W4. Discrete Classification II - Decision Trees

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Week 4

Outline

Issues in business decision making

- How to present complicated decision problems in a systematical, logic, and intuitive way?

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- Decision Tree

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- Expected monetary value (EMV)

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- Decision Tree
- Expected monetary value (EMV)
- Multi-stage decision problems

Motivating case: Warren agency

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- The client also requests:
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  - If A is sold within a month, then Warren will get the commission for A and the option of stopping at this point or trying to sell either the B or C next under the same conditions (i.e., sell within a month or no commission on the second property and no chance to sell the third).

Warren's decision problem

After the client left, Warren estimated the costs, revenues, and the probability to sell each property within a month:

	Cost	Price	Commission (4%)	Probability
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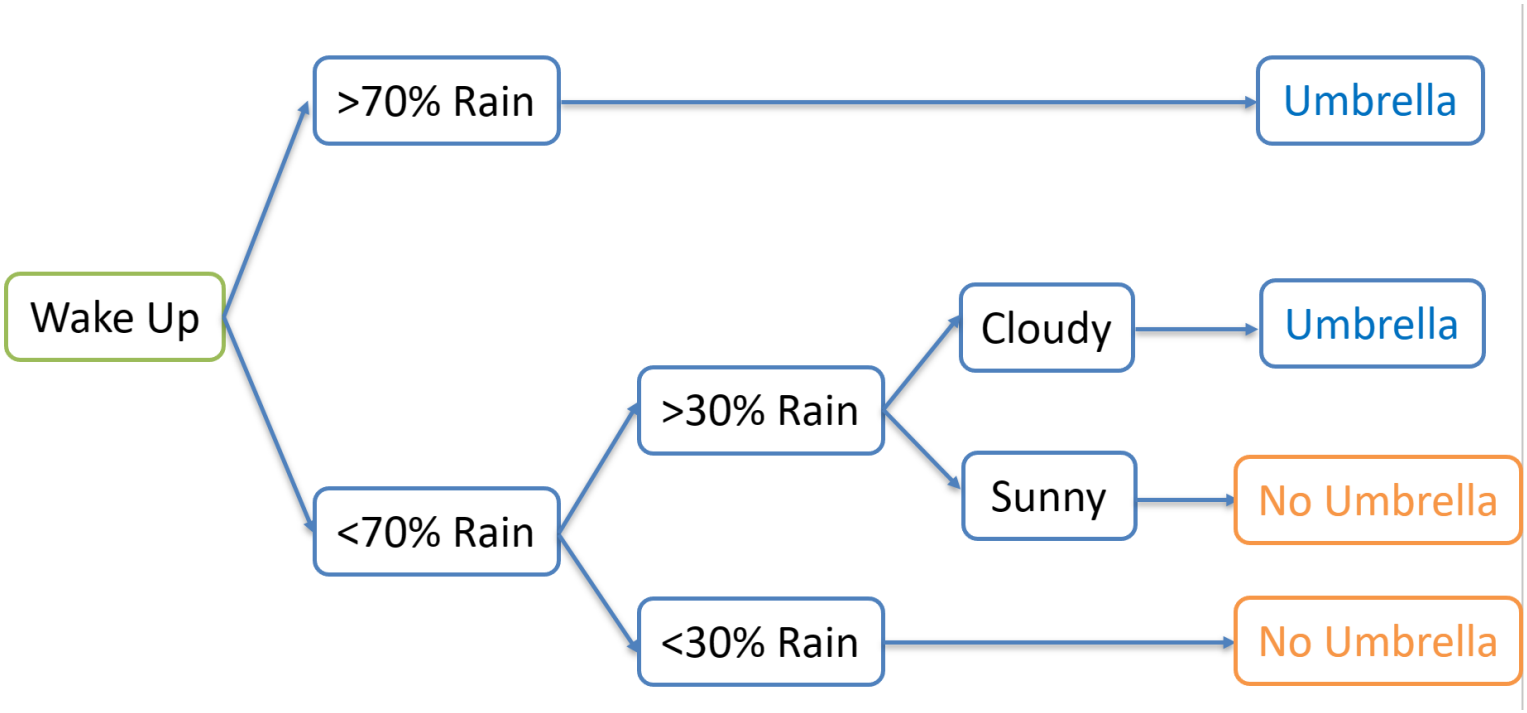
- On what basis would you make your decisions?

Can we present a complex decision problem in a logical and intuitive way?

- Using decision tree !

What is a decision tree ?



- Tree-logic uses a sequence of inquiries to come to a conclusion.

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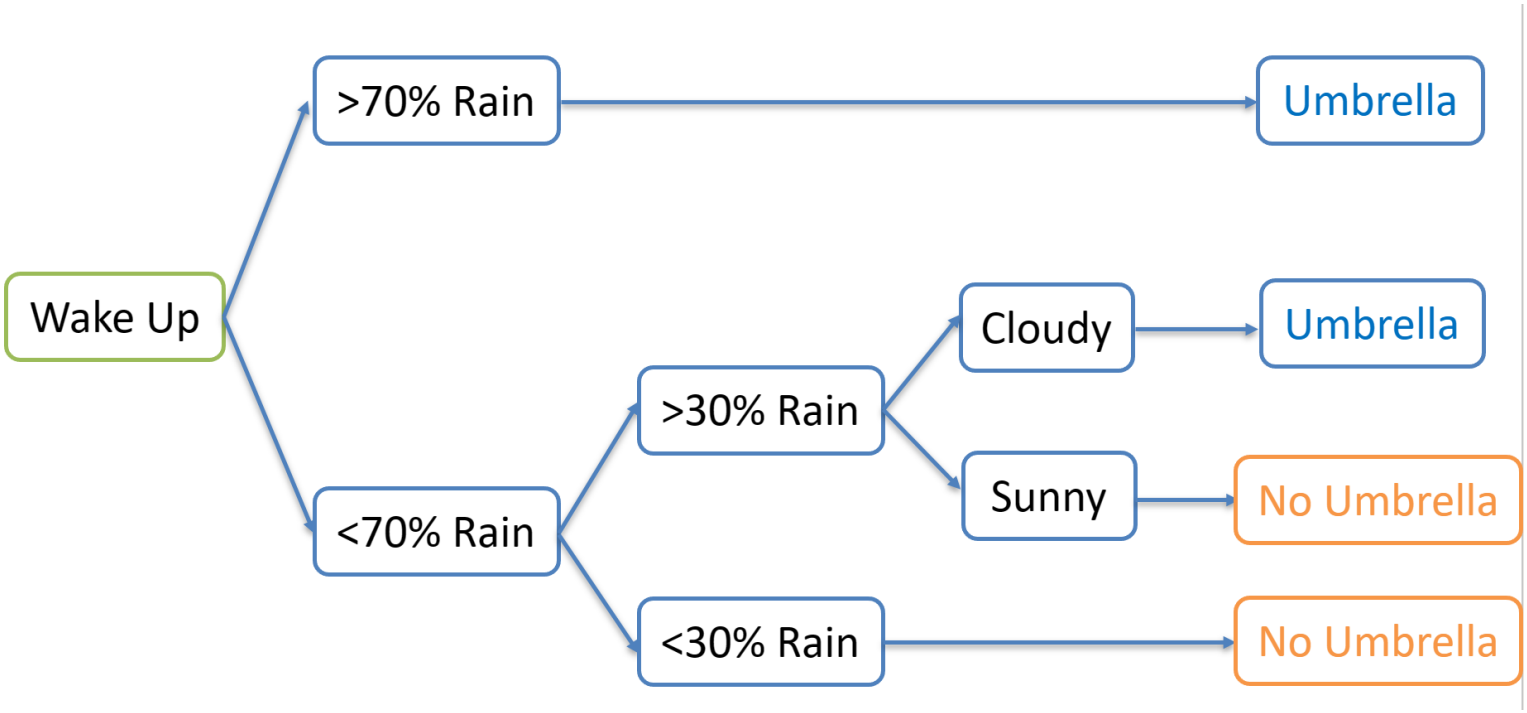

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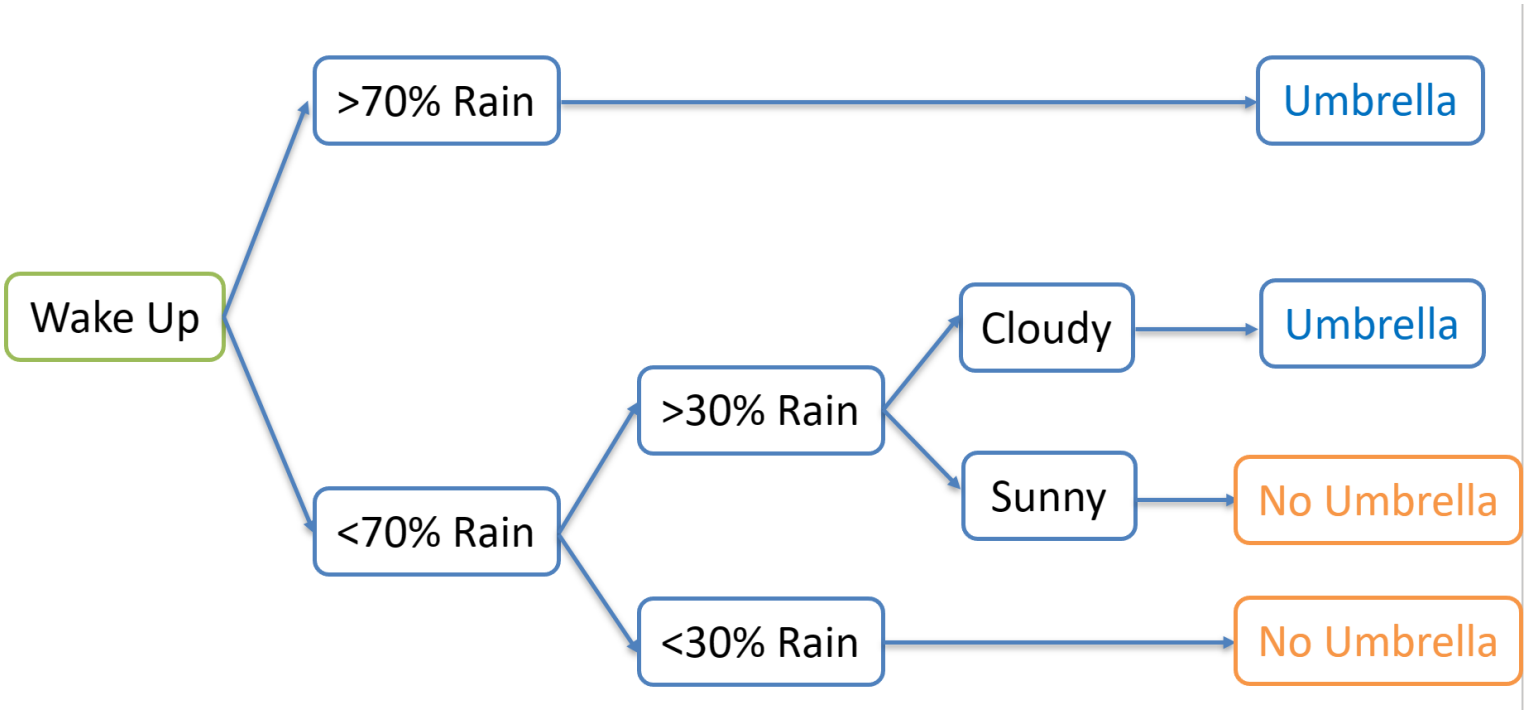

- Tree-logic uses a sequence of inquiries to come to a conclusion.
- The trick is to have mini-decisions combine for good choices.
- Each decision is a node, and the final prediction is a leaf node.

