Numerical Solution Method

Interactions and Coordination between Monetary and Macro-Prudential Policies

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These notes explain the method I use to solve the model numerically. The notes have two parts. The first part explains the numerical solution method for the economy with flexible prices. The second does so for the economy with sticky prices. In both parts, first, I consider an economy without macro-prudential policy intervention, and then, consider one with.

1 Flexible Price Economy...

...without Macro-Prudential Policy

Conditions (41)-(50) and (53)-(55) in the Online Appendix characterize analytically the solution to this economy. These conditions specify a second-order ordinary differential equation system (ODEs) for mappings $\{q/A, v\}$ in state η in implicit form. To solve the ODEs, I use spectral methods. Specifically, I interpolate q/A and v each with a linear combination of Chebyshev polynomials of the first kind, evaluating the interpolation at the Chebyshev nodes. I use a nonlinear solver to find the coefficients in the linear combinations. The nonlinear solver uses as its initial guess the Markov equilibrium in the frictionless economy.

...with Macro-Prudential Policy

For any given policy mapping Φ, conditions (42)-(50) and (53)-(56) in the Online Appendix characterize analytically the solution to this other economy. These conditions also specify a second-order ODEs for $\{q/A, v\}$ in η in implicit form. To solve this ODEs, I use the same method as in the previous economy except I use the solution to the previous ODEs as the initial guess for the nonlinear solver.

To derive the socially optimal policy, for simplicity, I restrict attention to mappings Φ that are a polynomial function of state η within intervention region $[\eta_L, \eta_H]$. That is:

$$\Phi(\eta) = \sum_{p=0}^{P} \frac{a_p}{\eta_H - \eta_L} (\eta - \eta_L)^p \, , \text{ for } \eta \in (\eta_L, \eta_H) \, , \tag{1}$$

with $\{a_p\}_{p=0}^{P}$ being real-valued constants, $P \geq 1$ the degree of the polynomial, and $\Phi(\eta_L) = \lambda v(\eta_L)$ and $\Phi(\eta_H)\eta_H = 1$.

Social welfare is $\int \tilde{U}(\eta) g(\eta) d\eta$, with value $\tilde{U}$ being analytically characterized in the paper by Hamilton-Jacobi-Bellman (HJB) equation (37). This equation specifies a second-order ODEs for $\tilde{U}$ in η in implicit form, which I solve using the same method as in the previous economy. In the codes, I find that beyond a polynomial degree of 5, i.e. $P \geq 5$, the social welfare level of the socially optimal policy increases only in a negligible manner as the degree of the polynomial does so.

2 Sticky Price Economy...

...without Macro-Prudential Policy

For a given policy mapping $\ln(l/l_E)$, mappings $\{q/A, v, \pi\}$ characterize analytically the solution to this economy. Conditions (41)-(50) and (53)-(55) in the Online Appendix specify a second-order ODEs for $\{q/A, v\}$ in state η in implicit form. This ODEs does not depend on inflation π, but it does depend on employment gap $\ln(l/l_E)$. To solve this ODEs, I use the same method as for the ODEs in the economy with flexible prices and without macro-prudential policy intervention. Conditions (60)-(63) in the Online Appendix specify a second-order partial differential equation system (PDEs) for mappings $\{M, N\}$ in states ω and η in implicit form. These mappings determine $\pi = \frac{\theta}{\varepsilon-1}\left[1 - \left(\frac{N}{M}\right)^{-(\varepsilon-1)}\right]$.

Because I restrict attention to employment gaps $\ln(l/l_E) \simeq 0$ that remain close to zero, I postulate that in the invariant distribution, price dispersion $\omega \simeq 1$ remains close to one. To simplify the solution method, I approximate $\{M, N\}$ with mappings that do not depend on state ω, and I solve the ODEs:

$$\theta\left(\frac{l}{l_E}\right)^{1+\psi}\frac{1}{N}+\varepsilon\pi-(\theta+\rho)+\varepsilon_N\mu_\eta+\frac{1}{2}\varepsilon_{N_\eta}\varepsilon_N\sigma_\eta^2=0\ , \tag{2}$$

$$\theta\frac{1}{M}+(\varepsilon-1)\,\pi-(\theta+\rho)+\varepsilon_M\mu_\eta+\frac{1}{2}\varepsilon_{M_\eta}\varepsilon_M\sigma_\eta^2=0\ , \tag{3}$$

for $\{M,N\}$ in η, with $\pi=\frac{\theta}{\varepsilon-1}\left[1-\left(\frac{N}{M}\right)^{-(\varepsilon-1)}\right]$. To do so, I use the method for the ODEs in the economy with flexible prices and without macro-prudential policy intervention.

To derive the socially optimal policy, for simplicity, I restrict attention to mappings $\ln\left(l/l_E\right)$ that are affine in state η. That is:

$$l\left(\eta\right)=l_E\exp\left\{a_0+a_1\eta\right\}\ , \tag{4}$$

with a_0 and a_1 being real-valued constants.

Social welfare is $\int\left[\tilde{U}\left(\eta\right)+\hat{U}\left(\omega,\eta\right)\right]g\left(\omega,\eta\right)d\left(\omega,\eta\right)$. To derive value $\tilde{U}$, I use the same method as in the economy with flexible prices. I decompose value $\hat{U}$ in two terms, $\hat{U}=\hat{U}_L+\hat{U}_P$, with

$$\rho\hat{U}_L=\alpha\ln l-\chi\frac{l^{1+\psi}}{1+\psi}+\frac{\partial\hat{U}_L}{\partial\eta}\mu_\eta\eta+\frac{1}{2}\frac{\partial^2\hat{U}_L}{\left(\partial\eta\right)^2}\left(\sigma_\eta\eta\right)^2\ , \tag{5}$$

$$\rho\hat{U}_P=\ln\frac{1}{\omega}+\frac{\partial\hat{U}_P}{\partial\omega}\mu_\omega\omega+\frac{\partial\hat{U}_P}{\partial\eta}\mu_\eta\eta+\frac{1}{2}\frac{\partial^2\hat{U}_P}{\left(\partial\eta\right)^2}\left(\sigma_\eta\eta\right)^2\ . \tag{6}$$

Condition (5) specifies a second-order ODEs for $\hat{U}_L$ in state η in implicit form, which I solve using the method for the ODEs in the economy with flexible prices and without macro-prudential policy intervention. I do not solve value $\hat{U}_P$. Rather, I compute $\int\hat{U}_P\left(\omega,\eta\right)g\left(\omega,\eta\right)d\left(\omega,\eta\right)$ as $\frac{1}{T-t}\sum_{s=t+1}^{T}\ln\frac{1}{\omega_s}$ by simulating the economy, with T being the number of time periods in the simulations and $t<T$ being a sufficiently large time period I set to mitigate the effect on the simulation of the initial state.

...with Macro-Prudential Policy

To solve this last economy, I combine the solution methods of the previous two.