Ideal Abstraction for Well-Structured Transition Systems

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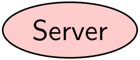
Example: client-server communication pattern



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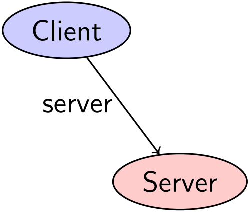

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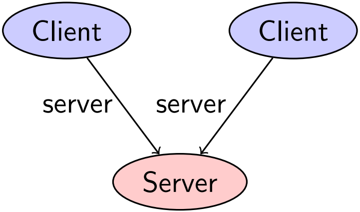

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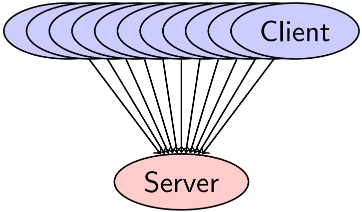

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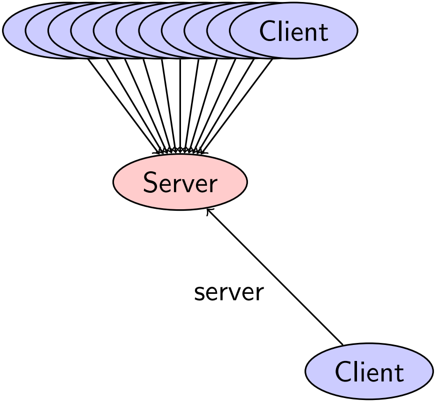

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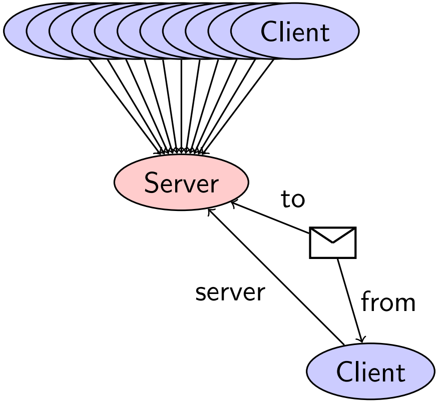

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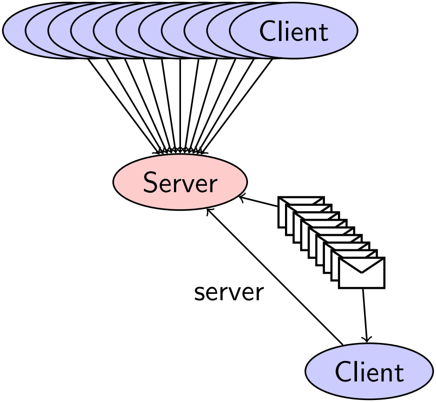

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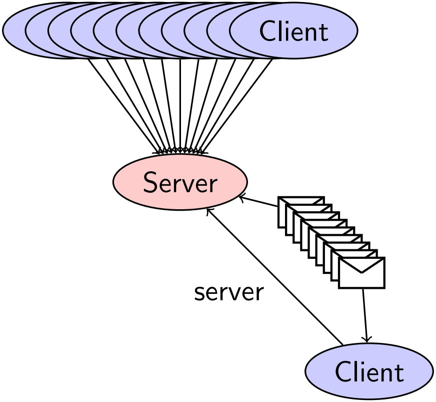
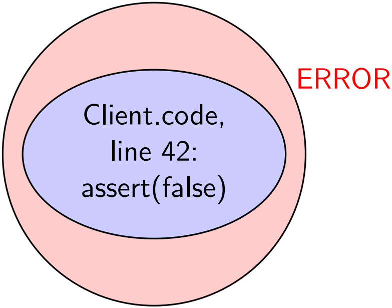


This example is a Depth-Bounded Process (DBP),
an instance of WSTS [Meyer, 2008, Wies et al., 2010].

What kind of properties are we looking at ?

Safety properties, more precisely the control-state reachability problem (aka covering problem).