Decision tree

- A graphical method to describe decision problems

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- Illustrate key decision variables, time sequence, and available information:

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- A graphical method to describe decision problems
- Illustrate key decision variables, time sequence, and available information:
  - Alternatives
  - Possible outcomes and associated probabilities for each alternative
  - Gains and losses of each possible outcome

Structure decision problems with a "Tree"

Decision trees have three types of nodes:

- A decision node (a square)

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- A probability node (a circle), also called "chance" or "event" node
  - When the result of an uncertain event becomes known.
- An end node (a triangle), also called "leaf" node
  - When the problem is completed (all decisions have been made, all uncertainty has been resolved, and all payoffs/costs have been incurred).

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  - Represent possible outcomes of an uncertain event
  - Decision maker has no control over which will occur, but can specify a probability for each branch

Structure decision problems with a "Tree"

Time proceeds from left to right:

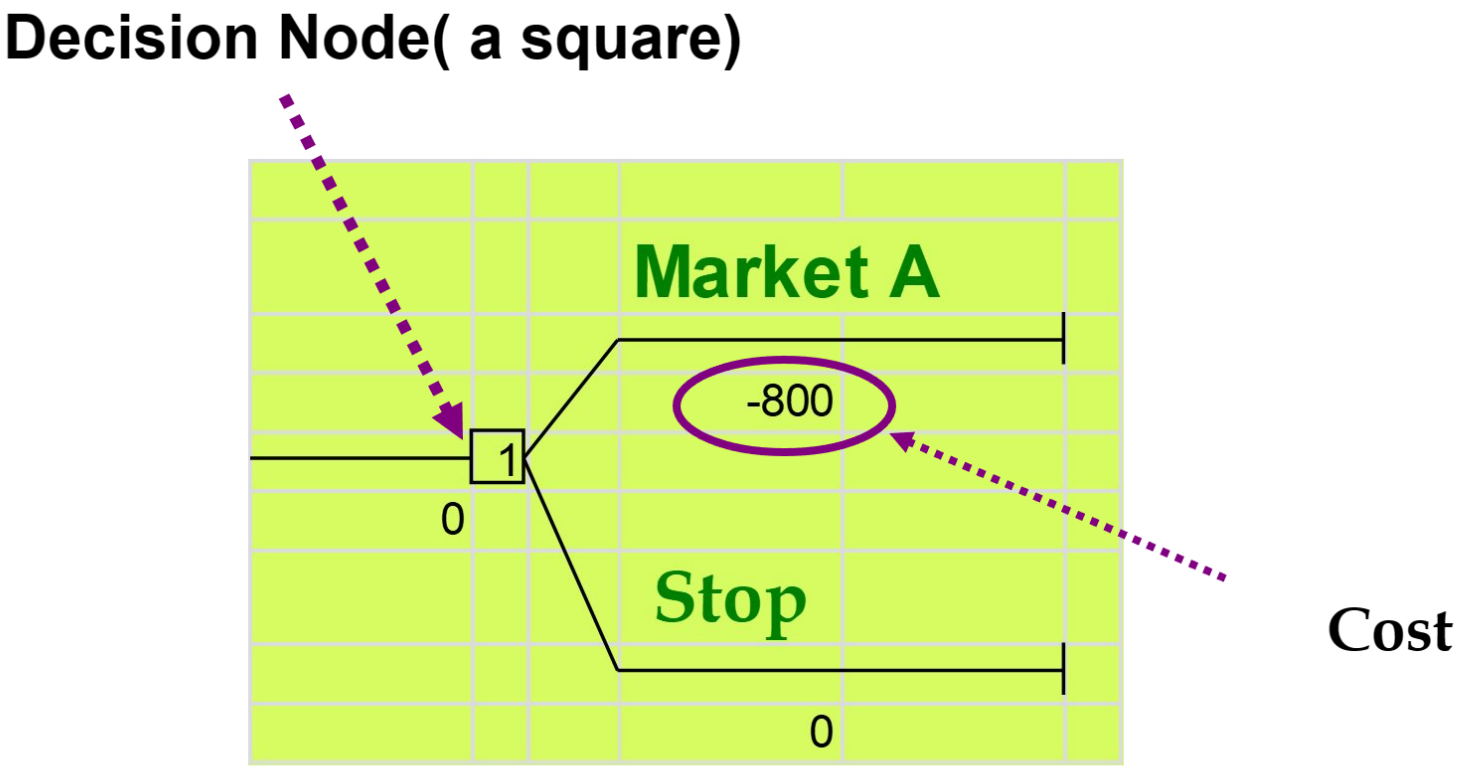
- Any branches leading into a node (from the left) have already occurred.

Structure decision problems with a "Tree"

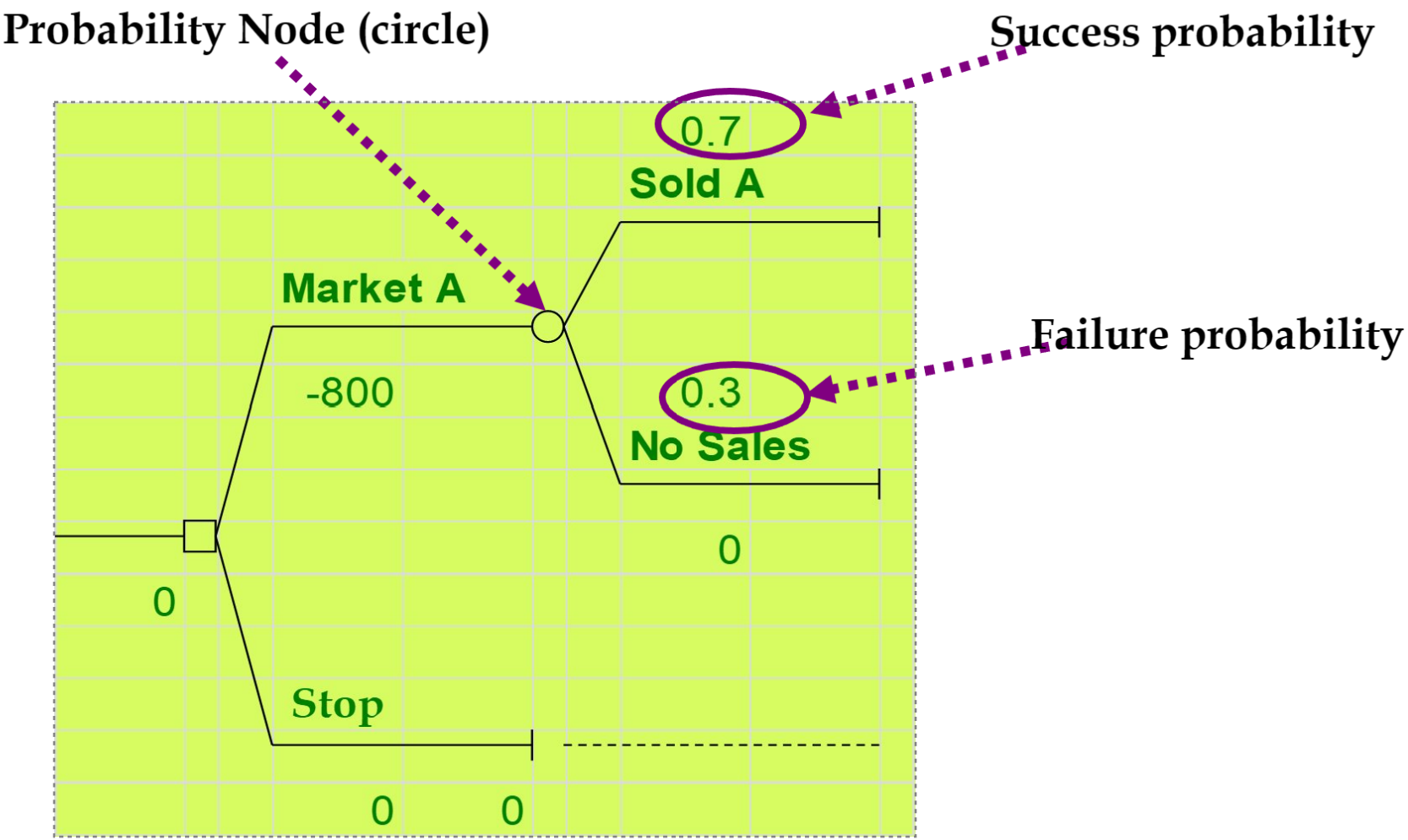
Time proceeds from left to right:

- Any branches leading into a node (from the left) have already occurred.
- Any branches leading out of a node (to the right) have not yet occurred.

