Lahcen, Mohammed Ait; Gomis-Porqueras, Pedro

Working Paper

A model of endogenous financial inclusion: Implications for inequality and monetary policy

This Version is available at:
https://hdl.handle.net/10419/195162

University of Zurich

Department of Economics

Working Paper Series

Working Paper No. 310

A Model of Endogenous Financial Inclusion: Implications for Inequality and Monetary Policy

Mohammed Ait Lahcen and Pedro Gomis-Porqueras

December 2018

A Model of Endogenous Financial Inclusion: Implications for Inequality and Monetary Policy

Mohammed Ait Lahcen*[†] Pedro Gomis-Porqueras*[‡]

University of Basel Deakin University

This Version: December 17, 2018

Abstract

We propose a monetary dynamic general equilibrium model with endogenous credit market participation to study the impact of financial inclusion on welfare and inequality. We find that significant consumption inequality can result from limited access to basic financial services. In this environment, monetary policy has distributional consequences as agents face different liquidity constraints. This heterogeneity generates a pecuniary externality which can result in overconsumption of financially included agents above the socially efficient level. We conduct a quantitative assessment for the case of India. Our simple model is able to account for approximately a third of the observed consumption inequality. We analyze various policies aimed at increasing financial inclusion. As a result of pecuniary externalities, interest rate policies can result in a decrease in welfare and an increase in consumption inequality. Moreover, we find that a direct benefit transfer to bank account owners is superior to interest rate policies as it can increase welfare and reduce consumption inequality despite a decrease in individual consumption.

Keywords: money; credit; banking; financial inclusion; inequality.

JEL classification: E40, E50.

1

1 Introduction

Financial exclusion (the lack of access to basic financial services) is a widely observed phenomenon in developing countries. According to Demirgüç-Kunt et al. (2015), 94% of the adult population in OECD countries owns an account at a formal financial institution while this proportion was only about 54% in developing economies. Within the latter, the numbers vary widely from 14% in the Middle East and North Africa to 69% in East Asia and the Pacific.[1] In figure 1 we plot different measures of financial inclusion against consumption inequality using data from 159 countries. All three panels depict a negative correlation between the two: higher levels of financial inclusion are accompanied by lower levels of consumption inequality. In addition, many microeconomic studies on economic development and poverty reduction suggest that improved access to finance reduces income inequality, poverty and increases food security.[2] Increasing financial inclusion can also affect the impact and effectiveness of monetary policy. This is the case as a wider access to saving vehicles makes consumers more reactive to changes in interest rates which improves the transmission of monetary policy.[3] These different findings highlight the inherent relationship between access to financial markets, inequality and monetary policy. Here we study such links.

We consider a monetary framework with endogenous financial market participation, where financial inclusion is an equilibrium outcome. In particular, agents face idiosyncratic preference shocks that determine their willingness to consume in a frictional goods market. All agents have access to a nominal asset, namely fiat money. Anonymity in the frictional goods market makes fiat money essential as a means of payment, as unsecured credit in this market is not incentive-compatible. As a consequence, the preference shock generates uncertainty about liquidity needs. Consumers can insure against this liquidity risk by accessing a competitive banking sector.[4] However, liquidity risk-sharing through banks requires the payment of a fixed cost. This feature

---

[1]According to Allen et al. (2016), higher financial inclusion is associated with lower fees, lower physical costs of accessing financial intermediaries, stronger legal rights and more political stability.

[2]We refer to Burgess and Pande (2005); Levine (2005); Beck et al. (2005, 2007); Marshall (2004); Sarma and Pais (2011); Laha et al. (2011) for more on these issues.

[3]See Mehrotra and Yetman (2014, 2015) for a related discussion.

[4]In our environment, banks provide basic loans and deposits by intermediating between liquidity-constrained and unconstrained agents.
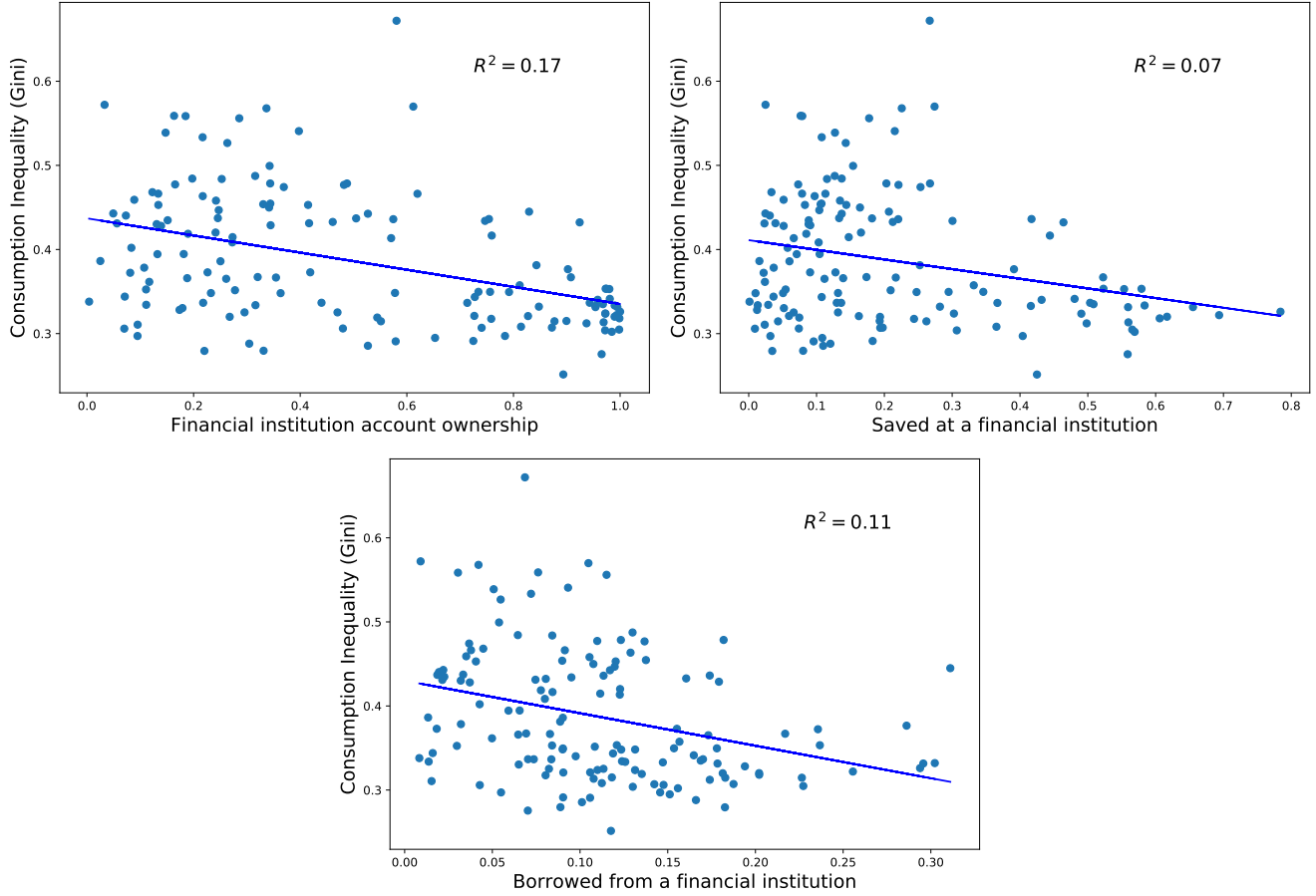
Figure 1: Consumption inequality and financial inclusion

captures the physical and informational costs that agents face when accessing banking services.[5] Since buyers face different costs, the measure of buyers who decide to do so determines endogenously the level of financial inclusion. Agents have also access to a frictionless competitive market where they can produce and trade the numeraire good, rebalance their portfolios and settle their financial obligations.

We find that significant consumption inequality can result from the limited access to basic financial services. Furthermore, the measure of financially included agents is non-monotonic in

---

[5]This type of costs has been emphasized by Allen et al. (2016) as one of the main factors influencing access to financial intermediaries.

inflation and the need for liquidity. Given that financially included and excluded agents coexist, monetary policy can have distributional consequences as agents face different liquidity constraints. Moreover, under competitive pricing in the frictional goods market this heterogeneity generates a pecuniary externality which can result in overconsumption of financially included agents above the socially efficient level. We conduct a quantitative assessment for the case of India. Our simple model is able to account for approximately a third of the observed consumption inequality. It accounts also for half of the share of consumer credit to GDP and 70% of the demand deposits to M1 ratio. We show that recent changes in the distribution of costs of accessing banking services can account for more than a third of the observed increase in financial inclusion in India. Finally, we analyze various policies aimed at increasing financial inclusion. As a result of pecuniary externalities, interest rate policies can result in a decrease in welfare and an increase in consumption inequality. We show that a borrowing interest rate subsidy is more distorting and costly than the one aimed at the deposit rate. Moreover, we find that a direct benefit transfer to bank account owners is superior to interest rate policies and can reduce consumption inequality as well as increase welfare even when individual consumption decreases. In light of these results and compared to the usual policy recommendations regarding financial inclusion, we highlight the importance of providing adequate returns on deposits and offer direct benefit transfer schemes to bank users as effective ways to improve access to the banking sector.

2   Related literature

This paper connects with three different strands of the literature. The one that explores the implications of limited access to financial markets. The literature that studies the coexistence of money and credit. Finally, this paper also contributes to the inequality and inflation literature.

The seminal papers of Chatterjee and Corbae (1992),Allen and Gale (1994) and Williamson (1994) study the consequences for the nature of equilibria of having endogenously segmented financial markets. In an environment where the demand for money results from transaction costs in other assets, Chatterjee and Corbae (1992) find that changes in the steady-state growth rate of

the money supply have a negative effect on real interest rates. Moreover, the authors show that there may be an equity-efficiency trade-off stemming from monetary deflation. Allen and Gale (1994) study the endogenous participation in asset markets in an environment based on Diamond and Dybvig (1983). These authors find that allowing for endogenous market participation, in an environment with arbitrarily small aggregate liquidity shocks, can cause significant price volatility and generate multiple equilibria. In a similar vein, Williamson (1994) considers an environment with a liquid asset traded without cost, and an illiquid asset subject to fixed transactions costs. The author shows that there exists a participation externality that tends to deliver under-provision of liquidity in equilibrium.[6] Relative to this literature we consider a monetary model where agents decide whether to participate in a credit market in the form of banking services and analyze its implications for inequality and monetary policy.

Frictions are necessary to generate an essential role for money as a medium of exchange.[7] However, some of these frictions, while making room for money, prevent the use of alternative payment instruments like credit. Using these insights, Monnet and Roberds (2008),Bencivenga and Camera (2011),Sanches and Williamson (2010), Chiu and Meh (2011), Sanches (2011), Rojas Breu (2013), Lotz and Zhang (2016), Gu et al. (2016), Chiu et al. (2018), among others, study the coexistence of money and credit in frictional environments.[8] Our paper is closely related to that of Rojas Breu (2013) and Chiu et al. (2018). Rojas Breu (2013) focuses on costless credit in an environment where uncertainty regarding the access of agents to credit generates a precautionary demand for money. Since some agents have access to credit while others don't, inflation makes consumption-risk sharing less efficient by increasing the wedge between the marginal rates of substitution of the two types of agents. Chiu et al. (2018) focus instead on costly credit in an environment with exogenous limited credit market participation. Both papers find the same effects of an increase in inflation. In addition, both show that an increase in credit market participation has an ambiguous effect on welfare. On the one hand it increases welfare by allowing more agents

---

[6]The same result is highlighted by Berentsen et al. (2014).

[7]Absence of double coincidence of wants, spatial separation, absence of a record keeping technology and absence of commitment have been advocated to explain the use of money to facilitate exchange.

[8]See Lagos et al. (2017) for a recent review of the New Monetarist literature and Rocheteau and Nosal (2017) for a textbook treatment.

to insure against liquidity risk. On the other hand, it generates a pecuniary externality which tightens the liquidity constraint on agents without access to credit. We contribute to the literature on money and credit, by endogenizing the participation in bank-intermediated credit markets and studying the resulting welfare and consumption inequality implications. By endogenizing the participation margin, the occurrence of the pecuniary externality is not limited to exogenous changes in credit market participation as in Chiu et al. (2018) and Rojas Breu (2013) but results from any policy that might affect the decision of agents to participate in credit markets. This allows us to analyze the impact on welfare and consumption inequality of several policies aimed at increasing financial inclusion.

Finally, there is a literature that has studied the relationship between inflation and inequality. Countries with a more unequal income distribution tend to have higher inflation.[9] There have been few attempts to rationalize this fact. Erosa and Ventura (2002) build a monetary growth model consistent with key features of cross-sectional household data and use this framework to study the distributional impact of inflation. Individuals hold money, although it is dominated in rate of return, because they value a large number of consumption goods and purchasing goods with credit is costly. If credit services exhibit economies of scale, inflation can work as a non-linear regressive consumption tax.[10] Gomis-Porqueras (2001) considers a monetary growth model where the use of discriminatory reserve requirements results in segmented financial markets where the high-skilled workers have access to better saving opportunities compared to the low-skilled ones. He shows that limiting the access of low-skilled workers to financial markets increases the demand for real balances and hence reduces inflation and the investment in physical capital. This in turn can increase welfare and reduce wage inequality between high and low-skilled workers. Cysne et al. (2005), instead, consider a shopping-time framework where agents have different productivity

---

[9]We refer to Beetsma and Van Der Ploeg (1996); Romer and Romer (1999); Easterly and Fischer (2001); Albanesi (2007) among others, for more details about such findings. For example, Romer and Romer (1999), using data for a large sample of countries from the 1970s and 1980s, find that a country with inflation one standard deviation above average is predicted to have a Gini coefficient 3.3 percentage points above average. Albanesi (2007) finds a positive correlation between average inflation tax and the Gini coefficient for a sample of 51 industrialized and developing countries, averaged over the time period from 1966 to 1990. This is empirically confirmed by.

[10]This is because high income households pay a higher fraction of their purchases with credit and hold less money as a fraction of total assets compared with low income households.

levels and differentiated access to financial asset markets.[11] The authors show, provided that the productivity of the interest-bearing asset in the transacting technology is high enough, there exists a positive correlation between inflation and income inequality. Along the same lines, Menna and Tirelli (2017) considers a DSGE model characterized by limited financial market participation. The authors show that a combination of higher inflation and lower income taxes reduces inequality. Another strand of the literature has used a political economy framework. For instance, Dolmas et al. (2000) consider an endowment overlapping generations economy where fiat money is the only storable asset. Since agents have different endowments voting in this environment illustrates how greater inequality leads to greater inflation. Along the same lines, Albanesi (2007) considers a monetary economy in which income inequality is an increasing function of exogenous differences in human capital and the nature of the transaction technology that makes low income households more vulnerable to inflation. The resulting political economy equilibrium is one where inflation is positively related to the degree of inequality in income.

