lb

Solution:

2 tons 216 pounds

Exercise:

Problem: ([link]) 7 qt

Solution:

1 gallon 3 quarts

For problems 14-18, perform the indicated operations. Simplify answers if possible.

Exercise:

Problem: ([link]) Add 6 wk 3 da to 2 wk 2 da.

Solution:

8 weeks 5 days

Exercise:

Problem: ([link]) Add 9 gal 3 qt to 4 gal 3 qt.

Solution:

14 gallons 2 quarts

Exercise:

Problem: ([link]) Subtract 3 yd 2 ft 5 in. from 5 yd 8 ft 2 in.

Solution:

2 yards 5 feet 9 inches

Exercise:

Problem: ([link]) Subtract 2 hr 50 min 12 sec from 3 hr 20 min 8 sec.

Solution:

29 minutes 56 seconds

Exercise:

Problem:

([link]) Subtract the sum of 3 wk 6 da and 2 wk 3 da from 10 wk.

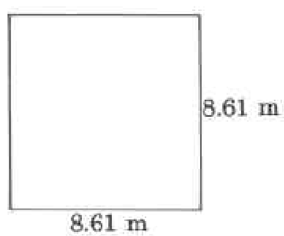
Solution:

3 weeks 5 days

For problems 19-30, find either the perimeter, circumference, area, or volume.

Exercise:

Problem: ([link]) Perimeter

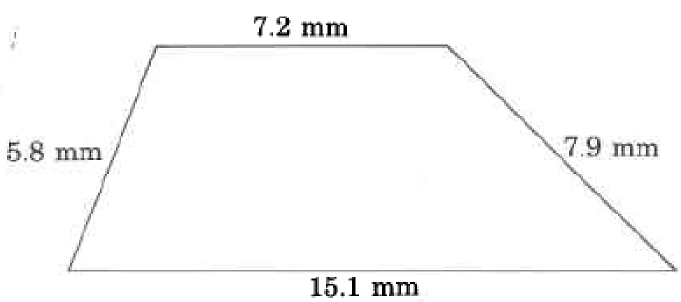


Solution:

34.44 m

Exercise:

Problem: ([link]) Perimeter

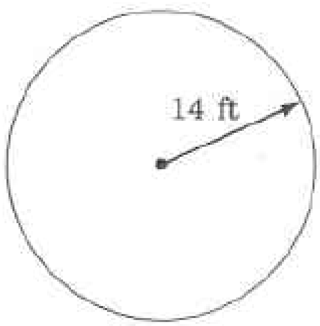


Solution:

36 mm

Exercise:

Problem: ([link]) Approximate circumference

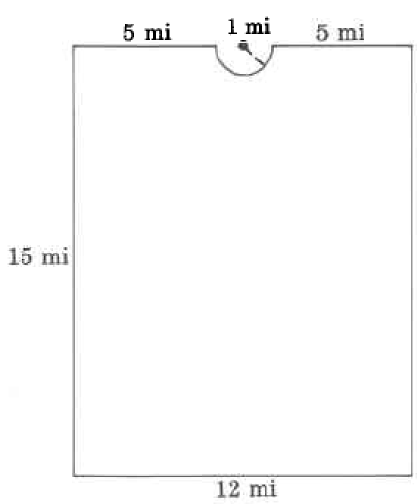


Solution:

87.92 feet

Exercise:

Problem: ([link]) Approximate perimeter

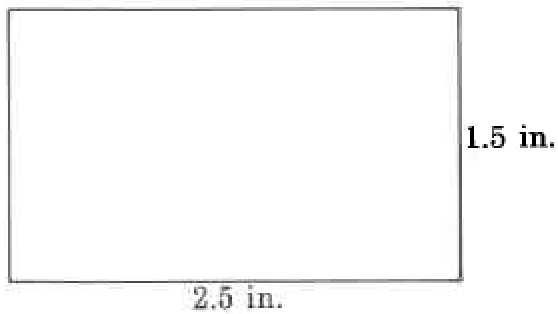


Solution:

55.14 miles

Exercise:

Problem: ([link]) Area

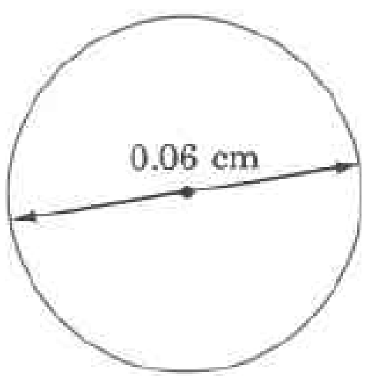


Solution:

3.75 sq in.

Exercise:

Problem: ([link]) Approximate area

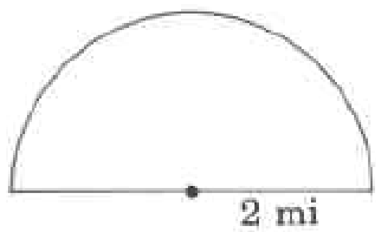


Solution:

6.002826 sq cm

Exercise:

Problem: ([link]) Approximate area

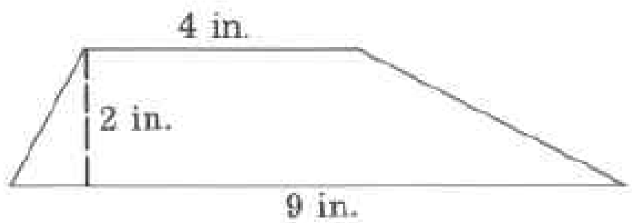


Solution:

6.28 sq miles

Exercise:

Problem: ([link]) Area

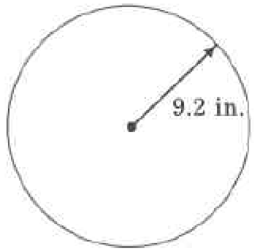


Solution:

13 sq in.

Exercise:

Problem: ([link]) Exact area

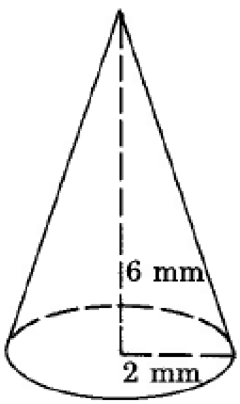


Solution:

84.64 sq in.

Exercise:

Problem: ([link]) Approximate volume

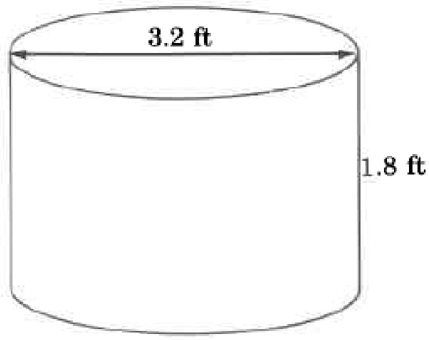


Solution:

25.12 cu mm

Exercise:

Problem: ([link]) Exact volume

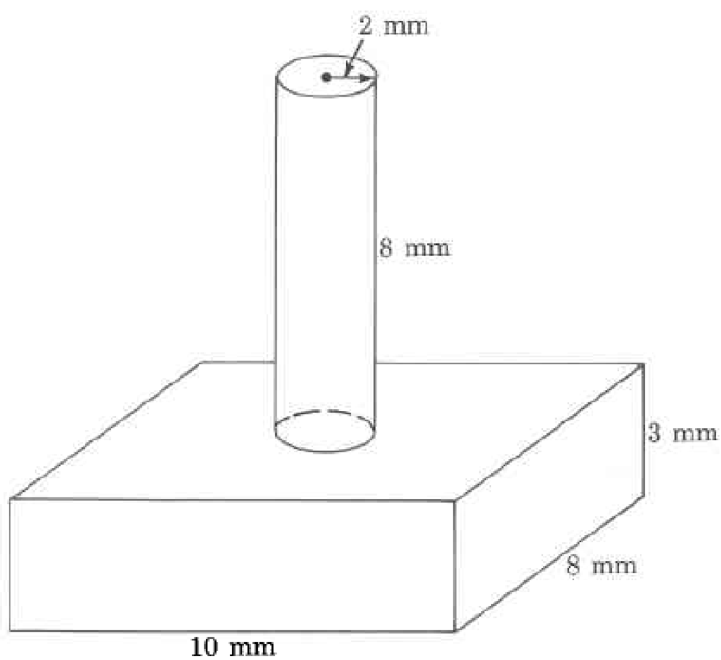


Solution:

4.608 cu ft

Exercise:

Problem: ([link]) Approximate volume



Solution:

340.48 cu mm

Objectives
This module contains the learning objectives for the chapter "Signed Numbers" from Fundamentals of Mathematics by Denny Burzynski and Wade Ellis, jr.

After completing this chapter, you should
Variables, Constants, and Real Numbers ([link])

- be able to distinguish between variables and constants
- be able to recognize a real number and particular subsets of the real numbers
- understand the ordering of the real numbers

Signed Numbers ([link])

- be able to distinguish between positive and negative real numbers
- be able to read signed numbers
- understand the origin and use of the double-negative product property

Absolute Value ([link])

- understand the geometric and algebraic definitions of absolute value

Addition of Signed Numbers ([link])

- be able to add numbers with like signs and with unlike signs
- be able to use the calculator for addition of signed numbers

Subtraction of Signed Numbers ([link])

- understand the definition of subtraction
- be able to subtract signed numbers
- be able to use a calculator to subtract signed numbers

Multiplication and Division of Signed Numbers ([link])

- be able to multiply and divide signed numbers
- be able to multiply and divide signed numbers using a calculator

Variables, Constants, and Real Numbers
This module is from Fundamentals of Mathematics by Denny Burzynski and Wade Ellis, Jr. This module discusses variables, constants, and real numbers. By the end of the module students should be able to distinguish between variables and constants, be able to recognize a real number and particular subsets of the real numbers and understand the ordering of the real numbers.

Section Overview

- Variables and Constants
- Real Numbers
- Subsets of Real Numbers
- Ordering Real Numbers

Variables and Constants

A basic distinction between algebra and arithmetic is the use of symbols (usually letters) in algebra to represent numbers. So, algebra is a generalization of arithmetic. Let us look at two examples of situations in which letters are substituted for numbers:

1. Suppose that a student is taking four college classes, and each class can have at most 1 exam per week. In any 1-week period, the student may have 0, 1, 2, 3, or 4 exams. In algebra, we can let the letter x represent the number of exams this student may have in a 1-week period. The letter x may assume any of the *various* values 0, 1, 2, 3, 4.
2. Suppose that in writing a term paper for a biology class a student needs to specify the average lifetime, in days, of a male housefly. If she does not know this number off the top of her head, she might represent it (at least temporarily) on her paper with the letter t (which reminds her of *time*). Later, she could look up the average time in a reference book and find it to be 17 days. The letter t can assume only the one value, 17, and no other values. The value t is *constant*.

Variable, Constant

1. A letter or symbol that represents any member of a collection of two or more numbers is called a **variable**.
2. A letter or symbol that represents one specific number, known or unknown, is called a **constant**.

In example 1, the letter x is a variable since it can represent any of the numbers 0, 1, 2, 3, 4. The letter t example 2 is a constant since it can only have the value 17.

Real Numbers

Real Number Line
The study of mathematics requires the use of several collections of numbers. The **real number line** allows us to visually display (graph) the numbers in which we are interested.

A line is composed of infinitely many points. To each point we can associate a unique number, and with each number, we can associate a particular point.

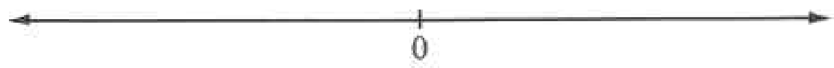
Coordinate
The *number* associated with a point on the number line is called the **coordinate** of the point.

Graph
The *point* on a number line that is associated with a particular number is called the **graph** of that number.

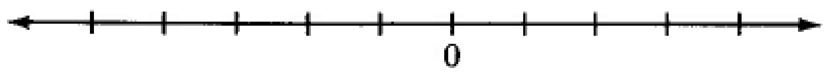
Constructing a Real Number Line
We construct a real number line as follows:

1. Draw a horizontal line.

2. **Origin**
   Choose any point on the line and label it 0. This point is called the **origin**.



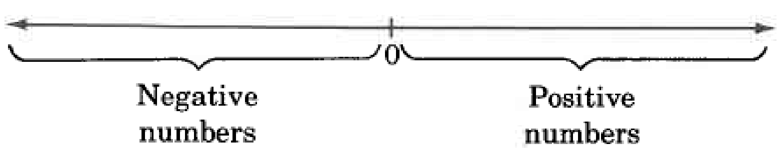
3. Choose a convenient length. Starting at 0, mark this length off in both directions, being careful to have the lengths look like they are about the same.



We now define a real number.

Real Number
A **real number** is any number that is the coordinate of a point on the real number line.

Positive Numbers, Negative Numbers
Real numbers whose graphs are to the right of 0 are called *positive real numbers*, or more simply, **positive numbers**. Real numbers whose graphs appear to the left of 0 are called *negative real numbers*, or more simply, **negative numbers**.

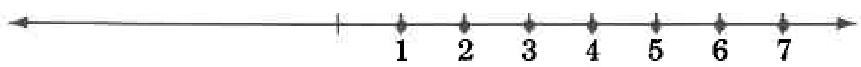


The number 0 is neither positive nor negative.

Subsets of Real Numbers

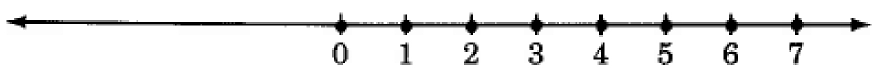
The set of real numbers has many subsets. Some of the subsets that are of interest in the study of algebra are listed below along with their notations and graphs.

Natural Numbers, Counting Numbers
The **natural** or **counting numbers** (N): 1, 2, 3, 4, . . . Read "and so on."



Whole Numbers

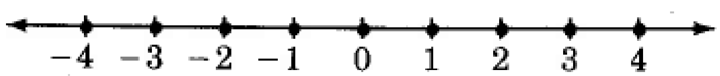
The **whole numbers** (W): 0, 1, 2, 3, 4, . . .



Notice that every natural number is a whole number.

Integers

The **integers** (Z): . . . -3, -2, -1, 0, 1, 2, 3, . . .



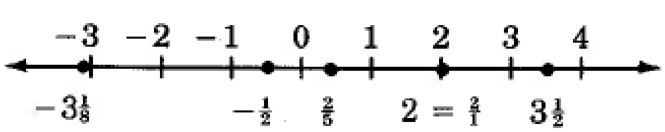
Notice that every whole number is an integer.

Rational Numbers (Fractions)

The **rational numbers** (Q): Rational numbers are sometimes called **fractions**. They are numbers that can be written as the *quotient* of two integers. They have decimal representations that either terminate or do not terminate but contain a repeating block of digits. Some examples are

$$\underset{\text{Terminating}}{\frac{-3}{4} = -0.75} \qquad \underset{\text{Nonterminating, but repeating}}{8\frac{11}{27} = 8.407407407\ldots}$$

Some rational numbers are graphed below.



Notice that *every integer is a rational number*.

Notice that there are still a great many points on the number line that have not yet been assigned a type of number. We will not examine these other types of numbers in this text. They are examined in detail in algebra. An

example of these numbers is the number π, whose decimal representation does not terminate nor contain a repeating block of digits. An approximation for π is 3.14.

Sample Set A

Example:
Is every whole number a natural number?
No. The number 0 is a whole number but it is not a natural number.

Example:
Is there an integer that is not a natural number?
Yes. Some examples are 0, -1, -2, -3, and -4.

Example:
Is there an integer that is a whole number?
Yes. In fact, every whole number is an integer.

Practice Set A

Exercise:

Problem: Is every natural number a whole number?

Solution:

yes

Exercise:

Problem: Is every whole number an integer?

Solution:

yes

Exercise:

Problem: Is every integer a real number?

Solution:

yes

Exercise:

Problem: Is there an integer that is a whole number?

Solution:

yes

Exercise:

Problem: Is there an integer that is not a natural number?

Solution:

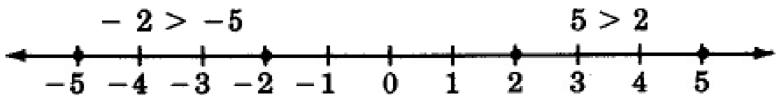
yes

Ordering Real Numbers

Ordering Real Numbers

A real number b is said to be *greater* than a real number a, denoted $b > a$, if b is to the right of a on the number line. Thus, as we would expect, $5 > 2$

since 5 is to the right of 2 on the number line. Also, $-2 > -5$ since -2 is to the right of -5 on the number line.



If we let a and b represent two numbers, then a and b are related in exactly one of three ways: Either

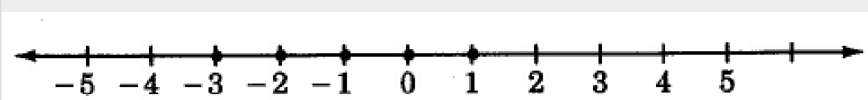
Equality Symbol
$a = b$ $\;$ a and b are equal $\;$ $(8 = 8)$

Inequality Symbols

$a > b$	a is greater than b	$(8 > 5)$
$a < b$	a is less than b	$(5 < 8)$
	Some variations of these symbols are	
$a \neq b$	a is not equal to b	$(8 \neq 5)$
$a \geq b$	a is greater than or equal to b	$(a \geq 8)$
$a \leq b$	a is less than or equal to b	$(a \leq 8)$

Sample Set B

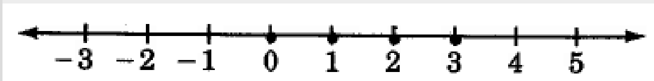
Example:
What integers can replace x so that the following statement is true?
$-3 \leq x < 2$



The integers are -3, -2, -1, 0, 1.

Example:

Draw a number line that extends from -3 to 5. Place points at all whole numbers between and including -1 and 3.



-1 is not a whole number

Practice Set B

Exercise:

Problem:

What integers can replace x so that the following statement is true?
$-5 \le x < 2$

Solution:

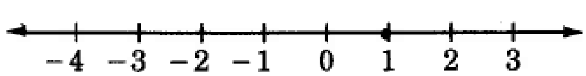
-5, -4, -3, -2, -1, 0

Exercise:

Problem:

Draw a number line that extends from -4 to 3. Place points at all natural numbers between, but not including, -2 to 2.

Solution:



Exercises

For the following 8problems, next to each real number, note all collections to which it belongs by writing N for natural number, W for whole number, or Z for integer. Some numbers may belong to more than one collection.

Exercise:

Problem: 6

Solution:

N, *W*, *Z*

Exercise:

Problem: 12

Exercise:

Problem: 0

Solution:

W, *Z*

Exercise:

Problem: 1

Exercise:

Problem: -3

Solution:

Z

Exercise:

Problem: -7

Exercise:

Problem: -805

Solution:

Z

Exercise:

Problem: -900

Exercise:

Problem:

Is the number 0 a positive number, a negative number, neither, or both?

Solution:

Neither

Exercise:

Problem:

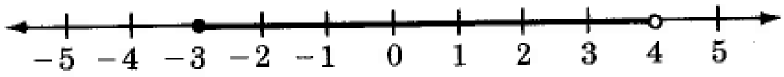
An integer is an even integer if it is evenly divisible by 2. Draw a number line that extends from -5 to 5 and place points at all negative even integers and all positive odd integers.

Exercise:

Problem:

Draw a number line that extends from -5 to 5. Place points at all integers that satisfy $-3 \leq x < 4$.

Solution:



Exercise:

Problem: Is there a largest two digit number? If so, what is it?

Exercise:

Problem: Is there a smallest two digit number? If so, what is it?

Solution:

Yes, 10

For the pairs of real numbers in the following 5 problems, write the appropriate symbol (<, >, =) in place of the □.

Exercise:

Problem: -7 □ -2

Exercise:

Problem: -5 □ 0

Solution:

<

Exercise:

Problem: -1 □ 4

Exercise:

Problem: 6 □ -1

Solution:

>

Exercise:

Problem: 10 □ 10

For the following 5 problems, what numbers can replace *m* so that the following statements are true?

Exercise:

Problem: $-1 \leq m \leq -5$, m an integer.

Solution:

{-1, 0, 1, 2, 3, 4, 5}

Exercise:

Problem: $-7 < m < -1$, m an integer.

Exercise:

Problem: $-3 \leq m < 2$, m a natural number.

Solution:

{1}

Exercise:

Problem: $-15 < m \leq -1$, m a natural number.

Exercise:

Problem: $-5 \leq m < 5$, m a whole number.

Solution:

{0, 1, 2, 3, 4}

For the following 10 problems, on the number line, how many units are there between the given pair of numbers?

Exercise:

Problem: 0 and 3

Exercise:

Problem: -4 and 0

Solution:

4

Exercise:

Problem: -1 and 6

Exercise:

Problem: -6 and 2

Solution:

8

Exercise:

Problem: -3 and 3

Exercise:

Problem: Are all positive numbers greater than zero?

Solution:

yes

Exercise:

Problem: Are all positive numbers greater than all negative numbers?

Exercise:

Problem: Is 0 greater than all negative number?

Solution:

yes

Exercise:

Problem: Is there a largest natural number?

Exercise:

Problem: Is there a largest negative integer?

Solution:

yes, -1

Exercises for Review

Exercise:

Problem: ([link]) Convert $6\frac{5}{8}$ to an improper fraction.

Exercise:

Problem: ([link]) Find the value: $\frac{3}{11}$ of $\frac{33}{5}$.