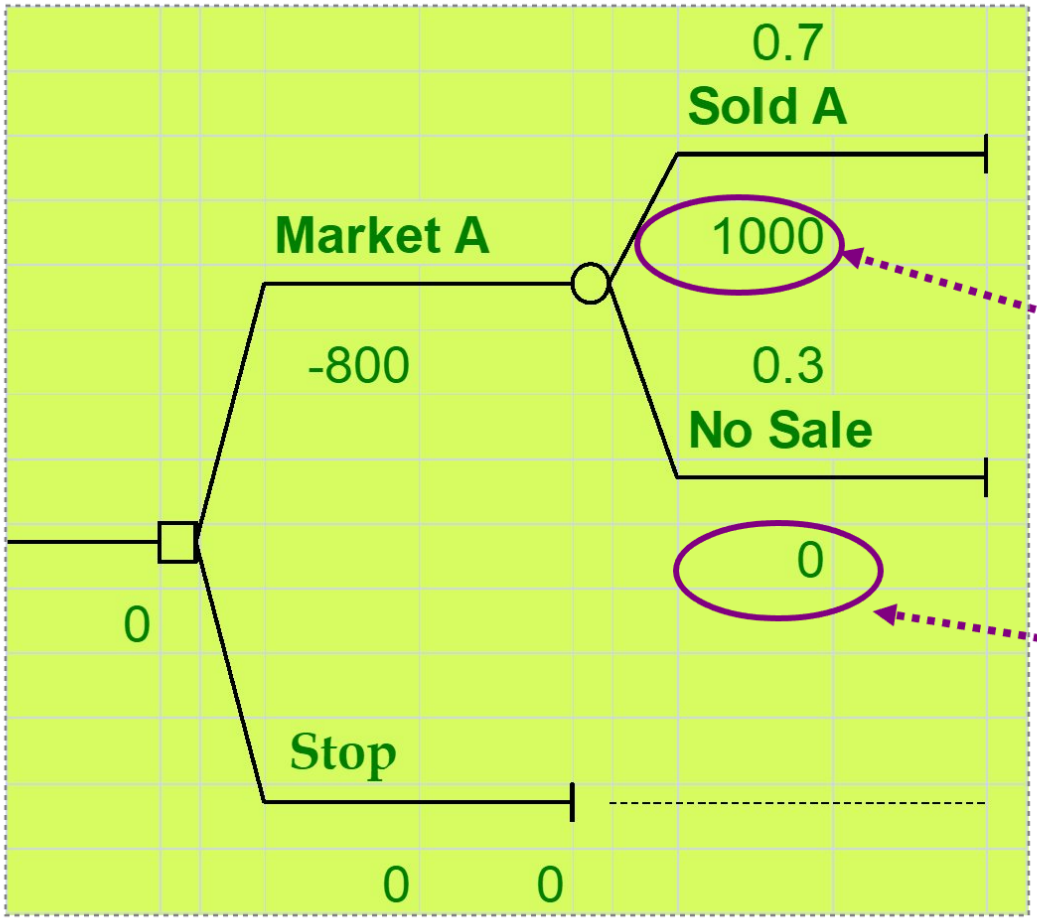
Construct a decision tree (decision node, cost)



Construct a decision tree (probability node, probability)



Construct a decision tree (all relative revenue and loss)



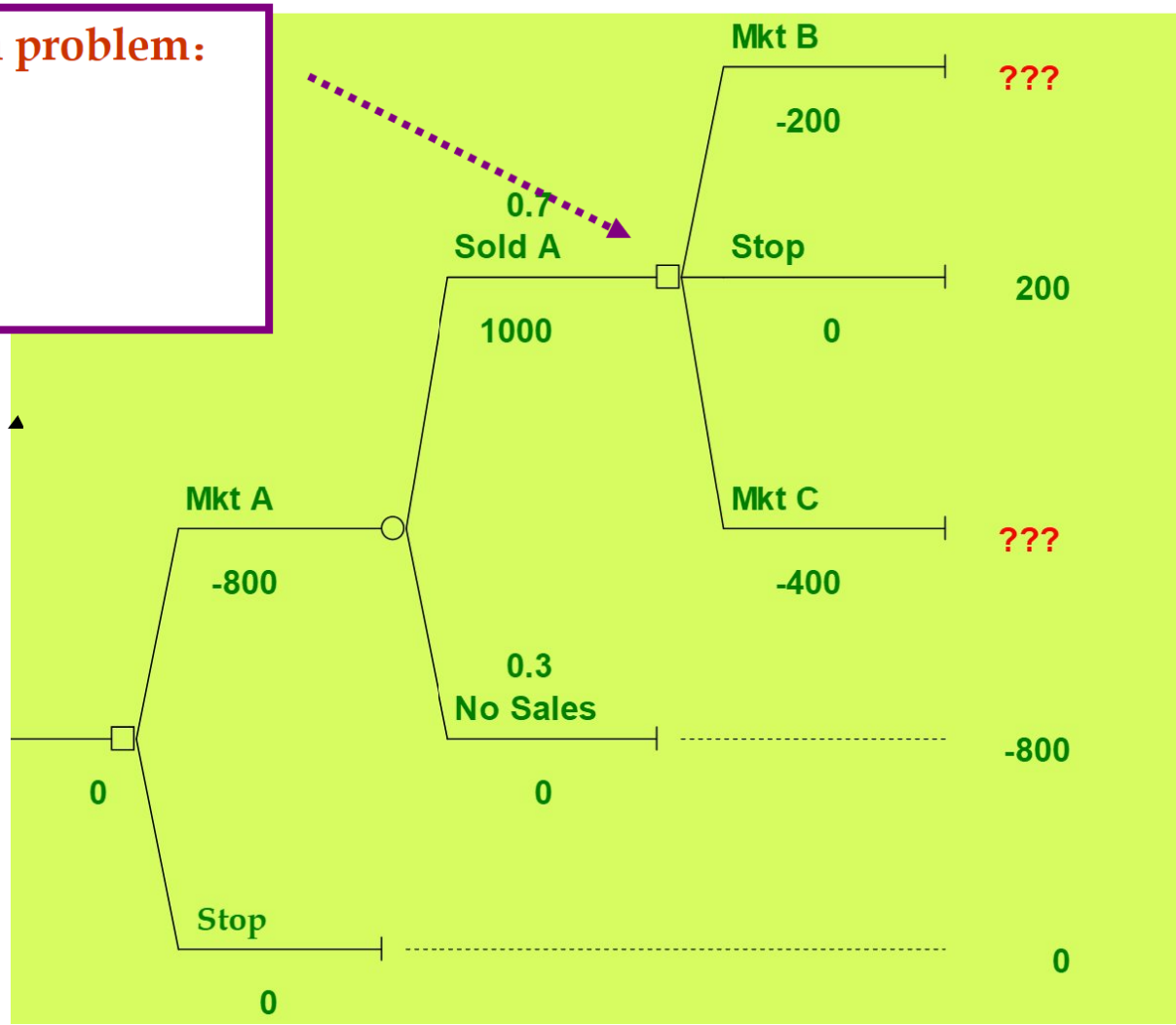
If sold, revenue
=(25,000)x(4%)
=1000

If no sales, no revenue & no right to sell other properties

Construct a decision tree (multiple phase decisions)

Multiple Decision problem:
After A is sold →

- **Market B?**
- **Stop?**
- **Market C?**



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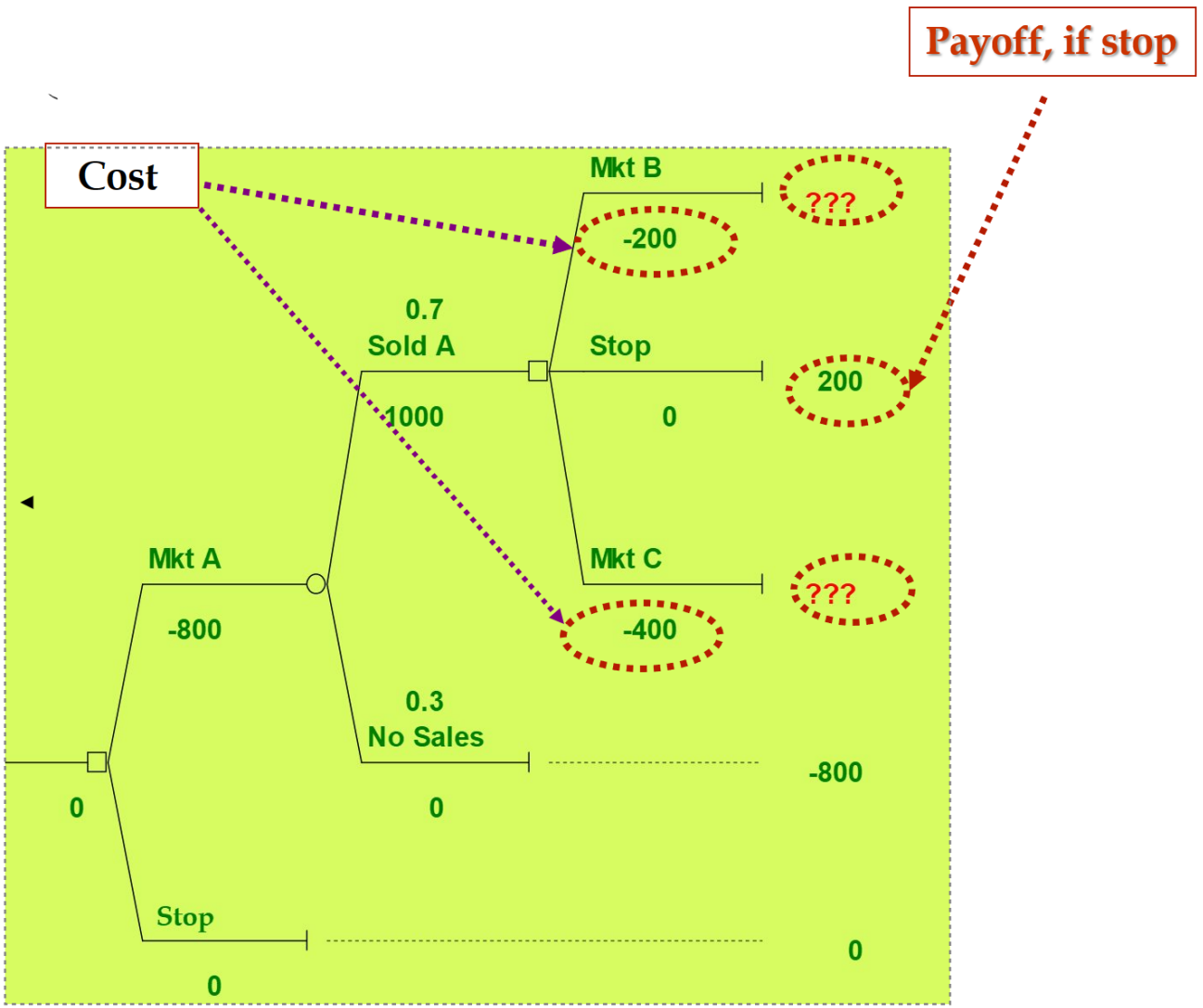

Group discussion 1

Grow a decision tree to describe

- win \$300 for sure

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- play a gamble in which you have an 80% chance of winning \$450, otherwise \$0.

