

Problem Set 3

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1 Fire Sales

1.(a) Solving for $q(\eta)$

We solve for:

$$q(\eta) = \frac{\frac{1}{\phi} + [\eta a^e + (1 - \eta) a^h]}{\frac{1}{\phi} + [\eta \rho^e + (1 - \eta) \rho^h]} \quad (1)$$

So, for $\eta = 0 \Rightarrow q(0) = \frac{1+a^h\phi}{1+\rho^h\phi}$

$\eta = 1 \Rightarrow q(1) = \frac{1+a^e\phi}{1+\rho^e\phi}$

1.(b,c,d,e,f,g)

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1 ##Main Code
2 clear all;
3 clc;
4 % Parameters and grid
5 a_e      = 0.11;    a_h      = 0.03;    % production rates
6 rho_0     = 0.04;    % time preference
7 rho_e_d   = 0.01; rho_h_d = 0.01;    % death rates
8 rho_e     = rho_0 + rho_e_d;    % experts discount rate
9 rho_h     = rho_0 + rho_h_d;    % households discount rate
10 zeta      = 0.05;    % probability of becoming an expert
11 delta     = 0.05; sigma     = 0.1;    % decay rate / volatility
12 phi       = 10;    % adjustment cost / equity constraint
13
14 N          = 10000;    % grid size
15 eta        = linspace(0.0001,0.9999,N)'; % grid for \eta
16
17 % Solution
18 % Solve for q (0)
19 q0 = (1 + a_h * phi) / (1 + rho_h * phi);
20
21 % Inner loop
22 [Q, SSQ, Kappa, Iota] = inner_loop_log_without_alpha(eta, q0, a_e, a_h,
23     rho_e, rho_h, sigma, phi);
24 S          = (Kappa - eta) .* SSQ;    % \sigma_{\eta^e} -- arithmetic
25     volatility of \eta^e
26 Sg_e       = S ./ eta;    % \sigma^{\eta^e} -- geometric
27     volatility of \eta^e
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26 Sg_h = -S ./ (1 - eta); % \sigma^{\eta^h} -- geometric
    volatility of \eta^h
27
28 VarS_e = Kappa ./ eta .* SSQ; % \varsigma^e -- experts price of
    risk
29 VarS_h = (1 - Kappa) ./ (1 - eta) .* SSQ; % \varsigma^h -- households
    price of risk
30
31 CN_e = rho_e; % experts consumption-to-networth
    ratio
32 CN_h = rho_h; % households consumption-to-
    networth ratio
33
34 MU = eta .* (1 - eta) .* ((VarS_e - SSQ) .* (Sg_e + SSQ) ...
35 - (VarS_h - SSQ) .* (Sg_h + SSQ) ...
36 - (CN_e - CN_h) ...
37 + (rho_h_d .* zeta .* (1 - eta) - rho_e_d .* (1 - zeta) .* eta) ...
38 ./ (eta .* (1 - eta))); % \mu_{\eta^e} -- arithmetic drift of \eta
    ^e
39
40
41 % Create figure with specific size FOR Price and Amplification
42 figure('Position', [100, 100, 900, 400]);
43
44 % Find transition point where kappa becomes >= 1
45 transition_idx = find(Kappa >= 1, 1, 'first');
46 if ~isempty(transition_idx)
47     transition_eta = eta(transition_idx);
48 else
49     transition_eta = 0.4; % fallback value
50 end
51
52 % Plot data
53 subplot(1,2,1)
54 plot(eta, Q, 'b-', 'LineWidth', 2);
55 hold on
56 % Add vertical dashed line at transition point
57 plot([transition_eta transition_eta], ylim, 'k:', 'LineWidth', 1)
58 hold off
59
60 xlabel('\eta', 'FontSize', 14)
61 ylabel('q', 'FontSize', 14)
62 title('Price of Capital', 'FontSize', 14, 'FontWeight', 'bold')
63 grid off
64 xlim([0 1])
65 ylim([0.9 1.4])
66
67 % Add region labels
68 text(0.15, 1.22, 'Crisis Region', 'FontSize', 8, 'Color', 'red', 'Rotation
    ', 0)
69 text(0.7, 1.22, 'Normal Region', 'FontSize', 8, 'Color', 'red', 'Rotation'
    , 0)
70
71 % Add kappa labels at top