Solution:

$\frac{9}{5}$ or $1\frac{4}{5}$ or 1.8

Exercise:

Problem: ([link]) Find the sum of $\frac{4}{5} + \frac{3}{8}$.

Exercise:

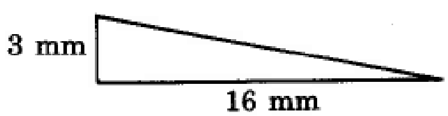
Problem: ([link]) Convert 30.06 cm to m.

Solution:

0.3006 m

Exercise:

Problem: ([link]) Find the area of the triangle.



Signed Numbers
This module is from Fundamentals of Mathematics by Denny Burzynski and Wade Ellis, Jr. This module discusses signed numbers. By the end of the module students be able to distinguish between positive and negative real numbers, be able to read signed numbers and understand the origin and use of the double-negative product property.

Section Overview

- Positive and Negative Numbers
- Reading Signed Numbers
- Opposites
- The Double-Negative Property

Positive and Negative Numbers

Positive and Negative Numbers
Each real number other than zero has a **sign** associated with it. A real number is said to be a **positive number** if it is to the right of 0 on the number line and **negative** if it is to the left of 0 on the number line.

Note:
THE NOTATION OF SIGNED NUMBERS
+ and − Notation
A number is denoted as **positive** if it is directly preceded by a plus sign or no sign at all.
A number is denoted as **negative** if it is directly preceded by a minus sign.

Reading Signed Numbers

The plus and minus signs now have *two meanings*:

The plus sign can denote the operation of addition *or* a positive number.

The minus sign can denote the operation of subtraction *or* a negative number.

To avoid any confusion between "sign" and "operation," it is preferable to read the sign of a number as "positive" or "negative." When "+" is used as an operation sign, it is read as "plus." When "−" is used as an operation sign, it is read as "minus."

Sample Set A

Read each expression so as to avoid confusion between "operation" and "sign."

Example:
-8 should be read as "negative eight" rather than "minus eight."

Example:
$4+(-2)$ should be read as "four plus negative two" rather than "four plus minus two."

Example:
$-6+(-3)$should be read as "negative six plus negative three" rather than "minus six plus minus three."

Example:
$-15-(-6)$should be read as "negative fifteen minus negative six" rather than "minus fifteen minus minus six."

Example:
$-5+7$ should be read as "negative five plus seven" rather than "minus five plus seven."

Example:
$0-2$ should be read as "zero minus two."

Practice Set A

Write each expression in words.

Exercise:

Problem: $6 + 1$

Solution:

six plus one

Exercise:

Problem: $2 + (-8)$

Solution:

two plus negative eight

Exercise:

Problem: $-7 + 5$

Solution:

negative seven plus five

Exercise:

Problem: $-10 - (+3)$

Solution:

negative ten minus three

Exercise:

Problem: $-1 - (-8)$

Solution:

negative one minus negative eight

Exercise:

Problem: $0 + (-11)$

Solution:

zero plus negative eleven

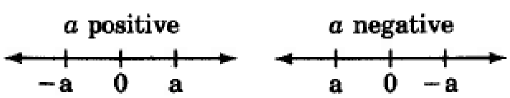
Opposites

Opposites
On the number line, each real number, other than zero, has an image on the opposite side of 0. For this reason, we say that each real number has an opposite. **Opposites** are the same distance from zero but have opposite signs.

The opposite of a real number is denoted by placing a negative sign directly in front of the number. Thus, if a is any real number, then $-a$ is its opposite.

Note: The letter "a" is a variable. Thus, "a" need not be positive, and "$-a$" need not be negative.

If a is any real number, $-a$ is opposite a on the number line.



The Double-Negative Property

The number a is opposite $-a$ on the number line. Therefore, $-(-a)$ is opposite $-a$ on the number line. This means that

$-(-a) = a$

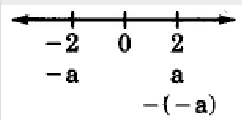
From this property of opposites, we can suggest the double-negative property for real numbers.

Double-Negative Property: $-(-a) = a$
If a is a real number, then
$-(-a) = a$

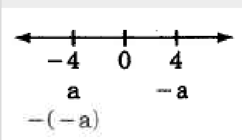
Sample Set B

Find the opposite of each number.

Example:
If $a = 2$, then $-a = -2$. Also, $-(-a) = -(-2) = 2$.



Example:
If $a = -4$, then $-a = -(-4) = 4$. Also, $-(-a) = a = -4$.



Practice Set B

Find the opposite of each number.

Exercise:

Problem: 8

Solution:

-8

Exercise:

Problem: 17

Solution:

-17

Exercise:

Problem: -6

Solution:

6

Exercise:

Problem: -15

Solution:

15

Exercise:

Problem: -(-1)

Solution:

-1

Exercise:

Problem: $-[-(-7)]$

Solution:

7

Exercise:

Problem: Suppose a is a positive number. Is $-a$ positive or negative?

Solution:

$-a$ is negative

Exercise:

Problem: Suppose a is a negative number. Is $-a$ positive or negative?

Solution:

$-a$ is positive

Exercise:

Problem:

Suppose we do not know the sign of the number k. Is $-k$ positive, negative, or do we not know?

Solution:

We must say that we do not know.

Exercises

Exercise:

Problem: A number is denoted as positive if it is directly preceded by .

Solution:

+ (or no sign)

Exercise:

Problem: A number is denoted as negative if it is directly preceded by .

How should the number in the following 6 problems be read? (Write in words.)
Exercise:

Problem: -7

Solution:

negative seven

Exercise:

Problem: -5

Exercise:

Problem: 15

Solution:

fifteen

Exercise:

Problem: 11

Exercise:

Problem: $-(-1)$

Solution:

negative negative one, or opposite negative one

Exercise:

Problem: $-(-5)$

For the following 6 problems, write each expression in words.

Exercise:

Problem: $5 + 3$

Solution:

five plus three

Exercise:

Problem: $3 + 8$

Exercise:

Problem: $15 + (-3)$

Solution:

fifteen plus negative three

Exercise:

Problem: $1 + (-9)$

Exercise:

Problem: $-7 - (-2)$

Solution:

negative seven minus negative two

Exercise:

Problem: $0 - (-12)$

For the following 6 problems, rewrite each number in simpler form.

Exercise:

Problem: $-(-2)$

Solution:

2

Exercise:

Problem: $-(-16)$

Exercise:

Problem: $-[-(-8)]$

Solution:

-8

Exercise:

Problem: $-[-(-20)]$

Exercise:

Problem: $7-(-3)$

Solution:

$7+3=10$

Exercise:

Problem: $6-(-4)$

Exercises for Review

Exercise:

Problem: ([link]) Find the quotient; $8 \div 27$.

Solution:

0.296

Exercise:

Problem: ([link]) Solve the proportion: $\frac{5}{9}=\frac{60}{x}$

Exercise:

Problem: ([link]) Use the method of rounding to estimate the sum: $5829+8767$

Solution:

$6{,}000+9{,}000=15\ 000\ \ (5{,}829+8{,}767=14\ 596)$ or $5{,}800+8{,}800=14\ 600$

Exercise:

Problem: ([link]) Use a unit fraction to convert 4 yd to feet.

Exercise:

Problem: ([link]) Convert 25 cm to hm.

Solution:

0.0025 hm

Absolute Value
This module is from Fundamentals of Mathematics by Denny Burzynski and Wade Ellis, Jr. This module discusses absolute value. By the end of the module students should understand the geometric and algebraic definitions of absolute value.

Section Overview

- Geometric Definition of Absolute Value
- Algebraic Definition of Absolute Value

Geometric Definition of Absolute Value

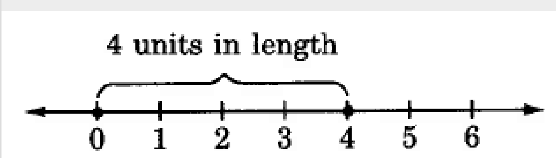
Absolute Value-Geometric Approach
Geometric definition of absolute value:
The **absolute value** of a number a, denoted $|\ a\ |$, is the distance from a to 0 on the number line.

Absolute value answers the question of "how far," and not "which way." The phrase "how far" implies "length" and *length is always a nonnegative quantity*. Thus, the absolute value of a number is a nonnegative number.

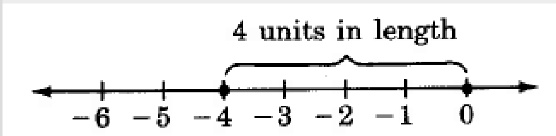
Sample Set A

Determine each value.

Example:
$|\ 4\ |= 4$



Example:
$|-4| = 4$



Example:
$|0| = 0$

Example:
$-|5| = -5$. The quantity on the left side of the equal sign is read as "negative the absolute value of 5." The absolute value of 5 is 5. Hence, negative the absolute value of 5 is -5.

Example:
$-|-3| = -3$. The quantity on the left side of the equal sign is read as "negative the absolute value of -3." The absolute value of -3 is 3. Hence, negative the absolute value of -3 is $-(3) = -3$.

Practice Set A

By reasoning geometrically, determine each absolute value.

Exercise:

Problem: $|7|$

Solution:

7

Exercise:

Problem: $\mid -3 \mid$

Solution:

3

Exercise:

Problem: $\mid 12 \mid$

Solution:

12

Exercise:

Problem: $\mid 0 \mid$

Solution:

0

Exercise:

Problem: $- \mid 9 \mid$

Solution:

-9

Exercise:

Problem: $- \mid -6 \mid$

Solution:

-6

Algebraic Definition of Absolute Value

From the problems in [link], we can suggest the following algebraic definition of absolute value. Note that the definition has two parts.

Absolute Value—Algebraic Approach
Algebraic definition of absolute value
The absolute value of a number a is

$$|a| = \begin{cases} a, & \text{if } a \geq 0 \\ -a, & \text{if } < 0 \end{cases}$$

The algebraic definition takes into account the fact that the number a could be either positive or zero $(a \geq 0)$ or negative $(a < 0)$.

1. If the number a is positive or zero $(a \geq 0)$, the upper part of the definition applies. The upper part of the definition tells us that if the number enclosed in the absolute value bars is a nonnegative number, the absolute value of the number is the number itself.
2. The lower part of the definition tells us that if the number enclosed within the absolute value bars is a negative number, the absolute value of the number is the opposite of the number. The opposite of a negative number is a positive number.

Note: The definition says that the vertical absolute value lines may be eliminated only if we know whether the number inside is positive or negative.

Sample Set B

Use the algebraic definition of absolute value to find the following values.

Example:
$\mid 8 \mid$. The number enclosed within the absolute value bars is a nonnegative number, so the upper part of the definition applies. This part says that the absolute value of 8 is 8 itself.
$\mid 8 \mid = 8$

Example:
$\mid -3 \mid$. The number enclosed within absolute value bars is a negative number, so the lower part of the definition applies. This part says that the absolute value of -3 is the opposite of -3, which is $-(-3)$. By the definition of absolute value and the double-negative property,
$\mid -3 \mid = -(-3) = 3$

Practice Set B

Use the algebraic definition of absolute value to find the following values.

Exercise:

Problem: $\mid 7 \mid$

Solution:

7

Exercise:

Problem: $\mid 9 \mid$

Solution:

9

Exercise:

Problem: $\mid -12 \mid$

Solution:

12

Exercise:

Problem: $\mid -5 \mid$

Solution:

5

Exercise:

Problem: $- \mid 8 \mid$

Solution:

-8

Exercise:

Problem: $- \mid 1 \mid$

Solution:

-1

Exercise:

Problem: $- \mid -52 \mid$

Solution:

-52

Exercise:

Problem: $- \mid -31 \mid$

Solution:

-31

Exercises

Determine each of the values.

Exercise:

Problem: $\mid 5 \mid$

Solution:

5

Exercise:

Problem: $\mid 3 \mid$

Exercise:

Problem: $\mid 6 \mid$

Solution:

6

Exercise:

Problem: $\mid -9 \mid$

Exercise:

Problem: $\mid -1 \mid$

Solution:

1

Exercise:

Problem: $\mid -4 \mid$

Exercise:

Problem: $- \mid 3 \mid$

Solution:

-3

Exercise:

Problem: $- \mid 7 \mid$

Exercise:

Problem: $- \mid -14 \mid$

Solution:

-14

Exercise:

Problem: $\mid 0 \mid$

Exercise:

Problem: $\mid -26 \mid$

Solution:

26

Exercise:

Problem: $-\mid -26\mid$

Exercise:

Problem: $-(-\mid 4\mid)$

Solution:

4

Exercise:

Problem: $-(-\mid 2\mid)$

Exercise:

Problem: $-(-\mid -6\mid)$

Solution:

6

Exercise:

Problem: $-(-\mid -42\mid)$

Exercise:

Problem: $\mid 5\mid -\mid -2\mid$

Solution:

3

Exercise:

Problem: $\mid -2 \mid^3$

Exercise:

Problem: $\mid -(2 \cdot 3) \mid$

Solution:

6

Exercise:

Problem: $\mid -2 \mid - \mid -9 \mid$

Exercise:

Problem: $(\mid -6 \mid + \mid 4 \mid)^2$

Solution:

100

Exercise:

Problem: $(\mid -1 \mid - \mid 1 \mid)^3$

Exercise:

Problem: $(\mid 4 \mid + \mid -6 \mid)^2 - (\mid -2 \mid)^3$

Solution:

92

Exercise:

Problem: $-[|-10|-6]^2$

Exercise:

Problem: $-\left\{-[-\mid -4\mid + \mid -3\mid]^3\right\}^2$

Solution:

-1

Exercise:

Problem:

A Mission Control Officer at Cape Canaveral makes the statement “lift-off, *T* minus 50 seconds.” How long is it before lift-off?

Exercise:

Problem:

Due to a slowdown in the industry, a Silicon Valley computer company finds itself in debt $2,400,000. Use absolute value notation to describe this company’s debt.

Solution:

$-\$\mid -2{,}400{,}000\mid$

Exercise:

Problem:

A particular machine is set correctly if upon action its meter reads 0. One particular machine has a meter reading of -1.6 upon action. How far is this machine off its correct setting?

Exercises for Review

Exercise:

Problem: ([link]) Find the sum: $\frac{9}{70} + \frac{5}{21} + \frac{8}{15}$.

Solution:

$\frac{9}{10}$

Exercise:

Problem: ([link]) Find the value of $\frac{\frac{3}{10} + \frac{4}{12}}{\frac{19}{20}}$.

Exercise:

Problem: ([link]) Convert $3.2\frac{3}{5}$ to a fraction.

Solution:

$3\frac{13}{50} \text{or} \frac{163}{50}$

Exercise:

Problem:

([link]) The ratio of acid to water in a solution is $\frac{3}{8}$. How many mL of acid are there in a solution that contain 112 mL of water?

Exercise:

Problem: ([link]) Find the value of $-6 - (-8)$.

Solution:

2

Addition of Signed Numbers
This module is from Fundamentals of Mathematics by Denny Burzynski and Wade Ellis, Jr. This module discusses how to add signed numbers. By the end of the module students should be able to add numbers with like signs and with unlike signs and be able to use the calculator for addition of signed numbers.

Section Overview

- Addition of Numbers with Like Signs
- Addition with Zero
- Addition of Numbers with Unlike Signs
- Calculators

Addition of Numbers with Like Signs

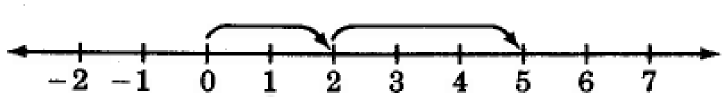
The addition of the *two positive numbers* 2 and 3 is performed on the number line as follows.

Begin at 0, the origin.

Since 2 is positive, move 2 units to the right.

Since 3 is positive, move 3 more units to the right.

We are now located at 5.



Thus, $2 + 3 = 5$.

Summarizing, we have

$(2 \text{ positive units}) + (3 \text{ positive units}) = (5 \text{ positive units})$

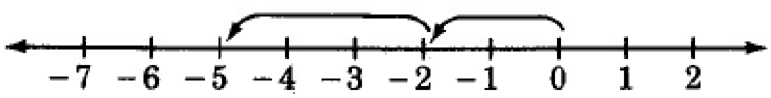
The addition of the *two negative numbers* -2 and -3 is performed on the number line as follows.

Begin at 0, the origin.

Since -2 is negative, move 2 units to the left.

Since -3 is negative, move 3 more units to the left.

We are now located at -5.



Thus, $(-2)+(-3)=-5$.

Summarizing, we have

$(2 \text{ negative units}) + (3 \text{ negative units}) = (5 \text{ negative units})$

Observing these two examples, we can suggest these relationships:

$(\text{positive number}) + (\text{positive number}) = (\text{positive number})$

$(\text{negative number}) + (\text{negative number}) = (\text{negative number})$

Adding Numbers with the Same Sign
Addition of numbers with like sign:
To add two real numbers that have the *same* sign, add the absolute values of the numbers and associate with the sum the common sign.

Sample Set A

Find the sums.

Example:
$3+7$
$$\left.\begin{array}{rcl}|3| & = & 3 \\ |7| & = & 7\end{array}\right\}$$ Add these absolute values.
$3+7=10$
The common sign is “+.”
Thus, $3+7=+10$, or $3+7=10$.

Example:
$(-4)+(-9)$
$$\left.\begin{array}{rcl}|-4| & = & 4 \\ |-9| & = & 9\end{array}\right\}$$ Add these absolute values.
$4+9=13$
The common sign is “$-$.“
Thus, $(-4)+(-9)=-13$.

Practice Set A

Find the sums.

Exercise:

Problem: $8+6$

Solution:

14

Exercise:

Problem: $41+11$

Solution:

52

Exercise:

Problem: $(-4)+(-8)$

Solution:

-12

Exercise:

Problem: $(-36)+(-9)$

Solution:

-45

Exercise:

Problem: $-14+(-20)$

Solution:

-34

Exercise:

Problem: $-\frac{2}{3}+\left(-\frac{5}{3}\right)$

Solution:

$-\frac{7}{3}$

Exercise:

Problem: $-2.8+(-4.6)$

Solution:

-7.4

Exercise:

Problem: $0+(-16)$

Solution:

-16

Addition With Zero

Addition with Zero
Notice that
$(0)+(\text{a positive number})=(\text{that same positive number}).$
$(0)+(\text{a negative number})=(\text{that same negative number}).$

The Additive Identity Is Zero
Since adding zero to a real number leaves that number unchanged, zero is called the **additive identity**.

Addition of Numbers with Unlike Signs

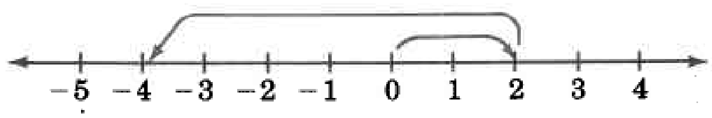
The addition $2+(-6)$, *two numbers with unlike signs*, can also be illustrated using the number line.

Begin at 0, the origin.

Since 2 is positive, move 2 units to the right.

Since -6 is negative, move, from 2, 6 units to the left.

We are now located at -4.



We can suggest a rule for adding two numbers that have *unlike signs* by noting that if the signs are disregarded, 4 can be obtained by subtracting 2 from 6. But 2 and 6 are precisely the absolute values of 2 and -6. Also, notice that the sign of the number with the larger absolute value is negative and that the sign of the resulting sum is negative.

Adding Numbers with Unlike Signs

Addition of numbers with unlike signs: To add two real numbers that have *unlike signs*, subtract the smaller absolute value from the larger absolute value and associate with this difference the sign of the number with the larger absolute value.

Sample Set B

Find the following sums.

Example:

$7 + (-2)$

$|7| = 7$ $\quad$ $|-2| = 2$

Larger absolute value. Sign is positive. $\quad$ Smaller absolute value.

Subtract absolute values: $7 - 2 = 5$.

Attach the proper sign: "+."

Thus, $7 + (-2) = +5$ or $7 + (-2) = 5$.

Example:

$3 + (-11)$

$|3| = 3$ $\quad$ $|-11| = 11$

Smaller absolute value. $\quad$ Larger absolute value. Sign is negative.

Subtract absolute values: $11 - 3 = 8$.
Attach the proper sign: "$-$."
Thus, $3 + (-11) = -8$.

Example:
The morning temperature on a winter's day in Lake Tahoe was -12 degrees. The afternoon temperature was 25 degrees warmer. What was the afternoon temperature?
We need to find $-12 + 25$.

$|-12| = 12$ $\quad$ $|25| = 25$

Smaller absolute value. $\quad$ Larger absolute value. Sign is positive.

Subtract absolute values: $25 - 12 = 16$.
Attach the proper sign: "+."
Thus, $-12 + 25 = 13$.

Practice Set B

Find the sums.

Exercise:

Problem: $4 + (-3)$

Solution:

1

Exercise:

Problem: $-3 + 5$

Solution:

2

Exercise:

Problem: $15 + (-18)$

Solution:

-3

Exercise:

Problem: $0 + (-6)$

Solution:

-6

Exercise:

Problem: $-26 + 12$

Solution:

-14

Exercise:

Problem: $35 + (-78)$

Solution:

-43

Exercise:

Problem: $15 + (-10)$

Solution:

5

Exercise:

Problem: $1.5 + (-2)$

Solution:

-0.5

Exercise:

Problem: $-8 + 0$

Solution:

-8

Exercise:

Problem: $0 + (0.57)$

Solution:

0.57

Exercise:

Problem: $-879 + 454$

Solution:

-425

Calculators

Calculators having the

$\boxed{+/-}$

key can be used for finding sums of signed numbers.