In contrast to the literature that delivers a positive relationship between inflation and inequality, we do not consider credit services that exhibit economies of scale nor an exogenous limited participation to the market for interest bearing assets nor political economy considerations. The resulting consumption inequality is a direct consequence of the endogenous choice to use costly financial services or not.

3   Environment

The general environment is based on Lagos and Wright (2005), Rocheteau and Wright (2005) and Berentsen, Camera, and Waller (2007). Time is discrete and continues forever. The economy is populated by two types of infinitely lived agents each of unit measure: buyers and sellers. Private agents trade in sequential goods markets that differ in terms of their frictions. In addition, agents have access to financial intermediaries operating in a competitive market to finance part of their consumption. Buyers and sellers discount the future at rate $\beta \in (0, 1)$.

---

[11] In particular, the poor only have access to currency to smooth their consumption.

Each period is divided into three consecutive sub-periods. In the first sub-period, buyers have access to a competitive banking sector, which offers loans and deposits. We call this market the BM. In the second sub-period, buyers and sellers trade a specialized perishable good in an anonymous competitive market, which we refer to as the AM. Finally, in the third sub-period, agents have access to a frictionless competitive market where they can produce and trade the numeraire perishable good, rebalance their portfolios and settle their financial obligations. We refer to this market as the CM.

Since buyers are anonymous and sellers do not have access to record-keeping services in the AM, a medium of exchange is essential for trades to take place. In contrast, since agents can produce and consume the CM numeraire good, a medium of exchange in this market is not essential. The only durable asset in this economy is an intrinsically useless object issued by the government; i.e, fiat money which we denote by M_t. The supply of money grows at a rate $\gamma > 1$ and is injected (withdrawn) through lump sum transfers (taxes) in the CM.

Preferences and technologies: Buyers are subject to an idiosyncratic preference shock that affects their marginal utility of consumption in the AM. In particular, with probability σ, a buyer gets utility $u(q)$ of consuming q AM goods, where $u'(q) > 0$, $u''(q) < 0$, $u'(0) = +\infty$ and $u'(+\infty) = 0$. With probability $1 - \sigma$, a buyer obtains no utility from consuming the AM good. This preference shock is independent across buyers and time. It results in heterogeneity among buyers in terms of liquidity needs.[12]

In the CM, all agents can consume and produce the CM good. By consuming x units of the CM good, the buyer obtains utility $U(x)$, where $U'(x) > 0$, $U''(x) \leq 0$, $U'(0) = +\infty$ and $U'(+\infty) = 0$. Agents derive linear disutility when producing the CM good. Thus the period utility of a buyer is given by

$$\mathcal{U}^b = \sigma u(q) + U(x) - x. \tag{1}$$

Sellers incur disutility $c(q)$ when producing q units of the AM good; where $c'(q) > 0$ and

---

[12]This setup is isomorphic to a model with decentralized trades and search frictions where the probability of finding a seller in the AM is σ.

$c''(q) \geq 0$. Similar to buyers, sellers can produce the numeraire good using a linear production technology, where one unit of labor produces one unit of the CM good. Hence, the sellers' period utility is given by

$$\mathcal{U}^s = -c(q) + U(x) - x. \tag{2}$$

As in Berentsen et al. (2007), financial intermediaries accept one-period nominal deposits and offer one-period nominal loans. From now on we refer to these intermediaries as banks. This is the case as they perform some of the banks' functions. Banks have access to a costless record keeping technology that allows them to register the identity of agents. Moreover, the government is able to enforce deposit and loan contracts in the CM. These two assumptions make financial intermediation possible. We rule out issues of commitment and assume borrowers do not default on their loans and banks are fully committed to pay their depositors.

At the end of each CM, every buyer faces an idiosyncratic, fixed and time-invariant cost ε of accessing the banking sector in the following BM. These costs are distributed according to $F(\varepsilon)$ with support $[\underline{\varepsilon}, \bar{\varepsilon}]$. They capture the physical and informational costs that buyers face when accessing financial services.
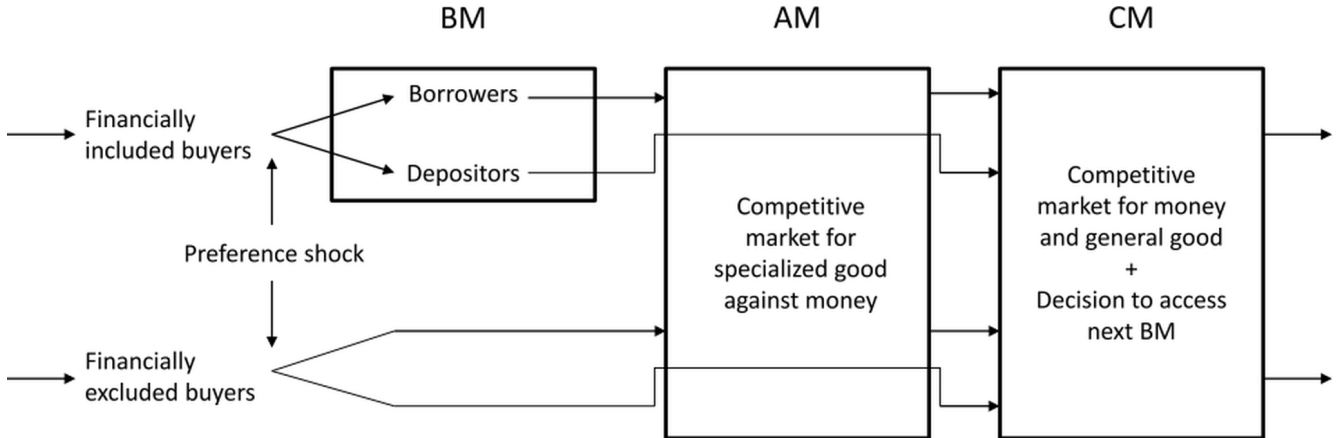
Figure 2: Timeline

Timing: The timeline of the model is depicted in figure 2. At the beginning of each period, buyers are subject to the preference shock σ. Once the shock is realized, buyers can access the BM. After this market closes, buyers and sellers trade in the AM and subsequently in the CM.

9

To simplify notation, we drop the time index for current period variables and index the next and previous periods' variables by $+1$ and -1 respectively.

4   Planner's solution

The social planner maximizes the expected life-time utility of buyers and sellers given by

$$(1 - \beta)\mathcal{W} = \sigma u(q_b) - c(q_s) + 2U(x) - 2x \tag{3}$$

subject to the resource constraint

$$\sigma q_b = q_s. \tag{4}$$

The efficient allocation is then given by

$$U'(x^*) = 1, \tag{5}$$

$$u'(q_b^*) = c'\left(\sigma q_b^*\right). \tag{6}$$

5   Decentralized solution

In what follows we describe agents' decision problems and determine the resulting equilibria. We focus on stationary monetary equilibria where aggregate real balances are constant over time. This implies that $\phi M = \phi_{+1} M_{+1}$; where ϕ is the value of money in units of the numeraire CM good.

Given the sequential nature of agents decisions, we first start with the CM problem then we study separately the optimal decisions for financially included and excluded agents when they trade in the AM and BM. Finally, we solve the banks' problem.

5.1 Sellers' problem

CM problem: In order to focus our analysis on buyers, we assume that sellers do not have access to banking services and hence solve the following optimization problem:[13]

$$W^s(m) = \max_{x,h,m_{+1}} U(x) - h + \beta V^s_{+1}(m_{+1}) \tag{7}$$

$$\text{s.t.} \quad x + \phi m_{+1} = h + \phi m + \mathcal{T} \tag{8}$$

where V^s and W^s denote the AM and CM value functions, respectively, $\mathcal{T}$ is the real monetary lump sum transfers. Substituting h from the budget constraint into the objective function, we can rewrite the seller's CM problem as follows

$$W^s(m) = \max_{x,m_{+1}} U(x) - x + \phi(m - m_{+1}) + \mathcal{T} + \beta V^b_{+1}(m_{+1}) \tag{9}$$

which yields the following first order and envelope conditions

$$U'(x) = 1 \tag{10}$$

$$\beta V^{s'}_{+1}(m_{+1}) = \phi \tag{11}$$

$$W^s_m = \phi. \tag{12}$$

AM problem: In the AM, sellers take the price of the AM good p as given and choose the quantity to be supplied, q_s, by solving

$$V^s(m) = \max_{q_s} \ -c(q_s) + W^s(m + pq_s) \tag{13}$$

which results in the following first order condition

$$c'(q_s) = \phi p. \tag{14}$$

---

[13]If allowed to participate in the banking sector, sellers will be indifferent in equilibrium as discussed in Rocheteau and Nosal (2017, chap. 8, p. 228). It is enough to assume they face an arbitrarily small cost of accessing banks to rule out their participation.

AM envelope condition: Taking the derivative of the seller's expected AM value function (13) with respect to money holdings, we have that

$$V_m^s = \frac{\partial V^s(m)}{\partial m} = W_m^s = \phi \tag{15}$$

where we replaced W_m^s by its value from (12). This last expression reflects the fact that sellers can only benefit from carrying an additional unit of money by spending it in the next CM since they don't consume in the AM.

5.2 Buyers' problem

Depending on their idiosyncratic cost of accessing banking services ε, some buyers will find it worthwhile to borrow or deposit in the BM, while others will exclusively use their money holdings to consume in the AM. We first start by solving for the optimal decisions of financially excluded buyers and then characterize the optimal choices of financially included buyers.

5.2.1 Financially excluded buyers

CM problem: The problem facing financially excluded buyers is similar to the one facing buyers in Rocheteau and Wright (2005). At the beginning of the third sub-period, a financially excluded buyer enters a frictionless competitive Walrasian market with m units of fiat money. In this market, buyers choose CM consumption and effort as well as fiat money holdings m_{+1} to bring forward to the next period. More formally, buyers solve the following optimization problem

$$W^b(\varepsilon, m) = \max_{x,h,m_{+1}} U(x) - h + \beta V(\varepsilon, m_{+1}) \tag{16}$$

$$\text{s.t. } x + \phi m_{+1} = h + \phi m + \mathcal{T} \tag{17}$$

where V^b and W^b denote the AM and CM value functions, respectively. Finally, ε represents the buyers' cost of accessing financial services, which is time invariant. For financially excluded buyers, since they choose not to use banking services, this cost is not incurred.

Substituting h from the budget constraint into the objective function, we can rewrite the agent's second sub-period problem as follows

$$W^b(\varepsilon, m) = \max_{x, m_{+1}} U(x) - x + \phi(m - m_{+1}) + \mathcal{T} + \beta V^b(\varepsilon, m_{+1}) \tag{18}$$

which yields the following first order and envelope conditions

$$U'(x) = 1 \tag{19}$$

$$\beta V^b_{m+1}(\varepsilon, m_{+1}) = \phi \tag{20}$$

$$W^b_m = \phi. \tag{21}$$

It is worth highlighting that the consumption of the CM good coincides with the efficient allocation. To determine whether the consumption of the AM good is efficient or not, we need to determine the value of bringing an additional unit of fiat money to the AM.

AM problem: The expected value function of a financially excluded buyer facing financial access cost ε and entering the AM with money holdings m is given by

$$V^b(\varepsilon, m) = \sigma \left[u(q_b) + W(\varepsilon, m - pq_b) \right] + (1 - \sigma) \left[W(\varepsilon, m) \right] \tag{22}$$

where q_b is the amount of the AM good demanded by the buyer.

A financially excluded buyer who wants to consume in the AM faces the following problem

$$\max_{q_b} u(q_b) + W^b(\varepsilon, m - pq_b, 0, 0) \quad \text{s.t.} \quad pq_b \leq m \tag{23}$$

which results in the following first order condition

$$\frac{u'(q_b)}{\phi p} = 1 + \frac{\lambda_m}{\phi} \tag{24}$$

where λ_m is the Lagrange multiplier associated with the cash feasibility constraint, whereby buyers

cannot spend more in AM goods than the amount of cash they have carried into the AM.

Using equation (14) this results in

$$\frac{u'(q_b)}{c'(q_s)} = 1 + \frac{\lambda_m}{\phi}. \tag{25}$$

It is worth noticing that if the cash feasibility constraint does not bind, such that $\lambda_m = 0$, then (25) reduces to $u'(q_b) = c'(q_s)$. However, if $\lambda_m > 0$ then we have that

$$\frac{u'(q_b)}{c'(q_s)} > 1 \tag{26}$$

which implies that a financially excluded buyer will be constrained by his money holdings.

AM envelope condition: Having characterized the resulting terms of trade in the AM, we can now establish the marginal value of bringing an additional unit of money to the AM for financially excluded buyers. If we take the derivative of the expected value function of the AM (given by equation (22)) with respect to money holdings, we have that

$$V_m^b(\varepsilon, m) = \frac{\partial V(\varepsilon, m)}{\partial m} = \sigma \left[u'(q_b)\frac{\partial q_b}{\partial m} + W_m(1 - p\frac{\partial q_b}{\partial m}) \right] + (1 - \sigma)W_m. \tag{27}$$

As long as holding money is costly (i.e. $\gamma > \beta$), we have $q_b = \frac{m}{p}$, hence $\frac{\partial q_b}{\partial m} = \frac{1}{p} = \frac{\phi}{c'(q_s)}$. From (21), we have $W_m = \phi$. Taking this into account, we can simplify the previous expression to

$$V_m^b(\varepsilon, m) = \phi \left[\sigma \frac{u'(q_b)}{c'(q_s)} + (1 - \sigma) \right]. \tag{28}$$

5.2.2 Financially included buyers

CM problem: The choices of financially included buyers are similar to Berentsen et al. (2007). They enter the CM with a portfolio of fiat money $\hat{m}$, nominal loans ℓ and nominal deposits d. In this market, buyers choose their CM consumption and effort as well as their fiat money holdings to bring forward to the next period. In addition, they have to incur a cost ε to have access to the

14

BM in the next period. Formally, financially included buyers solve

$$\hat{W}^b(\varepsilon, m, \ell, d) = \max_{x, h, \hat{m}_{+1}} U(x) - \varepsilon - h + \beta \hat{V}^b(\varepsilon, \hat{m}_{+1}) \tag{29}$$

$$\text{s.t. } x + \phi \hat{m}_{+1} = h + \phi \hat{m} + \phi(1 + i_d)d - \phi(1 + i_\ell)\ell + \mathcal{T} \tag{30}$$

where $\hat{V}^b$ and $\hat{W}^b$ denote the AM and CM value functions of financially included agents, respectively, i_d represents the interest rate earned on deposits and i_ℓ is the lending rate. In addition, financially included buyers incur the cost of access to financial services associated with the location ε in the CM. It is important to highlight that for financially included buyers the cost of financial inclusion is lower than its benefit such that the buyer is willing to access bank services so he can borrow ℓ or deposit d.

