Problem Set 3

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1 Fire Sales

1.(a)Solving for $q(\eta)$

We solve for:

$$q(\eta) = \frac{\frac{1}{\phi} + \left[\eta a^e + (1-\eta)a^h\right]}{\frac{1}{\phi} + \left[\eta \rho^e + (1-\eta)\rho^h\right]} \tag{1}$$

So, for $\eta = 0 \Rightarrow q(0) = \frac{1+a^h\phi}{1+\rho^h\phi}$
$\eta = 1 \Rightarrow q(1) = \frac{1+a^e\phi}{1+\rho^e\phi}$

1.(b,c,d,e,f,g)

```
##Main Code
clear all;
clc;
% Parameters and grid
a_e       = 0.11;   a_h     = 0.03;   % production rates
rho_0     = 0.04;                      % time preference
rho_e_d   = 0.01; rho_h_d = 0.01;     % death rates
rho_e     = rho_0 + rho_e_d;           % experts discount rate
rho_h     = rho_0 + rho_h_d;           % households discount rate
zeta      = 0.05;                      % probability of becoming an expert
delta     = 0.05; sigma   = 0.1;      % decay rate / volatility
phi       = 10;   % adjustment cost / equity constraint

N         = 10000;                      % grid size
eta       = linspace(0.0001,0.9999,N)';  % grid for \eta

% Solution
% Solve for q (0)
q0 = (1 + a_h * phi) / (1 + rho_h * phi);

% Inner loop
[Q, SSQ, Kappa, Iota] = inner_loop_log_without_alpha(eta, q0, a_e, a_h,
    rho_e, rho_h, sigma, phi);

S       = (Kappa - eta) .* SSQ;         % \sigma_{\eta^e} -- arithmetic
    volatility of \eta^e
Sg_e    = S ./ eta;                     % \sigma^{\eta^e} -- geometric
    volatility of \eta^e
```

```
Sg_h   = -S ./ (1 - eta);            % \sigma^{\eta^h} -- geometric volatility of \eta^h

VarS_e = Kappa ./ eta .* SSQ;          % \varsigma^e -- experts price of risk
VarS_h = (1 - Kappa) ./ (1 - eta) .* SSQ;  % \varsigma^h -- households price of risk

CN_e   = rho_e;                      % experts consumption-to-networth ratio
CN_h   = rho_h;                      % households consumption-to-networth ratio

MU = eta .* (1 - eta) .* ((VarS_e - SSQ) .* (Sg_e + SSQ) ...
     - (VarS_h - SSQ) .* (Sg_h + SSQ) ...
     - (CN_e - CN_h) ...
     + (rho_h_d .* zeta .* (1 - eta) - rho_e_d .* (1 - zeta) .* eta) ...
       ./ (eta .* (1 - eta)));  % \mu_{\eta^e} -- arithmetic drift of \eta^e


% Create figure with specific size FOR Price and Amplification
figure('Position', [100, 100, 900, 400]);

% Find transition point where kappa becomes >= 1
transition_idx = find(Kappa >= 1, 1, 'first');
if ~isempty(transition_idx)
    transition_eta = eta(transition_idx);
else
    transition_eta = 0.4; % fallback value
end

% Plot data
subplot(1,2,1)
plot(eta, Q, 'b-', 'LineWidth', 2);
hold on
% Add vertical dashed line at transition point
plot([transition_eta transition_eta], ylim, 'k:', 'LineWidth', 1)
hold off

xlabel('\eta', 'FontSize', 14)
ylabel('q', 'FontSize', 14)
title('Price of Capital', 'FontSize', 14, 'FontWeight', 'bold')
grid off
xlim([0 1])
ylim([0.9 1.4])

% Add region labels
text(0.15, 1.22, 'Crisis Region', 'FontSize', 8, 'Color', 'red', 'Rotation', 0)
text(0.7, 1.22, 'Normal Region', 'FontSize', 8, 'Color', 'red', 'Rotation', 0)

% Add kappa labels at top
```

```
text(transition_eta/2, 1.38, '\kappa_t^e␣<␣1', 'FontSize', 10, 'HorizontalAlignment', 'center')
text((transition_eta+1)/2, 1.38, '\kappa_t^e␣=␣1', 'FontSize', 10, 'HorizontalAlignment', 'center')

subplot(1,2,2)
plot(eta, SSQ-sigma, 'b-', 'LineWidth', 2);
hold on
% Add horizontal dashed line at y=0
plot([0 1], [0 0], 'k--', 'LineWidth', 1)
% Add vertical dashed line at transition point
plot([transition_eta transition_eta], ylim, 'k:', 'LineWidth', 1)
hold off

xlabel('\eta', 'FontSize', 14)
ylabel('\sigma^q', 'FontSize', 14)
title('Amplification', 'FontSize', 14, 'FontWeight', 'bold')
grid off
xlim([0 1])
ylim([-0.01 0.07])

% Add region labels
text(0.15, 0.04, 'Crisis␣Region', 'FontSize', 8, 'Color', 'red', 'Rotation', 0)
text(0.7, 0.04, 'Normal␣Region', 'FontSize', 8, 'Color', 'red', 'Rotation', 0)

% Add kappa labels at top
text(transition_eta/2, 0.065, '\kappa_t^e␣<␣1', 'FontSize', 10, 'HorizontalAlignment', 'center')
text((transition_eta+1)/2, 0.065, '\kappa_t^e␣=␣1', 'FontSize', 10, 'HorizontalAlignment', 'center')

% Adjust subplot spacing
set(gcf, 'PaperPositionMode', 'auto')
print('price_amplification_plot', '-dpdf', '-r300')


% Create figure with specific size FOR Iota and Kappa
figure('Position', [100, 100, 800, 400]);

% Plot data
subplot(1,2,1)
plot(eta, Iota, 'b-', 'LineWidth', 2);
hold on
% Add vertical dashed line at transition point
plot([transition_eta transition_eta], ylim, 'k:', 'LineWidth', 1)
hold off

xlabel('\eta', 'FontSize', 14)
ylabel('\iota', 'FontSize', 14)
title('Investment', 'FontSize', 14, 'FontWeight', 'bold')
grid off