Sample Set C

Use a calculator to find the sum of -147 and 84.

		Display Reads	
Type	147	147	
Press	$\boxed{+/-}$	-147	This key changes the sign of a number. It is different than −.
Press	+	-147	
Type	84	84	
Press	=	-63	

Practice Set C

Use a calculator to find each sum.

Exercise:

Problem: $673 + (-721)$

Solution:

-48

Exercise:

Problem: $-8{,}261 + 2{,}206$

Solution:

-6,085

Exercise:

Problem: $-1{,}345.6 + (-6{,}648.1)$

Solution:

-7,993.7

Exercises

Find the sums in the following 27 problems. If possible, use a calculator to check each result.

Exercise:

Problem: $4 + 12$

Solution:

16

Exercise:

Problem: $8+6$

Exercise:

Problem: $(-3)+(-12)$

Solution:

-15

Exercise:

Problem: $(-6)+(-20)$

Exercise:

Problem: $10+(-2)$

Solution:

8

Exercise:

Problem: $8+(-15)$

Exercise:

Problem: $-16+(-9)$

Solution:

-25

Exercise:

Problem: $-22+(-1)$

Exercise:

Problem: $0 + (-12)$

Solution:

-12

Exercise:

Problem: $0 + (-4)$

Exercise:

Problem: $0 + (24)$

Solution:

24

Exercise:

Problem: $-6 + 1 + (-7)$

Exercise:

Problem: $-5 + (-12) + (-4)$

Solution:

-21

Exercise:

Problem: $-5 + 5$

Exercise:

Problem: $-7+7$

Solution:

0

Exercise:

Problem: $-14+14$

Exercise:

Problem: $4+(-4)$

Solution:

0

Exercise:

Problem: $9+(-9)$

Exercise:

Problem: $84+(-61)$

Solution:

23

Exercise:

Problem: $13+(-56)$

Exercise:

Problem: $452+(-124)$

Solution:

328

Exercise:

Problem: $636 + (-989)$

Exercise:

Problem: $1{,}811 + (-935)$

Solution:

876

Exercise:

Problem: $-373 + (-14)$

Exercise:

Problem: $-1{,}211 + (-44)$

Solution:

-1,255

Exercise:

Problem: $-47.03 + (-22.71)$

Exercise:

Problem: $-1.998 + (-4.086)$

Solution:

-6.084

Exercise:

Problem:

In order for a small business to break even on a project, it must have sales of $21,000. If the amount of sales was $15,000, by how much money did this company fall short?

Exercise:

Problem:

Suppose a person has $56 in his checking account. He deposits $100 into his checking account by using the automatic teller machine. He then writes a check for $84.50. If an error causes the deposit not to be listed into this person's account, what is this person's checking balance?

Solution:

-$28.50

Exercise:

Problem:

A person borrows $7 on Monday and then $12 on Tuesday. How much has this person borrowed?

Exercise:

Problem:

A person borrows $11 on Monday and then pays back $8 on Tuesday. How much does this person owe?

Solution:

$3.00

Exercises for Review

Exercise:

Problem: ([link]) Find the reciprocal of $8\frac{5}{6}$.

Exercise:

Problem: ([link]) Find the value of $\frac{5}{12}+\frac{7}{18}-\frac{1}{3}$.

Solution:

$\frac{17}{36}$

Exercise:

Problem: ([link]) Round 0.01628 to the nearest tenth.

Exercise:

Problem: ([link]) Convert 62% to a fraction.

Solution:

$\frac{62}{100}=\frac{31}{50}$

Exercise:

Problem: ([link]) Find the value of $\mid -12 \mid$.

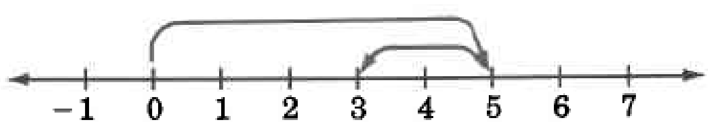
Subtraction of Signed Numbers
This module is from Fundamentals of Mathematics by Denny Burzynski and Wade Ellis, Jr. This module discusses how to subtract signed numbers. By the end of the module students should understand the definition of subtraction, be able to subtract signed numbers and be able to use a calculator to subtract signed numbers.

Section Overview

- Definition of Subtraction
- The Process of Subtraction
- Calculators

Definition of Subtraction

We know from experience with arithmetic that the subtraction $5-2$ produces 3, that is $5-2=3$. We can suggest a rule for subtracting signed numbers by illustrating this process on the number line.



Begin at 0, the origin.

Since 5 is positive, move 5 units to the right.

Then, move *2 units to the left* to get to 6. (This reminds us of addition with a negative number.)

From this illustration we can see that $5-2$ is the same as $5+(-2)$. This leads us directly to the definition of subtraction.

Definition of Subtraction
If a and b are real numbers, $a-b$ is the same as $a+(-b)$, where $-b$ is the opposite of b.

The Process of Subtraction

From this definition, we suggest the following rule for subtracting signed numbers.

Subtraction of Signed Numbers
To perform the subtraction $a - b$, add the opposite of b to a, that is, change the sign of b and add.

Sample Set A

Perform the indicated subtractions.

Example:
$5 - 3 = 5 + (-3) = 2$

Example:
$4 - 9 = 4 + (-9) = -5$

Example:
$-4 - 6 = -4 + (-6) = -10$

Example:
$-3 - (-12) = -3 + 12 = 9$

Example:

$0-(-15)=0+15=15$

Example:
The high temperature today in Lake Tahoe was 26°F. The low temperature tonight is expected to be -7°F. How many degrees is the temperature expected to drop?
We need to find the difference between 26 and -7.
$26-(-7)=26+7=33$
Thus, the expected temperature drop is 33°F.

Example:

$$\begin{aligned} -6-(-5)-10 &= -6+5+(-10) \\ &= (-6+5)+(-10) \\ &= -1+(-10) \\ &= -11 \end{aligned}$$

Practice Set A

Perform the indicated subtractions.

Exercise:

Problem: $9-6$

Solution:

3

Exercise:

Problem: $6-9$

Solution:

-3

Exercise:

Problem: $0 - 7$

Solution:

-7

Exercise:

Problem: $1 - 14$

Solution:

-13

Exercise:

Problem: $-8 - 12$

Solution:

-20

Exercise:

Problem: $-21 - 6$

Solution:

-27

Exercise:

Problem: $-6-(-4)$

Solution:

-2

Exercise:

Problem: $8-(-10)$

Solution:

18

Exercise:

Problem: $1-(-12)$

Solution:

13

Exercise:

Problem: $86-(-32)$

Solution:

118

Exercise:

Problem: $0-16$

Solution:

-16

Exercise:

Problem: $0 - (-16)$

Solution:

16

Exercise:

Problem: $0 - (8)$

Solution:

-8

Exercise:

Problem: $5 - (-5)$

Solution:

10

Exercise:

Problem: $24 - [-(-24)]$

Solution:

0

Calculators

Calculators can be used for subtraction of signed numbers. The most efficient calculators are those with a

$\boxed{+/-}$

key.

Sample Set B

Use a calculator to find each difference.

Example:
$3{,}187 - 8{,}719$

Display Reads		
Type	3187	3187
Press	-	3187
Type	8719	8719
Press	=	-5532

Thus, $3{,}187 - 8{,}719 = -5{,}532$.

Example:
$-156 - (-211)$
Method A:

Display Reads		
Type	156	156
Press	$\boxed{+/-}$	-156
Type	-	-156
Press	211	211
Type	$\boxed{+/-}$	-211
Press	=	55

Thus, $-156-(-211)=55$.
Method B:
We manually change the subtraction to an addition and change the sign of the number to be subtracted.
$-156-(-211)$ becomes $-156+211$

Display Reads		
Type	156	156
Press	$\boxed{+/-}$	-156

Press	+	-156
Type	211	211
Press	=	55

Practice Set B

Use a calculator to find each difference.

Exercise:

Problem: $44 - 315$

Solution:

-271

Exercise:

Problem: $12.756 - 15.003$

Solution:

-2.247

Exercise:

Problem: $-31.89 - 44.17$

Solution:

-76.06

Exercise:

Problem: $-0.797 - (-0.615)$

Solution:

-0.182

Exercises

For the following 18 problems, perform each subtraction. Use a calculator to check each result.

Exercise:

Problem: $8 - 3$

Solution:

5

Exercise:

Problem: $12 - 7$

Exercise:

Problem: $5 - 6$

Solution:

-1

Exercise:

Problem: $14 - 30$

Exercise:

Problem: $-6-8$

Solution:

-14

Exercise:

Problem: $-1-12$

Exercise:

Problem: $-5-(-3)$

Solution:

-2

Exercise:

Problem: $-11-(-8)$

Exercise:

Problem: $0-6$

Solution:

-6

Exercise:

Problem: $0-15$

Exercise:

Problem: $0-(-7)$

Solution:

7

Exercise:

Problem: $0 - (-10)$

Exercise:

Problem: $67 - 38$

Solution:

29

Exercise:

Problem: $142 - 85$

Exercise:

Problem: $816 - 1140$

Solution:

-324

Exercise:

Problem: $105 - 421$

Exercise:

Problem: $-550 - (-121)$

Solution:

-429

Exercise:

Problem: $-15.016-(4.001)$

For the following 4 problems, perform the indicated operations.

Exercise:

Problem: $-26+7-52$

Solution:

-71

Exercise:

Problem: $-15-21-(-2)$

Exercise:

Problem: $-104-(-216)-(-52)$

Solution:

164

Exercise:

Problem: $-0.012-(-0.111)-(0.035)$

Exercise:

Problem:

When a particular machine is operating properly, its meter will read 34. If a broken bearing in the machine causes the meter reading to drop by 45 units, what is the meter reading?

Solution:

-11

Exercise:

Problem:

The low temperature today in Denver was -4°F and the high was -42°F. What is the temperature difference?

Exercises for Review

Exercise:

Problem: ([link]) Convert $16.02\frac{1}{5}$ to a decimal.

Solution:

16.022

Exercise:

Problem: ([link]) Find 4.01 of 6.2.

Exercise:

Problem: ([link]) Convert $\frac{5}{16}$ to a percent.

Solution:

31.25%

Exercise:

Problem:

([link]) Use the distributive property to compute the product: $15 \cdot 82$.

Exercise:

Problem: ([link]) Find the sum: $16 + (-21)$.

Solution:

-5

Multiplication and Division of Signed Numbers
This module is from Fundamentals of Mathematics by Denny Burzynski and Wade Ellis, Jr. This module discusses how to multiply and divide signed numbers. By the end of the module students should be able to multiply and divide signed numbers and be able to multiply and divide signed numbers using a calculator.

Section Overview

- Multiplication of Signed Numbers
- Division of Signed Numbers
- Calculators

Multiplication of Signed Numbers

Let us consider first, the product of two positive numbers. Multiply: $3 \cdot 5$.

$3 \cdot 5$ means $5 + 5 + 5 = 15$

This suggests[footnote] that
In later mathematics courses, the word "suggests" turns into the word "proof." One example does not prove a claim. Mathematical proofs are constructed to validate a claim for all possible cases.

$(\text{positive number}) \cdot (\text{positive number}) = (\text{positive number})$

More briefly,

$(+)(+) = (+)$

Now consider the product of a positive number and a negative number. Multiply: $(3)(-5)$.

$(3)(-5)$ means $(-5) + (-5) + (-5) = -15$

This suggests that

$$(\text{positive number}) \cdot (\text{negative number}) = (\text{negative number})$$

More briefly,

$$(+)(-) = (-)$$

By the commutative property of multiplication, we get

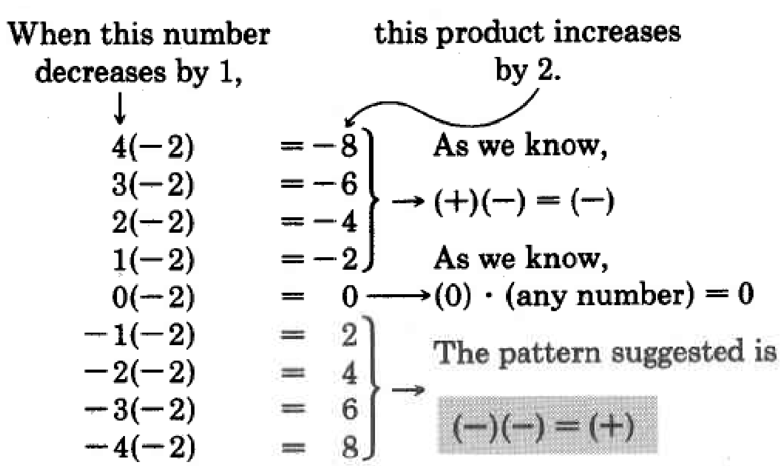
$$(\text{negative number}) \cdot (\text{positive number}) = (\text{negative number})$$

More briefly,

$$(-)(+) = (-)$$

The sign of the product of two negative numbers can be suggested after observing the following illustration.

Multiply -2 by, respectively, 4, 3, 2, 1, 0, -1, -2, -3, -4.



We have the following rules for multiplying signed numbers.

Rules for Multiplying Signed Numbers
Multiplying signed numbers:

1. To multiply two real numbers that have the *same sign*, multiply their absolute values. The product is positive.

$(+)(+) = (+)$
$(-)(-) = (+)$

2. To multiply two real numbers that have *opposite signs*, multiply their absolute values. The product is negative.
   $(+)(-) = (-)$
   $(-)(+) = (-)$

Sample Set A

Find the following products.

Example:
$8 \cdot 6$
$$\left.\begin{array}{rcl} |8| & = & 8 \\ |6| & = & 6 \end{array}\right\} \text{Multiply these absolute values.}$$
$8 \cdot 6 = 48$
Since the numbers have the same sign, the product is positive.
Thus, $8 \cdot 6 {=} {+}48$, or $8 \cdot 6 = 48$.

Example:
$(-8)(-6)$
$$\left.\begin{array}{rcl} |-8| & = & 8 \\ |-6| & = & 6 \end{array}\right\} \text{Multiply these absolute values.}$$
$8 \cdot 6 = 48$
Since the numbers have the same sign, the product is positive.
Thus, $(-8)(-6) {=} {+}48$, or $(-8)(-6) = 48$.

Example:
$(-4)(7)$

$$\left.\begin{array}{rcl} |-4| & = & 4 \\ |7| & = & 7 \end{array}\right\}$$ Multiply these absolute values.

$4 \cdot 7 = 28$

Since the numbers have opposite signs, the product is negative.

Thus, $(-4)(7) = -28$.

Example:

$6(-3)$

$$\left.\begin{array}{rcl} |6| & = & 6 \\ |-3| & = & 3 \end{array}\right\}$$ Multiply these absolute values.

$6 \cdot 3 = 18$

Since the numbers have opposite signs, the product is negative.

Thus, $6(-3) = -18$.

Practice Set A

Find the following products.

Exercise:

Problem: $3(-8)$

Solution:

-24

Exercise:

Problem: $4(16)$

Solution:

64

Exercise:

Problem: $(-6)(-5)$

Solution:

30

Exercise:

Problem: $(-7)(-2)$

Solution:

14

Exercise:

Problem: $(-1)(4)$

Solution:

-4

Exercise:

Problem: $(-7)7$

Solution:

-49

Division of Signed Numbers

To determine the signs in a division problem, recall that

$\frac{12}{3} = 4$ since $12 = 3 \cdot 4$

This suggests that

$$\frac{(+)}{(+)} = (+)$$
$$\frac{(+)}{(+)} = (+) \text{ since } (+) = (+)(+)$$

What is $\frac{12}{-3}$?

$-12 = (-3)(-4)$ suggests that $\frac{12}{-3} = -4$. That is,

$$\frac{(+)}{(-)} = (-)$$
$$(+) = (-)(-) \text{ suggests that } \frac{(+)}{(-)} = (-)$$

What is $\frac{-12}{3}$?

$-12 = (3)(-4)$ suggests that $\frac{-12}{3} = -4$. That is,

$$\frac{(-)}{(+)} = (-)$$
$$(-) = (+)(-) \text{ suggests that } \frac{(-)}{(+)} = (-)$$

What is $\frac{-12}{-3}$?

$-12 = (-3)(4)$ suggests that $\frac{-12}{-3} = 4$. That is,

$$\frac{(-)}{(-)} = (+)$$
$$(-) = (-)(+) \text{ suggests that } \frac{(-)}{(-)} = (+)$$

We have the following rules for dividing signed numbers.

Rules for Dividing Signed Numbers
Dividing signed numbers:

1. To divide two real numbers that have the *same sign*, divide their absolute values. The quotient is positive.

$\frac{(+)}{(+)} = (+)$ $\frac{(-)}{(-)} = (+)$

2. To divide two real numbers that have *opposite signs*, divide their absolute values. The quotient is negative.
   $\frac{(-)}{(+)} = (-)$ $\frac{(+)}{(-)} = (-)$

Sample Set B

Find the following quotients.

Example:
$\frac{-10}{2}$
$$\left.\begin{array}{rcr} |-10| & = & 10 \\ |2| & = & 2 \end{array}\right\}\text{Divide these absolute values.}$$
$\frac{10}{2} = 5$
Since the numbers have opposite signs, the quotient is negative.
Thus $\frac{-10}{2} = -5$.

Example:
$\frac{-35}{-7}$
$$\left.\begin{array}{rcr} |-35| & = & 35 \\ |-7| & = & 7 \end{array}\right\}\text{Divide these absolute values.}$$
$\frac{35}{7} = 5$
Since the numbers have the same signs, the quotient is positive.
Thus, $\frac{-35}{-7} = 5$.

Example:
$\frac{18}{-9}$

$$\left.\begin{aligned} |18| &= 18 \\ |-9| &= 9 \end{aligned}\right\} \text{Divide these absolute values.}$$

$\frac{18}{9} = 2$

Since the numbers have opposite signs, the quotient is negative.

Thus, $\frac{18}{-9} = 2$.

Practice Set B

Find the following quotients.

Exercise:

Problem: $\frac{-24}{-6}$

Solution:

4

Exercise:

Problem: $\frac{30}{-5}$

Solution:

-6

Exercise:

Problem: $\frac{-54}{27}$

Solution:

-2

Exercise:

Problem: $\frac{51}{17}$

Solution:

3

Sample Set C

Example:
Find the value of $\frac{-6(4-7)-2(8-9)}{-(4+1)+1}$.
Using the order of operations and what we know about signed numbers, we get,

$$\begin{aligned} \frac{-6(4-7)-2(8-9)}{-(4+1)+1} &= \frac{-6(-3)-2(-1)}{-(5)+1} \\ &= \frac{18+2}{-5+1} \\ &= \text{mfrac} \\ &= -5 \end{aligned}$$

Practice Set C

Exercise:

Problem: Find the value of $\frac{-5(2-6)-4(-8-1)}{2(3-10)-9(-2)}$.

Solution:

14

Calculators

Calculators with the

$\boxed{+/-}$

key can be used for multiplying and dividing signed numbers.

Sample Set D

Use a calculator to find each quotient or product.

Example:
$(-186) \cdot (-43)$
Since this product involves a $(\text{negative}) \cdot (\text{negative})$, we know the result should be a positive number. We'll illustrate this on the calculator.

		Display Reads
Type	186	186
Press	$\boxed{+/-}$	-186
Press	×	-186
Type	43	43

Press	$\boxed{+/-}$	-43
Press	=	7998

Thus, $(-186) \cdot (-43) = 7{,}998$.

Example:
$\frac{158.64}{-54.3}$. Round to one decimal place.

		Display Reads
Type	158.64	158.64
Press	÷	158.64
Type	54.3	54.3
Press	$\boxed{+/-}$	-54.3
Press	=	-2.921546961

Rounding to one decimal place we get -2.9.

Practice Set D

Use a calculator to find each value.
Exercise:

Problem: $(-51.3) \cdot (-21.6)$

Solution:

1,108.08

Exercise:

Problem: $-2.5746 \div -2.1$

Solution:

1.226

Exercise:

Problem: $(0.006) \cdot (-0.241)$. Round to three decimal places.