Substituting h from the budget constraint into the objective function, we can rewrite the agent's second sub-period problem as follows

$$\hat{W}^b(\varepsilon, \hat{m}, \ell, d) = \max_{x, \hat{m}_{+1}} U(x) - \varepsilon - x + \phi(\hat{m} - \hat{m}_{+1} + (1 + i_d)d - (1 + i_\ell)\ell) + \mathcal{T} + \beta \hat{V}^b(\varepsilon, \hat{m}_{+1})$$

$$\tag{31}$$

which yields the following first order and envelope conditions

$$U'(x) = 1 \tag{32}$$

$$\beta \hat{V}^b_{\hat{m}_{+1}}(\varepsilon, \hat{m}_{+1}, \ell, d) = \phi \tag{33}$$

$$\hat{W}^b_{\hat{m}} = \phi \tag{34}$$

$$\hat{W}^b_d = \phi(1 + i_d) \tag{35}$$

$$\hat{W}^b_\ell = -\phi(1 + i_\ell). \tag{36}$$

Again, to determine whether the consumption of the AM good is efficient or not, we need to determine the value of bringing an additional unit of fiat money to the next period.

AM and BM problem: Before the preference shock is realized, the expected value of a financially included buyer entering the BM and AM with money holdings m is given by

$$\hat{V}^b(\varepsilon, \hat{m}) = \sigma \left[u(\hat{q}_b) + \hat{W}^b(\varepsilon, \hat{m} + \ell - p\hat{q}_b, 0, \ell) \right] + (1 - \sigma)\hat{W}^b(\varepsilon, \hat{m} - d, d, 0) \tag{37}$$

where $\hat{q}_b$ is the quantity of AM goods consumed by financially included buyers. Note that, in principle, the amount of goods that buyers purchase in the AM can be different depending on whether they have access to financial intermediaries ($\hat{q}_b$) or not (q_b).

At the beginning of the period and after preference shocks are realized, banks open and offer their services to buyers. The latter can borrow fiat money to top up their real balances in order to increase the quantity of AM goods they can purchase. Alternatively, they can deposit their idle money holdings with the bank and earn some interest. Once banks close their doors, buyers and sellers trade the AM good for fiat money.

A financially included buyer that cannot consume in the AM decides how much they will deposit in the bank (d). Formally, the depositor's problem is given by

$$\max_{d} \quad \hat{W}^b(\varepsilon, \hat{m} - d, d, 0) \ \text{ s.t. } \ d \leq \hat{m} \tag{38}$$

where the constraint reflects the fact that the buyer cannot deposit more than the fiat money he has brought into the AM. It is easy to see that if $i_d > 0$, the buyer will deposit all his money holdings with the bank. This implies that the constraint holds with equality.

Using equations (34) and (35), we have that the first order condition for the choice of deposits is given by

$$\lambda_d = \phi i_d \tag{39}$$

where λ_d is the Lagrange multiplier on the constraint and represents the depositor's shadow value of depositing their idle money holdings into the bank. This implies that the depositor will always

deposit all his money holdings

$$d = \hat{m} \tag{40}$$

as long as money is valued ($\phi > 0$) and the interest rate earned on deposits is positive ($i_d > 0$).

A financially included buyer who consumes in the AM faces the following problem

$$\max_{\hat{q}_b, \ell} \quad u(\hat{q}_b) + \hat{W}^b(\varepsilon, \hat{m} + \ell - p\hat{q}_b, 0, \ell) \text{ s.t. } p\hat{q}_b \leq \hat{m} + \ell. \tag{41}$$

Using equations (14), (34) and (36), the optimal choices in AM can be summarized as follows

$$\ell = p\hat{q}_b - \hat{m} \tag{42}$$

$$\frac{u'(\hat{q}_b)}{c'(q_s)} = 1 + \frac{\hat{\lambda}_m}{\phi} \tag{43}$$

$$i_\ell = \frac{\hat{\lambda}_m}{\phi} \tag{44}$$

where $\hat{\lambda}_m$ is the Lagrangian corresponding to the cash feasibility constraint whereby the buyer cannot spend more in AM goods than the amount of cash they have carried from the previous CM and the cash loan they have borrowed from the bank.

It is worth noticing that if the cash feasibility constraint does not bind, such that $\hat{\lambda}_m = 0$, then (43) and (44) reduce to $u'(\hat{q}_b) = c'(q_s)$. However, if $\hat{\lambda}_m > 0$ then we have that

$$\frac{u'(\hat{q}_b)}{c'(q_s)} = 1 + i_\ell \tag{45}$$

which implies that a financially included buyer is constrained by his money holdings. As a result, they will borrow up to the point where the marginal benefit of borrowing is equal to its marginal cost $1 + i_\ell$. Their AM consumption will be $\hat{q}_b = \frac{\hat{m}+\ell}{p}$.

AM envelope condition: Having characterized the terms of trade in the AM, we can now establish the marginal value of bringing an additional unit of money to the AM for financially included buyers. If we now take the derivative of the expected value function of the AM (given

by equation (37)) with respect to money holdings, we have that

$$\hat{V}_{\hat{m}}^b(\varepsilon, \hat{m}) = \frac{\partial \hat{V}^b(\varepsilon, \hat{m})}{\partial \hat{m}} = \sigma \left[u'(\hat{q}_b)\frac{\partial \hat{q}_b}{\partial \hat{m}} + \hat{W}_m^b\left(1 + \frac{\partial \ell}{\partial \hat{m}} - p\frac{\partial \hat{q}_b}{\partial \hat{m}}\right) + \hat{W}_\ell^b\frac{\partial \ell}{\partial \hat{m}} \right] + (1-\sigma)\left[\hat{W}_m^b\left(1 - \frac{\partial d}{\partial \hat{m}}\right) + \hat{W}_d^b\frac{\partial d}{\partial \hat{m}} \right]. \tag{46}$$

From (34), (35) and (36) we have $\hat{W}_m^b = \phi$, $\hat{W}_d^b = \phi(1 + i_d)$ and $\hat{W}_\ell^b = -\phi(1 + i_\ell)$. In addition, we know that $\frac{\partial d}{\partial \hat{m}} = 1$ since the buyer will deposit all his money holdings as long as $i_d > 0$. Taking into account what precedes simplifies $\hat{V}^{b\prime}(\hat{m})$ to

$$\hat{V}_{\hat{m}}^b(\varepsilon, \hat{m}) = \sigma \left[u'(\hat{q}_b)\frac{\partial \hat{q}_b}{\partial \hat{m}} + \phi\left(1 + \frac{\partial \ell}{\partial \hat{m}} - p\frac{\partial \hat{q}_b}{\partial \hat{m}}\right) - \phi(1 + i_\ell)\frac{\partial \ell}{\partial \hat{m}} \right] + (1-\sigma)\phi(1 + i_d). \tag{47}$$

For $i_\ell > 0$, $p\hat{q}_b = \hat{m} + \ell$ which means $p\frac{\partial \hat{q}_b}{\partial \hat{m}} = 1 + \frac{\partial \ell}{\partial \hat{m}}$. Using this into the previous expression and rearranging terms we get

$$\hat{V}_{\hat{m}}^b(\varepsilon, \hat{m}) = \sigma \left[\frac{\partial \hat{q}_b}{\partial \hat{m}}(u'(\hat{q}_b) - p\phi(1 + i_\ell)) + \phi(1 + i_\ell) \right] + (1-\sigma)\phi(1 + i_d). \tag{48}$$

From (45) we have $u'(\hat{q}_b) = c'(q_s)(1 + i_\ell) = \phi p(1 + i_\ell)$ which yields the following:

$$\hat{V}_{\hat{m}}^b(\varepsilon, \hat{m}) = \phi\left[\sigma i_\ell + (1-\sigma)i_d + 1\right]. \tag{49}$$

5.2.3 Decision to access financial services

At the end of each CM, a buyer facing cost ε will choose whether to access to financial services in the next period or not. Given next period's monetary and financial conditions, his choice in the CM must satisfy

$$\max\{-\phi m_{+1} + \beta V_{+1}(\varepsilon, m_{+1}), -\varepsilon - \phi \hat{m}_{+1} + \beta \hat{V}_{+1}^b(\varepsilon, \hat{m}_{+1})\}. \tag{50}$$

It is easy to see that the money holdings and the value function of a financially excluded

18

buyer are independent of his cost ε, while the value function of a financially included buyer is monotonically decreasing in ε. Figure 3 describes the decision rule that buyers follow in the CM each period: those with cost $\varepsilon \leq \tilde{\varepsilon}$ will use banking services, whereas those with cost $\varepsilon \geq \tilde{\varepsilon}$ will remain financially excluded.
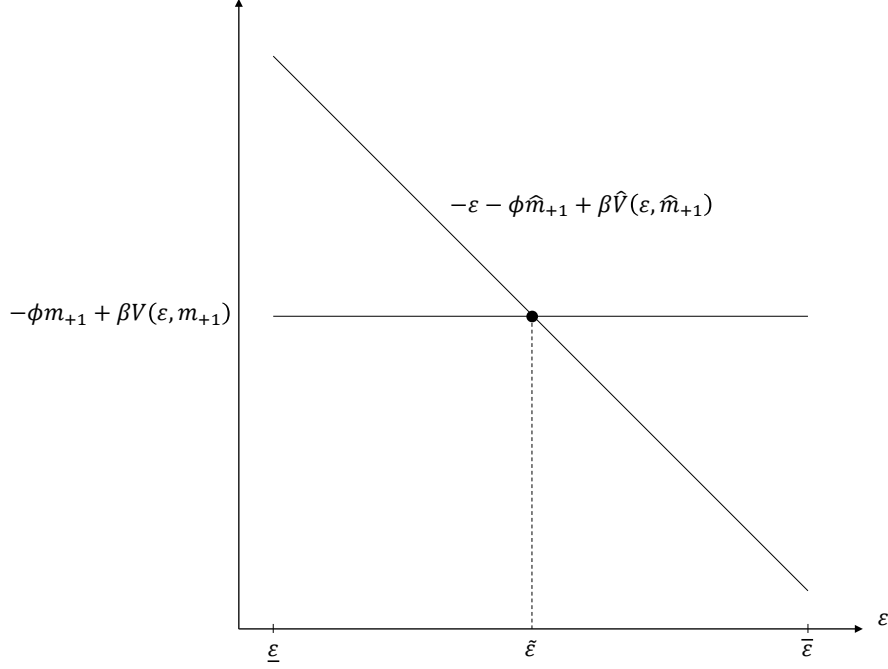
Figure 3: Buyer's financial access as a function of ε

It is straightforward to see that $\tilde{\varepsilon}$ satisfies the following indifference condition

$$- \phi m_{+1} + \beta V_{m+1}(\tilde{\varepsilon}, m_{+1}) = -\tilde{\varepsilon} - \phi \hat{m}_{+1} + \beta \hat{V}^b_{m+1}(\tilde{\varepsilon}, \hat{m}_{+1}). \tag{51}$$

Having characterized the decision to access financial services, we need to solve the banks' problem in order to determine the equilibrium interest rates and the resulting measures of financially included $(F(\tilde{\varepsilon}))$ and excluded $(1 - F(\tilde{\varepsilon}))$ buyers.

5.3 Banks

Banks trade both loans and deposits in perfectly competitive markets where they take interest rates as given. A bank accepts nominal deposits d, paying nominal interest rate i_d, and issues

19

loans ℓ, charging borrowers the nominal interest rate i_ℓ.

We restrict our attention to banking systems where a bank can only supply an amount of loans smaller or equal to the amount of deposits it demands. Each bank maximizes its profits by deciding the amount ℓ to lend per borrower subject to the deposit constraint. Since banks face free entry it follows that $i_\ell = i_d \equiv i$.

6 Stationary monetary equilibria

In a stationary monetary equilibrium, we know from equations (11) and (15) that sellers will be indifferent between carrying money across periods or not when the condition

$$\frac{\gamma - \beta}{\beta} \equiv \bar{\iota} = 0 \tag{52}$$

is satisfied. Recall that $\bar{\iota}$ represents the opportunity cost of holding money from one CM to the next.[14] From now on, we focus only on equilibria where $\bar{\iota} > 1$ (i.e. $\gamma > \beta$) such that sellers do not hold any money balances.

Financially excluded buyers' intertemporal equation resulting from (20) and (28) is given by

$$\bar{\iota} = \sigma \left[\frac{u'(q_b)}{c'(q_s)} - 1 \right] \tag{53}$$

where the left hand side of equation (53) describes the cost of holding one extra unit of money, while the right hand side represents the expected return. An extra unit of money allows a financially excluded buyer to consume an extra unit of the AM good.

The intertemporal trade-off facing financially included buyers results from equations (33) and (49) and can be summarized as follows

$$\bar{\iota} = \sigma i_\ell + (1 - \sigma) i_d \tag{54}$$

---

[14]If $\gamma = \beta$ holds, sellers will carry an indeterminate amount of money as discussed by Rocheteau and Wright (2005).
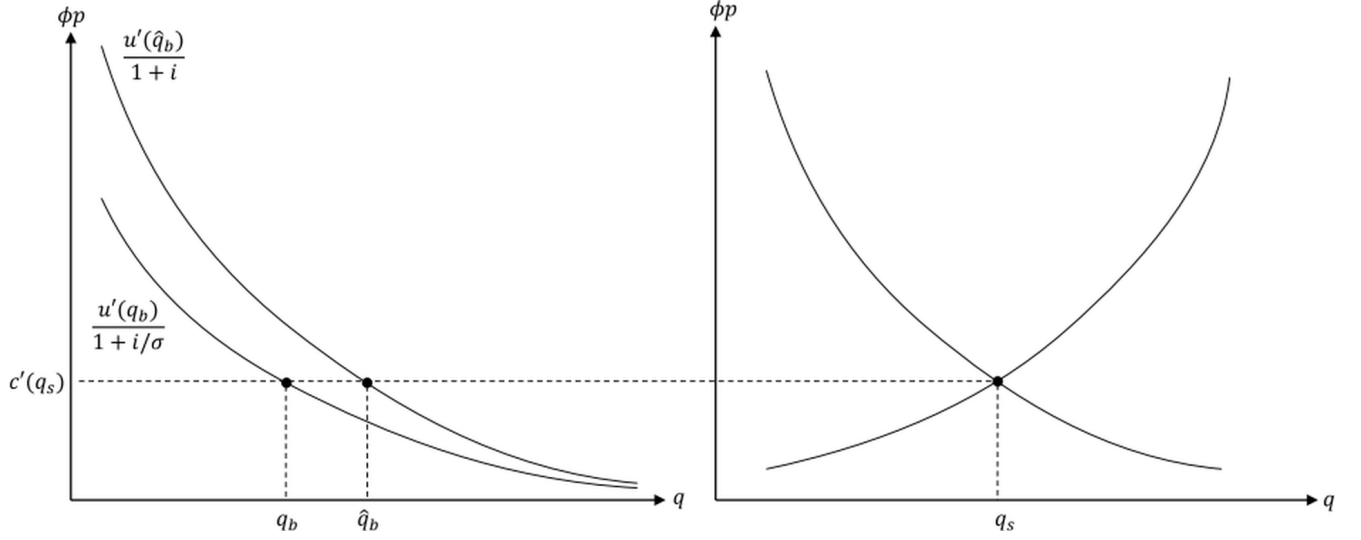
Figure 4: Equilibrium in the AM

where

$$i_\ell = \frac{u'(\hat{q}_b)}{c'(q_s)} - 1 \tag{55}$$

holds from equation (45). The left hand side of equation (54) describes the net cost of holding one extra unit of money to the next period while the right hand side represents the net expected return. An extra unit of money allows a borrower to reduce his costs by borrowing one unit less of money from the banking sector. For the depositor, taking one extra unit of money allows him to increase his money holdings through the interest bearing deposit account.