```

```matlab
subplot(1,2,2)
plot(eta, Kappa, 'b-', 'LineWidth', 2);
hold on

xlabel('\eta', 'FontSize', 14)
ylabel('\kappa', 'FontSize', 14)
title('Capital share', 'FontSize', 14, 'FontWeight', 'bold')
grid off


% Adjust subplot spacing
set(gcf, 'PaperPositionMode', 'auto')
print('Investment_CapitalShare_plot', '-dpdf', '-r300')


% Create figure with specific size FOR Debt issued
figure('Position', [100, 100, 600, 400]);

% Plot data
subplot(1,1,1)
plot(eta, Kappa-eta, 'b-', 'LineWidth', 2);

xlabel('\eta', 'FontSize', 14)
ylabel('$\frac{D^{e}_{t}}{q_{t}K_{t}}$','Interpreter', 'latex','FontSize', 14)
title('Debt Issued', 'FontSize', 14, 'FontWeight', 'bold')
grid off


% Adjust subplot spacing
set(gcf, 'PaperPositionMode', 'auto')
print('Debt_plot', '-dpdf', '-r300')


% Create figure with specific size FOR Drift and sigma
figure('Position', [100, 100, 900, 400]);

% Find transition point where kappa becomes >= 1
transition_idx = find(Kappa >= 1, 1, 'first');
if ~isempty(transition_idx)
    transition_eta = eta(transition_idx);
else
    transition_eta = 0.4; % fallback value
end

% Plot data
subplot(1,2,1)
plot(eta, MU, 'b-', 'LineWidth', 2);
hold on
% Add vertical dashed line at transition point
plot([transition_eta transition_eta], ylim, 'k:', 'LineWidth', 1)
hold off

xlabel('\eta', 'FontSize', 14)
```

```
ylabel('\mu_{\eta}', 'FontSize', 14)
title('Drift', 'FontSize', 14, 'FontWeight', 'bold')
grid off
xlim([0 1])
ylim([-0.01 0.03])

subplot(1,2,2)
plot(eta, S, 'b-', 'LineWidth', 2);
hold on
% Add horizontal dashed line at y=0
plot([0 1], [0 0], 'k--', 'LineWidth', 1)
% Add vertical dashed line at transition point
plot([transition_eta transition_eta], ylim, 'k:', 'LineWidth', 1)
hold off

xlabel('\eta', 'FontSize', 14)
ylabel('\sigma_{\eta}', 'FontSize', 14)
title('Volatility', 'FontSize', 14, 'FontWeight', 'bold')
grid off
xlim([0 1])
ylim([0 0.1])

% Adjust subplot spacing
set(gcf, 'PaperPositionMode', 'auto')
print('Drift␣and␣Vlatility', '-dpdf', '-r300')
```

```
function [Q, SSQ, Kappa, Iota] = inner_loop_log_without_alpha(eta, q0, a_e, a_h, rho_e, rho_h, sigma, phi)
    N    = length(eta);
    deta = [eta(1); diff(eta)];  % imposes the correct grid step for numerical derivative at   ^e = 0

    % variables
    Q     = ones(N,1);    % price of capital q
    SSQ   = zeros(N,1);   %    +   ^q
    Kappa = zeros(N,1);   % capital fraction of experts

    Rho   = eta * rho_e + (1 - eta) * rho_h;  % average consumption-to-networth ratio

    % Initiate the loop
    kappa   = 0;
    q_old   = q0;
    q       = q0;
    ssq     = sigma;

    % Iterate over eta
    % At each step apply N e w t o n s method to F(z) = 0 where z = [q, kappa, ssq]
    % Use chi = kappa
    for i = 1:N
        % Compute F(z_{n-1})
        F = [
            kappa * (a_e - a_h) + a_h - (q - 1)/phi - q * Rho(i);
```

```
            ssq * (q - (q - q_old)/deta(i) * (kappa - eta(i))) - sigma * q;
            a_e - a_h - q *(kappa - eta(i)) / (eta(i)*(1 - eta(i))) * ssq^2
        ];

        % Construct Jacobian J^{n-1}
        J = zeros(3,3);
        J(1,:) = [-1/phi - Rho(i), a_e - a_h, 0];
        J(2,:) = [
            ssq * (1 - (kappa - eta(i))/deta(i)) - sigma, ...
            -ssq * (q - q_old)/deta(i), ...
            q - (q - q_old)/deta(i) * (kappa - eta(i))
        ];
        J(3,:) = [
            -(kappa - eta(i)) / (eta(i)*(1 - eta(i))) * ssq^2, ...
            -q/ (eta(i)*(1 - eta(i))) * ssq^2, ...
            -2 * q * (kappa - eta(i)) / (eta(i)*(1 - eta(i))) * ssq
        ];

        % Iterate, obtain z_{n}
        z = [q; kappa; ssq] - J \ F;

        % If the new kappa is larger than 1, break
        if z(2) >= 1
            break;
        end

        % Update variables
        q     = z(1);
        kappa = z(2);
        ssq   = z(3);

        % save results
        Q(i)     = q;
        Kappa(i) = kappa;
        SSQ(i)   = ssq;
        q_old    = q;
    end

    % Set kappa = 1, use chi = kappa and compute the rest
    n1 = i;
    for i = n1:N
        q  = (1 + a_e * phi) / (1 + Rho(i) * phi);
        qp = (q - q_old) / deta(i);

        Q(i)      = q;
        Kappa(i)  = 1;
        SSQ(i)    = sigma / (1 - (1 - eta(i)) * qp / q);
        q_old     = q;
    end

    Iota = (Q - 1) / phi;
end
```

