Search Strategies

R&N: § 3.3, 3.4, 3.7

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Informatics 2D

Outline

- Uninformed search strategies use only information in problem definition
- Breadth-first search
- Depth-first search
- Depth-limited and Iterative deepening search

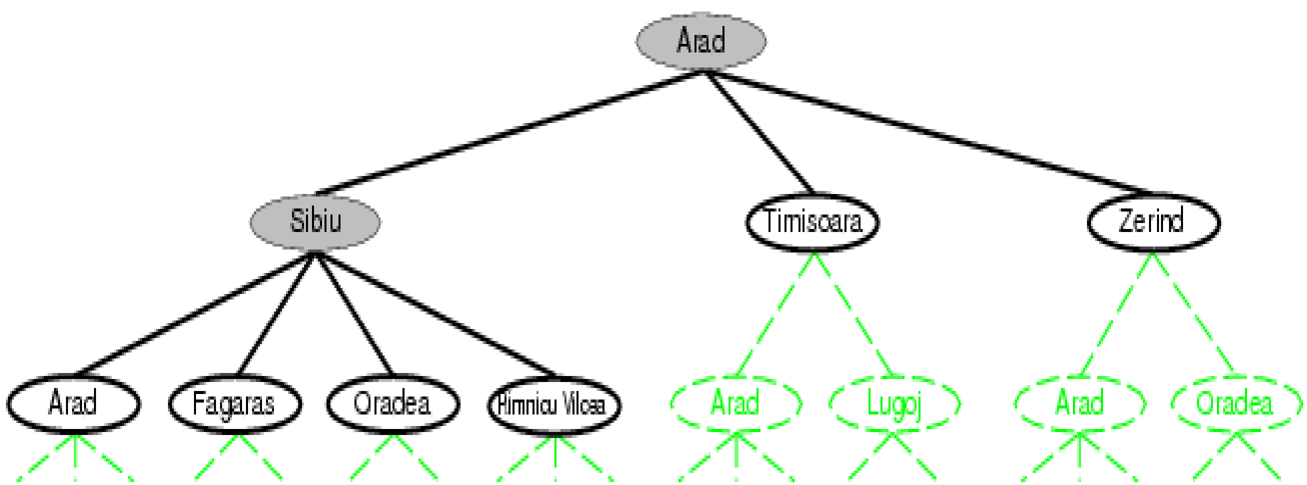
Search strategies

- A search strategy is defined by picking the order of node expansion – nodes are taken from the *frontier*
- Strategies are evaluated along the following dimensions:
  - completeness: does it always find a solution if one exists?
  - time complexity: number of nodes generated
  - space complexity: maximum number of nodes in memory
  - optimality: does it always find a least-cost solution?
- Time and space complexity are measured in terms of
  - *b:* maximum branching factor of the search tree
  - *d:* depth of the least-cost solution
  - *m*: maximum depth of the state space (may be ∞)

Recall: Tree Search

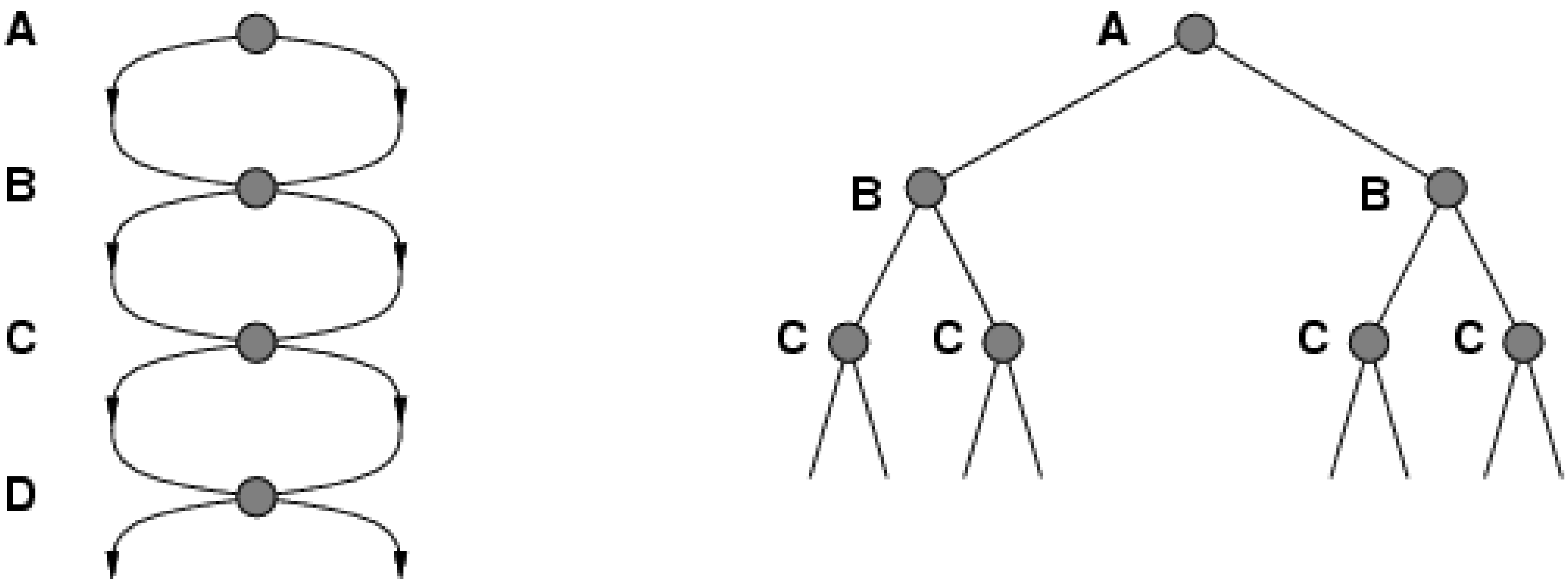
function TREE-SEARCH(*problem*) **returns** a solution, or failure
    initialize the frontier using the initial state of *problem*
    loop do
        if the frontier is empty **then return** failure
        choose a leaf node and remove it from the frontier
        if the node contains a goal state **then return** the corresponding solution
        expand the chosen node, adding the resulting nodes to the frontier

Repeated state



Repeated states

Failure to detect repeated states can turn a linear problem into an exponential one!



