

Bihan Q

Covariate, Pairwise, Anova, Tied Obs

Q1(a)

Consider the ChickWeight data in R. The body weights of the chicks were measured at birth (i.e., time=0) and every second day thereafter until day 20. They were also measured on day 21.

There were four groups of chicks on different protein diets.

1. Determine whether there is a significant difference in the mean weights of the four groups on Day 18:

a) Without adjusting for Birth Weight

```
In [46]: # packages
import pandas as pd
```

```
In [47]: # get data
import statsmodels.api as sm
ChickWeight = sm.datasets.get_rdataset('ChickWeight', 'datasets')
chick = pd.DataFrame(ChickWeight.data)
print(chick.shape)
chick.head(40)
```

(578, 4)

Out[47]:

	weight	Time	Chick	Diet
0	42	0	1	1
1	51	2	1	1
2	59	4	1	1
3	64	6	1	1
4	76	8	1	1
5	93	10	1	1
6	106	12	1	1
7	125	14	1	1
8	149	16	1	1
9	171	18	1	1
10	199	20	1	1
11	205	21	1	1
12	40	0	2	1
13	49	2	2	1
14	58	4	2	1
15	72	6	2	1
16	84	8	2	1
17	103	10	2	1
18	122	12	2	1
19	138	14	2	1
20	162	16	2	1
21	187	18	2	1
22	209	20	2	1
23	215	21	2	1
24	43	0	3	1
25	39	2	3	1
26	55	4	3	1
27	67	6	3	1
28	84	8	3	1
29	99	10	3	1
30	115	12	3	1
31	138	14	3	1
32	163	16	3	1
33	187	18	3	1

	weight	Time	Chick	Diet
34	198	20	3	1
35	202	21	3	1
36	42	0	4	1
37	49	2	4	1
38	56	4	4	1
39	67	6	4	1

In [48]: `chick['Diet'].value_counts()`

Out[48]:

```
1    220
2    120
3    120
4    118
Name: Diet, dtype: int64
```

In [49]: `chick.describe()`

Out[49]:

	weight	Time	Chick	Diet
count	578.000000	578.000000	578.000000	578.000000
mean	121.818339	10.717993	25.750865	2.235294
std	71.071960	6.758400	14.568795	1.162678
min	35.000000	0.000000	1.000000	1.000000
25%	63.000000	4.000000	13.000000	1.000000
50%	103.000000	10.000000	26.000000	2.000000
75%	163.750000	16.000000	38.000000	3.000000
max	373.000000	21.000000	50.000000	4.000000

In [50]:

```
chick_18 = chick[chick['Time'] == 18]
print(chick_18.shape)
chick_18.head(3)
```

(47, 4)

Out[50]:

	weight	Time	Chick	Diet
9	171	18	1	1
21	187	18	2	1
33	187	18	3	1

In [51]:

```
# with fixed time, weight is Y, diets are groups and independent
from statsmodels.formula.api import ols
model = ols('weight ~ Diet', data=chick_18).fit()
anova_table = sm.stats.anova_lm(model)

# Print the ANOVA table
```

```
print("One-Way ANOVA Table:")
print(anova_table)
```

One-Way ANOVA Table:

	df	sum_sq	mean_sq	F	PR(>F)
Diet	1.0	23263.992032	23263.992032	8.161704	0.006455
Residual	45.0	128267.284564	2850.384101	NaN	NaN