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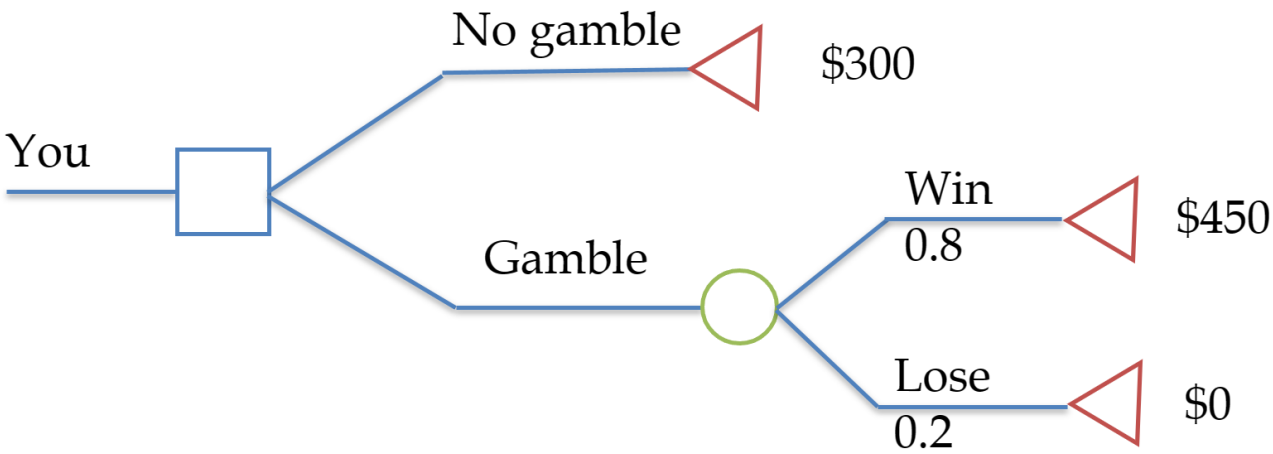

Group discussion 2

Grow a decision tree to describe

- play a gamble with a 75% chance of winning \$300, otherwise \$0

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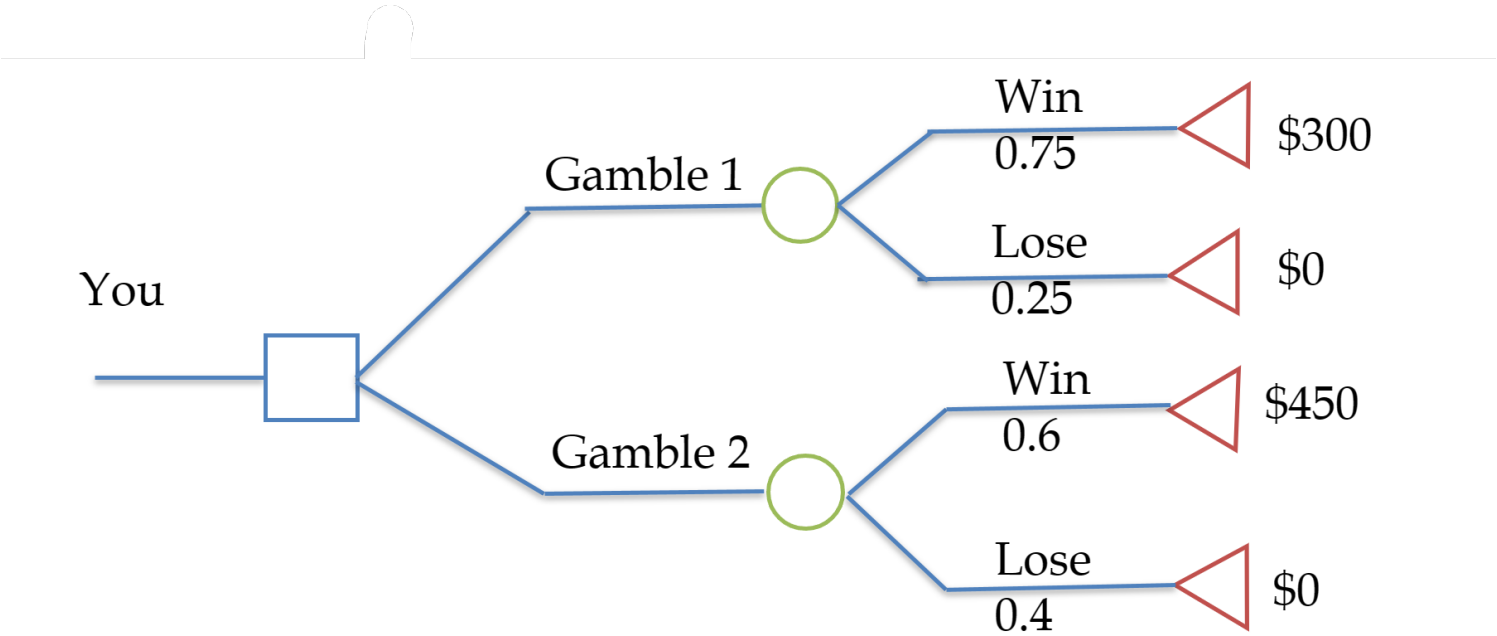

Warren's decision tree

Selling Rule:

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Please try to construct Warren's tree with your classmates

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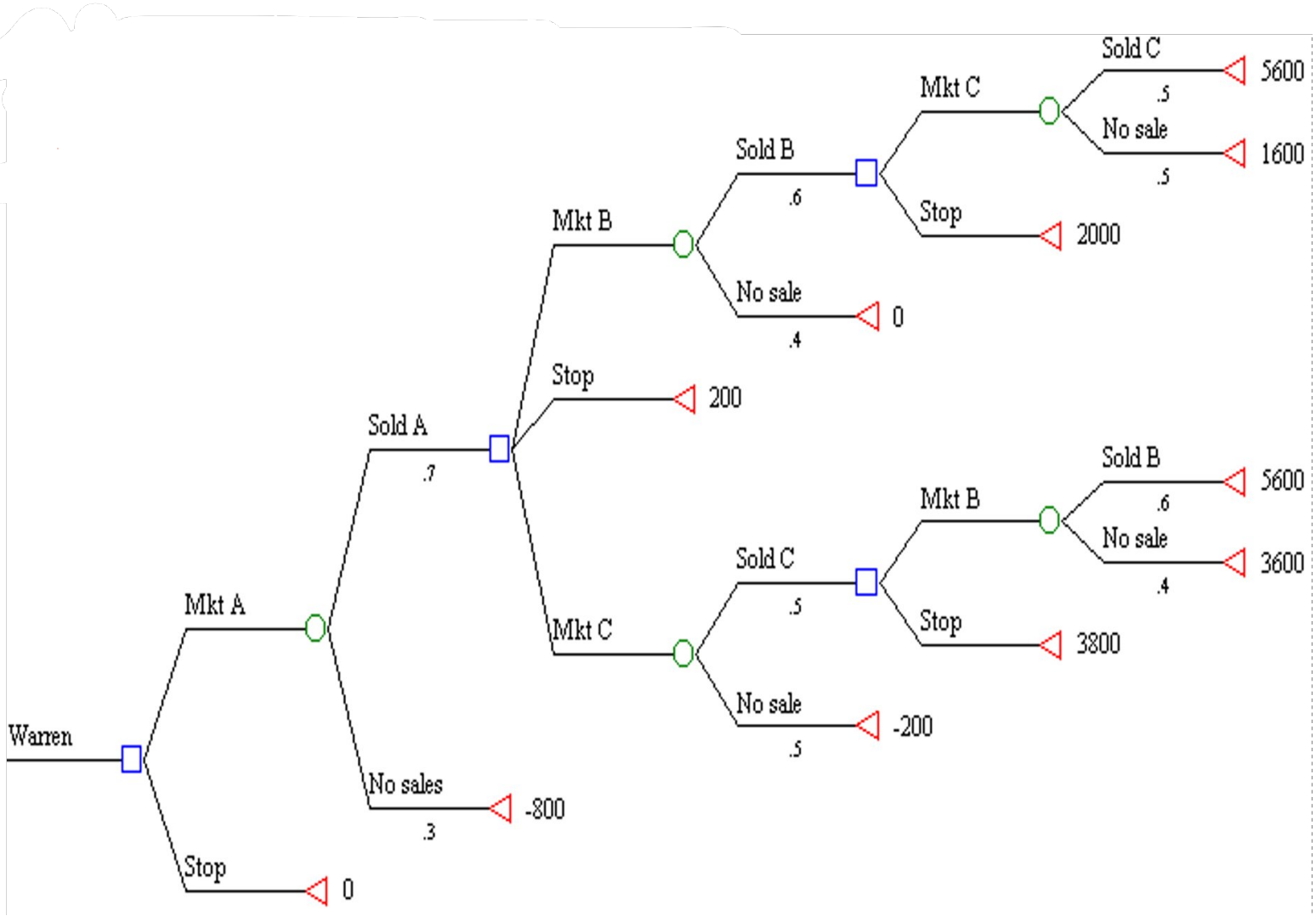

How to choose among multiple alternatives?

- Calculate Expected Monetary Value (EMV)!