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72 text(transition_eta/2, 1.38, '\kappa_t^e_{\leq 1}', 'FontSize', 10, '
    HorizontalAlignment', 'center')
73 text((transition_eta+1)/2, 1.38, '\kappa_t^e_{=1}', 'FontSize', 10, '
    HorizontalAlignment', 'center')
74
75 subplot(1,2,2)
76 plot(eta, SSQ-sigma, 'b-', 'LineWidth', 2);
77 hold on
78 % Add horizontal dashed line at y=0
79 plot([0 1], [0 0], 'k--', 'LineWidth', 1)
80 % Add vertical dashed line at transition point
81 plot([transition_eta transition_eta], ylim, 'k:', 'LineWidth', 1)
82 hold off
83
84 xlabel('\eta', 'FontSize', 14)
85 ylabel('\sigma^q', 'FontSize', 14)
86 title('Amplification', 'FontSize', 14, 'FontWeight', 'bold')
87 grid off
88 xlim([0 1])
89 ylim([-0.01 0.07])
90
91 % Add region labels
92 text(0.15, 0.04, 'Crisis_Region', 'FontSize', 8, 'Color', 'red', 'Rotation
    ', 0)
93 text(0.7, 0.04, 'Normal_Region', 'FontSize', 8, 'Color', 'red', 'Rotation'
    , 0)
94
95 % Add kappa labels at top
96 text(transition_eta/2, 0.065, '\kappa_t^e_{\leq 1}', 'FontSize', 10, '
    HorizontalAlignment', 'center')
97 text((transition_eta+1)/2, 0.065, '\kappa_t^e_{=1}', 'FontSize', 10, '
    HorizontalAlignment', 'center')
98
99 % Adjust subplot spacing
100 set(gcf, 'PaperPositionMode', 'auto')
101 print('price_amplification_plot', '-dpdf', '-r300')
102
103
104 % Create figure with specific size FOR Iota and Kappa
105 figure('Position', [100, 100, 800, 400]);
106
107 % Plot data
108 subplot(1,2,1)
109 plot(eta, Iota, 'b-', 'LineWidth', 2);
110 hold on
111 % Add vertical dashed line at transition point
112 plot([transition_eta transition_eta], ylim, 'k:', 'LineWidth', 1)
113 hold off
114
115 xlabel('\eta', 'FontSize', 14)
116 ylabel('\iota', 'FontSize', 14)
117 title('Investment', 'FontSize', 14, 'FontWeight', 'bold')
118 grid off
119

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120 subplot(1,2,2)
121 plot(eta, Kappa, 'b-', 'LineWidth', 2);
122 hold on
123
124 xlabel('\eta', 'FontSize', 14)
125 ylabel('\kappa', 'FontSize', 14)
126 title('Capital_share', 'FontSize', 14, 'FontWeight', 'bold')
127 grid off
128
129
130 % Adjust subplot spacing
131 set(gcf, 'PaperPositionMode', 'auto')
132 print('Investment_CapitalShare_plot', '-dpdf', '-r300')
133
134
135 % Create figure with specific size FOR Debt issued
136 figure('Position', [100, 100, 600, 400]);
137
138 % Plot data
139 subplot(1,1,1)
140 plot(eta, Kappa-eta, 'b-', 'LineWidth', 2);
141
142 xlabel('\eta', 'FontSize', 14)
143 ylabel('$\frac{D^e_{\{t\}}{q_{\{t\}}K_{\{t\}}}$', 'Interpreter', 'latex', 'FontSize',
144         14)
145 title('Debt_Issued', 'FontSize', 14, 'FontWeight', 'bold')
146 grid off
147
148 % Adjust subplot spacing
149 set(gcf, 'PaperPositionMode', 'auto')
150 print('Debt_plot', '-dpdf', '-r300')
151
152
153 % Create figure with specific size FOR Drift and sigma
154 figure('Position', [100, 100, 900, 400]);
155
156 % Find transition point where kappa becomes >= 1
157 transition_idx = find(Kappa >= 1, 1, 'first');
158 if ~isempty(transition_idx)
159     transition_eta = eta(transition_idx);
160 else
161     transition_eta = 0.4; % fallback value
162 end
163
164 % Plot data
165 subplot(1,2,1)
166 plot(eta, MU, 'b-', 'LineWidth', 2);
167 hold on
168 % Add vertical dashed line at transition point
169 plot([transition_eta transition_eta], ylim, 'k:', 'LineWidth', 1)
170 hold off
171
172 xlabel('\eta', 'FontSize', 14)