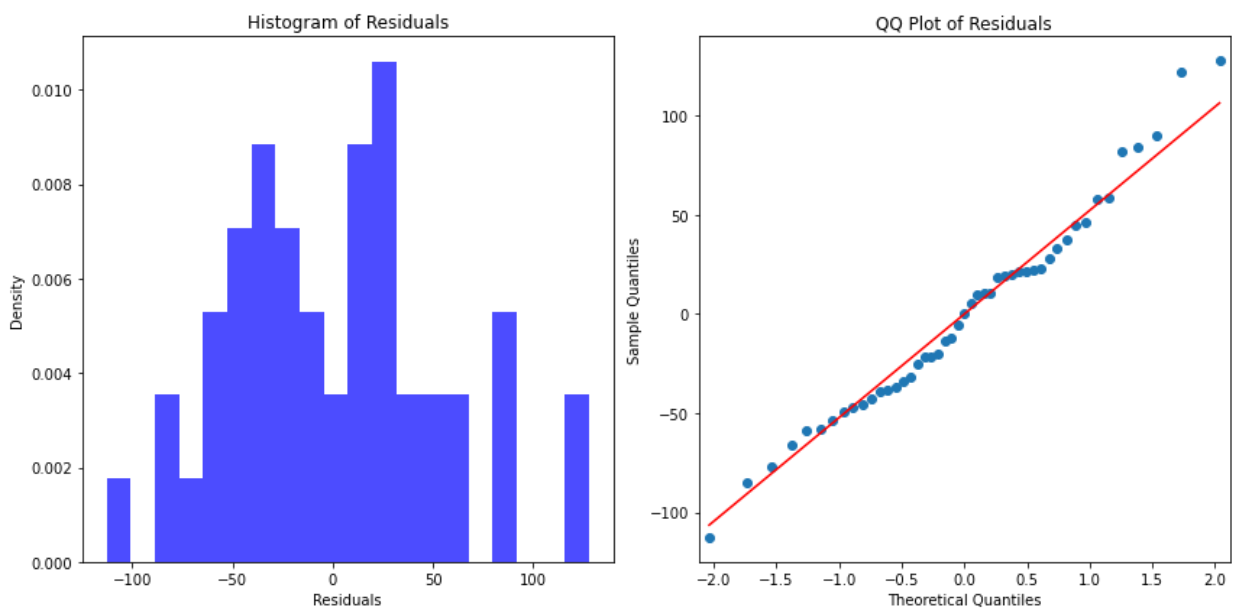
```
In [52]: #check non-normality
import matplotlib.pyplot as plt

# hist
model = ols('weight ~ Diet', data=chick_18).fit()
residuals = model.resid
plt.figure(figsize=(12, 6))
plt.subplot(1, 2, 1)
plt.hist(residuals, bins=20, density=True, color='b', alpha=0.7)
plt.title('Histogram of Residuals')
plt.xlabel('Residuals')
plt.ylabel('Density')

# qq
plt.subplot(1, 2, 2)
sm.qqplot(residuals, line='s', ax=plt.gca())
plt.title('QQ Plot of Residuals')

plt.tight_layout()
plt.show()

# Bartlett's test
import pingouin as pg
bartlett_test_result = pg.homoscedasticity(chick_18, dv='weight', group='Diet')
print("Bartlett's Test for Homogeneity of Variances:")
print(bartlett_test_result)
```



Bartlett's Test for Homogeneity of Variances:

	W	pval	equal_var
levene	1.058972	0.376444	True

```
In [53]: # shapiro
from scipy.stats import shapiro
```

```

model = ols('weight ~ Diet', data=chick_18).fit()
residuals_model_with_covariate = model.resid
shapiro_test_statistic, shapiro_test_pvalue = shapiro(residuals_model_with_covariate)

print("Shapiro-Wilk Test Statistic:", shapiro_test_statistic)
print("p-value:", shapiro_test_pvalue)

```

Shapiro-Wilk Test Statistic: 0.980424702167511
p-value: 0.6098649501800537

```

In [54]: # Bartlett's test
bartlett_test_result = pg.homoscedasticity(chick_18, dv='weight', group='Diet')
print("Bartlett's Test for Homogeneity of Variances:")
print(bartlett_test_result)

```

Bartlett's Test for Homogeneity of Variances:

	W	pval	equal_var
levene	1.058972	0.376444	True

Answer:

- Under satisfied normality and equal variance assumption, 26.098205 as F-stats and 4.418863e-07 as p-value indicate **a significant difference in the mean weights of the four groups on Day 18.**

Q1(b)

b) Adjusting for Birth Weight. Give the LS Means (i.e., adjusted for Birth Weight).

```

In [55]: # get initial weight
birth_weight = chick[chick['Time'] == 0][['Chick', 'weight']]
birth_weight.rename(columns={'weight': 'weight_initial'}, inplace=True)
chick_ajusted = pd.merge(chick, birth_weight, on='Chick', how='left')
chick_ajusted_18 = chick_ajusted[chick_ajusted['Time'] == 18]
print(chick_ajusted_18.shape[0]==chick_18.shape[0])
chick_ajusted_18.head(3)
chick_ajusted_18['weight_initial'].value_counts()

```

```

Out[55]: True
41    18
42    14
39     6
40     5
43     4
Name: weight_initial, dtype: int64

```

```

In [56]: chick_ajusted_18['Diet'].value_counts()

