AI HW2

Prob 1. CSP formulation

Note that there are other different formulations, any reasonable one will suffice.

x_{11}	x_{12}	x_{13}
x_{21}	x_{22}	x_{23}
x_{31}	x_{32}	x_{33}

(a) Variables : $x_{11}, x_{12}, x_{13},$

$x_{21}, x_{22}, x_{23},$

x_{31}, x_{32}, x_{33}

$r_1, r_2, r_3, c_1, c_2, c_3, d_1, d_2$

Domains : $x_{ij} \in D = \{1, 2, \cdots, 9\}$ for $i=1,2,3$; $j=1,2,3$

$r_i, c_i, d_j \in D' = \{6, 7, \cdots, 24\}$ for $i=1,2,3$; $j=1,2$

Constraints : ① All-different $(x_{11}, x_{12}, \cdots, x_{33})$

$$② \begin{cases} r_1 = r_2 = r_3 = c_1 = c_2 = c_3 = d_1 = d_2 \\ r_i = \sum_{j=1}^{3} x_{ij}, \text{ for } i = 1,2,3; \quad c_j = \sum_{i=1}^{3} x_{ij}, \text{ for } j = 1,2,3 \\ d_1 = x_{11} + x_{22} + x_{33}, \quad d_2 = x_{13} + x_{22} + x_{31} \end{cases}$$

(b) Variables : $x_1, x_2, \cdots x_n$

Domain : $x_i \in D = \{c_1, c_2, \cdots, c_n\}$ where c_i's are the cities

Constraints : ① All-different $(x_1, x_2, \cdots, x_n)$,

② Connected (x_i, x_{i+1}) for $i=1,2,\cdots, n-1$

(C) Formulation 1 :

$$\begin{array}{r} INT \\ \times \quad L \\ \hline AAAI \end{array}$$

Variables : I, N, T, L, A

Domain : $D = \{0, 1, \cdots, 9\}$

Constraint : ① All-different (I, N, T, L, A)

② $T \times L + N \times L \times 10 + I \times L \times 100 = 1110 \times A + I$

Formulation 2 :

Variables : I, N, T, L, A, I , $\underbrace{c_1}_{\text{carry}}, \underbrace{c_2}_{\text{carry}}$

Domain : $D = \{0, 1, \cdots, 9\}$

Constraint : ① All-different (I, N, T, L, A)