Using the free entry condition in the banking sector, $i_\ell = i_d \equiv i$, in (54) and (55) we get

$$\bar{\iota} = i \tag{56}$$

and

$$i = \frac{u'(\hat{q}_b)}{c'(q_s)} - 1 \tag{57}$$

where i, the interest rate prevailing in the BM, equals in equilibrium the Fisher equation nominal interest rate $\bar{\iota}$. Comparing equations (53) and (57) indicates that the quantity consumed by financially included buyers is always higher than the quantity consumed by financially excluded buyers since the former face a lower marginal cost of carrying money balances.

21

To close the model, markets have to clear. In particular, the amount of goods traded in the AM has to satisfy

$$\sigma \left((1 - F(\tilde{\varepsilon}))q_b + F(\tilde{\varepsilon})\hat{q}_b \right) = q_s. \tag{58}$$

as depicted in the right panel of figure 4.

In a symmetric equilibrium, all financially included buyers borrow and deposit the same amounts ℓ and d respectively. As a consequence, BM clearing implies the following

$$\sigma F(\tilde{\varepsilon})\ell = (1 - \sigma)F(\tilde{\varepsilon})d. \tag{59}$$

Combining the previous equilibrium conditions, we can simplify the cost threshold equation (A.8) derived in the appendix to get

$$\tilde{\varepsilon} = \beta\sigma \left[(u(\hat{q}_b) - u'(\hat{q}_b)\hat{q}_b) - (u(q_b) - u'(q_b)q_b) \right] \tag{60}$$

which simply states that the level of financial inclusion is determined by the discounted net utility gain of accessing banking services weighted by the probability of the preference shock. Buyers facing cost $\tilde{\varepsilon}$ are exactly indifferent between paying this cost or enjoying the utility gain of bank access.

Finally, the money demanded by buyers equals the money supplied by the government such that

$$\phi M = (1 - F(\tilde{\varepsilon}))\phi m + F(\tilde{\varepsilon})\phi\hat{m}. \tag{61}$$

Definition 1 *Given a nominal interest rate $\bar{\iota}$, a symmetric monetary equilibrium is a threshold $\tilde{\varepsilon}$, an interest rate on loans and deposits i, real balances $\{\phi m, \phi\hat{m}\}$ and AM quantities and real price $\{q_b, \hat{q}_b, q_s, \phi p\}$ that satisfy the optimal choices of agents and clear markets.*

To summarize, a symmetric monetary equilibrium with competitive banks satisfies the following equilibrium conditions

$$\sigma \left[\frac{u'(q_b)}{c'(q_s)} - 1 \right] = \bar{\iota}$$

22

$$\frac{u'(\hat{q}_b)}{c'(q_s)} - 1 = i$$

$$\bar{\iota} = i$$

$$\phi p = c'(q_s)$$

$$\phi m = \phi p q_b$$

$$\phi \hat{m} = \sigma \phi p \hat{q}_b$$

$$\sigma \left((1 - F(\tilde{\varepsilon})) q_b + F(\tilde{\varepsilon}) \hat{q}_b \right) = q_s$$

$$\tilde{\varepsilon} = \beta \sigma \left[(u(\hat{q}_b) - u'(\hat{q}_b)\hat{q}_b) - (u(q_b) - u'(q_b)q_b) \right].$$

In what follows we discuss the monetary equilibria resulting from different values taken by the model's parameters and in particular the money growth rate γ and the distribution of financial access costs $F(\varepsilon)$.[15]

6.1 Pure monetary equilibria

When money is costless to hold and/or when accessing banking services is too costly, the environment exhibits monetary equilibria where there is no demand for financial services. This can arise in two different circumstances.

Proposition 1 *As $\gamma \to \beta$, when the costs of accessing financial services are strictly positive buyers choose to remain financially excluded. When they are costless, buyers are indifferent. Thus banking services are always irrelevant for the real allocations and the equilibrium consumption coincides with the first-best allocation: $q_b = \hat{q}_b = q_b^*$.*

Proof. The proof can be found in appendix D.1. ∎

When the Friedman rule is satisfied ($\gamma \to \beta$), carrying money across periods is costless. This means the risk that real balances remain unused following a negative preference shock is irrelevant.

---

[15]We always operate under the assumption $\sigma < 1$.

Buyers can perfectly self-insure and there is no demand for financial intermediation. The BM is generically inactive.

Proposition 2 *If $\gamma > \beta$ and the costs of financial access are sufficiently high, buyers choose to remain financially excluded ($F(\tilde{\varepsilon}) = 0$). In this case, there exists a unique monetary equilibrium where the BM is inactive and consumption is below the first-best: $q_b < q_b^*$.*

Proof. The proof can be found in appendix D.2. ∎

It is worth highlighting that the equilibrium described in proposition 1 corresponds to the monetary equilibrium in Rocheteau and Wright (2005) under the Friedman rule while proposition 2 corresponds to the monetary equilibrium away from the Friedman rule.

6.2 Monetary and banking equilibria

Here we explore situations where equilibria with both financial intermediation and money can exist.

Proposition 3 *If $\gamma > \beta$ and the costs of financial access are not too high, a monetary equilibrium with limited BM participation ($1 > F(\tilde{\varepsilon}) > 0$) exists.*

Proof. Existence is shown using a numerical example in section 7. ∎

The previous proposition states that when carrying real balances across periods is costly and the cost of accessing banking services are not too high, a unique stationary monetary equilibrium exists where a measure of buyers chooses to access the BM while the rest of buyers chooses not to.

Proposition 4 *If $\gamma > \beta$ and $\varepsilon = 0 \ \forall \varepsilon \in [\underline{\varepsilon}, \bar{\varepsilon}]$, a unique monetary equilibrium with full BM participation exists. The equilibrium consumption is below the first-best allocation $\hat{q}_b < q_b^*$.*

Proof. A proof can be found in appendix D.3. ∎

When it is costless to access financial services, the resulting equilibria is the one in Berentsen et al. (2007). Since holding money is costly, buyers choose to be financially included in order to

24

insure against the idiosyncratic consumption risk. The cash loans for these buyers are financed by the deposits of buyers that obtain no utility from consuming AM goods.

7   Equilibrium properties

In this section, we focus on monetary equilibria where participation in the BM is limited, which corresponds to the equilibria described in proposition 3. This type of equilibria is of particular interest for two reasons. First, they describe what we observe in developing economies, in terms of the limited participation in credit markets. Second, from a more theoretical point of view, these equilibria involve two types of agents facing different liquidity constraints. This situation results in interesting interactions and non-trivial inefficiencies that can help explain some of the consumption inequality observed in developing countries.

Before we delve into the results, it is worth mentioning that any parameter or policy change that affects the trade-off facing buyers, when deciding whether to access banks or not, can have important consequences for welfare. Notice that when a buyer chooses to access banking services, he does not internalize the impact of his decision on the price of AM goods. This pecuniary externality, i.e. a situation in which the action of an agent affects another agent only through its effect on prices, always occurs in models where agents trade in a competitive market or more generally when prices faced by an agent depend on the choices of other agents.[16] As emphasized by Loong and Zeckhauser (1982) and Greenwald and Stiglitz (1986), in an economy with complete markets, pecuniary externalities do not generate inefficiencies.[17] However, when agents face incomplete markets, pecuniary externalities can result in substantial inefficiencies.[18] In our setting, agents face market incompleteness and limited participation in the credit market, which provides insurance against AM consumption risk. This partial access to insurance results in different liquidity constraints among agents. As a consequence, an increase in the AM price induced by higher

---

[16]Pecuniary externalities do not occur in markets where prices faced by individual agents are independent of aggregate quantities, e.g a market with bilateral trades where prices are bargained over between the two parties.

[17]As opposed to technological externalities which usually result in inefficiencies.

[18]For a related discussion in models with financial frictions see Dávila and Korinek (2017). Moreover, generically the direction of the inefficiency cannot be predicted (Loong and Zeckhauser, 1982; Dávila and Korinek, 2017).

Table 1: Parameter values

Parameter	Description	Value
β	Discount factor	0.95
γ	Growth rate of money	1.03
A	Parameter of AM utility function	1.20
a	Parameter of AM utility function	0.20
α	Parameter of AM cost function	0.10
σ	Probability of consuming in AM	0.50
B	Probability of CM utility	1.00
$F(\varepsilon)$	Distribution of financial access costs	Uniform $[0,1]$

financial inclusion can tighten buyers' liquidity constraint, with a potentially stronger effect on the financially excluded. When agents face different liquidity constraints, the welfare losses of one agent might not be canceled by welfare gains of others. Thus inefficiencies due to the pecuniary externality are possible in our setting.

To shed more light on the equilibrium properties of this model, in what follows we consider specific functional forms and parameter values. In particular, the AM utility and cost functions are given by $u(q) = A\frac{q^{1-a}}{1-a}$ and $c(q) = \frac{q^{1+\alpha}}{1+\alpha}$, respectively. The CM utility is given by $U(x) = B \log x$. We solve the model numerically using standard parameter values from the literature, which are summarized in Table 1. In a later section, we conduct a thorough calibration exercise for the case of India to better discipline our choice of parameters.

Regarding the effects of inflation and the liquidity risk we have the following result:

Result 1 *The AM consumption of financially excluded buyers q_b is decreasing in the money growth rate γ and increasing in the preference shock σ. The AM consumption of financially included buyers $\hat{q}_b$ is ambiguous in γ and decreasing in σ.*

The left-hand side panel of Figure 5 illustrates Result 1. As we can see, the difference between the two AM consumption levels is not constant as γ increases. This is the case as at the margin, financially included buyers are compensated against the liquidity risk. In contrast, the cost of holding money for financially excluded buyers is amplified by the liquidity risk.

A higher money growth rate γ increases the marginal cost of holding money across periods which makes AM consumption more costly and reduces real balances. This lowers the quantity
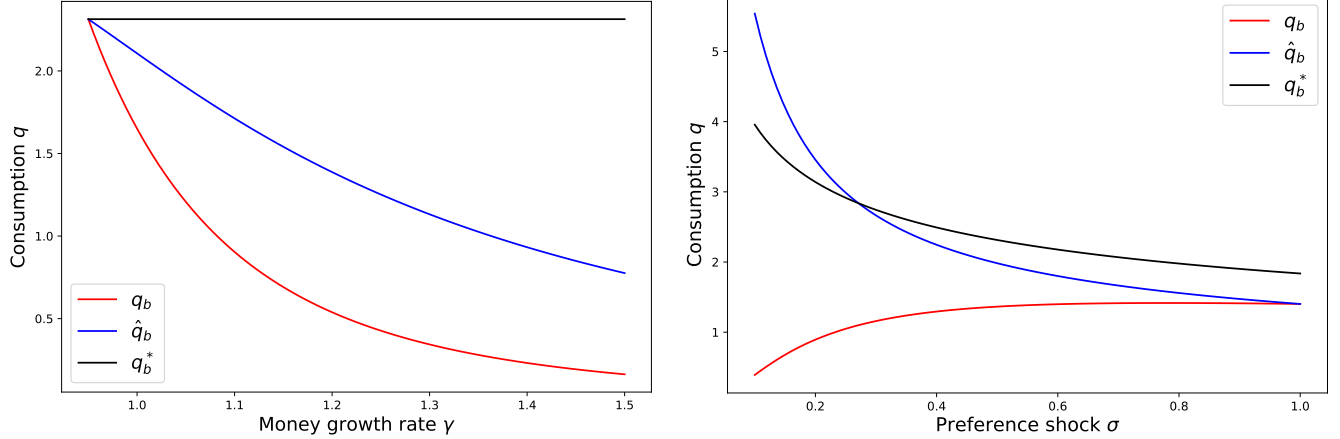
Figure 5: AM consumption as a function of γ and σ

consumed q_b and $\hat{q}_b$ as well as aggregate supply q_s. Assuming a strictly convex cost function in the AM ($c''(q_s) > 0$), lower q_s reduces the AM price which partially compensates buyers against the higher inflation. Since financially included buyers face a lower marginal cost in the AM because of the liquidity insurance provided by banks, an increase in γ affects them less relative to financially excluded buyers. For the former, the fall in the AM price can be so strong that it dominates the cost of higher inflation resulting in an increase $\hat{q}_b$. This is the case in particular for parameter values where the share of financially excluded agents and the liquidity risk are very high. However, aggregate welfare is invariantly decreasing in γ.

In addition, the differential effect of inflation on the two types of buyers results in changes in the measure of financially included buyers $F(\tilde{\varepsilon})$. This extensive margin effect is not present in models where the measure of agents with access to financial markets is exogenous. Changes in $F(\tilde{\varepsilon})$ produce a pecuniary externality which can have an additional effect on welfare. In the region of the parameter space where an increase in γ results in higher $F(\tilde{\varepsilon})$, the higher demand from the new financially included buyers puts upward pressure on the price of the AM good which reduces both q_b and $\hat{q}_b$. In the region of the parameter space where an increase in γ results in a decrease in $F(\varepsilon)$, the pecuniary externality operates in the opposite direction.

Result 1 highlights also the effect of changes in the liquidity risk resulting from the preference shock σ. An increase in σ implies a higher probability of AM consumption and hence a higher return of holding money across periods. As a consequence q_b is higher. In contrast, financially

included buyers are perfectly insured against the preference shock σ through banking services since they get the same return on money by either consuming or depositing their real balances.[19] This means that changes in σ do not affect $\hat{q}_b$ directly. Nevertheless, an indirect general equilibrium effect takes place whereby higher q_b increases the price of the AM good and reduces $\hat{q}_b$. A second general equilibrium effect works through the extensive margin: An increase in σ will reduce $F(\tilde{\varepsilon})$ and hence put downward pressure on the AM good price which increases both q_b and $\hat{q}_b$. However, the former effect always dominates and $\hat{q}$ is always decreasing in σ.

Result 2 *A high liquidity risk (low levels of σ) may result in overconsumption by the financially included buyers such that $\hat{q}_b > q_b^*$.*

Result 2 is illustrated in the right hand side panel of figure 5. We know already from Result 1 that $\hat{q}_b$ is decreasing in σ. What result 2 shows is that for very low values of σ, the liquidity constraint on financially excluded buyers can be so tight that it lowers q_b, q_s and the AM price enough to push $\hat{q}_b$ above the socially efficient quantity q^*.

