Matrix and Determinant : 10 marks

Matrix : A rectangular arrangement of number in rows and columns and enclosed by square and round bracket is known as matrix.

Matrices are denoted by capital letter A, B, C ... and their entries (or elements) are denoted by small letters. a, b, c ...

Example : $A = \begin{pmatrix} 1 & 4 \\ 2 & 5 \end{pmatrix}$

* Null or zero matrix

$0 = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$,

* Matrix Operation :

i) Addition or subtraction

Two matrices can be added or substracted if they have the same size (order)

Ex: If $A = \begin{bmatrix} 1 & 4 & 5 \\ 7 & 3 & 2 \end{bmatrix}_{2\times3}$ and $B = \begin{bmatrix} 1 & 5 & 0 \\ 2 & 6 & 1 \end{bmatrix}_{2\times3}$

size = row x column
= 2 x 3

$$A+B = \begin{pmatrix} 2 & 9 & 5 \\ 9 & 9 & 3 \end{pmatrix}$$

$$A-B = \begin{pmatrix} 0 & -1 & 5 \\ 5 & -3 & 1 \end{pmatrix}$$

ii) Matrix Multiplication

Case I : Multiplication of a matrix and scalar

Ex If $A = \begin{bmatrix} 2 & 4 & 5 \\ 6 & 7 & 8 \\ 3 & 0 & 0 \end{bmatrix}$

Then;

$$7A = \begin{bmatrix} 14 & 28 & 35 \\ 42 & 49 & 56 \\ 21 & 0 & 0 \end{bmatrix}$$

Case II : Multiplication between two matrices.

Two matrices can be modify iff (if and only sign, $\Rightarrow$) the numbers of columns of 1st matrix is equal to the no. of rows ~~and~~ of 2nd matrix.

Ex: If $A = \begin{pmatrix} 1 & 3 & 4 \\ 2 & 1 & 5 \end{pmatrix}$ and $B = \begin{pmatrix} 1 & 3 \\ 2 & 0 \\ 4 & 0 \end{pmatrix}$

Then $AB = \begin{pmatrix} 1 & 3 & 4 \\ 2 & 1 & 5 \end{pmatrix} \begin{pmatrix} 1 & 3 \\ 2 & 0 \\ 4 & 0 \end{pmatrix}$

$$= \begin{pmatrix} 1\times1 + 3\times2 + 4\times4 & 1\times3 + 3\times0 + 4\times0 \\ 2\times1 + 1\times2 + 5\times4 & 2\times3 + 0 + 0 \end{pmatrix}$$

$$\therefore \begin{bmatrix} 23 & 3 \\ 24 & 6 \end{bmatrix}$$

$$A = \begin{bmatrix} \overset{+}{a_{11}} & \overset{-}{a_{12}} \\ \overset{-}{a_{21}} & \overset{+}{a_{22}} \end{bmatrix}_{2\times2}$$

$$A = \begin{pmatrix} \overset{+}{a_{11}} & \overset{-}{a_{12}} & \overset{+}{a_{13}} \\ \overset{-}{a_{21}} & \overset{+}{a_{22}} & \overset{-}{a_{23}} \\ \overset{+}{a_{31}} & \overset{-}{a_{32}} & \overset{+}{a_{33}} \end{pmatrix}_{3\times3}$$

Transpose of a matrix

A new matrix obtained by interchanging the rows or columns of a given matrix A is known as transpose of that matrix A.

It is denoted by A', A^T, A^t

Example Let $A = \begin{pmatrix} 2 & 4 \\ 7 & 10 \end{pmatrix}$

$$A^T = \begin{pmatrix} 2 & 7 \\ 4 & 10 \end{pmatrix}$$

* Orthogonal

$AA^T = A^TA = I$ (identity)

$$I = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \quad \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

Properties :

i) A square matrix A is said to be orthogonal
$AA^T = A^TA = I$

ii) $(A^T)^T = A$

iii) $(A+B)^T = A^T + B^T$

iv) $(AB)^T = B^TA^T$

Symmetric matrix : A square matrix A is said to be symmetric if $A = A^T$ / $a_{ij} = a_{ji}$

Skew symmetric matrix : A square matrix A is said to be skew symmetric if $A = -A^T$

$$A = \begin{pmatrix} 0 & 2 & 3 \\ 2 & 0 & 4 \\ 1 & 3 & 0 \end{pmatrix}$$ → if in the diagonal its value 0 then it will be skew symmetric.

A square matrix is called diagonal matrix if its element expect principle diagonal are zero.

A square matrix in which all the elements of the principle diagonal are ones and all other elements are zeros. Is called Identity matrix.

Exercise 4.1

1) Let $A = \begin{bmatrix} 2 & 4 & 5 \\ 6 & 7 & 3 \\ 1 & -5 & 8 \end{bmatrix}$ and $B = \begin{bmatrix} 2 & 0 & 5 \\ 1 & 3 & 4 \\ 7 & 6 & 8 \end{bmatrix}$ write down the element

a) $a_{12} = 4$
$a_{23} = 3$
$a_{31} = 1$

b) $b_{21} = 1$
$b_{23} = 4$
$b_{31} = 7$

2) Construct a 3x3 matrix $A = a_{ij}$ whose elements a_{ij} are given by

a) $a_{ij} = 3i + 2j$

sol: Let the 3x3 matrix be $A = \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix}$

Given;
$a_{ij} = 3i + 2j$

For a_{11} $i=1, j=1$ $\therefore a_{11} = 3\times1 + 2\times1 = 5$
For a_{12} $i=1, j=2$ $\therefore a_{12} = 3\times1 + 2\times2 = 7$
$a_{13} = 3\times1 + 2\times3 = 9$
$a_{21} = 3\times2 + 2\times1 = 8$
$a_{22} = 3\times2 + 2\times2 = 10$
$a_{23} = 3\times2 + 2\times3 = 12$
$a_{31} = 3\times3 + 2\times1 = 11$
$a_{32} = 3\times3 + 2\times2 = 13$
$a_{33} = 3\times3 + 2\times3 = 15$

$\therefore$ Required matrix
$$A = \begin{pmatrix} 5 & 7 & 9 \\ 8 & 10 & 12 \\ 11 & 13 & 5 \end{pmatrix}$$

4) If $2\begin{bmatrix} 3 & 4 \\ 5 & x \end{bmatrix} + \begin{bmatrix} 1 & y \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 7 & 0 \\ 10 & 5 \end{bmatrix}$, find x and y.

Sol

$= 2\begin{pmatrix} 2 & 4 \\ 3 & x \end{pmatrix} + \begin{pmatrix} 1 & y \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 7 & 0 \\ 10 & 5 \end{pmatrix}$

or, $\begin{pmatrix} 6 & 8 \\ 10 & 2x \end{pmatrix} + \begin{pmatrix} 1 & y \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 7 & 0 \\ 10 & 5 \end{pmatrix}$

or, $\begin{pmatrix} 7 & 8+y \\ 10 & 2x+1 \end{pmatrix} = \begin{pmatrix} 7 & 0 \\ 10 & 5 \end{pmatrix}$

Since the matrices are equal in size

$\therefore 8+y = 0$ / $2x + 1 = 5$

$y = -8$ / $2x = 4$

$x = 2$

$\therefore \boxed{x = 2, y = -8}$

5) If $P = \begin{pmatrix} 1 & 4 & 2 \\ 4 & 0 & -1 \end{pmatrix}$ and $Q = \begin{pmatrix} 6 & -1 & 3 \\ 4 & 3 & 2 \end{pmatrix}$ find a matrix C such that $C = 2P + 3Q$

Sol:

$C = 2P + 3Q$

or, $2\begin{pmatrix} 1 & 4 & 2 \\ 4 & 0 & -1 \end{pmatrix} + 3\begin{pmatrix} 6 & -1 & 3 \\ 4 & 3 & 2 \end{pmatrix}$

or, $\begin{pmatrix} 2 & 8 & 4 \\ 8 & 0 & -2 \end{pmatrix} + \begin{pmatrix} 18 & -3 & 9 \\ 12 & 9 & 6 \end{pmatrix}$

$\begin{pmatrix} 20 & 5 & 13 \\ 20 & 9 & 4 \end{pmatrix}$

$\therefore C = \begin{pmatrix} 20 & 5 & 13 \\ 20 & 9 & 4 \end{pmatrix}$

6) If $P = \begin{pmatrix} 5 & 8 \\ -1 & 0 \\ 4 & 2 \end{pmatrix}$ and $Q = \begin{pmatrix} 1 & 0 \\ 3 & 6 \\ -2 & 7 \end{pmatrix}$, verify that

a) $P + Q = Q + P$

Sol:

Solving L.H.S

$P + Q$

$= \begin{pmatrix} 5 & 8 \\ -1 & 0 \\ 4 & 2 \end{pmatrix} + \begin{pmatrix} 1 & 0 \\ 3 & 6 \\ -2 & 7 \end{pmatrix}$

or $\begin{pmatrix} 6 & 8 \\ 2 & 6 \\ 2 & 9 \end{pmatrix}$

again;

Solving R.H.S

$Q + P$

$= \begin{pmatrix} 1 & 0 \\ 3 & 6 \\ -2 & 7 \end{pmatrix} + \begin{pmatrix} 5 & 8 \\ -1 & 0 \\ 4 & 2 \end{pmatrix}$

or, $\begin{pmatrix} 6 & 8 \\ 2 & 6 \\ 2 & 9 \end{pmatrix}$

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From above, we see that

$\boxed{P+Q = Q+P}$

Proved

b) $K(P+Q) = KP + KQ$, where $K=3$

Sol:

$= K(P+Q)$

or, $3\left[\begin{pmatrix} 5 & 8 \\ -1 & 0 \\ 4 & 2 \end{pmatrix} + \begin{pmatrix} 1 & 0 \\ 3 & 6 \\ -2 & 7 \end{pmatrix}\right]$

or, $3\begin{pmatrix} 6 & 8 \\ 2 & 6 \\ 2 & 9 \end{pmatrix}$

or, $\begin{pmatrix} 18 & 24 \\ 6 & 18 \\ 6 & 27 \end{pmatrix}$

again;

$KP + KQ$

$= 3\begin{pmatrix} 5 & 8 \\ -1 & 0 \\ 4 & 2 \end{pmatrix} + 3\begin{pmatrix} 1 & 0 \\ 3 & 6 \\ -2 & 7 \end{pmatrix}$

or, $\begin{pmatrix} 15 & 24 \\ -3 & 0 \\ 12 & 6 \end{pmatrix} + \begin{pmatrix} 3 & 0 \\ 9 & 18 \\ -6 & 21 \end{pmatrix}$

$\begin{pmatrix} 18 & 24 \\ 6 & 18 \\ 6 & 27 \end{pmatrix}$

from the above, we can see that

$\boxed{K(P+Q) = KP + KQ}$ proved

7) If $A = \begin{pmatrix} 2 & 4 \\ 3 & 2 \end{pmatrix}$ and $B = \begin{pmatrix} 1 & 3 \\ -2 & 9 \end{pmatrix}$ verify that $AB \neq BA$

Sol:

$AB \neq BA$

or, $AB = \begin{pmatrix} 2 & 4 \\ 3 & 2 \end{pmatrix}\begin{pmatrix} 1 & 3 \\ -2 & 9 \end{pmatrix}$

or, $\begin{pmatrix} 2-8 & 6+36 \\ 3-4 & 9+18 \end{pmatrix}$

or, $\begin{pmatrix} -6 & 42 \\ -1 & 27 \end{pmatrix}$

again;

$BA = \begin{pmatrix} 1 & 3 \\ -2 & 9 \end{pmatrix}\begin{pmatrix} 2 & 4 \\ 3 & 2 \end{pmatrix}$

or, $\begin{pmatrix} 2+9 & 4+6 \\ -4+27 & -8+18 \end{pmatrix}$

or, $\begin{pmatrix} 11 & 10 \\ 23 & 10 \end{pmatrix}$

from above, we see that

$\therefore \boxed{AB \neq BA}$

8) If $A = \begin{pmatrix} 0 & -1 & 5 \\ 6 & 3 & -4 \end{pmatrix}$ and $B = \begin{pmatrix} 4 & 3 \\ 1 & 0 \\ 7 & -6 \end{pmatrix}$ show that $(AB)^T = B^T A^T$

sol:

$$A^T = \begin{pmatrix} 0 & 6 \\ -1 & 3 \\ 5 & -4 \end{pmatrix}$$

$$B^T = \begin{pmatrix} 4 & 1 & 7 \\ 3 & 0 & -6 \end{pmatrix}$$

again;

$$AB = \begin{pmatrix} 0 & -1 & 5 \\ 6 & 3 & -4 \end{pmatrix} \times \begin{pmatrix} 4 & 3 \\ 1 & 0 \\ 7 & -6 \end{pmatrix}$$

or, $\begin{pmatrix} 0-1+35 & 0-0-30 \\ 24+3-28 & 18+0+24 \end{pmatrix}$

or, $\begin{pmatrix} 34 & -30 \\ -1 & 42 \end{pmatrix}$

$\therefore \boxed{AB^T = \begin{pmatrix} 34 & -1 \\ -30 & 42 \end{pmatrix}}$

again;

$$B^T A^T = \begin{pmatrix} 4 & 1 & 7 \\ 3 & 0 & -6 \end{pmatrix} \times \begin{pmatrix} 0 & 6 \\ -1 & 3 \\ 5 & -4 \end{pmatrix}$$

or, $\begin{pmatrix} 0-1+35 & 24+3-28 \\ 0-0-30 & 18+0+24 \end{pmatrix}$

or, $\begin{pmatrix} 34 & -1 \\ -30 & 42 \end{pmatrix}$

From above, we see that

$\therefore \boxed{(AB)^T = B^T A^T}$

10) If $A = \begin{bmatrix} 1 & -1 & 1 \\ -3 & 3 & -3 \\ -4 & 4 & -4 \end{bmatrix}$, show that $A^2 = 0$

sol:

$A^2 = A \times A$

or, $\begin{bmatrix} 1 & -1 & 1 \\ -3 & 3 & -3 \\ -4 & 4 & -4 \end{bmatrix} \times \begin{bmatrix} 1 & -1 & 1 \\ -3 & 3 & -3 \\ -4 & 4 & -4 \end{bmatrix}$

or, $\begin{pmatrix} 1+3-4 & -1-3+4 & 1+3-4 \\ -3-9+12 & 3+9-12 & -3-9+12 \\ -4-12+16 & 4+12-16 & -4-12+16 \end{pmatrix}$

or, $\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$

$\therefore A^2 = 0$

Proved

11) If $A = \begin{pmatrix} 1 & -1 & 1 \\ -3 & 3 & -3 \\ 5 & -5 & 5 \end{pmatrix}$ show that $A^2 = 9A$

Sol:

$$A^2 = \begin{pmatrix} 1 & -1 & 1 \\ -3 & 3 & -3 \\ 5 & -5 & 5 \end{pmatrix} \times \begin{pmatrix} 1 & -1 & 1 \\ -3 & 3 & -3 \\ 5 & -5 & 5 \end{pmatrix}$$

or, $\begin{pmatrix} 1+3+5 & -1-3-5 & 1+3+5 \\ -3-9-15 & 3+9+15 & -3-9-15 \\ 5+15+25 & -5-15-25 & 5+15+25 \end{pmatrix}$

or, $\begin{pmatrix} 9 & -9 & 9 \\ -27 & 27 & -27 \\ 45 & -45 & 45 \end{pmatrix}$

again;

$9A$

or, $9\begin{pmatrix} 1 & -1 & 1 \\ -3 & 3 & -3 \\ 5 & -5 & 5 \end{pmatrix}$

or, $\begin{pmatrix} 9 & -9 & 9 \\ -27 & 27 & -27 \\ 45 & -45 & 45 \end{pmatrix}$

$\therefore \boxed{A^2 = 9A}$ proved

12) Let $A = \begin{pmatrix} 1 & 2 \\ -3 & 6 \\ 0 & 1 \end{pmatrix}$ and $B = \begin{pmatrix} 0 & 3 \\ 5 & 7 \\ 1 & -4 \end{pmatrix}$, $k=2$ verify that

a) $(A^T)^T = A$

Given;

$$(A^T) = \begin{pmatrix} 1 & -3 & 0 \\ 2 & 6 & 1 \end{pmatrix}$$

again,

$$(A^T)^T = \begin{pmatrix} 1 & 2 \\ -3 & 6 \\ 0 & 1 \end{pmatrix}$$

$\therefore \boxed{(A^T)^T = A}$ proved

b) $(A+B)^T = A^T + B^T$

Given;

$$(A+B)^T = \left[\begin{pmatrix} 1 & 2 \\ -3 & 6 \\ 0 & 1 \end{pmatrix} + \begin{pmatrix} 0 & 3 \\ 5 & 7 \\ 1 & -4 \end{pmatrix}\right]^T$$

or, $\begin{pmatrix} 1 & 5 \\ 2 & 13 \\ 1 & -3 \end{pmatrix}^T$

or, $\begin{pmatrix} 1 & 2 & 1 \\ 5 & 13 & -3 \end{pmatrix}^T$

$\therefore (A+B)^T = \begin{pmatrix} 1 & 2 & 1 \\ 5 & 13 & -3 \end{pmatrix}$

again;

$= A^T + B^T$

or, $\begin{pmatrix} 1 & 2 \\ -3 & 6 \\ 0 & 1 \end{pmatrix}^T + \begin{pmatrix} 0 & 3 \\ 5 & 7 \\ 1 & -4 \end{pmatrix}^T$

or, $\begin{pmatrix} 1 & -3 & 0 \\ 2 & 6 & 1 \end{pmatrix} + \begin{pmatrix} 0 & 5 & 1 \\ 3 & 7 & -4 \end{pmatrix}$

or, $\begin{pmatrix} 1 & 2 & 1 \\ 5 & 13 & 3 \end{pmatrix}$

$\therefore \boxed{(A+B)^T = A^T + B^T}$ proved

c) $(KA)^T = KA^T$

Sol:

$A^T = \begin{pmatrix} 1 & -3 & 0 \\ 2 & 6 & 1 \end{pmatrix}$

now, $(KA)^T$

$= 2\begin{pmatrix} 1 & 2 \\ -3 & 6 \\ 0 & 1 \end{pmatrix}^T$

$= \begin{pmatrix} 2 & 4 \\ -6 & 12 \\ 0 & 2 \end{pmatrix}^T$ $\therefore (KA)^T = \begin{pmatrix} 2 & -6 & 0 \\ 4 & 12 & 2 \end{pmatrix}$

again;

KA^T

$= 2\begin{pmatrix} 1 & -3 & 0 \\ 2 & 6 & 1 \end{pmatrix}$

or, $\begin{pmatrix} 2 & -6 & 0 \\ 4 & 12 & 2 \end{pmatrix}$

$\therefore \boxed{(KA)^T = KA^T}$ proved

13) Let $A = \begin{pmatrix} 1 & 3 & -1 \\ 3 & -5 & -8 \\ -1 & 8 & 9 \end{pmatrix}$, Then verify

a) A and A^T are both symmetric

Give

$A = \begin{pmatrix} 1 & 3 & -1 \\ 3 & -5 & -8 \\ -1 & 8 & 9 \end{pmatrix}$

$A^T = \begin{pmatrix} 1 & 3 & -1 \\ 3 & -5 & -8 \\ -1 & 8 & 9 \end{pmatrix}$

$\therefore \boxed{A = A^T}$

b) $A + A^T$ is symmetric matrix

Sol:

$$A = \begin{pmatrix} 1 & 3 & -1 \\ 3 & -5 & 8 \\ -1 & 8 & 9 \end{pmatrix}$$

$$A^T = \begin{pmatrix} 1 & 3 & -1 \\ 3 & -5 & 8 \\ -1 & 8 & 9 \end{pmatrix}$$

again;

$= A + A^T$

or, $\begin{pmatrix} 1 & 3 & -1 \\ 3 & -5 & 8 \\ -1 & 8 & 9 \end{pmatrix} + \begin{pmatrix} 1 & 3 & -1 \\ 3 & -5 & 8 \\ -1 & 8 & 9 \end{pmatrix}$

or, $\begin{pmatrix} 2 & 6 & -2 \\ 6 & -10 & 16 \\ -2 & 16 & 18 \end{pmatrix}$

c) $A - A^T$ is skew sy-matrix

Sol:

$$A - A^T = \begin{pmatrix} 1 & 3 & -1 \\ 3 & -5 & 8 \\ -1 & 8 & 9 \end{pmatrix} - \begin{pmatrix} 1 & 3 & -1 \\ 3 & -5 & 8 \\ -1 & 8 & 9 \end{pmatrix}$$

$$= \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

14) If $A = \begin{pmatrix} 4 & x+3 \\ 2x-1 & -1 \end{pmatrix}$ be a sym-matrix find x

Sol:

$A = A^T$

$$\begin{pmatrix} 4 & x+3 \\ 2x-1 & -1 \end{pmatrix} = \begin{pmatrix} 4 & 2x-1 \\ x+3 & -1 \end{pmatrix}$$

Since matrices are equal;

$\therefore x+3 = 2x-1$

$x - 2x = -1-3$

$-x = -4$

$\therefore x = 4$

15) Express $\begin{pmatrix} 4 & 2 & -1 \\ 3 & 5 & 7 \\ 1 & -2 & 1 \end{pmatrix}$ as a sum of a symmetric and skew symmetric matrix.

Sol:

Let $A = \begin{pmatrix} 4 & 2 & -1 \\ 3 & 5 & 7 \\ 1 & -2 & 1 \end{pmatrix}$

we know that;

$$A = \frac{1}{2}(A + A^T) + \frac{1}{2}(A - A^T)$$

i.e A square matrix A can be expressed as the sum of symmetric $\left(\frac{A+A^T}{2}\right)$ matrix and skew symmetric $\left(\frac{A-A^T}{2}\right)$ matrix

$$A = \frac{1}{2}\left[\begin{pmatrix} 4 & 2 & -1 \\ 3 & 5 & 7 \\ 1 & -2 & 1 \end{pmatrix} + \begin{pmatrix} 4 & 3 & 1 \\ 2 & 5 & -2 \\ -1 & 7 & 1 \end{pmatrix}\right] + \frac{1}{2}$$

$$\left[\begin{pmatrix} 4 & 2 & -1 \\ 3 & 5 & 7 \\ 1 & -2 & 1 \end{pmatrix} - \begin{pmatrix} 4 & 3 & 1 \\ 2 & 5 & -2 \\ -1 & 7 & 1 \end{pmatrix}\right]$$

$$A = \frac{1}{2}\begin{pmatrix} 8 & 5 & 0 \\ 5 & 10 & 5 \\ 0 & 5 & 2 \end{pmatrix} + \frac{1}{2}\begin{pmatrix} 0 & -1 & -2 \\ 1 & 0 & 9 \\ 2 & -9 & 0 \end{pmatrix}$$

$$= \begin{pmatrix} 4 & 5/2 & 0 \\ 5/2 & 5 & 5/2 \\ 0 & 5/2 & 1 \end{pmatrix} + \begin{pmatrix} 0 & -1/2 & -1 \\ 1/2 & 0 & 9/2 \\ 1 & -9/2 & 0 \end{pmatrix}$$

16) If $A = \begin{pmatrix} 1 & 3 \\ -4 & 2 \end{pmatrix}$ and $f(x) = x^2 - 5x + 3$, find $f(A)$

Given;

$$f(x) = x^2 - 5x + 3$$
$$f(A) = A^2 - 5A + 3$$

$$= \begin{pmatrix} 1 & 3 \\ -4 & 2 \end{pmatrix} \times \begin{pmatrix} 1 & 3 \\ -4 & 2 \end{pmatrix} - 5\begin{pmatrix} 1 & 3 \\ -4 & 2 \end{pmatrix} + 3I$$

or, $\begin{pmatrix} 1-12 & 3+6 \\ -4-8 & -12+4 \end{pmatrix} - \begin{pmatrix} 5 & 5 \\ -20 & 10 \end{pmatrix} + 3\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

or, $\begin{pmatrix} -11 & 9 \\ -12 & -8 \end{pmatrix} - \begin{pmatrix} 5 & 5 \\ -20 & 10 \end{pmatrix} + \begin{pmatrix} 3 & 0 \\ 0 & 3 \end{pmatrix}$

or, $\begin{pmatrix} -16 & -6 \\ 8 & -18 \end{pmatrix} + \begin{pmatrix} 3 & 0 \\ 0 & 3 \end{pmatrix}$

or, $\begin{pmatrix} -13 & -6 \\ 8 & -15 \end{pmatrix}$

$\therefore f(A) = \begin{pmatrix} -13 & -6 \\ 8 & -15 \end{pmatrix}$

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3) Find the two matrices A and B such that

$A+B = \begin{bmatrix} 1 & -2 \\ 3 & 4 \end{bmatrix}$ and $A-B = \begin{pmatrix} 3 & 2 \\ -1 & 0 \end{pmatrix}$

Given;

$$A+B = \begin{pmatrix} 1 & -2 \\ 3 & 4 \end{pmatrix} \quad \text{— ①}$$

$$A-B = \begin{pmatrix} 3 & 2 \\ -1 & 0 \end{pmatrix} \quad \text{— ②}$$

2A

Adding ① and ②

$$2A = \begin{pmatrix} 1 & -2 \\ 3 & 4 \end{pmatrix} + \begin{pmatrix} 3 & 2 \\ -1 & 0 \end{pmatrix}$$

$$2A = \begin{pmatrix} 4 & 0 \\ 2 & 4 \end{pmatrix}$$

$$A = \frac{\begin{pmatrix} 4 & 0 \\ 2 & 4 \end{pmatrix}}{2}$$

$$A = \begin{pmatrix} 2 & 0 \\ 1 & 2 \end{pmatrix}$$

Applying A in ①

$$\begin{pmatrix} 2 & 0 \\ 1 & 2 \end{pmatrix} + B = \begin{pmatrix} 1 & -2 \\ 3 & 4 \end{pmatrix}$$

$$B = \begin{pmatrix} 1 & -2 \\ 3 & 4 \end{pmatrix} - \begin{pmatrix} 2 & 0 \\ 1 & 2 \end{pmatrix}$$

$$B = \begin{pmatrix} -1 & -2 \\ 2 & 2 \end{pmatrix} \text{ ans}$$

9) If $P = \begin{pmatrix} 4 & 2 \\ -1 & 1 \end{pmatrix}$ show that $P^2 - 5P + 6I = 0$

Given;

$$P = \begin{pmatrix} 4 & 2 \\ -1 & 1 \end{pmatrix}$$

L.H.S

$$P^2 - 5P + 6I$$

$$= P \times P - 5P + 6I$$

or $\begin{pmatrix} 4 & 2 \\ -1 & 1 \end{pmatrix} \times \begin{pmatrix} 4 & 2 \\ -1 & 1 \end{pmatrix} - 5\begin{pmatrix} 4 & 2 \\ -1 & 1 \end{pmatrix} + 6\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

or, $\begin{pmatrix} 16-2 & 8+2 \\ -4-1 & -2+1 \end{pmatrix} - \begin{pmatrix} 20 & 10 \\ -5 & 5 \end{pmatrix} + \begin{pmatrix} 6 & 0 \\ 0 & 6 \end{pmatrix}$

or, $\begin{pmatrix} 14 & 10 \\ -5 & -1 \end{pmatrix} - \begin{pmatrix} 20 & 10 \\ -5 & 5 \end{pmatrix} + \begin{pmatrix} 6 & 0 \\ 0 & 6 \end{pmatrix}$

or, $\begin{pmatrix} -6 & 0 \\ 0 & -6 \end{pmatrix} + \begin{pmatrix} 6 & 0 \\ 0 & 6 \end{pmatrix}$

or, $\begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$

$= 0$ R.H.S proved

V.V.I

Determinante

A square arrangement of number of rows and columns and enclosed between vertical bars is known as determinante.

Determinants are denoted by $|A|$ or $|B|$ or $|C|$ -- or $\det|A|$.

Case I : Expanding 2x2 order of determinante

Let $|A| = \begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix} = a_{11} \times a_{22} - a_{21} \times a_{12}$

Ex $|A| = \begin{vmatrix} -2 & -7 \\ 5 & 3 \end{vmatrix}$

$= -6 - (-35)$

$= -6 + 35$

$= 29$

Case II : Expanding 3x3 order of determinant

a) If $A = \begin{pmatrix} 2 & 1 & 3 \\ 4 & 5 & 0 \\ 5 & 7 & 8 \end{pmatrix}$ Find $|A|$

Given;

$$|A| = \begin{array}{c} \begin{array}{ccc} + & - & + \end{array} \\ \begin{vmatrix} 2 & 1 & 3 \\ 4 & 5 & 0 \\ 5 & 7 & 8 \end{vmatrix} \end{array}$$

Expanding along R_1,

or, $2\begin{vmatrix} 5 & 0 \\ 7 & 8 \end{vmatrix} - 1\begin{vmatrix} 4 & 0 \\ 5 & 8 \end{vmatrix} + 3\begin{vmatrix} 4 & 5 \\ 5 & 7 \end{vmatrix}$

or, $2(40-0) - 1(32-0) + 3(28-25)$

or, $80 - 32 + 9$

or, 57

* Expanding determinant (using sarrus rule)

$|A| = \begin{vmatrix} 2 & 1 & 3 \\ 4 & 5 & 0 \\ 5 & 7 & 8 \end{vmatrix}$

$= \begin{array}{ccccc} 2 & 1 & 3 & 2 & 1 \\ 4 & 5 & 0 & 4 & 5 \\ 5 & 7 & 8 & 5 & 7 \end{array}$

Its value $= (2 \times 5 \times 8) + (1 \times 0 \times 5) + (3 \times 4 \times 7) - (5 \times 5 \times 3) - (7 \times 0 \times 2) - (8 \times 4 \times 1)$

$= 80 + 0 + 84 - 75 - 0 - 32$

$= 57$

eg

* $|A| = \begin{vmatrix} 2 & 1 & 3 \\ 4 & 5 & 0 \\ 5 & 7 & 8 \end{vmatrix}$

Expanding along C_1,

$= 2\begin{vmatrix} 5 & 0 \\ 7 & 8 \end{vmatrix} - 4\begin{vmatrix} 1 & 3 \\ 7 & 8 \end{vmatrix} + 5\begin{vmatrix} 1 & 3 \\ 5 & 0 \end{vmatrix}$

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$= 2|40-0| - 4|8-21| + 5|0-15|$
$= 80 + 52 - 75$
$= 5 + 52$
$= 57$ ans

* If determinate of A is equal to zero then it is singular matrix. $[|A| = 0]$

* If determinante of A is not equal to zero then it is non-singular matrix. $|A| \neq 0$

* Inverse $(A^{-1}) = \dfrac{Adj.A}{|A|}$

* Inverse of two matrice formula
$\therefore AB = BA = I$

Q) Find the inverse of $\begin{pmatrix} 1 & 0 & -1 \\ 3 & 4 & 5 \\ 0 & -6 & -7 \end{pmatrix}$ if exists

Sol
Let given matrix be A

$$|A| = 1\begin{vmatrix} 4 & 5 \\ -6 & -7 \end{vmatrix} - 0\begin{vmatrix} 3 & 5 \\ 0 & 7 \end{vmatrix} + (-1)\begin{vmatrix} 3 & 4 \\ 0 & -6 \end{vmatrix}$$

$= 1|-28 \cancel{+30} -(-30)| - 0 - 1|-18 - 0|$
$= 1|2| - 0 - 1|-18|$
$= 2 + 18$
$= 20$

$\therefore A^{-1}$ exists

again;

$c(a_{11}) = +\begin{vmatrix} 4 & 5 \\ -6 & -7 \end{vmatrix} = +|-28 -(-30)| = +2$

$c(a_{12}) = -\begin{vmatrix} 3 & 5 \\ 0 & -7 \end{vmatrix} = -|-21 - 0| = 21$

$c(a_{13}) = +\begin{vmatrix} 3 & 4 \\ 0 & -6 \end{vmatrix} = +|-18 - 0| = -18$

$c(a_{23}) = -\begin{vmatrix} 1 & 0 \\ 0 & -6 \end{vmatrix} = \cancel{-|6+4| = -4}$
$-|-6-0| = 6$

$c(a_{31}) = +\begin{vmatrix} 0 & -1 \\ 4 & 5 \end{vmatrix} = +|0+4| = 4$

$c(a_{32}) = -\begin{vmatrix} 1 & -1 \\ 3 & 5 \end{vmatrix} = -|5-(-3)| = -8$

$c(a_{33}) = +\begin{vmatrix} 1 & 0 \\ 3 & 4 \end{vmatrix} = +|4-0| = 4$

again;
matrix of cofactors $= \begin{pmatrix} 2 & 21 & -18 \\ 6 & -7 & 6 \\ 4 & -8 & 4 \end{pmatrix}$

$Adj.A = \begin{bmatrix} 2 & 6 & 4 \\ 21 & -7 & -8 \\ -18 & 6 & 4 \end{bmatrix}$

We have;
$\therefore A^{-1} = \dfrac{Adj.A}{|A|}$

$= \dfrac{1}{20}\begin{pmatrix} 2 & 6 & 4 \\ 21 & -7 & -8 \\ -18 & 6 & 4 \end{pmatrix}$

$$= \begin{pmatrix} 2/20 & 6/20 & 4/20 \\ 23/20 & -3/20 & -8/20 \\ -18/20 & -6/20 & 4/20 \end{pmatrix}$$

$$= \begin{pmatrix} 1/10 & 3/10 & 1/5 \\ 23/20 & -3/20 & -2/5 \\ -9/10 & 3/10 & 1/5 \end{pmatrix}$$

a_1] Prove that $\begin{vmatrix} a-b-c & 2a & 2a \\ 2b & b-c-a & 2b \\ 2c & 2c & c-a-b \end{vmatrix} = (a+b+c)^3$

Sol:

Expanding $R_1 \to R_1 + R_2 + R_3$

$$= \begin{vmatrix} a-b-c+2b+2c & 2a+b-c-a+2c & 2a+2b+c-a-b \\ 2b & b-c-a & 2b \\ 2c & 2c & c-a-b \end{vmatrix}$$

or, $\begin{vmatrix} a+b+c & a+b+c & a+b+c \\ 2b & b-c-a & 2b \\ 2c & 2c & c-a-b \end{vmatrix}$

or, $(a+b+c)\begin{vmatrix} 1 & 1 & 1 \\ 2b & b-c-a & 2b \\ 2c & 2c & c-a-b \end{vmatrix}$

Expanding $C_1 \to C_1 - C_2$

$$= (a+b+c)\begin{vmatrix} 0 & \cancel{b-c-a}\ 1 & 1 \\ a+b+c & b-c-a & 2b \\ 0 & 2c & c-a-b \end{vmatrix}$$

$$= (a+b+c)^2 \begin{vmatrix} 0 & 1 & 1 \\ 1 & b-c-a & 2b \\ 0 & 2c & c-a-b \end{vmatrix}$$

or, $(a+b+c)^2 \left[0 - 1 \begin{vmatrix} 1 & 1 \\ 2c & c-a-b \end{vmatrix} + 0 \right]$

or, $(a+b+c)^2 . [-(c-a-b-2c)]$

~~or,~~

or, $(a+b+c)^2 . [-(-a-b-c)]$

or, $(a+b+c)^3$ ans

Exercise 4.2

1) Find the value of the determinante

a) $\begin{vmatrix} 5 & -3 \\ 2 & 5 \end{vmatrix}$

Let:

$|A| = \begin{vmatrix} 5 & -3 \\ 2 & 5 \end{vmatrix}$

$= |25+6|$

$= 31$

b) $\begin{vmatrix} -3 & 7 \\ -2 & -1 \end{vmatrix}$

Let:

$|A| = \begin{vmatrix} -3 & 7 \\ -2 & -1 \end{vmatrix}$

$= |3+14| = 17$ ans

c) $\begin{vmatrix} a+b & a \\ b & a-b \end{vmatrix}$

Let:

$|A| = \begin{vmatrix} a+b & a \\ b & a-b \end{vmatrix}$

$= a^2 - b^2 - ab$

d) $\begin{vmatrix} 4 & 5 & 6 \\ 7 & 8 & 9 \\ 2 & 1 & 3 \end{vmatrix}$

Let:

$|A| = \begin{vmatrix} 4 & 5 & 6 \\ 7 & 8 & 9 \\ 2 & 1 & 3 \end{vmatrix}$

or, $4\begin{vmatrix} 8 & 9 \\ 1 & 3 \end{vmatrix} - 5\begin{vmatrix} 7 & 9 \\ 2 & 3 \end{vmatrix} + 6\begin{vmatrix} 7 & 8 \\ 2 & 1 \end{vmatrix}$

or, $4|24-9| - 5|21-18| + 6|7-16|$

or, $4 \times 15 - 5 \times 3 + 6 \times (-9)$

or, $60 - 15 - 54$

or, -9 ans

2) Find the value of the determinant using ~~sarus~~ sarus rule

a) $\begin{vmatrix} 1 & 1 & 1 \\ 2 & 5 & 7 \\ 2 & 1 & -1 \end{vmatrix}$

Sol:

By sarus method;

$|A| = \begin{vmatrix} 1 & 1 & 1 \\ 2 & 5 & 7 \\ 2 & 1 & -1 \end{vmatrix}$

or, $\begin{array}{ccccc} 1 & 1 & 1 & 1 & 1 \\ 2 & 5 & 7 & 2 & 5 \\ 2 & 1 & -1 & 2 & 1 \end{array}$

or, $(1 \times 5 \times (-1)) + (1 \times 7 \times 2) + (1 \times 2 \times 1) - (2 \times 5 \times 1) - (1 \times 7 \times 1) - ((-1) \times 2 \times 1)$

or, $-5 + 14 + 2 - 10 - 7 - (-2)$

or, -4 ans

b) $\begin{vmatrix} 3 & 1 & 2 \\ 4 & 1 & 6 \\ 5 & 3 & 0 \end{vmatrix}$

Sol:

$= \begin{array}{ccccc} 3 & 1 & 2 & 3 & 1 \\ 4 & 1 & 6 & 4 & 1 \\ 5 & 3 & 0 & 5 & 3 \end{array}$

or, ~~(3+1×0)~~ $0 + 30 + 24 - 10 - 54 - 0$

or, -10 ans

3) Evaluate the following determinant by using their properties (without expanding).

a) $\begin{vmatrix} 48 & 1 & 6 \\ 32 & 5 & 4 \\ 16 & 3 & 2 \end{vmatrix}$

Sol.

We have;

$$\begin{vmatrix} 48 & 7 & 6 \\ 32 & 5 & 4 \\ 16 & 3 & 2 \end{vmatrix}$$

Taking common, 8 from C_1, we get

$$= 8\begin{vmatrix} 6 & 7 & 6 \\ 4 & 5 & 4 \\ 2 & 3 & 2 \end{vmatrix}$$

or, 8×0 [$\because C_1 = C_3$]

$= 0$ proved

Note: i) If any row and column are same then its value while be 0.

b) $$\begin{vmatrix} 1 & a & b+c \\ 1 & b & c+a \\ 1 & c & a+b \end{vmatrix} = 0$$

Sol:

Apply $C_2 \Rightarrow C_2 + C_3$

$$= \begin{vmatrix} 1 & a+b+c & b+c \\ 1 & a+b+c & c+a \\ 1 & a+b+c & a+b \end{vmatrix}$$

or, $a+b+c \begin{vmatrix} 1 & 1 & b+c \\ 1 & 1 & c+a \\ 1 & 1 & a+b \end{vmatrix}$

or, $a+b+c \times 0$

$= 0$ ans

c) $$\begin{vmatrix} b-c & b+c & b \\ c-a & c+a & c \\ a-b & a+b & a \end{vmatrix}$$

Sol:

Apply $C_1 \Rightarrow C_1 + C_2$

$$= \begin{vmatrix} b-c+b+c & b+c & b \\ c-a+c+a & c+a & c \\ a-b+a+b & a+b & a \end{vmatrix}$$

or, $$\begin{vmatrix} 2b & b+c & b \\ 2c & c+a & c \\ 2a & a+b & a \end{vmatrix}$$

Taking common, 2 from C_1 we have,

$$= 2\begin{vmatrix} b & b+c & b \\ c & c+a & c \\ a & a+b & a \end{vmatrix}$$

or, $2 \times 0 = 0$ proved

d) $$\begin{vmatrix} 1 & bc & a(b+c) \\ 1 & ca & b(c+a) \\ 1 & ab & c(a+b) \end{vmatrix} = 0$$

Sol:

Applying $C_2 \Rightarrow C_2 + C_3$

$$= \begin{vmatrix} 1 & ab+bc+ca & a(b+c) \\ 1 & ab+bc+ca & b(c+a) \\ 1 & ab+bc+ca & c(a+b) \end{vmatrix}$$

or $ab+bc+ca \begin{vmatrix} 1 & 1 & ab+ac \\ 1 & 1 & bc+ba \\ 1 & 1 & ac+bc \end{vmatrix}$

$= 0$ ans

e) $\begin{vmatrix} 1 & 1 & 1 \\ x & y & z \\ y+z & z+x & x+y \end{vmatrix}$

Sol

Taking x, y, z, common from ~~1~~ adding $R_2 \to R_2 + R_3$

$= \begin{vmatrix} 1 & 1 & 1 \\ x+y+z & x+y+z & x+y+z \\ y+z & z+x & x+y \end{vmatrix}$

or Taking $(x+y+z)$ common from R_2 we get

$(x+y+z)\begin{vmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ y+z & z+x & x+y \end{vmatrix}$

or, $(x+y+z) \times 0 \quad \therefore [R_1 = R_2]$

or, 0 <u>proved</u>

f) $\begin{vmatrix} 1/a & a^2 & bc \\ 1/b & b^2 & ca \\ 1/c & c^2 & ab \end{vmatrix} \quad \therefore \frac{1}{a} \times a = 1$

$= \begin{matrix} a \times \frac{1}{abc} \\ b \times \\ c \times \end{matrix} \begin{vmatrix} 1/a & a^2 & bc \\ 1/b & b^2 & ca \\ 1/c & c^2 & ab \end{vmatrix}$

or, $\frac{1}{abc}\begin{vmatrix} 1 & a^3 & abc \\ 1 & b^3 & abc \\ 1 & c^3 & abc \end{vmatrix}$

or, $\frac{1}{abc} \times abc \begin{vmatrix} 1 & a^3 & 1 \\ 1 & b^3 & 1 \\ 1 & c^3 & 1 \end{vmatrix}$

0 <u>ans</u>

Note: If we multiply any ~~row~~ number from outside we have to divide it also to make balance.

4) Prove of the following

a) $\begin{vmatrix} 1 & a & a^2-bc \\ 1 & b & b^2-ca \\ 1 & c & c^2-ab \end{vmatrix} = 0$

Sol:

$L.H.S = \begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix} - \begin{vmatrix} 1 & a & bc \\ 1 & b & ca \\ 1 & c & ab \end{vmatrix} \begin{matrix} \times a \\ \times b \\ \times c \end{matrix}$

or, $\begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix} - \frac{1}{abc}\begin{vmatrix} a & a^2 & abc \\ b & b^2 & abc \\ c & c^2 & abc \end{vmatrix}$

or, $\begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix} - \frac{1}{abc} \times abc \begin{vmatrix} a & a^2 & 1 \\ b & b^2 & 1 \\ c & c^2 & 1 \end{vmatrix}$

or, $\begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix} - \begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix}$

[: Interchanging 2 times]

$= 0$ R.H.S proved

b) $\begin{vmatrix} 0 & b & -c \\ -b & 0 & a \\ c & -a & 0 \end{vmatrix} = 0$

or 0 proved

Note : If 0 is in diagonal then the determinate will be directly 0.

c) $\begin{vmatrix} 1 & a^2 & a^3 \\ 1 & b^2 & b^3 \\ 1 & c^2 & c^3 \end{vmatrix} = \begin{vmatrix} bc & a & a^2 \\ ca & b & b^2 \\ ab & c & c^2 \end{vmatrix}$

sol:

$LHS = \begin{vmatrix} 1 & a^2 & a^3 \\ 1 & b^2 & b^3 \\ 1 & c^2 & c^3 \end{vmatrix}$

Multiply C_1 by abc

$\frac{1}{abc} \begin{vmatrix} abc & a^2 & a^3 \\ abc & b^2 & b^3 \\ abc & c^2 & c^3 \end{vmatrix}$

Taking a,b,c common factor R_1, R_2, R_3 respectively

$\frac{1}{abc} \times abc \begin{vmatrix} bc & a & a^2 \\ ac & b & b^2 \\ ab & c & c^2 \end{vmatrix}$

$= \begin{vmatrix} bc & a & a^2 \\ ac & b & b^2 \\ ab & c & c^2 \end{vmatrix}$ R.H.S proved

T.U

d) $\begin{vmatrix} -a^2 & ab & ac \\ ab & -b^2 & bc \\ ac & bc & -c^2 \end{vmatrix} = 4a^2b^2c^2$

sol:

Taking a,b,c common from R_1, R_2, R_3 respectively

$abc \begin{vmatrix} -a & b & c \\ a & -b & c \\ a & b & -c \end{vmatrix}$

Taking a,b,c common from C_1, C_2, C_3 respectively

$abc \times abc \begin{vmatrix} -1 & 1 & 1 \\ 1 & -1 & 1 \\ 1 & 1 & -1 \end{vmatrix}$

$a^2b^2c^2 \begin{vmatrix} -1 & 1 & 1 \\ 1 & -1 & 1 \\ 1 & 1 & -1 \end{vmatrix}$

Applying $R_1 \to R_2 + R_2$, we get

$$= a^2b^2c^2 \begin{vmatrix} 0 & 0 & 2 \\ 1 & -1 & 1 \\ 1 & 1 & -1 \end{vmatrix}$$

$$\text{or, } a^2b^2c^2 \left[0 - 0 + 2 \begin{vmatrix} 1 & -1 \\ 1 & 1 \end{vmatrix} \right]$$

$$\text{or, } a^2b^2c^2 [2(1+1)]$$

$$\text{or, } a^2b^2c^2 4$$

$$\text{or, } 4a^2b^2c^2 \quad \text{R.H.S proved}$$

Ins(e) $abc + 1 = 0$ show that $\begin{vmatrix} a & a^2 & a^3+1 \\ b & b^2 & b^3+1 \\ c & c^2 & c^3+1 \end{vmatrix} = 0$

Sol:

$$L.H.S = \begin{vmatrix} a & a^2 & a^3+1 \\ b & b^2 & b^3+1 \\ c & c^2 & c^3+1 \end{vmatrix}$$

$$\text{or, } \begin{vmatrix} a & a^2 & a^3 \\ b & b^2 & b^3 \\ c & c^2 & c^3 \end{vmatrix} + \begin{vmatrix} a & a^2 & 1 \\ b & b^2 & 1 \\ c & c^2 & 1 \end{vmatrix}$$

Taking a, b, c common from R_1, R_2, R_3 respectively

$$= \begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix} + (-) \begin{vmatrix} a & 1 & a^2 \\ b & 1 & b^2 \\ c & 1 & c^2 \end{vmatrix}$$

$$\text{or, } abc \begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix} + \begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix}$$

$$\text{or, } \begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix} (abc + 1)$$

$$\text{or, } \begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix} \times 0 \quad [\because abc + 1 = 0]$$

$= 0$ R.H.S proved

+ 2nd step

$$= \begin{vmatrix} a & a^2 & 1 \\ b & b^2 & 1 \\ c & c^2 & 1 \end{vmatrix}$$

$$\text{or, } - \begin{vmatrix} a & 1 & a^2 \\ b & 1 & b^2 \\ c & 1 & c^2 \end{vmatrix}$$

Note: While specifi shifting R_1 ow or column from 1 place to another, we have to take (-) common. In the above question we shift row 2 time so $(-) \times (-) = +$

$$\text{or, } (-) \times (-) \begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix}$$

$$\text{or, } + \begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix}$$

5) Prove that $\begin{vmatrix} 1 & 1 & 1 \\ a & b & c \\ a^3 & b^3 & c^3 \end{vmatrix} = (a-b)(b-c)(c-a)(a+b+c)$

Sol.

Applying $C_1 \to C_1 - C_2$, $C_2 \to C_2 - C_3$, we get

$$= \begin{vmatrix} 0 & 0 & 1 \\ a-b & b-c & c \\ a^3-b^3 & b^3-c^3 & c^3 \end{vmatrix}$$

or, $\begin{vmatrix} 0 & 0 & 1 \\ a-b & b-c & c \\ (a-b)(a^2+ab+b^2 & (b-c)(b^2+bc+c^2 & c^3 \end{vmatrix}$

or, Taking $(a-b)(b-c)$ common from C_1 and C_2 we get.

$$(a-b)(b-c)\begin{vmatrix} 0 & 0 & 1 \\ 1 & 1 & c \\ a^2+ab+b^2 & b^2+bc+c^2 & c^3 \end{vmatrix}$$

Expanding from R_1

$$= (a-b)(b-c)\left[0 - 0 + 1\begin{vmatrix} 1 & 1 \\ a^2+ab+b^2 & b^2+bc+c^2 \end{vmatrix}\right]$$

or, $(a-b)(b-c)[1(b^2+bc+c^2-(a^2+ab+b^2))]$

or, $(a-b)(b-c)[\cancel{b^2}+bc+c^2-a^2-ab-\cancel{b^2}]$

or, $(a-b)(b-c)[bc+-ab+c^2-a^2]$

or, $(a-b)(b-c)[b(c-a)+(c+a)(c-a)]$

or, $(a-b)(b-c)[(c-a)(b+c+a)]$

or, $(a-b)(b-c)(c-a)(a+b+c)$ proved

5)b) Prove that $\begin{vmatrix} 1 & 1 & 1 \\ a & b & c \\ bc & ca & ab \end{vmatrix} = (a-b)(b-c)(c-a)$

Sol:

Apply $C_1 \to C_1 - C_2$, $C_2 \to C_2 - C_3$ we have,

$$= \begin{vmatrix} 0 & 0 & 1 \\ a-b & b-c & c \\ bc-ca & ca-ab & ab \end{vmatrix}$$

or, $\begin{vmatrix} 0 & 0 & 1 \\ a-b & b-c & c \\ -c(a-b) & -a(b-c) & ab \end{vmatrix}$

Taking common $(a-b)(b-c)$ from C_1, C_2 respectively

or, $(a-b)(b-c)\begin{vmatrix} 0 & 0 & 1 \\ 1 & 1 & c \\ -c & -a & ab \end{vmatrix}$

Now; expanding

$$= (a-b)(b-c)\left[0-0+1\begin{vmatrix} 1 & 1 \\ -c & -a \end{vmatrix}\right]$$

or, $(a-b)(b-c)[+1(-a+c)]$

or, $(a-b)(b-c)(c-a)$ R.H.S proved.

7) Prove that $\begin{vmatrix} a & x & x \\ x & a & x \\ x & x & a \end{vmatrix} = (a+2x)(a-x)^2$

Sol:

Apply $C_1 \to C_1 + C_2 + C_3$; we get;

$$= \begin{vmatrix} a+2x & x & x \\ a+2x & a & x \\ a+2x & x & a \end{vmatrix}$$

or, $(a+2x)\begin{vmatrix} 1 & x & x \\ 1 & a & x \\ 1 & x & a \end{vmatrix}$

or, Apply $R_1 \to R_1 - R_2$, we get;

$$= (a+2x)\begin{vmatrix} 0 & x-a & 0 \\ 1 & a & x \\ 1 & x & a \end{vmatrix}$$

Expanding;

or, $(a+2x)\left(0 - (x-a)\begin{vmatrix} 1 & x \\ 1 & a \end{vmatrix} + 0\right)$

or, $(a+2x)(-(x-a)(a-x))$

or, $(a+2x)((a-x)(a-x))$

or, $(a+2x)(a-x)^2$

8) Prove that $\begin{vmatrix} 1+a & b & c \\ a & 1+b & c \\ a & b & 1+c \end{vmatrix} = 1+a+b+c$

Sol:

Applying $C_1 \to C_1 + C_2 + C_3$ we have;

$$\begin{vmatrix} 1+a+b+c & b & c \\ 1+a+b+c & 1+b & c \\ 1+a+b+c & b & 1+c \end{vmatrix}$$

Taking common $(1+a+b+c)$, from C_1, we have

$$(1+a+b+c)\begin{vmatrix} 1 & b & c \\ 1 & 1+b & c \\ 1 & b & 1+c \end{vmatrix}$$

Applying $R_1 \to R_1 - R_2$, we have

$$= (1+a+b+c)\begin{vmatrix} 0 & -1 & 0 \\ 1 & 1+b & c \\ 1 & b & 1+c \end{vmatrix}$$

or, $(1+a+b+c)\left[0 - (-1)\begin{vmatrix} 1 & c \\ 1 & 1+c \end{vmatrix} + 0\right]$

or, $(1+a+b+c)[1(1+c-c)]$

or, $(1+a+b+c) \times 1$

or, $(1+a+b+c)$ R.H.S proved

11) Prove that $\begin{vmatrix} x & y & z \\ x^2 & y^2 & z^2 \\ yz & zx & xy \end{vmatrix} = (x-y)(y-z)(z-x)(xy+yz+zx)$

Sol:

$$L.H.S = \begin{vmatrix} x & y & z \\ x^2 & y^2 & z^2 \\ yz & zx & xy \end{vmatrix}$$

Applying $C_1 \to C_1 - C_2$ or $C_2 \to C_2 - C_3$

$$= \begin{vmatrix} x-y & y-z & z \\ x^2-y^2 & y^2-z^2 & z^2 \\ -z(x-y) & -x(y-z) & xy \end{vmatrix}$$

$$= \begin{vmatrix} x-y & y & z \\ x^2-y^2 & y^2 & z^2 \\ yz-zx & zx-xy & xy \end{vmatrix}$$

$\therefore$ Note: $yz - zx$
$= -zx + yz$
$= -z(x-y)$

or, $$\begin{vmatrix} x-y & y-z & z \\ x^2-y^2 & y^2-z^2 & z^2 \\ -z(x-y) & -x(y-z) & xy \end{vmatrix}$$

Taking $(x-y)$ and $(y-z)$ common from C_1 and C_2 resp.

$$= (x-y)(y-z) \begin{vmatrix} 1 & 1 & z \\ x+y & y+z & z^2 \\ -z & -x & xy \end{vmatrix}$$

$C_1 \to C_1 - C_2$

$$= (x-y)(y-z) \begin{vmatrix} 0 & 1 & z \\ x-z & y+z & z^2 \\ z-z & -x & xy \end{vmatrix}$$

$\therefore x+y-(y+z)$
$x+y-y-z$
$x-z$

or, $$(x-y)(y-z) \begin{vmatrix} 0 & 1 & z \\ -(z-x) & y+z & z^2 \\ -(z-x) & -x & xy \end{vmatrix}$$

$\therefore -z-(-x)$
$-z+x$
$x-z$
$-(z-x)$

$$(x-y)(y-z)(z-x) \begin{vmatrix} 0 & 1 & z \\ -1 & y+z & z^2 \\ -1 & -x & xy \end{vmatrix}$$

$R_2 \to R_2 - R_3$

$$= (x-y)(y-z)(z-x) \begin{vmatrix} 0 & 1 & z \\ 0 & y+z+x & z^2-xy \\ -1 & -x & xy \end{vmatrix}$$

or, $$(x-y)(y-z)(z-x)\left[0 - 0 + (-1)\begin{vmatrix} 1 & z \\ y+z+x & z^2-xy \end{vmatrix}\right]$$

or, $(x-y)(y-z)(z-x)[-1(z^2-xy) - z(y+z+x)]$

or, $(x-y)(y-z)(z-x)[-1(z^2-xy-yz-z^2-zx)]$

or, $(x-y)(y-z)(z-x)(xy+yz+zx)$ R.H.S proved

9) Prove that $$\begin{vmatrix} 1+x & 1 & 1 \\ 1 & 1+y & 1 \\ 1 & 1 & 1+z \end{vmatrix} = xyz\left[1+\frac{1}{x}+\frac{1}{y}+\frac{1}{z}\right]$$

Sol:

$$\begin{vmatrix} 1+x & 1 & 1 \\ 1 & 1+y & 1 \\ 1 & 1 & 1+z \end{vmatrix} \begin{matrix} \div x \\ \div y \\ \div z \end{matrix}$$

$$= xyz \begin{vmatrix} 1+1/x & 1/x & 1/x \\ 1/y & 1+1/y & 1/y \\ 1/z & 1/z & 1+1/z \end{vmatrix}$$

note:
$\left[\because \frac{1+x}{x} = \frac{1}{x} + \frac{x}{x}\right.$
$\frac{1}{x} + 1 = 1 + \frac{1}{x}$

Applying $R_1 \to R_1 + R_2 + R_3$

$$= xyz \begin{vmatrix} 1+1/x+1/y+1/z & 1/x+1+1/y+1/z & 1/x+1/y+1+1/z \\ 1/y & 1+1/y & 1/y \\ 1/z & 1/z & 1+1/z \end{vmatrix}$$

Or, ~~xyz~~ (Taking $(1+1/x+1/y+1/z)$ common from R_1

$$\text{Or, } xyz(1+1/x+1/y+1/z) \begin{vmatrix} 1 & 1 & 1 \\ 1/y & 1+1/y & 1/y \\ 1/z & 1/z & 1+1/z \end{vmatrix}$$

~~$R_1 \to R_1 - R_2$~~

Expanding $C_1 \to C_1 - C_2$, $C_2 \to C_2 - C_3$

Note: $\frac{1}{y} - (1+\frac{1}{y})$

$$= xyz(1+1/x+1/y+1/z) \begin{vmatrix} 0 & 0 & 1 \\ -1 & 1 & 1/y \\ 0 & 0 & 1/z \end{vmatrix}$$

$\frac{1}{y} - 1 - \frac{1}{y} = -1$

$$\text{Or, } xyz(1+1/x+1/y+1/z)[0-0+1 \begin{vmatrix} -1 & 1 \\ 0 & 0 \end{vmatrix}$$

$$\text{Or, } xyz(1+1/x+1/y+1/z)$$

R.H.S proved

Note: If we divided any number from outside we have to multiply it also to make balance.