Solution:

-0.001

Exercises

Find the value of each of the following. Use a calculator to check each result.
Exercise:

Problem: $(-2)(-8)$

Solution:

16

Exercise:

Problem: $(-3)(-9)$

Exercise:

Problem: $(-4)(-8)$

Solution:

32

Exercise:

Problem: $(-5)(-2)$

Exercise:

Problem: $(3)(-12)$

Solution:

-36

Exercise:

Problem: $(4)(-18)$

Exercise:

Problem: $(10)(-6)$

Solution:

-60

Exercise:

Problem: $(-6)(4)$

Exercise:

Problem: $(-2)(6)$

Solution:

-12

Exercise:

Problem: $(-8)(7)$

Exercise:

Problem: $\frac{21}{7}$

Solution:

3

Exercise:

Problem: $\frac{42}{6}$

Exercise:

Problem: $\frac{-39}{3}$

Solution:

-13

Exercise:

Problem: $\frac{-20}{10}$

Exercise:

Problem: $\frac{-45}{-5}$

Solution:

9

Exercise:

Problem: $\frac{-16}{-8}$

Exercise:

Problem: $\frac{25}{-5}$

Solution:

-5

Exercise:

Problem: $\frac{36}{-4}$

Exercise:

Problem: $8-(-3)$

Solution:

11

Exercise:

Problem: $14-(-20)$

Exercise:

Problem: $20 - (-8)$

Solution:

28

Exercise:

Problem: $-4 - (-1)$

Exercise:

Problem: $0 - 4$

Solution:

-4

Exercise:

Problem: $0 - (-1)$

Exercise:

Problem: $-6 + 1 - 7$

Solution:

-12

Exercise:

Problem: $15 - 12 - 20$

Exercise:

Problem: $1 - 6 - 7 + 8$

Solution:

-4

Exercise:

Problem: $2 + 7 - 10 + 2$

Exercise:

Problem: $3(4 - 6)$

Solution:

-6

Exercise:

Problem: $8(5 - 12)$

Exercise:

Problem: $-3(1 - 6)$

Solution:

15

Exercise:

Problem: $-8(4 - 12) + 2$

Exercise:

Problem: $-4(1 - 8) + 3(10 - 3)$

Solution:

49

Exercise:

Problem: $-9(0-2)+4(8-9)+0(-3)$

Exercise:

Problem: $6(-2-9)-6(2+9)+4(-1-1)$

Solution:

-140

Exercise:

Problem: $\frac{3(4+1)-2(5)}{-2}$

Exercise:

Problem: $\frac{4(8+1)-3(-2)}{-4-2}$

Solution:

-7

Exercise:

Problem: $\frac{-1(3+2)+5}{-1}$

Exercise:

Problem: $\frac{-3(4-2)+(-3)(-6)}{-4}$

Solution:

-3

Exercise:

Problem: $-1(4+2)$

Exercise:

Problem: $-1(6-1)$

Solution:

-5

Exercise:

Problem: $-(8+21)$

Exercise:

Problem: $-(8-21)$

Solution:

13

Exercises for Review

Exercise:

Problem:

([link]) Use the order of operations to simplify $\left(5^2+3^2+2\right) \div 2^2$.

Exercise:

Problem: ([link]) Find $\frac{3}{8}$ of $\frac{32}{9}$.

Solution:

$\frac{4}{3} = 1\frac{1}{3}$

Exercise:

Problem:

([link]) Write this number in decimal form using digits: “fifty-two three-thousandths”

Exercise:

Problem:

([link]) The ratio of chlorine to water in a solution is 2 to 7. How many mL of water are in a solution that contains 15 mL of chlorine?

Solution:

$52\frac{1}{2}$

Exercise:

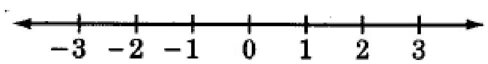
Problem: ([link]) Perform the subtraction $-8-(-20)$

Summary of Key Concepts
This module is from Fundamentals of Mathematics by Denny Burzynski and Wade Ellis, Jr. This module reviews the key concepts from the chapter "Signed Numbers."

Summary of Key Concepts

Variables and Constants ([link])
A **variable** is a letter or symbol that represents any member of a set of two or more numbers. A **constant** is a letter or symbol that represents a specific number. For example, the Greek letter π (pi) represents the constant 3.14159 . . . .

The Real Number Line ([link])
The **real number line** allows us to visually display some of the numbers in which we are interested.



Coordinate and Graph ([link])
The number associated with a point on the number line is called the **coordinate** of the point. The point associated with a number is called the **graph** of the number.

Real Number ([link])
A **real number** is any number that is the coordinate of a point on the real number line.

Types of Real Numbers ([link])
The set of **real numbers** has many subsets. The ones of most interest to us are:
The **natural numbers**: {1, 2, 3, 4, . . .}
The **whole numbers**: {0, 1, 2, 3, 4, . . .}
The **integers**: {. . . ,-3,-2,-1,0, 1, 2, 3, . . .}
The **rational numbers**: {All numbers that can be expressed as the quotient of two integers.}

Positive and Negative Numbers ([link])
A number is denoted as **positive** if it is directly preceded by a plus sign (+) or no sign at all. A number is denoted as **negative** if it is directly preceded by a minus sign (–).

Opposites ([link])
Opposites are numbers that are the same distance from zero on the number line but have opposite signs. The numbers a and $-a$ are opposites.

Double-Negative Property ([link])
$-(-a) = a$

Absolute Value (Geometric) ([link])
The **absolute value** of a number a, denoted $\mid a \mid$, is the distance from a to 0 on the number line.

Absolute Value (Algebraic) ([link])
$$|a| = \begin{cases} a, & \text{if } a \geq 0 \\ -a, & \text{if } a < 0 \end{cases}$$

Addition of Signed Numbers ([link])
To **add two numbers** with

1. **like signs**, add the absolute values of the numbers and associate with the sum the common sign.
2. **unlike signs**, subtract the smaller absolute value from the larger absolute value and associate with the difference the sign of the larger absolute value.

Addition with Zero ([link])
$0 + (\text{any number}) = \text{that particular number.}$

Additive Identity ([link])
Since adding 0 to any real number leaves that number unchanged, 0 is called the **additive identity**.

Definition of Subtraction ([link])
$a - b = a + (-b)$

Subtraction of Signed Numbers ([link])
To perform the **subtraction** $a - b$, add the opposite of b to a, that is, change the sign of b and follow the addition rules ([link]).

Multiplication and Division of Signed Numbers ([link])

$(+)(+) = (+)$ $\frac{(+)}{(+)} = (+)$ $\frac{(+)}{(-)} = (-)$

$(-)(-) = (+)$

$(+)(-) = (-)$ $\frac{(-)}{(-)} = (+)$ $\frac{(-)}{(+)} = (-)$

$(-)(+) = (-)$

Exercise Supplement
This module is from Fundamentals of Mathematics by Denny Burzynski and Wade Ellis, Jr. This module is an exercise supplement for the chapter "Signed Numbers" and contains many exercise problems. Odd problems are accompanied by solutions.

Exercise Supplement

Variables, Constants, and Real Numbers ([link])

For problems 1-5, next to each real number, note all subsets of the real numbers to which it belongs by writing *N* for natural numbers, *W* for whole numbers, or *Z* for integers. Some numbers may belong to more than one subset.

Exercise:

Problem: 61

Solution:

N, *W*, *Z*

Exercise:

Problem: -14

Exercise:

Problem: 0

Solution:

W, *Z*

Exercise:

Problem: 1

Exercise:

Problem: Write all the integers that are strictly between -4 and 3

Solution:

$\{-3, -2, -1, 0, 1, 2\}$

Exercise:

Problem:

Write all the integers that are between and including -6 and -1

For each pair of numbers in problems 7-10, write the appropriate symbol (<, >, =) in place of the □.

Exercise:

Problem: -5 □ -1

Solution:

<

Exercise:

Problem: 0 □ 2

Exercise:

Problem: -7 □ 0

Solution:

<

Exercise:

Problem: -1 □ 0

For problems 11-15, what numbers can replace *x* so that each statement is true?

Exercise:

Problem: $-5 \leq x \leq -1$, *x* is an integer

Solution:

$\{-5, -4, -3, -2, -1\}$

Exercise:

Problem: $-10 < x \leq 0$, *x* is a whole number.

Exercise:

Problem: $0 \leq x < 5$, *x* is a natural number.

Solution:

$\{1, 2, 3, 4\}$

Exercise:

Problem: $-3 < x < 3$, *x* is a natural number

Exercise:

Problem: $-8 < x \leq -2$, *x* is a whole number.

Solution:

none

For problems 16-20, how many units are there between the given pair of numbers?

Exercise:

Problem: 0 and 4

Exercise:

Problem: -1 and 3

Solution:

4

Exercise:

Problem: -7 and -4

Exercise:

Problem: -6 and 0

Solution:

6

Exercise:

Problem: -1 and 1

Exercise:

Problem:

A number is positive if it is directly preceded by a sign or no sign at all.

Solution:

+ (plus)

Exercise:

Problem: A number is negative if it is directly preceded by a sign.

Signed Numbers ([link])

For problems 23-26, how should each number be read?
Exercise:

Problem: -8

Solution:

negative eight

Exercise:

Problem: $-(-4)$

Exercise:

Problem: $-(-1)$

Solution:

negative negative one or opposite negative one

Exercise:

Problem: -2

For problems 27-31, write each expression in words.
Exercise:

Problem: $1+(-7)$

Solution:

one plus negative seven

Exercise:

Problem: $-2-(-6)$

Exercise:

Problem: $-1-(+4)$

Solution:

negative one minus four

Exercise:

Problem: $-(-(-3))$

Exercise:

Problem: $0-(-11)$

Solution:

zero minus negative eleven

For problems 32-36, rewrite each expression in simpler form.

Exercise:

Problem: $-(-4)$

Exercise:

Problem: $-(-15)$

Solution:

15

Exercise:

Problem: $-[-(-7)]$

Exercise:

Problem: $1-(-18)$

Solution:

19 or $1+18$

Exercise:

Problem: $0-(-1)$

Absolute Value ([link])

For problems 37-52, determine each value.

Exercise:

Problem: $\mid 9\mid$

Solution:

9

Exercise:

Problem: $\mid 16\mid$

Exercise:

Problem: $\mid -5 \mid$

Solution:

5

Exercise:

Problem: $\mid -8 \mid$

Exercise:

Problem: $- \mid -2 \mid$

Solution:

-2

Exercise:

Problem: $- \mid -1 \mid$

Exercise:

Problem: $-(- \mid 12 \mid)$

Solution:

12

Exercise:

Problem: $-(- \mid 90 \mid)$

Exercise:

Problem: $-(- \mid -16 \mid)$

Solution:

16

Exercise:

Problem: $-(- \mid 0 \mid)$

Exercise:

Problem: $\mid -4 \mid^2$

Solution:

16

Exercise:

Problem: $\mid -5 \mid^2$

Exercise:

Problem: $\mid -2 \mid^3$

Solution:

8

Exercise:

Problem: $\mid -(3 \cdot 4) \mid$

Exercise:

Problem: $\mid -5 \mid + \mid -2 \mid$

Solution:

7

Exercise:

Problem: $\mid -7 \mid - \mid -10 \mid$

Addition, Subtraction, Multiplication and Division of Signed Numbers ([link],[link],[link])

For problems 53-71, perform each operation.

Exercise:

Problem: $-6 + 4$

Solution:

-2

Exercise:

Problem: $-10 + 8$

Exercise:

Problem: $-1 - 6$

Solution:

-7

Exercise:

Problem: $8 - 12$

Exercise:

Problem: $0 - 14$

Solution:

-14

Exercise:

Problem: $5 \cdot (-2)$

Exercise:

Problem: $-8 \cdot (-6)$

Solution:

48

Exercise:

Problem: $(-3) \cdot (-9)$

Exercise:

Problem: $14 \cdot (-3)$

Solution:

-42

Exercise:

Problem: $5 \cdot (-70)$

Exercise:

Problem: $-18 \div -6$

Solution:

3

Exercise:

Problem: $72 \div -12$

Exercise:

Problem: $-16 \div -16$

Solution:

1

Exercise:

Problem: $0 \div -8$

Exercise:

Problem: $-5 \div 0$

Solution:

not defined

Exercise:

Problem: $\frac{-15}{-3}$

Exercise:

Problem: $\frac{-28}{7}$

Solution:

-4

Exercise:

Problem: $\frac{-120}{-\mid 2 \mid}$

Exercise:

Problem: $\frac{\mid -66 \mid}{-\mid -3 \mid}$

Solution:

-22

Proficiency Exam
This module is from Fundamentals of Mathematics by Denny Burzynski and Wade Ellis, Jr. This module is a proficiency exam to the chapter "Signed Numbers." Each problem is accompanied with a reference link pointing back to the module that discusses the type of problem demonstrated in the question. The problems in this exam are accompanied by solutions.

Proficiency Exam

Exercise:

Problem:

([link]) Write all integers that are strictly between –8 and –3.

Solution:

$\{-7, -6, -5, -4\}$

Exercise:

Problem:

([link]) Write all integers that are between and including –2 and 1.

Solution:

$\{-2, -1, 0, 1\}$

For problems 3-5, write the appropriate symbol (<, >, =) in place of the □ for each pair of numbers.

Exercise:

Problem: ([link]) –1 □ –1

Solution:

=

Exercise:

Problem: ([link]) 0 □ 3

Solution:

<

Exercise:

Problem: ([link]) –1 □ –2

Solution:

>

For problems 6 and 7, what numbers can replace x so that the statement is true?

Exercise:

Problem: ([link]) $-3 \leq x < 0$, x is an integer.

Solution:

$\{-3, -2, -1\}$

Exercise:

Problem: ([link]) $-4 \leq x \leq 0$, x is a natural number.

Solution:

$\{1, 2\}$

Exercise:

Problem: ([link]) How many units are there between –3 and 2?

Solution:

5

For problems 9-20, find each value.

Exercise:

Problem: ([link]) $|-16|$

Solution:

16

Exercise:

Problem: ([link]) $-|-2|$

Solution:

-2

Exercise:

Problem: ([link]) $-(-|-4|^2)$

Solution:

16

Exercise:

Problem: ([link]) $|-5|+|-10|$

Solution:

15

Exercise:

Problem: ([link]) $-8+6$

Solution:

-2

Exercise:

Problem: ([link]) $-3+(-8)$

Solution:

-11

Exercise:

Problem: ([link]) $0-16$

Solution:

-16

Exercise:

Problem: ([link]) $(-14)\cdot(-3)$

Solution:

42

Exercise:

Problem: ([link]) $(-5-6)^2$

Solution:

121

Exercise:

Problem: ([link]) $(-51) \div (-7)$

Solution:

$\frac{51}{7} \text{or} 7\frac{2}{7}$

Exercise:

Problem: ([link]) $\frac{-42}{-7}$

Solution:

6

Exercise:

Problem: ([link]) $\left|\frac{-32}{8} - \frac{-15-5}{5}\right|$

Solution:

0

Objectives
This module contains the learning objectives for the chapter "Algebraic Expressions and Equations" from Fundamentals of Mathematics by Denny Burzynski and Wade Ellis, jr.

After completing this chapter, you should

Algebraic Expressions ([link])

- be able to recognize an algebraic expression
- be able to distinguish between terms and factors
- understand the meaning and function of coefficients
- be able to perform numerical evaluation

Combining Like Terms Using Addition and Subtraction ([link])

- be able to combine like terms in an algebraic expression

Solving Equations of the Form $x + a = b$ and $x - a = b$ ([link])

- understand the meaning and function of an equation
- understand what is meant by the solution to an equation
- be able to solve equations of the form $x + a = b$ and $x - a = b$

Solving Equations of the Form $ax = b$ and $\frac{x}{a} = b$ ([link])

- be familiar with the multiplication/division property of equality
- be able to solve equations of the form $ax = b$ and $\frac{x}{a} = b$
- be able to use combined techniques to solve equations

Applications I: Translating Words to Mathematical Symbols ([link])

- be able to translate phrases and statements to mathematical expressions and equations

Applications II: Solving Problems ([link])

- be more familiar with the five-step method for solving applied problems
- be able to use the five-step method to solve number problems and geometry problems

Algebraic Expressions
This module is from Fundamentals of Mathematics by Denny Burzynski and Wade Ellis, Jr. This module discusses algebraic expressions. By the end of the module students should be able to recognize an algebraic expression, be able to distinguish between terms and factors, understand the meaning and function of coefficients and be able to perform numerical evaluation.

Section Overview

- Algebraic Expressions
- Terms and Factors
- Coefficients
- Numerical Evaluation

Algebraic Expressions

Numerical Expression
In arithmetic, a **numerical expression** results when numbers are connected by arithmetic operation signs (+, -, · , ÷). For example, $8+5$, $4-9$, $3 \cdot 8$, and $9 \div 7$ are numerical expressions.

Algebraic Expression
In algebra, letters are used to represent numbers, and an **algebraic expression** results when an arithmetic operation sign associates a letter with a number or a letter with a letter. For example, $x+8$, $4-y$, $3 \cdot x$, $x \div 7$, and $x \cdot y$ are algebraic expressions.

Expressions
Numerical expressions and algebraic expressions are often referred to simply as **expressions**.

Terms and Factors

In algebra, it is extremely important to be able to distinguish between terms and factors.

Distinction Between Terms and Factors
Terms are parts of *sums* and are therefore connected by + signs.
Factors are parts of *products* and are therefore separated by · signs.

Note: While making the distinction between sums and products, we must remember that subtraction and division are functions of these operations.

1. In some expressions it will appear that terms are separated by minus signs. We must keep in mind that subtraction is addition of the opposite, that is,
   $x-y=x+(-y)$
2. In some expressions it will appear that factors are separated by division signs. We must keep in mind that
   $\frac{x}{y}=\frac{x}{1} \cdot \frac{1}{y}=x \cdot \frac{1}{y}$

Sample Set A

State the number of terms in each expression and name them.

Example:
$x + 4$. In this expression, *x* and 4 are connected by a "+" sign. Therefore, they are terms. This expression consists of two terms.

Example:
$y - 8$. The expression $y - 8$ can be expressed as $y + (-8)$. We can now see that this expression consists of the two terms y and -8.
Rather than rewriting the expression when a subtraction occurs, we can identify terms more quickly by associating the + or - sign with the individual quantity.

Example:
$a + 7 - b - m$. Associating the sign with the individual quantities, we see that this expression consists of the four terms a, 7, $-b$, $-m$.

Example:
$5m - 8n$. This expression consists of the two terms, $5m$ and $-8n$. Notice that the term $5m$ is composed of the two factors 5 and m. The term $-8n$ is composed of the two factors -8 and n.

Example:
$3x$. This expression consists of one term. Notice that $3x$ can be expressed as $3x + 0$ or $3x \cdot 1$ (indicating the connecting signs of arithmetic). Note that no operation sign is necessary for multiplication.

Practice Set A

Specify the terms in each expression.
Exercise:

Problem: $x + 7$

Solution:

x, 7
Exercise:

Problem: $3m - 6n$

Solution:

$3m - 6n$
Exercise:

Problem: $5y$

Solution:

$5y$

Exercise:

Problem: $a + 2b - c$

Solution:

$a, 2b, -c$

Exercise:

Problem: $-3x - 5$

Solution:

$-3x, -5$

Coefficients

We know that multiplication is a description of repeated addition. For example,
$5 \cdot 7$ describes $7 + 7 + 7 + 7 + 7$

Suppose some quantity is represented by the letter x. The multiplication $5\mathrm{x}$ describes $x + x + x + x + x$. It is now easy to see that $5x$ specifies 5 of the quantities represented by x. In the expression $5x$, 5 is called the **numerical coefficient**, or more simply, the **coefficient** of x.

Coefficient
The **coefficient** of a quantity records how many of that quantity there are.

Since constants alone do not record the number of some quantity, they are not usually considered as numerical coefficients. For example, in the expression $7x + 2y - 8z + 12$, the coefficient of

$7x$ is 7. (There are 7 *x*'s.)
$2y$ is 2. (There are 2 *y* 's.)
$-8z$ is -8. (There are -8*z* 's.)

The constant 12 is not considered a numerical coefficient.

$1x = x$
When the numerical coefficient of a variable is 1, we write only the variable and not the coefficient. For example, we write x rather than $1x$. It is clear just by looking at x that there is only one.

Numerical Evaluation

We know that a variable represents an unknown quantity. Therefore, any expression that contains a variable represents an unknown quantity. For example, if the value of x is unknown, then the value of $3x + 5$ is unknown. The value of $3x + 5$ depends on the value of x.

Numerical Evaluation
Numerical evaluation is the process of determining the numerical value of an algebraic expression by replacing the variables in the expression with specified numbers.

Sample Set B

Find the value of each expression.

Example:

$2x + 7y$, if $x = -4$ and $y = 2$

Replace *x* with –4 and *y* with 2.

$$\begin{aligned} 2x + 7y &= 2(-4) + 7(2) \\ &= -8 + 14 \\ &= 6 \end{aligned}$$

Thus, when $x = -4$ and $y = 2$, $2x + 7y = 6$.

Example:

$\frac{5a}{b} + \frac{8b}{12}$, if $a = 6$ and $b = -3$.

Replace a with 6 and b with –3.

$$\begin{aligned} \frac{5a}{b} + \frac{8b}{12} &= \frac{5(6)}{-3} + \frac{8(-3)}{12} \\ &= \text{mfrac} + \text{mfrac} \\ &= -10 + (-2) \\ &= -12 \end{aligned}$$

Thus, when a = 6 and b = –3, $\frac{5a}{b} + \frac{8b}{12} = -12$.

Example:

$6(2a - 15b)$, if $a = -5$ and $b = -1$

Replace a with –5 and b with –1.

$$\begin{aligned} 6(2a - 15b) &= 6(2(-5) - 15(-1)) \\ &= 6(-10 + 15) \\ &= 6(5) \\ &= 30 \end{aligned}$$

Thus, when $a = -5$ and $b = -1$, $6(2a - 15b) = 30$.

Example:

$3x^2 - 2x + 1$, if $x = 4$

Replace *x* with 4.

$$\begin{aligned} 3x^2 - 2x + 1 &= 3(4)^2 - 2\left(4\right) + 1 \\ &= 3 \cdot 16 - 2(4) + 1 \\ &= 48 - 8 + 1 \\ &= 41 \end{aligned}$$

Thus, when $x = 4$, $3x^2 - 2x + 1 = 41$.

Example:

$-x^2 - 4$, if $x = 3$

Replace *x* with 3.

$$\begin{aligned} -x^2 - 4 &= -\frac{3}{2} - 4 \quad \text{Be careful to square only the 3. The exponent 2 is connected } \textit{only} \text{ to 3, not -3} \\ &= -9 - 4 \\ &= -13 \end{aligned}$$

Example:

$(-x)^2 - 4$, if $x = 3$.

