Inf2C - Computer Systems
Lecture 8
Logic Design

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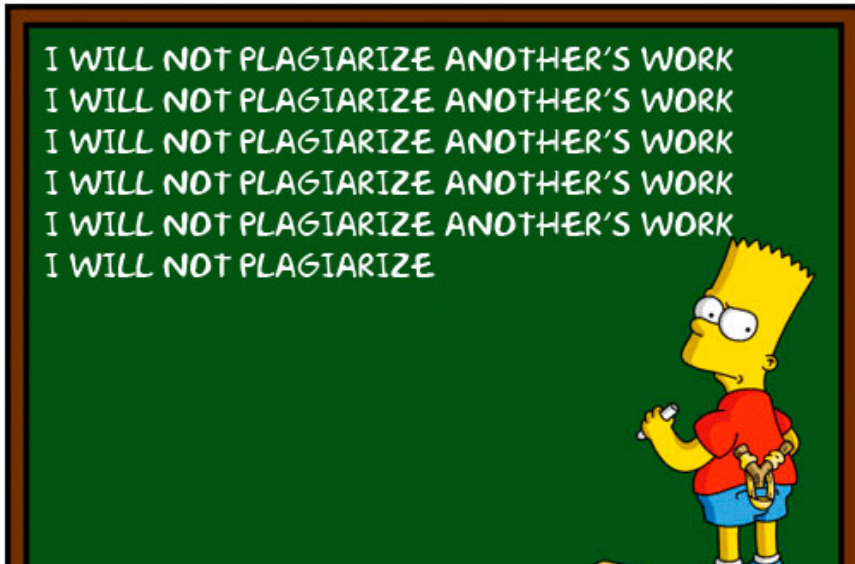
Reminder

- **Coursework 1: due Tues @ 4pm**
- Do:
  - Have correct code
    - Compiles/builds/runs without warnings or errors
    - MIPS & C syntax & semantics are followed
  - Have well-structured code
    - Use functions; no goto's
  - Have readable code
    - Meaningful comments
    - Meaningful names for functions, labels, C variables, etc.

Reminder

- **Coursework 1: due Tues @ 4pm**
- Don't:
  - Be late!
  - Ask me for extensions
    - UG2 organizer – Dr. Sharon Goldwater – handles these
  - Plagiarize!

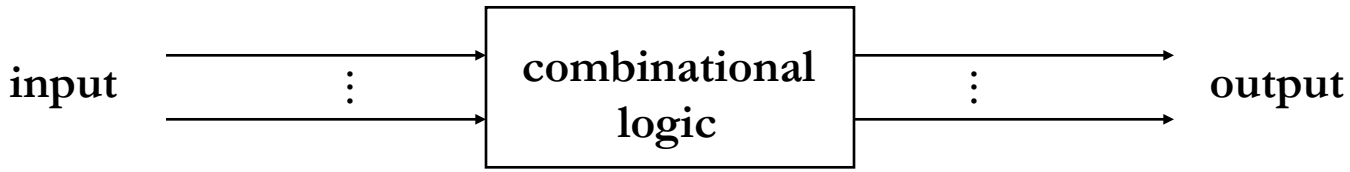


Logic design overview

Binary digital logic circuits:

- Two voltage levels (ground and supply voltage) for 0 and 1
  - Built from transistors used as on/off switches
  - Analog circuits not very suitable for generic computing
  - Digital logic with more than two states is not practical

Combinational logic: output depends only on the current inputs (no memory of past inputs)

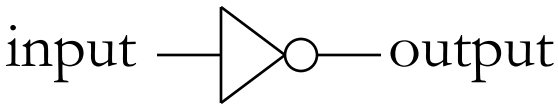


Sequential logic: output depends on the current inputs as well as (some) previous inputs → requires "memory"

Combinational logic circuits

- Inverter (or NOT gate): 1 input and 1 output

  "invert the input signal"