10) Prove that $\begin{vmatrix} a-b-c & 2a & 2a \\ 2b & b-c-a & 2b \\ 2c & 2c & c-a-b \end{vmatrix} = (a+b+c)^3$

Sol:

$$L.H.S = \begin{vmatrix} a-b-c & 2a & 2a \\ 2b & b-c-a & 2b \\ 2c & 2c & c-a-b \end{vmatrix}$$

Applying $R_1 \to R_1 + R_2 + R_3$

$$= \begin{vmatrix} a-b-c+2b+2c & 2a+b-c-a+2c & 2a+2b+c-a-b \\ 2b & b-c-a & 2b \\ 2c & 2c & c-a-b \end{vmatrix}$$

$$\text{Or, } \begin{vmatrix} a+b+c & a+b+c & a+b+c \\ 2b & b-c-a & 2b \\ 2c & 2c & c-a-b \end{vmatrix}$$

$$\text{Or, } (a+b+c) \begin{vmatrix} 1 & 1 & 1 \\ 2b & b-c-a & 2b \\ 2c & 2c & c-a-b \end{vmatrix}$$

Expanding $C_1 \to C_1 - C_2$

$$= (a+b+c) \begin{vmatrix} 0 & 1 & 1 \\ a+b+c & b-c-a & 2b \\ 0 & 2c & c-a-b \end{vmatrix}$$

Taking $(a+b+c)$ common from C_1

$$\text{Or, } (a+b+c)^2 \begin{vmatrix} 0 & 1 & 1 \\ 1 & b-c-a & 2b \\ 0 & 2c & c-a-b \end{vmatrix}$$

or, $(a+b+c)^2 \left[0 - 1 \begin{vmatrix} 1 & 1 \\ 2c & c-a-b \end{vmatrix} + 0 \right]$

or, $(a+b+c)^2 \cdot [-(c-a-b-2c)]$

or, $(a+b+c)^3$

12) Prove that $\begin{vmatrix} a^2 & bc & ac+c^2 \\ a^2+ab & b^2 & ac \\ ab & b^2+bc & c^2 \end{vmatrix} = 4a^2b^2c^2$

Sol:

Taking common of a, b, c from C_1, C_2, C_3, we get

$$abc \begin{vmatrix} a & c & a+c \\ a+b & b & a \\ b & b+c & c \end{vmatrix}$$

Applying $C_1 \to C_1 - C_2 - C_3$, we get

$$= abc \begin{vmatrix} -2c & c & a+c \\ 0 & b & a \\ -2c & b+c & c \end{vmatrix}$$

or, $abc \left[-2c \begin{vmatrix} b & a \\ b+c & c \end{vmatrix} - 2c \begin{vmatrix} c & a+c \\ b & a \end{vmatrix} \right]$

or, $abc\,[-2c(bc-ab-ac) - 2c(ac-ab-bc)]$

or, $abc\,[\cancel{-2bc^2} + \overset{2abc}{\cancel{2abc}} + \cancel{2ac^2} - \cancel{2ac^2} + 2abc + \cancel{2bc^2}]$

or, $abc \times 4abc$

or, $4a^2b^2c^2$ R.H.S proved

Formula

$a^3+b^3+c^3-3abc = (a+b+c)(a^2+b^2+c^2-ab-bc-ca)$

14) Prove that $\begin{vmatrix} b+c & a-b & a \\ c+a & b-c & b \\ a+b & c-a & c \end{vmatrix} = 3ab - a^3 - b^3 - c^3$

L.H.S

Applying $R_1 \to R_1 + R_2 + R_3$

$$= \begin{vmatrix} 2(a+b+c) & 0 & a+b+c \\ c+a & b-c & b \\ a+b & c-a & c \end{vmatrix}$$

or, $(a+b+c) \begin{vmatrix} 2 & 0 & 1 \\ c+a & b-c & b \\ a+b & c-a & c \end{vmatrix}$

or, $(a+b+c) \left[2 \begin{vmatrix} b-c & b \\ c-a & c \end{vmatrix} - 0 + 1 \begin{vmatrix} c+a & b-c \\ a+b & c-a \end{vmatrix} \right]$

or, $(a+b+c)[2(bc-c^2-bc+ab)+(c^2-a^2-(a+b)(b-c))]$

or, $(a+b+c)(2ab-2c^2+c^2-a^2-(ab-ca+b^2-bc)$

or, $(a+b+c)(2ab-c^2-a^2-ab+ca-b^2+bc$

or, $(a+b+c)(-a^2-b^2-c^2+ab+bc+ca)$

or, $(a+b+c)(a^2+b^2+c^2-ab-bc-ca)$

or, $-(a^3+b^3+c^3-3abc)$

or, $3abc - a^3 - b^3 - c^3$ R.H.S proved

15) Solve for x

a) $\begin{vmatrix} 3x-8 & 3 & 3 \\ 3 & 3x-8 & 3 \\ 3 & 3 & 3x-8 \end{vmatrix} = 0$

Sol:

Applying; $C_1 \to C_1 + C_2 + C_3$

or, $\begin{vmatrix} 3x-2 & 3 & 3 \\ 3x-2 & 3x-8 & 3 \\ 3x-2 & 3 & 3x-8 \end{vmatrix} = 0$

Taking common of $(3x-2)$ from C_1;

or, $(3x-2)\begin{vmatrix} 1 & 3 & 3 \\ 1 & 3x-8 & 3 \\ 1 & 3 & 3x-8 \end{vmatrix} = 0$

either $3x-2=0$

$\therefore x = \frac{2}{3}$

Or, $\begin{vmatrix} 1 & 3 & 3 \\ 1 & 3x-8 & 3 \\ 1 & 3 & 3x-8 \end{vmatrix} = 0$

$R_1 \to R_1 - R_2$

or, $\begin{vmatrix} 0 & 11-3x & 0 \\ 1 & 3x-8 & 3 \\ 1 & 3 & 3x-8 \end{vmatrix}$

Expanding along R_1

$0 = (11-3x)\begin{vmatrix} 1 & 3 \\ 1 & 3x-8 \end{vmatrix} + 0 = 0$

or, $(3x-11)(3x-8-3) = 0$

or, $(3x-11)(3x-11) = 0$

$\therefore 3x-11=0$ | $3x-11=0$

$x = \frac{11}{3}$ | $x = \frac{11}{3}$

$\therefore x = \frac{2}{3}, \frac{11}{3}$ ans

b) $\begin{vmatrix} x+1 & 3 & 5 \\ 2 & x+2 & 5 \\ 2 & 3 & x+4 \end{vmatrix} = 0$

Sol:

let $A = \begin{vmatrix} x+1 & 3 & 5 \\ 2 & x+2 & 5 \\ 2 & 3 & x+4 \end{vmatrix} = 0$

Applying $C_1 \to C_1 + C_2 + C_3$; we get

$= \begin{vmatrix} x+9 & 3 & 5 \\ x+9 & x+2 & 5 \\ x+9 & 3 & x+4 \end{vmatrix} = 0$

Taking common $(x+9)$ from C_1;

$$(x+9)\begin{vmatrix} 1 & 3 & 5 \\ 1 & x+2 & 5 \\ 1 & 3 & x+4 \end{vmatrix} = 0$$

Either,

$x+9=0$

$x=-9$

Or

$$\begin{vmatrix} 1 & 3 & 5 \\ 1 & x+2 & 5 \\ 1 & 3 & x+4 \end{vmatrix} = 0$$

Applying $R_1 \Rightarrow R_1 - R_2$

$$\begin{vmatrix} 0 & 1-x & 0 \\ 1 & x+2 & 5 \\ 1 & 3 & x+4 \end{vmatrix} = 0$$

Expanding along R_1;

$$= 0-(1-x)\begin{vmatrix} 1 & 5 \\ 1 & x+4 \end{vmatrix} + 0 = 0$$

or, $0 - 1 + x \,|\, x+4-5 \,|\, + 0 = 0$

or, $(x-1)(x-1)$

$\therefore x-1=0 \quad | \quad x-1=0$

$x=1 \quad | \quad x=1$

$\therefore x = 1, -9$ ans

14) Prove that $\begin{vmatrix} b+c & a-b & a \\ c+a & b-c & b \\ a+b & c-a & c \end{vmatrix} = 3abc - a^3 - b^3 - c^3$

Sol:

$$L.H.S = \begin{vmatrix} b+c & a-b & a \\ c+a & b-c & b \\ a+b & c-a & c \end{vmatrix}$$

$C_1 \Rightarrow C_1 + C_3$

$$= \begin{vmatrix} a+b+c & a-b & a \\ a+b+c & b-c & b \\ a+b+c & c-a & c \end{vmatrix}$$

or, $(a+b+c)\begin{vmatrix} 1 & a-b & a \\ 1 & b-c & b \\ 1 & c-a & c \end{vmatrix}$

$R_1 \Rightarrow R_1 - R_2 \quad | \quad R_2 \Rightarrow R_2 - R_3$

$$= (a+b+c)\begin{vmatrix} 0 & a-2b+c & a-b \\ 0 & a+b-2c & b-c \\ 1 & c-a & c \end{vmatrix}$$

or, $(a+b+c)\left[0 - 0 + 1 \begin{vmatrix} a-2b+c & a-b \\ a+b-2c & b-c \end{vmatrix}\right]$

or, $(a+b+c)[(b-c)(a-2b+c)-(a-b)(a+b-2c)]$

or, $(a+b+c)[ab-2b^2+bc-ca+2bc-c^2-a^2+ab-2ca-ab+b^2+2bc]$

or, $(a+b+c)(ab-2b^2+3bc-ca-c^2-a^2+2ca+b^2-2bc$

or, ~~$(a+b+c)(a^2-b^2-c^2+ab+bc+ca)$~~

~~or,~~

or, $(a+b+c)(-a^2-b^2-c^2+ab+bc+ca)$

or, $(a+b+c)(-(a^2+b^2+c^2-ab-bc-ca))$

or, $-(a^3+b^3+c^3-3abc)$

or, $3abc-a^3-b^3-c^3$ = R.H.S proved

Find the adjoint of the matrix

$$\begin{bmatrix} 1 & 2 & -2 \\ -1 & 3 & 0 \\ 0 & -2 & 1 \end{bmatrix}$$

sol:

$C(a_{11}) = +\begin{vmatrix} 3 & 0 \\ -2 & 1 \end{vmatrix} = 3$

$C(a_{12}) = -|-1-0| = 1$

$C(a_{13}) = 2$

$C(a_{21}) = -\begin{vmatrix} 2 & -2 \\ -2 & 1 \end{vmatrix} = -(2-4) = 2$

$C(a_{22}) = +\begin{vmatrix} 1 & -2 \\ 0 & 1 \end{vmatrix} = 1$

$C(a_{23}) = -\begin{vmatrix} 1 & 2 \\ 0 & -2 \end{vmatrix} = 2$

$C(a_{31}) = +|0+6| = 6$

$C(a_{32}) = -\begin{vmatrix} 1 & -2 \\ -1 & 0 \end{vmatrix} = -(0-2) = 2$

$C(a_{33}) = +\begin{vmatrix} 1 & 2 \\ -1 & 3 \end{vmatrix} = 3+2 = 5$

$\therefore$ Matrix of cofactors

$$\begin{pmatrix} 3 & 1 & 2 \\ 2 & 1 & 2 \\ 6 & 2 & 5 \end{pmatrix}$$

$$Adj.A = \begin{pmatrix} 3 & 2 & 6 \\ 1 & 1 & 2 \\ 2 & 2 & 5 \end{pmatrix}$$ ans

13) $\begin{vmatrix} a & a^2 & 1+a^3 \\ b & b^2 & 1+b^3 \\ c & c^2 & 1+c^3 \end{vmatrix} = (1+abc)\begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix}$

sol:

L.H.S ~~let~~ $|A| = \begin{vmatrix} a & a^2 & 1+a^3 \\ b & b^2 & 1+b^3 \\ c & c^2 & 1+c^3 \end{vmatrix}$

$= \begin{vmatrix} a & a^2 & 1+a^3 \\ b & b^2 & 1+b^3 \\ c & c^2 & 1+c^3 \end{vmatrix}$

or, $\begin{vmatrix} a & a^2 & 1 \\ b & b^2 & 1 \\ c & c^2 & 1 \end{vmatrix} + abc \begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix}$

or, $\begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix} + abc \begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix}$

or, $(1+abc) \begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix}$ R.H.S proved

Matrix Solution using methods

Q_1 If $A = \begin{pmatrix} 1 & 2 & 3 \\ 3 & 2 & 1 \end{pmatrix}$ and $B = \begin{pmatrix} 7 & 8 & 9 \\ 9 & 8 & 7 \end{pmatrix}$ find $A+B$ and $A-B$

Sol:

```
>>A=[1,2,3;3,2,1];

>>B=[7,8,9;9,8,7];

>>A+B,
and
>>A-B,
```

2) Size of matrix ⇒ >>size(A);

3) A^{-1} ⇒ >>inv(A);

4) ~~Adj of A (>>~~

4) Adj of A ⇒ >>det(A)* inv(A),

5) If $A = \begin{pmatrix} 2 & 3 \\ 3 & 0 \\ 1 & 5 \end{pmatrix}$ and $B = \begin{pmatrix} 4 & 2 & -1 \\ 3 & -2 & 1 \end{pmatrix}$

find AB and BA

Sol:

```
>>A=[2,3;3,0;1,5];

>>B=[4,2,-1;3,-2,1];

>>A*B,

>>B*A,
```

6) Find the size of matrix $A = \begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{pmatrix}$ size of A

Sol:

```
>>A=[1,2,3;4,5,6;7,8,9];
>>size(A);
```

7) If $A = \begin{pmatrix} 1 & 5 & -3 \\ 2 & 6 & -2 \\ 3 & 7 & 0 \end{pmatrix}$ find A^{-1} if possible

Sol:

```
>>a=[1,5,-3;2,6,-2;3,7,0]

>>inv(a)
```

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8) Let $y = x^2 - 5x + 6$; $x \in (-5, 10)$

⇒ >>x = [-5 : 10];
>>y = (x.^2 - 5 * x + 6);
>>plot (x,y)

* plot $y = e^x$ $x \in [-1, 1]$

>>x = [-1 : 1];
>>y = exp (x);
>> plot (x,y)

Solve the following using 'Matlab'.

1) Find the value of determinant $= \begin{pmatrix} 2 & 4 & 6 \\ 5 & 7 & 8 \\ 2 & 3 & 2 \end{pmatrix}$.

>> A = [2 4 6; 5 7 8; 2 3 2];
>> det (A)

2) Find the rank of matrix $\begin{pmatrix} 1 & 2 & 3 & 4 \\ 5 & 6 & 7 & 8 \\ 9 & 10 & 11 & 10 \end{pmatrix}$

⇒ >>rank (A)

3) Find the inverse of $\begin{pmatrix} 1 & 4 & 1 \\ 3 & 3 & -2 \\ 0 & -4 & 1 \end{pmatrix}$

= >> inv (A)

4) Solve $x + 2y + 3z = 14$
$3x + 4y + 2z = 17$
$2x + 3y + z = 11$

Sol:

>> a = [1,2,3; 3,4,2; 2,3,1];
>> b = [14, 17, 11];
>> x = a/b

5) Solve $2y - z = 3$
$5x - 2y + 7z - 3 = 0$
$2x + 7y = 7$

Sol:

>> a = [0, 2, -1; 5, -2, 7; 2, 7, 0];
>> b = [3, 3, 7]; (∵ 3 goes in the place of 0)
>> x = a/b

Graph and plotting

2D / 3D plotting

1) Plot the function $y = 6 - 6x^2 + x^4$, where $x \in [-6, 6]$

= >>x = [-6:6];
>>y = 6 - 6 * x.^2 + x.^4;
>>plot (x,y)

2) $y = 4x^3$, $x \in [-5, 5]$;

= >>x = [-5:5];
>>y = ~~4x~~ 4 * x.^3

for Rdiane use pi [-3, 1]

3) $y = e^{x^2}$; $x \in [-1,1]$

```
>>x = [-1:1];
>>y = exp(x);
>>plot(x,y)
```

$y = e^{x^2}$, $x \in [-1,1]$

```
>>x = [-1:1];
>>y = exp(x^2);
>>plot(x,y)
```

4) $y = \cos x$; $x \in [-\pi; \pi]$

```
>>x = [-pi:0.01;pi]
>>y = cos(x);
>>plot(x,y)
```

5) $y = \log^{2x}$; $x \in [-5,5]$

```
>>x = [-5:5];
>>y = log(2*x);
>>plot(x,y)
```

6) $y = -3 + (5+x)^2$; $x \in [-65, 55]$

```
>>x = [-65:55];
>>y = -3+(5+x).^2
>>plot(x,y)
```

7) $y = 3x^2 + 2x + 5$; $x \in [-3,4]$

```
>>x = [-3:4];
>>y = 3*x.^2 + 2*x + 5;
>>plot(x,y)
```

8) $y = x^3$; $x \in [-5,5]$

```
>>x = [-5:5];
>>y = x.^3;
>>plot(x,y)
```

9) plot : $x^2 + y^2 = 25$

~~>>ezplot('(x.^2)~~ Note $x^2 + y^2 - 25 = 0$

```
>>ezplot('x^2 + y^2 - 25');
```

10) $\frac{x^2}{9} + \frac{y^2}{16} - 1 = 0$

```
>>ezplot('(x^2)/9 + (y^2)/16 - 1');
```

11) $\frac{(x+1)^2}{12} - \frac{(y-2)^2}{18} = 1$

```
>>ezplot('(x+1)^2/12 - (y-2)^2/18 - 1');
```

12) $9x - y^2 = 0$

```
>>ezplot('9*x - y^2');
```

13) $y^2 - 8x = 0$

```
>>ezplot('y^2 - 8*x');
```

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14) $\left(\frac{x}{5}\right)^2 + \left(\frac{y}{3}\right)^2 = 1$

= ezplot('(x/5)^2 + (y/3)^2 - 1');

or

= ezplot('x^2/25 + y^2/9 - 1');

Note : Interval देकर power मा dot(.) नदिने
देकर होइन भने plot गर्दा power मा dot(.) लगाउने

Matlab

```
>>A+B
>>A-B
>>A*B
>>A/B
>>inv(A)
>>det(A)
>>inv(a)*det(a)
>>rank(a)
>>A'
```

Adjoint

Transpose

ex^2

```
>>y=exp(x^2)
```

Solving

1) Solve $x+3=0$

sol:

```
>>syms x;
>>solve(x+3==0,x)
```

2) $x^2-5x+6=0$

sol:

```
>>syms x;
>>solve(x^2-5*x+6==0,x)
```

3) $a^2+12a-6=0$

sol:

```
>>syms a;
>>solve(a^2+12*a-6==0,a)
```

4) $y^2+4=0$

sol:

```
>>syms y;
>>solve(y^2+4==0,y)
```

5) $x^3-7x^2+36=0$

sol:

```
>>syms x;
>>solve(x^3-7*x^2+36==0,x)
```

6) $4x^3-7x^2-9=0$

sol:

```
>>syms x;
>>solve(4*3^3-7*x^2-9==0,x)
```

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RANK

9) $x^2 - 9 = 0$

ans:

Ans: 2;
solve(x^2 - 9)

1) Find the adjoint and inverse of following matrices.

$$\begin{bmatrix} 3 & 2 \\ -1 & 6 \end{bmatrix}$$

ans:

Let $A = \begin{bmatrix} 3 & 2 \\ -1 & 6 \end{bmatrix}$

$|A| = \begin{vmatrix} 3 & 2 \\ -1 & 6 \end{vmatrix} = 18 - (-2) = 20$

again;

C.F of $a_{11} = +|6| = 6$
$C(a_{12}) = -[-1] = 1$
$C(a_{21}) = -|2| = -2$
$C(a_{22}) = +|3| = 3$

Matrices of Co-factor $= \begin{bmatrix} 6 & 1 \\ -2 & 3 \end{bmatrix}$

$\therefore Adj(A) = \begin{bmatrix} 6 & -2 \\ 1 & 3 \end{bmatrix}$

$A^{-1} = \frac{1}{20} \begin{bmatrix} 6 & -2 \\ 1 & 3 \end{bmatrix}$

$= \begin{bmatrix} 3/10 & -1/10 \\ 1/20 & 3/20 \end{bmatrix}$ ans

Rank / Range (A) = 1-5

A Rank is the size of the non-singular of the given matrix.

Questions:

i) $A = \begin{pmatrix} 2 & 7 \\ 4 & 8 \end{pmatrix}$ 2x2

sol:

$|A| = \begin{vmatrix} 2 & 7 \\ 4 & 8 \end{vmatrix} = 16 - 28 = -12 \neq 0$ (non-singular)

$r(A) \neq 2$

[2] [4] [2] [4]

[2] $= 2 \neq 0$

$r(A) = 1$

ii) $A = \begin{bmatrix} 2 & 2 \\ 4 & 4 \end{bmatrix}$

$|A| = \begin{vmatrix} 2 & 2 \\ 4 & 4 \end{vmatrix} = 8 - 8 = 0$ (singular)

$r(A) \neq 2$

[2] [4] [2] [4]

[2] $= 2 \neq 0$

$r(A) = 1$

iii) $A = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 3 & 4 \\ 4 & 10 & 18 \end{pmatrix}$

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$$|A| = 1\begin{vmatrix} 3 & 4 \\ 10 & 18 \end{vmatrix} - 2\begin{vmatrix} 2 & 4 \\ 4 & 18 \end{vmatrix} + 3\begin{vmatrix} 2 & 3 \\ 4 & 10 \end{vmatrix}$$

$$= 1|54 - 40| - 2|36 - 16| + 3|20 - 12|$$

$$= 14 - 40 + 24$$

$$= -2$$

A has rank 3.

iv) Find the rank of matrix

$$A = \begin{pmatrix} 1 & 2 & 3 & 1 \\ 2 & 5 & 7 & -1 \\ 2 & 1 & 0 & 5 \end{pmatrix}_{3\times 4}$$

sol:

$$\begin{vmatrix} 1^{+} & 2 & 3 & 1 \\ 2^{-} & 5 & 7 & -1 \\ 2^{+} & 1 & 0 & 5 \end{vmatrix}$$

Sub matrix

$$= \begin{pmatrix} 1 & 2 & 3 \\ 2 & 5 & 7 \\ 2 & 1 & 0 \end{pmatrix}, \begin{pmatrix} 2 & 3 & 1 \\ 5 & 7 & -1 \\ 7 & 0 & 5 \end{pmatrix}$$

or, $\begin{vmatrix} 1^{+} & 2^{-} & 3^{+} \\ 2 & 5 & 7 \\ 2 & 1 & 0 \end{vmatrix}$

or, $1\begin{vmatrix} 5 & 7 \\ 1 & 0 \end{vmatrix} - 2\begin{vmatrix} 2 & 7 \\ 2 & 0 \end{vmatrix} + 3\begin{vmatrix} 2 & 5 \\ 2 & 1 \end{vmatrix}$

$$= 1|0 - 7| - 2|0 - 14| + 3|2 - 10|$$

$$= 0 - 0 + 3 \times (-8)$$

$$= -24 \neq 0 \text{ (non-singular)}$$

$$r(A) = 3$$

Exercise 4.3

1) Find the adjoint and inverse matrix of the following matrix.

c) $\begin{pmatrix} 3 & 2 \\ -1 & 6 \end{pmatrix}$

sol:

$$A = \begin{pmatrix} 3 & 2 \\ -1 & 6 \end{pmatrix}$$

$$|A| = |3 \times 6 - (-1) \times 2| = |18 + 2| = |20| \neq 0$$

$\therefore A^{-1}$ exists

Co-factor of A

$$\begin{vmatrix} 3^{+} & 2^{-} \\ -1^{+} & 6^{+} \end{vmatrix} = \begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}$$

$C(a_{11}) = +|3| = 3$

$C(a_{12}) = -|2| = -2$

$C(a_{21}) = -|-1| = 1$

$C(a_{22}) = |6| = 6$

a) $\begin{bmatrix} 2 & -4 \\ -1 & 2 \end{bmatrix}$

sol:

$|A| = \begin{vmatrix} 2 & -4 \\ -1 & 2 \end{vmatrix}$

or, $|4-4| = 0$

$\therefore A^{-1}$ does not exist.

b) $\begin{bmatrix} 4 & 0 \\ 1 & 2 \end{bmatrix}$

sol:

$|A| = \begin{vmatrix} 4 & 0 \\ 1 & 2 \end{vmatrix} = 8-0$ or, $8 \neq 0$

A^{-1} exists.

$C(a_{11}) = +|2| = 2$
$C(a_{12}) = -|1| = -1$
$C(a_{21}) = -|0| = 0$
$C(a_{22}) = +|4| = 4$

Matrices of co-factor

$\begin{bmatrix} 2 & -1 \\ 0 & 4 \end{bmatrix}$

$Adj.(A) = \begin{bmatrix} 2 & 0 \\ -1 & 4 \end{bmatrix}$

$A^{-1} = \frac{Adj}{|A|} = \frac{1}{|A|} \cdot Adj.(A)$

$= \frac{1}{8}\begin{bmatrix} 2 & 0 \\ -1 & 4 \end{bmatrix}$

$= \begin{bmatrix} 1/4 & 0 \\ -1/8 & 1/2 \end{bmatrix}$

d) $\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$

sol:

$|A| = \begin{vmatrix} 1 & 2 \\ 3 & 4 \end{vmatrix} = 4-6 = -2 \neq 0$

A^{-1} exists

$C(a_{11}) = +|4| = 4$
$C(a_{12}) = -|3| = -3$
$C(a_{21}) = -|2| = -2$
$C(a_{22}) = +|1| = 1$

Matrics of Cofactor

$\begin{bmatrix} 4 & -3 \\ -2 & 1 \end{bmatrix}$

$Adj.(A) = \begin{bmatrix} 4 & -2 \\ -3 & 1 \end{bmatrix}$

$A^{-1} = \frac{1}{|A|} Adj.(A)$

$= \frac{1}{-2}\begin{bmatrix} 4 & -2 \\ -3 & 1 \end{bmatrix}$

$= \begin{bmatrix} -2 & 0 \\ -2/-3 & 1/-2 \end{bmatrix}$

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3) Let $A = \begin{bmatrix} 1 & 1 \\ 0 & 2 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 2 \\ 0 & 3 \end{bmatrix}$, verify that $(AB)^{-1} = B^{-1}A^{-1}$.

Sol

Given $A = \begin{bmatrix} 1 & 1 \\ 0 & 2 \end{bmatrix}$

For A^{-1}

$C(a_{11}) = +|2| = 2$
$C(a_{12}) = -|0| = 0$
$C(a_{21}) = -|1| = -1$
$C(a_{22}) = +|1| = 1$

$\therefore$ Matrices of cofactor $\begin{pmatrix} 2 & 0 \\ -1 & 1 \end{pmatrix}$

$|A| = \begin{bmatrix} 1 & 1 \\ 0 & 2 \end{bmatrix} = 2$

$Adj(A) = \begin{pmatrix} 2 & -1 \\ 0 & 1 \end{pmatrix}$

$A^{-1} = \frac{1}{|A|} adj.(A)$

$= \frac{1}{2}\begin{pmatrix} 2 & -1 \\ 0 & 1 \end{pmatrix}$

again;

$B = \begin{bmatrix} 1 & 2 \\ 0 & 3 \end{bmatrix}$

$C(a_{11}) = +|3| = 3$
$C(a_{12}) = -|0| = 0$
$C(a_{21}) = -|2| = -2$
$C(a_{22}) = +|1| = 1$

$\therefore$ Matrices of cofactor $= \begin{pmatrix} 3 & 0 \\ -2 & 1 \end{pmatrix}$

$Adj.(B) = \begin{pmatrix} 3 & -2 \\ 0 & 1 \end{pmatrix}$

$|B| = \begin{vmatrix} 1 & 2 \\ 0 & 3 \end{vmatrix} = 3 - 0 = 3$

$\therefore B^{-1} = \frac{1}{|A|} adj.(A)$

$= \frac{1}{3}\begin{pmatrix} 3 & -2 \\ 0 & 1 \end{pmatrix}$

For $(AB)^{-1}$

$AB = \begin{pmatrix} 1 & 1 \\ 0 & 2 \end{pmatrix}\begin{pmatrix} 1 & 2 \\ 0 & 3 \end{pmatrix}$

$= \begin{pmatrix} 1+0 & 2+3 \\ 0+0 & 0+6 \end{pmatrix}$

$\therefore AB = \begin{pmatrix} 1 & 5 \\ 0 & 6 \end{pmatrix}$

~~$Adj.(AB) = \begin{pmatrix} 1 & 0 \\ 5 & 6 \end{pmatrix}$~~

$|AB| = \begin{vmatrix} 1 & 5 \\ 0 & 6 \end{vmatrix} = 6 - 0 = 6$

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$(AB)^{-1} = \frac{1}{|AB|} \, Adj(AB)$

$= \frac{1}{6}\begin{pmatrix} 1 & -5 \\ 0 & 1 \end{pmatrix}$

$C(a_{11}) = +|6| = 6$
$C(a_{12}) = -|0| = 0$
$C(a_{21}) = -|5| = -5$
$C(a_{22}) = +|1| = 1$

$\therefore$ Matrix of cofactors $= \begin{pmatrix} 6 & 0 \\ -5 & 1 \end{pmatrix}$

$(AB)^{-1} = \frac{adj.(AB)}{|AB|}$

$= \frac{1}{6}\begin{pmatrix} 6 & -5 \\ 0 & 1 \end{pmatrix}$

$(AB)^{-1} = \begin{pmatrix} 1 & -5/6 \\ 0 & 1/6 \end{pmatrix}$

again,

$B^{-1}A^{-1} = \frac{1}{3}\begin{pmatrix} 3 & -2 \\ 0 & 1 \end{pmatrix} \cdot \frac{1}{2}\begin{pmatrix} 2 & -1 \\ 0 & 1 \end{pmatrix}$

$= \frac{1}{6}\begin{pmatrix} 3 & -2 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} 2 & -1 \\ 0 & 1 \end{pmatrix}$

$= \frac{1}{6}\begin{pmatrix} 6-0 & -3-2 \\ 0+0 & 0+1 \end{pmatrix}$

$= \frac{1}{6}\begin{pmatrix} 6 & -5 \\ 0 & 1 \end{pmatrix}$

$= \begin{pmatrix} 1 & -5/6 \\ 0 & 1/6 \end{pmatrix}$

$\therefore (AB)^{-1} = B^{-1}A^{-1}$

2) Prove that $\begin{bmatrix} 3 & -1 \\ 5 & -2 \end{bmatrix}$ and $\begin{bmatrix} 2 & -1 \\ 5 & -3 \end{bmatrix}$ are the inverse of each other.

Sol

$AB = BA = I$

now;

$AB = \begin{bmatrix} 3 & -1 \\ 5 & -2 \end{bmatrix} \times \begin{bmatrix} 2 & -1 \\ 5 & -3 \end{bmatrix}$

$= \begin{bmatrix} 6-5 & -3+3 \\ 10-10 & -5+6 \end{bmatrix}$

$= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$\therefore AB = I$

again;

$BA = \begin{bmatrix} 2 & -1 \\ 5 & -3 \end{bmatrix} \times \begin{bmatrix} 3 & -1 \\ 5 & -2 \end{bmatrix}$

$= \begin{bmatrix} 6-5 & -2+2 \\ 15-15 & -5+6 \end{bmatrix}$

$= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$\therefore BA = I$

$\therefore AB = BA = I$ proved

4) Find the adjoint matrix of $\begin{bmatrix} 1 & -1 & 0 \\ 2 & 3 & -2 \\ -2 & 0 & 1 \end{bmatrix}$

Sol:

Let $A = \begin{bmatrix} 1 & -1 & 0 \\ 2 & 3 & -2 \\ -2 & 0 & 1 \end{bmatrix}$

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Co-factor:

$C(a_{11}) = +\begin{vmatrix} 3 & -2 \\ 0 & 1 \end{vmatrix} = 3$

$C(a_{12}) = -\begin{vmatrix} 2 & -2 \\ -2 & 1 \end{vmatrix} = -|2-4| = -2$

$C(a_{13}) = +\begin{vmatrix} 2 & 3 \\ -2 & 0 \end{vmatrix} = |0+6| = 6$

$C(a_{21}) = -\begin{vmatrix} -1 & 0 \\ 0 & 1 \end{vmatrix} = -|-1+0| = 1$

$C(a_{22}) = +\begin{vmatrix} 1 & 0 \\ -2 & 1 \end{vmatrix} = 1$

$C(a_{23}) = -\begin{vmatrix} 1 & -1 \\ -2 & 0 \end{vmatrix} = -|0-2| = 2$

$C(a_{31}) = +\begin{vmatrix} -1 & 0 \\ 3 & -2 \end{vmatrix} = +|2-0| = 2$

$C(a_{32}) = -\begin{vmatrix} 1 & 0 \\ 2 & -2 \end{vmatrix} = -|-2| = 2$

$C(a_{33}) = +\begin{vmatrix} 1 & -1 \\ 2 & 3 \end{vmatrix} = 3+2 = 5$

matrices of co-factor $= \begin{pmatrix} 3 & 2 & 6 \\ 1 & 1 & 2 \\ 2 & 2 & 5 \end{pmatrix}$

Adj:(A) $= \begin{pmatrix} 3 & 1 & 2 \\ 2 & 1 & 2 \\ 6 & 2 & 5 \end{pmatrix}$

6) Find the inverse of the matrix $\begin{bmatrix} 2 & -3 & 4 \\ 1 & 0 & 1 \\ 0 & -1 & 4 \end{bmatrix}$

Sol:

Let $A = \begin{bmatrix} 2 & -3 & 4 \\ 1 & 0 & 1 \\ 0 & -1 & 4 \end{bmatrix}$

$|A| = -1\begin{vmatrix} -3 & 4 \\ -1 & 4 \end{vmatrix} + 0 - 1\begin{vmatrix} 2 & -3 \\ 0 & -1 \end{vmatrix}$

$= -1|-12+4| + 0 - 1|-2-0|$

$= -1|-8| + 0 + 2$

$= 8 + 0 + 2$

$= 10 \neq 0$

A^{-1} exist

Co-factor of matrices A

$C(a_{11}) = +\begin{vmatrix} 0 & 1 \\ -1 & 4 \end{vmatrix} = +|0+1| = 1$

$C(a_{12}) = -\begin{vmatrix} 1 & 1 \\ 0 & 4 \end{vmatrix} = -4$

$C(a_{13}) = +\begin{vmatrix} 1 & 0 \\ 0 & -1 \end{vmatrix} = -1$

$C(a_{21}) = -\begin{vmatrix} 0 & 1 \\ -1 & 4 \end{vmatrix} = -\begin{vmatrix} -3 & 4 \\ -1 & 4 \end{vmatrix} = -|-12+4| = \cancel{16}\ 8$

$C(a_{22}) = +\begin{vmatrix} 2 & 4 \\ 0 & 4 \end{vmatrix} = 8$

$C(a_{23}) = -\begin{vmatrix} 2 & -3 \\ 0 & -1 \end{vmatrix} = -|-2-0| = 2$

$C(a_{31}) = +\begin{vmatrix} -3 & 4 \\ 0 & 1 \end{vmatrix} = -3$

$C(a_{32}) = -\begin{vmatrix} 2 & 4 \\ 1 & 1 \end{vmatrix} = -|2-4| = 2$

$C(a_{33}) = +\begin{vmatrix} 2 & -3 \\ 1 & 0 \end{vmatrix} = \cancel{-3}\ 3$

Matrices of co-factor $= \begin{pmatrix} 1 & -4 & -1 \\ \cancel{16}\ 8 & 8 & 2 \\ -3 & 2 & 3 \end{pmatrix}$

$A^{-1} = \frac{1}{|A|} \text{Adj } A \quad (\text{Adj } A) = \begin{pmatrix} 1 & \cancel{16}\ 8 & -3 \\ -4 & 8 & 2 \\ -1 & 2 & 3 \end{pmatrix}$

$= \frac{1}{20}\begin{pmatrix} 1 & \cancel{16}\ 8 & -3 \\ \cancel{16}\ -4 & 8 & 2 \\ -1 & 2 & 3 \end{pmatrix}$

$= \begin{pmatrix} 1/20 & 4/5 & -3/20 \\ -2/5 & 4/5 & 1/5 \\ -1/20 & 1/5 & 3/20 \end{pmatrix}$

7) If $A = \begin{bmatrix} 1 & 2 & 5 \\ 2 & 3 & 1 \\ -1 & 1 & 1 \end{bmatrix}$ Then verify that $A^{-1}A = AA^{-1} = I$

Sol

Let $A = \begin{bmatrix} 1 & 2 & 5 \\ 2 & 3 & 1 \\ -1 & 1 & 1 \end{bmatrix}$

$|A| = 1\begin{vmatrix} 3 & 1 \\ 1 & 1 \end{vmatrix} - 2\begin{vmatrix} 2 & 1 \\ -1 & 1 \end{vmatrix} + 5\begin{vmatrix} 2 & 3 \\ -1 & 1 \end{vmatrix}$

$= 1|3-1| - 2|2+1| + 5|2+3|$

$= 2 - 6 + 25$

$= 21$

Co-factor of A

$C(a_{11}) = +\begin{vmatrix} 3 & 1 \\ 1 & 1 \end{vmatrix} = 2$

$C(a_{12}) = -\begin{vmatrix} 2 & 1 \\ -1 & 1 \end{vmatrix} = -|2+1| = -3$

$C(a_{13}) = +\begin{vmatrix} 2 & 3 \\ -1 & 1 \end{vmatrix} = 5$

$C(a_{21}) = -\begin{vmatrix} 2 & 5 \\ 1 & 1 \end{vmatrix} = -|2-5| = 3$

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$$C(a_{22}) = +\begin{vmatrix} 1 & 5 \\ -1 & 1 \end{vmatrix} = |1+5| = 6$$

$$C(a_{23}) = -\begin{vmatrix} 1 & 2 \\ -1 & 1 \end{vmatrix} = -|1-(-2)| = -3$$

$$C(a_{31}) = +\begin{vmatrix} 2 & 5 \\ 3 & 1 \end{vmatrix} = |2-15| = -13$$

$$-|1-10| = 9$$

$$C(a_{32}) = -\begin{vmatrix} 1 & 5 \\ -2 & 1 \end{vmatrix} = \cancel{-|1+5| = -6}$$

$$C(a_{33}) = +\begin{vmatrix} 1 & 2 \\ 2 & 3 \end{vmatrix} = |3-4| = -1$$

Matrices of co-factor

$$A = \begin{pmatrix} 2 & -3 & 5 \\ 3 & 6 & -3 \\ -13 & 9 & -1 \end{pmatrix}$$

$$Adj(A) = \begin{pmatrix} 2 & 3 & -13 \\ -3 & 6 & 9 \\ 5 & -3 & -1 \end{pmatrix}$$

$$A^{-1} = \frac{1}{|A|} Adj(A)$$

$$= \frac{1}{21}\begin{pmatrix} 2 & 3 & 13 \\ -3 & 6 & 9 \\ 5 & -3 & -1 \end{pmatrix}$$

$$= \begin{pmatrix} 1/11 & 3/22 & -13/22 \\ -3/22 & 3/11 & 9/22 \\ 5/22 & -3/22 & -1/22 \end{pmatrix}$$

again;

$$= A^{-1} \times A$$

or,

$$\frac{1}{21}\begin{bmatrix} 2 & 3 & -13 \\ -3 & 6 & 9 \\ 5 & -3 & -1 \end{bmatrix}\begin{bmatrix} 1 & 2 & 5 \\ 2 & 3 & 1 \\ -1 & 1 & 1 \end{bmatrix}$$

$$= \frac{1}{21}\begin{bmatrix} 2\times1+3\times2+(-13)\times(-1) & 2\times2+3\times3+(-13)\times1 & 2\times5+3\times1+(-13)\times1 \\ -3\times1+6\times2+9\times(-1) & -3\times2+6\times3+9\times1 & -3\times5+6\times1+9\times1 \\ 5\times1+(-3)\times2+(-1)\times(-1) & 5\times2+(-3)\times3+(-1)\times1 & 5\times5+(-3)\times1+(-1)\times1 \end{bmatrix}$$

$$= \frac{1}{21}\begin{bmatrix} 2+6+13 & 4+9-13 & 10+3-13 \\ -3+12-9 & -6+18+9 & -15+6+9 \\ 5-6+1 & 10-9-1 & 25-3-1 \end{bmatrix}$$

$$= \frac{1}{21}\begin{bmatrix} 21 & 0 & 0 \\ 0 & 21 & 0 \\ 0 & 0 & 21 \end{bmatrix}$$

$$\therefore A^{-1}A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

again;

$$A.A^{-1} = \begin{bmatrix} 1 & 2 & 5 \\ 2 & 3 & 1 \\ -1 & 1 & 1 \end{bmatrix}\frac{1}{21}\begin{bmatrix} 2 & 3 & -13 \\ -3 & 6 & 9 \\ 5 & -3 & -1 \end{bmatrix}$$

$$= \frac{1}{21}\begin{bmatrix} 1\times2+2\times(-3)+5\times5 & 1\times3+2\times6+5\times(-3) & 1\times(-13)+2\times9+5\times(-1) \\ 2\times2+3\times(-3)+1\times5 & 2\times3+3\times6+1\times(-3) & 2\times(-13)+3\times9+1\times(-1) \\ -1\times2+1\times(-3)+1\times5 & -1\times3+1\times6+1\times(-3) & -1\times(-13)+1\times9+1\times(-1) \end{bmatrix}$$

$$= \frac{1}{21}\begin{bmatrix} 2-6+25 & 3+12-5 & -13+18-5 \\ 4-9+5 & 6+18-3 & -26+27-1 \\ -2-3+5 & -3+6-3 & 13+9-1 \end{bmatrix}$$

$$= \frac{1}{21}\begin{bmatrix} 21 & 0 & 0 \\ 0 & 21 & 0 \\ 0 & 0 & 21 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Hence, $(\therefore A^{-1}A = AA^{-1} = I)$

proved

8) Find the rank of the following matrix.

a) $\begin{bmatrix} 3 & 4 \\ 2 & 5 \end{bmatrix}$

Sol

$$|A| = \begin{vmatrix} 3 & 4 \\ 2 & 5 \end{vmatrix} = 15 - 8 \text{ or, } 7 \neq 0$$

$\therefore$ A has rank two.

b) $\begin{bmatrix} 2 & 4 \\ 3 & 6 \end{bmatrix}$

Sol:

$$|A| = \begin{vmatrix} 2 & 4 \\ 3 & 6 \end{vmatrix} = 12 - 12 = 0$$

Here, A is 2x2 matrix and 2x2 minor is $\begin{vmatrix} 2 & 4 \\ 3 & 6 \end{vmatrix}$

$= 12 - 12 = 0$

$\therefore r(A) \neq 2$

Then, The square submatrices are [2], [4], [3], [6]

$|2| = 2 \neq 0$

$\therefore r(A) = 1$

9) Find the rank of the following matrices.

a) $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}$

Here, A is 3x3 matrix and 3x3 minor is

$$\begin{vmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{vmatrix} = 1\begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix} - 0 + 0$$

$$= 1|1-0|$$

$$\text{or, } 1 \neq 0$$

$\therefore$ The matrix A has rank 3

b) $\begin{bmatrix} 0 & 1 & 2 \\ 1 & 2 & 3 \\ 3 & 1 & 1 \end{bmatrix}$

Here, A is 3x3 matrix and 3x3 minor is

$$\begin{vmatrix} 0 & 1 & 2 \\ 1 & 2 & 3 \\ 3 & 1 & 1 \end{vmatrix}$$

$$= 0 - 1\begin{vmatrix} 1 & 3 \\ 3 & 1 \end{vmatrix} + 2\begin{vmatrix} 1 & 2 \\ 3 & 1 \end{vmatrix}$$

$$= -1|1-9| + 2|1-6|$$

$$= -1|-8| + 2|-5|$$

$$= 8 - 10 = -2$$

$\therefore$ The matrix A has rank 3

c) $\begin{bmatrix} 3 & 2 & 4 \\ 1 & -2 & 3 \\ -3 & -10 & 1 \end{bmatrix}$

Here, A is 3x3 matrix and 3x3 minor is

$$\begin{vmatrix} 3 & 2 & 4 \\ 1 & -2 & 3 \\ -3 & -10 & 1 \end{vmatrix}$$

$$= 3\begin{vmatrix} -2 & 3 \\ -10 & 1 \end{vmatrix} - 2\begin{vmatrix} 1 & 3 \\ -3 & 1 \end{vmatrix} + 4\begin{vmatrix} 1 & -2 \\ -3 & -10 \end{vmatrix}$$

$$= 3|-2+30| - 2|1+9| + 4|-10-6|$$

$$= 3|-28| - 2|10| + 4|-16|$$

$$= 84 - 20 - 64$$

$$= 84 - 84$$

$$= 0$$

$\therefore r(A) \neq 3$

Then the square sub matrices are:

$$\begin{bmatrix} 3 & 2 \\ 1 & -2 \end{bmatrix}, \begin{bmatrix} 2 & 4 \\ -2 & 3 \end{bmatrix}, \begin{bmatrix} 1 & -2 \\ -3 & -10 \end{bmatrix}, \begin{bmatrix} -2 & 3 \\ -10 & 1 \end{bmatrix}, \text{etc}$$

And the determinant of any one sub-square matrix is

$$\begin{vmatrix} 3 & 2 \\ 1 & -2 \end{vmatrix} = -6 - 2 = -8$$

$\therefore$ the matrix A has rank 2.

5) If $A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & -3 \\ 2 & -1 & 3 \end{bmatrix}$, verify that $A(\text{Adj}\,A) = (\text{Adj}\,A)A = |A|I$

Sol:

Given,

$$|A| = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & -3 \\ 2 & -1 & 3 \end{bmatrix}$$

or, $1\begin{vmatrix} 2 & -3 \\ -1 & 3 \end{vmatrix} - 1\begin{vmatrix} 1 & -3 \\ 2 & 3 \end{vmatrix} + 1\begin{vmatrix} 1 & 2 \\ 2 & -1 \end{vmatrix}$

or, $1(6-3) - 1(3+6) + 1(-1-4)$

or, $3 - 9 - 5$

or, -11

Cofactor:

$C(a_{11}) = +\begin{vmatrix} 2 & -3 \\ -1 & 3 \end{vmatrix} = |6-3| = 3$

$C(a_{12}) = -\begin{vmatrix} 1 & -3 \\ 2 & 3 \end{vmatrix} = -|3+6| = -9$

$C(a_{13}) = +\begin{vmatrix} 1 & 2 \\ 2 & -1 \end{vmatrix} = +|-1-4| = -5$

$C(a_{21}) = -\begin{vmatrix} 1 & 1 \\ -1 & 3 \end{vmatrix} = -|3+1| = 4$

$C(a_{22}) = +\begin{vmatrix} 1 & 1 \\ 2 & 3 \end{vmatrix} = 3-2 = 1$

$C(a_{23}) = -\begin{vmatrix} 1 & 1 \\ 2 & -1 \end{vmatrix} = -|-1-2| = 3$

$C(a_{31}) = +\begin{vmatrix} 1 & 1 \\ 2 & -3 \end{vmatrix} = |-3-2| = -5$

$C(a_{32}) = -\begin{vmatrix} 1 & 1 \\ 1 & -3 \end{vmatrix} = -|-3-1| = 4$

$C(a_{33}) = +\begin{vmatrix} 1 & 1 \\ 1 & 2 \end{vmatrix} = 2-1 = 1$

Matrice of cofactor $= \begin{pmatrix} 3 & -9 & -5 \\ -4 & 1 & 3 \\ -5 & 4 & 1 \end{pmatrix}$

$\text{Adj}(A) = \begin{pmatrix} 3 & -4 & -5 \\ -9 & 1 & 4 \\ -5 & 3 & 1 \end{pmatrix}$

again,

$$A.(\text{Adj}\,A) = \begin{pmatrix} 1 & 1 & 1 \\ 1 & 2 & -3 \\ 2 & -1 & 3 \end{pmatrix} \times \begin{pmatrix} 3 & -4 & -5 \\ -9 & 1 & 4 \\ -5 & 3 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} 3-9-5 & -4+1+3 & -5+4+1 \\ 3-18+15 & -4+2-9 & -5+8-3 \\ 6+9-15 & -8-1+9 & -10-4+3 \end{pmatrix}$$

or, $\begin{pmatrix} -11 & 0 & 0 \\ 0 & -11 & 0 \\ 0 & 0 & -11 \end{pmatrix}$

$$(\text{Adj } A)\cdot A = \begin{pmatrix} 3 & -4 & -5 \\ -9 & 1 & 4 \\ -5 & 3 & 1 \end{pmatrix} \times \begin{pmatrix} 1 & 1 & 1 \\ 1 & 2 & -3 \\ 2 & -1 & 3 \end{pmatrix}$$

$$= \begin{pmatrix} 3-4-10 & 3-8+5 & 3+12-15 \\ -9+1+18 & -9+2-4 & -9-3+12 \\ -5+3+2 & -5+6-1 & -5-9+3 \end{pmatrix}$$

or, $\begin{pmatrix} -11 & 0 & 0 \\ 0 & -11 & 0 \\ 0 & 0 & -11 \end{pmatrix}$

again,

$= |A| I$

or, $-11 \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$

or, $\begin{pmatrix} -11 & 0 & 0 \\ 0 & -11 & 0 \\ 0 & 0 & -11 \end{pmatrix}$

$\therefore A\cdot(\text{Adj } A) = (\text{Adj } A)\cdot A = |A| I$

I.m.p

A square matrix A is said to be orthogonal matrix if $[A^T A = AA^T = I$

i) Show that $\begin{pmatrix} 1/\sqrt{2} & 1/\sqrt{2} \\ 1/\sqrt{2} & -1/\sqrt{2} \end{pmatrix}$ is an orthogonal matrix

Sol:

$A^T = \begin{pmatrix} 1/\sqrt{2} & 1/\sqrt{2} \\ 1/\sqrt{2} & -1/\sqrt{2} \end{pmatrix}$

$= A^T A = \begin{pmatrix} 1/\sqrt{2} & 1/\sqrt{2} \\ 1/\sqrt{2} & -1/\sqrt{2} \end{pmatrix} \times \begin{pmatrix} 1/\sqrt{2} & 1/\sqrt{2} \\ 1/\sqrt{2} & -1/\sqrt{2} \end{pmatrix}$

or, $\begin{pmatrix} (1/\sqrt{2} \times 1/\sqrt{2}) + (1/\sqrt{2} \times 1/\sqrt{2}) & (1/\sqrt{2} \times 1/\sqrt{2}) + (1/\sqrt{2} \times -1/\sqrt{2}) \\ (1/\sqrt{2} \times 1/\sqrt{2}) + (-1/\sqrt{2} + 1/\sqrt{2}) & (1/\sqrt{2} \times 1/\sqrt{2}) + (-1/\sqrt{2} \times -1/\sqrt{2}) \end{pmatrix}$

or, $\begin{pmatrix} 1/2 + 1/2 & 1/2 - 1/2 \\ 1/2 - 1/2 & 1/2 + 1/2 \end{pmatrix}$

or, $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

$\therefore A^T A = I$ (Orthogonal matrix)

Transformation

A transformation (or mapping) T is linear if:

i) $T(u+v) = T(u) + T(v)$ for all u, v in the domain of T.

ii) $T(kU) = kT(U)$ where k is any scalar.

For all u, v in domain and scalar k such that:

$$T(u+v) = T(U) + T(V)$$
$$T(kU) = kT(U)$$

* Standard Matrices

$$e_1 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \quad e_2 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

$$e_1 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \quad e_2 = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}, \quad e_3 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

LET $T: \mathbb{R}^n \to \mathbb{R}^m$ be a linear transformation then there exists a unique matrix A such that $T(x) = Ax$ for all $x \in \mathbb{R}^n$ where $A = [T(e_1) ___ T(e_n)]$

T.U

A square matrix A is said to be orthogonal matrix if $[A^T A = AA^T = I]$

i) Show that $\begin{pmatrix} 1/\sqrt{2} & 1/\sqrt{2} \\ 1/\sqrt{2} & -1/\sqrt{2} \end{pmatrix}$ is an orthogonal matrix.

Sol:

$$A^T = \begin{pmatrix} 1/\sqrt{2} & 1/\sqrt{2} \\ 1/\sqrt{2} & -1/\sqrt{2} \end{pmatrix}$$

$$A^T A = \begin{pmatrix} 1/\sqrt{2} & 1/\sqrt{2} \\ 1/\sqrt{2} & -1/\sqrt{2} \end{pmatrix} \times \begin{pmatrix} 1/\sqrt{2} & 1/\sqrt{2} \\ 1/\sqrt{2} & -1/\sqrt{2} \end{pmatrix}$$

or, $\begin{pmatrix} (1/\sqrt{2} \times 1/\sqrt{2}) + (1/\sqrt{2} \times 1/\sqrt{2}) & ((1/\sqrt{2} \times 1/\sqrt{2}) + (1/\sqrt{2} \times -1/\sqrt{2})) \\ (1/\sqrt{2} \times 1/\sqrt{2}) + (-1/\sqrt{2} \times 1/\sqrt{2}) & ((1/\sqrt{2} \times 1/\sqrt{2}) + (-1/\sqrt{2} \times -1/\sqrt{2})) \end{pmatrix}$

or, $\begin{pmatrix} 1/2 + 1/2 & 1/2 - 1/2 \\ 1/2 - 1/2 & 1/2 + 1/2 \end{pmatrix}$

or, $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = I$

Exercise 4.4

1) Let $T: \mathbb{R}^2 \to \mathbb{R}^2$ be a linear transformation define by $T(x) = Ax$, where $A = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$. find the image under T of $u = \begin{bmatrix} 4 \\ 1 \end{bmatrix}, v = \begin{bmatrix} 2 \\ 3 \end{bmatrix}$ and $u+v = \begin{bmatrix} 6 \\ 4 \end{bmatrix}$

Sol:

Given $A = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$

let $T: \mathbb{R}^2 \to \mathbb{R}^2$ be a linear transformation define by $T(x) = Ax$

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$\therefore T(u) = Au$

$= \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} 4 \\ 1 \end{bmatrix}$

$= \begin{bmatrix} 0-1 \\ 4+0 \end{bmatrix} = \begin{bmatrix} -1 \\ 4 \end{bmatrix}$

$\therefore T(u) = Au$

$= \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} 2 \\ 3 \end{bmatrix} = \begin{bmatrix} 0-3 \\ 2+0 \end{bmatrix} = \begin{bmatrix} -3 \\ 2 \end{bmatrix}$ ans

$\therefore T(u+v) = A(u+v)$

$= \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} 6 \\ 4 \end{bmatrix}$

$= \begin{bmatrix} 0-4 \\ 6+0 \end{bmatrix} = \begin{bmatrix} -4 \\ 6 \end{bmatrix}$ ans

(agle page maa ka 5 ko)

5) Find the standard matrix A for linear transformation $T(x) = 3x$ of x in $\mathbb{R}^3$.

Sol:

Let $T(x) = 3x$ in $\mathbb{R}^3$. Then,

$e_1 = (1,0,0)$, $e_2 = (0,1,0)$, $e_3 = (0,0,1)$

$\therefore T(e_1) = 3(1,0,0) = 3(1,0,0) = (3,0,0)$

$\therefore T(e_2) = T(0,1,0) = 3(0,1,0) = (0,3,0)$

$T(e_3) = T(0,0,1) = 3(0,0,1) = (0,0,3)$

Now, the standard matrix A for $T(x) = 3x$ is

$A = [T(e_1), T(e_2), T(e_3)]$

$= \begin{bmatrix} 3 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 3 \end{bmatrix}$

2) Let $T: \mathbb{R}^2 \to \mathbb{R}^2$ defined by $T(u) = 3u$. Then show that T is linear.

Sol: ~~Tu+v~~

i) $T(u+v) = 3(u+v)$

$= 3u + 3v$

$= T(u) + T(v)$

$\therefore T(u+v) = T(u) + T(v)$

again;

ii) For scalar α

$T(\alpha u) = 3\alpha u = \alpha \cdot (3u) = \alpha (Tu)$

3) Show that the transformation T defined by $T(x_1, x_2) = (2x_1 - 3x_2, x_1 + 4, 5x_2)$ is not linear.

ans Sol:

put $x_1 = 0$, $x_2 = 0$

$T(0,0) = (2\times0 - 3\times0, 0+4, 5\times0)$

$T(0,0) = (0,4,0)$

$T(0) \neq 0$

$\therefore$ T is not linear

Note:

$T(0) = 0$ linear

$T(0) \neq 0$ non "

Note:

i) $R_2 = e_1 = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$, $e_2 = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$

ii) $R_3 = e_1 = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$, $e_2 = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$, $e_3 \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$

I.m.P 5) Find the standard matrix A for linear transformation $T(x) = 3x$ of x in $\mathbb{R}^3$.

Sol:

Let the std matrix $A = [T(e_1), T(e_2)\ T(e_3)]$

again;

$T(x) = 3x$

$T(e_1) = 3\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} 3 \\ 0 \\ 0 \end{pmatrix}$

$T(e_2) = 3e_2 = 3\begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ 3 \\ 0 \end{pmatrix}$

$T(e_3) = 3e_3 = 3\begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 3 \end{pmatrix}$

$\therefore$ Required matrix

$$A = \begin{pmatrix} 3 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 3 \end{pmatrix}$$

I.m.P

6) Show that the matrix $\begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$ is orthogonal.

Sol:

For orthogonal (we show $AA^T = A^TA = I$)

$$AA^T = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}$$

$$\text{or, } \begin{pmatrix} \cos^2\theta + \sin^2\theta & \cos\theta\sin\theta - \cos\theta\sin\theta \\ \sin\theta\cos\theta - \sin\theta\cos\theta & \sin^2\theta + \cos^2\theta \end{pmatrix}$$

$$\text{or, } \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = I$$

again;

$$A^TA = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}$$

$$\text{or, } \begin{pmatrix} \cos^2\theta + \sin^2\theta & -\cos\theta\sin\theta + \cos\theta\sin\theta \\ -\cos\theta\sin\theta + \cos\theta\sin\theta & \sin^2\theta + \cos^2\theta \end{pmatrix}$$

$$\text{or, } \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = I$$

From the given we get

$AA^T = A^TA = I$ so the given matrix is orthogonal.

The number of permutation of a set of "n" object taken "r" at a time. n = total number of possibility or event
"r" means taken at a time

Chapter 6 Permutation and Combination

Permutation (Arrangement)

The process of arranging the object denoted or things is called permutation. It is denoted by $P(n,r)$ or ${}^nP_r$

Case I : If the objects are different (no-repeate)

Total no. of ways $P(n,r) = \frac{n!}{(n-r)!}$

Case II : If the objects are not different (Repeated)

Total no of ways $= \frac{n!}{n_1! \times n_2! \times n_3! \text{ ----}}$

Case III : Circular Case

Total no of ways $= (n-1)!$

Case IV : Bracelet, Necklace, Garland

Total no. of ways $= \frac{(n-1)!}{2}$

Ex 6.2

1) Evaluate

a) ${}^{12}P_4$

Here,

$n = 12, r = 4$

${}^nP_r / P(n,r) = \frac{n!}{(n-r)!}$

$= \frac{12!}{(12-4)!}$

$= \frac{12!}{8!}$

$= \frac{\cancel{8!} \times 9 \times 10 \times 11 \times 12}{\cancel{8!}}$

$= 11880$ ans

b) ${}^8P_5$

Here;

$n = 8, r = 5$

${}^nP_r / P(n,r) = \frac{n!}{(n-r)!} = \frac{8!}{(8-5)!} = \frac{8!}{3!}$

$= \frac{\cancel{3!} \times 4 \times 5 \times 6 \times 7 \times 8}{\cancel{3!}}$

$= 6720$ ans

c) $P(6,4)$

Sol:

$n=6,\ r=4$

$P(n,r) = \frac{n!}{(n-r)!} = \frac{6!}{(6-4)!} = \frac{2! \times 3 \times 4 \times 5 \times 6}{2!} = 360$

2) Find the value of n, if

b) ${}^nP_6 = 42 \times {}^nP_4$

Given,

${}^nP_6 = 42 \times {}^nP_4$

$\Rightarrow \frac{n!}{(n-6)!} = 42 \times \frac{n!}{(n-4)!}$

$\Rightarrow \frac{1}{(n-6)!} = \frac{42}{(n-4)!}$

or, $42(n-6)! = (n-4)!$

$\Rightarrow \frac{42 \times (n-6)!}{(n-4)!} = 1$

$\Rightarrow \frac{42 \times \cancel{(n-6)!}}{\cancel{(n-6)!}(n-5)(n-4)!} = 1$

or, $(n-5)(n-4) = 42$

or, $n^2 - 4n - 5n + 20 = 42$

or, $n^2 - 9n + 20 - 42 = 0$

or, $n^2 - 9n - 22 = 0$

or, $n^2 - (11-2)n - 22 = 0$

or, $n^2 - 11n + 2n - 22 = 0$

or, $n(n-11) + 2(n-11) = 0$

or, $(n+2)(n-11) = 0$

$\therefore n = -2, 11$

$\therefore n = 11$ ans

c) ${}^{n-1}P_3 : {}^{n+1}P_3 = 5 : 12$

Given,

$= {}^{n-1}P_3 : {}^{n+1}P_3 = 5 : 12$

or, $\frac{(n-1)!}{(n-1-3)!} = \frac{5}{12}$
$\frac{(n+1)!}{(n+1-3)!}$

or, $\frac{(n-1)!}{(n-4)!} = \frac{5}{12}$
$\frac{(n+1)!}{(n-2)!}$

or, $\frac{(n-1)!}{(n-4)!} \times \frac{(n-2)!}{(n+1)!} = \frac{5}{12}$

or, $\frac{(n-1)! \times (n-3)!(n-3)!(n-2)!}{(n-4)! \times (n-1)!\ n!(n+1)!} = \frac{5}{12}$

or, $\frac{(n-3)(n-2)}{n(n+1)} = \frac{5}{12}$

or, $\frac{n^2 - 2n - 3n + 6}{n^2 + n} = \frac{5}{12}$

or, $12n^2 - 60n + 72 = 5n^2 + n$

or, $12n^2 - 60n + 72 - 5n^2 - 5n = 0$

or, $9n^2 - 65n + 72 = 0$

or, $9n^2 - (56 + 9)n + 72 = 0$

or, $9n^2 - 56n - 9n + 72 = 0$

or, $9n(n-8) - 9(n-8) = 0$

$= (9n-9)(n-8) = 0$

$\therefore n = \frac{9}{9}$ or, 8

$\therefore n = 8$ ans

Imp 4) There are 8 vacant seat in a room. In how many ways 5 persons can take the their seats.

ans Given;

$n = 8$

$r = 5$

Total no. of ways $= \frac{n!}{(n-r)!}$

$= \frac{8!}{(8-5)!}$

$= \frac{8!}{3!}$

$= \frac{3! \times 4 \times 5 \times 6 \times 7 \times 8}{3!}$

$= 6720$ ways.

