Adversarial Search

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Informatics 2D

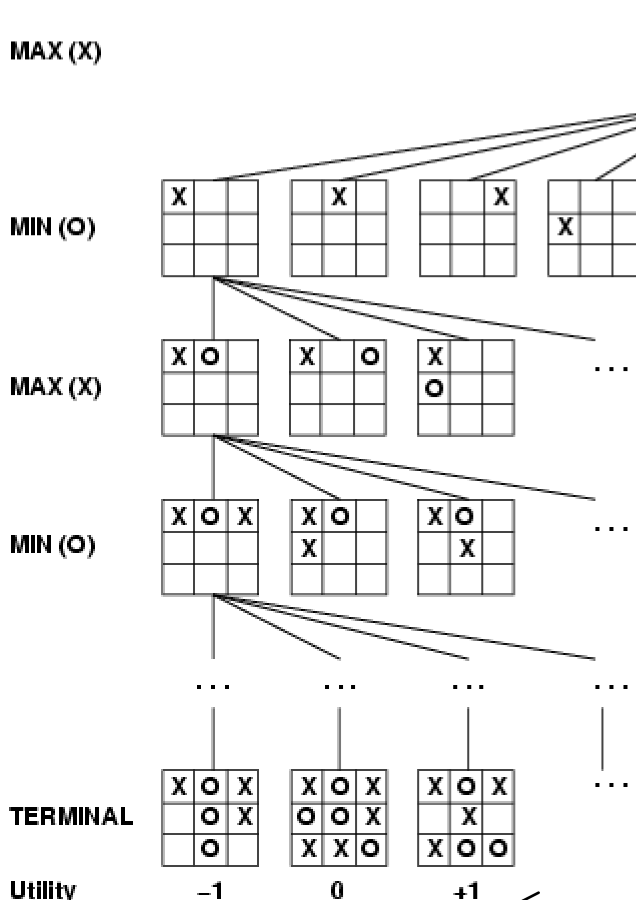
Outline

- Games
- Optimal decisions
- α-β pruning
- Imperfect, real-time decisions

Games vs. search problems

- We're (usually) interested in zero-sum games of perfect information
  - Deterministic, fully observable
  - Agents act alternately
  - Utilities at end of game are equal and opposite
- "Unpredictable" opponent → specifying a move for every possible opponent reply
- Time limits → unlikely to find goal, must approximate

Game tree (2-player, deterministic, turns)



- 2 players: MAX and MIN
- MAX moves first
- Tree built from MAX's POV

Utility of each terminal state from MAX's point of view.