IN	OUT
0	1
1	0

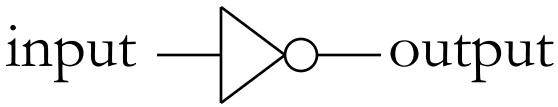
Truth table

$$\mathbf{OUT = \overline{IN}}$$

Combinational logic circuits

- Inverter (or NOT gate): 1 input and 1 output

  "invert the input signal"

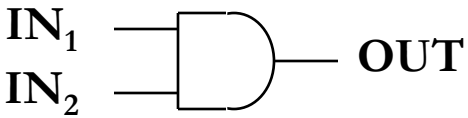


IN	OUT
0	1
1	0

$OUT = \overline{IN}$

- AND gate: 2 inputs and 1 output

  "output 1 only if both inputs are 1"

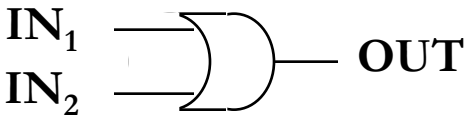


IN_1	IN_2	OUT
0	0	0
0	1	0
1	0	0
1	1	1

$OUT = IN_1 . IN_2$

Combinational logic circuits

- OR gate: "output 1 if at least one input is 1"

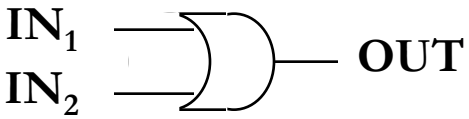


IN_1	IN_2	OUT
0	0	0
0	1	1
1	0	1
1	1	1

$OUT = IN_1 + IN_2$

Combinational logic circuits

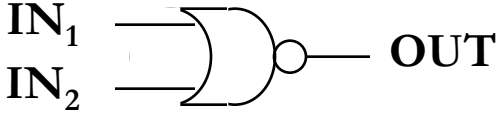
- OR gate: "output 1 if at least one input is 1"



IN_1	IN_2	OUT
0	0	0
0	1	1
1	0	1
1	1	1

$$OUT = IN_1 + IN_2$$

- NOR gate: "output 1 if no input is 1" (NOT OR)

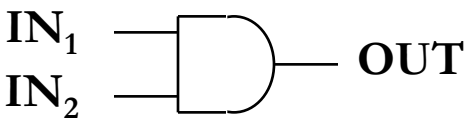


IN_1	IN_2	OUT
0	0	1
0	1	0
1	0	0
1	1	0

$$OUT = \overline{IN_1 + IN_2}$$

Combinational logic circuits

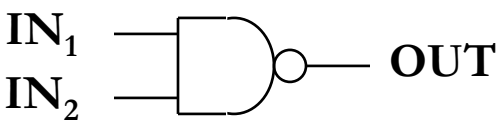
- AND gate: "output 1 if both inputs are 1"



IN_1	IN_2	OUT
0	0	0
0	1	0
1	0	0
1	1	1

$$OUT = IN_1 \,.\, IN_2$$

- NAND gate: "output 1 if both inputs are <u>not</u> 1" (NOT AND)

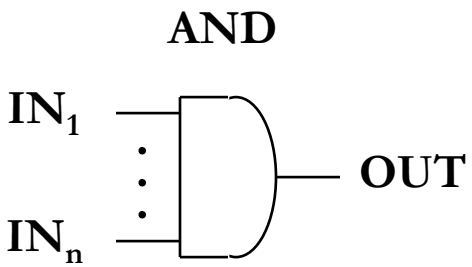


IN_1	IN_2	OUT
0	0	1
0	1	1
1	0	1
1	1	0

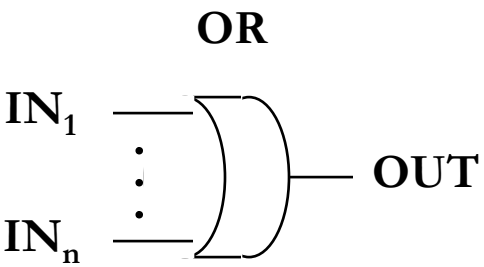
$$OUT = \overline{IN_1 \,.\, IN_2}$$

Combinational logic circuits

- Multiple-input gates:



OUT = 1 if <u>all</u> IN_i=1



OUT = 1 if <u>any</u> IN_i=1

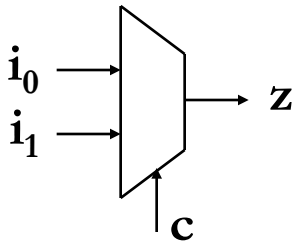
Combinational logic circuits

- Functional completeness:
  - Set of gates that is sufficient to express any boolean function

- Examples:
  - AND + OR + NOT
  - NAND
  - NOR

Multiplexer

- Multiplexer: a circuit for selecting one of multiple inputs



$$z = \begin{cases} i_0, \text{ if } c=0 \\ i_1, \text{ if } c=1 \end{cases}$$

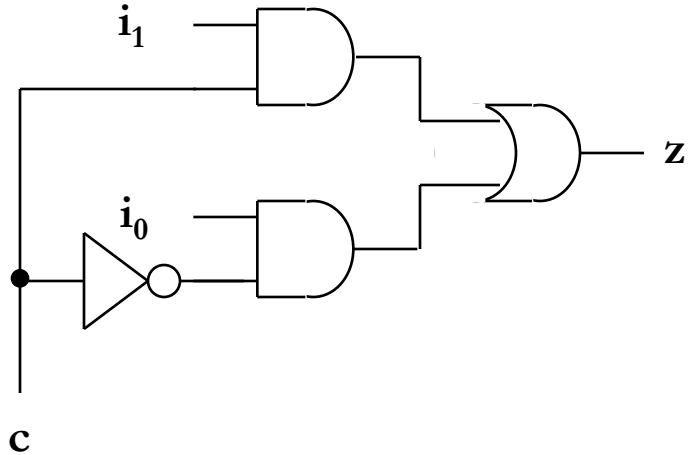
c	i_0	i_1	z
0	0	0	0
0	0	1	0
0	1	0	1
0	1	1	1
1	0	0	0
1	0	1	1
1	1	0	0
1	1	1	1

$$\begin{aligned} z &= \bar{c}.i_0.\bar{i}_1 + \bar{c}.i_0.i_1 + c.\bar{i}_0.i_1 + c.i_0.i_1 \\ &= \bar{c}.i_0.(\bar{i}_1 + i_1) + c.(\bar{i}_0 + i_0).i_1 \\ &= \bar{c}.i_0 + c.i_1 \end{aligned}$$

"sum of products form"

A multiplexer implementation

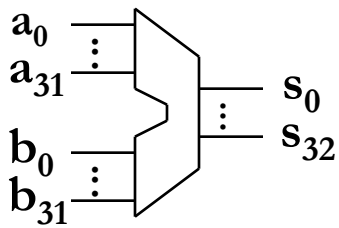
- Sum of products form: $\mathbf{i_1.c + i_0.\overline{c}}$
  - Can be implemented with 1 inverter, 2 AND gates & 1 OR gate:



- Sum of products is not practical for circuits with large number of inputs (n)
  - The number of possible products can be proportional to 2^n

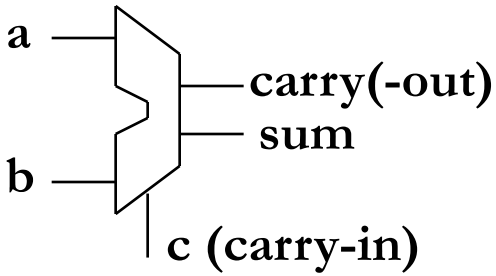
Arithmetic circuits

- 32-bit adder



64 inputs → too complex for sum of products

- Full adder:



