① Nominal Bonds as Safe Assets and Bubble Mining Seigniorage

Cont. of het. agents $\tilde{i} \in [0,1]$ choose $\tilde{c}^i_t, \theta^{\tilde{i}}_t, \iota^{\tilde{i}}_t$ s.t

$$\mathbb{E}\left[\int_0^\infty e^{-\rho t}(\log \tilde{c}^i_t)\,dt\right]$$

s.t
$$\frac{dn^{\tilde{i}}_t}{n^{\tilde{i}}_t} = -\frac{c^{\tilde{i}}_t}{n^{\tilde{i}}_t}dt + dr^B_t + (1-\theta^{\tilde{i}}_t)(dr^{K,\tilde{i}}_t(\iota^{\tilde{i}}_t) - dr^B_t)$$

q^B q^K positive, constant. $\tilde{\sigma}^2 > \rho$

No trans. cost $\Rightarrow$

$$y^{\tilde{i}}_t dt = (ak^{\tilde{i}}_t - \iota^{\tilde{i}}_t k^{\tilde{i}}_t)dt$$

$$\frac{dk^{\tilde{i}}_t}{k^{\tilde{i}}_t} = (\phi(\iota^{\tilde{i}}_t) - \delta)dt + \tilde{\sigma}\,d\tilde{Z}^{\tilde{i}}_t + d\Delta^{k,\tilde{i}}_t$$

$\uparrow$ Id. syn. risk.

Output tax $\Rightarrow$
$$\tau_t a k^{\tilde{i}}_t dt$$

no agg. risk dZ_t

$$(\mu^B - i)B_t + p_t K_t \tau_t a = 0$$

Using Ito on price processes $\Rightarrow$

$$dr^{K,\tilde{i}}_t(\iota) = \left[\frac{a(1-\tau_t) - \iota}{q^K_t} + \Phi(\iota) - \delta\right]dt + \tilde{\sigma}\,d\tilde{Z}^{\tilde{i}}_t \quad —①$$

$$dr^B_t = i_t dt + \frac{d(1/p_t)}{1/p_t}$$

Now, $\frac{1}{P_t} = \frac{q_t^B K_t}{B_t}$

Using Ito's $\Rightarrow$

(Since i, q^B, q^K constant)

(a)
$$\boxed{\begin{aligned} dr_t^B &= [\,i + \mu_t^K - \mu_t^B\,]\,dt \\ dr_t^k &= \left[\frac{a(1-\tau_t) - \iota}{q^k} + \Phi(\iota) - \delta\right]dt + \tilde{\sigma}\,d\tilde{Z}_t^i \end{aligned}}$$

(b),(c) $q^K = 1 + \phi\,\iota_t \iff q = 1 + \phi\,\iota \iff \iota_t = \frac{q^K - 1}{\phi}$

log-utility $\rightarrow$ $c_t^i = \rho\,\tilde{n}_t^i$

$$C_t = \rho N_t = \rho\,q_t K_t$$

So, $q_t = \frac{a - \iota_t}{\rho}$

$$(1-\vartheta_t)\,q_t = (1-\vartheta_t)\,\frac{(a-\iota_t)}{\rho}$$

$$\iff \iota_t = \frac{(1-\vartheta_t)\,a - \rho}{1-\vartheta_t + \phi\rho}$$

Imposing steady state $\rightarrow$

$$\mu_t^\vartheta = 0$$

$$\check{\mu}^B = -\rho$$

So, $q^B = \frac{s}{\rho}$ (By GBC)

$$\vartheta = \frac{s(1+\phi\rho)}{s+\rho(1+\phi a)}$$

$$q^k = \frac{1+\phi(a-s)}{1+\phi\rho}$$

$$\iota = \frac{a-s-\rho}{1+\phi\rho}$$

So, Inflation $\Rightarrow$ Growth rate of nom assets − Growth rate of cap.

$$= \mu^B - g$$

② From money valuation eq^n:

(a)

$$E_t[d\vartheta_t] = (\rho + \check{\mu}_t^B - ((1-\vartheta_t)^2\tilde{\sigma}^2))\vartheta_t dt$$

$$-\rho \le \check{\mu}^B < -\rho + \tilde{\sigma}^2 \quad \text{for } \mu_t^\vartheta = 0$$

$$-s = \frac{\check{\mu}^B(\tilde{\sigma} - \sqrt{\rho + \check{\mu}^B})a}{\rho\tilde{\sigma}}$$

[As $\phi \to \infty$, using L'Hospital rule]

So, $$\mathcal{S} := -s = \frac{\overset{\nearrow ①}{(\mu^B - i)}\,(\tilde{\sigma} - \overset{\nearrow ②}{\sqrt{\rho + (\mu^B - i)}})a}{\rho\tilde{\sigma}}$$

This is a laffer curve, since it's a tax-base v/s tax-rate situation. As $(\mu^B - i)$ increases, the ① term increases, but ② term decreases.

(b) $\mathcal{S}$ in concave, so there is a unique max., i.e. combination of $(\mu^B - i)$.

Let that be μ^*

$$\text{So,} \quad S^* = \frac{\mu^* (\tilde{\sigma} - \sqrt{\rho + \mu^*})\, a}{\rho \tilde{\sigma}}$$

$$\text{So,} \quad S^* = \frac{\mu^* a}{\rho} \left[1 - \frac{\sqrt{\rho + \mu^*}}{\tilde{\sigma}} \right]$$

$$\frac{\partial S^*}{\partial \tilde{\sigma}} = \frac{\mu^* a}{\rho} \cdot \frac{\sqrt{\rho + \mu^*}}{\tilde{\sigma}^2}$$

$$\left.\begin{aligned} &\text{So, if } \mu^B > i \text{ then } \frac{\partial S^*}{\partial \tilde{\sigma}} > 0 \\ &\qquad\ \ \mu^B < i \quad " \quad \frac{\partial S^*}{\partial \tilde{\sigma}} < 0 \end{aligned}\right\}.$$

(c) For seigniorage all that matters is $(\mu^B - i)$ so if inflation is arbitrarily large, i.e. $\mu^B - g$, then μ^B & i combination can be suitably chosen to extract desired level of seigniorage.