DMMR Tutorial sheet 1

Propositional Logic, Predicate Logic, Proof techniques

September 25, 2015

Some of the exercises for this tutorial are taken from Chapter 1 of the book: Kenneth Rosen, Discrete Mathematics and its Applications, 7th Edition, McGraw-Hill, 2012.

1. Construct the truth table for the formula $(A \rightarrow B) \rightarrow [((B \rightarrow C) \wedge \neg C) \rightarrow \neg A]$.

2. Let $P(m, n)$ be the statement "m divides n", where the domain for both variables consists of all positive integers. (By "m divides n" we mean that $n = km$ for some integer k.) Determine the truth values of each of these statements.

    (a) $P(4, 5)$

    (b) $P(2, 4)$

    (c) $\forall m \, \forall n \, P(m, n)$

    (d) $\exists n \, \forall m \, P(m, n)$

    (e) $\exists m \, \forall n \, P(m, n)$

    (f) $\forall n \, P(1, n)$

3. Prove by contraposition, that if m and n are integers and mn is even, then m is even or n is even.

4. Prove that the sum of an irrational number and a rational number is irrational.

5. Write the numbers $1, 2, \ldots, 2n$ on a blackboard, where n is an odd integer. Pick any two of the numbers, j and k write $|j - k|$ on the board and erase j and k. Continue this process until only one integer is written on the board. Prove that this integer must be odd.

    Hint consider what happens to the parity of the combined sum of the numbers that are on the blackboard at each stage.

Solutions (to the last question on the sheet) must be handed in on paper at the ITO by Wednesday, 30 September, 4:00pm.