Targeting Long Rates in a Model with Segmented Markets[†]

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This paper develops a model of segmented financial markets in which the net worth of financial institutions limits the degree of arbitrage across the term structure. The model is embedded into the canonical Dynamic New Keynesian (DNK) framework. We estimate the model using data on the term premium. Our principal results include the following. First, the estimated segmentation coefficient implies a nontrivial effect of central bank asset purchases on yields and real activity. Second, there are welfare gains to having the central bank respond to the term premium, e.g., including the term premium in the Taylor Rule. Third, a policy that directly targets the term premium sterilizes the real economy from shocks originating in the financial sector. A term-premium peg can have significant welfare effects. (JEL E12, E23, E31, E43, E44, E52, E58)

To restate my main point, I believe that measures of bond market risk premiums—for example, estimates of the expected excess returns on long-term Treasury securities relative to Treasury bills and on credit-risky bonds relative to Treasury securities—may turn out to be useful inputs into the monetary policy framework.

—FRB Governor Jeremy Stein, March 21, 2014, "Incorporating Financial Stability Considerations into a Monetary Policy Framework."

So what then might the Fed do if its target interest rate, the overnight federal funds rate, fell to zero? ... A more direct method, which I personally prefer, would be for the Fed to begin announcing explicit ceilings for yields on longer-maturity Treasury debt. The Fed could enforce these interest-rate ceilings by committing to make unlimited purchases of securities If this program were successful, not only would yields on medium-term Treasury securities fall, but yields on longer-term public and private debt would likely fall as well.

—FRB Governor Ben S. Bernanke, November 21, 2002, "Deflation: Making Sure 'It' Doesn't Happen Here."

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In the aftermath of the 2008 financial crisis many central banks have adopted unconventional policies, including outright purchases of long-term government debt. As suggested by the quotations from Governors Stein and Bernanke, these bond purchases raise a number of research questions for macro theory. Under what conditions can such purchases have aggregate effects? If they have aggregate consequences, how do term premia movements affect inflation and economic activity? Should policy respond to the term premium, and should the balance sheet move passively to hit long rate targets? To answer such questions, this paper develops a model of the term premium in which central bank purchases can affect the yield structure independently of the anticipated path of short-term interest rates. The model is embedded into an otherwise canonical medium-scale Dynamic New Keynesian (DNK) model where long-term bonds are necessary to finance investment purchases. This implies that both new and old policy questions can be examined in a unified framework.

The key features of the model include the following. First, the short-term bond market is segmented from the long-term bond market in that only financial intermediaries can purchase long-term debt. Households can access the long-term debt instruments indirectly by providing deposits to intermediaries, but the ability of intermediaries to arbitrage the yield gap between the short-term deposit rate and long-term lending rate is limited by net worth. That is, a simple hold-up problem constrains the amount of deposits that can be supported by a given level of intermediary net worth. Second, the intermediary faces adjustment costs in rapidly varying the size of its portfolio in the wake of shocks. These assumptions imply that central bank purchases of long-term bonds will have a significant effect on long yields. Finally, these long-term yields affect real economic activity because of our final assumption: Capital investment is financed by the issuance of long-term bonds which sell in the same market that absorbs long-term Treasury securities. Taken together, these assumptions imply that central bank purchases of long-term bonds will have a significant and persistent effect on long yields and real activity.

We use the model to consider the efficacy of alternative policies linked to the term premium. This is a natural policy in the context of the model as the distortion arising from market segmentation is, to a linear approximation, equal to the term premium. We show that there are significant welfare gains to including the term premium in a traditional Taylor rule operating on the short-term rate. We also consider policies that hold the term premium fixed by allowing the long bond holdings of the central bank to move passively. Such a long rate policy sterilizes the rest of the model economy from shocks originating in the financial system. This sterilization is directly analogous to the classic Poole (1970) result that a federal funds rate (FFR) target sterilizes the economy from money demand shocks. In summary, the policy focus of the paper is not on understanding the effect of bond purchases during unusual periods, but examining the role of monetary policy responses to the term premium during "normal" times.

The papers closest in spirit to the current work are Gertler and Karadi (2011, 2013) and Chen, Cúrdia, and Ferrero (2012). There are two crucial similarities between these papers and the present work. First, there is some friction that limits the ability to arbitrage across the short-term and long-term bond markets. This

implies that the long rate is not the expected average of short rates—i.e., there is a term premium. Second, the market segmentation has real effects because some portion of real activity is financed in the segmented market. Gertler and Karadi (2013) assume that the entire capital stock is refinanced each period by the purchase of equity claims in this market by intermediaries. Chen, Cúrdia, and Ferrero (2012) assume that a small subset of consumers finance their consumption in the segmented market. In contrast, the current paper assumes that new investment is financed in the segmented market with the issuance of long-term debt.

A complementary approach to modeling the term premium in a DSGE model has been pursued by Rudebusch and Swanson (2008, 2012). These authors study an otherwise standard DNK model with no market segmentation effects. To capture a time-varying term premium, they solve the model using third-order approximation. Rudebusch and Swanson (2008) use a specification for household preferences commonly used in the business cycle literature. In this case they find essentially no steady-state term premium and no time-variation in the premium. In contrast, Rudebusch and Swanson (2012) use Epstein-Zin (EZ) preferences. If the EZ risk aversion coefficient is set high enough, they are able to match a significant steadystate term premium and time variation in the premium. Importantly, in both of these papers, the higher order approximation and the EZ preferences have a trivial effect on the decision rules for real variables such as output and investment. That is, the model with EZ preferences is able to generate plausible movements in the term premium although the behavior of the real variables is largely unaltered by this time variation. Although Rudebusch and Swanson (2012) do not pursue policy issues, their results suggest that time variation in the term premium should have little consequence for monetary policy.

Finally, this paper is related to the literature on the welfare effects of adding credit spreads to simple monetary policy rules for the short-term interest rate e.g., Cúrdia and Woodford (2010) and Gilchrist and Zakrajšek (2011). Cúrdia and Woodford (2010) find—as we do—that lowering the policy rate in response to an increase in credit spreads is often welfare improving. But the size and sign of the optimal spread adjustment vary with the type of shock hitting the economy and its persistence in their model. One difference compared to our analysis is that Cúrdia and Woodford allow the baseline interest rate rule to depend on the natural rate of interest (computed in the absence of credit frictions), whereas we do not. The paper by Gilchrist and Zakrajšek (2011) analyzes a financial accelerator type model where financial shocks are capable of generating an economic contraction similar in magnitude to the Great Recession. They show that augmenting the Taylor rule, such that the policy rate is lowered one for one with credit spreads, performs remarkably well in reducing economic fluctuations in response to this shock. Although they do not search for the optimal adjustment coefficient, their finding is in line with our result that the welfare maximizing coefficient on credit spreads is minus 1.

The paper proceeds as follows. The next section develops the theoretical model. Section II presents our quantitative results, including our estimation of the key model

¹Fuerst and Mau (2016) demonstrate that the ability to generate significant variability in the term premium relies on a nonstandard Taylor rule.

parameters. Section II also focuses on how the segmentation affects the economy's response to shocks, as well as the efficacy of central bank policies that directly or indirectly target the term premium. Section III provides some sensitivity analysis and Section IV concludes.

I. The Model

The economy consists of households, financial intermediaries (FIs), and firms. Many of the ingredients are standard, with the chief novelty coming from our assumptions on household-FI interactions. We will focus our discussion on these issues.

A. Households

Each household maximizes the utility function

(1)
$$E_0 \sum_{s=0}^{\infty} \beta^s e^{m_{t+s}} \left\{ \ln(C_{t+s} - h C_{t+s-1}) - B \frac{H_{t+s}^{1+\eta}(j)}{1+\eta} \right\}$$

where C_t is consumption, h is the degree of habit formation, $H_t(j)$ is the labor input of household j, and e^{rn_t} is a shock to the discount factor. This intertemporal preference shock follows the stochastic process

$$(2) rn_t = \rho_{rn} r n_{t-1} + \varepsilon_{rn,t},$$

with $\varepsilon_{rn,t} \sim \text{i.i.d.} N(0, \sigma_{rn}^2)$. Each household is a monopolistic supplier of specialized labor, $H_t(j)$, as in Erceg et al. (2000). A large number of competitive employment agencies combine this specialized labor into a homogenous labor input sold to intermediate firms, according to

(3)
$$H_{t} = \left[\int_{0}^{1} H_{t}(j)^{1/(1+\lambda_{w,t})} dj \right]^{1+\lambda_{w,t}}.$$

The desired markup of wages over the household's marginal rate of substitution, $\lambda_{w,t}$, follows the exogenous stochastic process

(4)
$$\log \lambda_{w,t} = (1 - \rho_w) \log \lambda_w + \rho_w \log \lambda_{w,t-1} + \varepsilon_{w,t} - \theta_w \varepsilon_{w,t-1},$$

with $\varepsilon_{w,t}$ i.i.d. $N(0, \sigma_w^2)$. This is the wage markup shock. Profit maximization by the perfectly competitive employment agencies implies that the real wage (W_t) paid by intermediate firms for their homogenous labor input is

(5)
$$W_{t} = \left[\int_{0}^{1} W_{t}(j)^{-1/\lambda_{w,t}} dj \right]^{-\lambda_{w,t}}.$$

Every period a fraction θ_w of households cannot freely alter their nominal wage, so their real wage follows the indexation rule

(6)
$$W_{t}(j) = \frac{\prod_{t=1}^{L_{w}}}{\prod_{t}} W_{t-1}(j).$$

The remaining fraction of households chooses instead an optimal real wage $W_t(j)$ by maximizing

(7)
$$E_{t} \left\{ \sum_{s=0}^{\infty} \theta_{w}^{s} \beta^{s} \left[-e^{rn_{t+s}} B \frac{H_{t+s}(j)^{1+\psi}}{1+\psi} + \Lambda_{t+s} W_{t}(j) H_{t+s}(j) \right] \right\}$$

subject to the labor demand function coming from the employment agencies, and where Λ_{t+s} is the household's marginal utility of consumption. The existence of state-contingent securities ensures that household consumption (and thus Λ_{t+s}) is the same across all households. The household also earns income by renting capital to the intermediate goods firm.

