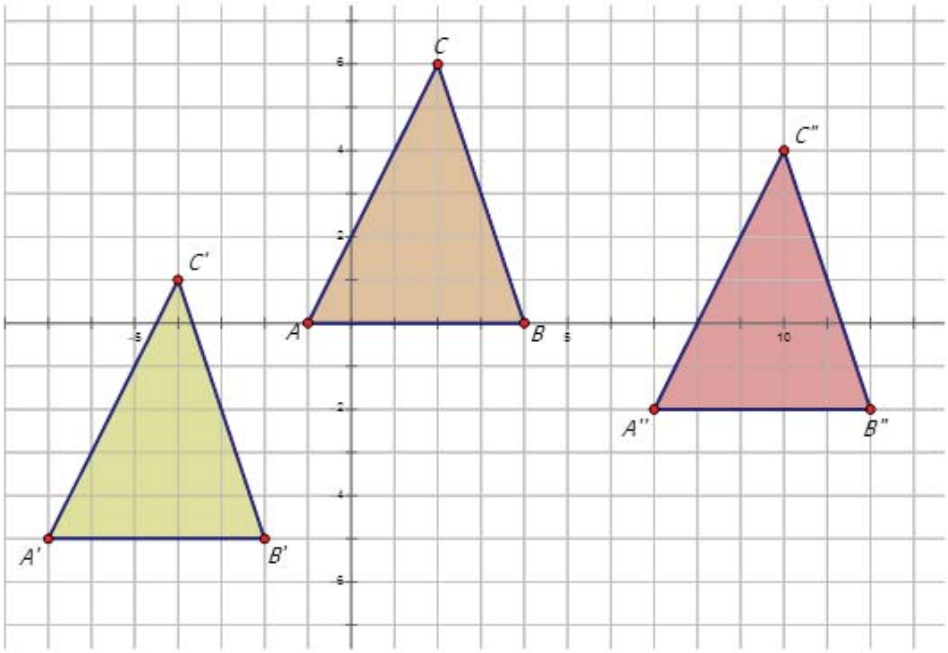
1.10 Composite Transformations

Here you will learn about composite transformations.

Look at the following diagram. It involves two translations. Identify the two translations of triangle *ABC*.



Watch This

First watch this video to learn about composite transformations.

MEDIA

Click image to the left for more content.

CK-12 FoundationChapter10CompositeTransformationsA

Then watch this video to see some examples.

MEDIA

Click image to the left for more content.

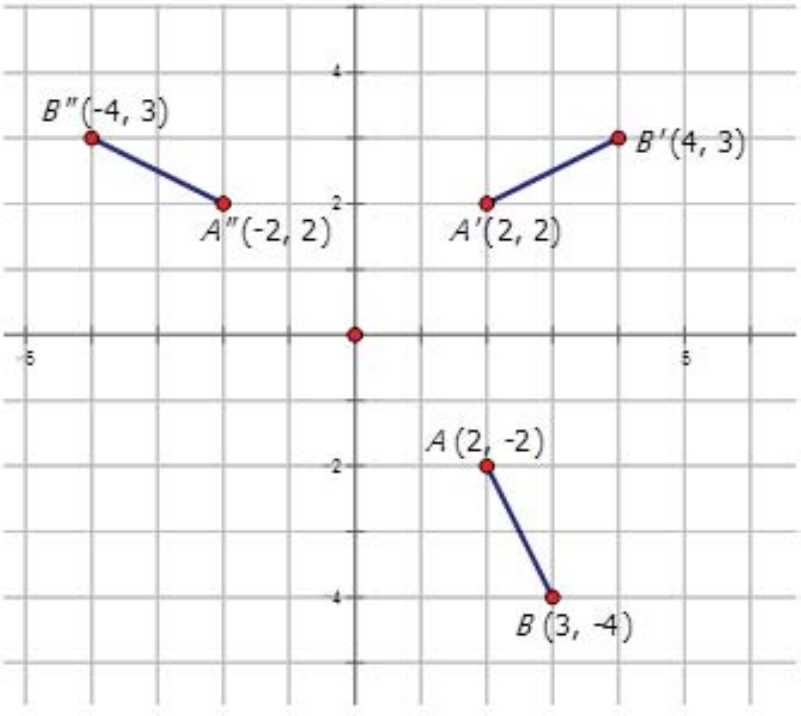
CK-12 FoundationChapter10CompositeTransformationsB

