Thesis Title

Institution Name

Author Name

Day Month Year

The Fluid-Structure Interaction problem can be seen in many parts of nature, from large industrial engineering complexes to the smallest blood vessels in the human body. A terrifying example is the collapse of the Tacoma Narrows Bridge that collapsed in 1940 only two months after being opened. The collapse was du to aero-elastic fluttering from strong winds. At the smaller scale, inter-cranial aneurysms which are balloon shaped geometries often occurring where a blood vessel splits into to two parts, due to weak vessel wall. Bursting of one of these aneurysms in the skull can have fatale consequences. We can therefore easily see the need to model these problems. The apparent difficulty however in both these problems is when fluid velocities reach speeds which gives turbulence. The solvers need then to handle these turbulences. The main goal of this master thesis is to build a framework to solve the FSI problem, looking at different approaches and schemes. The framework will be validated and verified through well known benchmarks.

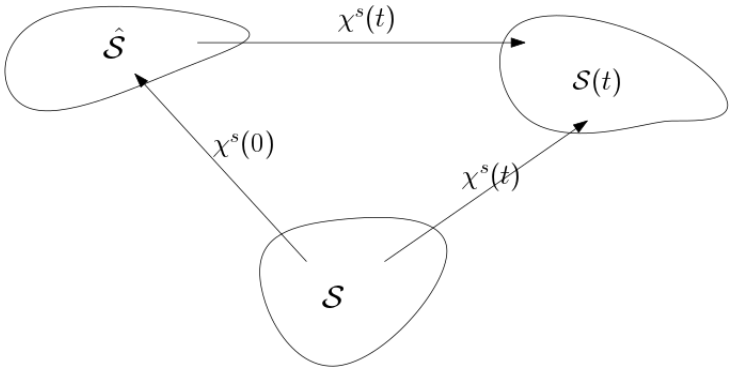
Solid Equations

The solid equations stated here will be in a Lagrangian description. This description fits a solid problem very nicely as the material particles are fixed with grid points. This we will see later is a very nice property when tracking the solid domain. The displacement vector will be the quantity describing the motion of solid.

Reference domain

Mapping and identites

To be able to state the solid equation in a Lagrangian reference configuration, we need to look at some mappings and identities:



We define $\hat{\mathcal{S}}$ as the initial stress free configuration of a given body. $\mathcal{S}$ as the reference and $\mathcal{S}(t)$ as the current configuration. We need to define a smooth mapping that maps from the reference configuration to the current configuration:

$$\chi^s(t) : \hat{\mathcal{S}} \rightarrow \mathcal{S}(t)$$

The solid mapping is set as $\chi^s(\mathbf{X}, t) = \mathbf{X} + d^s(\mathbf{X}, t)$ hence giving:

$$d^s(\mathbf{X}, t) = \chi^s(\mathbf{X}, t) - \mathbf{X}$$

$$w(\mathbf{X}, t) = \frac{\partial \chi^s(\mathbf{X}, t)}{\partial t}$$

where $\mathbf{X}$ denote a material point in the reference domain and χ^s denotes the mapping from the reference configuration. $d^s(\mathbf{X}, t)$ denotes the displacement field and w($\mathbf{X}$,t) is the domain velocity.

Deformation gradient

If $d(\mathbf{X}, t)$ is differentiable deformation field in a given body. We define the deformation gradient as:

$$F = \frac{\partial \chi}{\partial \mathbf{X}} = I + \nabla d(\mathbf{X}, t)$$

which denotes relative change of position under deformation in a Lagrangian frame of reference. The similar Eulerian viewpoint is defined as the inverse deformation gradient

$$\hat{\mathbf{F}} = I - \nabla d(\mathbf{X}, t)$$

J is det(F). In continuum mechanics relative change of location of particles is called strain and this is the fundamental quality that causes stress in a material. [godboka]. We say that stress is the internal forces between neighboring particles. E denotes the Green-Lagrangian strain tensor $E = F^T F - I$. This measures the squared length change under deformation.