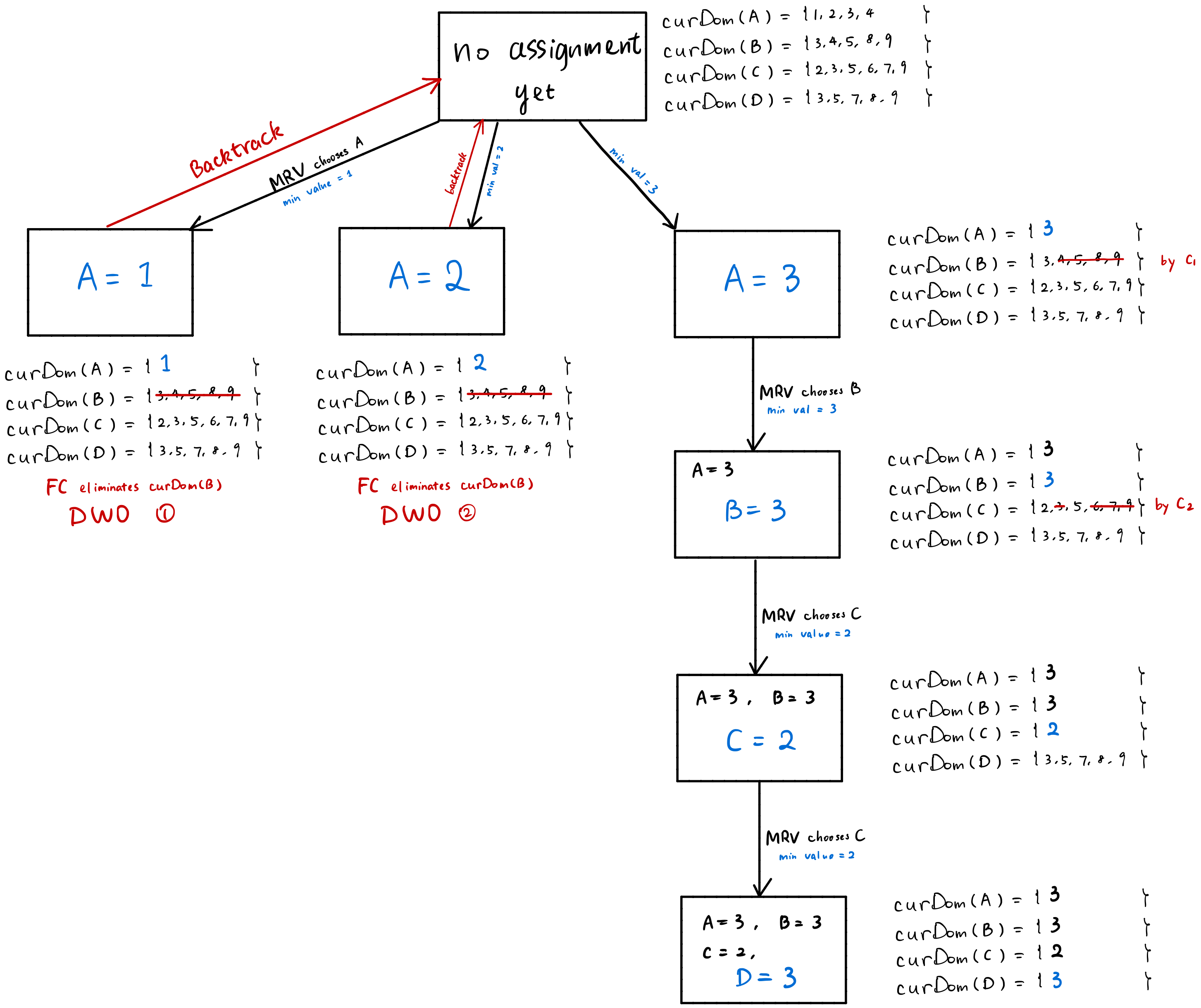
$$② \begin{cases} T \times L = c_1 \times 10 + I \\ N \times L + c_1 = c_2 \times 10 + A \\ I \times L + c_2 = A \times 10 + A \end{cases}$$

Note that the 2nd formulation is better than the 1st one, why ?

AI HW2

Prob 2 FC, GAC for CSP

(a) With the MRV heuristic and tie-breaking strategy, the solution is unique!

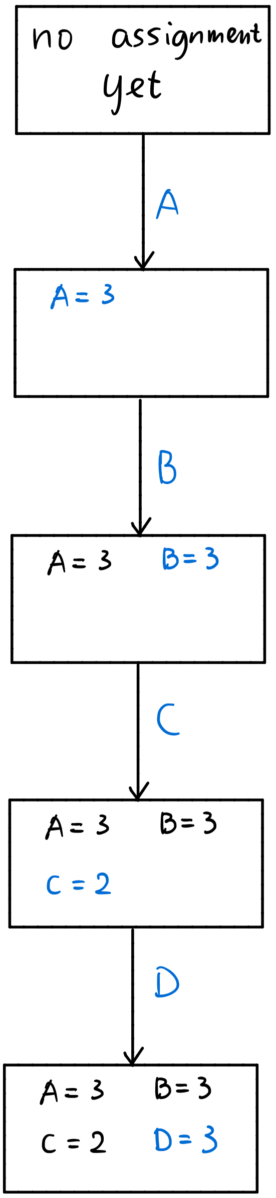


The first solution is A = 3, B = 3, C = 2, D = 3.