Optimal Decisions

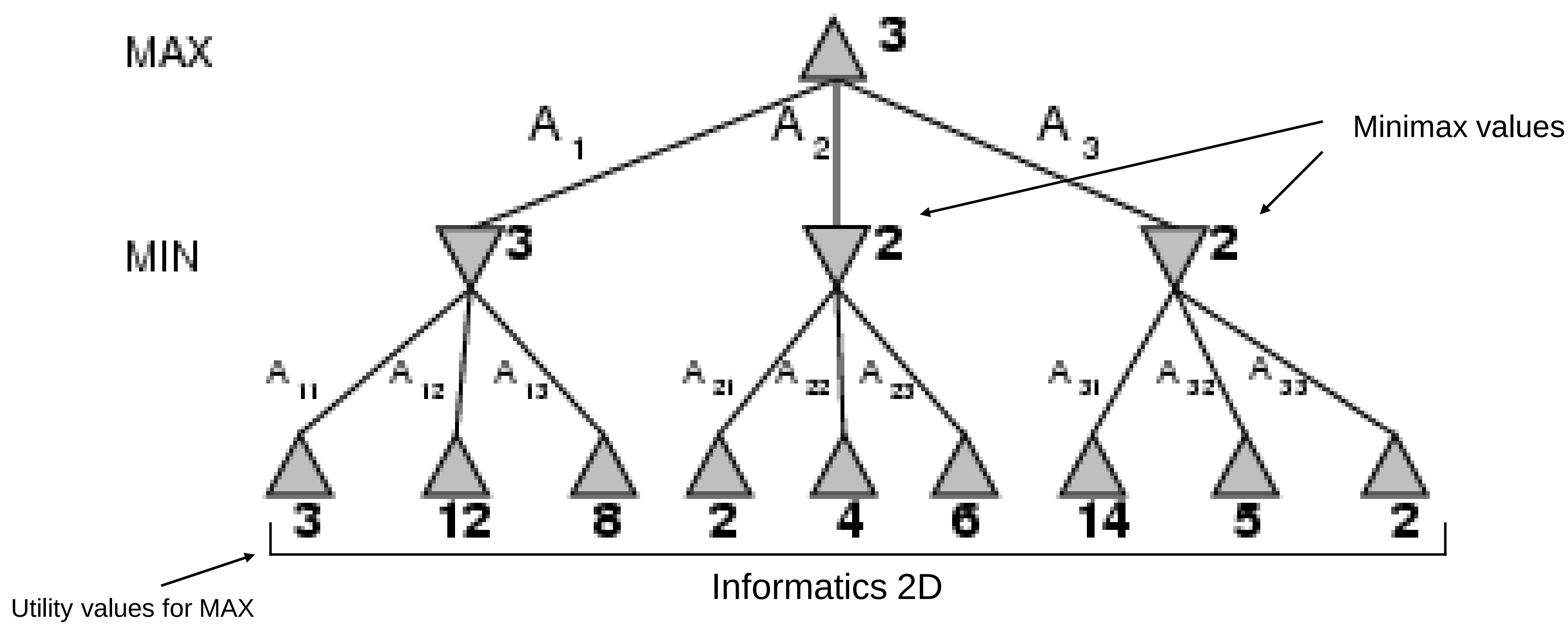
- Normal search: optimal decision is a sequence of actions leading to a goal state (i.e. a winning terminal state)
- Adversarial search:
  - MIN has a say in game
  - MAX needs to find a contingent strategy which specifies:
    - MAX's move in initial state then…
    - MAX's moves in states resulting from every response by MIN to the move then…
    - MAX's moves in states resulting from every response by MIN to all those moves, etc…

minimax value of a node = utility for MAX of being in corresponding state:

$$\text{MINIMAX}(s) = \begin{cases} \text{UTILITY}(s) & \text{if TERMINAL-TEST}(s) \\ \max_{a \in Actions(s)} \text{MINIMAX}(\text{RESULT}(s,a)) & \text{if PLAYER}(s) = \text{MAX} \\ \min_{a \in Actions(s)} \text{MINIMAX}(\text{RESULT}(s,a)) & \text{if PLAYER}(s) = \text{MIN} \end{cases}$$

Minimax

- Perfect play for deterministic games
- Idea: choose move to position with highest minimax value = best achievable payoff against best play
- Example: 2-ply game:



Minimax algorithm

```
function MINIMAX-DECISION(state) returns an action
   return arg max_{a ∈ ACTIONS(s)} MIN-VALUE(RESULT(state, a))

function MAX-VALUE(state) returns a utility value
   if TERMINAL-TEST(state) then return UTILITY(state)
   v ← −∞
   for each a in ACTIONS(state) do
      v ← MAX(v, MIN-VALUE(RESULT(s, a)))
   return v

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```

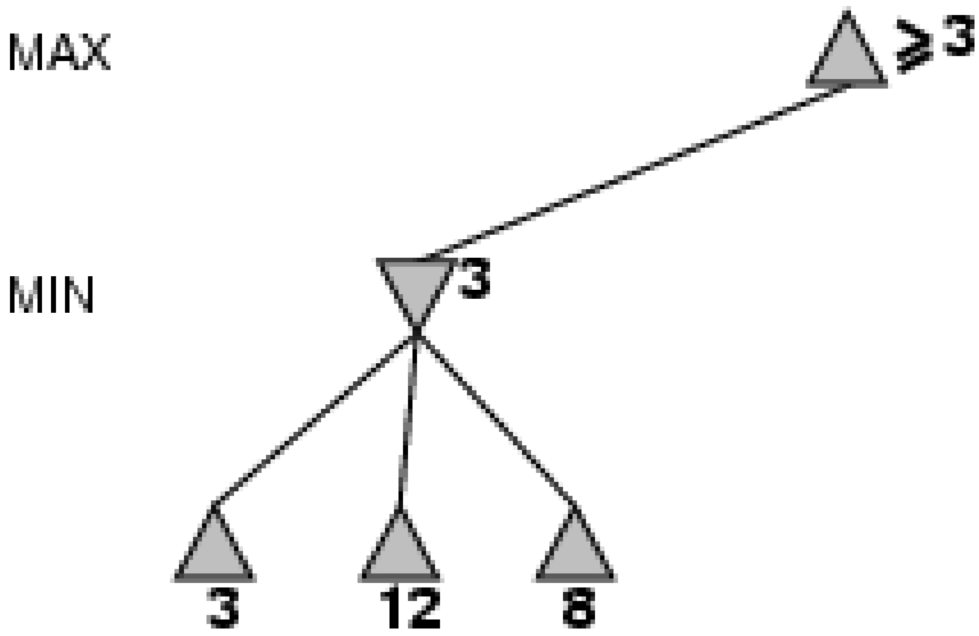
Idea: Proceed all the way down to the leaves of the tree then minimax values are backed up through tree