Result 3 *Consumption inequality measured by the ratio $\dfrac{\hat{q}_b}{q_b}$ is increasing in inflation γ and decreasing in the preference shock σ.*

Proof. The proof is available in appendix D.4. ■

Intuitively, buyers who access banking services are perfectly insured against AM consumption risk as opposed to financially excluded buyers. This limited access to insurance results in different marginal costs of holding money across periods. An increase in the money growth rate γ will then have a stronger effect on q_b compared to $\hat{q}_b$. As a result, we observe an increase in the ratio of AM consumption of financially included to excluded buyers.

In contrast to changes in inflation, which affect directly both types of buyers, banking users are insured against changes in σ. As explained above, any resulting change in $\hat{q}_b$ must arise from indirect general equilibrium effects through AM prices. A first effect occurs when the decrease in

---

[19]As in Berentsen et al. (2007) buyers that do not consume in AM obtain an interest rate on their deposits, which compensates them ex-post against the opportunity cost of holding money.

q_b, following a lower σ, results in a AM price fall and hence an increase in $\hat{q}_b$. A second effect is observed when the decrease in σ increases the measure of bank users $F(\tilde{\varepsilon})$ by making insurance through banks more attractive. This change in the extensive margin increases the AM price which results in a decrease along intensive margin for both types.

Figure 6 is a direct illustration of Result 3. It depicts the ratio of consumption $\frac{\hat{q}_b}{q_b}$ as a function of the money growth rate γ and the consumption risk σ. Notice that this inequality is not driven by differences in initial asset holdings or skills. It simply reflects disparities in the access to financial services which provide consumption risk sharing.
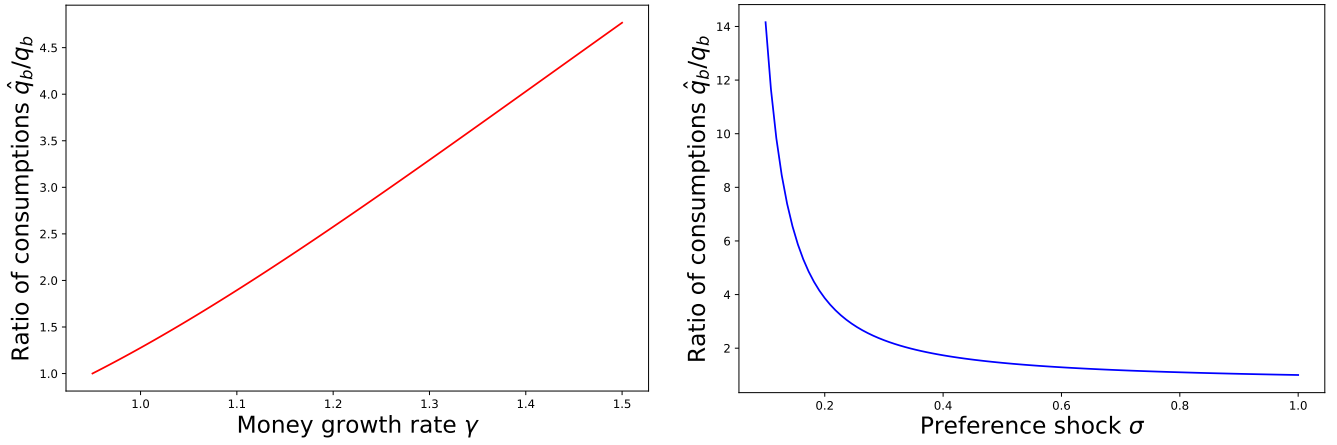
Figure 6: Ratio of AM consumptions as a function of γ and σ

Under the parametrization given in Table 1, we are able to establish the following results.

Result 4 *The measure of financially included buyers $F(\tilde{\varepsilon})$ is non-monotonic in the rate of money growth γ and the preference shock σ. Moreover, for a given $\sigma \in (0,1)$, there exists $\gamma > \beta$ such that financial inclusion is maximized.*

Figure 7 illustrates result 4. The left panel of Figure 7 depicts the measure of financially included buyers $F(\tilde{\varepsilon})$ as a function of the money growth rate γ. As the economy moves away from the Friedman rule, financial inclusion increases, reaches a maximum and then decreases. To understand this non-monotone relationship, remember that the interest rate earned on deposits perfectly compensates depositors for the opportunity cost of carrying money across periods. In contrast, the borrowing interest rate makes borrowers ex-post no better than financially excluded

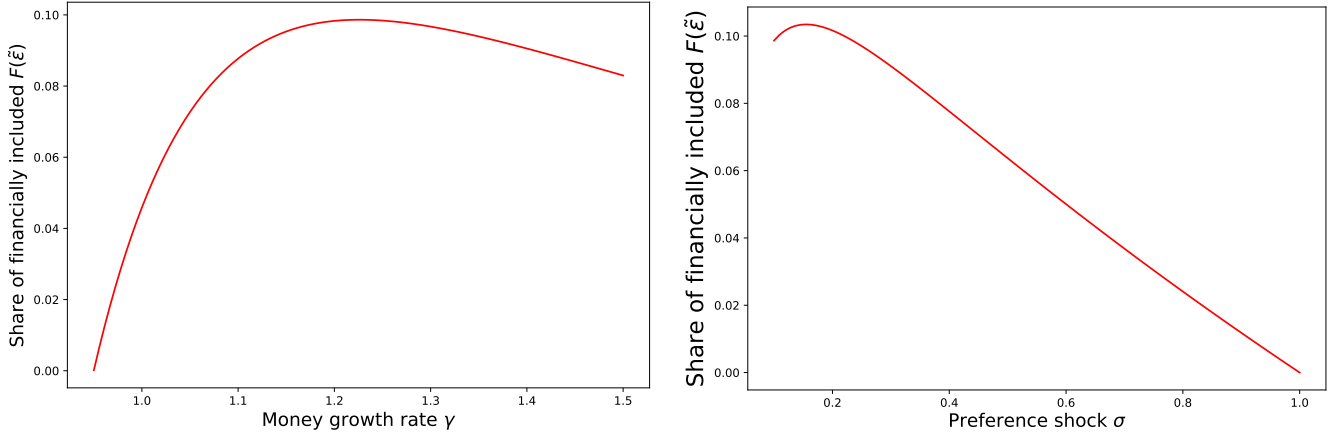Figure 7: Financial inclusion as a function of γ and σ

buyers. This means that all the welfare gain from accessing banks comes from the interests paid to depositors as emphasized by Berentsen et al. (2007). Given that, an increase in inflation has two effects on financially included buyers. First it reduces their real money balances and hence their consumption in case of a positive consumption shock (which occurs with probability σ). Second, it increases the value of the insurance provided by banks through the interest earned on their deposits in case of a negative consumption shock (which occurs with probability $1 - \sigma$). For low levels of inflation and nominal rates, the opportunity cost of holding money is small. This makes the cost of ending with idle balances (which occurs with probability $1 - \sigma$) negligible for buyers. As a consequence, only those with a low ε are willing to pay to insure against this risk. As inflation increases, the opportunity cost of holding money increases, which in turn raises the cost of ending with idle money balances. As a result, more and more buyers prefer paying the cost of financial access to earn an interest rate on their deposits. However, as inflation increases, real balances and AM consumption decrease. When γ is sufficiently high, the quantity of real balances held is so small that the gain from the insurance offered by banks does not justify the payment of ε.[20] This means that buyers with high ε start dropping out from the banking sector.

The impact of an increase in the liquidity risk is shown in the right panel of Figure 7. The relationship is also non-monotone as the same logic as above applies. When the probability of a negative preference shock increases, i.e. lower σ, the share of financially included buyers, who

---

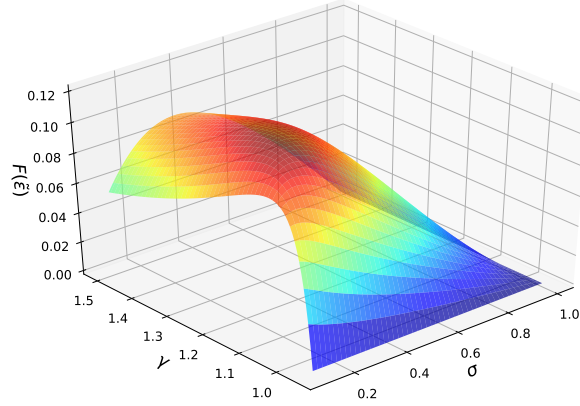[20]The cost of financial access ε is incurred in real terms.

Figure 8: Financial inclusion as a function of γ and σ

seek to insure against the risk of idle money balances, increases. However, as the liquidity risk becomes very high, i.e. very low σ, buyers with high banking costs, don't find it worthwhile to pay ε to obtain insurance as the amount of real balances they hold is pretty small.

To determine the robustness of the previous findings, we compute the equilibrium financial inclusion over a larger parameter space. Figure 8 depicts the measure of financially included buyers as a function of both γ and σ. As we can see, there is a global maximum when the liquidity risk is high (σ low) and inflation relatively moderate. This point corresponds to the maximal value of risk sharing provided by banks.

8 Quantitative analysis

8.1 Calibration

In order to evaluate the impact of various policies on financial inclusion, welfare and inequality as well as provide more discipline when choosing the parameter values, we calibrate the model to the Indian economy. We consider annual data over the period 1970 to 2016.[21]

We assume the utility of consumption takes the form $u(q) = A\frac{q^{1-a}}{1-a}$ for the AM good and $U(x) = B\log x$ for the CM good. The AM cost function is set as $c(q) = \frac{q^{1+\alpha}}{1+\alpha}$.

We divide the model's parameters into two groups: *(i)* independent parameters, $\{\beta, \gamma, F, \underline{\varepsilon}, \bar{\varepsilon}\}$,

---

[21] All the data used is publicly available from the Reserve Bank of India.

Table 2: Independent Parameters

Parameter	Description	Value
β	Discount factor	0.993
γ	Growth rate of money	1.079
i	Nominal interest rate	0.087
$F(\varepsilon)$	Distribution of financial access costs	Log-Normal

and *(ii)* jointly calibrated parameters, $\{\sigma, A, a, \alpha, B\}$. We set the money growth rate γ to match the average annual change in the Indian CPI over the period 1970 to 2016, the annual rate is 7.92%. For the nominal interest rate, i, we consider average call money interest rate, which is 8.67%. This then implies an average annual real interest rate of 0.75% and a discount factor β of 0.993.

In the absence of a direct measurements for the distribution of the cost of accessing banking services $F(\varepsilon)$, we use the number of bank branches per Indian state as a proxy.[22] By choosing this measure, we implicitly assume that a higher per capita number of bank branches implies a lower cost of accessing financial services, everything else being equal. Figure 9 plots the number of bank branches per 100'000 adults against the share of the adult population owning a bank account for 159 countries. Financial inclusion is positively correlated with bank branch density which provides some evidence backing our choice of proxy for $F(\varepsilon)$.

Panel 10a shows a normalized histogram of the number of bank branches per 100,000 individuals in each Indian state in 2011 weighted by the share of each state's population in the total population of India based on the 2011 population census. We define ε as the inverse of bank branch density and use a maximum likelihood estimation procedure to fit the parameters of a log-normal distribution to the population-weighted distribution of ε. Panel 10b depicts the implied log-normal distribution for India.[23]

The second group of parameters is calibrated jointly by matching three empirical moments: *(i)* the average money demand, *(ii)* the interest rate elasticity of money demand, and *(iii)* the level of financial inclusion. We include the first two targets in order to fit the empirical money demand

---

[22]Detailed data on bank branches per state and district is publicly available from the Reserve Bank of India.
[23]This results in a location parameter of 0, a scale of 0.129 and a standard deviation of 0.341.
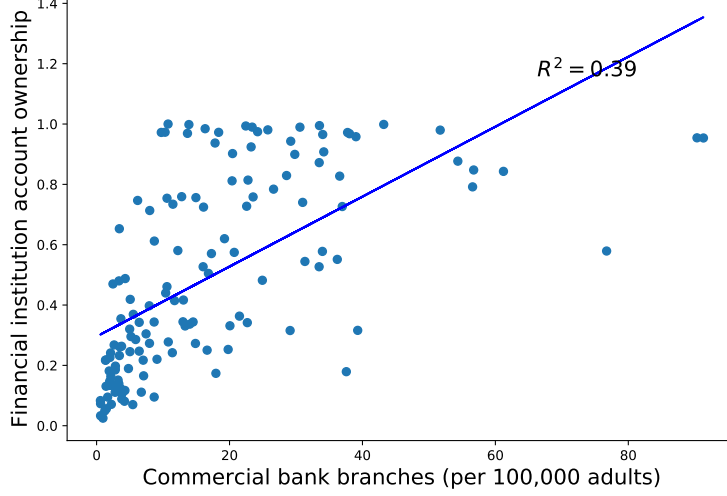
Figure 9: Financial inclusion and commercial bank branches density

Data sources: World Bank, Global Findex Database; IMF.

relationship following Lagos and Wright (2005).[24] In our model, the money demand relationship corresponds to

$$L \equiv \frac{M}{PY} = \frac{F(\tilde{\varepsilon})\phi\hat{m} + (1 - F(\tilde{\varepsilon}))\phi m}{\phi p q_s + 2B} \tag{62}$$

where the numerator represents the sum of real balances demanded by both financially included and excluded buyers, while the denominator represents the sum of output produced in the AM and CM.

To obtain the empirical estimates of the two money demand related moments we run, using annual data of the period 1970 to 2016, the following regression

$$\log M_t/P_t Y_t = \beta_1 + \beta_2 \log i_t + \nu_t. \tag{63}$$

As a measure of M, we choose the monetary aggregate M1 which includes currency and demand deposits but excludes time deposits. For the rest of the observables, we use nominal GDP as a measure of PY and the call money rate for i.[25] The OLS estimate of β_2 is used as a target for the

---

[24]Alternatively, one can use all the information in the data by solving for the parameter values which minimize the distance between the model-based money demand and the observed money demand for each observed interest rate. However this procedure is computationally more expensive and doesn't change the calibration results significantly.