- C 1 : A ≥ B
- C 2 : B > C or C – B = 2
- C 3 : C =\= D

(b) GAC

Note that, GAC is to be enforced even before the first assignment!
If the domain of X is changed, any affected constraint needs to be re-checked.



1^{st} GAC

	removed val	responsible constraints
curDom(A) = { ~~1~~, ~~2~~, 3, 4 }	A = 1, 2	C_1
curDom(B) = { 3, 4, ~~5~~, ~~8~~, ~~9~~ }	B = 5, 8, 9	C_1
curDom(C) = { 2, 3, 5, 6, ~~7~~, ~~9~~ }	C = 7, 9	C_2
curDom(D) = { 3, 5, 7, 8, 9 }		

2^{nd} GAC

	removed val	responsible constraints
curDom(A) = { 3 }		
curDom(B) = { 3, ~~4~~ }	B = 4	C_1
curDom(C) = { 2, ~~3~~, 5, ~~6~~ }	C = 3, 6	C_2
curDom(D) = { 3, 5, 7, 8, 9 }		

3^{rd} GAC

	removed val	responsible constraints
curDom(A) = { 3 }		
curDom(B) = { 3 }		
curDom(C) = { 2, 5, }		
curDom(D) = { 3, 5, 7, 8, 9 }		

4^{th} GAC

	removed val	responsible constraints
curDom(A) = { 3 }		
curDom(B) = { 3 }		
curDom(C) = { 2 }		
curDom(D) = { 3, 5, 7, 8, 9 }		

curDom(A) = { 3 }
curDom(B) = { 3 }
curDom(C) = { 2 }
curDom(D) = { 3 }

The first solution is A = 3, B = 3, C = 2, D = 3

AI HW2

Problem 3: Prove $S_1 \wedge S_2 \wedge S_3 \rightarrow Q$, where $Q = \forall x\, P(x,x)$

Proof: We prove by refutation, that is

(1) first transform $S_1 \wedge S_2 \wedge S_3 \wedge \neg Q$ into clausal form, denoted by S

(2) then use resolution to derive $S \vdash ()$

(1) Following the conversion procedure:

Conversion to Clausal Form

1. Eliminate Implications.
2. Move Negations inwards (and simplify ¬¬).
3. Standardize Variables.
4. Skolemize.
5. Convert to Prenex Form.
6. Distribute disjunctions over conjunctions.
7. Flatten nested conjunctions and disjunctions.
8. Convert to Clauses.

for example,

$$S_3 \equiv \forall x \exists y.\ P(x,y) \xrightarrow{①②③④} \forall t.\ P(t, f(t)) \xrightarrow{⑤⑥⑦⑧} P(t, f(t))$$

We have

1. $(\neg P(x,y),\ P(y,x))$ from S_1
2. $(\neg P(u,v),\ \neg P(v,w),\ P(u,w))$ from S_2
3. $P(t, f(t))$ from S_3
4. $\neg P(a, a)$ from $\neg Q$ note that a is a constant introduced by the Skolemization.