Solid equation

From the principles of conservation of mass and momentum, we get the solid equation stated in the Lagrangian reference system (Following the notation and theory from Richter, Godboka"):

$$\rho_s J \frac{\partial d^2}{\partial t^2} = \nabla \cdot (F\Sigma) + J\rho_s f \tag{1}$$

where f is the body force and Σ denotes the St. Venant Kirchhoff material law:

$$\Sigma = 2\mu_s E + \lambda_s tr(E)I$$

Multiplying Σ with F we get the 2nd Piola Krichhoff stress tensor. This gives us a non-linear stress tensor.

Locking

The problem og shear locking can happen FEM computations with certain elements. [mek4250 Kent] - Locking occurs if $\lambda >> \nu$ that is, the material is nearly incompressible. The reason is that all the elements discussed in this course are poor at approximating the divergence. Locking refers to the case where the displacement is to small because the divergence term essentially lock the displacement. It is a numerical artifact not a physical feature. [Verbatum]

Fluid equations

The Navier-Stokes equations are derived using principles of mass and momentum conservation. These equations describes the velocity and pressure in a given fluid continuum. They are here written in the time domain $\mathcal{F}$:

$$\rho \frac{\partial u}{\partial t} + \rho u \cdot \nabla u = \nabla \cdot \sigma_f + f \tag{2}$$

$$\nabla \cdot u = 0 \tag{3}$$

where ρ stands for constant density, and $\sigma_f = \mu_f(\nabla u + \nabla u^T) - pI$

Fluid Structure Interaction Problem formulation in ALE coordinates

The ALE formulation stands for Arbitrary Lagrangian Eulerian. This entails formulating the fluid equations in an Eulerian and the solid in a Lagrangian framework. This is as previously stated a very nice approach as it describes both fluid velocity and displacement in known configurations. The ALE method can be solved by moving the mesh for each time step following the structure body movements, and calculating on a new"mesh every time. This approach gives advantages as we can explicitly represent the fluid-structure interface, but problems arise when there are large deformations in the solid giving large deformations to the fluid domain. We will therefore approach the ALE from a reference frame. That is we solve the equation on a initial, stress free domain, and use a series of mappings to account for the movements of the domain. From a technical point of view, both formulations are equivalent. But the ease of computing on fixed mesh does so that we chose fixed approach.
Since the domain is moving with the structure movements, we will need to state the fluid in a little different.

Notation

u - Velocity in fluid and structure.
w- Velocity in the domain. It is the velocity of the mesh in the calculations. This will also be the velocity in structure when defined in the lagrangian formulation.
d - Displacement of the solid. The time derivative of the displacement will be the domain velocity.
p - Pressure in the fluid.
$\hat{\mathcal{S}}$ - Solid reference domain
$\mathcal{S}$ - Solid current domain
$\hat{\mathcal{F}}$ - Fluid reference domain
$\mathcal{F}$ - Fluid reference domain

Full FSI problem

Find $u \in \mathcal{F}, p \in \mathcal{F}$ and $d \in \mathcal{S}$ such that :

$$\rho_f \frac{\partial u}{\partial t} + (\nabla u)(u - \frac{\partial d}{\partial t}) + \nabla \cdot \sigma_f = 0 \text{ on } \mathcal{F} \tag{4}$$

$$\nabla \cdot u = 0 \text{ on } \mathcal{F} \tag{5}$$

$$\nabla^2 d = 0 \text{ on } \mathcal{F} \tag{6}$$

$$\rho_s \frac{\partial u}{\partial t} + \nabla \cdot \sigma_s = 0 \text{ on } \mathcal{S} \tag{7}$$

$$u - \frac{\partial d}{\partial t} = 0 \text{ on } \mathcal{S} \tag{8}$$

$$\sigma_f n_f = \sigma_s n_s \text{ on } \Gamma \tag{9}$$

Balance laws

We will formulate the equations in the Eulerian, Lagrangian and the ALE description.

