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A Model of Secular Stagnation: Theory and Quantitative Evaluation†

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This paper formalizes and quantifies the secular stagnation hypothesis, defined as a persistently low or negative natural rate of interest leading to a chronically binding zero lower bound (ZLB). Output-inflation dynamics and policy prescriptions are fundamentally different from those in the standard New Keynesian framework. Using a 56-period quantitative life cycle model, a standard calibration to US data delivers a natural rate ranging from −1.5 percent to −2 percent, implying an elevated risk of ZLB episodes for the foreseeable future. We decompose the contribution of demographic and technological factors to the decline in interest rates since 1970 and quantify changes required to restore higher rates. (*JEL* E12, E23, E31, E32, E43, E52)

The zero lower bound (ZLB) on the short-term nominal interest rate became a binding constraint in the United States in 2008 in the midst of the financial crisis. Accordingly, low interest rates are often tied to that event, and it may seem natural to presume that as the crisis of 2008 moves into the rear view mirror so too will the low interest rate environment. This perspective seems vindicated by the recent interest rate increases by the Federal Reserve in the fourth quarter of 2016. However, as shown in Figure 1, the low interest rate in 2008 was not just an anomaly that arose solely because of the financial crisis. Instead, it is the culmination of a 25-year trend across major industrial economies. In Japan, rates have been zero since the mid-1990s and remain there today. Furthermore, while recent increases in the Federal Funds rate may give rise to optimism in the United States, rates remain at zero in Europe as of this writing. The fact that rates remain so low should raise

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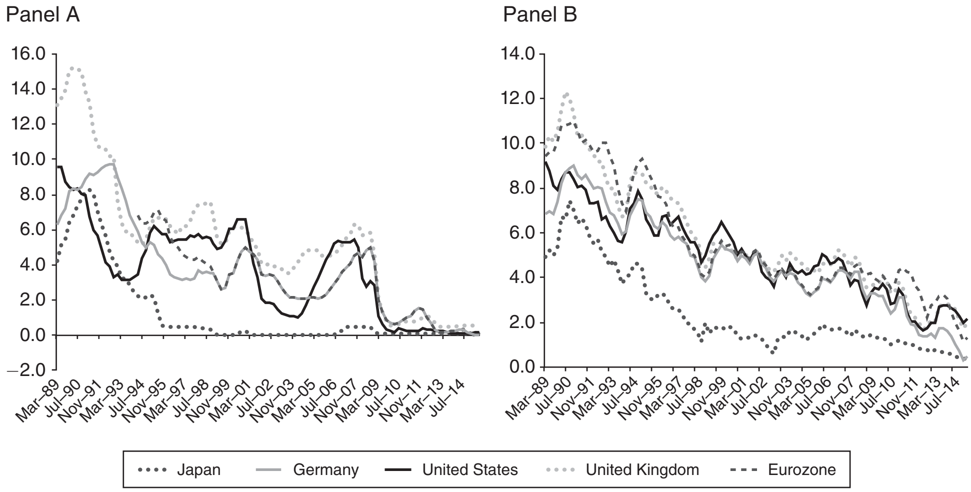

Figure 1. Short- and Long-Term Interest Rates

concern that the ZLB will remain a significant constraint on US monetary policy in the future should economic conditions deteriorate.

This paper contemplates the possibility that the fall in interest rates observed over the past 25 years in the industrialized world represents a permanent change—a "new normal." We propose a model that allows for ZLB episodes of arbitrary duration due to a persistent fall in interest rates (as observed in Figure 1) driven by slow-moving secular forces that are unlikely to reverse themselves. We also perform a quantitative assessment to ascertain the plausibility of a decline in the natural (full employment) interest rate to negative values using a quantitative life cycle model calibrated to current US data.

This paper should be read as a formalization of the secular stagnation hypothesis, originally proposed by Hansen (1939), who, a decade into the Great Depression with interest rates still at zero, argued that the US economy faced a permanent shortfall in aggregate demand. This idea was recently resurrected by Lawrence Summers, who argued that one should think of secular stagnation as the hypothesis that the natural rate of interest (the equilibrium real interest rate consistent with output at potential) is permanently negative (see Summers 2013, 2014). We formalize this idea by constructing a series of analytic and quantitative overlapping generation models (OLG) of varying degrees of complexity in which the steady-state, full-employment, real interest rate is permanently negative. This leads under certain conditions to a chronically binding ZLB, subpar growth, and inflation below target; we define these characteristics as a secular stagnation.

The policy implications of our framework differ in important ways from the standard ZLB literature. Perhaps most importantly, a policy of simply waiting for a ZLB episode to end is not a good strategy in a secular stagnation; there is no deus ex machina for recovery as presumed in the existing literature (see Krugman 1998 and Eggertsson and Woodford 2003, where the exogenous shocks that give rise to the ZLB must ultimately revert). The absence of an automatic recovery has fundamental implications for monetary policy. Unlike in Eggertsson and Woodford (2003), forward guidance loses most of its power, in part, because agents do not anticipate a date at which the ZLB ceases to be binding. Raising the inflation target may be an option to accommodate a negative natural rate of interest. This policy, however, suffers from two major drawbacks. First, an increase in the inflation target must be sufficiently large. For example, if the natural rate of interest is -4 percent, the inflation target must be 4 percent or higher. Small changes in the inflation target have no effect, capturing Krugman's observation of the "law of the excluded middle" or "timidity trap" when trying to explain why the Japanese economy might not respond to a higher inflation target announced by the Bank of Japan unless it was sufficiently aggressive (see Krugman 2014). Second, even with a large enough increase in the inflation target, the secular stagnation equilibrium is not eliminated. Hence, an increase in the inflation target *allows* for a better outcome, but it does not guarantee it because it cannot *exclude* a secular stagnation equilibrium.[1]

Fiscal policy is more effective in addressing the problems raised by secular stagnation, and an aggressive enough fiscal expansion does not suffer from multiplicity of steady states—it eliminates the secular stagnation steady state altogether. However, the effects of fiscal policy are more subtle than in the standard New Keynesian treatment of the ZLB. Increases in government spending can carry zero or negative multipliers in our model, depending on the distribution of taxes across generations. The key for successful fiscal policy is that it must reduce the oversupply of savings and raise the natural rate of interest. Fiscal policy that instead increases desired savings by, for example, reducing future disposable income through tax increases, can exacerbate a secular stagnation. In the standard NK model, government spending multipliers are above one, irrespective of the financing mechanism, and may be unboundedly large for reasons related to the forward guidance puzzle. These mechanisms are considerably attenuated in our framework.[2]

In addition to allowing for the possibility of arbitrarily long periods of negative interest rates, our model accounts for and can quantify a host of new forces that affect the natural rate of interest. These forces come naturally into play in our analysis since we abandon the representative agent framework of the standard NK model. Essentially any force that alters the relative supply of savings and investment can have an effect on the interest rate. We show how a slowdown in population growth or an increase in life expectancy puts downward pressure on the natural rate. Rising income inequality or a fall in the relative price of investment goods may also reduce the natural rate. A slowdown in productivity growth can also play an important role.[3]

Many existing narratives of the Great Recession in the United States have emphasized the effect of debt deleveraging caused by the housing crisis on

[1] In fact, theory provides no obvious way to determine whether the high or low steady state will be chosen because they both pass standard equilibrium selection tests, such as local determinacy.

[2] The reason ZLB in our model does not suffer from local indeterminacy, explosive deflations, or unboundedly large fiscal multipliers is subtle and is not only about finite lifetimes. The presence of debt-constrained households and/or fixed public debt and government spending ensures that an aggregate Euler equation of the type seen in representative agent models does not emerge. See Werning (2015), Proposition 2 for a discussion of conditions under which an aggregate Euler equation emerges.

[3] It should be noted that the elasticity of the interest rate to changes in productivity growth differs in an OLG setting relative to the standard representative agent model.

restraining aggregate demand and causing the ZLB to bind (see, for example, Mian and Sufi 2014; Eggertsson and Krugman 2012; Guerrieri and Lorenzoni 2011; and Mehrotra 2018). For this reason, we incorporate financial frictions in our baseline model. In earlier models, even a permanent shock that tightens credit to financially constrained households results in only a temporary ZLB episode. Once the deleveraging cycle runs its course, the natural rate of interest rises, aggregate demand rises, and the ZLB episode ends. In stark contrast, in our model a deleveraging cycle need not culminate in a rise in the natural rate, and there is no guarantee of a recovery once deleveraging ends. Indeed, our model predicts that, as the deleveraging cycle runs its course, interest rates continue to fall as the young borrowers who transition to saving in middle age have greater disposable income. Incorporating financial frictions in our model also gives some structure to the idea that secular stagnation conditions may have existed prior to 2008, but were masked by the tech bubble in the late 1990s and the subsequent housing bubble in the early 2000s, as suggested by Summers (2014).

While the first main contribution of this paper is an analytic framework that lays out the theoretical ingredients needed to characterize secular stagnation, the second main contribution is building a medium-scale life cycle model to explore whether persistently negative natural interest rates are quantitatively realistic. We build a 56-period OLG model with capital and calibrate it to match the US economy in 2015, assuming that the output gap at that time is either (i) zero, or (ii) corresponds to the deviation of output from its pre-2008 trend–two natural (and extreme) benchmarks. In our calibrated model, this gives us a range for the natural rate of interest from -1.47 percent to -2.20 percent.

Our quantitative model also includes a novel way of generating an economy with a negative interest rate that is also dynamically efficient. In OLG models, negative interest rates are generally associated with dynamic inefficiency, which in turn can lead to some undesirable properties, such as the existence of rational bubbles (see, e.g., Tirole 1985). We break this link by including markups in the model, which creates a wedge between the marginal product of capital and the interest rate. Thus in our economy the return on capital is high enough that it produces returns in excess of investment in the steady state, while the interest rate remains negative. The link between negative interest rates and dynamic inefficiency has been broken once before. Abel et al. (1989) show that including a risk premium on capital can lead to an economy in which the expected return on capital in excess of investment is positive, while the risk-free interest rate is, on average, negative. In that case the wedge between the marginal product of capital and the risk-free rate was the risk premium.[4]

Our quantitative model is able to generate a permanently *negative* natural rate of interest using parameters that are standard in the macro literature and match key moments from the US data. The main drivers of negative natural interest rates are an aging population, low fertility, and sluggish productivity growth. While this

[4] For a full discussion of dynamic efficiency in our model, see the conclusion. We suspect that if real interest rates remain negative in Europe and Japan it will become increasingly important to solve some of the technical problems that can arise when rates are permanently negative.

trend may reverse itself, if current projections for fertility and productivity hold, our analysis suggests that the natural rate of interest will be low or negative for the foreseeable future.[5] Though productivity growth has experienced unexpected periods of acceleration and deceleration since the 1970s, the demographic factors accounting for a low natural rate of interest are unlikely to abate.

We use our model to understand the decline in interest rates seen in the data. We take our 2015 calibration and revert observable demographic and productivity factors to their 1970s value. Over this period, our model generates a 4.02 percent decrease in the real interest rate from 1970 to 2015, which matches the actual decrease of 4.09 percent experienced in the real Federal Funds rate over that period. The reductions in fertility, mortality, and the rate of productivity growth play the largest role; each alone can account for a fall in the real interest rate of -1.84 percent, -1.92 percent, and -1.90 percent, respectively. The main factor that has tended to counterbalance these forces is an increase in government debt, which accounts for a 2.11 percent increase in the real interest rate. Changes in the labor share, the relative price of investment goods, and variation in consumer debt capacity play a quantitatively smaller role (the first two decrease rates by -0.50 percent and -0.44 percent, while the last raises rates by 0.13 percent).

We also evaluate quantitatively the assumptions under which one should expect real interest rates to revert to a more normal level of 1 percent—the assumption maintained by current Federal Reserve projections. At a 1 percent steady-state real interest rate, the ZLB is much less likely to pose a problem for business cycle stabilization (see, for example, Williams 2016). A key determinant of whether interest rates are likely to increase is whether the rate of productivity growth, which has slowed markedly since the 1970s, returns to its long-run rate of 2 percent per year. This experiment makes clear that the lively debate between Robert Gordon and others about the likely evolution of productivity is crucial in determining whether secular stagnation will remain a problem. Gordon (2012) takes a very pessimistic view of the evolution of future productivity, while Brynjolfsson and McAfee (2014), for example, take a more optimistic view. Our simulations suggest that the stakes are high in that debate for the future conduct of monetary policy, as it may determine the extent to which the ZLB remains an issue for macroeconomic stabilization.

Our paper is organized as follows. In Section I, we relate the paper to the existing literature. In Section II, we present our baseline model, starting with a simple endowment economy to explore interest rate determination in an OLG model with no nominal frictions. Section III adds price level determination. Sections IV and V incorporate nominal rigidities and show how negative real interest rates can create a demand recession. Section VI evaluates monetary and fiscal policy responses in a secular stagnation. Section VII presents a 56-generation life cycle model with capital accumulation and borrowing constraints to allow for a quantitative evaluation of the secular stagnation hypothesis.

[5] Of course, a negative natural rate of interest in steady state does not exclude the possibility that we may see a short-term rise in the nominal interest rate due to temporary business cycle factors. Instead, it suggests that there are plausible conditions under which one should expect recurrent and chronic ZLB episodes going forward that can be of arbitrary duration.

I. Related Literature

It may seem somewhat surprising that the idea of secular stagnation has not already been studied in detail in the relatively large literature on the liquidity trap. This literature already invites the possibility that the zero bound on the nominal interest rate is binding for some period of time due to a drop in the natural rate of interest. The reason for this omission, we suspect, is that secular stagnation does not emerge naturally from the current vintage of models in use in the literature. This, however, and perhaps unfortunately, has less to do with economic reality than with the limitations of these models. Most analyses of the current crisis take place within representative agent models (see, e.g., Krugman 1998; Eggertsson and Woodford 2003; Christiano, Eichenbaum, and Rebelo 2011; and Werning 2012; for a few well-known examples) where the long-run real interest rate is directly determined by the inverse of the discount factor of the representative agent. Zero lower bound episodes are caused by temporary shocks to the discount factor triggering temporary reductions in the real interest rate that eventually must revert back to a positive long-run level.[6] The second generation of these models, as in Eggertsson and Krugman (2012), that more explicitly incorporate financial frictions and deleveraging cannot generate permanently negative rates either, since the steady-state real interest rate is tied to the discount rate of the representative saver.

Our model differs fundamentally from this earlier generation of models. As has been understood since Samuelson (1958), with an OLG structure the discount factor of a representative saver is no longer the sole determinant of the natural rate of interest, and a negative natural rate that lasts for an arbitrarily long time is now a possibility.[7] Our contribution is to introduce the zero lower bound in a framework where the natural rate of interest is influenced by a broader set of factors than in the typical infinite-horizon model. As we argue, these differences are nontrivial for thinking about monetary and fiscal policy at the zero lower bound. Furthermore, an OLG structure allows for a richer set of factors to influence the real interest rate, which is essential for understanding the sources and potential persistence of the decline in the natural rate.