Replace *x* with 3.

$$\begin{aligned} (-x)^2 - 4 &= (-3)^2 - 4 \quad \text{The exponent is connected to -3, not 3 as in problem 5 above.} \\ &= 9 - 4 \\ &= -5 \end{aligned}$$

The exponent is connected to –3, not 3 as in the problem above.

Practice Set B

Find the value of each expression.

Exercise:

Problem: $9m - 2n$, if $m = -2$ and $n = 5$

Solution:

-28

Exercise:

Problem: $-3x - 5y + 2z$, if $x = -4$, $y = 3$, $z = 0$

Solution:

-3

Exercise:

Problem: $\frac{10a}{3\text{b}} + \frac{4\text{b}}{2}$, if $a = -6$, and $b = 2$

Solution:

-6

Exercise:

Problem: $8(3m - 5n)$, if $m = -4$ and $n = -5$

Solution:

104

Exercise:

Problem: $3[-40-2(4a-3b)]$, if $a=-6$ and $b=0$

Solution:

24

Exercise:

Problem: $5y^2+6y-11$, if $y=-1$

Solution:

-12

Exercise:

Problem: $-x^2+2x+7$, if $x=4$

Solution:

-1

Exercise:

Problem: $(-x)^2+2x+7$, if $x=4$

Solution:

31

Exercises

Exercise:

Problem: In an algebraic expression, terms are separated by signs and factors are separated by signs.

Solution:

Addition; multiplication

For the following 8 problems, specify each term.

Exercise:

Problem: $3m+7n$

Exercise:

Problem: $5x+18y$

Solution:

$5x, 18y$

Exercise:

Problem: $4a - 6b + c$

Exercise:

Problem: $8s + 2r - 7t$

Solution:

$8s, 2r, -7t$

Exercise:

Problem: $m - 3n - 4a + 7b$

Exercise:

Problem: $7a - 2b - 3c - 4d$

Solution:

$7a, -2b, -3c, -4d$

Exercise:

Problem: $-6a - 5b$

Exercise:

Problem: $-x - y$

Solution:

$-x, -y$

Exercise:

Problem: What is the function of a numerical coefficient?

Exercise:

Problem: Write $1m$ in a simpler way.

Solution:

m

Exercise:

Problem: Write $1s$ in a simpler way.

Exercise:

Problem: In the expression $5a$, how many a's are indicated?

Solution:

5

Exercise:

Problem: In the expression –7*c*, how many *c*’s are indicated?

Find the value of each expression.

Exercise:

Problem: $2m - 6n$, if $m = -3$ and $n = 4$

Solution:

-30

Exercise:

Problem: $5a + 6b$, if $a = -6$ and $b = 5$

Exercise:

Problem: $2x - 3y + 4z$, if $x = 1$, $y = -1$, and $z = -2$

Solution:

-3

Exercise:

Problem: $9a + 6b - 8x + 4y$, if $a = -2$, $b = -1$, $x = -2$, and $y = 0$

Exercise:

Problem: $\frac{8x}{3y} + \frac{18y}{2x}$, if $x = 9$ and $y = -2$

Solution:

-14

Exercise:

Problem: $\frac{-3m}{2n} - \frac{-6n}{m}$, if $m = -6$ and $n = 3$

Exercise:

Problem: $4(3r + 2s)$, if $r = 4$ and $s = 1$

Solution:

56

Exercise:

Problem: $3(9a-6b)$, if $a=-1$ and $b=-2$

Exercise:

Problem: $-8(5m+8n)$, if $m=0$ and $n=-1$

Solution:

64

Exercise:

Problem: $-2(-6x+y-2z)$, if $x=1$, $y=1$, and $z=2$

Exercise:

Problem: $-(10x-2y+5z)$ if $x=2$, $y=8$, and $z=-1$

Solution:

1

Exercise:

Problem: $-(a-3b+2c-d)$, if $a=-5$, $b=2$, $c=0$, and $d=-1$

Exercise:

Problem: $3[16-3(a+3b)]$, if $a=3$ and $b=-2$

Solution:

75

Exercise:

Problem: $-2[5a+2b(b-6)]$, if $a=-2$ and $b=3$

Exercise:

Problem: $-\{6x+3y[-2(x+4y)]\}$, if $x=0$ and $y=1$

Solution:

24

Exercise:

Problem: $-2\{19-6[4-2(a-b-7)]\}$, if $a=10$ and $b=3$

Exercise:

Problem: x^2+3x-1, if $x=5$

Solution:

39

Exercise:

Problem: $m^2 - 2m + 6$, if $m = 3$

Exercise:

Problem: $6a^2 + 2a - 15$, if $a = -2$

Solution:

5

Exercise:

Problem: $5s^2 + 6s + 10$, if $x = -1$

Exercise:

Problem: $16x^2 + 8x - 7$, if $x = 0$

Solution:

-7

Exercise:

Problem: $-8y^2 + 6y + 11$, if $y = 0$

Exercise:

Problem: $(y - 6)^2 + 3(y - 5) + 4$, if $y = 5$

Solution:

5

Exercise:

Problem: $(x + 8)^2 + 4(x + 9) + 1$, if $x = -6$

Exercises for Review

Exercise:

Problem: ([link]) Perform the addition: $5\frac{3}{8} + 2\frac{1}{6}$.

Solution:

$\frac{181}{24} = 7\frac{13}{24}$

Exercise:

Problem: ([link]) Arrange the numbers in order from smallest to largest: $\frac{11}{32}, \frac{15}{48}, \text{and } \frac{7}{16}$

Exercise:

Problem: ([link]) Find the value of $\left(\frac{2}{3}\right)^2 + \frac{8}{27}$

Solution:

$\frac{20}{27}$

Exercise:

Problem: ([link]) Write the proportion in fractional form: “9 is to 8 as *x* is to 7.”

Exercise:

Problem: ([link]) Find the value of $-3(2-6)-12$

Solution:

0

Combining Like Terms Using Addition and Subtraction
This module is from Fundamentals of Mathematics by Denny Burzynski and Wade Ellis, Jr. This module discusses how to combine like terms using addition and subtraction. By the end of the module students should be able to combine like terms in an algebraic expression.

Section Overview

- Combining Like Terms

Combining Like Terms

From our examination of terms in [link], we know that **like terms** are terms in which the variable parts are identical. Like terms is an appropriate name since terms with identical variable parts and different numerical coefficients represent different amounts of the same quantity. When we are dealing with quantities of the same type, we may combine them using addition and subtraction.

Simplifying an Algebraic Expression
An algebraic expression may be **simplified** by combining like terms.

This concept is illustrated in the following examples.

1. $8 \text{ records} + 5 \text{ records} = 13 \text{ records}.$

   Eight and 5 of the same type give 13 of that type. We have combined quantities of the same type.
2. $8 \text{ records} + 5 \text{ records} + 3 \text{ tapes} = 13 \text{ records} + 3 \text{ tapes}.$

   Eight and 5 of the same type give 13 of that type. Thus, we have 13 of one type and 3 of another type. We have combined only quantities of the same type.
3. Suppose we let the letter x represent "record." Then, $8x + 5x = 13x$. The terms $8x$ and $5x$ are like terms. So, 8 and 5 of the same type give 13 of that type. We have combined like terms.

4. Suppose we let the letter x represent "record" and y represent "tape." Then,

   $8x + 5x + 3y = 13x + 5y$

   We have combined only the like terms.

After observing the problems in these examples, we can suggest a method for simplifying an algebraic expression by combining like terms.

Combining Like Terms
Like terms may be combined by adding or subtracting their coefficients and affixing the result to the common variable.

Sample Set A

Simplify each expression by combining like terms.

Example:
$2m + 6m - 4m$. All three terms are alike. Combine their coefficients and affix this result to m: $2 + 6 - 4 = 4$.
Thus, $2m + 6m - 4m = 4m$.

Example:
$5x + 2y - 9y$. The terms $2y$ and $-9y$ are like terms. Combine their coefficients: $2 - 9 = -7$.
Thus, $5x + 2y - 9y = 5x - 7y$.

Example:
$-3a + 2b - 5a + a + 6b$. The like terms are

$-3a, -5a, a\ 2b, 6b$

$\underbrace{-3-5+1=-7}_{-7a} \quad \underbrace{2+6=8}_{8b}$

Thus, $-3a+2b-5a+a+6b=-7a+8b$.

Example:

$r-2s+7s+3r-4r-5s$. The like terms are

$$\underbrace{\underbrace{r, 3r, -4r}_{1+3-4=0}}_{0r} \qquad \underbrace{\underbrace{-2s, 7s, -5s}_{-2+7-5=0}}_{0s}$$

$$0r+0s=0$$

Thus, $r-2s+7s+3r-4r-5s=0$.

Practice Set A

Simplify each expression by combining like terms.

Exercise:

Problem: $4x+3x+6x$

Solution:

$13x$

Exercise:

Problem: $5a+8b+6a-2b$

Solution:

$11a+6b$

Exercise:

Problem: $10m - 6n - 2n - m + n$

Solution:

$9m - 7n$

Exercise:

Problem: $16a + 6m + 2r - 3r - 18a + m - 7m$

Solution:

$-2a - r$

Exercise:

Problem: $5h - 8k + 2h - 7h + 3k + 5k$

Solution:

0

Exercises

Simplify each expression by combining like terms.
Exercise:

Problem: $4a + 7a$

Solution:

$11a$

Exercise:

Problem: $3m + 5m$

Exercise:

Problem: $6h - 2h$

Solution:

$4h$

Exercise:

Problem: $11k - 8k$

Exercise:

Problem: $5m + 3n - 2m$

Solution:

$3m + 3n$

Exercise:

Problem: $7x - 6x + 3y$

Exercise:

Problem: $14s + 3s - 8r + 7r$

Solution:

$17s - r$

Exercise:

Problem: $-5m - 3n + 2m + 6n$

Exercise:

Problem: $7h + 3a - 10k + 6a - 2h - 5k - 3k$

Solution:

$5h + 9a - 18k$

Exercise:

Problem: $4x - 8y - 3z + x - y - z - 3y - 2z$

Exercise:

Problem: $11w + 3x - 6w - 5w + 8x - 11x$

Solution:

0

Exercise:

Problem: $15r - 6s + 2r + 8s - 6r - 7s - s - 2r$

Exercise:

Problem: $\mid -7 \mid m + \mid 6 \mid m + \mid -3 \mid m$

Solution:

$16m$

Exercise:

Problem: $\mid -2 \mid x + \mid -8 \mid x + \mid 10 \mid x$

Exercise:

Problem: $(-4 + 1)k + (6 - 3)k + (12 - 4)h + (5 + 2)k$

Solution:

$8h + 7k$

Exercise:

Problem: $(-5+3)a - (2+5)b - (3+8)b$

Exercise:

Problem: $5\star + 2\mathit{\Delta} + 3\mathit{\Delta} - 8\star$

Solution:

$5\mathit{\Delta} - 3\star$

Exercise:

Problem: $9\boxtimes + 10\boxplus - 11\boxtimes - 12\boxplus$

Exercise:

Problem: $16x - 12y + 5x + 7 - 5x - 16 - 3y$

Solution:

$16x - 15y - 9$

Exercise:

Problem: $-3y + 4z - 11 - 3z - 2y + 5 - 4(8-3)$

Exercises for Review

Exercise:

Problem: ([link]) Convert $\frac{24}{11}$ to a mixed number

Solution:

$2\frac{2}{11}$

Exercise:

Problem: ([link]) Determine the missing numerator: $\frac{3}{8} = \frac{?}{64}$.

Exercise:

Problem: ([link]) Simplify $\frac{\frac{5}{6} - \frac{1}{4}}{\frac{1}{12}}$.

Solution:

7

Exercise:

Problem: ([link]) Convert $\frac{5}{16}$ to a percent.

Exercise:

Problem: ([link]) In the expression $6k$, how many k's are there?

Solution:

6

Solving Equations of the Form x+a=b and x-a=b
This module is from Fundamentals of Mathematics by Denny Burzynski and Wade Ellis, Jr. This module discusses how to solve equations of the form $x+a=b$ and $x-a=b$. By the end of the module students should understand the meaning and function of an equation, understand what is meant by the solution to an equation and be able to solve equations of the form $x+a=b$ and $x-a=b$.

Section Overview

- Equations
- Solutions and Equivalent Equations
- Solving Equations

Equations

Equation
An equation is a statement that two algebraic expressions are equal.

The following are examples of equations:

$$\begin{array}{ccccccccc} x+6 & = & 10 & x-4 & = & -11 & 3y-5 & = & -2+2y \\ \text{This expression} & = & \text{This expression} & \text{This expression} & = & \text{This expression} & \text{This expression} & = & \text{This expression} \end{array}$$

Notice that $x+6$, $x-4$, and $3y-5$ are *not* equations. They are expressions. They are not equations because there is no statement that each of these expressions is equal to another expression.

Solutions and Equivalent Equations

Conditional Equations
The truth of some equations is conditional upon the value chosen for the variable. Such equations are called **conditional equations**. There are two additional types of equations. They are examined in courses in algebra, so we will not consider them now.

Solutions and Solving an Equation
The set of values that, when substituted for the variables, make the equation true, are called the **solutions** of the equation.
An equation has been **solved** when all its solutions have been found.

Sample Set A

Example:
Verify that 3 is a solution to $x+7=10$.
When $x=3$,

$$\begin{array}{rrcll} & x+7 & = & 10 & \\ \text{becomes} & 3+7 & = & 10 & \\ & 10 & = & 10 & \text{which is a } \mathit{true} \text{ statement, verifying that} \\ & & & & 3 \text{ is a solution to } x+7=10 \end{array}$$

Example:
Verify that -6 is a solution to $5y+8=-22$
When $y=-6$,

$$\begin{array}{rrcll} & 5y+8 & = & -22 & \\ \text{becomes} & 5(-6)+8 & = & -22 & \\ & -30+8 & = & -22 & \\ & -22 & = & -22 & \text{which is a } \mathit{true} \text{ statement, verifying that} \\ & & & & -6 \text{ is a solution to} 5y+8=-22 \end{array}$$

Example:
Verify that 5 is not a solution to $a-1=2a+3$.
When $a=5$,

$$\begin{array}{rrcll} & a-1 & = & 2a+3 & \\ \text{becomes} & 5-1 & = & 2\cdot 5+3 & \\ & 5-1 & = & 10+3 & \\ & 4 & = & 13 & \text{a } \mathit{false} \text{ statement, verifying that } 5 \\ & & & & \text{is not a solution to } a-1=2a+3 \end{array}$$

Example:
Verify that -2 is a solution to $3m-2=-4m-16$.
When $m=-2$,

$$\begin{array}{rrcll} & 3m-2 & = & -4m-16 & \\ \text{becomes} & 3(-2)-2 & = & -4(-2)-16 & \\ & -6-2 & = & 8-16 & \\ & -8 & = & -8 & \text{which is a } \mathit{true} \text{ statement, verifying that} \\ & & & & -2 \text{ is a solution to } 3m-2=-4m-16 \end{array}$$

Practice Set A

Exercise:

Problem: Verify that 5 is a solution to $m + 6 = 11$.

Solution:

Substitute 5 into $m + 6 = 11$.

$$\begin{aligned} 5 + 6 &\stackrel{?}{=} 11 \\ 11 &\stackrel{\checkmark}{=} 11 \end{aligned}$$

Thus, 5 is a solution.

Exercise:

Problem: Verify that -5 is a solution to $2m - 4 = -14$.

Solution:

Substitute -5 into $2m - 4 = -14$.

$$\begin{aligned} 2(-5) - 4 &\stackrel{?}{=} -14 \\ -10 - 4 &\stackrel{?}{=} -14 \\ -14 &\stackrel{\checkmark}{=} -14 \end{aligned}$$

Thus, -5 is a solution.

Exercise:

Problem: Verify that 0 is a solution to $5x + 1 = 1$.

Solution:

Substitute 0 into $5x + 1 = 1$.

$$\begin{aligned} 5(0) + 1 &\stackrel{?}{=} 1 \\ 0 + 1 &\stackrel{?}{=} 1 \\ 1 &\stackrel{\checkmark}{=} 1 \end{aligned}$$

Thus, 0 is a solution.

Exercise:

Problem: Verify that 3 is not a solution to $-3y + 1 = 4y + 5$.

Solution:

Substitute 3 into $-3y+1=4y+5$.

$$\begin{aligned} -3(3)+1 &\stackrel{?}{=} 4(3)+5 \\ -9+1 &\stackrel{?}{=} 12+5 \\ -8 &\neq 17 \end{aligned}$$

Thus, 3 is not a solution.

Exercise:

Problem: Verify that -1 is a solution to $6m-5+2m=7m-6$.

Solution:

Substitute -1 into $6m-5+2m=7m-6$.

$$\begin{aligned} 6(-1)-5+2(-1) &\stackrel{?}{=} 7(-1)-6 \\ -6-5-2 &\stackrel{?}{=} -7-6 \\ -13 &\stackrel{\checkmark}{=} -13 \end{aligned}$$

Thus, -1 is a solution.

Equivalent Equations

Some equations have precisely the same collection of solutions. Such equations are called equivalent equations. For example, $x-5=-1$, $x+7=11$, and $x=4$ are all equivalent equations since the only solution to each is $x=4$. (Can you verify this?)

Solving Equations

We know that the equal sign of an equation indicates that the number represented by the expression on the left side is the same as the number represented by the expression on the right side.

This number	is the same as	this number
↓	↓	↓
x	=	4

$x + 7$	=	11
$x - 5$	=	-1

Addition/Subtraction Property of Equality
From this, we can suggest the **addition/subtraction property of equality**.
Given any equation,

1. We can obtain an equivalent equation by *adding* the *same* number to *both* sides of the equation.
2. We can obtain an equivalent equation by *subtracting* the *same* number from *both* sides of the equation.

The Idea Behind Equation Solving
The idea behind **equation solving** is to isolate the variable on one side of the equation. Signs of operation (+, -, ·,÷) are used to associate two numbers. For example, in the expression $5 + 3$, the numbers 5 and 3 are associated by addition. An association can be *undone* by performing the opposite operation. The addition/subtraction property of equality can be used to undo an association that is made by addition or subtraction.

Subtraction is used to undo an addition.

Addition is used to undo a subtraction.

The procedure is illustrated in the problems of [link].

Sample Set B

Use the addition/subtraction property of equality to solve each equation.

Example:
$x + 4 = 6.$
4 is associated with x by addition. Undo the association by *subtracting* 4 from *both* sides.
$x + 4 - 4 = 6 - 4$

$x + 0 = 2$

$x = 2$
Check: When $x = 2$, $x + 4$ becomes

$$\begin{aligned} 2 + 4 &\stackrel{?}{=} 6 \\ 6 &\stackrel{\checkmark}{=} 6. \end{aligned}$$

The solution to $x + 4 = 6$ is $x = 2$.

Example:
$m - 8 = 5$. 8 is associated with m by subtraction. Undo the association by *adding* 8 to *both* sides.
$m - 8 + 8 = 5 + 8$

$m + 0 = 13$

$m = 13$
Check: When $m = 13$,
becomes

$$\begin{aligned} m - 8 &= 5 \\ 13 - 8 &\stackrel{?}{=} 5 \\ 5 &\stackrel{\checkmark}{=} 5 \end{aligned}$$

a true statement.
The solution to $m - 8 = 5$ is $m = 13$.

Example:
$-3 - 5 = y - 2 + 8$. Before we use the addition/subtraction property, we should simplify as much as possible.
$-3 - 5 = y - 2 + 8$
$-8 = y + 6$
6 is associated with y by addition. Undo the association by *subtracting* 6 from *both* sides.
$-8 - 6 = y + 6 - 6$

$-14 = y + 0$

$-14 = y$
This is equivalent to $y = -14$.
Check: When $y = -14$,
$-3 - 5 = y - 2 + 8$
becomes

$$\begin{aligned} -3 - 5 &\stackrel{?}{=} -14 - 2 + 8 \\ -8 &\stackrel{?}{=} -16 + 8 \\ -8 &\stackrel{\checkmark}{=} -8. \end{aligned}$$

,
a true statement.
The solution to $-3 - 5 = y - 2 + 8$ is $y = -14$.

Example:
$-5\text{a} + 1 + 6\text{a} = -2$. Begin by simplifying the left side of the equation.
$-5a + 1 + 6a = -2$

$-5+6=1$

$a + 1 = -2$ 1 is associated with a by addition. Undo the association by *subtracting* 1 from *both* sides.
$a + 1 - 1 = -2 - 1$

$a + 0 = -3$

$a = -3$
Check: When $a = -3$,
$-5\text{a} + 1 + 6\text{a} = -2$
becomes

$$\begin{aligned} -5(-3) + 1 + 6(-3) &\stackrel{?}{=} -2 \\ 15 + 1 - 18 &\stackrel{?}{=} -2 \\ -2 &\stackrel{\checkmark}{=} -2 \end{aligned}$$

,
a true statement.
The solution to $-5a + 1 + 6a = -2$ is $a = -3$.

Example:
$7k - 4 = 6k + 1$. In this equation, the variable appears on both sides. We need to isolate it on one side. Although we can choose either side, it will be more convenient to choose the side with the larger coefficient. Since 8 is greater than 6, we'll isolate k on the left side.
$7k - 4 = 6k + 1$ Since $6k$ represents $+6k$, subtract $6k$ from each side.
$7k - 4 - 6k = 6k + 1 - 6k$

$7-6=1$ $6-6=0$

$k - 4 = 1$ 4 is associated with k by subtraction. Undo the association by *adding* 4 to *both* sides.
$k - 4 + 4 = 1 + 4$

$k = 5$
Check: When $k = 5$,
$7k - 4 = 6k + 1$
becomes

$$\begin{aligned} 7 \cdot 5 - 4 &\stackrel{?}{=} 6 \cdot 5 + 1 \\ 35 - 4 &\stackrel{?}{=} 30 + 1 \\ 31 &\stackrel{\checkmark}{=} 31. \end{aligned}$$

a true statement.
The solution to $7k-4=6k+1$ is $k=5$.

