Online Summer School
Macro, Money, and Finance
Problem Set 4

June 10, 2025

Please submit your solutions to the dropbox link by 6/29/2025 23:59 (EDT).

1 Nominal Bonds as Safe Assets and Bubble Mining Seigniorage

Consider the following model, which is a variant of the money model covered in Lecture 10. The setup is as on Slide 7 of Lecture 10 with the following modifications:

- We call the liabilities issued by the government "bonds" and denote their nominal quantity by $\mathcal{B}_t$. Bonds are of infinitesimal duration and pay a floating nominal interest rate i_t.
- There are no transaction costs, $\mathfrak{T}_t \equiv 0$.
- There are no government expenditures, $\mathscr{G}_t \equiv 0$.
- Assume that the government holds the nominal bond growth rate $\mu^{\mathcal{B}}$ and the nominal interest rate i constant over time. Treat these two variables as parameters of the model.

In this problem, we focus on monetary steady state equilibria. You may therefore assume from the outset that $q^{\mathcal{B}}$ and q^K are positive and constant over time. Also, assume $\tilde{\sigma}^2 > \rho$.

1. Solve the model by performing the following steps:
   (a) Determine the return expressions for capital and bonds and simplify as much as possible using the assumptions provided above.
   (b) Characterize the optimal consumption, investment, and portfolio choice of households.
   (c) Combine the equations derived in (b) with market clearing to determine the equilibrium values for asset prices (q, q^K, $q^{\mathcal{B}}$), investment (ι), primary surpluses (s), the real risk-free rate, the output growth rate, and the inflation rate.
2. Investigate the potential for the government to extract seigniorage revenues from bubble mining due to the presence of idiosyncratic risk. For your answer, restrict attention to the special case $\phi \to \infty$ without physical investment. In this case, the capital stock grows at the exogenous rate $g = -\delta$.

(a) Derive a relationship between real seigniorage revenue per unit of K_t, $\mathscr{s} := -s$, and the policy variables i, $\mu^{\mathcal{B}}$. Explain intuitively why there is a Laffer curve.

(b) Show that there is a unique maximum $\mathscr{s}$, let's call it $\bar{\mathscr{s}}$, and determine how $\bar{\mathscr{s}}$ is affected by a change in $\tilde{\sigma}$?
Hint: You can do this without taking any first-order conditions by arguing that $\mathscr{s}$ is concave and by investigating how it is affected by a change in $\tilde{\sigma}$ for fixed $\mu^{\mathcal{B}}$ and i.

(c) Seigniorage is often depicted as an "inflation tax". Show that there is a policy choice that extracts seigniorage $\mathscr{s} = \bar{\mathscr{s}}$ for arbitrarily large inflation rates. Explain intuitively why high inflation does not necessarily erode the government's ability to extract seigniorage revenue.

2 Monetary Policy in Model with Bonds and Money

Consider a variant of the model from Lecture 10 with money (nominal quantity $\mathcal{M}_t$, nominal interest rate $i_t^{\mathcal{M}}$), nominal short-term bonds (nominal quantity $\mathcal{B}_t$, nominal interest rate $i_t^{\mathcal{B}}$), idiosyncratic risk and transaction services, but no fiscal policy ($\tau_t = \mathscr{G}_t \equiv 0$). Transaction services are modeled by a cash-in-advance constraint in production,

$$\mathcal{P}_t a k_t^{\tilde{i}} \leq \bar{\nu} m_t^{\tilde{i}},$$

where $k_t^{\tilde{i}}$ are the capital holdings and $m_t^{\tilde{i}}$ are the nominal money holdings of agent $\tilde{i}$. Importantly, only money relaxes this constraint, bonds do not.

The government starts with initial nominal liabilities $\mathcal{B}_0, \mathcal{M}_0 > 0$, and it sets a constant interest rate on reserves $i^{\mathcal{M}}$ and constant growth rates of nominal liabilities, $\mu^{\mathcal{B}} = \mu^{\mathcal{M}}$, to balance the budget. The nominal rate on bonds, $i_t^{\mathcal{B}}$, is not directly set by the government but left free to clear the bond market.

Throughout, restrict attention to the (unique) monetary steady-state equilibrium. For each of the policy experiments to be analyzed below, you may assume without justification that the economy immediately jumps to a new steady state.

1. Suppose the government unexpectedly announces one of the following policies at $t = 0$. For each of these policies, explain how this affects (i) the equilibrium value of ϑ, (ii) the equilibrium interest rate on bonds, $i^{\mathcal{B}}$, and inflation rate, π, (iii) the initial price level $\mathcal{P}_0$. Assume in all cases that the economy is initially in the "medium of exchange regime" (in which $\Delta i > 0$, analogous to left-hand column on Slide 32) and the policy change is sufficiently small such that the economy remains in this regime.

   - Policy 1: one-time Helicopter drop of money:
     The government prints $\Delta\mathcal{M} > 0$ units of money at $t = 0$ and distributes them lump-sum to households. Thereafter, it follows the same interest on reserve policy as before but possibly adjusts the growth rate of nominal liabilities to satisfy its budget constraint.
   - Policy 2: one-time Helicopter drop of bonds:
     The government prints $\Delta\mathcal{B} > 0$ units of bonds at $t = 0$ and distributes them lump-sum to households. Thereafter, it follows the same interest on reserve policy as before but possibly adjusts the growth rate of nominal liabilities to satisfy its budget constraint.
   - Policy 3: open market operation/bond auction:
     The government sells $\Delta\mathcal{B} > 0$ bonds and uses the proceeds to take money out of circulation.

Thereafter, it follows the same interest on reserve policy as before but possibly adjusts the growth rate of nominal liabilities to satisfy its budget constraint.

- Policy 4: increasing interest on reserves:
  The government raises the reserve rate $i^{\mathcal{M}}$ to a permanently higher level $i^{\mathcal{M}\prime}$. It adjusts the growth rate of nominal liabilities as needed to satisfy its budget constraint.

2. How do the answers to the previous question change if the economy is in the "store of value regime" (in which $\Delta i = 0$)? Which of the policies, if sufficiently aggressive, may move the economy out of this regime into the "medium of exchange regime"?