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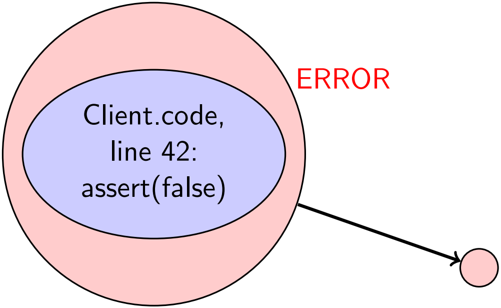

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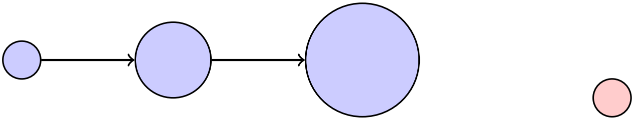

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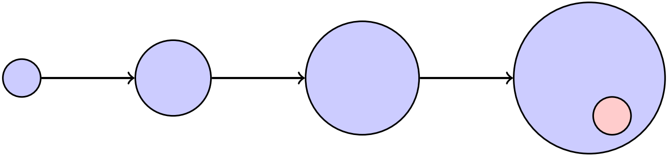

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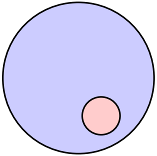

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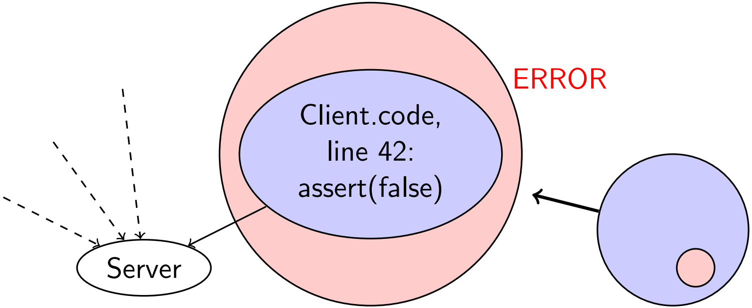


Outline

- Motivation
- Formalism
- Ideal abstraction
- Example of set-widening for ideal completion

Formal model: WSTS

A well-structured transition system (WSTS) is a transition system $\langle S, \rightarrow, \leq \rangle$ such that:

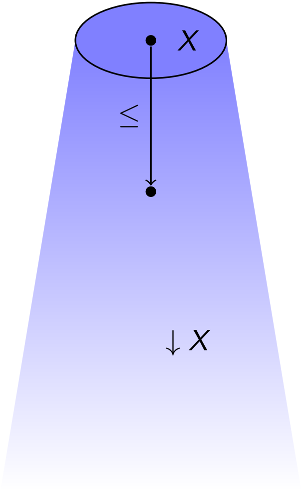
- $\leq$ is a well-quasi-ordering (wqo),
  i.e. well-founded + no infinite antichain.
- compatibility of $\leq$ w.r.t. $\rightarrow$

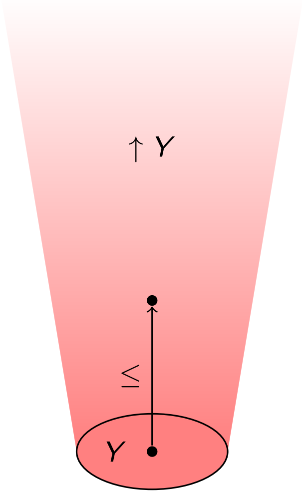
$$\forall \quad \begin{array}{ccc} t & \xrightarrow{*} & t' \\ \rotatebox{90}{$\geq$} & & \rotatebox{90}{$\geq$} \\ s & \longrightarrow & s' \end{array} \quad \exists$$

For more detail see:
[Finkel and Schnoebelen, 2001, Abdulla et al., 1996]

Downward and upward-closures

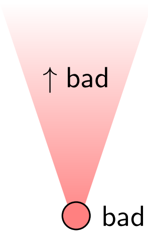
$$\downarrow X = \{\ x' \mid \exists x \in X.\ x' \leq x\ \}$$





$$\uparrow Y = \{\ y' \mid \exists y \in Y.\ y \leq y'\ \}$$

Backward algorithm for covering



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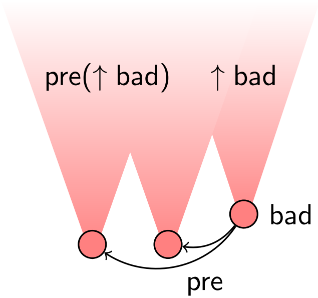