The household has two means of intertemporal smoothing: short-term deposits (D_t) in the FI and accumulation of physical capital (K_t) . Households also have access to the market in short-term government bonds ("T-bills"). But since T-bills are perfect substitutes with deposits, and the supply of T-bills moves endogenously to hit the central bank's short-term interest rate target, we treat D_t as the household's net resource flow into the FIs. To introduce a need for intermediation, we assume that all investment purchases must be financed by issuing new "investment bonds" that are ultimately purchased by the FI. We find it convenient to use the perpetual bonds suggested by Woodford (2001). In particular, these bonds are perpetuities with cash flows of 1, κ , κ^2 , etc. Let Q_t denote the time-t price of a new issue. Given the time pattern of the perpetuity payment, the new issue price Q_t summarizes the prices at all maturities, e.g., κQ_t is the time-t price of the perpetuity issued in period t-1. The duration and (gross) yield to maturity on these bonds are defined as: duration $=(1-\kappa)^{-1}$, gross yield to maturity $=Q_t^{-1}+\kappa$. Let CI_t denote the number of new perpetuities issued in time-t to finance investment. In time-t, the household's nominal liability on past issues is given by

(8)
$$F_{t-1} = CI_{t-1} + \kappa CI_{t-2} + \kappa^2 CI_{t-3} + \cdots$$

We can use this recursion to write the new issue as

(9)
$$CI_t = (F_t - \kappa F_{t-1}).$$

The representative's household constraints are thus given by

(10)
$$C_{t} + \frac{D_{t}}{P_{t}} + P_{t}^{k}I_{t} + \frac{F_{t-1}}{P_{t}} \leq W_{t}H_{t} + R_{t}^{k}K_{t} - T_{t} + \frac{D_{t-1}}{P_{t}}R_{t-1} + \frac{Q_{t}(F_{t} - \kappa F_{t-1})}{P_{t}} + div_{t};$$

$$(11) K_{t+1} \leq (1-\delta)K_t + I_t;$$

(12)
$$P_t^k I_t \leq \frac{Q_t(F_t - \kappa F_{t-1})}{P_t} = \frac{Q_t C I_t}{P_t},$$

where P_t is the price level; P_t^k is the real price of capital; R_{t-1} is the gross nominal interest rate on deposits; R_t^k is the real rental rate; T_t are lump-sum taxes; and div_t denotes the dividend flow from the FIs. The household also receives a profit flow from the intermediate goods producers and the new capital producers, but this is entirely standard so we dispense from this added notation for simplicity. The "loan-in-advance" constraint (12) will increase the private cost of purchasing investment goods. Although for simplicity we place capital accumulation within the household problem, this model formulation is isomorphic to an environment in which household-owned firms accumulate capital subject to the loan constraint. The first order conditions to the household problem include:

(13)
$$\Lambda_t = E_t \beta \Lambda_{t+1} \frac{R_t}{\Pi_{t+1}};$$

(14)
$$\Lambda_t P_t^k M_t = E_t \beta \Lambda_{t+1} [R_t^k + (1-\delta) P_{t+1}^k M_{t+1}];$$

(15)
$$\Lambda_t Q_t M_t = E_t \beta \Lambda_{t+1} \frac{\left[1 + \kappa Q_{t+1} M_{t+1}\right]}{\Pi_{t+1}},$$

where $\Pi_t \equiv \frac{P_t}{P_{t-1}}$ is gross inflation. Expression (13) is the familiar Fisher equation. The capital accumulation expression (14) is distorted relative to the familiar by the time-varying distortion M_t , where $M_t \equiv 1 + \frac{\vartheta_t}{\Lambda_t}$, and ϑ_t is the multiplier on the loan-in-advance constraint (12). The endogenous behavior of this distortion is fundamental to the real effects arising from market segmentation.

B. Financial Intermediaries

The FIs in the model are a stand-in for the entire financial nexus that uses accumulated net worth (N_t) and short-term liabilities (D_t) to finance investment bonds (F_t) and the long-term government bonds (B_t) . The FIs are the sole buyers of the investment bonds and long-term government bonds. We again assume that government debt takes the form of Woodford-type perpetuities that provide payments of 1, κ , κ^2 , etc. Let Q_t denote the price of a new-debt issue at time-t. The time-t asset value of the current and past issues of investment bonds is

(16)
$$Q_t C I_t + \kappa Q_t \left[C I_{t-1} + \kappa C I_{t-2} + \kappa^2 C I_{t-3} + \cdots \right] = Q_t F_t.$$

The FI's balance sheet is thus given by

(17)
$$\frac{B_t}{P_t}Q_t + \frac{F_t}{P_t}Q_t = \frac{D_t}{P_t} + N_t = L_t N_t,$$

where L_t denotes leverage. Note that on the asset side, investment lending and long-term bond purchases are perfect substitutes to the FI. Let $R_{t+1}^L \equiv \left(\frac{1+\kappa Q_{t+1}}{Q_t}\right)$. The FI's time-t profits are then given by

(18)
$$prof_{t} \equiv \frac{P_{t-1}}{P_{t}} [(R_{t}^{L} - R_{t-1}^{d}) L_{t-1} + R_{t-1}] N_{t-1}.$$

The FI will pay out some of these profits as dividends (div_t) to the household, and retain the rest as net worth for subsequent activity. In making this choice the FI discounts dividend flows using the household's pricing kernel augmented with additional impatience. The FI accumulates net worth because it is subject to a financial constraint: the FI's ability to attract deposits will be limited by its net worth. We will use a simple hold-up problem to generate this leverage constraint, but a wide variety of informational restrictions will generate the same constraint. We assume that leverage is taken as given by the FI. We will return to this below. The FI chooses dividends and net worth to solve

(19)
$$V_{t} \equiv \max_{N_{t}, div_{t}} E_{t} \sum_{j=0}^{\infty} (\beta \zeta)^{j} \Lambda_{t+j} div_{t+j}$$

subject to the financing constraint developed below and the following budget constraint:

(20)
$$div_t + N_t [1 + f(N_t)] \leq \frac{P_{t-1}}{P_t} [(R_t^L - R_{t-1}^d) L_{t-1} + R_{t-1}] N_{t-1}.$$

The function $f(N_t) \equiv \frac{\psi_n}{2} \left(\frac{N_t - N_{ss}}{N_{ss}}\right)^2$ denotes an adjustment cost function that dampens the ability of the FI to adjust the size of its portfolio in response to shocks. If we assumed no adjustment costs $(\psi_n = 0)$ and that the net worth solution is interior, the FI's value function is linear and given by

(21)
$$V_{t} = \frac{P_{t-1}}{P_{t}} \Lambda_{t} \left[\left(R_{t}^{L} - R_{t-1}^{d} \right) L_{t-1} + R_{t-1}^{d} \right] N_{t-1} \equiv X_{t} N_{t-1}.$$

But with convex adjustment cost in net worth accumulation, the FI's value function will include a time-varying additive term

$$V_t = X_t N_{t-1} + g_t,$$

where $g_{ss} = 0$. The term g_t is a function of aggregate variables that are exogenous to the FI (see the Appendix for details).

The hold-up problem works as follows. At the beginning of period t + 1, but before aggregate shocks are realized, the FI can choose to default on its planned

 $^{^2}$ In contrast, Gertler and Karadi (2011, 2013) assume that FIs only pay out dividends upon their exogenous death.

repayment to depositors. In this event, depositors can seize at most fraction $(1 - \Psi_t)$ of the FI's assets, where Ψ_t is a function of net worth and the other state variables. If the FI defaults, the FI is left with $\Psi_t R_{t+1}^L L_t N_t$, which it pays out to households and exits the world. To ensure that the FI will always repay the depositor, the time-t incentive compatibility constraint is thus given by

(22)
$$E_{t}V_{t+1} \geq \Psi_{t}L_{t}N_{t}E_{t}\Lambda_{t+1}\frac{P_{t}}{P_{t+1}}R_{t+1}^{L}.$$

We will calibrate the model so that this constraint is binding in the steady state (and thus binding for small shocks around the steady state). For a fixed Ψ_t , the presence of g_t in the value function implies that leverage will typically vary with net worth, e.g., leverage will be decreasing in net worth if $E_t g_{t+1} > 0$. For simplicity, we avoid this complication by assuming that Ψ_t is a function of net worth in a manner symmetric with the convexity in the adjustment cost function. Although theoretically convenient, this assumption is quantitatively unimportant (as $g_{ss} = 0$). In particular, we assume that the fraction of assets that the FI can keep in case of default is defined by

(23)
$$\Psi_t \equiv \Phi_t \left[1 + \frac{1}{N_t} \left(\frac{E_t g_{t+1}}{E_t X_{t+1}} \right) \right],$$

where Φ_t is an exogenous stochastic process that represents exogenous changes in the financial friction. For example, if $E_t g_{t+1} > 0$, assumption (23) implies that higher net worth makes the hold-up problem less severe. This decreased severity is chosen to counter the earlier implication that leverage would be decreasing in net worth. Assumption (23) implies that the binding incentive constraint (22) is given by

(24)
$$E_{t} \frac{P_{t}}{P_{t+1}} \Lambda_{t+1} \left[\left(\frac{R_{t+1}^{L}}{R_{t}^{d}} - 1 \right) L_{t} + 1 \right] = \Phi_{t} L_{t} E_{t} \Lambda_{t+1} \frac{P_{t}}{P_{t+1}} \frac{R_{t+1}^{L}}{R_{t}^{d}}.$$

As anticipated, leverage is a function of aggregate variables but is independent of each FI's net worth. This implies that only aggregate net worth is needed to describe the model as all FIs are scaled versions of one another (see the Appendix for details).

Log-linearizing expression (24), we have

(25)
$$\left(E_t r_{t+1}^L - r_t \right) = \nu l_t + \left[\frac{1 + L_{ss}(s-1)}{L_{ss} - 1} \right] \phi_t,$$

where $\nu \equiv (L_{ss}-1)^{-1}$, is the elasticity of the interest rate spread to leverage; s denotes the gross steady-state term premium; and the financial shock $\phi_t \equiv \ln(\Phi_t)$ follows an AR(1) process:

(26)
$$\phi_t = (1 - \rho_\phi)\phi_{ss} + \rho_\phi\phi_{t-1} + \varepsilon_{\phi,t}.$$

Increases in ϕ_t will exacerbate the hold-up problem, and thus are "credit shocks," which will increase the spread and lower real activity. Qualitatively the log-linearized expression (25) for leverage is identical to the corresponding relationship in the more complex costly state verification (CSV) environment of, for example, Bernanke, Gertler, and Gilchrist (1999). In a CSV model, the primitives include: (i) idiosyncratic risk, (ii) death rate, and (iii) monitoring cost. One typically chooses these to match values for: (i) leverage, (ii) interest rate spread, and (iii) default rate. The hold-up model has only two primitives: (i) the impatience rate ζ , and (ii) the fraction of assets that can be seized Φ . In comparison to the hold-up model, the extra primitive in the CSV framework thus allows it to match one more moment of the financial data (default rates). One important quantitative difference is that interest rate spreads are more responsive to leverage in our framework than in the CSV model calibrated to the same steady-state leverage. For example, suppose we calibrated a CSV model to a leverage of 6.0, a risk premium of 100 basis points (bp), and a quarterly default rate of 0.205 percent (the default rate in the hold-up model is 0 percent). This would imply $\nu = 0.097$. In the hold-up model analyzed here, a leverage of 6.0 implies $\nu = 0.20$, about twice as large as the CSV counterpart.

Since the incentive constraint (24) is now independent of net worth, the FI takes leverage as given. The FI's optimal accumulation decision is then given by

(27)
$$\Lambda_{t} \left[1 + N_{t} f'(N_{t}) + f(N_{t}) \right] = E_{t} \beta \zeta \Lambda_{t+1} \frac{P_{t}}{P_{t+1}} \left[\left(R_{t+1}^{L} - R_{t}^{d} \right) L_{t} + R_{t}^{d} \right].$$

Equations (24) and (27) are fundamental to the model as they summarize the limits to arbitrage between the return on long-term bonds and the rate paid on short-term deposits. The leverage constraint (24) limits the FI's ability to attract deposits and eliminate the arbitrage opportunity between the deposit and lending rate. Increases in net worth allow for greater arbitrage and thus can eliminate this market segmentation. Equation (27) limits this arbitrage in the steady-state by additional impatience ($\zeta < 1$) and dynamically by portfolio adjustment costs ($\psi_n > 0$). Since the FI is the sole means of investment finance, this market segmentation means that central bank purchases that alter the supply of long-term debt will have repercussions for investment loans because net worth and deposits cannot quickly sterilize the purchases.