[25]Due to limited data availability, we use the call money rate instead of the commercial paper rate used in the literature.
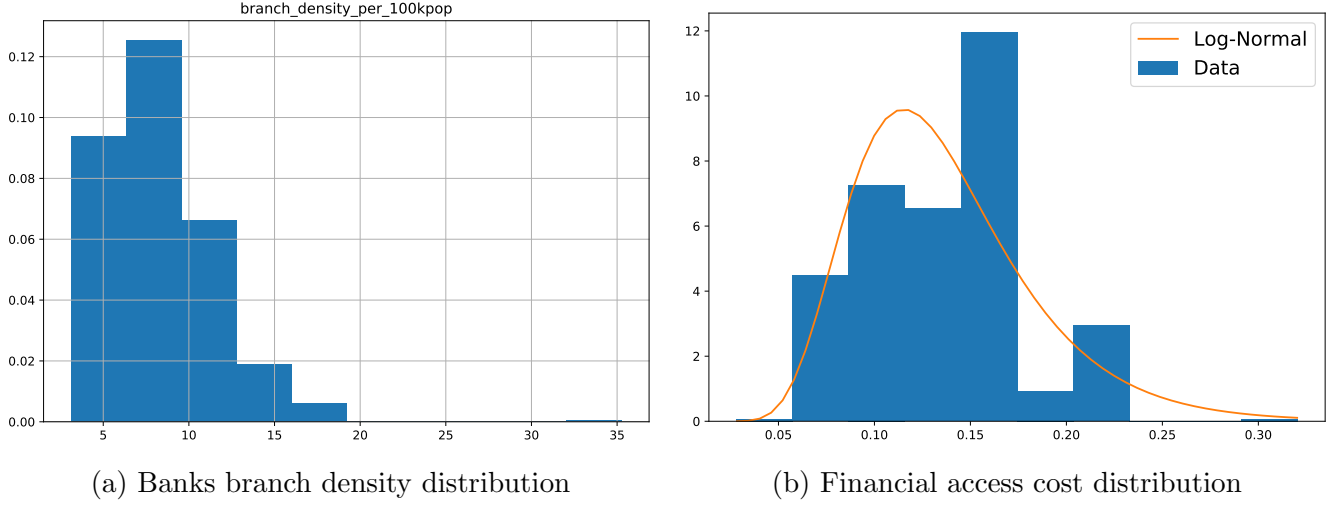
33

(a) Banks branch density distribution

(b) Financial access cost distribution

Figure 10: Measuring the cost of financial access using bank branch density

Table 3: Joint calibration results

Jointly calibrated parameters			
σ	Preference shock		0.643
a	Parameter of AM utility		0.013
A	Parameter of AM utility		2.528
α	Parameter of AM production function		2.649
B	Parameter of CM utility		7.629
Calibration targets		**Data**	**Model**
$L(\bar{i})$	Average Money Demand	0.175	0.175
ϵ	Elasticity of Money Demand to i	-0.195	-0.195
$F(\tilde{\varepsilon})$	Share of financially included buyers	0.352	0.352
Sum of squared residuals			0.000

model-based interest rate elasticity of money demand

$$\frac{\partial L(i)/L(i)}{\partial i/i} \tag{64}$$

while the average money demand in the data is used as a target for the level of money demand L at the average i.

To match the empirical level of financial inclusion, we target the percentage of the Indian adult population reported to own an account at a formal financial institution. This statistic is taken from the World Bank's Global Findex database and stood at 35.2% in the 2011 survey. The results of the joint calibration are presented in Table 3.
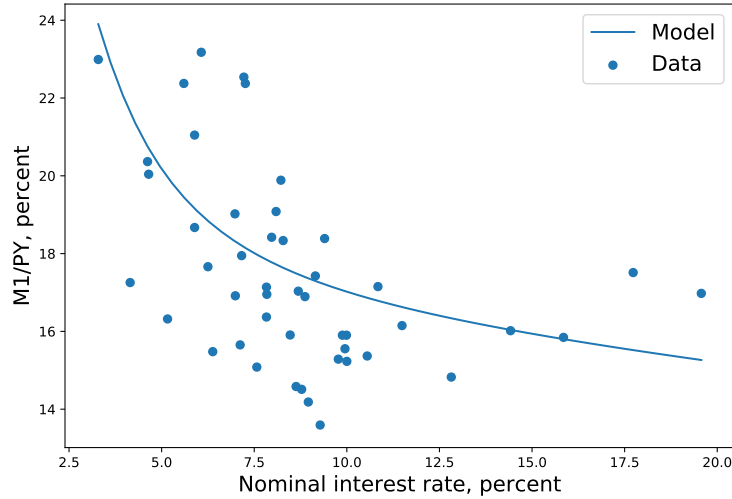
Figure 11: Money demand: model vs. data

As we can see, the model is able to exactly match the targeted moments using the calibrated parameters. It is also able to fit the money demand relationship in the data relatively well as shown in Figure 11.

We now use the benchmark calibration to determine how well the model performs. To do so we compute the resulting equilibrium consumption Gini coefficient, the demand to deposits to M1 ratio as well as the share of intra-period credit in total consumption. These are reported in Table 4. Our simple model is able to explain around a third of the observed consumption inequality in India. This is surprising since inequality in our model results merely from differences in the cost of access to liquidity insurance. Agents in our model are similar in all other aspects. The model is also able to explain two thirds of the average ratio of demand deposits to M1. If we interpret the AM in our model as the consumption of non-durables, we can also explain around half of the average share of non-durables consumer credit to GDP in India. We interpret these results as an encouraging external validation of our model and calibration. Given that, we present next some numerical experiments before delving into the policy analysis.

Table 4: Model validation

	Data	Model	% explained
Consumption Gini coefficient	0.38	0.11	29.0%
Demand deposits to M1 ratio	45.3%	32.5%	72.0%
Share of consumer credit in GDP	13.0%	6.3%	48.1%

35

8.2 Numerical experiments

8.2.1 Lower inflation

The first numerical experiment we conduct is to reduce inflation from the average level of 7.9% used in the calibration. Table 5 presents the results of a reduction of inflation of 1 pp and 5 pp.

Lowering inflation reduces the measure of financially included buyers. This means that the calibrated steady state is on the upward sloping part of the relationship between financial inclusion and inflation depicted in figure 7. Lowering inflation increases the consumption of both types with a much higher increase in the consumption of financially excluded buyers. This results in an increase in welfare and a decrease in inequality. In particular, increasing inflation by 5 pp increases welfare by 0.15% and reduces consumption inequality by 2.49 pp.

Table 5: Effect of reducing steady state inflation $\pi = 7.9\%$

	$\Delta F(\tilde{\varepsilon})$	Δq_b	$\Delta \hat{q}_b$	Δq_s	$\Delta \mathcal{W}$	Δ Gini
$\pi = 6.9\%$	-1.11 pp	42.97%	0.59%	0.35%	0.02%	-0.13 pp
$\pi = 2.9\%$	-16.84 pp	560.14%	4.39%	1.79%	0.15%	-2.49 pp

8.2.2 Lower costs of financial access

We now consider the effect of lowering the cost of financial access. India implemented an agressive policy of mandating banks to increase the number of their branches in remote areas in the years 2010s. Thus it is not surprising that the second World Bank's survey in 2014 reports a level of financial inclusion in India of 52.8% compared to 35.2% in 2011.[26]

Table 6: Effects of changing $F(\varepsilon)$

	$\Delta F(\tilde{\varepsilon})$	Δq_b	$\Delta \hat{q}_b$	Δq_s	$\Delta \mathcal{W}$	Δ Gini
F_{2014}	6.8 pp	-14.93%	-14.93%	0.08%	0.03%	-0.52 pp

To capture the impact of this policy, we reestimate the distribution $F(\varepsilon)$ using 2014 data on bank branches and the state-level population growth rates from 2011 to 2014. We then use

---

[26]These numbers exclude access to mobile banking which was reported by only 2.4% of the adult population in 2014. Statistics on mobile banking were not collected in the 2011 survey.
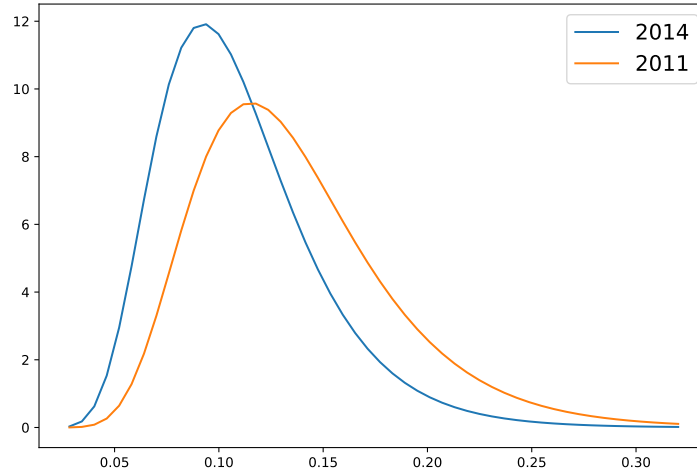
Figure 12: Distribution of estimated financial access costs $F(\varepsilon)$.

this distribution to calculate a new steady state equilibrium keeping the rest of the calibrated parameters constant. The resulting level of financial inclusion stands at 42%, an increase of 6.8 percentage points which represents 38.9% of the observed change. The impact of this change in F on equilibrium quantities is presented in table 6. Welfare increases by 0.03% while the consumption Gini coefficient falls by 0.52 percentage points.

8.3 Policy discussion

In this section we discuss policy instruments aimed at increasing financial inclusion and discuss their impact on welfare and consumption inequality. First, we discuss a policy under which the government provides direct monetary subsidies to the owners of bank accounts. Second, we examine the effects of interest rate subsidies to both the deposit and loan interest rates.

8.3.1 Transfer to bank account holders

We first consider direct monetary transfers to the owners of bank accounts. In practice, this policy can be implemented either by changing a pre-existing cash-based government transfer from cash handouts to bank account payments or through a new subsidy scheme. In the former case, it allow to leverage the disbursement of pensions and social benefits in order to increase financial inclusion. Notice that paying these transfers through bank accounts might represent the first

37

contact with the formal financial sector for a large part of the population. Similar policies have been implemented in Brazil, Colombia, India, Mexico and South Africa (Pickens et al., 2009). For example, India's Direct Benefit Transfer (DBT) program, operating since 2013, is recognized as the world's largest targeted benefit transfer scheme. The DBT program pays government subsidies directly to the beneficiaries through their bank accounts with the objective of increasing financial inclusion. This has the additional advantage of reducing corruption by bypassing the handling of cash by government officials.

Let us define the monetary subsidy by τ. We assume that such subsidy is the same for both depositors and borrowers. Monetary transfers are first given to banks, which subsequently pass them to all agents who use their financial services. We assume this subsidy is financed through lump-sum taxes in the CM. Note then that τ enters directly the AM value function of financially included buyers (37). The AM value function is now given by

$$\hat{V}^b(\varepsilon, \hat{m}) = \sigma \left[u(\hat{q}_b) + \hat{W}^b(\varepsilon, \hat{m} + \tau + \ell - p\hat{q}_b, 0, \ell) \right] + (1-\sigma)\hat{W}^b(\varepsilon, \hat{m} + \tau - d, d, 0). \qquad (65)$$

The amount deposited by liquidity-unconstrained buyers becomes $d = \hat{m} + \tau$, while the amount of loans that the liquidity-constrained buyers desire is $\ell = p\hat{q} - \hat{m} - \tau$. Note that this transfer scheme affects directly the cost threshold, $\tilde{\varepsilon}$ which is now given by

$$\tilde{\varepsilon} = \gamma\phi\tau + \beta\sigma \left[(u(\hat{q}_b) - u'(\hat{q}_b)\hat{q}_b) - (u(q_b) - u'(q_b)q_b) \right]. \qquad (66)$$

As one would expect, the equilibrium money holdings of financially excluded buyers remain unchanged, while the financially included buyers reduce their balances by exactly the amount of the transfer so that $\hat{m} = \sigma p\hat{q}_b - \tau$. The rest of the equilibrium conditions are the same as before.

Changes in τ translate into a pure shift in the share of financially included buyers $F(\tilde{\varepsilon})$. These changes affect welfare through a size and composition effects. We know that an increase in financial inclusion means the reallocation of some buyers from consuming q_b to consuming a higher quantity $\hat{q}_b$ and hence an increase in their welfare. However, for strictly convex AM cost functions, this

higher demand for the AM good puts upward pressure on the price and decreases consumption for both financially included and excluded buyers reducing their welfare. The net effect of the subsidy will depend on which of the two effects dominates. When the pecuniary externality is not present (e.g. with a constant marginal cost of production in the AM) the second effect is not present and welfare invariably increases.

8.3.2 Interest rate subsidies

We consider two additional policy instruments: a proportional subsidy τ^d to the interest rate earned by depositors and a proportional subsidy τ_ℓ to the interest rate paid by borrowers. Again, we assume that these subsidies are financed through lump-sum taxes in the CM.

Adding the two instrument τ^d and τ^ℓ, affects the intertemporal equilibrium condition of financially included agents which becomes

$$\frac{u'(\hat{q}_b)}{c'(q_s)} - 1 = i(1 - \tau^\ell), \tag{67}$$

and the interest rate on deposits and loans which becomes

$$i(\sigma(1 - \tau^\ell) + (1 - \sigma)(1 + \tau^d)) = \bar{\iota}, \tag{68}$$

while the rest of the equilibrium conditions are the same as in the baseline model. Notice interest rate subsidies do not directly affect financially excluded buyers. The only effect comes from the pecuniary externality i.e. changes in the price of the AM good as a reaction to changes in the quantities demanded by financially included buyers. In contrast, financially included buyers are directly affected by the two instruments. The borrowing rate subsidy affects the quantity consumed by borrowers both directly by reducing the wedge $1 + i(1 - \tau^\ell)$ between their marginal cost and marginal utility of consumption and indirectly through a general equilibrium effect by increasing the demand for loans which results on a higher i. The deposit rate subsidy τ^d affects consumption only through a general equilibrium effect by increasing the supply of deposits which decreases i.

8.3.3 Policy evaluation

Using the calibrated parameter values we calculate the welfare and inequality consequences of the different policies discussed above. To evaluate the impact of each policy, we measure welfare as

$$\mathcal{W} = \sigma F(\tilde{\varepsilon})u(\hat{q}_b) + \sigma(1 - F(\tilde{\varepsilon}))u(q_b) - c(q_s) + 2U(x^*) - 2x^* - \int_{\underline{\varepsilon}}^{\tilde{\varepsilon}} \varepsilon \, dF(\varepsilon) \tag{69}$$

where the integral in the RHS represents the sum of financial access costs incurred by financially included buyers. As a measure of consumption inequality, we use the consumption Gini coefficient (cf. Appendix B for details).[27] As opposed to the ratio of consumption levels used in the previous section, the Gini coefficient takes into account both the intensive and extensive margins.

Table 7 presents each policy's effect on consumption, output, welfare and consumption inequality of increasing financial inclusion by 1 percentage point from its actual calibrated level. As expected, an increase in financial inclusion reduces the consumption levels of both included and excluded buyers under the three policy interventions. Increasing the share of financially included buyers increases the demand for the AM good, which increases the output q_s and the price and lowers the quantities consumed by both types of agents. In the case of bank transfers, the decrease in consumption is the same for both financially included and excluded buyers since the policy affects only the credit market participation margin without generating other distortions. In contrast, policies based on interest rate subsidies result in a much higher decrease in consumption for financially excluded buyers compared to financially included buyers which tends to increase inequality. This is because interest rate subsidies not only increase participation in the credit market but also encourage financially included buyers to consume more, including buyers who already had access to banks before implementation of the policy. This is not the case with the direct benefit transfer which just increases participation without affecting the incentive of agents who had already access to banks before implementation of the policy. In conclusion, interest rate subsidies are not only subsidizing participation in the banking sector but also incentivizing agents

---

[27]The consumption Gini coefficient is twice the area under the consumption Lorenz curve which plots the cumulative share of the population against the cumulative share in total consumption.