Properties of minimax

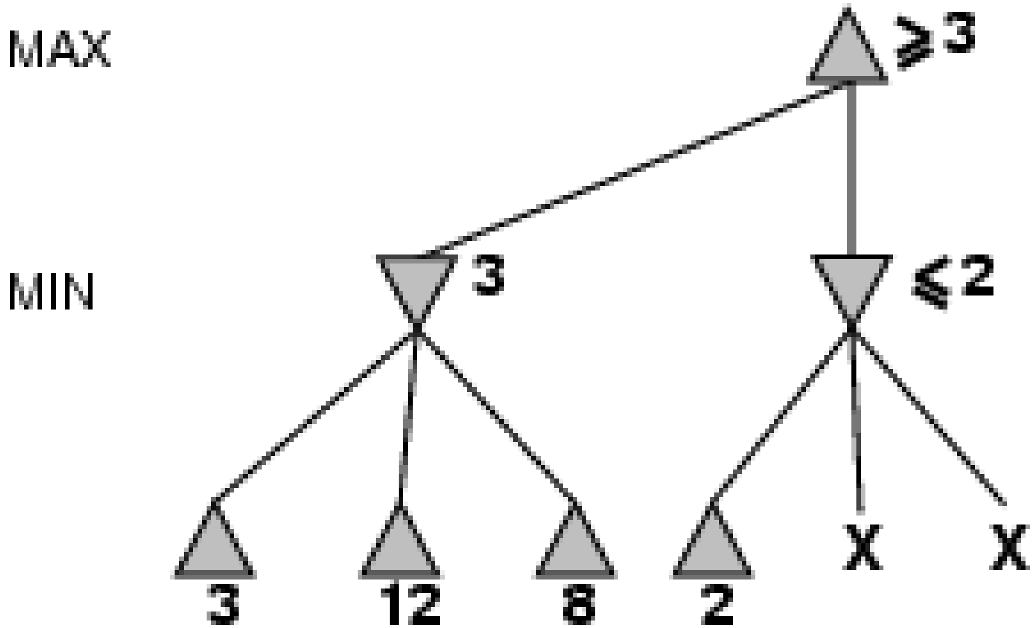
- Complete? Yes (if tree is finite)
- Optimal? Yes (against an optimal opponent)
- Time complexity? $O(b^m)$
- Space complexity? O(bm) (depth-first exploration)

- For chess, b ≈ 35, m ≈100 for "reasonable" games
  → exact solution completely infeasible!
  → would like to eliminate (large) parts of game tree

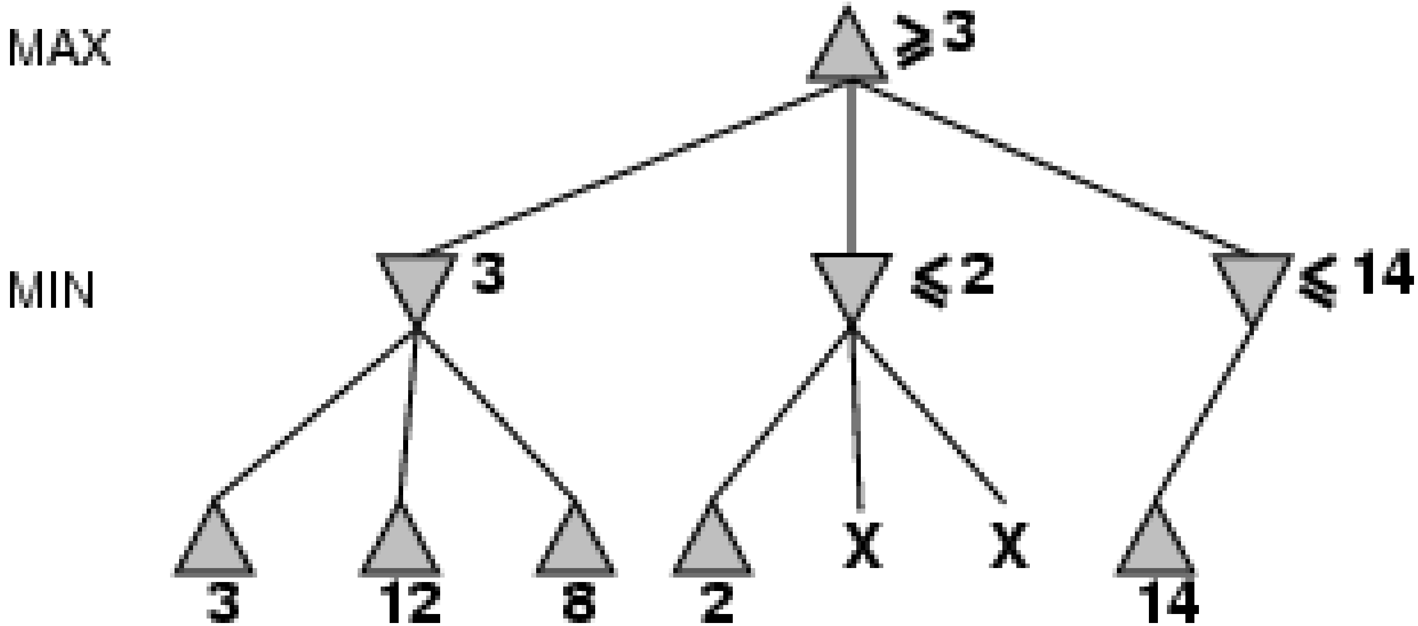
α-β pruning example



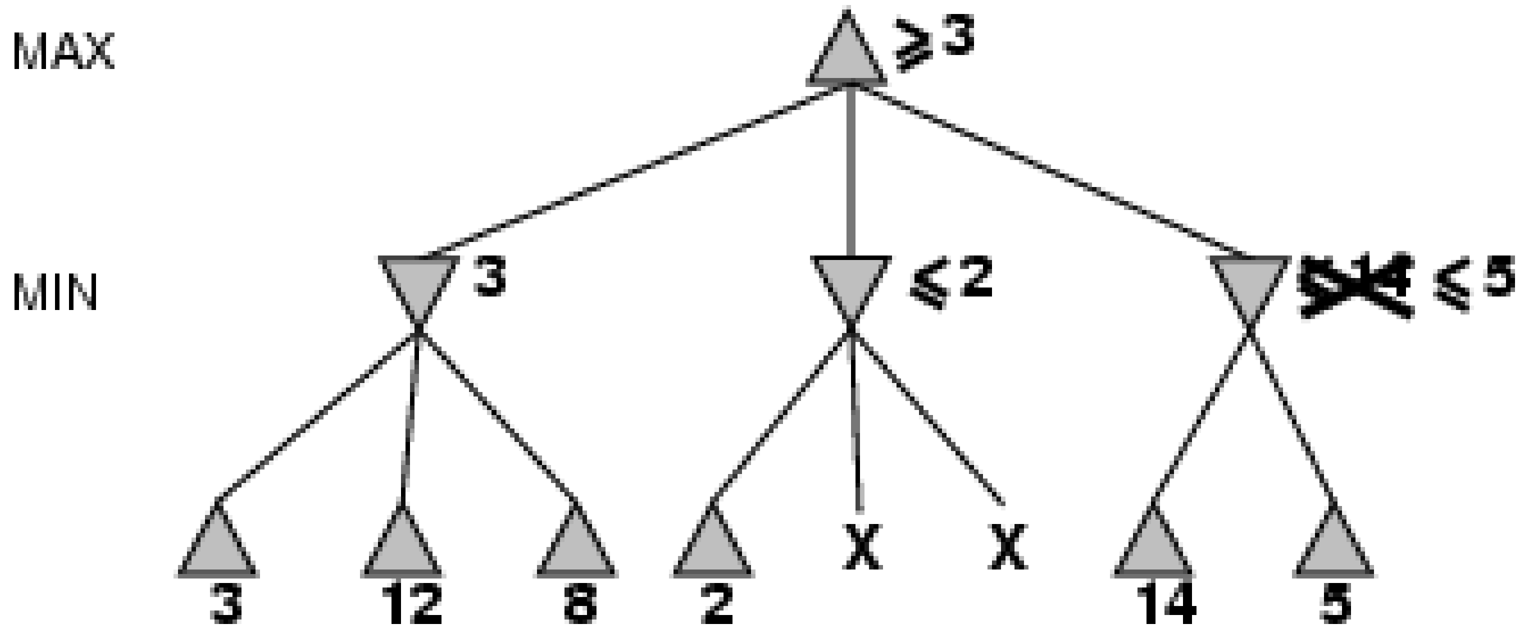
α-β pruning example



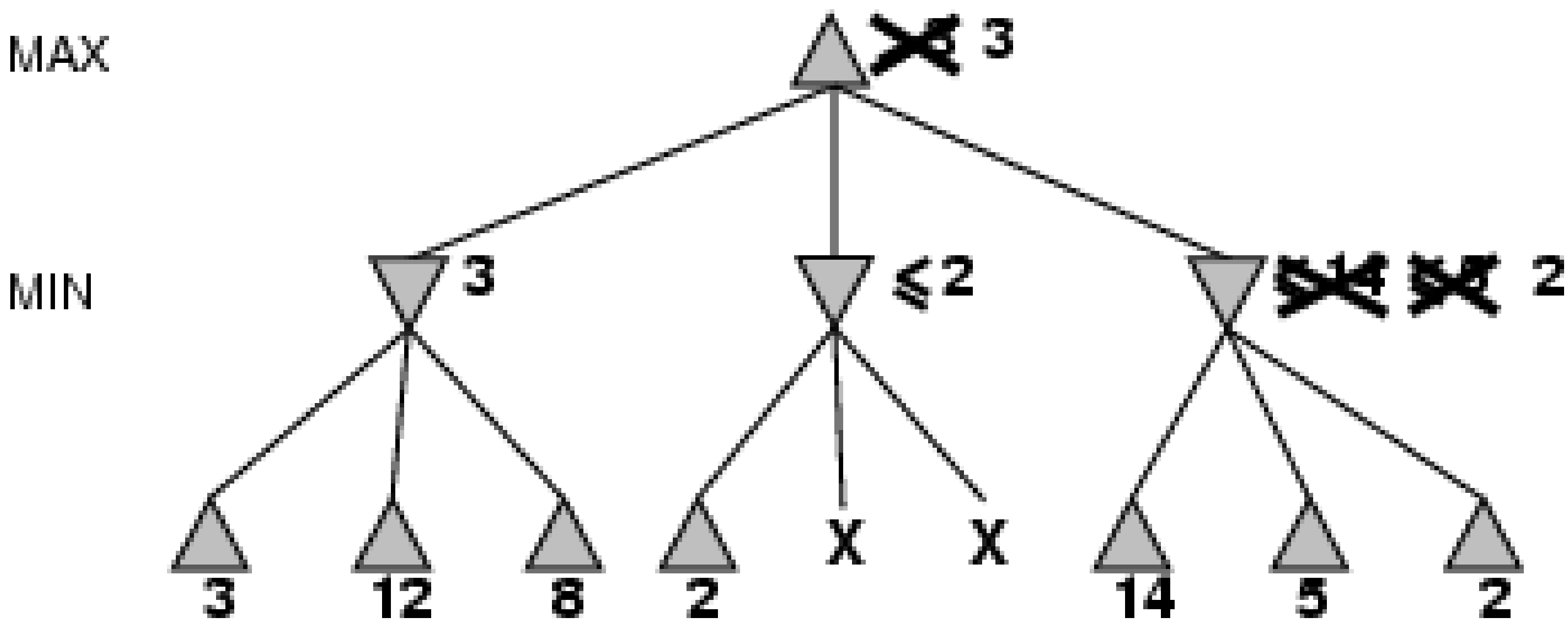