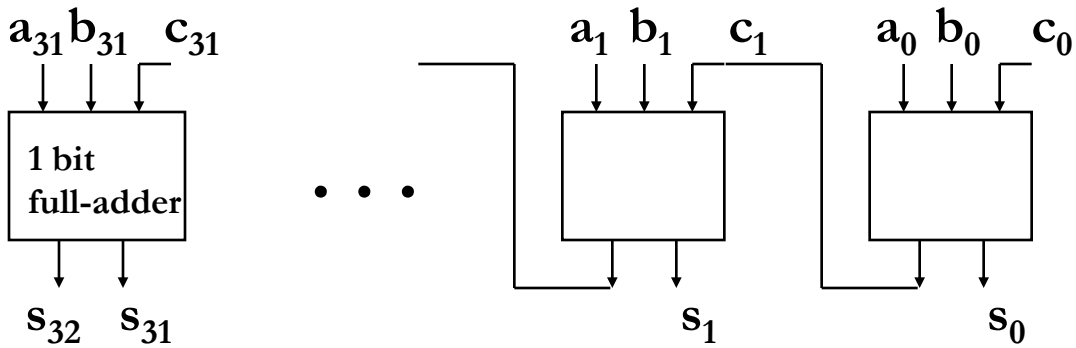
$$\text{sum} = \bar{a}.\bar{b}.c + \bar{a}.b.\bar{c} + a.\bar{b}.\bar{c} + a.b.c$$

$$\text{carry} = b.c + a.c + a.b$$

a	b	c	carry	sum
0	0	0	0	0
0	0	1	0	1
0	1	0	0	1
0	1	1	1	0
1	0	0	0	1
1	0	1	1	0
1	1	0	1	0
1	1	1	1	1

Ripple carry adder

- 32-bit adder: chain of 32 full adders



  - Carry bits c_i are computed in sequence $c_1, c_2, \ldots, c_{32}$ (where $c_{32} = s_{32}$), as c_i depends on c_{i-1}
  - Since sum bits s_i also depend on c_i, they too are computed in sequence

Propagation delays

- Propagation delay = time delay between input signal change and output signal change at the other end
- Delay depends on:
  1. technology (transistor parameters, wire capacitance, etc.)
  2. number of gates driven by a gate's output (**fan out**)
- e.g.: Half-adder circuit: 3 gate delays → fast! (inverter, AND, OR)
- e.g.: 32-bit ripple carry adder:
  - 65 gate delays → slow
  - 1 AND + 1 OR for each of 31 carries to propagate; followed by 1 inverter + 1 AND + 1 OR for S_{31}

Practice problem:

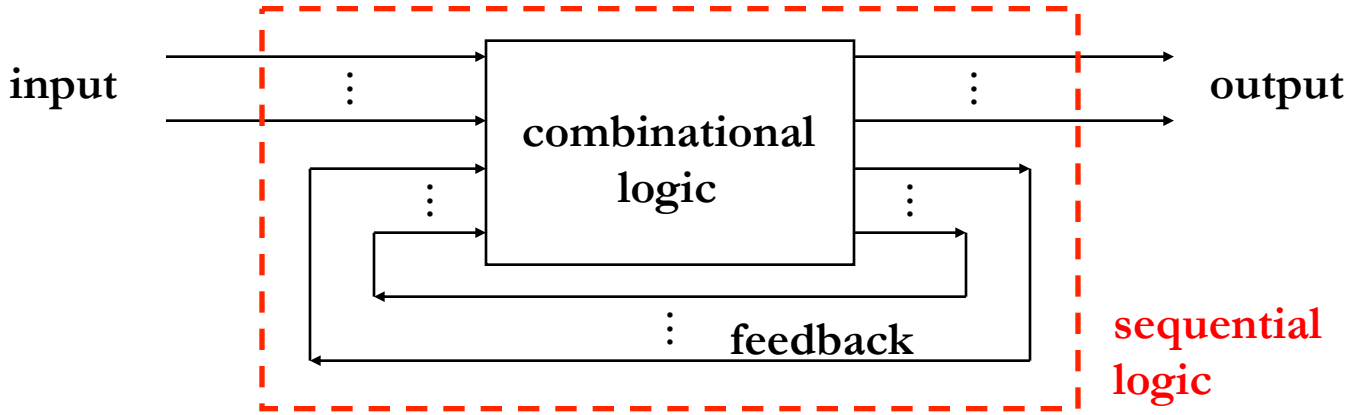
Design a circuit that, given a 4-bit hex character, outputs

1 - if the character is ≥9

0 - otherwise

What is the propagation delay of the circuit?