C. Final Good Producers

Perfectly competitive firms produce the final consumption good Y_t combining a continuum of intermediate goods according to the CES technology:

(28)
$$Y_{t} = \left[\int_{0}^{1} Y_{t}(i)^{1/(1+\epsilon_{p})} di \right]^{1+\epsilon_{p}}.$$

Profit maximization and the zero profit condition imply that the price of the final good, P_t , is the familiar CES aggregate of the prices of the intermediate goods.

D. Intermediate Goods Producers

A monopolist produces the intermediate good *i* according to the production function

(29)
$$Y_{t}(i) = A_{t}K_{t}(i)^{\alpha}H_{t}(i)^{1-\alpha},$$

where $K_t(i)$ and $H_t(i)$ denote the amounts of capital and labor employed by firm i. The variable $\ln A_t$ is the exogenous level of TFP and evolves according to

$$\ln A_t = \rho_A \ln A_{t-1} + \varepsilon_{a,t}.$$

Every period a fraction θ_p of intermediate firms cannot choose its price optimally, but instead resets it according to the indexation rule

(31)
$$P_{t}(i) = P_{t-1}(i) \prod_{t=1}^{t_p},$$

where $\Pi_t = \frac{P_t}{P_{t-1}}$ is gross inflation. The remaining fraction of firms chooses its price $P_t(i)$ optimally, by maximizing the present discounted value of future profits

(32)

$$E_{t} \left\{ \sum_{s=0}^{\infty} \theta_{p}^{s} \frac{\beta^{s} \Lambda_{t+s}/P_{t+s}}{\Lambda_{t}/P_{t}} \left[P_{t}(i) \left(\prod_{k=1}^{s} \Pi_{t+k-1}^{\iota_{p}} \right) Y_{t+s}(i) - W_{t+s} H_{t+s}(i) - P_{t+s} R_{t+s}^{k} K_{t+s}(i) \right] \right\},$$

where the demand function $Y_{t+s}(i)$ comes from the final goods producers.

E. New Capital Producers

New capital is produced according to the production technology that takes I_t investment goods and transforms them into $\mu_t \left[1 - S\left(\frac{I_t}{I_{t-1}}\right)\right] I_t$ new capital goods. The time-t profit flow is thus given by

$$(33) P_t^k \mu_t \left[1 - S\left(\frac{I_t}{I_{t-1}}\right) \right] I_t - I_t,$$

where the function S captures the presence of adjustment costs in investment, as in Christiano et al. (2005), and is given by $S\left(\frac{I_t}{I_{t-1}}\right) \equiv \frac{\psi_i}{2}\left(\frac{I_t}{I_{t-1}}-1\right)^2$. These firms are owned by households and discount future cash flows with Λ_t . The investment shock follows the stochastic process

$$\log \mu_t = \rho_\mu \log \mu_{t-1} + \varepsilon_{\mu,t},$$

where $\varepsilon_{\mu,t}$ is i.i.d. $N\left(0,\sigma_{\mu}^{2}\right)$.

F. Central Bank Policy

We assume that the central bank follows a familiar Taylor rule over the short rate (T-bills and deposits):

(35)
$$\ln(R_t) = (1 - \rho)\ln(R_{ss}) + \rho \ln(R_{t-1}) + (1 - \rho)(\tau_{\pi}\pi_t + \tau_{\nu}y_t^{gap}) + \epsilon_t^r,$$

where $y_t^{gap} \equiv (Y_t - Y_t^f)/Y_t^f$ denotes the deviation of output from its flexible price counterpart, and ϵ_t^r is an exogenous and auto-correlated policy shock with AR(1) coefficient ρ_r . We will think of this as the Federal Funds Rate (FFR). Below we will also investigate the efficacy of putting the term-premium into the Taylor rule. The supply of short-term bonds (T-bills) is endogenous, varying as needed to support the FFR target. As for the long-term bond policy, the central bank will choose between: (i) an exogenous path for the quantity of long-term debt available to FIs, or (ii) a policy rule that pegs the term premium and thus makes the level of debt endogenous. We will return to this below.

Fiscal policy is entirely passive. Government expenditures are set to zero. Lump sum taxes move endogenously to support the interest payments on the short and long debt.

G. Log-linearized Model

To gain further intuition and to derive the term premium, we first log-linearize the model. Let $b_t \equiv \ln\left(\frac{\overline{B}_t}{\overline{B}_{ss}}\right)$ and $f_t \equiv \ln\left(\frac{\overline{F}_t}{\overline{F}_{ss}}\right)$, where $\overline{B}_t \equiv Q_t \frac{B_t}{P_t}$ and $\overline{F}_t \equiv Q_t \frac{F_t}{P_t}$ denote the real market value of the bonds available to FIs. We will focus on bonds of ten-year maturities, so R_t^{10} will denote their gross yield. The variable L_{ss} denotes steady-state leverage. Using lower case letters to denote log deviations, the log-linearized model is given by the following:

(36)
$$\lambda_{t} = \frac{1}{(1 - \beta h)(1 - h)} E_{t} \left[\beta h c_{t+1} - (1 + \beta h^{2}) c_{t} + h c_{t-1} \right] + \frac{1}{1 - \beta h} (r n_{t} - \beta h E_{t} r n_{t+1})$$

$$(37) rn_t + \eta h_t - \lambda_t = mrs_t$$

(38)
$$\pi_{t}^{w} - \iota_{w} \pi_{t-1} = \kappa_{w} (mrs_{t} - w_{t}) + \beta (\pi_{t+1}^{w} - \iota_{w} \pi_{t}) + \epsilon_{t}^{w}$$

$$(39) w_t = w_{t-1} + \pi_t^w - \pi_t$$

$$\lambda_t = E_t \lambda_{t+1} + r_t - E_t \pi_{t+1}$$

$$(41) \quad \lambda_t + p_t^k + m_t = E_t \{ \lambda_{t+1} + [1 - \beta(1 - \delta)] r_{t+1}^k + \beta(1 - \delta) (p_{t+1}^k + m_{t+1}) \}$$

(42)
$$\lambda_t + q_t + m_t = E_t \lambda_{t+1} - E_t \pi_{t+1} + \beta \kappa E_t (q_{t+1} + m_{t+1})$$

$$(43) (1-\kappa)(p_t^k+i_t) = f_t - \kappa(f_{t-1}+q_t-q_{t-1}-\pi_t)$$

(44)
$$r_{t+1}^{L} = \frac{\kappa q_{t+1}}{R_{cc}^{L}} - q_{t}$$

$$r_t^{10} = -\left(\frac{R_{ss}^L - \kappa}{R_{ss}^L}\right) q_t$$

(46)
$$E_{t}(r_{t+1}^{L} - r_{t}) = \left(\frac{1}{L_{ss} - 1}\right)l_{t} + \left[\frac{1 + (s - 1)L_{ss}}{L_{ss} - 1}\right]\phi_{t}$$

(47)
$$\psi n_{t} = \left[\frac{sL_{ss}}{1 + L_{ss}(s-1)} \right] E_{t} (r_{t+1}^{L} - r_{t}) + \left[\frac{(s-1)L_{ss}}{1 + L_{ss}(s-1)} \right] l_{t}$$

$$(48) \qquad \qquad \frac{\overline{B}_{ss}}{L_{ss}N_{ss}}b_t + \left(1 - \frac{\overline{B}_{ss}}{L_{ss}N_{ss}}\right)f_t = n_t + l_t$$

$$(49) w_t = mc_t + mpl_t$$

$$(50) r_t^k = mc_t + mpk_t$$

(51)
$$\pi_t = \frac{\kappa_{\pi}}{1 + \beta \iota_n} mc_t + \frac{\beta}{1 + \beta \iota_n} E_t \pi_{t+1} + \frac{\iota}{1 + \beta \iota_n} \pi_{t-1} + \epsilon_t^p$$

(52)
$$p_t^k = \psi_i[(i_t - i_{t-1}) - \beta E_t(i_{t+1} - i_t)] - \mu_t$$

(53)
$$\left(1 - \frac{I_{ss}}{Y_{ss}}\right)c_t + \frac{I_{ss}}{Y_{ss}}i_t = a_t + \alpha k_t + (1 - \alpha)h_t$$

(54)
$$k_{t+1} = (1 - \delta)k_t + \delta(\mu_t + i_t)$$

$$(55) r_t = \rho r_{t-1} + (1-\rho)(\tau_\pi \pi_t + \tau_y y_t^{gap}) + \epsilon_t^r.$$

To close the model, we need one more equation outlining the policy rule for the long-term debt market. Before a discussion of these policy options, several comments are in order.

First, equation (41) highlights the economic distortion, m_t , arising from the segmented markets. Solving this forward, we have

(56)
$$p_t^k + m_t = E_t \sum_{i=0}^{\infty} [\beta(1-\delta)]^j \{ [1-\beta(1-\delta)] r_{t+j}^k - (r_{t+j} - \pi_{t+j+1}) \}.$$

As is clear from (56), the segmentation distortion, m_t , acts like a mark-up or excise tax on the price of new capital goods. What is this distortion? Using (42) and (44), we have

(57)
$$m_t = E_t \sum_{j=0}^{\infty} (\beta \kappa)^j \Xi_{t+j},$$

where

(58)
$$\Xi_{t+j} \equiv \beta \kappa \, q_{t+j+1} - q_{t+j} - r_{t+j} \approx r_{t+j}^L - r_{t+j}.$$

The distortion is thus the discounted sum of the future one-period loan to deposit spreads. As discussed above, this spread exists because of the assumed market segmentation.

Second, the market segmentation distortion is joined by the two familiar distortions in this familiar DNK model. The nominal price rigidity induces a time-varying wedge between rental rates and the marginal product of capital (50). The price and wage rigidities create a time-varying wedge or "labor distortion" between the marginal product of labor and the marginal rate of substitution (see (38) and (49)). To anticipate our empirical estimates, nominal wages are significantly more sticky than nominal prices so that this time-varying labor distortion is of greater importance than the wedge between the rental rate and marginal product of capital. Because of this our discussion below will focus on the interaction between the segmentation distortion and the labor distortion.

Third, the *term premium* can be defined as the difference between the observed yield on a ten-year bond (see (45)) and the corresponding yield implied by applying the expectation hypothesis (EH) of the term structure to the series of short rates. The price of this hypothetical EH bond satisfies

$$r_t = E_t \frac{\kappa q_{t+1}^{EH}}{R_{cc}} - q_t^{EH}$$

while its yield is given by

(60)
$$r_t^{EH,10} = \left(\frac{R_{ss} - \kappa}{R_{ss}}\right) q_t^{EH}.$$

Using these definitions, the term premium can be expressed as

$$(61) \quad \textit{term premium} \equiv \textit{tp}_{\textit{t}} \equiv \left(r_{\textit{t}}^{10} - r_{\textit{t}}^{\textit{EH},10}\right) = -\left(\frac{R_{\textit{ss}}^{\textit{L}} - \kappa}{R_{\textit{ss}}^{\textit{L}}}\right)q_{\textit{t}} + \left(\frac{R_{\textit{ss}} - \kappa}{R_{\textit{ss}}}\right)q_{\textit{t}}^{\textit{EH}}.$$

Solving the bond prices in terms of the future short rates, we have

(62)
$$tp_{t} = \left(\frac{R_{ss}^{L} - \kappa}{R_{ss}^{L}}\right) \sum_{j=0}^{\infty} \left(\frac{\kappa}{R_{ss}^{L}}\right)^{j} E_{t} r_{t+j}^{L} - \left(\frac{R_{ss} - \kappa}{R_{ss}}\right) \sum_{j=0}^{\infty} \left(\frac{\kappa}{R_{ss}}\right)^{j} E_{t} r_{t+j}$$
$$\approx \left(1 - \beta\kappa\right) \sum_{j=0}^{\infty} \left(\beta\kappa\right)^{j} E_{t} (r_{t+j}^{L} - r_{t+j}).$$

Comparing (57) and (62), the distortion m_t is essentially the term premium. Hence, a policy that eliminates fluctuations in the term premium will largely eliminate fluctuations in the market segmentation distortion.

