Exercise #1
ECON 8745
Numerical Methods in Economics

1. Consider the function

$$f(x) = \begin{cases} x^4 \exp\left(-\frac{1}{4}x^2\right) \sin\left(\frac{8}{x^3}\right) & \text{if } x \neq 0 \\ 0 & \text{if } x = 0 \end{cases}.$$

   (a) For $h = 10^{-n}$ for $n \in \{-1, ..., -16\}$, compute and plot the errors for a one-sided finite difference approximation to the derivative (the error is the absolute difference between the true value and the numerical approximation) at the point $x = 1$.

   (b) Do the same for a two-sided finite difference approximation.

   (c) Now use a four-point stencil.

   (d) Finally, use complex step differentiation.

2. Consider the calculation of $E\left[X^2 e^X\right]$, where X is a normal random variable. Using Gauss-Hermite quadrature, compute this expectation for $n \in \{2, 3, 5, 9, 11, 15, 31\}$. Then compute the same expectation using Monte Carlo integration with $T \in \{10, 100, 1000, 10000\}$. Assess the errors in your calculations.