Graph search

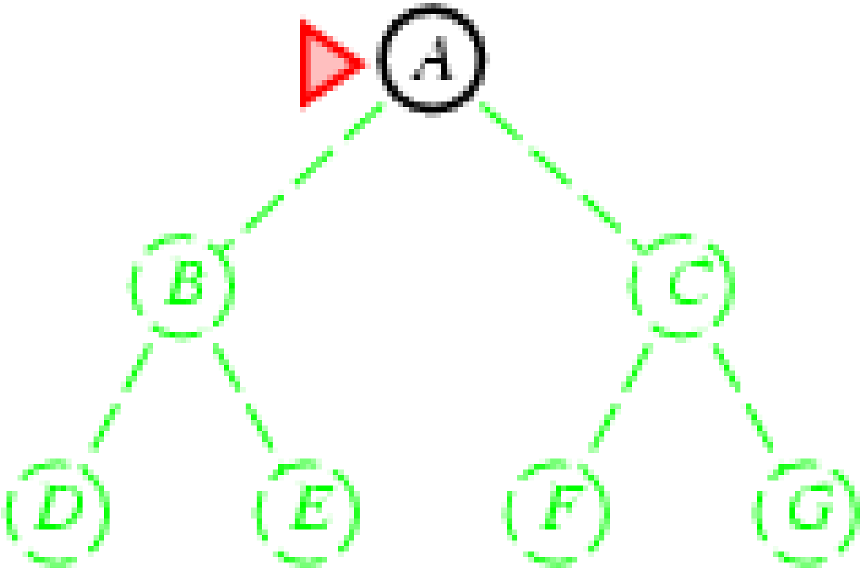
function GRAPH-SEARCH(*problem*) **returns** a solution, or failure
  initialize the frontier using the initial state of *problem*
  initialize the explored set to be empty
  loop do
    if the frontier is empty **then return** failure
    choose a leaf node and remove it from the frontier
    if the node contains a goal state **then return** the corresponding solution
    add the node to the explored set
    expand the chosen node, adding the resulting nodes to the frontier
      only if not in the frontier or explored set

Augment TREE-SEARCH with a new data-structure:

- the **explored set** (closed list), which remembers every expanded node
- newly expanded nodes already in explored set are discarded

Breadth-first search

- Expand shallowest unexpanded node
- Implementation:
  - *frontier* is a FIFO queue, i.e., new successors go at end



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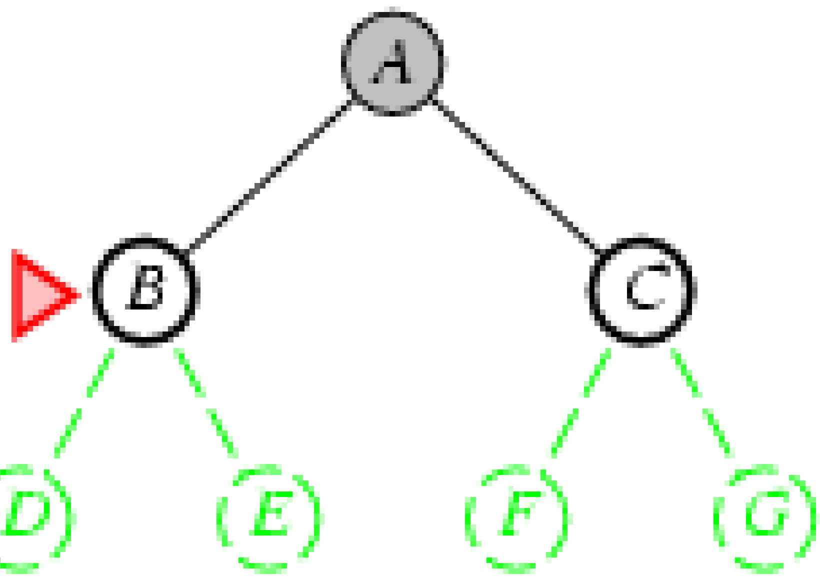

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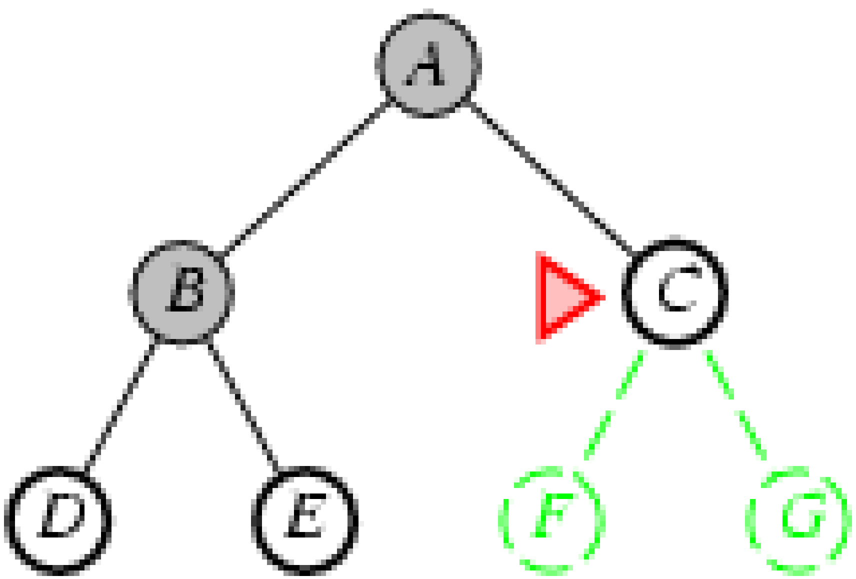