α-*β* pruning example



α-β pruning example



α-β pruning example



Are minimax value of root and, hence, minimax decision independent of pruned leaves?

Let pruned leaves have values u and v,
then MINIMAX(root) = max(min(3,12,8),min(2,u,v),min(14,5,2))
= max(3,min(2,u,v),2)
= max(3,z,2) where z ≤ 2
= 3

→ Yes!

Properties of *α-β*

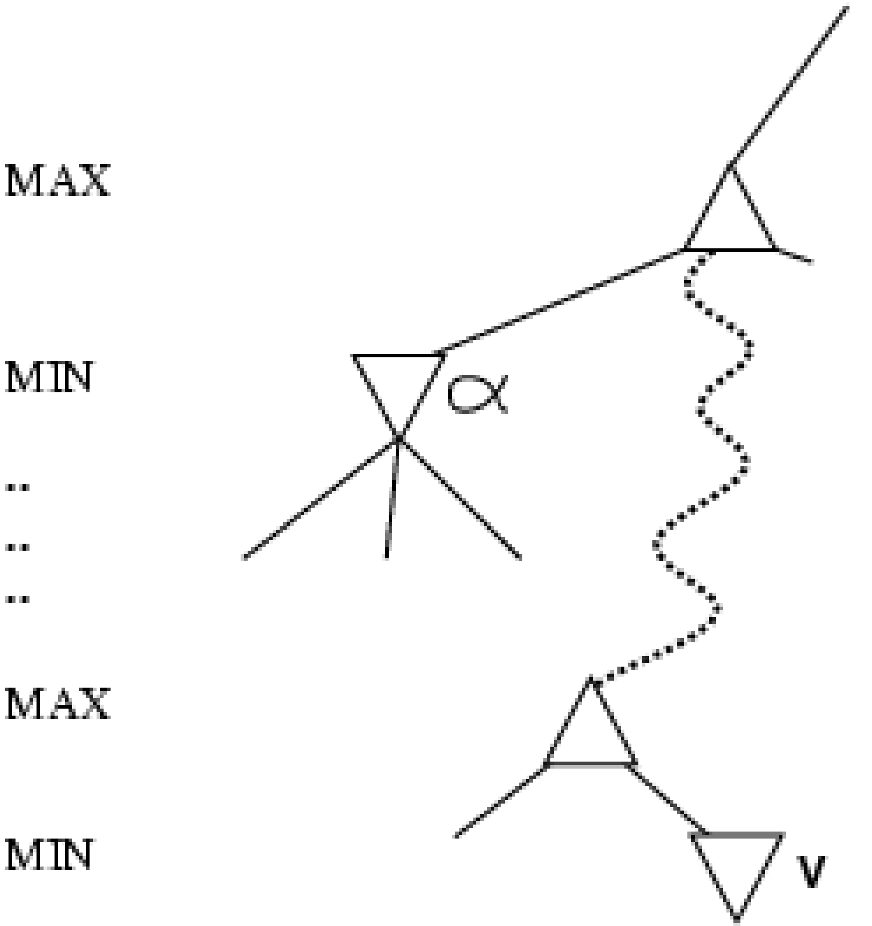
- Pruning does not affect final result (as we saw for example)
- Good move ordering improves effectiveness of pruning (How could previous tree be better?)
- With "perfect ordering", time complexity = $O(b^{m/2})$

  → branching factor goes from b to $\sqrt{b}$

  → (alternative view) doubles depth of search compared to minimax
- A simple example of the value of reasoning about which computations are relevant (a form of meta-reasoning)

Why is it called *α*-*β*?

- *α* is the value of the best (i.e., highest-value) choice found so far at any choice point along the path for MAX
- If *v* is worse than *α*, MAX will avoid it
  - → prune that branch
- Define *β* similarly for MIN



The α-β algorithm