Note or = +

and = x

5) There are 4 buses and two trains from city A to B. In how many ways can a person travel from A to B.

ans Given;

Bus $= {}^4P_1$

train $= {}^2P_1$

Total no. of ways $= {}^4P_1$ or ${}^2P_1$

$= \frac{4!}{(4-1)!} + \frac{2!}{(2-1)!}$

or, $\frac{3! \times 4}{3!} + \frac{1! \times 2}{1!}$

or, $4 + 2$

or, 6 ways

6) There are 4 gates in a college. In how many ways can a student enter into the college and exists from different gates.

ans Given;

Enter gate $= {}^4P_1$

Different exit gate $= {}^3P_1$

again;

Total no. of ways $= {}^4P_1 \times {}^3P_1$

$= \frac{4!}{(4-1)!} \times \frac{3!}{(3-1)}$

or, $\frac{3! \times 4}{3!} \times \frac{2! \times 3}{2!}$

or, 4×3

or, 12 ways

Imp

7) In how many ways can the letters of the following words be arranged?

a) Pencil

Given;

Pencil

$n = 6$

$r = 6$ (∴ If r is not given in non-repeated word or number we can use the total number)

again;

Total no. of ways $= \frac{n!}{(n-r)!} = \frac{6!}{(6-6)!} = \frac{6!}{0!}$

$= 720$

Another way;

$n = 6$ (non-repeated)

Total no. of ways $= n! = 6! = 720$

b) Careless

Sol:

$n = 8$

$n_1 = 2$

$n_2 = 2$

now;

Total no. of ways $= \frac{n!}{n_1! \times n_2!}$

$= \frac{8!}{2! \times 2!} = \frac{2 \times 3 \times 4 \times 5 \times 6 \times 7 \times 8}{2 \times 1 \times 2}$

$= \frac{20160}{2} = 10080$

Imp

c) Mississippi

Sol:

$n = 11$, $n_1 = 4$ (I's), $n_2 = 4$ (S's), $n_3 = 2$ (p's)

Total no. of ways $= \frac{n!}{n_1! \times n_2! \times n_3!}$ or, $\frac{11!}{4! \times 4! \times 2!}$

or, $\frac{4! \times 5 \times 6 \times 7 \times 8 \times 9 \times 10 \times 11}{4! \times 1 \times 2 \times 3 \times 4 \times 1 \times 2}$ or, 34650 ways

d) Examination

Sol:

$n = 11$ $n_2 = 2$

$n_1 = 2$ (a's) $n_3 = 2$

again,

Total number of ways $= \frac{n!}{n_1! \times n_2! \times n_3!}$

$= \frac{11!}{2! \times 2! \times 2!}$

$= \frac{39916800}{8}$

$= 4989600$ ways

e) Management

Sol:

$n = 10$, $n_1 = 2$, $n_2 = 2$, $n_3 = 2$, $n_4 = 2$

again;

Total no. of ways $= \frac{n!}{n_1! \times n_2! \times n_3! \times n_4!}$

$= \frac{10!}{2! \times 2! \times 2! \times 2!}$

$= \frac{2! \times 3 \times 4 \times 5 \times 6 \times 7 \times 8 \times 9 \times 10}{2! \times 1 \times 2 \times 1 \times 2 \times 1 \times 2} = \frac{1814400}{8} = 226800$

f) Calculus

sol:

$n = 8$, $n_1 = 2$, $n_2 = 2$, $n_3 = 2$

again

Total number of ways $= \frac{n!}{n_1! \times n_2! \times n_3!}$

$= \frac{8!}{2! \times 2! \times 2!}$

or, $\frac{\cancel{2} \times 3 \times \cancel{4} \times 5 \times \cancel{6} \times 7 \times 8}{\cancel{2!} \times 1 \times \cancel{2} \times 1 \times \cancel{2}}$

$= 5040$ ways.

g) Prism

sol:

$n = 5!$

Total number of ways $= 5! = 120$ ways.

8) a) How many different number can be formed from the digits 1, 2, 2, 4, 5!

sol:

$n = 5$

$n_1 = 2$

sol:

Total no. of ways $= \cancel{\frac{n!}{r!(n-r)!}}$ $\frac{n!}{n_1!}$

$= \frac{5!}{2!}$ or, $\frac{\cancel{2!} \times 3 \times 4 \times 5}{\cancel{2!}}$

$= 60$ ways.

b) ${}^nP_2 = 20$

now;

$n = n$ $x = 2$

$= \frac{n!}{(n-x)!} = 20$

or, $\frac{\cancel{(n-2)!}(n-1)\,n}{\cancel{(n-2)!}} = 20$

or, $n^2 - n - 20 = 0$

~~or, $n^2 - 5n + 4n$~~

or, $n^2 - (5-4)n - 20 = 0$

or, $n^2 - 5n - 4n - 20 = 0$

or, $n(n-5) + 4(n-5) = 0$

or, $(n+4)(n-5) = 0$

or, $n = -4, 5$

again;

$\therefore n = 5$

3) Find x if

a) ${}^5P_x : {}^4P_r = 5 : 2$

sol:

$= \frac{\frac{5!}{(5-x)!}}{\frac{4!}{(4-r)!}} = \frac{5}{2}$

or, $\frac{5!}{(5-x)!} \times \frac{(4-r)!}{4!} = \frac{5}{2}$

or, $\frac{\cancel{4!} \times 5}{\cancel{(4-r)!} \times (5-r)} \times \frac{\cancel{(4-r)!}}{\cancel{4!}} = \frac{5}{2}$

or, $\frac{5}{5-r} = \frac{5}{2}$

or, $10 = 5(5-r)!$

or, $10 = 25 - 5r$

or, $5r = 25 - 10$

or, $r = \frac{25}{5}$

or, $r = 3$ ans

b) ${}^{9}P_{r} = 20 \cdot {}^{9}P_{r-2}$

sol

$= \frac{9!}{(9-r)!} = 20 \times \frac{9!}{(9-(r-2))!}$

or, $\frac{9!}{(9-r)!} = 20 \times \frac{9!}{(9-r)!}$

or, $\frac{\cancel{9!}}{\cancel{(9-r)!}} = 20 \times \frac{\cancel{9!}}{\cancel{(9-r)}(8-r)(9-r)}$

or, $(8-r)(9-r) = 20$

or, $72 - 8r - 9r + r^2 = 20$

or, $r^2 - 17r + 72 - 20 = 0$

or, $r^2 - (13+4)r + 52 = 0$

or, $r^2 - 13r - 4r + 52 = 0$

or, $r(r-13) - 4(r-13) = 0$

or, $(r-4)(r-13) = 0$

now;

$r - 4 = 0$ / $r - 13 = 0$

$r = 4$ / $r = 13$

$\therefore r = 4$ ($r = 13$ is not possible because $n \geq r$)

c) $4 \cdot {}^{15}P_{r} = 3 \cdot {}^{16}P_{r}$

sol:

$= \frac{4 \times 15!}{(15-r)!} = 3 \times \frac{16!}{(16-r)!}$

or, $\frac{\cancel{4} \times \cancel{15!}}{\cancel{(15-r)!}} = 3 \times \frac{\cancel{15!} \times \cancel{16}\,4}{\cancel{(15-r)}(16-r)}$

or, $1 = \frac{12}{16-r}$

$= 16 - r = 12$

or, $16 - 12 = r$

$\therefore r = 4$ ans

3.b) How many number greater than 4000 can be formed with the digits 0, 1, 2, 5?

sol:

To make greater number than 4000 we have 5(1)as a fix number and other 3 number

Total number of ways $= 1 \times 3!$

$= 1 \times 1 \times 2 \times 3$

$= 6$ ways

c) How many numbers between 1000 and 2000 can be formed with the digits 1, 2, 3, 4, 5

sol:

Fixed digit 1

$n = 5$

$r = 3$ ($\therefore$ In 2000 there are last 3 0 so that $r = 3$)

now;

Total no. of ways $= 1 \times {}^{5}P_{3}$

$= 1 \times \frac{5!}{(5-3)!}$

$= 1 \times \frac{\cancel{2 \times 3} \times 4 \times 5}{\cancel{2!}}$

$= 60$ ways

d) If the numbers formed by using all the figures 1, 2, 3, 4, 5 once, how many are odd?

Sol:

Since 3 digits are odd (1, 3, 5) which is arrange by ${}^{3}P_{1} = \frac{3!}{(3-1)!} = \frac{\cancel{2} \times 3}{\cancel{2!}} = 3$

For other places we can fill up by 4! ways
$= 1 \times 2 \times 3 \times 4 = 24$ ways.

$\therefore$ Total no. of ways $= 3 \times 24$
$= 72$ ways

e) How many 10 digits mobile numbers can be formed with the digits 0, 1, 2 ... 9 if each number starts with 9841 and no digits is repeated

Sol:

Fixed 9841 = 1
$n = 6$, $r = 6$ (we have to substracted 4 digit from 10)

now;

$$1 \times \frac{6!}{(6-6!)} = 1 \times 6! = 720 \text{ ways}$$

9) a) In how many ways can 7 students be seated in a row if two students insist on sitting next to each other.

Sol:

The 2 students who seated in same ~~sit~~ seat can interchange there place 2! and other interchange the place = 6!

Total number of ways $= 2! \times 6!$
$= 1 \times 2 \times 720$
$= 1440$ ways.

Note:

Next to each other vanepaxi set of number bata 1 less hunxa.

b) A person invites 7 of his relatives for a party. In how many ways can he make this seating arrangements of the guests are to be seated.

i) in a row ii) in a round table

Sol:

i) $n = 7$

Total number of ways $= 7! = 5040$ ways

ii) Total number of ways $= (n-1)! = (7-1)! = 720$ ways

c) In how many ways can 5 BCA and 5 BBA students be arrange alternatively?

i) in a row ii) in a round table

Sol:

Total number of way(BCA) $= 5! = 120$
Total number of way (BBA) $= 5! = 120$
Alternativerly for two subject means first BBA, BCA and BCA, BBA so. total number of way $= 2! = 2$

Now;

i) Total number of ways $= 2! \times 5! \times 5!$
$= 2 \times 120 \times 120$
$= 28800$ ways

again;

ii) Total number of ways $= 2! (n-1)! \times (n-1)!$
$= 2! \times (5-1)! \times (5-1)!$
$= 2! \times 4! \times 4!$
$= 1152$

10) How many number of six digits, all different, can be formed from the integers 2,3,4,5,6 and 7? How many of these are divisible by 5? How many are not divisible by 5.

Sol:

$n = 6$

Total number of ways $= 6!$

$= 720$ ways

now:

For divisible of 5 = 5 is fixed place (1)

$n = 5$ divisible by 5.

Total number ~~of ways~~ $= 1 \times 5!$

$= 120$ ways

again;

Total no. which are not divisible by 5 is

= Total number − Divisible by 5

$= 720 - 120$

$= 600$

11) In how many ways letters of the word "PRECARIOUS" can be arranged so that all the vowels are always together?

Sol:

Vowel letters be a single letter (A,e,i,o,u)

~~To~~ $n = 5$

Total number of ways $= 5!$

$= 120$

again;

Total 6 letter by adding vowel letter with (P,R,C,R,S)

$n = 6$

$x_1 = 2$ (R's)

Total numbers of ways $= \frac{n!}{x_1!} = \frac{6!}{2!} = \frac{2! \times 3 \times 4 \times 5 \times 6}{2!}$

$= 360$ ways.

Now;

Total arrangement $= 120 \times 360$

$= 43200$ ways.

12) In how many ways can letters of the word "MONDAY" be arranged? How many of these arrangements don't begin with M? How many begin with M and don't end with Y?

Sol:

i) $n = 6$

$x = 6$

Total numbers of way $= \frac{n!}{(n-x)!} = \frac{6!}{(6-6)!} = 6! = 720$ ways.

ii) ~~Firs~~ let find the number of arrangement begin with M. M is fixed (1)

$= 1 \times 5! = 120$

Not begin with M.

= Total number of arrangement − Begin with M.

$= 720 - 120$

$= 600$ ways.

again;

iii) If we fixed M at begining and y at end

$P(4,4) = 4! = 24$

∴ Begin with m and don't end with y

$= 120 - 24 = 96$.

13) How many permutation of 9 object taken any 3 at a time be made so that.

i) One particular object never occur

sol:

For One object be never occur we have to remove that ~~Atum~~ object $= 9-1$

$n=8,\ r=3$

Total number of ~~r~~ ways $= {}^8P_3 = \frac{8!}{(8-3)!}$

$= \frac{\cancel{5!}\times6\times7\times8}{\cancel{5!}} = 336$

ii) One particular object always occur?

sol:

Total number of way $= {}^9P_3 = \frac{9!}{(9-3)!}$

$= \frac{\cancel{6!}\times7\times8\times9}{\cancel{6!}} = 504$

again;

~~one~~ Total number of ways particular objects always occur = Total no of way − never occur

$= 504-336$

$= 168$

14) Find the number of ways in which 4 boys and 3 girls be arranged in a circle.

sol:

boys = 4, girls = 3, Total $(n) = 4+3 = 7$

now;

Total number of ways $= (n-1)! = (7-1)!$

$= 6! = 720$ ways.

Note: (Clockwise and anticlockwise arrangement are same in circle)

b) In how many ways a necklace be made by using 5 different colored circle;

sol:

$n=5$

Total no. of ways $= \frac{(n-1)!}{2} = \frac{(5-1)!}{2} = \frac{4!}{2}$

$= \frac{\cancel{2!}\times3\times4}{\cancel{2!}} = 12$ ways

c) A boy has 5 pockets. In how many way can he put 3 marbles in his pocket?

sol:

Total no. of ways $= {}^5P_1 \times {}^5P_1 \times {}^5P_1$ (respectively for 1st, 2nd, 3rd marbles)

$= 5\times5\times5$

$= 120$ ways

15) In how many different ways can letter of the of the word "ARTICLE" be arranged so that these arrangement.

a) Begin with A

sol:

Fixed letter $A = 1$

Total no. of ways $= 1\times6! = 720$ ways

b) begin with A and do not end with E?

sol:

We take A and E for begin A and ~~do~~ end with E

$= 1!\times5!$

$= 120$

again:

Total no. of ways = begin with A − begin with A and end with e

$= 720 - 120$

$= 600$ ways

iii) The three vowel always together

sol:

vowels AIE (3 vowels to be single letter)

There are 5 letter (A, I, E), R, T, C, L altogether

now;

Total no. of ways $= \underbrace{3!}_{\text{(vowel)}} \times \underbrace{5!}_{\text{letter altogether}}$

$= 1\times2\times3 \times 1\times2\times3\times4\times5$

$= 720$ ways.

Combination

Properties

i) ${}^nC_r = {}^nC_{n-r}$

ii) ${}^nC_r + {}^nC_{r-1} = {}^{n+1}C_r$

iii) If ${}^nC_r = {}^nC_{r'}$ then either $r = r'$ or $r + r' = n$

[∴ If n is same and r is different, for different 'r' we use r'.

iv) ${}^nC_0 = {}^nC_n = 1$

v) ${}^nC_r = \dfrac{n!}{(n-r)!\,r!} = \dfrac{{}^nP_r}{r!}$

The process of selecting objects or things is called combination.

Exercise 6.3

1) Find the value

a) ${}^8C_5$

sol:

$n = 8,\ r = 5$

$C(n,r) = \dfrac{n!}{(n-r)!\times r!} = \dfrac{8!}{(8-5)!\times5!} = \dfrac{\cancel{5!}\times6\times7\times8}{1\times2\times3\times\cancel{5!}} = 56.$

b) ~~${}^5C_2$~~ ${}^8C_3 \times {}^5C_3$

sol:

$n = 8,\ r = 3$ and $n = 5,\ r = 3$

$C(n,r) = \dfrac{n!}{(n-r)!\times r!} \times \dfrac{n!}{(n-r)!\times r!}$

$= \dfrac{8!}{(8-3)!\times3!} \times \dfrac{5!}{(5-3)!\times3!}$

or, $\dfrac{\cancel{5!}\times6\times7\times8}{\cancel{5!}\times1\times2\times3} \times \dfrac{\cancel{3!}\times\cancel{4}^{2}\times5}{2\times\cancel{3!}}$

$= 56\times10$

$= 560$

2)a) If ${}^{16}C_r = {}^{16}C_{r+2}$, find ${}^rC_2$

sol:

$r + r + 2 = n$ (using of properties iii)

$2r + 2 = 16$

$2r = 16 - 2$

$r = \dfrac{14}{2} = 7$

now;

${}^rC_2 = {}^7C_2$

$$\frac{7!}{(7-2)! \times 2!} = \frac{\cancel{5!} \times \overset{3}{\cancel{6}} \times 7}{\cancel{5!} \times \cancel{2}} = 21 \text{ ans}$$

b) If ${}^{x}C_{x-2} = 28$, find x

Sol:

$n = x$

$r = x-2$

now;

$$C(n,r) \text{ if } {}^{n}C_{r} = \frac{n!}{(n-r)! \times r!}$$

$$= \frac{x!}{(x-(x-2))! \times (x-2)!} = 28$$

$$\text{or, } \frac{x!}{(x-x+2)! \times (x-2)!} = 28$$

$$\text{or, } \frac{x!}{2 \times (x-2)!} = 28$$

$$\text{or, } \frac{\cancel{(x-2)}(x-1)(x)}{2 \times \cancel{(x-2)}} = 28$$

$$\text{or, } \frac{x^2 - x}{2} = 28$$

or, $x^2 - x = 56$

or, $x^2 - x - 56 = 0$

or, $x^2 - (8-7)x - 56 = 0$

or, $x^2 - 8x + 7x - 56 = 0$

or, $x(x-8) + 7(x-8) = 0$

or, $(x+7)(x-8) = 0$

either	or
$x+7=0$	$x-8=0$
$x=-7$	$x=8$
Rejected	✓

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() If ${}^7P_r = 42$ and ${}^7C_r = 21$, find the value of r

Sol:

$${}^7P_r = 42 \qquad \Big/ \qquad {}^7C_r = 21$$

$$\Rightarrow \frac{7!}{(7-r)!} = 42 \quad \text{—①} \qquad \Big/ \qquad = \frac{7!}{(7-r)! \times r!} = 21$$

Apply ①

$$= \frac{7!}{(7-r)! \times r!} = 21$$

$$\text{or, } \frac{42}{r!} = 21$$

$$\text{or, } 42 = 21 r!$$

$$\text{or, } r! = 42/21$$

$$\text{or, } r! = 2$$

$$\text{or, } r! = 1 \times 2$$

$$\text{or, } r! = 2!$$

$$\therefore r = 2 \text{ ans}$$

3. a) If ${}^{2n}C_3 : {}^nC_2 = 12$, find n.

Sol:

$$= \frac{\frac{2n!}{(2n-3)! \times 3!}}{\frac{n!}{(n-2)! \times 2!}} = 12$$

$$\text{or, } \frac{2n!}{(2n-3)! \times 3!} \times \frac{(n-2)! \times 2!}{n!} = 12$$

$$\text{or, } \frac{\cancel{(2n-3)}(2n-2)(2n-1)\cancel{2n}}{\cancel{(2n-3)!} \times 1 \times \cancel{2} \times 3} \times \frac{\cancel{(n-2)} \times 1 \times 2}{\cancel{(n-2)}(n-1)(n)} = 12$$

$$\text{or, } \frac{(2n-2)(2n-1)n}{3} \times \frac{2}{(n-1)(n)} = 12$$

$= \frac{2(n-1)(2n-1)\times n}{3} \times \frac{2}{(n-1)\times n} = 12$

or, $\frac{4n-2}{3} \cdot 4(2n-1) = 36$

or, $2n-1 = \frac{36}{4}$

or, $2n-1 = 9$

or, $2n = 10$

or, $n = \frac{10}{2} = 5$ ans

b) If ${}^{n+2}C_4 = 6 \cdot {}^{n}C_2$, find the value of n

sol.

$= \frac{(n+2)!}{(n+2-4)! \times 4!} = 6 \cdot \frac{n!}{(n-2)! \times 2!}$

or, $\frac{(n+2)!}{(n-2)! \times 3 \times 4} = 6 \cdot \frac{n!}{(n-2)! \times 1 \times 2}$

or, $\frac{(n+2)(n+1)(n)}{12} = 6 \cdot n$

or, $n^2 + n + 2n + 2 = 72$

or, $n^2 + 3n + 2 - 72 = 0$

or, $n^2 + 10n - 7n - 70 = 0$

or, $n(n+10) - 7(n+10) = 0$

or, $(n-7)(n+10) = 0$

either	then
$n-7=0$	$n+10=0$
$n=7$	$n=-10$
	Rejected

$\therefore n = 7$

c) If ${}^{6}C_r + {}^{6}C_{r-1} = {}^{7}C_{2r-1}$, estimate the value of r.

Given;

${}^{6}C_r + {}^{6}C_{r-1} = {}^{7}C_{2r-1}$

or, ${}^{6+1}C_r = {}^{7}C_{2r-1}$ (properties ii)

or, ${}^{7}C_r = {}^{7}C_{2r-1}$ (properties iii)

$\therefore r = r'$

or, $r = 2r-1$

or, $1 = 2r - r$

or $1 = r$

$\therefore r = 1$ ans

I.m.e

4) In how many ways can a committe of 6 member be formed out of 5 students and 5 teachers

sol:

$n = 5+5 = 10$, $r = 6$

We know; that

$(C_{n,r}) = \frac{n!}{(n-r)! \times r!}$ or, $\frac{10!}{(10-6)! \times 6!}$

or, $\frac{6! \times 7 \times 8 \times 9 \times 10}{1 \times 2 \times 3 \times 4 \times 6!} = 210$ ways

5) ~~5)~~ A bag contains 10 red balls and 6 green balls. In how many ways can 4 red balls and 3 green balls be drawn

Given;

Out of 10 red ball 4 can be select by ${}^{10}C_4$ ways
Out of 6 green ball 3 can be selected by ${}^{6}C_3$ ways

$\therefore$ Total no. of ways $= {}^{10}C_4 \times {}^{6}C_3$

$$= \frac{10!}{(10-4)! \times 4!} \times \frac{6!}{(6-3)! \times 3!}$$

$$= \frac{6! \times 7 \times 8 \times 9 \times 10}{6! \times 1 \times 2 \times 3 \times 4} \times \frac{3! \times 4 \times 5 \times 6}{3! \times 1 \times 2 \times 3}$$

$= 210 \times 20$

$= 4200$ ways.

6) In an examination a student has to pass in each of the 6 papers. In how many different way the student can fail.

Sol:

Out of 5 papers, in exam the student can be failed by $= {}^{5}C_1$ or ${}^{5}C_2$ or ${}^{5}C_3$ or ${}^{5}C_4$ or ${}^{5}C_5$.

$$= \frac{5!}{(5-1)! \times 1!} + \frac{5!}{(5-2)! \times 2!} + \frac{5!}{(5-3)! \times 3!} + \frac{5!}{(5-4)! \times 4!} + \frac{5!}{(5-5)! \times 5!}$$

$$= \frac{4! \times 5}{4! \times 1} + \frac{3! \times 4 \times 5}{3! \times 1 \times 2} + \frac{3! \times 4 \times 5}{1 \times 2 \times 3!} + \frac{4! \times 5}{1 \times 4!} + \frac{5!}{5!} 1$$

$= 5 + 10 + 10 + 5 + 1$

$= 31$ ways.

W.3.Q.8

7) A commitee of 5 is to be constituted from 6 boys and 5 girls. In how many ways can this be done so as to include at least one boy and one girl?

Sol:

Boys = 6		Girls = 5	No. of ways
1	and	4	${}^{6}C_1 \times {}^{5}C_4 = 130$
2		3	${}^{6}C_2 \times {}^{5}C_3 = 150$
3		2	${}^{6}C_3 \times {}^{5}C_2 = 200$
4		1	${}^{6}C_4 \times {}^{5}C_1 = 75$

Total no. of committee $= ({}^{6}C_1 \times {}^{6}C_4) + ({}^{6}C_2 \times {}^{6}C_3) + ({}^{6}C_3 \times {}^{6}C_2) + ({}^{6}C_4 \times {}^{6}C_1)$

$= 130 + 150 + 200 + 75$

$= 455$ ways.

8) From 12 persons, in how many ways can a selection of 6 be made?

a) When two particular persons are always included?

Sol:

$n = 12 - 2 = 10$, $r = 6 - 2 = 4$

Total no. of ways $= {}^{10}C_4 = \frac{10!}{(10-4)! \times 4!} = \frac{6! \times 7 \times 8 \times 9 \times 10}{6! \times 1 \times 2 \times 3 \times 4}$

$= 210$ ways.

[Note: When question say included we have to substracted from both 'N' and 'R'.

b) When two particular persons are always excluded

Sol:

$n = 12 - 2 = 10$, $r = 6$

Total no. of ways $= {}^{10}C_6 = \frac{10!}{(10-6)! \times 6!} = \frac{6! \times 7 \times 8 \times 9 \times 10}{1 \times 2 \times 3 \times 4 \times 6!} = 210$ ways

[Note: When question says excluded we ~~on~~ only have to substracted of 'N' not from 'r'.

9) A candidate is required to answer 8 out of 10 question which are divided into two groups containing equal question and he is not ~~required~~ permitted to attempt more that 4 from any group. In how many ways can he makeup his choice.

Sol:

Group A (5)		Group B (5)	No. of ways
4	and	2	$^5C_4 \times {}^5C_2 = 50$
3		3	$^5C_3 \times {}^5C_3 = 100$
2		4	$^5C_2 + {}^5C_4 = 50$

Now;

Total number of ways $= (^5C_4 \times {}^5C_2) + (^5C_3 \times {}^5C_3) + (^5C_2 + {}^5C_4)$

$= 50 + 100 + 50$

$= 200$ ways.

10) A person has got 12 acquaintances of which 8 are relatives. In how many ways can he invite 7 guests so that 5 of them may be relatives?

Given;

Total no of acquaintances $= 12$

no. of relatives $= 8$

no. of non-relatives $= 12 - 8 = 4$

$\therefore$ Total no. of selection $= {}^8C_5 \times {}^4C_2$

$= \frac{8!}{(8-5)! \times 5!} \times \frac{4!}{(4-2)! \times 2!}$

$= \frac{\cancel{5!} \times 6 \times 7 \times 8}{1 \times \cancel{2} \times 3 \times \cancel{5!}} \times \frac{\cancel{2!} \times 3 \times \cancel{4!}\, 2}{\cancel{2!} \times 1 \times 2}$

$= 336$ ways.

11) From 7 boys and 4 girls, in how many ways a committe of 5 can be made so as to include at least 1 girl?

Sol:

No. of Boys = 7		No. of girls = 4	No. of ways
4	and	1	$^7C_4 \times {}^4C_1 = 140$
3		2	$^7C_3 \times {}^4C_2 = 210$
2		3	$^7C_2 \times {}^4C_3 = 84$
1		4	$^7C_1 \times {}^4C_4 = 7$

Now;

Total no. of ways $= (^7C_4 \times {}^4C_1) + (^7C_3 \times {}^4C_2) + (^7C_2 \times {}^4C_3) + (^7C_1 \times {}^4C_4)$

$= 140 + 210 + 84 + 7$

$= 441$

12) There are 10 questions in an examination paper out of which 3 particular questions are compulsory and candidate has to answer 7 in all. In how many ways can he select the questions?

Given;

$n = 10 - 3 = 7$, $r = 7 - 3 = 4$

Total no. of ways $= {}^7C_7 \times {}^7C_4$

$= \frac{7!}{(7-7)! \times 7!} \times \frac{7!}{(7-4)! \times 4!}$

$= 1 \times \frac{\cancel{4!} \times 5 \times \cancel{6} \times 7}{1 \times \cancel{2} \times \cancel{3} \times \cancel{4!}}$

$= 35$ ways.

13) There are 4 doctor in a hospital which is managed by 6 founder members. To make a well

co-ordination between them, a committee is to be formed consist of 2 doctors and 3 founder members. How many such committee can be formed.

Given,

~~and~~ For doctor : $n = 4, r = 2$

For founder : $n = 6, r = 3$

now,

Total no. of ways $= {}^4C_2 \times {}^6C_3$

$= \frac{4!}{(4-2)! \times 2!} \times \frac{6!}{(6-3)! \times 3!}$

$= \frac{\cancel{2!} \times 3 \times \cancel{4}2}{\cancel{2!} \times 1 \times \cancel{2}} \times \frac{\cancel{3!} \times 4 \times 5 \times \cancel{6}}{\cancel{3!} \times 1 \times \cancel{2} \times \cancel{3}}$

$= 6 \times 20$

$= 120$ ways.

14) A person has 6 friends. Find the number of ways in which he can invite.

a) One or more of them to a dinner.

sol:

Total no of ways $= \cancel{{}^6C_1 \text{ or } {}^6C_2 \text{ or } {}^6C_3 + {}^6C_4 + {}^6C_5 + {}^6C_6}$

$= {}^6C_1$ or ${}^6C_2$ or ${}^6C_3$ or ${}^6C_4$ or ${}^6C_5$ or ${}^6C_6$

$= {}^6C_1 + {}^6C_2 + {}^6C_3 + {}^6C_4 + {}^6C_5 + {}^6C_6$

$= 6 + 15 + 20 + 15 + 6 + 1$

$= 63$ ways.

b) more than four to a dinner.

Total number of ways $= {}^6C_5 + {}^6C_6$

$= \frac{6!}{(6-5)! \times 5!} + \frac{6!}{(6-6)! \times 6!}$

$= \frac{\cancel{5!} \times 6}{1! \times \cancel{5!}} + \frac{\cancel{6!}\ 1}{1 \times \cancel{6!}}$

$= 7$ ways.

c) at most four to a dinner.

sol:

Total number of ways $= {}^6C_0 + {}^6C_1 + {}^6C_2 + {}^6C_3 + {}^6C_4$

$= 1 + 6 + 15 + 20 + 15$

$= 57$ ways.

[Note : If question say at most we have to start from 0, we have to put $r = 0$, like ${}^6C_0$].

15) How many committee of 4 members can be formed from 5 boys and 4 girls if the committe consits at least two girls.

sol:

Total No. of boys(5)		No. of girls (4)	No. of ways
0	and	4	${}^5C_0 \times {}^4C_4 = 1$
1		3	${}^5C_1 \times {}^4C_3 = 20$
2		2	${}^5C_2 \times {}^4C_2 = 60$

Total No. of ways $= ({}^5C_0 \times {}^4C_4) + ({}^5C_1 \times {}^4C_3) + ({}^5C_2 \times {}^4C_2)$

$= 1 + 20 + 60$

$= 81$ ways.

Mark : 30 - 20

Chapter 3 Sequence and Series

Arithmetic Series (A.S)	Geometric Series (G.S)
$a, a+d, a+2d \ldots a(n-1)d$	a, ar, ar^2, ar^{n-1}
* i) $t_n = a + (n-1)d$ ↳ n^{th} term/General term/last term	i) $t_n = a \cdot r^{n-1}$
ii) Sum $S_n = \frac{n}{2}[2a+(n-1)d]$ (if t_n not given) or, $S_n = \frac{n}{2}[a+t_n]$ (if n given	ii) Sum, a) $S_n = \frac{a(r^n-1)}{r-1}$ if $r>1$ b) $\frac{a(1-r^n)}{1-r}$ if $r<1$ or, $\frac{lr-a}{r-1}$, a - l $r<1$

1) Find the 9th term of 4, 7, 10, 13 ....

Sol: Given sequence

4, 7, 10, 13 ...

$a = 4$

$d = t_2 - t_1 = 7 - 4 = 3$

$t_9 = ?$

we have,

$t_n = a + (n-1)d$

$= 4 + (9-1) \times 3$

$= 4 + 24$

$= 28$

Note:

i) 1, 2, 3, 4 ... to 10 terms (yesma $n = 10$ hunxa 'to' vane 'N' hunxa.

ii) 1, 2, 3, 4 ... 10 ['to' na vane 'tn' hunxa. $t_n = 10$

2) Find the sum of 4, 7, 10, 13, 16 ... 79

Given, Series, (A.S)

$4 + 7 + 10 + 13 + 16 + \ldots 79$

$a = 4$

$d = t_2 - t_1 = 7 - 4 = 3$

$n = 19$

$S_n = ?$

we have;

$S_n = \frac{n}{2}[2a + (n-1)d]$

$S_{19} = \frac{19}{2}[2 \times 4 + (19-1) \times 3]$

$= \frac{19}{2}(8 + 48)$

$= \frac{19 \times 56}{2} = 496$

3) Find the sum of $6, 5\frac{1}{3}, 4\frac{2}{3}, 4 \ldots$ to 10 terms

Given;

$6 + 16/3 + 14/3 + 4 + \ldots$ to 10 terms $[5\frac{1}{3} = \frac{15+1}{3} = \frac{16}{3}]$

$a = 6$

$d = t_2 - t_1 = 16/3 - 6 = \frac{16-18}{3} = \frac{-2}{3}$

$n = 10$

now;

$S_n = \frac{n}{2}[2a + (n-1)d]$

$S_{10} = \frac{10}{2}[2 \times 6 + (10-1) \times \frac{-2}{3}]$

$= 5(12 - 9 \times \frac{2}{3}) = 5(12 - 6)$

$= 30$

Mean:

i) Arithmetic Mean $= \frac{a+b}{2}$

Let two unequal number be a, b, then a, A, b in AS

$\Rightarrow t_2 - t_1 = t_3 - t_2$

$\Rightarrow A - a = b - A$

$\Rightarrow A + A = a + b$

$\Rightarrow 2A = a + b$

$\Rightarrow A = \frac{a+b}{2}$

ii) Geometric Mean $= \sqrt{ab}$

a, b two unequal number

$t_2/t_1 = t_3/t_2$

$\Rightarrow \frac{G}{a} = \frac{b}{G}$

$= G^2 = ab$

$\therefore G = \sqrt{ab}$

iii) $H.M = \frac{2ab}{a+b}$

Let a and b two unequal then a, H, b are in H.S

Then, $\frac{1}{a}, \frac{1}{H}, \frac{1}{b}$ are in A.S

$t_2 - t_1 = t_3 - t_2$

$\Rightarrow \frac{1}{H} - \frac{1}{a} = \frac{1}{b} - \frac{1}{H}$

$\Rightarrow \frac{1}{H} + \frac{1}{H} = \frac{1}{a} + \frac{1}{b}$

$\Rightarrow \frac{1+1}{H} = \frac{b+a}{ab}$

$\Rightarrow \frac{2}{H} = \frac{a+b}{ab}$

$\Rightarrow H(a+b) = 2ab$

$\therefore H = \frac{2ab}{a+b}$

Note:-

* Sequence : A sequence is an ordered set of numbers.

Ex ① 2, 4, 6, 8 ... (sequence in even number)

② 1, 3, 5, 7 ... (sequence in odd number)

* Series : When the terms of a sequence are connected with + or − signs, a series is formed.

Ex: $a_1 + a_2 + a_3 \ldots + a_n$ is a series of n terms.

* Finite sequence : A sequence is said to be finite if the number of element in that sequence is finite. In this case the last term is specified.

Ex : 1, 2, 3, 4, 5, 6, 7

* Infinite sequences : A sequence is said to be infinite if the number of elements is infinite.

Ex : 1, 2, 3, 4, 5 .....

* Types : There are 3 types of sequences

i) Arithmetic sequences (AS or AP)

ii) Geometric sequences (GS or GP)

iii) Harmonic sequences (HS or HP)

i) Arithmetic Sequence is a sequence if the difference between any two successive terms is constant. The constant difference is common difference (d).

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* A sequence of ~~a~~ number whose terms increase or decrease by a constant ratio is called Geometric Progression. The constant ratio is called common ratio (r).

* A sequence of numbers is called harmonic sequence if their reciprocal form of A.S.

(CU) (For 10 marks) Theorem : The A.M, G.M and H.M between two unequal positive (+ve) number satisfy the relation

i) $A.M \times H.M = (G.M)^2$ i.e $A \times H = G^2$

ii) $AM > GM > HM$ i.e $A > G > H$

Prove:

Let a and b be two unequal positive numbers we have, $A.M = \frac{a+b}{2}$, $G.M = \sqrt{ab}$, $HM = \frac{2ab}{a+b}$

We have to show $AM \times HM = (GM)^2$

Here;

$$A.M \times H.M = \frac{a+b}{2} \times \frac{2ab}{a+b}$$
$$= \frac{2ab}{2}$$
$$= ab$$
$$= (\sqrt{ab})^2 \quad \because (\sqrt{ab})^2 = ab$$
$$= (G.M)^2$$

$\therefore A.M \times H.M = (G.M)^2$

proved

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i.e) AM, GM and H.M are in GP

ii) $AM > GM > HM$

For $AM > GM$ (To ~~find~~ greater if $AM - GM$ came positive

$$AM - GM = \frac{a+b}{2} - \sqrt{ab}$$
$$= \frac{a+b-2\sqrt{a}.\sqrt{b}}{2}$$
$$= \frac{(\sqrt{a})^2 - 2\sqrt{a}.\sqrt{b} + (\sqrt{b})^2}{2} \quad \because (\sqrt{a})^2 = a$$
$$= \frac{(\sqrt{a} - \sqrt{b})^2}{2} \text{ (+ve)} > 0$$

$AM - GM > 0$, i.e $AM > GM$ — (i)

For $GM > HM$

We know,

$\Rightarrow AM \times HM = (GM)^2$

$\Rightarrow AM \times HM = GM \times GM$

$\Rightarrow \frac{AM}{GM} = \frac{GM}{HM}$ [We know $AM > GM$ then GM is also greater than HM]

Since, $AM > GM$, so

$GM > HM$ — (ii)

combining (i) and (ii)

$AM > GM > HM$

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Exercise 3.1

1) a) Check whether the sequence 3, 8, 13, 18, 23 ... form an A.S or not

sol:

Finding AS or not. the become AS it has to common difference

$\Rightarrow t_2 - t_1 = t_3 - t_2$

$\Rightarrow 8 - 3 = 13 - 8$

$\Rightarrow 5 = 5$

b) Find the 9th term of an arithmetic sequence 4, 7, 10, 13 ...

sol:

$A = 4$

$D = t_2 - t_1 = 7 - 4 = 3$

$t_9 = ?$

now;

$t_n = a + (n-1)d$

$t_9 = a + (9-1)d$

$= 4 + 8 \times 3$

$\therefore t_9 = 28$

c) Find the 13th term of an A.P 8, 3, -2, -7 ...

sol:

$A = 8$

$D = t_2 - t_1 = 3 - 8 = -5$

$t_{13} = ?$

now;

$t_n = a + (n-1)d$

$t_{13} = 8 + (13-1)(-5)$

$= 8 - 60$

$= -52$

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2. a) sol:

$A = -1$

$D = -3$

$t_{17} = ?$

$t_{21} = ?$

now;

$t_n = a + (n-1)d$

$t_{17} = -1 + (17-1)(-3)$

$= -1 - 48$

$t_{17} = -49$

$t_n = a + (n-1)d$

$t_{21} = -1 + (21-1)(-3)$

$= -1 - 60$

$\therefore t_{21} = -61$

b) sol:

$a = 21$

Common difference $(d) = t_2 - t_1 = 26 - 21 = 5$

last term $(t_n) = 136$

Sum of term $(S_n) = ?$ $n = ?$

now;

$S_n = \frac{n[2a - (n-1)d]}{2}$

$= \frac{136}{2}[2 \times 21 - (136-1)5]$

$= 68(42)$

$\Rightarrow t_n = a + (n-1)d$

$\Rightarrow 136 = 21 + (n-1)5$

$\Rightarrow 136 = 21 + 5n - 5$

$\Rightarrow 136 - 16 = 5n$

$\Rightarrow 5n = 120$

$\Rightarrow n = \frac{120}{5}$

$\therefore n = 24$

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3. a) Sol: We have,

$4^{th} = 11$ | $15^{th} = 44$

$t_4 = 11$ | $t_{15} = 44$

we know,

$t_n = a + (n-1)d$ / $t_n = a + (n-1)d$

$t_4 = a + 3d$ / $t_{15} = a + 14d$

$11 = a + 3d$ — ① / $44 = a + 14d$ — ②

Solving ① and ② we get

$a + 14d = 44$

$a + 3d = 11$

$- \quad - \quad -$

$11d = 33$

$\therefore d = 3$

Substituting the value of d in eqn ①, we get

$11 = a + 3 \times 3$

$a = 11 - 9$

$= 2$

Now,

The 19th term $= a + (n-1)d$

$= 2 + (19-1)3$

$= 2 + 54$

$= 56$

b) we know,

$\Rightarrow 7t_7 = 11t_{11}$

$\Rightarrow 7[a + (n-1)d] = 11[a + (n-1)d]$

$\Rightarrow 7[a + (7-1)d] = 11[a + (11-1)d]$

$\Rightarrow 7(a + 6d) = 11(a + 10d)$

$\Rightarrow 7a + 42d = 11a + 110d$

$\Rightarrow 7a + 42d - 11a - 110d = 0$

or, $-4a - 68d = 0$

or, $-4(a + 17) = 0$

or, $a + 17 = \frac{0}{-4} = 0$ — ①

again;

$\Rightarrow t_{18} = 0$

or, $a + (18-1)d = 0$

or, $a + 17d = 0$ — we have eqn (1)

4. a) Given;

$49 + 44 + 39 +$ to 12 term

$A = 49$

$D = t_2 - t_1 = 44 - 49 = -5$

$n = 12$

$S_n = ?$

Now;

$S_n = \frac{n}{2}[2a + (n-1)d]$

$= \frac{12}{2}[2 \times 49 + (12-1)(-5)]$

$= \frac{12}{2}[98 - 55]$

$= \frac{12}{2}(43)$

$= 258$

$t_n = a + (n-1)d$

$181 = 49 + (n-1)8$

$181 = 49 + 8n - 8$

$181 - 41 = 8n$

$n = 140/8$

$n = 17$

now;

$S_n = \frac{n}{2}(a + t_n)$

$= \frac{5}{2}(49 + 181)$

b) Given;

$5, 13, 21, \ldots 181$

$a = 49$

$D = t_2 - t_1 = 8$

$t_n = 181$

$S_n = ?$

c) 5, 13, 21, ... 181

Sol:

$a = 5$

$D = t_2 - t_1 = 8$

$t_n = 181$

$S_n = ?$

now;

$t_n = a + (n-1)d$

$181 = 5 + (n-1)8$

$181 = 5 + 8n - 8$

$\frac{184}{8} = n$

$n = 23$

again;

$S_n = \frac{n}{2}(a + t_n)$

$= \frac{23}{2}(5 + 181)$

$= \frac{23}{2}(93)$

$= 2139$

e) 0.7, 0.71, 0.72 ... to 100 terms

Sol:

$a = 0.7$

$D = t_2 - t_1 = 0.71 - 0.7 = 0.01$

$n = 100$

$S_n = ?$

now;

$S_n = \frac{n}{2}[2a + (n-1)d]$

$= \frac{100}{2}[2 \times 0.7 + (100-1)0.01]$

$= 50[1.4 + 1.4 + 0.99]$

$= 50 \times 2.39$

$= 119.5$

5. a) Sol:

12+16+20+ ...

$a = 12$

$d = t_2 - t_1 = 16 - 12 = 4$

$S_n = 208$,

$n = ?$

b.

18+16+14+ ...

$a = 18$

$d = t_2 - t_1 = 16 - 18 = -2$

$S_n = 0$

$n = ?$

Given;

$S_n = \frac{n}{2}[2a + (n-1)d]$

or, $208 = \frac{n}{2}[2 \times 12 + (n-1) \times 4]$

or, $208 = \frac{n}{2}(24 + 4n - 4)$

or, $208 = \frac{n}{2}(20 + 4n)$

or, $208 = \frac{n}{2}(10 + 2n)$

or, $208 = 10n + 2n^2$

or, $208 = 2(5n + n^2)$

or, $104 = 5n + n^2$

or, $n^2 + 5n - 104 = 0$

or, $n^2 + 13n - 8n - 104 = 0$

or, $n(n+13) - 8(n+13) = 0$

or, $(n+13)(n-8) = 0$

or, $n = -13, 8$ (note: n should be +ve and whole number)

$\therefore n = 8$

we know;

$S_n = \frac{n}{2}[2a + (n-1)d]$

$0 = \frac{n}{2}[2 \times 18 + (n-1)(-2)]$

$0 = \frac{n}{2}[36 - 2n + 2]$

$0 = \frac{n}{2}[38 - 2n]$

$0 = \frac{n}{2}(19 - n)$

$0 = n(19 - n)$

$0 = 19n - n^2$

$\cancel{19n - n^2 = 0}$

$n^2 = 19n$

$n = 19$

$\therefore n = 19$

c) Sol:

24+20+16+ ...

$a = 24$

$d = t_2 - t_1 = 20 - 24 = -4$

$S_n = 72$

$n = ?$

now,

$S_n = \frac{n}{2}[2a + (n-1)d]$

$72 = \frac{n}{2}[2 \times 24 + (n-1)(-4)]$

$72 = \frac{n}{2}[48 - 4n + 4]$

$72 = \frac{n}{2}[52 - 4n]$

$72 = \frac{n}{2}(26 - 2n)$

$72 = n(26 - 2n)$

or, $72 = 26n - 2n^2$

or, $2n^2 - 26n + 72 = 0$

or, $2(n^2 - 13n + 36) = 0$

or, $n^2 - 13n + 36 = 0$

or, $n^2 - 9n - 4n + 36 = 0$

or, $n(n-9) - 4(n-9) = 0$

or, $(n-4)(n-9) = 0$

Either	nor
$n - 4 = 0$	$n - 9 = 0$
$n = 4$	$n = 9$

3c) If 9th term of A.P is zero, prove that 29th term is double of 19th term

Given;

$t_9 = 0$

$= a + (9-1)d = 0$

$= a + 8d = 0$ — ①

$t_{29} = a + (29-1)d$

$= a + 28d$

We show;

$t_{29} = 2t_{19}$

now;

$2t_{19} = 2[a + (19-1)d]$

$= 2a + 36d$

$= (a + 28d) + (a + 8d)$

$= a + 28d + 0$ (applying ①)

$= a + 28d$

$= t_{29}$

$\therefore 2t_{19} = t_{29}$

Imp

6. a) ans

Sol:

The series from 100 to 200 which are divisible by 6 : $102 + 108 + 114 + \dots + 198$

$a = 102$

$d = t_2 - t_1 = 108 - 102 = 6$

$S_n = ?$, $t_n = 198$

We have;

$t_n = a + (n-1)d$

or, $198 = 102 + (n-1) \times 6$

or, $198 - 102 = 6n - 6$

or, $96 + 6 = 6n$

or, $6n = 102$

or, $n = 17$

Now;

$S_n = \frac{n}{2}[a + t_n]$

$= \frac{17}{2}[102 + 198]$

$= \frac{17}{2} \times 300^{150}$

$\therefore S_n = 2550$

b) Sol:

The series from 100 to 1000 which is divisible by 11 : $110 + 121 + 132 \dots + 990$

$a = 110$

$d = t_2 - t_1 = 121 - 110 = 11$

$t_n = 990$

$S_n = ?$

Now;

$t_n = a + (n-1)d$

or, $990 = 110 + (n-1)11$

or, $990 = 110 + 11n - 11$

or, $990 - 110 + 11 = 11n$

or, $891 = 11n$

or, $n = 81$

$S_n = \frac{n}{2}(a + t_n)$

$= \frac{81}{2}(110 + 990)$

$= \frac{81}{2} \times 1100$

$= 81 \times 550$

$= 44550$

* Note: Exclusive means we have to substract one from the full number like 5 to 100 inclusive, means 5 to 99.
* Inclusive means whole number like 5 to 100 exclusive means 5 to 100

c) Series from 5 to 100 which can be multiple by 5 : 5 + 10 + 15 ... + 100

$a = 5$

$d = t_2 - t_1 = 10 - 5 = 5$

$t_n = 100$

$S_n = ?$

now;

$t_n = a + (n-1)d$

$100 = 5 + (n-1)5$

$100 - 5 = 5n - 5$

$95 + 5 = 5n$

$100 = 5n$

$n = \frac{100}{5} = 20$

$S_n = \frac{n}{2}(a + t_n)$

$= \frac{20}{2}(5 + 100)$

$= \frac{20}{2} \times 105$

$= 10 \times 105$

$= 1050$

7.a) Let $a-d, a, a+d$ be three numbers which are in A.P.

Then,

$a - d + a + a + d = 30$

$3a = 30$

$a = \frac{30}{3} = 10$

again; by the second condition of the question

(a) $(a-d)(a)(a+d) = 990$

$(10)(10-d)(10+d) = 990$

$100 + 10d - 10d - d^2 = 990/10$

$100 - d^2 = 99$

$-d^2 = 99 - 100$

$+d^2 = +1$

$d^2 = 1$

$d^2 = 1^2$

$d = 1$.

when $d = 1$ then three number are 9, 10, 11

b) Find three numbers in an A.P whose sum is 21 and sum of their squares is 155

Sol:

Let $a-d, a, a+d$ be three numbers which are in A.P

then;

$a - d + a + a + d = 21$

$3a = 21$

$a = 21/3 = 7$

again;

By the second condition of the question

$(a-d)^2 + a^2 + (a+d)^2 = 155$

or, $(7-d)^2 + 7^2 + (7+d)^2 = 155$

or, $7^2 - 2.7.d + d^2 + 49 + 7^2 + 2.7.d + d^2 = 155$

or, $49 - 14d + d^2 + 49 + 49 + 14d + d^2 = 155$

or, $147 + 2d^2 = 155$

or, $2d^2 = 155 - 147$

or, $2d^2 = 8$

$\therefore d = \pm 2$

When $d = -2$, then three numbers are 9, 7, 5

When $d = 2$, then three numbers are 5, 7, 9

c.a) Let $a-d, a, a+d$ be the three number of AP

Then;

$a - d + a + a + d = 45$

$3a = 45$

$a = \frac{45}{3} = 15$

again;
By second side of question
$a \times (a+d) = 300$
or, $15 \times (15+d) = 300$
or, $15+d = 300/15$
or, $15+d = 20$
~~or~~ $\therefore d = 5$

When $d = 5$, the three numbers are 10, 15, 20

8. a) ans the given first ~~term~~ A.P 7, 12, 17, 22
Here, first ~~term~~ term $(a) = 7$
Common difference $(d) = t_2 - t_1 = 12 - 7 = 5$
We known,
n term $(t_n) = a + (n-1)d$
$= 7 + (n-1)5$
$= 7 + 5n - 5$
$= 2 + 5n$ — (i)

again;
the given second AP is 27, 30, 33, 36 ...
first ~~the~~ term $(a) = 27$
common difference $(d) = t_2 - t_1 = 30 - 20 = 3$
We know;
n^{th} term $(t_n) = a + (n-1)d$
$= 27 + (n-1)3$
$= 27 + 3n - 3$
$= 24 + 3n$ — (ii)

According to question, (i) and (ii) must equal
$2 + 5n = 24 + 3n$
$5n - 3n = 24 - 2$ $\therefore n = 11$

C. Ans

Sol:

For $t_{12} = 23$
$\Rightarrow a + (n-1)d = 23$
or, $a + (12-1)d = 23$
or, $a + 11d = 23$ — (i)

For $S_{22} = 484$
$\Rightarrow \frac{n}{2}[2a + (n-1)d] = 484$
or, $\frac{22}{2}[2a + (22-1)d] = 484$
or, $2a + 21d = 44$ — (ii)

solving (i) and (ii)
$2a + 22d = 46$ [$\because$ multiplying eqn (i) by 2 to cut a]
$2a + 21d = 44$
$\therefore d = 2$

Apply d in (i)
$a + 11d = 23$
or, $a + 11 \times 2 = 23$
or, $a = 23 - 22$
or, $\therefore a = 1$

again; $S_{30} = ?$
now; $a = 1$, $d = 2$
$S_{30} = \left[\frac{n}{2}[2a + (n-1)d]\right]$
$= \frac{30}{2}[2 \times 1 + (30-1)2]$
$= 15(2 + 58)$
$= 15 \times 60$
$\therefore S_{30} = 900$

9. a) ans

Sol:
let m_1, m_2, m_3, m_4, m_5 be 5 AM's.

2, m_1, m_2, m_3, m_4, m_5, 26

t_1	t_2	t_3	t_4	t_5	t_6	t_7
a	(a+d)	(a+2d)	(a+3d)	...	...	(a+6d)

now;
$a = 2$, $t_n = 7$, $t_7 = 26$
For d,
$t_n = a + (n-1)d$
$t_7 = 2 + (7-1)d$
$26 = 2 + 6d$

$26 - 2 = 6d$
$d = \frac{24}{6}$
$\therefore d = 4$

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∴ 5 AM's be

$m_1 = t_2 = a + d = 2 + 4 = 6$
$m_2 = t_3 = a + 2d = 2 + 2 \times 4 = 10$
$m_3 = t_4 = a + 3d = 2 + 3 \times 4 = 14$

$m_5 = t_6 = a + 5d = 2 + 5 \times 4 = 22$

b) Insert 6 A.M's between 5 and -9

Let $m_1, m_2, m_3 \ldots m_6$ be 6 A.M's

$5, m_1, m_2, m_3, m_4, m_5, m_6, -9$

now;
$a = 5, t_8 = -9$

again;
$t_n = a + (n-1)d$
$t_8 = 5 + (8-1)d$
$-9 - 5 = 7d$
$-14 = 7d$
$\therefore d = -14/7 = -2$

∴ 6 Am's be

$m_1 = t_2 = a + d = 5 + (-2) = 3$
$m_2 = t_3 = a + 2d = 5 - 4 = 1$
$m_3 = t_4 = a + 3d = 5 - 6 = -1$
⋮
$m_6 = t_7 = a + 6d = 5 - 12 = -7$

c) I. Insert 8 Am's between -42 and -15.

Sol:

Let $m_1, m_2, m_3, m_4 \ldots m_8$ be Am's
$-42, m_1, m_2 \ldots\ldots m_8, -15$ be 8 Am's

$a = -42$, $t_n = 10$, $t_{10} = -15$ ~~be in~~

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For d;

$t_n = a + (n-1)d$
or, $t_{10} = -42 + (10-1)d$
or, $-15 = -42 + 9d$
or, $-15 + 42 = 9d$
or, $9d = 27$
$\therefore d = 3$

∴ 8 Am's be

$m_1 = t_2 = a + d = -42 + 3 = -39$
$m_2 = t_3 = a + 2d = -42 + 2 \times 3 = -3$
⋮
$m_8 = t_9 = a + 8d = -42 + 8 \times 3 = -18$

10. a) ans

Sol:

First month saving $(a) = 50$
Succedding in every month $(d) = 5$
Sum of saving $(S_n) = 725$
Number of month $(n) = ?$

now;

$S_n = \frac{n[2a + (n-1)d]}{2}$

or, $725 = \frac{n[2 \times 50 + (n-1)5]}{2}$

or, $1450 = n(100 + 5n - 5)$
or, $1450 = 95n + 5n^2$
or, $95n + 5n^2 = 1450$
or, $5n^2 + 95n - 1450 = 0$
or, $5(n^2 + 19n - 290) = 0$
or, $n^2 + 19n - 290 = 0$
or, $n^2 + 29n - 10n - 290 = 0$
or, $n(n+29) - 10(n+29) = 0$
or, $(n-10)(n+29) = 0$

Either
$n - 10 = 0$, $n = 10$
$n + 29 = 0$, $n = -29$

n must be positive or whole number so the required months are 10.

11. a) ans

monthly salary (a) = 5000, ~~monthly~~ yearly salary (a) = 5000 × 12 = 60000
increasing monthly salary (d) = 100, increasing yearly salary (d) = 1200
withdraw in 10 year (S_{10}) = ?
now,

$$S_n = \frac{n}{2}[2a + (n-1)d]$$

or, $S_{10} = \frac{5}{2}[2 \times 60000 + (10-1) \times 1200]$

or, 5 (120000 + 10800)
or, 5 (130800)
∴ S_{10} = 654000

11. b) ans

Total amount (S_n) = 16500
number of year (n) = 10
Saving of each year (d) = 100
Salary of first year (a) = ?
now,

$$S_n = \frac{n}{2}[2a + (n-1)d]$$

or, $16500 = \frac{10}{2}[2a + (10-1)100]$

or, $16500 = 5[2a + 900]$
or, $3300 = 2a + 900$
or, $3300 - 900 = 2a$
or, $2400 = 2a$
or, $a = 2400/2$
$a = 1200$

c) ans

a = 8 × 12 = 96
D = 4
S_n = 168 × 12 = 2016
n = ?
now,

$$S_n = \frac{n}{2}[2a + (n-1)d]$$

or, $2016 = \frac{n}{2}[2 \times 96 + (n-1)4]$

or, $4032 = n(188 + 4n)$
or, $4032 = 188n + 4n^2$
or, $4n^2 + 188n - 4032 = 0$
or, $4(n^2 + 47n - 1008) = 0$
or, $n^2 + 47n - 1008 = 0$
or, $n(n+63) - 16(n+63) = 0$
or, $(n-16)(n+63) = 0$
Either
$n - 16 = 0$ / $n + 63 = 0$
$n = 16$ / $n + 63 = 0$

∴ n = 16

b. ans

Sol:
Starting value (a) = −2000
Common different (d) = 400
t_{12} = ?
now,

$t_n = a + (n-1)d$
$t_{12} = -2000 + (12-1)400$
$= -2000 + 4400$
$= 2400$

Gnal formula

If last or n^{th} term l (= tn = b) is given

$$= \frac{lr - a}{r - 1}$$

Exercise 3.2

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1. b) ans

Given; Series;

$$\frac{1}{4} - \frac{1}{2} + 1 \ldots$$

$a = \frac{1}{4}$

$r = \frac{t_2}{t_1} = \frac{-\frac{1}{2}}{\frac{1}{4}} = -\frac{1}{2} \times \frac{4}{1} = -2$

$t_8 = ?$, $n = 8$

We know;

$t_n = a \cdot r^{n-1}$

$t_8 = \frac{1}{4} \cdot (-2)^{8-1}$

$= \frac{1}{4} (-2)^7$

$= \frac{1}{4} \times -128$

$= -32$ ans

C. ans

Sol

$t_5 = 243$, $r = 3$, $a = ?$

now;

$t_n = a \cdot r^{n-1}$

$t_5 = a \cdot 3^{5-1}$

$243 = a \cdot 3^4$

$a \cdot 3^4 = 243$

$a = \frac{243}{81} = 3$

2. a) ans

$a = 1$, $r = \frac{t_2}{t_1} = \frac{\frac{1}{2}}{1} = \frac{1}{2}$, $t_n = \frac{1}{256}$, $n = ?$

now;

$t_n = a \cdot r^{n-1}$

$\frac{1}{256} = 1 \cdot \left(\frac{1}{2}\right)^{n-1}$

$\left(\frac{1}{2}\right)^8 = \left(\frac{1}{2}\right)^{n-1}$

$8 = n - 1$

$n = 8 + 1$

$n = 9^{th}$ ans

$t_n = 384$

C. ans

Given;

$t_6 = 64$

$\Rightarrow a \cdot r^{6-1} = 64$

or, $a \cdot r^5 = 64$ — (i)

$t_{13} = 8192$

$\Rightarrow a \cdot r^{13-1} = 8192$

or, $a \cdot r^{12} = 8192$ — (ii)

Dividing eqn (ii) by (i) we get

$\Rightarrow \frac{a \cdot r^{12}}{a \cdot r^5} = \frac{8192}{64}$

or, $r^7 = 128$

or, $r^7 = 2^7$

or, $r = 2$

Applying r in (i)

$a \times 2^5 = 64$

$a = \frac{64}{32}$

$\therefore a = 2$

Now;

$t_{11} = a \cdot r^{11-1}$

$= a \cdot r^{10}$

$= 2 \cdot (2)^{10}$

$= 2048$

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3. a) i) ans

Sol:

$a = 1$

$r = \frac{t_2}{t_1} = \frac{3}{1} = 3$

$n = 12$

$S_n = ?$

now;

$S_n = \frac{a(r^n - 1)}{r - 1}$

$S_{12} = \frac{1(3^{12} - 1)}{3 - 1}$

$= \frac{531440}{2}$

$= 265720$

ii) ans

Sol:

$a = 1$, $r = \frac{t_2}{t_1} = \frac{-\frac{1}{2}}{1} = \frac{-1}{2}$

$n = 9$, $S_n = ?$

now;

$S_n = \frac{a(r^n - 1)}{r - 1}$ $\quad \frac{a(1 - r^n)}{1 - r}$

$S_9 = \frac{1((-\frac{1}{2})^9 - 1)}{(-\frac{1}{2}) - 1}$

$S_9 = 1\left(\frac{1 - (-\frac{1}{2})^9}{1 - (-\frac{1}{2})}\right)$

$= \frac{1 + \frac{1}{512}}{1 + \frac{1}{2}}$

$= \frac{\frac{513}{512}}{\frac{3}{2}} = \frac{117}{256}$ $\therefore S_n = \frac{117}{256}$

$3 + 6 + 12 + 24 + 384$

now;

$a = 3$

$r = \frac{t_2}{t_1} = \frac{6}{3} = 2$

$t_n = 384$

$S_n = ?$

now;

$t_n = a \cdot r^{n-1}$

$384 = 3 \cdot 2^{n-1}$

$128 = 2^{n-1}$

$2^7 = 2^{n-1}$

$7 = n - 1$, $\therefore n = 8$

$S_n = \frac{a(r^n - 1)}{r - 1}$

$= \frac{3(2^8 - 1)}{2 - 1}$

$= \frac{3(256 - 1)}{1}$

$= 3 \times 255$

$= 765$

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iv) $-21 + 14 - \frac{28}{3} + \ldots$ to 6 terms.