Guidance

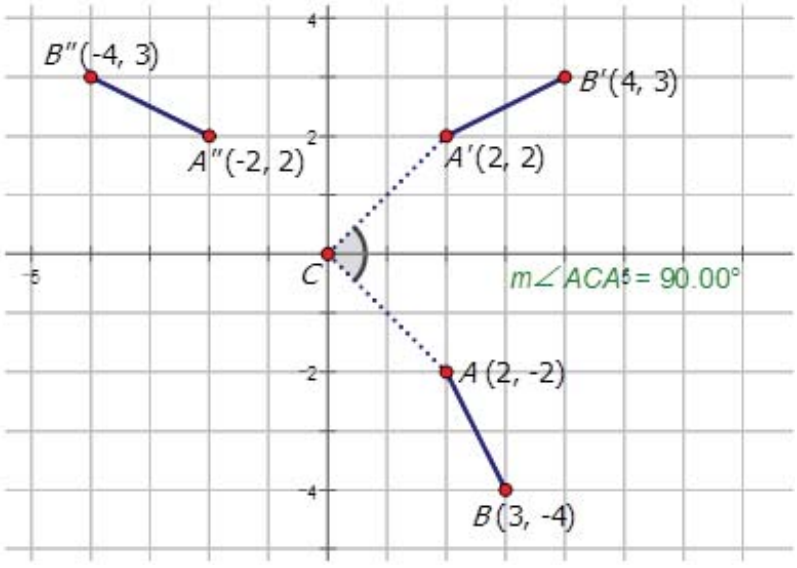
In geometry, a transformation is an operation that moves, flips, or changes a shape to create a new shape. A composite transformation is when two or more transformations are performed on a figure (called the preimage) to produce a new figure (called the image).

Example A

Describe the transformations in the diagram below. The transformations involve a reflection and a rotation.



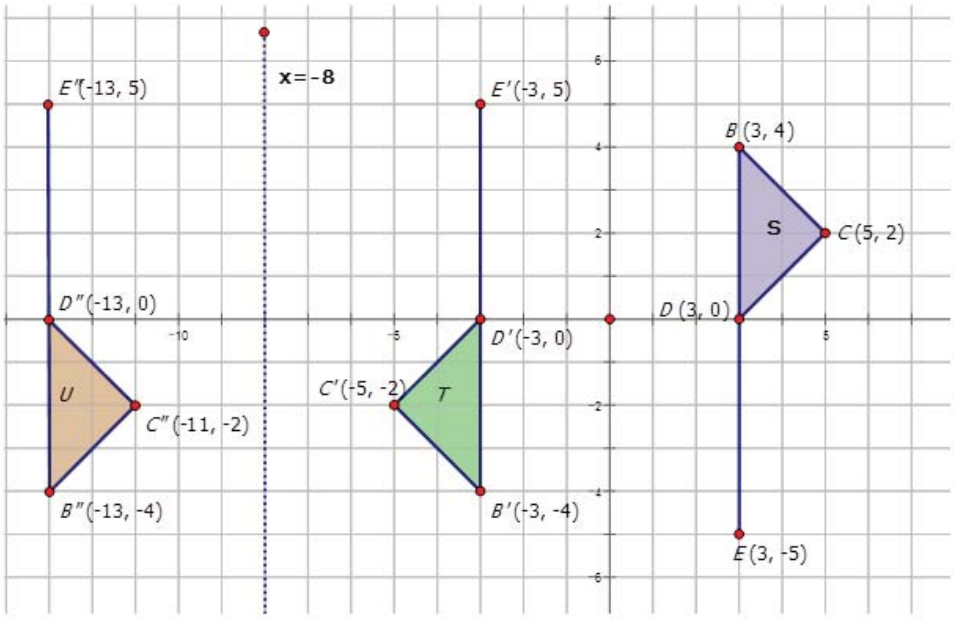
Solution: First line AB is rotated about the origin by 90°CCW.



Then the line $A'B'$ is reflected about the y-axis to produce line $A''B''$.

Example B

Describe the transformations in the diagram below.



Solution: The flag in diagram S is rotated about the origin 180° to produce flag T. You know this because if you look at one point you notice that both x- and y-coordinate points is multiplied by -1 which is consistent with a 180° rotation about the origin. Flag T is then reflected about the line $x = -8$ to produce Flag U.