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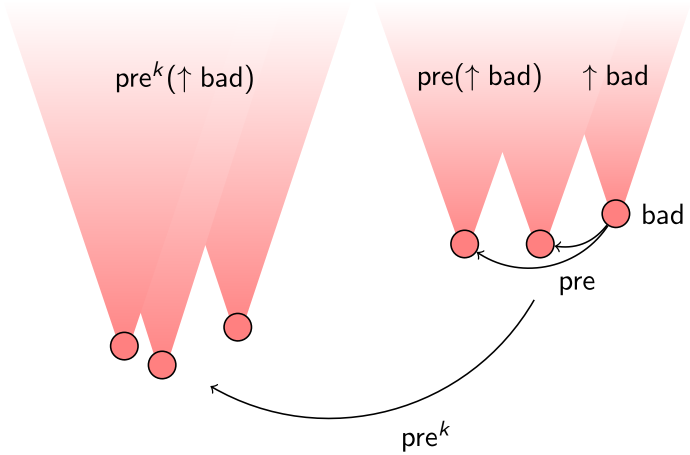
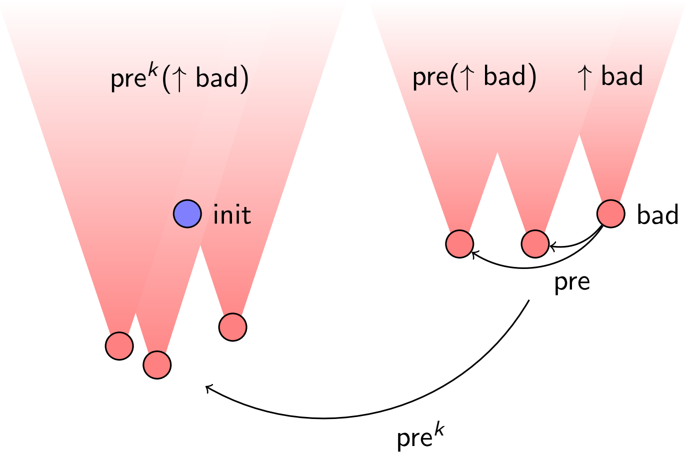
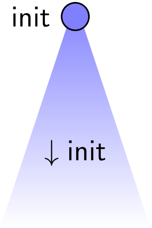




$\mathrm{pre}^k(\uparrow \mathrm{bad})$
init
$\mathrm{pre}(\uparrow \mathrm{bad})$
$\uparrow \mathrm{bad}$
bad
pre
pre^k

Forward algorithm for covering



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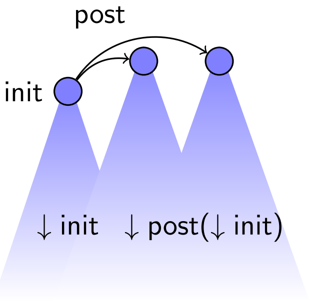

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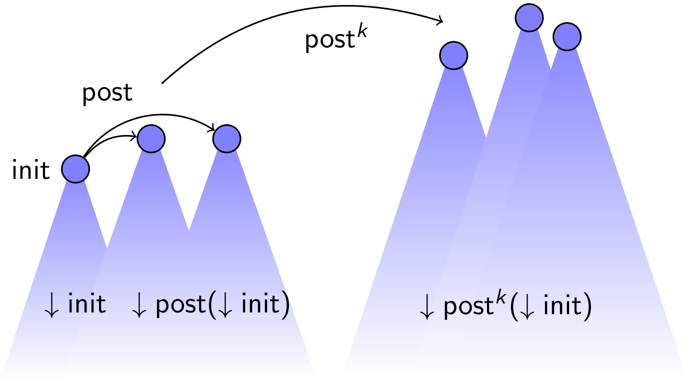

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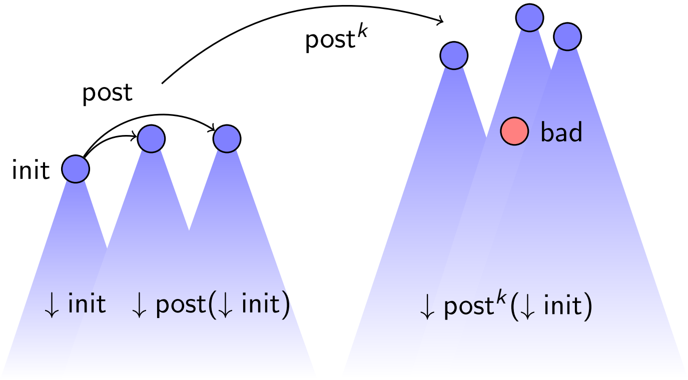

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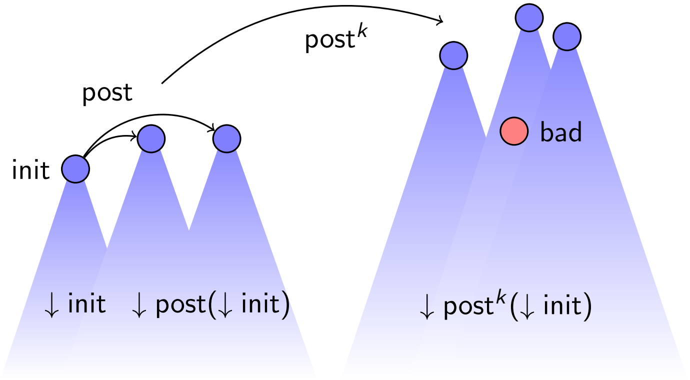


Computes the covering set rather than answering only a single coverability query.