Sequential logic circuits



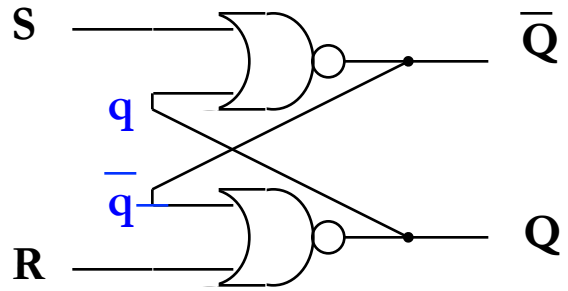
- Output depends on current AND past inputs
  - The circuit has memory
- Sequences of inputs generate sequences of outputs ⇒ sequential logic
  - With n feedback signals → up to 2^n stable states

SR Latch: the basic state element

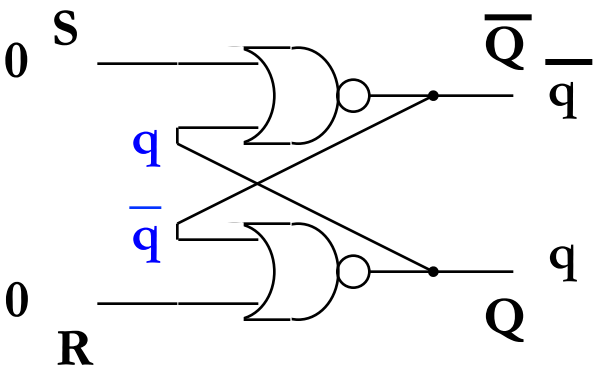
SR latch

- Inputs: R, S
- Feedback: q, q

- Output: Q



SR Latch

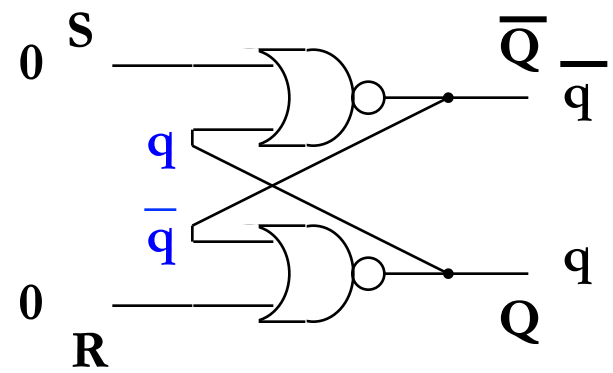


SR Latch

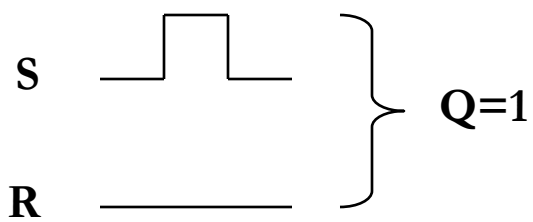
- Truth table:

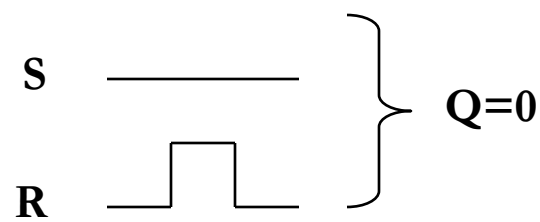
S	R	Q_i
0	0	Q_{i-1}
0	1	0
1	0	1
1	1	inv

inv=invalid



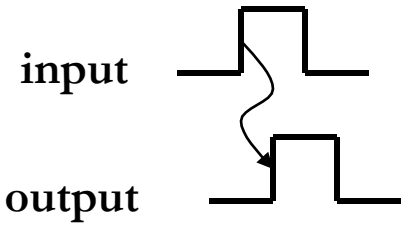
- Usage: 1-bit memory
  - Keep the value in memory by maintaining S=0 and R=0
  - Set the value in memory to 0 (or 1) by setting R=1 (or S=1) for a short time



