DMMR Tutorial sheet 6

Complexity, Binomial Coefficient

October 28, 2015

Some of the exercises for this tutorial are taken the book: Kenneth Rosen, Discrete Mathematics and its Applications, 7th Edition, McGraw-Hill, 2012.

1. Find the least integer n such that $f(x)$ is $O(x^n)$ for each of these functions.

   (a) $f(x) = 2x^3 + x^2 \log x$

   (b) $f(x) = 3x^3 + (\log x)^4$

   (c) $f(x) = (x^4 + x^2 + 1)/(x^3 + 1)$

2. Let $f_1(x)$ and $f_2(x)$ be functions from the set of real numbers to the set of positive real numbers. Show that if $f_1(x)$ and $f_2(x)$ are both $\Theta(g(x))$, where $g(x)$ is a function from the set of real numbers to the set of positive real numbers, then $f_1(x) + f_2(x)$ is $\Theta(g(x))$. Is this still true if $f_1(x)$ and $f_2(x)$ can take negative values?

3. Seven women and nine men are on the faculty in the mathematics department at a school.

   (a) How many ways are there to select a committee of five members of the department if at least one woman must be on the committee?

   (b) How many ways are there to select a committee of five members of the department if at least one woman and at least one man must be on the committee?

4. What is the coefficient C_8 of x^8y^9 in the expansion of $(3x + 2y)^{17}$ when it is rewritten as a polynomial of the form:

$$\sum_{i=0}^{17} C_i x^i y^{17-i}$$

5. Prove the following identity holds for all non-negative integers n,r and k, such that $r \leq n$, and $k \leq r$:

$$\binom{n}{r} \cdot \binom{r}{k} = \binom{n}{k} \cdot \binom{n-k}{r-k}$$

Solutions (to the last question on the sheet) must be handed in on paper at the ITO by Wednesday, 4 November, 4:00pm.