Representing downward-closed sets with ideals

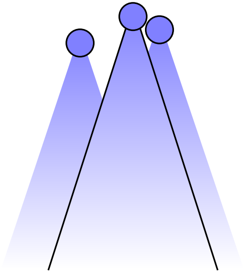
Given a wqo-set $(X, \leq)$.

A subset of X is directed if it is non-empty and closed under upper bounds.

An ideal of X is a directed downward-closed subset of X.

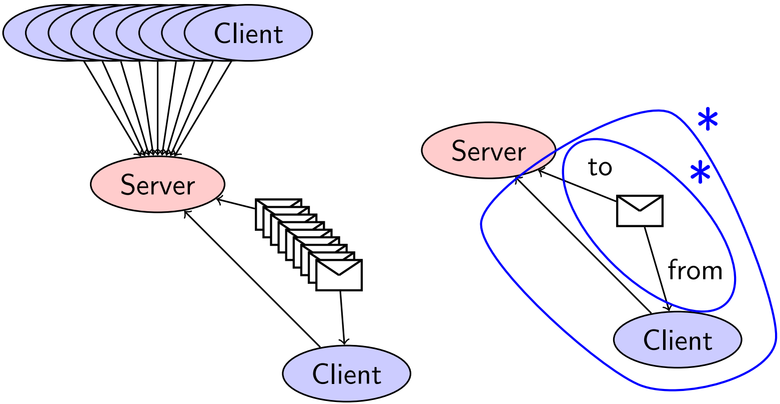
The ideal completion $Idl(X)$ of X is the set of all ideals of X.

A downward-closed subset is a finite union of ideals.



Ideal for representing downward-closed sets.

ADL: [Geeraerts et al., 2006]
Further developed in [Finkel and Goubault-Larrecq, 2009]
Applied to DBP in [Wies et al., 2010]



When does acceleration work ? (flat systems)

Forward algorithms are (usually) based on acceleration.
Acceleration *executes* loops infinitely many time (saturation).

Concretely, The algorithm terminates if the covering set can be generated by executing only simple loops. This condition is known as flattability [Bardin et al., 2005]. Acceleration *executes* traces of length $< \omega^2$.

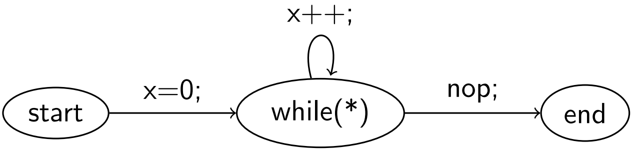
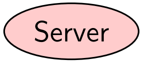
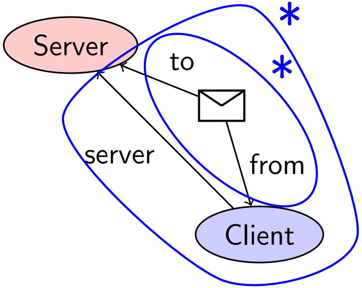


Figure: Example of a flat program

initial configuration:



covering set:

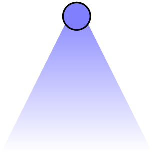


ω^2 steps from the initial state and the final state.

Nested loops are required to compute the covering set.

From acceleration to widening

Acceleration considers transitions - widening only states.



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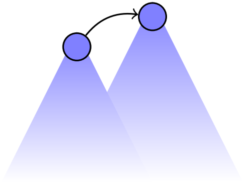

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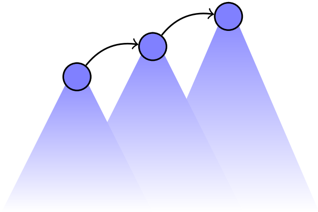

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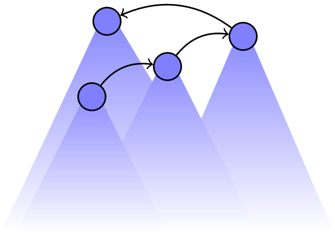

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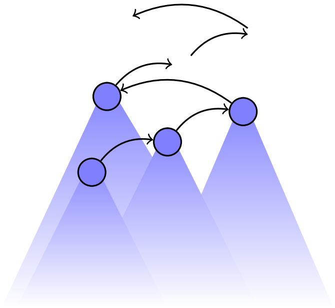