Expected Monetary Value (EMV)

- Example: Investment

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- 3 possible outcomes: A (bad), B(medium), C(good)

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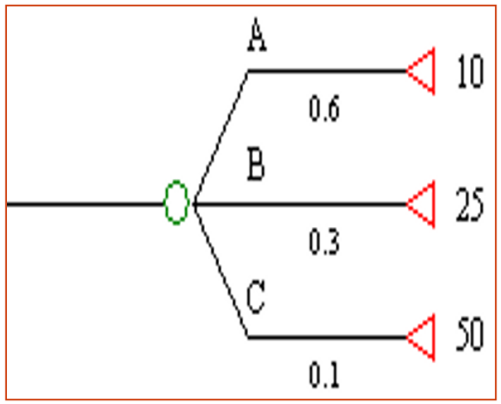

EMV:

$$\text{EMV} = \Sigma V_i P_i$$
$$= (10)(0.6) + (25)(0.3) + (50)(0.1)$$
$$= 18.5$$

Expected monetary value (EMV)

- Expected Monetary Value (EMV)—the weighted average of the possible outcomes, where the weights are the probabilities of those outcomes. Formally, if V_i is the monetary value corresponding to outcome i and P_i is its probability, then expected monetary value is defined as

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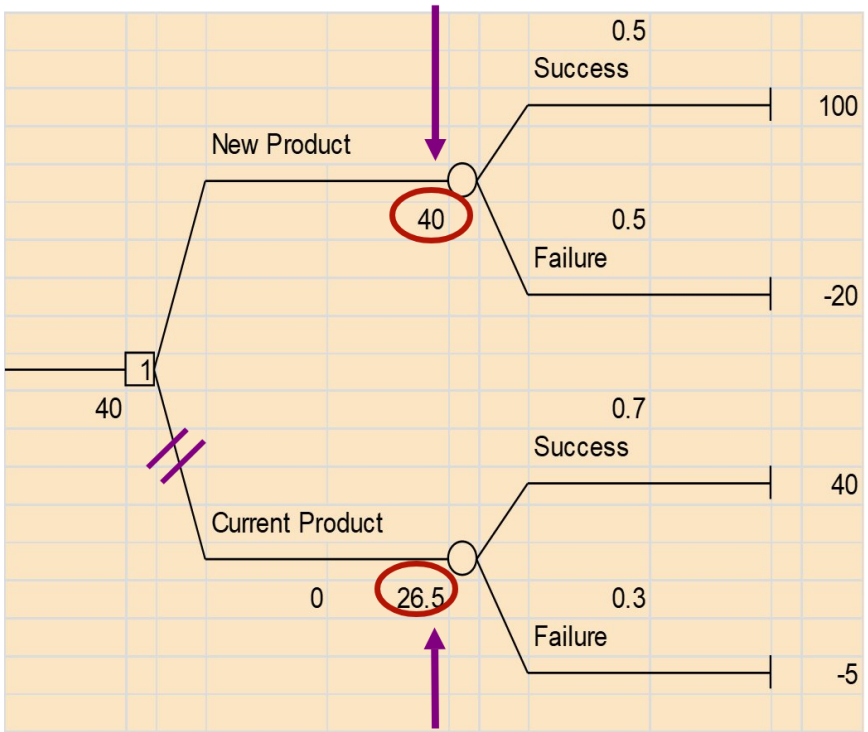
$$EMV = \sum V_i P_i.$$
- One common way to make the choice is to calculate EMV of each alternative and then choose the alternative with the largest EMV.

Making choice using EMV

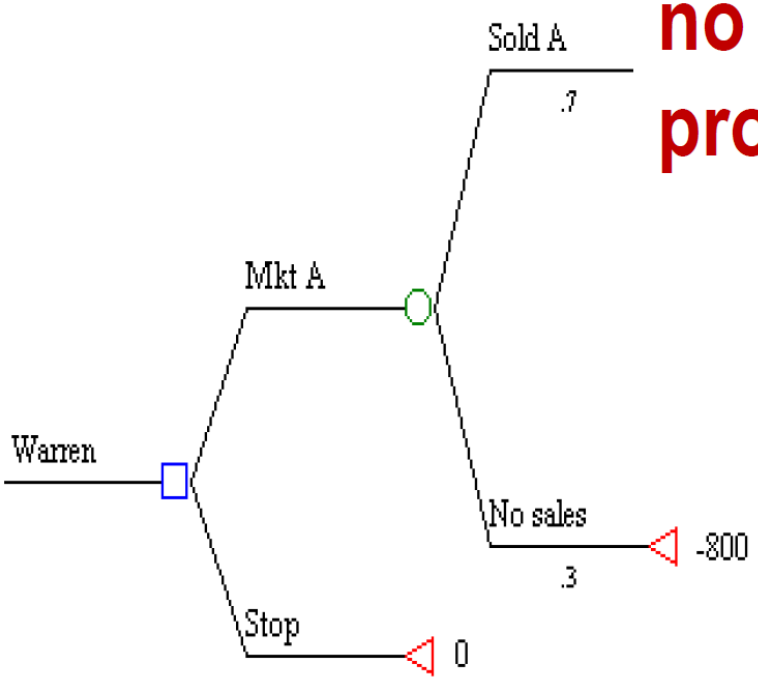
- Maximum EMV Criterion—using the information given in the payoffs together with the information given in the probabilities to determine the decision that has the highest expected value.

EMV=(100)*(0.5)+(-20)*(0.5)=40→optimal



EMV=(40)*(0.7)+(-5)*(0.3)=26.5

How to make a choice?



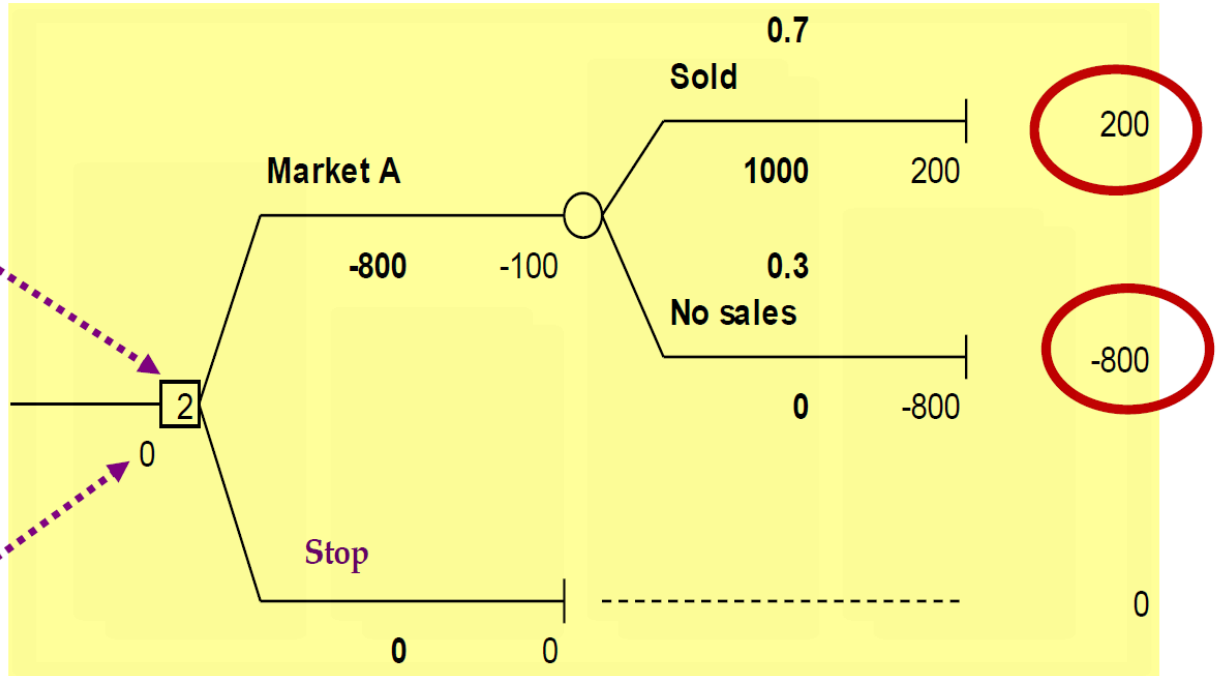
Let's first consider a simple single stage decision: no chance to sell other properties

Single stage decision

$$\text{EMV}=(25{,}000\times 4\%-800)(0.7)+(-800)(0.3)=\textbf{-100} \rightarrow \textbf{Loss!!}$$

The number in the decision node indicates which branch is optimal based on EMV.

The number before the decision node indicates EMV if following the optimal decision.



Group discussion 3

- Which to choose

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A multiple stage decision

- Warren's case $\rightarrow$ multiple stage problem:
  If A can be sold within a month $\rightarrow$ potential to earn profits!

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A multiple stage decision

- Warren's case $\rightarrow$ multiple stage problem:
  If A can be sold within a month $\rightarrow$ potential to earn profits!
  - How big is the chance?
  - Is the risk worth taking?
- "Folding back the tree" $\rightarrow$ Solve for the optimal strategy
- Decision software (such as TreePlan in Excel) automatically computes all decision branches' EMV, and identifies the optimal choice.

Solving a decision tree: folding back

- Start at the far right side of tree and move left

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- At chance nodes: calculate EMV