Solid

We express the solid balance laws in the Lagrangian formulation from the initial configuration

$$\rho_s \frac{\partial^2 d}{\partial t^2} = \nabla \cdot (J \sigma_s F^{-T}) \quad in \quad \hat{\mathcal{S}}$$

Fluid

The fluid equations are denoted from the initial configuration:

$$\rho_f J\big(\frac{\partial u}{\partial t} + ((\nabla u)F^{-1}(u - \frac{\partial d}{\partial t})\big) = \nabla \cdot (J\sigma_f F^{-T}) \quad on \ \hat{\mathcal{F}}$$

$$\nabla \cdot (JuF^{-T}) = 0 \quad on \ \hat{\mathcal{F}}$$

As we see the only difference from the usual N-S equations, is that in the convection term we have $u - \frac{\partial d}{\partial t}$ which is needed since the not only are the fluid particles moving but the domain, here denoted with $\frac{\partial d}{\partial t}$ as the domain velocity, is also moving. So $u - \frac{\partial d}{\partial t}$ will be the actual fluid velocity.

Harmonic extension

To bind together the computation of fluid and structure domain, we need a harmonic extension to the boundary values. The solid deformation d is extended from the interface into the fluid domain and is done to help deal with big deformations in fluid domain. These big deformations can then cause several challenges to the ALE mapping. For this purpose define the following harmonic extension equation in the fluid domain:

$$\nabla^2 d^f = 0 \quad in \ \hat{\mathbf{F}}$$

This equation is chosen for its good regularity and smoothing properties.
It is also possible to chose an harmonic extension with stiffening, which can give better control of the deformed meshes. This in practice behaves like a transport problem, transporting the deformation into the fluid domain. Another possibility is extension by pseudo-elasticity which defines the extension operator by means of the Navier-Lame equation. And lastly we can chose a biharmonic extension, that is of fourth order character, and thus will have a high computational cost. For now I will stick with the harmonic extension and maybe look at these in the future. [Godboka.]

Coupled Fluid Structure Interface conditions

This section is based on We introduce a global domain $\Omega \in \mathcal{S} \cup \mathcal{F}$ that is made up of the fluid and the structure and the interface. We define a global velocity function u that is the fluid velocity in the fluid domain and the structure velocity in the structure domain. This can be done due to the interface condition making the velocity field continuous over the entire domain. Then the interface will be $\Gamma \in \mathcal{S} \cap \mathcal{F}$ We need to define conditions that couple that motions of the fluid and structure together. These consist of:

- Kinematic condition: $u_f = u_s \quad on\Gamma$. The fluid and structure velocities need to be equal on the boundary.
  We later realize this later by setting $u - \frac{\partial d}{\partial t} = 0$ on the solid domain to strongly imply that the velocity on the solid is the derivative of the deformation.

- Dynamic condition: $\sigma_f n_f = \sigma_s n_s \quad on \ \Gamma$.
  This relates to Newtons third law of action and reaction. The forces on the interface area, here written as the normal stresses are balanced on the interface. These will be written in a Lagrangian formulation:
  $J\sigma_f F^{-T} n_f = F\Sigma n_s \quad on \ \Gamma$.
  This condition is often added to structure problem, since it is most often in FSI problem the fluid exerting force that cause deformation.

- Geometrical condition: This condition says that the fluid and structure domains do not overlap, but rather that elements connect so the functions needing to transfer force are continouos across the entire domain.

Boundary conditions

We usually have a Dirichlet condition on rigid walls.

$$u(x, y, t) = 0 \text{ on } \Gamma^D$$

FSI Problem in reference domain

Find $u \in \hat{\mathcal{F}}, p \in \hat{\mathcal{F}}$ and $d \in \hat{\mathcal{S}}$ such that :

$$\rho_f J \frac{\partial u}{\partial t} + (\nabla u) F^{-1}(u - \frac{\partial d}{\partial t}) + \nabla \cdot (J \sigma_f F^{-T}) = 0 \text{ on } \hat{\mathcal{F}} \tag{10}$$

$$\nabla \cdot (J u F^{-T})) = 0 \text{ on } \hat{\mathcal{F}} \tag{11}$$

$$\rho_s \frac{\partial u}{\partial t} + \nabla \cdot F S_s, = 0 \text{ on } \hat{\mathcal{S}} \tag{12}$$

$$\nabla^2 d = 0 \text{ on } \hat{\mathcal{F}} \tag{13}$$

$$u - \frac{\partial d}{\partial t} = 0 \text{ on } \hat{\mathcal{S}} \tag{14}$$

$$J \sigma_f F^{-T} n_f = \sigma_s n_s \text{ on } \Gamma \tag{15}$$

Finite Element FSI in ALE

Variational formulation

Reference domain

We use 3 testfunctions, ϕ, ψ, γ. As mentioned before we use a global velocity function u for both the solid and fluid.

