Notes on Powers & Roots

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Exponential Growth

- Case 1: +ve base < 1
  - E.g.:
    $$(\frac{1}{2})^1, (\frac{1}{2})^2, (\frac{1}{2})^3, (\frac{1}{2})^4, (\frac{1}{2})^5, (\frac{1}{2})^6 ...$$
    or
    $$\frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \frac{1}{16}, \frac{1}{32}, \frac{1}{64} ...$$
  - Higher the exponent smaller the number
- Case 2: -ve base < -1
  - E.g.:
    $$(-3)^1, (-3)^2, (-3)^3, (-3)^4, (-3)^5$$
    or
    $$-3, -9, -27, -81, -243$$
  - Absolute value increases
  - +,- alternates
- Case 3: -ve base between -1 & 0
  - E.g.:
    $$(-\frac{1}{2})^1, (-\frac{1}{2})^2, (-\frac{1}{2})^3, (-\frac{1}{2})^4, (-\frac{1}{2})^5, (-\frac{1}{2})^6 ...$$
    or
    $$-\frac{1}{2}, \frac{1}{4}, -\frac{1}{8}, \frac{1}{16}, -`\frac{1}{32}, \frac{1}{64} ...$$
  - Absolute value decreases
  - +,- alternates

Exponent Properties

- Inverse law:
  $$b^{-n} = \frac{1}{b^n}$$
  and
  $$(\frac{p}{1})^{-n} = (\frac{q}{p})^n$$

- Product law:
$$(ab)^n = a^n b^n$$
- Quotient law:
$$(\frac{a}{b})^n = \frac{a^n}{b^n}$$
- Factoring and Distributing:
$$P(a \pm b) = Pa \pm Pb$$
- Warning:
$$(a \pm b) \neq a^n \pm b^n$$
- Equating powers: If,
$$a^m = a^n$$
then,
$$m = n$$

Unit digit of Power

- Unit digit of any product is influenced by the unit digit of the two factors being multiplied. E.g.:
$$12\underline{3} \cdot 1\underline{6} = 3886\underline{8}$$
- What is the unit digit of
$$57^{123}$$
?
  1. Considering only one's place,
  $$7^1 = \underline{7}, 7^2 = ...\underline{9}, 7^3 = ...\underline{3}, 7^4 = ...\underline{1}, 7^5 = ...\underline{7}$$
  2. We restart pattern (1,7,9,3,1,7,9,3...) at multiples of 4. Therefore
  $$7^{120}$$
  must be 1
  3. Therefore,
  $$7^{120}$$
  has 1 in unit place,
  $$7^{121}$$
  has 7,
  $$7^{122}$$
  has 9 and
  $$7^{123}$$
  has 3
  4. Therefore, the unit digit of
  $$57^{123}$$
  is 3

Radicals

Radical Properties

- $$\sqrt{PQ} = \sqrt{P}.\sqrt{Q}$$
- $$\sqrt{\frac{P}{Q}} = \frac{\sqrt{P}}{\sqrt{Q}}$$
- $$b^{\frac{1}{2}} = \sqrt{b}$$
- $$b^{\frac{1}{m}} = \sqrt[m]{b}$$
- $$b^{\frac{m}{n}} = (b^m)^{\frac{1}{n}} = (b^{\frac{1}{n}})^m$$
- $$\sqrt{k^2} = k$$

  (only if
  $$k \geq 0$$
)
  - E.g.: if
  $$k = -4, \sqrt{-4^2} \neq -4$$
- Common radical-to-decimals:
  $$\sqrt{2} = 1.4; \sqrt{3} = 1.7; \sqrt{5} = 2.2$$

Rationalization

Always rationalize final value (eliminate radical from denominator) - E.g:

$$\frac{1}{\sqrt{5}} = \frac{1}{\sqrt{5}} \cdot \frac{\sqrt{5}}{\sqrt{5}} = \frac{\sqrt{5}}{25}$$

- Sometimes, use conjugate to rationalize the final value. E.g.:

$$a^2 \cdot b^2 = (a+b)(a-b)$$

Extraneous roots in Radical equations

In a quadratic equation there are 3 possibilities with the final root(s): - Both root work - Only 1 root works. E.g.:

$$\sqrt{x+3} = x - 3$$

has these roots: {1, 6}, but only x = 6 works not x =1 - Both roots do not work (Even if roots exist, they don't work)

Preservation of Order of Inequality

- If

$$b > 1$$

,

$$\sqrt{b} < b$$

- If

$$0 < b < 1$$

,

$$\sqrt{b} > b$$

- Roots preserve order of inequality
  - if

$$0 < a < b < c$$

    order is preserved when

$$0 < \sqrt[n]{a} < \sqrt[n]{b} < \sqrt[n]{c}$$

  - E.g.

$$\sqrt[4]{50}$$

    is between what two positive integers?
    * $16 < 50 < 81$

$$\equiv$$

$$0 < \sqrt[4]{16} < \sqrt[4]{50} < \sqrt[4]{81}$$

      So,

$$\sqrt[4]{50}$$

      is between 2 and 3