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173 ylabel('\mu_{\eta}', 'FontSize', 14)
174 title('Drift', 'FontSize', 14, 'FontWeight', 'bold')
175 grid off
176 xlim([0 1])
177 ylim([-0.01 0.03])
178
179 subplot(1,2,2)
180 plot(eta, S, 'b-', 'LineWidth', 2);
181 hold on
182 % Add horizontal dashed line at y=0
183 plot([0 1], [0 0], 'k--', 'LineWidth', 1)
184 % Add vertical dashed line at transition point
185 plot([transition_eta transition_eta], ylim, 'k:', 'LineWidth', 1)
186 hold off
187
188 xlabel('\eta', 'FontSize', 14)
189 ylabel('\sigma_{\eta}', 'FontSize', 14)
190 title('Volatility', 'FontSize', 14, 'FontWeight', 'bold')
191 grid off
192 xlim([0 1])
193 ylim([0 0.1])
194
195 % Adjust subplot spacing
196 set(gcf, 'PaperPositionMode', 'auto')
197 print('Drift_and_Vlatility', '-dpdf', '-r300')

```

```

1 function [Q, SSQ, Kappa, Iota] = inner_loop_log_without_alpha(eta, q0, a_e
, a_h, rho_e, rho_h, sigma, phi)
2     N = length(eta);
3     deta = [eta(1); diff(eta)]; % imposes the correct grid step for
numerical derivative at  $\eta_e = 0$ 
4
5     % variables
6     Q = ones(N,1); % price of capital q
7     SSQ = zeros(N,1); %  $\sigma^2$ 
8     Kappa = zeros(N,1); % capital fraction of experts
9
10    Rho = eta * rho_e + (1 - eta) * rho_h; % average consumption-to-
networth ratio
11
12    % Initiate the loop
13    kappa = 0;
14    q_old = q0;
15    q = q0;
16    ssq = sigma;
17
18    % Iterate over eta
19    % At each step apply Newtons method to  $F(z) = 0$  where  $z = [q, kappa$ 
, ssq]
20    % Use chi = kappa
21    for i = 1:N
22        % Compute  $F(z_{n-1})$ 
23        F = [
24            kappa * (a_e - a_h) + a_h - (q - 1)/phi - q * Rho(i);