```
function ALPHA-BETA-SEARCH(state) returns an action
   v ← MAX-VALUE(state, −∞, +∞)
   return the action in ACTIONS(state) with value v
```

```
function MAX-VALUE(state, α, β) returns a utility value
   if TERMINAL-TEST(state) then return UTILITY(state)
   v ← −∞
   for each a in ACTIONS(state) do
      v ← MAX(v, MIN-VALUE(RESULT(s,a), α, β))
      if v ≥ β then return v
      α ← MAX(α, v)
   return v
```

Prune as this value is worse for MIN and so won't ever be chosen by MIN!

α is value of the best i.e. highest-value choice found so far at any choice point along the path for MAX

β is value of the best i.e. lowest-value choice found so far at any choice point along the path for MIN

The α-β algorithm

function MIN-VALUE($state, \alpha, \beta$) **returns** *a utility value*
  if TERMINAL-TEST($state$) **then return** UTILITY($state$)
  $v \leftarrow +\infty$
  for each a **in** ACTIONS($state$) **do**
    $v \leftarrow$ MIN(v, MAX-VALUE(RESULT(s,a) $, \alpha, \beta$))
    if $v \leq \alpha$ **then return** v
    $\beta \leftarrow$ MIN(β, v)
  return v

Prune as this value is worse for MAX and so won’t ever be chosen by MAX!

Resource limits

Suppose we have 100 secs, explore 10^4 nodes/sec
→ 10^6 nodes per move

Standard approach:

- cutoff test:
  e.g., depth limit (perhaps add quiescence search, which tries to search interesting positions to a greater depth than quiet ones)
- evaluation function
  = estimated desirability of position

Evaluation functions

- For chess, typically linear weighted sum of features

  $$\text{EVAL}(s) = w_1 f_1(s) + w_2 f_2(s) + \ldots + w_n f_n(s)$$

  where each w_i is a weight and each f_i is a feature of state s
- Example
  - queen = 1, king = 2, etc.
  - f_i = number of pieces of type *i* on board
  - w_i = value of the piece of type *i*

Cutting off search

Minimax Cutoff is identical to *MinimaxValue* except

1. *TERMINAL-TEST* is replaced by CUTOFF
2. UTILITY is replaced by EVAL

Does it work in practice?

$b^m = 10^6$, $b=35 \rightarrow m=4$

4-ply lookahead is a hopeless chess player!

- 4-ply ≈ human novice
- 8-ply ≈ typical PC, human master
- 12-ply ≈ Deep Blue, Kasparov

Deterministic games in practice

- Checkers: Chinook ended 40-year-reign of human world champion Marion Tinsley in 1994. Used a precomputed endgame database defining perfect play for all positions involving 8 or fewer pieces on the board, a total of 444 billion positions.

- Chess: Deep Blue defeated human world champion Garry Kasparov in a six-game match in 1997. Deep Blue searches 200 million positions per second, uses very sophisticated evaluation, and undisclosed methods for extending some lines of search up to 40 ply.

- Othello: human champions refuse to compete against computers, who are too good.

- Go: human champions refuse to compete against computers, who are too bad. In Go, $b > 300$, so most programs use pattern knowledge bases to suggest plausible moves.

Summary

- Games are fun to work on!
- They illustrate several important points about AI
- perfection is unattainable → must approximate
- good idea to think about what to think about