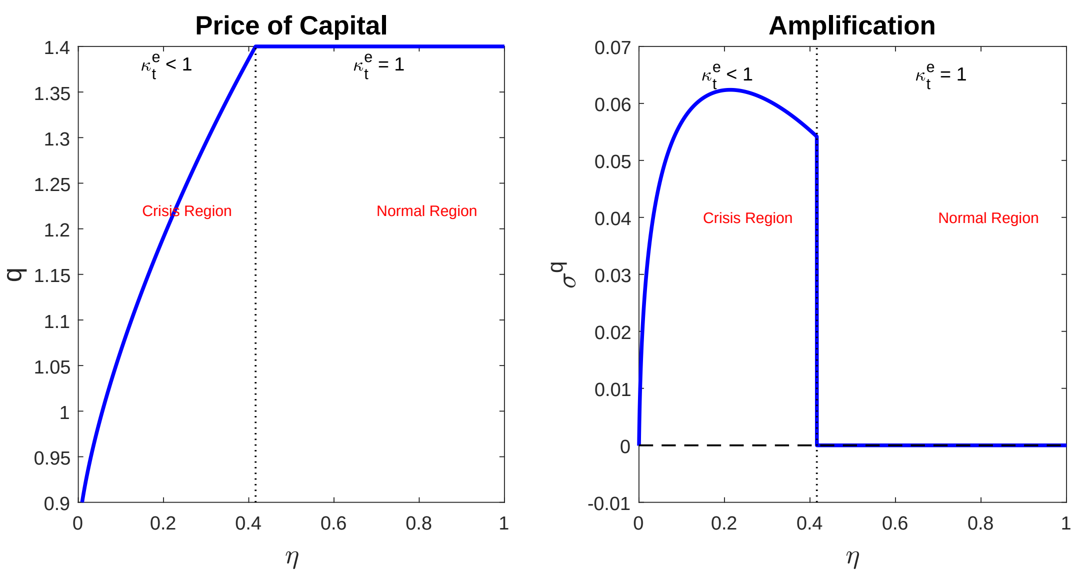

Figure 1: Price function $q(\eta)$ and volatility $\sigma^q(\eta)$

Yes, functions converge to the boundary solution for $\eta = 1$ obtained in (a) as $\eta \to 1$.