Example:
$-8+x=5$. -8 is associated with x by addition. Undo the by *subtracting* -8 from *both* sides. Subtracting -8 we get $-(-8)$=+8. We actually *add* 8 to both sides.
$-8+x+8=5+8$
$x=13$
Check: When $x=13$
$-8+x=5$
becomes

$$\begin{aligned} -8+13 &\stackrel{?}{=} 5 \\ 5 &\stackrel{\checkmark}{=} 5 \end{aligned}$$

,
a true statement.
The solution to $-8+x=5$ is $x=13$.

Practice Set B

Exercise:

Problem: $y+9=4$

Solution:

$y=-5$

Exercise:

Problem: $a-4=11$

Solution:

$a=15$

Exercise:

Problem: $-1+7=x+3$

Solution:

$x = 3$

Exercise:

Problem: $8m + 4 - 7m = (-2)(-3)$

Solution:

$m = 2$

Exercise:

Problem: $12k - 4 = 9k - 6 + 2k$

Solution:

$k = -2$

Exercise:

Problem: $-3 + a = -4$

Solution:

$a = -1$

Exercises

For the following 10 problems, verify that each given value is a solution to the given equation.

Exercise:

Problem: $x - 11 = 5$, $x = 16$

Solution:

Substitute $x = 4$ into the equation $4x - 11 = 5$.
$16 - 11 = 5$

$5 = 5$

$x = 4$ is a solution.

Exercise:

Problem: $y - 4 = -6, y = -2$

Exercise:

Problem: $2m - 1 = 1, m = 1$

Solution:

Substitute $m = 1$ into the equation $2m - 1 = 1$.

$$\begin{aligned} 2 - 1 &\stackrel{?}{=} 1 \\ 1 &\stackrel{\checkmark}{=} 1 \end{aligned}$$

$m = 1$ is a solution.

Exercise:

Problem: $5y + 6 = -14, y = -4$

Exercise:

Problem: $3\text{x} + 2 - 7\text{x} = -5\text{x} - 6, x = -8$

Solution:

Substitute $x = -8$ into the equation $3\text{x} + 2 - 7 = -5\text{x} - 6$.

$$\begin{array}{l} -24 + 2 - 7 \stackrel{?}{=} 40 - 6 \\ 34 \stackrel{\checkmark}{=} 34 \end{array}$$

$x = -8$ is a solution.

Exercise:

Problem: $-6a + 3 + 3a = 4a + 7 - 3a, a = -1$

Exercise:

Problem: $-8 + x = -8, x = 0$

Solution:

Substitute $x = 0$ into the equation $-8 + x = -8$.

$$-8 + 0 \stackrel{?}{=} -8$$
$$-8 \stackrel{\checkmark}{=} -8$$

$x = 0$ is a solution.

Exercise:

Problem: $8b + 6 = 6 - 5b$, $b = 0$

Exercise:

Problem: $4x - 5 = 6x - 20$, $x = \frac{15}{2}$

Solution:

Substitute $x = \frac{15}{2}$ into the equation $4x - 5 = 6x - 20$.

$$30 - 5 \stackrel{?}{=} 45 - 20$$
$$25 \stackrel{\checkmark}{=} 25$$

$x = \frac{15}{2}$ is a solution.

Exercise:

Problem: $-3y + 7 = 2y - 15$, $y = \frac{22}{5}$

Solve each equation. Be sure to check each result.

Exercise:

Problem: $y - 6 = 5$

Solution:

$y = 11$

Exercise:

Problem: $m + 8 = 4$

Exercise:

Problem: $k - 1 = 4$

Solution:

$k = 5$

Exercise:

Problem: $h - 9 = 1$

Exercise:

Problem: $a + 5 = -4$

Solution:

$a = -9$

Exercise:

Problem: $b - 7 = -1$

Exercise:

Problem: $x + 4 - 9 = 6$

Solution:

$x = 11$

Exercise:

Problem: $y - 8 + 10 = 2$

Exercise:

Problem: $z + 6 = 6$

Solution:

$z = 0$

Exercise:

Problem: $w - 4 = -4$

Exercise:

Problem: $x+7-9=6$

Solution:

$x=8$

Exercise:

Problem: $y-2+5=4$

Exercise:

Problem: $m+3-8=-6+2$

Solution:

$m=1$

Exercise:

Problem: $z+10-8=-8+10$

Exercise:

Problem: $2+9=k-8$

Solution:

$k=19$

Exercise:

Problem: $-5+3=h-4$

Exercise:

Problem: $3m-4=2m+6$

Solution:

$m=10$

Exercise:

Problem: $5a + 6 = 4a - 8$

Exercise:

Problem: $8b + 6 + 2b = 3b - 7 + 6b - 8$

Solution:

$b = -21$

Exercise:

Problem: $12h - 1 - 3 - 5h = 2h + 5h + 3(-4)$

Exercise:

Problem: $-4a + 5 - 2a = -3a - 11 - 2a$

Solution:

$a = 16$

Exercise:

Problem: $-9n - 2 - 6 + 5n = 3n - (2)(-5) - 6n$

Calculator Exercises

Exercise:

Problem: $y - 2.161 = 5.063$

Solution:

$y = 7.224$

Exercise:

Problem: $a - 44.0014 = -21.1625$

Exercise:

Problem: $-0.362 - 0.416 = 5.63m - 4.63m$

Solution:

$m = -0.778$

Exercise:

Problem: $8.078 - 9.112 = 2.106y - 1.106y$

Exercise:

Problem: $4.23k + 3.18 = 3.23k - 5.83$

Solution:

$k = -9.01$

Exercise:

Problem: $6.1185x - 4.0031 = 5.1185x - 0.0058$

Exercise:

Problem: $21.63y + 12.40 - 5.09y = 6.11y - 15.66 + 9.43y$

Solution:

$y = -28.06$

Exercise:

Problem: $0.029a - 0.013 - 0.034 - 0.057 = -0.038 + 0.56 + 1.01a$

Exercises for Review

Exercise:

Problem: ([link]) Is $\frac{\text{7calculators}}{\text{12students}}$ an example of a ratio or a rate?

Solution:

rate

Exercise:

Problem: ([link]) Convert $\frac{3}{8}\%$ to a decimal.

Exercise:

Problem: ([link]) 0.4% of what number is 0.014?

Solution:

3.5

Exercise:

Problem:

([link]) Use the clustering method to estimate the sum: $89+93+206+198+91$

Exercise:

Problem: ([link]) Combine like terms: $4x+8y+12y+9x-2y$.

Solution:

$13x+18y$

Solving Equations of the Form ax=b and x/a=b
This module is from Fundamentals of Mathematics by Denny Burzynski and Wade Ellis, Jr. This module discusses solving equations of the form $ax = b$ and $\frac{x}{a} = b$. By the end of the module students should be familiar with the multiplication/division property of equality, be able to solve equations of the form $\mathrm{ax} = b$ and $\frac{x}{a} = b$ and be able to use combined techniques to solve equations.

Section Overview

- Multiplication/ Division Property of Equality
- Combining Techniques in Equations Solving

Multiplication/ Division Property of Equality

Recall that the equal sign of an equation indicates that the number represented by the expression on the left side is the same as the number represented by the expression on the right side. From this, we can suggest the multiplication/division property of equality.

Multiplication/Division Property of Equality
Given any equation,

1. We can obtain an equivalent equation by *multiplying both sides* of the equation by the *same nonzero* number, that is, if $c \neq 0$, then $a = b$ is equivalent to
   $a \cdot c = b \cdot c$
2. We can obtain an equivalent equation by *dividing both sides* of the equation by the *same nonzero number*, that is, if $c \neq 0$, then $a = b$ is equivalent to
   $\frac{a}{c} = \frac{b}{c}$

The multiplication/division property of equality can be used to undo an association with a number that multiplies or divides the variable.

Sample Set A

Use the multiplication / division property of equality to solve each equation.

Example:
$6y = 54$
6 is associated with y by multiplication. Undo the association by *dividing both* sides by 6
$\frac{6y}{6} = \frac{54}{6}$

$\frac{\cancel{6}\,y}{\cancel{6}} = \frac{\overset{9}{\cancel{54}}}{\cancel{6}}$

$y = 9$
Check: When $y = 9$
$6y = 54$
becomes

$$6 \cdot 9 \stackrel{?}{=} 54$$
$$54 \stackrel{\checkmark}{=} 54$$

,
a true statement.
The solution to $6y = 54$ is $y = 9$.

Example:
$\frac{x}{-2} = 27.$
-2 is associated with x by division. Undo the association by *multiplying both* sides by -2.
$(-2)\frac{x}{-2} = (-2)27$

$$\left(\cancel{-2}\right)\frac{x}{\cancel{-2}} = (-2)27$$

$x = -54$

Check: When $x = -54$,

$$\frac{x}{-2} = 27$$

becomes

$$\frac{-54}{-2} \stackrel{?}{=} 27$$

$$27 \stackrel{\checkmark}{=} 27$$

a true statement.

The solution to $\frac{x}{-2} = 27$ is $x = -54$

Example:

$$\frac{3a}{7} = 6.$$

We will examine two methods for solving equations such as this one.

Method 1: Use of dividing out common factors.

$$\frac{3a}{7} = 6$$

7 is associated with a by division. Undo the association by *multiplying both* sides by 7.

$$7 \cdot \frac{3a}{7} = 7 \cdot 6$$

Divide out the 7's.

$$\cancel{7} \cdot \frac{3a}{\cancel{7}} = 42$$

$3a = 42$

3 is associated with a by multiplication. Undo the association by *dviding both* sides by 3.

$$\frac{3a}{3} = \frac{42}{3}$$

$$\frac{\cancel{3}\,a}{\cancel{3}} = 14$$

$a = 14$

Check: When $a = 14$,

$$\frac{3a}{7} = 6$$

becomes

$$\frac{3 \cdot 14}{7} \stackrel{?}{=} 6$$

$$\frac{42}{7} = 6$$

$$6 \stackrel{\checkmark}{=} 6$$

,

a true statement.

The solution to $\frac{3a}{7} = 6$ is $a = 14$.

Method 2: Use of reciprocals

Recall that if the product of two numbers is 1, the numbers are **reciprocals**.

Thus $\frac{3}{7}$ and $\frac{7}{3}$ are reciprocals.

$$\frac{3a}{7} = 6$$

Multiply *both* sides of the equation by $\frac{7}{3}$, the reciprocal of $\frac{3}{7}$.

$$\frac{7}{3} \cdot \frac{3a}{7} = \frac{7}{3} \cdot 6$$

$$\frac{\overset{1}{\cancel{7}}}{\underset{1}{\cancel{3}}} \cdot \frac{\overset{1}{\cancel{3}}\,a}{\underset{1}{\cancel{7}}} = \frac{7}{\underset{1}{\cancel{3}}} \cdot \frac{\overset{2}{\cancel{6}}}{1}$$

$1 \cdot a = 14$

$a = 14$

Notice that we get the same solution using either method.

Example:

$-8x = 24$

-8 is associated with x by multiplication. Undo the association by *dividing both* sides by -8.

$\frac{-8x}{-8} = \frac{24}{-8}$

$$\frac{-8x}{-8} = \frac{24}{-8}$$

$x = -3$

Check: When $x = -3$,

$-8x = 24$

becomes

$$\begin{aligned} -8(-3) &\stackrel{?}{=} 24 \\ 24 &\stackrel{\checkmark}{=} 24 \end{aligned}$$

,

a true statement.

Example:

$-x = 7.$

Since $-x$ is actually $-1 \cdot x$ and $(-1)(-1) = 1$, we can isolate x by multiplying *both* sides of the equation by -1.

$(-1)(-x) = -1 \cdot 7$

$x = -7$

Check: When $x = 7$,

$-x = 7$

becomes

$$\begin{aligned} -(-7) &\stackrel{?}{=} 7 \\ 7 &\stackrel{\checkmark}{=} 7 \end{aligned}$$

The solution to $-x = 7$ is $x = -7$.

Practice Set A

Use the multiplication/division property of equality to solve each equation. Be sure to check each solution.

Exercise:

Problem: $7x = 21$

Solution:

$x = 3$

Exercise:

Problem: $-5x = 65$

Solution:

$x = -13$

Exercise:

Problem: $\frac{x}{4} = -8$

Solution:

$x = -32$

Exercise:

Problem: $\frac{3x}{8} = 6$

Solution:

$x = 16$

Exercise:

Problem: $-y = 3$

Solution:

$y = -3$

Exercise:

Problem: $-k = -2$

Solution:

$k = 2$

Combining Techniques in Equation Solving

Having examined solving equations using the addition/subtraction and the multiplication/division principles of equality, we can combine these techniques to solve more complicated equations.

When beginning to solve an equation such as $6x - 4 = -16$, it is helpful to know which property of equality to use first, addition/subtraction or multiplication/division. Recalling that in equation solving *we are trying to isolate the variable* (disassociate numbers from it), it is helpful to note the following.

To *associate* numbers and letters, we use the order of operations.

1. Multiply/divide
2. Add/subtract

To *undo an association* between numbers and letters, we use the order of operations in reverse.

1. Add/subtract
2. Multiply/divide

Sample Set B

Solve each equation. (In these example problems, we will not show the checks.)

Example:
$6x - 4 = -16$
-4 is associated with x by subtraction. Undo the association by *adding* 4 to *both* sides.
$6x - 4 + 4 = -16 + 4$
$6x = -12$
6 is associated with x by multiplication. Undo the association by *dividing both* sides by 6
$\frac{6x}{6} = \frac{-12}{6}$
$x = -2$

Example:
$-8k + 3 = -45$.
3 is associated with k by addition. Undo the association by *subtracting* 3 from *both* sides.
$-8k + 3 - 3 = -45 - 3$
$-8k = -48$
-8 is associated with k by multiplication. Undo the association by *dividing both* sides by -8.
$\frac{-8k}{-8} = \frac{-48}{-8}$
$k = 6$

Example:
$5m - 6 - 4m = 4m - 8 + 3m$. Begin by solving this equation by combining like terms.

$m-6=7m-8$ Choose a side on which to isolate *m*. Since 7 is greater than 1, we'll isolate *m* on the right side.
Subtract *m* from *both* sides.
$-m-6-m=7m-8-m$
$-6=6m-8$
8 is associated with *m* by subtraction. Undo the association by *adding* 8 to *both* sides.
$-6+8=6m-8+8$
$2=6m$
6 is associated with m by multiplication. Undo the association by *dividing* *both* sides by 6.
$\frac{2}{6}=\frac{6m}{6}$ Reduce.
$\frac{1}{3}=m$
Notice that if we had chosen to isolate m on the left side of the equation rather than the right side, we would have proceeded as follows:
$m-6=7m-8$
Subtract $7m$ from *both* sides.
$m-6-7m=7m-8-7m$
$-6m-6=-8$
Add 6 to *both* sides,
$-6m-6+6=-8+6$
$-6m=-2$
Divide *both* sides by -6.
$\frac{-6m}{-6}=\frac{-2}{-6}$
$m=\frac{1}{3}$
This is the same result as with the previous approach.

Example:
$\frac{8x}{7}=-2$
7 is associated with x by division. Undo the association by *multiplying*

both sides by 7.

$$\cancel{7} \cdot \frac{8x}{\cancel{7}} = 7(-2)$$

$$7 \cdot \frac{8x}{7} = -14$$

$8x = -14$

8 is associated with x by multiplication. Undo the association by *dividing both* sides by 8.

$$\frac{\cancel{8}x}{\cancel{8}} = \frac{-7}{4}$$

$$x = \frac{-7}{4}$$

Practice Set B

Solve each equation. Be sure to check each solution.

Exercise:

Problem: $5m + 7 = -13$

Solution:

$m = -4$

Exercise:

Problem: $-3a - 6 = 9$

Solution:

$a = -5$

Exercise:

Problem: $2a + 10 - 3a = 9$

Solution:

$a = 1$

Exercise:

Problem: $11x - 4 - 13x = 4x + 14$

Solution:

$x = -3$

Exercise:

Problem: $-3m + 8 = -5m + 1$

Solution:

$m = -\frac{7}{2}$

Exercise:

Problem: $5y + 8y - 11 = -11$

Solution:

$y = 0$

Exercises

Solve each equation. Be sure to check each result.

Exercise:

Problem: $7x = 42$

Solution:

$x=6$

Exercise:

Problem: $8x=81$

Exercise:

Problem: $10x=120$

Solution:

$x=12$

Exercise:

Problem: $11x=121$

Exercise:

Problem: $-6a=48$

Solution:

$a=-8$

Exercise:

Problem: $-9y=54$

Exercise:

Problem: $-3y=-42$

Solution:

$y=14$

Exercise:

Problem: $-5a = -105$

Exercise:

Problem: $2m = -62$

Solution:

$m = -31$

Exercise:

Problem: $3m = -54$

Exercise:

Problem: $\frac{x}{4} = 7$

Solution:

$x = 28$

Exercise:

Problem: $\frac{y}{3} = 11$

Exercise:

Problem: $\frac{-z}{6} = -14$

Solution:

$z = 84$

Exercise:

Problem: $\frac{-w}{5}=1$

Exercise:

Problem: $3m-1=-13$

Solution:

$m=-4$

Exercise:

Problem: $4x+7=-17$

Exercise:

Problem: $2+9x=-7$

Solution:

$x=-1$

Exercise:

Problem: $5-11x=27$

Exercise:

Problem: $32=4y+6$

Solution:

$y=\frac{13}{2}$

Exercise:

Problem: $-5 + 4 = -8m + 1$

Exercise:

Problem: $3k + 6 = 5k + 10$

Solution:

$k = -2$

Exercise:

Problem: $4a + 16 = 6a + 8a + 6$

Exercise:

Problem: $6x + 5 + 2x - 1 = 9x - 3x + 15$

Solution:

$x = \frac{11}{2}$ or $5\frac{1}{2}$

Exercise:

Problem: $-9y - 8 + 3y + 7 = -7y + 8y - 5y + 9$

Exercise:

Problem: $-3a = a + 5$

Solution:

$a = -\frac{5}{4}$

Exercise:

Problem: $5b = -2b + 8b + 1$

Exercise:

Problem: $-3m + 2 - 8m - 4 = -14m + m - 4$

Solution:

$m = -1$

Exercise:

Problem: $5a + 3 = 3$

Exercise:

Problem: $7x + 3x = 0$

Solution:

$x = 0$

Exercise:

Problem: $7g + 4 - 11g = -4g + 1 + g$

Exercise:

Problem: $\frac{5a}{7} = 10$

Solution:

$a = 14$

Exercise:

Problem: $\frac{2m}{9} = 4$

Exercise:

Problem: $\frac{3x}{4} = \frac{9}{2}$

Solution:

$x = 6$

Exercise:

Problem: $\frac{8k}{3} = 32$

Exercise:

Problem: $\frac{3a}{8} - \frac{3}{2} = 0$

Solution:

$a = 4$

Exercise:

Problem: $\frac{5m}{6} - \frac{25}{3} = 0$

Exercises for Review

Exercise:

Problem: ([link]) Use the distributive property to compute $40 \cdot 28$.

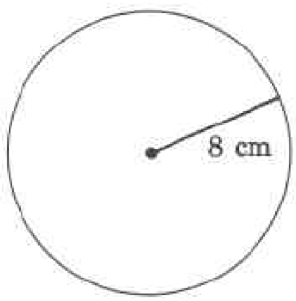
Solution:

$40(30 - 2) = 1200 - 80 = 1120$

Exercise:

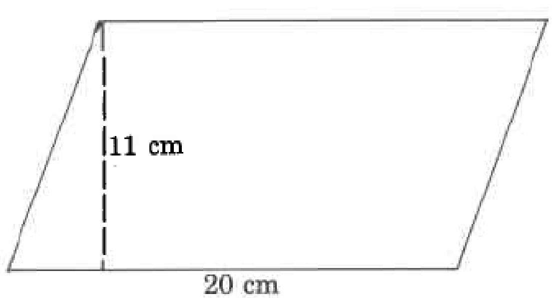
Problem:

([link]) Approximating π by 3.14, find the approximate circumference of the circle.



Exercise:

Problem: ([link]) Find the area of the parallelogram.



Solution:

220 sq cm

Exercise:

Problem: ([link]) Find the value of $\frac{-3(4-15)-2}{-5}$.

Exercise:

Problem: ([link]) Solve the equation $x - 14 + 8 = -2$.

Solution:

$x = 4$

Applications I: Translating Words to Mathematical Symbols
This module is from Fundamentals of Mathematics by Denny Burzynski and Wade Ellis, Jr. This module discusses how to translate word to mathematical symbols. By the end of the module students should be able to translate phrases and statements to mathematical expressions and equations.

Section Overview

- Translating Words to Symbols

Translating Words to Symbols

Practical problems seldom, if ever, come in equation form. The job of the problem solver is to translate the problem from phrases and statements into mathematical expressions and equations, and then to solve the equations.

As problem solvers, our job is made simpler if we are able to translate verbal phrases to mathematical expressions and if we follow the five-step method of solving applied problems. To help us translate from words to symbols, we can use the following Mathematics Dictionary.