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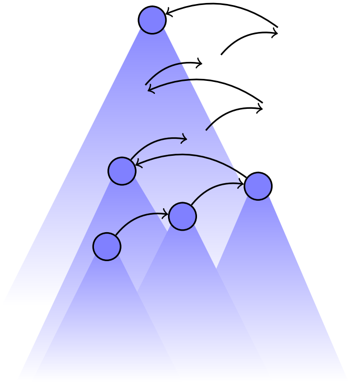

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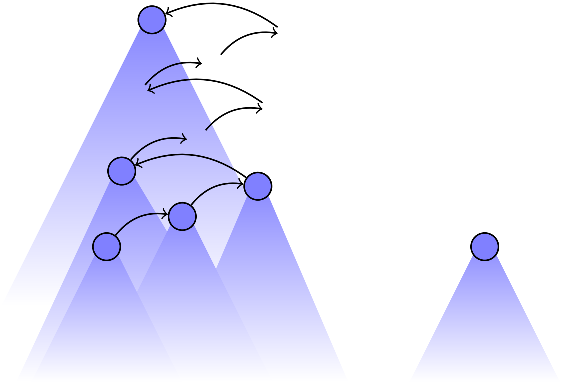

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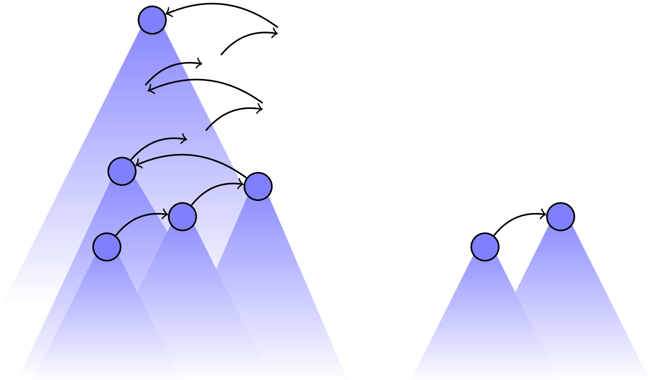

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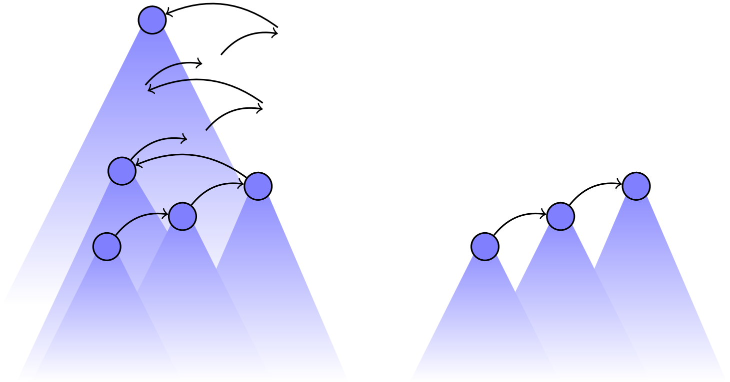

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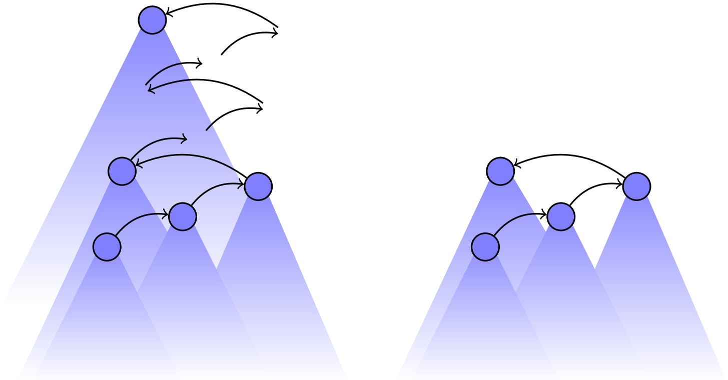

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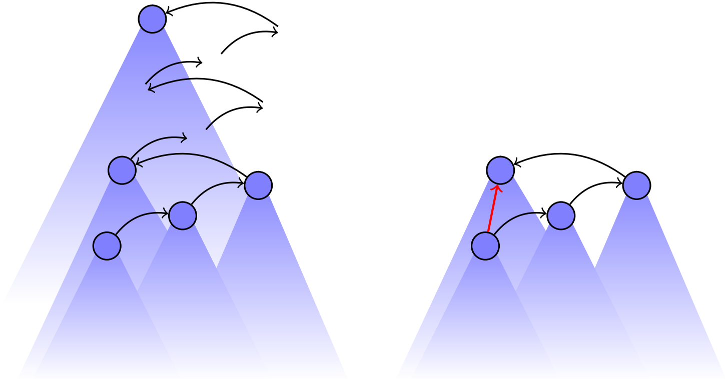