Sol:

$a = -21$, $r = \frac{t_2}{t_1} = \frac{14}{-21} = -\frac{2}{3}$, $n = 6$, $S_n = ?$

now;

$S_n = \frac{a(1 - r^n)}{1 - r}$

$= \frac{-21(1 - (-\frac{2}{3})^6)}{1 - (-\frac{2}{3})}$

$= -21 \cdot \frac{\left(1 - \frac{64}{729}\right)}{1 + \frac{2}{3}}$

$= -21 \cdot \frac{\frac{665}{729}}{\frac{5}{3}}$

$= -21 \cdot \frac{133}{243}$

$= -\frac{931}{81}$

3. b ans

$S_n = 728$, $r = 3$, $l = 486$, $a = ?$

now;

$S_n = \frac{lr - a}{r - 1}$

$728 = \frac{486 \times 3 - a}{3 - 1}$

$728 \times 2 = 1458 - a$

$1456 = 1458 - a$

$a = 1458 - 1456$

$a = 2$

→ let the G.P be $a + ar + ar^2$

C. ans

$t_3 = a^3$, $t_6 = 64$	$t_6 = 64$	$a^6 = 64$	Required G.S
Given;	$a \cdot r^6 - 1 = 64$	or, $a^6 = 2^6$	$2 + 2 \times 2 + 2 \times 2^3$
$t_3 = a^3$	$ar^5 = 64$	$\therefore a = 2$	i.e $2 + 4 + 8 + \ldots$
$= a \cdot r^{3-1} = a^3$	or, $a \cdot a^5 = 64$	$r = 2$	
or, $ar^2 = a^3$	(Taking $r = a$)		
$\therefore r = \pm a$			

4. a) ans

$a = 7$
$l = 448$
$S_n = 889$
$R = ?$

now,

$\Rightarrow S_n = \frac{lr - a}{r-1}$

or, $889 = \frac{448r - 7}{r-1}$

or, $889r - 889 = 448r - 7$
or, $889r - 889 - 448r + 7 = 0$
or, $441r - 882 = 0$
or, $441r = 882$
or, $r = \frac{882}{441}$
or, $\therefore r = 2$

b. ans

$t_2 = 24$
$a \cdot r^{n-1} = 24$
$a \cdot r^{2-1} = 24$
$ar = 24$ — ①

$t_5 = 81$
$a \cdot r^{n-1} = 81$
$a \cdot r^{5-1} = 81$
$a \cdot r^4 = 81$ — ②

now;

Equating ① and ②

$\frac{a \cdot r^4 = 81}{a \cdot r = 24}$

$= r^3 = \frac{27}{8}$

or, $r^3 = \left(\frac{3}{2}\right)^3$

or, $\therefore r = \frac{3}{2}$

Note: ~~a2~~ = equation ①. ~~maa ni~~ ~~same~~ maa ra ② maa same vaye ~~right~~ left side (a^2) (I) and (II) ko left side ~~equal~~ garaunu. right

C. ans

Sol:

Let the 3 terms of G.P be $a/r, a, ar$.

According to question;

$a \times a = 256$
$a^2 = 256r$ — ①

$a \times ar = 16$
$a^2 r = 16$
$a^2 = 16/r$ — ②

from ① and ②

$\Rightarrow 256r = 16/r$
or, $256r^2 = 16$
or, $r^2 = \frac{16}{256} = \frac{1}{16}$
or, $r^2 = \frac{1}{16}$
or, $r^2 = (1/4)^2$
or, $r = 1/4$

Applying r in ①
$\Rightarrow a^2 = 256 \times 1/4$
or, $a^2 = 64$
or, $a = 8$

now,

$t_5 = a \cdot r^{n-1}$

$= 8 \cdot \left(\frac{1}{4}\right)^{5-1}$
$= 8 \cdot \left(\frac{1}{4}\right)^4$
$= 8 \cdot \frac{1}{256}$
$= \frac{8}{256} = \frac{1}{32}$

$\therefore t_5 = \frac{1}{32}$

5. bars

Sol:

$a = 2,\ r = \frac{t_2}{t_1} = \frac{6}{2} = 3,\ S_n = 728,\ n = ?$

now;

$\Rightarrow S_n = \frac{a(r^n - 1)}{r-1}$

or, $728 = \frac{2(3^n - 1)}{3-1}$

or, $728 = \frac{\cancel{2}(3^n - 1)}{\cancel{2}}$

or, $728 = 3^n - 1$
or, $3^n = 729$
or, $3^n = 3^6$
$\therefore n = 6$ ans

$\therefore$ 3 number in G.P be $a/r, a, ar$

i.e $\frac{8}{4}, 8, 8 \times \frac{1}{4}$

i.e 32, 8, 2 — G.P

$\therefore a = 32,\ r = \frac{8}{32} = \frac{1}{4}$

$\therefore t_5 = ar^{n-1}$

$= 32 \times \left(\frac{1}{4}\right)^{5-1} = 32 \times \frac{1}{256} = \frac{1}{8}$

C.ω

Let the 3 number in G.P be $a/r, a, ar$

By question,

$\Rightarrow \frac{a}{r} + a + ar = 35$

or, $\frac{10}{r} + 10 + 10r = 35$

or, $\frac{10 + 10r + 10r^2}{r} = 35$

or, $10 + 10r + 10r^2 = 35r$

or, $10 + 10r + 10r^2 - 35r = 0$

or, $10r^2 - 25r + 10 = 0$

or, $5(2r^2 - 5r + 2) = 0$

or, $2r^2 - 5r + 2 = 0$

or, $2r^2 - 4r - r + 2 = 0$

or, $2r(r-2) - 1(r-2) = 0$

or, $(r-2)(2r-1) = 0$

$\therefore r = 2, \frac{1}{2}$

$\frac{a}{r} \times a \times ar = 1000$

$a^3 = 1000$

$\therefore a = 10$

$\therefore a = 10, r = 2, 1/2$

Required 3 numbers;

When $a = 10, r = 2$

$\frac{10}{2}, 10, 10 \times 2$

i.e 5, 10, 20

again;

When $a = 10, r = 1/2$

$\frac{10}{\frac{1}{2}}, 10, 10 \times 1/2$

i.e 20, 10, 5

6.a) Insert 3 geometric means between $2\frac{1}{4}$ and 36.

Sol:

Let G_1, G_2 and G_3 be three G.M's between $\frac{9}{4}$ and 36.

Then; $\frac{9}{4}, G_1, G_2, G_3, 36$ are in G.P

First term $(a) = 9/4$, last term $(b) = 36$, Number of means $(n) = 3$

We have common ration $(r) = \left(\frac{b}{a}\right)^{\frac{1}{n+1}} = \left(\frac{36}{\frac{9}{4}}\right)^{\frac{1}{3+1}}$

$= \left(\frac{144}{4}\right)^{\frac{1}{4}} = (16)^{\frac{1}{4}} = (2^4)^{1/4} = 2$

[A.P to method is good]

Now,

First mean $(G_1) = ar = \frac{9}{4} \times 2 = \frac{9}{2}$

Second mean $(G_2) = ar^2 = \frac{9}{4} \times 2^2 = 9$

Third mean $(G_3) = ar^3 = \frac{9}{4} \times 2^3 = 18$

$\therefore$ 3 means between $2\frac{1}{4}$ and 36 are $9/2, 9, 18$

b) Insert 5 G.M's between $3\frac{5}{9}$ and $40\frac{1}{2}$

Sol:

Let G_1, G_2, G_3, G_4, G_5 are 5 G.M's between $32/9$ and $81/2$

First term $(a) = 32/9$, last term $(b) = 41/2$, number of means $(n) = 5$

now,

Common ratio $(r) = \left(\frac{b}{a}\right)^{\frac{1}{n+1}} = \left(\frac{\frac{81}{2}}{\frac{32}{9}}\right)^{\frac{1}{5+1}} = \left(\frac{729}{64}\right)^{\frac{1}{6}}$

$= \left(\frac{3}{2}\right)^{\frac{1}{6} \times 6} = \frac{3}{2}$

now;

$G_1 = ar = \frac{32}{9} \times \frac{3}{2} = \frac{16}{3}$

$G_2 = ar^2 = \frac{32}{9} \times \left(\frac{3}{2}\right)^2 = \frac{32}{9} \times \frac{9}{4} = 8$

$\vdots$

$G_3 = ar^5 = \frac{32}{9} \times \left(\frac{3}{2}\right)^5 = \frac{32}{9} \times \frac{243}{32} = \frac{1024}{2187}$

$= 27$

note: 9 le multiply gare paxi divide gama-parxa balance gama.

7.a.i) ans

Let S_n be the sum to n term of the series

$S_n = 8 + 33 + 333 + \ldots$ to n terms

$= 3[1 + 11 + 111 + \ldots$ to n terms$]$

$= \frac{3}{9}[9 + 99 + 999 + \ldots$ to n terms$)$

$= \frac{1}{3}[(10-1) + (100-1) + (1000-1) + \ldots$ to n terms$)$

$= \frac{1}{3}[(10 + 100 + 1000 + \ldots$ to n terms$) - (1 + 1 + 1 \ldots$ terms$)]$

$a = 10,\ r = t_2/t_1 = 100/10 = 10 > 1$

$\therefore S_n = \frac{a(r^n - 1)}{r-1}$

$= \frac{1}{3}\left[\frac{10(10^n - 1)}{10-1} - 1 \cdot n\right]$ note: jedi matti 2+2+2 terms hun thiyo vane $-2n$ hunxa

$= \frac{1}{3}\left[\frac{10(10^n-1)}{9} - n\right] = \frac{10(10^n-1)}{27} - \frac{n}{3}$ ans

iv) $0.4 + 0.44 + 0.444 + \ldots$

Sol:

$0.4 + 0.44 + 0.444 \ldots$ to n terms

$= \frac{0.4}{10} + \frac{0.44}{100} + \frac{0.444}{1000} + \ldots$ to n terms

$= \frac{4}{10} + \frac{44}{100} + \frac{444}{1000} + \ldots$ to n terms

$= 4\left[\frac{1}{10} + \frac{11}{100} + \frac{111}{1000} + \ldots\right.$ to n terms$\left.\right]$

$= \frac{4}{9}\left[\frac{9}{10} + \frac{99}{100} + \frac{999}{1000} + \ldots\right.$ to n terms$\left.\right]$

$= \frac{4}{9}\left[\left(1 - \frac{1}{10}\right) + \left(1 - \frac{1}{100}\right) + \left(1 - \frac{1}{1000}\right) + \ldots\right.$ to n terms$\left.\right]$

$= \frac{4}{9}\left[(1 + 1 + 1 + \ldots$ to n terms$) - \left(\frac{1}{10} + \frac{1}{100} + \frac{1}{1000} + \ldots\right.$ to n terms$\left.\right)\right]$

$a = \frac{1}{10},\ r = \frac{t_2}{t_1} = \frac{\frac{1}{100}}{\frac{1}{10}} = \frac{1}{100} \times \frac{10}{1} = \frac{1}{10} < 1$

$\Rightarrow S_n = \frac{a(1 - r^n)}{1 - r}$

$= \frac{4}{9}\left[\frac{1 - \frac{1}{10}(1 - [1/10]^n)}{1 - \frac{1}{10}}\right]$

$= \frac{4}{9}\left[\frac{n - \frac{1}{10}\left(1 - \frac{1}{10^n}\right)}{\frac{9}{10}}\right]$

$= \frac{4n}{9} - \frac{4}{81}\left(1 - \frac{1}{10^n}\right)$ ans

→ v.v. imp

8.a) ans

Sol:

let the 3 nos in A.P be $a-d, a, a+d$

By question;

$(a-d) + a + (a+d) = 15$

$\Rightarrow a - d + a + a + d = 15$

$\Rightarrow 3a = 15$

$\boxed{a = 5}$

Since

$(a-d)+1,\ a+3,\ (a+d)+9$ be in GP

$\left(\frac{t_2}{t_1} = \frac{t_3}{t_2}\right)$

$\Rightarrow \frac{a+3}{a-d+1} = \frac{a+d+9}{a+3}$

$\Rightarrow \frac{5+3}{5-d+1} = \frac{5+d+9}{5+3}$

$\Rightarrow \frac{8}{6-d} = \frac{14+d}{8}$

$\Rightarrow (6-d)(14+d) = 64$

$\Rightarrow 84+6d-14d-d^2 = 64$

$\Rightarrow -d^2-8d+84-64=0$

$\Rightarrow -d^2-8d+20=0$

$\Rightarrow d^2+8d-20=0$

$\Rightarrow d^2+10d-2d-20=0$

$\Rightarrow d(d+10)-2(d+10)=0$

$\Rightarrow (d-2)(d+10)=0$

$\therefore d=2,-10$

Orginal 3 nos in A.P be

When $a=5$ $d=2$

$5-2, 5, 5+2$

i.e. 3, 5, 9

When $a=5$, $d=10$

$5-(-10), 5, 5+(-10)$

i.e. 15, 5, -5 ans

Q) Find the value of k such that $k+9$, $k-6$ and 4 form a G.P

Sol:

$k, k+9, k-6, 4$ be a G.P

$\Rightarrow \frac{t_2}{t_1} = \frac{t_3}{t_2}$

$\Rightarrow \frac{k-6}{k+9} = \frac{4}{k-6}$

$\Rightarrow (k-6)(k-6) = 4(k+9)$

$\Rightarrow k^2-6k-6k+36 = 4k+36$

$\Rightarrow k^2-12k+36-4k-36=0$

$\Rightarrow k^2-16k+0=0$

$\Rightarrow k(k-16)=0$

$\therefore k=0$, $k-16=0$

$k=16$ ans

9. a) ans

Sol:

$t_4 = 54$

$a \cdot r^{4-1} = 54$

$a \cdot r^3 = 54$ — ①

$t_6 = 486$

$a \cdot r^{6-1} = 486$

$a \cdot r^5 = 486$ — ②

Sol:

$\frac{ar^5}{ar^3} = \frac{486}{54}$

$\Rightarrow r^2 = 9$

$r = 3$

Apply r in ①

$a \times 3^3 = 54$

$a = \frac{54}{27}$

$\therefore a = 2$

again;

$t_8 = a \cdot r^{n-1}$

$= a \cdot r^{8-1}$

$= a \cdot r^7$

$= 2 \cdot (3)^7$

$= 4374$ ans

B. ans

We have;

Total Sum (S_n) = 4095

Number of Installments (n) = 12

Common ration (r) = 2

First term (a) = ?

last term (t_n) = ?

now;

$S_n = \frac{a(r^n-1)}{r-1}$

$4095 = \frac{a(2^{12}-1)}{2-1}$

$a(2^{12}-1) = 4095$

$a \cdot 4095 = 4095$

$a = \frac{4095}{4095}$ Rs 1

again;

$t_n = a \cdot r^{n-1}$

$= 1 \cdot 2^{12-1}$

$= 1 \cdot 2^{11}$

$= Rs\ 2048$

7.b) Find the sum of series $8+88+888+\ldots$ to 6 terms

Sol:

$8+88+888+\ldots$ to 6 terms

$= 8(1+11+111+\ldots$ to 6 terms$)$

$= \frac{8}{9}(9+99+999+\ldots$ to 6 terms$)$

$= \frac{8}{9}(10-1+100-1+1000-1+\ldots$ to 6 terms$)$

$= \frac{8}{9}[(10+100+1000+\ldots$ to 6 terms$)-(1+1+1\ldots$ to 6 terms$)$

$= \frac{8}{9}\left[\frac{10(10^6-1)}{10-1}-6\right]$

$= \frac{8}{9}\left[\frac{10\times 999999}{9}-6\right]$

$= \frac{8}{9}\left[\frac{9999990-54}{9}\right]$

$= \frac{8}{9}\left(\frac{9999936}{9}\right)$

$= \frac{8}{9}\times 111116$

$= \frac{888928}{9}$

$= 987648$

7. C. ans

Sol:

Let three terms in GP be $a/r, a, ar$.

From question,

$a/r + a + ar = 63$

$\frac{a+ar+ar^2}{r} = 63$

$a+ar+ar^2 = 63r$

$a(1+r+r^2) = 63r$

$a = \frac{63r}{1+r+r^2} \leftarrow (1)$

$\Rightarrow \frac{a+ar}{r} = \frac{77}{4}a$

$\Rightarrow \frac{a+ar^2}{r} = \frac{77}{4}a$

$\Rightarrow 4a+4ar^2 = 77ar$

$\Rightarrow 4ar^2+4a-77ar = 0$

$\Rightarrow 4ar^2-77ar+4a = 0$

$\Rightarrow a(4r^2-77r+4) = 0$

$\Rightarrow 4r^2-76r-1r+4 = 0$

$\Rightarrow 4r(r-4)-1(r-4) = 0$

$\Rightarrow (4r-1)(r-4) = 0$

either $4r-1=0$, $r = 1/4$ | or $r-4=0$, $r=4$

Apply r in (1)

$a = \frac{63\times 4}{1+4+4^2}$

$a = \frac{252}{21} = 12$

When $a=12$ and $r=4$

$a/r, a, ar$

$12/4, 12, 12\times 4$

i.e. $3, 12, 48$

When $a=12$ and $r=\frac{1}{4}$

$a/r, a, ar$

$12/\frac{1}{4}, 12, 12\times\frac{1}{4}$

i.e $48, 12, 3$

8. b ans

Sol:

Let three number in GP be $a/r, a, ar$

From question,

$a/r + a + ar = 14$

$\Rightarrow \frac{a+ar+ar^2}{r} = 14$

or, $a+ar+ar^2 = 14r$

or, $a(1+r+r^2) = 14r$

or, $a = \frac{14r}{1+r+r^2} \quad -(1)$

Applying a in (1)

$4 = \frac{14r}{1+r+r^2}$

$a/r \times a \times ar = 64$

$a^3 = 64$

$a^3 = 4^3$

$\therefore a = 4$

$4+4r+4r^2 = 14r$

or, $4r^2-14r+4r+4 = 0$

or, $4r^2-10r+4 = 0$

or, $4r^2-8r-2r+4 = 0$

or, $4r(r-2)-2(r-2) = 0$

or, $(4r-2)(r-2) = 0$

$\therefore r = \frac{1}{2}, 2$

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Exercise 3.3

Formula:

1) $S_\infty = \frac{a}{1-r}$ if $|r|<1$

S_∞ exists if $|r|<1$

2) Sum of 1st natural number

Let $S_n = 1+2+3+\dots+n$

$= \frac{n(n+1)}{2}$

3) Sum of square of n natural numbers

$S_n = 1^2+2^2+3^2+\dots+n^2$

$= \frac{n(n+1)(2n+1)}{3!} = \frac{n(n+1)(2n+1)}{6}$

4) Sum of cube of n natural number

$S_n = 1^3+2^3+3^3+\dots+n^3$

$= \left[\frac{n(n+1)}{2}\right]^2$

1.a) Decide which Infinite series have sum

i) $1+1/3+1/3^2+1/3^3+\dots$

Sol:

$|r| = \left|\frac{t_2}{t_1}\right| = \left|\frac{1/3}{1}\right| = \left|\frac{1}{3}\right| = \frac{1}{3} < 1$

S_∞ exists

$S_\infty = \frac{a}{1-r} = \frac{1}{1-\frac{1}{3}} = \frac{1}{\frac{3-1}{3}} = \frac{3}{2}$

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1.b) Find the sum of the series.

1) $1-1/3+1/9-1/27+\dots$

Given;

$a=1,\ r = t_2/t_1 = -1/3 / 1 = -1/3$

$|r| = |-1/3| = 1/3 < 1$

$\therefore S_\infty$ exists

We have,

$S_\infty = \frac{a}{1-r} = \frac{1}{1-(-\frac{1}{3})} = \frac{1}{1+1/3} = \frac{3}{4}$ ans

ii) $\sqrt{2}+1/\sqrt{2}+1/2\sqrt{2}+\dots$

Given;

$a=\sqrt{2},\ r = t_2/t_1 = \frac{1/\sqrt{2}}{\sqrt{2}} = \frac{1}{\sqrt{2}}\cdot\frac{1}{\sqrt{2}} = \frac{1}{2},\ r/|r| = |\frac{1}{2}| < 1$

We have;

$S_\infty = \frac{a}{1-r} = \frac{\sqrt{2}}{1-\frac{1}{2}} = \frac{\sqrt{2}}{\frac{2-1}{2}} = \frac{\sqrt{2}}{\frac{1}{2}} = 2\sqrt{2}$ ans

iii) $a+1+1/a+\dots$ (if $a>1$)

Sol:

$a=a,\ r = t_2/t_1 = 1/a,\ |r| = |1/a| < 1$

now;

$S_\infty = \frac{a}{1-r} = \frac{a}{1-\frac{1}{a}} = \frac{a}{\frac{a-1}{a}} = \frac{a^2}{a-1}$ ans

iv) $1+1/2+1/4+1/8+\dots$

Sol:

$a=1,\ r = \frac{t_2}{t_1} = \frac{\frac{1}{2}}{1} = \frac{1}{2},\ |r| = \left|\frac{1}{2}\right| = \frac{1}{2} < 1$

$S_\infty = \frac{a}{1-r} = \frac{1}{1-\frac{1}{2}} = \frac{1}{\frac{1}{2}} = \frac{2}{1} = 2$ ans

v) $0.9 + 0.03 + 0.001 + \ldots$ to ∞

Sol: $a = 0.9$, $r = t_2/t_1 = 0.03/0.9 = 1/30$

$S_\infty = \frac{a}{1-r} = \frac{0.9}{1-1/30} = \frac{0.9}{\frac{30-1}{30}} = \frac{0.9}{\frac{29}{30}} = \frac{27}{29}$

Formula: [G.S to ∞]

i) Sum of infinity of G.S, $S_\infty = \frac{a}{1-r}$ $[a + ar + ar^2 \ldots)$

ii) Sum of ∞ of G.S, $S_\infty = \frac{a^2}{1-r^2}$ $[a^2 + (ar)^2 + (ar^2)^2]$

iii) Sum of ∞ of G.S, $S_\infty = \frac{a^3}{1-r^3}$ $[a^3 + (ar)^3 + (ar^2)^3 + \ldots \infty)$

2. a) ans

Given,

Sum of infinity $(S_\infty) = 6$

first term $(a) = 2$

Common ration $(r) = ?$

Now,

$\Rightarrow S_\infty = \frac{a}{1-r}$

or, $6 = \frac{2}{1-r}$

or, $6 - 6r = 2$

or, $-6r = 2 - 6$

or, $+6r = +4$

or, $r = \frac{4}{6} = \frac{2}{3}$

B. ans ⇒ Imp

let $a + ar + ar^2 + \ldots$ to be a G.S

According to question,

$\frac{a}{1-r} = 20$

$a = 20(1-r)$ — ①

again;

$= \frac{a^2}{1-r^2} = 50$

or, $\frac{a \cdot a}{1-r^2} = 50$

or, $\frac{a}{1-r} \cdot \frac{a}{1+r} = 50$

or, $20 \cdot \frac{a}{1+r} = 50$

or, $\frac{a}{1+r} = \frac{50}{20}$

or, $a = \frac{5}{2}(1+r)$ — ②

From ① and ②

$20(1-r) = \frac{5}{2}(1+r)$

$40 - 40r = 5 + 5r$

or, $-40r - 5r = 5 - 40$

or, $-45r = -35$

or, $r = \frac{35}{45} = \frac{7}{9} = 7/9$

① becomes

$a = 20(1 - 7/9)$

$= 20 \cdot (2/9)$

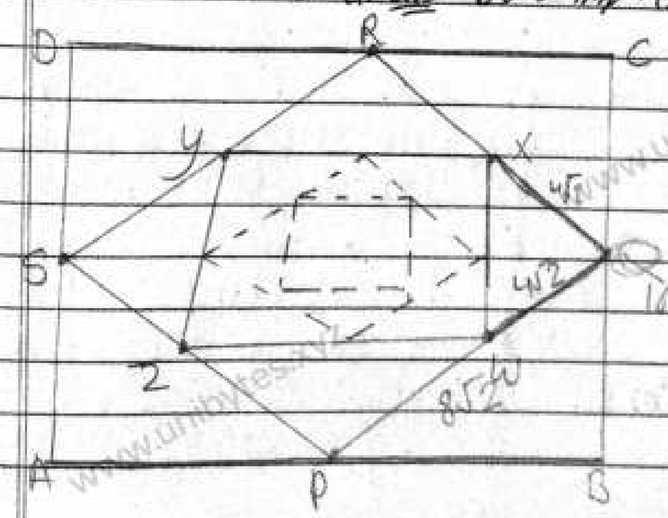
$\therefore a = \frac{40}{9}$

$a + ar + ar^2 + \ldots$ to ∞

i.e $40/9 + 40/9 \times 7/9 + \ldots$ to ∞

i.e $40/9 + 280/81 + \ldots$ to ∞

3. a) ans [v.v. imp. (TU)]



Let ABCD be a square whose each side is 16 cm similarly second, third etc. square are inscribed by square ABCD as show in fig.

For first square ABCD, perimeter $= 4 \times 16 = 64$

For 2nd square PQRS, $PQ = \sqrt{PB^2 + BQ^2}$ (Pythagoras theorem

$= \sqrt{8^2 + 8^2} = 8\sqrt{2}$

For 3rd square WXYZ, $WX = \sqrt{WQ^2 + QX^2}$

$= \sqrt{(4\sqrt{2})^2 + (4\sqrt{2})^2}$

$= \sqrt{32 + 32} = \sqrt{64} = 8$

$\therefore$ Perimeter squares $= 4 \times 16 + 4 \times 8\sqrt{2} + 4 \times 8 + \ldots$ to ∞

$= 64 + 32\sqrt{2} + 32 + \ldots$ to ∞

$a = 64,\ r = \frac{32\sqrt{2}}{64} = \sqrt{2}/2 = \sqrt{2}/(\sqrt{2})^2 = 1/\sqrt{2}$

$\therefore S_\infty = \frac{a}{1-r}$

$= \frac{64}{1 - 1/\sqrt{2}}$

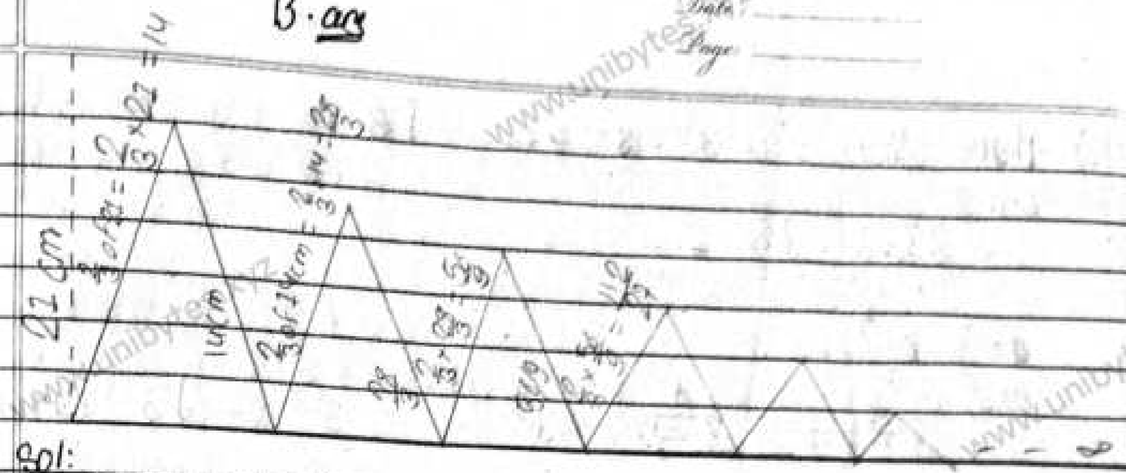
$= \frac{64}{\frac{\sqrt{2}-1}{\sqrt{2}}}$

$= \frac{64\sqrt{2}}{\sqrt{2}-1} \times \frac{\sqrt{2}+1}{\sqrt{2}+1}$

$= \frac{64 \times 2 + 64\sqrt{2}}{2 - 1}$

$= 64(2 + \sqrt{2})$ cm

B. ans


21 cm
14 cm
$\frac{28}{3}$ cm
$\frac{56}{9}$ cm
$\frac{112}{27}$ cm
$\frac{224}{81}$ cm
$\frac{448}{243}$ cm
$\frac{896}{729}$ cm
$\frac{1792}{2187}$ cm
$\frac{3584}{6561}$ cm
$\frac{7168}{19683}$ cm
$\frac{14336}{59049}$ cm
$\frac{28672}{177147}$ cm
$\frac{57344}{531441}$ cm
$\frac{114688}{1594323}$ cm
$\frac{229376}{4782969}$ cm
$\frac{458752}{14348907}$ cm
$\frac{917504}{43046721}$ cm
$\frac{1835008}{129140163}$ cm
$\frac{3670016}{387420489}$ cm
$\frac{7340032}{1162261467}$ cm
$\frac{14680064}{3486784401}$ cm
$\frac{29360128}{10460353203}$ cm
$\frac{58720256}{31381059609}$ cm
$\frac{117440512}{94143178827}$ cm
$\frac{234881024}{282429536481}$ cm
$\frac{469762048}{847288609443}$ cm
$\frac{939524096}{2541865828329}$ cm
$\frac{1879048192}{7625597484987}$ cm
$\frac{3758096384}{22876792454961}$ cm
$\frac{7516192768}{68630377364883}$ cm
$\frac{15032385536}{205891132094649}$ cm
$\frac{30064771072}{617673396283947}$ cm
$\frac{60129542144}{1853020188851841}$ cm
$\frac{120259084288}{5559060566555523}$ cm
$\frac{240518168576}{16677181699666569}$ cm
$\frac{481036337152}{50031545098999707}$ cm
$\frac{962072674304}{150094635296999121}$ cm
$\frac{1924145348608}{450283905890997363}$ cm
$\frac{3848290697216}{1350851717672992089}$ cm
$\frac{7696581394432}{4052555153018976267}$ cm
$\frac{15393162788864}{12157665459056928801}$ cm
$\frac{30786325577728}{36472996377170786403}$ cm
$\frac{61572651155456}{109418989131512359209}$ cm
$\frac{123145302310912}{328256967394537077627}$ cm
$\frac{246290604621824}{984770902183611232881}$ cm
$\frac{492581209243648}{2954312706550833698643}$ cm
$\frac{985162418487296}{8862938119652501095929}$ cm
$\frac{1970324836974592}{26588814358957503287787}$ cm
$\frac{3940649673949184}{79766443076872509863361}$ cm
$\frac{7881299347898368}{239299329230617529590083}$ cm
$\frac{15762598695796736}{717897987691852588770249}$ cm
$\frac{31525197391593472}{2153693963075557766310747}$ cm
$\frac{63050394783186944}{6461081889226673298932241}$ cm
$\frac{126100789566373888}{19383245667680019896796723}$ cm
$\frac{252201579132747776}{58149737003040059690390169}$ cm
$\frac{504403158265495552}{174449211009120179071170507}$ cm
$\frac{1008806316530991104}{523347633027360537213511521}$ cm
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$\frac{83808349429934695030013835770545044123821861280415744}{11607267643568076029800863740835157145948581426107335960080465829988129998909764641}$ cm
$\frac{167616698859869390060027671541090088247643722560831488}{34821802930704228089402591222505471437845744278322007880241397489964389996729293923}$ cm
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$\frac{670466795439477560240110686164360352990574890243325952}{313396226376338052804623320002549242940611698504898070921172577409679509970563645307}$ cm
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$\frac{5363734363515820481920885489314882823924599121946607616}{8461698112152127425724829640068829559396515859632247914871659590052346769205218423289}$ cm
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$\frac{5492463988240200173486986741058440011698789500873326198784}{499654811824470972361625465416424310091804864993237107125256626132998024374799942676792161}$ cm
$\frac{10984927976480400346973973482116880023397579001746652397568}{1498964435473412916884876396249272930275414594979711321375769878398994073124399828030376483}$ cm
$\frac{21969855952960800693947946964233760046795158003493304795136}{4496893306420238750654629188747818790826243784939133964127309635196982219373199484090949449}$ cm
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$\frac{87879423811843202775791787856935040187180632013973219180544}{40472039757782148755891662698730369117436194064452205677145786716772839974358795356818545041}$ cm
$\frac{175758847623686405551583575713870080374361264027946438361088}{121416119273346446267674988096191107352308582193356617031437360150318519923076386070455635123}$ cm
$\frac{351517695247372811103167151427740160748722528055892876722176}{364248357820039338803024964288573322056925746580069851094312080450955559769229158211366905369}$ cm
$\frac{703035390494745622206334302855480321497445056111785753444352}{1092745073460118016409074892865719966170777239740209553282936241352866679307687474634100716107}$ cm
$\frac{1406070780989491244412668605710960642994890112223571506888704}{3278235220380354049227224678597159898512331719220628659848808724058600037923062423902302148321}$ cm
$\frac{2812141561978982488825337211421921285989780224447143013777408}{9834705661140062147681674035791479695537995157661885979546426172175800113769187271706906444963}$ cm
$\frac{5624283123957964977650674422843842571979560448894286027554816}{29504116983420186443045022107374439086613985472985657938639278516527400341307561815120719334889}$ cm
$\frac{11248566247915929955301348845687685143959120897788572055109632}{88512350950260559329135066322123317259841956418956973815917835549582201023922685445362157004667}$ cm
$\frac{22497132495831859910602697691375370287918241795577144110219264}{265536052850781677987405198966369951779525869256870921447753506648746603071768056335086471013999}$ cm
$\frac{44994264991663719821205395382750740575836483591154288220438528}{796608158552345033962215596899109855338577607770612764343260519946239809215304169005259413041997}$ cm
$\frac{89988529983327439642410790765501481151672967182308576440877056}{2389824475656035101886646790697329566015732823311838293029781559838719427645912507015778239125991}$ cm
$\frac{179977059966654879284821581530002962303345934364617152881754112}{7169473426968105305659940372091988698047198469935514879089344679516158282937737521047334717377973}$ cm
$\frac{359954119933309758569643163060005924606691868729234305763508224}{21508420280904315916979820116275966094141595409806544637268034038548474848813212563142004152133919}$ cm
$\frac{719908239866619517139286326120011849213383737458468611527016448}{64525260842712947750939460348827898282424786229419633911804102115645424546439637689426012456401757}$ cm
$\frac{1439816479733239034278572652240023698426767474916937223054032896}{193575782528138843252818381046483694847274358688258901735412306346936273639318913068278037369205271}$ cm
$\frac{2879632959466478068557145304480047396853534949833874446108065792}{580727347584416529758455143139451084541823076064776705206236919040808820917956739204834111107615813}$ cm
$\frac{5759265918932956137114290608960094793707069899667748892216131584}{1742182042752249589275365429418353253625469228194330115618710757122426462753870217614502333322847439}$ cm
$\frac{11518531837865912274228581217920189587414139799335497784432263168}{5226546128256748767826096288255059760876407684582990346856132271367279388261610652843506999968542317}$ cm
$\frac{23037063675731824548457162435840379174828279598670995568864526336}{15679638384770246303478288864765179282629222953748970040568396814101838164784831958530520999905626951}$ cm
$\frac{46074127351463649096914324871680758349656559197341991137729052672}{47038915154310738910434866594295537847887668861246910121705190442305514494354495875591562999716880853}$ cm
$\frac{92148254702927298193828649743361516699313118394683982275458105344}{141116745462932216731304599782886613543662006583740730365115571326916543483063487626774688999150642559}$ cm
$\frac{184296509405854596387657299486723033398626236789367964550916210688}{423350236388796650193913799348659840630986019751222191095346713980749630449190462880324066997451927677}$ cm
$\frac{368593018811709192775314598973446066797252473578735929101832421376}{1270050709166389950581741398045979521892958059253666573286040141942248891347571388640972200992355783031}$ cm
$\frac{737186037623418385550629197946892133594504947157471858203664842752}{3810152127499169851745224194137938565678874177760999719858120425826746674042714165922916602977067349093}$ cm
$\frac{1474372075246836771101258395893784267188989894314943716407329685504}{11430456382497509555235672582413815696936622533282999159574361277480239022128142497768749808931202047279}$ cm
$\frac{2948744150493673542202516791787568534377979788629887432814659371008}{34291369147492528665706017747241446090809867599848997478723083832440717066384427493306249426793606141837}$ cm
$\frac{5897488300987347084405033583575137068755959577259774865629318742016}{102874107442477585997118053241724338272429602799546992436169251497322151199153282479918748280380818425511}$ cm
$\frac{11794976601974694168810067167150274137511919154519549731258637484032}{308622322327432757991354159725173014817288808398640977308507754491966453597459847439756244841142455076533}$ cm
$\frac{23589953203949388337620134334300548275023838309039099462517274968064}{925866966982298273974062479175519044451866425195922931925523263475899360792379542319268734523427365229599}$ cm
$\frac{47179906407898776675240268668601096550047676618078198925034549936128}{2777600900946894821922187437526557133355599275587768795776569790427698082377138626957204103570096695688797}$ cm
$\frac{94359812815797553350480537337202193100095353236156397850069099872256}{8332802702840684465766562312579671400066797826763306387329709371283094247131415880871612310710290087066391}$ cm
$\frac{188719625631595106700961074674404386200190706472312795700138199744512}{24998408108522053397299686937739014200200393480289919161989128113849282741394247642614836932130870261199173}$ cm
$\frac{377439251263190213401922149348808772400381412944625591400276399489024}{74995224325566160191898060813217042600601180440869757485967384341547848224182742927844510796392610783597519}$ cm
$\frac{754878502526380426803844298697617544800762825889251182800552798978048}{224985672976698480575694182439651127801803541322609272457902152024643544672548228783533532389177832350792557}$ cm
$\frac{1509757005052760853607688597395235089601525651778502365601105597956096}{674957018930095441727082547318953383405410623967827817373706456073930634017644686350600597167533497052377671}$ cm
$\frac{3019514010105521707215377194790470179203051303557004731202211195912192}{2024871056790286325181247641956860150216231871903483452121119368221791902052934058051801791502600491157133013}$ cm
$\frac{6039028020211043414430754389580940358406102607114009462404422391824384}{6074613170370858975543742925870580450648695615710450356363358104665375706158802174155405374507801473471399039}$ cm
$\frac{12078056040422086828861508779161880716812205214228018924808844783648768}{18223839511112576926631228777611741351946086847131350069090074313996127118476406522466216123523404420414197117}$ cm
$\frac{24156112080844173657723017558323761433624410428456037849617689567297536}{54671518533337730779893686332835224055838260541394050207270222941988381355429219567398648370570213261242591351}$ cm
$\frac{48312224161688347315446035116647522867248820856912075699235379134595072}{164014555599013192339680058998505672167514781624182150621810668825965144066287658702195945111710639783727774053}$ cm
$\frac{96624448323376694630892070233295045734497641713824151398470758269190144}{492043666797039577018040176995517016502544344872546451865431996477895432198862976106587835335131919351183322159}$ cm
$\frac{193248896646753389261784140466590091468995283427648302796941516538380288}{1476131000391118731054120530986551049507633034617639355596295989433686296596588928319763505005395758053549966477}$ cm
$\frac{386497793293506778523568280933180182937990566855296605593883033076760576}{4428393001173356193162361592959653148522899103852918066788887968301058889789766784958290515016187274160649899431}$ cm
$\frac{772995586587013557047136561866360365875981133710593211187766066153521152}{13285179003520068579487084778878959445568697311558754200366663904903176669369300352874871545048561822481949698293}$ cm
$\frac{1545991173174027114094273123732720731751962267421186422375532132307042304}{39855537010560205738461254336636878336706091934676262601099991714709529008107901058624614635145685467445848894879}$ cm
$\frac{3091982346348054228188546247465441463503924534842372844751064264614084608}{119566611031680617215383762009910635009118275804028787803299975144128587024323703175873843905437056402337546684637}$ cm
$\frac{6183964692696108456377092494930882926007849069684745689502128529228169216}{358699833095041851646151286029731905027354827412086363409899925432385761072971109527621531716311169207012640053911}$ cm
$\frac{12367929385392216912754184989861765852015698139369491379004257058456338432}{1076099499285125554938453858089195715082064482236259090229699776297157283218913328582864595148933507621037920161733}$ cm
$\frac{24735858770784433825508369979723531704031396278738982758008514116912676864}{3228298497855376664815361574267587145246193446708777270689099328891471849656739985748593785446800522863113760485199}$ cm
$\frac{49471717541568867651016739959447063408062792557477965516017028233825353728}{9684895493566129994446084722802761435738580340126331812067297986674415548970219957245781356340401568589341281455597}$ cm
$\frac{98943435083137735302033479918894126816125585114955931032034056467650707456}{29054686480698389983338254168408284307215740020379995436201893960023246646910659871737343068021204705767023844366791}$ cm
$\frac{197886870166275470604066959837788253632251170229911862064068112935301414912}{87164059442095169950014762505224852921647220061139986308605681880069739940731979615212029204063614117301071533100373}$ cm
$\frac{395773740332550941208133919675576507264502340459823724128136225870602829824}{261492178326285509850044287515674558764941660183419958925817045640209219822195938845636087612190842351903214599301119}$ cm
$\frac{791547480665101882416267839351153014529004680919647448256272451741205659648}{784476534978856529550132862547023676294824980550259876777451136920627659466587816536908262836572527055709643797903357}$ cm
$\frac{1583094961330203764832535678702306029058009361839294896512544903482411319296}{2353429604936569588650398587641070928884474941650779630332353410761882978399763449610724788509717581167128931393710071}$ cm
$\frac{3166189922660407529665071357404612058116018723678589793025089806964822638592}{7060288814809708765951195762923212786653424824952338890996060232285648935199290348832174365528152743501386794181130213}$ cm
$\frac{6332379845320815059330142714809224116232037447357179586050179613929645277184}{21180866444429126297853587288769638359960274474857016672988180696856946805597871046496523096584458229504160382543390639}$ cm
$\frac{12664759690641630118660285429618448232464074894714359172100359227859290554368}{63542599333287378893560761866308915079880823424571050018964542090570840416793613139489569289753374688512481147630171917}$ cm
$\frac{25329519381283260237320570859236896464928149789428718344200718455718581108736}{190627797999862136680682285598926745239642470273713150056893626271712521250380839418468707869260124065537443442890515751}$ cm
$\frac{50659038762566520474641141718473792929856299578857436688401436911437162217472}{571883393999586410042046856796780235718927410821139450170680878815137563751142518255406123607780372196612330328671547253}$ cm
$\frac{101318077525133040949282283436947585859712599157714873376802873822874324434944}{1715650181998759230126140570390340707156782232463418350512042636445412691253427554766218370323340116589836990986014641759}$ cm
$\frac{202636155050266081898564566873895171719425198315429746753605747645748648869888}{5147950545996277690378421711171022121470346697390255051536127909336238073760282664298655110970020349769510972958043925277}$ cm
$\frac{405272310100532163797129133747790343438850396630859493507211495291497297739776}{15443851637988833071135265133513066364411040092170765154608383727008714221280848992895965332910061049308532918874131775831}$ cm
$\frac{810544620201064327594258267495580686877700793261718987014422990582994595479552}{46331554913966499213405795400539199093233120276512295463825151181026142663842546978687895998730183147925598756622395327493}$ cm
$\frac{1621089240402128655188516534991161373755401586523437974028845981165988190959104}{138994664741899497640217386201617597279699360829536886391475453543078427991527640936063687996190549443776796269867185982479}$ cm
$\frac{3242178480804257310377033069982322747510803173046875948057691962331976381918208}{416983994225698492920652158604852791838098082488610659174426360629235283974582922808191063988571648331330388809601557947437}$ cm
$\frac{6484356961608514620754066139964645495021606346093751896115383924663952763836416}{1250951982677095478761956475814558375514294247465831977523279081887705851923748768424573191965714944993991166428804673842311}$ cm
$\frac{12968713923217029241508132279929290990043212692187503792230767849327905527672832}{3752855948031286436285869427443675126542882742397495932569837245663117555771246305273719575897144834981973499286412021526933}$ cm
$\frac{25937427846434058483016264559858581980086425384375007584461535698655811055345664}{11258567844093859308857608282330025379628648227192487797709511736989352667313738915820158727691434504945920497859236064580799}$ cm
$\frac{51874855692868116966032529119717163960172850768750015168923071397311622110691328}{33775703532281577926572824846990076138885944681577463393128535210968057001941216747460476183074303514837761493577708193742397}$ cm
$\frac{103749711385736233932065058239434327920345701537500030337846142794623244221382656}{101327110596844733779718474540970228416657833044732390179385605632904171005823650242381428549222910544513284480733124581227191}$ cm
$\frac{207499422771472467864130116478868655840691403075000060675692285589246488442765312}{303981331790534201339155423622910685249973499134197170538156816898712513017470950727144285647668731633539853442199373743681573}$ cm
$\frac{414998845542944935728260232957737311681382806150000121351384571178492976885530624}{911943995371602604017466270868731055749920497402591511614470450696137539052412852181432856942006194900619560326598121230044719}$ cm
$\frac{829997691085889871456520465915474623362765612300000242702769142356985953771061248}{2735831986114807812052398812606193167249761492207774534843411352088412617157238556544298570826018584701858680979794363690134157}$ cm
$\frac{1659995382171779742913040931830949246725531224600000485405538284713971907542122496}{8207495958344423436157196437818579501749284476623323604530234056265237851471715669632895712478055754105575042939382091070402471}$ cm
$\frac{3319990764343559485826081863661898493451062449200000970811076569427943815084244992}{24622487875033270308471589313455738505247853429869970813590702168795713554415146908898687137434167262316725128818146273211207413}$ cm
$\frac{6639981528687118971652163727323796986902124898400001941622153138855887630168489984}{73867463625099810925414767940367215515743560289609912440772106506387140663245440726696061412302501786950175386454438819633622239}$ cm
$\frac{13279963057374237943304327454647593973804249796800003883244306277711775260336979968}{221602390875299432776244303821101646547230680868829737322316319519161421989736322180088184236907505360850526159363316458900866717}$ cm
$\frac{26559926114748475886608654909295187947608499593600007766488612555423550520673959936}{664807172625898298328732911463304939641692042606489211966948958557484265969208966540264552710722516082551578478089949376702600151}$ cm
$\frac{53119852229496951773217309818590375895216999187200015532977225110847101041347919872}{1994421517877694894986198734389914818925076127819467635900846875672452797907626899620793658132167548247654735434269848130107800453}$ cm
$\frac{106239704458993903546434619637180751790433998374400031065954450221694202082695839744}{5983264553633084684958596203169744456775228383458402907702540626017358393722880698861380974396502644742962205302809544390323401359}$ cm
$\frac{212479408917987807092869239274361503580867996748800062131908900443388404165391679488}{17949793660899252054875788609509233370325685150375208723107621878052075181168642096584142923189507934228886615908428633170970203077}$ cm
$\frac{424958817835975614185738478548723007161735993497600124263817800886776808330783358976}{53849380982697756164627365828527700110977055450125626169322865634156225543505926289752428769568523802686659847725285899512910609231}$ cm
$\frac{849917635671951228371476957097446014323471986995200248527635601773553616661566717952}{161548142948093268493882097485583100332931166350376878507968596902468676630517778869257286308705571408059979543175857698538731827693}$ cm
$\frac{1699835271343902456742953914194892028646943973990400497055271203547107233323133435904}{484644428844279805481646292456749300998793499051130635523905790707406029891553336607771858926116714224179938629527573095616195482079}$ cm
$\frac{3399670542687804913485907828389784057293887947980800994110542407094214466646266871808}{1453933286532839416444938877370247902996380497153391906571717372122218089674660009823315578778350142672539815888582719286848586446237}$ cm
$\frac{6799341085375609826971815656779568114587775895961601988221084814188428933292533743616}{4361799859598517249334816632110743708989141491460175719715152116366654269023980029469946736335050427017619447665748157860545758338711}$ cm
$\frac{13598682170751219653943631313559136229175551791923203976442169628376857866585067487232}{13085399578795551747004449896332231126967424474380527158145456349099962807071940088409840209005151281052858343997244473581636275016133}$ cm
$\frac{27197364341502439307887262627118272458351103583846407952884339256753715733170134974464}{39256198736386655241013349688996693380902273423141581474436369047299888421215820265229520627015453843158575031991733420744908825048399}$ cm
$\frac{54394728683004878615774525254236544916702207167692815905768678513507431466340269948928}{117768596209159965723040049066990080142706820269424744423309107141899665263647460795588561881046361529475725095975200262234726475145197}$ cm
$\frac{108789457366009757231549050508473089833404414335385631811537357027014862932680539897856}{353305788627479897169120147200970240428120460808274233269927321425698995790942382386765685643139084588427175287925600786704179425435591}$ cm
$\frac{217578914732019514463098101016946179666808828670771263623074714054029725865361079795712}{1059917365882439691507360441602910721284361382424822699809781964277096987372827147160297056929417253765281525863776802359112538306306773}$ cm
$\frac{435157829464039028926196202033892359333617657341542527246149428108059451730722159591424}{3179752097647319074522081324808732163853084147274468099429345892831290962118481441480891170788251761295844577591330407077337614918920319}$ cm
$\frac{870315658928078057852392404067784718667235314683085054492298856216118903461444319182848}{9539256293941957223566243974426196491559252441823404298288037678493872886355444324442673512364755283887533732773991221231012844756760957}$ cm
$\frac{1740631317856156115704784808135569437334470629366170108984597712432237806922888638365696}{28617768881825871670698731923278589474677757325470212894864113035481618659066332973328020537094265851662601198319973663693038534270282871}$ cm
$\frac{3481262635712312231409569616271138874668941258732340217969195424864475613845777276731392}{85853306645477614012096195769835768424033272976410636684592339106444855977198998919984061611282797554987803594959920990079115602810848613}$ cm
$\frac{6962525271424624462819139232542277749337882517464680435938390849728951227691554553462784}{257559919936432842036288587309506305272099818929231910053776317319334567931596996759952184833848392664963410784879762970237346808432545839}$ cm
$\frac{13925050542849248925638278465084555498675765034929360871876781699457902455383109106925568}{772679759809298526108865761928518915816299456787695730161328951958003703794790990279856554501545177994890232354639288910711040425297637517}$ cm
$\frac{27850101085698497851276556930169110997351530069858721743753563398915804910766218213851136}{2318039279427895578326597285785556747448898370363087190483986855874011111384372970839569663504635533984670696963917866732133121275892912551}$ cm
$\frac{55700202171396995702553113860338221994703060139717443487507126797831609821532436427702272}{6954117838283686734979791857356670242346695111089261571451960567622033334153118912518708990513906601953012090891753600196399363827678737653}$ cm
$\frac{111400404342793991405106227720676443989406120279434886975014253595663219643064872855404544}{20862353514850060204939375572069010727040085333267784714355881702866100002459356737555126971541719805859036272675260800589198091482036212959}$ cm
$\frac{

4.a) Prove that $3^{\frac{1}{3}} \cdot 3^{\frac{1}{9}} \cdot 3^{\frac{1}{27}} \cdots = \sqrt{3}$

L.H.S

$= 3^{\frac{1}{3}} \cdot 3^{\frac{1}{9}} \cdot 3^{\frac{1}{27}} \cdots$

$= 3^{\frac{1}{3}+\frac{1}{9}+\frac{1}{27}+\cdots \text{ to } \infty}$

$a = \frac{1}{3}, r = \frac{\frac{1}{9}}{\frac{1}{3}} = \frac{1}{3}$

now;

$3^{\frac{\frac{1}{3}}{1-\frac{1}{3}}} = 3^{\frac{\frac{1}{3}}{\frac{3-1}{3}}} = 3^{\frac{1}{2} \times \frac{3}{2}} = 3^{\frac{1}{2}} = \sqrt{3}$ R.H.S proved

sol: B. ans

First year income $(t_1) = 64$

Second year income $(t_2) = \frac{3}{4} \times 64 = 48$

Third year income $(t_3) = \frac{3}{4} \times 48 = 36$

Common ratio $(r) = \frac{t_3}{t_2} = 36/48 = 3/4$

$S_\infty = ?$

now;

$S_\infty = \frac{a}{1-r} = \frac{64}{1-\frac{3}{4}} = \frac{64}{\frac{4-1}{4}} = \frac{64}{\frac{1}{4}} = Rs256$

5. a) i. ans

i) $1.3 + 3.5 + 5.7 + \cdots$

sol:

p^{th} term of $1, 3, 5 \ldots$

$a = 1, d = 3-1 = 2$

$t_n = a + (n-1)d$

$= 1 + (n-1).2$

$= 1 + 2n - 2$

$= 2n - 1$

n^{th} term of $3, 5, 7 \ldots$

$a = 3, d = 5-3 = 2$

$\therefore t_n = a + (n-1)d$

$= 3 + (n-1) \times 2$

$= 3 + 2n - 2$

$= 2n + 1$

Note:

$\Sigma_1 = n$

$\Sigma_2 = 2n$

$\therefore n^{th}$ term of $1.3 + 3.5 + 5.7 + \cdots$

$= (2n-1)(2n+1)$

$\therefore t_n = 4n^2 - 1$ ans

Sum,

$S_n = \Sigma t_n$

$= \Sigma(4n^2 - 1)$

$= \Sigma 4n^2 - \Sigma 1$

$= 4\Sigma n^2 - n$

$= \frac{\cancel{4}^{2} \times n(n+1)(2n+1)}{\cancel{6}\,3} - n$

$= \frac{2n(2n^2 + n + 2n + 1) - 3n}{3}$

$= \frac{4n^3 + 6n^2 + 2n - 3n}{3}$

$= \frac{1}{3}(4n^3 + 6n^2 - n)$

$= \frac{n}{3}(4n^2 + 6n - 1)$

c) Sum of infinity : $a/x + b/x^2 + a/x^3 + b/x^4 + \cdots \infty$ where $|x| > 1$

sol:

$a/x + b/x^2 + a/x^3 + b/x^4 \cdots \infty$

$(a/x + a/x^3 + \cdots \infty) + (b/x^2 + b/x^4 + \cdots \infty)$

$\therefore a = a/x$ $\qquad \therefore a = b/x^2$

$r = \frac{a/x^3}{a/x} = \frac{1}{x^2}$ $\qquad r = \frac{t_2}{t_1} = \frac{b/x^4}{b/x^2} = \frac{1}{x^2}$

Now;

$S_\infty = \left(\frac{1}{a-r}\right) + \left(\frac{1}{a-r}\right)$

$$= \left(\frac{a/x}{1-\frac{1}{x^2}}\right) + \left(\frac{b/x^2}{1-\frac{1}{x^2}}\right)$$

$$= \frac{a/x}{\frac{x^2-1}{x^2}} + \frac{b/x^2}{\frac{x^2-1}{x^2}}$$

$$= \frac{a \times x}{x^2-1} - \frac{b}{x^2-1}$$

$$= \frac{ax}{x^2-1} + \frac{b}{x^2-1}$$

$$= \frac{ax+b}{x^2-1}$$

5.b iii) ans

Given;

$(x+b)+(x^2+2b)+(x^3+3b)\ldots$

$(x+x^2+x^3\ldots$ to n term$)$ + $(b+2b+3b\ldots$ to n term$)$

$a=x,\ r=t_2/t_1=x^2/x=x$ + ~~a=b~~, $b(1+2+3\ldots$ to n term$)$

$a=1,\ d=t_2-t_1=2-1=1$

now;

$S_n = \frac{a(r^n-1)}{r-1}$ + ~~b~~ $b.\ S_n = \frac{n}{2}[2a+(n-1)d]$

$= \frac{x(x^n-1)}{x-1} + b.\frac{n}{2}[2\times1+(n-1)1]$

$= \frac{x(x^n-1)}{x-1} + b.\frac{n}{2}(2+n-1)$

$= \frac{x(x^n-1)}{x-1} + b.\frac{n}{2}(n+1)$ ans

c) If $|x|<1$, $y=x+x^2+x^3+\ldots\infty$, prove that $x=\frac{y}{1+y}$

Sol:

$a=x,\ r=t_2/t_1=x^2/x=x$

now;

$S_\infty = \frac{a}{1-r}$

$\Rightarrow y = \frac{x}{1-x}$

or, $y-xy=x$

or, $-xy-x=-y$

or, $+(xy+x)=+y$

or, $x(1+y)=y$

or, $x=\frac{y}{1+y}$ R.H.S

proved

Exercise 3.4 ⇒ five marks fixed

1. a) ~~ans~~ T.U

Since H is the H.M between a and b

$$H = \frac{2ab}{a+b}$$

L.H.S

$$= \frac{1}{H-a} + \frac{1}{H-b}$$

$$= \frac{1}{\frac{2ab}{a+b}-a} + \frac{1}{\frac{2ab}{a+b}-b}$$

$$= \frac{1}{\frac{2ab-a(a+b)}{a+b}} + \frac{1}{\frac{2ab-b(a+b)}{a+b}}$$

$$= \frac{1\times a+b}{2ab-a^2-ab} + \frac{1\times a+b}{2ab-ab-b^2}$$

$$= \frac{a+b}{ab-a^2} + \frac{a+b}{ab-b^2}$$

$$= \frac{a+b}{-a(a-b)} + \frac{a+b}{b(a-b)}$$

$$= \frac{b(a+b)+(-a)(a+b)}{-ab(a-b)}$$

$$= \frac{ab+b^2-a^2-ab}{-ab(a-b)}$$

$$= \frac{+(a^2+b^2)}{+ab(a-b)}$$

$$= \frac{(a+b)(a-b)}{ab(a-b)}$$

$$= \frac{a+b}{ab}$$

$$= \frac{a}{ab} + \frac{b}{ab}$$

$$= \frac{1}{b} + \frac{1}{a} = \frac{1}{a} + \frac{1}{b}$$ R.H.S proved

1 b. ~~ans~~

Since A and H be the A.M and H.M between the quantities a and b. So. $A = \frac{a+b}{2}$, $H = \frac{2ab}{a+b}$

L.H.S $\frac{a-A}{a-H} \times \frac{b-A}{b-H}$

$$= \frac{a-\left(\frac{a+b}{2}\right)}{a-\left(\frac{2ab}{a+b}\right)} \times \frac{b-\left(\frac{a+b}{2}\right)}{b-\left(\frac{2ab}{a+b}\right)}$$

Formula

$A.M \times H.M = G.M^2$

$$\Rightarrow \frac{\frac{2a-a-b}{2}}{\frac{a^2+ab-2ab}{a+b}} \times \frac{\frac{2b-a-b}{2}}{\frac{ab+b^2-2ab}{a+b}}$$

$$\Rightarrow \left(\frac{\cancel{a-b}}{2} \times \frac{a+b}{a(\cancel{a-b})}\right) \times \left(\frac{\cancel{a-b}}{2} \times \frac{a+b}{b(\cancel{a-b})}\right)$$

$$\Rightarrow \frac{\cancel{a+b}}{2} \times \frac{1}{a} \times \frac{\cancel{a+b}}{2} \times \frac{1}{b}$$

$$\Rightarrow \frac{a+b}{2} \times \frac{a+b}{2ab}$$

$$\Rightarrow A \times \left(\frac{2ab}{a+b}\right)^{-1} = A \times \frac{1}{H} = \frac{A}{H}$$ R.H.S

C. (TU)

Let a and b be two numbers. then $AM = \frac{a+b}{2}$, $GM = \sqrt{ab}$, $HM = \frac{2ab}{a+b}$

According to question

$AM = GM + 4$	$GM = HM + 3.2$
$GM = AM - 4$	$HM = GM - 3.2$

We know that;

$A.M \times H.M = (G.M)^2$

$\Rightarrow (GM+4)+(GM-3.2) = (G.M)^2$

or, $(G.M)^2 - 3.2GM + 4GM - 12.8 = G.M^2$

$0.8\,GM = 12.8$

$GM = \frac{12.8}{0.8}$

$GM = 16$

$\sqrt{ab} = 16$

Squaring

$ab = 256$, $b = \frac{256}{a}$

$A.M = G.M + 4$

$A.M = 16 + 4$

$\frac{a+b}{2} = 20$

$a+b = 40$

Applyin ①

$a + \frac{256}{a} = 40$

or, $\frac{a^2+256}{a} = 40$

or, $a^2 + 256 = 40a$

or, $a^2 - 40a + 256 = 0$

or, $a^2 - 32a - 8a + 256 = 0$

or, $a(a-32) - 8(a-32) = 0$

or, $(a-32)(a-8) = 0$

$\therefore a = 32, 8$

When; $a = 32$, $b = 256/32 = 8$

When; $a = 8$, $b = 256/8 = 32$

$\therefore$ Required number are:-

32, 8 ~~and~~ or 8, 32

2. a) i) ans

Since a, b, c are in GP

Then; $G.M(b) = \sqrt{ac}$

$Am(x) = \frac{a+b}{2}$

$Am(y) = \frac{b+c}{2}$

LHS

$= a/x + c/y$

$= \frac{a}{\frac{a+b}{2}} + \frac{c}{\frac{b+c}{2}}$

$= \frac{2a}{a+b} + \frac{2c}{b+c}$

$= 2\left[\frac{a}{a+b} + \frac{c}{b+c}\right]$

$= 2\left(\frac{a(b+c) + c(a+b)}{(a+b)(b+c)}\right)$

$= 2\frac{(ab+ac+ac+bc)}{ab+ac+b^2+bc}$

$= 2 \cdot \frac{ab+2ac+bc}{ab+ac+ac+bc}$ [$\because b = \sqrt{ac}$]

$= 2 \cdot \frac{ab+2ac+bc}{ab+2ac+bc}$

$= 2 \cdot 1$

$= 2$ proved

$b = \sqrt{ac}$ ii) ans

$b^2 = ac$, $x = \frac{a+b}{2}$, $y = \frac{b+c}{2}$

L.H.S $= \frac{1}{x} + \frac{1}{y}$

$= \frac{1}{\frac{a+b}{2}} + \frac{1}{\frac{b+c}{2}}$

$= \frac{2}{a+b} + \frac{2}{b+c}$

$= 2\left[\frac{b+c+a+b}{(a+b)(b+c)}\right]$

$= \frac{2(a+2b+c)}{ab+ac+b^2+bc}$

$= \frac{2(a+2b+c)}{ab+2b^2+bc}$

$= \frac{2(a+2b+c)}{b(a+2b+c)}$

$= \frac{2}{b}$ ans

3. b) ans

Sol:

$b = \frac{a+c}{2}$, $y = \sqrt{xz}$ $\therefore y^2 = xz$

LHS

$= x^{b-c} \cdot y^{c-a} \cdot z^{a-b}$

$= x^{\frac{a+c}{2}-c} \cdot y^{c-a} \cdot z^{a-\frac{a+c}{2}}$

$= x^{\frac{a-c}{2}} \cdot y^{c-a} \cdot z^{\frac{a-c}{2}}$

$= (xz)^{\frac{a-c}{2}} \cdot y^{c-a}$

$= (y^2)^{\frac{a-c}{2}} \cdot y^{c-a}$

$= y^{a-c+c-a}$

$= y^0$

$= 1$ proved

3. a) TU

$\frac{a}{b+c}, \frac{b}{c+a}, \frac{c}{a+b}$ are in H.P

Sol:

Since a, b, c are in H.P

then, H.M $(b) = \frac{2ac}{a+c} \Rightarrow 2ac = b(a+c)$

$= ab + bc$

we show,

$\frac{a}{b+c}, \frac{b}{c+a}, \frac{c}{a+b}$ are in HP.