$$\rho_f J \big(\frac{\partial u}{\partial t} + (\nabla u) F^{-1}(u - \frac{\partial d}{\partial t}), \phi \big)_{\hat{\mathbf{F}}} + (J \sigma_f F^{-T}, \nabla \phi)_{\hat{\mathbf{F}}} = 0 \tag{16}$$

$$\big(\nabla \cdot (J u F^{-T}), \gamma \big)_{\hat{\mathbf{F}}} = 0 \tag{17}$$

$$\big(\rho_s \frac{\partial u}{\partial t}, \phi \big)_{\hat{\mathbf{S}}} + \big(F S_s, \nabla \phi \big)_{\hat{\mathbf{S}}} = 0 \tag{18}$$

$$\big(\nabla d, \nabla \psi \big)_{\hat{\mathbf{F}}} = 0 \tag{19}$$

$$\big(u - \frac{\partial d}{\partial t}, \psi \big)_{\hat{\mathbf{S}}} = 0 \tag{20}$$

Equation (5) has not been addressed and is added since we use a global function for velocity we need to force that the structure velocity is the time derivative of the deformation in the structure domain.

Spaces and Elements

The velocity and pressure copling in the fluid domain must satisfy the inf-sup condition. If not stabilization has to added. We here need to define some spaces that will have these desired properties. We denote $u_h \in V_h$ and $d_h \in W_h$, here the finite element pair og pressure and velocity mush satisfy the inf-sup condition given in ALE coordinates:

$$\inf_{p_h \in L_{h,f}} \sup_{v_h \in V_{h,f}} \frac{(p_h, div(J_f F_f^{-1} u_h))_{\mathcal{F}}}{\| \| J^{\frac{1}{2}} p_h \| \|_{\mathcal{F}} \| \| J_f^{\frac{1}{2}} \nabla u_h F_f^{-T} \| \|_{\mathcal{F}}} \geq \gamma$$

A good choice of spaces will be P2-P2-P1 for velocity, displacement and fluid pressure respectively.

Introduction

Here we will look at the partitioned approach to solving the FSI problem. This means splitting our scheme into parts where we solve the fluid, structure and extension problem in different steps. This is to greatly reduce the size of the computational cost, and hopefully increase speed. So far the methods for coupling of the fluid and structure, has led to unconditional numerical instabilites and a large added-mass effect. Here we look at a new approach to explicit coupling, first proposed by Fernadez, which uses a Robin-Neumann explicit treatment of the interface first for thin walled structure [1] but later with an extension to thick walled structures[2]. This combined with a lumped mass approximation of the solid problem ensures added-mass free stability. [Generalized R-N explicit coupling schemes]

Robin-Neumann Interface

The Robin-Neumann treatment of the interface proposed by Fernandez uses a boundary operator $B_h : \Lambda_{\Sigma,h} \to \Lambda_{\Sigma,h}$ which is used together with the known coupling of stresses:

$$J^n \sigma^f(u^n, p^n)(F^n)^{-T} n^f + \frac{\rho^s}{\tau} B_h u^n = \frac{\rho^s}{\tau} B_h(\dot{d}^{n-1} + \tau \partial_t \dot{d}^*) - \Pi^* n^s$$

- The explicit treatment of the solid ensures uncoupling of the fluid and solid computations. Giving a genuine partitioned system.
- Treating the left hand side solid tensor implicitly ensures added-mass free stability

The fluid domain is computed using a generalized Robin condition on the interface, and the solid is computed with the familiar Neumann condition on the interface, equalling the stresses from the fluid and structure.
The general r-order extrapolation x^* is defined:

$$x^* = \begin{cases} 0, & \text{if } r = 0 \\ x^{n-1}, & \text{if } r = 1 \\ 2x^{n-1} - x^{n-2}, & \text{if } r = 2 \end{cases} \tag{21}$$