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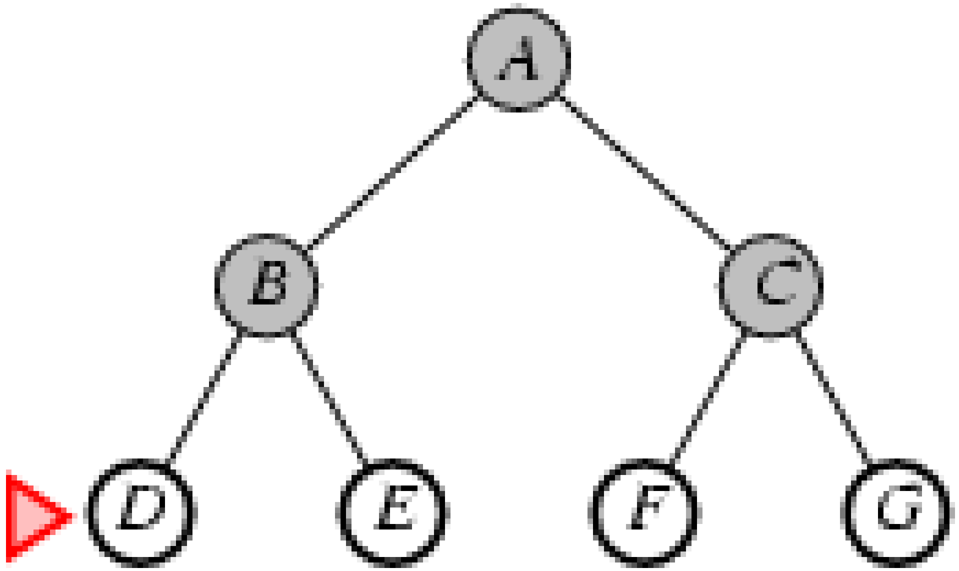


Breadth-first search algorithm

```
function BREADTH-FIRST-SEARCH(problem) returns a solution, or failure
  node ← a node with STATE = problem.INITIAL-STATE, PATH-COST = 0
  if problem.GOAL-TEST(node.STATE) then return SOLUTION(node)
  frontier ← a FIFO queue with node as the only element
  explored ← an empty set
  loop do
     if EMPTY?(frontier) then return failure
     node ← POP(frontier)   /* chooses the shallowest node in frontier */
     add node.STATE to explored
     for each action in problem.ACTIONS(node.STATE) do
        child ← CHILD-NODE(problem, node, action)
        if child.STATE is not in explored or frontier then
           if problem.GOAL-TEST(child.STATE) then return SOLUTION(child)
           frontier ← INSERT(child, frontier)
```

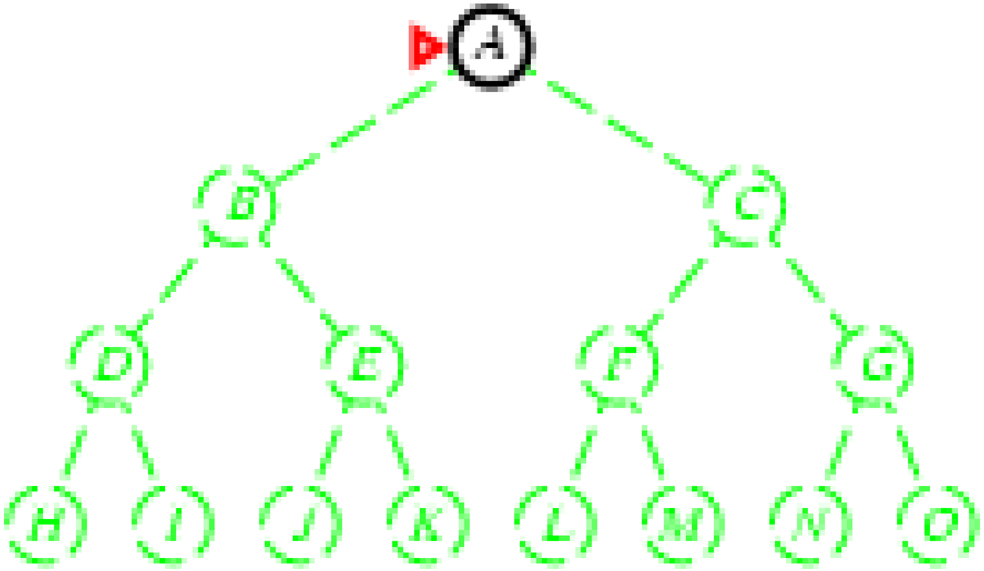
Properties of breadth-first search

- Complete? Yes (if b is finite)
- Time? $b+b^2+b^3+\ldots+b^d = O(b^d)$ (worst-case)
- Space? $O(b^d)$ (keeps every node in memory)
- Optimal? Yes (if cost = 1 per step)

Space is the bigger problem (more than time)

Depth-first search

- Expand deepest unexpanded node
- Implementation:
  - *frontier* = LIFO queue, i.e., put successors at front



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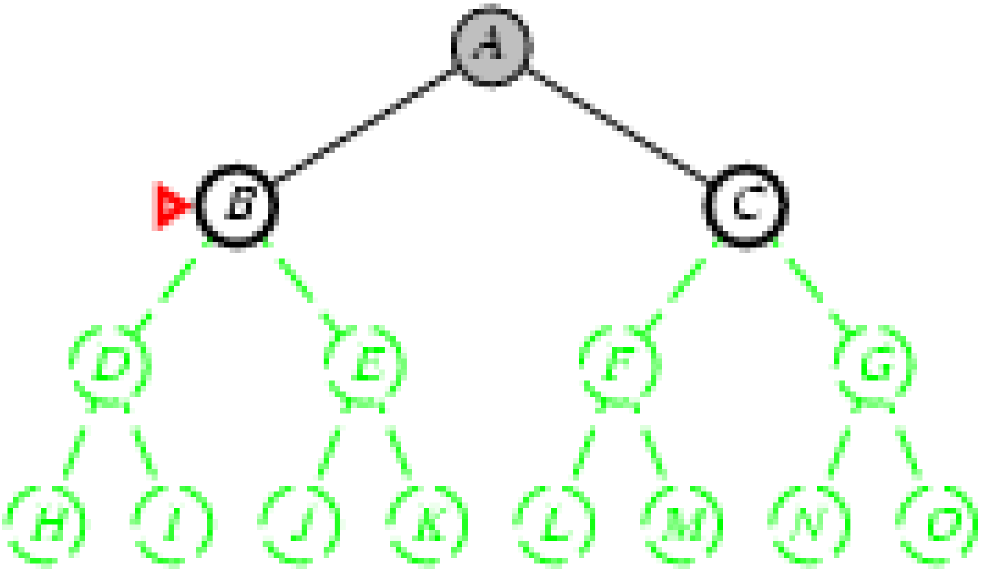