Example C

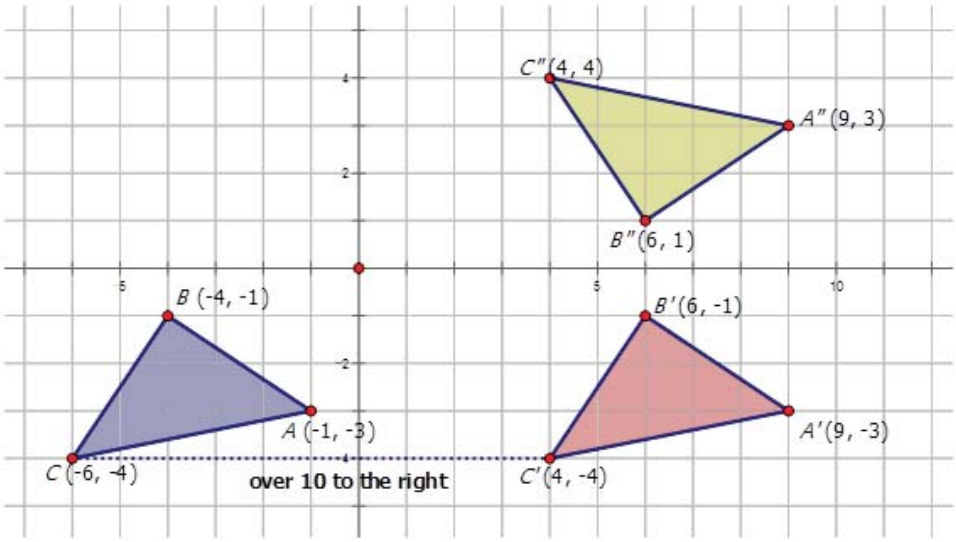
Triangle ABC where the vertices of ΔABC are $A(-1,-3)$, $B(-4,-1)$, and $C(-6,-4)$ undergoes a composition of transformations described as:

a) a translation 10 units to the right, then

b) a reflection in the x-axis.

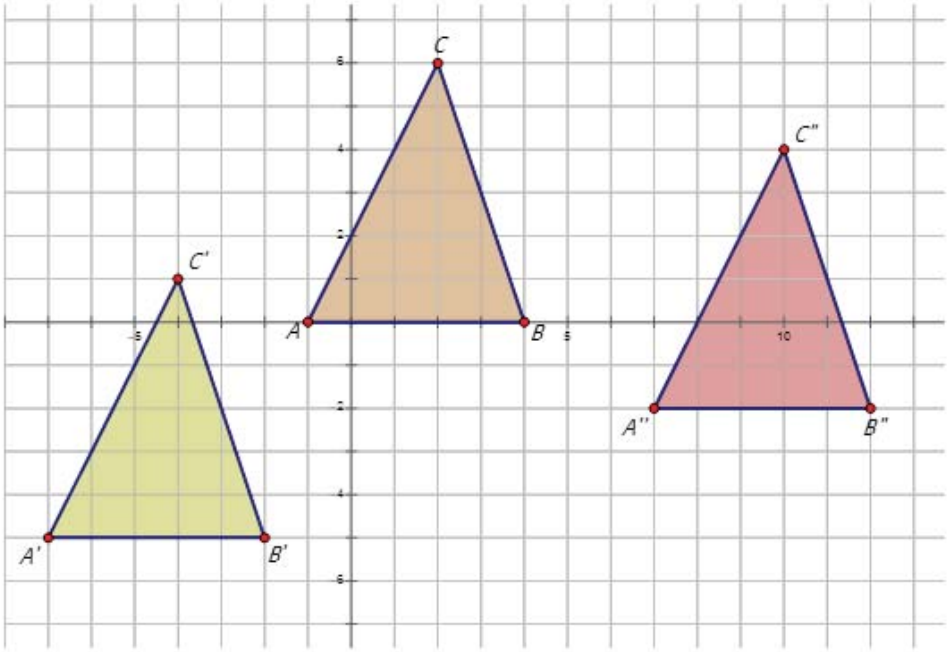
Draw the diagram to represent this composition of transformations. What are the vertices of the triangle after both transformations are applied?

Solution:

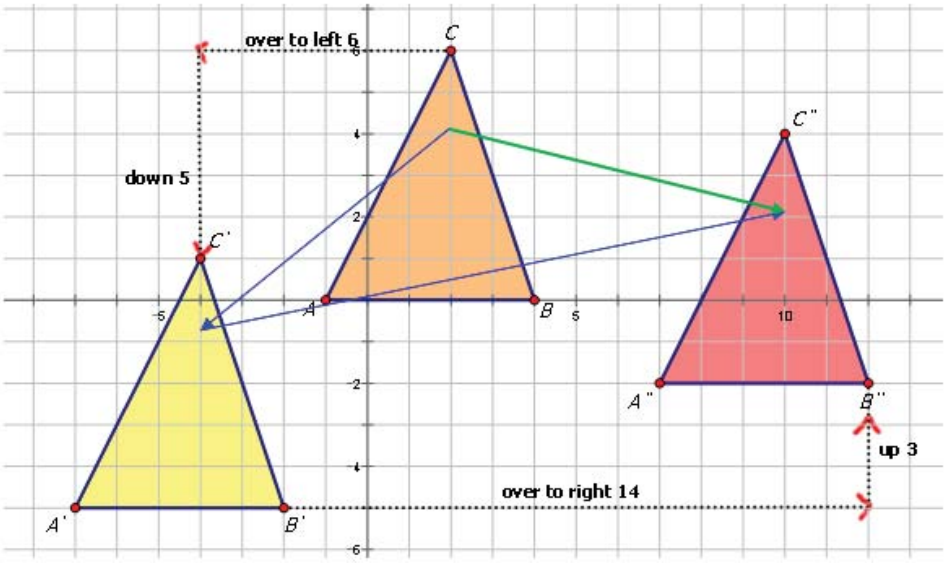


Triangle $A''B''C''$ is the final triangle after all transformations are applied. It has vertices of $A''(9,3)$, $B''(6,1)$, and $C''(4,4)$.

Concept Problem Revisited



ΔABC moves over 6 to the left and down 5 to produce $\Delta A'B'C'$. Then $\Delta A'B'C'$ moves over 14 to the right and up 3 to produce $\Delta A''B''C''$. These translations are represented by the blue arrows in the diagram.



All together ΔABC moves over 8 to the right and down 2 to produce $\Delta A''B''C''$. The total translations for this movement are seen by the green arrow in the diagram above.

Vocabulary

Image

In a transformation, the final figure is called the ***image***.

Preimage

In a transformation, the original figure is called the ***preimage.***

Transformation

A ***transformation*** is an operation that is performed on a shape that moves or changes it in some way. There are four types of transformations: translations, reflections, dilations and rotations.

Dilation

A ***dilation*** is a transformation that enlarges or reduces the size of a figure.

Translation

A ***translation*** is an example of a transformation that moves each point of a shape the same distance and in the same direction. Translations are also known as **slides**.

Rotation

A ***rotation*** is a transformation that rotates (turns) an image a certain amount about a certain point.

Reflection

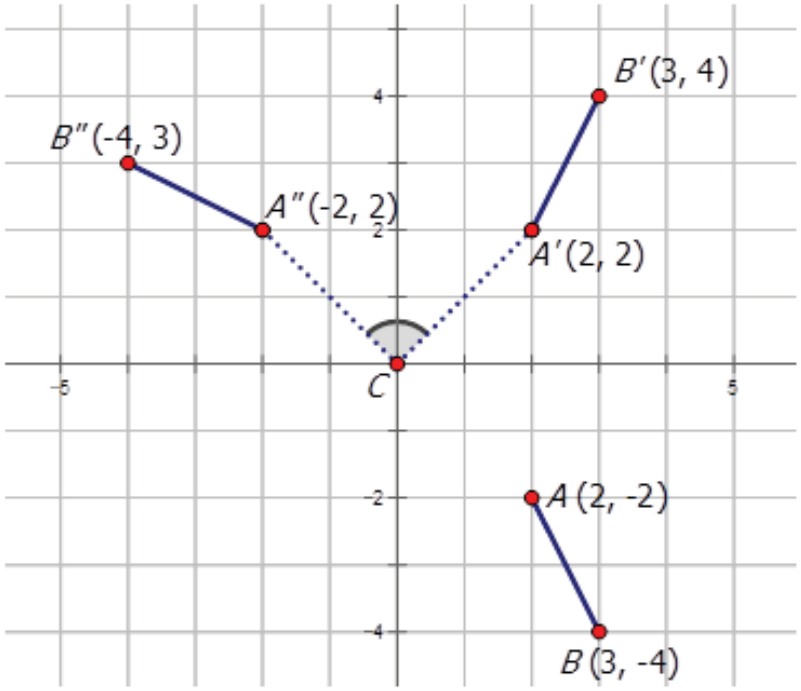
A ***reflection*** is an example of a transformation that flips each point of a shape over the same line.

Composite Transformation

A ***composite transformation*** is when two or more transformations are combined to form a new image from the preimage.