MATHEMATICS DICTIONARY	
Word or Phrase	Mathematical Operation
Sum, sum of, added to, increased by, more than, and, plus	+
Difference, minus, subtracted from, decreased by, less, less than	-

Product, the product of, of, multiplied by, times, per	$\cdot$
Quotient, divided by, ratio, per	$\div$
Equals, is equal to, is, the result is, becomes	$=$
A number, an unknown quantity, an unknown, a quantity	x (or any symbol)

Sample Set A

Translate each phrase or sentence into a mathematical expression or equation.

Example:
$\underbrace{\text{Nine}}_{9}\underbrace{\text{more than}}_{+}\underbrace{\text{some number}}_{x}.$
Translation: $9 + x$.

Example:
$\underbrace{\text{Eighteen}}_{18}\underbrace{\text{minus}}_{-}\underbrace{\text{a number}}_{x}.$
Translation: $18 - x$.

Example:
$\underbrace{\text{A quantity}}_{y}\underbrace{\text{less}}_{-}\underbrace{\text{five}}_{5}.$
Translation: $y - 5$.

Example:
$\underbrace{\text{Four}}_{4}\underbrace{\text{times}}_{\cdot}\underbrace{\text{a number}}_{x}\ \underbrace{\text{is}}_{=}\ \underbrace{\text{sixteen}}_{16}.$
Translation: $4x = 16$.

Example:
$\underbrace{\text{One fifth}}_{\frac{1}{5}}\ \underbrace{\text{of}}_{\cdot}\ \underbrace{\text{a number}}_{n}\ \underbrace{\text{is}}_{=}\ \underbrace{\text{thirty}}_{30}.$
Translation: $\frac{1}{5}n = 30$, or $\frac{n}{5} = 30$.

Example:
$\underbrace{\text{Five}}_{5}\ \underbrace{\text{times}}_{\cdot}\ \underbrace{\text{a number}}_{x}\ \underbrace{\text{is}}_{=}\ \underbrace{\text{two}}_{2}\ \underbrace{\text{more than}}_{+}\ \underbrace{\text{twice}}_{2\cdot}\ \underbrace{\text{the number}}_{x}.$
Translation: $5x = 2 + 2x$.

Practice Set A

Translate each phrase or sentence into a mathematical expression or equation.

Exercise:

Problem: Twelve more than a number.

Solution:

$12 + x$

Exercise:

Problem: Eight minus a number.

Solution:

$8 - x$

Exercise:

Problem: An unknown quantity less fourteen.

Solution:

$x - 14$

Exercise:

Problem: Six times a number is fifty-four.

Solution:

$6x = 54$

Exercise:

Problem: Two ninths of a number is eleven.

Solution:

$\frac{2}{9}x = 11$

Exercise:

Problem:

Three more than seven times a number is nine more than five times the number.

Solution:

$3 + 7x = 9 + 5x$

Exercise:

Problem:

Twice a number less eight is equal to one more than three times the number.

Solution:

$2x - 8 = 3x + 1$ or $2x - 8 = 1 + 3x$

Sample Set B

Example:
Sometimes the structure of the sentence indicates the use of grouping symbols. We’ll be alert for *commas*. They set off terms.

$\underbrace{\text{A number}}_{(x}\ \underbrace{\text{divided by}}_{\div}\ \underbrace{\text{four,}}_{4)}\ \underbrace{\text{minus}}_{-}\ \underbrace{\text{six,}}_{6}\ \underbrace{\text{is}}_{=}\ \underbrace{\text{twelve}}_{12}$

Translation: $\frac{x}{4} - 6 = 12.$

Example:
Some phrases and sentences do not translate directly. We must be careful to read them properly. The word *from* often appears in such phrases and sentences. The word **from** means “a point of departure for motion.” The following translation will illustrate this use.

Translation: $x - 20.$

The word *from* indicated the motion (subtraction) is to begin at the point of "some number."

Example:
Ten less than some number. Notice that *less than* can be replaced by *from*.
Ten from some number.
Translation: $x - 10$.

Practice Set B

Translate each phrase or sentence into a mathematical expression or equation.

Exercise:

Problem: A number divided by eight, plus seven, is fifty.

Solution:

$$\frac{x}{8} + 7 = 50$$

Exercise:

Problem:

A number divided by three, minus the same number multiplied by six, is one more than the number.

Solution:

$$\frac{2}{3} - 6x = x + 1$$

Exercise:

Problem: Nine from some number is four.

Solution:

$x - 9 = 4$

Exercise:

Problem: Five less than some quantity is eight.

Solution:

$x - 5 = 8$

Exercises

Translate each phrase or sentence to a mathematical expression or equation.

Exercise:

Problem: A quantity less twelve.

Solution:

$x - 12$

Exercise:

Problem: Six more than an unknown number.

Exercise:

Problem: A number minus four.

Solution:

$x - 4$

Exercise:

Problem: A number plus seven.

Exercise:

Problem: A number increased by one.

Solution:

$x + 1$

Exercise:

Problem: A number decreased by ten.

Exercise:

Problem: Negative seven added to some number.

Solution:

$-7 + x$

Exercise:

Problem: Negative nine added to a number.

Exercise:

Problem: A number plus the opposite of six.

Solution:

$x + (-6)$

Exercise:

Problem: A number minus the opposite of five.

Exercise:

Problem: A number minus the opposite of negative one.

Solution:

$x - [-(-1)]$

Exercise:

Problem: A number minus the opposite of negative twelve.

Exercise:

Problem: Eleven added to three times a number.

Solution:

$3x + 11$

Exercise:

Problem: Six plus five times an unknown number.

Exercise:

Problem: Twice a number minus seven equals four.

Solution:

$2x - 7 = 4$

Exercise:

Problem: Ten times a quantity increased by two is nine.

Exercise:

Problem:

When fourteen is added to two times a number the result is six.

Solution:

$14 + 2x = 6$

Exercise:

Problem: Four times a number minus twenty-nine is eleven.

Exercise:

Problem: Three fifths of a number plus eight is fifty.

Solution:

$\frac{3}{5}x + 8 = 50$

Exercise:

Problem: Two ninths of a number plus one fifth is forty-one.

Exercise:

Problem:

When four thirds of a number is increased by twelve, the result is five.

Solution:

$\frac{4}{3}x + 12 = 5$

Exercise:

Problem:

When seven times a number is decreased by two times the number, the result is negative one.

Exercise:

Problem:

When eight times a number is increased by five, the result is equal to the original number plus twenty-six.

Solution:

$8x + 5 = x + 26$

Exercise:

Problem:

Five more than some number is three more than four times the number.

Exercise:

Problem:

When a number divided by six is increased by nine, the result is one.

Solution:

$\frac{x}{6} + 9 = 1$

Exercise:

Problem: A number is equal to itself minus three times itself.

Exercise:

Problem: A number divided by seven, plus two, is seventeen.

Solution:

$$\frac{x}{7} + 2 = 17$$

Exercise:

Problem:

A number divided by nine, minus five times the number, is equal to one more than the number.

Exercise:

Problem: When two is subtracted from some number, the result is ten.

Solution:

$$x - 2 = 10$$

Exercise:

Problem:

When four is subtracted from some number, the result is thirty-one.

Exercise:

Problem:

Three less than some number is equal to twice the number minus six.

Solution:

$$x - 3 = 2x - 6$$

Exercise:

Problem:

Thirteen less than some number is equal to three times the number added to eight.

Exercise:

Problem:

When twelve is subtracted from five times some number, the result is two less than the original number.

Solution:

$5x - 12 = x - 2$

Exercise:

Problem:

When one is subtracted from three times a number, the result is eight less than six times the original number.

Exercise:

Problem:

When a number is subtracted from six, the result is four more than the original number.

Solution:

$6 - x = x + 4$

Exercise:

Problem:

When a number is subtracted from twenty-four, the result is six less than twice the number.

Exercise:

Problem:

A number is subtracted from nine. This result is then increased by one. The result is eight more than three times the number.

Solution:

$9 - x + 1 = 3x + 8$

Exercise:

Problem:

Five times a number is increased by two. This result is then decreased by three times the number. The result is three more than three times the number.

Exercise:

Problem:

Twice a number is decreased by seven. This result is decreased by four times the number. The result is negative the original number, minus six.

Solution:

$2x - 7 - 4x = -x - 6$

Exercise:

Problem:

Fifteen times a number is decreased by fifteen. This result is then increased by two times the number. The result is negative five times the original number minus the opposite of ten.

Exercises for Review

Exercise:

Problem: ([link]) $\frac{8}{9}$ of what number is $\frac{2}{3}$?

Solution:

$$\frac{3}{4}$$

Exercise:

Problem: ([link]) Find the value of $\frac{21}{40} + \frac{17}{30}$.

Exercise:

Problem: ([link]) Find the value of $3\frac{1}{12} + 4\frac{1}{3} + 1\frac{1}{4}$.

Solution:

$$8\frac{2}{3}$$

Exercise:

Problem: ([link]) Convert $6.11\frac{1}{5}$ to a fraction.

Exercise:

Problem: ([link]) Solve the equation $\frac{3x}{4} + 1 = -5$.

Solution:

$$x = -8$$

Applications II: Solving Problems
This module is from Fundamentals of Mathematics by Denny Burzynski and Wade Ellis, Jr. This module discusses how to solve algebraic problems. By the end of the module students should be more familiar with the five-step method for solving applied problems and be able to use the five-step method to solve number problems and geometry problems.

Section Overview

- The Five-Step Method
- Number Problems
- Geometry Problems

The Five Step Method

We are now in a position to solve some applied problems using algebraic methods. The problems we shall solve are intended as logic developers. Although they may not seem to reflect real situations, they do serve as a basis for solving more complex, real situation, applied problems. To solve problems algebraically, we will use the five-step method.

Strategy for Reading Word Problems
When solving mathematical word problems, you may wish to apply the following "**reading strategy**." Read the problem quickly to get a feel for the situation. Do not pay close attention to details. At the first reading, too much attention to details may be overwhelming and lead to confusion and discouragement. After the first, brief reading, read the problem carefully in *phrases*. Reading phrases introduces information more slowly and allows us to absorb and put together important information. We can look for the unknown quantity by reading one phrase at a time.
Five-Step Method for Solving Word Problems

1. Let x (or some other letter) represent the unknown quantity.
2. Translate the words to mathematical symbols and form an equation. Draw a picture if possible.
3. Solve the equation.
4. Check the solution by substituting the result into the original statement, not equation, of the problem.
5. Write a conclusion.

If it has been your experience that word problems are difficult, then follow the five-step method carefully. Most people have trouble with word problems for two reasons:

1. They are not able to translate the words to mathematical symbols. (See [link].)
2. They neglect step 1. After working through the problem phrase by phrase, to become familiar with the situation,

INTRODUCE A VARIABLE

Number Problems

Sample Set A

Example:
What number decreased by six is five?

Let n represent the unknown number.
Translate the words to mathematical symbols and construct an equation. Read phrases.

What number:	n	
decreased by:	$-$	
six:	6	$n-6=5$
is:	$=$	
five:	5	

Solve this equation.

$n-6=5$ Add 6 to *both* sides.

$n-6+6=5+6$

$n=11$

Check the result.

When 11 is decreased by 6, the result is $11-6$, which is equal to 5. The solution checks.

The number is 11.

Example:

When three times a number is increased by four, the result is eight more than five times the number.

Let $x=$ the unknown number.

Translate the phrases to mathematical symbols and construct an equation.

When three times a number:	$3x$	
is increased by:	$+$	
four:	4	
the result is:	$=$	$3x+4=5x+8$
eight:	8	
more than:	$+$	
five times the number:	$5x$	

$3x+4=5x+8.$	Subtract $3x$ from *both* sides.
$3x+4-3x=5x+8-3x$	
$4=2\text{x}+8$	Subtract 8 from *both* sides.
$4-8=2\text{x}+8-8$	
$-4=2\text{x}$	Divide *both* sides by 2.
$-2=x$	

Check this result. Three times -2 is -6. Increasing -6 by 4 results in $-6+4=-2$. Now, five times -2 is -10. Increasing -10 by 8 results in $-10+8=-2$. The results agree and the solution checks.

The number is -2

Example:

Consecutive integers have the property that if

$$\begin{aligned} n &= \text{the smallest integer, then} \\ n+1 &= \text{the next integer, and} \\ n+2 &= \text{the next integer, and so on.} \end{aligned}$$

Consecutive odd or even integers have the property that if

$$\begin{aligned} n &= \text{the smallest integer, then} \\ n+2 &= \text{the next odd or even integer (since odd or even numbers differ by 2), and} \\ n+4 &= \text{the next odd or even integer, and so on.} \end{aligned}$$

The sum of three consecutive odd integers is equal to one less than twice the first odd integer. Find the three integers.

$$\begin{aligned} \text{Let} \quad n &= \text{the first odd integer. Then,} \\ n+2 &= \text{the second odd integer, and} \\ n+4 &= \text{the third odd integer.} \end{aligned}$$

Translate the words to mathematical symbols and construct an equation. Read phrases.	The sum of:	add some numbers	
	three consecutive odd integers:	n,$n+2$,$n+4$	
	is equal to:	$=$	$n+(n+2)+(n+4)=2n-1$
	one less than:	subtract 1 from	
	twice the first odd integer:	$2n$	

$n+n+2+n+4=2n-1$	
$3n+6=2n-1$	Subtract $2n$ from *both* sides.
$3n+6-2n=2n-1-2n$	
$n+6=-1$	Subtract 6 from *both* sides.
$n+6-6=-1-6$	
$n=-7$	The first integer is -7.
$n+2=-7+2=-5$	The second integer is -5.
$n+4=-7+4=-3$	The third integer is -3.

Check this result. The sum of the three integers is

$$\begin{aligned} -7+(-5)+(-3) &= -12+(-3) \\ &= -15 \end{aligned}$$

One less than twice the first integer is $2(-7)-1=-14-1=-15$. Since these two results are equal, the solution checks.

The three odd integers are -7, -5, -3.

Practice Set A

Exercise:

Problem: When three times a number is decreased by 5, the result is -23. Find the number.

Let $x=$
Check:
The number is.

Solution:

-6

Exercise:

Problem:

When five times a number is increased by 7, the result is five less than seven times the number. Find the number.

Let$n =$
Check:
The number is.

Solution:

6

Exercise:

Problem: Two consecutive numbers add to 35. Find the numbers.

Check:
The numbers areand.

Solution:

17 and 18

Exercise:

Problem:

The sum of three consecutive even integers is six more than four times the middle integer. Find the integers.

Let$x =$smallest integer.= next integer.= largest integer.
Check:
The integers are,, and.

Solution:

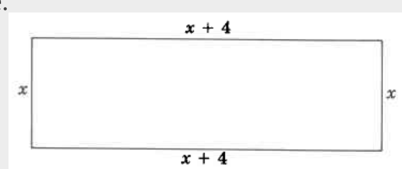
-8, -6, and -4

Geometry Problems

Sample Set B

Example:
The perimeter (length around) of a rectangle is 20 meters. If the length is 4 meters longer than the width, find the length and width of the rectangle.

Let$x =$the width of the rectangle. Then,$x + 4 =$the length of the rectangle.
We can draw a picture.



The length around the rectangle is

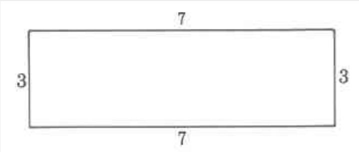
$$\underbrace{x}_{\text{width}} + \underbrace{(x+4)}_{\text{length}} + \underbrace{x}_{\text{width}} + \underbrace{(x+4)}_{\text{length}} = 20$$

$x + x + 4 + x + x + 4 = 20$

$4x + 8 = 20$	Subtract 8 from *both* sides.
$4x = 12$	Divide *both* sides by 4.
$x = 3$	Then,

$x + 4 = 3 + 4 = 7$

Check:



$3 + 7 + 3 + 7 \stackrel{?}{=} 20$

$20 \stackrel{\checkmark}{=} 20$

The length of the rectangle is 7 meters.The width of the rectangle is 3 meters.

Practice Set B

Exercise:

Problem:

The perimeter of a triangle is 16 inches. The second leg is 2 inches longer than the first leg, and the third leg is 5 inches longer than the first leg. Find the length of each leg.

Let$x =$length of the first leg.= length of the second leg.= length of the third leg.
We can draw a picture.
Check:
The lengths of the legs are,, and.

Solution:

3 inches, 5 inches, and 8 inches

Exercises

For the following 17 problems, find each solution using the five-step method.

Exercise:

Problem: What number decreased by nine is fifteen?

Let$n =$the number.
Check:
The number is.

Solution:

24

Exercise:

Problem: What number increased by twelve is twenty?

$n =$

Let the number.
Check:
The number is.

Exercise:

Problem: If five more than three times a number is thirty-two, what is the number?

Let $x =$ the number.
Check:
The number is.

Solution:

9

Exercise:

Problem: If four times a number is increased by fifteen, the result is five. What is the number?

Let $x =$
Check:
The number is.

Exercise:

Problem:

When three times a quantity is decreased by five times the quantity, the result is negative twenty. What is the quantity?

Let $x =$
Check:
The quantity is.

Solution:

10

Exercise:

Problem:

If four times a quantity is decreased by nine times the quantity, the result is ten. What is the quantity?

Let $y =$
Check:
The quantity is.

Exercise:

Problem:

When five is added to three times some number, the result is equal to five times the number decreased by seven. What is the number?

Let $n =$

Check:
The number is.

Solution:

6

Exercise:

Problem:

When six times a quantity is decreased by two, the result is six more than seven times the quantity. What is the quantity?

Let $x =$
Check:
The quantity is.

Exercise:

Problem:

When four is decreased by three times some number, the result is equal to one less than twice the number. What is the number?

Check:

Solution:

1

Exercise:

Problem:

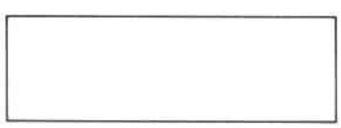
When twice a number is subtracted from one, the result is equal to twenty-one more than the number. What is the number?

Exercise:

Problem:

The perimeter of a rectangle is 36 inches. If the length of the rectangle is 6 inches more than the width, find the length and width of the rectangle.

Let $w =$the width.= the length.
We can draw a picture.



Check:
The length of the rectangle isinches, and the width isinches.

Solution:

Length=12 inches, Width=6 inches

Exercise:

Problem:

The perimeter of a rectangle is 48 feet. Find the length and the width of the rectangle if the length is 8 feet more than the width.

Let$w =$the width.= the length.
We can draw a picture.

Check:
The length of the rectangle isfeet, and the width isfeet.

Exercise:

Problem: The sum of three consecutive integers is 48. What are they?

Let$n =$the smallest integer.= the next integer.= the next integer.
Check:
The three integers are,, and.

Solution:

15, 16, 17

Exercise:

Problem: The sum of three consecutive integers is -27. What are they?

Let$n =$the smallest integer.= the next integer.= the next integer.
Check:
The three integers are,, and.

Exercise:

Problem: The sum of five consecutive integers is zero. What are they?

Let$n =$
The five integers are,,,, and.

Solution:

-2, -1, 0, 1, 2

Exercise:

Problem: The sum of five consecutive integers is -5. What are they?

Let$n =$

The five integers are,,,, and.

Continue using the five-step procedure to find the solutions.

Exercise:

Problem:

The perimeter of a rectangle is 18 meters. Find the length and width of the rectangle if the length is 1 meter more than three times the width.

Solution:

Length is 7, width is 2

Exercise:

Problem:

The perimeter of a rectangle is 80 centimeters. Find the length and width of the rectangle if the length is 2 meters less than five times the width.

Exercise:

Problem:

Find the length and width of a rectangle with perimeter 74 inches, if the width of the rectangle is 8 inches less than twice the length.

Solution:

Length is 15, width is 22

Exercise:

Problem:

Find the length and width of a rectangle with perimeter 18 feet, if the width of the rectangle is 7 feet less than three times the length.

Exercise:

Problem:

A person makes a mistake when copying information regarding a particular rectangle. The copied information is as follows: The length of a rectangle is 5 inches less than two times the width. The perimeter of the rectangle is 2 inches. What is the mistake?

Solution:

The perimeter is 20 inches. Other answers are possible. For example, perimeters such as 26, 32 are possible.

Exercise:

Problem:

A person makes a mistake when copying information regarding a particular triangle. The copied information is as follows: Two sides of a triangle are the same length. The third side is 10 feet less than three times the length of one of the other sides. The perimeter of the triangle is 5 feet. What is the mistake?

Exercise:

Problem:

The perimeter of a triangle is 75 meters. If each of two legs is exactly twice the length of the shortest leg, how long is the shortest leg?

Solution:

15 meters

Exercise:

Problem:

If five is subtracted from four times some number the result is negative twenty-nine. What is the number?

Exercise:

Problem: If two is subtracted from ten times some number, the result is negative two. What is the number?

Solution:

$n = 0$

Exercise:

Problem:

If three less than six times a number is equal to five times the number minus three, what is the number?

Exercise:

Problem:

If one is added to negative four times a number the result is equal to eight less than five times the number. What is the number?

Solution:

$n = 1$

Exercise:

Problem: Find three consecutive integers that add to -57.

Exercise:

Problem: Find four consecutive integers that add to negative two.

Solution:

-2, -1, 0, 1

Exercise:

Problem: Find three consecutive even integers that add to -24.

Exercise:

Problem: Find three consecutive odd integers that add to -99.

Solution:

-35, -33, -31

Exercise:

Problem:

Suppose someone wants to find three consecutive odd integers that add to 120. Why will that person not be able to do it?

Exercise:

Problem:

Suppose someone wants to find two consecutive even integers that add to 139. Why will that person not be able to do it?

