Probability I: Assignment 7

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Submit solutions to Problems 1, 3, 5 and 7 on Moodle by 12th December, 10 PM. Write down the probability space in all questions clearly before writing down the solutions.

1. Compute the variances of the uniform random variable on $[n]$, the Binomial and the hypergeometric random variable.
2. Show that $\mathsf{VAR}[aX + b] = a^2\mathsf{VAR}[X]$.
3. Two fair dice are rolled independently. Find the pmf, mean and variance of the following random variables - (1) The sum of the two dice. (2) The maximum among the two dice.
4. Balls are thrown one after another (uniformly at random) into two bins. Each throw is independent of the previous throw. The experiment stops when there is no empty bin. Let X be the total number of balls thrown. Find $\mathbb{P}(X \geq n)$ for all $n \geq 1$.
5. Let X be the number of empty cells corresponding to Maxwell-Boltzmann distribution. Compute the pmf, mean and variance of X.
6. Let X be the number of empty cells corresponding to Fermi-Dirac distribution. Compute the pmf, mean and variance of X.
7. Prove Markov's inequality and Chebyshev's inequality just using pmf of a random variable.
8. In a population of size N, m people prefer candidate A. In an opinion poll, n people are chosen at random and asked their preferences. Let Y denote the proportion of people who prefer candidate A among the n randomly chosen people. Find t (depending on N, n, m) such that $\mathbb{P}(|Y - \frac{m}{N}| \geq t) \leq 10^{-2}$. [1]
9. Suppose two permutations of $[n]$ are chosen at random and independently. Let X denote the number of matches between the two random permutations i.e., the number of co-ordinates at which the permutations match. Compute the pmf and mean of X.

[1] EXTRA : Can you find n such that $t \leq 10^{-4}$.

10. Let a standard fair die be rolled. Suppose the die shows the number i, then we choose a coin with probability of heads being $i/6$. Now this coin is tossed independently and repeatedly until we get a heads. Let the random variable X be the number of tosses. Find the probability $\mathbb{P}(X = n)$ for all $n \geq 1$.