Fourth, the loan-deposit spread arises because of the segmentation effects summarized in (46)–(47). Equation (46) expresses the endogenous response of leverage to higher expected returns on intermediation, while equation (47) summarizes the FI's desire to accumulate more net worth in response to the profit opportunity of the spread. The model's dynamics collapse to the familiar DNK model if we set $\psi_n = 0$, so that net worth can move instantaneously to eliminate all arbitrage opportunities. But if $\psi_n > 0$, then the segmentation acts like an endogenous adjustment cost to investment. That is, increases in investment necessitate an increase in investment bonds (48), but this drives up the one-period spread (46) and thus, m_t . The net worth adjustment cost (47) implies that this effect cannot be entirely undone by movements in net worth.

Fifth, the previous suggests that a policy that stabilizes the term premium will likely be welfare improving (unless the interaction with labor distortion is significant). This suggests the efficacy of a central bank including the term premium in a Taylor type rule. But we can take this argument one step further by targeting the term premium. Under a policy that directly targets the term premium the supply of long debt held by FIs will be endogenous. In particular, (48) separates out from the rest of the model, and defines the behavior of long bonds that move endogenously to support the long rate target. This implies that "credit shocks," those proxied by ϕ_t , will have no effect on real activity or inflation. That is, a long rate policy sterilizes the real economy from financial shocks. This is analogous to the classic result of Poole (1970) in which an interest rate target sterilizes the real economy from shocks to money demand.

Sixth and finally, the assumption that the long bonds are nominal implies that monetary policy shocks will have real effects even in a flexible price model. This is seen most clearly in (43). Innovations in inflation will erode the existing real value of investment debt thus making increased issuance less costly. This effect disappears if the debt is only one period ($\kappa=0$), or if the debt is indexed to inflation (κ is a real payment).

H. Debt Market Policies

To close the model, we need one more restriction that will pin down the behavior in the long debt market. We will consider two different policy regimes for this market: exogenous debt, and endogenous debt. We will discuss each in turn.

Exogenous Debt.—The variable b_t denotes the real value of long-term government debt on the balance sheet of FIs. There are two distinct reasons why this variable could fluctuate. First, the central bank could engage in long bond purchases ("quantitative easing," or QE). Second, the fiscal authority could alter the mix of short debt to long debt in its maturity structure. We will model both of these scenarios as exogenous movements in long debt. Under either scenario, the long yield r_t^{10} will be endogenous. Our benchmark experiments will hold long debt fixed at

steady state, $b_t = 0$. We will also explore QE policies. To model a persistent and hump-shaped QE policy shock we will use an AR(2):

(63)
$$b_t = \rho_1^b b_{t-1} + \rho_2^b b_{t-2} + \epsilon_t^b.$$

Within such an exogenous debt regime, we will also consider policies in which the Taylor rule for the short rate responds to some measure of the term premium:

(64)
$$r_t = \rho r_{t-1} + (1 - \rho) (\tau_\pi \pi_t + \tau_\nu y_t^{gap} + \tau_{tp} t p_t),$$

where the term premium (tp_t) is defined as in (61). As noted earlier, there are reasons to think that such a policy may be welfare-improving.

Endogenous Debt.—The polar opposite scenario is a policy under which the central bank pegs the term premium $tp_t = 0$. Under this policy regime the level of long debt b_t will be endogenous. Under a term-premium peg the asset value of the intermediary will remain fixed, while composition of assets will vary. That is, any increase of FI holdings of investment debt is achieved via the central bank purchasing an equal magnitude of government bonds. The proceeds from this sale effectively finances loans for investment.

II. Quantitative Results

A. Estimation and Calibration

We calibrate several parameters to match long-run features of US data, features that are not well identified by the high frequency business cycle data. We interpret a model period as a quarter, and set $\beta=0.99$. The production parameters are given by $\alpha=0.33$ and $\delta=0.025$. The elasticity parameters are set at $\epsilon_p=\epsilon_w=5$, implying a 20 percent mark-up in both price and wage-setting. We use evidence on interest rate spreads and leverage to pin down two primitive parameters. The steady-state loan-deposit spread and leverage ratio are given by

$$\zeta = \left(\frac{R_{ss}^{10}}{R_{ss}^{10,EH}}\right)^{-1};$$

$$L_{ss} = \left[1 + (\Phi_{ss} - 1)\left(\frac{R_{ss}^{10}}{R_{ss}^{10,EH}}\right)\right]^{-1}.$$

We will choose the parameters ζ and Φ_{ss} to match a term premium of 100 annual basis points, and a leverage level of $L_{ss}=6$. This is the same calibration as in Gertler and Karadi (2013). The government and investment bonds will both be calibrated to a duration of 40 quarters, $(1-\kappa)^{-1}=40$. We also need to calibrate the balance sheet proportion, $\frac{\overline{B}_{ss}}{N_{ss}}$. This is proportional to the fraction of FI assets held as long term debt: $\frac{\overline{B}_{ss}}{N_{ss}}=\frac{\overline{B}_{ss}}{\overline{F}_{ss}+\overline{B}_{ss}}\times L_{ss}$. Given our interpretation of the FIs as the financial

sector, we set the ratio of government securities to total FI assets to $\frac{\overline{B}_{ss}}{\overline{F}_{ss} + \overline{B}_{ss}} = 40\%$, comparable to data on outstanding government debt and investment spending.

The remaining parameters are estimated using familiar Bayesian techniques. The model includes seven exogenous shocks so that we need at least seven observables. We treat as observables the growth rates of real GDP, real gross private domestic investment, real wages, and the PCE index. Real wages are given by nominal compensation per hour in the non-farm business sector, divided by the consumption deflator. Labor input is measured as average weekly hours in the non-farm business sector. The two interest rate measures are the effective federal funds rate, and the series on the term premium on a ten-year Treasury estimated by Adrian, Crump, and Moench (2013). This term-premium series is derived from an empirical methodology that assumes frictionless arbitrage, an assumption that contrasts with the segmentation model developed here. But all that is needed for our identification is that this empirical measure of the term premium is significantly correlated with the true unobserved term premium. The term-premium series is available starting in 1962:1. We end our estimation before the zero bound, so our estimation period is 1962:1–2008:4. All data is de-meaned.

Distribution functions and priors on the estimated coefficients are outlined in Table 1. These largely follow the literature (see Carlstrom, Fuerst, and Paustian 2014). The key parameter of interest is the portfolio adjustment cost parameter ψ_n . For this we assume a diffuse uniform prior between 0 and 10. Recall that the model collapses to the standard DNK model if $\psi_n = 0$. The net worth elasticity is given by $1/\psi_n$, so that our priors imply an elasticity between 0.1 and infinity. In this estimation we assume exogenous debt.³ The coefficient estimates are reported in Table 1.

A few comments are in order. First, the estimates include modest levels of wage and price indexation. But the estimates imply a much higher degree of wage stickiness compared to price stickiness. Although nominal wages are indexed to overall price inflation ($\iota_w = 0.51$), only 4 percent of nominal wages are freely reset each period compared to 25 percent of nominal prices ($\theta_w = 0.96$, $\theta_p = 0.75$). As noted above, this suggests that the labor wedge will be of central importance.

Second, the four main drivers of output are shocks to TFP, the investment technology, credit shocks, and natural rate shocks. The fraction of output variance explained by these four shocks is 21 percent, 36 percent, 28 percent, and 13 percent, respectively. Hence, we will concentrate on these four shocks below.

Third, and finally, the point estimate of the adjustment cost parameter is $\psi_n = 0.79$, or a net worth elasticity of 1.27. The 90 percent confidence window on this parameter implies an estimated elasticity between 0.81 and 2.95. We will conduct sensitivity analysis on this key parameter below.

³Debt is likely to be endogenous since the Treasury's stated objective is to structure its portfolio to minimize interest costs. This will likely bias our adjustment cost parameter down.

TABLE 1—MODEL ESTIMATION

	Prior			Posterior			
Coefficient	Prior density	Prior mean	pstdev	Post. mean	5%	95%	
$\overline{\eta}$	G	2.00	0.75	2.0259	1.2673	2.7526	
h	В	0.60	0.10	0.6225	0.5760	0.6687	
ψ_n	U	5.00	2.89	0.7850	0.3389	1.2394	
φ	G	3.00	1.00	3.2821	2.1857	4.3639	
$ au_{\pi}$	N	1.50	0.10	1.4202	1.2828	1.5493	
$ au_{ m v}$	N	0.50	0.10	0.4906	0.3566	0.6292	
ρ_i	В	0.80	0.10	0.7712	0.7309	0.8109	
ι_p	В	0.60	0.10	0.4175	0.2752	0.5610	
ι_w	В	0.60	0.10	0.5110	0.4085	0.6205	
κ_{pc}	В	0.20	0.10	0.0860	0.0104	0.1544	
κ_w	В	0.20	0.10	0.0002	0.0001	0.0004	
ρ_a	В	0.60	0.20	0.9921	0.9841	0.9997	
$ ho_{\mu}$	В	0.60	0.20	0.8695	0.8281	0.9122	
$ ho_arphi$	В	0.60	0.20	0.9821	0.9682	0.9963	
$ ho_{mk}$	В	0.60	0.20	0.6650	0.4945	0.8405	
$ ho_{mkw}$	В	0.60	0.20	0.2059	0.1036	0.3027	
ρ_m	В	0.60	0.20	0.1564	0.0646	0.2515	
$ ho_{rn}$	В	0.60	0.20	0.9483	0.9212	0.9751	
σ_a	I	0.50	1.00	0.6481	0.5936	0.7030	
σ_i	I	0.50	1.00	7.3454	5.5735	9.2124	
σ_{mp}	I	0.10	1.00	0.2151	0.1935	0.2368	
σ_{mk}	I	0.10	1.00	0.2442	0.1830	0.3049	
σ_{mkw}	I	0.10	1.00	0.4840	0.4103	0.5569	
σ_{rn}	I	0.10	1.00	0.1588	0.1179	0.2000	
σ_{ψ}	I	0.50	1.00	2.7196	1.9449	3.4826	

Notes: N stands for Normal, B-Beta, G-Gamma, U-Uniform, and I-Inverted-Gamma distribution. pstdev stands for prior standard deviations. Posterior percentiles are from 2 chains of 100,000 draws generated using a Random Walk Metropolis algorithm. We discard the initial 50,000 and retain one every 5 subsequent draws.

B. QE Shock

Although not our primary focus, it is natural to first consider a QE-type shock in the estimated model. Figure 1 graphs the change in the Fed's bond portfolio relative to the government debt in the hands of the domestic public.

The QE policies are quite apparent. We will consider a QE shock that decreases b_t by 6.5 percent, comparable to the magnitude in Figure 1 (roughly \$300 billion).⁴ To match the persistent nature of this expansion, we set $\rho_1^b = 1.8$, and $\rho_2^b = -0.81$. Empirical estimates of the response of the ten year yield to these QE shocks vary from no effect to over 45 bp (e.g., the evidence discussed in Chen, Cúrdia, and Ferrero 2012). Using the estimated model parameters, the impulse response to the

⁴Recall that b_t is the amount of government debt held by the FIs, so that a QE shock is a decrease in b_t .

