Computational Physics; January–May 2020
Assignment 3
Due: Monday, 18 May 2020

Instructions

- When you are asked to write a code, submit your code by posting it to Github and sending in the Github link.
- When you are asked to solve something manually, or when you are asked for a number or a plot as an answer, submit your response by email.

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1. Write a Python code to compute the Fourier transform of the sinc function

$$f(x) = \begin{cases} \sin x/x, & \text{if } x \neq 0 \\ 1, & \text{otherwise,} \end{cases} \tag{1}$$

using Numpy. Make a plot comparing your numerical result with the analytical result.

2. Write a C code to compute the Fourier transform of the sinc function using FFTW. Put the result on the plot you made for the above exercise to show that your computation is correct.

3. Write a C code to compute the Fourier transform of the sinc function using GSL. Put the result on the plot you made for the above exercises to show that your computation is correct.

4. Write a C code to compute the Fourier transform of the Gaussian function

$$f(x) = \exp\left(-x^2\right) \tag{2}$$

using FFTW. Make a plot comparing your numerical result with the analytical result.

5. Take n numbers and write a Python code to compute their DFT using direct computation, i.e., without using FFT. Now compute the same DFT using `numpy.fft.fft`. Measure the time taken by the two methods. Now change the number n over a wide range of values, say from 4 to 100, and repeat the exercise. Make a plot showing the time taken by the two DFT methods as a function of n.

6. Write a Python code to compute the Fourier transform of a constant function.

7. Show that FFT is an $\mathcal{O}(n \log_2 n)$ algorithm.

8. Compute the two-dimensional Fourier transform of the Gaussian function

$$f(x, y) = \exp\left(-[x^2 + y^2]\right) \tag{3}$$

using `numpy.fft.fft2`. Make a three-dimensional plot comparing your numerical result with the analytical result.

9. Write a Python code to compute the convolution of the box function

$$f(x) = \begin{cases} 1 & \text{if } -1 < x < 1 \\ 0, & \text{otherwise.} \end{cases} \tag{4}$$

with itself. Make a plot that shows the result as well as the above box function.

10. An experiment has reported several independent measurements of a quantity. These measurements are listed in `http://theory.tifr.res.in/~kulkarni/noise.txt`. Plot these measurements. Plot their DFT. Compute their power spectrum using the periodogram. Make a plot of the power spectrum. Bin the power spectrum in ten k bins. Plot the binned power spectrum.