Benchmark

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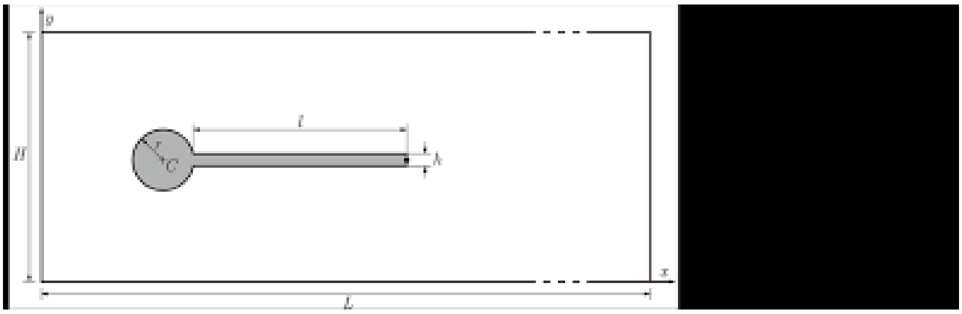
Problem Defintion

Domain



The computational domain resembles the classic cfd benchmark with an added bar, with dimensions: The box: L = 2.5, H = 0.41 The bar: l = 0.35, h = 0.02 The circle is positioned at (0.2, 0.2) making it 0.05 of center from bottom to top, this is done to induce oscillations to a otherwise laminar flow.

Fluid Structure Interaction Problem formulation in ALE coordinates

To get a monolithic ALE formulation, we need to define a mapping, in the solid and fluid domain:

$$\chi^s(t) : \hat{\mathcal{S}} \to \mathcal{S}(t)$$

$$\chi^f(t) : \hat{\mathcal{F}} \to \mathcal{F}(t)$$

The solid mapping is set as $\chi^s(\mathbf{X}, t) = \mathbf{X} + d^s(\mathbf{X}, t)$ hence giving:

$$d^s(\mathbf{X}, t) = \chi^s(\mathbf{X}, t) - \mathbf{X}$$

$$w(\mathbf{X}, t) = \frac{\partial \chi^s(\mathbf{X}, t)}{\partial t}$$

where $\mathbf{X}$ denote a material point in the reference domain and χ^s denotes the mapping from the reference configuration. We denote Γ^1 as the "ceiling"and floor"and the circle and $\Gamma^{2,3}$ as the inlet and outlet. The velocity in the fluid is denoted $u(\mathbf{X}, t)$ We define the deformation gradient $F = I + \nabla d$ and $J = det(F)$ We express the solid balance laws in the Lagrangian formulation from the initial configuration

$$J\rho_s \frac{\partial^2 d}{\partial t^2} = \nabla \sigma_s(d) in \hat{\mathcal{S}}$$

The fluid equations are denoted from the initial configuration:

$$\rho_s J\big(\frac{\partial u}{\partial t} + (\nabla u)F^{-1}(u - w)\big) = \nabla \cdot (J\sigma_f F^{-T}) \quad in \hat{\mathcal{F}}$$

$$\nabla \cdot (JuF^{-T}) = 0 \quad in \ \hat{\mathcal{F}}$$

$$\nabla^2 d = 0 \quad in \ \hat{\mathcal{F}}$$

Boundary conditions:

$$u = u0 \quad on \ \Gamma^2$$

$$u = 0 \quad on \ \Gamma^1$$

$$\sigma_f n_s = \sigma_s n_f \quad on \ \Gamma^0 (interface)$$

Variational formulation

We use 4 testfunctions, $\phi, \psi, \epsilon, \gamma$.

$$\rho_s J\big(\frac{\partial u}{\partial t} + (\nabla u)F^{-1}(u-w), \phi\big)_{\hat{\mathcal{F}}} + (J\sigma_f F^{-T}, \nabla\phi)_{\hat{\mathcal{F}}} = 0$$

$$\big(\nabla \cdot (JuF^{-T}), \gamma\big)_{\hat{\mathcal{F}}} = 0$$

$$\big(J\rho_s \frac{\partial^2 d}{\partial t^2}, \psi\big)_{\hat{\mathcal{S}}} + \big(F\sigma_s(d), \nabla\psi\big)_{\hat{\mathcal{S}}} = 0$$

$$\big(\nabla d, \nabla\epsilon\big)_{\hat{\mathcal{F}}} = 0$$

$$\big(w - \frac{\partial d}{\partial t}, \epsilon\big)_{\hat{\mathcal{S}}} = 0$$

Or we can change the last two with: ??????

$$\big(\nabla(dt \cdot w + d^0), \nabla\epsilon\big)_{\hat{\mathcal{F}}} = 0$$