$\therefore \frac{b}{c+a} = \frac{2 \cdot \frac{a}{b+c} \cdot \frac{c}{a+b}}{\frac{a}{b+c} + \frac{c}{a+b}}$ $\left[\because b = \frac{2(a).(c)}{(a)+(c)}\right]$

$= \frac{2ac}{\frac{(b+c)(a+b)}{a(a+b)+c(b+c)}}$

$= \frac{2ac}{\frac{(b+c)(a+b)}{a(a+b)+c(b+c)}}$

$= \frac{2ac}{a(a+b)+c(b+c)}$

$= \frac{2ac}{a^2+ab+bc+c^2}$

$= \frac{b(a+c)}{a^2+2ac+c^2}$

$= \frac{b(a+c)}{(a+c)^2}$

$= \frac{b}{a+c}$

$\therefore \frac{b}{c+a} = \frac{2 \cdot \frac{a}{b+c} \cdot \frac{c}{a+b}}{\frac{a}{b+c} + \frac{c}{a+b}}$

$\therefore \frac{a}{b+c}, \frac{b}{c+a}, \frac{c}{a+b}$ are in H.P

Sol: 2.b) ans

Since p^{th}, q^{th}, r^{th} term of a H.P be a, b, c

$\therefore p^{th}, q^{th}, r^{th}$ term of A.P be $\frac{1}{a}, \frac{1}{b}, \frac{1}{c}$

$t_p = \frac{1}{a}$ | $t_q = 1/b$ | $t_r = 1/c$

$t_1 + (p-1)d = 1/a$ — ① | $t_1 + (q-1)d = 1/b$ — ② | $t_1 + (r-1)d = 1/c$ — ③

Subtracting ① by ②

$t_1 + (p-1)d = 1/a$

$-\ t_1 + (q-1)d = 1/b$

$(p-1)-(q-1)d = \frac{1}{a} - \frac{1}{b}$

$\Rightarrow (p-q)d = \frac{b-a}{ab}$

Or, $(p-q) = \frac{b-a}{abd}$ — Ⓐ

Subtracting ③ by ②

$t_1 + (q-1)d = 1/b$

$-\ t_1 + (r-1)d = 1/c$

$(q-1)(r-1)d = \frac{1}{b} - \frac{1}{c}$

$\Rightarrow (q-r)d = \frac{c-b}{bc}$

$\Rightarrow (q-r) = \frac{c-b}{bcd}$

(III) − ①

$t_1 + (r-1)d = 1/c$

$-\ t_1 + (p-1)d = 1/a$

$(r-1)(p-1)d = \frac{1}{c} - \frac{1}{a}$

$(r-p) = \frac{a-c}{cad}$ — (c)

L.H.S

$bc(q-r) + ca(r-p) + ab(p-q) = 0$

$= \cancel{bc} \cdot \frac{c-b}{\cancel{bc}d} + \cancel{ca} \cdot \frac{a-c}{\cancel{ca}d} + \cancel{ab} \cdot \frac{b-a}{\cancel{ab}d}$

$= \frac{c-b+a-c+b-a}{d} = \frac{0}{d} = 0$ R.H.S

Given:

$p = \frac{q+r}{2}$

$2p = q+r$

C. ans

$q = \sqrt{rp}$

We have to show;

$r = \frac{2pq}{p+q}$

R.H.S $= \frac{2pq}{p+q}$

$= \frac{(q+r)q}{p+q}$

$= \frac{q^2+qr}{p+q}$

$= \frac{rp+qr}{p+q}$

$= \frac{r\cancel{(p+q)}}{\cancel{(p+q)}}$

$= r$ L.H.S

Used letter (AFE) = 3!

Other remaining place (AFE), R, C, N, G

$5! = 60$

$2!$

$= 60 \times 6 = 360$

Given;

3. a)ii) ans

$\frac{b+a}{b-a} + \frac{b+c}{b-c}$

$\frac{(b+a)(b-c) + (b+c)(b-a)}{(b-a)(b-c)}$

$= \frac{b^2 - \cancel{bc} + \cancel{ab} - ac + b^2 - \cancel{ab} + \cancel{bc} - ac}{(b-a)(b-c)}$

$b = \frac{2ac}{a+c}$

$b(a+c) = 2ac$

$= \frac{2b^2 - 2ac}{b^2 - bc - ab + ac}$

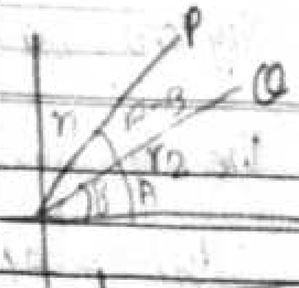
$= \frac{2(b^2 - ac)}{b^2 - b(c+a) + ac}$

$= \frac{2(b^2 - ac)}{b^2 - 2ab + ac}$

$\frac{2(b^2 - ac)}{b^2 - ac}$

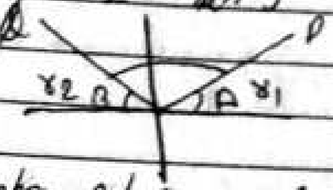
$= 2 \cdot 1$

$= 2$ R.H.S



proved $\boxed{\vec{OQ} \times \vec{OP}}$

$\vec{OP} \times \vec{OQ}$



3.c) If three positive number a,b,c are in H.P; then show that

Sol: $a^4 + c^4 > 2b^4$

Since a,b,c are in HP, $\therefore b = \frac{2ac}{a+c}$

Firstly, we prove that $b^2 < ac$

For this, $ac - b^2 = ac - \left(\frac{2ac}{a+c}\right)^2 =$

$= \frac{ac(a+c)^2 - 4a^2c^2}{(a+c)^2}$

$= \frac{ac(a+c)^2 - 4ac}{(a+c)^2}$

$= ac\left(\frac{a-c}{a+c}\right)^2 > 0$

$\therefore b^2 < ac$ (i.e $ac > b^2$)

Now;

A.M between a^4 and c^4 > G.M between a^4 and c^4

i.e $\frac{a^4+c^4}{2} > \sqrt{a^4c^4}$

$\therefore \frac{a^4+c^4}{2} > a^2c^2$

$= a^4 + c^4 > 2(ac)^2$

$= \cancel{a^4+c^4 > 2(ac)^2}$

$\frac{\vec{P} \cdot \vec{Q}}{|\vec{P}||\vec{Q}|}$

$|\vec{OQ} \times \vec{OP}|$

$|OQ||OP|$

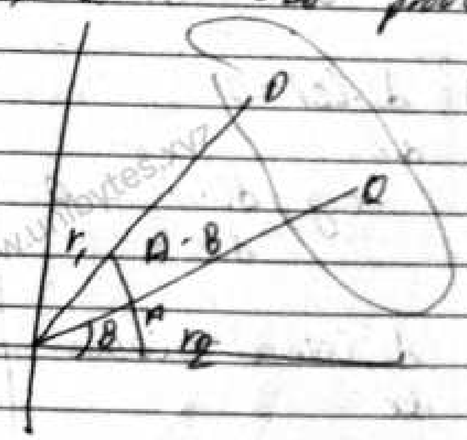
$\vec{OP} = (r_1\cos A, r_1\sin A, 0)$

$\vec{OQ} = \cancel{(-r_2\sin B}$

$\cancel{r_2\cos B, r_2\sin B}$

$\Rightarrow a^4 + c^4 > 2(b^2)^2$

$a^4 + c^4 > 2b^4$ proved



$(r_1\cos A, r_1\sin A)(r_2\cos B, r_2\sin B)$

Taylors Series

The process of differentation is called derivative.

$$\frac{d(y)}{dx} = \frac{df(x)}{dx}$$

Formula

Power Rule; $\frac{d}{dx}(x^n) = n.x^{n-1}$

e.g; $\frac{d(x^{10})}{dx} = 10x^9$

$\frac{d}{dx}(c.x^n) = c.\frac{d(x^n)}{dx} = c.n.x^{n-1}$

e.g; $\frac{d}{dx}(2.x^4) = 2.\frac{d}{dx}x^4 = 8x^3$

1) $\frac{d}{dx}(c) = 0$

e.g; $\frac{d(5)}{dx} = 0$

$\frac{d}{dx}(\sin x) = \cos x$

$\frac{d}{dx}(\cos x) = -\sin x$

- $\frac{d}{dx}(\tan x) = \sec^2 x$
- $\frac{d}{dx}(\sec x) = \tan x \cdot \sec x$
- $\frac{d}{dx}(\operatorname{cosec} x) = -\operatorname{cosec} x \cdot \cot x$
- $\frac{d}{dx}(\cot x) = -\operatorname{cosec}^2 x$

- $\frac{d}{dx}(u+v) \Rightarrow \frac{d}{dx}(u) + \frac{d}{dx}(v)$
- $\frac{d}{dx}\left(\frac{u}{v}\right) \Rightarrow \frac{v.\frac{d(u)}{dx} - u\frac{d(v)}{dx}}{v^2}$

Examples: $\frac{d}{dx}(5x^2 - 2x^3 + 3x^2)$

$\Rightarrow \frac{d5x^2}{dx} - \frac{d(2x^3)}{dx} + \frac{d(3x^2)}{dx}$

$\Rightarrow 10x - 6x^2 + 6x$

$\Rightarrow 16x - 6x^2$

5) Product rule:

$\frac{d}{dx}(u \cdot v) = u.\frac{d(v)}{dx} + v.\frac{d}{dx}(u)$

6) i) $\frac{d}{dx}(e^x) \Rightarrow e^x$

ii) $\frac{d}{dx}e^{mx} \Rightarrow m.e^{mx}$

8) i) $\frac{d}{dx}\log x = \frac{1}{x}$

ii) $\frac{d}{dx}\log^{mx} \Rightarrow m.\frac{1}{mx}$

* If $y = f(x) = 4x^4 - 7x^3 + 8x - 5$

i) $\frac{dy}{dx} = f'(x) = 16x^3 - 21x^2 + 8$

ii) $\frac{d^2y}{dx^2} = f''(x) = 48x^2 - 42x$

iii) $\frac{d^3y}{dx^3} = f'''(x) = 96x - 42$

iv) $\frac{d^4y}{dx^4} = f^{iv}(x) = 96$

v) $\frac{d^5y}{dx^5} = f^{v}(x)\ 0$

Taylor's Series at $x = a$

$f(x) = f(a) + (x-a)f'(a) + (x-a)^2 \cdot \frac{f''(a)}{2!} + \ldots$

Taylor's Series at $x = 0$ (Maclaurin Series).

$f(x) = f(0) + xf'(0) + \frac{x^2}{2!}f''(0) + \frac{x^3}{3!}f'''(0) + \ldots$

Q) Find the maclaurin's series of the function $f(x) = \sin x$

Sol:

$f(x) = \sin x \quad \therefore f(0) = 0$

$f'(x) = \cos x \quad \therefore f'(0) = 1$

$f''(x) = -\sin x \quad \therefore f''(0) = 0$

$f'''(x) = -\cos x \quad \therefore f'''(0) = -1$

$f^{iv}(x) = \sin x \quad \therefore f^{iv}(0) = 0$

$f^{v}(x) = \cos x \quad \therefore f^{v}(x) = 1$

Using Maclaurin's Series

$\Rightarrow f(x) = f(0) + xf'(0) + \frac{x^2}{2!}f''(0) + \frac{x^3}{3!}f'''(0) + \frac{x^4}{4!}f^{iv}(0) + \frac{x^5}{5!}f^{v}(0) + \ldots$

$\Rightarrow \sin x = 0 + x \times 1 + \frac{x^2}{2!} \times 0 + \frac{x^3}{3!} \times (-1) + \frac{x^4}{4!} \times 0 + \frac{x^5}{5!} \times 1 + \ldots$

$\Rightarrow \sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} \ldots$ ans

The Taylor Series about $x = a$. $f(x) = f(a) + (x-a)f'(a) + \frac{(x-a)^2}{2!}f''(a) + \ldots$ (Infinite Formula $= a$)

The Taylor series about $x = a$

$\Rightarrow f(x) = f(0) + xf'(0) + \frac{x^2}{2!}f''(0) + \frac{x^3}{3!}f'''(0) + \ldots$

$(a = 0)$ maclaurin series.

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Exponential Series: A series of the form

$$e^x = 1 + \frac{x}{1!} + \frac{x^2}{2!} + \frac{x^3}{3!} + \cdots + \frac{x^n}{n!}$$ is called an exponential series.

Exercise 3.5

1.a) Find the value of $\frac{e - e^{-1}}{2}$

we have,

$$e^x = 1 + \frac{x}{1!} + x^2/2! + x^3/3! + x^4/4! + \cdots$$

Putting $x = 1$ and -1

$$e^1 - e^{-1} = \left(1^2 + \frac{1^1}{1!} + \frac{1^2}{2!} + \cdots\right) - \left(1^2 + \frac{(-1)^1}{1!} + \frac{(-1)^2}{2!} + \cdots\right)$$

$$= \left(1 + \frac{1^1}{1!} + \frac{1^2}{2!} + \cdots\right) - \left(1 - \frac{1}{1!} + \frac{1}{2!} - \frac{1}{3!} + \cdots\right)$$

$$= \cancel{1} + \frac{1}{1!} + \frac{\cancel{1}}{2!} + \cdots - \cancel{1} + \frac{1}{1!} - \frac{\cancel{1}}{2!} + \frac{1}{3!}$$

$$= \frac{2}{1!} + \frac{2}{3!} + \frac{2}{5!} + \cdots$$

$$= 2\left(\frac{1}{1!} + \frac{1}{3!} + \frac{1}{5!} \cdots\right)$$

$$\therefore \frac{e^1 - e^{-1}}{2} = \frac{1}{1!} + \frac{1}{3!} + \frac{1}{5!} + \cdots \text{ ans}$$

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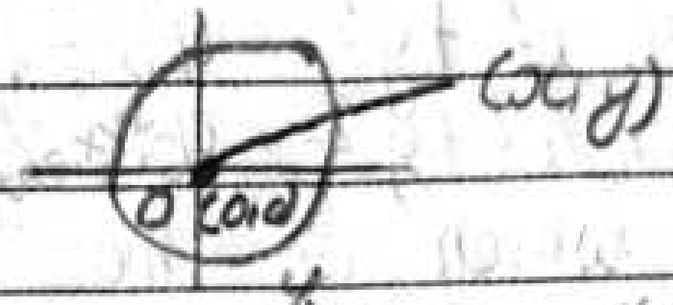
Chapter 5 Analytical Geometry

Circle : Equation of Circle (For more see the book)

i) Centre at Origin (0,0)

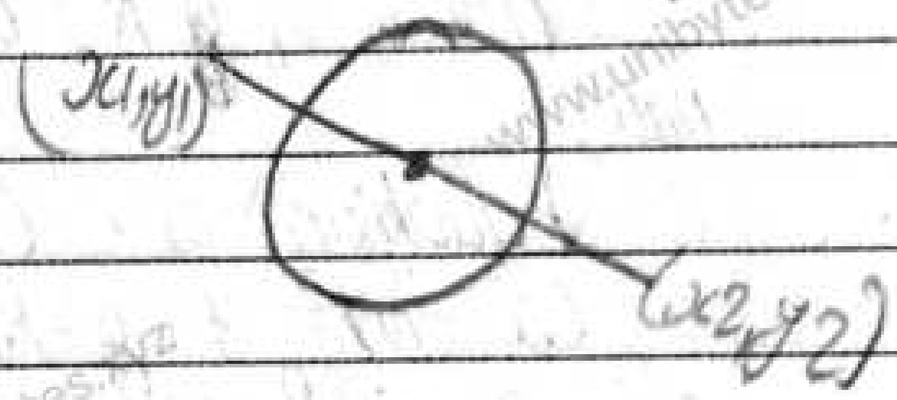


$\Rightarrow x^2+y^2=r^2$

ii) Center at (h,k)

$\Rightarrow (x-h)^2+(y-k)^2=r^2$

iii) Diameter form



$\Rightarrow (x-x_1)(x-x_2)+(y-y_1)(y-y_2)$

iv) General Form

$\rightarrow$ ~~x^2+y^2+~~ $x^2+y^2+2gx+2fy+c=0$

where centre $(h,k)=(-g,-f)$ and radius $(r)=\sqrt{g^2+f^2-c}$

v) To find the distance $=r=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$

Exercise 5.1 $\rightarrow$ V.V. Imp

1) Find the equation of the circle under the following conditions.

a) Centre at (2,1) and radius = 3

Sol:

Centre $(h,k)=(2,1)$, radius $(r)=3$

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Equation of circle
$(x-h)^2 + (y-k)^2 = r^2$
or, $(x-2)^2 + (y-1)^2 = 3^2$
or, $x^2 - 4x + 4 + y^2 - 2y + 1 = 9$
or, $x^2 + y^2 - 4x - 2y + 5 - 9 = 0$
or, $x^2 + y^2 - 4x - 2y - 4 = 0$
which is the required equation of circle.

b) Centre at (a,b) and diameter $= \sqrt{a^2+b^2}$
sol:
$h = a$, $k = b$, $r = \frac{d}{2} = \frac{\sqrt{a^2+b^2}}{2}$
now;
Equation of circle
$= (x-h)^2 + (y-k)^2 = r^2$
or, $(x-a)^2 + (y-b)^2 = \left(\frac{\sqrt{a^2+b^2}}{2}\right)^2$
or, $x^2 - 2xa + a^2 + y^2 - 2yb + b^2 = \frac{a^2+b^2}{4}$
or, $4x^2 - 8xa + 4a^2 + 4y^2 - 8yb + 4b^2 - a^2 + b^2 = 0$
or, $4x^2 + 4y^2 - 8xa - 8yb + 3a^2 + 3b^2 = 0$

c) Centre at $(-3,2)$ and passing through the point $(1,0)$
sol:
$h = -3$, $k = 2$,
$x_1 = -3$, $y_1 = 2$, $x_2 = 1$, $y_2 = 0$
now,
radius $(r) = \sqrt{(x_2-x_1)^2 + (y_2-y_1)^2}$
$= \sqrt{(1-(-3))^2 + (0-2)^2}$
$= \sqrt{16+4}$
$= \sqrt{20}$

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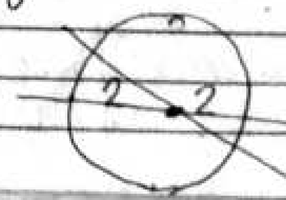
Equation of circle:
$(x-h)^2 + (y-k)^2 = r^2$
or, $(x-(-3))^2 + (y-2)^2 = (\sqrt{20})^2$
or, $x^2 + 6x + 9 + y^2 - 4y + 4 = 20$
or, $x^2 + y^2 + 6x - 4y - 7 = 0$

d) Center at $(0,3)$ and touching the x-axis
sol:
$h = 0$, $k = 3$, $r = k = 3$
now;
$= (x-h)^2 + (y-k)^2 = r^2$
or, $(x-0)^2 + (y-3)^2 = 3^2$
or, $x^2 + y^2 - 6y + 9 - 9 = 0$
or, $x^2 + y^2 - 6y = 0$

e) Center at $(5,6)$ and touching the y-axis
sol:
$h = 5$, $k = 6$, $r = h = 5$
again;
$= (x-h)^2 + (y-k)^2 = r^2$
or, $(x-5)^2 + (y-6)^2 = 5^2$
or, $x^2 - 10x + 25 + y^2 - 12y + 36 = 25$
or, $x^2 + y^2 - 10x - 12y + 61 - 25 = 0$
or, $x^2 + y^2 - 10x - 12y + 36 = 0$

Imp
f) two diameters are $x + 2y = 3$ and $2x - y = 1$ and diameter $= 4$
sol:
Solving for diameter



$x+2y=3$, $2x-y=1$

solving;

$2x+4y=6$

$-2x-y=1$

$5y=5$

$y=5/5=1$

putting value of y

$x+2y=3$

or, $x+2\times1=3$

or, $x=1$

centre $(h,k)=(1,1)$

radius $(R)=d/2=4/2=2$

Equation of circle

$(x-h)^2+(y-k)^2=r^2$

$(x-1)^2+(y-1)^2=2^2$

or, $x^2-2x+1+y^2-2y+1-4=0$

or, $\therefore x^2+y^2-2x-2y-2=0$

g) Two diameter are $x+2y=3$ and $2x-y=1$ and diam

h) Concentric with the circle $x^2+y^2-9x+5y-3=0$ and passing through the point $(1,1)$

sol:

$x^2+y^2-9x+5y-3=0$

or, $x^2+y^2+2\cdot(-9/2)x+2\cdot(5/2)y+(-3)=0$

Comparing it with $x^2+y^2+2gx+2fy+c=0$

Centre $(h,k)=(-g,-f)=\left(-\left(\frac{-9}{2}\right),\frac{-5}{2}\right)$

$=\left(\frac{9}{2},\frac{-5}{2}\right)$

$\therefore$ Radius $(r)=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$

$=\sqrt{\left(\frac{9}{2}-1\right)^2+\left(-\frac{5}{2}-1\right)^2}$

$=\sqrt{49/4+49/4}$

$=\sqrt{98/4}$

$=\frac{\sqrt{2\times7\times7}}{\sqrt{4}}$

$=\frac{7\sqrt{2}}{2}$

$=\frac{7\sqrt{2}}{\sqrt{2}\times\sqrt{2}}=\frac{7}{\sqrt{2}}$

Eqn of circle

$=(x-h)^2+(y-k)^2=r^2$

$=\left(x-\frac{9}{2}\right)^2+\left(y+\frac{5}{2}\right)^2=\left(\frac{7}{\sqrt{2}}\right)^2$

or, $\left(x^2-2x\cdot\frac{9}{2}+\left(\frac{9}{2}\right)^2+y^2+2y\cdot\frac{5}{2}+\left(\frac{5}{2}\right)^2\right.$

$=49/2$

or, $x^2-9x+\frac{81}{4}+y^2+5y+\frac{25}{4}-\frac{49}{2}=0$

or, $x^2+y^2-9x+5y+\frac{81}{4}+\frac{25}{4}-\frac{49}{2}=0$

or, $x^2+y^2-9x+5y+\frac{106}{4}-\frac{49}{2}=0$

or, $x^2+y^2-9x+5y+\frac{106-98}{4}=0$

or, $x^2+y^2-9x+5y+2=0$

2) Find the centre and radius of the circle

a) $x^2+y^2=36$

sol:

$(x-0)^2+(y-0)^2=6^2$

comparing it with $(x-h)^2+(y-k)^2=r^2$

$(h,k)=(0,0)$ and $r=6$

b) $x^2+(y-2)^2=9$

sol:

$(x-0)^2+(y-2)^2=3^2$

Comparing it with $(x-h)^2+(y-k)^2=r^2$

$(h,k)=(0,2)$, $r=3$

c) $(x+2)^2+(y-1)^2=16$

sol:

$(x+(-2))^2+(y-1)^2=4^2$

Comparing it ...

$(h,k)=(-2,1)$, $r=4$

= balance gama $+5^2 - 5^2$ di huma.

d) $x^2 + y^2 - 10x + 16 = 0$

sol:

$= x^2 - 10x + y^2 + 16 = 0$

or, $x^2 - 2.5.x + 5^2 - 5^2 + y^2 + 16 = 0$

or, $(x-5)^2 + (y-0)^2 - 25 + 16 = 0$

or, $(x-5)^2 + (y-0)^2 = 9$

or, $(x-5)^2 + (y-0)^2 = 3^2$

Comparing it with $(x-h)^2 + (y-k)^2 = r^2$

$(h,k) = (5,0)$ and $r = 3$

e) $2x^2 + 2y^2 - 8x + 4y - 3 = 0$

sol:

$= 2(x^2 + y^2 - 4x + 2y - 3/2) = 0$

or, $x^2 + y^2 - 4x + 2y - 3/2 = 0$

or, $x^2 + y^2 + 2.(-2)x + 2.(1).y + (-3/2) = 0$

which is formula of

$x^2 + y^2 + 2gx + 2fy + c = 0$

$\therefore g = -2, f = 1$

again

Center $(h,k) = (-g, -f) = (-(-2), -1) = (2, -1)$

Radius $= \sqrt{g^2 + f^2 - c} = \sqrt{(-2)^2 + (1)^2 - (-3/2)}$

or, $\sqrt{4 + 1 + 3/2}$

or, $\sqrt{5 + 3/2}$

or, $\sqrt{8/2}$

3) Find the equation of the circle if the ends of the diameters are

a) $(0,0)$ and $(2,4)$

sol:

$(x_1, y_1) = (0,0)$ and $(x_2, y_2) = (2,4)$

now,

$= (x - x_1)(x - x_2) + (y - y_1) + (y - y_2) = 0$

or, $(x-0)(x-2) + (y-0) + (y-4) = 0$

or, $x^2 - 2x + y^2 + 4y = 0$

$\therefore x^2 + y^2 - 2x + 4y = 0$

b) $(3,5)$ and $(5,-1)$

sol:

$(x_1, y_1) = (3,5)$ and $(x_2, y_2) = (5,-1)$

now,

$= (x - x_1)(x - x_2) + (y - y_1)(y - y_2) = 0$

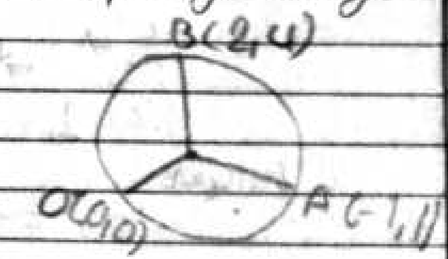
or, $(x-3)(x-5) + (y-5)(y-(-1)) = 0$

or, $x^2 - 5x - 3x + 15 + y^2 + y - 5y - 5 = 0$

or, $x^2 - 8x + 15 + y^2 - 4y - 5 = 0$

or $\therefore x^2 + y^2 - 8x - 4y + 10 = 0$ ~~are~~

4) Find the equation of the circle passing through the points.



b) $(0,0)$, $(1,-1)$ and $(2,4)$

sol:

$OP = AP = BA$

now,

$OP = AP$

Using distance formula

$\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$= \sqrt{x^2+y^2} = \sqrt{}$

$= \sqrt{(x-0)^2+(y-0)^2} = \sqrt{(x+1)^2+(y-1)^2}$

$= \sqrt{x^2+y^2} = \sqrt{x^2+2x+1+y^2-2y+1}$

squaring both side;

$= x^2+y^2 = x^2+2x+1+y^2-2y+1$

or, $0 = 2x-2y+2$

or, $2x-2y+2=0$

$x-y+1=0$

$\therefore x-y=-1$ — ①

again,

$AP = BP$

$= \sqrt{(x+1)^2+(y-1)^2} = \sqrt{(x-2)^2+(y-4)^2}$

or, $x^2+2x+1+y^2-2y+1 = x^2-4x+4+y^2-8y+16$

or, $2x-2y+2+4x+8y-20=0$

or, $6x+6y-18=0$

or, $6(x+y-3)=0$

$\therefore x+y=3$ — ②

solving ① and ②

$x-y=-1$

$x+y=3$

$2x=2$

$x=1$

① become

$1-y=-1$

$-y=-2$

$y=2$

centre $(h,k)=(1,2)$

Radius $r=OP=\sqrt{(1-0)^2+(2-0)^2}$

$=\sqrt{1+4}$

$=\sqrt{5}$

again;

Eqn of circle:

$=(x-h)^2+(y-k)^2=r^2$

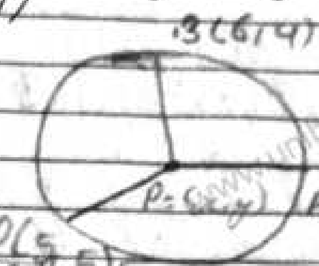
$=(x-1)^2+(y-2)^2=(\sqrt{5})^2$

$=x^2-2x+1+y^2-4y+4=5$

$=x^2+y^2-2x-4y=0$

* Find the equation of circle passing through the point (5,5) (6,4) (-2,4)

Sol:



$CP = AP = BP$

Solve

$CP = AP$

$= \sqrt{(5-x)^2+(5-y)^2} = \sqrt{(x+2)^2+(y-4)^2}$

or, $\sqrt{25-10x+x^2+25-10y+y^2} = \sqrt{x^2+4x+4+y^2-8y+16}$

or, $\sqrt{50-10x+x^2-10y+y^2} = \sqrt{20+x^2+4x+y^2-8y}$

squaring both side;

$= 50-10x+x^2-10y+y^2 = 20+x^2+4x+y^2-8y$

or, $50-10x-10y = 20+4x-8y$

or, $50-10x-10y-4x-20+8y=0$

or, $30-14x-2y=0$

or, $2(15-7x-y)=0$

or, $-(7x+y)=-15$

or, $7x+y=15$

or, $x+y=15/7$ — ①

again,

$AP = BP$

$= \sqrt{(x+2)^2+(y-4)^2} = \sqrt{(6-x)^2+(4-y)^2}$

or, $\sqrt{x^2+4x+4+y^2-8y+16} = \sqrt{6^2-12x+x^2+4^2-8y+y^2}$

or, $\sqrt{x^2+4x+y^2-8y+20} = \sqrt{52-12x+x^2-8y+y^2}$

squaring both side,

$= x^2+4x+y^2-8y+20 = 52-12x+x^2-8y+y^2$

or, $4x+20-52+12x=0$

or, $16x=32$

$x=32/16=2$ — ①

putting value of x in eqn (1)

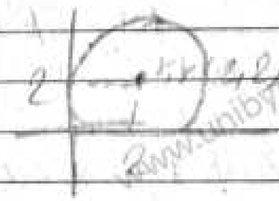
$7x + y = 15$
$7 \times 2 + y = 15$
$y = 1$

Center $(h,k) = (2,1)$
Radius $(OP) = \sqrt{(2-0)^2 + (1-0)^2}$
$= \sqrt{4+1}$
$= \sqrt{5}$

again;
Eqn of circle
$= (x-h)^2 + (y-k)^2 = r^2$
or, $(x-2)^2 + (y-1)^2 = (\sqrt{5})^2$
or, $x^2 - 4x + 4 + y^2 - 2y + 1 = 5$
or, $x^2 + y^2 - 4x - 2y \cancel{+0} = 0$

Q) Find the eqn of circle when radius = 2 and circle touches both axis and lies in axis 1st quardan.

sol:



$(h,k) = (2,2)$, $r = 2$
$= (x-h)^2 + (y-k)^2 = r^2$
$= (x-2)^2 + (y-2)^2 = 2^2$
or, $x^2 - 4x + 4 + y^2 - 4y + 4 - 4 = 0$
or, $x^2 + y^2 - 4x - 4y + 4 = 0$

Vector

* Scalar projection of $\vec{b}$ on $\vec{a} = \dfrac{\vec{a} \cdot \vec{b}}{|\vec{a}|}$

* Vector projection of $\vec{b}$ on $\vec{a} = \left(\dfrac{\vec{a} \cdot \vec{b}}{|\vec{a}|^2}\right)\vec{a}$

Q) Find the direction cosin of following
$\vec{a} = (2, 1, 2)$

sol:

$\cos\alpha = \dfrac{a_1}{|\vec{a}|} = \dfrac{2}{\sqrt{2^2+1^2+2^2}} = \dfrac{2}{\sqrt{9}} = \dfrac{2}{3}$

$\cos\beta = \dfrac{a_2}{|\vec{a}|} = \dfrac{1}{3}$

$\cos\gamma = \dfrac{a_3}{|\vec{a}|} = \dfrac{2}{3}$

∴ Direction cosines be $\cos\alpha, \cos\beta, \cos\gamma$ i.e $2/3, 1/3, 2/3$

Note:
i) orthogonal $\Rightarrow \vec{a} \cdot \vec{b} = 0$
ii) Parallel by ~~(x)~~ = cross $(\times) \Rightarrow \vec{a} \times \vec{b} = 0$
iii) Area of triangle $= \dfrac{1}{2} |\vec{a} \times \vec{b}|$

iv) Parallel by $(\cdot) \Rightarrow \vec{a} \cdot \vec{b} = 1$
v) Area $= \vec{a} \times \vec{b}$
vi) Volume $= \vec{a} \cdot (\vec{b} \times \vec{c})$

* Unit vector is a vector which has a magnitude 1.

* area of parallelogram = $|\vec{a} \times \vec{b}|$

Let $\vec{a} = (a_1, a_2, a_3)$
$\vec{b} = (b_1, b_2, b_3)$

i) Unit vector along the given vector $\vec{a}$

$$\hat{a} = \frac{\vec{a}}{|\vec{a}|} \Big/ \frac{\vec{a} \times \vec{b}}{|\vec{a} \times \vec{b}|}$$

ii) If θ be the angle between two vector $\vec{a}$ and $\vec{b}$

$$\cos\theta = \frac{\vec{a} \cdot \vec{b}}{|\vec{a}||\vec{b}|} \Big/ \cancel{\text{Or or}}$$

iii) Sine of the angle between the two vectors.

$$\sin\theta = \frac{|\vec{a} \times \vec{b}|}{|\vec{a}||\vec{b}|}$$

iv) Two vector $\vec{a}$ and $\vec{b}$ are perpendicular if
$\vec{a} \cdot \vec{b} = 0$

Exercise 5.6

1d Given the vector $\vec{a} = (2, 3, 1)$, $\vec{b} = (2, -1, 1)$, compute

i) $\vec{a} + \vec{b}$
$= (2, 3, 1) + (2, -1, 1)$
$= (4, 2, 2)$

iv) $-3\vec{b}$
$= -3(2, -1, 1)$
$= (-6, 3, -3)$

Note: Area of triangle and area of parallelogram are different.

(b) If $\vec{a} = \vec{i} + 5\vec{j} - \vec{k}$ and $b = 2\vec{i} + \vec{k}$, find $3\vec{a} + 5\vec{b}$ and its magnitude.

Sol:
$= 3\vec{a} + 5\vec{b}$
or, $3(\vec{i} + 5\vec{j} - \vec{k}) + 5(2\vec{i} + 0 + \vec{k})$
or, $(3\vec{i} + 15\vec{j} - 3\vec{k}) + (10\vec{i} + 0 + 5\vec{k})$
$= 13\vec{i} + 15\vec{j} + 2\vec{k}$

now;
magnitude of $3\vec{a} + 5\vec{b}$
$= |3\vec{a} + 5\vec{b}| = \sqrt{(13)^2 + (15)^2 + (2)^2}$
$= \sqrt{169 + 225 + 4}$
$= \sqrt{398}$ unit.

3. Find the unit vector along the vector $\vec{a} + 3\vec{b}$ if

(b) $\vec{a} = 2\vec{i} - \vec{j}$ and $b = \vec{j} - 2\vec{k}$

Sol:
$\vec{a} = (2, -1, 0)$, $b = (0, 1, -2)$

now;
$\vec{a} + 3\vec{b} = (2, -1, 0) + 3(0, 1, -2)$
$= (2, -1, 0) + (0, 3, -6)$
$= (2, 2, -6) = 2\vec{i} + 2\vec{j} - 6\vec{k}$

$|\vec{a} + 3\vec{b}| = \sqrt{(2)^2 + (2)^2 + (6)^2} = \sqrt{44}$

now;
Unit vector $= \frac{\vec{a} + 3\vec{b}}{|\vec{a} + 3\vec{b}|} = \frac{2\vec{i} + 2\vec{j} - 6\vec{k}}{\sqrt{44}}$

$= \frac{2\vec{i} + 2\vec{j} - 6\vec{k}}{\sqrt{11 \times 2 \times 2}} = \frac{2\vec{i} + 2\vec{j} - 6\vec{k}}{\sqrt{11} \times 2} = \cancel{2}\left(\frac{i + j - 3\vec{k}}{\sqrt{11}\ \cancel{2}}\right)$ ans

4) i) Find the length of the vector i) $\vec{a} = 3\vec{i} - 2\sqrt{3}\vec{j} + 2\vec{k}$

Sol:

$\vec{a} = 3\vec{i} - 2\sqrt{3}\vec{j} + 2\vec{k} = (3, -2\sqrt{3}, 2)$

$a = |\vec{a}| = \sqrt{(3)^2 + (-2\sqrt{3})^2 + (2)^2} = \sqrt{25} = 5$ units

ii) Find the distance between two points.

i) $A(2\sqrt{3}, 3, -1)$ and $B(5\sqrt{2}, 4, 2)$

Sol:

$\vec{AB} = OB - OA$

$= (5\sqrt{2}, 4, 2) - (2\sqrt{3}, 3, -1)$

$= (3\sqrt{2}, 1, 3)$

$AB = |\vec{AB}| = \sqrt{(3\sqrt{2})^2 + (1)^2 + (3)^2} = \sqrt{28} = \sqrt{4 \times 2} = 2\sqrt{7}$ units

ii) P and Q whose position vector are $3\vec{i} + 4\vec{j} - \vec{k}$ and $4\vec{i} + 2\vec{j} - 3\vec{k}$ respectively.

Sol:

Let O be the origin, then

$\vec{OP} = 3\vec{i} + 4\vec{j} - \vec{k} = (3, 4, -1)$

$\vec{OQ} = 4\vec{i} + 2\vec{j} - 3\vec{k} = (4, 2, -3)$

now,

$\vec{PQ} = OQ - OP$

$= (4, 2, -3) - (3, 4, -1)$

$= (1, -2, -2)$

$PQ = |\vec{PQ}|$

$= \sqrt{1^2 + (-2)^2 + (-2)^2} = \sqrt{9} = 3$ unit

$\therefore$ distance between P on Q = 3 unit

5) Given the vector $\vec{a} = 2\vec{i} - \vec{j} + 5\vec{k}$ and $b = 2\vec{j} + \vec{k}$ find the scalar product $\vec{a} \cdot \vec{b}$. Also, find $|\vec{a}||\vec{b}|$ and projection of $\vec{b}$ on $\vec{a}$.

Sol:

$\vec{a} = (2\vec{i} - \vec{j} + 5\vec{k}) = (2, -1, 5)$

$\vec{b} = (2\vec{j} + \vec{k} + 0\vec{i}) = (2, 1, 0)$

now,

$\vec{a} \cdot \vec{b} = (2, -1, 5) \cdot (2, 1, 0) = (4 - 1) = 3$.

again,

$|\vec{a}||\vec{b}| = \sqrt{(2)^2 + (-1)^2 + (5)^2} \sqrt{(2)^2 + (1)^2 + 0}$

$= \sqrt{30}\sqrt{5} = \sqrt{30 \cdot 5}$

now,

projection of $\vec{b}$ on $\vec{a} = \dfrac{\vec{a} \cdot \vec{b}}{|\vec{a}|} = \dfrac{3}{\sqrt{30}}$

6) Are the following vector are perpendicular or not.

Note: if the dotted (·) product of any two vector is 0 then it is perpendicular

a) $\vec{a} = 4\vec{i} + 3\vec{j} - 5\vec{k}$ and $\vec{b} = 3\vec{i} + \vec{j} + 3\vec{k}$

Sol:

$\vec{a} = 4\vec{i} + 3\vec{j} - 5\vec{k} = (4, 3, -5)$

$\vec{b} = 3\vec{i} + \vec{j} + 3\vec{k} = (3, 1, 3)$

now,

$\vec{a} \cdot \vec{b} = (4, 3, -5) \cdot (3, 1, 3)$

$= (12 + 3 - 15)$

$= (15 - 15)$

$= 0$

$\therefore \vec{a}$ ~~and b~~ is perpendicular on $\vec{b}$

7) If $\vec{u} = (3, 8, -2)$ and $\vec{v} = (k, 2, 5)$, find the value of k so that the two vectors are orthogonal.

Sol:

According to question

$\vec{u} \cdot \vec{v} = 0$

$\Rightarrow (3, 8, -2) \cdot (k, 2, 5) = 0$

or, $3k + 16 - 10 = 0$

or, $3k = -6$

or, $k = -6/3 = -2$

8) Find the angle between the following pair of vectors.

b) $\vec{a} = 2\vec{i} + 5\vec{j} - \vec{k}$ and $\vec{b} = 5\vec{i} + 2\vec{j} + \vec{k}$

Sol:

$\vec{a} = (2\vec{i} + 5\vec{j} - \vec{k}) = (2, 5, -1)$

$\vec{b} = (5\vec{i} + 2\vec{j} + \vec{k}) = (5, 2, 1)$

Sol:

$\cos\theta = \frac{\vec{a} \cdot \vec{b}}{|\vec{a}||\vec{b}|} = \frac{(2, 5, -1) \cdot (5, 2, 1)}{\sqrt{(2)^2 + (5)^2 + (-1)^2}\sqrt{(5)^2 + (2)^2 + (1)^2}}$

$= \frac{10 + 10 - 1}{\sqrt{4 + 25 + 1}\sqrt{4 + 25 + 1}} = \frac{19}{\sqrt{30}\sqrt{30}} = \frac{19}{30}$

$\therefore \cos^{-1}\frac{19}{30}$

9) Given,

a) $\vec{a} = (1, 2, 0)$ and $\vec{b} = (3, 1, 1)$

i) Book

i) $\vec{a} \times \vec{b} = \begin{vmatrix} i & j & k \\ 1 & 2 & 0 \\ 3 & 1 & 1 \end{vmatrix}$

$= i\begin{vmatrix} 2 & 0 \\ 1 & 1 \end{vmatrix} - j\begin{vmatrix} 1 & 0 \\ 3 & 1 \end{vmatrix} + k\begin{vmatrix} 1 & 2 \\ 3 & 1 \end{vmatrix}$

or, $i(2 - 0) - j(1 - 0) + k(1 - 6)$

$= 2\vec{i} - 1\vec{j} - 5\vec{k}$

$= (2, -1, -5)$

Unit vector perpendicular to $\vec{a}$ and $\vec{b} = \frac{\vec{a} \times \vec{b}}{|\vec{a} \times \vec{b}|}$

For $|\vec{a} \times \vec{b}| = \sqrt{(2)^2 + (-1)^2 + (-5)^2} = \sqrt{4 + 1 + 25} = \sqrt{30}$

now,

$= \frac{\vec{a} \times \vec{b}}{|\vec{a} \times \vec{b}|}$ or, $\frac{2}{\sqrt{30}}, \frac{-1}{\sqrt{30}}, \frac{-5}{\sqrt{30}}$

ii) Since the area of the parallelogram determined by the vector $\vec{a}$ and $\vec{b}$ is given by

$|a| = \sqrt{1^2 + (2)^2 + 0} = \sqrt{5}$ | $|b| = \sqrt{3^2 + 1^2 + 1^2} = \sqrt{11}$

$\sin\theta = \frac{|\vec{a} \times \vec{b}|}{|\vec{a}||\vec{b}|} = \frac{\sqrt{30}}{\sqrt{5}\sqrt{11}} = \frac{\sqrt{30}}{\sqrt{55}} = \sqrt{\frac{30}{55}} = \sqrt{\frac{6}{11}}$

$= \sqrt{\frac{6}{11}}$

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(b) $\vec{a} = 2i + \vec{j} - \vec{k}$ and $\vec{b} = \vec{j} + 3\vec{k}$

sol:

$\vec{a} = (2, 1, -1)$ and $\vec{b} = (0, 1, 3)$

now;

$$\vec{a} \times \vec{b} = \begin{vmatrix} i & j & k \\ 2 & 1 & -1 \\ 0 & 1 & 3 \end{vmatrix}$$

$$= i\begin{vmatrix} 1 & -1 \\ 1 & 3 \end{vmatrix} - j\begin{vmatrix} 2 & -1 \\ 0 & 3 \end{vmatrix} + k\begin{vmatrix} 2 & 1 \\ 0 & 1 \end{vmatrix}$$

$$= i(3+1) - j(6-0) + k(2-0)$$

$$= 4\vec{i} - 6\vec{j} + 2\vec{k} = (4, -6, 2)$$

~~= (4~~ now;

for $|\vec{a} \times \vec{b}| = \sqrt{(4)^2 + (-6)^2 + (2)^2} = \sqrt{16+36+4} = \sqrt{56}$

now;

Unit vector perpendicular to $\vec{a}$ and $\vec{b}$

$$= \frac{\vec{a} \times \vec{b}}{|\vec{a} \times \vec{b}|} = \frac{4\vec{i} - 6\vec{j} + 2\vec{k}}{\sqrt{56}} = \frac{4\vec{i} - 6\vec{j} + 2\vec{k}}{\sqrt{14 \times 2 \times 2}}$$

$$= \frac{4\vec{i} - 6\vec{j} + 2\vec{k}}{2\sqrt{14}} = \frac{2(2\vec{i} - 3\vec{j} + k)}{2\sqrt{14}}$$

$$= \frac{1}{14}(2\vec{i} - 3\vec{j} + k)$$

ii) Now the area of the parallelogram determined by the vector $\vec{a}$ and $\vec{b}$ is given by

$|\vec{a} \times \vec{b}| = \sqrt{56}$ units

$|\vec{a}| = \sqrt{(2)^2 + (1)^2 + (-1)^2} = \sqrt{6}$

$|\vec{b}| = \sqrt{0 + (1)^2 + (3)^2} = \sqrt{10}$

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$$\sin\theta = \frac{|\vec{a} \times \vec{b}|}{|\vec{a}||\vec{b}|} = \frac{\sqrt{56}}{\sqrt{6}\sqrt{10}} = \sqrt{\frac{56}{60}} = \sqrt{\frac{14}{15}}$$ ans

10. Find the area of the parallelogram determined by the vectors.

(a) $\vec{a} = 2\vec{i} + \vec{k}$ and $\vec{b} = -3\vec{i} + \vec{j}$

sol:

$\vec{a} = 2\vec{i} + \vec{k} = (2, 0, 1)$

$\vec{b} = -3\vec{i} + \vec{j} = (-3, 1, 0)$

now;

area of parallelogram $= |\vec{a} \times \vec{b}|$

for,

$$\vec{a} \times \vec{b} = \begin{vmatrix} i & j & k \\ 2 & 0 & 1 \\ -3 & 1 & 0 \end{vmatrix}$$

$$= i\begin{vmatrix} 0 & 1 \\ 1 & 0 \end{vmatrix} - j\begin{vmatrix} 2 & 1 \\ -3 & 0 \end{vmatrix} + k\begin{vmatrix} 2 & 0 \\ -3 & 1 \end{vmatrix}$$

$$= -i - 3j + 2k$$

~~now; = (-1~~

$$|\vec{a} \times \vec{b}| = \sqrt{(-1)^2 + (-3)^2 + (2)^2}$$
$$= \sqrt{14}$$

11. If $\vec{a} + \vec{b} + \vec{c} = 0$, show that $\vec{a} \times \vec{b} = \vec{b} \times \vec{c} = \vec{c} \times \vec{a}$.

sol:

$\vec{a} + \vec{b} + \vec{c} = 0$

Taking cross product of 'a' on both side

Note: $\vec{a}\times\vec{a}=0$
$\vec{a}\times\vec{b}+\vec{b}\times\vec{a}+\vec{a}\times\vec{b}=1$ | $\vec{b}\cdot\vec{b}=1$

$= \vec{a}\times(\vec{a}+\vec{b}+\vec{c}) = \vec{a}\times 0$
or, $\vec{a}\times\vec{a}+\vec{a}\times\vec{b}+\vec{a}\times\vec{c}=0$
or, $0+\vec{a}\times\vec{b}+\vec{a}\times\vec{c}=0$
or, $\vec{a}\times\vec{b}=(\vec{a}\times\vec{c})$
$\therefore \vec{a}\times\vec{b}=-(\vec{a}\times\vec{c})$
$\therefore \vec{a}\times\vec{b}=\vec{c}\times\vec{a}$ —①

again;
Taking cross product of 'b' on both side.
$= \vec{b}\times(\vec{a}+\vec{b}+\vec{c})=\vec{b}\times 0$
or, $\vec{a}\times\vec{b}+\vec{b}\times\vec{b}+\vec{c}\times\vec{b}=0$
or, $\vec{a}\times\vec{b}+0=\vec{c}\times\vec{b}$
or, $\vec{a}\times\vec{b}=-(\vec{b}\times\vec{c})$
or, $\vec{a}\times\vec{b}=\vec{b}\times\vec{c}$ —②

now;
Combining ① and ②

$\vec{a}\times\vec{b}=\vec{b}\times\vec{c}=\vec{c}\times\vec{a}$

Exercise 5.7

i) Scalar triple product $=(\vec{a},\vec{b},\vec{c})=a\cdot(\vec{b}\times\vec{c})=$ volume of parallelopiped.

ii) Vector triple product $=\vec{a}\times(\vec{b}\times\vec{c})$

iii) $a\cdot(\vec{b}\times\vec{c})=\vec{b}\cdot(\vec{a}\times\vec{c})=\vec{c}\cdot(a\cdot\vec{b})$

iv) $\vec{a}\times(\vec{b}\times\vec{c})\neq\vec{b}(\vec{a}\times\vec{c})$

1.b) If $\vec{a}=(-1,2,3)$, $\vec{b}=(0,1,-2)$ and $\vec{c}=(3,0,-1)$ find $\vec{a}\cdot(\vec{b}\times\vec{c})$, $(\vec{a}\times\vec{b})\cdot\vec{c}$ and $\vec{a}\times(\vec{b}\times\vec{c})$.

Given;
$\vec{a}=(-1,2,3)$
$\vec{b}=(0,1,-2)$
$\vec{c}=(3,0,-1)$

Sol:
$\vec{a}\cdot(\vec{b}\times\vec{c})=[\vec{a},\vec{b},\vec{c}]=\begin{vmatrix}-1 & 2 & 3\\ 0 & 1 & -2\\ 3 & 0 & -1\end{vmatrix}$

$=-1\begin{vmatrix}1 & -2\\ 0 & -1\end{vmatrix}-2\begin{vmatrix}0 & -2\\ 3 & -1\end{vmatrix}+3\begin{vmatrix}0 & 1\\ 3 & 0\end{vmatrix}$
$=-1(-1)-2(0+6)+3(0-3)$
$=1-12-9$
$=-20$ ans

again;
$(\vec{a}\times\vec{b})\cdot\vec{c}=\vec{c}\cdot(\vec{a}\times\vec{b})=[\vec{c},\vec{a},\vec{b}]$
$=\begin{vmatrix}3 & 0 & -1\\ -1 & 2 & 3\\ 0 & 1 & -2\end{vmatrix}=3\begin{vmatrix}2 & 3\\ 1 & -2\end{vmatrix}-0-1\begin{vmatrix}-1 & 2\\ 0 & 1\end{vmatrix}$
$=3(-4-3)-0-1(-1-0)$
$=3(-7)-1(-1)$
$=-21+1$
$=-20$ ans

now;
$\vec{a}\times(\vec{b}\times\vec{c})$

now;
$\vec{b}\times\vec{c}=\begin{vmatrix}i & j & k\\ 0 & 1 & 2\\ 3 & 0 & -1\end{vmatrix}$

$$= i\begin{vmatrix} 1 & 2 \\ 0 & -1 \end{vmatrix} - j\begin{vmatrix} 0 & 2 \\ 3 & -1 \end{vmatrix} + k\begin{vmatrix} 0 & 1 \\ 3 & 0 \end{vmatrix}$$

$$= i(-1-0) - j(0-6) + k(0-3)$$

$$= -1i + 6j - 3\vec{k} = (-1, 6, -3)$$

again,

$$\vec{a}\times(\vec{b}\times\vec{c}) = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ -1 & 2 & 3 \\ -1 & 6 & -3 \end{vmatrix} = i\begin{vmatrix} 2 & 3 \\ 6 & -3 \end{vmatrix} - \vec{j}\begin{vmatrix} -1 & 3 \\ -1 & -3 \end{vmatrix} + \vec{k}\begin{vmatrix} -1 & 2 \\ -1 & 6 \end{vmatrix}$$

$$= 12\vec{i} - 6\vec{j} + 8\vec{k}$$

$$= (12, -6, 8)$$

2) If $\vec{a} = \vec{i} - \vec{j} - \vec{k}$, $\vec{b} = 2\vec{i} + \vec{j}$ and $\vec{c} = 3\vec{k}$, show $\vec{a}\times(\vec{b}\times\vec{c}) \neq (\vec{a}\times\vec{b})\times\vec{c}$

sol:

$$\vec{a} = \vec{i} - \vec{j} - \vec{k} = (1, -1, -1)$$
$$\vec{b} = 2\vec{i} + \vec{j} = (2, 1, 0)$$
$$\vec{c} = 3\vec{k} = (0, 0, 3)$$

sol:

$$\vec{b}\times\vec{c} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 2 & 1 & 0 \\ 0 & 0 & 3 \end{vmatrix}$$

$$= i\begin{vmatrix} 1 & 0 \\ 0 & 3 \end{vmatrix} - j\begin{vmatrix} 2 & 0 \\ 0 & 3 \end{vmatrix} + k\begin{vmatrix} 2 & 1 \\ 0 & 0 \end{vmatrix}$$

$$= 3i - 6j + 0$$

now,

$$\vec{a}\times(\vec{b}\times\vec{c}) = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 1 & -1 & -1 \\ 3 & -6 & 0 \end{vmatrix}$$

$$= i\begin{vmatrix} -1 & -1 \\ -6 & 0 \end{vmatrix} - j\begin{vmatrix} 1 & -1 \\ 3 & 0 \end{vmatrix} + k\begin{vmatrix} 1 & -1 \\ 3 & -6 \end{vmatrix}$$

$$= -6i \quad \therefore \vec{a}\times(\vec{b}\times\vec{c}) = -6i - 3j - 3k$$

again,

$$\vec{a}\times\vec{b} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 1 & -1 & -1 \\ 2 & 1 & 0 \end{vmatrix}$$

$$= \vec{i}\begin{vmatrix} -1 & -1 \\ 1 & 0 \end{vmatrix} - \vec{j}\begin{vmatrix} 1 & -1 \\ 2 & 0 \end{vmatrix} + \vec{k}\begin{vmatrix} 1 & -1 \\ 2 & 1 \end{vmatrix}$$

$$= 1\vec{i} - 2j + 3\vec{k}$$

now,

$$(\vec{a}\times\vec{b})\times\vec{c} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 1 & -2 & 3 \\ 0 & 0 & 3 \end{vmatrix}$$

$$= \vec{i}\begin{vmatrix} -2 & 3 \\ 0 & 3 \end{vmatrix} - \vec{j}\begin{vmatrix} 1 & 3 \\ 0 & 3 \end{vmatrix} + \vec{k}\begin{vmatrix} 1 & -2 \\ 0 & 0 \end{vmatrix}$$

$$= -6\vec{i} - 3\vec{j} \quad \underline{\underline{\text{ans}}}$$

$$\therefore \vec{a}\times(\vec{b}\times\vec{c}) \neq (\vec{a}\times\vec{b})\times\vec{c}$$

3) Find the volume of the parallelopiped whose adjacent edges are represented by following vector.