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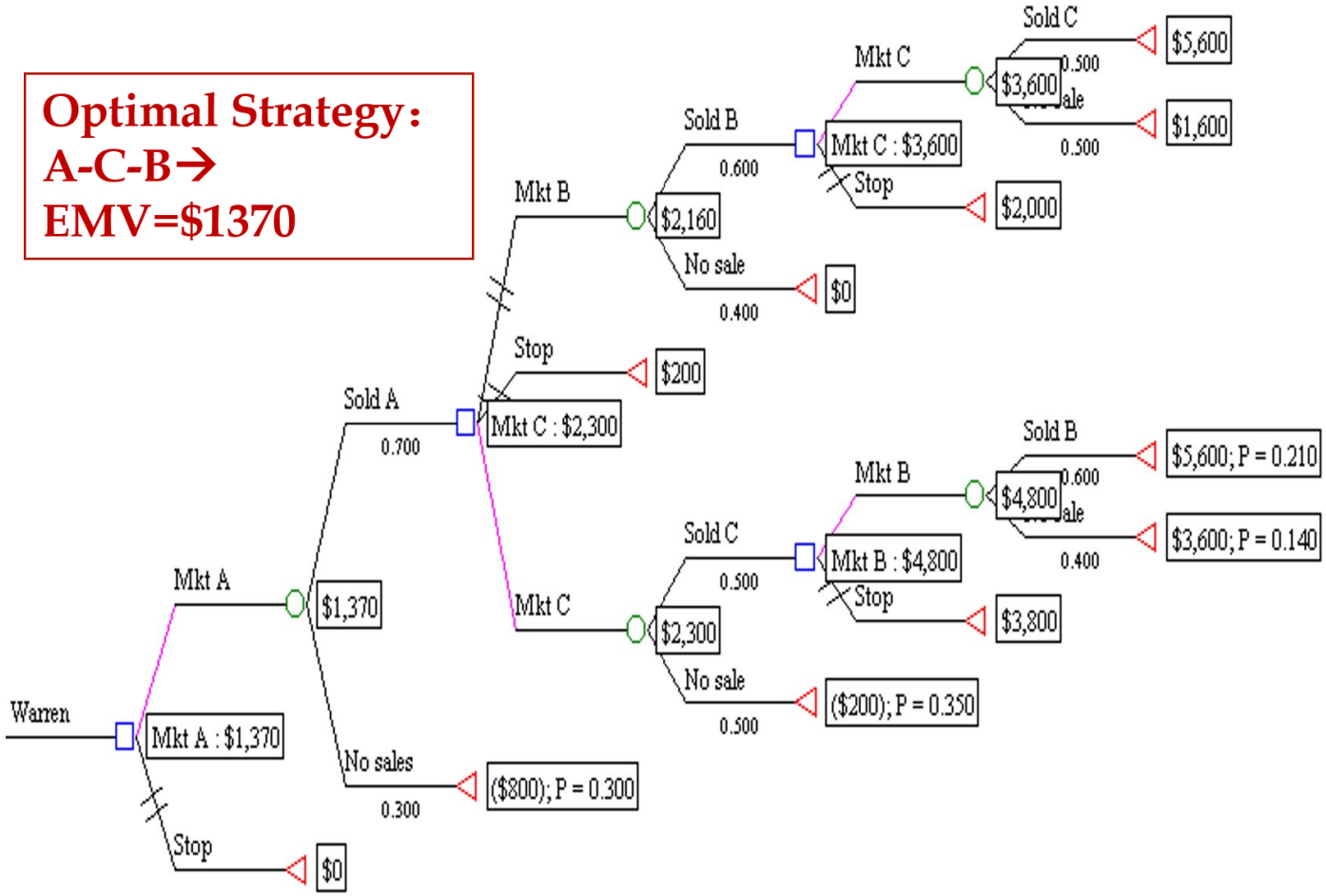
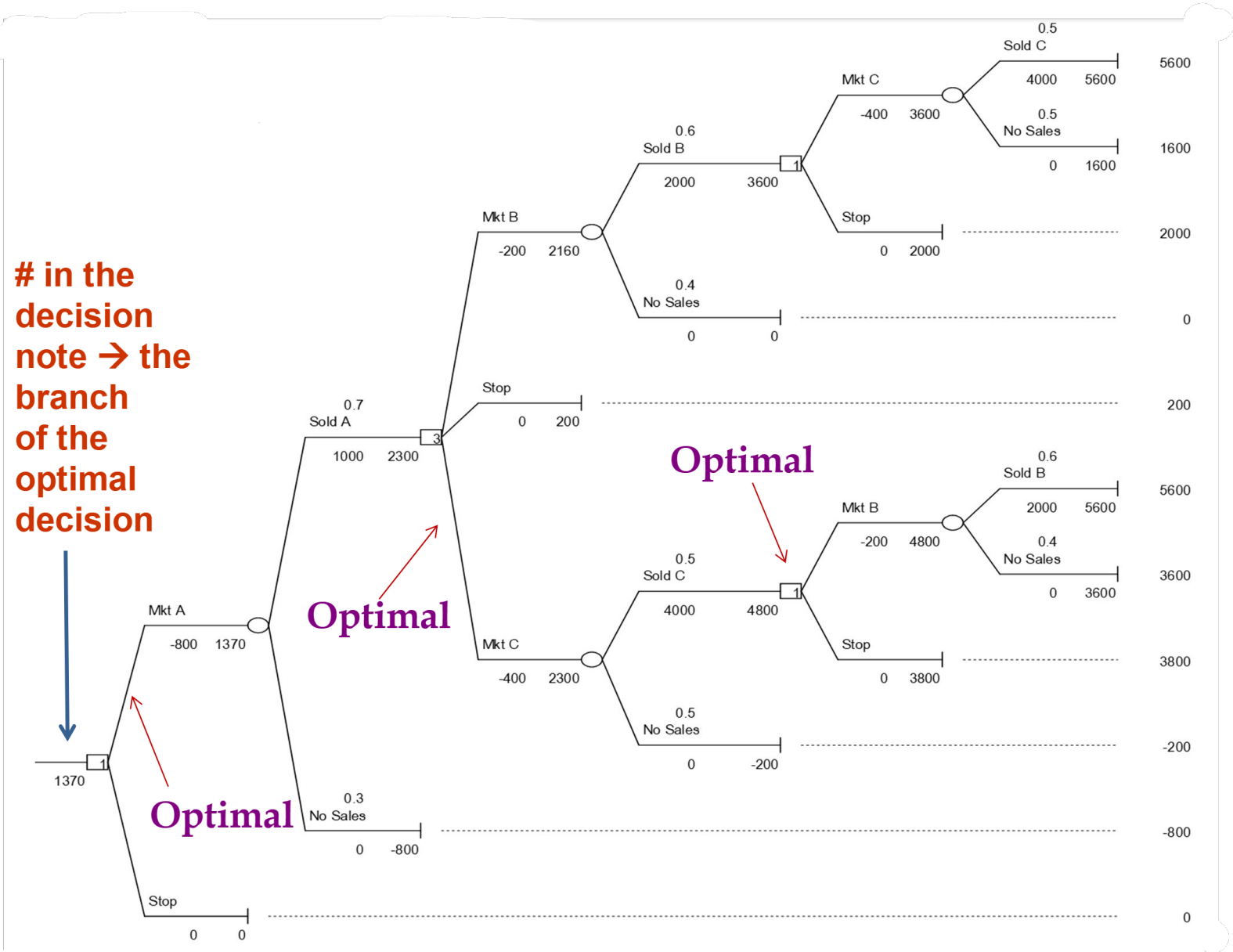
  - Select the alternative with the highest EMV

"Folding back the tree"



Building A decision tree using TreePlan



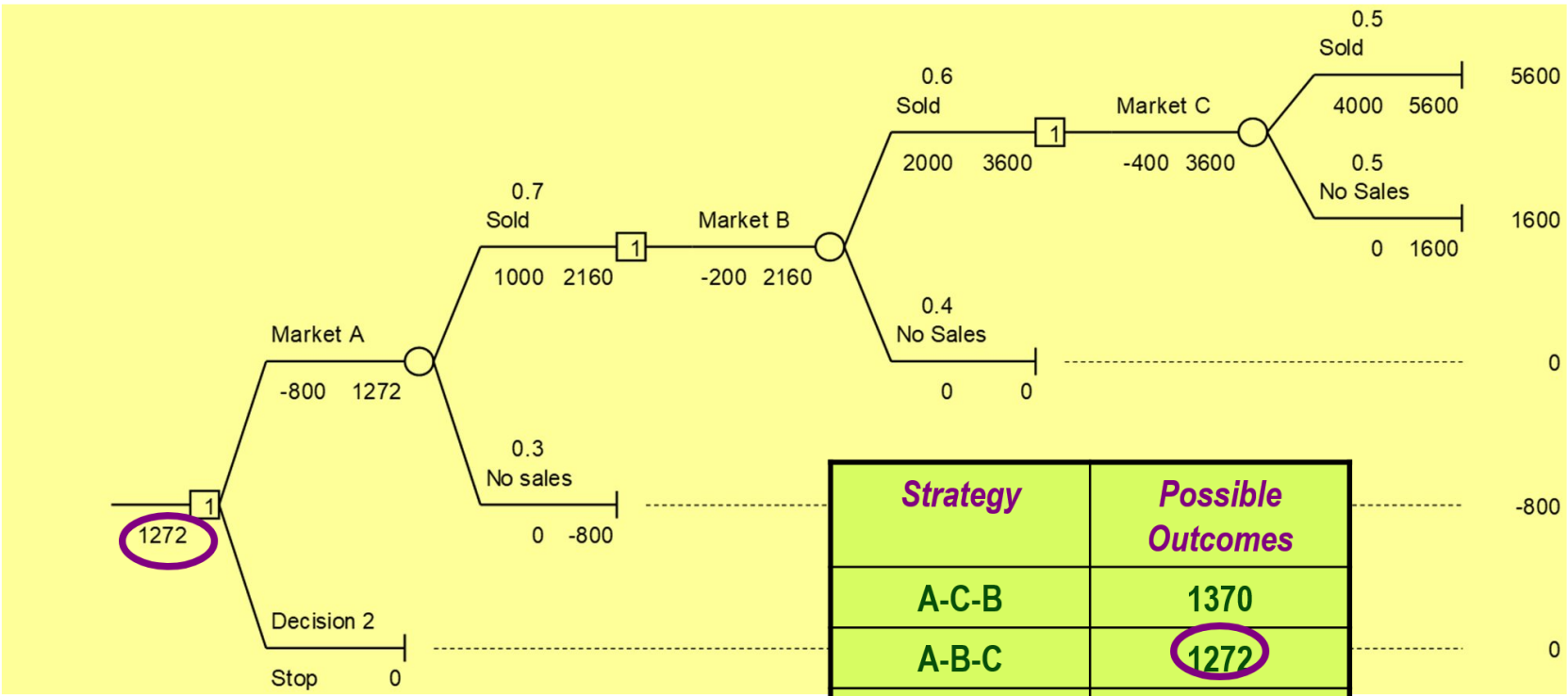
Warren's decisions

- Compare EMV of different alternative strategies

Strategy	*EMV*
A-C-B	**1370**
A-B-C	**1272**
A-C-Stop	**1020**
A-B-Stop	**600**
A-Stop	**-100**

Group discussion: EMV for other alternatives?

- please verify "A-B-C" strategy: EMV = $ 1272



Strategy	Possible Outcomes
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A-B-C	1272
A-C-Stop	1020
A-B-Stop	600
A-Stop	-100

Case: new product introduction

Product A

- A problem with the production system has not yet been solved: P(delay) $= 0.05$

Case: new product introduction

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- Price: High or Low (The price would not be set until just before the product is to be introduced.)