Guided Practice

1. Describe the transformations in the diagram below. The transformations involve a rotation and a reflection.



2. Triangle XYZ has coordinates $X(1,2)$, $Y(-3,6)$ and $Z(4,5)$.The triangle undergoes a rotation of 2 units to the right and 1 unit down to form triangle $X'Y'Z'$. Triangle $X'Y'Z'$ is then reflected about the y-axis to form triangle $X''Y''Z''$. Draw the diagram of this composite transformation and determine the vertices for triangle $X''Y''Z''$.

3. The coordinates of the vertices of ΔJAK are $J(1,6)$, $B(2,9)$, and $C(7,10)$.

a) Draw and label ΔJAK.

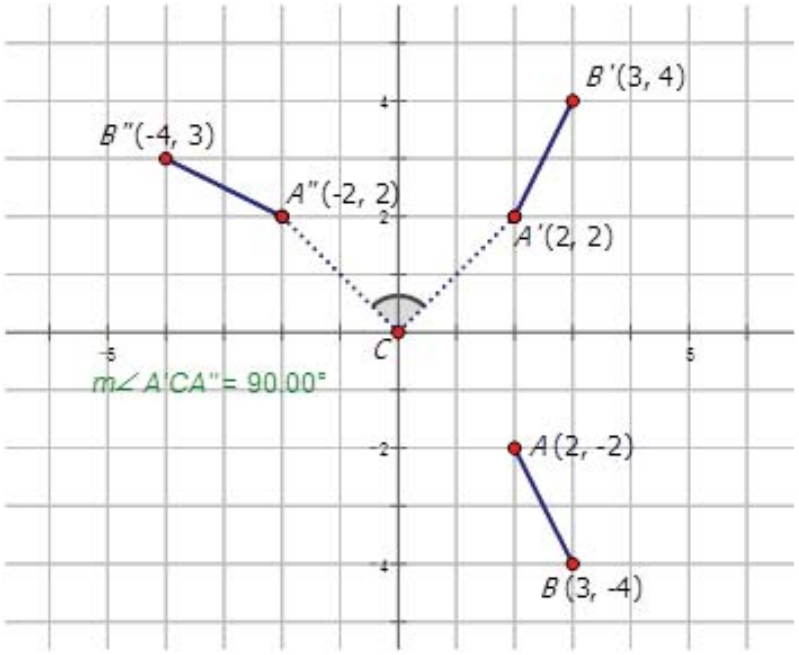
b) ΔJAK is reflected over the line $y = x$. Graph and state the coordinates of $\Delta J'A'K'$.

c) $\Delta J'A'K'$ is then reflected about the x-axis. Graph and state the coordinates of $\Delta J''A''K''$.

d) $\Delta J''A''K''$undergoes a translation of 5 units to the left and 3 units up. Graph and state the coordinates of $\Delta J'''A'''K'''$.

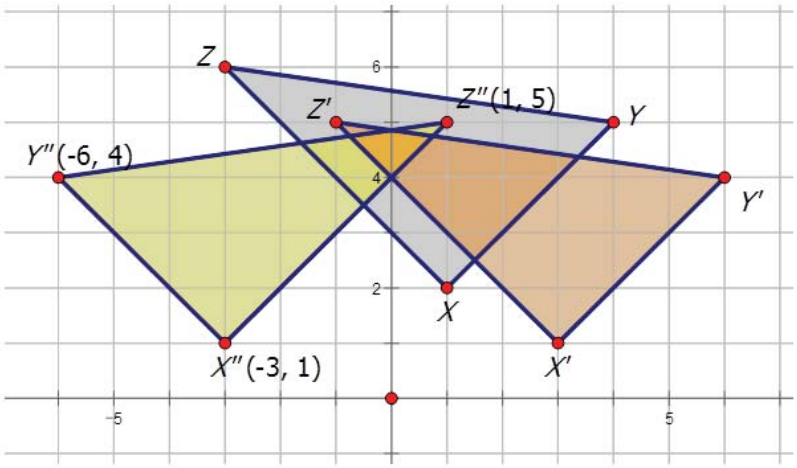
Answers:

1. The transformations involve a reflection and a rotation. First line AB is reflected about the y-axis to produce line $A'B'$.