(b) $5\vec{i} + 2\vec{j} - 3\vec{k}$, $2\vec{i} + 3\vec{j} - 4\vec{k}$ and $3\vec{i} - \vec{j} + 2\vec{k}$

sol:

$$\vec{a} = (5\vec{i} + 2\vec{j} - 3\vec{k}) = (5, 2, -3)$$
$$\vec{b} = (2\vec{i} + 3\vec{j} - 4\vec{k}) = (2, 3, -4)$$
$$\vec{c} = (3\vec{i} - \vec{j} + 2\vec{k}) = (3, -1, 2)$$

volume of parallelopiped $= \vec{a}\cdot(\vec{b}\times\vec{c})$

$$= \begin{vmatrix} 5 & 2 & -3 \\ 2 & 3 & -4 \\ 3 & -1 & 2 \end{vmatrix}$$

$$= 5\begin{vmatrix} 3 & -4 \\ -1 & 2 \end{vmatrix} - 2\begin{vmatrix} 2 & -4 \\ 3 & 2 \end{vmatrix} - 3\begin{vmatrix} 2 & 3 \\ 3 & -1 \end{vmatrix}$$

$= 5(6-4) - 2(4+12) - 3(-2-9)$

$= 10 - 32 + 33$

$= 11$ cu units

IU 2018

4) Find the value of m so that the volume of the parallelopiped determined by three concurrent edges represented by the vectors $2\vec{i} + 3\vec{j} - m\vec{k}$, $\vec{i} - 2\vec{j}$ and $3\vec{i} + \vec{j} - 2\vec{k}$ is 28 cu unit

Sol: let,

$\vec{a} = (2, 3, -m)$

$\vec{b} = (1, -2, 0)$

$\vec{c} = (3, 1, -2)$

Since the volume of parallelpiped $= 28$ cu. unit

now,

$\vec{a}\cdot(\vec{b}\times\vec{c}) = 28$

or, $\begin{vmatrix} 2 & 3 & -m \\ 1 & -2 & 0 \\ 3 & 1 & -2 \end{vmatrix} = 28$

or, $2\begin{vmatrix} -2 & 0 \\ 1 & -2 \end{vmatrix} - 3\begin{vmatrix} 1 & 0 \\ 3 & -2 \end{vmatrix} - m\begin{vmatrix} 1 & -2 \\ 3 & 1 \end{vmatrix} = 28$

or, $2(4-0) - 3(-2-0) - m(1+6) = 28$

or, $8 + 6 - 7m = 28$

or, $m = \frac{14}{-7} = -2$ ans

~~Parabola~~

$\vec{a}^2 = |\vec{a}|^2 = a^2$

IU)

Q) If $\vec{a}$ and $\vec{b}$ are two vectors of unit length and θ is the angle between them show that

$\frac{1}{2}|\vec{a} - \vec{b}| = \sin\frac{\theta}{2}$

Sol:

Since $\vec{a}$ and $\vec{b}$ are two vector of unit vector length $\therefore |\vec{a}| = 1$, $|\vec{b}| = 1$

now,

$|\vec{a} - \vec{b}|^2 = (\vec{a} - \vec{b})^2$

$= \vec{a}^2 - 2\vec{a}\cdot\vec{b} + \vec{b}^2$

$= 1^2 - 2\vec{a}\cdot\vec{b} + 1^2$

$= 2 - 2\vec{a}\cdot\vec{b}$ $\quad \because \vec{a}\cdot\vec{b} = |\vec{a}||\vec{b}|\cos\theta$

$= 2(1 - \vec{a}\cdot\vec{b})$

$= 2[1 - |\vec{a}||\vec{b}|\cos\theta]$ $\quad \because \cos\theta = \frac{\vec{a}\cdot\vec{b}}{|\vec{a}||\vec{b}|}$

$= 2[1 - (1)(1)\cos\theta]$

$= |\vec{a} - \vec{b}|^2 = 2\left(2\sin^2\frac{\theta}{2}\right)$

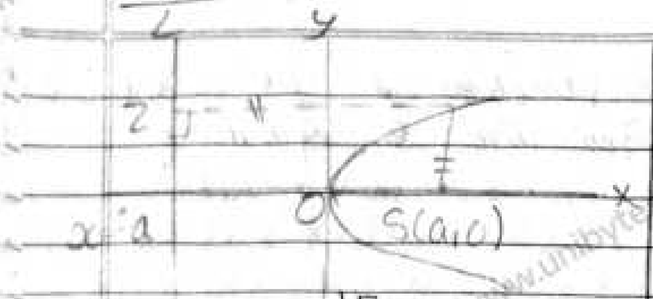
or, $|\vec{a} - \vec{b}|^2 = 4\sin^2\theta/2$

or, $|\vec{a} - \vec{b}|^2 = 2(\sin\theta/2)^2$

or, $|\vec{a} - \vec{b}| = 2\sin\theta/2$

$\Rightarrow \frac{1}{2}|\vec{a} - \vec{b}| = \sin\frac{\theta}{2}$ proved

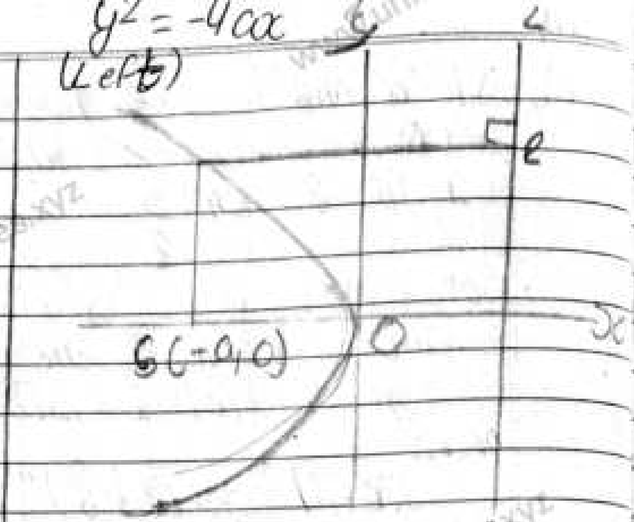
Parabola



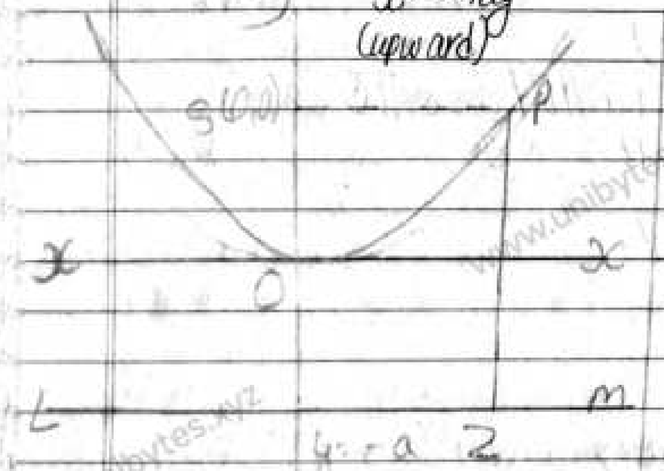
$[y^2 = 4ax]$ (Right)

$O(0,0)$ = vertex
$S(a,0)$ = Focus
LM = Directrix (st. line)

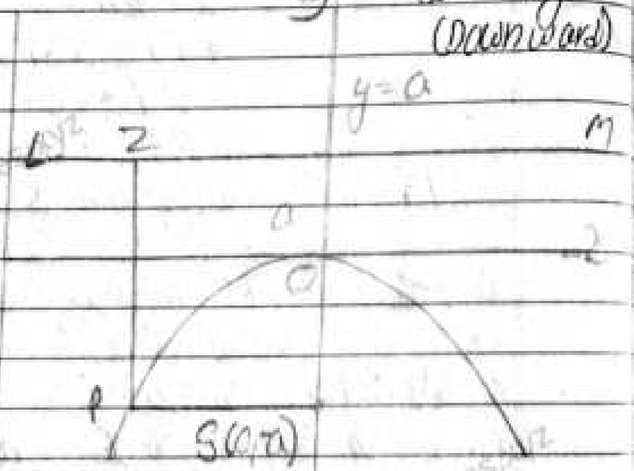
$y^2 = -4ax$ (Left)



$x^2 = 4ay$ (upward)



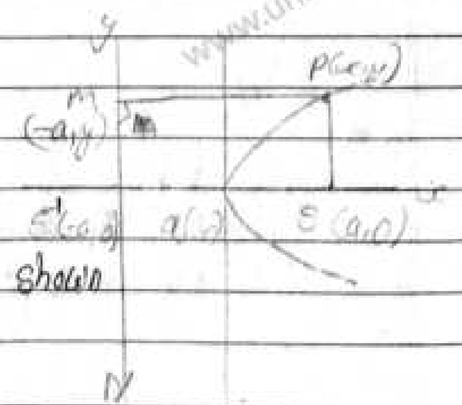
$x^2 = -4ay$ (Downward)



* Derive $y^2 = 4ax$

Sol:

Let $S(a,0)$ be the focus, O be the vertex and LN be the directrix of parabola as shown in fig $OS = OS' = a$



Let $P(x,y)$ be any point on the Parabola.

We known,

$$PS = PM$$

$$\Rightarrow \sqrt{(a-x)^2 + (0-y)^2} = \sqrt{(-a-x)^2 + (y-y)^2}$$

$$\Rightarrow a^2 - 2ax + x^2 + y^2 = a^2 + 2ax + x^2$$

$$\Rightarrow y^2 = 4ax$$

Parabola Formula:

	Equation	Vertex	Focus	Directrix	Axis	Length of Latus Rectum
x-axis	i) $y^2 = 4ax$	$(0,0)$	$(a,0)$	$x = -a$	$y = 0$	$4a$
y-axis	ii) $x^2 = 4ay$	$(0,0)$	$(0,a)$	$y = -a$	$x = 0$	$4a$
x-axis	iii) $(y-k)^2 = 4a(x-h)$	(h,k)	$(h+a,k)$	$x = h-a$	$y = k$	$4a$
y-axis	iv) $(x-h)^2 = 4a(y-k)$	(h,k)	$(h,k+a)$	$y = k-a$	$x = h$	$4a$

1 Exercise 5.2

1) Find the equation of the parabola in the following case

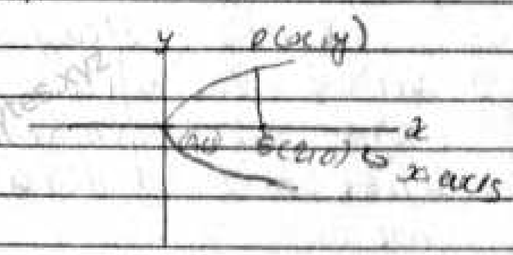
a) Vertex at $(0,0)$ and Focus $(2,0)$

Given;

Vertex $= (0,0)$

Focus $(a,0) = (2,0)$

$\therefore a = 2$



$\therefore$ Equation of parabola

$= y^2 = 4ax$

or, $y^2 = 4 \cdot 2 \cdot x$

or, $y^2 = 8x$ ans

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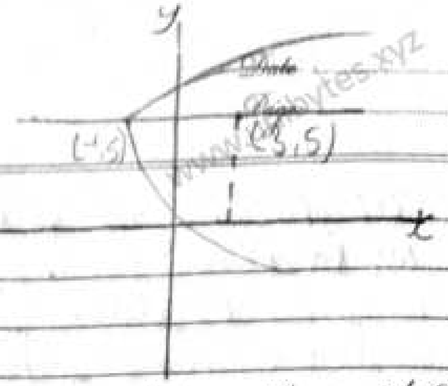


b) Given; Sol

Vertex $= (-1, 5)$

$(h,k) = (-1, 5)$

$h = -1,\ k = 5$

Parabola Parallel to x - axis and vertex at (h,k)

Eqn of parabola

$$(y-k)^2 = 4a(x-h)$$

vertex

$(h,k) = (-1,5)$

$h = -1,\ k = 5$

Focus

$(h+a, k) = (3,5)$

$h + a = 3$

$-1 + a = 3,\ a = 4$

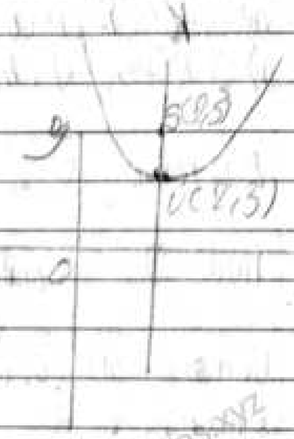
now;

$\Rightarrow (y-k)^2 = 4a(x-h)$

or, $(y-5)^2 = 4 \cdot 4(x-(-1))$

or, $y^2 - 10y + 25 = 16x + 16$ ans

~~or~~, or, $y^2 - 10y - 16x =$



c) Vertex $= (2,3)$

Focus $= (2,5)$

Axis of Parabola // to y axis

$= (x-h)^2 = 4a(y-k)$

or, vertex $= 2, 3$

$(h,k) = (2,3)$

$h = 2,\ k = 3$

Focus $= (2,5)$

$(h, k+a) = (2,5)$

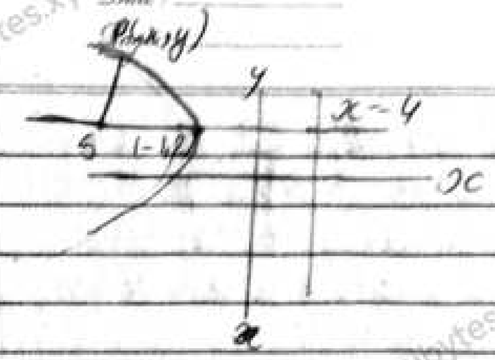
$k + a = 5,\ 3 + a = 5,\ a = 2$

now,

$(x-h)^2 = 4a(y-k)$

$(x-2)^2 = 4 \cdot 2(y-3)$

$x^2 - 4x + 4 = 8y - 24$ ans



d) Vertex $= (-1, 2)$

Directrix $x = 4$

vector $(h,k) = (-1, 2)$

$h = -1,\ k = 2$

Directrix $x = 4$

$x = h - a$

$h - a = 4$

$-1 - a = 4$

$-a = 5$

$a = -5$

Parabola axis // to x axis

$= (y-k)^2 = 4a(x-h)$

or, $(y-2)^2 = 4 \cdot (-5)(x - (-1))$

or, $y^2 - 4y + 2^2 = -20(x + 5)$

or, $y^2 - 4y + 4 = -100$

or, $y^2 - 4y = -100 - 4$

or, $y^2 - 4y = -104$ ans

e) Vertex at $(2,0)$ Focal width 16 and axis parallel to x - axis.

Sol:

Vertex $= (2,0)$

$h = 2,\ k = 0$

Focal width $(4a) = 16$

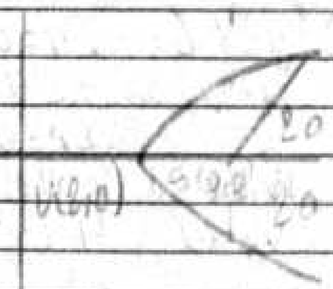
(Lactus rectum) $a = 4$

Since axis of parabola is // to x - axis

Eqn of parabola: $(y-k)^2 = 4a(x-h)$

$\Rightarrow (y-0)^2 = 4 \times 4(x-2)$

$\Rightarrow y^2 = 16(x-2)$



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2.a) $y^2 = 16x$

$y^2 = 4(4)x$

which is in the form

$y^2 = 4ax$ (std. form)

$a = 4$

vertex $(h,k) = O(0,0)$

Focus $(a,0) = S(4,0)$

Eq of Directrix $x = a$

i.e $x = -4$ ans

b) $y^2 = 12x$

$y^2 = 4(-3)x$

Comparing it with

$y^2 = 4ax$

$a = -3$

vertex $= (0,0)$

Focus $= (-3,0)$

Directrix $x = a$

$x = -(-3) = 3$ ans

c) $x^2 = 4y$

$x^2 = 4 \cdot 1 y$

which is in the form

$x^2 = 4ay$

$a = 1$

vertex $(h,k) = O(0,0)$

Focus $(0,a) = (0,1)$

Equation of Directrix $y = -a$

$= -1$

d) $y^2 + 6x - 24 = 0$

$y^2 = -6x + 24$

$y^2 = -6(x - 4)$

Reduce it in the form $(y-k)^2 = 4a(x-h)$

$(y-0)^2 = 4 \cdot (-3/2)(x-4)$

$\therefore k = 0,\ h = 4,\ a = -3/2$

vertex $(h,k) = (4,0)$

Focus $= (h+a, k)$

$= (4 + (-3/2), 0)$

$= 5/2, 0$

$\Rightarrow$ Eqn of directrix

$x = h - a$

$\Rightarrow x = 4 - (-3/2)$

$= 4 + 3/2$

$= 11/2$

e) $y^2 = 6y + 8x + 7$

$\Rightarrow y^2 - 6y = 8x + 7$

or, $y^2 - 2 \cdot 3y + 3^2 - 3^2 = 8x + 7$

or, $(y-3)^2 = 8x + 9 + 7$

or, $(y-3)^2 = 8x + 16$

or, $(y-3)^2 = 8(x+2)$

or, $(y-3)^2 = 4 \cdot 2(x+2)$

Comparing it with

$(y-k)^2 = 4a(x-h)$

$k = 3,\ a = 2,\ h = -2$

vertex $= (h,k) = (-2,3)$

Focus $= (h+a, k) = (-2+2, 3)$

$= (0,3)$

Directrix, $x = h - a$

$x = -2 - 2$

$x = -4$ ans

f) $(x+1)^2 - 8y + 16 = 0$

$\Rightarrow (x+1)^2 = 8y - 16$

or, $(x+1)^2 = 8(y-2)$

or, $(x+1)^2 = 4 \cdot 2(y-2)$

Comparing;

$(x-h)^2 = 4a(y-k)$

$h = -1,\ a = 2,\ k = 2$

vertex $(h,k) = (-1,2)$

Focus $(h, k+a) = (-1, 4)$

Directrix $y = k - a$

$y = 2 - 2$

$y = 0$ ans

Ellipse $(e < 1)$

Equation of ellipse

1) $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$; $a > b$ (along x-axis)

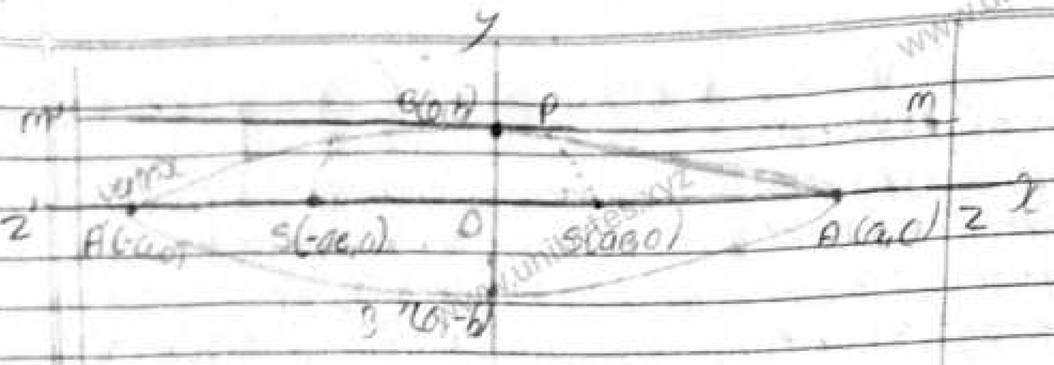
Centre to Focus distances $c = \sqrt{a^2 - b^2}$

Foci $= (\pm c, 0)$, $c = ae$

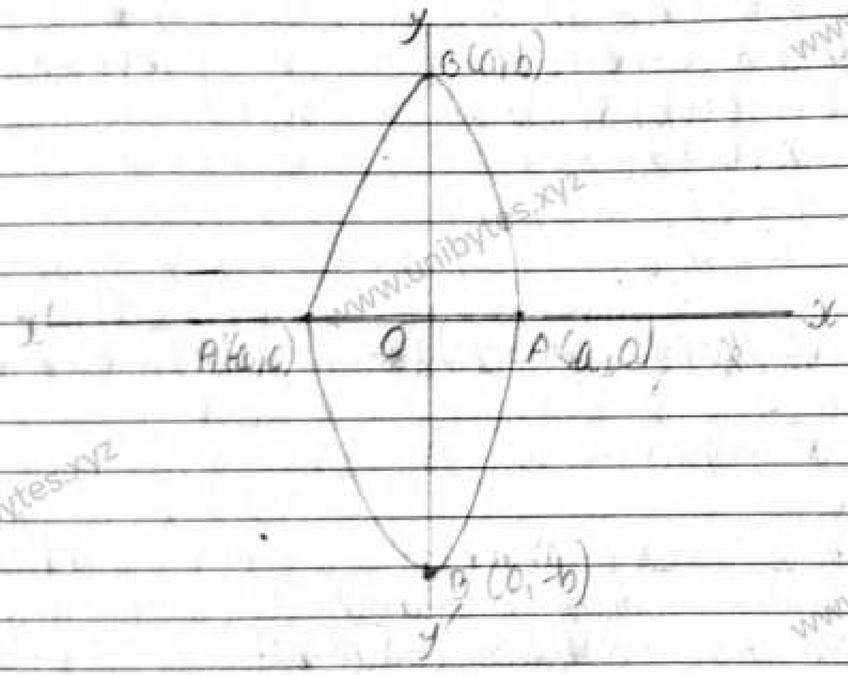
vertices $= (\pm a, 0)$

lengths of major axis $= 2a$

" " minor " $= 2b$



vertex $= (\pm a, 0)$
Focus $= (\pm ae, 0)$



* Formula

i) $a > b$

- $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, centre $= (0,0)$, vertex $= (\pm a, 0)$,
Focus $= (\pm ae, 0)$, $e = \sqrt{1 - \frac{b^2}{a^2}}$, Directrix $= x = \pm \frac{a}{e}$

ii) $b > a$

- $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, centre $= (0,0)$, vertex $= (0, \pm b)$, Focus
$= (0, \pm be)$, $e = \sqrt{1 - \frac{a^2}{b^2}}$, Directrix $= \pm b/e$

iii) $\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$, centre $= (h,k)$, vertex $= (h \pm a, k)$,
Focus $= (h \pm ae, k)$, $e = \sqrt{1 - \frac{b^2}{a^2}}$, Directrix $x = h \pm \frac{a}{e}$
$(a > b)$ (Horizontal)

iv) $a < b$ (vertical)

$\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$, centre $= (h,k)$, vertex $= (h, k \pm b)$,
Focus $= (h, k \pm be)$, $e = \sqrt{1 - \frac{a^2}{b^2}}$, Directrix $y = k \pm \frac{b}{e}$

Exercise 5.3

1.a) $\frac{x^2}{36} + \frac{y^2}{9} = 1$

Sol:

$$\frac{x^2}{6^2} + \frac{y^2}{3^2} = 1$$

Comparing it with

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

$\therefore a = 6, \; b = 3$

$a > b$ (ellipse is elongated along x-axis)

i) Centre $= (0, 0)$

ii) Vertices $= (\pm a, 0) = (\pm 6, 0)$

iii) Eccentricity $(e) = \sqrt{1 - \frac{b^2}{a^2}}$

$= \sqrt{1 - \frac{3^2}{6^2}} = \sqrt{\frac{3}{2}}$

iv) Foci $= (\pm ae, 0) = (\pm 6 \times \frac{\sqrt{3}}{2}, 0)$

$= (\pm 3\sqrt{3}, 0)$

v) Length of major axis $= 2a$

$= 2 \times 6 = 12$

vi) Length of minor axis $= 2b$

$= 2 \times 3 = 6$

b) $9x^2 + 4y^2 = 72$

Given,

$$9x^2 + 4y^2 = 72$$

$$= \frac{9x^2}{72} + \frac{4y^2}{72} = \frac{72}{72}$$

or, $\frac{x^2}{8} + \frac{y^2}{18} = 1$

~~x-axis~~ or, $\frac{x^2}{(\sqrt{8})^2} + \frac{y^2}{(\sqrt{18})^2} = 1$

Comparing it with $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

$\therefore a = \sqrt{8} = 2\sqrt{2}$

$\therefore b = \sqrt{18} = 3\sqrt{2} \; (\because a < b)$

$a < b$ ellipse elongates along y-axis

i) Centre $= (0, 0)$

ii) Vertices $= (0, \pm b) = (0, \pm 3\sqrt{2})$

iii) $E = \sqrt{1 - \frac{a^2}{b^2}} = \sqrt{1 - \frac{8}{18}} = \sqrt{\frac{5}{9}}$

iv) Foci $= (0, \pm be) = (0, \pm 3\sqrt{2} \times \sqrt{\frac{5}{9}})$

$= (0, \pm\sqrt{10})$

iv) Length of major axis $= 2b = 2 \times 3\sqrt{2} = 6\sqrt{2}$

v) Length of minor axis $= 2a = 2 \times 2\sqrt{2} = 4\sqrt{2}$

c) $\frac{x^2}{25} + \frac{(y-1)^2}{16} = 1$

$= \frac{(x-0)^2}{5^2} + \frac{(y-1)^2}{4^2} = 1$

Comparing it with $\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$

$\therefore h = 0, \; k = 1, \; a = 5, \; b = 4$

$\therefore a > b$ (ellipse is elongated parallel to x-axis)

i) Centre $= (h, k) = 0, 1$

ii) Vertices $= (h \pm a, k) = (0 \pm 5, 1) = (\pm 5, 1)$

iii) Eccentricity $(e) = \sqrt{1 - \frac{b^2}{a^2}} = \sqrt{1 - \frac{4^2}{5^2}} = \sqrt{\frac{25-16}{25}} = \frac{3}{5}$

iv) Foci $= (h \pm ae, k) = (0 \pm 5 \times \frac{3}{5}, 1) = (\pm 3, 1)$

v) Length of major axis $= 2a = 2 \times 5 = 10$

vi) Length of minor axis $= 2b = 2 \times 4 = 8$

d) $\frac{(x-2)^2}{8} + \frac{(y+1)^2}{20} = 1$

$= \frac{(x-2)^2}{(\sqrt{8})^2} + \frac{(y-(-1))^2}{(\sqrt{20})^2} = 1$

Comparing it with $\frac{(x-h)^2}{a} + \frac{(y-k)^2}{b} = 1$

$\therefore h = 2, \; k = -1, \; a = \sqrt{8} = 2\sqrt{3}, \; b = \sqrt{20} = 2\sqrt{5}$

$\therefore a < b$ (ellipse is elongated parallel to y-axis).

i) Centre $= (h, k) = (2, -1)$

ii) Vertices $= (h, k \pm b) = (2, -1 \pm 2\sqrt{5}$

iii) $E = \sqrt{1 - \frac{a^2}{b^2}} = \sqrt{1 - \frac{(2\sqrt{3})^2}{(2\sqrt{5})^2}} = \sqrt{\frac{20-12}{20}} = \sqrt{\frac{3}{5}}$

iv) Foci $= (h, k \pm be) = (2, -1 \pm 2\sqrt{5} \cdot \sqrt{\frac{3}{5}}) = (2, -1 \pm 2\sqrt{3})$

Length of major axis $= 2b = 2\times 2\sqrt{5} = 4\sqrt{5}$
" " minor " $= 2a = 2\times 2\sqrt{3} = 4\sqrt{3}$

e) $5x^2 + 9y^2 - 30x = 0$

Given,

$5x^2 + 9y^2 - 30x = 0$

or, $5(x^2 - 6x) + 9y^2 = 0$

or, $(x^2 - 2\cdot 3x + 3^2 - 3^2) + 9y^2 = 0$

or, $(x-3)^2 + 9y^2 = 9$

or,

or, $5(x^2 - 6x + \frac{9}{5}y^2) = 0$

or, $x^2 - 6x + \frac{9}{5}y^2 = 0$

or, $(x)^2 - 2\cdot x\cdot 3 + (3)^2 - (3)^2 + \frac{9}{5}y^2 = 0$

or, $(x-3)^2 + \frac{9}{5}y^2 = 9$

or, $\frac{(x-3)^2}{9} + \frac{9}{5\cdot 9}y^2 = \frac{9}{9}$

or, $\frac{(x-3)^2}{9} + \frac{y^2}{5} = 1$

or, $\frac{(x-3)^2}{3^2} + \frac{y^2}{(\sqrt{5})^2} = 1$

Comparing it with $\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$

$\therefore h = 3,\ k = 0,\ a = 3,\ b = \sqrt{5}$

$\therefore$ [$a > b$] (ellipse is elongated parallel to x-axis)

i) Centre $= (h, k) = (3, 0)$

ii) Vertices $= (h \pm a, k) = (3 \pm 3, 0) = (3-3, 0) = (0, 0)$
$= (3+3, 0) = (6, 0)$

iii) Eccentricity $(e) = \sqrt{1 - \frac{b^2}{a^2}} = \sqrt{1 - \frac{(\sqrt{5})^2}{3^2}} = \sqrt{1 - \frac{5}{9}} = \sqrt{\frac{9-5}{9}} = \frac{2}{3}$

iv) Foci $= (h \pm ae, k) = (3 \pm 3\cdot\frac{2}{3}, 0) = (3 \pm 2, 0)$

v) Length of major axis $= 2a = 2\times 3 = 6$

vi) Length of minor axis $= 2b = 2\times\sqrt{5} = 2\sqrt{5}$

f) $9x^2 + 4y^2 - 18x - 16y - 11 = 0$

Sol:

$9x^2 - 18x + 4y^2 - 16y - 11 = 0$

$\Rightarrow 9x^2 - 18x + 4y^2 - 16y = 11$

$\Rightarrow 9(x^2 - 2x) + 4(y^2 - 4y) = 11$

or, $9\{x^2 - 2\cdot x\cdot 1 + 1^2 - 1^2\} + 4\{y^2 - 2\cdot y\cdot 2 + 2^2 - 2^2\} = 11$

or, $9\{(x-1)^2 - 1\} + 4\{(y-2)^2 - 4\} = 11$

or, $9(x-1)^2 - 9 + 4(y-2)^2 - 16 = 11$

or, $9(x-1)^2 + 4(y-2)^2 = 11 + 9 + 16$

or, $9(x-1)^2 + 4(y-2)^2 = 36$

or, $\frac{9(x-1)^2}{36} + \frac{4(y-2)^2}{36} = \frac{36}{36}$

or, $\frac{(x-1)^2}{4} + \frac{(y-2)^2}{9} = 1$

or, $\frac{(x-1)^2}{(2)^2} + \frac{(y-2)^2}{(3)^2} = 1$

Comparing with $\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$

$\therefore a = 2,\ b = 3,\ h = 1,\ y = 2$

$\therefore a < b$ (Axis of ellipse is // to y-axis)

Ellipse

$a > b \rightarrow$ Lactus rectum $= \frac{2b^2}{a}$

$a < b \rightarrow$ lactus rectum $= \frac{2a^2}{b}$

i) Centre of ellipse $= (h, k) = (1, 2)$

ii) vertex $= (h, k \pm b) = (1, 2 \pm 3) = (1, 2+3) = (1, 5)$
$= (1, 2-3) = (1, -1)$

iii) $e = \sqrt{1 - \frac{a^2}{b^2}} = \sqrt{1 - \frac{2^2}{3^2}} = \sqrt{\frac{9-4}{9}} = \frac{\sqrt{5}}{3}$

iv) Foci of the ellipse $= (h, k \pm be) = (1, 2 \pm 3\frac{\sqrt{5}}{3})$
$= (1, 2 \pm \sqrt{5})$

v) Equation of directrix $= k \pm \frac{b}{e} = 2 \pm \frac{3}{\frac{\sqrt{5}}{3}}$
$= 2 \pm \frac{3 \times 3}{\sqrt{5}} = 2 \pm \frac{9}{\sqrt{5}}$

* Find the eqn of ellipse where lactus rectum is 3 and $e = 1/\sqrt{2}$

Sol:

Let the eqn of ellipse be $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

Lactus rectum $= 3$
$\Rightarrow \frac{2b^2}{a} = 3$

$e = 1/\sqrt{2}$
$= \sqrt{1 - \frac{b^2}{a^2}} = \frac{1}{\sqrt{2}}$

or, Squaring both side
$1 - \frac{b^2}{a^2} = \frac{1}{2}$

or, $1 - \frac{b^2}{a^2} = \frac{1}{2} - 1$

or, $-\frac{b^2}{a^2} = \frac{1-2}{2}$

or, $\frac{+b^2}{a^2} = \frac{+1}{2}$

or, $2b^2 = a^2$ — ①

Lactus rectum $= 3$
$\Rightarrow \frac{2b^2}{a} = 3$

Applying eqn ①
$\frac{a^2}{a} = 3$

$a = 3$.

again,
$\Rightarrow 2b^2 = a^2$

or, $2b^2 = 3^2$

or, $b^2 = \frac{9}{2}$

now,
Eqn of ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

$= \frac{x^2}{9} + \frac{y^2}{9/2} = 1$

$= \frac{x^2}{9} + \frac{2y^2}{9} = 1$

$= \frac{x^2 + 2y^2}{9} = 1$

$\therefore x^2 + 2y^2 = 9$ ans.

* Find the e and foci of ellipse $25x^2 + y^2 = 100$

Sol:

$= \frac{25x^2}{100} + \frac{y^2}{100} = \frac{100}{100}$

or, $\frac{x^2}{4} + \frac{y^2}{100} = 1$

or, $\frac{(x-0)^2}{2^2} + \frac{y^2}{10^2} = 1$

$\therefore b > a$ (axis of ellipse is // to y-axis).

$e = \sqrt{1 - \frac{a^2}{b^2}} = \sqrt{1 - \frac{2^2}{10^2}}$

$= \sqrt{1 - \frac{4}{100}} = \sqrt{\frac{100-4}{100}} = \sqrt{\frac{96}{100}} = \frac{4\sqrt{6}}{10} = \frac{2\sqrt{6}}{5}$ ans

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2. Find the equation of the ellipse under the following conditions.

a) Length of major and minor axes are 8 and 6 respectively.

Sol:

major axis = 8
$2a = 8$
$a = 4$

minor axis = 6
$2b = 6$
$b = 3$

now,

$$\text{equation} = \frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$
$$= \frac{x^2}{4^2} + \frac{y^2}{3^2} = 1$$
$$= \frac{9x^2 + 16y^2}{144} = 1$$
$$\therefore 9x^2 + 16y^2 = 144 \text{ ans}$$

b) Vertex at (5,0) and focus at (-2,0)

Sol:

$a = \pm 5$, $f = (-2, 0)$

now,
$f = (-2, 0)$
$(\pm ae, 0) = (-2, 0)$
$\pm 5 \cdot e = -2$
$e = \frac{-2}{\pm 5}$

now,
$e = \frac{-2}{\pm 5}$

sqr.
$e^2 = \left(\frac{-2}{\pm 5}\right)^2$

$1 - \frac{b^2}{a^2} = \frac{4}{25}$
$1 - \frac{b^2}{5^2} = \frac{4}{25}$
$\frac{25 - b^2}{25} = \frac{4}{25}$
$625 - 25b^2 = 100$
$b^2 = \frac{525}{25} = 21$

Equations
$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$
$\frac{x^2}{25} + \frac{y^2}{21} = 1$ ans

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c) vertex at (0,6) and focus at (0, ±3)

now,

vertex = (0,6)
$(a,b) = (0,6)$
$\therefore b = 6$

now,
Focus = (0, ±3)
$(0, be) = (0, \pm 3)$
$be = \pm 3$
$\pm 6 \cdot e = \pm 3$
$\therefore e = \frac{\pm 3}{\pm 6} = \frac{\pm 1}{\pm 2}$

$e = \frac{\pm 1}{\pm 2}$
~~±b~~ squaring
$= 1 - \frac{a^2}{b^2} = \frac{1}{4}$
$= 1 - \frac{a^2}{36} = \frac{1}{4}$
$= \frac{36 - a^2}{36} = \frac{1}{4}$

$= 144 - 4a^2 = 36$
or, $-4a^2 = 36 - 144$
or, $-4a^2 = -108$
$= a^2 = 27$

now,
$$\text{equation} = \frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$
$$= \frac{x^2}{27} + \frac{y^2}{36} = 1 \text{ ans}$$

d) Foci at (±2, 0) and eccentricity = $\frac{1}{3}$

Sol:

$(\pm ae, 0) = (\pm 2, 0)$
$= \pm a \cdot e = \pm 2$
$= \pm a \cdot \frac{1}{3} = \pm 2$
or, $\pm a = \frac{\pm 2}{\frac{1}{3}}$
or, $\pm a = \pm 6$

now,
eccentricity = $\frac{1}{3}$

squaring both side

$1 - \frac{b^2}{a^2} = \frac{1}{9}$

$= 1 - \frac{b^2}{36} = \frac{1}{9}$
or, $\frac{36 - b^2}{36} = \frac{1}{9}$
or, $324 - 9b^2 = 36$
or, $324 - 36 = 9b^2$
or, $288 = 9b^2$
$\therefore b^2 = 32$

now,
$$\text{equation} = \frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$
$$= \frac{x^2}{36} + \frac{y^2}{32} = 1 \text{ ans}$$

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Hyperbola

Equation of hyperbola $(a>b)$

i) ~~Foci~~ on x - axis $= \frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$

Centre of focus distance $c = \sqrt{a^2+b^2}$

Vertices $= (\pm a, 0)$

Foci $= (\pm c, 0) = (\pm ae, 0)$

Asymptes $= \frac{x^2}{a^2} - \frac{y^2}{b^2} = 0$

$(a<b)$

ii) Equation of hyperbola on y - axis $= \frac{x^2}{a^2} - \frac{y^2}{b^2} = -1$

Centre of focus distance $(c) = \sqrt{a^2+b^2}$

Foci $= (0, \pm c) = (0, \pm be)$

Vertices $= (0, \pm b)$

Asymptes $= \frac{y^2}{b^2} - \frac{x^2}{a^2} = 0$

iii) Equation of hyperbola when centre is in (h, k)

$= \frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1$

Vertex $= (h \pm a, k)$

Foci $= (h \pm ae, k)$

Note: Eccentricity is the ratio of distance from focus to moving point and moving point to distance

Exercise 5.4

1) Find the centre, vertices, eccentricity and foci of the following hyperbola.

a) $\frac{x^2}{16} - \frac{y^2}{9} = 1$

sol:

$\frac{x^2}{16} - \frac{y^2}{9} = 1 \ (a>b)$

$\therefore$ Centre $= (0,0)$

Vertices $= (\pm a, 0) = (\pm 4, 0)$

$e = \sqrt{1 + \frac{b^2}{a^2}} = \sqrt{1 + \frac{9}{16}} = \frac{5}{4}$

Foci $= (\pm ae, 0)$

$= (\pm \cancel{4} \times \frac{5}{\cancel{4}}, 0)$

$= (\pm 5, 0)$ ans

b) $\frac{x^2}{36} - \frac{y^2}{64} = 1$

sol:

$\frac{x^2}{6^2} - \frac{y^2}{8^2} = 1$

yedi $\frac{x^2}{a^2} - \frac{y^2}{b^2} = -1$ vayo vane balla $b>a$ ho rahecha

Comparing with the formula $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$

$a = 6, \ b = 8 \ (a<b)$

Centre $= (0,0)$

vertices $= (\pm a, \pm b) = \cancel{(0, \pm 8)} = (\pm 6, 0)$

$e = \sqrt{1 + \frac{b^2}{a^2}} = \sqrt{1 + \frac{8^2}{6^2}} = \sqrt{\frac{\cancel{64+36}}{64}} =$

$= \sqrt{\frac{36+64}{36}} = \frac{\cancel{10}}{\cancel{6}} = \frac{5}{3}$

Foci $= (\pm ae, 0)$

$= (\pm \frac{5}{3}, 0) = (\pm 10, 0)$

c) $4x^2 - 3y^2 = -24$

sol:

$= \frac{4x^2}{24} - \frac{3y^2}{24} = \frac{-24}{-24}$

or, $\frac{x^2}{6} - \frac{y^2}{8} = -1$

or, $\frac{x^2}{(\sqrt{6})^2} - \frac{y^2}{(\sqrt{8})^2} = -1$

Comparing it with $= \frac{x^2}{a^2} - \frac{y^2}{b^2} = -1$

Centre $= (0, 0)$

vertices $= (0, \pm b) = (0, \pm\sqrt{8}) = (0, \pm 2\sqrt{2})$

~~eccentricity $= (0, \pm be) = (0, \pm$~~

eccentricity $= \sqrt{1 + \frac{a^2}{b^2}} = \sqrt{1 + \frac{6}{8}} = \sqrt{\frac{8+6}{8}} = \frac{\sqrt{14}}{2\sqrt{2}} = \frac{\sqrt{7}}{2}$

Foci $= (0, \pm be) = (0, 2\sqrt{2} \cdot \frac{\sqrt{7}}{2}) = (0, \pm\sqrt{14})$

d) $\frac{(x+1)^2}{12} - \frac{(y-2)^2}{18} = 1$ (yeha ma di same last mark kaiho c 35 ma 1 aye kaha 2 c ho.)

sol:

$= \frac{(x+1)^2}{(\sqrt{12})^2} - \frac{(y-2)^2}{(3\sqrt{2})^2} = 1$

Comparing it with $\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1$

$h = -1$, $k = 2$, $a = 2\sqrt{3}$, $b = 3\sqrt{2}$ $(b > a)$

i) centre $(h, k) = (-1, 2)$

~~ii) vertices $= (h, k \pm b) = (-1, 2 \pm 3\sqrt{2})$~~

ii) Vertices $= (h \pm a, k) = (-1 \pm 2\sqrt{3}, 2)$

iii) eccentricity $= \sqrt{1 + \frac{b^2}{a^2}} = \sqrt{\frac{12+18}{12}} = \sqrt{\frac{30}{12}} = \sqrt{\frac{5}{2}}$ ans

iv) Foci $= (h \pm ae, k)$

~~$= -1 \pm 2\sqrt{3} \times \sqrt{\frac{5}{12}}, 2)$~~

$= (-1 \pm 2\sqrt{3} \cdot \sqrt{\frac{5}{2}}, 2)$

$= (-1 \pm \sqrt{30}, 2)$

e) $x^2 - 5y^2 - 6x = 11$

sol:

$= x^2 - 6x - 5y^2 = 11$

or, $x^2 - 2.3.x + 3^2 - 3^2 - 5y^2 = 11$

or, $(x-3)^2 - (5y-0)^2 = 11 + 9$

or, $\frac{(x-3)^2}{20} - \frac{5(y)^2}{20} = \frac{20}{20}$

or, $\frac{(x-3)^2}{20} - \frac{(y-0)^2}{4} = 1$

or, $\frac{(x-3)^2}{\sqrt{20}} - \frac{(y-0)^2}{2} = 1$

Comparing it with $\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1$

$\therefore h = 3,\ k = 0,\ a = \sqrt{20},\ b = 2$

i) Centre $= (h, k) = (3, 0)$

ii) Vertices $= (h \pm a, k) = (3 \pm \sqrt{20}, 0)$

iii) $e = \sqrt{1 + \frac{b^2}{a^2}} = \sqrt{1 + \frac{4}{20}} = \frac{3}{2\sqrt{5}} \sqrt{\frac{24}{20}} = \sqrt{\frac{6}{5}}$

iv) Foci $= (h \pm ae, k) = (3 \pm \sqrt{20} \cdot \sqrt{\frac{6}{5}}, 0)$

or, $\left(3 \pm 2\sqrt{5} \cdot \frac{\sqrt{6}}{\sqrt{5}}, 0\right) \therefore (3 \pm 2\sqrt{6}, 0)$ ans

2) Find the equation of hyperbola under the following condition

a) length of transverse axis = 12 and length of conjugate axis = 8.

sol:

length of transverse axis = 12

$2a = 12$

$a = 6$

Length of conjugate axis = 8

$2b = 8$

$b = 4$

now;

Equation;

$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$

or, $\frac{x^2}{36} - \frac{y^2}{16} = 1$

or, $\frac{16x^2 - 9y^2}{576} = 1$

or, $4(4x^2 - 9y^2) = 576$

or, $4x^2 - 9y^2 = 144$ ans

b) Focus at $(\pm 5, 0)$ and vertex at $(2, 0)$

sol:

$(\pm ae, 0) = (\pm 5, 0)$

$\pm ae = \pm 5$

$2 \cdot e = 5$

$e = 5/2$

vertex $= (2, 0)$

$(a, 0) = 2, 0$

$a = 2$

now;

$e = \sqrt{1 + \frac{b^2}{a^2}}$

squaring both side

$= \frac{25}{4} = 1 + \frac{b^2}{4}$

or, $\frac{25}{4} = \frac{4 + b^2}{4}$

or, $25 = 4 = b^2$

or, $b^2 = 21$

now;

Equation of Hyperbola

$= \frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$

or $\frac{x^2}{4} - \frac{y^2}{21} = 1$ ans

c) Focus at $(0,2)$ and vertex at $(0,-1)$

Sol:

Focus $= (0,2)$ | Vertex $= (0,-1)$

$(0, \pm be) = (0,2)$ | $(0, \pm b) = (0,-1)$

$1e = 2$ | $-b = -1$

$e = 2$ | $b = 1$

now;

$$e = \sqrt{1 + \frac{a^2}{b^2}}$$

squaring both side

$$2^2 = 1 + \frac{a^2}{1}$$

$$4 = 1 + a^2$$

$$a^2 = 3$$

Equation of hyperbola

$$= \frac{x^2}{a^2} - \frac{y^2}{b^2} = -1$$

$$\text{or, } \frac{x^2}{3} - \frac{y^2}{1} = -1$$

$$\text{or, } x^2 - 3y^2 = -3 \text{ ans}$$

d) Focus at $(\pm 4, 0)$ and eccentricity $= \frac{4}{3}$

Sol:

$(\pm ae, 0) = (\pm 4, 0)$ and $e = \frac{4}{3}$

$ae = 4$

$a \cdot \frac{4}{3} = 4$

$a4 = 12$

$a = \frac{12}{4}$

$a = 3$

now;

$$b^2 = a^2(e^2 - 1)$$

$$= 3^2\left(\left(\tfrac{4}{3}\right)^2 - 1\right)$$

$$= 9\left(\frac{16}{9} - 1\right)$$

$$= 9\left(\frac{16-9}{9}\right)$$

$$= 9\left(\frac{7}{9}\right) = \left(\frac{63}{9}\right) = 7$$

Equation of Hyperbola

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$

$$\text{or, } \frac{x^2}{9} - \frac{y^2}{7} = 1$$

$$\text{or, } \frac{7x^2 - 9y^2}{63} = 1$$

$$\therefore 7x^2 - 9y^2 = 63 \text{ ans}$$

e) Focus at $(3,0)$ and length of transverse axis $= 4$.

Sol:

length of transverse axis $= 4$

$2a = 4$

$a = 2$

now;

$(\pm a, e) = (\pm 3, 0)$

$ae = 3$

$e = \frac{3}{2}$

$b^2 = a^2(e^2 - 1)$

$= 2^2((\frac{3}{2})^2 - 1)$

$= 4(\frac{9}{4} - 1)$

$= 4(\frac{9-4}{4})$

$\therefore b^2 = 5$

Equation of hyperbola

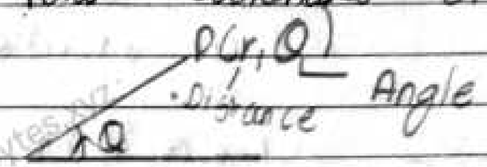
$= \frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$

or, $\frac{x^2}{2^2} - \frac{y^2}{5} = 1$

or, $\frac{5x^2 - 4y^2}{20} = 1$

$\therefore 5x^2 - 4y^2 = 20$ ans

Polar Coordinates of Point



$x = r\cos\theta$

$y = r\sin\theta$

$r^2 = x^2 + y^2$

$\tan\theta = \frac{y}{x}$

* Hyperbola, parabola and ellipse are known as conic section.

Ellipse, Parabola and Hyperbola in polar cordinates

$r = \frac{ke}{1 + e\cos\theta}$

This is an ellipse if $e < 1$

This is an parabola if $e = 1$

This is an Hyperbola if $e > 1$

Eqn of circle in parabola form

i) Eqn of circle at origin $r = 2a\cos(\theta - \theta_0)$

ii) Centre on x-axis $r = 2a\cos\theta$

iii) Centre on y-axis $r = 2a\sin\theta$

Exercise 5.5

1) Classify the following conic section whose polar equation are given below:

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a) $r = \frac{10}{5+5\cos\theta}$

sol:
Reduce it into $r = \frac{ke}{1+e\cos\theta}$

$\Rightarrow r = \frac{5\times 2}{5(1+1.\cos\theta)}$

$r = \frac{2.1}{1+1.\cos\theta}$

$k = 2$ [$\therefore x = k$]

$e = 1$

$\therefore$ It is parabola.

b) $r = \frac{8}{2-2\sin\theta}$

sol:
Reduce it into $r = \frac{ke}{1-e\sin\theta}$

$\Rightarrow r = \frac{2\times 4}{2(1-1.\sin\theta)}$

or, $\frac{4.1}{(1-1.\sin\theta}$

$\therefore y = -k$
$= -4$

$e = 1$

$\therefore$ It is parabola

c) $r = \frac{6}{3+\cos\theta}$

sol:
Reduce it into $r = \frac{ke}{1+e\cos\theta}$

$r = \frac{\overset{6}{\cancel{18}}\times\frac{1}{3}}{\cancel{3}(1+\frac{1}{3}\cos\theta)}$

$r = \frac{6\times\frac{1}{3}}{1+\frac{1}{3}\cos\theta}$

$e = 1/3$

$x = k = 6$

$\therefore$ This is ellipse

d) $r = \frac{100}{25-20\sin\theta}$

sol:
Reduce it into $r = \frac{ke}{1-e\sin\theta}$

$\Rightarrow r = \frac{20\times 4}{25(1-\frac{20}{25}\sin\theta)}$

or, $\frac{4}{1-\frac{4}{5}\sin\theta}$

or, $\frac{5\times 4/5}{1-4/5\sin\theta}$

$e = \frac{4}{5}$, $y = -k$
$= -5$

$\therefore$ It is ellipse

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e) $r = \frac{20}{5-10\cos\theta}$

Reduce it into $\frac{ke}{1-e\cos\theta}$

$\Rightarrow \frac{5\times 4}{5(1-2\cos\theta)}$

or, $r = \frac{4}{1-2\cos\theta}$

or, $r = \frac{2.2}{1-2\cos\theta}$

$\therefore e = 2$, $\cancel{x} = -\cancel{k}$
$y = -2$

$\therefore$ This is Hyperbola.

f) $r = \frac{36}{12+18\sin\theta}$

Reduce it into $\frac{ke}{1+e\sin\theta}$

$r = \frac{12\times 3}{12(1+\frac{3}{2}\sin\theta)}$

or, $r = \frac{3}{1+\frac{3}{2}\sin\theta}$

or, $r = \frac{2\times 3/2}{1+\frac{3}{2}\sin\theta}$

$\therefore e = 3/2$, $y = k$
$= 2$

$\therefore$ This is ellipse

Imp

2) Given below the eccentricity and the equation of the directrix, find the polar equation of conic section.

a) $e = 1$, $x = 3$
(Directrix $x = k$)

So polar eqn of conic section is

$r = \frac{ke}{1+e\cos\theta}$

$r = \frac{3\times 1}{1+1.\cos\theta}$

$r = \frac{3}{1+1\cos\theta}$ ans

b) $e = 1$, $y = -2$

Sol:

(Directrix $y = -k$

$+2 = +k \therefore k = 2$

now;

$$r = \frac{ke}{1 - e\sin\theta}$$

$$= \frac{2 \cdot 1}{1 - 1 \cdot \sin\theta}$$

$$= \frac{2}{1 - \sin\theta} \text{ ans}$$

c) $e = \frac{2}{3}$, $x = -4$

(Directrix $x = -k$

$= 4$)

now,

$$r = \frac{ke}{1 - e\cos\theta}$$

$$\Rightarrow \frac{4 \cdot 2/3}{1 - 2/3 \cos\theta}$$

$$\Rightarrow \frac{\frac{12 \times 2}{3}}{3 - 2\cos\theta}$$

$$\Rightarrow \frac{8}{3 - 2\cos\theta} \text{ ans}$$

d) $e = \frac{1}{5}$, $y = 6$

Sol:

(Directrix $y = k$)

now;

$$r = \frac{ke}{1 + e\sin\theta}$$

$$= \frac{6 \cdot 1/5}{1 + 1/5 \sin\theta}$$

$$= \frac{6}{5 + \sin\theta}$$

d) $e = 2$, $x = 2$

(Directrix $x = k$

$= 2$)

now;

$$r = \frac{ke}{1 + e\cos\theta}$$

$$= \frac{2 \cdot 2}{1 + 2 \cdot \cos\theta}$$

$$\Rightarrow \frac{4}{1 + 2\cos\theta} \text{ ans}$$

f) $e = 4/3$, $y = 5$

Sol:

(Directrix $y = k$

$y = 5$)

now;

$$r = \frac{ke}{1 + e\sin\theta}$$

$$= \frac{5 \cdot 4/3}{1 + 4/3 \sin\theta}$$

$$= \frac{20}{3 + 4\sin\theta} \text{ ans}$$

3) Express the following polar equations in cartesian form.

a) $r\cos\theta = 3$

Sol:

$x = 3$

b) $r = -2\cos\theta$

$$r = \frac{2}{\sin\theta}$$

$r\sin\theta = -2$

$y = -2$ ans

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d) $r = 5\sec(\theta - \pi/6)$

Sol:

i. $r = 5\sec(\theta - \pi/6)$

ii. or, $r \cdot \frac{1}{\cos(\theta - \pi/6)} = 5$

iii. or, $r \cdot \cos(\theta - \pi/6) = 5$

iv. or, $r \cdot \left(\cos\theta \cdot \cos\frac{\pi}{6} + \sin\theta \cdot \sin\frac{\pi}{6}\right) = 5$

v. or, $r \cdot \left(\cos\theta \cdot \frac{\sqrt{3}}{2} + \sin\theta \cdot \frac{1}{2}\right) = 5$

or, $r\cos\theta \cdot \frac{\sqrt{3}}{2} + r\sin\theta \cdot \frac{1}{2} = 5$

or, $x \cdot \frac{\sqrt{3}}{2} + y \cdot \frac{1}{2} = 5$

$\therefore \sqrt{3}x + y = 10$

4) Express the following cartesian equation into polar form.

a) $x = 2$

$r\cos\theta = 2$

c) $\sqrt{3}x - y = 2$

$\Rightarrow \sqrt{3} \cdot r\cos\theta - r\sin\theta = 2$

or, $r\left(\frac{\sqrt{3}}{2}\cos\theta - \frac{1}{2}\sin\theta\right) = 2$

or, $r\left(\cos\frac{\pi}{6} \cdot \cos\theta - \sin\frac{\pi}{6} \cdot \sin\theta\right) = 1$

$\therefore r\cos\left(\theta + \frac{\pi}{6}\right) = 1$

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b) $(x-2)^2 + y^2 = 4$

Sol:

$(r\cos\theta - 2)^2 + (r\sin\theta)^2 = 4$

$\Rightarrow r^2\cos^2\theta - 4r\cos\theta + 4 + r^2\sin^2\theta = 4$

$\Rightarrow r^2(\cos^2\theta + \sin^2\theta) - 4r\cos\theta = 0$

or, $r^2 \cdot 1 - 4r\cos\theta = 0$

or, $r^2 = 4r\cos\theta$

$\therefore r = 4\cos\theta$

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Note: Explain book bata padni

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Chapter 2 Relation, Function and Graphs

* Order Pair : A pair of elements a and b is said to be an ordered pair if 'a' is fixed as the first element and b as the second element. It is denoted by (a,b) since $(a,b) \neq (b,a)$

* Cartesian Products : Given two non-empty sets A and B. The cartesian product of A and B is denoted by $A \times B$ (read as A cross B) and is defined as the collection of all ordered pairs (x,y), $x \in A$, $y \in B$.

Symbolically : $A \times B = \{(x,y) : x \in A, y \in B\}$

either A or B is empty then $A \times B = \emptyset$

* Relation (R)

Let A and B be two sets. Any subset of $A \times B$ is called a relation from A to B. It is denoted by R, or $\boxed{aRb}$, $a \in A$, $b \in B$. The relation from A to A is called a relation on A.

Domain and Range.

The set of all first elements of each ordered pair of R is said to be the domain of the relation R and the set of all second elements of each pair of R is called range.

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Exercise 2.1

1) Find the value of x and y if

a) $(x-y, y) = (2, 3)$

Sol: $(x-y, y) = (2, 3)$

Equating the corresponding elements,

$x - y = 2$, $y = 3$

$x - 3 = 2$

$x = 2 + 3$

$= 5$

b) $(2x+3y, -1) = (5, x-2y)$

Sol: Equating the corresponding elements,

$2x + 3y = 5$, $-1 = x - 2y$

or, $2x = 5 - 3y$, or, $-1 = \frac{5-3y}{2} - 2y$

or, $x = \frac{5-3y}{2}$ ① , or, $-1 = \frac{5-3y-4y}{2}$

or, $-1 = \frac{5-7y}{2}$

or, $-2 = 5 - 7y$

or, $7y = 5 + 2$

or, $y = \frac{7}{7} = 1$ $\therefore y = 1$

Putting the value of y in eqn ①

$x = \frac{5-3y}{2}$

$x = \frac{5-3\times1}{2} = \frac{5-3}{2} = \frac{2}{2} = 1$

$\therefore x = 1$

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2) Symmetric

A relation R on a set A is called symmetric if $(x,y) \in R$

$\Rightarrow (y,x) \in R$ for every $x, y \in A$

or, $[x R y \Rightarrow y R x]$ for every $x, y \in A$

3) Transitive

A relation R on a set A is called transitive if $(x,y) \in R$, $(y,z) \in R$ $\Rightarrow (x,z) \in R$ ~~from~~ for every $x, y, z \in A$

or, $[x R y, y R z \Rightarrow x R z]$ for every $x, y, z \in A$

4) Antisymmetric

A relation R on A is called antisymmetric if $x, y \in A$, $x R y$, $y R y$, $y R x \Rightarrow x = y$

i.e $x, y \in A$, $\boxed{\begin{matrix}(x,y) \in R \\ (y,x) \in R\end{matrix} \Rightarrow x = y}$

* A Relation is the set of order pairs which is taken from Cartesian product under certain mathematical function.

* The subset of cartisan product $A \times B$ is called Relation.

Equivalence Relation
- Reflexive $(x,x) \in R$ For all $x \in A$
- Symmetric $(x,y) \Rightarrow (y,x) \in R$ For all $x,y \in A$
- Transitive $(x,y), (y,z) \Rightarrow (x,z) \in R$ For all $x,y,z \in A$

Antisymmetric

$$(x,y)\,(y,x) \Rightarrow x=y \in R$$

F.Q

7) Let $A=\{1,2,3,4,5\}$ and relation $R=\{(1,1),(2,2),(3,3),(4,4),(5,5),(3,4),(4,5)\}$. Show that the relation R is reflexive but neither symmetric nor transitive.

Given;

$A=\{1,2,3,4,5\}$

$R=\{(1,1),(2,2),(3,3),(4,4),(5,5),(3,4),(4,5)\}$

For Reflexive:

For all $1,2,3,4,5 \in A$

$\therefore (1,1),(2,2),(3,3),(4,4),(5,5) \in R$

$\therefore xRx$, i.e $(x,x) \in R$

$\therefore R$ is Reflexive

For Symmetric:

For all $3,4,5 \in A$

$(3,4) \Rightarrow (4,5) \notin R \; [\because (x,y) \not\Rightarrow (y,x)]$

$\therefore R$ is not symmetric

For transitive:

For all $3,4,5 \in A$

$\left.\begin{matrix}(3,4)\\(4,5)\end{matrix}\right\} \Rightarrow (3,5) \notin R$

8) Show that the relation xRy on A defined by $x-y=0$, $x,y \in A$ is an equivalence relation on A.

ans

Sol:

Relation (R) is defined by

$x-y=0 \quad x,y \in A$

i) For Reflexive,

For $x \in A \Rightarrow x-x=0$

$\therefore R$ is reflexive $[\because (x,x) \in R]$

ii) For Symmetric.

For $x,y \in A$,

$x-y=0 \Rightarrow y-x=0$

$\therefore R$ is symmetric

i.e $(x,y) \in R \Rightarrow (y,x) \in R \quad \therefore \boxed{(x,y) \in R \Rightarrow (y,x) \in R}$

iii) For transitive

For $x,y,z \in A$

$\left.\begin{matrix}x-y=0\\y-z=0\end{matrix}\right\} \Rightarrow x-z=0$

i.e $\left.\begin{matrix}(x,y) \in R\\(y,z) \in R\end{matrix}\right\} \Rightarrow (x,z) \in R$

$\therefore R$ is transitive

So R is reflexive sym and transitive and hence it is equivalence relation on A.

9) Show that the relation (is parallel to) over the set of all straight lines in a plane is equivalence relation,

Sol:

For reflexive: Since $x \parallel x$ (parallel) for all $x \in A$

So R is reflexive

iv) For Symmetric, for $x, y \in A$

$x \parallel y \Rightarrow y \parallel x$

$\therefore$ R is symmetric

iii) For transitive, for $x, y, z \in A$

$\left.\begin{array}{l} x \parallel y \\ y \parallel z \end{array}\right\} \Rightarrow \cancel{x \nparallel z}\ x \parallel z$

$\therefore$ R is transitive

$\therefore$ 'Parallel' is equivalence relation on A.