Boundary interface operator

Using the notation of [Generalized robin-neumann explicit coupling scheme] We denote $(.,.)_{\mathcal{S},h}$ as the lumped mass approximation of the inner product $(.,.)_{\mathcal{S}}$. We will consider a solid and fluid sided discrete lifting operator $\mathcal{L}_h^s : \Lambda_{\Sigma,h} \to \mathcal{S}$, lifting values from the interface into the solid domain. If $\xi_h, \lambda_h \in \Lambda_{\Sigma,h}$ then $\mathcal{L}_h^s|_\Sigma = \mathcal{L}_h^f|_\Sigma = \xi_h$. We use this to define the boundary operator: $B_h = (\mathcal{L}_h^s)^* \mathcal{L}_h^s$, mapping from interface to interface $B_h : \Lambda_{\Sigma,h} \to \Lambda_{\Sigma,h}$. Where stars stands for the adjoint operator of $\mathcal{L}_h^s$. We can then write:

$$(B_h \xi_h, \lambda_h)_\Sigma = (\mathcal{L}_h^s \xi_h, \mathcal{L}_h^s \lambda_h)_{\mathcal{S},h}$$

Explicit Robin-Neumann scheme:
Step 1: Fluid domain update

$$\begin{aligned} d^{f,n} &= Ext(d^{n-1}) \\ w^n &= \frac{\partial d^{f,n}}{\partial t} \\ with F &= I + \nabla d, J = det(F) \end{aligned}$$

Step 2: Fluid step: find u^n, p^n:

$$\begin{aligned} \rho^f \big(\frac{\partial u^n}{\partial t} + (u^{n-1} - w^n) \cdot \nabla u^n\big) - \nabla \cdot \sigma(u^n, p^n) &= 0 \in \mathcal{F} \\ \nabla \cdot u &= 0 \in \mathcal{F} \\ \sigma(u^n, p^n) n^f &= f \\ J^n \sigma(u^n, p^n)(F^n)^{-T} n^f + \frac{\rho^s}{\tau} B_h u^n &= \frac{\rho^s}{\tau} B_h(\dot{d}^{n-1} + \tau \partial_t \dot{d}) - \Pi^* n^s \end{aligned}$$

Step 3: Solid Step: find d^n

$$\begin{aligned} \rho^s \partial_t \dot{d}^n + \alpha \rho^s \dot{d}^n - \nabla \cdot \Pi^n &= 0 \in \mathcal{S} \\ \dot{d} &= \partial_t d^n \\ d^n = 0, \beta \dot{d}^n &= 0 \in \Gamma^d \\ \Pi^n n^s &= 0 \in \Gamma^n \\ \Pi^n n^s &= -J^n \sigma(u^n, p^n)(F^n)^{-T} n^f \in \Sigma \end{aligned}$$

The solid stress tensor is given as $\Pi^n = \pi(d^n) + \beta \pi^?(d^{n-1}) \dot{d}^n$

Monolithic FSI Code

The first FSI-code we look at is the monolithic version. Where all the equations are solved simultaneously.
We start by looking at the function spaces. We need two vectorfunction spaces for u, d and a function space for p, made from "Continuous Galerkin" elements . These are put together in a mixed space to allow us to solve them all at once.

```
V1 = VectorFunctionSpace(mesh, "CG", v_deg) # Fluid velocity
V2 = VectorFunctionSpace(mesh, "CG", d_deg) # displacement
Q  = FunctionSpace(mesh, "CG", p_deg)       # Fluid Pressure
VVQ = MixedFunctionSpace([V1, V2, Q])
```

From this mixed space we make three testfunctions for u, p and d named ϕ, ψ and γ. We also make the functions u, p and d, these are made in the mixed space and then split up:

```
phi, psi, gamma = TestFunctions(VVQ)
udp = Function(VVQ)
u, d, p  = split(udp)
```

The boundaries are specified with facetfunctions on the boundary and subdomains to specify where the boundaries are, and these are used to make the Dirichlet conditions:

```
Inlet = AutoSubDomain(lambda x: "on_boundary" and near(x[0],0))
boundaries = FacetFunction("size_t",mesh)
Inlet.mark(boundaries, 3)
u_wall    = DirichletBC(VVQ.sub(0), ((0.0, 0.0)), boundaries, 2)
```