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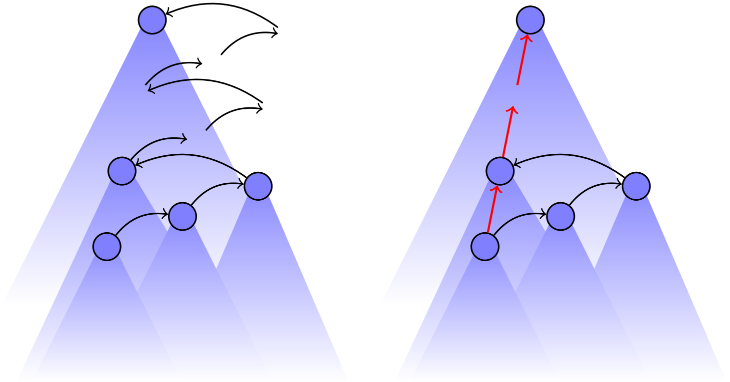


Ideal abstraction

Rephrase analysis in terms of abstract interpretation [Cousot and Cousot, 1977].

- Concrete domain: sets of configurations ($\mathcal{P}(S)$)
- Abstract domain: ideal completion of $(S, \leq)$ ($\mathcal{P}_{\mathit{finite}}(\mathit{Idl}(S))$)

- Concretization function γ: identity
- Abstraction function α: downward-closure

(α, γ) is a Galois connection.

Covering set is abstract fixed point: $\mu X.\alpha(\mathit{init}) \cup \alpha \cdot \mathit{post} \cdot \gamma(X)$

To guarantee termination we need a widening operator.

Widening (1)

Goal: try to mimic acceleration (when possible), and force termination

A set-widening operator (∇) [Cousot and Cousot, 1992] for a poset S is partial function ($\mathcal{P}(S) \to S$) that satisfies:

Covering : for all $X \subseteq S$, $x \in X \Rightarrow x \leq \nabla(X)$;

Termination : widening of any ascending chain stabilizes.

Why set-widening rather than the usual pair-widening.
Reason of using a set-widening operator: we need the history.

Widening (2)

Keeping it simple: need only set-widening operator on $Idl(S)$.
It can be lifted to $\mathcal{P}_{finite}(Idl(S))$, using a general construction (details in the paper).

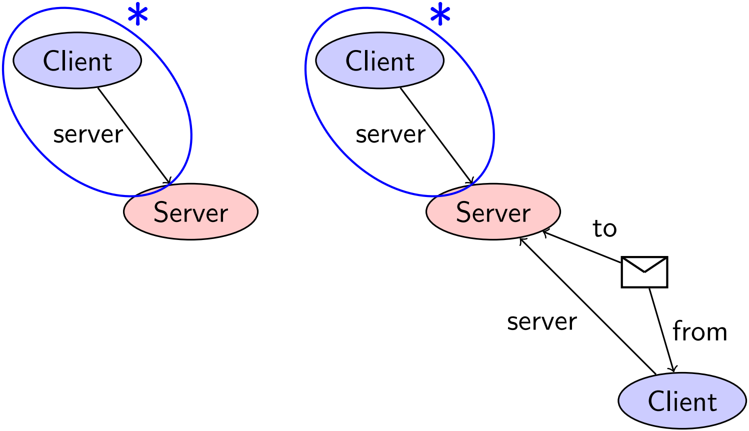
We assume that the ordering S is a better-quasi-ordering (bqo).
$\Rightarrow$ Thus, $Idl(S)$ is also a bqo.
$\Rightarrow$ No infinite antichain in $Idl(S)$.

We still need to define the widening on ideals.

We provide concrete operators for

- Petri nets (and monotonic extensions),
- Lossy channel systems [Abdulla and Jonsson, 1993],
- Depth-bounded processes (DBP).

