Probability I: Assignment 5

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November 11, 2021

Submit solutions to Problems 4, 5, 9 and 10 on Moodle by 18th November, 10 PM. Write down the probability space in all questions clearly before writing down the solutions.

1. Ten fair dice were thrown. Given that at least one of them produced one, what is the probability that two or more dice produced one.

2. Let B be an event such that $\mathbb{P}(B) > 0$. Show that $\mathbb{P}(\cdot|B)$ is a probability distribution on B.

3. A lie detector is known to be reliable 80 percent of the time when the person is guilty, and 90 percent reliable when the person is innocent. A suspect is chosen at random from a group of suspects of whom only 1 percent have ever committed a crime. If the test indicates that the suspect is guilty, what is the probability that the suspect is innocent?

4. There are three boxes, each with two drawers. Box 1 has a gold coin in each drawer and box 2 has a silver coin in each drawer. Box 3 has a silver coin in one drawer and a gold coin in the other. One box is chosen at random, and then a drawer is chosen at random from the box. Find the probability that box 1 is chosen, given that the chosen drawer yields a gold coin.

5. A man possesses five coins, two of which are double-headed, one is double-tailed, and two are normal. He shuts his eyes, picks a coin at random, and tosses it. (a) What is the probability that the lower face of the coin is a head? (b) He opens his eyes and sees that the coin is showing heads; what is the probability that the lower face is a head? He shuts his eyes again, and tosses the coin again. (c) What is the probability that the lower face is a head?

6. A standard deck of cards if shuffled at random. Compute the (conditional) probabilities of the following events : (1) The first spade to appear is an ace. (2) Ace spade is the third card given that there is a spade in the first five cards and (3) Ace spade is not in the first 40 cards given that it is not there in the first 30 cards.

7. Consider allocation of hash codes to n people. Each of the n people are assigned an hash code from $[N]$ uniformly at random. Denoting by

$(h(1), \ldots, h(n))$ the vector of hash codes of the n people, find the probability that at least one of the hash codes from $[N]$ is not assigned to any person. Further, what is the probability that none of the other hash codes from $h(2), \ldots, h(n)$ are same as $h(1)$.

8. Among the families with two children, one of them is chosen at random and it is reported that one of them is a boy and was born on a sunday. What is the probability that the other is a girl ?

9. In a bolt factory machines A, B, C manufacture $25, 35$ and 40 percent of the total, respectively. Of their output $5, 4$ and 2 per cent (respectively) are defective bolts. A bolt is drawn at random from the produce and is found defective. What are the probabilities that it was manufactured by machines A, B and C?

10. Consider the Monty Hall problem from Lecture 12. Suppose that after Monty Hall shows one of the other doors with a zonk, you toss a coin (with probability p of heads) and you change your choice if it is heads. For each $p \in [0, 1]$, compute the probability of winning without changing. [1]

[1] Extra Question (not for submission) : Is there a choice of p which yields a better probability of winning than changing always ?