A recent literature has emphasized that existing New Keynesian models of the zero lower bound tend to generate inflation and output dynamics at odds with recent ZLB episodes; NK models predict a sharp collapse in output and deflationary spirals for particularly long-lasting ZLB episodes (see, e.g., Cochrane 2016). By contrast, our model does not suffer from these counterfactual dynamics. In a secular stagnation steady state, inflation is persistently below target and output falls below trend, which matches the observed dynamics of output, inflation, and interest rates in the United States, Eurozone, and Japan. Similarly, our model also does not suffer from the "forward guidance puzzle," the idea that interest

[6] A permanent shock to the household's discount factor is not possible since the maximization problem of the representative household is no longer well defined. One possible alternative in the representative agent framework is a rise in uncertainty large enough to make the risk-free rate negative, as in Abel et al. (1989).

[7] For a more recent reference that provides a good overview of the results in the literature, see Blanchard and Weil (2001).

rate changes very far in the future have implausibly large effects on output and inflation today, as discussed in Del Negro, Giannoni, and Patterson (2012) and McKay, Nakamura, and Steinsson (2016). The IS equation displays discounting in the Euler equation and, when combined with debt-constrained agents, our model does not admit an aggregate Euler equation representation of the form discussed in Werning (2015).

A literature related to our work that explores the deflation steady state in a standard NK model is found in Benhabib, Schmitt-Grohé, and Uríbe (2001); Schmitt-Grohé and Uríbe (2017); and Benigno and Fornaro (2015). A key difference to this strand of the literature is that, in these papers, the ZLB is binding due to self-fulfilling expectations rather than the fundamental factors we consider here. Moreover, the ZLB steady state in these models is locally indeterminate. In contrast, the secular stagnation equilibrium in our model satisfies normal determinacy conditions. This allows us to consider well-defined comparative statics, and our steady state is immune from criticism as in Christiano, Eichenbaum, and Johannsen (2016), who argue that indeterminate ZLB episodes driven by self-fulfilling expectations can be ruled out on the grounds that they are not "learnable," and, hence, it is unclear how expectations can coordinate on this steady state (see Gibbs 2017).

Other closely related work includes Kocherlakota (2013), who also considers a permanent liquidity trap. The focus in that paper is on falling land prices as a trigger for the crisis, while our focus is on the forces usually associated with secular stagnation: population dynamics, income inequality, a decline in the relative price of investment goods, and a debt deleveraging shock. In the context of the Japanese crisis, Krugman (1998) suggested that population dynamics might be driving some of the decline in the natural rate of interest, although he did not explore this possibility explicitly. Carvalho and Ferrero (2014) quantify this force in a medium-scale DSGE model in the Japanese context and argue that demographic pressures were a significant contributor. Other recent contributions in the same vein, but focusing on US aging, are Gagnon, Johannsen, and Lopez-Salido (2016) and Jones (2016).

Alternatively, Caballero and Farhi (2014) characterize a long-lasting ZLB episode resulting from a scarcity of safe assets. This work is more closely focused on explaining falling rates during the Great Recession as opposed to long-term interest rate trends, the focus of the secular stagnation hypothesis. It is worth stressing, however, that the safety trap mechanism is not inconsistent with the forces we emphasize as driving secular stagnation. Indeed, building on an earlier version of this paper, Caballero and Farhi (2014) explicitly incorporate the supply side of our model and show that many of the same policy conclusions apply.

In separate work, the simple model in this paper is extended to an open economy setting (Eggertsson et al. 2016), where the open economy dimensions of secular stagnation are analyzed. Open economy factors are particularly relevant when thinking about the global savings glut identified in Bernanke (2005) as a source of low interest rates in the early 1990s, as well as in understanding policy spillovers from current account imbalances and monetary/fiscal policy interactions across countries in a global secular stagnation (see also Coeurdacier, Guibaud, and Jin 2015 for the role of demographics and low interest rates in a global context).

II. Endowment Economy

We start by considering a simple overlapping-generations economy to analyze the determination of interest rates in an OLG setting. Households live for three periods. They are born in period 1 (young), enter middle age in period 2 (middle age), and retire in period 3 (old). Consider the case in which no aggregate saving is feasible (that is, there is no physical capital) but that generations can borrow and lend to one another. Moreover, imagine that only the middle-aged and old generations receive any income in the form of an endowment: Y_t^m and Y_t^o. In this case, the young will borrow from the middle-aged households, which, in turn, will save for retirement. The old will not save, but will fully consume their remaining income and assets. We assume, however, that there is a limit on the amount of debt the young can borrow. Generally, we would like to think of this as reflecting some sort of incentive constraint, but for the purposes of this paper it will simply take the form of an exogenous time-varying constant D_t (as in the "debtors" in Eggertsson and Krugman 2012).

More concretely, consider the representative household of a cohort born at time t. This household has the following utility function:

$$\max_{C_t^y,\, C_{t+1}^m,\, C_{t+2}^o} E_t\left\{\log\left(C_t^y\right) + \beta \log\left(C_{t+1}^m\right) + \beta^2 \log\left(C_{t+2}^o\right)\right\},$$

where C_t^y is the consumption of the household when young, C_{t+1}^m its consumption when middle aged, and C_{t+2}^o its consumption while old. We assume that borrowing and lending take place via one-period riskless bonds denoted B_t^i, where $i = y, m, o$ at an interest rate r_t. Given this structure, we can write the budget constraints facing households born at time t in each period as

$$C_t^y = B_t^y, \tag{1}$$

$$C_{t+1}^m = Y_{t+1}^m - (1 + r_t) B_t^y + B_{t+1}^m, \tag{2}$$

$$C_{t+2}^o = Y_{t+2}^o - (1 + r_{t+1}) B_{t+1}^m, \tag{3}$$

$$(1 + r_t) B_t^i \leq D_t, \tag{4}$$

where equation (1) corresponds to the budget constraint for the young, where consumption is financed by borrowing. Equation (2) gives the budget constraint of the middle-aged household that receives the endowment Y_t^m, repays what was borrowed, and borrows B_{t+1}^m (equivalently, saves $-B_{t+1}^m$ for retirement). Finally, equation (3) corresponds to the budget constraint when the household is old, consuming savings and interest and any endowment received in the last period.[8]

[8] The addition of a warm-glow bequest motive would not affect our qualitative results because the steady-state real interest may remain negative. Thwaites (2015) shows that bequests do not materially impact determination of real interest rates in an OLG setting. However, in the full life-cycle model, we include an explicit bequest motive.

The inequality equation (4) corresponds to the exogenous borrowing limit (as in Eggertsson and Krugman 2012). We assume that the collateral constraint is tight enough so that it binds for the young:[9]

$$C_t^y = B_t^y = \frac{D_t}{1+r_t}, \tag{5}$$

where the amount of debt that the young can borrow depends on their ability to repay in the middle period and, therefore, includes interest payments. As a result, a drop in the real interest rate increases borrowing by the young.

The old at any time t are hand-to-mouth and will consume their income:

$$C_t^o = Y_t^o - (1+r_{t-1})\, B_{t-1}^m. \tag{6}$$

The middle-aged, however, are at an interior solution, and their consumption-saving choices satisfy a standard Euler equation given by

$$\frac{1}{C_t^m} = \beta E_t \frac{1+r_t}{C_{t+1}^o}. \tag{7}$$

We assume that the size of each generation is given by N_t. Let us define the growth rate of the new cohort by $1+g_t = N_t/N_{t-1}$. Equilibrium in the bond market requires that borrowing of the young equals the savings of the middle-aged, so that $N_t B_t^y = -N_{t-1} B_t^m$ or

$$(1+g_t)B_t^y = -B_t^m. \tag{8}$$

An equilibrium is now defined as a set of stochastic processes $\{C_t^y, C_t^o, C_t^m, r_t, B_t^y, B_t^m\}$ that solve equations (1), (2), (5), (6), (7), and (8) given an exogenous process for $\{D_t, g_t\}$. All variables are normalized by the size of the middle generation population.

To analyze equilibrium determination, let us focus on equilibrium in the market for savings and loans given by equation (8) using the notation L_t^s and L_t^d; the left-hand side of equation (8) denotes the demand for loans, L_t^d, and the right-hand side its supply, L_t^s. Hence, the demand for loans (using equation (5)) can be written as

$$L_t^d = \frac{1+g_t}{1+r_t} D_t, \tag{9}$$

[9] For the constraint to be binding, it must be the case that $\frac{Y_t^m + Y_{t+1}^o/(1+r_t)}{1+\beta(1+\beta)} > D_{t-1}$. We check in our numerical experiments that the relevant condition is satisfied.

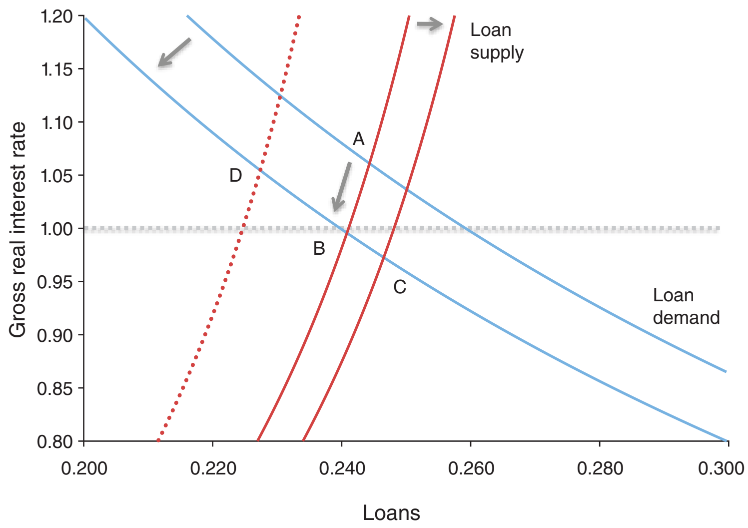

Figure 2. Equilibrium in the Asset Market

while an expression for loan supply, assuming perfect foresight, can be derived by combining the household budget constraints and the middle generation Euler equation:

$$L_t^s = \frac{\beta}{1+\beta}(Y_t^m - D_{t-1}) - \frac{1}{1+\beta}\frac{Y_{t+1}^o}{1+r_t}. \tag{10}$$

The real interest rate depicted in Figure 2, is then determined by the intersection of the loan demand, L_t^d, and loan supply, L_t^s:

$$1 + r_t = \frac{1+\beta}{\beta}\frac{(1+g_t)D_t}{Y_t^m - D_{t-1}} + \frac{1}{\beta}\frac{Y_{t+1}^o}{Y_t^m - D_{t-1}}. \tag{11}$$

In contrast to the standard representative agent model, the real interest rate will now, in general, depend on a host of factors in addition to the discount factor: the income profile over the life cycle, the debt limit, and population growth all influence the real interest rate.

A. *Productivity, Population Growth, and Inequality*

In our OLG setting, any factor that affects the relative supply or demand for loans changes the interest rate. Unlike in the standard representative agent model used in business cycle analysis, these forces can have permanent effects on the interest rate, and we should expect these dynamics to play out over an extended period. Importantly, there is nothing that prevents the real interest rate in expression (2) from being either positive or negative. We offer a few examples of the forces that may affect the equilibrium interest rate; this list is far from exhaustive, and in the quantitative section we take a firmer stand on which of these forces best account for declining US interest rates.

Let us first consider a potential fall in total factor productivity growth that has been commonly associated with discussions of secular stagnation (this hypothesis is most forcefully articulated by Gordon 2015). For this exercise, we assume that the income of the middle-aged and the old is proportional to the aggregate endowment Y_t, which in turn is proportional to productivity such that $Y_t = A_t\tilde{Y}$. Moreover, as the debt limit reflects the extent to which the middle-aged agents can replay their debt, we assume that it grows with the middle-aged income so that the debt limit relevant to the young at time t is given by $D_t = A_{t+1}\tilde{D}$. The loan demand and supply can then be written in terms of renormalized variables, as shown.[10]

Consider now the effect of a slowdown in productivity. First, lower productivity growth shifts out the supply of savings since lower expected future income induces the middle-aged generation to increase retirement savings. Second, the expectation of lower future productivity tightens the borrowing constraint of the young, leading to a backward shift in the demand for loans. The new equilibrium is shown in point C in Figure 2, which now depicts the renormalized variables. It should be noted that, unlike in the representative agent model, it is not required for productivity growth to be negative in order for the real interest rate to become negative.[11] Instead, interest rates depend on how income is distributed over the life cycle, as well as on the interaction between productivity growth and the income distribution.

The mechanism by which a reduction in population growth lowers the interest rate is straightforward and can be seen directly by inspecting the expression for loan demand. As the number of young decreases relative to the middle-aged (a decline in g_t), loan demand falls, shifting back the L_t^d curve and lowering the real interest rate to point B in Figure 2. In the quantitative model, we will also consider changes in mortality risk, which have a relatively intuitive effect on the real interest rate. As mortality risk decreases, individuals save more for retirement, increasing the supply of savings and lowering the interest rate.

Our model can also be used to consider the impact of an increase in inequality on interest rates. Generically, the effect of an increase in inequality is ambiguous. There are plausible conditions, however, under which higher inequality will in fact reduce the natural rate of interest. We provide one such example in online Appendix B,

[10] The demand and supply for loans can now be written as

$$\tilde{L}_t^d = \frac{1+g_t}{1+r_t}\frac{A_t}{A_{t-1}}\tilde{D}, \tag{12}$$

$$\tilde{L}_t^s = \frac{\beta}{1+\beta}\left(\tilde{Y}^m - \tilde{D}\right) - \frac{1}{1+\beta}\frac{\tilde{Y}^o}{1+r_t}\frac{A_{t+1}}{A_t}, \tag{13}$$

where *tilde* denotes that the original variable has been divided by productivity.

[11] In the representative household model (without population growth) the real interest rate is given by

$$1 + r_t = \beta^{-1}\frac{A_{t+1}}{A_t},$$

so productivity would need to decline at a rate greater than the inverse of the discount factor for the interest rate to be negative. While productivity growth is low in many advanced countries, it is not negative.

in which a fraction of the middle-aged population are credit constrained. In this case, shifting income from the low-skilled, credit-constrained households to the high-skilled, middle-aged households reduces the real interest rate. In general, the condition needed for inequality to reduce rates is that those with higher incomes at a given age save more than those with lower incomes.[12]

A fourth force often associated with secular stagnation is a persistent fall in the relative price of investment goods. As investment goods become cheaper, less savings is needed to finance a given level of investment in the capital stock, thereby reducing overall demand. Since our example does not include capital, we will defer this discussion to Section VII.

B. *Deleveraging*

One of the main narratives about the source of the 2008 crisis is that it was triggered by a debt deleveraging shock (see, e.g., Mian and Sufi 2014). Households took on excessive levels of mortgage debt in the mid-2000s, and the 2008 crisis corresponded to an abrupt correction whereby households were forced to pay down high levels of debt, often referred to as a "Minsky moment." Eggertsson and Krugman (2012—henceforth, EK) model this as permanent tightening in the household collateral constraint D_t. A key prediction of EK and related models is that once agents in the economy pay down their debt to the new sustainable level, the debt deleveraging ends and interest rates return to their precrisis level. However, in our framework, this prediction is overturned, and the end of a debt deleveraging cycle does not lead to a normalization of interest rates.

