DMMR Tutorial sheet 2

Sets, Relations

September 30, 2015

Some of the exercises for this tutorial are taken from Chapter 2 and 9 of the book: Kenneth Rosen, Discrete Mathematics and its Applications, 7th Edition, McGraw-Hill, 2012.

1. Show that the subset relation $\subseteq$ is reflexive.
2. The absorption law for sets is $A \cup (A \cap B) = A$. Give a proof that $A \cup (A \cap B) = A$.
3. For each of the following relations on the set of all real numbers, determine whether it is reflexive, symmetric, antisymmetric, and/or transitive, where (x, y) are related if and only if

   (a) $x - y$ is a rational number.

   (b) $x = 2y$.

   (c) $xy \geq 0$.

   (d) $xy = 0$.

   (e) $x = 1$.

   (f) $x = 1$ or $y = 1$.

4. Let A, B, C be sets. Derive a formula for $|A \cup B \cup C|$, which only uses the cardinality $|\cdot|$, intersection $\cap$ and arithmetic operators.
5. Many program analysis methods rely on call graphs. A call graph is a relation R_C and a pair (f, g) of function names is in R_C, iff the body of function f calls the function g. For example for the function f

```
function f() {
  g();
  h()
}
```

   the pairs (f, g) and (f, h) are in the relation R_C.

   The *transitive closure* of the relation R_C written as $Trans(R_C)$ is a new relation, which contains a pair (f, f'), if there is a chain $(f, f_2), (f_2, f_3), ..., (f_{n-1}, f')$ of pairs all contained in R_C. Formally this can be defined as

$$Trans(R_C) = \{(f, f') | \, \exists (f_1, ..., f_n) \text{ with } f = f_1 \wedge f' = f_n \wedge \forall i \in \{1, ..., (n-1)\} (f_i, f_{i+1}) \in R_C\}$$

   The *symmetric closure* of the relation R_C is a new relation written as $Sym(R_C)$, which contains all pairs (f, g) from R_C along with their corresponding pairs (g, f).

   (a) Prove that $Trans(R_C)$ is transitive

   (b) Explain what information the relations $Trans(Sym(R_C))$ and $Sym(Trans(R_C))$ contain about the program.

(c) Decide which of the two relations $Trans(Sym(R_C))$ and $Sym(Trans(R_C))$ subsumes the other, give a formal proof of your claim and show an example relation R_C and a pair which is contained in exactly one of them.

Solutions (to the last question ONLY on the sheet) must be handed in on paper at the ITO by Wednesday, 7 October, 4:00pm. Please post it into the grey metal box on the wall outside the ITO