(2) Resolution derivation steps

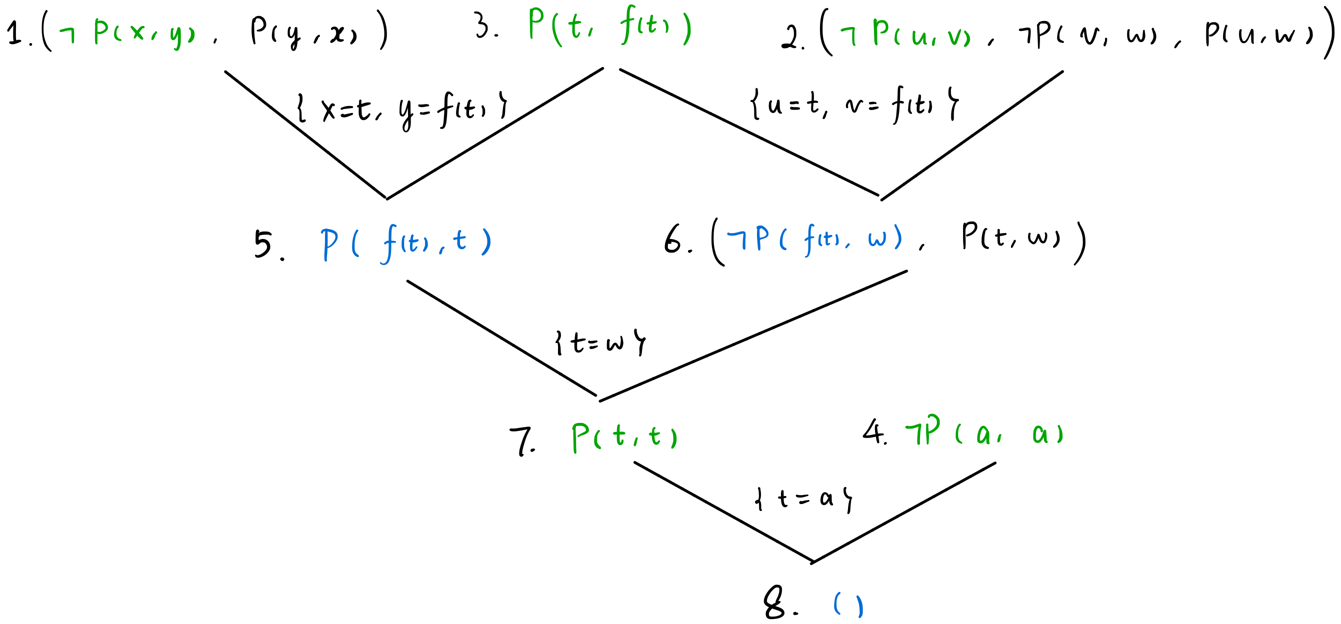
$R[1, 3]\ \{x=t,\ y=f(t)\}$ 5. $P(f(t), t)$

$R[2, 3]\ \{u=t,\ v=f(t)\}$ 6. $(\neg P(f(t), w),\ P(t,w))$

$R[5, 6]\ \{t=w\}$ 7. $P(t,t)$

$R[4, 7]\ \{t=a\}$ 8. $()$

Or in tree form



Problem 4 (a)

- Let $a = Alex, b = Peter, c = Michael$ be 3 constants
- $M(x)$ = x is the murderer
- $F(x)$ = x is a friend of the victim's
- $Al(x)$ = x has an alibi.

- Alex says: $F(b) \wedge \neg F(c)$
- Peter says: $Al(b) \wedge \neg F(b)$
- Michael says: $\neg Al(a) \wedge \neg Al(b)$

Note that we may have different formalization, but the reasoning should give us the unique answer: Peter is the murderer.

Also, the formalization could also be in propositional logic.
Answer extraction could also be used resolution.

- Formalize the facts:
- Single murderer
- $M(a) \vee M(b) \vee M(c)$
- $M(a) \rightarrow \neg M(b) \wedge \neg M(c)$
- $M(b) \rightarrow \neg M(a) \wedge \neg M(c)$
- $M(c) \rightarrow \neg M(a) \wedge \neg M(b)$
- Statements
- $\neg M(a) \rightarrow F(b) \wedge \neg F(c)$
- $\neg M(b) \rightarrow Al(b) \wedge \neg F(b)$
- $\neg M(c) \rightarrow \neg Al(a) \wedge \neg Al(b)$
- $\forall x. Al(x) \rightarrow \neg M(x)$
- Query: there is a murderer
- $\exists x. M(x)$

- Convert into clausal forms
- Eliminate the implication

$$M(a) \rightarrow \neg M(b) \wedge \neg M(c)$$
$$\equiv \neg M(a) \vee \left(\neg M(b) \wedge \neg M(c)\right)$$
$$\equiv (\neg M(a) \vee \neg M(b)) \wedge (\neg M(a) \vee \neg M(c))$$

