

**Tutorial 0 – practice counting and calculus problems**

**Tutorial Date: Monday, 11 January, 2016**

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1. Let  $f(x) = 3x^4 - 2x^2 + 4\sin(x)$ ,  $g(x) = e^{2x^3-4x}$  and  $h(x) = e^x / \cos(x)$ . Compute the derivatives  $f'(x)$ ,  $g'(x)$  and  $h'(x)$  of these functions.

2. A student can select one of 6 different mathematics books, one of 3 different chemistry books and one of 4 different science books. In how many different ways can a student select a book of mathematics, a book of chemistry and a book of science?

3. Let  $f(x) = 4x^7 - 2x^3 + 1$ ,  $g(x) = \sin(3x)$  and  $h(x) = x/(1+x^2)$ . Compute the integrals

$$\int_0^2 f(x) dx, \quad \int_0^\pi g(x) dx, \quad \text{and} \quad \int_0^4 h(x) dx.$$

4. There are 3 different roads from city A to city B and 2 different roads from city B to city C. In how many ways can someone go from city A to city C passing by city B?

5. Let

$$f(x) = x^2 \cos(2x + x^3), \quad g(x) = xe^{-x^2 + \sin(2x)}, \quad h(x) = \frac{\sin(e^{3x-x^4})}{3x^5 - 2x^3 + 2}.$$

In each case, find the derivatives  $f'(x)$ ,  $g'(x)$  and  $h'(x)$  of these functions.

6. In a company, ID cards have 5 digit numbers.

- a) How many ID cards can be formed if repetition of the digit is allowed?
- b) How many ID cards can be formed if repetition of the digit is not allowed?

7. Let  $f(x) = e^{-2x}$ ,  $g(x) = x \sin(2\pi x^2)$ , and  $h(x) = x \cos(2\pi x)$ . Compute

$$\int_0^1 f(x) dx, \quad \int_0^1 g(x) dx, \quad \text{and} \quad \int_0^1 h(x) dx.$$

For the last integral, use the integration by parts formula

$$\int_a^b j(x)k'(x) dx = - \int_a^b j'(x)k(x) dx + j(x)k(x) \Big|_{x=a}^{x=b}$$

which follows from the product rule of differentiation  $d/dx[j(x)k(x)] = j'(x)k(x) + j(x)k'(x)$  and the fundamental theorem of calculus:

$$j(x)k(x) \Big|_{x=a}^{x=b} = \int_a^b \frac{d}{dx}[j(x)k(x)] dx = \int_a^b j'(x)k(x) dx + \int_a^b j(x)k'(x) dx.$$

8. Three coins are tossed and a die is rolled. What is the total number of all possible outcomes?