Point B in Figure 2 shows the equilibrium level of the real interest rate on impact after a deleveraging shock. As we can see, the shock leads directly to a reduction in the demand for loans since the demand curve shifts inward. The supply of loanable funds is unchanged since the debt repayment of the middle-aged generation depends on the lagged value of collateral constraint, D_{t-1}.

On impact, the young are now spending less at a given interest rate, while the middle-aged and old are spending the same. Since the endowment must be fully consumed in our economy, this fall in spending by the young then needs to be made up by inducing some agents to spend more. This adjustment takes place via a reduction in the real interest rate that induces more spending.[13] So far, the

[12] Most realistic models of bequests, for example, argue that the preference for leaving a bequest increases with income, implying an increase in bequests is associated with higher inequality. See Dynan, Skinner, and Zeldes (2004) for a discussion of other mechanisms, such as the persistence of skill types across generations. We have also experimented with a production structure where skill-biased technological change (as in Krusell et al. 2000) increases inequality and found plausible conditions under which skill-biased technical change puts downward pressure on the real interest rate. We leave a fuller analysis of this subject for future research.

[13] The drop in the real interest rate stimulates spending via two channels. First, as equation (11) shows, a fall in the real interest rate makes consumption today more attractive to the middle aged, thus increasing their spending. Observe that the L_t^s curve is upward sloping in the real interest rate; in general, the slope of the savings curve depends upon the elasticity of intertemporal substitution. If preferences exhibit very weak substitution effects and if the endowment is received only by the middle-aged generation, it is possible to have a downward sloping L_t^s curve. However, our view is that the empirically relevant case is when savings are increasing in the interest rate. Second, for the credit-constrained young generation, a reduction in the real interest rate relaxes their borrowing constraint. A lower interest rate allows them to take on more debt, B_t^y, for any given value of D_t. Since borrowing is limited by

mechanism described in our model is exactly the same as in EK. A deleveraging shock triggers a drop in spending by borrowers at the existing rate of interest. The real interest rate then falls to keep the aggregate level of spending the same.

In EK, the economy reaches a new steady state in the next period in which, once again, the real interest rate is determined by the discount factor of the representative savers in the economy. In that setting, the loan supply curve shifts back so that the real interest rate is exactly the same as before (as seen in point D). Loan supply shifts back in EK because borrower deleveraging reduces interest income accruing to savers, which implies that their supply of savings falls in equilibrium. In contrast, in our model there is no representative saver; instead, households are both borrowers and savers at different stages in their lives. The fall in the borrowing of young households in period t then implies that in the next period, when that agent becomes a saver, each household has more resources to save since disposable income is higher. At time $t+1$, the supply of savings L_t^s shifts outward as shown in Figure 2. In sharp contrast to EK, where the economy settles back into the old steady state after a one-period transition, the economy in our model reaches a new steady state with a permanently lower real rate of interest. The new interest rate may be negative, depending on the size of the shock. This process can serve as a powerful and persistent propagation mechanism for the original deleveraging shock. More generally, even if the drop in D_t is not permanent, the natural rate of interest will inherit the dynamics of the drop in D_t, which may be of arbitrary duration. One interesting implication of these dynamics is that policies that are implemented in the midst of a deleveraging crisis, such as macroprudential policies, run the risk of permanently depressing the natural rate of interest.

III. Price Level Determination

We now introduce perfectly flexible nominal prices. We will show that this introduces a kink in the model: If the steady-state equilibrium interest rate is negative, then there is no equilibrium consistent with inflation below a certain level. For example, if the real interest rate is -3 percent, then inflation must be greater than 3 percent in a steady state that respects the 0 lower bound on nominal interest rates. This will have fundamental implications when we introduce realistic nominal frictions; the unwillingness or inability of the central bank to accommodate a high enough inflation rate will result in a demand shortfall and output contraction.

As is standard in the literature, we introduce a nominal price level by assuming that one-period nominal debt denominated in money is traded and that the government controls the nominal rate of return on this asset.[14] The saver in our economy (the middle generation household) now has access to risk-free nominal debt that is

their ability to repay in the next period, changes in the interest rate that affect the size of the total repayment affect borrowing today.

[14] There are various approaches to microfounding a demand for money by using money in the utility function or cash-in-advance constraints. For completeness, we explicitly add money in the utility function to our model in online Appendix F.

indexed in dollars, in addition to one-period, risk-free real debt.[15] This assumption gives rise to a consumption Euler equation, which is the nominal analog of the Euler equation (7):

$$\frac{1}{C_t^m} = \beta E_t \frac{1}{C_{t+1}^o}(1+i_t)\frac{P_t}{P_{t+1}}, \tag{14}$$

where i_t is the nominal rate and P_t is the price level. We impose a nonnegativity constraint on nominal rates. Implicitly, we assume that the existence of money precludes the possibility of a negative nominal rate. At all times,

$$i_t \geq 0. \tag{15}$$

Equations (7) and (14) imply (assuming perfect foresight) the standard Fisher relation

$$1 + r_t = (1+i_t)\frac{P_t}{P_{t+1}}, \tag{16}$$

where again the equilibrium real interest rate r_t is determined by equation (11). The Fisher equation simply states that the real interest rate should be equal to the nominal rate deflated by the growth rate of the price level (inflation).[16]

From equation (15) and equation (16), it follows that if the real rate of interest is permanently negative, then there is no equilibrium consistent with a stable price level. To see this, assume such an equilibrium and that $P_{t+1} = P_t = P^*$. Then the Fisher equation implies that $i_t = r_t < 0$, violating the 0 bound. Hence, a constant price level, price stability—cannot be sustained when r_t is negative.[17]

Let us denote the growth rate of the price level (inflation) by $\Pi_t = P_{t+1}/P_t = \bar{\Pi}$. The 0 bound and the Fisher equation then imply that, for an equilibrium with constant inflation to satisfy the ZLB, there is a bound on the inflation rate given by $\Pi(1+r) = 1 + i \geq 1$ or

$$\bar{\Pi} \geq \frac{1}{1+r}, \tag{17}$$

[15] For simplicity, we assume that this asset trades in zero net supply, so that, in equilibrium, the budget constraints already considered are unchanged. However, we relax this assumption once we incorporate fiscal policy. Strictly speaking, what is needed, once money is explicitly incorporated, as in online Appendix F, with explicit fiscal policy rules, is that net government nominal debt (inclusive of money) be in zero supply. As we show, our conclusions are unchanged when one considers a strictly positive level of government debt.

[16] Again, we can define an equilibrium as a collection of stochastic processes $\{C_t^y, C_t^o, C_t^m, r_t, i_t, B_t^y, B_t^m, P_t\}$ that solve equations (1), (2), (5), (6), (7), and (8) and now also (14) and (15) given an exogenous process for $\{D_t, g_t\}$ and a specification for monetary policy like an interest rate rule.

[17] For this result, it is important that we rule out bubbles and that fiscal policy keeps the real value of government liabilities constant. With passive fiscal policy and positive government liabilities, the price level could fall sharply on impact to increase the real value of debt and raise the natural rate of interest back to zero. After impact, the nominal rate will be at zero and the central bank will keep prices stable. This possibility will become more transparent once fiscal policy is explicitly introduced. We thank Jaume Ventura for pointing this out.

which implies that steady-state inflation is bounded from below by the real interest rate due to the zero bound. We term this the natural lower bound on inflation.

At a positive real interest rate, the natural bound is of little relevance. If, as is common in the literature using representative agent models, the real interest rate in steady state is equal to the inverse of the discount factor, then the natural bound says that $\Pi \geq \beta$. Under a standard calibration, this implies a bound on the steady-state inflation rate of about -2 percent to -4 percent. This is, of course, well below the inflation target of most central banks, making this bound of little empirical relevance.

With permanently negative real rates, however, this bound takes on a greater practical significance. If the real interest rate is negative, it implies that, under flexible prices, steady-state inflation must be positive. Suppose, for example, that the natural rate of interest is -4 percent in Europe; this implies the natural lower bound on inflation is 4 percent. But what happens if the European Central Bank (ECB) refuses to allow inflation to rise above 2 percent? With flexible prices, this possibility cannot be contemplated in the model; it does not admit an equilibrium. If the real interest rate is negative, the inflation rate *has to be positive*. The ECB must target inflation above 4 percent in the example above.

This is an unappealing result. A key question we want to ask our model is, what happens if the natural rate of interest is negative and the central bank does not provide enough inflation, causing the zero lower bound to become binding? So far, our model is not able to answer this question because there is simply no equilibrium under this scenario. A straightforward solution, and one that fits the data, is to employ nominal rigidities in the price system. Once we introduce nominal frictions, then the central bank can indeed set inflation below the natural lower bound on inflation. In this case, however, the refusal of the central bank to allow inflation to rise to or overshoot its target results in an output gap and the nominal interest rate stuck permanently at zero.

IV. Aggregate Supply

In this section, we incorporate nominal rigidities into our baseline model. The presence of nominal rigidities carries important implications for adjustment when the natural rate falls negative. We show that if the natural rate of interest is negative and the central bank does not set a sufficiently high inflation target, an employment and output gap emerges.

While it is common for models to feature short-term pricing rigidities, our model will incorporate a form of long-run rigidity. Since our modeling choice is uncommon, some initial discussion is warranted before we proceed to our exact specification. There is a broad consensus among economists that, with permanently high levels of inflation, expectations about future inflation will ultimately adjust so there is no long-run trade-off between inflation and unemployment. This empirical prediction was indeed largely borne out during the 1970s in the United States. Our model will incorporate this neutrality of high inflation in the form of a long-run vertical aggregate supply curve when inflation is sufficiently high (see the top half of the aggregate supply curve in Figure 3).

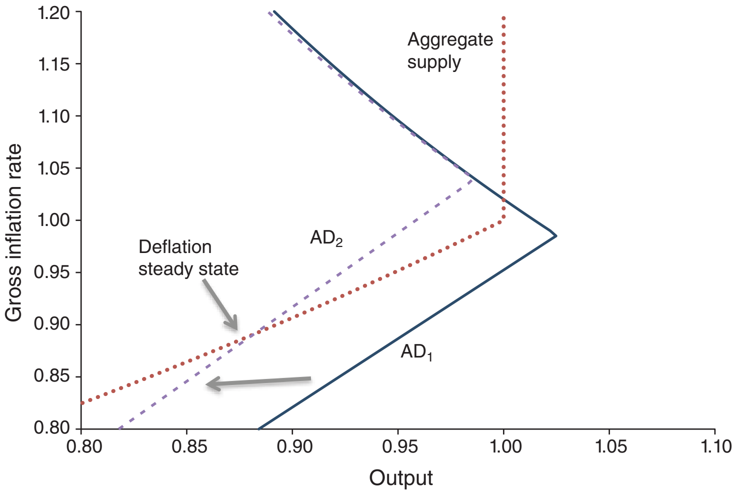

Figure 3. Steady-State Aggregate Demand and Aggregate Supply Curves

In contrast, a similar consensus has not emerged for long-run neutrality in a low inflation/deflation environment. Tobin (1972) argued that, during the Great Depression, firms were reluctant to cut nominal wages despite high unemployment. Tobin argued that this behavior suggests a permanent, long-run trade-off between inflation and unemployment at low levels of inflation. Others have built on and formalized Tobin's analysis in the form of an upward sloping long-run Phillips curve (see, for example, Akerlof, Dickens, and Perry 1996; Kim and Ruge-Murcia 2009; Fagan and Messina 2009; Benigno and Ricci 2011; Coibion, Gorodnichenko, and Wieland 2012; and Daly and Hobijn 2014). We incorporate this notion of aggregate supply in our model, leading to an upward sloping Phillips curve (see the bottom half of the aggregate supply curve in Figure 3).

Alternative microfoundations for introducing nominal rigidities will provide similar qualitative results as long as the models feature a long-run trade-off between inflation and output. Here, we opt for capturing this trade-off by introducing downward nominal wage rigidity; this specification has the virtue of capturing the neutrality of inflation when it is high, yet simultaneously giving rise to meaningful trade-offs at low inflation rates.

A large body of evidence documents the presence of downwardly nominally rigid wages even in the face of high unemployment. This notion dates back as far as Malthus who noted that "it very rarely happens that the nominal price of labour universally falls" (Malthus 1798). Bewley (1999) interviewed firm executives directly and explicitly documented their reluctance to cut nominal wages. More recently, substantial nominal wage rigidity has been studied in US administrative data by Fallick, Lettau, and Wascher (2011), in worker surveys by Barattieri, Basu, and Gottschalk (2014), and in cross-country data by Schmitt-Grohé and Uríbe (2016). Our specification is closely related to the supply side of Schmitt-Grohé and Uríbe (2016), and our calibrated level of wage rigidity in our quantitative analysis (see Section VII) matches closely their estimates for the degree of wage rigidity.

Our specification does imply that countercyclicality of real wages in secular stagnation episodes. Recent evidence from Beraja, Hurst, and Ospina (2016) uses regional variation to show that real wages fell more in regions with higher unemployment. Over the business cycle, real wages are typically acyclical. What is relevant to the mechanism is not wage rigidity per se; price rigidities as with Calvo pricing could also deliver a secular stagnation as we show in online Appendix E. Some combination of price and wage rigidities are likely needed to explain quantitatively the behavior of both inflation and real wages in the Great Recession. As emphasized in Christiano, Eichenbaum, and Evans (2005), both wage and price rigidities are needed to quantitatively explain the empirical dynamics of real wages in response to a monetary policy shock.[18]

We simplify our exposition by assuming that only the middle-aged generation receives income, now as payment for labor supplied instead of as an exogenous endowment.The budget constraint for young agents is again given by equations (1) and (4), but now we replace the budget constraint of the middle-aged generation equation (2) and old generation equation (3) with the following:

$$C^m_{t+1} = \frac{W_{t+1}}{P_{t+1}} L_{t+1} + \frac{Z_{t+1}}{P_{t+1}} - (1+r_t) B^y_t + B^m_{t+1}, \tag{18}$$

$$C^o_{t+2} = -(1+r_{t+1}) B^m_{t+1}, \tag{19}$$

where W_{t+1} is the nominal wage rate, P_{t+1} the aggregate price level, L_{t+1} the labor supply of the middle-aged generation, and Z_{t+1} the profits of the firms. For simplicity, we assume that the middle-aged generation will supply a constant level of labor $\bar{L}$ inelastically. Note that if the firms do not hire all available labor supplied, then labor demand L_t may be lower than labor supply $\bar{L}$ due to rationing. Under these assumptions, each of the generations' consumption-saving decisions remains the same as before.[19]

On the firm side, we assume that firms are perfectly competitive and take prices as given. They hire labor to maximize period-by-period profits. Their problem is given by

$$Z_t = \max_{L_t} P_t Y_t - W_t L_t \tag{20}$$

subject to

$$Y_t = L_t^{\alpha}. \tag{21}$$

[18] Wages tend to rise slightly after a monetary policy shock but only sticky wages would predict a fall. Nevertheless, wage rigidity remains the most important friction in their estimated model.

