DMMR Tutorial sheet 3

Functions, Sequences

October 7, 2015

Some of the exercises for this tutorial are taken from Chapter 2 and 8 of the book: Kenneth Rosen, Discrete Mathematics and its Applications, 7th Edition, McGraw-Hill, 2012.

1. Determine whether the function $f : \mathbb{Z} \times \mathbb{Z} \to \mathbb{Z}$ is surjective if

   (a) $f(m, n) = m^2 + n^2$
   (b) $f(m, n) = m$
   (c) $f(m, n) = |n|$
   (d) $f(m, n) = m - n$

2. (a) Prove that a strictly decreasing function from $\mathbb{R}$ to itself is one-to-one.
   (b) Give an example of a decreasing function from $\mathbb{R}$ to itself that is not one-to-one.

3. Determine (and prove) whether each of these sets is countably infinite or uncountable. For those that are countably infinite, exhibit a one-to-one correspondence (i.e., bijection) between the set of positive integers and that set.

   (a) the odd negative integers
   (b) the real numbers in the open interval $(0, 2)$
   (c) the irrational numbers in the open interval $(0, 2)$
   (d) the set $A \times \mathbb{Z}^+$ where $A = \{2, 3\}$

4. A vending machine dispensing books of stamps accepts only \$1 coins, \$1 bills and \$5 bills.

   (a) Find a recurrence relation for the number of ways to deposit n dollars in the vending machine, where the order in which the coins and bills are deposited matters.
   (b) What are the initial conditions?
   (c) How many ways are there to deposit \$10 for a book of stamps?

5. Show that the set of functions from positive integers to the set $\{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\}$ is uncountable.

   Solutions (to the last question ONLY on the sheet) must be handed in on paper to the ITO by Wednesday, 14 October, 4:00pm. Please post it into the grey metal box on the wall outside the ITO.