```

```

Out[56]: 1    17
2    10
3    10
4    10
Name: Diet, dtype: int64

```

```
In [57]: # two-anova (python)
model2 = ols('weight ~ Diet+weight_initial', data=chick_adjusted_18).fit()
anova_table2 = sm.stats.anova_lm(model2)
print(anova_table2)
```

	df	sum_sq	mean_sq	F	PR(>F)
Diet	1.0	23263.992032	23263.992032	8.573318	0.005384
weight_initial	1.0	8871.784187	8871.784187	3.269457	0.077419
Residual	44.0	119395.500377	2713.534099	NaN	NaN

```
In [58]: # anova (r)
from IPython.display import Image
image_file_path = f"C:/Users/11139/Desktop/STAT5391/anova2.png"
Image(filename=image_file_path)
```

```
Out[58]: 300 # anova
301 library(car)
302 m <- aov(data=chick_adjusted_18, weight ~ Diet+weight_initial)
303 summary(m)
304 ^
```

	Df	Sum Sq	Mean Sq	F value	Pr(>F)
Diet	3	36690	12230	4.729	0.00623 **
weight_initial	1	6229	6229	2.409	0.12818
Residuals	42	108612	2586		

Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

```
In [59]: # pairwise test
import pingouin as pg
from statsmodels.stats.anova import AnovaRM
# ls_means = pg.pairwise_ttests(data=chick_adjusted_18, dv='weight', between='Diet', co
from IPython.display import Image
image_file_path = f"C:/Users/11139/Desktop/STAT5391/pairwise_test.png"
Image(filename=image_file_path)
```

```

Out[59]: 292 library(stats)
293 library(stats)
294 pairwise_tests <- pairwise.t.test(
295   x = chick_adjusted_18$weight,
296   g = chick_adjusted_18$Diet,
297   p.adjust.method = "fdr",
298   covariates = chick_adjusted_18$weight_initial
299 )
300
301 # Print the results
302 print(pairwise_tests)
303
304

```

Pairwise comparisons using t tests with pooled SD

data: chick_adjusted_18\$weight and chick_adjusted_18\$Diet

	1	2	3
2	0.2379	-	-
3	0.0049	0.1119	-
4	0.1119	0.5142	0.2379

P value adjustment method: fdr

```

In [60]: # LSM
image_file_path = f"C:/Users/11139/Desktop/STAT5391/LSM.png"
Image(filename=image_file_path)

```


```

Out[60]: 314 model <- lm(weight ~Diet+weight_initial, chick_adjusted_18)
315 anova_result <- anova(model)
316 lsmeans_result <- lsmeans(model, "Diet", cov.reduce=TRUE)
317 lsmeans_result
318

```

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Console

Background Jobs 



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4	202	16.1	42	169	234
---	-----	------	----	-----	-----

Confidence level used: 0.95

```
> lsmeans_result <- lsmeans(model, "Diet", cov.reduce=TRUE)
```

```
> lsmeans_result
```

Diet	lsmean	SE	df	lower.CL	upper.CL
1	164	12.8	42	138	190
2	183	16.3	42	150	216
3	230	16.2	42	197	262
4	202	16.1	42	169	234

Confidence level used: 0.95

```
> |
```

Answer:

Two-way Anova:

1. Compared to before adjusted, after we add initial_birth_weight as second variable, diet with p-value 0.00623 indicates that **there is a statistically difference in the mean weights among diet groups on Day18.**
2. From p-value = 0.12818, **different initial weight groups does not have a significant difference on mean weight on Day18.**
3. Python and R gives little different numbers, but similar in result significance.

Pairwise:

1. Only p-value for the comparison between **Group 1 and Group 3** is 0.0049. **There is statistically significant difference in the mean weights between Group 1 and Group 3 after adjusting for the covariate 'weight_initial' at Day18.** For other groups, there are no significant difference.

LSM:

1. Compared to original weights for four groups 1-4 (158.94,187.70,233.10,202.90), we get new adjusted weight for group 1-4 is 164,183,230,and 202. **Four groups of diet are different.**

1(c)

c)Check the validity of your assumptions, including parallelism. Suggest measures that you would take if the assumptions are not satisfied.

```
In [61]: # Shapiro-Wilk
# from scipy.stats import shapiro
# shapiro_test_statistic, shapiro_test_pvalue = shapiro(residuals_model_with_covariate)
image_file_path = f"C:/Users/11139/Desktop/STAT5391/shapiro.png"
Image(filename=image_file_path)
```

```
Out[61]: 300 # anova
301 library(car)
302 m <- aov(data=chick_adjusted_18, weight ~ Diet+weight_initial)
303 summary(m)
304
305 shapiro.test(resid(aov(data=chick_adjusted_18, weight ~ Diet+weight_initial)))
306 ^
```