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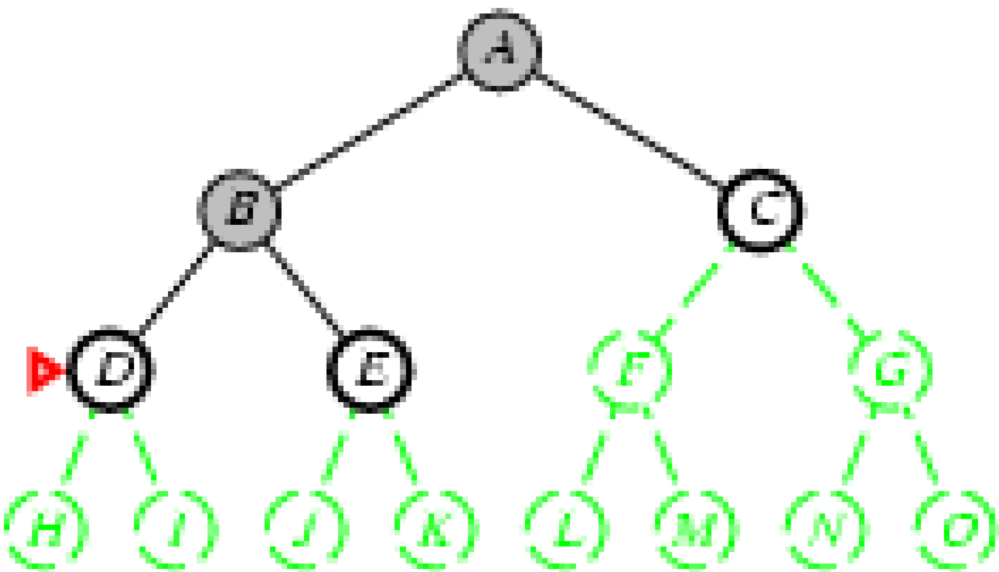

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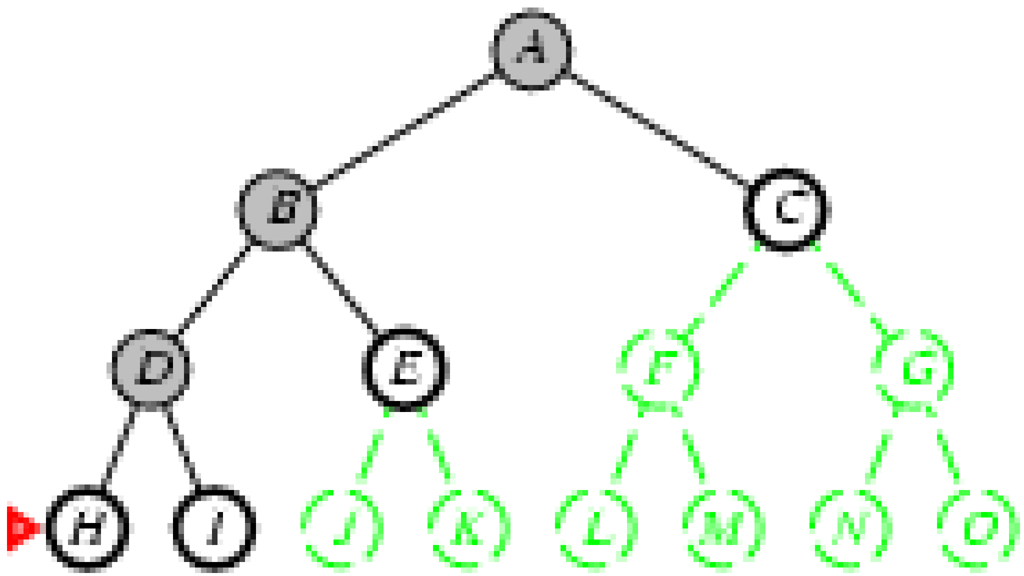

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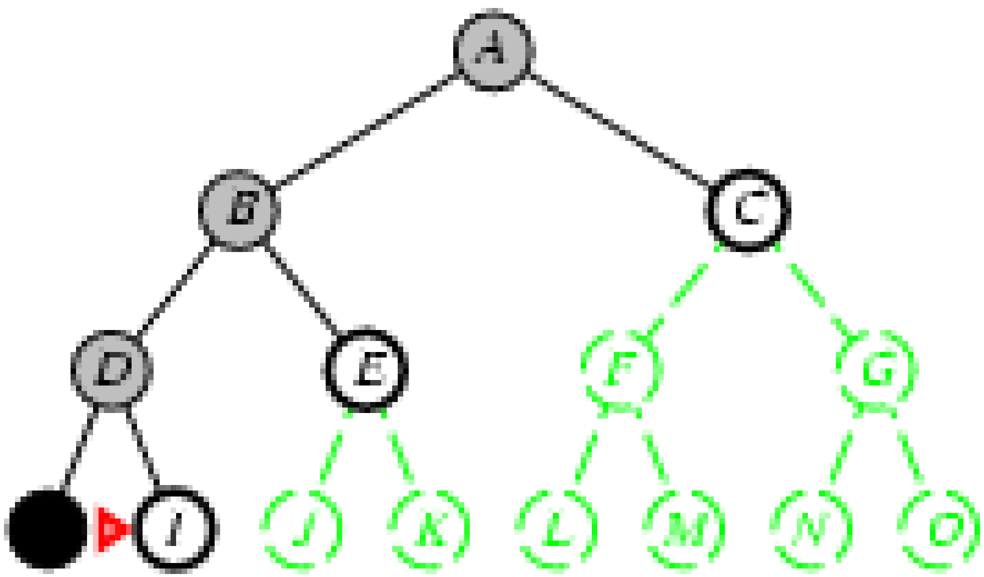