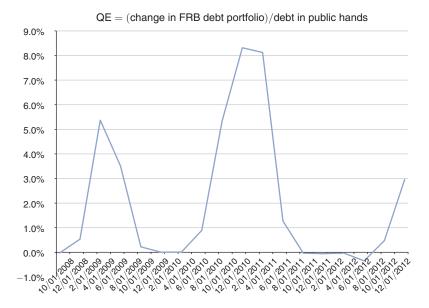


FIGURE 1. CHANGE IN FRB DEBT PORTFOLIO

QE shock is exhibited in Figure 2. The policy shock has a modest (19 bp at its peak) but persistent effect on the ten-year yield, with most of this movement being driven by changes in the term premium. This term-premium effect dissipates as net worth responds and segmentation returns to steady-state levels, so that the long rate is eventually driven by the path of the short rate. The policy has a persistent and significant effect on investment and output, while consumption at first declines modestly before rising subsequently. The increased output leads to a policy-induced increase in the funds rate. The funds rate eventually overshoots its long-run level, thus leading to a persistent decline in the long rate. Sensitivity analysis on the QE experiment is reported in Table 2.

The first observation is that the quantitative results are only modestly affected by the size of the adjustment costs on net worth, ψ_n . The peak investment response only varies from 3.1 percent to 5.2 percent as we vary the adjustment cost parameter from one end of the 90 percent confidence interval to the other end. A key parameter is the calibrated duration of the investment bonds. Longer maturities for the investment bonds lead to much larger effects on both real activity and the long bond term premium. For example, as we move from 1-year bonds to 20-year bonds, the peak investment response increases from 1.6 percent to 5.7 percent. As the maturity of the investment bonds decreases, more of the QE purchase is absorbed by movements in the yield and not movements in real activity. Recall that without a borrowing constraint for investment, changes in the term premium would have no real effects. 5

⁵The duration of the government debt is irrelevant for the model's dynamics.

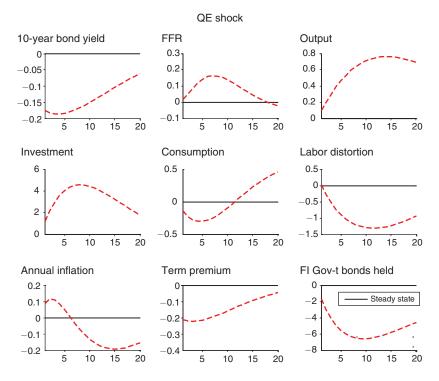


FIGURE 2. QE EXPERIMENT, BASELINE PARAMETER VALUES

Notes: All variables are in percentage points and all rates are annualized. The variable "Labor Distortion" is the ratio of the marginal product of labor to the marginal rate of substitution.

Parameter value	Peak investment response	Peak ten-year yield response	Peak inflation response
$\psi_n = 0$	0	0	0
$\psi_n = 0.34$	3.14	-0.14	0.06
$\psi_n = 0.78$ (baseline)	4.57	-0.19	0.12
$\psi_n = 1.24$	5.23	-0.21	0.15
Duration $= 4 q$	1.57	-0.31	0.01
Duration = 20 q	3.56	-0.21	0.06
Duration = 80 q	5.71	-0.18	0.18

TABLE 2—SENSITIVITY ANALYSIS OF QE SHOCK

Notes: All parameter values are held at their estimated or calibrated values unless otherwise noted. Duration is duration of the investment bond.

How do these QE effects compare to the empirical findings on the effects of the Federal Reserve's Large Scale Asset purchases on long bond yields? Such a comparison is naturally rather difficult, because the effects of asset purchases depend on a number of factors that are hard to quantify, such as the perceived persistence of the program, signaling about the path of the future short rate that is associated with bond purchases as documented in Bauer and Rudebusch (2014), as well as any information that market participants might infer about the underlying state of the economy. Nevertheless the evidence reported in Chen, Cúrdia, and Ferrero (2012) suggests

that the impact on the ten year treasury yield per \$100 billion is in the range of -3 to -15 basis points, with a median value of -5 basis points. This compares to an estimated effect of about -6 basis points per 100 billion in our model.

C. Other Shocks under Exogenous and Endogenous Debt Policies

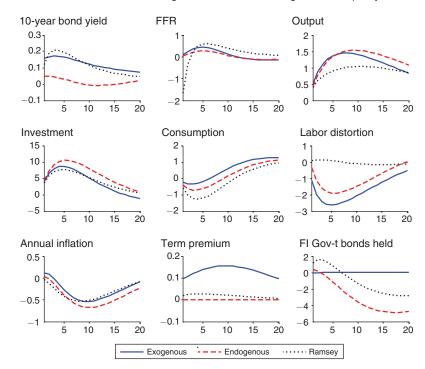
Figures 3–6 look at the effect of the four key estimated shocks (shocks to TFP, the investment technology, credit shocks, and natural rate shocks). Each figure considers three different policy scenarios: (i) the "exogenous debt" case in which the long bond portfolio is held fixed and the short rate is conducted according to a familiar Taylor rule, (ii) the "endogenous debt" case in which the bond portfolio responds endogenously to hold the term premium fixed (and the short rate follows the same Taylor rule), and (iii) the "Ramsey" policy, which we will discuss below. The "exogenous debt" case is the assumption used in estimating the model. But as noted earlier, the term-premium peg is of interest because it largely stabilizes the market segmentation distortion so that the IRFs to these shocks will mirror their DNK counterparts.

The Ramsey policy is the choice of a planner who is subject to the model's equilibrium conditions but can otherwise freely choose the short-term interest rate and the level of long-term debt (one can alternatively think of the planner choosing the term premium) to maximize lifetime household utility. The planner uses these two instruments to manage three distortions: the mark-up of prices over marginal cost, the mark-up of wages over the marginal rate of substitution, and the market segmentation distortion. To focus on the response of the planner to shocks we assume that the planner has access to three constant subsidies (on wages, capital rental rates, and the price of capital) so that the model's steady state is Pareto efficient. Hence, the planner's steady-state levels of price inflation, wage inflation, and term premium, match those of the market equilibrium.⁶ But our interest will be in how the planner adjusts his policy instruments in response to shocks. In particular, how does the Ramsey planner vary the term premium in response to shocks?

As noted earlier, the shocks driving output variability include shocks to investment, TFP, credit markets, and the natural rate, respectively. We will review the shocks in this order. For the TFP and investment shocks in Figures 3–4, the term premium barely moves with the Ramsey planner and is thus similar to the endogenous debt policy.

But for exogenous debt things are different. The term premium increases because the shocks increase the demand for investment (a demand-side channel) and increase the real value of existing investment debt (a supply-side channel) thus making it more costly to absorb additional debt. This increase in the term premium dampens investment and thus output behavior. By altering investment behavior, the increase

 $^{^6}$ If the Ramsey planner did not have access to these subsidies, his steady state would include zero price and nominal wage inflation, and a term premium of -360 annual bp. This negative term premium subsidizes capital accumulation and thus helps overcome the steady state distortion from market power in the product and labor markets. But the response of this Ramsey planner to shocks is qualitatively and quantitatively similar to the planner who has access to constant subsidies.



Investment shock: Exogenous debt versus endogenous debt policy

FIGURE 3. ONE STANDARD DEVIATION INVESTMENT SHOCK UNDER EXOGENOUS AND ENDOGENOUS DEBT POLICIES

Notes: All variables are in percentage points and all rates are annualized. The variable "Labor Distortion" is the ratio of the marginal product of labor to the marginal rate of substitution.

in the term premium also changes the composition of output, shifting it more toward consumption.

Impulse responses to the credit shock are charted in Figure 5.

In this linear world, these shocks can be interpreted in two ways. First, and as modeled above, they can be thought of as shocks to the hold-up problem. Second, they can instead be interpreted as shocks to the FI's impatience parameter. In both cases, they imply an exogenous increase in the term premium. As with the previous shocks, the increase in the term premium increases the relative cost of investment, leading to an increase in consumption but a decline in overall output. As discussed above, these shocks are entirely sterilized under a term-premium peg. The central bank passively engages in QE purchases of an order of magnitude comparable to the QE experiment in Figure 2.

Finally, the shock to the natural rate is something of a pure demand shock in that there is an increase in the desire for current consumption (see Figure 6).

This naturally crowds out investment, but leads to a sustained increase in output. The decline in the term premium comes from a decline in desired investment, and a decline in the real value of existing investment debt. As with the credit shock, the term premium is countercyclical for natural rate shocks. In contrast, the term premium is procyclical for the TFP and investment shocks. For the estimated model as

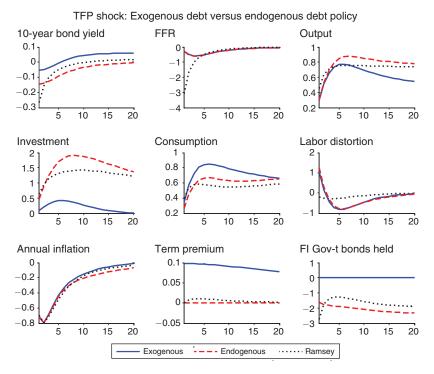
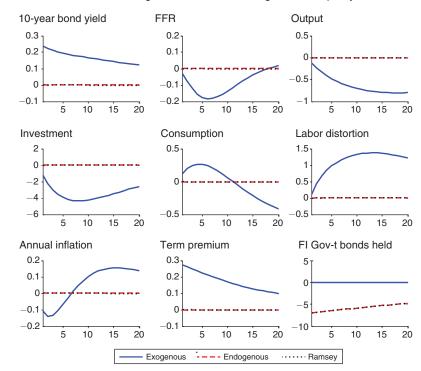


FIGURE 4. ONE STANDARD DEVIATION TFP SHOCK UNDER EXOGENOUS AND ENDOGENOUS DEBT POLICIES

Notes: All variables are in percentage points and all rates are annualized. The variable "Labor Distortion" is the ratio of the marginal product of labor to the marginal rate of substitution.

a whole, the implied term premium is mildly procyclical (the correlation with output is 0.12). Similarly, FI leverage is procyclical for the estimated model (the correlation with output is 0.57).

There are three important features of the Ramsey planner's behavior in response to all four shocks that deserve mention. First, the planner tolerates very little movement in the term premium. That is, the planner does not use movements in the market segmentation distortion to counter movements in the labor distortion, but instead essentially pegs the term premium. Second, the planner chooses a high level of volatility in the federal funds rate, e.g., a movement of 300 bp in the wake of a 1 standard deviation TFP shock. This is in sharp contrast to the estimated Taylor rule that hardly varies in response to shocks. This aggressive policy choice implies that the planner can dramatically dampen movements in the labor market distortion. Essentially, the planner uses a term-premium peg to neutralize the market segmentation distortion, and an aggressive federal funds rate policy to stabilize the labor market distortion. A third important feature of Ramsey behavior is that the ten-year yield's volatility is comparable to both the exogenous and endogenous debt policies. This is a consequence of a highly variable short rate with little movement in the term premium. But this implies that there is no rationale for a policy that pegs or smooths the ten-year rate. That is, it is welfare-enhancing to smooth the term premium, but not to smooth the ten-year rate. This is analogous to the desirability of smoothing the output gap but not output in a simpler DNK framework.