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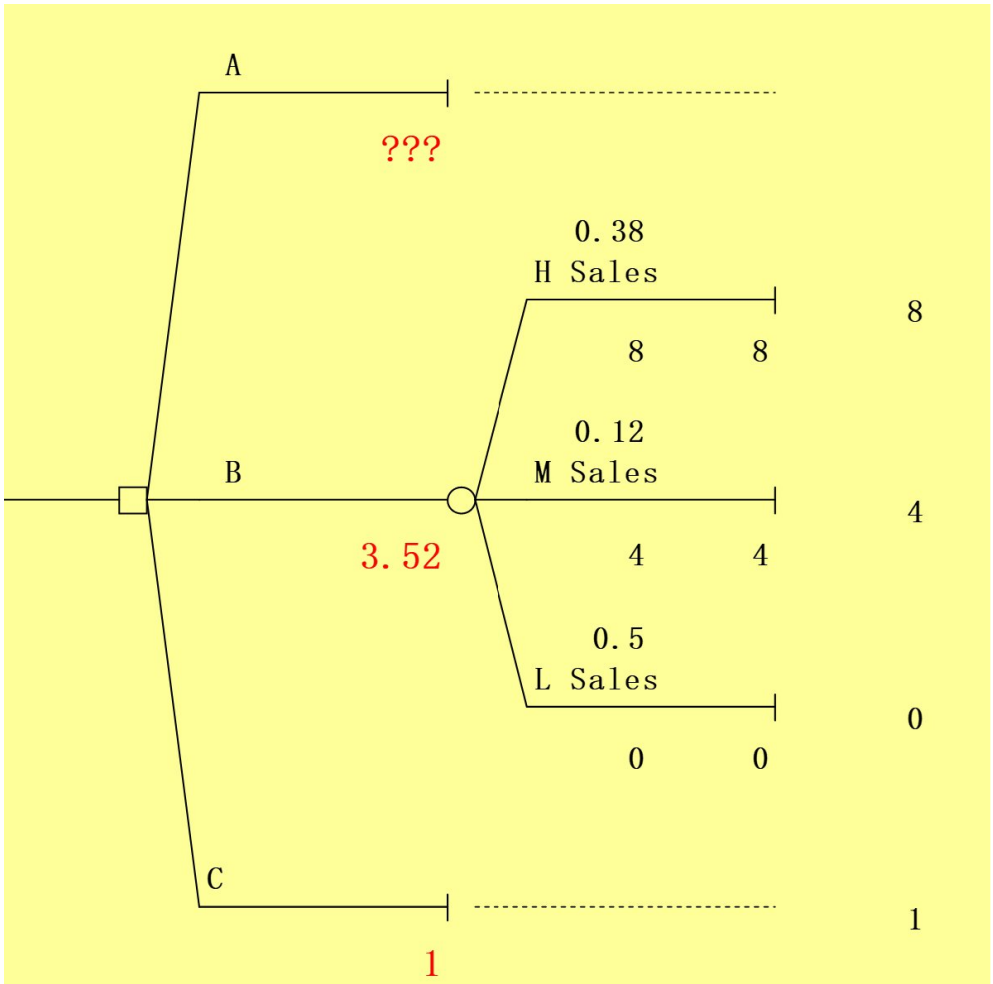
	Price	Payoff if High Sales (chance)	Payoff if Low Sales (chance)
Delay	High	5.0 (30%)	-0.5 (70%)
	Low	3.5 (50%)	1.0 (50%)
No Delay	High	12 (40%)	0.0 (60%)
	Low	4.5 (50%)	1.5 (50%)

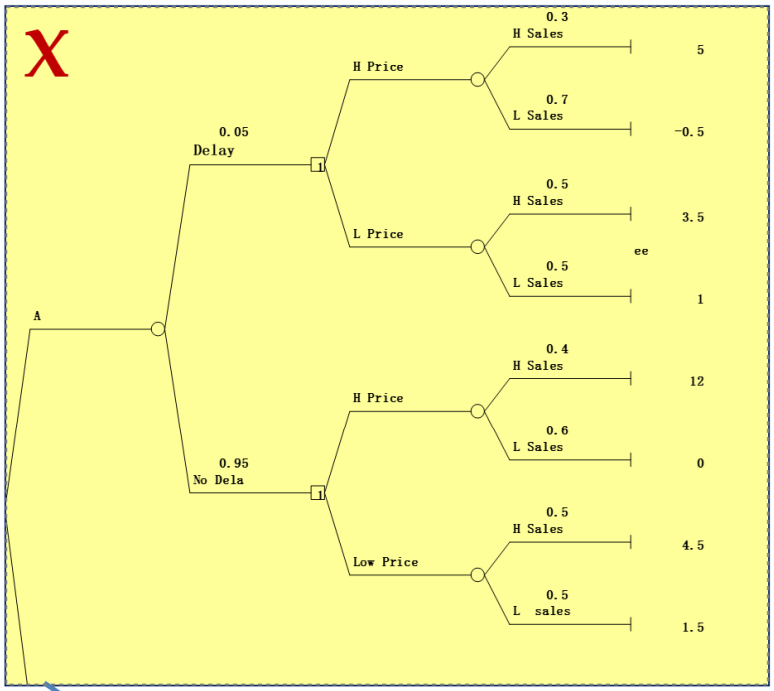
Case: new product introduction

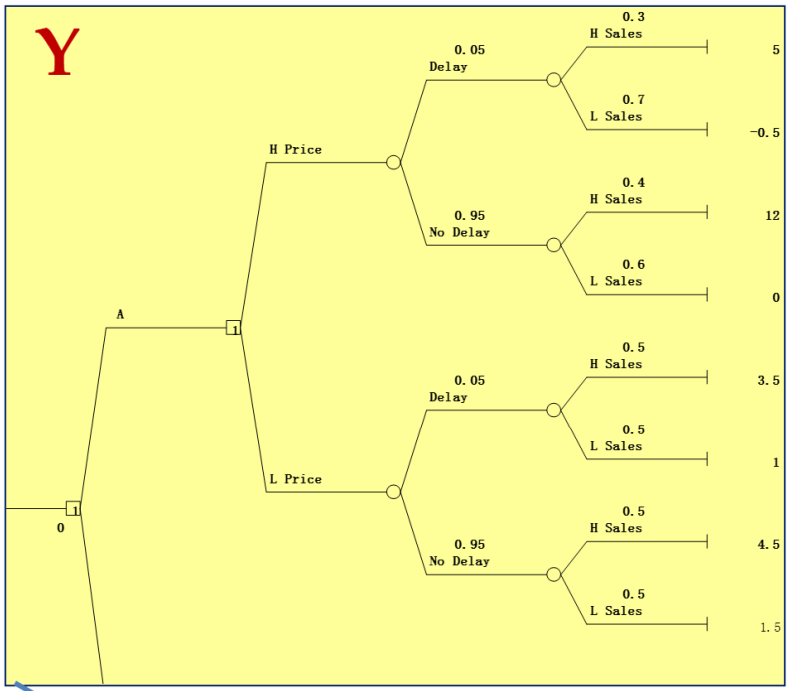
- Which product should be introduced?



Decision tree for product A

- Are the two trees the same?





Make decision with information

Make decision without information

Which tree is correct?

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  - If you do not have such information $\rightarrow Y$ is correct (chance node follows the decision node).

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- A tidal barrier is being constructed at the mouth of the river, but the Hartland River Authority has to decide how to provide flood protection in the two years before the barrier is ready.
- If flooding occurs in any one year, the Authority will have to pay out compensation of about $2 million for that year.

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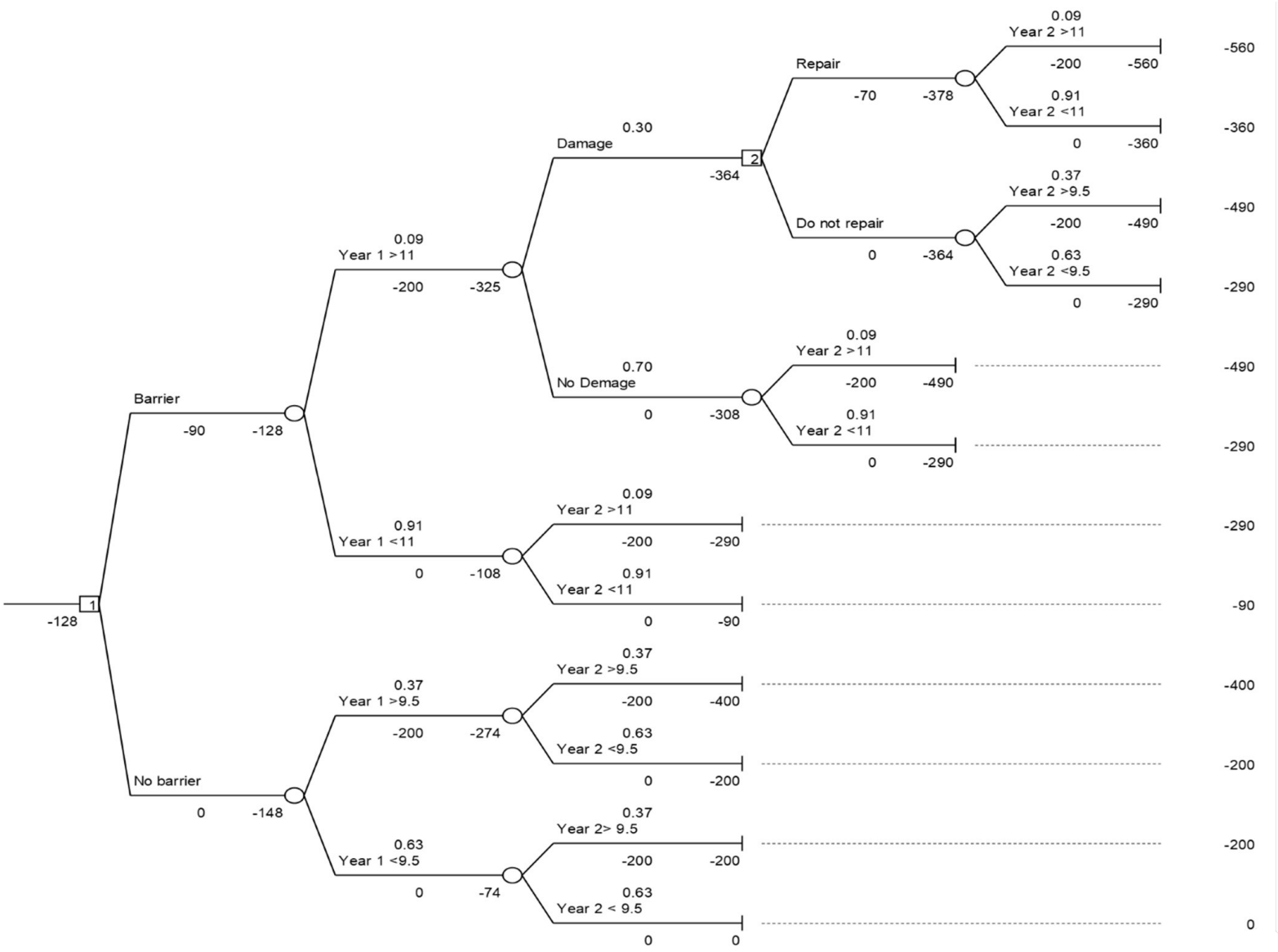
  - If damage, one can repair the barrier at a cost of \$0.7 million or leave the river unprotected for the second year

Case: the risk of flooding

after class exercise!