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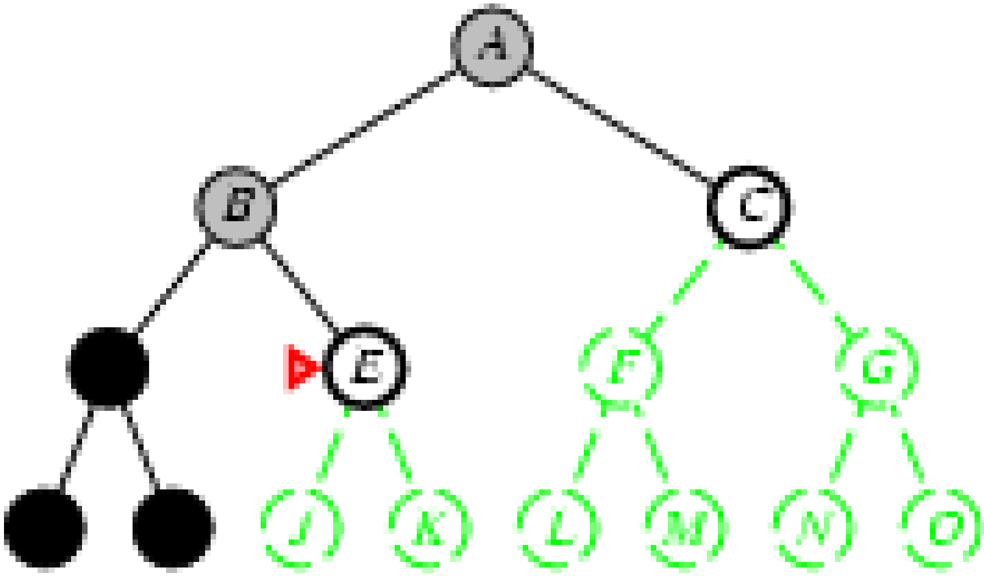

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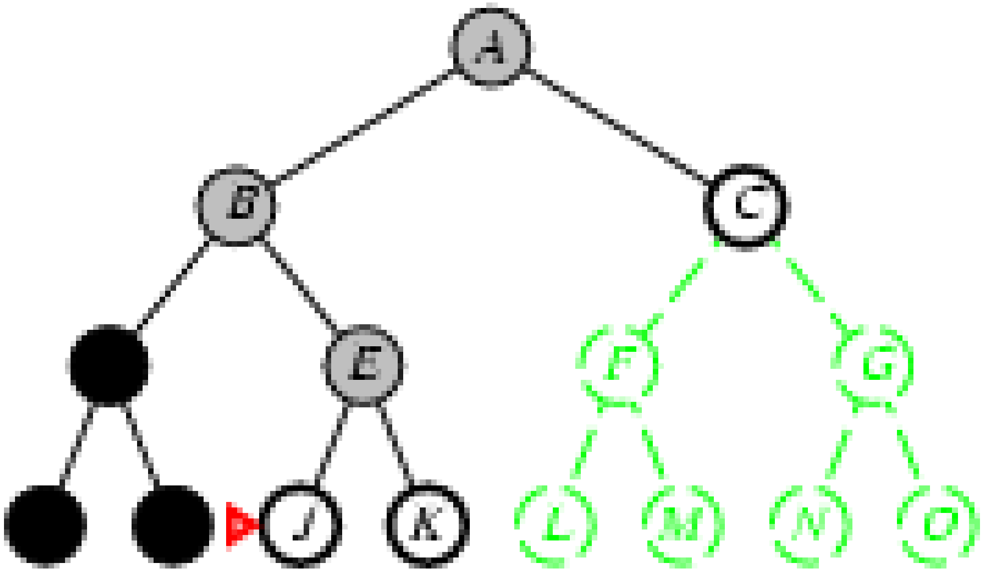

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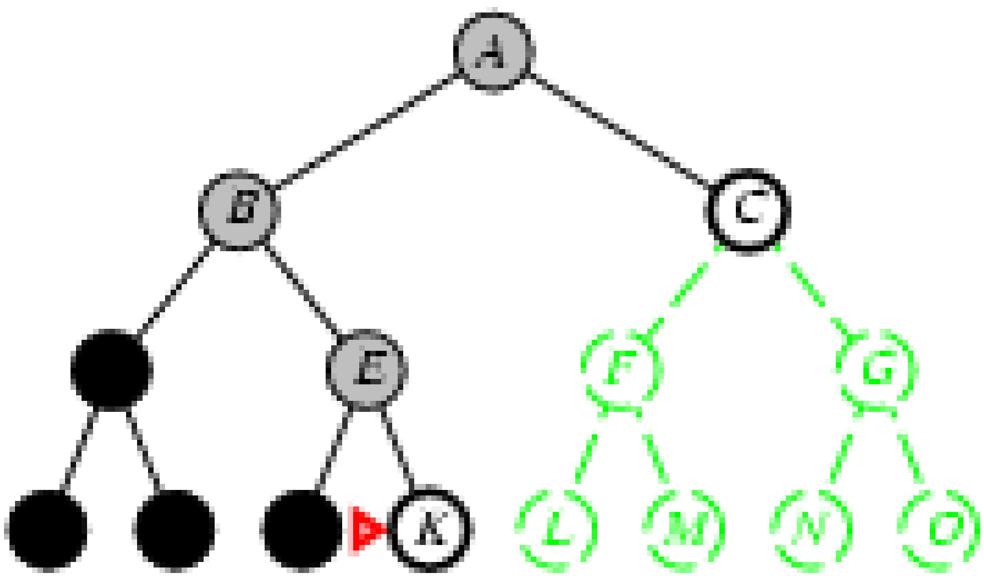

