Probability I: Assignment 1

Yogeshwaran D.

September 29, 2021

Submit solutions to Q.2, Q.4, Q.5(b) and Q.6 by October 4th on Moodle.

1. Compute the cardinalities of the sample spaces in all the examples in Sep28 note.

2. Describe the following events in the sample spaces : In Eg. 6^1 (positions of labelled particles), Eg. 7 ($\Omega_{r,n}$ - configuration of unlabelled particles) and Eg. 8 ($\Omega^*_{r,n}$ -configuration of electrons), describe the event such that no two consecutive sites/cells consist of particles.

3. In a communication channel, messages are transmitted in the form of n bits (i.e., 0's or 1's). Describe the sample space of all possible messages and also the event that the number of 0's is always larger than that of number of 1's in a message.

4. Consider the previous exercise. An encoder takes as input a binary string (i.e., a sequence of $0, 1$ bits) of length n and repeats each bit k times and then sends it across the communication channel. The sample space consists of all possible binary strings obtained via the encoding. Describe the sample space.

5. Find the sample space / space of outcomes in the following experiments and also describe the events specified :

   (a) There is a box containing 1 coupon labelled 1, 2 coupons labelled 2, 3 coupons labelled 3 and so on upto 10 coupons labelled 10. The experiment consists of drawing two distinct coupons from the box. Describe the event that the labels of both the coupons are even and the event that the sum of labels of both the coupons is an even number.

   (b) There is a group of five people some of whom are friends. The sample space consists of all pairs of people who are friends i.e., every element is a set consisting of all pairs of people who are friends. Describe the sample space mathematically and also the event that there are three people such that each of them is a friend of the other. [Here the experiment can be thought of as choosing pairs of people who are friends.]

---

[1]as per Sep28 note

(c) The experiment consists of placing the black and white kings on a chessboard such that they cannot take out each other in a single move.

(d) Three players A,B,C play each other. The sample space consists of all possible winners of the three matches.

(e) A film survey agency lists 4 movies (say A,B,C,D) and asks respondents to answer that in each of the following pairs of movies which one do they prefer : A vs B, A vs C, A vs D, B vs C, B vs D, C vs D. The respondents indicate only the movies they prefer in each pair and might not be consistent in their choices. Describe the outcome space of the possible survey answers. Can you describe the above outcome space in terms of matrices ? Can you describe the event in the outcome space where one of the movies is a clear winner (i.e., preferred over the others) ? Can you describe the event in the outcome space where there is no clear winner or clear loser ?

6. A jar contains a large collection of red, green and white marbles. Marbles are drawn from the jar one at a time. The color of the marble is recorded and it is put back into the jar. This is repeated n times. Let Ω be the sample space. Let R_k denote the event that the kth marble is red and similary G_k be the event that the kth marble is green. Describe the following events in terms of R_k, G_k's.

(a) The first marble drawn is white

(b) The first marble is green and the second marble is not white.

(c) The first and last marble are of the same color.

(d) The first three marbles are of different colors.