(6)(a) Give an example of a relation which is transitive but not reflexive and symmetric

Sol:

Let $A = \{1, 2, 3, 4\}$

Relation R defined by $\{(1,2), (2,3), (1,3)\}$

For Reflexive: For $1 \in A$ $(1,1) \notin R$

For $2 \in A$ $(2,2) \notin R$

For $3 \in A$ $(3,3) \notin R$ and so on

So R is not reflexive

For Symmetric For $1, 2 \in A$,

$(1,2) \in R \not\Rightarrow (2,1) \in R$

So R is not symmetric

For transitive For $1, 2, 3 \in A$

$\left.\begin{array}{l} \therefore (1,2) \in R \\ (2,3) \in R \end{array}\right\} \Rightarrow (1,3) \in R$

$\therefore$ R is transitive

(b) Give an example of relation which is reflexive but not symmetric

Sol:

Let $A = \{1, 2, 3\}$

Relation $R = \{(1,1), (2,2), \cancel{(2,1)}, (1,3)\}$

For Reflexive For $1, 2 \in A$

$(1,1) \in R$

$(2,2) \in R$

So R is Reflexive

For Symmetric;

For $1, 2 \in R$

$(1,2) \in R \not\Rightarrow (2,1) \notin R$

So, R is not symmetric

(11) Check whether the relation R define in the set $\{1, 2, 3, 4\}$ defined by $R = \{(x,y) : x < y\}$ is reflexive, symmetric or transitive

$\Rightarrow$ Let $A = \{1, 2, 3, 4\}$

So $A \times A = \{1, 2, 3, 4\} \times \{1, 2, 3, 4\}$

$= \{(1,1), (1,2), (1,3), (1,4), (2,1), (2,2), (2,3), (2,4)$
$(3,1), (3,2), (3,3), (3,4), (4,1), (4,2), (4,3), (4,4)\}$

$\therefore$ Relation define by $x < y$

$R = \{(1,2), (1,3), (1,4), (2,3), (2,4), (3,4)\}$

i) For Reflexive : For $1,2,3,4 \in A$

$\therefore (1,1) \notin R$
$(2,2) \notin R$
$(3,3) \notin R$
$(4,4) \notin R$

$\therefore$ R is not reflexive.

ii) For symmetric : For $1,2 \in A$

$(1,2) \in R \not\Rightarrow (2,1) \in R$

i.e $xRy \not\Rightarrow yRx$

So R is not symmetric.

iii) For Transitive : For $1,2,3 \in A$

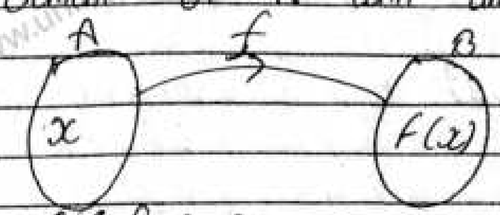
$\left.\begin{array}{l}(1,2) \in R \\ (2,3) \in R\end{array}\right\} \Rightarrow (1,3) \in R$

i.e xRy any $yRz \Rightarrow xRz$

So R is transitive

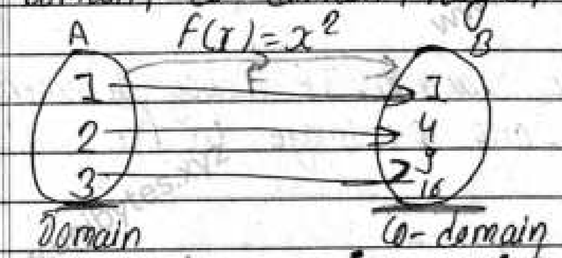
FUNCTION (T.U)

Let A and B be two sets. A function f. $f : A \to B$ is a relation which associates each element of A with unique element of B.



$f : A \to B$

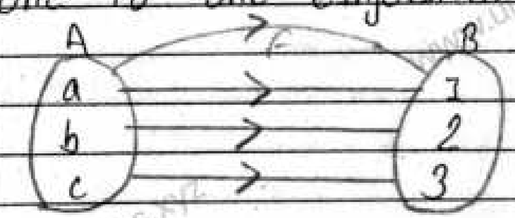
Domain, Co-domain, range, pre-image



Range /image = $\{1, 4, 9\}$
Pre-image = $\{1, 2, 3\}$

* Types of function

i) One to one (Injective)



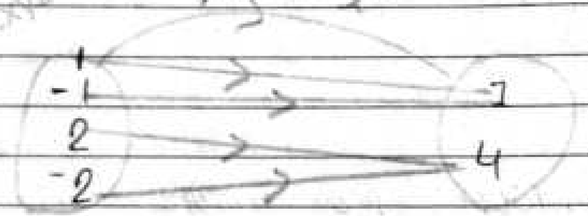
let $f : A \to B$ be a function. The function

f is said to be one-one function if different elements of A have different images in B.

For, $x_1, x_2 \in A$, $\boxed{\begin{array}{l} f(x_1) = f(x_2) \\ \Rightarrow x_1 = x_2 \end{array}}$

ii) Onto function (surjective)

$f(x) = x^2$

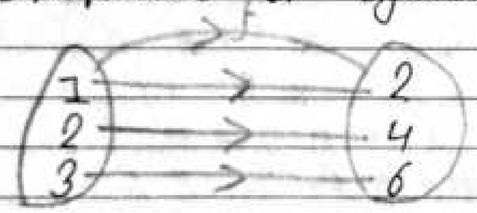


Let $f: A \to B$ be a function. If every element in B has at least one pre-image in A.

$\boxed{f(x) = y}$

iii) One to one onto function (bijective)

A function $f: A \to B$ is both one to one and onto, then it is called a one to one correspondence or bijective function.



Q) Show that the function $f: \mathbb{R} \to \mathbb{R}$ defined by $f(x) = x+1$ is bijective.

Sol: For f is one to one

Let $x_1, x_2 \in \mathbb{R}$ (domain) such that $f(x_1) = f(x_2)$

$\Rightarrow x_1 + 1 = x_2 + 1$

$\Rightarrow x_1 = x_2$

$\therefore f$ is one to one

For f is onto

Let $y \in \mathbb{R}$ (co-domain), ~~then~~ we need to show $\exists\, x \in \mathbb{R}$ such that $f(x) = y$

As, $f(x) = y$ or, $y = x + 1$

$\Rightarrow x + 1 = y$ $\quad x = y - 1$

$\Rightarrow x = y - 1 \in \mathbb{R}$ Clearly $x = y - 1 \in \mathbb{R}$

Hence f is onto $\therefore f(x) = f(y-1) = (y-1)+1 = y$

$\therefore f$ is onto.

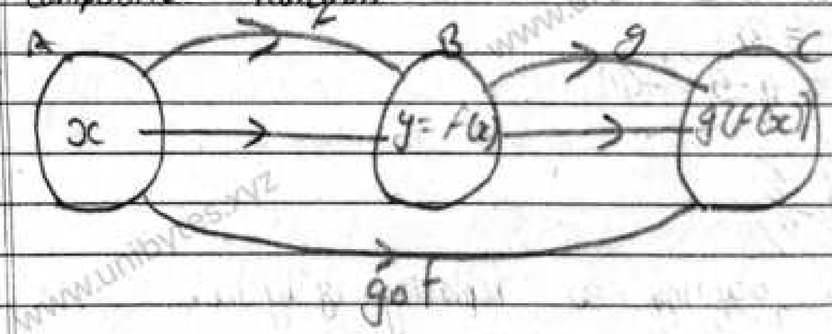
* Inverse function.

$f^{-1}(y) = x$ iff $f(x) = y$

eg: if $f = \{(1,a), (2,b), (3,c)\}$ then,

$f^{-1} = \{(a,1), (b,2), (c,3)\}$.

* Composite function



$(gof)(x) = g(f(x))$ for all $x \in A$

1-mp (a) Let $f: R \to R$ defined by $f(x) = 3x+5$ Show that f is bijective find f^{-1}

Sol: For bijective, we show f is one to one and onto

f is one to one	f is onto
let $x_1, x_2 \in R$ (Domain)	let y be an element of R (co-domain) i.e $y \in R$
Now, $f(x_1) = f(x_2)$	Then; $y = 3x+5$
$\Rightarrow 3x_1+5 = 3x_2+5$	$x = y-5/3$
$\Rightarrow 3x_1 = 3x_2$	Clearly, $x = \frac{y-5}{3} \in R$
$\therefore x_1 = x_2$	
$\therefore f$ is one to one	

and $f(x) = f\left(\frac{y-5}{3}\right) = 3\left(\frac{y-5}{5}\right)+5 = y$

~~f is onto~~

$\therefore f(x) = y$ $\therefore f$ is onto

Hence, f is bijective

For f^{-1}

Here,

$y = f(x) = 3x+5$

We have, $[y = f(x)]$ — ①

again,

$y = 3x+5$

$y-5 = 3x$

i.e $x = \frac{y-5}{3}$ — ②

from ① and ②

$f^{-1}(y) = y-5/3$

Since y is dummy, so replacing y by x.

$\therefore f^{-1}(x) = \frac{x-5}{3}$ Ans

Exercise 2.2

1)(a) If $f(x) = 4x-1$, find $f(0), f(-2), f(-1/2)$

We have,

$f(x) = 4x-1$

$f(0) = 4\times 0 - 1 = -1$

$f(-2) = 4\times(-2) - 1 = -9$

$f(-1/2) = \cancel{4}^{2} \times \left(\frac{-1}{\cancel{2}}\right) - 1 = -3$

(b) Let $f(x) = 3x+1$ be function defined on the closed interval $-2 \le x \le 2$. Find $f(2), f(-2), f(0), f(3)$.

Give;

$x \in [-2, 2]$

$f(x) = 3x+1$

$f(2) = 3\times 2 + 1 = 7$

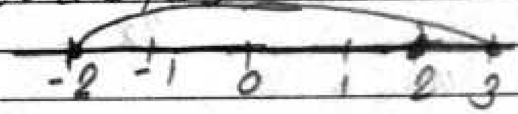
$f(-2) = 3\times(-2)+1 = -5$

$f(0) = 3\times 0 + 1 = 1$

$f(3)$ is not possible since $3 \notin [-2, 2]$

$-2 \le x \le 2$

$x \in [-2, 2]$



c) Let $f(x) = \frac{|x-4|}{x-4}$, find $f(0), f(2), f(-2), f(4), f(-4)$

Given;

$f(x) = \frac{|x-4|}{x-4}$

$f(0) = \frac{|0-4|}{0-4} = \frac{|-4|}{-4} = \frac{4}{-4} = -1$

$f(2) = \frac{|2-4|}{2-4} = \frac{|-2|}{-2} = \frac{2}{-2} = -1$

$f(-2) = \frac{|-2-4|}{-2-4} = \frac{|-6|}{-6} = \frac{6}{-6} = -1$

$f(4) = \frac{|4-4|}{4-4} = \frac{|0|}{0}$ = undefined

$f(-4) = \frac{|x-4|}{x-4} = \frac{|-4-4|}{-4-4} = \frac{|-8|}{8} = \frac{8}{-8} = -1$

2.a) If the function $f: R \to R$ be defined by

$$f(x) = \begin{cases} 2x+3 & \text{if } x < -2 \\ x^2-2 & \text{if } -2 \le x \le 3 \\ 3x-1 & \text{if } x > 3 \end{cases}$$

. find $f(2)$, $f(4)$, $f(-1)$ and $f(-3)$. find $f(2)$, $f(4)$, $f(-1)$ and $f(-3)$.

Given;

$$f(x) = \begin{cases} 2x+3 \; ; & x < -2 \\ x^2-2 \; ; & -2 \le x \le 3 \\ 3x-1 \; ; & 3 > x \end{cases} \quad \begin{cases} x \in (-\infty, -2) \\ x \in [-2, 3] \\ x \in (3, \infty) \end{cases}$$

now;

$f(2) = x^2 - 2 = 2^2 - 2 = 2$

$f(4) = 3x - 1 = 3 \times 4 - 1 = 11$

$f(-1) = x^2 - 2 = 1^2 - 2 = 1 - 2 = -1$

$f(-3) = 2x + 3 = 2 \times -3 + 3 = -6 + 3 = -3$

b) Let $f: R \to R$ be defined by $f(x) = \begin{cases} 3 - 2x & \text{for } -3 \le x \le 0 \\ 3 + 2x & x > 0, \end{cases}$

Find the images of the elements -1 and 2

Given;

$$f(x) = \begin{cases} 3 - 2x \; ; & -3 \le x \le 0 \\ 3 + 2x \; ; & x > 0 \end{cases}$$

now;

$f(-1) = 3 - 2 \times -1 = 3 + 2 = 5$

$f(2) = 3 + 2 \times 2 = 3 + 4 = 7$

$\therefore$ The image are 5, 7.

c) Let the function $f: R \to R$ be defined by

$$f(x) = \begin{cases} 4x - 2 & \text{for } x \ge 1 \\ 2x & \text{for } x < 1 \end{cases}$$

. find $f(2), f(1), f(0), f(-1)$

$\frac{f(1+h) - f(1)}{h}$, $h > 1$

sol:

$$f(x) = \begin{cases} 4x - 2 \; ; & x \ge 1 \\ 2x \; ; & x < 1 \end{cases}$$

$\frac{f(1+h) - f(1)}{h}$, $h > 1$

now;

$f(2) \Rightarrow 4x - 2 = 4 \times 2 - 2 = 6$

$f(1) \Rightarrow 4x - 2 = 4 \times 1 - 2 = 2$

$f(0) \Rightarrow 4x - 2 = 4 \times 0 - 2 = -2$

$f(-1) \Rightarrow 2x = 2 \times -1 = -2$

again;

$= \frac{f(1+h) - f(1)}{h}$

$= \frac{4h + 2 - 2}{h}$

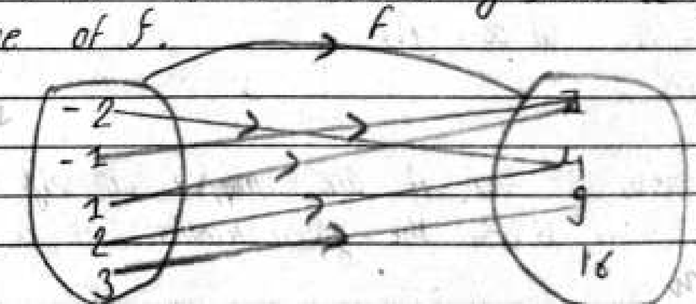
$= 4$

$f(1+h) = 4x - 2 = 4(1+h) - 2$
$= 4 + 4h - 2 = 4h + 2$

arko page ma xa

3.a) let $A = \{-2, -1, 1, 2, 3\}$ and $B = \{1, 4, 9, 16\}$. If a function $f: A \to B$ be defined by $f(x) = x^2$, what is the range of f.

sol:



range = $\{1, 4, 9\}$

Q Let $A = \{-1, 0, 2, 4, 6\}$ and a function $f: A \to R$ is defined by $y = f(x) = \frac{x}{x+2}$. Find the range of f.

We have;

$$f(x) = \frac{x}{x+2}, \quad A = \{-1, 0, 2, 4, 6\}$$

When $x = -1$, $f(-1) = \frac{-1}{-1+2} = \frac{-1}{1} = -1$

When $x = 0$, $f(0) = \frac{0}{0+2} = \frac{0}{2} = 0$

When $x = 2$, $f(2) = \frac{2}{2+2} = \frac{\cancel{2}\,1}{\cancel{4}\,2} = \frac{1}{2}$

When $x = 4$, $f(4) = \frac{4}{4+2} = \frac{\cancel{4}\,2}{\cancel{6}\,3} = \frac{2}{3}$

When $x = 6$, $f(6) = \frac{6}{6+2} = \frac{\cancel{6}\,3}{\cancel{8}\,4} = \frac{3}{4}$

Hence, range of $f = \left\{-1, 0, \frac{1}{2}, \frac{2}{3}, \frac{3}{4}\right\}$

4) Let $f = \{(1,1), (2,3), (3,5), (5,7)\}$ be a relation from A to B. Is this relation a function from A to B? If so, what are the values of m and n if $f(x) = mx + n$ for all $x \in A$.

⇒ Here, $A \times B$

All the elements of set A are mapped uniquely with the elements of set B. So the given relation is a function

Since

$f(x) = mx + n \; \forall A$

For $x = 1$, $f(1) = m \times 1 + n$

$1 = m + n$ — ①

For $x = 2$, $f(2) = m \times 2 + n$

$3 = 2m + n$ — ②

~~Start putting~~ Applying ① and ②

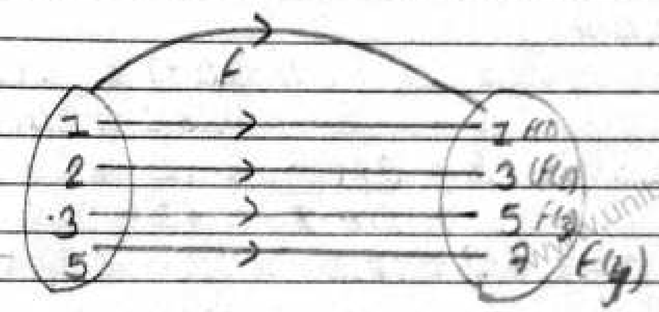
$$\begin{aligned} m + n &= 1 \\ 2m + n &= 3 \\ \hline -m &= -2 \end{aligned}$$

$\therefore m = 2$

Put $m = 2$ in ①

$1 = 2 + n$

$\therefore n = -1$



3.a) We have,

$f(x) = x^2$; $A = \{-2, -1, 1, 2, 3\}$ and $B = \{1, 4, 9, 16\}$

When $x = -2$, $f(-2) = (-2)^2 = 4$

When $x = -1$, $f(-1) = (-1)^2 = 1$

When $x = 1$, $f(1) = (1)^2 = 1$

When $x = 2$, $f(2) = 2^2 = 4$

When $x = 3$, $f(3) = 3^2 = 9$

Hence range of $f = \{4, 1, 9\}$

Note

In function one to many is not possible. But ~~Not~~ many to one is possible. (as (5))

For e.g: $R_1 = \{(a,p), (a,r), (b,r), (c,s)\}$ (✗), $R_2 = \{(a,s), (b,s), (c,s)\}$ (✓)

Q) Let a function $f: A \to B$ be defined by $f(x) = \frac{x+1}{2x-1}$ with $A = \{-1, 0, 1, 2, 3, 4\}$ and $B = \{-1, 0, 4/5, 5/7, 1, 2, 3\}$. Find the range of f. Is the f one to one and onto both? If not, how can the function can be made one to one and onto both.

Sol:

$f(x) = \frac{x+1}{2x-1}$, $A = \{-1, 0, 1, 2, 3, 4\}$ and $B = \{-1, 0, 4/5, 5/7, 1, 2, 3\}$

For range:

put $x = -1$, $f(-1) = \frac{x+1}{2x-1} = \frac{-1+1}{2\times(-1)-1} = \frac{0}{-3} = 0$

put $x = 0$, $f(0) = \frac{0+1}{2\times 0-1} = \frac{1}{-1} = -1$

put $x = 1$, $f(1) = \frac{1+1}{2\times 1-1} = \frac{2}{2-1} = \frac{2}{1} = 2$

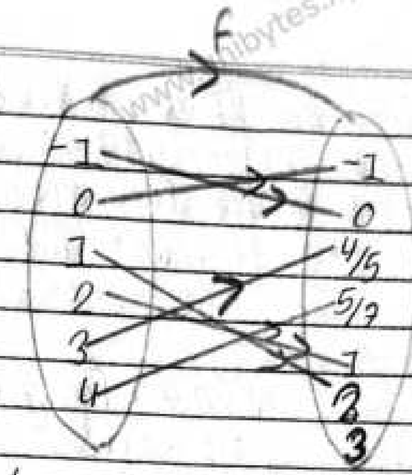
put $x = 2$, $f(2) = \frac{2+1}{2\times 2-1} = \frac{3}{4-1} = \frac{3}{3} = 1$

put $x = 3$, $f(3) = \frac{3+1}{2\times 3-1} = \frac{4}{5}$

put $x = 4$, $f(4) = \frac{4+1}{2\times 4-1} = \frac{5}{7}$

$\therefore$ Range of $f = \{0, -1, 2, 1, \frac{4}{5}, \frac{5}{7}\}$

Diagram



All the elements of set A have unique and distinct images in set B. So it is one to one. But $3 \in B$ has no pre-image in set A. So f is not onto. [ONTO: Every elements of B has at least one pre-image in A]

To make it one to one, onto, we should remove 3 from B or we can add the corresponding pre-image of 3 in A.

Winter 2019

7.a) Prove that a function $f: R \to R$ defined by $f(x) = 3x-1$ is one to one onto (i.e bijective)

Sol: For one to one

let $x_1, x_2 \in R$

then, $f(x_1) = f(x_2)$

$\Rightarrow 3x_1 - \cancel{1} = 3x_2 - \cancel{1}$

$\Rightarrow \cancel{3}x_1 = \cancel{3}x_2$

$\Rightarrow x_1 = x_2$

$\therefore f$ is one to one

Let $y \in R$
such that;

$y = f(x) = 3x - 1$

$\Rightarrow y = 3x - 1$

$\Rightarrow 3x - 1 = y$

$\Rightarrow 3x = y + 1$

$\therefore x = \frac{y+1}{3} \in R$

now; $f(x) = f\left(\frac{y+1}{3}\right)$

$= \cancel{3} \times \frac{y+1}{\cancel{3}} - 1$

$= y + 1 - 1$

$= y$

$\therefore f(x) = y$

$\therefore f$ is onto

Now: → [illegible] point line

one to one:

For all $x_1, x_2 \in$ Domain such that $f(x_1) = f(x_2)$

ii) Onto: For $y \in$ codomain

$\Rightarrow y = f(x)$

$\Rightarrow y = \ldots$

Solve for x

$f = f$

$f(x) = y$

Imp

⑥ Show that the function $f: R \to R$ defined by $f(x) = x^3 + 1$ for all $x \in R$ is bijective.

Sol:

For one to one,

For all $x_1, x_2 \in R$ such that $f(x_1) = f(x_2)$

$\Rightarrow x_1^3 + 1 = x_2^3 + 1$

$\Rightarrow x_1^3 = x_2^3$

$\Rightarrow x_1 = x_2$

$\therefore f$ is one to one

again;

$y \in R$

such that $y = f(x) = x^3 + 1$

$\Rightarrow y = x^3 + 1$

$\Rightarrow x^3 + 1 = y$

$\Rightarrow x^3 = y - 1$

$\Rightarrow x = (y-1)^{1/3} \in R$

now,

$f(x) = ((y-1)^{1/3})^3 + 1$

$= y - 1 + 1$

$= y$

$\therefore$ Both condition are thus it is bijective

⑦ Show that function $f: N \to N$ defined by $f(x) = 2x - 1$ is one to one but not onto

Sol: For one to one

let $x_1, x_2 \in N$

Then $f(x_1) = f(x_2)$

$\Rightarrow 2x_1 - 1 = 2x_2 - 1$

$\Rightarrow 2x_1 = 2x_2$

$\Rightarrow x_1 = x_2$

$\therefore f$ is one to one

now,

let $y \in R$

$\Rightarrow y = f(x) = 2x - 1$

$\Rightarrow 2x - 1 = y$

$\Rightarrow 2x = y + 1$

$\Rightarrow x = \frac{y+1}{2}$

again;

$f(x) = f\left(\frac{y-1}{2}\right)$

$\Rightarrow \cancel{2} \times \left(\frac{y-1}{\cancel{2}}\right) - 1$

$\Rightarrow y - 1 - 1$

$\Rightarrow y - 2$

$\therefore f(x) \neq y$

That why it is not onto function.

Imp

8(a) When does an inverse of function exist? If $f: R \to R$ be defined by $f(x) = x^3 - 2$, find the formula that defines f^{-1}.

Ans Inverse function exists iff the function is bijective

$y = f^{-1}(x)$

Sol.

Here, $f(x) = y = x^3 - 2$

Interchanging the role of x and y

$\Rightarrow x = y^3 - 2$

or, $y^3 - 2 = x$

or, $y^3 = x + 2$

or, $y = \sqrt[3]{x+2}$

$\therefore f^{-1}(x) = \sqrt[3]{x+2}$

T.U. [illegible]

① Let $f: R \to R$ be defined by $f(x) = cx + d$, $(c \neq 0)$ and d are real numbers. Find f^{-1} and show that $f^{-1}(f(x)) = f(f^{-1}(x)) = x$

Given;

$$f(x) = y = cx + d$$

Interchanging the role of x and y

$$x = cy + d$$

or, $cy + d = x$

or, $cy = x - d$

or, $y = \frac{x-d}{c}$

$\therefore f^{-1}(x) = \frac{x-d}{c}$

Solving L.H.S

$= f^{-1}(f(x))$

$= f^{-1}(cx+d)$ $\quad \therefore [f^{-1}(x) \bullet]$

$= \frac{cx+d-d}{c}$ $\quad [\therefore f^{-1}(x) = \frac{x-d}{c}]$

$= x$

Solving Mid Term

$= f(f^{-1}(x))$

$= f\left(\frac{x-d}{c}\right)$ $\quad [\therefore f(x)]$

$= c\left(\frac{x-d}{c}\right) + d$ $\quad [\therefore f(x) = cx + d$

$= x$

$\therefore f^{-1}(f(x)) = f(f^{-1}(x)) = x$

T.U. almost fix

* Show that the function $f: R \to R$ defined by $f(x) = x^3 + 7$ for all $x \in R$ is bijective. Find $f^{-1}(x)$

Given;

$$y = f(x) = x^3 + 7$$

For one to one

Let $x_1, x_2 \in R_{domain}$

Such that $f(x_1) = f(x_2)$

$\Rightarrow x_1^3 + 7 = x_2^3 + 7$

$\Rightarrow x_1^3 = x_2^3$

$\Rightarrow x_1 = x_2$ $\quad \therefore f$ is one to one

For onto

Let $y \in R_{codomain}$

$y = f(x) = x^3 + 7$

$y = x^3 + 7$

Solve for x.

$\Rightarrow x^3 + 7 = y$

$\Rightarrow x^3 = y - 7$

$\Rightarrow x = (y-7)^{1/3} \in R$

$f(x) = ((y-7)^{1/3})^3 + 7 \in R$

$= y - 7 + 7$

$= y \in R$

$\therefore f(x) = y$

$\therefore f$ is onto

$\therefore f$ is one to one and onto, so bijective.

For f^{-1} Given,

$$y = x^3 + 7$$

Interchanging the role of x and y

$$x = y^3 + 7$$

Solve for y.

$$y^3 + 7 = x$$
$$\Rightarrow y^3 = x - 7$$
$$\Rightarrow y = \sqrt[3]{x-7}$$
$$f^{-1}(x) = \sqrt[3]{x-7}$$

9) a) Let f and g be real value functions defined as $f(x) = 4x+7$, $g(x) = 5x-2$, find $(fog)(x)$ and $gof(-2)$.

Ans Given,

$$f(x) = 4x+7$$
$$g(x) = 5x-2$$

now,

$$(fog)(x) = f(g(x))$$
$$= f(5x-2) \quad [\because f(x)$$
$$= 4\times(5x-2)+7 \quad [\because f(x) = 4x+7]$$
$$= 20x - 8 + 7$$
$$= 20x - 1$$

and,

$$(gof)(x) = g(f(x))$$
$$= g(4x+7)$$
$$= 5(4x+7) - 2$$
$$= 20x + 35 - 2$$
$$= 20x + 33$$

again;

$$(gof)(-2) = g(f(-2)) = g(4\times(-2)+7 = g(-1)$$
$$= g(-1) = 5\times(-1) - 2 = -5 - 2 = -7 \text{ ans}$$

b) Let $f: R \to R$ be defined by $f(x) = 3x+1$ and $g: R \to R$ be defined by $g(x) = x^2 - 1$. Find the value of fof and gog.

Sol:

$$f(x) = 3x+1$$
$$g(x) = x^2 - 1$$

Now;

$$fof = f(f(x)) = f(3x+1) = 3(3x+1)+1$$
$$= 9x + 3 + 1 = 9x + 4$$

again;

$$gog = g(g(x))$$
$$= g(x^2-1)$$
$$= (x^2-1)^2 - 1$$
$$= \cancel{x^4} - 2(x^2)^2 - 2.x^2.1 + 1^2 - 1$$
$$= x^4 - 2x^2$$
$$= x^2(x^2-2) \text{ ans}$$

10) Find the domain of the functions ($x = ?$)

a) $f(x) = 2x - 3$

Sol:

Since for all $x \in R$, $f(x)$ is defined $\therefore x \in (-\infty, \infty)$

$\therefore$ Domain of $f = D(f) = R = (-\infty, \infty)$

b) $f(x) = 2x^2 + 7$

Sol.

Since for all $x \in R$, $f(x)$ is defined $\therefore x \in (-\infty, \infty)$

$\therefore$ Domain of $f = D(f) = R = (-\infty, \infty)$

c) $f(x) = \frac{1}{x-1}$

Clearly, $f(x)$ is not defined when $x-1=0$
$\Rightarrow x = 1$

To define, $f(x)=y$, we should take $x-1 \neq 0$
$x \neq 1$

$\therefore D(f) = \mathbb{R} - \{1\}$

d) $f(x) = \sqrt{x+1}$

Sol:

To define $f(x)$, we should have $x+1 \geq 0$
(If $(x+1) < 0$, it will be imaginary)

$\therefore x \geq -1$ $\quad \therefore x \in [-1, \infty)$

$\therefore$ Domain of $f = [-1, \infty)$

e) $f(x) = \frac{1}{\sqrt{16-x^2}}$

Clearly, $f(x)$ is not defined if $16-x^2 < 0$ and $(16-x^2) = 4$

To define $f(x)$ or y we should take $16-x^2 > 0$
or, $-x^2 > -16$
or, $-(-x^2 < -16)$
or, $x^2 < 16$
or, $x^2 < 4^2$
or $-4 < x < 4$

But another condition also to apply here
$16-x^2 \neq 0$
or, $x^2 \neq 16$
or, $x^2 \neq (\pm 4)^2$
$\therefore x \neq \pm 4$

$\therefore$ Domain $-4 < x < 4$ [$\therefore x \in (-4, 4)$]
i.e $D(f) = (-4, 4)$

Imp

11. Find the domain and range of the following functions defined in the set of real numbers.

a) $f(x) = -x^2 + 4x - 3$

For Domain
$f(x)$ can be defined for all $x \in \mathbb{R}$
$\therefore D(f) = (-\infty, \infty)$

For Range
$f(x) = y = -x^2 + 4x - 3$
or, $x^2 - 4x = -y - 3$
or, $x^2 - 2.2.x + 2^2 - 2^2 = -y - 3$
or, $\cancel{x^2 - 2 \cdot 2}$ $(x-2)^2 = -y - 3 + 2^2$
or, $(x-2)^2 = -y + 1$

Since $(x-2)^2 \geq 0$, then $-y+1 \geq 0$
$\Rightarrow -y + 1 \geq 0$
$\Rightarrow -y \geq -1$
$\Rightarrow -(y \leq 1)$
$\therefore y \leq 1$

$\therefore$ Range of $f = (-\infty, 1]$

b) $y = \sqrt{x-2}$

For domain
y or $f(x)$ is defined if $x-2 \geq 0$
i.e. $x \geq 2$
$\therefore D(f) = [2, \infty)$

For range
$y = \sqrt{x-2}$

Squaring both side
$y^2 = x - 2$
$x - 2 = y^2$ $\quad x = y^2 + 2$

Since $x \geq 2$, $y^2 + 2 \geq 2$

$\Rightarrow y^2 \geq 2 - 2$

$\Rightarrow y^2 \geq 2 - 2$

$\Rightarrow y^2 \geq 0$

$\Rightarrow \sqrt{y^2} \geq \sqrt{0}$

$\Rightarrow y \geq 0$

Now;

$y \geq 0$

Range$(f) = [0, \infty)$

c) $f(x) = \sqrt{4 - x^2}$

For domain,

$f(x)$ can be defined if $4 - x^2 \geq 0$

or, $-x^2 \geq -4$

or, $x^2 < (\pm 2)^2$

or, $-2 \leq x \leq 2$

$D(f) = [-2, 2]$

For Range

$y = \sqrt{4 - x^2}$

Squaring both side

$y^2 = (\sqrt{4 - x^2})^2$

$y^2 = 4 - x^2$

$\cancel{4 - x^2} = \cancel{y^2}$

$x^2 = 4 - y^2$

Since $x^2 \geq 0$, $4 - y^2 \geq 0$

$\Rightarrow -y^2 \geq -4$

$\Rightarrow y^2 \leq 4$

$\Rightarrow y^2 \leq (\pm 2)^2$

$\Rightarrow -2 \leq y \leq 2$

But y is the square root

so, $0 \leq y \leq 2$

$\therefore$ Range$(f) = R(f)$

$= [0, 2]$

d) $f(x) = \sqrt{2 - x - x^2}$

For Domain,

$f(x) = \sqrt{2 - x - x^2}$

$= \sqrt{-(x^2 + x - 2)}$

$= \sqrt{-\{(x)^2 + 2 \cdot x \cdot \frac{1}{2} + (\frac{1}{2})^2 - (\frac{1}{2})^2 - 2\}}$

$= \sqrt{-\{(x + \frac{1}{2})^2 - \frac{1}{4} - 2}$

$= \sqrt{-\{(x + \frac{1}{2})^2 - \frac{1 - 8}{4}\}}$

$= \sqrt{-\{(x + 1/2)^2 - 9/4\}}$

$= \sqrt{9/4 - (x + 1/2)^2}$

$f(x)$ can be defined only if $9/4 - (x + 1/2)^2 \geq 0$

$\Rightarrow -(x + 1/2)^2 \geq -9/4$

$\Rightarrow (x + 1/2)^2 \leq 9/4$

$\Rightarrow \sqrt{(x + 1/2)^2} \leq \sqrt{(\pm \frac{3}{2})^2}$

$\Rightarrow |x + 1/2| \leq |\pm 3/2|$

$\Rightarrow -3/2 \leq x + 1/2 \leq 3/2$

$\Rightarrow -3/2 \leq \frac{2x + 1}{2} \leq 3/2$

$\Rightarrow -3 \leq 2x + 1 \leq 3$

$\Rightarrow -3 - 1 \leq 2x + 1 - 1 \leq 3 - 1$

$\Rightarrow -4 \leq 2x \leq 2$

$\Rightarrow -2 \leq x \leq 1$

$x \in [-2, 1]$

$\therefore D(f) = [-2, 1]$

For Range:

$$y = f(x) = \sqrt{9/4 - (x + 7/2)^2}$$

Squaring both side;

$\Rightarrow y^2 = \left(\sqrt{9/4 - (x + 7/2)^2}\right)^2$

$\Rightarrow y^2 = 9/4 - (x + 7/2)^2$

$\Rightarrow (x + 7/2)^2 = 9/4 - y^2$

As $(x + 7/2)^2 \geq 0$ for $x \in R$

$\Rightarrow \frac{9}{4} - y^2 \geq 0$

$\Rightarrow y^2 \leq 9/4$

$\Rightarrow |y|^2 \leq (3/2)^2$

$\Rightarrow |y| \leq 3/2$

$\Rightarrow -3/2 \leq y \leq 3/2$

Another method $\frac{9}{4} - y^2 \geq 0$

$\Rightarrow y^2 \leq 9/4$

$\Rightarrow (y)^2 \leq (\pm 3/2)^2$

$\therefore -\frac{3}{2} \leq y \leq \frac{3}{2}$

But $f(x)$ being ~~always~~ +ve square root

so,

$0 \leq y \leq \frac{3}{2}$

$\therefore R(f) = [0, 3/2]$

e) $f(x) = \sqrt{-x^2 - 4x + 21}$

For Domain,

$$f(x) = \sqrt{-x^2 - 4x + 21}$$
$$= \sqrt{21 - (x^2 + 4x)}$$
$$= \sqrt{21 - [x^2 + 2.x.2 + 2^2 - 2^2]}$$
$$= \sqrt{21 - [(x+2)^2 - 4]}$$
$$= \sqrt{21 - (x+2)^2 + 4}$$
$$= \sqrt{25 - (x+2)^2}$$

$f(x)$ can be defined only if $25 - (x+2)^2 \geq 0$

$\Rightarrow -(x+2)^2 \geq -25$

$\Rightarrow -(-(x+2)^2 \leq 25)$

$\Rightarrow (x+2)^2 \leq 25$

$\Rightarrow (x+2)^2 \leq 5^2$

$\Rightarrow x+2 \leq 5$

$\Rightarrow -5 \leq x+2 \leq 5$

$\Rightarrow -5-2 \leq x+2-2 \leq 5-2$

$\Rightarrow -7 \leq x \leq 3$

$\therefore D(f) = [-7, 3]$

For Range:

$$y = f(x) = \sqrt{25 - (x+2)^2}$$

squaring both side;

$\Rightarrow y^2 = \left(\sqrt{25 - (x+2)^2}\right)^2$

$\Rightarrow y^2 = 25 - (x+2)^2$

$\Rightarrow (x+2)^2 = 25 - y^2$

As, $(x+2)^2 \geq 0$ for $x \in R$

$\Rightarrow 25 - y^2 \geq 0$

$\Rightarrow -y^2 \geq -25$

$\Rightarrow -(y^2 \leq 25)$

$\Rightarrow (y)^2 \leq (\pm 5)^2$

$\therefore -5 \leq y \leq 5$

But $f(x)$ being a +ve square root

so,

$0 \leq y \leq 5$

$\therefore R(f) = [0, 5]$

Function and their Graphs

Exponential function: A function of the form $y = a^x$ is called exponential function where $a > 0$, $a \neq 1$ and $x \in \mathbb{R}$.

Logarithmic function: If $y = a^x$, $a > 0$, $a \neq 1$ then, x is called logarithmic function of the number y to the base 'a' and is written as $x = \log_a y$.
OR, It is the inverse of an exponential f^n.

Formula

$$\boxed{\log_a x = y \rightleftarrows x = a^y}$$

i) $a^0 = 1$
ii) $\log_a 1 = 0$
iii) $\log_a a = 1$
iv) $\log e = 1$
v) $\log_a (a^0) = 0$

v) $\log(xy) = \log x + \log y$
vi) $\log\left(\frac{x}{y}\right) = \log x - \log y$
vii) $\log x^n = n \cdot \log x$

$\Rightarrow 0 \cdot \log x^a$ → (applying vii)
$\Rightarrow 0 \cdot 1$
$\Rightarrow 0$

Exercise 2.3

1) Solve for x:

a) $3^{x+5} = 27^{-2x+1}$
$\Rightarrow 3^{x+5} = 3^{3(-2x+1)}$
or, $x+5 = 3(-2x+1)$
or, $x+5 = -6x+3$
or, $x+6x = 3-5$
or, $7x = -2$
or, $x = -\frac{2}{7}$

b) $18^{2x} \times 3^{-2x} = 6$
$\Rightarrow (2 \cdot 3^2)^{2x} \times 3^{-2x} = 6$
or, $2^{2x} \cdot 3^{2 \times 2x} \times 3^{-2x} = 6$
or, $2^{2x} \cdot 3^{4x} \times 3^{-2x} = 6$
or, $2^{2x} \cdot 3^{4x-2x} = 6$
or, $2^{2x} \cdot 3^{2x} = 6$
or, $6^{2x} = 6$
or, $2x = 1$
or, $x = 1/2$

c) $\log_3 x = 3$
Sol:
$x = 3^3$ [$\because x = a^y$]
$x = 27$

d) $\log_4 x = 1/-2$
Sol:
$x = (4)^{1/-2}$
$x = 2^{2 \times 1/-2}$
$= 2^{-1}$
$= 1/2$

2) Solve:

a) $5\log_5 x - 3\log_3 9 = 2\log_5 x$
Sol:
$\Rightarrow 5\log_5 x - 3\log_3 9 = 2\log_5 x$
or, $5\log_5 x - 2\log_5 x = 3\log_3 9$
or, $3\log_5 x = 3\log_3 3^2$
or, $3\log_5 x = 3 \cdot 2\log_3 3$ [$\log a^a = 1$]
or, $3\log_5 x = 6 \cdot \log 1$
or, $3\log_5 x = 6$
or, $\log_5 x = \frac{6}{3}$
or, $\log_5 x = 2$
or, $x = 5^2$
$\therefore x = 25$ ans

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(b) $2\log x = \log(5x-6)$

$\Rightarrow \log x^2 = \log(5x-6)$

$\Rightarrow x^2 = 5x-6$

$\Rightarrow x^2 - 5x + 6 = 0$

$\Rightarrow x^2 - (3+2)x + 6 = 0$

$\Rightarrow x^2 - 3x - 2x + 6 = 0$

$\Rightarrow x(x-3) - 2(x-3) = 0$

$\Rightarrow (x-2)(x-3) = 0$

Solving;

$x-2=0$ / $x-3=0$

$x=2$ / $x=3$

3) Prove that:

(a) $\log_3 \sqrt[3]{9} + \log_4 \sqrt[4]{64} - \log_5 \sqrt[5]{625} = 37/60$

Sol: L.H.S

$= \log_3 \sqrt[3]{9} + \log_4 \sqrt[4]{64} - \log_5 \sqrt[5]{625}$

$= \log_3 (9)^{1/3} + \log_4 (64)^{1/4} - \log_5 (625)^{1/5}$

$= \log_3 (3^2)^{1/3} + \log_4 (4^3)^{1/4} - \log_5 (5^4)^{1/5}$

$= \log_3 (3)^{2/3} + \log_4 (4)^{3/4} - \log_5 (5)^{4/5}$

$= \frac{2}{3}\log_3 3 + \frac{3}{4}\log_4 4 - \frac{4}{5}\log_5 5$

$= \frac{2}{3}\cdot 1 + \frac{3}{4}\cdot 1 - \frac{4}{5}\cdot 1$

$= \frac{40+45-48}{60}$ [∵ In first we divide 60 by 3 then multiply it by 2.

$= \frac{37}{60}$ proved

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(b) $\log_a x^2 - 2\log_a \sqrt{x} = \log_a x$

Taking L.H.S

$= \log_a x^2 - 2\log_a \sqrt{x}$

$= 2\log_a x - 2\log_a (x)^{1/2}$

$= 2\log_a x - \log_a x^{2\times 1/2}$

$= 2\log_a x - \log_a x$

$= \log_a x$ proved

$(\log m)^2 - (\log n)^2 = \log(mn)\log\left(\frac{m}{n}\right)$

Taking R.H.S

$= \log(m.n).\log(m/n)$

$= (\log m + \log n).(\log m - \log n)$

$= (\log m)^2 - (\log n)^2$

c) $(\log m)^2 - (\log n)^2 = \log(mn).\log(m/n)$

Taking R.H.S

$= \log m$

e) $(yz)^{\ln(y/z)}.(zx)^{\ln(z/x)}.(xy)^{\ln(x/y)} = 1$

Taking L.H.S

let $a = (yz)^{\ln(y/z)}.(zx)^{\ln(z/x)}.(xy)^{\ln(x/y)}$

$\Rightarrow a = (yz)^{\ln y - \ln z}.(zx)^{\ln z - \ln x}.(xy)^{\ln x - \ln y}$

Taking 'ln' on both sides

$\Rightarrow \ln a = \ln\left[(yz)^{\ln y - \ln z}.(zx)^{\ln z - \ln x}.(xy)^{\ln x - \ln y}\right]$

$\Rightarrow \ln a = \ln(yz)^{\ln y - \ln z} + \ln(zx)^{\ln z - \ln x} + \ln(xy)^{\ln x - \ln y}$

$\Rightarrow \ln a = (\ln y - \ln z)\ln(yz) + (\ln z - \ln x)\ln(zx) + (\ln x - \ln y)\ln(xy)$

$\Rightarrow \ln a = (\ln y - \ln z)(\ln y + \ln z) + (\ln z - \ln x)(\ln z + \ln x) + (\ln x - \ln y).(\ln x + \ln y)$

$\Rightarrow \ln a = (\ln y)^2 - (\ln z)^2 + (\ln z)^2 - (\ln x)^2 + (\ln x)^2 - (\ln y)^2$

$\Rightarrow \ln a = 0$

$\Rightarrow \ln a = \ln(1)$

$\Rightarrow a = 1$ proved

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f) $2\log_a x + \log_a y^3 + \log_a z = \log_a x^2y^3z$

Taking L.H.S

$= 2\log_a x + \log_a y^3 + \log_a z$

$= \log_a x^2 + \log_a y^3 + \log_a z$

$= \log_a x^2y^3z$ proved

T.U

5(a) If $x = \log_a bc$, $y = \log_b ca$, $z = \log_c ab$, prove that

$$\frac{1}{x+1} + \frac{1}{y+1} + \frac{1}{z+1} = 1$$

Sol:

$x+1 = \log_a bc + 1$

$= \log_a bc + \log_a a$

$x+1 = \log_a abc$

Or,

$\frac{1}{x+1} = \frac{1}{\log_a abc} = \log_{abc} (a)$ [$\because \log_a m = \frac{1}{\log_m (a)}$]

Again;

$y+1 = \log_b ca + \log_b b$

$= \log_b abc$

Or,

$\frac{1}{y+1} = \frac{1}{\log_b abc} = \log_{abc} (b)$

Now,

$z+1 = \log_c ab + \log_c c$

$= \log_c abc$

Or,

$\frac{1}{z+1} = \frac{1}{\log_c abc} = \log_{abc} c$

alternative way

Similarly,

$\frac{1}{y+1} = \log_{abc} (b)$

$\frac{1}{z+1} = \log_{abc} (c)$

Taking L.H.S

$= \frac{1}{x+1} + \frac{1}{y+1} + \frac{1}{z+1}$

$= \log_{abc} (a) + \log_{abc} (b) + \log_{abc} (c)$

$= \log_{abc} abc$

$= 1$

T.U

6) If $f(x) = \log \frac{1-x}{1+x}$ $(-1 < x < 1)$, show that $f\left(\frac{2ab}{1+a^2b^2}\right) = 2f(ab)$ where $|ab| < 1$

Sol:

Here $f(x) = \log \frac{1-x}{1+x}$

L.H.S

$f\left(\frac{2a}{1+a^2b^2}\right) = \log\left(\frac{1 - \frac{2ab}{1+a^2b^2}}{1 + \frac{2ab}{1+a^2b^2}}\right)$

$= \log\left(\frac{\frac{1+a^2b^2-2ab}{1+a^2b^2}}{\frac{1+a^2b^2+2ab}{1+a^2b^2}}\right)$

$= \log\left(\frac{1^2 - 2ab + a^2b^2}{1^2 + 2\cdot 1\cdot ab + a^2b^2}\right)$

$= \log\frac{(1-ab)^2}{(1+ab)^2}$

$= \log\left(\frac{1-ab}{1+ab}\right)^2$

$= 2\cdot\log\frac{1-ab}{1+ab}$

$= 2f(ab)$ RHS

Other questions

Imp

1) If $f(x) = \log \frac{1+x}{1-x}$ $(-1 < x < 1)$ show that $f(a) + f(b) = f\left(\frac{a+b}{1+ab}\right)$, where $|a| < 1, |b| < 1$

Sol:

$f(x) = \log \frac{1+x}{1-x}$

$f(a) = \log \frac{1+a}{1-a}$

$f(b) = \log \frac{1+b}{1-b}$

Now $f(a) + f(b) = \log \frac{1+a}{1-a} + \log \frac{1+b}{1-b}$

$= \log\left(\frac{1+a}{1-a} \times \frac{1+b}{1-b}\right)$ $\because \log x + \log y = \log xy$

$= \log\left(\frac{1+a+b+ab}{1-a-b+ab}\right)$

$= \log\left[\frac{(1+ab)+(a+b)}{(1+ab)-(a+b)}\right]$

$= \log\left(\frac{(1+ab)+(a+b)}{1+ab} \Big/ \frac{(1+ab)-(a+b)}{(1+ab)}\right)$

$= \log\left(\frac{1+\frac{a+b}{1+ab}}{1-\frac{a+b}{1+ab}}\right)$

$= f\left(\frac{a+b}{1+ab}\right)$

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2) If $x^2 + y^2 = 11xy$, prove that $\ln \frac{x-y}{3} = \frac{1}{2}(\ln x + \ln y)$

Sol:

$x^2 + y^2 = 11xy$

$x^2 + y^2 - 2xy = x^2 - 2xy + y^2 + 2xy = 11xy$

$\Rightarrow (x-y)^2 = 11xy - 2xy$

or, $(x-y)^2 = 9xy$

or, $\left(\frac{x-y}{3}\right)^2 = xy$

Taking 'ln' on the both sides

$\ln\left(\frac{x-y}{3}\right)^2 = \ln(xy)$

or, $2 \cdot \ln\left(\frac{x-y}{3}\right) = \ln(xy)$

or, $\ln \frac{x-y}{3} = \frac{1}{2} \ln(xy)$

$\therefore \ln \frac{x-y}{3} = \frac{1}{2}(\ln x + \ln y)$

Q) If $f(x) = \log \frac{1+x}{1-x}$ $(-1 < x < 1)$ show that $f(a) + f(b) = f\left(\frac{a+b}{1+ab}\right)$ where $|a| < 1, |b| < 1$.

Sol:

$f(x) = f\left(\frac{a+b}{1+ab}\right)$

$= \log \frac{1+\left(\frac{a+b}{1+ab}\right)}{1-\left(\frac{a+b}{1+ab}\right)}$

$= \log$

$$= \log \frac{1 + ab + a + b}{1 + ab}$$

$$\frac{1 + ab - a - b}{1 + ab}$$

$$= \log \frac{(1+ab) + (a+b)}{(1+ab) - (a+b)} = \log \frac{(1 + a + b + ab)}{(1 - a - b + ab)}$$

$$L.H.S = f(a) + f(b)$$

$$= \log \frac{(1+a)}{(1-a)} + \log \frac{(1+b)}{(1-b)}$$

$$= \log \left(\frac{1+a}{1-a} \cdot \frac{1+b}{1-b} \right)$$

$$= \log \frac{1(1+b) + a(1+b)}{1(1-b) - a(1-b)}$$

$$= \log \frac{1 + b + a + ab}{1 - b - a + ab}$$

$$= \log \frac{1 + a + b + ab}{1 - a - b + ab}$$

$\therefore$ LHS = RHS proved

2.i) $\log(x+2) + \log x = \log(x+12)$

sol: $= \log((x+2)x) = \log(x+12)$

or, $x^2 + 2x = x + 12$

or, $x^2 + 2x - x - 12 = 0$

or, $x^2 + x - 12 = 0$

or, $x^2 + (4-3)x - 12 = 0$

or, $x^2 + 4x - 3x - 12 = 0$

or, $x(x+4) - 3(x+4) = 0$

or, $(x-3)(x+4) = 0$

~~equating~~ solving

$x - 3 = 0$ | $x + 4 = 0$

$x = 3$ | $x = -4$

we should take +ve number

$\therefore x = 3$

4) If $3^x = 5^y = 25^z$, show that $\frac{y}{z} = \frac{1}{2} - \frac{1}{x}$

sol:

Taking 1st two

$3^x = 5^y$

$\Rightarrow 3 = 5^{y/x}$ — ①

Taking last two

$\Rightarrow 5^y = 25^z$

$\Rightarrow 25^z = 5^y$

$\Rightarrow (3 \times 5^2)^z = 5^y$

$\Rightarrow 3 \times 5^{2z} = 5^y$

$\Rightarrow 3 = \frac{5^y}{5^{2z}}$

$\Rightarrow 3 = 5^{y/z - 2}$ — ②

From ① and ②

$5^{y/x} = 5^{y/z - 2}$

$\frac{y}{x} = \frac{y}{z} - 2$

V.V. imp

4) If $3^x = 5^y = 75^z$, show that $\frac{2}{y} = \frac{1}{z} - \frac{1}{x}$

Sol: Given,

$3^x = 5^y = 75^z$

Taking 1st and 2nd terms

$3^x = 5^y$

$3 = 5^{y/x}$ — ①

Taking 1st and 3rd terms

$3^x = 75^z$

$\Rightarrow 3^x = (3 \times 5^2)^z$

$\Rightarrow 3^x = 3^z \cdot 5^{2z}$

$\Rightarrow \frac{3^x}{3^z} = 5^{2z}$

$\Rightarrow 3^{x-z} = 5^{2z}$

$\Rightarrow 3 = 5^{2z/x-z}$ — ⑪

From ① and ⑪

$5^{y/x} = 5^{2z/x-z}$

$\Rightarrow \frac{y}{x} = \frac{2z}{x-z}$

$\Rightarrow 2zx = xy - yz$

Dividing each term by xyz

$\Rightarrow \frac{2zx}{xyz} = \frac{xy}{xyz} - \frac{yz}{xyz}$

$\therefore \frac{2}{y} = \frac{1}{z} - \frac{1}{x}$

3d) $m^{\ln(\frac{n}{p})} \cdot n^{\ln(\frac{p}{m})} \cdot p^{\ln(\frac{m}{n})} = 1$

Sol:

Taking ln on both sides

$\ln(m^{\ln(n/p)} \cdot n^{\ln(p/m)} \cdot p^{\ln(m/n)}) = \ln u$

or, $\ln m^{\ln(n/p)} + \ln n^{\ln(p/m)} + \ln p^{\ln(m/n)} = \ln u$

or, $\ln \frac{n}{p} \ln m + \ln \frac{p}{m} \cdot \ln n + \ln \frac{m}{n} \cdot \ln p = \ln u$

or, $(\ln n - \ln p) \cdot \ln m + \ln p - \ln m \cdot \ln n + \ln m - \ln n \cdot \ln p = \ln u$

or, $(\ln n - \ln p) \cdot \ln m + (\ln p - \ln m) \cdot \ln n + (\ln m - \ln n) \cdot \ln p = \ln u$

or, $\ln n \ln m - \ln p \ln m + \ln p \ln n - \ln m \ln n + \ln m \ln p - \ln n \ln p = \ln u$

or, $0 = \ln u$

$\ln u = \ln 1$

$u = 1$

7) If $\frac{\ln x}{y-2} = \frac{\ln y}{z-x} = \frac{\ln z}{x-y}$, prove that $x^x y^y z^z = 1$

Sol:

$\frac{\ln x}{y-2} = \frac{\ln y}{z-x} = \frac{\ln z}{x-y} = k$ (say)

or, $\frac{\ln x}{y-2} = k$

or, $\ln x = k(y-2)$

multiply both side by x

$x \ln x = xk(y-2)$ — ①

$x \ln x = k(xy - xz)$

$\frac{\ln y}{z-x} = k$

or, $\ln y = k(z-x)$

multiply y

$y \ln y = yk(z-x)$ — ①

$y \ln y = k(zy - xy)$

$\frac{\ln z}{x-y} = k$

or, $\ln z = k(x-y)$

multiply by z

$z \ln z = zk(x-y)$ — ⑪

$z \ln z = k(zx - yz)$

Adding ①, ① and ⑪

$= x \ln x + y \ln y + z \ln z = k(xy - xz) + k(zy - xy) + (k(zx - yz)$

or, $\ln x^x + \ln y^y + \ln z^z = \cancel{kxy} - \cancel{kxz} + \cancel{kzy} - \cancel{kxy} + \cancel{kzx} - kyz$

or, $\ln(x^x y^y z^z) = 0$

or, $x^x y^y z^z = e^0$

or, $x^x y^y z^z = 1$

R.H.S proved

Chapter 1 Set Theory, Real Number and Complex Number

A set is define as a well defined collection of objects.

Examples:

i) The set of BCA students of TU

ii) The set of Prime number = {2, 3, 5, 7, 11 ...}

iii) The set of Natural number = {1, 2, 3, 4, 5, 6, ...}

The set are denoted by Capital letters A, B, C ... X, Y, Z, and the elements of sets are denoted by small letters a, b, c ... x, y, z.

Types

1) Finite and Infinite sets:

A set having finite numbers of elements is called finite set.

Ex: A = {1, 2, 3}

A set having infinite numbers of elements is called infinite ~~system~~ set

Example A = {x : x is a star in the sky}

B = {x : x is a prime number} (set of builder form)

= {2, 3, 5, 7 ...} (Roster Form)

2) Null set or Empty set or void set:

A set having no elements is called null set it is denoted by ∅ or { }

Note: Ven-diagram phone gallery.

3) Singleton or Unit Set:

A set having only one element is called singleton set. Ex: A = {0}

B = the set of even prime number = {2}

4) Universal Set

Universal set is the totality of elements under the consideration as elements of any set. It is denoted by U.

Operation on sets:

1) Union: Example i) If A = {a, b, c, d} and B = {c, d, e, f}

Then, A∪B = {a, b, c, d, e, f}

2) Intersection: The common elements on the set is called Intersection.

E.g. If A = {a, b, c, d} and B = {e, f, g}

A∩B = ∅

3) Difference: It is the set of all the elements that belongs to A not to B. It denoted by A − B read as A difference B.

For example: If A = {a, b, c, d}, B = {c, d, e, f, g} Then

A − B = {a, b, c, d} − {c, d, e, f, g}

= {a, b}

Similarly,

B − A = {c, d, e, f, g} − {a, b, c, d}

= {e, f, g}

4) Compliment : The set of all elements that belong to U but not to A. It is denoted by, A', $\bar{A}$ or A^c

Eg: If $U = \{a,b,c,d,e,f,g,h\}$ or $A = \{a,c,e,g\}$

$A^c = U - A = \{a,b,c,d,e,f,g,h\} - \{a,c,e,g\}$

$= \{b,d,f,h\}$.

5) Symmetric Difference : The symmetric difference of two sets A and B denoted by $A \Delta B$ (read as A delta B) is define by

Imp

$$A \Delta B = (A-B) \cup (B-A)$$

eg:

$A = \{1,2,3,4\}$ and $B = \{3,4,5,6\}$

Then, $A \Delta B = (A-B) \cup (B-A)$

$= \{1,2\} \cup \{5,6\}$

$= \{1,2,5,6\}$

6) Cardinality of a set

The number of elements in a finite set A is called the cardinality of the set A. It is denoted by $n(A)$ or $|A|$

Ex: If $A = \{1,2,3\}$, Then $n(A) = 3$.

De-Morgan's law : Let A and B be two set ~~A is called the cardinality of set A. It is denoted by n(A) or |A|~~, Then

a) $(A \cup B)^c = A^c \cap B^c$

b) $(A \cap B)^c = A^c \cup B^c$

Either or = $\cup$

Both = $\cap$

Neither nor = $\overline{A \cup B}$

A but not B = $A - B$

B but not A = $B - A$

At least = $\cup$

Formula :

i) $n(A \cup B) = n(A) + n(B) - n(A \cap B)$ [If A and B are joint]

ii) $n(A \cup B) = n(A) + n(B)$, [If A and B are disjoint]

iii) $n(A \cup B) = n_o(A) + n_o(B) + n(A \cap B)$

$n_o(A)$ = no. of element in A only.

$n_o(B)$ = no. of elements in B only.

iv) $n(A \cap B) = n(A) + n(B) - n(A \cup B)$

v) $n(A^c) = n(U) - n(A)$

vi) $n_o(A) = n(A) - n(A \cap B) = n(A \cup B) - n(B)$

vii) $n(A-B) = n(A) - n(A \cap B)$

viii) $n(A \cup B \cup C) = n(A) + n(B) + n(C) - n(A \cap B) - n(B \cap C) - n(C \cap A) + n(A \cap B \cap C)$

or,

$n(A \cup B \cup C) = n_o(A) + n_o(B) + n_o(C) + n_o(A \cap B) + n_o(B \cap C) + n_o(C \cap A) + n(A \cap B \cap C)$

If A, B, C are disjoint sets, Then

$n(A \cup B \cup C) = n(A) + n(B) + n(C)$.

$n(A \cup B \cup C) = n(P) + n(C) + n(M) - n(P \cap C) - n(C \cap M) - n(P \cap M) + n(P \cap C \cap M)$

2) What is the cardinality of the set $A = \{a,e,i,o,u\}$.