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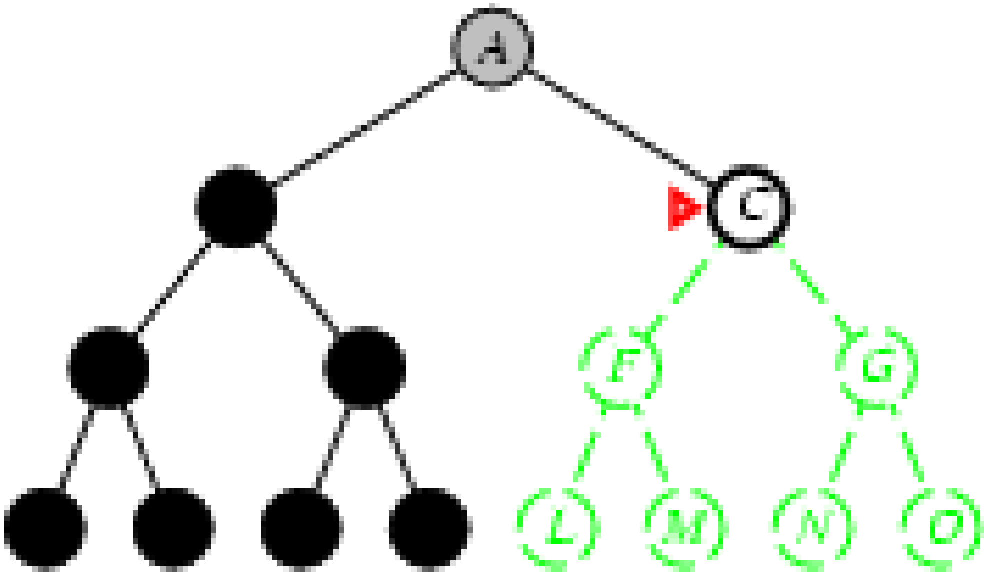

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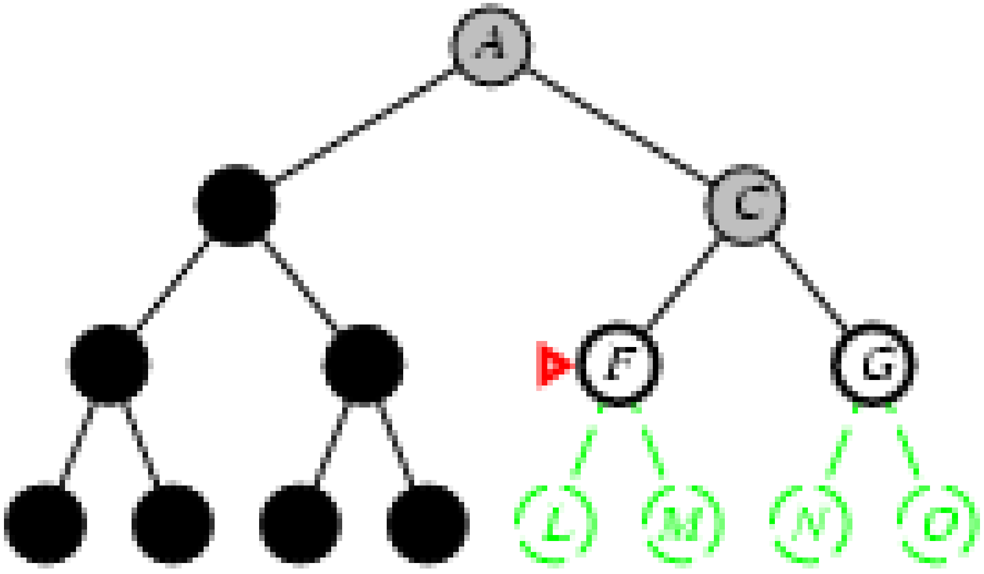

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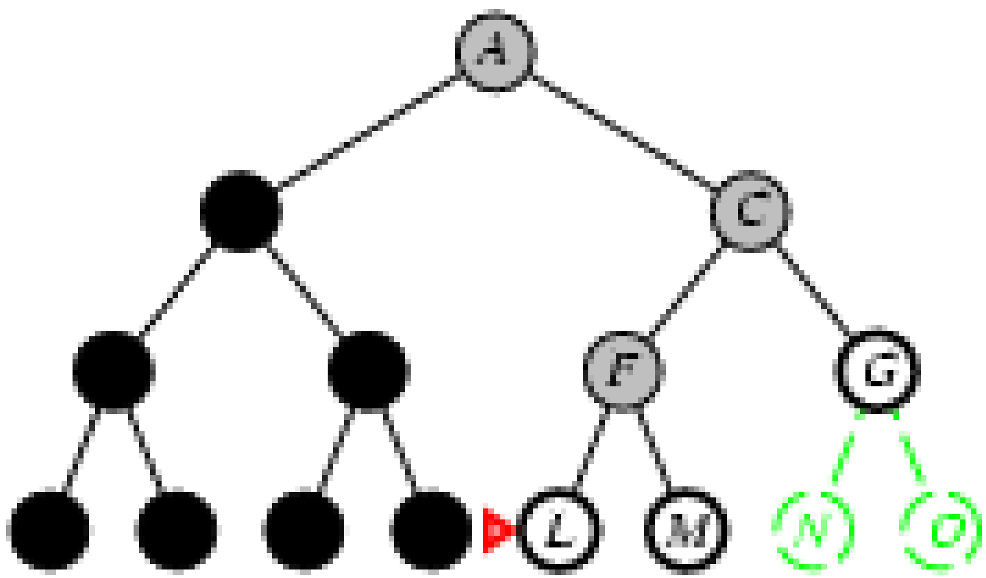

