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T04 Machine learnning

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目录

1 Q1

(a) $Gain(lawers) = B(7/12) - [1/2\ B(2/3) + 1/2\ B(1/6)] = 0.980 - 0.784 = 0.196$

(b)

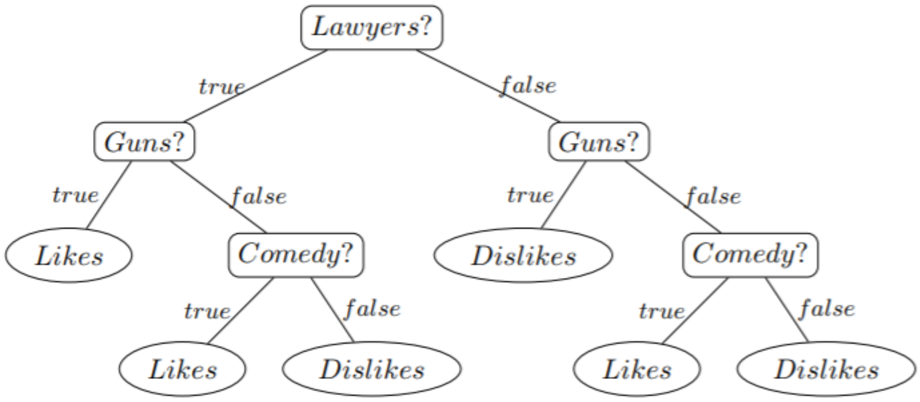

图 1: 决策树

(1) 计算决策树第二层右节点属性:

$Gain(Guns) = B(1/6) - [1/2\ B(0) + 1/2\ B(1/3)] = 0.650 - 0.459 = 0.191$

$Gain(Doctors) = B(1/6) - [1/3\ B(0) + 2/3\ B(1/4)] = 0.650 - 0.541 = 0.109$

$Gain(Comedy) = B(1/6) - [1/2\ B(0) + 1/2\ B(1/3)] = 0.650 - 0.459 = 0.191$

选择属性 $Guns$

(2) 计算决策树第二层左节点属性:

$Gain(Guns) = B(2/3) - [1/3\ B(0) + 2/3\ B(1/2)] = 0.252$

$Gain(Doctors) = B(2/3) - [1/2\ B(1/3) + 1/2\ B(1/3)] = 0$

$Gain(Comedy) = B(2/3) - [1/3\ B(0) + 2/3\ B(1/2)] = 0.252$

选择属性 $Guns$

(3) 计算决策树第三层左节点属性:

$Gain(Doctors) = B(1/2) - [1/2\ B(1/2) + 1/2\ B(1/2)] = 0$

$Gain(Comedy) = B(1/2) - [1/2\ B(0) + 1/2\ B(1)] = 1$

选择属性 $Comedy$

(4) 计算决策树第三层右节点属性:

$Gain(Doctors) = B(1/3) - [1/3\ B(0) + 2/3\ B(1/2)] = 0$

$Gain(Comedy) = B(1/2) - [1/3\ B(0) + 2/3\ B(1)] = 0.918$

选择属性 $Comedy$

2 Q2

(a) 由 $P_r(H|d) = \alpha P_r(d|H)P_r(H)$ 得，

当 $missingflavor = cherry$ 时，令 $\alpha = 1$，则

$P_r(h_1|d) = P_r(d|h_1)P_r(h_1) = 0$

$P_r(h_2|d) = P_r(d|h_2)P_r(h_2) = 0.00234375$

$P_r(h_3|d) = P_r(d|h_3)P_r(h_3) = 0.0125$

$P_r(h_4|d) = P_r(d|h_4)P_r(h_4) = 0.00527$

$P_r(h_5|d) = P_r(d|h_5)P_r(h_5) = 0$

$h_{max} = h_3$

当 $missingflavor = lime$ 时，令 $\alpha = 1$，则

$P_r(h_1|d) = P_r(d|h_1)P_r(h_1) = 0$

$P_r(h_2|d) = P_r(d|h_2)P_r(h_2) = 0.000586$

$P_r(h_3|d) = P_r(d|h_3)P_r(h_3) = 0.0125$

$P_r(h_4|d) = P_r(d|h_4)P_r(h_4) = 0.0158$

$P_r(h_5|d) = P_r(d|h_5)P_r(h_5) = 0$

$h_{max} = h_4$

所以，$missing\ flavor = cherry$

(b) 使用贝叶斯预测：

令 $\alpha = 1$，则

$P(lime|d) = \sum_i P(lime|h_i)P(h_i|d) = 0.0107884375$

$P(cherry|d) = \sum_i P(cherry|h_i)P(h_i|d) = 0.0093253125$

$P(lime|d) > P(cherry|d)$，故预测下一颗糖口味为 $lime$

使用极大似然预测：

由 $h_{ML} = argmax_h P(d|h)$，$P(X|d) = P(X|h_{ML})$，得

$h_{ML} = h_3$

$P(lime|d) = 0.5 = P(cherry|d)$

故下一颗糖两种口味都有可能

3 Q3

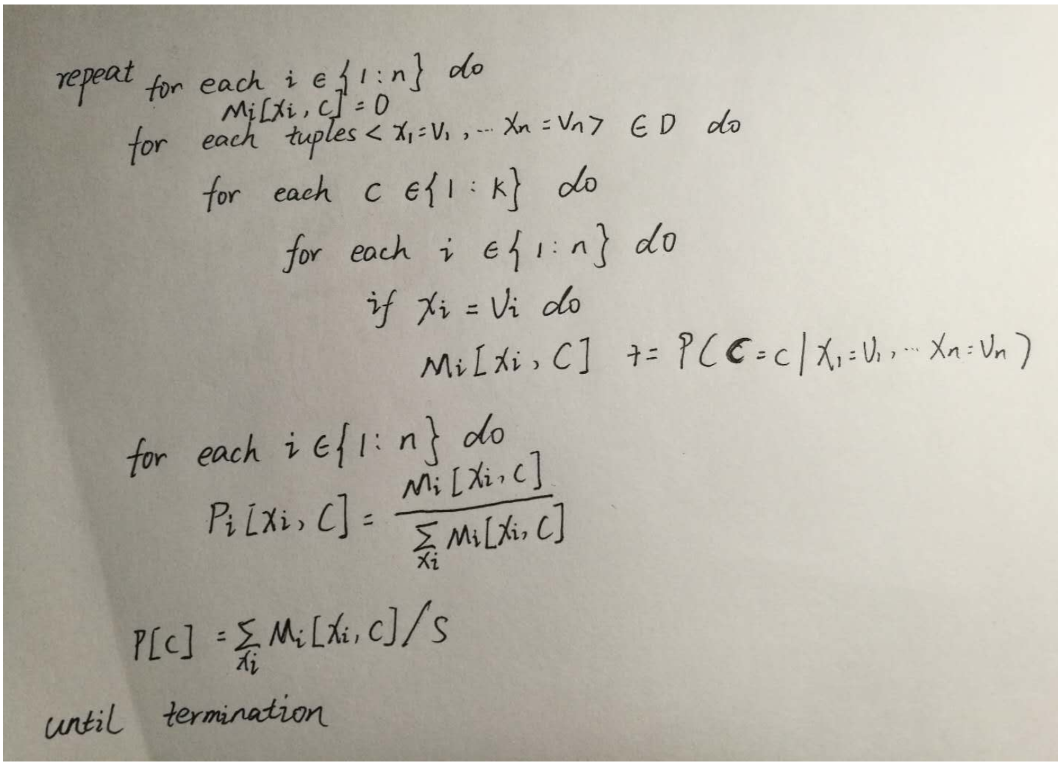

图 2: EM 算法

4 Q4

(a)

要证明收敛到真 $Qvalue$ 值，即证 $\sum_{k=1}^{\infty}\alpha_k=\infty, \sum_{k=1}^{\infty}\alpha_k^2<\infty$