[19] The rationing approach followed here and in Schmitt-Grohé and Uríbe (2016) is essentially of the same form as in the general disequilibrium model of Barro and Grossman (1971); for a review of this literature, see Bénassy (1993).

The firms' labor demand condition is then given by

$$\frac{W_t}{P_t} = \alpha L_t^{\alpha-1}. \tag{22}$$

So far we have described a perfectly frictionless production side, and, if this were the end, our model would be analogous to what we have already considered in the endowment economy. Output would be given by $Y_t = L_t^{\alpha} = \bar{L}^{\alpha}$ and the labor demand condition equation (22) would fix the real wage.

Now, consider a world in which households will never accept working for wages that fall below their wage in the previous period, so nominal wages at time t cannot be lower than what they were at time $t-1$. Or, slightly more generally, imagine that the household would never accept lower wages than a wage norm given by $\tilde{W}_t = \gamma W_{t-1} + (1-\gamma) W_t^{flex}$, where

$$W_t^{flex} = P_t \alpha \bar{L}^{\alpha-1}. \tag{23}$$

If $\gamma = 1$, the wage norm is simply last period's nominal wages and wages are perfectly downwardly rigid; if $\gamma = 0$, we obtain the flex-price nominal wage. Nominal wages in our economy can never fall below the wage norm $\tilde{W}_t$. If labor market clearing requires nominal wages lower than the previous nominal wage rate, then the labor market will not clear, and labor is rationed.[20]

To close the model, we specify a monetary policy rule. We posit that the central bank sets the nominal rate according to a standard Taylor rule:

$$1 + i_t = \max\left(1, (1+i^*)\left(\frac{\Pi_t}{\Pi^*}\right)^{\phi_\pi}\right), \tag{24}$$

where $\phi_\pi > 1$; Π^* and i^* are parameters of the policy rule that we hold constant. This rule states that the central bank attempts to keep inflation at the inflation target Π^* and the nominal rate at i^* so long as the nominal rate implied by the rule is not constrained by the zero bound.[21]

It should be noted that the results we present in this paper do not depend on the particulars of the Taylor rule. We are interested in exploring what happens in an inflation-targeting regime when the natural rate of interest is negative and the central bank's inflation target is low enough that it cannot be reached even at a zero nominal rate. A simpler way of thinking about the policy regime we have in mind—an equivalent one for our purposes to the Taylor rule specified above—is that we are assuming that the central bank will set inflation equal to its target Π^* at all times (without needing to concern ourselves with how exactly this is accomplished) *except* for when this implies a negative nominal interest rate. In this case, the nominal

[20] If labor market clearing requires nominal wages higher than the past nominal wage rate, nominal wages will rise to their market clearing level, equating labor demand and supply.

[21] In online Appendix F, we introduce money explicitly into our model and show that our results are not affected so long as fiscal policy keeps consolidated government liabilities constant.

interest rate is zero and inflation falls below target (see Eggertsson et al. 2016 for an example of this specification of the policy regime).[22]

Equilibrium in the economy with nominal rigidities is defined as follows.

DEFINITION 1: *A competitive equilibrium is a sequence of quantities* $\{C_t^y, C_t^o, C_t^m, B_t^y, B_t^m, L_t, Y_t, Z_t\}$ *and prices* $\{P_t, W_t, W_t^{flex}, r_t, i_t\}$ *that satisfy equations* (1), (5), (6), (7), (8), (14), (15), (18), (20), (21), (22), (23), (24), *and the wage norm given an exogenous process for* $\{D_t, g_t\}$ *and initial values for* W_{-1} *and* B_{-1}^m.

We start by characterizing the steady state of the model, which can be shown graphically based on two relationships between output and inflation in a steady state: aggregate supply and aggregate demand. The aggregate supply specification of the model consists of 2 regimes: one in which the real wage equals the market-clearing real wage (if $\Pi \geq 1$) and the other when the bound on nominal wages is binding ($\Pi < 1$). Intuitively, positive inflation in the steady state means that wages behave as if they are flexible, since nominal wages must rise to keep real wages constant. If $\Pi \geq 1$, then labor demand equals the exogenous level of labor supply $\bar{L}$, defining the full-employment level of output Y^f:

$$Y = \bar{L}^{\alpha} = Y^f \quad \text{for } \Pi \geq 1. \tag{25}$$

This is shown in Figure 3 as a solid vertical segment.

If there is deflation in the steady state ($\Pi < 1$), the wage norm binds ($W = \tilde{W}$) and the real wage exceeds the market-clearing real wage.[23] We can then derive a relationship between output and inflation tracing the lower segment of the AS curve:

$$\frac{\gamma}{\Pi} = 1 - (1-\gamma)\left(\frac{Y}{Y^f}\right)^{\frac{1-\alpha}{\alpha}} \quad \text{for } \Pi < 1. \tag{26}$$

Equation (26) is simply a nonlinear Phillips curve. The intuition is straightforward; as inflation increases, real wages fall and firms hire more labor. Importantly, this Phillips curve relationship is not a short-run relationship; instead, it describes the behavior of steady-state inflation and output. The aggregate supply curve is shown

[22] The key assumption here is that, at the ZLB, we do not allow for the possibility that the inflation rate is higher than the inflation target. Essentially, the central bank would not tolerate such an outcome since it could always raise the nominal interest rate in that scenario. The only equilibria then consistent with the policy regime would be those in which inflation is below target because the central bank is constrained by the ZLB. Of course, there are equilibria consistent with higher inflation targets that do accommodate a negative natural rate of interest. We see these equilibria as those that emerge from different policy regimes that are willing to accept a higher inflation target (see Section VI).

[23] With a steady-state deflation rate of Π, we can use the wage norm to find the steady-state wage rate of $w = \frac{(1-\gamma)\alpha\bar{L}^{\alpha-1}}{1-\gamma\Pi^{-1}}$.

in Figure 3, with the vertical segment given by equation (25), the upward sloping segment given by equation (26), and the kink point at $\Pi = 1$.[24]

Turning to the aggregate demand relation, we again have two regimes: one in which the zero bound is not binding and the other in which it is binding. When the nominal rate is unconstrained, $Y^o = 0$ so that $Y = Y^m$, we obtain the AD curve by combining the real interest rate equation, Fisher relation, and monetary policy rule—equations (11), (16), and (24)—to get

$$Y = D + \frac{(1+\beta)(1+g)D\Gamma^*}{\beta}\frac{1}{\Pi^{\phi_\pi - 1}} \quad \text{for } i > 0, \tag{27}$$

where $\Gamma^* \equiv (1+i^*)^{-1}(\Pi^*)^{\phi_\pi}$ is the composite policy parameter in the monetary policy reaction function. The upper portion of the AD curve in Figure 3 depicts this relationship. As inflation increases, the central bank raises the nominal interest rate by more than one for one (since $\phi_\pi > 1$), which, in turn, increases the real interest rate and reduces demand.

At the 0 lower bound, we combine the same set of equations, but now impose $i = 0$. We obtain the following expression relating output and inflation:

$$Y = D + \frac{(1+\beta)(1+g)D}{\beta}\Pi \quad \text{for } i = 0. \tag{28}$$

In this case, the AD curve is upward sloping. The logic should again be relatively straightforward for those familiar with the literature on the liquidity trap: As inflation increases, the nominal interest rate remains constant, thus reducing the real interest rate. This change in the real rate raises consumption demand, as shown by the bottom portion of the AD curve in Figure 3.

The kink in the aggregate demand curve occurs at the inflation rate at which monetary policy is constrained by the zero lower bound. That is, the AD curve depicted in Figure 3 will become upward sloping when the inflation rate is sufficiently low that the implied nominal rate the central bank would like to set is below zero. Mathematically, we can derive an expression for this kink point by solving for the inflation rate that equalizes the two arguments in the max operator of equation (24):

$$\Pi_{kink} = \left(\frac{1}{1+i^*}\right)^{\frac{1}{\phi_\pi}}\Pi^*. \tag{29}$$

The location of the kink in the AD curve depends on both parameters of the policy rule: the inflation target Π^* and the targeted nominal interest rate i^*.

[24] The AS and AD diagrams and numerical examples in this section assume the following parameter values: $\beta = 0.985$, $\gamma = 0.94$, $\alpha = 0.7$, $\Pi^* = 1.01$, $\phi_\pi = 2$, $D = 0.28$, and $g = 0.9$ percent. The AD curve is linear, but the AS curve is nonlinear (and becomes more nonlinear as $\gamma \rightarrow 0$. However, as we show in Proposition 1, under mild conditions, we can establish uniqueness of the ZLB steady state.

In what follows, it will be useful to define the natural rate of interest—the interest rate at which output is at its full-employment level. The natural rate can be obtained by evaluating equation (11) at the full-employment level of output Y^f:

$$1 + r_t^f = \frac{1+\beta}{\beta} \frac{(1+g_t) D_t}{Y^f - D_{t-1}}.$$

It is useful to note that the full-employment interest rate corresponds exactly to the real interest rate we derived in the endowment economy. Hence, any of the forces that we showed affect the real interest rate in the endowment economy will directly affect the full-employment real interest rate in the more general setup.

V. Full Employment and Secular Stagnation

Equilibrium output and inflation is determined by the intersection of the aggregate demand and aggregate supply curves. We first consider a normal equilibrium at which the natural rate of interest is positive. We assume that the central bank aims for a positive inflation target (that is, $\Pi^* > 1$) and that the nominal interest rate target is consistent with the inflation target: $1 + i^* = \left(1 + r^f\right)\Pi^*$. With $r_f > 0$, the aggregate demand curve intersects the aggregate supply curve on the vertical segment of the AS curve. The exact intersection point is determined by the inflation target.[25] This full-employment equilibrium is displayed in Figure 3. Under our assumed policy rule, the equilibrium depicted in Figure 3 is unique for a small enough inflation target and a high enough γ.[26]

The making of a secular stagnation equilibrium is shown in Figure 3. Here, we illustrate the effect of a tightening in the collateral constraint D_t.[27] We assume the shock is large enough to move the natural rate of interest negative and, indeed, below the inflation target: $1 + r_t^f < (\Pi^*)^{-1}$. As can be seen from equation (27), a reduction in D reduces output for any given inflation rate. This fall in output stems from the decline in consumption by the young households, who cannot borrow as much as before to finance their consumption early in life. In the normal equilibrium, this drop in spending would be compensated by a drop in the real interest rate, restoring spending to its pre-shock level. However, the zero lower bound prevents this adjustment. Hence, the shock moves the economy off the full-employment segment of the AS curve to a deflationary steady state where the nominal interest rate is zero. Here, steady-state deflation raises steady-state real wages above their market-clearing level, thus depressing demand for labor and contracting output.[28]

[25] If the central bank targets zero inflation ($\Pi^* = 1$), the intersection is at the kink of the AS curve.

[26] Uniqueness is guaranteed if $\gamma = 1$. As γ approaches zero, more equilibria are possible. We discuss these additional equilibria in Section VI when they appear in a more general setting.

[27] Or, equivalently, we could consider a fall in the rate of population or productivity growth.

[28] As we show, outright deflation is not central to the mechanism. If wages are indexed to the inflation target, then an inflation rate that falls below target results in real wages that exceed the market-clearing level and output falls below the full-employment level.

PROPOSITION 1. *If* $\gamma > 0, \Pi^* = 1$, *and* $i^* = r^f < 0$, *then there exists a unique, locally determinate secular stagnation equilibrium.*

PROOF:

See online Appendix D. ∎

As shown in online Appendix C, if we linearize the model around the unique secular stagnation steady state, the dynamic system is locally determinate. The determinacy of the secular stagnation steady state in our model stands in contrast to the indeterminacy of the deflation steady state analyzed in Schmitt-Grohé and Uríbe (2012) and Benigno and Fornaro (2015).[29] Indeterminacy of equilibrium has been one justification for disregarding the deflation steady state and restricting analysis to the determinate positive interest rate steady state in New Keynesian models. Importantly, local determinacy implies that an economy in secular stagnation may continue to experience well-defined (unique bounded) business cycle fluctuations in unemployment and output around a permanently depressed growth path.

The emergence of a locally determinate secular stagnation steady state stems from the fact that the AD curve in Figure 3 is steeper than the AS curve at the steady state. Indeed, this relative slope condition is always satisfied under the conditions stated in Proposition 1. Intuitively, the AD curve intersects the *x*-axis at some positive level of output while the AS curve intersects the *y*-axis at some positive level $\Pi = \gamma$, ensuring both existence and that the relative slope condition is satisfied. By contrast, the steady state considered in, for example, Schmitt-Grohé and Uríbe (2012) does not satisfy this relative slope condition since the AD curve is horizontal with $\Pi = \beta$. Likewise, determinacy of the ZLB steady state is *not* due to the way the supply side is modeled—our specification is essentially identical to Schmitt-Grohé and Uríbe (2012). Rather, the combination of the demand side and supply side deliver the slope condition that ensures determinacy.

Our model delivers a permanent steady-state slump with no pull toward full employment. What forces can move the economy back to full employment, absent any government intervention? First, and perhaps obviously, if the shock that pushes the natural rate negative is only temporary, then the full-employment equilibrium can be attained. To the extent that the shocks leading to negative real interest rates are slow moving (such as demographic factors) there is less reason to be optimistic about this adjustment mechanism.

A key friction in triggering unemployment was the fact that wages were downwardly rigid. A natural question, then, is whether an increase in the flexibility of the wage adjustment process will push the economy back to full employment. The answer to this question is, surprisingly, no. This result emerges because an increase in price/wage flexibility triggers a drop in expected inflation, increasing the real interest rate. This cannot be offset by interest rate cuts due to the zero bound. As wages become more flexible (a decrease in the parameter γ), the slope of the AS curve steepens (see the left-hand panel of Figure 4). For any given deflation steady

[29] For further discussion of the stability properties in Benhabib, Schmitt-Grohé, and Uríbe (2001), see Bullard (2005).

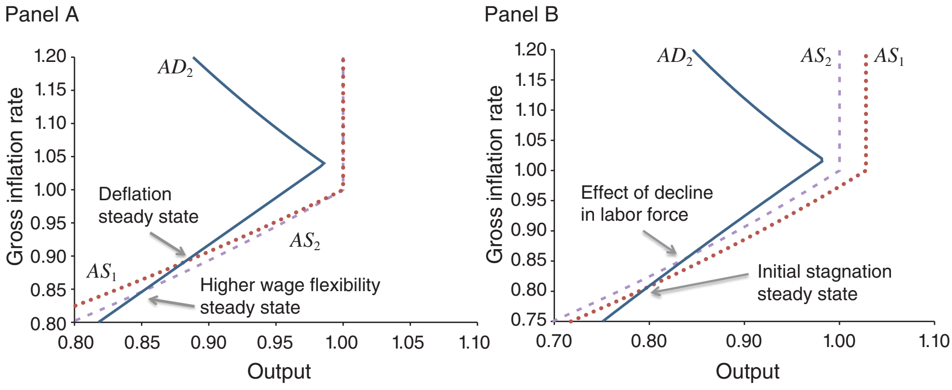

Figure 4. Adjustment Mechanisms: Wage Flexibility and Hysteresis

state, a decrease in γ will shift the steady state along the AD curve, increasing the rate of deflation, raising real wages, and therefore increasing the shortfall in output. This result echoes the paradox of flexibility shown in Eggertsson and Krugman (2012) and considered in a more general setting in Bhattarai, Eggertsson, and Schoenle (2014).