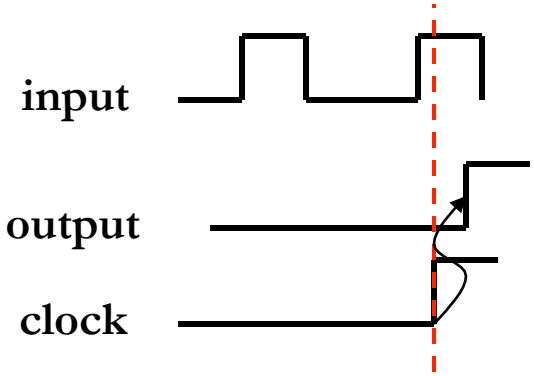


Timing of events

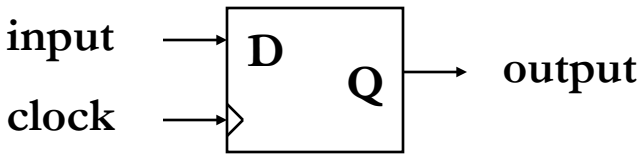
- Asynchronous sequential logic
  - State (and possibly output) of circuit changes whenever inputs change

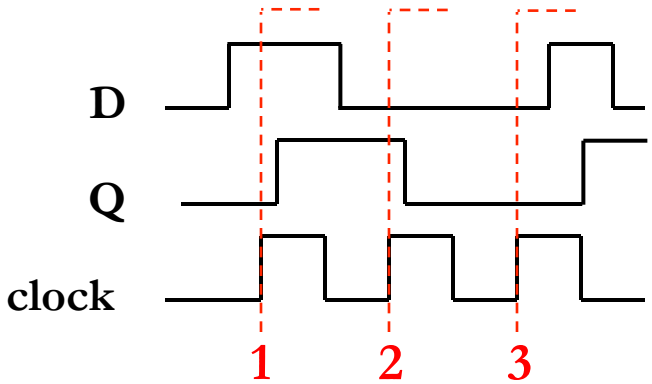


- Synchronous sequential logic
  - State (and possibly output) can only change at times synchronized to an external signal → the **clock**



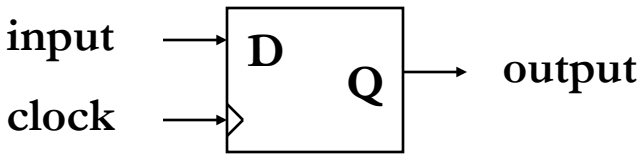
Using clock to build a D latch

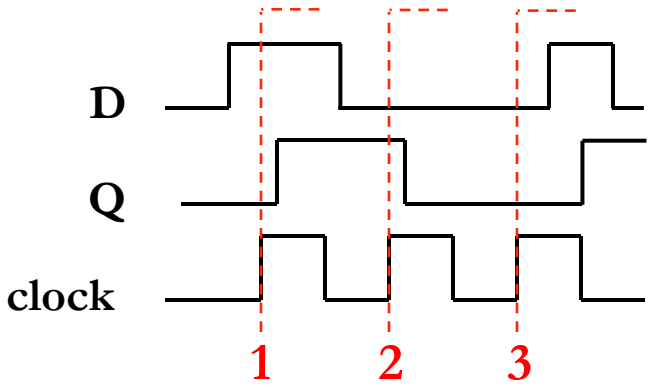




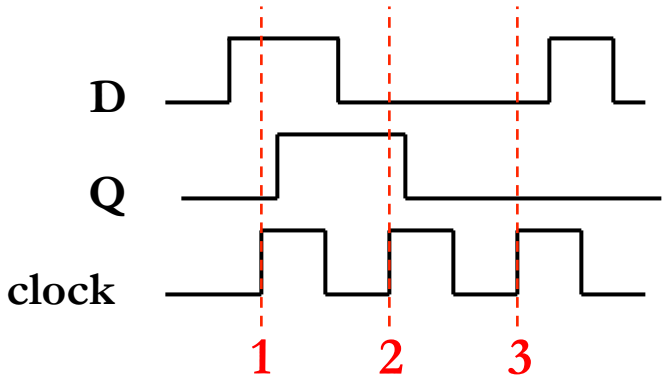
- Level-triggered latch: whenever clock is 1, D is propagated to Q