Credit shock: Exogenous debt versus endogenous debt policy

FIGURE 5. ONE STANDARD DEVIATION CREDIT SHOCK UNDER EXOGENOUS AND ENDOGENOUS DEBT POLICIES

Notes: All variables are in percentage points and all rates are annualized. The variable "Labor Distortion" is the ratio of the marginal product of labor to the marginal rate of substitution.

D. Welfare Consequences of a Taylor Rule Including the Term Premium

In this section, we consider the effect of a central bank including the term premium in its FFR Taylor rule. In particular, suppose that the Taylor Rule is given by

(65)
$$r_t = \rho r_{t-1} + (1 - \rho) (\tau_\pi \pi_t + \tau_y y_t^{gap} + \tau_{tp} t p_t),$$

where the term premium (tp_t) is defined above. As an initial experiment, we set the remainder of the Taylor rule at the estimated parameter values in Table 1, and consider the welfare consequences of alternative values for τ_{tv} .

The first step in the analysis is to ensure equilibrium determinacy. Figure 7 looks at equilibrium determinacy for the Taylor rule that includes the term premium.

For determinacy under a term-premium rule, the response of the short rate to the term premium cannot be too large. At the baseline calibration for the Taylor rule, this restriction is $\tau_{tp} < 0.74$. To understand the intuition for this upper bound, consider the long run response of the policy rate to a permanent shift in inflation:

(66)
$$\frac{dr}{d\pi} = \frac{1}{(1+\tau_{tp})} \left(\tau_{\pi} + \tau_{y} \frac{dy^{gap}}{d\pi} + \tau_{tp} \frac{dr^{L}}{d\pi} \right),$$

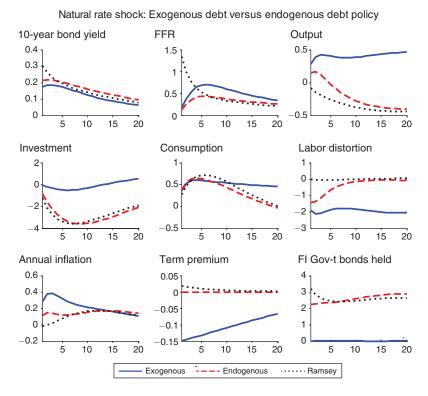


FIGURE 6. ONE STANDARD DEVIATION NATURAL RATE SHOCK UNDER EXOGENOUS AND ENDOGENOUS DEBT POLICIES

Notes: All variables are in percentage points and all rates are annualized. The variable "Labor Distortion" is the ratio of the marginal product of labor to the marginal rate of substitution.

where we have used the fact that the long-run level of the term premium is just the loan-deposit spread. The Taylor Principle is that this policy rate response must exceed unity. How does the loan rate respond to an innovation in inflation? An innovation to inflation leads to an erosion of the real value of the existing debt, which lowers the hold-up problem and thus the long rate, i.e., $\frac{dr^L}{d\pi} < 0$. Hence, if the central bank responds positively to movements in the term premium, then it is effectively lowering its response to an innovation in inflation. By this logic, a negative response to the term premium is consistent with determinacy. Further, Figure 7 implies that if this response is not too negative, then the response to inflation can be significantly below unity.

Our measure of welfare is given by the discounted lifetime utility of the household. This is calculated with a second-order approximation of the value function and then evaluated at the model's steady state. These welfare numbers are then scaled by the steady-state marginal utility of consumption and level of consumption so that the reported values are in consumption perpetuity units.

Figure 8 displays the welfare consequence of alternative τ_{tp} for the estimated parameter values.

The preferred response is negative, so we only plot the welfare function for $\tau_{tp} < 0$. The optimal response occurs at $\tau_{tp} = -1.0$, implying that a 50 bp increase

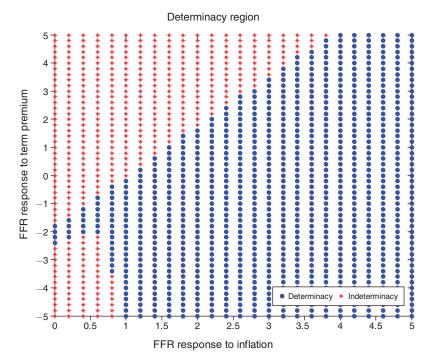


FIGURE 7. EQUILIBRIUM DETERMINACY UNDER A TAYLOR RULE WITH A RESPONSE TO THE TERM PREMIUM

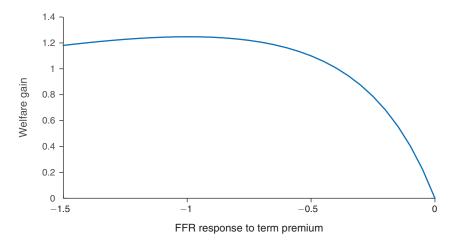


Figure 8. Welfare Consequences of a Taylor Rule with a Response to the Term Premium

Notes: The units are in consumption perpetuities, i.e., 0.5 means a perpetual increase in consumption equal to 0.5 percent of steady-state consumption, or a one-time increase of 50 percent. The welfare change is on the vertical axis, and the term premium coefficient in the Taylor Rule is on the horizontal axis. The peak welfare gain occurs at $\tau_{prem} = -1.0$.

in the term premium should lead the central bank to lower its funds rate target by 50 bp. The welfare gain, relative to not responding to the term premium, is significant: 1.25 percent of consumption in perpetuity.

TABLE 3—COMPARING TWO STARK POLICIES

Here we consider two extreme policy choices: holding the balance sheet fixed $(b_t = 0)$, versus a te	m premium
$peg, (tp_t = 0).$	

	Credit shocks only	Investment shocks only	TFP shocks only	Discount shocks only	All four shocks	
Welfare gain of term premium peg	0.47	0.12	0.13	1.63	2.35	
All four shocks	$b_t = 0$	$tp_t = 0$	$b_t = 0$ (with subsidies)	$tp_t = 0$ (with subsidies)	$b_t = 0$ (flexible wages)	$tp_t = 0$ (flexible wages)
Welfare gain of term premium peg Mean of segmentation distortion	1.08	2.35 1.06	1.02	1.89	 1.14	1.80 1.05

Notes: The welfare units are in consumption perpetuities, i.e., 2.35 means a perpetual increase in consumption equal to 2.35 percent of steady-state consumption, or a one-time increase of 235 percent. In the non-stochastic steady state without subsidies, the segmentation distortion is $M_{ss} = 1.07$. The labor distortion is the ratio of the marginal product of labor to the marginal rate of substitution. In the non-stochastic steady state without subsidies, the labor distortion is equal to 1.56. In the case of subsidies, we have $M_{ss} = 1$, and the labor distortion is equal to 1.

Finally, Table 3 considers two stark policies. In both cases, the central bank uses the estimated Taylor rule (without a response to the term premium). In terms of long debt policy, we consider two extremes: (i) the level of long debt in circulation is held fixed (so that the term premium is endogenous), versus (ii) the term premium is pegged (so that the level of long debt is endogenous). The overall welfare gain is quite large, 2.35 percent of aggregate consumption in perpetuity. Almost all of this gain comes from the credit and natural rate shocks.

Why are credit and natural rate shocks so important here? There are three underlying distortions in the model: (i) the mark-up of prices over marginal cost creates a wedge between the marginal product of capital and the rental rate, (ii) the combined price and wage mark-up creates a labor wedge between the marginal product of labor and the marginal rate of substitution, and (iii) the market segmentation distortion that creates a wedge between the price of investment goods and the production cost $(M_t > 1)$. Because the estimated frequency of price adjustment is significantly larger than wage adjustment, the latter two distortions are key. Natural rate and credit shocks are "demand" shocks in that output and inflation move together. An increase in inflation leads to a decline in the two key distortions as real wages fall, and inflation erodes the value of existing investment debt and thus leads to a decline in the term premium. Hence, the labor distortion and segmentation distortion positively co-move with credit shocks and shocks to the natural rate. But this is disastrous from a welfare perspective. Under a term premium peg, this co-movement goes trivially to zero. In the nonlinear model movements in the labor distortion also exacerbate its average distortion. The average labor wedge falls from 1.74 with exogenous debt to 1.56 with endogenous debt.

The second panel of Table 3 provides some sensitivity analysis on these welfare costs. If we include steady-state subsidies so that the steady state is efficient, the welfare gain of the term-premium peg falls by about one-fifth, to 1.89 percent. The decline in the welfare gain arises because inferior policies are less costly if the

model is centered at an efficient steady state. The importance of sticky wages is also evident in Table 3. If we assumed that nominal wages are perfectly flexible, then the welfare cost again falls by about one-fifth. With flexible wages, the model's inflation rate becomes more volatile implying that the real level of investment bonds becomes more volatile, thus leading to greater variability and a higher mean of the segmentation distortion.

The Ramsey policy will, of course, welfare-dominate the term-premium peg ("endogenous debt" policy). But the term-premium peg does remarkably well. The welfare gain of the Ramsey planner over the term-premium peg is only 0.15 percent (assuming that steady state subsidies make the steady states coincide). Compared to the welfare cost of the fixed debt policy, the term-premium peg thus achieves over 90 percent of the welfare gain possible.

We have also explored endogenous debt policies in which the central bank pegs the ten-year rate at its steady-state value (and thus allows the term premium to move endogenously). The practical advantage of such a policy is that the ten-year rate is directly observable. But the problem is that the underlying distortion is linked to the term premium, and a peg of the long-term interest rate can exacerbate movements in this distortion. For example, consider the natural rate shock in Figure 6. Under an exogenous debt policy, the term premium declines while the ten-year rate rises. If the central bank pegs the ten-year rate, then the term premium will endogenously decline by even more in response to a natural rate shock. From a welfare perspective this is again disastrous. Compared to an exogenous debt policy, the welfare cost of a ten year rate peg is over 1.03 percent in perpetuity.

III. Sensitivity Analysis

A. Alternative Observables

In this section, we consider an alternative estimation in which we treat the term premium as unobserved and instead use the ten-year Treasury rate as an observable. Figure 9 plots the de-meaned ten-year rate and ten-year term premium for the sample period of our estimation. The two series are highly correlated in levels (correlation = 0.77), but less so in first differences (correlation = 0.24). The ten-year rate is more volatile reflecting the high inflation years in the middle of the sample.

Our baseline estimation uses the term premium as an observable. We make this choice because the term premium reflects the key underlying distortion captured in the model and is particularly helpful in identifying the key net worth elasticity ψ_n . But as a form of sensitivity analysis, we re-estimate the model using the ten-year rate instead of the term premium as an observable. The parameter estimates are presented in Table 4. Figure 10 plots the model's predicted term premium versus the term premium measured in the data. The model matches the data surprisingly well. The correlation in levels is 0.90, and 0.45 in first differences.

There are only two significant changes in the parameter estimates reported in Table 4. First, the implied net worth parameter is roughly half of the baseline estimate, $\psi_n = 0.39$. As suggested by Table 2, this difference will have only a modest effect on our quantitative results. The second key estimation difference is in the size



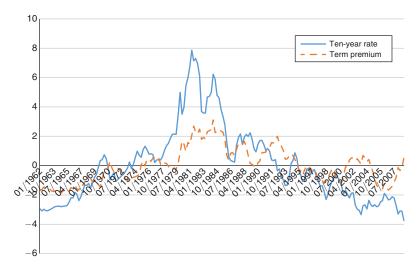


FIGURE 9. TEN-YEAR RATE VERSUS TERM PREMIUM

of the credit shocks, a standard deviation of 7.28, more than double the baseline estimate of 2.72. As noted, these shocks are entirely sterilized by a term-premium peg, so that this estimate will reinforce the welfare gain of an endogenous debt policy that stabilizes the term premium.