Set-widening for DBP (1)



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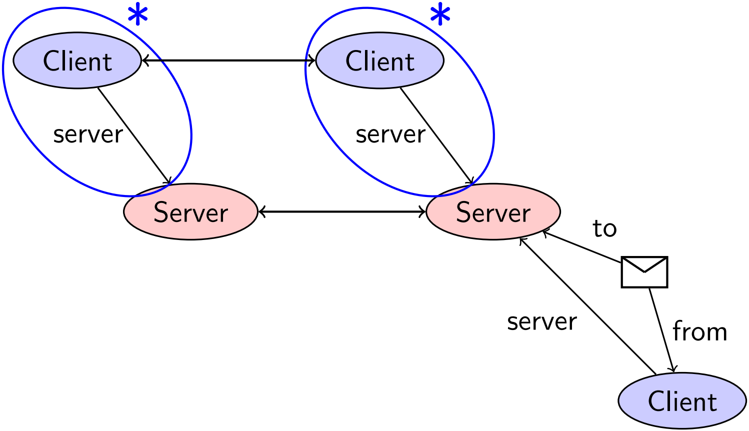

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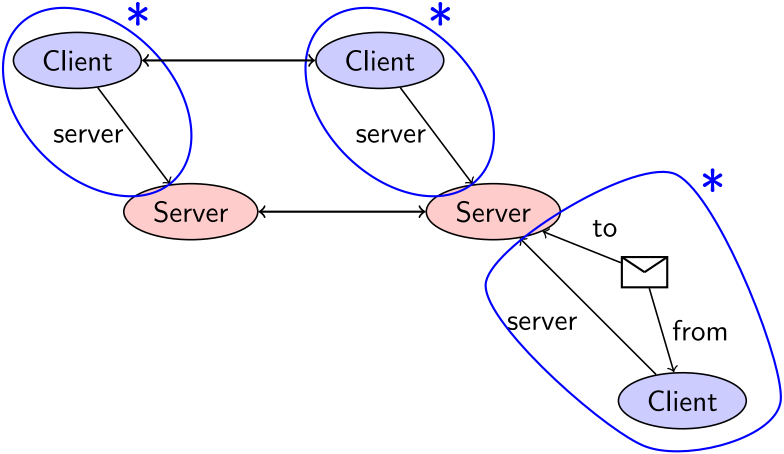
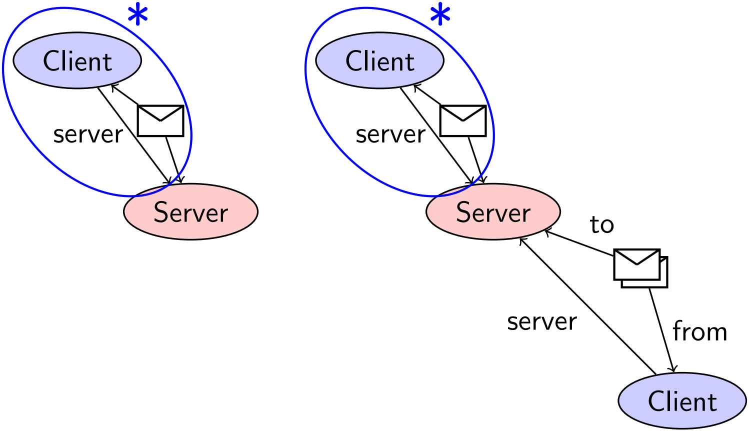


Set-widening for DBP (2)



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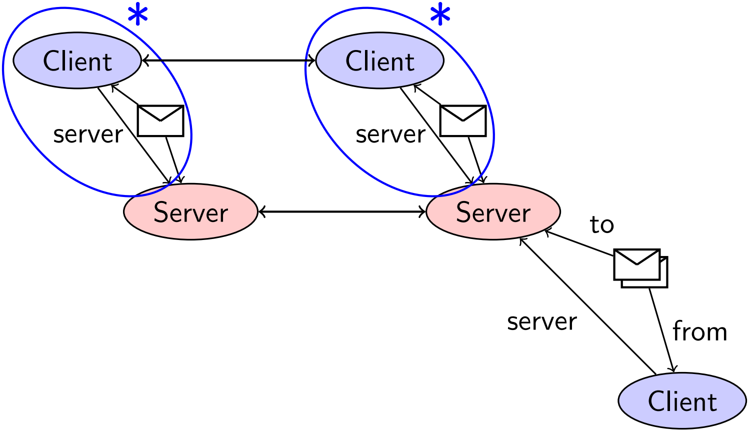