Table 7: Effect of increasing financial inclusion by 1 pp using different policies

	Δ	Δq_b	$\Delta \hat{q}_b$	Δq_s	$\Delta \mathcal{W}$	Δ Gini
$\phi\tau$	0.004	-2.50%	-2.50%	0.01%	0.004%	-0.07 pp
τ_d	0.199	-33.84%	-0.35%	0.20%	-0.009%	0.13 pp
τ_ℓ	0.166	-33.84%	-0.35%	0.20%	-0.009%	0.13 pp
γ	0.016	-41.55%	-0.64%	-0.54%	-0.031%	0.10 pp

to borrow and deposit more. If the objective of policymakers is just to increase participation in the banking sector without distorting the incentives of already banked individuals, the direct benefit transfer is the appropriate policy.

Using inflation to push agents to the banking sector requires increasing money growth rate by 1.6pp for a 1pp increase in financial inclusion. This results in lower consumption for both financially included and excluded agents, lower welfare and higher inequality. Since it directly reduces the consumption of both groups as opposed to interest rate subsidies, inflation results in a lower increase in inequality. However, the reduction in welfare is almost three times higher.

8.3.4 Positive interest rate margin

We extend the baseline model by adding a linear cost of transforming deposits into loans. This cost $k\ell$ can be interpreted as the cost of managing a volume ℓ of loans. We assume k is the same for all banks. As in section 5.3, assuming perfect competition and free entry in the banking sector yields zero profits such that $i_\ell = i_d + k$. Combining that with the intertemporal equation of financially included buyers (54) we get the following expressions for the interest rates on deposits and loans as a function of $\bar{\iota}$ and the model's parameters:

$$i_d = \bar{\iota} - \sigma k \tag{70}$$

$$i_\ell = \bar{\iota} + (1 - \sigma)k \tag{71}$$

where i_ℓ (i_d) is increasing (decreasing) in the costs of financial intermediation k. The rest of the equilibrium conditions is listed in appendix C. Notice that setting $k = 0$ reduces the model to the baseline case. This shortcut allows us to explore the effect of imperfect competition on
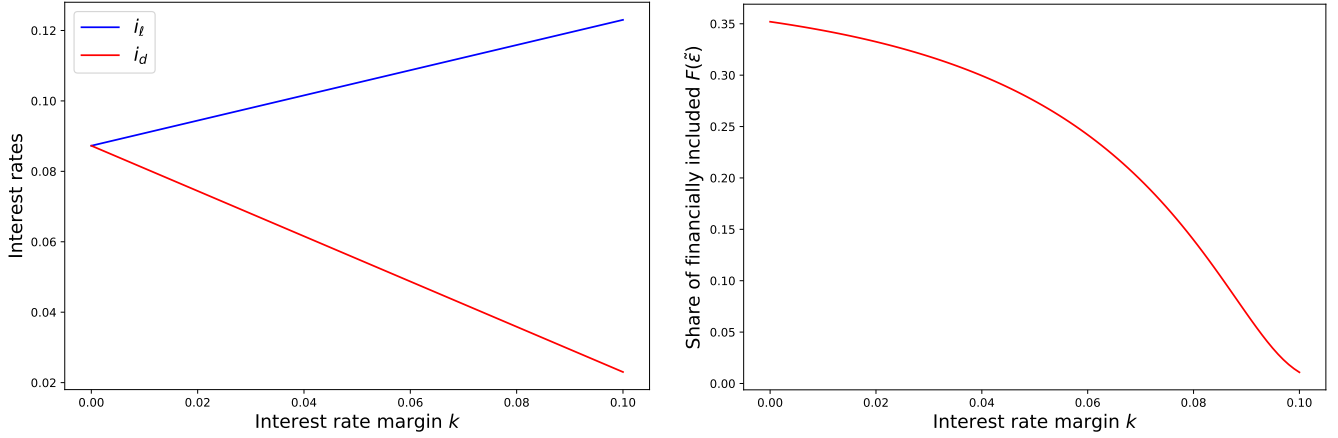
Figure 13: Effect of increasing the interest rate margin

financial inclusion in a reduced form using k as a proxy without modeling explicitly the industrial organization of the banking sector.

The positive interest rate margin $i_\ell - i_d$ can be interpreted as a proxy for the level of competition in the banking sector. In order to analyze the impact of increasing k on interest rates, financial inclusion and welfare we start at a steady state where $k = 0$ and the rest of the parameters are calibrated as before and gradually increase k. The results are presented in figure 13. As expected, the level of financial inclusion is decreasing in the interest rate margin. Surprisingly, an interest rate margin of approximately 10pp is enough to drive all households away from the banking sector. This points to a positive effect of competition in the banking sector on financial inclusion which stems from a higher pass-through from the interest rate on loans to the interest rate paid to depositors.

9   Conclusion

In this paper we present a theoretical framework that sheds some light on financial inclusion and how it relates to welfare and consumption inequality. We consider an environment where informational frictions make money essential as a means of exchange. Agents face idiosyncratic preference shocks, inducing heterogeneous liquidity needs among agents. Banks operate as intermediaries in the credit market by providing a risk sharing mechanism against this shock. To access financial

services, agents need to pay an idiosyncratic fixed cost. Agents that trade with banks are perfectly insured against the liquidity risk while financially excluded agents are not. Hence the latter face a higher opportunity cost of holding money. As a result, consumption inequality between agents based solely on differentiated access to liquidity insurance is observed. Moreover, equilibria with overborrowing and overconsumption above the socially efficient level can arise when financially excluded agents are severely liquidity-constrained. We study various policies to remedy such problems. Depending on the policy used, it turns out that higher participation in the credit market can lead to a decrease in welfare and an increase in consumption inequality. This is a direct result of the pecuniariy externality caused by agents' participation decision. An increase in the proportion of agents with access to the banking sector increases the quantities demanded and hence the price of the traded good. This price increase tightens the liquidity constraint on all agents and in particular on agents without access to credit who see their consumption decrease.

Our paper shows how financial inclusion is a complex phenomenon which depends on the cost of financial access, on monetary policy, the degree of liquidity insurance offered by banks as well as on the reaction of producers to changes in aggregate demand. Moreover it presents several testable implications that we think are worth examining. As far as monetary policy is concerned, the level of financial inclusion can have non-trivial consequences on the stabilization policy of the central bank. We leave this issue and others to future research.

Appendix A Cost threshold $\tilde{\varepsilon}$ derivation

Agents facing a cost $\tilde{\varepsilon}$ are indifferent between accessing banking services and remaining financially excluded such that

$$-\phi m_{+1} + \beta V_{+1}(\tilde{\varepsilon}, m_{+1}) = -\tilde{\varepsilon} - \phi \hat{m}_{+1} + \beta \hat{V}^b_{+1}(\tilde{\varepsilon}, \hat{m}_{+1}) \tag{A.1}$$

Shifting one period backward and simplifying yields:

$$\begin{aligned}
\tilde{\varepsilon} = &-\phi_{-1}(\hat{m} - m) + \beta \sigma \left[u(\hat{q}_b) - u(q_b) + \hat{W}^b(\tilde{\varepsilon}, \hat{m} + \ell - p\hat{q}_b, 0, \ell) - W(\tilde{\varepsilon}, m - pq_b, 0, 0) \right] \\
&+ \beta(1 - \sigma) \left[\hat{W}^b(\tilde{\varepsilon}, \hat{m} - d, d, 0) - W(\tilde{\varepsilon}, m, 0, 0) \right]
\end{aligned} \tag{A.2}$$

For buyers who do not consume in the AM, we have

$$\begin{aligned}
\hat{W}(\tilde{\varepsilon}, \hat{m} - d, d, 0) - W(\tilde{\varepsilon}, m, 0, 0) = &-\tilde{\varepsilon} + \phi(\hat{m} - d - \hat{m}_{+1} + (1 + i_d)d) \\
&+ \beta \hat{V}^b_{+1}(\tilde{\varepsilon}, \hat{m}_{+1}) - \phi(m - m_{+1}) - \beta V_{+1}(\tilde{\varepsilon}, m_{+1})
\end{aligned} \tag{A.3}$$

Using equation (A.1) and $d = \hat{m}$ for $i_d \geq 0$ we get:

$$W(\tilde{\varepsilon}, \hat{m} - d, d, 0) - W(\tilde{\varepsilon}, m, 0, 0) = (1 + i_d)\phi \hat{m} - \phi m \tag{A.4}$$

For buyers consuming in the AM we have:

$$\begin{aligned}
\hat{W}(\tilde{\varepsilon}, \hat{m} + \ell - p\hat{q}_b, 0, \ell) - W(\tilde{\varepsilon}, m - pq_b, 0, 0) = &-\tilde{\varepsilon} + \phi(\hat{m} + \ell - p\hat{q}_b - \hat{m}_{+1} - (1 + i_\ell)\ell) + \beta \hat{V}^b_{+1}(\tilde{\varepsilon}, \hat{m}_{+1}) \\
&- \phi(m - pq_b - m_{+1}) - \beta V_{+1}(\tilde{\varepsilon}, m_{+1})
\end{aligned} \tag{A.5}$$

which in turn simplifies to

$$\hat{W}(\tilde{\varepsilon}, \hat{m} + \ell - p\hat{q}_b, 0, \ell) - W(\tilde{\varepsilon}, m - pq_b, 0, 0) = -(1 + i_\ell)\phi \ell \tag{A.6}$$

Putting all the above together we get:

$$\tilde{\varepsilon} = -\phi_{-1}(\hat{m} - m) + \beta\sigma\left[u(\hat{q}_b) - u(q_b) - (1 + i_\ell)\phi\ell\right] + \beta(1 - \sigma)\left[(1 + i_d)\phi\hat{m} - \phi m\right] \quad (A.7)$$

which can be rewritten as

$$\tilde{\varepsilon} = -(\gamma - \beta)\phi(\hat{m} - m) + \beta\left[(1 - \sigma)i_d + \sigma i_\ell\right]\phi\hat{m} + \beta\sigma\left[(u(\hat{q}_b) - (1 + i_\ell)\phi p\hat{q}_b) - (u(q_b) - \phi p q_b)\right]$$

$$(A.8)$$

Appendix B Derivation of the consumption Gini coefficient

Given the distribution of consumption in table 8 the resulting consumption Gini coefficient is

$$G = 1 - \frac{x^* + \sigma^2\left((1 - F(\tilde{\varepsilon}))^2 q_b + 2F(\tilde{\varepsilon})(1 - F(\tilde{\varepsilon}))q_b + (F(\tilde{\varepsilon}))^2\hat{q}_b\right)}{q_s + x^*} \quad (B.1)$$

Notice that we do not include the CM consumption of sellers in our calculations. Since we consider output in our money demand calibration as the sum of the AM and CM consumptions of both buyers and sellers, one can think of the CM consumption of sellers as equivalent to the share of investment in output which is consistent with its exclusion from the consumption Gini coefficient.

Table 8: Buyers' consumption distribution

Population share	Consumption
$\sigma F(\tilde{\varepsilon})$	$\hat{q}_b + x^*$
$\sigma(1 - F(\tilde{\varepsilon}))$	$q_b + x^*$
$1 - \sigma$	x^*
Total	$q_s + x^*$

Appendix C   Equilibrium with interest rate margin k

The resulting equilibrium equations are

$$\sigma \left[\frac{u'(q_b)}{c'(q_s)} - 1 \right] = \bar{\imath}$$

$$\frac{u'(\hat{q}_b)}{c'(q_s)} - 1 = i_\ell$$

$$\bar{\imath} = \sigma i_\ell + (1 - \sigma) i_d$$

$$i_\ell = i_d + k$$

$$\phi p = c'(q_s)$$

$$\phi m = \phi p q_b$$

$$\phi \hat{m} = \sigma \phi p \hat{q}_b$$

$$\sigma \left((1 - F(\tilde{\varepsilon})) q_b + F(\tilde{\varepsilon}) \hat{q}_b \right) = q_s$$

$$\tilde{\varepsilon} = \beta \sigma \left[(u(\hat{q}_b) - u'(\hat{q}_b)\hat{q}_b) - (u(q_b) - u'(q_b)q_b) \right].$$

Appendix D   Proofs

D.1   Proof of Proposition 1

Proof. From (56), when $\gamma \to \beta$, the interest rate prevalent in the banking sector $i \to 0$. Using that in (53) and (57) results in $\hat{q}_b \to q_b^*$ and $q_b \to q_b^*$ respectively. Replacing in (60), we get $\tilde{\varepsilon} \to 0$.

■

D.2 Proof of Proposition 2

Proof. When the costs of financial access are sufficiently high, we have

$$- \phi m_{+1} + \beta V_{+1}(\varepsilon, m_{+1}) > -\varepsilon - \phi \hat{m}_{+1} + \beta \hat{V}^b_{+1}(\varepsilon, \hat{m}_{+1}), \ \forall \varepsilon \in [\underline{\varepsilon}, \bar{\varepsilon}] \tag{D.1}$$

which implies $F(\tilde{\varepsilon}) = 0$ as $\tilde{\varepsilon} < \underline{\varepsilon}$. In this case, the equilibrium is as follows:

$$\sigma \left[\frac{u'(q_b)}{c'(q_s)} - 1 \right] = \bar{\iota}$$

$$\phi p = c'(q_s)$$

$$\phi m = \phi p q_b$$

$$\sigma q_b = q_s$$

where $q_b < q_b^*$. Combining the consumption Euler equation and the goods market clearing, I get

$$\frac{u'(q_b)}{c'(\sigma q_b)} = 1 + \frac{\bar{\iota}}{\sigma}$$

Under the usual assumptions on the utility function $(u'(q) > 0, u''(q) < 0, \lim_{q \to 0} u'(q) = +\infty, \lim_{q \to \infty} u'(q) = 0)$ and cost function $(c'(q) > 0, c''(q) \geq 0, , \lim_{q \to 0} c'(q) = 0, \lim_{q \to \infty} c'(q) = +\infty)$ we have

$$\frac{\partial \frac{u'(q_b)}{c'(\sigma q_b)}}{\partial q_b} < 0 \tag{D.2}$$

$$\lim_{q_b \to 0} \frac{u'(q_b)}{c'(\sigma q_b)} = +\infty > 0 \text{ for } \sigma \in (0, 1], \tag{D.3}$$

$$\lim_{q_b \to +\infty} \frac{u'(q_b)}{c'(\sigma q_b)} = 0. \tag{D.4}$$

It follows that q_b exists and is unique. ∎

D.3 Proof of Proposition 4

Proof. When the distribution F is degenerate such that $\forall \varepsilon \in [\underline{\varepsilon}, \bar{\varepsilon}]$ we have $\varepsilon = 0$, buyers are the same and face no costs of accessing banks. Therefore, in a symmetric equilibrium all buyers will make the same decision of accessing or not the banking sector. To determine which one will be chosen we have to compare the lifetime utility of a buyer under each choice. We start first with AM consumption. Under a pure monetary equilibrium without bank access, the AM consumption solves

$$\frac{u'(q_b)}{c'(\sigma q_b)} = 1 + \frac{i}{\sigma} \tag{D.5}$$

while in an economy where all agents access banks it solves

$$\frac{u'(\hat{q}_b)}{c'(\sigma \hat{q}_b)} = 1 + i. \tag{D.6}$$

As long as $\sigma < 1$ and $\bar{\imath} > 0$ (i.e. $\gamma > \beta$), we have $\hat{q}_b > q_b$.