How to measure the effectiveness of a classifier ?

- Use ROC and AUC !

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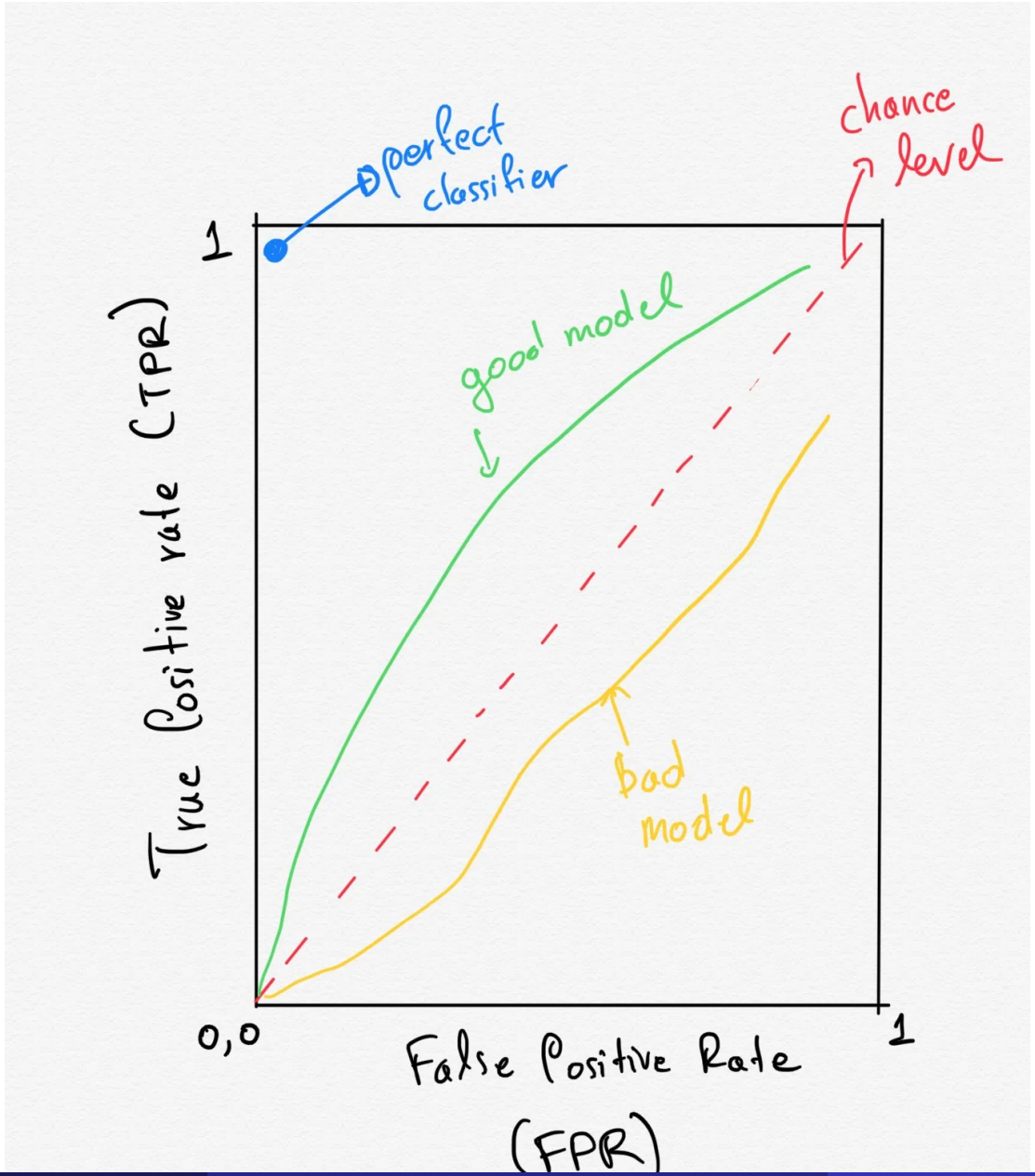
$$FPR = \frac{FP}{FP + TN}.$$

An example of ROC curve



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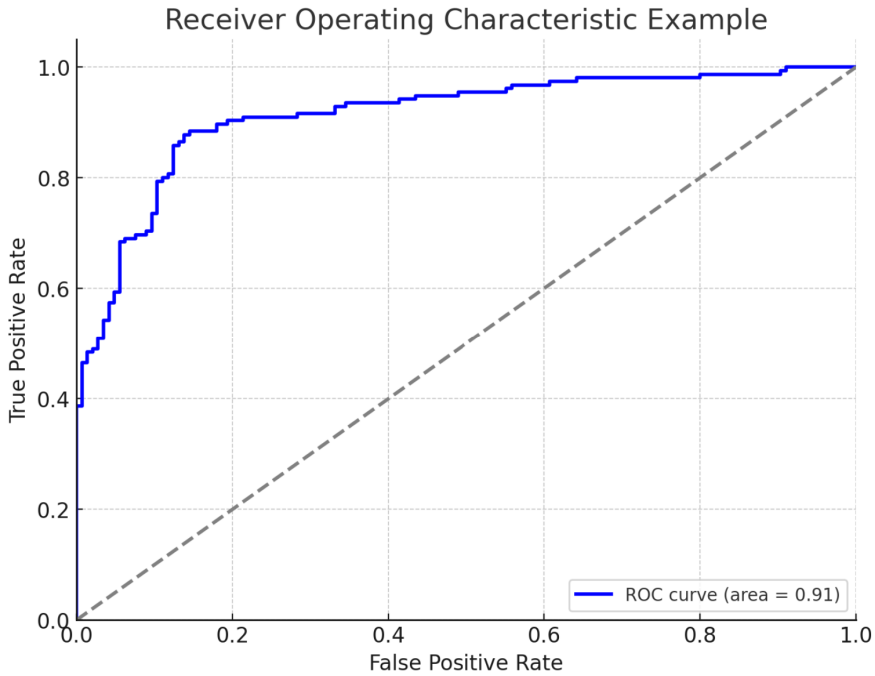
  - A model whose predictions are 100% correct has an AUC of 1.0.
  - AUC values between 0 and 1 represent the degree of separability the model has. The higher the AUC, the better the model is at distinguishing between the positive and negative classes.

An example of ROC and AUC



- The blue line represents the ROC curve for a Logistic Regression model trained on a binary classification dataset. The area under the curve (AUC) is 0.91. The gray dashed line represents a baseline model that randomly guesses the class (AUC = 0.5). The closer the ROC curve follows the top-left border, the more accurate the model.

ROC and AUC in Decision Trees - Practical Example

- Suppose we're analyzing data from a medical study to predict the risk of a certain disease. We'll use one key health metric as our feature and the presence or absence of the disease as our label.

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- Feature: "Blood Pressure Level" (a continuous scale from 0.1 to 1.0, where higher values indicate higher blood pressure).
- Label: Diagnosis of a specific disease (0 = No disease, 1 = Disease present).

ROC and AUC in Decision Trees - Practical Example

Table: Patient Data

Blood Pressure Level	Disease Diagnosis
0.10	0
0.40	0
0.35	1
0.80	1
0.70	1
0.30	0
0.60	1
0.55	1
0.50	0
0.45	0

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- Let's choose a threshold of 0.5 for the "Blood Pressure Level" to calculate a point on the ROC curve. This means we classify a patient as having the disease (positive) if their blood pressure level is equal to or higher than 0.5.

ROC and AUC in Decision Trees - Practical Example

Calculation Process:

- True Positives (TP): Count of patients who have the disease (Label $=$ 1) and their blood pressure level is ≥ 0.5.

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Formulas:

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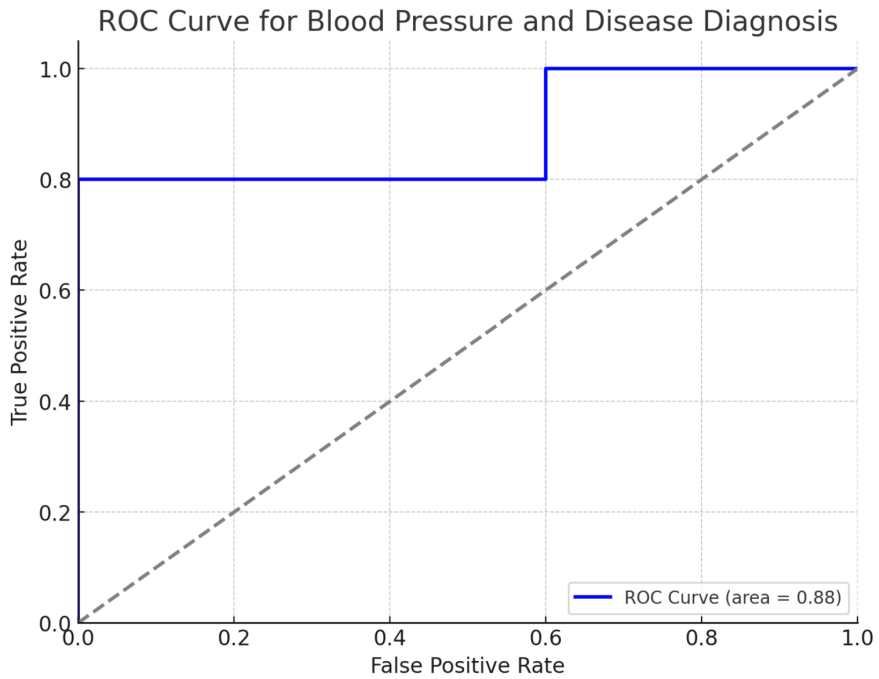
- $\text{FPR} = \text{FP}/(\text{FP} + \text{TN}) = 0.2$

ROC and AUC in Decision Trees - Practical Example

- By calculating all the points, we get the ROC Curve.



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- Verify AOC = 0.88 = 1-0.6*0.2