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25         ssq * (q - (q - q_old)/deta(i) * (kappa - eta(i))) - sigma * q
26         ;
27         a_e - a_h - q *(kappa - eta(i)) / (eta(i)*(1 - eta(i))) * ssq
28         ^2
29     ];
30     % Construct Jacobian J^{n-1}
31     J = zeros(3,3);
32     J(1,:) = [-1/phi - Rho(i), a_e - a_h, 0];
33     J(2,:) = [
34         ssq * (1 - (kappa - eta(i))/deta(i)) - sigma, ...
35         -ssq * (q - q_old)/deta(i), ...
36         q - (q - q_old)/deta(i) * (kappa - eta(i))
37     ];
38     J(3,:) = [
39         -(kappa - eta(i)) / (eta(i)*(1 - eta(i))) * ssq^2, ...
40         -q/ (eta(i)*(1 - eta(i))) * ssq^2, ...
41         -2 * q * (kappa - eta(i)) / (eta(i)*(1 - eta(i))) * ssq
42     ];
43     % Iterate, obtain z_{n}
44     z = [q; kappa; ssq] - J \ F;
45
46     % If the new kappa is larger than 1, break
47     if z(2) >= 1
48         break;
49     end
50
51     % Update variables
52     q = z(1);
53     kappa = z(2);
54     ssq = z(3);
55
56     % save results
57     Q(i) = q;
58     Kappa(i) = kappa;