Next, we compare their equilibrium production and consumption in the CM. We start first with a financially excluded buyer. With probability σ he consumed in the AM and he enters the CM with no money holdings. In this case hours worked is

$$h = x^* + \phi m_{+1} - T = x^* + \gamma c'(\sigma q_b)q_b - T \tag{D.7}$$

We have

$$T = \phi(M - M_{-1}) = (\gamma - 1)\phi M_{-1} = (\gamma - 1)\frac{\phi_{-1}}{\gamma}M_{-1} = \frac{\gamma - 1}{\gamma}c'(\sigma q_b)q_b \tag{D.8}$$

which gives us:

$$h = x^* + \left(\frac{\gamma^2 - \gamma + 1}{\gamma}\right)c'(\sigma q_b)q_b. \tag{D.9}$$

If the financially excluded buyer didn't consume in the previous AM we have

$$h = x^* + \left(\frac{\gamma^2 - \gamma + 1}{\gamma}\right)c'(\sigma q_b)q_b - c'(\sigma q_b)q_b = x^* + \frac{(\gamma - 1)^2}{\gamma}c'(\sigma q_b)q_b. \tag{D.10}$$

48

Combining the two we have

$$(1-\beta)V = \sigma(u(q_b)+U(x^*)-x^* - (\frac{\gamma^2-\gamma+1}{\gamma})c'(\sigma q_b)q_b) + (1-\sigma)(U(x^*)-x^* - \frac{(\gamma-1)^2}{\gamma}c'(\sigma q_b)q_b).$$

(D.11)

which simplifies to

$$(1-\beta)V = U(x^*) - x^* + \sigma u(q_b) - \frac{\gamma^2+(\sigma-2)\gamma+1}{\gamma}c'(\sigma q_b)q_b.$$

(D.12)

For financially included buyers who consumed in the AM we have

$$h = x^* + \phi\hat{m}_{+1} + \phi(1+i)\ell - T$$

(D.13)

The lump-sum transfer is

$$T = \phi(M-M_{-1}) = (\gamma-1)\phi M_{-1} = (\gamma-1)\frac{\phi_{-1}}{\gamma}M_{-1} = \frac{\gamma-1}{\gamma}\sigma c'(\sigma\hat{q}_b)\hat{q}_b$$

(D.14)

Using the above and $\phi\ell = (1-\sigma)c'(\sigma\hat{q}_b)\hat{q}_b$ gives us:

$$h = x^* + \gamma\sigma c'(\sigma\hat{q}_b)\hat{q}_b + (1+i)(1-\sigma)c'(\sigma\hat{q}_b)\hat{q}_b - \frac{\gamma-1}{\gamma}\sigma c'(\sigma\hat{q}_b)\hat{q}_b$$

(D.15)

$$= x^* + (\frac{\gamma^2-\gamma+1}{\gamma}\sigma + (1+i)(1-\sigma))c'(\sigma\hat{q}_b)\hat{q}_b$$

(D.16)

For financially included buyers who didn't consume in the AM we have

$$h = x^* + \phi\hat{m}_{+1} - \phi(1+i)d - T$$

(D.17)

which simplifies to

$$h = x^* + \gamma\sigma c'(\sigma\hat{q}_b)\hat{q}_b - (1+i)\sigma c'(\sigma\hat{q}_b)\hat{q}_b - \frac{\gamma-1}{\gamma}\sigma c'(\sigma\hat{q}_b)\hat{q}_b$$

(D.18)

$$= x^* + (\frac{\gamma^2-\gamma+1}{\gamma}\sigma - (1+i)\sigma)c'(\sigma\hat{q}_b)\hat{q}_b$$

(D.19)

Combining the two we have

$$(1-\beta)\hat{V} = \sigma(u(q_b) + U(x^*) - x^* - (\frac{\gamma^2 - \gamma + 1}{\gamma}\sigma + (1+i)(1-\sigma))c'(\sigma\hat{q}_b)\hat{q}_b) \tag{D.20}$$

$$+ (1-\sigma)(U(x^*) - x^* - (\frac{\gamma^2 - \gamma + 1}{\gamma}\sigma - (1+i)\sigma)c'(\sigma\hat{q}_b)\hat{q}_b). \tag{D.21}$$

which simplifies to

$$(1-\beta)\hat{V} = U(x^*) - x^* + \sigma u(\hat{q}_b) - \frac{\gamma^2 - \gamma + 1}{\gamma}\sigma c'(\sigma\hat{q}_b)\hat{q}_b. \tag{D.22}$$

By comparing the two value functions one can show that

$$\sigma u(\hat{q}_b) - \frac{\gamma^2 - \gamma + 1}{\gamma}\sigma c'(\sigma\hat{q}_b)\hat{q}_b \geq \sigma u(q_b) - \frac{\gamma^2 + (\sigma - 2)\gamma + 1}{\gamma}c'(\sigma q_b)q_b \tag{D.23}$$

always holds for $\sigma \leq 1$ with a strict inequality for $\sigma < 1$.

As a consequence, $\hat{V}(0,\hat{m}) > V(0,m)$ and all agents choose to access banks and consume $\hat{q}_b$. The equilibrium conditions reduce to equations (56), (57) and the market clearing condition $\sigma\hat{q}_b = q_s$.

Under the usual assumptions on the utility function ($u'(q) > 0, u''(q) < 0, \lim_{q\to 0} u'(q) = +\infty, \lim_{q\to\infty} u'(q) = 0$) and cost function ($c'(q) > 0, c''(q) \geq 0, , \lim_{q\to 0} c'(q) = 0, \lim_{q\to\infty} c'(q) = +\infty$) we have

$$\frac{\partial \frac{u'(\hat{q}_b)}{c'(\sigma\hat{q}_b)}}{\partial \hat{q}_b} < 0 \tag{D.24}$$

$$\lim_{\hat{q}_b\to 0} \frac{u'(\hat{q}_b)}{c'(\sigma\hat{q}_b)} = +\infty > 0 \text{ for } \sigma \in (0,1], \tag{D.25}$$

$$\lim_{\hat{q}_b\to +\infty} \frac{u'(\hat{q}_b)}{c'(\sigma\hat{q}_b)} = 0. \tag{D.26}$$

It follows that $\hat{q}_b$ exists and is unique. Furthermore, since $\bar{\iota} > 0$, $\hat{q}_b < q^*$. ■

D.4 Proof of Result 3

Proof. It is easy to show that $\frac{\hat{q}_b}{q_b}$ is increasing in γ and decreasing in σ by solving for the ratio of marginal utilities $\frac{u'(\hat{q}_b)}{u'(q_b)}$. To do that start from (53) and (57) to get:

$$\frac{u'(\hat{q}_b)}{u'(q_b)} = \frac{\gamma}{\beta + \frac{\gamma - \beta}{\sigma}}. \tag{D.27}$$

It is straightforward to show that $\frac{\partial(u'(\hat{q}_b)/u'(q_b))}{\partial \gamma} < 0$ and $\frac{\partial(u'(\hat{q}_b)/u'(q_b))}{\partial \sigma} > 0$. As a result, $\frac{\partial(\hat{q}_b/q_b)}{\partial \gamma} > 0$ and $\frac{\partial(\hat{q}_b/q_b)}{\partial \sigma} < 0$. ∎

References

Stefania Albanesi. Inflation and inequality. *Journal of Monetary Economics*, 54(4):1088–1114, 2007.

Franklin Allen and Douglas Gale. Limited market participation and volatility of asset prices. *American Economic Review*, 84(4):933–55, 1994.

Franklin Allen, Asli Demirgüç-Kunt, Leora Klapper, and María Soledad Martínez Pería. The foundations of financial inclusion: Understanding ownership and use of formal accounts. *Journal of Financial Intermediation*, 2016.

Thorsten Beck, Asli Demirguc-Kunt, and Ross Levine. Smes, growth, and poverty: cross-country evidence. *Journal of economic growth*, 10(3):199–229, 2005.

Thorsten Beck, Asli Demirguc-Kunt, and Maria Soledad Martinez Peria. Reaching out: Access to and use of banking services across countries. *Journal of Financial Economics*, 85(1):234–266, 2007.

Roel MWJ Beetsma and Frederick Van Der Ploeg. Does inequality cause inflation?: The political economy of inflation, taxation and government debt. *Public Choice*, 87(1-2):143–162, 1996.

Valerie R Bencivenga and Gabriele Camera. Banking in a matching model of money and capital. *Journal of Money, Credit and Banking*, 43:449–476, 2011.

Aleksander Berentsen, Gabriele Camera, and Christopher Waller. Money, credit and banking. *Journal of Economic Theory*, 135(1):171–195, 2007.

Aleksander Berentsen, Samuel Huber, and Alessandro Marchesiani. Degreasing the wheels of finance. *International economic review*, 55(3):735–763, 2014.

Robin Burgess and Rohini Pande. Do rural banks matter? evidence from the indian social banking experiment. *American Economic Review*, 95(3):780–795, June 2005.

doi: 10.1257/0002828054201242. URL `http://www.aeaweb.org/articles?id=10.1257/0002828054201242`.

Satyajit Chatterjee and Dean Corbae. Endogenous market participation and the general equilibrium value of money. *Journal of Political Economy*, 100(3):615–646, 1992.

Jonathan Chiu and Césaire A Meh. Financial intermediation, liquidity, and inflation. *Macroeconomic Dynamics*, 15(S1):83–118, 2011.

Jonathan Chiu, Mei Dong, and Enchuan Shao. On the welfare effects of credit arrangements. *International Economic Review*, 2018.

Rubens P Cysne, Wilfredo L Maldonado, and Paulo Klinger Monteiro. Inflation and income inequality: A shopping-time approach. *Journal of Development Economics*, 78(2):516–528, 2005.

Eduardo Dávila and Anton Korinek. Pecuniary externalities in economies with financial frictions. *The Review of Economic Studies*, page rdx010, 2017.

Asli Demirgüç-Kunt, Leora F Klapper, Dorothe Singer, and Peter Van Oudheusden. The global findex database 2014: Measuring financial inclusion around the world. 2015.

Douglas W Diamond and Philip H Dybvig. Bank runs, deposit insurance, and liquidity. *Journal of Political Economy*, 91(3):401–419, 1983.

Jim Dolmas, Gregory W Huffman, and Mark A Wynne. Inequality, inflation, and central bank independence. *Canadian Journal of Economics/Revue canadienne d'économique*, 33(1):271–287, 2000.

William Easterly and Stanley Fischer. Inflation and the poor. *Journal of Money, Credit and Banking*, pages 160–178, 2001.

Andrés Erosa and Gustavo Ventura. On inflation as a regressive consumption tax. *Journal of Monetary Economics*, 49(4):761–795, 2002.

Pere Gomis-Porqueras. When Should Bank Regulation Favor the Wealthy? *Journal of Economic Theory*, 99(1-2):327–337, July 2001. URL `https://ideas.repec.org/a/eee/jetheo/v99y2001i1-2p327-337.html`.

Bruce C Greenwald and Joseph E Stiglitz. Externalities in economies with imperfect information and incomplete markets. *The Quarterly Journal of Economics*, 101(2):229–264, 1986.

Chao Gu, Fabrizio Mattesini, and Randall Wright. Money and credit redux. *Econometrica*, 84(1): 1–32, 2016. ISSN 1468-0262. doi: 10.3982/ECTA12798. URL `http://dx.doi.org/10.3982/ECTA12798`.

Ricardo Lagos and Randall Wright. A unified framework for monetary theory and policy analysis. *Journal of Political Economy*, 113(3):463–484, 2005.

Ricardo Lagos, Guillaume Rocheteau, and Randall Wright. Liquidity: A new monetarist perspective. *Journal of Economic Literature*, 55(2):371–440, 2017.

Arindam Laha, D Kuri, and P Kumar. Determinants of financial inclusion: A study of some selected districts of west bengal, india. *Indian journal of finance*, 5(8):29–36, 2011.

Ross Levine. Finance and growth: theory and evidence. *Handbook of Economic Growth*, 1:865–934, 2005.

Lee Hsien Loong and Richard Zeckhauser. Pecuniary externalities do matter when contingent claims markets are incomplete. *The Quarterly Journal of Economics*, 97(1):171–179, 1982.

Sebastien Lotz and Cathy Zhang. Money and credit as means of payment: A new monetarist approach. *Journal of Economic Theory*, 164:68–100, 2016.

J Neill Marshall. Financial institutions in disadvantaged areas: a comparative analysis of policies encouraging financial inclusion in britain and the united states. *Environment and Planning A*, 36(2):241–261, 2004.

Aaron Mehrotra and James Yetman. Financial inclusion-issues for central banks. 2015.

Aaron N Mehrotra and James Yetman. Financial inclusion and optimal monetary policy. 2014.

Lorenzo Menna and Patrizio Tirelli. Optimal inflation to reduce inequality. *Review of Economic Dynamics*, 24:79–94, 2017.

Cyril Monnet and William Roberds. Optimal pricing of payment services. *Journal of Monetary Economics*, 55(8):1428–1440, 2008.

Mark Pickens, David Porteous, and Sarah Rotman. Banking the poor via g2p payments. *CGAP Focus Note, World Bank*, (58), 2009.

Guillaume Rocheteau and Ed Nosal. *Money, Payments, and Liquidity*. MIT Press, 2017.

Guillaume Rocheteau and Randall Wright. Money in search equilibrium, in competitive equilibrium, and in competitive search equilibrium. *Econometrica*, 73(1):175–202, 2005.

Mariana Rojas Breu. The welfare effect of access to credit. *Economic Inquiry*, 51(1):235–247, 2013.

Christina D Romer and David H Romer. Monetary policy and the well-being of the poor. *Economic Review, Federal Reserve Bank of Kansas City*, 84(1):21, 1999.

Daniel Sanches. A dynamic model of unsecured credit. *Journal of Economic Theory*, 146(5):1941–1964, 2011.

Daniel Sanches and Stephen Williamson. Money and credit with limited commitment and theft. *Journal of Economic Theory*, 145(4):1525–1549, 2010.

Mandira Sarma and Jesim Pais. Financial inclusion and development. *Journal of international development*, 23(5):613–628, 2011.

Stephen D Williamson. Liquidity and market participation. *Journal of Economic Dynamics and Control*, 18(3):629–670, 1994.