

# **Complex Networks: Quiz #14**

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**RUOPENG XU**  
**18M38179**

**Problem 1**

Consider a configuration model network that has vertices of degree 1, 2, and 3 only, in fractions  $p_1$ ,  $p_2$  and  $p_3$ , respectively

**Answer 1**

In fraction  $p_1$ , if we want the degree distribution equals to 1, we must only use the vertices of degree 1, and the degree distribution will be 1 forever.

In fraction  $p_2$ , if we want the degree distribution equals to 2, the number of vertex(degree 1) must equals to the number of vertex(degree 3) and the number of vertex(degree 2) can be any number.

In fraction  $p_3$ , same to the first situation, if we want the degree distribution equals to 3, we must only use the vertices of degree 3, and the degree distribution will be 3 forever.

## Problem 2

Find the value of the critical vertex occupation probability  $\phi_c$  at which site percolation take place

### Answer 2

$$\phi_c = \frac{\langle k \rangle}{\langle k^2 \rangle - \langle k \rangle} = \frac{\text{sum}(z)/\text{len}(z)}{\text{sum}(z^2)/\text{len}(z) - \text{sum}(z)/\text{len}(z)} = \frac{\text{sum}(z)}{\text{sum}(z^2) - \text{sum}(z)} = \frac{p_1 + 2p_2 + 3p_3}{2p_2 + 6p_3}$$

If  $p_1, p_2$  and  $p_3$  must exist (not equal to 0), to make the  $\phi_c$  minimum, we need to set  $p_1$  to 1,  $p_2$  to 1 and the final result is 0.540.

### Problem 3

Show that there is no giant cluster for any value of the occupation probability  $\phi$  if  $p_1 > 3p_3$ . Why does this result not depend on  $p_2$ ?

### Answer 3

In problem 2 we can see that :

$$\phi_c = \frac{\langle k \rangle}{\langle k^2 \rangle - \langle k \rangle} = \frac{\text{sum}(z)/\text{len}(z)}{\text{sum}(z^2)/\text{len}(z) - \text{sum}(z)/\text{len}(z)} = \frac{\text{sum}(z)}{\text{sum}(z^2) - \text{sum}(z)} = \frac{p_1 + 2p_2 + 3p_3}{2p_2 + 6p_3}$$

In this equation, make  $p_2 = 10 - p_1 - p_3$

So  $\phi_c = \frac{20 - p_1 + p_3}{20 - 2p_1 + 4p_3}$ . In this equation, if  $p_1 = 3p_3$ ,  $\phi_c = 1$ , which means the model will be a giant cluster if no vertices is removed. If  $p_1 > 3p_3$ ,  $\phi_c > 1$ , so there is no giant cluster for any value of  $\phi$ .

This result not depend on  $p_2$  because  $p_2$  can be represented as the combination of  $p_1$  and  $p_3$ , so it will not appear in the equation of the final result.

## Problem 4

Please explain why your friends have more friends than you do

### Answer 4

There are two distributions of friendship, one is friend of individuals (the normal number of friends we count) and the other is friends of friends (some of the same individuals over and over). When we use calculate the friends of friends, many people contribute to this distribution more than once. So it is a different distribution from the number of friends among individuals.

In general, if the original distribution has  $n$  individuals with  $x_i$  friends, the mean is  $\sum x_i / n$ . However, the distribution of friends has  $\sum x_i$  cases (for all of the friends) and they have a total of  $\sum x_i^2$  friends, since each individual is counted as many times as she or he has friends,  $x_i$ , and that individual has  $\sum x_i$  friends. Thus, the mean number of friends among the friends is  $\sum x_i^2 / \sum x_i$ , it is bigger (or equal) than  $\sum x_i$  according to the slides, which shows that the mean among friends is always at least as great as the mean among individuals.[1]

[1] Feld, S. L. (1991). *Why your friends have more friends than you do*. *American Journal of Sociology*, 96(6), 1464-1477.