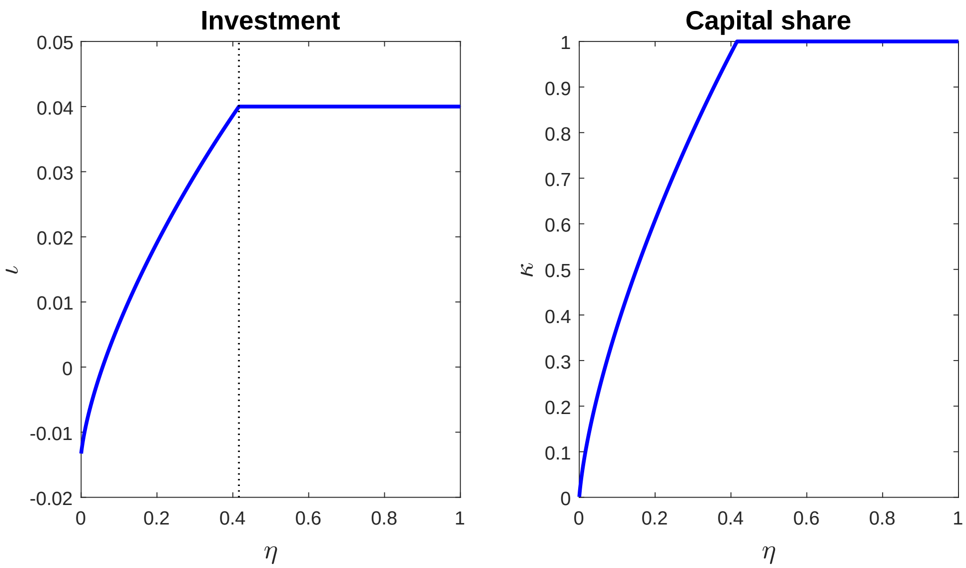

Figure 2: Investment and Capital share

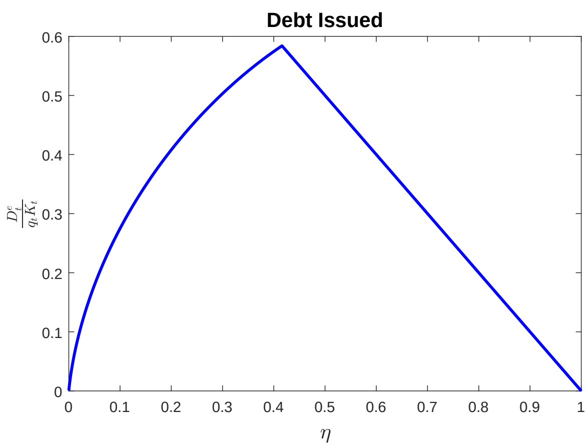

Figure 3: Debt dynamics

2.(a,b)

We know the fraction of wealth consumed is constant so $\frac{C}{N}$ terms will be replaced by ρ and gets a simplification for risk sharing following a ratio of χ_t to the wealth share, so the price of risk is equal to the volatility of net worth.

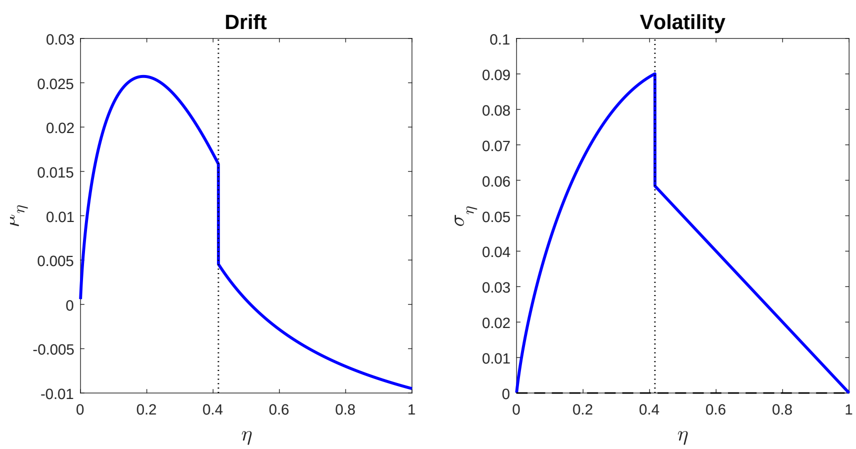

Figure 4: Drift and Volatility