(1) $\alpha_k = 1/k$

对 α_k,

$$\begin{aligned} S_n &= 1+1/2+1/3+\cdots+1/n \\ &= 1+1/2+(1/3+1/4)+(1/5+1/6+1/7+1/8)+\cdots+ \\ &\left(\frac{1}{2^{k-1}+1}+\frac{1}{2^{k-1}+2}+\cdots+\frac{1}{2^{k-1}+2^{k-1}}\right)+\frac{1}{2^k+1}+\cdots+\frac{1}{n} \\ &\geq \frac{1}{2}+\frac{1}{2}+\left(\frac{1}{4}+\frac{1}{4}\right)+\left(\frac{1}{8}+\frac{1}{8}+\frac{1}{8}+\frac{1}{8}\right)+\cdots+\left(\frac{1}{2^k}+\cdots+\frac{1}{2^k}\right) \\ &= \frac{k+1}{2} \end{aligned} \tag{1}$$

因 k 可以取任意大，故 S_n 无上界，故 $\sum_{k=1}^{\infty}\alpha_k=\infty$

对 α_k^2,

$$\begin{aligned} S_n &= 1 + 1/2^2 + 1/3^2 + \cdots + 1/n^2 \\ &= 1 + (\frac{1}{2^2} + \frac{1}{3^2}) + (\frac{1}{4^2} + \frac{1}{5^2} + \frac{1}{6^2} + \frac{1}{7^2}) + \cdots + \\ &[\frac{1}{(2^{k-1})^2} + \frac{1}{(2^{k-1}+1)^2} + \cdots + \frac{1}{(2^k-1)^2}] + \\ &\frac{1}{(2^k)^2} + \frac{1}{(2^k+1)^2} + \cdots + \frac{1}{n^2} \\ &\le 1 + \frac{2}{2^2} + \frac{4}{4^2} + \cdots + \frac{2^{k-1}}{(2^{k-1})^2} + \frac{2^k}{(2^{k2})} \\ &= 1 + \frac{1}{2^{2-1}} + (\frac{1}{2^{2-1}})^2 + \cdots + (\frac{1}{2^{2-1}})^k \\ &= \frac{1-(\frac{1}{2^{2-1}})^{k+1}}{1-\frac{1}{2^{2-1}}} \le \frac{1}{1-\frac{1}{2^{2-1}}} = \frac{2^{2-1}}{2^{2-1}-1} = 2 \end{aligned} \tag{2}$$

故 $\sum_{k=1}^{\infty} \alpha_k^2 < \infty$, 可收敛。

(2) $\alpha_k = 10/(9+k)$

对 α_k, 因为 $\frac{10}{9+k} - \frac{1}{k} = \frac{9(k-1)}{(9+k)K} \ge 0, k \ge 1$

所以，$\sum_{k=1}^{\infty} \frac{10}{9+k} \ge \sum_{k=1}^{\infty} \frac{1}{k}$

对 α_k^2, 因为 $\frac{1}{(9+k)^2} < \frac{1}{k^2}, k \ge 1$

所以，$\sum_{k=1}^{\infty} (\frac{10}{9+k})^2 \le 100 \sum_{k=1}^{\infty} \frac{1}{k^2}$

故 $\sum_{k=1}^{\infty} \alpha_k = \infty, \sum_{k=1}^{\infty} \alpha_k^2 < \infty$，可收敛

(3) $\alpha_k = 0.1$

对 α_k^2，因为 $\sum_{k=1}^{\infty} 0.01 = \infty$，所以无法收敛到真值。

(4) $\alpha_k = 0.1, 0.01, \ldots$

对 α_k,

$$\begin{aligned} \sum_{k=1}^{\infty} \alpha_k &= 1000 + 100 + 10 + 1 + \cdots \\ &= 1111.111 \cdots < 1112 < \infty \end{aligned} \tag{3}$$

故无法收敛到真值。

(b)

(c)