D flip-flop



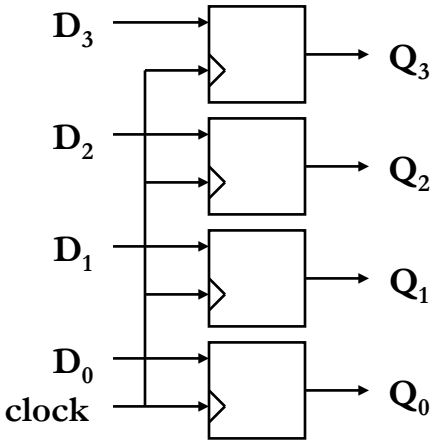


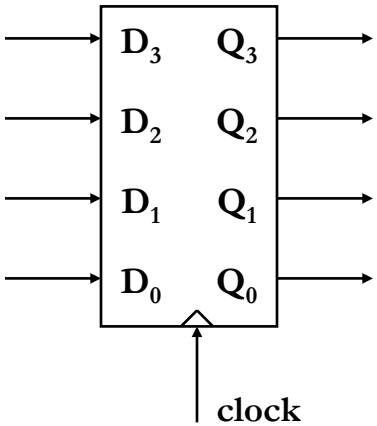
- Level-triggered latch: whenever clock is 1, D is propagated to Q
- Edge-triggered flip-flop: on a positive clock edge, D is propagated to Q



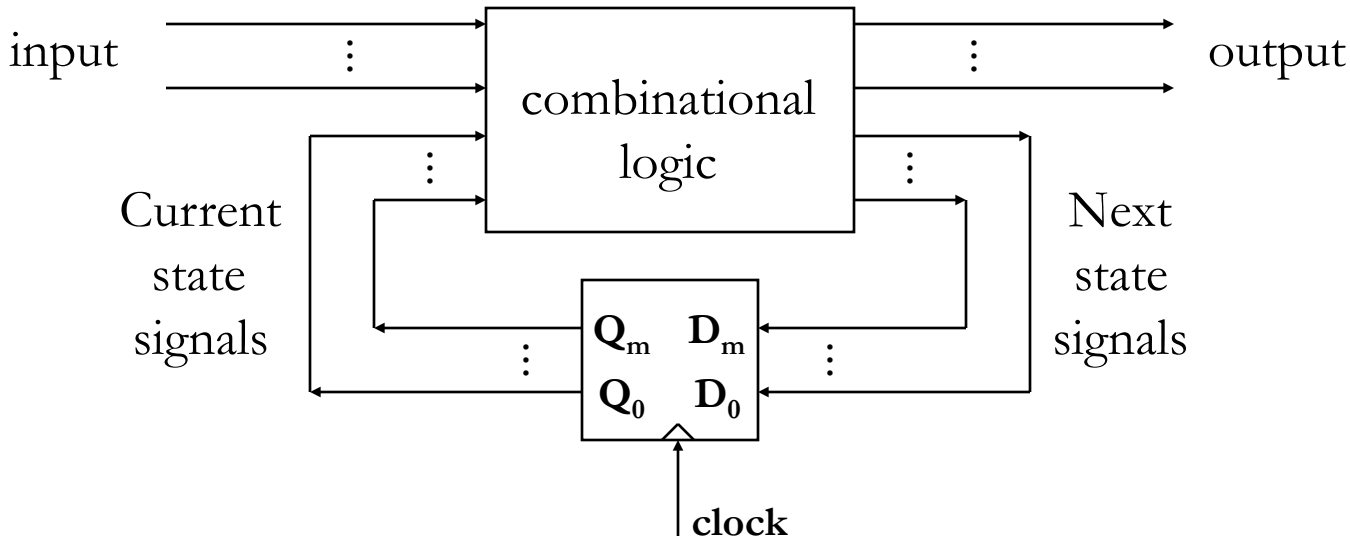
Registers out of flip-flops

- Tie multiple D flip-flops together using a common clock
- E.g., 4-bit register:





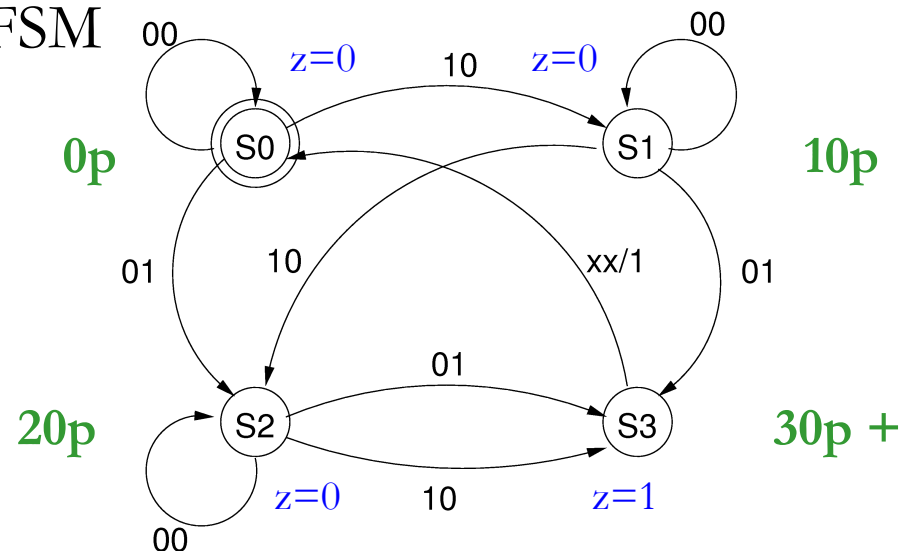
General sequential logic circuit



- Operation:
  - At every rising clock edge next state signals are propagated to current state signals
  - Current state signals plus inputs work through combinational logic and generate output and next state signals

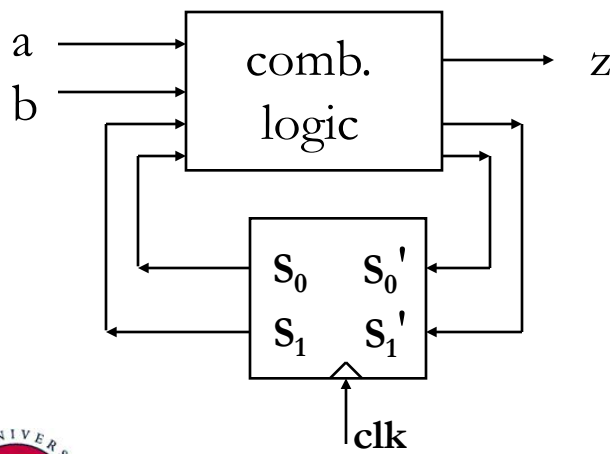
Hardware FSM

- A sequential circuit is a (deterministic) Finite State Machine – FSM
- Example: Vending machine
  - Accepts 10p, 20p coins, sells one product costing 30p, no change given
  - Coin reader has 2 signals: *a, b* for 10p, 20p coins respectively. These are the inputs to our FSM
  - Output *z* asserted when 30p or more has been paid in



FSM implementation

- Methodology:
  - Choose encoding for states, e.g S0=00, …, S3=11
  - Build truth table for the next state s_1', s_0' and output z
  - Generate logic equations for s_1', s_0', z
  - Design comb logic from logic equations and add state-holding register



s_1	s_0	a	b	s_1'	s_0'	z
0	0	0	0	0	0	
0	0	0	1	1	0	0
0	0	1	0	0	1	
0	1	0	0	0	1	
0	1	0	1	1	1	0
0	1	1	0	1	0	
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