Finally, a third adjustment mechanism is a reduction in the labor force led by discouraged workers (or workers whose skills have deteriorated) after prolonged spells of unemployment. Labor force participation fell markedly in the United States, and the duration of unemployment remains elevated despite recent reductions in the unemployment rate. A reduction in the labor force reduces downward pressures on wages and weakens deflationary pressure. If the contraction of the labor force proceeds far enough, the output gap is eliminated, and the AD curve intersects the AS curve at the new, lower full-employment level of output.[30]

In Figure 5, we present data on the evolution of output, inflation, and the short-term nominal interest rate in the United States, Japan, and the Eurozone. We calibrate our simple three-period model and show how our model generates dynamics consistent with the observed behavior of these macro aggregates. The key message of the figure is that our model has, in principle, no difficulty generating a persistent fall in inflation, interest rates at the ZLB, and output persistently below trend. These features are in stark contrast to the standard New Keynesian model in which the model explodes (i.e., a determinate equilibrium no longer exists) if the natural rate of interest is expected to be negative for a long enough period (see, e.g., Eggertsson and Singh 2016). Even though highly stylized, a straightforward calibration of our simple model does a good job of capturing the transition dynamics of output and interest rates in these stagnation

[30] Under hysteresis mechanisms, the AS curve shifts inward along the transition path. Unemployment falls and inflation rises back toward target, while nominal interest rates stay at the ZLB. Relative to the pre-stagnation trend, output remains depressed. In this extension, there is no returning to the pre-trend level of employment. Hysteresis mechanisms need not work solely through the labor market. See Garga and Singh (2016) for a model with hysteresis effects on productivity growth.

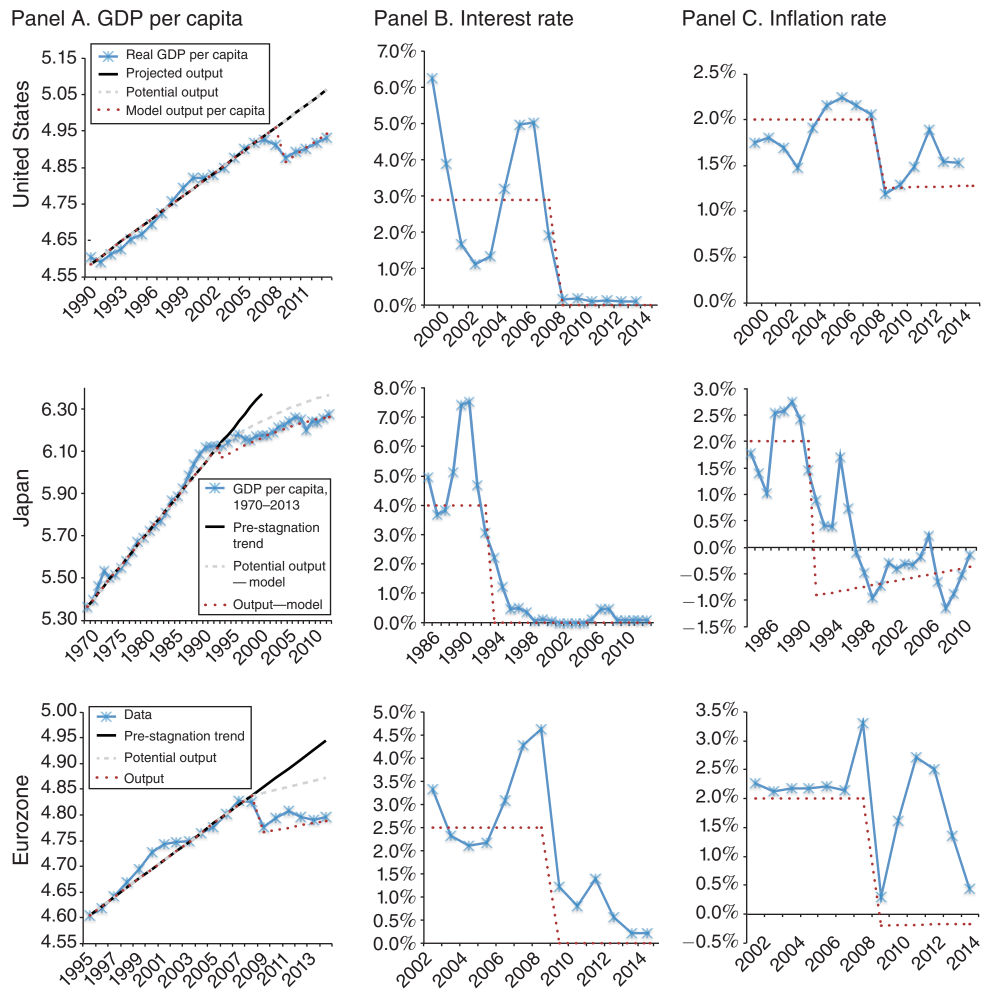

Figure 5. Data versus Model Transition Paths: United States, Japan, and the Eurozone

episodes. To conserve space, we defer the description of the calibration underlying Figure 5 to online Appendix H, which allows us to focus on a richer quantitative life cycle model.[31]

VI. Monetary and Fiscal Policy

We now consider monetary and fiscal policy in a secular stagnation. As we show in this section, the policy implications of our model for both monetary and fiscal

[31] To capture the slowdown in GDP per capita growth and the absence of outright deflation in the United States and eurozone, we extend our basic model to incorporate hysteresis effects on productivity growth and a more general wage norm where nominal wages are indexed to productivity growth and the inflation target. See online Appendix G.

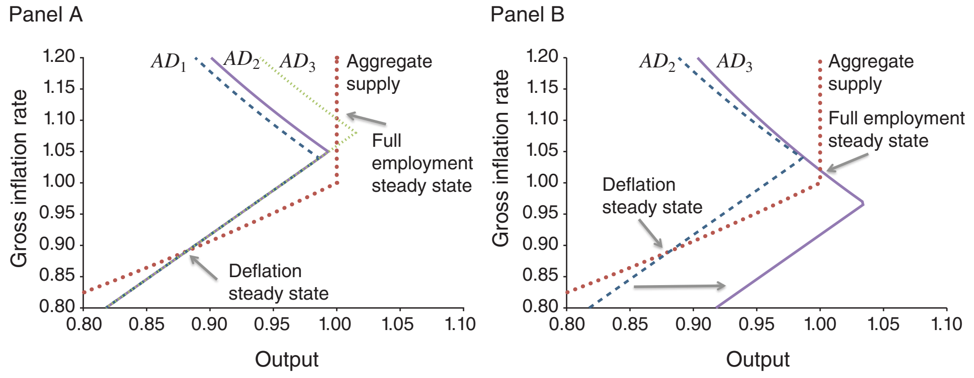

FIGURE 6. MONETARY AND FISCAL POLICY RESPONSES

policy are starkly different from those of existing New Keynesian models of the zero lower bound.

Let us first consider the effect of an increase in the inflation target Π^*. This change has no effect on the AS curve but instead shifts the AD curve. Specifically, a rise in the inflation target shifts out the kink point in the AD curve, as shown in the left-hand panel of Figure 6. In this figure, the initial inflation target is set at 1 percent. As the inflation target increases, the kink point moves upward, effectively shifting up the downward sloping portion of the AD curve. The curve AD_1 shows the original aggregate demand curve with a unique secular stagnation steady state. The curve AD_2 shows the effect of a modest increase in the inflation target, while AD_3 shows the effect of a large increase in the inflation target. Notice that AD_3 now intersects the aggregate supply curve at three points.

Notice, AD_2 illustrates the perils of too small an increase in the inflation target. For a sufficiently negative natural rate of interest, a small increase in the inflation target will not shift the AD curve enough to intersect the full-employment line. The contention that a small increase in the inflation target will be ineffective has been labeled by Krugman variously as the "timidity trap" or, in reference to Japan in the late 1990s, the "law of the excluded middle." Our framework readily captures this idea. Formally, the inflation target needs to be high enough so that $\left(1 + r^f\right)\Pi^* \geq 1$; otherwise, the kink point in the AD curve occurs to the left of the full-employment line.

With a sufficiently large increase in the inflation rate (as shown by AD_3), our model admits three distinct steady states. The first steady state at the top intersection of the two curves is the normal, full-employment equilibrium at which point inflation is equal to the inflation target of the central bank Π^*. At this point the nominal interest rate is positive because the inflation target is large enough to accommodate the negative natural rate—that is, $\left(1 + r^f\right)\Pi^* > 1$. However, there is another equilibrium at full employment that is consistent with the policy rule. This is the second intersection of the two curves, where $i = 0$ and $\Pi < \Pi^*$. This steady state, however, is locally indeterminate.[32] Importantly, even with a sufficiently

[32] This steady state is akin to the deflation steady state in Benhabib, Schmitt-Grohé, and Urîbe (2001).

large increase in the inflation target, the secular stagnation steady state remains; an increase in the inflation target does not eliminate this equilibrium. Furthermore, this steady state is locally determinate.

This multiplicity shows that monetary policy is less effective in our environment than in models that feature temporary liquidity traps, such as those in Krugman (1998) or Eggertsson and Woodford (2003). In those models, a permanent increase in the inflation target will always have an effect because, by assumption, one can always reach the higher inflation target at some point in the future. Working backward, a commitment of this sort will always have expansionary effects during a liquidity trap, and, provided the inflation target is high enough, it may even eliminate the demand slump altogether. Since the trap is permanent in our model, however, this backward induction breaks down; there is no future date at which one can be certain that the higher inflation target is reached (even if the policy regime is fully credible in the sense that people do not expect the government to deviate from the policy rule). For the same reason, a commitment to keep nominal rates low for a long period in a secular stagnation is of limited use and does not by itself guarantee a recovery. Indeed, interest rate commitments of the type pursued by the Federal Reserve during the crisis (often referred to as "forward guidance") would be irrelevant in shifting the economy out of the deflationary equilibrium since households are expecting rates to stay at zero forever. Even if a recovery were anticipated, the expansionary effect of a commitment to keep interest rates lower in the future is far less effective, given discounting in the Euler equation due to finite lifetimes.[33] Though an increase in the inflation target could make a full-employment steady state feasible, our model is silent on how a government could coordinate expectations on the favored full-employment equilibrium.

The fact that we have two locally determinate steady states suggests there is no obvious reason why a higher inflation target should bring about full employment for an economy at the secular stagnation steady state.[34] Moreover, as shown by Gibbs (2017), both the full-employment and secular stagnation steady state are learnable and survive under deviations from rational expectations.[35]

Given the drawbacks of monetary policy, we turn to fiscal policy. We extend our model to incorporate taxes and denote taxes on each generation by T_t^i, where

[33] See online Appendix C, equation (A.24) for the linearized IS curve in the vicinity of the secular stagnation steady state. This IS equation displays discounting in the Euler equation that dampens the response of current output to changes in expected future output.

[34] One suggestion, proposed by John Cochrane in discussing this paper, is that the government can select between the two determinate steady states by raising nominal interest rates since, with a high enough inflation target, the nominal interest rate is positive. The main problem with this interpretation is that it presumes the Taylor rule is a structural policy regime that can be changed by increasing the nominal rate, so that increasing interest rates today will necessarily imply an increase in the nominal interest rate forever. If one instead interprets the Taylor rule as a reduced-form representation of a policy regime that aims at targeting a certain level of inflation Π^*, then an increase in the nominal interest rate at time t need not imply anything about future policy commitment. If an increase in the nominal interest rate at time t only changes the interest rate at that time, without affecting expectations, then it will have a contractionary effect via the aggregate demand channel. This could be modeled more formally as optimal policy under discretion (Markov Perfect Equilibrium), for example, whereby the policy objective of the government is to minimize the deviation of inflation from a target.

[35] Another consideration against raising the inflation target and accommodating a negative natural rate of interest is that very low rates could spur asset-price bubbles and raise financial stability concerns. See Galí (2014) and Asriyan et al. (2016) for further discussion.

$i = y, m$, or o. We first consider the effect of fiscal policy on the natural rate (i.e., in the endowment economy) before reincorporating nominal frictions. The budget constraints can now be written taking taxes into account:

$$C_t^y + T_t^y = B_t^y, \tag{30}$$

$$C_{t+1}^m + (1+r_t)B_t^y = Y_{t+1}^m - T_{t+1}^m - B_{t+1}^m, \tag{31}$$

$$C_{t+2}^o = Y_{t+2}^o + (1+r_{t+1})B_{t+1}^m - T_{t+2}^o. \tag{32}$$

Public debt now enters the asset market-clearing condition:

$$-N_{t-1}B_t^m = N_t B_t^y + N_{t-1}B_t^g, \tag{33}$$

where B_t^g is government debt (normalized in terms of the size of the middle-aged generation). In previous sections, the only demand for borrowing came from the young households; now the government also may want to borrow so that loan demand is given by the right-hand side of equation (33):

$$L^d = \frac{1+g}{1+r}D + B^g. \tag{34}$$

We have omitted the time subscript to indicate that we are evaluating the steady state. Meanwhile, the supply of loans can be derived in exactly the same way as before, yielding

$$L^s = -B^m = \frac{\beta}{1+\beta}(Y^m - D - T^m) - \frac{1}{1+\beta}\frac{Y^o - T^o}{1+r}. \tag{35}$$

Relative to the earlier specification, loan supply must now also track tax payments. The government's budget constraint is the final equation needed to determine asset market equilibrium:

$$T^m + B^g + \frac{1}{1+g}T^o + (1+g)T^y = G + (1+r)\frac{1}{1+g}B^g, \tag{36}$$

where G is government spending (normalized in terms of the size of the middle-aged generation). The equilibrium real interest rate is once again the interest rate that equalizes loan supply L^s and loan demand L^d and takes the same form as we saw previously in Section II and as was illustrated by Figure 2. The key difference is that now the real interest rate can be affected by fiscal policy, shifting the loan supply and loan demand curves in Figure 2. A fiscal policy regime corresponds to a choice of the level and distribution of taxation and government spending (T^o, T^m, T^y, G, B^g) subject to the government's budget constraint. The overall effect of fiscal policy on the real interest rate then depends on the contribution of all the fiscal variables. Equating equations (34) and (35), and taking account of the budget constraint equation (36), we have two equations and six unknown variables (T^o, T^m, T^y, G, B^g, and $1 + r$). Hence, we need four restrictions on the fiscal policy instruments to determine the interest rate. Previously, we implicitly assumed that $T^o = T^m = T^y = 0, G = 0$, which implies $B^g = 0$ from the government budget constraint, leaving equations (34)–(35) to pin down the real interest rate $1 + r$.