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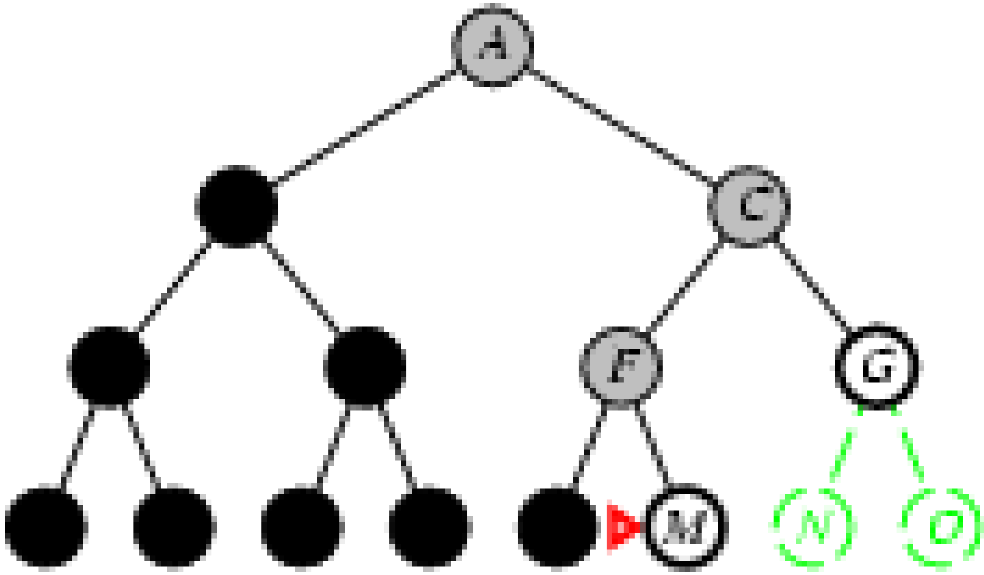


Properties of depth-first search

- Complete? No: fails in infinite-depth spaces, spaces with loops
  - Modify to avoid repeated states along path
    - complete in finite spaces
- Time? $O(b^m)$: terrible if m is much larger than d
  - but if solutions are dense, may be much faster than breadth-first
- Space? $O(bm)$, i.e., linear space!
- Optimal? No

Mid-Lecture Exercise

- Compare breadth-first and depth-first search.
  - When would breadth-first be preferable?
  - When would depth-first be preferable?

Solution

- Breadth-First:
  - When completeness is important.
  - When optimal solutions are important.
- Depth-First:
  - When solutions are dense and low-cost is important, especially space costs.

Depth-limited search

This is depth-first search with depth limit *l*, i.e., nodes at depth *l* have no successors

Recursive implementation:

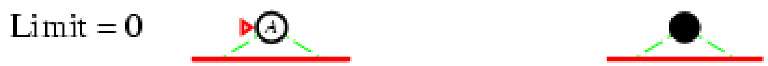
```
function DEPTH-LIMITED-SEARCH(problem, limit) returns a solution, or failure/cutoff
  return RECURSIVE-DLS(MAKE-NODE(problem.INITIAL-STATE), problem, limit)

function RECURSIVE-DLS(node, problem, limit) returns a solution, or failure/cutoff
  if problem.GOAL-TEST(node.STATE) then return SOLUTION(node)
  else if limit = 0 then return cutoff
  else
      cutoff_occurred? ← false
      for each action in problem.ACTIONS(node.STATE) do
          child ← CHILD-NODE(problem, node, action)
          result ← RECURSIVE-DLS(child, problem, limit − 1)
          if result = cutoff then cutoff_occurred? ← true
          else if result ≠ failure then return result
      if cutoff_occurred? then return cutoff else return failure
```

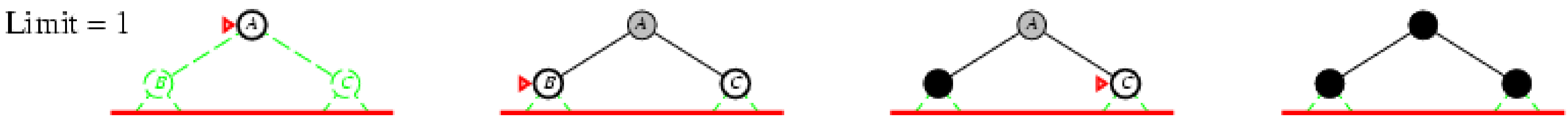
Iterative deepening search

```
function ITERATIVE-DEEPENING-SEARCH(problem) returns a solution, or failure
   for depth = 0 to ∞ do
      result ← DEPTH-LIMITED-SEARCH(problem, depth)
      if result ≠ cutoff then return result
```

