Theorem Prover

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The Objective

We have been given a statement in propositional logic.

The program needs to check whether the statement is a theorem or not.

The use of semantics is not allowed.

Only syntax rules and the deduction theorem is used.

Points of Note

We assume that we have proven the following statements to be theorems:

p -> p

(p -> q) <-> (~q -> ~p) (Contrapositive)

F -> p

This is reasonable as their proofs are easily obtainable using the deduction theorem.

The Method

What we do first is to repeatedly apply deduction theorem on the statement S to obtain something of the following form:

$H_1, H_2, \ldots\ldots, H_n \models F$

Hence we have a set of hypothesis from which we wish to derive F.

The Method (Contd.)

First we append the contrapositives of all Hi to the set of the hypothesis.

Next we repeatedly apply Modus Ponens and take contrapositives to get a maximal set of hypothesis on which no more Modus Ponens can be applied.

If at this point we obtain F we stop.

The Method (Contd.)

Let the maximal set be

M1, M2,…., Ms

To prove S using these, we can try proving the negation of any of the statements.

If that statement is of the form of an axiom we are done.

The Method (Contd.)

Let us say we are trying to prove K. If K is of the form ((A -> F) -> F) we remove the double not.

We assume each statement to be of the form (LHS -> RHS) if an implication is present.

We search for K in the RHS of all the statements and if we find it, then we attempt to prove the LHS.

The Method (Contd.)

Also if K is the form of (A -> B) we can prove either (~B -> ~A) or B to finish the job.

This because of contrapositives being equivalent to the original statement and Axiom 1 [(B -> (A -> B)) and B give (A -> B)]

We use Axiom 2 in a similar manner. If K is of the form ((A -> B) -> (A -> C)) we try proving (A -> (B -> C)).

The Method (Contd.)

We keep going backwards in this manner until we get stuck at a hypothesis.

Lets say we got stuck at P1, ..., Pg

Now we can assume either to finish the proof or trying proving one of them. We ask for the user's input here to choose one of them.

Now we attempt to prove it.

The Method (Contd.)

Let us say the user choose Pj
then we have to prove

The set of statements |= Pj

We apply the deduction theorem to get F on the RHS and repeat our procedure.

User Input

If the process gets stuck(no new hypotheses are created) then the program signals an error.

It will ask the user to add a new hypotheses, verify a possible hypotheses or let the program assume a hypothesis and try to solve using that hypothesis

Thank You