59     SSQ(i) = ssq;
60     q_old = q;
61 end
62
63 % Set kappa = 1, use chi = kappa and compute the rest
64 n1 = i;
65 for i = n1:N
66     q = (1 + a_e * phi) / (1 + Rho(i) * phi);
67     qp = (q - q_old) / deta(i);
68
69     Q(i) = q;
70     Kappa(i) = 1;
71     SSQ(i) = sigma / (1 - (1 - eta(i)) * qp / q);
72     q_old = q;
73 end
74
75 Iota = (Q - 1) / phi;
76 end

```

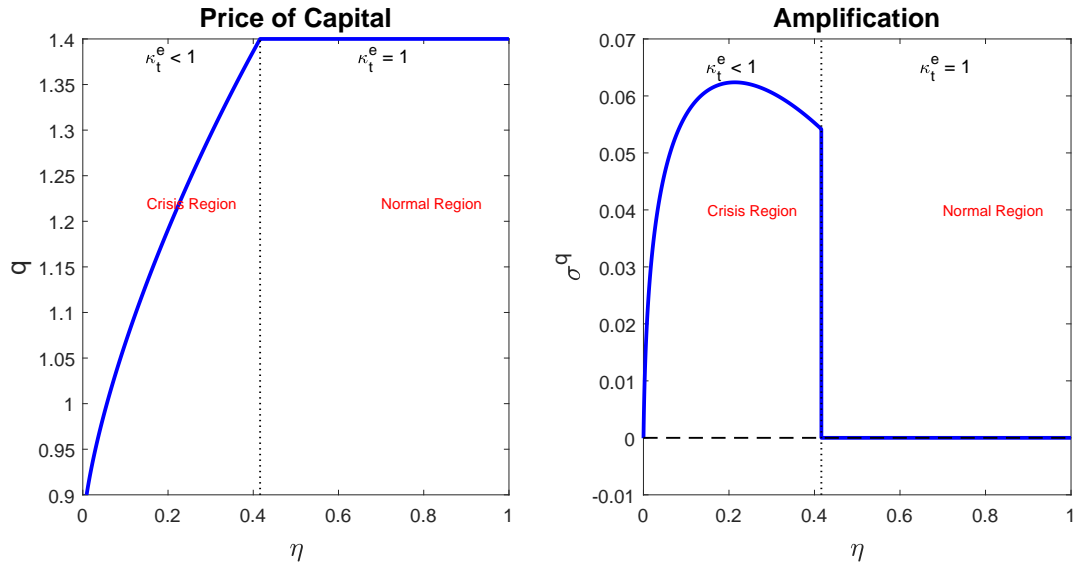


Figure 1: Price function $q(\eta)$ and volatility $\sigma^q(\eta)$

Yes, functions converge to the boundary solution for $\eta = 1$ obtained in (a) as $\eta \rightarrow 1$.

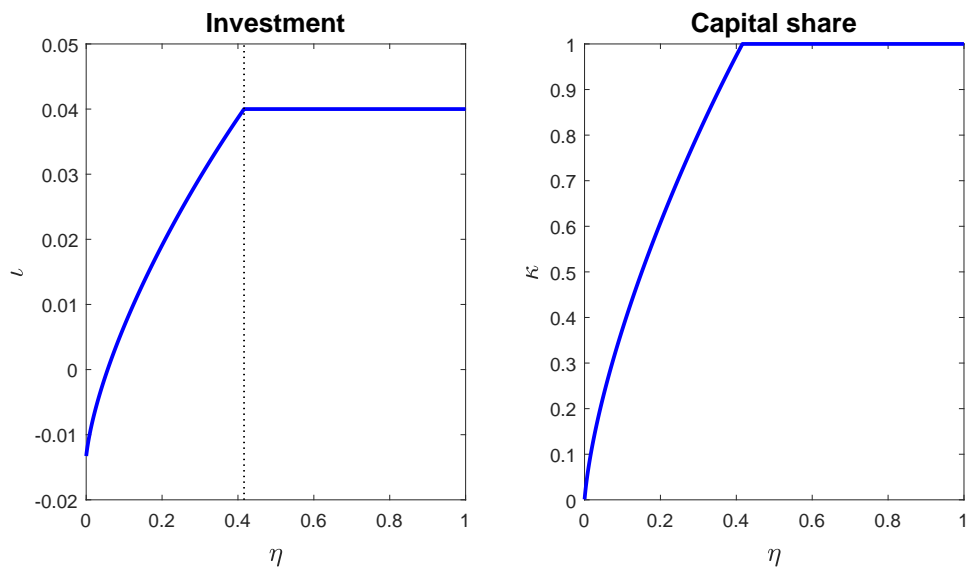


Figure 2: Investment and Capital share

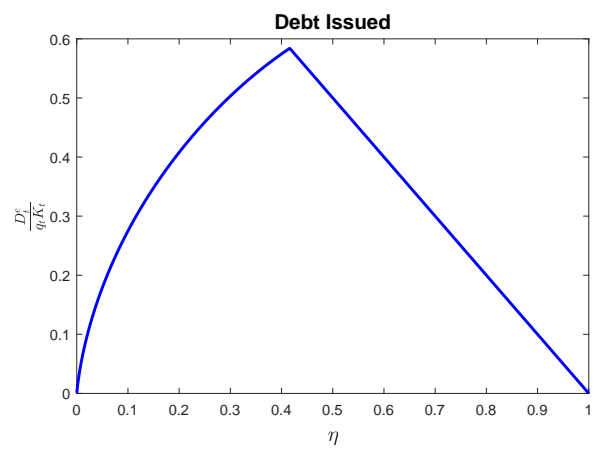


Figure 3: Debt dynamics

2.(a,b)

We know the fraction of wealth consumed is constant so $\frac{C}{N}$ terms will be replaced by ρ and gets a simplification for risk sharing following a ratio of χ_t to the wealth share, so the price of risk is equal to the volatility of net worth.

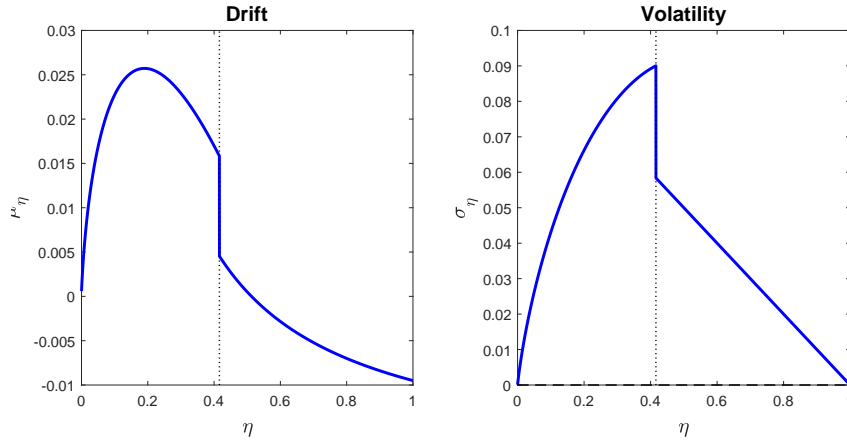


Figure 4: Drift and Volatility