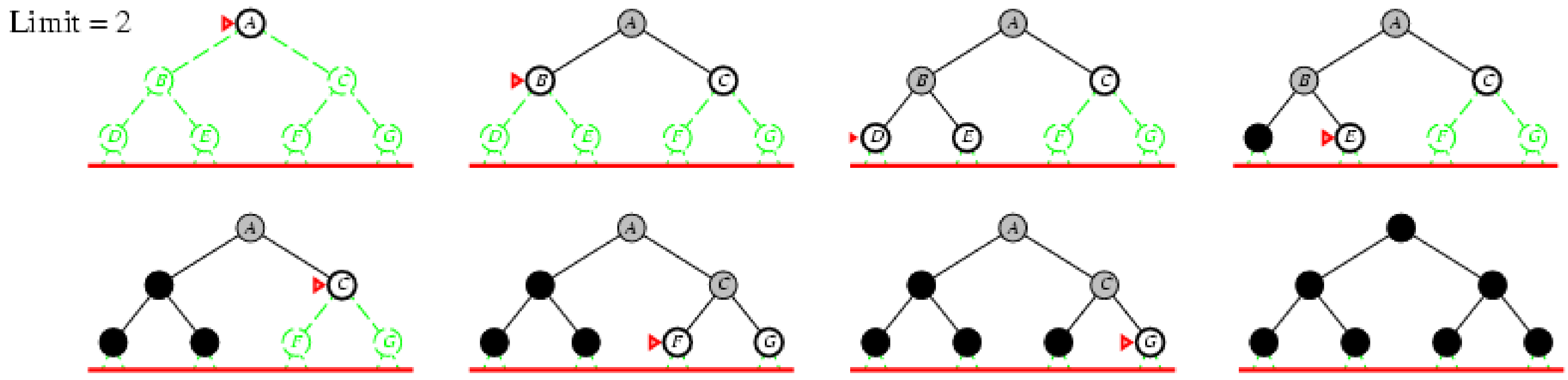
Iterative deepening search l =0



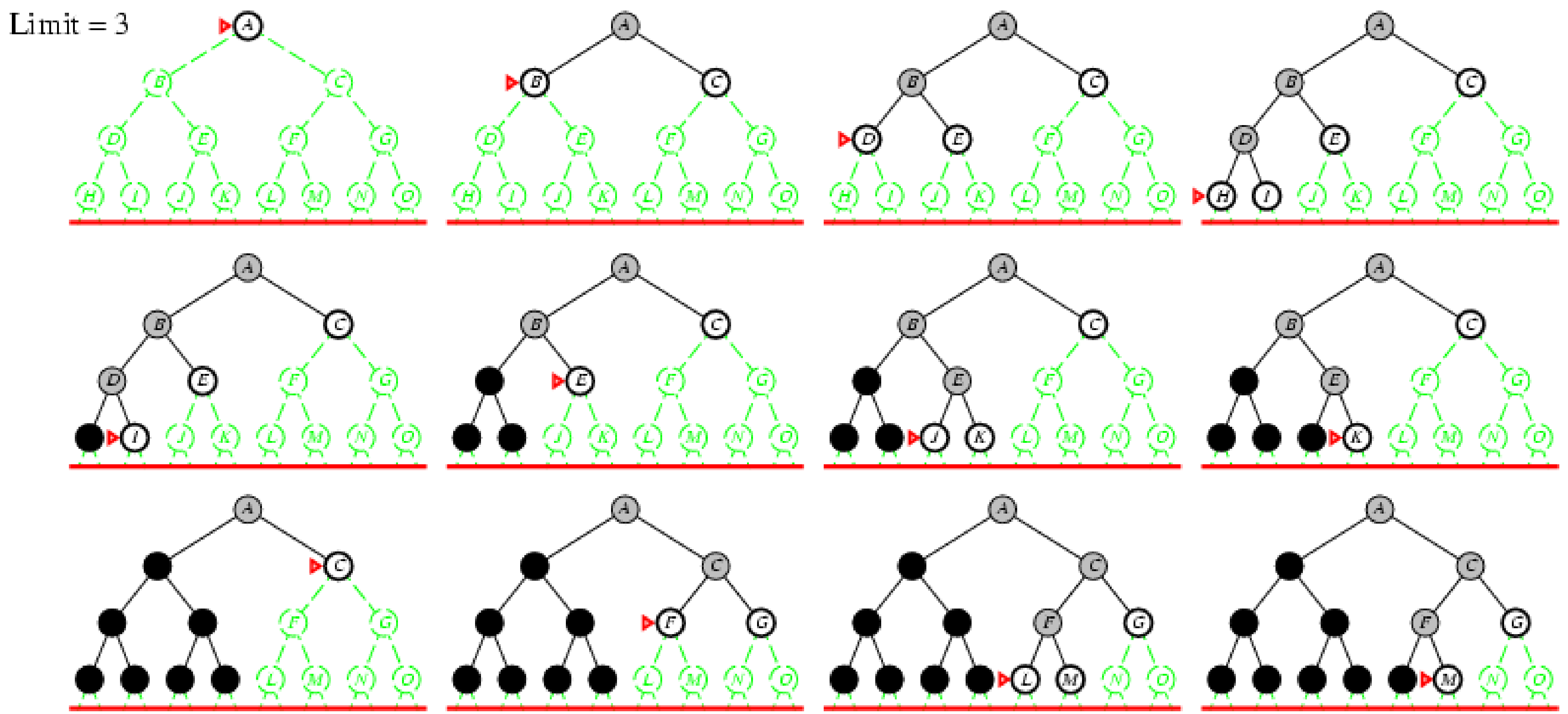
Iterative deepening search *l* =1



Iterative deepening search l =2



Iterative deepening search l =3



Iterative deepening search

- Number of nodes generated in an iterative deepening search to depth *d* with branching factor *b*:

$$N_{IDS} = (d)b + (d\text{-}1)\, b^2 + \ldots + (2)b^{d-1} + (1)b^d$$

- Some cost associated with generating upper levels multiple times
- Example: For *b = 10*, *d = 5*,
  - N_{BFS} = 10 + 100 + 3,000 + 10,000 + 100,000 = 111,110
  - N_{IDS} = 50 + 400 + 3,000 + 20,000 + 100,000 = 123,450
- Overhead = (123,450 - 111,110)/111,110 = 11%

Properties of iterative deepening search

- Complete? Yes
- Time? $(d)b + (d\text{-}1)b^2 + \ldots + (1)b^d = O(b^d)$
- Space? $O(bd)$
- Optimal? Yes, if step cost = 1

Summary of algorithms

Criterion	Breadth-First	Uniform-Cost	Depth-First	Depth-Limited	Iterative Deepening
Complete?	Yes	Yes	No	No	Yes
Time	$O(b^d)$	$O(b^{\lceil C^*/\epsilon \rceil})$	$O(b^m)$	$O(b^l)$	$O(b^d)$
Space	$O(b^d)$	$O(b^{\lceil C^*/\epsilon \rceil})$	$O(bm)$	$O(bl)$	$O(bd)$
Optimal?	Yes	Yes	No	No	Yes

Summary

- Variety of uninformed search strategies:
  - breadth-first, depth-first, iterative deepening
- Iterative deepening search uses only linear space and not much more time than other uninformed algorithms