Consider now a more general policy regime that will help us clarify a number of results.[36] The tax on the young and government spending are exogenously given by $T^y = T^*$ and $G = G^*$. Similarly, the overall level of *real* government debt is exogenously given by B^g_*. Finally, we assume that taxes on the middle aged and the old satisfy the following relation:

$$T^m = \frac{1}{\beta}\frac{1}{1+r}T^o = T, \tag{37}$$

where the level of taxation T adjusts so that the budget constraint equation (36) is satisfied. The distribution of taxation condition given by equation (37) ensures that now there is no effect of fiscal policy on the supply of loans given by equation (35).

We first consider an increase in public debt B^g_*. Under the fiscal rule we have considered, an increase in public debt only affects loan demand, shifting out the demand for debt and raising the natural rate. In this respect, increasing government debt is a natural way of avoiding a secular stagnation.[37] Who receives the proceeds from this increase in government debt? The conditions above show that this does not matter so long as equation (37) is satisfied. The increase in government debt could be directed to the young or toward government spending, or be distributed to the middle aged and the old in accordance with the fiscal rule (37). The effect of public debt also shows that if a central bank kept the *nominal* level of money constant in a secular stagnation with deflation, eventually the natural rate would have to rise (see online Appendix F for more details). We can similarly explore the effect of increasing government spending, funded via taxes, or various tax redistribution schemes. The effect of those policies can be gauged by analyzing how the policy shifts loan demand and loan supply in Figure 2. We will be a bit more specific about these types of policy experiments in the full life cycle model.

A critical reason for why an increase in government debt raised the natural rate of interest is that it is expected to be permanent. To make this clear, let us consider the following policy regime: $G_t = T^y_t = B^g_{t-1} = 0$ and

$$T^m_t = -B^g_t,$$

$$T^o_{t+1} = (1 + r_t)B^g_t.$$

[36] While this particular policy regime is clearly special, we think it helps illustrate how relatively cleanly fiscal policy operates in this environment.

[37] The ability of an increase in the public debt in an OLG economy to undo the effect of credit frictions (that is, the effect of borrowing constraints) is similar to examples presented in Woodford (1990).

The thought experiment here is that an increase in the public debt results in a lump-sum transfer to the middle aged in period t. The old are then taxed by the same amount in the next period (plus interest) to return the debt to its original level.[38] A temporary increase in the public debt is completely irrelevant for the interest rate: The increase in debt in period t is met by an increase in the middle-aged savings to pay off the future tax. The point is that the effect of an increase in the public debt considered in the previous policy regime depends critically on agents' expectations about future fiscal policy. In particular, it depends upon the expectation of the middle aged that they will not be taxed to pay down the debt in the future.[39]

So far, we have only considered the effect of fiscal policy on the real interest rate in the endowment economy. Our results, however, carry over to the full model with endogenous production. Fiscal policy that leads to a change in the real interest rate in the endowment economy is equivalent to a policy that changes the natural rate of interest in the model with production. It therefore corresponds directly to a shock that shifts the AD curve, as shown in the right-hand panel of Figure 6, displaying the effect of an increase in government spending via debt issuance. Thus, fiscal policies that raise the natural rate of interest correspond to an outward shift in the aggregate demand curve, while policies that reduce the natural rate correspond to an inward shift of the AD curve. Importantly, in contrast to monetary policy, where an increase in the inflation target only allowed for the possibility of a "good equilibria" and suffered from multiple steady states, fiscal policy can eliminate the secular stagnation equilibrium since it shifts the entire AD curve (see Figure 6).

To derive some analytic results for the effect of fiscal policy on output, let us generalize equation (28) by combining equations (34) and (35), together with equations (16) and (24), and assuming $i = 0$, to yield

$$Y = D + T^m + \frac{1+\beta}{\beta} B^g + \left(\frac{(1+g)(1+\beta)}{\beta} D - \frac{1}{\beta} T^o \right) \Pi, \tag{38}$$

where we now see how fiscal instruments directly affect aggregate demand at a zero interest rate. Tracing out exactly how aggregate demand shifts requires being specific about the policy regime. In Table 1, we derive analytically the steady-state multiplier of government spending at the zero lower bound under different financing conditions, under the policy regime we specified before (except we relax equation (37) when government spending is financed by the middle aged or old).[40]

[38] For simplicity, we set population growth to zero.

[39] Since money and public debt are perfect substitutes at the zero lower bound, a *temporary* helicopter drop would also have the same effect. A helicopter drop is only effective to the extent that the rise in consolidated government liabilities is permanent.

[40] The parameters κ and ψ, the slope of the AS and AD curves, respectively, are given by

$$\kappa = \frac{1-\alpha}{\alpha}\frac{1-\gamma}{\gamma},$$

$$\psi = \frac{1+\beta}{\beta}(1+g)D.$$

Table 1—Government Purchases Multiplier at the Zero Lower Bound

Financing	Multiplier	Value
Increase in public debt	$\frac{1+\beta}{\beta}\frac{1}{1-\kappa\psi}$	>2
Tax on the young	0	0
Tax on the middle-aged	$\frac{1}{1-\kappa\psi}$	>1
Tax on the old	$-\frac{1+g}{\beta}\frac{1}{1-\kappa\psi}$	<0

Observe that, away from the ZLB, the multiplier is zero since labor is supplied inelastically; once all workers are employed, government purchases will reduce private consumption one to one without any effect on output. At the zero bound, however, the multiplier is generally different from 0 as seen in Table 1 and critically depends on the way in which government spending is financed.[41]

First, we consider the case in which spending is financed via an issuance of public debt. Financing via an increase in the public debt results in the largest multiplier shown in Table 1. Using the formula in Table 1, we see that because β is less than 1, and κ can be no lower than 0 (when wages are perfectly fixed), this multiplier has to be larger than 2. As we increase the value of κ, the multiplier becomes larger and can even be unboundedly large—a result similar to that in Christiano, Eichenbaum, and Rebelo (2011) in the context of the standard New Keynesian model.

However, in contrast to the NK model, government spending multipliers are not always positive at the ZLB. The sign of the multiplier depends on whether fiscal expansions reduce the saving glut in a secular stagnation, which, in turn, depends on how government spending is financed. If instead of being financed by increasing debt, spending is financed via a tax on young households, the multiplier is zero. The collateral constraint is binding in equilibrium so that the young will cut their consumption by exactly the same amount as they are taxed. An increase in government purchases then simply substitutes for existing consumption by the young, leaving output unchanged. If an increase in spending is financed via a tax on middle-generation households, the purchases multiplier is still positive, but smaller than if spending is financed via debt as shown in Table 1. We see from the analytic expression that this multiplier always has to be greater than one. Finally, if government purchases are financed via a tax on the old, the multiplier is negative. This negative multiplier obtains because the old generation will cut their spending one for one with the tax (thus offsetting the higher spending by the government). Meanwhile, the middle aged will now increase their saving, having anticipated higher future taxes, which reduces aggregate demand.

Nevertheless, overall the model suggests a relatively positive picture of fiscal policy as it can lead to an increase in demand via either debt policy or tax and spending actions.

[41] In computing the multiplier, we consider only a small increase in spending (so the zero bound is still binding) and a linear approximation of the model around a zero inflation steady state.

VII. A Quantitative Life-Cycle Model

We now turn to a medium-scale quantitative version of the baseline model where we can incorporate a variety of additional features.[42] The economy consists of a large number of households with identical utility functions. Households enter economic maturity at age 26, after which they work, consume, have children, and trade in asset markets. Households pass away with certainty at age J, which we take to be 81 years. Households have children at age 26, and the population growth rate is determined by the total fertility rate (Γ) of every family. Households face a probability of dying stochastically before reaching maximum age J. The probability of surviving between age j and $j+1$ is denoted by s_j.[43] The unconditional probability of reaching age j is denoted with a superscript s^j.[44]

Households receive utility from two sources: (i) consumption, which is given by a time-separable, constant elasticity of substitution (CES) utility function $u(\cdot)$ with elasticity parameter ρ; and (ii) bequests that are divided equally among all descendants. The bequest motive is also characterized by a CES function $v(\cdot)$ whose argument is the amount of bequests left *per descendent*, denoted by x.[45] The utility function for bequests is multiplied by a parameter $\mu \geq 0$ that determines the strength of the bequest motive. Denoting the consumption of households of age j at time t by $c_{j,t}$ and the discount rate by β, a household that enters economic maturity at time t has lifetime expected utility given by the expression

$$U_t = \sum_{j=26}^{J} s^j \beta^j u\big(c_{j,t+j-1}\big) + s^J \beta^J \mu v\big(x_{J,t+J-1}\big).$$

A household of age j can trade in a real asset $a_{j,t}$ at time t, which is used as productive capital. At time $t+1$, capital will pay a return of r^k_{t+1}, which is the rental rate of capital, and has a resell value (net of depreciation) of $\xi_{t+1}(1-\delta)$, where ξ_{t+1} is the exogenous relative price of capital in terms of the consumption good. Each household has an identical exogenous labor productivity process, or human capital profile, denoted by hc_j, which changes with age. Households receive no wage income after retirement, which, in our model, occurs after age 65. We assume an inelastic labor supply; hence, wage income is equal to the wage multiplied by the household's age-specific labor productivity hc_j net of labor taxes $(1-\tau^w)$.

Households also receive income from the pure profits of firms, denoted by $\pi^f_{j,t}$, and we assume that profits are distributed proportionally to labor income. Finally, the household may receive a bequest $q_{j,t}$. Individuals receive bequests q the year after their parents die; thus descendants receive bequests at age $J-24$. For example, if we take J to be 81, individuals would receive bequests at age 57. Bequests made (x) are zero at all times except in the final year of life at age J.

[42] The model is based on Auerbach and Kotlikoff (1987) and Ríos-Rull (1996).

[43] Age-specific survival rates may also vary over time t; however, for notational simplicity, we omit these additional subscripts.

[44] This can be calculated as the product of the one-period survival probabilities: $s^j = \prod_{m=26}^{j} s_m$.

[45] Thus, the total size of the bequest left by households is the bequest x multiplied by the fertility of the household.

Following Ríos-Rull (1996), we suppose that agents insure themselves against the idiosyncratic risk of early death via one-period annuity contracts.

The flow budget constraint of a household of age j at time t is

$$\begin{aligned} & c_{j,t} + \xi_t a_{j+1,t+1} + \Gamma_{26,t-j+26} \cdot x_{j,t} \\ & \quad = (1-\tau^w) w_t hc_j + \pi^f_{j,t} + \left(r^k_t + \xi_t(1-\delta)\right)\left(a_{j,t} + q_{j,t} + \frac{1-s_j}{s_j} a_{j,t}\right). \end{aligned}$$

Households can borrow against future income, and we impose a borrowing constraint of the same form as in our simple model of Section II[46]:

$$a_{j,t} \geq \frac{D_t}{1+r_t}.$$

There are two types of firms: producers of final goods and producers of intermediate goods. The final goods firms produce a differentiated good Y^f_t. The final good composite is the CES aggregate:

$$Y_t = \left(\int_0^1 \left(Y^f_t\right)^{\frac{\theta_t-1}{\theta_t}} df\right)^{\frac{\theta_t}{\theta_t-1}}.$$

Each final goods producer utilizes Y^m_t of intermediate goods to produce output via a linear technology: $Y^f_t = Y^m_t$. The presence of monopolistically competitive final goods firms allows for a time-varying markup given by $\theta_t/(\theta_t - 1)$; pure profits due to monopoly rents are returned to the households.

There is a perfectly competitive intermediate goods sector that sells its production to the final goods sector. These firms hire workers at wage rate w_t and rent capital at rate r^k_t. They operate a CES production function in labor and capital with an elasticity of substitution σ. The production function is

$$Y^m_t = \left(\alpha K_t^{\frac{\sigma-1}{\sigma}} + (1-\alpha)(A_t L_t)^{\frac{\sigma-1}{\sigma}}\right)^{\frac{\sigma}{\sigma-1}},$$

where A_t is an exogenous, labor-augmenting technological progress, and $0 < \alpha < 1$.

The government purchases some level of output G_t and may issue debt. The budget constraint for the government becomes

$$b_{g,t} = G_t + (1+r_t) b_{g,t-1} - T_t,$$

[46] We will assume, in addition, that the borrowing constraint D_t grows at the rate of productivity growth and the household's earning potential over the life cycle. In contrast to Section II, the financial constraint is now expressed on asset accumulation.

where taxes are collected on labor income. We economize on notation by omitting real and nominal bonds as assets above; these assets enter in the same way as in the simpler model (Section IV) so there is both a well-defined real interest rate r_t on a risk-free one-period real bond and a nominal interest rate i_t on nominal bonds.

Monetary policy and wage rigidity are modeled as in Section IV. The full nonlinear model is solved numerically; we solve for both the stationary equilibrium and perfect foresight transition paths. We outline the computational details and the numerical solution algorithm in online Appendix J.[47]

A. *Calibration*

We first calibrate our model to match the US economy in 2015. Considerable uncertainty remains as to the size of the output gap in 2015. We consider two polar cases: (i) a zero output gap in 2015 based on Stock and Watson (2012) who argue that the slow recovery largely reflects a slow recovery in labor force growth, and (ii) an alternative calibration, where the output gap is -15 percent, which is what Hall (2017) estimates as the deviation of output from its pre-recession trend in 2015. We consider this latter case as a sensible upper bound on the possible size of the output gap.[48]

We first consider the case of the zero output gap. We calibrate our model to match the 2015 real interest rate in the United States, which was -1.47 percent. Since we are assuming a zero output gap, this implies that the natural rate of interest is also -1.47 percent. We will thus be on the upper section of the aggregate demand and aggregate supply curves and can continue with our parameterization without reference to nominal frictions.

Our parameters come from three main sources. The first is statistical data about US demographics and the economy that we can match directly, such as mortality rates, fertility levels, productivity growth, and the size of government debt. The second source of parameters we take directly from the related literature that has previously estimated or calibrated these parameters. The third set is chosen to match key moments in the data, such as the investment-to-output ratio. The moments will be matched by minimizing an objective function. We discuss each in turn.

Panel A of Table 2 shows the first category of parameters that are taken directly from observed data. We use mortality data from the Centers for Disease Control (CDC) to directly match US survival tables. The total fertility rate is taken from UN fertility data, and the retirement length is chosen to match the average years of retirement. Government debt to GDP and government spending to GDP are also chosen to match current values.[49] The rate of productivity growth is a key determinant of the real interest rate. Our baseline uses a productivity growth rate of

[47] The equilibrium conditions consist of roughly 170 equations in the stationary equilibrium. Transition dynamics are more complicated as the birthrate over the past 25 years becomes a state variable in that case and agents form expectations over the entire transition path.

[48] To be clear, Hall (2017) does not interpret this gap as reflecting the output gap of the kind we see in the model, but we still think that computing deviation from trend provides a convenient benchmark for contrasting with the opposite extreme—an economy with no output gap.