Since we are solving an FSI problem we need to specify where the fluid and solid domains exist. This is done using Cellfunctions and marked as before. For ease of writing we make two subdomains that represent the different domains.

```
domains = CellFunction("size_t",mesh)
Bar_area.mark(domains,2)

dx = Measure("dx",subdomain_data=domains)
dx_f = dx(1,subdomain_data=domains)
dx_s = dx(2,subdomain_data=domains)
```

All the mappings and stresstensors are made using functions, I will just show some to understand the code later on:

```
def F_(U):
        return (I + grad(U))
def J_(U):
        return det(F_(U))

def sigma_f(v,p):
        return 2*mu_f*sym(grad(v)) - p*Identity(2)

def sigma_f_hat(v,p,u):
        return J_(u)*sigma_f(v,p)*inv(F_(u)).T
```

The variational form can be written directly into FEniCS. We write all the forms and add them together to make one big form to be calculated in the upcoming timeloop:

```
# Fluid variational form
F_fluid = (rho_f/k)*inner(J_(d)*(u - u0), phi)*dx_f \
        + rho_f*inner(J_(d)*inv(F_(d))*grad(u)*(u - ((d-d0)/k)), phi)*dx_f \
        + inner(sigma_f_hat(u,p,d), grad(phi))*dx_f \
        - inner(div(J_(d)*inv(F_(d).T)*u), gamma)*dx_f

# Structure variational form
F_structure = (rho_s/k)*inner(u-u0,phi)*dx_s + inner(P1(d),grad(phi))*dx_s

# Setting w = u on the structure using (d-d0)/k = w
F_w = delta*((1.0/k)*inner(d-d0,psi)*dx_s - inner(u,psi)*dx_s)

# Laplace
F_laplace =  (1./k)*inner(d-d0,psi)*dx_f +inner(grad(d), grad(psi))*dx_f

F = F_fluid + F_structure + F_w + F_laplace
```

The equation $u = \frac{\partial d}{\partial t} in \mathcal{S}$, is weighted with a delta value which we will see later on is important when solving monolithic.
To solve a non-linear problem we need make a newton solver [Mikael kompendium]
In the time loop we call on the solver and update the functions u, d for each round:

```
while t <= T:
    print "Time t = %.5f" % t
    time_list.append(t)
    if t < 2:
        inlet.t = t;
    if t >= 2:
        inlet.t = 2;

    #Reset counters
    atol = 1e-6;rtol = 1e-6; max_it = 100; lmbda = 1.0;

    udp = Newton_manual(F, udp, bcs, atol, rtol, max_it, lmbda,udp_res,VVQ)

    u,d,p = udp.split(True)

    #plot(u)
    if counter%step==0:
        u_file << u
        d_file << d
        p_file << p

        Dr = -assemble((sigma_f_hat(u,p,d)*n)[0]*ds(6))
        Li = -assemble((sigma_f_hat(u,p,d)*n)[1]*ds(6))
        Dr += -assemble((sigma_f_hat(u('-'),p('-'),d('-'))*n('-'))[0]*dS(5))
        Li += -assemble((sigma_f_hat(u('-'),p('-'),d('-'))*n('-'))[1]*dS(5))
        Drag.append(Dr)
        Lift.append(Li)

        dsx = d(coord)[0]
        dsy = d(coord)[1]
        dis_x.append(dsx)
```

```
        dis_y.append(dsy)

        if MPI.rank(mpi_comm_world()) == 0:
            print "t = %.4f " %(t)
            print 'Drag/Lift : %g %g' %(Dr,Li)
            print "dis_x/dis_y : %g %g "%(dsx,dsy)

    u0.assign(u)
    d0.assign(d)
    p0.assign(p)
    t += dt
    counter +=1
```

Kapittel 1

Verification and validation.

The goal of this is section is to verify and validate the different numerical schemes implemented. To solve a real world problem FSI-problem we need to know that we are solving the right equations and that the equations are solved right. By verification of code means that we make sure we are solving the given equation in the right fashion. This can be done with a convergence test, using the method of manufactured solution for instance. We can then check with mathematical theory to see if our solution converges with decreasing time-step or increasing number of cells in our computation.