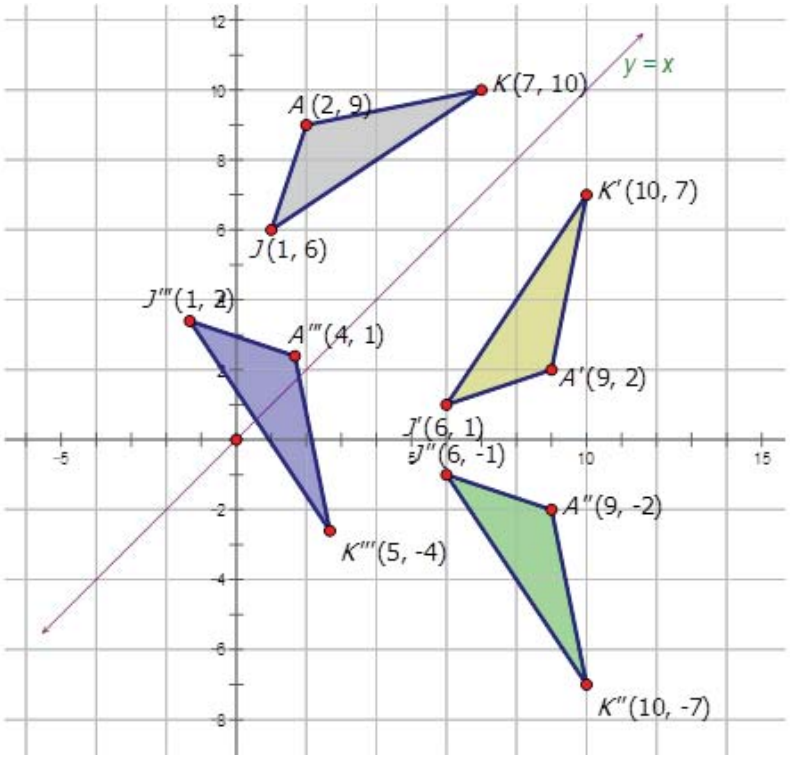


Then the line $A'B'$ is rotated about the origin by 90°CCW to produce line $A''B''$.

2.



3.