[49] We set government debt as the sum of federal, state, and local debt as reported by the Council of Economic Advisers and the Census Bureau.

Table 2—Parameters Taken from the Data and Related Literature

	Symbol	Value	Source
Panel A. Data			
Mortality profile	$s_{j,t}$		US mortality tables, CDC
Income profile	hc_j		Gourinchas and Parker (2002)
Total fertility rate	n	1.88	UN fertility data
Productivity growth	g	0.65%	Fernald (2012)
Government spending (percent of GDP)	G	21.3%	CEA
Public debt (percent of GDP)	b_g	118%	Flow of Funds
Panel B. Related literature			
Elasticity of intertemporal substitution	ρ	0.75	Gourinchas and Parker (2002)
Capital/labor elasticity of substitution	σ	0.6	Antras (2004)
Depreciation rate	δ	12%	Jorgenson (1996)

0.65 percent per year, which we have taken from Fernald (2012). The wage profile hc_j is chosen to match the earnings profile estimated from the data by Gourinchas and Parker (2002).

Panel B of Table 2 shows the second category of parameters taken from the related literature. We set the intertemporal elasticity of substitution ρ equal to 0.75. This parameter has been estimated widely in the literature, with ranges between 0.25 and 1.[50] The depreciation rate comes from Jorgenson (1996) and BEA (2004), which have extensive estimates of the depreciation rate of private and governmental nonresidential equipment.[51] The value of the production elasticity of substitution, σ, has also been estimated widely in the literature, and generally falls between 0.4 and 1.[52] As these parameters are not directly observed in the data, we report in online Appendix L how our results change for different parameter choices and we will also comment on them below.

We choose the remaining parameters to match five key data moments as of 2015: a real interest rate of -1.47 percent, an investment-to-output ratio of 15.9 percent, a consumer-debt-to-output ratio of 6.3 percent, a labor share of 66.0 percent, and a bequest-to-output ratio of 3 percent.[53, 54] The parameters chosen this way are the rate of time preference β, the debt limit D, the bequest parameter μ, the capital share

[50] For example, Cooley and Prescott (1995) set $\rho = 1$, while Auerbach and Kotlikoff (1987) and Ríos-Rull (1996) set $\rho = 0.25$.

[51] Since our model does not explicitly incorporate housing, our preferred specification uses depreciation applicable to equipment investment, which generally is higher than for housing. We consider a lower value in the sensitivity analysis in the online Appendix.

[52] For example, Antras (2004) estimates it is between 0.4 and 0.9, while Oberfield and Raval (2014) estimate an elasticity of 0.7 and Klump, McAdam, and Willman (2007) estimate a value between 0.5 and 0.6.

[53] Sources are listed in Table 3.

[54] We choose to target the investment-to-output ratio, rather than the capital output ratio, as this ratio does not require adjustment of the past capital stock for changes in the relative price of capital goods. Investment to output is just a ratio of two nominal, readily measured quantities.

Table 3—Parameters Chosen to Match Targets

	Model/Data	Source
Panel A. Targets		
Natural rate of interest	-1.47%	Federal Reserve
Investment-to-output ratio	15.9%	NIPA
Consumer-debt-to-output ratio	6.3%	Flow of Funds
Labor share	66.0%	Elsby, Hobijn, and Sahin (2013)
Bequests-to-output ratio	3.0%	Hendricks (2001)
	Symbol	Value
Panel B. Parameters chosen to match targets		
Rate of time preference	β	0.98
Borrowing limit (percent of annual income)	D	23.4%
Bequests parameter	μ	21.6
Retailer elasticity of substitution	θ	4.9
Capital share parameter	α	0.24

parameter α, and the retailer elasticity of substitution θ. We select the parameters by minimizing a loss function whose value is the sum of the squared differences between the moments of the model and those we take from the data.

Generically, there is no one-to-one mapping between these remaining parameters and the targets. Hence, we jointly choose all parameters to match the model output to the targets. Nevertheless, each of the parameters above corresponds relatively closely with one of the key moments we are trying to match. The rate of time preference β has a direct effect on the real interest rate; as β increases, the real interest rate falls. Not unexpectedly, the debt limit most directly affects the level of consumer debt to output, while the bequest parameter μ mostly directly affects the bequests-to-output ratio. Finally, the capital share parameter α determines the investment-to-output ratio, while inverse profit share θ controls the labor share.

B. *Negative Real Interest Rates*

Table 3 shows the results of the calibration procedure with the zero output gap. With the parameters selected by our minimization, each of the model moments perfectly matches those in the data. We document in online Appendix N that the parameters in Table 3 fall directly within the range of parameter values reported in the existing literature. Therefore, we argue that a relatively standard OLG model with capital accumulation calibrated to match US data can generate permanently negative real interest with standard parameter values.[55]

[55] In online Appendix L, we show the optimal consumption path for households over their life cycle. Consumption tracks income over the early part of the life cycle due to borrowing constraints, then declines gradually as households save for retirement. This leads to the classic hump-shaped consumption profile. For comparison purposes, this figure also includes the estimated consumption profile from the Consumer Expenditure Survey, as estimated by Gourinchas and Parker (2002). We also compare the model's population pyramid in online Appendix L to the

That a standard, conservatively calibrated OLG model can deliver a substantially negative natural rate of interest suggests a reevaluation of the standard estimate of the natural rate for the United States. Indeed, there is no reason a priori to expect a normalization of interest rates in the United States. Pre-recession estimates for the long-run neutral real interest rate remained between 2 percent and 3 percent (see Williams 2016). As long-term rates have continued to fall, the Federal Reserve has adjusted downward its estimates of the neutral rate to between 0.5 percent and 1.8 percent according to FOMC projections (see FOMC 2016). Our calibration, however, suggests that, given current productivity and demographic trends, these estimates for the long-run neutral rate could be too optimistic. Our benchmark calibration has the steady-state real interest rate at –1.47 percent, or 2.27 percent lower than the FOMC estimate as of 2016.

Our results are robust to a variety of additional specifications; in particular, we focus on three parameters taken from the literature. Online Appendix L calculates the same moments in Table 3 for three alternate specifications: (i) setting depreciation to 8 percent, (ii) setting the intertemporal elasticity of substitution ρ to 1, and (iii) setting the production elasticity parameter σ to 1 (Cobb-Douglas). For each of these specifications, we again minimize the objective function to find β, α, μ, D, and θ to match the targets from the data. The results for these alternate specifications are similar to our main specification. The model still hits the targets, but with different values for the calibration parameters that remain well within the ranges from the literature. We report results under these alternative parameterizations for other experiments in online Appendix L.

While the model can clearly generate permanently negative real interest rates, it is also of interest to explore if it can explain the reduction observed in the real interest rate over time. Before getting there, however, it is worth asking if our model can also replicate a scenario in which the natural rate of interest is negative enough that the zero bound is binding with output below potential.

C. *Secular Stagnation in a Quantitative Life-Cycle Model*

We next turn to our calibration for the US economy with a substantial output gap. We keep the first group of parameters (those taken directly from the data) and the second group of parameters (those taken from the literature) the same as in our initial calibration. For our third group of parameters (those we choose to match moments), we keep all parameters unchanged save for the rate of time preference β. In addition, we must choose the wage rigidity parameter γ.

We jointly choose β and γ to match two moments of the US economy in 2015: the inflation rate and the output gap. We model the economy as in a secular stagnation, with an inflation rate of 1.62 percent as in the data and an output gap of -15 percent, corresponding to the deviation of output from its precrisis trend in 2015 as documented in Hall (2017). A value of γ of 0.91 and a discount rate of β

current US pyramid. The fact that we are considering the stationary equilibrium is not innocuous. It does not take into account the dynamics of the aging of the baby boom and the effect on interest rates as the boom filters through the population pyramid. To study these effects, we need to consider transition dynamics, which we turn to shortly.

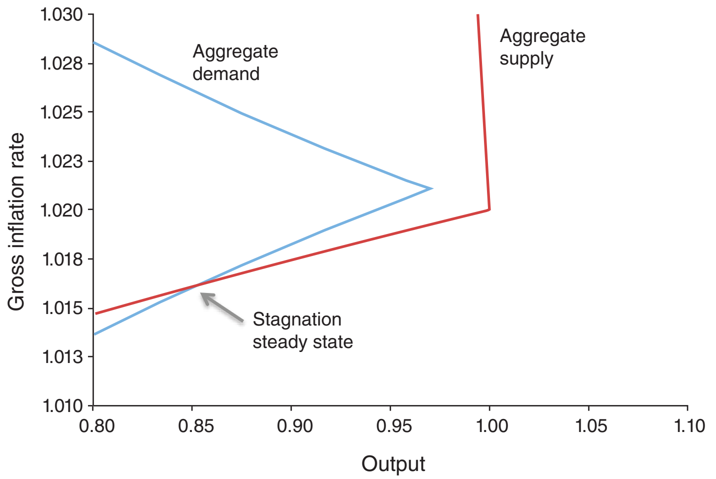

Figure 7. Secular Stagnation Equilibrium

of 0.99 match these moments. Our calibrated value of γ falls within the range wage rigidity estimated by Schmitt-Grohé and Uríbe (2016). The value β is also consistent with common estimates from the OLG literature (see online Appendix N).

Figure 7 shows the aggregate demand and aggregate supply curves of the secular stagnation stationary equilibrium. The two curves cross at an output gap of -15 percent. The natural interest rate in this calibration is -2.2 percent—lower than the -1.47 percent in the calibration without an output gap. A lower natural rate is needed so that an inflation target of 2 percent is too low to allow the nominal interest rate to the track the natural rate. As a result, the economy is drawn into the secular stagnation steady state. The equilibrium real interest is -1.62 percent and the inflation rate is 1.62 percent—40 basis points below target. Investment to output, bequests to output, labor share, and consumer debt to output continue to closely match the US moments.[56]

Figure 7 represents the second key quantitative finding of the paper. It suggests that the model can replicate a permanent stagnation with a standard parameterization of precisely the same form we have shown in the simple model. As in the three-period model, a surplus of savings over investment opportunities pushes the equilibrium interest rate negative. If the inflation target is insufficient to prevent the zero lower bound from binding and nominal wage rigidities bind, a secular stagnation emerges.

D. *Decline in Interest Rates since 1970*

We use our model to quantitatively analyze the fall in the real interest rate observed since 1970 and to assess the relative contribution of different factors.

[56] The investment-to-output ratio, consumer debt-to-output ratio, labor share, and bequests-to-output remain very close to their values in Table 3; hence, we choose not to reparameterize the model for μ, α, D, and θ relative to the benchmark calibration.

1970 is a natural starting point, for it is the point at which a number of economic trends began shifting. Briefly, they are (i) children of the baby boom generation (born beginning in 1945) begin to enter the labor force; (ii) mortality rapidly falls with life expectancy increasing from an average of 70.8 in 1970 to 78.7 in 2010; (iii) according to Gordon (2016), the fall in the rate of productivity growth began in the 1970s; (iv) Greenwood, Hercowitz, and Krusell (1997) and Fernald (2012) have documented that the relative price of investment goods has fallen by 30 percent since 1970; and (v) Karabarbounis and Neiman (2014) have documented a significant drop in the labor share since 1970. Also, this period saw a significant rise in government and personal debt. For a full description of the changes in the economy since 1970, see online Appendix M. For the data series used for these variables, see the online Data Appendix.

We begin by targeting our calibration of the US economy to 1970 as shown in Table 4. For all type 1 parameters (those taken directly from the data), we adjust to their 1970 counterparts (see panel A), with the exception of the life-cycle income profile, which remains unchanged. All type 2 parameters (those taken from the literature) are kept the same with the exception of the relative price of investment goods (panel B). For the type 3 parameters (those we take to match moments), we keep the utility and production parameters (β, α, μ) unchanged. We adjust the collateral constraint D, to match the lower consumer-debt-to-output ratio observed in 1970, and the profit share θ to match the higher labor share observed in the 1970s (panel C). The respective 1970 values for these two parameters are then $D = 0.144$ and $\theta = 7.93$.

Table 5 shows the results of the simulation. The simulated 1970 real interest rate is 2.55 percent, extremely close to the 2.62 percent observed in the data. This is an important quantitative result; our model is able to match the entire size of the decline in real interest rates over the past 45 years. The model predicts an investment-to-output ratio (which we did not explicitly target) slightly higher than what is observed in the data. By our choice of D and θ the model matches the consumer-debt-to-output ratio exactly as well as the labor share. Overall, the model does a reasonable job of explaining the fall in the real interest rate observed over the past 45 years by using observed changes in productivity, demographics, the relative price of investment goods, credit constraints, and changes in the labor share.

Table 6 decomposes the contribution of each of these factors to the decline in interest rates. We change each parameter from its steady-state value in 2015 to its steady-state value in 1970, holding all other parameters constant. We then examine the effect of this change on the real interest rate. For example, changing productivity growth from its 2015 level of 0.65 percent per year to its 1970 level of 2.02 percent results in an increase in the steady-state real interest rate of 1.9 percentage points. The table shows a decomposition of the relative importance of all the other factors.[57] The reductions in fertility, mortality, and the rate of productivity

[57] The decomposition is calculated by first adding up the total change in the real interest rate from all these factors and then dividing the effect of changing a particular forcing variable by the entire change. Numbers may not sum to unity due to interaction effects.

Table 4—Change in Parameters from 1970 to 2015

	1970	2015
Panel A. Data		
Life expectancy	70.7	78.7
Total fertility rate	2.8	1.9
Productivity growth	2.02%	0.65%
Government debt (percent of GDP)	42%	118%
Panel B. Relative price of investment		
Relative price of investment (index 100 = 2015)	130	100
Panel C. Change in targets		
Consumer-debt-to-output ratio	4.2%	6.3%
Labor share	72.4%	66.0%

Table 5—Simulation Results for 1970

Moment	Model	US data
Natural rate of interest	2.55%	2.62%
Investment-to-output ratio	19.0%	16.8%
Consumer-debt-to-output ratio	4.2%	4.2%
Labor share	72.4%	72.4%

Table 6—Decomposition of the Decline in the Natural Rate of Interest: 1970–2015

Forcing variable	Δ in r	Percent of total Δ
Total interest rate change	−4.02%	100
Mortality rate	−1.82	43
Total fertility rate	−1.84	43
Productivity growth	−1.90	44
Government debt (percent of GDP)	+2.11	−49
Labor share	−0.52	12
Relative price of investment goods	−0.44	10
Change in debt limit	+0.13	−3

growth play the largest role in the decrease in real interest rates. The main factor that has tended to counterbalance these forces is an increase in government debt. Changes in the labor share and the relative price of investment goods play a smaller role in explaining the decline in real interest rates. Similarly, the increase in the consumer debt limit does not have a significant effect on the evolution of the interest rate in this parameterization.[58]

[58] As our calibration takes account of only consumer debt, we suspect that this force may play a substantially larger role once one takes account of housing purchases and the associated debt, along with firms borrowing and lending. Together, these forces would have tended to increase the real interest rate in the past 45 years and lowered

Table 7—Raising the Natural Rate of Interest to 1 Percent

Forcing variable	2015 value	Counterfactual value
Total fertility rate	1.88	3.28
Government debt (percent of GDP)	118%	215%
Productivity growth	0.65%	2.43%
Relative price of investment goods	1.00	2.43

E. *Raising the Natural Rate of Interest*

If the natural rate of interest is indeed -1.47 percent (the level in our baseline analysis), this poses a challenge for policymakers. With a 2 percent inflation target, small decreases in the natural rate will cause the 0 lower bound to bind, implying that downturns may be sharper and more persistent. We now consider an alternative thought experiment, taking as given that the natural rate is -1.47 percent in 2015. We ask, what economic conditions would be needed to increase the steady-state real interest rate to a positive territory of 1 percent? While this target is somewhat arbitrary, we find it to be a useful benchmark. With the Federal Reserve's inflation target of 2 percent, a natural rate of 1 percent would give policymakers a reasonable amount of room to respond to negative shocks that otherwise would lead to a binding ZLB. It also corresponds with the current expectations of the FOMC about long-run real interest rates. An alternative way of formulating our quantitative experiment is to ask, which forcing variables in 2015 would one need to change to be consistent with the current FOMC projections?