B. Indexed Debt and More Aggressive Taylor Rules

An important assumption is that the bonds used to finance investment are nominal. This creates a debt-deflation effect in which unanticipated decreases in inflation lead to an increase in the real value of the stock of investment debt (see equation (43)). Given the portfolio adjustment costs faced by the FIs, the surge in debt holdings drives up the term premium, making it more costly to issue more investment debt. This suggests two sensitivity analyses: debt indexed to inflation and a central bank that more aggressively responds to movements in inflation. We pursue both experiments here.

For the experiment "indexed debt," we assume that nominal investment debt is indexed so that innovations in the inflation rate have no effect on the real stock of investment debt held by the FIs. For the experiment "inflation hawk," we double the estimated Taylor rule coefficient on inflation from the estimated 1.42 to 2.84. We then redo our welfare calculations under these alternative scenarios. Table 5 summarizes our results.

If debt is indexed to inflation, then the welfare advantage of smoothing the term premium is roughly cut in half: the gain of a term-premium peg falls from 1.89 percent to 0.99 percent (assuming steady state subsidies). Similarly, with debt indexed there is a much smaller welfare gain to including the term premium in the Taylor rule (1.25 percent compared to just 0.26 percent). In summary, if nominal debt is indexed to innovations in inflation, the advantage of term-premium policies is

TABLE 4—MODEL ESTIMATION WITH TEN-YEAR RATE OBSERVED

	Prior			Posterior			
Coefficient	Prior density	Prior mean	pstdev	Post. mean	5%	95%	
$\overline{\eta}$	G	2.00	0.75	2.1030	1.1301	3.0384	
h	В	0.60	0.10	0.6883	0.6329	0.7456	
ψ_n	U	5.00	2.89	0.3860	0.1500	0.6488	
φ	G	3.00	1.00	3.2485	1.6601	4.5667	
$ au_{\pi}$	N	1.50	0.10	1.6524	1.5194	1.7877	
$ au_{ m v}$	N	0.50	0.10	0.1862	0.1253	0.2439	
ρ_i	В	0.80	0.10	0.8289	0.7991	0.8600	
ι_p	В	0.60	0.10	0.3892	0.2625	0.5142	
ι_w	В	0.60	0.10	0.6407	0.4974	0.7973	
κ_{pc}	В	0.20	0.10	0.1420	0.0688	0.2149	
κ_w	В	0.20	0.10	0.0028	0.0004	0.0064	
ρ_a	В	0.60	0.20	0.9963	0.9928	0.9997	
$ ho_{\mu}$	В	0.60	0.20	0.9628	0.9402	0.9874	
$ ho_{arphi}$	В	0.60	0.20	0.9489	0.9203	0.9755	
ρ_{mk}	В	0.60	0.20	0.7809	0.6435	0.9148	
ρ_{mkw}	В	0.60	0.20	0.2585	0.1184	0.4077	
ρ_m	В	0.60	0.20	0.1250	0.0422	0.2035	
ρ_{rn}	В	0.60	0.20	0.9304	0.8799	0.9806	
σ_a	I	0.50	1.00	0.6606	0.6038	0.7157	
σ_i	I	0.50	1.00	6.2766	3.9322	8.2807	
σ_{mp}	I	0.10	1.00	0.1977	0.1794	0.2156	
σ_{mk}	I	0.10	1.00	0.2341	0.1719	0.2961	
σ_{mkw}	I	0.10	1.00	0.5014	0.4244	0.5781	
σ_{rn}	I	0.10	1.00	0.1941	0.1204	0.2699	
σ_{ψ}	I	0.50	1.00	7.2795	5.2344	9.9904	

Notes: N stands for Normal, B-Beta, G-Gamma, U-Uniform, and I-Inverted-Gamma distribution. Posterior percentiles are from 2 chains of 100,000 draws generated using a Random Walk Metropolis algorithm. We discard the initial 50,000 and retain one every 5 subsequent draws.

sharply curtailed, but still significant. The term-premium peg remains advantageous as it insulates the economy from credit shocks, and accommodates the sharp movements in investment coming from the natural rate shocks.

In contrast, the advantage of term-premium policies are sharply curtailed under the inflation hawk. The gain to a term-premium peg falls from 1.89 percent to 0.35 percent, and the gain of including the term premium in the policy rule is largely eliminated (just 0.09 percent). That is, if a central bank more aggressively targets inflation, there is little reason to include the term premium in a traditional Taylor rule. As discussed above, the welfare gains of a term-premium peg come largely from the credit and natural rate shocks, "demand" shocks in that output and inflation move together. With nominal debt these movements in inflation cause the labor and segmentation distortions to move together. An inflation hawk will dampen these movements in inflation and thus mitigate the need for a term-premium peg. In short,

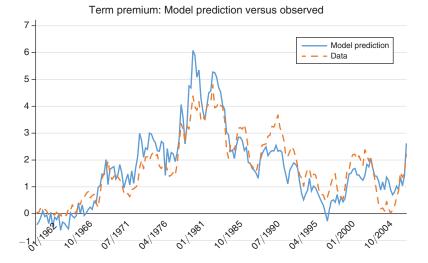


FIGURE 10. TERM PREMIUM MODEL PREDICTION VERSUS DATA

TABLE 5—SENSITIVITY ANALYSIS ON WELFARE RESULTS

	Baseline	Indexed debt	Inflation hawk
Welfare gain of term premium peg (with steady-state subsidies)	1.89	0.99	0.35
Optimal coefficient on term premium	-1	-0.6	-0.85
Welfare gain of optimal coefficient on term premium	1.25	0.26	0.09

Note: The welfare units are in consumption perpetuities, i.e., 1.89 means a perpetual increase in consumption equal to 1.89 percent of steady-state consumption, or a one-time increase of 189 percent.

if welfare costs come from demand shocks, a good policy response is to increase the Taylor rule coefficient on inflation. Although smaller in size, the advantage of using the central bank's portfolio to smooth the term premium remains: a 0.35 percent consumption perpetuity is a nontrivial welfare gain.

IV. Conclusion

This paper is motivated by the Quantitative Easing policy used by the Fed during the recent financial crisis. The paper contributes to the literature by constructing and estimating a segmented markets model that helps understand policy during these events. At the core of any such model is an assumption about market segmentation between the short-term money market and the long-term bond market. In the present context, we assume that (i) net worth limits the ability of intermediaries to arbitrage the return differentials across markets, and (ii) adjustment costs make it difficult for intermediaries to quickly scale up net worth. The novelty of the current work is to use this estimated model to think about policy issues outside of crisis periods. That is, if markets are segmented during crisis times, they may also suffer from segmentation during normal times. Hence, we use data over a long period to estimate the segmentation model, and then use the estimated model to answer policy questions.

In the estimated model, the real impact of this segmentation is meaningful. These real effects arise because the assumed segmentation introduces a time-varying wedge or distortion on the cost of investment goods. But any wedge needs a remedy. We emphasize two results. First, a monetary policy that includes the term premium in a Taylor rule can dampen movements in the market segmentation distortion. In particular, welfare is improved when the short-term rate responds negatively to the term premium. Second, a policy that makes the balance sheet endogenous by directly targeting the term premium will sterilize credit shocks. The advantage of this sterilization depends quite naturally on the importance of credit shocks in the business cycle.

The financial sector in the model is highly stylized, and there are many caveats to our results. First, we have assumed that government and private sector bonds are perfect substitutes. When government bonds are purchased from intermediaries, they respond by replacing public with private debt one for one. In practice, this link is less strong because of imperfect substitutability. Hence, our model is likely to give an upper bound on the impact of asset purchases. Second, we assume that all new investment must be financed with the issuance of long-term debt, the so-called "loan-in-advance" constraint. This is similar to the assumption in Gertler and Karadi (2013) that the entire capital stock is refinanced each period by intermediaries. Both assumptions are wildly unrealistic and again suggest that the model is an upper bound on these segmentation effects. An interesting extension for future work is to estimate the duration of the investment bonds, or allow firms an alternative means to finance investment. We leave this for future work.

APPENDIX

A. Nonlinear Equilibrium Conditions

(A1)
$$\Lambda_{t} = \frac{b_{t}}{C_{t} - h C_{t-1}} - E_{t} \frac{\beta h b_{t+1}}{C_{t+1} - h C_{t}}$$

(A2)
$$\Lambda_t = E_t \beta \Lambda_{t+1} \frac{R_t}{\Pi_{t+1}}$$

(A3)
$$\left(W_t^*\right)^{(1+\epsilon_w\eta)} = \frac{\epsilon_w}{\epsilon_w - 1} \frac{G_{1t}}{G_{2t}}$$

(A4)
$$G_{1t} = W_t^{\epsilon_w(1+\eta)} b_t B H_t^{1+\eta} + \beta \theta_w \left(\frac{\Pi_{t+1}}{\Pi_t^{t_w}}\right)^{\epsilon_w(1+\eta)} G_{1t+1}$$

(A5)
$$G_{2t} = \Lambda_t W_t^{\epsilon_w} H_t + \beta \theta_w \left(\frac{\Pi_{t+1}}{\Pi_t^{t_w}} \right)^{(\epsilon_w - 1)} G_{2t+1}$$

⁷The duration of government bonds is irrelevant. All that is essential is that government bonds (of whatever duration) are perfect substitutes for investment bonds.

$$(A6) W_t^{1-\epsilon_w} = (1-\theta_w)(W_t^*)^{1-\epsilon_w} + \theta_w \left(\frac{\Pi_{t-1}^{\iota_w}}{\Pi_t}\right)^{1-\epsilon_w} W_{t-1}^{1-\epsilon_w}$$

(A7)
$$\Lambda_{t} P_{t}^{k} M_{t} = E_{t} \beta \Lambda_{t+1} [R_{t+1}^{k} + (1 - \delta) P_{t+1}^{k} M_{t+1}]$$

(A8)
$$\Lambda_{t}Q_{t}^{I}M_{t} = E_{t}\beta\Lambda_{t+1}\frac{\left[1 + \kappa_{I}Q_{t+1}^{I}M_{t+1}\right]}{\Pi_{t+1}}$$

(A9)
$$V_t^h = b_t \left\{ \ln \left(C_t - h C_{t-1} \right) - d_t^w B \frac{H_t^{1+\eta}}{1+\eta} \right\} + \beta E_t V_{t+1}^h$$

$$(A10) R_t^k = MC_t MPK_t$$

$$(A11) W_t = MC_t MPL_t$$

(A12)
$$\Pi_t^* = \frac{\epsilon_p}{\epsilon_p - 1} \frac{X_{1t}}{X_{2t}} \Pi_t$$

(A13)
$$X_{1t} = MC_t \Lambda_t Y_t + \beta \theta_p \Pi_t^{-\iota_p \epsilon_p} \Pi_{t+1}^{\epsilon_p} X_{1t+1}$$

(A14)
$$X_{2t} = \Lambda_t Y_t + \beta \theta_p \Pi_t^{\iota_p (1 - \epsilon_p)} \Pi_{t+1}^{\epsilon_p - 1} X_{2t+1}$$

(A15)
$$\Pi_t^{1-\epsilon_p} = (1-\theta_p)(\Pi_t^*)^{1-\epsilon_p} + \theta_p \Pi_{t-1}^{\iota_p(1-\epsilon_p)}$$

(A16)
$$d_t = \Pi_{tp}^{\epsilon_p} \left[(1 - \theta_p) (\Pi_t^*)^{-\epsilon_p} + \theta_p \Pi_{t-1}^{-\iota_p \epsilon_p} d_{t-1} \right]$$