Solution:

…because the sum of any even number (in this case, 2) o even integers (consecutive or not) is even and, therefore, cannot be odd (in this case, 139)

Exercise:

Problem:

Three numbers add to 35. The second number is five less than twice the smallest. The third number is exactly twice the smallest. Find the numbers.

Exercise:

Problem:

Three numbers add to 37. The second number is one less than eight times the smallest. The third number is two less than eleven times the smallest. Find the numbers.

Solution:

2, 15, 20

Exercises for Review

Exercise:

Problem: ([link]) Find the decimal representation of $0.34992 \div 4.32$.

Exercise:

Problem:

([link]) A 5-foot woman casts a 9-foot shadow at a particular time of the day. How tall is a person that casts a 10.8-foot shadow at the same time of the day?

Solution:

6 feet tall

Exercise:

Problem: ([link]) Use the method of rounding to estimate the sum: $4\frac{5}{12} + 15\frac{1}{25}$.

Exercise:

Problem: ([link]) Convert 463 mg to cg.

Solution:

46.3 cg

Exercise:

Problem:

([link]) Twice a number is added to 5. The result is 2 less than three times the number. What is the number?

Summary of Key Concepts

This module is from Fundamentals of Mathematics by Denny Burzynski and Wade Ellis, Jr. This module reviews the key concepts from the chapter "Algebraic Expressions and Equations."

Summary of Key Concepts

Numerical Expression ([link])
A **numerical expression** results when numbers are associated by arithmetic operation signs. The expressions $3+5$, $9-2$, $5 \cdot 6$ and $8 \div 5$ are numerical expressions.

Algebraic Expressions ([link])
When an arithmetic operation sign connects a letter with a number or a letter with a letter, an **algebraic expression** results. The expressions $4x+1$, $x-5$, $7x \cdot 6y$, and $4x \div 3$ are algebraic expressions.

Terms and Factors ([link])
Terms are parts of **sums** and are therefore separated by addition (or subtraction) signs. In the expression, $5x-2y$, $5x$ and $-2y$ are the terms. **Factors** are parts of products and are therefore separated by multiplication signs. In the expression $5a$, 5 and a are the factors.

Coefficients ([link])
The **coefficient** of a quantity records how many of that quantity there are. In the expression $7x$, the coefficient 7 indicates that there are seven x's.

Numerical Evaluation ([link])
Numerical evaluation is the process of determining the value of an algebraic expression by replacing the variables in the expression with specified values.

Combining Like Terms ([link])
An algebraic expression may be simplified by combining like terms. **To combine like terms**, we simply add or subtract their coefficients then affix the variable. For example $4x+9x=(4+9)x=13x$.

Equation ([link])
An **equation** is a statement that two expressions are equal. The statements $5x + 1 = 3$ and $\frac{4x}{5} + 4 = \frac{2}{5}$ are equations. The expressions represent the same quantities.

Conditional Equation ([link])
A **conditional equation** is an equation whose truth depends on the value selected for the variable. The equation $3x = 9$ is a conditional equation since it is only true on the condition that 3 is selected for x.

Solutions and Solving an Equation ([link])
The values that when substituted for the variables make the equation true are called the **solutions** of the equation.
An equation has been **solved** when all its solutions have been found.

Equivalent Equations ([link])
Equations that have precisely the same solutions are called **equivalent equations**. The equations $6y = 18$ and $y = 3$ are equivalent equations.

Addition/Subtraction Property of Equality ([link])
Given any equation, we can obtain an equivalent equation by

1. adding the same number to both sides, or
2. subtracting the same number from both sides.

Solving $x + a = b$ and $x - a = b$ ([link])
To **solve** $x + a = b$, *subtract* a from both sides.

$$x + a = b$$
$$x + a - a = b - a$$
$$x = b - a$$

To **solve** $x - a = b$, *add* a to both sides.

$$x - a = b$$
$$x - a + a = b + a$$
$$x = b + a$$

Multiplication/Division Property of Equality ([link])
Given any equation, we can obtain an **equivalent equation** by

1. multiplying both sides by the same nonzero number, that is, if $c \neq 0$, $a = b$ and $a \cdot c = b \cdot c$ are equivalent.
2. dividing both sides by the same nonzero number, that is, if $c \neq 0$, $a = b$ and $\frac{a}{c} = \frac{b}{c}$ are equivalent.

Solving $ax = b$ and $\frac{x}{a} = b$ ([link])
To **solve** ax $= b$, $a \neq 0$, *divide both sides by* a.

$$\begin{aligned} ax &= b \\ \frac{ax}{a} &= \frac{b}{a} \\ \frac{\cancel{a}x}{\cancel{a}} &= \frac{b}{a} \\ x &= \frac{b}{a} \end{aligned}$$

To **solve** $\frac{x}{a} = b$, $a \neq 0$, *multiply* both sides by a.

$$\begin{aligned} \frac{x}{a} &= b \\ a \cdot \frac{x}{a} &= a \cdot b \\ \cancel{a} \cdot \frac{x}{\cancel{a}} &= a \cdot b \\ x &= a \cdot b \end{aligned}$$

Translating Words to Mathematics ([link])
In solving applied problems, it is important to be able to translate phrases and sentences to mathematical expressions and equations.

The Five-Step Method for Solving Applied Problems ([link])
To solve problems algebraically, it is a good idea to use the following **five-step procedure**.
After working your way through the problem carefully, phrase by phrase:

1. Let x (or some other letter) represent the unknown quantity.
2. Translate the phrases and sentences to mathematical symbols and form an equation. Draw a picture if possible.
3. Solve this equation.
4. Check the solution by substituting the result into the original statement of the problem.
5. Write a conclusion.

Exercise Supplement
This module is from Fundamentals of Mathematics by Denny Burzynski and Wade Ellis, Jr. This module is an exercise supplement for the chapter "Algebraic Expressions and Equations" and contains many exercise problems. Odd problems are accompanied by solutions.

Exercise Supplement

Algebraic Expressions ([link])

For problems 1-10, specify each term.

Exercise:

Problem: $6a - 2b + 5c$

Solution:

$6a$, $-2b$, $5c$

Exercise:

Problem: $9x - 6y + 1$

Exercise:

Problem: $7m - 3n$

Solution:

$7m$, $-3n$

Exercise:

Problem: $-5h + 2k - 8 + 4m$

Exercise:

Problem: $x + 2n - z$

Solution:

x, $2n$, $-z$

Exercise:

Problem: $y - 5$

Exercise:

Problem: $-y - 3z$

Solution:

$-y$, $-3z$

Exercise:

Problem: $-a - b - c - 1$

Exercise:

Problem: -4

Solution:

-4

Exercise:

Problem: -6

Exercise:

Problem: Write $1k$ in a simpler way.

Solution:

k

Exercise:

Problem: Write $1x$ in a simpler way.

Exercise:

Problem: In the expression $7r$, how many r's are indicated?

Solution:

7

Exercise:

Problem: In the expression $12m$, how many m's are indicated?

Exercise:

Problem: In the expression $-5n$, how many n's are indicated?

Solution:

-5

Exercise:

Problem: In the expression $-10y$, how many y's are indicated?

For problems 17-46, find the value of each expression.

Exercise:

Problem: $5a - 2s$, if $a = -5$ and $s = 1$

Solution:

-27

Exercise:

Problem: $7n - 3r$, if $n = -6$ and $r = 2$

Exercise:

Problem: $9x + 2y - 3s$, if $x = -2$, $y = 5$, and $s = -3$

Solution:

1

Exercise:

Problem: $10a - 2b + 5c$, if $a = 0$, $b = -6$, and $c = 8$

Exercise:

Problem: $-5s - 2t + 1$, if $s = 2$ and $t = -2$

Solution:

-5

Exercise:

Problem: $-3m - 4n + 5$, if $m = -1$ and $n = -1$

Exercise:

Problem: $m - 4$, if $m = 4$

Solution:

0

Exercise:

Problem: $n = 2$, if $n = 2$

Exercise:

Problem: $-x + 2y$, if $x = -7$ and $y = -1$

Solution:

5

Exercise:

Problem: $-a + 3b - 6$, if $a = -3$ and $b = 0$

Exercise:

Problem: $5x - 4y - 7y + y - 7x$, if $x = 1$ and $y = -2$

Solution:

18

Exercise:

Problem: $2\text{a} - 6\text{b} - 3\text{a} - a + 2\text{b}$, if $a = 4$ and $b = -2$

Exercise:

Problem: $a^2 - 6a + 4$, if $a = -2$

Solution:

20

Exercise:

Problem: $m^2 - 8m - 6$, if $m = -5$

Exercise:

Problem: $4y^2 + 3y + 1$, if $y = -2$

Solution:

11

Exercise:

Problem: $5a^2 - 6a + 11$, if $a = 0$

Exercise:

Problem: $-k^2 - k - 1$, if $k = -1$

Solution:

-1

Exercise:

Problem: $-h^2 - 2h - 3$, if $h = -4$

Exercise:

Problem: $\frac{m}{6} + 5m$, if $m = -18$

Solution:

-93

Exercise:

Problem: $\frac{a}{8} - 2a + 1$, if $a = 24$

Exercise:

Problem: $\frac{5x}{7} + 3x - 7$, if $x = 14$

Solution:

45

Exercise:

Problem: $\frac{3k}{4} - 5k + 18$, if $k = 16$

Exercise:

Problem: $\frac{-6a}{5} + 3a + 10$, if $a = 25$

Solution:

55

Exercise:

Problem: $\frac{-7h}{9} - 7h - 7$, if $h = -18$

Exercise:

Problem: $5(3a + 4b)$, if $a = -2$ and $b = 2$

Solution:

10

Exercise:

Problem: $7(2y - x)$, if $x = -1$ and $y = 2$

Exercise:

Problem: $-(a-b)$, if $a=0$ and $b=-6$

Solution:

-6

Exercise:

Problem: $-(x-x-y)$, if $x=4$ and $y=-4$

Exercise:

Problem: $(y+2)^2-6(y+2)-6$, if $y=2$

Solution:

-14

Exercise:

Problem: $(a-7)^2-2(a-7)-2$, if $a=7$

Combining Like Terms Using Addition and Subtraction ([link])

For problems 47-56, simplify each expression by combining like terms.

Exercise:

Problem: $4a+5-2a+1$

Solution:

$2a+6$

Exercise:

Problem: $7x + 3x - 14x$

Exercise:

Problem: $-7b + 4m - 3 + 3n$

Solution:

$-4n + 4m - 3$

Exercise:

Problem: $-9k - 8h - k + 6h$

Exercise:

Problem: $-x + 5y - 8x - 6x + 7y$

Solution:

$-15x + 12y$

Exercise:

Problem: $6n - 2n + 6 - 2 - n$

Exercise:

Problem: $0m + 3k - 5s + 2m - s$

Solution:

$3k + 2m - 6s$

Exercise:

Problem: $|-8|a + |2|b - |-4|a$

Exercise:

Problem: $| \, 6 \, | \, h- \, | -7 \, | \, k+ \, | -12 \, | \, h+ \, | \, 4 \, | \cdot | -5 \, | \, h$

Solution:

$38h - 7\text{k}$

Exercise:

Problem: $| \, 0 \, | \, a - 0a + 0$

Equations of the Form $ax = b$ and $\frac{x}{a} = b$, Translating Words to Mathematical Symbols , and Solving Problems ([link],[link],[link])

For problems 57-140, solve each equation.

Exercise:

Problem: $x + 1 = 5$

Solution:

$x = 4$

Exercise:

Problem: $y - 3 = -7$

Exercise:

Problem: $x + 12 = 10$

Solution:

$x = -2$

Exercise:

Problem: $x - 4 = -6$

Exercise:

Problem: $5x = 25$

Solution:

$x = 5$

Exercise:

Problem: $3x = 17$

Exercise:

Problem: $\frac{x}{2} = 6$

Solution:

$x = 12$

Exercise:

Problem: $\frac{x}{-8} = 3$

Exercise:

Problem: $\frac{x}{15} = -1$

Solution:

$x = -15$

Exercise:

Problem: $\frac{x}{-4}=-3$

Exercise:

Problem: $-3x=9$

Solution:

$x=-3$

Exercise:

Problem: $-2x=5$

Exercise:

Problem: $-5x=-5$

Solution:

$x=1$

Exercise:

Problem: $-3x=-1$

Exercise:

Problem: $\frac{x}{-3}=9$

Solution:

$x=-27$

Exercise:

Problem: $\frac{a}{-5} = 2$

Exercise:

Problem: $-7 = 3y$

Solution:

$$y = -\frac{7}{3}$$

Exercise:

Problem: $-7 = \frac{x}{3}$

Exercise:

Problem: $\frac{m}{4} = \frac{-2}{5}$

Solution:

$$m = -\frac{8}{5}$$

Exercise:

Problem: $4y = \frac{1}{2}$

Exercise:

Problem: $\frac{-1}{3} = -5x$

Solution:

$$x = \frac{1}{15}$$

Exercise:

Problem: $\frac{-1}{9} = \frac{k}{3}$

Exercise:

Problem: $\frac{-1}{6} = \frac{s}{-6}$

Solution:

$$s = 1$$

Exercise:

Problem: $\frac{0}{4} = 4s$

Exercise:

Problem: $x + 2 = -1$

Solution:

$$x = -3$$

Exercise:

Problem: $x - 5 = -6$

Exercise:

Problem: $\frac{-3}{2}x = 6$

Solution:

$x = -4$

Exercise:

Problem: $3x + 2 = 7$

Exercise:

Problem: $-4x - 5 = -3$

Solution:

$x = -\frac{1}{2}$

Exercise:

Problem: $\frac{x}{6} + 1 = 4$

Exercise:

Problem: $\frac{a}{-5} - 3 = -2$

Solution:

$a = -5$

Exercise:

Problem: $\frac{4x}{3} = 7$

Exercise:

Problem: $\frac{2x}{5} + 2 = 8$

Solution:

$x = 15$

Exercise:

Problem: $\frac{3y}{2} - 4 = 6$

Exercise:

Problem: $m + 3 = 8$

Solution:

$x = 5$

Exercise:

Problem: $\frac{1x}{2} = 2$

Exercise:

Problem: $\frac{2a}{3} = 5$

Solution:

$a = \frac{15}{2}$

Exercise:

Problem: $\frac{-3x}{7} - 4 = 4$

Exercise:

Problem: $\frac{5x}{-2} - 6 = -10$

Solution:

$x = \frac{8}{5}$

Exercise:

Problem: $-4k - 6 = 7$

Exercise:

Problem: $\frac{-3x}{-2} + 1 = 4$

Solution:

$x = 2$

Exercise:

Problem: $\frac{-6x}{4} = 2$

Exercise:

Problem: $x + 9 = 14$

Solution:

$x = 5$

Exercise:

Problem: $y+5=21$

Exercise:

Problem: $y+5=-7$

Solution:

$y=-12$

Exercise:

Problem: $4x=24$

Exercise:

Problem: $4w=37$

Solution:

$$w=\frac{37}{4}$$

Exercise:

Problem: $6y-11=13$

Exercise:

Problem: $-3x+8=-7$

Solution:

$x=5$

Exercise:

Problem: $3z + 9 = -51$

Exercise:

Problem: $\frac{x}{-3} = 8$

Solution:

$x = -24$

Exercise:

Problem: $\frac{6y}{7} = 5$

Exercise:

Problem: $\frac{w}{2} - 15 = 4$

Solution:

$w = 38$

Exercise:

Problem: $\frac{x}{-2} - 23 = -10$

Exercise:

Problem: $\frac{2x}{3} - 5 = 8$

Solution:

$x = \frac{39}{2}$

Exercise:

Problem: $\frac{3z}{4} = \frac{-7}{8}$

Exercise:

Problem: $-2 - \frac{2x}{7} = 3$

Solution:

$x = -\frac{35}{2}$

Exercise:

Problem: $3 - x = 4$

Exercise:

Problem: $-5 - y = -2$

Solution:

$y = -3$

Exercise:

Problem: $3 - z = -2$

Exercise:

Problem: $3x + 2x = 6$

Solution:

$x = \frac{6}{5}$

Exercise:

Problem: $4x + 1 + 6x = 10$

Exercise:

Problem: $6y - 6 = -4 + 3y$

Solution:

$y = \frac{2}{3}$

Exercise:

Problem: $3 = 4a - 2a + a$

Exercise:

Problem: $3m + 4 = 2m + 1$

Solution:

$m = -3$

Exercise:

Problem: $5w - 6 = 4 + 2w$

Exercise:

Problem: $8 - 3a = 32 - 2a$

Solution:

$a = -24$

Exercise:

Problem: $5x - 2x + 6x = 13$

Exercise:

Problem: $x + 2 = 3 - x$

Solution:

$$x = \frac{1}{2}$$

Exercise:

Problem: $5y + 2y - 1 = 6y$

Exercise:

Problem: $x = 32$

Solution:

$$x = 32$$

Exercise:

Problem: $k = -4$

Exercise:

Problem: $\frac{3x}{2} + 4 = \frac{5x}{2} = 6$

Solution:

$$x = -2$$

Exercise:

Problem: $\frac{x}{3} + \frac{3x}{3} - 2 = 16$

Exercise:

Problem: $x - 2 = 6 - x$

Solution:

$x = 4$

Exercise:

Problem: $\frac{-5x}{7} = \frac{2x}{7}$

Exercise:

Problem: $\frac{\text{2x}}{3} + 1 = 5$

Solution:

$x = 6$

Exercise:

Problem: $\frac{-3x}{5} + 3 = \frac{2x}{5} + 2$

Exercise:

Problem: $\frac{3x}{4} + 5 = \frac{-3x}{4} - 11$

Solution:

$$x = \frac{-32}{3}$$

Exercise:

Problem: $\frac{3x}{7} = \frac{-3x}{7} + 12$

Exercise:

Problem: $\frac{5y}{13} - 4 = \frac{7y}{26} + 1$

Solution:

$$y = \frac{130}{3}$$

Exercise:

Problem: $\frac{-3m}{5} = \frac{6m}{10} - 2$

Exercise:

Problem: $\frac{-3m}{2} + 1 = 5m$

Solution:

$$m = \frac{2}{13}$$

Exercise:

Problem: $-3z = \frac{2z}{5}$

Proficiency Exam
This module is from Fundamentals of Mathematics by Denny Burzynski and Wade Ellis, Jr. This module is a proficiency exam to the chapter "Algebraic Expressions and Equations." Each problem is accompanied with a reference link pointing back to the module that discusses the type of problem demonstrated in the question. The problems in this exam are accompanied by solutions.

Proficiency Exam

For problems 1 and 2 specify each term.
Exercise:

Problem: ([link]) $5x + 6y + 3z$

Solution:

$5x$, $6y$, 3z

Exercise:

Problem: ([link]) $8m - 2n - 4$

Solution:

$8m$, $-2n$, -4

Exercise:

Problem: ([link]) In the expression $-9a$, how many a's are indicated?

Solution:

-9

For problems 4-9, find the value of each expression.
Exercise:

Problem: ([link]) $6a-3b$, if $a=-2$, and $b=-1$.

Solution:

-9

Exercise:

Problem: ([link]) $-5m+2n-6$, if $m=-1$ and $n=4$.

Solution:

7

Exercise:

Problem: ([link]) $-x^2+3x-5$, if $x=-2$.

Solution:

-15

Exercise:

Problem: ([link]) y^2+9y+1, if $y=0$.

Solution:

1

Exercise:

Problem: ([link]) $-a^2+3a+4$, if $a=4$.

Solution:

0

Exercise:

Problem:

([link]) $-(5-x)^2+7(m-x)+x-2m$, if $x=5$ and $m=5$.

Solution:

-5

For problems 10-12, simplify each expression by combining like terms.

Exercise:

Problem: ([link]) $6y+5-2y+1$

Solution:

$4y+6$

Exercise:

Problem: ([link]) $14a-3b+5b-6a-b$

Solution:

$8a+b$

Exercise:

Problem: ([link]) $9x+5y-7+4x-6y+3(-2)$

Solution:

$13x-y-13$

For problems 13-22, solve each equation.

Exercise:

Problem: ([link]) $x+7=15$

Solution:

$x=8$

Exercise:

Problem: ([link]) $y-6=2$

Solution:

$y=8$

Exercise:

Problem: ([link]) $m+8=-1$

Solution:

$m=-9$

Exercise:

Problem: ([link]) $-5+a=-4$

Solution:

$a=1$

Exercise:

Problem: ([link]) $4x=104$

Solution:

$x=26$

Exercise:

Problem: ([link]) $6y+3=-21$

Solution:

$y=-4$

Exercise:

Problem: ([link]) $\frac{5m}{6}=\frac{10}{3}$

Solution:

$m=4$

Exercise:

Problem: ([link]) $\frac{7y}{8}+\frac{1}{4}=\frac{-13}{4}$

Solution:

$y=-4$

Exercise:

Problem: ([link]) $6x+5=4x-11$

Solution:

$x=-8$

Exercise:

Problem: ([link]) $4y-8-6y=3y+1$

Solution:

$$y = \frac{-9}{5}$$

Exercise:

Problem:

([link] and [link]) Three consecutive even integers add to -36. What are they?

Solution:

-14, -12, -10

Exercise:

Problem:

([link] and [link]) The perimeter of a rectangle is 38 feet. Find the length and width of the rectangle if the length is 5 feet less than three times the width.

Solution:

$l = 13, w = 6$

Exercise:

Problem:

([link] and [link]) Four numbers add to -2. The second number is three more than twice the negative of the first number. The third number is six less than the first number. The fourth number is eleven less than twice the first number. Find the numbers.

Solution:

6, -9, 0, 1