Table 7 provides the results of this experiment and shows that substantial changes in the underlying fundamentals are needed in order to increase the natural rate to 1 percent. Given current demographic trends, it is implausible that fertility will reverse its decline and dramatically increase to 3.28 births per woman. An increase in immigration could make up a portion of the difference. It would also be challenging to increase productivity growth to 2.43 percent per year given the headwinds to productivity noted by Gordon (2016) and the fact that productivity growth has rarely exceeded 2 percent since 1970.

Of particular interest is the potential impact of an increase in government debt on interest rates, given that a higher public debt is an obvious policy lever. As Table 7 shows, government debt would need to double to roughly 215 percent of GDP to increase the natural rate to 1 percent. Such a large level of debt raises questions about the feasibility of this policy, for we have not modeled any costs or limits on the government's ability to issue risk-free debt—an assumption that may be strained at such high levels. While these results suggest that several reforms would tend to increase the natural rate of interest, the menu of options does not paint a particularly rosy picture relative to the alternative of raising the inflation target of the central bank.

rates during the crisis. The increase in inequality observed during this period may have also been working to reduce interest rates; we have not taken account of this factor in the quantitative section and leave it for future research.

F. *Transition Dynamics*

So far, we have confined our analysis to stationary equilibria. While this is one natural benchmark, we can also consider transition dynamics. This requires taking a strong stance on agents' expectations during the transition. Here, we document numerical experiments in which it is assumed that the economy was at a stationary equilibrium in 1970, and then project the model forward, assuming agents have perfect foresight about the path of exogenous processes.

We feed into the model the dynamic paths for each of the forcing variables in Table 6 and calculate the resulting transition path. We document the time series for each forcing variable in the online Data Appendix. On impact in 1970, agents have perfect foresight about each of the exogenous and endogenous variables. For example, in 1970 all living agents will realize that there will be a productivity slowdown over the next 40 years and will adjust their optimal decisions accordingly. The forcing variables are set to their final steady-state values after the year 2015.

Since agents enter the labor force at the age of 26, the birthrates from 1945 to the 1970s are key state variables that will affect the real interest rate beginning in the 1970s. Since individuals entering the labor force in 1970 (age 26) were born in 1945, we use as model inputs data on fertility from 1945 onward to measure the size of each incoming generation. The birthrate data from 1945 are reflected below the solid line in the age pyramid for the US economy in Figure 9, but, in 1970, those generations are economically inactive, entering the labor force only gradually.

Figure 8 shows the full transition path for the real interest rate. The 1970 real interest rate is 2.55 percent and declines non-monotonically throughout the period until it reaches -1.0 percent in 2015. The interest rate exhibits a brief recovery in the mid-to-late 1990s as productivity growth increases, but this subsides by the mid-2000s. Note that the model projects that the interest rate will continue to decrease until it hits a nadir in 2020. After 2020, there are cycles in the real interest rate due to the echo effects of the baby boom. The economy gradually converges to the final steady-state interest rate of -1.47 percent.[59]

One interesting observation is that our model predicts a more rapid decline in the real interest rate than that observed in the data. A possible explanation for this, which we leave for future research, is offered by Summers (2014). He hypothesizes that the decline in the natural rate of interest since the late 1990s was masked by both the tech and housing bubble so that the true natural interest rate has only been observed post crisis. This could be modeled as exogenous fluctuations in the collateral constraint D in our model.

One interesting aspect of our model is that it does not generate an investment boom even though real interest rates are declining. In fact, investment actually falls in our model, from 19.0 percent in 1970 to 15.9 percent in 2015. Although lower interest rates will tend to lead to a higher capital-to-output ratio and thus a higher investment

[59] One factor we do not focus on in these transition paths is debt deleveraging. While the evidence and theory certainly suggest there was a significant deleveraging shock in 2008, our model as of yet does not include a housing sector. Since housing debt is a significant portion of consumer debt, we leave the effect of debt deleveraging on interest rates to future quantitative work.

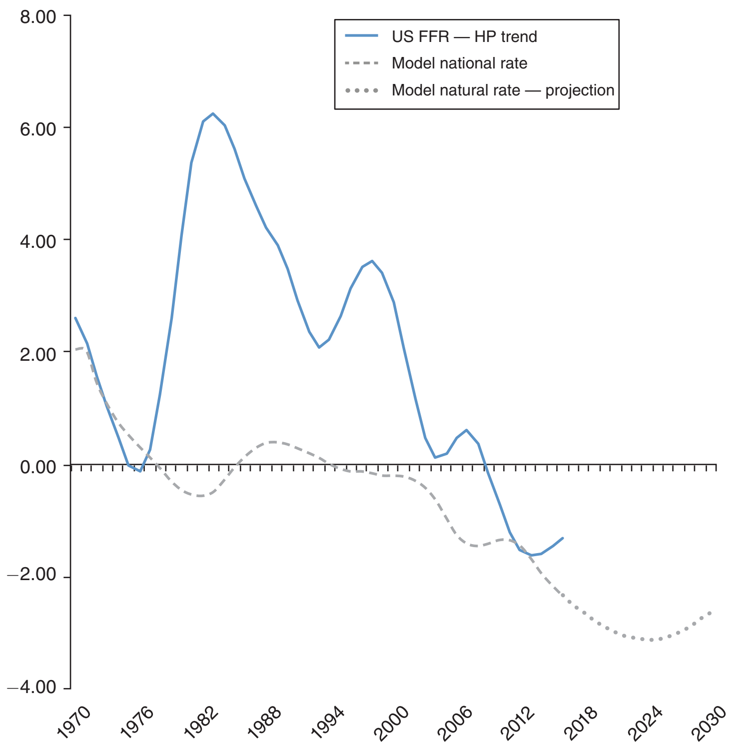

Figure 8. Transition Path of the Natural Rate of Interest

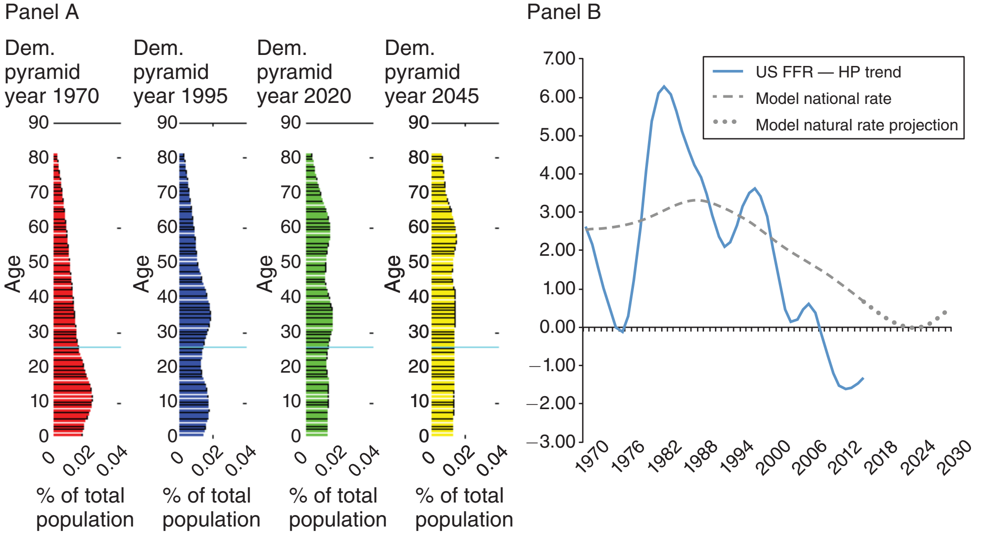

Figure 9. Baby Boom and Bust

rate, there are several counterbalancing forces in the model. In particular, decreases in the rate of population growth and productivity growth lead directly to a lower investment-to-output ratio, holding constant the capital-to-output ratio. Finally, our relatively inelastic production CES parameter means that the steady-state K/Y ratio stays relatively constant for a given interest rate, which also dampens the effect on investment.

G. *Effects of the Baby Boom*

We now study the effects of the baby boom on real interest rates. The baby boom started in 1945, so the children of the baby boom did not enter economic maturity until 1970, when they reached age 26. Thus, we are able to look at the impact of these demographic changes on the evolution of interest rates starting from our model economy in 1970.

Panel A of Figure 9 shows the demographic pyramid for the model during the transition years. In 1970, the economically active population is assumed to be in steady state; thus the pyramid above the solid line is a triangle. The younger generation of the baby boom, not yet in the labor force, bulges the distribution and will soon enter the labor force. Twenty-five years later, in 1995, the bulge now shows up in the working-age population. Fifty years later, in 2020, the baby boom generation is retired. There is also a second bulge in the population pyramid due to the children of the baby boomers (the millennials). By the year 2170, the population pyramids are approaching steady state.

Panel B of Figure 9 shows the interest rate transition for the baby boom simulation. From 1970 until 2000, the baby boom tends to increase real interest rates owing to an increase in the rate of population growth. Following the baby boom comes the baby bust, and fertility rates drop as the boomers enter retirement, which leads to a sharp drop in real interest rates. The echo effects result in cycles in the real interest rate until it reaches its final steady-state value of around 1 percent.

VIII. Conclusion

This paper builds a quantitative theory of negative interest rates and secular stagnation. We show how a low natural rate of interest can lead to a secular stagnation: a persistent output slump, inflation below target, and a chronically binding ZLB. Our message is not that the ZLB will be binding forever with certainty. A world of low natural rates admits business cycles in which the short-term rate can still be temporarily positive. Instead, it is a world characterized by a "new normal," in which real interest rates need on average to be negative to achieve full employment. As in the case of Hansen's prediction in 1938, we hope that this gloomy outlook will prove to be wrong. If negative rates persist, our analysis has identified several measures that could eliminate secular stagnation via appropriate policy.

A serious challenge, however, is that our policy recommendations advocate in favor of policies that were considered vices rather than virtues in macroeconomic theory: a higher inflation target, persistent increases in the debt-to-GDP ratio, or even more generous pay-as-you-go Social Security. This poses a conundrum for policymakers who cannot know with certainty if we are indeed in a "new normal" or just in a prolonged period of low interest rates that will abate in the near future. If the "new normal" hypothesis is incorrect, then those very policies that are desirable in order to eliminate secular stagnation are likely to be as counterproductive and costly as existing economic theory suggests. Given the uncertainty about the future, this creates new trade-offs for optimal policy, which we have not touched upon but hope that future research considers.

In closing, we briefly discuss some of the main arguments against the hypothesis of permanently negative interest rates, leaving a more detailed analysis to future research. The first argument, stated by Paul Samuelson and recently recapitulated by Ben Bernanke, is that negative real interest rates are a theoretical impossibility:

> *As…Paul Samuelson taught me in graduate school at MIT, if the real interest rate were expected to be negative indefinitely, almost any investment is profitable. For example, at a negative (or even zero) interest rate, it would pay to level the Rocky Mountains to save even the small amount of fuel expended by trains and cars that currently must climb steep grades. It's therefore questionable that the economy's equilibrium real rate can really be negative for an extended period.*

Samuelson's argument did not take into account two factors: monopoly rents and risk. In particular, under monopolistic competition, the rental rate of capital is given by $r^k = (1/markup) \cdot MPK$. With positive markups, the marginal product of capital is higher than the rental rate of capital. Thus, in equilibrium, there can be positive *social* returns to capital (even net of depreciation) while the rental rate (net of depreciation) and hence the real interest rate is negative. This is indeed the case in our quantitative model. Moreover, even though our calibrated model produces negative real interest rates, it can be shown that the economy is not dynamically inefficient.[60]

Adding risk to our model is another rejoinder to Samuelson's argument. As Abel et al. (1989) show, adding aggregate risk can lead to a negative *risk-free* interest rate, while the average and marginal return from capital (net of depreciation) remains *positive*. While adding aggregate risk to our economy is beyond the scope of this paper, we can illustrate this mechanism in a reduced-form way. In online Appendix K, we extend the model by including an additional financial friction: all borrowing and lending must now occur through a bank, which charges a spread between the borrowing and lending rates of interest. With a spread of 2 percent, for example, it is possible to have a borrowing rate of interest that is positive while the lending rate of interest is negative. Since firms borrow at the higher rate, the marginal product of capital in excess of depreciation is positive in this economy.

The second argument against the presence of secular stagnation conditions in the United States is that the measured return on capital is stable (see, for example, Gomme, Ravikumar, and Rupert 2015). The argument is that the most relevant measure of return is not the return on government debt but the return on productive

[60] The condition for dynamic inefficiency, following Abel et al. (1989), is that the economy spends more on investment in capital than it gets out of it in terms of production: $I > MPK \times K$.

capital. One problem with that argument is that business income does not just measure capital income but also pure profits.[61] And over the past decades, there is some evidence that competition has decreased and monopoly rents have risen.[62] If an increase in monopoly rents cancels out the decrease in competitive returns to capital, this would lead to a stable measured average return on capital.[63] This tends to increase the measured average return in our model through two channels: higher pure profits and a higher marginal product of capital relative to the rental rate of capital. Although our economy exhibits a decline in the marginal product of capital over time, profits increase enough to almost completely offset this decline, and thus the measured average return on capital stays constant. Future research will consider alternative measures of the return on capital and evidence from firm-level data on whether returns have fallen over time.

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[61] It is also possible that the NIPA accounts systematically understate the capital stock by mismeasuring intangible capital, as discussed in McGrattan and Prescott (2010). To the extent that this bias has increased over time, the true return on capital may be declining as our model would suggest.

[62] See, for example, Gutiérrez and Philippon (2016), CEA (2016), and Decker et al. (2016).

[63] An increase in monopoly rents is one possible explanation for the measured decrease in the labor share over the past 30 years. See in particular Barkai (2016), who finds that an increase in markups is responsible for a decrease in both the labor and the capital share.

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