Ans $n(A) = 5$

3) Write down the power set of the set $\{a,b,c\}$

Ans $P(A) = \{\emptyset, \{a\}, \{b\}, \{c\}, \{a,b\}, \{b,c\}, \{a,c\}, \{a,b,c\}\}$

4) If $U = \{1,2,3 \ldots 8,9,10\}$, $A = \{1,2,3,4\}$, $B = \{3,5,7,8\}$ and $C = \{1,3,5,7\}$ Find the following.

a) $A \cup B$

$U = \{1,2,3,4,5,6,7,8,9,10\}$
$A = \{1,2,3,4\}$
$B = \{3,6,7,8\}$
$C = \{1,3,5,7\}$

a) $A \cup B$
$= \{1,2,3,4\} \cup \{3,6,7,8\}$
$= \{1,2,3,4,5,7,8\}$

b) $A \cap C$
$= \{1,2,3,4\} \cap \{3,5,7,8\}$
$= \{3\}$

c) $(A \cup C)$
$= U - (A \cup C)$
$= \{1,2,3 \ldots 10\} - \{1,2,3,4,5\} \{1,3,5,7\}$
$= \{1,2,3 \ldots 10\} - \{1,2,3,4,5,7\}$
$= \{6,8,9,10\}$

d) $(A \cap B) - C$
$= \{3\} - \{1,3,5,7\}$
$\emptyset$

e) $A - (B \cup C)$
$= \{1,2,3,4\} - (\{3,5,7,8\} \cup \{1,3,5,7\}$
$= \{1,2,3,4\} - \{1,3,5,7,8\}$
$= \{2,4\}$

f) $\overline{A} \cup \overline{C}$
$= (U-A) \cup (U-C)$
$= (\{1,2,3 \ldots 10\} - \{1,2,3,4\}) \cup (\{1,2,3 \ldots 10\} - \{1,3,5,7\}$
$= \{5,6,7,8,9,10\} \cup \{2,4,6,8,9,10\}$
$= \{2,4,5,6,7,8,9,10\}$

g) $(A \cap B) \cap C$
$= \{3\} \cap \{1,3,5,7\}$
$= \{3\}$

5. a) Given $A = \{1,2,3\}$ and $B = \{3,4,5,6\}$, show that $A-(A-B) = A \cap B$

Sol: Given;
$A = \{1,2,3\}$
$B = \{3,4,5,6\}$

LHS: We have to show $A-(A-B) = A \cap B$
$A-(A-B)$
$= \{1,2,3\} - (\{1,2,3\} - \{3,4,5,6\}$
$= \{1,2,3\} - \{1,2\}$
$= \{3\}$

R.H.S $= A \cap B$
$= \{1,2,3\} \cap \{3,4,5,6\}$
$= \{3\}$

b) If $A = \{x : x = 2n+1, n \le 6, n \in N\}$ and $B = \{x : x = 3n-2, n \le 3, n \in N\}$, find

Sol:
$A = \{x : x = 2n+1, n \le 6, n \in N\}$
$= \{3,5,7,9,11,13\}$

$\leq$ = vane 0 hudina ani number jati xa teti Gamma janxa.

$B = \{x : x = 3n - 2, n \leq 3, n \in N\}$
$= \{1, 4, 7\}$

i) $A \cap B$
$= \{3, 5, 7, 9, 11, 13\} \cap \{1, 4, 7\}$
$= \{7\}$

ii) $A - B$
$= \{3, 5, 7, 9, 11, 13\} - \{1, 4, 7\}$
$\{3, 5, 9, 11, 13\}$

iii) $A \cup B$
$= \{3, 5, 7, 9, 11, 13\} \cup \{1, 4, 7\}$
$= \{1, 3, 4, 5, 7, 9, 11, 13\}$

1. a Given $U = \{x : x$ is a natural number upto $20\}$, $A = \{x : x \geq 6\}$, $B = \{x : x \leq 8\}$ and $C = \{x : 10 < x < 15\}$. find $A \cap B$, $B \cup C$, $A - C$ and $\overline{A \cup B}$.

Given;
$U = \{1, 2, 3, 4, 5 \ldots 20\}$
$B = \{1, 2, 3, 4, 5, 6, 7, 8\}$
$A = \{6, 7, 8, 9, 10 \ldots 20\}$
$C = \{11, 12, 13, 14\}$

i) $A \cap B$
$= \{6, 7, 8, 9, 10 \ldots 20\} \cap \{1, 2, 3, 4, 5, 6, 7, 8\}$
$= \{6, 7, 8\}$

ii) $B \cup C$
$= \{1, 2, 3, 4, 5, 6, 7, 8\} \cup \{11, 12, 13, 14\}$
$= \{1, 2, 3, 4, 5, 6, 7, 8, 11, 12, 13, 14\}$

iii) $A - C$
$= \{6, 7, 8, 9, 10 \ldots 20\} - \{11, 12, 13, 14\}$
$= \{6, 7, 8, 9, 10, 15, 16, 17, 18, 19, 20\}$

iv) $\overline{A \cup B}$
$= U - (A \cup B)$
$= \{1, 2, 3, 4, 5 \ldots 20\} - (\{6, 7, 8, 9, 10 \ldots 20\} \cup \{1, 2, 3, 4, 5, 6, 7, 8\})$
$= \{1, 2, 3, 4, 5 \ldots 20\} - \{1, 2, 3, 4, 5, 6, 7, 8 \ldots 20\}$
$= \emptyset$

(b) If $U = \{x : -1 \leq x - 2 \leq 7\}$, $A = \{x : x$ is a prime number$\}$ and $B = \{x : x$ is an odd number$\}$, find

Sol:
$U = \{x : -1 + 2 \leq x - 2 + 2 \leq 7 + 2\}$
$= \{x : 1 \leq x \leq 9\}$
$= \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$
$A = \{x : x$ is a prime number$\}$
$= \{2, 3, 5, 7\}$
$B = \{x : x$ is an odd number$\}$
$= \{1, 3, 5, 7, 9\}$

i) The set of elements which are either prime or odd
$= A \cup B$
$= \{2, 3, 5, 7\} \cup \{1, 3, 5, 7, 9\}$

$= \{1, 2, 3, 5, 7, 9\}$

ii) Prime as well as odd

$= A \cap B$

$= \{2, 3, 5, 7\} \cap \{1, 3, 5, 7, 9\}$

$= \{3, 5, 7\}$

iii) Prime but not odd

$= A - B$

$= \{2, 3, 5, 7\} - \{1, 3, 5, 7, 9\}$

$= \{2, 9\}$

7) Let $U = \{a, b, c, d, e, f, g, h\}$, $A = \{a, b, c, d\}$, $B = \{c, d, e, f\}$ and $C = \{d, e, f, g, h\}$. verify the following

Given,

$U = \{a, b, c, d, e, f, g, h\}$

$A = \{a, b, c, d\}$

$B = \{c, d, e, f\}$

$C = \{d, e, f, g, h\}$

a) $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

Sol: L.H.S

$= A \cup (B \cap C)$

$= \{a, b, c, d\} \cup (\{c, d, e, f\} \cap \{d, e, f, g, h\})$

$= \{a, b, c, d\} \cup \{d, e, f\}$

$= \{a, b, c, d, e, f\}$

R.H.S

$= (A \cup B) \cap (A \cup C)$

$= (\{a, b, c, d\} \cup \{c, d, e, f\}) \cap (\{a, b, c, d\} \cup \{d, e, f, g, h\})$

$= \{a, b, c, d, e, f\} \cap \{a, b, c, d, e, f, g, h\}$

$= \{a, b, c, d, e, f\}$

b) $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

Sol: L.H.S

$= A \cap (B \cup C)$

$= \{a, b, c, d\} \cap (\{c, d, e, f\} \cup \{d, e, f, g, h\})$

$= \{a, b, c, d\} \cap \{c, d, e, f, g, h\}$

$= \{c, d\}$

R.H.S

$= (A \cap B) \cup (A \cap C)$

$= (\{a, b, c, d\} \cap \{c, d, e, f\}) \cup (\{a, b, c, d\} \cap \{d, e, f, g, h\})$

$= \{c, d\} \cup \{d\}$

$= \{c, d\}$

c) $\overline{(A \cup B)} = (\overline{A} \cap \overline{B})$

$=$ R.H.S

$= \overline{(A \cup B)}$

$= U - (A \cup B)$

$= \{a, b, c, d, e, f, g, h\} - \{a, b, c, d, e, f\}$

$= \{g, h\}$

L.H.S

$= (\overline{A} \cap \overline{B})$

$= (U - A) \cap (U - B)$

$= (\{a, b, c, d, e, f, g, h\} - \{a, b, c, d\}) \cap (\{a, b, c, d, e, f, g, h\} - \{c, d, e, f\})$

$= \{e, f, g, h\} \cap \{a, b, g, h\}$

$= \{g, h\}$

d) $A-(B\cup C) = (A-B)\cap(A-C)$

R.H.S

$= A-(B\cup C)$

$= \{a,b,c,d\} - \{c,d,e,f,g,h\}$

$= \{a,b\}$

L.H.S

$= (A-B)\cap(A-C)$

$= \{a,b,c,d\} - \{c,d,e,f\} \cap \{a,b,c,d\} - \{d,e,f,g,h\}$

$= \{a,b,e,f\} \cap \{a,b,c,\}$

$= \{a,b\}$

8) If $n(A) = 40$, $n(B) = 55$ and $A \subset B$, find $n(A\cup B)$, $n(A\cap B)$, $n(B-A)$ and $n(A-B)$

Given;

$n(A) = 40$

$n(B) = 55$

Now;

$n(A\cup B) = n(B) = 55$

$n(A\cap B) = n(A) = 40$

$n(B-A) = n(B) - n(A\cap B)$

$= 55 - 40 = 15$

$n(A-B) = n(A) - n(A\cap B)$

$= 40 - 40$

$= 0$

Note:

$A \subset B$ means A is subset of B. So that in B, ~~we~~ A is already contain. So $n(A\cup B)$ is equal to $n(B)$ Because $n(B)$ acts as a universal set.

9) In a committee, 50 people speak Nepali, 20 speak English and 10 speak both Nepali and English. How many speak at least one of these two languages.

Sol: Let A = Nepali and B = English

$n(A) = 50$

$n(B) = 20$

$n(A\cap B) = 10$

$n(A\cup B) = ?$

now;

$n(A\cup B) = n(A) + n(B) - n(A\cap B)$

$= 50 + 20 - 10$

$= 60$

10) In a group of 50 students, 25 play hockey, 30 play football and 8 play neither hockey nor football. And the number of students who play both games.

Sol: Let A = Hockey, B = Football

$n(U) = 50$, $n(A) = 25$, $n(B) = 30$, $n(\overline{A\cup B}) = 8$,

$n(A\cap B) = ?$

Now;

$= n(A\cup B) = n(A) + n(B) - n(A\cap B)$

or, $n(U) - n(\overline{A\cup B}) = 25 + 30 - n(A\cap B)$

or, $50 - 8 = 55 - n(A\cap B)$

or, $42 = 55 - n(A\cap B)$

or, $n(A\cap B) = 55 - 42$

or, $n(A\cap B) = 13$ ans

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Bu not means only one.
like drink tea but not coffe means only tea

11) Sol: A = Tea and B = Coffe

$n(U) = 65$, $n(\overline{A \cup B}) = 15$

$n_o(A) = 20$, $n(A \cap B) = ?$

$n(A) = 38$, $n_o(B) = ?$

now,

$\therefore n_o(A) = n(A) - n(A \cap B)$

or, $20 = 38 - n(A \cap B)$

or, $n(A \cap B) = 38 - 20$

or, $n(A \cap B) = 18$

again,

$= n(\overline{A \cup B}) = n_o(A) + n_o(B) + n(A \cap B)$

or, $n(U) - n(\overline{A \cup B}) = n_o(A) + n_o(B) + n(A \cap B)$

or, $65 - 15 = 20 + n_o(B) + 18$

or, $50 = 38 + n_o(B)$

or, $n_o(B) = 38 - 50$

or, $n_o(B) = -12$

$\therefore n_o(B) = 12$

12. Sol:

$n(P) = 18$

$n(C) = 23$

$n(M) = 24$

$n(C \cap M) = 13$

$n(P \cap C) = 12$

$n(M \cap P) = 11$

$n(P \cap C \cap M) = 6$

$n(P \cup C \cup M) = ?$

$n_o(M) = ?$

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$n(P \cup C \cup M) = n(P) + n(C) + n(M) - n(P \cap C) - n(C \cap M) - n(P \cap M) + n(P \cap C \cap M)$

$= 18 + 23 + 24 - 12 - 13 - 11 + 6$

$= 35$

ii) Math but not chemistry

$n_o(M) = n(M) - n(M \cap C)$

$= 24 - 13$

$= 11$

c) Exactly one of three subjects

= Physics only or chemistry only or math only

For physics only

$n_o(P) = n(P) - n(P \cap C) - n(P \cap M) + n(P \cap C \cap M)$

$= 18 - 12 - 11 + 6$

$= 1$

For chemistry only

$n_o(C) = n(C) - n(P \cap C) - n(C \cap M) + n(P \cap C \cap M)$

$= 23 - 12 - 13 + 6$

$= 4$

~~For~~

For math only

$n_o(M) = n(M) - n(C \cap M) - n(P \cap M) + n(P \cap C \cap M)$

$= 24 - 13 - 11 + 6$

$= 6$

Total number $= n_o(P)$ or $n_o(C)$ or $n_o(M)$

$= 1 + 4 + 6$

$= 11$

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13) B = Basketball , H = Hockey , F = football

Given,

$n(B) = 21$

$n(H) = 26$

$n(F) = 29$

$n(H \cap B) = 14$

$n(H \cap F) = \cancel{14}\ 15$

$n(B \cap F) = 12$

$n(B \cap H \cap F) = 8$

i) $n(B \cup H \cup F) = n(B) + n(H) + n(F) - n(H \cap B) - n(H \cap F) - n(B \cap F) + n(B \cap H \cap F)$

$= 21 + 26 + 29 - 14 - 15 - 12 + 8$

$= 43$

ii) Football only $\rightarrow n_o(F)$

$n_o(F) = n(F) - n(H \cap F) - n(B \cap F) + n(B \cap H \cap F)$

$= 29 - 12 - 15 + 8$

$= 10$

c) No. of players who play Football and hockey only

$n_o(B \cap H) = n(B \cap H) - n(B \cap H \cap F)$

$= 14 - 8 = 6$

No. of players who play basketball and football only

$n_o(B \cap F) = n(B \cap F) - n(B \cap H \cap F)$

$= 12 - 8 = 4$

No. of players who play Hockey and Football only

$n_o(H \cap F) = n(H \cap F) - n(B \cap H \cap F)$

$= 15 - 8$

$= 7$

$\therefore$ Total players in exactly two teams

$= 6 + 4 + 7$

$= 17$

14) Sol:

$n(A) = 60\%$	$n(B \cap C) = 20\%$
$n(B) = 50\%$	$n(C \cap A) = 25\%$
$n(C) = 40\%$	$n(A \cap B \cap C) = 10\%$
$n(A \cap B) = 30\%$	$n(U) = 100\%$

a) Only A $\rightarrow n_o(A)$

$n_o(A) = n(A) - n(A \cap B) - n(C \cap A) + n(A \cap B \cap C)$

$= 60\% - 30\% - 25\% + 10$

$= 15\%$

b) Only B $\rightarrow n_o(B)$

$n_o(B) = n(B) - n(A \cap B) - n(B \cap C) + n(A \cap B \cap C)$

$= 50\% - 30\% - 20\% + 10\%$

$= 10\%$

c) $n(\overline{A \cup B \cup C}) = n(U) - n(A \cup B \cup C)$

$= n(U) - n(A) - n(B) - n(C) - n(A \cap B) - n(B \cap C) - n(C \cap A) + n(A \cap B \cap C)$

$= 100\% - 60\% + 50\% + 40\% - 30\% - 20\% - 25\% + 10\%$

$= 100\% - 85\%$

$= 15\%$

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* 32 students play basketball and 25 students play volleyball. It is found that 20 students playing ~~at least~~ one both n game. Find the no. of students playing at least one game. Also find total no. of students. If 13 students play none of these games.

Sol

$n(B) = 32$	$n(\overline{A \cup B}) = 13$
$n(V) = 25$	$n(A \cup B) = ?$
$n(B \cap V) = 20$	$n(U) = ?$

Now;

$$n(A \cup B) = n(B) + n(V) - n(B \cap V)$$
$$= 32 + 25 - 20$$
$$= 37$$

again;

$$= n(\overline{A \cup B}) = n(U) - n(A \cup B)$$
or, $13 = n(U) - 37$
or, $13 + 37 = n(U)$
or, $n(U) = 50$

2019) In a class of 100 students, 40 students failed in mathematics, 70 failed in english and 20 failed in both. Find
a) How many students passed in both subjects?
b) How many students passed in mathematics only
c) How many students failed in mathematics only.

sol:

$n(U) = 100$	$n(\overline{M \cup E}) = ?$
$n(M) = 40$	$n_o(M) = ?$
$n(E) = 70$	
$n(M \cap E) = 20$	

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a) $n(\overline{M \cup E}) = n(U) - n(M \cup E)$
$= n(U) - [n(M) + n(E) - n(M \cap E)]$
$= 100 - [40 + 70 - 20]$
$= 100 - 90$
$= 10$

b) Passed in mathematics only
No. of students failed in English only $= n_o(E)$
$= n(E) - n(M \cap E)$
$= 50$

$n(U) - n_o(E) = 100 - 50 = 50$.

c) $n_o(M) = n(M) - n(M \cap E)$
$= 40 - 20 = 20$

*) In a city of 26000 population, 5000 read English newspaper, 12000 read Nepal newspaper and 1000 read both what percentage read neither English nor Nepali newspaper?

sol:

$n(U) = 26000$
$n(E) = 5000$
$n(N) = 12000$
$n(E \cap N) = 1000$
$n(\overline{E \cup N}) = ?$

now;

$$n(\overline{E \cup N}) = n(U) - n(E \cup N)$$
$$= n(U) - [n(E) + n(N) - n(E \cap N)]$$
$$= 26000 - [5000 + 12000 - 1000]$$
$$= 26000 - 16000$$
$$= 10000$$

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Converting into percentage

$\frac{n(A \cap B)}{n(U)} \times 100 = \frac{[illegible]}{[illegible]} \times 100 = 39.46\%$

2) Out of 500 people, 285 like tea, 195 like coffee, 115 like lemon juice, 45 like tea and coffee, 70 like tea and juice, 50 like juice and coffee. If 50 donot like any drink.

* Given,

$n(U) = 500$ | $n(L) = 115$
$n(T) = 285$ | $n(T \cap C) = 45$
$n(C) = 195$ | $n(T \cap L) = 70$
$n(L \cap C) = 50$ | $n(\overline{T \cup C \cup L}) = 50$

i) How many of people like all 3 drinks ?

$n(A \cap B \cap C) = ?$

~~$n(A \cup B \cup C) = n(U) - n$~~

$= n(A \cup B \cup C) = n(T) + n(C) + n(L) - n(T \cap C) - n(T \cap C) - n(T \cap L) + n(A \cap B \cap C)$

or, $n(U) - n(\overline{T \cup C \cup L}) = 285 + 195 + 115 - 45 - 50 - 70 + n$ ~~$(A \cap B \cap C)$~~ $(T \cap C \cap L)$

or, $500 - 50 = 430 +$ ~~$n(A \cap B \cap C)$~~ $n(T \cap C \cap L)$

or, $450 = 430 +$ ~~$n(A \cap B \cap C)$~~ $n(T \cap C \cap L)$

or, $n(T \cap C \cap L) =$ ~~450~~ 20

ii) How many people like only one drink.

* For only Tea

$n_o(T) = n(T) - n(T \cap C) - n(T \cap L) + n(T \cap C \cap L)$
$= 285 - 45 - 70 + 20$
$= 190$

* For only Coffee

$n_o(C) = n(C) - n(L \cap C) - n(T \cap C) + n(T \cap C \cap L)$
$= 195 - 50 - 45 + 20$
$= 120$

* For only lemon Juice

$n_o(L) = n(L) - n(L \cap C) - n(T \cap L) + n(T \cap C \cap L)$
$= 115 - 50 - 70 + 20$
$= 15$

Total number $= n_o(T) + n_o(C) + n_o(L)$
$= 190 + 120 + 15$
$= 325$

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Number system

Intervals:

i) $[a,b]$ = $\bullet_a$ —— $\bullet_b$ = closed interval $(a \le x \le b)$

ii) $(a,b]$ = $\circ_a$ —— $\bullet_b$ = half open $(a < x \le b)$

iii) $[a,b)$ = $\bullet_a$ —— $\circ_b$ = half closed $(a \le x < b)$

iv) (a,b) = $\circ_a$ —— $\circ_b$ = Open $(a < x < b)$

Note:

$\bullet$ = means also contain that number

$\circ$ = means does not contain that number

4) If $x = 2, y = -3$ verify each of the following

a) $|x+y| \le |x| + |y|$

L.H.S	R.H.S
$= \lvert x+y \rvert$	$= \lvert x \rvert + \lvert y \rvert$
$= \lvert 2 + (-3) \rvert$	$= \lvert 2 \rvert + \lvert -3 \rvert$
$= \lvert 2-3 \rvert$	$= 2+3$
$= \lvert -1 \rvert$	$= 5$
$= 1$	$\therefore \lvert x+y \rvert < \lvert x \rvert + \lvert y \rvert$

b) $|x-y| \ge |x| - |y|$

L.H.S	R.H.S
$= \lvert x-y \rvert$	$= \lvert x \rvert - \lvert y \rvert$
$= \lvert 2-(-3) \rvert$	$= \lvert 2 \rvert - \lvert -3 \rvert$
$= \lvert 2+3 \rvert$	$= 2-3$
$= \lvert 5 \rvert$	$= -1$
$= 5$	$\therefore \lvert x-y \rvert > \lvert x \rvert - \lvert y \rvert$

If $-a < x < a$
Then $|x| < a$

or

$|x| < a \Leftrightarrow |x| \le a \Rightarrow -a \le x \le a$

$\Leftarrow$ (if)

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c) $|xy| = |x||y|$

L.H.S	R.H.S
$= \lvert xy \rvert$	$= \lvert x \rvert \lvert y \rvert$
$= \lvert 2 \times -3 \rvert$	$= \lvert 2 \rvert \lvert -3 \rvert$
$= \lvert -6 \rvert$	$= 2 \times 3$
$= 6$	$= 6$ $\quad \therefore \lvert xy \rvert = \lvert x \rvert \lvert y \rvert$

d) $\left|\frac{x}{y}\right| = \frac{|x|}{|y|}$

L.H.S	R.H.S
$= \left\lvert \frac{x}{y} \right\rvert$	$= \frac{\lvert x \rvert}{\lvert y \rvert}$
$= \left\lvert \frac{2}{-3} \right\rvert$	$= \frac{\lvert 2 \rvert}{\lvert -3 \rvert}$
$= \lvert 2/-3 \rvert$	$= \frac{2}{3}$
$= \lvert -0.6667 \rvert$	$= 0.6667$
$= 0.6667$	$\therefore \left\lvert \frac{x}{y} \right\rvert = \frac{\lvert x \rvert}{\lvert y \rvert}$

5) Rewrite the following without using absolute value sign and simplify

a) $|x-3| < 2$

Sol:

$= -2 < x - 3 < 2$

$= -2+3 < x-3+3 < 2+3$

$= 1 < x < 5$

$\therefore x \in (1,5)$

b) $|2x+1| \le 5$

Sol:

$= -5 \le 2x+1 \le 5$

$= -5-1 \le 2x+1-1 \le 5-1$

$= -6 \le 2x \le 4$

$= -6/2 \le 2x/2 \le 4/2$

$= -3 \le x \le 2$

$\therefore x \in [-3, 2]$

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c) $|3x+2| < 1$

Sol:

i.e $-1 < 3x+2 < 1$

i.e $-1-2 < 3x+2-2 < 1-2$

i.e $-3 < 3x < -1$

i.e $-3/3 < 3x/3 < -1/3$

i.e $-1 < x < -1/3$

$\therefore x \in (-1, -1/3)$

Q) Solve the following inequations.

a) $|x-1| \le 2$

i.e $-2 \le x-1 \le 2$

i.e $-2+1 \le x-1+1 \le 2+1$

i.e $-1 \le x \le 3$

$\therefore x \in [-1, 3]$

b) $|2x+3| \le 1$

i.e $-1 \le 2x+3 \le 1$

i.e $-1-3 \le 2x+3-3 \le 1-3$

i.e $-4 \le 2x \le -2$

i.e $\frac{-4}{2} \le \frac{2x}{2} \le \frac{-2}{2}$

i.e $-2 \le x \le -1$ $\therefore x \in [-2, -1]$

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c) $|1-2x| \ge 3$

we know;

$$|x| = \begin{cases} +x, & x \ge 0 \\ -x, & x \le 0 \end{cases}$$

Taking +ve sign

i.e $|1-2x| \ge 3$

i.e $(1-2x-1) \ge 3-1$

i.e $-2x \ge 2$

i.e $\frac{-2x}{-2} \ge \frac{2}{-2}$

Note: while we divide number by (-) then we have to change sign.

i.e $x \le -1$

$x \in (-\infty, -1]$

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Taking -ve sign

i.e $-|1-2x| \ge 3$

i.e $-1+2x \ge 3$

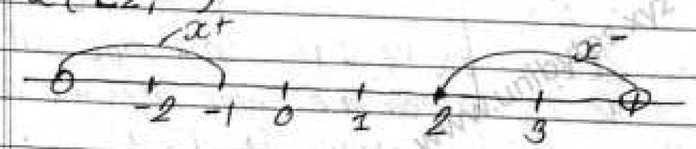
i.e $-1+2x+1 \ge 3+1$

i.e $2x \ge 4$

i.e $\frac{2x}{2} \ge \frac{4}{2}$

i.e $x \ge 2$

$x \in [2, \infty)$



Absolute value (modulus)

The absolute value or modulus of a real no. x denoted by $|x|$ is defined as

$$|x| = \begin{cases} x, & \text{for } x \ge 0 \\ -x, & \text{for } x < 0 \end{cases}$$

$\rightarrow \boxed{\begin{matrix} -a < x < a \\ |x| < a \end{matrix}}$

7) Rewrite the following ~~inequation~~ Inequalities using absolute value sign.

a) $-5 < x < 7$

i.e $-5-1 < x-1 < 7-1$

i.e $-6 < x-1 < 6$

i.e $|x-1| < 6$

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(b) $-3 \le x \le -1$

i.e ~~$-3+1 \le x \le -1+1$~~

i.e $-3+2 \le x+2 \le -1+2$

i.e $-1 \le x+2 \le 1$

i.e $|x+2| \le 1$

$\frac{-3+(-1)}{2} = \frac{-4}{2} = -2$

Note: If the number after simplification came in (-) sign then we have to it in (+) or came with (+) sign make (-) sign

c) $-4 \le x \le -1$

i.e $\frac{-4+5}{2} \le \frac{x+5}{2} \le \frac{-1+5}{2}$

i.e $\frac{-8+5}{2} \le \frac{2x+5}{2} \le \frac{-2+5}{2}$

i.e $-3 \le 2x+5 \le 3$

i.e $|2x+5| \le 3$

$\frac{-4+(-1)}{2} = \frac{-5}{2}$

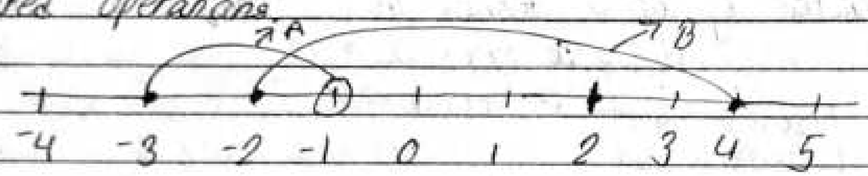
Note: if in divide all the number are same the we can remove it (2)

Note: $-a \le x < a$ condition (tasify vaye pou sluve na gam)

Sme

8) If $A = [-3, 1)$ and $B = [-2, 4]$, perform the indicated operations.

Graph:



a) $A \cup B$

$= [-3, 1) \cup [-2, 4]$

$= \{x : -3 \le x < 1\} \cup \{x : -2 \le x \le 4\}$ → This is not essential

$= \{x : -3 \le x \le 4\}$

$\therefore A \cup B = [-3, 4]$

i.e

x

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(b) $A \cap B$

sol: $[-3, 1) \cap [-2, 4]$

$= \{x : -3 \le x < 1\} \cap \{x : -2 \le x \le 4\}$

$= \{x : -2 \le x < 1\}$

$\therefore A \cap B = [-2, 1)$

c) $A - B$

$\{x : -3 \le x < 2\}$

$\therefore A - B = [-3, 2)$

d) $B - A$

$\{x : 1 \le x \le 4\}$

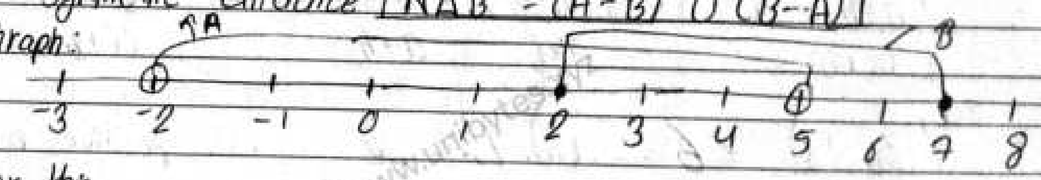
$\therefore B - A = [1, 4]$

9) If $A = (-2, 5]$ and $B = [2, 7]$, find the symmetric difference between A and B.

sol:

Symmetric difference $[A \Delta B = (A - B) \cup (B - A)]$

Graph:



For this

$A - B = (-2, 5] - [2, 7]$

$= \{x : -2 < x < 5\} - \{x : 2 \le x \le 7\}$

$= \{x : -2 < x < 2\}$

$= (-2, 2)$

$B - A = [2,7] - (-2,5)$

$= \{x : 2 \le x \le 7\} - \{x : -2 < x < 5\}$

$= \{x : 5 \le x \le 7\}$

$= [5,7]$

$A \Delta B = (A - B) \cup (B - A)$

$= (-2,2) \cup [5,7]$ ans

10) Prove that: $\frac{5a+6b}{a+b}$ where a and b are positive rational numbers lies between 5 and 6

Sol:

$\frac{5a+6b}{a+b} = \frac{5a+5b+b}{a+b} = \frac{5(a+b)+b}{a+b}$

$= \frac{5(a+b)}{a+b} + \frac{b}{a+b}$ so; $\frac{5a+6b}{a+b} > 5$ ---(i) [$\because \frac{b}{a+b}$ is positive number]

again;

$\frac{5a+6b}{a+b} = \frac{5a+a+6b-a}{a+b} = \frac{6a+6b-a}{a+b}$

$= \frac{6(a+b)-a}{a+b} = \frac{6(a+b)}{a+b} - \frac{a}{a+b} = 6 - \frac{a}{a+b}$

so,

$\frac{5a+6b}{a+b} < 6$ ---(ii) [$\because \frac{a}{a+b}$ is positive number]

From eqn (i) and (ii) we have

$5 < \frac{5a+6b}{a+b} < 6$

i.e $\frac{5a+6b}{a+b}$ lies between 5 and 6 proved

Complex Number

Any number which can be expressed in the form (a,b) or $(a+ib)$ where a is the real part and b is the imaginary part is called complex number. It is denoted by z or w.

An order pair (a,b) where a is the real part and b is the imaginary part is called complex no.

$z = (x,y)$ or $x + iy$

$z = (-2,3)$ or $-2 + 3i$

i = imaginary unit, $i = \sqrt{-1}$, $i^2 = -1$

Let $\boxed{z = x + iy}$ be a complex

Then;

Conjugate : $\bar{z} = x - iy$ [Note: Sign change garnu pa'i ho]

modulus : $|z| = \sqrt{(x)^2 + (y)^2}$

$= \sqrt{(\text{Real part})^2 + (\text{Imaginary part})^2}$

Absolute value

Multiplicative inverse $= \frac{1}{z}$ or z^{-1}

Addition

If $z_1 = a + ib$, $z_2 = c + id$

Then;

$z_1 + z_2 = (a+ib) + (c+id)$

$= (a+c) + (b+d)i$

Note: $5 = 5+0i = (5,0)$

iff = (if and only if)

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Subtraction

$$z_1 - z_2 = (a+ib) - (c+id)$$
$$= (a-c) + (b-d)i$$

Properties:

i) $|z| = |\bar{z}|$

ii) $|z| = 0$ iff $z = 0$

iii) $|zw| = |z||w|$ for two complex no

iv) $\left|\frac{z}{w}\right| = \frac{|z|}{|w|}$, if $|w| \neq 0$

v) $z\bar{z} = |z|^2$

vi) $Re(z) \leq |z|$ and $img(z) \leq |z|$

The additive inverse of $w = 2+3i$ is $-w = -2-3i$.

$z + (-z) = 0$. Then no $-z$ is called additive inverse of z.

2) Evaluate

a) $(1,0)^2$
$= (1+0i)^2$
$= (1)^2$
$= 1$

b) $(0,1)^5$
$= (0+1i)^5$
$= (i)^5$
$= (i^2)^2 \times i$
$\cancel{(-1)} \times (-1^2)^2 \times i$
$1 \times i$
$= i$

3) Simplify

a) $\sqrt{-9} + \sqrt{-25} - \sqrt{-49}$
$= \sqrt{(-1)\times(3)^2} + \sqrt{(-1)\times(5)^2} - \sqrt{(-1)\times(7)^2}$
$= \sqrt{i^2\times 3^2} + \sqrt{i^2\times 5^2} - \sqrt{i^2\times 7^2}$
$= \sqrt{(3i)^2} + \sqrt{(5i)^2} - \sqrt{(7i)^2}$
$= 3i + 5i - 7i$
$= 8i - 7i$
$= i$
$= 0 + 1i$
$= (0,1)$

b) $3i^2 + i^3 + 9i^4 - i^7$
$= 3\times(-1) + (-1)\times i + 9\times 1 - (1)\times(-1)\times i$
$= -3 - 1i + 9 + 1i$
$= 6$
$= (6 + 0i) = (6,0)$

c) $\frac{1}{i} + \frac{1}{i^2} + \frac{1}{i^3} + \frac{1}{i^4}$
$= \frac{1}{i} + \frac{1}{(-1)} + \frac{1}{(-1)\times i} + \frac{1}{(-1)^2}$
$= \frac{\cancel{1}}{i} - \frac{\cancel{1}}{1} - \frac{\cancel{1}}{i} + \frac{\cancel{1}}{1}$
$= 0$ ans

4) Find the values of x and y in each of the following

a) $(x,y) = (3,2) - (1,1)$

Sol: $(x+iy) = 3+2i - (1+1i)$

or, $x + iy = 3 + 2i - 1 - 1i$

or, $x + iy = 2 + 1i$

equality of the real and imaginary part we get:

$x = 2$ / $iy = 1i$
$y = 1$

b) $(x,y) = (1,1)(2,3)$

Sol: $(x+iy) = (1+1i)(2+3i)$

or, $(x+iy) = (2+3i) + (2i+3i^2)$

or, $(x+iy) = 2+3i+2i+3i^2$

or, $(x+iy) = 2+3i+2i-3$

or, $(x+iy) = -1+5i$

$\therefore x = -1$ / $iy = 5i$
$y = 5$

c) $(x,y) = \frac{(1,1)}{(3,4)}$

sol: $x+iy = \frac{(1+1i)}{(3+4i)}$

or, $x+iy = \frac{1+i}{3+4i} \times \frac{3-4i}{3-4i}$

or, $x+iy = \frac{3-4i+3i-4i^2}{(3)^2-(4i)^2}$

or, $x+iy = \frac{3-i+4}{9-16i^2}$

or, $x+iy = \frac{7-i}{9+16}$

or, $x+iy = \frac{7-i}{25}$

or, $x+iy = \frac{7}{25} - \frac{1}{25}i$

equating

$x = \frac{7}{25}$ / $y = \frac{-1}{25}$

3) Express each of the following in the form of $a+ib$.

i) $\frac{3-7i}{-2+4i}$

sol: $\frac{3-7i}{-2+4i} \times \frac{-2-4i}{-2-4i}$

$= \frac{-6-12i+14i+28i^2}{(-2)^2-(4i)^2}$

$= \frac{-6+2i-28}{4+16}$

$= \frac{-34+2i}{20}$

$= \frac{2(-17+i)}{20}$

$= \frac{-17}{10} + \frac{1}{10}i$

c) $(x-1)i + (y+1) = (1+i)(4-3i)$

sol: $(y+1) + (x-1i) = 4-3i+4i-3i^2$

or, $(y+1)(x-1i) = 4+1-3(-1)$

or, $(y+1)(x-1)i = 4+1+3$

or, $(y+1)(x-1)i = 7+1$

equating;

$y+1 = 7$ / $x-1 = 1$

$y = 6$ / $x = 2$

Note: $(-a-b)^2$

$\downarrow$

$(a+b)^2$

5.a) $(1+2i)(2+3i)$

sol:

Let $z = (1+2i)(2+3i)$

$= 2+3i+4i+6i^2$

$= 2+7i-6 \quad [\because i^2=-1]$

$\therefore z = -4+7i$

which is in the form of $a+ib$ where $a=-4$, $b=7$

b) $(-1-2i)^2$

Let $z = (-1-2i)^2$

$= (1+2i)^2$

$= 1^2 + 2.1.2i + (2i)^2$

$= 1+4i+4i^2$

$= 1+4i-4$

$= -3+4i$

which is in the form of $a+ib$

$\therefore a=-3$, $b=4$

c) $\frac{3+2i}{4+5i}$

Let $z = \frac{3+2i}{4+5i}$

$= \frac{3+2i}{4+5i} \times \frac{4-5i}{4-5i}$

$= \frac{12-15i+8i-10i^2}{(4)^2-(5i)^2}$

$= \frac{12-7i+10}{16-(25\times-1)}$

$= \frac{22-7i}{16-(-25)}$

$= \frac{22-7i}{16+25}$

$= \frac{22-7i}{41}$

$= \frac{22}{41} - \frac{7i}{41}$

Which is in the form of $a+ib$ where

$a = \frac{22}{41}$, $b = \frac{-7}{41}i$

6) If $z_1 = 3-4i$, $z_2 = 2-3i$ verify that

a) $\overline{z_1+z_2} = \overline{z_1} + \overline{z_2}$

sol:

$z_1+z_2 = (3-4i)+(2-3i)$

$= 3-4i+2-3i$

$= 5-7i$

L.H.S

$\overline{z_1+z_2} = \overline{5-7i}$

$= 5+7i$

R.H.S

$\overline{z_1} + \overline{z_2} = (3+4i)+(2+3i)$

$= 3+4i+2+3i$

$= 5+7i$

$\therefore \overline{z_1+z_2} = \overline{z_1} + \overline{z_2}$

b) $\overline{z_1 . z_2} = \overline{z_1} . \overline{z_2}$

sol:

$z_1 . z_2 = (3-4i).(2-3i)$

$= 6-9i-8i+12i^2$

$= 6-17i-12$

$= -6-17i$

L.H.S

$\overline{z_1 . z_2} = \overline{-6-17i}$

$= -6+17i$

R.H.S

$= \overline{z_1} . \overline{z_2}$

$= (3+4i)(2+3i)$

$= 6+9i+8i+12i^2$

$= 6+17i-12$

$= -6+17i$

$\therefore \overline{z_1 . z_2} = \overline{z_1} . \overline{z_2}$ verified

Note: If $z = x+iy$ then $|z| = |x+iy| = \sqrt{(x)^2+(y)^2}$

$z = x-iy$ then $|z| = |x-iy| = \sqrt{(x)^2+(y)^2}$

c) $\overline{\left(\frac{z_1}{z_2}\right)} = \frac{\overline{z_1}}{\overline{z_2}}$

sol:

$\frac{z_1}{z_2} = \frac{3-4i}{2-3i}$, $\frac{3-4i}{2-3i} \times \frac{2+3i}{2+3i}$, $\frac{6+9i-8i-12i^2}{(2)^2-(3i)^2}$

$= \frac{6+i+12}{4+9}$, $= \frac{18+i}{13} = \frac{18}{13} + \frac{1}{13}$

L.H.S

$\overline{\left(\frac{z_1}{z_2}\right)} = \overline{\left(\frac{18+i}{13}\right)} = \left(\frac{18-i}{13}\right) = \frac{18}{13} - \frac{1}{13}$

R.H.S

$\frac{\overline{z_1}}{\overline{z_2}} = \frac{3+4i}{2+3i}$, $= \frac{3+4i}{2+3i} \times \frac{2-3i}{2-3i}$, $= \frac{6-9i+8i-12i^2}{(2)^2-(3i)^2}$

$= \frac{6-i+12}{4+9} = \frac{18-i}{13} = \frac{18}{13} - \frac{1}{13}$

$\therefore \overline{\left(\frac{z_1}{z_2}\right)} = \frac{\overline{z_1}}{\overline{z_2}}$ verified

d) $|z_1+z_2| \leq |z_1| + |z_2|$

L.H.S

$= |z_1+z_2|$

$= |3-4i+2-3i|$

$= |5-7i|$

$= \sqrt{(5)^2+(-7)^2}$

$= \sqrt{25+49}$

$= \sqrt{74}$

$= 8.60$

R.H.S

$= |z_1| + |z_2|$

$= |3-4i| + |2-3i|$

$= \sqrt{(3)^2+(-4)^2} + \sqrt{(2)^2+(-3)^2}$

$= \sqrt{9+16} + \sqrt{4+9}$

$= \sqrt{25} + \sqrt{13}$

$= 5+\sqrt{13}$

$= 8.60$

$\therefore |z_1+z_2| \leq |z_1| + |z_2|$ verified

7) Find the modulus of

a) $(3-2i)(2-3i)$

Sol: $z = (3-2i)(2-3i)$

$= 6 - 9i - 4i + 6i^2$

$= 6 - 13i - 6$

$z = 0 - 13i$

now

$|z| = |0 - 13i|$

$= \sqrt{0 + (-13)^2}$

$= \sqrt{169}$

$= 13$

b) $\frac{3+4i}{12-5i}$

Sol: $|z| = \frac{|3+4i|}{|12-5i|}$

$= \frac{\sqrt{(3)^2 + (4)^2}}{\sqrt{(12)^2 + (5)^2}}$

$= \frac{\sqrt{9+6}}{\sqrt{144+25}}$

$= \frac{\sqrt{25}}{\sqrt{169}}$

$= \frac{\sqrt{(5)^2}}{\sqrt{(13)^2}}$

$= \frac{5}{13}$

8) Find the multiplicative inverse of

a) $3 + 4i$

Let $z = 3+4i$

$\therefore$ Multiplicative inverse $= 1/z$

$= \frac{1}{3+4i}$

$= \frac{1}{3+4i} \times \frac{3-4i}{3-4i}$

$= \frac{3-4i}{(3)^2 - (4i)^2}$

$= \frac{3-4i}{9+16}$

$= \frac{3-4i}{25}$

$\therefore \frac{3}{25} - \frac{4}{25}i$

Additive inverse $= -z$

multiplicative inverse $= z^{-1} = 1/z$

Additive inverse $= -z$

$= (3+4i)$

$= -3-4i$ ans

9 - 13 imp

b) $\frac{1}{(3-2i)^2}$

let $z = 1/(3-2i)^2$

As we know,

Multiplicative $= 1/z$

$= \frac{1}{\frac{1}{(3-2i)^2}}$

$= \frac{1}{9 - 2.3.2i + (2i)^2}$

$= \frac{1}{9 - 12i - 4}$

$= \frac{1}{5 - 12i}$

$\therefore$ The multiplicative inverse of z is given by $z^{-1} = 1/z$

$= 5 - 12i$

9) If $x = 3+2i$, $y = 2+3i$, find the value of

a) $x^2 - y^2$

Given;

$x = 3+2i$

$y = 2+3i$

now;

$x^2 - y^2 = (3+2i)^2 - (2+3i)^2$

$= \{3^2 + 2.3.2i + (2i)^2\} - \{2^2 + 2.2.3i + (3i)^2\}$

$= \{9 + 12i - 4\} - \{4 + 12i - 9\}$

$= 9 + 12i - 4 - 4 - 12i + 9$

$= 10$ ans

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(b) $x^2 + xy + y^2$

$= (3+2i)^2 + (3+2i)(2+3i) + (2+3i)^2$

$= (3^2 + 2\cdot3\cdot2i + 2i^2) + (6 + 9i + 4i + 6i^2) + (2^2 + 2\cdot2\cdot3i + 3i^2)$

$= 9 + 12i - 4 + 6 + 13i - 6 + 4 + 12i - 9$

$= 37i$

10) If $x - iy = \frac{2-3i}{2+3i}$ prove that $x^2 + y^2 = 1$

1st method

$\Rightarrow x - iy = \frac{2-3i}{2+3i}$

$\Rightarrow x - iy = \frac{2-3i}{2+3i} \times \frac{2-3i}{2-3i}$

$\Rightarrow x - iy = \frac{4-6i-6i+9i^2}{(2)^2 - (3i)^2}$

$\Rightarrow x - iy = \frac{4-12i-9}{4+9}$

$\Rightarrow x - iy = \frac{-5-12i}{13}$

$x - iy = -\frac{5}{13} - \frac{12}{13}i$

Equating the real part and imaginary

$\therefore x = \frac{-5}{13}, y = \frac{12}{13}$

Taking L.H.S $x^2 + y^2$

$= \left(\frac{-5}{13}\right)^2 + \left(\frac{12}{13}\right)^2$

$= \frac{25}{169} + \frac{144}{16}$

$= \frac{169}{169} = 1$

$\therefore x^2 + y^2 = 1$

2nd Method

Give

$x - iy = \frac{2-3i}{2+3i}$

Taking modulus on both side

$= |x - iy| = \left|\frac{2-3i}{2+3i}\right|$

$= |x - iy| = \frac{|2-3i|}{|2+3i|}$

$\sqrt{(x)^2 + (-y)^2} = \frac{\sqrt{(2)^2 + (-3)^2}}{\sqrt{(2)^2 + (3)^2}}$

$\Rightarrow \sqrt{x^2 + y^2} = \frac{\sqrt{4+9}}{\sqrt{4+9}}$

$\Rightarrow \sqrt{x^2 + y^2} = \frac{\sqrt{13}}{\sqrt{13}}$

$\Rightarrow \sqrt{x^2 + y^2} = 1$

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Squaring both side;

$(\sqrt{x^2 + y^2})^2 = 1^2$

$\therefore x^2 + y^2 = 1$

11) If $x + iy = \sqrt{\frac{1+i}{1-i}}$, show that $x^2 + y^2 = 1$

Given;

$\Rightarrow x + iy = \sqrt{\frac{1+i}{1-i}}$

$\Rightarrow x + iy = \sqrt{\frac{1+i}{1-i} \times \frac{1+i}{1+i}}$

$\Rightarrow x + iy = \frac{\sqrt{(1+i)^2}}{\sqrt{(1)^2 - (i)^2}}$

$\Rightarrow x + iy = \frac{1+i}{\sqrt{1+1}}$

$\Rightarrow x + iy = \frac{1+i}{\sqrt{2}} = \frac{1}{\sqrt{2}} + \frac{i}{\sqrt{2}}$

equating

$x = \frac{1}{\sqrt{2}}, y = \frac{1}{\sqrt{2}}i$

L.H.S

$= x^2 + y^2$

$= \left(\frac{1}{\sqrt{2}}\right)^2 + \left(\frac{1}{\sqrt{2}}\right)^2$

$= \frac{1}{2} + \frac{1}{2}$

$= 1$

$\therefore x^2 + y^2 = 1$

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12) If $\sqrt{a-ib} = x - iy$, prove that $\sqrt{a+ib} = x + iy$

Given: $\sqrt{a-ib} = x - iy$

To prove: $\sqrt{a+ib} = x + iy$

Now,

$$\sqrt{a-ib} = x - iy$$

Squaring on both side

$\Rightarrow (\sqrt{a-ib})^2 = (x-iy)^2$

$\Rightarrow a - ib = x^2 - 2xiy + (yi)^2$

$\Rightarrow a - ib = x^2 - 2xiy + y^2 \cdot i^2$

$\Rightarrow a - ib = x^2 - 2xiy + y^2 \cdot -1$

$\Rightarrow a - ib = x^2 - 2xiy - y^2$

$\Rightarrow a - ib = (x^2 - y^2) - 2xiy$

equating

$$a = x^2 - y^2$$
$$b = 2xy$$

$L.H.S = \sqrt{a+ib}$

$= \sqrt{x^2 - y^2 + 2xiy}$

$= \cancel{\sqrt{x^2 + 2xy}}$

$= \sqrt{x^2 + (iy)^2 + 2xiy}$

$= \sqrt{x^2 + 2xiy + (iy)^2}$

$= \sqrt{(x+iy)^2}$

$= x + iy$

$\therefore \sqrt{a+ib} = x + iy$

13) If $z = \frac{3+i}{3-i}$, find the values of $\frac{1}{z}$ and $z + \frac{1}{z}$

Given

$$z = \frac{3+i}{3-i}$$

now;

$\Rightarrow \frac{1}{z} = \frac{1}{\frac{3+i}{3-i}}$

$= \frac{3-i}{3+i}$

again;

$\Rightarrow z + \frac{1}{z}$

$= \frac{3+i}{3-i} + \frac{3-i}{3+i}$

$= \frac{(3+i)(3+i) + (3-i)(3-i)}{(3)^2 - (i)^2}$

$= \frac{(3+i)^2 + (3-i)^2}{9 - (-1)}$

$= \frac{3^2 + 2.3.i + i^2 + 3^2 - 2.3.i + i^2}{9+1}$

$= \frac{9 + \cancel{6i} - 1 + 9 - \cancel{6i} - 1}{10}$

$= \frac{9 - 1 + 9 - 1}{10}$

$= \frac{\cancel{16}\ 8}{\cancel{10}\ 5}$

$\therefore z + \frac{1}{z} = \frac{8}{5}$

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Class assigment

* Rewrite the following without absolute value sign.

i) $|3x-2| \le 7$

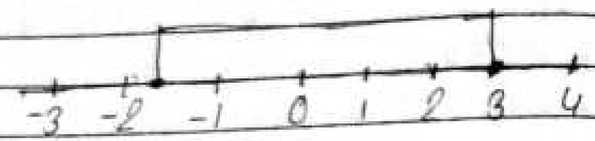
or $-7 \le 3x-2 \le 7$

$= -7+2 \le 3x-2+2 \le 7+2$

$= \frac{-5}{3} \le \frac{3x}{3} \le \frac{9}{3}$

$= -\frac{5}{3} \le x \le 3$

$x \in [-\frac{5}{3}, 3]$



ii) $|2x-3| \ge 1$

Taking positive

$= |2x-3| \ge 1$

$= (2x-3+3) \ge 1+3$

$= 2x \ge 4$

$= x \ge 2$

$x \in [2, \infty)$

Taking negative

$-|2x-3| \ge 1$

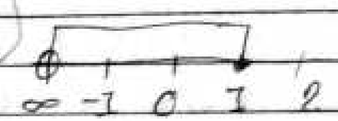
$(-2x+3) \ge 1$

$(-2x+3-3) \ge 1-3$

$\frac{-2x}{-2} \ge \frac{-2}{-2}$

$\Rightarrow x \le 1$

$x \in (-\infty, 1]$



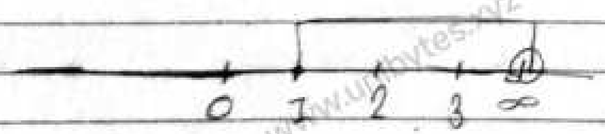
iii) $\left|\frac{x+1}{2}\right| \ge 1$

$= x+1 \ge 2$

$= x+1-1 \ge 2-1$

$= x \ge 1$

$= x \in [1, \infty]$



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3) Prove that $\frac{3+4i}{1-i} + \frac{3-4i}{1+i}$ is a real no.

Given,

$= \frac{3+4i}{1-i} + \frac{3-4i}{1+i}$

$= \frac{(3+4i)(1+i) + (3-4i)(1-i)}{(1)^2 - (i)^2}$

$= \frac{3+3i+4i+4i^2 + 3-3i-4i+4i^2}{1+1}$

$= \frac{3+\cancel{7i}-4+3-\cancel{7i}-4}{2}$

$= \frac{-1+1}{2} = \frac{-2}{2} = -1$ ans

4) If $x+iy = \frac{a+ib}{a-ib}$ prove that $x^2+y^2=1$

given;

$= x+iy = \frac{a+ib}{a-ib}$

$= x+iy = \frac{a+ib}{a-ib} \times \frac{a+ib}{a+ib}$

$= x+iy = \frac{(a+ib)^2}{(a)^2-(ib)^2}$

$\Rightarrow x+iy = \frac{a^2+2aib+(ib)^2}{a^2+b^2}$

$\Rightarrow x+iy = \frac{a^2+2aib-b^2}{a^2+b^2}$

$\Rightarrow x+iy = \frac{a^2-b^2}{a^2+b^2} + \frac{2aib}{a^2+b^2}$

$x = \frac{a^2-b^2}{a^2+b^2} / y = \frac{2ab}{a^2+b^2}$

L.H.S

x^2+y^2

$= \left(\frac{a^2-b^2}{a^2+b^2}\right)^2 + \left(\frac{2ab}{a^2+b^2}\right)^2$

$= \frac{(a^2-b^2)^2 + 4a^2b^2}{(a^2+b^2)^2}$

$= \frac{a^4-2a^2b^2+b^4+4a^2b^2}{(a^2+b^2)^2}$

$= \frac{a^4-2a^2b^2+b^4}{(a^2+b^2)^2}$

$= \frac{(a^2)^2+2a^2b^2+(b^2)^2}{(a^2+b^2)^2}$

$= \frac{\cancel{(a^2+b^2)^2}}{\cancel{(a^2+b^2)^2}} = 1$

$\therefore x^2+y^2 = 1$

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Another method

$x + iy = \frac{a+ib}{a-ib}$

Taking modulus

$\Rightarrow |x + iy| = \left|\frac{a+ib}{a-ib}\right|$

$\Rightarrow |x+iy| = \frac{|a+ib|}{|a-ib|}$

$\Rightarrow \sqrt{(x)^2+(y)^2} = \frac{\sqrt{(a)^2+(b)^2}}{\sqrt{(a)^2+(-b)^2}}$

$\Rightarrow \sqrt{x^2+y^2} = \cancel{\sqrt{a}}\ 1$

Squaring both side

$(\sqrt{x^2+y^2})^2 = 1$

$\therefore x^2+y^2 = 1$

5) If $x - iy = \frac{5-6i}{5+6i}$, prove that $x^2+y^2 = 1$

Taking modulus

$\Rightarrow |x - iy| = \left|\frac{5-6i}{5+6i}\right|$

$\Rightarrow |x-iy| = \frac{|5-6i|}{|5+6i|}$

$\Rightarrow \sqrt{(x)^2+(-y)^2} = \frac{\cancel{|5-6i|}\sqrt{(5)^2+(-6)^2}}{\sqrt{(5)^2+(6)^2}}$

$\Rightarrow \sqrt{x^2+y^2} = \frac{\sqrt{25+36}}{\sqrt{25+36}}$

$\Rightarrow \sqrt{x^2+y^2} = 1$

Squaring both side

$(\sqrt{x^2+y^2})^2 = 1^2$

$x^2+y^2 = 1$

6) If $x = 2-3i$, prove that $x^2-4x+13=0$

Given;

$x = 2-3i$

Taking L.H.S

$= x^2-4x+13$

$= (2-3i)^2 - 4(2-3i)+13$

$= 2^2 - 2.2.3i + (3i)^2 - 8 + 12i + 13$

$\Rightarrow 4 - \cancel{12i} - 9 - 8 + \cancel{12i} + 3$

$\Rightarrow 0$ proved

7) If $z = -2-5i$, find $\bar{z}, |z|, |\bar{z}|$

Given;

$z = -2-5i$

i) $\bar{z} = \overline{(-2-5i)}$

$= -(-2+5i)$

$= -2-5i$

ii) $|z| = |-2-5i|$

$= \sqrt{(-2)^2+(-5)^2}$

$= \sqrt{4+25}$

$= \sqrt{29}$

iii) $|\bar{z}| = |\overline{-2-5i}|$

$= |-2+5i|$

$= \sqrt{(-2)^2+(5)^2}$

$= \sqrt{4+25}$

$= \sqrt{29}$

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8) If A and B be two subsets of U. student $n(U) = 350$, $n(A) = 100$, $n(B) = 150$ and $n(A \cap B) = 50$, find $n(\bar{A} \cap \bar{B})$.

→ Given;

$n(U) = 350$

$n(A) = 100$

$n(B) = 150$

$n(A \cap B) = 50$

$n(\bar{A} \cap \bar{B}) = ?$

De Morgan's law

i) $n(\overline{A \cap B}) = n(\bar{A} \cup \bar{B})$

ii) $n(\overline{A \cup B}) = n(\bar{A} \cap \bar{B})$

We have;

De More $n(\bar{A} \cap \bar{B}) = n(\overline{A \cup B})$

$= n(U) - n(A \cup B)$

$= n(U) - [n(A) + n(B) - n(A \cap B)]$

$= 350 - 100 - 150 + 50$

$= 150.$ ans

MATLAB Complex No.

i) Find the real and imaginary part of the given complex no $z = 3 - 2i$

sol:
```
>> z = 3-2*i;
>> real(z);
>> imag(z);
```
ans -2

ii) Find the conjugate and absolute value of the given complex No. $z = 5 + 12i$

```
>> z = 5+12*i;
>> conj(z);        = 13
>> abs(z);
```

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3) Using Matlab Create the Script file to find i) $z + w$, ii) $z - w$, iii) zw, iv) z/w, v) $1/z$ vi) $|z/w|$ vii) $|zw|$ of the complex no $z = 5 + 12i$, $w = 3 + 4i$

sol:
```
>> z = 5+12*i;
>> w = 3+4*i;
>> z+w;  = 8+16i
>> z-w;  = 2+8i
>> z*w;  = -33+56i
>> z/w;  = 2.52+0.16i
>> 1/z;  = 0.0296-0.0710i
>> abs(z/w);  = 2.6
>> abs(z*w);  = 65
```

* Vectors and Scalars

i) Calculate the magnitude of $8\vec{i} - \vec{j} + 4\vec{k}$

sol:
```
>> u = [8, -1, 4];
>> norm(u);
  = 16.1245
```

ii) Find a unit vector that has the same direction as given vector $4\vec{i} + 2\vec{j} - 3\vec{k}$

```
>> u = [4, 2, -3]
   -4  2  -3
>> u/norm(u)
 = 0.7428   0.3714   -0.5571
```

iii) Find the dot (scalar) product of $\vec{a}$ and $\vec{b}$ of given vectors $\vec{a} = 2\vec{i} + 3\vec{j}$, $\vec{b} = 4\vec{i} - 6\vec{j}$

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Sol: >> a = [2, 3] ; = 2 3
>> b = [4, -6] ; = 4 -6
>> dot (a, b) ; = -10

iv) find the cross product of two vectors
$\vec{a} = (3, 4, -6)$ $\vec{b} = (2, 4, -3)$
Sol: >> a = [3 4 6] ; = 3 4 6
>> b = [2 4 -3] ; = 2 4 -3
>> cross (a, b) ; = -36 21 4

v) If $\vec{a} = -4\vec{i} - 3\vec{j} + 2\vec{k}$, $\vec{b} = 2\vec{i} - 7\vec{i} + 4\vec{k}$, find $\vec{a} \cdot \vec{b}$ and $\vec{a} \times \vec{b}$
Sol: >> a = [-4, -3, 2] ; = -4 -3 2
>> b = [2, -7, 4] ; = 2 -7 4
>> dot (a, b) = 21
>> cross (a, b) = 2 20 34

vi) find the scalar triple product and vector triple product of $\vec{a} = 2\vec{i} - 3\vec{j} + 7\vec{k}$, $\vec{b} = 2\vec{j} - 3\vec{k}$, $\vec{c} = 3\vec{i} + 8\vec{k}$
Sol: >> a = [2, -3, 7] ; = 2 -3 7
>> b = [0, 2, -3] ; = 0 2 3
>> c = [3, 0, 8] ; = 3 0 8
>> dot (a, cross (b, c)) ⇒ 17
>> cross (a, cross (b, c)) ⇒ 81 124 30

Note:
(Dot) scalar triple product = $\vec{a} \cdot (\vec{b} \times \vec{c})$
(cross) vector triple product = $\vec{a} \times (\vec{b} \times \vec{c})$