Shapiro-Wilk normality test

data: resid(aov(data = chick_adjusted_18, weight ~ Diet + weight_initial))
W = 0.98613, p-value = 0.8441

```
In [62]: # Levene's and Box's test
image_file_path = f"C:/Users/11139/Desktop/STAT5391/Box.png"
Image(filename=image_file_path)
```



```
Out[62]: 319 leveneTest(weight ~Diet, chick_adjusted_18)
320 fligner.test(weight ~Diet, chick_adjusted_18) #alternative to Box, robust to normality
321
```

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```
> leveneTest(weight ~Diet, chick_adjusted_18)
Levene's Test for Homogeneity of Variance (center = median)
      Df F value Pr(>F)
group  3   1.059 0.3764
      43

> fligner.test(weight ~Diet, chick_adjusted_18) #alternative to Box, robust to normality

      Fligner-Killeen test of homogeneity of variances

data: weight by Diet
Fligner-Killeen:med chi-squared = 2.6903, df = 3, p-value = 0.4419
```

```
In [63]: # Two-way ANOVA with interaction
two_way_anova_result = sm.stats.anova_lm(ols('weight ~ Diet * weight_initial' , data=chick_wt))
print("Two-Way ANOVA with Interaction:")
print(two_way_anova_result)
```

Two-Way ANOVA with Interaction:

	df	sum_sq	mean_sq	F	PR(>F)
Diet	1.0	23263.992032	23263.992032	8.393622	0.005901
weight_initial	1.0	8871.784187	8871.784187	3.200930	0.080642
Diet:weight_initial	1.0	215.522671	215.522671	0.077760	0.781693
Residual	43.0	119179.977706	2771.627389	NaN	NaN

Answer:

Shapiro-Wilk:

- Shapiro-Wilk p-value (0.8441) shows normality assumption is **satisfied**.

Bartlett's test:

- Assumption of constant variance is **satisfied**.

Interaction(parallelism):

- The p-value for the interaction term (0.781693), suggesting **no interaction** effect between diet and weight_initial.

2(a)

2.For 1a), perform pairwise comparisons among the 4 groups using each of the following, and comment on the results a)Bonferroni method

```
In [64]: from IPython.display import Image
image_file_path = f"C:/Users/11139/Desktop/STAT5391/Bonferroni.png"
Image(filename=image_file_path)
```

Out[64]:

```
335
336 # bonferroni
337 model <- aov(weight ~ Diet, data = chick_18)
338 pairwise_results <- pairwise.t.test(chick_18$weight, chick_18$Diet, p.adjust.method =
  "bonferroni")
339 print(pairwise_results)
340
341
```

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```
> print(pairwise_results)
```

Pairwise comparisons using t tests with pooled SD

data: chick_18\$weight and chick_18\$Diet

	1	2	3
1			
2	1.0000	-	-
3	0.0049	0.3358	-
4	0.2313	1.0000	1.0000

P value adjustment method: bonferroni

```
> |
```

Answer:

- There is significant difference in the mean weights of the diet1 and diet3 on Day 18.
- All other groups are not significantly different from others on Day18..

2(b)

2.For 1a), perform pairwise comparisons among the 4 groups using each of the following, and comment on the results b)Tukey method

In [65]:

```
from statsmodels.stats.multicomp import pairwise_tukeyhsd

post_hoc = pairwise_tukeyhsd(chick_18['weight'], chick_18['Diet'], alpha=0.05)
#table
print("ANOVA Table:")
print(anova_table)
print("\nPairwise Comparisons (Bonferroni Correction):")
print(post_hoc)
```

ANOVA Table:

	df	sum_sq	mean_sq	F	PR(>F)
Diet	1.0	23263.992032	23263.992032	8.161704	0.006455
Residual	45.0	128267.284564	2850.384101	NaN	NaN

Pairwise Comparisons (Bonferroni Correction):

Multiple Comparison of Means - Tukey HSD, FWER=0.05

group1	group2	meandiff	p-adj	lower	upper	reject
1	2	28.7588	0.5085	-26.2809	83.7986	False
1	3	74.1588	0.0044	19.1191	129.1986	True
1	4	43.9588	0.1586	-11.0809	98.9986	False
2	3	45.4	0.2173	-16.3638	107.1638	False
2	4	15.2	0.9123	-46.5638	76.9638	False
3	4	-30.2	0.5638	-91.9638	31.5638	False

Answer:

- Same as Bonferroni correction, by Tukey, there is significant difference in the mean weights of the diet1 and diet3 on Day 18.