- Eliminate the quantifiers
  - From $\forall x. Al(x) \rightarrow \neg M(x)$ to $\neg Al(x) \vee \neg M(x)$
  - Negate the query and convert into clausal form
  - $\neg \exists x. M(x) \equiv \forall x. \neg M(x)$, then drop the universal quantifier:

$$\neg M(x)$$

- Finally the KB and the negated query in clauses
- 1. $M(a) \vee M(b) \vee M(c)$
- 2. $\neg M(a) \vee \neg M(b)$
- 3. $\neg M(a) \vee \neg M(c)$
- 4. $\neg M(b) \vee \neg M(c)$
- 5. $M(a) \vee F(b)$
- 6. $M(a) \vee \neg F(c)$
- 7. $M(b) \vee Al(b)$
- 8. $M(b) \vee \neg F(b)$
- 9. $M(c) \vee \neg Al(a)$
- 10. $M(c) \vee \neg Al(b)$
- 11. $\neg Al(x) \vee \neg M(x)$
- 12. $\neg M(x)$

- Derivation of Resolution
- R[7, 10]

  13. $M(b) \vee M(c)$
- R[5, 8]

  14. $M(a) \vee M(b)$
- R[3, 13]

  15. $\neg M(a) \vee M(b)$
- R[14, 15]

  16. $M(b)$
- R[12, 16]$\{\boldsymbol{x} = b\}$
- 17. ()

- 7. $M(b) \vee Al(b)$
- 10.$M(c) \vee \neg Al(b)$
- 5. $M(a) \vee F(b)$
- 8. $M(b) \vee \neg F(b)$
- 3. $\neg M(a) \vee \neg M(c)$
- 13. $M(b) \vee M(c)$
- 14. $M(a) \vee M(b)$
- 15. $\neg M(a) \vee M(b)$
- 12. $\neg M(x)$
- 16. $M(b)$

Problem 4 (b)

(b) Suppose we discover that we were wrong – we cannot assume that there was only a single murderer (there may have been a conspiracy). Show that in this case the facts do not support anyone's guilt. <u>In other words, for each suspect, present a logical interpretation that supports all the facts but where that suspect is innocent and the other two are guilty.</u>

- For Alexander, find an interpretation in which the other two are guilty. Similarly for Peter and Michael.

- Since there may be a conspiracy, we need to update the KB.

- Suspects (now conspiracies are possible)
- $M(a) \lor M(b) \lor M(c)$
- $\cancel{M(a) \to \neg M(b) \land \neg M(c)}$
- $\cancel{M(b) \to \neg M(a) \land \neg M(c)}$
- $\cancel{M(c) \to \neg M(a) \land \neg M(b)}$
- Statements
- $\neg M(a) \to F(b) \land \neg F(c)$
- $\neg M(b) \to Al(b) \land \neg F(b)$
- $\neg M(c) \to \neg Al(a) \land \neg Al(b)$
- $\forall x. Al(x) \to \neg M(x)$

- Consider the case: only Alex is innocent
- A logical interpretation $< D, I >$
  - Domain: $D = \{Alex, Peter, Michael\}$
  - Interpretation mapping:

1. Constants: $I(a) = Alex, I(b) = Peter, I(c) = Michael$
2. Predicates:
   - $I(M) = \{Peter, Michael\}$
   - $I(F) = \{Peter\}$
   - $I(Al) = \{Alex\}$

- $< D, I > \vDash KB$, and Alex is innocent
- The cases for Peter and Michael are similar.

The Updated KB

- $M(a) \vee M(b) \vee M(c)$
- $\neg M(a) \rightarrow F(b) \wedge \neg F(c)$
- $\neg M(b) \rightarrow Al(b) \wedge \neg F(b)$
- $\neg M(c) \rightarrow \neg Al(a) \wedge \neg Al(b)$
- $\forall x. Al(x) \rightarrow \neg M(x)$