1.1 Verification

1.2 Validation

After the code has been verified to see that we are indeed computing in the right fashion. We have to see that it is the right equations that are being solved. This is achieved using known benchmark tests. These tests supply us with a problem setup, initial and boundary conditions, and lastly results that we can compare with. In the following we will look at tests for the fluid solvers both alone, testing laminar to turbulent flow, and with solid. We will test the solid solver, and lastly the entire coupled FSI problem.

1.2.1 Taylor-Green vortex

1.2.2 Fluid-Structure Interaction between an elastic object and laminar incompressible flow

Problem Defintion

Domain

The computational domain resembles the classic cfd benchmark with an added bar, with dimensions:
The box: L = 2.5, H = 0.41
The bar: l = 0.35, h = 0.02
The circle is positioned at (0.2, 0.2) making it 0.05 of center from bottom to top, this is done to induce oscillations to an otherwise laminar flow.
Boundary conditions:
The fluid velocity has a parabolic profile on the inlet that changes over time:

$$u(0,y) = 1.5u_0 \frac{y(H-y)}{(\frac{H}{2})^2}$$
$$u(0,y,t) = u(0,y)\frac{1-cos(\frac{\pi}{2}t)}{2} \text{ for } t < 2.0$$
$$u(0,y,t) = u(0,y) \text{ for } t \leq 2.0$$

We set no slip on the floor"and "ceilingso to speak.
On the fluid solid interface the boundary conditions are set to:

$$\sigma_f n_f = \sigma_s n_s \quad on \ \ \Gamma^0(interface)$$

In our variational form we leave this out and so implying that they are equal.

CSM test

Parameters

Tabell 1.1: My caption

Parameters	CSM1	CSM2	CSM3
$\rho_f[10^3 \frac{kg}{m^3}]$	1	1	1
$\nu_f[10^{-3} \frac{m^2}{s}]$	1	1	1
u_0	0	0	0
$\rho_s[10^3 \frac{kg}{m^3}]$	1	1	1
ν_s	0.4	0.4	0.4
$\mu_s[10^6 \frac{m^2}{s}]$	0.5	2.0	0.5
g	2	2	2

FSI test

Tabell 1.2: Parameters

Parameters	FSI1	FSI2	FSI3
$\rho_f[10^3 \frac{kg}{m^3}]$	1	1	1
$\nu_f[10^{-3} \frac{m^2}{s}]$	1	1	1
u_0	0.2	1	2
Re $= \frac{Ud}{\nu_f}$	20	100	200
$\rho_s[10^3 \frac{kg}{m^3}]$	1	10	1
ν_s	0.4	0.4	0.4
$\mu_s[10^6 \frac{m^2}{s}]$	0.5	0.5	2

Results: In my monolithic

Tabell 1.3: FSI 1

Cells	Dofs	ux of A $[x10^{-3}]$	uy of A $[x10^{-3}]$	Drag	Lift	Spaces
2698	7095	0.0234594	0.797218	14.4963	0.915801	P1-P1-P1 stab= 0.01
2698	23563	0.02271	0.80288	14.1736	0.787891	P2-P2-P1
2698	23563	0.00581116	0.000000738678	12.07	0.02345	P2-P2-P1 without weighting
10792	92992	0.0227341	0.808792	14.1855	0.801044	P2-P2-P1
43168	369448	0.227352	0.812595	14.227	0.797242	P2-P2-P1
ref	**ref**	**0.0227**	**0.8209**	**14.295**	**0.7638**	**ref**

Bibliografi

[1] Miguel A. Fernández, Jimmy Mullaert, and Marina Vidrascu. Explicit robin-neumann schemes for the coupling of incompressible fluids with thin-walled structures. *Computer Methods in Applied Mechanics and Engineering*, 267:566–593, 2013.

[2] Miguel A. Fernández, Jimmy Mullaert, and Marina Vidrascu. Generalized Robin-Neumann explicit coupling schemes for incompressible fluid-structure interaction: Stability analysis and numerics. *International Journal for Numerical Methods in Engineering*, 101(3):199–229, 2015.