Q3

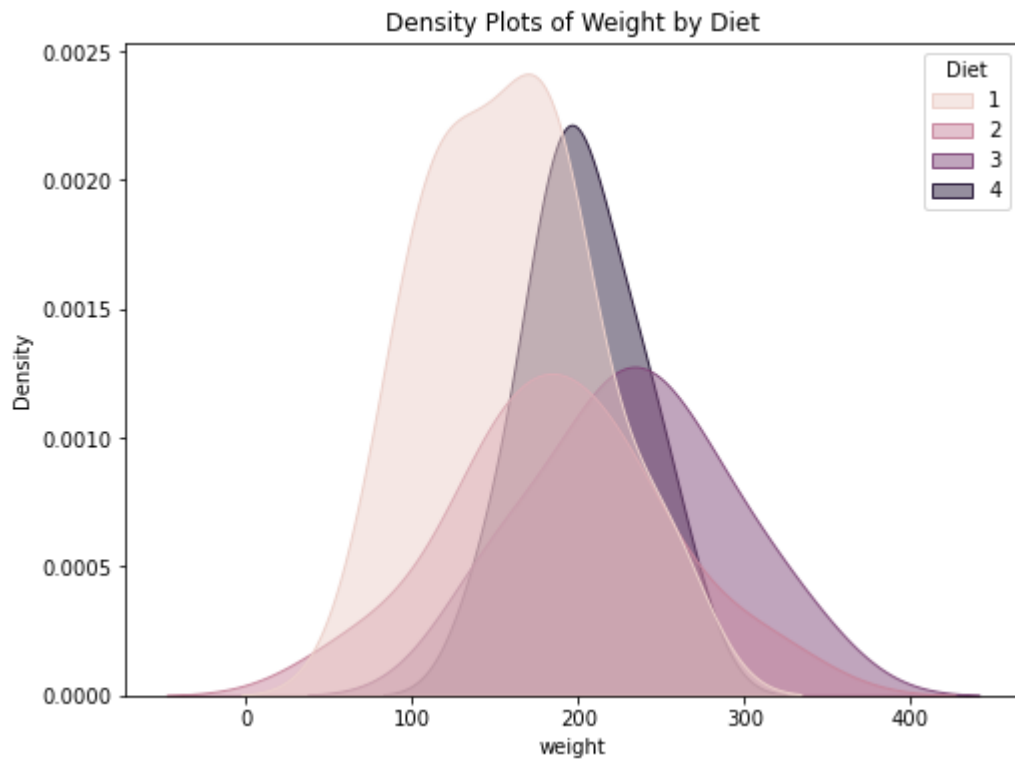
Repeat 1a) using the Kurskal-Wallis test

```
In [66]: # exam of tie data
chick_18['weight'].value_counts()
```

```
Out[66]: 187    2
          163    2
          234    2
          146    2
          184    2
          185    2
          171    1
          227    1
          204    1
          263    1
          294    1
          332    1
          232    1
          157    1
          230    1
          214    1
          262    1
          198    1
          174    1
          210    1
          261    1
          151    1
          231    1
          207    1
          205    1
          154    1
          199    1
          160    1
          250    1
          134    1
          100    1
          112    1
           81    1
          248    1
          123    1
          120    1
          107    1
          307    1
          148    1
           72    1
          203    1
          Name: weight, dtype: int64
```

```
In [67]: # density plot for each group
import matplotlib.pyplot as plt
import seaborn as sns

plt.figure(figsize=(8, 6))
sns.kdeplot(data=chick_18, x='weight', hue='Diet', fill=True, alpha=0.5)
plt.title('Density Plots of Weight by Diet')
plt.show()
```



```
In [68]: import pandas as pd
from scipy.stats import kruskal
result = kruskal(*[group["weight"] for name, group in chick_18.groupby("Diet")])
print(result)
```

KruskalResult(statistic=10.623444357500071, pvalue=0.01394646461394125)

Answer:

Discussion for assumption of Kruskal:

- Total sample size larger than 5.
- We test with numerical data weight.
- Deal with tied values, correction automatically applied.
- The distributions are similar across groups from density plot.

Result:

- The result of Kruskal test suggests that there is a significant difference in the mean weights of the four groups on Day 18.