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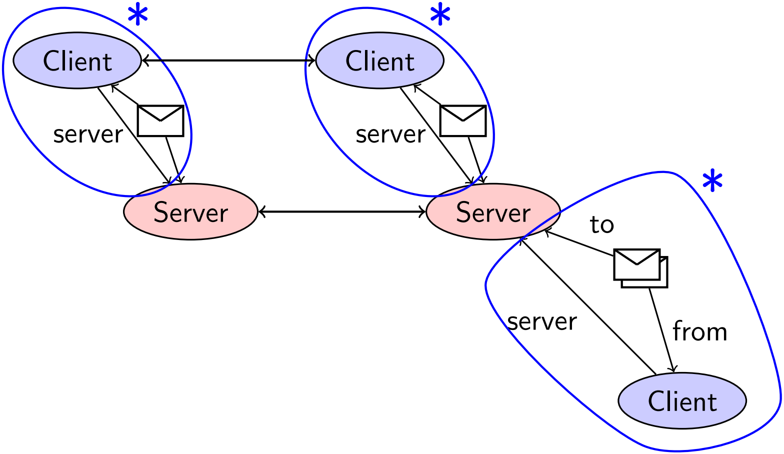
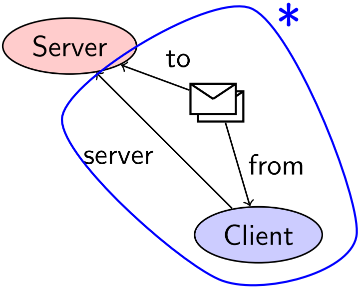
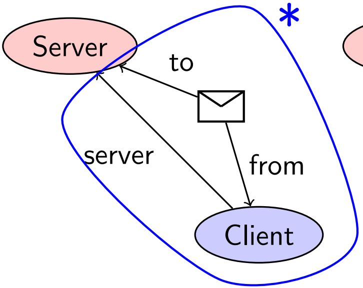


Set-widening for DBP (3)



Set-widening for DBP (3)



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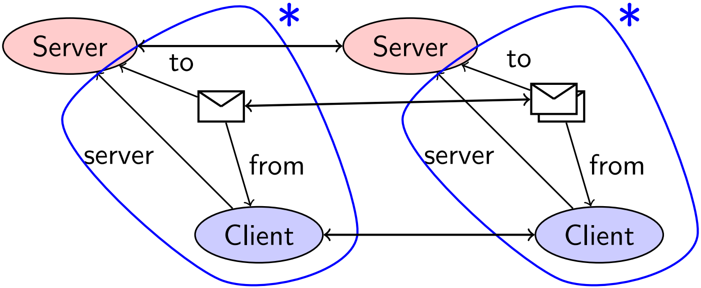

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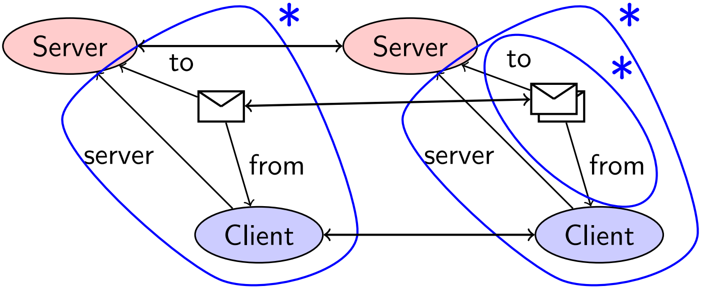

Implementation: Picasso

Picasso: Pi-Calculus-based Static Software Analyzer

Picasso implements

- Ideal abstraction domain
- Widening for DBP
- Trace partitioning domain [Rival and Mauborgne, 2007] (similar to Karp-Miller Tree)

Target: Scala actor programs

Input: manually extracted models (soon a compiler plug-in)

Experimental results in the paper

Available at `http://pub.ist.ac.at/~zufferey/picasso/`.

Name	tree size	cov. set size	time
`ping-pong`	17	14	0.6 s
`client-server`	25	2	1.9 s
`client-server-with-TO`	184	5	12.8 s
`genericComputeServer`	57	4	4.6 s
`genericComputeServer-fctAsActor`	98	8	14.8 s
`liftChatLike`	1846	21	1830.9 s
`round_robin_2`	830	63	48.8 s
`round_robin_3`	3775	259	737.8 s

Further related work:

[Abdulla et al., 1996] Complete backward algorithm for coverability

[Geeraerts et al., 2006] Complete algorithm for coverability based on ideal completions

[Ganty et al., 2006] Complete algorithm for coverability based on abstract interpretation but not ideal completions

[Finkel and Goubault-Larrecq, 2009] Acceleration-based algorithm for computing covering sets

Conclusion

- Many verification problems for concurrent systems can be phrased in terms of coverability
- Coverability is decidable for well-structured transition systems but with high complexity
- Ideal Abstraction: a generic framework for computing an approximation of the covering set.
  - promises precise but more scalable analysis of WSTS
  - we provide instantiations of the framework for common classes of WSTS
  - Picasso: implementation of our framework for the analysis of Scala actor programs

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