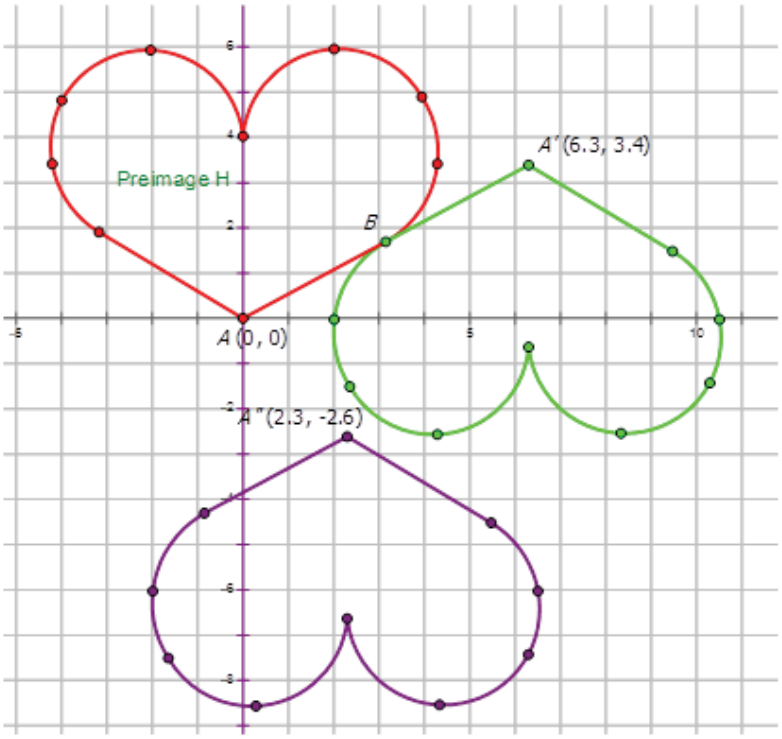


Practice

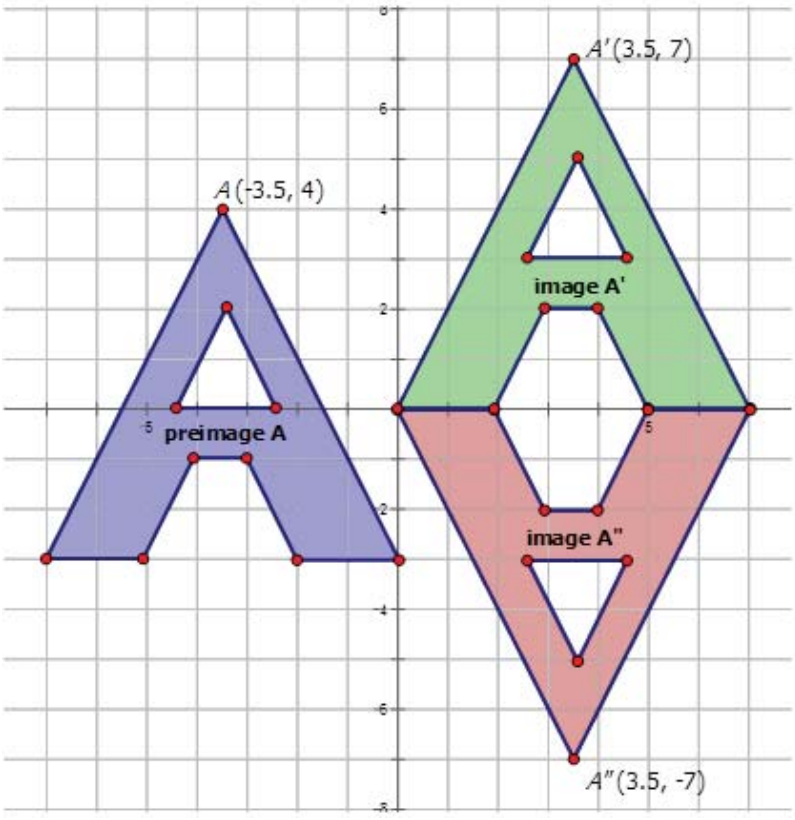
1. A point X has coordinates (-1, -8). The point is reflected across the y-axis to form X'. X' is translated over 4 to the right and up 6 to form X''. What are the coordinates of X' and X''?
2. A point A has coordinates (2, -3). The point is translated over 3 to the left and up 5 to form A'. A' is reflected across the x-axis to form A''. What are the coordinates of A' and A''?
3. A point P has coordinates (5, -6). The point is reflected across the line $y = -x$ to form P'. P' is rotated about the origin 90°CW to form P''. What are the coordinates of P' and P''?
4. Line JT has coordinates $J(-2,-5)$ and $T(2,3)$. The segment is rotated about the origin 180° to form $J'T'$. $J'T'$ is translated over 6 to the right and down 3 to form $J''T''$. What are the coordinates of $J'T'$ and $J''T''$?
5. Line SK has coordinates $S(-1,-8)$ and $K(1,2)$. The segment is translated over 3 to the right and up 3 to form $S'K'$. $S'K'$ is rotated about the origin 90°CCW to form $S''K''$. What are the coordinates of $S'K'$ and $S''K''$?
6. A point K has coordinates (-1, 4). The point is reflected across the line $y = x$ to form K'. K' is rotated about the origin 270°CW to form K''. What are the coordinates of K' and K''?

Describe the following composite transformations:

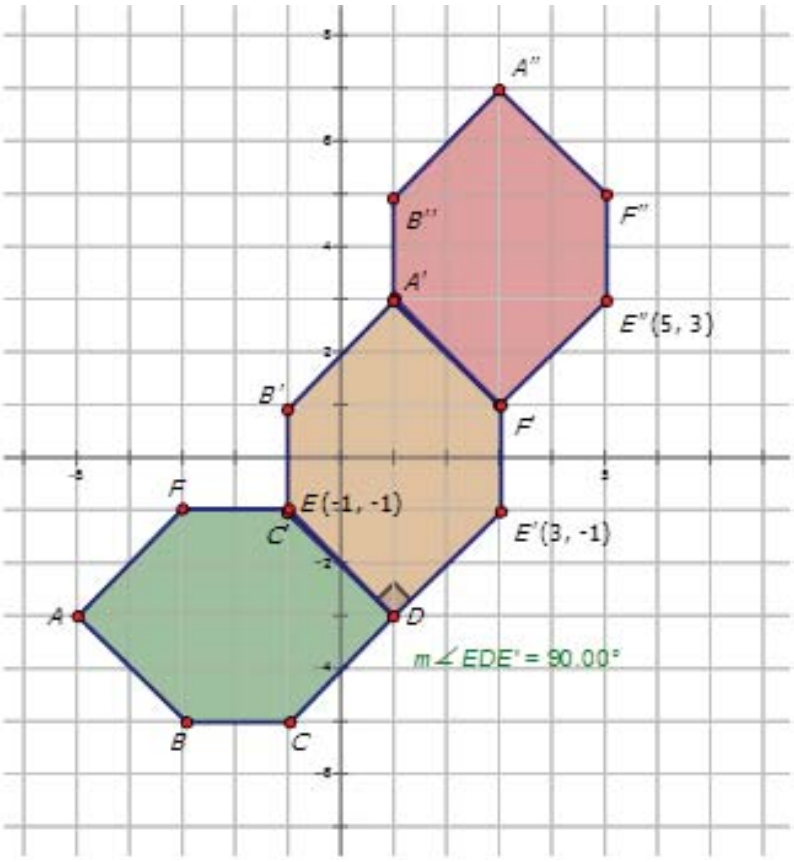
7.