(A17)
$$d_t^w = (1 - \theta_w) \left(\frac{W_t^*}{W_t}\right)^{-\epsilon_w(1+\eta)} + \theta_w \left(\frac{W_{t-1}}{W_t}\right)^{-\epsilon_w(1+\eta)} \left(\frac{\Pi_t}{\Pi_{t-1}^{\iota_w}}\right)^{\epsilon_w(1+\eta)} d_{t-1}^w$$

$$(A18) C_t + I_t = Y_t$$

$$(A19) Y_t = \frac{A_t K_t^{\alpha} L_t^{1-\alpha}}{d_t}$$

(A20)
$$K_t = (1 - \delta) K_{t-1} + \mu_t \left[1 - S\left(\frac{I_t}{I_{t-1}}\right) \right] I_t$$

$$(A21) P_t^k \mu_t \left[1 - S\left(\frac{I_t}{I_{t-1}}\right) - \frac{I_t}{I_{t-1}} S'\left(\frac{I_t}{I_{t-1}}\right) \right] = 1 - \frac{\beta \Lambda_{t+1}}{\Lambda_t} P_{t+1}^k \mu_{t+1} \left(\frac{I_{t+1}}{I_t}\right)^2 S'\left(\frac{I_{t+1}}{I_t}\right)$$

$$(A22) \overline{B}_t + \overline{F}_t \leq N_t L_t$$

(A23)
$$L_{t} = \frac{E_{t} \frac{\Lambda_{t+1}}{\Pi_{t+1}}}{\left[E_{t} \frac{\Lambda_{t+1}}{\Pi_{t+1}} + (\Phi_{t} - 1) E_{t} \frac{\Lambda_{t+1}}{\Pi_{t+1}} \left(\frac{R_{t+1}^{L}}{R_{t}^{d}}\right)\right]}$$

$$(A24) P_t^k I_t \le \overline{F}_t - \kappa \frac{\overline{F}_{t-1}}{\Pi_t} \frac{Q_t}{Q_{t-1}}$$

(A25)
$$\Lambda_{t}[1 + N_{t}f'(N_{t}) + f(N_{t})] = E_{t}\beta\zeta\Lambda_{t+1}\frac{P_{t}}{P_{t+1}}[(R_{t+1}^{L} - R_{t}^{d})L_{t} + R_{t}^{d}]$$

(A26)
$$R_{t+1}^{L} = \frac{1 + \kappa Q_{t+1}}{Q_{t}}$$

(A27)
$$R_t^{10} = Q_t^{-1} + \kappa,$$

where

$$f(N_t) \equiv \frac{\psi_n}{2} \left(\frac{N_t - N_{ss}}{N_{ss}}\right)^2$$
 $S\left(\frac{I_t}{I_{t-1}}\right) \equiv \frac{\psi_i}{2} \left(\frac{I_t}{I_{t-1}} - 1\right)^2$ $\overline{B}_t \equiv Q_t \frac{B_t}{P_t}$ $\overline{F}_t \equiv Q_t \frac{F_t}{P_t}$.

B. Steady State

We choose B so that $H_{ss}=1$. We also normalize $\mu_{ss}=A_{ss}=1$.

$$\Lambda_{ss} = \frac{(1 - \beta h)}{(1 - h) C_{ss}}$$

$$1 = \beta R_{ss}$$

$$B = W_{ss}\Lambda_{ss}$$

$$R_{ss}^{k} = \frac{M_{ss}[1 - \beta(1 - \delta)]}{\beta}$$

$$M_{ss} = \frac{\beta}{\left(1 - \beta \kappa\right) Q_{ss}^{I}}$$

$$R_{ss}^k = MC_{ss}MPK_{ss}$$

$$W_{ss} = MC_{ss}MPL_{ss}$$

$$1 = \frac{\epsilon_p}{\epsilon_p - 1} \frac{X_{1ss}}{X_{2ss}}$$

$$X_{1ss} = \frac{MC_{ss}\Lambda_{ss}Y_{ss}}{1 - \beta\theta_p}$$

$$X_{2ss} = \frac{\Lambda_{ss} Y_{ss}}{1 - \beta \theta_p}$$

$$d_{ss} = 1$$

$$G_{1ss} = rac{W_{ss}^{\epsilon_w(1+\eta)}BH_{ss}^{1+\eta}}{1-eta heta_w}$$

$$G_{2ss} = \frac{\Lambda_{ss} W_{ss}^{\epsilon_w} H_{ss}}{1 - \beta \theta_w}$$

$$d_{ss}^{w} = 1$$

$$W_{ss} = \frac{\epsilon_w}{\epsilon_w - 1} \frac{BH_{ss}^{\eta}}{\Lambda_{ss}}$$

$$C_{ss} + I_{ss} = Y_{ss}$$

$$Y_{ss} = K_{ss}^{\alpha}$$

$$\delta K_{ss} = I_{ss}$$

$$P_{ss}^k = 1$$

$$\overline{B}_{ss} + \overline{F}_{ss} = N_{ss} \left[1 + (\Phi_{ss} - 1) \left(\frac{R_{ss}^L}{R_{ss}} \right) \right]^{-1}$$

$$I_{ss} = \overline{F}_{ss} (1 - \kappa)$$

$$1 = \beta \zeta R_{ss}^L$$

$$Q = \left(R_{ss}^L - \kappa \right)^{-1}$$

$$R_{ss}^{10} = R_{ss}^L$$

Some simplifications:

$$M_{ss} = rac{\left(eta R_{ss}^L - eta \kappa
ight)}{\left(1 - eta \kappa
ight)} > 1$$
 $MC_{ss} = rac{\epsilon_p - 1}{\epsilon_p} < 1$
 $rac{K_{ss}}{Y_{ss}} = rac{eta lpha MC_{ss}}{M_{ss} [1 - eta (1 - \delta)]}$
 $K_{ss} = \left(rac{K_{ss}}{Y_{ss}}
ight)^{1/(1 - lpha)}$
 $C_{ss} = Y_{ss} - \delta K_{ss}$

C. Calibration

$$\beta = 0.99, \quad \beta R_{ss}^L = 1.0025, \quad L_{ss} = 6.$$

$$(1 - \Phi_{ss})(1.0025) = \frac{L_{ss} - 1}{L_{ss}}$$

Duration =
$$40 = (1 - \kappa)^{-1}$$

$$\frac{\overline{B}_{ss}}{\overline{B}_{ss} + \overline{F}_{ss}} = 40\%.$$

D. Details of the FI's Value Function

The FI's problem is given by

(A28)
$$V_{t} \equiv \max_{N_{t}, div_{t}} E_{t} \sum_{i=0}^{\infty} (\beta \zeta)^{j} \Lambda_{t+j} div_{t+j}$$

(A29)
$$div_t + N_t[1 + f(N_t)] \leq X_t N_{t-1},$$

where

(A30)
$$X_{t} \equiv \frac{P_{t-1}}{P_{t}} [(R_{t}^{L} - R_{t-1}^{d}) L_{t-1} + R_{t-1}^{d}].$$

The function $f(N_t) \equiv \frac{\psi_n}{2} \left(\frac{N_t - N_{ss}}{N_{ss}}\right)^2$, denotes an adjustment cost function that dampens the ability of the FI to adjust the size of its portfolio in response to shocks. We assume that leverage is given exogenously to the FI. We will return to this below. Assuming an interior solution, the FI's accumulation choice is given by

(A31)
$$[N_t f'(N_t) + f(N_t)] = \frac{E_t \beta \zeta \Lambda_{t+1} X_{t+1} - \Lambda_t}{\Lambda_t}.$$

This implies that N_t is a function only of the forecasted market spread:

(A32)
$$N_t = h(z_t), \text{ where } z_t \equiv \frac{E_t \beta \zeta \Lambda_{t+1} X_{t+1} - \Lambda_t}{\Lambda_t}.$$

Note that $N_{ss} = h(0)$. We now conjecture the form of the value function:

$$(A33) V_t = \Lambda_t X_t N_{t-1} + g_t,$$

where g_t is independent of net worth. We check this conjecture by putting it into the Bellman equation. The Bellman equation is given by

(A34)
$$V_t = \Lambda_t X_t N_{t-1} - \Lambda_t N_t \left[1 + f(N_t) \right] + \beta \zeta E_t V_{t+1}.$$

Substituting in the conjectured value function, we have

(A35)
$$g_{t} = -\Lambda_{t} N_{t} [1 + f(N_{t})] + \beta \zeta E_{t} (\Lambda_{t+1} X_{t+1} N_{t} + g_{t+1}).$$

Using (A31), we have

(A36)
$$g_t = \Lambda_t N_t^2 f'(N_t) + \beta \zeta E_t g_{t+1}.$$

Using (A32), we have

(A37)
$$g_t = \Lambda_t [h(z_t)]^2 f'[h(z_t)] + \beta \zeta E_t g_{t+1},$$

or

(A38)
$$g_t = E_t \sum_{j=0}^{\infty} (\beta \zeta)^j \Lambda_{t+j} [h(z_{t+j})]^2 f'[h(z_{t+j})].$$

Hence, g_t is a function of the current and forecasted market spreads z_t , independent of N_{t-1} , and satisfies $g_{ss} = 0$.

Let us now turn to leverage. We assumed that leverage was exogenous to the FI. This means that the FI cannot alter his leverage level by acquiring more net worth (the only FI-specific variable). The hold-up constraint is given by

(A39)
$$E_{t}\Lambda_{t+1}X_{t+1}N_{t}^{i} + E_{t}g_{t+1} \geq \Psi_{t}^{i}L_{t}N_{t}^{i}E_{t}\Lambda_{t+1}\frac{P_{t}}{P_{t+1}}R_{t+1}^{L}.$$

Note that we have introduced a FI-specific index i on net worth. We will calibrate the model so (A12) is binding in the steady state (and thus binding for small shocks around the steady state). We need to show that leverage does not depend upon individual net worth. Since the value function includes a linear term, we need to reverse-engineer the hold-up function Ψ_t^i . This is straightforward. Let us assume that the hold-up function is given by

$$(A40) \quad \Psi_t^i \equiv \Phi_t \left[1 + \frac{1}{N_t^i} \left(\frac{E_t g_{t+1}}{E_t \Lambda_{t+1} X_{t+1}} \right) \right] = \Phi_t \frac{E_t \Lambda_{t+1} X_{t+1} N_t^i + E_t g_{t+1}}{E_t \Lambda_{t+1} X_{t+1} N_t^i}.$$

Assumption (A13) implies that the binding incentive constraint (A12) is given by

(A41)
$$E_t \Lambda_{t+1} \frac{P_t}{P_{t+1}} \left[\left(\frac{R_{t+1}^L}{R_t^d} - 1 \right) L_t + 1 \right] = \Phi_t L_t E_t \Lambda_{t+1} \frac{P_t}{P_{t+1}} \frac{R_{t+1}^L}{R_t^d}.$$

As planned, leverage is now a function only of aggregate variables that are outside the control of the FI. That is, leverage is independent of individual net worth. Expression (A14) is the one used in the paper.

To develop intuition for (A13), suppose that $E_t g_{t+1} > 0$. If the hold-up function were a constant $\Psi^i_t = \Psi$, then (A12) implies that leverage would be a decreasing function of net worth (because $\frac{E_t V^i_{t+1}}{N^i_t}$ is decreasing in net worth). To counter this effect, assumption (A13) implies that the hold-up problem becomes less severe with higher levels of net worth.

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