Preimage H
A (0, 0)
B
A' (6.3, 3.4)
A" (2.3, -2.6)

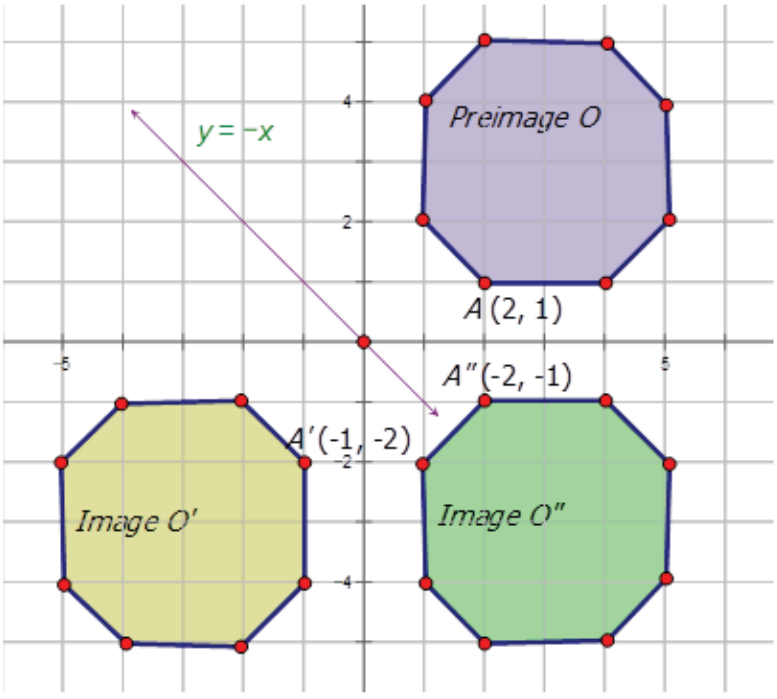
8.

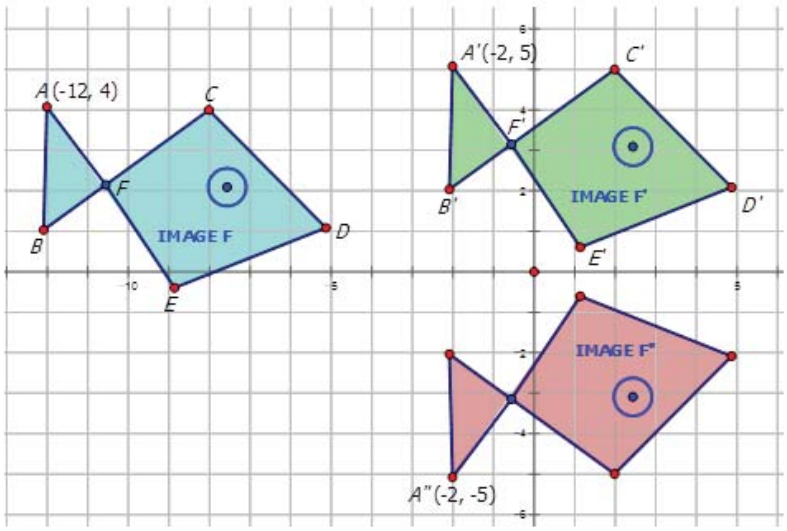

A (-3.5, 4)
preimage A
A' (3.5, 7)
image A'
image A"
A" (3.5, -7)

9.


A''
B''
F''
A'
E''(5, 3)
B'
F'
F
E(-1, -1)
C'
E'(3, -1)
A
D
m∠EDE' = 90.00°
B
C

10.


y = −x
Preimage O
A(2, 1)
A''(-2, -1)
A'(-1, -2)
Image O'
Image O''

11. 

12. Explore what happens when you reflect a shape twice, over a pair of parallel lines. What one transformation could have been performed to achieve the same result?
13. Explore what happens when you reflect a shape twice, over a pair of intersecting lines. What one transformation could have been performed to achieve the same result?
14. Explore what happens when you reflect a shape over the x-axis and then the y-axis. What one transformation could have been performed to achieve the same result?
15. A composition of a reflection and a translation is often called a glide reflection. Make up an example of a glide reflection. Why do you think it's called a **glide** reflection?